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Globally stable adaptive pole placement

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GLOBALLY STABLE ADAPTIVE POLE PLACEMENT

BY

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This dissertation, written by Ali Abdelkader Cherid under the direction of his Dissertation Advisor and approved by his Dissertation Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering.

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Dedication

Dedicated to my dearest parents, wife, and daugthers Sarah and Safa

Acknowledgement

Praise be to Allah the Almighty for his help and guidance to complete this thesis.

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Abstract

Name: **Ali Abdelkader Cherid**

Subject Title: **Globally Stable Adaptive Pole Placement**

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The problem of global stability of indirect adaptive pole assignment for time-invariant system forms the issue addressed in this thesis. Two different algorithms are discussed. The first one is based on the right model representation (r.m.r.), and the second on the left model representation (l.m.r.). The adaptive control based on (r.m.r.) as well as (l.m.r.) and its associated stability analysis are discussed in detail for the SISO case. The key idea to establish global stability is to show that any possible unbounded signal is observable from the equation error. Linear boundedness of a partial state vector derived from the equation error is then used to establish boundedness of all signals. Moreover the equation error is also shown to converge asymptotically to zero.

The result of global stability is applicable to a wide class of estimation schemes. It is proved that uniform boundedness of system signals and convergence of the equation error to zero is not conditional on convergence of the estimated parameters to their true values. This results in a minimum number of required assumptions. In particular neither the assumption on the block length N nor on the persistency of excitation of the reference signal is required.

To obtain convergence of the system parameters to their true values, knowledge of the system order n is required, together with the persistency of excitation of the reference signal. Furthermore, if the idea of block processing is used, bounds on the rate of convergence could be obtained. The analysis includes a number of estimation schemes.

The global stability of adaptive pole assignment based on the (l.m.r.) is extended to MIMO systems. It is shown that the resultant closed-loop system is globally stable with convergence of the system parameters to their true values when driven by a persistently exciting external reference signal or signals. Fast exponential convergence is obtained for some version of least-squares algorithms which gives some robustness properties to the adaptive algorithms. It is also shown that uniform boundedness of all signals is independent from the convergence of the system parameters to their desired values.

Several computer simulations and results on a real experimental set-up using microcomputers are presented to validate the effectiveness of the theory presented.

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خلاصة الرسالة

اسم الطالب الكامل : علي عبد القادر شريد

عنوان الدراســـة : اثبات التوازن الشامل لتعيين الأقطاب بطريقة التكيف

التخصـــــــص : هندسة كهربائية

تاريخ الشهــادة : يناير ١٩٨٨م .

ان مشكلة التوازن لتعيين الأقطاب بطريقة التكيف لأشكال الأنظمة التي لا تتغير مع الزمن ، هي المحور الرئيسي الذي يدور حوله هذا البحث . وقد ناقشت منهجين مختلفين ، يعتمد المنهج الأول على عرض التمثيل الأيمن ، كما يعتمد المنهج الثاني على عرض التمثيل الأيسر .

وقد ناقشت التوازن الشامل للتحكم المرن الذي يعتمد على الطريقتين سابقتي الذكر ، وتنبع الفكرة في تحقيق التوازن الشامل من اثبات أن كل اشارة غير محدودة ، ملاحظة تماما عن طريق الخطأ ، لهذا تم استعمال المحدودية الخطية لمتجه الحالة الجزئي من خطأ المعادلة لتحقيق المحدودية الخطية لكل الاشارات . وكذلك فقد أثبت أن خطأ المعادلة يتجه نحو الصفر بطريقة المحور التقاربي .

وقد شمل التطبيق نتيجة التوازن الشامل مجموعة من المنهجيات التقديرية . لقد أثبت أن المحدودية المنتظمة لاشارات النظم وتقارب خطأ المعادلة الى الصفر ليس شرطا لتقارب القيم الخاصة المحسوبة من قيمتها الحقيقية . كما نتج عن هذا أقل عدد من الافتراضات المطلوبة . أنه ليس من المطلوب افتراض طول الفترة الزمنية لأثبات الاشارة للاشارة الرجعية .

يتطلب الحصول على تقارب القيم المحسوبة للنظام من قيمته الحقيقية معرفة درجة النظام مع الثبات على اشارة الإشارة الرجعية . أما اذا أستعملت فكرة المعالجة الكتلييمكن الحصول على حدود معدل التقارب .

ان التحليل يضم عددا من المخططات التقيمية . لقد وسعت نطاق التوازن الشامل التكييفي في تعيين القطب والذي يعتمد على عرض التمثيل الأيمن يشمل الأنظمة المتعددة المداخل والمخارج . كما أثبت أن النظام المغلق هو شامل التوازن مع تقارب قيم النظام الخاص نحو قيمته الحقيقية اذا دفعت بواسطة اشارة أو اشارات مرجعية مثارة باستمرار من الخارج . لقد أكملت الحصول على تقارب سريع لبعض الصيغ التربيعية الصغرى والتي تعطي خاصيات التحمل للطرق التكييفية . كما أثبت أن المحدودية المنسجمة بالنسبة الى الاشارات ليست مشروطة بتقارب قيم النظام الخاصة نحو القيمة الحقيقية .

لقد عرض العديد من التمثيلات بواسطة الحاسب الآلي والنتائج التجريبية باستعمال الحاسبات لاثبات فعالية النظرية المعروضة .

درجة الدكتوراة في الفلسفة

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

التاريخ : يناير ١٩٨٨م

CHAPTER ONE

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

In many practical situations a system which is only inaccurately known, or is operating over a wide range of different operating conditions is to be controlled. In such situations, the usual fixed-gain controller may be unsatisfactory and controller gains that suitably adapt to operating points may have to be used.

From this, the fundamental adaptive control concepts arise. Given a Multi-Input Multi-Output (MIMO) linear time-invariant system, the parameters of which are completely unknown, the problem is to find a control scheme, which will appropriately adjusts its own controller gains from available input-output information, while at the same time controlling the unknown system.

In reality, adaptive control is of real interest when the parameters of the system are time-varying; in such case, the above principle of adaptive control is also applicable. The only major difference is that now the parameter estimator must be capable of continuously tracking the time varying parameters.

The simplest conceptual scheme is when the system is parameterized in a natural way (i.e. left or right fraction decomposition), and the design calculations are carried out based on the estimated system model. The adaptive control algorithm reduces now to an appropriate selection of an estimation scheme and a control strategy. The control strategy that is being used is based on pole assignment. The idea of pole assignment is to relocate the closed-loop poles to any arbitrary desired locations. The adaptive control problem is solved by estimating the system parameters on-line and calculating the corresponding feed-back law. This class of adaptive algorithms is commonly called 'indirect'.

It is important to have a comprehensive theory of these indirect approaches, not only to provide guidelines for designs, but also to point the pitfalls and limitations of the adaptive algorithms. Both of these are essential to successful practical applications. A key question in adaptive pole placement concerns the stability of the resultant closed-loop system. Also, it is highly desirable that the adaptive algorithms be simple and easy to implement, since the adaptive control requires on-line estimation procedures to update the parameters (at least the parameters of the system), within the time span between successive samples. Also, there exists a close link between the convergence theory and the performance of algorithms in practice. In this thesis, approaches are given to solve some of these problems.

1.2 Literature review

The past decade has seen a considerable growth in theories as well as applications of adaptive control systems. During this period, some significant progress in designing globally stable adaptive control schemes for unknown time invariant Single-Input Single-Output (SISO) systems was observed.

Many apparently different approaches to adaptive control have been propased in the literature [1]–[20]. Two schemes in particular have attracted much interest: Model Reference Adaptive Control (MRAC) and Self-Tuning Regulator (STR).

In MRAC algorithm, the controller gains are directly computed (direct approach) in order that the unknown system asymptotically behaves as a given reference model. Only recently, considerable progress in obtaining fairly general results and rigorous stability proofs have been made [12,16,17,21]. However, the model reference adaptive control approach is associated with the short-comings that the unknown system must be known to be minimum-phase which appears to be inherent in the model reference concept, and that further prior knowledge such as the sign of the gain and the relative degree of the system transfer function must be available. This substantially limits the practical applicability of the method.

The other alternative approach that has attracted much interest is the self-tuning regulator [22]–[25]. The basic idea of the self-tuning regulator was presented in [24]. The idea was further developed by the use of least-squares parameter estimation with a minimum variance controller [25]. This technique is equivalent to one-step ahead optimal control as shown in [25].

These schemes are commonly classified as direct approaches, in the sense that there is no explicit estimation of the system parameters. Instead, the system is parameterized directly in terms of the control law parameters. In both of these algorithms the system is required to be stably invertable.

Another adaptive control technique which has attracted researchers is to identify the parameters of the unknown system and to use the current parameter estimates for synthesizing suitable controller gains, while the control system is in operation[3,5,7,8,11,14,15]. Because the controller gains are determined indirectly via identifications of the unknown system parameters, this approach is called indirect. Although this indirect approach has been applied successfully in practical applications, one of the unsolved problems associated with it has been to obtain algorithms that yield global stable closed-loop systems under relatively weak assumptions, (such as no prior boundedness of input-output models), and that applies to a wide class of estimation schemes.

Many researchers have developed local convergence results of indirect adaptive control systems such as Goodwin and Sin [20] and Kreisselmeier [2] etc. However, it is only recently that global stability and convergence of indirect adaptive control of SISO systems have been studied under relatively general assumptions as in [3,8,14].

The proof of Kreisselmeier [8] is established for cases where a priori bounds on the unknown plant parameters are known and where for each set of parameter values with these bounds the plant has no unstable pole-zero cancellation. Anderson and Johnstone [14] addressed the discrete adaptive pole placement problem and established global convergence under certain general assumptions. Their goal was to first guarantee boundedness of the system input-output data and then show persistency of excitation. Although their scheme allows for a more relaxed definitions of persistency of excitation of the reference signal, a prior knowledge of a lower bound on the Sylvester resultant determinant associated with numerator and denominator polynomials in the transfer function is required. Furthermore, the use of arbitrary controller during the so-called special strategy deteriorates the performance of the transient.

Elliott, Christi and Das [3] also established global stability with some restriction on the exciting reference signal. Their adaptive scheme make use of block processing in the sense that N data samples are taken, and N iterations of the estimation algorithm are performed between control parameter updates. The procedure is to show persistency of excitation without requiring bounded data. Boundedness of all signals follows indirectly from proof of parameter convergence.

The MIMO systems have seen less significant progress in designing global stable adaptive control schemes than their SISO counterparts. The extension of SISO idea to MIMO has been slowly progressing. It is only during the past few years that some insight has been gained into the problems associated with the design of multi-variable model-matching adaptive controllers for minimum phase systems [9,16,23,26].

The most general class of MIMO systems were first considered by Elliott and Wolovich [27]. They have shown that the design of model reference adaptive control for the general class of MIMO system of minimum phase is possible if and only if one had a priori knowledge of the system interactor matrix. Goodwin and Long [26] extended the work presented in [9] to the general class of MIMO minimum phase systems. Of course, the minimum phase restriction yields these approaches impractical in many applications. Thus it is necessary to consider other strategies that applies to nonminimum phase systems.

The situation for nonminimum phase system is much worse. In fact the extension of SISO pole assignment algorithms to the MIMO case proved to be quite challenging. Elliott, Wolovich and Das [7] have used a direct strategy which does not require the system to be minimum phase. It is based on arbitrary pole assignment. However, the complete proof of global stability remained unresolved, because of the restriction imposed on the reference input.

This lack of global stability analysis can be explained in two ways. First, much of the SISO theory deals with direct algorithms. More importantly, extension of indirect SISO strategies to the MIMO case is not straight-forward and can lead to algorithms which require complex real-time numerical calculations in order to map estimated process parameters into controller parameters.

Inspite of all this, in this research the indirect methods have been selected for the following reasons. First, by using a new algorithm for polynomial matrix division which has been presented in [28] and requires essentially only real multiplications, additions and subtractions, it is now possible to develop computationally attractable indirect multi-variable strategies. Second, indirect strategies are more likely to lead to a reduction in the number of parameters to be estimated. Also, it is natural to expect prior knowledge regarding physical quantities in a system to be more easily mapped into prior knowledge of parameters in an input-output model of the process, than into prior knowledge regarding controller parameters. Simplification of the estimated problem by use of prior knowledge is critical to the practical application of multi-variable adaptive control.

1.3 Problem formulation and proposed approach

In general, if a system which is not completly known is required to be controlled, adaptive controller may suitably solve the problem. In this thesis, the choice of an adaptive control approach is based on the indirect adaptive pole assignment.

Suppose an MIMO linear system which is known to be controllable and observable is given. Such systems can be represented in operator form by either a right or left matrix fraction decomposition (i.e. the transfer function can either be written by $R(z)P_R^{-1}(z)$ or $P_L^{-1}(z)Q(z)$).

Although in adaptive control problem the parameters are assumed unknown, it will be assumed that the system is controllable and observable. In addition the observability indices ν_i and the controllability index μ are assumed to be known. The first part of the design concerns the estimation of the parameters. As mentioned earlier, in the left representation, the output can be expressed as a linear combination of past outputs and past inputs. There are various estimation schemes available in the literature that can be applied directly to the above model for estimation of parameters such as least-squares and its variants and projection algorithms [29]. However, the emphasis will be given to the least-squares and its variants for the following reasons. First, generally the least-squares schemes have much faster convergence than the projection algorithms. Second, it has been shown that the least-squares algorithm can be used essentially unaltered with noisy signals as mentioned in [29].

Based on the output of the parameter estimator, it is now possible to design the control law depending on the control strategy adopted with emphasis on pole assignment. The calculations of the feedback law from the parameters of the system to assign arbitrarily the poles of the closed-loop require the solution of the diophantine equation [30]. It is of interest to note that in Wolovich and Antsaklis [31], a new computational algorithm is given for solving polynomial matrix diophantine equations which involves the inversion of only a single real matrix of dimension equal to the system order n.

It was felt necessary to first consider the SISO system before embarking to the MIMO case. The reasons are two-fold. First, in analyzing SISO, more understanding of the convergence of the adaptive algorithms will be gained. Second, the indirect adaptive algorithm based on the right fraction decomposition for SISO are in general not convenient for MIMO case. This is because the control strategy used requires knowledge of the parameter of the right fraction representation. Specifically, the algorithm given in [3] is not convenient to MIMO systems.

So, the first part of the research deals with global stability of indirect adaptive control algorithms of SISO systems with minimum required assumptions. It is shown that global stability (i.e. boundedness of all system signals and convergence of the equation error to zero) is derived without a persistency of excitation requirement. For a generalization of the proof, the analysis will include indirect adaptive algorithms with controller parameters updated at different time frame from the system parameter updates, in a similar manner as given in [3].

The use of the idea of block processing in adaptive control algorithms has the following practical advantages:

(1) It separates the bandwidth of the control law from that of the system.

(2) It will be shown that it is possible to ensure persistency of excitation condition on the input-output signals from persistency of excitation of an external reference signal. Moreover, it is possible to obtain fast

exponential convergence if certain conditions are satisfied.

(3) The system parameters are first calculated from input-output data, then from these estimates the control law is designed online. If the complete processing cannot be done fast enough one may have to maintain the previous controller till the new one is computed.

The proof is derived for arbitrary values of N. Global asymptotic convergence with specified bounds on the rate of convergence is guaranteed if the persistency of excitation of the reference signal and the block length satisfy some assumptions.

The global convergence of the indirect adaptive pole assignment based on left fraction representation is also given. This later approach can be easily extended to MIMO case.

The global stability analysis includes a large variety of least-squares algorithms that are available in the literature and summarized in [29]. One reason is that, some of the algorithms are faster than others in particular applications. Moreover, if exponential convergence can be obtained for some of them, then, certain robustness properties are automatically guaranteed, in particular, certain type of noise can be accommodated.

Second, as typically done in analysis of adaptive systems, the unknown plant is assumed to be time-invariant. Toward this, the analysis is also carried out for several modified version of least-squares that retain rapid initial convergence while ensuring that the gain does not go to zero, namely least-squares with exponential data weighting, covariance resetting and covariance modification. Finally the theoretical analysis will be confirmed by simulation of some selected systems (test cases) to verify the validity of the algorithms.

One more point of practical importance is the investigation of the rate of convergence of these different algorithms. A performance comparison between different estimation schemes when used in indirect adaptive algorithms is also given.

1.4 Contributions

The contributions of this work are summarized in the following main points:

- A novel proof of global stability of indirect adaptive pole assignment of SISO systems is presented. It is shown that global stability is independent from the convergence of the system parameters to their true values. The key idea is to show boundedness of all signals and convergence of the equation error to zero without persistency of excitation requirement. This result is valid for any arbitrary value of N if block processing is used.

- Bounds on the rate of convergence of the parameters to their true values are obtained for a number of estimation schemes namely standard least-squares and its variants, and projection algorithms. In particular exponential convergence is obtained for certain least-squares algorithms with forgetting factors. This result is based on the use of block processing together with appropriate choice of the block length N and the persistency of excitation of the reference signal.

- Extension of the global stability to MIMO systems. The result obtained is similar to SISO case, in the sense that uniform boundedness of all signals and convergence of the equation error to zero are obtained with similar assumptions. Also bounds on rate of convergence are obtained for different estimation schemes if persistency of excitation, together with the use of block processing are assumed.

CHAPTER TWO

BASIC CONCEPTS FOR MODELING ESTIMATIONS AND CONTROL OF LINEAR SYSTEMS

2.1 Introduction

This chapter is intended to discuss a number of mathematical concepts related to modeling, estimation and control of linear system, and introduce certain key mathematical notations which will be used throughout the rest of this thesis.

To start, certain key mathematical notation is given. Then models for linear deterministic finite dimensional systems are discussed. In particular difference operator representation and autoregressive moving-average models are given. This is important, since the choice of the model is often the first step toward the estimation and control of a process. An appropri-

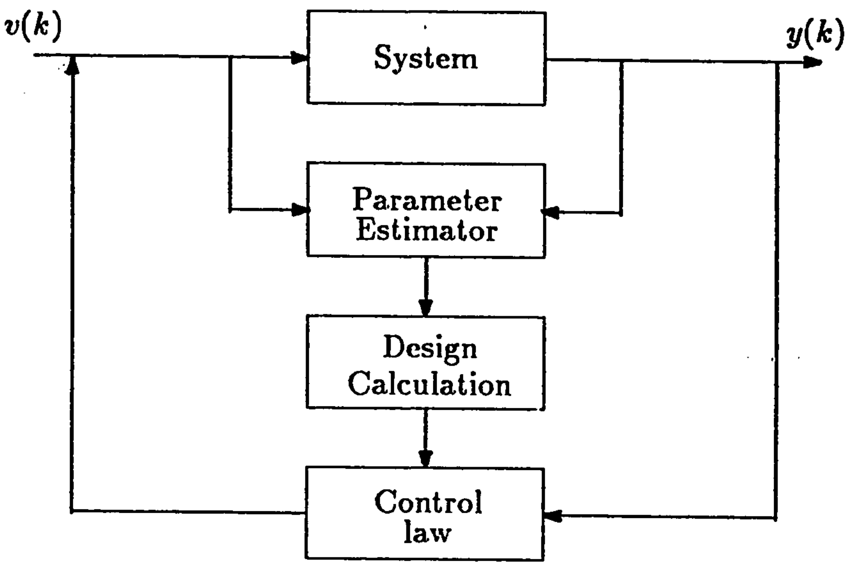

Figure 2.1: General Block Diagram of Adaptive Control

ately chosen model structure can greatly simplify the parameter estimation procedure and facilitate the design of control algorithms for the process. As the basic structure of an adaptive controller of Fig. 2.1 shows that underlying each of the problems of adaptive control, there is some form of parameters estimator.

In fact parameter estimators form an integral part of any adaptive scheme. The discussion includes the gradient algorithms, standard least-squares and its variants. Again there are a vast array of design techniques for generating control strategies when the model of the system is known. The present approach which is of interest relies on the pole-assignment, with one based on the left model representation and the other on the right one. Most of the material presented in this chapter can be obtained from [29,30] and wherever possible the original notations were retained.

2.2 Preliminary concepts from polynomial matrix theory

Certain preliminary concepts associated with the class of matrices whose elements are finite degree polynomials with coefficients in the field of real numbers ($\Re$) are introduced in the following sections:

2.2.1 Degree of a polynomial matrix

The degree of a square polynomial matrix $P(z)$, denoted by $\partial[P(z)]$ is defined to be the degree of its determinant. The degree of the ith column (ith row) of $P(z)$, denoted as $\partial_{c_i}[P(z)]$ ($\partial_{r_i}[P(z)]$) is defined to be the degree of polynomial element of highest degree in the ith column (ith row) of $P(z)$. We will denote by $\Gamma_c[P(z)]$ ($\Gamma_r[P(z)]$) the constant matrix with elements consisting of the coefficients of the highest degree terms in each column (row) of $P(z)$.

2.2.2 Row proper and column proper matrices

A polynomial matrix $P(z)$ is said to be column (row) proper if and only if $\Gamma_c[P(z)]$ ($\Gamma_r[P(z)]$) has full rank. It is thus clear that a square polynomial matrix $P(z)$ is column (row) proper if and only if $\det \Gamma_c[P(z)]$ ($\det \Gamma_r[P(z)]$) is not equal to zero.

2.2.3 Relative prime polynomial matrices

A pair of polynomial matrices { $P_R(z)$, $R(z)$ } which have the same number of columns are said to be relatively right prime (r. r. p.) if and only if their greatest common right divisor (g. c. r. d.) is a unimodular matrix. Similarly, a pair { $P_L(z)$, $Q(z)$ } of polynomial matrices which have the same number of rows are said to be relatively left prime (r. l. p.) if and only if their greatest common left divisor (g. c. l. d.) is a unimodular matrix.

2.3 System modeling

Throughout the remainder of this thesis, we will consider l-input, m-output linear shift-invariant discrete time systems. The result given below applies also to SISO systems as a special case.

2.3.1 State space representation

The general state space representation of a discrete linear dynamical system can be written as

$$x(k+1) = Ax(k) + Bu(k) \tag{2.1}$$

$$y(k) = Cx(k) + Hu(k) \tag{2.2}$$

where $u(k)$ is the $(l \times 1)$ system input vector sequence, $y(k)$ the $(\ m \times 1\ )$ output vector sequence, and $x(k)$ the $(\ n \times 1\ )$ state vector sequence. For this state space representation, let the controllability indices μ_i, $1 \leq i \leq l$, and observability indices ν_i, $1 \leq i \leq m$, be defined in the standard manner by sequencing through the columns of the controllability and transposed observability matrices from left to right finding the first n independent columns [29].

2.3.2 Difference operator representation

As shown in [29], Eqns 2.1 and 2.2 can also be represented by the operator equations

$$P(z)w(k) = Q(z)u(k) \tag{2.3}$$

$$y(k) = R(z)w(k) \tag{2.4}$$

where $w(k)$ denotes the system partial state and with $P(z)$, $Q(z)$ and $R(z)$ polynomial matrices in the delay operator z with appropriate dimensions. To ensure existence and uniqueness of the solution to Eqns 2.3 and 2.4, we require $P(z)$ to be square and nonsingular (det $P(z) \neq 0$ for almost all z). The difference operator representation includes the state-space model as a special case.

Here we shall be particularly interested in two special forms of the difference operator representation. Right difference representation and left difference representation.

In a right difference operator representation, the model of Eqns 2.3 and 2.4, if it is controllable, takes the following form:

$$P_R(z)w(k) = u(k) \quad (2.5)$$

$$y(k) = R(z)w(k) \quad (2.6)$$

This is an equivalent description to a controllable state-space model and is completely controllable.

In a left difference operator representation, the model of Eqns 2.3 and 2.4, if it is observable, takes the following form:

$$P_L(z)y(k) = Q(z)u(k) \quad (2.7)$$

This form turns out to be equivalent to an observable state-space model and is completely observable. $P_R(z)$ ($P_L(z)$) is a column proper (row proper) polynomial matrix and $\partial_{c_i}[P(z)] = \mu_i$ ($\partial_{r_i}[P(z)] = \nu_i$) [29]. It can be assumed without loss of generality that $\Gamma_c[P(z)]$ is upper ($\Gamma_r[P(z)]$ is lower) triangular with ones on the main diagonal.

2.3.3 Controlability and observability

Consider the general difference operator representation given in Eqns 2.3 and 2.4, then the system is said to be controllable (observable) if there are no zero-pole cancellation between $R(z)$ and $P_R(z)$ ($Q(z)$ and $P_L(z)$).

A system is called stabilizable (detectable) if all uncontrollable (unobservable) modes have corresponding eigenvalues strictly inside the unit circle [30].

2.3.4 Deterministic autoregressive moving-average models

In this section an alternative model format of Eqn 2.7 is introduced in which the current output vector is expressed as a linear combination of past outputs $y(k)$, and past inputs $u(k)$. Eqn 2.7 can be rewritten as:

$$P_L^0 y(k+\nu) = -\sum_{j=1}^{\nu-1} P_L^j y(k+j) + \sum_{j=0}^{\nu-1} Q^j u(k+j) \qquad (2.8)$$

where P_L^i are square matrices containing the coefficients of the matrix polynomial $P_L(z)$, and Q^i are matrices containing the coefficients of the matrix polynomial $Q(z)$. The dimension of $y(k)$ and $u(k)$ are m and l respectively and ν is the observability index. The model of Eqn 2.8 is termed the deterministic autoregressive moving average (DARMA) model [29].

Moreover, from the structure of the left model representation, it follows that P_L^0 can be assumed to be lower triangular with ones on the main diagonal without loss of generality. In general, Eqn 2.8 can be normalized so that $P_L^0 = I$ by multiplying both sides by the inverse of P_L^0. With

$P_L^0 = I$, the DARMA model can be expressed as

$$y(k+\nu) = \theta_o^T \phi(k+\nu-1) \tag{2.9}$$

where θ_o^T is an $(m \times p)$ matrix of parameters in $P_L(z)$ and $Q(z)$ and $\phi(k)$ is $(p \times 1)$ vector containing past values of the output and input vectors. We shall find models of the form Eqn 2.9 particularly convenient in the subsequent development. For the remaining part of the thesis the subscript R and L will be droped from P to simplify the notations.

2.4 Parameter estimation for deterministic systems

Online algorithms for estimating the system parameters are of principal importance in adaptive control. These schemes deals with sequential data, which requires that the parameter estimates be recursively updated within the time limit imposed by the sampling period.

Based on the model Eqn 2.9, which is linear in the parameters, one can introduce the following online parameter estimation schemes [29]

2.4.1 Projection algorithm

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\phi(k-1)}{c+\phi(k-1)^T\phi(k-1)}[y(k) - \hat{\theta}(k-1)^T\phi(k-1)]^T \tag{2.10}$$

with $\hat{\theta}(0)$ arbitrary initial estimate, and $c > 0$. The vector $\hat{\theta}(k)$ is an estimate of θ_o and $\phi(k)$ is a regression vector containing all past inputs and outputs. Introducing the following notation:

$$\tilde{\theta}(k) = \hat{\theta}(k) - \theta_o \tag{2.11}$$

$$e(k) = y(k) - \hat{\theta}(k-1)^T\phi(k-1) \tag{2.12}$$

$$= \tilde{\theta}(k-1)^T\phi(k-1) \tag{2.13}$$

elementary properties of the projection algorithm necessary for our analysis are summarized in the following lemma.

Lemma 2.1 *for the algorithm Eqn 2.10 and subject to Eqn 2.9, it follows that*

$$(i) \quad \|\hat{\theta}(k) - \theta_o\|^2 \leq \|\hat{\theta}(k-1) - \theta_o\|^2 \leq \|\hat{\theta}(0) - \theta_o\|^2, \quad k \geq 1 \tag{2.14}$$

$$(ii) \qquad \lim_{l\to\infty} \sum_{k=1}^{l} \frac{e(k)^T e(k)}{c+\phi(k-1)^T\phi(k-1)} < \infty \tag{2.15}$$

and this implies

$$(a) \quad \lim_{k\to\infty} \frac{e(k)^T e(k)}{c+\phi(k-1)^T\phi(k-1)} = 0 \tag{2.16}$$

$$(b) \quad \lim_{k\to\infty} \|\hat{\theta}(k) - \hat{\theta}(k-l)\|^2 = 0 \quad \textit{for finite } l \tag{2.17}$$

where the symbol $\|.\|$ *is defined as* $\|M\| = trace(M^T M)$

Note that these properties do not guarantee the convergence of $\hat{\theta}(k)$ to the true value θ_o, nor the convergence of the output error $e(k)$ to zero. However, these properties are of great importance since they have been derived under extremely weak assumptions. Such as no prior knowledge of the exact order of the system is assumed, as well as no prior boundedness of the input output models is required.

Property(i) ensures that $\hat{\theta}(k)$ is never further from θ_o than $\hat{\theta}(0)$ is.

Property(ii) implies that the modeling error, $e(k)$, when appropriately normalized is square sumable.

Property(iib) shows that the parameter estimates error is non-increasing as $k \longrightarrow \infty$.

Considerable use of these properties are made in the subsequent development.

2.4.2 Least-squares algorithm

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-2)\phi(k-1)}{1+\phi(k-1)^{\,T}P(k-2)\phi(k-1)}[y(k) - \hat{\theta}(k-1)^{\,T}\phi(k-1)\,]^T \tag{2.18}$$

$$P(k-1) = P(k-2) - \frac{P(k-2)\phi(k-1)\,\phi(k-1)^{\,T}P(k-2)}{1+\phi(k-1)^{\,T}P(k-2)\phi(k-1)} \tag{2.19}$$

with $\hat{\theta}(0)$ arbitrary initial estimate, and $P(-1)$ is any positive definite matrix.

The basic convergence properties of the least-squares algorithms necessary for the subsequent analysis can be summarized in the following lemma:

Lemma 2.2 *for the algorithm Eqns 2.18 and 2.19 and subject to Eqn 2.9, it follows that*

$$(i) \qquad ||\hat{\theta}(k) - \theta_o||^2 \leq \kappa||\hat{\theta}(0) - \theta_o||^2 \qquad k \geq 1 \tag{2.20}$$

$$\textit{where} \quad \kappa = \textit{condition number of } [P(-1)^{-1}] = \frac{\lambda_{max}[P^{-1}(-1)]}{\lambda_{min}[P^{-1}(-1)]} \tag{2.21}$$

$$(ii) \qquad \lim_{l\to\infty} \sum_{k=1}^{l} \frac{e(k)^T e(k)}{1+\phi(k-1)^T P(k-2)\phi(k-1)} < \infty \tag{2.22}$$

and this implies

$$(a) \quad \lim_{k\to\infty} \frac{e(k)^T e(k)}{[1+\kappa\phi(k-1)^T\phi(k-1)]} = 0 \tag{2.23}$$

$$(b) \qquad \lim_{k\to\infty} ||\hat{\theta}(k) - \hat{\theta}(k-l)|| = 0 \qquad \textit{for finite } l \tag{2.24}$$

There are variant least-squares algorithms that have the above properties. In particular these are the least-squares with exponential data weighting, covariance resetting and covariance modification. An important effect of the modification of the latter three cases is that they render the least-squares algorithm applicable to time-varying system.

CHAPTER THREE

INDIRECT ADAPTIVE POLE PLACEMENT FOR SISO LINEAR SYSTEMS

3.1 Introduction

In this chapter we turn our attention to the control of linear systems whose parameters are unknown. Essentially, the approach adopted is to combine the parameter estimation scheme of section 2.4 with the well known pole assignment control strategy presented in [3]. The parameters of the system are estimated explicitly, then based on these estimates, feedback controllers are calculated. This leads to the name of indirect adaptive pole assignment.

In this chapter we will focus on the SISO case. We begin our discussion of adaptive pole assignment by developing an adaptive version of the pole assignment. The approach considered is to first estimate the parameters on a given model (right or left model representation) for the system, then these are subsequently used to generate the feedback control law via intermediate calculations.

For a SISO system, parameters of the right model representation are directly mapped to the left model representation.Thus one can use then left representation for estimation purposes then, extract the parameters of the equivalent right model representation. The final step is to combine the estimated parameters with the corresponding control strategy.

A detailed stability analysis of the adaptive control with the assumptions required for a global convergence result are given. The present derivation eliminates the persistency of excitation requirement. The key idea in this proof is to show that any unbounded variation in the input-output data is observable from the equation error by exploiting the detectability property of the system obtained from the closed-loop and the identification error. The proof will be carried out for the standard least-squares and its variants, namely least-squares with exponential data weighting, covariance reset, and covariance modification. These schemes have in general superior convergence over the standard least-squares, and can handle system with time varying parameters [29].

The organization of this chapter is as follows: Starting with a brief introduction to modeling and problem formulation, we proceed next to the fixed control strategy that achieves the desired closed-loop pole assignment.

The assumptions required for global result together with the algorithm is then presented. It is then followed by some technical lemmas that are needed in subsequent derivations. Finally the issue of global stability is discussed in details.

3.2 System modeling and problem formulation

Let us consider the problem of controlling a linear shift invariant discrete time system characterized by either of the following SISO models:

$$p(z)w(k-n+1) = u(k-n+1) \tag{3.1}$$

$$y(k-n+1) = r(z)w(k-n+1) \tag{3.2}$$

$$\text{or} \quad p(z)y(k-n+1) = q(z)u(k-n+1) \tag{3.3}$$

where all the notations possess their usual meanings as discussed in chapter 2. For the (l.m.r.) of Eqn 3.3, let the transfer function be represented as

$$T(z) = \frac{q(z)}{p(z)} \tag{3.4}$$

In the development to follow we assume that the transfer function is strictly proper. Thus, it will be assumed that $p(z)$ and $q(z)$ have the following general forms:

$$p(z) = z^n + \sum_{i=0}^{n-1} p_i z^i \tag{3.5}$$

$$q(z) = \sum_{i=0}^{n-1} q_i z^i \tag{3.6}$$

in addition we assume that the order of the system is known. The pole assignment problem now consists of designing a suitable feedback for the system represented by Eqns 3.1–3.2 or Eqn 3.3.

3.3 Fixed control strategy

The indirect pole assignment control strategy depends on the estimated model (left or right model representation). Although, only parameters of the left model can be estimated directly by use of one of the standard estimation schemes available in the literature, the pole assignment control based on right model representation does not create a problem because the parameters of the left model representation are directly mapped into the parameter of the right one.

3.3.1 Pole assignment control strategy based on left model representation

Consider the fixed control strategy [27]

$$u(k-n+1) = (h(z)/c(z))y(k-n+1) + v(k-n+1) \qquad (3.7)$$

where

$$c(z) = q^{*}(z) - k(z) \qquad (3.8)$$

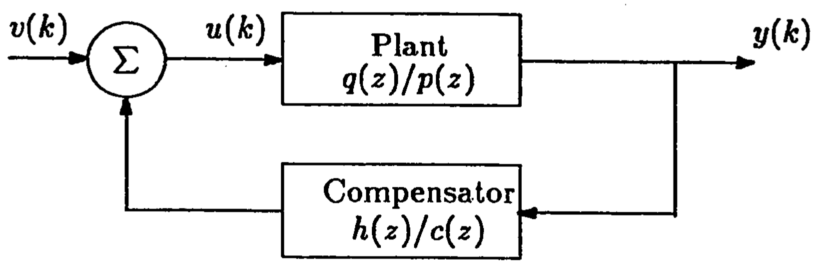

Figure 3.1: Block diagram of pole assignment controller based on (l.m.r.)

and $q^*(z)$ is arbitrary stable polynomial of degree $n-1$, $h(z)$ and $k(z)$ are controller polynomial of degree $n-1$ and $n-2$ respectively. Defining

$$y(k-n+1) = c(z)w(k-n+1) \tag{3.9}$$

Using Eqn 3.7 as a feedback to control the system model Eqn 3.3, the closed-loop system becomes

$$[q^*(z)p(z) - h(z)q(z) - k(z)p(z)]w(k-n+1) = q(z)v(k-n+1) \tag{3.10}$$

Solving the following equation, one can then obtain a unique pair of polynomials $k(z)$ and $h(z)$ such that

$$q^*(z)p(z) - h(z)q(z) - k(z)p(z) = p^*(z)q^*(z) \tag{3.11}$$

for arbitrary polynomials $p^*(z)$ and $q^*(z)$. As a result, the new desired closed-loop polynomial becomes $p^*(z)q^*(z)$. The general block diagram of pole assignment controller based on (l.m.r.)is shown in Fig. 3.1.

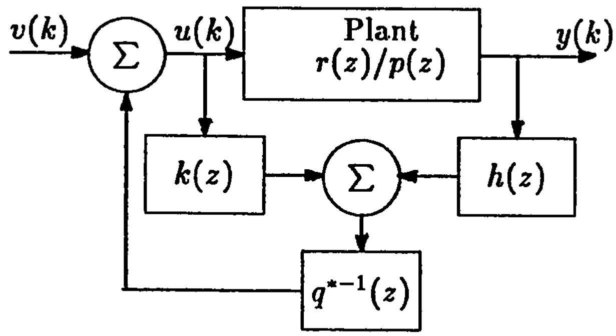

Figure 3.2: Block diagram of feedback controller based on (r.m.r.)

3.3.2 Pole assignment control strategy based on right model representation

Following the scheme illustrated in [3,29], consider the fixed control strategy

$$q^*(z)u(k-n+1) = h(z)y(k-n+1) + k(z)u(k-n+1) + q^*(z)v(k-n+1) \tag{3.12}$$

where $v(k)$ is an external reference signal. The control law depicted in Fig. 3.2 can be interpreted as a combination of input dynamics and linear state variable feedback via asymptotic state estimation, where roots of $q^*(z)$ represent the stable arbitrary uncontrollable poles.

Using this control strategy the closed-loop system becomes

$$[q^*(z)p(z) - h(z)r(z) - k(z)p(z)]w(k-n+1) = q^*(z)v(k-n+1) \tag{3.13}$$

$$y(k-n+1) = r(z)w(k-n+1) \tag{3.14}$$

choosing $q^*(z)$ and $p^*(z)$ as stable polynomials of degree $n-1$ and n respectively of the following form:

$$q^*(z) = z^{n-1} + \sum_{i=0}^{n-2} q_i^* z^i \tag{3.15}$$

$$p^*(z) = z^n + \sum_{i=0}^{n-1} p_i^* z^i \tag{3.16}$$

and defining $h(z)$ and $k(z)$ as the solutions to

$$h(z)r(z) + k(z)p(z) = q^*(z)[p(z) - p^*(z)] \tag{3.17}$$

the closed-loop system simplifies to

$$q^*(z)p^*(z)w(k-n+1) = q^*(z)v(k-n+1) \tag{3.18}$$

$$y(k-n+1) = r(z)w(k-n+1) \tag{3.19}$$

As readily seen from Eqn 3.18, the closed-loop poles are the roots of $q^*(z)p^*(z)$. The roots of $q^*(z)$ are interpreted as uncontrollable poles, and those of $p^*(z)$ are the new assigned pole locations. Let the polynomials $h(z)$ and $k(z)$ satisfying Eqn 3.17 take the following forms:

$$h(z) = \sum_{i=0}^{n-1} h_i z^i \tag{3.20}$$

$$k(z) = \sum_{i=0}^{n-2} k_i z^i \tag{3.21}$$

where the degrees of $h(z)$ and $k(z)$ are chosen to guarantee uniqueness of the solution of the diophantine equation.

3.4 Assumptions and adaptive control algorithm

In this section we list all the assumptions required for a global result, together with the algorithm. The algorithm makes use of block processing. It is used in the sense that N data samples are taken, and N iterations of the estimation scheme are performed between control parameter updates. The final estimate at each interval is used to generate the control parameters $\hat{h}(k, z)$ and $\hat{k}(k, z)$, which in turns are used during the interval which follows. The block processing which also implies the block length N is not a necessary condition to establish the global stability proof. However, it has been introduced for practical reasons that has been clearly presented in chapter 1.

The proof derived in [3] is applicable only with the constraint that the block length N is fixed by the number of component frequencies available in the reference signal. This constraint is associated with the shortcoming that the reference should have a finite number of frequency components which is not always desirable. Moreover, less a priori assumptions are required to establish global stability (i.e. boundedness of all signals and convergence of the equation error to zero). The global stability results obtained does not depend on the convergence of the system parameters to their true values.

Let us define the time axis as a set of intervals I_j of length N. Then we have

$$I_j = \{k | k_j < k \leq k_j + N (= k_{j+1})\} \quad \text{k integer,}$$

The choice of the positive integer N will be discussed later. Assuming that the control parameters are updated only at time k_j, using the fixed control strategy which is based on the left model representation, the feedback control law becomes:

$$u(k-n+1) = \frac{\hat{h}\ (k_j, z)}{q^*(z) - \hat{k}\ (k_j, z)} y(k-n+1) + v(k-n+1) \qquad (3.22)$$

where $\hat{h}(k_j, z)$ and $\hat{k}(k_j, z)$ are estimates of $h(z)$ and $k(z)$ at time k_j and satisfying the following diophantine equation:

$$\hat{h}\ (k_j, z)\hat{q}\ (k_j, z) + \hat{k}\ (k_j, z)\hat{p}\ (k_j, z) = q^*(z)[\hat{p}\ (k_j, z) - p^*(z)] \qquad (3.23)$$

the polynomials $\hat{q}\ (k_j, z)$ and $\hat{p}\ (k_j, z)$ are estimates of the polynomials $q(z)$ and $p(z)$ using the estimation scheme given in section (2.4). When there is pole cancellation between $\hat{q}\ (k_j, z)$ and $\hat{p}\ (k_j, z)$ i. e. the Sylvester resultant determinant of $\hat{q}\ (k_j, z)$ and $\hat{p}\ (k_j, z)$ defined as $\det M(k_j)$ is zero, the algorithm will run into difficulty. A solution to this problem is to assume a lower bound on the $\det M(k_j)$, then if the $\det M(k_j)$ is less than this lower bound value, the control parameters are not updated [3]. The algorithm is summarized as follows:

Algorithm 3.1 *The identifier is run N times during the interval I_j, at the end of which the parameters are updated if* $\det M(k_j) \neq 0$ *using Eqn 3.23. When* $\det M(k_j) = 0$ *the control parameters remain unchanged during the interval I_{j+1}.*

In the case of the right model representation, if we assume similarly that the parameters are updated only at time instants k_j, the control law becomes

$$q^*(z)u(k-n+1) = \hat{h}\ (k_j, z)y(k-n+1) + \hat{k}\ (k_j, z)u(k-n+1) + q^*(z)v(k-n+1) \tag{3.24}$$

with $\hat{h}\ (k_j, z)$ and $\hat{k}\ (k_j, z)$ are estimate of $h(z)$ and $k(z)$ obtained from the solution of the following diophantine equation

$$\hat{h}\ (k_j, z)\hat{r}\ (k_j, z) + \hat{k}\ (k_j, z)\hat{p}\ (k_j, z) = q^*(z)[\hat{p}\ (k_j, z) - p^*(z)] \tag{3.25}$$

Before proceeding, the following theorem is needed for succeeding results [29].

Theorem 3.1 *Two polynomials $(a(z), b(z))$ of order n are relatively prime if and only if the Sylvester resultant determinant* $\det M$ *is not equal to zero,* *where M is defined to be the following $(2n \times 2n)$ matrix*

$$M = \begin{pmatrix} a_0 & & & b_0 & & \\ a_1 & \ddots & & \vdots & \ddots & \\ \vdots & & a_0 & \vdots & & b_0 \\ a_n & & a_1 & b_n & & \\ & \ddots & \vdots & & \ddots & \vdots \\ & & a_n & & & b_n \end{pmatrix} \tag{3.26}$$

where

$$a(z) = \sum_{i=0}^{n} a_i z^{n-i}$$
$$b(z) = \sum_{i=0}^{n} b_i z^{n-i}$$

It is clear that the values of $\hat{p}\ (k_j, z)$ and $\hat{q}\ (k_j, z)$ that give rise to an exact pole-zero cancellation and hence exact singularity of M are on set of measure zero [3]. Thus Eqn 3.23 is solvable with probability 1. However, in the analysis to follow, one require $\hat{h}\ (k_j, z)$ and $\hat{h}\ (k_j, z)$ to have bounded coefficients and hence near-singularity of Eqn 3.23 must be avoided.

The assumptions required to establish global stability i.e. boundedness of all signals and convergence of the equation error to zero are:

A1: $p(z)$ and $q(z)$ are relatively prime polynomials.

A2: order of the system n is known

A3: The desired closed-loop characteristic polynomial $q^*(z)p^*(z)$ has all roots strictly inside the unit circle.

Moreover, asymptotic convergence with specified bounds on the rate of convergence of the parameters to their desired values require the following additional assumptions:

A4: $v(k)$ is persistently exciting.

A5: The number of sample $N \geq 2n - 2 + L$ (where $L \geq 3n - 2$).

A6: The covariance matrix P in the least-squares algorithm does not vanish to zero.

Remark 3.1 *A precise definition of persistently exciting signal is given subsequently, roughly the signal must be frequency rich (i.e. the reference signal should have enough frequency components to excite all the modes of the system).*

Remark 3.2 *The minimum required value of the block length N to establish the global convergence (i.e. convergence of the system parameters to their true values) is not necessary but sufficient. Therefore, values of N less than the minimum required condition still yield global stability (i.e. boundedness of all system signals with convergence of the equation error converging to zero). The value of L in the expression of N will later be elaborated.*

Remark 3.3 *There are a number of variants of least-squares algorithm which have the property that the covariance matrix does not vanish to zero.*

As mentioned earlier this makes the least-squares algorithms applicable to time varying systems.

3.5 Technical lemmas

In order to analyze the stability of the overall adaptive control system we need the following lemmas which appeared frequently in the literature.

Lemma 3.1 *[29] If the following conditions are satisfied for the given sequences $\{s(k)\}$, $\{\sigma(k)\}$, $\{a(k)\}$, and $\{b(k)\}$*

$$(1) \qquad \lim_{k\to\infty} \frac{s(k)^T s(k)}{a(k)+b(k)\sigma(k)^T\sigma(k)} = 0 \tag{3.27}$$

where $\{a(k)\}$, and $\{b(k)\}$ are scalar sequences and $\{\sigma(k)\}$ and $\{s(k)\}$, are real vector sequences.

(2) Uniform boundedness condition, i.e.

$$0 < a(k) < K < \infty, \quad 0 < b(k) < K < \infty, \ \textit{for all } k \geq 1 \tag{3.28}$$

(3) Linear boundedness condition,

$$||\sigma(k)|| \leq C_1 + C_2 \max_{0\leq j\leq k} |s(j)| \tag{3.29}$$

where C_1 and C_2 are finite positive numbers, then follows that:

(i) $\lim_{k\to\infty} s(k) = 0$

(ii) $\{||\sigma(k)||\}$ *is bounded*

Proof is available in appendix.

Definition 3.1 *[14] The sequence $v(k)$ is persistently exciting, of order r and persistency interval L if there exists positive ϵ_1, ϵ_2 such that for all j*

$$\epsilon_1 I \leq \sum_{k=j}^{j+L-1} \begin{pmatrix} v(k) \\ v(k-1) \\ \vdots \\ v(k-r+1) \end{pmatrix} \begin{pmatrix} v(k) & v(k-1) & \cdots & v(k-r+1) \end{pmatrix} \leq \epsilon_2 I \tag{3.30}$$

Remark 3.4 *[14] It is not hard to secure satisfaction of this condition. The signal $v(k)$ should contain at least r complex frequencies.*

Lemma 3.2 *[14] Consider the plant $p(z)y(k) = q(z)u(k)$ with $p(z)$ and $q(z)$ coprime, and suppose that a controller of the form Eqn 3.22 is used. Furthermore, suppose that $y(k)$ and $u(k)$ are bounded and $v(k)$ is persistently exciting of order $3n-2$ i. e.*

$$\epsilon_1 I \leq \sum_{k=j}^{j+L-1} \begin{pmatrix} v(k+2n-1) \\ v(k+2n-2) \\ \vdots \\ v(k-n+1) \end{pmatrix} \begin{pmatrix} v(k+2n-1) & v(k+2n-2) & \cdots & v(k-n+1) \end{pmatrix} \leq \epsilon_2 I \tag{3.31}$$

For some positive ϵ_1 and ϵ_2, some integer L and any arbitrary j. Then for $\phi(k)$ defined as

$$\phi(k) = \begin{pmatrix} y(k) & y(k-1) & \cdots & y(k-n+1) & u(k) & \cdots & u(k-n+1) \end{pmatrix} \tag{3.32}$$

there holds

$$\beta_1 I \leq \sum_{k=j}^{j+L+2n-2} \phi(k)\ \phi(k)^{\ T} \leq \beta_2 I \tag{3.33}$$

for some positive β_1 *and* β_2.

Proof is available in appendix.

From lemma 3.2, it is now clear that the regression vector $\phi(k)$ is persistently exciting if the reference input is persistently exciting It is also clear that Eqn 3.31 is satisfied if $L \geq 3n-2$. This implies that if the reference input signal is persistently exciting, the input-output signals are also persistently exciting over a different time interval. Hence, to ensure persistency of excitation of the regression vector one requires the use of block processing of length $N = 2n-2+L$. The $2n-1$ term in N is contributed by the delay in the persistency of excitation between the reference input and the system signals. Therefore if $N \geq 5n-3$ it is possible to ensure the persistency of excitation of the regression vector from the reference input. It is important to note that this condition will only be used to establish global asymptotic convergence of the parameters to their desired values. Moreover, bounds on the rate of convergence are also obtained.

3.6 Global stability analysis

The global stability proof to be given is without the use of persistency of excitation of the reference signal. Moreover, if block processing is used, then global stability is guaranteed for any value of N. This results from the fact that boundedness of all signals does not depend on the convergence of the system parameters. This reduces the assumptions to minimum. Also, it is possible to obtain asymptotic stability with bounds on the rate of convergence if the idea of block processing is used with appropriate choice of N together with the assumption of persistency of excitation.

The first part of the analysis tackles in detail the global stability of the adaptive pole assignment based on the (l.m.r.). The analysis for (r.m.r.) follows later.

3.6.1 Global stability of adaptive pole assignment based on (l.m.r.)

To start, the indirect adaptive control based on (l.m.r.) is given in detail. The assumptions stated earlier will be used whenever required in the proof. The following theorem states the complete result.

Theorem 3.2 *Given the assumptions* **A1–A3** *and either one of the algorithm statement of section 2.4, then* $e(k) \to 0$ *as* $k \to \infty$. *The sequences* $u(k)$ *and* $y(k)$ *remain bounded.*

Moreover if assumptions **A4–A6** *are satisfied, then* $\hat{q}(z) \to q(z)$, $\hat{p}\,(z) \to p(z)$ *exponentially fast as* $k \to \infty$. *Also the closed-loop* $[q^*(z) - \hat{k}\,(k_j, z)]p(z) - \hat{h}\,(k_j, z)q(z) \to q^*(z)p^*(z)$ *exponentially fast.*

For clarity of the proof, some intermediate results are first presented before establishing the final proof. The procedure can be summarized as follows: we first establish a linear boundedness of an equivalent state vector of the resultant closed-loop in terms of the equation error $e(k)$, which later, will be converted into a linear boundedness of the regression vector $\phi(k)$ in terms of the equation error. Then, the key technical lemmas of section 3.5 can be used to establish boundedness of the input-output data as well as the convergence of the equation error to zero. This global stability result requires only the assumptions **A1–A3**. To obtain asymptotic convergence, the assumptions **A4–A6** need to used. The remaining statement of the theorem 3.2 follows in a straightforward manner.

Given Eqn 3.3, the system equation, and Eqn 3.22 the feedback control law, the closed loop system becomes

$$[q^*(z)p(z) - \hat{h}\,(k_j, z)q(z) - \hat{k}\,(k_j, z)p(z)]w(k - n + 1) = q(z)v(k - n + 1) \tag{3.34}$$

$$\begin{aligned} y(k - n + 1) &= [q^*(z) - \hat{k}(k_j, z)]w(k - n + 1) \\ &= \hat{c}(z)w(k - n + 1) \end{aligned} \tag{3.35}$$

The input can then be expressed as follows:

$$u(k) = \hat{h}(k_j, z)w(k-n+1) + v(k-n+1) \tag{3.36}$$

with k_j fixed during the interval I_j and where $w(k-n+1)$ represents the partial state of the closed-loop system. The degree of the closed-loop system is $2n-1$.

Next we shall rewrite $e(k+1)$ which we recall from Eqn 2.12 as follows

$$e(k+1) = [\hat{p}\ (k,z) - p(z)]y(k-n+1) - [\hat{q}\ (k,z) - q(z)]u(k-n+1) \tag{3.37}$$

Substituting $y(k)$ and $u(k)$ by their respective expressions of Eqns 3.35 and 3.36 in Eqn 3.37, we obtain

$$\begin{aligned} e(k+1) &= M(k,z)w(k-n+1) - \hat{q}\ (k,z)v(k-n+1) \\ \text{where } M(k,z) &= [\hat{p}\ (k,z)q^*(z) - \hat{p}\ (k,z)\hat{k}\ (k_j,z) - \hat{q}\ (k,z)\hat{h}\ (k_j,z)] \end{aligned} \tag{3.38}$$

where k is variable and k_j is fixed during I_j.

The key words now are to find an equivalent state space realization of the system obtained from Eqns 3.34 and 3.38, where $e(k+1)$ is assumed to be the output of the system, and $w(k-n+1)$ is its partial state. One then, can argue that the state vector of the equivalent system is detectable from $e(k+1)$. Prior to this, one needs to establish the detectability property. This will take us to the next lemma.

Lemma 3.3 *Recalling the definitions of $M(k,z)$, together with the observation that $M(k_j,z) = q^*(z)p^*(z)$, there exists a finite time k_J beyond which the roots of $M(k,z)$ remain inside the unit circle.*

Proof: Observing that $M(k_j,z) = q^*(z)p^*(z)$, then by lemmas 2.1 or 2.2, $||\hat{\theta}(k) - \hat{\theta}(k_j)|| \rightarrow 0$ for $k \in I_j$, and $j \rightarrow \infty$. Thus given any arbitrary $\delta > 0$, there exists a k_J such that for all $j \geq J$ and $k \in I_j$

$$||\hat{\theta}(k) - \hat{\theta}(k_j|| < \delta \tag{3.39}$$

Also, since the roots of a polynomial are continuous function of the polynomial coefficients and using, the observation that the output polynomial $M(k_j,z)$ has all roots inside the unit circle, therefore there exists a region in the parameter space centered on $\hat{\theta}(k_j)$ and having radius $\bar{\delta} > 0$ sufficiently small such that if

$$||\hat{\theta}(k) - \hat{\theta}(k_j)|| < \bar{\delta} \tag{3.40}$$

it implies that the roots of $M(k,z)$ remain inside the unit circle during the whole interval I_j. Thus by choosing $\delta = \bar{\delta}$, we guarantee that for $k_j \geq k_J$, all roots of the output polynomial remain inside the unit circle. This completes the proof.

It is now possible to state that the system obtained from the resultant closed-loop and identification error equations is detectable because any possible pole-zero cancellation is a root of the desired closed-loop polynomial which is stable by definition.

Having established a detectability property we shall next find an equivalent state space realization.

Lemma 3.4 *Given the assumptions* **A1–A3**, *and the above definition Eqn 3.38 of* $e(k+1)$, *then defining* $D(z) = [q^*(z) - \hat{k}\ (k_j, z)]p(z) - \hat{h}\ (k_j, z)q(z)$ *and* $Q(z) = q(z)$ *during the interval* I_j, *the closed-loop system of Eqn 3.34 has an equivalent state space realization of the form:*

$$x(k+1) = Ax(k) + bv(k-n+1) \tag{3.41}$$

with

$$x(k) = t_D(z)w(k-n+1) + t_Q(z)v(k-n+1) \tag{3.42}$$

$$t_D(z)^T = [t_D^{2n-1}(z) \cdots t_D^1(z)] \tag{3.43}$$

$$t_Q(z)^T = [t_Q^{2n-1}(z) \cdots t_Q^1(z)] \tag{3.44}$$

The polynomials $t_D^i(z)$ *and* $t_Q^i(z)$ *are the Tchirnhausen Polynomials*[1] *for* $D(z)$ *and* $Q(z)$, *respectively defined as*

$$t_D^i(z) = \sum_{j=0}^{2n-i-1} D_j z^{2n-i-j-1} \tag{3.45}$$

[1] the state space model obtained for $x(k)$ is in observer form. Thus the coefficient of the resulting state space model appear directly in the corresponding left difference model [29].

$$t_Q^i(z) = \sum_{j=1}^{2n-i-1} Q_j z^{2n-i-j-1} \qquad (3.46)$$

$$\textit{with} \quad D(z) = z^{2n-1} + \sum_{j=1}^{2n-2} D_j z^{2n-j-1} \qquad (3.47)$$

$$\textit{and} \quad Q(z) = q(z) = \sum_{j=1}^{2n-2} Q_j z^{2n-j-1}$$

It is possible to show that the entries of the matrix A and vector b are directly related to the coefficients of the polynomials $D(z)$, $Q(z)$ as follows:

$$A = \begin{pmatrix} -D_1 & 1 & 0 & \cdots & 0 \\ -D_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -D_{2n-2} & 0 & 0 & \cdots & 1 \\ -D_{2n-1} & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$b^T = \begin{pmatrix} Q_1 & Q_2 & \cdots & Q_{2n-1} \end{pmatrix}$$

Moreover the output error $e(k+1)$ can be expressed as

$$e(k+1) = cx(k) + d(z)v(k-n+1) \qquad (3.48)$$

for some vector c and polynomial $d(z)$, with the equivalence relation valid over the interval I_j. Similarly, the entries of the row vector c, and polynomial coefficient of $d(z)$, have simple relation with the coefficients of $D(z)$ and $Q(z)$.

Proof: From [29], defining $x(k)$ in Eqn 3.42 as state vector, then the resultant closed-loop system Eqn 3.34 is equivalent to Eqn 3.41, with the equivalence relation holding over the interval I_j. Next, let us define

$$t_D(z) = T_D \Psi(z) \qquad (3.49)$$

where

$$\Psi(z)^T = \begin{pmatrix} 1 & z & \cdots & z^{2n-2} \end{pmatrix} \tag{3.50}$$

A comparison between Eqns 3.43 and 3.49, reveals that T_D is a square non-singular matrix of the following form:

$$T_D = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ D_1 & 1 & 0 & \cdots & 0 \\ D_2 & D_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D_{2n-2} & D_{2n-3} & D_{2n-4} & \cdots & 1 \end{pmatrix} \tag{3.51}$$

substituting in Eqn 3.42, we obtain:

$$\Psi(z)w(k-n+1) = T_D^{-1}x(k) - T_D^{-1}t_Q(z)v(k-n+1) \tag{3.52}$$

With the definition of $\bar{\Psi}(z)$ as $\begin{pmatrix} 1 & z & \cdots & z^{2n-1} \end{pmatrix}$, the polynomials $M(k, z)$ and $D(k_j, z)$ can be rewritten as:

$$\begin{aligned} M(z) &= \begin{pmatrix} M_{2n-1} & M_{2n-2} & \cdots & M_0 \end{pmatrix} \bar{\Psi}(z) \\ &= \bar{M}\Psi(z) + M_0 z^{2n-1} \\ \text{and} \qquad D(z) &= \begin{pmatrix} D_{2n-1} & D_{2n-2} & \cdots & 1 \end{pmatrix} \bar{\Psi}(z) \\ &= \bar{D}\Psi(z) + z^{2n-1} \end{aligned}$$

From Eqn 3.38, the error $e(k+1)$ is a linear functional of $\Psi(z)w(k-n+1)$, the reference input $v(k)$ and the $w(k+n)$ term which can be written as

$$\begin{aligned} e(k+1) &= \{\bar{M}\Psi(z) + M_0 z^{2n-1}\}w(k-n+1) - \hat{q}(k,z)v(k-n+1) \\ &= \bar{M}\Psi(z)w(k-n+1) + M_0 w(n+1) - \hat{q}(k,z)v(k-n+1) \end{aligned}$$

From Eqn 3.34, we have

$$w(k+n) = -\bar{D}\Psi(z)w(k-n+1) + q(z)v(k-n+1)$$

Thus, the equation error becomes:

$$e(k+1) = (\bar{M} - M_0\bar{D})\Psi(z)w(k-n+1) + (M_0 q(z) - \hat{q}(k,z))v(k-n+1) \tag{3.53}$$

Using Eqn 3.52, it is now possible to express $e(k+1)$ in the form Eqn 3.48 as follows:

$$e(k+1) = (\bar{M} - M_0\bar{D})T_D^{-1}x(k) + \{M_0 q(z) - \hat{q}(k,z) - (\bar{M} - M_0\bar{D})T_D^{-1}t_Q(z)\}v(k-n+1)$$

This ends the proof of lemma 3.4.

Using the results of lemmas 3.3-3.4, we conclude that the state vector $x(k)$ mapping the input $v(k)$ to the output error $e(k+1)$ remains detectable while $k \geq k_J$.

The next lemma makes use of the detectability property to establish a linear boundedness of the state vector $x(k)$ in terms of the output error $e(k+1)$.

Lemma 3.5 *Assume the same hypotheses as lemma 3.4, together with the definition Eqn 3.38 of* $e(k+1)$*, and with the result of Lemma 3.3 and 3.4, (i.e. that the closed-loop system is detectable), then*

$$||x_m(j)|| \leq C_1 \max_{0 \leq i \leq j} |e_m(i)| + C_2 \tag{3.54}$$

with $x_m(j) = \max_{k \in I_j} x(k)$ *and* $e_m(j) = \max_{k \in I_j} e(k+1)$, *and where* C_1 *and* C_2 *are positive constants.*

Proof: Let us consider the equivalent state space model given by Eqns 3.41 and 3.48 and assume $\bar{n}$ zero-pole cancellations, then there exists a linear transformation, $x(k) = Q\bar{x}(k)$, which transforms the system of Eqns 3.41 and 3.48 into the following state space form:

$$\begin{pmatrix} \bar{x}^{\,u}(k+1) \\ -- \\ \bar{x}^{\,o}(k+1) \end{pmatrix} = \begin{pmatrix} A_{11} & | & A_{12} \\ -- & & -- \\ 0 & | & A_{22} \end{pmatrix} \begin{pmatrix} \bar{x}^{\,u}(k) \\ -- \\ \bar{x}^{\,o}(k) \end{pmatrix} + \begin{pmatrix} b_1 \\ -- \\ b_2 \end{pmatrix} v(k-n+1) \tag{3.55}$$

where $\bar{x}^u$ denotes the unobservable state vector of order $(\bar{n} \times 1)$, and $\bar{x}^o$ denotes the observable state vector of order $(2n-1-\bar{n} \times 1)$, and

$$e(k+1) = \begin{pmatrix} 0 & c_2 \end{pmatrix} \begin{pmatrix} \bar{x}^{\,u}(k) \\ --- \\ \bar{x}^{\,o}(k) \end{pmatrix} + d(z)v(k-n+1) \tag{3.56}$$

we proceed to the analysis by induction. It is possible to verify that Eqn 3.54 is true for the interval I_0 when $\bar{x}(0)$ is bounded because the system can change only by a finite amount during any finite time interval. Let us assume that Eqn 3.54 is true for $j-1$, next we need to prove that, it is also true for j. The observable states $\bar{x}^{\,o}$ of $\bar{x}\,(k)$ can be obtained for an interval of suitable length by observing the output $e(k+1)$ and reference input $v(k)$ over the same interval.

Hence during the interval of time I_j, there exists positive constants K_1 and $\bar{K}_2$ such that

$$||\bar{x}\,{}^{o}_{m}(j)|| \leq K_1|e_m(j)| + \bar{K}_2 \max|v(k)| \qquad (3.57)$$

Since the sequence $\{v(k)\}$ is bounded then

$$||\bar{x}\,{}^{o}_{m}(j)|| \leq K_1|e_m(j)| + K_2\ ,\ K_2 > 0 \qquad (3.58)$$

For the unobservable states, we can derive similar result, if one observes that by successive substitution of Eqn 3.55, the solution of $\bar{x}\,{}^{u}(k)$ for $k \in I_j$ given $\bar{x}(k_j)$ is

$$\bar{x}\,{}^{u}(k) = A_{11}^{k-k_j}\bar{x}^{u}(k_j) + \sum_{i=k_j+1}^{k} A_{11}^{k-i}b_1\bar{v}(i-1) + \sum_{i=k_j+1}^{k} A_{11}^{k-i}A_{12}\bar{x}^{o}(i-1) \qquad (3.59)$$

where $v(k-n+1)$ is replaced by $\bar{v}(k)$ to simplify the notation. Then taking the norm of both sides we have:

$$\begin{aligned} ||\bar{x}\,{}^{u}(k)|| \leq\ & ||A_{11}^{k-k_j}||||\bar{x}^{u}(k_j)|| + \sum_{i=k_j+1}^{k} ||A_{11}^{k-i}||||b_1|||\bar{v}(i-1)| + \\ & \sum_{i=k_j+1}^{k} ||A_{11}^{k-i}||||A_{12}||||\bar{x}^{o}(i-1)|| \\ \leq\ & K_3||\bar{x}^{u}(k_j)|| + K_6 \sum_{i=k_j+1}^{k} \lambda^{k-i} \max|\bar{v}(i-1)| + K_7 \sum_{i=k_j+1}^{k} \lambda^{k-i}||\bar{x}^{o}(i-1)|| \end{aligned} \qquad (3.60)$$

where K_3, K_6 and K_7 are finite real positive numbers.

We have used the fact that A_{11} is asymptotically stable, then $||A_{11}^i|| \leq K\lambda^i$ where $0 \leq \lambda < 1$ and $0 \leq K < \infty$. Eqn 3.60 can be simplified to give

$$\begin{aligned} ||\bar{x}^u(k)|| &\leq K_3||\bar{x}^u(k_j)|| + K_6 \sum_{i=k_j+1}^{k} \lambda^{k-i} + K_7 \max_{k\in I_j} ||\bar{x}^o(i-1)|| \sum_{i=k_j+1}^{k} \lambda^{k-i} \\ &\leq K_3||\bar{x}^u(k_j)|| + K_5 + K_4||\bar{x}_m^o(j)|| \qquad (3.61) \end{aligned}$$

where K_4 and K_5 are all positive constants. Since $k_j \in I_{j-1}$, then using Eqns 3.61 and 3.58 we have

$$\begin{aligned} ||\bar{x}_m(j)|| &\leq ||\bar{x}_m^o(j)|| + ||\bar{x}_m^u(j)|| \\ &\leq \bar{C}_1 \max_{0\leq i\leq j} |e_m(i)| + \bar{C}_2 \qquad (3.62) \end{aligned}$$

where $\bar{C}_1$ and $\bar{C}_2$ are positive constants. Since the state vector $x(k)$ is related to $\bar{x}(k)$ through a linear transformation, therefore Eqn 3.54 follows from Eqn 3.62.

Our real interest is to find a linear boundedness on the regression vector rather than on the state vector. The next lemma shows how the result carries over to $\phi(k)$.

Lemma 3.6 *Assume the same hypotheses as lemma 3.4 and recalling the definition Eqn 3.41 of $x(k)$, and the result of lemma 3.5, then there exists constants C_3 and C_4 such that*

$$||\phi(k)|| \leq C_3 \max_{0\leq i\leq j} |e_m(i)| + C_4 \qquad (3.63)$$

when $k \in I_j$ and $\phi(k)$ is as defined in Eqn 3.32.

Proof: If one recalls that the terms involving the output of the controlled system in $\phi(k)$ can be represented using Eqn 3.35 as follows:

$$y(k-l) = \hat{c}(z)w(k-l) = \sum_{i=0}^{n-1} \hat{c}_i w(k-l+i) \qquad (3.64)$$
$$\text{for } l = 0 \cdots n-1$$

For a compact expression of the entries of the regression vector that involves the outputs in terms of the state vector one can consult appendix A.4.

From Eqn 3.64 and the definition Eqn 3.32 of $\phi(k)$, we observe that all terms of $\phi(k)$ involving the output are functional of $w(k+n-1)$ to $w(k-n+1)$. Hence the entries of the vector $\phi(k)$ can be obtained from the state vector $x(k)$ through a linear functional as seen from Eqn 3.52, plus a term involving the reference input $v(k)$. Thus the model outputs contained in $\phi(k)$ are linearly bounded by $e(k+1)$. Similarly we proceed for the remaining term of $\phi(k)$ involving the input which can be represented using Eqn 3.36 as follows:

$$\begin{aligned} u(k-l) &= \hat{h}(z)w(k-l) + v(k-l) \\ &= \sum_{i=0}^{n-1} \hat{h}_i w(k-l+i) + v(k-l) \qquad (3.65) \\ &\text{for } \ l = 0 \cdots n-1 \end{aligned}$$

For a compact expression of the entries of the regression vector that involves the inputs in terms of the state vector one can consult appendix

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A.4.

Then for $l = 0$ to $n-1$ the input involves terms of $x(k)$ plus terms involving the reference input $v(k)$, and hence the result follows immediately.

Corollary 3.1 *Assume the same hypotheses as lemma 3.4, then as* $k \to \infty$, $e(k) \to 0$, *and* $||\phi(k)||$ *remains bounded.*

Proof: Using the result of the lemma 3.1 and 3.6 one concludes that $e(k+1) \to 0$ as $k \to \infty$. Also we conclude that $||\phi(k)||$ remains bounded.

This completes the proof of the theorem 3.2 that deals with the input-output boundedness and the convergence of the equation error to zero. The only assumptions that were used so far are **A1–A3**. However, nothing has been said about the convergence of the system parameters to the true values. Therefore it is important to note that the global convergence result is unconditional on the convergence of the parameters to their desired values and hence is independent on the persistency of excitation requirement. To prove the remaining part of the theorem 3.2, we need the use of the remaining assumptions, namely the length of the block length $N \geq 5n-3$ and a persistently exciting reference signal.

Lemma 3.7 *Given the assumptions* **A1–A5** *and algorithm statement 3.1, then* $\hat{\theta}(k) \to \theta_o$ *as* $k \to \infty$ *asymptotically. If* **A6** *is satisfied, then the convergence is exponential.*

Proof: The proof will first be given for standard least-squares.

Case 1 Standard least-squares:

To show the convergence for the case of standard least-squares, define the Lyapunov function [29]

$$V(k) = \tilde{\theta}(k)\ ^T P(k-1)^{-1}\tilde{\theta}(k) \tag{3.66}$$

where $\tilde{\theta}(k) = \hat{\theta}(k) - \theta_o$ and $P(-1)^{-1} = \sigma I$, $\sigma > 0$

Applying the matrix inversion lemma to the covariance matrix formula [29], we can write

$$V(k_j) = \tilde{\theta}(k_j)^T[(P(-1)^{-1} + \sum_{k=0}^{k_j} \phi(k)\phi(k)^T)]\tilde{\theta}(k_j) \tag{3.67}$$

Dividing the time axis into two time frame, one over each interval of length N and the other over the union of these intervals, one can write:

$$V(k_j) = \tilde{\theta}(k_j)^T[(P(-1)^{-1} + \sum_{i=0}^{j}\sum_{l=0}^{N-1} \phi(Ni+l)\phi(Ni+l)^T)]\tilde{\theta}(k_j) \tag{3.68}$$

Using the lower bound condition of Eqn 3.33 which is satisfied if $N \geq 5n-3$, it is possible to write:

$$V(k_j) \geq (\sigma + j\beta_1)||\tilde{\theta}(k_j)||^2 \tag{3.69}$$

Since $V(k)$ is nonnegative, nonincreasing (see appendix A.2), hence it is bounded. Thus, the subsequence $\{||\tilde{\theta}(k_j)||, j = 0, 1, \cdots\}$ must converge to zero as j increases. Since $V(k)$ is nonincreasing, it can be readily seen from Eqn 3.66 that $\{ ||\tilde{\theta}(k_j + i)||^2 \leq ||\tilde{\theta}(k_j)||^2$ for $i = 1, 2, \ldots N\}$. Thus we have that the sequence $\{||\tilde{\theta}(k)||, \ k = 0, 1, \ldots\}$ converges to zero.

Case 2 least-squares with covariance resetting

For the case of covariance reset, define $V(k)$ as before, but observe that in this case, at the reset instances k' (for simplicity assume k' as subset of the set of instance k_j), the scalar function $V(k)$ becomes

$$V(k') = \sigma||\tilde{\theta}(k')||^2 \tag{3.70}$$

Applying the matrix inversion lemma to the covariance matrix formula [29], we can write

$$V(k_j) = \tilde{\theta}(k_j)^T[P(k_{j-1})^{-1} + \sum_{l=1}^{N} \phi(k_{j-1} + l)\phi(k_{j-1} + l)^T]\tilde{\theta}(k_j) \tag{3.71}$$

Assuming that $k_{j-1} \leq k' < k_j$ then, using Eqn 3.33, which is satisfied if $N \geq 5n - 3$ (assumption **A5**) and the persistency requirement of the reference signal (assumption **A4**), the definition of $V(k)$, we have

$$V(k_j) \geq (\sigma + \beta_1)||\tilde{\theta}(k_j)||^2 \tag{3.72}$$

Now using the monotonicity of $V(k)$, and observing that $k_{j-1} \leq k' < k_j$ we have

$$V(k_j) \leq V(k') \leq \sigma||\tilde{\theta}(k_{j-1})||^2 \tag{3.73}$$

combining Eqns 3.72 and 3.73 yields

$$||\tilde{\theta}(k_j)||^2 \leq \frac{\sigma}{\sigma + \beta_1}||\tilde{\theta}(k_{j-1})||^2 \tag{3.74}$$

Since $\beta_1 > 0$, we can conclude that the subsequence $\{||\tilde{\theta}(k_j)||, j = 0, 1, \ldots\}$ is exponentially convergent to zero. Moreover, since $V(k)$ is nonincreasing, it is possible to show that $\{||\tilde{\theta}(k_j + i)||^2 \leq ||\tilde{\theta}(k_j)||^2$ for $i = 1, 2, \ldots N\}$. Thus we have that the sequence $\{||\tilde{\theta}(k)||, k = 0, 1, \ldots\}$ converges exponentially fast to zero.

For the case of least squares with exponential data weighting or the least squares with covariance modification we have

$P^{-1}(k) \geq \sigma I$ for any arbitrary time k.

Hence the proof follows in a manner similar to the least squares with covariance reset.

Recalling the result of Lemma 3.7 and Corollary 3.1, the complete proof of Theorem 3.2 is established.

This completes the global stability proof of the first case namely the adaptive pole assignment based on (l.m.r.).

3.6.2 Global stability of adaptive pole assignment based on (r.m.r.)

To avoid repetition of what has been presented in the case of (l.m.r.), we limit ourselves to the discussion of the main differences between the two adaptive algorithms. Boundedness of all system signals will be established first together with the convergence of the the equation error to zero. The asymptotic convergence of the parameters to their desired values is then given.

Recalling Eqn 2.12, the equation error can be rewritten as

$$\begin{aligned} e(k+1) &= [\hat{p}\ (k_j, z) - p(z)]y(k-n+1) + [r(z) - \hat{r}\ (k_j, z)]u(k-n+1) \\ &= M(k,z)w(k-n+1) \qquad (3.75) \\ M(k,z) &= \hat{r}\ (k,z)p(z) - \hat{h}\ (k,z)r(z) \qquad (3.76) \end{aligned}$$

Similarly, the key words now is to find an equivalent state space realization of the combined system obtained from the output error $e(k+1)$ and the resultant closed-loop obtained from use of the feedback law Eqn 3.24. The next lemma shows how this can be obtained.

Lemma 3.8 *By applying the feedback control law Eqn 3.24 to the system defined by Eqns 3.1 and 3.2, the following resultant closed-loop system is obtained:*

$$D(k_j, z)w(k-n+1) = q^*(z)v(k-n+1) \qquad (3.77)$$

$$D(k_j, z) = p(z)q^*(z) - p(z)\hat{k}\ (k_j, z) - r(z)\hat{h}\ (k_j, z) \qquad (3.78)$$

The resultant closed-loop system has an equivalent state space realization of the form Eqn 3.41 where the state vector is defined by:

$$\begin{aligned} x(k) &= t_D(z)w(k-n+1) + t_Q(z)v(k-n+1) \qquad (3.79) \\ t_D^i(z) &= \sum_{j=0}^{2n-i-1} D_j z^{2n-i-j-1} \qquad (3.80) \\ t_Q^i(z) &= \sum_{j=0}^{2n-i-1} Q_j z^{2n-i-j-1} \qquad (3.81) \end{aligned}$$

with $Q(z) = q^*(z)$ *and where* $t_D^{(l)}(z)$ *and* $t_Q^{(l)}(z)$ *are the Tchirnhausen polynomials of* $D(z)$ *and* $Q(z)$ *respectively. Moreover the error* $e(k+1)$ *in Eqn 3.75 can be expressed as*

$$e(k+1) = cx(k) + d(z)v(k-n+1) \tag{3.82}$$

for some row vector c *and polynomial* $d(z)$ *depending only on the coefficients of* $D(k_j, z)$*, and the desired closed-loop polynomials* $p^*(z)$ *and* $q^*(z)$ *coefficients, with the equivalence relation valid over the interval* I_j.

Proof The proof of this lemma follows the same manner as discussed in lemma 3.4. The first part of the proof follows directly if we observe that the system Eqn 3.77 has similar structure to system Eqn 3.34, and hence an equivalent state space realization of the form Eqn 3.41 can be obtained by using the transformation of Eqn 3.79.

The second part of the proof that involves writing the equation error in term of the state space vector and the reference input follows in a similar manner by observing that the expression of the equation error in 3.75 is similar to Eqn 3.38. Hence, the equation error will have the following form:

$$e(k+1) = (\bar{M} - M_0\bar{D})T_D^{-1}x(k) + \{M_0q^*(z) - (\bar{M} - M_0\bar{D})T_D^{-1}t_Q(z)\}v(k-n+1) \tag{3.83}$$

where $\bar{M}$, $\bar{D}$, and M_0 retain the same maining as given in the proof of lemma 3.4, but with $M(z)$ and $D(z)$ given by Eqns 3.76 and 3.78. This completes the proof of the lemma.

The next step is to establish detectability property of the system obtained from the closed-loop and output polynomials over the interval $k_j \geq k_J$. This results from the fact that the output polynomial $M(k, z)$ and the resultant closed-loop polynomial $D(k_j, z)$ have possible pole zero cancellation only inside the unit circle.

Lemma 3.9 *Given the definitions of $D(k_j, z)$, $M(k_j, z)$, and the diophantine equation $[q^*(z) - \hat{k}\ (k_j, z)]\hat{p}(k_j, z) - \hat{h}\ (k_j, z)\hat{r}(k_j, z) = q^*(z)p^*(z)$ then all possible exact pole-zero cancellations lie inside the unit circle.*

Moreover, there exist a lower bound distance between each root of $D(k_j, z)$ and the roots of $M(k_j, z)$ outside the unit circle.

Proof: Let us consider z_o as a common root for both the output polynomials $M(k_j, z)$ and the resultant closed-loop $D(k_j, z)$ defined by Eqns 3.76 and 3.78 respectively. Then, we have

$$\hat{p}\ (k_j, z_o)r(z_o) - \hat{r}\ (k_j, z_o)p(z_o) = 0 \tag{3.84}$$

$$\hat{h}\ (k_j, z_o)r(z_o) - [q^*(z_o) - \hat{k}\ (k_j, z_o)]p(z_o) = 0 \tag{3.85}$$

Now either $p(z_o)$ or $r(z_o)$ is non-zero since $p(z)$ and $r(z)$ are coprime by assumption **A1**, hence we get:

$$\frac{r(z_o)}{p(z_o)} = \frac{\hat{r}\ (k_j, z_o)}{\hat{p}\ (k_j, z_o)} = \frac{q^*(z_o) - \hat{k}\ (k_j, z_o)}{\hat{h}\ (k_j, z_o)} \tag{3.86}$$

The last two terms of Eqn 3.86 together with Eqn 3.25 yield:

$$[q^*(z_o) - \hat{k}\ (k_j, z_o)]\hat{p}(k_j, z_o) - \hat{h}\ (k_j, z_o)\hat{r}(k_j, z_o) = q^*(z_o)p^*(z_o) = 0 \tag{3.87}$$

It implies that z_o is a root of either $q^*(z_o)$ or $p^*(z_o)$ which are stable by (assumption **A3**).

We shall next prove that there exist a lower bound distance separation between all roots of the output and the resultant closed-loop polynomials that are outside the unit circle.

Suppose the claim is not true. Let us assume that $M(k_j, z_o) = 0$, then there exist an infinite sequence z_i with limit z_o such that $\lim_{z_i \to z_o} D(k_j, z_i) = 0$. Since $r(z)$ and $p(z)$ are prime polynomials, with lower bound distance between each root of $r(z)$ and all roots of $p(z)$, therefore z_o and the limit of the infinite sequence z_i cannot be common roots of both $r(z)$ and $p(z)$.

Moreover, since the roots of these polynomials are continuous function of their coefficients, therefore we have

$$\frac{r(z_o)}{p(z_o)} = \lim_{z_i \to z_o} \frac{r(z_i)}{p(z_i)} \tag{3.88}$$

This implies that

$$\hat{h}\ (k_j, z_o)\hat{r}(k_j, z_i) - [q^*(z_o) - \hat{k}\ (k_j, z_o)]\hat{p}(k_j, z_i) = 0 \tag{3.89}$$

Taking the $\lim_{z_i \to z_o}$ of the left hand side of Eqn 3.89, it is possible to show that z_o should be a root of the desired closed-loop polynomial which is by definition inside the unit circle.

This yields an immediate contradiction to the assumption that z_o is outside the unit circle. This completes the proof.

It is possible to show that the output and closed loop polynomials have possible pole zero cancellations only inside the unit circle, beyond a finite time instant k_J, in a way similar to the discussion of the (l.m.r).

The next lemma establishes the detectability of the state vector from the output error $e(k+1)$ beyond some finite time instant k_J.

Lemma 3.10 *Given the system Eqns 3.75 and 3.77 and its equivalent state space given by Eqns 3.41 and 3.82; the state variable $x(k)$ defined by Eqn 3.79 mapping the input $v(k)$ into $e(k+1)$, is detectable.*

Proof: Since the system defined by Eqns 3.75 and 3.77 is equivalent to Eqns 3.41 and 3.82 (by Lemma 3.8), then from [30], the state $x(k)$ is detectable from $e(k+1)$. This completes the proof of the lemma.

The remaining part of the proof follows, in a straight forward manner, the analysis of the (l.m.r.) given in detail in section 3.6.1.

It is important to mention that the analysis of the proof of global stability when the projection estimation scheme is used follows in a similar manner to the proof given above for the least-squares. This follows from the fact that the stability analysis given here is only based on the common properties of these estimation schemes.

To conclude this chapter, I would like to summarize the important points. The global stability of indirect pole assignment based on the (l.m.r.) and (r.m.r.) has been presented under relatively weak assumptions in comparison to [3,14]. The global stability obtained is unconditional on the convergence of the parameters to their desired values. This results is obtained under the assumptions **A1–A3**. So, uniform boundedness of all signals and the convergence of the equation error is guaranteed for arbitrary values of N if block processing is used.

However, to obtain asymptotic convergence with bounds on the rate of convergence we require the use of block processing with values of $N > 5n - 3$, as well as persistency of excitation of the reference signal.

CHAPTER FOUR

INDIRECT ADAPTIVE POLE PLACEMENT FOR MULTIVARIABLE SYSTEMS

4.1 Introduction

The adaptive algorithms discussed in the previous chapter namely, indirect adaptive approaches with control strategies based either on the (l.m.r.) or (r.m.r.) cannot be extended in a straightforward manner to the MIMO case. In fact, the adaptive control based on the right model representation requires the estimation of the parameters of the right model representation. These in general cannot be estimated directly using the standard estimation schemes. This was not a problem in the SISO case because the parameters of both representations are related in simple manner.

As pointed out earlier, the indirect adaptive control has some advantages over the direct approach in the sense that it can handle non-minimum phase systems. Also, it is natural to expect prior knowledge regarding physical quantities in a system to be more easily mapped into prior knowledge of parameters in an input-output model of the process, than into prior knowledge regarding control parameters. This simplification is critical to practical applications of multivariable adaptive control. The algorithm presented here requires less number of estimated parameters to find the feedback control in comparison to the algorithm given in reference [7].

In this chapter we consider the problem of adaptive control of multivariable systems, using the indirect adaptive pole assignment algorithm that is based on the left model representation. As pointed out in the previous chapter, the usual approach in tackling the scalar version of this problem involves inversion of $(2n \times 2n)$ dimensional Sylvester resultant matrix. In extending the same idea to the multivariable case, one faces the uphill task of finding the pseudo inverse of $\nu(l+m) \times (n+l\nu)$ dimensional generalized Sylvester resultant matrix, where n is the system older, ν the observability index, while l and m denote the number of inputs and outputs of the plant respectively.

A new algorithm that requires only the inversion of an $(n \times n)$ matrix has been given in [31] to solve the diophantine equation. So the on-line solution of the diophantine equation does not really represent a problem for large order systems.

The organization of this chapter is as follows. Starting with a brief introduction to the modeling assumptions and a fixed control strategy that achieves the desired closed-loop pole assignment, we then proceed to the technical lemmas required for stability analysis. The issue of global stability is discused in section 4.5. We have included a few remark regarding the implementation of the proposed scheme.

4.2 Modeling and fixed control strategy

Consider the control of an l-input, m-output linear time-invariant system modeled by the state equation

$$x(k+1) = Ax(k) + Bu(k) \tag{4.1}$$

$$y(k) = Cx(k) \tag{4.2}$$

where $u(k)$ is the $(l \times 1)$ input vector, $y(k)$ is the $(m \times 1)$ output vector, $x(k)$ is the $(n \times 1)$ state vector. It is assumed that the system represented by Eqns 4.1 and 4.2 is both controllable and observable. For this state space representation, let the controllability indices, μ_i, $1 \leq i \leq l$, and the observability indices ν_i, $1 \leq i \leq m$, be defined in the standard manner by sequencing through the columns of the controllability and transposed observability matrices from left to right finding the first n independent columns [29].

As shown in [29,30], and explained earlier, systems of the form of Eqns 4.1 and 4.2 can also be represented in the operator form as follows:

$$P(z)y(k) = Q(z)u(k) \tag{4.3}$$

Without loss of generality, it can be assumed that $\partial_{r_i}[P(z)] = \nu_i$ and that $\Gamma_r[P(z)] = \Gamma$ is a lower triangular matrix with ones on the main diagonal, and where $\partial_{r_i}[P(z)]$ denotes the highest polynomial degree in the ith row of $P(z)$, and the real $(m \times m)$ matrix $\Gamma_r[P(z)]$ consists of the coefficients of z^{ν_i} terms in each row of $P(z)$. In the proposed algorithm $P(z)$ and $Q(z)$ are assumed to be unknown, but the system observability indices ν_i (all of which are equal), and an upper bound μ on the system controllability index, are assumed to be known.

4.2.1 Fixed control structure

Consider the following control structure [27]

$$u(k) = H(z)C(z)^{-1}y(k) + v(k) \tag{4.4}$$

$$\text{with} \qquad C(z) = Q^*(z) - K(z), \tag{4.5}$$

defining,

$$y(k) = [Q^*(z) - K(z)]w(k) \tag{4.6}$$

then using the representation of Eqn 4.3 and the control structure of Eqn 4.4, the closed-loop equation becomes

$$D(z)w(k) = Q(z)v(k) \tag{4.7}$$

$$\text{where } D(z) = P(z)Q^*(z) - P(z)K(z) - Q(z)H(z) \tag{4.8}$$

Using the dual result of the diophantine equation given in [30], one can then obtain a unique pair of polynomials matrices $K(z)$ and $H(z)$ such that

$$P(z)Q^*(z) - P(z)K(z) - Q(z)H(z) = P^*(z)Q^*(z) \tag{4.9}$$

for arbitrary polynomial matrices $P^*(z)$ and $Q^*(z)$ (desired closed-loop matrix polynomials).

In order to ensure that the solution of Eqn 4.9 exists (which also guarantees that the control law Eqn 4.4 is physically realizable), constraints must be imposed on the structure of $H(z)$, $K(z)$, $Q^*(z)$, $P^*(z)$ [30]. A set of sufficient conditions that meet these requirements can be obtained in particular by setting:

$$Q^*(z)^T = \begin{pmatrix} z^{\mu-1} & 0 & \dots & q_{m1}(z) \\ -1 & z^{\mu-1} & \dots & q_{m2}(z) \\ 0 & -1 & \dots & q_{m3}(z) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z^{\mu-1} + q_{mm}(z) \end{pmatrix} \tag{4.10}$$

where

$$q_{mi} = \sum_{j=0}^{\mu-2} a_{(i-1)(\mu-1)+j}\, z^j \tag{4.11}$$

for $i = 1, 2, \ldots m$

$$H(z) = \sum_{i=0}^{\mu-1} H_i z^i \tag{4.12}$$

$$K(z) = \sum_{i=0}^{\mu-2} K_i z^i \tag{4.13}$$

$$\Gamma_r[P^*(z)] = I \tag{4.14}$$

Remark 4.1 *evaluating* $\det Q^*(z)$ *by last row minor, it follows that*

$$\det Q^*(z) = a_0 + a_1 z + \ldots + a_{m\mu-m-1}\, z^{m\mu-m-1} + z^{m\mu-m} \tag{4.15}$$

and therefore, any arbitrary polynomial of degree $m\mu - m$ *can be chosen as* $\det Q^*(z)$ *through appropriate selection of* $q_{mi}(z)$.

Remark 4.2 *It is of interest to note that in [31], a new computational algorithm is given for solving the polynomial matrix diophantine equation, which involves the inversion of only a single real matrix of dimension n, where n is the system order.*

Remark 4.3 *In [28], a left division algorithm for polynomial matrices, that need real matrix multiplications alone, is presented. Thus when used along with the algorithm of [31] that solves the diophantine equation, calculations can be tremendously simplified.*

4.3 Technical lemma

The analysis of discrete-time multivariable adaptive control algorithm require the following technical results. The result of the following lemma is not a necessary condition for global stability i.e. boundedness of all signals with convergence of the equation error to zero. It is only required to obtain asymptotic convergence of the system parameters with specified bounds on the rate of convergence.

Definition 4.1 *[32] A sequence of vectors $f(k)$ of finite dimension is said to be persistently exciting of order r if there exists positive ϵ_1, ϵ_2, and an integer L such that for all j*

$$\epsilon_1 I \leq \sum_{k=j}^{j+L-1} \begin{pmatrix} f(k) \\ f(k-1) \\ \vdots \\ f(k-r+1) \end{pmatrix} \begin{pmatrix} f(k) & f(k-1) & \cdots & f(k-r+1) \end{pmatrix} \leq \epsilon_2 I \tag{4.16}$$

Lemma 4.1 *Consider the plant Eqn 4.3 with $Q(z)$ and $P(z)$ left coprime. Suppose further that the controller Eqn 4.4 is used. Then if the external reference signal is persistently exciting of order r i.e.*

$$\epsilon_1 I \leq \sum_{k=j}^{j+L-1} \begin{pmatrix} v(k+r-1) \\ v(k+r-2) \\ \vdots \\ v(k) \end{pmatrix} \begin{pmatrix} v(k+r-1) & v(k+r-2) & \cdots & v(k) \end{pmatrix} \leq \epsilon_2 I \tag{4.17}$$

with $(r = 2n + m\mu + \mu + \nu - m - 2)$, *for some positive* ϵ_1 *and* ϵ_2, *some integers* L *and* j. *Then with* $\phi(k+\nu-1)$ *defined as*

$$\phi(k+\nu-1)^T = \begin{pmatrix} y(k+\nu-1)^T & \cdots & y(k)^T & u(k+\nu-1)^T & \cdots & u(k)^T \end{pmatrix} \tag{4.18}$$

the following is true

$$\delta_1 I \leq \sum_{k=j}^{j+L-1+2n+m\mu-m} \phi(k+\nu-1)\phi(k+\nu-1)^T \leq \delta_2 I \tag{4.19}$$

for some positive δ_1 *and* δ_2.

Proof: The upper bound is trivial if one assume that the sequences $\{u(k)\}$ and $\{y(k)\}$ are bounded. Suppose now the lower bound fails i.e. there exists a vector $\begin{pmatrix} \alpha_1^0 & \alpha_2^0 & \cdots & \alpha_m^0 & \alpha_1^1 & \cdots & \alpha_m^{\nu-1} & \beta_1^0 & \cdots & \beta_l^{\nu-1} \end{pmatrix}^T$ denoted by $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ of unit length such that:

$$\begin{pmatrix} \alpha^T & \beta^T \end{pmatrix} \phi(k+\nu-1) = 0 \tag{4.20}$$

$$\text{for } k = j, j+L-1+2n+m\mu-m$$

If now one sets $\alpha(z)^T = \begin{pmatrix} \alpha_1(z) & \alpha_2(z) & \cdots & \alpha_m(z) \end{pmatrix}$ and $\beta(z)^T = \begin{pmatrix} \beta_1(z) & \beta_2(z) & \cdots & \beta_m(z) \end{pmatrix}$ with $\alpha_i(z) = \sum_{j=0}^{\nu-1} \alpha_i^{(j)} z^j$, and $\beta_i(z) = \sum_{j=0}^{\nu-1} \beta_i^{(j)} z^j$, then Eqn 4.20 is equivalent to

$$\alpha(z)^T y(k) + \beta(z)^T u(k) = 0 \qquad \text{for } k = j, j+L-1+2n+m\mu-m \tag{4.21}$$

Using Eqn 4.21 and the fact that $P(z)$ and $Q(z)$ in Eqn 4.3 are left prime, there exist right prime polynomial matrices $\bar{Q}(z)$ and $\bar{P}(z)$ of dimension

$(m \times l)$ and $(l \times l)$ respectively, such that $P(z)^{-1}Q(z) = \bar{Q}(z)\bar{P}(z)^{-1}$. Thus Eqn 4.21 can be rewritten as

$$[\alpha(z)^T \bar{Q}(z) + \beta(z)^T \bar{P}(z)]\bar{P}(z)^{-1}u(k) = 0 \qquad (4.22)$$

$$\text{for } k = j, j + L - 1 + 2n + m\mu - m$$

Let us define $\lambda_P(z) = \det[\bar{P}(z)]$, then

$$\bar{P}(z)^{-1} = \frac{1}{\lambda_{\bar{P}}(z)}\bar{P}_{adj}(z) \qquad (4.23)$$

where $\bar{P}_{adj}(z)$ is the adjoint of $\bar{P}(z)$. Hence Eqn 4.22 becomes

$$[\alpha(z)^T \bar{Q}(z) + \beta(z)^T \bar{P}(z)]\bar{P}_{adj}(z)u(k) = 0 \qquad (4.24)$$

$$\text{for } k = j, j + L - 1 + n + m\mu - m$$

Our interest is to find a condition of persistency on $v(k)$ rather than $u(k)$. Using Eqns 4.4 and 4.6, then Eqn 4.24 can be rewritten as

$$[\alpha(z)^T \bar{Q}(z) + \beta(z)^T \bar{P}(z)]\bar{P}_{adj}(z)[H(z)D(z)^{-1}Q(z) + I]v(k) = 0 \quad (4.25)$$

$$\text{for } k = j, j + L - 1 + n + m\mu - m$$

Let us now define $D_{adj}(z)$ as the adjoint of the matrix polynomial $D(z)$, and $\lambda_D(z) = \det[\bar{D}(z)]$, then then Eqn 4.25 becomes:

$$[\alpha(z)^T \bar{Q}(z) + \beta(z)^T \bar{P}(z)]\bar{P}_{adj}(z)[H(z)D_{adj}(z)Q(z) + \lambda_D(z)I]v(k) = 0$$

$$(4.26)$$

$$\text{for } k = j, j + L - 1$$

Since $v(k)$ is persistently exciting, therefore we have:

$$[\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)]\bar{P}_{adj}(z)[H(z)D_{adj}(z)Q(z)+\lambda_D(z)I]=0 \quad (4.27)$$

Observing that the elements of the polynomial matrix $H(z)D_{adj}(z)Q(z)$ are of degree $< n+m\mu-m$, thus the degree of the determinant of the matrix polynomial $H(z)D_{adj}(z)Q(z)+\lambda_D(z)I$ is of the same order as $[\lambda_D(z)]^l$. Since $\bar{P}_{adj}(z)$ and $H(z)D_{adj}(z)Q(z)+\lambda_D(z)I$ are nonsingular matrices, then the left side of Eqn 4.27 implies that $\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)=0$.

To complete the proof we need to show that the polynomial vector $\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)$ cannot be made null for any arbitrary unit vector $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$. The polynomial vector $\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)$ can be represented as

$$\begin{pmatrix}\alpha^T & \beta^T\end{pmatrix} M\Psi(z)=\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z) \quad (4.28)$$

where $\Psi(z)^T=$ Block diag $\begin{pmatrix}1 & z & \cdots & z^{\mu+\nu-1}\end{pmatrix}$, and M is the eliminant matrix of the matrix pair $\bar{P}(z)$ and $\bar{Q}(z)$ which are prime. The polynomial vector $\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)$ cannot be made null for any arbitrary unit vector $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ for the following reason:

Since $\bar{P}(z)$ and $\bar{Q}(z)$ are right prime, the rank of M is $n+l\nu$ [30]. Hence $\begin{pmatrix}\alpha^T & \beta^T\end{pmatrix}M=0$ cannot be true if $\left\|\begin{pmatrix}\alpha\\ \beta\end{pmatrix}\right\|=1$. It follows that $\alpha(z)^T\bar{Q}(z)+\beta(z)^T\bar{P}(z)$ is a polynomial vector with elements of degree $\leq \nu+\mu-1$. Thus Eqn 4.26 is in contradiction with Eqn 4.17, which implies that the assumption we have made is wrong. Hence Eqn 4.19 is true. This completes the proof of the lemma.

Eqn 4.19 is relevant for system identification with asymptotic convergence. It is therefore clear that for the result to be of interest, we require the value of $L \geq lr$.

4.4 Assumption and proposed algorithm

The assumptions required for a global result, as well as the algorithm are listed below.

Assumptions:

A1: The plant has strictly proper transfer function and is minimal.

A2: An upper bound of the the controllability indices (μ_i, $i = 1, 2 \cdots l$) and the observability indices (ν_i, $i = 1, 2 \cdots m$), all of which are equal, are known.

A3: The desired closed-loop characteristic, i.e. $\det[P^*(z)Q^*(z)]$ has all roots inside the unit circle $|z| < 1$.

Similar to the SISO case, the following additional assumptions are required to establish fast exponential convergence:

A4: The reference inputs $v^{(i)}(k)$, $i = 1, 2 \cdots l$ are persistently exciting.

A5: The integer $N \geq 2n + m\mu - m + L - 1$

A6: The covariance matrix $P(k)$ in the least-squares algorithm does not vanish to zero.

We will first establish global stability i.e. boundedness of all signals and convergence of the equation error to zero. This result requires the assumptions **A1–A3**. We will assume that the control law parameters are updated in different time frame from that of the system parameters in a similar manner to SISO case. The assumptions **A4–A6** are required to establish global stability with fast exponential convergence.

Then during the time interval I_j, the control structure of Eqn 4.4 becomes

$$
\begin{aligned}
u(k) &= \hat{H}\ (k_j, z)\hat{C}(k_j, z)^{-1}y(k) + v(k) \\
&= \hat{H}\ (k_j, z)w(k) + v(k) \qquad (4.29) \\
y(k) &= \hat{C}(k_j, z)w(k) \qquad (4.30)
\end{aligned}
$$

where $\hat{C}(k_j, z) = Q^*(z) - \hat{K}\ (k_j, z)$. The matrix polynomials $\hat{H}\ (k_j, z)$ and $\hat{K}\ (k_j, z)$ are designed based on the matrix polynomials $\hat{P}(k, z)$ and $\hat{Q}(k, z)$ which are estimates of polynomial matrices $P(z)$ and $Q(z)$ respectively using the estimation scheme of section 2.4. The algorithm can be stated as follows:

Algorithm 4.1 *The identifier is run N-times during the interval I_j, at the* *end of which the control parameters are updated using Eqn 4.9, if* $\hat{P}(k_j, z)$ *and* $\hat{Q}(k_j, z)$ *are prime. When* $\hat{P}(k_j, z)$ *and* $\hat{Q}(k_j, z)$ *are not prime the control parameters are left unchanged during the interval* I_j.

The next section gives the global stability proof of the indirect adaptive control algorithm.

4.5 Stability analysis

In this section, the global stability of the indirect adaptive pole placement scheme is derived. The statement of the global result is summarized in the following theorem.

Theorem 4.1 *Given the assumptions* **A1–A3**, *and the above mentioned algorithm statement, then* $e(k) \to 0$, *and the vector sequences* $\{u(k)\}$ *and* $\{y(k)\}$ *remain bounded. Moreover, if assumptions* **A4–A6** *are satisfied then,* $\hat{P}(k, z) \to P(z)$ *and* $\hat{Q}(k, z) \to Q(z)$ *exponentially fast. Also the closed-loop* $P(z)[Q^*(z) - \hat{K}(k_j, z)] - Q(z)\hat{H}(k_j, z) \to P^*(z)Q^*(z)$ *exponentially fast.*

The proof of the theorem is complicated. We will proceed in a similar manner to the SISO case. To start, we first need to establish a linear boundedness of an equivalent state space vector of the resultant closed-loop system in terms of the equation error $e(k)$, which will later be converted into a relation between the regression $\phi(k)$ vector and the equation error

$e(k)$. Boundedness of all signals with convergence of the equation error to zero follows by making use of technical lemma 3.1. This result requires only assumptions **A1–A3**. The persistency of excitation with the condition on the block length N is not required for this result. The asymptotic convergence of the system parameter to their true values with specified bounds on the rate of convergence is guaranteed if persistency of excitation and condition on the block length N are satisfied. This final result will be obtained using lemma 4.1.

Combining the system representation Eqn 4.3 with the control structure Eqn 4.29, and substituting $y(k)$ by its expression in Eqn 4.30, the closed-loop equation becomes:

$$D(k_j, z)w(k) \;=\; Q(z)v(k) \tag{4.31}$$

where

$$D(k_j, z) \;=\; P(z)[Q^*(z) - \hat{K}\,(k_j, z)] - Q(z)\hat{H}\,(k_j, z) \tag{4.32}$$

with k_j fixed during the interval I_j.

It is verified that the closed-loop polynomial matrix $D(k_j, z)$ is row reduced of degree $n + m\mu - m$, and satisfying $\partial_{r_i} D(k_j, z) = \mu + \nu - 1$. Recalling Eqn 2.12, the equation error can be rewritten as

$$e(k + \nu) = [\hat{P}(k, z) - P(z)]y(k) + [Q(z) - \hat{Q}(k, z)]u(k) \tag{4.33}$$

To assist in obtaining the linear boundedness of the regression in terms of the equation error, we need to define

$$\bar{e}(k+\nu) = [\hat{P}\ (k_j,z) - P(z)]y(k) + [Q(z) - \hat{Q}\ (k_j,z)]u(k) \qquad (4.34)$$

It is possible to express the equation error in terms of the partial state vector $w(k)$. To do so, substitute $u(k)$ and $y(k)$ by their respective expressions in Eqns 4.29 and 4.30, we obtain

$$\begin{aligned} \bar{e}\ (k+\nu) &= [\hat{P}\ (k_j,z)Q^*(z) - \hat{P}\ (k_j,z)\hat{K}\ (k_j,z) - \hat{Q}\ (k_j,z)\hat{H}\ (k_j,z)]w(k) \\ &\qquad -\hat{Q}\ (k_j,z)v(k) \\ &= P^*(z)Q^*(z)w(k) - \hat{Q}\ (k_j,z)v(k) \end{aligned} \qquad (4.35)$$

From [29], any differential operator representation given by Eqns 4.31 and 4.35 has an equivalent state-space realization of the form Eqns 2.1 and 2.2. The next lemma shows how an equivalent state space representation can be obtained.

Lemma 4.2 *Given the system Eqn 4.31, with $D(k_j,z)$ row reduced and of degree $n+m\mu-m$, then for $k \in I_j$, an equivalent state space realization of the form:*

$$x(k+1) = Ax(k) + Bv(k) \qquad (4.36)$$

can be obtained by defining the $n+m\mu-m$ state vector

$$x(k) = t_D(z)w(k) + t_Q(z)v(k) \qquad (4.37)$$

with

$$t_D(z) = \begin{pmatrix} t_{D11}^{(\nu+\mu-1)}(z) & t_{D12}^{(\nu+\mu-1)}(z) & \cdots & t_{D1m}^{(\nu+\mu-1)}(z) \\ \vdots & \vdots & \ddots & \vdots \\ t_{D11}^{(1)}(z) & t_{D12}^{(2)}(z) & \cdots & t_{D1m}^{(1)}(z) \\ t_{D21}^{(\nu+\mu-1)}(z) & t_{D22}^{(\nu+\mu-1)}(z) & \cdots & t_{D2m}^{(\nu+\mu-1)}(z) \\ \vdots & \vdots & \ddots & \vdots \\ t_{Dm1}^{(1)}(z) & t_{Dm2}^{(2)}(z) & \cdots & t_{Dmm}^{(1)}(z) \end{pmatrix} \tag{4.38}$$

$$t_Q(z) = \begin{pmatrix} t_{Q11}^{(\nu+\mu-1)}(z) & t_{Q12}^{(\nu+\mu-1)}(z) & \cdots & t_{Q1l}^{(\nu+\mu-1)}(z) \\ \vdots & \vdots & \ddots & \vdots \\ t_{Q11}^{(1)}(z) & t_{Q12}^{(2)}(z) & \cdots & t_{Q1l}^{(1)}(z) \\ t_{Q21}^{(\nu+\mu-1)}(z) & t_{Q22}^{(\nu+\mu-1)}(z) & \cdots & t_{Q2l}^{(\nu+\mu-1)}(z) \\ \vdots & \vdots & \ddots & \vdots \\ t_{Qm1}^{(1)}(z) & t_{Qm2}^{(2)}(z) & \cdots & t_{Qml}^{(1)}(z) \end{pmatrix} \tag{4.39}$$

where $t_{D_{ij}}^{(l)}(z)$ *and* $t_{Q_{ij}}^{(l)}(z)$ *are the Tchirnhausen polynomials of* $D_{ij}(z)$ *and* $Q_{ij}(z)$ *respectively. The polynomials* $D_{ij}(z)$ *and* $Q_{ij}(z)$ *denote the entries of the polynomial matrices* $D(z)$ *and* $Q(z)$*. The pair* (A, B) *is in observer form.*

Moreover the error $\bar{e}(k)$ *in Eqn 4.35 can be expressed as*

$$\bar{e}(k+\nu) = Cx(k) + E(z)v(k) \tag{4.40}$$

for some matrices C *and* $E(z)$ *depending only on* $D(z)$*,* $Q(z)$ *and the desired closed-loop polynomial* $P^*(z)Q^*(z)$*, with the equivalence relation valid over the interval* I_j*.*

Proof: Since $D(z)$ is row reduced and $\partial_{ri}[D(k,z)] \geq \partial_{ri}[Q(z)]$, then from [29], the system has an equivalent state space realization of the form

Eqn 4.36 which is in observer form. The state vector can also be written as:

$$x(k) = T\Psi(z)w(k) + t_Q(z)v(k) \tag{4.41}$$

where the $(n + m\mu - m) \times m$ matrix $\Psi(z)$ is given by

$$\Psi(z)^T = \text{block diag}\begin{pmatrix} 1 & z & \cdots & z^{\nu+\mu-2} \end{pmatrix} \tag{4.42}$$

and where the matrix T is block element such that

$$T = [T_{ij}] \tag{4.43}$$

$$T_{ii} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ * & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ * & * & \cdots & 1 \end{pmatrix}$$

$$\text{for } i > j \ T_{ij} = \begin{pmatrix} * & 0 & \cdots & 0 \\ * & * & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ * & * & \cdots & * \end{pmatrix}$$

$$\text{for } i < j \ T_{ij} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ * & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ * & * & \cdots & 0 \end{pmatrix}$$

where $*$ represents possible non-zero element.

Now, in view of the special structure of the square matrix T of dimension $(n+m\mu-m)$, the rank of the matrix T is $(n+m\mu-m)$. Thus, it is possible to uniquely determine the vector

$$\Psi(z)w(k) = T^{-1}x(k) - T^{-1}t_Q(z)v(k) \tag{4.44}$$

With the definition of $\bar{\Psi}(z)$ as:

$$\bar{\Psi}(z)^T = \text{Block diag}\left(1 \quad z \quad \cdots \quad z^{\nu+\mu-1} \right) \tag{4.45}$$

the polynomials $P^*(z)Q^*(z)$ and $D(k_j, z)$ can be rewritten as:

$$\begin{aligned} P^*(z)Q^*(z) &= G\bar{\Psi}(z) \\ &= \bar{G}\Psi(z) + \bar{\bar{G}}z^{\nu+\mu-1} \\ \text{and} \qquad D(z) &= D\bar{\Psi}(z) \\ &= \bar{D}\Psi(z) + \bar{\bar{D}}z^{\mu+\nu-1} \end{aligned}$$

with $\bar{G}$, $\bar{\bar{G}}$, $\bar{D}$, and $\bar{\bar{D}}$ are constant matrices of appropriate dimensions, with $\bar{\bar{D}}$ square and nonsingular.

Since the vector $\bar{\Psi}(z)w(k)$ contains the vector $\Psi(z)w(k)$ plus $w(k+\nu+\mu-1)$, therefore the equation error $\bar{e}(k)$ as given by Eqn 4.35 can be written as

$$\begin{aligned} \bar{e}(k+1) &= (\bar{G}\Psi(z) + \bar{\bar{G}}z^{\nu+\mu-1})w(k) - \hat{Q}(k_j, z)v(k) \\ &= \bar{G}\Psi(z)w(k) + \bar{\bar{G}}w(k+\nu+\mu-1) - \hat{Q}(k_j, z)v(k) \end{aligned}$$

From Eqn 4.31, it follows that:

$$w(k+\mu+\nu-1) = -\bar{\bar{D}}^{-1}\bar{D}\Psi(z)w(k) + Q(z)v(k) \tag{4.46}$$

Thus, the equation error becomes:

$$\bar{e}(k+1) = (\bar{G} - \bar{\bar{G}}\bar{\bar{D}}^{-1}\bar{D})\Psi(z)w(k) + (\bar{\bar{G}}Q(z) - \hat{Q}(k_j, z))v(k) \tag{4.47}$$

Using Eqn 4.44, it is now possible to express $e(k+1)$ in the form Eqn 4.40 as follows:

$$\bar{e}(k+1) = (\bar{G}-\bar{\bar{G}}\bar{\bar{D}}^{-1}\bar{D})T^{-1}x(k)+\{\bar{\bar{G}}Q(z)-\hat{Q}(k_j,z)+(\bar{G}-\bar{\bar{G}}\bar{\bar{D}}^{-1}\bar{D})T^{-1}t_Q(z)\}v(k) \tag{4.48}$$

This ends the proof of lemma 4.2.

The key idea is to prove that the state variable system $x(k)$ mapping $v(k)$ into $\bar{e}(k)$ is detectable.

Lemma 4.3 *Given the assumptions* **A1–A3**, *the system Eqns 4.31 and 4.35 and its equivalent state space given by Eqns 4.36 and 4.40, the state variable $x(k)$, defined by Eqn 4.37 and mapping the input $v(k)$ into $e(k)$, is detectable.*

Proof: The equivalent state space representation given by Eqns 4.36 and 4.40 is detectable because, any unobservable modes of the system defined by Eqns 4.31 and 4.35 are roots of $\det[P^*(z)Q^*(z)]$ which are stable by (assumption **A3**). This completes the proof.

The whole point of proving the detectability is to allow us to obtain a linear boundedness condition of $x(k)$ in terms of $\bar{e}(k+\nu)$.

Lemma 4.4 *With the same hypothesis as lemma 4.3, and the definition of Eqn 4.40 of $\bar{e}(k)$, there exists K_1 and K_2 positive constants such that*

$$||x_m(j)|| \leq K_1 \max_{0\leq i\leq j} ||\bar{e}_m(i)|| + K_2 \tag{4.49}$$

where $x_m(j) = \max_{k \in I_j} x(k)$ *and* $\bar{e}_m(j) = \max_{k \in I_j} \bar{e}(k+\nu)$

Proof: Assume that the system defined by Eqns 4.36 and 4.40 have $\bar{n}$ pole-zero cancellations ($\bar{n}$ unobserved modes), then there exists a linear transformation $x(k) = Q\bar{x}(k)$, which transforms the system Eqns 4.36 and 4.40 into the following state space form:

$$\begin{pmatrix} \bar{x}^u(k+1) \\ --- \\ \bar{x}^o(k+1) \end{pmatrix} = \begin{pmatrix} A_{11} & | & A_{12} \\ -- & | & -- \\ 0 & | & A_{22} \end{pmatrix} \begin{pmatrix} \bar{x}^u(k) \\ --- \\ \bar{x}^o(k) \end{pmatrix} + \begin{pmatrix} B_1 \\ -- \\ B_2 \end{pmatrix} \quad (4.50)$$

$$\bar{e}(k+\nu) = \begin{pmatrix} 0 & c_2 \end{pmatrix} \begin{pmatrix} \bar{x}^u(k) \\ \bar{x}^o(k) \end{pmatrix} + \bar{E}(z)v(k) \quad (4.51)$$

where $\bar{x}^u$ and $\bar{x}^o$ denotes the unobservable and the observable state vectors respectively. Similar to the SISO case, we will proceed to the proof by induction.

It is not difficult to show that Eqn 4.49 is true for I_0. We will assume next that this is true for I_{j-1} and show that it is also true for I_j.

The idea of observability is that one can determine the state over any finite time interval of suitable length from complete knowledge of the system input and output over the same interval. Hence, there exists two constants K_3 and $\bar{K}_4$ such that

$$||\bar{x}^o_m(j)|| \leq K_3||\bar{e}_m(i))|| + \bar{K}_4 \max_{k \in I_j} ||v(k)|| \quad (4.52)$$

The reference vector signal $v(k)$ being bounded, Eqn 4.52 becomes then:

$$||\bar{x}^o_m(j)|| \leq K_3||\bar{e}_m(j)|| + K_4, \; K_4 > 0 \quad (4.53)$$

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Similar to the SISO case, solving for the unobservable state $\bar{x}^u(k)$ using Eqn 4.50, one guarantees the existence of positive constant K_5 and K_6 such that

$$||\bar{x}^u_m(j)|| \leq K_5 \max_{0 \leq i \leq j} ||\bar{e}_m(i)|| + K_6 \tag{4.54}$$

In the above, the fact that A_{11} is asymptotically stable has been used. Now using Eqns 4.53 and 4.54, and recalling that $\bar{x}^u(k)$ and $\bar{x}^o(k)$ are subvectors of $\bar{x}(k)$. One then obtains

$$||\bar{x}_m(j)|| \leq K_1 \max_{0 \leq i \leq j} ||\bar{e}_m(i)|| + K_2 \tag{4.55}$$

Finally, the fact that $x(k)$ is related to $\bar{x}(k)$ by linear transformation is used to establish the result Eqn 4.49

Our real interest is in $e(k)$ rather than $\bar{e}(k)$. The following lemma shows how the result of lemma 4.4 carries over to $e(k)$.

Lemma 4.5 *With the same hypothesis as lemma 4.3, and the definition Eqn 4.33 of the error* $e(k)$, *there exists constants* C_1 *and* C_2 *such that*

$$||x_m(j)|| \leq C_1 \max_{0 \leq i \leq j} ||e_m(i)|| + C_2 \tag{4.56}$$

$x_m(j) = \max_{k \in I_j} x(k)$ *and* $\bar{e}_m(j) = \max_{k \in I_j} \bar{e}(k + \nu)$

Proof: by lemmas 2.1 or 2.2, $||\hat{\theta}(k) - \hat{\theta}(k_j)|| \to 0$, when $k \in I_j$ and $j \to \infty$. Hence given any arbitrary $\delta > 0$, there exists a k_J such that for $k_j \geq k_J$ and $k \in I_j$,

$$||\hat{\theta}(k) - \hat{\theta}(k_j)|| < \delta \tag{4.57}$$

Since the roots of $\det[\hat{P}(k,z)\hat{C}(k_j,z) - \hat{Q}(k,z)\hat{H}(k_j,z)]$ are continuous functions of their polynomial matrix coefficients, and by (assumption **A3**) $\det[P^*(z)Q^*(z)]$ has roots inside the unit circle, therefore there exists a region in the parameter space centered on $\hat{\theta}(k_j)$ and having radius $\bar{\delta} > 0$ sufficiently small such that if

$$||\hat{\theta}(k) - \hat{\theta}(k_j)|| < \bar{\delta} \tag{4.58}$$

it implies that the roots of the $\det[\hat{P}(k,z)\hat{C}(k_j,z) - \hat{Q}(k,z)\hat{H}(k_j,z)]$ are inside the unit circle during the whole interval I_j. Thus by choosing $\delta = \bar{\delta}$, one guarantees that for $k \geq k_J$, the state vector $x(k)$ remains detectable from the equation error. Since $e(k_j+\nu) = \bar{e}(k_j+\nu)$ and the error can only change by a finite amount during any finite interval of time; the inequality Eqn 4.49 still holds when $e(k+\nu)$ replaces $\bar{e}(k+\nu)$.

Lemma 4.6 *Assume the same hypothesis as lemma 4.3 and the definition Eqn 4.33 of* $e(k+\nu)$. *Recalling also the definition Eqn 4.18 of* $\phi(k+\nu-1)$, *then the result of lemma 4.5 will still be valid when* $x(k)$ *in Eqn 4.56 is replaced by* $\phi(k+\nu-1)$.

Proof: From Eqns 4.29 and 4.30, we note that $y(k)$ and $u(k)$ are linear functional of $w(k), w(k+1), \cdots w(k+\mu-1)$. Thus the vector $\phi(k+\nu-1)$ is linearly related to $w(k), w(k+1), \cdots w(k+\mu+\nu-1)$ plus a term functional of $v(k)$. Thus the vector $\phi(k+\nu-1)$ can be obtained from the state vector $x(k)$ by a linear transformation plus terms involving only $v(k)$. For a detailed expression of the regression vector in terms of the state vector one may consult appendix A.5.

Finally by use of the result of lemma 4.5 the proof is completed.

Corollary 4.1 *Assume the same hypothesis as lemma 4.3 together with the result of lemma 4.6, we conclude that* $e(k) \to 0$ *when* $k \to \infty$, *and* $\{\|\phi(k)\|\}$ *remains bounded.*

Proof: Use lemmas 3.1 and 4.6 to conclude that $e(k) \to 0$ as $k \to \infty$, and $\{\|\phi(k)\|\}$ remains bounded.

The global stability of the indirect adaptive algorithm for MIMO systems is now established. This result is derived with minimum number of assumptions.

The next lemma makes use of the persistency of excitation property of $v(k)$ assumption **A4**, together with the condition on N assumption **A5** to complete the proof of theorem 4.1.

Lemma 4.7 *Given the assumptions* **A1–A6**, *together with the result of corollary 4.1 and the algorithm statement mentioned earlier. Then* $\hat{Q}(k,z) \to Q(z)$ *and* $\hat{P}(k,z) \to P(z)$ *exponentially fast. Also the closed-loop* $P(z)[Q^*(z) - \hat{K}(k_j,z)] - Q(z)\hat{H}(k_j,z) \to P^*(z)Q^*(z)$ *exponentially fast.*

Proof: The proof of this lemma follows the procedure described in lemma 3.7.

For the case of covariance reset, define the following lyapunov function [29]

$$V(k) = ||\tilde{\theta}(k)P(k-1)^{-1}\tilde{\theta}(k)|| \tag{4.59}$$

where $||.||$ is defined as $||M|| = \text{trace}(M^T M)$.

Assume that at the reset instances k' (for simplicity assume k' as subset of the set of instance k_j), the scalar function $V(k)$ becomes

$$V(k') = \sigma||\tilde{\theta}(k')||^2 \tag{4.60}$$

Applying the matrix inversion lemma to the covariance matrix formula [29], we can write

$$V(k_j) = \tilde{\theta}(k_j)^T[P(k_{j-1})^{-1} + \sum_{l=1}^{N}\phi(k_{j-1}+l)\phi(k_{j-1}+l)^T]\tilde{\theta}(k_j) \tag{4.61}$$

Assuming that $k_{j-1} \leq k' < k_j$ then, using Eqn 4.19, which is satisfied if $N \geq 5n-3$ (assumption **A5**) and the persistency requirement of the reference signal (assumption **A4**), the definition of $V(k)$, we have

$$V(k_j) \geq (\sigma + \delta_1)||\tilde{\theta}(k_j)||^2 \tag{4.62}$$

Now using the monotonicity of $V(k)$, and observing that $k_{j-1} \leq k' < k_j$ we have

$$V(k_j) \leq V(k') \leq \sigma||\tilde{\theta}(k_{j-1})||^2 \tag{4.63}$$

Combining Eqns 4.62 and 4.63 yields

$$||\tilde{\theta}(k_j)||^2 \leq \frac{\sigma}{\sigma+\delta_1}||\tilde{\theta}(k_{j-1})||^2 \tag{4.64}$$

Since $\delta_1 > 0$, we can conclude that the subsequence $\{||\tilde{\theta}(k_j)||, j = 0, 1, \ldots\}$ is exponentially convergent to zero. Moreover, since $V(k)$ is nonincreasing, it is possible to show that $\{||\tilde{\theta}(k_j + i)||^2 \leq ||\tilde{\theta}(k_j)||^2$ for $i = 1, 2, \ldots N\}$. Thus we have that the sequence $\{||\tilde{\theta}(k)||, k = 0, 1, \ldots\}$ converges exponentially fast to zero.

For the case of least squares with exponential data weighting or the least squares with covariance modification we have

$P^{-1}(k) \geq \sigma I$ for any arbitrary time k.

Hence the proof follows in a manner similar to the least squares with covariance reset. This completes the proof of the lemma.

The complete proof of theorem 4.1 follows by invoking lemma 4.7 and corollary 4.1.

CHAPTER FIVE

SIMULATION RESULTS

5.1 Introduction

The purpose of this chapter is to investigate the performance of the globally stable adaptive pole assignment algorithms developed in the previous chapters using digital computer simulations. It has been proved that the convergence of the system parameters to the true values is exponentially fast when using the projection algorithm or least-squares algorithms that are known to have fast convergence rate, namely least-squares with covariance reset or least-squares with exponential data weighting. Also, the equation error was proved to converge exponentially fast to zero. In view of these theoretical results gained from the global stability analysis of the indirect adaptive algorithms, the simulation studies prove valuable in providing a better insight into the nature of indirect adaptive controller. Extensive simulations studies are of selected test cases were carried out for further assertion of the analysis as well as to get a better understanding of the performance of the algorithms. The present simulation studies use the least-squares with covariance resetting in the estimation of the parameters of the system because it is relatively simple, in the sense that it requires less space and computations. These properties make it more preferable over other algorithms for real time applications.

The simulations of the adaptive system on the digital computer indicate that the adaptive control procedure outlined in the previous chapters does result in a robust controller with parameters converging to the true value at rates which are practical.

The simulations carried out were for:

(i) adaptive pole assignment algorithm of SISO systems based on the (r.m.r.).

(ii) adaptive pole assignment algorithm of SISO and MIMO systems based on the (l.m.r.).

The evolution of the output error $e(k)$, together with the parameters and feedback controllers are given for all systems simulated, including unstable nonminimum phase system, in order to show the performance of the adaptive algorithm. For the sake of completeness, the input and output signals are also shown.

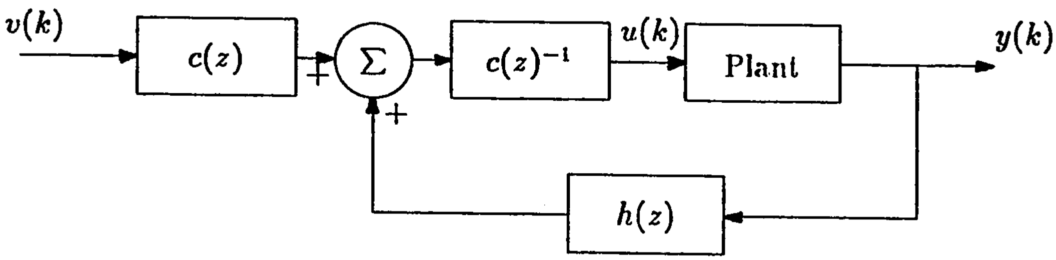

Figure 5.1: Configuration of a feedback controller based on (l.m.r.)

5.2 Simulation examples of indirect adaptive algorithms for SISO systems

In this section, we present some simulation studies of the indirect adaptive algorithms for SISO systems. The adaptive control based on the two control strategies described in chapter 3 are simulated. The results given, include all typical model signals of the system together with the parameters trajectories. The system parameters are updated using the least-squares estimation scheme.

The idea of block processing is introduced as described in chapter 3 to confirm the results of the analysis. The simulations consist of three selected test cases, including These simulations consider unstable nonminimum phase systems. To start, we consider the case of SISO system based on (l.m.r.). The configuration used in the adaptive control simulation for the (l.m.r.) is shown in Fig. 5.1.

5.2.1 Simulation 1 for SISO

The first simulation considers the case of a second order system. The system to be controlled has the following transfer function.

$$T(z) = \frac{0.4z + 0.61}{z^2 + 2.1z + 0.2} \tag{5.1}$$

Thus the system zero is at $(z = -1.52)$ and the system poles are at $(z = -2.0, -0.1)$. The desired closed-loop polynomial is chosen to be $p^*(z) = z^2 - 0.7$. It is thus required that the new pole location be at $(z = +0.84, -0.84)$. The polynomial $q^*(z)$ is assumed to have a root at the origin. The initial conditions on the plant parameters and the controller were all set to zero. The reference input $v(k)$ is taken to be a square wave with an amplitude of 0.5 units and a period of 40 time samples. A least-squares estimator is used with an initial covariance matrix $P(-1) = 10^{10}I$. Typical system signals are given in Fig. 5.2–5.8. As shown in Fig. 5.7 and 5.8, the plant parameters converge to the true value in a period of five time samples. In practice, after the identifier has reached the steady state, one can stop the identification process as long as there are no changes in the plant parameters. The desired feedback controller is obtained in the same period of time as Fig. 5.5 and 5.6 indicate. Also the plant input and output signals do not exercise large transients before the adaptation occurs as it is shown from Fig. 5.2 and 5.3. For completeness, the equation error is also given in Fig. 5.4.

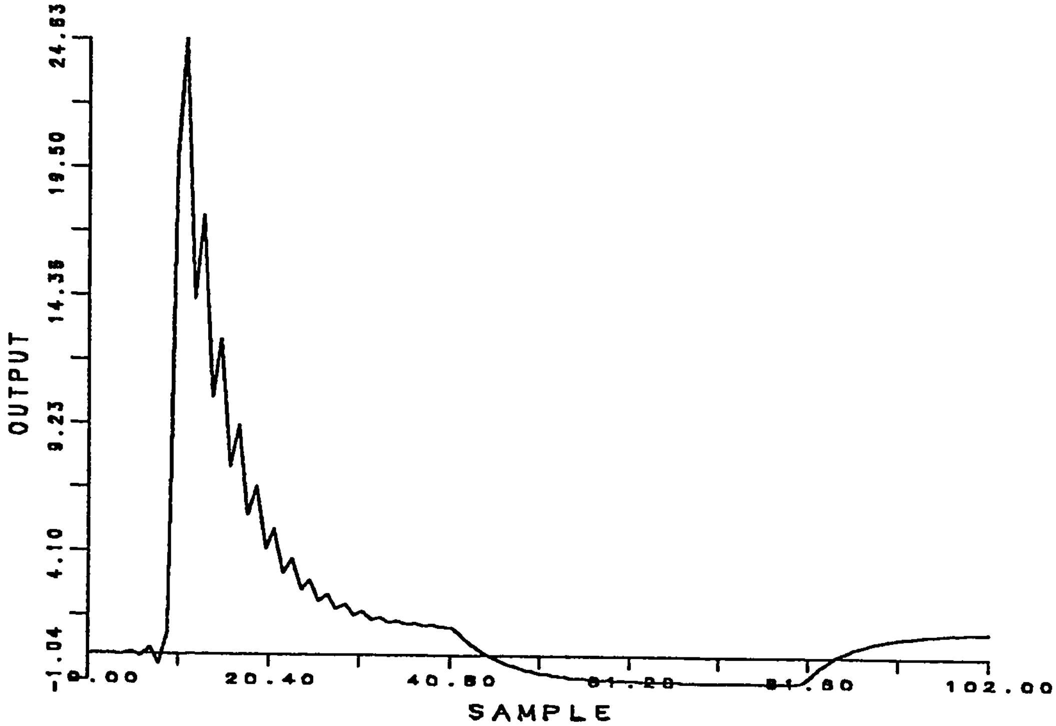

Figure 5.2: Output trajectory of second order system

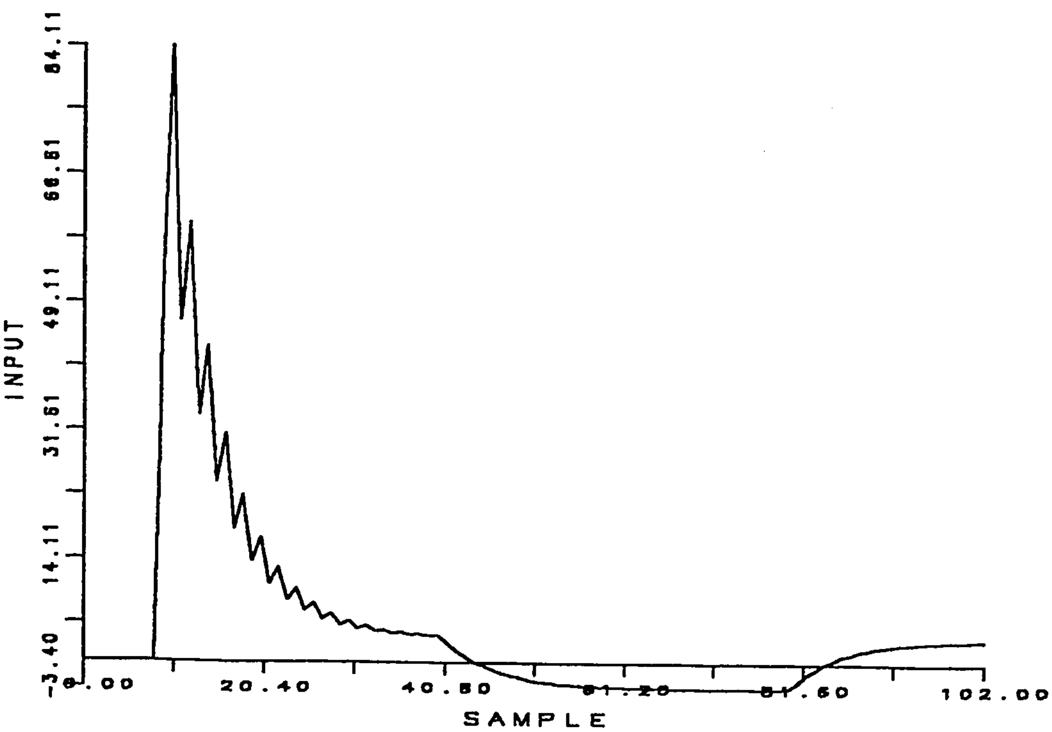

Figure 5.3: Input trajectory of second order system

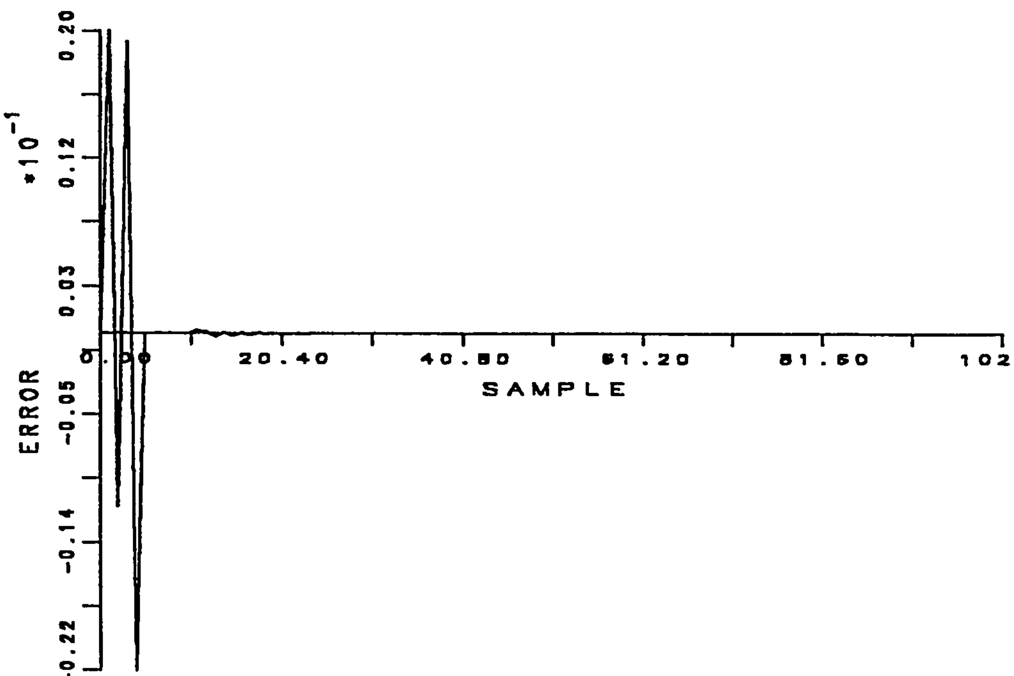

Figure 5.4: Error trajectory of second order system

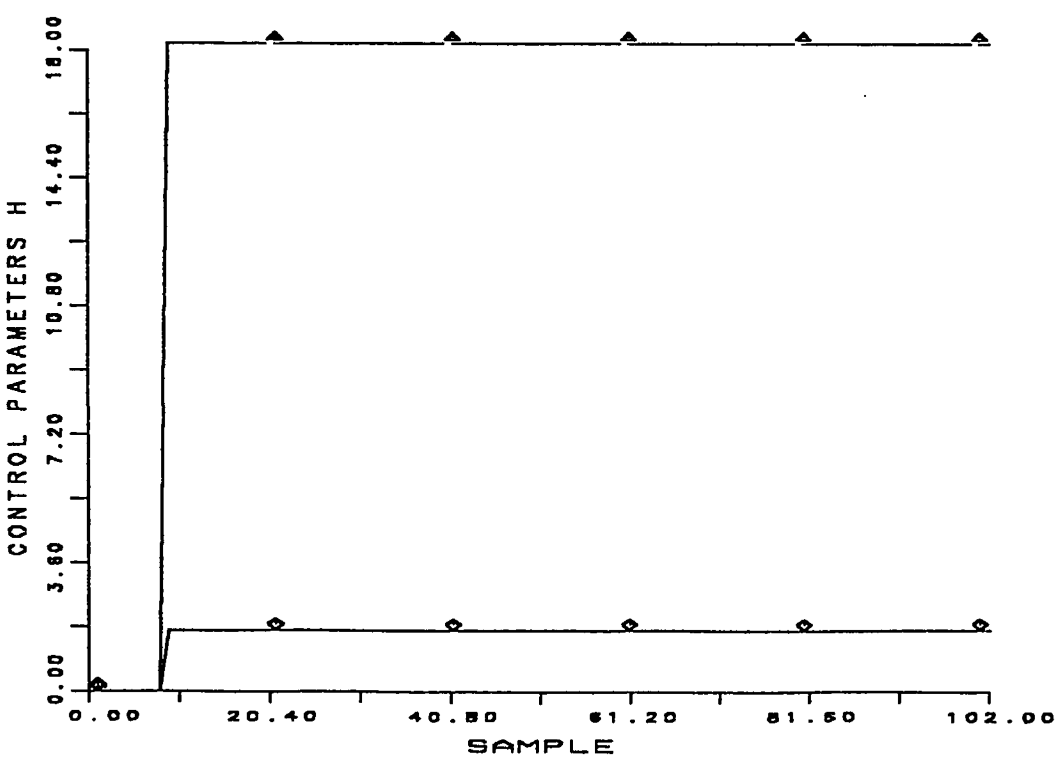

Figure 5.5: Controller parameters trajectory of second order system

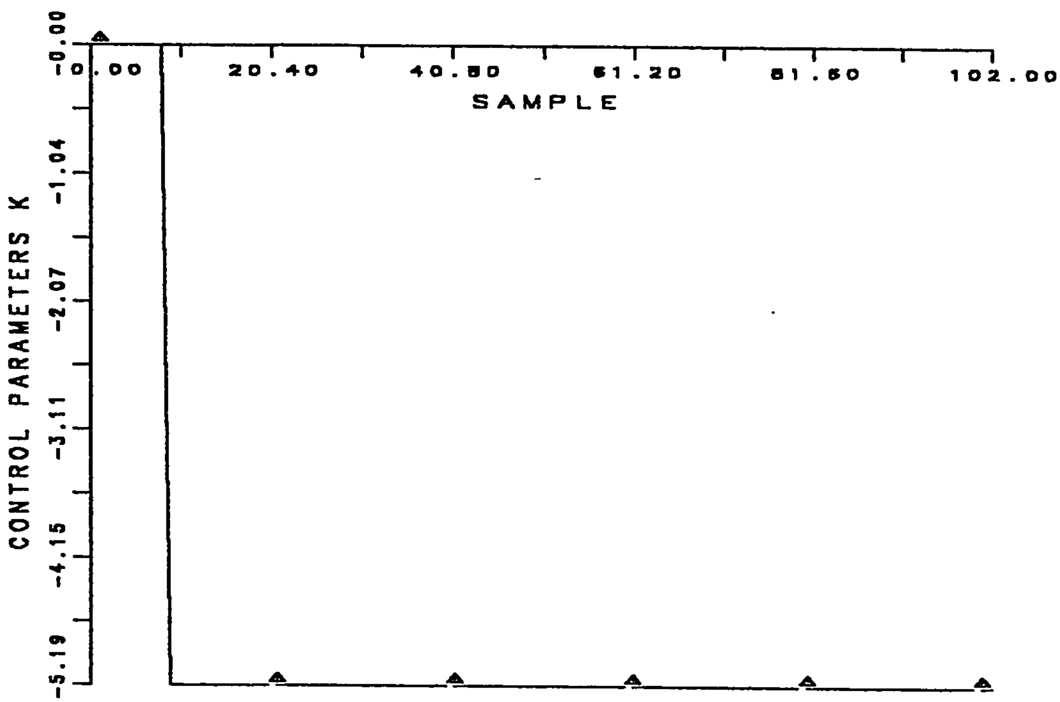

Figure 5.6: Controller parameters trajectory of second order system

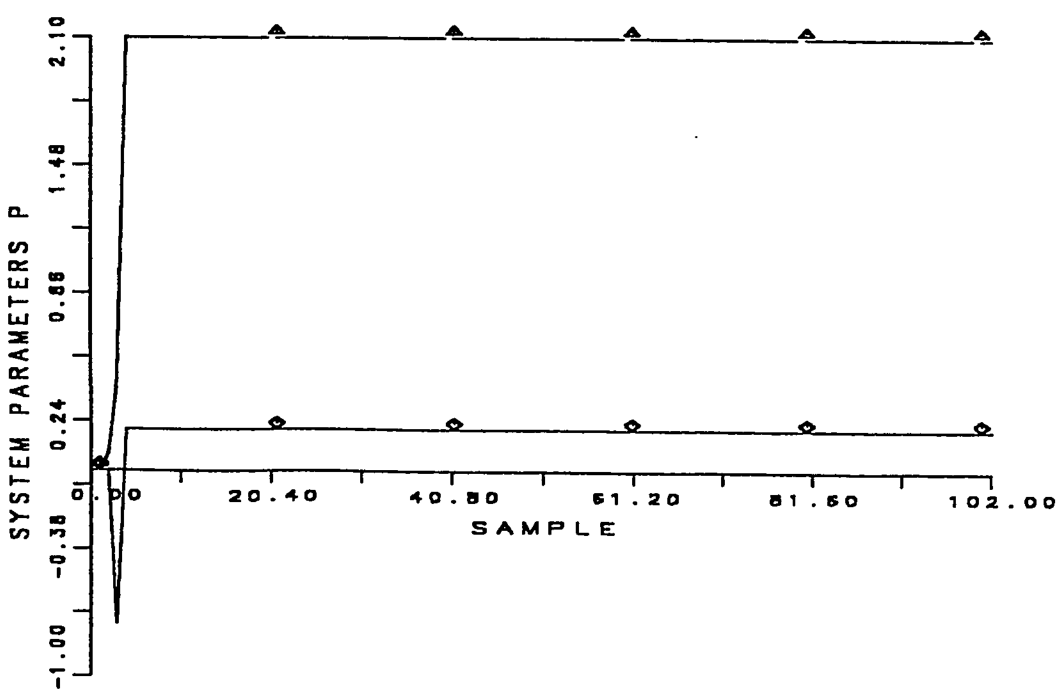

Figure 5.7: System parameters trajectory of second order system

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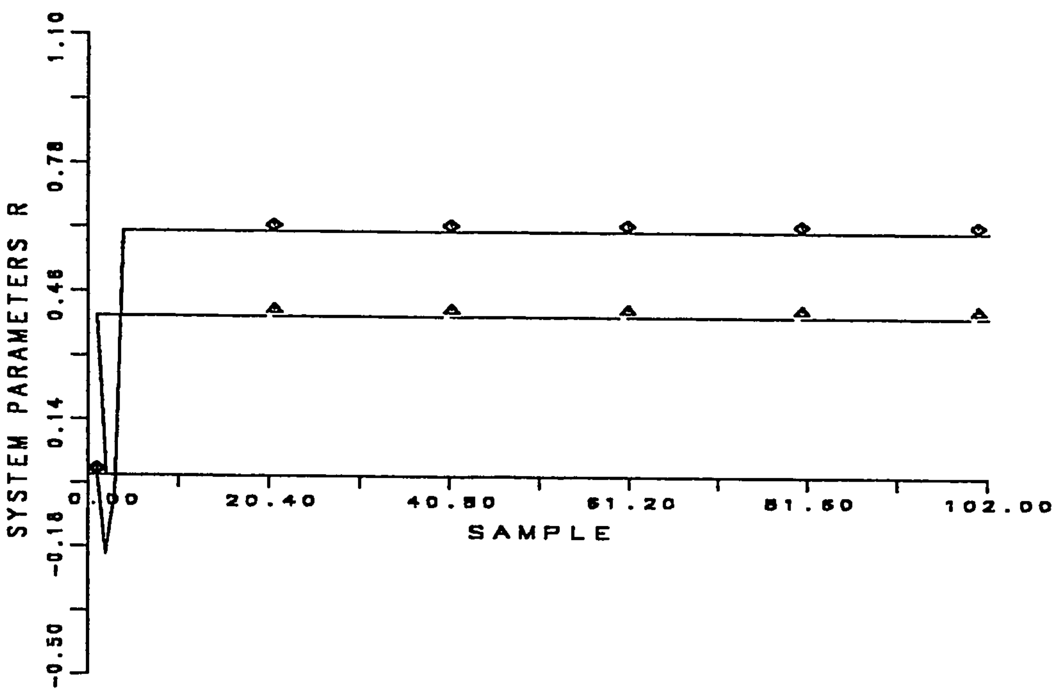

Figure 5.8: System parameters trajectory of second order system

5.2.2 Simulation 2 for SISO

In this simulation a third order system is considered. The system has the following transfer function

$$T(z) = \frac{0.2z^2 + 0.27z + 0.04}{z^3 + 2.1z^2 + 0.2z} \tag{5.2}$$

The system zeros are at $(z = -0.34, -2.36)$ and the system poles are at $(z = 0, -0.1, -2)$. The desired closed-loop poles are selected at $(z = 0, 0, 0, 0.85, 0.75)$. The initial conditions on the plant parameters and controllers were all set to zero. The reference input $v(k)$ is given as a square wave of period 40 samples. All simulation results are shown on Fig. 5.9–5.15. The plant parameters converge to the true values in approximately 18 time samples. As it is clear from Fig. 5.9, the system output signal does not undergo large transients.

5.2.3 Simulation 3 for SISO

This simulation considers a fifth order system with the following transfer function

$$T(z) = \frac{z^2 + 1.5z - 1}{z^5 + 0.134z^3 - 0.144z} \tag{5.3}$$

The system zeros are at $(z = 0.5, -2.0)$. The desired closed-loop poles are selected at $(z = \pm 0.9, 0.5 \pm j0.4, 0.1)$. The polynomial $q^*(z)$ has all four roots at the origin. The initial conditions on the plant parameters and the

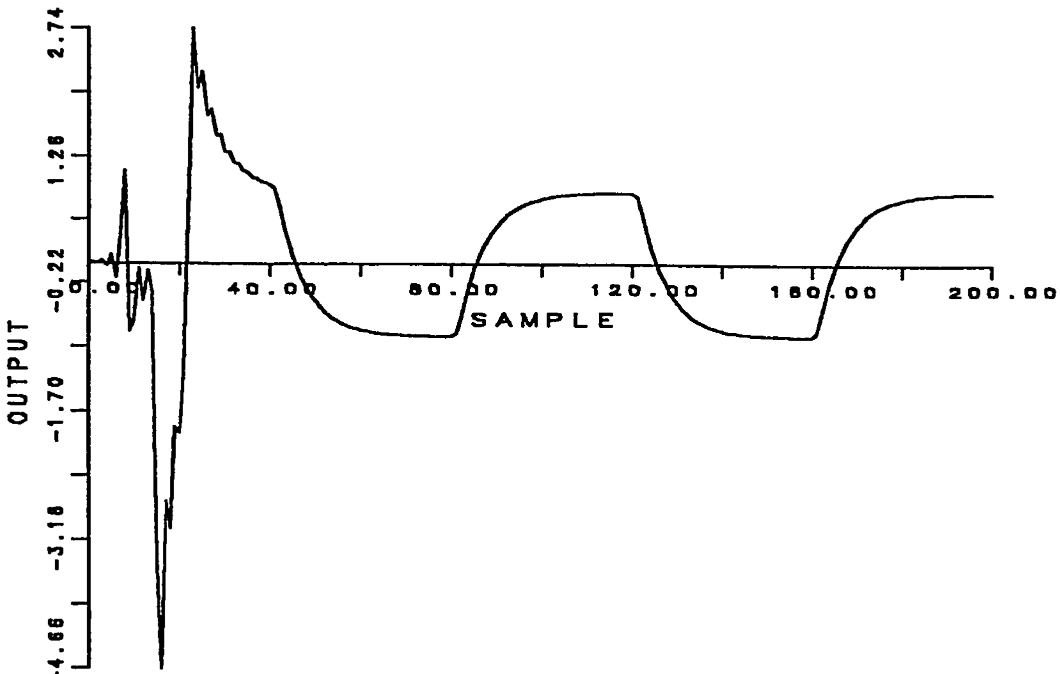

Figure 5.9: Output trajectory of third order system

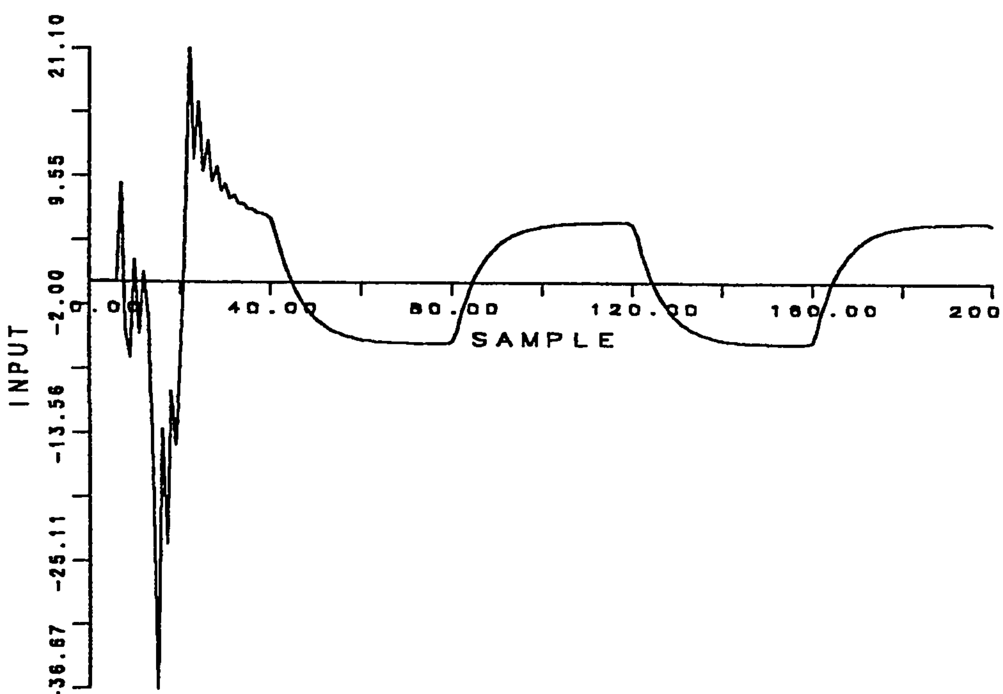

Figure 5.10: Input trajectory of third order system

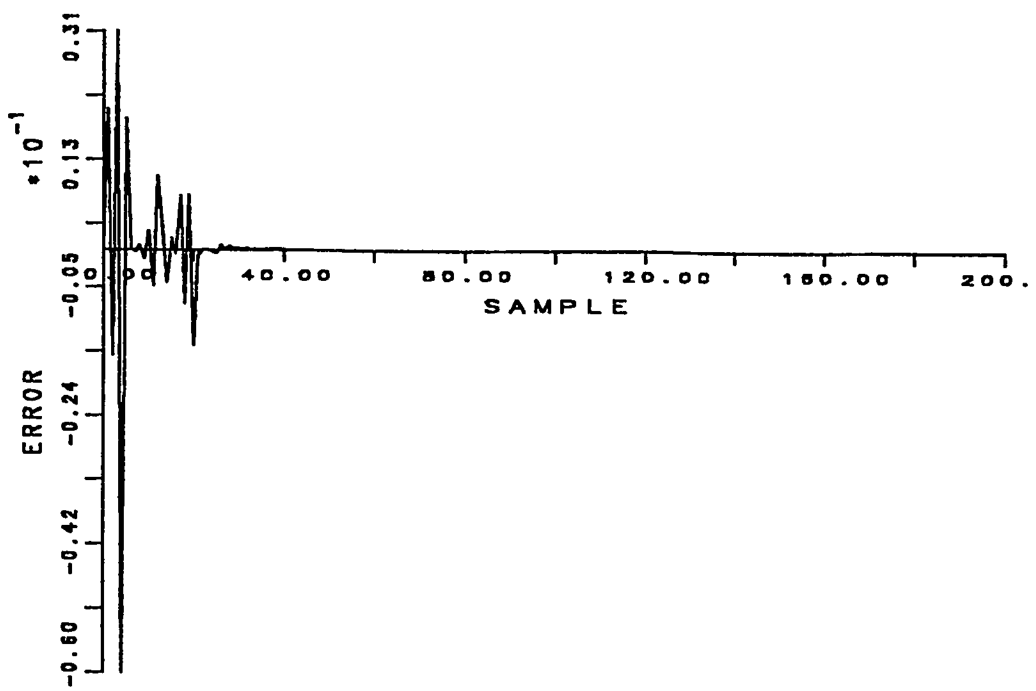

Figure 5.11: Error trajectory of third order system

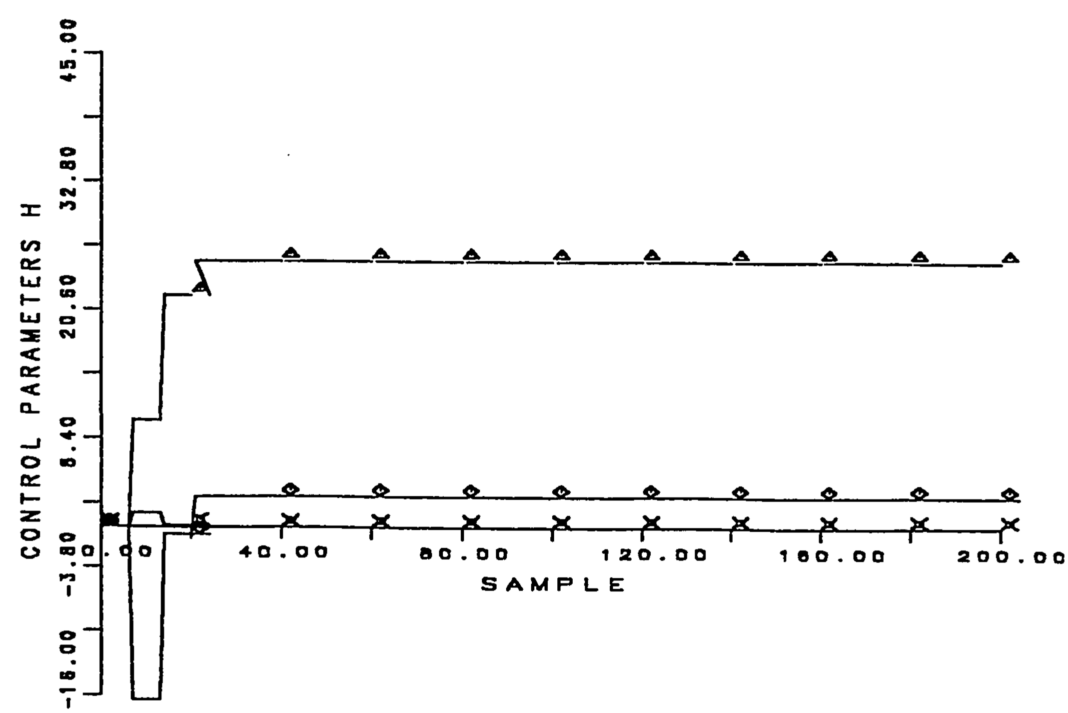

Figure 5.12: Controller parameters trajectory of third order system

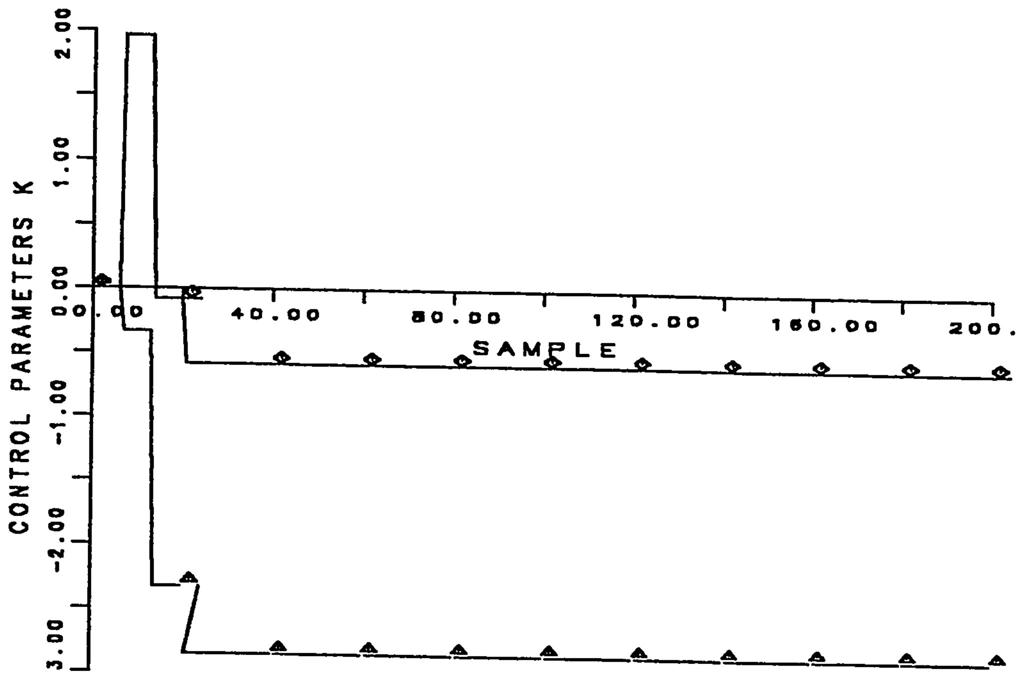

Figure 5.13: Controller parameters trajectory of third order system

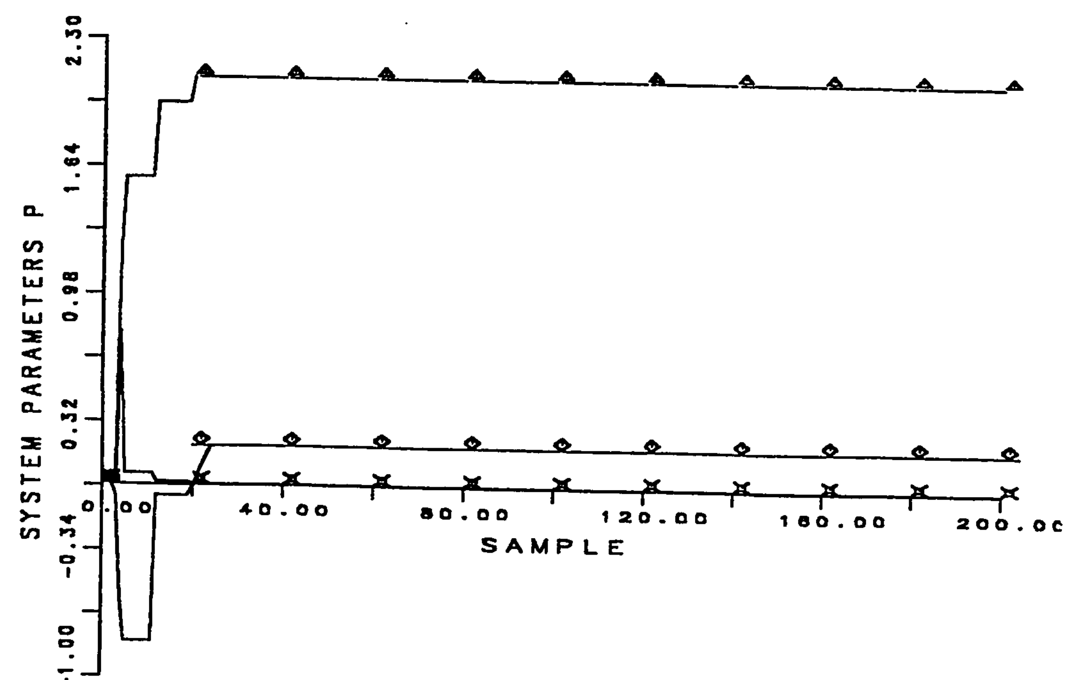

Figure 5.14: System parameters trajectory of third order system

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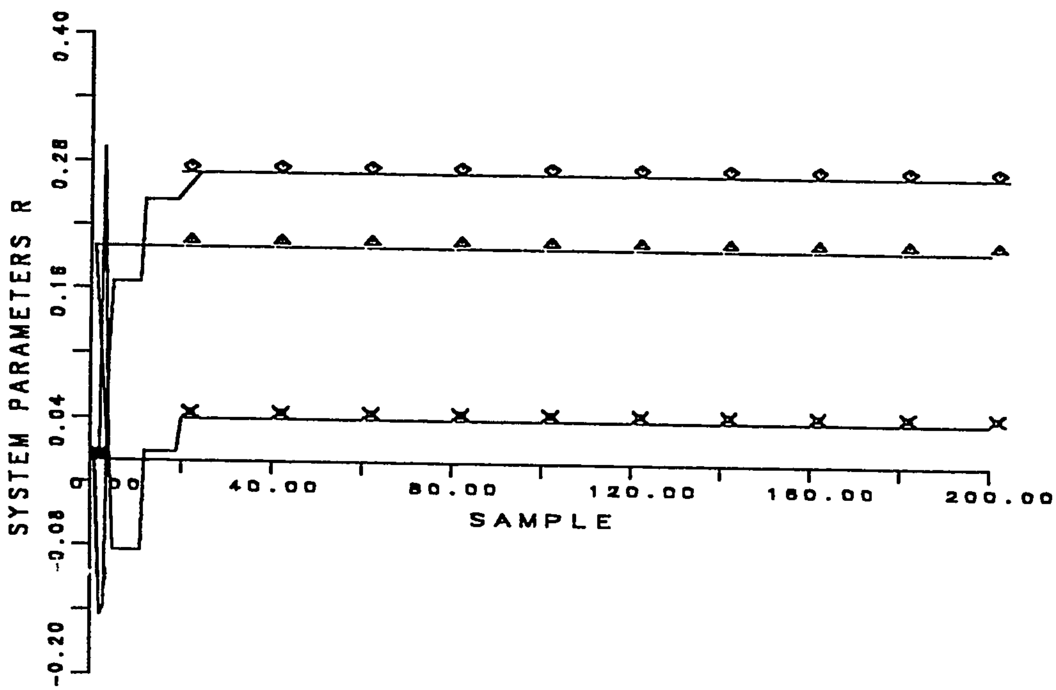

Figure 5.15: System parameters trajectory of third order system

controller were all set to zero. The reference input $v(k)$ is taken to be a sum of 100 distinct frequency components. The adaptive control algorithm based on the (l.m.r.) is used with a block length $N = 25$. The trajectories of all signals are shown in Fig. 5.16-5.22.

5.2.4 Simulation 4 for SISO

This simulation considers a fifth order system with the following transfer function

$$T(z) = \frac{z^2 + 1.5z - 1}{z^5 + 0.6z^3 - 0.55z} \tag{5.4}$$

The system zeros are at $(z = 0.5, -2.0)$ and system poles are at $(z = 0, \pm j1.05, \pm 0.707)$. The desired closed-loop poles are selected at $(z = \pm 0.9, 0.5 \pm j0.4, 0.1)$. The polynomial $q^*(z)$ has roots at $(z = \pm 0.1, \pm j0.2)$. The initial conditions on the plant parameters and the controller were all set to zero. The reference input $v(k)$ is taken to be a sum of 100 distinct frequency components. The adaptive control algorithm based on the (l.m.r.) is used with a block length $N = 25$. The trajectories of all signals are shown in Fig. 5.23-5.29.

Similar to the (l.m.r.) the adaptive pole assignment based on the (r.m.r.) has been simulated for the first two mentioned systems for comparison with (l.m.r.). The configuration used in the adaptive control simulation is shown

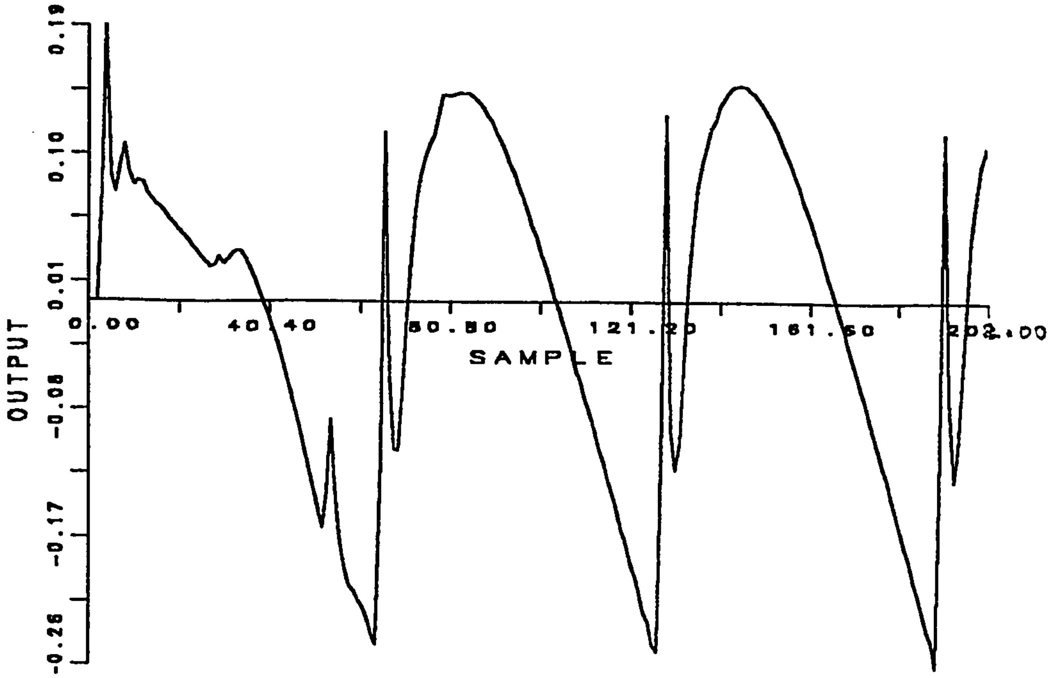

Figure 5.16: Output trajectory of fifth order system

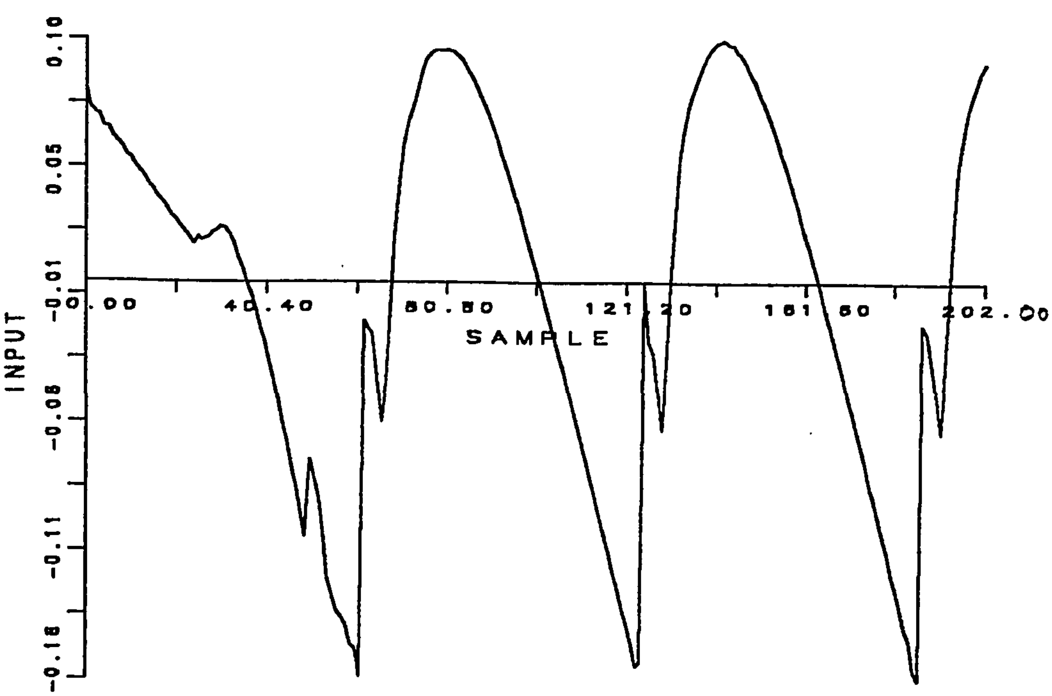

Figure 5.17: Input trajectory of fifth order system

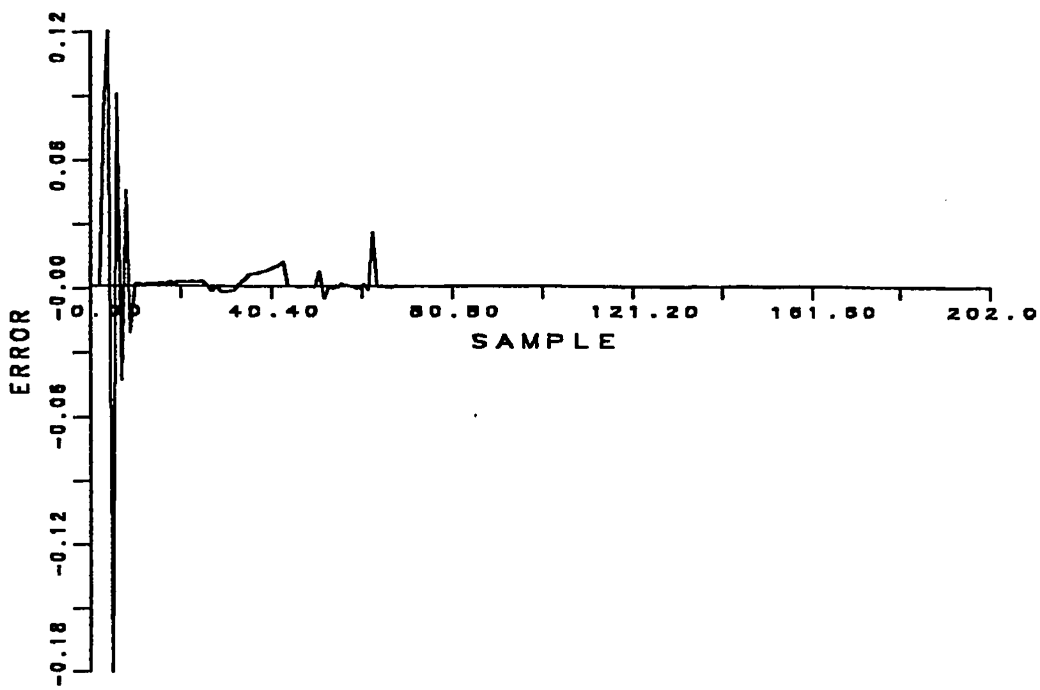

Figure 5.18: Error trajectory of fifth order system

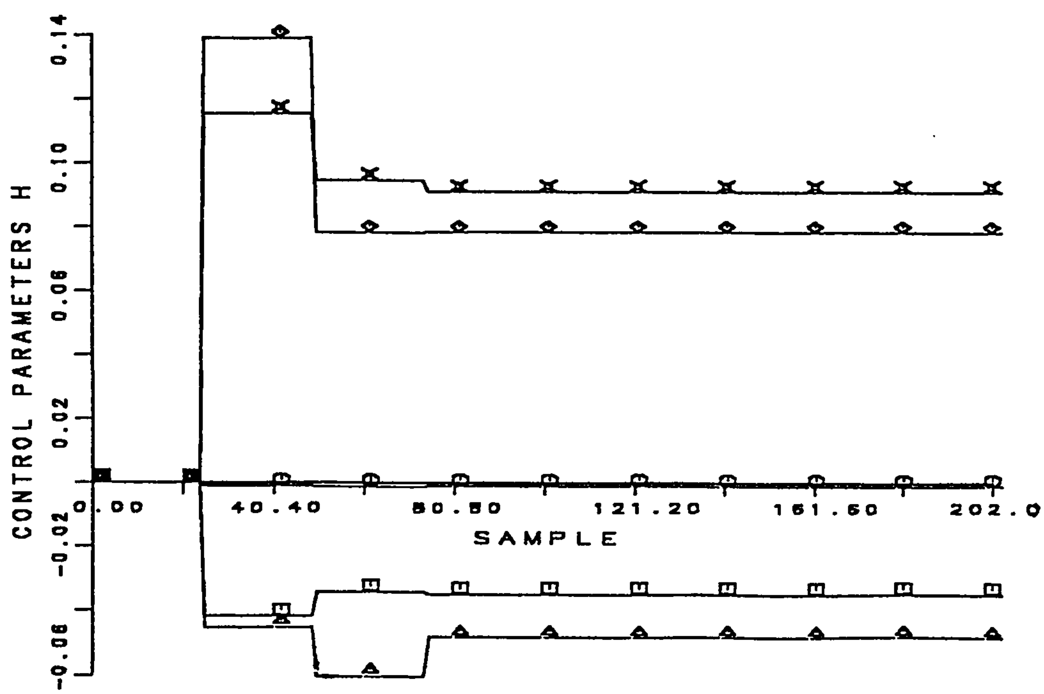

Figure 5.19: Controller parameters trajectory of fifth order system

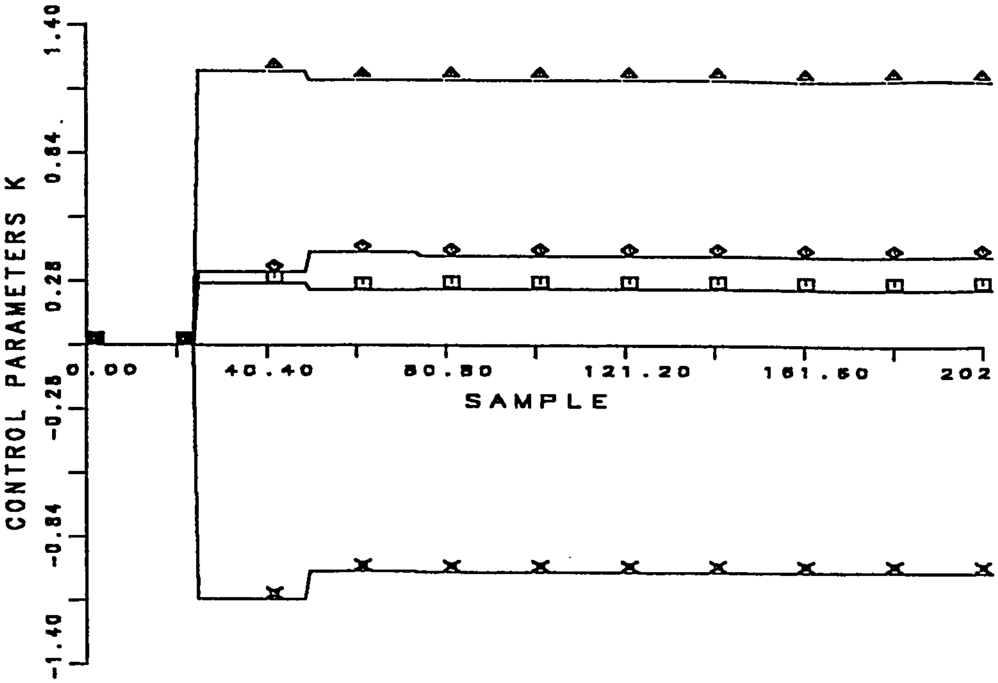

Figure 5.20: Controller parameters trajectory of fifth order system

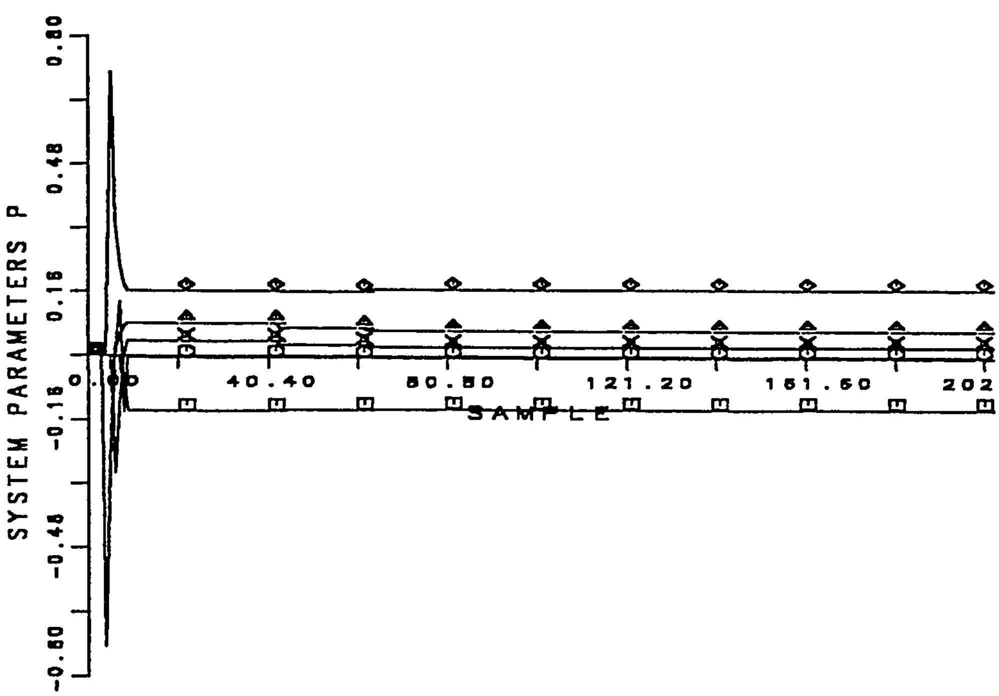

Figure 5.21: System parameters trajectory of fifth order system

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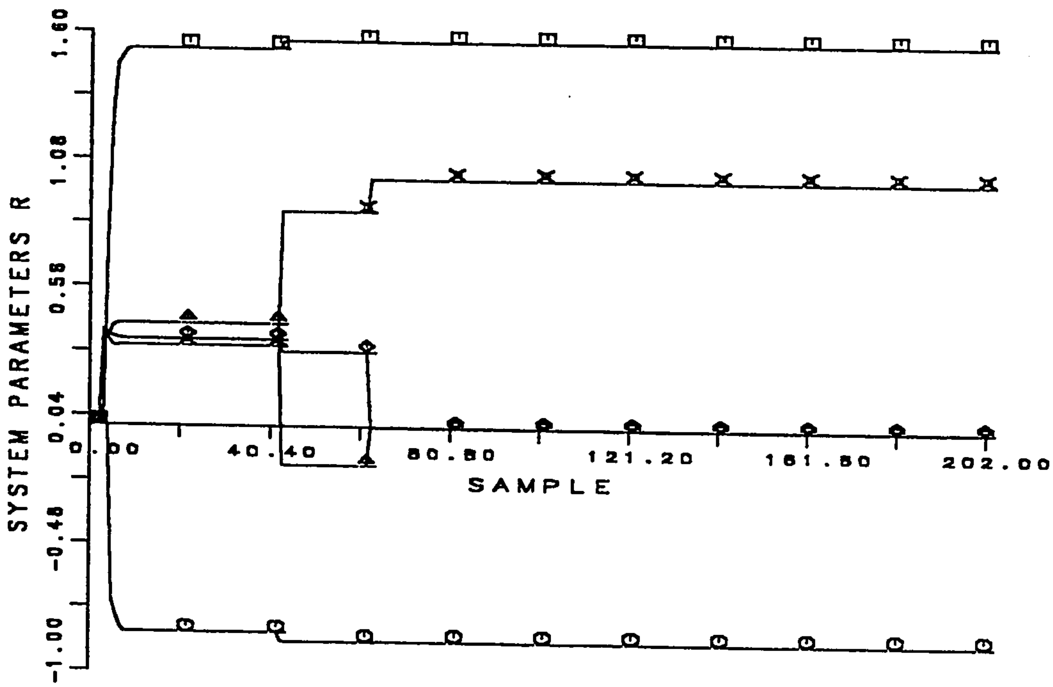

Figure 5.22: System parameters trajectory of fifth order system

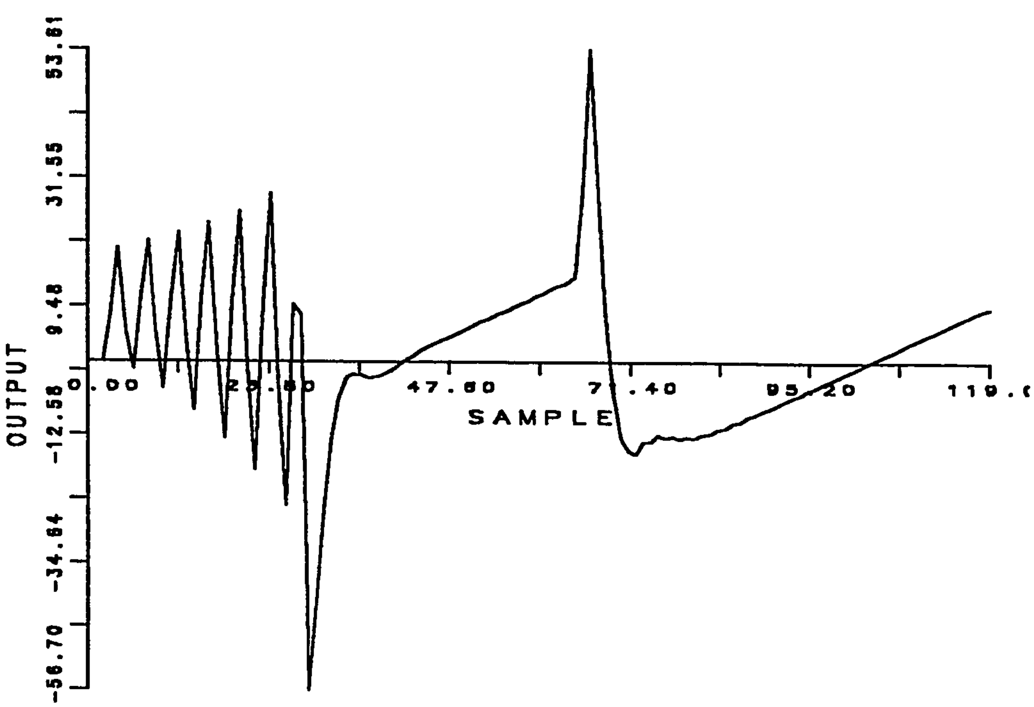

Figure 5.23: Output trajectory of fifth order system

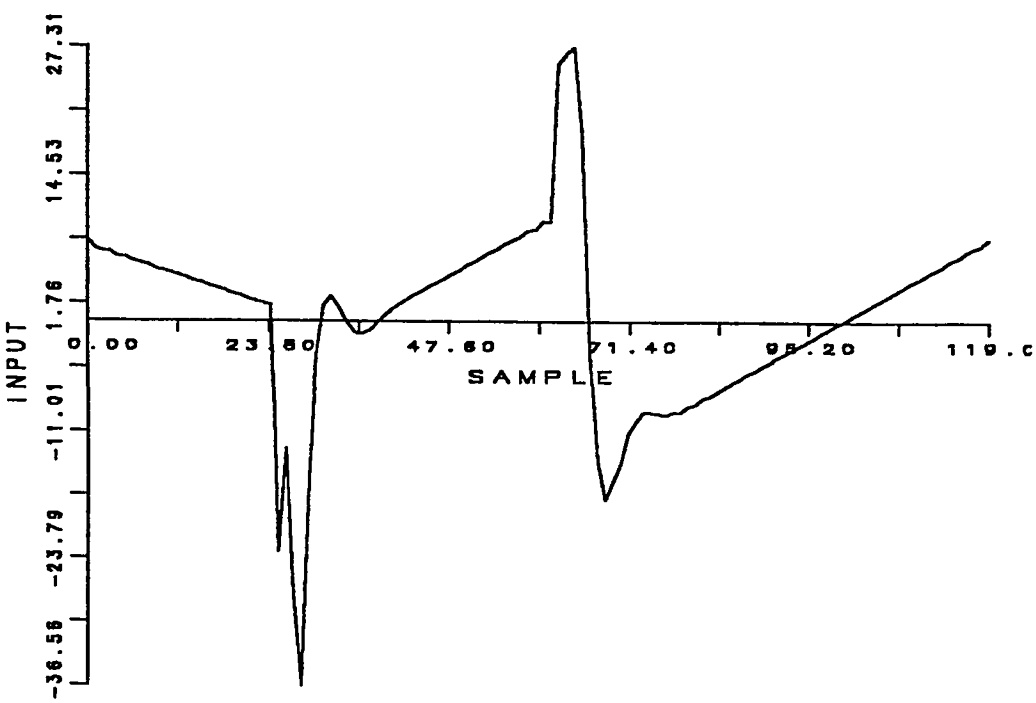

Figure 5.24: Input trajectory of fifth order system

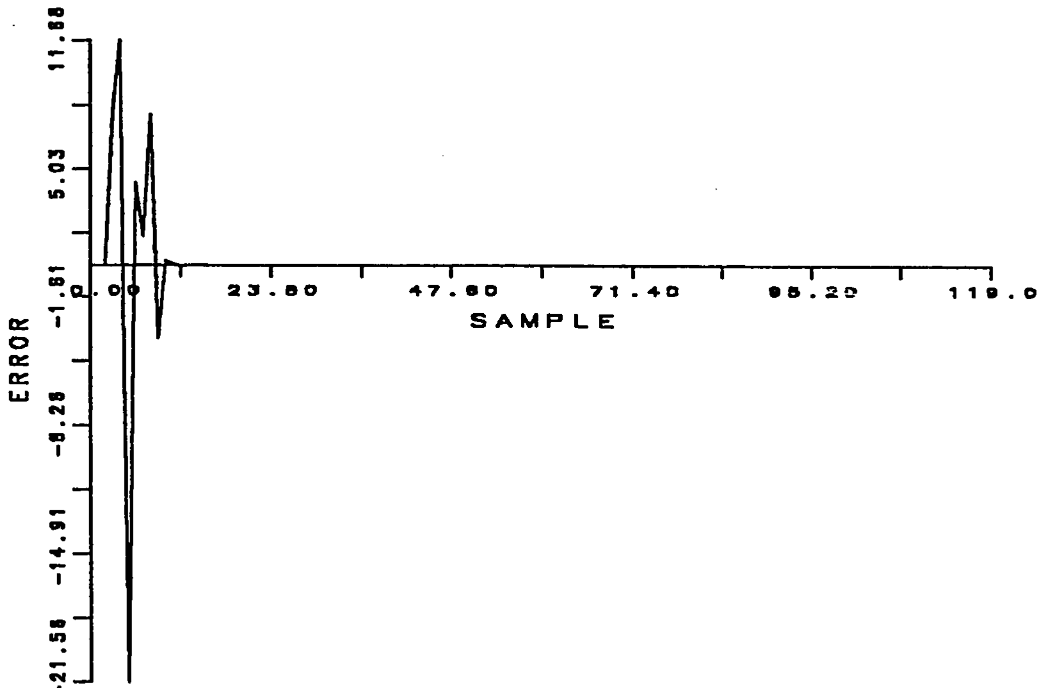

Figure 5.25: Error trajectory of fifth order system

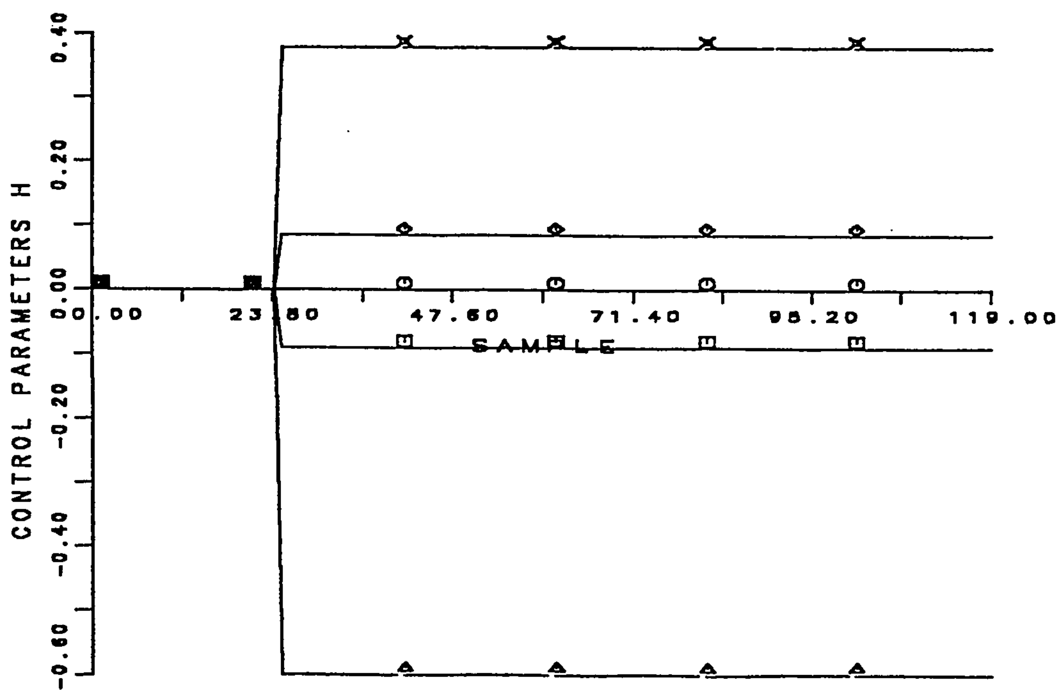

Figure 5.26: Controller parameters trajectory of fifth order system

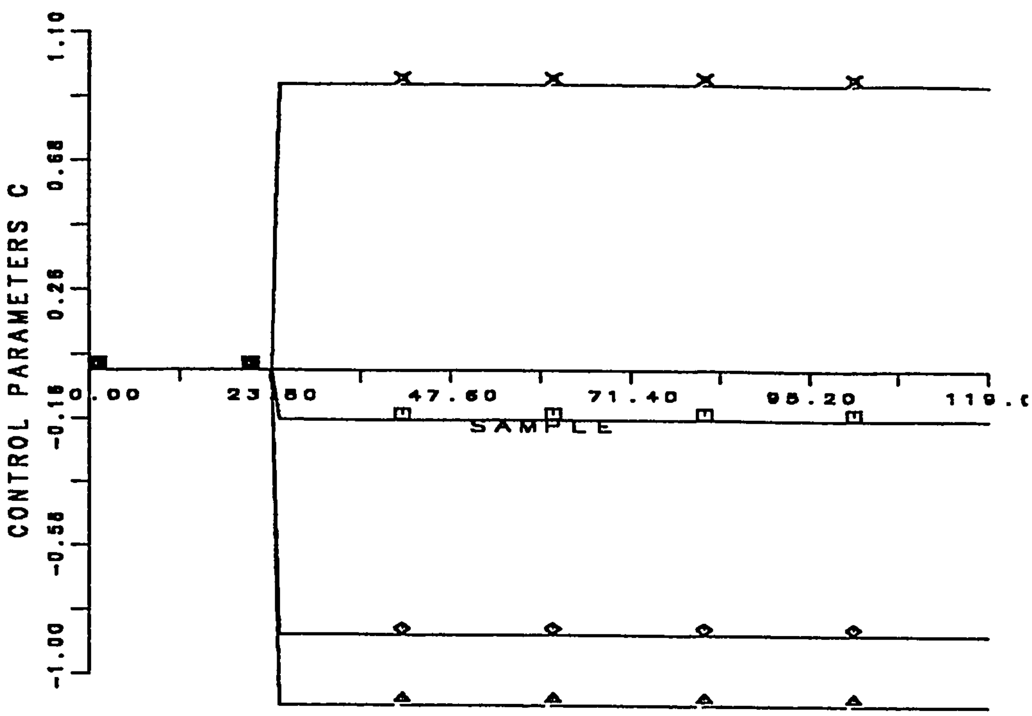

Figure 5.27: Controller parameters trajectory of fifth order system

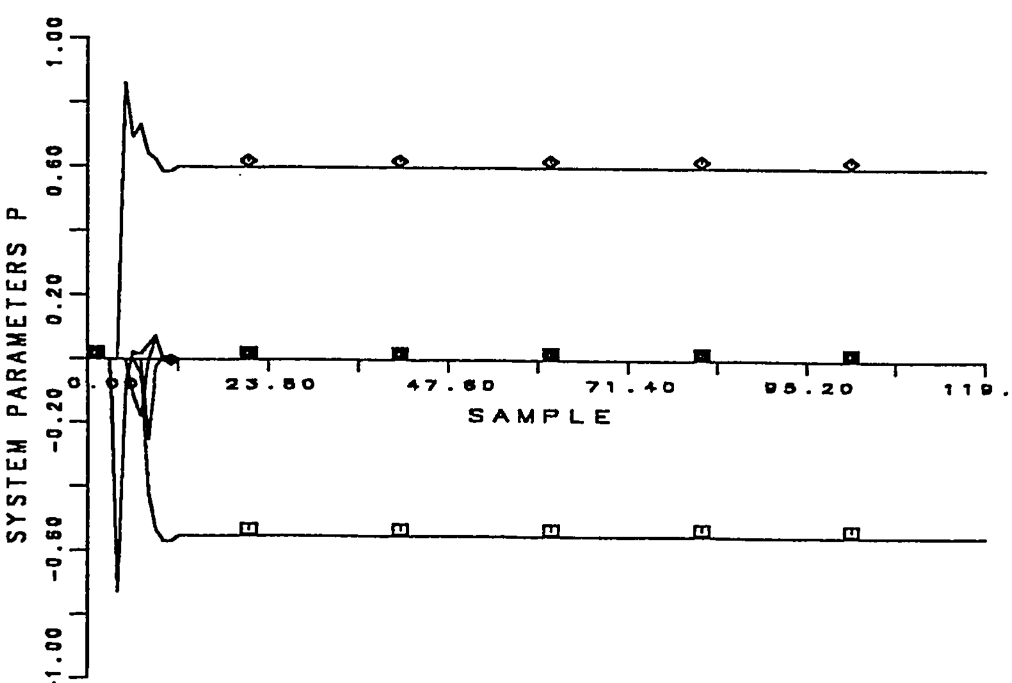

Figure 5.28: System parameter trajectories of fifth order system

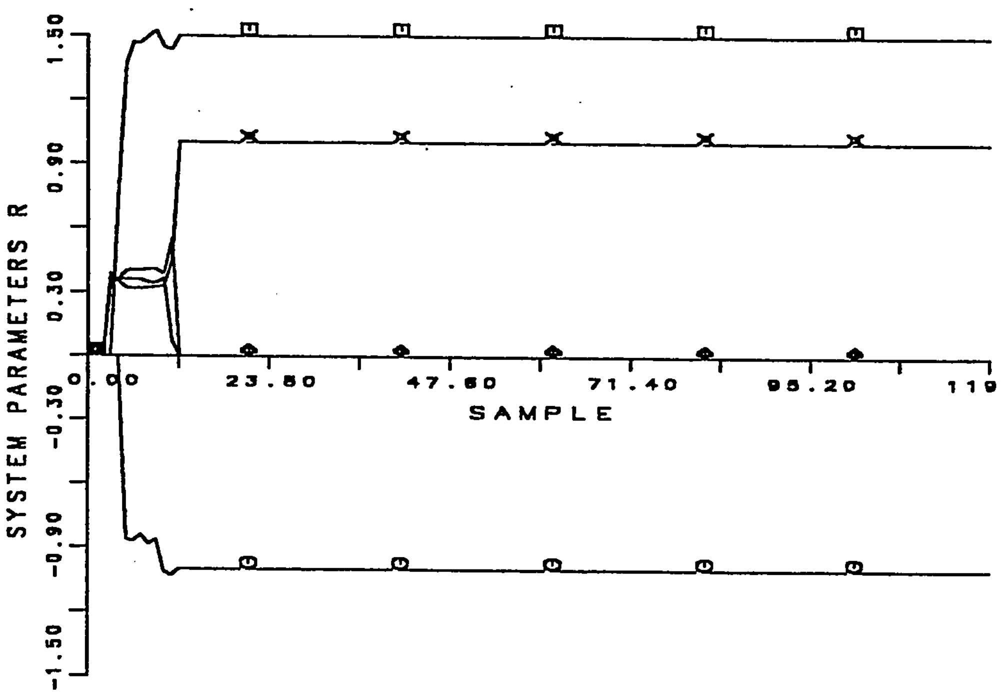

Figure 5.29: System parameter trajectories of fifth order system

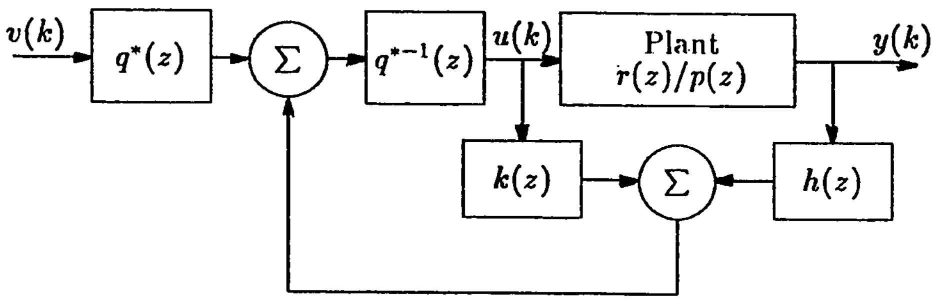

Figure 5.30: Configuration of a feedback controller based on (r.m.r.)

in Fig. 5.30.

The simulation results are given in Fig. 5.31–5.37 for the SISO second order system. The third order system has also been simulated and result are shown in Fig. 5.38–5.44.

5.3 Simulation examples of indirect algorithms for MIMO systems

In this section, we present some simulation studies of the indirect adaptive pole assignment based on the left fraction decomposition discussed in chapter 4. The least-squares estimation scheme is used in estimating the system parameters. The idea of block processing is used to update the

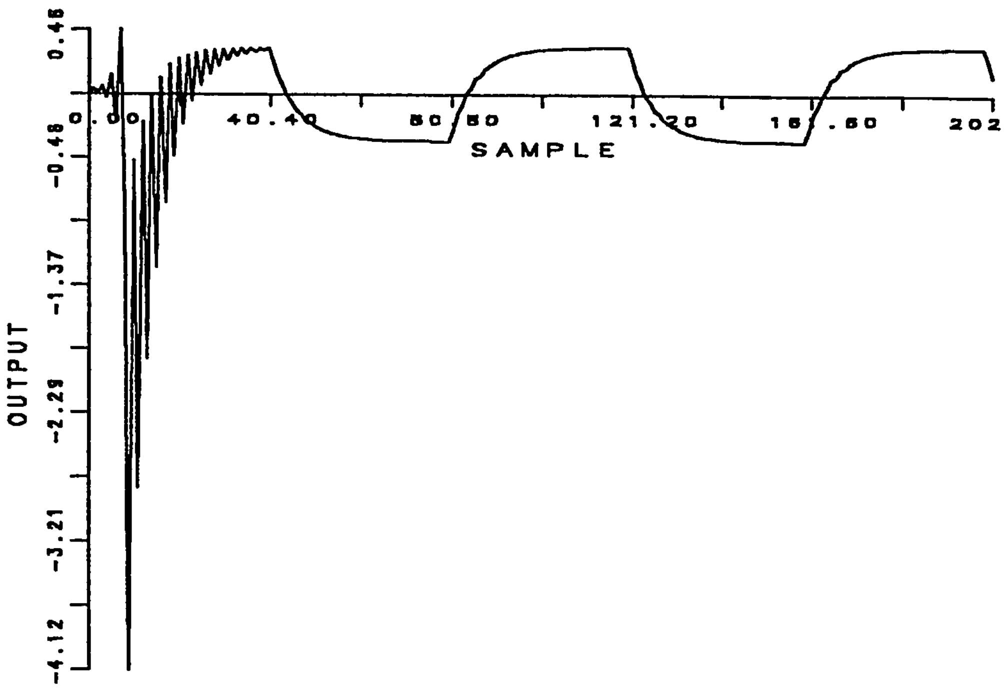

Figure 5.31: Output trajectory of second order system

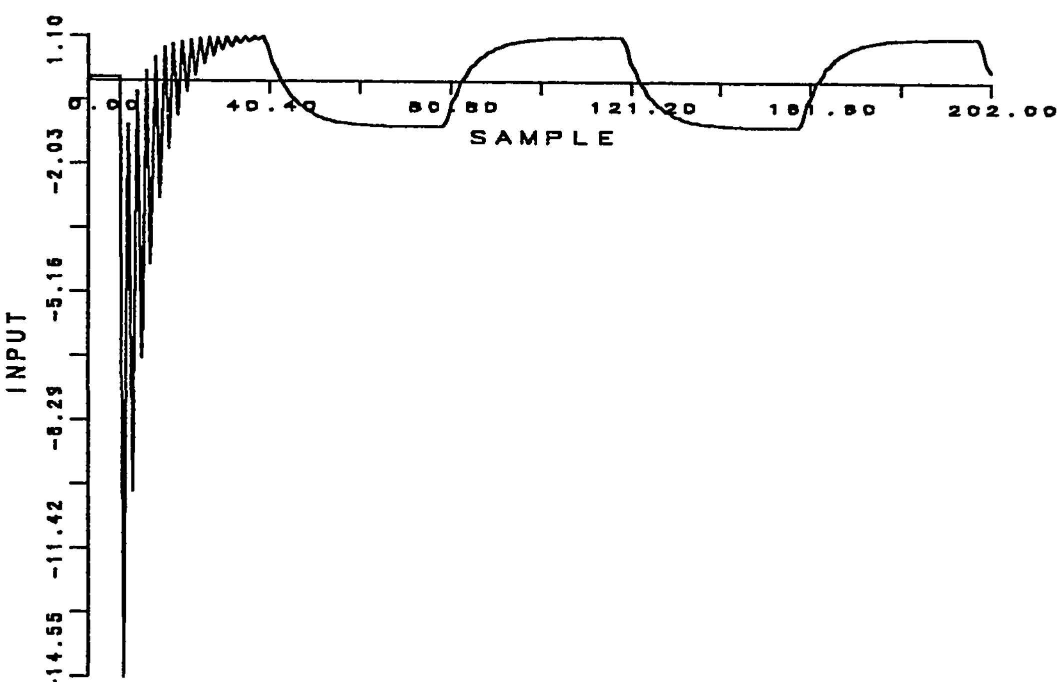

Figure 5.32: Input trajectory of second order system

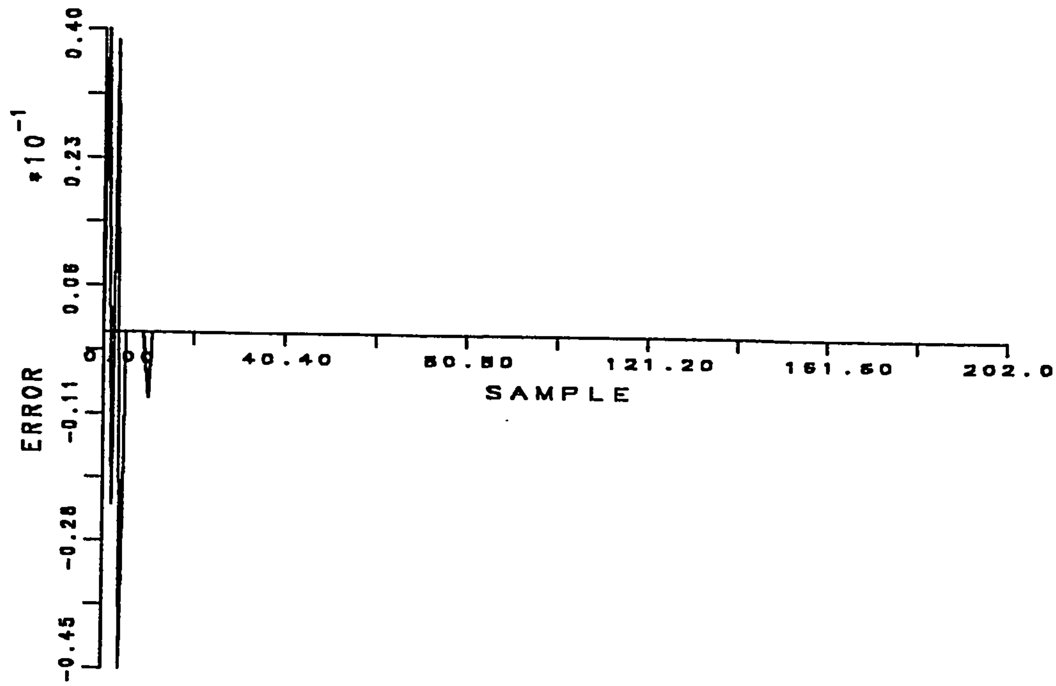

Figure 5.33: Error trajectory of second order system

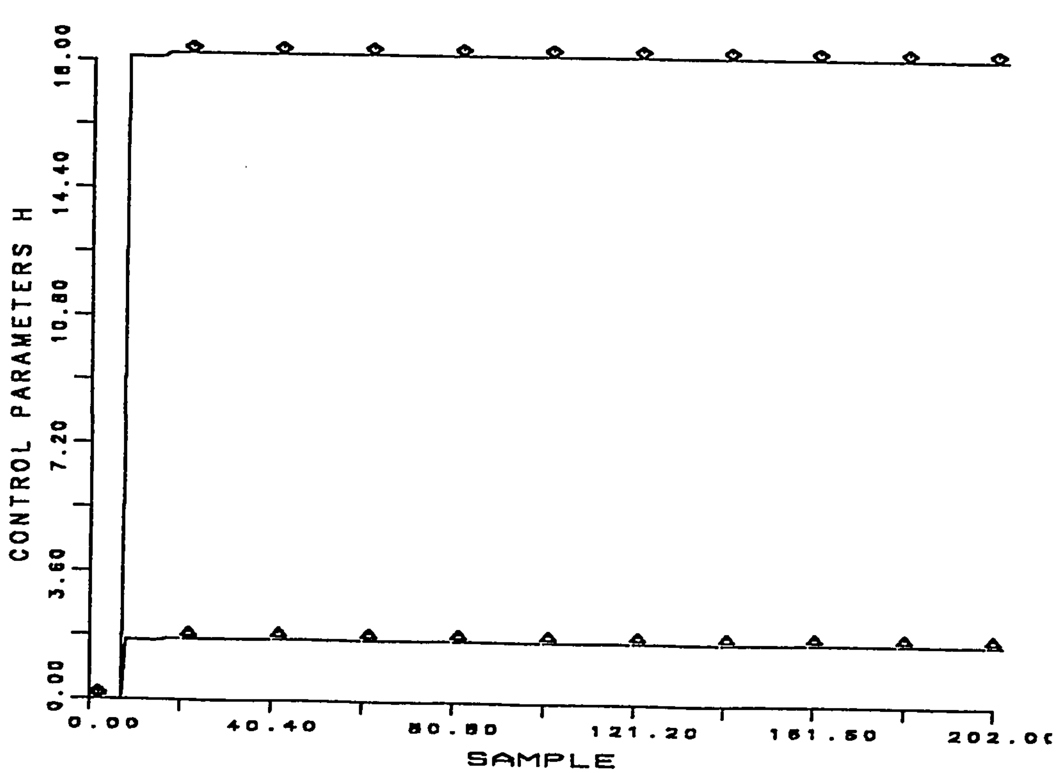

Figure 5.34: Controller parameters trajectory of second order system

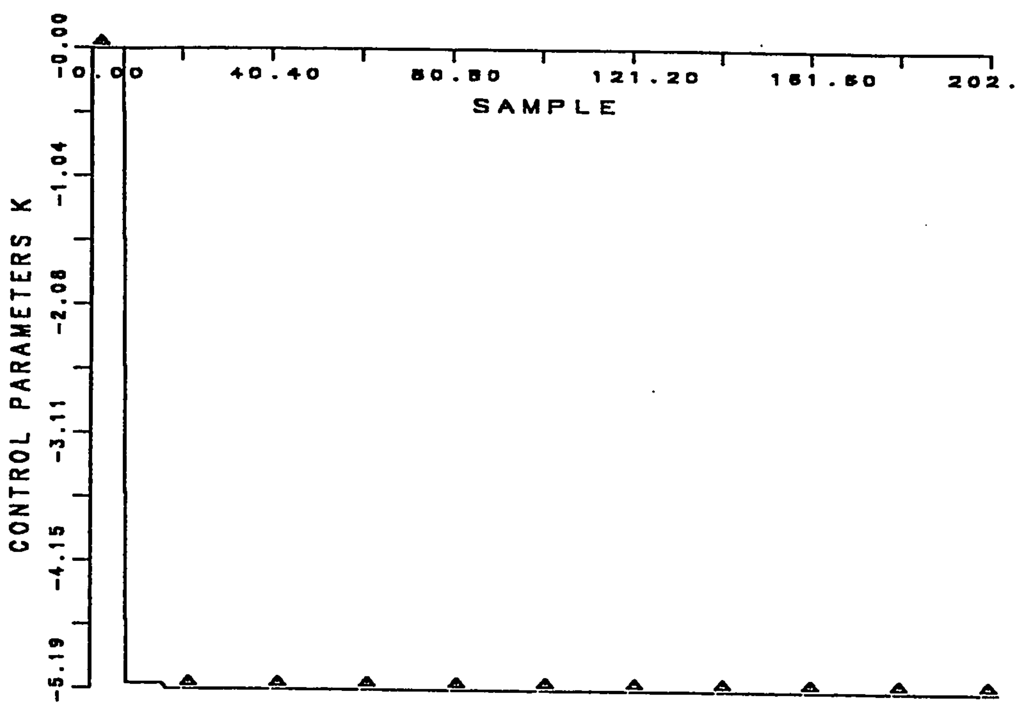

Figure 5.35: Controller parameters trajectory of second order system

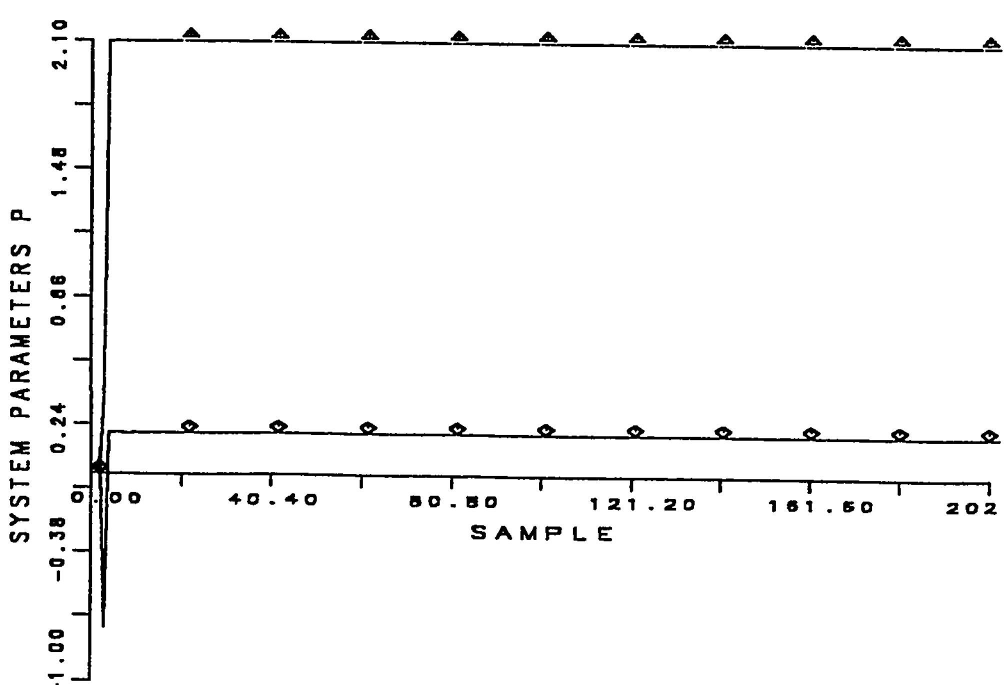

Figure 5.36: System parameters trajectory of second order system

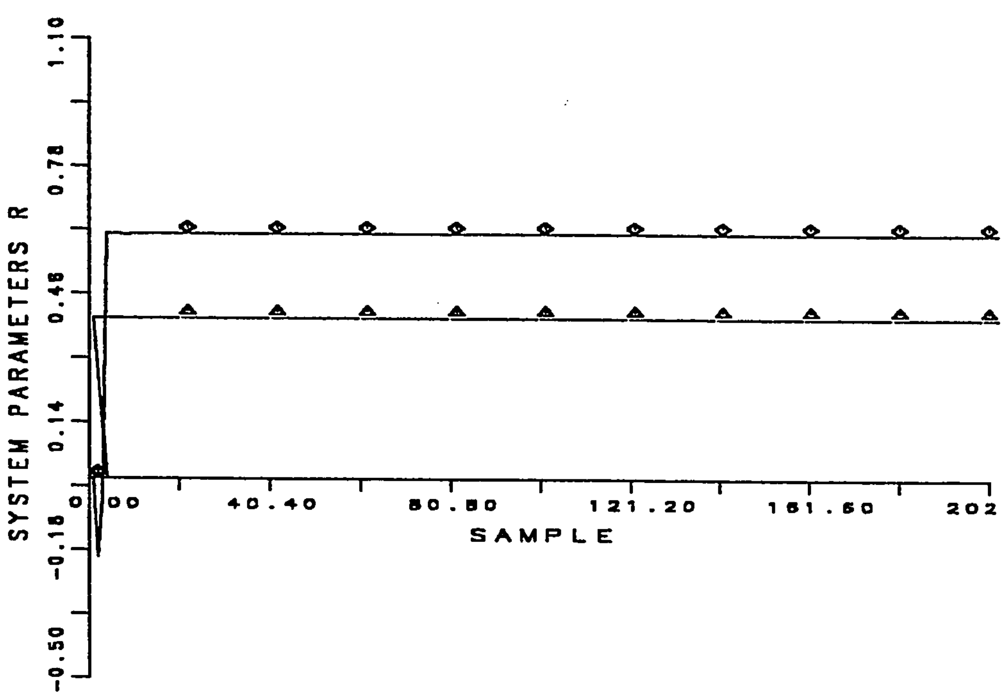

Figure 5.37: System parameters trajectory of second order system

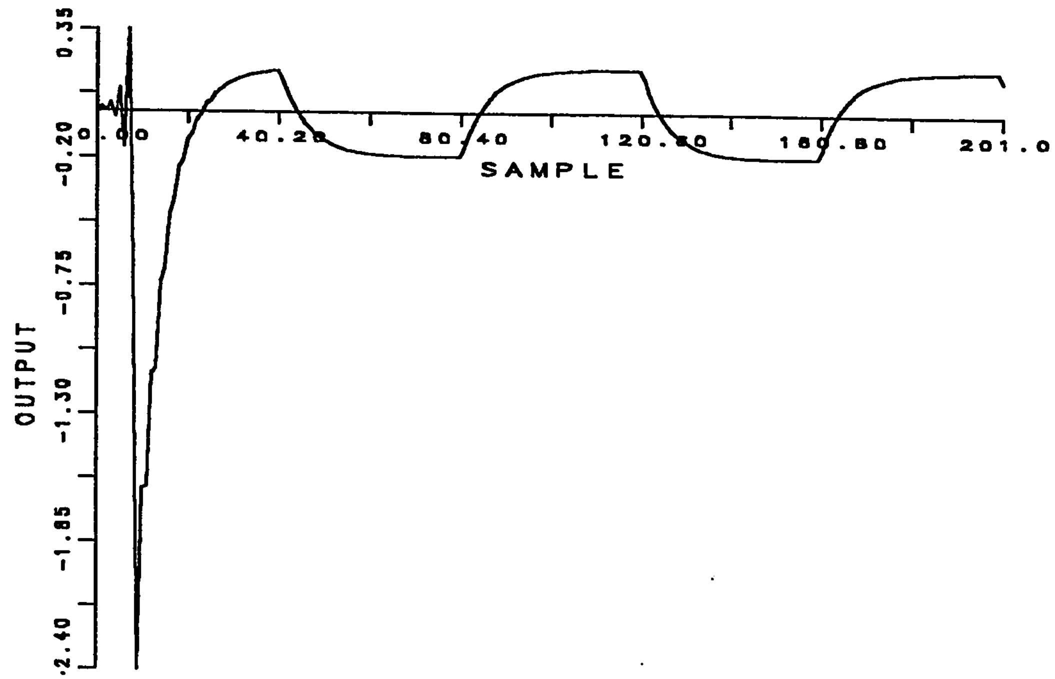

Figure 5.38: Output trajectory of third order system

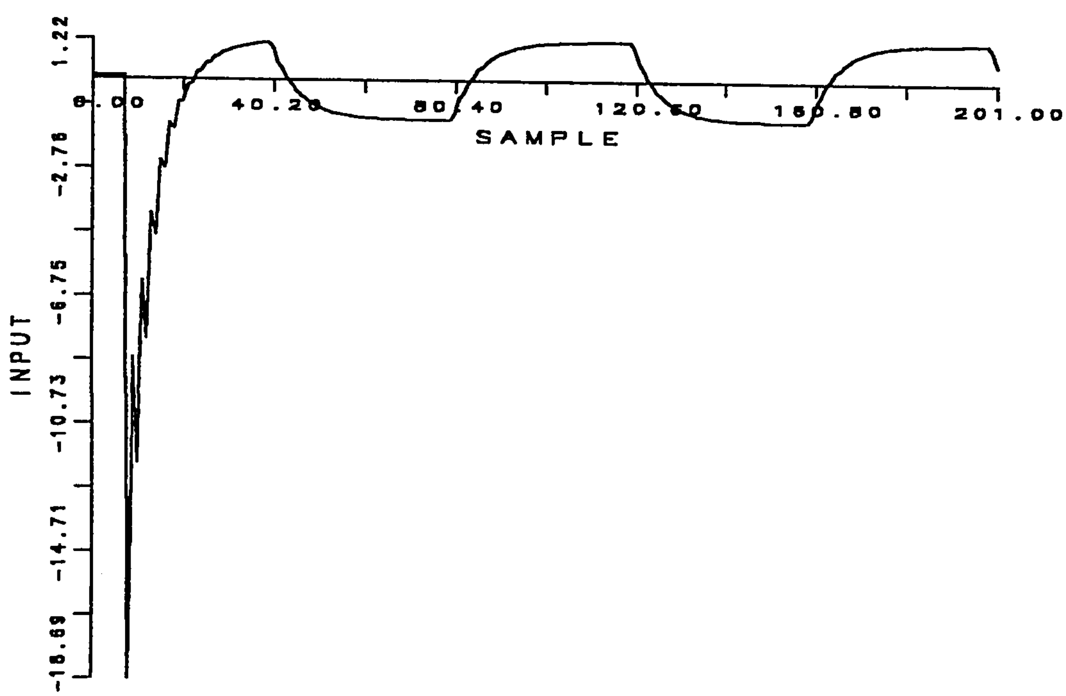

Figure 5.39: Input trajectory of third order system

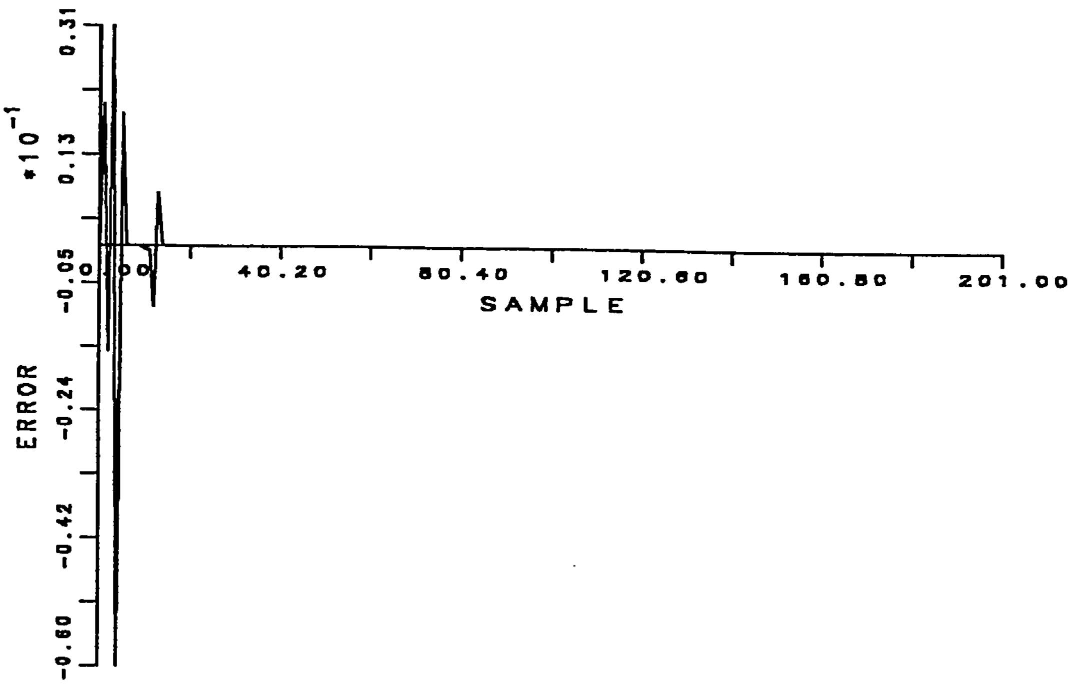

Figure 5.40: Error trajectory of third order system

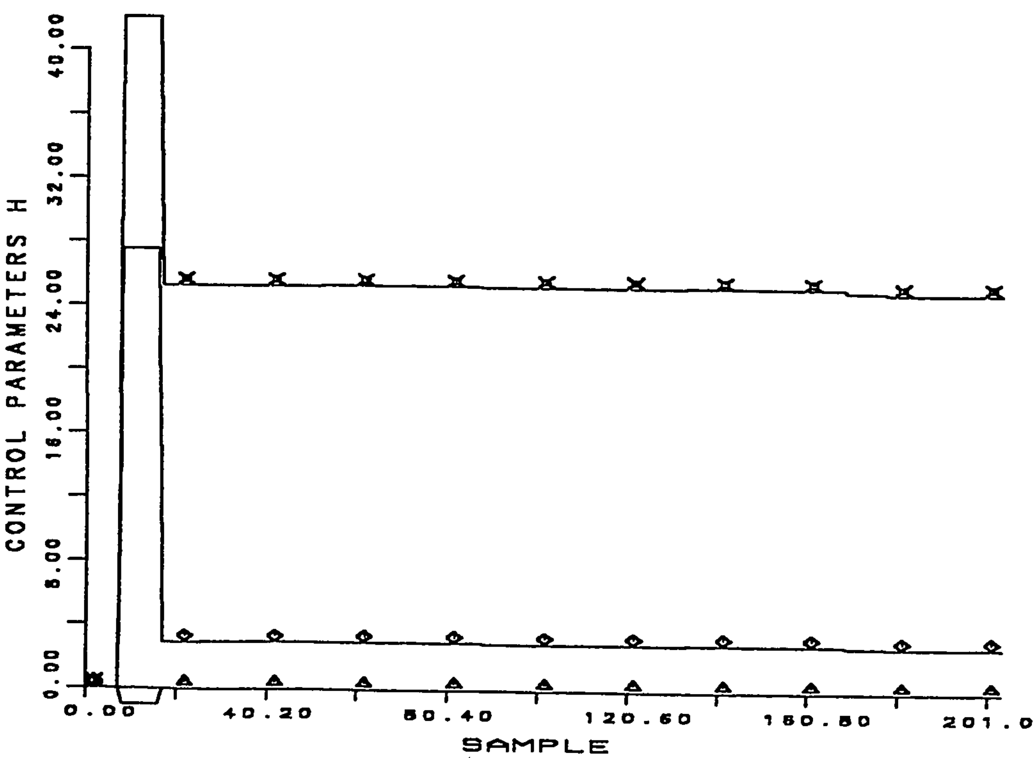

Figure 5.41: Controller parameters trajectory of third order system

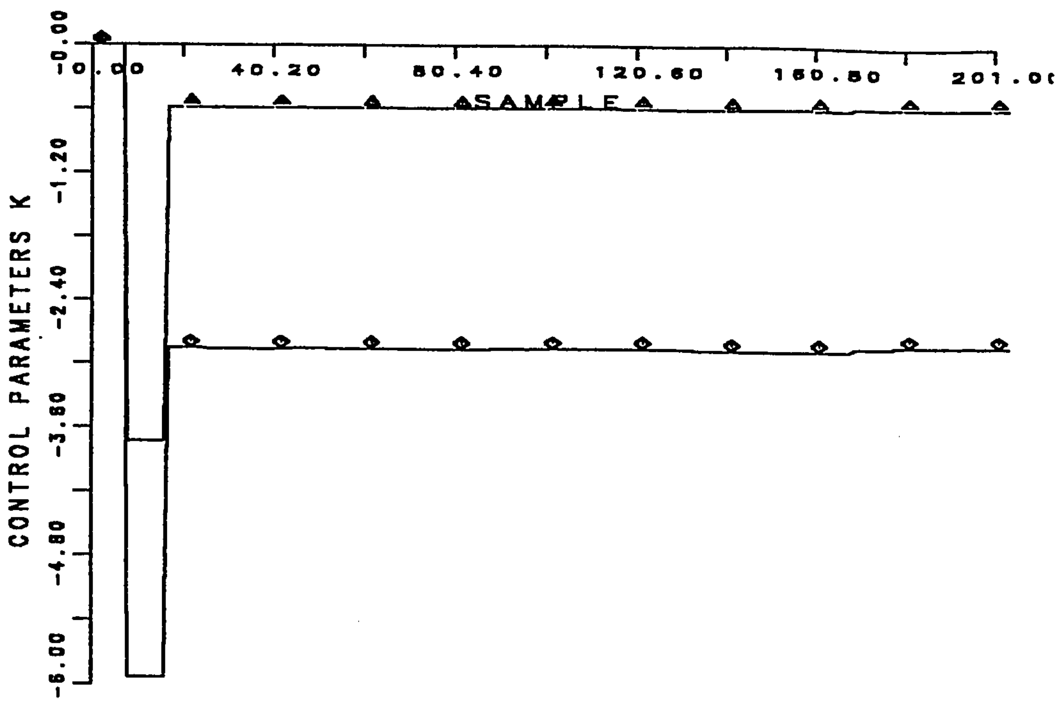

Figure 5.42: Controller parameters trajectory of third order system

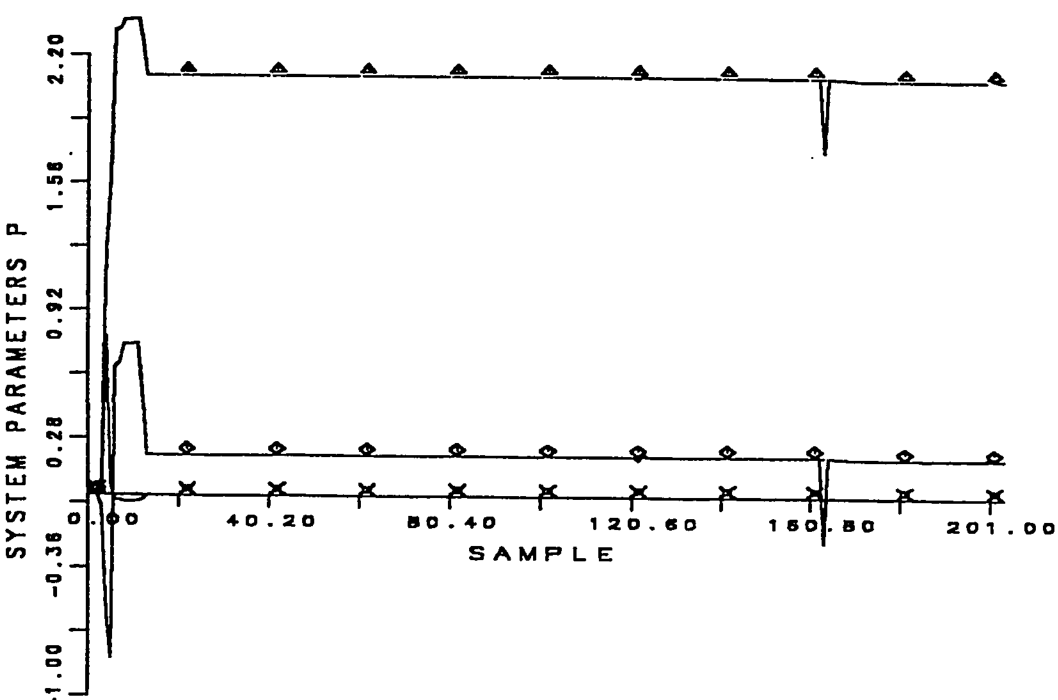

Figure 5.43: System parameters trajectory of third order system

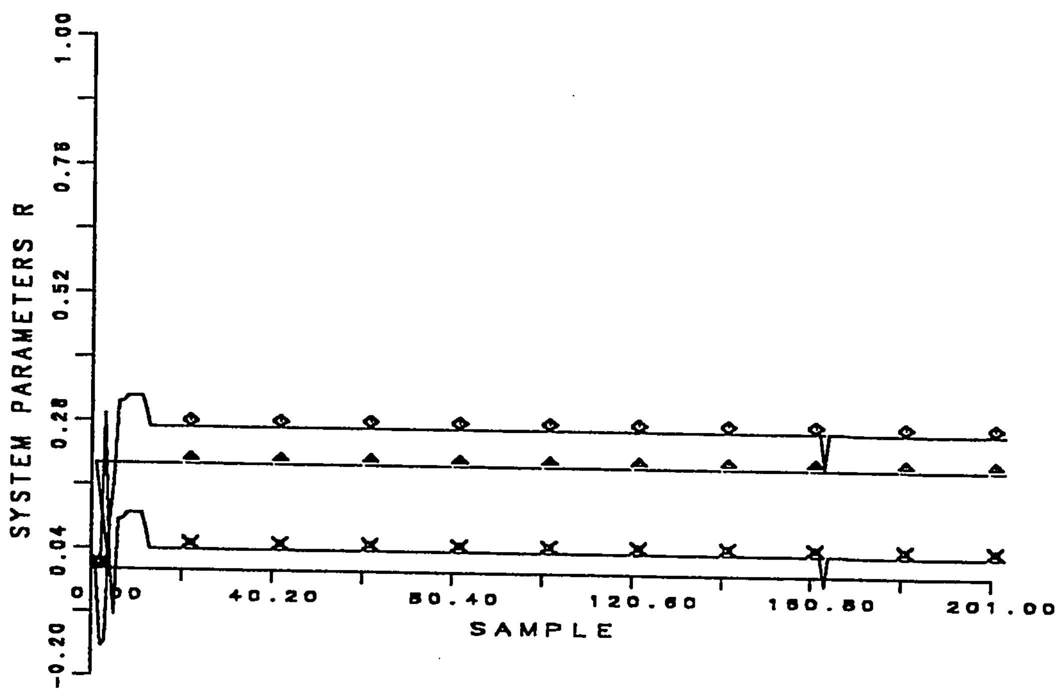

Figure 5.44: System parameters trajectory of third order system

controller parameters. The results of the simulations show that the convergence of the system parameters to their true values is within a peactical finite number of samples.

5.3.1 Simulation 1 for MIMO

This simulation considers a fourth order system. The system to be controlled has the following matrix transfer function:

$$
\begin{aligned}
T(z) &= P^{-1}(z)Q(z) \\
&= \begin{pmatrix} z^2+1.6z+0.15 & 0 \\ 0 & z^2+z \end{pmatrix}^{-1} \begin{pmatrix} 0.61 & 0.6z+0.76 \\ 0.6z+0.5 & 0.2z+0.1 \end{pmatrix}
\end{aligned} \tag{5.5}
$$

with the system zeros at ($z = -0.881 \pm j0.333$). The system poles are at ($z = -1.5, -0.1, -1.0, 0.0$). The observability indices of the system are $\nu_1 = \nu_2 = 2$. For this example the desired closed-loop polynomial matrices are characterized by:

$$
P^*(z) = \begin{pmatrix} z^2+0.3z+0.4 & 0.0 \\ 0.0 & z^2-0.8 \end{pmatrix}
$$

$$
\text{and } Q^*(z) = \text{Diag } (z)
$$

The external input signal was taken as follows

$$
\begin{aligned}
v_1(k) &= \sum_{i=1}^{10} \alpha_i \sin(i/100k) \\
v_2(k) &= \sum_{i=1}^{10} \beta_i \sin(k/100)
\end{aligned}
$$

where α_i and β_i are coefficients different from zero. The initial conditions on the plant parameters and the feedback controller were all set to zero. Finally the simulation was performed using the least-squares sequential estimator with initial covariance matrix $P = 10^{10} I$. All system signals are shown in Fig. 5.45-5.54.

After 20 time samples, the estimated polynomial matrices converge to the true values as shown in Fig. 5.48–5.51. At $k = 20$ we have

$$\begin{aligned} P(z) &= \begin{pmatrix} z^2 + 1.6001z + 0.1502 & -0.0001z - 0.0002 \\ 0.0 & z^2 + 1.0004z + 0.0001 \end{pmatrix} \\ Q(z) &= \begin{pmatrix} 0.6097 & 0.6z + 0.7609 \\ 0.6z + 0.5003 & 0.2z + 0.0997 \end{pmatrix} \end{aligned}$$

From Fig. 5.52–5.54, the estimated feedback controller obtained at $k = 30$ is as follows:

$$\begin{aligned} H(z) &= \begin{pmatrix} -2.9z - 0.024 & 0.3522z - 0.052 \\ 3.042z + 0.123 & 1.865z + 0.262 \end{pmatrix} \\ K(z) &= \begin{pmatrix} -0.52 & -1.118 \\ 1.157 & 0.416 \end{pmatrix} \end{aligned}$$

The trajectories of the input vector $u(k)$ and output vector $y(k)$ are shown in Fig. 5.45–5.46. For completeness the identification error vector is given in Fig. 5.47.

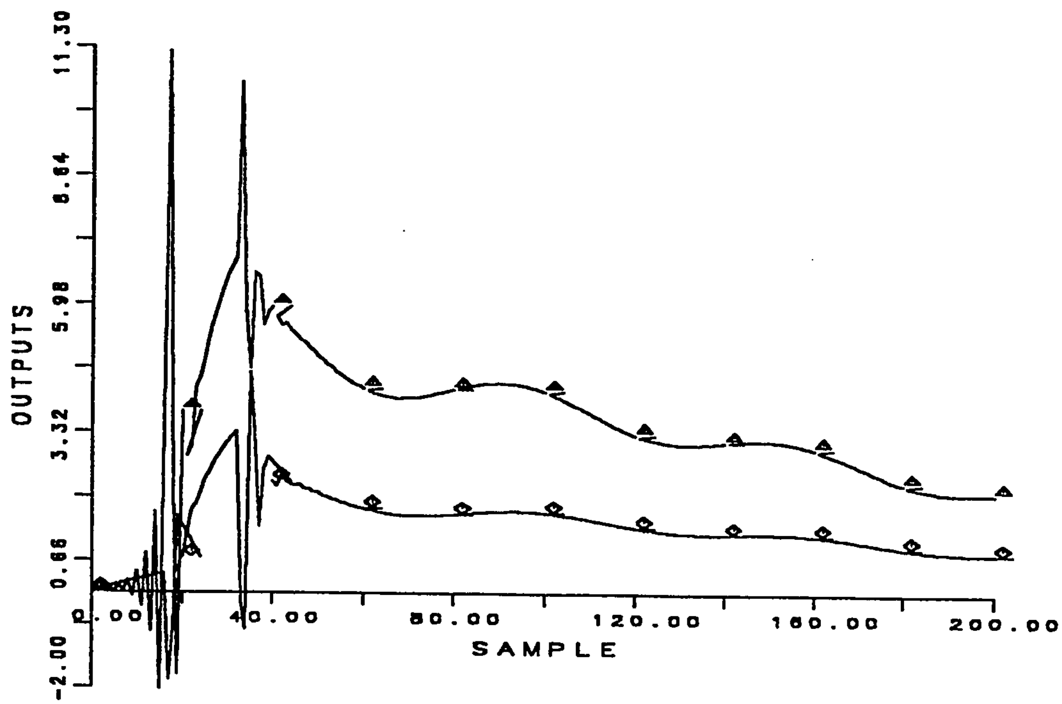

Figure 5.45: Output trajectories of the (2 × 2) MIMO fourth order system

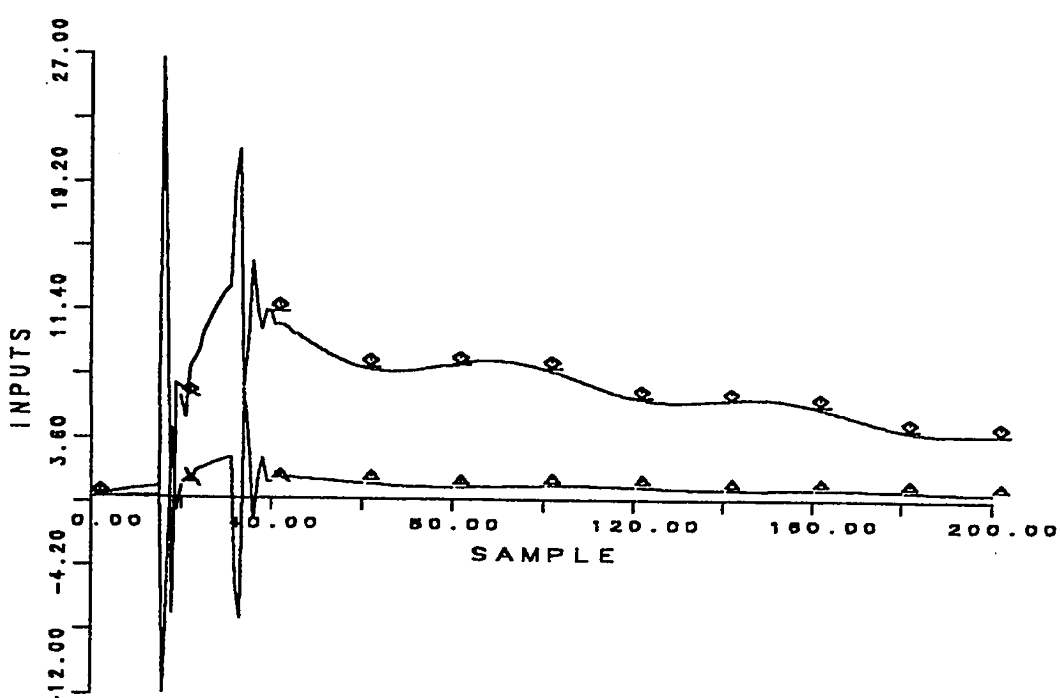

Figure 5.46: Input trajectories of the (2 × 2) MIMO fourth order system

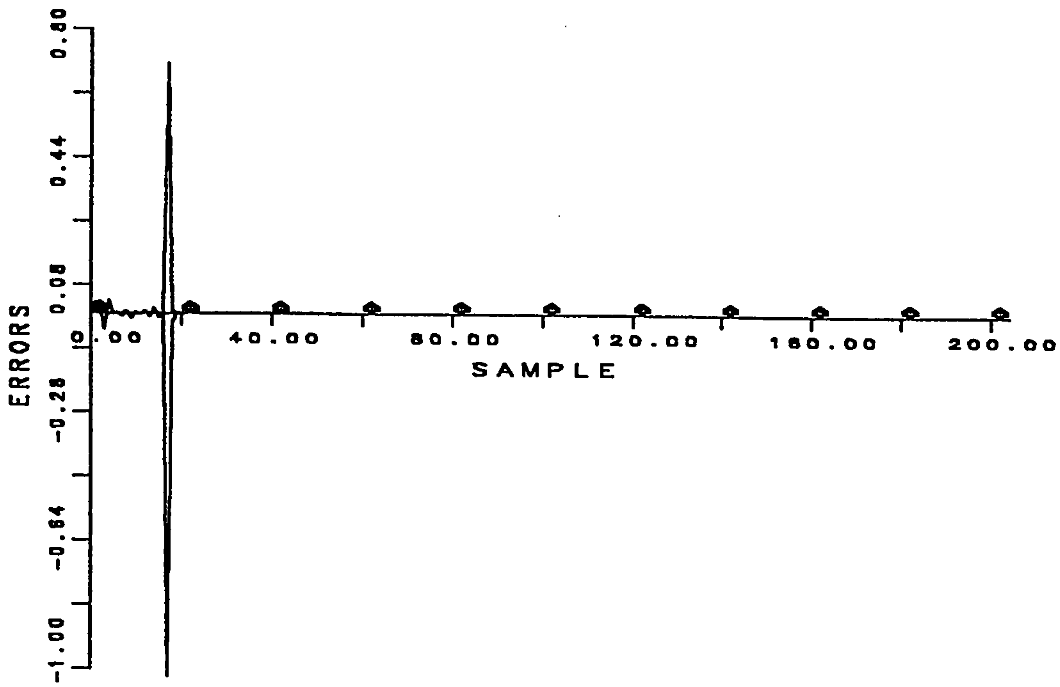

Figure 5.47: Error trajectories of the (2 × 2) MIMO fourth order system

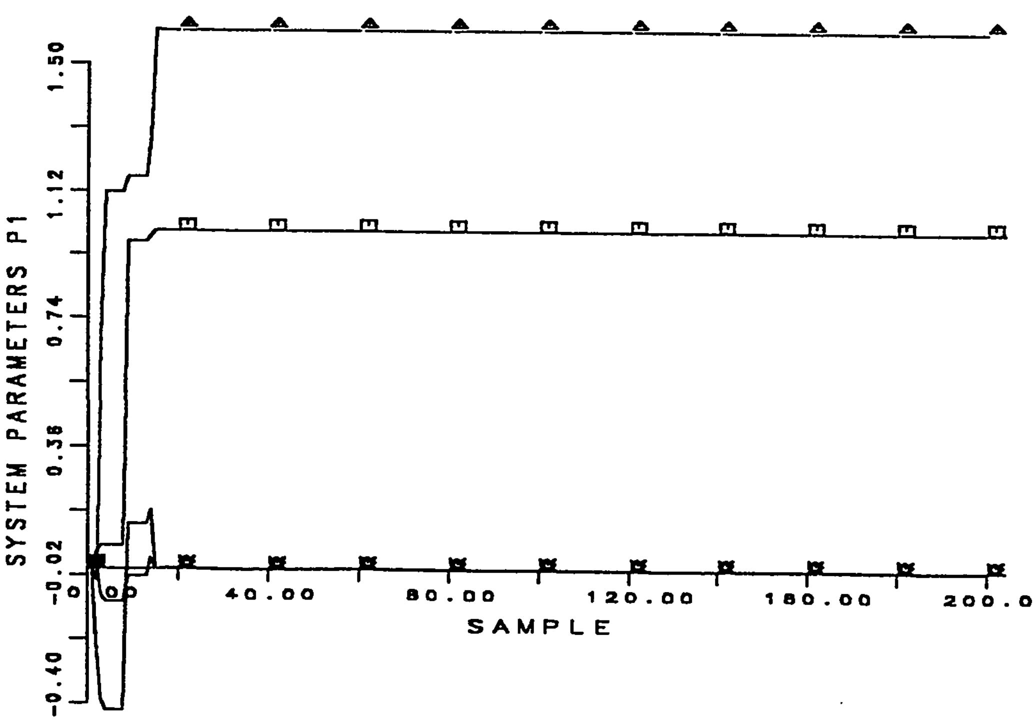

Figure 5.48: System parameters trajectories of the (2 × 2) MIMO fourth order system

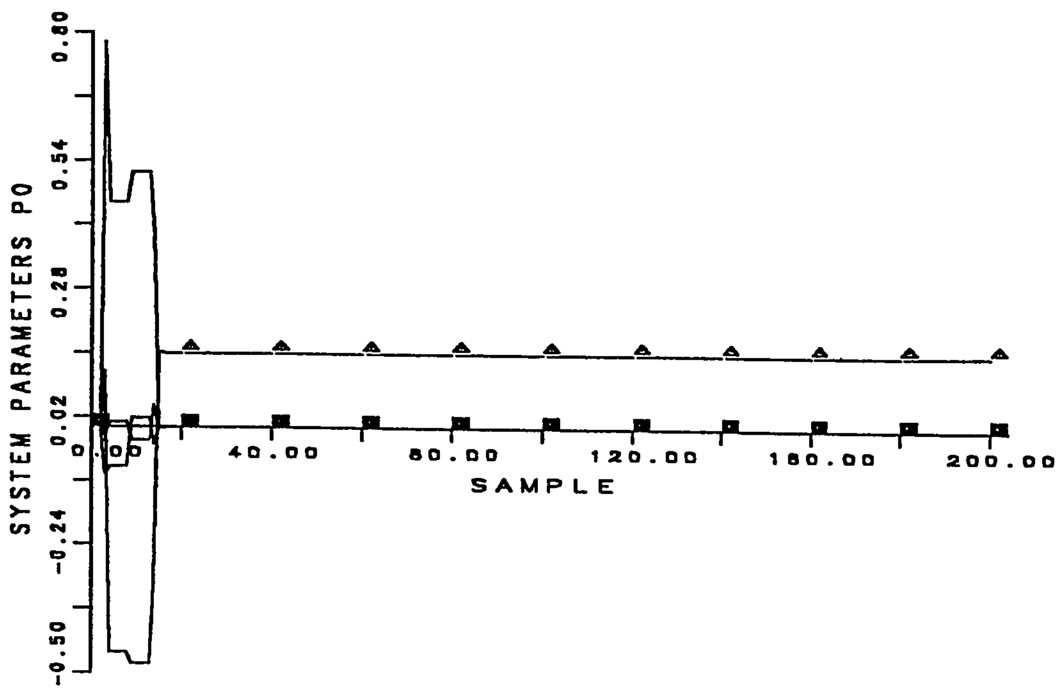

Figure 5.49: System parameters trajectories of the (2 × 2) MIMO fourth order system

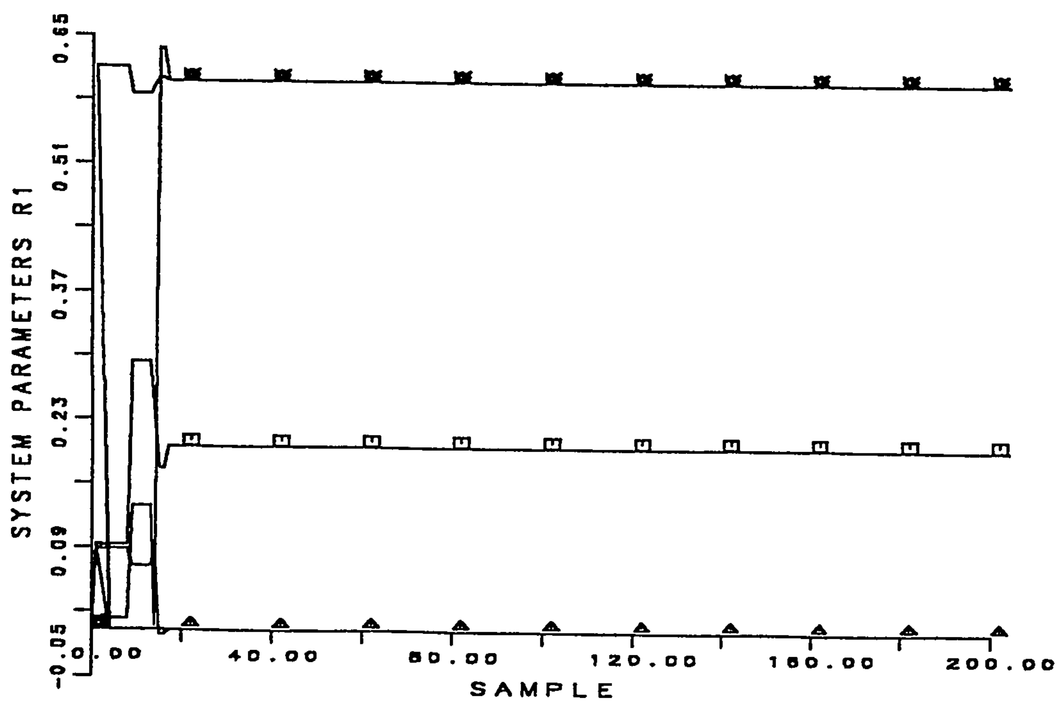

Figure 5.50: System parameters trajectories of the (2 × 2) MIMO fourth order system

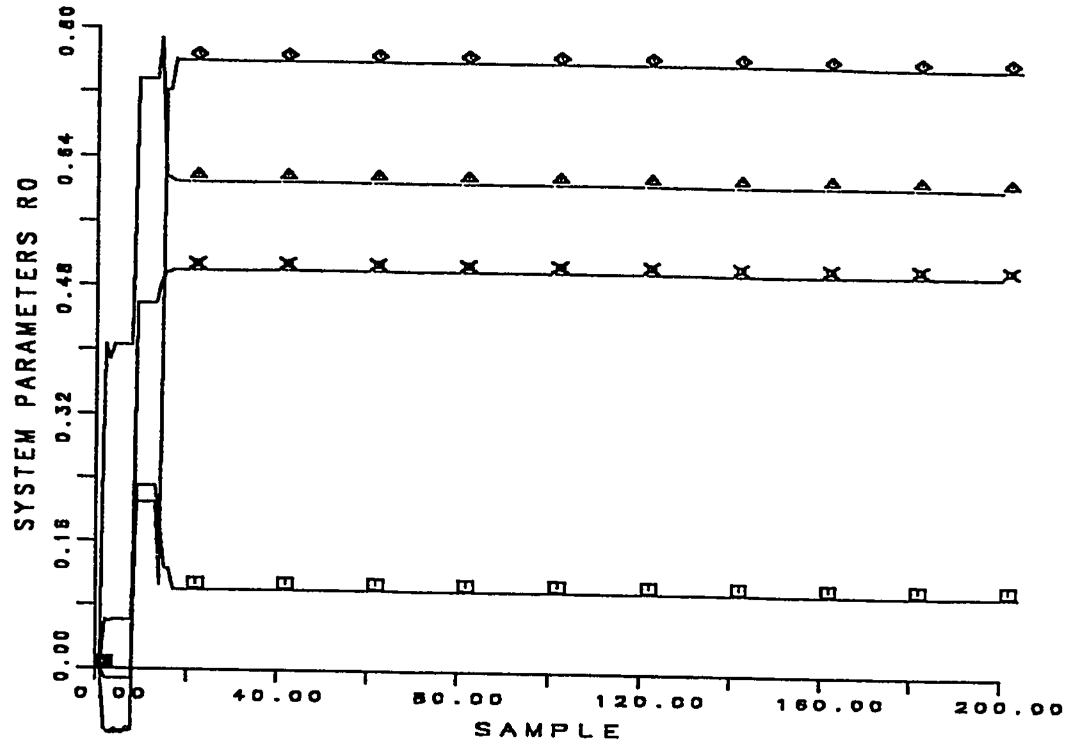

Figure 5.51: System parameters trajectories of the (2 × 2) MIMO fourth order system

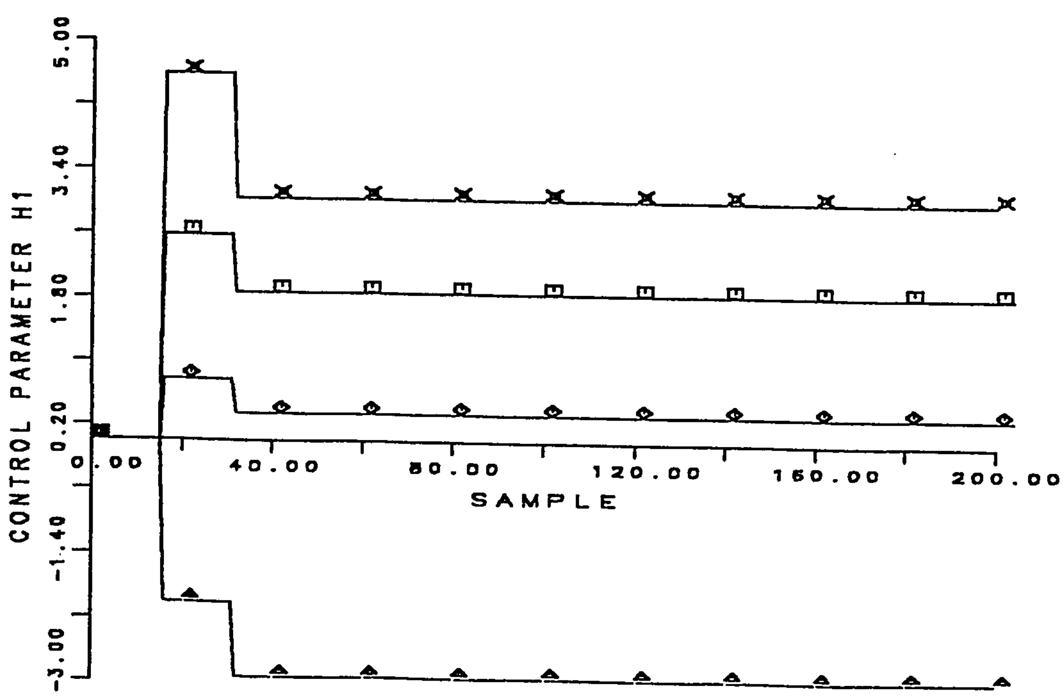

Figure 5.52: Controller parameters trajectories of the (2 × 2) MIMO fourth order system

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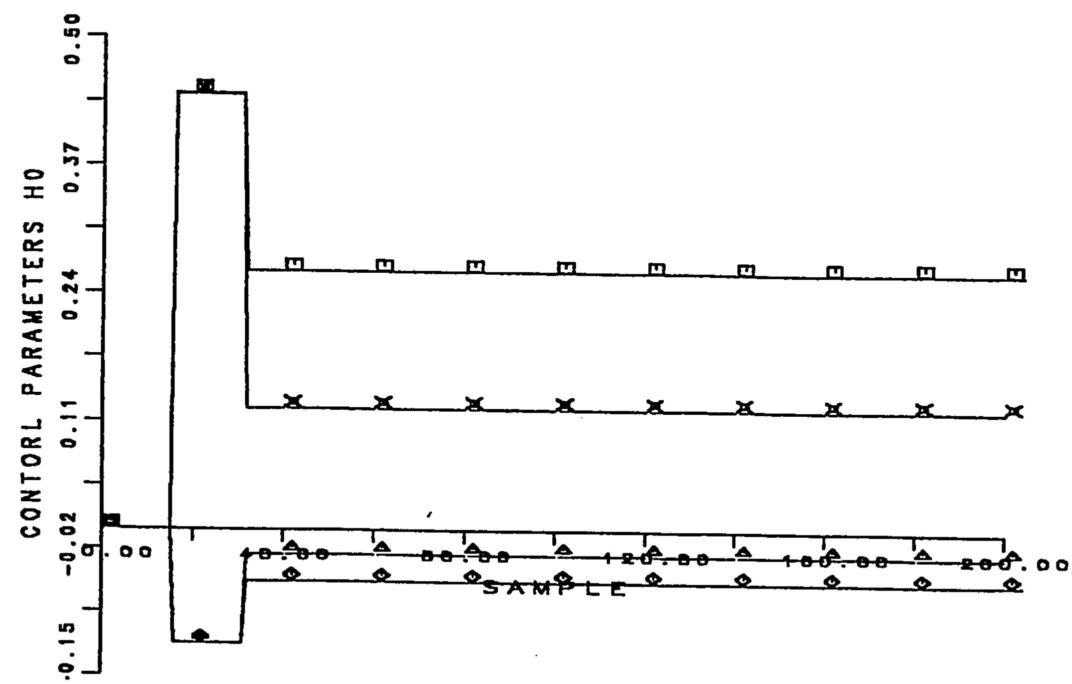

Figure 5.53: Controller parameters trajectories of the (2×2) MIMO fourth order system

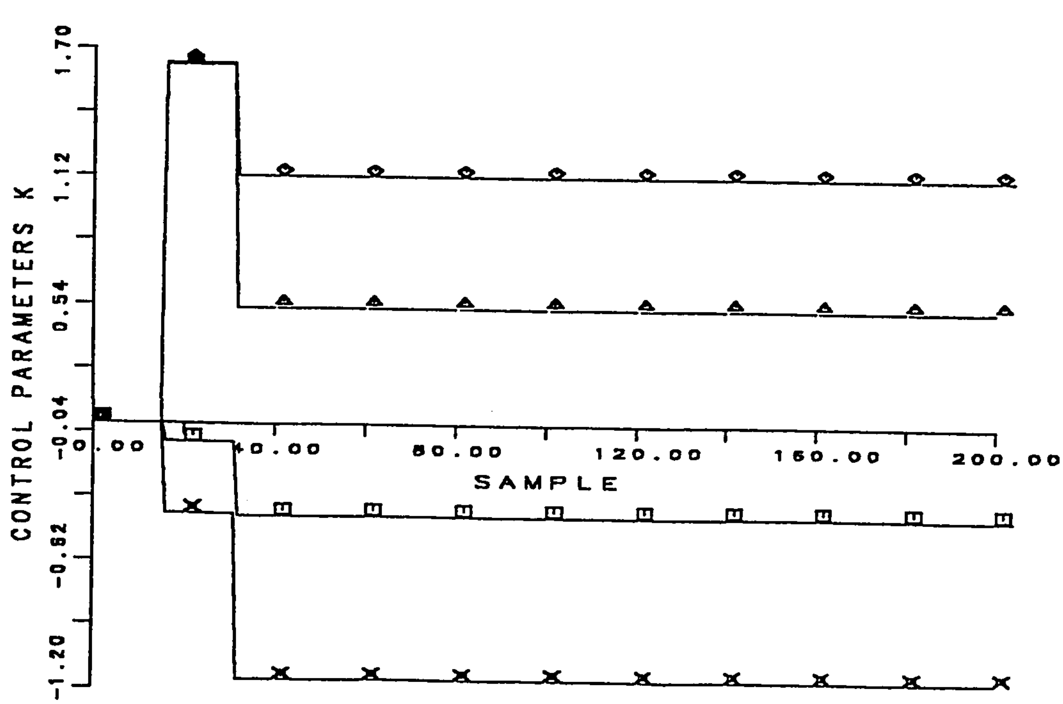

Figure 5.54: Controller parameters trajectories of the (2×2) MIMO fourth order system

5.3.2 Simulation 2 for MIMO

This simulation considers a 3×2 MIMO system of order 6 with the following transfer function:

$$
\begin{aligned}
T(z) &= P(z)^{-1}Q(z) \\
&= \begin{pmatrix} z^2 - 1.1z & 0.1 & 0.2z + 0.01 \\ 0 & z^2 - 0.8z - 0.48 & 0 \\ 0 & 0 & z^2 - 1.8z + 0.82 \end{pmatrix}^{-1} \\
&\qquad \begin{pmatrix} 0.1 & 0.03 \\ 0.3 & 0.1z - 0.3 \\ 0.4z + 0.1 & 0.3 \end{pmatrix}
\end{aligned}
\tag{5.6}
$$

The system zeros are at ($z = -2, 1, 5$) and the system poles are at ($z = 0.0, 1.1, -0.4, 1.2, 0.9 \pm j0.1$). The desired pole locations are at ($z = 0.0, 0.99, -0.1 \pm j0.95, \pm 0.9$) with the following closed-loop polynomial matrices:

$$
\begin{aligned}
P^*(z) &= \begin{pmatrix} z^2 - 0.99z & 0.0 & 0.0 \\ 0 & z^2 - 0.2z + 0.91 & 0 \\ 0 & 0 & z^2 - 0.81 \end{pmatrix} \\
\text{and } Q^*(z) &= \begin{pmatrix} z^2 & 0 & 0 \\ 0 & z^2 & 0 \\ 0 & 0 & z^2 - 0.1z \end{pmatrix}
\end{aligned}
$$

The external exciting signals applied to the system are the sum of 100 sinusoids of distinct frequencies. Again the simulation assumes zero initial conditions for the system and the controller parameters. Block processing is used with block length $N = 30$. The simulation results are shown in Fig. 5.55–5.66.

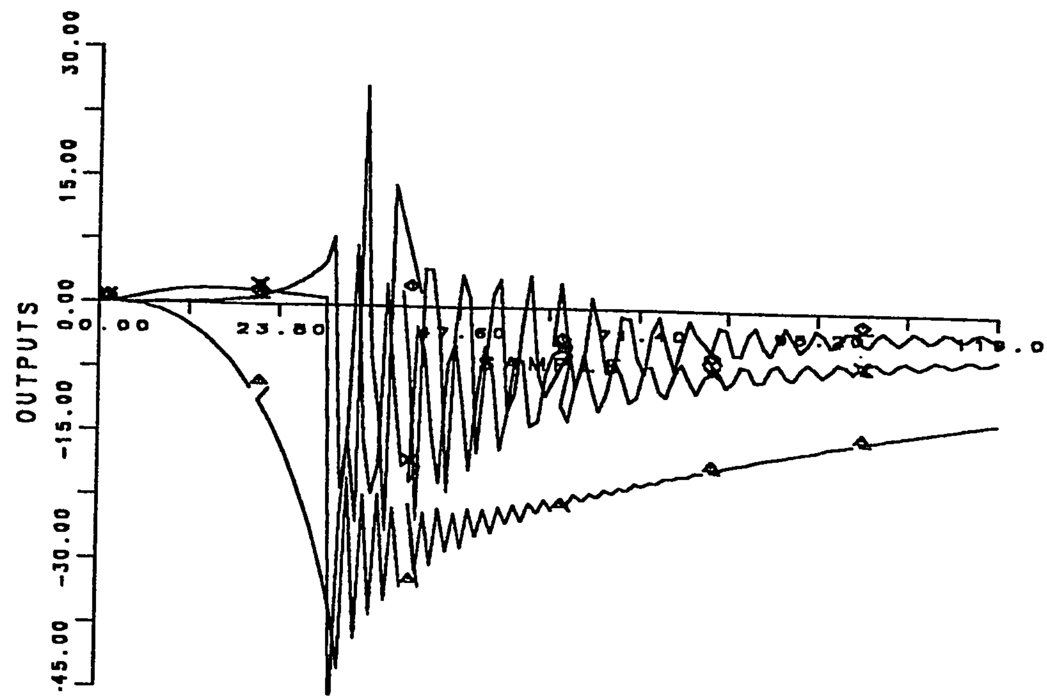

Figure 5.55: Output trajectories of the (3 × 2) MIMO sixth order system

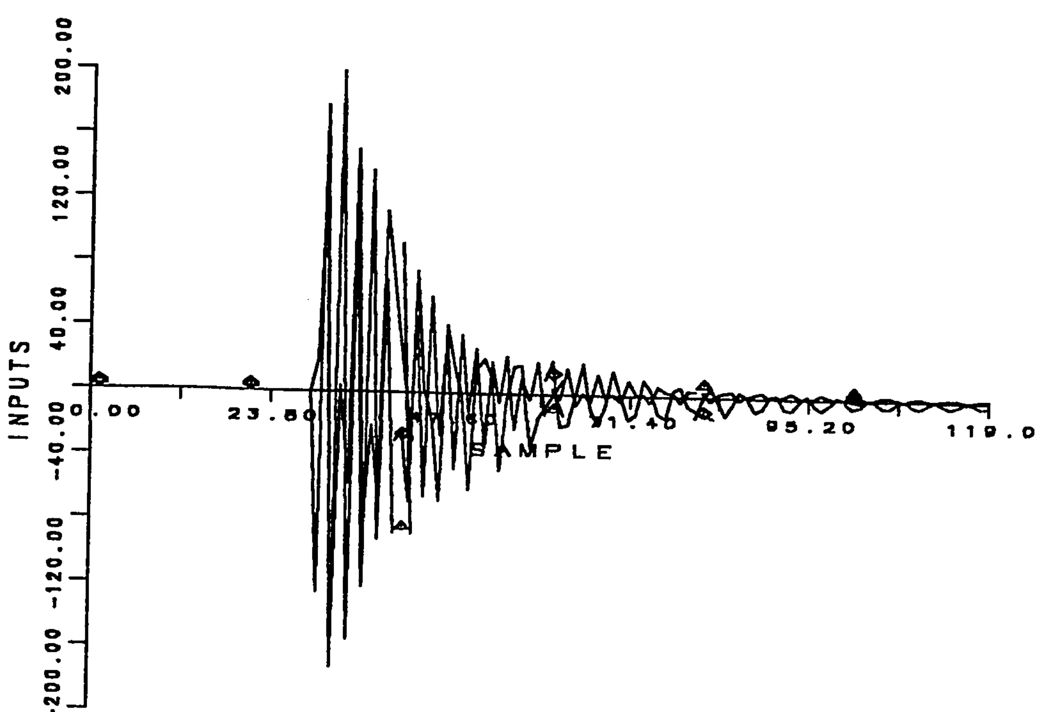

Figure 5.56: Input trajectories of the (3 × 2) MIMO sixth order system

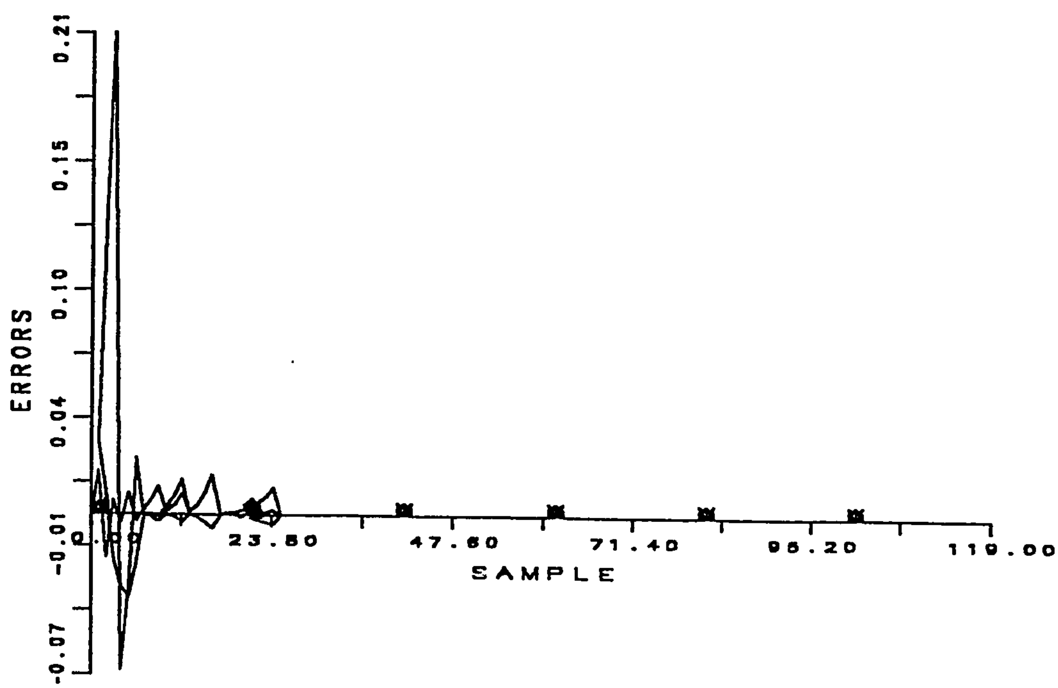

Figure 5.57: Error trajectories of the (3 × 2) MIMO sixth order system

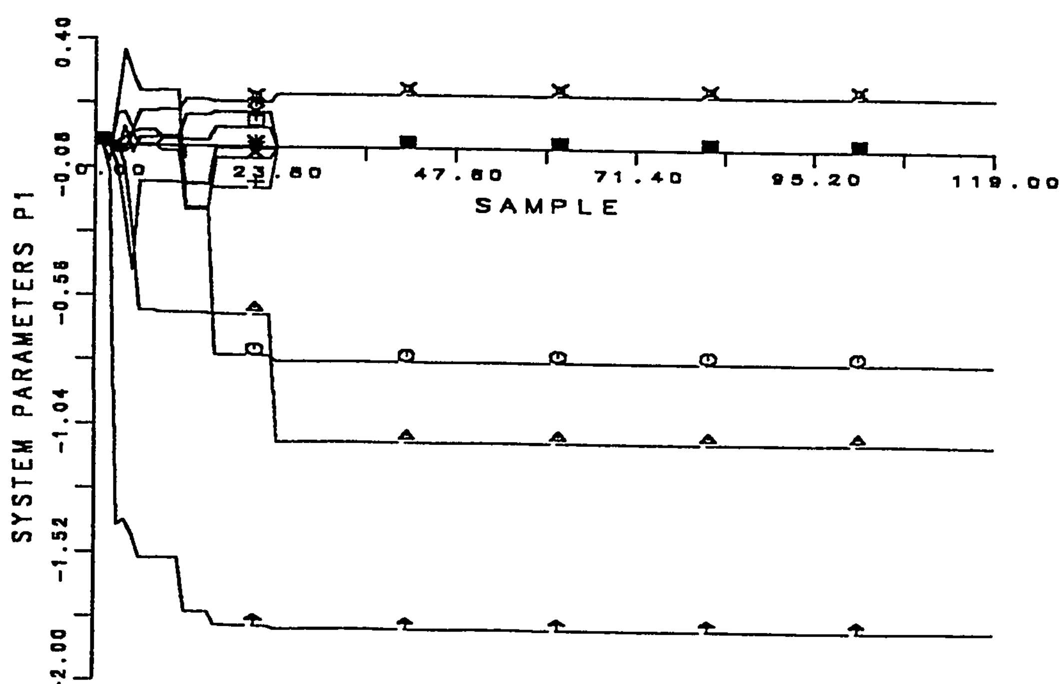

Figure 5.58: System parameters trajectories of the (3 × 2) MIMO sixth order system

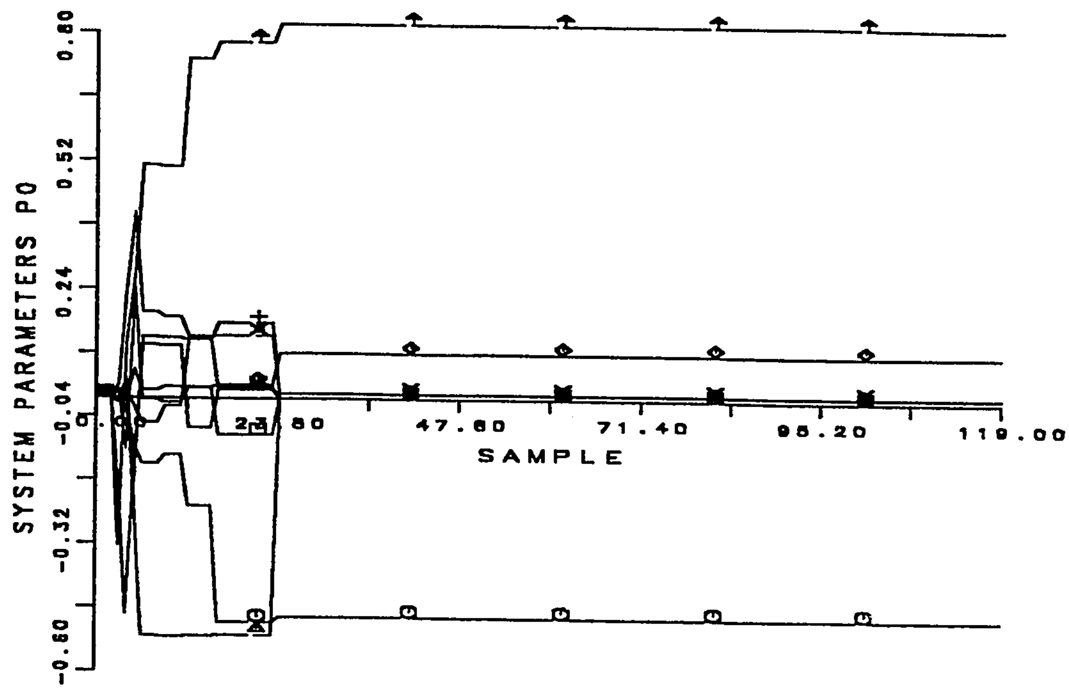

Figure 5.59: System parameters trajectories of the (3 × 2) MIMO sixth order system

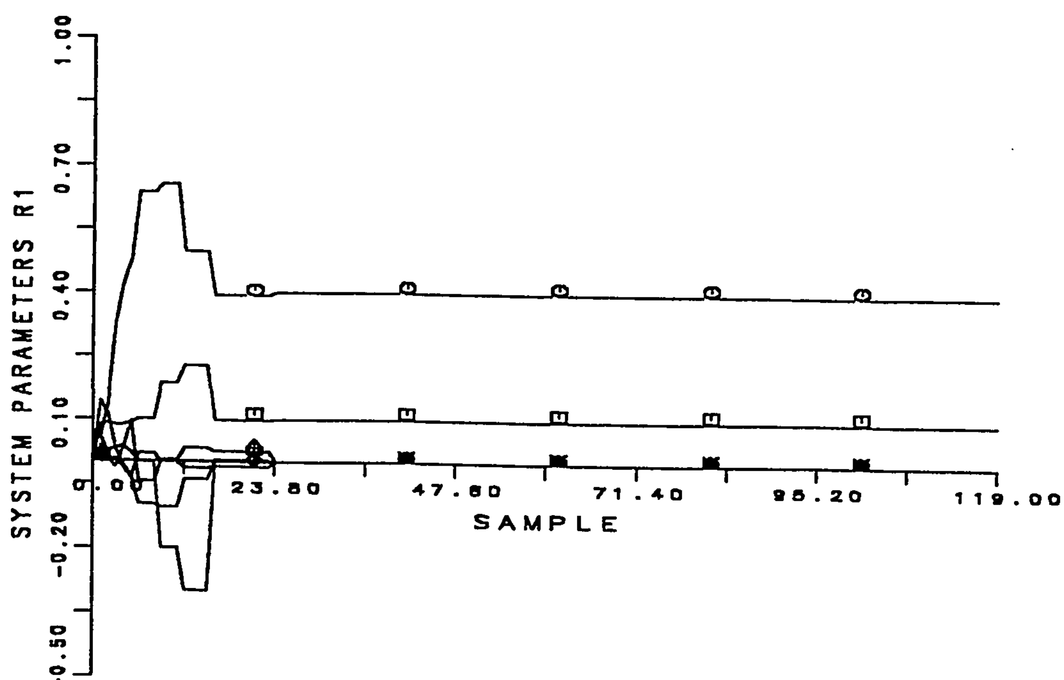

Figure 5.60: System parameters trajectories of the (3 × 2) MIMO sixth order system

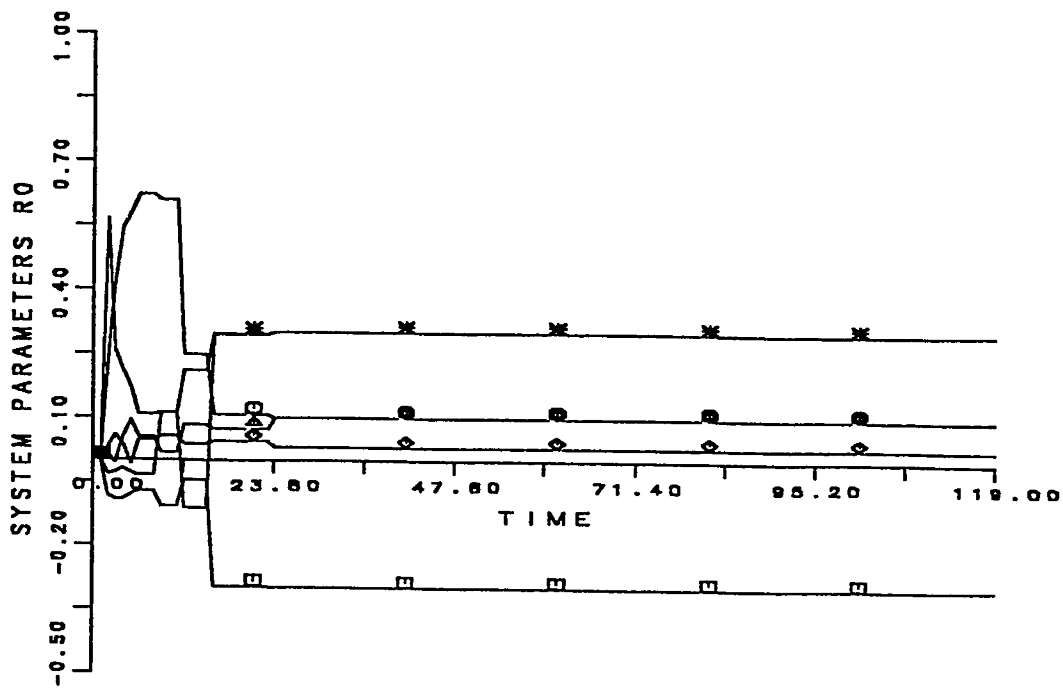

Figure 5.61: System parameters trajectories of the (3 × 2) MIMO sixth order system

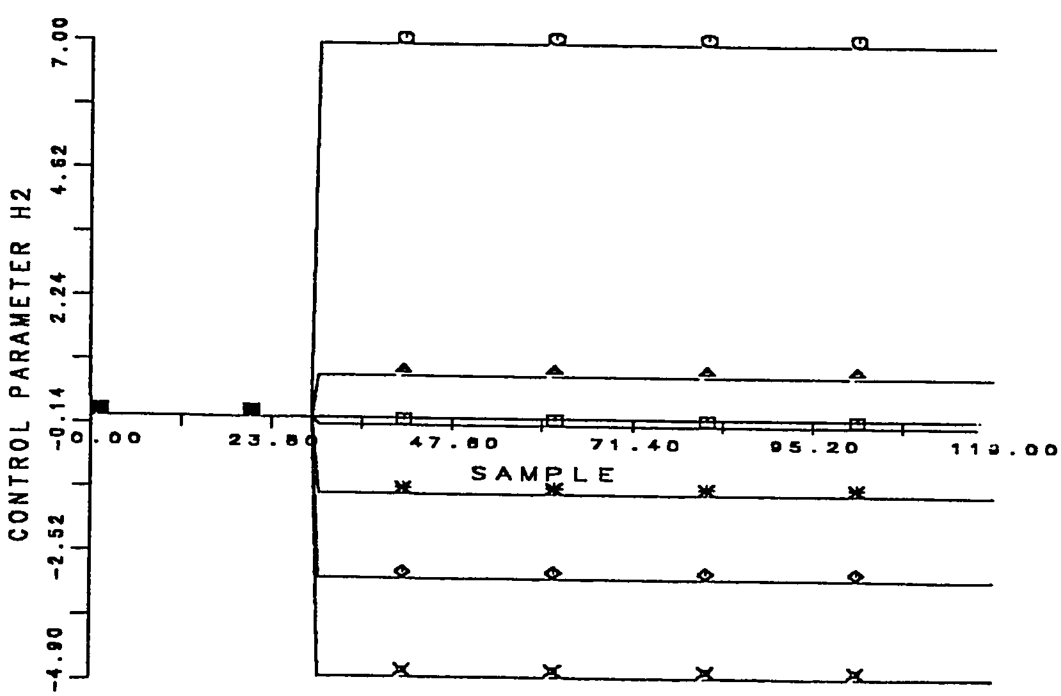

Figure 5.62: Controller parameters trajectories of the (3 × 2) MIMO sixth order system

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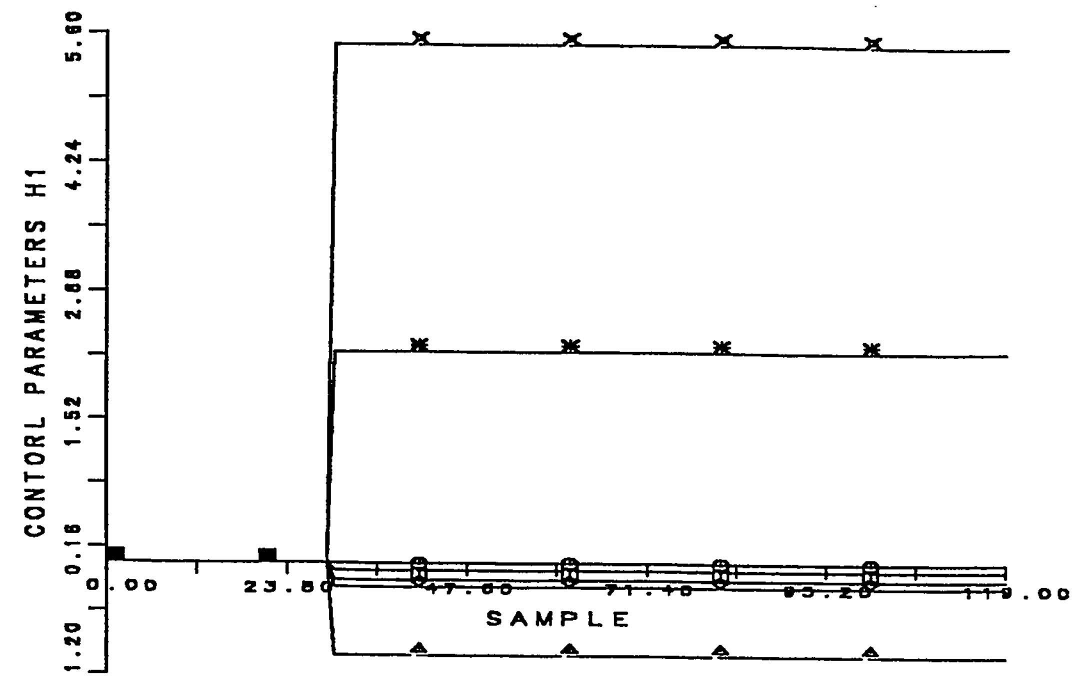

Figure 5.63: Controller parameters trajectories of the (3 × 2) MIMO sixth order system

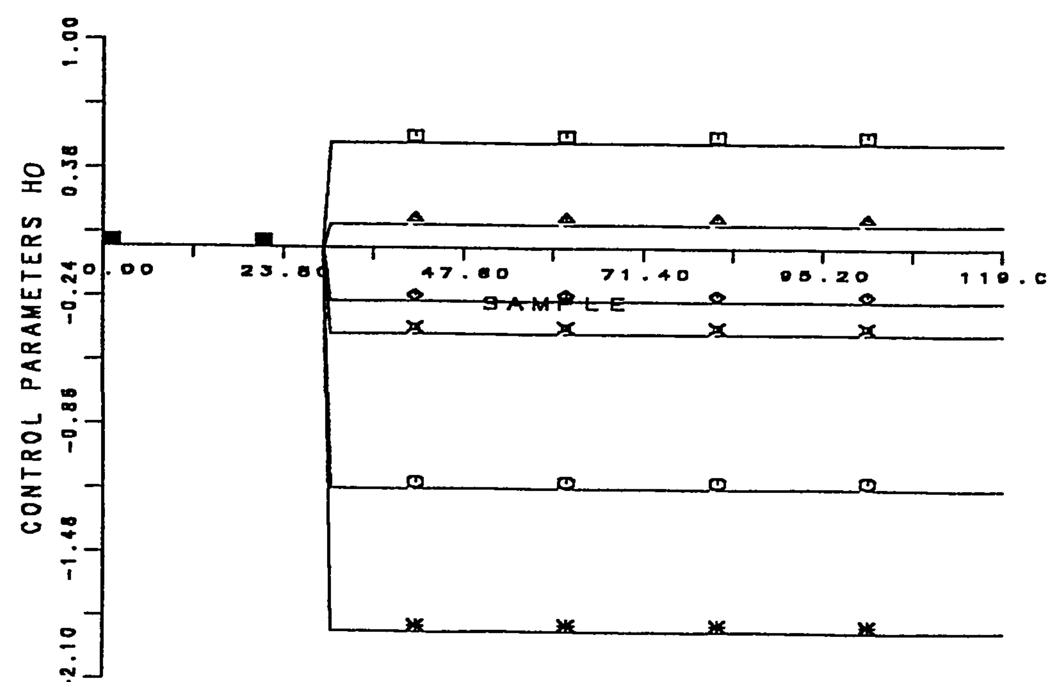

Figure 5.64: Controller parameters trajectories of the (3 × 2) MIMO sixth order system

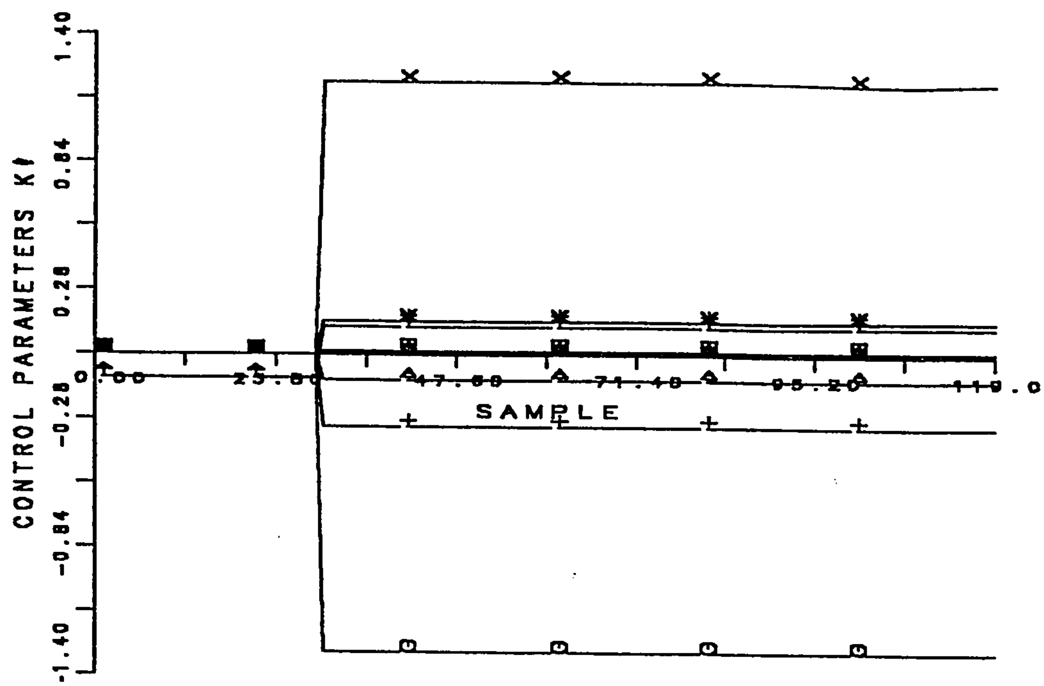

Figure 5.65: Controller parameters trajectories of the (3 × 2) MIMO sixth order system

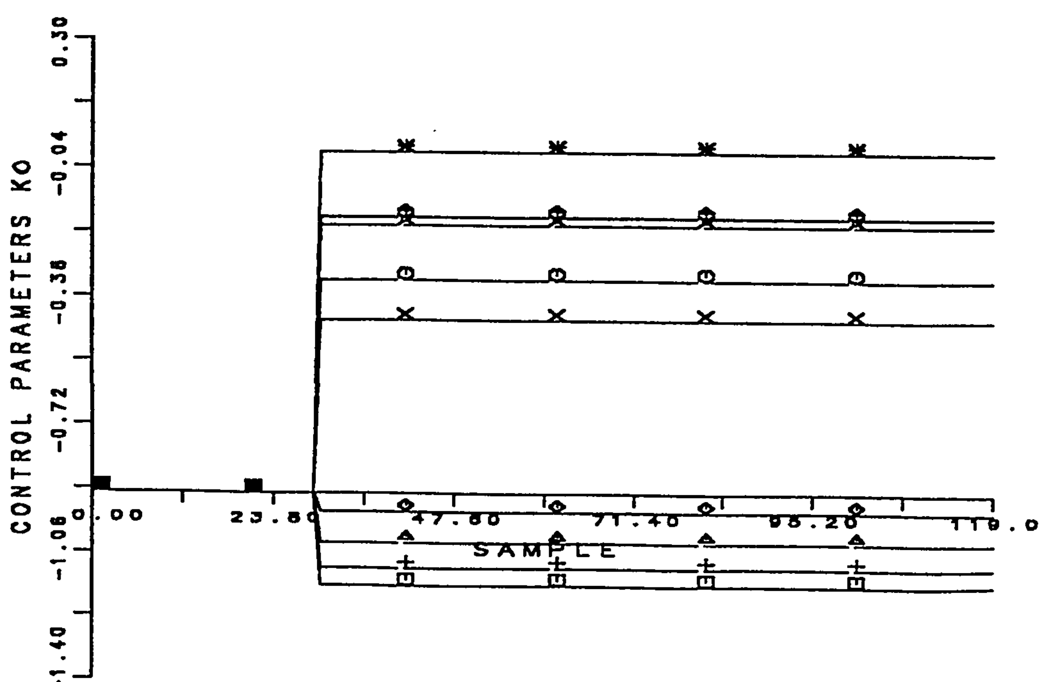

Figure 5.66: Controller parameters trajectories of the (3 × 2) MIMO sixth order system

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5.4 Effect of the block length N on the rate of convergence

In this section we shall investigate the convergence properties of the indirect adaptive control algorithm of linear discrete time shift invariant systems. From the global stability proof given in chapters 3 and 4, if certain assumptions are satisfied, one ensures for all initial states, and arbitrary initial parameter estimates, the input-output system model remain bounded for all time, and the closed-loop poles are effectively assigned, as time tends to infinity. An attractive feature of these results, is that asymptotic exponential convergence is obtained when the adaptive control algorithm make use of least-squares estimation schemes with forgetting factors. The above mentioned convergence implies various robustness properties. By simulating some test cases, the effect of N on the rate of convergence is investigated.

For the purpose of comparison, the theoretical results obtained on the bounds of the rate of convergence are summarized below:

Standard least-squares: The upper and lower bounds on the rate of convergence of the system parameters to their true values are $[N/(\sigma + \beta_2 k)]^{1/2}$ and $[N/(\sigma + \beta_1 k)]^{1/2}$ respectively, where β_1 and β_2 are as given in previous chapters.

Least-squares with forgetting factors: The upper and lower bounds

The length of N (no. of samples).	The no. of samples taken for the system parameters to converge to their true values.	The no. of samples taken for the controller parameters to converge to their desired values.
5	13	16
10	13	16
15	14	16
20	14	21
25	14	26
30	14	31
35	14	36
40	14	41
45	14	46
50	14	51

Table 5.1: Effect of N on the rate of convergence when tested on a SISO system

on the rate of convergence are $[\sigma/(\sigma+\beta_2)]^{k/2N}$ and $[\sigma/(\sigma+\beta_1)]^{k/2N}$ respectively.

Note that these expressions are valid under the condition that N is greater than some minimum value. The effect of N is investigated by experimental simulation for two test cases.

Case 1: In this case, the system of Eqn 5.4 is considered. The block length N is varied in the range of 5 to 50. The results are shown in Table 5.1.

Case 2: This simulation considers the system given by Eqn 5.6. Similar

The length of N (no. of samples).	The no. of samples taken for the system parameters to converge to their true values.	The no. of samples taken for the controller parameters to converge to their desired values.
5	11	11
10	11	11
15	14	16
20	14	21
25	14	26
30	14	31
35	14	36
40	14	41
45	14	46
50	14	51

Table 5.2: Effect of N on the rate of convergence when tested on an MIMO system

to the previous case, the block length N is varied over the range 5 to 50. Table 5.2 shows the results of the simulations.

The simulation results show that for all values of N, the convergence is very fast for both cases. Hence, it is not possible to draw any conclusion about the effect of N on the rate of convergence. However, it is believed that the effect of N could be observed if the system is non-deterministic.

CHAPTER SIX

ADAPTIVE POLE ASSIGNMENT OF A DC MOTOR

6.1 Introduction

The purpose of this chapter is to apply the globally stable adaptive pole assignment technique described in chapter 3 to a practical system. The design of the controller is based on linearized model of the plant dynamics. The design employs only input-output model measurements to estimate and control the system. The adaptive control algorithm consists of an estimation scheme for the purpose of identification and a pole assignment controller. The system parameters are estimated using the least-squares algorithm. Based on the output of the estimator, the controllers are then estimated by solving a diophantine type equation online to relocate the closed-loop poles at appropriate locations. The optimal controllers are

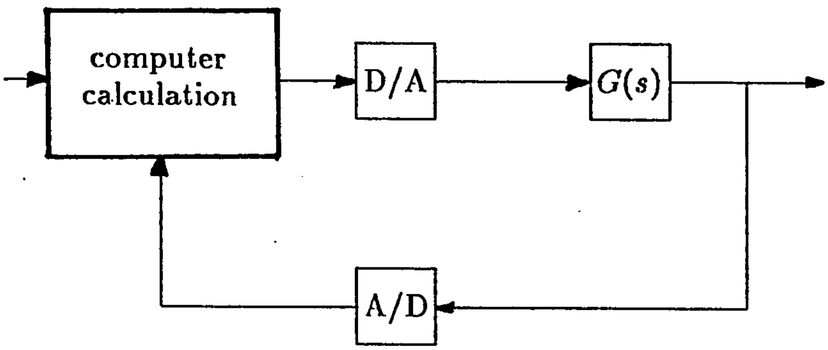

Figure 6.1: Basic digital feedback control

designed online using the idea of block processing. The general block diagram of the adaptive control system is shown in Fig. 6.1.

The adaptive algorithm has been used to control the speed of a laboratory model first order DC motor. The performance of the adaptive control algorithm obtained is satisfactory.

6.2 Computer hardware and software

An IBM PC (XT) has been used to identify the system parameter and to synthesize the forced inputs. The IBM PC includes a floating point processor (8087), a 64 KB of RAM, one floppy disc drive, a lab master, and a hard disc. The IBM lab master combines all the functions necessary for most data manipulations on one multifunction board. The board contains 2 D/A channels, 16 A/D channels, timer counters, and a zero-order hold. The software consists of the FORTRAN machine code, and assembler-language subroutines. The FORTRAN program consists of the least-squares estimation scheme and the control synthesizer. The assembler-language routines are divided as follows:

(i) A/D conversion

(ii) D/A conversion

(iii) Initialization

The real time assembler-language A/D conversion subroutine forms an interrupt service which is entered each time the line clock sends a conversion pulse to the A/D unit.

6.3 System modeling and control

The adaptive control algorithm is used to control the speed of the DC servomotor. The system consists of the Servo-Amplifier Unit SA150D plus a DC motor MT150F. The complete system can be represented by a first order system for most practical purposes if the input is assumed to be greater than 3.0 volts. The system hence can be represented in continuous-time as

$$G(s) = \frac{K}{sT + 1} \qquad \text{for } u > 3 \text{ volts} \tag{6.1}$$

Its discrete-time transfer function is

$$G_D(z) = \frac{q}{z+p} \tag{6.2}$$

where q and p are the unknown parameters. The first part of the adaptive algorithm consists then of the system parameter estimation which in this case are q and p. For this purpose the standard sequential least-squares estimation scheme is used. Then, based on this estimate the controller is designed by solving a discrete time diophantine type equation. We can think of this design as mapping the continuous-time desired pole location into discrete-time domain and then do a discrete-time pole assignment. This design is equivalent on performing the design entirely in the continuous time and then make an approximate discrete time implementation when the sampling rate is relatively fast. The sampling rate was chosen as $33Hz$ which is suitable for the system considered. The time constant of the DC motor is about (1 to 2 sec). The discrete time control law required to relocate the placement of the poles is given by:

$$c(z)u(k) = h(z)[y_{ref}(k) - y(k)] \tag{6.3}$$

with $c(z) = q^*(z) - k(z)$. In the first case the order of the system is one, and hence $h(z)$ is of zero order. The closed-loop thus obtained is of second order. the purpose is not only to achieve the desired transient performance, but also to satisfy the steady state condition. To eliminate the steady state

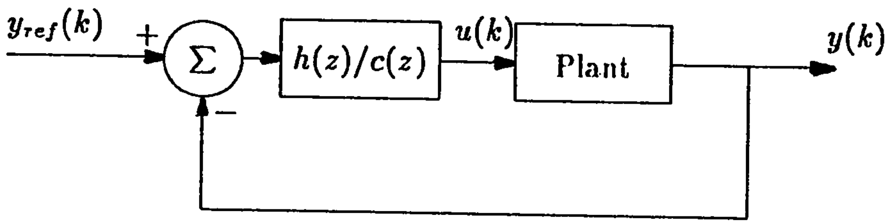

Figure 6.2: Feedback controller based on (l.m.r.)

error for constant desired output, one can simply introduce a pole into the control at $z = 1$. The complete control configuration is given in Fig. 6.2.

The closed-loop characteristic polynomial becomes

$$k(z)p(z) + (-h(z))q(z) = q^*(z)[p(z) - p^*(z)] \tag{6.4}$$

where $q^*(z)p^*(z)$ is the desired third order polynomial. The degree of the polynomials $h(z)$ and $k(z)$ has now increased by one. Resolving the pole assignment equation for $h(z)$ and $k(z)$ using the estimated parameter generated from the estimator one obtain the adaptive control law. In the first case, the desired discrete closed-loop poles were chosen at 0.90, 0.95 and 0. The corresponding continuous time desired closed-loop poles of the system are approximately at -1.55 and -3.19.

The system was run and at the end of the test the data held in memory is transfered to floppy disc and hard copies of plots is obtained. The continuous time response is shown in Fig. 6.3-6.6.

In the second case, the desired closed-loop poles are selected at 0.95, 0 and 0. The corresponding pole location in continuous time is at -1.5. The

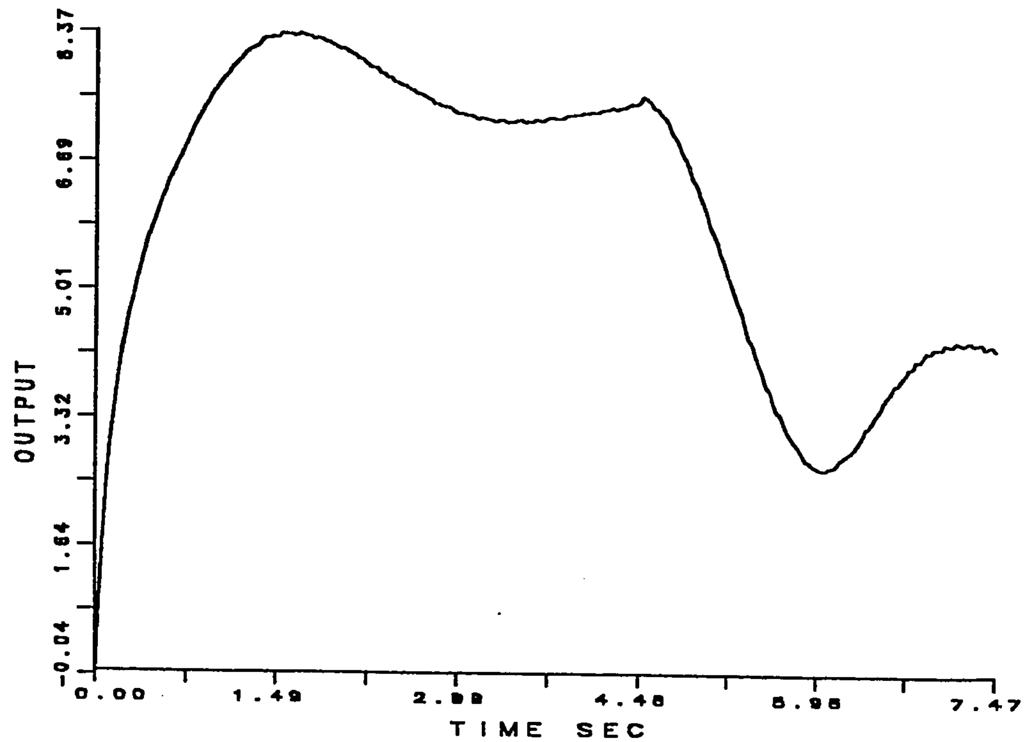

Figure 6.3: DC motor speed trajectory for the case of desired closed-loop poles 0.9, 0.95, and 0

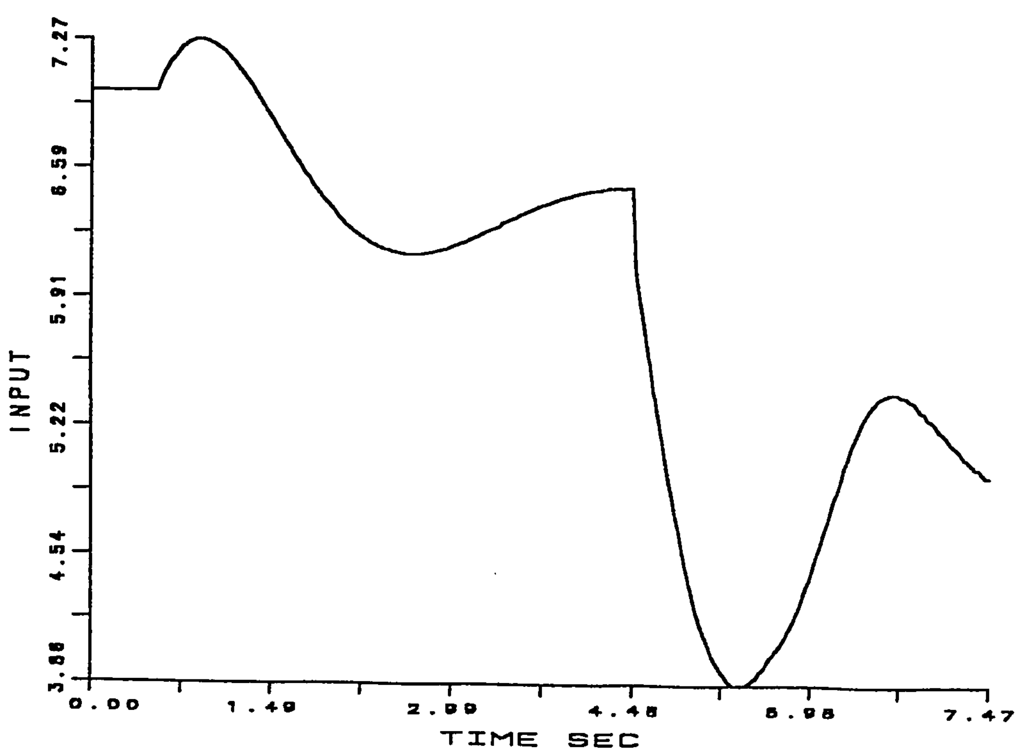

Figure 6.4: Armature input voltage for the case of desired closed-loop poles 0.9, 0.95, and 0

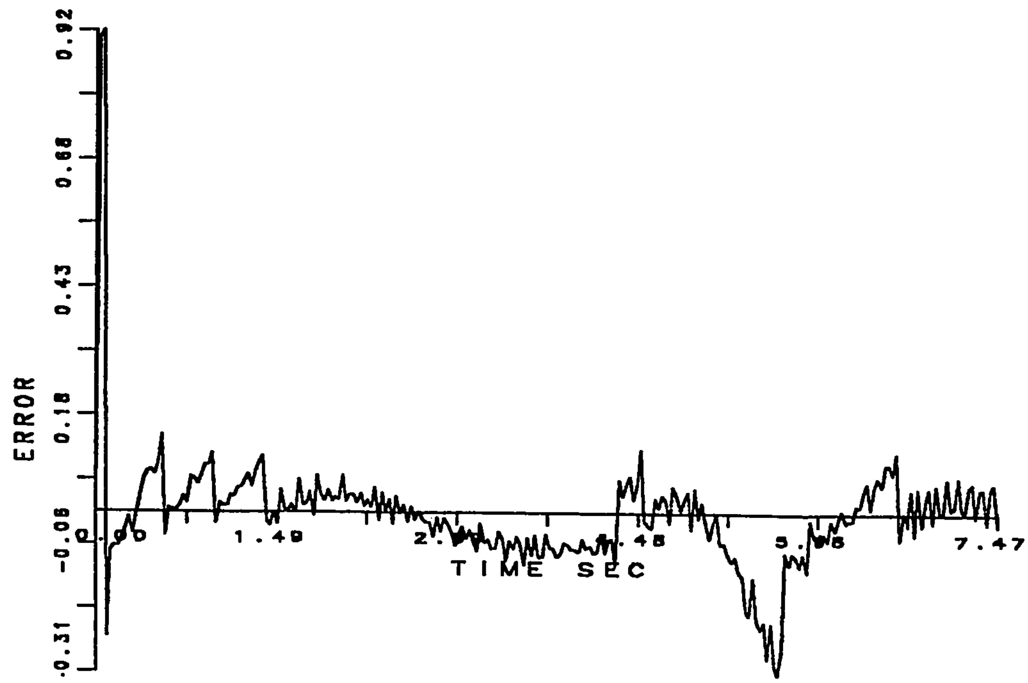

Figure 6.5: Error identification of the speed adaptive control system for the case of desired closed-loop poles 0.9, 0.95, and 0

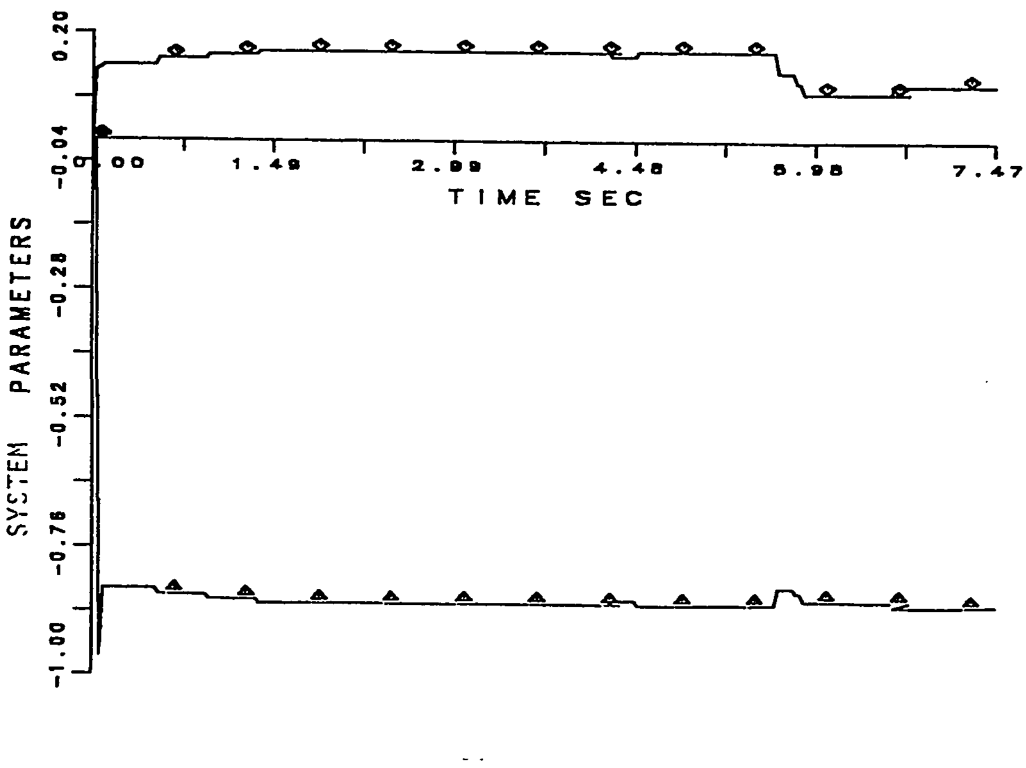

Figure 6.6: System parameter trajectories of speed control system for the case of desired closed-loop poles 0.9, 0.95, and 0

result of continuous time system is shown in Fig. 6.7-6.10. In both cases, we have assumed zero initial conditions for both the system and control parameters. Block processing was used with a block length $N = 10$. The initial covariance matrix was selected as $P = 1000I$. The system output and input were constrained to remain between -10 volts, and 10 volts.

The reference input is selected as:

$$y_{ref}(t) = \begin{cases} 7.0 & \text{for } 0 \leq t \leq t_1 \\ 4.0 & \text{for } t > t_1 \end{cases} \tag{6.5}$$

The output is shown to follow the desired reference signal in the two cases. Moreover, during transients, the system response is governed by the assigned closed-loop poles as shown in Fig. 6.3 and 6.7. In the first case, the speed of the DC motor was desired to follow a system with dominant poles at -1.5 and -3.9. In the second case, it was desired that the speed of the DC motor behaves as system with dominant pole at -1.5. The results shown in Fig. 6.3 and 6.7 prove that the adaptive control can achieved good performance. The adaptive pole assignment algorithm gives what appears to be excellent performances. The time histories of the system signals indicate the performances of the adaptive control algorithm. The transient responses obtained also indicate that the adaptive controller is quite stable.

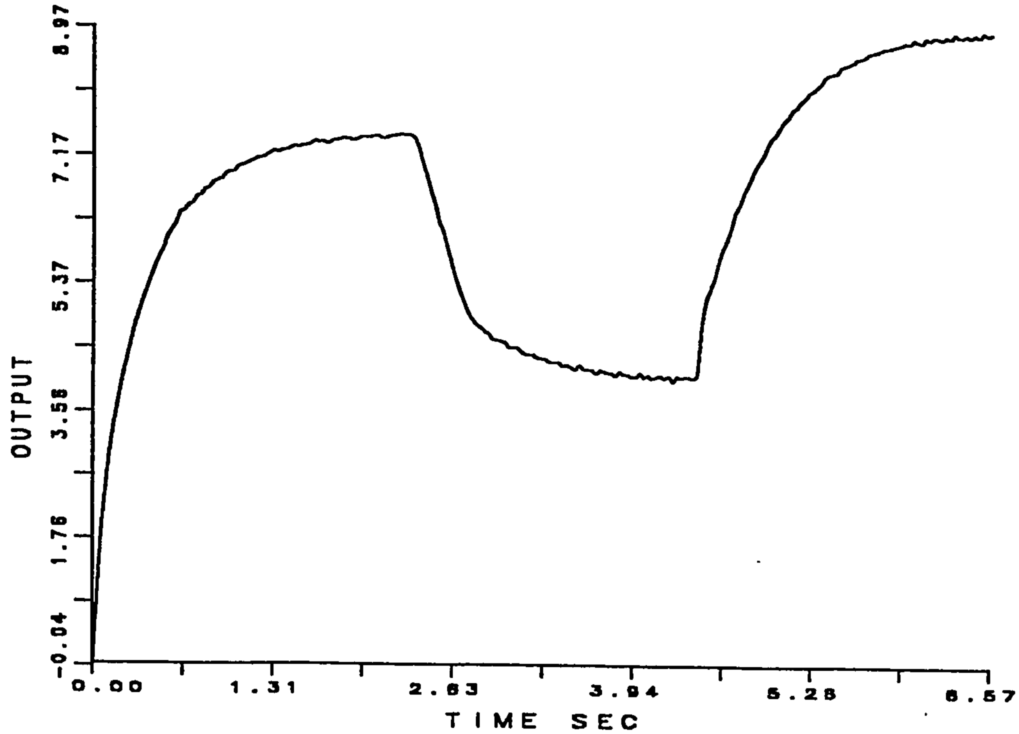

Figure 6.7: DC motor speed trajectory for the case of desired closed-loop poles 0.9, 0, and 0

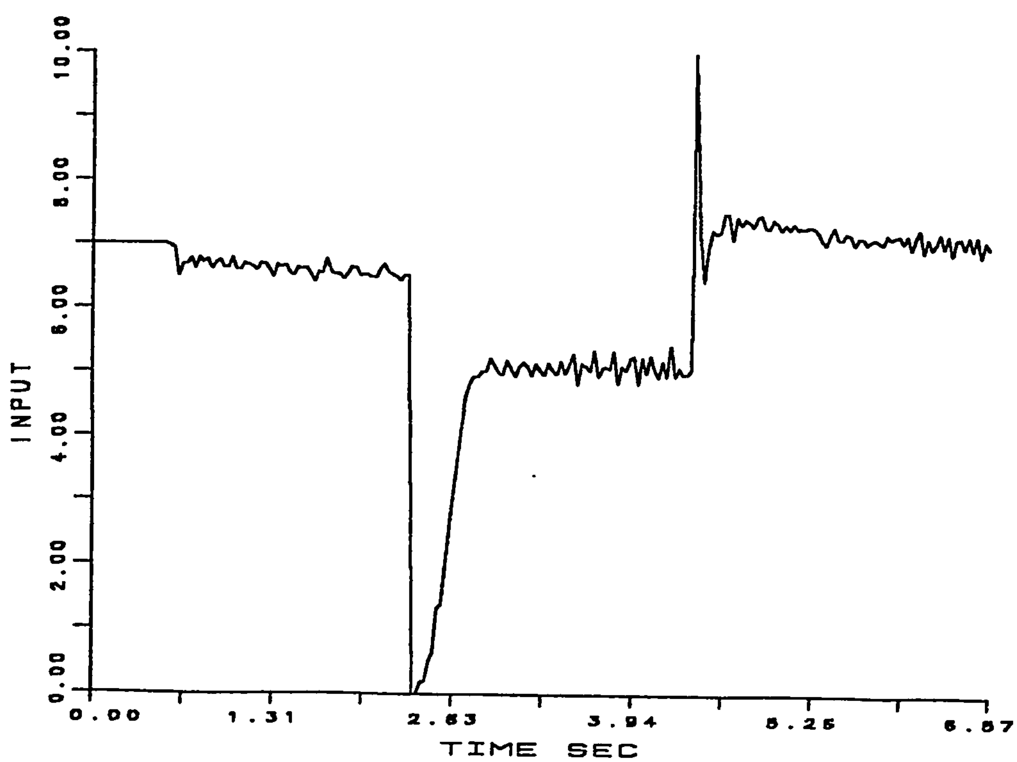

Figure 6.8: Armature input voltage for the case of desired closed-loop poles 0.9, 0, and 0

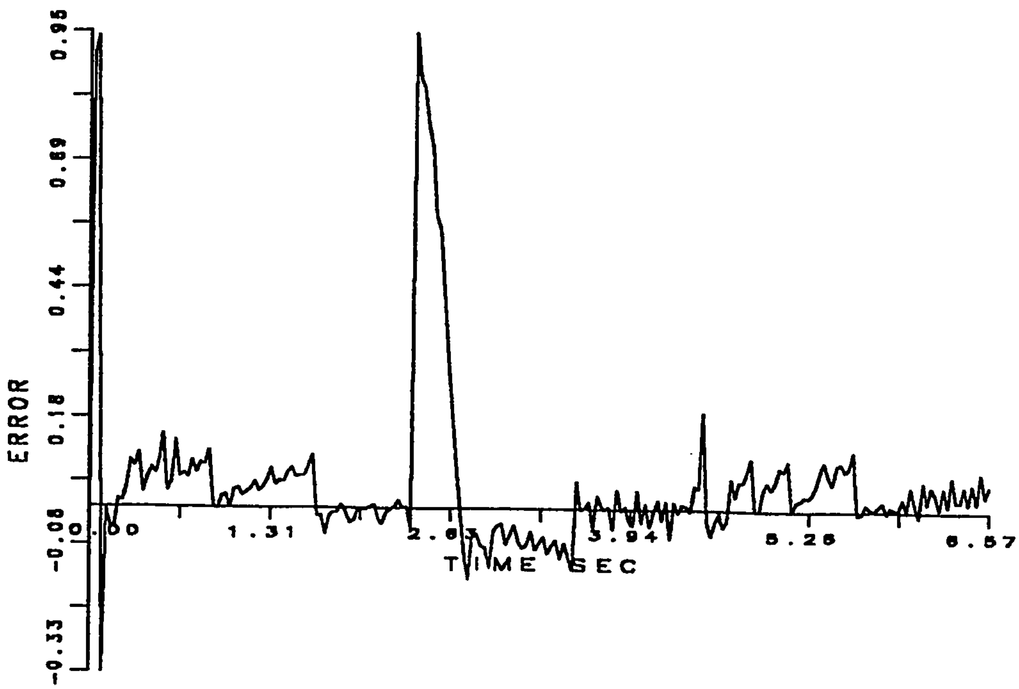

Figure 6.9: Error identification of the speed adaptive control system for the case of desired closed-loop poles 0.9, 0, and 0

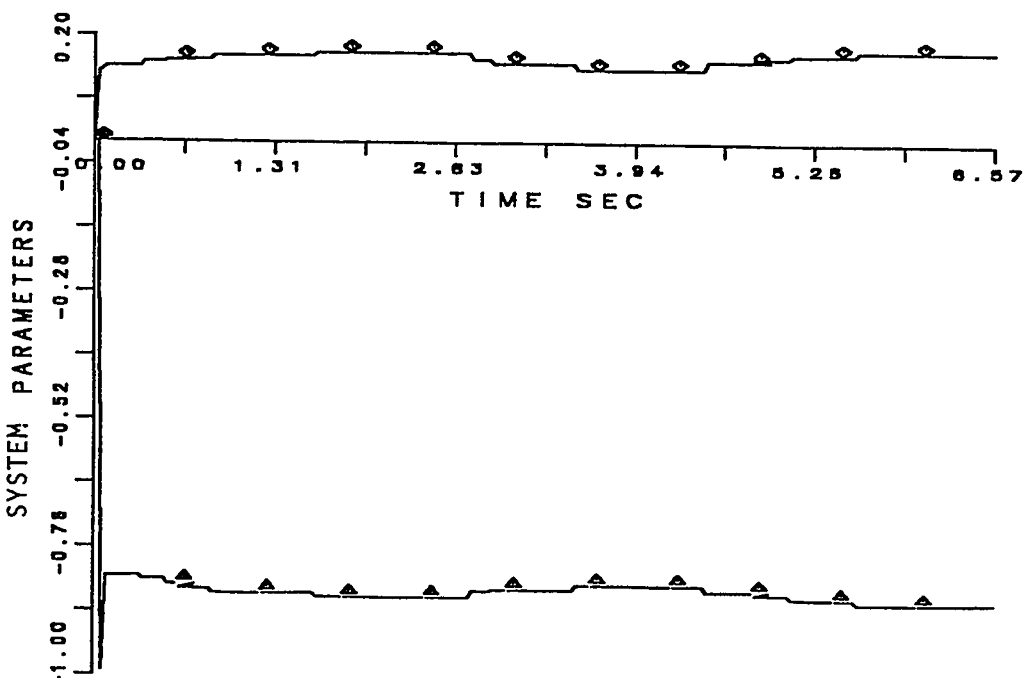

Figure 6.10: System parameter trajectories of speed control system for the case of desired closed-loop poles 0.9, 0, and 0

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CHAPTER SEVEN

CONCLUSION

The global stability of indirect adaptive pole assignment has been established for both SISO and MIMO nonminimum phase systems under relatively weak assumptions. It has been shown that the adaptive control algorithm is globally stable unconditional on the convergence of the system parameters to their true values. This new result is important because it reduces the required assumptions to a minimum. In fact, this result requires only the prior knowledge of the system order n, and that the desired closed-loop poles are strictly located inside the unit circle.

The global stability analysis is given for the indirect adaptive pole assignment of SISO system with a control strategy based on (l.m.r.) and (r.m.r.). The key idea to prove the global convergence was to show first that any possible unbounded signal is observable from the equation error. This property was then converted into a linear boundedness of the partial state in terms of the equation error. Thereafter the linear boundedness was used to establish uniform boundedness of the input-output system signals. This result is unconditional on the convergence of the parameters of the system to their true values.

The global stability with asymptotic convergence of the system parameters to their true values requires the a priori knowledge of the system order n and the persistency of the reference signal. Bounds on the rate of convergence of the system and controller parameters to their desired values is also given for a number of estimation schemes. The discussion included in particular, the standard least-squares and its variants namely least-squares with covariance resetting and least-squares with exponential data weighting. This result also applies to the projection algorithm and its modified versions.

It is particularly important to find algorithms that yield fast exponential convergence rate. The reason is that it is preferable to use adaptive algorithms in practical application that have fast convergence rate such as asymptotic exponential convergence. It is interesting to note that these fast exponential convergence schemes give some robustness properties to the adaptive control system. In fact, it was shown that least-squares with forgetting factors do have this property.

The fast exponential convergence is obtained under the conditions that the adaptive algorithm make use of block processing together with the condition that the reference input signal is persistently exciting. It has been shown that the block length N has to be greater than some finite values for the result to be valid.

Global stability of MIMO systems has also been established for indirect adaptive pole assignment with a control strategy based on (l.m.r.), if prior knowledge of the observability indices and the controllability index are assumed. This result requires the system to be minimal. Bounds on the rates of convergence of the system parameters to their true values, is guaranteed provided the reference signal is persistently exciting and that the block length N is greater than the required minimum value. In particular, exponential convergence is also obtained when the algorithm makes use of the estimation schemes with forgetting factors. This new generalized result is important because it applies to both minimum and nonminimum phase systems.

Finally, extensive simulations of the adaptive control algorithm were performed on the digital computer which confirmed the theoretical analysis given in chapter 3 and 4. The simulation results show that the adaptive control algorithm do results in a robust controller with the equation error tending to zero at rates which are practical. It was also noted that the system and controller parameters converged in a finite number of samples. For further assessment of the results and its application, the adaptive controller was applied to a physical system. The results indicate that the algorithm can be of considerable importance for some practical applications.

From the investigation of robustness it has been seen that the adaptive pole assignment algorithms are robust. This was shown analytically as well as by means of simulations for a number of estimation schemes.

The following suggestions are give for future work:

(1) The stability analysis of adaptive algorithms when the order of the system is not exactly known (i.e. smaller or greater than the order of the system) and the effect on the performance of the control algorithms. This is important in practical application because in general the controlled system always contains unmodeled dynamics.

(2) A new direction is the global stability analysis of these adaptive algorithms in the presence of noise.

APPENDIX

A.1 Proof of lemma 2.1

(i) Subtracting θ_o from both sides of Eqn 2.10 and using Eqn 2.9, we have

$$\tilde{\theta}\,(k) = \tilde{\theta}\,(k-1) - \frac{\phi(k-1)}{c+\phi(k-1)\,^T\phi(k-1)}\tilde{\theta}\,(k-1)^T\phi(k-1) \qquad \text{(A.1)}$$

Hence, using Eqn 2.13,

$$||\tilde{\theta}\,(k)||^2 - ||\tilde{\theta}\,(k-1)||^2 = \left[-2 + \frac{\phi(k-1)\,^T\phi(k-1)}{c+\phi(k-1)\,^T\phi(k-1)}\right]\frac{e(k)^T e(k)}{c+\phi(k-1)\,^T\phi(k-1)} \qquad \text{(A.2)}$$

where $||M|| = \text{trace}[M]$

Now since $c > 0$, we have

$$[-2 + \frac{\phi(k-1)\,^T\phi(k-1)}{c+\phi(k-1)\,^T\phi(k-1)}] < 0 \qquad \text{(A.3)}$$

and then Eqn 2.14 follows from Eqn A.2

(ii) We observe that $||\tilde{\theta}\,(k)||^2$ is bounded nonincreasing function, and by summing Eqn A.2, we have

$$||\tilde{\theta}\,(k)||^2 = ||\tilde{\theta}\,(0)||^2 + \sum_{j=1}^{k}\left[-2 + \frac{\phi(j-1)\,^T\phi(j-1)}{c+\phi(j-1)\,^T\phi(j-1)}\right]\frac{e(j)^T e(j)}{c+\phi(j-1)\,^T\phi(j-1)} \qquad \text{(A.4)}$$

Since $||\tilde{\theta}\,(k)||$ is nonnegative, and since Eqn A.3 holds, we can conclude Eqn 2.15.

(a) Eqn 2.16 follows immediately from Eqn 2.15

(b) It is clear that

$$||\hat{\theta}(k) - \hat{\theta}(k-l)||^2 = ||\hat{\theta}(k) - \hat{\theta}(k-1) + \hat{\theta}(k-1) - \hat{\theta}(k-2) \ldots \hat{\theta}(k-l+1) - \hat{\theta}(k-l)||^2 \tag{A.5}$$

Then, using the schwartz inequality,

$$||\hat{\theta}(k) - \hat{\theta}(k-l)||^2 \leq l(||\hat{\theta}(k) - \hat{\theta}(k-1)||^2 + \ldots + ||\hat{\theta}(k-l+1) - \hat{\theta}(k-l)||^2) \tag{A.6}$$

Hence using this result together with Eqn 2.10 and 2.16 the final result follows.

A.2 Proof of lemma 2.2

(i) Subtracting θ_o from both sides of of Eqn 2.18 and using Eqn 2.9, we obtain

$$\tilde{\theta}(k) = \tilde{\theta}(k-1) + \frac{P(k-2)\phi(k-1)\,\phi(k-1)^T\tilde{\theta}(k-1)}{1+\phi(k-1)^T P(k-2)\phi(k-1)} \tag{A.7}$$

Then, using Eqn 2.20 the matrix inversion lemma, we have

$$\tilde{\theta}(k) = P(k-1)P(k-2)^{-1}\tilde{\theta}(k-1) \tag{A.8}$$

Hence introducing $V(k) = \tilde{\theta}(k)^T P(k-1)^{-1}\tilde{\theta}(k)$, we have

$$\begin{aligned} V(k) - V(k-1) &= [\tilde{\theta}(k) - \tilde{\theta}(k-1)]^T P(k-2)^{-1}\tilde{\theta}(k-1) \\ &= -\frac{\tilde{\theta}(k-1)^T\phi(k-1)\,\phi(k-1)^T\tilde{\theta}(k-1)}{1+\phi(k-1)^T P(k-2)\phi(k-1)} \end{aligned} \tag{A.9}$$

Thus $V(k)$ is a nonnegative, nonincreasing function and hence

$$\tilde{\theta}\ (k)^T P(k-1)^{-1}\tilde{\theta}\ (k) \leq \tilde{\theta}\ (0)^T P(-1)^{-1}\tilde{\theta}\ (0) \qquad \text{(A.10)}$$

Now from the matrix inversion lemma

$$P(k)^{-1} = P(k-1)^{-1} + \phi(k)\ \phi(k)\ ^T \qquad \text{(A.11)}$$

It follows that

$$\begin{aligned} \lambda_{min}[P(k)^{-1}] &\geq \lambda_{min}[P(k-1)^{-1}] \\ &\geq \lambda_{min}[P(-1)^{-1}] \qquad \text{(A.12)} \end{aligned}$$

Eqn A.12 implies

$$\begin{aligned} \lambda_{min}[P(k)^{-1}]\|\tilde{\theta}\ (k)\|^2 &\leq \lambda_{min}[P(k-1)^{-1}]\|\tilde{\theta}\ (k)\|^2 \\ &\leq \|\tilde{\theta}\ (k)P(k-1)^{-1}\tilde{\theta}\ (k)\| \\ &\leq \|\tilde{\theta}\ (0)P(-1)^{-1}\tilde{\theta}\ (0)\| \qquad \text{(A.13)} \\ &\leq \lambda_{max}[P(-1)^{-1}]\|\tilde{\theta}\ (0)\|^2 \qquad \text{(A.14)} \end{aligned}$$

This establishes part (i).

(ii) Returning to Eqn A.9 and summing from 1 to j gives

$$V(k) = V(0) - \sum_{j=1}^{k} \left\| \frac{\tilde{\theta}\ (j-1)^T \phi(j-1)\ \phi(j-1)\ ^T \tilde{\theta}\ (j-1)}{1 + \phi(j-1)\ ^T P(j-2)^{-1}\phi(j-1)} \right\| \qquad \text{(A.15)}$$

Since $V(k)$ is nonnegative, we immediately have Eqn 2.22.

(a) Eqn 2.23 follows immediately from Eqn 2.22 and A.12, which implies that

$$\lambda_{max}[P(k)^{-1}] \leq \lambda_{max}[P(k-1)^{-1}] \leq \lambda_{max}[P(-1)^{-1}] \qquad \text{(A.16)}$$

(b) Noting that

$$\frac{e(k)^T e(k)}{1+\phi(k-1)\,^T P(k-2)\phi(k-1)} = \frac{1+\phi(k-1)\,^T P(k-2)\phi(k-1)\; e(k)^T e(k)}{[1+\phi(k-1)\,^T P(k-2)\phi(k-1)\,]^2} \qquad \text{(A.17)}$$

The result then follows by use of the Schwartz inequality, as in the above proof of part (b) of lemma 2.1.

A.3 Proof of lemma 3.1

If $\{s(k)\}$ is a bounded sequence, then by Eqn 3.27 $\{||\sigma(k)||\}$ is a bounded sequence. Then by Eqn 3.28 and 3.27 it follows that

$$\lim_{k\to\infty} s(k) = 0 \qquad \text{(A.18)}$$

Now assume that $\{s(k)\}$ is unbounded. It follows that there exists a subsequence $\{k_i\}$ such that

$$\lim_{k_i\to\infty} s(k) = \infty \qquad \text{(A.19)}$$

and

$$||s(k)|| \leq ||s(k_i|| \text{ for } k \leq k_i \qquad \text{(A.20)}$$

Now along the subsequence $\{k_i\}$ and recalling Eqn 3.28

$$\begin{aligned}\left\|\frac{s(k_i)}{[a(k_i)+b(k_i)\sigma(k_i)^T\sigma(k_i)]^{1/2}}\right\| &\geq \frac{\|s(k_i)\|}{[K+K\|\sigma(k_i)\|^2]^{1/2}} \\ &\geq \frac{\|s(k_i)\|}{K^{1/2}+K^{1/2}\|\sigma(k_i)\|} \\ &\geq \frac{\|s(k_i)\|}{K^{1/2}+K^{1/2}[C_1+C_2\|s(k_i)\|]}\end{aligned} \tag{A.21}$$

using Eqn 3.29 and A.20. Hence

$$\left\|\frac{s(k_i)}{[a(k_i)+b(k_i)\sigma(k_i)^T\sigma(k_i)]^{1/2}}\right\| \geq \frac{1}{k^{1/2}c_2} \tag{A.22}$$

But this contradicts Eqn 3.27 and hence the assumption that $\{s(k)\}$ is unbounded is false and the result follows immediately. The upper bound is trivial. Suppose the lower bound fails. Then there exists a vector

$$(\gamma^T\ \delta^T) = \begin{pmatrix}\gamma_{n-1} & \gamma_{n-2} & \cdots & \gamma_0 & \delta_{n-1} & \cdots & \delta_0\end{pmatrix} \tag{A.23}$$

of unit length such that

$$(\gamma^T\ \delta^T)\phi(k) = 0 \tag{A.24}$$

for $k = j$ to $j+L-1+2n-1$. Set $\gamma(z) = \sum_{i=0}^{n-1}\gamma_i z^{n-i}$, $\delta(z) = \sum_{i=1}^{n-1}\delta_i z^{n-i}$, then Eqn A.24 is equivalent to

$$\gamma(z)y(k-n+1)+\delta(z)u(k-n+1) = 0 \tag{A.25}$$

for $k = j$ to $j+L-1+2n-1$. Now the plant and control equations together imply that

$$
\begin{aligned}
u(k) &= \frac{p(z)c(z)}{p(z)c(z) - q(z)h(z)} v(k) && \text{(A.26)} \\
y(k) &= \frac{q(z)c(z)}{p(z)c(z) - q(z)h(z)} v(k) && \text{(A.27)} \\
\text{where } c(z) &= q^*(z) - k(z)
\end{aligned}
$$

The degree of the closed-loop denominator denoted by $d_w(z)$ is $2n - 1$. If both sides of Eqn A.25 are multiplied by the closed-loop polynomial then we have

$$
d_w(z)[\gamma(z)y(k-n+1) + \delta(z)u(k-n+1)] = 0 \qquad \text{(A.28)}
$$

Using Eqn A.26 and A.27, it can be simplified further to

$$
[\gamma(z)q(z) + \delta(z)p(z)]c(z)v(k-n+1) = 0 \qquad \text{(A.29)}
$$

for $k = j$ to $j + L - 1$.

Now observe that $\epsilon(z) = \gamma(z)q(z) + \delta(z)p(z)$ must be nonzero. Otherwise, $q(z)/p(z) = -\delta(z)/\gamma(z)$ with $\gamma(z)$ of lower degree than $p(z)$, contradicting the coprimeness of $p(z)$ and $q(z)$. Let $||\epsilon||^2$ denote the sum of the squares of the coefficients of $\epsilon(z)$.

Note that $\inf_{||\gamma||^2+||\delta||^2=1} ||\epsilon||^2$ is nonzero; otherwise we could construct a bounded sequence of $\gamma_i(z)$, $\delta_i(z)$ with $||\gamma||^2+||\delta||^2 = 1$ such that $\lim_{i\to\infty} ||\epsilon||^2 = 0$. There would be a convergent subsequence of the $\gamma_i(z)$, $\delta_i(z)$ converging to say $\bar{\gamma}(z)$, $\bar{\delta}(z)$, with $\bar{\gamma}(z)q(z) + \bar{\delta}(z)p(z) = \bar{\epsilon}(z) = 0$, a contradiction.

Noting that $\epsilon(z)$ has degree $2n-1$, we see that Eqn A.29 implies a contradiction to Eqn 2.22.

A.4

Defining $\phi(k)^T = \begin{pmatrix} \phi_y(k) & \phi_u(k) \end{pmatrix}$ where $\phi_y(k)$ and $\phi_u(k)$ denotes sub-vectors containing the outputs and inputs respectively. From Eqn 3.64 we have

$$\phi_y(k) = \begin{pmatrix} \hat{c}_{n-1} & \hat{c}_{n-2} & \cdots & \hat{c}_0 & 0 & \cdots & 0 \\ 0 & \hat{c}_{n-1} & \cdots & \hat{c}_1 & \hat{c}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{c}_{n-1} & \hat{c}_{n-2} & \cdots & \hat{c}_0 \end{pmatrix} \Psi(z)w(k-n+1) \tag{A.30}$$

Using Eqn 3.42 we have

$$\phi_y(k) = \begin{pmatrix} \hat{c}_{n-1} & \hat{c}_{n-2} & \cdots & \hat{c}_0 & 0 & \cdots & 0 \\ 0 & \hat{c}_{n-1} & \cdots & \hat{c}_1 & \hat{c}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{c}_{n-1} & \hat{c}_{n-2} & \cdots & \hat{c}_0 \end{pmatrix} \{T_D^{-1}x(k)+q(z)v(k-n+1)\} \tag{A.31}$$

Similar expression can be obtained for $\phi_u(k)$. Therefore, from Eqn 3.65 we have

$$\phi_u(k) = \begin{pmatrix} \hat{h}_{n-1} & \hat{h}_{n-2} & \cdots & \hat{h}_0 & 0 & \cdots & 0 \\ 0 & \hat{h}_{n-1} & \cdots & \hat{h}_1 & \hat{h}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{h}_{n-1} & \hat{h}_{n-2} & \cdots & \hat{h}_0 \end{pmatrix} \Psi(z)w(k-n+1) + \begin{pmatrix} v(k) \\ v(k-1) \\ \vdots \\ v(k-n+1) \end{pmatrix} \tag{A.32}$$

Using Eqn 3.42 we have

$$\phi_u(k) = \begin{pmatrix} \hat{h}_{n-1} & \hat{h}_{n-2} & \cdots & \hat{h}_0 & 0 & \cdots & 0 \\ 0 & \hat{h}_{n-1} & \cdots & \hat{h}_1 & \hat{h}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{h}_{n-1} & \hat{h}_{n-2} & \cdots & \hat{h}_0 \end{pmatrix} \{T_D^{-1} x(k) + q(z) v(k-n+1)\} + \begin{pmatrix} v(k) \\ v(k-1) \\ \vdots \\ v(k-n+1) \end{pmatrix}$$

A.5

Defining $\phi(k)^T = \begin{pmatrix} \phi_y(k) & \phi_u(k) \end{pmatrix}$ where $\phi_y(k)$ and $\phi_u(k)$ denotes subvectors containing the outputs and inputs respectively. Define also $\hat{C}(k_j, z) = \{\hat{c_{ij}}(z)\}$, with $\hat{c_{ij}}(z) = \sum_{l=0}^{\mu-1} \hat{c_{ij}}^l z^{\mu-l-1}$ then, using Eqn 4.30 the entries of $\phi_y(k)$ can be written as:

$$y(k) = \begin{pmatrix} \hat{c}_{11}^0 & \hat{c}_{11}^1 & \cdots & \hat{c}_{11}^{\mu-1} & \hat{c}_{12}^0 & \cdots & \hat{c}_{1m}^{\mu-1} \\ \hat{c}_{21}^0 & \hat{c}_{21}^1 & \cdots & \hat{c}_{21}^{\mu-1} & \hat{c}_{22}^0 & \cdots & \hat{c}_{2m}^{\mu-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{c}_{m1}^0 & \hat{c}_{m1}^1 & \cdots & \hat{c}_{m1}^{\mu-1} & \hat{c}_{m2}^0 & \cdots & \hat{c}_{mm}^{\mu-1} \end{pmatrix} \begin{pmatrix} w_1(k+\mu-1) \\ w_1(k+\mu-2) \\ \vdots \\ w_1(k) \\ w_2(k+\mu-1) \\ \vdots \\ w_m(k) \end{pmatrix}$$

$$\vdots$$

$$y(k+\nu-1) = \begin{pmatrix} \hat{c}^0_{11} & \hat{c}^1_{11} & \cdots & \hat{c}^{\mu-1}_{11} & \hat{c}^0_{12} & \cdots & \hat{c}^{\mu-1}_{1m} \\ \hat{c}^0_{21} & \hat{c}^1_{21} & \cdots & \hat{c}^{\mu-1}_{21} & \hat{c}^0_{22} & \cdots & \hat{c}^{\mu-1}_{2m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{c}^0_{m1} & \hat{c}^1_{m1} & \cdots & \hat{c}^{\mu-1}_{m1} & \hat{c}^0_{m2} & \cdots & \hat{c}^{\mu-1}_{mm} \end{pmatrix} \begin{pmatrix} w_1(k+\mu+\nu-2) \\ w_1(k+\mu+\nu-3) \\ \vdots \\ w_1(k+\nu-1) \\ w_2(k+\mu+\nu-2) \\ \vdots \\ w_m(k+\nu-1) \end{pmatrix}$$

where $w_i(k)$ denotes the i th element of the partial state vector $w(k)$. Therefore the vector $\phi_y(k+\nu-1)$ can be rewritten as

$$\phi_y(k+\nu-1) = C\Psi(z)w(k) \tag{A.33}$$

where the entries of the matrix C consists of $\hat{c}^l_{ij}$ elements. Hence, we can now write

$$\phi_y(k+\nu-1) = CT_D^{-1}x(k) - CT^{-1}t_Q(z)v(k) \tag{A.34}$$

Similar expression can be obtained for $\phi_u(k)$. Defining $\hat{H}(k_j, z) = \{\hat{h}_{ij}(z)\}$, with $\hat{h}_{ij}(z) = \sum_{l=0}^{\mu-1} \hat{h}_{ij}^{\,l} z^{\mu-l-1}$ then, using Eqn 4.29, the entries of $\phi_u(k)$ can be written as:

$$u(k) = \begin{pmatrix} \hat{h}^0_{11} & \hat{h}^1_{11} & \cdots & \hat{h}^{\mu-1}_{11} & \hat{h}^0_{12} & \cdots & \hat{h}^{\mu-1}_{1m} \\ \hat{h}^0_{21} & \hat{h}^1_{21} & \cdots & \hat{h}^{\mu-1}_{21} & \hat{h}^0_{22} & \cdots & \hat{h}^{\mu-1}_{2m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{h}^0_{m1} & \hat{h}^1_{m1} & \cdots & \hat{h}^{\mu-1}_{m1} & \hat{h}^0_{m2} & \cdots & \hat{h}^{\mu-1}_{mm} \end{pmatrix} \begin{pmatrix} w_1(k+\mu-1) \\ w_1(k+\mu-2) \\ \vdots \\ w_1(k) \\ w_2(k+\mu-1) \\ \vdots \\ w_m(k) \end{pmatrix} + v(k)$$

$$\vdots$$

$$u(k+\nu-1) \;=\; \begin{pmatrix} \hat{h}_{11}^{0} & \hat{h}_{11}^{1} & \cdots & \hat{h}_{11}^{\mu-1} & \hat{h}_{12}^{0} & \cdots & \hat{h}_{1m}^{\mu-1} \\ \hat{h}_{21}^{0} & \hat{h}_{21}^{1} & \cdots & \hat{h}_{21}^{\mu-1} & \hat{h}_{22}^{0} & \cdots & \hat{h}_{2m}^{\mu-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{h}_{m1}^{0} & \hat{h}_{m1}^{1} & \cdots & \hat{h}_{m1}^{\mu-1} & \hat{h}_{m2}^{0} & \cdots & \hat{h}_{mm}^{\mu-1} \end{pmatrix} \begin{pmatrix} w_1(k+\mu+\nu-2) \\ w_1(k+\mu+\nu-3) \\ \vdots \\ w_1(k+\nu-1) \\ w_2(k+\mu+\nu-2) \\ \vdots \\ w_m(k+\nu-1) \end{pmatrix} + v(k+\nu-1)$$

Thus we can write

$$\begin{aligned} \phi_u(k+\nu-1) &= H\Psi(z)w(k) + \begin{pmatrix} v(k+\nu-1) \\ v(k+\nu-2) \\ \vdots \\ v(k) \end{pmatrix} \\ &= HT^{-1}x(k) - HT^{-1}t_Q(z)v(k) + \begin{pmatrix} v(k+\nu-1) \\ v(k+\nu-2) \\ \vdots \\ v(k) \end{pmatrix} \end{aligned}$$

where the entries of the matrix H consists of $\hat{h}_{ij}^{l}$ elements.

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GLOSSARY

(l.m.r.):	Left model representation
(r.m.r.):	Right model representation
SISO:	Single-input single-output
MIMO:	Multi-input multi-output
$u(k)$:	System input
$y(k)$:	System output
$v(k)$:	Reference input signal
$w(k)$:	Partial state vector
$x(k)$:	System state vector
$e(k)$:	Equation error
n:	Order of the system
N:	Length of the block processing
$\phi(k)$:	Regression vector
θ_o:	Vector or matrix containing true values of the process parameters
$\hat{\theta}(k)$:	Vector or matrix containing the estimate of the system parameters
$\tilde{\theta}(k)$:	Vector or matrix containing the difference between the true and the estimated system parameters

$P(k)$:	Covariance matrix of the least-squares estimation schemes
z:	Forward shift operator
$p_L(z)$, $q(z)$:	SISO system polynomials of the (l.m.r.) of order n and $n-1$ respectively
$\hat{p}_L(k,z)$, $\hat{q}(k,z)$:	Estimates of the SISO system polynomials of the (l.m.r.)
$p_R(z)$, $r(z)$:	SISO system polynomials of the (r.m.r.) of order n and $n-1$ respectively
$\hat{p}_R(k,z)$, $\hat{r}(k,z)$:	Estimates of the SISO system polynomials of the (r.m.r.)
$P_L(z)$, $Q(z)$:	MIMO system matrix polynomials of the (l.m.r.)
$\hat{P}_L(k,z)$, $\hat{Q}(k,z)$:	Estimates of the MIMO system matrix polynomials of the (l.m.r.)
$P_R(z)$, $R(z)$:	MIMO system matrix polynomials of the (r.m.r.)
$p^*(z)$, $q^*(z)$:	Desired closed-loop polynomials
$P^*(z)$, $Q^*(z)$:	Desired closed-loop matrix polynomials
$k(z)$, $h(z)$:	Scalar polynomials controller
$\hat{k}(k,z)$, $\hat{h}(k,z)$:	Estimates of the SISO polynomials controller
$H(z)$, $K(z)$:	Matrix polynomials controller
$\hat{K}(k,z)$, $\hat{H}(k,z)$:	Estimates of the MIMO matrix polynomials controller
$t_D(z)$:	Vector (matrix) polynomial of chirnhaussen scalar (matrix) polynomials

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