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Modes of nonlinear optical fibers

Hassan, Mohamed Adel, Ph.D.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1993
MODES OF NONLINEAR OPTICAL FIBERS

BY

Mohamed Adel Hassan

A Thesis Presented to the
FACULTY OF THE COLLEGE OF GRADUATE STUDIES
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

In

Electrical Engineering

January 1993
This Dissertation, written by Mohamed Adel Hassan Aly, under the direction of his Dissertation Advisor and approved by his Dissertation Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY DEGREE in ELECTRICAL ENGINEERING.

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Date 16-5-93
Dedicated to the memory

of my parents
ACKNOWLEDGEMENT

All praises and thanks be to the Almighty, Allah who blessed me with courage and wisdom to accomplish this research work.

I greatly owe my success to my beloved family, my wife, son, daughter, brother and sisters for their patience, encouragement and continuous prayers.

Acknowledgement is due to King Fahad University of Petroleum and Minerals and the department of electrical engineering for supporting this work.

I wish to express my deep appreciation to Dr. Samir J. Al-Bader, my advisor for his patience, careful attention, advice, guidance and continuous help.

My thanks and appreciation extend to Dr. Hussain A. Jamid, thesis committee member for his valuable assistance and suggestions during the study. I am also grateful to the other thesis committee members Dr. Samir H. Abdul-Jauwad, Dr. Essam E. M. Hassan and Dr. Mohamed A. Chaudary for their support and helpful suggestions.

Thanks are also due to Mr. M. Imtar, King Faisal University, for writing the computer subroutines used for evaluating Bessel’s functions.

Last, but of course not least, I am sincerely grateful to all my colleagues and friends who enriched me with moral support, inspiration and motivation.
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DISSECTI0N ABSTRACT

FULL NAME OF STUDENT: Mohamed Adel Hassan Hassan Aly

TITLE OF STUDY: Modes of Nonlinear Optical Fibers

MAJOR FIELD: Electrical Engineering

DATE OF DEGREE: January 1993

The research work presented in this thesis solves the nonlinear wave equation for electromagnetic fields and finds the guided modes in the nonlinear optical fiber. The nonlinear materials in optoelectronics are characterized by dielectric constants which are functions of the light intensity. The dispersion characteristics of the nonlinear fiber are accordingly functions of the light intensity. The solution of the problem has been carried out using a multilayer numerical approach in which the nonlinear wave equation is piecewise linearized by dividing the fiber into a large number of layers. In each layer the local field is determined and used to evaluate the dielectric constant of that layer. The dispersion relations, field profiles and dielectric constant distributions have been evaluated for different types of polarization including TE, TM and hybrid modes. Two waveguide structures have been considered for which either the core or cladding material is taken to be nonlinear while the other is linear. The nonlinear fiber results have been checked against the linear ones in the limiting cases of very low and very high power. Another numerical scheme based on a self-consistent solution of electromagnetic field, has also been used to confirm the results obtained using the multilayer scheme. The stability of the nonlinear guided modes has been discussed based on the resultant dispersion relations. The recursive scheme has also been used to study the metal-clad optical fiber consisting of two, three or four layers. The attenuation characteristics of these fiber structures have been found. These results are relevant in the design of mode filters and fiber polarizers.

DOCTOR OF PHILOSOPHY DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran, Saudi Arabia

January 1993

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خلاصة الرسالة

اسم الطالب الكامل: محمد عادل حسن علي
عنوان الدراسة: الأشياء الموجبة في الألياف البصرية غير الخطية
التخصص: هندسة كهربائية
تاريخ الشهادة: يناير 1993م

يتناول البحث المقدم في هذه الرسالة حل المعادلة الموجبة غير الخطية للمجال البصري الكهروكيميائية لإيجاد الأسس الموجبة داخل الألياف البصرية غير الخطية.

إن المواد غير الخطية المستخدمة في الإلكترونيات البصرية تتميز بعملية إزاحة ضوئية يعتمد على شدة الضوء المرور في تلك المواد، وعلى ذلك فإن الخواص الموجبة للألياف البصرية الفائقة من تلك المواد تكون أيضاً معتدلة على شدة الضوء المرور داخل هذه الألياف البصرية. إن طريقة الحل المستخدمة تعتمد على تقسيم أحد هذه الألياف البصرية إلى عدد كبير من الحلقات التي تتلف فيها شدة الضوء ثم استخدام شدة الضوء المرور لكل حلقة لإيجاد مجال الإلكترام الخاص بهذه الحلقة، ومن ثم يتم توحيد المعادلة الموجبة غير الخطية إلى معادلة موجبة خليطية تختلف من طبقة إلى أخرى بسما إيجاد معامل الإزاحة الضوئي. هذه المعادلات الموجبة يمكن حلها بالطرق المعرفة.

قد تم إيجاد وحساب الخواص الموجبة وتوزيعات شدة الضوء المرور في الألياف البصرية وكذلك تغير معامل الإزاحة داخل هذه الألياف نتيجة لإنتاج الناتج على شدة الضوء. وقد خرجت الدراسة الأدوار المختلفة للمجال البصري الكهروكيميائية في وقائد النواحي الإستثنائية والتي تشمل النواحي ذات استثناء المجال الكهربائي، وأخرى ذات استثناء المجال المصطلحي وتلفة مختلفة إستثناء. وقد خرجت الدراسة نموذج مختلف لقواعد النواحي غير الخليطية يكون في إجمالي مادة اللب للفائدة غير خليطية بينما مادة الفضاء المليئ خليطية. وفي النموذج الآخر يكون اللب مصنعاً من مادة خليطية بينما الفضاء المليئ من مادة خليطية.

هذا وقد تم تحقيق النتائج المحسوبة بإستخدام طريقة رقمية أخرى للحل تعتمد على الطرق الثنائية للمجال البصري الكهروكيميائية. وقد ساعدت النتائج المستندة للخواص الموجبة في تحديد مدى إستقرار النواحي أثناء إنتاجها داخل الألياف البصرية غير الخطية. وقد تم أيضاً استخدام نموذج حل التفاعلي معتمد للنواحي في دراسة قواعد النواحي الإستثنائية المطلقة خليطية ببطيئة من المعدلة والمكونة من طبيعتين أو ثلاث أو أربع طبقات. ومن المعروف أن قواعد النواحي هذه أهمية خاصة في تصميم مشاريع النواحي الأحذية الإستثناء.

درجة الدكتوراه في الفلسفة
جامعة الملك فيهد للبترول والمعادن
الظهران - المملكة العربية السعودية
بتاريخ 1993م
CHAPTER 1

NONLINEAR SLAB AND CYLINDRICAL WAVEGUIDES

1.1 INTRODUCTION

The nonlinear effects in optical fibers and dielectric waveguides have received a growing interest in the last two decades. The nonlinear guided and surface waves have been investigated in many different respects particularly in planar waveguides.

A complete study of nonlinear phenomena in optical fibers and fiber devices requires the solution of nonlinear wave equations subject to the appropriate boundary conditions.

For the slab waveguide, the nonlinear wave equation in rectangular coordinates can be solved analytically for certain types of nonlinearities such as the Kerr-like nonlinearity. However, in fibers there have been no analytical solutions of the nonlinear wave equation and numerical techniques must be used to find the solution.

The research work proposed here uses a multilayered technique for solving nonlinear optical fiber problems. The nonlinearity under
consideration will be the third order Kerr-like nonlinearity with a single frequency input and a single frequency output. The saturation limit of the dielectric constant will also be considered.

The applications of nonlinear optical effects and nonlinear guided waves have been shown to cover wave mixing process, harmonic generation, parametric amplification, self-focusing, self-defocusing and a wide range of devices including bistable switches, logic gates, optical limiters, bandpass power filters, periodic structures, and optical filters.

In this thesis, we are interested in the guided modes of optical fibers. A considerable part of the recent work in guided waves is now directed to study the nonlinear fiber. The nonlinear fiber may be one of the three configurations, linear core-nonlinear cladding, nonlinear core-linear cladding or nonlinear core-nonlinear cladding.

In the following sub-sections we will study the research work related to the nonlinear guided waves in optical fibers as well as in slab waveguides. It can be shown that some of the nonlinear dispersion characteristics are common in both fiber and slab waveguide. The field distributions will also be investigated for both cases. The multi-layer technique adopted in this thesis is not only applicable to the nonlinear waveguide problems, but it is also applicable to any arbitrary refractive index profile. Thus it can be used to solve graded-index profiles and waveguide structures that involve a metallic
layer as the outer layer. In chapter 4 of this thesis an extensive work has been done to study the attenuation characteristics of metal-clad cylindrical waveguide. The objective of such a study is to check the recursive scheme in complicated waveguide structures and at the same time to report some of the important attenuation characteristics of cylindrical metal-clad waveguide. These attenuation characteristics have not been studied in literature as will be seen in chapter 4.

It can be seen from the literature review of section 1.3 that the nonlinear optical fiber has not been investigated in full, specially when the nonlinearity is of saturable type which is more practical and realistic than the Kerr nonlinearity. The research study presented in this thesis is devoted to the investigation of guided modes in nonlinear cylindrical waveguide. The nonlinearity of the waveguide media is considered of saturable type. There are two different structures of the nonlinear optical fiber. The first one consists of a nonlinear core and a linear cladding while the second structure is formed of a linear core and a nonlinear cladding. TE, TM and hybrid modes are to be studied in the proposed work. In the case of the fundamental $HE_{11}$ mode all the six field components are utilized by the recursive scheme. It is thus considered more rigorous than those used in literature which involve the use of linearly polarized modes. In such LP approximation the problem reduces to only three field components and can
be treated using the recursive scheme in a manner similar to the TE modes.

The solution of the problem is carried out numerically using two different numerical schemes. The first scheme is a multilayer one in which the nonlinear fiber is divided into a large number of linear sub-layers. In each layer the wave equation is solved linearly by considering the effect of the local field on the dielectric constant. The second scheme is, however, based on the self-consistent behavior of electromagnetic fields. In this approach the solution of the nonlinear wave equation is obtained through a successive number of solutions for a linear graded-index profile problem. The two scheme will be explained in details in chapter 3.

1.2 THE NONLINEAR SLAB WAVEGUIDE

The nonlinear effects in slab waveguides have received a large attention in the last few years. Because our main objective is to study nonlinear guided modes in fibers, we will only concentrate on the relevant slab work which serves our purpose. In 1987, Al-Bader and Jamid [1] studied the nonlinear TE waves in a nonlinear thin film. The waveguide structure consists of a nonlinear self-focusing film cladded in both sides by linear media. The TE wave dispersion relations have been obtained for both symmetric and asymmetric solutions. Both Kerr and saturable nonlinearities were considered. The saturation limit has been considered in [1] for the first time. The research work preceding [1] accounts only for Kerr-nonlinearity.
The dispersion characteristics are shown in fig.(1.1a). The symmetric solutions (broken lines) show a multivalued dependence of the effective index on surface intensity. This is the case for relatively high saturation levels. This is an important feature of nonlinear guided waves. For low saturation levels the dependence of the effective index on surface intensity is monotonic. Only the fundamental TE mode has been studied. The relation between guided power and effective index has also been obtained for different saturation levels, as shown in fig.(1.1b). In this figure all curves start at the same asymptotic line which corresponds to the minimum value of the effective index. The field distribution corresponding to these dispersion characteristics are evaluated. The problem of Kerr nonlinearity has been solved analytically and numerically. The two solutions are consistent. For the saturable nonlinearity there is no analytical solutions available and only the numerical technique is used.

The TM polarized waves in a nonlinear thin film bounded by linear media is investigated in [2]. A multilayered scheme is introduced to study the problem. The nonlinear medium is divided into a large number of layers. Each layer is characterized by an isotropic linear dielectric constant. The value of the dielectric constant is determined from the information of the field distribution of the previous layer. Therefore, the nonlinear wave equation is treated for each layer as a linear wave equation with the dielectric constant calculated locally from the field information. A similar recursive multilayered
FIG. 1.1

(a) Mode Index versus Surface Intrinsic for Two Saturation Levels

(b) Mode Index versus Guided Power for Four Saturation Levels

1.55
1.56
1.57

n

0.0
0.1
0.2
0.3
0.4
0.5

a

0.0
0.1
0.2
0.3

a

0.0
0.1
0.2
0.3
0.4
0.5

a

0.0
0.1
0.2
0.3
0.4
0.5

a
scheme is used to study the nonlinear fiber in our proposed work.

It must be noted that most of the research work in nonlinear guided waves has been devoted to the Kerr nonlinearity in which the dielectric constant is given by:

\[ \varepsilon = \varepsilon_{bg} + \alpha |E|^2 \] (1.1)  

Where \( \varepsilon_{bg} \) is the background dielectric constant with no field applied, \( \alpha \) is the nonlinear coefficient of the material and \( |E|^2 \) is the local field intensity. The kerr model is relatively simple to analyse and the problem with Kerr nonlinearity can be solved analytically for some cases [2].

However, this model is not realistic. The nonlinear material always responds to the applied field up to a certain limit. Any further increase in the applied field will not affect the value of the dielectric constant. It may cause a complete breakdown of the dielectric material. The use of a saturable dielectric function is a result of this fact. The saturable expressions of the dielectric materials are more realistic and based on the fact that the dielectric function cannot grow indefinitely but must level off at a maximum value that depends on the nature of the nonlinear material. Two models have been shown in [3] as follows:
\[ \varepsilon = \varepsilon_{bg} + \frac{\alpha |E|^2}{1 + g\alpha |E|^2} \]  
(1.2)

\[ \varepsilon = \varepsilon_{bg} + \frac{1}{g} \left( 1 - e^{-\alpha |E|^2} \right) \]  
(1.3)

where the maximum increase in the dielectric function is equal to \( \frac{1}{g} \) for an infinitely intense field. These two models of saturation have also been used in [1].

The dispersion relations for the fundamental TM mode are obtained in terms of the mode index versus both guided power and surface intensity. They are shown in figs. (1.2.a) and (1.2.b) respectively. For low values of saturation level the mode index is a monotonic function of the intensity. For the Kerr nonlinearity and high level of saturation the relation is generally a multivalued i.e. for the same value of surface intensity there are two values of the effective index.

In the above citations we have considered one configuration of nonlinear planar waveguide in which the nonlinear thin film is surrounded on both sides by linear media. Another important nonlinear planar waveguide structure has been given by Sang-Yang Shin et al. [4] in 1989. This
Fig.(1.2.a) The dispersion characteristic as a plot of $n_e$ versus scaled intensity for different saturation levels. Solid lines, the nonlinearity due to thermal effect. Broken lines, the nonlinearity due to electronic distortion [2].
Fig.(1.2.b) The dispersion characteristic as a plot of $n_e$ versus guided power for different saturation levels. Solid lines for, the nonlinearity due to thermal effect. Broken lines, the nonlinearity due to electronic distortion [2].
waveguide consists of two parallel dielectric thin films separated by a layer of linear medium and bounded on both sides by a nonlinear Kerr medium. It is therefore considered a five layers structure forming a nonlinear directional coupler. In the nonlinear directional coupler the coupling between the two waveguide channels is controlled by the input power. This has an important application in the design of optical logic gates.

The technique of solution used in [4], is based on integrating the wave equation in the nonlinear regions and imposing the condition that the transverse field vanishes at infinity. The wave equation in the nonlinear region is solved directly in terms of trigonometric functions. The solutions are then matched at each interface to obtain the propagation constant and the corresponding field profiles.

The dispersion characteristics for this directional coupler are shown in fig.(1.3.a) for a structure similar to that described above and in fig.(1.3.b) for a different structure in which one of the nonlinear outer layers is replaced by a linear medium. Figure (1.3.a) shows that both $TE_1$ and $TM_1$ modes have two solutions, symmetric and asymmetric. For the symmetric solution The field distribution has even symmetry with respect to the axis of the waveguide. At low power there are two field maxima, one in each guiding film. This is the linear state of the problem. As the power increases, each maximum moves towards the adjacent nonlinear interface. A further increase of the
Fig. (1.3)
a) The power- $n_e$ diagram for $TE_0$ nonlinear guided mode in a symmetric nonlinear coupled waveguide. Branch A, symmetric solution. Branch B, asymmetric solution.
b) The power- $n_e$ diagram for $TE_0$ and $TE_1$ wave solutions in a nonlinear coupled waveguide [4].
power forces these maxima to cross the interface and lie in the nonlinear regions. The dispersion relations shown in fig.(1.3.b) exhibits a peak power for $TE_1$ mode at a certain value of effective index. for the $TE_0$ the effective index varies monotonically with the power.

1.3 THE NONLINEAR FIBER

The work done to study wave propagation in nonlinear optical fibers is limited. The reason for this is that the theoretical analysis is difficult compared to the slab waveguide. The solution of the optical fiber problems in cylindrical coordinates system involves the use of Bessel functions which are more complicated compared to the trigonometric functions resulting from the solution of the slab waveguide problems. The simplest structure of a nonlinear fiber consists of two layers, a core and an unlimited cladding. Three configurations are considered, nonlinear core - linear cladding, linear core - nonlinear cladding and nonlinear core - nonlinear cladding. With the availability of fast computers and efficient numerical techniques, those analytical difficulties can be overcome.

1.3.1 Fiber with nonlinear core and linear cladding

The first attempt to find the modes of nonlinear cylindrical waveguides has been carried out by Garmire et al, in 1964 [5]. The wave equation with a nonlinear Kerr term is solved numerically for a single medium in cylindrical
coordinates. The conventional waveguide which has a core and a cladding is not treated here. The waveguide structure consists of unlimited core. The self-focusing action and the beam diameter will determine the physical dimension of the waveguide. The work in this reference is devoted to self-trapping of optical beam. The self trapping is a state in which the effects of self-focusing and scattering balance each other under a certain critical value of the applied power. This will be discussed in more details in chapter 2. The field profile for this waveguide resembles the fundamental LP mode of an optical fiber. It has a peak intensity at the beam axis and decays in the radial direction.

This work was followed by another investigation by Haus et al. in 1966, which includes the higher order modes [6]. The first five modes are analysed numerically. The field distributions corresponding to these modes have been evaluated and plotted. These field profiles exhibit self-focused rings. The number of rings is less than the radial mode number by one. For example, the fifth order mode has four self-focusing rings.

The chromatic dispersion of a nonlinear fiber has been investigated in [7] by Okamoto and Marcatili in 1989. The term chromatic dispersion refers to the dependence of the group velocity on the optical frequency. It arises from the interaction of an electromagnetic wave with the bound electrons of the dielectric [8]. Chromatic dispersion is used to determine the chirping
properties of an optical pulse. Pulses at different wavelengths propagate at different speeds inside the fiber because of the group velocity mismatch.

The chromatic dispersion characteristics are closely related to the index profile and hence they are expected to be different from linear fiber. The technique used in [7] is based on solving the nonlinear wave equation using a variational method. Two values of the total power flow have been chosen to represent the linear and nonlinear states of the fiber. These values are 1 mW and 200 KW respectively. The dispersion relations for the nonlinear fiber are compared to those of the linear fiber. The wave equation is assumed to have a Kerr nonlinearity and the fiber is considered circularly symmetric with no azimuthal dependence. The analysis is therefore an approximate one valid only for weakly guiding fibers. The scheme used in [7] divides the fiber into a nonlinear region, surrounding the core and spreading over a radial distance D equals five times the core radius, and a linear region for the radial distance greater than D . The nonlinear region is subdivided into N subregions. The finite element method is used to find the solution of this part. The solution of the linear region is given in terms of modified Bessel function.

The dispersion characteristics obtained in [7] as a plot of the normalized propagation constant versus the fiber V number have only a slight difference between the linear and the nonlinear waveguides shown in fig (1.4). For the
Fig. (1.4)
a) Dispersion characteristics of step-index fiber with core nonlinearity.
b) Dispersion characteristics of step-index fiber with cladding nonlinearity.
b(v) is the normalized propagation constant, d(v) is the normalized group delay and g(v) is the waveguide dispersion [7].
nonlinear fiber with nonlinear core and linear cladding, the nonlinear propagation constant is relatively greater than the corresponding linear case for the same V number.

1.3.2 Fiber with linear core and nonlinear cladding

The case of linear core and nonlinear cladding is treated by Boardman et al. [9] in 1986. The field solutions for the fundamental mode $HE_{11}$ are obtained from the scaler wave equation with no azimuthal dependence for a radially symmetric fiber. A new class of radially symmetric waves is reported. This class of waves creates self-focusing rings of energy flowing parallel to the fiber axis.

The investigation is done for the Kerr nonlinearity where the nonlinear cladding is assumed to have a refractive index of the form $n = n_c + \alpha |E|^2$. The wave equation used for this investigation is simplified representing only the weakly guiding fiber. The modes of such a fiber are linearly polarized. The linearly polarized (LP) modes are characterized by an electric field vector pointing in one direction in the transverse plane and a magnetic field vector perpendicular to that direction. The axial components are neglected with respect to the transverse component. Therefore LP modes are similar to TEM waves in parallel plates transmission lines. If the nonlinear term added to the dielectric constant is very small compared to the the background
value, the LP approximations can be used to solve the wave equation and find the nonlinear guided modes. On the other hand if the nonlinear term is relatively high, a complete rigorous solution of the wave equation should be obtained.

The important conclusion of this work is that unlike the linear fiber the field can have a maximum which takes place in the nonlinear region. Also the mode index can exceed the core value. For this case the guided waves are transformed into surface waves. The stability of these nonlinear waves is given in terms of the slope of the power versus mode index curve. If the slope is positive the solution is stable. If it is negative the solution needs more investigation to decide about its stability.

Akhamdiev et al. have investigated the linear cylindrical waveguide surrounded by nonlinear medium [10] in 1985. Only TE modes have been reported. The nonlinearity of the cladding is also Kerr-like with dielectric constant of the form \( \varepsilon = \varepsilon_0 + \alpha |E|^2 \). It has been shown in [10] that the dispersion relations as a plot of the energy flux versus the normalized mode index for this fiber configuration behave as an N curve. This means that for a single value of the field intensity there are three values for the mode index. For higher modes we may have more than one N curve depending on the mode number. Also for this investigation the nonlinearity has been assumed not to have a saturation limit i.e., Kerr-like nonlinearity.
An important observation here is that the nodes of the solution which represent the crossings of the field profile to the fiber axis can go beyond the core of the waveguide. This is a unique characteristic of the nonlinear waveguide and has no linear counterpart. The technique of solution used in [10] is based on solving Maxwell's equations for the linear core in terms of Bessel functions and finding the solution of the wave equation for the nonlinear cladding using numerical methods. The boundary conditions are matched at the core cladding interface. The investigation also covers the case when the dielectric constant of the waveguide is smaller than or equal to the linear cladding value. In the linear sense, waveguide modes do not exist in such fiber configuration. In a linear waveguide, guidance takes place only if the core refractive index is greater than the cladding refractive index which can fulfill the condition for total internal reflection. However, for a nonlinear surrounding medium, waveguide modes exist only for values of power greater than a certain threshold. The main conclusion shows that a cylindrical waveguide immersed in a medium with a nonlinear permittivity has some important characteristics, in particular the multi-valued dependence of optical power on effective index. This nonlinear waveguide is used in the design of switches in optical communication devices, where the light intensity is used to switch the fiber between two guiding states or between a cutoff and a guiding state.
The waveguide of linear core and nonlinear cladding has also been studied by Akhmediev et al. in 1990 [11]. The dielectric function in [11] can account for both saturable and nonsaturable nonlinearities. To the best of our knowledge this is the first time a saturable dielectric is treated in optical fiber beside the ordinary Kerr nonlinearity. Two solutions are obtained, symmetrical and asymmetrical. For the asymmetrical solution the field distributions have no axial symmetry. The asymmetrical solutions occur only for values of effective index greater than a minimum value \( n_0 \). Below this value only the symmetric solutions exist. Above \( n_0 \) the asymmetric solutions branches from the symmetric ones.

For the fiber with linear core and nonlinear cladding studied in [7] the dispersion relation is almost the same for the linear and the nonlinear states.
CHAPTER 2

NONLINEAR MATERIAL IN OPTOELECTRONICS

2.1 INTRODUCTION

Many characteristics of extremely low-loss fibers, whose development points towards the feasibility of long-distance wide-band transmission systems, enhances the relevance of nonlinear phenomena in optical waveguide propagation. These characteristics are the long interaction length provided by the fiber itself, the small fiber diameter pertinent to monomode operation and the existence of narrow-linewidth single frequency lasers. In particular, the product of the fiber length L and the intensity $\frac{P}{(\pi a^2)}$, associated with a core radius $a$ and an input power $P$ can become large enough, compared with nonlinear propagation in an unbounded medium, to balance the intrinsically small nonlinearity of silica glass at relatively low power.

The third order nonlinear susceptibility $\chi^{(3)}$ is an important optical property of material because of its contribution to numerous nonlinear optical processes. Some of these processes are the self-focusing, self-defocusing,
self-trapping and self-bending of light. Degenerate four-wave mixing, and phase conjugation are also results of the third order nonlinear susceptibility.

2.2 CHARACTERISTICS OF NONLINEAR MATERIALS

Nonlinear optical processes are usually described in terms of the polarization vector $\vec{P}$ which is related to the $\vec{D}$ and the $\vec{E}$ vectors by the following equation:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad (2.1)$$

The polarization can be formally expanded in a power series of the electric field as $ith$ component by:

$$P_i = \chi_{ij} E_j + 2\chi_{ijk} E_j E_k + 4\chi_{ijkl} E_j E_k E_l + \ldots \quad (2.2)$$

where $P_i$ is the $ith$ component of $\vec{P}$ and $\chi$ is the susceptibility.

For linear materials only the first order susceptibility $\chi_{ij}$ is important. High rank susceptibility terms are negligible. For this type of materials the relative dielectric constant is represented by

$$\varepsilon_r = 1 + \chi_{ij} \quad (2.3)$$

For nonlinear materials the higher order susceptibilities are not negligible.
specially in the presence of strong electric field.

The second order nonlinear term \(2\chi_{ijk}E_iE_jE_k\) is responsible for the phenomena of second harmonic generation, parametric amplification and oscillation. It is however negligible in glasses because of inversion symmetry of the material, so that in practice all the nonlinear effects taking place inside the glass optical fiber are associated with the third term of the right hand side of in equation (2.2). These nonlinear effects can be roughly divided into two classes, according to whether the induced polarization vibrates at the same frequency as the incident field or not. The second class includes stimulated Raman scattering, stimulated Brillouin scattering and four-wave mixing, while the first concerns the so called induced field effects which can be described in terms of a nonlinear refractive index eg. the optical Kerr effect. Table 2.1 lists the values of \(\chi^{(n)}\) for some optical nonlinear materials. It can be seen from the table that the value of \(\chi^{(n)}\) is about \(10^{-12}\) for most of the material at different optical wavelengths. Therefore, a field strength of the order \(10^5 \text{ (V/m)}\) can make an increase of the dielectric constant equal to 0.01. The above field strength is roughly corresponding to a power density of \(10^4 \text{ (W/cm}^2\)\). This power density can be obtained form a Helium-Neon or a Yag laser source.
### TABLE 2.1

Values of nonlinear polarizability $\chi^{(3)}$ of some nonlinear materials [12], [13].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$ (µm)</th>
<th>$\chi^{(3)}$ $V^{-2}m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polydiacetylene</td>
<td>0.602</td>
<td>3.6(-9)</td>
</tr>
<tr>
<td>Poly-4-BCMU</td>
<td>0.585</td>
<td>3.6(-9)</td>
</tr>
<tr>
<td>Polythiophene</td>
<td>0.602</td>
<td>9.0(-9)</td>
</tr>
<tr>
<td>Polyacetylene</td>
<td>1.060</td>
<td>3.6(-9)</td>
</tr>
<tr>
<td>Polyacetylene</td>
<td>0.602</td>
<td>9.0(-9)</td>
</tr>
<tr>
<td>Polysilane</td>
<td>1.064</td>
<td>1.35(-11)</td>
</tr>
<tr>
<td>Polygermane</td>
<td>1.064</td>
<td>1.02(-10)</td>
</tr>
<tr>
<td>Poly(PBNSPhs)</td>
<td>1.064</td>
<td>4.68(-11)</td>
</tr>
<tr>
<td>Poly(PDN4S)</td>
<td>1.064</td>
<td>1.98(-11)</td>
</tr>
<tr>
<td>Poly(PDN14S)</td>
<td>1.064</td>
<td>4.68(-11)</td>
</tr>
<tr>
<td>silica</td>
<td>0.6328</td>
<td>4.68(-12)</td>
</tr>
<tr>
<td>fused silica</td>
<td>0.6328</td>
<td>2.1(-12)</td>
</tr>
</tbody>
</table>
2.3 SELF-INDUCED NONLINEAR EFFECTS

The third order polarizability corresponding to the third term of the right hand side of equation (2.2) can be written by assuming the fiber material to be isotropic and its nonlinear response to be dominated by the fast response of electronic processes.

\[ P^{(3)} = \varepsilon_0 \chi^{(3)} E \cdot E \] (2.4)

where the assumption that \( P \) and \( E \) are oscillating at the same frequency \( \omega \) is considered. Equations (2.2) and (2.4) show that the \( \chi^{(n)} \) is in general a fourth rank tensor which can be represented by 81 components in the three dimensional coordinate system. However, because of the isotropy of the nonlinear materials and other symmetry properties, the forth rank tensor can be reduced to a second rank tensor having 9 components and can be put in a \( 3 \times 3 \) matrix. For a weakly guiding fiber where the transverse fields components are large compared to the longitudinal components, the \( 3 \times 3 \) matrix is reduced to a \( 2 \times 2 \) matrix. By further assuming that the fiber is a polarization-maintaining one, it can be shown that the susceptibility is only a function of a single field \([3], \ and \ [8] \). The nonlinear dielectric constant under the third order Kerr effect is given for homogeneous isotropic medium by:

\[ \varepsilon = \varepsilon_{bg} + \alpha |E|^2 \] (2.5)
where \( \varepsilon_{bg} \) is the background dielectric constant. It is in fact the linear part of the dielectric constant obtained at weak fields. The parameter \( \alpha \) is the nonlinear coefficient of the material and \( E \) is the applied electric field strength. It has in general three components: \( E_x, E_y \), and \( E_z \). The Kerr model is relatively simple to analyse and the problem with Kerr nonlinearity can be solved analytically for a slab waveguide for lossless media. However, the model given by equation (2.5) is not realistic. The nonlinear material always responds to the applied field in a limited way. At high applied electric field intensities saturation effects ensue. Moreover, high field intensities may cause complete breakdown of the dielectric material. Saturation effects suggest the use of more realistic model of the dielectric function. The model is Phenomenological and is based on the fact that the dielectric function cannot grow indefinitely but must level off at a maximum value that depends on the nature of the nonlinear material. Two forms have been proposed in [3] to account for saturation effects. They are given by the following expressions:

\[
\varepsilon = \varepsilon_{bg} + \frac{\alpha |E|^2}{1 + g \alpha |E|^2} \quad (2.6)
\]

\[
\varepsilon = \varepsilon_{bg} + \frac{1}{g} (1 - e^{-\alpha |E|^2}) \quad (2.7)
\]
where it is seen that the maximum increase in the dielectric function is seen to be equal to $\frac{1}{g}$ at very high field intensity. It must also be noted that for a weak electric field, equations (2.6) and (2.7) approximate the Kerr behavior.

The nonlinear effects have been widely used in the slab waveguide. There are important devices which have been implemented and tested in integrated optics such as directional coupler, Mach-Zender interferometers and prism coupler. These devices can be used in all-optical signal processing when at least one of the waveguide media exhibits an intensity independent refractive index.

2.4 PHYSICAL DESCRIPTION OF SELF-FOCUSING PHENOMENA

Self-focusing is an induced lens effect. It results from a wavefront distortion inflicted on the beam by itself while traversing a nonlinear medium. Consider a single mode laser beam with a Gaussian transverse profile propagating into a medium with refractive index $n$ given by $n = n_0 + \Delta n(|E|^2)$ where $\Delta n(|E|^2)$ is the optical-field-induced refractive index change. If $\Delta n$ is positive, the central part of the beam having a higher intensity will experience a larger refractive index than the edge and therefore travel at a slower velocity than the edge. Consequently, as the beam travels in the medium, the original plane wavefront gets progressively more distorted as shown in
fig. (2.1) [14]. The distortion is similar to that imposed on the beam by a positive lens. Since the optical ray propagation is in the direction perpendicular to the wavefront, the beam appears to focus by itself. However, a beam with a finite cross section should also diffract. Only when the self-focusing is greater than diffraction will the beam self-focus. The self-focusing is proportional to $\Delta n(|E|^2)$, while the diffraction action is inversely proportional to the square of the beam radius. Therefore, as the beam shrinks in self-focusing action the diffraction action become stronger. At some point, diffraction overcomes self-focusing and the beam after reaching a minimum cross section (the focal point) will diffract. The self trapping is a special case when both self-focusing action and diffraction action on the input beam just balance each other. It must be mentioned that self-defocusing effects also occur in third order nonlinear materials. These are associated with materials with negative third order nonlinearity.
Fig.(2.1) Distortion of the wavefront of a laser beam leading to self-focusing in the nonlinear medium [14].
CHAPTER 3

THEORETICAL STUDY OF MULTILAYER OPTICAL FIBER

3.1 INTRODUCTION

In this chapter Maxwell's equations are used to develop the general vector wave equation for the electric and magnetic fields. The simplified scalar wave equation is deduced from the vector wave equation under certain conditions. We always assume monochromatic field quantities. This means that the input and output frequencies are the same.

Although in the case of third order Kerr nonlinearity the interacting field components generate another field at the third harmonic of the input frequency [8], our interest in this thesis is concerned only with the fundamental frequency component. The basic theory of guided waves is developed for TE and TM guided modes as well as the fundamental $HE_{11}$ hybrid mode. The approach used in this research is based on the linearization of the wave equation. The linearization is achieved by dividing the nonlinear fiber into a large number of layers. Each layer is treated as a linear, isotropic and homogeneous medium.
The electromagnetic theory-in this chapter is developed using references [15]–[18]. The notations used for Bessel functions and their important properties are taken from [19] and [20].

3.2 MAXWELL’S EQUATIONS

The following four equations govern the propagation of electromagnetic fields in different media:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  \hspace{1cm} (3.1)

\[ \nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J} \]  \hspace{1cm} (3.2)

\[ \nabla \cdot \vec{B} = 0 \]  \hspace{1cm} (3.3)

\[ \nabla \cdot \vec{D} = \rho \]  \hspace{1cm} (3.4)

where \( \vec{E} \) and \( \vec{H} \) are the electric and magnetic fields respectively, \( \vec{B} \) is the magnetic flux density, \( \frac{\partial \vec{D}}{\partial t} \) is the displacement current density, \( \vec{J} \) is the conduction current density and \( \rho \) is the electric charge density. The bar on any quantity denotes a vector quantity.
Maxwell's equations can explain all macroscopic phenomenon in electromagnetic. There are four field vectors $\vec{E}, \vec{H}, \vec{B}, \vec{D}$, each of which has three components. Therefore, twelve scalar equations are needed to solve for all fields. These twelve equations are obtained by expanding the two vector curl equations and the two vector constitutive relations $\vec{D} = \varepsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$ which will be discussed in a subsequent section of this chapter.

Under monochromatic time dependence all field quantities vary as $e^{-i\omega t}$.

For vacuum and dielectric media both $\vec{J}$ and $\rho$ are zeros.

Therefore Maxwell's equations can be written as:

\[ \nabla \times \vec{E} = -i\omega \vec{B} \quad (3.5) \]

\[ \nabla \times \vec{H} = i\omega \vec{D} \quad (3.6) \]

\[ \nabla \cdot \vec{B} = 0 \quad (3.7) \]

\[ \nabla \cdot \vec{D} = 0 \quad (3.8) \]

Because Maxwell's equations are not all independent, the constitutive relations must be used for a complete description of electromagnetic fields. This should be accomplished by applying the proper boundary conditions which is the subject of the next section.
3.3 BOUNDARY CONDITIONS

To solve Maxwell's equations in different media we must apply the appropriate boundary conditions at the interface separating any two different media. These boundary conditions are as follows:

\[
\hat{n} \times (\overrightarrow{E}_1 - \overrightarrow{E}_2) = 0 \quad (3.9)
\]

\[
\hat{n} \times (\overrightarrow{H}_1 - \overrightarrow{H}_2) = \overrightarrow{J}_s \quad (3.10)
\]

\[
\hat{n} \cdot (\overrightarrow{B}_1 - \overrightarrow{B}_2) = 0 \quad (3.11)
\]

\[
\hat{n} \cdot (\overrightarrow{D}_1 - \overrightarrow{D}_2) = \rho_s \quad (3.12)
\]

where \( \hat{n} \) is the unit normal at the interface pointing outwards of medium 2. \( \rho_s \) is the surface charge density and \( J_s \) is the surface current density. They vanish for a source free medium.

Maxwell's equations do not include the properties of the medium under consideration. However, these properties are accounted for by the following constitutive relations:

\[
\overrightarrow{D} = \varepsilon \overrightarrow{E} \quad (3.13)
\]

\[
\overrightarrow{B} = \mu \overrightarrow{H} \quad (3.14)
\]
\[ \mathbf{J} = \sigma \mathbf{E} \]  \hspace{1cm} (3.15)

where \( \varepsilon \) is the permittivity, \( \mu \) is the magnetic permeability and \( \sigma \) is the conductivity. For nonmagnetic materials \( \mu \) is equal to \( \mu_0 \), the free space value.

The dielectric constant is written in the following form for nonmagnetic media:

\[ \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 n^2 \]  \hspace{1cm} (3.16)

where \( \varepsilon_r \) is the relative permittivity and \( n \) is the refractive index of the medium. The refractive index is real for a nonabsorbing (lossless) medium and complex for an absorbing or a lossy medium. For such lossy medium the dielectric constant and the refractive index are written as

\[ \varepsilon_r = \varepsilon_1 + i\varepsilon_2 = n^2 \]  \hspace{1cm} (3.17)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the real and the imaginary parts of the dielectric constant and \( n \) is the complex refractive index.

When the constitutive relations are substituted in Maxwell’s equations, the following forms are obtained:
\[ \nabla \times \vec{E} = -i\omega \mu_0 \vec{H} \]  
(3.18)

\[ \nabla \times \vec{H} = i\omega \varepsilon_0 n^2 \vec{E} \]  
(3.19)

\[ \nabla \cdot \vec{H} = 0 \]  
(3.20)

\[ \nabla \cdot \vec{E} = 0 \]  
(3.21)

These equations are valid for a nonmagnetic, isotropic, linear and source-free medium. Throughout this work, the nonlinear wave equation will be locally linearized by using a multi-layer structure. All layers share the same properties of the medium except for the value of the dielectric constant which is different for each layer. It is thus possible to use the known linear solution of the above Maxwell’s equations for each layer.

### 3.4 THE VECTOR WAVE EQUATION

To derive the vector wave equation, either \( \vec{E} \) or \( \vec{H} \) must be eliminated from the above 4 equations. By applying the curl operator to equation (3.18) and using equation (3.19) the following equation is obtained:

\[ \nabla^2 \vec{E} + \nabla(\vec{E} \cdot \nabla \ln n^2) + k_0^2 n^2 \vec{E} = 0 \]  
(3.22)

where the vector identity:
\[ \nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \]  
(3.23)

has been used and \( k_0 = \left( \omega^2 \mu_0 \epsilon_0 \right)^{\frac{1}{2}} \) is the free space wave number.

Equation (3.22) is the vector wave equation for the \( \vec{E} \) field. In a similar manner the vector wave equation for the \( \vec{H} \) field can be derived as:

\[ \nabla^2 \vec{H} + \nabla (\ln n^2) \times (\nabla \times \vec{H}) + k_0^2 n^2 \vec{H} = 0 \]  
(3.24)

These two vector wave equations are valid for any coordinate system. In a nonlinear medium the permittivity is a function of the electric field and hence it is also a function of space coordinates. When the multilayer approach is used, the dielectric function is evaluated locally at each layer and treated as constant for the entire layer. Under this condition the second terms of equations (3.22) and (3.24) vanish and the two vector wave equations for both \( \vec{E} \) and \( \vec{H} \) fields reduce to:

\[ \nabla^2 \vec{E} + k_0^2 n^2 \vec{E} = 0 \]  
(3.25)

\[ \nabla^2 \vec{H} + k_0^2 n^2 \vec{H} = 0 \]  
(3.26)

In cylindrical coordinates the vector Laplacian operator \( \nabla^2 \) appearing in equations (3.25) and (3.26) is given by:
\[ \tilde{\nabla}^2 A = \left[ \nabla^2 A_r - \frac{2}{r^2} \frac{\partial A_r \partial \phi}{\partial \phi} - \frac{A_r}{r^2} \right] \hat{r} + \left[ \nabla^2 A_\phi + \frac{2}{r^2} \frac{\partial A_\phi \partial \phi}{\partial \phi} - \frac{A_\phi}{r^2} \right] \hat{\phi} \]
\[ + \nabla^2 A_z \hat{z} \]  

(3.27)

where we introduce the bar in the left hand side to distinguish between the vector Laplace operator and the scalar Laplace operator on the right hand side of the equation. The scalar operator is given by:

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \]  

(3.28)

Therefore, the z-component of the two vector wave equations (3.25) and (3.26) reduces to the same scalar form given by:

\[ \nabla^2 \psi + k^2 n^2 \psi = 0 \]  

(3.29)

where \( \psi \) is either \( E_z \) or \( H_z \). The variation in the z-direction is taken as \( e^{i \beta z} \), where \( \beta \) is the propagation constant. Equation (3.29) now reduces to the form:

\[ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} + (k_e n_z^2 - \beta^2) \psi = 0 \]  

(3.30)

The solution of equation (3.30) is sought in the form:
\[ \Psi(r, \phi) = R(r)e^{\imath \phi} \]  

(3.31)

where \( \nu \) is an integer representing the azimuthal periodicity of the field solution. Substituting equation (3.31) into equation (3.30) the following equation is obtained:

\[ \frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + \left( k_0^2 n^2 - \beta^2 - \frac{\nu^2}{r^2} \right) R = 0 \]  

(3.32)

Defining the parameter \( \gamma \) as:

\[ \gamma^2 = k_0^2 n^2 - \beta^2 \]  

(3.33)

Equation (3.32) is the standard Bessel differential equation. The solution of this equation is given by:

\[ R(r) = A J_{\nu}(\gamma r) + B Y_{\nu}(\gamma r) \]  

(3.34)

where \( J_{\nu} \) and \( Y_{\nu} \) are Bessel functions of the first and the second kind respectively and \( A \) and \( B \) are constants to be determined from the boundary conditions. Using Maxwell’s equations, the transverse field components are expressed in terms of the axial components as:

\[ E_r = -\frac{1}{\gamma^2} \left( \frac{\nu \omega \mu_0}{r} H_z - i\beta \frac{\partial E_z}{\partial r} \right) \]  

(3.35)
\[ E_\varphi = -\frac{1}{\gamma^2} (i\omega \mu_0 \frac{\partial H_z}{\partial r} + \beta \frac{E_z}{r}) \quad (3.36) \]

\[ H_r = \frac{1}{\gamma^2} \left( \frac{\nu n^2 \varepsilon_\varphi \omega}{r} E_z + i\beta \frac{\partial H_z}{\partial r} \right) \quad (3.37) \]

\[ H_\varphi = \frac{1}{\gamma^2} \left( i\nu n^2 \varepsilon_\varphi \omega \frac{\partial E_z}{\partial r} - \frac{\nu \beta}{r} H_z \right) \quad (3.38) \]

For a multilayer structure equations (3.34)-(3.38) are valid for each layer where A,B and n are replaced by the appropriate values corresponding to that layer.

The above analysis is general and can be used to cover different types of polarization, namely the transverse electric (TE), the transverse magnetic (TM) and the hybrid modes. In the next sections each of the above cases is studied and the corresponding eigenvalue equation is derived.

**3.5 TE EIGENVALUE EQUATION**

For TE waves the electric field \( \vec{E} = (0, E_\varphi, 0) \) has only one field component which is transverse and the magnetic field \( \vec{H} = (H_r, 0, H_\varphi) \) has one axial component and one transverse component. Also the value of \( \nu \) for TE waves is zero which means that the fields do not have azimuthal dependence.
Fig. (3.1) shows the multilayer structure under investigation.

For any arbitrary layer, (except for the innermost layer), the field components are written in the form:

\[ H_z^l = A_lJ_0(\gamma_l r) + B_l Y_0(\gamma_l r) \quad d_{k-1} \leq r \leq d_l \quad (3.39) \]

\[ E_p^l = \frac{i\omega \mu_0}{\gamma_l} (A_lJ_1(\gamma_l r) + B_l Y_1(\gamma_l r)) \quad d_{k-1} \leq r \leq d_l \quad (3.40) \]

\[ H_r^l = -\frac{j\beta}{\gamma_l} (A_lJ_1(\gamma_l r) + B_l Y_1(\gamma_l r)) \quad d_{k-1} \leq r \leq d_l \quad (3.41) \]

where \( \gamma_l \) is given by

\[ \gamma_l = \sqrt{\varepsilon_l k_0^2 - \beta^2} \quad (3.42) \]

To evaluate \( \varepsilon \), we define the general nonlinear dielectric function as:

\[ \varepsilon(r,E) = \varepsilon_{bg}(r) + \frac{\alpha |E|^2}{1 + g\alpha |E|^2} \quad (3.43) \]

where \( \varepsilon_{bg}(r) \) is the field-independent part of \( \varepsilon \). The values of the constants \( \alpha \) and \( g \) depend on the dielectric material. The above representation of the dielectric function gives a maximum change of \( \varepsilon \) equal to \( \frac{1}{g} \). The term \( |E|^2 \)
Fig.(3.1) The multi-layer structure in a cylindrical waveguide.
is given by

$$|E|^2 = |E_z|^2 + |E_r|^2 + |E_\varphi|^2$$  \hspace{1cm} (3.44)

The technique for calculating the dielectric constant for any layer is based on evaluating the electric field at the inner boundary of that layer and then calculate the dielectric constant using equation (3.43). This can be represented mathematically as:

$$\varepsilon_l = \varepsilon_{bg} + \frac{\alpha |E_{l-1}|^2}{1 + g\alpha |E_{l-1}|^2}$$  \hspace{1cm} (3.45)

where $E_{l-1}$ is the electric field for the $(l-1)^{th}$ layer evaluated at $r = d_l$. For TE waves, there is only one electric field component $E_\varphi$ contributing to the dielectric function.

It is well known that the standard Bessel differential equation (equation (3.32)) can be solved in terms of J and Y functions as given in equation (3.33) or in terms of the modified Bessel functions I and K as follows

$$R(r) = Cl_\nu(i\gamma r) + DK_\nu(i\gamma r)$$  \hspace{1cm} (3.46)

where I and K are the modified Bessel functions of the first and the second kind respectively.
The recursive relations are expressed in terms of $J$ and $Y$ functions for TE guided waves. These recursive relations are used to derive the effective index and the modal fields of a given TE mode of the nonlinear optical fiber. The $I$ and $K$ functions are also used in the TM case.

To start the recursive scheme, the dielectric constant must be evaluated for the most inner layer (layer 1). The fields components for the first layer are given by:

$$H_z^1 = A_1 J_0(\gamma_1 r) \quad 0 \leq r \leq d_1 \quad (3.47)$$

$$E_{\varphi}^1 = \frac{i \omega \mu_0}{\gamma_1} A_1 J_1(\gamma_1 r) \quad 0 \leq r \leq d_1 \quad (3.48)$$

$$H_r^1 = -\frac{i \beta}{\gamma_1} A_1 J_1(\gamma_1 r) \quad 0 \leq r \leq d_1 \quad (3.49)$$

where the coefficient $B_1$ has been set equal to zero because the $Y$ function is infinite at the origin. The electric field of equation (3.48) is evaluated at the axis of the fiber and substituted into equation (2.45) to obtain the value of $\epsilon_1$.

The value of this field is zero since $J_1(\gamma r)$ vanishes at $r = 0$. This is also necessary to insure the continuity of $E_{\varphi}$ at $r = 0$. The evaluated value of the dielectric constant $\epsilon_1$ is used to calculate the field at position $r = d_1$. which
will in turn be used to evaluate $\varepsilon_2$, the dielectric constant of layer 2 and so on.

Equations (3.39)-(3.41) can be repeated for the $(l+1)^{th}$ layer as follows:

$$H_{z}^{l+1} = A_{l+1}J_{0}(\gamma_{l+1}r) + B_{l+1}Y_{0}(\gamma_{l+1}r) \quad d_{l} \leq r \leq d_{l+1} \quad (3.50)$$

$$E_{\phi}^{l+1} = \frac{i\omega\mu_0}{\gamma_{l+1}} \left( A_{l+1}J_{1}(\gamma_{l+1}r) + B_{l+1}Y_{1}(\gamma_{l+1}r) \right) \quad d_{l} \leq r \leq d_{l+1} \quad (3.51)$$

$$H_{r}^{l+1} = \frac{-i\beta}{\gamma_{l+1}} \left( A_{l+1}J_{1}(\gamma_{l+1}r) + B_{l+1}Y_{1}(\gamma_{l+1}r) \right) \quad d_{l} \leq r \leq d_{l+1} \quad (3.52)$$

At the interface $r = d_{l}$, the continuous tangential fields $H_{z}$ and $E_{\phi}$ are matched on both sides of the interface in order to express the field amplitudes $A_{l+1}$ and $B_{l+1}$ in terms of $A_{l}$ and $B_{l}$. In so doing, the following two equations are obtained

$$A_{l}J_{0}(\gamma_{l}d_{l}) + B_{l}Y_{0}(\gamma_{l}d_{l}) = A_{l+1}J_{0}(\gamma_{l+1}d_{l}) + B_{l+1}Y_{0}(\gamma_{l+1}d_{l}) \quad (3.53)$$

$$\frac{1}{\gamma_{l}} \left( A_{l}J_{1}(\gamma_{l}d_{l}) + B_{l}Y_{1}(\gamma_{l}d_{l}) \right) = \frac{1}{\gamma_{l+1}} \left( A_{l+1}J_{1}(\gamma_{l+1}d_{l}) + B_{l+1}Y_{1}(\gamma_{l+1}d_{l}) \right) \quad (3.54)$$

Equations (3.53) and (3.54) are now solved for $A_{l+1}$ and $B_{l+1}$ in terms of $A_{l}$ and $B_{l}$. The two recurrence relations for the amplitude coefficients are given by:
\[ A_{i+1} = \frac{C_1}{C_2} A_i + \frac{C_3}{C_2} B_i \]  

(3.55)

where \( C_1, C_2 \) and \( C_3 \) are given by:

\[ C_1 = \frac{\gamma_{i+1}}{\gamma_i} J_1(\gamma_i d_i) Y_0(\gamma_{i+1} d_i) - J_0(\gamma_i d_i) Y_1(\gamma_{i+1} d_i) \]

\[ C_2 = J_1(\gamma_{i+1} d_i) Y_0(\gamma_{i+1} d_i) - J_0(\gamma_{i+1} d_i) Y_1(\gamma_{i+1} d_i) \]

\[ C_3 = \frac{\gamma_{i+1}}{\gamma_i} Y_1(\gamma_i d_i) Y_0(\gamma_{i+1} d_i) - Y_0(\gamma_i d_i) Y_1(\gamma_{i+1} d_i) \]

After determining \( A_{i+1} \) using equation (3.55). \( B_{i+1} \) is obtained from equation (3.53):

\[ B_{i+1} = \frac{1}{Y_0(\gamma_{i+1} d_i)} \left( A_i J_0(\gamma_i d_i) + B_i Y_0(\gamma_i d_i) - A_{i+1} J_0(\gamma_{i+1} d_i) \right) \]

(3.56)

Considering an N-layers fiber surrounded by an unlimited cladding as shown in fig.(3.1), the eigenvalue equation is obtained by writing the fields of the last layer (layer N) as:

\[ H_z^N = A_N J_0(\gamma_N r) + B_N Y_0(\gamma_N r) \quad \text{for} \quad d_{N-1} \leq r \leq d_N \]  

(3.57)
\[ E_\psi^N = \frac{i\omega \mu_0}{\gamma_N} \left( A_N J_1(\gamma_N r) + B_N Y_1(\gamma_N r) \right) \quad d_{N-1} \leq r \leq d_N \]  

(3.58)

For the open cladding region the fields should be a decaying function of \( r \).
This is satisfied by the modified Bessel function of the second kind (the \( K \) function). The tangential fields are given by:

\[ H_z^c = A_c K_0(\gamma_c r) \quad d_N \leq r \leq \infty \]  

(3.59)

\[ E_\psi^c = \frac{i\omega \mu_0}{\gamma_c} A_c K_1(\gamma_c r) \quad d_N \leq r \leq \infty \]  

(3.60)

The subscript \( c \) denotes the cladding. Equating the tangential fields \( H_z \) and \( E_\psi \) at the cladding interface \( r = d_N \), the required eigenvalue equation is obtained as follows:

\[ A_N J_0(\gamma_N d_N) + B_N Y_0(\gamma_N d_N) = A_c K_0(\gamma_c d_N) \quad r = d_N \]  

(3.61)

\[ \frac{1}{\gamma_N} \left( A_N J_1(\gamma_N d_N) + B_N Y_1(\gamma_N d_N) \right) = -\frac{1}{\gamma_c} A_c K_1(\gamma_c d_N) \]  

at \( r = d_N \)  

(3.62)

where \( \gamma_c \) is given by:

\[ \gamma_c = \left( \beta^2 - n_c^2 k_0^2 \right)^{1/2} \]  

(3.63)
where \( n_e \) is the cladding refractive index. Eliminating \( A_e \), the following equation is obtained

\[
K_e(\gamma_c d_N)\left[A_N J_1(\gamma_N d_N) + B_N Y_1(\gamma_N d_N)\right] + \frac{\gamma_N}{\gamma_c} K_1(\gamma_c d_N) \\
\left[A_N J_0(\gamma_N d_N) + B_N Y_0(\gamma_N d_N)\right] = 0 \quad \text{at} \quad r = d_N \tag{3.64}
\]

Equation (3.64) is based on assuming a given axial electric field amplitude at the axis of the fiber and then, propagating this field recursively through the fiber by expressing the field amplitudes of any layer in terms of the amplitudes of the previous layer. At \( r = d_N \), the tangential fields are matched at the core cladding interface to obtain this equation. The only unknown in this equation is the propagation constant \( \beta \) or equivalently the mode effective index \( n_e \). Equation (3.64) can be solved numerically using a zero finding routine which can handle both real and complex functions. In our computer work, the routine based on Muller's method is used \([21]\).

It must be noted that our recursive scheme is similar to that reported by C. Yeh and G. Lindgren in 1977 \([22]\) and applied to a linear fiber. The two schemes are based on stratifying the fiber into a large number of layers. The latter utilizes matrix operations for eliminating all field coefficients and can handle any linear arbitrary refractive index profile. Our scheme is however applicable to linear as well as nonlinear fibers. For the nonlinear case, we
use the electric field intensity at the boundary of the innermost layer \((r=d_1)\) as a parameter. The propagation constant in this case, will be a function of the electric field at \(r=d_1\).

### 3.6 TM EIGENVALUE EQUATION

The field components associated with the TM polarization are \(E_z, E_r\), and \(H_\theta\). There are therefore, two electric field components contributing to the dielectric function of the nonlinear medium given by equation (3.43). The analysis of the TM case is carried out in a manner similar to that used for the TE guided waves. Modified Bessel functions are used in region \(r>d_1\) for the description of the field components. The tangential field components \(E_z\) and \(H_\theta\) are expressed for the different layers as:

\[
E_z^1 = A_1 J_\gamma (\gamma_1 r) \quad 0 \leq r \leq d_1 \quad (3.65)
\]

\[
E_z^I = A/I_\gamma (\gamma r) + B/K_\gamma (\gamma r) \quad d_{N-1} \leq r \leq d_1 \quad (3.66)
\]

\[
E_z^c = A_c K_\gamma (\gamma_c r) \quad d_N \leq r \leq \infty \quad (3.67)
\]

The parameters \(\gamma_1\) and \(\gamma_c\) are the same as those given in equations (3.42) and (3.63) respectively. While \(\gamma_i\) is given by:
\[ \gamma_l = \left( \beta^2 - \kappa_n^2 n_n^2 \right)^{\frac{1}{2}} \]  

(3.68)

Also the tangential magnetic field \( H_\phi \) is given by:

\[ H^1_\phi = \frac{-ie_\circ n_1^2 \omega}{\gamma_1} A_1 J_1(\gamma_1 r) \quad 0 \leq r \leq d_1 \]  

(3.69)

\[ H^l_\phi = \frac{ie_\circ n_1^2 \omega}{\gamma_1} \left( A_1 J_1(\gamma_f r) - B K_1(\gamma_f r) \right) \quad d_1 \leq r \leq d_l \]  

(3.70)

\[ H^c_\phi = \frac{-ie_\circ n_c^2 \omega}{\gamma_c} A_c K_1(\gamma_c r) \quad d_c \leq r \leq \infty \]  

(3.71)

The radial component of electric field \( E_r \) is written as:

\[ E^1_r = \frac{-i\beta}{\gamma_1} A_1 J_1(\gamma_1 r) \quad 0 \leq r \leq d_1 \]  

(3.72)

\[ E^l_r = \frac{-i\beta}{\gamma_l} r \left( A_1 J_1(\gamma_f r) - B K_1(\gamma_f r) \right) \quad d_1 \leq r \leq d_l \]  

(3.73)

\[ E^c_r = \frac{-i\beta}{\gamma_c} A_c K_1(\gamma_c r) \quad d_c \leq r \leq \infty \]  

(3.74)

To start the recursive scheme, the dielectric function must be evaluated for the first layer. By examining the two electric field components at the axis of
the fiber, it is clear that $E_r$ vanishes at $r = 0$ and $E_z$ is maximum at $r = 0$ and equal to $A_1$. Therefore, $\varepsilon_1$ is given by:

$$
\varepsilon_1 = \varepsilon_{bg} + \frac{\alpha |A_1|^2}{1 + g\alpha |A_1|^2}
$$

$0 \leq r \leq d_1$ (3.75)

Following the same procedure used in the TE case and matching the tangential field components $E_z$ and $H_z$, we derive the recurrence relations at an arbitrary interface ($r = d_i$). These relations are given by:

$$
A_{i+1} = \frac{X_1}{X_2} A_i + \frac{X_3}{X_2} B_i
$$

(3.76)

where $X_1, X_2, X_3$ are given as follows:

$$
X_1 = \frac{\gamma_{i+1}}{\gamma_i} \frac{\varepsilon_i}{\varepsilon_{i+1}} l_1(\gamma_i d_i) K_0(\gamma_{i+1} d_i) + l_0(\gamma_i d_i) K_1(\gamma_{i+1} d_i)
$$

$$
X_2 = l_1(\gamma_{i+1} d_i) K_0(\gamma_{i+1} d_i) - l_0(\gamma_{i+1} d_i) K_1(\gamma_{i+1} d_i)
$$

$$
X_3 = K_0(\gamma_i d_i) K_1(\gamma_{i+1} d_i) - \frac{\gamma_{i+1}}{\gamma_i} \frac{\varepsilon_i}{\varepsilon_{i+1}} K_1(\gamma_i d_i) K_0(\gamma_{i+1} d_i)
$$

Using equation (3.76) for $A_{i+1}$, the amplitude coefficient $B_{i+1}$ is then evaluated.
It is given by:

\[
B_{i+1} = \frac{1}{K_0(\gamma_{i+1}d_i)} \left( A_j I_0(\gamma j d_i) + B_j K_0(\gamma j d_i) - A_{i+1} I_0(\gamma_{i+1}d_i) \right)
\]

(3.77)

The boundary conditions are applied at \( r = d_n \) to obtain the following equation:

\[
\frac{\varepsilon_N \gamma_c}{\varepsilon_c \gamma_N} K_0(\gamma_c d_N) [A_N I_1(\gamma_N d_N) - B_N K_1(\gamma_N d_N)] + K_1(\gamma_c d_N) \left[ A_N' I_0(\gamma_N d_N) + B_N K_0(\gamma_N d_N) \right] = 0 \quad \text{at } r = d_N \tag{3.78}
\]

After assuming the amplitude of the electric field \( E_z \) at the axis of the optical fiber, the only unknown in equation (3.78) is the effective index \( n_e \).

Therefore, it is considered as the TM eigenvalue equation for an N-layer fiber. If the fiber is linear and weak guidance is assumed, equations (3.63) and (3.78) will result in almost the same eigenvalues. In this case the TE modes and their corresponding TM modes become almost degenerate. However, for the nonlinear fiber the eigenvalues depend on the field intensity as well as the refractive index difference.
3.7 FUNDAMENTAL $HE_{11}$ EIGENVALUE EQUATION

The derivation of the eigenvalue equation for the hybrid modes requires a complete solution of Maxwell’s equations assuming that both $E_z$ and $H_z$ exist. The other four field components $E_r, H_r, E_r$, and $H_r$ are evaluated as usual from equations (3.35)-(3.38). The strategy for solving the problem is based on assuming different coefficients for $E_z$ and $H_z$ at $r=0$. The next step is to find the recurrence relations of all field coefficients in terms of these two coefficients. The last step is to relate the coefficient of $H_z$ to that of $E_z$ through the computer program. The evaluation of $\varepsilon_1$ for the innermost layer is discussed in the end of this section. The axial electric field components in the different layers are given by:

$$E^1_z = A_1 J_1(\gamma_1 r) \cos\varphi \quad 0 \leq r \leq d_1 \quad (3.79)$$

$$E^l_z = (A_1 J_1(\gamma_l r) + B K_1(\gamma_l r)) \cos\varphi \quad d_1 \leq r \leq d_l \quad (3.80)$$

$$E^c_z = A_c K_1(\gamma_c r) \cos\varphi \quad d_l \leq r \leq \infty \quad (3.81)$$

and the axial magnetic field components are given as follows:

$$H^1_z = C_1 J_1(\gamma_1 r) \sin\varphi \quad 0 \leq r \leq d_1 \quad (3.82)$$
\[ H_z^l = (C_1(\gamma r) + D K_1(\gamma r)) \sin \varphi \quad d_{l-1} \leq r \leq d_1 \quad (3.83) \]

\[ H_z^c = C_c K_1(\gamma cr) \sin \varphi \quad d_N \leq r \leq \infty \quad (3.84) \]

The other field components \( E_\varphi, H_\varphi \) and \( E_r \) are given for the different layers by:

\[ E_\varphi^l = -\frac{i}{\gamma_1^2} \left( -\frac{\beta}{r} A_1 J_1(\gamma_1 r) - \omega \mu_0 C_1 \gamma_1 J_1'(\gamma_1 r) \right) \sin \varphi \quad 0 \leq r \leq d_1 \quad (3.85) \]

\[ E_\varphi^l = i \frac{\beta}{\gamma_1^2} r \left[ A_1 J_1(\gamma r) - B K_1(\gamma r) \right] \sin \varphi + i \omega \frac{\mu_0}{\gamma_1} \left[ C_1' J_1(\gamma r) + D K_1'(\gamma r) \right] \sin \varphi \quad d_{l-1} \leq r \leq d_1 \quad (3.86) \]

\[ E_\varphi^c = -\frac{i}{\gamma_c^2} \left( -\frac{\beta}{r} A_c K_1(\gamma_c r) - \omega \mu_0 C_c \gamma_c K_1'(\gamma_c r) \right) \sin \varphi \quad d_N \leq r \leq \infty \quad (3.87) \]

\[ H_\varphi^l = -\frac{i}{\gamma_1^2} \left( \frac{\beta}{r} C_1 J_1(\gamma_1 r) + \omega \mu_0 n_1^2 C_1 J_1'(\gamma_1 r) \right) \cos \varphi \quad 0 \leq r \leq d_1 \quad (3.88) \]
\[ H_\phi' = -i \frac{\beta}{\gamma_i^2 r} \left[ C_{l_1}(\gamma r) + D K_{l_1}(\gamma r) \right] \cos \varphi + i \omega \frac{n_i^2}{\gamma_i} d_{l_1 < r < d_1} \] (3.89)

\[ H_\phi^c = -i \frac{\beta}{\gamma_c^2} \left[ \frac{C_c}{r} K_{l_1}(\gamma_c r) + \omega \frac{n_c^2}{\gamma_c} A_c K_{l_1}(\gamma_c r) \right] \sin \varphi d_{N < r < \infty} \] (3.90)

\[ E_r^i = -i \frac{\beta}{\gamma_i^2} \left[ A_{l_1}'(\gamma r) + B K_{l_1}'(\gamma r) \right] \cos \varphi + \frac{i \omega \mu_i}{r \gamma_i^2} \left[ C_{l_1}(\gamma r) + D K_{l_1}(\gamma r) \right] \cos \varphi d_{l_1 < r < d_1} \] (3.91)

\[ E_r^l = -i \frac{\beta}{\gamma_i} \left[ A_{l_1}'(\gamma r) + B K_{l_1}'(\gamma r) \right] \cos \varphi + i \omega \frac{\mu_i}{r \gamma_i^2} \] (3.92)

\[ E_r^c = -i \frac{\beta}{\gamma_c^2} \left[ \beta \gamma_c A_c K_{l_1}(\gamma_c r) + \omega \frac{n_c^2}{\gamma_c} C_c K_{l_1}(\gamma_c r) \right] \sin \varphi d_{N < r < \infty} \] (3.93)

\[ H_r^i = i \frac{\beta}{\gamma_i^2} \left[ \beta \gamma_i C_{l_1} J_{l_1}(\gamma_i r) + \omega \frac{\mu_i}{r \gamma_i^2} n_i^2 A_{l_1} J_{l_1}(\gamma_i r) \right] \cos \varphi 0 < r < d_1 \] (3.94)
\[ H'_r = -i \frac{B}{\gamma_i} \left[ C_{1}'(\gamma r) + D K_{1}'(\gamma r) \right] \cos \varphi + \frac{i \omega \epsilon_0 n^2}{r \gamma_i^2} \left[ A_{1}'(\gamma r) + B K_{1}'(\gamma r) \right] \cos \varphi \quad d_{i-1} \leq r \leq d_i \quad (3.95) \]

\[ H^c_r = -i \frac{1}{\gamma_c^2} \left\{ \beta \gamma_c C_{c} K_{1}'(\gamma_c r) + \frac{\omega \epsilon_0 n_c^2}{r} A_c K_{1}(\gamma_c r) \right\} \sin \varphi \quad d_{i+1} \leq r \leq \infty \quad (3.96) \]

In equations (3.85)-(3.96) the prime on Bessel functions denotes a differentiation with respect to the argument. After writing the field components for the different layers, the next step is to derive the recurrence relations for the amplitude coefficients and the eigenvalue equation. In order to simplify the notation of the next section the abbreviation \( \Psi_x(m) \), is introduced where \( \Psi \) is used to denote any field component while \( x \) denotes the coordinate of that field component and \( m \) indicates that this field is evaluated for the \( m^{th} \) layer at a distance \( r = d_m \) from the axis of the fiber. Choosing an arbitrary interface \( r = d_i \) and matching the four tangential fields components \( E_z, H_z, E_\phi \) and \( H_\phi \), we obtain the four recurrence relations which relate the coefficients of the \((l+1)^{th}\) layer to those of the \( l^{th} \) layer as follows:

\[ A_{l+1} = \frac{X_1}{X_2} \quad \text{at} \quad r = d_i \quad (3.97) \]
where $X_1$ and $X_2$ are given by:

$$
X_1 = i \gamma_1^2 H_y(l) K_1(\gamma_{l+1} d_l) - \omega \varepsilon_0 n_{l+1}^2 \gamma_{l+1} E_z(l) K_1'(\gamma_{l+1} d_l) \\
\quad - \frac{\beta}{d_l} H_z(l) K_1(\gamma_{l+1} d_l)
$$

$$
X_2 = \omega \varepsilon_0 n_{l+1}^2 \gamma_{l+1} \left( l_1'(\gamma_{l+1} d_l) K_1(\gamma_{l+1} d_l) - l_1(\gamma_{l+1} d_l) K_1'(\gamma_{l+1} d_l) \right)
$$

After determining $A_{l+1}$, the second electric field coefficient $B_{l+1}$ is obtained from the following equation

$$
B_{l+1} = \frac{1}{K_1(\gamma_{l+1} d_l)} \left( E_z(l) - A_{l+1} l_1(\gamma_{l+1} d_l) \right)
$$

(3.98)

The magnetic field coefficients are determined in a similar way as:

$$
C_{l+1} = \frac{X_3}{X_3} \quad \text{at } r = d_l
$$

(3.99)

where $X_3$ and $X_4$ are given by:

$$
X_3 = i \gamma_1^2 E_y(l) K_1(\gamma_{l+1} d_l) + \omega \mu_0 \gamma_{l+1} H_z(l) K_1'(\gamma_{l+1} d_l) \\
\quad + \frac{\beta}{d_l} E_z(l) K_1(\gamma_{l+1} d_l)
$$
\[ X_4 = -\omega \mu_0 \gamma_{l+1} \left( l_1 \gamma_{l+1} d_l \right) K_1(\gamma_{l+1} d_l) - l_1(\gamma_{l+1} d_l) K_1(\gamma_{l+1} d_l) \]

After determining \( C_{l+1} \), the second magnetic field coefficient \( D_{l+1} \) is found to be:

\[ D_{l+1} = \frac{1}{K_1(\gamma_{l+1} d_l)} \left( H_2(l) - C_{l+1} l_1(\gamma_{l+1} d_l) \right) \quad (3.100) \]

For the N-layer fiber shown in fig.(3.1), matching the azimuthal tangential field components \( E_\phi \) and \( H_\phi \) at the core/cladding interface results in the following two equations:

\[ E_\phi(N) + \frac{i}{\gamma_c^2} \left( -\frac{\beta}{d_N} E_\phi(N) - \omega \mu_0 \gamma_c H_2(N) \frac{K_1(\gamma_c d_N)}{K_1(\gamma_c d_N)} \right) = 0 \quad (3.101) \]

\[ H_\phi(N) + \frac{i}{\gamma_c^2} \left( \frac{\beta}{d_N} H_\phi(N) + \omega \varepsilon_0 n_c^2 \gamma_c E_\phi(N) \frac{K_1(\gamma_c d_N)}{K_1(\gamma_c d_N)} \right) = 0 \quad (3.102) \]

It is to be noted that equations (3.101) and (3.102) must give the same eigenvalue if the ratio of the axial electric field to the axial magnetic field coefficients, \( A_1/C_1 \) assumed for the first layer is correct. To start the recursive scheme, an initial value is taken to be the same as that of the
weakly guiding fiber namely, $\frac{\eta_0}{n_1}$, where $\eta_0$ is the free space intrinsic impedance equals to $120\pi$ and $n_1$ is the core refractive index.

The first run will produce two different eigenvalues. By changing the ratio $\frac{A_1}{C_1}$ in small increments, the two eigenvalues can be made as close to each other as desired until the proper ratio $\frac{A_1}{C_1}$ corresponding to the solution is captured. It is thus seen that the recursive scheme for the $HE_{11}$ mode, not only solves the effective index but also results in the proper electric to magnetic field ratio.

In order to start the recursive scheme for the case of nonlinear core and linear cladding, the dielectric constant of the innermost layer $\varepsilon_1$ must be evaluated at the axis of the fiber. By examining the three electric field components contributing to the dielectric function, the following value for electric field intensity is obtained at $r = 0$

$$|E_\theta|^2 = |E_r|^2 + |E_\phi|^2 = \left(\frac{\beta A_1}{\gamma_1}\right)^2 + \left(\frac{\gamma_0 C_1}{\gamma_1}\right)^2 \quad r=0 \quad (3.103).$$

It is noted that $E_\phi$ vanishes at $r = 0$. The dielectric constant $\varepsilon_1$ is found
from equation (3.45) as

$$
\varepsilon_1 = \varepsilon_{bg} + \frac{\alpha |E_\theta|^2}{1 + g\alpha |E_\theta|^2} \\
0 \leq r \leq d_1 \quad (3.104)
$$

It must be noted that $\varepsilon_1$ is also embedded in $\gamma_1$. After substituting and arranging the terms, the following quadratic equation is obtained:

$$
\gamma_1^2 \varepsilon_1 (\gamma_1^2 + F_1) = \gamma_1^2 \varepsilon_{bg} (\gamma_1^2 + F_1) + F_2 \quad (3.105)
$$

Where $F_1, F_2$ and $\gamma_1$ are given by:

$$
F_1 = g\alpha \left[ \left( \frac{\beta A_1}{\gamma_1} \right)^2 + \left( \frac{\omega \mu_s C_1}{\gamma_1} \right)^2 \right]
$$

$$
F_2 = \alpha \left[ \left( \frac{\beta A_1}{\gamma_1} \right)^2 + \left( \frac{\omega \mu_s C_1}{\gamma_1} \right)^2 \right]
$$

$$
\gamma_1 = (\varepsilon_1 k_0^2 - \beta^2)^{1/2}
$$

By solving this quadratic equation, the value of $\gamma_1$ is found. This value is necessary for starting the recursive scheme. To check the results obtained using the multilayer scheme, another numerical scheme is to be used. This
is the self-consistent scheme which is represented in the next section.

3.8 THE SELF-CONSISTENT SOLUTION NUMERICAL SCHEME

Because the solution of the nonlinear fiber adopted in this thesis is purely numerical, it is necessary to use another numerical scheme to compare the results. It is also of interest to show the advantages and disadvantages of each numerical scheme. The convergence and computational time are two important factors in comparing the two schemes.

The numerical scheme presented here is based on the method of the self-consistent solution of the nonlinear wave equation. This scheme has been reported by Fedrico Dois et al in 1989 [23]. It makes use of the self-consistent nature of the electromagnetic fields. The application of this method requires first the solution of the linear wave equation. The eigen field produced by this solution is taken to modify the dielectric function of the nonlinear fiber through the relation

$$\varepsilon(r) = \varepsilon_{bg} + \frac{\alpha |E(r)|^2}{1 + g\alpha |E(r)|^2} \quad (3.106)$$

This results in a linear graded index profile which can be solved by a suitable multilayer scheme. The eigen field of the graded index fiber is then taken to produce another graded index profile which can be solved linearly. After
a certain number of iterations; the field distribution and the effective index are stabilized because of the self-consistent nature of the field. The final field is taken as the solution of the nonlinear problem. The method is simple and straightforward, but it suffers from some drawbacks. From the point of view of computational time the scheme requires much more time than the recursive scheme. The ratio between the two computation times is roughly equal to the number of iterations required for the self-consistent scheme to converge.

The convergence in the recursive scheme depends mainly on the number of layers. We found that 100 layers give good convergence (six significant digits) in most cases. The self-consistent routine requires at least 10 iterations for convergence in most cases. When the dispersion relation is multivalued (interface intensity versus mode index), the convergence of the self-consistent scheme is slow in the negative slope region. Because there are three values of $n_e$ sharing the same intensity value, The routine sometimes fails to converge to the required value of $n_e$ (located in the negative slope region) which is the closest solution to the input field. To overcome this difficulty the increment between intensity values should be very small. Another difficulty arises when the fiber with nonlinear core and linear cladding is cut-off at low power ($r_{bg} < r_c$). For such a case there is no linear solution corresponding to the cut-off fiber to start with. However, the
dispersion curve in this case can be traced from the saturable solution corresponding to \( c_i = \varepsilon_{bg} + \frac{1}{g} \). The turning points on the dispersion curve, when the slope changes its sign, are also difficult to obtain.

In each run of the self-consistent routine, the effective index and the corresponding field distribution are obtained for a certain value of surface intensity. If this field profile is not stored it cannot be retrieved from knowledge of the effective index and surface intensity. In our recursive scheme, knowing a point on the dispersion relation curve is enough to obtain its corresponding field profile. The recursive scheme is also superior in the case of multivalued solution. The reason for this comes from the fact that the recursive scheme is governed by the value of the input field at the fiber axis which produces a unique value of \( n_0 \).
CHAPTER 4

DISPERSION CHARACTERISTICS OF METAL-CLAD OPTICAL FIBER

4.1 INTRODUCTION

Having developed the mathematical formulation of the recursive scheme in chapter 3 and checked it in the solution of eigenvalue equation for different types of lossless fiber structures consisting of two or three layers, the scheme can be applied to more complicated structures containing a metal as the outer layer. These structures are of importance particularly because of their attenuation that leads to application in mode filtering and the design of polarizers.

The fiber configurations that will be studied are: 1) hollow metallic step-index metal-clad fiber, 2) step index metal-clad fiber with a buffer layer between the core and the metal cladding and 3) a four-layer structure which has a very thin buffer layer separating the cladding layer form the metal cladding. The first structure is covered in section (4.5.1). Section (4.5.2) is devoted to the second three-layer structure. The four layer structure which exhibits a thin dielectric buffer layer between the cladding and the outer
metallic jacket is investigated in section (4.5.3). The two important cases of a low-index buffer and a high-index buffer will be considered.

For the two layer step-index metal-clad fiber, the TE and the TM lowest order modes are to be investigated as well as the hybrid $HE_{11}$ mode. However, for the three and four layer structures the study essentially concentrates on the TE and the TM modes because the main practical objective of this investigation is to use the waveguide as a polarizer or a mode filter. Also, there are some basic results reported for the planar metal-clad waveguide with structures similar to those mentioned for the fiber. We expect the cylindrical structure to behave in some respects the same as the planar waveguide specially when the fiber radius is large enough compared to the wavelength. When the radius of the cylindrical waveguide becomes very large it can be treated as a planar waveguide.

The hybrid mode $HE_{11}$ is to be studied also in a multilayer structure for a special case of a single TE mode fiber. This fiber also supports $TM_{01}$ as well as $HE_{11}$. In so doing, it is possible to compare the attenuation of the lowest order modes having different polarizations. This is the subject of section (4.5.4) of this chapter.

In the next sub-section (4.2), some of the important optical properties of metals are discussed. The investigation of metal-clad slab and cylindrical
waveguides will follow in sections (4.3) and (4.4) respectively.

4.2 OPTICAL PROPERTIES OF METALS

The permittivity of a lossy dielectric or a metal can be represented by:

$$\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \left( \varepsilon' + \frac{i\sigma}{\omega \varepsilon_0} \right) = \varepsilon_0 \left( \varepsilon' + i \varepsilon'' \right)$$

(4.1)

where $\sigma$ is conductivity, $\varepsilon_r$ is the relative permittivity and $\varepsilon'$ and $\varepsilon''$ are the real and imaginary parts of $\varepsilon_r$ respectively. The refractive index is defined in terms of the relative permittivity as:

$$n = \frac{1}{\varepsilon_r} = n' + i n''$$

(4.2)

where $n'$ and $n''$ are the real and the imaginary parts of $n$. The extinction factor of the metal is represented by $n''$. The values of the dielectric constant and refractive index for some of the most commonly used metals are listed in table 4.1 at $\lambda = 0.6328 \mu m$ [24]. It is clear from the table that most of the metals exhibit a permittivity with a negative real part at the optical frequencies. The factor $n''$ compared to $n'$ determines how lossy the metal is. It is shown in [24] that the three-layer structure consisting of a dielectric substrate, a dielectric core and a metal cladding can support guided modes when the metal has a permittivity with a positive real part,
### TABLE 4.1

The relative permittivities and refractive indices of some important metals at a wavelength = 0.6328 μm [24].

<table>
<thead>
<tr>
<th>Metal</th>
<th>$\varepsilon'$</th>
<th>$\varepsilon''$</th>
<th>$n'$</th>
<th>$n''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>-47.0</td>
<td>16.3</td>
<td>1.172</td>
<td>6.955</td>
</tr>
<tr>
<td>Copper</td>
<td>-10.0</td>
<td>1.0</td>
<td>0.158</td>
<td>3.166</td>
</tr>
<tr>
<td>Gold</td>
<td>-10.3</td>
<td>1.0</td>
<td>0.156</td>
<td>3.213</td>
</tr>
<tr>
<td>Silver</td>
<td>-16.3</td>
<td>0.51</td>
<td>0.063</td>
<td>4.04</td>
</tr>
<tr>
<td>Chromium</td>
<td>5.07</td>
<td>14.9</td>
<td>3.226</td>
<td>2.309</td>
</tr>
<tr>
<td>Germanium</td>
<td>17.4</td>
<td>15.3</td>
<td>4.504</td>
<td>1.698</td>
</tr>
</tbody>
</table>
under certain conditions. However, in this chapter only metals having permittivities with negative real parts are to be considered. This will enable guidance to occur with no restrictions on the metal used.

### 4.3 PLANAR METAL-CLAD WAVEGUIDE

For the metal-clad slab waveguide the extensive reported work covers different structures involving the metal as the outer layer. The three-layer waveguide consisting of a dielectric core, a metal cladding and a dielectric or air substrate exhibits a strong attenuation difference between TE and TM modes. The TM modes are attenuated about one order of magnitude greater than the TE modes. Also the higher order modes are strongly attenuated in comparison to lower order modes. The choice of the metal affects the attenuation characteristics. Metals with a large value of $n''$ lead to modes that have higher attenuation factors than those with a small value of $n''$. Mode attenuation can also be increased by reducing the guide thickness.

A technique to reduce the guided-mode attenuation is obtained by introducing a thick buffer layer between the core layer and the metallic cladding. For this case the guided energy absorbed by the metal is decreased because of the buffer layer. The thickness of the buffer layer controls the amount of attenuation for the waveguide. It should be chosen in such a way that the metal has a considerable effect on guided modes within
the core region. Reduction in the attenuation of the three-layer waveguide is also obtainable by increasing the waveguide thickness.

One important waveguide structure was studied by Yamamoto et al. in 1978 [25]. The waveguide is shown in fig. (4.1.a). The thickness of the buffer layer is very small compared to waveguide thickness and its refractive index is less than the guide indices $n_1$ and $n_2$. The attenuation of this waveguide depends on four factors, namely the waveguide thickness, the buffer layer thickness, the refractive index of each layer and the type of metal used as a cladding. Some of these factors, for instance, the guide thickness and the type of metal used have been discussed in the above sections. However, the thickness of the buffer layer has a great importance in the attenuation process. It will be studied for slab waveguide in the next paragraph and for the fibers in section 4.5.3.

It has been shown in [25] that the attenuation of the $TM_0$ guided mode exhibits an absorption peak at a specific thickness of the buffer layer. This feature is important in the design of TE-TM mode filter with a high extinction ratio. Focusing attention on the $TE_0$ and $TM_0$ modes, at the absorption peak, the TM losses are about three orders of magnitude greater than the TE losses. This is shown in fig.(4.1.b). The losses of the TE modes are decreasing with the increase of the buffer layer thickness. Also, higher order
Fig.(4.1)
a) Multilayer metal-clad planar waveguide
b) Variation of attenuation factor with buffer thickness [25].
modes have higher attenuation than lower order modes. Another four-layer structure different from that of reference [25] was reported by Jamid and Al-Bader in 1987 [26]. The waveguide structure uses a high-index buffer-layer instead of the low-index buffer used in reference [25]. It has been shown that this high-index buffer structure operates in dual mode as that of [25] and in fact discriminates against TE polarization.

4.4 METAL-CLAD OPTICAL FIBER

The reported work on metal-clad optical fibers is much less than that on planar optical waveguides. The reasons for this is that the optical fiber are normally used for long distance transmission with lossless materials used for core and cladding. Lossy jackets are kept at appreciable distance from the fiber axis for the purpose of loss reduction.

Most of the reported work on three-layer fiber structures treats the third layer (jacket) as a lossy dielectric [27]–[30]. The attenuation produced by a lossy dielectric is much less than that of a real metal, and therefore, perturbation analysis can be used to find the attenuation characteristics of these fiber structures [27] and [28].

The picture is completely different if the metal-clad fiber is to be used for specific applications involving mode filtering and the designing of TM or TE polarizers [31]–[32]. In this case, a real metal with considerable attenuation
properties must be used as a direct cladding or outer jacket. In such a case, the metallic jacket should be close enough to the core to influence the propagational characteristics of the waveguide. A complete exact solution of the guided modes must be obtained for the multilayer cylindrical waveguide without applying any approximation such as perturbation theory.

In 1989 Blok and Gorbulsy studied the hollow metallic waveguide [33]. They approximated the metal permittivity by a negative real number and considered a model having the following parameters: \( \varepsilon_{c0} = 1, \varepsilon_{cl} = -1 \) and \( \lambda = 2\pi a \) where \( a \) is the core radius. The dispersion characteristics for the proposed model have been obtained as a relation between the real propagation constant \( \beta \) and the fiber \( V \) number defined as:

\[
V = ak_0 (n_1^2 - n_2^2)^{1/2}
\]

(4.3)

Where \( a \) is the core radius, \( k_0 \) is the free space wavenumber, \( n_1 \) is the core refractive index and \( n_2 \) is the cladding refractive index. The attenuation characteristics for such waveguide structure cannot be studied in this approximation. A real metal should be assumed which has both a negative real part and an imaginary part. It has been shown in [33] that the hybrid mode \( HE_{11} \) for this hypothetical structure has a cut-off frequency different from zero. This is contrary to the case of a dielectric-clad fiber where the
fundamental mode $HE_{11}$ has a zero cutoff frequency. The fundamental mode in [33] is the $TM_{01}$ mode which has the lowest cutoff frequency. Both the hybrid mode $HE_{11}$ and the $TE_{01}$ mode have the same cutoff frequency which is higher than that of $TM_{01}$. Also the cutoff values of the guided modes are shifted towards the higher frequency region (higher V number).

The three-layer radially stratified metal-clad optical fiber has also been studied by Charles Y. H. et al. in 1989 [34]. The eigenvalue equation for this structure is derived and solved numerically for TM and TE modes. This investigation facilitates the solution of D-shaped metal-clad fiber which can be reduced under conformal mapping transformation to the three layer cylindrical waveguide. The results obtained in this investigation show that for the D-shaped metal-clad fiber the attenuation of TM modes is greater than the attenuation of TE modes while the effective index of a certain TE mode is greater than that of the corresponding TM mode. These conclusions are common in both the planar and the cylindrical waveguides.

4.5 NUMERICAL RESULTS AND CONCLUSIONS

In this section, all the attenuation characteristics of metal-clad cylindrical waveguide are developed. The section is divided into three sub-sections covering two-layer, three-layer and four-layers metal-clad cylindrical waveguide. The different waveguide structures are shown in fig.(4.2)
Fig.(4.2)  

a) Two-layer metal-clad optical fiber

b) Three-layer metal-clad optical fiber

c) Four-layer metal-clad optical fiber
4.5.1 Dielectric-Metal Cylindrical Waveguide

The first cylindrical structure investigated is shown in fig.(4.2.a). The waveguide consists of dielectric core surrounded by a metal cladding. The waveguide parameters are as follows: \( n_1^2 = (1.51)^2 \), \( \lambda = 0.6328 \mu m \), \( a = 6 \mu m \) and \( n_2^2 = -10.3 + j1 \) (gold).

The dispersion and attenuation characteristics of this waveguide for fundamental TE, TM and HE modes have been studied. The results obtained by solving the direct eigenvalue equation and by using the recursive scheme are identical. These values are shown in table 4.2. The reason for this consistence arises from the fact that the recursive scheme indirectly generates the same eigenvalue of linear fiber regardless of the number of layers used. Fig.(4.3) shows a plot of the effective index versus the fiber radius. It can be seen that the fundamental mode is \( HE_{11} \). Unlike the dielectric-clad fiber, the \( HE_{11} \) mode has a nonzero finite cut-off frequency. Another observation drawn from fig.(4.3) is the shift of cut-off frequencies of both \( TE_{01} \) and \( TM_{01} \) towards the lower frequency range (smaller core radius). For example, if the metal-clad of fig.(4.2.a) is replaced by a dielectric with \( n_2^2 = (1.50)^2 \), the resultant cutoff core radius of both \( TE_{01} \) and \( TM_{01} \) is 1.394 \( \mu m \), which is equivalent to a V number equal to 2.405. It can be shown from
**TABLE 4.2**

The effective indices of the $TE_{01}$, $TM_{01}$, and $HE_{11}$ modes in a two-layer metal-clad fiber, obtained from the direct solution of the eigenvalue equation and the recursive scheme.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Solution</th>
<th>$n_e'$</th>
<th>$n_e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{01}$</td>
<td>recursive scheme</td>
<td>1.508654</td>
<td>0.497621(-6)</td>
</tr>
<tr>
<td>$TE_{01}$</td>
<td>eigenvalue equation</td>
<td>1.508654</td>
<td>0.497621(-6)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>recursive scheme</td>
<td>1.508581</td>
<td>0.358786(-5)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>eigenvalue equation</td>
<td>1.508581</td>
<td>0.358786(-5)</td>
</tr>
<tr>
<td>$HE_{11}$</td>
<td>recursive scheme</td>
<td>1.509456</td>
<td>0.806624(-6)</td>
</tr>
<tr>
<td>$HE_{11}$</td>
<td>eigenvalue equation</td>
<td>1.509456</td>
<td>0.806624(-6)</td>
</tr>
</tbody>
</table>
fig.(4.3) that these values for a metal-clad fiber are 0.22 μm and 0.27 μm for the \(TE_{01}\) and the \(TM_{01}\) respectively. The third conclusion that can be drawn from fig.(4.3), is that the effective index for the three investigated modes has an asymptotic value equal to the core index value as the fiber radius becomes very large. This feature is similar to that of the dielectric-clad fiber. One of the above conclusions has been obtained in [33], which is the nonzero cutoff frequency of the \(HE_{11}\) mode. All other conclusions contradict the results obtained in [33]. As has been mentioned in the previous section they reported in [33] that the fundamental mode is \(TM_{01}\) and the cutoff values move towards higher frequency region. Also the dispersion characteristic obtained in [33], shows that the effective index can exceed the core index value for all modes. The reason for these discrepancies is the use of hypothetical model in [33] which does not represent a real metal.

The attenuation characteristics of the dielectric-metal cylindrical waveguide are shown in fig.(4.4) as a plot of \(n_\varepsilon''\) versus the core radius. The actual attenuation coefficient is defined as \(\alpha = k_0 n_\varepsilon''\). All field components are attenuated with rate \(e^{-\alpha z}\). However the attenuation coefficient for power is \(2\alpha\) since the power is proportional to the square of electric or magnetic field. Throughout the work of this chapter the attenuation will be represented by \(n_\varepsilon''\) which is considered as the normalized attenuation factor.

It is seen from fig.(4.4) that the attenuation of the \(TM_{01}\) mode is larger than
Fig. (4.4) Variation of $n_e''$ with fiber radius
that corresponding to $TE_{01}$ mode by about an order of magnitude. The $HE_{11}$
mode has a moderate attenuation which is greater than that of the $TE_{01}$ mode
and smaller than that of the $TM_{01}$ mode. In addition the attenuation is a
decreasing function of the fiber radius. The interaction between the metallic
cladding and the guided field inside the core, decreases as the radius
becomes larger.

4.5.2 THE THREE-LAYER METAL-CLAD FIBER

The waveguide is shown in fig.(4.2.b). The metal is used as a jacket
surrounding the dielectric cladding. The waveguide parameters are as
follows: $n_1^2 = (1.51)^2$, $n_2^2 = (1.50)^2$, $n_3^2 = -10.3 + j1$ (gold), $\lambda = 0.6328 \, \mu m$, $a = 6 \, \mu m$ and $b = 7.2 \, \mu m$.

The modes of this waveguide have in general a smaller attenuation
compared to the corresponding modes of the previous section. The reason
for this reduction in mode attenuation is the reduced interaction between the
guided field (which resides mostly in the core) and the metallic layer. A
parameter $c = b/a$ has been defined in the description of the performance of
the structure. There are two limiting cases of this parameter. The first
one for $c = 1$, the fiber is reduced to the two layer cylindrical waveguide of
section (4.4.1). The second limiting value is $c = \infty$ and the fiber in this case
reduces to dielectric step-index fiber. In practice $c$ should not be much
greater than 1 in order that the influence of metal on guided modes be of appreciable magnitude. The value of c has been chosen to be 1.2 in the study of the three-layer cylindrical waveguide. The direct eigenvalue equation derived by Unger [15] has been solved numerically for the above fiber parameters. Solution of the problem has also been obtained by using the recursive scheme. Table 4.3 lists the values of the effective index of the two solutions for both $TE_{01}$ and $TM_{01}$ modes. It is seen from table 4.3 that the two methods of solution exhibit agreement to six significant digits. The recursive scheme generates indirectly the same closed form eigenvalue equation.

4.5.3 THE FOUR-LAYER STRUCTURE

It must be stated from the outset that the objective of the study of threelayer structure shown in fig.(4.2.b) is in order to check our recursive scheme against the solution of the exact eigenvalue equation. Confidence in the recursive scheme enables us to apply it in the solution of more important waveguide structures. The four-layer waveguide shown in fig.(4.2.c) contains a thin dielectric buffer layer between the cladding and the metallic layer. An eigenvalue equation can be derived for this four layers structure.

However, the effort involved is large. The three-layer structure can, however, be represented by $8 \times 8$ matrix [15]. The representation of the four-layer structure requires a $12 \times 12$ matrix containing different kinds of
TABLE 4.3

The effective indices of the $TE_{01}$ and $TM_{01}$ modes in a three-layer metal-clad fiber, obtained from the direct solution of the eigenvalue equation and the recursive scheme.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Solution</th>
<th>$n_e'$</th>
<th>$n_e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{01}$</td>
<td>recursive scheme</td>
<td>1.508867</td>
<td>0.240928(-7)</td>
</tr>
<tr>
<td>$TE_{01}$</td>
<td>eigenvalue equation</td>
<td>1.508867</td>
<td>0.240928(-7)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>recursive scheme</td>
<td>1.508860</td>
<td>0.282922(-6)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>eigenvalue equation</td>
<td>1.508860</td>
<td>0.282922(-6)</td>
</tr>
</tbody>
</table>
Bessel functions. Also, numerical techniques are required to solve the resulting $12 \times 12$ determinant to obtain the eigenvalues of the structure.

The recursive scheme adopted here relies on the elimination of field coefficients for each layer. As we have seen in chapter 3, this reduces the mathematical effort to dealing with $2 \times 2$ matrix operations for TE and TM modes and a $4 \times 4$ matrix operations for the hybrid modes irrespective of the number of layers of the waveguide structure. It must be noted that the same recursive scheme can be used to solve inhomogeneous fiber problems as well. In such a case, the inhomogeneous index profile is used to generate the values of $\epsilon$ for each layer.

As indicated in the case of the slab waveguide, the factors involving in the guided mode attenuation are many. The most important one for the four-layer structure is the buffer layer thickness. For this reason all waveguide parameters are going to be fixed and only the thickness of the buffer layer will be varied in order that its effect on the attenuation characteristics be indicated.

4.5.3.1 THE FOUR-LAYER STRUCTURE WITH A LOW-INDEX BUFFER LAYER

The waveguide model is shown in fig.(4.2.c). The metal used is gold and the fiber parameters are as follows: $n_1^2 = (1.51)^2$, $n_2^2 = (1.50)^2$. 
\[ n_3^2 = (1.30)^2, \quad n_4^2 = -10.3 + j1(gold), \quad a = 6 \mu m, \quad c = 1.2 \text{ and } \lambda = 0.6328 \mu m. \]

The mode attenuation represented by the imaginary part of the effective index \( n_e'' \) is plotted versus the buffer-layer thickness for the three lowest order TE modes, see fig.(4.5). It is maximum for zero buffer thickness and decreases with increasing buffer-layer thickness. However, the TM modes for the same waveguide structure have a different attenuation characteristics. These attenuation characteristics are shown in fig.(4.6) as a plot of \( n_e'' \) versus the buffer thickness. Each mode exhibits an absorption peak at a specific critical buffer layer thickness. This absorption peak is caused by resonance coupling to the lossy TM surface mode. At these absorption peaks the attenuation of the TM modes is about 2-3 orders of magnitude greater than the TE modes. In such cases the extinction ratio will be large enough to design a good TE pass polarizer. Numerical values of extinction ratio for different fiber configurations, with the corresponding transmission losses will be given at the end of this chapter.

It must be noted that the type of metal used affects the amount of attenuation for both TE and TM modes. For example, if the gold (which has been used in our investigation) is replaced by Aluminium the attenuation will increase because Aluminium has permittivity with a relatively large imaginary part as shown in table 4.1. However, the general characteristics of figs.(4.5)
Fig. 4.5: Variation of n with buffer-layer thickness for TE modes.
FIG. 4.6 Variation of \( n^0 \) with buffer-layer thickness for TM modes.
and (4.6) will remain unchanged. The attenuation plots will in this case exhibit a shift of the attenuation peak to higher values. Another important factor in designing polarizers is the insertion loss which should be minimized to have an efficient polarizer. The insertion loss is mainly the sum of three losses, two of them are due to the coupling at the input and output sections of the polarizer and the third one is the transmission loss through the length of the polarizer. The coupling losses are generally difficult to calculate. The insertion loss is usually measured experimentally. The coupling losses can be minimized when the polarizer used is of in-line type. This type of polarizer will be explained in some details at the end of this chapter. Therefore, the transmission loss can only be considered. As a matter of fact the TE normalized attenuation factor at the TM absorption peaks are between $10^{-5}$ and $10^{6}$ which enable the TE waves to move a relatively large distance through the guide before being substantially attenuated. Table 4.4 shows the values of $n_3$ for the first three lowest order TM modes at the absorption peaks and the corresponding TE values.

4.5.3.2 THE FOUR-LAYER STRUCTURE WITH A HIGH-INDEX BUFFER LAYER

The model used for this case is similar to the low-index buffer case except the value $n_3$ is now $(1.7)^2$. For this waveguide structure the attenuation of both TE and TM waves exhibit an oscillating behavior over the
**TABLE 4.4**

The effective indices of the three lowest order TM modes at the absorbing peaks and the corresponding TE values for the same low-buffer thickness in a four-layer waveguide structure.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta$ ($\mu m$)</th>
<th>$n_e'$</th>
<th>$n_e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TM_{01}$</td>
<td>0.105</td>
<td>1.508853</td>
<td>0.250770(-4)</td>
</tr>
<tr>
<td>$TM_{02}$</td>
<td>0.110</td>
<td>1.506175</td>
<td>0.110831(-3)</td>
</tr>
<tr>
<td>$TM_{03}$</td>
<td>0.120</td>
<td>1.502076</td>
<td>0.373227(-3)</td>
</tr>
<tr>
<td>$TE_{01}$</td>
<td>0.105</td>
<td>1.508859</td>
<td>0.919616(-8)</td>
</tr>
<tr>
<td>$TE_{02}$</td>
<td>0.110</td>
<td>1.506198</td>
<td>0.427356(-7)</td>
</tr>
<tr>
<td>$TE_{03}$</td>
<td>0.120</td>
<td>1.502097</td>
<td>0.150879(-6)</td>
</tr>
</tbody>
</table>
range of buffer layer thickness investigated between 0 and 1 \( \mu m \). Fig.(4.7) illustrates this oscillating behavior for the \( TE_{01} \) and \( TM_{01} \) modes. There are two absorption peaks on each side of a minimum for this range of buffer-layer thickness. The important observation is that the peaks of the TE and the TM absorption are well separated. It is thus possible to discriminate against TE or TM polarization by the appropriate choice of the buffer-layer thickness.

This oscillating feature of the loss has been reported in [26] for the slab waveguide. In [26] the buffer-layer thickness has been varied to cover the first absorption peak of both \( TE_0 \) and \( TE_1 \) modes as well as the minimum absorption of the \( TM_0 \) mode. In our investigation the buffer thickness has been extended to include two maximum attenuation points on each side of a minimum attenuation point for both \( TE_{01} \) and \( TM_{01} \) modes.

It must be noted that the attenuation of the slab structure mentioned above is about two orders of magnitude greater than that of the cylindrical waveguide. The reason for this is the strong influence of the metal on the guided fields inside the thin film compared to the cylindrical waveguide. The attenuation behavior of higher order modes is similar to that of the first mode shown in fig.(4.7). Table 4.5 gives the values of \( n_e'' \) at the first two absorption peaks and the corresponding high-index buffer-thicknesses for the first three lowest order TE and TM modes.
FIG. 4.7 Variation of $n^0$ with buffer-layer thickness for TE$^0$ and TM$^0$ modes.
TABLE 4.5

The effective indices of the three lowest order TE and TM modes at the first two absorption peaks and the corresponding high buffer thickness in the four-layer waveguide structure.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta$ ((\mu)m)</th>
<th>$n_e'$</th>
<th>$n_e''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{01}$</td>
<td>0.200</td>
<td>1.50848</td>
<td>0.24561(-3)</td>
</tr>
<tr>
<td>$TE_{02}$</td>
<td>0.605</td>
<td>1.50865</td>
<td>0.10218(-3)</td>
</tr>
<tr>
<td>$TE_{02}$</td>
<td>0.195</td>
<td>1.50543</td>
<td>0.19482(-3)</td>
</tr>
<tr>
<td>$TE_{02}$</td>
<td>0.595</td>
<td>1.50671</td>
<td>0.11239(-3)</td>
</tr>
<tr>
<td>$TE_{03}$</td>
<td>0.185</td>
<td>1.50320</td>
<td>0.15340(-3)</td>
</tr>
<tr>
<td>$TE_{03}$</td>
<td>0.585</td>
<td>1.50340</td>
<td>0.15854(-3)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>0.320</td>
<td>1.50892</td>
<td>0.10736(-3)</td>
</tr>
<tr>
<td>$TM_{01}$</td>
<td>0.725</td>
<td>1.50890</td>
<td>0.14218(-3)</td>
</tr>
<tr>
<td>$TM_{02}$</td>
<td>0.320</td>
<td>1.50589</td>
<td>0.16168(-3)</td>
</tr>
<tr>
<td>$TM_{02}$</td>
<td>0.715</td>
<td>1.50660</td>
<td>0.29195(-3)</td>
</tr>
<tr>
<td>$TM_{03}$</td>
<td>0.300</td>
<td>1.50286</td>
<td>0.26584(-3)</td>
</tr>
<tr>
<td>$TM_{03}$</td>
<td>0.700</td>
<td>1.50300</td>
<td>0.30735(-3)</td>
</tr>
</tbody>
</table>
4.5.4 THE FUNDAMENTAL HYBRID MODE IN THE FOUR-LAYER STRUCTURE

To complete the study of loss of the lowest order TE and TM modes it is important to study the influence of the buffer layer on the $HE_{11}$ mode. This mode is particularly important because it is the fundamental mode of the optical fiber. We have chosen a fiber radius of $3 \mu m$ which at $\lambda = 0.6328 \mu m$ allows the support of $TE_{01}$ and $TM_{01}$ modes in addition to the $HE_{11}$ mode.

For the case of low-index buffer, the $HE_{11}$ mode behaves in a manner similar to that of $TM_{01}$ mode. Only the attenuation of the hybrid mode is slightly less than that of the $TM_{01}$ mode. This result is illustrated in fig.(4.8). It is also clear that the attenuation of both $TM_{01}$ and $HE_{11}$ modes is much higher than of that of $TE_{01}$ mode. The attenuation difference reaches its maximum value when either $TM_{01}$ mode or $HE_{11}$ mode exhibit an absorption peak.

For the high-index buffer case, the attenuation factor ($n''$) is shown in fig.(4.9). The $HE_{11}$ exhibits an oscillating behavior similar to the TE and the TM modes. For a buffer thickness between 0.0 and 0.3 $\mu m$ the $HE_{11}$ has an absorption peak which is located very close to the absorption peak of the $TE_{01}$ mode. It can also be shown that there are some points of equal attenuation
FIG. 4.8: Variation of $n^e$ with buffer-layer thickness for $\text{HE}^{11}$, $\text{TE}^{01}$, and $\text{TM}^{01}$ modes.
Fig. 4.9 Variation of $n_e''$ with buffer-layer thickness for $HE_{11}$, $TE_{01}$, and $TM_{01}$ modes.
for different modes. For example, the $HE_{11}$ and the $TE_{01}$ modes have the same attenuation for some specific buffer thicknesses (0.05 $\mu$m and 0.26 $\mu$m).

The maximum attenuation occurs for The $TE_{01}$ at a buffer thickness ($\delta = 0.19 \, \mu m$). The imaginary part of effective index $n_e''$ for this maximum attenuation is 0.12380(-3) which means an equivalent loss of 106.77 db/cm at a wavelength of 0.6328 $\mu$m.

4.6 APPLICATIONS OF THE METAL-CLAD OPTICAL FIBER

The design of metal-clad fiber polarizers has received a large attention in the last few years [27]-[37]. The technique used in these references is essentially based on forming a metal-clad D-fiber. The polarizer is formed by grinding off the cladding on one side of the fiber and evaporating metals onto the polished surface. Figure (4.10) shows the cross-section and side view of such a fiber polarizer. It is called an in-line fiber polarizer because the metal-cladded area is formed on the main fiber. This helps to reduce the coupling losses. The structure shown in fig.(4.10) works as a polarizer by utilizing the attenuation difference between the $TM_{01}$ and $TE_{01}$. It can also work as a TM-pass polarizer by utilizing the cutoff of the TE mode. The operation of some polarizers is based on the interaction between $TM_{01}$ and the surface plasma waves inside the metallic coated region [40]-[42]. It is known that a single mode fiber can support two different perpendicular polarizations for
Fig.(4.10)
Structure of in-line D-fiber polarizer
a) cross-section       b) Side view [37].
the $HE_{11}$ mode. The D-fiber can easily separate these two polarizations by
the proper orientation of the flat edge of the D. This is important feature of
the D-fiber. However the recursive scheme adopted in this thesis is only
applied for circularly symmetric waveguide. In such a waveguide the two
$HE_{11}$ polarizations exist. The design of all the D-fiber polarizers depends on
experimental testing. There is no rigorous analytical solution for the D-fiber
configuration. The expected attenuation characteristics are taken
qualitatively from the similar slab structure. Our recursive scheme is
capable of solving different configurations of metal-clad optical fiber. This is
an advantage which facilitates the choice of polarizer parameters to achieve
certain design requirements. The extinction ratio and the transmission loss
for different fiber structures have been calculated and listed in table 4.6. The
extinction ratio is defined as the ratio between the attenuation of two different
modes. The polarizer length is taken to be 3 cm for calculating the
transmission loss. It is seen from table 4.6 that the low-index buffer
structure is only suitable for a TE-pass polarizer with a high extinction ratio
and a very low transmission loss. However, the high-index buffer structure
can work as a TE-pass or TM-pass polarizer.
TABLE 4.6

The Extinction ratio and transmission loss of a four-layer metal-clad fiber

a) Low-index buffer-layer

<table>
<thead>
<tr>
<th>Polarizer</th>
<th>$\delta$ (µm)</th>
<th>$n_e''$</th>
<th>E.R. (db)</th>
<th>T.L. (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE-pass</td>
<td>0.10</td>
<td>0.411779(-6)</td>
<td>53.776</td>
<td>1.065</td>
</tr>
</tbody>
</table>

b) High-index buffer-layer

<table>
<thead>
<tr>
<th>Polarizer</th>
<th>$\delta$ (µm)</th>
<th>$n_e''$</th>
<th>E.R. (db)</th>
<th>T.L. (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE&lt;sub&gt;01&lt;/sub&gt;-pass</td>
<td>0.30</td>
<td>0.197180(-5)</td>
<td>24.195</td>
<td>5.102</td>
</tr>
<tr>
<td>TM&lt;sub&gt;01&lt;/sub&gt;-pass</td>
<td>0.20</td>
<td>0.721767(-5)</td>
<td>9.265</td>
<td>3.823</td>
</tr>
<tr>
<td>HE&lt;sub&gt;11&lt;/sub&gt;-pass</td>
<td>0.10</td>
<td>0.147750(-5)</td>
<td>9.727</td>
<td>18.674</td>
</tr>
</tbody>
</table>
CHAPTER 5

TE AND TM NONLINEAR GUIDED WAVES IN
AN OPTICAL FIBER

5.1 INTRODUCTION

In this chapter and the next, the results on the behavior of the nonlinear guided modes in an optical fiber are given. The TE and TM nonlinear guided modes are presented in chapter 5 while the fundamental hybrid mode $HE_{11}$ is investigated in chapter 6. The present chapter is divided into two main parts covering the lowest order TE and TM modes respectively.

In sections 5.2 and 5.3, the TE nonlinear guided waves are investigated. Two important fiber geometries are considered. In the first the core is nonlinear while the cladding is linear and in the second the core is linear while the cladding is nonlinear. In all cases, only self-focusing materials are considered. For these materials, the nonlinear coefficient is positive which means that the refractive index of the nonlinear material increases with the applied power.

In the case of a the cutoff fiber, the fiber will not transmit guided power at
low power levels because the background core refractive index is less than or equal to the cladding refractive index. This case is investigated in section 5.2. Guidance for this fiber can only take place when the applied field intensity exceeds a certain threshold value. The interface (surface) intensity means the value of light intensity evaluated at the core-cladding interface. It will be used frequently in this chapter and the next chapter.

When the cladding is nonlinear, the behavior of guided waves is considerably changed. The number of modes which can be supported by the fiber at low power increases with the applied intensity. The characteristics of the guided waves can be modified for this fiber with the shifting of the peak value of the field towards the nonlinear cladding.

The second part of this chapter (sections 5.4 and 5.5) is dedicated to TM nonlinear guided waves. All the investigations made for TE modes are repeated for the TM case. For the TM waves, there are two field components, $E_z$ and $E_r$, contributing to the nonlinear dielectric constant while in the TE case only one electric field, $E_{ph}$, is affecting the dielectric constant.

All the results of this chapter have been obtained by the recursive scheme and checked using the self-consistent numerical scheme discussed in chapter 3. This is shown in section 5.6. The use of two numerical approaches is necessary because of the absence of analytical solution of the
problems at hand.

5.2 TE WAVES IN OPTICAL FIBERS WITH A NONLINEAR CORE AND A LINEAR CLADDING

The model used for this fiber structure is shown in fig.(5.1) where the nonlinear dielectric function is given by equation (2.43). Accordingly the dielectric function of the core material can be written as:

\[ \varepsilon_1 = \varepsilon_{bg} + \Delta \varepsilon_{NL} \]  

(5.1)

where \( \Delta \varepsilon_{NL} \) is the increment in the dielectric function due to the action of the field. This increment has a maximum value of \( \frac{1}{g} \) for very large field. Three different cases have been studied. These are the fiber with \( \varepsilon_{bg} > \varepsilon_2 \), the cutoff fiber with \( \varepsilon_{bg} = \varepsilon_2 \) and the cutoff fiber with \( \varepsilon_{bg} < \varepsilon_2 \).

The first case investigated is a fiber which supports guided modes at low power. The fiber parameters used in the numerical investigation are as follows: \( \varepsilon_{bg} = 2.255 \), \( \varepsilon_2 = 2.25 \), \( \alpha = 10^8 \text{ V}^2 \text{m}^2 \), \( g = 20 \), \( \lambda = 0.6328 \mu \text{m} \) and \( a = 4 \mu \text{m} \). The values of \( \alpha \) and \( \lambda \) are kept fixed at the above mentioned values for all investigations. The remaining parameters are changed for the individual cases of nonlinear optical fibers. For the present case, however, the fiber is made to be of the TE_{01} mode regardless of the amount of power...
Fig. (5.1) Structure of nonlinear cylindrical waveguide
applied. Fig.(5.2) illustrates the dispersion characteristics for this fiber structure. The mode index is plotted versus the the surface intensity $|E_y|^2$ at the core cladding interface. As seen from the figure, there are three distinct regions constituting the so called N curve. For low values of field intensity (represented here by the value at the core-cladding interface), there is only a single valued solution. There is one value of the mode index corresponding to one value of the interface intensity. As seen from the plot of fig.(5.2), the high intensity region has also a single valued solution. At intermediate values of surface intensity a multivalued solution is obtained. In this case a single value of surface intensity corresponds to three values of mode index.

The field profiles for interface value of $E_y$ equals to 410 V/m and corresponding to three mode index values are shown in figure (5.3). The plot shows the variation of $E_y$ across the fiber radius. The actual values of the electric field $E_y$ have been multiplied by a scaling factor SF to have the same value for all field maxima. This makes the comparison between field profiles more feasible. To retrieve the actual field profiles the plotted field values should be divided by the corresponding scaling factor SF. This method of comparing field profiles will frequently be used in this chapter and the next chapter. It is seen from fig.(5.3) that as the mode index increases, the field becomes better guided and confined in the core region. The
Fig. (5.2) Variation of $n_e$ with the interface intensity for TE mode.
Fig. (5.3) Three field plots corresponding to three values of \( n_e \) taken from fig. (5.2)

<table>
<thead>
<tr>
<th>( n_e )</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50087</td>
<td>7.622</td>
</tr>
<tr>
<td>1.51087</td>
<td>1.000</td>
</tr>
</tbody>
</table>
distributions of the dielectric functions corresponding to these values of mode index are shown in fig.(5.4). The dielectric constant increases gradually from the background value at \( r=0 \) to a maximum value inside the core. It is then decreases gradually to the linear cladding value.

The self-focusing action is clearly illustrated by the plot of fig.(5.3). As the applied intensity increases, the value of the refractive index increases and the peak intensity moves towards the center of the fiber where the refractive index has a maximum value. This means that the width of the light beam becomes smaller as the applied field intensity increases.

For very low intensity values, the fiber operates as a linear step-index fiber with \( \varepsilon_1 = \varepsilon_{bp} \). The value of the mode index of the \( TE_{01} \) mode obtained by the recursive scheme, has been checked by the direct solution of the eigenvalue equation:

\[
\frac{J_1(u)}{uJ_0(u)} + \frac{K_1(w)}{wK_0(w)} = 0 \tag{5.2}
\]

where \( u \) and \( w \) are defined as:

\[
u = a(n_{co}^2k_0^2 - \beta^2)^{\frac{1}{2}}, \quad w = a(\beta^2 - n_{cl}^2k_0^2)^{\frac{1}{2}}
\]

where \( a \) is the core radius and \( n_{co} \) and \( n_{cl} \) are the core and cladding refractive
Fig (5.4) Variation of the dielectric constant with the radial distance for three values of $n$. 

$\nu = 1.51067$

$\nu = 1.50499$

$\nu = 1.50097$
indices respectively. Values obtained by the two solutions are found to be in agreement to six significant digits as shown in table 5.1. At the high intensity end of the dispersion characteristics, the fiber becomes saturated and behaves as a linear step-index fiber with \( c_1 = c_{bg} + \Delta c_{NL,max} \). This can also be seen in fig.(5.5). The value of \( n_z \) for the saturated fiber is checked against the one obtained from the closed form eigenvalue equation. The two values are in very good agreement as shown in table 5.1. Fig.(5.5) shows two plots of the nonlinear dielectric constant distribution for very high and very low intensity values. The fiber in these cases acts as a linear step-index fiber.

It must be noted that the same nonlinear guided wave characteristics have been obtained for the planar waveguide in the case of a nonlinear film surrounded by a linear medium on both sides [1].

The second case is devoted to a fiber which is cutoff for guided modes at low power The fiber parameters for this model are the same as the previous case except \( c_{bg} \) is now changed to 2.25, thus making \( c_{bg} = \varepsilon_z \). In the linear sense, this fiber cannot support guided waves. In fact at low power level it resembles a uniform medium. However, if the core material is nonlinear and the applied field intensity exceeds a threshold value, the fiber can support nonlinear guided waves. To achieve this the recursive scheme starts by assuming a very low magnetic field \( (H_z(0)) \) value at the center of the
TABLE 5.1

Checking the values of $n_e$ in the linear limit of the recursive scheme against the direct solution of the closed form eigenvalue equation for $TE_{01}$ mode

| Structure       | Figure | Method               | $n_e$  | $A_{HE_s}(0)$ | $|E_s|^2$ |
|-----------------|--------|----------------------|--------|---------------|----------|
| $NL_{co} - L_{cl}$ | (5.2)  | recursive            | 1.500194 | 0.001    | 6.75     |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation  | 1.500194 |           |          |
| $NL_{co} - L_{cl}$ | (5.2)  | recursive            | 1.515730 | 100      | 68.42(3) |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation  | 1.515732 |           |          |
| $NL_{co} - L_{cl}$ | (5.6)  | recursive            | 1.514105 | 100      | 1.3(9)   |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation  | 1.514107 |           |          |
| $NL_{co} - L_{cl}$ | (5.7)  | recursive            | 1.512483 | 200      | 2.31(10) |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation  | 1.512484 |           |          |
| $L_{co} - NL_{cl}$ | (5.10) | recursive            | 1.506044 | 0.1      | 10797    |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation  | 1.506043 |           |          |
| $L_{co} - NL_{cl}$ | (5.15) | recursive            | 1.501447 | 0.01     | 296.9    |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation  | 1.501447 |           |          |
FIG. 6.5 (a) The dielectric constant distribution for two values of $n$.

Radial Distance $r$ (μm)

$-n_e = 1.51573$

$-n_e = 1.50019$
fiber. This will not probably lead to a solution of equation (3.64). By increasing the value of the magnetic field slowly, a solution of equation (3.64) is obtained when this magnetic field exceeds a threshold value.

The dispersion curve for this fiber is shown in fig.(5.6). The three operating regions discussed in section 5.2 in the case of the guiding fiber (fig.(5.2)) are also observed in this case. However, the low intensity region of fig.(5.2) is seen to be confined to a very small portion of the curve. There is a minimum value of intensity required to switch the fiber on. The mode index increases with the intensity at the core-cladding interface but soon the intensity undergoes a decrease with a further increase of the mode index.

The value of the mode index of a saturated fiber has been checked by the direct solution of the eigenvalue equation of the step-index fiber with \( \varepsilon_1 = \varepsilon_{bg} + \Delta \varepsilon_{NLmax} \). The results are compared in table 5.1. It must be noted that the fully saturable nonlinear fiber requires an infinitely large field. If the field is not large enough there will be some discrepancies between the eigenvalues obtained using the closed form eigenvalue equation and those obtained using the the recursive scheme.

The third case is for a fiber which is cutoff at low field intensity. For this fiber structure \( \varepsilon_{bg} \) is taken to be 2.245 and all other fiber parameters
FIG. 6.6 Variation of $n$ with the Interface Intensity for $TE_{01}$
remain the same as those used in previous case. The structure is cutoff in the linear sense. By increasing the applied power above a threshold value, the fiber will be turned on and the $TE_{01}$ mode is supported. The dispersion characteristic of this mode is illustrated in fig.(5.7). It is seen that there are two main regions identified in the dispersion curve, the low input power region (above the threshold value) and the high input power region. In the low power region, as the power exceeds the threshold to turn the fiber on, the resulting field distribution has a large value of intensity at the core-cladding interface but relatively low peak value. This is indeed a feature of fibers working close to cutoff. By increasing the input power the field becomes more confined in the core region, causing the interface intensity to decrease and the peak value to increase.

For the second portion of the dispersion curve where the fiber is approaching saturation, both the interface intensity and the peak field value increase with the input power. It must be noted that in all of the dispersion curves, the ratio of the peak value of intensity to the interface intensity increases with the mode index. The value of the mode index for saturable medium with $c_1 = c_{og} + \Delta c_{HL_{max}}$ has also been checked against the direct solution of the eigenvalue equation (5.2). The results are shown in table 5.1.

The dispersion of the cutoff nonlinear fiber as a relation between the total propagated power and the mode index $n_e$ has been studied for the first TE
FIG. 6.7 Variation of $\theta_0$ with the Interface Intensity for $TE_0$ ($r_c = 6\epsilon_r^0$)
mode for two different saturation levels. Fig.(5.8.a) shows this relation. The first portion of the dispersion curve has a negative slope which is considered unstable. However, the second portion has a positive slope. This portion is characterized by stable operation where \( n_e \) increases with the input power.

The two values for the saturation parameter \( g \) are taken to be 20 and 40. This makes a contribution to the dielectric constant equal to 0.05 and 0.025 respectively. The amount of power required to turn the fiber on is not the same for both levels of saturation. It can be seen from Fig.(5.8.a) that the high saturation level requires less power for the fiber to support guided modes. The power can be calculated from the integral of the Poynting vector as follows:

\[
P = \frac{1}{2} \text{Re} \int_{-\infty}^{\infty} E \times H^\dagger \; d\bar{S}
\]

(5.3)

where \( d\bar{S} \) is the element of area given in cylindrical coordinates by \( d\bar{S} = rd\varphi dr \). In the case of TE modes this expression reduces to

\[
P = -\frac{1}{2} \text{Re} \int_0^{2\pi} \int_0^\infty E_y H_x^* d\varphi dr
\]

(5.4)

The direction of propagation in equation (5.4) is the positive \( z \) direction. Figure (5.8.b) shows the dispersion relation as a plot of interface intensity...
Having two different saturation levels

\[ \text{Fig. 5.8.4 Variation of } n^2 \text{ with the total power for } T_{\text{e}0} \text{ mode in a waveguide} \]

Mode index

\[ \begin{align*}
\text{Power (W)} & \\
10^{-02} & \\
10^{-03} & \\
10^{-04} & \\
10^{-05} & \\
10^{-06} & \\
10^{-07} & \\
\end{align*} \]

\[ \theta = 20^\circ \quad \phi = 40^\circ \]
Fig. (5.8.2) Variation of \( I_{0} \) with the interface intensity for TE01 mode in a waveguide having two different saturation levels.

Intensity \( |E|^2 \) at \( r = a \) (V/m)^2

\[ \theta = \frac{\pi}{4} \]

\[ \theta = \frac{\pi}{2} \]
versus the mode index, for the same two saturation levels. As a matter of fact the threshold interface intensity depends on the difference between $\varepsilon_{bg}$ and $\varepsilon_2$ as well as the maximum level of saturation used.

The important application of the cutoff fiber is the limiting process where the intensity of light acts as a limiter to control the operation of the fiber. If this intensity is above the threshold value, the fiber is on and and supports guided modes. and if it is below the threshold the fiber is cutoff.

It is important to have an insight into the behavior of higher order modes. For this purpose the radius of the fiber has been increased to 5 $\mu m$. This radius can support the first three lowest order TE modes $TE_{01}$, $TE_{02}$ and $TE_{03}$ for high intensity values. The dispersion relation of this waveguide as a plot of power versus mode index is shown in fig.(5.9.a). It can be seen from this figure that the higher order modes require more power to be supported by the waveguide. However, the general behavior of the dispersion curve is unchanged. Supporting higher order modes depends on the way of excitation and coupling the fiber to the laser source. It is also of interest to show the field profiles and dielectric constant distributions for the three lowest order modes. Figure (5.9.b) illustrates three field profiles corresponding to an input magnetic field amplitude of 5.1 A/m. For this value of magnetic field the first mode $TE_{01}$ is approaching saturation while the third mode $TE_{03}$ is close to cutoff. The field profiles are seemed to be
Figure 5.4 (a): Variation of $n^2$ with the modal power for $TE_0^1$, $TE_0^2$, and $TE_0^3$ modes.
Fig. (5.9.d) Three field plots for $TE_{01}^0$, $TE_{02}^0$, and $TE_{03}^0$ modes corresponding to the same input value of axial magnetic field $H_z$.

Radial Distance $r$ (nm)

Electric Field $E_r$ (V/m)
similar to their linear counterpart. As the mode index increases, the confinement of the light improves in the nonlinear core. The dielectric constant distribution corresponding to the same input magnetic field is shown in fig.(5.9.c). It is clearly seen that the dielectric constant for higher order modes drops to the background value \( c_{bg} \) wherever the corresponding field profile has a node (zero crossing).

5.3 TE WAVES IN OPTICAL FIBERS WITH A LINEAR CORE AND A NONLINEAR CLADDING

The model studied for this case is shown in fig.(5.1). The cladding dielectric function has the general form given by equation (3.43). Two important cases are to be considered. The first one is characterized by \( c_{bg} + \Delta c_{NL, max} < c_1 \) and the second case has \( c_{bg} + \Delta c_{NL, max} = c_1 \).

The first case is a fiber which supports guided modes at low power. The fiber parameters used in this case are as follows: \( c_1 = 2.275 \), \( c_{bg} = 2.25 \), \( \alpha = 10^8 V^{-2} m^2 \), \( g = 50 \), \( \lambda = 0.6328 \mu m \) and \( a = 4 \mu m \). The optical fiber in this case supports the TE_{01} mode regardless of the amount of the applied power. The dispersion relation of the TE_{01} is shown in fig (5.10) where \( n_a \) is seen to increase with the interface field intensity. The effect of the nonlinearity is to modify the field profiles. For low intensity levels \( \Delta c_{NL} \)
FIG. 5(c) Variation of the dielectric constant with the radial distance for TE\textsuperscript{00}, TE\textsuperscript{02}, and TE\textsuperscript{04} modes.
Fig. 6.10 Variation of \( n^2 \) with the Intensity Intensity for TE\(^0\) in a Waveguide

**Intensity** \( \left| \mathbf{E} \right|^2 \) at \( r = a \) (V/m)

-1.5 x 10^-6 to 1.5 x 10^-4

-1.5 x 10^-2 to 1.5 x 10^0

-1.5 x 10^1 to 1.5 x 10^3

**Mode Index** \( n^2 \)

-1.5 x 10^-6 to 1.5 x 10^-4

-1.5 x 10^-2 to 1.5 x 10^0

-1.5 x 10^1 to 1.5 x 10^3
is small and the field distribution represents the linear solution of the fiber. However, for high intensity levels the peak of the first TE mode moves towards the nonlinear cladding. This behavior illustrates the self-focusing action of the nonlinear medium. Figure (5.11) shows three field distributions for three values of the mode index to show the shift of the peak of field profile. The first value of \( n_e \) is the linear value where \( \Delta e_{NL} = 0 \) and it is corresponding to curve (A). The second value corresponding to curve (B) is taken for \( \Delta e_{NL} = 0.5 \Delta e_{NL,max} \). The third value for fully saturated medium where \( \Delta e_{NL} = \Delta e_{NL,max} \). It is represented by curve (C). As the intensity increases the field maximum moves towards the nonlinear cladding. This makes the field less confined in the core region. A considerable part of propagated power is concentrated in the cladding region. It must be noted that the peak of the field will not cross the core cladding interface as long as \( e_{ng} + \Delta e_{NL,max} < e_1 \). The distribution of the dielectric function versus the radial distance for the three values of the mode index used is shown in fig.(5.12). The distribution for low intensity resemble the step-index profile. For the moderate and high intensity values there is a graded-index decaying portion between \( e_{ng} + \Delta e_{NL} \) and \( e_{ng} \).

It is important to explain the application of the numerical recursive scheme for this fiber structure. The linear core is taken as the first layer and the field at the core cladding interface (evaluated in the linear region) is used
Fig (5.11) Three field plots corresponding to three values of $n_0$, taken from fig (5.10).
Values of \( n\) taken from Fig. 6.10

Fig. 6.12: Variation of the dielectric constant with the radial distance for three dielectric mediums.

- \( n = 1.50677\)
- \( n = 1.50615\)
- \( n = 1.50604\)

Radial Distance \( R \) (\( \mu m \))

Dielectric Constant
to modify the first nonlinear layer. The cladding is divided into a large number of layers. The boundary condition is applied at a distance far enough into the cladding such that the effect of the field is neglected. Accordingly the nonlinear medium is treated as a linear one with a dielectric constant equal \( \varepsilon_{le} \). However, this boundary point should not be taken far away from core-cladding interface to have a solution of equation (3.64). If the fields values at the boundary point are too small, the zero finding routine will not be able to obtain the correct eigenvalue. In such a case, the expression for evaluating the mode index will suffer from overflow or underflow. If the increment added to the dielectric constant due to the field is equal to or less than \( 10^{-4} \), the effect of the field is considered negligible.

The second case represents a fiber which supports variable number of modes according to the value of intensity. The parameters of this optical fiber are: \( \varepsilon_1 = 2.26 \), \( \varepsilon_{le} = 2.25 \), \( \alpha = 10^{4} m^{-2} \), \( g = 100 \), \( \lambda = 0.6328 \mu m \) and \( a = 4 \mu m \). For this case the fiber can support only the \( TE_{01} \) mode for low power values. As the applied field increases the peak value of the field distribution of the \( TE_{01} \) mode moves towards the nonlinear medium and crosses the core cladding interface. Three field distributions are shown in fig.(5.13) for low, intermediate and high intensity levels. It is clear that the field maximum corresponding to the highest chosen surface intensity (curve C), is located in the cladding region. This is a feature of nonlinear guided waves in this
Fig. 6.13) Three field plots corresponding to low, moderate and high intensity values.

Radial Distance R (in m)

Electric Field $E_r$ (V/m)

<table>
<thead>
<tr>
<th>SF</th>
<th>$J_r = 3.48 (V/m^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.069</td>
<td>$I_r = 7.95 (V/m^2)$</td>
</tr>
<tr>
<td>1.000</td>
<td>$I_r = 2.30 (V/m^2)$</td>
</tr>
<tr>
<td>0.307</td>
<td>$I_r = 3.48 (V/m^2)$</td>
</tr>
</tbody>
</table>
waveguide structure and has no linear counterpart. For such a case most of the propagated power is located in the cladding region. Eventually the $TE_{01}$ mode will be mainly guided by the nonlinear cladding. Another important conclusion can be drawn from fig.(5.13) (curve C), when the fiber approaches the saturation, the physical radius of the fiber becomes greater than the actual core radius (4 μm). The higher order modes of weak intensities can be supported by the dielectric constant profile corresponding to $TE_{01}$ mode. However, if their intensities are large the modes will interact and the wave propagation process inside the fiber will be complicated. The dielectric constant variations corresponding to the three field distributions of fig.(5.13) are shown in fig.(5.14). When the fiber approaches saturation ($c_{bg} + \Delta c_{NL} \approx c_1$), the refractive index profile resembles a step-index profile with a physical radius of about 25 μm. This can be seen from fig.(5.14). This physical radius is about six times the actual core radius (4 μm). The next higher order mode $TE_{02}$ has also been investigated. This mode is cut-off at low power values. However, for high power values the saturated fiber supports this mode among the higher order modes. The fiber in this case resembles a linear fiber with a large radius and a refractive index difference between the core and the cladding equal to $c_1 - c_{ho}$. The fiber thus has a large value of V number which allows the support of many modes.
The dielectric constant distributions for the three intensity values used in Fig. 6.13 are shown in the graph. The graph plots the radial distance (µm) against the dielectric constant. The labels A, B, and C are used to identify different regions of the graph.
The $TE_{02}$ mode reaches cutoff as the applied intensity becomes lower than a threshold value. The value of the threshold intensity depends on the physical parameters of the fiber, mode order, and the nonlinear coefficient of the cladding material. An important application for the fiber of this type is as a power limiter. At high power levels, the fiber can support higher order modes in addition to the $TE_{01}$ mode. However, these modes may interact if their intensity is large. As the power decreases gradually the higher order modes are made cut-off and eventually only the $TE_{01}$ mode will be supported by the fiber. The dispersion curve for this fiber is shown in fig.(5.15.a) where the power is plotted versus the mode index. for the first order mode $TE_{01}$ It can be seen that, for low power values, the fiber behaves as a linear fiber and the mode index is power independent. However, for high power values the mode index increases with the power. The variation of the $n_e$ of the $TE_{02}$ with the interface intensity is shown in fig.(5.15.b). It is seen that there is a small threshold value above which the mode is supported and below this value the mode is cut-off.

5.4 TM WAVES IN FIBERS WITH A NONLINEAR CORE AND A LINEAR CLADDING

TM guided waves are characterized by the field vectors $\vec{E}=(E_r, 0, E_z)$ and $\vec{H}=(0, H_r, 0)$. There are thus two electric fields components contributing to
Fig. 6.5.2. Variation of $n^2$ with the total power for $TE_1^1$ mode in a waveguide.
Waveguide having linear core and nonlinear cladding

**Fig. 6.15(b)** Variation of \( n^* \) with the interface value of \( E^* \) for TE\(_0^a\) mode in a

Intensity \( |E| \) at \( r = a \) (V/m)

\( n^* \)
the dielectric function of the nonlinear medium. The wave equation is solved, in this case, for $E_z$ and $H_z$ is obtained from $E_z$ using equation (3.38). The dispersion relations are described for convenience in terms of the tangential magnetic field intensity $|H_x|^2$ measured at the core cladding interface. The magnetic field $H_z$ in the TM modes is considered the dual of the electric field $E_z$ in the TE modes. Also it is the only magnetic field component and at the same time it is transverse to the fiber axis. Therefore, it is the component meant by transverse magnetic. As all boundaries, this magnetic field component is also continuous at the core cladding interface. It is noted that most of the dispersion characteristics of TM modes of nonlinear fiber are similar from the outset to their TE counterparts. This will be clearly shown in the next subsections.

However, the distribution of the dielectric constant of the nonlinear medium is different from that of TE case. In the case of TE modes in nonlinear waveguide (nonlinear core-linear cladding) the value of $e_1$ at the fiber axis ($r=0$) is constant independent of the field $E_z$ and equal to the background value $e_{bg}$. However, for TM case it is field dependent and given by equation (3.75). Two cases are to be studied corresponding to whether the fiber is supporting guided modes at low power or not.

The first case is a fiber which supports guided modes at low power. The
parameters used for this case are as follows: \( \varepsilon_{bg} = 2.255 \), \( \varepsilon_2 = 2.25 \), \( \alpha = 10^{-3} \text{ V}^{-2} \text{m}^2 \), \( g = 20 \), \( \lambda = 0.6328 \mu m \) and \( a = 4 \mu m \). The dispersion characteristic is shown in fig.(5.16) as a plot of mode index versus the magnetic field intensity \( |H_s|^2 \) evaluated at the core-cladding interface. As in the TE case there are three regions of interest for the first order TM mode \( (TM_{01}) \) depending on the field intensity. The first region has a positive slope and represent the low intensity behavior. In this region the mode index becomes an increasing function of the surface intensity. A similar behavior is also obtained in the high intensity region. The mode index becomes independent of the the applied field and equal to the linear case corresponding to \( \varepsilon_1 = \varepsilon_{sat} \), as the intensity becomes very large. The high intensity and low intensity results obtained by the recursive scheme have been checked by solving the eigenvalue equation given below

\[
\frac{\varepsilon_{co} J_1(u)}{u J_0(u)} + \frac{\varepsilon_{cr} K_1(w)}{w K_0(w)} = 0
\]

(5.3)

where \( u, w \) are defined in equation (5.2) and \( \varepsilon_{co} \) is taken equal to \( \varepsilon_{bg} \) for the low power case and equal to \( \varepsilon_{bg} + \Delta \varepsilon_{NL, max} \) for the high power case. These results are shown in table 5.2. The agreement between the two results is up to the sixth digit.
Fig (5.16) Variation of $n_o$ with the interface intensity for $TM_0$ mode.
TABLE 5.2

Checking the values of $n_e$ in the linear limit of the recursive scheme against the direct solution of the closed form E.V.E. for $TM_{o1}$ mode. scheme.

| Structure     | Figure | Method                  | $n_e$    | $A_{H_z}(0)$ | $|H_z|^2$  |
|---------------|--------|-------------------------|---------|--------------|------------|
| $NL_{co} - L_{cl}$ | (5.16) | recursive               | 1.500194 | 0.01         | 1.7(-7)    |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation     | 1.500194 |              |            |
| $NL_{co} - L_{cl}$ | (5.16) | recursive               | 1.515722 | 10000        | 11767.9    |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation     | 1.515709 |              |            |
| $NL_{co} - L_{cl}$ | (5.20) | recursive               | 1.512473 | 30000        | 130610     |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation     | 1.512475 |              |            |
| $L_{co} - NL_{cl}$ | (5.22) | recursive               | 1.506038 | 10           | 2.2(-2)    |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation     | 1.506038 |              |            |
| $L_{co} - NL_{cl}$ | (5.25) | recursive               | 1.501446 | 10           | 7.497(-2)  |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation     | 1.501445 |              |            |
The second region of the dispersion relation shown in fig.(5.16) is characterized by a negative slope where the mode index increases with the surface intensity. The multivalued solution is clear. For the same value of surface intensity there are three values of the mode index. The field profiles in fig.(5.17) are shown for three values of \( n_e \) covering different region of fig.(5.16) and chosen in the region of multivalued solution. As the modal power increases, the magnetic field becomes more confined in the core region and the field maximum moves towards the axis of the fiber. The distributions of the dielectric constant corresponding to the three values of \( n_e \) is illustrated in fig.(5.18). As has been mentioned before, the value of \( \varepsilon \) at \( r=0 \) is not equal to the background value as TE case, but it has an increment due to the value of \( E_z \) at \( r=0 \). This fact is clearly seen in fig.(5.18).

Figure (5.19) shows this dispersion relation as a plot of mode index versus total power. The power is calculated from equation (5.3). For the TM modes the expression for calculating the power is given by:

\[
P = \frac{1}{2} \text{Re} \int_0^{2\pi} \int_0^{\infty} E_r H_r' r \, dr \, d\varphi \quad (5.5)
\]

The dispersion curve has three regions of operation, corresponding to low, moderate and high power. The first region is the low power region in which, \( n_e \) is almost constant and independent of power. This is also the case for the high power (saturation) region. For moderate values of power,
FIG. 6.17) Three field plots corresponding to three values of $\mu$. Taken from Fig. 5.16.

<table>
<thead>
<tr>
<th>$u^0$</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5078</td>
<td>3.875</td>
</tr>
<tr>
<td>1.5054</td>
<td>1.000</td>
</tr>
<tr>
<td>1.5106</td>
<td>0.101</td>
</tr>
</tbody>
</table>
Fig (5.18) Variation of the dielectric constant with the radial distance for the three values of $n_0$, taken from fig (5.16).
FIG. 6.19 Variation of $n^0$ with the total power for TM$^0_1$ in a waveguide having nonlinear core and linear cladding.
it can be seen from this figure that \( n_e \) increases with the modal power. This is shown in the second region of the dispersion curve.

The second case is devoted to an optical fiber which is cut-off at low power. This fiber is characterized by \( \varepsilon_{bg} \leq \varepsilon_2 \). Two cases similar to those discussed in section 5.2, (second and third cases), for the TE\(_{01}\) mode have been investigated and studied. All the fiber parameters are also the same as those used in section 5.2. The dispersion characteristic for the fiber with \( \varepsilon_{bg} < \varepsilon_2 \) is shown in fig.(5.20) as a plot of \( n_e \) versus \( |H_e|^2 \). Similar to TE case the cutoff fiber requires a minimum surface intensity to be turned on. This threshold intensity depends on the refractive index difference, the nonlinear coefficient and the mode order. For the case of \( \varepsilon_{bg} = \varepsilon_2 \), the dispersion characteristic is similar to fig.(5.6) of the TE case.

5.5 TM WAVES IN OPTICAL FIBERS WITH A LINEAR CORE AND A NONLINEAR CLADDING

Two cases are to be considered. The first one is the fiber which supports the TM\(_{01}\) for all intensity values and characterized by \( \varepsilon_1 > \varepsilon_{bg} + \Delta \varepsilon_{NL \text{max}} \). The second case is devoted to a fiber which also supports the same mode, however, the saturation limit is such that \( \varepsilon_1 = \varepsilon_{bg} + \Delta \varepsilon_{NL \text{max}} \).
Waveguide having non-linear core and linear cladding.

Figure 6.20: Variation of $n$ with the interface intensity for $T_m^0$ mode in $a (A/m)^2$ at $r = a$.

Intensity $|H|^2$ vs. $n$.

- $1.5000$ to $1.5060$
- $1.5060$ to $1.5080$
- $1.5080$ to $1.5100$
- $1.5100$ to $1.5120$
- $1.5120$ to $1.5140$
- $1.5140$ to $1.5160$
- $1.5160$ to $1.5120$
- $1.5120$ to $1.5140$
- $1.5140$ to $1.5160$
- $1.5160$ to $1.5080$
- $1.5080$ to $1.5060$
- $1.5060$ to $1.5040$
- $1.5040$ to $1.5020$
- $1.5020$ to $1.5000$
- $1.5000$ to $1.5020$
- $1.5020$ to $1.5040$
- $1.5040$ to $1.5060$
- $1.5060$ to $1.5080$
- $1.5080$ to $1.5100$
- $1.5100$ to $1.5120$
- $1.5120$ to $1.5140$
- $1.5140$ to $1.5160$
- $1.5160$ to $1.5080$
- $1.5080$ to $1.5060$
- $1.5060$ to $1.5040$
- $1.5040$ to $1.5020$
- $1.5020$ to $1.5000$
- $1.5000$ to $1.5020$
- $1.5020$ to $1.5040$
- $1.5040$ to $1.5060$
- $1.5060$ to $1.5100$
- $1.5100$ to $1.5120$
- $1.5120$ to $1.5140$
- $1.5140$ to $1.5160$
- $1.5160$ to $1.5080$
- $1.5080$ to $1.5060$
- $1.5060$ to $1.5040$
- $1.5040$ to $1.5020$
- $1.5020$ to $1.5000$
- $1.5000$ to $1.5080$
In the first case, the fiber parameters are the same as those of section (5.3). The reason for choosing the same parameters is to have a basis of comparison between different polarization, particularly between \( TE_{01} \) and \( HE_{11} \) modes which will be shown in the next chapter. The dispersion characteristics are expressed here in two ways. In the first representation the total power is plotted against \( n_e \) as shown in fig.(5.21). The second one is illustrated in fig.(5.22) as a plot of surface intensity versus the mode index. Comparing the two plots of fig.(5.10) and fig.(5.22), there is a clear similarity between TE and TM dispersion characteristics. In fig.(5.22), for very low and very high intensity values the mode index is independent of the interface magnetic field intensity. The transition between these two states occurs monotonically. This is also the case in fig.(5.10).

It is seen from fig.(5.21) that the fiber can support the \( TE_{01} \) mode for any value of the input intensity. Three mode indices covering different regions of fig.(5.22) have been chosen and their corresponding field profiles are shown in fig.(5.23). The shift of maximum field amplitude towards the nonlinear medium is clear.

Unlike the fiber with a nonlinear core and a linear cladding in which most of the field energy is concentrated in the core, a considerable part of the field energy is propagated in the cladding where the core is linear and the cladding is nonlinear. This is also the case of TE modes. It is seen from
Linear core and nonlinear cladding

Figure 6.21: Variation of $n$ with the total power for TM₀₁ in a waveguide having

![Graph showing variation of mode index $n$ with power.](image)
Figure 6.22. Variation of $n^2$ with the interface intensity for $TM_{01}^0$ in a waveguide.
Fig. 6.23: Three field plots corresponding to three values of $\mu$, taken from Fig. 6.22.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\delta F$ = 1.51659 SF = 0.569</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.56039 SF = 1.000</td>
</tr>
<tr>
<td>3.0</td>
<td>1.56039 SF = 3.235</td>
</tr>
</tbody>
</table>
fig.(5.23) that as the medium is approaching saturation, the field peak becomes more closer to the core cladding interface.

The distribution of the dielectric constant corresponding to the above three values of mode index is illustrated in fig.(5.24). For this fiber structure the index profile is similar to that of the TE case shown in fig.(5.12). As can be seen from fig.(5.24), there is a graded-index distribution in the cladding region depending of the amount of power applied, the larger the power the more penetration of the graded-index profile in the cladding region.

As has been done for the TE case, the solutions of the linear states of the waveguide are checked against the corresponding direct solution of the eigenvalue equation. These two linear states are characterized by $\varepsilon_2 = \varepsilon_{bg}$ and $\varepsilon_2 = \varepsilon_{bg} + \Delta \varepsilon_{NL_{max}}$ respectively. The results are identical as shown in table 5.2.

The second case under consideration is characterized by $\varepsilon_1 = \varepsilon_{bg} + \Delta \varepsilon_{NL_{max}}$. This optical fiber supports the $TM_{01}$ mode for low intensity levels which makes $\varepsilon_1 > \varepsilon_{bg} + \Delta \varepsilon_{NL}$. However, for high input power, the fiber approaches saturation for which $\varepsilon_1 \approx \varepsilon_{bg} + \Delta \varepsilon_{NL_{max}}$. In this case the fiber resembles a linear step-index with a large radius. This enables the fiber to support the next higher order modes. The parameters taken for this
Three values of \( n \) taken from Fig. 6.22.

Fig. 6.24: Variation of the dielectric constant with the radial distance for the

Radial Distance \( r \) (\( \mu m \))

Dielectric Constant

\( \epsilon = 1.50659 \)

\( \epsilon = 1.50839 \)

\( \epsilon = 1.5069 \)
waveguide are similar to those given in section (5.3). The dispersion characteristic is shown in fig.(5.25) as a plot of the modal power of the $TM_{01}$ mode versus the mode index. It is seen that $n_e$ is independent of power for low power values. However, for moderate and high power values, $n_e$ increases with power until the fiber reaches saturation. Three values of the mode index are chosen corresponding to close to linear, close to saturation, and an intermediate value. The field profiles associated with these three values are shown in fig.(5.26). It is clear that as the nonlinearity becomes stronger, the field is guided mainly in the nonlinear region (cladding) and only a small amount of energy is confined in the core region. This unique characteristic of the waveguide with nonlinear cladding also occurs in the TE case. The distribution of the dielectric constant for the same values of the mode indices is shown in fig.(5.27). It is similar to fig.(5.14) of the TE case. The physical radius of the fiber increases at saturation to about five times the actual radius.

5.6 CHECKING THE RESULTS USING THE SELF-CONSISTENT SCHEME

After applying the recursive scheme to different nonlinear waveguide structure, it is instructive to check the obtained results using the self-consistent routine. The strategy for doing this is based on choosing three different values of the mode index from the dispersion curve. The intensity profiles for these values of $n_e$ are known from the recursive solution. The
Figure 5.25: Variation of n° with the total power for TM_{01} in a waveguide having linear core and nonlinear cladding.
FIG. (6.26) Three field plots corresponding to three values of \( \eta \). Taken from FIG (6.25)
Values of $n^2$ taken from Fig. (6.25)

Fig. (6.27) Variation of the dielectric constant with the radial distance for three

Radial Distance $r$ (mm)

Dielectric Constant

$v^2 = 1.5017$

$v^2 = 1.5010$

$v^2 = 1.5015$
intensity $|E|^2$ has the general definition given in equation (3.44). For each value of the $n_e$, the corresponding intensity profile is used as an input to the self-consistent scheme. This intensity profile modifies the dielectric constant according to equation (3.45) and generates a graded-index profile. The self-consistent scheme solves linearly the graded-index profile and produce the corresponding mode index and its associated intensity profile. If the input to the recursive scheme is a correct solution to the nonlinear problem, the first run of the scheme will produce the same mode index and the same intensity profile. This will guarantee a quick convergence of the self-consistent scheme. Table 5.3 lists the results obtained from the two methods in the case of TE waves. The figure number shown in the second column gives the dispersion curve, which has been checked. The fifth column in the table is assign to the amplitude of $H_z$ at $r=0$. This amplitude is necessary to start the recursive scheme. The value of $n_e$, the field profile and the surface intensity will be function of this amplitude. However, in the self-consistent routine, the solution is independent of this field amplitude. The same field amplitude has been used in both routines in order to obtain the same surface intensity $|E_s|^2$ directly without normalizing the results. The values of $n_e$ in both methods are the same since the self-consistent scheme is initiated with the proper solution of the nonlinear problem. Table 5.4 is similar to table 5.3 and is devoted to TM waves. To start the TM
**TABLE 5.3**

The effective indices of the fundamental TE mode obtained from the recursive solution and the solution using self-consistent scheme.

| Structure | Figure | Method   | $n_e$   | $A_{y_2}(0)$ | $|E_x|^2$ |
|-----------|--------|----------|--------|-------------|---------|
| $NL_{co} - L_{cl}$ | (5.2) | recursive | 1.500969 | 0.2 | 167329 |
| $NL_{co} - L_{cl}$ |          | self-cons. | 1.500969 | 0.2 | 167329 |
| $NL_{co} - L_{cl}$ | (5.2) | recursive | 1.507101 | 1.0 | 101049 |
| $NL_{co} - L_{cl}$ |          | self-cons. | 1.507101 | 1.0 | 101049 |
| $NL_{co} - L_{cl}$ | (5.2) | recursive | 1.515087 | 5.0 | 1.165(7) |
| $NL_{co} - L_{cl}$ |          | self-cons. | 1.515087 | 5.0 | 1.165(7) |
| $L_{co} - NL_{cl}$ | (5.10) | recursive | 1.506063 | 0.5 | 286475 |
| $L_{co} - NL_{cl}$ |          | self-cons. | 1.506063 | 0.5 | 286475 |
| $L_{co} - NL_{cl}$ | (5.10) | recursive | 1.506311 | 2.0 | 8923126 |
| $L_{co} - NL_{cl}$ |          | self-cons. | 1.506311 | 2.0 | 8923126 |
| $L_{co} - NL_{cl}$ | (5.10) | recursive | 1.506655 | 5.0 | 1.184(8) |
| $L_{co} - NL_{cl}$ |          | self-cons. | 1.506655 | 5.0 | 1.184(8) |
TABLE 5.4

The effective indices of the first order TM mode obtained from the recursive solution and the self-consistent scheme.

| Structure   | Figure | Method   | $n_e$    | $A_{E_x}(0)$ | $|E_z|^2$ |
|-------------|--------|----------|---------|--------------|----------|
| $NL_{co} - L_{ci}$ | (5.16) | recursive | 1.500958 | 50           | 166011   |
| $NL_{co} - L_{ci}$ |        | self-cons. | 1.500958 | 50           | 166011   |
| $NL_{co} - L_{ci}$ | (5.16) | recursive | 1.503624 | 120          | 216292   |
| $NL_{co} - L_{ci}$ |        | self-cons. | 1.503624 | 120          | 216292   |
| $NL_{co} - L_{ci}$ | (5.16) | recursive | 1.512957 | 600          | 759226   |
| $NL_{co} - L_{ci}$ |        | self-cons. | 1.512957 | 600          | 759226   |
| $L_{co} - NL_{cl}$ | (5.22) | recursive | 1.506088 | 200          | 790890   |
| $L_{co} - NL_{cl}$ |        | self-cons. | 1.506088 | 200          | 790890   |
| $L_{co} - NL_{cl}$ | (5.22) | recursive | 1.506227 | 400          | 4616806  |
| $L_{co} - NL_{cl}$ |        | self-cons. | 1.506227 | 400          | 4616806  |
| $L_{co} - NL_{cl}$ | (5.22) | recursive | 1.506509 | 800          | 3.583(7) |
| $L_{co} - NL_{cl}$ |        | self-cons. | 1.506509 | 800          | 3.583(7) |
recursive scheme, the value of $E_z$ at $r=0$ is required. This value is given in the fifth column. The same comment on the TE case is valid for the TM case.

It must be noted that the self-consistent scheme can be used to solve the nonlinear optical fiber completely. However, it is very time consuming. The convergence of the routine differs from one region to another on the dispersion curve. For example, in fig.(5.2), the convergence of the single-valued solution regions is faster than that of the multi-valued region. Also the convergence is slower in the negative slope region than that in the positive slope regions. The objective of using the self-consistent routine is only to check the obtained results using the recursive scheme.
CHAPTER 6

THE FUNDAMENTAL HYBRID MODE IN A NONLINEAR OPTICAL FIBER

6.1 INTRODUCTION

The hybrid $HE_{11}$ mode is the most important mode of an optical fiber. It has a zero cut-off frequency and therefore can be supported by any optical fiber regardless of its core size and refractive index difference. Single mode fibers which are characterized by a fiber V number less than 2.405 have superior dispersion performance in communication systems than multimode fibers. This results in the single mode optical fiber having a much higher bandwidth compared to multimode optical fibers.

The nature of the hybrid mode $HE_{11}$ is explained for a linear optical fiber and then is considered in a nonlinear fiber. As in the case of TE and TM modes discussed in chapter 5, the field profiles of a nonlinear fiber are similar to those of a linear fiber except for a power dependent change in the field profile. This results in a change of the distribution of total power over the core and the cladding regions.
For the fundamental mode in saturable media, there is no shift of the field maximum towards the nonlinear region, because it is always located at the origin. However, the distribution of power is changed.

In step-index optical fiber, the hybrid modes are designated HE modes and EH modes. The designation HE modes and EH modes has been explained by Unger [10] as follows: $HE_{11}$ and $EH_{11}$ modes are the lowest order hybrid modes in optical waveguide. One of them $HE_{11}$ gives field profiles inside the core which are similar to those of the round hollow metallic waveguide operating in the $H_{11}$ ($TE_{11}$) mode. Because the $H_{11}$ is the fundamental mode in the metallic waveguide, the fundamental mode of the fiber has been designated $HE_{11}$. The hybrid nature of the HE modes results from the fact that the mode is not a pure H-mode, but has also an axial $E_z$ component. Accordingly HE modes are H-like modes or ($TE$-like modes) when related to the equivalent metallic waveguide modes. As there is another mode sharing the same subscript 11 with the $HE_{11}$ mode the designation $EH_{11}$ has been allotted to it. This designation fixes the classification of higher order modes.

The fundamental Hybrid mode $HE_{11}$ is characterized by some important features. For instance, it is linearly polarized when the refractive index difference between the core and cladding is very small. The optical fiber in
this case, is called a weakly guiding fiber. It is well-known that the linearly polarized modes of the weakly guiding fiber are in general a superposition of degenerate modes which share the same propagation constant $\beta$. However, the $HE_{11}$ is linearly polarized by itself which is a distinguishing characteristic of all $HE_{1m}$ modes. The linear polarization feature means briefly that the important electric and magnetic fields are in the transverse plane normal to each other while the axial fields components are negligible compared to the transverse components. This makes the field inside the fiber similar to that of TEM plane wave.

By examining the values of the field components for $HE_{11}$ (equations (3.85)-(3.96)) in the case of weak guidance ($n_1 \approx n_2$), two important conclusions are obtained. The first one is that the transverse electric fields $E_\phi$ and $E_r$ are equal in magnitude and in their radial dependence, but are different in their azimuthal dependence. If one of them is taken to vary as $\cos \phi$ the other varies as $\sin \phi$. This feature is also found for the transverse magnetic field components $H_\phi$ and $H_r$. The second feature is that the axial field components $E_z$ and $H_z$ have the same radial dependence. Their azimuthal variations differ by $\frac{\pi}{2}$. The amplitudes ratio $\frac{A_{E_z}}{A_{H_z}}$ is the ratio between the amplitudes of axial electric field $E_z$ and the amplitude of the axial
magnetic field $H_z$. It is equal to the ratio $\frac{A_1}{C_1}$ obtained from equations (3.79) and (3.82). This ratio is also equal to the intrinsic impedance of the core material. This fact is considered because for weak guidance, both the core and the cladding have almost the same refractive index. Wave propagation in this case is similar to the one in a single unbounded medium. In the subsections (6.2) and (6.3) the dispersion characteristics of the hybrid mode $HE_{11}$ in a nonlinear optical fiber are presented. The nonlinear optical fiber with nonlinear core and linear cladding is discussed in section 6.2. Section 6.3 is devoted to the optical fiber with linear core and nonlinear cladding. Checking of the results obtained by the recursive scheme in sections 6.2 and 6.3 is made by using the self-consistent scheme (discussed in section (3.8)). The results are given in section 6.4.

6.2 THE OPTICAL FIBER WITH A NONLINEAR CORE AND A LINEAR CLADDING

Two cases have been studied, the first one is for a fiber which supports guided modes at low power and has $r_{bg} > r_2$. The second one is the case of a cut-off fiber characterized by $r_{bg} < r_2$. For the first case the fiber parameters are as follows: $r_{bg} = 2.255$, $r_2 = 2.25$, $\alpha = 10^9 V^{-2} m^2$, $g = 20$, $\lambda = 0.6328 \mu m$ and $a = 4 \mu m$. The dispersion characteristics are expressed
as plots of the mode index, \( n_e \) versus the tangential field component \( |E_e|^2 \) evaluated at the core-cladding interface as has been done in the TE case. The dispersion plots can alternatively be given as plots of \( |E|^2 \) versus the mode index or plots of mode power versus the mode index.

Figure (6.1) shows the dispersion relation of the \( HE_{11} \) mode of this waveguide as a plot of surface intensity \( |E|^2 \) versus \( n_e \) for two saturation levels. In the case of high saturation level (\( g = 20 \)), the multivalued dependence of \( n_e \) on the surface intensity is clear for a range of the surface intensity. However, if the surface intensity is outside this range, the relation is single valued. For the case of low saturation level (\( g = 100 \)), there is no optical bistability and the relation is monotonic. It must be noted that in the regions where the dependence is single valued for \( g = 20 \), the fiber behaves in a manner similar to that of a linear fiber with \( \varepsilon_1 = \varepsilon_{bg} \) or \( \varepsilon_1 = \varepsilon_{bg} + \Delta \varepsilon_{NL_{max}} \) depending on whether the intensity is very low or very high respectively. The values of \( n_e \) for these two cases have been compared with the results of the solution of the eigenvalue equation given in reference [10]. Table 6.1 lists the eigenvalues obtained by the two methods.

Figure (6.2) shows the field profiles for three values of the effective index covering different regions of operation. ( points A, B and C in fig.(6.1) ). It can be seen from fig.(6.2) that as the input power increases the field becomes
Fig 16.1. Variation of $n_e$ with the surface intensity for $HE_{11}$ mode in a waveguide having two different saturation levels.

Intensity $|E|^2$ at $r = a$ (V/m)$^2$

$g = 20$

$g = 100$
The effective indices of the fundamental HE mode obtained from the recursive solution and the closed form eigenvalue equation

| Structure     | Figure | Method            | $n_e$  | $A_{e_z}(0)$ | $|E_z|^2$ |
|---------------|--------|-------------------|--------|--------------|----------|
| $NL_{co} - L_{cl}$ | (6.2)  | recursive         | 1.501033 | 1.0          | 335.40   |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation | 1.501029 |              |          |
| $NL_{co} - L_{cl}$ | (6.2)  | recursive         | 1.517209 | 10000        | 15.44(7) |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation | 1.517237 |              |          |
| $NL_{co} - L_{cl}$ | (6.4)  | recursive         | 1.505779 | 5000         | 17.92(8) |
| $NL_{co} - L_{cl}$ |        | eigenvalue equation | 1.515780 |              |          |
| $L_{co} - NL_{cl}$ | (6.8)  | recursive         | 1.507409 | 10           | 5654.15  |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation | 1.507409 |              |          |
| $L_{co} - NL_{cl}$ | (6.8)  | recursive         | 1.507645 | 1000         | 2.817(8) |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation | 1.507677 |              |          |
| $L_{co} - NL_{cl}$ | (6.12) | recursive         | 1.502564 | 1.0          | 152.96   |
| $L_{co} - NL_{cl}$ |        | eigenvalue equation | 1.502564 |              |          |
Fig. (6.2) Three field plots corresponding to three values of $n^0$, taken from Fig. (6.1):

- $n^0 = 1.51704$, SF = 0.49
- $n^0 = 1.60358$, SF = 1.00
- $n^0 = 2.715$, SF = 1.5112
more confined in the core region and the field penetration in the cladding decreases. The corresponding dielectric constant distributions for points A, B and C is illustrated in fig.(6.3).

It can be shown by comparing fig.(6.3) to the corresponding TE and TM cases in figs.(5.4) and (5.18), that the $HE_{11}$ has a different dielectric constant profile. In the case of the $HE_{11}$ mode, the dielectric constant has a maximum at the axis of the fiber and is in general a graded index distribution. However, in both the TE and the TM cases, the dielectric constant has a distribution which is completely different form that occurring in the $HE_{11}$ case. The maximum value of the dielectric constant occurs inside the core but, not at the center of the fiber. The profile in general resembles the letter M.

The cutoff fiber investigated here is similar to that used in section 5.2 with all waveguide parameters are taken the same. The dispersion relation for this fiber is shown in fig.(6.4) as a plot of interface value of $|E_z|^2$ versus $n_e$. The fiber requires a minimum amount of intensity to support the $HE_{11}$ mode. As the guidance takes place with more power propagated along the fiber the interface intensity decreases and the field maximum increases which represents the negative slope part of fig.(6.4). For the positive slope part of the same figure, both interface intensity and $n_e$ increase with the mode power.
Values of $n^2$ taken from Fig. 6.1

Fig. 6.3 Variation of the dielectric constant with the radial distance for three cases.

$A$, $B$, and $C$ correspond to different values of $n^2$. The graph shows a decrease in dielectric constant with increasing radial distance for each case.
Fig. (6.4) Variation of $n^0$ with the Interface Intensity for HE mode in a waveguide having $\epsilon_{oo} > \epsilon_z$.

Intensity $|E|^2$ at $r = a \text{ (V/m)}^2$.
and the fiber is driven into saturation. The same dispersion characteristic is illustrated in fig.(6.5.a) with the surface intensity replaced by the modal power. The power can be calculated form the integral of the Poynting vector given in equation (5.3). For the \( HE_{11} \) mode, this integral reduces to:

\[
P = \frac{1}{2} \text{Re} \int_0^{2\pi} \int_0^\infty (E_r H_e^* - E_e H_r^*) \ r \ dr \ d\phi
\]

In equation (6.1), the power flow is considered in the positive \( z \) direction. The minimum threshold power required for the fiber to support the \( HE_{11} \) mode can be seen from fig.(6.5.b). There is a small negative slope part corresponding to unstable operation. The remaining part of the dispersion curve with positive slope is considered stable. When the fiber approaches saturation the effective index \( n_e \) becomes constant independent of the modal power.

Figure (6.5.b) is similar to fig.(6.5.a) and showing at the same time the behavior of the \( TE_{01} \) mode compared to the \( HE_{11} \) mode. The comparison is important because it gives the power range in which the fiber can support only the \( HE_{11} \) mode and works as a single mode fiber. This type of fiber is commonly used in long distance communication and has low dispersion compared to multimode fiber.
Fig. (6.5a) Variation of $n_0$ with the guided power for $HE_{11}$ mode.
Fig. 6.5 (b) Variation of $n^2$ with the modal power for $HE_{11}$ and $TE_{01}$ modes.
Two field distributions are shown in fig (6.6) corresponding to points A and B in fig.(6.5.a). The confinement of light increases with the mode index. This improves the performance of the waveguide since the light intensity is concentrated in the core. The dielectric constant distribution corresponding to the same two points is shown in fig.(6.7). For point B, the nonlinear optical fiber approaches saturation and the distribution is close to step-index profile. However, the profile corresponding to point A is similar to W-fiber.

6.3 OPTICAL FIBER WITH A LINEAR CORE AND A NONLINEAR CLADDING

The nonlinearity can affect this fiber structure in two ways: the first one is to modify the field profile according to the amount of input power. A considerable part of the propagated energy will be concentrated in the nonlinear cladding. The second effect of the nonlinearity is to support the higher order modes which are cutoff at low power levels. This case has been investigated in [8] for LP modes. The field profile nodes of the higher order modes are located in the nonlinear cladding which is in contrast with the linear case in which all the field nodes are in the core region. Another important feature which has been shown in [9] is the existence of surface modes when the value of the nonlinear cladding refractive index exceeds the linear core value. The peak of such modes is located at the core-cladding interface.
Fig. 6.6 Two field plots for $E_r$, corresponding to low and high power values.
Fig. (67). The dielectric constant distribution for low and high power values.
The first case to be treated here is the fiber which supports the $HE_{11}$ mode regardless of the amount of power propagated. In this case the nonlinearity changes the features of the field. The saturation level is taken such that the saturable cladding refractive index is less than linear core refractive index. The parameters chosen are the same as those used for the TE case in section 5.3. The dispersion curve is shown in fig.(6.8). The surface intensity is plotted versus $n_e$. It is similar to those of TE and TM waves shown in figs.(5.10) and (5.19) respectively. The waveguide behaves as linear waveguide for low and high intensity. The effect of nonlinearity is clear for moderate values of intensity, where the mode index increases with the interface intensity. Figure (6.9) shows two fields profiles corresponding to two different points A and B. As the nonlinearity increases the field becomes less confined in the core region. In this case a considerable amount of the propagated energy is located in the cladding region. The dielectric constant distributions for point A and B are given in fig.(6.10). The index profile is similar to the corresponding TE and TM cases. It must be noted that the dispersion relation in fig.(6.9) can also be expressed as a plot of modal power versus mode index. This plot is shown in fig.(6.11). The mode index is independent of power for low and high power values and increases with power for moderate power values.

The second case under consideration is the fiber which supports the $HE_{11}$
Fig. 16.8: Variation of $n_e$ with the interface intensity for $HE_{11}$ mode in a waveguide having linear core and nonlinear cladding.

$\text{Intensity } |E|^2 \text{ at } r = a (\text{V/m})^2$

1E+04
1E+05
1E+06
1E+07
1E+08
1E+09

Mode Index $n_e$
Fig. (6.3) Two field plots corresponding to two values of \( n \), taken from fig. (6.8).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n_0 = 1.50762 )</th>
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<tbody>
<tr>
<td>SF</td>
<td>1.000</td>
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<td>SF</td>
<td>0.347</td>
</tr>
</tbody>
</table>
Values of $n$ taken from Fig. (6.8).

Fig. (6.10) Variation of the dielectric constant with the radial distance for the

Radial Distance $r$ (μm)

Dielectric Constant

- $n^2 = 1.50762$
- $n^2 = 1.50763$
Fig. (6.11) Variation of $n^2$ with the total power for HE$^{11}$ mode in a waveguide.
mode at low power and its saturation limit is such that \( \varepsilon_{bg} + \Delta \varepsilon_{NLmax} = \varepsilon_1 \).

The fiber parameters used are identical to those mentioned in section 5.3. The dispersion characteristic is shown in fig.(6.12) as a power-mode index relationship. The mode index increases with power until it reaches its saturation value. Any further increase in power will not cause an increase in the mode index. The field profiles corresponding to three points (A, B and C) on the dispersion curve are plotted in fig.(6.13). The penetration of the field in the nonlinear cladding can be seen clearly. A considerable part of the propagated power is concentrated in the cladding region. The dielectric function distributions for the same three points A, B and C are given in fig.(6.14). Point C represents the fiber approaching saturation. The physical radius of the fiber increases when the waveguide approaches saturation. The profile in this case resemble a step-index profile. However, the physical radius of the fiber in this case is several times the actual core radius (4 \( \mu m \)). This enables the fiber to support a large number of higher order modes.

6.4 VERIFYING THE RESULTS USING THE SELF-CONSISTENT SCHEME

The results obtained form the recursive scheme have been checked against the self consistent scheme and tabulated in table 6.2. The recursive scheme is applied the same way explained in section 5.6. The agreement between the schemes can be seen clearly from table 6.2. There is a very
Fig.(6,12) Variation of $n_0$ with the total power for $HE_{11}$ mode.
FIG. 6.19 Three field plots corresponding to three values of $n^2$ taken from Fig. 6.12.

<table>
<thead>
<tr>
<th>$n^2$</th>
<th>SF</th>
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<tbody>
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<td>1.50314</td>
<td>0.425</td>
</tr>
<tr>
<td>1.50282</td>
<td>1.000</td>
</tr>
<tr>
<td>1.50260</td>
<td>2.875</td>
</tr>
</tbody>
</table>
Fig. 1.4) Variation of the dielectric constant with the radial distance for three values of $n^o$.}

- $n^o = 1.50314$
- $n^o = 1.60282$
- $n^o = 1.50260$
### TABLE 6.2

The effective indices of the fundamental HE mode obtained from the recursive solution and the self-consistent scheme.

| Structure     | Figure | Method  | $n_e$    | $A_{e^z}(0)$ | $|E_z|^2$ |
|---------------|--------|---------|----------|--------------|----------|
| $NL_{co} - L_{cl}$ | (6.2)  | recursive | 1.501297 | 10           | 15810    |
| $NL_{co} - L_{cl}$ | self-cons. | 1.501297 | 10       | 158101       |
| $NL_{co} - L_{cl}$ | (6.2)  | recursive | 1.503384 | 100          | 7489     |
| $NL_{co} - L_{cl}$ | self-cons. | 1.503384 | 100     | 7489         |
| $NL_{co} - L_{cl}$ | (6.2)  | recursive | 1.516834 | 800          | 4445478  |
| $NL_{co} - L_{cl}$ | self-cons. | 1.516834 | 800     | 4445478      |
| $L_{co} - NL_{cl}$ | (6.8)  | recursive | 1.507425 | 100          | 640137   |
| $L_{co} - NL_{cl}$ | self-cons. | 1.507425 | 100     | 640137       |
| $L_{co} - NL_{cl}$ | (6.8)  | recursive | 1.507562 | 400          | 2.7(7)   |
| $L_{co} - NL_{cl}$ | self-cons. | 1.507562 | 400     | 2.7(7)       |
| $L_{co} - NL_{cl}$ | (6.8)  | recursive | 1.507633 | 800          | 1.674(7) |
| $L_{co} - NL_{cl}$ | self-cons. | 1.507633 | 800     | 1.674(7)     |
good agreement between the two results. It must be noted that the amplitude of the electric field \( A_{\varepsilon z}(0) \) is required to start the recursive scheme which produces the corresponding eigenvalue and eigen field. However, this amplitude is arbitrary in the self-consistent scheme because the problem is linear and the solution does not depend on the input filed amplitude.

To show the effect of nonlinearity on the guided field, a comparison between linear and nonlinear waveguides has been made. The nonlinear fiber is identical to the one studied in the above section. The linear fiber has the same parameters with the nonlinear coefficient \( \alpha \) equal to zero. The recursive computer program has been ran two times, the first one for nonlinear fiber with input \( A_{\varepsilon z}(0) = 170 \mu m \). The second run is made for the linear fiber with the input unchanged. The resultant field profiles are shown in fig.(6.15). Because the linear mode index is much less than the nonlinear one, the field amplitude of the linear guide is less than that of the nonlinear guide. However the linear field is more confined in the core region. Also the power propagated in the linear guide is less than that of the nonlinear guide although they have the same input \( A_{\varepsilon z}(0) \).
Fig. 6.16: Comparison between two field plots of linear and nonlinear waveguides having the same axial field amplitude $A_z(0)$ at $r = 0$. The radial distance $r$ (in m) is plotted on the horizontal axis, while the electric field $E_r$ (V/m) is plotted on the vertical axis. Two curves are shown:

- **Nonlinear**: $SF = 1.000$
- **Linear**: $SF = 2.033$
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

CONCLUSIONS

The checking of the multilayer scheme against the closed form eigenvalue equation shows a complete consistency between the eigenvalues obtained from the two methods. The multilayer scheme reduces indirectly to the same eigenvalue equation in the case of linear fiber. For a fiber consisting of more than three layers, the use of the recursive scheme is essential to obtain the eigenvalues. The utilization of complex Bessel function is preferable to overcome the problems arise when the argument of Bessel function changes from real to imaginary or vice versa. Also it has another advantage in treating the problems involving metallic layer or lossy dielectric.

The metal-clad optical fiber studied in chapter 4 exhibits some important features. For the two-layer fiber consisting of a dielectric core and a metal-clad, the hybrid mode $HE_{11}$ is the fundamental mode. However it has a finite cutoff frequency different from zero which is the case of the dielectric-clad. For the $TE_{01}$ and $TM_{01}$ the cutoff frequencies are shifted towards the low
frequency region compared to the dielectric-clad fiber. The attenuation of the $TM_{01}$ is greater than that of the $TE_{01}$ by about an order of magnitude. The attenuation of the hybrid mode $HE_{11}$ is located between the two and slightly close to that of the $TM_{01}$. In general the attenuation is a decreasing function of the core radius. The four-layer structures involving a thin dielectric buffer layer between the dielectric-clad and the metallic jacket have some important attenuation characteristics. In the case of low-index buffer, the TM modes exhibit an absorption peak located at some critical buffer thickness. However the attenuation of the TE modes is a decreasing function of the buffer thickness. In general the highest order modes are attenuated more than the lowest order modes. In the case of high-index buffer, the attenuation of both TE and TM modes has an oscillating behavior function of the buffer thickness. The absorption peaks are well separated which facilitate the design of a good fiber polarizer with high extinction ratio.

The dispersion relations of nonlinear optical fiber are similar for $TE_{01}$, $TM_{01}$, and $HE_{11}$. The following discussion is valid for any one of these modes unless mentioned otherwise. The dispersion relation is obtained as a plot of the mode index versus the surface intensity measured at the core-cladding interface. For a fiber with nonlinear core and linear cladding, the shape of the dispersion curve depends on the initial state of the fiber (at zero applied intensity). If the fiber is initially guiding, the dispersion curve will be similar
to that shown in fig.(5.2). It exhibits a negative slope region for which a single value of surface intensity corresponds to three values of the mode index. If the fiber is initially cutoff the dispersion curve will be similar to fig.(5.7). The fiber in this case requires a threshold surface intensity to support guided modes. The switching action in this case can be used in the design of logic gates and limiters. In both cases of guiding and cutoff fiber the nonlinearity will cause the field to be more confined in the core region. It must be noted that the distribution of the dielectric constant over the core region is not the same for the three modes under consideration. For the \( HE_{11} \) mode, it has a maximum value at the center and decreases gradually with distance. This gives in general a graded index distribution as shown in fig.(6.3). However for the \( TE_{01} \) and \( TM_{01} \) modes, the maximum value of the dielectric constant is located between the center of the fiber and the surface of the core. The general distribution is shown in fig.(5.4). The field profiles of the \( HE_{11} \) mode forms a spot of light which has a maximum value at the center and decreases with the radial distance. However for both \( TE_{01} \) and \( TM_{01} \) the field profile resembles a ring of light which has a zero intensity at the origin and a maximum value inside the core.

The other important waveguide structure is a fiber with linear core and nonlinear cladding. For this fiber, the dispersion relation is monotonic. The mode index increases with the surface intensity. This will cause the
peak of both $TE_{01}$ and $TM_{01}$ to move towards the nonlinear cladding. If the
induced dielectric constant of the nonlinear cladding is less than the core
value, the peak of the field will remain inside the core region. However if
$\varepsilon_{nL} > \varepsilon_{co}$, the peak of the field will cross the core cladding interface and
position in the cladding region. The nonlinear field in this case is different
from its linear counterpart. Although they have the same field profiles, the
nonlinear field is mainly guided by the nonlinear cladding and its peak is
located in the cladding region. This reduces the effect of the linear core on
the guidance process. It also causes most of the field energy to concentrate
in the nonlinear cladding. For the $HE_{11}$, the peak of the field remains at the
origin. However the confinement of the field inside the core region
decreases as the intensity increases.

The nonlinear fiber models used throughout the thesis can be tested
experimentally. The value of nonlinear coefficient $\alpha$ used in the numerical
examples is larger than the practical values of the nonlinear materials (table
2.1). However, this can be compensated through the term $\alpha|E|^2$. The
important observation is that the total power required is limited and can be
delivered by practical laser sources.

FUTURE WORK

The recursive scheme developed in this thesis is applicable to graded-
index as well as nonlinear fiber. This solves a large number of fiber problems in which the core or the cladding can be taken nonlinear, linear or graded-index. A combination of nonlinearity and inhomogeneity can also be treated with the same algorithm after making the necessary modification in the array containing the dielectric constant.

Another class of important problems is the one in which the metal is used as a cladding or an outer jacket, with the core considered nonlinear. In this thesis different linear structures have been covered including two, three and four layers cylindrical waveguide. The future investigation should replace one of the dielectric layers by a nonlinear materials and study the effect of nonlinearity on the attenuation characteristics of metal-clad optical fiber.

This thesis covers mainly the first three lowest order modes $HE_{11}$, $TE_{01}$ and $TM_{01}$. Although the higher order TE modes have been investigated in some cases, the work should be extended to cover highest order modes. In the nonlinear optical fiber, light intensity will control the number of modes supported by the fiber.

The nonlinear materials used in this thesis are characterized by a self-focusing effect, in which the nonlinear coefficient is positive. This gives rise to the dielectric constant as the intensity increases. The same computer programs can be used to treat materials featuring a self-defocusing effect. It
is of interest to study a fiber in which the core material is self-focusing and the cladding material is self-defocusing. A fiber with this configuration is expected to have field profiles which are very well confined in the core region.
REFERENCES


12) "Material for Nonlinear Optics - Chemical Perspectives" Edited by S. R.


APPENDICES

C THE FOLLOWING 3 PROGRAMS ARE USED TO OBTAIN THE RESULTS OF CHAPTER 4
C
C
C
PROGRAM 1 TEMC
C
C
C
C THIS PROGRAM SOLVES FOR THE COMPLEX MODE EFFECTIVE INDEX IN A 4 LAYER
C CIRCULAR WAVEGUIDE STRUCTURE. THE OUTER LAYER IS FORMED OF METAL.
C WITH COMPLEX PERMITTIVITY. THE WAVEGUIDE PARAMETERS ARE DEFINED
C AS FOLLOWS
C R1 = THE RADIUS OF THE INNERMOST LAYER
C C = RATIO BETWEEN THE RADIUS OF THE SECOND LAYER TO THAT OF THE
C INNERMOST LAYER.
C DELR = THICKNESS OF THE BUFFER LAYER
C REFSQ = THE SQUARE OF THE EFFECTIVE INDEX
C EPSCO = DIELECTRIC CONSTANT OF THE INNERMOST LAYER
C EPSCL = DIELECTRIC CONSTANT OF THE SECOND LAYER
C EPSCL2 = DIELECTRIC CONSTANT OF THE THIRD LAYER (THE BUFFER LAYER)
C EPSCL0 = DIELECTRIC CONSTANT OF THE OUTER METALLIC LAYER.
C THE PROGRAM IS DEVELOPED TO WORK ON PC WITH MS FORTRAN VER. 5.
C THE SUBROUTINE FN IS USED FOR CALCULATING THE EIGENVALUE EQUATION
C THE SUBROUTINE MULLER IS USED TO FIND THE EIGENVALUE. IT IS CALLED
C BY THE MAIN PROGRAM. AN INITIAL GUESS IS REQUIRED BY MULLER
C TO FIND THE ROOT OF THE EIGENVALUE EQUATION.
C THE FUNCTIONS BKN, BNK, BNN ARE COMPLEX BESSEL FUNCTION. THESE
C FUNCTION ARE CALLED BY THE SUBROUTINE FN.
C THE SAME PROGRAM CAN BE USED TO SOLVE THE 3-LAYER STRUCTURE (DELR=0)
C AND 2-LAYER STRUCTURE (C=1, DELR=0).
C THE PROGRAM IS WRITTEN FOR TE MODES.
C IT CAN ALSO BE USED TO PLOT THE FIELD PROFILE (EIGEN FUNCTION)
C CORRESPONDING TO CERTAIN EIGENVALUE.
$DEBUG
$LARGE
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2),VALUE(2),NE
EXTERNAL FN
LOGICAL FNREAL
C FNREAL = .FALSE.
FNREAL = .TRUE.
C ENTER GUESS
WRITE(*,') 'ENTER GUESS'
READ*,*ZEROS(1)
OPEN(UNIT=6,STATUS='OLD',FILE='OUT1')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
MAXIT = 15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
EP1 = 1.0D-30
EP2 = 1.0D-40
CALL MULLER (FN,FNREAL,ZEROS,N,NPREV,MAXIT,EP1,EP2)
DO 312 I=1,N
CALL FN(ZEROS(I),VALUE(I))
WRITE('^')
WRITE('^,ZEROS(I),VALUE(I)
NE=CD(SQRT(ZEROS(I))
WRITE('^,NE
312 CONTINUE
STOP
END
C SUBROUTINE FN(REFSQ,FY)
C SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL'8 (A-H,O-Z)
COMPLEX'16 B1(200),B2(200),ALPHA(200),ARG(200)
COMPLEX'16 BJ01(200),BJ11(200),BJ02(200),BJ12(200)
COMPLEX'16 EPS(200),H2(200),HR(200)
COMPLEX'16 BY0(200),BY11(200),BY02(200),BY12(200)
COMPLEX'16 EPS(200),COKSQ(200),REFSQ,BJN,BKN
COMPLEX'16 EPS(200),Betasq,beta,bs2(3),bs1(3),bs3(3)
COMPLEX'16 EPS(200),EPS(200),H2,H3,H4,H5
COMPLEX'16 FZ,X1,X2,X3,X4,BK1A,BK0A,G
COMPLEX'16 T1,T2,T12,T22,T23,T28,T29,F9
DIMENSION DIS(200)
PI = 2.0D*DASIN(1.0D)
R1 = 3.0D-6
C = 1.2D0
K = 200
L = 50
WL = 0.6328D-6
C0 = 3.0D8
UB = PI*4.0D-7
DELR = 0.220D-6
G = DCMLX(0.0D0,1.0D0)
R2 = C*R1 + DELR
EPSCL = DCMLX(2.250D0,0.0D0)
C EPSCL2 = DCMLX(1.69D0,0.0D0)
EPSCL2 = DCMLX(2.890D0,0.0D0)
EPSCO = DCMLX(2.2801D0,0.0D0)
EPSCLO = DCMLX(-10.30D0,1.0D0)
C EPSCLO = DCMLX(-47.00D0,16.3D0)
YY = L
XX = R1/YY
W = 2.0D0*PI*(CD/WL)
FSKSQ = (W/CO)^2.0D0
FSK = DSQRT(FSKSQ)
BETASQ = REFSQ*FSKSQ
BETA = CD/SQRT(BETASQ)
B1(1) = DCMLX(0.1D0,0.0D0)
B2(1) = DCMLX(0.0D0,0.0D0)
DO 55 I = 1, 50
EPS(I) = EPSCO
55 CONTINUE
DO 65 I = 51, 60
EPS(I) = EPSCL
65 CONTINUE
DO 75 I = 1, 60
DIS(I) = XX + (I-1)*XX
75 CONTINUE
EPS(61) = EPSCL2
DIS(61) = DIS(60) + DELR
COKSQ(1) = FSKSQ*EPS(1)
ALPHA(1) = COKSQ(1)-BETASQ
ARG(1) = CD/SQRT(ALPHA(1))
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 2 & K
Z2 = ARG(1)*DIS(1)
C CALL DCSBJNS(Z2,3,BS1)
BS1(1) = BJN(0,22)
BS1(2) = BJN(1,22)
BJ01(1) = BS1(1)
BJ11(1) = BS1(2)
HZ(1) = B1(1)*BJ01(1)
EPHI(1) = -(G*W*U0*ARG(1))*(B1(1)*BJ11(1))
HR(1) = -(BETA*ARG(1))*(B1(1)*BJ11(1))
WRITE(6,*)(DIS(1),DIMAG(EPHI(1))
C DO 24 M=2,K
  COKS(M) = FSKSQ*EPS(M)
  ALPHA(M) = BETASQ-COKS(M)
  ARG(M) = CDSQRT(ALPHA(M))
  Z1 = ARG(M)**DIS(M-1)

C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
  BS2(1) = BIN(0,Z1)
  BS2(2) = BIN(1,Z1)
  H2 = BKN(0,Z1)
  H3 = BKN(1,Z1)
  BJ02(M) = BS2(1)
  BJ12(M) = BS2(2)
  BV02(M) = H2
  BV12(M) = H3
  T1 = HZ(M-1)**BY12(M) - (ARG(M)**(G**W**U00)**BY02(M)**EPII(M))
  T2 = BJ12(M)**BY02(M) + BJ02(M)**BY12(M)
  B1(M) = T1/T2
  B2(M) = (HZ(M-1)-B1(M)**BJ02(M))/BY02(M)
  Z3 = ARG(M)**DIS(M)

C CALLING BESSEL FUNCTION I OF ORDER 0 AND 1
  BS3(1) = BIN(0,Z3)
  BS3(2) = BIN(1,Z3)
  H4 = BKN(0,Z3)
  H5 = BKN(1,Z3)
  BJ01(M) = BS3(1)
  BJ11(M) = BS3(2)
  BY01(M) = H4
  BY11(M) = H5
  HZ(M) = B1(M)**BJ01(M) + B2(M)**BY01(M)
  EPHI(M) = -(G**W**U0*ARG(M))**B1(M)**BJ11(M)-B2(M)**BY11(M))
  HR(M) = -(BETA/ARG(M))**(B1(M)**BJ11(M)-B2(M)**BY11(M))
  WRITE(6,**DIS(M),DIMAG(EPHI(M))
  IF(DABS(DIS(M)-R2),LT,1.0D-10) GO TO 57

24 CONTINUE

C 57 FR = BETASQ_EPSCL0*FSKSQ
  Z3 = CDSQRT(FR)
  Z2 = Z8**DIS(M)

C CALLING BESSEL FUNCTION K OF ORDERS 0,1 FOR THE LAST LAYER
  BKDA = BKN(0,Z3)
  BK1A = BKN(1,Z3)

C SOLVING THE EIGENVALUE EQUATION FOR TE MODES
X1 = B1(M) + B01(M) + B2(M) + BY01(M)
X2 = B1(M) + B11(M) - B2(M) + BY11(M)
X3 = 2*B0A
X4 = ARG(M) + BK1A
FZ = 1.000 + (X2*X3)(X1*X4)
C FZ = (X2*X3)+(X1*X4)
C FZ = (X1/X3)+(X2/X4)
WRITE(*)FZ
RETURN
END

C SUBROUTINE MULLER DETERMINES UP TO N ZEROS OF THE
C FUNCTION SPECIFIED BY FN, USING QUADRATIC INTERPOLATION.
C I.E., MULLER'S METHOD.
C LIST OF DEFINITION
C FN = NAME OF A SUBROUTINE, OF THE FORM FN(Z,FZ) WHICH,
C FOR A GIVEN Z, RETURNS F(Z), MUST APPEAR IN AN EXTERNAL
C STATEMENT IN THE CALLING PROGRAM.
C FNREAL A LOGICAL VARIABLE. IF TRUE, ALL APPROXIMATIONS
C ARE TAKEN TO BE REAL, ALLOWING THIS ROUTINE TO BE
C USED EVEN IF F(Z) IS ONLY DEFINED FOR REAL Z.
C ZEROS(1),...,ZEROS(NPREV) CONTAINS PREVIOUSLY FOUND
C ZEROS (IF NPREV.GT.0)
C ZEROS(NPREV+1),...,ZEROS(N) CONTAINS FIRST GUESS FOR
C THE ZEROS TO BE FOUND.
C MAXIT MAXIMUM NUMBER OF FUNCTION EVALUATIONS ALLOWED.
C PER ZERO.
C EP1 ITERATION IS STOPPED IF ABS(FZ) .LT. EP1 'ABS(ZR).
C WITH H = LATEST CHANGE IN ZEROS ESTIMATE ZERO.
C EP2 ALTHOUGH THE EP1 CRITERION IS NOT MET, ITERATION
C IS STOPPED IF ABS(FZ) .LT. EP2.
C N TOTAL NUMBER OF ZEROS TO BE FOUND.
C NPREV NUMBER OF ZEROS TO BE FOUND.
C ZEROS(NPREV+1),...,ZEROS(N) APPROXIMATIONS TO ZEROS.

SUBROUTINE MULLER(FN,FNREAL,ZEROS,N,NPREV,MAXIT,EP1,EP2)
    IMPLICIT REAL*8(A,H,O-Z)
    EXTERNAL FN
    LOGICAL FNREAL
    INTEGER MAXIT,N,NPREV,KOUNT
    COMPLEX*16 ZEROS(N),C,DEN,DIVDF1,DIVDF2,DIVDF1P,FZR
    COMPLEX*16 FZRDFL,FZRPRV,H,ZERO,SQR
    C COMPLEX*16 FZRDFL,FZRPRV,H,ZERO,SQR
    EPS1 = DMAX1(EP1, 1.0D-30)
    EPS2 = DMAX1(EP2, 1.0D-40)
DO 200 I = NPREV + 1, N
KOUNT = 0
1 ZERO = ZEROS(I)
C COMPUTE FIRST THREE ESTIMATES FOR ZERO AS
C ZEROS(I) + .5D-6, ZEROS(I) -.5D-6, ZEROS(I).
H = .5D-6
CALL DFLATE(FN, ZERO + .5D-6, I, KOUNT, FZR, DVDF1P, ZEROS, *1)
C CALL DFLATE(FN, ZERO -.5D-6, I, KOUNT, FZR, DVDF1P, ZEROS, *1)
CALL DFLATE(FN, ZERO, I, KOUNT, FZR, FZRPRV, ZEROS, *1)
HPREV = -1.0D-6
DVDF1P = (FZRPRV - DVDF1P) / HPREV
CALL DFLATE(FN, ZERO, I, KOUNT, FZR, FZRDFL, ZEROS, *1)
40 DVDF1 = (FZRDFL - FZRPRV) / H
DVDF2 = (DVDF1 - DVDF1P) / (H + HPREV)
HPREV = H
DVDF1P = DVDF1
C = DVDF1 + H * DVDF2
SQR = C * C - 4.0 * FZRDFL * DVDF2
IF (FNREAL .AND. REAL(SQR) .LT. 0.) SQR = 0.
SQR = CDGRG(SQR)
IF (REAL(C) * REAL(SQR) + DIMAG(SQR) .LT. 0.) THEN
   DEN = C + SQR
ELSE
   DEN = C + SQR
END IF
IF (CDABS(DEN) .LE. 0.0D0) DEN = 1.0D0
H = -2.0 * FZRDFL / DEN
FZRPRV = FZRDFL
ZERO = ZERO + H
IF (KOUNT .GT. MAXIT) GO TO 99
70 CALL DFLATE(FN, ZERO, I, KOUNT, FZR, FZRDFL, ZEROS, *1)
C CHECK FOR CONVERGENCE.
IF (CDABS(H) .LT. EPS1 * CDABS(ZERO)) GO TO 99
IF (DMAX1(CDABS(FZR), CDABS(FZRDFL)) .LT. EPS2) GO TO 99
C CHECK FOR CONVERGENCE.
IF (CDABS(FZRDFL) .GE. 10.0 * CDABS(FZRPRV)) THEN
   H = H / 2.
   ZERO = ZERO - H
ELSE
   GO TO 70
END IF
99 ZEROS(I) = ZERO
200 CONTINUE
RETURN
END
SUBROUTINE DFLATE (FN,ZERO,I,KOUNT,FZERO,FZRDFL,ZEROS)
C TO BE CALLED BY MULLER.
IMPLICIT REAL*8 (A-H,O-Z)
EXTERNAL FN
INTEGER I,KOUNT,J
COMPLEX*16 FZERO,FZRDFL,ZERO,ZEROS(I), DEN
KOUNT = KOUNT + 1
CALL FN (ZERO,FZERO)
FZRDFL = FZERO
IF (I .LT. 2) RETURN
DO 10 J = 2, I
DEN = ZERO-ZEROS(J-1)
IF (CABS(DEN), EQ, 0.000) THEN
   ZEROS(I) = ZERO*1.001
   RETURN1
ELSE
   FZRDFL = FZRDFL/DEN
END IF
10 CONTINUE
RETURN
END
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
COMPLEX*16 FUNCTION BJN(N,Z)
COMPLEX*16 Z,T1,T2
REAL *8 FCTN
T1 = -Z*Z/4
K = 0
T2 = 1
BJN = T2
200 K = K + 1
T2 = T2*T1/K/(N+K)
BJN = BJN + T2
IF (CABS(T2), LT, 1.0-19) GO TO 120
GO TO 200
120 IF (N .GT. 0) THEN
   FCTN = 1.0
   DO 130 J = 1, N
   FCTN = FCTN*T
   BJN = BJN*(Z/2)**N/FCTN
   END IF
   RETURN
END
C

COMPLEX'16 FUNCTION BIN(N,Z)
COMPLEX'16 Z,T1,T2
REAL '8 FCTN
T1=Z'/2/4
K=0
T2=1
BIN=T2
200 K=K+1
T2=T2*T1/K/(N+K)
BIN=BIN+T2
IF(CABS(T2),LT,1,5E-19)GO TO 120
GO TO 200
120 IF(N,GT,0)THEN
FCTN=1.00
DO 130 I=1,N
130 FCTN=FCTN*I
BIN=BIN*(Z'/2)'*N/FCTN
END IF
RETURN
END

C

COMPLEX'16 FUNCTION BKN(N,Z)
COMPLEX'16 Z,T,T1,T2,S1,S2,S3,Z2N,BIN
REAL '8 FCTN,PK,PNK,P1,GAM
P1=3.14159265358979323846D0
GAM=0.57721 58645 69689 61279 00532 86060 0
Z2N=(Z'/2)'*N
T=Z'/2/4
IF(N,EQ,0)THEN
C
C ORDER 0
C
K=1
T1=T
S1=T1
PK=1
30 K=K+1
T1=T1*(T)/K/K
PK=PK+1.00/K
S1=S1+PK'T1
IF(CABS(T1),LT,1.0E-19)GO TO 40
GO TO 30
40 BKN=-(CDLOG(Z'/2)+GAM)*BIN(0,Z')/S1
ELSE
C
C ORDER N
C S2,S3
C
K=0
T2=1
S2=T2
PK=-GAM
PNK=-GAM
DO 50 I=1,N
50 PNK=PNK+1.D0/I
S3=PK+PNK
200 K=K+1
T2=T2*T/K/(N+K)
PK=PK+1.D0/K
PNK=PNK+1.D0/(N+K)
S2=S2+T2
S3=S3+(PK+PNK)*T2
IF(CABS(T2),LT,1.E-19)GO TO 120
GO TO 200
120 FCTN=1.D0
DO 130 I=1,N
130 FCTN=FCTN*I
C
C S1
C
T1=FCTN/N
S1=T1
IF(N.GT.1)THEN
DO 150 I=1,N-1
T1=T1*(T-I)/I*(N-I)
150 S1=S1+T1
END IF
C
C FIND BKN
C
BKN=S1/2.D0/Z2N+(-1)**(N+1))*CLOG(Z/2)**2N/FCTN**5D
& +(-1)**N/2.D0/Z2N/FCTN**S3
END IF
RETURN
END
C
C
C PROGRAM 2 TMMC
C
C THIS SUBROUTINE IS SIMILAR TO THE SUBROUTINE FN USED IN THE FIRST
C PROGRAM EXCEPT THAT IT IS WRITTEN FOR TM MODES. IT MUST BE USED
C IN CONJUNCTION WITH MULLER’S SUBROUTINE AND BESSEL FUNCTIONS AS
C EXPLAINED IN THE PREVIOUS PROGRAM.
C
SUBROUTINE FN(REFSQ,FZ)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 B1(200),B2(200),ALPHA(200),ARC(200)
COMPLEX*16 BJ1(200),BJ11(200),BJ02(200),BJ12(200)
COMPLEX*16 HPHI(200),EZ(200),ER(200)
COMPLEX*16 BY1(200),BY11(200),BY12(200),BY13(200),BY14(200)
COMPLEX*16 EPS(200),COKSQ(200),REFSQ,BJN,BIN,BKN
COMPLEX*16 EPSCL,BETASQ,BETASQ,BS2(3),BS1(3),BS3(3)
COMPLEX*16 EPSCO,EPSCL,H2,H3,H4,H5
COMPLEX*16 FZ,X1,X2,X3,X4,BK1A,BK2A,G
COMPLEX*16 T1,T2,T3,T4,T5,F9
DIMENSION DIS(200)
PI=2.0D0*DASIN(1.0D0)
R1=6.0D0-6
C=1.2D0
DELR=0.000D-6
R2=C*R1+DELR
K=200
L=50
WL=0.6328D-6
C0=3.0D8
UD=PI*4.0D7
G=DCMPLX(0.0D0,1.0D0)
EPSCL=DCMPLX(2.2500D0,0.0D0)
EPSCL2=DCMPLX(2.8900D0,0.0D0)
EPSCL3=DCMPLX(2.890100D0,0.0D0)
EPSCL4=DCMPLX(-10.3000D0,1.0D0)
YY=L
XX=R1/YY
W=2.0D0*PI*(C0/WL)
FSKSQ=((W/C0)**2.0D0)
FSK=DSQRT(FSKSQ)
BETASQ=REFSQ*FSKSQ
BETA=CDSQRT(BETASQ)
B1(1) = DCMPLX(0,1D0,0,0D0)
B2(1) = DCMPLX(0,0D0,0,0D0)
DO 55 I = 1,50
EPS(I) = EPSCO
55 CONTINUE
DO 65 I = 51,60
EPS(I) = EPSCL
65 CONTINUE
DO 75 I = 1,60
DIS(I) = XX + (I-1)**XX
75 CONTINUE
EPS(61) = EPSCL2
DIS(61) = DIS(60) + DELR
COKSQ(1) = FSKSQ*EPS(1)
ALPHA(1) = COKSQ(1)-BETASQ
ARG(1) = CDSQR(ALPHA(1))
C
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 2 & K

Z2 = ARG(1)**DIS(1)

C CALL BESSEL FUNCTION J OF ORDER 0.1
BS1(1) = BJN(0,2Z)
BS1(2) = BJN(1,2Z)
BJ0(1) = BS1(1)
BJ1(1) = BS1(2)
EZ(1) = B1(1)**BJ0(1)
HPHI(1) = ((C**COKSQ(1))/((W**U0**ARG(1)))*B1(1)**BJ1(1))
ER(1) = -(BETA**ARG(1))**B1(1)**BJ1(1))
WRITE (6,** DIS(1),DIMAG(HPHI(1))

C
DO 24 M = 2,K
COKSQ(M) = FSKSQ*EPS(M)
ALPHA(M) = BETASQ-COKSQ(M)
ARG(M) = CDSQR(ALPHA(M))
Z1 = ARG(M)**DIS(M-1)

C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0.1
BS2(1) = BIN(0,Z1)
BS2(2) = BIN(1,Z1)
H2 = BKH(0,Z1)
H3 = BKH(1,Z1)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3
T1 = EZ(M-1)*BY12(M) + ((ARG(M)**W**U0)/((C**COKSQ(M)))*BY02(M)**H1*M 1)
T2 = B12(M) \times BY02(M) + BJ02(M) \times BY12(M)
B1(M) = T1/T2
B2(M) = (EZ(M) - B1(M) \times BJ02(M))/BY02(M)
Z3 = ARG(M) \times DIS(M)

CALLING BESSEL FUNCTIONS I AND K OF ORDER 0, 1
BS3(1) = B1N(0,Z3)
BS3(2) = B1N(1,Z3)
H4 = BK1N(0,Z3)
H5 = BK1N(1,Z3)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
EZ(M) = B1(M) \times BJ01(M) + B2(M) \times BY01(M)
HPHI(M) = ((G^COKSQ(M))/(W^U0^ARG(M))) \times (B1(M) \times BJ11(M) - R2(M) \times BY11(M))
ER(M) = (BETA/ARG(M)) \times (B1(M) \times BJ11(M) - B2(M) \times BY11(M))
WRITE (6,*) DIS(M), DIMAG(HPHI(M))
IF(DABS(DIS(M) - R2), LT, 1.0D-10) GO TO 57

24 CONTINUE

57 F9 = BETASQ - EPSCL0^2*FSK0^2
Z8 = COSQRT(F9)
Z9 = Z8*DIS(M)

CALLING BESSEL FUNCTIONS K OF ORDERS 0, 1 FOR THE LAST LAYER
BK0A = BK1N(0,Z9)
BK1A = BK1N(1,Z9)

CALLING THE EIGENVALUE EQUATION FOR TE MODES
X1 = B1(M) \times BJ01(M) + B2(M) \times BY01(M)
X2 = EPS(M) \times (B1(M) \times BJ11(M) - B2(M) \times BY11(M))
X3 = Z8*BK0A
X4 = EPSCL0^2*ARG(M) \times BK1A
FZ = 1.0D0 + (X1*Z4)/(X2*Z3)
C FZ = 1.0D0 + (X2*Z3)/(X1*Z4)
C FZ = (X2*Z3)/(X1*Z4)
C FZ = (X1/X3) + (X2/X4)
WRITE (*,*) FZ
RETURN
END
C THIS PROGRAM IS USED TO FIND THE COMPLEX EFFECTIVE INDEX OF THE HE11
C MODE IN A 4-LAYER STRUCTURE SIMILAR TO THAT GIVEN IN PROGRAMS 1,2
C GIVEN ABOVE. IT IS USED IN CONJUNCTION WITH MULLER'S SUBROUTINE.
C BESSEL FUNCTION J AND I USED IN THIS PROGRAM ARE CALLED FROM THE
C IMSL LIBRARY OF THE DPC IN KFUPM. THE K FUNCTION IS CALLED FROM THE
C SUBROUTINE BESK WHICH IS SHOWN BELOW. THE WAVEGUIDE PARAMETERS
C ARE DEFINED IN THE ABOVE TWO PROGRAMS (1,2). THIS PROGRAM REQUIRE
C AN INITIAL GUESS FOR THE EIGENVALUE. IT ALSO NEEDS AN INITIAL VALUE
C OF THE AMPLITUDE OF THE MAGNETIC FIELD H2 (B1(1)). THIS AMPLITUDE IS
C DEFINED BY CHANGING THE VALUE OF DELTA BY SMALL INCREMENTS, WHEN
C THE EIGENVALUES OBTAINED USING THE MATCHING OF EPHI AND THE MATCHING
C HPHI ARE THE SAME, THIS IS CONSIDERED THE SOLUTION FOR THE HE11 MODE
C AND THE AMPLITUDE B1(1) IS CONSIDERED THE PROPER AMPLITUDE OF THE
C MAGNETIC FIELD CORRESPONDING TO THE INITIAL VALUE OF THE ELECTRIC
C FIELD A1(1).
C
C IMPLICIT REAL*8 (A-H,O-Z)
C COMPLEX*16 ZEROS(20),VALUE(20),HH,NE
C EXTERNAL FN
C LOGICAL FNREAL
C FNREAL=.FALSE.
C
C ENTER GUESS
C WRITE(7,25)
25 FORMAT(// 'ENTER GUESS')
READ(*,*)HH
ZEROS(1)=HH
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN.
NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
MAXIT=20
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
EP1=10.0**(-30)
EP2=10.0**(-40)
CALL MULLER (FN,FNREAL,ZEROS,N,NPREV,MIXIT,EP1,EP2)
DO 312 I=1,N
CALL FN(ZEROS(I),VALUE(I))
WRITE(7,*)
WRITE(7,*)ZEROS(I),VALUE(I)
NE=CDSQRZ(ZEROS(I))
WRITE(7,*)NE
312 CONTINUE
STOP
END

C SUBROUTINE RN(REFSQ,FY)

C SUBROUTINE RN (REFSQ,FZ)

SUBROUTINE RN (REFSQ,FZ)

IMPLICIT REAL*8 (A-H,O-Z)

COMPLEX*16 A1(200),A2(200),B1(200),B2(200),BY1P(200),BY1P(200)

COMPLEX*16 B1(200),B2(200),ALPHA(200),ARG(200)

COMPLEX*16 B1(200),B2(200),BY1P(200),BY1P(200)

COMPLEX*16 EPSCL,BETASQ,BETA,BS2(3),BS1(3),BS3(3)

COMPLEX*16 EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL,EPSCL

EPSCL=EPSCL(0.0D0,1.0D0)

EPSCL=EPSCL(2.2601D0,0.0D0)

EPSCL=EPSCL(2.2500D0,0.0D0)

EPSCL=EPSCL(2.2800D0,0.0D0)

EPSCL=EPSCL(-1.0300D0,1.0D0)

ETA=120.000*PI

EN=CDSSQRT(REFSQ)

YY=L

XX=R1/YY

W=2.000*PI*(CO/WL)

FSDK=(W/CO)**2.000

FSDK=DSQRT(FSDK)

BETASQ=REFSQ*FSDK

BETA=CDSSQRT(BETASQ)

C INTER AMPLITUDE OF ELECTRIC FIELD EZ

V1=100.000
A1(1) = DCMPLX(V1,0.0D0)
A2(1) = DCMPLX(0.0D0,0.0D0)
C INTER A GUESS FOR THE AMPLITUDE OF MAGNETIC FIELD HZ
V2 = V1*1.5D0/ETA
DELTA = 0.0022D0
B1(1) = DCMPLX(V2,0.00D0) + DELTA
B2(1) = DCMPLX(0.0D0,0.0D0)
DO 55 I = 1,L
EPS(I) = EPSCO
55 CONTINUE
DO 65 I = L+1,L+2
EPS(I) = EPSCL
65 CONTINUE
DO 75 I = 1,12
DIS(I) = XX + (I-1)*XX
75 CONTINUE
EPS(13) = EPSBF
DIS(13) = DIS(12) + DELR
COKSQ(1) = FSKSQ*EPS(1)
ALPHA(1) = COKSQ(1)-BETASQ
ARG(1) = CDQSRT(ALPHA(1))
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
Z2 = ARG(1)*DIS(1)
CALL DCBJNS(Z2,3,BS1)
CALL BESK(Z2,H0,H1)
BJ01(1) = BS1(1)
BJ11(1) = BS1(2)
BY01(1) = H0
BY11(1) = H1
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 1 & K
EZ(1) = A1(1)*BJ11(1) + A2(1)*BY11(1)
HZ(1) = B1(1)*BJ11(1) + B2(1)*BY11(1)
C CALCULATIONS OF THE DERIVATIVES OF J & K FUNCTION
BJ1P(1) = BS1(1)-(BS1(2)/Z2)
BY1P(1) = -H0*(H1/Z2)
EPHI(1) = (G/ALPHA(1))'*(BETA'*EZ(1)/DIS(1) + W'*U0*B1(1)'*ARG(1)*R1P(1))
ER(1) = -(G/ALPHA(1))'*W'*U0*HZ(1)/DIS(1) + BETA'*ARG(1)'*A1(1)'*R1P(1)
HPHI(1) = -(G/ALPHA(1))'* (BETA' * HZ(1) / DIS(1)) + COKSQ(1)'*A1(1)'*R1P(1)'*ARG(1)'(W'*U0))
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
DO 24 M = 2,K
J = M-1
COKSQ(M) = FSKSQ*EPS(M)
ALPHA(M) = BETASQ-COKSQ(M)
ARG(M) = CDSQRT(ALPHA(M))
Z1 = ARG(M) * DIS(M-1)
CALL DCBINS(Z1,3,BS2)
CALL BESK(Z1,H2,H3)
C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3
C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTION
BJ1P(M) = BS2(1) + (BS2(2)/Z1)
BY1P(M) = -H2 - (H3/Z1)
T1 = -(G*ALPHA(M) * HPHI(M-1) + (BETA * HZ(M-1) * DIS(M-1))) * BY12(M)
T2 = BJ1P(M) * BY12(M) - BJ12(M) * BY1P(M)
T3 = (COKSQ(M) * ARG(M) * EZ(M-1) * BY1P(M) / W * U0)
T4 = -(G*ALPHA(M) * EPHI(M-1) - (BETA * EZ(M-1) * DIS(M-1))) * BY12(M)
T5 = (W * U0 * ARG(M) * HZ(M-1) * BY1P(M))
A1(M) = (T1 - T3) / (T2 * COKSQ(M) * ARG(M) / (W * U0))
A2(M) = (EZ(M-1) - A1(M) * BJ12(M)) / BY12(M)
B1(M) = -T4 + T5 / (T2 * ARG(M) * W * U0)
B2(M) = (HZ(M-1) - B1(M) * BJ12(M)) / BY12(M)
Z3 = ARG(M) * DIS(M)
CALL DCBINS(Z3,3,BS3)
CALL BESK(Z3,H4,H5)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTIONS
BJ2P(M) = BS3(1) + (BS3(2)/Z3)
BY2P(M) = -H4 - (H5/Z3)
EZ(M) = -(A1(M) * BJ11(M) + A2(M) * BY11(M))
HZ(M) = -(B1(M) * BJ11(M) + B2(M) * BY11(M))
EPHI(M) = -(G/ALPHA(M)) * ((BETA / DIS(M)) * EZ(M) + W * U0 * ARG(M)) * (G1(M) * RDP * M + B2(M) * BY2P(M))
HPHI(M) = + (G/ALPHA(M)) * ((COKSQ(M) * ARG(M) / (W * U0)) * (A1(M) * RDP(M) + A2(M) * BY2P(M)) * W
* M) * BY2P(M) + (BETA / DIS(M)) * HZ(M))
ER(M) = + (G/ALPHA(M)) * ((BETA * ARG(M)) * (A1(M) * BJ1P(M) + A2(M) * BY1P(M)) * W
* U0)) * HZ(M) / DIS(M))
IF (DBS(DS(M)-R2).LT.1.0D-10) GO TO 57
24 CONTINUE
C
57 F9 = BETASQ * EPSCLQ * FSK3Q
F10 = EPSCLQ * FSK3Q
Z8 = CDSQRT(F9)
Z9 = Z8 * DIS(M)
S1 = CDSQRT(EPSCL0)

CALLING BESSEL FUNCTIONS K OF ORDERS 0, 1 FOR THE LAST LAYER
CALL BESK(Z9, H6, H7)

CALCULATIONS OF THE DERIVATIVE OF K FUNCTION
BKP = H6 - H7 / Z9
XY = -1.0DD0 / 1.0DD0 / Z9

SOLVING THE EIGENVALUE EQUATION FOR HE11 MODE
X1 = -(G/F9) * ((BETA/DIS(M))^2 * EZ(M) + WU01 * Z81 * HZ(M) * BKP * H7)
X2 = -(G/F9) * ((BETA/DIS(M))^2 * HZ(M) + (F10 * Z81 * EZ(M) * BKP * H7 * WU10))

MATCHING OF EPRI

FZ = 1.0DD0 * (EPHI(M) / X1)
FZ = EPRI(M) * X1

MATCHING OF HPHI

FZ = 1.0DD0 * (HPHI(M) / X2)
FZ = HPHI(M) + X2

WRITE('T', FZ)
RETURN
END

SUBROUTINE BESK(Z, BK0, BK1)

IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 Z, T, GO2, G1Z, BK0, BK1, T2, T3, SUM2, SUM3, C2
DIMENSION HS(12)
C = 0.57721566490
PI = 4.0DD0 * DATAN(1.0DD0)
T = 1.0DD0
IF(CDABS(Z), LE, 1.0DD0) GO TO 20

GO2 = 1.253314DD0 + 0.456864DD0 * T + 0.087128DD0 * (T ** 2) + 0.091369DD0 * (T ** 3)
** 3) + 0.134458DD0 * (T ** 4) - 0.229538DD0 * (T ** 5) + 0.379210DD0 * (T ** 6) - 0.254
** 7) * T7 * T ** 7 + 0.557356DD0 * (T ** 8) - 0.428623DD0 * (T ** 9) + 0.218451DD0 * T
** 10) - 0.8688097DD0 * (T ** 11) + 0.0913953DD0 * (T ** 12)

C = 1.253314DD0 + 0.456864DD0 * T + 0.087128DD0 * (T ** 2) + 0.091369DD0 * (T ** 3)
** 3) + 0.134458DD0 * (T ** 4) - 0.229538DD0 * (T ** 5) + 0.379210DD0 * (T ** 6) - 0.254
** 7) * T7 * T ** 7 + 0.557356DD0 * (T ** 8) - 0.428623DD0 * (T ** 9) + 0.218451DD0 * T
** 10) - 0.8688097DD0 * (T ** 11) + 0.0913953DD0 * (T ** 12)
***10)+0.07880001D0*(T**11)-0.0108242000D0*(T**12)
BK0=CDEXP(-Z)*CDSQRT(T)*G02
BK1=CDEXP(-Z)*CDSQRT(T)*G12
GO TO 30
20 SUM1=0.0D0
DO 77 L=1,10
   T1=1.0D0/V
   SUM1=SUM1+T1
   HS(L)=SUM1
77 CONTINUE
   C2=(C+CDLOG(0.5D0*Z))
   SUM2=DCMPLX(0.0D0,0.0D0)
   DO 88 J=1,8
      T2=((0.5*Z)**(2*J)+HS(J)-C2)/((DFAC(J))**2)
      SUM2=SUM2+T2
88 CONTINUE
   BK0=-C2+SUM2
   SUM3=DCMPLX(0.0D0,0.0D0)
   DO 44 I=1,8
      T3=((0.5*Z)**(2*I-1)+HS(I))/(C2+HS(I))/((DFAC(I))**2)
      SUM3=SUM3+T3
44 CONTINUE
   BK1=T+SUM3
30 RETURN
END

THE FOLLOWING 4 PROGRAMS ARE USED TO OBTAIN THE RESULTS OF CHAPTER 5

PROGRAM 4 TENL RECURSIVE

THIS PROGRAM IS USED TO FIND THE EIGENVALUES AND EIGEN FUNCTIONS
OF TE MODES IN A FIBER HAVING NONLINEAR CORE AND LINEAR CLADDING.
THE PROGRAM IS DEVELOPED USING THE RECURSIVE SCHEME APPROACH.
z IS USED IN CONJUNCTION WITH MULLER'S SUBROUTINE AND THE COMPLEX
BESSEL FUNCTIONS. IT REQUIRES THE MAGNETIC FIELD AMPLITUDE OF 1HE
INNERMOST LAYER IN ADDITION TO THE INITIAL GUESS FOR THE EIGENVALUE
THE PROGRAM ALSO CALCULATES THE TOTAL MODAL POWER PROPAGATED.
THE PARAMETERS USED ARE DEFINED AS FOLLOWS:
EPSCO = THE CORE DIELECTRIC CONSTANT (THE BACKGROUND VALUE)
EPSCL = THE LINEAR CLADDING DIELECTRIC CONSTANT
C  C = THE NONLINEAR COEFFICIENT
C   P = THE SATURATION PARAMETER
C   R1 = THE CORE RADIUS
C   L = THE NUMBER OF LAYERS IN THE NONLINEAR REGION
C   THE PROGRAM CAN BE MODIFIED TO SOLVE FOR THE EIGENVALUES AND
C   EIGEN FUNCTION IN THE CASE OF FIBER WITH LINEAR CORE AND NONLINEAR
C   CLADDING. THE ONLY MODIFICATIONS REQUIRED IS TO REPLACE THE
C   THE DIS(i) AND EPS(j) THEIR NEW VALUES. THESE VALUES ARE LISTED
C   DIRECTLY BELOW THE OLD VALUES AND MARKED BY A COMMENT SIGN (C)
C   ALSO THE IF STATEMENT WHICH TERMINATES THE PROGRAM AT THE CORE-
C   CLADDING INTERFACE SHOULD BE REPLACED BY ANOTHER ONE WHICH
C   TERMINATES THE PROGRAM AT DISTANCE LARGE ENOUGH TO MAKE THE EFFECT
C   OF THE ELECTRIC FIELD ON THE DIELECTRIC CONSTANT NEGIGIBLE.
C   THIS DISTANCE IS VARIABLE AND IT DEPENDS ON THE INPUT FIELD
C   AMPLITUDE FOR THE SECOND FIBER STRUCTURE, THE PROGRAM CALCULATES THE
C   ONLY THE POWER PROPAGATED IN THE NONLINEAR CLADDING. TO OBTAIN
C   THE TOTAL POWER OF THE MODE THE POWER PROPAGATED IN THE LINEAR
C   CORE SHOULD BE CALCULATED AND ADDED TO THAT OF THE CLADDING.
$DEBUG
$LARGE
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2),VALUE(2),NE
EXTERNAL FN
COMMON X
LOGICAL FNREAL
C   FNREAL=.FALSE.
   FNREAL=.TRUE.
C ENTER AMPLITUDE OF HZ
   WRITE(*,'('')' ENTER AMP,(HZ)
   READ(*,')X
C ENTER GUESS
   WRITE(*,'()' ENTER GUESS'
   READ(*,)ZEROS(1)
   OPEN(UNIT=6,STATUS='OLD',FILE='OUT3')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
   N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
   NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
   MAXIT = 15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
   EP1 = 10.0D-30
   EP2 = 10.0D-40
   CALL MULLER (FN,FNREAL,ZEROS,N,NPREV(MAXIT,EP1,EP2)
   DO 312 I=1,N
CALL FN(ZEROS(I),VALUE(I))
WRITE(\*,\*)
WRITE(\*,\*)ZEROS(I),VALUE(I)
NE=CDQRT(ZEROS(I))
WRITE(\*,\*)NE
312 CONTINUE
STOP
END

C SUBROUTINE FN(REFSQ,FY)
C
SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 B1(300),B2(300),ALPHA(300),ARG(300)
COMPLEX*16 BJ0(300),BJ1(300),BJ2(300),BJ3(300)
COMPLEX*16 EPHI(300),H2(300),HR(300)
COMPLEX*16 BY0(300),BY1(300),BY2(300),BY3(300)
COMPLEX*16 EPS(300),COKSQ(300),REFSQ,BJN,BKN,BIN
COMPLEX*16 EPSCL,BETA,SQ,BETA,BS2(300),BS1(300),BS3(300)
COMPLEX*16 EPSC1,H2,H3,H4,H5,FZ
COMPLEX*16 X1,X2,X3,X4,BK1A,BK0A,G
COMPLEX*16 T1,T2,T1,22,23,26,29,F9,210,H8,H9
DIMENSION DIS(300),P1(300),ETSQ(300)
COMMON X
PI=2.0*DASIN(1.0D0)
R1=4.0D-6
K=300
L=100
WL=0.528D-8
C0=3.0D8
U0=PI*4.0D-7
G=DCMPLX(0.0D0,0.0D0)
C=1.0D-8
P=20.0D0
EPSCL=DCMPLX(2.250000D0,0.0D0)
EPSCL=DCMPLX(2.250000D0,0.0D0)
YY=L
XX=R1/YY
W=2.0D0*PI*(C0/WL)
FSKSQ=((W/C0)**2.0D0).
FSK=DSQRT(FSKSQ)
BETASQ=REFSQ*FSJKSQ
BETA=CDQRT(BETASQ)
B1(1)=DCMPLX(X,0.0D0)
EPS(1) = EPSCO
DO 75 I = 1, K
  DI(S(I)) = XX + (I-1)*XX
C  DI(S(I)) = R1 + (I-1)*XX
75 CONTINUE
  COKSQ(1) = FSKSQ*EPS(1)
  ALPHA(1) = COKSQ(1)-BETASQ
  ARG(1) = CDOSQR(ALPHA(1))
C = CALCULATIONS FOR THE LAYERS BETWEEN 1 & K
Z2 = ARG(1)*DI(S(1))
C  CALLING BESSEL FUNCTION I OF ORDER 0,1
  BS1(1) = BJN(0, Z2)
  BS1(2) = BJN(1, Z2)
  BJ01(1) = BS1(1)
  BJ11(1) = BS1(2)
  HZ(1) = B1(1)*BJ01(1)
  EPHI(1) = -(G*W'-U0/ARG(1))*B1(1)*BJ11(1))
  HR(1) = -(BETA/ARG(1))*B1(1)*BJ11(1))
C = DO 24 M = 2, K
J = M-1
  ETSQ(J) = (CDABS(EPHI(J)))*2
  EPS(M) = EPSCL + (C*ETSQ(J))/(1.0D0 + (C*ETSQ(J))
C  EPS(M) = EPSCL + (C*ETSQ(J))/(1.0D0 + (C*ETSQ(J))
  COKSQ(M) = FSKSQ*EPS(M)
  ALPHA(M) = BETASQ-COKSQ(M)
  ARG(M) = CDOSQR(ALPHA(M))
  Z1 = ARG(M)*DI(S(M-1))
C  CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
  BS2(1) = BKN(0, Z1)
  BS2(2) = BKN(1, Z1)
  H2 = BKN(0, Z1)
  H3 = BKN(1, Z1)
  BJ02(M) = BS2(1)
  BJ12(M) = BS2(2)
  BY02(M) = H2
  BY12(M) = H3
  T1 = HZ(M-1)*BY12(M) - (ARG(M)/(G*W'-U0))*BY02(M)*EPHI(M-1)
  T2 = BJ12(M)*BY02(M) + BJ02(M)*BY12(M)
  B1(M) = T1/T2
  B2(M) = (HZ(M-1) - B1(M)*BJ02(M))/BY02(M)
  Z3 = ARG(M)*DI(S(M))
C  CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
BS3(1) = BIN(0,23)
BS3(2) = BIN(1,23)
H4 = BKN(0,23)
H5 = BKN(1,23)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
H2(M) = B1(M) * BJ01(M) + B2(M) * BY01(M)
EPHI(M) = -(G'W'U'D/ARG(M))' * [B1(M) * BJ11(M) - B2(M) * BY11(M)]
HR(M) = -(BETA/ARG(M)) * (B1(M) * BJ11(M) - B2(M) * BY11(M))
ETSQ(M) = (CDABS(EPHI(M)))^2
IF(DABS(DIS(M) - R1), LT.1.0D-10) GO TO 57
C IF(DABS(DIS(M) - 2.0D0*R1), LT.1.0D-10) GO TO 57
24 CONTINUE
C
57 F9 = BETASQ * EPSCL * FSKSQ
Z8 = CDSQRT(F9)
Z9 = Z8 * DIS(M)
C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYFR
BK0A = BKN(0.29)
BK1A = BKN(1.29)
C
C SOLVING THE EIGENVALUE EQUATION FOR NONLINEAR TE MODES
X1 = B1(M)' * BJ01(M) + B2(M)' * BY01(M)
X2 = B1(M)' * BJ11(M) - B2(M)' * BY11(M)
X3 = Z8 * BK0A
X4 = ARG(M)' * BK1A
FZ = 1.0D0 + (X2' X3) / (X1' X4)
C FZ = 1.0D0 + (X1' X4) / (X2' X3)
C FZ = X2' X3 + X1' X4
C FZ = (1.0D0 / (X2' X3) + 1.0D0 / (X1' X4))
C FZ = (X3 / X4) + (X1 / X2)
C FZ = (X4 / X3) + (X2 / X1)
DO 32 J = 1,100
WRITE(*,*) DIS(J)'(1.0D6), -DIMAG(EPHI(J))
32 CONTINUE
WRITE(*,*) FZ
C
C CALCULATION OF POWER
C
R = DIS(M)
DO 51 I = M + 1, M + 100
K2 = I - M
DELR = XX'0.10D0'K2
R = R + DELR
DIS(I) = R
Z10 = R'28
C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
H8 = BKN(0,Z10)
H9 = BKN(1,Z10)
HZ(I) = HZ(M)'H8/BK0A
EPhi(I) = EPhi(M)'H9/BK1A
HR(I) = HR(M)'H9/BK1A
R = R + DELR
51 CONTINUE
DO 52 I = 1,M + 100
P1(I) = (CDABS(EPhi(I))'CDABS(HR(I))'DIS(I)
52 CONTINUE
SUM = 0.0D0
DO 53 I = 1,M + 100 - 1
P2 = (P1(I) + P1(I + 1))'DIS(I - 1) - DIS(I)'0.5D0
SUM = SUM + P2
53 CONTINUE
PT = SUM'P1
WRITE(6,'(8,*)
WRITE(6,'(8,*)
RETURN
END
C
C
C PROGRAM 5 TENL SELF-CONSISTENT
C
C
C THIS PROGRAM IS USED TO FIND THE EIGENVALUES AND THE EIGEN FUNCTIONS
C OF A FIBER HAVING NONLINEAR CORE AND LINEAR CLADDING.
C THE PROGRAM IS DEVELOPED USING THE SELF-CONSISTENT APPROACH.
C IT IS USED IN CONJUNCTION WITH MULLER'S SUBROUTINE AND THE COMPLEX
C BESSEL FUNCTIONS GIVEN IN PROGRAM 1. TO START THE PROGRAM AN INITIAL
C INTENSITY PROFILE IS REQUIRED IN ADDITION TO THE GUESSED FOR THE MODE
C EFFECTIVE INDEX. THE PRODUCED FIELD PROFILE IS USED INTERMEDIATELY TO
C THE PROGRAM TO MODIFY THE REFRACTIVE INDEX PROFILE AND PRODUCF
C ANOTHER FIELD PROFILE AND SO ON. THE NUMBER OF ITERATION REQUIRED
C IS CHOSEN THROUGH THE INDEX OF THE DO LOOP WHICH IS 10 FOR THE
C EXAMPLE (DO 75 ...). THE PROGRAM CAN ALSO BE USED FOR THE FIBER
C STRUCTURE HAVING LINEAR CORE AND NONLINEAR CLADDING. THE REQUIRED
C MODIFICATION ARE SIMILAR TO THOSE MENTIONED IN PROGRAM 4 IN
C ADDITION TO CHANGING THE INTERFACE VALUES FROM E(1001) TO E(11)
C AND FROM ETSQ(100) TO ETSQ(1)
$DEBUG
$LARGE
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2),VALUE(2),NE
DIMENSION E(300),ETSQ(300)
C COMMON E,ET
COMMON E,ETSQ
EXTERNAL FN
LOGICAL FNREAL
C FNREAL=.FALSE.
FNREAL=.TRUE.
C ENTER GUESS
WRITE(*,*)'ENTER GUESS'
READ(*,*)ZEROS(1)
OPEN(UNIT=5,STATUS='OLD',FILE='DATA')
OPEN(UNIT=6,STATUS='OLD',FILE='OUT')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
MAXIT=15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
EP1=10.0D-30
EP2=10.0D-40
DO 74 I=1,100
READ(5,*E(I))
74 CONTINUE
DO 75 I=1,10
CALL MULLER(FN,FNREAL,ZEROS,N,PREV,MAXIT,EP1,EP2)
CALL FN(ZEROS(1),VALUE(1))
NE=CDSQRT(ZEROS(1))
WRITE(*,*)ZEROS(1),VALUE(1)
WRITE(*,*)NE
WRITE(*,*)
DO 31 M=1,100
E(M)=ETSQ(M)
31 CONTINUE
ZEROS(1)=NE**2
75 CONTINUE
DO 76 I=1,100
WRITE(6,*E(I))
76 CONTINUE
STOP
END

C SUBROUTINE FN(REFSQ,FY)

SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 B1(300),B2(300),ALPHA(300),ARG(300)
COMPLEX*16 BJ1(300),BJJ1(300),B102(300),BJJ12(300)
COMPLEX*16 EPHI(300),H(300),HR(300)
COMPLEX*16 BY01(300),BY11(300),BY02(300),BY12(300)
COMPLEX*16 EPS(300),CO(300),X,REFSQ,REN,BN,BKN
COMPLEX*16 EPSL,BETA,S,S1(3),S3(3)
COMPLEX*16 EPSCO,H2,H3,H4,H5,FZ,ET(300)
COMPLEX*16 X1,X2,X3,X4,BK1A,BK0A,G
COMPLEX*16 T1,T2,T3,T4,T5,T6,T7,T8,F9
DIMENSION DIS(300),E(300),ETSQ(300)
COMMON E,ETSQ
PI = 3.141592653589793
ES = 3.20096
R1 = 4.696
K = 300
L = 100
WL = 0.83286
C0 = 3.0
U0 = PI/4.0
G = DCMPLX(0.0,0.0)
C = 1.0
P = 4.0
EPSCL = DCMPLX(2.25000,0.0)
EPSCO = DCMPLX(2.24000,0.0)
YY = L
XX = R1
W = 2.000*PI/(C0/WL)
FSK = (W/C0)**2.0
FSK = SQRT(FSK)
BETA = REFSQ*FSK
BETA = CDSQRT(BETA)
B1(1) = DCMPLX(2.0,0.0)
EPS(1) = EPSCO
DO 75 I = 1,L
C DIS(I) = R1 + (I-1)*XX
DIS(I) = XX + (I-1)*XX
75 CONTINUE
DO 72 M = 1,100
J = M + 1
ET(M) = E(M)*((ES/EE(100))
C ET(M) = E(M)*((ES/EE(1)))
C EPS(J) = EPSCL + (C*ET(M))/(1.0D0 + (C*F*ET(M)))
EPS(J) = EPSCO + (C*ET(M))/(1.0D0 + (C*F*ET(M))
72 CONTINUE
C ===
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 1 & K
Z2 = ARG(1)*DIS(1)
C CALLING BESSEL FUNCTION J OF ORDER 0.1
B3(1) = BQN(0,22)
B3(2) = BQN(1,22)
BJ01(1) = BS1(1)
BJ11(1) = BS1(2)
HZ(1) = BJ01(1)*BJ11(1)
C菲律(1) = (G*W*U0*ARG(1))*(B3(1)*BJ11(1))
HR(1) = (BETA*ARG(1)))*(B1(1)*BJ11(1))
C ===
DO 24 M = 2,K
J = M - 1
COKSQ(M) = FSQSQ*EPS(M)
ALPHA(M) = BETASQ-COKSQ(M)
ARG(M) = CDSQRT(ALPHA(M))
Z1 = ARG(M)*DIS(M - 1)
C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
BS2(1) = BKn(0,21)
BS2(2) = BKn(1,21)
H2 = BKn(0,21)
I3 = BKn(1,21)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3
T1 = H2(M - 1)*BY12(M)*(ARG(M)/(G*W*U000))/BY02(M)*/FFFF/MM - 1
T2 = BJ12(M)*BY02(M) + BJ02(M)*BY12(M)
B1(M) = T1/T2
B2(M) = (H2(M - 1) - B1(M)*BJ02(M))/BY02(M)
Z3 = ARG(M)*DIS(M)
C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
BS3(1) = BKn(0,23)
BS3(2) = BIN(1,Z3)
H4 = BKN(0,Z3)
H5 = BKN(1,Z3)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
HZ(M) = B1(M)*BJ01(M) + B2(M)*BY01(M)
EPHI(M) = -(G*W*U0/ARG(M))*(B1(M)*BJ11(M)-B2(M)*BY11(M))
HR(M) = -(BETA/ARG(M))*(B1(M)*BJ11(M)-B2(M)*BY11(M))
IF(DABS(DIM(S(M))<1.0D0)*R1,LT.1.0D0-10) GO TO 57
24 CONTINUE
57 DO 78 I = 1,100
   ETSQ(I) = (CDABS(EPHI(I)))**2
78 CONTINUE
DO 79 I = 1,100
C WRITE(6,*)DIS(I)*1.006*DIMAG(EPHI(I)),DREAL(EPHI(I))
   ETSQ(I) = ETSQ(I)*(ES/ETSQ(100))
C WRITE(6,*)(ETSQ(I)*(ES/ETSQ(100))-E(I)) / ETSQ(I)
70 CONTINUE
   F9 = BETASQ-EPSC1*FSKSQ
   Z8 = CDSQRT(F9)
   Z9 = Z8*DIS(M)
C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
   BK0A = BKN(0,Z9)
   BK1A = BKN(1,Z9)
C SOLVING THE EIGENVALUE EQUATION FOR NONLINEAR TE MODES
   X1 = B1(M)*BJ01(M)+B2(M)*BY01(M)
   X2 = B1(M)*BJ11(M)-B2(M)*BY11(M)
   X3 = Z8*BK0A
   X4 = ARG(M)*BK1A
   FZ = 1.0D0/(X2*X3)*(X1*X4)
C FZ = 1.0D0/(X1*X4)/(X2*X3)
C FZ = (X2*X3)/(X1*X4)
C FZ = (X1*X2)/(X4*X3)
C WRITE(6,*)FZ
C WRITE(6,*)RETURN
C END
C PROGRAM 6 TMNL, RECURSIVE
C
C THIS PROGRAM IS USED TO FIND THE EIGENVALUES AND EIGEN FUNCTIONS
C OF TM MODES IN A FIBER HAVING NONLINEAR CORE AND LINEAR CLADDING.
C THE PROGRAM IS DEVELOPED USING THE RECURSIVE SCHEME APPROACH.
C IT IS USED IN CONJUNCTION WITH MULLER'S SUBROUTINE AND THE COMPLEX
C BESSEL FUNCTIONS. THE COMMENTS ON THIS PROGRAM ARE THE SAME AS
C THOSE STATED FOR PROGRAM 4.
$DEBUG
$LARGE
  IMPLICIT REAL*8 (A-H,O-Z)
  COMPLEX*16 ZEROS(2),VALUE(2),NE
  EXTERNAL FN
  LOGICAL FNREAL
C FNREAL=.FALSE.
C FNREAL=.TRUE.
C ENTER GUESS
  WRITE(*,'(A)')'ENTER GUESS'
  READ(*,'(A)')ZEROS(1)
  OPEN(UNIT=6,STATUS='OLD',FILE='OUT1')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
  N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
  NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO:
  MAXIT=15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
  EP1=10.0D-30
  EP2=10.0D-40
  CALL MULLER (FN,FNREAL,ZEROS,N,NPREV,MAXIT,EP1,EP2)
DO 312 I=1,N
  CALL FN(ZEROS(I),VALUE(I))
  WRITE(*,'(A)')
  WRITE(*,'(A)')ZEROS(I),VALUE(I)
  NE=CDVSQRT(ZEROS(I))
  WRITE(*,'(A)')NE
312 CONTINUE
  STOP
END
C SUBROUTINE FN(REFSQ,FY)
C SUBROUTINE FN (REFSQ,F2)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 B1(300),B2(300),ALPHA(300),ARG(300)
COMPLEX*16 BJ01(300),BJ02(300),BJ12(300)
COMPLEX*16 HPHY(300),EZ(300),ER(300),ETSQ(300)
COMPLEX*16 BV01(300),BY11(300),BY22(300),BY12(300)
COMPLEX*16 EPS(300),COKSQ(300),REFSQ,BKN,BIN,BIN
COMPLEX*16 EPSCL,BETASQ,BETA,BS2(3),BS1(3),BS3(3)
COMPLEX*16 EPSCO,H2,H3,H4,H5,ET
COMPLEX*16 FZ,X1,X2,X3,X4,G
COMPLEX*16 T1,T2,Z1,Z2,Z3,Z8,Z9,F9,H6,H7,Z10,H8,H9
DIMENSION DIS(300),P1(300)
P1 = 2.000D0/DASIN(1.000D0)
R1 = 4.00D-8
K = 300
L = 100
WL = 0.6328D-5
C0 = 3.008
U0 = P1*4.00D-7
G = DCMLXY(0.00D0,1.00D0)
C = 1.00D-8
P = 50.00D0
EPSCL = DCMLXY(2.250D0,0.00D0)
EPSCO = DCMLXY(2.255D0,0.00D0)
VV = L
XX = R1/YY
W = 2.00D0*PI*(C0/WL)
FSKSQ = ((W/C0)**2.00D0)
FSK = DSQRT(FSKSQ)
BETASQ = REFSQ*FSKSQ
BETA = DCQSRT(BETASQ)
B1(1) = DCMLXY(290.00D0,0.00D0)
ET = (CDABS(B1(1))**2)
C EPS(1) = EPSCL + ((C*ET)/1.00D0 + (C*P*ET))
EPS(1) = EPSCO
DO 75 I = 1,K
C DIS(I) = XX + (I-1)**XX
C DIS(I) = R1 + (I-1)**XX
DIS(I) = R1 + (I-1)**XX * 2.00D0
75 CONTINUE
COKSQ(1) = FSKSQ*EPS(1)
ALPHA(1) = COKSQ(1)-BETASQ
ARG(1) = DCQSRT(ALPHA(1))
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 2 & K
Z2 = ARG(1)' * DIS(1)

CALL DCBJNS(Z2,3,BS1)
BS1(1) = BJSN(0,Z2)
BS1(2) = BJSN(1,Z2)
BJ01(1) = BS1(1)
BJ11(1) = BS1(2)
EZ(1) = B1(1)' * BJ01(1)
HPHI(1) = ((G' * COKSQ(M))/(|W'U0' ARG(1)|)) * (B1(1)' * BJ11(1))
ER(1) = -(BETA*ARG(1))^2 * (B1(1)' * BJ11(1))

DO 24 M = 2, K
J = M - 1
ETSQ(J) = ((CDABS(EZ(J)))^2 + (CDABS(ER(J)))^2)

EPS(M) = EPSCO + (C' * ETSQ(J))/(1.0D0 + (C' * EPS(M)))
EPS(M) = EPSCL + (C' * ETSQ(J))/(1.0D0 + (C' * EPS(M)))
COKSQ(M) = FSKSQ * EPS(M)
ALPHA(M) = BETASQ * COKSQ(M)
ARG(M) = CDSQRT(ALPHA(M))
Z1 = ARG(M)' * DIS(M - 1)

CALLING BESSEL FUNCTIONS I & X OF ORDERS 0, 1

BS2(1) = BIN(0,Z1)
BS2(2) = BIN(1,Z1)
H2 = BKN(0,Z1)
H3 = BKN(1,Z1)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3
T1 = EZ(M - 1)' * BY12(M) + ((ARG(M)' * W'U0)/(|G' * COKSQ(M)|)) * BY02(M) * HPHI(M - 1)
T2 = BJ12(M)' * BY02(M) + BJ02(M)' * BY12(M)
B1(M) = T1/T2
B2(M) = (EZ(M - 1) - B1(M)' * BJ02(M))/BY02(M)
Z3 = ARG(M)' * DIS(M)
BS3(1) = BIN(0,Z3)
BS3(2) = BIN(1,Z3)
H4 = BKN(0,Z3)
H5 = BKN(1,Z3)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
EZ(M) = B1(M)' * BJ01(M) + B2(M)' * BY01(M)
HPHI(M) = (G' * COKSQ(M))/(|W'U0' ARG(M)|) * (B1(M)' * BJ11(M) - R2(M)' * RY11(M))
ER(M) = -(BETA*ARG(M))^2 * (B1(M)' * BJ11(M) - B2(M)' * BY11(M))
ETSQ(M) = ((CDABS(EZ(M)))^2) + ((CDABS(ER(M)))^2)

C IF(DABS(DIS(M)-R1).LT.1.0D-10) GO TO 57
    IF(DABS(DIS(M)-4.60D0*R1).LT.1.0D-10) GO TO 57
24   CONTINUE
C
57  DO 71 I=1,M
    WRITE(6,*)DIS(I)^1E6,HPHI(I)
71   CONTINUE
C F9 = BETASQ-EPSC1^FSKSQ
C Z3 = CDVQT(F9)
C Z9 = Z8^DIS(M)
C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
C H6 = BKN(0,Z9)
C H7 = BKN(1,Z9)
C
C SOLVING THE EIGENVALUE EQUATION FOR NONLINEAR TM MODE
C X1 = B1(M)^B101(M)+B2(M)^BY01(M)
C X2 = EPS(M)*(B1(M)^B111(M)-B2(M)^BY11(M))
C X3 = Z8^H6
C X4 = EPSCL^ARG(M)^H7
C FZ = 1.0D0 + (X2^X3)/(X1^X4)
C FZ = 1.0D0 + (X1^X4)/(X2^X3)
C FZ = (X2^X3) + (X1^X4)
C FZ = (X2^X4) + (X1^X3)
    WRITE('(',*) FZ
C
C CALCULATION OF POWER
C
C 51   CONTINUE
DO 52 I=1,M+100
C R = DIS(M)
K2 = I-M
DELR = XX^0.03D0*K2
R = R + DELR
DIS(I) = R
EPS(I) = EPSC1
Z10 = R^Z8
H8 = BKN(0,Z10)
H9 = BKN(1,Z10)
EZ(I) = EZ(M)^H8/H6
HPHI(I) = HPHI(M)^H9/H7
ER(I) = ER(M)^H9/H7
52   CONTINUE

P1(I) = (CDABS(HPHI(I))^2)(CDABS(ER(I))^2)*DIS(I)
52 CONTINUE
SUM=0.0D0
DO 53 I=1,M+100-1
P2=(P1(I)+P1(I+1))'*(DIS(I+1)-DIS(I))'0.5D0
SUM=SUM+P2
53 CONTINUE
PT=SUM*PI
WRITE(6,'*)PT
WRITE(6,'*)
RETURN
END

C

PROGRAM 7 TMNL SELF-CONSISTENT
C

THIS PROGRAM IS USED TO FIND THE EIGENVALUES AND THE EIGENFUNCTIONS
FOR TM MODES OF A FIBER HAVING NONLINEAR CORE AND LINEAR CLADDING
THE PROGRAM IS DEVELOPED USING THE SELF-CONSISTENT APPROACH, IT IS
USED WITH MULLER'S SUBROUTINE AND THE COMPLEX BESSEL FUNCTIONS.
ALL THE COMMENTS STATED IN PROGRAM 5 ARE VALID FOR THIS PROGRAM.
The ONLY DIFFERENCE IS THAT THE OUTPUT INTENSITY PROFILE
IS WRITTEN IN AN EXTERNAL FILE AND SHOULD BE TRANSFERRED TO THE INPUT
FILE EXTERNALLY. THIS MAKES THE PROGRAM A LITTLE MORE TIME
CONSUMING BUT IT ENABLE THE USER TO LOOK AT THE FIELD IN EACH
ITERATION WHICH IS NOT POSSIBLE FOR PROGRAM 5
$DEBUG
$LARGE
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2),VALUE(2),NE,E(200)
EXTERNAL FN
COMMON E
LOGICAL FNREAL
FNREAL= .TRUE.
C ENTER GUESS
WRITE(*,*)'ENTER GUESS'
READ(*,*)ZEROS(1)
OPEN(UNIT=5,STATUS='OLD',FILE='DATA1')
OPEN(UNIT=6,STATUS='OLD',FILE='OUT1')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
MAXIT = 15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
   EP1 = 10.00D-30
   EP2 = 10.00D-40
   DO 77 I = 1,100
      READ(*,*)E(I)
   77 CONTINUE
   DO 312 I = 1,N
      CALL MULLER (FN,FNREAL,ZEROS,N,NPREV,MAXIT,EP1,EP2)
      CALL FN(ZEROS(I),VALUE(I))
      WRITE(*,*)
      WRITE(*,*)ZEROZ(I),VALUE(I)
      NE = CDSQRT(ZEROS(I))
      WRITE(*,*)NE
   312 CONTINUE
   STOP
   END
C SUBROUTINE FN(REFSQ,FY)
C
SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 B1(300),B2(300),ALPHA(300),ARG(300)
COMPLEX*16 BJ01(300),BJ11(300),BJ02(300),BJ12(300)
COMPLEX*16 HPHI(300),EZ(300),ER(300),ETSQ(300)
COMPLEX*16 BY01(300),BY11(300),BY02(300),BY12(300)
COMPLEX*16 EPS0(300),C0SQ0(300),REFSQ,BK1,BK2,BIN
COMPLEX*16 EPSCL,ETSQ,BETA,BK2(3),BK1(3),BS3(3)
COMPLEX*16 EPS0,EPS00,H2,H3,H4,H5,ET(200)
COMPLEX*16 FZ,X1,X2,X3,X4,G,E(200)
COMPLEX*16 T1,T2,T11232,T8,29,60,H6,H7,Z10,J18,H9
DIMENSION DIS(300),PI(300)
COMMON E
ES = 3.583007
PI = 2.00D*0DASIN(1.00D0)
R1 = 4.00D-6
K = 300
L = 100
WL = 0.6328D-6
C0 = 3.0D8
U0 = PI*4.00D-7
G = DCMPLX(0.00D0,1.00D0)
C = 1.00D-8
P = 50.00D0
EPSCL = DCMLPX(2.250D0,0.0D0)
EPSCO = DCMLPX(2.275D0,0.0D0)
YY = L
XX = RI/YY
W = 2.0D0*P***(CD/WL)
FSKSQ = ((W/CO)**2.0D0)
FSK = DSQRT(FSKSQ)
BETASQ = REFSQ*FSKSQ
BETA = CDSQRT(BETASQ)
B1(1) = DCMLPX(800.0D0,0.0D0)
C
EPS(1) = EPSCO + ((C*E(1))/(1.0D0 + (C*P*E(1))))
EPS(1) = EPSCO
C DO 75 J = 1,K
C DIS(I) = XX + (I-1)*XX
DIS(I) = RI + (I-1)*XX
75 CONTINUE
DO 73 M = 1,100
J = M + 1
C ET(M) = (E(M))**((ES/E(100)))
ET(M) = (E(M))**((ES/E(1)))
C EPS(J) = EPSCO + ((C*ET(M))/(1.0D0 + (C*P*ET(M))))
EPS(J) = EPSCL + ((C*ET(M))/(1.0D0 + (C*P*ET(M))))
73 CONTINUE
COKSQ(1) = FSKSQ*EPS(1)
ALPHA(1) = COKSQ(1)*BETASQ
ARG(1) = CDSQRT(ALPHA(1))
C
C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 2 & K
Z2 = ARG(1)*DIS(1)
BS1(1) = BJN(0,22)
BS1(2) = BJN(1,22)
BJ01(1) = BS1(1)
BJ11(1) = BS1(2)
EZ(1) = B1(1)*BJ01(1)
HPHI(1) = ((G*COKSQ(1))/((W*U0*ARG(1))**(1/11)))*RI11(1)
ER(1) = (BETA/ARG(1))**(1/11)*RI11(1)
C DO 24 M = 2,K
J = M + 1
ETSQ(J) = (((CDABS(EZ(J)))**2) + (((CDABS(ER(J)))**2)
C EPS(M) = EPSCO + ((C*ETSQ(J))/(1.0D0 + (C*P*ETSQ(J))))
COKSQ(M) = FSKSQ*EPS(M)
ALPHA(M) = BETASQ*COKSQ(M)
ARG(M) = CDSQRT(ALPHA(M))

Z1 = ARG(M) * DIS(M-1)

C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1

BS2(1) = BIN(0, Z1)
BS2(2) = BIN(1, Z1)
H2 = BKN(0, Z1)
H3 = BKN(1, Z1)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3
T1 = EZ(M-1) * BY12(M) + ((ARG(M) * W * U0) / (G * COKSQ(M))) * BY02(M) * HPHI(M-1)
T2 = BJ12(M) * BY02(M) + BJ02(M) * BY12(M)
B1(M) = T1 / T2
B2(M) = (EZ(M-1) - B1(M) * BJ02(M)) / BY02(M)
Z3 = ARG(M) * DIS(M)
BS3(1) = BIN(0, Z3)
BS3(2) = BIN(1, Z3)
H4 = BKN(0, Z3)
H5 = BKN(1, Z3)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5
EZ(M) = B1(M) * BJ01(M) + B2(M) * BY01(M)
HPHI(M) = (G * COKSQ(M)) / (W * U0 * ARG(M)) * (B1(M) * BJ11(M) - B2(M) * BY11(M))
ER(M) = (BETA * ARG(M)) * (B1(M) * BJ11(M) - B2(M) * BY11(M))
ETSQ(M) = ((CDABS(EZ(M))) * Z3) + ((CDABS(ER(M))) * Z3)
IF(DABS(DIS(M) - 2.000 * R11) LT 0.001) GO TO 57

24 CONTINUE

C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYFR

H6 = BKN(0, Z9)
H7 = BKN(1, Z9)

C SOLVING THE EIGENVALUE EQUATION FOR NONLINEAR TM MODES

X1 = B1(M) * BJ01(M) + B2(M) * BY01(M)
X2 = EPS(M) * (B1(M) * BJ11(M) - B2(M) * BY11(M))
\[ X_3 = 2^8 \times H_6 \]
\[ X_4 = \text{EPSCL} \cdot \text{ARG}(M)^\dagger H_7 \]

\[ F_Z = D\text{REAL}(X_2 \times X_3 + X_1 \times X_4) + D\text{IMAG}(X_2 \times X_3 + X_1 \times X_4) \]
\[ F_Z = 1.000 + (X_2 \times X_3)/(X_1 \times X_4) \]
\[ F_Z = (X_2 \times X_3) + (X_1 \times X_4) \]
\[ F_Z = D\text{REAL}(X_2 \times X_3 + X_1 \times X_4) \]
\[ F_Z = D\text{IMAG}(X_2 \times X_3 + X_1 \times X_4) \]
\[ F_Z = (X_2 \times X_3)/(X_1 \times X_4) \]
\[ \text{WRITE}(.*, F_Z) \]
\[ \text{WRITE}(6, *) \]
\[ \text{RETURN} \]
\[ \text{END} \]

---

**THE FOLLOWING 2 PROGRAMS ARE USED TO OBTAIN THE RESULTS OF CHAPTER 6**

**PROGRAM 8 HE11NL RECURSIVE**

**THIS PROGRAM IS USED TO FIND THE EIGENVALUE AND THE EIGFN FUNCTION**

OF THE HE11 MODE IN A WAVEGUIDE HAVING A NONLINEAR CORE AND A LINEAR CLADDING. THE PROGRAM IS USED IN CONJUNCTION WITH MULTI-FR'S

SUBROUTINE AND THE COMPLEX BESSEL FUNCTIONS.

THE MATCHING TECHNIQUE BETWEEN THE TWO RESULTANT VALUES OF THE EFFECTIVE INDEX IS SIMILAR TO THAT EXPLAINED IN PROGRAM 3.

HOWEVER, THIS PROGRAM IS WRITTEN TO BE MORE INTERACTIVE AND MORE FASTER THAN PROGRAM 3. THE PROGRAM CAN BE MODIFIED TO WORK FOR A WAVEGUIDE HAVING LINEAR CORE AND NONLINEAR CLADDING. THE SAME MODIFICATION MENTIONED IN PROGRAM 4 ARE APPLIED TO THIS PROGRAM IN ADDITION TO OMITTING ALL THE STEPS USED TO CALCULATE EPS11(1), EPS11(1) FOR THIS CASE IS EQUAL TO EPSCO.


SDEBUG

SLARGE

IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2), VALUE(2), NE
EXTERNAL FN
COMMON L1, C1, C2
LOGICAL FREAL
C FNREAL=.FALSE.
FNREAL=.TRUE.
C ENTER AMP(E2)
   WRITE(*,*)'ENTER AMP.(EZ)'
   READ(*,*)C1
C ENTER DELTA
   WRITE(*,*)'ENTER DELTA'
   READ(*,*)C2
C ENTER FUNCTION
   WRITE(*,*)'ENTER FUNCTION'
   READ(*,*)L1
C ENTER GUESS
   WRITE(*,*)'ENTER GUESS'
   READ(*,*)ZEROS(1)
   OPEN(UNIT=6,STATUS='OLD',FILE='OUT1')
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
   N=1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
   NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO
   MAXIT=15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
   EP1=1.0D-30
   EP2=1.0D-40
   CALL MULLER(FN,FNREAL,ZEROS,N,PREV,MAXIT,EP1,EP2)
   DO 312 I=1,N
      CALL FN(ZEROS(I),VALUE(I))
      WRITE(*,*)
      WRITE(*,*)ZEROS(I),VALUE(I)
      NE=CDSQRT(ZEROS(I))
      WRITE(*,*)NE
   CONTINUE
312 CONTINUE
   STOP
   END
C SUBROUTINE FN(REFSQ,FZ)
C==================================================================================================
SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL*8 (A-I,O-Z)
COMPLEX*16 A1(210),A2(210),BJ1P(210),BY1P(210)
COMPLEX*16 BJ2P(210),BY2P(210),ER(300),BHN,BKN,BIN
COMPLEX*16 BI(210),B2(210),ALPHA(210),ARG(210)
COMPLEX*16 BJ10(210),BJ11(210),BJ20(210),BJ12(210)
COMPLEX*16 EZ(300),EPHI(300),HZ(300),HPHI(300),HR(300)
COMPLEX*16 BY01(210),BY11(210),BY02(210),BY12(210)
COMPLEX*16 EPS(210),COKSQ(210),REFSQ,Y2,Y3,Y4,Y5
COMPLEX*16 EPSCL,BETASQ,BETA,BS2(3),BS1(3),BS3(3)
COMPLEX*16 EPSCO,H2,H3,H4,H5,H6,H7
COMPLEX*16 X1,X2,BKP,G,T3,T4,T5,FZ,P3(300),P4(300)
COMPLEX*16 T1,T2,Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,Z10,H8,H9,F10
DIMENSION DIS(210),ETSQ(210),P1(210)
COMMON L1,C1,C2
PI=2.0D0*DASIN(1.0D0)
R1=4.0D-6
K=200
L=100
WL=0.0328D-6
C0=3.0D8
U0=PI*4.0D-7
G=DCMPLX(0.0D0,1.0D0)
C=1.0D-6
P=20.0D0
EPSCO=DCMPLX(2.2550D0,0.0D0)
EPSCL=DCMPLX(2.2350D0,0.0D0)
ETA=120.0D0*PI
YY=L
XX=R1/YY
W=2.0D0*PI*(C0/WL)
FSKSQ=(IW/C0)**2.0D0
FSK=DSQRT(FSKSQ)
BETASQ=REFSQ*FSKSQ
BETA=DSQRT(BETASQ)
C INTER AMPLITUDE OF ELECTRIC FIELD EZ
  V1=C1
  A1(1)=DCMPLX(V1,0.0D0)
  A2(1)=DCMPLX(0.0D0,0.0D0)
C INTER A GUESS FOR THE AMPLITUDE OF MAGNETIC FIELD HZ
  CC=(DREAL(REFSQ))**0.5D0
  V2=(V1**1.5D0/ETA)+C2
  B1(1)=DCMPLX(V2,0.0D0)
  B2(1)=DCMPLX(0.0D0,0.0D0)
C CALCULATION OF EPS(1)
C == == == == == == == == == == == == == == == == == == == == == ==
  Y1=(CDABS(W**U0*B1(1)))**2.0D0+(CDARN(BETA*A1(1)))**2.0D0
  Y2=(C*Y1/FSK)+EPSCO-REFSQ
  Y3=EPSCO*REFSQ*(C*Y1/FSK)+1.0D0
  Y4=(Y2+DSQRT(Y2**2.0D0*Y3))/2.0D0
\[ Y_5 = (-y^2 - \text{CDSQRT}(y^2 + 2.0 - \text{d}y^3) + 2.0d0) \]

\[
\text{EPS}(1) = Y_4
\]

C WRITEx='EPS(1),Y5
C == == == == == == == == == == == == == == == == == == == == == == == == == == ==
C EPS(1) = EPSCO
DO 75 I = 1,K
DIS(I) = XX + (I-1)*XX
C DIS(I) = R1 + (I-1)*XX
75 CONTINUE
C COKSQ(1) = FSOKSQ'EPS(1)
ALPHA(1) = COKSQ(1)'BETASQ
ARG(1) = CDSQRT(ALPHA(1))
C == == == == == == == == == == == == == == == == == == == == == == == == == == ==
Z2 = ARG(1)'DIS(1)
BJ01(1) = Bjin(0,22)
BJ11(1) = Bjin(1,22)
BS1(1) = BJ01(1)
BS1(2) = BJ11(1)
C CALCULATIONS OF THE FIELDS BETWEEN 1 & K
EZ(1) = A11(1)'BJ11(1)
HZ(1) = B11(1)'BJ11(1)
C CALCULATIONS OF THE DERIVATIVES OF J & K FUNCTION
BJ1P(1) = BS1(1)'(BS1(2)/22)
EPHI(1) = (G/ALPHA(1))'(BETA'EZ(1)'DIS(1) + W'U0'B1(1)'ARG(1)'BJ1P(1))
HR(1) = -(COKSQ(1)'(W'U0))'EZ(1)'DIS(1) + BETA'B1(1)'ARG(1)'BJ1P(1))
**'(G/ALPHA(1))
ER(1) = -(G/ALPHA(1))'(W'U0'HZ(1)'DIS(1) + BETA'ARG(1)'A1(1)'RJ1P(1))
HPHI(1) = -(G/ALPHA(1))'(BETA'HZ(1)'DIS(1) + COKSQ(1)'A1(1)'RJ1P(1))
'ARG(1)'(W'U0)
ETSQ(1) = ((CDAB(EZ(1)))'2.0) + ((CDAB(EPHI(1)))'2.0) + ((CDAB(ER(1))'2.0)
C == == == == == == == == == == == == == == == == == == == == == == == == == == ==
DO 24 M = 2,K
J = M-1
EFS(M) = EPSCO + ((C'ETSQ(J))/(1.0D0 + C'P'ETSQ(J)))
C EPS(M) = EPSCL + ((C'ETSQ(J))/(1.0D0 + C'P'ETSQ(J)))
COKSQ(M) = FSOKSQ'EFS(M)
ALPHA(M) = BETASQ-COKSQ(M)
ARG(M) = CDSQRT(ALPHA(M))
Z1 = ARG(M)'DIS(M-1)
C CALLING BESSEL FUNCTIONS I,K OF ORDERS 0,1
BS2(1) = BIN(0,Z1)
BS2(2) = BIN(1,Z1)
HZ = BKHN(0,Z1)
H3 = BKN(1,2,1)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3

C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTION
BJ1P(M) = BS2(1)-(BS2(2)/Z1)
BY1P(M) = H2-(H3/Z1)
T1 = -(G' \times \text{ALPHA}(M'))\times\text{PHI}(M-1) + (\text{BETA}' HZ(M-1)/\text{DIS}(M-1)) \times BY12(M)
T2 = BJ1P(M) \times BY12(M) - BJ12(M) \times BY1P(M)
T3 = (COKSQ(M) \times \text{ARG}(M') \times EZ(M-1) \times BY1P(M) / W' U0)
T4 = -(G' \times \text{ALPHA}(M') \times EPHI(M-1) - (\text{BETA}' EZ(M-1)/\text{DIS}(M-1)) \times BY12(M)
T5 = (W' U0' \times \text{ARG}(M') \times HZ(M-1) \times BY1P(M))
A1(M) = (T1 - T3) / (T2 * COKSQ(M) \times \text{ARG}(M) / W' U0)
A2(M) = (EZ(M-1) - A1(M) \times BJ12(M)) / BY12(M)
B1(M) = -(T4 + T5) / (T2 * ARG(M') \times W' U0)
B2(M) = (HZ(M-1) - B1(M) \times BJ12(M)) / BY12(M)
Z3 = ARG(M) \times DIS(M)
BS3(1) = BIN(0,23)
BS3(2) = BIN(1,23)
H4 = BKN(0,23)
H5 = BKN(1,23)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5

C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTIONS
BJ2P(M) = BS3(1) - (BS3(2)/Z3)
BY2P(M) = H4 - (H5/Z3)
EZ(M) = (A1(M) \times BJ11(M) + A2(M) \times BY11(M))
H2(M) = (B1(M) \times BJ11(M) + B2(M) \times BY11(M))
EPHI(M) = -(G/\text{ALPHA}(M')) \times (\text{BETA}/\text{DIS}(M)) \times EZ(M) + W' U0' \times \text{ARG}(M') \times \text{R1}(M) \times R1P'(M') + B2(M) \times BY2P(M))
HPHI(M) = +(G/\text{ALPHA}(M')) \times (COKSQ(M) \times \text{ARG}(M)/W' U0) \times A1(M) \times R1P'(M') \times A2(M) \times BY2P(M) + W' U0' \times HZ(M-1)/DIS(M)
ER(M) = +(G/\text{ALPHA}(M')) \times (\text{BETA}' \times \text{ARG}(M') \times A1(M) \times R1P(M) \times A2(M) \times BY2P(M) + W' U0' \times HZ(M-1)/DIS(M)
HR(M) = (COKSQ(M) \times W' U0') \times EZ(M) / DIS(M) + R1A1(M) \times R1P(M) \times A2(M) \times BY2P(M) + B2(M) \times BY2P(M)) \times G/\text{ALPHA}(M))
ETSQ(M) = ((\text{CDABS}(EZ(M)))^2 + ((\text{CDABS}(EPHI(M)))^2)^2)^2 + ((\text{CDABS}(FR1(M)))^2)^2)^2)
IF(DABS(DIS(M)-R1),LT,1.0D-10) GO TO 57
IF(DABS(DIS(M)-2.0000*R1),LT,1.0D-10) GO TO 57
CONTINUE
CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
H6 = BKN(0, Z9)
H7 = BKN(1, Z9)
C CALCULATIONS OF THE DERIVATIVE OF K FUNCTION
BKP = -H6*(H7/Z9)
C SOLVING THE EIGENVALUE EQUATION FOR THE HE11 MODE
X1 = (G/F9)^((BETA/DIS(M))*EZ(M) + W*U0*Z8*102(M)*BKP/H7)
X2 = (G/F9)^((BETA/DIS(M))*H2(M) + (F10*Z8*EZ(M)*BKP/(H7*W*101)))
C IF (L1.EQ.1) THEN
C MATCHING OF EPb
C FZ = DREAL(EPh(M)-X1) + DIMAG(EPh(M)-X1)
C FZ = EPh(M)-X1
C ELSE
C MATCHING OF HPh
C FZ = 1.0D0 + (HPh(M))/(X2)
C FZ = DREAL(HPh(M)-X2)+DIMAG(HPh(M)+X2)
C FZ = HPh(M)+X2
C END IF
C DO 32 I = 1, L
WRITE(6,'(ETSQ(I))')
C WRITE(6,'(ETSQ(I))')
C CONTINUE
C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
R = DIS(M)
DO 51 I = M+1, M+100
K2 = I - M
DELR = XX'0.1000'K2
R = R + DELR
DIS(I) = R
EPS(I) = EPSCL
Z10 = R*Z8
H8 = BKN(O, Z10)
H9 = BKN(1, Z10)
HZ(I) = HZ(M)*H9/H7
EZ(I) = EZ(M)*H9/H7

51 CONTINUE
END

52 CONTINUE
SUM = 0.D0
DO 53 I = 1, M + 100 - 1
P2 = (P(I) + P(I + 1))'0.5D0
SUM = SUM + P2

53 CONTINUE
PT = SUM*'PI
WRITE(6, *)
WRITE(6, *) PT
RETURN
END

== PROGRAM 9 HE11NL SELF-CONSISTENT ==
== THIS PROGRAM IS USED TO FIND THE EIGENVALUE AND THE EIGENFUNCTION ==
== OF THE HE11 MODE IN A WAVEGUIDE HAVING A NONLINEAR CORE AND A LINFA ==
== CLADDING. THE PROGRAM IS BASED ON THE SELF-CONSISTENT APPROACH. ==
== IT IS USED WITH MULLER'S ROUTINE AND THE COMPLEX BESSEL FUNCTIONS. ==
== THE MATCHING TECHNIQUE BETWEEN THE TWO RESULTANT VALUES ==
C OF THE EFFECTIVE INDEX IS SIMILAR TO THAT EXPLAINED IN PROGRAM 8.
C THE PROGRAM CAN BE MODIFIED TO WORK FOR A WAVEGUIDE HAVING LINEAR
C CORE AND NONLINEAR CLADDING IN A MANNER SIMILAR TO THAT EXPLAINED
C IN THE PROGRAM 8. IN ADDITION TO THE MODIFICATION OF PROGRAM 8.
C THE INTERFACE INTENSITY VALUES E(100) AND ETSQ(100) SHOULD BE
C REPLACED BY THE NEW INTERFACE VALUES E(1) AND ETSQ(1)
C THE INPUTS FOR THIS PROGRAM ARE: THE AMPLITUDE OF THE AXIAL ELECTRIC
C FIELD, THE VALUE OF DELTA REQUIRED FOR THE AMPLITUDE OF THE AXIAL
C MAGNETIC FIELD, THE FUNCTION WHICH IS TO BE SOLVED FOR ITS ZEROS
C (WHETHER THE ONE RESULTING FROM MATCHING EPPI OR THAT RESULTING
C FROM MATCHING HPHI) AND THE GUESS REQUIRED FOR THE EFFECTIVE INDEX.
C AN INITIAL INTENSITY PROFILE IS REQUIRED TO START THE PROGRAM AND
C PRODUCE THE FIRST EIGENVALUE AND THE FIRST EIGENFIELD. THIS EIGEN
C FIELD IS FED BACK TO THE MAIN PROGRAM TO PRODUCE A NEW FIELD AND
C SO ON. THE PROGRAM IS TERMINATED AFTER NUMBER OF RUNS ENOUGH FOR
C THE EIGEN VALUE TO BE STABILIZED.
$DEBUG
$LARGE
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 ZEROS(2),VALUE(2),NE
DIMENSION E(300),ETSQ(300)
COMMON E,ETSQ,L1,C1,C2
EXTERNAL FN
LOGICAL FNREAL
FNREAL=.TRUE.
OPENUNIT = 5,STATUS = 'OLD',FILE = 'DATA1'
OPENUNIT = 6,STATUS = 'OLD',FILE = 'OUT1'
DO 74 I = 1,100
READ(5,*)E(I)
74 CONTINUE
C ENTER AMP(EZ)
   WRITE(*,*)'ENTER AMP(EZ)'
   READ(*,*)C1
C ENTER DELTA
   WRITE(*,*)'ENTER DELTA'
   READ(*,*)C2
C ENTER FUNCTION
   WRITE(*,*)'ENTER FUNCTION'
   READ(*,*)L1
C ENTER GUESS
   WRITE(*,*)'ENTER GUESS'
   READ(*,1)ZEROS(1)
C ENTER NUMBER OF ZEROS TO BE SEARCHED FOR.
    N = 1
C ENTER THE NUMBER OF ZEROS PREVIOUSLY KNOWN
NPREV=0
C ENTER THE MAXIMUM NUMBER OF FUNCTION EVALUATION FOR EACH ZERO.
MAXIT = 15
C ENTER THE MAXIMUM ERROR CONTROL PARAMETERS
EP1=10.0D-30
EP2=10.0D-40
DO 75 I=1,10
CALL MULLER(FN,FNREAL,ZEROS,N,NPREV,MAXIT,EP1,EP2)
CALL FN(ZEROS(1),VALUE(1))
NE=CDSORT(ZEROS(1))
WRITE(*,*) ZEROS(1),VALUE(1)
WRITE(*,*) NE
WRITE(*,*)
DO 31 M=1,100
E(M)=ETSQ(M)
31 CONTINUE
ZEROS(1)=NE**2
75 CONTINUE
DO 76 I=1,100
WRITE(6,*)(E(I))
76 CONTINUE
STOP
END
C SUBROUTINE FN(REFSQ, FY)
C SUBROUTINE FN (REFSQ,FZ)
IMPLICIT REAL '8 (A-H,O-Z)
COMPLEX'16 A(210),A2(210),BJ1P(210),BY1P(210)
COMPLEX'16 BJ2P(210),BY2P(210),ER(300),BJN,BKN,BIN
COMPLEX'16 B1(210),B2(210),ALPHA(210),ARG(210)
COMPLEX'16 BJ01(210),BJ11(210),BJ02(210),BJ12(210)
COMPLEX'16 EZ(300),EPHI(300),Hz(300),PHI(300),HR(300)
COMPLEX'16 BY01(210),BY11(210),BY02(210),BY12(210)
COMPLEX'16 EPS(210),COKSQ(210),REFSQ
COMPLEX'16 EPSCL,BETASQ,BETA,B52(3),R51(3),R53(3)
COMPLEX'16 EPSQ,H2,H4,H5,H6,H7
COMPLEX'16 X1,X2,BKP,G,T3,T4,T5,FZ,ET(300)
COMPLEX'16 T1,T2,Z1,Z2,Z3,Z4,Z9,F9,F10
DIMENSION DIS(210),ETSQ(300),E(300)
COMMON E,ETSQ,L1,L1,C1,C2
PI=2.0D0*DASIN(1.0D0)
ES=3953.0D0
R1 = 4.0D-6
K = 200
L = 100
WL = 0.6328D-6
C0 = 3.0D8
U0 = P1*4.0D-7
G = DCMPLEX(0,0,0,0,1,0)
C = 1.0D-8
P = 20.0D0
EPSCL = DCMPLEX(2.255D0,0,0,0)
ETA = 120.0D0*PI
YY = L
XX = R1/YY
W = 2.0D0*PI*(C0/WL)
FSKSQ = (W/C0)**2.0D0
FSK = DSQRT(FSKSQ)
BETASQ = REFSQ*FSKSQ
BETA = DSQRT(BETASQ)
C INTER AMPLITUDE OF ELECTRIC FIELD EZ
V1 = C1
A1(I) = DCMPLEX(V1,0.0D0)
A2(I) = DCMPLEX(0.0D0,0.0D0)
C INTER A GUESS FOR THE AMPLITUDE OF MAGNETIC FIELD HZ
CC = (DRSEZ/REFSQ)**0.5D0
V2 = (V1*CC/ETA) + C2
C V2 = (V1*1.5D0/ETA) + C2
B1(I) = DCMPLEX(V2,0.0D0)
B2(I) = DCMPLEX(0.0D0,0.0D0)
C DO 75 I = 1,K
75 CONTINUE
DO 72 M = 1,100
J = M+1
ET(M) = (E(M))^2/(E/(E100))
C ET(M) = EPSCL + (C*ET(M))/(1.0D0+C*P*ET(M))
C EPS(J) = EPSCL + (C*ET(M))/(1.0D0+C*P*ET(M))
EPS = EPSCL + (C*ET(M))/(1.0D0+C*P*ET(M))
72 CONTINUE
C CALCULATION OF EPS(1)
C EPS(1) = EPSCL + (C*ET(1))/(1.0D0+C*P*ET(1))
COKSQ(1) = FSKSQ' EPS(1)
ALPHA(1) = COKSQ(1) - BETASQ
ARG(1) = CDSQRT(ALPH0A(1))
Z2 = ARG(1) * DIS(1)
BJ01(1) = BJN(0, Z2)
BJ11(1) = BJN(1, Z2)
BS1(1) = BJ01(1)
BS(2) = BJ11(1)

C CALCULATIONS OF THE FIELDS FOR LAYERS BETWEEN 1 & K
EZ(1) = A1(1) * BJ11(1)
H2(1) = B1(1) * BJ11(1)

C CALCULATIONS OF THE DERIVATIVES OF J & K FUNCTION
BJ1P(1) = BS1(1) - BS(2) / Z2

C BJ1P(1) = BJ01(1) - BJ11(1) / Z2

EPIH(1) = (G/ALPHA(1)) * (BETA' * EZ(1) / DIS(1) + W'U0 * B1(1) * ARG(1)' R1P(1))
HRI(1) = -(COKSQ(1) / W'U0) * EZ(1) / DIS(1) + BETA' * B1(1)' * ARG(1)' B1P(1))
*((G/ALPHA(1))
EER(1) = -(G/ALPHA(1)) * (W'U0 * HZ(1) / DIS(1) + BETA' * ARG(1)' A1(1) * BJ1P(1))
HPHI(1) = -(G/ALPHA(1)) * ((BETA' * HZ(1) / DIS(1)) + COKSQ(1) * A1(1) * R1P(1)')
* ARG(1) / W'U0)
ETSQ(1) = (ICDABS(EZ(1))) * 2.0 + (ICDABS(EPIH(1))) * 2.0 + (ICDABS(EER(1)) * 2.0)

C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =

DO 24 M = 2, K
J = M - 1
COKSQ(M) = FSKSQ' EPS(M)
ALPHA(M) = BETASQ - COKSQ(M)
ARG(M) = CDSQRT(ALPH0A(M))
Z1 = ARG(M) * DIS(M - 1)

C CALLING BESSEL FUNCTIONS I, K OF ORDERS 0, 1
BS2(1) = BIN(0, Z1)
BS2(2) = BIN(1, Z1)
H2 = BKN(0, Z1)
H3 = BKN(1, Z1)
BJ02(M) = BS2(1)
BJ12(M) = BS2(2)
BY02(M) = H2
BY12(M) = H3

C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTION
BJ1P(M) = BS2(1) - BS2(2) / Z1
BY1P(M) = -H2 - H3 / Z1
T1 = (G/ALPHA(M)) * HPHI(M - 1) + (BETA' * HZ(M - 1) / DIS(M - 1)) * BY12(M)
T2 = BJ1P(M) * BY12(M) - BJ12(M) * BY1P(M)
T3 = (COKSQ(M) * ARG(M) * EZ(M - 1) * BY1P(M)) / (W'U0)
T4 = (G' ALPHA(M)) EPHI(M-1) - (BETA' EZ(M-1)/DIS(M-1)) BY12(M)
T5 = (W' U0' ARG(M) HZ(M-1) BY1P(M))
A1(M) = (T1-T3)/(T2'COKSQ(M) ARG(M)/W'U0))
A2(M) = (EZ(M-1) A1(M) BJ12(M))/BY12(M)
B1(M) = -[T4 + T5]/(T2' ARG(M) W'U0)
B2(M) = (HZ(M-1) B1(M) BJ12(M))/BY12(M)
Z3 = ARG(M) DIS(M)
BS3(1) = BKN(0,23)
BS3(2) = BKN(1,23)
H4 = BKN(0,23)
H5 = BKN(1,23)
BJ01(M) = BS3(1)
BJ11(M) = BS3(2)
BY01(M) = H4
BY11(M) = H5

C CALCULATIONS OF THE DERIVATIVES OF I & K FUNCTIONS
BJ2P(M) = BS3(1) - (BS3(2)/Z3)
BY2P(M) = H4 - (H5/Z3)
EZ(M) = (A1(M) BJ11(M) + A2(M) BY11(M))
HZ(M) = (B1(M) BJ11(M) + B2(M) BY11(M))
EPHI(M) = -(G' ALPHA(M)) *((BETA/DIS(M))' FZ(M) + W'U0' ARG(M) R1(M) R2P
'M) + B2(M)' BY2P(M))
HPHI(M) = + (G' ALPHA(M)) *((COKSQ(M) ARG(M)/W'U0))' ((A1(M) R2P(M) - A2P
'M) BY2P(M)) + (BETA/DIS(M))' HZ(M))
ER(M) = + (G' ALPHA(M)) *((BETA' ARG(M))' ((A1(M) BJ2P(M) + A2(M) BY2P(M)) + W
'U0' HZ(M)/DIS(M))
HR(M) = (COKSQ(M) W'U0)' EZ(M) DIS(M) + BETA' ARG(M) (B1(M) BJ2P(M) +
B2(M) BY2P(M))' (G' ALPHA(M))
ETSQ(M) = (CDABS(EZ(M)))'**2.0 + ((CDABS(EPHI(M)))'**2.0 + ((CDAIS(ER(M'
')))'**2.0)
IF(DABS(DIS(M)-R1).LT.1.0D-10) GO TO 57
C IF(DABS(DIS(M)-2.000'R1).LT.1.0D-10) GO TO 57
24 CONTINUE
C == == == == == == == == == == == == == == == == == == == == == == == == == == == ==

57 F9 = BETA S-Q EPSCL* FSKSQ
 F10 = EPSCL* FSKSQ
 Z8 = CDSQRT(F9)
 Z9 = Z8*DIS(M)
 S1 = CDSQRT(EPSCL)

C CALLING BESSEL FUNCTIONS K OF ORDERS 0,1 FOR THE LAST LAYER
H6 = BKN(0,Z9)
H7 = BKN(1,Z9)

C CALCULATIONS OF THE DERIVATIVE OF K FUNCTION
BKPK = -H6/(H7/Z9)
C SOLVING THE EIGENVALUE EQUATION FOR THE HE11 MODE
X1 = -(G/F9)*((BETA/DIS(M))^BZ(M)+W'U'O'ZB'HZ(M)'^BKP/H7)
X2 = -(G/F9)*((BETA/DIS(M))^BZ(M)^H2(M)+(F10'ZB'EZ(M)'^BKP/H7W'U'O'))
IF (L1.EQ.1) THEN
   FZ = 1.D0-(EPHI(M)/X1)
   END IF
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
ELSE
   FZ = 1.D0+(HPHI(M)/X2)
   END IF
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
DO 32 I = 1,L
   ETSQ(I) = ETSQ(I)*(ES/ETSQ(100))
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
WRITE(6,'(ETSQ(I))')
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
32 CONTINUE
WRITE(*,*)FZ
RETURN
END