Optimal Design of Multicharacteristics Inspection Plan Under Inspection Errors and Statistical Dependency

by

Abbi Moghaiyera Hassan

A Thesis Presented to the

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In Partial Fulfillment of the
Requirements for the Degree of

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In

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كلية الدراسات العليا
COLLEGE OF GRADUATE STUDIES

Dedicated to

My Parents

whose patience and perseverance

led to this accomplishment

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Abstract

Name: Abbi Moghaiyera Hassan

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Complete inspection plans have become increasingly important in the area of quality control due to the growth in modern manufacturing systems that makes complete inspection inexpensive and reliable. In the process of inspection, an inspector is likely to commit Type I and Type II errors in his judgement about the product quality. In this thesis the effect of Type I and Type II errors are investigated on multicharacteristic repeat inspection plans for critical components. The practicality of the inspection models are enhanced by modifying them for the case where the defective rates of the characteristics are statistically dependent. The results indicate that the effect of errors and statistical dependency are significant and should be incorporated in the design of the inspection plans. Since the error probabilities are a function of incoming quality, the suggested procedure for estimating Type I and Type II errors for a given incoming quality is utilized to incorporate the dynamic behavior of inspection errors into the repeat inspection models and their effect is studied. It is noticed that varying inspection errors have a significant effect on the plans in terms of expected total cost. Then a factorial experiment is conducted to find the factors and interactions that have a significant effect on the performance measures of the inspection plan.

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خلاصة الرسالة

إسم الطالب بالكامل : آبي مغيرة حسن

عنوان الدراسة : أفضل تصاميم لحفظ الفحص المتكرر في حالات أخطاء الفحص والإعتماد الإحصائي .

التخصص : هندسة النظم

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أصبحت خطط الفحص المتكرر ذات أهمية قصوى في ممارسات ضبط الجودة وذلك لنمو أنظمة التصنيع التي تجعل عمليات الفحص رخيصة وسهلة وذات إعتمادية عالية . وخلال عمليات الفحص يرتكب الفاحص أخطاء من النوع الأول وذلك حين يصنف منتج سليم بأنه غير ذلك وايضاً يرتكب خطأ من النوع الثاني حينما يصنف منتج سليم بأنه غير سليم .

وفي هذه الأطروحة ندرس تأثير هذان النوعان من الأخطاء على خطط الفحص المتكرر للمُركّبات ذات الخواص المتعددة ونطور هذه الخطط لتعالج الحالات التي يكون فيها خواص المرُكّبات معتمدة على بعضها إحصائياً ، ودلت نتائج الدراسة ان الأخطاء من النوع الأول والثاني وكذلك اعتماد الخواص على بعضها إحصائياً له تأثير كبير على خطط الفحص ولابد من وضعها في الاعتبار عند تصميم هذه الخطط . وطورت النماذج الرياضية للخطط بإدخال دوال تعبر عن الأخطاء بواسطة جودة المنتج وبذلك يتم مكاملة عملية نمذجة خطط الفحص وجعلها أكثر واقعية . وبعد ذلك صممت تجربة إحصائية لمعرفة أثار العوامل في خطط الفحص على معايير الأداء لتلك الخطط ووضحت التجربة العوامل ذات التأثير الفعّال على خطط الفحص المتكرر .

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ديسمبر ١٩٩٦م

Chapter 1

Introduction

1.1 Introduction

Statistical Quality Control (SQC) is being widely used these days to improve the quality of the products used by our society. These products consist of manufactured goods such as automobiles, computers and clothing as well as services such as electrical energy, public transportation, banking and health care. Quality Control can be broadly classified into three categories: *Statistical process control (SPC), Quality Engineering and Product control.* Product control can be broadly classified into: *acceptance sampling plans, 100% inspection and repeat inspection.*

Some of the well known sampling plans are : MIL STD 105D, ANSI/ASQC Z1.4, double sampling plans, multiple sampling plans, sequential sampling plans, chain sampling plans and skip-lot sampling plans. Acceptance sampling simply ac-

cepts or rejects lots and provides some sort of quality assurance, however acceptance sampling does not build quality in the product or the process.Acceptance sampling is mostly useful in the following situations :

1. When testing is destructive.

2. When the cost of 100% inspection is extremely high.

3. When 100% inspection is not technologically feasible or would require so much time and as a result production scheduling would be seriously impacted.

4. When there are many items to be inspected and the inspector error rate is sufficiently high that 100% inspection might cause a higher percentage of the defective units to be passed than would occur with the use of a sampling plan.

5. When the vendor quality history is excellent and some reduction of 100% inspection is desired, but the vendor's process-capability ratio is sufficiently low to make no inspection an unsatisfactory alternative.

In 100% inspection every item of the lot is inspected removing all defective units found (defectives may be returned to the vendor, re-worked, replaced with known good items or discarded). 100% inspection plans are used in situations where the component is extremely critical and passing any defective would result in loss which may be high or catastrophic or when the vendors's process capability is inadequate to meet specifications.

Recent advances in automation and computer control in manufacturing are changing the fundamental role and functions of quality control/assurance. Com-

plete inspection plans have become increasingly important in the area of quality control due to the growth in modern manufacturing systems that makes complete inspection inexpensive and reliable [1, 2, 3]. In particular, the use of automatic test equipment (ATE) has greatly increased inspection speed and accuracy [4]. Consequently, screening (100% inspection) is becoming an attractive practice for removing non-conforming items, and it has been suggested that inspection will essentially become an inherent part of modern manufacturing processes. Although nowadays the emphasis is on process control and quality engineering, inspection sampling will still play an important part in quality control [5].

The concept of repeat inspection is proposed for critical multicharacteristic components. Critical components are those whose failure results in catastrophe, serious hazard or very high costs. The repeat inspection guards against Type I and Type II errors with more emphasis on Type II error. This thesis will focus on extending a repeat inspection plan proposed by Duffuaa and Al-Najjar [6] for critical multicharacteristic components.

The rest of this chapter is organized as follows : Section 2 states the multicharacteristic inspection problem followed by the objective of the thesis in section 3. Section 4 outlines the organization of the thesis.

1.2 The Multicharacteristic Inspection Problem

Repeat inspection plans have been proposed for critical multicharacteristic components. Such components usually have several characteristics and failure of any one of them results in component failure. The quality requirement for such components are tight and field failure must be kept to a minimum level. Such components can be part of an aircraft, a space shuttle, a nuclear reactor or a complex gas ignition system. All such components need to be inspected for all of their quality characteristics, and one inspection of each characteristic may not be sufficient or economical. The reason simply is that inspection is never perfect. There is always a possibility of false rejection (Type I error) or false acceptance (Type II error). Although both errors have cost but in case of critical components the cost of false acceptance is much higher than the cost of false rejection, because falsely accepted components may result in system failure which may involve very high cost as well as human life losses. It is likely that repeat inspection reduces the cost of errors and increases the cost of inspection. However the expected total cost which is the sum of the cost of inspection, cost of false acceptance and the cost of false rejection is likely to be reduce.

Inspection plans in the literature and the models representing them were developed for components having several characteristics ($i = 1, 2, \ldots, N$) with known incoming quality P_i for the i-th characteristic. A component is classified as non-

defective only if all the characteristics meet the quality specifications. The probability of Type I error and Type II error are assumed to be known for all the characteristics. There are three different types of costs that are considered : cost of inspection for each characteristic C_i, cost of false acceptance C_a and the cost of false rejection C_r. It is also assumed that the estimates of these costs are available in the industry. The objective is to find the optimal number of repeat inspections n for each characteristic in order to minimize the expected total cost.

Raouf et al [7] proposed a repeat inspection plan which is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process and all the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the component is finally accepted. Here n is the optimal number of inspections necessary to minimize the cost per accepted component or maximize the probability of accepting a non-defective component. Recently Duffuaa and Al-Najjar [8] have proposed a new inspection plan for critical components which is shown in figure 1.1 and is applied as follows : an inspector performs n_1 repeat inspections on the first characteristic and passes all the accepted components of this first stage to the inspector of the second stage to perform n_2 repeat inspections on the second characteristic. This chain of inspection continues until

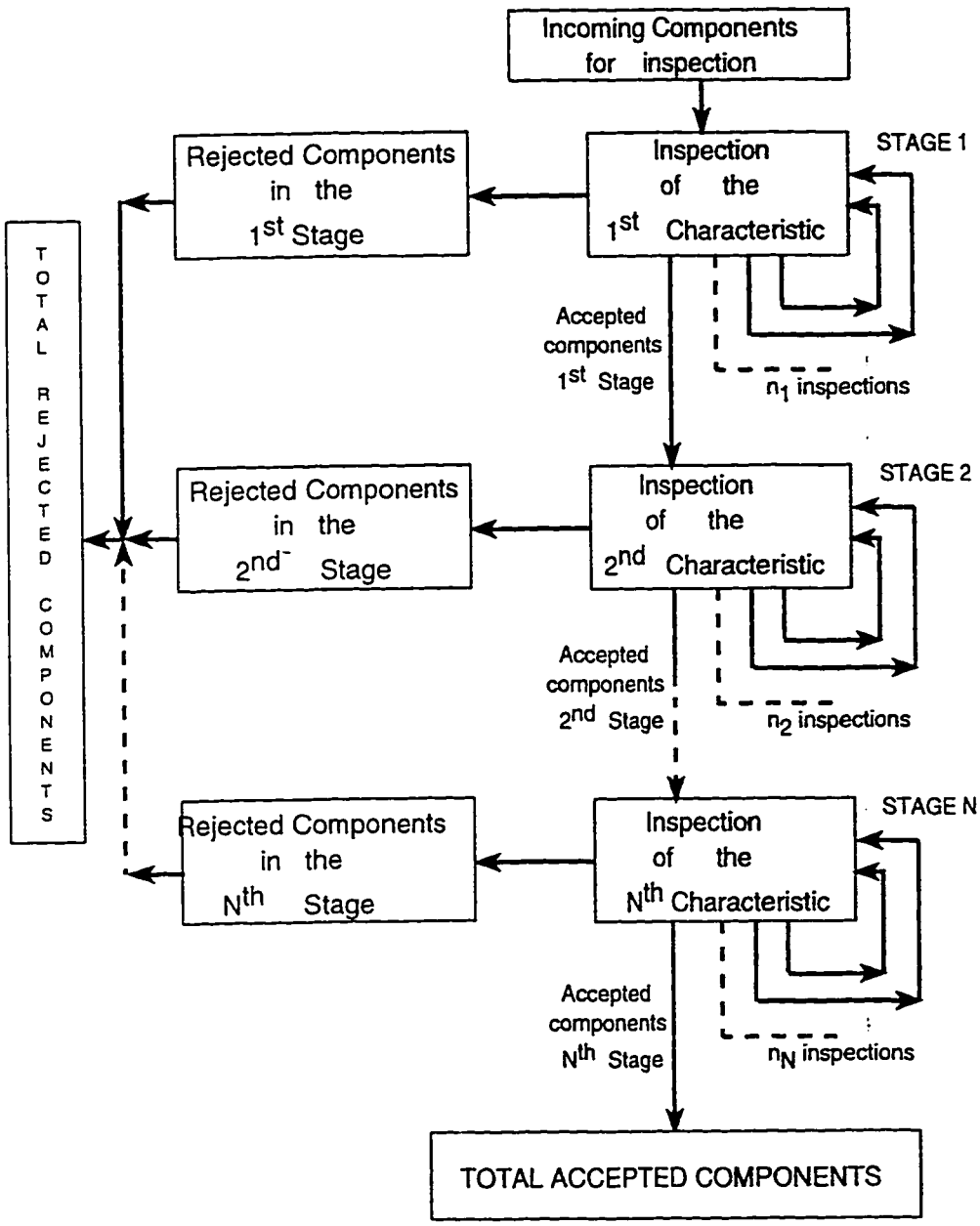

Figure 1.1: General Inspection Plan

all the characteristics have been inspected $n_1, n_2, \ldots, n_N$ times for characteristics $i = 1, 2, \ldots, N$ respectively. Here, n_i is the optimal number of repeat inspection for characteristic i needed to minimize the expected total cost per accepted component. Stage i has n_i cycles of inspection. Finally, the accepted components will be those which are accepted at the N-th stage, and the rejected components are the sum of those rejected in the $1st, 2nd, \ldots, N$-th stages. Duffuaa and Al-Najjar [8] proposed another model which is a special case of the general plan given above where each characteristic is inspected equal number of times n.

The main purpose of this thesis is to extend the model for repeat inspection plan by incorporating issues such as variability in inspection errors and statistical dependency between characteristics defective rates. Specially these extensions will be made to the model that depicts the plan proposed by Duffuaa and Al-Najjar [8].

1.3 Objectives of the Thesis

1. To investigate the effects of inspection errors on the plan proposed by Duffuaa and Al-Najjar [6].

2. To generalize the model by Duffuaa and Al-Najjar [6] for the case where characteristics defective rates are statistically dependent.

3. To examine the effect of inspection errors for the case of dependency.

4. To extend the models (independent and dependent) for varying inspection errors .

1.4 Thesis Organization

This thesis is organized in seven chapters. An overview of the literature relevant to the present work is presented in chapter 2. The models for repeat multicharacteristic inspection along with their development are also discussed in this chapter. Chapter 3 is devoted to study the effect of inspection errors on the performance measures of the complete inspection plans such as expected total cost, average total inspection and average outgoing quality by varying Type I and Type II errors. In chapter 4, the multicharacteristic inspection plans given in chapter 2 are extended to situation where the characteristics defective rates are statistically dependent. Also in this chapter the algorithm to find the optimal number of repeat inspections is obtained for the developed models. Then the effect of inspection errors on the models in chapter 4 is also investigated. Chapter 5 presents a procedure for the estimation of inspection errors as a function of incoming quality. The obtained relationship is used to incorporate the effect of varying inspection errors in the inspection models. In chapter 6, a factorial experiment is conducted to study the effects and sensitivity of the input parameters on the performance measures of the inspection plan. Finally, in chapter 7 we provide the conclusion and directions for future research.

Chapter 2

Inspection Models and Literature Review

2.1 Introduction

Inspection plans provide decision-making procedures for controlling and improving lot quality through accept-reject decision. Frequently, the users of such plans have erroneously assumed that inspection process yields accurate results. In reality an inspector usually commits Type I and Type II errors in the judgement about product quality. The error of judgement made by human observers have been studied for a long time by human factor specialists. Not until 1970, quality control researchers started considering inspection errors and began to model their implications.

The purpose of this chapter is to review the literature which is relevant to the

theme of this thesis which is the optimal design of multicharacteristic inspection plans under statistical dependency and inspection errors and is presented in section 2. Then the models for multicharacteristic inspection plan which will be extended in this thesis are outlined in section 3. Finally, section 4 gives the conclusion to this chapter.

2.2 Literature Review

The literature review has been divided into two parts : error modelling and repeat inspection plans.

2.2.1 Error Modelling

Harris [9] was the first to examine the effect of incoming quality on inspection accuracy. He came to the conclusion that the inspection accuracy decreases with the reduction in defect rate. Green and Swets [10] studied the effect of inspection errors by using Signal Detection Theory (SDT), and by using a graph called the Receiver Operating Characteristic (ROC) curve. Ayoub et al [11] defined mean inspector error to be the average number of defectives classified as good item by the inspector. They presented a formula for Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) for a single sampling plan under inspection error. Collins et al [12] relaxed the assumption of perfect inspection of replacement and allowed defective

replacement in the formula for *AOQ* and *ATI*. They also considered the effect of inspection errors on the design of a single sampling plan based on the measures, lot tolerance percent defective (LTPD) and acceptable quality level (AQL). Bennet et al [13] investigated the effect of inspection errors on cost based single sampling plan with known incoming quality distribution. Case et al [14] showed the effect of errors on quality control systems using *AOQ* and *ATI* as quality cost measure. They also presented economic comparison, of with and without errors, under simulated (*OC*) curve behaviors. Mei et al [15] assumed that the measurement error distribution (often normal) has a known mean (bias) and a standard deviation (imprecision). The lot distribution and the measurement error distribution are assumed to be independent. A method is presented whereby the variable sampling plan may be designed to explicitly compensate for measurement error and provide the desired *OC* curve. Dorris and Foote [16] surveyed the effect of inspection errors on standard quality control procedures. It is seen that when inspection errors are included in the model, the (*OC*) curve shape changes as does the protection that an (*OC*) may afford for a sampling plan. Raouf and Elfeturi [17] conducted a study to investigate the factors which effect inspector accuracy and concluded that Type II error is more realistic criterion for measuring inspector accuracy. Maghsoodloo [18] provides the expression for performance measures (probability of acceptance P_a, *AOQ*, *ATI*) in the presence of inspection error in a multistage sampling plan. Tang and Schneider [19] investigated the economic and statistical effect of inspection error

on the complete inspection plan. In order to determine the expected quality cost per item the authors use Taguchi's concept of quadratic loss function. Dhavale [20] advocates that the distribution of defectives due to errors in 100% inspected lots follow negative binomial distribution. The technique to estimate the parameter of the negative binomial is presented. Shor and Raz [21] identified the human factors that cause inspection errors. Suich [22] investigated the effect of inspection errors on acceptance sampling when one is inspecting for the number of nonconformities per item. Formulas are derived for the *AOQ* and *ATI* for situations when these errors are present. Shin and Lingayat [23] studied the effect of varying inspection errors on single sampling plans. They assumed inspection errors to be a function of the sequence of the item number in the sample or the lot, and used *AOQ* and *ALIC* (Average Lot Inspection Cost) as performance to evaluate the effects of the errors.

2.2.2 Repeat inspection Plans

Raouf et al [7] were the first to develop a model for determining the optimal number of repeat inspections for multicharacteristic components to minimize the total expected cost per accepted component due to Type I error, Type II error and cost of inspection. Garcia-Diaz et al [24] presented a dynamic programming (DP) model for repeat 100% inspection. Elmaghraby [25] further analysed the model of Garcia-Diaz et al and presented an alternative condition for the applicability of the *DP* model. Jaraeidi et al [26] presented a model to determine *AOQ* for a prod-

uct which has multiple quality characteristics and which is subjected to multiple 100% inspections where the inspection is subjected to errors. Lee [27] presented a simplified version of the cost-minimization model developed by Raouf et al [7] to capture the cost implication of false rejection, false acceptance and inspection of the components. Optimality of the sequence of the characteristics to be inspected was also obtained. Duffuaa and Raouf [28] developed three mathematical optimization models for multicharacteristic repeat inspection. The first model (cost minimization model) minimizes the total cost due to inspections, Type I error and Type II error to determine the optimal number of repeat inspections. The second model (probability minimization model) minimizes the probability of accepting a defective component. The third model (satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and for the probability of accepting a defective component. Duffuaa and Raouf [29] established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf et al [7]. Duffuaa and Nadeem [30] developed an extension of the model proposed in Raouf et al [7] for components whose characteristic's defective rates are statistically dependent. Duffuaa and Al-Najjar [8] proposed a new inspection plan for critical multicharacteristic components. They proposed an algorithm to determine the optimal number of repeat inspections and sequence characteristics for inspection in order to minimizes the total expected cost .

2.3 Models for Repeat Multicharacteristic Inspection

We have seen in the previous section that many models were developed in the area of complete repeat inspection of critical components.These models were developed to determine the optimal number of repeat inspections for critical components having several characteristics. The objective of the model is to minimize the total cost per accepted component due to the cost of false acceptance, the cost of false rejection and the cost of inspection.

It is assumed that components have N characteristics, $i = 1, 2, \ldots, N$. Characteristic i has a probability P_i of being defective. Inspectors commit Type I error E_{1i} and Type II error E_{2i} for each characteristic i. The cost of inspection C_i for characteristic i, cost of false acceptance C_a and cost of false rejection C_r constitutes the total cost of inspection. These data are assumed to be known.

Recently Duffuaa and Al-Najjar [6] have developed two inspection plans for critical components. However, the second plan is a special case of the first plan where characteristics are inspected equal number of times. The purpose of this thesis is to extend the models proposed by Duffuaa and Al-Najjar [8] by incorporating issues such as variability in inspection errors and statistical dependency between characteristics defective rates. Therefore, it is felt appropriate to present these models.

2.3.1 Model 1 of Duffuaa and Al-Najjar

The model is developed for components described in the previous section and the inspection plan is shown in *Figure* 2.1. In the nomenclature, i ranges from 1 to N and j ranges from 1 to n.

Nomenclature

M	Number of components to be inspected.
M_i	Number of components entering the i-th stage of inspection.
$M_{i,j}$	Number of components entering the j-th cycle of stage i.
N	Number of characteristics in each component to be inspected.
n	Optimal number of repeat inspections.
C_i	Cost of inspection of characteristic i.
C_a	Cost of acceptance per bad component.
C_r	Cost of rejection per good component.
P_i	Probability of the i-th characteristic being defective on entering the inspection.
$P_{i,j}$	Probability of the i-th characteristic in the sequence of inspection being defective on entering the j-th cycle of inspection.
E_{1i}	Probability of classifying the i-th non-defective characteristic in the sequence of inspection as defective (Type I error).
E_{2i}	Probability of classifying the i-th defective characteristic in the sequence

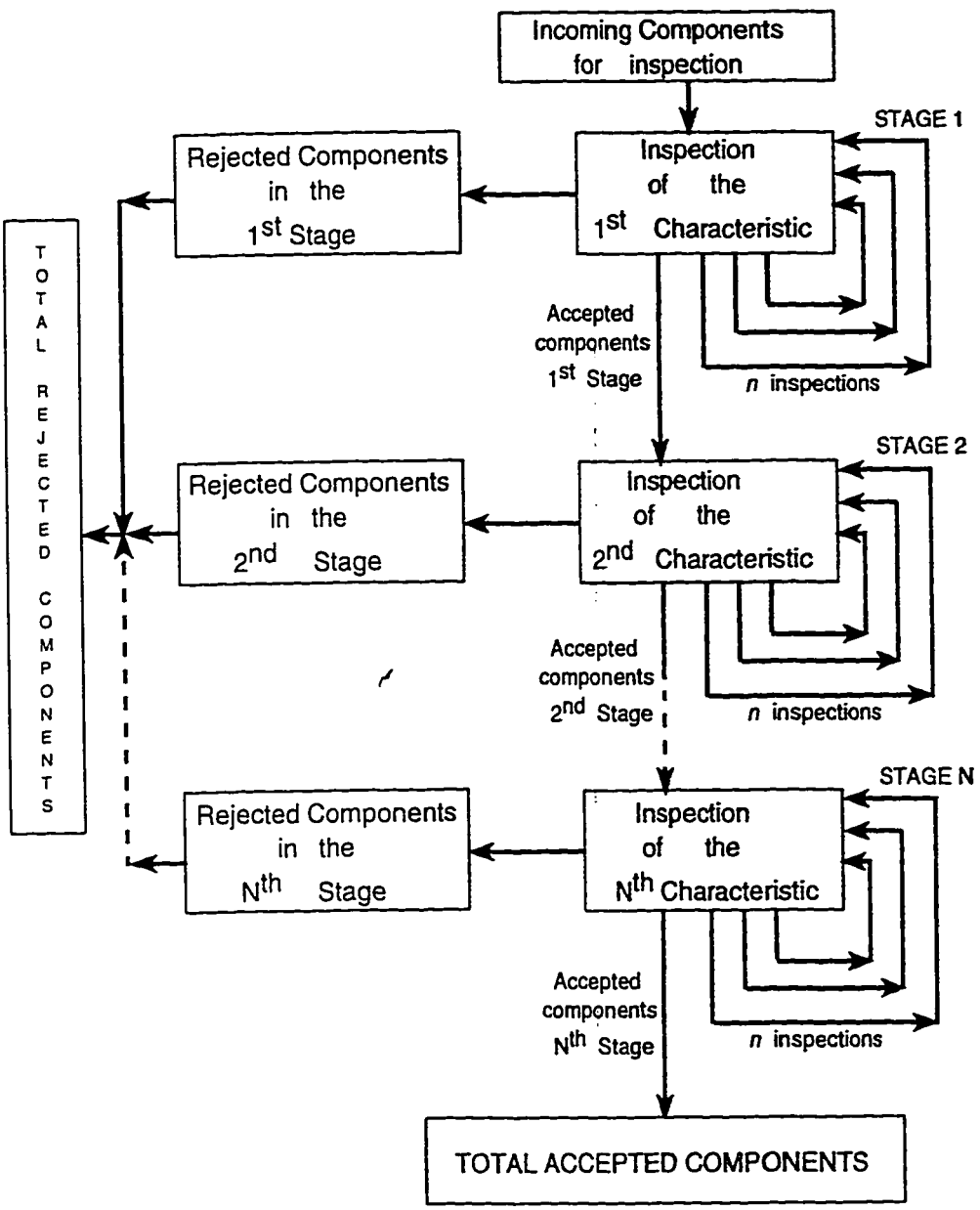

Figure 2.1: The Inspection Plan

of inspection as non-defective (Type II error).

PG Probability of a component being non-defective on entering the inspection process.

$PG_{i,j}$ Probability of the component being non-defective on entering the j-th cycle of the i-th stage of inspection.

$FR_{i,j}$ Expected number of falsely rejected components in the j-th cycle of the i-th stage .

$FA_{i,j}$ Expected number of falsely accepted components in the j-th cycle of the i-th stage .

$CA_{i,j}$ Expected number of correctly accepted components in the j-th cycle of the i-th stage .

$R_{i,j}$ Rate of rejection of components due to the i-th characteristic in the sequence of inspection of the j-th cycle .

A_i Expected number of accepted components in the i-th stage.

CFR_i Cost of false rejection in the i-th stage.

CFA_i Cost of false acceptance in the i-th stage.

CI_i Cost of inspection in the i-th stage.

$TCFR$ Total cost of false rejection.

$TCFA$ Total cost of false acceptance.

TCI Total cost of inspection.

TA Total number of accepted components.

$E(tc)|_i$ Expected total cost per accepted component after the i-th stage of inspection.

Basic Relationships in the Model

The probability of the i-th characteristic being defective will vary from cycle to cycle.The relationship between $P_{i,j}$ and P_i will be as follows [6]

$$P_{i,1} = P_i \tag{2.1}$$

Using Bayes theorem, we get

$$P_{i,2} = \frac{P_i E_{2i}}{[P_i E_{2i} + (1 - P_i)(1 - E_{1i})]} \tag{2.2}$$

and

$$P_{i,3} = \frac{P_{i,2} E_{2i}}{[P_{i,2} E_{2i} + (1 - P_{i,2})(1 - E_{1i})]} \tag{2.3}$$

Substituting for $P_{i,2}$ from 2.2 in the above formulae gives, after simplification,

$$P_{i,3} = \frac{P_i E_{2i}^2}{[P_i E_{2i}^2 + (1 - P_i)(1 - E_{1i})^2]} \tag{2.4}$$

Similarly,

$$P_{i,4} = \frac{P_i E_{2i}^3}{[P_i E_{2i}^3 + (1 - P_i)(1 - E_{1i})^3]} \tag{2.5}$$

In general from the symmetry of expression 2.3, 2.4 and 2.5 the following is deduced:

$$P_{i,j} = \frac{P_i E_{2i}^{j-1}}{[P_i E_{2i}^{j-1} + (1 - P_i)(1 - E_{1i})^{j-1}]} \tag{2.6}$$

The probability of a characteristic being defective changes in each cycle, hence the probability of a component being non-defective also changes. The probability of a component being non-defective when entering inspection is

$$PG = \prod_{i=1}^{N}(1 - P_i) \tag{2.7}$$

The probability of a component being non-defective after inspecting characteristic 1, n times is :

$$PG_{i,n+1} = \left[\prod_{i=2}^{N}(1 - P_i)\right][(1 - P_{1,n+1})] \tag{2.8}$$

The probability of a component being non-defective after inspecting all characteristic n times is :

$$PG_{N,n+1} = \prod_{i=1}^{N}(1 - P_{i,n+1}) \tag{2.9}$$

The probability of a component being non-defective after inspecting characteristic 1 through i - 1, n times and characteristic i, k times, with the other characteristics

from $i + 1$ through N not inspected, is given by [6] :

$$PG_{i,k} = \left[\prod_{m=1}^{i-1}(1 - P_{m,n+1})\right][(1 - P_{i,k+1})]\left[\prod_{m=i+1}^{N}(1 - P_m)\right] \tag{2.10}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost due to false acceptance of defective components and is given by:

$$E(tc)|_{j=o} = C_a(1 - PG) \tag{2.11}$$

The expected total cost per accepted component, after inspecting each characteristic n times will be

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{2.12}$$

Cost Minimization Model

In order to determine $TCFR, TCFA, TCI$ and TA, an analysis of different stages of inspection is necessary.

Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristic in each component. Each stage has n cycles. Following is the first cycle of inspection.

Cycle (1)

Number of components entering cycle (1) is

$$M_{1,1} = M_1 \tag{2.13}$$

The probability of a component being non-defective is

$$PG_{1,1} = PG \tag{2.14}$$

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1}PG_{1,1}E_{11} \\ &= M_1PGE_{11} \end{aligned} \tag{2.15}$$

E (number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1E_{21} + (1 - PG_{1,1} - P_1)(1 - E_{11})] \\ &= M_1[P_1E_{21} + (1 - PG - P_1)(1 - E_{11})] \end{aligned} \tag{2.16}$$

E (number of correctly accepted components) is

$$\begin{aligned} CA_{1,1} &= M_{1,1}PG_{1,1}(1 - E_{11}) \\ &= M_1PG(1 - E_{11}) \end{aligned} \tag{2.17}$$

All accepted components in this cycle go to the first inspector again to inspect the first characteristics for the second time for each component.

Cycle (2)

$$\begin{aligned} M_{1,2} &= FA_{1,1} + CA_{1,1} \\ &= M_1[P_1E_{21} + (1 - PG - P_1)(1 - E_{11})] + M_1PG(1 - E_{11}) \\ &= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})] \end{aligned} \tag{2.18}$$

$$\begin{aligned} PG_{1,2} &= CA_{1,1}/M_{1,2} \\ &= PG(1 - E_{11})/[P_1E_{21} + (1 - P_1)(1 - E_{11})] \end{aligned} \tag{2.19}$$

$$\begin{aligned} FR_{1,2} &= M_{1,2}PG_{1,2}E_{11} \\ &= M_1PGE_{11}(1 - E_{11}) \end{aligned} \tag{2.20}$$

$$\begin{aligned} FA_{1,1} &= M_{1,2}[P_{1,2}E_{21} + (1 - PG_{1,2} - P_{1,2})(1 - E_{11})] \\ &= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})] \\ &\times [P_{1,2}E_{21} + (1 - PG_{1,2} - P_{1,2})(1 - E_{11})] \end{aligned} \tag{2.21}$$

$$
\begin{aligned}
CA_{1,2} &= M_{1,2}PG_{1,2}(1-E_{11}) \\
&= M_1PG(1-E_{11})^2
\end{aligned}
\tag{2.22}
$$

Similarly,

Cycle (n)

$$
M_{1,n} = M_1 \prod_{j=1}^{n-1} [P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})] \tag{2.23}
$$

$$
PG_{1,n} = \frac{PG(1-E_{11})^{n-1}}{\left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]} \tag{2.24}
$$

$$
FR_{1,n} = M_1PGE_{11}(1-E_{11})^{n-1} \tag{2.25}
$$

$$
\begin{aligned}
FA_{1,n} &= M_1 \left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \\
&\times [P_{1,n}E_{21} + (1-PG_{1,n}-P_{1,n})(1-E_{11})]
\end{aligned}
\tag{2.26}
$$

$$
CA_{1,n} = M_1PG(1-E_{11})^n \tag{2.27}
$$

This completes one stage of inspection.

Results of Stage (1)

E (number of accepted components) is

$$\begin{aligned} A_1 &= FA_{1,n} + CA_{1,n} \\ &= M_1 \left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\ &\times [P_{1,n}E_{21} + (1 - PG_{1,n} - P_{1,n})(1 - E_{11})] + M_1[PG(1 - E_{11})^n] \end{aligned} \quad (2.28)$$

where $PG_{1,n}$ is given in equation 2.24.

Cost of false rejection after stage (1) is completed

$$\begin{aligned} CFR_1 &= C_r \sum_{j=1}^{n} FR_{1,j} \\ &= C_r M_1 PGE_{11} \sum_{k=1}^{n} (1 - E_{11})^{k-1} \end{aligned} \quad (2.29)$$

Cost of false acceptance after stage (1) is completed

$$\begin{aligned} CFA_1 &= C_a FA_{1,n} \\ &= C_a M_1 \left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\ &\times [P_{1,n}E_{21} + (1 - PG_{1,n} - P_{1,n})(1 - E_{11})] \end{aligned} \quad (2.30)$$

Cost of inspection after stage (1) is completed

$$\begin{aligned} CI_1 &= C_1 \sum_{j=1}^{n} M_{1,j} \\ &= C_1 M_1 \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \end{aligned} \tag{2.31}$$

E(total cost per accepted component after one stage of inspection) is

$$E(tc)|_{i=1} = [CFR_1 + CFA_1 + CI_1]/A_1$$

where A_1, CFR_1, CFA_1, and CI_1 are given by equation 2.28, 2.29, 2.30 and 2.31 respectively.

Analysis of Stage (2)

Number of components entering stage (2) is $M_{2,1} = A_1$, where A_1 is given by equation 2.28.

Cycle (1)

$$\begin{aligned} M_{2,1} &= FA_{1,n} + CA_{1,n} \\ &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \end{aligned}$$

$$PG_{2,1} = \frac{PG(1 - E_{11})^n}{\left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right]}$$

$$FR_{2,1} = M_1 PG E_{12} (1 - E_{11})^n$$

$$
\begin{aligned}
FA_{2,1} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\
&\times \; [P_{2,2}E_{22} + (1 - PG_{2,1} - P_2)(1 - E_{12})] \\
&+ \; M_1 PG(1 - E_{11})^n (1 - E_{1,2})
\end{aligned}
$$

$$
CA_{2,1} = M_1 PG(1 - E_{11})^n (1 - E_{1,2})
$$

Cycle (2)

$$
\begin{aligned}
M_{2,2} &= FA_{2,1} + CA_{2,1} \\
&= M_1 \left[\prod_{j=1}^{n} P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11}) \right] \\
&\times \; [P_{2,2}E_{22} + (1 - P_2)(1 - E_{12})]
\end{aligned}
$$

$$
\begin{aligned}
PG_{2,2} &= \frac{PG(1 - E_{11})^n (1 - E_{1,2})}{\left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right]} \\
&\times \; [P_{2,2}E_{22} + (1 - PG_{2,1} - P_2)(1 - E_{12})]
\end{aligned}
$$

$$
FR_{2,2} = M_1 PGE_{12}(1 - E_{11})^n (1 - E_{1,2})
$$

$$
\begin{aligned}
FA_{2,2} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\
&\times \; [P_{2,2}E_{22} + (1 - P_2)(1 - E_{12})] \\
&\times \; [P_{2,2}E_{22} + (1 - PG_{2,2} - P_{2,2})(1 - E_{12})]
\end{aligned}
$$

$$CA_{2,2} = M_1 PG(1 - E_{11})^n (1 - E_{1,2})^2$$

Cycle (n)

$$\begin{aligned} M_{2,n} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\ &\times \left[\prod_{j=1}^{n-1} [P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12}) \right] \end{aligned}$$

$$\begin{aligned} PG_{2,n} &= \frac{PG(1 - E_{11})^n (1 - E_{1,2})^{n-1}}{\left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right]} \\ &\times \prod_{j=1}^{n-1} [P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})] \end{aligned}$$

$$FR_{2,n} = M_1 PG E_{12} (1 - E_{11})^n (1 - E_{1,2})^{n-1}$$

$$\begin{aligned} FA_{2,2} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\ &\times \prod_{j=1}^{n-1} [P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})] \\ &\times [P_{2,n} E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})] \end{aligned}$$

$$CA_{2,n} = M_1 PG(1 - E_{11})^n (1 - E_{1,2})^n$$

Results of Stage (2)

$$
\begin{aligned}
A_2 &= FA_{2,n} + CA_{2,n} \\
&= M_1 \left[\prod_{j=1}^{n-1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \right] \times \left[\prod_{j=1}^{n-1} \{P_{2,j}E_{22} + (1 - P_{2,j})(1 - E_{12})\} \right] \\
&\times [P_{2,n}E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})] + M_1[PG(1 - E_{11})^n(1 - E_{1,2})^n]
\end{aligned}
$$

$$
\begin{aligned}
CFR_2 &= C_r \sum_{j=1}^{n} FR_{2,j} \\
&= C_r M_1 PGE_{12}(1 - E_{11})^n \sum_{k=1}^{n} (1 - E_{12})^{k-1}
\end{aligned}
$$

$$
\begin{aligned}
CFA_2 &= C_a FA_{2,n} \\
&= C_a M_1 \left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times \left[\prod_{j=1}^{n-1} [P_{2,j}E_{22} + (1 - P_{2,j})(1 - E_{12})] \right] \\
&\times [P_{2,n}E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})]
\end{aligned}
$$

$$
\begin{aligned}
CI_2 &= C_1 \sum_{j=1}^{n} M_{2,j} \\
&= C_1 M_1 \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} \{P_{2,j}E_{22} + (1 - P_{2,j})(1 - E_{12})\} \right] \left[\prod_{j=1}^{n} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \right]
\end{aligned}
$$

Results needed to compute expected total cost

The total number of accepted components after completing N stages of inspection, i.e. after inspecting the N-th characteristic is given as :

$$A_N = M\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n}[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})]\right] \times \left[\prod_{j=1}^{n-1}[P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})]\right]$$

$$\times [P_{N,n}E_{2N} + (1 - PG_{N,n} - P_{N,n})(1-E_{1N})] + M\left[PG\prod_{k=1}^{N}(1-E_{1k})]\right] \qquad (2.32)$$

The cost of false rejection at each stage i, $i = 1, \ldots, N$, is given as :

$$CFR_i = [C_r \times M \times PG \times E_{1i}]\left[\prod_{k=1}^{i-1}(1-E_{1k})^n\right] \times \left[\sum_{k=1}^{n}(1-E_{1i})^{k-1}\right] \qquad (2.33)$$

The cost of false acceptance after completing N stages of inspection is given as :

$$\begin{aligned} CFA_N &= C_a M\left[\prod_{k=1}^{N-1}(\prod_{j=1}^{n}[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})]\right] \\ &\times \left[\prod_{j=1}^{n-1}[P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})]\right] \\ &\times [P_{N,n}E_{2N} + (1 - PG_{N,n} - P_{N,n})(1-E_{1N})] \end{aligned} \qquad (2.34)$$

The cost of inspection at each stage i, $i = 1, \ldots, N$, is given as :

$$CI_i = C_i M \times \left[\prod_{k=1}^{i-1}\prod_{j=1}^{n}\{P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\}\right]$$

$$\times \quad \left[\sum_{k=1}^{n}\left\{\prod_{j=1}^{k-1}[P_{i,j}E_{2i}+(1-P_{i,j})(1-E_{1i})]\right\}\right] \tag{2.35}$$

Now, in order to determine the general expression for the expected total cost per accepted component, we must determine the total cost of false rejection $TCFR$, the total cost of false acceptance $TCFA$, the total cost of inspection TCI and the total number of components finally accepted TA.

$$TCFR = \sum_{i=1}^{N} CFR_i \tag{2.36}$$

$$TCFA = CFA_N = CaFA_{N,n} \tag{2.37}$$

$$TCI = \sum_{i=1}^{N} CI_i \tag{2.38}$$

$$TA = A_N = FA_{N,n} + CA_{N,n} \tag{2.39}$$

$$E(tc)|_{j=n} = \frac{TCFR + TCFA + TCI}{TA} \tag{2.40}$$

The objective is to find the value of n which provides the minimum of $E(tc)|_{j=n}$. The probability of a component being non-defective entering the n-th cycle of inspection of the N-th characteristic is given by

$$PG_{N,n} = \left[\prod_{i=1}^{N-1}(1-P_{i,n+1})\right](1-P_{N,n}) \tag{2.41}$$

Number of components entering the j-th inspection of stage i is given by

$$\begin{aligned} M_{i,j} &= M\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n}[P_{k,j}E_{2k}+(1-P_{k,j})(1-E_{1k})]\right] \\ &\times \left[\prod_{k=1}^{j-1}\{P_{i,k}E_{2i}+(1-P_{i,k})(1-E_{1i})\}\right] \end{aligned} \tag{2.42}$$

Determining the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which the characteristics are ordered for inspection, i.e., the order of stages. The following rule provides the optimal sequence of inspection for the characteristics, when each characteristic is inspected j times.

$$Let \quad r_i = \frac{C_i f_1(R_{i,j})}{1-f_2(R_{i,j})} \quad i=1,2,\ldots,N. \quad j=1,2,\ldots,n. \tag{2.43}$$

where

$$R_{i,j} = P_{i,j}(1-E_{2,i})+(1-P_{i,j})E_{1,i}$$

$$f_1(R_{i,j}) = \sum_{j=1}^{n}\left[\prod_{k=1}^{j}(1-R_{i,k-1})\right]$$

$$f_2(R_{i,j}) = \prod_{k=1}^{n}(1-R_{i,k})$$

The rule is that the characteristic with the lowest ratio r_i be inspected first, next higher, second, next higher, third, and so on, and the characteristic with the highest

ratio is the N-th characteristic to be inspected. The optimality of this rule follows from the proof given by Duffuaa and Raouf [29]. Next the computational procedure is presented to find the optimal n.

Computational Procedure

STEP 1: Set $j = 0$, compute PG and $E(tc)|_j = 0$ using equation 2.7, 2.11 respectively.

STEP 2: Let $j = j + 1$, sequence the characteristics according to equation 2.43.

STEP 3: Compute $P_{i,j}, PG_{N,n}, A_N, CFR_i, CFA_N, CI_i$ for $i = 1, 2, \ldots, N$ from equations 2.6, 2.41, 2.32, 2.33, 2.34 and 2.35 respectively.

STEP 4: Compute $TCFR, TCFA, TCA, TA$ and $E(tc)_j$ from equations 2.36, 2.37, 2.38, 2.39 and 2.40, respectively.

STEP 5: If $E(tc)_j < E(tc)_{j-1}$, go to STEP 2, otherwise STOP, $n = j - 1$.

2.3.2 Model 2 of Duffuaa and Al-Najjar

Model 2 is developed by relaxing the assumption of the equality of the number of repeat inspections needed for each characteristic. In this plan it is suggested that the inspector inspects the first characteristic n_1 times, passes the accepted components to the second inspector who inspects the second characteristic n_2 times, until all the characteristics are inspected $n_1, n_2, \ldots, n_N$ times each. The relaxed inspection plan is shown in figure 1.1.

Basic Relationships in the Model

The assumptions for this model are the same as that of Model 1. The probability of the i-th characteristic being defective is the same as developed in the previous section, which is

$$P_{i,j} = \frac{P_i E_{2i}^{j-1}}{[P_i E_{2i}^{j-1} + (1 - P_i)(1 - E_{1i})^{j-1}]} \tag{2.44}$$

The probability of a component being non-defective when entering inspection is

$$PG = \prod_{i=1}^{N} (1 - P_i) \tag{2.45}$$

The probability of a component being non-defective after inspecting characteristic 1, n_1 times is :

$$PG_{i,n_1+1} = \left[\prod_{i=2}^{N} (1 - P_i)\right] [(1 - P_{i,n_1+1})] \tag{2.46}$$

The probability of a component being non-defective after inspecting each characteristic n_i times is :

$$PG_{N,n_i+1} = \prod_{i=1}^{N} (1 - P_{i,n_i+1}) \tag{2.47}$$

The probability of a component being non-defective after inspecting characteristic 1 through i - 1, n_i times and characteristic i, k times, with the other characteristics

from $i + 1$ through N not inspected, is given by [6] :

$$PG_{i.k} = \left[\prod_{k=1}^{i-1}(1 - P_{m,n_k+1})\right] [(1 - P_{i,k+1})] \left[\prod_{k=i+1}^{N}(1 - P_k)\right] \tag{2.48}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost due to false acceptance of defective components and is given by:

$$E(tc)|_{j=o} = C_a(1 - PG) \tag{2.49}$$

The expected total cost per accepted component, after inspecting each characteristic n times will be

$$E(tc) = [TCFR + TCFA + TCI]/TA \tag{2.50}$$

Cost Minimization Model

In order to determine $TCFR, TCFA, TCI$ and TA, an analysis of different stages of inspection is necessary.

Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or non-defective.

Cycle (n_1)

$$M_{1,n_1} = M_1 \prod_{j=1}^{n_1-1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \tag{2.51}$$

$$PG_{1,n_1} = \frac{PG(1 - E_{11})^{n_1-1}}{\left[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right]} \tag{2.52}$$

$$FR_{1,n_1} = M_1 PGE_{11}(1 - E_{11})^{n_1-1} \tag{2.53}$$

$$\begin{aligned} FA_{1,n_1} &= M_1 \left[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \\ &\times [P_{1,n_1}E_{21} + (1 - PG_{1,n_1} - P_{1,n_1})(1 - E_{11})] \end{aligned} \tag{2.54}$$

$$CA_{1,n_1} = M_1 PG(1 - E_{11})^{n_1} \tag{2.55}$$

This completes one stage of inspection.

Results of Stage (1)

E (number of accepted components) is

$$\begin{aligned} A_1 &= FA_{1,n_1} + CA_{1,n_1} \\ &= M_1 \left[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \\ &\times [P_{1,n_1}E_{21} + (1 - PG_{1,n_1} - P_{1,n_1})(1 - E_{11})] \\ &+ M_1[PG(1 - E_{11})^{n_1}] \end{aligned} \tag{2.56}$$

where PG_{1,n_1} is given in equation 2.52.

Cost of false rejection after stage (1) is completed.

$$\begin{aligned} CFR_1 &= C_r \sum_{j=1}^{n_1} FR_{1,j} \\ &= C_r M_1 PGE_{11} \sum_{k=1}^{n_1} (1 - E_{11})^{k-1} \end{aligned} \tag{2.57}$$

Cost of false acceptance after stage (1) is completed.

$$\begin{aligned} CFA_1 &= C_a FA_{1,n_1} \\ &= C_a M_1 \left[\prod_{j=1}^{n_1-1} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \\ &\times \ [P_{1,n_1} E_{21} + (1 - PG_{1,n_1} - P_{1,n_1})(1 - E_{11})] \end{aligned} \tag{2.58}$$

Cost of inspection after stage (1) is completed.

$$\begin{aligned} CI_1 &= C_1 \sum_{j=1}^{n_1} M_{1,j} \\ &= C_1 M_1 \left[\sum_{k=1}^{n_1} \prod_{j=1}^{k-1} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \end{aligned} \tag{2.59}$$

E(total cost per accepted component after one stage of inspection) is

$$E(tc)|_{i=1} = [CFR_1 + CFA_1 + CI_1]/A_1$$

where A_1, CFR_1, CFA_1, and CI_1 are given by equation 2.56, 2.57, 2.58 and 2.59 respectively.

Analysis of Stage (2)

All the accepted components from stage (1) go to the second inspector who inspects the second characteristic in each component in order to classify it as defective or non-defective.

Cycle (n_2)

$$\begin{aligned} M_{2,n_2} &= M_1 \left[\prod_{j=1}^{n_2} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\} \right] \\ &\times \left[\prod_{j=1}^{n_2-1} [P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})] \right] \end{aligned}$$

$$\begin{aligned} PG_{2,n_2} &= \frac{PG(1-E_{11})^{n_2}(1-E_{1,2})^{n_2-1}}{\left[\prod_{j=1}^{n_2} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]} \\ &\times \prod_{j=1}^{n_2-1} [P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})] \end{aligned}$$

$$FR_{2,n_2} = M_1 PGE_{12}(1-E_{11})^{n_2}(1-E_{1,2})^{n_2-1}$$

$$
\begin{aligned}
FA_{2,n_2} &= M_1 \left[\prod_{j=1}^{n_2} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\} \right] \\
&\times \left[\prod_{j=1}^{n_2-1} [P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})] \right] \\
&\times [P_{2,n_2}E_{22} + (1-PG_{2,n_2} - P_{2,n_2})(1-E_{12})]
\end{aligned}
$$

$$
CA_{2,n_2} = M_1 PG(1-E_{11})^{n_2}(1-E_{1,2})^{n_2}
$$

Results of Stage (2)

$$
\begin{aligned}
A_2 &= FA_{2,n_2} + CA_{2,n_2} \\
&= M_1 \left[\prod_{j=1}^{n_2-1} [P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11}) \right] \\
&\times \left[\prod_{j=1}^{n_2-1} \{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\} \right] \\
&\times [P_{2,n_2}E_{22} + (1-PG_{2,n_2} - P_{2,n_2})(1-E_{12})] \\
&+ M_1[PG(1-E_{11})^{n_2}(1-E_{1,2})^{n_2}]
\end{aligned}
$$

$$
\begin{aligned}
CFR_2 &= C_r \sum_{j=1}^{n_2} FR_{2,j} \\
&= C_r M_1 PGE_{12}(1-E_{11})^{n_2} \sum_{k=1}^{n_2} (1-E_{12})^{k-1}
\end{aligned}
$$

$$
\begin{aligned}
CFA_2 &= C_a FA_{2,n_2} \\
&= C_a M_1 \left[\prod_{j=1}^{n_2-1} \{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\} \right] \\
&\times \left[\prod_{j=1}^{n_2-1} [P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12}) \right] \\
&\times [P_{2,n_2}E_{22} + (1-PG_{2,n_2} - P_{2,n_2})(1-E_{12})]
\end{aligned}
$$

$$
\begin{aligned}
CI_2 &= C_1 \sum_{j=1}^{n_2} M_{2,j} \\
&= C_1 M_1 \left[\sum_{k=1}^{n_2} \prod_{j=1}^{k-1} \{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\} \right] \\
&\times \left[\prod_{j=1}^{n_2} [P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11}) \right]
\end{aligned}
$$

Results needed to compute expected total cost

The total number of accepted components after inspecting the N-th characteristic is given as :

$$
A_N = M \left[\prod_{k=1}^{N-1} \prod_{j=1}^{n_k} [P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})] \right] \times \left[\prod_{j=1}^{n_N-1} [P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})] \right]
$$

$$
\times [P_{N,n_N}E_{2N} + (1-PG_{N,n_N} - P_{N,n_N})(1-E_{1N})] + M \left[PG \prod_{k=1}^{N} (1-E_{1k}) \right] \quad (2.60)
$$

The cost of false rejection at each stage i, $i = 1, \ldots, N$, is given as :

$$CFR_i = [C_r \times M \times PG \times E_{1i}] \left[\prod_{k=1}^{i-1}(1 - E_{1k})^{n_k}\right] \times \left[\sum_{k=1}^{n_i}(1 - E_{1i})^{k-1}\right] \tag{2.61}$$

The cost of false acceptance after completing N stages of inspection is given as :

$$\begin{aligned} CFA_N &= C_a M \left[\prod_{k=1}^{N-1}(\prod_{j=1}^{n_k}[P_{k,j}E_{2k} + (1 - P_{k,j})(1 - E_{1k})]\right] \\ &\times \left[\prod_{j=1}^{n_N-1}[P_{N,j}E_{2N} + (1 - P_{N,j})(1 - E_{1N})]\right] \\ &\times [P_{N,n_N}E_{2N} + (1 - PG_{N,n_N} - P_{N,n_N})(1 - E_{1N})] \end{aligned} \tag{2.62}$$

The cost of inspection at each stage i, $i = 1, \ldots, N$, is given as :

$$\begin{aligned} CI_i &= C_i M \times \left[\prod_{k=1}^{i-1}\prod_{j=1}^{n_k}\{P_{k,j}E_{2k} + (1 - P_{k,j})(1 - E_{1k})\}\right] \\ &\times \left[\sum_{k=1}^{n_i}\left\{\prod_{j=1}^{k-1}[P_{i,j}E_{2i} + (1 - P_{i,j})(1 - E_{1i})]\right\}\right] \end{aligned} \tag{2.63}$$

Now, in order to determine the general expression for the expected total cost per accepted component, we must determine the total cost of false rejection $TCFR$, the total cost of false acceptance $TCFA$, the total cost of inspection TCI and the total number of components finally accepted TA.

$$TCFR = \sum_{i=1}^{N} CFR_i \tag{2.64}$$

$$TCFA = CFA_N = CaFA_{N,n_N} \tag{2.65}$$

$$TCI = \sum_{i=1}^{N} CI_i \tag{2.66}$$

$$TA = A_N = FA_{N,n_N} + CA_{N,n_N} \tag{2.67}$$

$$E(tc)|_{j=n} = \frac{TCFR + TCFA + TCI}{TA} \tag{2.68}$$

The objective is to find the value of $n_i, i = 1, 2, \ldots, N$ which provides the minimum of $E(tc)$.

The probability of a component being non-defective entering the n-th cycle of inspection of the N-th characteristic is given by

$$PG_{N,n_N} = \left[\prod_{i=1}^{N-1}(1 - P_{i,n_i+1}\right](1 - P_{N,n_N}) \tag{2.69}$$

Number of components entering the j-th inspection of stage i is given by

$$\begin{aligned} M_{i,j} &= M\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n_k}[P_{k,j}E_{2k} + (1 - P_{k,j})(1 - E_{1k})]\right] \\ &\times \left[\prod_{k=1}^{j-1}\{P_{i,k}E_{2i} + (1 - P_{i,k})(1 - E_{1i})\}\right] \end{aligned} \tag{2.70}$$

Determining the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which the characteristics are ordered for inspection, i.e., the order of stages. The following rule provides the

optimal sequence of inspection for the characteristics, when each characteristic is inspected j times.

$$r_i = \frac{C_i f_1(R_{i,j})}{1 - f_2(R_{i,j})} \quad i = 1, 2, \ldots, N. \quad j = 1, 2, \ldots, n_i. \tag{2.71}$$

where

$$R_{i,j} = P_{i,j}(1 - E_{2,i}) + (1 - P_{i,j})E_{1,i}$$

$$f_1(R_{i,j}) = \sum_{j=1}^{n_i} \left[\prod_{k=1}^{j} (1 - R_{i,k-1}) \right]$$

$$f_2(R_{i,j}) = \prod_{k=1}^{n_i} (1 - R_{i,k})$$

The characteristic with the lowest ratio is inspected first, next higher, second, next higher, third, and so on, and the characteristic with the highest ratio is the N-th characteristic to be inspected.

Computational Procedure

The computational procedure for this model depends on the concept of steepest decent.

At iteration i we have inspected characteristic i, r_i times. The total cost of inspection is

$$ETC(r_1, r_2, \ldots, r_N)$$

The decent in the direction i, from $(r_1, r_2, \ldots, r_i, \ldots, r_N)$ to $(r_1, r_2, r_i + 1, \ldots, r_N)$ is given by

$$DIN(i) = ETC(r_1, r_2, r_i + 1, \ldots, r_N) - ETC(r_1, r_2, \ldots, r_N)$$

at each point the decent is computed in all directions. Then a move is made in the direction which has the largest decent. Suppose we are at the stage where each characteristic is inspected r_i times. The steps of the algorithm are :

STEP (1) : Compute $ETC(r_1, r_2, \ldots, r_N)$.

STEP (2) : Find $DIN(i)$ for $i = 1, 2, \ldots, N$.

If $DIN \geq 0$ for all i, go to STEP (6). Otherwise proceed.

STEP (3) : Find $\{\max_i |DIN(i)| \mid DIN(i) < 0\} = DIN(k)$.

STEP (4) : Inspect characteristic k and compute $ETC(r_1, r_2, r_k + 1, r_{k+1}, \ldots, r_N)$.

STEP (5) : Go to STEP (2).

STEP (6) : STOP. The optimal inspection is $(r_1, r_2, \ldots, r_N)$ and the optimal total expected cost of inspection is $ETC(r_1, r_2, \ldots, r_N)$.

The above stated algorithm is expected to provide a local minimum.

2.4 Conclusion

In this chapter the literature relevant to this thesis is reviewed and a description of the models which will be extended in this thesis are given. The algorithms for

obtaining the optimal number of repeat inspections are also presented. However, in this thesis Hooke and Jeeves is used to obtain the optimal number of repeat inspections n_i for characteristic i to be performed in order to minimize the expected total cost. The plans can be extended for the case where the characteristic are statistically dependent. It is indicated in the literature [17] that inspection error varies with the change in the incoming quality. No work has been done to incorporate the varying inspection errors in the plans proposed by Duffuaa and Al-Najjar [6]. These will be the subjects discussed in the following chapters of this thesis.

Chapter 3

The Effect of Inspection Errors on Repeat Inspection Plans

3.1 Introduction

Inspection is often used to appraise the quality of purchased or manufactured items. Studies have shown that inspection tasks are not error free. On the contrary, these tasks are often error prone [9, 31]. Many researchers in the past have considered the effect of inspection errors on the overall effectiveness of various quality control schemes. The main focus of the existing work evaluates the individual effect of factors like deterioration in the statistical properties of the inspection plan [12, 11, 32], behavior of cost measures [13], and indicators like Average Total Inspection (ATI) and Average Outgoing Quality (AOQ) [12] due to inspection errors.

Inspection errors have clearly been shown to have adverse effects on the result desired from a quality assurance sampling plan. The impact of inspection errors are more adverse when dealing with critical components with several characteristics. In order to guard against these inspection errors, repeat inspection has been instituted. In this chapter the effect of inspection errors on Model 1 and Model 2 proposed by Duffuaa and Al-Najjar [6] is illustrated. The study is conducted by observing the effects of inspection errors on performance measures such as AOQ, ATI and ETC of the repeat inspection plan. This is accomplished by varying Type I error and Type II error for a given situation and observing its effects on the performance measures.

3.2 Impact of inspection errors

The models stated in the previous chapter depict the inspection plan proposed for repeat inspection and are employed to investigate the impact of inspection errors on ATI, ETC and AOQ. To examine the effect of Type I and Type II errors, a batch of 100 components, each with three characteristics were used in the models and the algorithms mentioned in the previous chapter are used to determine the optimal inspection plan for both Model 1 and Model 2. The probability of each characteristic being defective P_i is taken to be 0.1, 0.2, 0.3, for $i = 1, 2, 3$ respectively.The cost of inspection $C_i = 100$ for $i = 1, 2, 3$. The cost of false acceptance is 100000 and the cost of false rejection is 500. Type I and Type II errors are varied from 0.00 to

0.15. The algorithms that determine the optimal number of repeat inspections of the model was implemented on a Pentium machine under UNIX environment and the model was run 36 times using 6 different values of Type I and Type II errors. For each run Type I and Type II error are assumed the same for all the characteristics and referred to as E_1 and E_2. The models determine the optimal number of repeat inspections and simulates the inspection process for each pair of errors. The results of the 36 runs are shown in Table 3.1 for Model 1 and in Table 3.2 for Model 2. Both Table 1 and Table 2 show the pair of errors and the optimal values ETC, AOQ and A_N for each pair of errors. The tables also show the total cost of inspection CI, the cost of false acceptance CFA and the cost of false rejection CFR. All the costs are costs per accepted component. Table 3.1 and Figures 3.1, 3.2, 3.5, 3.6, 3.9 and 3.10 are examined to determine the impact of errors on the performance measures of the inspection plan proposed in Model 1. Table 3.2 and Figures 3.3, 3.4, 3.7, 3.8, 3.11 and 3.12 are examined to determine the impact of errors on the performance measures of the inspection plan proposed in Model 2.

3.2.1 Effects of Inspection Error on Average Total Inspection (ATI)

The Average Total Inspection (ATI) is the total number of inspections performed until the optimal parameters of the plan are determined. It is also a function of the

Table 3.1: The effect of inspection errors on Model 1 of Duffuaa and Al-Najjar

$No.$	E_1	E_2	$1-PG$	ETC	CI	CFR	CFA	$A(n)$	n^*
1	0.00	0.00	0.00000	448.41	448.41	0.00	0.00	50	1
2	0.00	0.01	0.00008	807.26	799.36	0.00	7.90	50	2
3	0.00	0.03	0.00071	872.06	801.02	0.00	71.04	50	2
4	0.00	0.05	0.00197	999.52	802.38	0.00	197.15	50	2
5	0.00	0.10	0.00079	1237.62	1158.69	0.00	78.92	50	3
6	0.00	0.15	0.00266	1429.55	1163.53	0.00	266.02	51	3
7	0.01	0.00	0.00000	473.73	458.43	15.31	0.00	49	1
8	0.01	0.01	0.00008	870.39	831.26	31.08	8.06	47	2
9	0.01	0.03	0.00072	936.54	833.00	31.06	72.48	47	2
10	0.01	0.05	0.00201	1066.58	834.42	31.02	201.14	48	2
11	0.01	0.10	0.00081	1354.01	1225.37	47.30	81.34	46	3
12	0.01	0.15	0.00274	1551.91	1230.56	47.21	274.14	46	3
13	0.03	0.00	0.00000	527.40	479.56	47.84	0.00	46	1
14	0.03	0.01	0.00008	1009.49	900.85	100.25	8.39	42	2
15	0.03	0.03	0.00075	1078.44	902.76	100.18	75.50	42	2
16	0.03	0.05	0.00210	1213.86	904.30	100.05	209.51	42	2
17	0.03	0.10	0.00086	1619.76	1375.73	157.56	86.47	38	3
18	0.03	0.15	0.00291	1830.34	1381.69	157.24	291.42	38	3
19	0.05	0.00	0.00000	585.45	502.27	83.18	0.00	43	1
20	0.05	0.01	0.00009	1168.04	979.12	180.17	8.75	37	2
21	0.05	0.03	0.00079	1239.98	981.23	180.05	78.71	37	2
22	0.05	0.05	0.00218	1381.11	982.91	179.79	218.41	37	2
23	0.05	0.10	0.00092	1937.83	1552.72	293.07	92.04	32	3
24	0.05	0.15	0.00310	2162.19	1559.59	292.43	310.17	32	3
25	0.10	0.00	0.00000	752.97	567.10	185.87	0.00	37	1
26	0.10	0.01	0.00872	1620.14	563.88	184.25	872.01	37	1
27	0.10	0.03	0.00088	1753.42	1225.28	440.45	87.69	27	2
28	0.10	0.05	0.00243	1910.44	1227.36	439.77	243.31	27	2
29	0.10	0.10	0.00968	2634.58	1229.77	436.57	968.23	27	2
30	0.10	0.15	0.00365	3316.07	2163.71	787.70	364.65	20	3
31	0.15	0.00	0.00000	960.20	646.03	314.17	0.00	31	1
32	0.15	0.01	0.00923	1876.30	642.07	311.27	922.97	31	1
33	0.15	0.03	0.00098	2486.48	1563.26	824.92	98.30	19	2
34	0.15	0.05	0.00273	2662.08	1565.88	823.48	272.72	19	2
35	0.15	0.10	0.01085	3469.60	1568.23	816.78	1084.59	19	2
36	0.15	0.15	0.02417	4787.49	1564.63	805.78	2417.09	19	2

Table 3.2: The effect of inspection errors on Model 2 of Duffuaa and Al-Najjar

No.	E_1	E_2	$1-PG$	ETC	CI	CFR	CFA	$A(n)$	n_1^*, n_2^*, n_3^*
1	0.00	0.00	0.00000	448.41	448.41	0.00	0.00	50	1, 1, 1
2	0.00	0.01	0.00008	807.26	799.36	0.00	7.90	50	2, 2, 2
3	0.00	0.03	0.00071	872.06	801.02	0.00	71.04	50	2, 2, 2
4	0.00	0.05	0.00197	999.52	802.38	0.00	197.15	50	2, 2, 2
5	0.00	0.10	0.00179	1236.27	1057.53	0.00	178.73	50	2, 3, 3
6	0.00	0.15	0.00266	1429.55	1163.53	0.00	266.02	51	3, 3, 3
7	0.01	0.00	0.00000	473.73	458.43	15.31	0.00	49	1, 1, 1
8	0.01	0.01	0.00119	866.80	722.04	25.74	119.02	48	1, 2, 2
9	0.01	0.03	0.00072	936.54	833.00	31.06	72.48	47	2, 2, 2
10	0.01	0.05	0.00201	1066.58	834.42	31.02	201.14	48	2, 2, 2
11	0.01	0.10	0.00183	1336.73	1111.89	41.79	183.06	47	2, 3, 3
12	0.01	0.15	0.00274	1551.91	1230.56	47.21	274.14	46	3, 3, 3
13	0.03	0.00	0.00000	527.40	479.56	47.84	0.00	46	1, 1, 1
14	0.03	0.01	0.00122	976.60	772.83	82.15	121.62	43	1, 2, 2
15	0.03	0.03	0.00075	1078.44	902.76	100.18	75.50	42	2, 2, 2
16	0.03	0.05	0.00210	1213.86	904.30	100.05	209.51	42	2, 2, 2
17	0.03	0.10	0.00192	1562.91	1233.04	137.70	192.17	40	2, 3, 3
18	0.03	0.15	0.00291	1830.34	1381.69	157.24	291.42	38	3, 3, 3
19	0.05	0.00	0.00000	585.45	502.27	83.18	0.00	43	1, 1, 1
20	0.05	0.01	0.00124	1099.42	829.09	146.00	124.33	39	1, 2, 2
21	0.05	0.03	0.00079	1239.98	981.23	180.05	78.71	37	2, 2, 2
22	0.05	0.05	0.00218	1381.11	982.91	179.79	218.41	37	2, 2, 2
23	0.05	0.10	0.00202	1828.58	1373.46	253.16	201.96	34	2, 3, 3
24	0.05	0.15	0.00310	2162.19	1559.59	292.43	310.17	32	3, 3, 3
25	0.10	0.00	0.00000	752.97	567.10	185.87	0.00	37	1, 1, 1
26	0.10	0.01	0.00405	1445.86	779.60	261.02	405.24	33	1, 1, 2
27	0.10	0.03	0.00088	1753.42	1225.28	440.45	87.69	27	2, 2, 2
28	0.10	0.05	0.00243	1910.44	1227.36	439.77	243.31	27	2, 2, 2
29	0.10	0.10	0.00503	2581.83	1536.44	542.63	502.75	24	2, 2, 3
30	0.10	0.15	0.00620	3119.90	1842.35	657.43	620.13	22	2, 3, 3
31	0.15	0.00	0.00000	960.20	646.03	314.17	0.00	31	1, 1, 1
32	0.15	0.01	0.00429	1806.68	921.48	455.88	429.33	26	1, 1, 2
33	0.15	0.03	0.00475	2322.43	1223.76	623.90	474.77	22	1, 2, 2
34	0.15	0.05	0.00273	2662.08	1565.88	823.48	272.72	19	2, 2, 2
35	0.15	0.10	0.01085	3469.60	1568.23	816.78	1084.59	19	2, 2, 2
36	0.15	0.15	0.01347	4429.45	2036.99	1045.41	1347.05	16	2, 2, 3

sequence of inspection at each inspection cycle and is expressed as

$$
\begin{aligned}
ATI &= M_1 \sum_{i=1}^{N} \left[\sum_{j=2}^{n} \prod_{k=1}^{j-1} \{P_{i,k} E_{2i} + (1 - P_{i,k})(1 - E_{1i})\} \right] \\
&\times \left[\prod_{k=1}^{i-1} \prod_{j=1}^{n} \{P_{k,j} E_{2k} + (1 - P_{k,j})(1 - E_{1k})\} \right] \qquad (3.1)
\end{aligned}
$$

The relationship between E_1 and ATI and E_2 and ATI are shown in figures 3.1 and 3.2 for Model 1. Figure 3.1 shows that the average total inspection (ATI) decreases with the increase in Type I error. This is obvious because as E_1 increases, more and more non-defective components are rejected and hence the number of components inspected in the later inspection cycles is reduced. For higher values of E_1 and E_2 the rate of decrease in ATI is faster as seen in the figures. Looking closely in table 3.1 and 3.2 it is seen that at higher values of E_1 and E_2 the number of repeat inspection instituted by the plan is less. Since the number of inspection performed is less so the inspection load decreases. ATI increases with the increase in Type II error and is shown in figure 3.2. For a particular value of E_1 as E_2 increases the inspection load almost remains constant and it increases only when E_2 reaches a certain value where the cost due to false acceptance becomes so high that it is economical to go for more inspection thereby increasing the inspection load. Again as E_2 increases the increase in cost due to false acceptance is not enough to force more inspection keeping the inspection load almost the same. Therefore, the curve

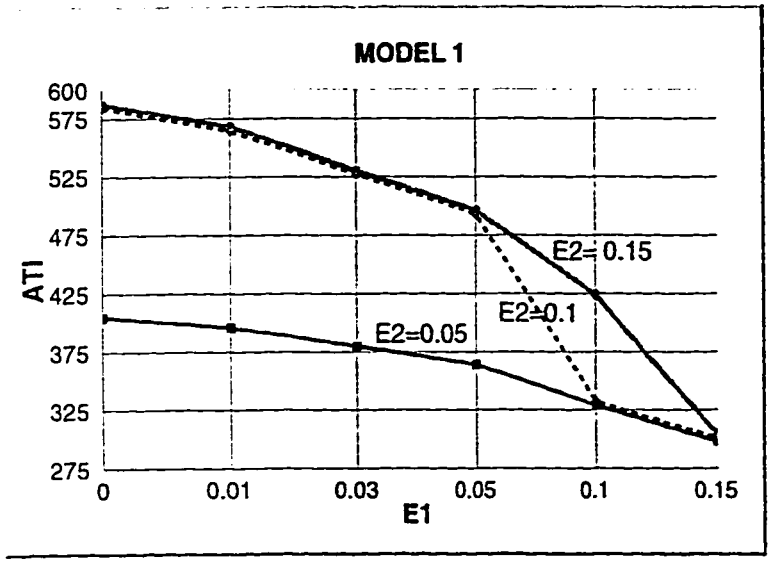

Figure 3.1: Relationship between E1 and ATI

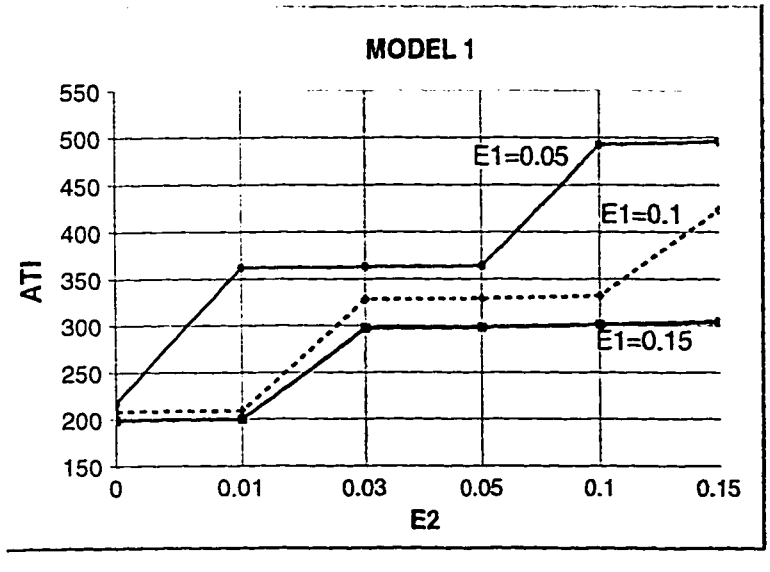

Figure 3.2: Relationship between E2 and ATI

remains almost constant and then increases for one value of E_2 and again remains constant and then again increases.

In case of Model 2 the relationship between Type I error and ATI and between Type II error and ATI exhibits an almost similar behavior as that for Model 1 and is shown in figure 3.3 and figure 3.4 respectively. As seen in figure 3.3 there is a sharp decrease in ATI for high values of E_1 and E_2. For Model 2 unlike Model 1there is a continuous increase in ATI with the increase in E_2. In Model 2 one can inspect the characteristics different number of times. So with the increase in E_2 it is economical to inspect one or two characteristic one more time to screen out falsely accepted components rather than inspecting all the characteristics one more time as in Model 1.

3.2.2 Effects of Inspection Error on Expected Total Cost (ETC)

The Expected Total Cost (ETC) includes the cost of false acceptance, the cost of false rejection and the cost of inspection per accepted component. There is a significant effect of inspection errors on the ETC per inspection.

The relationship between E_1 and ETC and E_2 and ETC are shown in figure 3.5 and 3.6 for Model 1. ETC increases as Type I error increases but the increase is slow for lower values of E_1 because the cost due to false rejection increases very

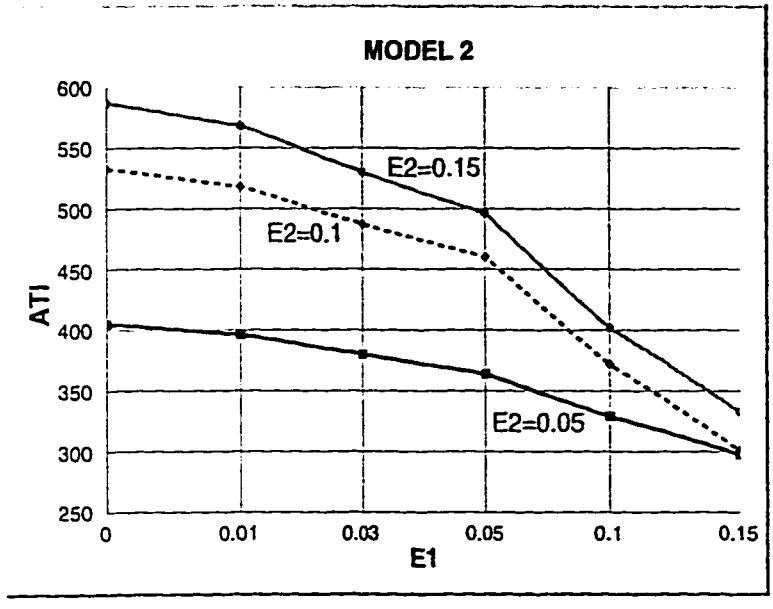

Figure 3.3: Relationship between E1 and ATI

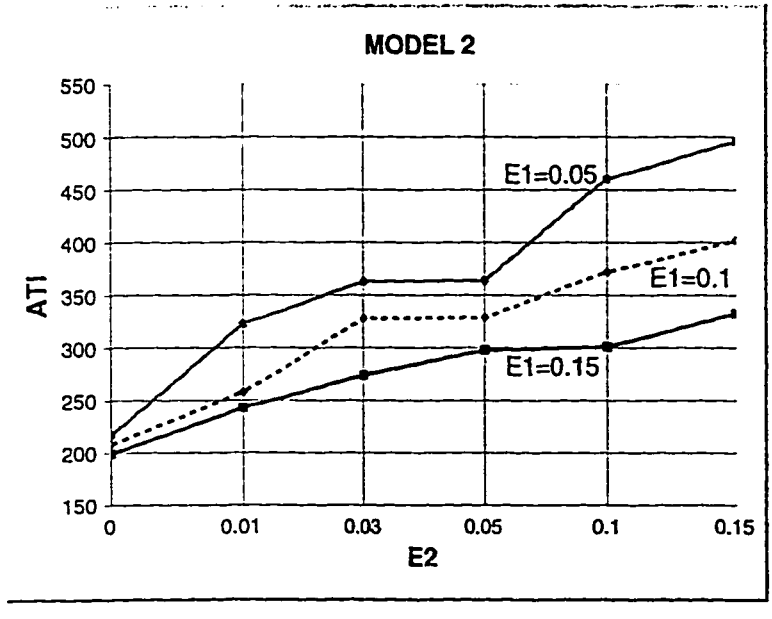

Figure 3.4: Relationship between E2 and ATI

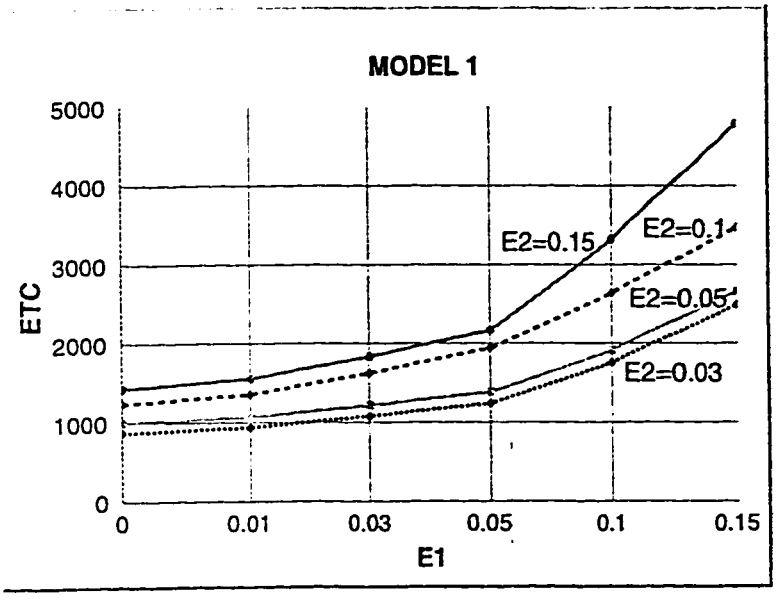

Figure 3.5: Relationship between E1 and ETC

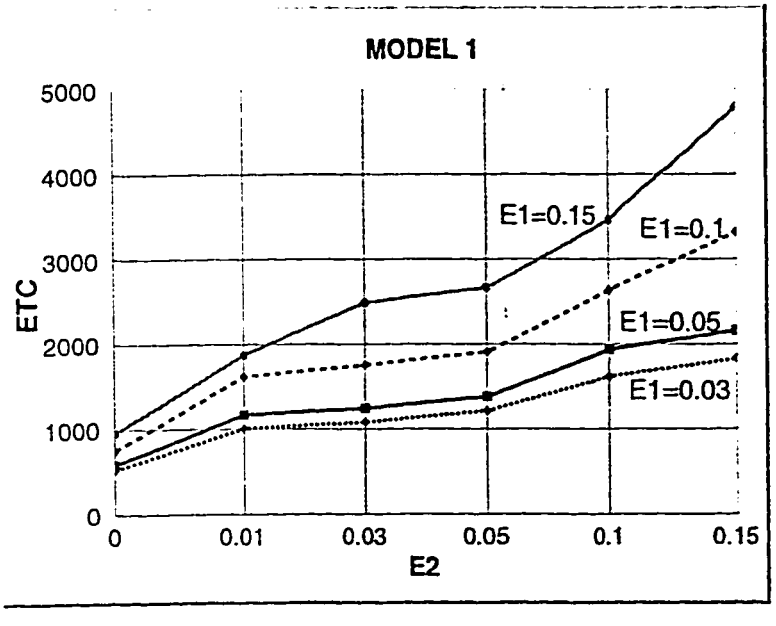

Figure 3.6: Relationship between E2 and ETC

slowly as the increase in the number of good components rejected is low. But for higher vales of E_1 and E_2 ($E_2 = 0.15$), ETC sharply increases. When both E_1 and E_2 are high, the number of components accepted after inspection decreases sharply while both the cost of false rejection increases with increase in E_1 and the cost of false acceptance increases with increase in E_2. So when the total cost increases and the number of components accepted decreases then the cost per accepted component increases sharply.

Similarly, ETC increases as Type II error increases as can be seen in figure 3.5. This is intuitively justified because the cost of accepting a defective component is very high so as we accept more bad components ETC increases. However, Type II error has more drastic effect on ETC as can be seen from the slope of the graph. This is expected because the cost of accepting a defective component is very very high compared to the cost of rejecting a non-defective component.

The relationships between Type I error and ETC and between Type II error and ETC for Model 2 are shown in figure 3.7 and figure 3.8 respectively. They exhibit an almost similar behavior as that for Model 1 which can be seen in the figures. However, Model 2 performs better than Model 1 in terms of the expected total cost. The characteristics can be inspected different number of times in Model 2 unlike Model 1 where all the characteristics must be inspected equal number of times. It is economical to inspect some characteristic less number of times and hence Model 2 performs better in terms of having less total expected cost.

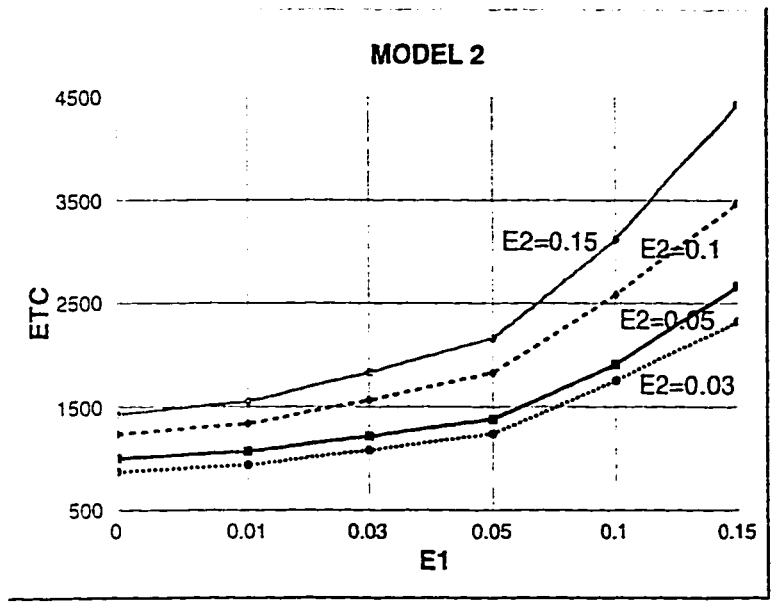

Figure 3.7: Relationship between E1 and ETC

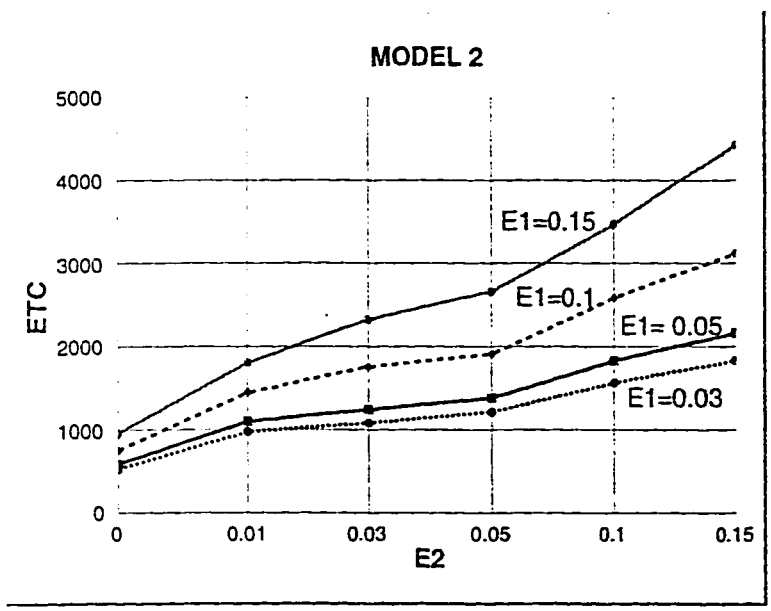

Figure 3.8: Relationship between E2 and ETC

3.2.3 Effects of Inspection Error on Average Outgoing Quality (AOQ)

Average Outgoing Quality (AOQ) is defined by the ratio

$$AOQ = \frac{\text{expected number of defective components remaining after inspection}}{\text{total number of components in the lot}} \tag{3.2}$$

The effect of Type I error and Type II error on AOQ for Model 1 is shown in figure 3.9 and 3.10 and that for Model 2 is shown in figure 3.11 and 3.12 respectively. It can be seen from the figures 3.9 for Model 1 and 3.11 for Model 2 that the AOQ remains almost the same for lower values of E_1 because the number of defective components accepted is low but increases drastically for higher values of E_1 when E_2 is also high. The total number of accepted components in the lot reduces as E_1 is high while among the components accepted after inspection the number of defective components is also high since E_2 is high. Hence, AOQ sharply increases which is the ratio of the number of defective components remaining to the total number of components. Also for Model 2 it is seen in figure 3.11 that AOQ for $E_2 = 0.1$ is less than that for for $E_2 = 0.05$. This is because for $E_2 = 0.1$ the cost due to false acceptance forces more inspection to screen out the defective component thus lowering AOQ.

From figures 3.10 and 3.12 it can be seen that AOQ increases and then decreases

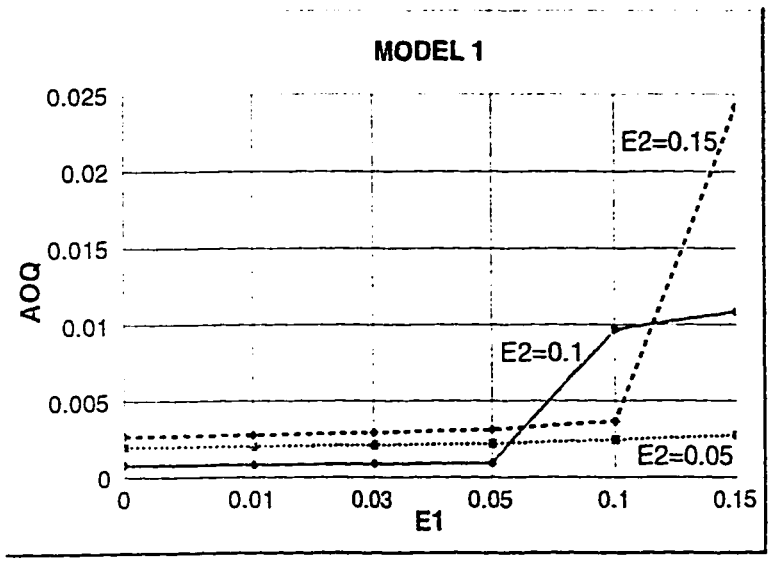

Figure 3.9: Relationship between E1 and AOQ

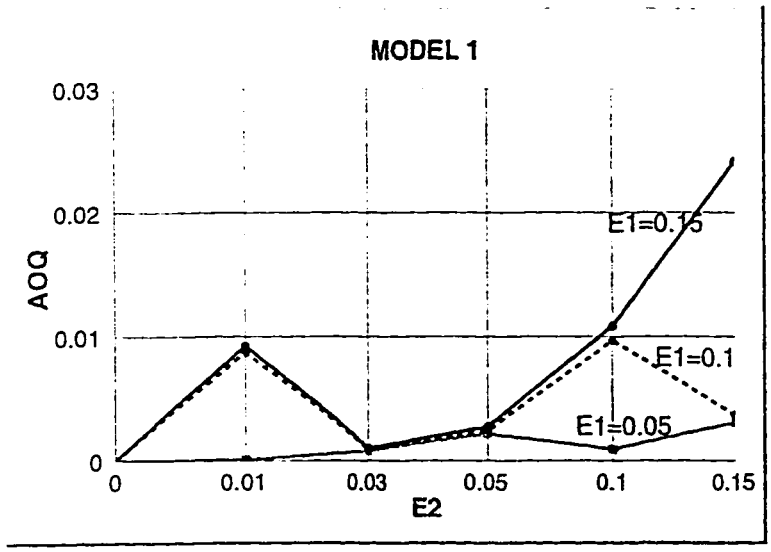

Figure 3.10: Relationship between E2 and AOQ

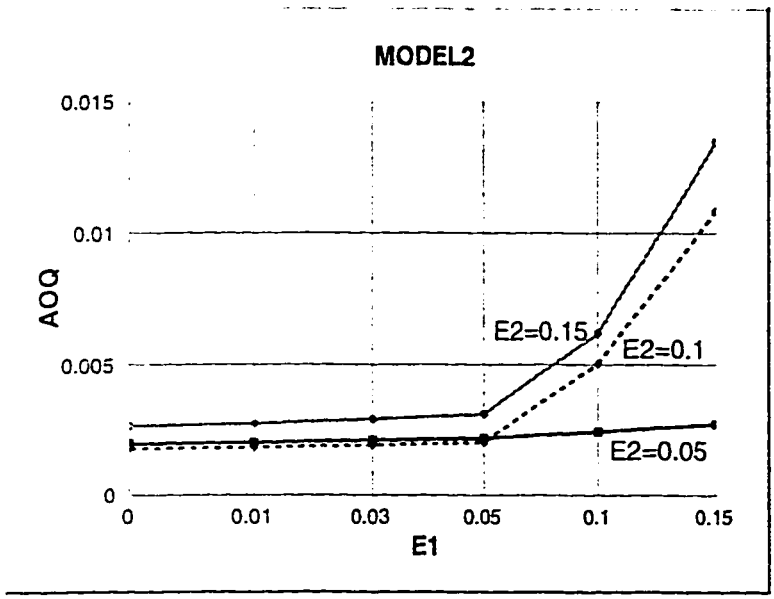

Figure 3.11: Relationship between E1 and AOQ

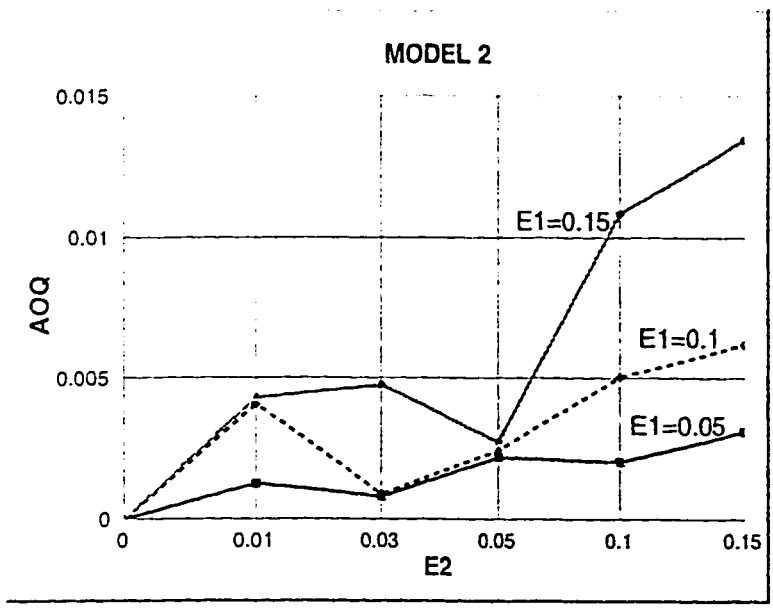

Figure 3.12: Relationship between E2 and AOQ

with the increase in Type II error. This was intuitively expected because as E_2 increases we accept more defective components which increase the cost as well as the average outgoing quality. This pattern is followed with the increase in E_2 until a point is reached where ETC increases so high that the inspection plan forces to perform more inspection to screen out the falsely accepted component hence lowering AOQ as well as the ETC. Looking closely in Table 3.1 and 3.2 and the plot we can see that AOQ decreases after increasing at points where more inspection is instituted by the plan.

Figure 3.12 shows that as E_2 increases beyond a certain value, more inspection is performed which screens out more defective components hence decreasing the AOQ. Again as E_2 increases, AOQ keeps on increasing because the probability of accepting a defective component is increasing, until it increase the ETC so high that the model forces more inspection which lowers the AOQ as more bad components are screened out.

3.3 Conclusion

The major purpose of this chapter is to present the effects, both statistical and economic, that Type I error and Type II error have on both the inspection plans presented by Duffuaa and Al-Najjar. Type I and Type II errors were varied and their effect on the performance measures such as ETC, ATI and AOQ for both

models were studied. It was seen that both the inspection plans exhibited an almost identical behavior. With the increases in Type II error, ETC increases at a faster rate than in the case of Type I error. This indicates that E_2 has a larger effect on ETC than E_1. For both the inspection plans the AOQ remains almost same for lower values of E_1 but drastically increases for higher values of E_1. Also the AOQ increases as E_2 increases but then decreases for a certain value of E_2 because at this point more inspection is instituted to screen out falsely accepted components which increase the cost. After another repeat inspection the number of defective components remaining in the lot becomes low thus decreasing the AOQ. Again AOQ keeps on increasing until more inspection is done which again lowers AOQ. Nevertheless, the combined effect of increasing E_1 and E_2 elevates the AOQ values much more than E_1 and E_2 does it alone.

The average total inspection is illustrated as a function of Type I error and Type II error. Again for both the models, ATI decreases as Type I error increases. This is expected since an increase in the probability of rejecting a good component reduces the number of components to be inspected in the subsequent cycle. It is also seen that for higher values of E_1 and E_2 the rate of decrease of ATI is faster. However, for both the models, ATI increases with the increase in Type II error and then remains constant for some values of Type II error and then again increases. This is also justified because as E_2 increases the inspection load almost remains constant and it increases only when E_2 reaches a certain value where the cost due to false

acceptance becomes so high that it is economical to go for more inspection thereby increasing the inspection load.

Chapter 4

A Statistically Dependent Multicharacteristic Inspection Plan

4.1 Introduction

The models in the area of multicharacteristic inspection plans assume that the defective rates of the characteristics are statistically independent. However, in many situations components have characteristics whose defective rates are statistically dependent on each other. For example, a welded joint has four key quality characteristics : toughness, fatigue strength, tensile strength and fracture toughness. The quality of the four characteristics are dependent on one another. Another ex-

ample, in military fitness medical screening, blood pressure, weight and speed are dependent on each other.

In such a situation the inspection models need to be modified for the case where the characteristic defective rates are statistically dependent. In order to do so we must have knowledge about the joint probability mass function ($j.p.m.f.$) of the random variables depicting the defective rates of the characteristics. Knowing the $j.p.m.f.$ we can obtain the marginal probability mass function ($m.p.m.f.$) of each characteristic. Since the $j.p.m.f.$ varies from cycle to cycle, the values for the individual marginal mass function must be updated using Bayes theorem. Since the characteristics defective rate are statistically dependent, after the inspection of the first characteristic, the marginal of the others must be updated prior to inspecting them. Using the above concepts, both models presented in chapter 2 will be modified to incorporate the dependency of the characteristics.

In this chapter both the models of Duffuaa and Al-Najjar [8] given in chapter 2 are extended for the dependency case and rules for updating the quality of the dependent characteristic are proposed. These rules are consistent with the basic probability rules. In section 2, the dependency of the characteristics is incorporated in Model 1 and an algorithm for obtaining the optimal number of inspections is presented. An example is also presented to demonstrate the results of the model. In section 3, Model 2 is modified to handle the dependency case. In section 4, the effects of inspection errors on the performance measures, ATI, ETC, and AOQ, for

both extended models are studied.

4.2 Model 1

The model is developed for components with several characteristics which are statistically dependent. A component is accepted if all of its characteristics meet the quality specifications. We denote the random variable X_i which takes the value 0 if characteristic i is defective and 1 if it is non-defective. The joint probability density function of the multivariate random variable $X = (x_1, x_2, \ldots, x_N)$ is assumed to be known. The values of E_{1i} and E_{2i} are also assumed to be known. The expected total cost involves the cost of false acceptance, the cost of false rejection and the cost of inspection, which are also assumed to be known. The inspection plan is shown in figure 2.1 where each characteristic is inspected equal number of times. In the nomenclature given below, i ranges from 1 to N and j ranges from 1 to n.

Nomenclature

X_i	A discrete random variable which takes value 0 if characteristic i is defective and 1 if it is non-defective.
$P_i(x_i)$	The marginal probability mass function of the random variable X_i.
$P_i(0)$	Probability of the i-th characteristic being defective on

entering the inspection process.

${}^{j}P_i(0)$	Probability of the i-th characteristic being defective on entering the j-th inspection cycle.
$P(x_1, x_2, \ldots, x_N)$	The joint probability mass function of the random variables X_i, $i = 1, 2, \ldots, N$.
${}^{i}P(x_1, x_2, \ldots, x_N)$	The joint probability mass function of the random variables X_i for the component entering the i-th stage of inspection.

4.2.1 Basic Relationships of the Model

At the start of inspection we know the joint probability mass function (*j.p.m.f.*) of the random variables $(X_1, X_2, \ldots, X_N)$. So, using the *j.p.m.f.* we can obtain the individual marginal probability mass functions:

$$P_i(x_i) = \sum_{x_1}\sum_{x_2}\sum_{x_3}\cdots\sum_{x_{i-1}}\sum_{x_{i+1}}\cdots\sum_{x_N} P(x_1, x_2, \ldots, x_N) \tag{4.1}$$

The marginal mass functions will vary from cycle to cycle

$${}^{1}P_i(0) = P_i(0) \tag{4.2}$$

Using Bayes theorem

$$^{2}P_i(0) = \frac{P_i(0)E_{2i}}{[P_i(0)E_{2i} + (1 - P_i(0))(1 - E_{1i})]} \tag{4.3}$$

$$^{2}P_i(1) = 1 -^{2} P_i(0) \tag{4.4}$$

The updated values for the individual random variable marginal mass function can be obtained using equation 4.3 and 4.4. In general, the marginal probability mass function for the i-th characteristic at the j-th cycle of inspection is :

$$^{j}P_i(0) = \frac{^{j-1}P_i(0)E_{2i}}{[^{j-1}P_i(0)E_{2i} + (1 -^{j-1} P_i(0))(1 - E_{1i})]} \tag{4.5}$$

$$^{j}P_i(1) = 1 -^{j} P_i(0) \tag{4.6}$$

Owing to the inspection the joint and the marginal mass function must be updated after the n inspection of the characteristics at each stage. Because of the statistical dependency between characteristic i and the other characteristics, the marginal of the other characteristics must be updated prior to inspecting them. The updated values of the joint probability mass function will be obtained using Bayes theorem. After inspecting characteristic i, n times at the first stage of inspection the rule for updating the joint probability mass function is

$$^{1}P(x_1, x_2, \ldots, x_N) \;=\; P(x_1, x_2, \ldots, x_N) \tag{4.7}$$

$$^{2}P(x_1, x_2, \ldots, x_N) \;=\; {}^{1}P(x_1, x_2, \ldots, x_N)\frac{^{n+1}P_i(x_i)}{^{1}P_i(x_i)}$$

i.e. we multiply the old joint probability mass function by the updated marginal mass function for characteristic i (the characteristic which is just inspected n times at the first stage) and divided by the old marginal mass function of the inspected characteristic. It can be seen from Bayes theorem that the updated function is a probability mass function. After obtaining the updated joint probability mass function we can find the marginal for each characteristic. Then we can inspect the second characteristic n times and so on until we inspect all characteristics n times. At the end of each stage, we can compute the probability of the component being non-defective, which is given by

$$PG_{1,n+1} =^{1} P(1,1,1,\ldots,1)\frac{^{n+1}P_i(1)}{^{1}P_i(1)} \tag{4.8}$$

The probability of a component being non-defective entering the n-th cycle of inspection of the N-th characteristic is given by

$$PG_{N,n} =^{N} P(1,1,1,\ldots,1)\frac{^{n}P_i(1)}{^{1}P_i(1)} \tag{4.9}$$

If no inspection is performed we incur only the cost of false acceptance

$$E(tc)|_{j=0} = C_a[1 - P(1,1,1,\ldots,1)] \tag{4.10}$$

The expected total cost per accepted component, after inspecting each characteristic n times will be

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{4.11}$$

The analysis of the different stages of inspection and the results needed to compute the expected total cost remains the same as given in chapter 2 section 2.3.1. The only change that has to be made is that, in all the equations $P_{i,j}$ have to be replaced by ${}^{j}P_i(0)$.

4.2.2 Algorithm to determine the number of repeat inspections

STEP 1 : Set $j = 0$, find $E(tc)|_{j=0}$ using equation 4.10.

STEP 2 : Find the ratio of the characteristics according to equation 2.43. Inspect the characteristic with the lowest ratio j times.

STEP 3 : Update the $j.p.m.f$ of the characteristics remaining to be inspected using equation 4.8. Find the ratio of the characteristics remaining to be inspected using equation 2.43. Inspect the characteristic with the lowest ratio j times. Repeat this until all the characteristics are inspected.

STEP 4 : Compute ${}^{j}P_i(0)$, ${}^{j}P_i(1), PG_{N,n}, A_N, CFR_i, CFA_N, CI_i$ for $i = 1, 2, \ldots, N$ from equations 4.5 , 4.6, 4.9, 2.32, 2.33, 2.34 and 2.35 respectively.

STEP 5 : Compute $TCFR, TCFA, TCI, TA$ and $E(tc)_j$ from equations 2.36, 2.37,

2.38, 2.39 and 2.40, respectively.

STEP 6 : If $E(tc)_j < E(tc)_{j-1}$, set $j = j + 1$ and go to STEP 2, otherwise STOP, $n = j - 1$.

4.3 Model 2

Model 2 is developed by relaxing the assumption of the equality of the number of repeat inspections needed for each characteristic. In this plan it is suggested that the inspector inspects the first characteristic n_1 times, passes the accepted components to the second inspector who inspects the second characteristic n_2 times, until all the characteristics are inspected $n_1, n_2, \ldots, n_N$ times each.

4.3.1 Basic Relationships of the Model

At the start of inspection we know the joint probability mass function (j.p.m.f) of the random variables $(X_1, X_2, \ldots, X_N)$. So, using the j.p.m.f. we can obtain the individual marginal probability mass functions:

$$P_i(x_i) = \sum_{x_1}\sum_{x_2}\sum_{x_3}\cdots\sum_{x_{i-1}}\sum_{x_{i+1}}\cdots\sum_{x_N} P(x_1, x_2, \ldots, x_N) \tag{4.12}$$

In general, the marginal probability mass function for the i-th characteristic at the j-th cycle of inspection is :

$$^{j}P_i(0) = \frac{^{j-1}P_i(0)E_{2i}}{[^{j-1}P_i(0)E_{2i} + (1 - ^{j-1}P_i(0))(1 - E_{1i})]} \tag{4.13}$$

$$^{j}P_i(1) = 1 - ^{j}P_i(0) \tag{4.14}$$

After inspecting characteristic i, n_i times at the first stage of inspection the rule for updating the joint probability mass function is

$$^{1}P(x_1, x_2, \ldots, x_N) = P(x_1, x_2, \ldots, x_N) \tag{4.15}$$

$$^{2}P(x_1, x_2, \ldots, x_N) = {}^{1}P(x_1, x_2, \ldots, x_N)\frac{^{n_i+1}P_i(x_i)}{^{1}P_i(x_i)} \tag{4.16}$$

At the end of each stage, we can compute the probability of the component being non-defective, which is given by

$$PG_{1,n_1+1} = ^{1}P(1,1,1,\ldots,1)\frac{^{n_i+1}P_i(1)}{^{1}P_i(1)} \tag{4.17}$$

The probability of a component being non-defective entering the n-th cycle of inspection of the N-th characteristic is given by

$$PG_{N,n_N} = ^{N}P(1,1,1,\ldots,1)\frac{^{n_N}P_i(1)}{^{1}P_i(1)} \tag{4.18}$$

4.3.2 Algorithm to determine the number of repeat inspections

The algorithm uses the method of Hooke and Jeeves to find the optimal number of inspections n_i for each characteristic i. At any iteration, if characteristic i is inspected n_i number of times then the total cost of inspection is $ETC(n_1, n_2, \ldots, n_N)$, for $i = 1, 2, \ldots, N$.

Initialization Step:

Let $(d_1, d_2, \ldots, d_N)$ be the coordinate directions. Choose a scalar $\epsilon > 0$ to be used for terminating the algorithm. We suggest to choose $\epsilon = 1$ because the least number of times a characteristic can be inspected is one. Choose an initial integer step size of $\Delta \geq \epsilon$. Choose a starting point say z_1, let $y_1 = z_1$, let $k = i = 1$, go to the main step.

Main Step:

1. If $ETC(y_i + \Delta d_i) < ETC(y_i)$, put $y_{i+1} = y_i + \Delta d_i$, and go to step 3.

2. If however, $ETC(y_i + \Delta d_i) \geq ETC(y_i)$, then if $y_i > \Delta$ put $y_{i+1} = y_i - \Delta d_i$, otherwise $y_{i+1} = y_i$ and go to step 3.

3. If $i < N$, replace i by $i + 1$, go to step 1. Otherwise go to step 4 if $ETC(y_{N+1}) < ETC(z_k)$, and go to step 5 if $ETC(y_{N+1}) \geq ETC(z_k)$.

4. Let $z_{k+1} = y_{N+1}$ and let $y_1 = z_{k+1} + (z_{k+1} - z_k)$. Replace k by $k + 1$, let $i = 1$, and go to step 1.

5. If $\Delta \leq \epsilon$ stop: z_k is the solution. Otherwise, replace Δ by $\Delta/2$. Let $y_1 = z_k, z_{k+1} = z_k$, replace k by $k+1$, let $i = 1$ and go to step 1.

4.4 Example

In this section we provide an example to demonstrate the developed models and the proposed algorithms. The same example is solved using Duffuaa and Al-Najjar [8] model which assumes independence of the characteristics. The models presented in section 4.2 and 4.3 depict the process of repeat inspection for critical components with multicharacteristics having dependent defective rates. The output of the algorithms is an optimal inspection plan. A program is developed implementing the algorithms stated above. In the example, we have 100 components, each with three characteristics. The joint probability mass function for the defective rates is $P(0,0,0) = 0.05$, $P(0,0,1) = 0.05$, $P(0,1,0) = 0.05$, $P(1,1,0) = 0.15$, $P(1,0,0) = 0.05$, $P(1,0,1) = 0.05$, $P(0,1,1) = 0.1$, $P(1,1,1) = 0.5$. Other input values are, $C_a = 100{,}000$, $C_r = 500$, $C_i = 100$, $E_{1i} = 0.01$ and $E_{2i} = 0.015$. The results of the above example problem are given in Table 4.1 and 4.2 for Model 1 and Model 2 respectively. The models in this chapter suggests an inspection plan with three stages and two cycles of inspection. However, the probability of the accepted component being non-defective is high and hence the AOQ is low for the models suggested in this chapter, therefore it is more reliable.The results indicate that the effect of statistical

Table 4.1: Results obtained for Model 1

Parameters	Independent characteristics	Dependent characteristics
n*	2	2
ETC	880.93	909.43
PG	0.99982	0.99862
ATI	394	392
AOQ	0.00018	0.000138

Table 4.2: Results obtained for Model 2

Parameters	Independent characteristics	Dependent characteristics
n_1^*, n_2^*, n_3^*	2, 2, 2	2, 2, 2
ETC	880.93	909.43
PG	0.99982	0.99862
ATI	394	392
AOQ	0.00018	0.000138

dependency is significant and must be incorporated in the inspection plan.

4.5 Effects of Errors for the Dependency Case

In this section we examine the effect of inspection errors on the performance measures of the inspection plans such as ATI, ETC and AOQ. The Average Total Inspection (ATI) is the total number of inspections performed until the optimal parameters for the inspection plan are determined. The Expected Total Cost (ETC) includes the cost of false acceptance, the cost of false rejection and the cost of inspection per accepted component. Average Outgoing Quality (AOQ) is defined as the ratio of the expected number of defective components remaining after inspection to the total number of components in the lot. The effect on these performance measures are studied by varying E_1 and E_2. from 0.00 to 0.15, on a batch of 100 components, each with three characteristics. The joint probability mass function for the defective rates is $P(0,0,0) = 0.05$, $P(0,0,1) = 0.05$, $P(0,1,0) = 0.05$, $P(1,1,0) = 0.15$, $P(1,0,0) = 0.05$, $P(1,0,1) = 0.05$, $P(0,1,1) = 0.1$, $P(1,1,1) = 0.5$. The values of $C_a = 100.000$ per component, $C_r = 500$ per component and $C_i = 100$ for each characteristics were fixed. The effects of inspection errors on ATI, ETC and AOQ for both the models for the dependency is studied and the obtained values are shown in table 4.3 for Model 1 and in table 4.4 for Model 2. Figures 4.1 to 4.12 also illustrates the effects of inspection errors on the models measure of performance.

Table 4.3: The effect of inspection errors on dependent Model 1

$No.$	E_1	E_2	$1-PG$	ETC	CI	CFR	CFA	$A(n)$	n_1^*, n_2^*, n_3^*
1	0.00	0.00	0.00000	450.00	450.00	0.00	0.00	50	1
2	0.00	0.01	0.00006	806.97	800.97	0.00	6.00	50	2
3	0.00	0.03	0.00054	856.76	802.77	0.00	54.00	50	2
4	0.00	0.05	0.00150	954.31	804.35	0.00	149.96	50	2
5	0.00	0.10	0.00060	1220.61	1160.62	0.00	59.99	50	3
6	0.00	0.15	0.00202	1368.39	1165.95	0.00	202.43	50	3
7	0.01	0.00	0.00000	490.69	460.08	30.61	0.00	49	1
8	0.01	0.01	0.00006	901.30	833.02	62.15	6.12	47	2
9	0.01	0.03	0.00055	952.12	834.90	62.12	55.09	47	2
10	0.01	0.05	0.00153	1051.63	836.56	62.06	153.01	47	2
11	0.01	0.10	0.00062	1384.03	1227.59	94.61	61.83	46	3
12	0.01	0.15	0.00209	1536.42	1233.32	94.47	208.63	46	3
13	0.03	0.00	0.00000	577.01	481.33	95.68	0.00	46	1
14	0.03	0.01	0.00006	1109.82	902.93	200.51	6.38	42	2
15	0.03	0.03	0.00057	1162.80	905.01	200.41	57.39	42	2
16	0.03	0.05	0.00159	1266.40	906.82	200.20	159.38	42	2
17	0.03	0.10	0.00637	1746.44	910.20	199.24	637.00	42	2
18	0.03	0.15	0.00222	1921.73	1385.24	314.69	221.79	38	3
19	0.05	0.00	0.00000	670.54	504.18	166.35	0.00	43	1
20	0.05	0.01	0.00631	1298.68	502.47	165.30	630.91	43	1
21	0.05	0.03	0.00060	1403.87	983.88	360.16	59.83	37	2
22	0.05	0.05	0.00166	1511.80	985.87	359.78	166.16	37	2
23	0.05	0.10	0.00664	2011.56	989.50	357.98	664.08	37	2
24	0.05	0.15	0.00236	2385.52	1564.14	585.29	236.09	32	3
25	0.10	0.00	0.00000	941.15	569.41	371.74	0.00	36	1
26	0.10	0.01	0.00666	1602.48	567.30	369.27	665.92	37	1
27	0.10	0.03	0.00067	2176.99	1229.25	881.09	66.66	27	2
28	0.10	0.05	0.00185	2296.95	1231.78	880.04	185.13	27	2
29	0.10	0.10	0.00740	2851.10	1236.13	875.15	739.82	27	2
30	0.10	0.15	0.01662	3766.58	1237.61	867.02	1661.94	27	2
31	0.15	0.00	0.00000	1277.18	648.85	628.33	0.00	31	1
32	0.15	0.01	0.00705	1975.16	646.21	623.90	705.04	31	1
33	0.15	0.03	0.00075	3294.15	1569.19	1650.23	74.73	19	2
34	0.15	0.05	0.00208	3428.03	1572.45	1648.04	207.54	19	2
35	0.15	0.10	0.00829	4044.68	1577.62	1637.77	829.29	19	2
36	0.15	0.15	0.01863	5061.70	1578.44	1620.71	1862.56	19	2

Table 4.4: The effect of inspection errors on dependent Model 2

No.	E_1	E_2	$1-PG$	ETC	CI	CFR	CFA	$A(n)$	n_1^*, n_2^*, n_3^*
1	0.00	0.00	0.00000	450.00	450.00	0.00	0.00	50	1, 1, 1
2	0.00	0.01	0.00105	805.68	700.77	0.00	104.91	50	2, 1, 2
3	0.00	0.03	0.00054	856.76	802.77	0.00	54.00	50	2, 2, 2
4	0.00	0.05	0.00150	954.31	804.35	0.00	149.96	50	2, 2, 2
5	0.00	0.10	0.00150	1209.56	1059.57	0.00	149.98	50	3, 2, 3
6	0.00	0.15	0.00202	1368.39	1165.95	0.00	202.43	50	3, 3, 3
7	0.01	0.00	0.00000	490.69	460.08	30.61	0.00	49	1, 1, 1
8	0.01	0.01	0.00106	882.29	724.79	51.48	106.02	48	2, 1, 2
9	0.01	0.03	0.00055	952.12	834.90	62.12	55.09	47	2, 2, 2
10	0.01	0.05	0.00153	1051.63	836.56	62.06	153.01	47	2, 2, 2
11	0.01	0.10	0.00154	1351.34	1114.20	83.59	153.55	46	3, 2, 3
12	0.01	0.15	0.00209	1536.42	1233.32	94.47	208.63	46	3, 3, 3
13	0.03	0.00	0.00000	577.01	481.33	95.68	0.00	46	1, 1, 1
14	0.03	0.01	0.00108	1049.23	776.60	164.33	108.31	43	2, 1, 2
15	0.03	0.03	0.00057	1162.80	905.01	200.41	57.39	42	2, 2, 2
16	0.03	0.05	0.00159	1266.40	906.82	200.20	159.38	42	2, 2, 2
17	0.03	0.10	0.00161	1672.52	1235.99	275.48	161.05	39	3, 2, 3
18	0.03	0.15	0.00222	1921.73	1385.24	314.69	221.79	38	3, 3, 3
19	0.05	0.00	0.00000	670.54	504.18	166.35	0.00	43	1, 1, 1
20	0.05	0.01	0.00319	1204.07	657.84	227.01	319.22	41	1, 1, 2
21	0.05	0.03	0.00060	1403.87	983.88	360.16	59.83	37	2, 2, 2
22	0.05	0.05	0.00166	1511.80	985.87	359.78	166.16	37	2, 2, 2
23	0.05	0.10	0.00368	1984.54	1186.62	430.39	367.53	35	2, 2, 3
24	0.05	0.15	0.00446	2334.13	1382.80	505.08	446.25	33	3, 2, 3
25	0.10	0.00	0.00000	941.15	569.41	371.74	0.00	36	1, 1, 1
26	0.10	0.01	0.00666	1602.48	567.30	369.27	665.92	37	1, 1, 1
27	0.10	0.03	0.00388	2088.30	1009.35	690.82	388.13	30	2, 1, 2
28	0.10	0.05	0.00185	2296.95	1231.78	880.04	185.13	27	2, 2, 2
29	0.10	0.10	0.00740	2851.10	1236.13	875.15	739.82	27	2, 2, 2
30	0.10	0.15	0.00973	3594.11	1540.99	1080.14	972.99	24	2, 2, 3
31	0.15	0.00	0.00000	1277.18	648.85	628.33	0.00	31	1, 1, 1
32	0.15	0.01	0.00705	1975.16	646.21	623.90	705.04	31	1, 1, 1
33	0.15	0.03	0.01097	2909.63	906.62	905.64	1097.37	26	1, 1, 2
34	0.15	0.05	0.00760	3245.18	1241.33	1244.22	759.63	22	2, 1, 2
35	0.15	0.10	0.00829	4044.68	1577.62	1637.77	829.29	19	2, 2, 2
36	0.15	0.15	0.01863	5061.70	1578.44	1620.71	1862.56	19	2, 2, 2

4.5.1 Effects of Inspection Error on Average Total Inspection (ATI)

The relationship between E_1 and ATI and E_2 and ATI are shown in figure 4.1 and 4.2 respectively for Model 1. Figure 4.1 shows that with an increase in E_1, ATI decreases. This is in line with intuition, since when E_1 increases, more and more good components are rejected and hence the number of characteristics inspected in the later cycles of inspection is decreased. For lower values of E_2 (0.05) the decrease in ATI with the increase in E_1 is very slow mainly because the number of repeat inspections for increasing values of E_1 remains the same. However, for higher values of E_2 (0.1, 0.15) there is a sharp decrease in ATI at a particular value of E_1. Looking in table 4.3 it can be seen that at these particular value of E_1 the number of repeat inspection decreases and hence the inspection load decreases.

Figure 4.2 shows that with an increase in E_2, ATI increases. This is reasonable and can be explained as follows : an increase in E_2 will result in higher expected total cost since C_a is very high and therefore in order to reduce the cost additional inspection is required and that increases the inspection load. However, it can be seen that with the increase in E_2 the inspection load almost remains the same for some values of E_2 and then increases at a particular value of E_2(0.03) and then again remains constant. With the increase in E_2 the number of defective components accepted increases thereby increasing the cost of false acceptance increases. But

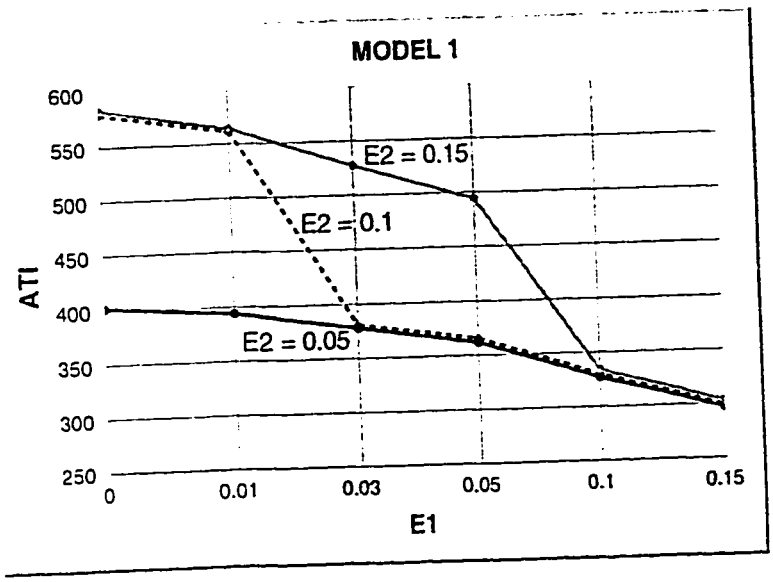

Figure 4.1: Relationship between E1 and ATI

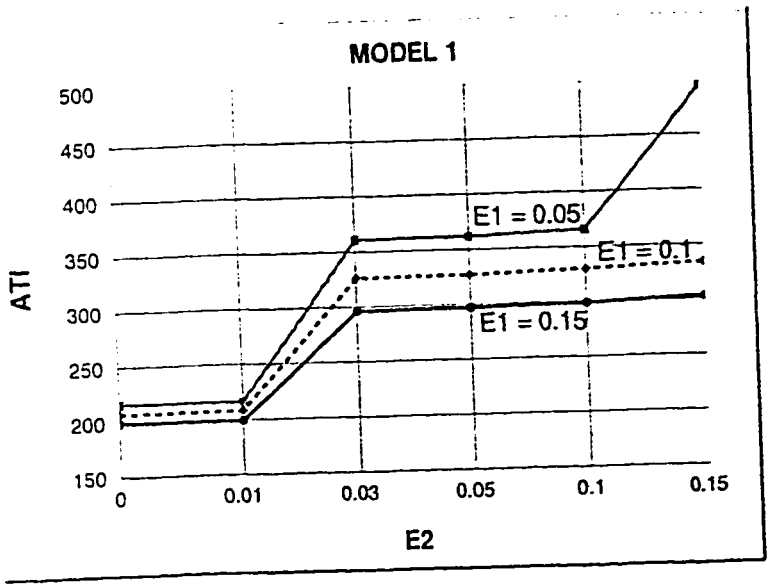

Figure 4.2: Relationship between E2 and ATI

the increase in cost is not enough to force another inspection and hence the ATI remains constant. The inspection plan forces us to do another repeat inspection only when it is economical thus increasing the inspection load. This happens when the cost of false acceptance becomes high because of the increase in E_2.

The relationships between E_1 and ATI and E_2 and ATI for Model 2 is shown in figure 4.3 and 4.4 respectively. With the increase in E_1, ATI decreases while the opposite happens with the increase in E_2. However, the rate of decrease of ATI is high for high values of E_1. Since $E_1$is high the number of falsely rejected components is high and hence the decrease in ATI is high. Whereas ATI increases with the increase in E_2 because number of falsely accepted component increases and so another repeat inspection has to be done in order to screen them out.

4.5.2 Effects of Inspection Error on Expected Total Cost (ETC)

The relationships between E_1 and ETC and E_2 and ETC for Model 1 are shown in figures 4.5 and 4.6 respectively. The expected total cost increase with the increase in E_1 as well as with the increase in E_2. But the increase in ETC is more when E_2 increases than with the increase in E_1. This is reasonable because an increase in E_2 will result in higher ETC since the cost of accepting a defective component is very high. Figures 4.5 and 4.6 shows that the combined effect of E_1 and E_2 is more

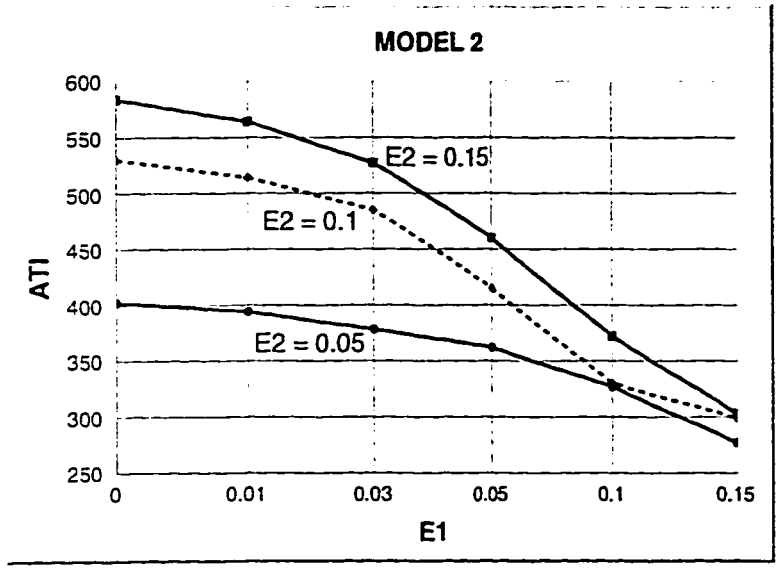

Figure 4.3: Relationship between E1 and ATI

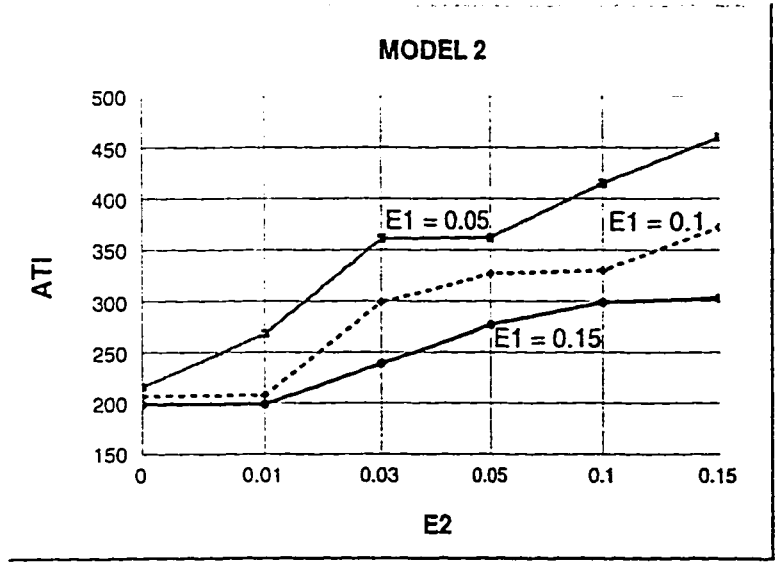

Figure 4.4: Relationship between E2 and ATI

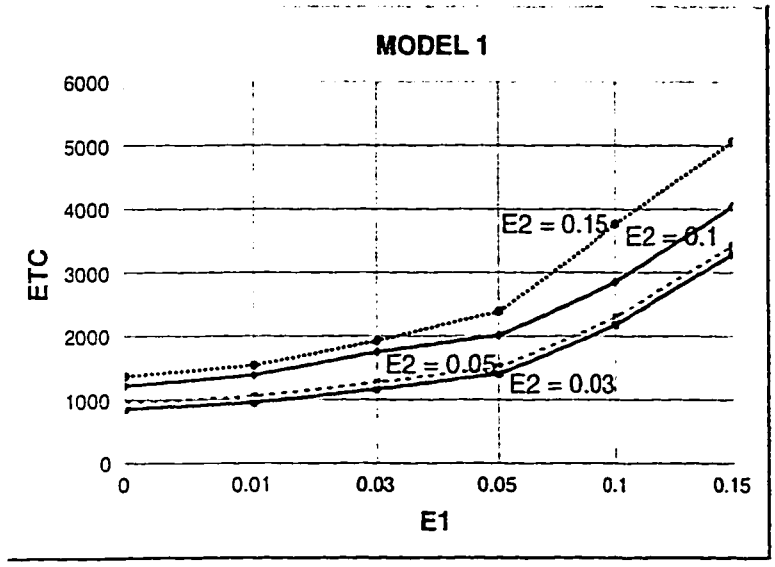

Figure 4.5: Relationship between E1 and ETC

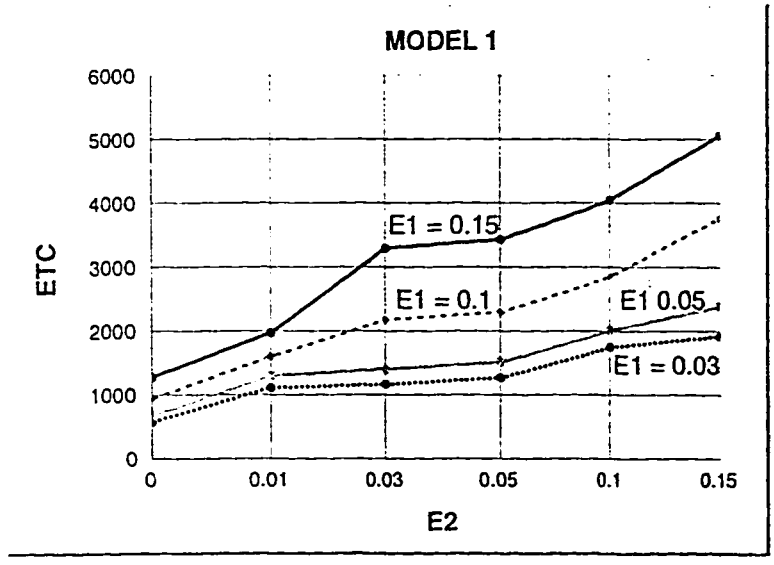

Figure 4.6: Relationship between E2 and ETC

serious as ETC elevates in an almost linear trend for higher values of E_1 and E_2. This is because when both E_1 and E_2 are high, the number of components accepted after inspection decreases sharply while both the cost of false rejection and the cost of false acceptance increases with increase in E_1 and E_2 respectively and hence the cost per accepted component increases sharply.

The relationship between E_1 and ETC and E_2 and ETC E_2 for Model 2 is shown in figure 4.7 and 4.8 respectively. The slope of ETC in case of E_2 is higher than that of E_1. However, the performance of Model 2 is better than Model 1 in terms of the expected total cost. This is because it is economical to inspect the characteristics different number of times as in Model 2.

4.5.3 Effects of Inspection Error on Average Outgoing Quality (AOQ)

The relationship between E_1 and AOQ and E_2 and AOQ is shown in figure 4.9 and 4.10 respectively for Model 1.

For low values of E_2 (0.05) it can be seen that AOQ almost remains the same with the increase in E_1. As E_2 is low the increase in the number of falsely accepted component is low which prevents the need for more inspection to screen out defective components and hence the number of inspection remains the same. Looking in figure 4.9 closely it is seen that for values of E_1 equal to 0.01, AOQ for $E_2 = 0.1$ is less

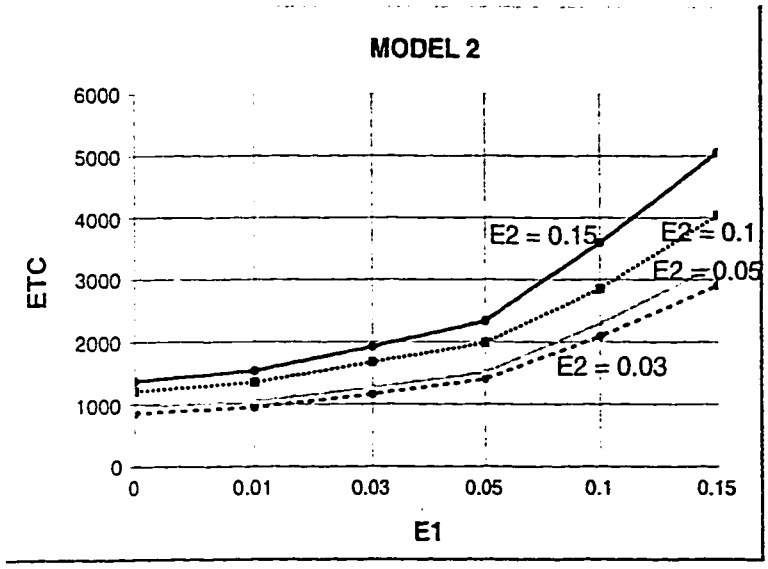

Figure 4.7: Relationship between E1 and ETC

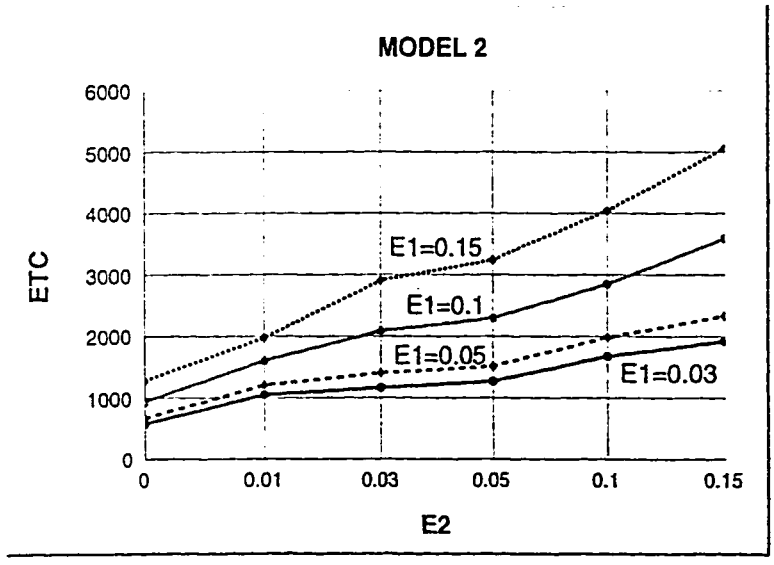

Figure 4.8: Relationship between E2 and ETC

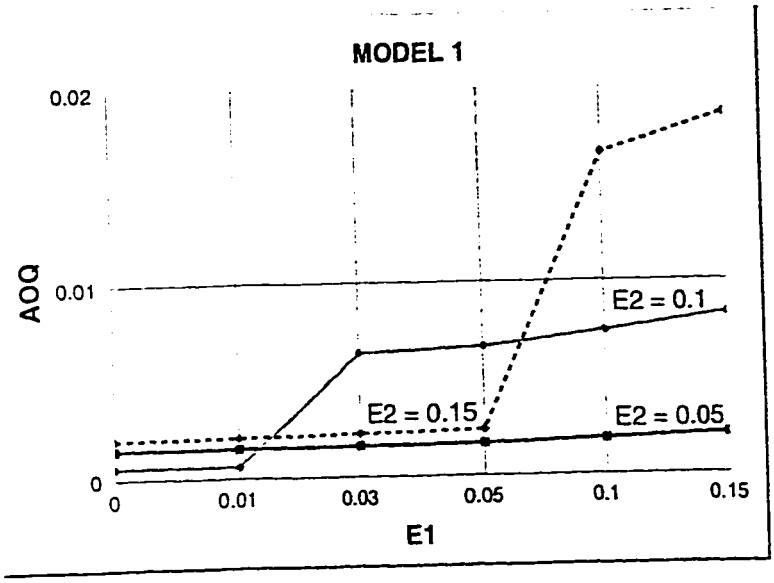

Figure 4.9: Relationship between E1 and AOQ

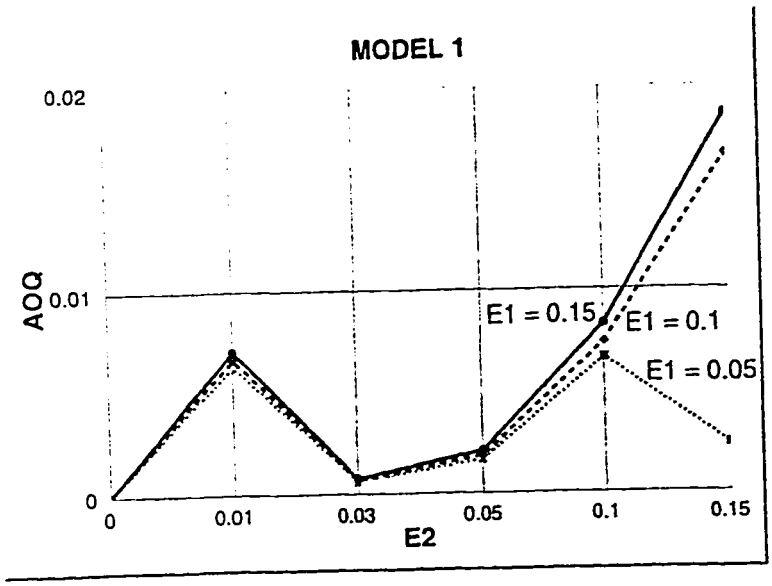

Figure 4.10: Relationship between E2 and AOQ

than that for $E_2 = 0.05$. This is because the number of repeat inspection at $E_2 = 0.1$ is more which screens out the defective components thus lowering AOQ. The average outgoing quality increase when the number of inspection instituted by the inspection plan decreases and then it remains almost constant. When the values of E_1 and E_2 is high the effect is more significant.

Figure 4.10 shows that as E_2 increases beyond a certain value, more inspection is performed which screens out more defective components hence decreasing the AOQ. Again as E_2 increases, AOQ keeps on increasing because the probability of accepting a defective component is increasing, until it increases the ETC so high that the model forces more inspection which lowers the AOQ as more defective components are screened out. This can be observed when looking closely in Table 4.3 and 4.4.

The relationship between E_1 and ETC and E_2 and ETC is shown in figure 4.11 and 4.12 respectively for Model 2. The average outgoing quality increases with the increase in E_1. The trend of average outgoing quality with the increase in E_2 is similar to that of Model 1.

4.6 Conclusion

In this chapter the models developed by Duffuaa and al-Najjar [6] have been extended to the case where the characteristics defective rates are statistically depen-

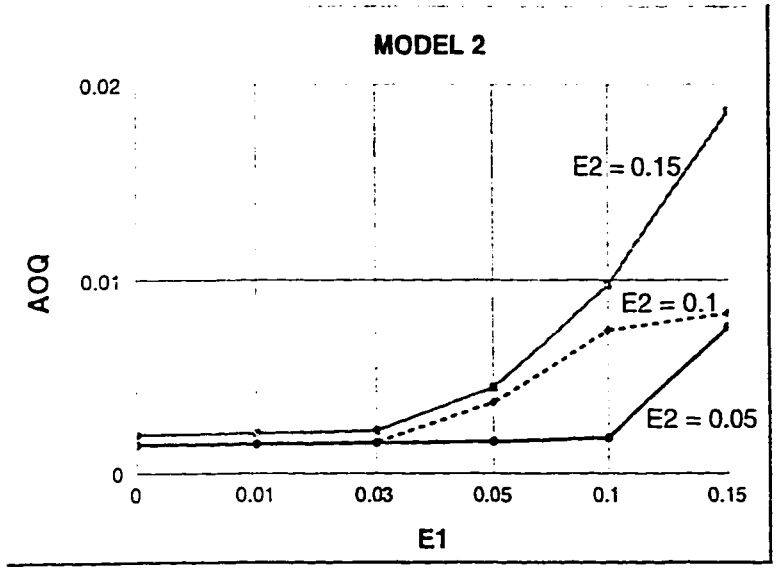

Figure 4.11: Relationship between E1 and AOQ

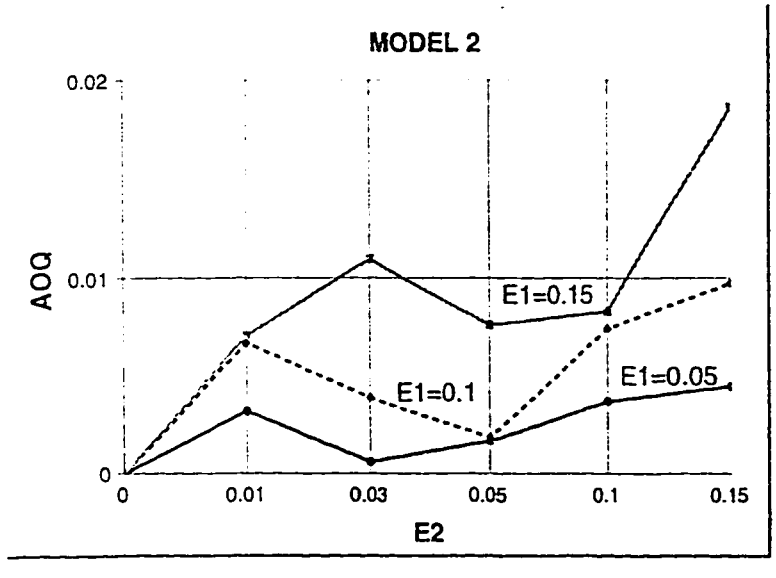

Figure 4.12: Relationship between E2 and AOQ

dent. The algorithms to determine the optimal number of repeat inspections for each characteristic is also suggested. However, the algorithm guarantees a local minima only. The necessary software to run the models is also developed. The result of the model is an optimal inspection plan, where the characteristics are dependent, which can be used for controlling the quality of critical components. As shown in this chapter, the effect of dependency is significant and must be incorporated in the inspection plan. The application of the model has been demonstrated by an example. The effect of Type I and Type II errors on ETC, ATI and AOQ is also investigated. ETC increases as Type I error and Type II error increases, however the increase is higher when Type II error increases. This is because the cost of committing Type II error is very high compared to the cost of committing Type I error. The average total inspection decreases with the increase in Type I error, the decrease is sharp for high values of E_1 and E_2. With the increase in E_2 the ATI remains for some values of E_2 and then increases at a particular value of E_2 and then again remains constant for some increase in E_2. As E_2 increases the cost of false acceptance increases but the number of inspection and hence the inspection load increases only when the number of falsely accepted component is high enough that another inspection is required to screen out the defective components falsely accepted. AOQ increase with the increase in E_1,however, the combined effect of higher values of E_1 and E_2 is more significant because the rate of increase is sharp. When E_2 increases, the AOQ increases, however if E_2 becomes very high, the AOQ

becomes low because more inspection takes place which screens some of the falsely accepted component. The results obtained in chapter 3 and 4 are consistent with the ones in the literature obtained for sampling plans.

Chapter 5

Inspection Plans Under Varying Inspection Errors

5.1 Introduction

It has been shown in the literature [17] that the error probabilities, Type I error and Type II error, are a function of the incoming quality. Also, in [9] it has been demonstrated that an increase in incoming quality results in a decreased Type II error and increased Type I error. In repeat inspection plans, the defective rate of a component changes each time it is inspected. Since, the error probabilities changes with the incoming quality, therefore, error probabilities must be updated after each inspection. In order to do so we must establish the relationship between the defect rates and the two types of error probabilities.

Jaraiedi [33] has presented two procedures for the estimation of error probabilities using signal detection theory (SDT) and the receivers operating characteristic (ROC) curve. The ROC curve is used to quantify the inspection error probabilities and the cost of committing these errors. Using the general equation of the ROC curve, we can compute the relationship between fraction defective and Type I error and Type II error. The functional relationships developed between P and E_1 and P and E_2 can be used to incorporate the effect of varying inspection errors into the repeat inspection models which is the subject of this chapter.

The rest of the chapter is organized as follows : section 2 presents an overview of the signal detection theory followed by error estimation as given by Jaraiedi in section 3. Section 4 outlines the procedure for estimation of error probabilities employed in this thesis. In section 5 we provide the results of the independent and the dependent inspection plans under varying inspection errors, together with comparison of the results of the inspection plans with and without varying inspection errors. Section 6 concludes this chapter.

5.2 Overview of Signal Detection Theory

The development of signal detection theory started around 1945. The fundamentals of the theory were formally developed and introduced by Peterson, Birdsall, and Fox [34]. Since then, the theory has found its usefulness in such diverse areas as

radar communications, medical diagnoses, the psychophysics of human perception, and industrial quality control.

According to the signal detection model, an observer bases his decision on the information available prior to the presentation of the stimulus, the information content of the stimulus, the sensory analyzer mechanism, and the consequences of each decision. A stimulus may contain noise alone or it may contain a signal super imposed on the noise. The observer's response is "yes" or "no", corresponding to his belief that a signal has been present or not. This situation results in a two-by two layout as shown in table 5.1. The stimulus presented is mapped into a decision axis

Table 5.1: Two-by-two Layout

		Response	
		Yes	No
Stimulus	Signal-plus-noise	Correct decision (HIT)	Miss
	Noise alone	False alarm	Correct decision

on which the probability distributions for the noise alone and signal plus noise are defined. An optimal observer chooses a criterion point on this axis as the boundary between "yes" or "no" responses in order to optimize an objective function.

The detection performance of an observer can be evaluated by a graph called the Receiver Operating Characteristic (ROC) curve. The vertical axis on the ROC

curve is "Probability of Hit", P(HIT) and the horizontal axis is the "Probability of False Alarm", P(FA). The (*ROC*) curve for an observer can be generated either by a two-choice experiment in which the payoff matrix and prior probabilities are manipulated or by a rating method in which the observer expresses his degree of confidence in the stimulus which is a result of the noise alone or a signal-plus noise. Figure 5.1 shows the chance diagonal on which P(HIT) = P(FA) representing random performance where the decision is independent of the stimulus presented.

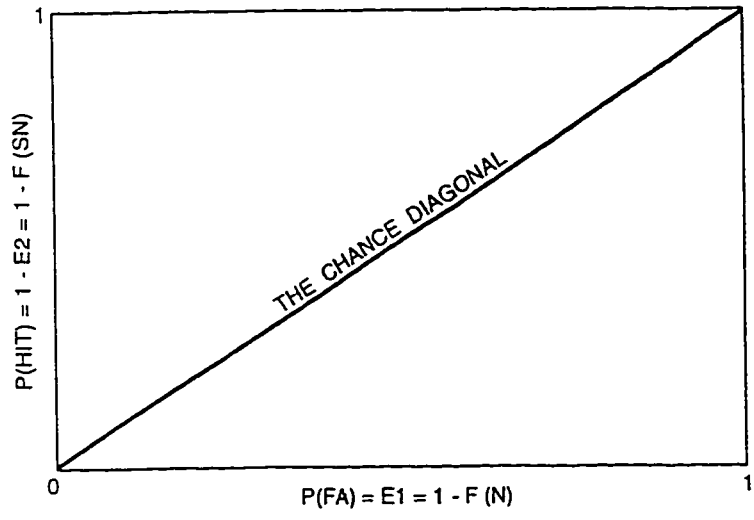

Figure 5.1: The Chance Diagonal

5.3 Error Estimation as given by Jaraiedi

Jaraiedi [33] used Receivers Operating Characteristic (*ROC*) curve to develop a functional relationship between defective rate (P) and probability of Type I error

and Type II error.

The ROC analysis of the detection task, where the observation interval is constant, has revealed that a change in P results in a significant change in the level of detection of defects [35] and causes the observation to shift along the ROC curve due to criterion adjustments made by inspectors. In such a case one can assume that the length of the ROC curve travelled due to change in P is proportional to the amount of the change. An increase in P causes a shift towards the upper corner of the ROC space while a decrease in P has the opposite effect. Based on this assumption a method for estimation of two types of error probabilities is developed. Given two known defect rates and their corresponding points on the ROC curve, we see that the problem is how to find the point on the curve which corresponds to a value of P between the two known points. The following notation is needed

S_{x_1,x_2} = the length of the ROC curve between points corresponding to p_1 and p_2.

S_{x_1,x_p} = the length of the ROC curve between points corresponding to p_1 and p.

In general the length of any ROC curve, $Y = f(X)$, between two points (X_1, Y_1) and (X_2, Y_2) can be expressed as

$$S = \int_{(x_1,x_2)} \left[1 + \left[\frac{dY}{dX}\right]^2\right]^{.5} dX \tag{5.1}$$

If the ROC curve is of the form

$$Y = A_1 X^{1/n_1} + A_2 X^{1/n_2}$$

then

$$\left[\frac{dY}{dX}\right]^2 = \frac{A_1^2}{n_1^2} X^{[(2-2n_1)/n_1]} + \frac{A_2^2}{n_2^2} X^{[(2-2n_2)/n_2]} + \frac{2A_1A_2}{n_1n_2} X^{[(n_1+n_2-2n_1n_2)/n_1n_2]}$$

Hence

$$S_{x_1,x_2} = \frac{1}{n_1n_2} \int_{(x_1,x_2)} [n_1^2 n_2^2 + g(X)]^{.5} dX \tag{5.2}$$

where

$$g(X) = \left[A_1 n_2 X^{[(1-n_1)/n_1]} + A_2 n_1 X^{[(1-n_2)/n_2]}\right]^2$$

Ideally, if the integral function for S was computable by analytical methods, the length of the ROC curve in the closed interval $[x_1, x_2]$ would be a real monotonic function, say h(.), of x_1 and x_2. In such a case the following is true

$$S_{x_1,x_2} = h(x_2) - h(x_1)$$

To estimate the Type I and Type II error probabilities at a certain incoming fraction defective P which is between the two known points P_1 and P_2, the length of the curve between x_1 corresponding to P_1 and x_2 corresponding to P_2 can be

used. Using the assumption that distance travelled on the curve is proportional to the change in signal rate it can be seen from figure 5.2 that

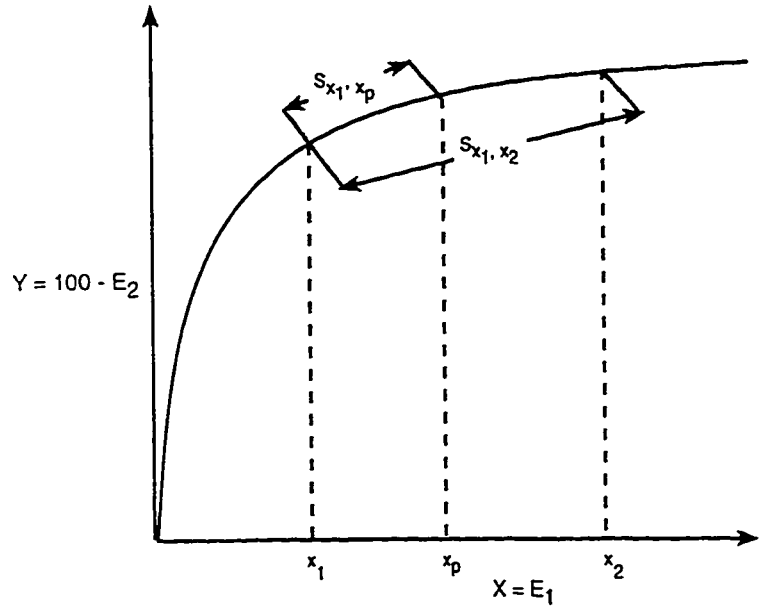

Figure 5.2: Computation of unknown error probabilities

$$\frac{S_{x_1,x_2}}{S_{x_1,x_p}} = \frac{p_2 - p_1}{p - p_1}$$

$$\frac{h(x_2) - h(x_1)}{h(x_p) - h(x_1)} = \frac{p_2 - p_1}{p - p_1}$$

Solving for $h(x_p)$ the result is

$$h(x_p) = \frac{p_2 - p_1}{p - p_1} \times [h(x_2) - h(x_1)] + h(x_1)$$

Hence the Type I error probability E_1, becomes:

$$E_1 = h^{-1}(x_p)$$

and given the equation for ROC curve, the Type II error probability is given as:

$$E_2 = 100 - f(E_1)$$

where f is the functional form of the ROC curve.

5.4 Procedure for Estimation of Error Probabilities

The first step in developing the functional relationship between incoming quality and the error probabilities is to estimate the functional form of the ROC curve. The ROC curve is fitted between probability of false alarm (Type I error) and the probability of hit (1 - Type II error). In this fit the properties of the ROC curve are employed to develop a constrained regression model to fit the functional form of the ROC curve. The steps of the procedure are as follows [36]:

1. Use a constrained regression model for fitting the ROC curve. The functions selected for the fit should reflect the properties of the ROC curve which are

$$f'(E_1) \geq 0 \quad \text{for} \;\; 0 < E_1 < 100$$

and

$$f''(E_1) \leq 0 \quad \text{for} \;\; 0 < E_1 < 100$$

where $f'(E_1)$ (first derivative) guarantees that the curve does not peak in the (0,100) range and $f''(E_1)$ (second derivative) ensures that it is a concave down in the same region and E_1 and E_2 are in percentages.

2. Use the method in section 5.3 to generate data for E_1, E_2 and P from ROC curve.

3. Develop a constrained regression model to estimate E_1 and E_2 as a function of P. The model should reflect the expected behavior of E_1 and E_2 versus P.

The problem was formulated as a constrained regression model. It is a non-linear programming problem :

$$Min \sum_{i=1}^{n} \{(1 - E_{2i}) - f(E_{1i})\}^2$$

subject to

$$f'(E_{1i}) \geq 0$$

$$f''(E_{1i}) \leq 0$$

$$f(0) = 0$$

$$f(100) = 100$$

The function used to find the best fit of the *ROC* curve was of the form :

$$P(\text{Hit}) = A_1 P(FA)^{1/n_1} + A_2 P(FA)^{1/n_2}$$

or

$$(1 - E_2) = A_1 E_1^{1/n_1} + A_2 E_1^{1/n_2}$$

The data set used to find the best fit for the *ROC* curve was adopted from the results of the experiment reported by Harris [9] and is given in Appendix A. The following parameter estimates were obtained giving the best fit using *GINO* (General INteractive Optimizer) :

$$A_1 = 36.67 \qquad n_1 = 18.31$$

$$A_2 = 41.17 \qquad n_2 = 18.44$$

Hence, the *ROC* equation becomes

$$f(E_1) = 36.67 E_1^{1/18.31} + 41.17 E_1^{1/18.44}$$

After the ROC curve has been fitted, many points corresponding to E_1 and E_2 can be generated following the method presented in the previous section. A program that uses the numerical integration method is used to generate the desired points for a given defective rate as given in table 5.2. The following non-linear programming

Table 5.2: Corresponding values of E_1 and E_2 for a given P

P	E_1	E_2
0.01	0.00470	0.243895
0.02	0.5322	0.240275
0.025	0.56611	0.236669
0.03	0.60197	0.236669
0.04	0.68000	0.233082

model was used to find the estimates of the functional relationship between P and E_1 and P and E_2.

Model for E_1 :

$$Min \sum_{i=1}^{n} [E_{1i} - f_1(P_i)]^2$$

subject to

$$f_1'(P_i) \geq 0 \quad \text{[increasing function]}$$

$$f_1(P_i) \leq 0.5$$

$$f_1(P_i) \geq 0$$

Model for E_2 :

$$Min \sum_{i=1}^{n} [E_{2i} - f_2(P_i)]^2$$

subject to

$$f_2'(P_i) \leq 0 \quad \text{[decreasing function]}$$

$$f_2(P_i) \leq 0.5$$

$$f_2(P_i) \geq 0$$

f_1 and f_2 are restricted for the class of monotone polynomial functions. Using the data in table 5.2 and $GINO$, the following two polynomials have been found the best to describe the data:

$$E_1 = 0.016956 + 0.184380p + 1.207995p^2 + 1.269504p^3 + 1.283867p^4 \qquad (5.3)$$

and

$$E_2 = 0.283998 - 1.220556p - 0.291845p^2 - 0.014420p^3 + 0.000632p^4 \qquad (5.4)$$

These functions describe the relationship between the error probabilities and the incoming quality.

5.5 Inspection Plans Under Varying Inspection Errors

The models developed previously and discussed in this thesis assume fixed Type I error and Type II error throughout the inspection process. However, it has been found in the literature that the errors are a function of incoming quality [17]. In general, Type I error increases as incoming quality P increases and Type II error decreases with the increase in incoming quality [9]. In all the inspection plans, each time a characteristic is inspected, the incoming quality for the next inspection changes. The probability of the component being defective is updated after every inspection using Bayes theorem. Since the incoming quality is updated so must be the inspection errors. Therefore, we add in the models, that after each characteristic is inspected the errors are estimated as a function of the updated P. Then the obtained value of the inspection errors are used for the next inspections.

The effect of incorporating the varying inspection errors in the repeat inspection plan can be evaluated by looking at the results obtained from the plan. The functional relationship that is used to update the Type I error and Type II error is given in equation 5.3 and 5.4 respectively. The rest of the equations and the algorithms are the same as given in the previous chapters. The results of the example obtained for the independent models with and without varying inspection errors is shown in Table 5.3 and that for the dependent models is shown in Table 5.4.

Table 5.3: Results obtained for statistically independent characteristics

Parameters	Without Varying errors	With Varying errors
n^*	2	2
ETC	880.93	1236.75
PG	0.99982	0.99650
ATI	394	392
AOQ	0.00018	0.00350

Table 5.4: Results obtained for statistically dependent characteristics

Parameters	Without Varying errors	With Varying errors
n^*	2	2
ETC	909.43	12311.88
PG	0.999862	0.996483
ATI	392	371
AOQ	0.000138	0.003517

The comparison of the results obtained in this chapter with those obtained in the previous chapters underlines the significance of the varying inspection errors. It is clear that there is substantial increase in the expected total cost. This is due to higher cost of false acceptance, since the probability of Type II error increases as the probability of the component being defective decreases with the ongoing inspection. The other change that is noticeable is that the average outgoing quality under varying inspection errors is higher. This was expected because an increase in Type II error would decelerate the decreasing rate of P, and the net effect would be a slower growth of PG (Probability of a good component entering the inspection cycle). In a nutshell, the varying inspection errors posted adverse effects on the inspection plans, nonetheless, it presented a close-to-real picture of the business of repeat inspection plans.

5.6 Conclusion

In this chapter we first presented an overview of the signal detection theory .Then a procedure to estimate the probability of Type I error and Type II error from the ROC curve has been outlined. These estimates are used to develop the functional relationships between the incoming quality and Type I error and the incoming quality and Type II error. These functional relationships are then used to incorporate the dynamic effect of error probabilities on the repeat inspection plans. Comparing

the obtained result from the repeat inspection plans with and without varying errors showed a significant change in the expected total cost and the average outgoing quality. The results indicate varying inspection errors have a dominating effect on the multicharacteristic inspection plans. Hence, the varying inspection error should be incorporated in the design of the inspection plans to reflect the real nature of repeat inspection and the dynamic behavior in the inspection process.

Chapter 6

Factor Effects and Sensitivity Analysis

6.1 Introduction

Factorial designs are widely used in experiments involving several factors where it is necessary to study the joint effect of several factors on the response. In this chapter we conduct a factorial experiment to study the effects and sensitivities of the input parameters on the performance measures of the inspection plans. The input parameters are the cost of false acceptance C_a, the cost of false rejection C_r, the cost of inspection C_i, incoming quality P_i, Type I error $E_1$and Type II error E_2 and the performance measures are the expected total cost ETC, the average total inspection ATI and the average outgoing quality AOQ.

This experiment will help us to identify the factors and their interactions that significantly affect the response and hence are important. The experiment is conducted on the models of repeat inspection plans. It has been found that the factors effects and most of the two factor interactions effects are significant on the performance measures of the inspection plan. The experiment has also helped in identifying the levels of some factors which will reduce the effect of the others. The rest of the chapter is organized as follows : The experimental design along with the sensitivity analysis on the three models are given in section 2. Section 3 describes the developed computer software. Section 4 presents the conclusion to this chapter.

6.2 The Experimental Design

A factorial experiment is conducted to identify the factor effects and their interactions that have a significant effect on the performance measures of repeat inspection plans. Also the experiment will help in studying the sensitivity of the model to each factor. There are six input parameters and three levels are assigned to each input parameters in the factorial experimental design, these are low, medium and high. These levels are designated by the digits 0 (low), 1 (medium), and 2 (high). Table 6.1 shows the input parameters and their assigned levels.

It should be noted that if the levels of the input parameters are varied over different ranges, the experimental results may be different. However, the levels used

for the study are not unrealistic.

The factorial experiment design used for the sensitivity analysis is 3^k design. A total of 3^6 or 729 runs are required to conduct the experiment. Since the solution algorithm used by the repeat inspection models are deterministic, these experiment can not be replicated. Thus, there is no estimate of error.

The approach to solve this problem is to assume that certain high-order interactions are negligible and combine their sum of squares to estimate the error. This approach is justified by the fact that most systems are dominated by some of the main effects and low-order interactions and most high-order interactions are negligible.

Therefore, in the analysis of variance (ANOVA) table, we calculate the F-ratio values for the main effects and the their two factor interactions and consider the three order interactions and above as residuals. One should note that the residual carried out in the experiments is not the pure error but the effect of higher order (three factor and above) interactions is used to compare the effects of the main factors and two factor interactions.

6.2.1 Analysis of the Experimental Results

In order to do the sensitivity analysis input parameters were varied over different ranges. The analysis of variance presented here is for the levels shown in table 6.1. It should be noted that when the levels of the input parameters were varied

Table 6.1: Input parameters and levels used in the experiment

Parameters	Factors	Levels		
		Low	Medium	High
Cost of False Acceptance C_a	A	1000	100000	1000000
Cost of False Rejection C_r	B	100	500	10000
Cost of Inspection C_i	C	10	100	500
Incoming Quality P_i	D	0.01	0.05	0.10
Type I Error E_{1i}	E	0.01	0.05	0.10
Type II Error E_{2i}	F	0.01	0.05	0.10

over different ranges, the experimental results obtained were different. However, the significance levels and the F-ratios in the ANOVA table give us some insight about the factors and their interactions that have a significant impact on the response.

The analysis of variance for ETC, ATI and AOQ on Model 2 of Duffuaa and Al-Najjar is shown in table 6.2, 6.3 and 6.4 respectively. Looking at the significance levels, all the factors and two factor interaction except CE are significant for ETC. But looking at the F-ratios we can say that factor A has the largest effect on ETC followed by factor B and C. Since, our aim is to guard against the possibility of accepting a defective component hence factor A should have the largest impact on the response. Among the two factor interactions AB, AC and BE have high F-ratios and hence are more significant than the others.

Similarly, looking at the significance levels of the ANOVA table for ATI all the factors except BD and BF are significant at 5% significance level. However, the F-ratios indicate that factor A is the most significant followed by factor C and E

Table 6.2: ANOVA of *ETC* for Model 2 of Duffuaa and Al-Najjar

Source of variation	SS	df	MS	F - ratio	Sig. level
Main Effects					
A: C_a	1.2241E9	2	6.1204E8	1034.809	0.0000
B: C_r	5.3826E8	2	2.6913E8	455.029	0.0000
C: C_i	5.8602E8	2	2.9301E8	495.410	0.0000
D: P	1.5238E8	2	7.6188E7	128.814	0.0000
E: E_1	2.5838E8	2	1.2919E8	218.429	0.0000
F: E_2	9.7293E7	2	4.8647E7	82.249	0.0000
Interactions					
AB	3.4493E8	4	86233672	145.799	0.0000
AC	3.4255E8	4	85638298	144.793	0.0000
AD	5.6092E7	4	14023049	23.709	0.0000
AE	1.8036E8	4	45090836	76.237	0.0000
AF	6.7919E7	4	16979789	28.709	0.0000
BC	1.1140E7	4	2784937	4.709	0.0009
BD	4.0699E7	4	10174827	17.203	0.0000
BE	2.3682E8	4	59204867	100.100	0.0000
BF	1.9199E7	4	4799844	8.115	0.0000
CD	4.4358E7	4	11089406	18.749	0.0000
CE	1.8176E6	4	454411	0.768	0.5461
CF	2.2327E7	4	5581773	9.437	0.0000
DE	2.7578E7	4	6894535	11.657	0.0000
DF	1.0543E7	4	2635792	4.456	0.0015
EF	1.3624E7	4	3405975	5.759	0.0001
Residual	3.8799E8	656	591454.30		
Total (Corrected)	4.6644E9	728			

SS = Sum of Squares

df = degrees of freedom

MS = Mean Sum of Squares

Sig. level = Significance level

Table 6.3: ANOVA of ATI for Model 2 of Duffuaa and Al-Najjar

Source of variation	SS	df	MS	F - ratio	Sig. level
Main Effects					
A: C_a	25602069	2	12801035	3756.489	0.0000
B: C_r	1331340	2	665670	195.342	0.0000
C: C_i	2693726	2	1346863	395.240	0.0000
D: P	1114707	2	557354	163.557	0.0000
E: E_1	1605306	2	802653	235.540	0.0000
F: E_2	1366625	2	683312	200.519	0.0000
Interactions					
AB	53233.44	4	13308.36	3.905	0.0038
AC	111969.68	4	27992.42	8.214	0.0000
AD	66693.16	4	16673.29	4.893	0.0007
AE	590342.64	4	147585.66	43.309	0.0000
AF	923662.64	4	230915.66	67.763	0.0000
BC	734898.63	4	183724.66	53.914	0.0000
BD	1416.19	4	354.05	0.104	0.9811
BE	56914.90	4	14228.73	4.175	0.0024
BF	31326.21	4	7831.55	2.298	0.0576
CD	99958.70	4	24989.67	7.333	0.0000
CE	274174.38	4	68543.59	20.114	0.0000
CF	106269.98	4	26567.50	7.796	0.0000
DE	38862.35	4	9715.59	2.851	0.0232
DF	48303.51	4	12075.88	3.544	0.0072
EF	154698.77	4	38674.69	11.349	0.0000
Residual	2235459.6	656	3407.7128		
Total (Corrected)	39241958	728			

Table 6.4: ANOVA for AOQ for Model 2 of Duffuaa and Al-Najjar

Source of variation	SS	df	MS	F - ratio	Sig. level
Main Effects					
A: C_a	2.0581195	2	1.0290597	1734.863	0.0000
B: C_r	0.0491284	2	0.0245642	41.412	0.0000
C: C_i	0.1180446	2	0.0590223	99.504	0.0000
D: P	0.4158282	2	0.2079141	350.517	0.0000
E: E_1	0.0052487	2	0.0026244	4.424	0.0123
F: E_2	0.0013197	2	0.0006599	1.112	0.3294
Interactions					
AB	0.0621964	4	0.0155491	26.214	0.0000
AC	0.1979975	4	0.0494994	83.450	0.0000
AD	0.7929725	4	0.1982431	334.212	0.0000
AE	0.0035271	4	0.0008818	1.487	0.2046
AF	0.0005783	4	0.0001446	0.244	0.9135
BC	0.0443204	4	0.0110801	18.680	0.0000
BD	0.0252197	4	0.0063049	10.629	0.0000
BE	0.0030240	4	0.0007560	1.275	0.2786
BF	0.0000065	4	0.0000016	0.003	1.0000
CD	0.0694543	4	0.0173636	29.273	0.0000
CE	0.0025553	4	0.0006388	1.077	0.3669
CF	0.0002407	4	0.0000602	0.101	0.9820
DE	0.0019267	4	0.0004817	0.812	0.5177
DF	0.0004342	4	0.0001086	0.183	0.9472
EF	0.0000805	4	0.0000201	0.034	0.9978
Residual	0.3891162	656	5.93165E-4		
Total (Corrected)	4.2413395	728			

among the main effects. Among the two factor interactions AE, BF and BC have high F-ratios and hence can be said to have a dominating effect on ATI.

Some insight about the effect of interactions of the factors on ATI can also be gained by simplifying equation 3.1. On doing so it is noticed that there are two and three factor interaction terms between P, E_1 and E_2 in the equation. However, it must be noted that since the value of the factors are very small, their product term is even smaller and hence some of the interaction effects is not significant in the ANOVA table.

The ANOVA table for AOQ indicates that except factors F, AE, AF, BE, BF, CD, CE, CF, DE, DF and EF all main effects and two factor interactions are significant. Intuitively, factor F should have be significant for AOQ. When the levels of factor A were decreased and that of factor F were increased simultaneously, factor F became significant. Thus, it could be deduced that whether a factor has a significant effect or not largely depends on the levels of the factors. In the presented results, factor F is not significant mainly because factor A which is the cost of false acceptance is very high because our purpose is to guard against the possibility of accepting a defective component and hence it over-shadows the effect of factor F. This was justified when the levels of factor were varied over different ranges. This may help in setting the value of factor A in order to reduce the effect of factor F. It should be set to a value that will make the effect of factor F insignificant.

The plots of interaction will assist us in the practical interpretation of the ex-

periment. Figures 6.1 to 6.5 shows the interaction plots on the response. The plot of interactions for factors A and F on ATI is shown in figure 6.1. Note that 0, 1 and 2 lines are not parallel, indicating that factors A and F interact significantly. The significance level in table 6.3 also indicates that there is a significant interaction between factors A and F. The plot of interaction for factors C and D on AOQ is shown in figure 6.2. Again it can be seen that the lines are not parallel indicating significant interaction between the factors. It can also be deduced that when factor D is at low level there is not much difference in AOQ for any level of factor C. However, at high level of factor D there is significant difference in AOQ for different levels of factor C. Figures 6.3 and 6.4 shows the plot of interaction for factors B and F and for factors D and F on AOQ. In both the interaction plots it can be seen that the lines are almost parallel, indicating that the factors do not interact significantly. The significance levels presented in the tables 6.2, 6.3 and 6.4 also indicates the same results as indicated by the interaction plots.

The ANOVA table provides us with a framework for ranking the factors in terms of their effects on the performance measures of the plan. Also they show the factors to which the plan is very sensitive and what level to set some factor to reduce the effect of others. For example, setting factor A at a certain level will make the effect of factor F insignificant and hence reducing the effect of Type II error which is one of the main reasons for this type of model.

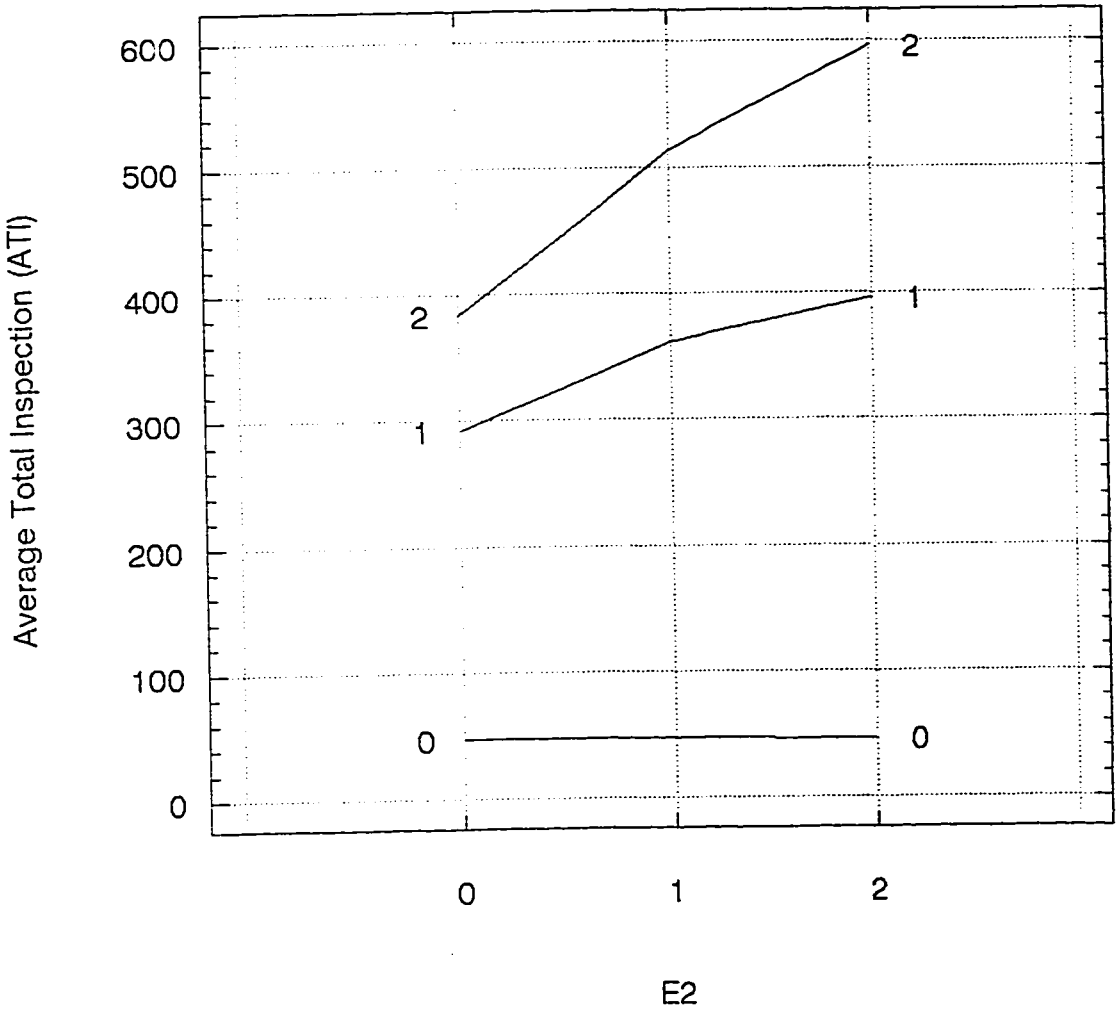

Figure 6.1: Plot of Interactions for C_a by E_2 on Average Total Inspection

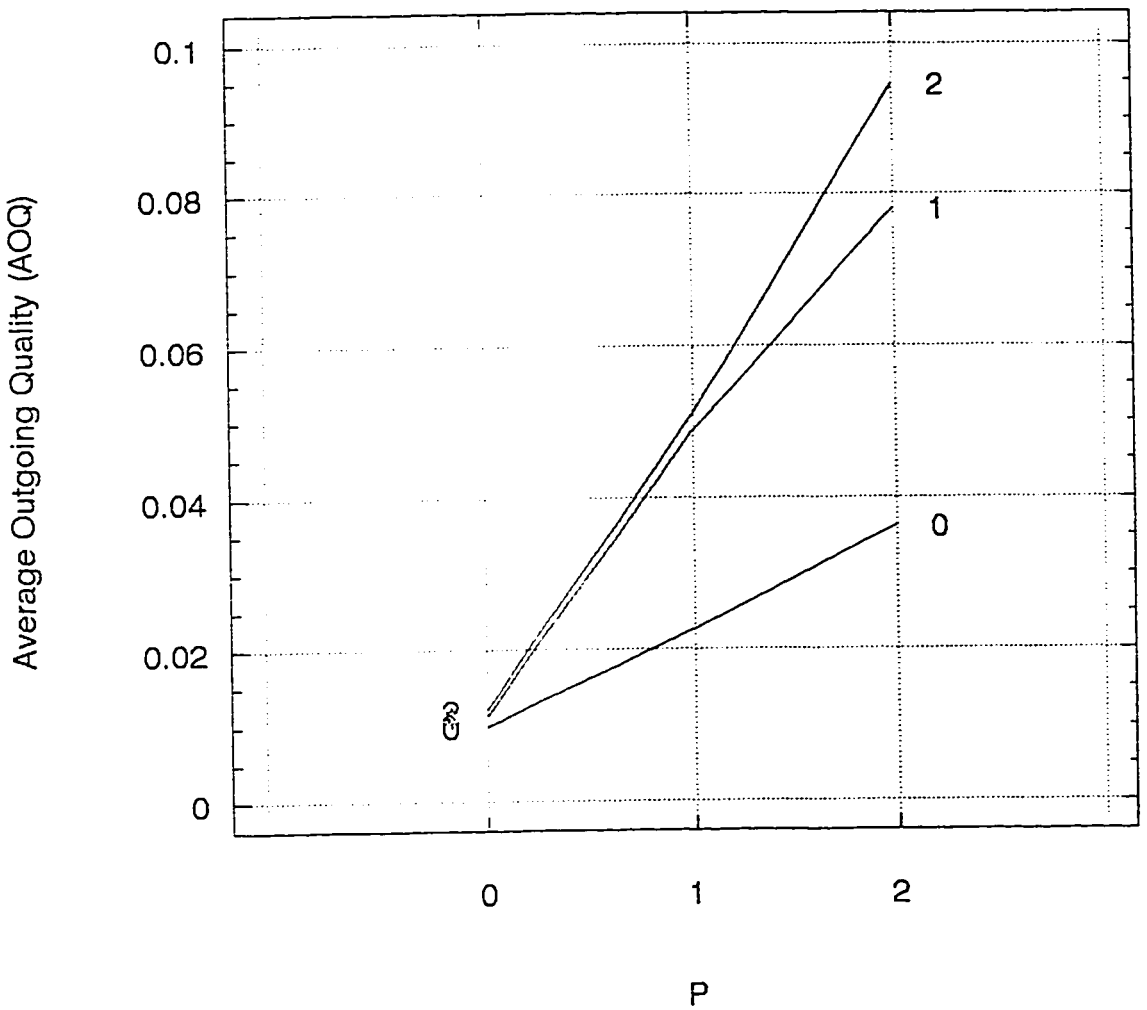

Figure 6.2: Plot of Interactions for C_i by P on Average Outgoing Quality

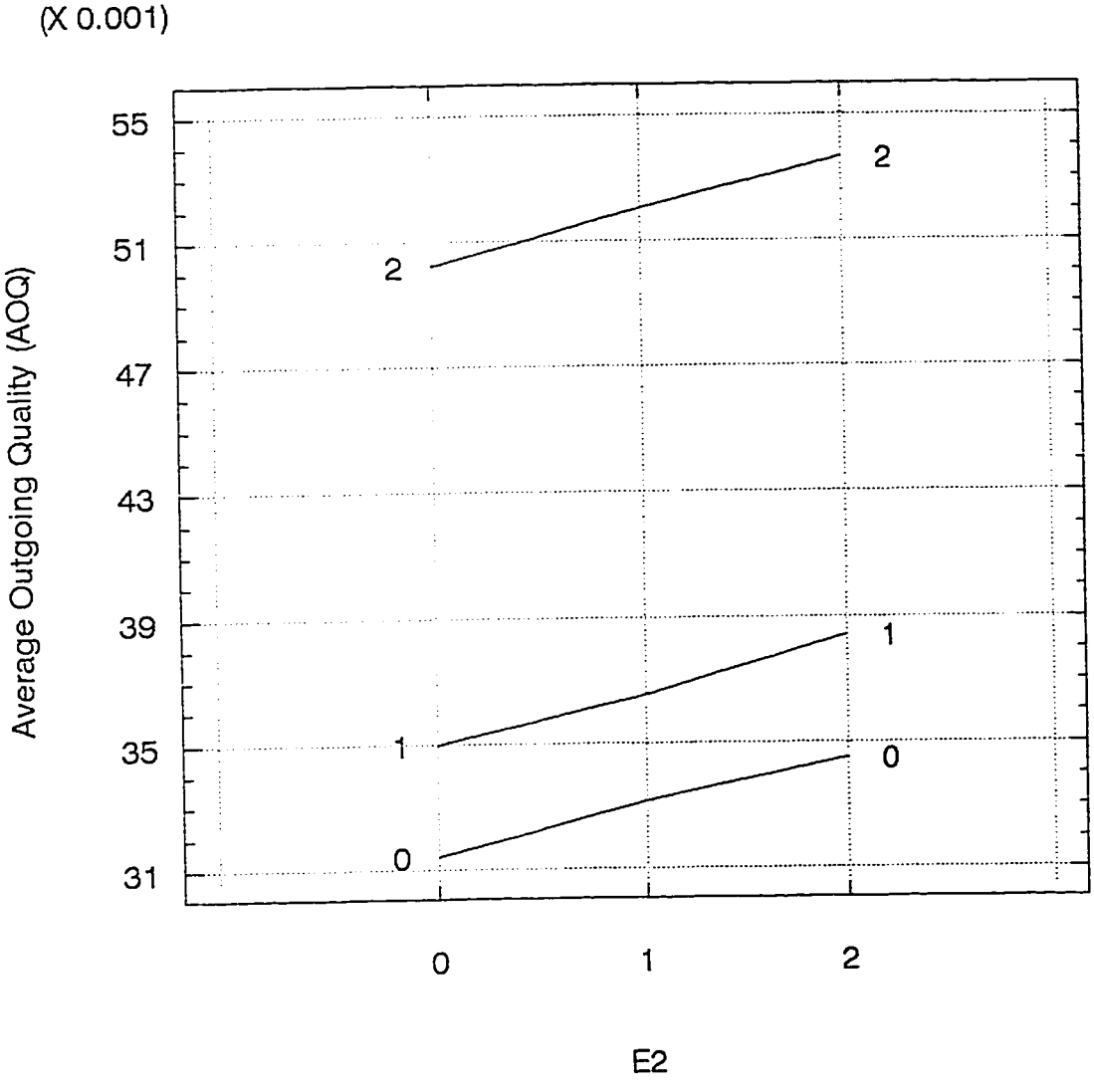

Figure 6.3: Plot of Interactions for C_r by E_2 on Average Outgoing Quality

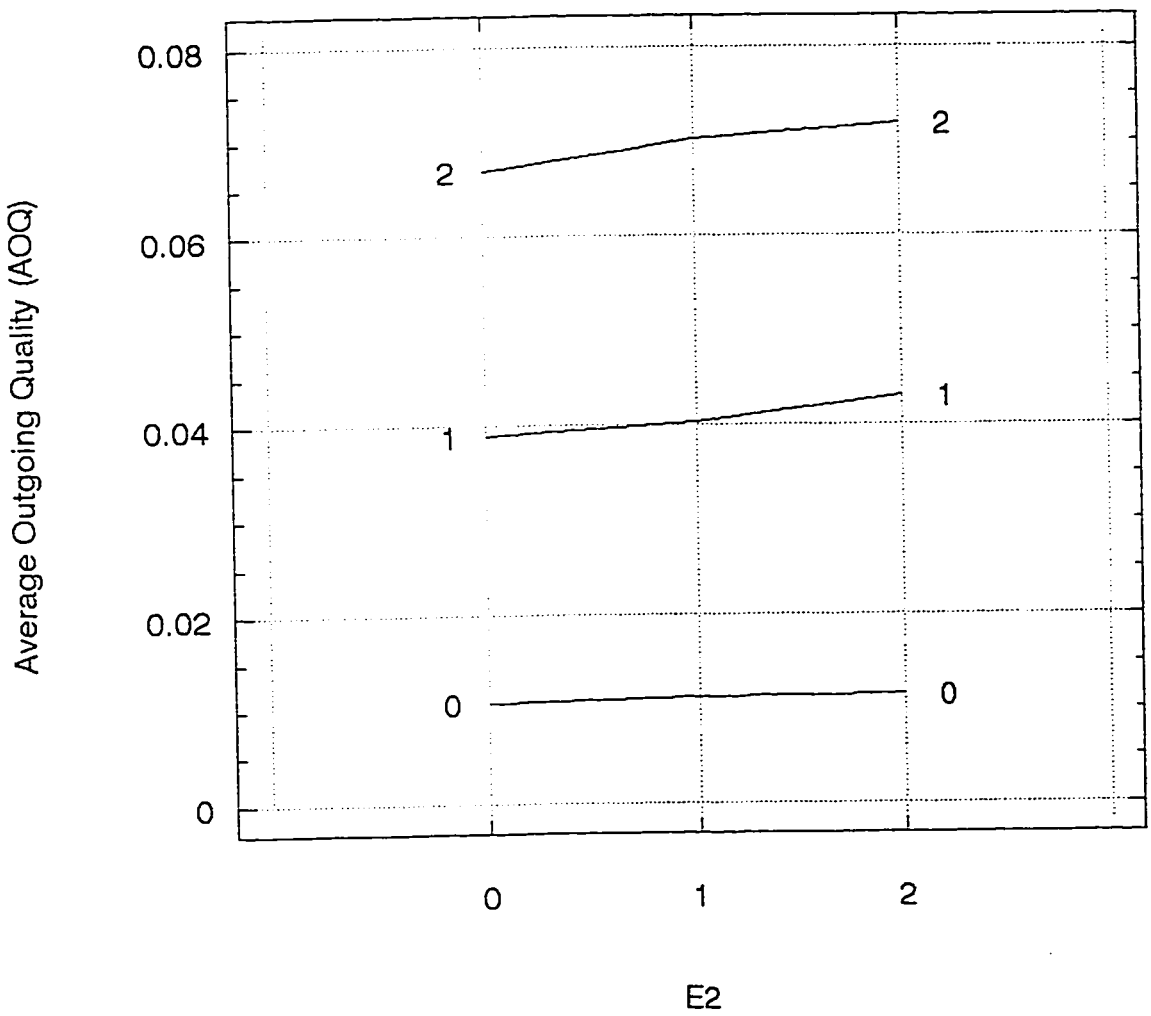

Figure 6.4: Plot of Interactions for P by E_2 on Average Outgoing Quality

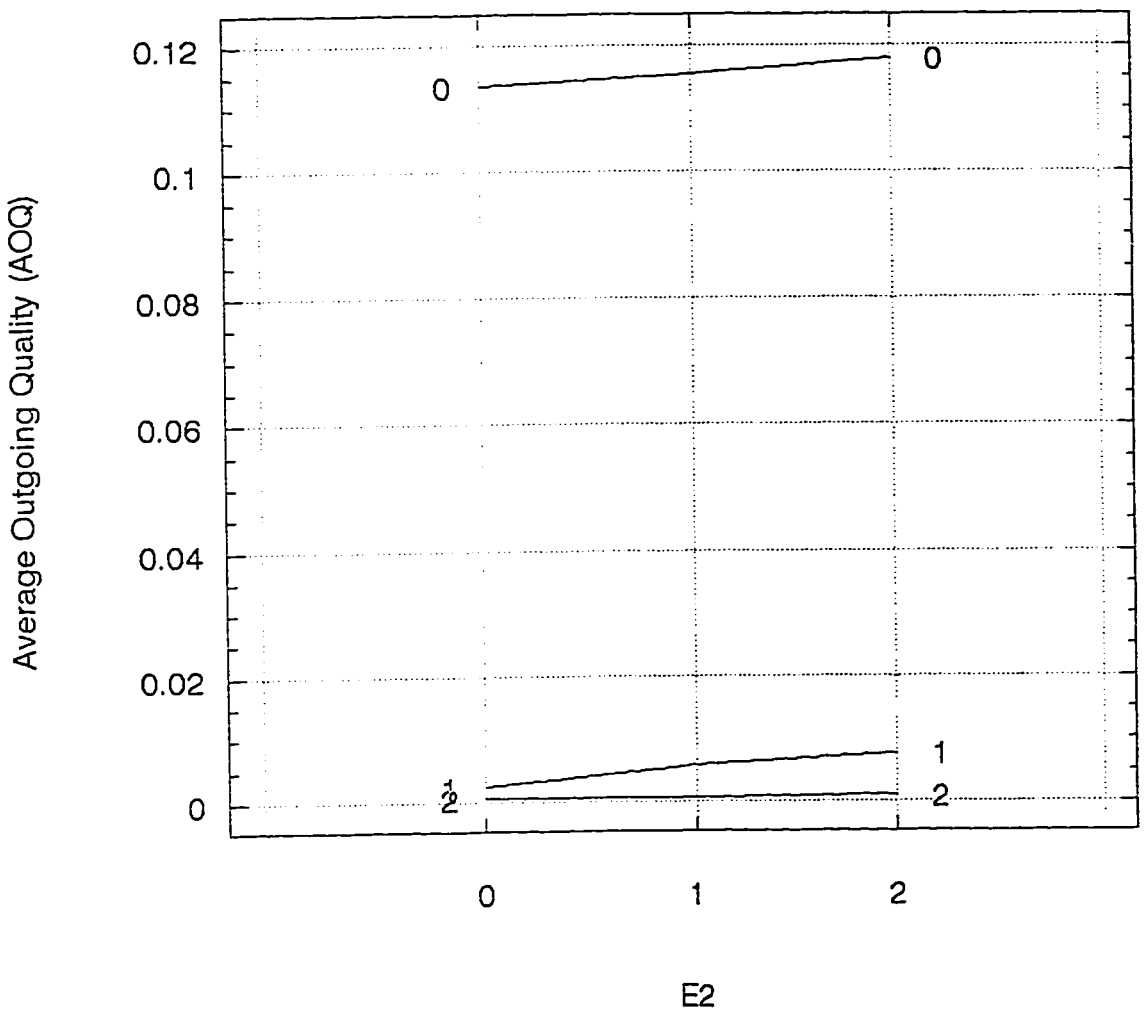

Figure 6.5: Plot of Interactions for C_a by E_2 on Average Outgoing Quality

6.3 Computer Software

A software package is also developed for the existing models for multicharacteristic inspection plans. The software is written in C. The software can be used to find the optimal number of inspections that must be performed in order to minimize the expected total cost. The software includes the independent as well as the dependent models. The independent models included in the software are :

- Raouf et al model [7]
- Model 1 of Duffuaa and Al-Najjar [6]
- Model 2 of Duffuaa and Al-Najjar [6]
- Model 1 of Duffuaa and Al-Najjar [6] with varying inspection errors
- Model 2 of Duffuaa and Al-Najjar [6] with varying inspection errors

The dependent models included are :

- Duffuaa and Nadeem model [30]
- Dependent model 1 presented in this thesis
- Dependent model 2 presented in this thesis
- Dependent model 1 with varying inspection errors presented in this thesis
- Dependent model 2 with varying inspection errors presented in this thesis

The software package consists of a main calling program and several subroutines and function sub-programs. The main programme prompts the user to identify which model he wants to work with. After selecting the model, the user will be asked for certain input such as the number of characteristic, the size of the lot, the value of P_i, E_{1i} and E_{2i}. It also asks for the cost of accepting a defective component, the cost of rejecting a good component and the cost of inspecting each characteristic. Given the input parameters, the software finds the optimal plan and it also gives the probability of a component being defective after inspection along with the average total inspection. It also gives the optimal sequence in which the characteristics of the component must be inspected.

The software can be utilized for situations where the cost of committing inspection errors is substantial. It will help to find the optimal number of repeat inspections which should be instituted.

6.4 Conclusion

In this chapter the effect of input parameters and their interactions on the performance measures of the inspection plans have been investigated. It is clear that these factors and their interactions have a dominating effect on ATI, ETC and AOQ. However, it is noticed that there was some difference in the results when the factors are varied over different ranges. Nonetheless, the experiment has helped us

to identify the factors that have a dominant effect on the response. Factor A has the most significant effect on the response. Also the experiment aided in identifying the level of some factors which will reduce the effect of the others. For example factor A value will have an impact on factor F. Then a C based computer package is presented for finding the optimal number of inspections using different multicharacteristic inspection models present in the literature and the models developed in this thesis.

Chapter 7

Conclusions

7.1 Summary and Conclusions

The objective of this research was to model and evaluate the effect of Type I and Type II errors on the multicharacteristic repeat inspection plans and to extend the models in the literature by incorporating issues such as variability in inspection errors and statistical dependency between characteristics defective rates. This work was motivated by the realization that the inspection process is error prone and the assumption of perfect inspection in most practical application is not valid. Also, since the probability of the Type I and Type II errors are a function of the incoming quality so the errors must also be updated after each inspection as the probability of the component being defective changes. The implications of these errors could be catastrophic in the event of a critical component failure.

This thesis is an extension of the inspection models developed by Duffuaa and Al-Najjar [6]. The effect of Type I and Type II errors were investigated on the performance measures such as ETC, ATI and AOQ of the inspection plans. In general, it was observed that the increase in ETC is more when E_2 increase than with the increase in E_1. Also there is a sharp increase in the cost for high values of E_1 and E_2. However, ATI increased as E_2 was increased but it showed the opposite effect in case of E_1. Again the rate of decrease in ATI for higher values of the Type I error was high. The AOQ increase with the increase in E_1, however the increase is more for higher values of E_1 and E_2. With the increase in E_2, the AOQ keeps on increasing until it increase the ETC so high that the model forces more inspection which lowers the AOQ as more bad components are screened out.

The practicality of the developed models were enhanced by modifying them for the case where the characteristics defective rates are statistically dependent. This was achieved by using the knowledge of the joint probability mass function ($j.p.m.f$) of the random variables representing the characteristics defective rates. The rules for updating the quality of dependent characteristics were proposed which are consistent with the basic probability rules. The effect of inspection errors on the dependency model was evaluated by observing the behaviors of the performance measures, ETC, ATI and AOQ at different levels of error. The results were in line with the one in the literature obtained for the other multicharacteristic dependent model [30].

From the literature it is indicated that the error probabilities, E_1 and E_2, are

a function of the incoming quality. Signal Detection Theory (SDT) can be used to evaluate the performance of an inspector .The receivers operating characteristic (ROC) can be used to estimate the relationship between the incoming quality and the two error probabilities. The GINO computer package was used to find the parameters of the ROC curve which was later used to find Type I and Type II errors for a corresponding value of incoming quality. It was assumed that the distance travelled on the ROC curve due to the change in P is proportional to the amount of change that has taken place. A computer program was written to perform the necessary estimation procedure via numerical method. The developed functional relationship between incoming fraction defective and the two types of errors were incorporated in the independent as well as the dependent model to evaluate the effect of varying inspection errors.

Finally, factorial design experiments were conducted to study the effect of the input parameters on the performance measures of the inspection plan. The performance measures include the expected total cost, the average total inspection and the average outgoing quality. This study was performed on the plans proposed by Raouf et al [7] and the models proposed by Duffuaa and Al-Najjar [6]. In general the analysis revealed that all the main effects and most of the two factor interactions have a significant effect on the performance measures. However, it was noticed that factor A(C_a) had the most significant effect on all the performance measures. Some of the interaction plots have also been shown to assist in the interpretation of

the results. It was also observed that when the levels of the input parameters were varied over different ranges, the results obtained were different. Nonetheless, it is clear from the study of effects and sensitivities, that the input parameters and two factor interactions are critical for the performance measures of the inspection plans.

A software package is also developed for the existing models and multicharacteristic inspection plans. The software can be used to choose among the existing inspection plans. Giving the input values to the various models and comparing the number of repeat inspections, the expected total cost, the average total inspection or the average outgoing quality one can choose a particular inspection plan. This will give us an estimate of the performance measures before we go on to do the inspection.

The results of this research can be applied to many practical situations where the cost of committing inspection errors is substantial and therefore a need for repeat inspection is required.The performance of inspectors can be compared based on the estimated ROC curve. This can be used in the selection and training of inspectors when inspection tasks are being designed or evaluated.

7.2 Future Research

The research in this thesis focussed on extending repeat inspection plans for the cases where inspection errors is varying and is a function of the incoming quality

and when the characteristics defective rates are statistically dependent. More research is needed to investigate other issues related to repeat inspection plans. Some of the possible research directions in the future are as follows :

1. Practical use of repeat inspection plans and their designs should be addressed in detail.

2. Other inspection plans could be developed combining Raouf et al plan with Duffuaa and Al-Najjar plan. Models for such plan could be developed.

3. Other objective functions besides minimizing the expected total cost, maximizing of the probability of a component being non-defective can be considered. In other words multi-objective optimization approaches can be used to design such types of plan.

4. The solution methodology proposed for the second model could be enhanced to provide global minimum.

5. The number of data points used in this research to estimate the parameters of the *ROC* curve was too few. Collection of more data points under controlled experimental conditions. in a laboratory or in a plant, will enhance the accuracy of parameter estimation for the *ROC* function and assessment of error probabilities.

6. Much work remains to be done in the area of estimation of error probabilities for a given *ROC* curve. The effect of job-related factors (such as task pace, product type, training of inspectors etc) need to be investigated.

Appendix A

Table A.1: Data set from the results of a visual inspection experiment reported by Harris

P	Defects Detected	False Reports
0.0025	0.575	0.850
0.01	0.715	0.395
0.04	0.750	0.179
0.16	0.840	0.075

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