Identification and Control of a Class of Nonlinear Systems

by

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Dedicated to my family, especially to

my mother, late father,
brothers and sisters.

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Nomenclature

Notations and Symbols

t, k	integer time index
$u(k)$	input signal
$y(k)$	output signal
$y_r(k)$	reference signal
$y_d(k)$	desired signal
$\hat{y}(k)$	predicted output signal
$e(k)$	white noise (a sequence of independent random variables)
λ^2	variance of white noise
q^{-1}	backward shift operator
θ	parameter vector
$\hat{\theta}$	estimate of parameter vector
n_θ	dimension of parameter vector

N	number of data points
$\varepsilon(t, \theta)$	prediction error corresponding to the parameter vector θ
P	variance of $\hat{\theta}$
$K(t)$	weighting or gain factor

Abbreviations

NL	Non-linear
G-Model	General Model
LS	Least Squares
RLS	Recursive Least Squares
NLS	Nonlinear Least Squares
SVD	Singular Value Decomposition
PRBS	Pseudo Random Binary Sequence
SISO	Single Input Single Output
MIMO	Multi Input Multi Output
GA	Genetic Algorithm
ANN	Artificial Neural Network
PE	Prediction Error Method
MFNN	Multilayer Feedforward Neural Network
AR	Auto Regressive
ARMA	Auto Regressive Moving Average

N-way	Multiway
PLS	Partial Least Squares
N-PLS	Multiway Partial Least Squares
NLMS	Normalized Least Mean Squares
FIR	Finite Impulse Response
LTI	Linear Time Invariant

THESIS ABSTRACT

Name: SYED OMER FAROOQ

Title: IDENTIFICATION AND CONTROL OF A CLASS OF NONLINEAR SYSTEMS

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There are many systems in the field of biological systems, control systems and communication systems that can be represented by non-linear models. Identification of non-linear systems is a fundamental problem in the fields of control and communication systems. Many investigations of non-linear systems have been carried out from the viewpoint of parameter identification. A class of nonlinear systems such as the Hammerstein, the Wiener and the Hammerstein-Wiener are considered in this work. The purpose of this work is to use the Recursive Least Squares (RLS) algorithm and the Multiway algorithm tools for parameter estimation of a class of nonlinear (NL) systems. It is also intended that we develop an adaptive control scheme for the same class of NL systems.

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خلاصة الرسالة

الاسم : سيد عمر فاروق

عنوان الدراسة : تحديد مجموعة من الأنظمة غير الخطية والتحكم بها

التخصص : هندسة النظم

تاريخ التخرج : جون 2003

تمثل عملية تحديد الأنظمة غير الخطية مشكلة أساسية في مجالات الأنظمة والاتصالات. تم إجراء العديد من عمليات التحقق من الأنظمة الغير خطية من جهة تحديد المعاملات. تم التطرق في هذا البحث إلى مجموعة من النظم غير الخطية مثل هاميرشتان و ونر وهاميرشتان - ونر. يهدف هذا البحث إلى استخدام خوارزمية المربعات الصغرى التعاقبية وخوارزمية الطرق المتعددة لتقدير المعاملات لمجموعة من الأنظمة غير الخطية. كما يهدف أيضاً إلى تطوير طرق للتحكم التكيفي لمجموعة الأنظمة غير الخطية ذاتها. تم اختبار هذه الطرق على العديد من الأمثلة وأثبتت مقدرتها على التعرف على المعاملات والتحكم في الأنظمة.

درجة الماجستير في العلوم
جامعة الملك فهد للبترول والمعادن
جون 2003

Chapter 1

Introduction

System identification is the field of modelling dynamic systems from experimental data [2]. The bulk of the work done in system identification starts with representing the process as a black box [3]. We may have access to the inputs and outputs, but the internal mechanisms are assumed to be totally unknown to us. The problem in system identification is to construct a model which would mimic the *inner mechanism* of the system, using the input/output data. The usual procedure is to select a model structure with some unknown parameters and then to estimate the model parameters. The last step is to check whether the model obtained is adequate.

Historically, system identification has been motivated by the need to design better control of dynamic systems [4]. It enables more insight into the system and thus possible to make the system more controllable and observable [2]. Very frequently we are faced with the necessity of experimentally determining some important physical

parameters such as heat transfer coefficient, chemical reaction rate, damping factor, and so on. The need for highly accurate system models has been intensified by the development of optimal and adaptive control theories.

1.1 Nonlinear system identification

The identification of nonlinear systems is a topic which has received considerable attention over the last two decades [5, 6, 7, 8]. Nonlinear behavior is a rule, rather than exception, in the dynamic behavior of physical systems. Most physical systems have nonlinear characteristics outside a limited linear range. A well known method that can represent a wide class of nonlinear systems is the well known Volterra series expansion. In some particular cases, nonlinear systems can be separated into linear subsystems that include zero-memory nonlinearities in various combinations.

One approach for understanding the nonlinear behavior is to form a mathematical model of the process. To achieve this, a mathematical model of each unit operation has to be formed by making some simplifying assumptions, and then these models are combined to obtain a model representing the complete system [3]. In the last decades, many research activities have been carried out on modelling, identification and control design of nonlinear systems [9]. The reason is that nonlinear models describe most of physical processes and naturally the use of nonlinear analysis increases accuracy and performance of the system behavior.

1.2 Input Signals

The input signal used in an identification experiment have a significant influence on the resulting parameter estimates [2]. Some of the input signals generally used in identification are:

- Step function
- Pseudorandom binary sequence
- Autoregressive moving average process
- Sum of sinusoids

In our simulation work, generally we use the sum of sinusoids as an input function.

1.2.1 A sum of sinusoids

In this class of input signal, $u(k)$ is given by:

$$u(k) = \sum_{j=1}^{r} a_j \sin(w_j k + \varphi_j) \tag{1.1}$$

Where the angular frequencies w_j are distinct,

$$0 \leq w_1 < w_2 < \ldots < w_r \leq \pi \tag{1.2}$$

For a sum of sinusoids the user has to choose the amplitudes a_i, the frequencies and the phases φ_i. Fig. 1.1 illustrates a typical example for the input signal using a sum

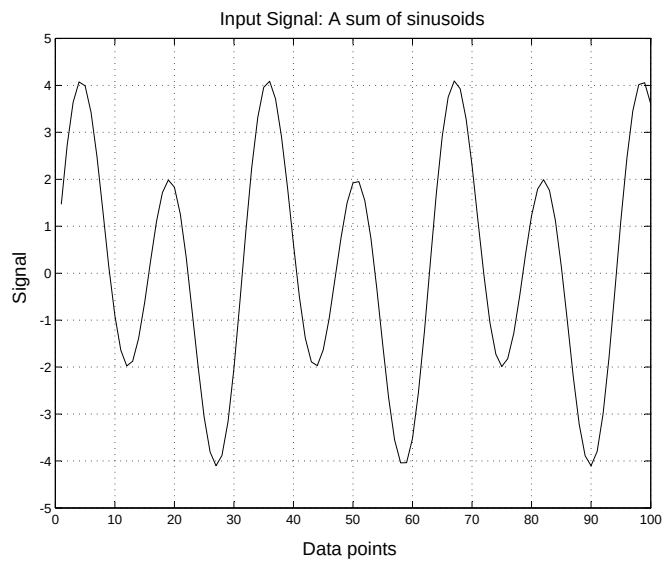

Figure 1.1: A sum of two sinusoids

of two sinusoids.

1.3 Persistent Excitation

Consider a finite impulse response (FIR) model:

$$y(k) = b_1 u(k-1) + b_2 u(k-2) + ... + b_n u(k-n) \tag{1.3}$$

It is obvious that the parameters of the model (1.3) cannot be determined unless some conditions are imposed on the input signal $u(k)$ [10]. It follows from the

condition for uniqueness of the least-squares estimate explained in (3.10)-(3.11), that the minimum is unique if the matrix

$$\Phi^T\Phi = \begin{pmatrix} \sum_{n+1}^{t} u^2(k-1) & \sum_{n+1}^{t} u(k-1)u(k-2) & \dots & \sum_{n+1}^{t} u(k-1)u(k-n) \\ \sum_{n+1}^{t} u(k-1)u(k-2) & \sum_{n+1}^{t} u^2(k-2) & \dots & \sum_{n+1}^{t} u(k-2)u(k-n) \\ \vdots & & & \\ \sum_{n+1}^{t} u(k-1)u(k-n) & & & \sum_{n+1}^{t} u^2(k-n) \end{pmatrix} \tag{1.4}$$

has full rank. This condition is called an *excitation* condition. For long data sets the end effects are negligible, and all sums in (1.4) can be taken from 1 to t. We then get

$$\mathbf{C_n} = \lim_{\mathbf{t}\to\infty} \frac{\mathbf{1}}{\mathbf{t}} \mathbf{\Phi^T\Phi} = \begin{pmatrix} c(0) & c(1) & \dots & c(n-1) \\ c(1) & c(0) & \dots & c(n-2) \\ \vdots & & & \\ c(n-1) & c(n-2) & \dots & c(0) \end{pmatrix} \tag{1.5}$$

where $c(k)$ are the empirical covariances of the input, i.e.,

$$c(k) = \lim_{t\to\infty} \frac{1}{t} \sum_{i=1}^{t} u(i)u(i-k) \tag{1.6}$$

For long data sets the condition for uniqueness can thus be expressed as the matrix in (1.5) being positive definite. This leads to the definition that "A square summable signal u is called *persistently exciting* (PE) of order n if the matrix C_n given in (1.5) is positive definite".

1.4 Singular Value Decomposition

The singular value decomposition (SVD) is a powerful technique in many matrix computations and analysis. Using the SVD of a matrix in computations, rather than the original matrix, has the advantage of being more robust to numerical error. Additionally, the SVD exposes the geometric structure of a matrix, an important aspect of many matrix calculations. A matrix can be described as a transformation from one vector space to another. The components of the SVD quantify the resulting change between the underlying geometry of those vector spaces. The SVD is employed in a variety of applications, from least square problems to solving systems of linear equations. Each of these applications exploit key properties of the SVD, its relation to the rank of a matrix and its ability to approximate matrices of a given rank. Many fundamental aspects of linear algebra rely on determining the rank of a matrix, making the SVD an important and widely used technique.

Theorem 1 [11]: Let $A \in R^{n\times m}$ have rank r. Then there exists

$$U \in R^{n\times n}, \quad \Sigma \in R^{n\times m}, \quad \text{and} \quad V \in R^{m\times m} \tag{1.7}$$

such that U and V are orthogonal, Σ has the form

$$\mathbf{\Sigma} = \left[\begin{array}{cccc|c} \sigma_1 & & 0 & & \\ & \sigma_2 & & & 0 \\ & & \ddots & & \\ & 0 & & \sigma_r & \\ \hline & \underbrace{\qquad 0 \qquad} & & & 0 \end{array}\right]$$

and

$$A = U\Sigma V^T \tag{1.8}$$

Theorem 2 [11]

Let $A \in R^{n\times m}$ have rank r. Let $\sigma_1, ..., \sigma_r$ be the nonzero singular values of A, with associated right and left singular vectors $v_1, ..., v_r$ and $u_1, ..., u_r$, respectively. Then

$$A = \sum_{j=1}^{r} \sigma_j u_j v_j^T \tag{1.9}$$

1.5 Objectives of the Thesis

The objectives of this thesis are the following:

1. To develop a recursive version of the Bai's method [1]. This will be achieved by using the RLS in the Bai's scheme.

2. To test the resulting RLS for different systems of each case of Hammerstein,

Wiener and the Hammerstein-Wiener nonlinear systems.

3. To formulate the identification problem using the multiway partial least squares (N-PLS) method.

4. To test the N-PLS based formulation scheme on different classes of the nonlinear dynamic systems.

5. To develop an adaptive scheme for control of nonlinear systems, based on the developed RLS method.

6. Simulating the adaptive scheme using different nonlinear systems, such as Hammerstein, Wiener and Hammerstein-Wiener systems each for different reference commands.

7. To develop an normalized least mean squares (NLMS) based adaptive control scheme for stable nonlinear plants.

1.6 Organization of the Thesis

The first chapter of this thesis is used to introduce and define the problem, state the thesis objective and outline the organization. The second chapter covers the selected literature in the areas of nonlinear system identification, multiway analysis and adaptive control. In the third chapter the RLS method for the Bai's system is presented and extended for higher order nonlinear systems. In the fourth chapter the

idea of multiway analysis method is used for system identification of the nonlinear systems. In the fifth chapter the adaptive control schemes for compensation of nonlinear systems with different approaches such as the noise estimation scheme and the prediction based adaptive control scheme, are presented and finally the sixth chapter presents the summary, conclusion and recommendations for the possible future work.

Chapter 2

Literature Review

2.1 Hammerstein and Wiener models

The behavior of many systems can be approximated by a static nonlinearity cascaded with a linear part in a particular form. These models are known as Hammerstein and Wiener block cascade models. These models are used to model several classes of nonlinear systems. The literature on these block cascaded models is extensive and reflects considerable current activity [12]. Their flexibility lies in having the nonlinearity entirely separate from the common and easily realizable linear parts. The choice of the nonlinear element is virtually unlimited and for the linear element, a variety of options, for it's identification, are available including the step response and the transfer function-based models [13]. The main idea of the block oriented approach is that the identified system consists of simple subsystems such as linear-

dynamic subsystem and nonlinear-memoryless subsystem. The main objective of this identification approach is to recover descriptions of all subsystems from observations taken at input and output of the whole system [14]. The block-oriented models provide simple architecture to the nonlinear models. Moreover, complex structures can be constructed from simple blocks via the parallel-series approach [15].

2.1.1 Hammerstein model

The Hammerstein Model can be described by a memoryless nonlinear element followed by a linear dynamical system, illustrated in Fig. 2.1, where the static nonlinear element scales the input $u(k)$ and transforms it to $x(k)$, and the dynamics are modeled by a linear transfer function, whose output is $y(k)$. Despite it's relative

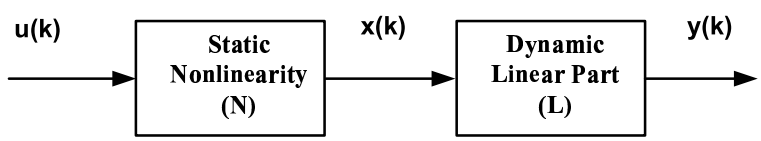

Figure 2.1: The Hammerstein Model

simplicity this model is perceived to be quite adequate in describing some real systems, such as chemical processes, distillation column, electric heat exchanger etc. [3]. The Hammerstein model models the nonlinear effects as an input-dependent gain nonlinearity. The slope of the nonlinearity at a certain operating point is the instantaneous gain of the system. The Hammerstein model can be described by the

following Eq. 2.1[3].

$$\begin{aligned} y(k) &+ a_1 y(k-1) + \cdots + a_n y(k-n) \\ &= b_1 x(k-1) + b_2 x(k-2) + \cdots + b_m x(k-m) \end{aligned} \tag{2.1}$$

Or,

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{j=1}^{m} b_j x(k-j) \tag{2.2}$$

Where $$x(k) = f(u(k))$$

$u(k)$ and $y(k)$ are the input and output of the system, respectively and $x(k)$ is the nonlinear function of the input $u(k)$. $x(k)$ cannot be measured, but it can be eliminated from the equation.

If we introduce the shifting operator q defined by

$$q^{-1} y(k) = y(k-1) \tag{2.3}$$

and the polynomials

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{2.4}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_m q^{-m} \tag{2.5}$$

Then (2.2) can be written in the following form, such that the intermediate variable

$x(t)$ has been removed.

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} f(u(k)) \tag{2.6}$$

If $f(.)$ is assumed to be approximated by a polynomial of order l, then:

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} \sum_{i=1}^{l} \gamma_i u^i(k) \tag{2.7}$$

Many identification methods have been developed to identify the *Hammerstein model*. All of the techniques largely depend on the prior knowledge of the system. Narendra and Gallman [16] for the first time in 1966 used an iterative method for the identification of Hammerstein model with a polynomial nonlinearity. The method utilizes the alternate adjustment of the parameters of the linear and NL parts of the model while minimizing the error function. Chang and Luus [17] used a non-iterative technique for the identification of a Hammerstein system where the transfer function may have zeros. Zhu [6] proposed a relaxation iteration scheme by making use of a model structure in which the error is bilinear in the parameters. The order of the linear part and NL part are determined by looking at an output error related criterion which is control relevant. Rangan *et al.* [18] proposed an approach for multivariable Hammerstein identification for state-space and linear-FIR models with white noise excitation. Moreover they have shown that in the linear-FIR case with white noise input, the standard iterative procedure is guaranteed to converge and thus provides an asymptotically optimal estimator. Daniel-Berhe and Unbehauen [19] presented a

frequency weighted least squares (FWLS) based formulation method to identify the parameters of Hammerstein-type nonlinear continuous-time systems based on input and noise contaminated output data observed over a finite time interval. The FWLS and the hartley modulating functions (HMF) algorithms are applied to the system using the Monte Carlo simulation. Billings and Fakhouri [20] presented an identification algorithm for the systems having the structure of a Hammerstein model using the cross-correlation techniques to decouple the identification of the linear dynamics from the characterization of the nonlinear element. Li [21] implemented the GA for identification, and used the piecewise linear approximation of the memoryless nonlinear characterisation of the Hammerstein models. The algorithm has been proven to be robust, globally stable and powerful. Kristinsson and Dumont [22] demonstrated the usefulness of GA to estimate both continuous and discrete time systems and for identifying poles and zeros or physical parameters of a system. Hatanaka and Uosaki [23] proposed a novel approach for identification using the genetic programming and determined the nonlinearity of the system. Tobin *et al.* [24] used a point-slope parameterization of the static nonlinearity that leads to a computationally tractable optimization problem. The identification method considered, using a Hammerstein feedback model with piecewise linear static maps simultaneously approximates the linear dynamic and static nonlinear blocks of the nonlinear feedback model. Eskinat and Johnson [3] used the identification methods for simulated distillation columns and to an experimental heat exchanger process.

Ngia [25] proposed separable nonlinear least-squares methods for on-line estimation of neural nets Hammerstein models. Boutayeb *et al.* [26] have given an algorithm which transforms the nonlinear model into a model which is linear in parameters and then they derived the pseudo-inverse technique, leading to a consistent estimator or the initial realisation as well the model of the noise. Al-Amer and AL-Sunni [27] proposed a new iterative procedure to identify Hammerstein models. The algorithm minimizes the infinity norm of the deviation between the true model and identified model. Gomez and Baeyens [9] presented a non iterative algorithm for the simultaneous identification of the linear and nonlinear parts of the multivariate Hammerstein systems. The proposed algorithm is based on LS and SVD. Voros [28] presented a new approach to the parameter identification of nonlinear dynamic system using the Hammerstein model with two-segment nonlinearity. Sun [29] proposed a new least squares type of identification algorithm for the Hammerstein model based on the over-sampling technique. Tingqi [30] proposed a novel approach based on dynamical separation technology for a class of Hammerstein systems. Westwick and Kearney [31] proposed separable least squares (SLS) optimization methods as a mean for simultaneously estimating both the linear and the nonlinear elements, in an exact least squares frame-work. Belforte and Gay [32] considered the problem of optimal input design for Hammerstein system identification when the linear dynamic part of the model is an FIR and the measurement errors are unknown but bounded. Giarre and Zappa [33] proposed an identification algorithm based on a relaxation

technique and proved its consistency. Duwaish *et al.* [34] developed a new method for identification and control. This method consists of a MFNN in series with an ARMA model. A recursive identification algorithm has been derived to update the weights of the MFNN and the parameters of the ARMA.

2.1.2 Wiener model

The Wiener model of a nonlinear systems is constructed by a nonlinear gain cascaded after a linear subsystem. The block cascade structure of Wiener model is shown in Fig. 2.2. Where the intermediate variable $x(k)$ is the output of the linear dynamic

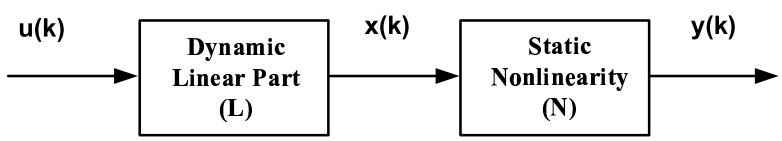

Figure 2.2: The Wiener Model

part given by:

$$x(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) \tag{2.8}$$

$u(k)$ is the input to the system, $A(q^{-1})$ and $B(q^{-1})$ are as defined in (2.4) and (2.5), respectively.

The observed output $y(k)$ of the system is

$$y(k) = f(x(k)) \tag{2.9}$$

Where $f(x(k))$ is the static nonlinear function of $x(k)$. Thus, the problem of the Hammerstein and Wiener model identification is to estimate the coefficients of the linear part and the parameters of the nonlinearity, from the input-output data.

The *Wiener model*, as in Fig. 2.2, is constructed by a nonlinear gain cascaded after a linear system. Ikonen *et al.* [35] suggested a Wiener model for the identification of a MISO steady-state static model with linear FIR dynamics for each input. Unit steady state gain is obtained by using a reduced FIR model, consisting of a unit gain plus finite step response (FSR) dynamics. Zhu and Control [36] studied the identification of SISO Wiener model. They extended the *assymertic* (ASYM) method which provides the systematic solutions to the problem of identification for both open-loop and closed-loop operation. While the parameter estimation, the bilinear-in-the-parameters property of the high order model is used to derive relaxation algorithm. Wigren [37] derived a recursive prediction error identification algorithm for the Wiener model. Where the linear dynamic block is modelled as a SISO transfer function operator and the static nonlinearity is approximated with a piecewise linear function. Kalafatis [38] proposed an approach for the identification of Wiener systems in a noisy environment. The estimated models are represented in terms of the frequency response of the linear subsystem and the inverse of the static nonlinearity. Chou and Verhaegen [39] presented a method to identify Wiener models with a general disturbance configuration in closed-loop using the indirect approach. They used a disturbance structure such that the noise

enters both at the output side and in between the LTI and NL block. Hu and Wang [40] identified the impulse response functions of the linear part and the polynomial coefficients of the nonlinear part of the discrete-time Wiener model using the three-level-pseudorandom-sequences with different amplitudes as input signals. Greblicki [41] proposed a nonparametric approach to recover the nonlinearity in Wiener systems. Verhaegen and Westwick [42] showed that the multivariable output error state space model (MOESP) class of subspace model identification (SMI) schemes can be extended to identify Wiener systems. Westwick and Kearney [43] demonstrated that the multiple input Wiener systems can be identified when the inputs are driven by independent Gaussian signals. They presented an iterative algorithm for identification. Greblicki [44] identified a continuous time Wiener system. The impulse response of the linear part is recovered by a correlation method, while the nonlinear characteristic is estimated with the help of the nonparametric kernel regression method. Kalafatis *et al.* [38] proposed an iterative least squares procedure to remove the bias in the parameter estimates in the presence of severe output noise. Zhu [45] studied the identification of MISO Wiener models and extended the so called ASYM method providing systematic solutions to the problems of identification for both open-loop and closed-loop tests. The method is used to identify two distillation columns. Ikonen and Najim [46] suggested a Wiener model structure for the identification of MISO steady-state static systems with linear unit steady-state gain output error (OE) dynamics for each input. Duwaish *et al.* [47] proposed an

identification model which uses a hybrid model consisting of a linear ARMA model in cascade with a MFNN. A two step procedure is proposed to estimate the linear and the nonlinear parts separately.

2.1.3 Wiener-Hammerstein model

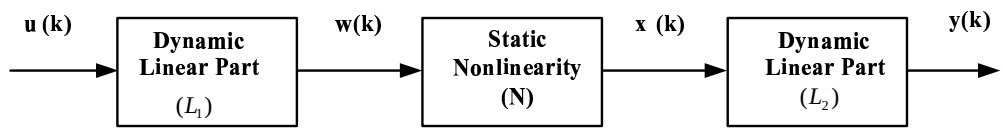

Figure 2.3: The Wiener-Hammerstein model

Fig. 2.3 is a block diagram of the well known Wiener-Hammerstein model or the General model (G-model), which consists of a linear system L_1 (with impulse response $h_1(k)$) in cascade with a static zero-memory nonlinear element N and a linear system L_2 (with impulse response $h_2(k)$). Many chemical and other industrial processes may be represented by this type [48].

For a SISO system, the Box-Jenkins model of the Wiener-Hammerstein nonlinear system, shown in Fig. 2.3, can be represented as:

$$\begin{aligned} y(k) &= \frac{D(q^{-1})}{C(q^{-1})} \sum_{i=1}^{p} c_i g_i \Big[\frac{B(q^{-1})}{A(q^{-1})} u(k) \Big] \\ &= \frac{D(q^{-1})}{C(q^{-1})} \ \mathbf{N} \Big[\frac{B(q^{-1})}{A(q^{-1})} u(k) \Big] \\ &= \frac{D(q^{-1})}{C(q^{-1})} \mathbf{N}[w(k)] \end{aligned}$$

$$= \frac{D(q^{-1})}{C(q^{-1})}x(k) \tag{2.10}$$

Where $g_i(\bullet)$ are the nonlinear functions used to describe the nonlinear zero memory subsystem $N(\bullet)$ of order p and k is an integer time index. A typical choice for the functions $g_i(\bullet)$ is $g_i(w(k)) = w^i(k)$, in which case the nonlinear subsystem is said to be a block of polynomial nonlinearity. The pair of polynomials $(B(q^{-1}), A(q^{-1}))$ and $(D(q^{-1}), C(q^{-1}))$ are related to the LTI blocks L_1 and L_2 respectively. Where the polynomials $B(q^{-1})$ and $A(q^{-1})$ are as given in 2.5 and 2.4, respectively and the polynomials $D(q^{-1})$ and $C(q^{-1})$ are given such that:

$$D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_p q^{-p} \tag{2.11}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_q q^{-q} \tag{2.12}$$

Very few efforts have been made to study the *Wiener-Hammerstein* model (or the G-model) when compared to the work done for the Hammerstein and the Wiener models [5]. Billings and Fakhouri [48] proposed an identification algorithm for Wiener-Hammerstein model based on correlation analysis by applying the cross-correlation techniques to decouple the identification of the linear dynamics from the characterisation of the nonlinear element when the input is white Gaussian signal. Boutayeb and Darouach [49] proposed a method for recursive identification of MISO Wiener-Hammerstein model and by means of a transformation, they showed that

parameters to be estimated are those of each subsystem of the initial and unique realization. Yoshine and Ishii [50] developed a new identification method for the G-model in the discrete time domain, based on the input-output causality and further developed a new formula for the estimation of nonlinear parameters in the G-model. Talaq [14] discussed the structure identification problem in the time and the frequency domains for all the three types of the above block cascade models. Vandersteen and Shoukens [8] showed that it is possible to estimate the nonparametric frequency response functions (FRFs) of the linear dynamic elements of a Wiener-Hammerstein type of system. The identification is carried without measuring the signals over the static nonlinearity.

2.1.4 Hammerstein-Wiener model

Hammerstein-Wiener model may be considered as the system where two static nonlinear elements N_1 and N_2 surround a linear block L, as in Fig. 2.4.

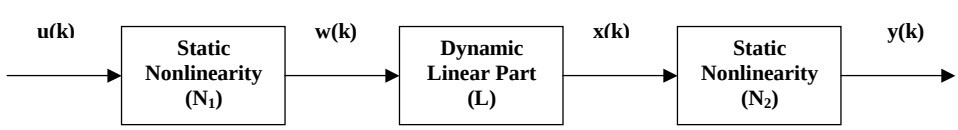

Figure 2.4: The Hammerstein-Wiener model

Here the model is given as:

$$\begin{aligned} y(k) &= \sum_{j=1}^{q} d_j f_j \left[\frac{B(q^{-1})}{A(q^{-1})} \sum_{i=1}^{p} c_i g_i \Big[u(k) \Big] \right] \\ &= \sum_{j=1}^{q} d_j f_j \left[\frac{B(q^{-1})}{A(q^{-1})} \ \mathbf{N}_1(u(k)) \right] \\ &= \sum_{j=1}^{q} d_j f_j \Big[x(k) \Big] \\ &= \mathbf{N}_2(x(k)) \end{aligned} \tag{2.13}$$

Where $g_i(\bullet)(i = 1...p)$ and $f_j(\bullet)(j = 1...q)$ are the nonlinear functions for the nonlinear blocks N_1 and N_2, respectively.

Representation for a class of Hammerstein-Wiener type of nonlinear systems is given in [1], where the discrete time nonlinear dynamic system considered is:

$$y(k) = \sum_{i=1}^{p} a_i \{ \sum_{l=1}^{q} d_l g_l [y(k-i)] \} + \sum_{j=1}^{n} b_j \{ \sum_{t=1}^{m} c_t f_t [u(k-j)] \} + e(k) \tag{2.14}$$

In (2.14), $u(k)$, $y(k)$ and $e(k)$ are the system input, output and noise at time k respectively. The $g_l(.)s$ and $f_t(.)s$ are nonlinear functions and

$$a = (a_1, ..., a_p)', \quad b = (b_1, ..., b_n)', \quad c = (c_1, ..., c_m)', \quad \text{and} \quad d = (d_1, ..., d_q)'$$

denote the system parameter vectors. The model given in (2.14) may be considered as the system where two static nonlinear elements N_1 and N_2 surround a linear

block, Fig. 2.4.

In models (2.10) and (2.13), the purpose of identification is to estimate unknown parameter vectors a,b,c and d, from the observed input-output measurements. The functions f_t $(t = 1, 2, ...m)$ and g_l $(l = 1, 2, ...q)$ are assumed to be *a priori known smooth* nonlinear functions and the orders q, n, p and m are assumed to be known as well.

Bai [1] presented an optimal two stage identification algorithm for the *Hammerstein-Wiener* nonlinear system where two static nonlinear elements surround a linear block. The algorithm consists of two steps, the first is to find the least squares (LS) estimation of the product of parameters and the second is to extract the parameters of the system. AL-Rumaih and AL-Sunni [51] proposed an identification scheme of the Bai's system [1]. The two stage algorithm developed in the literature was used to formulate the identification problem as an optimization problem possible to use the GA for optimizing.

2.2 Multiway Analysis

2.2.1 Multiway Data

Any set of data for which the elements can be arranged as

$$x_{ijk...} \qquad i = 1...I, j = 1...J, k = 1...K, ... \tag{2.15}$$

where the number of indices may vary, is a multiway array [52]. With only one index the array will be a one way or first-order array-a vector, with two indices it will be a two-way array-a matrix and with three indices it will be a three-way array-a cube.

As shown in Fig. 2.5 the data with three indices can be arranged geometrically in a box(cube).

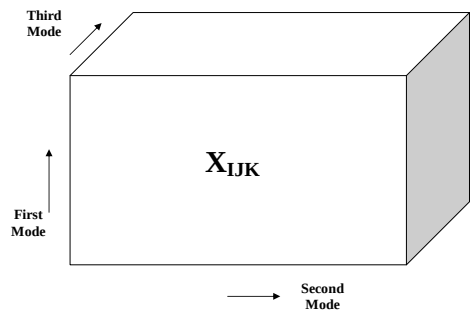

Figure 2.5: A graphical representation of a three-way data array

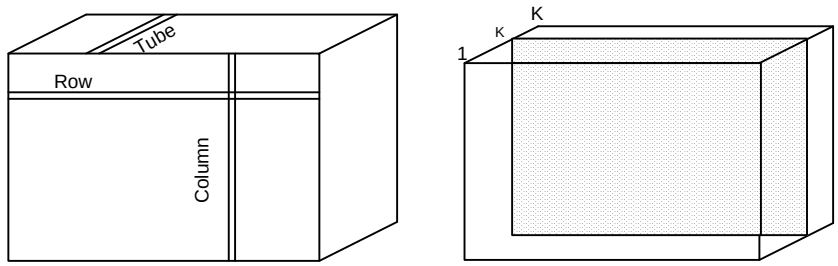

Figure 2.6: Definition of row, column and tube(left), and the k'th frontal slab(right).

For two-way matrices the terms rows and columns are used. Vectors in the third mode are called tubes, as shown in Fig. 2.6. Fixing the third mode index, the grey area is called slab, layer of slice of the array. In this case the slab is called a frontal slab as opposed to vertical and horizontal slabs.

In Multiway analysis [52], scalars are designated using lowercase italics, e.g. x, vectors are generally interpreted as column vectors and designated using bold lower case $\mathbf{x}$. Matrices are shown in bold uppercase, $\mathbf{X}$, and all higher-way arrays are shown as bold underline capitals, $\underline{\mathbf{X}}$. The characters I, J and K are reserved for indicating the dimensions of an array. A three-way array will be assumed to be of size $I \times J \times K$ and elements of $\underline{\mathbf{X}}$ by x_{ijk}

The *Multiway Analysis* is the natural extension of multivariate analysis, when data are arranged in three or higher way arrays [52]. It is a logical and advantageous tool in many different situations. This instrumental development makes it possible to obtain information that more adequately describes the intrinsic multivariate and complex reality. Bro [52] comprehensively studied the multiway Analysis. Stahle [53] developed an algorithm for linear three-way decomposition (LTD) of three-way tables for the case of one dependent block of variables and one independent, predicted block. Bro [54] presented a new multi-way regression method, called N-way partial least squares (N-PLS), given an emphasis on the three-way PLS version (tri-PLS). Smilde [55] extended it by giving more comprehensive notations and established the LS properties of the proposed multilinear PLS. De Jong [56] obtained the expressions to compute the regression coefficients. He also proposed a novel algorithm for PLS regression, which calculates the PLS factors directly as linear combinations of the original variables. De Jong [57] determined the PLS factors such as to maximize a covariance criterion while obeying certain orthogonality and normalization

restrictions . Qin [58] presented a recursive PLS regression approach for dynamic system identification. The method used, removes the correlation by projecting the original variable space to an orthogonal latent space.

2.3 Adaptive Control of Nonlinear Systems

The Adaptive control problem has attracted the attention of many researchers in the field for a long time [59]. Adaptive control systems have evolved as an attempt to avoid degradation of the dynamic performance of a control system when environmental variations occur [60]. In the adaptive identification or control, a set of parameters are adjusted so that the output of the given plant and that of a model approach each other asymtotically. Adaptive approach is an iterative probabilistic approach in which a set of parameters are iteratively updated using the current information [59]. The block diagram of an adaptive system is shown in Fig. 2.7. The

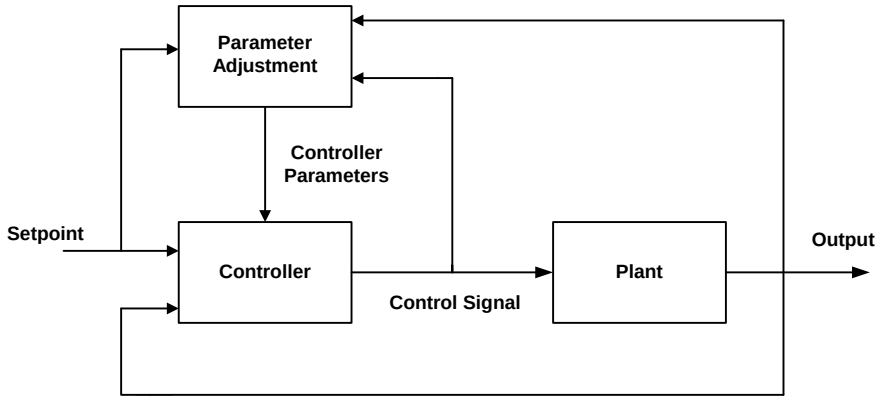

Figure 2.7: Block Diagram of an Adaptive system

adaptive control system can be thought of as having two loops. One loop is a normal feedback with the process and the controller. The other loop is the parameter adjustment loop. The parameter adjustment loop is often slower than the normal feedback loop.

2.3.1 Model Reference Adaptive Systems

Among the various types of adaptive system configurations, model reference adaptive systems (MRAS) are important since they lead to relatively easy-to-implement systems with a high speed of adaptation which can be used in a variety of situations [59]. The block diagram of a typical MRAS is shown in Fig. 2.8. The controller can

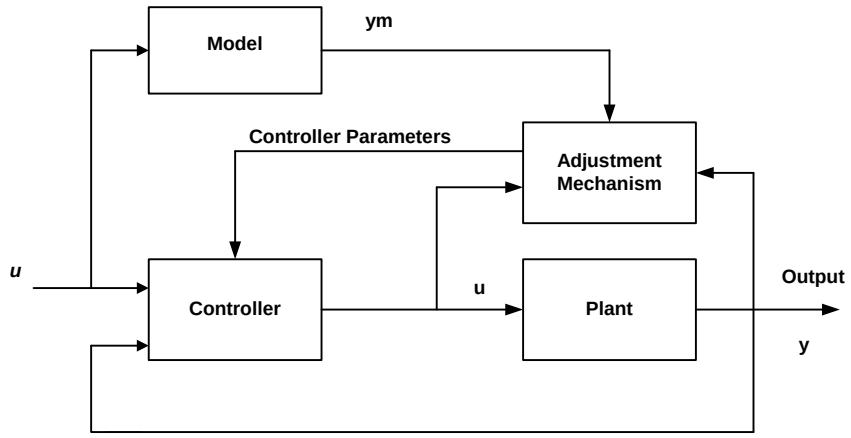

Figure 2.8: Block Diagram of a model reference adaptive system

be thought of as consisting of two loops. The inner loop is an ordinary feedback loop composed of the process and the controller. The outer loop adjusts the controller parameters in such a way that the error, which is the difference between process

output y and model output y_m, is small. This model tells how the process output ideally should respond to the command signal [10, 61].

An adaptive regulator is a regulator that can modify its behavior in response to changes in the dynamics of the process and the disturbances [10].

Ding and Xie [62] presented the inverse operator method of nonlinear systems with a nonlinearity and established the adaptive control algorithm of this type of nonlinear systems. Al-Naemi and Phillips [63] investigated the control of nonlinear plant by a conventional adaptive controller based on the identification of time varying parameters of the proposed affine model. Qin [64] presented the solution of the adaptive control of nonlinear systems with unkown(certain or uncertain) parameters, using the theory of nonlinear H_∞.

Literature discussing about the compensation and linearization of the nonlinear systems is available. Carini [65] presented a theory for the exact and the p-th order equalization or linearization of nonlinear systems with known recursive or nonrecursive polynomial input-output relationships. Fernando and Sesay [66] proposed a Hammerstein type decision feedback equalizer (HDFE) that compensates for both the nonlinear distortion and the linear dispersion of a Wiener type system. Tsimbinos and Lever [67] presented a technique for composing and inverting orthogonal representation and then using this technique they derived a nonlinear compensation method that is based on the removal of specific terms in the orthogonal series. Kang *et al.* [68] derived an adaptive algorithm for Wiener system, adjusting the pa-

rameters of the precompensator, structured by a Hammerstein model by using the stochastic gradient method. Lim *et al.* [69] discussed the linearization of a nonlinear system by connecting a nonlinear Volterra prefilter tandemly with the nonlinear system and by adaptively adjusting the coefficients of the prefilter. Bodson *et al.* [70] proposed an active noise control algorithm for sinusoidal periodic disturbances of unknown frequency using a feedback system. Sankar and Demir [71] presented a novel method of spectral estimation in the presence of sinusoidal noise interference based on structural signal representation and by using the techniques for estimating the frequencies of single and multiple sinusoids, a sinusoidal noise cancellation algorithm is developed and implemented. Shafiq [72] proposed an NLMS based stable adaptive controller for stable plants which may be non-minimum phase, using an adaptive FIR in the feedback loop. Casco [73] proposed a two step size NLMS adaptive filter algorithm in which the step size adjustment is controlled by using the square of the output error. Tsuda [74] studied and modified the NLMS algorithm so that the error sequence is used to design the step size parameter at each iteration.

Chapter 3

Recursive Identification of the Bai's [1] System

3.1 Least Squares Estimation

The least squares (LS) technique provides us with a mathematical procedure by which a model can achieve a best fit to experimental data in the sense of minimum-error-squares [4].

The linear regression is the simplest type of parametric model [2]. The corresponding model structure can be written as

$$y(k) = \varphi^T(k)\theta \tag{3.1}$$

Where $y(k)$ is a measurable quantity, $\varphi(k)$ is an n-vector of known quantities and θ is an n-vector of unknown parameters. The elements of the vector $\varphi(k)$ are often called regression variables or regressors while $y(k)$ is called the regressed variable. θ is called the parameter vector and the variable k takes integer values.

The identification problem is to find an estimate $\hat{\theta}$ of the parameter vector θ from measurements of, say, N data set points $y(1), \varphi(1), ..., y(N), \varphi(N)$. Given these measurements, a system of linear equations is obtained, namely

$$\begin{aligned} y(1) &= \varphi^T(1)\theta \\ y(2) &= \varphi^T(2)\theta \\ &\vdots \\ y(N) &= \varphi^T(N)\theta \end{aligned}$$

This can be written in the matrix notation as:

$$Y = \Phi\theta \tag{3.2}$$

Where,

$$\mathbf{Y} = \begin{pmatrix} y(1) \\ \vdots \\ y(N) \end{pmatrix}$$

$$\mathbf{\Phi} = \begin{pmatrix} \varphi^T(1) \\ \vdots \\ \varphi^T(N) \end{pmatrix}$$

Now introduce the equation error as

$$\varepsilon(k) = y(k) - \varphi^T\theta \tag{3.3}$$

and stack these in a vector ε defined as

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon(1) \\ \vdots \\ \varepsilon(N) \end{pmatrix}$$

In the statistical literature, the equation errors are often called *residuals*. The *least squares estimate* of θ is defined as the vector $\hat{\theta}$ that minimizes the loss function

$$V(\theta) = \frac{1}{2}\sum_{t=1}^{N} \boldsymbol{\varepsilon}^2(k) = \frac{1}{2}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} = \frac{1}{2}\|\boldsymbol{\varepsilon}\|^2 \tag{3.4}$$

Where $\|.\|$ denotes the Euclidean vector norm.

Now we have

$$\varepsilon = Y - \Phi\theta \tag{3.5}$$

and

$$\begin{aligned} V(\theta) &= \frac{1}{2}[Y - \Phi\theta]^T[Y - \Phi\theta] && (3.6) \\ &= \frac{1}{2}[\theta^T\Phi^T\Phi\theta - \theta^T\Phi^T Y - Y^T\Phi\theta + Y^T Y] && (3.7) \end{aligned}$$

Hence

$$\begin{aligned} V(\theta) &= \frac{1}{2}[\theta - (\Phi^T\Phi)^{-1}\Phi^T Y]^T(\Phi^T\Phi)[\theta - (\Phi^T\Phi)^{-1}\Phi^T Y] \\ &+ \frac{1}{2}[Y^T Y - Y^T\Phi(\Phi^T\Phi)^{-1}\Phi^T Y] && (3.8) \end{aligned}$$

The matrix $\Phi^T\Phi$ is by construction always nonnegative definite [2], the first term in (3.8) is always greater than or equal to zero. Thus $V(\theta)$ can be minimized by setting the first term to zero.

Setting the gradient to zero,

$$0 = \frac{dV(\theta)}{d\theta} = -Y^T\Phi + \theta^T(\Phi^T\Phi) \tag{3.9}$$

Thus the function $V(\theta)$ given in (3.6) is minimal for parameters $\hat{\theta}$, such that

$$(\Phi^T\Phi)\hat{\theta} = \Phi^T Y \tag{3.10}$$

if the matrix $\Phi^T\Phi$ is nonsingular, the minimum is unique and given by

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \tag{3.11}$$

or in an equivalent form it can be written as,

$$\hat{\theta} = \left[\sum_{k=1}^{N} \varphi(k)\varphi^T(k)\right]^{-1}\left[\sum_{k=1}^{N} \varphi(k)y(k)\right] \tag{3.12}$$

3.2 Recursive Least Squares method

The RLS method can be used for the parameter estimation of NL systems [2]. In recursive identification method, the parameter estimates are computed recursively in time. If there is an estimate $\hat{\theta}(k-1)$ based on data upto time $k-1$, then $\hat{\theta}(k)$ is computed by some simple modification of $\hat{\theta}(k-1)$.

Recursive identification methods have the following general features [2]:

- They are a central part of adaptive systems (used, for example, for control or signal processing) where the (control, filtering, etc.) action is based on the most recent model.

- Their requirement on primary memory is quite modest, since not all data are stored.

- They can be easily modified into real-time algorithms, aimed at tracking time-varying parameters.

- They can be the first step in a fault detection algorithm, which is used to find out whether the system has changed significantly.

Define $P(k) = (\Phi^T\Phi)^{-1}$, where Φ is the *regressor vector* and $P(k)$ is the variance of $\hat{\theta}(k)$.

The summarized version of RLS algorithm is given by [75]

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\varepsilon(k) \tag{3.13}$$

$$K(k) = P(k)\varphi(k) \tag{3.14}$$

$$\varepsilon = y(k) - \varphi^T(k)\hat{\theta}(k-1) \tag{3.15}$$

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{[1+\varphi^T(k)P(k-1)\varphi(k)]} \tag{3.16}$$

Where $\varphi^T(k)\hat{\theta}(k-1)$ is one step ahead predicted output and ε is the prediction error to be minimized.

The LS method used by Bai [1] is extended for the case of online RLS approach for parameter estimation. The different models of Hammerstein, Wiener and Hammerstein-Wiener are considered to test the proposed identification scheme.

The identification scheme for the Hammerstein-Wiener model (2.14), proposed in [1] is a batch estimation scheme. The algorithm consists of two stages.

Stage 1: The parameter vector $\hat{\theta}$ is obtained using the LS method for N values of the input and output data set points.

Stage 2: The SVD is used to extract the parameters from the bi-linear in parameters vector, $\hat{\theta}$.

In our proposed approach, we extend it to the case of online identification using RLS and SVD. While SVD is used in every step of recursive identification.

In sections 3.1-3.3 we present the proposed scheme for the Hammerstein-Wiener, the Hammerstein and the Wiener models, respectively.

3.3 Hammerstein-Wiener model

Consider the Hammerstein-Wiener system (2.14). Using the RLS, the parameter vector $\hat{\theta}$ is obtained in each iteration such that:

$$\hat{\theta} = (\hat{a}_1\hat{d}_1, \ldots, \hat{a}_1\hat{d}_q \ldots, \hat{a}_p\hat{d}_1, \ldots, \hat{a}_p\hat{d}_q, \hat{b}_1\hat{c}_1, \ldots, \hat{b}_1\hat{c}_m, \ldots \hat{b}_n\hat{c}_1, \ldots, \hat{b}_n\hat{c}_m)^T \tag{3.17}$$

Now $\hat{\theta}$ is splitted as:

$$\hat{\theta}_{ad} = \hat{a}\hat{d}^T = \begin{pmatrix} \hat{a}_1\hat{d}_1, \ldots\ldots, \hat{a}_1\hat{d}_q \\ \hat{a}_2\hat{d}_1, \ldots\ldots, \hat{a}_2\hat{d}_q \\ \vdots \\ \hat{a}_p\hat{d}_1, \ldots\ldots, \hat{a}_p\hat{d}_q \end{pmatrix} \tag{3.18}$$

and

$$\hat{\theta}_{bc} = \hat{b}\hat{c}^T = \begin{pmatrix} \hat{b}_1\hat{c}_1, \ldots\ldots, \hat{b}_1\hat{c}_m \\ \hat{b}_2\hat{c}_1, \ldots\ldots, \hat{b}_2\hat{c}_m \\ \vdots \\ \hat{b}_n\hat{c}_1, \ldots\ldots, \hat{b}_n\hat{c}_m \end{pmatrix} \tag{3.19}$$

Using the SVD property let $\hat{\theta}_{ad}$ and $\hat{\theta}_{bc}$ be decomposed, such that:

$$\hat{\theta}_{ad} = \sum_{i=1}^{min(p,q)} \delta_i \xi_i \zeta_i^T, \quad \text{and} \quad \hat{\theta}_{bc} = \sum_{i=1}^{min(n,m)} \sigma_i \mu_i \nu_i^T \tag{3.20}$$

Where $\xi_i(i = 1, 2, ..., p)$, $\zeta_i(i = 1, 2, ..., q)$, $\mu_i(i = 1, 2, ..., n)$, and $\nu_i(i = 1, 2, ..., m)$, are p, q, n, m-dimensional orthonormal vectors and $\delta_i(i = 1, 2, ..., min(p, q))$ and $\sigma_i(i = 1, 2, ..., min(n, m))$ are the nonzero singular values of $\hat{\theta}_{ad}$ and $\hat{\theta}_{bc}$ respectively. Now the parameter vectors can be extracted as:

$$\hat{a} = s_\xi \xi_1, \quad \hat{d} = s_\xi \delta_1 \zeta_1, \quad \hat{b} = s_\mu \mu_1, \quad \hat{c} = s_\mu \sigma_1 \nu_1 \tag{3.21}$$

Where, s_ξ and s_μ denote the sign of the first non-zero element of ξ_1 and μ_1, respectively.

Uniqueness assumption

Consider the system (2.14). Assume that ad^T and bc^T are not both zero. Moreover, assume that $\|a\|_2 = 1$ and $\|b\|_2 = 1$ ($\|.\|_2$ stands for the 2-norm) and the signs of the first nonzero elements of a and b are positive. Under the uniqueness assumption, it can be easily verified that the parametrization of system (2.14) is unique.

Convergence theorem[1]

Let the disturbance $\eta(k)$ be white with zero mean and finite variance and independent of $u(k)$. Suppose the input $u(k)$ is bounded and the regressor $\varphi(k)$ is persistently exciting (PE), i.e.

$$\alpha_2 I \;\geq\; \sum_{k=k_0}^{k_0+l_0} \varphi(k)\varphi^T(k) \;\geq\; \alpha_1 I \;>\; 0 \tag{3.22}$$

for any $k_0 \geq 0$ and some $l_0 > 0$. Then, with probability 1 as $N \to 0$,

$$\hat{a}(N) \to a, \quad \hat{b}(N) \to b, \quad \hat{c}(N) \to c, \quad \hat{d}(N) \to d.$$

3.3.1 Algorithm of the proposed method for the Hammerstein-Wiener model

The algorithm below explains the proposed method for the Hammerstein-Wiener nonlinear model.

1. Use the RLS identification scheme and obtain the parameter vector $\hat{\theta}$ for the given model.

2. Split the obtained vector $\hat{\theta}$ into two matrices $\hat{\theta}_{ad}$ and $\hat{\theta}_{bc}$ as in (3.18) and (3.19), respectively.

3. Use the SVD property to decompose the obtained matrices into their singular values and orthogonal vectors, as in (3.20).

4. Now, using the results, obtained the individual parameter vectors $\hat{a}$,$\hat{d}$,$\hat{b}$ and $\hat{c}$, as in (3.21).

5. Go to step 1, till the N data set points.

Example 3.1a

Consider a system of 12 parameters, as shown below:

$$\begin{aligned} y(k) &= \sum_{i=1}^{3}\Big[a_i\Big(d_1\cos(y(k-i)) + d_2\sin(y(k-i) + d_3\tan(y(k-i))\Big)\Big] \\ &+ \sum_{i=1}^{3}\Big[b_i\Big(c_1\sin(u(k-i)) + c_2(u(k-i)^2) + c_3\cos(u(k-i))\Big)\Big] + e(k) \end{aligned} \tag{3.23}$$

Following the method for RLS, (3.13)-(3.16) and for SVD, (3.17)-(3.20), the parameters are estimated each time for the data set $[u(k), y(k)]_{k=1}^{N}$. Where N is the maximum number of data set points obtained using the experiment. $u(k)$ is the input and is a sum of sinusoids, as explained in section 1.2, satisfying the persistent excitation (PE) condition. Table 3.1 shows the simulation results for the actual values and the end values of the estimated parameters of model (3.23). Fig. 3.1 gives the trajectories for the online convergence of the estimated parameters with dashed line plots for their respective actual/true values. The range for the plant output noise is shown on the plots.

Parameters	a_1	a_2	a_3	d_1	d_2	d_3
Actual	0.300	0.400	0.866	0.500	0.150	-0.100
Estimated	0.2972	0.3994	0.8673	0.4999	0.1524	-0.1000
Parameters	b_1	b_2	b_3	c_1	c_2	c_3
Actual	0.250	0.550	0.797	1.000	0.100	-0.250
Estimated	0.2534	0.5518	0.7945	1.0009	0.1008	-0.2425

Table 3.1: Hammerstein-Wiener system with 12 parameters: using the proposed method

In this case the system (3.23) is simulated with the LS method and the results are compared. The table below shows the estimated values of the parameters using the LS method. It is observed that our results obtained, as in the table (reftable:hwr333) are comparable to them and both the approaches gives rise to close convergence to the actual parameter values.

Parameters	a_1	a_2	a_3	d_1	d_2	d_3
Actual	0.300	0.400	0.866	0.500	0.150	-0.100
Estimated	0.2962	0.3991	0.8693	0.4991	0.1504	-0.100
Parameters	b_1	b_2	b_3	c_1	c_2	c_3
Actual	0.250	0.550	0.797	1.000	0.100	-0.250
Estimated	0.254	0.5511	0.7745	1.0005	0.101	-0.2421

Table 3.2: Hammerstein-Wiener system with 12 parameters: using LS method

Example 3.1b

The formulation is carried such that the number of parameters are 20 as given in the following system.

$$\begin{aligned} y(k) &= \sum_{i=1}^{5}\Big[a_i(d_1\sin(y(k-i)) + d_2\sec(y(k-i)) + d_3\tan(y(k-i)) + d_4\cos(y(k-i)) \\ &+ d_5\text{atan}(y(k-i)))\Big] + \sum_{i=1}^{5}\Big[b_i(c_1\text{atan}(u(k-i)) + c_2(u(k-i)^2) + c_3\cos(u(k-i)) \\ &+ c_4\tan(u(k-i)) + c_5\sin(u(k-i)))\Big] + e(k) \end{aligned} \tag{3.24}$$

For N data set points, such that N=300 with input $u(k)$, similar to as mentioned in example 3.1a, the convergence is shown in Fig. 3.2 and the end values for the estimates are given in Table 3.3.

Parameters	a_1	a_2	a_3	a_4	a_5	d_1	d_2	d_3	d_4	d_5
Actual	0.100	0.300	-0.400	0.500	0.700	0.500	0.250	-0.100	0.100	-0.250
Estimated	0.092	0.296	-0.401	0.509	0.695	0.499	0.249	-0.100	0.098	-0.254
Parameters	b_1	b_2	b_3	b_4	b_5	c_1	c_2	c_3	c_4	c_5
Actual	0.250	-0.450	0.550	-0.650	-0.316	1.000	0.100	-0.250	0.400	-0.500
Estimated	0.241	-0.423	0.541	-0.627	-0.298	1.039	0.105	-0.255	0.417	-0.524

Table 3.3: Hammerstein-Wiener system with 20 parameters

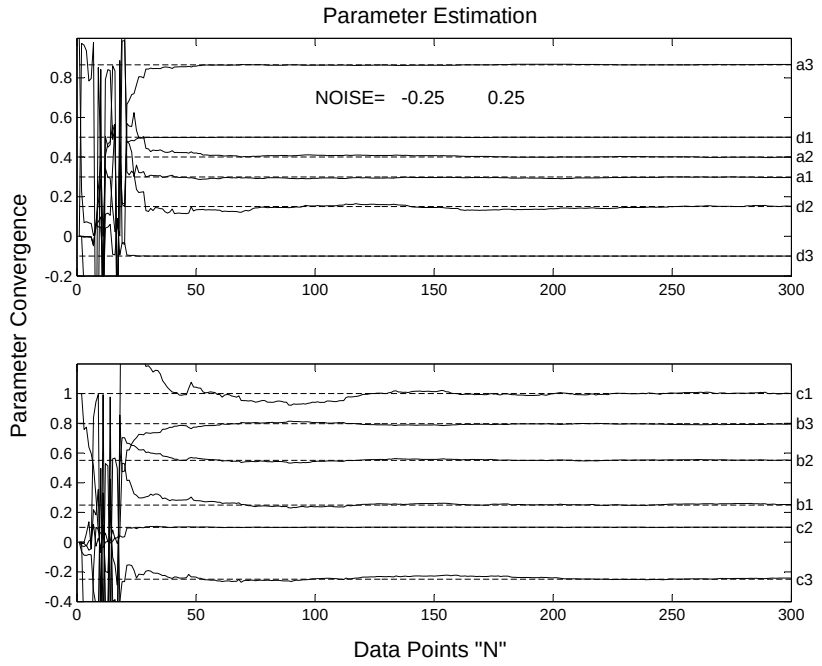

Figure 3.1: RLS with SVD for Hammerstein-Wiener system with 12 parameters

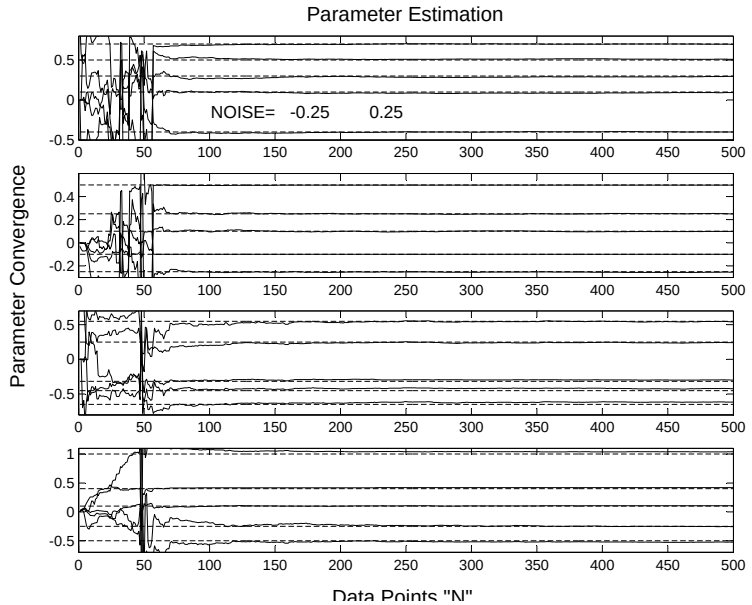

Figure 3.2: RLS with SVD for Hammerstein-Wiener system with 20 parameters

3.4 Hammerstein model

The Hammerstein Model is extracted from the system in (2.14), such that the parameter vector d does not appear [32]. Thus the system is

$$y(k) = \sum_{i=1}^{p} a_i y(k-i) + \sum_{j=1}^{n} b_j \{\sum_{t=1}^{m} c_t f_t[u(k-j)]\} + e(k) \tag{3.25}$$

Here the parameter vector is given as:

$$\hat{\theta} = (\hat{a}_1, ..., \hat{a}_p, \hat{b}_1\hat{c}_1, ..., \hat{b}_1\hat{c}_m, ..., \hat{b}_n\hat{c}_1, ..., \hat{b}_n\hat{c}_m)^T \tag{3.26}$$

Then $\hat{\theta}$ is divided as:

$$\hat{\theta}_a = \hat{a} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_p)^T \tag{3.27}$$

and

$$\hat{\theta}_{bc} = \hat{b}\hat{c}^T = \begin{pmatrix} \hat{b}_1\hat{c}_1, \ldots\ldots, \hat{b}_1\hat{c}_m \\ \hat{b}_2\hat{c}_1, \ldots\ldots, \hat{b}_2\hat{c}_m \\ \vdots \\ \hat{b}_n\hat{c}_1, \ldots\ldots, \hat{b}_n\hat{c}_m \end{pmatrix} \tag{3.28}$$

Clearly now, the SVD is to be applied only for $\hat{\theta}_{bc}$.

3.4.1 Algorithm of the proposed method for the Hammerstein model

1. Use the RLS identification scheme and obtain the parameter vector $\hat{\theta}$ for the given model.

2. Split the obtained vector $\hat{\theta}$ into a vector $\hat{\theta}_a$ and matrix $\hat{\theta}_{bc}$ as in (3.27) and (3.28), respectively.

3. Use the SVD property to decompose $\hat{\theta}_{bc}$ into it's singular values and orthogonal vectors.

4. Now with the results obtained, get individual parameter vectors $\hat{a}$,$\hat{b}$ and $\hat{c}$.

5. Go to step 1, till the N data set points.

Example 3.2a

Consider a Hammerstein system with 7 parameters. Here the SVD matrix is a square matrix of size 2x2 with the parameter vectors b and c.

$$\begin{aligned} y(k) &= a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) \\ &+ b_1\Big(c_1 u(k-1) + c_2 u(k-1)^2\Big) + b_2\Big(c_1 u(k-2) + c_2 u(k-2)^2\Big) + e(k) \end{aligned} \tag{3.29}$$

The parameter convergence is shown in Fig. 3.3 and their estimated end values in Table 3.4. Simulation results are obtained while using the input as a sum of

sinusoidal function. The noise range is shown on the plot and the data set point $N = 100$.

Parameters	a_1	a_2	a_3	b_1	b_2	c_1	c_2
Actual	1.0000	0.1000	-0.2000	0.4400	-0.8944	-0.1000	2.0000
Estimated	1.001	0.0721	-0.2001	0.4425	-0.8968	-0.1074	1.9963

Table 3.4: Hammerstein system with 7 parameters

Example 3.2b

Now, consider the system with 13 parameters as shown in the following equation.

$$\begin{aligned} y(k) &= a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3) + \sum_{i=1}^{5} \Big[b_i \Big(c_1 \mathrm{atan}(u(k-i)) + c_2 u(k-i)^2 \\ &+ c_3 \sin(u(k-i)) + c_4 \cos(u(k-i)) + c_5 \tan(u(k-i)) \Big) \Big] + e(k) \end{aligned}$$

The trajectories for convergence with the given noise range is shown in Fig. 3.4 and the end values in Table 3.5.

Parameters	a_1	a_2	a_3	b_1	b_2	b_3	b_4
Actual	1.000	0.100	-0.200	0.100	0.300	-0.400	0.500
Estimated	1.0000	0.1052	-0.1998	0.0966	0.2990	-0.4029	0.4972
Parameters	b_5	c_1	c_2	c_3	c_4	c_5	
Actual	0.700	0.500	0.780	-0.150	1.2000	-1.5000	
Estimated	0.7012	0.4938	0.7808	-0.1599	1.1935	-1.5010	

Table 3.5: Hammerstein system with 13 parameters

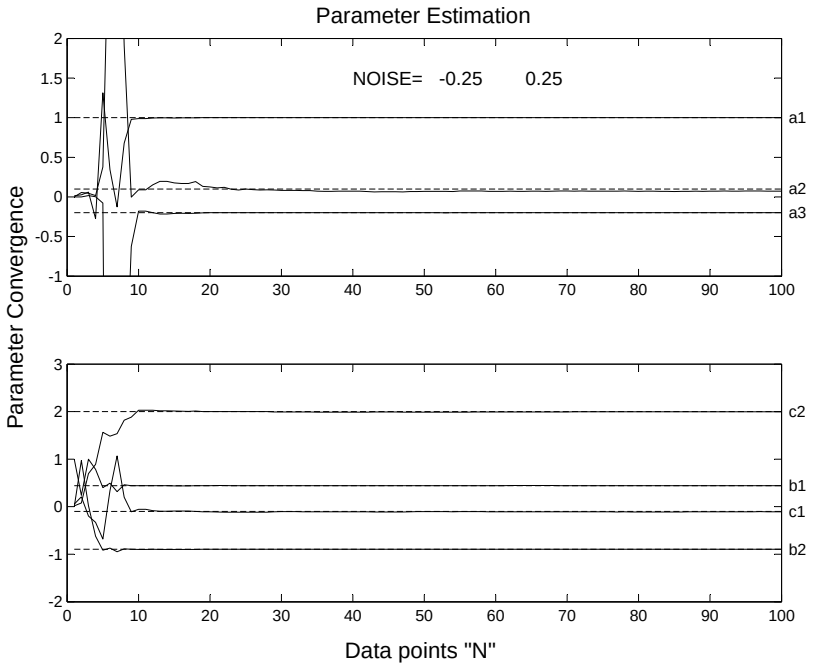

Figure 3.3: RLS with SVD for Hammerstein system with 7 parameters

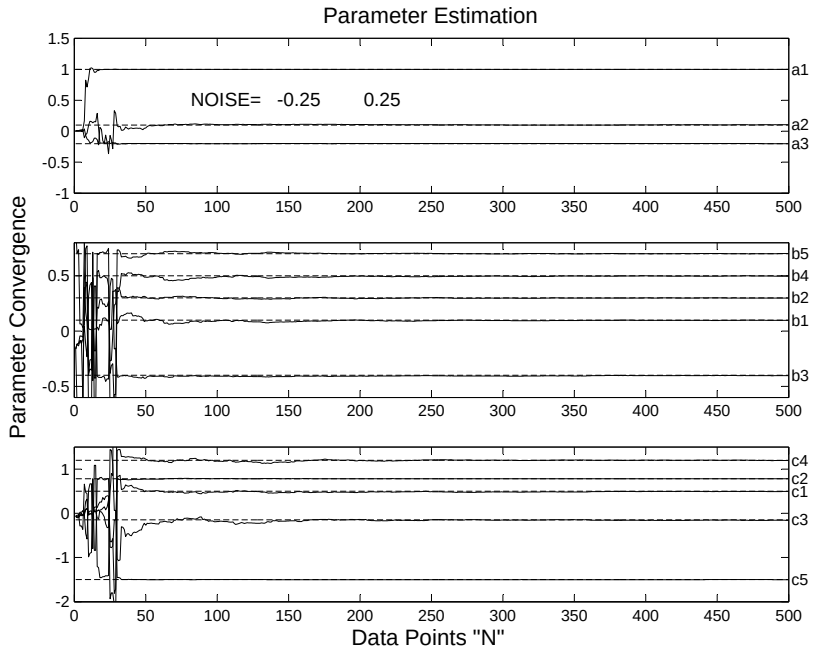

Figure 3.4: RLS with SVD for Hammerstein system with 13 parameters

3.5 Wiener model

The Wiener model is a block cascade model in which the linear block follows the zero memory nonlinear system. In this system, which is also extracted from (2.14), such that,

$$y(k) = \sum_{i=1}^{p} a_i \{\sum_{l=1}^{q} d_l g_l[y(k-i)]\} + \sum_{j=1}^{n} b_j u(k-j) + e(k) \tag{3.30}$$

Where the parameter vector c does not appear. The SVD matrix consists of the parameters a and d. Define:

$$\hat{\theta} = (\hat{a}_1\hat{d}_1, ..., \hat{a}_1\hat{d}_q, ..., \hat{a}_p\hat{d}_1, ..., \hat{a}_p\hat{d}_q, \hat{b}_1, ..., \hat{b}_n)^T \tag{3.31}$$

$$\hat{\theta}_{ad} = \hat{a}\hat{d}^T = \begin{pmatrix} \hat{a}_1\hat{d}_1, \ldots\ldots, \hat{a}_1\hat{d}_q \\ \hat{a}_2\hat{d}_1, \ldots\ldots, \hat{a}_2\hat{d}_q \\ \ldots \\ \hat{a}_p\hat{d}_1, \ldots\ldots, \hat{a}_p\hat{d}_q \end{pmatrix} \tag{3.32}$$

and

$$\hat{\theta}_b = \hat{b} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)^T \tag{3.33}$$

Here SVD is applied only for $\hat{\theta}_{ad}$:

3.5.1 Algorithm of the proposed method for the Wiener model

1. Use the RLS identification scheme and obtain the parameter vector $\hat{\theta}$ for the given model.

2. Split the obtained vector $\hat{\theta}$ into a matrix $\hat{\theta}_{ad}$ and a vector $\hat{\theta}_b$ as in (3.32) and (3.33), respectively.

3. Use the SVD property to decompose $\hat{\theta}_{ad}$ into it's singular values and orthogonal vectors.

4. Now with the results obtained, get individual parameter vectors $\hat{a}$,$\hat{d}$ and $\hat{b}$.

5. Go to step 1, till the N data set points.

Example 3.3a

The formulation is first carried for a system of 6 parameters with the SVD matrix of size 2x2 with parameter vectors a and d.

$$\begin{aligned} y(k) &= a_1\Big(d_1\cos(y(k-1)) + d_2\sin(y(k-1))\Big) + a_2\Big(d_1\cos(y(k-2)) + d_2\sin(y(k-2))\Big) \\ &+ b_1u(k-1) + b_2u(k-2) + e(k) \end{aligned} \tag{3.34}$$

The convergence is shown in Fig. 3.5 and the end values in Table 3.6.

Parameters	a_1	a_2	d_1	d_2	b_1	b_2
Actual	1.0000	-0.2000	0.5000	0.2500	0.4461	-0.8931
Estimated	0.9801	-0.1987	0.5221	0.2388	0.4453	-0.9066

Table 3.6: Wiener system with 6 parameters

Example 3.3b

Now increasing the number of parameters in the above NL system. Consider a system with 13 parameters.

$$\begin{aligned} y(k) &= \sum_{i=1}^{5} \Big[a_i \Big(d_1 \sin(y(k-i)) + d_2 \sec(y(k-i)) + d_3 \tan(y(k-i)) + d_4 \cos(y(k-i)) \\ &+ d_5 \text{atan}(y(k-i)) \Big) \Big] + b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) + e(k) \end{aligned} \tag{3.35}$$

The convergence is shown in Fig. 3.6 and the end values in Table 3.7. The convergence is quite satisfactory and the percentage error is typically $\leq 10\%$.

Parameters	a_1	a_2	a_3	a_4	a_5	d_1	d_2
Actual	0.1000	0.3000	-0.4000	0.5000	0.7000	0.5000	2.250
Estimated	0.1010	0.3014	-0.4008	0.4975	0.7006	0.5007	2.244
Parameters	d_3	d_4	d_5	b_1	b_2	b_3	
Actual	-0.1000	0.1000	-0.2500	0.2500	-0.4500	0.5500	
Estimated	-0.100	0.1063	-0.2463	0.2477	-0.4483	0.5500	

Table 3.7: Wiener system with 13 parameters

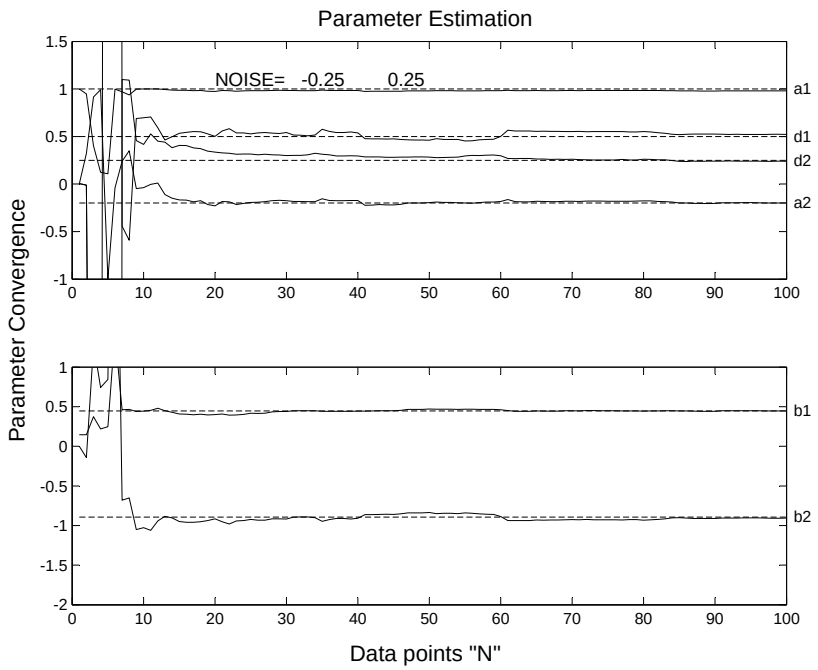

Figure 3.5: RLS with SVD for Wiener system with 6 parameters

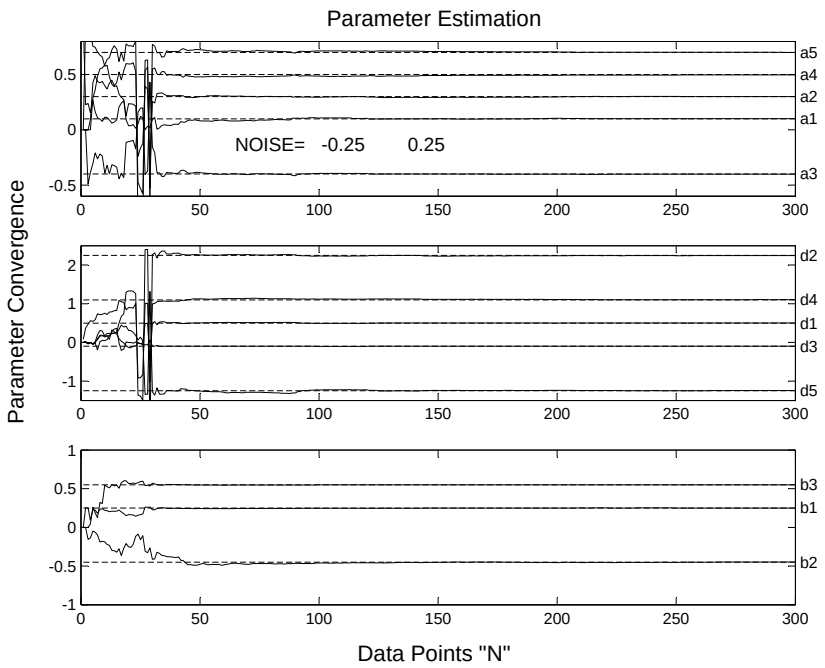

Figure 3.6: RLS with SVD for Wiener system with 13 parameters

Chapter 4

The Multiway Partial Least Squares method

The multiway partial least squares (N-PLS) method is used to estimate the parameters of the class of the discrete time NL dynamic systems mentioned in section 1.2.

In partial least squares (PLS) a regression model is formed between an independent block X and a dependent variable block y [52, 54, 53]. PLS consists of two steps : a decomposition of the calibration array (independent block X) and establishing a relation(regression) between the decomposed array of independent variables and the dependent variable(s)[54]. PLS can separate noise (irrelevant information) from the information sought for and it can also handle the correlation between the variables in the model. PLS ignores any relation of elements within blocks of interest and

only deals with the relations between blocks and is capable of investigating multiple contrasts in whole volume data.

The advantages of tri-PLS (X of order three) are manifold. The main feature of the algorithm is, that it produces score vectors that, in a tri-linear sense, have maximum covariance with the unexplained part of the dependent variable. The algorithm is easy to interpret compared to unfolding methods [53]. This is especially important when the number of variables is high.

If $\underline{\mathbf{X}}$ is a three-way matrix $(I \times J \times K)$ and y is a univariate $(I \times 1)$, with typical elements x_{ijk} and y_i respectively, $\underline{\mathbf{X}}$ is decomposed into one score vector $t(I \times 1)$ and two weight vectors $w^J(J \times 1)$ and $w^K(K \times 1)$. The general idea is to find w^J and w^K so that the covariance between t and y is maximized:

$$\max_{w^J w^K} \left[\sum_{i=1}^{t} t_i y_i \middle| t_i = \sum_{j=1}^{j} \sum_{k=1}^{k} x_{ijk} w_j^J w_k^K \quad \text{and} \quad ||w^J|| = ||w^K|| = 1 \right] \tag{4.1}$$

Now w^J and w^K are our intermediate results to obtain the regression coefficient matrix b_{NPLS}, $y(I\text{x}1)$ is the output vector. $T(I\text{x}A)$, $W^J(J\text{x}A)$ and $W^K(K\text{x}A)$ are formed with $t_i s$, $w^J s$ and $w^K s$, respectively for A components each. If $z_{jk} = y_i x_{ijk}$, then using (4.1) the weights W^J and W^K are given by:

$$\max_{w^J w^K} [(W^J)^T Z W^K] \qquad \Rightarrow \qquad (W^J, W^K) = SVD(Z) \tag{4.2}$$

Having obtained the weight matrix W, scores T is given in terms of X as:

$$T = XW \tag{4.3}$$

Regressing y on the component scores T gives

$$\hat{y} = Tb \tag{4.4}$$

with

$$b = (T^T T)^{-1} T^T y \tag{4.5}$$

Combining (4.3) and (4.4)

$$\hat{y} = XWb \tag{4.6}$$

Hence the regression coefficients vector b_{NPLS} is given as:

$$b_{NPLS} = Wb \tag{4.7}$$

In this work we use the multiway partial least squares (N-PLS) regression method for identification of a class of nonlinear systems. PLS is a method for building regression models between independent variable block X and dependent variable y [76]. PLS regression is able to give a robust solution in case of collinear or correlated input variables, where the ordinary least squares regression gives rise to ill-conditioned

problem [58].

The *tri-PLS* can be expressed as the method of finding suitable unit-length weight vectors w^J and w^K [52] that minimize the covariance between the score vector t and the dependent variable y.

We use the method proposed by Smilde [55], to find the regression coefficients (parameters of the nonlinear model). We extend it by using the RLS along with SVD for the same approach and plot each time the convergence of the regression coefficients by dividing the interval after every few steps. Different nonlinear models such as Hammerstein, Wiener and Hammerstein-Wiener are considered for simulation. The results are plotted which shows convergence of the parameters for the same class of dynamic models. The results show close resemblance to the actual values of the parameters.

4.1 Proposed approach

Our approach now can be divided in two stages as under:

1. For any instant in time $k \in [1, N]$, assume input to the system as zero while the only excitation signal is the noise $e(k)$. Using this *zero input response* or the *natural response*, estimate the parameters related to the output regressor terms. Now for estimation, RLS is used in case of the Hammerstein system and the N-PLS is used in case of the Wiener and the Hammerstein-Wiener

systems.

2. Pick the estimated values of the parameters. A new output vector can be designed using difference of the actual output at any instant in time k and the terms of the estimated parameters in the true model. Using the input u and the so formed output with a different plant output noise, the remaining parameters can be obtained by applying RLS in case of Wiener system and N-PLS in case of Hammerstein and Hammerstein-Wiener systems.

The flow chart in Fig. 4.1 describes the procedure for different class of nonlinear systems and the proposed approach is explained in case of a Hammerstein-Wiener system as in (4.8)-(4.11).

4.2 Proposed approach for the Hammerstein-Wiener model

Consider the Hammerstein-Wiener nonlinear dynamic system (2.14).

- Assuming first the zero input condition, we have an AR model of the form

$$y(k) = \sum_{i=1}^{p} a_i \{\sum_{l=1}^{q} d_l g_l[y(k-i)]\} + e(k) \tag{4.8}$$

Defining the three way independent variable block X as $X_{lki} = g_l[y(k-i)]$ use the N-PLS method (4.1)-(4.7). Having obtained the regression coefficient

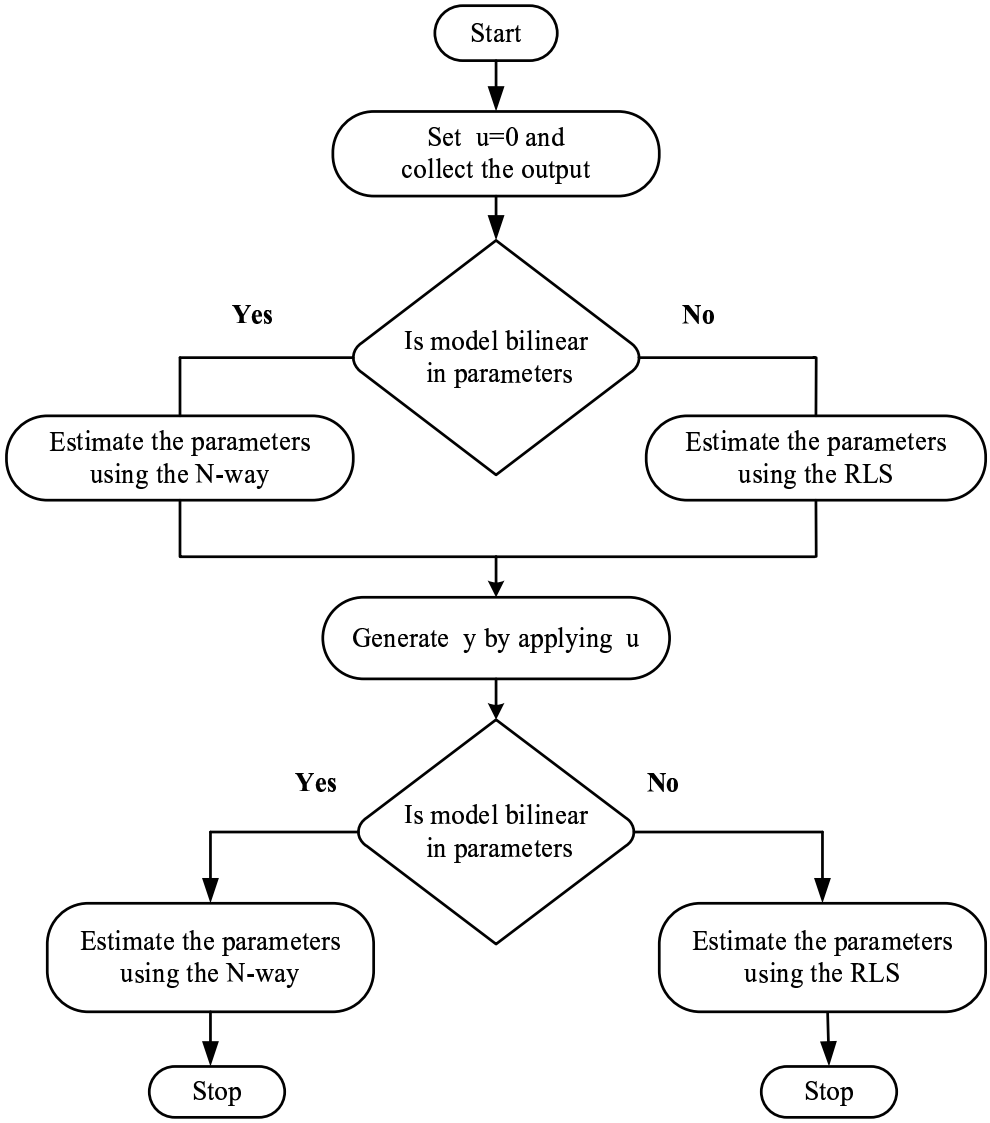

Figure 4.1: Flow chart describing the approach for different nonlinear systems such as the Hammerstein, the Wiener and the Hammerstein-Wiener

vector (4.7), define:

$$\hat{b}_{NPLS_{\hat{a}\hat{d}}} = \hat{a}\hat{d}^T = \begin{pmatrix} \hat{a}_1\hat{d}_1, \hat{a}_1\hat{d}_2, \ldots\ldots, \hat{a}_1\hat{d}_q \\ \hat{a}_2\hat{d}_1, \hat{a}_2\hat{d}_2, \ldots\ldots, \hat{a}_2\hat{d}_q \\ \ldots \\ \ldots \\ \hat{a}_p\hat{d}_1, \hat{a}_p\hat{d}_2, \ldots\ldots, \hat{a}_p\hat{d}_q \end{pmatrix} \tag{4.9}$$

Use the SVD [77]:

$$\hat{b}_{NPLS_{\hat{a}\hat{d}}} = \sum_{i=1}^{min(p,q)} \delta_i \xi_i \zeta_i^T \quad \Rightarrow \quad \hat{a} = s_\xi \xi_1 \quad \text{and} \quad \hat{d} = s_\xi \delta_1 \zeta_1$$

Where $\xi_i (i = 1, 2, ..., p)$, and $\zeta_i (i = 1, 2, ..., q)$, are p, q-dimensional orthonormal vectors, s_ξ denote the sign of the first non-zero element of ξ_1 and $\delta_i (i = 1, 2, ..., min(p, q))$ are the nonzero singular values of $\hat{b}_{NPLS_{\hat{a}\hat{d}}}$.

- Now using the estimated parameters formulate the problem as:

$$y(k) - \sum_{i=1}^{p} \hat{a}_i \{\sum_{l=1}^{q} \hat{d}_l g_l[y(k-i)]\} = \sum_{j=1}^{n} b_j \{\sum_{t=1}^{m} c_t f_t[u(k-j)]\} + \eta(k) \tag{4.10}$$

$$\text{or,} \qquad yn(k) = \sum_{j=1}^{n} b_j \{\sum_{t=1}^{m} c_t f_t[u(k-j)]\} + \eta(k) \tag{4.11}$$

Where $yn(k)$ is the formulated output. Apply the N-PLS method (4.1)-(4.7) again and define:

$$\hat{b}_{NPLS_{\hat{b}\hat{c}}} = \hat{b}\hat{c}^T = \begin{pmatrix} \hat{b}_1\hat{c}_1, \hat{b}_1\hat{c}_2, \ldots\ldots, \hat{b}_1\hat{c}_m \\ \hat{b}_2\hat{c}_1, \hat{b}_2\hat{c}_2, \ldots\ldots, \hat{b}_2\hat{c}_m \\ \ldots \\ \ldots \\ \hat{b}_n\hat{c}_1, \hat{b}_n\hat{c}_2, \ldots\ldots, \hat{b}_n\hat{c}_m \end{pmatrix} \tag{4.12}$$

Using the SVD:

$$\hat{b}_{NPLS_{\hat{b}\hat{c}}} = \sum_{i=1}^{min(n,m)} \sigma_i \mu_i \nu_i^T \qquad \Rightarrow \qquad \hat{b} = s_\mu \mu_1, \quad \text{and} \quad \hat{c} = s_\mu \sigma_1 \nu_1,$$

Where $\mu_i (i = 1, 2, ..., n)$, $\nu_i (i = 1, 2, ..., m)$, are n, m-dimensional orthonormal vectors, s_μ denote the sign of the first non-zero element of μ_1 and $\sigma_i (i = 1, 2, ..., min(n, m))$ are the nonzero singular values of $\hat{b}_{NPLS_{\hat{b}\hat{c}}}$.

4.2.1 Algorithm of the proposed method for the Hammerstein-Wiener model

1. Obtain the zero input response for the given model.

2. Define the three way independent variable block X and use the N-PLS method to obtain the parameter matrix $\hat{a}\hat{d}^T$.

3. Use the SVD method and obtain the parameter vectors $\hat{a}$ and $\hat{d}$.

4. Now apply the input $u(k)$ to the plant, formulate the N-PLS problem again while using the results obtained in step 3.

5. Use the SVD method again and obtain the parameter vectors $\hat{b}$ and $\hat{c}$.

Different examples similar to the ones in (3.23) and (3.24) are considered in formulation. The trajectories for the parameter convergence is shown in Figs. 4.2-4.3 and for the corresponding examples, the values of the parameters, at the instant, while the mismatch error norm of the actual parameter vector θ and the estimated parameter vector $\hat{\theta}$ is the least, are given in tables 4.1-4.2. Similar plotting and tabulation is obtained for the Hammerstein models and the Wiener models. The range for the plant output noise is given in each plot.

Parameters	a_1	a_2	d_1	d_2	b_1	b_2	c_1	c_2
Actual	0.440	0.898	-0.150	-0.320	0.302	-0.953	-0.270	0.450
Estimated	0.4572	0.8894	-0.2054	-0.3250	0.2642	-0.9645	-0.2573	0.4374

Table 4.1: Hammerstein-Wiener system with 8 parameters

Parameters	a_1	a_2	a_3	d_1	d_2	d_3
Actual	0.5800	-0.6700	0.4634	0.3000	0.0100	-0.8660
Estimated	0.5797	-0.6700	0.4636	0.3005	0.0099	-0.8666
Parameters	b_1	b_2	b_3	c_1	c_2	c_3
Actual	0.2000	-0.4500	0.8703	0.2000	-0.1500	0.0400
Estimated	0.1969	-0.4734	0.8585	0.1984	-0.1523	0.0209

Table 4.2: Hammerstein-Wiener system with 12 parameters

4.3 Proposed approach for the Hammerstein model

Consider the Hammerstein model

$$y(k) = \sum_{i=1}^{p} a_i y(k-i) + \sum_{j=1}^{n} b_j \{\sum_{t=1}^{m} c_t f_t[u(k-j)]\} + e(k) \tag{4.13}$$

Here following the same two stage procedure as above, we apply the RLS in the first stage to arrive with the estimates of vector a, use it to form the new output and then apply the N-PLS for the rest of the coefficient vectors b and c. Different examples similar to the ones in (3.29) and (3.30) are considered in formulation. The parameter convergence is as shown in Figs. 4.4-4.5 and the values of the parameters at which the mismatch of the parameter vector θ and $\hat{\theta}$ is the least are given in tables 4.3-4.4.

Parameters	a_1	a_2	b_1	b_2	c_1	c_2
Actual	1.0000	0.5000	0.4400	-0.8944	0.9998	0.0194
Estimated	0.9998	0.5232	0.4305	-0.9026	1.0009	0.0178

Table 4.3: Hammerstein system with 6 parameters

Parameters	a_1	a_2	a_3	b_1	b_2	b_3
Actual	1.0000	0.5000	-0.2000	0.5000	0.3500	-0.4500
Estimated	0.9125	0.4974	-0.2047	0.5438	0.3602	-0.4151
Parameters	b_4	c_1	c_2	c_3	c_4	
Actual	0.6519	0.0500	-0.2000	0.4000	0.2500	
Estimated	0.6343	0.0265	-0.1701	0.3765	0.2367	

Table 4.4: Hammerstein system with 11 parameters

4.4 Proposed approach for the Wiener model

Consider the Wiener system, as mentioned in (4.14), where the parameters to be estimated are the vectors a, d and b.

$$y(k) = \sum_{i=1}^{p} a_i \{\sum_{l=1}^{q} d_l g_l[y(k-i)]\} + \sum_{j=1}^{n} b_j u(k-j) + e(k) \tag{4.14}$$

In this case apply the N-PLS in the first stage and then the RLS in the next stage of the proposed algorithm. Here, examples similar to the ones in (3.34) and (3.35) are considered in formulation. The parameter convergence is as shown in Figs. 4.6-4.7 and their values at the instant of least mismatch error for θ and $\hat{\theta}$ are given in tables 4.5-4.6.

Parameters	a_1	a_2	d_1	d_2	b_1	b_2	b_3
Actual	0.4400	-0.8980	0.9731	-0.3020	-0.2000	0.4500	0.8000
Estimated	0.5040	-0.8637	1.0140	-0.3338	-0.2128	0.4742	0.8361

Table 4.5: Wiener system with 7 parameters

Parameters	a_1	a_2	a_3	d_1	d_2	d_3	b_1	b_2	b_3
Actual	0.580	-0.670	0.464	0.300	0.010	-0.866	0.200	-0.450	0.874
Estimated	0.5799	-0.6700	0.4635	0.1302	0.0113	-0.8666	0.1998	-0.4514	0.8684

Table 4.6: Wiener system with 9 parameters

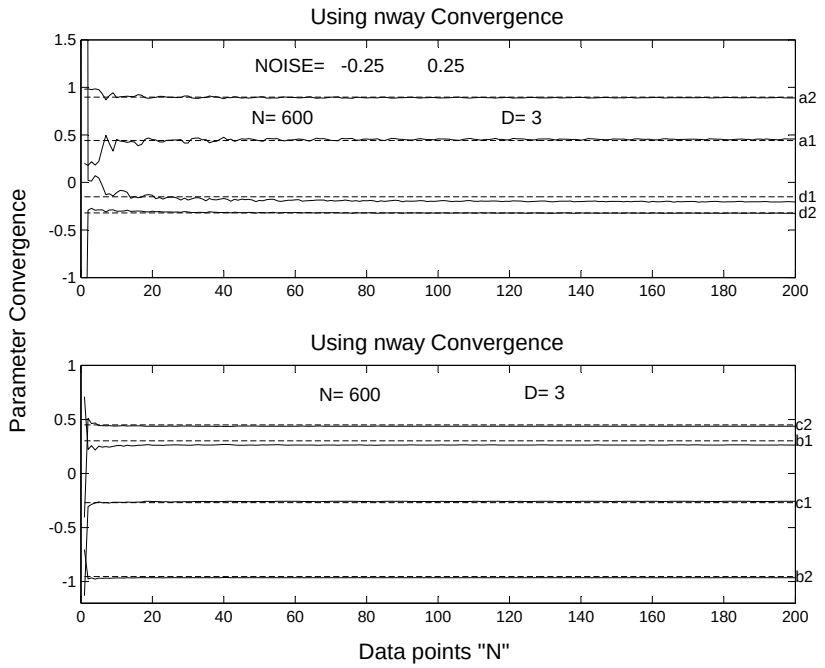

Figure 4.2: N-PLS for Hammerstein-Wiener model with 8 parameters

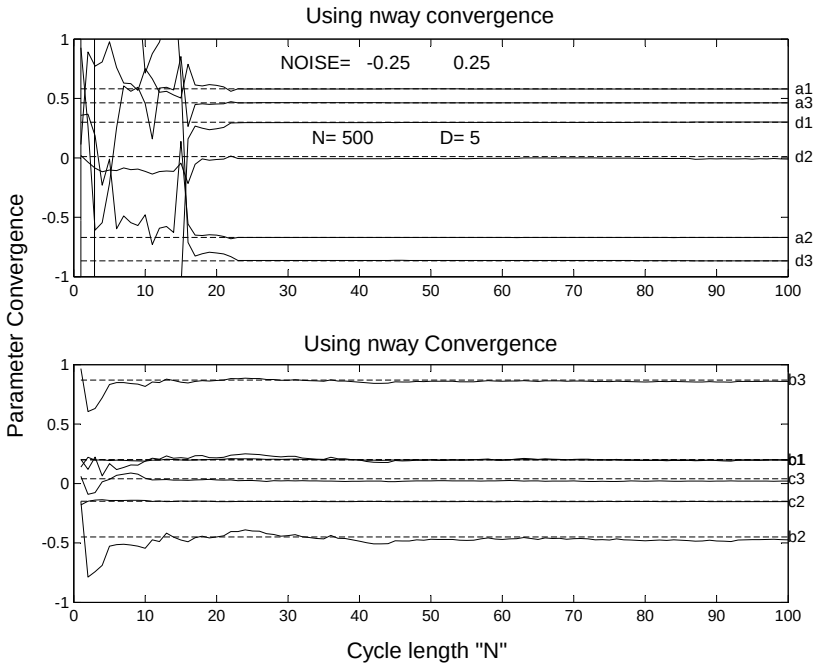

Figure 4.3: N-PLS for Hammerstein-Wiener model with 12 parameters

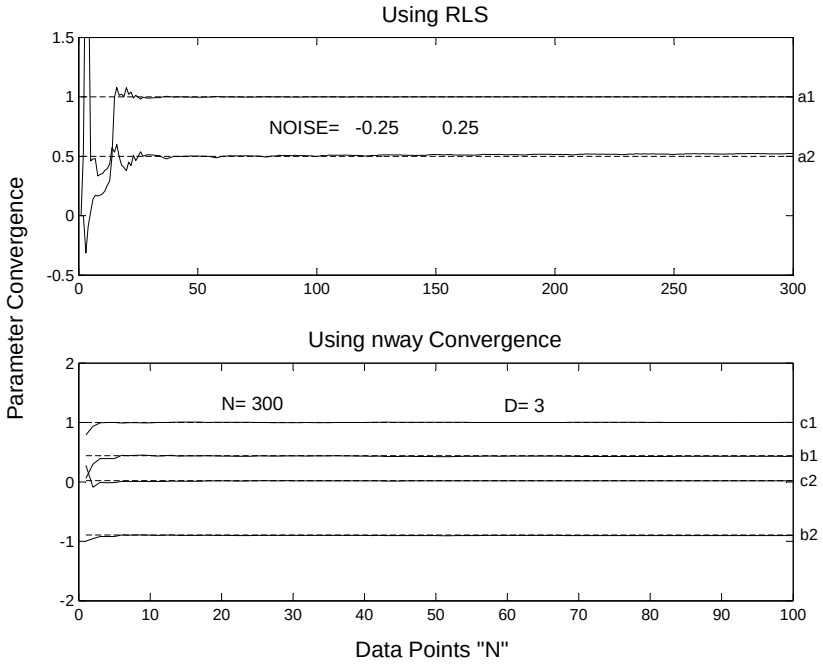

Figure 4.4: N-PLS for Hammerstein model with 6 parameters

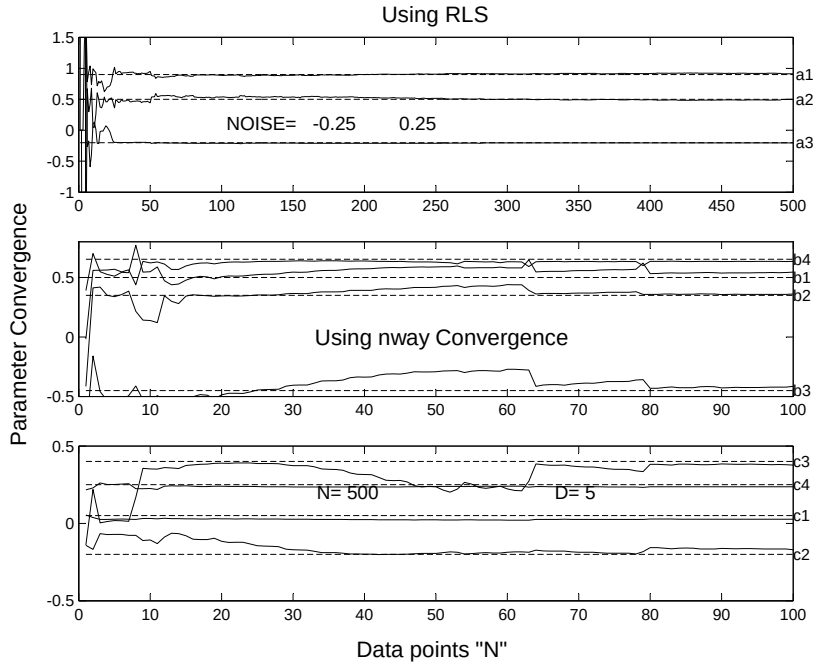

Figure 4.5: N-PLS for Hammerstein model with 11 parameters

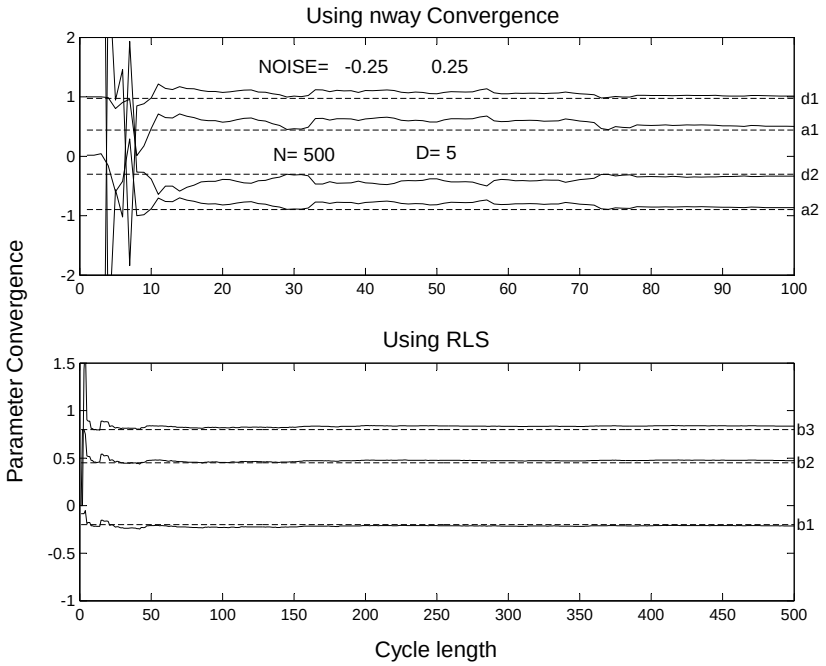

Figure 4.6: N-PLS for Wiener model with 7 parameters

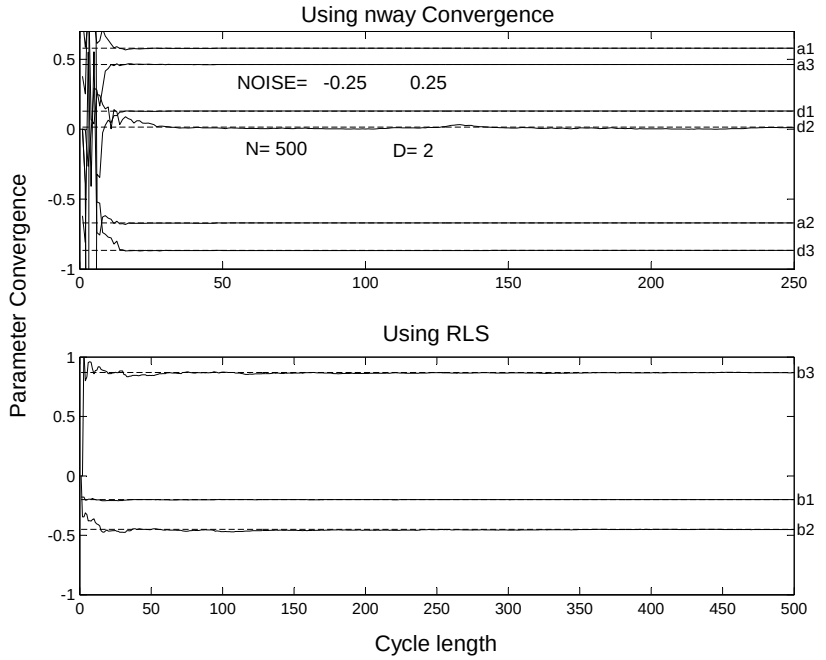

Figure 4.7: N-PLS for Wiener model with 9 parameters

Chapter 5

Adaptive Control Schemes

5.1 An approximate inverse based adaptive control

We develop an adaptive control scheme for a class of nonlinear systems. The class of systems belongs to Hammerstein, Wiener and Hammerstein-Wiener nonlinear systems. The technique developed is presented and examples are given in illustration.

It is desired that the output of the overall system follow a desired signal. The desired signal will be injected at the precompensator side and using an approximate inverse of the system, the system is forced to track the desired signal. The block diagram for the proposed adaptive control scheme is as shown in Fig. 5.1 and the flow chart in Fig. 5.2.

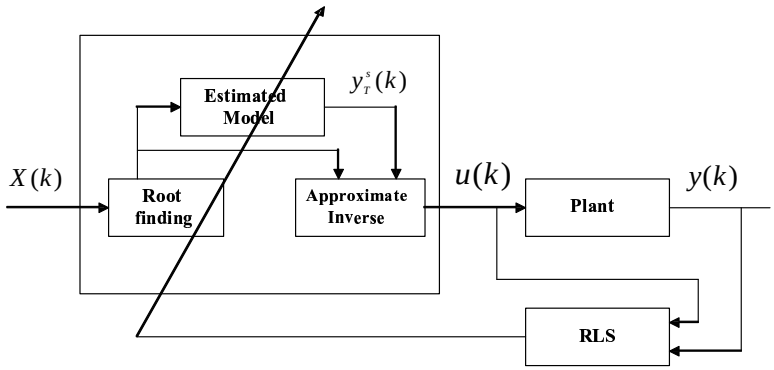

Figure 5.1: Block diagram for the precompensator technique

Consider the Hammerstein-Wiener model (2.14) and consider the inverse problem of finding $u(k)$ to achieve a given $X(k)$. Given the estimates $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ at the precompensator, the description is given by:

$$X(k) = \sum_{i=1}^{p} \hat{a}_i \{\sum_{l=1}^{q} \hat{d}_l g_l[X(k-i)]\} + \sum_{j=1}^{n} \hat{b}_j \{\sum_{t=1}^{m} \hat{c}_t f_t[u(k-j)]\} \tag{5.1}$$

The terms with the regressors of $X(k)$ are known and can be used to obtained a function in terms of the unknowns $u(k-j)$ for $j = 1...n$. This is given by:

$$X(k) - \sum_{i=1}^{p} \hat{a}_i \{\sum_{l=1}^{q} \hat{d}_l g_l[X(k-i)]\} = \sum_{j=1}^{n} \hat{b}_j \{\sum_{t=1}^{m} \hat{c}_t f_t[u(k-j)]\} \tag{5.2}$$

This is now solved for $u(k)$. Since the solution amounts to the problem of finding the roots of (5.2), we need to find among the roots, the root that will result in a value closest to the given $X(k)$ signal.

To do this, the roots are substituted into the model with the actual parameters:

$$y_T^s(k) = \sum_{i=1}^{p} a_i \{\sum_{l=1}^{q} d_l g_l[y_T^s(k-i)]\} + \sum_{j=1}^{n} b_j \{\sum_{t=1}^{m} c_t f_t[u(k-j)]\} \tag{5.3}$$

Where s stands for the number of the root. Now a root $u_s(k-j)$ is selected such that it gives closest value of the output $y_T^s(k)$ to the injected signal $X(k)$. The pair of parameters are updated using the RLS.

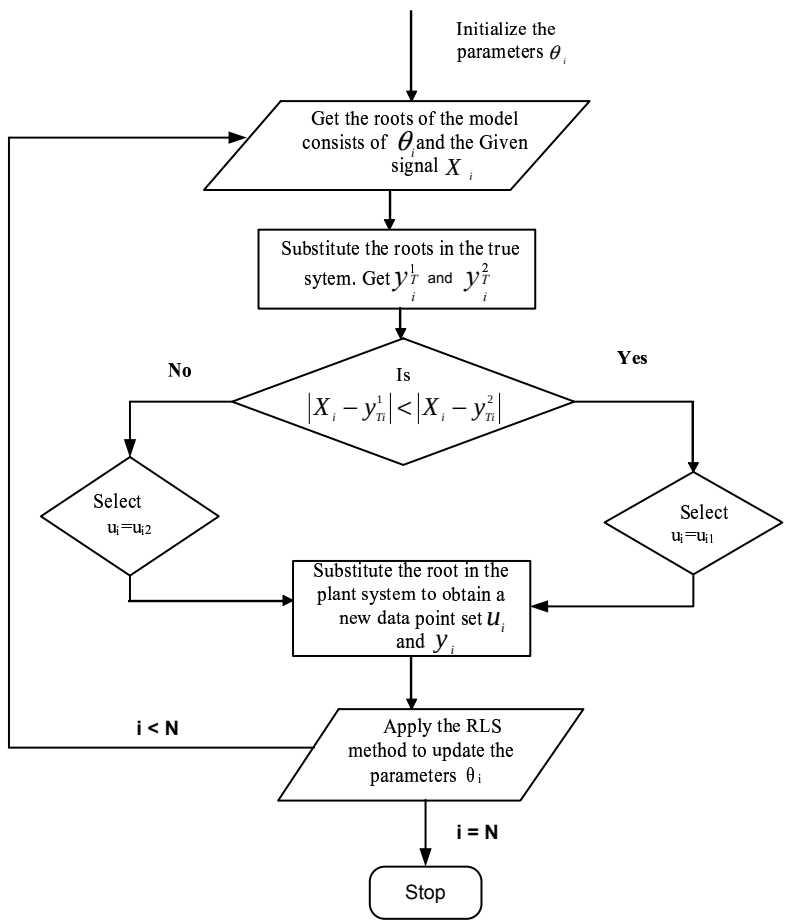

Figure 5.2: Flow chart for the precompensator technique with a second order polynomial nonlinear function

5.1.1 The proposed adaptive control scheme

The flow chart in Fig. 5.2 describes the proposed adaptive control scheme.

1. The desired output, $X(k)$ is injected into the inverse system, the precompensator block with some initialized estimates of the parameters.

2. Now the problem at the precompensator side is a root finding problem. Calculate the roots, the input to the actual system.

3. Substitute the obtained values in the true system and obtain the output $y_T^s(k)$ for each of the $'s'$ roots.

4. Compare $y_T^s(k)$ to the injected signal $X(k)$. A root is selected such that it gives the output value $y_T^s(k)$ closest to $X(k)$.

5. Using the input-output pair $u(k)$ and $y(k)$, apply the RLS method to update the parameters.

6. The procedure is carried for N data points and the formulation is obtained.

5.1.2 Proposed algorithm for the Hammerstein-Wiener model

1. Using the desired output, find the output of the precompensator block, which is a root finding problem.

2. Select the best of the roots using the plant model.

3. Using the obtained controller signal $u(k)$ and the plant output, the parameters are updated using the RLS.

4. The updated parameters are provided to the precompensator block and the procedure is carried online.

5.1.3 Examples

Example 5.1.3a

Consider the Hammerstein-Wiener system:

$$\begin{aligned} y(k) &= a_1\Big(d_1\cos(y(k-1)) + d_2\sin(y(k-1))\Big) + a_2\Big(d_1\cos(y(k-2)) + d_2\sin(y(k-2))\Big) \\ &+ b_1\Big(c_1u(k-1) + c_2u(k-1)^2\Big) + e(k) \end{aligned} \tag{5.4}$$

This is a nonlinear system with 7 parameters. Given the desired output, we can re-arrange (5.4) to get

$$\begin{aligned} X(k) &- \hat{a}_1\Big(\hat{d}_1\cos(X(k-1)) + \hat{d}_2\sin(X(k-1))\Big) - \hat{a}_2\Big(\hat{d}_1\cos(X(k-2)) + \hat{d}_2\sin(X(k-2))\Big) \\ &= b_1\Big(c_1u(k-1) + c_2u(k-1)^2\Big) \end{aligned} \tag{5.5}$$

We need to solve this for $u(k)$ and the obtained roots are substituted into the true model to decide on the value of $u(k)$ which provides $y(k)$ closest to $X(k)$.

While considering the zero noise case the results for the end values of the parameters are as shown in Table 5.1 and the plots for parameter convergence, tracking

and mismatch are shown in Figs. 5.3-5.4. In case of the noise in range [-0.15 0.15] the end values at N iterations are shown in Table 5.2, the parameter convergence plot in Fig. 5.5 and the mismatch error in Fig. 5.6.

In case of zero noise, as shown in Figs. 5.3-5.4, it is observed that the tracking is fast and perfect, and the parameter convergence is good. While in case of noise, for both the examples, the effect of noise appearing in tracking of the plant dynamics. The sinusoidal case in example 5.1.3b has less steady state error when compared to the square wave case in example 5.1.3a.

Parameters	a_1	a_2	d_1	d_2	b_1	c_1	c_2
Actual	0.8944	-0.44	0.4375	0.6614	1	-0.1042	0.005
Estimated	0.8973	-0.4414	0.4368	0.6596	1	-0.1041	0.005
% Error	0.33	0.31	0.16	0.42	0	0.05	0.02

Table 5.1: Hammerstein-Wiener system with 7 parameters: Zero noise

Parameters	a_1	a_2	d_1	d_2	b_1	c_1	c_2
Actual	0.8944	-0.44	0.4375	0.6614	1	-0.1042	0.005
Estimated	0.8923	-0.4514	0.3926	0.707	1	-0.1063	0.0052
% Error	0.23	2.58	10.27	10.42	0	2.02	3.64

Table 5.2: Hammerstein-Wiener system with 7 parameters: noise range [-0.15 0.15]

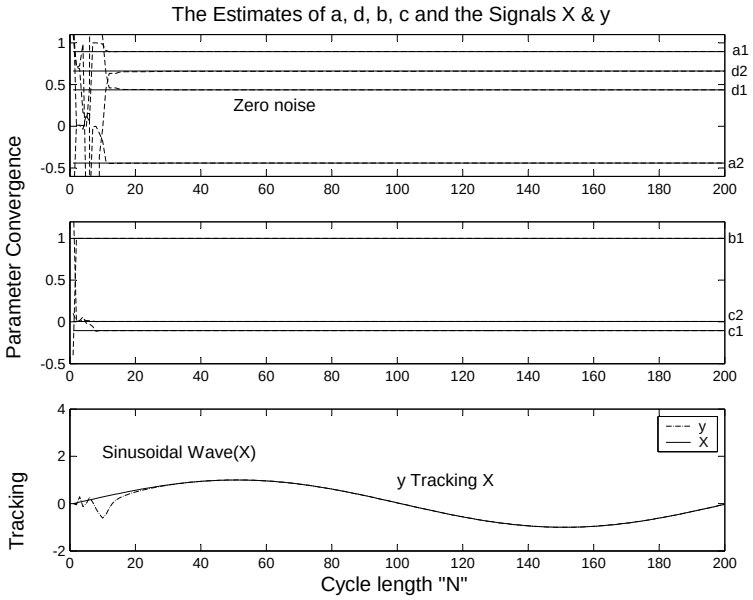

Figure 5.3: Parameter convergence and tracking signal for Hammerstein-Wiener model with 7 parameters, with zero noise: Example 5.1.3a

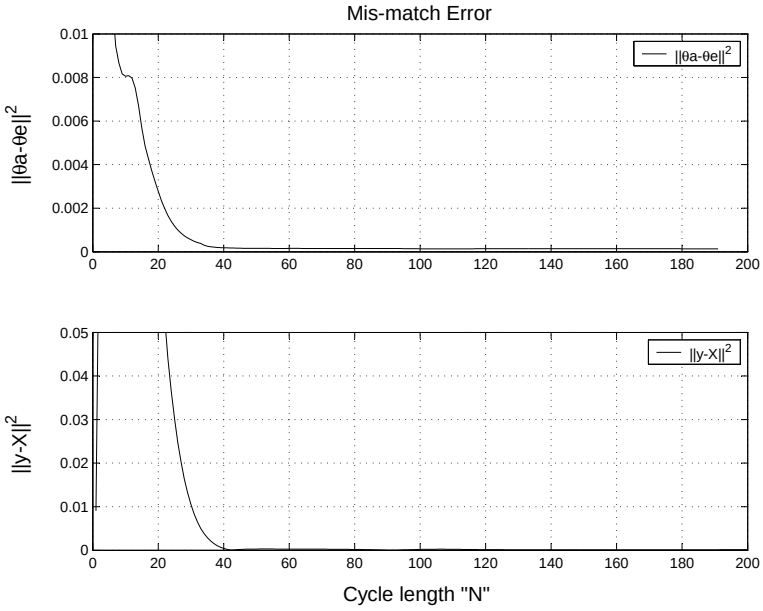

Figure 5.4: Mis-match error with zero noise: Example 5.1.3a

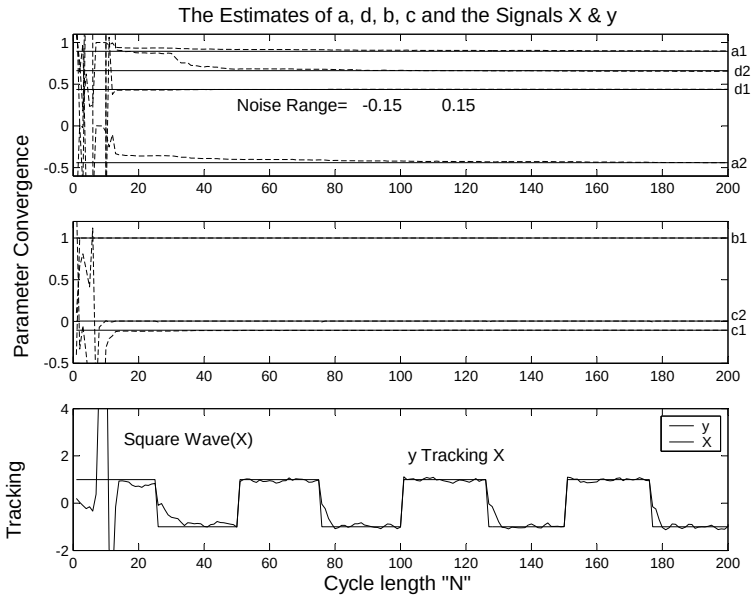

Figure 5.5: Parameter convergence and tracking signal for Hammerstein-Wiener model with 7 parameters, with noise=[-0.15 0.15]: Example 5.1.3a

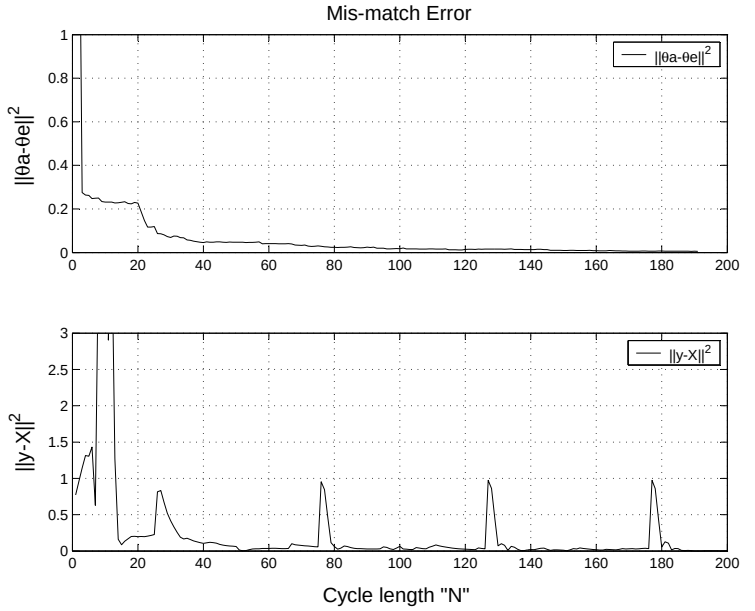

Figure 5.6: Mis-match error with noise=[-0.15 0.15]: Example 5.1.3a

Example 5.1.3b

Consider now the nonlinear system given below with 8 parameters.

$$\begin{aligned} y(k) &= a_1\Big(d_1\tan(y(k-1)) + d_2\sin(y(k-1)) + d_3\cos(y(k-1))\Big) \\ &+ a_2\Big(d_1\tan(y(k-2)) + d_2\sin(y(k-2)) + d_3\cos(y(k-2))\Big) \\ &+ b_1\Big(c_1u(k-1) + c_2\sin(u(k-1))\Big) + e(k) \end{aligned} \tag{5.6}$$

Parameters	a_1	a_2	d_1	d_2	d_3	b_1	c_1	c_2
Actual	0.8944	-0.44	0.4375	0.8883	0.14	1	-0.1042	0.05
Estimated	0.8956	-0.4449	0.4301	0.9036	0.1557	1	-0.1041	0.0481
% Error	0.13	1.12	1.69	1.72	11.21	0	0.09	3.85

Table 5.3: Hammerstein-Wiener system with 8 parameters and noise range [-0.15 0.15]: Example 5.1.3b

Here the end values at N iterations are shown in Table 5.3 for noise in the range [-0.15 0.15]. The parameter convergence is shown in Fig. 5.7 and the mis-match error in Fig. 5.8. Thus it can be inferred from the above two examples that the tracking is achieved and the overall system tracks the reference signal X, precisely.

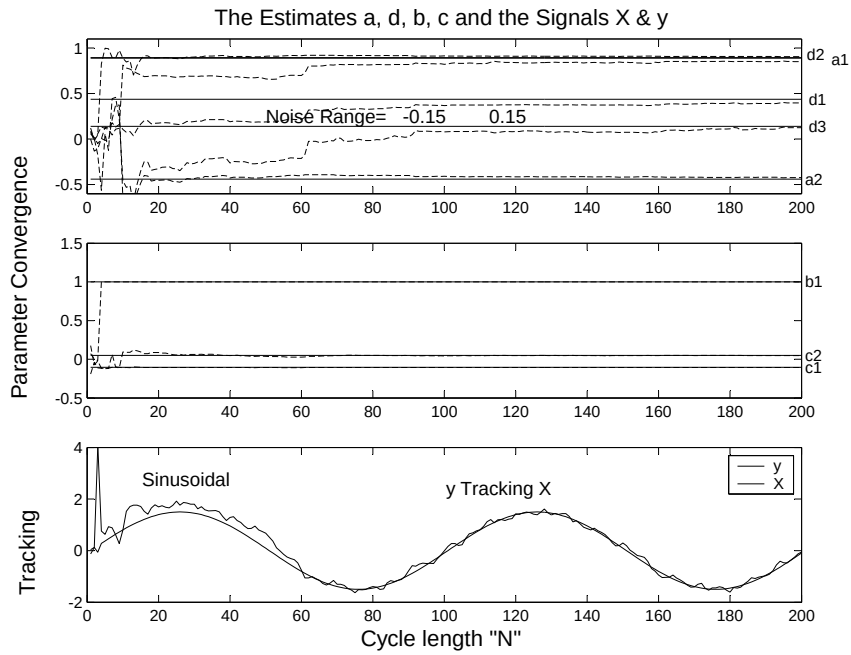

Figure 5.7: Parameter convergence and tracking signal for Hammerstein-Wiener model with 8 parameters, with noise=[-0.15 0.15]: Example 5.1.3b

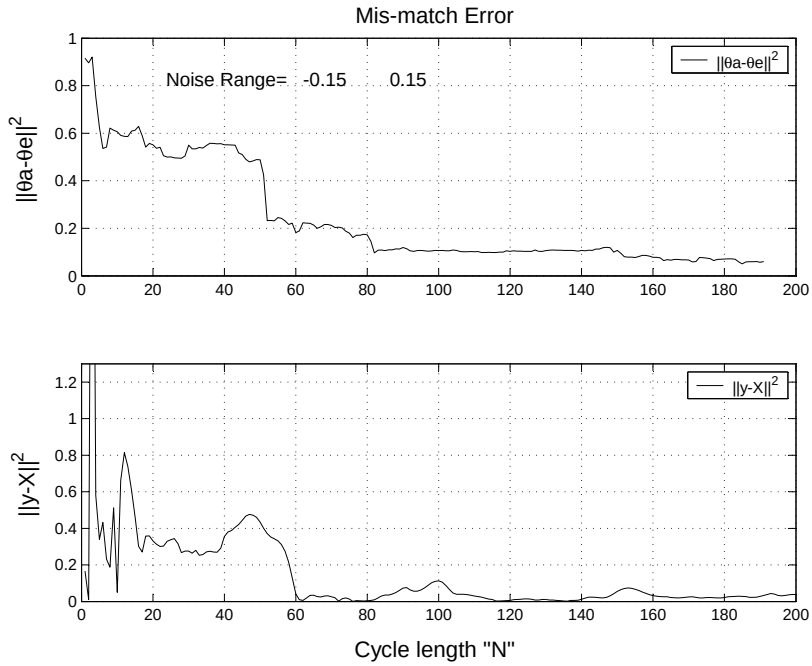

Figure 5.8: Mis-match error with noise=[-0.15 0.15]: Example 5.1.3b

5.2 Noise estimation based Adaptive Control

The elimination of noise and the extraction of signal are important issues in adaptive control techniques and is a subject of much scientific research in recent years. Literature is available discussing about some particular type of noise, such as a step noise and a sinusoidal noise [70]-[71] with practical examples.

Our scheme mentioned before, scheme 1, is enhanced here by using the feedback loop for the estimate of the plant noise. This scheme can be summarized as follows:

1. The plant P is identified using the online identification scheme.

2. The compensator block is updated based upon the information from the online identification for the plant with noise $e(k)$.

3. An estimate of the plant noise is obtained using the actual plant output $y(k)$ and an identified plant output $\hat{y}(k)$.

4. The compensator block is provided with the information of the plant noise using the estimate of the noise and thus the reference signal $y_r(k)$ is tracked.

Fig. 5.9 shows the block diagram for the technique used. Where $\hat{P}$ is the estimated plant model. Different types of noise signals such as step, sinusoidal and random are used for simulation.

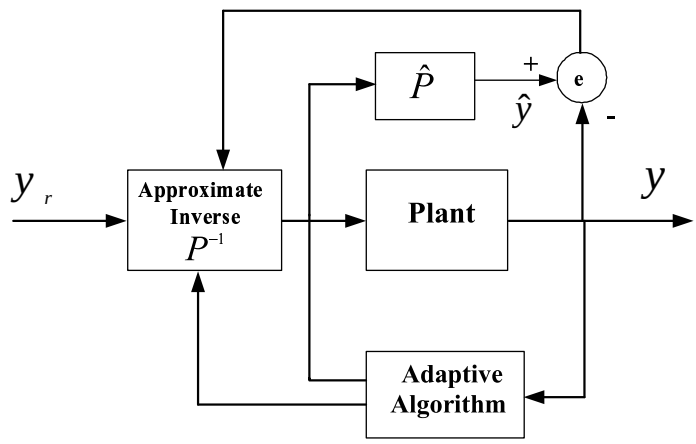

Figure 5.9: Block diagram of the adaptive compensation technique with the feedback of the estimated plant noise

5.2.1 An Example

Consider a Hammerstein model with 6 parameters, as shown below:

$$y(k) = a_1 y(k-1) + a_2 y(k-2)$$
$$+b_1\Big(c_1 u(k-1) + c_2 u(k-1)^2\Big) + b_2\Big(c_1 u(k-2) + c_2 u(k-2)^2\Big) + e(k) \quad (5.7)$$

Simulations are carried for different conditions of noise such as no noise, step noise, sinusoidal noise and random noise. The results are plotted each time for the parameter convergence, the reference signal, the plant output, the control signal and the mismatch performance. Results are compared for the case while not using the noise estimates in the FB loop, scheme 1 (section 5.1).

The plots in Figs. 5.10-5.11 show the noise free performance of the plant in the

example above. Figs. 5.12-5.13 and Figs. 5.16-5.17 show the performance for scheme 1 with step noise and sinusoidal noise respectively. While Figs. 5.14-5.15 and 5.18-5.19 show the performance for the scheme with FB of estimated noise (scheme 2), with step noise and sinusoidal noise respectively. Comparing the results obtained, it can be inferred that an improved performance, hence better tracking is obtained in scheme 2, while we use the information of the estimated noise in the FB loop. The effect of the above mentioned noises still remain with the plant output in scheme 1 while it is almost all removed in scheme 2 because of the added information of the plant noise. In case of the random noise, this scheme does not perform well, as shown in Figs. 5.20-5.21. The tracking is not so perfect and the absolute value plot for the mismatch error for the reference $y_r(k)$ and the plant output $y(k)$ has a higher range than the absolute value plot for the plant noise $e(k)$. Hence the corresponding variance values, shown in Fig. 5.21 has a higher value for the mismatch.

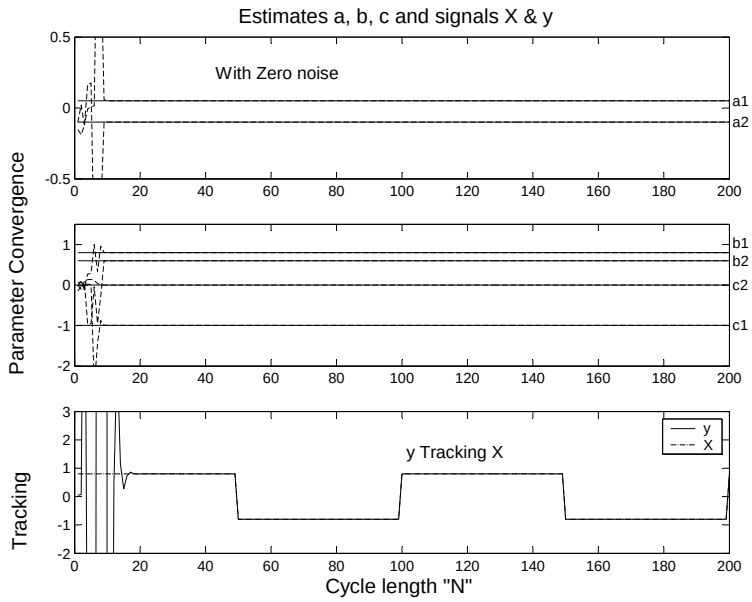

Figure 5.10: Parameters and tracking: model with the estimated noise in the feedback , in case of zero noise

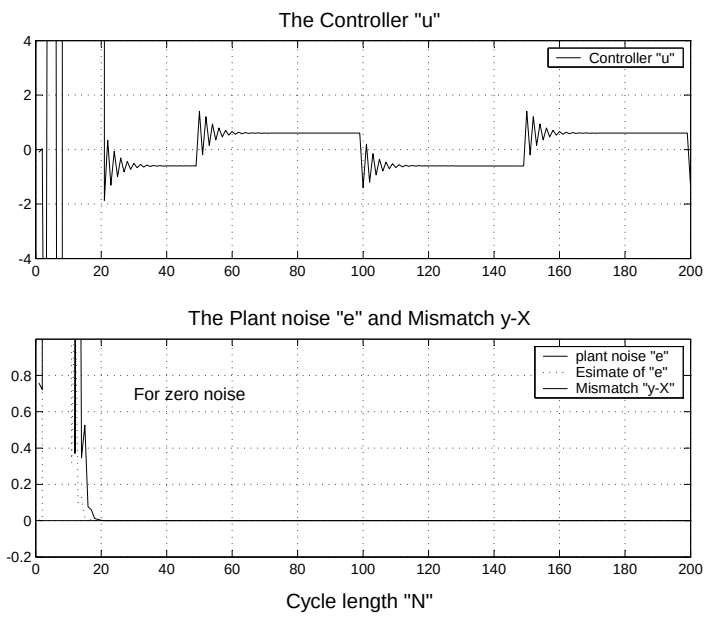

Figure 5.11: Controller u and Mismatch performance: model with the estimated noise in the feedback , in case of zero noise

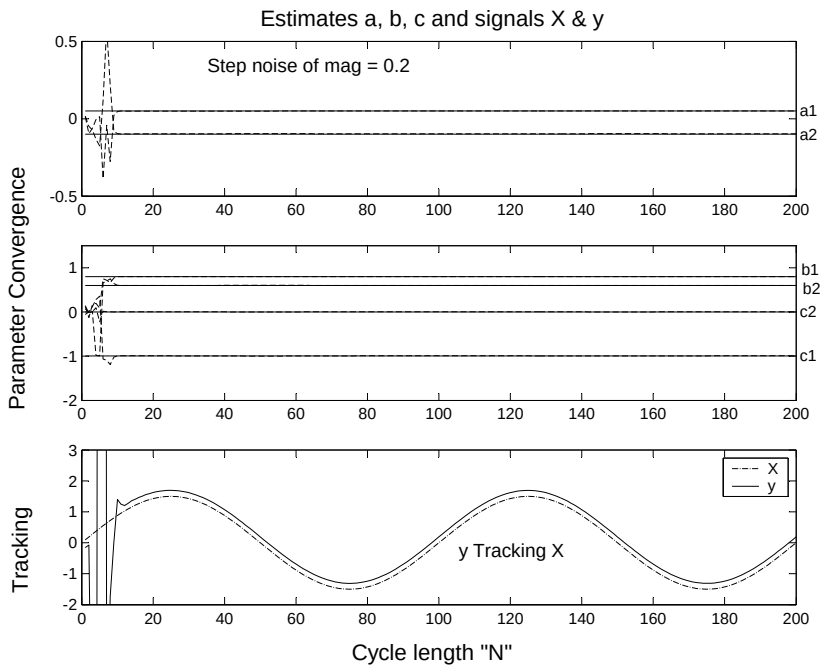

Figure 5.12: Parameters and tracking: model with out the estimated noise in the feedback , in case of a step noise

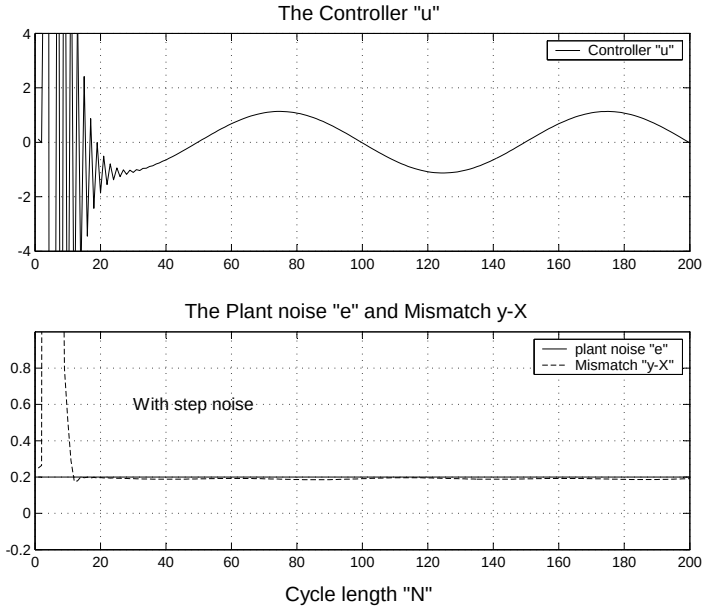

Figure 5.13: Controller u and Mismatch performance: model with out the estimated noise in the feedback , in case of a step noise

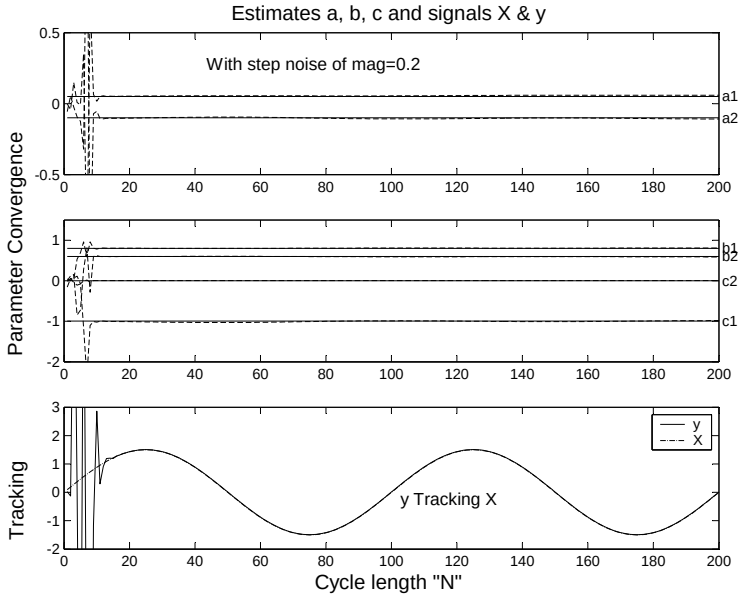

Figure 5.14: Parameters and tracking: model with the estimated noise in the feedback , in case of a step noise

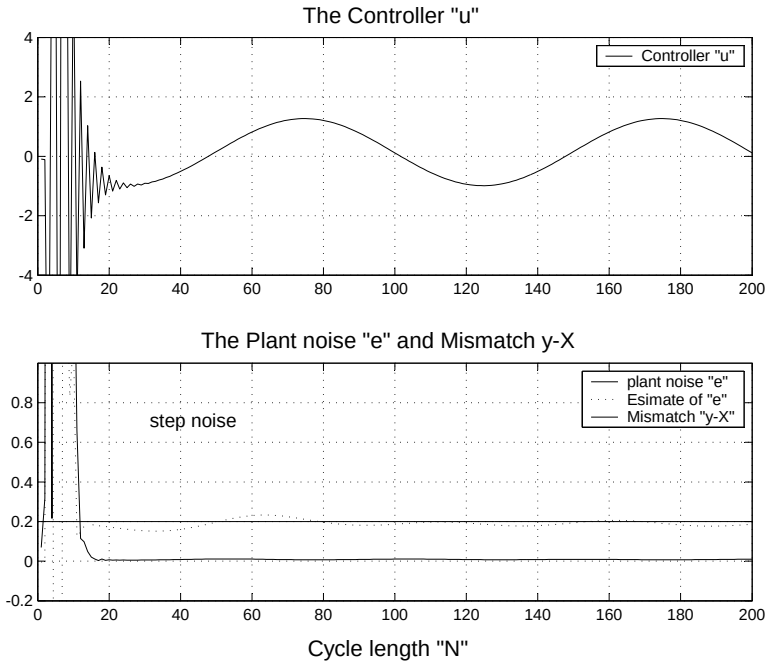

Figure 5.15: Controller u and Mismatch performance: model with the estimated noise in the feedback, in case of a step noise

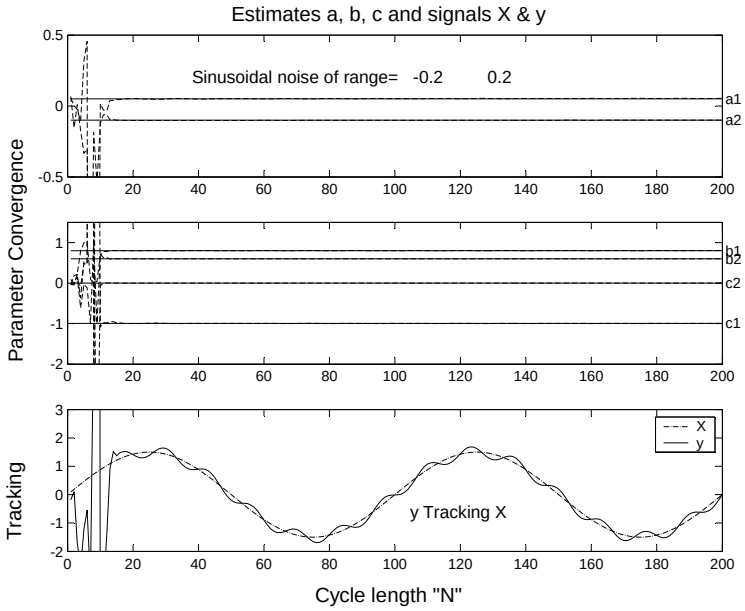

Figure 5.16: Parameters and tracking: model with out the estimated noise in the feedback , in case of a sinusoidal noise

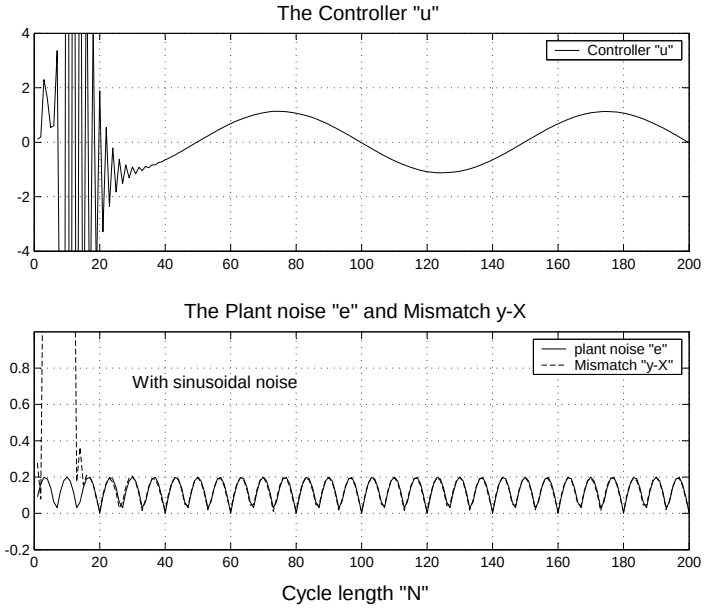

Figure 5.17: Controller u and Mismatch performance with absolute values plot: model with out the estimated noise in the feedback, in case of a sinusoidal noise

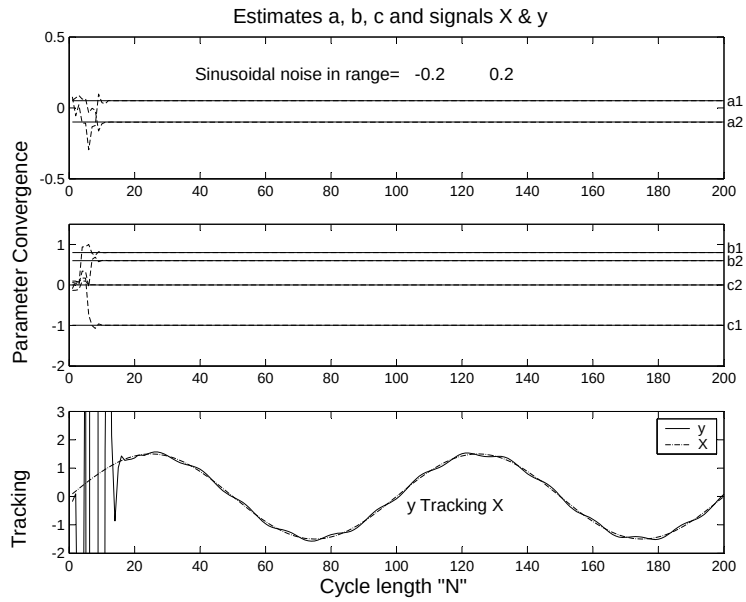

Figure 5.18: Parameters and tracking: model with the estimated noise in the feedback , in case of a sinusoidal noise

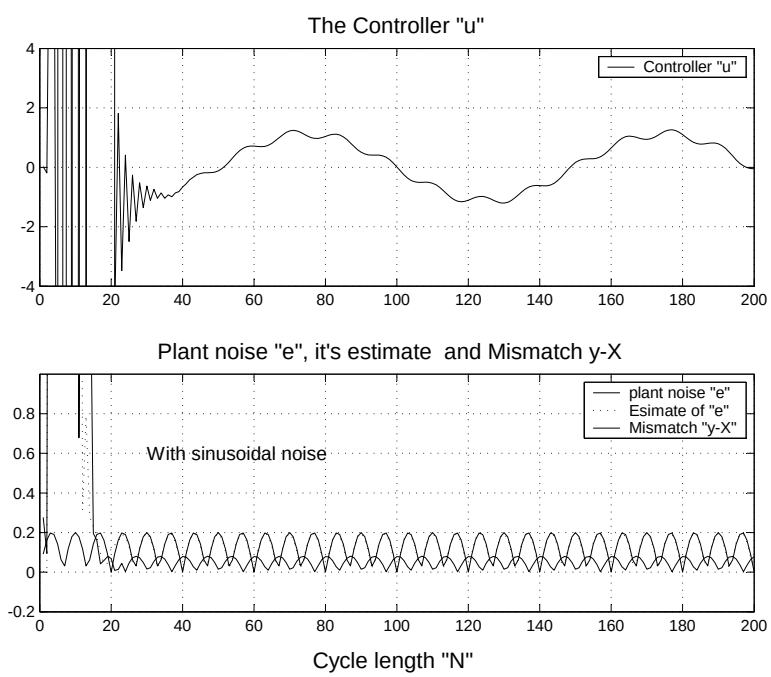

Figure 5.19: Controller u and Mismatch performance with absolute values plot: model with the estimated noise in the feedback, in case of a sinusoidal noise

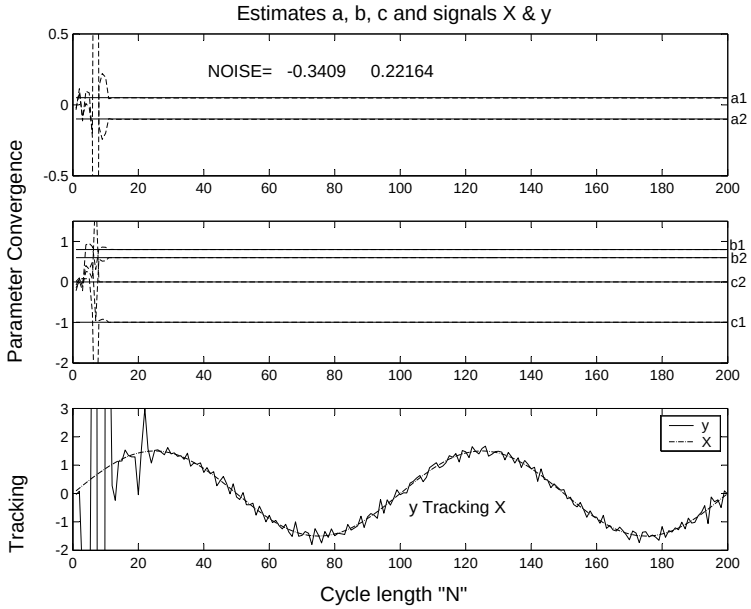

Figure 5.20: Parameters and tracking: model with the estimated noise in the feedback , in case of a random noise

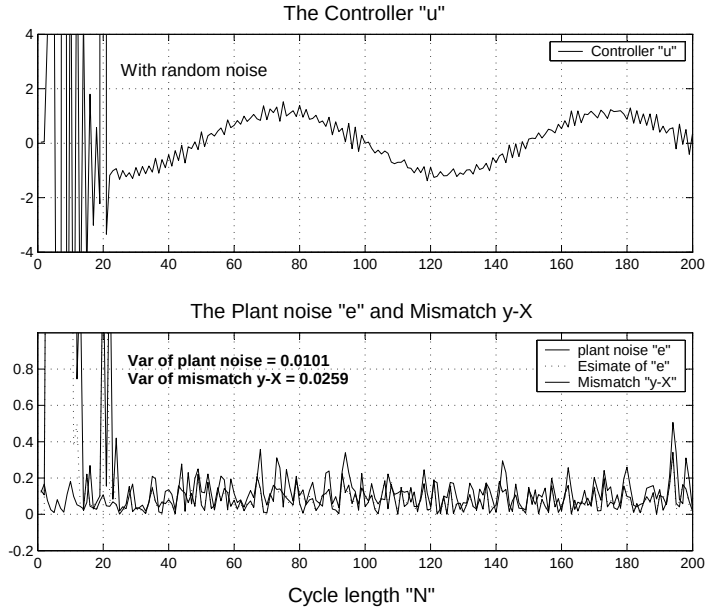

Figure 5.21: Controller u and Mismatch performance with absolute values plot: model with the estimated noise in the feedback, in case of a random noise

5.3 NLMS based Adaptive Control

Here, we propose a stable adaptive controller for stable discrete time nonlinear plant, which may be non-minimum phase. We estimate the inverse of a stable nonlinear plant using the normalized least mean squares (NLMS) technique by an adaptive FIR filter. Then, we copy this filter in the feedback loop as the compensator of the plant. The incorporation of this filter in the feed-forward path to the plant gives approximately a delay as open loop transfer function. The adaptive FIR is designed online as an L-delay approximate inverse system of the nonlinear plant. If the plant is unstable, then first the plant should be stabilized using any robust control technique and then the controller suggested here can be used to achieve the tracking property of the overall system [61]. The proposed idea is depicted as in Fig. 5.22.

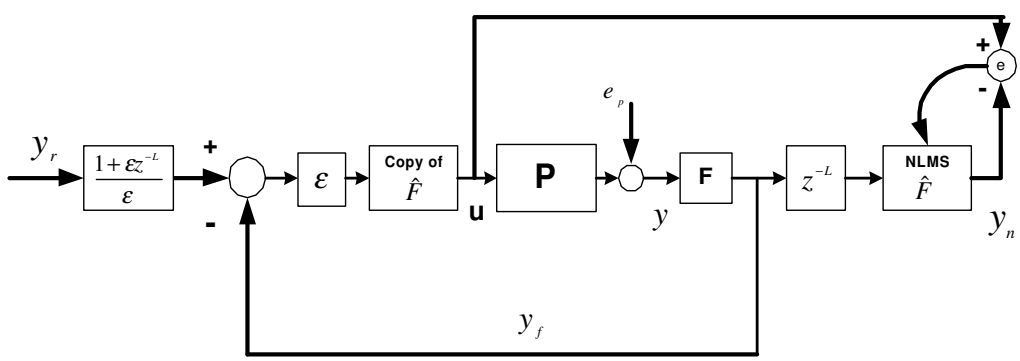

Figure 5.22: Block diagram of the proposed method

5.3.1 The proposed scheme

As shown in Fig. 5.22, $u(k)$ and $y(k)$ are measurable input and output of the nonlinear plant, respectively and $e(k)$ is the plant noise. The objective here is to synthesize a bounded control input $u(k)$ using the controller such that the plant ouput $y(k)$ tracks the desired bounded output sequence $y_r(k)$. The FIR filter in the feedback loop $\hat{F}(q)$ is a copy of the filter $\hat{F}_N(q)$. While $\hat{F}_N(q)$ is obtained using the NLMS error criteria. The mismatch error input to the NLMS filter is the difference of the plant input $u(k)$ and the output of the NLMS filter, $y_n(k)$, as shown in Fig 5.22. The filter taps are updated such that this error is minimized. $\hat{F}_N(q)$ is an L-delay approximate inverse system of the plant and at every instant in time estimated using NLMS estimator, like the adaptive equalization problem [78].

The control input $u(k)$ can now be expressed as:

$$u(k) = \hat{F}(q)[\Gamma(q)y_r(k) - \varepsilon y_f(k)] \tag{5.8}$$

$\hat{F}(q)$ is a polynomial satisfying the approximation:

$$q^{-d}P\hat{F}(q) \approx q^{-L} \tag{5.9}$$

Where P is the plant model given by (2.6), (2.9) and (2.13) for Hammerstein, Wiener and Hammerstein-Wiener models, respectively. d is the plant delay and L is a

positive integer such that $L \geq d$. The characteristic polynomial $\Gamma(q) = 1 + \varepsilon q^{-L}$ is an FIR filter and ε a small positive real number selected such that the close loop remain stable. Moreover as the adaptive filter and the FIR filter are inherently stable, our controller will remain stable. In order to cancel out the plant noise a first order low pass butterworth filter F is used immediately following the plant and the filtered output y_f is used in the feedback, as shown in the Fig. 5.22.

5.3.2 An Example

Consider a plant with a Hammerstein type of nonlinear model [32]:

$$y(k) = a_1 y(k-1) + b_1 c_1 u(k-1) + b_2 c_1 \cos(u(k-2)) + b_3 c_1 \sin(u(k-3)) + e(k) \quad (5.10)$$

Where $e(k)$ is the plant noise. The computer simulation is carried and the real-time results for the proposed scheme are shown. Simulation results are obtained for different conditions of noise such as zero noise, step noise, sinusoidal noise and random noise.

First considering a noise free plant, the reference input and the plant output for this case are in Fig. 5.23. The control input $u(k)$, for the same case is plotted in Fig. 5.24, showing that a perfect tracking is achieved and the controller is stable and bounded.

Similarly results are obtained while considering the step noise and the sinusoidal

noise in the plant model. For a step noise Figs. 5.25-5.26 describe it's performance and Figs. 5.27-5.28 describe the performance for a sinusoidal noise. It can be inferred from the plots that the tracking is good same as in case of our scheme 2 (section 5.2), which uses the estimate of noise in the FB loop.

Lastly the performance with a random noise is evaluated as shown in Figs. 5.29-5.30 concluding that the controller $u(k)$ is bounded. The variance of plant noise and the mismatch are shown in Fig. 5.29 which shows that the NLMS scheme works better than the scheme 2, and we achieve the performance with much better accuracy.

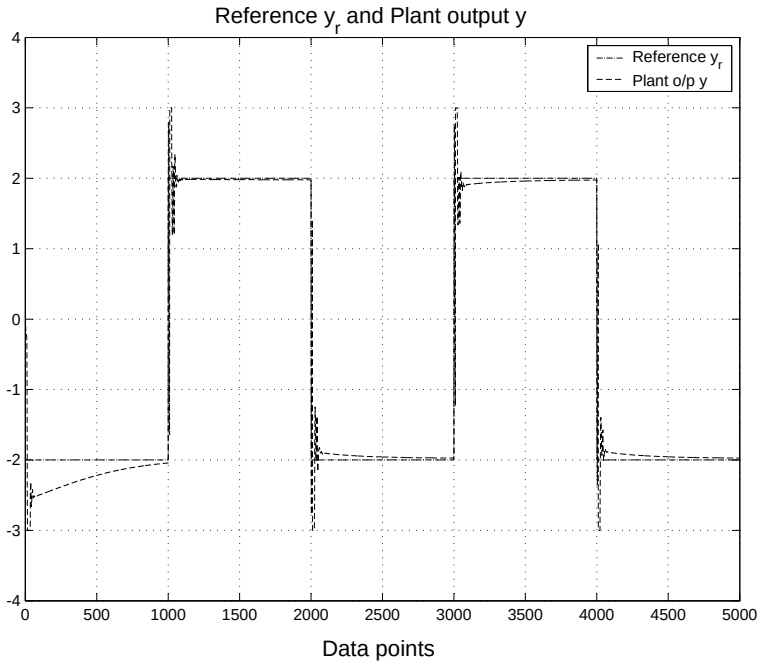

Figure 5.23: Reference signal and the plant output with zero noise: square reference

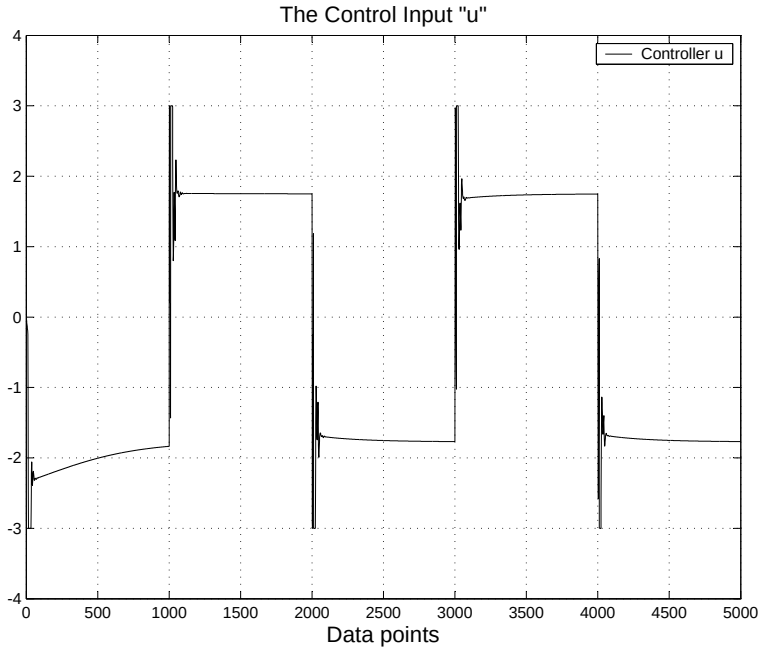

Figure 5.24: Controller signal u, zero noise case: square reference

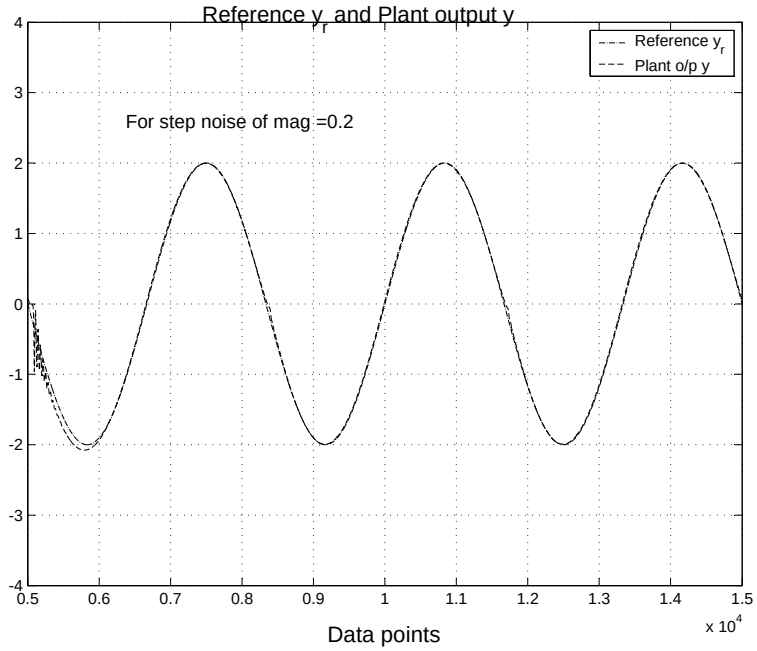

Figure 5.25: Reference signal and the plant output: with step noise

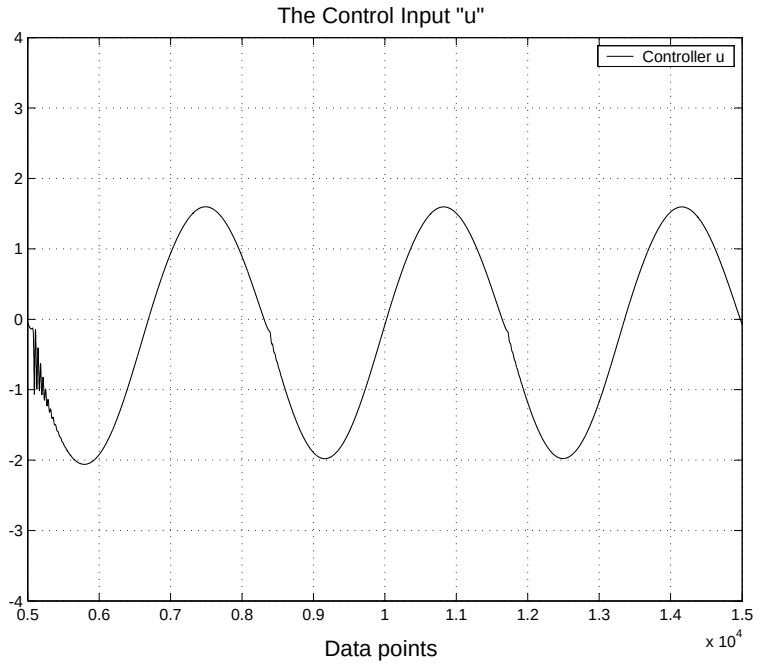

Figure 5.26: Controller signal u: with step noise

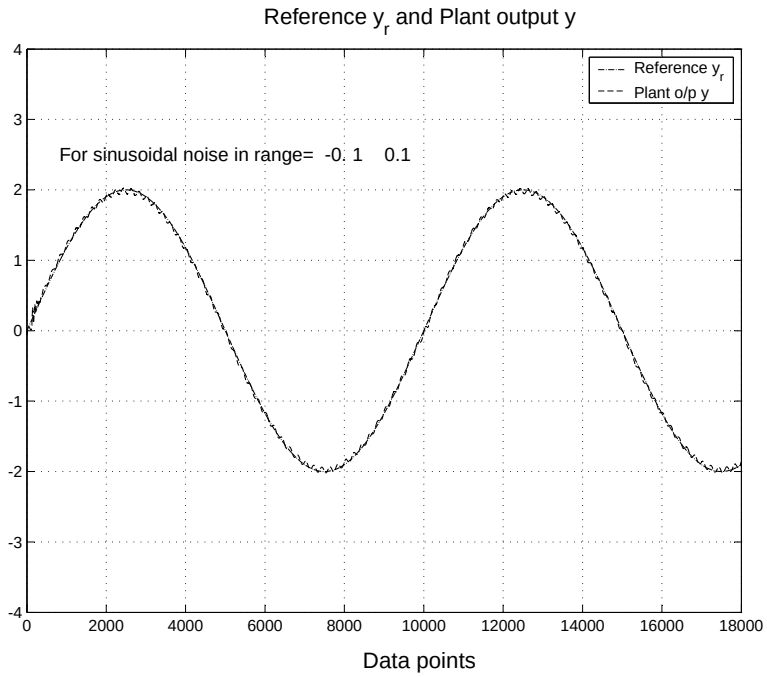

Figure 5.27: Reference signal and the plant output: with sinusoidal noise

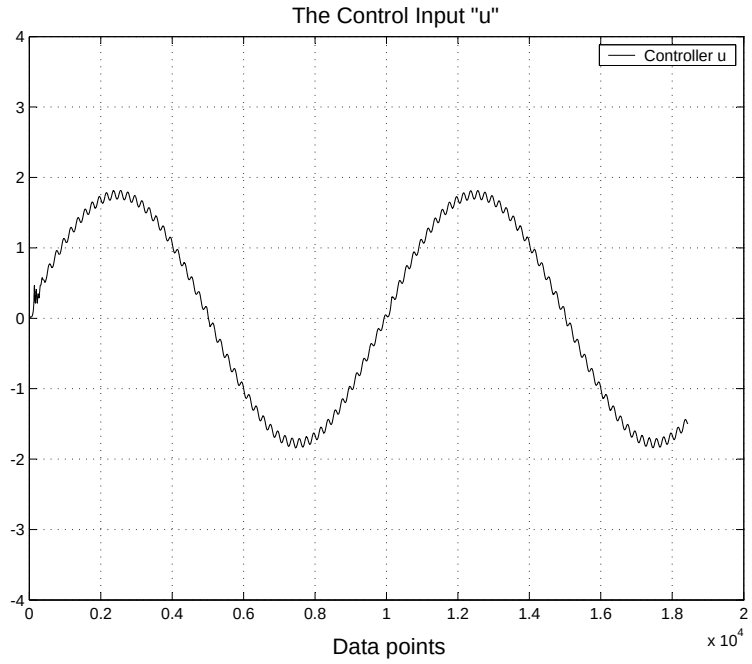

Figure 5.28: Controller signal u: with sinusoidal noise

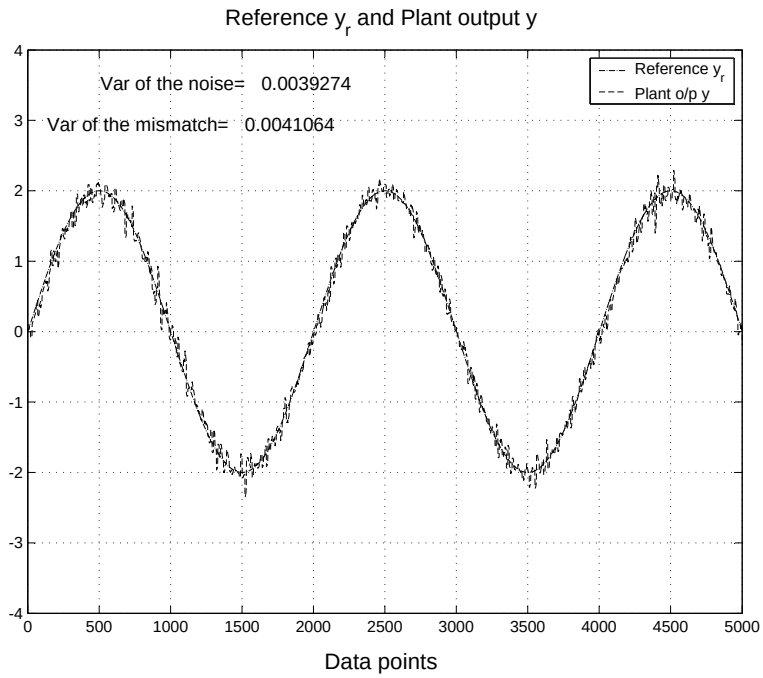

Figure 5.29: Reference signal and the plant output with given values of variances: with random noise

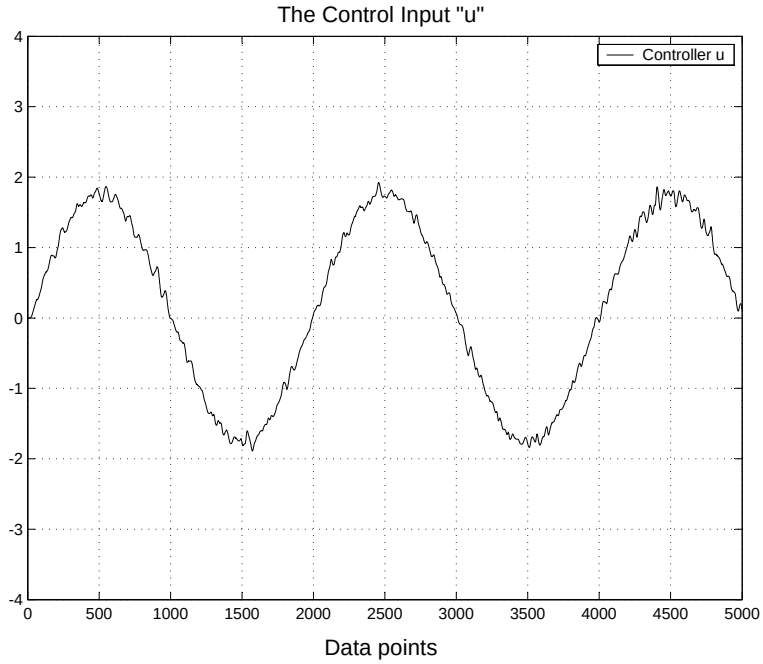

Figure 5.30: Controller signal u: with random noise

Chapter 6

Conclusions, Summary and recommendations for Future Work

This chapter concludes the thesis by presenting the conclusions, summarizing important contributions and highlights some of the future avenues of research that can be originated from the work.

6.1 Conclusions

In this thesis, the LS method used in [1] is extended for the case of online RLS method for the parameter estimation. The different models of Hammerstein, Wiener and Hammerstein-Wiener are considered to test the identification each for different number of parameters with higher order nonlinear plants.

The concept of Multiway Analysis is used while formulation of the identification problem. The results show close convergence of the parameters to their actual values. While convergence, the initial variations of the parameter values are reduced Figs. 4.4-4.3. The algorithm is fast in convergence and the dimensions of the independent block X can be increased further to define the parameter vectors of some higher length. The multiway PLS method overcomes the limitations of the LS method specially in case if the input is correlated. It is observed that the algorithm is quite sensitive to the type of nonlinearity present in the system. Smooth nonlinear functions tend to give good results. Using the multiway the number of parameters can be increased conveniently as in the case of the RLS method.

We have developed adaptive controllers for a class of nonlinear systems. Simulation for the tracking of the desired signal is carried for the same class of nonlinear systems, which shows that the output follows the reference signal precisely. The method of using the estimated noise in the feedback loop is implemented and thus obtaining the plant output noise cancellation. The adaptive controller is applied with the prediction method using the NLMS adaptation of FIR filter for stable plants and the results are presented. The tracking of the plant dynamics is achieved with better accuracy, stable controller is designed and it is observed that the designed control signal for the given stable plant is bounded.

6.2 Summary

The main contributions to the thesis are:

- A recursive version of the Bai's method [1] is developed. This is achieved by using the RLS in the Bai's scheme.
- The resulting RLS is tested for different systems in case of Hammerstein, Wiener and the Hammerstein-Wiener nonlinear systems.
- The identification problem is formulated using the N-PLS method.
- The N-PLS based formulation scheme is tested on different class of the nonlinear dynamic systems.
- An adaptive scheme is developed for the control of nonlinear systems, based on the developed RLS method.
- The adaptive scheme is simulated using different nonlinear systems, such as Hammerstein, Wiener and Hammerstein-Wiener systems each for different reference commands with approximate inverse method and noise estimation method.
- An NLMS based adaptive control scheme is developed for stable nonlinear plants.

6.3 Recommendations for Future Work

Scientific research is an ongoing process and there is always some room for improvement. The following is a brief list of suggestions for possible future work in this area.

- In this thesis, constant parameter models are considered. Including the time-delays and time-varying parameters can be an extension to this work.
- Multivariable versions of the nonlinear models can be considered.
- Different recursive/online methods can be applied instead of the RLS and the results to be compared.
- The nonlinear models can be considered for some very complex type of nonlinearities and for some more general type of models.
- The adaptive noise cancellation methods can be explored with other techniques, such as to remove the noise more precisely and perfectly.

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