Integrated Production, Quality and Maintenance Models for Multistage Production-Inventory Systems

by

Abdul Rahim Nasir Ahmed

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEMS ENGINEERING

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This thesis, written by **ABDUL RAHIM NASIR AHMED**

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Dedicated to

My Brother *Abdul Karim*

whose presence relieved me from many worries

and whose absence is felt all the time.

Acknowledgments

All praise be to Allah, the Lord of the worlds. May peace and blessings be upon Mohammad, the last messenger, and his family. I thank Allah for His limitless help and guidance at every instant in my life.

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offer my unending gratitude and love. Thanks to all my siblings whose love, support, encouragement and belief in me has been invaluable.

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Abstract (English)

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TITLE: Integrated Production, Quality and Maintenance Models For Multi-Stage Production-Inventory Systems.

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Multi-stage production-inventory systems (MS-PIS) are very common in any manufacturing environment. The research in this area is really challenging both from modelling and application point of view. Production, quality and maintenance are three important and interrelated functions in any industrial process. These aspects are traditionally treated as three separate problems even for single-stage production systems which is likely to render sub-optimal solutions. Here we give an overall review of research literature dealing with lot sizing decisions in series MS-PIS with its deficiencies and limitations to facilitate the scope of future research and applications. We formulate various mathematical models so as to integrate quality and maintenance with the lot sizing decisions for series MS-PIS. Both uniform lot sizing and variable lot sizing models are considered. In the case of imperfect production processes, the screening of defective items between stages and errors in screening inspections are also considered. In uniform lot sizing model, the effect of maintenance-inspection and process restorations is taken into account. A fractional factorial experiment is designed and analysed to check the sensitivity of various response variable against various system parameters. While, in the case of variable lot sizing model, the effect of preventive maintenance is also considered beside process restorations. Mathematical models are coded in 'C' for comparison purposes. Several numerical examples are used to demonstrate the usefullness of the developed models.

MASTER OF SCIENCE DEGREE
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December, 1997

ملخص

الإسم: عبد الرحيم نصير أحمد

العنوان: نماذج متكاملة للإنتاج والجودة و الصيانة لنظم الإنتاج و التخزين متعددة و متتابعة المراحل

التخصص: هندسة النظم

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إن نظم الإنتاج و التخزين متتابعة المراحل هي من وسائل الإنتاج المستعملة بكثرة في الصناعة. كما إن الإنتاج والجودة النوعية و الصيانة هي ثلاثة عناصر رئيسة في أي عملية صناعية. و لكن بصفة عامة تمت دراسة هذه النواحي كل على حدة خاصة في مجال النظم متعددة المراحل. بعد استعراض البحوث المتعلقة بإيجاد كميات الإنتاج الاقتصادية لنظم الإنتاج و التخزين متعددة و متتابعة المراحل نبين مواطن القصور في هذا المجال تمهيدا لاقتراح نماذج رياضية لدمج الإنتاج والجودة النوعية و الصيانة معا. هذه الرسالة تطور عدة نماذج رياضية تشمل في آن واحد كميات الإنتاج الاقتصادية، جدول الصيانة و اعتبارات الجودة النوعية لنظم الإنتاج و التخزين متعددة و متتابعة المراحل. كما أن النماذج المقترحة تأخذ في الاعتبار أيضا أخطاء الفحص بين مراحل الإنتاج. نماذج الجزء الأول من الرسالة مخصصة لكميات الإنتاج الموحدة بين جميع المراحل، و يعالج الجزء الثاني كميات إنتاج مختلفة في النظم ذات المرحلتين. كما تم اقتراح طرق إيجاد الحل الأمثل الملائمة لحل النماذج الرياضية المطورة. وقد تم تطوير برامج حاسب آلي لحل عدة أمثلة استخدمت لتوضيح مزايا النماذج المقترحة. كما استخدمت هذه البرامج لتحليل حساسية النماذج للقيم المتغيرة.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول و المعادن

الظهران – المملكة العربية السعودية

Chapter 1

Introduction

1.1 General Background

Multi-stage production-inventory systems (MS-PIS) are very frequently encountered in the real life. Many models dealing with multi-stage lot-sizing are present in the existing research literature; but, even the single-stage systems are so complex and complicated that no model can capture all the elements into consideration. The inter-stage interaction and existence of work-in-process inventories in MS-PIS further complicates the issue.

Production, quality and maintenance are three important aspects in any industrial process. In manufacturing, new strategies that favor a variety of products are replacing the older ones of mass production of a uniform product. Swiftly changing markets and the requirement to confront the explosion of products diversity have augmented the necessity of automation and complex equipment which, consequently, increased the cost of manufacturing processes and, hence, need of bringing all three

aforementioned aspects into consideration.

Production-inventory control is very important and serves several important functions. It adds a great deal of flexibility in the operations of a firm. Some of the uses of inventory are:

1. The decoupling function.
2. Storing of resources.
3. A hedge against inflation.
4. Smooth functioning in the presence of irregular supply and demand.
5. Quantity discounts.
6. Avoiding stockouts and shortages.

The efficiency of long production runs, a widely accepted doctrine just a few decades ago, has been challenged by EPQ (Economic Production Quantity) models. These models are applicable when a single item is produced intermittently to satisfy a continuous demand.

Since the development of basic EOQ (Economic Order Quantity) model, attributed to Harris [47] by Hadley and Whitin [46], inventory lot sizing models have been embellished considerably through the incorporation of several complicated and complex real world considerations. Still, this field is wide open for further research especially for multi-stage production systems.

The classical EPQ is valid only when the product is manufactured by a single operation or process. It overstates the optimal lot size when misused for multi-stage production-inventory systems; since, the work-in-process inventory warrants smaller production runs. Considerable savings can be yielded by replacing the single-stage EPQ model with a multi-stage model when suitable.

The classical EPQ model assumes that the output of the production system is of perfect quality; but, in most of the cases, this is not a realistic assumption. Generally, product quality is a function of the state of the production process. When the process is in good condition the quality may be high or perfect. With the passage of time, the process may deteriorate resulting in production of items which may contain items which are defective or of sub-standard quality.

The link between quality improvement and productivity is very well establised. Some of the major ways that quality affects a manufacturing organization are:

1. *Loss of business* due to the damages done to the organization's image by poor quality leading to a decreased share of the market.

2. *Legal liabilities* due to the damages or injuries resulting from poor quality.

3. *Productivity* and quality are often closely related. Poor quality can adversely affect productivity during the manufacturing process if parts are defective and have to be reworked or if assembler has to try a number of parts before finding the one that fits properly. Similarly, poor quality in tools and equipment can

lead to injuries and defective output, which must be reworked or scrapped, thereby reducing the amount of usable output for a given amount of input.

4. Poor quality increases certain *quality costs* incurred by the organization.

The quality costs can be classified into three categories:

1. *Failure costs* incurred by defective parts or products. *Internal failures* are those that are discovered during the production process; *external failures* are those that are discovered after delivery to the customer. The costs of internal failure include lost production time, scrap and rework, investigation costs, possible equipment damage, and possible employee injury. External failure results in costs including warranty work, handling of complaints, replacements, liability/litigation, and loss of customer goodwill.

2. *Appraisal costs* relate inspection, testing, and other activities intended to uncover defective products; or, to ensure that there are no defectives. These include costs of inspectors, testing, test equipment, labs, quality audits, field testing, and the like.

3. *Prevention costs* relate to attempts to prevent defects from occurring. These include such costs as planning and administration systems, working with vendors, training, quality control procedures, and extra attention in both the design and production phases to decrease the probability of producing nonconforming products or services.

The problem of improving the product quality in production systems and its integration with the determination of production quantity in single-stage production-inventory systems has received considerable attention in the literature.

In many industrial processes the mean shifts during production and if it is not reset after a specific period of operation a large proportion of nonconforming items are produced. Industrial organizations recognize that an effective maintenance function is a crucial element for the success of manufacturing operations. Maintenance is the total of activities aiming to retain the technical systems in or restore them to the state necessary for fulfilment of the production function. Processes are producing certain products that must meet certain specifications. Any shift in a process affects the quality characteristics of the products and, hence, more non-conforming items. In order to get the best out of expensive manufacturing processes and meet the quality challenge, production processes have to be maintained in good operating conditions so that they can fulfil their function. Decision makers have two basic options with respect to maintenance. One option is *reactive:* It is to deal with breakdowns or other problems when they occur; this is also referred to as *breakdown maintenance.* The other option is *proactive:* It is to reduce breakdowns through a program of lubrication, adjustment, cleaning, inspection, repair, and replacement of worn parts. Decision makers try to forge out a trade-off between these two basic options that will result in minimizing their combined cost.

Attention has shifted from increasing efficiency by means of the so-called economies of scale and internal specialization to meeting market conditions in terms of flexibility, delivery performance and quality. Ideally, this trend towards 'just-in-time' production implies working without inventories at all. Consequently, unplanned unavailability of means of production will directly result in serious delivery problems.

One way of dealing with unplanned unavailability is to eliminate it as much as possible. This can be achieved by modification of the means of production underlying failures and/or by prescription of preventive maintenance transforming unplanned unavailability into planned unavailability. The process failure may also have catastrophic consequences.

These facts have shifted the focus to maintenance and the need for efficant maintenance policies which are effective sources to cut costs. When the process is in a good condition, the items produced would be of high or acceptable quality; but, with the passage of time the process may shift to a state in which it starts producing non-conforming (or unacceptable level of non-conforming) items. To avoid or, at least, delay the shift of the process from an in-control state to an out-of-control state preventive maintenance procedures are adopted. Maintenance by inspection of the production process or equipment has also been an important means of controlling the quality of products manufactured. Maintenance by process inspection is useful in diagnosing the out-of-control state of the production process. The premise is that, in general, the sooner we realize that the process is out-of-control, the less costly it would be to repair and restore it. Preventive maintenance is the action taken prior to the incidence of failures to avoid associated costs. The preventive maintenance activities have the dual effect of reducing the amount of nonconforming items produced and the cost of restoring the machines back to the in-control state.

The purpose of adopting preventive maintenance (PM) procedures can be summarized as follows:

1. Preventing the deterioration of the production process by keeping the production process in the "good" condition and to produce high quality items (i.e. to increase the reliability of the system over the long term by staving off the aging effects of tool wear, corrosion, fatigue, and related phenomena). If the production process is left to deteriorate without any remedial action then many defective items will be produced and extra cost will be incurred.

2. Maximizing the availability of plant facilities in a good operating condition. In effect this permits maximum performance.

3. Minimizing emergency maintenance.

4. Preserving plants and equipment to provide good service life.

5. Providing a safe environment for all workers which is, no doubt, a productive environment.

6. Assisting in providing a clean environment which enhances favorable conditions which lead to more satisfied workers.

7. Conserving energy usage.

The maintenance costs are highly controllable because maintenance is highly subjective and labor-intensive. The age and condition of facilities and equipment, the degree of technology involved , the type of production process, and similar factors enter into the decision of how much maintenance is desirable. The optimum level of maintenance is that level where the combined costs of maintenance, down-time, scrap, work-overs, and untimely deterioration are minimal. The problem of maintenance procedures have been well studied in the literature (Hsu [50], Danny [25]);

but, the relationship between lot sizes, inspection and PM has not been adequately addressed.

Production quantity, production quality and maintenance planning/scheduling of the production system are interrelated problems. Although there were some attempts to develop models integrating production, quality and maintenance for single-stage production-inventory systems, these problems have been treated traditionally as three separate problems which is likely to give sub-optimal solutions. The interdependence between these three important aspects has motivated many attempts to develop models incorporating more than one of these for single-stage production-inventory systems. Makhdoum[62] gives a detailed review of models integrating two or more of the aforementioned aspects of single-stage production system.

Though the importance of integrating production, quality and maintenance into a single model for multi-stage production-inventory systems can not be overemphasized, no significant attempt has been made for so far. In this thesis we look at various mathematical models on series multi-stage production-inventory system lot-sizing decisions integrating production, quality and maintenance for multi-sage production-inventory systems. We consider imperfect quality of production and incorporated errors in product inspections. We assume that the production processes start in in-control states; but, with the passage of time one or more production process(es) may shift to out-of-control state(s). The time elapsed before a shift occurred is considered to be a random variable. In variable lot-size model, we have considered a two-stage series production system lot-sizing decisions and incorporated quality

and maintenance into that model.

1.2 Problem Definition

We develop generalized integrated models incorporating quality and maintenance into the lot sizing decisions in the context of multi-stage production-inventory systems. We consider both uniform lot sizing models and variable lot sizing models.

1.2.1 Multi-stage Production System with Uniform Lot-Size

We consider a multi-stage system in which the inventory items are arranged in hierarchical manner . The finished goods comprise the top most level, intermediate goods at various prior stages occupy the subsequent levels and raw materials and other inputs lie at the lowest level of the hierarchy. According to our notational scheme, in an n-stage production system, the top level is labelled *1* and the subsequent levels are labelled $2, 3, \cdots, n+1$. Thus, at the initial manufacturing stage $(n+1)^{th}$ level items (input items) are processed to obtain WIP at level n and so on, until at stage 1 the level *2* semi-finished good is transformed to the final product. Each unit is passed to the subsequent stage as soon as processing at the present stage is complete without waiting for the completion of the whole batch.

Banerjee and Burton [6] developed a general approach for tackling lot sizing problem for single-stage and multi-stage production systems taking into account:

- Work-in-process (WIP) inventories.
- The notion of gradual transformation of input to output at each production stage.

The model proposed by Banerjee and Burton [6] implicitly assumes perfect quality of output. While, studies from Rosenblatt and Lee [65] [66], Lee and Rosenblatt [58] and Porteus [63] for single-stage prodcution-inventory systems suggest that the optimal EPQ is smaller than that determined by classical EPQ model. Obviously, a larger lot size requires a longer production cycle, and hence, likely to contain more defective items.

We incorporate the following into the Banerjee and Burton's [6] model in order to get a generalized mathematical model for total expected cost:

1. Imperfect quality of production.
2. Inspection errors in rejecting non-conforming items.
3. Random shift of the processes from in-control to out-of-control states following a general distribution.

For the sake of simplicity, we consider the production of a single end product under deterministic conditions i.e., production rates, demand rate and lead times, etc., are all constant and known.

1.2.2 Two-Stage Production System with Varying Lot-Sizes

Increasing automation in manufacturing processes has appreciably decreased the number of manufacturing operations required for simple products and for parts of complex products in most industries. Despite the fact that many manufacturing stages are still required in large number of cases, the use of two-stage production systems is increasing quite fast.

Earlier models in the literature incorrectly assume that the lot-size of a stage must not be larger than that of a preceding stage as lots move from initial to the final stage (which satisfies demand); and that the lot-size at a given stage must be a positive integer multiple of the lot-size at any following stage. This integrality assumption was claimed to be optimal by Crowston *et al.* [23] for a special and rather impractical case where infinite production rates were assumed. Williams [92] and Szendrovits [84],[85] refuted the integrality assumption by giving both counter examples and analytical proofs.

We consider a two stage production process is considered where several smaller lots are produced at the final stage and one larger lot is produced at the initial stage. An infinite time horizon, deterministic and constant demand and production rates are assumed. Fixed costs per lot, including constant setup costs and transshipment costs, are considerd to be deterministic and known. Inventory holding costs are also assumed to be deterministic and linear in time. Shipments to the next stage is not permitted until the entire lot at the preceding stage is completed. We assume that this cost of holding one unit of inventory increases with the value of the product.

Unrestricted capacity at each stage is assumed. Setup and transportation times are considered insignificant, hence, ignored. The product is assumed to be infinitely divisible, thus lot sizes are not required to be integers. The end of production at stage 1 in a given production run coincide with the depletion of inventory from the previous production run to zero. We incorporate the following in order to get a more generalized mathematical model for total expected cost:

1. Imperfect quality of production.
2. Inspection errors in rejecting non-conforming items.
3. Random shift of the processes from in-control to out-of-control states following a general distribution with increasing hazard rates.
4. Preventive maintenance into the developed mathematical model.

For the sake of simplicity, we have considered the production of a single end product under deterministic conditions i.e., production rates, demand rate and lead times, etc., are all constant and known.

1.2.3 Thesis Objectives

To develop and test mathematical models integrating production, quality and maintenance aspects for series multi-stage production-inventory systems. The following are the objectives of this thesis:

1. Develop a integrated mathematical models series multistage production-inventory systems with identical lot sizes. The proposed models incorporate quality

costs, inspection errors in screening nonconforming items, maintenance-inspection and restorations into the lot-sizing model.

2. Develop a integrated mathematical models series multistage production-inventory systems with variable lot sizes. The proposed models incorporate quality costs, inspection errors in screening nonconforming items, process restorations and preventive maintenance into the lot-sizing model.

3. Suggest optimization procedures for solving these models.

4. Coding the models to get numerical results for the purpose of comparison and analysis.

1.2.4 Thesis Organization

This thesis presentation is organized in five chapters. Chapter 2 presents a comprehensive survey of the relevant literature. Chapter 3 presents the development of integrated production, quality and maintenance model for a series MS-PIS. Chapter 4 presents the development of integrated production, quality, and maintenance model for a two-stage production-inventory system. Chapter 5 concludes the thesis.

Chapter 2

Literature Survey

2.1 Introduction

A **multi-stage production-inventory system (MS-PIS)** is one in which each stage receives its input from one or more immediate predecessors and supplies its output to one or more immediate successors. MS-PIS are, in essence, the most common case in any manufacturing environment. The work-in-process inventory is inherent in any MS-PIS when the product is manufactured through several stages. Forrester's [30] study of the cyclical variation of stocks in large production-distribution chains marked the start of a period of increasing interest in multi-stage inventory systems.The purpose of this chapter is to give a comprehensive survey of relevant literature.

2.2 Lot-Sizing Problem for Multi-Stage Production-Inventory Systems

The control of process inventory, and especially its functional relationship with the cycle time has received a lot of attention in recent years. The larger the production lot-size, the longer the manufacturing cycle which, in turn, increases the process inventory. This relationship is modelled to varying degrees in different models for different production systems configurations.

Multi-stage production-inventory systems can be broadly classified on the basis of system configurations as follows:

1. **General Network** is one in which each stage obtains supplies from a set of predecessors and supplies its output to a set of successors. Because of the complexity of general network structures, very limited progress can be found in this area and the existing models are too complicated and time-consuming for huge product structures to use.

2. An **Arborescent/Distribution System** is a special case of multi-stage production-inventory system in which each stage obtains its supply from a unique immediate predecessor and supplies its output to a set of immediate successors. Stages without immediate system predecessors obtain their supplies from outside the system; stages without immediate system successors supply outside customer demand.

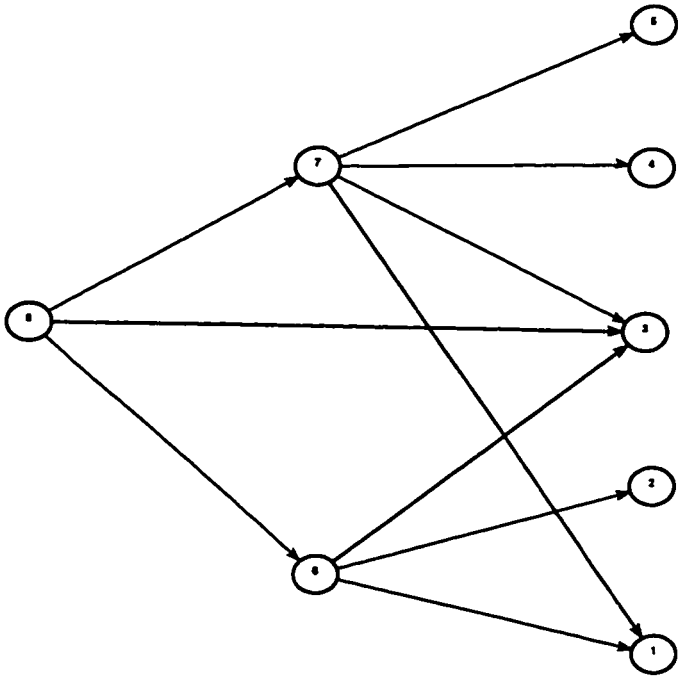


Figure 2.1: General Network

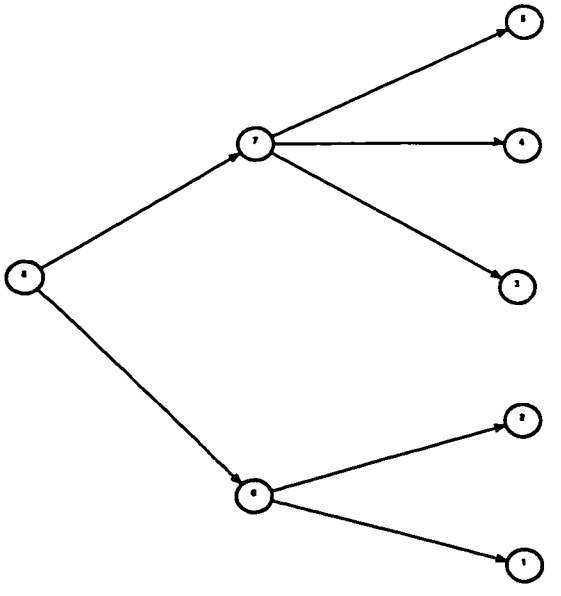


Figure 2.2: Arborescent/Distribution System

3. An **Assembly System** is a special case of general arborescent system and, in a naive sense, is "upside-down" i.e. inverseorconvergent general arborescent system . In assembly system each stage obtains its supply from a set of immediate predecessors and supplies its output to a unique immediate successor. Stages without immediate system predecessors obtain their supplies from outside the system; stage without immediate system successor supply outside customer demand. More research has been done in the assembly type than in the other multi-stage systems. This is the result of popularity of MRP (Materials Requirement Planning). In a multi-stage assembly system the production of an item, which itself is a component of a single parent item, requires a certain number of components. The final step satisfies external demand.

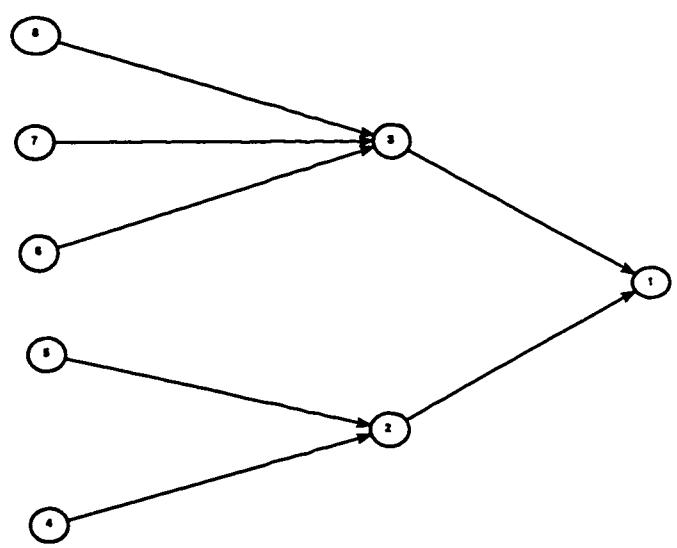


Figure 2.3: Assembly System

4. A**Series System** is a special case of both general arborescent system and assembly system. Methods for either systems can be directly applied to series-systems. In a series system each stage obtains its supply from a unique im-

mediate predecessor and supplies its output to a unique immediate successor. Stage without immediate system predecessor obtain its supplies from outside the system; stage without immediate system successor supply outside customer demand. Because of relative simplicity of series structures, researchers have attempted to find optimal solutions to this category of problem. Rosling [68] and Houtum *et al.* [49], in particular, established equivalence properties with series-systems, also discussed by Langenhoff *et al.* [57].

Figure 2.4: Series System

A real production system is, however, so complex that no model can capture all the elements under consideration and the research in this area is really challenging from application point of view. Therefore, a review of the literature on MS-PIS facilitates the scope for further research and its applications. Many models dealing with multi-stage lot sizing are present in the literature.

The purpose of this section is to give a brief literature survey of research papers dealing with multi-stage series-structure lot-sizing models for a single product under deterministic conditions and finite production rate.

2.3 Review of Lot-Sizing Models for Series Multi-Stage Production-Inventory Systems

Deterministic lot-size models for multi-stage production-inventory systems present in the literature include many variations in detail. Review of such papers are important for future researchers.

Koenigsberg [55] reviews the basic problems associated with efficient operations of production lines. Clark [20] reviews literature on multi-echelon inventory problems. Aggarwal [4] reviews the lot-sizing techniques under stationary demand conditions, with emphasis on pure inventory situations, and also discussed the literature on MS-PIS. The literature on management of production-inventories are reviewed and a comparison of material requirements planning and statistical inventory control has been made by Fortuin [31]

Silver [75] reviews the existing theory in inventory management. De Bodt *et al.* [26] review the lot-sizing under dynamic demand conditions in production-inventory situations. Chikan [14, 15] presents a review of inventory theory and models. Chikan [17] gives comparison of various classification systems of inventory models. Clark [22] gives an account of events leading to one of the earliest studies of multi-echelon production-inventory systems by Clark *et al.* [18]. Subsequent extensions are mentioned and areas of current and future applications are discussed.

Gupta *et al.* [44] give an extensive review of the literature dealing with multi-stage

lot-sizing . Since one of the most common approaches is to apply single-stage lot-sizing models, they also review single-stage static lot-sizing models and their performance when applied to multi-stage lot-sizing.

We will concetrate mainly on lot-sizing models for series multi-stage production-inventory systems for a single product under deterministic conditions and finite production rate. Two main classes that can be identified for series MS-PIS lot-sizing models are as follows:

1. The **uniform lot-size models** have constant lot-sizes across all stages, thereby incurring only one setup cost at all stages during on production cycle. They allow portions of a lot to be transported to next stage in batches.

2. The **variable/unequal lot-size** models allow different lot-sizes across stages. Overproduction from a larger lot -size is carried as work-in-process inventory (WIP) until it is consumed by smaller lot-sizes produced with repeated setup costs at the next production stage, and only full lots are transported between stages.

In both classes the recurring production cycle time is the time period during which the continuous demand consumes the largest lot size. Depending on the particular set of parameters that are used, one model may involve less cost than the other. Szendrovits and Golden [86] compare the costs of two representative models for a large number of simulated practical problems. The results show that the uniform lot-size model with batch shipment generally yields a lower cost than the variable lot-size model. This cost saving becomes more pronounced as the number of stages increases. However, the advantages of unequal lot-sizes are discussed by Goyal [35],

Szendrovits and Drenzer [83], and Goyal and Szendrovits [36]. Administrative problems also affect the selection of the model.

2.3.1 Uniform Lot-Size Models

Szendrovits [76] refines the concept of Eilon *et al.* [29] for determinig the manufacturing cycle time as a function of the lot-size in a MS-PIS. Szendrovits [76] evaluates th EPQ (Economic Production Quantity) using the functional realtionship.

Goyal [33] develops a model to minimize the manufacturing cycle time of a production lot. The movement of items between stages is done in sub-batches of equal sizes as propopsed by Szendrovits [76]. Szendrovits [76], however, ignored the effect of number of sub-batches on the EPQ. Goyal [33] takes this into account. Also, a more general expression for determining average investment in the process inventory has been given. Goyal [33] presents an exact solution approach.

Szendrovits [77] discusses the practice of optimization of number of sub-batches and assumes that the number of sub-batches will be selected to suit the technical feasibility of transportation.

Tanaka *et al.* [?] analyze the relationship among the number of set ups, lead time, total elapsed time, in-process waiting and tardiness time on two-stage models. They propose a procedure to determine the optimal number of set ups and lead time in a lot production system.

Goyal [34] proposes a model and considers the effect of number of sub-batches transshipped between stages on the economic production quantity. Conventional optimization by differentiating the cost function with respect to Q is used to obtain optimal solution.

Szendrovits *et al.* [82] present two basic, deterministic, infinite horizon models for series multi-stage production-inventory systems. The first model assumes that the lot-size is maintained through all the production stages while transportation of equal size batches is allowed between stages. The second model has varying lot-sizes with lot-sized intershipments assuming certain integrality restrictions on splitting of lots and monotonically increasing stage inventory cost. Batch splitting reduces the inventory costs due to queueing of batches between stages. Comparison of the two models provide some insight into the characteristics of multi-stage production-inventory systems and reflect various basic assumptions of multi-stage production-inventory systems. Szendrovits *et al.* [82] present an optimal solution procedure for one of his models and a good approximation to the optimal for the other one.

Transportation cost of batches for a stage is related to the load capacity of transport equipment used at that stage. Szendrovits *et al.* [83] relax the assumption of having same number of equal-sized batches for transshipments in Szendrovits [76],[77]. Relaxation of this assumption considerably reduces the work-in-process inventories and, hence, cost. The greater the variation among the transportation costs for a

batch at various stages the more significant the cost reduction. Szendrovits *et al.* [83] propose an efficient heuristic which renders optimal solution in a large number of cases. An optimal solution procedure is also given which uses the heuristic solution in developing the lower and upper bounds. They use a scanning procedure which requires just a few iterations as compared to a large number of iterations required for complete enumeration.

Deterministic lot-sizing models tacitly assume that the facility is used for another product when the lot is completed and that a new setup is required for recurring production of the lot. When the plant utilization is rather low or when resuming the production of the lot occurs in within a relatively short interval of time, this assumption may be invalid. Interruption of the production process generates idle-time but reduces the process inventory. Interrupting the production temporarily are economical when the additional costs generated by idle-time and restarting the machines are offset by the reduced frequency of major setup costs and by decreased inventory holding costs. Szendrovits [87] considers a lot-sizing model with equal-sized inter-stage batch shipments for MS-PIS permitting temporary interruption of the production at certain stages. He ignored the effect of work-in-process inventories. He examins the models intuitively and gives optimal and heuristic solution procedures.

Banerjee *et al.* [6] develop a general approach for tackling the lot-sizing problems in single-stage and multi-stage systems taking into account work-in-process inventories and, more importantly, incorporating the notion of the gradual transfor-

mation of input to output at every production stage. They also establish parametric conditions under which the impact of work-in-process inventories on the lot-sizing decisions become critical from a cost stand point. They use an exact optimization (conventional optimization by differntiating the cost function with respect to Q) technique by realizing the convex nature of the objective function.

Grubbstrom *et al.* [39] consider a model involving the initial order quantities and associated backlogs. They give a dynamic programming approach to determine optimal solutions. Before each entire batch of end products is completed, any demand appearing from the start of the production period cannot be met when there is a lack of initial inventory. in order to reduce lost sales, smaller batches and/or larger backlogs than the stationary ones may be preferred,; but, this would incur additional costs. It is known and shown that the optimal production sequence consists of two parts: an initial transient sequence (in which batches will increase and backlogged amounts decrease) followed by a stationary phase(in which both remain constant).

2.3.2 Variable/Unequal Lot-Size Models

Taha *et al.* [88] build lot-size intershipments as well as backlogging into the model. This model include process inventory and is one of the first in this regard. This model allow any number of intershipments between stages, ignoring the shipment costs. They use complete enumeration and some limiting assumptions to arrive at the solution. They assume that in an optimal schedule the lot-size at any given stage is an integral multiple of the lot-size at its immediate successor stage (the so

called 'integrality assumption').

Jensen *et al.* [52] treat the continuous production case similar to Taha *et al.* [88] without backlogging. They assume that each stage will be periodically shut-down and restarted so that the average production rate at that stage is equal to the demand rate. They develop deterministic simulation algorithms for finding startup delays.

Schwarz [70] present a model similar to that in Taha *et al.* [88], while incorporating echelon inventory costs, for assembly type structure. The concept of *echelon inventory* allows the value-added inventory holding cost to be considered into the model and is suggested by Clark *et al.* [19]. They propose system myopic heuristic policies and an efficient Branch and Bound algorithm (for finding optimal policy) under continuous review over an infinite time horizon. They are first to propose system myopic policies. In a system myopic policy optimization is done with respect to only two stages neglecting the interstage interaction.They present theoretical and practical evidences for near optimality of system myopic policies. They conclude that system-myopic policies give following advantages : (1) easy to understand; (2) require less information; and (3) fast and easy to compute.

Schwarz *et al.* [70] model assumes that for optimal policies in a multi-echelon production-inventory system the lot-size at a particular stage must not be larger than that of a predecessor stage as lots move from the initial stage to the final stage. Szendrovits [84] points out , using examples, that setup costs and inventory

holding costs may be such that the optimal policies exist when smaller lot sizes are produced intermittently at a stage, with an overlap, to feed the continuous production of a larger lot size at a proceeding stage. It is also shown, by an example, that the optimality of solution does not always require that the lot-size at each stage be an integer multiple of lot-size at the successor stage, denouncing the integrality assumption.

Szendrovits *et al.* [79] points out a potential flaw in the model proposed by Schwarz *et al.* [70]. Schwarz *et al.* [71] clarify the ambiguity.

Crowston *et al.* [23],[24] prove a potentially important characteristic of assembly systems, i.e. the lot-size at a particular stage has to be a positive integer multiple of the lot-size at the preceding stage. This property is popularly known as 'integrality assumption' and is used by scores of researchers to supplement solution techniques for getting optimal and near optimal solutions. The theorem has been said to apply for uncapacitated assembly systems having final product demand over infinite time horizon with no backlogs or lost sales permitted, where lot-sizes must be rational numbers and time invariant. The validity of this theorem for general assembly structures is challenged by some researchers at later stages. Crowston *et al.* [24] offer a dynamic programming algorithm for optimization purposes.

Graves *et al.* [38] modify the solution approaches proposed by Schwarz *et al.* [70] to suit the cases of arborescent production-inventory systems. These start with myopic solutions and then attempt to improve them. They describe some char-

acteristics of optimal policies an analyze optimal and heuristic stationary policies. 'Stationary policy' means one wherein each stage produces the same lot-size each time it produces. They show that under fairly mild conditions (e.g. zero inventroy) the optiaml stationary policy is a 'single-cycle policy' (one in which each time any stage produces, all of its successors also produce). The results given by Schwarz *et al.* [70] and Graves *et al.* [38] show that the system myopic policies are, in general, close to optimal. They also propose and test an efficient Branch and Bound algorithm for finding optimal policy.

Kumar *et al.* [56] determine the optimum level of inventory of finished goods considering work-in-process at different stages of production.

Szendrovits *et al.* [82] present two basic, deterministic, infinite horizon models for series multi-stage production-inventory systems. The first model assumes that the lot-size is maintained through all the production stages while transportation of equal size batches is allowed between stages. The second model has varying lot-sizes with lot-sized intershipments assuming certain integrality restrictions on splitting of lots and monotonically increasing stage inventory cost. Batch splitting reduces the inventory costs due to queueing of batches between stages. Comparison of the two models provide some insight into the characteristics of multi-stage production-inventory systems and reflect various basic assumptions of multi-stage production-inventory systems. Szendrovits *et al.* [82] present an optimal solution procedure for one of his models and a good approximation to the optimal for the other one.

Szendrovits [84] shows by counter examples that for finite production rate the integrality restriction (as concluded by Crowston *et al.* [23]) is not necessarily optimal.

William [92] questions the well known theorem reported by Crowston *et al.*[23]. While theorem supposedly applied to the case of both the finite production rates and infinite production rates, the proof dealt only with the case of instantaneous production. Also, the proof implicitly assumed that the lots were processed at a given stage only after unchanging spans of time. Williams [92] shows that with or without this implicit assumption, the theorem does not hold true for the special case of instantaneous processing in a two-level assembly systems (i.e. configurations where there is only one successor stage in the entire system). However, he also shows by counter example that without this implicit assumption the theorem is invalid for more general assembly structures. Also, the proof by Crowston *et al.* [23] was defective at the point where they extended their results to more general assembly structures. Though the theorem is valid for special cases, it is an open question that whether or not the theorem is valid for all assembly structures when processing at any particular stage must occur only after unchanging spans of time. This theorem is still used by researchers for special cases of assembly structures, e.g. Andijani [3] has used it to abate optimizations efforts.

Blackburn *et al.* [11] develop and test modifications to existing lot-sizing heuristics for a multi-stage assembly process managed on periodic-review basis. Simulation experiments are done to test those heuristics.

William [93] gives an excellent review of papers exploring possible solutions to simultaneous scheduling of production-distribution systems.

Szendrovits [85] discuses a two-stage production-inventory system in which smaller lots are produced at one stage and one larger lot is produced at the other stage. For the class of multi-stage production-inventory systems discussed in the noteworthy findings are reported. Szendrovits [85] show that even in a two-stage systems non-integer lot ratios can be also be optimal. They conclude that depending on the problem parameters different inventory patterns represent optimal policies. They also present efficient optimization procedures for various inventory patterns.

Moily [61] considers component lot-splitting in which the lot-size of a component item may cover only a fraction of its parent item's lot-size. Optimal and heuristic solution procedures are provided with their performance comparisons.

Billington *et al.* [10] formulate the multilevel lot sizing problem when there is a bottleneck facility as an integer programming problem and present a heuristic method based on Langrangian relaxation . A series of Langrangian relaxations is embedded in a branch and bound procedure.

Babu *et al.*[5] attempt to compare the performance of integrated production- inventory (IPI) model for a serial batch production system and the EPQ approach. In IPI models the stages are linked through integer multiples and order sizes for raw

materials are also linked via integer multiples to the batch size of concerned product. The results show a good scope for application of heuristic policies in IPI models.

Most models, however, fail to incorporate the effects of work-in-process inventories. Schwarz [72] makes efforts to overcome this deficiency for infinite production rate case which has given useful ideas for ones working on finite production rate case. But, this and other papers did not consider the effects of gradual conversion of input to output at finite rate within each production stage.

Gunasekaran *et al.* [40] present a mathematical model for a multi-stage, multi-facility and multi-product production-inventory system to study the effect of machine unreliability on EPQ and total system cost.

Gunasekaran *et al.* [41] present a mathematical model for determining the EPQ at each facility in a multi-stage, multi-facility, and multi-product production-inventory system with significant setup operation time.

Toklu *et al.* [89] solve the problem with a bottleneck, as in Billington *et al.* [10], using a very simple and efficient heuristic, simple enough that it does not even require a computer routine. Previously, such a problem used to be formulated as an integer programming model and solved using Lagrangian relaxation embedded within branch and bound procedure. They also use Linear Programming and Branch and Bound procedures to find solution to ILP formulation. LP solution is used to

give optimal solution; while, Branch and Bound process had to be stopped after a specified large amount of computer time had elapsed and further effort appeared unproductive. The heuristic solutions are found to be close to optimal. Simulation results show that solutions of reasonable quality can be obtained very efficiently.

Voros *et al.* [90] analyse the problem with backlogging case and suggest a cost modification structure with the help of which the performance of level-by-level heuristics can be improved. The performance of the improved heuristic is also investigated and examples show that when simple level-by-level heuristic show extremely bad results these improved heuristics give satisfactory results.

Gunasekaran *et al.* [42] propose a mathematical model to establish relationship among the quality at source, work-in-process inventories and lot-sizes in a multi-stage JIT production-inventory system. A search method is used to determine the EPQ.

Gupta *et al.* [45] examine the performance of various lot-sizing procedures under rolling horizon environment using simulation. It is shown that Silver-Meal [73] outperforms other procedures, like Wagner-Whitin [91] and incremental approach (Boel *et al.* [12]) in most of the cases. The performance of incremental approach and Silver-Meal are found better than their respective counterparts Gaither's rule [32] and modified Silver-Meal (Silver *et al.* [74]).

Ding [27] investigates the operation procedure of initial order quantities in a se-

ries multi-stage production-inventory system allowing overproduction at different a stages as an extension to Grubbstrom *et al.* [39]. He shows that allowing batch-size to vary from stage to stage gives an opportunity to improve production performance by producing a better balance between lost sales and additional costs incurred from producing smaller batches.

Gunasekaran *et al.* [43] present a mathematical model to study the implications of lot-sizing with respect to each stage, setup cost reduction and process control in a multi-stage production system.

2.3.3 Integrated Models for Production, Quality and Maintenance in Single-Stage Production-Inventory Systems

In the recent past, research has been done to incorporate quality and maintenance into the single-stage production control models. Rosenblatt and Lee [65][66], Lee and Rosenblatt [58] and Porteus [63] have studied the effect of the presence of defective items on the EMQ in an imperfect production process. These studies have shown that the the optimal EMQ was smaller than that determined by classical EMQ model (obviously, a larger lot-size requires a longer production cycle, and hence, likely to contain more defective items).

Lee and Rosenblatt[58] addressed the problem of joint control of production cy-

cles and scheming inspection. When scheming inspection is adopted, the condition that the optimal inspection intervals are equally spaced is shown.

Rosenblatt and Lee [67] considered the joint problem of production planning and scheming inspection under the assumption that the cost of process restoration is a function of detection delay and the existence of shortages in the process. The "detection delay" is defined as the time elapsed since the production process has deteriorated until it is identified by some inspection procedure and restored.

All the aforementioned studies have assumed that the deterioration of the process is a random variable which is exponentially distributed. While the exponential distribution is demonstrated to provide good approximations to lifetime distributions, it often criticized for its robustness and memoryless nature. Lin *et al.* [59] studied the Lee and Rosenblatt[58] model under the assumption that the deterioration of the process follows a general distribution. Their model assumed constant restoration cost and did not allow for the existence of shortages in the process. They did not explore the optimality of the successive inspection at equal intervals.

Huang and Chiu [51] have also studied the Lee and Rosenblatt[58] model under the assumption that the deterioration of the process follows a general distribution. They studied their model under two policies: one with PM procedures and other without PM procedures. They assumed the restoration cost and percent of defective items produced in an out-of-control state as functions of detection delay. Their studies suggested that it is beneficial to perform a PM procedure when the restoration

cost is greater than the PM cost.

2.3.4 Limitations of the Existing Lot-sizing Models for Series MS-PIS

A review of series MS-PIS lot-sizing models, classified in two broad categories, and the solution methodoligies adopted has been summarized. The review of models in MS-PIS in general, and series MS-PIS in particular, shows following limitations and deficiencies of the existing research literature in MS-PIS lot-sizing decisions with finite production rates (Goyal *et. al* [37]):

1. Machine/process failures or drifts are usually not considered in MS-PIS lot-sizing models.

2. No consideration is given to preventive/corrective maintenance of MS-PIS.

3. Quality aspect of the production system is usually ignored by assuming perfect quality of output.

4. Setup times of batches are usually neglected. While, in real life finite setup times are common.

5. In most of the cases, Transportation times of batches is also ignored.

6. Generally, in most of lot-sizing models of MS-PIS, the constraints on batch sizes and capacity of the production facility are not considered. The aspects are important for practical system.

7. In the most existing MS-PIS models, queueing inventory between batches is neglected due to the modelling difficulty of such complicated production systems.

These problems are important for researchers as the future research work. In this thesis an attempt is made to incorporate the quality and maintenance aspects of MS-PIS lot sizing models.

Chapter 3

Multistage Production Lot Sizing Model With Uniform Lot Size

The purpose of this chapter is to model the effect of imperfect production processes on lot sizing decisions for the single end item in context of uniform lot size multi-stage production systems with work-in-process inventories, under deterministic conditions.

3.1 Introduction

The issue of modelling the effect of imperfect production processes on lot sizing decisions does not seem to have been adequately addressed in the literature for MS-PIS. Also, the presence of imperfect production processes in an MS-PIS framework calls for screening of non-conforming items in order to prevent unnecessary processing at the subsequent stages and reduce waste. In such a case, the issue of erroneous product inspections during screening of nonconforming items, if present, must also be addressed. Maintenance procedures, including corrective and preventive mainte-

nance, should be used to reduce quality costs.

The classical EPQ model assumes that the output of the production system is of perfect quality; but, in most of the cases, this is not a realistic assumption. Generally, product quality is a function of the state of the production process. When the process is in good condition the quality may be high or perfect. With the passage of time, the process may deteriorate resulting in production of items which may contain items which are defective or of sub-standard quality.

Studies from Rosenblatt and Lee [65],[66], Lee and Rosenblatt [58] and Porteus [63] have shown that for imperfect production processes the optimal EPQ was smaller than that determined by classical EMQ model. Obviously, a larger lot size requires a longer production cycle, and hence, likely to contain more defective items.

Banerjee and Burton [6] developed a general approach for tackling lot sizing problem for single-stage and multi-stage production systems, taking into account work-in-process (WIP) inventories and, more importantly, incorporating the notion of gradual transformation of input to output at each production stage. The work-in-process at any stage other than input stage is characterized by a single semi-finished good, eventually emerging as the finished good after the completion of processing at the last manufacturing stage. The phenomena of a finite conversion rate is captured by specifying an inter-stage transfer batch size of one unit. This implies that, even though we are dealing with a batch production system, each unit is passed to the next stage as soon as processing at the current stage is completed. Such production

systems are common in practice.

Starting with the inventory pattern tackled by Banerjee and Burton [6], we develop following general models in the subsequent sections for series MS-PIS with uniform lot sizes:

1. Multi-stage lot sizing model for perfect production processes.
2. Multi-stage lot sizing model for deteriorating production processes and screening of nonconforming items between stages.
3. Multi-stage lot sizing model with deteriorating production processes, screening of nonconforming items between stages, and errors in screening inspections.
4. Multi-stage lot sizing model with deteriorating production processes, screening of nonconforming items between stages,errors in screening inspections, and maintenance inspections and process restorations.

3.2 Problem Definition

The inventory items in the multi-stage system are arranged in hierarchical manner. The finished goods comprise the top most level, intermediate goods at various prior stages occupy the subsequent levels and raw materials and other inputs lie at the lowest level of the hierarchy. According to our notational scheme, in an n-stage production system, the top level is labelled *1* and the subsequent levels are labelled $2, 3, \cdots, n+1$. Thus, at the initial manufacturing stage $(n+1)^{th}$ level items (input items) are processed to obtain WIP at level n and so on, until at stage 1 the level

2 semi-finished good is transformed to the final product. This is shown in Figure: 3.1. Each unit is passed to the subsequent stage as soon as processing at the present stage is completed without waiting for the completion of the whole batch.

Assumptions

It is assumed that the processes are in an in-control state at start of the production cycle, producing items of acceptable quality. However, after some time a production process may shift to an out-of-control state. The elapsed time (t) for which the process remains in the in-control state, before a shift occurs, is considered to be a random variable following a known probability distribution. Once in the out-of-control state, the process starts producing a fixed percentage of defective items and stays in that state till either the end of production run or some restoration action. Each setup of a process brings the process back to the in-control and as-new state. These production systems are not uncommon in practice. Consequently, in the case of perfect production processes, a uniform lot size of Q units is processed at any production stage.

In the presence of deteriorating production processes, nonconforming items must be screened so that they are not passed to subsequent stage to avoid unnecessary processing. Also, some remedial actions must be taken in order to prevent and/or delay the shift in a production process.

For the sake of simplicity, we consider the production of a single end product under deterministic conditions i.e., production rates, demand rate etc., are all constant

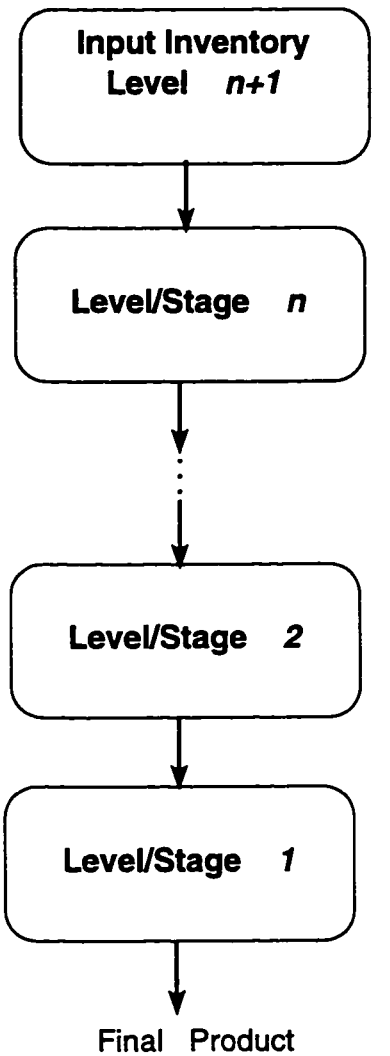


Figure 3.1: Hierarchichal arrangement of inventories in an n-stage PIS.

and known; and, the production rate at any stage exceeds the demand rate, i.e. $P_j > D,\ j = 1, 2, ..., n$. Backlogging is not permitted. Beyond the $(n+1)^{th}$ level, the work-in-process (WIP) at any subsequent level is characterized by a single semi-finished item, until it becomes the final product after being processed at the final level. The $(n+1)^{th}$ level items (the raw materials and other input products) are of perfect quality.

We further assume that below level *1*, all lot sizing decisions for processing intermediate goods or procuring material and other inputs are based on the lot-for-lot technique. This implies that lot size is Q units at any production stage. The lot-for-lot assumption also allows us to easily handle cases where multiple input items may be required at the final stage of production. In such a situation, all the input items are treated as a single composite item, whose cost parameters are obtained by adding the respective individual costs.

Notations

Following are the notations used in the development of mathematical model:

General notation

Q = The production lot size for the end product in *units*.

Q_j = The production lot size for the product at the jth stage in *units*.

D = The demand rate for the end products in *units/unit time*.

r = The fractional inventory carrying cost in $/$/*unit time* (expressed as a fraction of production cost).

$\bar{I}_j$ = The average inventory of the items at the jth level.

P_j = The Production rate in *units/unit time* at which the $(j+1)^{th}$ level item is converted to the jth level inventory item.

A_j = The fixed setup or ordering cost of the inventory item at the jth level item in $\$/unit$.

C_j = The total production or procurement cost (excluding setup/ordering costs) of the jth level item in $\$/unit$.

ETC= Total cost per unit time in $\$/unit\ time$.

Notation related to quality

s_j = The Quality cost of producing a defective item at the jth level in $\$/unit\ defective$ produced.

α_j = The fraction of defective units produced at the jth level after the process has shifted from an in-control state to an out-of-control state.

N_j = The expected number of defective items produced at the jth level.

$\bar{N}_j$ = The expected average number of defective items produced at the jth level.

$\bar{I}_j$ = The expected average inventory of the items at the jth level.

η_j = The number of equal length inspection intervals of the process at the jth level.

t_j = Total production time per production cycle required by the jth level item.

$F_j(t)$ = The failure or shift distribution for the process at the jth level.

$f_j(t)$ = The p.d.f. of elapsed time before shift at the jth level.

τ = The detection Delay at the jth stage, i.e. the time elapsed between the

occurrence of a shift and end of production at the jth stage.

Notation related to screening inspecion errors

$E_{1,j}$ = The probability of committing Type I error in the product inspections at the jth stage.

$E_{2,j}$ = The probability of committing Type II error in the product inspections at the jth stage.

$\pi_{1,j}$ = The quality cost of incorrectly rejecting a conforming item at the jth stage.

$\pi_{2,j}$ = The quality cost of incorrectly accepting a non-conforming item at the jth stage.

Notation related to maintenance-inspection and restorations

η_j = The number of maintenance inspections in an equal inspection interval maintenance-inspection plan.

m_j = The cost of a single maintenance inspection.

IC = The total cost of maintenance inspections per unit time.

$R_j(\tau)$= The cost of restoring the process at the jth stage.

R_{jc} = The constant component of the restoration cost .

R_{jv} = The component of the restoration cost dependent upon the detection delay τ.

$RC_{i,j}$= The expected restoration cost for the process at the jth stage during the

i^{th} inspection interval.

$TRC=$ The total expected restoration cost per unit time.

3.3 Multistage Lot Sizing with Work-In-Process Inventory and Perfect Production Processes

Here we reproduce the work by Banerjee and Burton [6], to make this thesis self-contained, considered for further treatment in the following sections.

3.3.1 Costs Involved

Following are the costs involved in determining the total cost:

1. Setup costs.

2. Inventory holding costs.

Setup Costs

The setup cost includes the cost of preparing and placing orders for replenishing inventories, the cost of handling and shipments, the cost of machine setups for production runs (by adjusting machines, changing tools and fixtures, etc.), the cost of inspection of received and produced orders of inventory, and all costs that do not vary with the size of the order. Sometimes it is difficult to determine these costs in detail; therefore, analysts usually combine costs. Since we are assuming that a process must start in an as-new state upon each setup, the setup costs also include

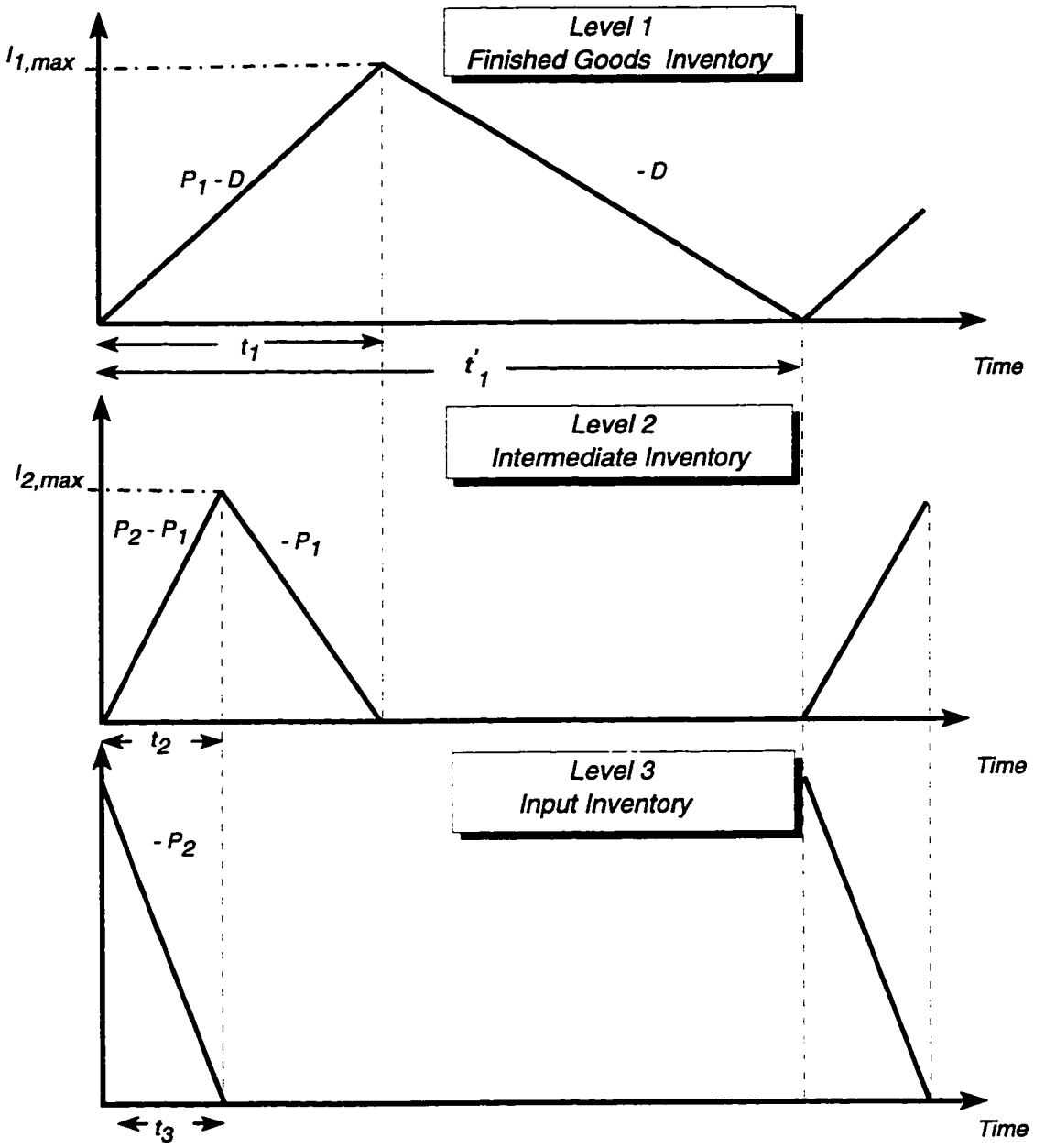


Figure 3.2: Inventory time plots for a 2-stage system; case I: $P_1 < P_2$

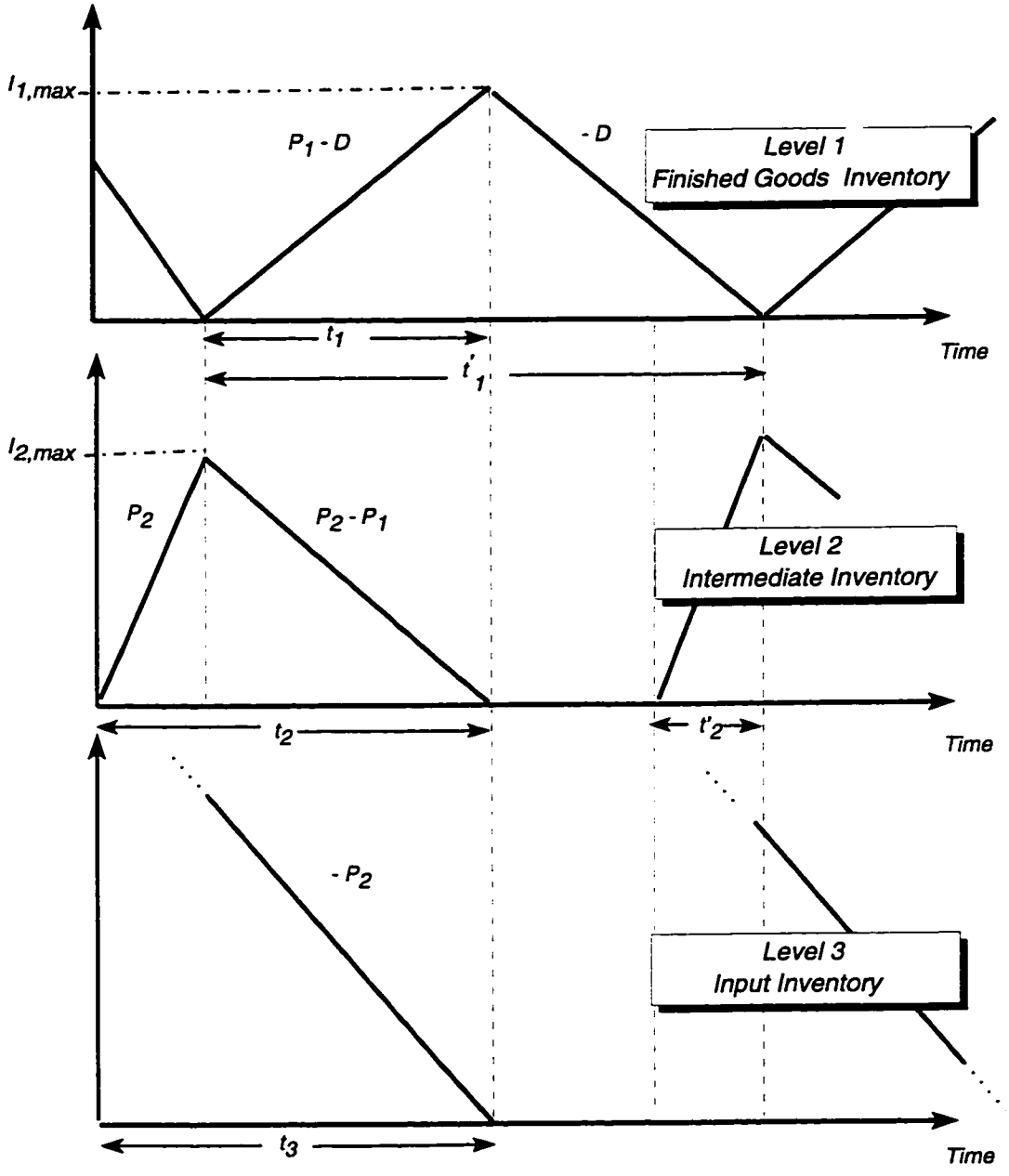


Figure 3.3: Inventory time plots for a 2-stage system; case II: $P_1 > P_2$

process restoration costs.For inventory models developed in this thesis, it is assumed that the setup cost does not vary with the size of EPQ. Total Setup Cost per unit time for an n-stage, system considered in this chapter, is:

$$SC = \frac{D}{Q}\sum_{j=1}^{n+1} A_j \tag{3.1}$$

Inventory Carrying Costs

The cost of carrying or holding inventory can be broken down into several components:

1. The *opportunity cost* of money being tied up in inventory; e.g., interest forgone because that money is not placed in an interest-bearing account.

2. *Storage and space charges* representing the cost of providing space, as well as its cost of maintenance (including the cost of handling units and the cost of Information System that is used to keep track of inventory).

3. Cost of *taxes and insurance* .

4. The cost of *physical deterioration and its prevention.*

5. The *cost of obsolescence* due to technological changes, as in the fields of computers and communication equipment.

For systematic development in order to find average inventories, we first consider the situation involving only two production stages, where the inventory hierarchy consists of *3* levels: end item(level *1*), intermediate good(level *2*) and input materials(level *3*). Two possible distinct cases are discussed below.

Case I ($P_1 < P_2$)

This case implies that the production of level *2* and level *1* (end items) can start at the same instant and that $t_2 \leq t_1$. Figure 3.2 shows the inventory time plots for this case.

From Figure 3.2 we can find average inventories at every level:

$$\bar{I}_1 = \frac{Q}{2}\left(1 - \frac{D}{P_1}\right) \tag{3.2}$$

$$\bar{I}_3 = \frac{DQ}{2P_2} \tag{3.3}$$

$$\bar{I}_2 = \frac{1}{2}t_1(P_2 - P_1)t_2\frac{D}{Q}$$

$$\bar{I}_2 = \frac{D}{2Q}\frac{Q}{P_1}(P_2 - P_1)\frac{Q}{P_2}$$

$$\bar{I}_2 = \frac{DQ(P_2 - P_1)}{2P_1P_2} \tag{3.4}$$

Case II ($P_1 \geq P_2$)

This case implies that it is necessary for the production of level *2* items to begin some time (say t_2' periods) prior to starting the production of the level 1 items. This is shown in the inventory time plot of fig. 3.3.

It can be seen that $t_2 \geq t_1$ and $t_2' = t_2 - t_1$. Since $t_1 = Q/P_1$ and $t_2 = Q/P_2$, the values of $\bar{I}_1$ and $\bar{I}_3$ are algebraically the same as in the case I, but for level *2*, the average inventory is given by:

$$\bar{I}_2 = \frac{1}{2}t_2P_2t_2'\frac{D}{Q}$$

$$\bar{I}_2 = \frac{DQ(P_1 - P_2)}{2P_1P_2} \tag{3.5}$$

General Case

Comparing the average inventories from both cases, we can generalize these results obtained in the *2*-stage case for a general n-stage case:

$$\begin{aligned} \bar{I}_1 &= \frac{Q}{2}\left(1-\frac{D}{P_1}\right) \\ \bar{I}_3 &= \frac{DQ}{2P_2} \end{aligned} \tag{3.6}$$

Therefore,

$$\begin{aligned} \bar{I}_{n+1} &= \frac{DQ}{2P_n} \\ \bar{I}_2 &= \frac{DQ \mid P_2 - P_1 \mid}{2P_2P_1} \end{aligned} \tag{3.7}$$

Therefore,

$$\bar{I}_j = \frac{DQ \mid P_j - P_{j-1} \mid}{2P_jP_{j-1}} \qquad \forall\, j = 2, 3, \cdots, n. \tag{3.8}$$

Which is true regardless of the relative magnitudes of P_{j-1} and P_j.

Using Equations (3.6), (3.7) and (3.8); the total inventory holding cost per unit time, for the system considered in this chapter, is given by:

$$HC = \frac{rQ}{2}\left\{C_1\left(1-\frac{D}{P_1}\right) + D\sum_{j=2}^{n}\frac{C_j \mid P_j - P_{j-1} \mid}{P_jP_{j-1}} + \frac{C_{n+1}D}{P_n}\right\} \tag{3.9}$$

Expected Total Cost

The Total Cost per unit time can be found by adding up all the costs from Equation (3.1) and(3.9):

$$ETC(Q) = \frac{D}{Q}\sum_{j=1}^{n+1}A_j + \frac{rQ}{2}\left\{C_1\left(1-\frac{D}{P_1}\right) + D\sum_{j=2}^{n}\frac{C_j \mid P_j - P_{j-1} \mid}{P_jP_{j-1}} + \frac{C_{n+1}D}{P_n}\right\} \tag{3.10}$$

Optimization Procedure

Differentiating Equation (3.10) w.r.t Q:

$$ETC' = -\frac{D}{Q^2}\sum_{j=1}^{n+1} A_j + \frac{r}{2}\left\{C_1\left(1-\frac{D}{P_1}\right) + D\sum_{j=2}^{n}\frac{C_j \mid P_j - P_{j-1} \mid}{P_j P_{j-1}} + \frac{C_{n+1}D}{P_n}\right\}$$

Setting $ETC' = 0$ we get:

$$Q^* = \sqrt{\frac{2D\sum_{j=1}^{n+1} A_j}{r\left\{C_1\left(1-\frac{D}{P_1}\right) + D\sum_{j=2}^{n}\frac{C_j|P_j-P_{j-1}|}{P_jP_{j-1}} + \frac{C_{n+1}D}{P_n}\right\}}} \tag{3.11}$$

If we differentiate ETC' w.r.t. Q we can see that:

$$ETC'' = \frac{2D}{Q^3}\sum_{i=1}^{n+1} A_j > 0$$

i.e. ETC'' is always positive showing that ETC is a convex function in Q and hence Q^* give the global minimum of the Expected Total Cost. It can be readily seen from Equation (3.11) that the optimal Q for an imperfect production system is smaller than that determined by assuming perfect production quality.

3.4 Lot Sizing Model for Imperfect Production Processes

If we consider a model in which the nonconforming items produced at one stage are screened before proceeding the lot of the intermediate good to the next stage the model will become complicated as at every stage the lot size will be different. If Q_j is the lot size produced at the jth stage then, obviously, $Q_j \geq Q_{j-1} \quad \forall\, j = 2, 3, \cdots, n$ i.e. the lot size produced at the preceding stage is greater than or equal to the that at the current stage. This also calls for 100% inspection of lots between stages which is

made possible by automated inspection mechanisms in the modern industries. Here we assume that screening of nonconforming items is done by some prefect inspector or automated process. This means no inspection errors are involved in it. This assumption will be relaxed in the subsequent section.

It can be noted that:

$$Q_{j-1} = Q_j - N_j \tag{3.12}$$

$$Q_n = Q + \sum_{j=1}^{n} N_j \tag{3.13}$$

$$Q_{n+1} = Q_n \tag{3.14}$$

The Equation (3.12) can be used to find $Q_j's$, recursively. The Equation (3.14) implies that the input supplied to stage n (i.e. the level $n+1$ inventory) is free from nonconforming items.

3.4.1 Expected Number of Defective Items

If N_j represents the expected number of defective items produced at the jth stage, then:

$$N_j = \int_0^{t_j} \alpha_j P_j(t_j - t) f_j(t) dt$$

where t is a random variable representing the elapsed time for which the process remains in the in-control state, before the occurrence of a shift. We assume that once a shift has occurred, the process stays in the out-of-control state. The above equation for finding expected number of nonconforming items produced the jth stage

can also be given by:

$$
\begin{aligned}
N_j &= \alpha_j P_j \left\{ \frac{Q_j}{P_j} \left\{ F_j\left(\frac{Q_j}{P_j}\right) - F_j(0) \right\} - \int_0^{\frac{Q_j}{P_j}} t f_j(t) dt \right\} \\
N_j &= \alpha_j P_j \left\{ \frac{Q_j}{P_j} \left\{ F_j\left(\frac{Q_j}{P_j}\right) - 1 \right\} - \int_0^{\frac{Q_j}{P_j}} t f_j(t) dt \right\} \qquad (3.15)
\end{aligned}
$$

The exponential distribution is undoubtedly the most commonly used distribution in reliability applications. But, this distribution fails to account for the age of the process or the equipment. This means that the process or the equipment experiences no effects of wear out. This property of exponential distribution is, sometimes, referred to as 'no memory' property. The hazard function of exponential distribution is constant $\frac{1}{\theta}$. If t is exponentially distributed, for the jth stage, with mean θ_j:

$$
\begin{aligned}
F_j(t) &= 1 - e^{\frac{-t}{\theta_j}} \qquad (3.16) \\
N_j &= \int_0^{t_j} \frac{\alpha_j P_j (t_j - t) e^{\frac{-t}{\theta_j}}}{\theta_j} dt \\
N_j &= \alpha_j P_j \left(t_j + \theta_j e^{\frac{-t_j}{\theta_j}} - \theta_j \right)
\end{aligned}
$$

The above equation can be used directly for finding N_j; but, our preliminary studies have shown that this equation can be approximated without any significant loss of accuracy by using McLaurin's Series:

$$
e^{\frac{-t}{\theta_j}} \simeq 1 - \frac{t}{\theta_j} + \frac{\left(\frac{t}{\theta_j}\right)^2}{2}
$$

We get:

$$
\begin{aligned}
N_j &= \frac{\alpha_j P_j t_j^2}{2\theta_j} \\
N_j &= \frac{\alpha_j {Q_j}^2}{2 P_j \theta_j} \qquad (3.17)
\end{aligned}
$$

The last equation follows from the fact that : $t_j = Q_j/P_j$ and cycle length $= Q/D$. Hence, the expected average number of defective items is:

$$\bar{N}_j = \frac{DQ_j{}^2\alpha_j}{2QP_j\theta_j} \tag{3.18}$$

3.4.2 Costs Involved

Following are the costs involved in determining the total cost:

1. Setup costs.
2. Inventory holding costs.
3. Expected quality costs.

Setup Costs

The total Setup Cost per unit time for an n-stage system is:

$$SC = \frac{D}{Q}\sum_{j=1}^{n+1} A_j \tag{3.19}$$

Inventory Carrying Costs

For systematic development in order to find average inventories, we once again consider the situation involving only two production stages, where the inventory hierarchy consists of *3* levels: end item(level *1*), intermediate good(level *2*) and input materials(level *3*). We have assumed that each unit is passed to the next stage as soon as processing at the current stage is completed in order to capture the phenomena of finite conversion rate. If we further assume that each item undergoes quality inspection before it is transferred to the next stage then this would render

Figures 3.2 and 3.3 still valid for the ongoing considerations. The difference is that the screening of defective items needs to be considered in the depletion rate at which the corresponding stage's inventory is consumed while producing. We here assume that the difference in the production rates of any two successive stages is so large that the rate of screening the nonconforming items does not become critical.

Two possible distinct cases are discussed below.

Case I ($P_1 < P_2$)

This case implies that the production of level *2* and level *1*(end items) can start at the same instant and that $t_2 \leq t_1$. Figure 3.2 shows the inventory time plots for this case. If t_1' be the time required to deplete stage 1 inventory by the outside demand and rejection of defective items and $I_{i,max}$ be the maximum inventory level at the ith stage then the average inventory for stage 1 can be found by calculating the area under the triangle. We can see from Figure 3.2 that:

$$\begin{aligned} I_{1,max} &= (P_1 - D)\, t_1 - N_1 \\ I_{1,max} &= \left(1 - \frac{D}{P_1}\right) Q_1 - N_1 \end{aligned} \tag{3.20}$$

$$\begin{aligned} t_1' &= \frac{P_1 t_1 - N_1}{D} \\ t_1' &= \frac{Q_1 - N_1}{D} \\ t_1' &= \frac{Q}{D} \end{aligned} \tag{3.21}$$

Therefore, the average inventory at stage 1 can be given as:

$$\begin{aligned} \bar{I}_1 &= \frac{1}{2} \frac{t_1' I_{1,max}}{Q/D} \\ \bar{I}_1 &= \frac{1}{2} \left\{ \left(1 - \frac{D}{P_1}\right) Q_1 - N_1 \right\} \end{aligned}$$

$$\bar{I}_1 = \frac{1}{2}\left\{\left(1-\frac{D}{P_1}\right)Q_1 - \frac{\alpha_1 Q_1^2}{2P_1\theta_1}\right\} \tag{3.22}$$

This last equation is obtained by substituting for N_1 from Equation (3.17). Another way of expressing $\bar{I}_1$ is by realizing that $N_1 = Q_1 - Q$:

$$\begin{aligned}\bar{I}_1 &= \frac{1}{2}\left\{\left(1-\frac{D}{P_1}\right)Q_1 - (Q_1 - Q)\right\} \\ \bar{I}_1 &= \frac{1}{2}\left\{Q - \frac{DQ_1}{P_1}\right\} \end{aligned} \tag{3.23}$$

Similarly, for level 3:

$$\begin{aligned} I_{3,max} &= P_2 t_2 \\ I_{3,max} &= P_2 \frac{Q_2}{P_2} \\ I_{3,max} &= Q_2 \end{aligned} \tag{3.24}$$

$$\begin{aligned} t_3 &= t_2 \\ t_3 &= \frac{Q_2}{P_2} \end{aligned} \tag{3.25}$$

Therefore, the average inventory at level 3 can expressed as:

$$\begin{aligned}\bar{I}_3 &= \frac{1}{2}\frac{t_3 I_{3,max}}{Q/D} \\ \bar{I}_3 &= \frac{DQ_2^2}{2QP_2} \end{aligned} \tag{3.26}$$

Similarly for stage 2:

$$I_{2,max} = (P_2 - P_1)\,t_2 - N_2$$

Therefore, the average inventory at stage 2 can be calculated as:

$$\bar{I}_2 = \frac{1}{2}\frac{t_1 I_{2,max}}{Q/D}$$

$$\begin{aligned}
\bar{I}_2 &= \frac{DQ_1}{2QP_1}\left\{(P_2 - P_1)\frac{Q_2}{P_2} - N_2\right\} \\
\bar{I}_2 &= \frac{DQ_1}{2QP_1}\left\{(P_2 - P_1)\frac{Q_2}{P_2} - \frac{\alpha_2 Q_2^2}{2P_2\theta_2}\right\} \\
\bar{I}_2 &= \frac{DQ_1Q_2}{2QP_1P_2}\left\{P_2 - P_1 - \frac{\alpha_2 Q_2^2}{2\theta_2}\right\} \qquad (3.27)
\end{aligned}$$

This last equation is obtained by substituting for N_2 from Equation (3.17). Another way of expressing $\bar{I}_2$ is by realizing that $N_2 = Q_2 - Q1$:

$$\begin{aligned}
\bar{I}_2 &= \frac{DQ_1}{2QP_1}\left\{\left(1 - \frac{P_1}{P_2}\right)Q_2 - (Q_2 - Q_1)\right\} \\
\bar{I}_2 &= \frac{DQ_1}{2QP_1}\left\{Q_1 - \frac{P_1}{P_2}Q_2\right\} \\
\bar{I}_2 &= \frac{DQ_1}{2Q}\left\{\frac{Q_1}{P_1} - \frac{Q_2}{P_2}\right\} \qquad (3.28)
\end{aligned}$$

Case II ($P_1 > P_2$)

This case implies that it is necessary for the production of level *2* items to begin some time (say t_2' periods) prior to starting the production of the level 1 items. This is shown in the inventory time plot of Figure 3.3.

It can be seen that $t_2 \geq t_1$ and $t_2' = t_2 - t_1$. Since $t_1 = Q_1/P_1$ and $t_2 = Q_2/P_2$, the values of $\bar{I}_1$ and $\bar{I}_3$ are algebraically the same as in the case I, but for level *2*:

$$\begin{aligned}
I_{2,max} &= P_2(t_2 - t_1) - N_{2,t_2'} \\
I_{2,max} &= P_2(t_2 - t_1) - \frac{\alpha_2 P_2 (t_2 - t_1)^2}{2\theta_2}
\end{aligned}$$

Therefore,

$$\bar{I}_2 = \frac{1}{2}\frac{t_2 I_{2,max}}{Q/D}$$

$$\begin{aligned}
\bar{I}_2 &= \frac{DQ_2}{2P_2Q}\left\{P_2\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)-\frac{\alpha_2 P_2}{2\theta_2}\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)^2\right\} \\
\bar{I}_2 &= \frac{DQ_2}{2Q}\left\{\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)-\frac{\alpha_2}{2\theta_2}\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)^2\right\} \\
\bar{I}_2 &= \frac{DQ_2}{2Q}\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)\left\{1-\frac{\alpha_2}{2\theta_2}\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)\right\} \qquad (3.29)
\end{aligned}$$

General Case

Comparing the average inventories from both cases, the results obtained in the *2*-stage can be generalized for a general n-stage case:

$$\bar{I}_1 = \frac{1}{2}\left\{Q-\frac{DQ_1}{P_1}\right\} \qquad (3.30)$$

$$\bar{I}_{n+1} = \frac{DQ_n^2}{2QP_n} \qquad (3.31)$$

Comparing Equation (3.28) and Equation (3.29) the average inventory of stage j $\forall\, j = 2, 3, \cdots, n$ is given by:

$$\bar{I}_j = \begin{cases} \frac{DQ_{j-1}}{2Q}\left\{\frac{Q_{j-1}}{P_{j-1}}-\frac{Q_j}{P_j}\right\} & \text{if } P_{j-1} < P_j \\ \\ \frac{DQ_j}{2Q}\left(\frac{Q_j}{P_j}-\frac{Q_{j-1}}{P_{j-1}}\right)\left\{1-\frac{\alpha_j}{2\theta_j}\left(\frac{Q_j}{P_j}-\frac{Q_{j-1}}{P_{j-1}}\right)\right\} & \text{if } P_{j-1} > P_j \end{cases} \qquad (3.32)$$

Using Equations (3.23), (3.26) and (3.32); the total inventory holding cost per unit time is given by:

$$HC = r\sum_{j=1}^{n+1} C_j \bar{I}_j \qquad (3.33)$$

Quality Cost

Using Equation (3.17), the expected quality cost (of producing defective items) is given by:

$$QC = \frac{D}{2Q}\sum_{j=1}^{n}\frac{s_j\alpha_j {Q_j}^2}{2\theta_j P_j} \qquad (3.34)$$

Total Cost

The Total Cost per unit time can be found by adding up all the costs from Equations (3.19), (3.33) and (3.34):

$$ETC(Q) = \frac{D}{Q}\sum_{j=1}^{n+1} A_j + r\sum_{j=1}^{n+1} C_j\bar{I}_j + \frac{D}{2Q}\sum_{j=1}^{n} \frac{s_j\alpha_j {Q_j}^2}{2\theta_j P_j} \tag{3.35}$$

Decision Variable

The decision variable in this case is the optimal lot size at the nth stage (Q_n).

3.4.3 Numerical Example

Numerical example is presented in Table 3.1 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hooke and Jeeves optimizatiom algorithm [7] is used to find solutions to the proposed model.

3.5 Inspection Errors in Screening of Nonconforming Items Between Stages

In the last model we have assumed that the screening inspections are done by a perfect inspector. In this section we relax this assumption. Two types of errors can be committed in the screening inspections:

1. Rejecting a conforming item as a non-conforming one, at the jth stage, with a probability $E_{1,j}$. This is generally known as Type I error (sometimes referred to as *producer's risk*).

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
n	3	C_1	\$10/unit
D	10,000 units/year	C_2	\$5/unit
P_1	50,000 units/year	C_3	\$2/unit
P_2	40,000 units/year	C_4	\$1/unit
P_3	100,000 units/year	r	0.25
A_1	\$100/setup	α_j	0.05
A_2	\$35/setup	θ_j	0.05
A_3	\$20/setup	s_j	C_j
A_4	\$15/setup		
Optimal Solution			
Variable	Value	Variable	Value
Q	759 units	ETC	\$4092.3
Q_1	764 units	SC	\$2042.2
Q_2	771 units	HC	\$1337.3
Q_3	773 units	QC	\$712.9

Table 3.1: Solved Example 3.1

2. Accepting a non-conforming item as a conforming one, at the jth stage, with a probability $E_{2,j}$. This is generally known as Type II error (sometimes referred to as *consumer's risk*).

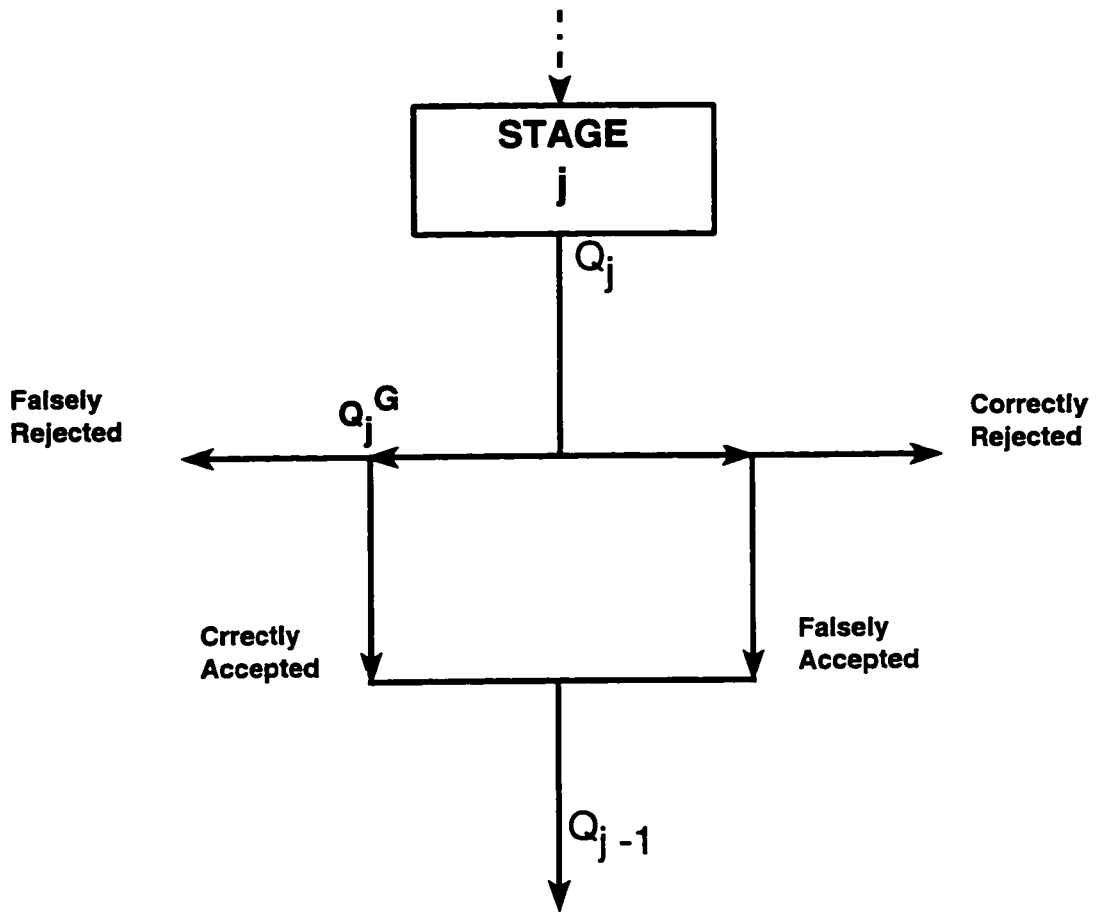


Figure 3.4: Errors in Product Inspections

From the Figure3.4 it can be seen that:

$$
\begin{aligned}
Q_{j-1} &= (Q_j - N_j)(1 - E_{1,j}) + N_j E_{2,j} \\
Q_{j-1} &= Q_j(1 - E_{1,j}) - (1 - E_{1,j} - E_{2,j})N_j
\end{aligned}
\tag{3.36}
$$

$$Q_{j-1} = Q_j(1 - E_{1,j}) - \frac{(1 - E_{1,j} - E_{2,j})\alpha_j Q_j^2}{2P_j\theta_j} \tag{3.37}$$

3.5.1 Costs Involved

Following are the costs involved in determining the total cost:

1. Setup costs.
2. Inventory holding costs.
3. Expected quality costs.

Setup Costs

The total Setup Cost per unit time for an n-stage system is:

$$SC = \frac{D}{Q}\sum_{j=1}^{n+1} A_j \tag{3.38}$$

Inventory Carrying Costs

If $E_{2,j}$ is very small then an item which is incorrectly accepted at the jth stage and passed to the $(j-1)^{th}$ stage will be detected after the processing at the $(j-1)^{th}$ stage with a probability very close to 1. So, it would be reasonable to consider that an incorrectly accepted non-conforming item will be subject to unnecessary processing at only and exactly one stage. This would greatly simplify the procedure of, and reduce the computational efforts for, determining I_j's without any loss of accuracy. Here, we also assume that an incorrectly accepted non-conforming item cannot become a conforming one while being processed at the proceeding stage. Two possible distinct cases considered for calculating average inventories are discussed below.

Case I ($P_1 < P_2$)

This case implies that the production of level *2* and level *1*(end items) can start at the same instant and that $t_2 \leq t_1$. Figure 3.2 shows the inventory time plots for this case. If t_1' be the time required to deplete stage 1 inventory by the outside demand and rejection of defective items and $I_{i,max}$ be the maximum inventory level at the ith stage then th average inventory for stage 1 can be found by calculating the area under the triangle. We can see from Figure 3.2 that:

$$\begin{aligned} I_{1,max} &= (P_1 - D)t_1 - \{N_1(1 - E_{2,1}) + (Q_1 - N_1)E_{1,1}\} \\ I_{1,max} &= \left(1 - \frac{D}{P_1}\right) Q_1 - \{N_1(1 - E_{2,1}) + (Q_1 - N_1)E_{1,1}\} \end{aligned}$$

It can be seen from Figure 3.4 that:

$$N_j(1 - E_{2,j}) + (Q_j - N_j)E_{1,j} = Q_j - Q_{j-1}$$

Therefore,

$$\begin{aligned} I_{1,max} &= \left(1 - \frac{D}{P_1}\right) Q_1 - (Q_1 - Q) \\ I_{1,max} &= \left(Q - \frac{DQ_1}{P_1}\right) \end{aligned} \tag{3.39}$$

And, the time required to deplete the inventory at stage 1 by outside demand and screening of items deemed non-conforming upon inspection is given by:

$$\begin{aligned} t_1' &= \frac{P_1 t_1 - \{N_1(1 - E_{2,1}) + (Q_1 - N_1)E_{1,1}\}}{D} \\ t_1' &= \frac{Q_1 - (Q_1 - Q)}{D} \\ t_1' &= \frac{Q}{D} \end{aligned} \tag{3.40}$$

Therefore, the average inventory at stage 1 will remain same as in Equation (3.23) and given as:

$$\bar{I}_1 = \frac{1}{2}\left\{Q - \frac{DQ_1}{P_1}\right\} \tag{3.41}$$

Similarly for stage 2:

$$\begin{aligned} I_{2,max} &= (P_2 - P_1)\,t_2 - \{N_2(1 - E_{2,2}) + (Q_2 - N_2)E_{1,2}\} \\ I_{2,max} &= (P_2 - P_1)\,t_2 - (Q_2 - Q_1) \end{aligned}$$

Therefore, the average inventory at stage 1 will remain same as in Equation (3.28) and given as:

$$\bar{I}_2 = \frac{DQ_1}{2Q}\left\{\frac{Q_1}{P_1} - \frac{Q_2}{P_2}\right\} \tag{3.42}$$

Case II($P_1 > P_2$)

This case implies that it is necessary for the production of level *2* items to begin some time (say t_2' periods) prior to starting the production of the level 1 items. This is shown in the inventory time plot of Figure 3.3. It can be seen that $t_2 \geq t_1$ and $t_2' = t_2 - t_1$. Since $t_1 = Q_1/P_1$ and $t_2 = Q_2/P_2$, the values of $\bar{I}_1$ and $\bar{I}_3$ are algebraically the same as in the case I, but for level *2* it is different. If ζ_2 is the number of nonconforming items produced at stage 2 during the interval t_2' then:

$$I_{2,max} = P_2 t_2' - \{\zeta_2(1 - E_{2,2}) + (P_2 t_2' - \zeta_2)E_{1,2}\} \tag{3.43}$$

$$\begin{aligned} I_{2,max} &= P_2 t_2' - \{(1 - E_{1,2} - E_{2,2})\zeta_2 + P_2 t_2' E_{1,2}\} \\ I_{2,max} &= P_2\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right) - \left\{(1 - E_{1,2} - E_{2,2})\frac{\alpha_2 P_2}{2\theta_2}\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right)^2 + P_2\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right)E_{1,2}\right\} \\ I_{2,max} &= P_2\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right) - \left\{1 - E_{1,2} - (1 - E_{1,2} - E_{2,2})\frac{\alpha_2}{2\theta_2}\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right)\right\} \end{aligned} \tag{3.44}$$

$$\bar{I}_2 = \frac{1}{2}\frac{t_2 I_{2,max}}{Q/D}$$
$$\bar{I}_2 = \frac{DQ_2}{2Q}\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right)\left\{1 - E_{1,2} - (1 - E_{1.2} - E_{2,2})\frac{\alpha_2}{2\theta_2}\left(\frac{Q_2}{P_2} - \frac{Q_1}{P_1}\right)\right\} \tag{3.45}$$

Which reduces to the Equation (3.29) for the case of perfect product inspections.

General Case

Comparing the average inventories from both cases, the results obtained in the *2*-stage can be generalized for a general n-stage case:

$$\bar{I}_1 = \frac{1}{2}\left\{Q - \frac{DQ_1}{P_1}\right\} \tag{3.46}$$
$$\bar{I}_{n+1} = \frac{DQ_n^2}{2QP_n} \tag{3.47}$$

Comparing Equations (3.42) and (3.45) the average inventory of stage j $\forall$ $j = 2, 3, \cdots, n$ is given by:

$$\bar{I}_j = \begin{cases} \frac{DQ_{j-1}}{2Q}\left\{\frac{Q_{j-1}}{P_{j-1}} - \frac{Q_j}{P_j}\right\} & \text{if } P_{j-1} < P_j \\ \\ \frac{DQ_j}{2Q}\left(\frac{Q_j}{P_j} - \frac{Q_{j-1}}{P_{j-1}}\right)\left\{1 - E_{1,j} - (1 - E_{1,j} - E_{2,j})\frac{\alpha_j}{2\theta_j}\left(\frac{Q_j}{P_j} - \frac{Q_{j-1}}{P_{j-1}}\right)\right\} & \text{if } P_{j-1} > P_j \end{cases} \tag{3.48}$$

Therefore, the total expected inventory holding cost is given by:

$$HC = \sum_{j=1}^{n+1} rC_i\bar{I}_j \tag{3.49}$$

Quality Costs

The Equation (3.34) gives the quality costs under the assumption of perfect inspections. In the presence of inspection errors this equation is modified (in accordance

to Figure 3.4) to the following:

$$
\begin{aligned}
QC &= \frac{D}{Q}\sum_{j=1}^{n}\{\pi_{1,j}(Q_j - N_j)E_{1,j} + \pi_{2,j}N_jE_{2,j} + s_jN_j(1-E_{2,j})\} \\
QC &= \frac{D}{Q}\sum_{j=1}^{n}\left\{\pi_{1,j}E_{1,j}\left(Q_j - \frac{\alpha_jQ_j^2}{2P_j\theta_j}\right) + \frac{\pi_{2,j}\alpha_jQ_j^2E_{2,j}}{2P_j\theta_j}\right. \\
&\quad \left. + \frac{s_j\alpha_j{Q_j}^2(1-E_{2,j})}{2P_j\theta_j}\right\} \qquad (3.50)
\end{aligned}
$$

Total Cost

Using Equations (3.19), (3.46), (3.47), (3.48), and (3.50); the total cost per unit time can be written as:

$$
\begin{aligned}
ETC(Q) &= \frac{D}{Q}\sum_{j=1}^{n+1}A_j + r\sum_{j=1}^{n+1}C_j\bar{I}_j \\
&\quad + \frac{D}{Q}\sum_{j=1}^{n}\left\{\pi_{1,j}E_{1,j}\left(Q_j - \frac{\alpha_jQ_j^2}{2P_j\theta_j}\right) + \frac{\pi_{2,j}\alpha_jQ_j^2E_{2,j}}{2P_j\theta_j} + \frac{s_j\alpha_j{Q_j}^2(1-E_{2,j})}{2P_j\theta_j}\right\} \qquad (3.51)
\end{aligned}
$$

Decision Variable

The decision variable in this case is the optimal lot size at the nth stage (Q_n).

3.5.2 Numerical Example

Numerical example is presented in Table 3.2 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hooke and Jeeves optimizatiom algorithm [7] is used to find solutions to the proposed model.

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
n	3	C_1	\$10/unit
D	10,000 units/year	C_2	\$5/unit
P_1	50,000 units/year	C_3	\$2/unit
P_2	40,000 units/year	C_4	\$1/unit
P_3	100,000 units/year	$\pi_{1,j}$	$0.5C_j$
A_1	\$100/setup	$\pi_{2,1}$	$1.5C_1$
A_2	\$35/setup	$\pi_{2,2}$	C_1
A_3	\$20/setup	$\pi_{2,3}$	C_2
A_4	\$15/setup	s_j	C_j
r	0.25/setup	$E_{1,j}$	0.01
α_j	0.05	$E_{2,j}$	0.01
θ_j	0.05		
Optimal Solution			
Variable	Value	Variable	Value
Q	736 units	ETC	\$5001.7
Q_1	748 units	SC	\$2106.0
Q_2	762 units	HC	\$2208.2
Q_3	772 units	QC	\$687.7

Table 3.2: Solved Example 3.2

3.6 Maintenance Inspections and Process Restorations

In this section we model and explore the effect of incorporating maintenance-inspections of the processes and their restoration from out-of-control states to in-control states. If an out-of control signal is detected then the process restoration procedures are undertaken. We assume that each maintenance-inspection will result in reduction of the age of the process to zero and restoration costs are dependent on detection delays. Also, maintenance-inspections and process restorations are considered instantaneous and does not cause disruption of the production process. This means that the Figure 3.2 and Figure 3.3 are to be considered in calculating the expected inventories.

3.6.1 Costs Involved

Following are the costs involved in determining the total cost:

1. Setup Cost
2. Expected Inventory Holding Cost
3. Expected Quality Cost
4. Maintenance Inspection Cost
5. Expected Process Restoration Cost

Setup Costs

Total Setup Cost per unit time for an n-stage system is:

$$\begin{aligned} SC &= \sum_{j=1}^{n+1} A_j/(Q/D) \\ SC &= \frac{D}{Q}\sum_{j=1}^{n+1} A_j \end{aligned} \tag{3.52}$$

Expected Quality Cost

If η_j is the number of maintenance inspections in an equal inspection interval maintenance inspection plan, then the length of each inspection interval is given by:

$$\begin{aligned} t_{i,j} &= \frac{t_j}{\eta_j} \\ t_{i,j} &= \frac{Q_j}{P_j\eta_j} \end{aligned} \tag{3.53}$$

And, the expected number of non-conforming items per production cycle is given by:

$$\begin{aligned} N_{i,j} &= \frac{\alpha_j P_j}{2\theta_j} t_{i,j}^2 \\ N_j &= \sum_{i=1}^{\eta_j} \frac{\alpha_j P_j}{2\theta_j} t_{i,j}^2 \\ N_j &= \eta_j \frac{\alpha_j P_j}{2\theta_j} \frac{Q_j^2}{P_j^2 \eta_j^2} \\ N_j &= \frac{\alpha_j Q_j^2}{2P_j\theta_j\eta_j} \end{aligned} \tag{3.54}$$

And, the expected average number of defective items per unit time can be found as:

$$\bar{N}_j = \frac{D\alpha_j Q_j^2}{2QP_j\theta_j\eta_j} \tag{3.55}$$

The Equation (3.34) gives the quality costs under the assumption of perfect inspections. In the presence of inspection errors this equation is modified (in accordance

to Figure 3.4) to the following:

$$
\begin{aligned}
QC &= \frac{D}{Q}\sum_{j=1}^{n}\{\pi_{1,j}(Q_j - N_j)E_{1,j} + \pi_{2,j}N_jE_{2,j} + s_jN_j(1-E_{2,j})\} \\
QC &= \frac{D}{Q}\sum_{j=1}^{n}\left\{\pi_{1,j}E_{1,j}\left(Q_j - \frac{\alpha_jQ_j^2}{2P_j\theta_j\eta_j}\right) + \frac{\pi_{2,j}\alpha_jQ_j^2E_{2,j}}{2P_j\theta_j\eta_j}\right. \\
&\quad \left. + \frac{s_j\alpha_jQ_j{}^2(1-E_{2,j})}{2P_j\theta_j\eta_j}\right\} \qquad (3.56)
\end{aligned}
$$

Expected Inventory Holding Cost

It is evident that the expressions for $\bar{I}_1$, $\bar{I}_n$ will remain same as those calculated for in the previous section and given by Equations (3.46) and (3.47) respectively. The expression for $\bar{I}_2$ will also not change for case I and III. But, the expression for $\bar{I}_2$ in case II will be different from the previous expression.

Case II ($P_1 > P_2$)

If the number of nonconforming items produced at stage 2 during the interval t_2', represented by ζ_2, is given by:

$$
\zeta_2 = \sum_{k=1}^{g_2-1}\frac{\alpha_2P_2}{2\theta_2}\left(\frac{t_2}{\eta_2}\right)^2 + \frac{\alpha_2P_2}{2\theta_2}\left\{t_2' - \frac{t_2}{\eta_2}(g_2-1)\right\}^2
$$

Where, g_2 is the smallest integer such that $\frac{t_2}{\eta_2}g_2 \geq t_2'$.

$$
\begin{aligned}
\zeta_2 &= \frac{\alpha_2P_2}{2\theta_2}\frac{Q_2^2}{P_2^2\eta_2^2}(g_2-1) + \frac{\alpha_2P_2}{2\theta_2}\left\{\frac{Q_2}{P_2} - \frac{Q_1}{P_1} - \frac{Q_2}{P_2\eta_2}(g_2-1)\right\}^2 \\
\zeta_2 &= \frac{\alpha_2P_2}{2\theta_2}\left\{\left(\frac{Q_2}{P_2\eta_2}\right)^2(g_2-1) + \left\{\frac{Q_2}{P_2} - \frac{Q_1}{P_1} - \frac{Q_2}{P_2\eta_2}(g_2-1)\right\}^2\right\} \qquad (3.57)
\end{aligned}
$$

$$
\begin{aligned}
I_{2,max} &= P_2t_2' - \{\zeta_2(1-E_{2,2}) + (P_2t_2' - \zeta_2)E_{1,2}\} \\
I_{2,max} &= P_2t_2' - \{(1-E_{1,2}-E_{2,2})\zeta_2 + P_2t_2'E_{1,2}\}
\end{aligned}
$$

$$
\begin{aligned}
I_{2,max} &= P_2\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)-(1-E_{1,2}-E_{2,2})\frac{\alpha_2 P_2}{2\theta_2}\left\{\left(\frac{Q_2}{P_2\eta_2}\right)^2(g_2-1)\right. \\
&\quad \left.+\left\{\frac{Q_2}{P_2}-\frac{Q_1}{P_1}-\frac{Q_2}{P_2\eta_2}(g_2-1)\right\}^2\right\}-P_2\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)E_{1,2} \qquad (3.58)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\bar{I}_2 &= \frac{1}{2}\frac{t_2 I_{2,max}}{Q/D} \\
\bar{I}_2 &= \frac{DQ_2}{2Q}\left[\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)-(1-E_{1,2}-E_{2,2})\frac{\alpha_2}{2\theta_2}\left\{\left(\frac{Q_2}{P_2\eta_2}\right)^2(g_2-1)\right.\right. \\
&\quad \left.\left.+\left\{\frac{Q_2}{P_2}-\frac{Q_1}{P_1}-\frac{Q_2}{P_2\eta_2}(g_2-1)\right\}^2\right\}-\left(\frac{Q_2}{P_2}-\frac{Q_1}{P_1}\right)E_{1,2}\right] \qquad (3.59)
\end{aligned}
$$

General Case

Comparing the average inventories from both cases, the results obtained in the *2*-stage case be generalized for a general n-stage case:

$$
\bar{I}_1 = \frac{1}{2}\left\{Q-\frac{DQ_1}{P_1}\right\} \qquad (3.60)
$$

$$
\bar{I}_{n+1} = \frac{DQ_n^2}{2QP_n} \qquad (3.61)
$$

Comparing Equations (3.46) and (3.59) the average inventory of stage j for $j = 2, 3, \cdots, n$ is given by:

$$
\bar{I}_j = \begin{cases}
\frac{DQ_{j-1}}{2Q}\left\{\frac{Q_{j-1}}{P_{j-1}}-\frac{Q_j}{P_j}\right\} & \text{if } P_{j-1} < P_j \\
\frac{DQ_j}{2Q}\left[\left(\frac{Q_j}{P_j}-\frac{Q_{j-1}}{P_{j-1}}\right)-(1-E_{1,j}-E_{2,j})\frac{\alpha_j}{2\theta_j}\left\{\left(\frac{Q_j}{P_j\eta_j}\right)^2(g_j-1)\right.\right. & \\
\left.\left.+\left\{\frac{Q_j}{P_j}-\frac{Q_{j-1}}{P_{j-1}}-\frac{Q_j}{P_j\eta_j}(g_j-1)\right\}^2\right\}-\left(\frac{Q_j}{P_j}-\frac{Q_{j-1}}{P_{j-1}}\right)E_{1,j}\right] & \text{if } P_{j-1} > P_j
\end{cases}
$$

(3.62)

Where, g_j is the smallest integer such that $\frac{t_j}{\eta_j} g_j > t'_j$. Using Equations (3.60), (3.61) and (3.62); the total inventory holding cost per unit time is given by:

$$HC = r \sum_{j=1}^{n+1} C_j \bar{I}_j \tag{3.63}$$

Maintenance Inspection Cost

If m_j is the cost of a single maintenance inspection and η_j is number of maintenance-inspection intervals at stage 1, then there will a total of $(\eta_j - 1)$ maintenance-inspections at the jth stage. The total cost of maintenance inspections per unit time is given by:

$$IC = \frac{D}{Q} \sum_{j=1}^{n} (\eta_j - 1)\, m_j \tag{3.64}$$

Expected Process Restoration Cost

It is assumed that the processes always start in an in-control state and no inspection is done at the end of the production run. Also, every maintenance inspection of a process reduces the age of that process to zero. Let $R_j(\tau)$ is the cost of restoring the jth stage process comprising on two components: one R_{jc} constant and the other R_{jv} dependent upon the detection delay τ. Hence, the cost of restoring the jth stage process can be expressed as:

$$R_j(\tau) = R_{jc} + R_{jv}\tau$$

The expected restoration cost for the jth stage during the ith inspection interval is given by:

$$RC_{i,j} = \int_0^{t_{i,j}} R_j(\tau) f_j(t) dt$$

$$
\begin{aligned}
RC_{i,j} &= \int_0^{t_{i,j}} \{R_{jc} + R_{jv}\tau\} f_j(t)dt \\
RC_{i,j} &= \int_0^{t_{i,j}} \{R_{jc} + R_{jv}(t_{i,j} - t)\} \frac{1}{\theta_j} e^{\frac{-t}{\theta_j}} dt
\end{aligned} \tag{3.65}
$$

Knowing that:

$$
\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) \tag{3.66}
$$

Few simple manipulations would reduce the Equation (3.65) to:

$$
\begin{aligned}
RC_{i,j} &= \left[R_{jc} + R_{jv}\left\{\theta_j + t_{i,j}\left(1 + \theta_j^2\right)\right\}\right] e^{\frac{-t_{i,j}}{\theta_j}} - \left[R_{jc} + R_{jv}\left\{t_{i,j} + \theta_j\right\}\right] \\
RC_{i,j} &= \left[R_{jc} + R_{jv}\left\{\theta_j + \frac{Q_j}{P_j\eta_j}\left(1 + \theta_j^2\right)\right\}\right] e^{\frac{-Q_j}{P_j\eta_j\theta_j}} \\
&\quad - \left[R_{jc} + R_{jv}\left\{\frac{Q_j}{P_j\eta_j} + \theta_j\right\}\right]
\end{aligned} \tag{3.67}
$$

Therefore, the total expected restoration cost per unit time is given as:

$$
TRC = \frac{D}{Q}\sum_{j=1}^{n}\sum_{i=1}^{\eta_j} RC_{i,j} \tag{3.68}
$$

Expected Total Cost

The Total Cost per unit time can be found by adding up all the costs from Equations (3.52), (3.63), (3.56), (3.64) and (3.68):

$$
ETC(Q) = SC + HC + QC + IC + TRC \tag{3.69}
$$

Decision Variables

The decision variables in this case are the optimal lot size at the nth stage (Q_n) and number of maintenance-inspection intervals (η_j) at the each production stage.

3.6.2 Solution Procedure and Numerical Example

Numerical example is presented in Table 3.3 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hooke and Jeeves optimizatiom algorithm [7] is used to find solutions to the proposed model.

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
n	3	C_1	\$10/unit
D	10,000 units/year	C_2	\$5/unit
P_1	50,000 units/year	C_3	\$2/unit
P_2	40,000 units/year	C_4	\$1/unit
P_3	100,000 units/year	$\pi_{1,j}$	$0.5C_j$
A_1	\$100/setup	$\pi_{2,1}$	$1.5C_1$
A_2	\$35/setup	$\pi_{2,2}$	C_1
A_3	\$20/setup	$\pi_{2,3}$	C_2
A_4	\$15 /setup	s_j	C_j
r	0.25	m_j	$0.05A_j$
α_j	0.10	R_{cj}	$0.15A_j$
θ_j	0.05	R_{vj}	$0.01A_j$
$E_{1,j}$	0.01		
$E_{2,j}$	0.01		
Optimal Solution			
Variable	Value	Variable	Value
Q	1346 units	ETC	\$5227.6
Q_1	1366 units	SC	\$1263.0
Q_2	1387 units	HC	\$1259.4
Q_3	1407 units	QC	\$1820.9
η_1	5	IC	\$228.5
η_2	6	TRC	\$655.8
η_3	3		

Table 3.3: Solved Example 3.3

Design and Analysis of Experiment

A fractional factorial experiment 2^{12-7} with resolution IV is also designed and analyzed to realize the sensitivity of various response variables against various system parameters. The Table 3.6 summarizes the effects considered in the experiment. The values for other parameters are same as in Table 3.4. ANOVA Tables 3.7-3.16 are included to summarize the results. We have neglected interactions of order 3 or more. Since it is a Resolution IV design, no main effect is confounded with any other main effect or two factor interaction. Whereas, two factor interactions are confounded with other two factor interactions. Hence, conclusions for main effects can be deduced directly; while, further experiments would be required for before concluding something about two factor interactions. Each ANOVA Table is followed by a plot of main effects and Pareto chart for the corresponding response variable. The most significant factors can be identified directly from these plots. The analysis can be summarized as follows:

- The optimal lot size (Q) is most sensitive to the demand rate (D), the setup cost at stage 1 (A_1), and the production rate at stage 3 (P_3).

- The expected total setup cost per unit time (SC) is most sensitive to the demand rate (D), the setup cost at stage 1 (A_1), and the production rate at stage 2 (P_2).

- The expected total quality cost per unit time (QC) is most sensitive to the demand rate (D), the fraction of defective items produced while the process is in an out-of-control state (α_j), the exponential distribution parameter (θ_j), the setup cost at stage 1 (A_1) and stage 2 (A_2), the constant term of the

restoration cost (R_{cj}), and the preventive maintenance cost (m_j).

- The expected total inspection cost per unit time (IC) is most sensitive to the demand rate (D), the fraction of defective items produced while the process is in an out-of-control state (α_j), the exponential distribution parameter (θ_j), the setup cost at stage 1 (A_1) and stage 3 (A_3), the constant term of the restoration cost (R_{cj}), the preventive maintenance cost (m_j), and the production rate at stage 1 (P_1).

- The expected total restoration cost per unit time (TRC) is most sensitive to the demand rate (D), the fraction of defective items produced while the process is in an out-of-control state (α_j), the exponential distribution parameter (θ_j), the production rate at stage 1 (P_1), the setup cost at stage 1 (A_1), and the preventive maintenance cost (m_j).

- The expected total holding cost per unit time (HC) is most sensitive to the demand rate (D), the setup costs at each stage (A_j), the production rate at stage 1 (P_1), and the preventive maintenance cost (m_j).

- The expected total cost per unit time (TC) is most sensitive to the demand rate (D), the fraction of defective items produced while the process is in an out-of-control state (α_j), the setup costs at each stage (A_j), the exponential distribution parameter (θ_j), the constant term of the restoration cost (R_{cj}), and the preventive maintenance cost (m_j). The production rates at various stages seem to have negligible effect on the expected total cost per unit time.

- The number of maintenance-inspection intervals at stage 1 (η_1) is most sensitive to the exponential distribution parameter (θ_j), the fraction of defective

items produced while the process is in an out-of-control state (α_j), the demand rate (D), the production rate at stage 1 (P_1), and the constant term of the restoration cost (R_{cj}).

- The number of maintenance-inspection intervals at stage 2 (η_2) is most sensitive to the exponential distribution parameter (θ_j), the fraction of defective items produced while the process is in an out-of-control state (α_j), the demand rate (D), the production rate at stage 2 (P_2), the setup cost at stage 1 (A_1), and the constant term of the restoration cost (R_{cj}).

- The number of maintenance-inspection intervals at stage 3 (η_3) is most sensitive to the exponential distribution parameter (θ_j), the fraction of defective items produced while the process is in an out-of-control state (α_j), the demand rate (D), the production rate at stage 3 (P_3), and the constant term of the restoration cost (R_{cj}).

3.7 Conclusion

The results for the various models discussed in this chapter are summarized in Table 3.4 and 3.5. The values used for this example are same as given in Table 3.3. The possibility of the shift of the process to an out-of-control state and presence of defective items result in larger lot sizes at higher stages. The lot sizes and expected total cost are sensitive to the proportion of defective items produced at different levels. Errors in product inspections also result in increased costs. The total expected cost is also sensitive to changes in Type I and Type II errors.

Perfect Production Process Q = 1254 ETC(Q) = 2711					
Imperfect Production Process Without Errors in Product Inspections $\theta_j = 0.05; \quad j = 1,2,3$					
α	Q	Q_1	Q_2	Q_3	$ETC(Q)$
0.05	759	764	771	773	4092.3
0.10	584	590	598	601	5282.5
0.25	391	398	408	412	7901.5
0.40	308	315	325	329	9891.4
Imperfect Production Process With Errors in Product Inspections $\theta_j = 0.05, \alpha_j = 0.1, E_{1,j} = 0.01; \quad j = 1,2,3$					
$E_{2,j}$	Q	Q_1	Q_2	Q_3	$ETC(Q)$
0.005	576	588	602	608	6163.7
0.010	577	589	603	612	6224.7
0.050	562	573	586	620	6728.5
Imperfect Production Process With Errors in Product Inspections $\theta_j = 0.05, \alpha_j = 0.1, E_{2,j} = 0.01; \quad j = 1,2,3$					
$E_{1,j}$	Q	Q_1	Q_2	Q_3	$ETC(Q)$
0.025	559	579	602	611	7497.1
0.050	553	588	628	638	9746.4
0.100	516	579	653	664	14755.9

Table 3.4: Effect of α and inspection errors.

Imperfect Production Process With Maintenance Inspections and Restorations $\theta_j = 0.05, \alpha_j = 0.1, E_{1,j} = 0.01, E_{2,j} = 0.01, m_j = 0.05A_j, R_{cj} = 0.15A_j; \quad j = 1,2,3$					
R_{vj}	Q	η_1	η_2	η_3	$ETC(Q)$
$0.01A_j$	1346	5	6	3	5227.6
$0.1A_j$	1438	5	7	4	5246.9
$0.25A_j$	1401	5	6	3	5283.0
Imperfect Production Process With Maintenance Inspections and Restorations $\theta_j = 0.05, \alpha_j = 0.1, E_{1,j} = 0.01, E_{2,j} = 0.01, m_j = 0.05A_j, R_{vj} = 0.01A_j; \quad j = 1,2,3$					
R_{cj}	Q	η_1	η_2	η_3	$ETC(Q)$
$0.15A_j$	1346	5	6	3	5227.6
$0.25A_j$	1429	4	6	3	5598.8
$0.35A_j$	1501	4	5	3	5913.6
Imperfect Production Process With Maintenance Inspections and Restorations $\theta_j = 0.05, \alpha_j = 0.1, E_{1,j} = 0.01, E_{2,j} = 0.01, R_{cj} = 0.15A_j, R_{vj} = 0.01A_j; \quad j = 1,2,3$					
m_j	Q	η_1	η_2	η_3	$ETC(Q)$
$0.05A_j$	1346	5	6	3	5227.6
$0.1A_j$	1351	5	6	3	5427.6
$0.15A_j$	1390	4	5	3	5608.6

Table 3.5: Effect of maintenance-inspection and process restorations.

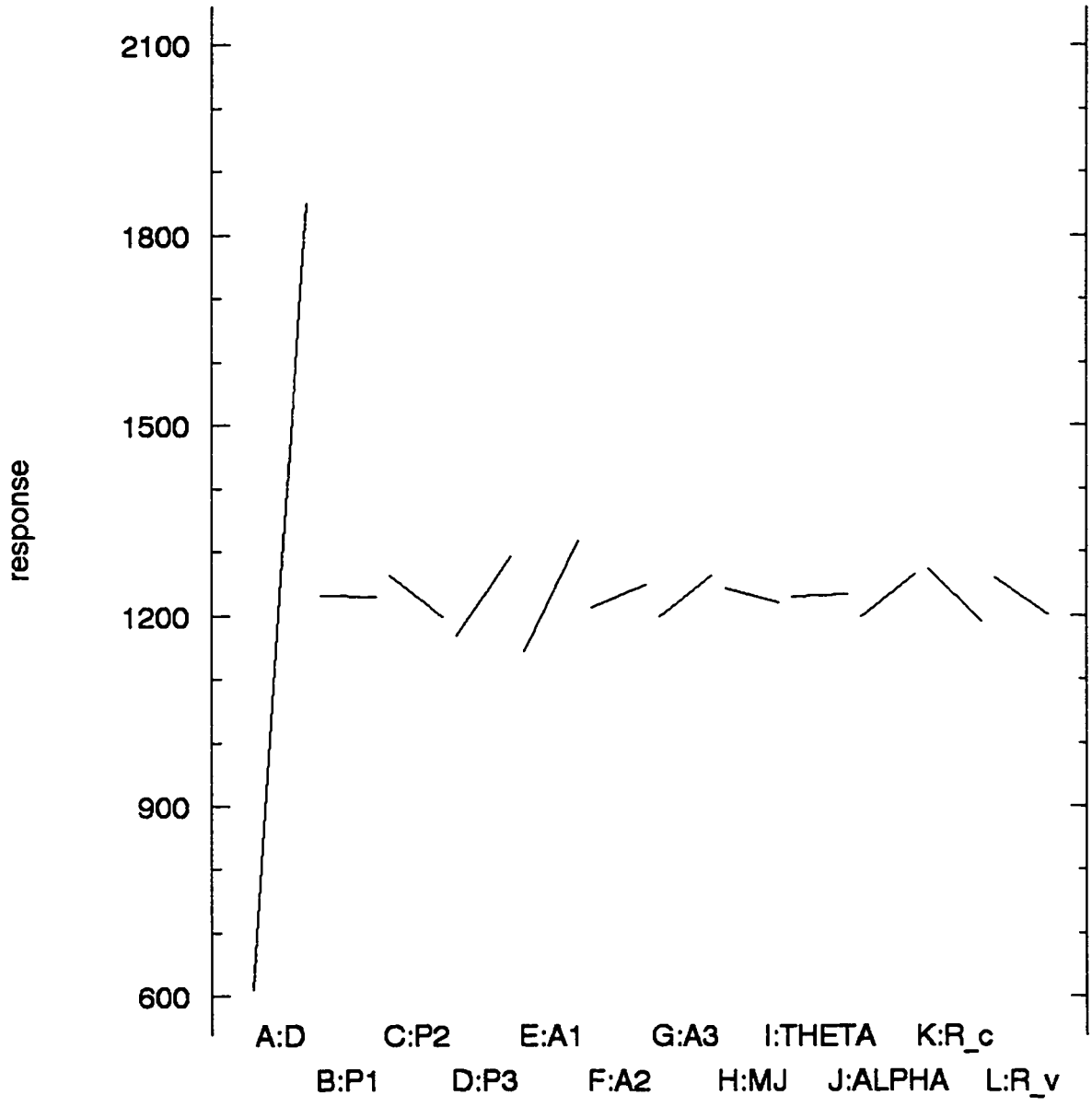
Name	Description	Low	High
A	Outside Demand (D) in *units/unit time*	10000	15000
B	Production rate at stage 1 (P_1) *units/unit time*	50000	75000
C	Production rate at stage 2 (P_2) *units/unit time*	40000	60000
D	Production rate at stage 3 (P_3) *units/unit time*	100000	140000
E	Setup cost for stage 1 (A_1) $/*setup*	100	150
F	Setup cost for stage 2 (A_2) $/*setup*	35	50
G	Setup cost for stage 3 (A_3)$/*setup*	20	30
H	Cost of mt. inspection as a fraction of setup cost (m_j)	0.05	0.07
I	Exponential distribution parameter	0.05	0.1
J	Fraction of defectives produced in an out-of-control state (α_j)	0.05	0.1
K	Constant cost of restoration as a fraction of setup cost ($R_{c,j}$)	0.15	0.25
L	Variable cost of restoration as a fraction of setup cost ($R_{v,j}$)	0.01	0.05

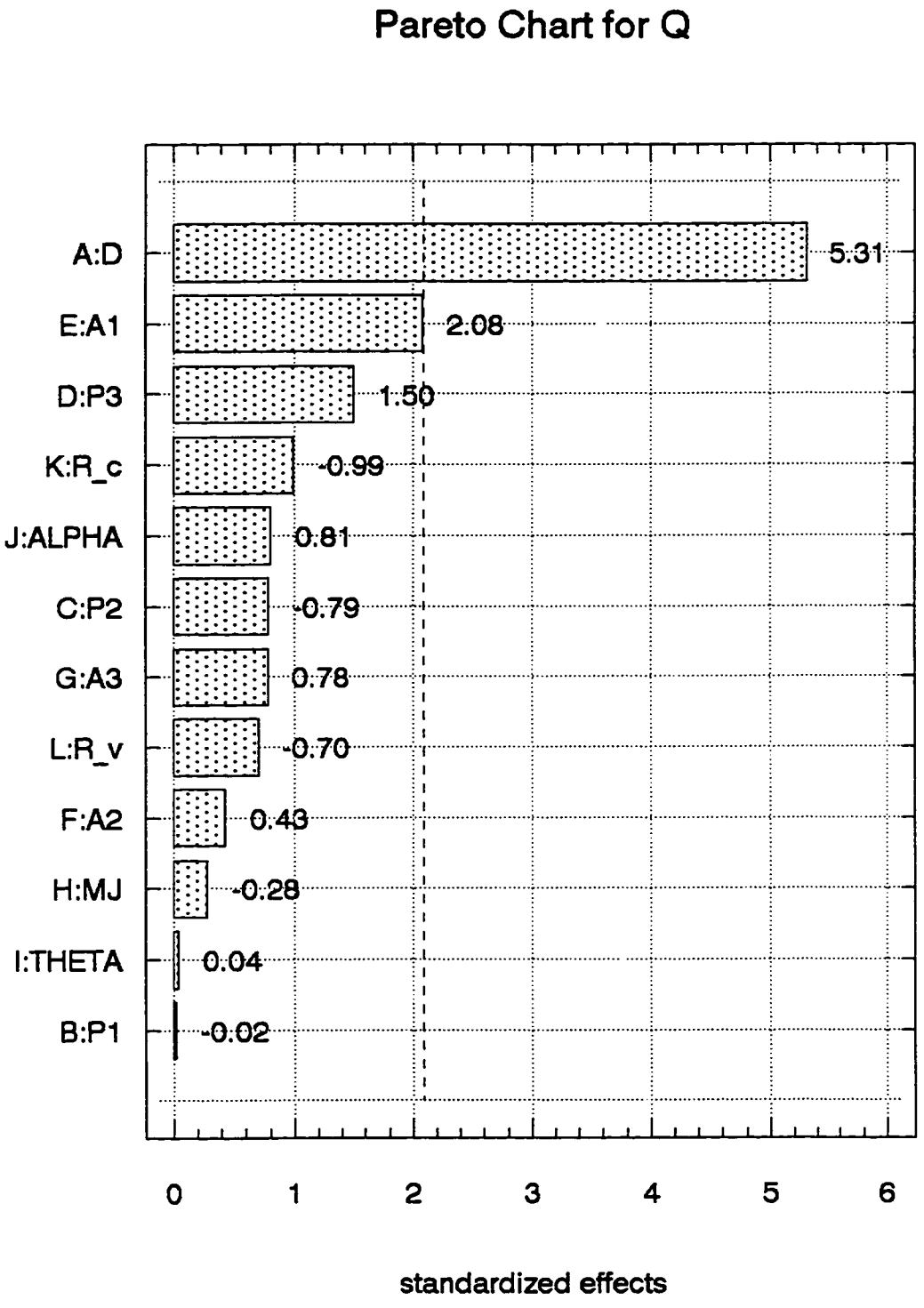
Table 3.6: Effects considered in the experimental design

Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	1568663.28	1	1568663.3	26.90
B	30.03	1	30.0	0.00
C	34518.78	1	34518.8	0.59
D	124625.28	1	124625.3	2.14
E	241686.28	1	241686.3	4.15
F	10260.28	1	10260.3	0.18
G	33865.03	1	33865.0	0.58
H	4301.28	1	4301.3	0.07
I	101.53	1	101.5	0.00
J	36247.78	1	36247.8	0.62
K	55029.03	1	55029.0	0.94
L	27319.53	1	27319.5	0.47
AB + CK + DL + FH + GI	45375.78	1	45375.8	0.78
AC + BK + EH + GJ	26507.53	1	26507.5	0.45
AD + BL + EI + FJ	230690.28	1	230690.3	3.96
AE + CH + DI + FK + GL	65250.78	1	65250.8	1.12
AF + BH + DJ + EK	2128.78	1	2128.8	0.04
AG + BI + CJ + EL	16698.78	1	16698.8	0.29
AH + BF + CE + JL	4584.03	1	4584.0	0.08
AI + BG + DE + JK	6526.53	1	6526.5	0.11
AJ + CG + DF + HL + IK	6873.78	1	6873.8	0.12
AK + BC + EF + IJ	88936.53	1	88936.5	1.53
AL + BD + EG + HJ	45225.28	1	45225.3	0.78
BE + CF + DG + HK + IL	30814.03	1	30814.0	0.53
BJ + CI + DH + FL + GK	35979.03	1	35979.0	0.62
CD + FG + HI + KL	123380.28	1	123380.3	2.12
CL + DK + EJ + FI + GH	94721.28	1	94721.3	1.62
Total Error	233221.38	4	58305.3	
Total	3193562.22	31		

Table 3.7: ANOVA Table for Optimal Lot Size (Q)

Plot of Main Effects for Q

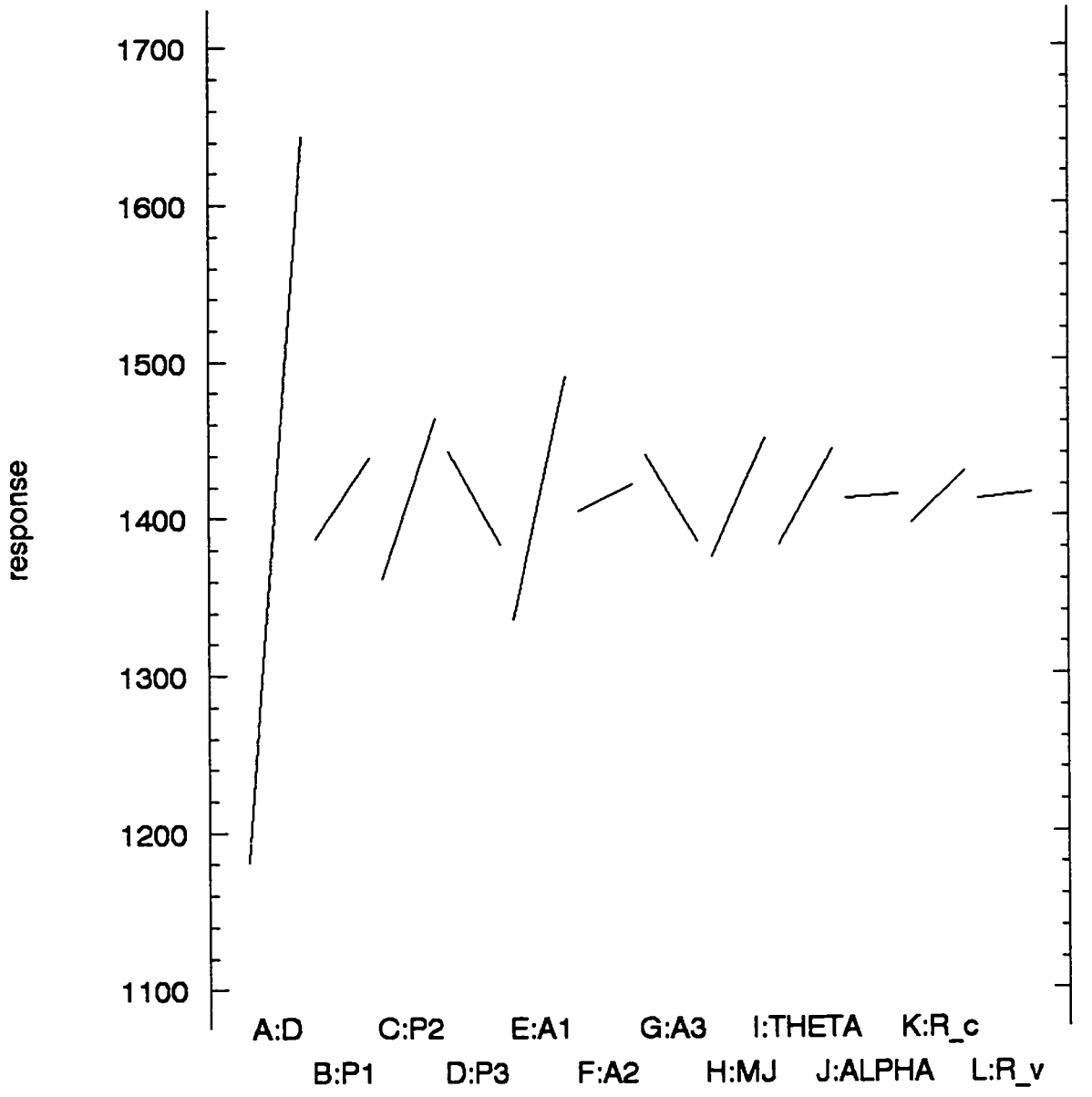


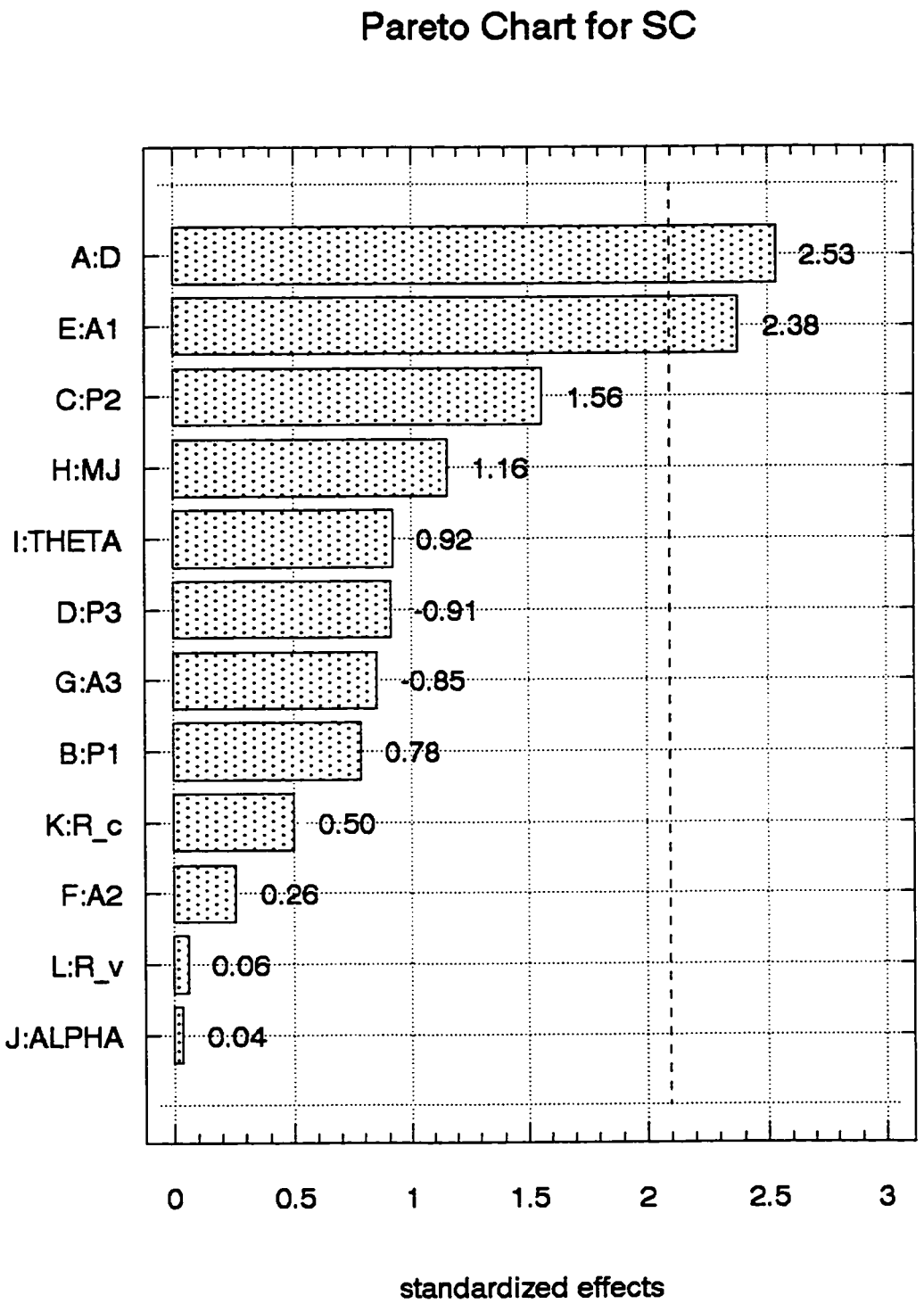

Pareto Chart for Q
A:D
E:A1
D:P3
K:R_c
J:ALPHA
C:P2
G:A3
L:R_v
F:A2
H:MJ
I:THETA
B:P1
5.31
2.08
1.50
-0.99
0.81
-0.79
0.78
-0.70
0.43
-0.28
0.04
-0.02
0
1
2
3
4
5
6
standardized effects

Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	217925.42	1	217925.42	13.48
B	20922.40	1	20922.40	1.29
C	82686.18	1	82686.18	5.12
D	82686.18	1	82686.18	1.74
E	28165.14	1	28165.14	11.90
F	2248.52	1	2248.52	0.14
G	24628.68	1	24628.68	1.52
H	45508.43	1	45508.43	2.82
I	28965.84	1	28965.84 S	1.79
J	45.98	1	45.98	0.00
K	8592.29	1	8592.29	0.53
L	113.85	1	113.85	0.01
AB + CK + DL + FH + GI	9192.32	1	9192.32	0.57
AC + BK + EH + GJ	4578.29	1	4578.29	0.28
AD + BL + EI + FJ	98874.60	1	98874.60	6.12
AE + CH + DI + FK + GL	98652.38	1	98652.38	6.10
AF + BH + DJ + EK	37837.25	1	37837.25	2.34
AG + BI + CJ + EL	1629.63	1	1629.63	0.10
AH + BF + CE + JL	65645.14	1	65645.14	4.06
AI + BG + DE + JK	15547.90	1	15547.90	0.96
AJ + CG + DF + HL + IK	14695.84	1	14695.84	0.91
AK + BC + EF + IJ	20795.76	1	20795.76	1.29
AL + BD + EG + HJ	2808.00	1	2808.00	0.17
BE + CF + DG + HK + IL	526.18	1	526.18	0.03
BJ + CI + DH + FL + GK	151905.21	1	151905.21	9.40
CD + FG + HI + KL	35601.79	1	35601.79	2.20
CL + DK + EJ + FI + GH	22385.16	1	22385.16	1.39
Total Error	64649.742	4	16162.44	
Total	1297427.14	31		

Table 3.8: ANOVA Table for Total Setup Cost (SC)

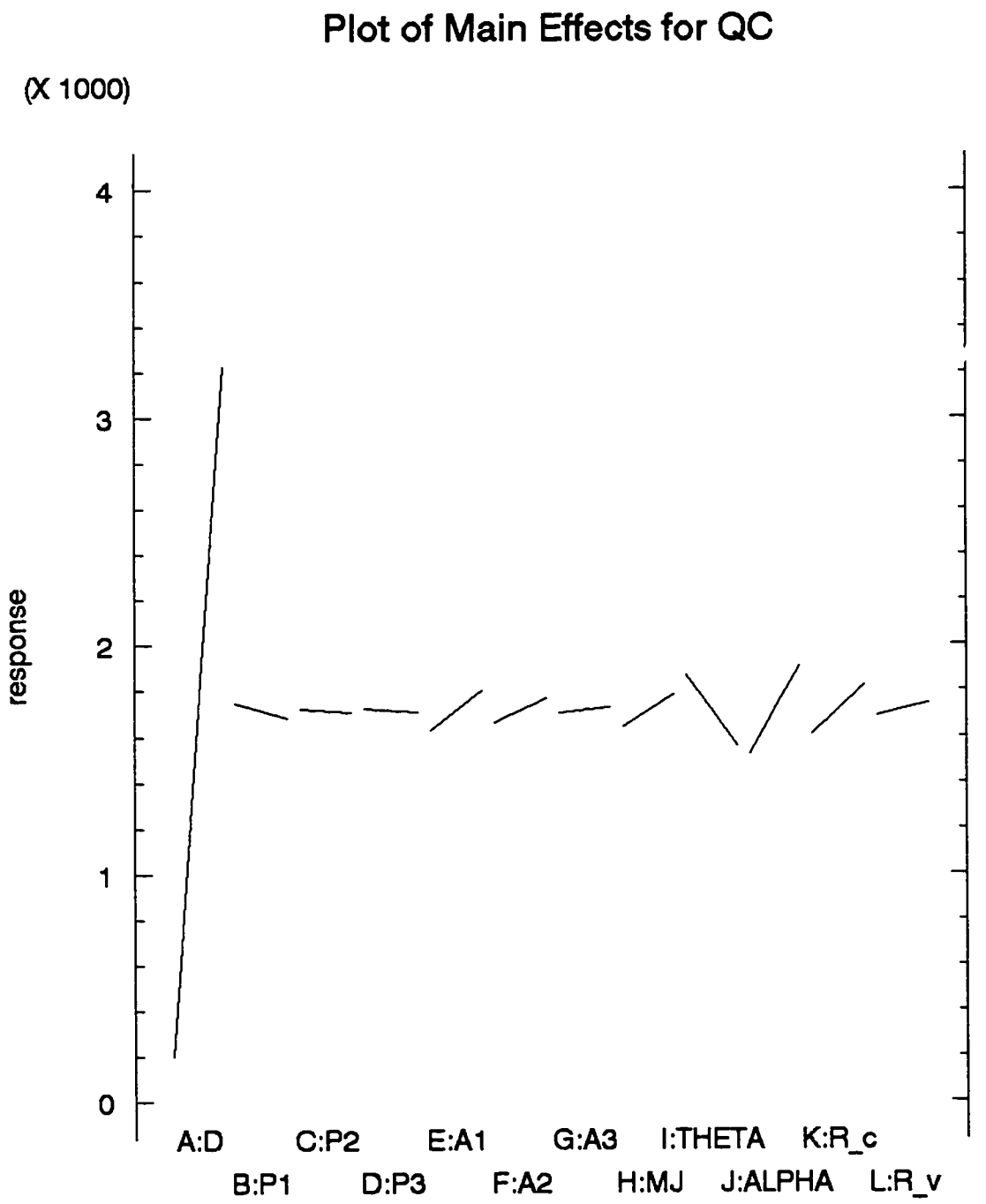
Plot of Main Effects for SC

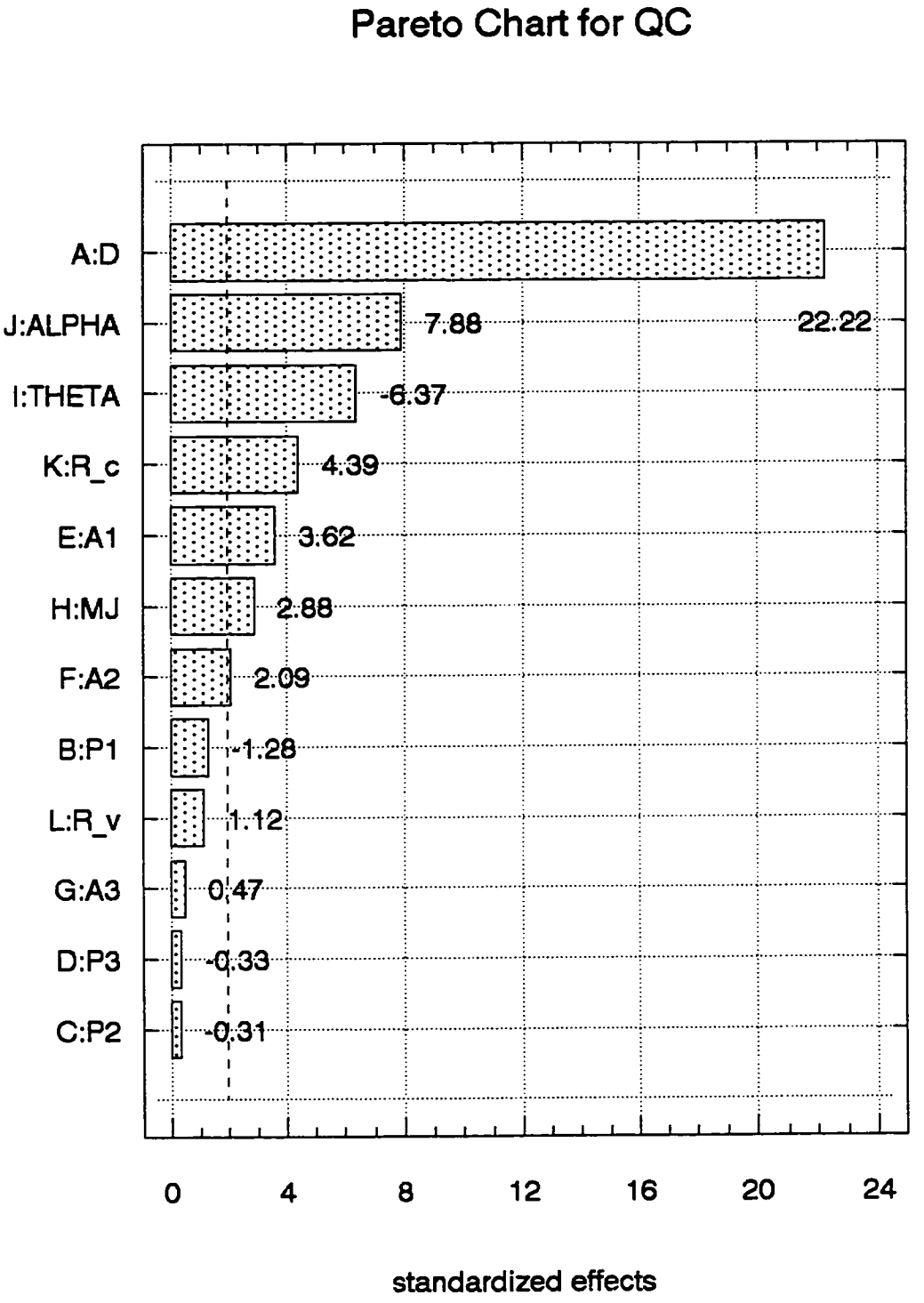



Pareto Chart for SC
A:D
E:A1
C:P2
H:MJ
I:THETA
D:P3
G:A3
B:P1
K:R_c
F:A2
L:R_v
J:ALPHA
2.53
2.38
1.56
1.16
0.92
0.91
0.85
0.78
0.50
0.26
0.06
0.04
0
0.5
1
1.5
2
2.5
3
standardized effects

Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	932.6988.48	1	932.6988.48	870.43
B	31131.36	1	31131.36	2.91
C	1771.61	1	1771.61	0.17
D	2014.54	1	2014.54	0.19
E	248177.74	1	248177.74	23.16
F	82225.26	1	82225.26	7.67
G	4092.86	1	4092.86	0.38
H	156786.00	1	156786.00	14.63
I	766291.05	1	766291.05	71.51
J	1173550.30	1	1173550.30	109.52
K	364679.35	1	364679.35	34.03
L	23539.08	1	23539.08	2.20
AB + CK + DL + FH + GI	9587.66	1	9587.66	0.89
AC + BK + EH + GJ	561.96	1	561.96	0.05
AD + BL + EI + FJ	4467.49	1	4467.49	0.42
AE + CH + DI + FK + GL	24084.64	1	24084.64	2.25
AF + BH + DJ + EK	41608.91	1	41608.91	3.88
AG + BI + CJ + EL	24981.71	1	24981.71	2.33
AH + BF + CE + JL	70697.40	1	70697.40	6.60
AI + BG + DE + JK	6730.90	1	6730.90	0.63
AJ + CG + DF + HL + IK	70687.60	1	70687.60	6.60
AK + BC + EF + IJ	8.10	1	8.10	0.00
AL + BD + EG + HJ	24870.08	1	24870.08	2.32
BE + CF + DG + HK + IL	5834.70	1	5834.70	0.54
BJ + CI + DH + FL + GK	1429.79	1	1429.79	0.13
CD + FG + HI + KL	338.65	1	338.65	0.03
CL + DK + EJ + FI + GH	30128.99	1	30128.99	2.81
Total Error	42861.58	4	10715.4	
Total	SS	31		

Table 3.9: ANOVA Table for Total Quality Cost (QC)

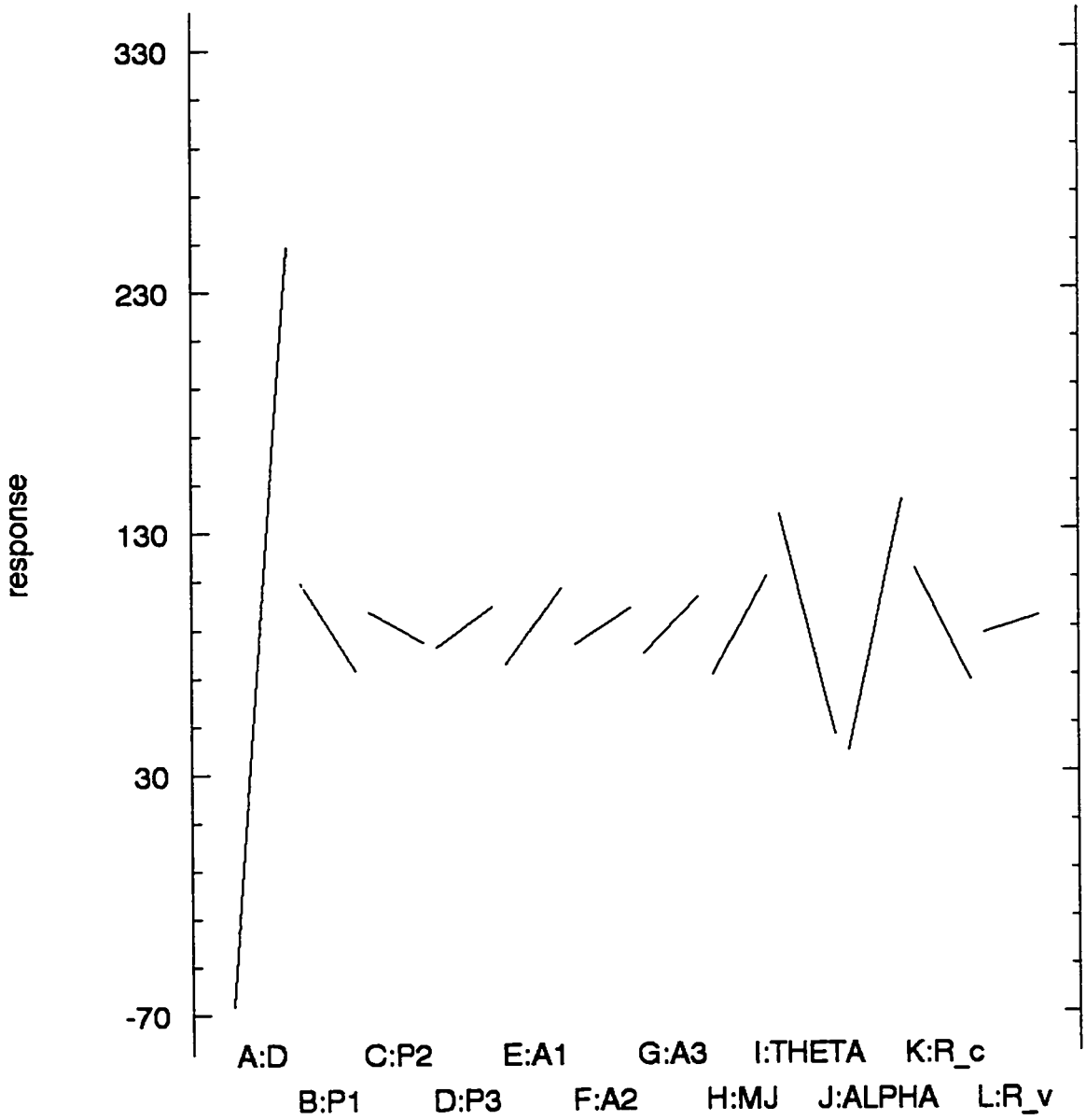

Plot of Main Effects for QC
(X 1000)
response
4
3
2
1
0
A:D
B:P1
C:P2
D:P3
E:A1
F:A2
G:A3
H:MJ
I:THETA
J:ALPHA
K:R_c
L:R_v


Pareto Chart for QC
A:D
J:ALPHA
I:THETA
K:R_c
E:A1
H:MJ
F:A2
B:P1
L:R_v
G:A3
D:P3
C:P2
22.22
7.88
-6.37
4.39
3.62
2.88
2.09
-1.28
1.12
0.47
-0.33
-0.31
0
4
8
12
16
20
24
standardized effects

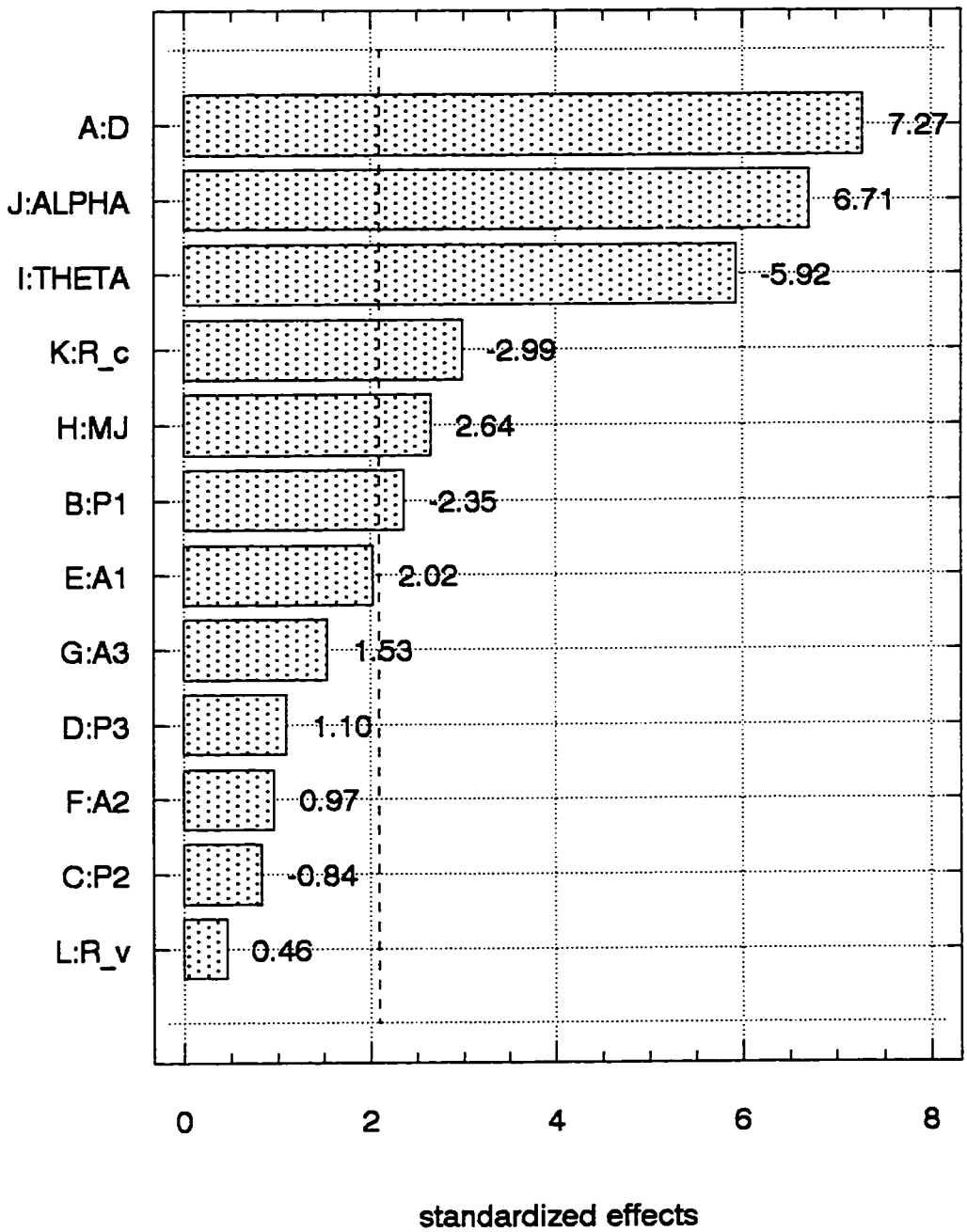
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	101065.13	1	101065.13	123.26
B	10564.91	1	10564.91	12.83
C	1335.36	1	1335.36	1.63
D	2291.04	1	2291.04	2.79
E	7787.40	1	7787.40	9.50
F	1780.31	1	1780.31	2.17
G	4490.77	1	4490.77	5.48
H	13353.21	1	MS	16.29
I	66995.94	1	66995.94	81.71
J	85904.71	1	85904.71	104.77
K	17079.03	1	17079.03	20.83
L	401.04	1	401.04	0.49
AB + CK + DL + FH + GI	1642.56	1	1642.56	2.00
AC + BK + EH + GJ	2710.25	1	2710.25	3.31
AD + BL + EI + FJ	3729.37	1	3729.37	4.55
AE + CH + DI + FK + GL	3962.81	1	3962.81	4.83
AF + BH + DJ + EK	1.51	1	1.51	0.00
AG + BI + CJ + EL	1362.11	1	1362.11	1.66
AH + BF + CE + JL	540.35	1	540.35	0.66
AI + BG + DE + JK	1918.65	1	1918.65	2.34
AJ + CG + DF + HL + IK	6520.36	1	6520.36	7.95
AK + BC + EF + IJ	3296.40	1	3296.40	4.02
AL + BD + EG + HJ	2642.21	1	2642.21	3.22
BE + CF + DG + HK + IL	167.70	1	167.70	0.20
BJ + CI + DH + FL + GK	3364.13	1	3364.13	4.10
CD + FG + HI + KL	150.93	1	150.93	0.18
CL + DK + EJ + FI + GH	993.96	1	993.96	1.21
Total Error	3279.69	4	819.92	
Total	SS	31		

Table 3.10: ANOVA Table for Total Inspection Cost (IC)

Plot of Main Effects for IC



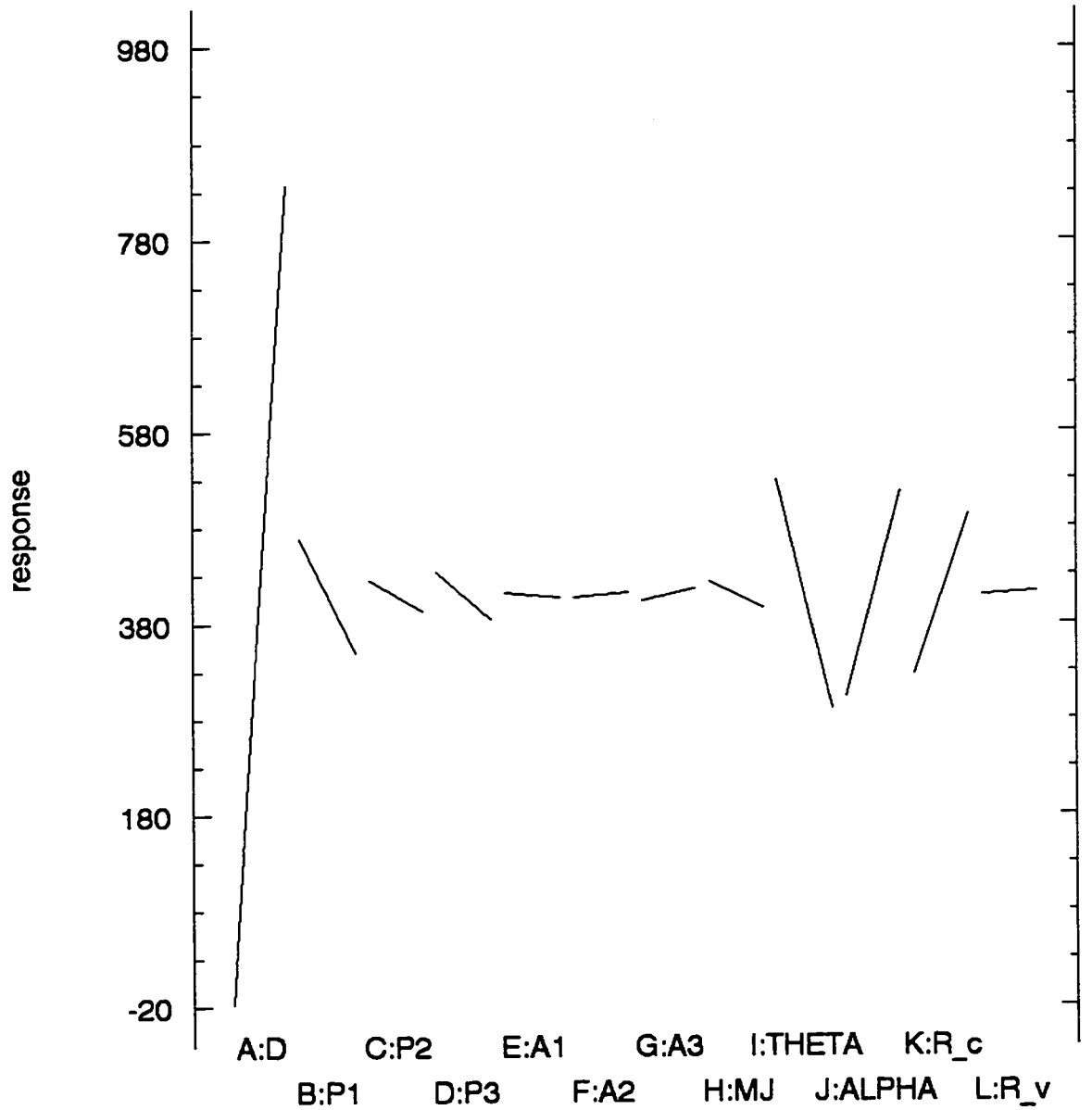
Pareto Chart for IC



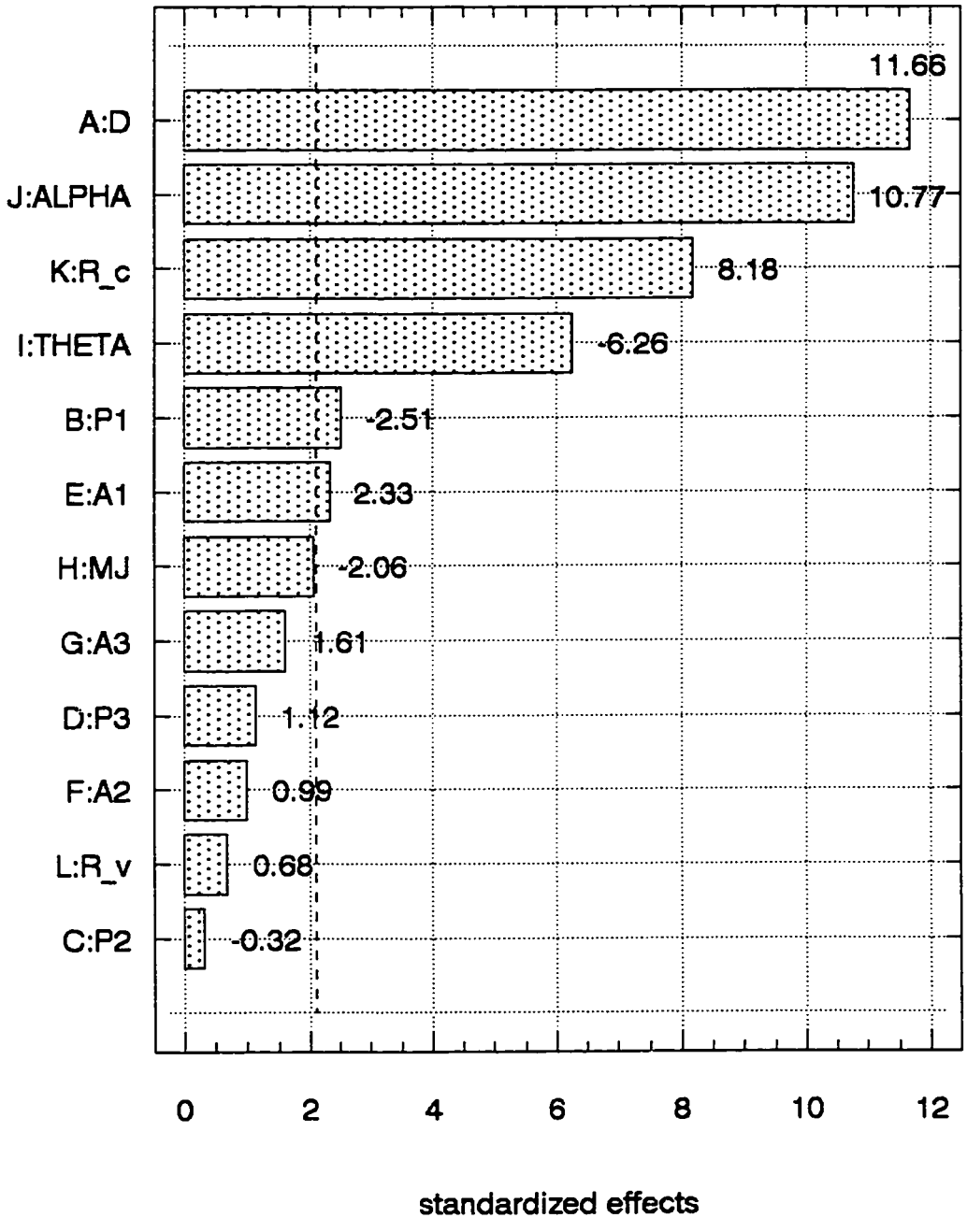
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	744200.00	1	744200.00	185.95
B	34584.50	1	34584.50	8.64
C	561.13	1	561.13	0.14
D	6903.13	1	6903.13	1.72
E	29768.00	1	29768.00	7.44
F	5356.13	1	5356.13	1.34
G	14196.13	1	14196.13	3.55
H	23220.13	1	23220.13	5.80
I	214840.13	1	214840.13	53.68
J	635064.50	1	635064.50	158.68
K	365940.13	1	365940.13	91.43
L	2556.13	1	2556.13	0.64
AB + CK + DL + FH + GI	7021.13	1	7021.13	1.75
AC + BK + EH + GJ	1352.00	1	1352.00	0.34
AD + BL + EI + FJ	15138.00	1	15138.00	3.78
AE + CH + DI + FK + GL	10441.13	1	10441.13	2.61
AF + BH + DJ + EK	1058.00	1	1058.00	0.26
AG + BI + CJ + EL	2312.00	1	2312.00	0.58
AH + BF + CE + JL	1800.00	1	1800.00	0.45
AI + BG + DE + JK	13944.50	1	13944.50	3.48
AJ + CG + DF + HL + IK	11476.13	1	11476.13	2.87
AK + BC + EF + IJ	5202.00	1	5202.00	1.30
AL + BD + EG + HJ	480.50	1	480.50	0.12
BE + CF + DG + HK + IL	21.13	1	21.13	0.01
BJ + CI + DH + FL + GK	3916.13	1	3916.13	0.98
CD + FG + HI + KL	13041.13	1	13041.13	3.26
CL + DK + EJ + FI + GH	820.13	1	820.13	0.20
Total Error	16009.00	4	4002.25	
Total	2181221.88	31		

Table 3.11: ANOVA Table for Total Restoration Cost (TRC)

Plot of Main Effects for RC



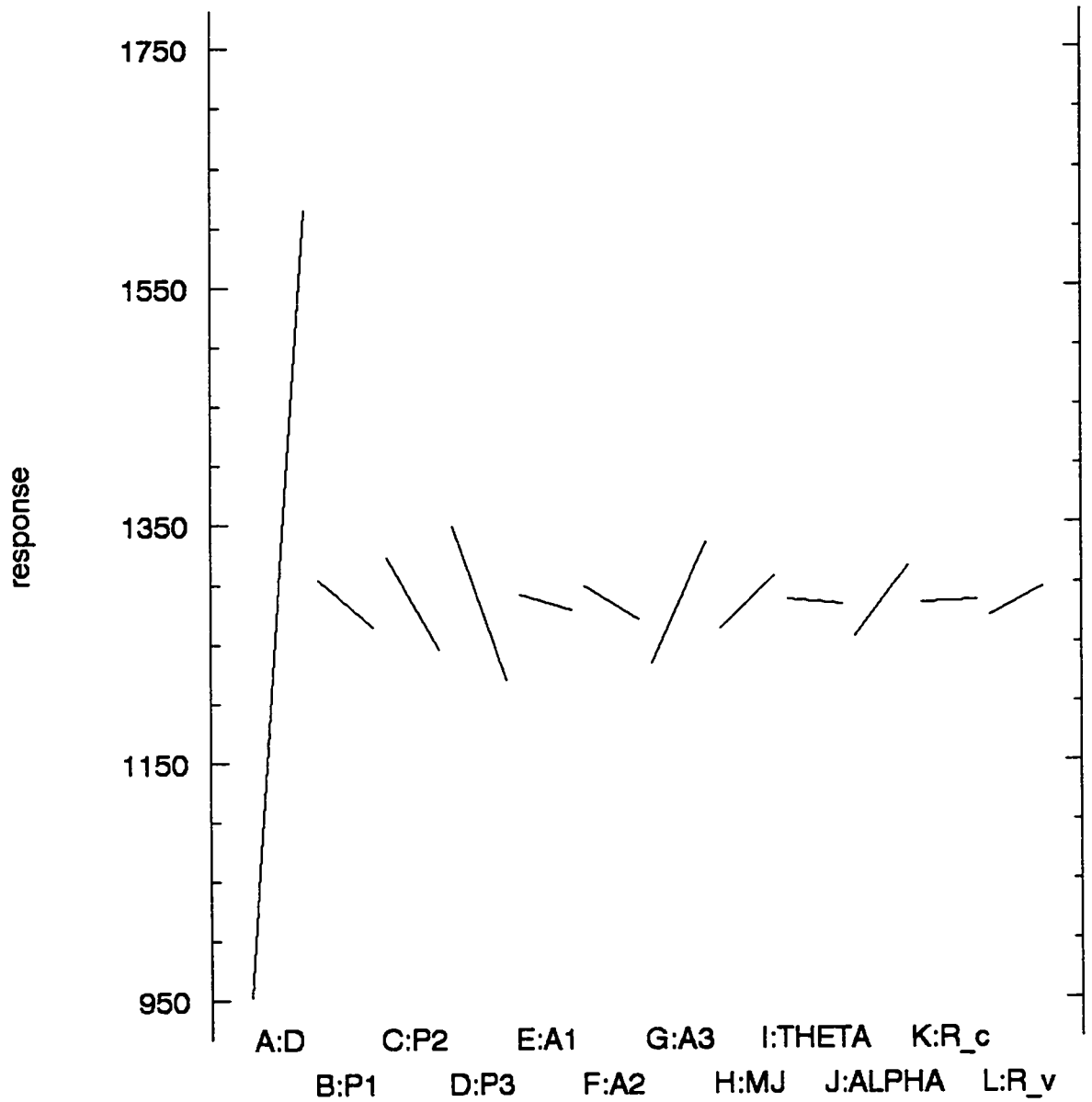
Pareto Chart for RC



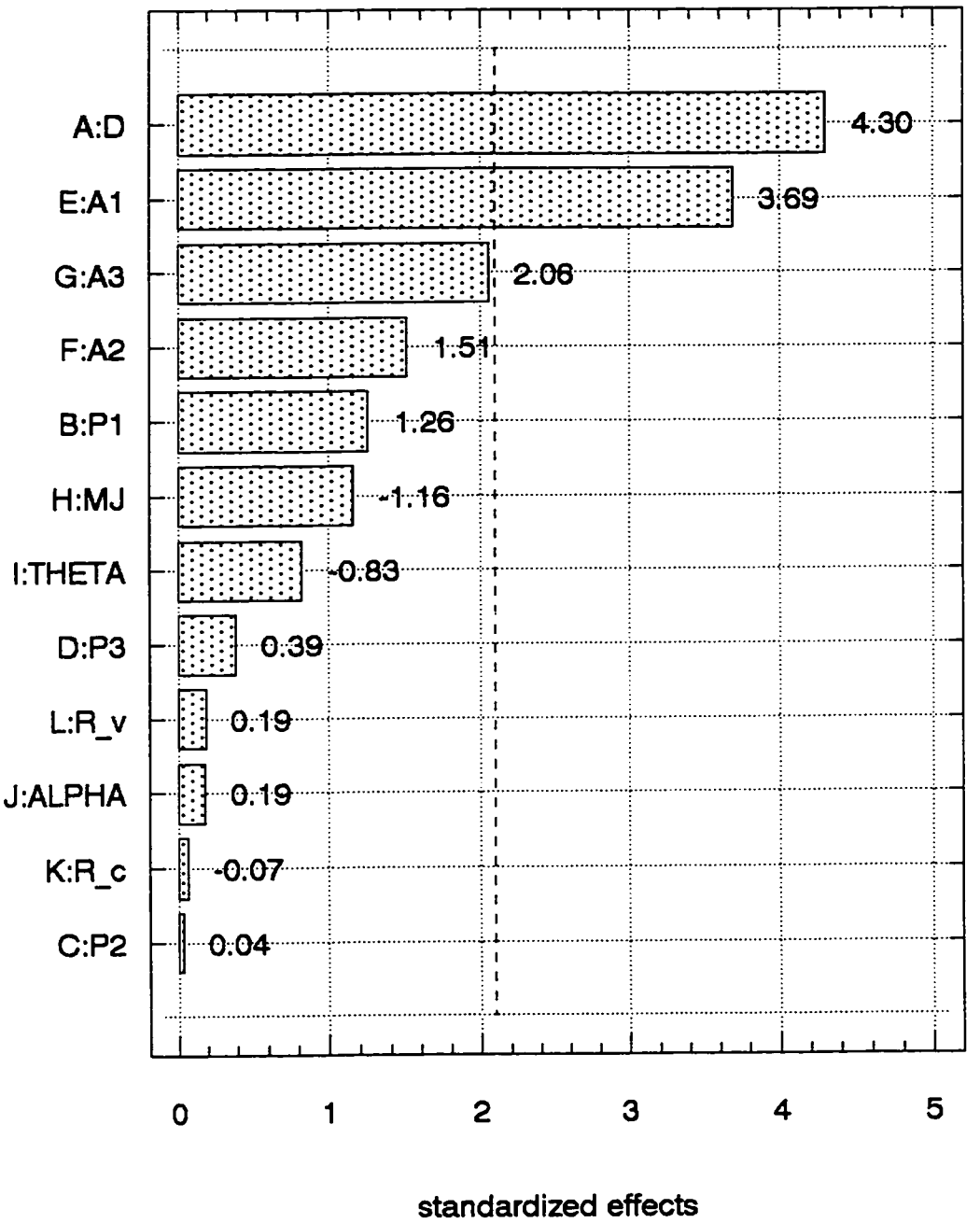
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	446748.78	1	446748.78	54.92
B	38157.03	1	38157.03	4.69
C	34.031	1	34.031	0.00
D	3633.78	1	3633.78	0.45
E	329469.03	1	329469.03	40.50
F	55361.28	1	55361.28	6.81
G	102491.28	1	102491.28	12.60
H	32576.28	1	32576.28	4.00
I	16607.53	1	16607.53	2.04
J	830.28	1	830.28	0.10
K	108.78	1	108.78	0.01
L	850.78	1	850.78	0.10
AB + CK + DL + FH + GI	29100.78	1	29100.78	3.58
AC + BK + EH + GJ	16065.28	1	16065.28	1.98
AD + BL + EI + FJ	56196.28	1	56196.28	6.91
AE + CH + DI + FK + GL	115080.03	1	115080.03	14.15
AF + BH + DJ + EK	30442.78	1	30442.78	3.74
AG + BI + CJ + EL	331.53	1	331.53	0.04
AH + BF + CE + JL	29100.78	1	29100.78	3.58
AI + BG + DE + JK	4347.78	1	4347.78	0.53
AJ + CG + DF + HL + IK	5859.03	1	5859.03	0.72
AK + BC + EF + IJ	69.03	1	69.03	0.01
AL + BD + EG + HJ	413.28	1	413.28	0.05
BE + CF + DG + HK + IL	1785.03	1	1785.03	0.22
BJ + CI + DH + FL + GK	108927.78	1	108927.78	13.39
CD + FG + HI + KL	17813.28	1	17813.28	2.19
CL + DK + EJ + FI + GH	11438.28	1	11438.28	1.14
Total Error	32536.63	4	8134.16	
Total	1486376.47	31		

Table 3.12: ANOVA Table for Total Holding Cost (HC)

Plot of Main Effects for HC



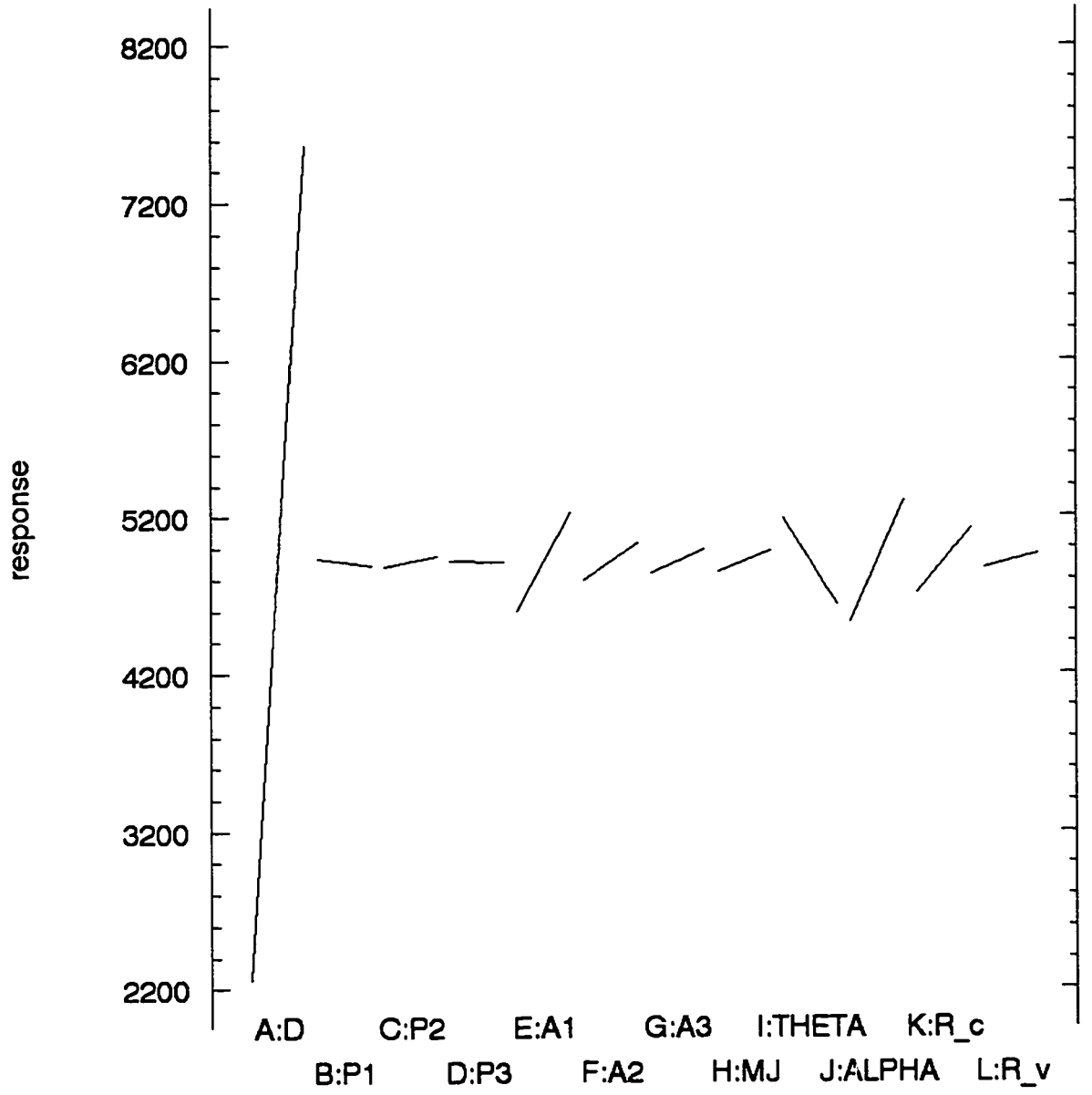
Pareto Chart for HC



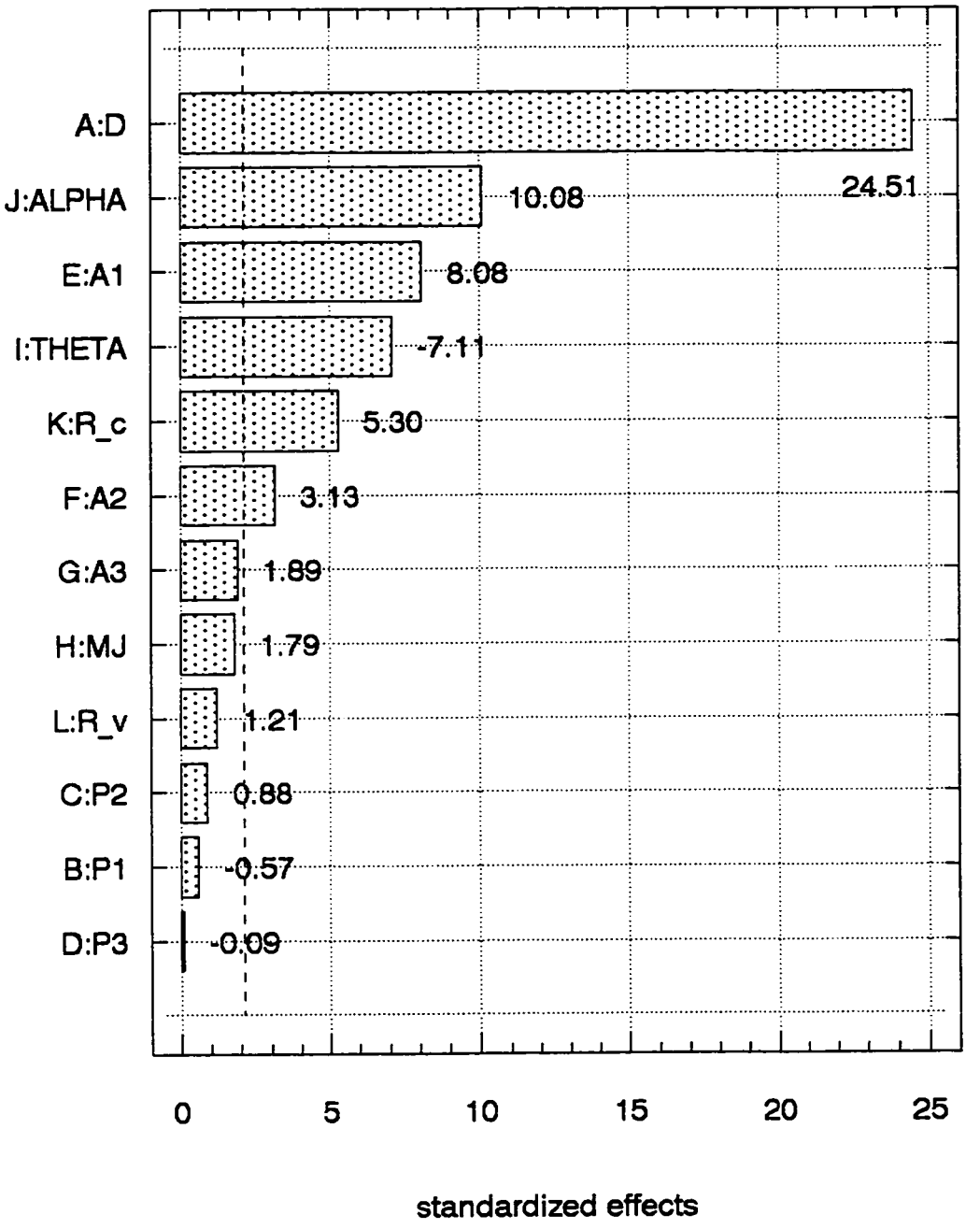
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	28836316.5	1	28836316.5 S	978.45
B	15797.5	1	15797.5	0.54
C	36788.3	1	36788.3	1.25
D	427.8	1	427.8	0.01
E	3138138.8	1	3138138.8	106.48
F	469722.8	1	469722.8	15.94
G	170966.3	1	170966.3	5.80
H	153596.5	1	153596.5	5.21
I	2424952.5	1	2424952.5	82.28
J	4878907.0	1	4878907.0	165.55
K	1348492.5	1	1348492.5	45.76
L	69844.5	1	69844.5	2.37
AB + CK + DL + FH + GI	10117.5	1	10117.5	0.34
AC + BK + EH + GJ	29342.5	1	29342.5	1.00
AD + BL + EI + FJ	1554.0	1	1554.0	0.05
AE + CH + DI + FK + GL	119194.0	1	119194.0	4.04
AF + BH + DJ + EK	46588.8	1	46588.8	1.58
AG + BI + CJ + EL	48906.3	1	48906.3	1.66
AH + BF + CE + JL	110567.5	1	110567.5	3.75
AI + BG + DE + JK	2574.0	1	2574.0	0.09
AJ + CG + DF + HL + IK	248336.3	1	248336.3	8.43
AK + BC + EF + IJ	21892.8	1	21892.8	0.74
AL + BD + EG + HJ	69471.3	1	69471.3	2.36
BE + CF + DG + HK + IL	17719.0	1	17719.0	0.60
BJ + CI + DH + FL + GK	472.8	1	472.8	0.02
CD + FG + HI + KL	30938.3	1	30938.3	1.05
CL + DK + EJ + FI + GH	36652.8	1	36652.8	1.24
Total Error	117885.4	4	29471.4	
Total	42456164.5	31		

Table 3.13: ANOVA Table for Expected Total Cost (ETC)

Plot of Main Effects for TC



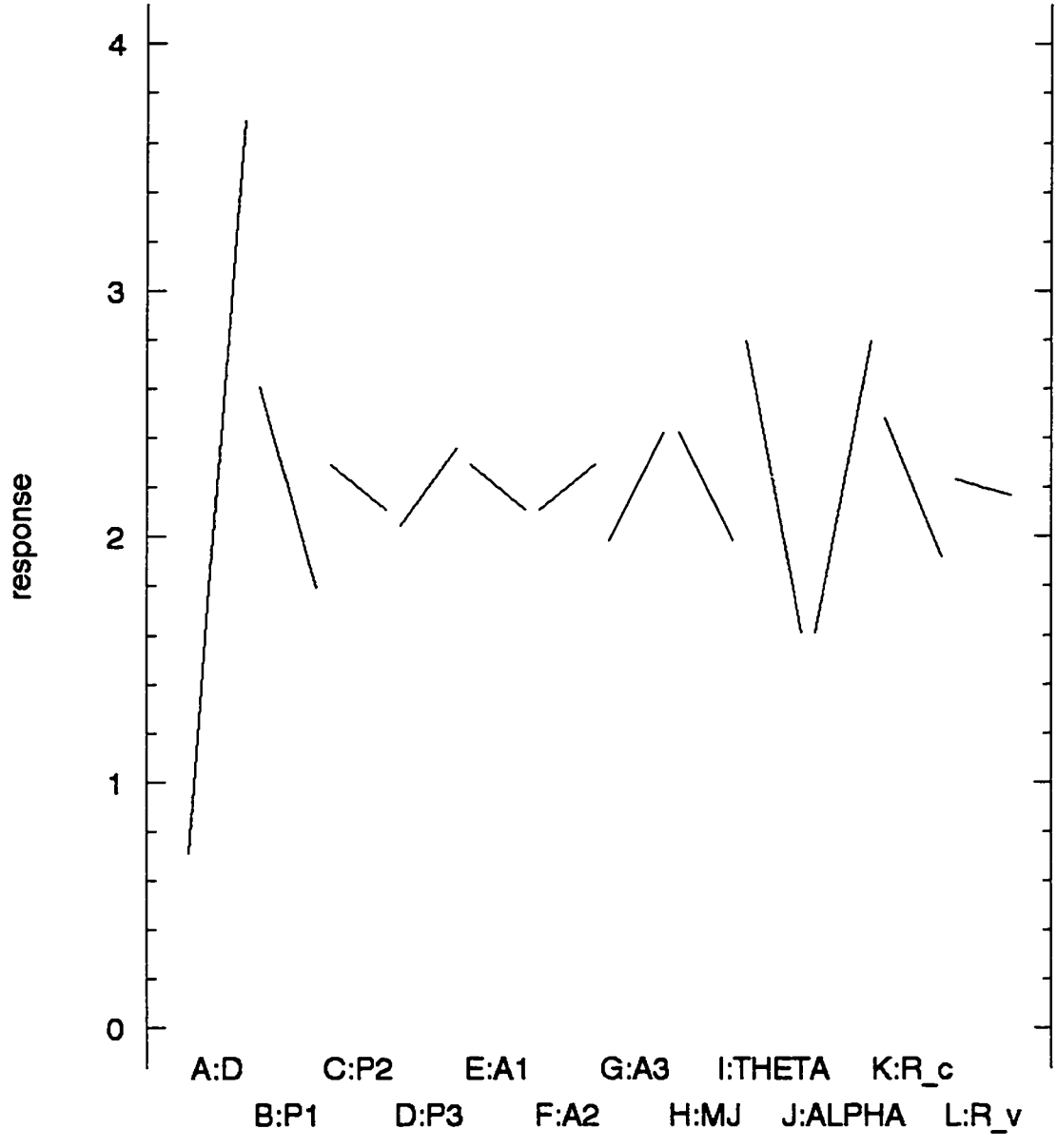
Pareto Chart for TC



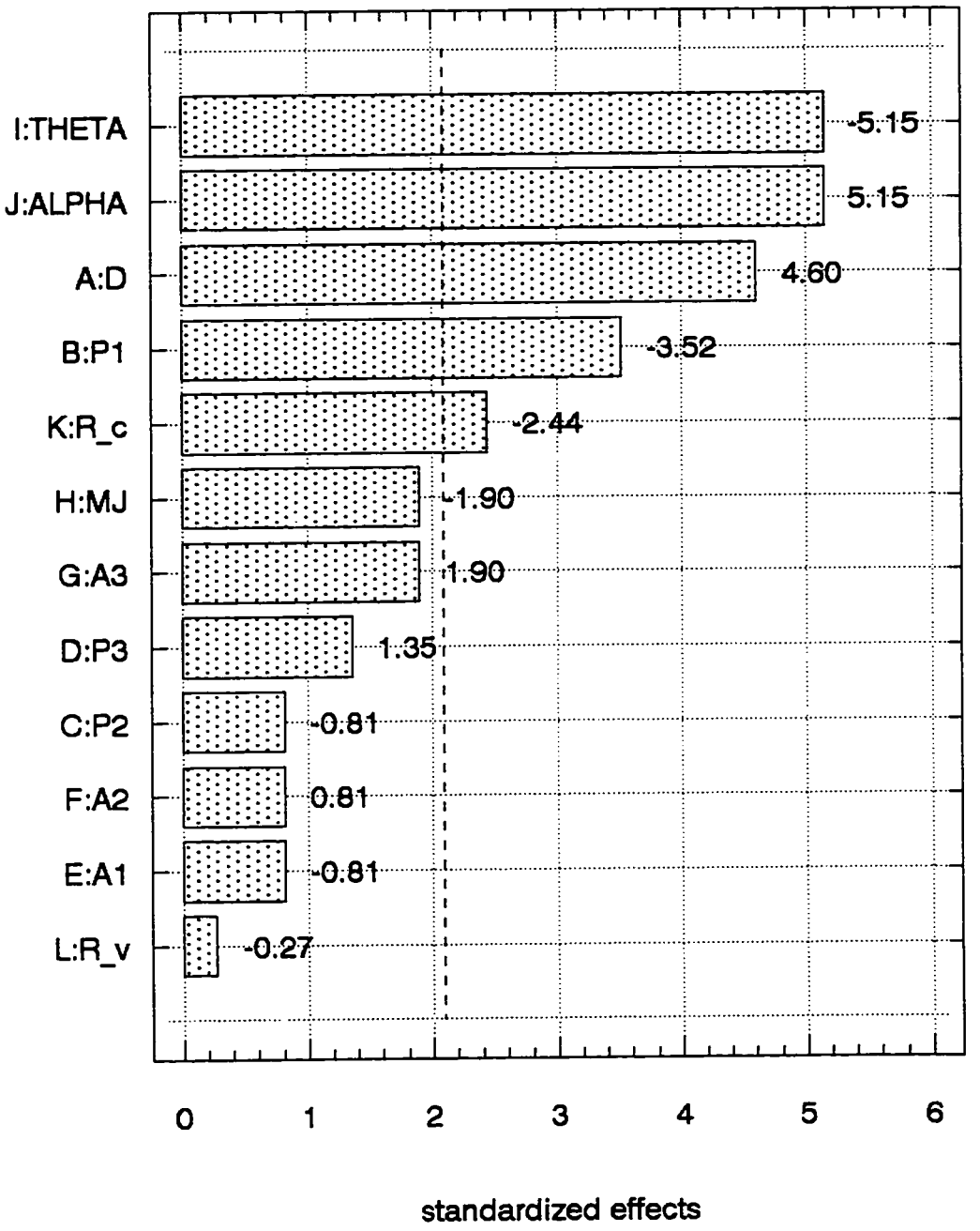
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	9.031	1	9.031	96.33
B	5.281	1	5.281	56.33
C	0.281	1	0.281	3.00
D	0.781	1	0.781	8.33
E	0.281	1	0.281	3.00
F	0.281	1	0.281	3.00
G	1.531	1	1.531	16.33
H	1.531	1	1.531	16.33
I	11.281	1	11.281	120.33
J	11.281	1	11.281	120.33
K	2.531	1	2.531	27.00
L	0.031	1	0.031	0.33
AB + CK + DL + FH + GI	0.281	1	0.281	3.00
AC + BK + EH + GJ	0.781	1	0.781	8.33
AD + BL + EI + FJ	2.531	1	2.531	27.00
AE + CH + DI + FK + GL	0.781	1	0.781	8.33
AF + BH + DJ + EK	0.281	1	0.281	3.00
AG + BI + CJ + EL	0.281	1	0.281	3.00
AH + BF + CE + JL	0.281	1	0.281	3.00
AI + BG + DE + JK	0.281	1	0.281	3.00
AJ + CG + DF + HL + IK	0.281	1	0.281	3.00
AK + BC + EF + IJ	0.781	1	0.781	8.33
AL + BD + EG + HJ	0.031	1	0.031	0.33
BE + CF + DG + HK + IL	0.281	1	0.281	3.00
BJ + CI + DH + FL + GK	0.781	1	0.781	8.33
CD + FG + HI + KL	0.031	1	0.031	0.33
CL + DK + EJ + FI + GH	0.031	1	0.031	0.33
Total Error	0.375	4	0.094	
Total	52.219	31		

Table 3.14: ANOVA Table for No. of Inspections at Stage 1 (N_1)

Plot of Main Effects for N1



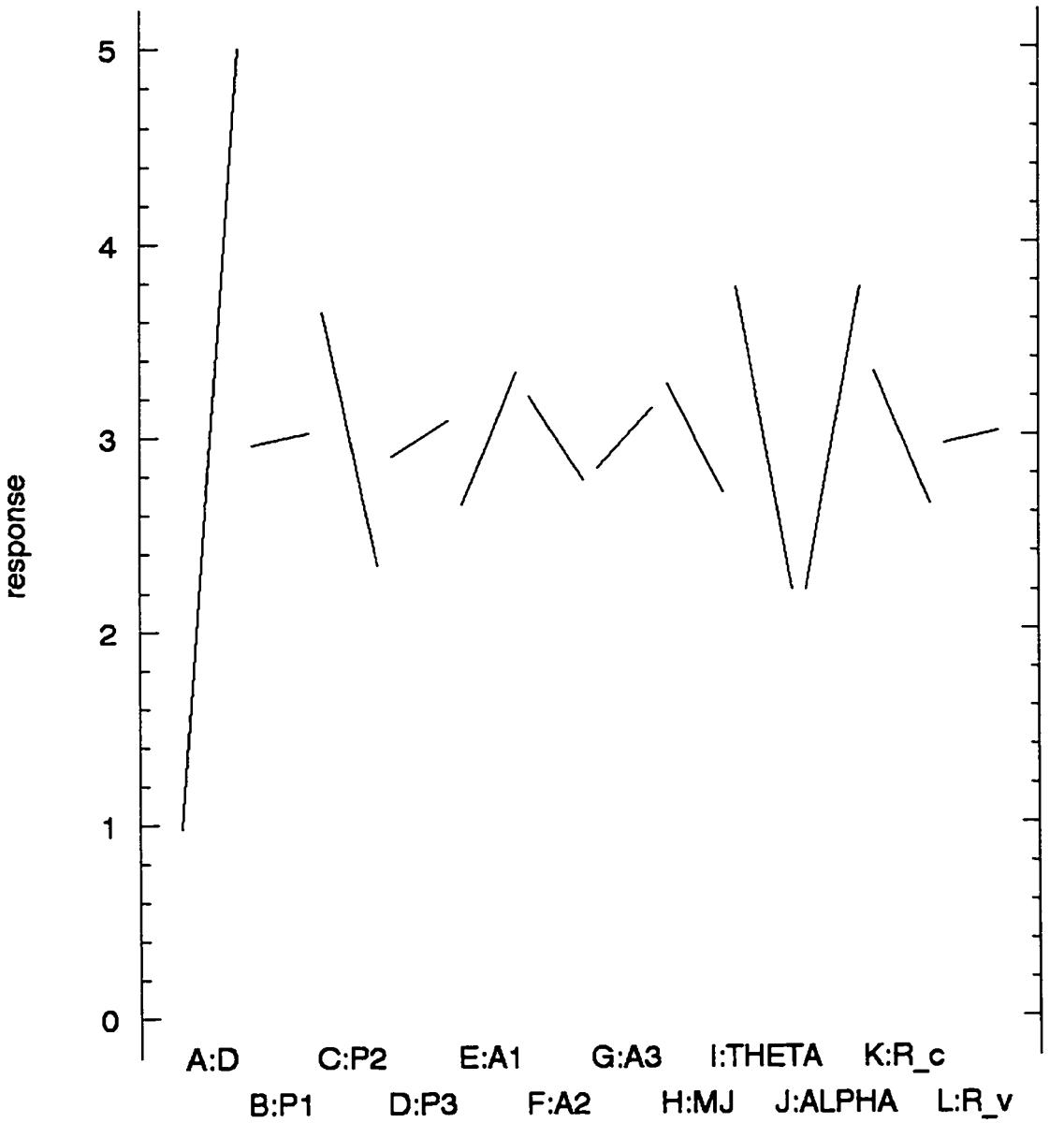
Pareto Chart for N1

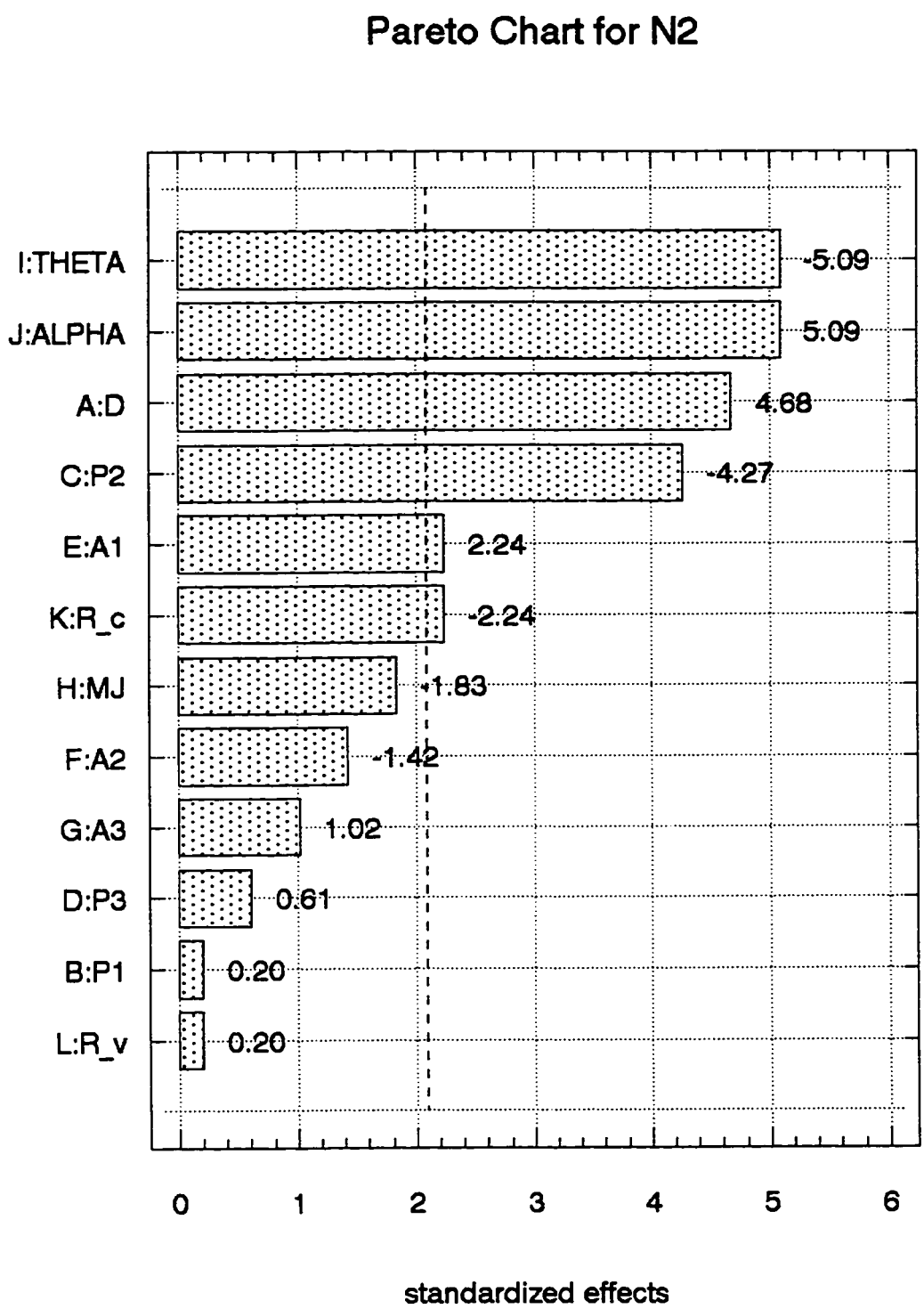


Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	16.531	1	16.531	48.09
B	0.031	1	0.031	0.09
C	13.781	1	13.781	40.09
D	0.281	1	0.281	0.82
E	3.781	1	3.781	11.00
F	1.531	1	1.531	4.45
G	0.781	1	0.781	2.27
H	2.531	1	2.531	7.36
I	19.531	1	19.531	56.82
J	19.531	1	19.531	56.82
K	3.781	1	3.781	11.00
L	0.031	1	0.031	0.09
AB + CK + DL + FH + GI	0.281	1	0.281	0.82
AC + BK + EH + GJ	0.031	1	0.031	0.09
AD + BL + EI + FJ	0.031	1	0.031	0.09
AE + CH + DI + FK + GL	2.531	1	2.531	7.36
AF + BH + DJ + EK	0.281	1	0.281	0.82
AG + BI + CJ + EL	0.031	1	0.031	0.09
AH + BF + CE + JL	1.531	1	1.531	4.45
AI + BG + DE + JK	1.531	1	1.531	4.45
AJ + CG + DF + HL + IK	1.531	1	1.531	4.45
AK + BC + EF + IJ	0.781	1	0.781	2.27
AL + BD + EG + HJ	0.031	1	0.031	0.09
BE + CF + DG + HK + IL	0.281	1	0.281	0.82
BJ + CI + DH + FL + GK	2.531	1	2.531	7.36
CD + FG + HI + KL	0.031	1	0.031	0.09
CL + DK + EJ + FI + GH	1.531	1	1.531	4.45
Total Error	1.375	4	0.344	
Total	96.469	31		

Table 3.15: ANOVA Table for No. of Inspections at Stage 2 (N_2)

Plot of Main Effects for N2

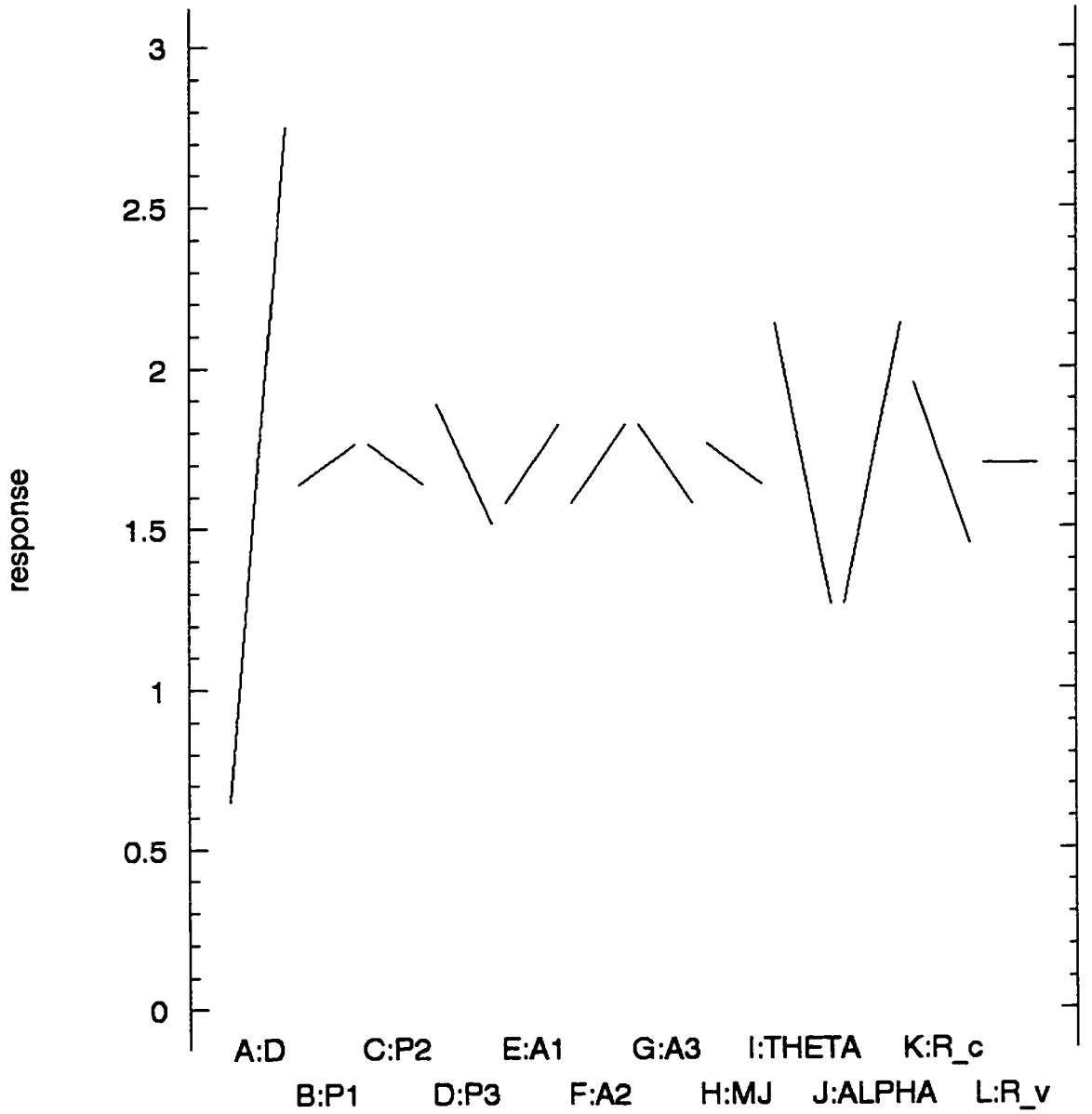



Pareto Chart for N2
I:THETA
J:ALPHA
A:D
C:P2
E:A1
K:R_c
H:MJ
F:A2
G:A3
D:P3
B:P1
L:R_v
-5.09
5.09
4.68
-4.27
2.24
-2.24
-1.83
-1.42
1.02
0.61
0.20
0.20
0
1
2
3
4
5
6
standardized effects

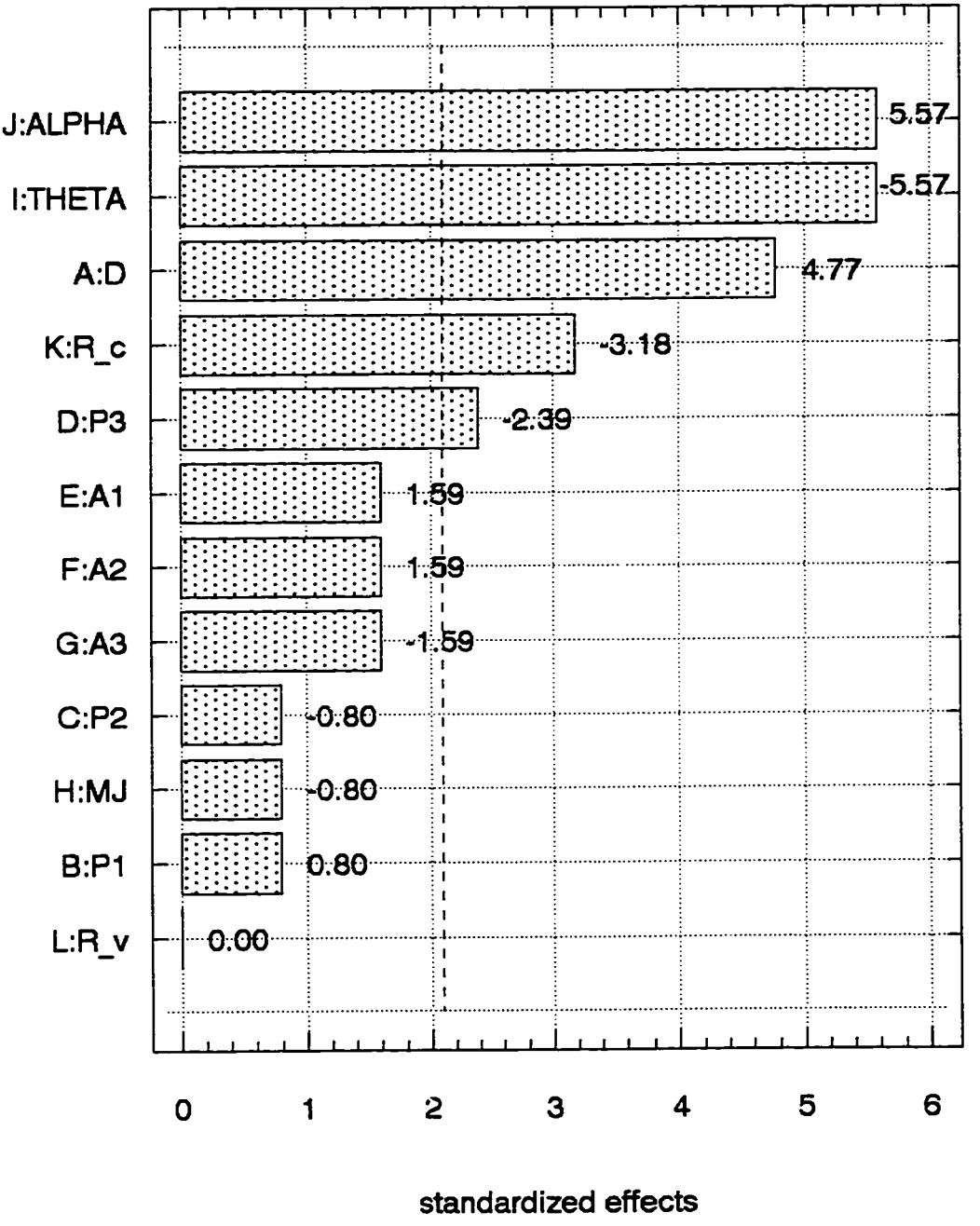
Effect	Sum of Squares	DF	Mean Sq.	F-ratio
A	4.500	1	4.500	72.0
B	0.125	1	0.125	2.0
C	0.125	1	0.125	2.0
D	1.125	1	0.125	18.0
E	0.500	1	0.500	8.0
F	0.500	1	0.500	8.0
G	0.500	1	0.500	8.0
H	0.125	1	0.125	2.0
I	6.125	1	6.125	98.0
J	6.125	1	6.125	98.0
K	2.000	1	2.000	32.0
L	0.000	1	0.000	0.0
AB + CK + DL + FH + GI	0.125	1	0.125	2.0
AC + BK + EH + GJ	0.125	1	0.125	2.0
AD + BL + EI + FJ	0.125	1	0.125	2.0
AE + CH + DI + FK + GL	2.000	1	2.000	32.0
AF + BH + DJ + EK	0.000	1	0.000	0.0
AG + BI + CJ + EL	0.000	1	0.000	0.0
AH + BF + CE + JL	0.125	1	0.125	2.0
AI + BG + DE + JK	0.125	1	0.125	2.0
AJ + CG + DF + HL + IK	0.125	1	0.125	2.0
AK + BC + EF + IJ	0.000	1	0.000	0.0
AL + BD + EG + HJ	0.000	1	0.000	0.0
BE + CF + DG + HK + IL	0.125	1	0.125	2.0
BJ + CI + DH + FL + GK	0.500	1	0.500	8.0
CD + FG + HI + KL	0.000	1	0.000	0.0
CL + DK + EJ + FI + GH	0.125	1	0.125	2.0
Total Error	0.250	4	0.063	
Total	25.500	31		

Table 3.16: ANOVA Table for No. of Inspections at Stage 3 (N_3)

Plot of Main Effects for N3



Pareto Chart for N3



Chapter 4

Two-Stage Production Lot Sizing Model With Varying Lot Sizes

The purpose of this chapter is to model the effect of imperfect production processes on the lot sizing decisions for a single end item in context of varying lot size two-stage production systems under deterministic conditions.

4.1 Introduction

In most industries, increasing automation in manufacturing processes has appreciably decreased the number of manufacturing operations required for simple products and for parts of complex products. Despite the fact that many manufacturing stages are still required in large number of cases, the use of two-stage production systems is increasing quite fast.

This chapter deals with a two-stage imperfect production-inventory system where

one larger lot at the initial stage and several smaller lots at the final stage are produced. Earlier models in the literature incorrectly assume that the lot size at a stage must not be larger than that at a preceding stage as lots move from initial to the final stage (which satisfies demand); and, that the lot size at a given stage must be a positive integer multiple of the lot size at any following stage. This integrality assumption was claimed to be optimal by Crowston *et al.* [23] for a special and rather impractical case where infinite production rates were assumed. Williams [92] and Szendrovits [84, 85] refuted the applicability of this integrality theorem for general MS-PIS by giving both counter examples and analytical proofs.

As discussed in chapter 2, the issue of modelling the effect of imperfect production processes on lot sizing decisions does not seem to have been adequately addressed in the literature for MS-PIS. Also, the presence of imperfect production processes in an MS-PIS framework calls for screening of non-conforming items in order to prevent unnecessary processing at the subsequent stages and reduce waste. In such a case, the issue of erroneous product inspections during screening of nonconforming items, if present, must also be addressed. Maintenance procedures, including corrective and preventive maintenance, should be used to reduce quality costs.

The classical EPQ models unrealistically assume that the output of the production-inventory system is of perfect quality. Generally, product quality is a function of state of the production process. When the process is in good condition the quality may be high or perfect. But with the passage of time, the process may deteriorate resulting in production of lot containing items which are nonconforming or of sub-

standard quality.

Studies from Rosenblatt and Lee [65],[66], Lee and Rosenblatt [58] and Porteus [63] have shown that for imperfect production processes the optimal EPQ was smaller than that determined by classical EMQ model. Obviously, a larger lot size requires a longer production cycle and, hence, likely to contain more nonconforming items.

Previous studies regarding maintenance planning models for even single-stage production-inventory systems assumed:

- Equally spaced inspection and maintenance intervals which merely renders administrative convenience, but does not promise optimal solutions; though, in some cases sufficient conditions under which such a schedule is optimal are provided in the literature e.g. [65].
- Instantaneous maintenance and restorations without disruption of the production process.
- Reduction of the age of the equipment to zero upon each maintenance; i.e. rendering it as-good-as-new.

A more general and realistic version can be developed by incorporating one or more of the following into the lot sizing decision models:

- Variable maintenance-inspection intervals.
- Reduction of the age of the process proportional to the level of the preventive maintenance activities.

- Interruption of the process for the sake of maintenance.
- Interruption of the process for the sake of restoration.

Szendrovits [85] presented various inventory patterns for two-stage production-inventory system. This chapter is an extension to the work by Szendrovits [85] incorporating quality, maintenance and restoration aspects into the model. This chapter contains the following lot sizing models:

1. Two-stage lot sizing model with perfect production processes.
2. Two-stage lot sizing model with deteriorating production processes, screening of nonconforming items between stages and process restorations.
3. Two-stage lot sizing model with deteriorating production processes, screening of nonconforming items between stages, errors in screening inspections and process restorations.
4. Two-stage lot sizing model with deteriorating production processes, screening of nonconforming items between stages, errors in screening inspections, PMs and process restorations.

4.2 Problem Definition

A two stage production process is considered where several smaller lots are produced at the final stage and one larger lot is produced at the initial stage. An infinite time horizon, deterministic and constant demand and production rates are assumed. Fixed costs per lot, including constant setup costs and transshipment

costs, are considered to be deterministic and known. Inventory holding costs are also assumed to be deterministic and linear in time. Shipments to the next stage is not permitted until the entire lot at the preceding stage is completed. We assume that this cost of holding one unit of inventory increases with the value of the product. Unrestricted capacity at each stage is assumed. Setup and transportation times are considered insignificant, hence, ignored. The product is assumed to be infinitely divisible, thus lot sizes are not required to be integers. The end of production at stage 1 in a given production run coincide with the depletion of inventory from the previous production run to zero.

At the start of the production cycle, the processes are in an in-control state, producing items of acceptable quality. However, after some time a production process may shift to an out-of-control state. The elapsed time (t) for which the process remains in the in-control state, before a shift occurs, is considered to be a random variable following a general probability distribution with increasing hazard rate. Once in the out-of-control state, the process starts producing a fixed percentage of nonconforming items and stays in that state till the end of production run or some restoration action. Each setup of a process brings the process back to the in-control and as-new state. These production systems are not uncommon in practice.

In the presence of deteriorating production processes, nonconforming items must be screened so that they are not passed to the subsequent stage. Also, some remedial action must be taken in order to prevent and/or delay the shift in a production process so that the quality rates of these processes are improved.

For the sake of simplicity, we consider the production of a single end product under deterministic conditions i.e., production rates, demand rate etc., are all constant and known; and, the production rate at any stage exceeds the demand rate, i.e. $P_j > D,\ j = 1, 2$. Backlogging is not permitted.

4.2.1 Notations

Following are the notations used for the development of mathematical models:

General Notations

A_j = Fixed setup and transportation costs per lot at stage j.

D = Demand rate of the final product(at stage 1).

P_j = Production rate at stage j (note that $P_j > D$)

Q = The total no. of the end-product items produced during a single production cycle.

Q_1 = The larger lot size produced during the longer production run at stage 1.

Q_2 = The total no. of the items produced at stage 2 during a single production cycle.

k = Lot size ratio; $k = Q_2/Q_1$ (note that $k \geq 1$ and is continuous).

$k_\uparrow$ = the smallest integer which is greater than k.

$k_\downarrow$ = The largest integer which is less than k.

$Q_{1,s}$ = The smaller lot size produced during the longer production run at stage 1 $= (k - k_1)Q_1$.

C_j = Total economic value added to an item after being processed at stage j (note that $C_2 \leq C_1$).

r_j = Inventory carrying cost expressed as a fraction of the economic value added (C_j) at the jth stage.

I_j = The total inventory of items at the jth stage.

HC = The total holding cost for one production cycle.

T = The recurring production cycle length.

$T_{1,L}$ = The length of larger production run at stage 1.

$T_{1,s}$ = The length of shorter production run at stage 1.

Notations related to imperfect processes with restorations

Q'_j = The expected no. of the items shipped from the jth stage to the $(j-1)^{th}$ stage during the production cycle.

α_j = The fraction of nonconforming units produced at the jth stage when the process shifts to an out-of-control state.

s_j = The quality cost of producing a nonconforming item at the jth stage.

$E(N_{2,i})$ = Expected number of nonconforming items produced during the ith interval at the j^{th}stage .

$E(N_j)$ = The number of nonconforming items produced at the jth stage.

$E(N_{1,L})$ = The number of nonconforming items produced at the jth stage during longer production run.

$E(N_{1,s})$= The number of nonconforming items produced at the jth stage during shorter production run.

QC = Total expected quality cost of producing nonconforming items during a single production cycle.

η_2 = Maximum number of inspections intervals at the stage 2 during a single production cycle.

C_{I_2} = Cost of a single inspection done at stage 2.

IC = Total expected process inspection cost for one production cycle.

$F_j(t)$ = Failure or shift distribution for the jth stage.

$f_j(t)$ = p.d.f. of elapsed time before shift at the jth stage.

$Z_j(t)$ = Hazard function for the process at the jth stage.

τ = Detection delay, i.e. the time elapsed between the occurrence of a shift and end of production.

TRC = Total expected restoration cost for one production cycle.

TQC = Total expected quality cost for one production cycle given as the sum of all quality, inspection, restoration and preventive maintenance costs.

$h_{2,i}$ = Length of ith inspection interval at stage 2.

$y_{2,i}$ = Expected age of the process at stage 2 immediately before i^{th} inspection.

$y_{2,i} = w_{2,i-1} + h_{2,i}$

$w_{2,i}$ = Expected age of the process at stage 2 immediately after i^{th} inspection.

Notations related to inspection errors

$E_{1,j}$ = The probability of committing Type I error in the product inspections at the jth stage.

$E_{2,j}$ = The probability of committing Type II error in the product inspections at the jth stage.

$\pi_{1,j}$ = The quality cost of incorrectly rejecting a conforming item at the jth stage.

$\pi_{2,j}$ = The quality cost of incorrectly accepting a non-conforming item at the jth stage.

Notations related to preventive maintenance

$C^o_{pm_2}$ = Cost of Maintenance to bring stage 2 process back to the as-new condition, which is also the maximum PM level.

C_{pm_2} = Cost of a single PM done at stage 2.

$t^o_{pm_2}$ = Time taken to complete the maintenance activities required to bring stage 2 process back to the as-new condition, which is also the maximum PM level.

t_{pm_2} = Time taken to complete the actual maintenance activities at stage 2.

ν_2 = Frequency of PMs done at stage 2 in terms of inspection intervals.

γ_2 = Fraction used to compute reduction in age of the process at stage 2 upon PM, and is a function of $C^o_{pm_2}$, C_{pm_2} and ϵ_2.

ϵ_2 = Value used to compute γ_2.

CPM = Total expected PM cost for one production cycle.

All parameters and variables are greater than zero. In this chapter we will discuss the case in which $Q_2 > Q_1$.

4.3 Two-Stage Perfect Production System with Varying Lot Sizes

This section reproduces the work by Szendrovits [85], for the inventory pattern considered in this chapter for further treatment, in order to make this thesis self-contained.

4.3.1 Expected Cycle Length

The expected recurring production cycle length can be given by the time required to consume the largest lot size.

$$T = \frac{Q_2}{D} \tag{4.1}$$

4.3.2 Expected Inventories

The time-weighted inventory of units produced at stage 1 and stage 2 are shown in Figure 4.1. The expected total inventory I_j for the jth stage can be found by using the algebraic expression for the appropriate area. While, the average inventory can be calculated by dividing the I_j by the expected recurring production cycle length ($T = Q/D$). The total inventory at stage 1 during a production cycle is given by the following expression:

$$I_1 = k_\downarrow \left(\frac{Q_1^2}{2P_1} + \frac{Q_1^2}{2D} \right) + \left(\frac{Q_{1,s}^2}{2P_1} + \frac{Q_{1,s}^2}{2D} \right)$$

Since $Q_1 = Q_2/k$, therefore:

$$I_1 = \left[k_\downarrow + (k - k_\downarrow)^2 \right] \left(\frac{1}{P_1} + \frac{1}{D} \right) \frac{Q_2^2}{2k^2} \tag{4.2}$$

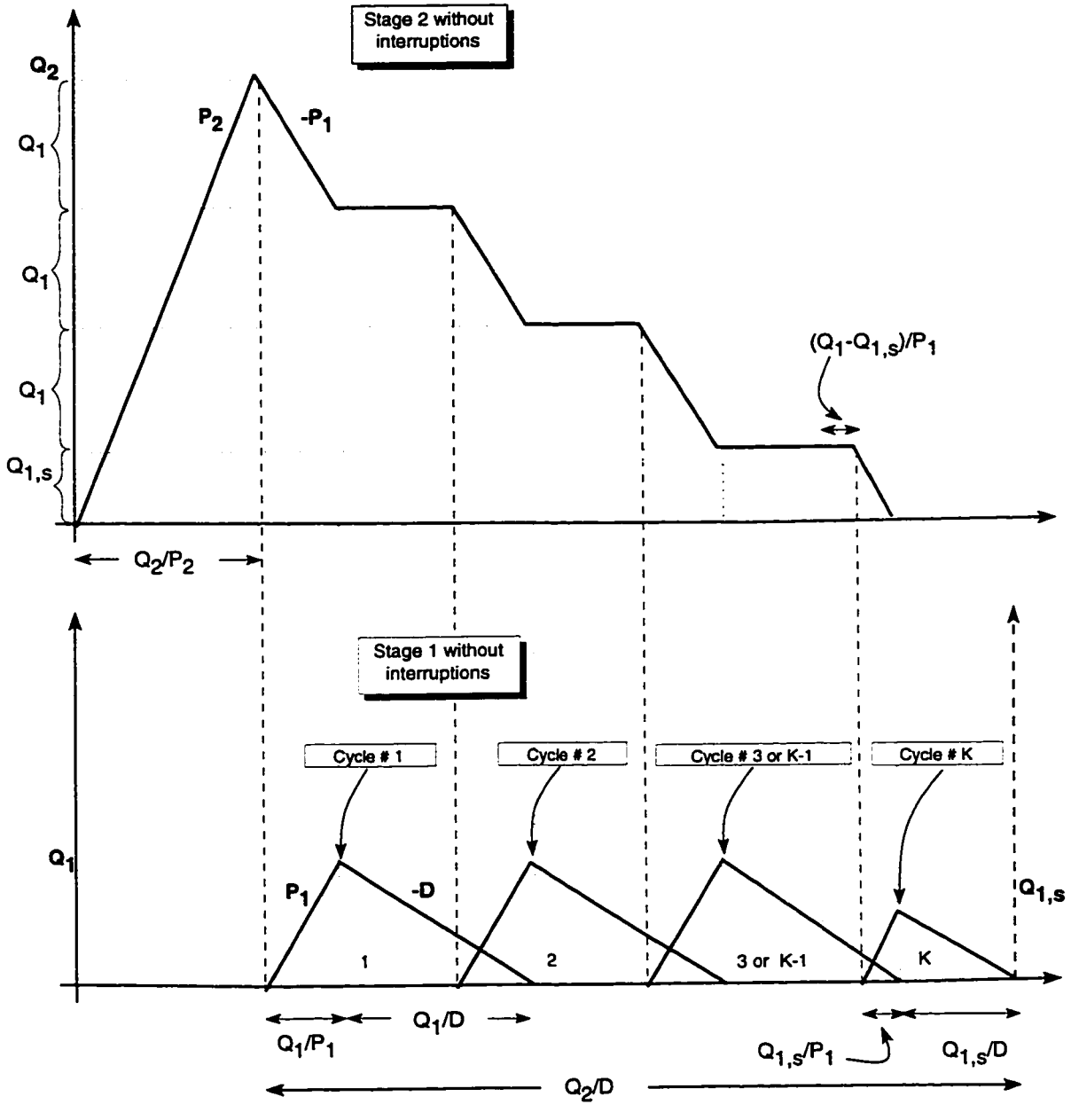


Figure 4.1: Inventory time plots for a perfect two-stage production process.

The total inventory at stage 2 during a production cycle is given by the following expression:

$$I_2 = \frac{Q_2^2}{2P_2} + k_\downarrow \frac{Q_1^2}{2P_1} + \frac{Q_{1,s}^2}{2P_1} + \frac{k_\downarrow(k_\downarrow - 1)}{2}\frac{Q_1^2}{D} + k_\downarrow Q_{1,s}\frac{Q_1}{D} + \left[\frac{Q_1 - Q_{1,s}}{P_1}\right] Q_{1,s}$$

Since $Q_1 = Q_2/k$ and $Q_{1,s} = (k - k_\downarrow)Q_1$, therefore:

$$I_2 = Q_2^2\left[\frac{1}{2P_2} + \frac{2k - k_\downarrow - (k - k_\downarrow)^2}{2k^2 P_1} + \frac{k_\downarrow(2k - k_\downarrow - 1)}{2k^2 D}\right] \tag{4.3}$$

4.3.3 Costs Involved

Following costs are involved in determining the total cost per unit time:

- Setup cost.
- Inventory holding cost.

Therefore, the total cost per unit time can be found using Equation (4.2) and (4.3):

$$TC = \frac{[k_\uparrow A_1 + A_2] + r\sum_{j=1}^{2} C_j I_j}{T} \tag{4.4}$$

4.3.4 Solution Procedure and Numerical Example

Since most of the proposed models result in non-linear cost equations, some Nonlinear Optimization technique must be applied for getting optimal values of decision variables. A good review of such optimization techniques can be found in [7] and [2] We have used a Hybrid Tabu Search algorithm for global optimization algorithm , proposed by Al-Sultan and Al-Fawzan [1], for this very purpose. The algorithm uses the optimization technique of Hooke and Jeeves where the search directions are

generated randomly and a line search is performed along each direction to determine the optimal step size. The best nontabu improving point (or the best nontabu nonimproving point) is selected and its associated direction is stored in the tabu list. This procedure is repeated and controlled by tabu search.

Numerical example is presented in Table 4.1 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hybrid Tabu Search optimization procedure proposed by Al-Sultan and Al-Fawzan [1] is used to find solutions to the proposed model.

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
D	10,000 units/year	C_1	\$15/unit
P_1	20,000 units/year	C_2	\$5/unit
P_2	50,000 units/year	s_j	C_j
A_1	\$250/setup	r_j	0.20
A_2	\$3000/setup		
Optimal Solution			
Variable	Value	Variable	Value
Q_1	1236.83 units	Q	4947.32 units
Q_2	4947.32 units	ETC	\$16186.42
k	4.0	T	0.4947 years

Table 4.1: Solved Example 4.1

4.4 Screening of Nonconforming Items between Stages and Process Restorations

Here we consider an imperfect production system in which processes start in in-control states; but, with the passage of time one or more processes may shift to an out-of-control state producing a fixed fraction of nonconforming items. Regular inspections of the processes are done to detect a shift in the process. Restorations are done in response to to the detection of an out-of-control state, which renders the process in as-new state.

Following additional assumptions are made for the development of mathematical model:

- Once the production at stage 2 has started, inspections are done at regular intervals in such a way that the integrated hazard rate remains constant for all intervals throughout the production run.
- The process at stage 2 is disrupted and restored whenever an out-of-control state is detected.
- The production at stage 1 is done in (k_1+1) runs. Each run requires a separate setup. The process is restored to the as-new state after each production run; so, the costs of process restoration can be considered as a part of setup cost.
- The restoration of the process brings the process back into the as-new state.
- Process inspections are assumed to be perfect.

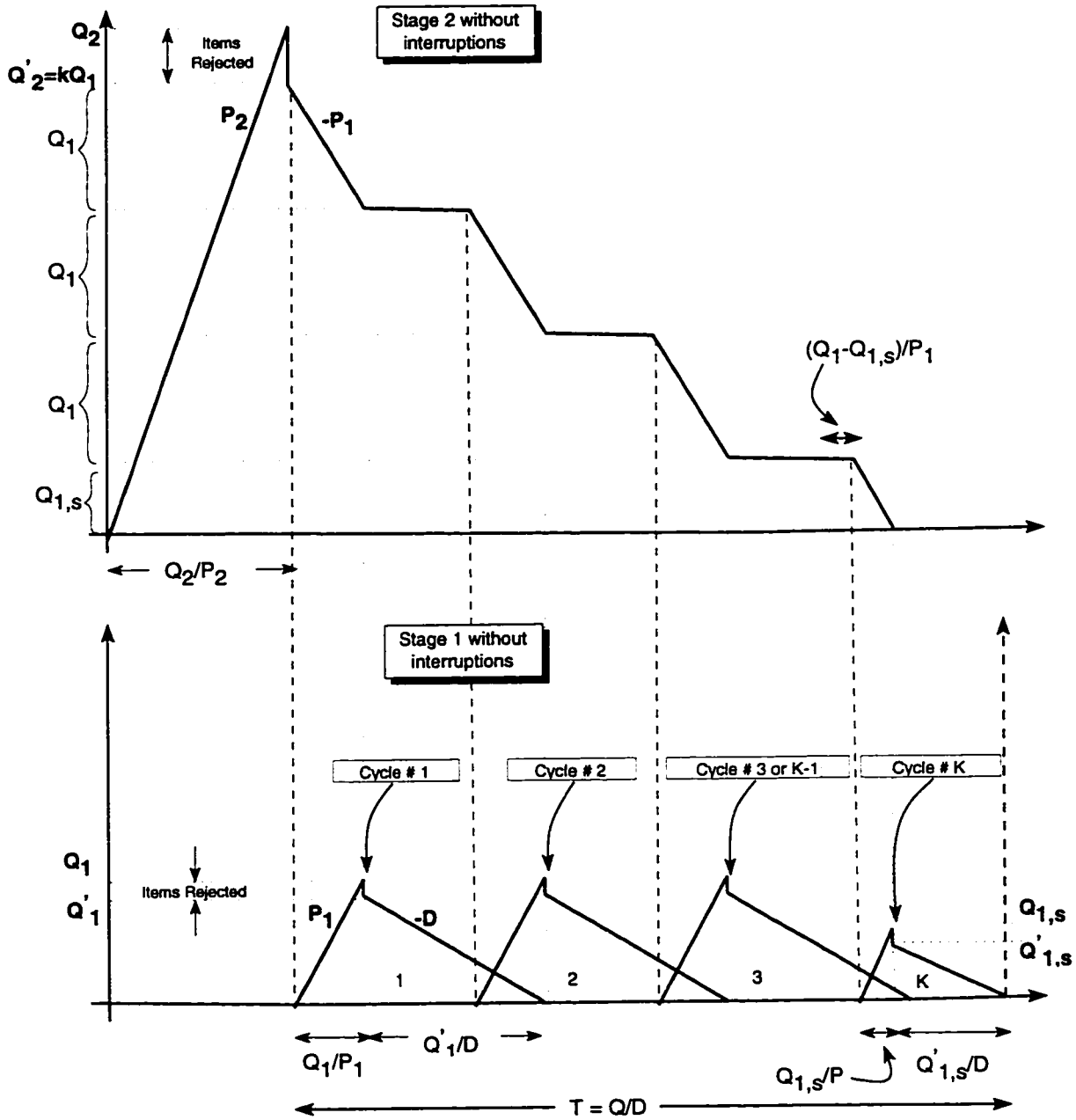


Figure 4.2: Inventory time plots for an imperfect two-stage production process.

- The lot is inspected for nonconformities just before the shipment and items found to be nonconforming are discarded.

4.4.1 Expected Recurring Production Cycle Length

The expected recurring production cycle length can be given by the time required to consume the largest lot size. In our case, the presence and rejection of items deemed nonconforming requires the recurring production cycle length to be the time required to consume the total output of the production system in a single production cycle by the outside demand (i.e. Q). Therefore,

$$\begin{aligned} T &= \frac{Q}{D} \\ T &= \frac{k_{\downarrow} Q'_1 + Q'_{1,s}}{D} \end{aligned}$$

It can be seen that:

$$\begin{aligned} Q'_2 &= Q_2 - N_2 \\ Q'_2 &= kQ_1 && (4.5) \\ Q_{1,s} &= (k - k_{\downarrow})\, Q_1 && (4.6) \\ Q'_1 &= Q_1 - N_{1,L} && (4.7) \\ Q'_{1,s} &= Q_{1,s} - N_{1,s} \\ Q'_{1,s} &= (k - k_{\downarrow})Q_1 - N_{1,s} && (4.8) \end{aligned}$$

Therefore,

$$\begin{aligned} T &= \frac{k_{\downarrow}(Q_1 - N_{1,L}) + \{(k - k_{\downarrow})Q_1 - N_{1,s}\}}{D} \\ T &= \frac{kQ_1 - k_{\downarrow}N_{1,L} - N_{1,s}}{D} && (4.9) \end{aligned}$$

4.4.2 Lengths of Successive Inspection Intervals

We have assumed that once the production at stage 2 has started, inspections are done at regular intervals in such a way that the integrated hazard rate remains constant for all intervals throughout the production run. This implies that:

$$\int_0^{y_{2,1}} Z_2(t)dt = \int_{w_{2,i-1}}^{y_{2,i}} Z_2(t)dt \tag{4.10}$$

And, inspections are done at $h_{2,1}$, $h_{2,1} + h_{2,2}$, ..., $h_{2,1} + h_{2,2} + \ ... \ + \ h_{2,i}$; where $Z_2(t)$ is the hazard function and $h_{2,i}$ is the length of ith inspection interval at the 2nd stage. For Weibull distribution we can derive a general expression for finding the length of ith inspection interval, given by:

$$h_{2,i} = \{h_{2,1}^{\beta} + w_{i-1}^{\beta}\}^{1/\beta} - w_{i-1} \tag{4.11}$$

Where β is the shape parameters for the Weibull distribution.

4.4.3 Expected Ages of the Processes

The expected age of the process at stage 2 immediately before the ith inspection is given by:

$$y_{2,i} \ = \ w_{2,i-1} + h_{2,i} \tag{4.12}$$

Since, we have assumed that upon detection of an out-of-control signal the process is restored back to the in-control state; therefore, the expected age of the process at stage 2 immediately after the ith inspection is:

$$w_{2,i} \ = \ [1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}]\, y_{2,i} \tag{4.13}$$

Where $[1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}]$ is the probability of detecting an in-control signal upon i^{th} inspection.

4.4.4 Expected Inventories

We have assumed that a production lot is inspected for nonconformities before shipment to the proceeding stage. It implies that items deemed nonconforming after quality inspection are discarded only just before shipments; hence, nonconforming items are included in the process inventories and must be considered in the calculation of holding costs. The inventory pattern for this case is given in the Figure 4.2. The expected total inventory at stage 1 during a production cycle is given by the following expression:

$$
\begin{aligned}
I_1 &= k_{\downarrow}\left[\frac{Q_1^2}{2P_1}+\frac{Q_1'^{\,2}}{2D}\right]+\left[\frac{Q_{1,s}^2}{2P_1}+\frac{Q_{1,s}'^{\,2}}{2D}\right] \\
I_1 &= \left\{k_{\downarrow}+(k-k_{\downarrow})^2\right\}\frac{Q_1^2}{2P_1}+\frac{1}{2D}\left[\{Q_1-N_{1,L}\}^2+\{(k-k_{\downarrow})Q_1-N_{1,s}\}^2\right]
\end{aligned}
$$

The expected total inventory at stage 2 without interruptions, during a production cycle, is given by the following expression:

$$
\begin{aligned}
I_2 &= \frac{Q_2^2}{2P_2}+k_{\downarrow}\frac{Q_1^2}{2P_1}+\frac{Q_{1,s}^2}{2P_1}+\frac{k_{\downarrow}(k_{\downarrow}-1)}{2}Q_1\frac{Q_1'}{D}+k_{\downarrow}Q_{1,s}\frac{Q_1'}{D}+\left[\frac{Q_1-Q_{1,s}}{P_1}\right]Q_{1,s} \\
I_2 &= \frac{Q_2^2}{2P_2}+\left[2k-k_{\downarrow}-(k-k_{\downarrow})^2\right]\frac{Q_1^2}{2P_1}+k_{\downarrow}(2k-k_{\downarrow}-1)\frac{Q_1^2}{2D} \\
&\quad -k_{\downarrow}(2k-k_{\downarrow}-1)\frac{N_{1,L}}{2D} \qquad (4.14)
\end{aligned}
$$

If $t_{r,2}$ is the finite and constant time required to restore the system from an out-of-control state to an in-control state, then:

$$
\begin{aligned}
I_2 &= \frac{Q_2^2}{2P_2}+P_2t_{r,2}\sum_{i=1}^{m-1}\left[F_2(y_{2,i})-F_2(w_{2,i-1})\right]\sum_{l=1}^{i}h_{2,l} \\
&\quad +\left[2k-k_{\downarrow}-(k-k_{\downarrow})^2\right]\frac{Q_1^2}{2P_1}+k_{\downarrow}(2k-k_{\downarrow}-1)\frac{Q_1^2}{2D}- \\
&\quad k_{\downarrow}(2k-k_{\downarrow}-1)\frac{N_{1,L}}{2D} \qquad (4.15)
\end{aligned}
$$

Where:

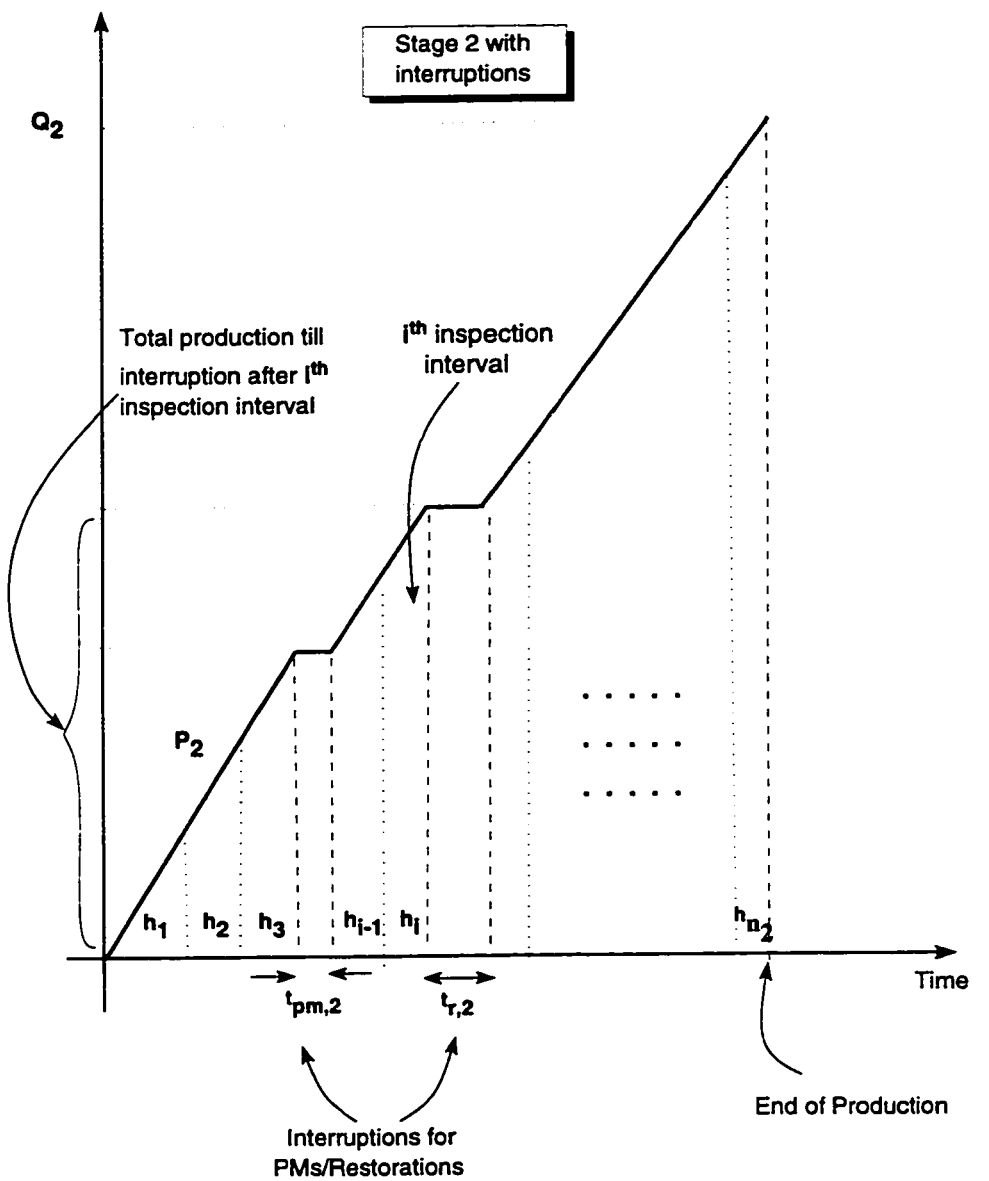


Figure 4.3: Interruptions in the production process.

$F_2(y_{2,i}) - F_2(w_{2,i-1})$ = Probability that a failure will occur in the ith inspection interval.

$\sum_{l=1}^{i} h_{2,l}$ = Total active production time till the ith inspection.

$\sum_{l=1}^{i} h_{2,l} P_2$ = Total number of items produced till the ith inspection.

4.4.5 Costs Involved

Following costs are involved in determining the total cost per unit time:

- Setup cost.
- Inventory holding cost.
- Quality cost (cost of nonconforming items+process inspection cost + restoration cost).

Expected Total Restoration Cost

Here we consider that the restoration cost is linearly related to the detection delay, i.e.:

$$R(\tau) = R_0 + R_1 \tau$$

The expected cost of restoration done at the end of ith inspection interval at stage 2 is given by:

$$\begin{aligned} RC_{2,i} &= \int_{w_{2,i-1}}^{y_{2,i}} R(\tau) f_2(t) dt \\ RC_{2,i} &= \int_{w_{2,i-1}}^{y_{2,i}} R\left(y_{2,i} - t\right) f_2(t) dt \\ RC_{2,i} &= \int_{w_{2,i-1}}^{y_{2,i}} \left[R_0 + R_1\left(y_{2,i} - t\right)\right] f_2(t) dt \end{aligned}$$

$$
\begin{aligned}
RC_{2,i} &= (R_0 + R_1 y_{2,i})\left[F_2(y_{2,i}) - F_2(w_{2,i-1})\right] - R_1 \int_{w_{2,i-1}}^{y_{2,i}} t f_2(t)dt \\
TRC_2 &= \sum_{i=1}^{\eta_2 - 1} \left[(R_0 + R_1 y_{2,i})\left\{F_2(y_{2,i}) - F_2(w_{2,i-1})\right\} - R_1 \int_{w_{2,i-1}}^{y_{2,i}} t f_2(t)dt \right]
\end{aligned}
\tag{4.16}
$$

Total Cost of Process Inspections

The cost of inspecting the process at stage 1 can be considered as a part of fixed costs. While, there are η_2 inspection intervals at stage 2 and no inspection is done at the end of production run; therefore, the total cost of cost during a production cycle is given by:

$$
IC = C_{I2}(\eta_2 - 1) \tag{4.17}
$$

Expected Cost of Producing Nonconforming Items

The expected number of nonconforming items produced during one longer production run at stage 1 is given by:

$$
\begin{aligned}
E(N_{1,L}) &= \int_0^{T_{1,L}} (T_{1,L} - t)\, \alpha_1 P_1 f_1(t)dt \\
E(N_{1,L}) &= \alpha_1 P_1 \left[T_{1,L}\left\{F_1(T_{1,L}) - F_1(0)\right\} - \int_0^{T_{1,L}} t f_1(t)dt \right] \\
E(N_{1,L}) &= \alpha_1 P_1 \left[T_{1,L} F_1(T_{1,L}) - \int_0^{T_{1,L}} t f_1(t)dt \right]
\end{aligned}
\tag{4.18}
$$

Similarly, the expected number of nonconforming items produced during the shorter production run at stage 1 is given by:

$$
E(N_{1,s}) = \alpha_1 P_1 \left[T_{1,s} F_1(T_{1,s}) - \int_0^{T_{1,s}} t f_1(t)dt \right] \tag{4.19}
$$

Therefore, the expected number of nonconforming items produced at stage 1 during a production cycle can be found by using Equations (4.18) and (4.19) and is given

by:

$$E(N_1) = k_1 E(N_{1,L}) + E(N_{1,s}) \tag{4.20}$$

The expected number of nonconforming items produced during i^{th} inspection interval at stage 2 is given by:

$$\begin{aligned} E(N_{2,i}) &= \int_{w_{2,i-1}}^{y_{2,i}} (y_{2,i} - t)\, \alpha_2 P_2 f_2(t) dt \\ E(N_{2,i}) &= \alpha_2 P_2 \left[y_{2,i} \left\{ F_2(y_{2,i}) - F_2(w_{2,i-1}) \right\} - \int_{w_{2,i-1}}^{y_{2,i}} t f_2(t) dt \right] \end{aligned}$$

The total expected number of nonconforming items produced at stage 2 during a production cycle cab be found by summing the number of nonconforming items produced during each inspection interval and is given by:

$$E(N_2) = \alpha_2 P_2 \sum_{i=1}^{n_2} \left[y_{2,i} \left\{ F_2(y_{2,i}) - F_2(w_{2,i-1}) \right\} - \int_{w_{2,i-1}}^{y_{2,i}} t f_2(t) dt \right] \tag{4.21}$$

In the absence of errors in screening inspections, the expected total quality cost is given by:

$$QC = s_1 E(N_1) + s_2 E(N_2)$$

Expected Total Quality Costs

The expected total quality cost is given by:

$$TQC = QC + TRC + IC \tag{4.22}$$

Where TRC, IC and QC are given by 4.16,4.17, and 4.22.

Expected Total Cost

The expected total cost per unit time is given by the following equation:

$$ETC = \frac{[k_{\uparrow}A_1 + A_2] + r\sum_{j=1}^{2} C_j I_j + TQC}{T} \tag{4.23}$$

Where I_j is found by using Equations (4.14) and (4.15); whereas, TQC is given by Equation (4.22)

4.4.6 Numerical Example

Numerical example is presented in Table 4.2 to illustrate the model developed in this section. The effect of α on the lot size and ETC is shown in Table 4.3. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hybrid Tabu Search optimization procedure proposed by Al-Sultan and Al-Fawzan [1] is used to find solutions to the proposed model.

4.5 Errors in the Screening Inspections

In the previous model we have assumed that the screening inspections are done by a perfect inspector or automated process. Here we relax this assumption. The following two types of errors can be committed in the screening inspections:

1. Rejecting a conforming item as a non-conforming one, at the jth stage, with a probability $E_{1,j}$. This is generally known as Type I error (sometimes referred to as *producer's risk*).

2. Accepting a non-conforming item as a conforming one, at the jth stage, with a probability $E_{2,j}$. This is generally known as Type II error (sometimes referred

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
D	10,000 units/year	C_1	\$15/unit
P_1	20,000 units/year	C_2	\$5/unit
P_2	50,000 units/year	s_j	C_j
A_1	\$250/setup	C_{I_2}	\$3
A_2	\$3000/setup	R_{c2}	\$1500
r_j	0.20	R_{v2}	$250A_2$
α_j	0.20	$t_{r,2}$	1 *day*
θ_j	0.1		
β_j	2.0		
Optimal Solution			
Variable	Value	Variable	Value
Q	3920.44 units	ETC	\$21345.7
Q_1	791.99 units	SC	\$10840.6
Q_2	3962.80 units	HC	\$6243.2
η_2	62	QC	\$1584.6
k	5.0	IC	\$466.8
$h_{2,1}$	0.009341 years	TRC	\$2210.5
T	0.392044 years	TQC	\$4261.8

Table 4.2: Solved Example 4.2

Effect of α on the lot size						
α	k	Q	Q_1	Q_2	TQC	ETC
0.10	5.0	4273.96	859.81	4300.71	3682.9	20409.5
0.20	5.0	3920.44	791.99	3962.80	4261.8	21345.65
0.30	5.0	3666.56	743.16	3719.60	4728.0	22173.5

Table 4.3: Effect of α on the lot size

to as *consumer's risk*).

4.5.1 Expected Recurring Production Cycle Length

The expected recurring production cycle length can be given by the time required to consume the largest lot size. In our case, the presence and rejection of items deemed nonconforming requires the recurring production cycle length to be the time required to consume the total output of the production system in a single production cycle by the outside demand (i.e. Q). Therefore,

$$\begin{aligned} T &= \frac{Q}{D} \\ T &= \frac{k_{\downarrow}Q'_1 + Q'_{1,s}}{D} \end{aligned}$$

It can be seen that:

$$Q_1 = \frac{Q'_2}{k} \tag{4.24}$$

$$Q_{1,s} = (k - k_{\downarrow})\, Q_1 \tag{4.25}$$

$$\begin{aligned} Q'_1 &= (1 - E_{1,1})(Q_1 - N_{1,L}) + E_{2,1}N_{1,L} \\ Q'_1 &= (1 - E_{1,1})Q_1 - (1 - E_{1,1} - E_{2,1})N_{1,L} \end{aligned} \tag{4.26}$$

$$\begin{aligned} Q'_{1,s} &= (1 - E_{1,1})Q_{1,s} - (1 - E_{1,1} - E_{2,1})N_{1,s} \\ Q'_{1,s} &= (1 - E_{1,1})(k - k_{\downarrow})Q_1 - (1 - E_{1,1} - E_{2,1})N_{1,s} \end{aligned} \tag{4.27}$$

Therefore,

$$T = \frac{k(1 - E_{1,1})Q_1 - k_{\downarrow}(1 - E_{1,1} - E_{2,1})N_{1,L} - (1 - E_{1,1} - E_{2,1})N_{1,s}}{D} \tag{4.28}$$

4.5.2 Expected Inventories

We have assumed that a production lot is inspected for nonconformities before shipment to the proceeding stage. It implies that items deemed nonconforming after quality inspection are discarded only just before shipments; hence, included in the process inventories and must be considered in calculating holding costs. Using Figure 3.4 and Equations (4.25-4.27), the expected total inventory at stage 1 during a production cycle is given by the following expression:

$$
\begin{aligned}
I_1 &= k_{\downarrow}\left[\frac{Q_1^2}{2P_1}+\frac{{Q'_1}^2}{2D}\right]+\left[\frac{Q_{1,s}^2}{2P_1}+\frac{{Q'_{1,s}}^2}{2D}\right] \\
I_1 &= \left\{k_{\downarrow}+(k-k_{\downarrow})^2\right\}\frac{Q_1^2}{2P_1}+\frac{1}{2D}\left[\{(1-E_{1,1})Q_1-(1-E_{1,1}-E_{2,1})N_{1,L}\}^2\right. \\
&\quad \left.+\{(k-k_{\downarrow})(1-E_{1,1})Q_1-(1-E_{1,1}-E_{2,1})N_{1,s}\}^2\right] \qquad (4.29)
\end{aligned}
$$

The expected total inventory at stage 2 during a production cycle is given by the following expression:

$$
\begin{aligned}
I_2 &= \frac{Q_2^2}{2P_2}+k_{\downarrow}\frac{Q_1^2}{2P_1}+\frac{Q_{1,s}^2}{2P_1}+\frac{k_{\downarrow}(k_{\downarrow}-1)}{2}Q_1\frac{Q'_1}{D}+k_{\downarrow}Q_{1,s}\frac{Q'_1}{D}+\left[\frac{Q_1-Q_{1,s}}{P_1}\right]Q_{1,s} \\
I_2 &= \frac{Q_2^2}{2P_2}+\left[2k-k_{\downarrow}-(k-k_{\downarrow})^2\right]\frac{Q_1^2}{2P_1}+k_{\downarrow}(2k-k_{\downarrow}-1)(1-E_{1,1})\frac{Q_1^2}{2D} \\
&\quad -k_{\downarrow}(2k-k_{\downarrow}-1)(1-E_{1,1}-E_{2,1})\frac{N_{1,L}}{2D}
\end{aligned}
$$

If $t_{r,2}$ is the finite and constant time required to restore the system from an out-of-control state to an in-control state, then treatment similar to that done in previous sections would result in the following:

$$
\begin{aligned}
I_2 &= \frac{Q_2^2}{2P_2}+P_2t_{r,2}\sum_{i=1}^{n-1}\left[F_2(y_{2,i})-F_2(w_{2,i-1})\right]\sum_{l=1}^{i}h_{2,l} \\
&\quad +\left[2k-k_{\downarrow}-(k-k_{\downarrow})^2\right]\frac{Q_1^2}{2P_1}+k_{\downarrow}(2k-k_{\downarrow}-1)(1-E_{1,1})\frac{Q_1^2}{2D}
\end{aligned}
$$

$$-k_{\downarrow}(2k - k_{\downarrow} - 1)(1 - E_{1,1} - E_{2,1})\frac{N_{1,L}}{2D} \tag{4.30}$$

For the case of perfect inspections the above equation reduces to Equation (4.15).

4.5.3 Costs Involved

Following costs are involved in determining the total cost per unit time:

- Setup cost.
- Inventory holding cost.
- Quality cost (cost of nonconforming items+process inspection cost + restoration cost).

Expected Cost of Producing Nonconforming Items

Considering errors in product inspections the expected total quality cost can be given by the sum of costs resulting from correct rejection, incorrect rejection and incorrect acceptance.

$$QC = \sum_{j=1}^{2} [\pi_{1,j} \{Q_j - E(N_j)\} E_{1,j} + \pi_{2,j} E(N_j) E_{2,j} + s_j E(N_j)(1 - E_{2,j})] \tag{4.31}$$

The expected total quality cost is given by:

$$TQC = QC + TRC + IC \tag{4.32}$$

Where TRC, IC and QC are given by 4.16, 4.17, and 4.31.

Expected Total Cost

Using Equations (4.32), (4.29) and (4.30), the expected total cost per unit time is given by:

$$ETC = \frac{[k_{\uparrow}A_1 + A_2] + r\sum_{j=1}^{2} C_j I_j + TQC}{T} \tag{4.33}$$

4.5.4 Numerical Example

Numerical example is presented in Table 4.4 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hybrid Tabu Search optimization procedure proposed by Al-Sultan and Al-Fawzan [1] is used to find solutions to the proposed model.

4.6 Preventive Maintenance

The objective of PM activities is to prevent or delay the onset of failure or shift in the process. Here we assume that the failure rate of the equipment is decreased after every PM. This corresponds to a reduction in the age of the equipment. The change in the age of the equipment will affect the time-to-shift distribution; and consequently, change the the length of the production cycle and EPQ. The amount of PM efforts which results in the least expected total cost corresponds to the optimal PM-level.

Following additional assumptions are made for the development of the mathematical model:

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
D	$10,000$ units/year	C_1	\$15/unit
P_1	$20,000$ units/year	C_2	\$5/unit
P_2	$50,000$ units/year	$\pi_{1,j}$	$0.5C_j$
A_1	\$250/setup	$\pi_{2,j}$	$1.5C_j$
A_2	\$3000/setup	s_j	C_j
r_j	0.20	C_{I_2}	\$3
α_j	0.20	R_{c2}	\$1500
θ_j	0.1	R_{v2}	$250A_2$
β_j	2.0	$t_{r,2}$	1 *day*
$E_{1,j}$	0.005		
$E_{2,j}$	0.05		
Optimal Solution			
Variable	Value	Variable	Value
Q	3897.09 units	ETC	\$ 22121.1
Q_1	790.81 units	SC	\$ 10905.6
Q_2	3976.60 units	HC	\$ 6247.8
η_2	63	QC	\$ 2261.0
k	5.0	IC	\$ 477.3
$h_{2,1}$	0.009295 years	TRC	\$ 2229.4
T	0.389709 years	TQC	\$ 4967.7

Table 4.4: Solved Example 4.3

Effect of inspection errors on the lot size						
Effect of E_1 on the lot size $E_2 = 0.05$						
E_1	k	Q	Q_1	Q_2	TQC	ETC
0.001	5.0	3914.788	791.230	3962.803	4431.7	21527.2
0.005	5.0	3897.090	790.809	3976.596	4967.7	22121.1
0.01	5.0	3879.398	790.200	3988.677	5653.3	22872.9
Effect of E_2 on the lot size $E_1 = 0.005$						
E_1	k	Q	Q_1	Q_2	TQC	ETC
0.01	5.0	3904.59	792.69	3986.16	4946.2	22094.0
0.005	5.0	3897.09	790.81	3976.6	4967.7	22121.1
0.1	5.0	3896.21	790.22	3973.45	5002.7	2219.0

Table 4.5: Effect of inspection errors on the lot size

Effect of mean time before shift on the lot size						
$MTBS$	k	Q	Q_1	Q_2	TQC	ETC
$\theta = 0.05$; $\beta = 1.5$; 0.0451	9.0	4332.98	495.69	4488.89	10310.4	29066.0
$\theta = 0.10$; $\beta = 2.0$; 0.0866	5.0	3897.09	790.81	3976.606	4967.7	22121.1
$\theta = 0.15$; $\beta = 2.5$; 0.1331	5.0	4474.00	901.69	4532.98	2501.0	19040.5

Table 4.6: Effect of mean time before shift on the lot size

- PM is done at stage 2 after every ν_2 inspection intervals.
- Each PM reduces the age of the equipment by a factor which is proportional to the level of PM done.
- The level of PM is kept constant throughout the time horizon under consideration.

4.6.1 Expected Age After PM

We have assumed that upon detection of an out-of-control signal the process is restored back to the in-control state. Therefore, the expected age of the process at stage 2 immediately after the ith inspection, considering only process restorations, is given by:

$$w_{2,i} = [1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}]\, y_{2,i}$$

Let us consider that the parameter γ_2 is represented by the following function:

$$\gamma_2 = \left[1 - \left(\frac{C_{pm}}{C_{pm}^o}\right)^{\epsilon_2}\right] \tag{4.34}$$

Where ϵ_2 is a constant, varying from process to process, used to define the shape of the function γ_2.

Since the PM activities are undertaken upon detection of an in-control signal and are assumed to affect the age of the equipment proportional to the level of those activities, the age of the equipment after PM is related to its age before PM as

follows:

$$w_{2,i} = [1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}]\, \gamma_{2i} y_{2,i} \tag{4.35}$$

where

$$\gamma_{2i} = \begin{cases} \gamma_2 & \text{if } i = \nu_2, 2\nu_2, 3\nu_2, \cdots \\ 0 & \text{otherwise} \end{cases}$$

Where $[1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}]$ is the probability of detecting an in-control signal upon i^{th} inspection.

4.6.2 Expected Total Inventories

If each PM takes a finite and constant time $t_{pm_2} = \gamma_2 t^o_{pm_2}$ then Equation (4.30) can be extended for this case as follows:

$$\begin{aligned} I_2 = & \frac{Q_2^2}{2P_2} + P_2 t_{r,2} \sum_{i=1}^{\eta_2 - 1} [F_2(y_{2,i}) - F_2(w_{2,i-1})] \sum_{l=1}^{i} h_{2,l} \\ & + P_2 \sum_{i=1}^{\eta_2 - 1} t_{pm_{2i}} [1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}] \sum_{l=1}^{i} h_{2,l} \\ & + \left[2k - k_\downarrow - (k - k_\downarrow)^2\right] \frac{Q_1^2}{2P_1} + k_\downarrow (2k - k_\downarrow - 1)(1 - E_{1,1}) \frac{Q_1^2}{2D} \\ & - k_\downarrow (2k - k_\downarrow - 1)(1 - E_{1,1} - E_{2,1}) \frac{N_{1,L}}{2D} \end{aligned} \tag{4.36}$$

where

$$t_{pm_{2i}} = \begin{cases} t_{pm_2} & \text{if } i = \nu_2, 2\nu_2, 3\nu_2, \cdots \\ 0 & \text{otherwise} \end{cases}$$

4.6.3 Costs Involved

Expected Total PM Cost

Since PM is done after every ν_2 inspection intervals only upon detection of an in-control signal, the expected total cost of PM at stage 2 is given by:

$$CPM = \sum_{i=1}^{\eta_2-1} C_{pm_{2i}} \left[1 - \{F_2(y_{2,i}) - F_2(w_{2,i-1})\}\right] \tag{4.37}$$

where

$$C_{pm_{2i}} = \begin{cases} C_{pm_2} & \text{if } i = \nu_2, 2\nu_2, 3\nu_2, \cdots \\ \\ 0 & \text{otherwise} \end{cases}$$

Expected Total Quality Costs

The expected total quality cost is given by:

$$TQC = QC + IC + TRC + CPM \tag{4.38}$$

Where TRC, IC, QC, and CPM are given by 4.16, 4.17, 4.31, and 4.38.

Expected Total Cost

Following costs are involved in determining the total cost per unit time:

- Setup cost.
- Inventory holding cost.
- Quality cost (cost of nonconforming items+process inspection cost + restoration cost + PM cost).

Therefore, the expected total cost per unit time is give by:

$$ETC = \frac{[k_1 A_1 + A_2] + r\sum_{j=1}^{2} C_j I_j + TQC}{T} \tag{4.39}$$

Where I_1, I_2, and TQC are given by Equations (4.29), (4.36), and (4.38).

4.6.4 Numerical Example

Numerical example is presented in Table 4.7 to illustrate the model developed in this section. The optimum values of the lot sizes at different stages and the total expected cost. A 'C' code implementing the Hybrid Tabu Search optimization procedure proposed by Al-Sultan and Al-Fawzan [1] is used to find solutions to the proposed model. The effect of various PM level on ETC is shown in Table 4.8. It should be noted that γ_2 is a discrete variable in real practice as there are only some feasible levels of PM. It can be seen from Table 4.8 that RC and QC decrease continuously with the increase in CPM resulting in a decline in ETC. But, after a certain PM level ETC starts increasing as increase in CPM overtakes the decrease in RC and QC.

4.7 Conclusion

In this chapter we have developed generalized models for uniform lot size MS-PIS by incorporating quality and maintenance functions into the lot sizing models. The superiority of the generalized models is illustrated by examples.

The lot sizes at each stage and expected total cost per unit time is found sensitive to the fraction of nonconforming items produced in the out-of-control state as can be seen from Table 4.3. These are also found to be sensitive to errors in screening

Data Used In The Solved Example			
Parameter	Value	Parameter	Value
D	10,000 units/year	C_1	\$15/unit
P_1	20,000 units/year	C_2	\$5/unit
P_2	50,000 units/year	$\pi_{1,j}$	$0.5C_j$
A_1	\$250/setup	$\pi_{2,j}$	$1.5C_j$
A_2	\$3000/setup	s_j	C_j
r_j	0.20	ϵ_2	1.0
α_j	0.20	R_{c2}	\$1500
θ_j	0.1	R_{v2}	$250A_2$
β_j	2.0	$t_{r,2}$	1 *day*
$E_{1,j}$	0.005	$C^o_{pm,2}$	\$150
$E_{2,j}$	0.05	$t_{pm,2}$	$\gamma_2 t_{r,2}$
C_{I_2}	\$3		
Optimal Solution			
Variable	Value	Variable	Value
Q	4078.32 units	ETC	\$ 21579.1
Q_1	828.32 units	SC	\$ 10389.2
Q_2	4166.38 units	HC	\$ 6886.5
k	5.0	IC	\$ 418.3
η_2	72	QC	\$ 1096.8
γ_2	0.5	TRC	\$ 1428.6
ν_2	5	CPM	\$ 1345.7
$h_{2,1}$	0.00925 years	TQC	\$ 4297.4

Table 4.7: Solved Example 4.4

Effect of Change in PM Level					
γ_2	RC	QC	CPM	TQC	ETC
0.00	2229.4	2261.0	0.0	4967.7	22121.1
0.25	1908.3	1598.4	425.6	4363.5	21654.3
0.50	1428.6	1096.8	1345.7	4297.4	21579.1
0.75	1201.1	1054.8	1857.1	4525.2	21795.6
1.00	1116.2	1010.7	2107.6	4644.8	22009.1

Table 4.8: Effect PM on the expected total cost per unit time.

inspections as evident from Table 4.5. The mean time before shift occurs in a process affects not only lot sizes and expected total cost per unit time; but also, the ratio of lot sizes k as evident from Table 4.6. The introduction of preventive maintenance activities results in reduction of overall expected total cost per unit time.

Chapter 5

Conclusion

5.1 Summary

In this thesis we develop and test generalized integrated mathematical models incorporating quality and maintenance into the lot sizing decisions in the context of multi-stage production-inventory systems. We consider both uniform lot sizing models and variable lot sizing models. Errors in product inspections while screening nonconforming items between stages are also incorporated in the proposed models. We suggest appropriate optimization procedures for solving these models. Optimal lot sizes, number of maintenance-inspections, investment in process restoration and preventive maintenance is determined by minimizing the expected total cost per unit time. Models are coded in 'C' to get numerical results for the sake of comparison and analysis. The analysis reported demonstrates the usefullness of the

approaches considered. The results reported indicate the extent to which imperfectness of production processes, errors in screening inspection, maintenance and process restorations can affect the lot sizing decisions.

5.2 Contributions

This thesis contributes the following to the multistage lot sizing models:

- Mathematical models for determining Uniform lot-size for series MS-PIS are formulated incorporating:
    1. Imperfect quality of production and screening of nonconforming items between stages.
    2. Errors in screening inspections.
    3. Maintenance-inspections.
    4. Process restorations.
- Mathematical models for determining Varying lot-sizes for series MS-PIS are formulated incorporating:
    1. Imperfect quality of production and screening of nonconforming items between stages.
    2. Errors in screening inspections.
    3. Process restorations.
    4. Preventive Maintenance.

- Models are coded in 'C' for comparison purposes. Hooke & Jeeves and Al-Sultan & Al-Fawzan's Hybrid Tabu Search algorithm is used for optimization . Numerical results are given to illustrate various aspects of the proposed models.

5.3 Future Research

The models presented in this thesis can be extended by incorporating several real life considerations including the following:

- Effect of errors committed in process inspections, preventive maintenance activities and process restorations.
- Effect of setup times, transportation times, constraints on batch sizes, constraints on production capacities and time taken for process restorations in the uniform lot size models.
- Extesion of the variable lot size model for a general n-stage system.

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Vita

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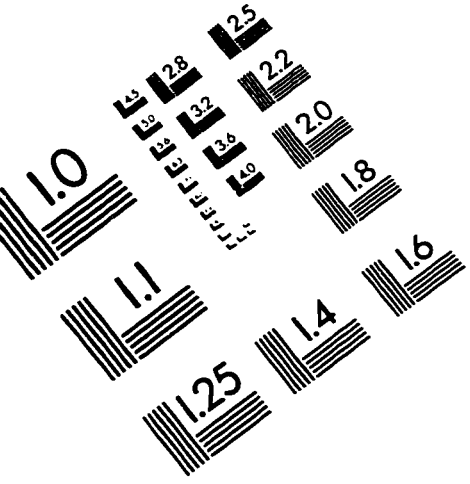
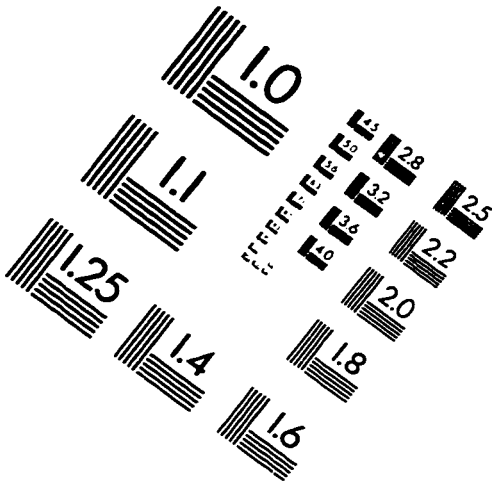
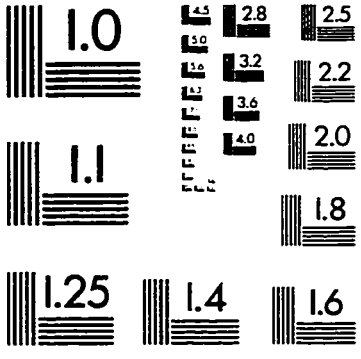
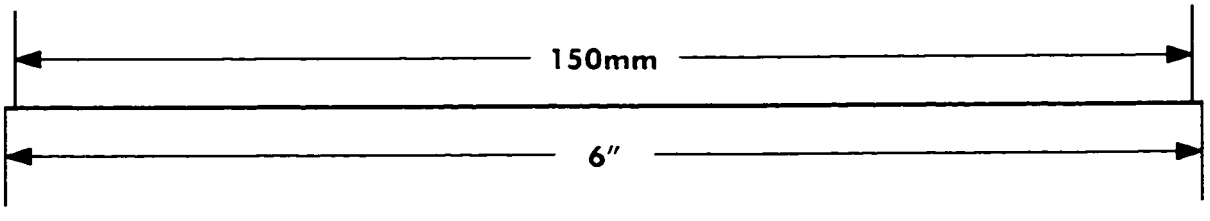
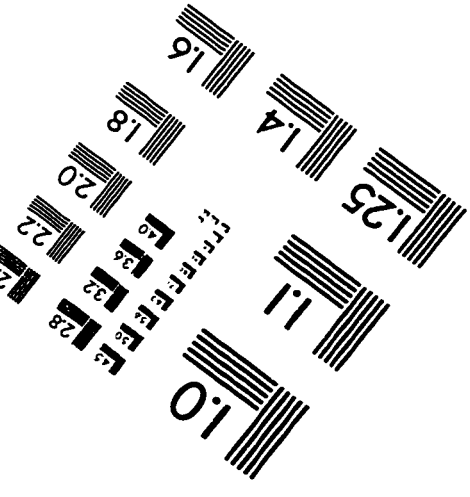
IMAGE EVALUATION TEST TARGET (QA-3)

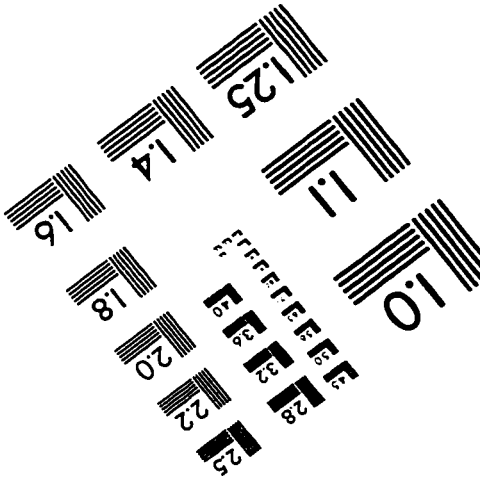












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