# Capacity of Slotted Aloha under Nakagami Fading

by

Nayyar Grami

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

**MASTER OF SCIENCE**

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# KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
# DHAHRAN, SAUDI ARABIA

## COLLEGE OF GRADUATE STUDIES

*This thesis, written by*

**Nayyar Grami**

*under the direction of his Thesis Advisor, and approved by his Thesis committee, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirements for the degree of*

## MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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Date: 26-12-98

Dedicated to My Parents

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## Thesis Abstract

**Name:** Nayyar Grami

**Title:** Capacity of Slotted ALOHA under Nakagami Fading

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*Random Access Schemes for the mobile communication systems are widely investigated. The choice of a particular random access scheme changes the way in which the mobile terminals contend for getting hold of the channel. Slotted ALOHA is one such scheme which, despite being simple, is shown to offer a good performance over mobile radio channels. The present thesis is focussed on the study of slotted ALOHA under a generalized fading model taking into account various physical effects of a mobile radio channel. Through this study generalized results are obtained which provide better mathematical insight to model the real world changing transmission scenario.*

*Keywords: Mobile Communications, Slotted ALOHA, Nakagami Fading*

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King Fahd University of Petroleum and Minerals, Dhahran.

December 1998

# خلاصة الرسالة


**اسم الطالب** :نـــــيرجــــــرامي

**عنوان الرسالة** : سعةالوهاالمقطعةبتأثيراضمحلالناكاقامي

**التخصص**: الهندسةالكهربائية

**تاريخ الشهادة** : يونيو(ديسمبر)١٩٩٨م–رمضان١٤١٩هـ

إن طرق الدخول العشوائي في الاتصالات اللاسلكية درست بتوسع فاختيار طريقه الدخول العشوائي تغير طريقة السماح للجهاز الجوال يحجر القناة . والوها المقطعة عبارة عن طريقه من طرق الدخول العشـــوائي وهي على الرغم من كون بسيطة فهي تتميز بأداء غالى في قنوات الراديو الجوال.

وتركز هذه الرسالة إلى دراسة الوها المقطعة بتأثير نموذج الاضمحلال العام الـــذي يـــأخذ بالجنســـيات التأثيرات الفيزيائية المختلفة في قنوات الراديو الجوال. فمن خلال هذه الدراسة حصلنا إلى نتـــائج معمـــه تعطي مدلول رياضي أفضل للنماذج التي تمثل العالم الحقيقي المتغير للاتصالات.

مفتاح الكلمات :الوهاالمقطعة،اضمحلالناكاقامي،الاتصالات اللاسلكية


**درجة الماجستير في العلوم**

جامعة الملك فهد للبترول و المعادن

الظهران،المملكةالعربيةالسعودية

**يونيو(ديسمبر)١٩٩٨م–رمضان١٤١٩هـ**

# Chapter 1

# Introduction

Slotted ALOHA is a class of random access schemes, which belong to a broader category of what is known as 'Multiple Access Schemes'. Multiple access refers to the remote sharing of the fixed communications resource. This sharing may take place due to the efficiency considerations or due to the nature of the application. In the realm of the mobile communications, multiple access schemes are used to allow many mobile users to simultaneously share a finite amount of the radio spectrum. The sharing of the spectrum is required to achieve a high capacity by simultaneously allocating the available bandwidth or the available amount of the channels to multiple users. Also due to the bursty nature of transmission, common to the mobile terminals, the use of multiple access schemes results in more effective channel utilization. The key to all multiple access schemes is that the various signals share a communication resource without causing unmanageable interference to each other in the detection process. The allowable limit of such interference is that signals on one communication channel should not significantly increase the probability of error in another channel.

Frequency division multiple access (FDMA), time division multiple access (TDMA) and code division multiple access (CDMA) are the three major access techniques that are used to share the available bandwidth in a wireless communication system. Random access schemes are another way of multiple access which are also characterized as the demand–assignment or packet radio multiple access schemes. They give the station access, only when it requests access and are generally used when the traffic generated by the users is bursty.

## 1.1 Random Access Schemes

In random access schemes, many subscribers attempt to access a single channel in an uncoordinated or minimally coordinated manner. In these schemes the following characteristics are common:

1. The network is composed of the independent users attempting to communicate with the single communication channel or perhaps even multiple channels but the number of channels is much smaller than the number of users.

2. The data from each of the users accessing the network is bursty; i.e. transmissions from a single user are separated by periods of silence (usually much

greater than the period of the transmission).

3. The number of the users active on the network is a random variable usually unknown to the receiver.

Random access schemes capitalize on the fact that actual demand rarely equals the peak demand. If a system's capacity is equal to the total peak demand and if the traffic is bursty, the system will be underutilized most of the time. However by using buffers and demand assignment techniques a system with the reduced average capacity can handle the bursty traffic at the expense of some queuing delay.

In these schemes collisions from simultaneous transmissions of multiple transmitters are detected at the base station receiver; in which case an ACK or NAK is broadcasted by the base station to alert the desired user (and all other users) of the received transmission. The ACK signal indicates an acknowledgment of a received burst from a particular user by the base station , and a NAK (negative acknowledgment) indicates that the previous burst was not received correctly by the base station. By using ACK and NAK signals, such schemes employ a perfect feedback. The performance of these techniques is evaluated by the throughput (average number of the messages successfully transmitted per unit time) and the average delay experienced by a typical message burst.

The various protocols falling under this class are as follows:

1. **Pure ALOHA**

The first system that employs the pure ALOHA technique was the ALOHA system computer network at the University of Hawaii in 1970 [1]. The system concept is extremely simple. A user/station accesses the channel as soon as the message is ready to be transmitted. After the message transmission, the user waits for an acknowledgement (ACK) from the receiver. In case of simultaneous reception of more than one message a collision is declared and a negative acknowledgement (NAK) is issued to the user. On receiving the NAK the user retransmits after a random delay. If after first transmission, the user doesn't receive either ACK or NAK then the user retransmits the message [2].

2. **Slotted ALOHA**

The pure ALOHA scheme is improved by inserting a slight amount of coordination among the transmitting stations. The time is divided into equal time slots of length greater than the packet duration . The users only transmit at the beginning of the slot which prevents partial collision resulting in doubling the efficiency as compared to pure ALOHA [3].

3. **Carrier Sense Multiple Access (CSMA)**

ALOHA protocols don't listen to the channel before transmitting and so don't exploit the information about the other users. By listening to the channel before engaging in the transmission, greater efficiencies can be achieved. CSMA protocols are based on the fact that each terminal on the network is able to monitor the status of the channel before transmitting information. If the channel is idle (i.e. no carrier is detected) the user is allowed to transmit a packet based on the particular algorithm that is common to all the transmitters on the network.

4. **Inhibit Sense Multiple Access(ISMA)**

   In mobile radio networks, CSMA might not work as a mobile terminal might not be able to sense a transmission by another terminal. ISMA avoids this problem as the central (base) station sends a busy signal to all the other stations on the network once it is engaged in transmission with one of the stations. This inhibits all other stations from transmitting. A disadvantage of the ISMA is the necessity of the real-time feedback channel [4].

5. **Reservation ALOHA**

   The basic concept of the reservation ALOHA is that the data units may reserve exclusive use of a shared channel by transmitting short reservation requests, sent using a slotted ALOHA protocol on the same shared channel [5]. Slots can be permanently reserved or can be reserved on the request. For high traffic conditions, reservation on request offers better throughput. In one type of reservation ALOHA, the terminal making a successful transmission reserves a slot permanently until it completes its transmission. Another scheme allows a user to transmit a request on a sub-slot which is reserved in each frame. If the transmission is successful (i.e. no collisions are detected), the terminal is allocated the next regular slot in the frame for data transmission [2].

6. **Packet Reservation Multiple Access**

   PRMA uses a discrete packet time technique similar to reservation ALOHA and combines the cyclical frame structure of the TDMA in a manner that allows each TDMA time slot to carry either voice or data, where voice is given priority. PRMA was proposed as a means of integrating bursty data and human speech. It has an advantage in that it can utilize the discontinuous nature of speech with the help of a voice activity detector to increase capacity of the radio channel [2].

Like all engineering solutions the choice of a random access scheme is a trade off between the performance and complexity.

## 1.2 Slotted ALOHA

Out of the various random access techniques, this thesis is focussed on the analysis of the "slotted ALOHA". The reason is twofold.

1. The scheme is simple and easier to analyze than most of its successors but provides the basic framework for the analysis of all the random access schemes.

2. With the advent of the mobile communications there is a renewed interest in the scheme and is currently employed in the GSM systems for handling users requests for channel assignment.

Motivated by these facts we decided to study the protocol under a generalized fading model. Before going into the details of the analysis, a brief review of the slotted ALOHA scheme is first presented; a task best achieved by analyzing the pure ALOHA protocol in a little more detail.

In ALOHA, users have the following modes [6]:

1. **Transmission Mode:**

   Users transmit at any time they desire, encoding their transmission with an error correction code.

2. **Listening Mode:**

   After a message transmission, a user listens for an acknowledgment (ACK) from the receiver. Transmissions from different users will sometimes overlap in time, causing reception errors in data in each of the contending messages. The condition is interpreted as a collision occurrence. In such cases, the errors are detected and a negative acknowledgement(NAK) is sent to the receiver. The error detection for the wired local area networks is by verifying a known check sum [3]. For mobile networks the check sum approach might not be feasible as the bit errors can occur even without any interference (e.g. due to fading). An approach based on the received power levels of the transmitting stations is more appropriate. The base station can sense the average power level of each terminal, in case of interference the joint power level will be higher than the individual power level thus signalling an interference occurrence.

3. **Retransmission mode:**

   When a NAK is received, the messages are simply re-transmitted after a random delay so as to decrease the chances of collision on the next attempt.

4. **Time out mode:**

   If after transmission the user doesn't receive either an acknowledgment (ACK) or a negative acknowledgment(NAK) within a specified time, the user retransmits the message.

In the standard ALOHA , the probability of '$n$' requests or packets generation within a packet duration is generally modelled by the Poisson point process as:

$$R_n = \frac{G^n e^{-G}}{n!} \tag{1.1}$$

where '$G$' is the mean no. of requests generated per packet time. The probability of no packet reception during a given packet time is given as '$exp(-G)$'. For a successful transmission, once a packet is generated, there should be no other packet generation in an interval corresponding to the duration of two packet-time duration. In an interval of two packet-time duration, the mean number of packets generated are '$2G$'. The probability of no other traffic during the same time will be given as

$$R_o = exp(-2G). \tag{1.2}$$

The corresponding successful transmission rate (throughput) will then be given by

$$S = G\ exp(-2G). \tag{1.3}$$

The maximum throughput occurs when $G = 0.5$; that is, on the average there is only one request per two packets generated.

In slotted ALOHA the time is divided into discrete slots where each slot corresponds to the length of one packet. This requires to introduce some co-ordination among the transmitting entities which maybe achieved by broadcasting a sequence of synchronization pulses at the beginning of each time slot. Messages are required to be sent in the slot time between synchronization pulses and can be started only at the beginning of the slot time [3].

With the introduction of timing synchronization the slotted ALOHA protocol doubles the throughput over pure ALOHA as the collision window is now halved. The throughput is thus given by

$$S = G\ exp(-G). \tag{1.4}$$

The retransmission strategy is also changed slightly that if a negative acknowledgment is received, the user transmits after the random delay of an integer number of time slots as shown in Fig. 1.1.

If the throughput expressions of pure and slotted ALOHA are plotted then it can be clearly seen in Fig. 1.2 that the maximum throughput for the two systems is limited to 18.4% and 36.8%, respectively. Apart from their maximum throughput limitations the schemes also suffer from the instability problem. For the slotted ALOHA, as shown in Fig. 1.2, an input traffic of 6 Erlangs reduces the throughput to almost zero implying total blockage, that is, most of the users are in the retransmission mode attempting randomly but failing to transmit.

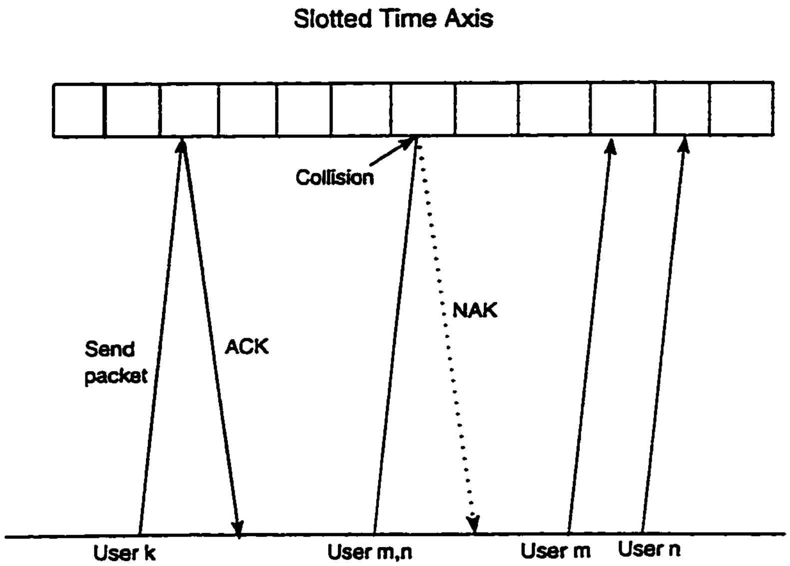



Figure 1.1: Slotted ALOHA Operation

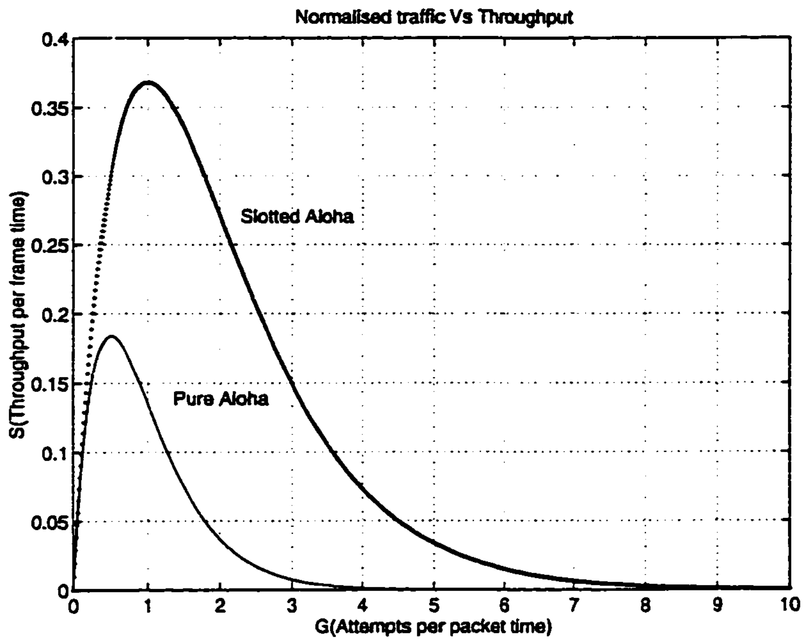



Figure 1.2: Comparison of the Throughput of Pure and Slotted ALOHA

## 1.3 Mobile Radio Slotted ALOHA Networks

Despite its poor throughput and stability problems, the slotted ALOHA protocol is still used in many mobile systems, such as the GSM. This fact motivates us to investigate further, the effect of a mobile radio channel on the slotted ALOHA scheme.

In a mobile radio channel the transmission path between the transmitter and the receiver can vary from simple direct line of sight to one that is severely obstructed by buildings and foliage. The speed of motion impacts how rapidly the signal level varies as a mobile terminal moves in space. Unlike wired channels that are stationary and predictable, radio channels are quite random. The randomness in the behavior of a mobile radio channel is due to the physical environment in which the channel exists. Most cellular radio systems operate in urban areas where there is no direct line of sight path between the transmitter and the receiver, and the presence of high rise buildings causes severe diffraction losses. Because of the multiple reflections from various objects, the electromagnetic waves travel along different paths of varying lengths. The interaction between these waves causes "**multi-path fading**" at a specific location and the strengths of the waves decrease as the distance between the transmitter and the receiver increases.

For most practical channels, where the signal propagation takes place in the atmosphere and near the ground; the free space model (i.e. the attenuation of RF energy between the transmitter and the receiver follows an inverse square law) is not adequate to predict the channel behavior and system performance. For mobile radio applications this is even more pronounced. Not only the channel suffers from multi-path fading but is also time variant as the relative motion between the transmitter and the receiver results in a change in propagation path with time. The rate of change of these propagation conditions accounts for fading rapidity. Frequency selective behavior of fading is not an issue here as all the terminals share the whole frequency band in the slotted ALOHA protocol.

The behavior of the mobile radio channels is represented by various statistical models. These models have traditionally focussed on predicting average received signal strength at a given distance from the transmitter, as well as the variability of the signal strength in spatial proximity to a particular location. A statistical representation of the mean signal strength for an arbitrary transmitter-receiver separation distance is useful in estimating the radio coverage area of the transmitter, whereas radio modem design issues, such as antenna diversity and coding require models that predict the signal variability over a very small distance. This thesis is primarily focussed on predicting the protocol performance under small signal variations (that is, multipath fading). The large signal variations are also considered but in a more subtle way.

Several fading models are used to analyze the performance of the slotted ALOHA. Some of the popular models are considered in the discussion to follow.

- **Rayleigh Fading Model**

  The Rayleigh fading model is the most commonly used distribution to model multipath fading and is used by the number of researchers [7, 8, 9, 10, 11] for the analysis of the slotted ALOHA. It characterizes the reception of a signal through a large number of multiple reflective paths with no line of sight signal component. The envelope of the received signal ($R_i$) is then statistically described by the Rayleigh probability density function (p.d.f); i.e.

$$f_{R_i}(r) = \frac{2r}{P_o} exp(\frac{-r^2}{P_o}) \tag{1.5}$$

  where $P_o$ is the mean signal power.

- **Rician Fading Model**

  The use of the Rayleigh fading model in the literature for the analysis of the slotted ALOHA is due to its analytical tractability though it's not a good model under all conditions. One reason for considering Rayleigh fading so widely in the literature is that it is supposed to model the cellular environment quite well; but it is true for the case of the Macro cells. However when the cell size is made smaller or in the presence of the flat environments without large obstacles there exists a substantial line of sight component and it is more appropriate to model the channel with the Rician Fading model [12]. The Rician Fading model is a more generalized fading model that also reduces to the Rayleigh model under special conditions. The p.d.f of the received signal

$R_i$ is given as [2]:

$$f_{R_i}(r) = \frac{2r}{P_o} exp\left(\frac{-(r^2 + A^2)}{P_o}\right) I_o\left(\frac{2Ar}{P_o}\right) \tag{1.6}$$

where $A$ is the line of sight (LOS) signal component and $I_o$ is the zero order modified Bessel function of the first kind. The Rician distribution is also defined in terms of a parameter $K$ called the Rice factor which is defined as the ratio between the power associated with the LOS component and the scattered component; i.e.

$$K = \frac{A^2}{P_o}. \tag{1.7}$$

Rayleigh fading and the absence of fading correspond to $K = 0$ and $K \rightarrow \infty$, respectively. It is shown in [12] and [13] that the maximum throughput of the slotted ALOHA under Rician fading is substantially larger than that considered for the ideal slotted ALOHA in non-fading environments.

- **Nakagami Fading Model**

The Nakagami Fading model has been recently used for analyzing the behavior of the slotted ALOHA in cellular environments [14, 15]. The Nakagami model is becoming more prevalent in performance studies related to mobile communications, because it is a parameterized distribution which has the flexibility to describe different fading environments by judicious choice of parameters [16]. The p.d.f of the received signal $R_i$ is described as [14]:

$$f_{R_i}(r) = \frac{2}{\Gamma(m)} \frac{m}{P_o} r^{2m-1} exp(-\frac{mr^2}{P_o}). \tag{1.8}$$

Nakagami Fading is characterized by the two parameters, namely, the mean power ($P_o$) and the fade figure ($m$) . The variation of the p.d.f with the change in the fading parameter $m$ is shown in Fig. 1.3. For $m = 1$ we have a Rayleigh fading channel, and as $m \rightarrow \infty$ we have a channel that becomes non fading (as p.d.f tends to an impulse function). At the other extreme, for $m = 0.5$ we have a one sided Gaussian fading distribution.

The Nakagami model is shown to provide a closer match to the experimental data than the Rayleigh and Rician distributions. The more popular Rayleigh and Rician distributions correspond to the physical model which assumes a sum of sufficiently large number of equal power multipath components with different phases. The Nakagami distribution arises as an approximate solution to this general problem of the distribution of the magnitude of the sum of random vectors; whereas Rayleigh and the Rician distributions are particular solutions to the same problem [17].

The equivalence between the Rice and the Nakagami distributions can be obtained through the functional relationship of the parameters of the two distributions. In [17] it is shown that the Nakagami and the Rician distributions are related by

$$m = \frac{1}{1 - \frac{K^2}{(1+K)^2}}. \tag{1.9}$$

It is further shown that for values of '$K \geq 2$' a linear relationship exists between the '$m$' and '$K$', given by,

$$m = s_m K + m_o \tag{1.10}$$

where $s_m$ is the slope and $m_o$ is the m−axis intercept. So it is clear that '$m$' is a parameter which is similar in physical interpretation as that of '$K$' i.e. the amount of specular power in the received signal with respect to the random power. An added advantage of Nakagami model is the presence of the closed form expression of characteristic function whereas no closed form expressions for the characteristic function of the Rician and Rayleigh random variables are known [18].

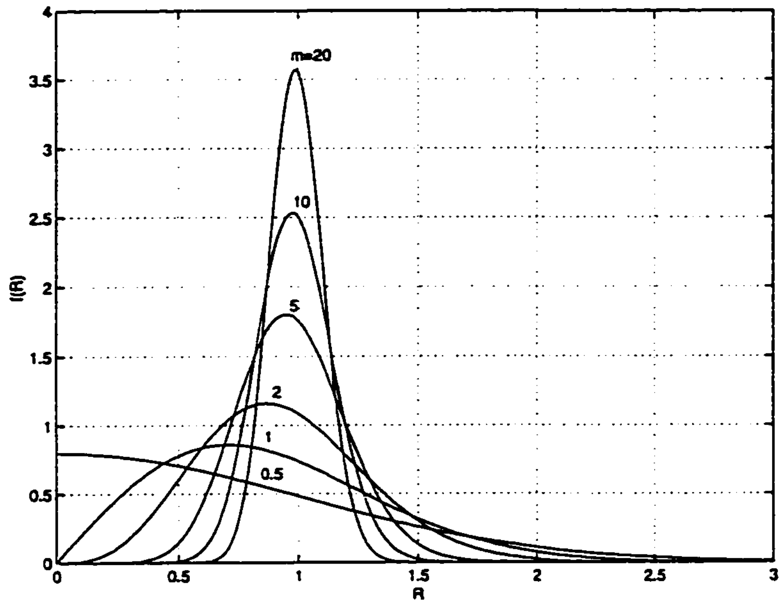



Figure 1.3: Nakagami-m density function

## 1.4 Capture Effect

Mobile-radio slotted ALOHA benefits from the natural division of the signals in various power classes accounting for a phenomenon called the " **capture effect**". Due to the capture effect it is possible for the strongest user to successfully capture the intended receiver, even when many other users are transmitting.

Capture of a strong signal in the presence of the other weaker signals dates back to the days of analog frequency modulation (FM) receivers and portable police radios [19]. The capture effect was first considered for the ALOHA systems by Roberts [20], when he examined the throughput and delay for a capture ALOHA system and discussed the cases of no capture and perfect capture. Metzner [21] showed that by dividing the users into two power groups – one transmitting at high power and the other at low power– the maximum throughput of the slotted ALOHA can be increased to about 53 percent. These observations were then extended in the literature. Most of the researchers [7, 12, 22, 23] assume that the test packet can capture the receiver if the difference in the signal strengths of two or more colliding packets is above a certain threshold (capture ratio ); then receiver will be able to decode one packet correctly; the other packets would simply cause some interference to the desired packet. Capture of a packet is thus a probabilistic phenomenon which based on the above definition can be mathematically expressed as:

$$P_{capt}(n) = Pr\left(\frac{P_s}{P_n} > z_o\right) \tag{1.11}$$

where $P_{capt}(n)$ is the probability of capture of a test packet with power $P_s$ in the presence of $n$ interfering packets with joint interference power $P_n = \sum_{i=1}^{n} P_i$. The value of capture parameter ($z_o$) depends on the propagation property of the transmission and the modulation and coding techniques used in the network. A typical value is $z_o = 6dB$ which characterizes the reception capability of a typical narrow band FM receiver [24]. In radio packet switching with mobile terminals, signal power variations arise automatically due to the different distances between the terminals and the receiver (path loss) and due to shadowing or fading. Different power levels will be received and therefore there is a possibility of reading one of the packets correctly despite receiving several packets in the same time slot .

Another definition of capture assumes that the strongest packet will capture the receiver when the ratio of its power to the power of next to the strongest packet is above the capture ratio [8, 20, 25]. The capture probability is then expressed as:

$$P_{capt}(n) = Pr\left(\frac{P_s}{P_i} > z_o, \quad \forall\, i, 0 < i \leq \; n\right). \tag{1.12}$$

Leung [26] included the effects of modulation and coding in the above capture definitions. In [11] it is asserted that the packet is captured if its bit error probability is above a certain preset threshold or a packet is received successfully if its address header is received correctly at the base station. In [9, 27, 24] it is assumed that a packet captures the receiver only when the bit sequence detected by the receiver entirely matches the bit sequence in the test packet.

In this thesis the popular capture definition of (1.11) is considered. This definition is only modified when considering the effects of modulation and coding where a definition similar to the one proposed by [26] is used. Following [7] the probability of successful transmission for the given capture parameter ($z_o$) is given as:

$$P_{success}(z_o) = 1 - \sum_{n=1}^{\infty} R_n \; Pr\left(\frac{P_s}{P_n} < z_o\right). \qquad (1.13)$$

The throughput is thus determined by

$$S = G \; P_{success}(z_o). \qquad (1.14)$$

In the limit $z_o \rightarrow \infty$ the above equation reduces to (1.4), but for any finite value of $z_o$ the throughput would exceed this value.

## 1.5 Objective and Organization of the Thesis

Most of the research [7, 8, 9, 10, 23, 27, 24, 28, 29] on the slotted ALOHA scheme has so far been done with the assumption that the channel can be sufficiently described by the Rayleigh fading model. Though this assumption is generally true and provides a close match to the experimental data collected from the dense urban environment, there are situations, e.g. a rural environment or an urban micro cellular environment, where fading conditions are not as severe as assumed in the Rayleigh fading model. As asserted earlier, the Nakagami fading model is a generalized fading model which reduces to the Rayleigh fading model for $m = 1$ and can also accommodate fading conditions that are more or less severe than the Rayleigh fading. So it seems quite interesting if one could generalize the results obtained for the popular random access scheme like the slotted ALOHA under this model. Slotted ALOHA, despite its simplicity, provides the basic framework for the study of more complicated of the random access schemes e.g. CSMA or ISMA. The utility of the scheme is not only limited to a channel access protocol (which is the primary concern here) but with the introduction of the mobile data or multimedia communications (characterized by their burstiness and demands for the large communication capacity in a short period of time) the slotted ALOHA provides a good alternative to the fixed assignment multiple access schemes. The performance measure of such a scheme is generally the throughput– which determines the average rate of successful transmission through the channel. The capacity of the channel is determined by the maximum throughput it can offer to the communicating entities.

This thesis is focussed on the determination of the capacity and the throughput behavior of the slotted ALOHA under generalized fading conditions through the use of the Nakagami fading model. The apparent benefit of this study is the generalization of the effects that are so far studied for more particular fading models with few exceptions [14, 15]. As we'll show during the course of the study that the present thesis gives more general results that give us better mathematical insight, closer to the real world changing fading and shadowing conditions.

The work done in [14] has considered a fade only environment in which all the packet signals reach the receiver with uncorrelated slow Nakagami fading and equal mean power. The effects of the thermal noise are also neglected. For evaluating the capture probability, the interfering signals are incoherently summed up (i.e. their powers are added) at the receiver and then the ratio of the signal power to the joint interference power is compared to the capture ratio. The throughput expression is found out to be:

$$S = G\left[1 - \Sigma_{n=1}^{\infty} \frac{{z_o}^m G^n exp(-G)}{n!} \frac{\Gamma(nm+m)}{\Gamma(m)\Gamma(nm)} \times \; F(nm+m, m, m+1, -z_o)\right] \tag{1.15}$$

The throughput expression is shown to be simplified to the one reported in [7] for m=1.

Hafez [15] presents an extension of the work in [14] for arbitrary fading parameters and received power levels and also includes correlated Lognormal shadowing and the spatial distribution of the interfering packets for a dual slope path loss model. Due to the assumed perfect correlation between the interfering packets, a single interfering packet is assumed to represent the behavior of all the interfering packets. It is shown that the throughput is less sensitive to the severity of the fading of the desired packet than that of the interferers. The results also show that the presence of shadowing and the spatial distribution increase the throughput.

In this thesis an extension of the work undertaken in [14] and [15] is presented. After briefly introducing the subject in this chapter, a generalized study of the slotted ALOHA is presented in an environment where fading is considered as the soul contributor to the random power fluctuations. With a different approach around the problem, we arrive at the results which are expressed in terms of the incomplete beta functions, providing a simpler alternative to the hypergeometric functions reported in [14] and [15]. In chapter 3 the distribution of the users accessing the central receiver and the statistically independent shadowing effects of the obstacles are also taken into account, which yields a further improvement in the channel capacity. The influence of the use of modulation, thus the packet length and noise, on the system performance and channel utilization is the subject of chapter 4. The numerical results justify the intuition that the inclusion of such deleterious effects like the white noise and the probability of bit error, decreases the throughput as compared to the ideal case. Finally the effect of changing fading conditions on the system stability is studied using a finite user markov model. The results are in agreement with those reported in the literature for the Rayleigh fading model and confirm that the effect of the capture is to increase the system stability. we then conclude our discussion with some suggestions for the future research.

# Chapter 2

# Nakagami Fade only Environment

## 2.1 Introduction

The channel throughput of the slotted ALOHA in a Nakagami fading environment is evaluated in this chapter for three different cases. In the first case all the packets are assumed to reach a central receiver under uncorrelated Nakagami fading with equal mean power . The second case corresponds to the assumption that the mean power of the arbitrarily selected test packet is different from all the other packets which form the interference. Finally, assuming that the test packet and the interfering packets have different fading statistics, an alternative expression to [15] is derived. In contrast to the cumbersome hypergeometric functions reported in [14] and [15], all the results are expressed with a much simpler incomplete beta function.

## 2.2 Transmitters having Equal Mean Power Level

In a Nakagami fading environment, the signal magnitude is characterized statistically as (1.8):

$$f_{R_i}(r) = \frac{2}{\Gamma(m)}\frac{m}{P_o}r^{2m-1}exp(-mr^2/P_o). \tag{2.1}$$

The test packet power ($P_s$) is then statistically described by a gamma distributed random variable with probability density function (pdf) $f_{P_s}(p_S)$ given by [30]:

$$f_{P_s}(p_s) = (\frac{m}{\bar{p_s}})^m \frac{{p_s}^{m-1}}{\Gamma(m)} exp(-\frac{m}{\bar{p_s}}p_s), \quad p_s > 0 \tag{2.2}$$

where $m \geq 0.5$ and $\bar{p_s}$ is the average power of the test packet.

Assume that there are '$n$' independent identically distributed interferers sending packets to the central base station with the equal mean power. The pdf of the joint interference power $P_n = \Sigma_{i=1}^{n} P_i$ is then given as [31]:

$$f_{P_n}(y) = (\frac{m}{\bar{y}})^{nm} \frac{y^{nm-1}}{\Gamma(nm)} exp(-\frac{m}{\bar{y}}y), \quad y > 0 \tag{2.3}$$

where $\bar{y}$ is the average power of each interferer. With $\bar{p_s} = P_o$ and all the packets with equal mean power we have $\bar{p_n} = nP_o$. The analysis in [14] evaluates the ratio of $P_s$ and $P_n$ and compares them to the receiver capture parameter($z_o$). For the case of the Nakagami fading channel model, both $P_s$ and $P_n$ are gamma distributed with parameters $(m, m/P_o)$ and $(nm, m/P_o)$. It can be proved that their ratio i.e. $P_s/P_n$ will have a beta distribution of the second kind. As a result the CDF i.e.

$$Pr\left(\frac{P_s}{P_n} < z_o\right) \tag{2.4}$$

can be expressed as an incomplete beta function.

Define $P_s = X$ and $P_n = Y$ then it is required to evaluate

$$Pr\left(\frac{X}{Y} < z_o\right). \tag{2.5}$$

since X and Y are independent random variables therefore their joint probability density function will be the product of their individual probability density functions i.e.

$$f_{XY}(x,y) = f_X(x) \times f_Y(y). \tag{2.6}$$

Referring to the individual probability distributions (2.2) and (2.3) we have:

$$f_{XY}(x,y) = \frac{\lambda^m}{\Gamma[m]} x^{m-1} exp(-\lambda x) \frac{\lambda^{nm}}{\Gamma[nm]} y^{nm-1} exp(-\lambda y) \tag{2.7}$$

where $\lambda \equiv \frac{m}{P_o}$.

Let us make the following transformations:

$$u = x + y \quad and \quad z = \frac{x}{y} \tag{2.8}$$

so that

$$x = u[1 - \frac{1}{(z+1)}] = \frac{uz}{z+1} \tag{2.9}$$

and

$$y = \frac{u}{1+z} \tag{2.10}$$

Then it can be shown that

$$f(u,z) = \underbrace{\frac{\lambda^{nm+m}}{\Gamma(nm+m)} u^{nm+m-1} exp(-\lambda u)}_{f_U(u)} \underbrace{\frac{\Gamma(nm+m)}{\Gamma(nm)\Gamma(m)} \frac{z^{m-1}}{(1+z)^{nm+m}}}_{f_Z(z)}. \tag{2.11}$$

The first term is the convolution of the individual pdf's resulting in the density function for the $U$ (which is also a Gamma distribution with parameters '$nm + m, m/P_o$') and the second term gives the pdf for $Z$. Thus

$$f_Z(z) = \frac{\Gamma(nm+m)}{\Gamma(nm)\Gamma(m)} \frac{z^{m-1}}{(1+z)^{nm+m}} \tag{2.12}$$

which is the beta distribution of the 2nd kind. The CDF of this distribution gives an incomplete beta function ($I_x(m, nm)$) defined as in [32]:

$$I_x(m, nm) = \frac{\Gamma(nm+m)}{\Gamma(m)\Gamma(nm)} \int_0^x u^{m-1}(1-u)^{nm-1} du \tag{2.13}$$

where $x = z_o/(z_o + 1)$. As a result the throughput expression

$$S = G[1 - \sum_{n=1}^{\infty} R_n F_{Z_n}(z_o)] \tag{2.14}$$

where

$$R_n = (\frac{G^n}{n!}) exp(-G) \tag{2.15}$$

and

$$F_{Z_n}(z_o) = Pr\left(\frac{P_s}{P_n} < z_o\right) \tag{2.16}$$

can be written as

$$S = G[1 - \Sigma_{n=1}^{\infty} R_n I_{\frac{z_o}{(z_o+1)}}(m, nm)]. \tag{2.17}$$

The result is also verified using simulation. The values of probability distribution function $F_{Z_n}(z_o)$ which determines the probability of capture (thus improvement in the throughput) are shown. Fig. 2.1 shows how closely simulation matches the exact result.

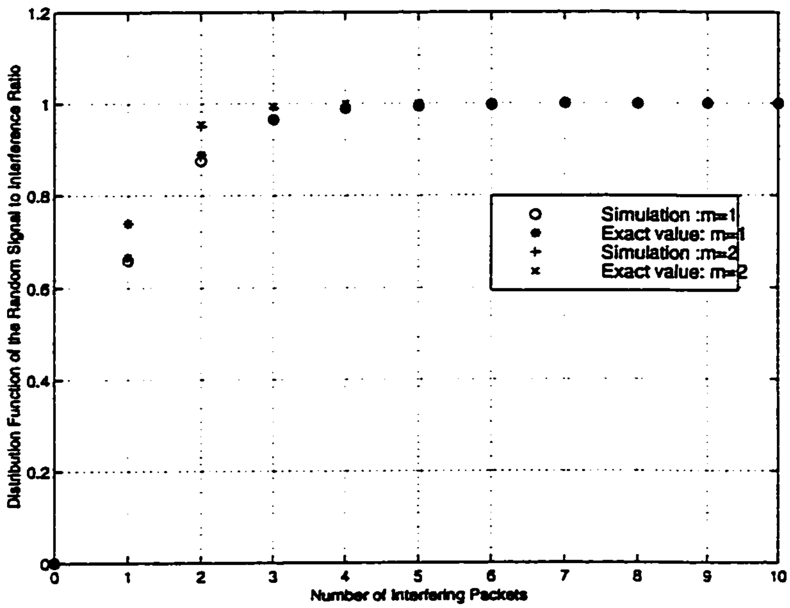



Figure 2.1: Distribution function for the signal to interference ratio assuming equal power with $z_o$= 3dB and m=1,2.

Using the exact result the throughput of the slotted ALOHA is plotted under varying fading conditions. Fig. 2.2 and Fig. 2.3 confirm the results reported in [14] but with an alternative simplified expression for the channel throughput as indicated above. The results are reported considering the fade figure $m$ as a parameter. Comparing Fig. 2.2 and Fig. 2.3 it can be noticed that as the capture ratio increases the throughput goes down due to the reduction in the capture probability. Also for a particular capture ratio, Fig. 2.2 indicates that the throughput decreases with the increase in the value of $m$ (that is, with the reduction in fading depth). Referring to Fig. 2.3, when the capture ratio is large the fading conditions do not significantly affect the throughput.

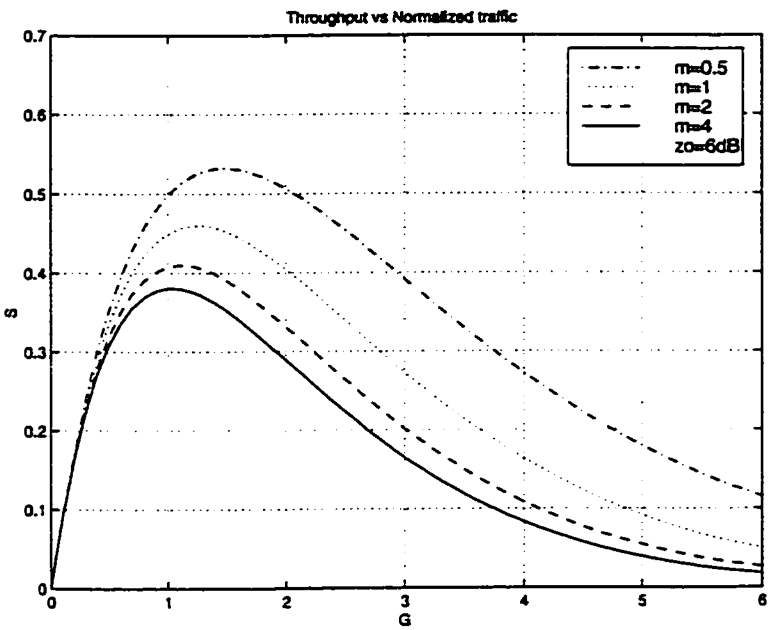



Figure 2.2: Throughput curves for slotted ALOHA assuming ring model with $z_o$= 6dB

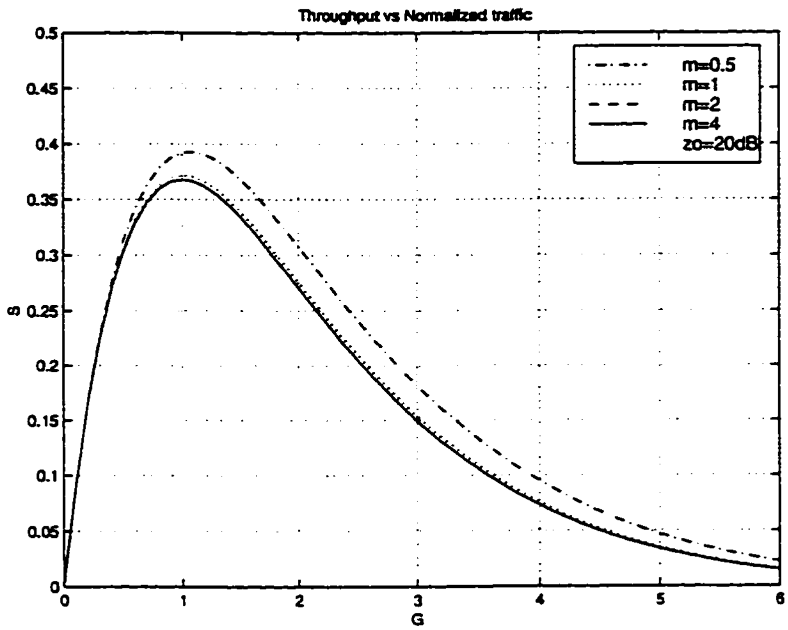



Figure 2.3: Throughput curves for slotted ALOHA assuming ring model with $z_o$= 20dB

## 2.3 Transmitters having Unequal Mean Power Level

The probability density function of the test packet is given by (2.2) and that of the joint interference power by (2.3) where the average powers of the test packet and the interferers are assumed different. Defining $Z_n = \frac{P_s}{P_n}$ then [14]:

$$f_{Z_n,y}(z,y) = f_{P_s}(zy)\ f_{P_n}(y)\ y. \tag{2.18}$$

The marginal distribution of $Z$ is thus given by :

$$f_{Z_n}(z) = \int_0^\infty f_{P_s}(zy)\ f_{P_n}(y)\ y\ dy \tag{2.19}$$

and the required outage probability (or the probability distribution function) will then be given by:

$$F_{Z_n}(z_o) = \int_0^{z_o} f_{Z_n}(z)dz. \tag{2.20}$$

Referring to (2.2) we've:

$$f_{P_s}(zy) = \left(\frac{m}{\bar{p_s}}\right)^m \frac{z^{m-1}y^{m-1}}{\Gamma(m)} exp\left(-\frac{mzy}{\bar{p_s}}\right). \tag{2.21}$$

$$f_{Z_n,y}(z,y) = \left(\frac{m}{\bar{p_s}}\right)^m \frac{z^{m-1}y^{m-1}}{\Gamma(m)} exp\left(-\frac{mzy}{\bar{p_s}}\right) \times \left(\frac{m}{\bar{y}}\right)^{nm} \frac{y^{nm-1}y^{m-1}}{\Gamma(m)} exp\left(-\frac{my}{\bar{y}}\right)\ y. \tag{2.22}$$

$$f_{Z_n,y}(z,y) = Cy^{nm+m-1}\ exp\left(-\left(\frac{mz}{\bar{p_s}} + \frac{m}{\bar{y}}\right) y\right) \tag{2.23}$$

where

$$C = \frac{m^{nm+m} z^{m-1}}{\bar{p_s}^m \bar{y}^{nm} \Gamma(nm)\Gamma(m)} \tag{2.24}$$

$$f_{Z_n}(z) = C \int_0^\infty y^{nm+m-1}\, exp\left(-\left(\frac{mz}{\bar{p_s}} + \frac{m}{\bar{y}}\right) y\right)\, dy. \tag{2.25}$$

Using integral 3.351-3 of [33]

$$f_{Z_n}(z) = C\ \Gamma\left(nm+m\right)\left(\frac{mz}{\bar{p_s}} + \frac{m}{\bar{y}}\right)^{-(nm+m)}. \tag{2.26}$$

So finally

$$F_{Z_n}(z_o) = \frac{m^{nm+m}}{\bar{p_s}^m \bar{y}^{nm}} \frac{\Gamma(nm+m)}{\Gamma(nm)\Gamma(m)} \int_0^{z_o} z^{m-1}\left(\frac{mz}{\bar{p_s}} + \frac{m}{\bar{y}}\right)^{-(nm+m)} dz \tag{2.27}$$

which using Eq. 3.194-1 of [33] can be evaluated as:

$$F_{Z_n}(z_o) = C' \frac{\bar{y}}{m}^{nm+m} \frac{z_o{}^m}{m}\ F\left(nm+m, m; 1+m; -\frac{\bar{y}z_o}{\bar{p_s}}\right) \tag{2.28}$$

where

$$C' = \frac{m^{nm+m}}{\bar{p_s}^m \bar{y}^{nm}} \frac{\Gamma(nm+m)}{\Gamma(nm)\Gamma(m)} \tag{2.29}$$

Using the transformations [32] :

$$F\left(\alpha, \beta; \gamma; z\right) = (1-z)^{-\beta}\ F\left(\beta; \gamma-\alpha; \gamma; \frac{z}{z-1}\right). \tag{2.30}$$

$$F(a, 1-b; a+1; x) = \frac{a\ B_x\left(a, b\right)}{x^a}. \tag{2.31}$$

It can be shown that

$$F\left(nm+m, m; 1+m; -\frac{\bar{y}z_o}{\bar{p_s}}\right) = \left(1+\frac{\bar{y}z_o}{\bar{p_s}}\right)^{-m}\left(1+\frac{\bar{p_s}z_o}{\bar{y}}\right)^m\ B_x\left(m, nm\right) \tag{2.32}$$

which when substituted in (2.28) gives the final result:

$$F_{Z_n}(z_o) = I_x\left(m, nm\right) \tag{2.33}$$

where $x = \frac{1}{1+\frac{SIR}{x_o}}$ and :

$$SIR = \frac{average\ signal\ power}{average\ single\ interferer\ power} = \frac{\bar{p_s}}{\bar{y}} \tag{2.34}$$

The above result is also reported in [31], where it is utilized for the study of the co-channel interference.

For the sake of comparison we observe that the simulation results for the probability distribution functions very closely match those which are obtained from the exact calculations. Comparing the results with Fig. 2.1 it is evident that increase of the average signal power over the average interference power results in the improvement in the capture probabilities, which correspondingly increases the throughput. If the average signal power is different from the average power of an interferer then Fig. 2.5 and Fig. 2.6 show that for a particular capture ratio the throughput increases with increasing SIR. Fig. 2.7 compares the throughput for different SIR under similar fading condition which (as expected) shows the increased throughput for higher SIR.

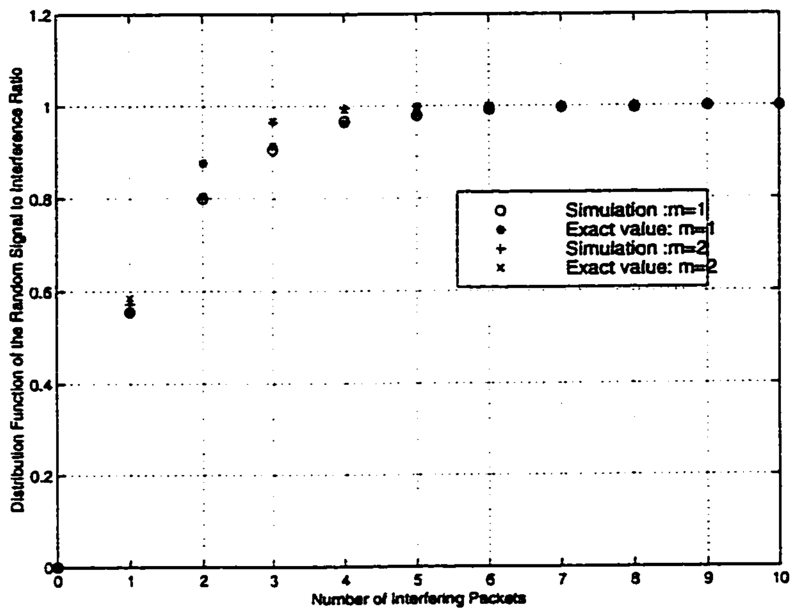



Figure 2.4: Distribution function for the signal to interference ratio assuming unequal mean power (SIR=2dB) with $z_o$= 3dB and m=1,2.

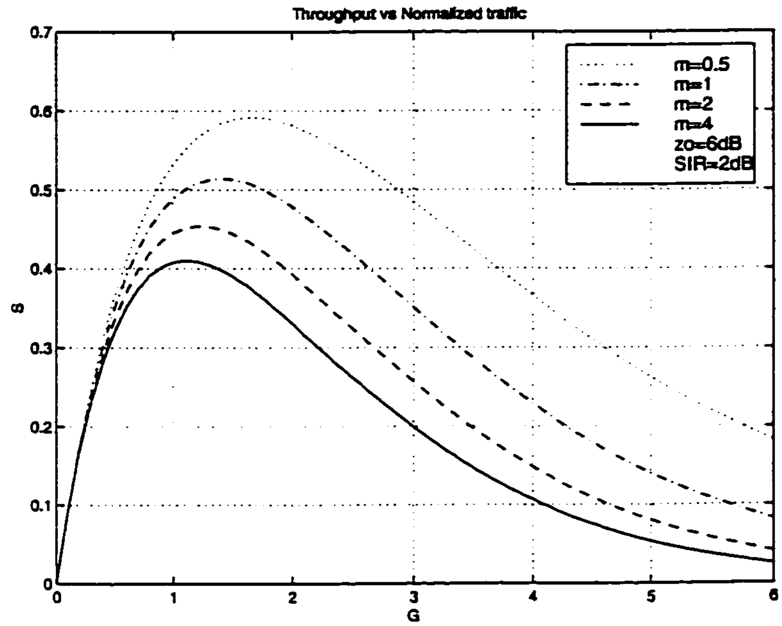



Figure 2.5: Throughput curves for slotted ALOHA assuming unequal mean power ($SIR = 2dB$) with $z_o$= 6dB

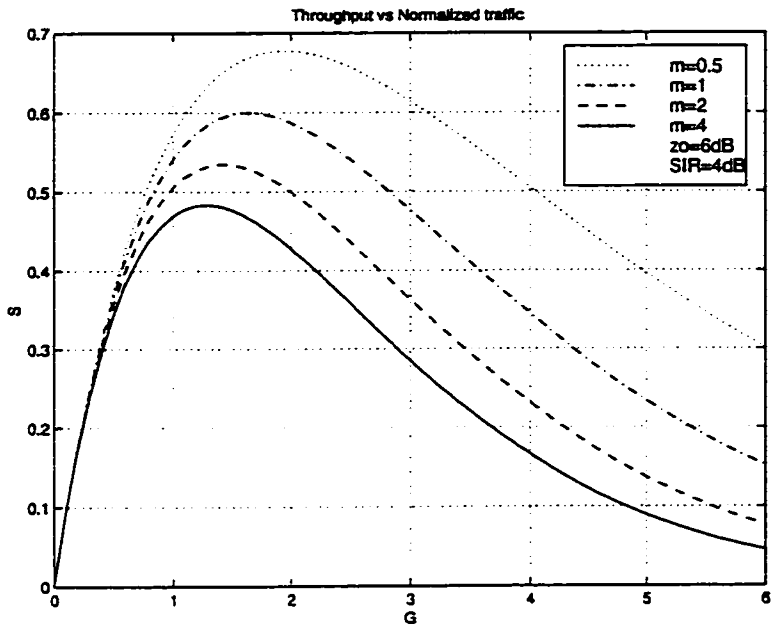



Figure 2.6: Throughput curves for slotted ALOHA assuming unequal mean power ($SIR = 4dB$ ) with $z_o$= 6dB

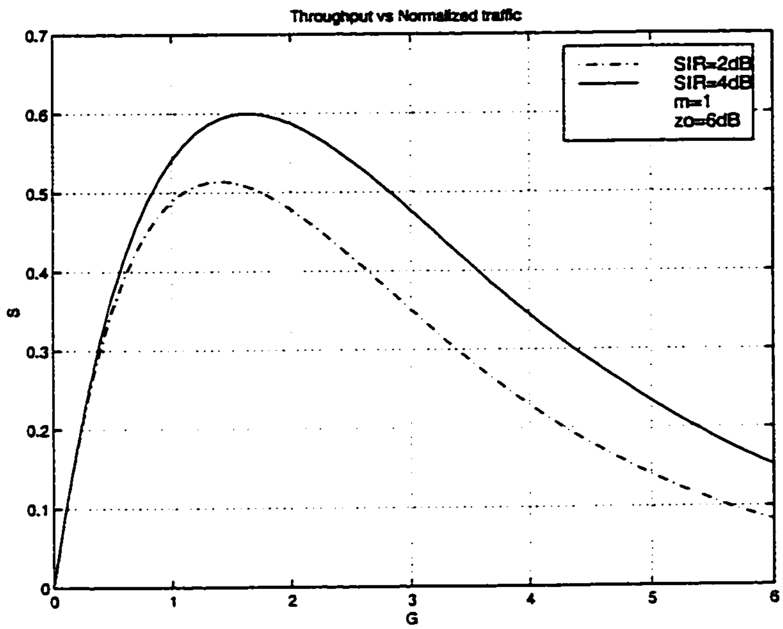



Figure 2.7: Throughput curves for slotted ALOHA for two different $SIRs$' with $z_o$= 6dB

## 2.4 Transmitters having Different Fading Statistics

If the fading parameter of the interferers is assumed to be different from that of the test packet then it can be shown that the distribution function can still be expressed in terms of a simple incomplete beta function and is given as:

$$F_{Z_n}(z_o) = I_x(m_o, nm_u) \tag{2.35}$$

where $m_o$ is the fading parameter for the test packet, $m_u$ is the fading parameter associated with the interferers, and $x = \frac{1}{1+\frac{m_u SIR}{z_o m_o}}$. $SIR$ is as defined in (2.34) and $z_o$ is the capture ratio.

The final result for the probability of outage reported in [15] is given as:

$$F_{Z_n}(z_o) = 1 - \frac{\Gamma(nm_u + m_o)}{nm_u \Gamma(nm_u)\Gamma(m_o)} \frac{\left(\frac{z_o m_o}{m_u R_u}\right)^{m_o}}{\left(1 + \frac{z_o m_o}{m_u R_u}\right)^{m_o + nm_u}} F\left(1, nm_u + m_o; nm_u + 1; \frac{1}{1 + \frac{z_o m_o}{m_u R_u}}\right). \tag{2.36}$$

where

$$R_u = SIR = \frac{\bar{p_s}}{\bar{y}}. \tag{2.37}$$

Here we'll follow a different approach to prove our point. Instead of rigorously rederiving the result following the approach of the section 2.3, we simplified the above result using simple transformations on the hypergeometric functions. Referring to [33] we have:

$$F(\alpha, \beta; \gamma; z) = (1-z)^{\gamma-\alpha-\beta} F(\gamma-\alpha; \gamma-\beta; \gamma; z). \tag{2.38}$$

Based on the above (2.38) we can express:

$$
\begin{aligned}
F\left(1, nm_u + m_o; nm_u + 1; \frac{1}{1 + \frac{z_o m_o}{m_u R_u}}\right) &= \left(1 - \frac{1}{1 + \frac{z_o m_o}{m_u R_u}}\right)^{-m_o} \times \\
F\left(nm_u; 1 - m_o; nm_u + 1; \frac{1}{1 + \frac{z_o m_o}{m_u R_u}}\right) & \qquad (2.39)
\end{aligned}
$$

which based on (2.31) can be further simplified to

$$
= \left(\frac{\frac{z_o m_o}{m_u R_u}}{1 + \frac{z_o m_o}{m_u R_u}}\right)^{-m_o} \frac{nm_u}{\frac{1}{1+\frac{z_o m_o}{m_u R_u}}} B_x\left(nm_u, m_o\right) \qquad (2.40)
$$

where

$$
x \equiv \frac{1}{1 + \frac{z_o m_o}{m_u R_u}}. \qquad (2.41)
$$

Substituting the result in (2.36)

$$
\begin{aligned}
F_{Z_n}(z_o) &= 1 - \frac{\Gamma(nm_u + m_o)}{\Gamma(nm_u)\Gamma(m_o)} B_x\left(nm_u, m_o\right) \\
&= 1 - \frac{B_x\left(nm_u, m_o\right)}{B\left(nm_u, m_o\right)} \\
&= 1 - I_x\left(nm_u, m_o\right) \\
&= I_{1-x}\left(m_o, nm_u\right). \qquad (2.42)
\end{aligned}
$$

The result is more general than the one reported in [15] in the sense that it reduces to prevoius two cases for specific parameters without the need of any further simplifications. Figures 2.8,2.9,2.10 and 2.11 show the effect of fading parameters on the throughput of the slotted ALOHA when the users are transmitting with equal mean power. The throughput decreases as the fading parameter of the interfering packets $m_u$ increases as seen in Fig. 2.8 and Fig. 2.9. As observed through Fig. 2.10 and

Fig. 2.11 the effect of the change of the fading parameter of the test packet has a less significant effect on the throughput. So it can be concluded that the throughput is less sensitive to the severity of the fading of the desired packet than that of the interferers.

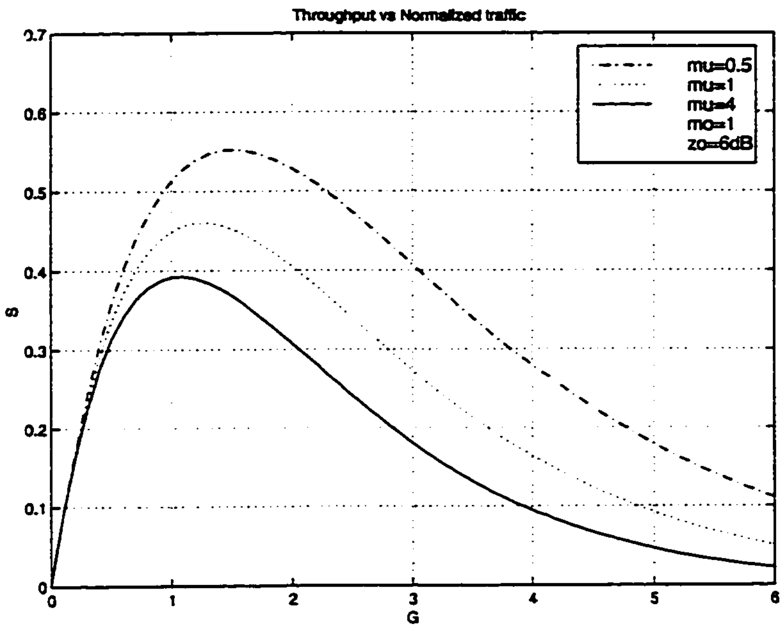



Figure 2.8: Effect of fading parameters on the throughput of the slotted ALOHA for capture ratio $z_o = 6dB$

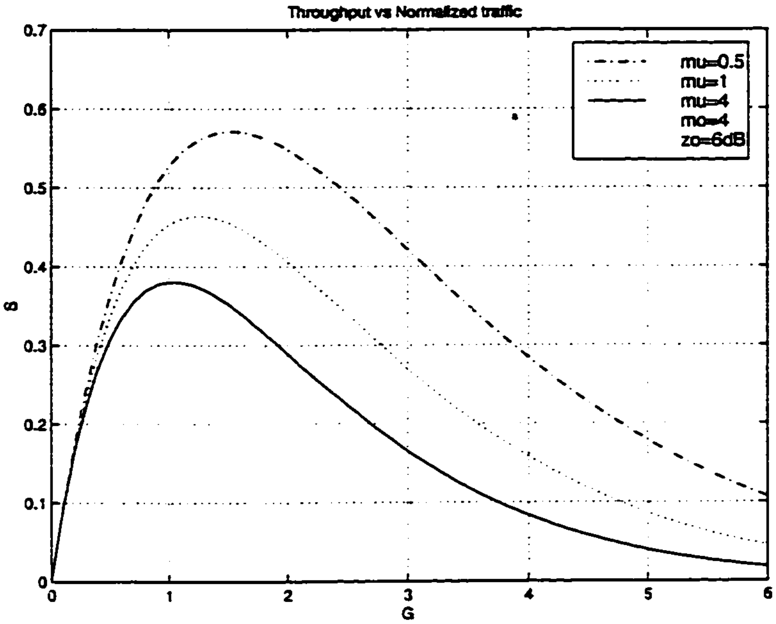



Figure 2.9: Effect of fading parameters on the throughput of the slotted ALOHA for capture ratio $z_o = 6dB$

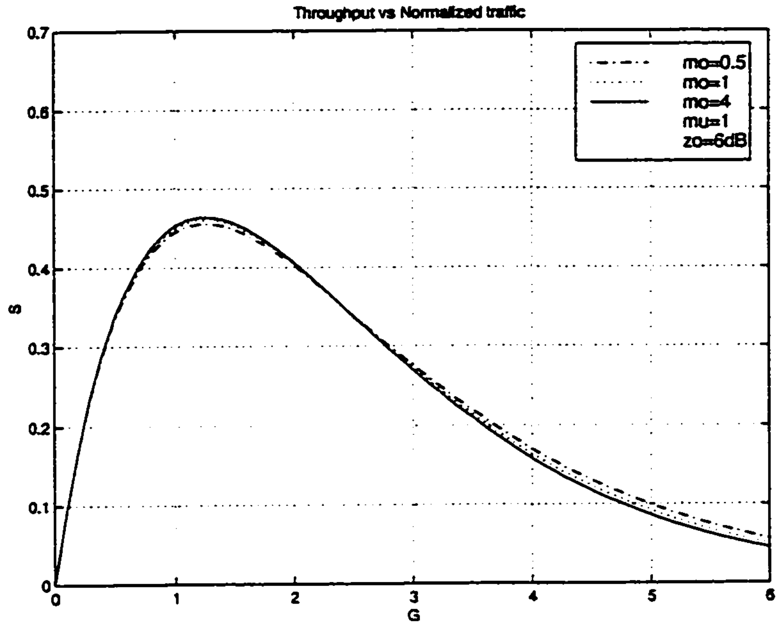



Figure 2.10: Effect of fading parameters on the throughput of the slotted ALOHA for capture ratio $z_o = 6dB$

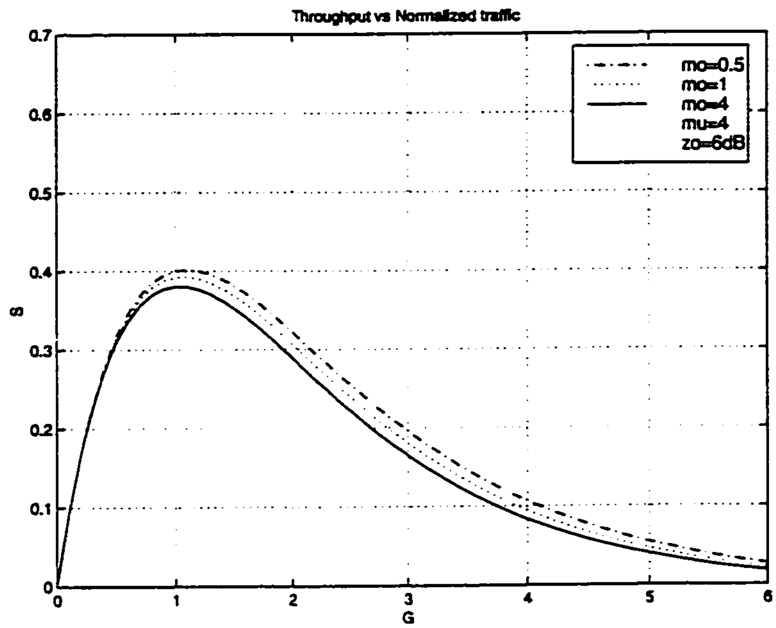



Figure 2.11: Effect of fading parameters on the throughput of the slotted ALOHA for capture ratio $z_o = 6dB$

# Chapter 3

# Effect of the Spatial Distribution and Shadowing

## 3.1 Introduction

In chapter 2 we have assumed that all the users of the mobile network are either at a constant distance from the base station or the distance of all the interferers is same from the base station. This results in either the constant average power for all the contending terminals or same average power level of all the interfering packets. The assumptions are suitable for the local area network with the uniform attenuation for all terminals and also for a mobile radio network with the adaptive control of the transmit power of all terminals or with these moving at a fixed distance from the common base station (e.g. on a circular ring) without any shadowing object (e.g. buildings or hills). An increase in the throughput is observed due to the fading phenomenon where the nature randomly attaches priority to certain transmission packets. In this chapter the effect of the distribution of the transmitting users and power level variations caused by shadowing is studied. The study of the effect of the spatial distribution in the presence of the Nakagami fading is undertaken first and then we have investigated the affect of packet shadowing on the throughput of the slotted ALOHA.

## 3.2 Effect of the Spatially Distributed Users

As observed in chapter 2 the power differences increase the throughput for all mobile transmitters but these don't benefit from the propagation phenomenon to the same extent. When the spatial distribution of the users is taken into account, then the slotted ALOHA systems suffer from the **near-far effect**, where a user transmitting closer to the central receiver has a better chance of capturing the receiver. So the near-far effect apparently seems unfavorable to the users near the boundary of the cell in a cellular network. The weaker packets from distant transmitters experience a higher probability of loss due to the interference from the contending stronger packets near to the base station. The necessary retransmissions result in a relatively more traffic offered to the channel by distant terminals resulting in the saturation of the ALOHA networks. But unlike standard slotted ALOHA networks this saturation doesn't mean total blockage. In case of saturation the traffic from the nearest users will be handled first, while the remaining terminals are served with some delay. So if the fading channel is saturated this means that the demanded throughput can not be achieved immediately, rather than the total blockage.

A basic model to analyze the distribution of the packet traffic over the cellular area was proposed by Abramson in 1977 [1]. The model assumed that the generation of packets in a given area depends only on the distance from the central receiver and is independent of the direction $\theta$. It was further assumed that a test packet transmitted at a distance $\rho$ can capture the receiver, if and only if, no other packet is generated simultaneously inside a critical circle of radius $a\rho$, where $a \geq 1$ is a constant which defines the critical distance to which capture can take place. Competition from one or more packet signals generated outside this circle is not taken into account. Arnbak and Van Blitterswijk [7] presented a refined model, including Rayleigh fading and cumulation of interference from several colliding packets. Terminals are assumed distributed over the circular area such that the offered traffic density is quasi–uniform. Some other models [11, 15, 27] assuming uniform traffic density are also considered in the literature. In [29, 34] a Lognormal probability density function for modelling the real distributions of the users around a central base station is presented.

### 3.2.1 Modelling of the Near–Far Effect

The near–far effect is modelled by assuming that the mean received power from a mobile terminal at a distance $r$ from the receiving base station is of the general form

$$\bar{p_s} = \alpha_i r^{-\beta} \tag{3.1}$$

where the exponent $\beta$ gives the attenuation law for the channel under consideration and range between $2 \leq \beta \leq 5$ [7]. For the cellular networks with terminals transmitting in UHF band a typical value is $\beta = 4$. The quantity $\alpha_i$ depends on the transmitted power, gains and heights (above ground) of the transmitter and the receiver antennas defined as

$$\alpha_i = P_{T_i} G_{T_i} G_R H_{T_i}^2 H_R^2 \tag{3.2}$$

where $P_{T_i}, G_{T_i}$ and $H_{T_i}$ are the transmitter power, antenna gain and antenna height, respectively of the mobile terminal sending slot packet $i$. $G_R$ and $H_R$ are the gain and height of the base station antenna. For explicitly demonstrating the effects of near–far phenomenon we've neglected the shadowing effects. If all the users are taken to be identical and use omni directional antennas with radiation pattern maxima in the horizontal plane then we can set $\alpha_i$ to any suitable constant $\alpha_o$ as the receiver capture is determined by the ratio of signal powers. Let's define $\alpha_o = r_{max}^{\beta}$ where $r_{max}$ is the radius of the circular area (centred on the base station) in which the associated mobile terminals are expected to move. Thus we can write:

$$\bar{p_s} = \alpha_o r^{-\beta}$$

$$= \rho^{-\beta}. \tag{3.3}$$

The random distribution of the participating terminals will then be in a circular area with radius $0 < \rho \leq 1$, where:

$$\rho = \frac{r}{r_{max}} = r(P_{T_i} G_{T_i} G_R H^2_{T_i} H^2_R)^{-1/\beta}. \tag{3.4}$$

Invoking Abramson's assumption [1] that the mean number of packets generated per time slot be Poisson distributed and are the function of $\rho$ only, we can define:

$G(\rho)$= Packets per time slot offered per unit area at a normalized distance $\rho$.

$S(\rho)$= Packets per time slot received per unit area at a normalized distance $\rho$.

Following Arnbak [7] the total traffic offered to and captured by the receiver respectively becomes:

$$G_t = 2\pi \int_0^\infty G(\rho)\rho d\rho \tag{3.5}$$

$$S_t = 2\pi \int_0^\infty S(\rho)\rho d\rho. \tag{3.6}$$

The spatial distribution function for the random generation of the packets is given by:

$$\begin{aligned} F_\rho(\rho) &= Prob(Packets\ generated\ within\ distance\ \rho) \\ &= \frac{2\pi}{G_t}\int_0^\rho G(x)x dx \end{aligned} \tag{3.7}$$

The corresponding p.d.f is:

$$f_\rho(\rho) = \frac{2\pi}{G_t} G(\rho)\rho. \tag{3.8}$$

Using random variable transformation [35], the p.d.f for the mean received power is given by

$$f_{\bar{P}_s}(\bar{p}_s) = f_\rho(\rho)\left|\frac{d\rho}{d\bar{p}_s}\right|. \tag{3.9}$$

Finally substituting $\bar{p}_s = \rho^{-\beta}$ in (3.9) we get:

$$f_{\bar{P}_s}(\bar{p}_s) = \frac{2\pi}{\beta\ G_t}\bar{p}_s{}^{-(1+2/\beta)}G(\bar{p}_s{}^{-1/\beta}). \tag{3.10}$$

From the above equation we can determine the p.d.f of the mean power of the test packet given the distribution of the traffic. The p.d.f of the mean power of the sum of interfering packets is obtained by convolving the test packet distribution $n$ times where '$n$' is the number of the interfering packets.

With the above development we are interested in observing the impact of the spatial distribution of the users on the total throughput under Nakagami fading and the variation of the throughput with the transmission distance.

Due to the use of the omni directional antenna at the base station it is required to obtain the distribution of the users over the distance rather than the plane as the angle of the user's position accessing the base station would become immaterial. The choice of the probability density for the user distribution is restricted by the near-far phenomenon. We should not allow the colocation of the receiver and a transmitter, otherwise the former will be swamped by the latter. Furthermore a significant number of users should not be allowed at very large distances from the receiver because it imposes limitations on the frequency reuse and the connectivity parameters of the network [8]. Based on the above criterion, of the various distributions that we discussed, we've selected a popular traffic model proposed by [7] and used by a number of researchers [10, 24, 28] for the Rayleigh fading channels. This model assumes a quasi-constant density of the cellular traffic. The spatial density is given by:

$$G(\rho) = \frac{G_t}{\pi} exp(-\frac{\pi}{4}\rho^4), \quad 0 \le \rho < \infty \tag{3.11}$$

where $\rho = r/r_{max}$ is the normalized radius and $G_t$ is the total traffic load offered to the base station. This model assumes almost constant traffic density inside a unit circle and falls off rapidly as we move outside as shown in Fig. 3.1. The corresponding pdf of the distance '$\rho$' between the mobile and the base station is

$$f_\rho(\rho) = 2\rho \; e^{-\pi\rho^4/4} \qquad 0 \le \rho < \infty. \tag{3.12}$$

The model can be compared with the original traffic model proposed by Abramson [1] in which he assumed a step function traffic distribution. In contrast here due to the capture effect this model has softer transition allowing finite traffic outside the unit circle.

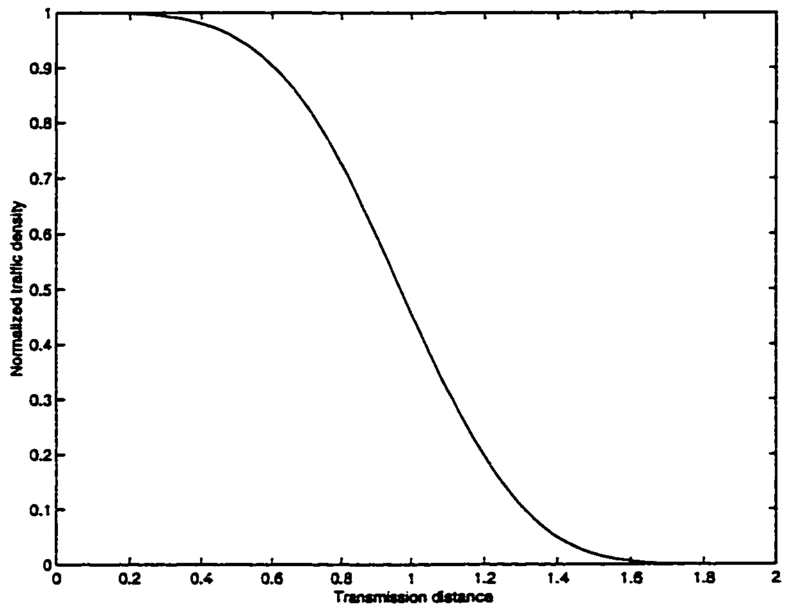



Figure 3.1: Shape of the traffic density, modelling a constant density of the offered packets in a radio cell

### 3.2.2 Evaluation of the Channel Throughput

As discussed above the influence of the spatial distribution of the users can be accounted for by determining the fluctuations in the mean power of the received packets. The mean power level variations are statistically described by the corresponding probability density functions. In chapter 2 where the average test packet power ($\bar{p_s}$) is assumed constant the p.d.f of the test packet is given by (2.2); when $\bar{p_s}$ is not constant and instead has the p.d.f $f_{\bar{P_s}}(\bar{p_s})$ then

$$f_{P_s}(p_s) = \int_0^\infty (\frac{m}{\bar{p_s}})^m \frac{{p_s}^{m-1}}{\Gamma(m)} exp(-\frac{m}{\bar{p_s}} p_s) f_{\bar{P_s}}(\bar{p_s}) d\bar{p_s}. \tag{3.13}$$

For the bell shaped function for the spatial density of the mobiles as given by (3.11); the p.d.f of the mean power ($f_{\bar{P_s}}(\bar{p_s})$) is obtained by (3.10)

$$f_{\bar{P_s}}(\bar{p_s}) = \frac{1}{2} \bar{p_s}^{-3/2} exp(-\frac{\pi}{4\bar{p_s}}) \tag{3.14}$$

which when substituted in (3.13) gives:

$$f_{p_s}(p_s) = \frac{1}{2\sqrt{m}} \frac{\Gamma(m+0.5)}{\Gamma(m)} \frac{{p_s}^{m-1}}{(p_s + \frac{\pi}{4m})^{m+0.5}}. \tag{3.15}$$

The p.d.f of the joint interference power assuming $\bar{p_n} = n\bar{y}$ (i.e. the constant mean power for all the signals) is given as

$$f_{P_n}(p_n) = (\frac{mn}{\bar{p_n}})^{nm} \frac{p_n^{nm-1}}{\Gamma(nm)} exp(-\frac{nm}{\bar{p_n}} p_n). \tag{3.16}$$

But when the users are spatially distributed then

$$f_{P_n}(p_n) = \int_0^\infty (\frac{mn}{\bar{p_n}})^{nm} \frac{p_n^{nm-1}}{\Gamma(nm)} exp(-\frac{nm}{\bar{p_n}} p_n) f_{\bar{P_n}}(\bar{p_n}) d\bar{p_n} \tag{3.17}$$

The joint p.d.f of the mean interference power obtained by the $n-$fold convolution of (3.14) is given as:

$$f_{\bar{P_n}}(\bar{p_n}) = \frac{n}{2}\bar{p_n}^{-3/2}exp(-\frac{\pi n^2}{4\bar{p_n}}) \tag{3.18}$$

which when substituted in (3.17) gives the p.d.f of the joint interference power as:

$$f_{P_n}(p_n) = 0.5\sqrt{\frac{\pi}{m}}\frac{\Gamma(nm+0.5)}{\Gamma(nm)}\frac{p_n^{nm-1}}{(p_n+\frac{\pi\ n}{4m})^{nm+0.5}} \tag{3.19}$$

Finally with these p.d.fs' $F_{Z_n}(z_o)$ is evaluated to be:

$$F_{Z_n}(z_o) = K\int_0^\infty x^{nm+m-1}(x+\frac{\pi\ n}{4m})^{-(nm+0.5)}F(m+0.5, m, m+1, \frac{-4mz_ox}{\pi})dx \tag{3.20}$$

where

$$K = 2^{2m-1}\frac{\Gamma(m+0.5)\Gamma(nm+0.5)m^{m-3/2}n^{1/2}z_o{}^m}{\Gamma(m)\Gamma(nm)\pi^{m+0.5}}$$

Also (3.20) reduces to

$$F_{Z_n}(z_o) = 1 - \frac{1}{\sqrt{\pi}}\frac{\Gamma(n+0.5)}{\Gamma(n)}B(n,1)F(0.5, n, n+1, 1-nz_o) \tag{3.21}$$

for $m = 1$.

The above integral is well behaved and can be evaluated for various values of '$n$' where evaluation requires few seconds for every point on the Sun Ultra Sparc machines . The evaluated values of $F_{Z_n}(z_o)$ are substituted in (2.14) to determine the average throughput behavior of the channel. From the results (refer to Figures 3.2, 3.3 and 3.4) it can be observed that when the users spatial distribution is taken into account then it is observed in general that the throughput is largely a function of the capture ratio of the receiver and for a particular capture ratio there is no significant variation due to the change in the fading parameter $m$. So it can be inferred that for the spatially distributed users the division of users into different power classes is mostly dependent on the location of the users accessing the central noise free receiver and is not affected significantly by the fading conditions.

Another rather close observation of Fig. 3.2 and Fig. 3.3 reveals that fading doesn't necessarily improve the throughput especially when the capture ratio is low. Only for higher values of the capture ratio (e.g. $z_o = 6dB$) the benefit of fading is observed and that too only for the low traffic conditions.

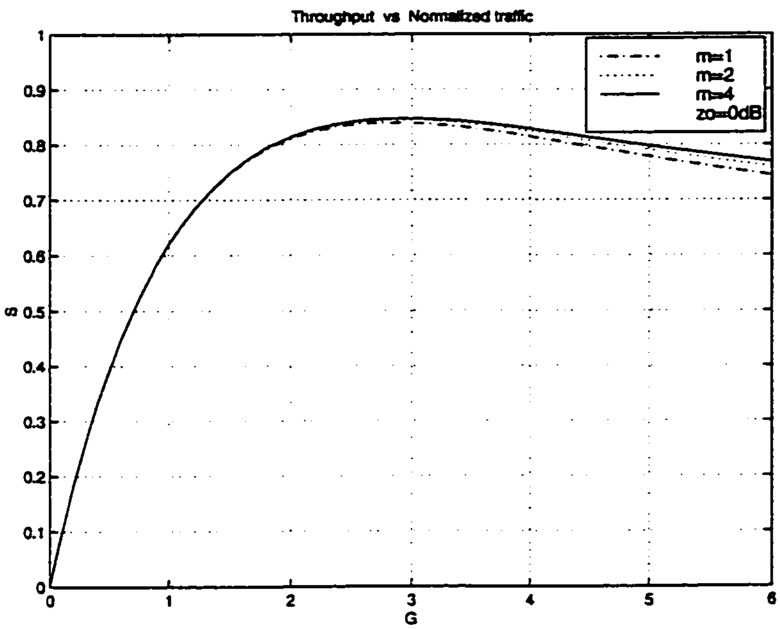



Figure 3.2: Throughput curves for slotted ALOHA assuming spatially distributed users with $z_o$= 0dB

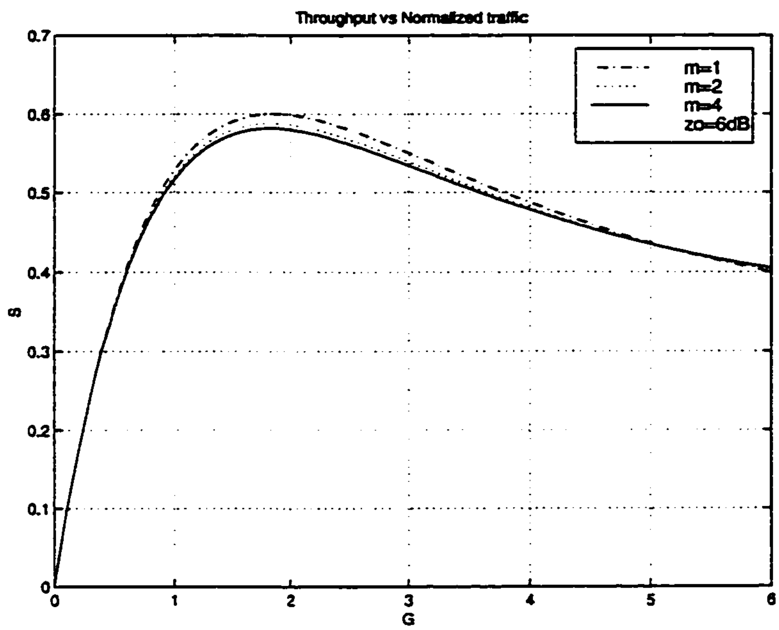



Figure 3.3: Throughput curves for slotted ALOHA assuming spatially distributed users with $z_o$= 6dB

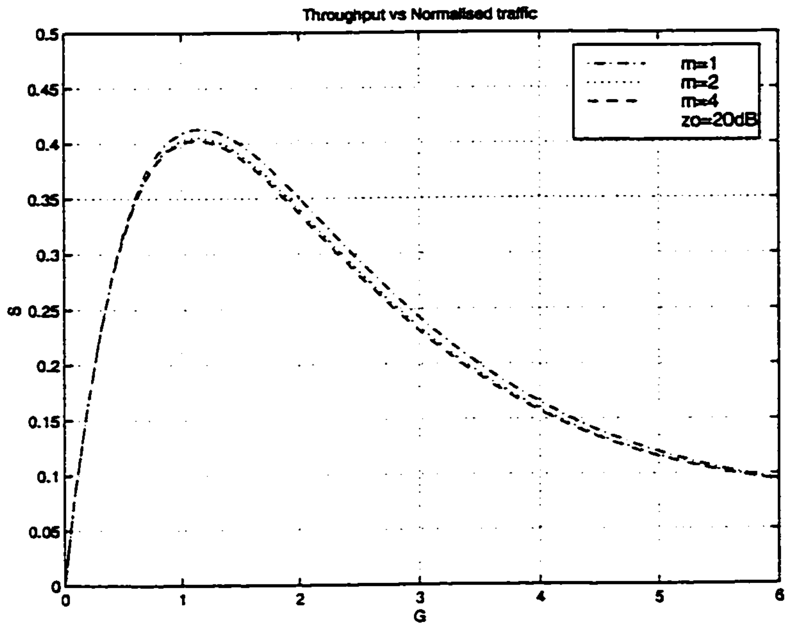



Figure 3.4: Throughput curves for slotted ALOHA assuming spatially distributed users with $z_o$= 20dB

For observing the effects on the throughput when a transmitting user moves away from the base station we determine the throughput as a function of the normalized transmission distance ($\rho$). In section 2.3 we've shown that for a test packet and the interferers with constant but unequal average power, the probability of outage($F_{Z_n}(z_o)$) is evaluated to be $I_x(m, nm)$ where $x = \frac{1}{1+\frac{\bar{p_s}}{\bar{y} z_o}}$. $\bar{p_s}$ and $\bar{y}$ denote the average power of the test packet and the interferers simultaneously. When the users are spatially distributed then the p.d.f of average power is given by (3.14) which is determined by the traffic distribution (3.11) . The n–fold convolution of this p.d.f generates the p.d.f for the '$n$' interferers given by (3.18). With these probability density functions the outage probability, $F_{Z_n}(z_o)$, can be evaluated to be:

$$F_{Z_n}(z_o) = \int_0^\infty \int_0^\infty f_{\bar{P_s}}(\bar{p_s}) f_{\bar{P_n}}(\bar{p_n}) F_{Z_n}(z_o|\bar{P_s}, \bar{P_n}) d\bar{p_s} d\bar{p_n} \tag{3.22}$$

where

$$F_{Z_n}(z_o|\bar{P_s}, \bar{P_n}) = I_{\left(1+\frac{n\bar{p_s}}{\bar{p_n} z_o}\right)^{-1}}.(m, nm) \tag{3.23}$$

The distribution function $F_{Z_n}(z_o)$ is given by (3.20). Here we are interested in determining this function as as the function of the transmission distance of the test user from the base station. With the substitution of the above p.d.fs' and further substituting $\bar{p_s} = \rho^{-4}$ we determine this function say $Q_n(\rho)$ defined as:

$$Q_n(\rho) = \frac{n}{2} \int_0^\infty y^{-3/2} exp\left(-\frac{\pi n^2}{4y}\right) I_{\left(1+\frac{n}{\rho^4 y z_o}\right)^{-1}} (m, nm) . \tag{3.24}$$

The probability of the successful transmission as the function of the normalized transmission distance is then given by:

$$\frac{S(\rho)}{G(\rho)} = \left[1 - e^{-G_t \sum_{n=1}^{\infty} \frac{G_t{}^n}{n!} Q_n(\rho)}\right] \tag{3.25}$$

where $G_t$ is the total traffic directed towards the base station. Equation 3.25 is numerically evaluated for different transmission distances. It is seen that for a particular capture ratio ($z_o$) and the fading parameter ($m$) the probability of successful packet reception goes down with the increasing packet transmission distance. The lowest value of probability of success is determined by the total input traffic directed towards the central receiver. For the out-of-cell users ($\rho > 1$) the success probability is almost constant. When the traffic is high, this value is almost zero. This fact can be explained by considering that when the traffic is low there is always a finite probability of capturing the receiver by a test packet due to power level fluctuations caused by fading and spatial distribution; no matter how large a distance it is supposed to travel across.

Another interesting observation is made when the success probability is determined for varying fading conditions '$m$'. It is seen that all the curves converge to a common point the location of which is determined by the total traffic ($G_t$) and the value of capture parameter ($z_o$). For the fixed $z_o$ and changing total traffic, it is noticed (referring to Fig. 3.6 and Fig. 3.7) that the curves converge at almost the same value of the transmission distance from the central receiver. When the value of $z_o$ is increased then from Fig. 3.8 it is seen that the convergence point shifts to a lower packet transmission distance. The benefit offered by the fading to the test packet occur only when the test user is farther from the base station and it occurs after a threshold transmission distance. For a user transmitting very close to the base station, fading decreases the throughput. The result appears very natural because when the test user is near to the base station the path loss would be low and the chances of the test packet having a higher power than the joint interference power is mainly determined by the position of the user. On the other hand for the user very far from the base station the path loss will be quite high and the only chance of test user power exceeding over the joint interference power is due to the random power fluctuations due to fading.

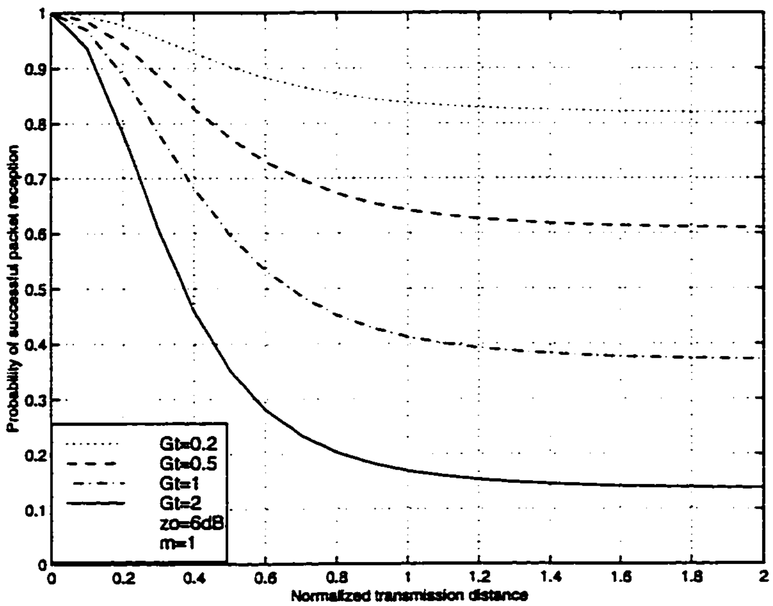



Figure 3.5: Probability of successful packet reception versus transmission distance for different values of total traffic($G_t$), ($z_o = 6dB$ and $m = 1$)

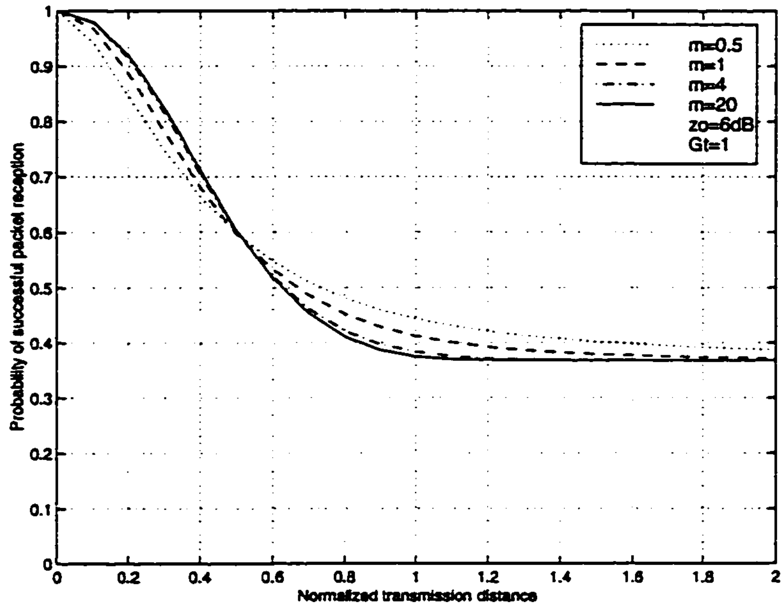



Figure 3.6: Probability of successful packet reception versus transmission distance for different values of $m$ ($z_o = 6dB$ and $G_t = 1$)

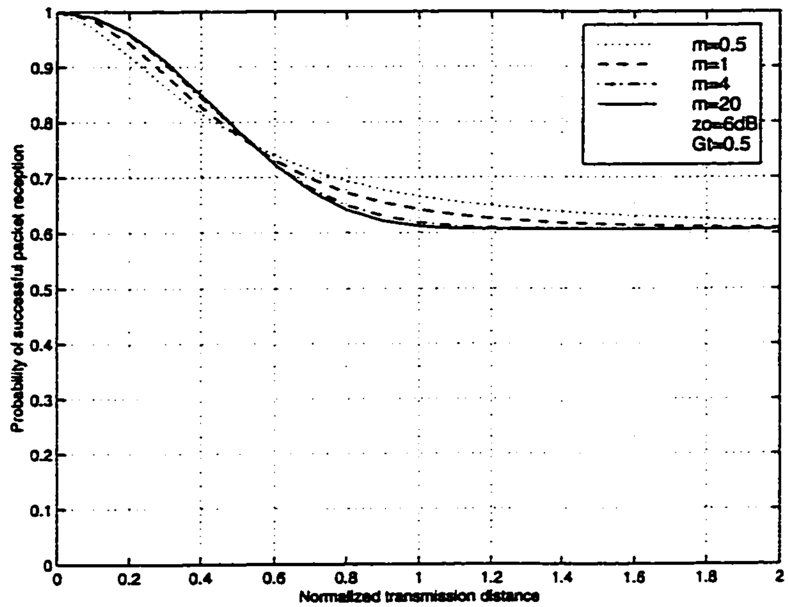



Figure 3.7: Probability of successful packet reception versus transmission distance for different values of $m$ ($z_o = 6dB$ and $G_t = 0.5$)

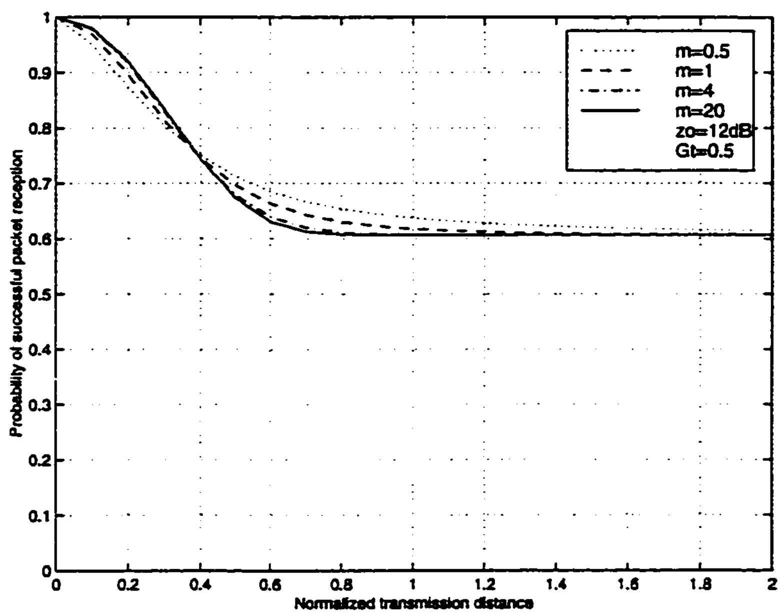



Figure 3.8: Probability of successful packet reception versus transmission distance for different values of $m$ ($z_o = 12dB$ and $G_t = 0.5$)

## 3.3 Shadowing Effects on the Throughput

In the mobile radio systems the received signal may suffer from both fading and shadowing. As discussed in chapter 1 fading is due to the multi-path propagation ; shadowing on the other hand is due to the topographical variations of the transmission path. Shadowing of the radio signal by buildings and hills leads to the gradual change in the local mean level with the result that the local mean ($\bar{P}_s$) is lognormally distributed within an area which is at roughly constant distance from the transmitter. By Lognormal it is meant that the local mean, expressed in dB, is normally distributed about the area mean. The Lognormal distribution has been confirmed in a number of surveys [36] .

The probability density function (p.d.f) of the lognormally distributed signal is given by

$$f_{S_d}(s_d) = \frac{1}{\sqrt{2\pi}\sigma_s} exp\left(-\frac{(s_d - s_{nd})^2}{2\sigma_s{}^2}\right) \tag{3.26}$$

where $s_d$ is given by

$$s_d = 10 log \bar{p}_s. \tag{3.27}$$

The shadowing parameter $\sigma_s$ is the standard deviation of the corresponding distribution of the signal power. The quantity $s_{nd}$ reflects the median logarithmic attenuation in the signal. It is defined in [31] as

$$s_{nd} = 10 log(C s_n) \tag{3.28}$$

where $C = exp(0.0265\sigma^2)$ and $s_n$ is the area mean signal power produced by the transmitter, following a propagation law given by (3.1) i.e.

$$s_n = \frac{k}{r^\beta} \tag{3.29}$$

where the distance between the mobile station and the base station is '$r$' . The propagation parameter, $\beta$, as mentioned earlier is typically between 2 and 5, and $k$ is the propagation constant .

In the literature [8, 15, 23, 34, 37] the natural shadowing effects are shown to benefit the throughput of the slotted ALOHA. In the present work we've incorporated the effects of shadowing in the presence of Nakagami fading. The effects are studied through the MonteCarlo simulations.

### 3.3.1 Simulation Model

The traffic model is assumed to be Poisson with the mean offered traffic of '$G$' packets per time slot. The same capture criterion is used that is the test packet is assumed to capture the receiver, if its instantaneous power $P_s$ sufficiently exceeds the joint interference power $P_n$ of the $n$ contenders

$$\frac{P_s}{P_n} > z_o \tag{3.30}$$

where $z_o$ is the capture ratio of the receiver, which is assumed noise-free. The channel throughput is

$$S = G\left[1 - \Sigma_{n=1}^{\infty} F_{Z_n}(z_o)\frac{G^n}{n!}exp(-G)\right] \tag{3.31}$$

where

$$F_{Z_n}(z_o) = Pr\left(\frac{P_s}{P_n} < z_o\right) \tag{3.32}$$

There is no closed form pdf of the sum of '$n$' lognormally distributed random variables . The methods employed are based on the approximation that the sum also demonstrates a Lognormal behavior [38] with mean and variance determined by different approaches [38, 39, 40]. Fenton's method is suitable for small dB spread ($\sigma$) while the method proposed by Schwartz and Yeh is more appropriate for $\sigma$ lying between 4 and 12 dB, a range of interest for the mobile communication channels. In [40] a comparative study of different approaches used for calculating the distribution of the sum of correlated Lognormal random variables is presented. In [23, 37] the effect of independent shadowing is studied using the model proposed by [39] but it is shown in [39] that for higher values of dB spread ($\sigma > 10dB$) the approximation error can be quite large especially if the number of interferers is large. In [8] another way around the problem is selected using an alternative definition for the capture which simplifies the analytical procedure but that too doesn't reflect the exact phenomenon. Instead of picking a simplified model we've preferred here to resort to simulations to ascertain the true behavior and compare the results with those obtained by [8] and [23]. Only the distribution function (3.32) is simulated using the appropriate random variables as needed to model a particular fading and shadowing distribution.

Three different cases are considered. Firstly a pure log normal shadowing environment is simulated. The combined effects of shadowing along with Rayleigh fading are then studied. Finally we present results of the effects of shadowing on the slotted ALOHA protocol when the signal undergoes Nakagami fading. The shadowing effects are considered statistically independent.

### 3.3.2 Effect on the Throughput under Shadowing only Environment

To quantify the benefit to the throughput of the slotted ALOHA due to the Lognormal shadowing, the study of the shadowing alone is first done through the simulations. The simulation model is the same as formulated above. It is observed that with the increasing standard deviation of the interfering signals power (the so called dB spread) the chances of the survival of a test packet are increased. This is an anticipated behavior as there will be large variations in the interfering signals power so that the probability that all reach high values simultaneously will off course be low. The throughput of the slotted ALOHA is shown for the various values of the standard deviations of the competing signals with two different capture ratios. The choice of the parameters is governed by the need to draw a comparison with the exact results reported in [23]. When compared with the exact results, the simulated values are seen to be in close agreement. The results show that the Lognormal shadowing enhances the capacity of a randomly accessed data channel more than do

Rayleigh fading alone, as the Lognormal pdf show a larger number of the upfades on the average than the Rayleigh fading, so the probability of the test packet power exceeding that of the joint interference is higher.

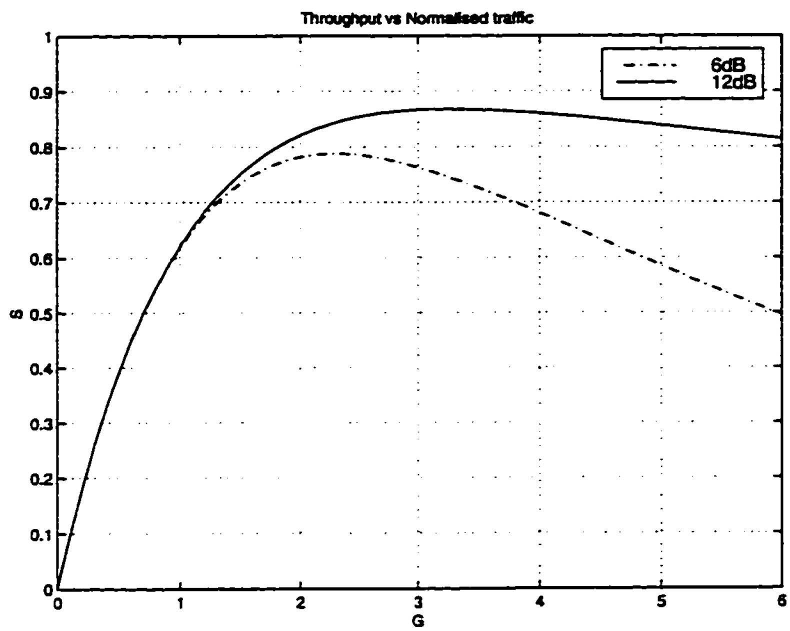



Figure 3.9: Throughput for different values of $\sigma$ and $z_o = 1(0dB)$

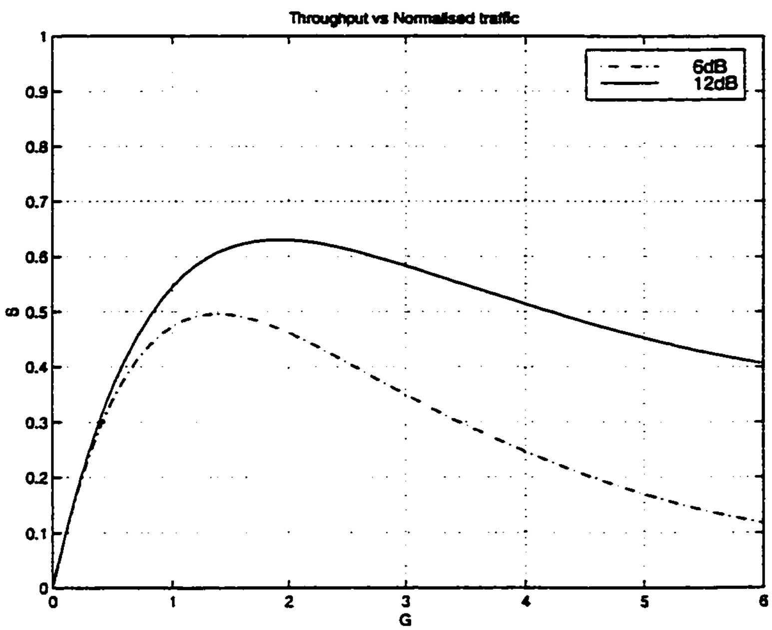



Figure 3.10: Throughput for different values of $\sigma$ and $z_o = 4(6dB)$

### 3.3.3 Effect on the Throughput under Shadowing along with Rayleigh Fading

When there are large number of multi–path components then the signal exhibits Rayleigh fading. If there exists buildings or hills around the transmitting mobile then it is possible that the average power received at the central (base) station shows a Lognormal behavior. The combined effects of the Rayleigh fading and the Lognormal shadowing on the throughput of the slotted ALOHA are reported by a number of researchers [8, 37]. For the purpose of completeness and the verification of our simulation results we've also studied the same for various values of the dB spread. The results are in agreement with [37] and show that the shadowing when superimposed on fading produces higher throughput than the one obtained in only fading or shadowing environment.

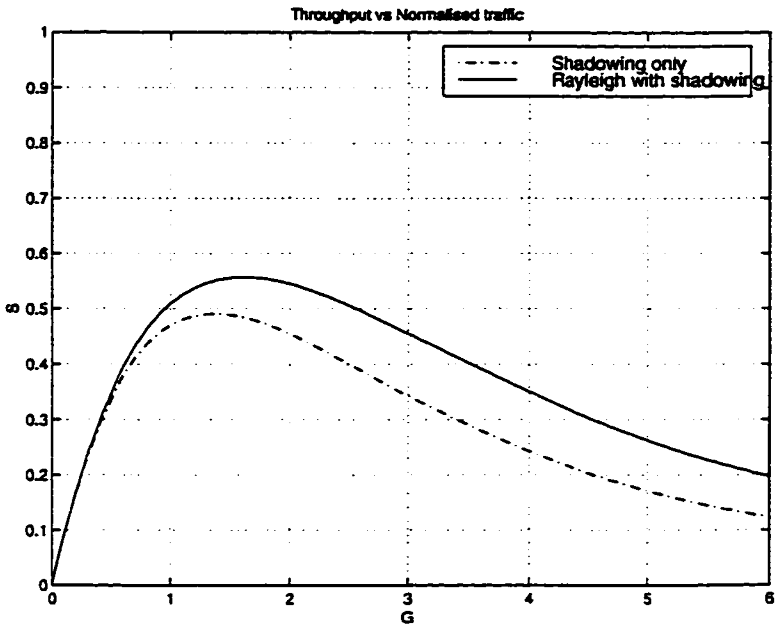



Figure 3.11: Throughput with $z_o = 4(6dB)$ for incoherent interference addition and a shadowing only environment with $\sigma = 6dB$

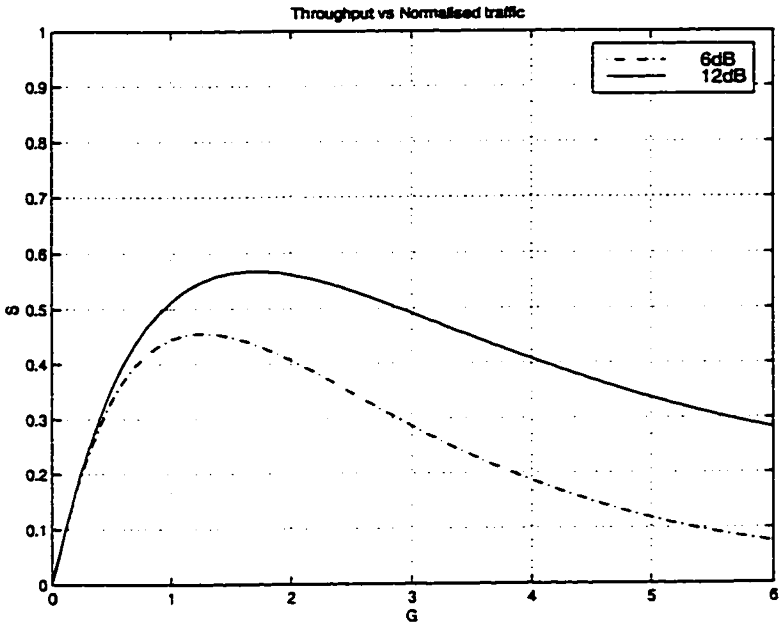



Figure 3.12: Throughput for incoherent interference addition with $z_o = 10$, for $\sigma = 6$ *and* $12dB$

### 3.3.4 Effect on the Throughput under Shadowing along with Nakagami Fading

Our primary interest here is to study the effect of Nakagami fading with the Lognormal shadowing. The analytical study of this effect is recently undertaken by [15]. The analysis is done for a perfectly correlated interfering packets that is all the interfering packets are assumed to have the same average power. In our simulations we've considered independent shadowing effects where the mean powers of the packets are independent random variables.

It is asserted in [31] that for multiple Nakagami interferers, in the presence of the Lognormal shadowing, the evaluation of the capture probability employing multiple integrals have inherent round off and verification problems. So the study based on the simulations is a better alternative. The area mean power is assumed to be unity so the effects of path loss are not taken into account. We believe that the incorporation of such effects will increase the throughput as reported for Rayleigh channels in [8, 24, 29]. Comparing figures 3.13 and 2.2 it can be seen that the throughput improvement due to Lognormal shadowing is substantial under all fading conditions. As anticipated the throughput seems to be decreasing with the reduction in the fading depth i.e. with the increased value of m. Also when compared with the results reported in [37] for the Rayleigh fading, a close agreement is observed with the analytical results reported for $m = 1$. Referring to Fig. 3.10 and 3.13 it can also be seen that with the increase in the value of $m$ the benefits due to the fading are diminishing and the channel is behaving as a pure Lognormal channel.

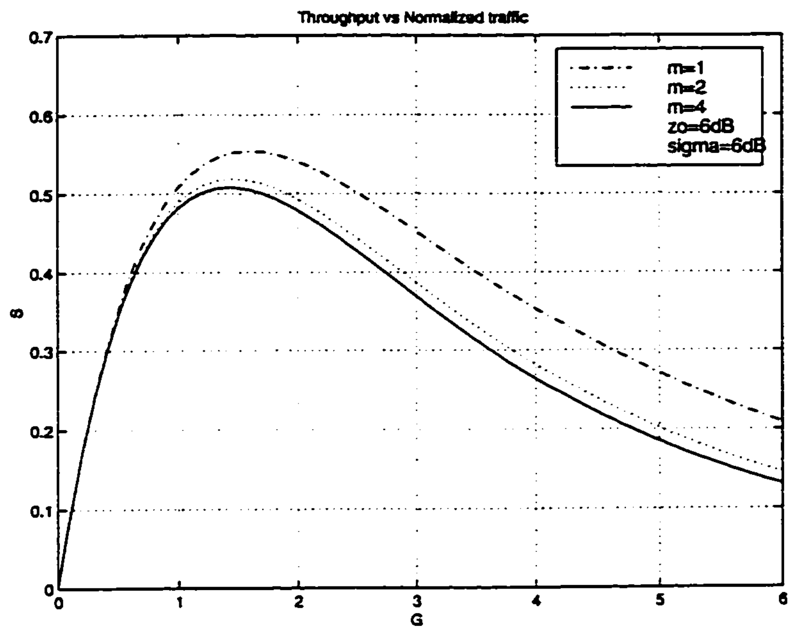



Figure 3.13: Throughput for different values of m with $z_o$=6dB and $\sigma$=6dB

# Chapter 4

# Effect of Modulation and Noise

## 4.1 Introduction

Modulation and Coding also affect the performance of slotted ALOHA. A number of researchers [8, 9, 10, 11, 27, 24] investigated these effects under Rayleigh fading environments. The results indicate that the throughput increases with improved modulation and coding techniques. In [27] block codes are shown to perform better than convolutional codes for the Rayleigh fading environment. The enhancement in the system throughput for the coded systems is achieved at the expense of the bandwidth expansion. In [10] an exact model and two bounds for the throughput are presented. The correlation between the different bits of the same packet is also accounted for in their calculations. Both slow and fast fading are considered. Linnartz [24] has extended the same model presenting several different capture models based on the synchronization of the receiver with the test packet or an interferer. Lau and Leung [26] proposed a capture condition in which similar to the foregoing analysis the successful decoding of the packet is shown to be the probabilistic function of the test signal to interference ratio. The model was developed for the non-fading conditions. In our analysis we've followed their approach and extended their model to incorporate the effect of modulation and noise under Nakagami fading environment.

The performance is analyzed under slow fading (which is more applicable for practical systems) where the fade statistics remain same for all the bits of a packet.

## 4.2 Throughput Evaluation for a NCFSK system

Lau and Leung [26] introduced a capture condition in which the successful decoding of a test packet is the probabilistic function of the test signal to interference ratio. We've slightly modified their definition and following [9, 13] also included the gaussian noise in the ratio; thus defining:

$$Z = \frac{P_s}{P_n + \frac{N_o}{2}} \tag{4.1}$$

where $P_s$ is the instantaneous power of the test packet, $P_n$ is the joint interference power and $\frac{N_o}{2}$ is the variance of the received noise. The indiscriminate addition of noise variance to the joint interference power makes sense for the slowly fading channel. Also the packet length is short so it can be assumed that the net noise power is constant over the length of the packet. For a given value of $z$, the probability that a test packet from a mobile is successfully decoded is given by:

$$P(S|z) = \begin{cases} 0 & z < z_o \\ g(z) & z \geq z_o \end{cases}$$

where $g(z)$ is a function that depends on the type of modulation and coding used and $z_o$ is the receiver capture ratio whose value depends on the synchronization capabilities of the receiver [24]. As asserted in chapter 1, the practical narrow band receivers require a signal (carrier) to interference ratio of at least $6dB$ to perform reliable detection and synchronization.

We assume that the users transmit their signals using Non-Coherent Frequency Shift Keying (NCFSK). The received signal at the base station will then be given by

$$r_0(t) = \alpha_0 s_0(t) + \sum_{i=1}^{n} \alpha_i s_i(t) + n(t) \tag{4.2}$$

where $s_i(t)$'s are the transmitted signals and $n(t)$ is the additive white gaussian noise (AWGN). The $\alpha_i$'s are Nakagami distributed and their averages are functions of the locations of the transmitting users (shadowing is neglected). When there is no collision then the received signal is the sum of the reference signal and the AWGN. If, due to the simultaneous transmission of the packets, a collision occurs then at the sampling instant the signal value will be given as $r_0(kT) = \alpha_0 a_0 + \sum_{i=1}^{n} \alpha_i a_i + n_r$

where $a_i$ is the data sample from packet $i$ and $T$ is the symbol time.

The decision variable is thus given by:

$$\xi_0 = \alpha_0 a_0 + \sum_{i=1}^{n} \alpha_i a_i + n_r \tag{4.3}$$

where $\alpha_0$ is Nakagami distributed bit from the test packet and $a_i$'s are assumed to be identically distributed taking on 1 and 0 with equal probability.

The bit error rate for NCFSK under AWGN as a function of $z$ is given by [26]:

$$p_b = \frac{1}{2} e^{-z/2}. \tag{4.4}$$

The bit errors are assumed to be independent and the probability that a packet of length $L$ doesn't contain any bit errors is given by:

$$P(S|z) = (1 - p_b)^L \tag{4.5}$$

where $P(S|z)$ is the probability of success given $z$, $L$ is the length of the test packet and $p_b$ is defined by (4.4). If the terminals are assumed to be moving very slowly in a cellular network then the signal to noise plus interference ratio for all the bits of the packets will be the same. In case of fading all the bits will experience the same fade and the effect of fading on the capture probability can be determined by averaging the probability of success given by (4.5) over the distribution of $z$ that is:

$$P_{capt}(n) = \int_{z_o}^{\infty} \left(1 - \frac{1}{2} e^{-z/2}\right)^L f_Z(z) dz \tag{4.6}$$

where $P_{capt}(n)$ is the probability of capturing the receiver in the presence of '$n$'

interfering packets. The probability density function $f_Z(z)$ is determined by the fading phenomenon and the spatial distribution of the terminals in the area.

### 4.2.1 Effect of Modulation for the Nakagami Fade only Environment

The p.d.f of '$z$' for the Nakagami fading when all the users transmit with equal mean power is given by (2.12). The $P(S|z)$ given by (4.5) can be averaged over this distribution to get the corresponding probability of capture which then determines the throughput of the slotted ALOHA protocol. The probability of capture is thus given by:

$$P_{capt}(n) = \frac{\Gamma(nm+m)}{\Gamma(m)\Gamma(nm)} \int_{z_o}^{\infty} \left(1 - \frac{1}{2}e^{-z/2}\right)^L \frac{z^{m-1}}{(1+z)^{nm+m}} dz. \tag{4.7}$$

The effect of noise is neglected in the above equation such that $z = \frac{p_s}{p_n}$. With the above formulation we determine the probability of capture and the throughput for different values of fading depths ($m$), packet lengths ($L$) and capture ratios ($z_o$). The throughput ($S$) is determined by the (2.14) where $F_{Z_n}(z_o)$ is the distribution function related to the probability of capture by:

$$F_{Z_n}(z_o) = 1 - P_{capt}(n). \tag{4.8}$$

Referring to Fig. 4.1 it is observed that with the increase in packet length the throughput decreases as there is an increased probability of occurrence of a bit error. The probability of outage thus increases which lowers the maximum throughput of the slotted ALOHA. The effect of increase in fading is to decrease the outage probability. This can be understood by referring to (4.4) and (4.5) where the argument $z$ increases on average with the decrease in the value of fading parameter $m$. The overall impact is the increase in the probability of success (due to the decrease in the bit error rate) which when averaged over the p.d.f of $z$ yields a higher capture probability. In Fig. 4.3 as anticipated the throughput goes down when the capture ratio of the receiver is increased. The probability of the existence of $g(z)$ is the function of the capture ratio and it will decrease as the value of $z_o$ increases.

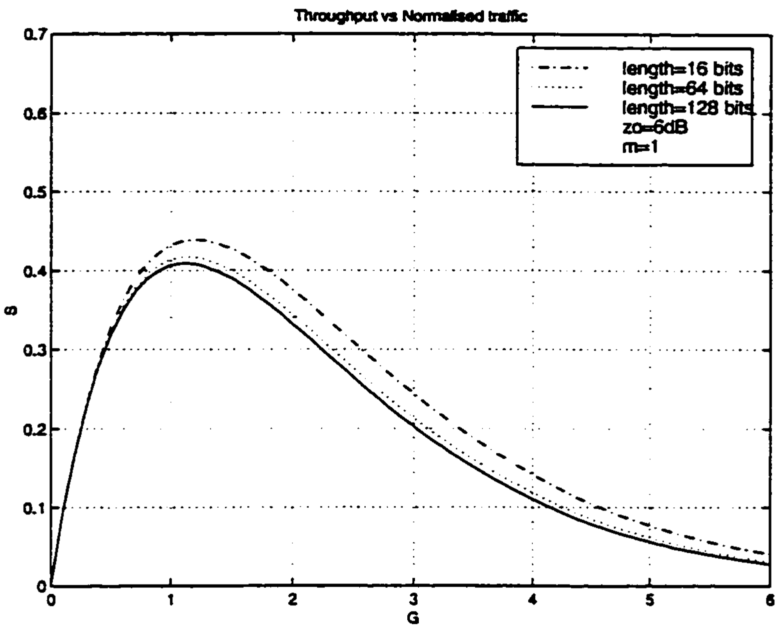



Figure 4.1: Effect of packet length on the throughput under Rayleigh fading ($m = 1$) with $z_o$= 6dB

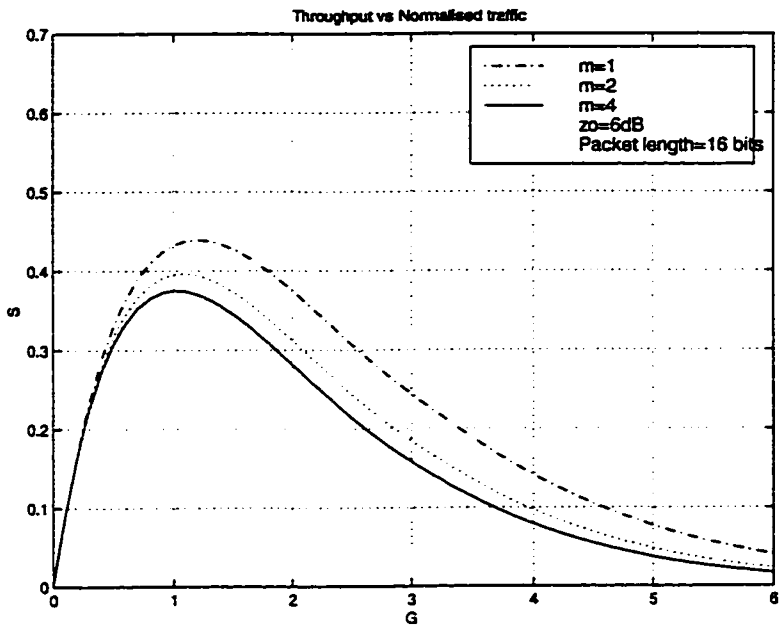



Figure 4.2: Effect of fading condition on the throughput with $z_o = 6dB$ and packet length=$16bits$

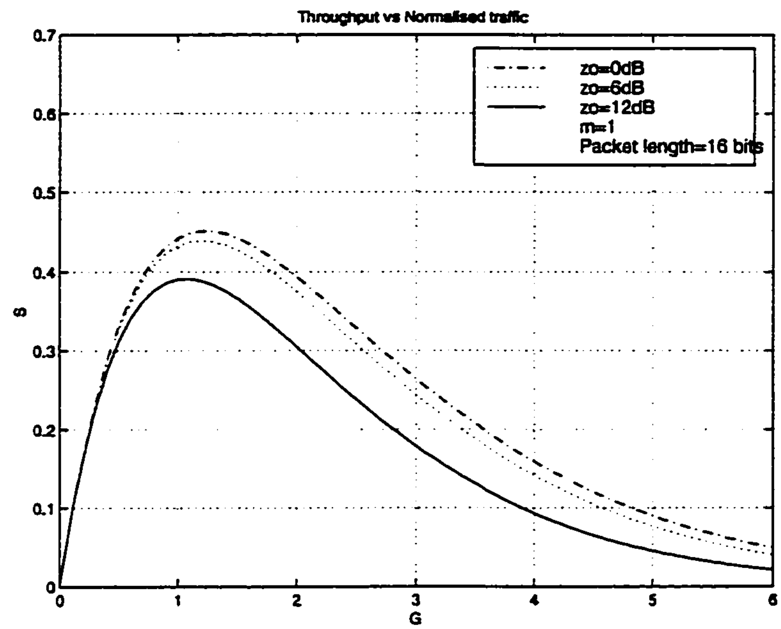



Figure 4.3: Effect of capture ratio on the throughput under Rayleigh fading ($m = 1$) with packet length=16*bits*

### 4.2.2 Effect of Modulation under Nakagami Fading with Noise

When the white noise is also included along with the interference then the probability of capture is given by (4.6) where the probability density function $f_Z(z)$, in the presence of uncorrelated Nakagami fading, is modified to:

$$
\begin{aligned}
f_Z(z) &= \int_{N_o/2}^{\infty} f_{P_s}(zw) f_{P_n}\left(w - \frac{N_o}{2}\right) w dw \\
&= \frac{m^{nm+m} z^{m-1}}{\Gamma(m)\Gamma(nm)} \int_{N_o/2}^{\infty} w^m (w - \frac{N_o}{2})^{nm-1} \\
&\quad \times exp\left(-m(zw + w - \frac{N_o}{2})\right) dw \qquad (4.9)
\end{aligned}
$$

where we've assumed that $w = p_n + \frac{N_o}{2}$.

Instead of getting a closed form result of the above integral (which is too big to tackle) we've determined the probability of capture by numerically integrating (4.6).

The results indicate the same trend as was observed in the absence of noise. The effect of the finite signal to noise ratio is to decrease the capture probability for all cases considered and thus an overall decrease in the throughput is observed when compared to the corresponding curves without including the effect of noise.

In the Fig. 4.8 it is observed that for a suitable $\frac{E_b}{N_o}$, the noise doesn't significantly affect the throughput and it substantiates the claim made by several researchers [26, 27, 29] that the performance of the ALOHA systems is basically limited by the contention among the transmitting users.

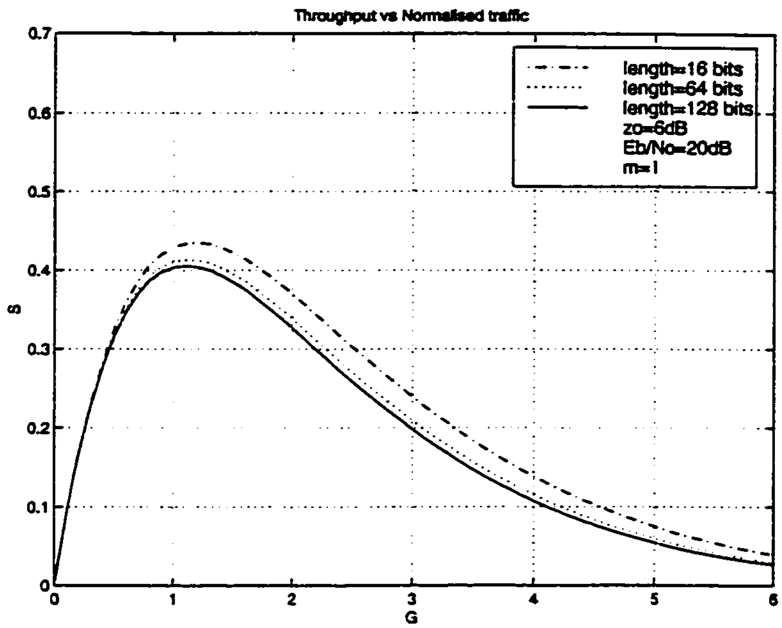



Figure 4.4: Effect of packet length on the throughput under Rayleigh fading ($m = 1$) with $z_o = 6dB$ and $\frac{E_b}{N_o} = 20dB$

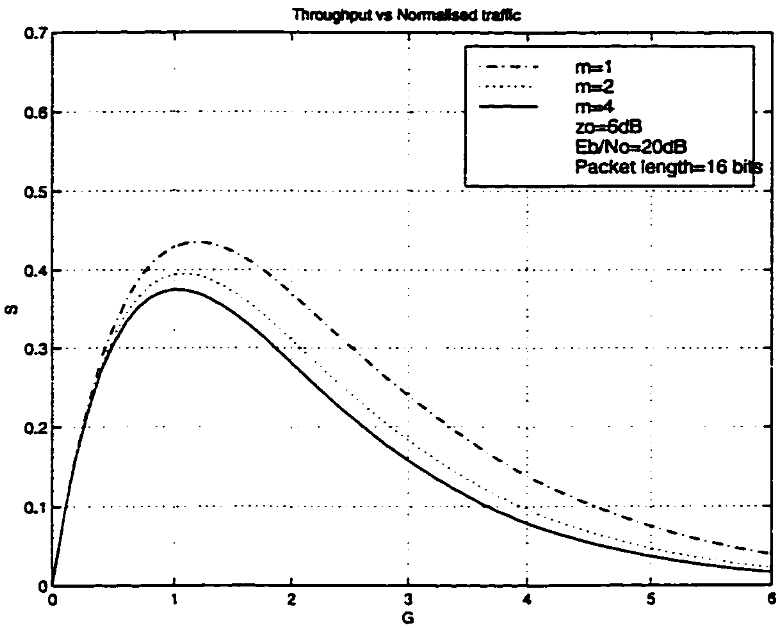



Figure 4.5: Effect of fading condition on the throughput with $z_o = 6dB$, packet length=16 *bits* and $\frac{E_b}{N_o} = 20dB$

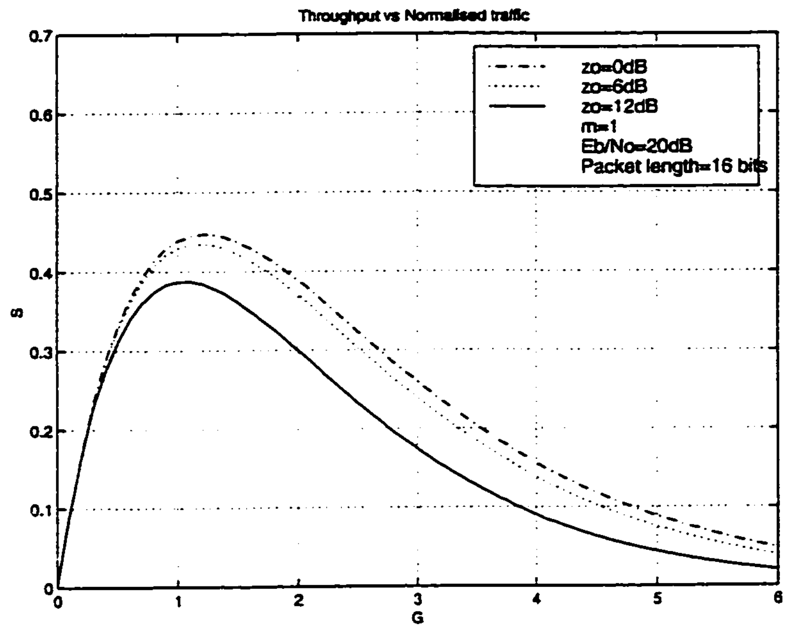



Figure 4.6: Effect of capture ratio on the throughput under Rayleigh fading ($m = 1$), packet length=16 *bits* and $\frac{E_b}{N_o} = 20dB$

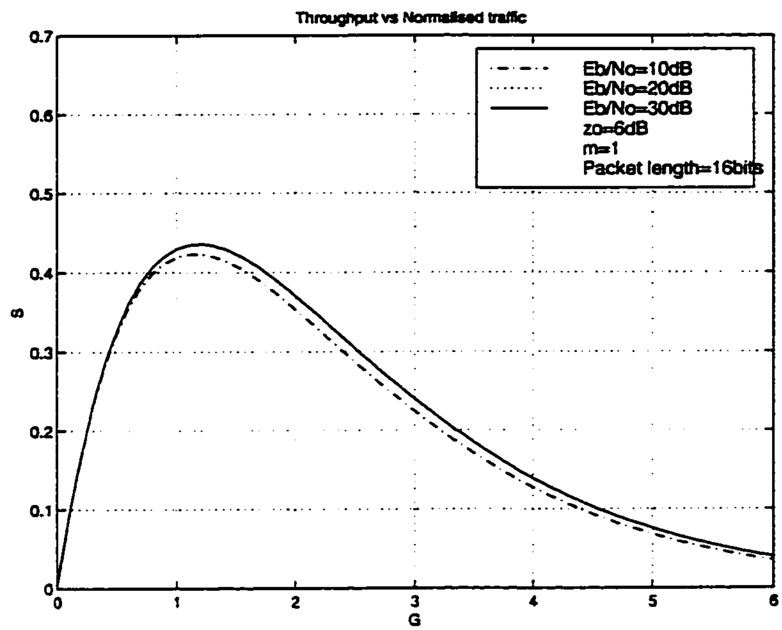



Figure 4.7: Effect of $\frac{E_b}{N_o}$ on the throughput under Rayleigh fading ($m = 1$), packet length=16 *bits*

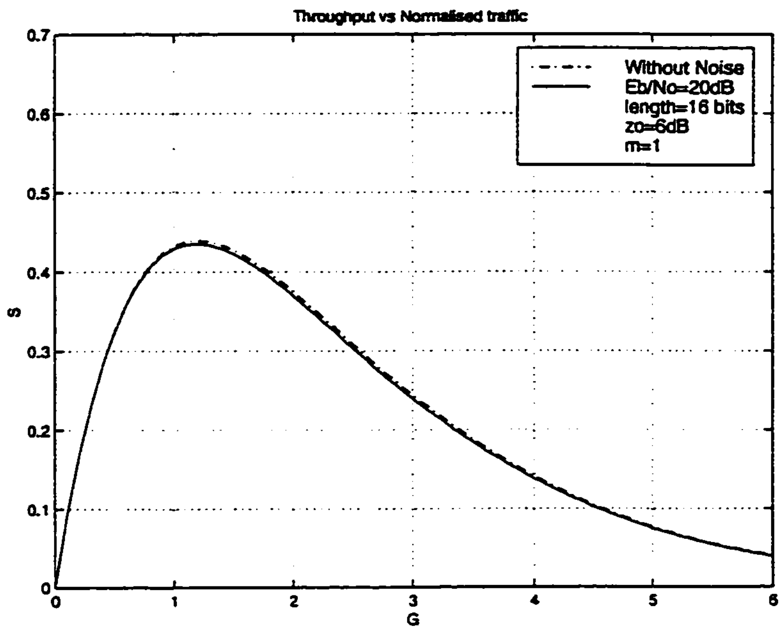



Figure 4.8: Comparison of the system performance with and without noise

In Fig. 4.9 the NCFSK modulation is compared with the ideal case of chapter 2 where all the users transmit with the same average power. The packet length of 16 bits is considered for the modulated bits. It can be seen in Fig. 4.9 that when the probability of bit error is accounted for, then the probability of capture decreases slightly so does the throughput.

This can also be explained mathematically. The probability of capture for the test packet of length $L$ in the presence of $n$ interfering packets is given by:

$$P_{capt}(n) = \int_{z_o}^{\infty} (1 - p_b)^L f_Z(z)dz. \tag{4.10}$$

Using the binomial expansion and neglecting the higher powers of $p_b$ we'll have:

$$P_{capt}(n) = \underbrace{\int_{z_o}^{\infty} f_Z(z)dz}_{Ideal\ case,\ p_b=0} - \underbrace{Lp_b \int_{z_o}^{\infty} f_Z(z)dz}_{Decrease\ due\ to\ consideration\ of\ p_b} \tag{4.11}$$

where the first term corresponds to the ideal case (where the probability of bit error is assumed to be zero) and the second term incorporates the degradation with the consideration of the probability of bit error. It can also be noted that with the increasing packet length the capture probability would decrease which consequently decreases the throughput as observed in Fig. 4.4.

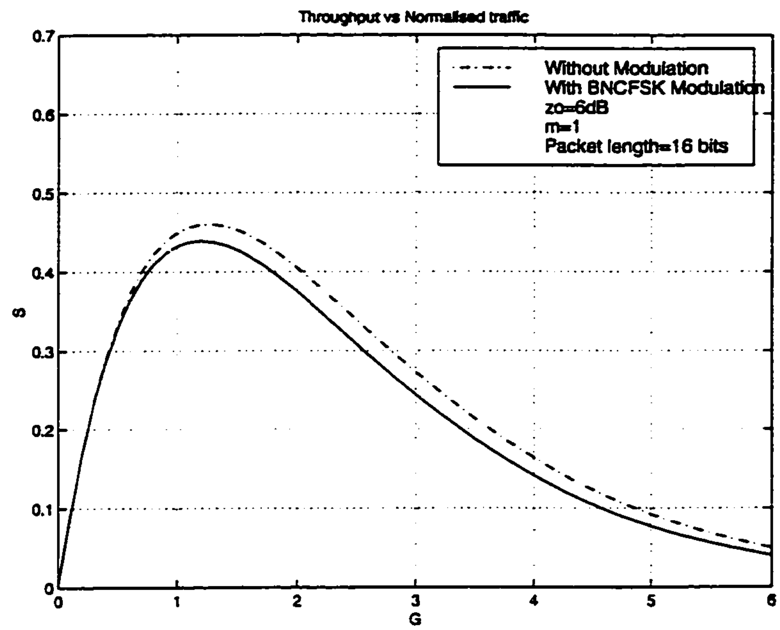



Figure 4.9: Comparison of the system performance with and without Modulation

## 4.3 Modulation Effects with Spatially Distributed Users

The effect of the modulation is also considered for the case of the spatially distributed users having a quasi–uniform traffic density of Fig. 3.1. It is required to determine the p.d.f of $z$ when the spatial distribution of the users is included. In chapter 2 we have evaluated the p.d.fs' of test signal $P_s$ and the interference power $P_n$ including the mean power variations due to the spatial distribution of the users. Using these p.d.fs' given by (3.15) and (3.19) we can evaluate the p.d.f of $z$. Once the p.d.f of $z$ is determined the probability of capture can be determined by averaging (4.5) over this p.d.f. The p.d.f of $z$ is given by:

$$f_{Z_n}(z) = 0.25\frac{\sqrt{n}\Gamma(m+0.5)\Gamma(nm+0.5)}{m\Gamma(m)\Gamma(nm)}z^{-1.5}\int_0^\infty \frac{w^{nm+m-1}}{(w+\frac{\pi n}{4m})^{nm+0.5}(w+\frac{\pi}{4mz})^{m+0.5}}dw. \tag{4.12}$$

The closed form result of the above integral is too complicated to tackle. Finally (4.12) is substituted in (4.5) to determine the capture probability.

Referring to Fig. 4.10 it can be seen that like in the ideal case the throughput decreases when the length of the packet and the bit error rate are taken into account. When the throughput is determined for the varying fading conditions then it can be seen in Fig. 4.11 that the modulated signal show little sensitivity to the fading as was observed in Fig. 3.3 for the idealized packets with the spatial distribution. In most of the analysis a packet length of $L =$ 16 *bits* is considered which, as advocated by

[24], seems to make sense for the underlying assumption that the mobile terminals use this scheme for placing requests at the base station for establishing a longer duration communication sessions in separate channels. Also the use of the capture ratio ($z_o = 6\ dB$) which as pointed out earlier, is a good choice to model the real narrow band receiver capture capability.

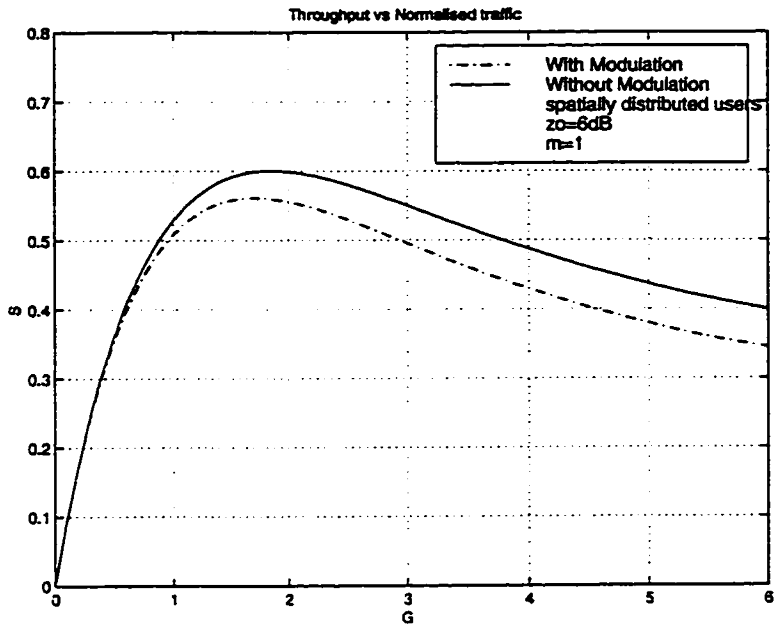



Figure 4.10: Comparison of throughput with and without modulation, $m = 1$ and $z_o = 6dB$

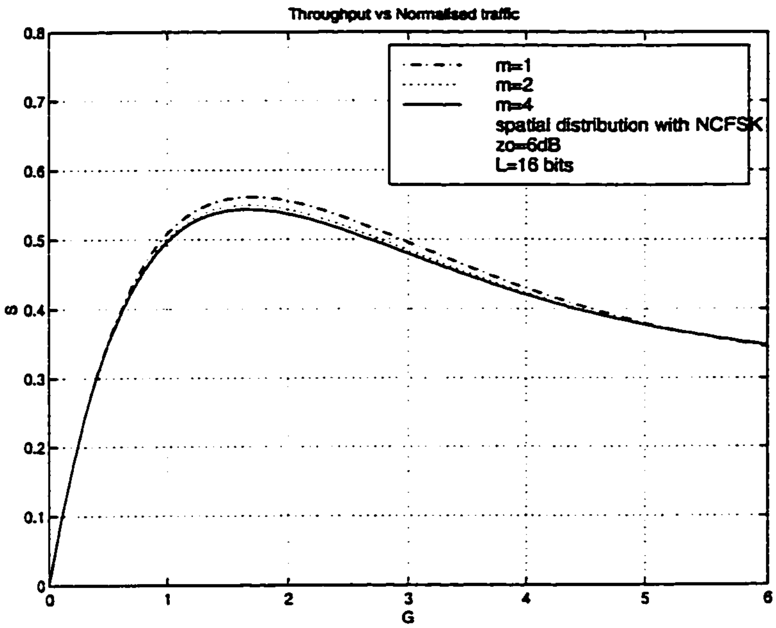



Figure 4.11: Comparison of throughput for different values of $m$, $z_o = 6dB$ and packet length=16 bits

# Chapter 5

# Stability Analysis

## 5.1 Introduction

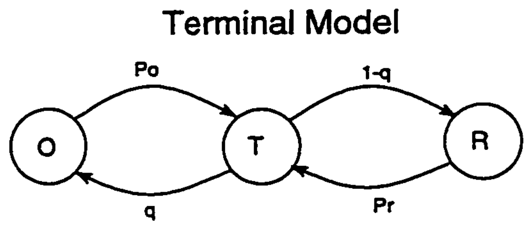
The analysis considered so far is an average analysis that doesn't take into account the dynamic behavior of the network. The parameter $G$ of the Poisson distribution is the average offered traffic load to the network. For considering the dynamic behavior of the network and to be able to deal with issues like the stability and the drift of the network we switch to a discrete Markov model of the slotted ALOHA network [22, 34, 41].

The term instability defines the behavior of the network where all the users are failing to transmit despite repeated random attempts suffering unbounded delay. In other words, at some finite point in time, the system enters a deadlock situation and is not able to recover. Classical slotted ALOHA is shown [41] to have a bistable behavior. In [42] it is mathematically argued that for all the range of transmission parameters the slotted ALOHA can have only one stable equilibrium point or three equilibrium points with the first and third one stable. Onozato [43] applied the concepts of catastrophe theory to study the problem of the thrashing encountered in conventional slotted ALOHA. To achieve this end the markovian model of [41] is used and with the concept of bifurcation set thrashing is shown to occur for a range of transmission parameters. In mobile ALOHA networks due to the presence of the capture effect the stability problem is not that severe. In [22] it is claimed that the mobile slotted ALOHA networks show a mono–stable behavior and the network throughput degrades rather gracefully under overload; also the mean delay is shown to increase slowly. In [44] the stability problem is studied by dividing the users into two different power groups. Conditions are obtained when the lower power group shows a stable behavior. It is shown in [29] that the bistability of the ALOHA networks occur only for a limited range of transmission parameters. It is also shown that with the increasing probability of capture there are increased chances of recovering the network from high backlog. A heuristic analysis of the stability of the slotted ALOHA system is presented by [12] which claimed that due to the negative average drift the system recover quickly from the backlogged state. Moreover if packet dropping is also considered (a characteristic of the real mobile systems) then it has a further stabilizing effect on the system performance.

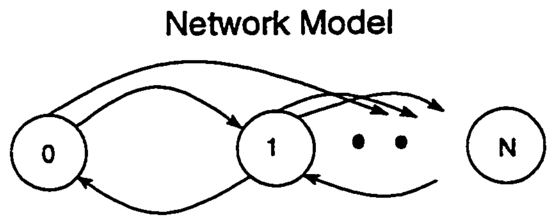
## 5.2 System Model

Our analysis in this chapter is based on the finite user Markov model as presented by [22]. The slotted ALOHA network is assumed to be consisted of N identical independently operating mobile terminals. The behavior of each of the mobile is described by the finite Markov chain. All the terminals share the same frequency band for packet communication with the centrally located base station. All the terminals can be in any of the three states. In the origination state (O) a packet is generated and transmitted in the next time slot with probability $p_o$. In the transmission state (T), the terminal is busy with either transmitting a new packet or retransmitting a previously collided packet. The mobile terminal returns to the O state if it receives a positive acknowledgement at the end of the time slot during which the packet is transmitted; otherwise it enters a retransmission or backlogged state (R). Packet generation is inhibited when the terminal is in the retransmission mode. A transition from retransmission state (R) to transmission state (T) occurs with probability $p_r$. Fig. 5.1(a) represents this behavior.

We can extend this model of single terminal to the entire network as shown in Fig. 5.1(b). With $N$ terminals we'll have $N + 1$ states, defined as the number of terminals in state $R$ at the beginning of a time slot. Suppose $q_i$ is the probability of successfully receiving one packet when $i$ packets are simultaneously transmitted. By definition $q_o = 0$ and in a well designed system $q_1$ is close to one. Otherwise




(a)




(b)

Figure 5.1: Markov chain model for the (a) individual terminals and (b) the entire network

the main reason for the unsuccessful transmission is the collision which occurs for $i > 1$. Namilso [22] expressed the state transition probabilities for a network with capture in terms of the number of participating terminals $N$ and the probabilities of generation $p_o$, capture $q_i$ and retransmission $p_r$. Recently Metzner [45] pointed out a minute error in the original formula and proposed a a slight modification to the expression of the state transition probability which can be given as:

$$\pi_{n,m} = \begin{cases} 0 & m < n-1 \\ (1-p_o)^{N-n} \sum_{i=1}^{N} C_i^n p_r^i (1-p_r)^{n-i} q_i & m = n-1 \\ C_{m-n+1}^{N-n} (1-p_o)^{N-m-1} p_o^{m-n+1} \sum_{i=0}^{n} C_i^n (1-p_r)^{n-i} {p_r}^i q_{i+m-n+1} & \\ \quad + C_{m-n}^{N-m} {p_o}^{m-n} (1-p_o)^{N-m} \sum_{i=0}^{n} C_i^n {p_r}^i (1-p_r)^{n-i} (1-q_{i+m-n}), & m \geq n \\ p_o^{m-n} \sum_{i=0}^{n} C_i^n {p_r}^i (1-p_r)^{n-i} (1-q_{i+N-n}) & m = N \end{cases} \tag{5.1}$$

where $C_i{}^n = \begin{pmatrix} n \\ i \end{pmatrix}$.

The overall average throughput of the system is

$$S = \sum_{n=0}^{N} \pi_n S_n \tag{5.2}$$

where $S_n$ is the expected throughput when the system is in state $n$, given by

$$S_n = \sum_{k=0}^{N-n} C_{N-n-k}^{N-n} {p_o}^k (1-p_o)^{N-n-k} \sum_{j=0}^{n} C_j{}^n {p_r}^j (1-p_r)^{n-j} q_{j+k} \tag{5.3}$$

and $\pi_n$ is the equilibrium state occupation probability which can be calculated using $\pi_{nm}$ with the procedure outlined in the appendix.

The expected delay of the system in terms of the packets is given by the expected backlog (expected state) divided by the expected throughput i.e.,

$$D = \frac{\sum_{i=1}^{N} i\pi_i}{S}. \tag{5.4}$$

The expected drift ($d_n$) in each state that determines the stability of the system is defined as the difference between the expected input and output traffic

$$d_n = (N - n)p_o - S_n. \tag{5.5}$$

A system is considered in an equilibrium state when the expected drift (5.5) crosses zero with negative derivative [29] . The effect of capture is to reduce the drift in each state thus providing improved stabilization of the system.

## 5.3 Stability under Nakagami Fading Model

In [29] it is claimed that the Rayleigh fading contributes little to the improvement of the network stability when considered alone. Here we'll further investigate their assertion and present a generalized picture using Nakagami fading model. For the determination of the expected drift and delay we'll consider the model formulated above . A requirement of the above model is the choice of the origination and retransmission probabilities, which determines the delay involved in transmitting a particular packet. For a good system the average wait time for a retransmission should be short, usually much shorter than the average time between new packet originations. So the probability of retransmission should be larger usually an order than the packet origination probability.

For our analysis, following [29] we have considered $po = 0.0055$ and $pr = 0.08$. The approach also permits us to draw comparison of our results with the results reported in [29] for the Rayleigh fading channels.

### 5.3.1 Fade only Environment

For the case of the finite user Markov model the capture probabilities are determined by [22], given as:

$$q_i = i Prob\left(\frac{P_s}{\sum_{j=1}^{i} P_j} > z_o\right) \qquad for\ i > 1 \tag{5.6}$$

where $P_s$ is the test signal power level and $P_i = \sum_{j=1}^{i} P_j$ is the joint interference power. The determination of the capture probabilities will follow the approach discussed in section 2.2. With these capture probabilities, we determine the expected drift and the state occupation probabilities following the approach discussed in section 5.1, for the various values of fading parameter $(m)$.

Referring to Fig. 5.2 it can be observed that for the parameters studied stability of the system is affected by the fading conditions. For Rayleigh fading the result is same as reported in [29], but when the fading is worse than Rayleigh (e.g. m=0.5) then the system shows a bistable behavior. A stable condition (with reasonable throughput and short delays) exist when most of the terminals are in the origination mode. The other stable condition would have most terminals in the retransmission mode, thus blocking the flow of any traffic. For Rayleigh fading and non–fading environments a mono stable condition with high backlog is shown to exist. A comparison of the drift and the equilibrium state occupation probabilities ($\pi_n$) for different values of $m$ is shown in the Fig. 5.3. It can be seen, though the network shows bistability for $m = 0.5$ (as clear from the drift curve), but under equilibrium the probability of the occupation of the states with higher throughput and low delay is approximately zero.

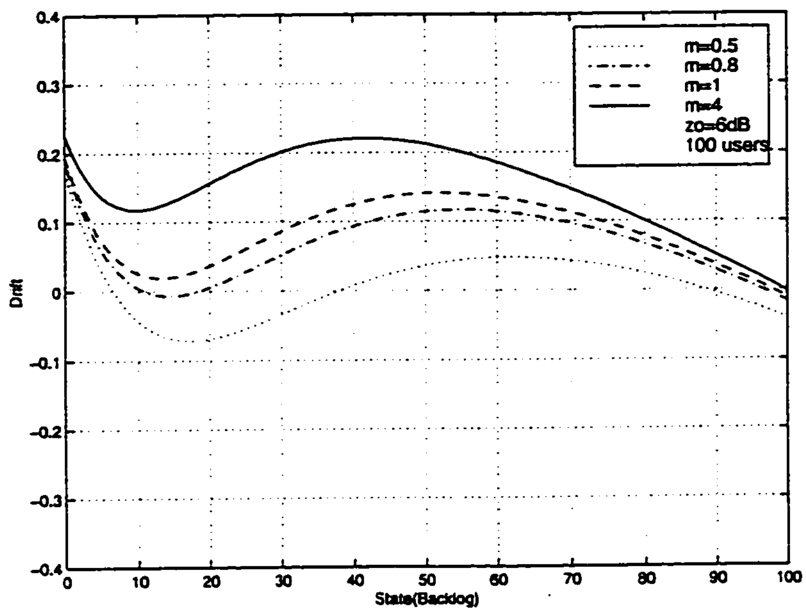



Figure 5.2: Effect of fading condition on the drift of the slotted ALOHA for capture ratio $z_o = 6dB$

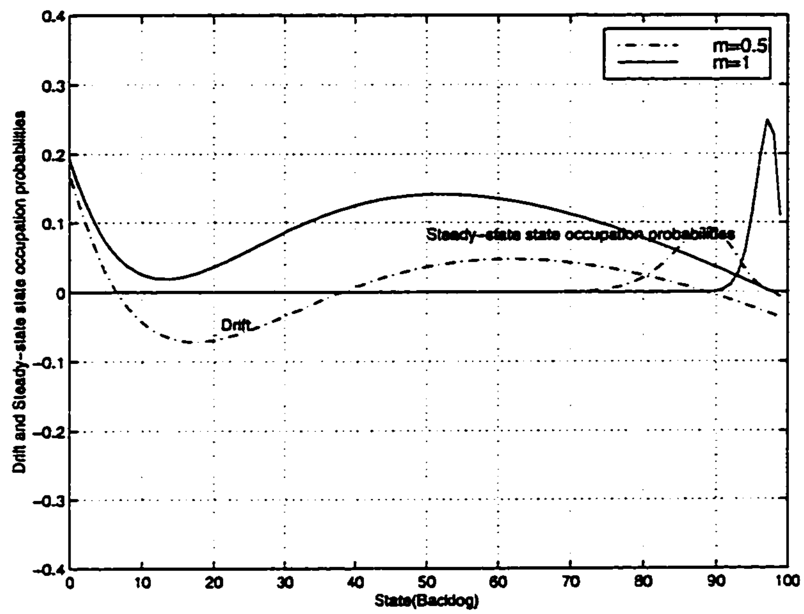



Figure 5.3: Comparison of the drift and the state occupation probabilities for different values of $m$ with capture ratio $z_o = 6dB$

### 5.3.2 Fading along with the Spatial Distribution

The effect on the stability of the spatially distributed users around the base station is also studied. It is observed in chapter 3 that for the spatially distributed users the capture probabilities are quite high as compared to the case when the users transmit with equal average power.

When the drift is determined for different values of $m$ incorporating spatial distribution then as expected the stability of the system is improved showing a mono–stable behavior. The drift for all values of $m$ under high backlog is negative as observed in Fig. 5.4. Due to the high negative drift if the system happens to start under high backlog then it will quickly move towards the achievement of the stable state. Under stable condition the system is shown to have a low backlog thus low delay and high throughput. The equilibrium state occupation probabilities are also considered for the spatially distributed users in Fig. 5.5 which also show that the highest steady–state state occupation probabilities occur when the system is in a stable state. In [29] it is shown that the system stability can also be improved by directly controlling the system parameters like the retransmission probabilities and the number of users that are allowed to be signed on at one time.

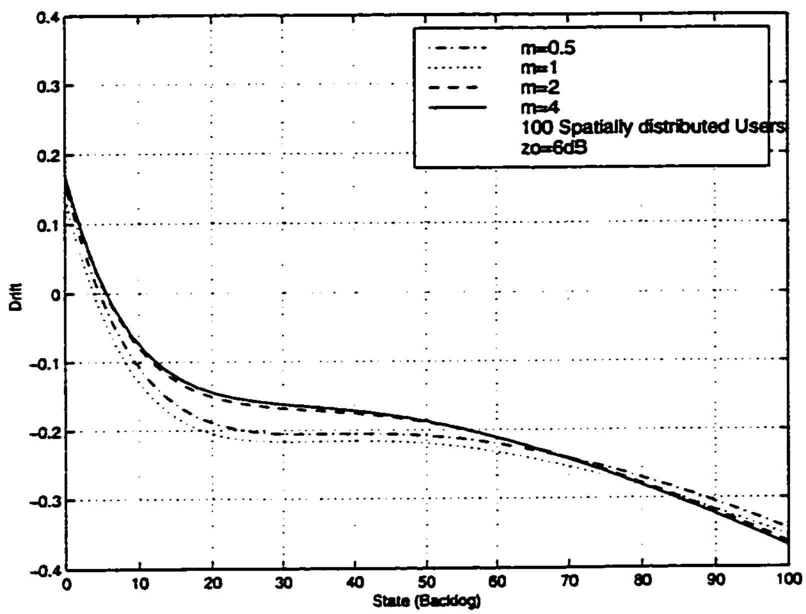



Figure 5.4: Effect of fading along with spatial distribution on the drift of the slotted ALOHA for capture ratio $z_o = 6dB$

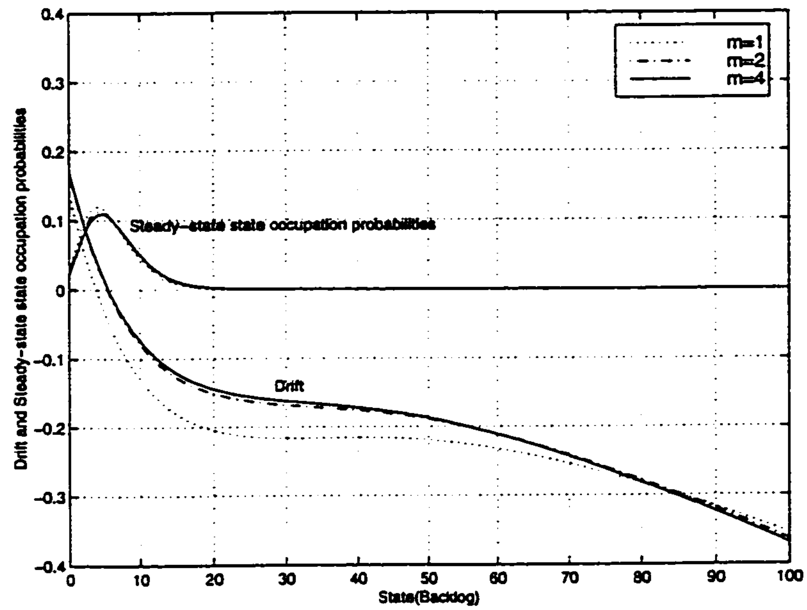



Figure 5.5: Comparison of the drift and the state occupation probabilities for different values of $m$ with capture ratio $z_o = 6dB$ and spatially distributed users

# Chapter 6

# Conclusion and Future Research

In this thesis, a popular random access scheme is analyzed under a generalized fading model, with the view of application in channels like mobile communication channel. The scheme is treated in detail with regard to several effects which are encountered in practical mobile communication channels.

With the use of the simple incomplete beta function we obtained closed form results for capture probability under several different fading conditions. The results are quite simplified compared to their previously reported forms in literature. Also in the literature the spatial distribution of the users around the central receiver is shown to have profound effect on the probability of correctly receiving a packet in case of the several colliding packets for the Rayleigh fading channels. This effect is generalized for the variety of fading conditions using the parameter $m$ of the Nakagami fading model. For quasi−uniform distribution of the transmitting terminals it is shown that the increase in fading depth doesn't necessarily enhance the throughput. For a particular fading condition the throughput is shown to be strongly related to the capture parameter (receiver sensitivity). For varying fading conditions and a practical receiver sensitivity ($z_o = 6dB$) the maximum achievable throughput increases as the fading depth increases.

Also the mobility of the users around the base station is investigated which reveals that the fading increases the success probability only when the test user is farther from the base station. For a user transmitting very close to the base station the effect of fading is detrimental.

The Lognormal shadowing exhibits a higher probability of the upfades of the signal power, hence the probability that one out of many contending signals is sufficiently stronger than the joint interference is high as compared to the fade only environment. The result is the enhancement in the throughput which increases with the increasing spread of the powers of the contending signals. We studied this effect through simple computer simulations. The results are in agreement with the analytical results for the special case of Rayleigh fading reported in the literature.

For more realistic transmission conditions the effects of the modulation, length of the packet and the presence of the additive white gaussian noise (AWGN) are also considered in chapter 4. The more practical case of slow fading is only considered. It is shown that the incorporation of the realistic conditions degrades the throughput than the one observed for the idealized systems. The noise is shown to be of little concern for the scheme and the throughput is shown to be mainly limited by the contention.

Finally we studied the dynamic behavior of the network using a markovian model. System stability is studied through the drift and the state occupation probabilities. For the parameters studied, it is observed that the changing fading conditions don't affect the stability significantly but the inclusion of the spatial distribution enhances the stability and reduces the overall delay.

## 6.1 Suggestions for the Future Work

1. The primary concern of the present thesis is to study the slotted ALOHA as the random access scheme which is used for establishing initial connection and then the traffic transmission takes place using either FDMA (AMPS), TDMA (GSM) or CDMA (IS-95) multiple access schemes. Some researchers have proposed the slotted ALOHA as a primary multiple access scheme even for voice communication. It would be interesting to generalize such studies using Nakagami fading model.

2. More sophisticated random access schemes like Carrier sense multiple access (CSMA), Inhibit sense multiple access (ISMA) or Cellular packet division multiple access schemes (CPDA) can also be studied using the basic frame work outlined for the slotted ALOHA.

3. The capacity of the TDMA systems used in the GSM systems is hard limited by the fact that once a user is in the possession of the channel then the particular time slots are reserved for his transmission whether he transmits in that slot or not. A proposal is the use of the scheme like reservation ALOHA in which the optimum utilization of the user can be made. Such a scheme can also be studied under Nakagami fading model.

# Appendix A

# Drift and Steady-State State Occupation Probabilities

In this appendix a formulation of the drift equation given by (5.5) is presented. Also the procedure for the evaluation of the steady state occupation probabilities following [22] is reiterated for the sake of completion.

With the network expressed in terms of the Markov chain, the system can have $N+1$ possible states which correspond to the number of terminals in the retransmission mode at the beginning of the time slot. The transition from one state to another can be expressed with the help of the state transition matrix $P = [\pi_{n,m}]$ where $\pi_{n,m} = Pr\left[X(t+1) = m | X(t) = n\right]$ (due to the memoryless assumption) is given by (5.1). For $0 < p_o < 1$ and $0 < p_r < 1$, the Markov chain is irreducible, aperiodic and positive recurrent, with a vector of the long term state occupation probabilities

$\mu = (\mu_o, \mu_1, \mu_2, ..., \mu_N)$ satisfying the equation $\mu^T = \mu^T P$ [41]. The expected drift of the system from state $n$ can be obtained by [41]

$$d_n = \sum_{m=0}^{N} (m-n)\, \pi_{n,m}. \tag{A.1}$$

From the above equation we can arrive at (5.5) as follows:

Let $S_n$ be the system's expected output flow per time slot at state $n$, then [41]

$$\begin{aligned} S_n &= \pi_{n,n-1} + (N-n)p_o(1-p_o)^{N-n-1}(1-p_r)^n \\ &= (1-p_o)^{N-n-1}(1-p_r)^{n-1}\left[(N-n)p_o(1-p_r) + np_r(1-p_o)\right]. \end{aligned} \tag{A.2}$$

Assuming $m - n = i$ and since $\pi_{n,m} = 0$ for $m < n-1$ [(5.1)] we can express (A.1) as

$$\begin{aligned} d_n &= \sum_{i=-1}^{N-n} i\pi_{n,n+i} \\ &= -\pi_{n,n-1} + \pi_{n,n+1} + \sum_{i=2}^{N-n} i \begin{pmatrix} N-n \\ i \end{pmatrix} {p_o}^i(1-p_o)^{N-n-i}. \end{aligned} \tag{A.3}$$

The last term is equal to the mean of the binomial distribution minus the quantity $(N-n)p_o(1-p_o)^{N-n-1}$ i.e., $(N-n)p_o - (N-n)p_o(1-p_o)^{N-n-1}$. Substituting the values of $\pi_{n,n-1}$ and $\pi_{n,n+1}$ from (5.1, where it is assumed that $i = 1$ i.e. the effect of capture is neglected) we arrive at (5.5). In our case the capture effect is taken care of while evaluating $S_n$ so the expression is valid even when capture is considered.

From (5.1) the equilibrium state occupation probabilities $\pi_n$ can be calculated.

Starting with an arbitrary positive constant $\pi_o{}^*$ recursively one can obtain

$$\pi_n{}^* = \frac{1}{\pi_{n,n-1}} \left( \pi^*_{n-1} - \sum_{i=0}^{n-1} \pi^*_i . \pi_{n,n-1} \right) \tag{A.4}$$

and after normalizing we get the desired steady state occupation probabilities given as

$$\pi_n = \frac{\pi^*_n}{\sum_{i=0}^{N} \pi^*_i}. \tag{A.5}$$

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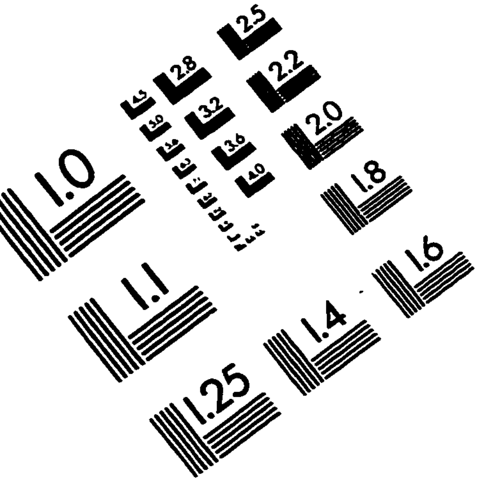
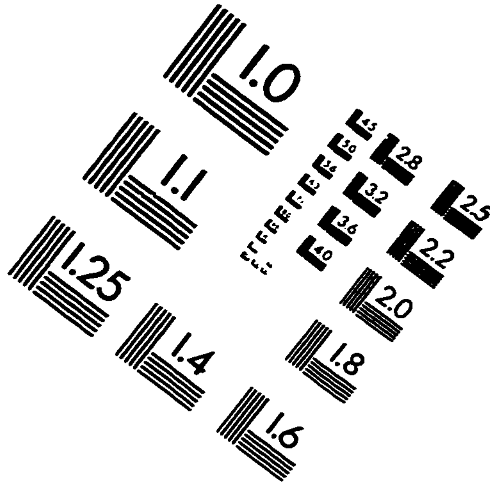
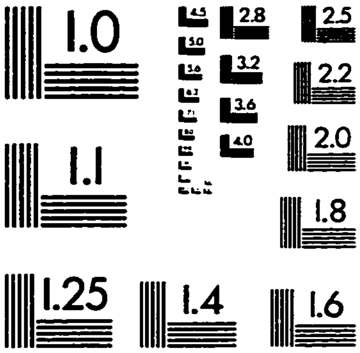
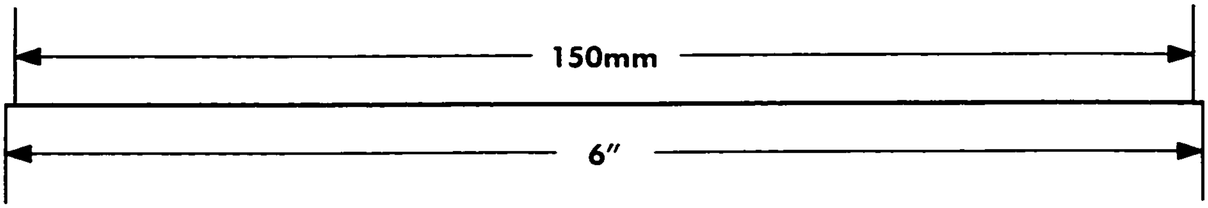
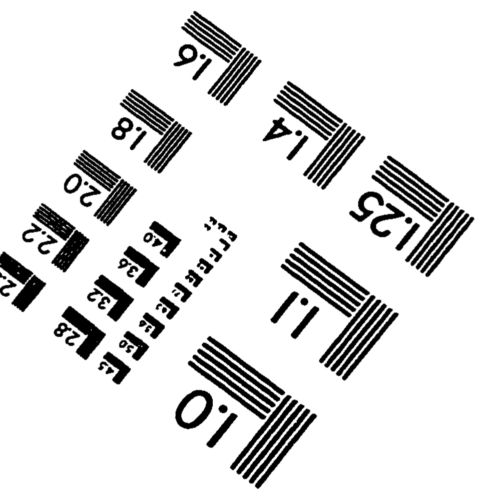
# IMAGE EVALUATION TEST TARGET (QA-3)

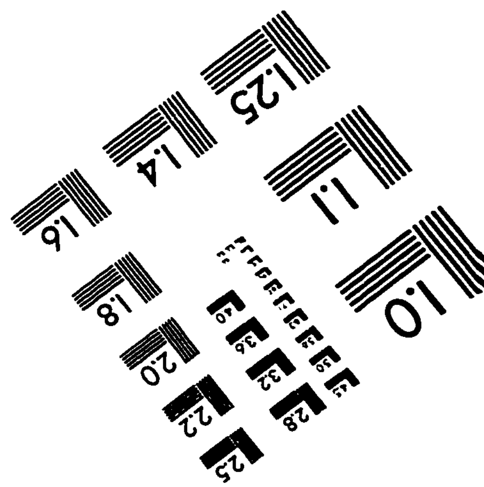








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