Chapter 1

INRODUCTION AND LITERATURE REVIEW

1.1 Introduction

The manufacturing process of any product, no matter how well designed or carefully maintained, always involves a certain amount of variation in the production conditions. This natural fluctuation, often called a “stable system of chance causes,” is the cumulative effect of many small, essentially uncontrollable factors. Under these conditions we say that the manufacturing process is in a state of *statistical control.* Sometimes in the output of a production process, other forms of non-natural variability occur, usually from three main sources. These include improper adjustments in machines, operator errors, or defective raw material. This non-natural variability is called “assignable causes,” and is generally larger than the natural variability. It represents an unacceptable level of process performance and such process is said to be *out of control*.

A control chart is a graphical technique used for continuous monitoring whether the manufacturing process is in a state of statistical control or not. Its primary objective is to quickly detect the formation of assignable causes of process shifts so that investigation of the process and corrective measure may be taken before many nonconforming units are manufactured. Generally, control chart is an effective tool in eliminating process variability as well as estimating the parameters of the production process.

Since the first work of Shewhart [65], the construction of quality control charts has undergone series of modifications with new methods being suggested. Most of the methods reported in the literature are based on simple random sampling (SRS), which to certain extent is considerably less effective in estimating the population mean as compared to new sampling technique such as ranked set sampling (RSS) and its modifications, see McIntyre [34], Takahashi and Wakimoto [76], Muttlak [42], Samawi et al [60]. The use of ranked set sampling (RSS) and median ranked set sampling (MRSS) as a sampling plan to develop control charts for monitoring the process mean was first suggested by Salazar and Sinha [59]. They showed that the new charts were substantially better than those based on the traditional SRS. Also, Muttlak and Al-Sabah [47] developed control charts based on RSS, MRSS and other modifications of RSS namely: the extreme ranked set sampling (ERSS), paired ranked set sampling (PRSS) and selected ranked set sampling (SRSS) and showed that all these charts dominates the classical SRS control charts for means.

The performance of such control charts using RSS and some of its modifications lead us to assume without loss of generality that further modifications of RSS could produce better control charts than the traditional SRS. The emergence of double ranked set sampling (DRSS), see Al-Saleh and Al-Kadiri [6], and other proposed sampling techniques namely: median double ranked set sampling (MDRSS), double median ranked set sampling (DMRSS) and extreme double ranked set sampling (EDRSS) which have all proven to be more effective in estimating the population mean as compared to SRS, RSS, ERSS, PRSS and SRSS may play significant role in the monitoring a characteristic of manufacturing process. This is the primary interest that motivates us to investigate this problem. Other things that lead to this study are the need to investigate how fast a shift in process variability could be detected using these new sampling techniques.

1.2 Literature Review

In this section, we review some of the works in the area of statistical quality control as well as the ranked set sampling (RSS) and classified them into two separate groups.

1.2.1 Statistical Quality Control

Although quality control has been with us since when manufacturing began and competition accompanied manufacturing but, its scientific foundation with respect to how many sample units to inspect and what conclusion to draw from the result and the eventual extension to statistical quality control took place relatively late. The beginning of statistical quality control dates back to 1924, when Shewhart [65] introduced his first control chart for the fractional nonconforming units. His first control chart monitors whether the nonconforming fraction of a product remains within the control limits during the time of observation or not.

After over twenty five years from the original work of Shewhart [65], Aroian and Levene [3] proposed the first trials to determine the three decision parameters of a control chart namely; sample size, control limit, and time between sampling. With the aim of minimizing the number of product units when the process is out of control, they noted that the frequency of the false alarms which depends on the time interval between samples plays a greater role in the determination of the control limits than the probability of those false alarms per sample.

Weiler [81], used sample size in constructing a model to minimize the average amount of inspection before a process shift occurs. In his work, he had completely avoided the time interval between samples and the probability of detecting the effect of process shift. In other words, the average run length (ARL) when the process is out of control was neglected.

None of these works had so far taken into consideration the costs related to false alarm and defectives incurred while the manufacturing process is out of control. These cost related problems and the frequency of shift between two processes were considered in the works of Duncan [23], Barnard [10] and Barish & Hauser [9]. All these works have a common goal of pursuing control strategies that will effectively minimize the average total cost per time unit of the respective manufacturing and control system. But while Duncan [23] considered models with only a single point out of control, Barnard [10] and Barish & Hauser [9] works on models with numerous numbers of points which are out of control.

Crowder [19] presented a numerical procedure for the computation of average run lengths (ARL) of a control chart using the combination of individual measurement and moving range chart based on two consecutive measurements. He supplied the exact expression for ARL in integral form and its approximation in numerical form. He also gave ARL values for several settings of control limits and shifts in the process mean and standard deviation.

On the effect of non-normality on $\overline{x}$ and $\overline{R}$, Chan, et al [15] used some symmetric distribution to study the departure from normality by comparing the probabilities of when $\overline{x}$ and $\overline{R}$ lies outside the 3-standrard-deviation and 2-standrard-deviation control limits. They reported that when the tails of the underlying distribution are thin and tick, then the control charts based on the assumption of normality will produce inaccurate results.

Champ and Woodall [14] suggested the use of Markov chain to obtain run length of Shahwart control charts with supplementary runs rules. They presented the average run length for the Shahwart $\overline{x}$ charts with supplementary runs rules, Shahwart $\overline{x}$ charts, and the cumulative sum (CUSUM) chart. It was observed that although the supplementary runs rules had made the traditional Shawhart charts to be more effective, but not as sensitive as CUSUM charts.

Cryer and Ryan [20] studied the estimation of sigma for individual observations control charts using the moving range and observed that the method is not as effective as to the use of sample standard deviation when the observations are independent and normally distributed. With aid of some real chemical data, they showed that the moving range approach could produce poor results when the observations are correlated.

On a study of detecting the shifts in the process mean using the control chart for averages, Palm [55] studied how sensitive a chart is to a process mean shift using the distribution of the run length. He produced a table of percentile values for the distribution of the measurement carried out on outgoing products. In a related study, Walker and Philpot [80] observed that although the run lengths are effective in detecting shift problems, they however increase the probability of a false signal.

Saniga [62] presented a FORTRAN program for determining the parameters of control limits as well as the sample size for designing an $\bar{X}$ and R charts. The program was based on a statistical criterion that can be stated in terms average run length, or probabilities of type I and type II errors.

In his study of shift in process mean, Costa [17] observed that the use of $\bar{x}$ with variable sample interval (VSI) or / and variable sample size (VSS) to detect the process shift in mean is much faster as compared to the traditional $\bar{x}$ charts. He extended his work, Costa [18], to the cases where both the $\bar{x}$ and R charts are used in detecting shifts and observed that the new VSI and VSS based charts have improved the rate at which the shifts in mean and / or variance are detected.

Amin and Wolff [2] studied the average run length (ARL) properties of some control procedure for monitoring the mean and variance of a process. Considering a situation where the underlying distribution is a mixture of normal distribution, they computed the ARL values for the $\bar{X}$, R, and Extreme-value charts and show that the later is the most efficient of the three charts when it is targeted at detecting the presence of mixture alternatives.

Roes and Does [58] discussed the use of an analysis of variance model in constructing control charts with smaller variance. Using different estimators of variability, they developed control charts for the mean and linear contrasts and also provided a lead for the construction and evaluation of the charts.

Salazar and Sinha [59] constructed $\bar{X}$ - control chart based on ranked set samples considering normal population and various shift values. Using visual comparison and Monte Carlo simulation for the computation of average run length, they show that RSS and median ranked set sampling (MRSS), based control charts for means were considerably better in detecting a shift in process mean than that of the classical Shewhart $\bar{X}$ chart with same sample size. In their work, they had considered both the cases where ranking can and cannot be performed without error in ranking with equal and unequal allocations. In other words, perfect and imperfect ranking were considered.

Reynold and Stoumbos [56] investigated control charts for monitoring a process to detect changes in the mean and / or variance for individual observations taken at sampling intervals. They evaluated the $\bar{x}$ chart, moving range (MR) chart and the exponentially weighted moving average (EWMA) charts and noted that the combination of $\bar{x}$ and R chart is not as effective in detecting small shifts as compared to EWMA charts. Also observed is the effect of variable sample interval (VSI) on the combination of $\bar{x}$ and EWMA chart and note there is significant improvement on the time required to detect shift in process parameters.

Muttlak and Al-Sabah [47] went further beyond the work of Salazar and Sinha [59] by considering further modifications of RSS namely: extreme ranked set sampling (ERSS), paired ranked set sampling (PRSS) and selected ranked set sampling (SRSS). Using normal population and various shift values, they computed various ARL values with an aid of computer simulation and showed that all the control charts for means based on the above sampling techniques were better than those of classical Shewhart charts.

1.2.2 Ranked Set Sampling

The method of ranked set sampling (RSS) was first proposed by McIntyre [34] in estimation of mean pasture yield. He noted that RSS is considerably more efficient in the estimation of a population mean than the standard simple random sampling (SRS). Although with no mathematical theory for McIntyre [34] scheme over the next decade, Halls and Dell [25] applied it on the estimation of forage yield. A major breakthrough in terms of necessary mathematical theory in support of McIntyre [34]'s work were given by Takahasi and Wakimoto [76]. Through an independent work, they proved that the sample mean of the ranked set sampling (RSS) is an unbiased estimator of the population mean with smaller variance as compared to sample mean of SRS with same sample size.

In just about a year after the work of Takahasi and Wakimoto [76], Takahasi [73] this time around alone, reconsidered the problem in situation where the elements within each set are correlated. In his work, he proposed a model and an estimator of the population mean. The relative efficiencies of his estimators for some distribution were also computed. Takahasi [74] went further with the modification of RSS by considering a situation where elements are randomly selected and measured before their position in a rank is determined.

Where the earlier works were assuming perfect ranking, Dell and Cluster [22] studied the case in which ranking may not be perfect. They showed that regardless of the error in ranking, mean of RSS is an unbiased estimator of the population mean and that the efficiency of the RSS estimator decrease with increasing ranking errors. Also noted in their work is that even with the error in ranking, the RSS estimator is still more efficient than that of the SRS using same sample size. In other words

$$\frac{Var\left(\bar{X}_{srs}\right)}{Var\left(\bar{X}_{rss}\right)} \geq 1, \tag{1.1}$$

where $\bar{X}_{srs}$ and $\bar{X}_{rss}$ are the estimators of the population mean based on SRS and RSS respectively. Equality holds in situation where judgment ranking is very poor to produce random sample.

The selection of elements for the estimation of the population mean using a procedure known as selective probability matrix (SPM) was proposed by Yanagawa and Shirahata [78]. The SPM is an n by m matrix of probabilities $\left\{P_{ij} : i = 1,2,\ldots,n;\ j = 1,2,\ldots,m\right\}$ satisfying the condition $\sum_{j=1}^{m} P_{ij} = 1$ for $i = 1,2,\ldots,n$. They showed that their estimator for the population mean is a generalization of the estimator proposed by Takahasi and Wakimoto [76] and that it is an unbiased estimator if SPM satisfies

$$\sum_{i=1}^{n} P_{ij} = \frac{m}{n} \quad ; \quad j = 1,2,\ldots,m \tag{1.2}$$

Stokes [68] studied a situation where the variable of interest X may not easily be measured or ordered but there is a concomitant variable Y which is correlated with the variable of interest X that can readily be ordered. A sampling method based on concomitant variable Y was proposed and observed that the precision of a population mean estimator depends on how strong the relationship between the $X's$ and $Y's$ is. She noted that the mean estimator is equivalent to McIntyre [34] estimator if the correlation coefficient $\rho = 1$ and equals SRS estimator if $\rho = 0$.

Stokes [69] in her study of population variance σ^2 using RSS data proposed an estimator which she showed to be asymptotically unbiased for large sample size. Because of the difficulty in ranking the units for very large sample size, large number of cycles was suggested. She also proved that the estimator of the population variance based on RSS is more efficient than that of SRS using sample size. In other words

$$\frac{Var\left(s^2\right)}{MSE\left(\hat{\sigma}^2_{rss}\right)} \geq 1, \tag{1.3}$$

where s^2 and $\hat{\sigma}^2_{rss}$ are the estimators of the population variance using SRS and RSS respectively. In this case, equality holds when judgment in ranking the units is so poor as to produce random sample.

Discussing the unpublished work of Miller and Griffiths further, Yanagawa and Chen [77] studied their procedure which is similar to that of Yanagawa and Shirahata [78] and produced an estimator for a population mean which they showed to be an unbiased when

$$\sum_{i=1}^{r}\left\{P_{ij} + P_{i(m+1-j)}\right\} = \frac{m}{n} \tag{1.4}$$

for $j = 1, 2, \ldots, m$. Where $n = 2r$ in Yanagawa and Shirahata [78] procedure and n & m are not necessarily equal. They showed that the new procedure is considerably more efficient than those of McIntyre [34] and Yanagawa and Shirahata [78] when n and m are not equal. However, they become the same with the equality of n and m.

Ridout and Cobby [57] observed that apart from errors involved in ranking the variable of interest, another source of error due to non-random selection of sets can arise in the practical implementation of RSS. The effects of such error on the relative efficiency of RSS estimators were studied and with an aid of example were able to show that the relative precision reduces more rapidly with increasing non-randomness in sampling as compared to errors in ranking the variable of interest.

In the study of the estimation of the cumulative distribution function (cdf) of a population, Stokes and Sagar [71] proposed an empirical distribution function for RSS which was shown to be an unbiased estimator for the population cdf. Even with errors in ranking, they show that the RSS estimator for cdf is relatively more efficient than that of SRS. The need and how to improve the existing SRS confidence interval for cdf using RSS empirical cumulative distribution and the Kolmogorov-Smirnov statistic were also discussed.

Muttlak and McDolnald [49] considered a two-phase sampling procedure where, in the first phase, units are selected with the probability proportional to size for each unit, and in the second phase, units are selected using the procedure of RSS. They showed that the efficiency of their estimators for the population mean and size were considerably more effective than those of SRS irrespective of whether there is error in ranking or not.

Muttlak and McDolnald [50] proposed a two-stage sampling procedure using the line intercept method to select the units in the first stage and for the second stage, the RSS procedure in combination with size biased probability was employed to select the units. They suggested estimators based on RSS for density, cover and total amount of some variables of interest and proved that their estimators were unbiased, and with an aid of practical example show that their estimators dominate those of the regular SRS.

Bohn and Wolfe [13] in the study of two-sample location problem for RSS data developed a nonparametric test for ranked set samples using their empirical distribution function. They proposed estimation and testing procedures which were independent of known distributions and showed that an improved form of the standard Mann-Whitney-Wilcoxon scheme can readily be achieved using their approach as compared to the regular case base on SRS.

Kvam and Samaniego [31] pointed out that RSS may occur naturally in life testing experiments and suggested some circumstances under which the RSS estimators could be improved uniformly. Also suggested were the RSS estimators for the unbalanced cases as well as sufficient conditions for inadmissibility.

On the correlation between the variable of interest X and its concomitant variable Y, Patil et al [51] compared the RSS and the regression estimator assuming that both X and Y follow a bivariate normal distribution. They showed that the RSS estimator is more efficient if the correlation coefficient $\rho \leq 0.85$ and that the regression estimator has a upper hand if $\rho \geq 0.85$.

In the study of RSS from a finite population, Patil et al [52] supplied explicit expressions for the variance and relative precision of the RSS estimator for several set sizes when the population follows a linear or quadratic trend. They compared the performance of RSS with that of systematic and stratified random sampling and noted that the RSS was more superior in some cases.

Kvam and Samaniego [32] studied and prove the existence and uniqueness of the nonparametric maximum likelihood estimator for a distribution function and gave a general numerical procedure which converges to their proposed estimator. While the procedure supplied by Stokes and Sager [71] does not apply to situation where the RSS is unbalanced, they modified their method to suit this case and went ahead to show the superiority of their procedure over those proposed by Stokes and Sager [71] when RSS is balanced.

Patil et al [53] classified the work carried out in area of RSS into three groups namely: theory, methods and application. The review of these various aspects in a single unified notation was carried out with the performance of RSS compared to that of SRS in determining the level of contamination at a hazardous waste site was illustrated. They also demonstrated the use of RSS methods for improving the formation of composite samples.

Based on the improved estimators of the normal mean μ given by Sinha et al [66], Shen [64] used RSS to derive tests for μ when the when the variance is known. He showed that under this scheme, several improved tests can be constructed, all of which are more powerful than the traditional normal test.

In a similar study of two-stage sampling plan involving RSS combined with line intercept suggested by Muttlak and McDolnald [50], Muttlak [37] applied the procedure for the estimation of coverage, density and total number of stems per unit area of rose rock (*Cistus Villosns*) in a study area in Jordan.

Stoke [72] considered the location scale distribution, $F\left[(x-\mu)/\sigma\right]$ in which she estimated the population mean μ and standard deviation σ using the methods of maximum likelihood estimation (MLE) and best linear unbiased estimation (BLUE). She studied a general method for finding BLUE of these parameters using RSS and found them to be as efficient as MLE for some distribution, and poor for some cases.

In his study of parameter estimation in simple linear regression using RSS, Muttlak [38] showed that ranking either on the dependent or independent variables increases the reliability of RSS estimators as compared to SRS estimators. He also showed that if ranking is on independent variable and the correlation between the dependent and independent variables is low, $\rho < 0.25$, then the RSS procedures are not important.

Bohn [12] studied some nonparametric procedures for RSS data which includes: empirical distribution function, the two-sample location setting, the sign test, and the signed-rank test. He considered the estimation of the distribution function in a more general setting, and for each of the above settings, she discussed the similarities and differences in the property of RSS procedures.

Muttlak [40] proposed a modification of RSS namely; paired ranked set sampling (PRSS). He suggested that the procedure could be used in some areas of application instead of RSS to increase the efficiency of the estimators relative to SRS. Estimators for the population mean under this sampling plan were proposed and shown to more efficient than those of SRS.

Sinha et al [66] in their study estimated the parameters of the normal and exponential distributions using RSS and some of its modifications. A best linear unbiased estimator for full and partial RSS was proposed for each of the parameter. For partial RSS, the least number of cycles for which the proposed estimators dominate the SRS estimators were found.

Abu-Dayyeh and Muttlak [4] in their work showed that the hypothesis tests based on RSS are much better than uniformly most powerful test (UMPT) and the likelihood ratio test (LRT) incase of exponential distribution under SRS. Same conclusion was drawn for UMPT in case of uniform distribution.

Koti and Buba [30] studied the sign test using RSS and for some continuous distribution, showed that this test based on RSS is much better than a similar test using SRS. The effects of imperfect judgment on the test were discussed and concluded that it may lead to greater percentage of the probability of type I error for RSS sign test than the SRS sign test.

On the model of one-way layout, Muttlak [39] used the RSS method to increase the efficiency of the parameter estimators relative to SRS. Muttlak [41] showed that using RSS again, the estimators of the parameters of a multiple regression model are more efficient than the corresponding SRS parameter estimators. In the case of ratio estimator, Samawi and Muttlak [61] demonstrated the result of Muttlak [41].

Samawi et al [60] introduced an extreme ranked set sampling (ERSS). They noted this procedure could readily be applied in practical situation as compared to RSS. Estimators for the population mean were proposed and they showed that the efficiency of this method is greater than that of the SRS.

Muttlak [42] proposed another modification of RSS called median ranked set sampling (MRSS) to overcome or reduce the loss of efficiency in RSS due to errors in ranking the units observed by Dell and Cluster [22]. He suggested estimator for the population mean which is unbiased for symmetric distributions and biased for others. He noted that his estimator for the population mean does better than the McIntyre [34] estimator for some distributions. The effects of errors in ranking in reducing the efficiency of the estimators under MRSS were also studied.

Bohj [11] proposed linear unbiased estimators of the location and scale parameters of the extreme value distribution under RSS and showed that these estimators are better than the ordered least square estimator. He noted that his estimator for the population mean performed better than the usual RSS estimator.

Patil et al [54] examines the effect of the set size on the performance of RSS for estimation of sample mean. He showed that the performance of RSS is monotone increasing with the set size for the wide class of ranking models that satisfy coherence, the ranking on a set is consistence with the ranking on every superset.

On the problem of estimation of the variance of a normal population based on balanced or unbalanced RSS, Yu et al [79] proposed several methods for estimating the population variance. They pointed out that some proposed estimators were better than the ordinary Skokes-modified unbiased estimator for single cycle with multiple cycles achieving the smallest variance.

Considering samples drawn from a finite population without replacement, Takahasi and Futasuya [75] studied the concepts of likelihood ratio dependence and negatively regression dependence. The dominance of RSS estimators over those of regular SRS was demonstrated.

Muttlak and Abu-Dayyeh [46] studied the testing of some hypothesis about the mean μ and variance σ^2 of the normal distribution under RSS. They showed that the normal mean and variance using RSS were more powerful than those based on SRS.

Employing the method of concomitant variable in ranking, Muttlak [44] used MRSS to estimate the population mean for the variable of interest and showed that the approach is more efficient than using the method of RSS. He also showed that MRSS estimators dominate the regression estimators for most case unless if the correlation between the auxiliary variable and the variable of interest in the regression model is more than 0.9.

Muttlak [43] considered the problem of two-phase sampling procedure in Muttlak and McDonald [49] using MRSS. He noted that the MRSS could be used to reduce the error in ranking and pointed out that the relative efficiency for estimating the population mean as well as the population size are generally better than those of SRS and also dominates RSS for some distributions.

On the estimation of the population mean and variance using paired ranked set sampling (PRSS), Hossain and Muttlak [27] showed that PRSS estimators dominates those of the SRS, RSS and minimum variance linear unbiased estimators (MVLUE). They also showed that even with error in ranking the variables of interest, the estimators using PRSS method are more efficient than the above mentioned methods for normal distribution.

Al-Saleh and Al-Kadiri [6] suggested an extension of RSS namely; double ranked set sampling (DRSS). They proposed an estimator for the population mean and showed that DRSS estimator dominates the SRS and RSS estimators. Using the idea of degree of distinguishability between the sample observations, they showed that ranking in the second stage is much easier than in the first stage.

In his study of regression-type estimators based on MRSS and ERSS for estimating the population mean of variable of interest, Muttlak [45] considered the RSS based regression-type estimators proposed by Yu and Lam [79]. He showed that when the concomitant variable and the variable of interest jointly follow a normal distribution, then the regression-type estimator of the population mean using ERSS is more efficient than those of SRS, RSS and MRSS.

Kaur et al [29] suggested the RSS version of the sign test for testing the hypothesis concerning the quantiles of a population characteristic. Considering both equal and unequal allocations, they obtained the relative performance of different allocations in terms of Pitman’s asymptotic relative efficiency. Noting the allocation that maximizes the efficacy for each quantile, they showed that it is independent of the population size.

Al-Saleh and Al-Omari [7] proposed a generalization of RSS that increases its efficiency for a fixed sample size, the multistage ranked set sampling (MSRSS). They pointed out that the steady state efficiency and the limiting efficiency as the number of stages goes to infinity, varies from distribution to distribution. They showed that the relative efficiency based on their proposed procedure is always greater than one.

1.3 Thesis Organization

The rest of this thesis is organized in the following way. In chapter 2, we present the fundamental concepts on which all our work in this thesis is built. In chapter 3, some modifications to the new sampling technique -double ranked set sampling (DRSS) were suggested. In chapter 4, attempts were made to construct control charts for monitoring a shift in process mean using DRSS and its proposed modifications. In chapter 5, we develop control charts for monitoring process standard deviation using some modifications of RSS as well as charts for detecting both shift in process mean and standard deviation. In chapter 6, we gave the implementation of some of the newly suggested control chart using real life data. Finally in the last chapter, we summarized and discussed the results of the whole thesis.

Chapter 2

SAMPLING METHODS AND PRELIMINARIES

2.1 Introduction

In this chapter, we discuss some basic sampling techniques -simple random sampling, ranked set sampling, median ranked set sampling, extreme ranked set sampling and double ranked set sampling as well as quality control chart preliminaries that form bases for our work in this thesis.

2.2 Sampling Methods

Most often, the direct observation of every individual in the target population or every output of industrial machines, etc, is laborious, expensive, time consuming and sometimes even impossible. Because of this, researchers often collect representative units from a subset of the target population – *a random sample* - and use those observations to make inferences about the entire population. Such a process of selecting only a part of the population under study is known as random sampling. In that case, the researcher's conclusions from the sample are applicable to the entire population.

2.2.1 Simple Random Sampling (SRS)

Simple random sampling can be defined as a sampling technique which involves the drawing of n units from a population of size N in such a way that every possible sample of the population has the same chance of being selected. The sample thus obtained is called a simple random sample. See Scheaffer et al [63] for more detail.

To draw a simple random sample of size n from a population of size N, the units of the entire population are listed from 1 to N. A unit of the population is selected to be included in the sample based on the outcome from the table of random numbers or a computer program that produces such a table. In other words, a unit is chosen if the selected random number coincides with the list number of the unit. Sampling could be with replacement or without replacement.

Let $X_1, X_2, \ldots, X_n$ be a simple random sample of size n. Then the unbiased estimator of the population mean, see Scheaffer et al [63], is defined as

$$\bar{X}_{srs} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{2.1}$$

and the variance of $\bar{X}_{srs}$ for infinite population is given by

$$Var\left(\bar{X}_{srs}\right) = \frac{\sigma^2}{n}, \tag{2.2}$$

where σ^2 is the population variance and is usually estimated by the sample variance

$$s^2 \frac{1}{(n-1)}\sum_{i=1}^{n}\left(X_i - \bar{X}\right) \tag{2.3}$$

2.2.2 Ranked Set Sampling (RSS)

As proposed by McIntyre [34], the method of ranked set sampling (RSS) can be summarized as follows. Select a random sample of size n^2 units from target population and randomly partition the sample into n sets each of size n as shown in Figure 2.1a. The units within each set are then ranked with respect to a variable of interest. Then the n measurements are obtained by taking the smallest unit from the first set, second smallest from the second set. The procedure continues in this manner until the largest unit is been selected from n^{th} set. The diagonal of Figure 2.1c represent our *single-cycled* (i.e. $m = 1$) ranked set samples in this case. The cycle may be repeated m times until nm units have been measured. Thus, the nm units form the RSS sample data.

Let $X_{(i:n)j}$ denote the i^{th} order statistic from the i^{th} sample of size n in the j^{th} cycle, then the unbiased estimator for the population mean, see Takahasi and Wakimoto [76], is defined as

$$\bar{X}_{rss} = \frac{1}{nm} \sum_{j=1}^{m} \sum_{i=1}^{n} X_{(i:n)j} \tag{2.4}$$

and the variance of $\bar{X}_{rss}$ is given by

$$Var\left(\bar{X}_{rss}\right) = \frac{1}{nm} \sum_{i=1}^{n} \sigma^2_{(i:n)}, \tag{2.5}$$

where $\sigma^2_{(i:n)} = E\left(\left(X_{(i:n)} - E\left(X_{(i:n)}\right)\right)^2\right)$ is the population variance of the i^{th} order statistic.

Right from the early works of McIntyre [34], Takahasi and Wakimoto [76] and subsequent adjustments discussed in section 1.2.2, it is noted that the variance of the RSS mean $Var(\bar{X}_{rss})$ is smaller than that of the SRS, $Var(\bar{X}_{srs})$. In other words, the population mean estimated by the RSS mean $\bar{X}_{rss}$ is more efficient than the one estimated by SRS mean $\bar{X}_{srs}$.

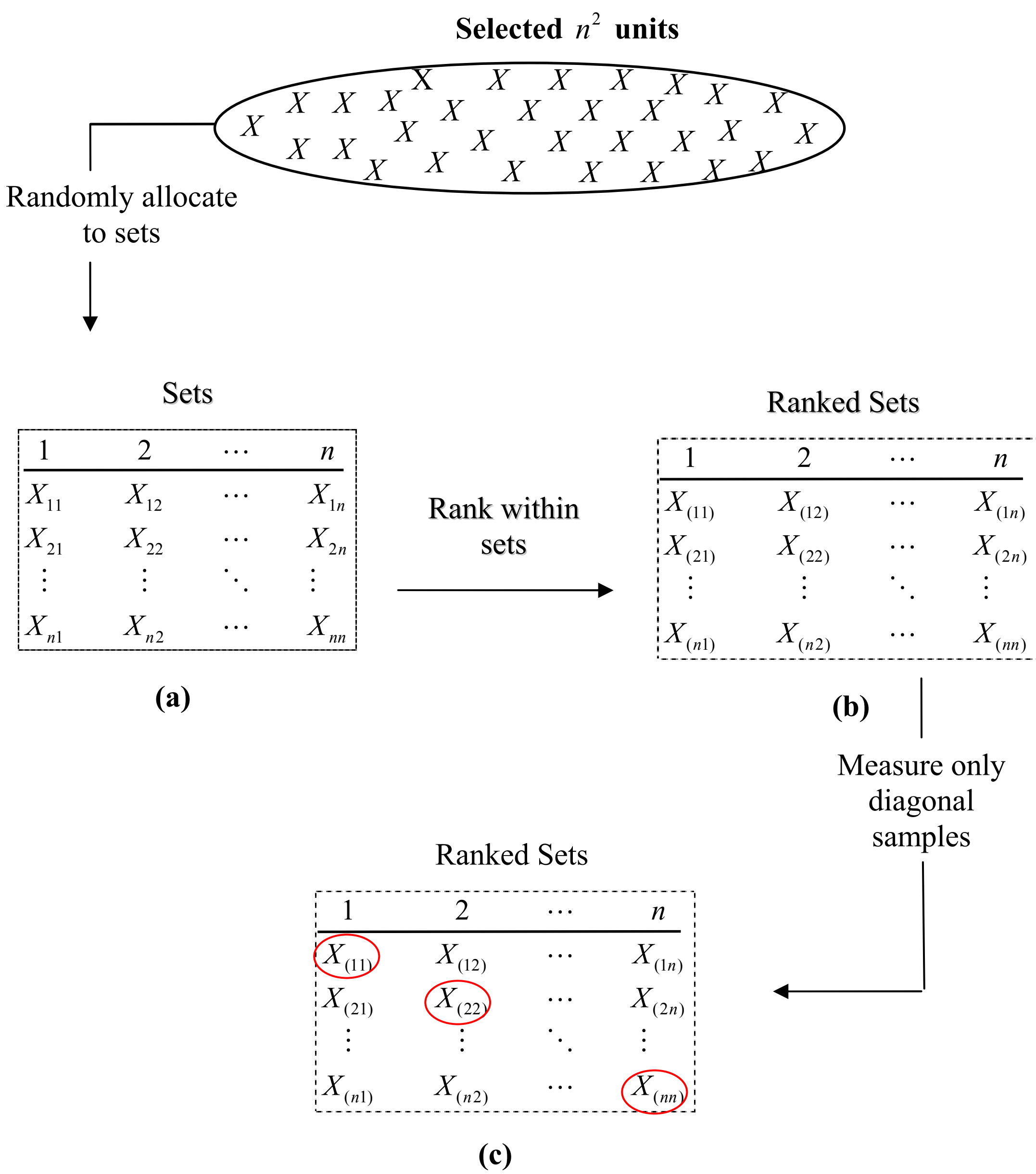

Figure 2.1: Setup of a Ranked Set Sampling scheme.

2.2.3 Median Ranked Set Sampling (MRSS)

The method of median ranked set sampling (MRSS) proposed by Muttlak [42] can be summarized as follows. Randomly select a sample of size n^2 units from target population and partition the sample into n sets each of size n and rank the units of each set with respect to a variable of interest. The n measurements are then obtained depending on whether the set size is even or odd. For odd set sizes, select the median value for measurement from each ranked set (i.e. the $\left((n+1)/2\right)^{th}$ smallest rank). And for the even set sizes, select the $\left(n/2\right)^{th}$ smallest element from the first $n/2$ sets and select $\left((n+2)/2\right)^{th}$ smallest element from the remaining $n/2$ sets. The cycle may be repeated m times until nm units have been measured. Thus, the nm units form the MRSS sample data.

Let $X_{(i:m)j}$ represent the i^{th} median from the i^{th} set of size n in the j^{th} cycle if the set size is odd. Also let the same notation represent the $\left(n/2\right)^{th}$ order statistic the i^{th} set of size n $(i=1,2,\ldots,k=n/2)$ and the $\left((n+2)/2\right)^{th}$ order statistic the i^{th} set of size n $(i=k+1,k+2,\ldots,n)$ in the j^{th} cycle if the set size is even. Then the estimator for the population mean and its variance are respectively given by, see Muttlak [42]

$$\bar{X}_{mrss} = \frac{1}{nm}\sum_{j=1}^{m}\sum_{i=1}^{n} X_{(i:m)j} \tag{2.6}$$

$$Var\left(\bar{X}_{mrss}\right) = \frac{1}{nm}\sum_{i=1}^{n}\sigma^2_{(i:m)}, \tag{2.7}$$

where $\sigma^2_{(i:m)} = E\left[\left(X_{(i:m)} - E\left(X_{(i:m)}\right)\right)^2\right]$ is the population variance of the i^{th} order statistic.

The variance of MRSS mean $Var\left(\bar{X}_{mrss}\right)$ is less $Var\left(\bar{X}_{srs}\right)$, variance of simple random sample mean if the underlying distribution is symmetric. If however the distribution under consideration is not symmetric, then the mean square error of $\bar{X}_{mrss}$ defined by equation (2.8) is less than $Var\left(\bar{X}_{srs}\right)$.

$$MSE\left(\bar{X}_{mrss}\right) = Var\left(\bar{X}_{mrss}\right) + \left(bias\right)^2 \tag{2.8}$$

where $bias = \mu - E\left(\bar{X}_{mrss}\right)$, see Muttlak [42].

2.2.4 Extreme Ranked Set Sampling (ERSS)

The extreme ranked set sampling (ERSS) as studied by Samawi et al [60] can be summarized in the following way. Randomly select n^2 units from the population under consideration and divide the sample into n sets each of size n and rank the units of each set with respect to a variable of interest. Here also, the n measurements are obtained depending on whether the set size is even or odd. For even set sizes, select the smallest unit from the first $n/2$ sets and largest unit from the other $n/2$ sets for measurement. If on the other hand the set size is odd then, select the smallest unit from the first $(n-1)/2$ sets and largest unit from the second $(n-1)/2$ sets and finally the median from the remaining set for measurement. The cycle may be repeated m times until nm units have been measured. The nm units thus, form the ERSS sample data.

Let $X_{(i:e)j}$ stand for the smallest of the i^{th} set $(i = 1, 2, \ldots, v = n/2)$ and the largest of the i^{th} set $(i = v+1, v+2, \ldots, n)$ of size n in the j^{th} cycle if the set size is even. Let the same notation also stand for the smallest of the i^{th} set $(i = 1, 2, \ldots, w = (n-1)/2)$, the largest of the i^{th} set $(i = w+1, w+2, \ldots, n)$ and the median of the i^{th} set $(i = (n+1)/2)$ of size n in the j^{th} cycle if the set size is odd. The estimator of the population mean and its variance, see Samawi et al [60] are given respectively by

$$\bar{X}_{erss} = \frac{1}{nm}\sum_{j=1}^{m}\sum_{i=1}^{n} X_{(i:e)j} \tag{2.9}$$

$$Var\left(\bar{X}_{erss}\right) = \frac{1}{nm}\sum_{i=1}^{n}\sigma^2_{(i:e)}, \tag{2.10}$$

where $\sigma^2_{(i:e)} = E\left[\left(X_{(i:e)} - E\left(X_{(i:e)}\right)\right)^2\right]$.

The variance of $\bar{X}_{erss}$ have also been shown to less than that of $\bar{X}_{srs}$ if the underlying distribution is symmetric. And for the case of asymmetric distribution, the mean square error of $\bar{X}_{erss}$ given by

$$MSE\left(\bar{X}_{erss}\right) = Var\left(\bar{X}_{erss}\right) + \left(bias\right)^2 \tag{2.11}$$

where $bias = \mu - E\left(\bar{X}_{erss}\right)$, is less than $Var\left(\bar{X}_{srs}\right)$. See Samawi, et al [60].

2.2.5 Double Ranked Set Sampling (DRSS)

The method of double ranked set sampling (DRSS) as proposed by Al-Saleh and Al-Kadiri [6] can be summarized as follows. Assuming the cycle is repeated only once, i.e. m = 1, randomly select n^3 elements from the target population and divide them randomly into *n* sets each of size n^2 elements as shown in Figure 2.2a. The procedure of ranked set sampling (RSS) is then applied on each of the set to obtain the n sets of ranked set samples of size n each, Figure 2.2b. These ranked set samples are collected together to form n set of elements each of size n, as can be seen in Figure 2.2c. The RSS procedure is then applied again on this set to obtain a second stage RSS. The whole cycle may be repeated m times to yield a sample size nm. These nm units thus, form the double ranked set samples.

Let $Y_{(i:n)j}$ denote the i^{th} order statistic from the i^{th} sample of size n of a RSS data in the j^{th} cycle of size m. Then the unbiased estimator of the population mean using DRSS data based on j^{th} cycle as proposed by Al-Saleh and Al-Kadiri [6] is given by

$$\overline{Y}_{drssj} = \frac{1}{n}\sum_{i=1}^{n} Y_{(i:n)j} \; ; \qquad j = 1, 2, \ldots m. \tag{2.12}$$

And the variance of $\overline{Y}_{drssj}$ is given to be

$$Var\left(\overline{Y}_{drssj}\right) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma^{*2}_{(i:n)} \tag{2.13}$$

where $\sigma^{*2}_{(i:n)} = E\left[\left(Y_{(i:n)} - E\left(Y_{(i:n)}\right)\right)^2\right]$ is the population variance of the i^{th} order statistic from RSS data.

The relative precision of DRSS with respect to both simple random sampling (SRS) as well as ranked set sampling (RSS) have been proven to be greater than or equal to one even without increasing the sample size n, see Al-Saleh and Al-Kadiri [6]. That is

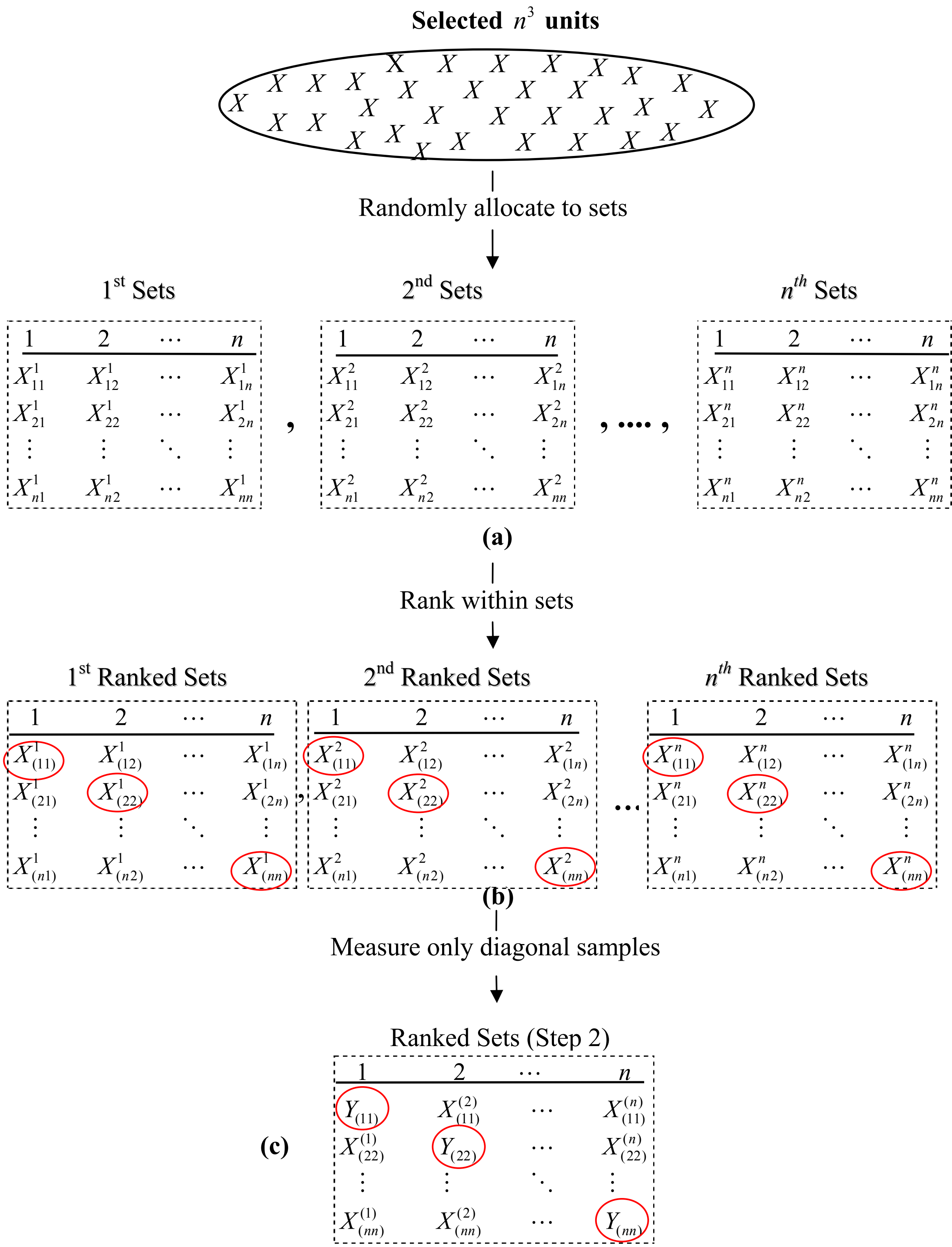

Figure 2.2: Setup of a Double Ranked Set Sampling scheme.

$$RP = \frac{Var\left(\bar{X}_{srs}\right)}{Var\left(\bar{Y}_{drss}\right)} \geq \frac{Var\left(\bar{X}_{rss}\right)}{Var\left(\bar{Y}_{drss}\right)} \geq 1. \tag{2.14}$$

Equality holds in cases where judgment ranking is poor enough to produce a simple random sample. Ranking in the second stage to obtain DRSS data have been shown to be much easier than ranking in the first stage which yields the RSS data. And that the new method is cost effective and yields accurate estimator for the population mean.

2.3 Control Chart Preliminaries

The application and success of a control chart largely depends on a good sampling method as it involves drawing of samples of fixed size n from a production process at regular sampling intervals. The values $X_1, X_2, \ldots, X_n$ that can be observed from the quality characteristic that is been monitored are usually summarized in the sample vector X, which are either used in their original form or are condensed to a sample statistic such as the sample mean, sample range or sample standard deviation.

A control chart consists of three horizontal lines as shown in Figure 2.3. The *center line* (CL) represents the average value of the quality characteristics taken from a pre-run of the manufacturing process in state of statistical control. The other two lines are called the *upper control limit* (UCL) and *lower control limit* (LCL). The UCL and LCL are often calculated in such a way that nearly all the sample points are between the two lines when the process is in the state of control. Most often, the sample points on a control chart are connected with straight lines for easy visualization of over time evolvement of sequence of points.

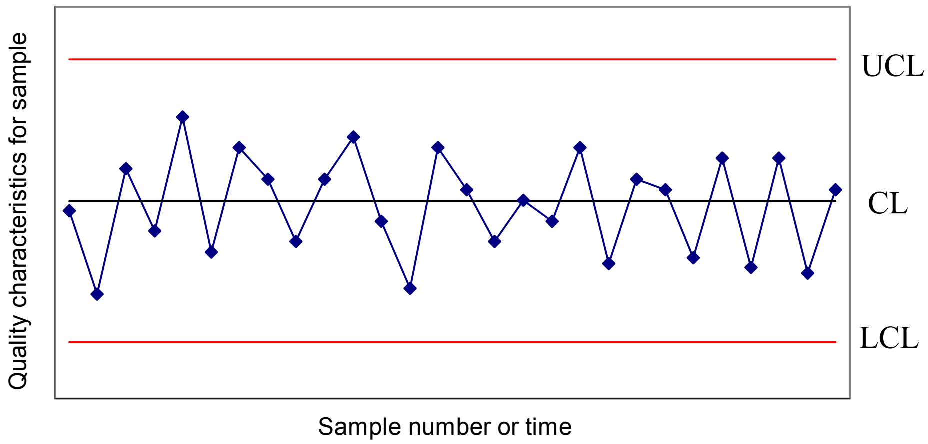

Figure 2.3: A typical control chart

In applying a control chart, there are three different possible outcomes for each sample. It is either the observed value lies within the warning limits (i.e. inner limits usually at 2-sigma) between the warning limits and control limits or outside the control limits. But because the use of warning limits increase risk of false alarm, see Montgomery [36], the following decision rules are often used in real life situations.

- Rule 1: *The sample points lies between the control limits*

  Here the manufacturing process is assumed to be in state of statistical control, and as such, it is not necessary to take any form of action. Thus, the process is allowed to continue as it was.

- Rule 2: *The sample points lies on or outside the control limits*

  If this happen, it will serve as evidence that the manufacturing process is no longer in a state of statistical control and an immediate intervention is necessary.

In other words, investigation and corrective action is required to find an assignable cause or causes responsible for this abnormality.

In addition to these decision rules, if the sample points are nonrandom in nature, even if they all lie within the control limits could be an indication that the process is out of control, see Mittag and Rinne [35].

2.3.1 Average Run Length (ARL)

The performance of a control chart can be measured using the average run length (ARL). It is the average number of points that must be plotted before an out-of-control signal is observed. For a classical chart, Figure 2.3, the ARL for in-control process often denoted by ARL_0 is given by

$$ARL_0 = \frac{1}{\alpha} \tag{2.15}$$

where α is the false alarm rate (i.e. *probability of type I error*). For example, the ARL for a stable in-control for a normally distributed process is expected to be approximately 370. That is, an average of 370 control points must be plotted before an out-of-control signal is observed.

Now if there is a shift in the process, then we expect the probability of an out-of-control signal to increase. Since, the probability of not getting an out-of-control signal if the process has shifted is β (i.e. *probability of type II error*) then the probability of getting an out-of-control signal would be $1 - \beta$. Thus, the ARL for an out of control process often denoted by ARL_1 is given by

$$ARL_1 = \frac{1}{1-\beta} \tag{2.16}$$

The $1 - \beta$ is usually called the power of statistical procedure. See Alwan [1] for more detail.

2.3.2 Variable Control Chart

Control charts can be classified into a pair of six categories, see Mittag and Rinne [35]. Since our interest on the measurement of quality characteristic is on a numerical scale, we make use of control chart for variables. The variable control charts are used when the quality is measure as variables, for example, length, weight, tensile strength etc. They have wider application in the monitoring of process mean and standard deviation. The monitoring of the shift in process means is often carried out using the control chart for mean $\bar{X}$ chart, $\bar{X} - S$ chart or $\bar{X} - R$ chart. While the process standard deviation can be monitored using the S chart, or a control chart for the range, called the R chart.

Chapter 3

SOME MODIFICATIONS TO DOUBLE RANKED SET SAMPLING

3.1 Introduction

In this chapter, we attempt to introduce some alternative sampling techniques to double ranked set sampling (DRSS) method which could be much easier to apply in practical situations. The suggested methods are median double ranked set sampling (MDRSS), double median ranked set sampling (DMRSS) and extreme double ranked set sampling (EDRSS).

3.2 Proposed Sampling Techniques

3.2.1 Median Double Ranked Set Sampling

In the MDRSS procedure, select n random samples each of size n^2 units from the population and apply the RSS procedure on each set to obtain n sets of ranked set samples of size n each. The procedure of MRSS is then applied on the resultant n samples of size n units. The whole process may be repeated m times to obtain a measurement of nm units. These nm units obtained form a MDRSS data of size n.

3.2.2 Double Median Ranked Set Sampling

The procedure of double median ranked set sampling (DMRSS) can be described as follows: Select n random samples each of size n^2 units from the population and apply the procedure of median ranked set sampling (MRSS) on each set of size n to obtain n sets of median ranked set sampling data of n size each. The same procedure is then re-applied on the newly formed median ranked set samples to obtain a second stage median ranked set samples. The whole process may be repeated m times to obtain a measurement of nm units. These nm units thus, form a double median ranked set sampling data of size n.

3.2.3 Extreme Double Ranked Set Sampling

The procedure of EDRSS can be summarized in the following way. Draw n random samples of size n^2 units from the population under consideration. Using the procedure of RSS on each of the set, results in n sets of ranked set samples each of size n. The procedure for ERSS is then applied on the resultant RSS data obtained in the first stage sampling. The cycle may be repeated m times to obtain nm elements. Thus, these nm samples form EDRSS data

3.3 Notations and Some Definitions

Suppose that the variable of interest X has probability density function $f(x)$, with absolute continues distribution function $F(x)$, mean μ and variance σ^2. Let

$X_1, X_2, \ldots, X_n$ be a simple random sample drawn from the continuous distribution $F(x)$. and let assume that the ranking is perfect, so that $X_{(i:n)j}$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$, is the i^{th} order statistic in j^{th} cycle of $F(x)$. Then the distribution of $X_{(i:n)j}$ which depends on the rank order i but not on cycle j, has a probability distribution function (pdf) and cumulative distribution function (cdf) given respectively by

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\left(F(x)\right)^{i-1}\left(1-F(x)\right)^{n-i} f(x), \tag{3.1}$$

$$F_{i:n}(x) = \int_{-\infty}^{x} f_{i:n}(y)dy. \tag{3.2}$$

Let $Y_{(i:n)j}$ $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$ denotes a random variable MDRSS, DMRSS or EDRSS samples of size n in j^{th} cycle. Suppose that $Y_{(i:d)j}$ has density function $g_{i:n}(x)$, with cumulative distribution function $G_{i:n}(x)$ where

$$\sum_{i=1}^{n} g_{i:n}(x) = nf(x) \text{ and } \sum_{i=1}^{n} G_{i:n}(x) = nF(x) \tag{3.3}$$

then, the $Y_{(i:d)j}$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$ are independent but not identically distributed, see Al-Saleh and Al-Kadiri [6] for more detail.

3.4 Median Double Ranked Set Sampling

As the method of MDRSS involves the measurement of the elements in the middle in step 2 sampling (i.e. step-one RSS and step-two MRSS.), it will be easy to apply in real life situation and is prone to less error in ranking when compared to DRSS.

3.4.1 Efficiency of MDRSS

The efficiency of the MDRSS in estimating the population mean will be compared to other methods discussed in Chapter 2. Let assume that the cycle is repeated only once, i.e. $m=1$, let $Y_{i((n+1)/2)}$ represent the median of the i^{th} ranked set sample $(i=1,2,\dots,n)$ when the set size is odd. If on the other hand the sample size is even, let $Y_{i:(n/2)}$ and $Y_{i((n+2)/2)}$ represent the $(n/2)^{th}$ and $((n+2)/2)^{th}$ order statistic of the i^{th} ranked set samples $(i=1,2,\dots,k=n/2)$ and $(i=k+1,k+2,\dots,n)$ respectively.

Let $\bar{X}_{srs}$, and $\bar{X}_{rss}$ be the sample means of simple random sampling (SRS) and ranked set sampling (RSS) respectively, all with the same sample sizes. The estimators of the population mean based on MDRSS may be defined in cases of odd and even sample sizes respectively as

$$\bar{Y}_{mdrss1}=\frac{1}{n}\sum_{i=1}^{n}Y_{i((n+1)/2)} \tag{3.4}$$

$$\bar{Y}_{mdrss2}=\frac{1}{n}\left(\sum_{i=1}^{k}Y_{i(n/2)}+\sum_{i=k+1}^{n}Y_{i((n+2)/2)}\right) \tag{3.5}$$

where $k=n/2$. The following are properties of the above estimators

(i) If the distribution is symmetric about the population mean μ then,

$\mathrm{E}\left(\bar{Y}_{mdrss1}\right)=\mu$, and $\mathrm{E}\left(\bar{Y}_{mdrss2}\right)=\mu$.

(ii) $Var\left(\bar{Y}_{mdrss1}\right)\le Var\left(\bar{X}_{rss}\right)$, and $Var\left(\bar{Y}_{mdrss2}\right)\le Var\left(\bar{X}_{rss}\right)$

(iii) If the distribution is not symmetric about the mean μ then,

$$MSE\left(\overline{Y}_{mdrss1}\right) \le Var\left(\overline{X}_{srs}\right) \text{ and } MSE\left(\overline{Y}_{mdrss2}\right) \le Var\left(\overline{X}_{srs}\right).$$

Proof:

To prove **(i)**: It is obvious that $\mathrm{E}\left(\overline{Y}_{mdrss1}\right) = \mu$ since the distribution is symmetric about μ. To show the second part, we consider

$$\mathrm{E}\left(\overline{Y}_{mdrss2}\right) = \mathrm{E}\left\{\frac{1}{n}\left(\sum_{i=1}^{k} Y_{i(n/2)} + \sum_{i=k+1}^{n} Y_{i((n+2)/2)}\right)\right\}$$

$$= \frac{1}{n}\left(\sum_{i=1}^{k} \mathrm{E}\left(Y_{i(n/2)}\right) + \sum_{i=k+1}^{n} \mathrm{E}\left(Y_{i((n+2)/2)}\right)\right)$$

$$= \frac{1}{n}\left(\sum_{i=1}^{k} \mu_{i(n/2)} + \sum_{i=k+1}^{n} \mu_{i((n+2)/2)}\right) \tag{3.6}$$

If the distribution is symmetric about μ, then $\mu_{i(n/2)} = \mu - c$ and $\mu_{i((n+2)/2)} = \mu + c$ for a fixed constant c. Therefore $\mathrm{E}\left(\overline{Y}_{mdrss2}\right) = \mu$. See Muttlak [42] and Al-Saleh and Al-Kadiri [6].

To prove **(ii)**, consider

$$Var\left(\overline{Y}_{mdrss1}\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var\left(Y_{i((n+1)/2)}\right) \tag{3.7}$$

and let $Z_1, Z_2, \ldots, Z_{md}, \ldots, Z_{n-1}, Z_n$ be the a RSS data, with Z_{md} denoting its median value. Then

$$Var\left(\overline{X}_{rss}\right) = Var\left(\frac{1}{n}\sum_{i=1}^{n} Z_i\right)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n} Var\left(Z_i\right) + \frac{1}{n^2}\sum_{i \ne r}^{n} Cov\left(Z_i, Z_r\right)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n} Var\left(Z_{(i)}\right) + \frac{1}{n^2}\sum_{i\neq r}^{n} Cov\left(Z_{(i)}, Z_{(r)}\right)$$

but $Var\left(Z_{(md)}\right) \leq Var\left(Z_{(i)}\right)$ for any i = 1, 2,…., *md*, …., *n*-1, *n*, see Sinha, et al [66]. Therefore,

$$Var\left(\bar{X}_{rss}\right) \geq \frac{1}{n^2}\sum_{i=1}^{n} Var\left(Z_{(md)}\right) + \frac{1}{n^2}\sum_{i\neq r}^{n} Cov\left(Z_{(i)}, Z_{(r)}\right)$$

$$\geq \frac{1}{n^2}\sum_{i=1}^{n} Var\left(Y_{i((n+1)/2)}\right) + \frac{1}{n^2}\sum_{i\neq r}^{n} Cov\left(Z_{(i)}, Z_{(r)}\right)$$

$$\geq Var\left(\bar{Y}_{mdrss1}\right) + \frac{1}{n^2}\sum_{i\neq r}^{n} Cov\left(Z_{(i)}, Z_{(r)}\right) \qquad (3.8)$$

But, $Cov\left(Z_{(i)}, Z_{(r)}\right) \geq 0$. See Lehmann [33] and Essary et al [24]. Thus,

$$Var\left(\bar{Y}_{mdrss1}\right) \leq Var\left(\bar{X}_{rss}\right) \qquad (3.9)$$

We can similarly show that

$$Var\left(\bar{Y}_{mdrss2}\right) \leq Var\left(\bar{X}_{rss}\right). \qquad (3.10)$$

To prove **(iii),** we consider

$$MSE\left(\bar{Y}_{mdrss1}\right) = Var\left(\bar{Y}_{mdrss1}\right) + \left(bias\right)^2$$

$$= Var\left(\bar{Y}_{mdrss1}\right) + \left(\mu - \mathrm{E}\left(\bar{Y}_{mdrss1}\right)\right)^2$$

$$\leq Var\left(\bar{X}_{rss}\right) + \left(\mu - \mathrm{E}\left(\bar{Y}_{mdrss1}\right)\right)^2 \qquad \text{(using equation 3.9)}$$

$$\leq \left\{Var\left(\bar{X}_{srs}\right) - \frac{1}{n^2}\sum_{i=1}^{n}\left(E\left(X_{(i:n)}\right) - \mu\right)^2\right\} + \left(\mu - E\left(\bar{Y}_{mdrss1}\right)\right)^2$$

But, the inequality

$$\frac{1}{n^2}\sum_{i=1}^{n}\left(E\left(X_{(i:n)}\right)-\mu\right)^2 \le \left(\mu-E\left(\bar{Y}_{mdrss1}\right)\right)^2 \tag{3.11}$$

holds for almost all the distribution if the sample size is small. This can be confirmed from the results in Table 3.1, see Muttlak [42]. Thus,

$$MSE\left(\bar{Y}_{mdrss1}\right) \le Var\left(\bar{X}_{srs}\right). \tag{3.12}$$

Similar argument for even case proves the second part.

3.4.2 Examples

Assume that the order statistics $X_{(i:n)}$, $(i=1,2,\dots,n)$ are from a distribution with pdf $f(x)$ and cdf $F(x)$. Then for a sample size of $n = 2$, the distribution of $Y_{(1:2)}$ and $Y_{(2:2)}$ are given respectively by, see Al-Saleh and Al-Omari [7]

$$\begin{aligned} G_{1:2}(x) &= 1-\left(1-F_{1:2}(x)\right)\left(1-F_{2:2}(x)\right)=2F(x)-2F^3(x)+F^4(x) \\ G_{2:2}(x) &= F_{1:2}(x)F_{2:2}(x)=2F^3(x)-F^4(x) \end{aligned} \tag{3.13}$$

The expected value and the variance of $Y_{(1:2)}$ and $Y_{(2:2)}$, are respectively given by

$$\begin{aligned} \mathrm{E}\left(Y_{(1:2)}\right) &= \int_{-\infty}^{\infty} x g_{(1:2)}(x)dx \\ \mathrm{E}\left(Y_{(2:2)}\right) &= \int_{-\infty}^{\infty} x g_{(2:2)}(x)dx \end{aligned} \tag{3.14}$$

$$\begin{aligned} Var\left(Y_{(1:2)}\right) &= \int_{-\infty}^{\infty} x^2 g_{(1:2)}(x)dx - \left\{\mathrm{E}\left(\bar{Y}_{(1:2)}\right)\right\}^2 \\ Var\left(Y_{(2:2)}\right) &= \int_{-\infty}^{\infty} x^2 g_{(2:2)}(x)dx - \left\{\mathrm{E}\left(\bar{Y}_{(2:2)}\right)\right\}^2 \end{aligned} \tag{3.15}$$

For $n = 3$, the distributions of $Y_{(1:3)}$, $Y_{(2:3)}$ and $Y_{(3:3)}$ are given by

$$\begin{aligned}
G_{1:3}(x) &= 1-\left(1-F_{1:3}(x)\right)\left(1-F_{2:3}(x)\right)\left(1-F_{3:3}(x)\right)\\
&= 3F(x)-9F^{3}(x)+12F^{4}(x)-9F^{5}(x)+12F^{6}(x)-15F^{7}(x)+9F^{8}(x)-2F^{9}(x)\\
G_{2:3}(x) &= 3F(x)-G_{1:3}(x)-G_{3:3}(x)\\
&= 9F^{3}(x)-12F^{4}(x)+9F^{5}(x)-21F^{6}(x)+30F^{7}(x)-18F^{8}(x)+4F^{9}(x)\\
G_{3:3}(x) &= F_{1:3}(x)F_{2:3}(x)\,F_{3:3}(x)=9F^{6}(x)-15F^{7}(x)+9F^{8}(x)-2F^{9}(x)
\end{aligned} \tag{3.16}$$

Again the expected values and the variances of $Y_{(1:3)}$, $Y_{(2:3)}$ and $Y_{(3:3)}$, are respectively

$$\begin{aligned}
\mathrm{E}\left(Y_{(1:3)}\right) &= \int_{-\infty}^{\infty} x g_{(1:3)}(x)dx\\
\mathrm{E}\left(Y_{(2:3)}\right) &= \int_{-\infty}^{\infty} x g_{(2:3)}(x)dx\\
\mathrm{E}\left(Y_{(3:3)}\right) &= \int_{-\infty}^{\infty} x g_{(3:3)}(x)dx
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
Var\left(Y_{(1:3)}\right) &= \int_{-\infty}^{\infty} x^{2} g_{(1:3)}(x)dx \ -\left\{\mathrm{E}\left(\overline{Y}_{(1:3)}\right)\right\}^{2}\\
Var\left(Y_{(2:3)}\right) &= \int_{-\infty}^{\infty} x^{2} g_{(2:3)}(x)dx \ -\left\{\mathrm{E}\left(\overline{Y}_{(2:3)}\right)\right\}^{2}\\
Var\left(Y_{(3:3)}\right) &= \int_{-\infty}^{\infty} x^{2} g_{(3:3)}(x)dx \ -\left\{\mathrm{E}\left(\overline{Y}_{(3:3)}\right)\right\}^{2}
\end{aligned} \tag{3.18}$$

We now compute the efficiency of proposed estimators for the population mean using MDRSS method with respect to SRS estimator given by

$$Eff\left(\overline{X}_{srs},\overline{X}_{mdrss}\right)=Var\left(\overline{X}_{srs}\right)\big/Var\left(\overline{X}_{mdrss}\right) \tag{3.19}$$

for three distributions namely: normal, uniform and exponential. Note that if the underlying distribution is not symmetric, we replace $Var\left(\overline{X}_{mdrss}\right)$ by $MSE\left(\overline{X}_{mdrss}\right)$ in equation (3.19).

1. Uniform Distribution, U(0,1)

Using the above relations for $Y_{(1:2)}$ and $Y_{(2:2)}$, i.e. when n = 2 we have approximately to five decimals, $[E\left(Y_{(1:2)}\right), E\left(Y_{(2:2)}\right)]$ = (0.3000, 0.7000) and $[Var\left(Y_{(1:2)}\right), Var\left(Y_{(2:2)}\right)]$ = (0.0433, 0.0433). Thus the efficiency of MDRSS with respect to SRS is 1.9231.

For n = 3, we have

$[E\left(Y_{(1:3)}\right), E\left(Y_{(2:3)}\right), E\left(Y_{(3:3)}\right)]$ = (0.2107, 0.5000, 0.7893) and $[Var\left(Y_{(1:3)}\right),\ Var\left(Y_{(2:3)}\right)$ $Var\left(Y_{(3:3)}\right)]$ = (0.02400, 0.0346, 0.02340) and the corresponding efficiency for the median is 2.4063.

2. Normal Distribution, N(0,1)

We have for n = 2, $[E\left(Y_{(1:2)}\right), E\left(Y_{(2:2)}\right)]$ = (-0.6632, 0.6632) and $[Var\left(Y_{(1:2)}\right),\ Var\left(Y_{(2:2)}\right)]$ = (0.5602, 0.5602). Hence the efficiency of MDRSS with respect to SRS is 1.7852 (using numerical integration)

For n = 3, we have $[E\left(Y_{(1:3)}\right), E\left(Y_{(2:3)}\right), E\left(Y_{(3:3)}\right)]$ = (-0.9646, 0.0000, 0.9646) and $[Var\left(Y_{(1:3)}\right),\ Var\left(Y_{(2:3)}\right)\ \ Var\left(Y_{(3:3)}\right)]$ = (0.4313, 0.2767, 0.4313) and the corresponding efficiency for median is 3.6145 (using numerical integration).

3. Exponential Distribution, Exp(1)

With the same formula, we have for n = 2, $[E\left(Y_{(1:2)}\right), E\left(Y_{(2:2)}\right)]$ = (0.4167, 1.5833) and $[Var\left(Y_{(1:2)}\right), Var\left(Y_{(2:2)}\right)]$ = (0.1458, 1.1736). Hence the efficiency of MDRSS with respect to SRS is 1.5158 (using numerical integration).

For n = 3, we have $[E\left(Y_{(1:3)}\right), E\left(Y_{(2:3)}\right), E\left(Y_{(3:3)}\right)]$ = (0.2599, 0.7802, 1.9599) and $[Var\left(Y_{(1:3)}\right),\ Var\left(Y_{(2:3)}\right)\ \ Var\left(Y_{(3:3)}\right)]$ = (0.0521, 0.2054, 1.2250). The corresponding efficiency for MDRSS is 2.8536. See Section 3.7 for more efficiency of MDRSS and those of other sampling methods.

3.5 Double Median Ranked Set Sampling

This new method has to do with the measurement of the elements in the middle both in first and second stage sampling. In other words, measure of the median of the medians. This proposed method will be easy to apply in practical situations and will also save time spent on ranking the units with respect to the variables of interest.

3.5.1 Efficiency of DMRSS

The efficiency of the DMRSS in estimating the population mean will be compared to other methods discussed in Chapter 2. Let assume $m=1,$ and let $Y^{*}_{i((n+1)/2)}$ be the median of the i^{th} median ranked set sample (i = 1, 2, …, n) when the set size is odd. In other words, the $((n+1)/2)^{th}$ order statistic of the i^{th} order median ranked set sample denotes DMRSS. If the set size is even, let $Y^{*}_{i(n/2)}$ and $Y^{*}_{i((n+2)/2)}$ be the $(n/2)^{th}$ and $((n+2)/2)^{th}$ order statistic of the i^{th} median ranked set samples (i = 1, 2, …, k=1/n) and (i = k+1, k+2, …, n) respectively.

The estimators of the population mean μ using DMRSS can be defined for the cases odd and even sample sizes respectively as

$$\overline{Y}_{dmrss1} = \frac{1}{n}\sum_{i=1}^{n} Y^{*}_{i((n+2)/2)} \tag{3.20}$$

$$\overline{Y}_{dmrss2} = \frac{1}{n}\left(\sum_{i=1}^{k} Y^{*}_{i(n/2)} + \sum_{i=k+1}^{n} Y^{*}_{i((n+2)/2)}\right) \tag{3.21}$$

where $k = n/2$. The following properties hold for estimators given in equations (3.20) and (3.21). If the distribution is symmetric about μ, then $\mathrm{E}\left(\overline{Y}_{dmrss1}\right)$ and $\mathrm{E}\left(\overline{Y}_{dmrss2}\right)$ are unbiased estimators population mean μ.

(i) $Var\left(\overline{Y}_{dmrss1}\right) \le Var\left(\overline{X}_{rss}\right)$, and $Var\left(\overline{Y}_{dmrss2}\right) \le Var\left(\overline{X}_{rss}\right)$

(ii) If the distribution is not symmetric about μ, then

$MSE\left(\overline{Y}_{dmrss1}\right) \le Var\left(\overline{X}_{srs}\right)$, and $MSE\left(\overline{Y}_{dmrss2}\right) \le Var\left(\overline{X}_{srs}\right)$

The proof of these properties follows immediately from Section 3.4.1.

3.5.2 Examples

Considering the case when $n = 3$, the distributions of $Y^{*}_{(1:3)}$, $Y^{*}_{(2:3)}$ and $Y^{*}_{(3:3)}$ are given respectively by

$$\begin{aligned} G_{1:3}(x) &= 3F_{1:3}(x) - 3F_{1:3}^{2}(x) + F_{1:3}^{3}(x) \\ &= 9F(x) - 36F^{2}(x) + 84F^{3}(x) - 126F^{4}(x) + 126F^{5}(x) \\ &= -84F^{6}(x) + 36F^{7}(x) - 9F^{8}(x) + F^{9}(x) \end{aligned}$$

$$G_{3:3}(x) = F_{3:3}^{3}(x) = F^{9}(x). \tag{3.22}$$

$$\begin{aligned} G_{2:3}(x) &= 3F_{2:3}^{2}(x) - 2F_{2:3}^{3} \\ &= 12F^{4}(x) - 36F^{5}(x) - 42F^{6}(x) + 108F^{7}(x) - 72F^{8}(x) + 16F^{9}(x) \end{aligned}$$

The expected values and the variance of $Y^*_{(1:3)}$, $Y^*_{(2:3)}$ and $Y^*_{(3:3)}$ are related to those in the previous example. And on the computation of the efficiency of proposed estimators for the population mean using DMRSS method with respect to SRS estimator given by

$$Eff\left(\bar{X}_{srs}, \bar{X}_{dmrss}\right) = Var\left(\bar{X}_{srs}\right) / Var\left(\bar{X}_{dmrss}\right) \tag{3.23}$$

where $Var\left(\bar{X}_{dmrss}\right)$ is replaced by $MSE\left(\bar{X}_{dmrss}\right)$ for asymmetric distribution, we consider as before three distributions: normal, uniform and exponential.

1. Uniform Distribution, U(0,1)

If we use the formula for the distributions of $Y^*_{(1:3)}$, $Y^*_{(2:3)}$ and $Y^*_{(3:3)}$, i.e. for the case $n = 3$ we have approximately the following results:

$[E\left(Y^*_{(1:3)}\right), E\left(Y^*_{(2:3)}\right), E\left(Y^*_{(3:3)}\right)]$ = (0.1000, 0.5000, 0.9000) and $[Var\left(Y^*_{(1:3)}\right)$, $Var\left(Y^*_{(2:3)}\right)$ $Var\left(Y^*_{(3:3)}\right)]$ = (0.0082, 0.0266, 0.0082). Measuring only the median value, the corresponding efficiency is 3.1301. See Section 3.7 for more detail.

2. Normal Distribution, N(0,1)

For $n = 3$, we have

$[E\left(Y^*_{(1:3)}\right), E\left(Y^*_{(2:3)}\right), E\left(Y^*_{(3:3)}\right)]$ = (-1.4453, 0.0000, 1.4453) and $[Var\left(Y^*_{(1:3)}\right)$, $Var\left(Y^*_{(2:3)}\right)$ $Var\left(Y^*_{(3:3)}\right)]$ = (0.4436, 0.2003, 0.4436) and the corresponding efficiency if only the median is measured is 4.9889.

3. Exponential Distribution, Exp(1)

For $n = 3$, we have $[E\left(Y^*_{(1:3)}\right), E\left(Y^*_{(2:3)}\right), E\left(Y^*_{(3:3)}\right)]$ = (0.1111, 0.7564, 2.8290) and $[Var\left(Y^*_{(1:3)}\right),\ Var\left(Y^*_{(2:3)}\right)\ Var\left(Y^*_{(3:3)}\right)]$ = (0.0124, 0.1428, 1.5398). The corresponding efficiency if only the median is measured is 3.1160. More efficiency of DMRSS is given in Section 3.7.

3.6 Extreme Double Ranked Set Sampling

The method of EDRSS can be carried out with less error in ranking, as it is always easy to identify the largest and the smallest elements within a sample. Performing such a task in the second stage (i.e. step-one RSS and step-two ERSS), will considerably reduce the amount of errors in ranking the units of the variable of interest.

3.6.1 Efficiency of EDRSS

The efficiency of EDRSS in estimating the population mean will be compared to other methods discussed in Chapter 2. Assuming $m = 1$, let $Y_{2i-1(1)}$ be the smallest of the i^{th} set of ranked set samples $(i = 1, 2, \ldots, n/2)$ and $Y_{2i(n)}$ be the largest of the i^{th} set of ranked set samples $(i = 1, 2, \ldots, n/2)$ for even set size n of samples. And for the case of odd sample sizes, let $Y_{i(1)}$ be the smallest of the i^{th} set of ranked set samples $(i = 1, 2, \ldots, w = (n-1)/2)$, $Y_{i(n)}$ be the largest of the i^{th} set of ranked set samples $(i = w+1, w+2, \ldots, n-1)$, and $Y_{n((n+1)/2)}$ be the median of the n^{th} ranked set sample.

If we let $\overline{X}_{srs}$, and $\overline{X}_{rss}$ denotes the sample means for SRS and RSS respectively, and from equal sample sizes, then the estimator of population mean using EDRSS can be defined for the cases of even and odd sample sizes respectively by

$$\overline{Y}_{edrss1} = \frac{1}{n}\left(\sum_{i=1}^{k} Y_{2i-1(1)} + \sum_{i=1}^{k} Y_{2i(n)} \right) \tag{3.24}$$

$$\overline{Y}_{edrss2} = \frac{1}{n}\left(\sum_{i=1}^{w} Y_{i(1)} + \sum_{i=w+1}^{n-1} Y_{i(n)} + Y_{n((n+1)/2)} \right). \tag{3.25}$$

where $k = n/2$ and $w = (n-1)/2$. If the underlying distribution is symmetric about μ then, we can easily show that $\overline{Y}_{edrss1}$ and $\overline{Y}_{edrss2}$ are unbiased estimators of the population mean μ. Table 3.1 indicates that $Var\left(\overline{Y}_{edrss1;2}\right) \le Var\left(\overline{X}_{rss}\right)$ if the underlying distribution is uniform or normal for both the odd and even cases.

3.6.2 Examples

Suppose that we have the same setup of Example 3.4.2, then we compute the efficiencies of the EDRSS estimators for the population mean with respect to SRS estimator i.e.

$$Eff\left(\overline{X}_{srs}, \overline{X}_{edrss}\right) = Var\left(\overline{X}_{srs}\right) / Var\left(\overline{X}_{edrss}\right) \tag{3.26}$$

where $MSE\left(\overline{X}_{edrss}\right)$ replaces $Var\left(\overline{X}_{edrss}\right)$ for asymmetric distribution. The earlier three distributions are once again considered and the summaries of the results for efficiencies of EDRSS for $n = 2,3,4,5$ are given in the next section.

3.7 Comments on the Efficiency of Proposed Sampling Methods

The use of DRSS for the estimation of the population mean will always increase the relative efficiency better than the RSS, see Al-Saleh and Al-Kadiri [6]. But its practical application most especially for sample size greater then five will not be an easy task because of the difficulty in ranking the units for the variables of interest. In other words, it is prone to errors in ranking and this could reduce its efficiency. For these reason, we introduce MDRSS, DMRSS and EDRSS, which will be easy to implement in the field.

The efficiencies of these new methods: MDRSS, DMRSS, EDRSS and those of RSS, MRSS, ERSS and DRSS are given in Table 3.1 for three distributions namely: uniform, normal and exponential. From the results in Table 3.1, we can deduce the following:

1. If the underlying distribution is uniform, then there is a gain in the efficiency of MDRSS, DMRSS and EDRSS estimators for different values of n. Observe that the DMRSS and EDRSS estimators dominates the estimators for the rest of the methods including DRSS. For example, if $n = 5$ the relative efficiency for estimating the population mean using DMRSS is 6.925, EDRSS is 6.998, as compared to 5.816 of DRSS.
2. For the case of normal distribution with mean zero and variance one, a general increase in efficiency for the method of MDRSS, DMRSS and EDRSS is observed. Also, the efficiency of DMRSS based estimator is twice the efficiency

of each of remaining methods, DRSS inclusive but except MDRSS. Example: If n = 5, the efficiency of DMRSS is 12.226 as compared to DRSS which has 4.462.

3. If the underlying distribution is not symmetric, as in the exponential distribution, there is a loss in the efficiency of the MDRSS, DMRSS and EDRSS estimators as the sample size increases. The method appears not to be doing better than the DRSS but better than SRS with MDRSS and DMRSS doing as well as RSS.

Distribution	N	Sampling Methods						
		RSS	MRSS	ERSS	DRSS	MDRSS	DMRSS	EDRSS
	2	1.500	1.500	1.500	1.923	1.923	1.923	1.923
Uniform	3	2.000	1.667	2.000	3.026	2.406	3.130	3.026
U(0,1)	4	2.500	2.083	3.125	4.711	4.073	5.514	5.587
	5	3.000	2.333	3.621	5.816	4.352	6.925	6.998
	2	1.467	1.467	1.467	1.785	1.785	1.785	1.785
Normal	3	1.914	2.229	1.787	2.633	3.615	4.992	2.633
N(0,1)	4	2.347	2.774	2.034	3.526	5.045	7.632	2.710
	5	2.770	3.486	2.234	4.462	7.323	12.226	3.421
	2	1.333	1.333	1.333	1.516	1.516	1.516	1.923
Exponential	3	1.636	2.250	1.636	2.024	2.854	3.116	2.024
Exp(1)	4	1.920	2.441	1.170	2.374	2.601	4.824	1.225
	5	2.190	2.230	1.444	3.375	2.189	2.226	1.601

Table 3.1: Relative efficiency for three distributions, for estimating the population mean using RSS, MRSS, ERSS, DRSS, MDRSS, DMRSS and EDRSS methods.

Chapter 4

CONTROL CHART FOR MONITORING THE PROCESS MEAN

4.1 Introduction

In this chapter, an attempt is made to develop control charts based on double ranked set sampling and its suggested modifications for monitoring a process to detect changes in the mean. The average run length (ARL) performance of these charts will be investigated and compared to the traditional control charts for the mean using simple random sampling (SRS) and other sampling techniques.

4.2 Shift in Process Mean

The average run length (ARL) assumes that the process is in the state of statistical control with mean μ_0 and standard deviation σ_0, and at certain point in time the process start to get out of statistical control with a shift in mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0 / \sqrt{n}$, Figure 4.1, see Montgomery [36] for more detail. Now, assuming that the process follows a normal distribution with mean μ_0 and variance σ_0^2 when the process is in the state of statistical control, the shift on the process mean is given by $\delta = \sqrt{n}\left|\mu_1 - \mu_0\right| / \sigma_0$. Note that if a point is outside the control limits when the process is in state of control i.e. $\delta = 0$, then it is a false alarm.

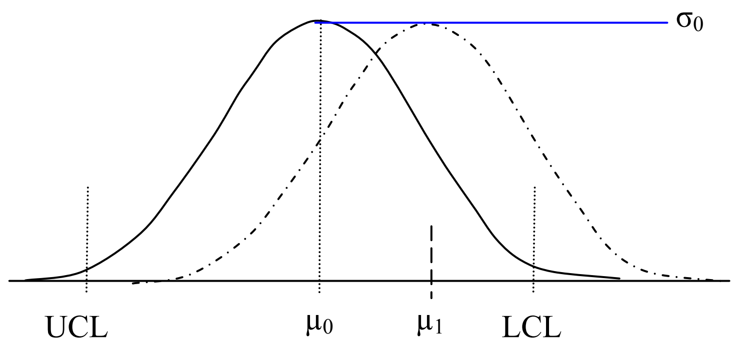

Figure 4.1: Shift in process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0 / \sqrt{n}$

4.3 Control Chart for Mean using SRS

Considering the Shewhart [65] control chart for mean using SRS, let X_{ij} for i = 1, 2,…, n and j = 1, 2,…., m denote the m samples each of size n and from a normal distribution with mean μ and variance σ^2. If both the population mean μ and variance σ^2 are known, then the sample mean is given by

$$\bar{X}_j = \frac{1}{n}\sum_{i=1}^{n} X_{ij} \; ; \qquad j = 1, 2, \ldots, m \tag{4.1}$$

can be plotted on the chart for mean

$$\begin{aligned} UCL &= \mu + 3\frac{\sigma}{\sqrt{n}} \\ CL &= \mu \\ LCL &= \mu - 3\frac{\sigma}{\sqrt{n}} \end{aligned} \tag{4.2}$$

where UCL, CL and LCL are the upper central limit, central limit and lower central limit respectively. The average run length (ARL) for this chart is as described in Section 2.3.1 of Chapter 2, see Salazar and Sinha [59].

4.4 Control Chart for Mean using RSS, MRSS or ERSS

The RSS, MRSS or ERSS mean of the j^{th} cycle denoted by $\bar{X}_{ssj}$, can be plotted on the control chart for mean based on their respective data as suggested by Muttlak and Al-Sabah [47], and Salazar and Sinha [59] as follows

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{X}_{ss}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\bar{X}_{ss}} \end{aligned} \tag{4.3}$$

where $\sigma_{\bar{X}_{ss}} = \sqrt{(1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:n)}}$ with the values of $\sigma^2_{(i:n)}$ being obtained from the table of order statistics for the standard normal distribution, see for example Harter and Balakrishnan [26].

4.5 Control Chart for Mean using DRSS

Let the $\bar{Y}_{drssj}$ represent the mean of the j^{th} cycle of DRSS we want to plot on the control chart for mean based on DRSS data. Assuming the process is following the normal distribution $N(\mu, \sigma^2)$, with a known variance $\sigma^2_{(i:n)}$ of i^{th} order statistic for RSS then, the control chart based on DRSS data are given by

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{Y}_{drss}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\bar{Y}_{drss}} \end{aligned} \tag{4.4}$$

where $\sigma_{\overline{Y}_{drss}} = \sqrt{(1/n^2)\sum_{i=1}^{n} \sigma^{*2}_{(i:n)}}$ and $\sigma^{*2}_{(i:n)} = E\left[\left(Y_{(i:n)} - E\left(Y_{(i:n)}\right)\right)^2\right]$ is the variance for i^{th} order statistic using DRSS method which is calculated using numerical integration.

4.5.1 Visual Comparison of DRSS with SRS for Mean Chart

Assuming that X_{ij} $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$ are from stable normal distribution with mean μ and variance σ^2. Using a sample of size $n = 3$ with a run length of $m = 50$, a simulation for the above process with $\mu = 0$ and $\sigma^2 = 1$ was carried out for the SRS (Figure 4.2) based control chart for means. The means of DRSS data was also plotted on the same chart to see their pattern. Figure 4.2 indicates that the means estimated by DRSS have less variability as compared to those estimated by SRS.

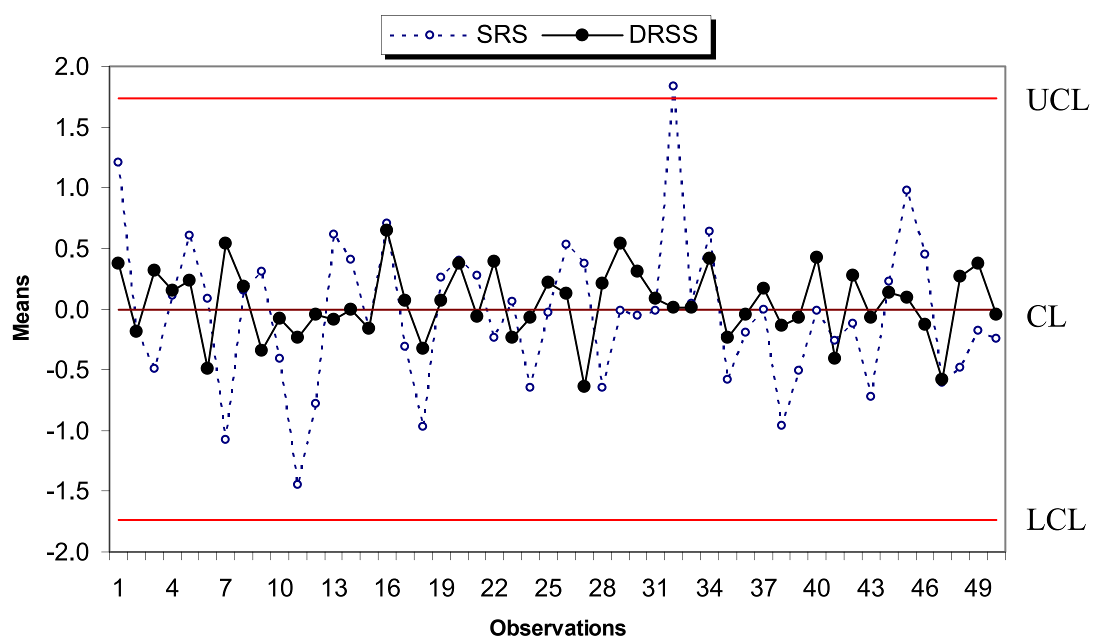

Figure 4.2: Control chart for mean using SRS & DRSS for same process.

4.5.2 ARL Comparison of DRSS with other Mean Chart

In support of our visual comparison that the control charts based on DRSS has less variability than the classical SRS chart, we make use of the average run length (ARL). As with the works of Muttlak and Al-Sabah [47] and Salazar and Sinha [59], we considered only simulation for the first rule (a point out of control limits) and for each shift, 1,000,000 iterations were simulated. The control limits, equation (4.4), of the DRSS based control chart for means are computed using numerical integration.

Considering only the case for perfect ranking i.e. when ranking the variable of interest without error in ranking the units, we run computer simulations for various values of δ, $\delta = 0.0, 0.1, 0.2, 0.3, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, \textit{and } 3.2.$, when the sample sizes are $n = 2, 3, 4, 5$. For a better comparison of the performance of DRSS with SRS, it is advisable to match the ARL to the accepted false alarm rate in the traditional SRS chart, ARL_0 = 370.40. See Champ & Woodall [14], and Salazar & Sinha [59] for a detail. Hence, we open the control limits up to $3.072\sigma_{\bar{Y}_{drss}}$ and the results are displayed in Table 4.1. The ARL values for the classical SRS chart reported in Table 4.1 are independent of the sample size. See Champ & Woodall [14]. Also, the simulation results based on the usual $3\sigma_{\bar{Y}_{drss}}$ are presented in section 4.9.

Table 4.1 indicates that the new charts based on DRSS gives better ARL performance as compared to the SRS. Observe that for $n \geq 2$ and $1.2 \leq \delta \leq 2.4$ the ARL value of DRSS are less than half the corresponding values for the SRS and is even less than one-quarter of the corresponding values of SRS when $n \geq 5$ and $0.8 \leq \delta \leq 2.0$.

δ	DRSS				SRS
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	
0.0	370.64	370.37	370.37	370.10	370.54
0.1	332.68	343.17	320.31	306.56	354.14
0.2	269.40	256.02	228.21	207.13	312.50
0.3	203.17	176.74	147.67	123.98	253.90
0.4	148.54	116.40	89.56	73.54	200.92
0.8	40.56	26.09	17.34	12.24	71.49
1.2	13.55	7.78	4.91	3.42	27.84
1.6	5.58	3.17	2.11	1.61	12.38
2.0	2.84	1.75	1.33	1.14	6.30
2.4	1.77	1.25	1.08	1.02	3.64
2.8	1.32	1.08	1.02	1.00	2.37
3.2	1.12	1.02	1.00	1.00	1.72

Table 4.1: ARL values for mean charts, matched to 370, using DRSS.

Most often in practical situations there is always need to estimate μ and $\sigma_{\overline{Y}_{drss}}$ using the DRSS data since they are not known. The unbiased estimator for μ using DRSS is given by Al-Saleh and Al-Kadiri [6] to be

$$\overline{Y}_{drss} = \frac{1}{m}\sum_{j=1}^{m}\overline{Y}_{drssj} \tag{4.5}$$

As analogue to Muttlak and Al-Sabah [47], we proposed the estimator for $\sigma_{\overline{Y}_{drss}}$ to be

$$\hat{\sigma}_{\overline{Y}_{drss}} = \sqrt{\frac{1}{n}\left(\hat{\sigma}^2_{drss} - \frac{1}{n}\sum_{i=1}^{n}\left(\tilde{Y}_{(i)} - \overline{Y}_{drss}\right)^2\right)} \tag{4.6}$$

where $\hat{\sigma}^2_{drss} = \frac{1}{nm-1}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{(i:n)j} - \overline{Y}_{drss}\right)^2$ and $\tilde{Y}_{(i)} = \frac{1}{m}\sum_{j=1}^{m}Y_{(i:n)j}$ are the estimators for the variance of DRSS and population mean of i^{th} order statistic respectively.

Our control charts may now be constructed based on $\overline{Y}_{drss}$ and $\hat{\sigma}_{\overline{Y}_{drss}}$ as follows:

$$\begin{aligned} UCL &= \overline{Y}_{drss} + 3\hat{\sigma}_{\overline{Y}_{drss}} \\ CL &= \overline{Y}_{drss} \\ LCL &= \overline{Y}_{drss} - 3\hat{\sigma}_{\overline{Y}_{drss}} \end{aligned} \tag{4.7}$$

Clearly, the proposed estimator ($\hat{\sigma}_{\overline{Y}_{drss}}$) is biased estimator for $\sigma_{\overline{Y}_{drss}}$, and hence a need to investigate the level of its biasness. We employ computer simulation in this direction. Simulations were carried out using data from standard normal distribution for sample sizes $n = 2,3,4,5$ at different values of m using 50,000 iterations. The values of $\hat{\sigma}_{\overline{Y}_{drss}}$ obtained together with the bias are tabulated in Table 4.2 and from the table, we can see that the bias become very small as number of replications m increases.

m	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	$\hat{\sigma}_{\overline{Y}_{drss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{drss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{drss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{drss}}$	Bias
2	0.4944	0.0348	0.3396	0.0162	0.2564	0.0098	0.2053	0.0064
5	0.5107	0.0185	0.3460	0.0098	0.2600	0.0061	0.2074	0.0042
10	0.5193	0.0099	0.3507	0.0051	0.2627	0.0034	0.2096	0.0020
20	0.5238	0.0054	0.3531	0.0027	0.2644	0.0018	0.2108	0.0008
30	0.5256	0.0036	0.3539	0.0019	0.2650	0.0012	0.2112	0.0005
50	0.5269	0.0023	0.3545	0.0012	0.2655	0.0007	0.2114	0.0002
75	0.5277	0.0015	0.3549	0.0009	0.2657	0.0004	0.2115	0.0001
100	0.5281	0.0011	0.3552	0.0006	0.2659	0.0002	0.2117	0.0000
200	0.5286	0.0006	0.3555	0.0003	0.2661	0.0001	0.2118	0.0001

Table 4.2: Values of $\hat{\sigma}_{\overline{Y}_{drss}}$ and bias for different n and replications m.

4.6 Control Chart for Mean using MDRSS

Assuming the process is following a normal distribution $N(\mu, \sigma^2)$, with a known variance. Then the MDRSS mean of the j^{th} cycle denoted by $\overline{Y}_{mdrssj}$ can be plotted on the following control chart for mean based on MDRSS data

$$\begin{aligned} UCL &= \mu + 3\sigma_{\overline{Y}_{mdrss}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\overline{Y}_{mdrss}} \end{aligned} \tag{4.8}$$

where $\sigma_{\overline{Y}_{mdrss}} = \sqrt{(1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:md)}}$ and $\sigma^2_{(i:md)} = E\left[\left(Y_{(i:md)} - E\left(Y_{(i:md)}\right)\right)^2\right]$ is the i^{th} variance for MDRSS calculated using numerical integration.

4.6.1 Visual Comparison of MDRSS with SRS for Mean Chart

Assume that the MDRSS data are from normal distribution with mean μ and variance σ^2. Using a sample of size $n = 3$ with a run length of $m = 50$, we simulate the above process with $\mu = 0$ and $\sigma^2 = 1$ for the SRS based mean chart, Figure 4.3. We also plot the means of MDRSS data on same chart to see how much two charts vary. From Figure 4.3 we observe that the mean estimated by MDRSS have less variability as compared to those estimated using SRS.

4.6.2 ARL Comparison of MDRSS with other Mean Chart

Again, considering simulation for a point out of control limits only, we carried out simulations for various values of δ (as in Section 4.5.2) in 1,000,000 iterations.

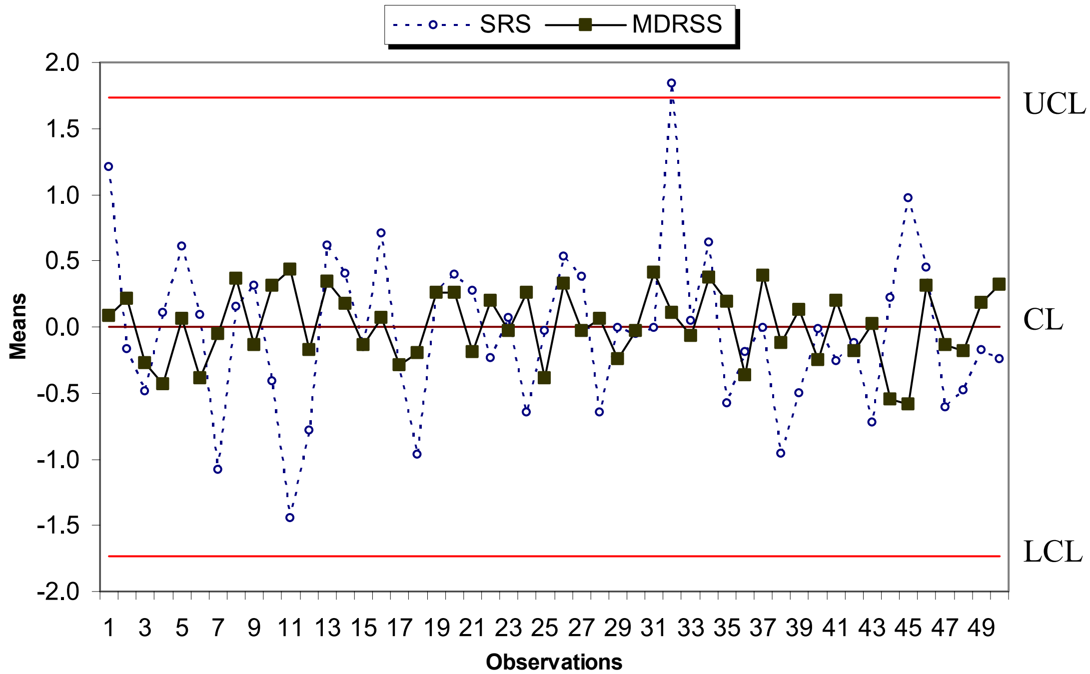

Figure 4.3: Control chart for mean using SRS & MDRSS for same process.

The control limits of the MDRSS based chart for the mean in equation 4.8 are computed using numerical integration. Here again, we considered only the case for perfect ranking and run computer simulations for various values of δ, when the sample size are n = 2, 3, 4 and 5. The control limits were open up to $3.021\sigma_{\overline{Y}_{mdrss}}$ to match the ARL to the accepted false alarm rate in the classical SRS chart, and the results are in Table 4.3.

From Table 4.3 we can see that the ARL performance of new charts based on MDRSS are better as compared to the SRS method. Also from the table, we see that the ARL values of MDRSS are less than half the corresponding values for the SRS when $n \geq 3$ and $0.4 \leq \delta \leq 2.8$ while for $n \geq 5$ and $0.4 \leq \delta \leq 2.0$ the ARL values of MDRSS is one-fifth less than those of the corresponding SRS.

δ	MDRSS				SRS
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	
0.0	370.10	370.37	370.64	370.64	370.54
0.1	332.67	317.66	297.80	267.52	354.14
0.2	269.25	217.01	184.84	143.18	312.50
0.3	203.00	130.51	100.77	69.46	253.90
0.4	148.46	81.87	57.71	36.06	200.92
0.8	40.55	15.01	9.03	5.01	71.49
1.2	13.55	4.37	2.69	1.68	27.84
1.6	5.58	1.96	1.39	1.10	12.38
2.0	2.84	1.28	1.08	1.01	6.30
2.4	1.77	1.06	1.01	1.00	3.64
2.8	1.32	1.01	1.00	1.00	2.37
3.2	1.12	1.00	1.00	1.00	1.72

Table 4.3: ARL values for mean chart, matched to 370, using MDRSS.

In practical situations, we need to estimate the μ and $\sigma_{\overline{Y}_{mdrss}}$ using the MDRSS data since we do not know them. But since the underlying distribution is normal, then from chapter 3 we see that the unbiased estimator for μ using MDRSS is given by

$$\overline{Y}_{mdrss} = \frac{1}{m}\sum_{j=1}^{m}\overline{Y}_{mdrssj} \tag{4.9}$$

We proposed the estimator for $\sigma_{\overline{Y}_{mdrss}}$ to be

$$\hat{\sigma}_{\overline{Y}_{mdrss}} = \sqrt{\frac{1}{n(n-1)}\left(\sum_{j=1}^{m}\sum_{i=1}^{n}\left(Y_{(i:md)j} - \overline{Y}_{mdrss}\right)^{2}\right)} \tag{4.10}$$

where $Y_{(i:md)j}$ denotes the j^{th} observation of the i^{th} median of a RSS of size n if the set size is odd or the $(n/2)^{th}$ and $((n+1)/2)^{th}$ order statistic of the i^{th} set (i = 1, 2,…,k = n/2) and ($I = k$+1, k+2,…,n) of RSS respectively if the sample size is even. We use $\overline{Y}_{mdrss}$ and $\hat{\sigma}_{\overline{Y}_{mdrss}}$ to construct the control charts as follows

$$\begin{aligned} UCL &= \overline{Y}_{mdrss} + 3\hat{\sigma}_{\overline{Y}_{mdrss}} \\ CL &= \overline{Y}_{mdrss} \\ LCL &= \overline{Y}_{mdrss} - 3\hat{\sigma}_{\overline{Y}_{mdrss}} \end{aligned} \tag{4.11}$$

We used computer simulation to investigate the level of biasness of $\hat{\sigma}_{\overline{Y}_{mdrss}}$. Simulations were carried out for a data from standard normal distribution for sample sizes $n = 2,3,4,5$ at different valves of m using 50,000 iterations. The values of $\hat{\sigma}_{\overline{Y}_{mdrss}}$ obtained as well as the bias are given in Table 4.4 and we can see that the bias is negligible for any value of set size n which become very clear with the increase in number of replications m.

m	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	$\hat{\sigma}_{\overline{Y}_{mdrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{mdrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{mdrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{mdrss}}$	Bias
2	0.4944	0.0348	0.2884	0.0153	0.2147	0.0079	0.1607	0.0046
5	0.5107	0.0185	0.2977	0.0060	0.2196	0.0030	0.1637	0.0015
10	0.5193	0.0099	0.3008	0.0029	0.2213	0.0013	0.1644	0.0009
20	0.5238	0.0054	0.3023	0.0014	0.2220	0.0006	0.1649	0.0003
30	0.5256	0.0036	0.3028	0.0009	0.2223	0.0003	0.1651	0.0002
50	0.5269	0.0023	0.3031	0.0006	0.2223	0.0003	0.1652	0.0001
75	0.5277	0.0015	0.3034	0.0003	0.2224	0.0002	0.1652	0.0001
100	0.5281	0.0011	0.3035	0.0002	0.2225	0.0001	0.1652	0.0000
200	0.5286	0.0006	0.3036	0.0001	0.2226	0.0000	0.1653	0.0000

Table 4.4: Values of $\hat{\sigma}_{\overline{Y}_{mdrss}}$ and bias for different n and replications m.

4.7 Control Chart for Mean using DMRSS

We assume that the process follows a normal distribution with mean μ and variance σ^2. Suppose that the variance is known then, the DMRSS mean of the j^{th} cycle denoted by $\overline{Y}_{dmrssj}$ can be plotted on the control chart based on DMRSS data as follows

$$\begin{aligned} UCL &= \mu + 3\sigma_{\overline{Y}_{dmrss}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\overline{Y}_{dmrss}} \end{aligned} \tag{4.12}$$

where $\sigma_{\overline{Y}_{dmrss}} = \sqrt{(1/n^2)\sum_{i=1}^{n}\sigma^2_{(i:dm)}}$ and $\sigma^2_{(i:dm)} = E\left[\left(Y_{(i:dm)} - E\left(Y_{(i:dm)}\right)\right)^2\right]$ is the variance for i^{th} order statistic using DMRSS method obtained using numerical integration.

4.7.1 Visual Comparison of DMRSS with SRS for Mean Chart

We assume that the DMRSS data are from $N(\mu,\sigma^2)$ and using same values of *n*, *m*, in previous section, we simulate the SRS mean chart for above process with μ = 0 and $\sigma^2 = 1$, and also plot the means of DMRSS data on the same chart to see their variability, Figure 4.4. We see from Figure 4.4 that the means estimated using DMRSS have less variability as compared to those estimated by SRS which means that they may detect shift in process mean faster as compared to SRS.

4.7.2 ARL Comparison of DMRSS with other Mean Chart

We run computer simulations for various values of δ, *n* in 1,000,000 repetitions considering only when a point is out of control limits. The control limits of the DMRSS

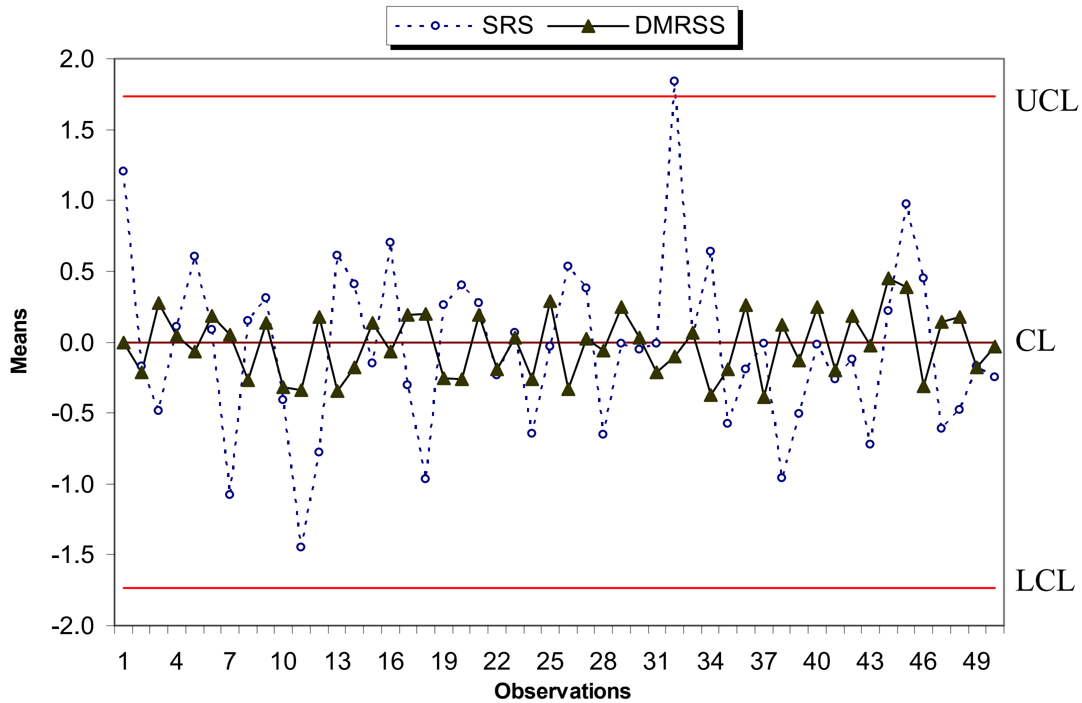

Figure 4.4: Control chart for mean using SRS & DMRSS for same process.

based control chart for means in equation 4.12 is computed using numerical integration with perfect ranking. We again open the control limits up to $3.017\sigma_{\overline{Y}_{dmrss}}$ to match the ARL to the accepted false alarm rate in the classical SRS chart, ARL = 370.40, and the results are supplied in Table 4.5.

Table 4.5 suggests that the ARL performance of the DMRSS based charts have less variability as compared to the one based on traditional SRS. Table 4.5 also shows that the ARL values of DMRSS are less than one-third the corresponding values for the SRS when $n \geq 3$ and $0.4 \leq \delta \leq 2.4$ while for $n \geq 5$ and $0.4 \leq \delta \leq 2.0$ the ARL values of DMRSS is one-sixth less than those of the corresponding SRS.

δ	DMRSS				SRS
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	
0.0	370.64	370.10	370.64	370.93	370.54
0.1	332.67	293.08	288.85	221.53	354.14
0.2	269.40	188.04	147.67	92.99	312.50
0.3	203.17	101.81	74.27	39.18	253.90
0.4	148.54	58.17	39.40	18.24	200.92
0.8	40.56	9.13	5.37	2.38	71.49
1.2	13.55	2.72	1.76	1.13	27.84
1.6	5.58	1.40	1.12	1.00	12.38
2.0	2.84	1.08	1.01	1.00	6.30
2.4	1.77	1.01	1.00	1.00	3.64
2.8	1.32	1.00	1.00	1.00	2.37
3.2	1.12	1.00	1.00	1.00	1.72

Table 4.5: ARL values for mean chart, matched to 370, using DMRSS.

The values of μ and $\sigma_{\overline{Y}_{dmrss}}$ are usually unknown when dealing with real life situations, so we estimate them using the DMRSS data. From chapter 3, we see that the unbiased estimator for μ using DMRSS is

$$\overline{Y}_{dmrss} = \frac{1}{m}\sum_{j=1}^{m}\overline{Y}_{dmrssj} \tag{4.13}$$

We proposed the estimator for $\sigma_{\overline{Y}_{dmrss}}$ to be

$$\hat{\sigma}_{\overline{Y}_{dmrss}} = \sqrt{\frac{1}{n(n-1)}\left(\sum_{j=1}^{m}\sum_{i=1}^{n}\left(Y_{(i:dm)j} - \overline{Y}_{dmrss}\right)^2\right)} \tag{4.14}$$

where $Y_{(i:dm)j}$ is the j^{th} observation of the i^{th} median of a MRSS of size n if the set size is odd or the $(n/2)^{th}$ and $((n+1)/2)^{th}$ order statistic of the i^{th} set ($i = 1, 2,\ldots,k = n/2$) and ($i =$

k+1, k+2,…,n) of MRSS respectively if the sample size is even. We can now construct our control charts using $\overline{Y}_{dmrss}$ and $\hat{\sigma}_{\overline{Y}_{dmrss}}$ as follows

$$\begin{aligned} UCL &= \overline{Y}_{dmrss} + 3\hat{\sigma}_{\overline{Y}_{dmrss}} \\ CL &= \overline{Y}_{dmrss} \\ LCL &= \overline{Y}_{dmrss} - 3\hat{\sigma}_{\overline{Y}_{dmrss}} \end{aligned} \tag{4.15}$$

The bias of $\hat{\sigma}_{\overline{Y}_{dmrss}}$ as an estimator for $\sigma_{\overline{Y}_{dmrss}}$ was investigated using computer simulation for sample sizes $n = 2,3,4,5$ at different valves of m using 50,000 iterations for a data from standard normal distribution. The values of $\hat{\sigma}_{\overline{Y}_{dmrss}}$ obtained and the bias for different values of m and n are given in Table 4.6. It is clear from the table that the bias of $\hat{\sigma}_{\overline{Y}_{dmrss}}$ becomes very negligible for any value of n and with increase in number of replications m.

m	$n = 2$		$n = 3$		$n = 4$		$n = 5$	
	$\hat{\sigma}_{\overline{Y}_{dmrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{dmrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{dmrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{dmrss}}$	Bias
2	0.4944	0.0348	0.2464	0.0120	0.1741	0.0069	0.1242	0.0037
5	0.5107	0.0185	0.2537	0.0047	0.1784	0.0025	0.1266	0.0013
10	0.5193	0.0099	0.2561	0.0023	0.1798	0.0012	0.1271	0.0008
20	0.5238	0.0054	0.2573	0.0011	0.1806	0.0004	0.1276	0.0003
30	0.5256	0.0036	0.2577	0.0007	0.1807	0.0003	0.1277	0.0002
50	0.5269	0.0023	0.2579	0.0005	0.1808	0.0002	0.1278	0.0001
75	0.5277	0.0015	0.2582	0.0002	0.1809	0.0001	0.1278	0.0001
100	0.5281	0.0011	0.2583	0.0001	0.1809	0.0001	0.1279	0.0000
200	0.5286	0.0006	0.2584	0.0000	0.1810	0.0000	0.1279	0.0000

Table 4.6: Values of $\hat{\sigma}_{\overline{Y}_{dmrss}}$ and bias for different n and replications m.

4.8 Control Chart for Mean using EDRSS

Suppose that the process we are considering is following a normal distribution with mean μ and variance σ^2 and that the variance is known. Then, the EDRSS mean of the j^{th} observation $\bar{Y}_{edrssj}$ can be plotted on the control chart based on EDRSS as follows

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{Y}_{edrss}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\bar{Y}_{edrss}} \end{aligned} \tag{4.16}$$

where $\sigma_{\bar{Y}_{edrss}} = \sqrt{(1/n^2)\sum_{i=1}^{n}\sigma_{(i:ed)}^2}$ and $\sigma_{(i:ed)}^2 = E\left[\left(Y_{(i:ed)} - E\left(Y_{(i:ed)}\right)\right)^2\right]$ is the i^{th} variance for EDRSS.

4.8.1 Visual Comparison of EDRSS with SRS for Mean Chart

Using data from the standard normal distribution $N(0,1)$, a computer simulation was performed for a sample of size $n = 4$ with forty replications for the SRS, Figure 4.5, using control chart for means. The means of EDRSS data are also graphed within the same chart to see the two patterns. Figure 4.5 shows that the means estimated using EDRSS have less variability as compared to those estimated using SRS.

4.8.2 ARL Comparison of EDRSS with other Chart

As in previous sections, computer simulations were carried out for the same values of δ, and n in 1,000,000 repetitions considering only when a point is out of control limits. Control limits of EDRSS based control chart for means in equation 4.16 is calculated using numerical integration considering only perfect ranking.

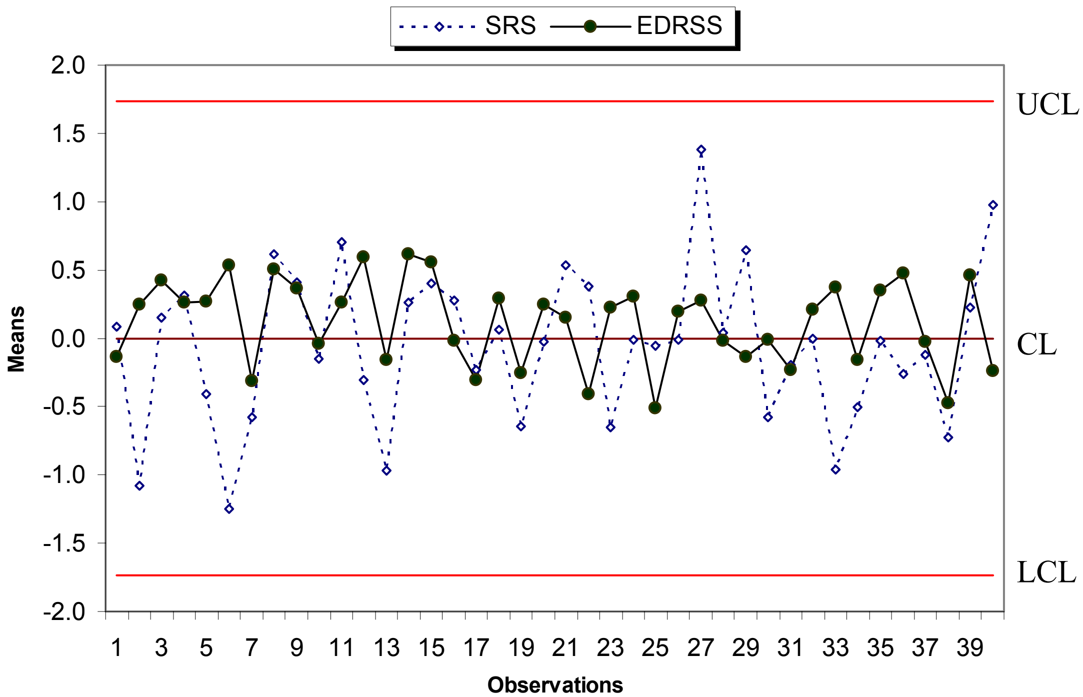

Figure 4.5: Control chart for mean using SRS & EDRSS for same process.

The limits were open up to $3.082\sigma_{\overline{Y}_{edrss}}$ to give approximately the in-control ARL value of SRS control chart for mean and results are tabulated in Table 4.7.

Considering the results in Table 4.7, it can be observed that the EDRSS based control charts have smaller ARL values as than those based on usual SRS. We can also see that for $n \geq 3$ and $0.8 \leq \delta \leq 2.8$ the ARL values of EDRSS are less than half of the corresponding values for the SRS and that when $n \geq 5$ and $0.8 \leq \delta \leq 2.4$, the ARL values of EDRSS is one-third less than those of the corresponding SRS.

δ	n				SRS
	2	3	4	5	
0.0	370.64	370.37	370.40	370.37	370.54
0.1	332.68	343.17	340.46	329.38	354.14
0.2	269.40	256.02	248.38	239.69	312.50
0.3	203.17	176.74	168.40	156.35	253.90
0.4	148.54	116.39	107.09	94.80	200.92
0.8	40.56	26.09	23.72	18.62	71.49
1.2	13.55	7.78	6.51	5.20	27.84
1.6	5.58	3.17	3.05	2.22	12.38
2.0	2.84	1.75	1.71	1.36	6.30
2.4	1.77	1.25	1.23	1.10	3.64
2.8	1.32	1.08	1.07	1.02	2.37
3.2	1.12	1.02	1.02	1.00	1.72

Table 4.7: ARL values for mean chart, matched to 370, using EDRSS.

The values of μ and $\sigma_{\overline{Y}_{edrss}}$ are usually unknown in real life situations, we estimate them using the EDRSS data. The unbiased estimator for μ using EDRSS is given by

$$\overline{Y}_{edrss} = \frac{1}{m}\sum_{j=1}^{m}\overline{Y}_{edrssj} \tag{4.17}$$

We proposed the estimator for $\sigma_{\overline{Y}_{edrss}}$ to be

$$\hat{\sigma}_{\overline{Y}_{edrss}} = \sqrt{\frac{1}{n(n-1)}\left(\sum_{j=1}^{m}\sum_{i=1}^{n}\left(Y_{(i:ed)j} - \overline{Y}_{edrss}\right)^2\right)} \tag{4.18}$$

where $Y_{(i:ed)j}$ is the j^{th} observation of the EDRSS data for both the odd and even cases. Using $\overline{Y}_{edrss}$ and $\hat{\sigma}_{\overline{Y}_{edrss}}$ the control charts based on EDRSS now becomes

$$\begin{aligned} UCL &= \overline{Y}_{edrss} + 3\hat{\sigma}_{\overline{Y}_{edrss}} \\ CL &= \overline{Y}_{edrss} \\ LCL &= \overline{Y}_{edrss} - 3\hat{\sigma}_{\overline{Y}_{edrss}} \end{aligned} \tag{4.19}$$

We investigate the bias of $\hat{\sigma}_{\overline{Y}_{edrss}}$ as an estimator for $\sigma_{\overline{Y}_{edrss}}$ using computer simulation for sample sizes $n = 2,3,4,5$ at different valves of m in 50,000 repetitions for a data from standard normal distribution. The values of $\hat{\sigma}_{\overline{Y}_{edrss}}$ obtained and the bias for different values m and n are given in Table 4.8 and it can be deduce from the table that the bias is negligible as number of replications m increases.

m	$n = 2$		$n = 3$		$N = 4$		$n = 5$	
	$\hat{\sigma}_{\overline{Y}_{edrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{edrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{edrss}}$	Bias	$\hat{\sigma}_{\overline{Y}_{edrss}}$	Bias
2	0.4944	0.0348	0.3396	0.0162	0.2915	0.0123	0.2312	0.0106
5	0.5107	0.0185	0.3460	0.0098	0.2990	0.0047	0.2375	0.0043
10	0.5193	0.0099	0.3507	0.0051	0.3012	0.0025	0.2378	0.0040
20	0.5238	0.0054	0.3531	0.0027	0.3023	0.0014	0.2393	0.0025
30	0.5256	0.0036	0.3539	0.0019	0.3027	0.0010	0.2400	0.0018
50	0.5269	0.0023	0.3545	0.0012	0.3031	0.0006	0.2410	0.0008
75	0.5277	0.0015	0.3549	0.0009	0.3032	0.0005	0.2412	0.0006
100	0.5281	0.0011	0.3552	0.0006	0.3033	0.0004	0.2414	0.0004
200	0.5286	0.0006	0.3555	0.0003	0.3035	0.0002	0.2415	0.0003

Table 4.8: Values of $\hat{\sigma}_{\overline{Y}_{edrss}}$ and bias for different *n* and replications m.

4.9 Comparing the New Charts Based on the Standard 3-Sigma

In order to compare the new charts with SRS and RSS based on the standard 3σ, simulations were carried for various values of δ and n in 1,000,000 replications for the rule: a point out of control limits, see Salazer and Sinha [59]. Using numerical integration, the control limits for DRSS, MDRSS and DMRSS based control charts for mean were computed. The average run length (ARL) for the SRS, RSS, MRSS and the new methods using the standard 3σ are given in Tables 4.9 – 4.12. And from the table, we can draw the following conclusions:

1. The ARL performance of the new charts based on DRSS, MDRSS, DMRSS and EDRSS are generally better as compared with those based on SRS, RSS, MRSS and ERSS even without increase in sample size n.
2. As the sample size n increases and the process starts to go out of control (i.e. $\delta \geq 0$), the ARL performance of EDRSS appears to be just as good as the corresponding values of MRSS but still dominates SRS, RSS and ERSS. For example, if $n = 5$ and $\delta = 0.4$, the ARL values of EDRSS is 77.95 as compared to 81.58 of MRSS, 98.45 of RSS, 112.41 of ERSS and 200.01 of SRS.
3. Among the new charts, EDRSS appears to be the least in terms of ARL performance as n increases and $\delta \geq 0$. It is followed by DRSS and then MDRSS with DMRSS dominating all the charts. For example, if $n = 4$ and $\delta = 0.4$, the ARL value of DMRSS is 29.42 as compared to 55.56 of MDRSS, 76.59 of DRSS and 94.70 of EDRSS.

4. Although, the new charts have proven to be better than the SRS, RSS, MRSS, and ERSS. There is great price to pay as they all increase the false alarm rate when a process is in a state of statistical control (i.e. δ = 0). But as the sample size increase, the ARL value of DMRSS seems to be matching up with the accepted false alarm rate in the traditional SRS chart for mean (i.e. ARL = 370.40). For example, if n = 5 and δ = 0, the ARL value of DMRSS is 369.34.

δ	Sampling Methods							
	SRS	RSS	MRSS	ERSS	DRSS	MDRSS	DMRSS	EDRSS
0.0	371.55	348.43	348.43	348.43	322.79	322.79	322.79	322.79
0.1	355.31	318.37	318.37	318.37	285.21	285.21	285.21	285.21
0.2	308.12	278.67	278.67	278.67	234.49	234.49	234.49	234.49
0.3	251.86	208.31	208.31	208.31	178.01	178.01	178.01	178.01
0.4	200.40	155.27	155.27	155.27	130.10	130.10	130.10	130.10
0.8	72.03	46.96	46.96	46.96	36.69	36.69	36.69	36.69
1.2	27.79	16.44	16.44	16.44	12.43	12.43	12.43	12.43
1.6	12.38	6.98	6.98	6.98	5.21	5.21	5.21	5.21
2.0	6.31	3.56	3.56	3.56	2.70	2.70	2.70	2.70
2.4	3.64	2.16	2.16	2.16	1.72	1.72	1.72	1.72
2.8	2.37	1.53	1.53	1.53	1.29	1.29	1.29	1.29
3.2	1.72	1.23	1.23	1.23	1.11	1.11	1.11	1.11

Table 4.9: ARL values when n = 2 using different sampling techniques.

δ	Sampling Methods							
	SRS	RSS	MRSS	ERSS	DRSS	MDRSS	DMRSS	EDRSS
0.0	370.54	343.20	361.56	343.06	301.15	351.00	355.08	300.84
0.1	354.14	326.80	326.80	316.14	271.89	287.84	285.05	272.32
0.2	312.50	251.37	251.37	249.85	206.99	203.28	173.50	207.58
0.3	253.90	175.48	175.48	184.70	144.55	126.29	98.37	152.02
0.4	200.92	130.87	118.89	135.87	98.47	78.06	56.50	99.77
0.8	71.49	34.09	28.15	34.09	22.52	14.42	8.91	22.94
1.2	27.84	11.14	8.83	11.14	6.93	4.25	2.61	7.01
1.6	12.38	4.65	3.70	4.65	2.98	1.93	1.39	2.99
2.0	6.30	2.45	2.02	2.45	1.67	1.27	1.08	1.70
2.4	3.64	1.60	1.39	1.60	1.22	1.06	1.01	1.23
2.8	2.37	1.23	1.13	1.23	1.06	1.01	1.00	1.07
3.2	1.72	1.08	1.04	1.08	1.02	1.00	1.00	1.02

Table 4.10: ARL values when $n = 3$ using different sampling techniques

δ	Sampling Methods							
	SRS	RSS	MRSS	ERSS	DRSS	MDRSS	DMRSS	EDRSS
0.0	368.22	337.50	361.08	340.37	301.50	352.67	356.95	294.99
0.1	356.57	310.52	321.92	311.92	259.05	282.69	283.83	265.91
0.2	314.34	234.63	230.22	243.55	185.03	171.91	103.40	201.54
0.3	252.13	166.55	152.74	177.44	120.03	97.09	54.09	140.36
0.4	199.88	112.80	99.99	123.95	76.59	55.56	29.42	94.70
0.8	71.92	26.38	20.94	31.44	15.03	8.75	4.49	21.65
1.2	28.01	8.17	6.33	10.17	4.43	2.63	1.61	6.63
1.6	12.41	3.44	2.71	4.23	1.99	1.38	1.09	2.82
2.0	6.29	1.90	1.59	2.27	1.29	1.07	1.01	1.60
2.4	3.64	1.33	1.19	1.51	1.07	1.01	1.00	1.20
2.8	2.38	1.11	1.05	1.19	1.01	1.00	1.00	1.06
3.2	1.73	1.03	1.01	1.06	1.00	1.00	1.00	1.01

Table 4.11: ARL values when $n = 4$ using different sampling techniques

δ	Sampling Methods							
	SRS	RSS	MRSS	ERSS	DRSS	MDRSS	DMRSS	EDRSS
0.0	369.04	347.11	365.44	338.07	297.42	363.41	369.34	292.47
0.1	353.23	300.66	310.06	304.05	248.45	263.98	221.17	255.51
0.2	303.58	226.96	209.79	232.75	166.07	138.55	91.61	183.81
0.3	257.50	153.63	132.34	162.41	100.89	69.04	38.99	121.83
0.4	200.01	98.45	81.58	112.41	61.08	36.12	18.31	77.95
0.8	70.58	21.02	15.16	25.06	10.63	4.96	2.38	15.69
1.2	27.91	6.35	4.47	7.82	3.13	1.67	1.13	4.66
1.6	12.37	2.72	2.02	3.32	1.54	1.10	1.00	2.07
2.0	6.30	1.59	1.30	1.85	1.12	1.01	1.00	1.31
2.4	3.65	1.19	1.07	1.30	1.02	1.00	1.00	1.08
2.8	2.37	1.05	1.01	1.10	1.00	1.00	1.00	1.02
3.2	1.72	1.01	1.00	1.03	1.00	1.00	1.00	1.00

Table 4.12: ARL values when n = 5 using different sampling techniques

Chapter 5

CONTROL CHART FOR MONITORING THE PROCESS MEAN AND STANDARD DEVIATION

5.1 Introduction

In this chapter, we construct control charts based on some modifications of ranked set sampling for monitoring a process to detect shifts in mean and standard deviation. We will also investigate the average run length (ARL) performance of these charts and compare them with the corresponding control charts using simple random sampling (SRS) and other sampling techniques.

5.2 Control Chart for Monitoring the Process Standard Deviation

The average run length (ARL) assumes that the process is in the state of statistical control with mean μ_0 and standard deviation σ_0. But at certain point in time, the process begin to go out of statistical control with a shift in standard deviation from σ_0 to $\sigma_1 \geq \sigma_0$, see Figure 5.1. If we assume that the process follows a normal distribution with mean μ_0 and variance σ_0^2 when the process is in the state of statistical control then, the shift in the process standard deviation is given by σ_1/σ_0. To monitor such a shift, we implore the control chart for range (*R* chart).

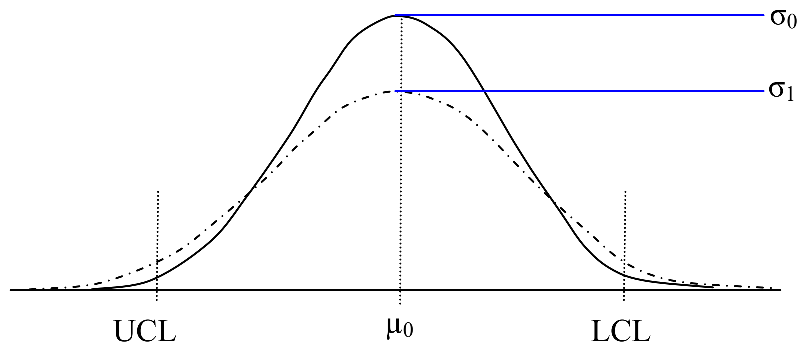

Figure 5.1: Shift in standard deviation from σ_0 to $\sigma_1 \geq \sigma_0$

5.2.1 Control Chart for Range using SRS

The process standard deviation can be estimated using the range *R* when the sample is normally distributed. *R* is said to be closely related σ when small sample size *n*, is used, see Amin and Wolff [2] for more detail. The estimator of σ using the range is given by

$$\hat{\sigma} = \frac{\overline{R}}{d_2} \tag{5.1}$$

where d_2 is a function of sample size *n* and $\overline{R}$ is the average of the ranges given by

$$\overline{R} = \frac{1}{m}\sum_{j=1}^{m} R_j \; . \tag{5.2}$$

Because of such a relationship between R and σ the process variability may be controlled by R_j for $j = 1, 2, \ldots, m$ on a control chart, called R chart. Thus, the control limits for the of the R chart are given by

$$\begin{aligned} UCL &= \overline{R} + 3\hat{\sigma}_R \\ CL &= \overline{R} \\ LCL &= \overline{R} - 3\hat{\sigma}_R \end{aligned} \tag{5.3}$$

where $\hat{\sigma}_R = d_3 R / d_2$ is an estimator of the range standard deviation σ_R, while d_2 and d_3 are known function of *n*, see Montgomery [36] for more detail.

5.2.2 Control Chart for Range using ERSS

The ERSS range $R_{erssj} = \left(X_{(n)} - X_{(1)} \right)$ based on any two sets from the j^{th} cycle can be plotted on the control chart using ERSS data as follows

$$\begin{aligned} UCL &= \overline{R}_{erss} + 3\sigma_{R_{erss}} \\ CL &= \overline{R}_{erss} \\ LCL &= \overline{R}_{erss} - 3\sigma_{R_{erss}} \end{aligned} \tag{5.4}$$

where $\overline{R}_{erss} = \frac{1}{m}\sum_{j=1}^{m} R_{erssj}$ and $\sigma_{R_{erss}} = \sqrt{Var\left(X_{(n)}\right) + Var\left(X_{(1)}\right)}$ which is computed from the table of order statistics for the standard normal distribution, for example see Harter and Balakrishnan [26].

5.2.2.1 Visual Comparison of ERSS with SRS *R* Chart

Suppose that X_{ij} $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$ are from stable normal distribution with mean μ and variance σ^2. Using a sample of size $n = 4$ with a run length of $m = 50$, a simulation for the above process with $\mu = 0$ and $\sigma^2 = 1$ was carried out for the SRS, Figure 5.2, based control chart for ranges. The ranges of ERSS data were also plotted on the same chart to see their variability. From Figure 5.2 we can see that the ranges estimated by ERSS have less variability as compared to those estimated by SRS.

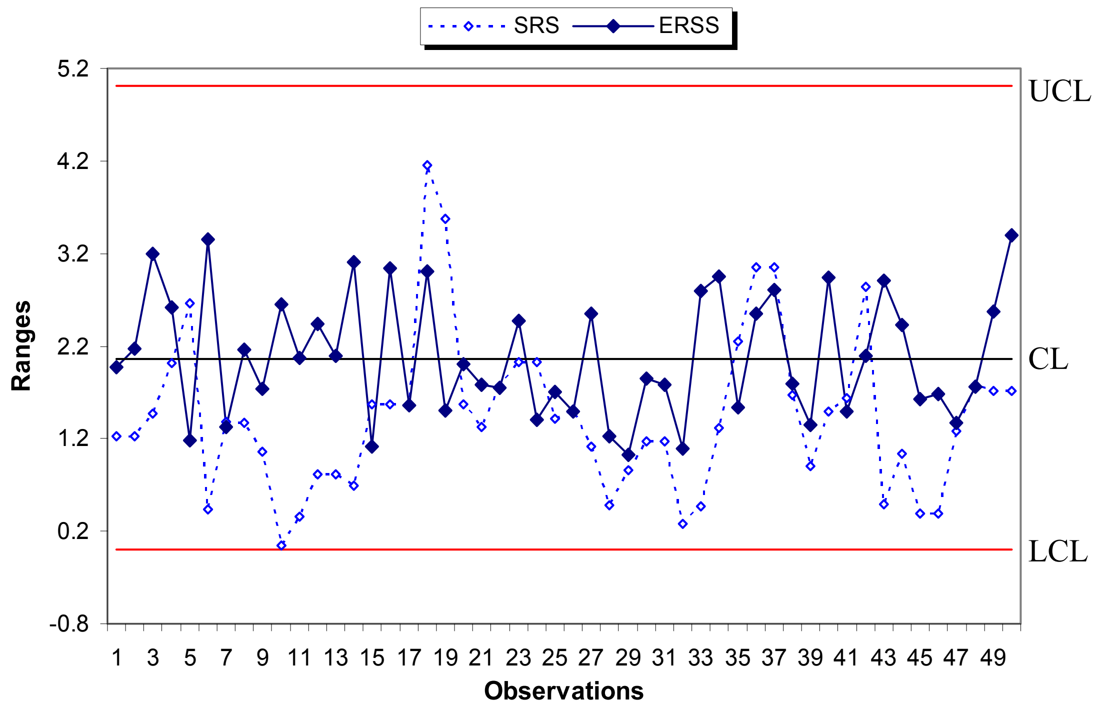

Figure 5.2: Control chart for range using SRS & ERSS for same process.

5.2.2.2 ARL Comparison of ERSS with SRS for *R* Charts

We carry out computer simulations considering only perfect ranking for various values of σ_1/σ_0 = 1.0, 1.4, 1.8, 2.2, 2.6, 3.0, 3.4, 3.8, 4.2, 4.8, 5.0 and $n = 2,3,4,5,6$. The control limits, equation (5.4), of ERSS based control chart for ranges were calculated using the table of order statistics, see Harter and Balakrishnan [26]. We simulate 500,000 iterations matching the ARL to the accepted false alarm rate in the classical SRS chart, ARL = 370.40, for a fair comparison of the performance of ERSS with SRS considering only perfect ranking. See Reynolds and Stoumbos [56] for a detail. The control limits of *R* chart were opened up to $3.093\sigma_R$ and the results are shown in Table 5.1. Note that the SRS based *R* chart in Table 5.1 is independent of sample size *n*.

Table 5.1 indicates that the ARL values for *R* control charts based on ERSS are smaller than those based on the classical SRS as the process begins to go out of control signifying better performance of ERSS *R* chart over the traditional SRS *R* chart. Such good performance of ERSS increases with the increase in sample size *n*. For example, if $n = 3$ and $1.4 \leq \sigma_1/\sigma_0 \leq 4.2$, the ARL values of the *R* chart based on ERSS are less than two-third of the corresponding values of SRS and when $n = 6$ and $1.4 \leq \sigma_1/\sigma_0 \leq 3.0$, the ARL values of SRS are thrice those of the ERSS.

σ_1/σ_0	ERSS					SRS
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	
1.0	370.78	370.64	370.51	370.51	370.37	370.37
1.4	23.10	18.74	9.30	7.49	4.32	31.93
1.8	7.36	5.59	2.68	2.20	1.45	10.61
2.2	4.07	3.10	1.63	1.41	1.10	5.87
2.6	2.88	2.24	1.31	1.18	1.03	4.04
3.0	2.31	1.84	1.17	1.09	1.01	3.17
3.4	1.99	1.61	1.10	1.05	1.00	2.66
3.8	1.78	1.47	1.07	1.03	1.00	2.34
4.2	1.64	1.38	1.04	1.02	1.00	2.10
4.6	1.54	1.32	1.03	1.01	1.00	1.95
5.0	1.47	1.27	1.02	1.01	1.00	1.83

Table 5.1: ARL values for *R* Chart with deferent set size using ERSS and SRS matched to 370

5.2.3 Control Chart for Range using EDRSS

The EDRSS range $R_{edrssj} = \left(Y_{(n)} - Y_{(1)}\right)$ based on any two sets from the j^{th} cycle can be plotted on the control chart using EDRSS data as follows

$$\begin{aligned} UCL &= \overline{R}_{edrss} + 3\sigma_{R_{edrss}} \\ CL &= \overline{R}_{edrss} \\ LCL &= \overline{R}_{edrss} - 3\sigma_{R_{edrss}} \end{aligned} \tag{5.5}$$

where $\overline{R}_{edrss} = \frac{1}{m}\sum_{j=1}^{m} R_{edrssj}$ and $\sigma_{R_{edrss}} = \sqrt{Var\left(Y_{(n)}\right) + Var\left(Y_{(1)}\right)}$. The value of $\sigma_{R_{edrss}}$ is computed using numerical integration.

5.2.3.1 Visual Comparison of EDRSS with SRS *R* Chart

As in Section 5.2.2.1, we assume that data are from stable normal distribution with mean μ and variance σ^2 and using a sample of size n = 4 with a run length of m = 50, we simulate the *R* chart for the above process with $\mu = 0$ and $\sigma^2 = 1$ using SRS, Figure 5.3. The ranges of EDRSS data were also plotted on same chart to see their pattern. From Figure 5.3, we observe that the ranges estimated by EDRSS have less variability as compared to those estimated by SRS, which means that the EDRSS based control charts may detect changes in process standard deviation quicker than those based on SRS for same process.

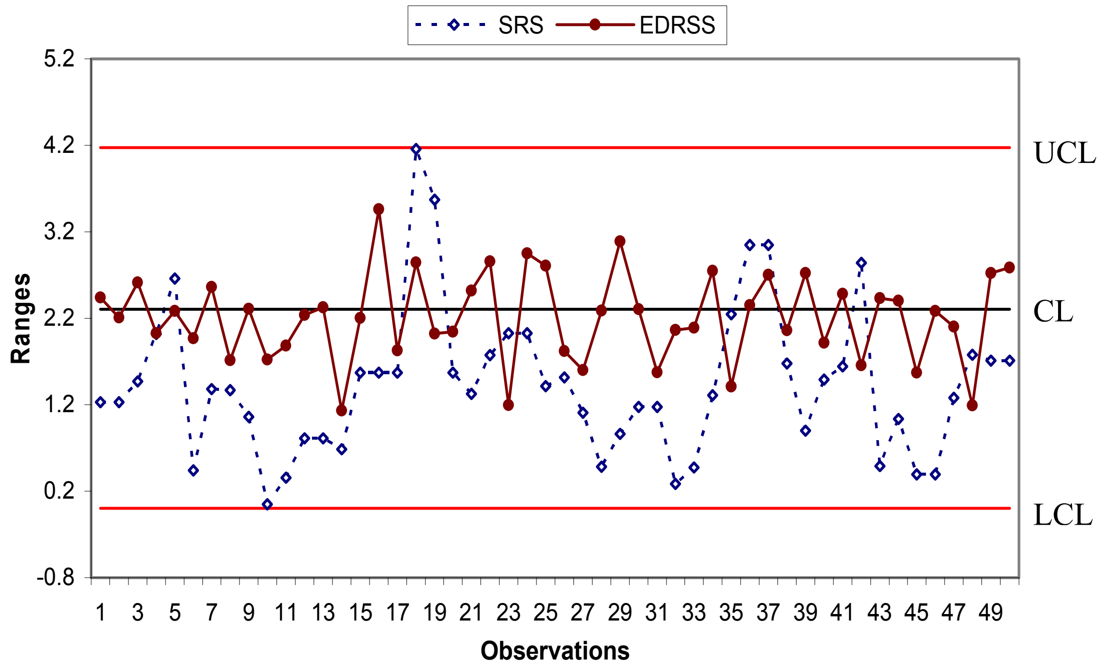

Figure 5.3: Control chart for range using SRS & EDRSS for same process.

5.2.3.2 ARL Comparison of EDRSS with Other *R* Charts

As we did in Section 5.4.2 for the case of ERSS, we use the same values of σ_1/σ_0 and $n = 2,3,4,5$ to run simulations for 500,000 iterations to compute the ARL values based on EDRSS for perfect ranking. The control limits, equation (5.5), of EDRSS based control chart for ranges were calculated using numerical integration. We matched the ARL values to the accepted false alarm rate in the classical SRS chart by opening up control limits of *R* chart to $3.152\sigma_R$. The results for *R* charts using EDRSS and SRS are displayed in Table 5.2. The *R* chart in base on SRS in Table 5.2 is independent of sample size *n*. See Reynolds and Stoumbos [56] for a detail.

σ_1/σ_0	EDRSS				SRS
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	
1.0	370.37	370.37	370.37	370.64	370.37
1.4	23.05	17.29	7.48	5.97	31.93
1.8	7.10	4.97	2.12	1.71	10.61
2.2	3.87	2.72	1.34	1.17	5.87
2.6	2.73	1.96	1.13	1.05	4.04
3.0	2.19	1.61	1.06	1.01	3.17
3.4	1.88	1.43	1.02	1.01	2.66
3.8	1.69	1.31	1.02	1.00	2.34
4.2	1.56	1.24	1.01	1.00	2.10
4.6	1.47	1.19	1.01	1.00	1.95
5.0	1.40	1.16	1.00	1.00	1.83

Table 5.2: ARL values for *R* Chart with different set size using EDRSS and SRS matched to 370

Considering Table 5.2, we see that the ARL values for *R* control charts based on EDRSS are doing better than those based on SRS as the process starts to go out of control. Also, the ARL values of EDRSS based *R* chart appears to be smaller than those of the ERSS with the same sample size *n*, although not very much. Generally, the performance of EDRSS based control charts increases with the increase in sample size. For example, if $n = 2$ and $\sigma_1/\sigma_0 = 2.2$, the ARL values of the *R* chart based on EDRSS is 3.87 as compared to 4.06 of ERSS and 5.87 of SRS while for $n = 5$ and $\sigma_1/\sigma_0 = 1.4$, the ARL values of EDRSS is 5.97 as compared to 7.57 of ERSS and 31.92 of SRS.

5.3 Monitoring Both Mean and Standard Deviation

Here we develop control charts for controlling both the process mean as well as the variability. As in Section 5.2, the average run length (ARL) assumes that the process is in the state of statistical control with mean μ_0 and standard deviation σ_0 and that at some point, the process begins to go out of control with a shift in mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0/\sqrt{n}$, and/ or shift in standard deviation from σ_0 to $\sigma_1 \geq \sigma_0$, see Figure 5.4. If we assume that the process follows a normal distribution with mean μ_0 and variance σ_0^2 when the process is in the state of control, then the shift on the process mean and increase in variance are respectively given by $\delta = \sqrt{n}\left|\mu_1 - \mu_0\right|/\sigma_0$ and σ_1/σ_0.

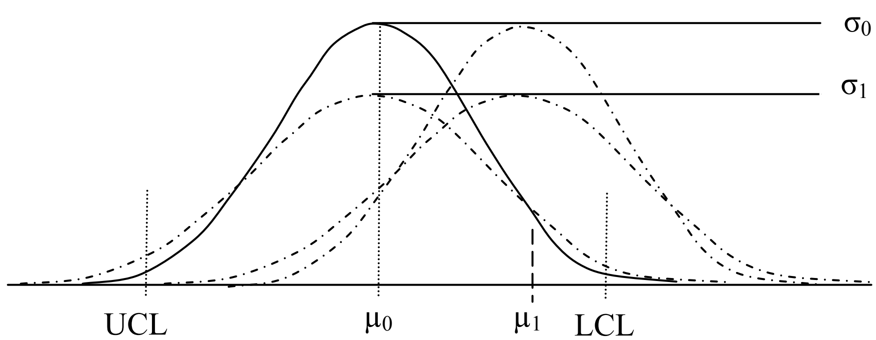

Figure 5.4: Shift in mean from $\mu_1 \geq \mu_0$ and / or standard deviation from $\sigma_1 \geq \sigma_0$

5.3.1 Control Chart for Mean and Range with RSS and its Modifications

Using the control limits for RSS, MRSS, ERSS base chart for mean, Chapter 2, and the ERSS based *R* chart, Section 5.2.2, we run simulations to compute the ARL values in order to compare the control charts. Considering only perfect ranking, we run computer simulations for mixed values of δ = 0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, σ_1/σ_0 = 1.0, 1.4, 1.8, 2.2, 2.6, 3.0, 3.4, 3.8, and n = 2,3,4,5,6 in 500,000 replications. And for fair comparison, the ARL of all the charts were matched to the accepted false alarm rate in classical SRS chart for mean, see Reynolds and Stoumbos [56]. Hence for the control chart for means, RSS control limits were opened up to $3.0293\,\sigma_{\overline{X}_{rss}}$, MRSS to $3.013\,\sigma_{\overline{X}_{mrss}}$, and ERSS to $3.0296\,\sigma_{\overline{X}_{erss}}$ while the control limits for ERSS based R chart was opened up to $3.0930\,\sigma_{R_{erss}}$. The results are shown in Tables 5.3 – 5.7 and following conclusion can be made

1. The SRS based R chart is not as good as the control chart for means in detecting increases in the process standard deviation and is completely not effective in detecting shift in process mean. For example consider n = 2 (Table 5.3) and δ = 2.0 and σ_1/σ_0 = 1.0, the ARL value of the classical SRS chart for mean is 6.31 as compared to the 363.77 of its corresponding R chart.
2. The RSS, MRSS, and ERSS based control charts for means are all doing better than the corresponding SRS chart for means as long as there is a shift in the process mean. The MRSS is dominating all the charts. See for example the case when n = 5 (Table 5.6) , δ = 0.8 and σ_1/σ_0 = 1.4, the ARL value of SRS is 16.23 as compared to 8.82 of RSS, 7.13 of MRSS and 9.81 of ERSS.
3. While the control charts for the means using RSS, MRSS, and ERSS are just as good as SRS counterpart when δ = 0 and $\sigma_1/\sigma_0 \geq 1.0$, the ERSS based R chart is more effective in detecting increases in the process standard deviation for as long

as there in no shift in the process mean. For example, see $n = 6$ (Table 5.7) when $\delta = 0.0$ and $\sigma_1 / \sigma_0 = 1.40$, the ARL value of R chart using ERSS is 4.32 as compared to 31.47 of RSS, 31.43 of MRSS and 31.34 of ERSS.

4. As the sample sizes increases, MRSS based control chart for the means appears to be more effective in detecting large shift and increases in the process mean and variance respectively. While for large increase in variance but small shift in process mean, the ERSS based R is more effective than all other charts, see Tables 5.3-5.7.

δ	σ_1/σ_0	Chart for Means SRS	RSS	MRSS	ERSS	*R* Chart ERSS	SRS
0.00	1.00	370.69	370.10	370.10	370.10	370.78	370.64
0.40	1.00	196.85	162.60	162.60	162.60	368.60	368.73
0.80	1.00	72.29	48.46	48.46	48.46	365.00	365.10
1.20	1.00	27.50	16.83	16.83	16.83	364.30	364.30
1.60	1.00	12.38	7.12	7.12	7.12	362.45	364.03
2.00	1.00	6.31	3.61	3.61	3.61	361.53	363.77
2.40	1.00	3.64	2.18	2.18	2.18	358.04	362.84
2.80	1.00	2.37	1.54	1.54	1.54	354.62	355.98
0.00	1.40	31.93	31.31	31.31	31.31	23.10	30.54
0.00	1.80	10.61	10.60	10.60	10.60	7.36	10.38
0.00	2.20	5.87	5.86	5.86	5.86	4.07	5.72
0.00	2.60	4.04	4.08	4.08	4.08	2.88	4.00
0.00	3.00	3.17	3.18	3.18	3.18	2.31	3.13
0.00	3.40	2.66	2.67	2.67	2.67	1.99	2.64
0.00	3.80	2.34	2.35	2.35	2.35	1.78	2.33
0.80	1.40	16.23	13.47	13.47	13.47	23.10	30.54
0.80	1.80	7.82	7.08	7.08	7.08	7.36	10.38
0.80	2.20	4.97	4.74	4.74	4.74	4.07	5.72
0.80	2.60	3.69	3.59	3.59	3.59	2.88	4.00
1.20	1.40	9.89	7.52	7.52	7.52	23.10	30.54
1.20	1.80	5.93	5.01	5.01	5.01	7.36	10.38
1.20	2.20	4.25	3.85	3.85	3.85	4.07	5.72
1.20	2.60	3.36	3.15	3.15	3.15	2.88	4.00
1.60	1.40	6.26	4.53	4.53	4.53	23.10	30.54
1.60	1.80	4.47	3.60	3.60	3.60	7.36	10.38
1.60	2.20	3.56	3.09	3.09	3.09	4.07	5.72
1.60	2.60	2.99	2.72	2.72	2.72	2.88	4.00
2.00	1.40	4.19	2.97	2.97	2.97	23.10	30.54
2.00	1.80	3.43	2.68	2.68	2.68	7.36	10.38
2.00	2.20	2.97	2.50	2.50	2.50	4.07	5.72
2.00	2.60	2.64	2.33	2.33	2.33	2.88	4.00

Table 5.3: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 2$ using SRS, RSS, MRSS and ERSS.

δ	σ_1/σ_0	Chart for Means				*R* Chart	
		SRS	RSS	MRSS	ERSS	ERSS	SRS
0.00	1.00	370.69	370.37	370.78	370.37	370.64	370.64
0.40	1.00	196.85	140.53	121.20	140.53	368.61	368.73
0.80	1.00	72.29	35.82	28.24	35.82	364.92	365.10
1.20	1.00	27.50	11.54	8.90	11.54	364.06	364.30
1.60	1.00	12.38	4.79	3.73	4.79	363.26	364.03
2.00	1.00	6.31	2.51	2.03	2.51	359.81	363.77
2.40	1.00	3.64	1.62	1.39	1.62	358.12	362.84
2.80	1.00	2.37	1.24	1.14	1.24	346.36	355.98
0.00	1.40	31.93	31.83	31.14	31.83	18.74	30.54
0.00	1.80	10.61	10.60	10.48	10.60	5.59	10.38
0.00	2.20	5.87	5.82	5.82	5.82	3.10	5.72
0.00	2.60	4.04	4.00	4.05	4.00	2.24	4.00
0.00	3.00	3.17	3.11	3.17	3.11	1.84	3.13
0.00	3.40	2.66	2.60	2.66	2.60	1.61	2.64
0.00	3.80	2.34	2.31	2.34	2.31	1.47	2.33
0.80	1.40	16.23	11.48	10.07	11.48	18.70	30.54
0.80	1.80	7.82	6.48	5.96	6.48	5.62	10.38
0.80	2.20	4.97	4.50	4.29	4.50	3.10	5.72
0.80	2.60	3.69	3.49	3.38	3.49	2.24	4.00
1.20	1.40	9.89	6.04	5.18	6.04	18.70	30.54
1.20	1.80	5.93	4.38	3.92	4.38	5.62	10.38
1.20	2.20	4.25	3.51	3.27	3.51	3.10	5.72
1.20	2.60	3.36	2.97	2.84	2.97	2.24	4.00
1.60	1.40	6.26	3.55	3.03	3.55	18.70	30.54
1.60	1.80	4.47	3.05	2.71	3.05	5.62	10.38
1.60	2.20	3.56	2.73	2.52	2.73	3.10	5.72
1.60	2.60	2.99	2.50	2.35	2.50	2.24	4.00
2.00	1.40	4.19	2.34	2.02	2.34	18.70	30.54
2.00	1.80	3.43	2.25	2.01	2.25	5.62	10.38
2.00	2.20	2.97	2.18	2.00	2.18	3.10	5.72
2.00	2.60	2.64	2.11	1.97	2.11	2.24	4.00

Table 5.4: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 3$ using SRS, RSS, MRSS and ERSS.

δ	σ_1/σ_0	Chart for Means SRS	RSS	MRSS	ERSS	*R* Chart ERSS	SRS
0.00	1.00	370.69	370.23	370.64	370.23	370.51	370.64
0.40	1.00	196.85	119.22	102.51	133.83	367.11	368.73
0.80	1.00	72.29	27.98	21.33	33.34	365.07	365.10
1.20	1.00	27.50	8.59	6.41	10.66	362.58	364.30
1.60	1.00	12.38	3.55	2.73	4.39	361.12	364.03
2.00	1.00	6.31	1.94	1.59	2.32	360.36	363.77
2.40	1.00	3.64	1.35	1.19	1.53	357.23	362.84
2.80	1.00	2.37	1.11	1.05	1.20	345.58	355.98
0.00	1.40	31.93	31.58	31.57	31.61	9.30	30.54
0.00	1.80	10.61	10.52	10.56	10.74	2.68	10.38
0.00	2.20	5.87	5.84	5.82	5.86	1.63	5.72
0.00	2.60	4.04	4.01	4.00	4.04	1.31	4.00
0.00	3.00	3.17	3.12	3.11	3.16	1.17	3.13
0.00	3.40	2.66	2.63	2.61	2.63	1.10	2.64
0.00	3.80	2.34	2.33	2.32	2.33	1.07	2.33
0.80	1.40	16.23	10.05	8.55	11.14	9.30	30.54
0.80	1.80	7.82	5.99	5.43	6.34	2.68	10.38
0.80	2.20	4.97	4.30	4.05	4.47	1.63	5.72
0.80	2.60	3.69	3.40	3.25	3.48	1.31	4.00
1.20	1.40	9.89	5.08	4.25	5.79	9.30	30.54
1.20	1.80	5.93	3.89	3.45	4.23	2.68	10.38
1.20	2.20	4.25	3.25	2.99	3.45	1.63	5.72
1.20	2.60	3.36	2.82	2.66	2.94	1.31	4.00
1.60	1.40	6.26	2.94	2.48	3.38	9.30	30.54
1.60	1.80	4.47	2.66	2.35	2.93	2.68	10.38
1.60	2.20	3.56	2.49	2.26	2.67	1.63	5.72
1.60	2.60	2.99	2.32	2.16	2.45	1.31	4.00
2.00	1.40	4.19	1.96	1.69	2.22	9.30	30.54
2.00	1.80	3.43	1.97	1.75	2.16	2.68	10.38
2.00	2.20	2.97	1.96	1.79	2.12	1.63	5.72
2.00	2.60	2.64	1.94	1.79	2.06	1.31	4.00

Table 5.5: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 4$ using SRS, RSS, MRSS and ERSS.

δ	σ_1/σ_0	Chart for Means SRS	RSS	MRSS	ERSS	*R* Chart ERSS	SRS
0.00	1.00	370.69	370.23	370.23	370.51	370.51	370.64
0.40	1.00	196.85	107.92	84.49	118.72	368.00	368.73
0.80	1.00	72.29	22.38	15.41	27.18	363.65	365.10
1.20	1.00	27.50	6.66	4.53	8.28	363.10	364.30
1.60	1.00	12.38	2.80	2.03	3.43	360.33	364.03
2.00	1.00	6.31	1.62	1.30	1.89	358.49	363.77
2.40	1.00	3.64	1.20	1.08	1.32	357.58	362.84
2.80	1.00	2.37	1.05	1.01	1.10	343.84	355.98
0.00	1.40	31.93	31.71	31.41	31.82	7.49	30.54
0.00	1.80	10.61	10.59	10.58	10.60	2.20	10.38
0.00	2.20	5.87	5.85	5.85	5.87	1.41	5.72
0.00	2.60	4.04	4.01	4.00	4.02	1.18	4.00
0.00	3.00	3.17	3.15	3.16	3.17	1.09	3.13
0.00	3.40	2.66	2.63	2.61	2.65	1.05	2.64
0.00	3.80	2.34	2.32	2.33	2.33	1.03	2.33
0.80	1.40	16.23	8.82	7.13	9.81	7.51	30.54
0.80	1.80	7.82	5.56	4.85	5.93	2.20	10.38
0.80	2.20	4.97	4.12	3.78	4.28	1.41	5.72
0.80	2.60	3.69	3.30	3.11	3.39	1.18	4.00
1.20	1.40	9.89	4.34	3.43	4.97	7.51	30.54
1.20	1.80	5.93	3.50	2.97	3.83	2.20	10.38
1.20	2.20	4.25	3.03	2.69	3.22	1.41	5.72
1.20	2.60	3.36	2.69	2.47	2.81	1.18	4.00
1.60	1.40	6.26	2.52	2.02	2.87	7.51	30.54
1.60	1.80	4.47	2.38	2.02	2.62	2.20	10.38
1.60	2.20	3.56	2.28	2.00	2.45	1.41	5.72
1.60	2.60	2.99	2.18	1.97	2.31	1.18	4.00
2.00	1.40	4.19	1.71	1.43	1.92	7.51	30.54
2.00	1.80	3.43	1.77	1.52	1.93	2.20	10.38
2.00	2.20	2.97	1.80	1.59	1.94	1.41	5.72
2.00	2.60	2.64	1.81	1.63	1.92	1.18	4.00

Table 5.6: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 5$ using SRS, RSS, MRSS and ERSS.

δ	σ_1/σ_0	Chart for Means SRS	RSS	MRSS	ERSS	*R* Chart ERSS	SRS
0.00	1.00	370.69	370.37	370.23	370.23	370.37	370.64
0.40	1.00	196.85	94.11	73.06	118.47	366.30	368.73
0.80	1.00	72.29	18.13	12.28	27.12	364.50	365.10
1.20	1.00	27.50	5.34	3.61	8.27	364.36	364.30
1.60	1.00	12.38	2.32	1.71	3.43	363.90	364.03
2.00	1.00	6.31	1.41	1.18	1.89	357.11	363.77
2.40	1.00	3.64	1.12	1.03	1.32	357.24	362.84
2.80	1.00	2.37	1.02	1.00	1.10	347.30	355.98
0.00	1.40	31.93	31.47	31.43	31.34	4.32	30.54
0.00	1.80	10.61	10.60	10.60	10.62	1.45	10.38
0.00	2.20	5.87	5.82	5.81	5.85	1.10	5.72
0.00	2.60	4.04	4.03	4.01	4.04	1.03	4.00
0.00	3.00	3.17	3.16	3.10	3.17	1.01	3.13
0.00	3.40	2.66	2.63	2.61	2.64	1.00	2.64
0.00	3.80	2.34	2.34	2.34	2.34	1.00	2.33
0.80	1.40	16.23	7.84	6.25	9.81	4.32	30.54
0.80	1.80	7.82	5.17	4.48	5.91	1.45	10.38
0.80	2.20	4.97	3.93	3.56	4.29	1.10	5.72
0.80	2.60	3.69	3.21	3.00	3.39	1.03	4.00
1.20	1.40	9.89	3.79	2.97	4.96	4.32	30.54
1.20	1.80	5.93	3.19	2.69	3.82	1.45	10.38
1.20	2.20	4.25	2.83	2.49	3.23	1.10	5.72
1.20	2.60	3.36	2.57	2.33	2.81	1.03	4.00
1.60	1.40	6.26	2.21	1.78	2.87	4.32	30.54
1.60	1.80	4.47	2.16	1.83	2.62	1.45	10.38
1.60	2.20	3.56	2.11	1.85	2.46	1.10	5.72
1.60	2.60	2.99	2.06	1.85	2.31	1.03	4.00
2.00	1.40	4.19	1.53	1.30	1.92	4.32	30.54
2.00	1.80	3.43	1.62	1.40	1.93	1.45	10.38
2.00	2.20	2.97	1.67	1.47	1.94	1.10	5.72
2.00	2.60	2.64	1.70	1.52	1.92	1.03	4.00

Table 5.7: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 6$ using SRS, RSS, MRSS and ERSS.

5.3.2 Control Chart for Mean and Range with DRSS and its Modifications

As in Section 5.3.1 for the cases of RSS, MRSS and ERSS, we use control limits for DRSS, MDRSS, DMRSS, EDRSS based chart for means, Chapter 4, and that of EDRSS base *R* chart in Section 5.2.3, to compute the ARL values. Perfect ranking was considered again and computer simulations were run for mixed values of δ, σ_1/σ_0 and n as in Section 5.3.1 using 500,000 iterations. The ARL values were matched to the in-control chart for means, to allow fair comparison. See Reynolds and Stoumbos [56]. Thus, the control limits for the means charts using DRSS were opened up to $3.047\,\sigma_{\overline{Y}_{drss}}$, MDRSS to $3.021\,\sigma_{\overline{Y}_{mdrss}}$, DMRSS to $3.132\,\sigma_{\overline{Y}_{dmrss}}$ and EDRSS to $3.0824\,\sigma_{\overline{Y}_{edrss}}$ while the control limits for the *R* chart using EDRSS was opened up to $3.1520\,\sigma_{R_{edrss}}$. The results are displayed in Tables 5.8 – 5.11 and the following can be deduced from the tables

1. The control charts for means using DRSS, MDRSS, DMRSS and EDRSS are all doing better in detecting both the increases in standard deviation and shift in mean than the corresponding SRS. For example consider n = 4 (Table 5.10) , δ = 0.8 and σ_1/σ_0 = 1.40, the ARL value of SRS mean chart is 16.23 as compared to 7.74 of DRSS, 5.24 of MDRSS, 3.81 of DMRSS and 9.69 of EDRSS.
2. The DRSS, MDRSS, DMRSS and EDRSS charts for means have smaller ARL values as compared to RSS, MRSS and ERSS with same sample size, δ and σ_1. But as the sample size increases, the MRSS control chart for means appears to be doing better job than the corresponding EDRSS. See for example the case when

n = 5 (Table 5.11), $\delta = 0.4$ and $\sigma_1/\sigma_0 = 1.0$ the ARL value of DRSS is 73.54 as compared to 107.92 (Table 5.6) of RSS.

3. The DMRSS based control charts is move effective in detecting increases in standard deviation and shift in process mean than any other control chart for mean and such dominancy increases with the increase in sample size.
4. The control charts for the means using RSS, MRSS, ERSS, DRSS, MDRSS, DMRSS and EDRSS appears to be just as good as SRS counterpart when there is no shift in mean but increase in standard deviation. In such cases, the *R* chart based on EDRSS dominates all other charts. See the case when n = 5 (Table 5.11), $\delta = 0.0$ and $\sigma_1/\sigma_0 = 1.40$, the ARL value of EDRSS is 5.91 as compared to 31.50 for others.
5. If there is no shift in the process mean, i.e. $\delta = 0$, the EDRSS based *R* charts is more effective in detecting increases in the standard deviation than the *R* chart based ERSS. The *R* chart is however not sensitive in detecting shifts in mean.

δ	σ_1/σ_0	Chart for Means SRS	DRSS	MDRSS	DMRSS	EDRSS	*R* Chart EDRSS	SRS
0.00	1.00	370.69	370.64	370.64	370.64	370.64	370.37	370.64
0.40	1.00	196.85	148.54	148.54	148.54	148.54	368.72	368.73
0.80	1.00	72.29	40.56	40.56	40.56	40.56	363.97	365.10
1.20	1.00	27.50	13.55	13.55	13.55	13.55	363.14	364.30
1.60	1.00	12.38	5.58	5.58	5.58	5.58	362.73	364.03
2.00	1.00	6.31	2.84	2.84	2.84	2.84	362.30	363.77
2.40	1.00	3.64	1.77	1.77	1.77	1.77	362.03	362.84
2.80	1.00	2.37	1.32	1.32	1.32	1.32	355.00	355.98
0.00	1.40	31.93	31.58	31.58	31.58	31.58	22.91	30.54
0.00	1.80	10.61	10.03	10.03	10.03	10.03	7.14	10.38
0.00	2.20	5.87	5.88	5.88	5.88	5.88	3.88	5.72
0.00	2.60	4.04	4.00	4.00	4.00	4.00	2.73	4.00
0.00	3.00	3.17	3.17	3.17	3.17	3.17	2.19	3.13
0.00	3.40	2.66	2.63	2.63	2.63	2.63	1.89	2.64
0.00	3.80	2.34	2.34	2.34	2.34	2.34	1.69	2.33
0.80	1.40	16.23	12.56	12.56	12.56	12.56	22.91	30.54
0.80	1.80	7.82	6.87	6.87	6.87	6.87	7.14	10.38
0.80	2.20	4.97	4.68	4.68	4.68	4.68	3.88	5.72
0.80	2.60	3.69	3.60	3.60	3.60	3.60	2.73	4.00
1.20	1.40	9.89	6.68	6.68	6.68	6.68	22.91	30.54
1.20	1.80	5.93	4.69	4.69	4.69	4.69	7.14	10.38
1.20	2.20	4.25	3.67	3.67	3.67	3.67	3.88	5.72
1.20	2.60	3.36	3.08	3.08	3.08	3.08	2.73	4.00
1.60	1.40	6.26	3.92	3.92	3.92	3.92	22.91	30.54
1.60	1.80	4.47	3.27	3.27	3.27	3.27	7.14	10.38
1.60	2.20	3.56	2.86	2.86	2.86	2.86	3.88	5.72
1.60	2.60	2.99	2.60	2.60	2.60	2.60	2.73	4.00
2.00	1.40	4.19	2.54	2.54	2.54	2.54	22.91	30.54
2.00	1.80	3.43	2.40	2.40	2.40	2.40	7.14	10.38
2.00	2.20	2.97	2.28	2.28	2.28	2.28	3.88	5.72
2.00	2.60	2.64	2.19	2.19	2.19	2.19	2.73	4.00

Table 5.8: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 2$ using SRS, DRSS, MDRSS, DMRSS and EDRSS.

δ	σ_1/σ_0	Chart for Means SRS	DRSS	MDRSS	DMRSS	EDRSS	*R* Chart EDRSS	SRS
0.00	1.00	370.69	370.37	370.37	370.10	370.37	370.37	370.64
0.40	1.00	196.85	116.39	81.87	58.17	116.39	368.74	368.73
0.80	1.00	72.29	26.09	15.01	9.13	26.09	366.01	365.10
1.20	1.00	27.50	7.78	4.37	2.72	7.78	363.12	364.30
1.60	1.00	12.38	3.17	1.96	1.40	3.17	362.14	364.03
2.00	1.00	6.31	1.75	1.28	1.08	1.75	361.89	363.77
2.40	1.00	3.64	1.25	1.07	1.01	1.25	361.22	362.84
2.80	1.00	2.37	1.08	1.01	1.00	1.08	349.47	355.98
0.00	1.40	31.93	31.60	31.63	31.59	31.60	17.34	30.54
0.00	1.80	10.61	10.32	10.62	10.61	10.32	4.99	10.38
0.00	2.20	5.87	5.88	5.87	5.86	5.88	2.72	5.72
0.00	2.60	4.04	4.03	4.03	4.02	4.04	1.96	4.00
0.00	3.00	3.17	3.17	3.16	3.16	3.17	1.62	3.13
0.00	3.40	2.66	2.67	2.65	2.64	2.67	1.43	2.64
0.00	3.80	2.34	2.34	2.34	2.34	2.34	1.32	2.33
0.80	1.40	16.23	9.75	7.03	5.26	9.75	17.34	30.54
0.80	1.80	7.82	5.95	4.80	3.98	5.95	4.99	10.38
0.80	2.20	4.97	4.33	3.74	3.30	4.33	2.72	5.72
0.80	2.60	3.69	3.43	3.11	2.86	3.43	1.96	4.00
1.20	1.40	9.89	4.79	3.35	2.47	4.79	17.34	30.54
1.20	1.80	5.93	3.76	2.93	2.34	3.76	4.99	10.38
1.20	2.20	4.25	3.19	2.66	2.25	3.19	2.72	5.72
1.20	2.60	3.36	2.79	2.45	2.17	2.79	1.96	4.00
1.60	1.40	6.26	2.74	1.98	1.53	2.74	17.34	30.54
1.60	1.80	4.47	2.54	1.98	1.61	2.54	4.99	10.38
1.60	2.20	3.56	2.39	1.97	1.66	2.39	2.72	5.72
1.60	2.60	2.99	2.26	1.95	1.69	2.26	1.96	4.00
2.00	1.40	4.19	1.82	1.41	1.18	1.82	17.34	30.54
2.00	1.80	3.43	1.86	1.50	1.27	1.86	4.99	10.38
2.00	2.20	2.97	1.87	1.56	1.34	1.87	2.72	5.72
2.00	2.60	2.64	1.87	1.61	1.40	1.87	1.96	4.00

Table 5.9: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 3$ using SRS, DRSS, MDRSS, DMRSS and EDRSS.

δ	σ_1/σ_0	Chart for Means					*R* Chart	
		SRS	DRSS	MDRSS	DMRSS	EDRSS	EDRSS	SRS
0.00	1.00	370.69	370.37	370.64	370.64	370.37	370.37	370.64
0.40	1.00	196.85	89.56	57.71	39.40	116.90	368.66	368.73
0.80	1.00	72.29	17.34	9.03	5.37	25.72	365.32	365.10
1.20	1.00	27.50	4.91	2.69	1.76	7.51	364.44	364.30
1.60	1.00	12.38	2.11	1.39	1.12	3.05	363.09	364.03
2.00	1.00	6.31	1.33	1.08	1.01	1.71	362.97	363.77
2.40	1.00	3.64	1.08	1.01	1.00	1.23	362.31	362.84
2.80	1.00	2.37	1.02	1.00	1.00	1.07	350.07	355.98
0.00	1.40	31.93	32.02	31.92	31.71	32.04	7.47	30.54
0.00	1.80	10.61	10.64	10.62	10.47	10.45	2.12	10.38
0.00	2.20	5.87	5.86	5.86	5.84	5.87	1.35	5.72
0.00	2.60	4.04	4.04	4.03	4.01	4.04	1.14	4.00
0.00	3.00	3.17	3.17	3.16	3.16	3.17	1.06	3.13
0.00	3.40	2.66	2.65	2.65	2.64	2.66	1.03	2.64
0.00	3.80	2.34	2.34	2.34	2.33	2.34	1.02	2.33
0.80	1.40	16.23	7.74	5.24	3.81	9.69	7.47	30.54
0.80	1.80	7.82	5.14	3.97	3.20	5.90	2.12	10.38
0.80	2.20	4.97	3.94	3.31	2.84	4.31	1.35	5.72
0.80	2.60	3.69	3.22	2.85	2.56	3.43	1.14	4.00
1.20	1.40	9.89	3.61	2.45	1.81	4.71	7.47	30.54
1.20	1.80	5.93	3.09	2.33	1.85	3.70	2.12	10.38
1.20	2.20	4.25	2.77	2.25	1.87	3.16	1.35	5.72
1.20	2.60	3.36	2.53	2.16	1.87	2.78	1.14	4.00
1.60	1.40	6.26	2.08	1.52	1.23	2.68	7.47	30.54
1.60	1.80	4.47	2.06	1.59	1.32	2.49	2.12	10.38
1.60	2.20	3.56	2.04	1.65	1.40	2.36	1.35	5.72
1.60	2.60	2.99	2.00	1.69	1.46	2.24	1.14	4.00
2.00	1.40	4.19	1.45	1.17	1.05	1.78	7.47	30.54
2.00	1.80	3.43	1.54	1.26	1.11	1.83	2.12	10.38
2.00	2.20	2.97	1.60	1.33	1.17	1.85	1.35	5.72
2.00	2.60	2.64	1.64	1.40	1.23	1.85	1.14	4.00

Table 5.10: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 4$ using SRS, DRSS, MDRSS, DMRSS and EDRSS.

δ	σ_1/σ_0	Chart for Means					*R* Chart	
		SRS	DRSS	MDRSS	DMRSS	EDRSS	EDRSS	SRS
0.00	1.00	370.69	370.10	370.64	377.93	370.37	370.64	370.64
0.40	1.00	196.85	73.54	36.06	18.25	94.80	367.67	368.73
0.80	1.00	72.29	12.24	5.01	2.39	18.62	365.24	365.10
1.20	1.00	27.50	3.42	1.68	1.13	5.20	364.29	364.30
1.60	1.00	12.38	1.61	1.10	1.00	2.22	363.00	364.03
2.00	1.00	6.31	1.14	1.01	1.00	1.36	362.14	363.77
2.40	1.00	3.64	1.02	1.00	1.00	1.10	361.71	362.84
2.80	1.00	2.37	1.00	1.00	1.00	1.02	344.87	355.98
0.00	1.40	31.93	31.79	31.50	31.36	31.87	5.91	30.54
0.00	1.80	10.61	10.57	10.51	10.42	10.60	1.72	10.38
0.00	2.20	5.87	5.86	5.84	5.80	5.86	1.18	5.72
0.00	2.60	4.04	4.04	4.04	4.02	4.04	1.05	4.00
0.00	3.00	3.17	3.16	3.16	3.16	3.16	1.02	3.13
0.00	3.40	2.66	2.66	2.65	2.65	2.66	1.01	2.64
0.00	3.80	2.34	2.34	2.33	2.33	2.35	1.00	2.33
0.80	1.40	16.23	6.25	3.65	2.26	8.01	5.91	30.54
0.80	1.80	7.82	4.47	3.10	2.19	5.24	1.72	10.38
0.80	2.20	4.97	3.61	2.78	2.14	4.01	1.18	5.72
0.80	2.60	3.69	3.04	2.53	2.07	3.26	1.05	4.00
1.20	1.40	9.89	2.88	1.76	1.25	3.75	5.91	30.54
1.20	1.80	5.93	2.62	1.81	1.34	3.16	1.72	10.38
1.20	2.20	4.25	2.46	1.83	1.42	2.83	1.18	5.72
1.20	2.60	3.36	2.31	1.83	1.47	2.56	1.05	4.00
1.60	1.40	6.26	1.70	1.21	1.03	2.15	5.91	30.54
1.60	1.80	4.47	1.76	1.30	1.08	2.11	1.72	10.38
1.60	2.20	3.56	1.79	1.38	1.14	2.08	1.18	5.72
1.60	2.60	2.99	1.81	1.43	1.19	2.03	1.05	4.00
2.00	1.40	4.19	1.26	1.04	1.00	1.49	5.91	30.54
2.00	1.80	3.43	1.35	1.10	1.01	1.57	1.72	10.38
2.00	2.20	2.97	1.42	1.16	1.04	1.63	1.18	5.72
2.00	2.60	2.64	1.48	1.21	1.07	1.66	1.05	4.00

Table 5.11: ARL values for Mean and *R* Chart for $\delta \geq 0.0$ and/or $\sigma_1/\sigma_0 \geq 1.0$ with $n = 5$ using SRS, DRSS, MDRSS, DMRSS and EDRSS

Chapter 6

APPLICATIONS

6.1 Introduction

To see how well and efficient our proposed control charts could readily be applied to real life situations, an attempt is made in this chapter to construct some control charts introduced in Chapter 4 and 5 using real data set.

6.2 Data Collection

Here we use the sets of data from Muttlak and Al-Sabah [47] collected from a *filling bottle with soft drink* production line of the Pepsi Cola production company in Al-Khobar, Saudi Arabia. The data were collected by measuring the unfilled part of the bottle using SRS, RSS, MRSS and ERSS sampling techniques with perfect ranking as well as imperfect ranking for sample sizes n = 3 and 4. In all, 69 random samples of set size n = 3 and 54 random samples of n = 4 were collected based on the above mentioned sampling methods. And with the permission of Muttlak and Al-Sabah [47], we make use of the data sets for the case of perfect ranking and applied our new sampling techniques on them to obtain the required DRSS, MDRSS, DMRSS and EDRSS data sets. See Muttlak and Al-Sabah [47] for the original data set.

6.3 Construction of Control Charts using Real Data

6.3.1 Control Charts Using RSS Data

Here, the colleted data set based on SRS and RSS sampling methods for sample sizes n = 3 and 4 were used to construct control charts for the mean. And for proper comparison, both SRS and RSS mean points were plotted on the same control chart. Figures 6.1 and 6.2 shows the mean charts for n = 3 and 4 respectively.

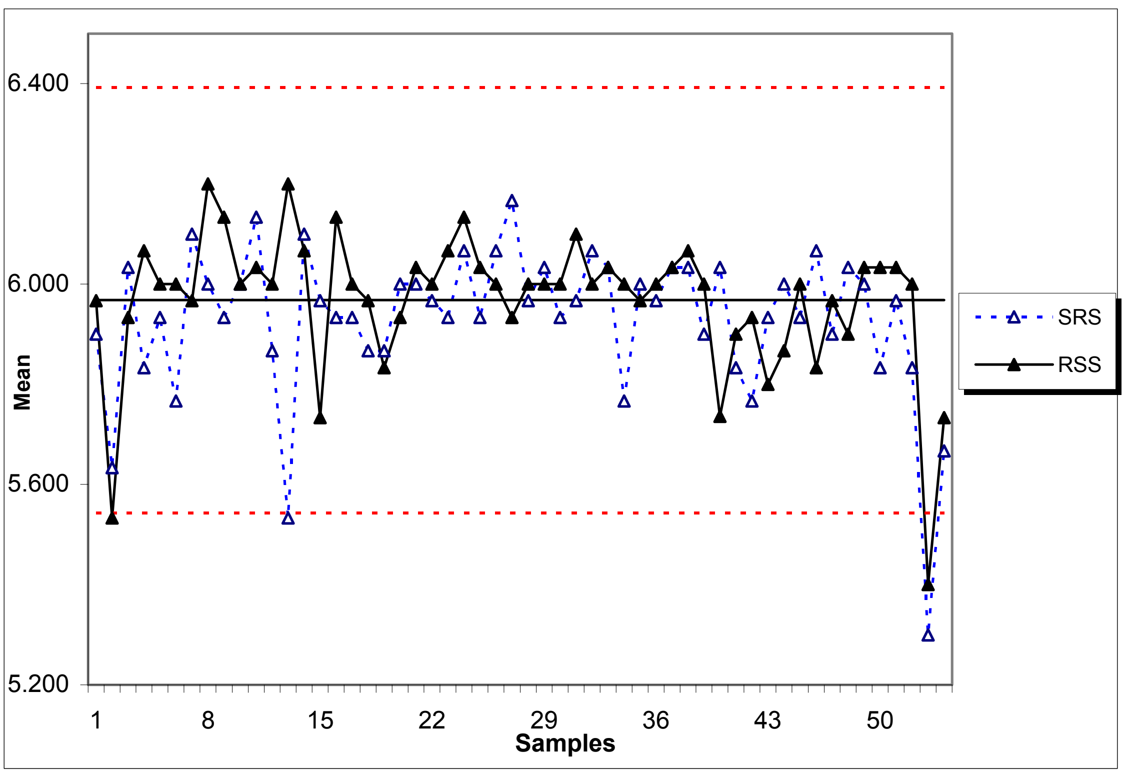

Figure 6.1: Control chart for mean using SRS and RSS with n = 3

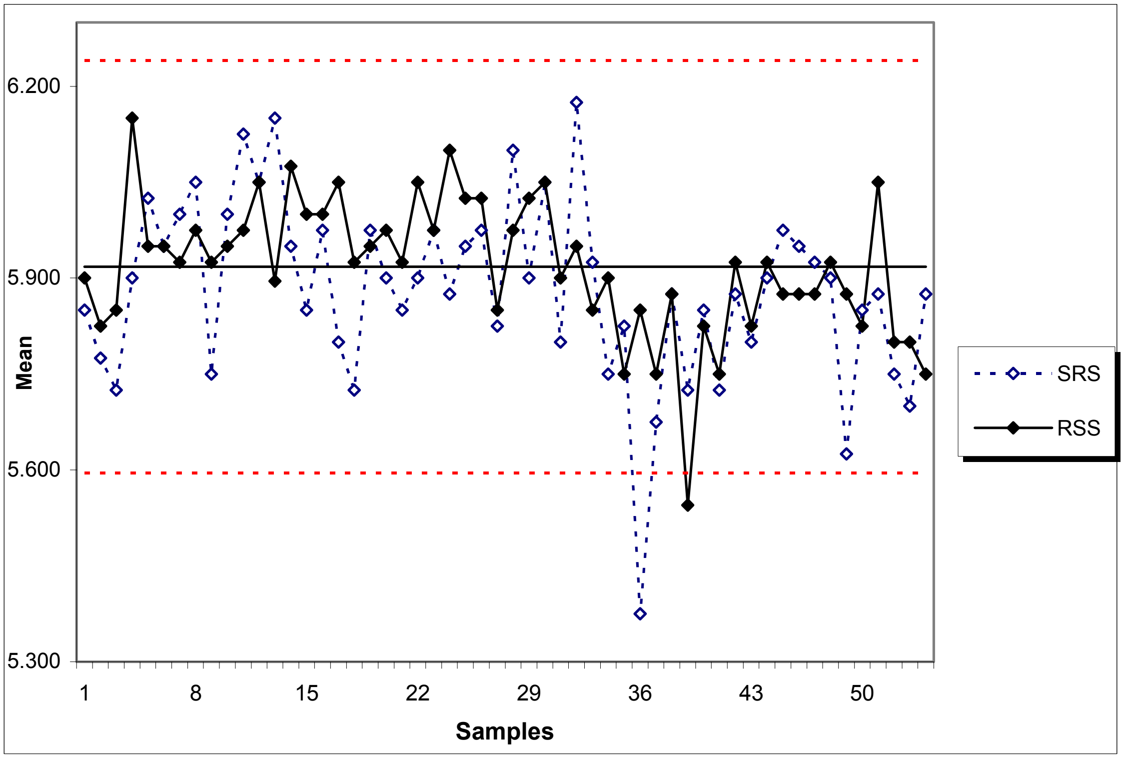

Figure 6.2: Control chart for mean using SRS and RSS with $n = 4$

6.3.2 Control Charts Using ERSS and MRSS Data

The data sets collected based on MRSS method for $n = 3$ and ERSS for $n = 4$ together with their corresponding SRS data were used to construct the control charts for mean as shown in Figures 6.3 and 6.4 respectively. Also, the ERSS data for $n = 4$ and its SRS counterpart were used to construct the control chart for range and is illustrated in Figure 6.5.

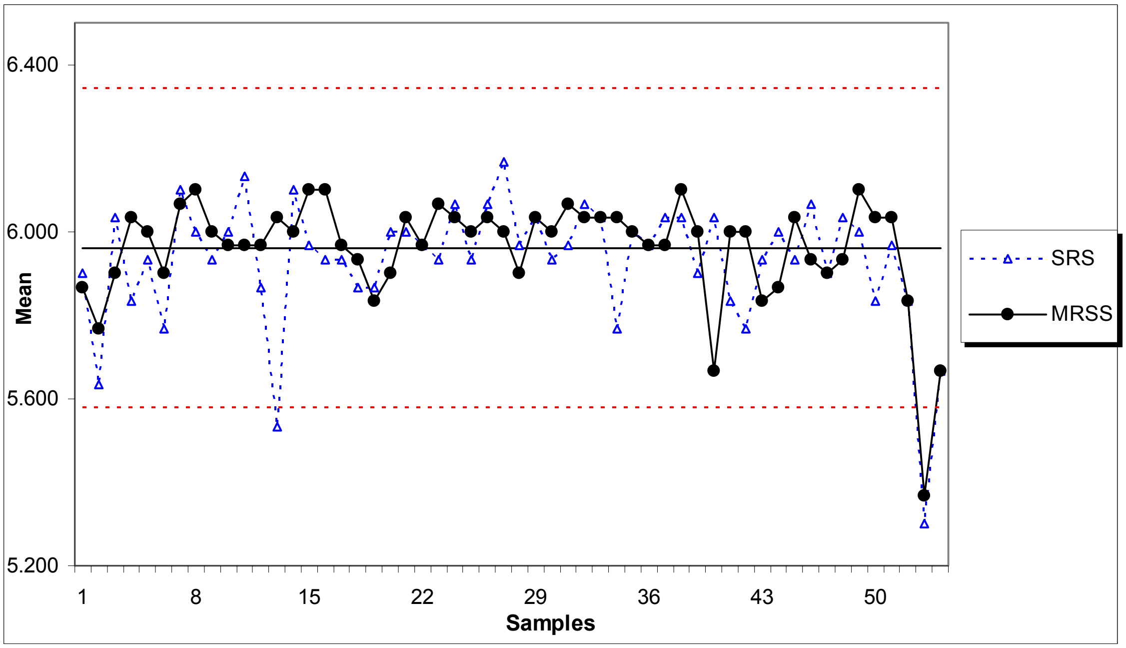

Figure 6.3: Control chart for mean using SRS and MRSS with $n = 3$

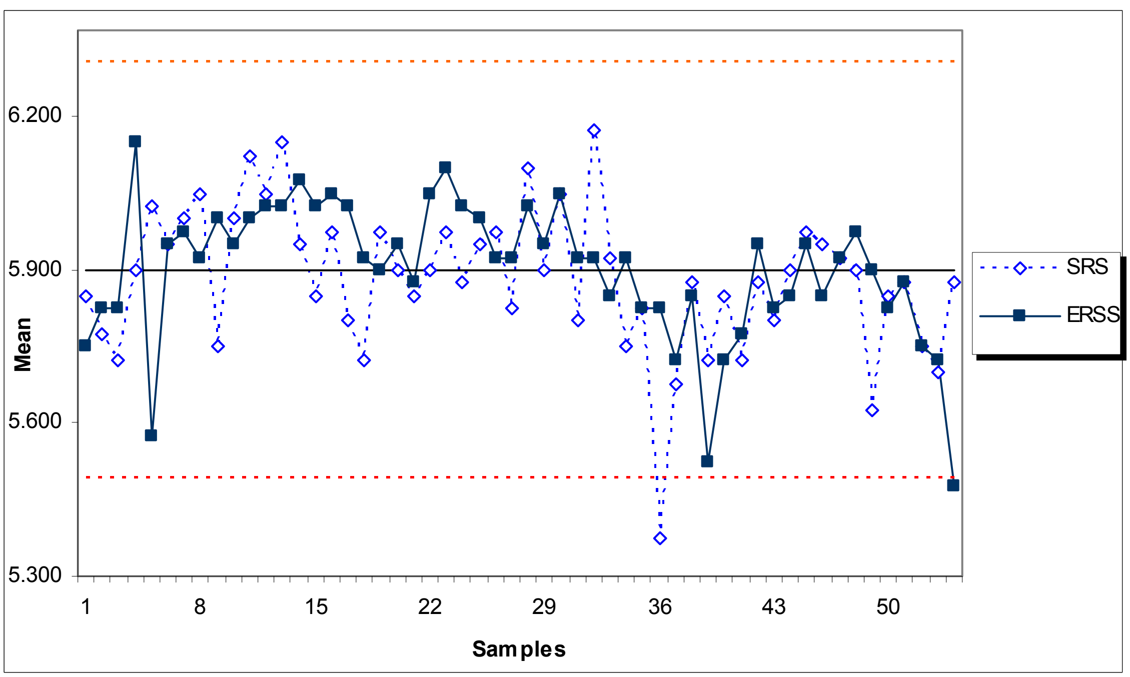

Figure 6.4: Control chart for mean using SRS and ERSS with $n = 4$

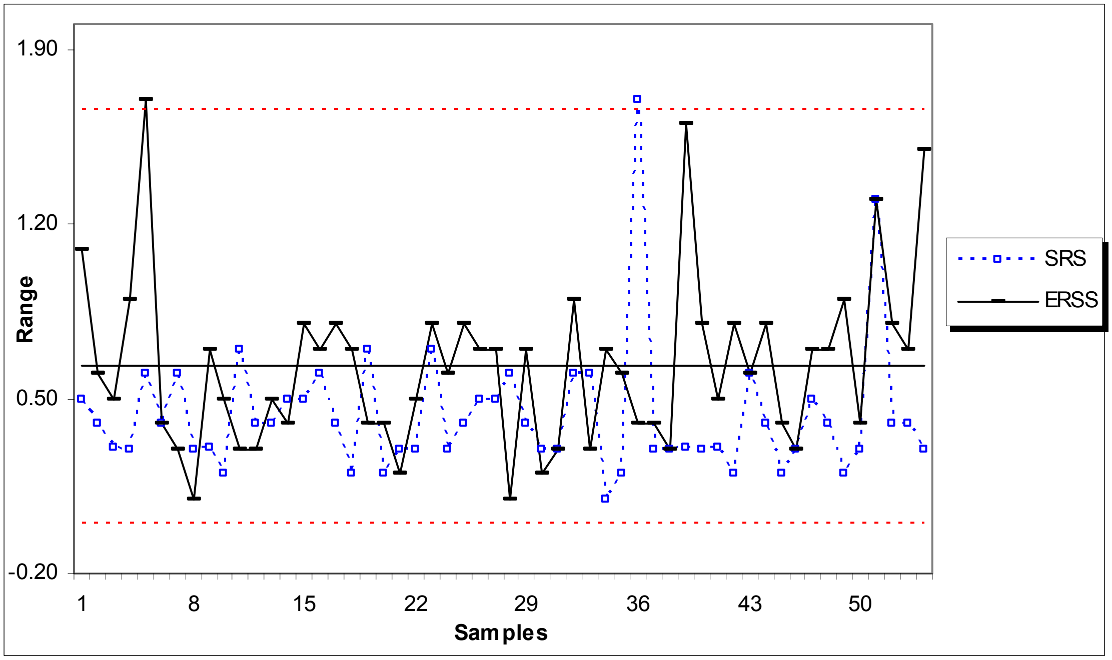

Figure 6.5: Control chart for range using SRS and ERSS with $n = 4$

6.3.3 Control Charts Using DRSS Data

To compare the control charts based on DRSS method with SRS, RSS, MRSS and ERSS, we used the data sets collected using these sampling techniques to construct the charts. The data sets obtained when DRSS method was applied to RSS data for $n = 3$ was used to construct a control chart for mean and for fair comparison, the SRS and RSS counterparts were also plotted on the same charts as shown in Figure 6.6. Similarly, Figure 6.8 gives same chart for mean when $n = 4$. The control charts for the combinations of SRS, MRSS, and DRSS when $n = 3$ and SRS, ERSS, DRSS for $n = 4$ are given respectively in Figures 6.7 and 6.9 respectively.

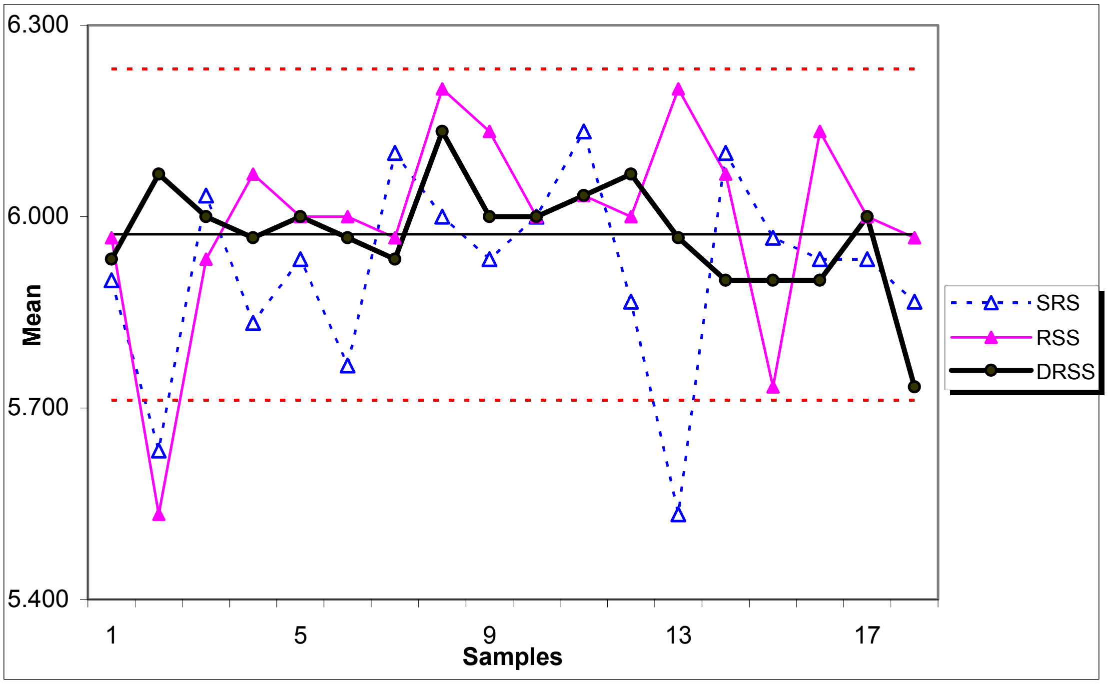

Figure 6.6: Control chart for mean using SRS, RSS and DRSS with $n = 3$

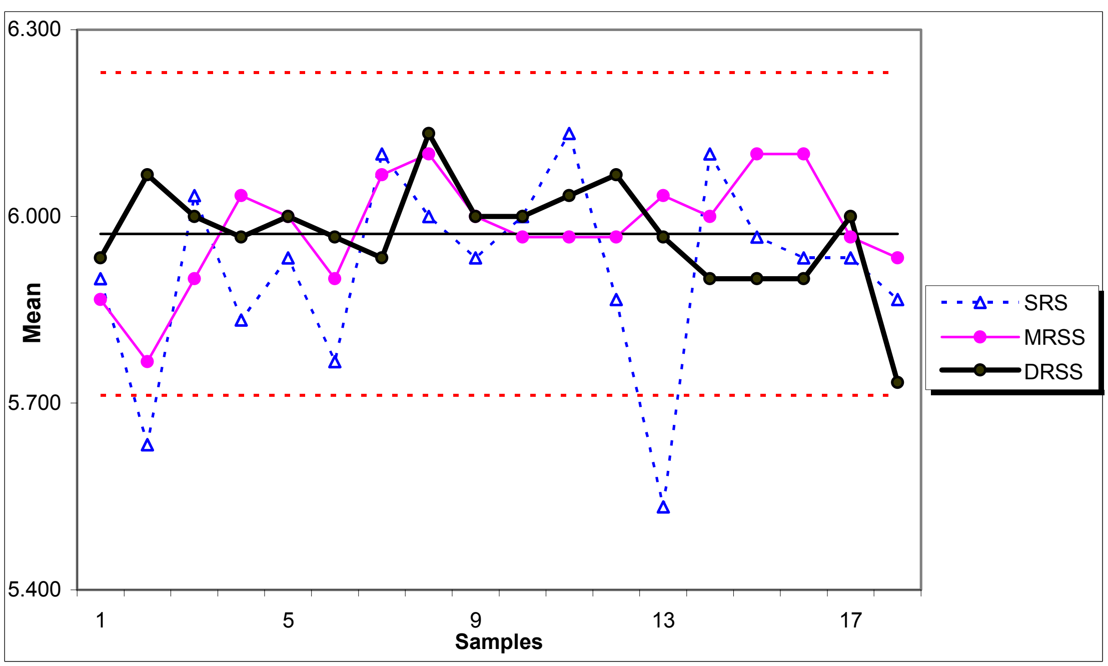

Figure 6.7: Control chart for mean using SRS, MRSS and DRSS with $n = 3$

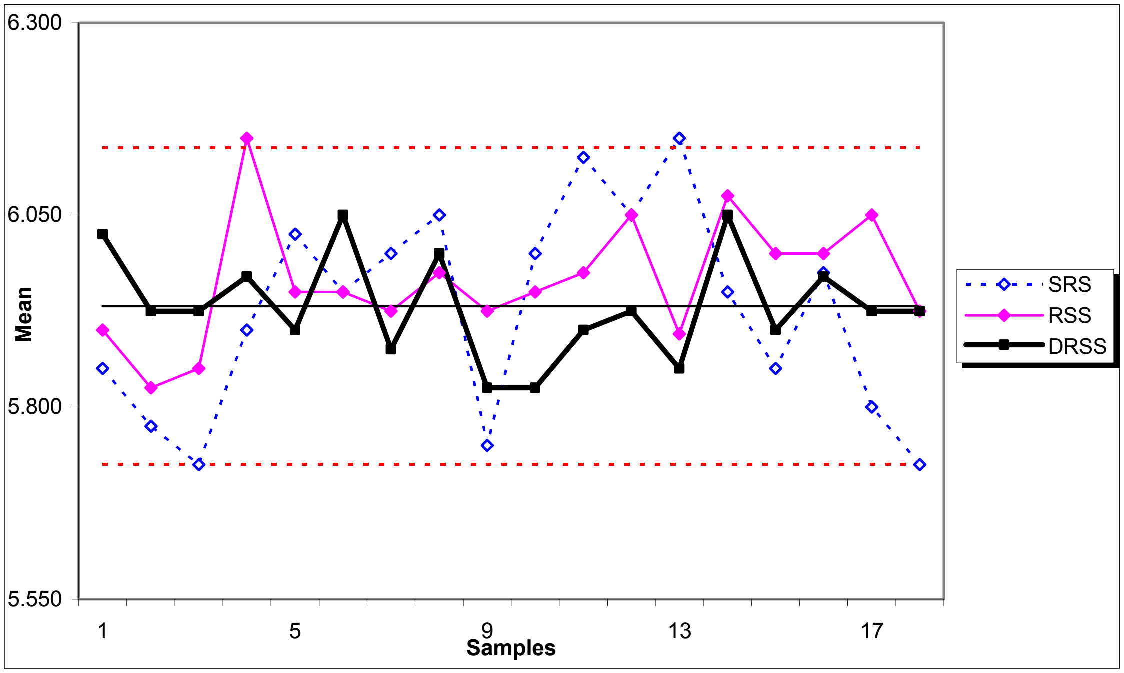

Figure 6.8: Control chart for mean using SRS, RSS and DRSS with $n = 4$

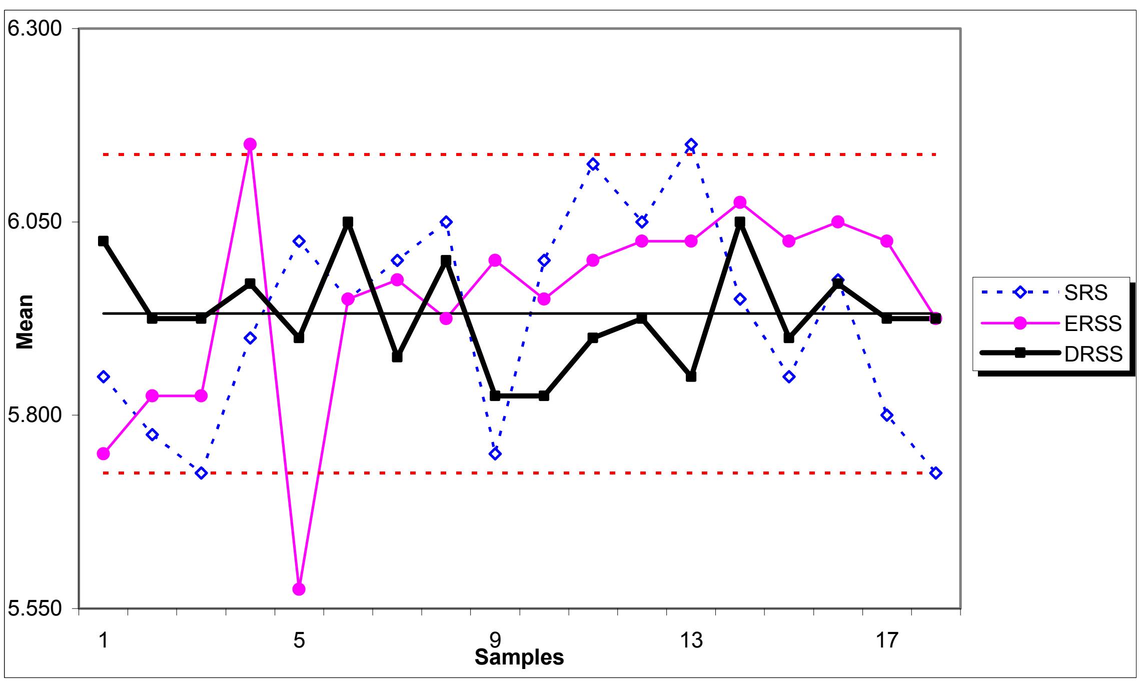

Figure 6.9: Control chart for mean using SRS, ERSS and DRSS with $n = 4$

6.3.4 Control Charts Using MDRSS Data

Using the data sets collected from applying MRSS on RSS data to obtain MDRSS, we construct the control charts for mean using the combinations of sampling methods SRS, RSS, MDRSS and SRS, MRSS, MDRSS for set size n = 3 as shown in Figures 6.10 and 6.11 respectively. For the set size n = 4, the control charts using the combinations of sampling methods SRS, RSS, MDRSS and SRS, ERSS, MDRSS are respectively given in Figures 6.12 and 6.13.

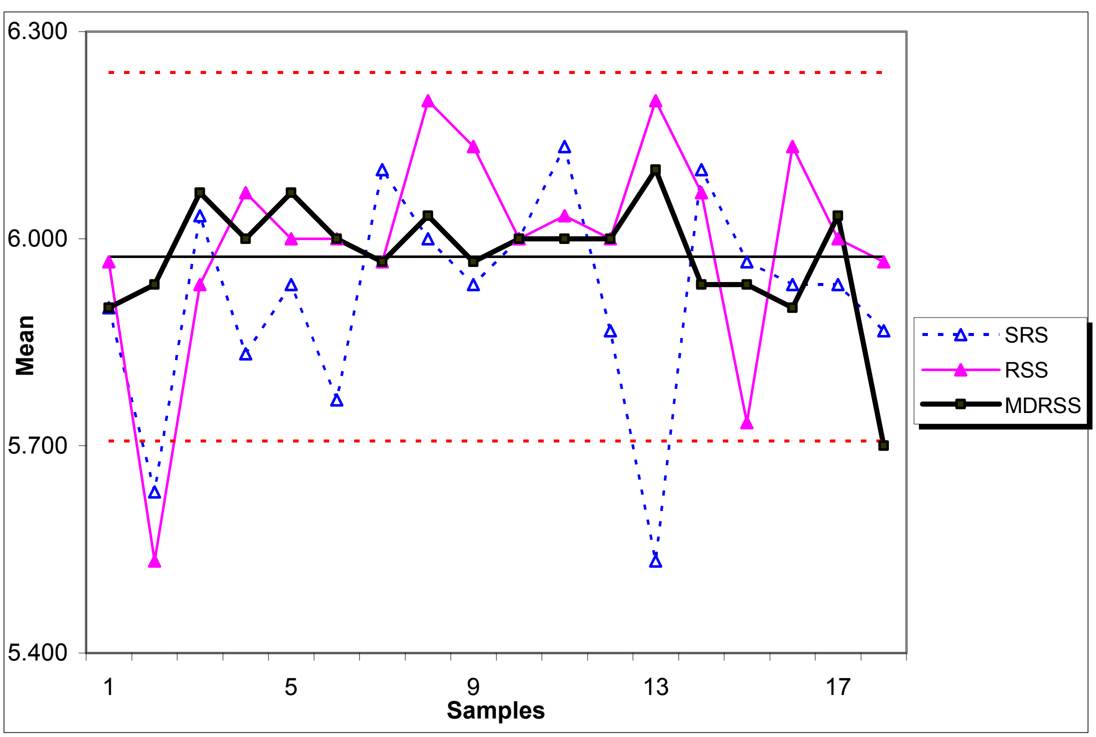

Figure 6.10: Control chart for mean using SRS, RSS and MDRSS with n = 3

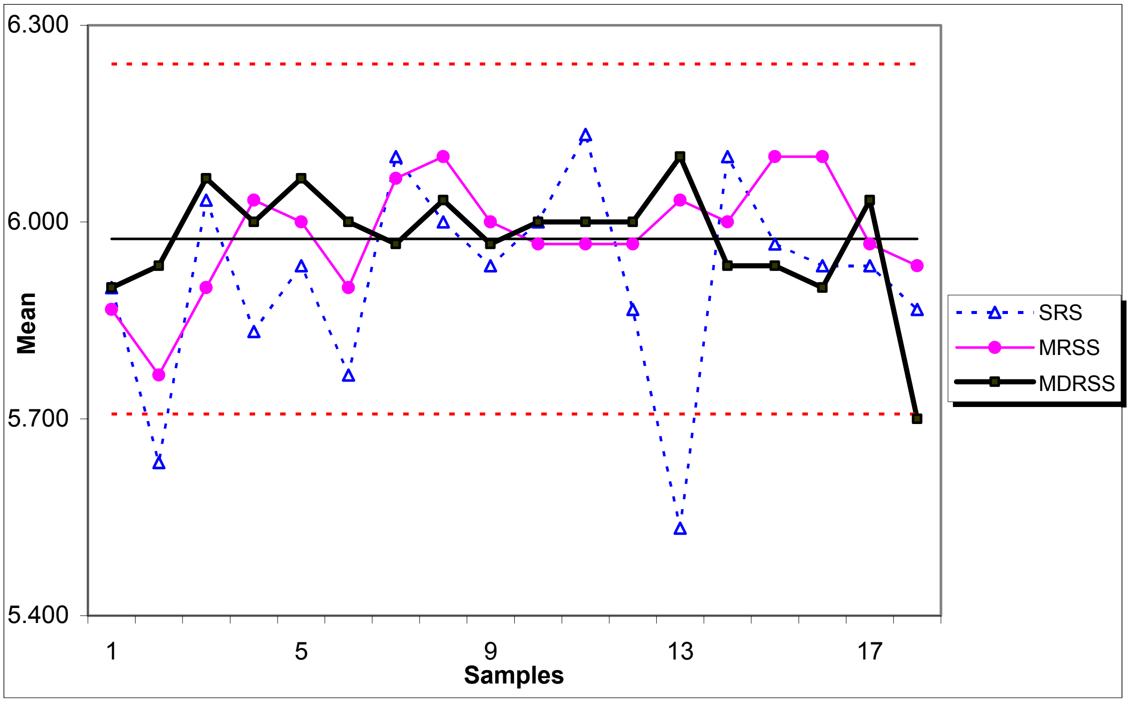

Figure 6.11: Control chart for mean using SRS, MRSS and MDRSS with n = 3

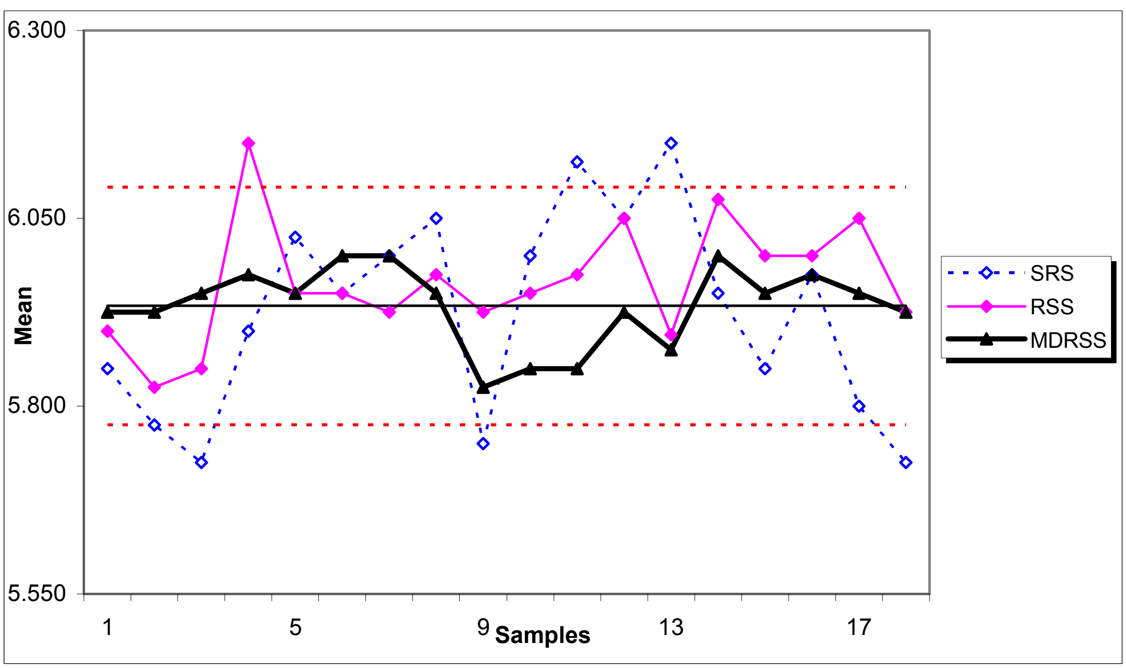

Figure 6.12: Control chart for mean using SRS, RSS and MDRSS with n = 4

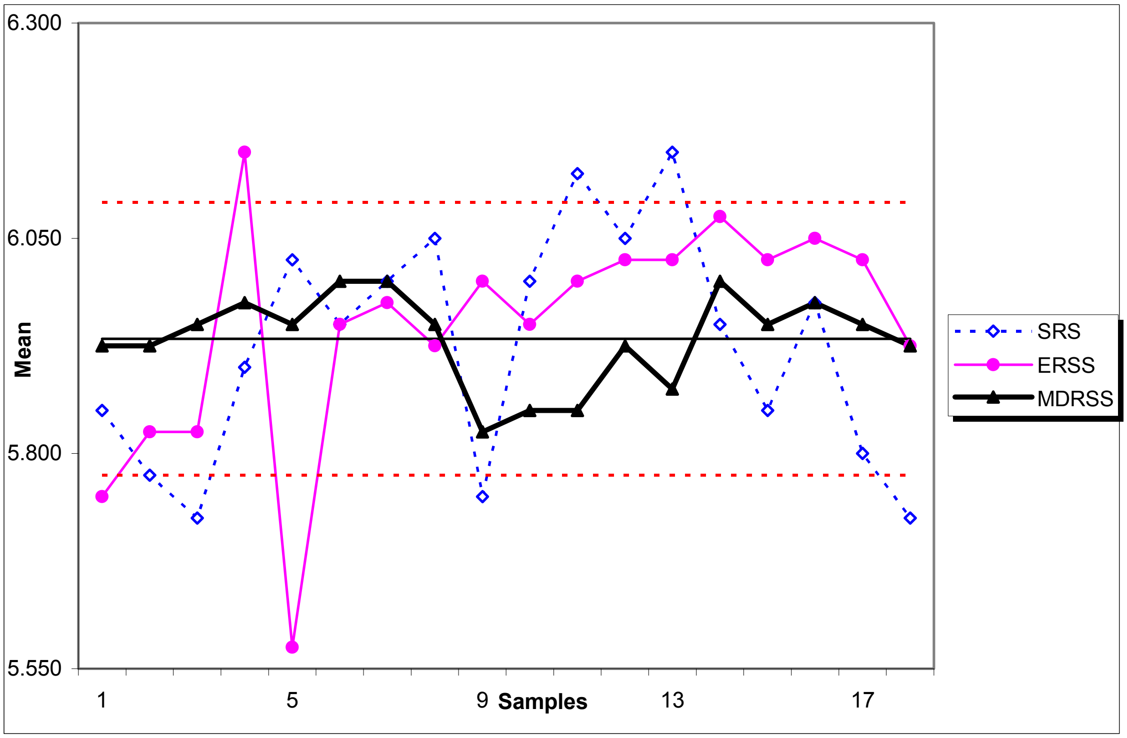

Figure 6.13: Control chart for mean using SRS, ERSS and MDRSS with $n = 4$

6.3.5 Control Charts Using DMRSS Data

Here the sets of data from DMRSS method obtained by applying MRSS on the MRSS data when $n = 3$ with perfect ranking are used to develop quality control charts for mean. Figure 6.14 give the mean control charts using DMRSS method as well as SRS and RSS for same process. And the combinations of sampling methods namely: SRS, MRSS and DMRSS are plotted in Figure 6.15.

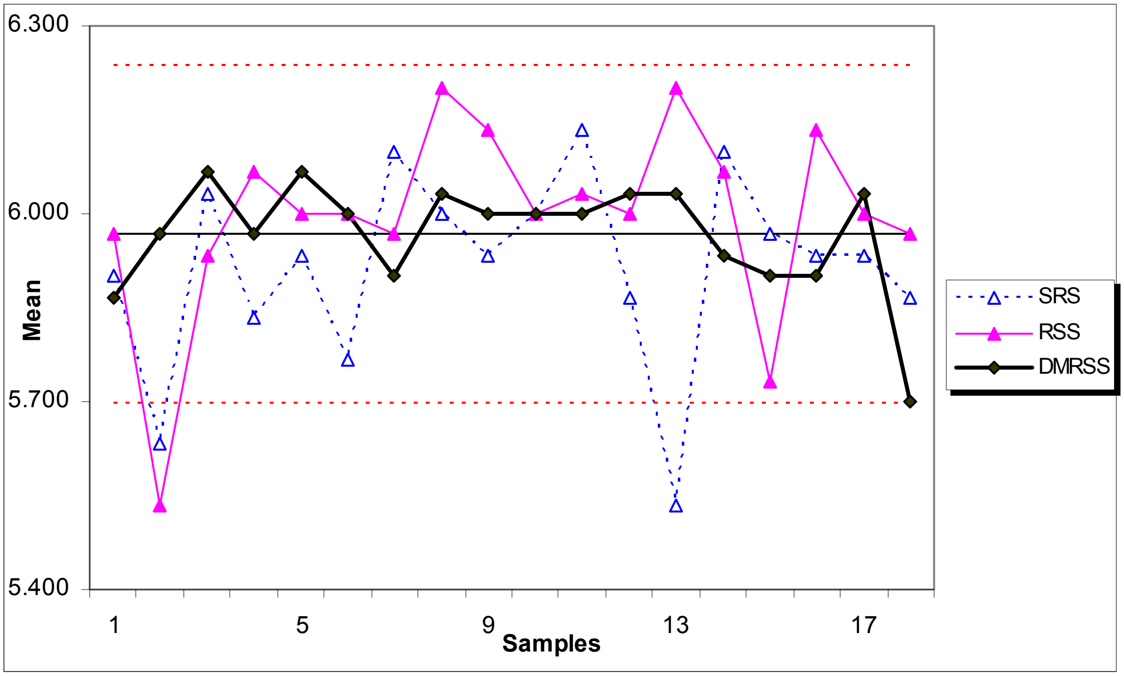

Figure 6.14: Control chart for mean using SRS, RSS and DMRSS with $n = 3$

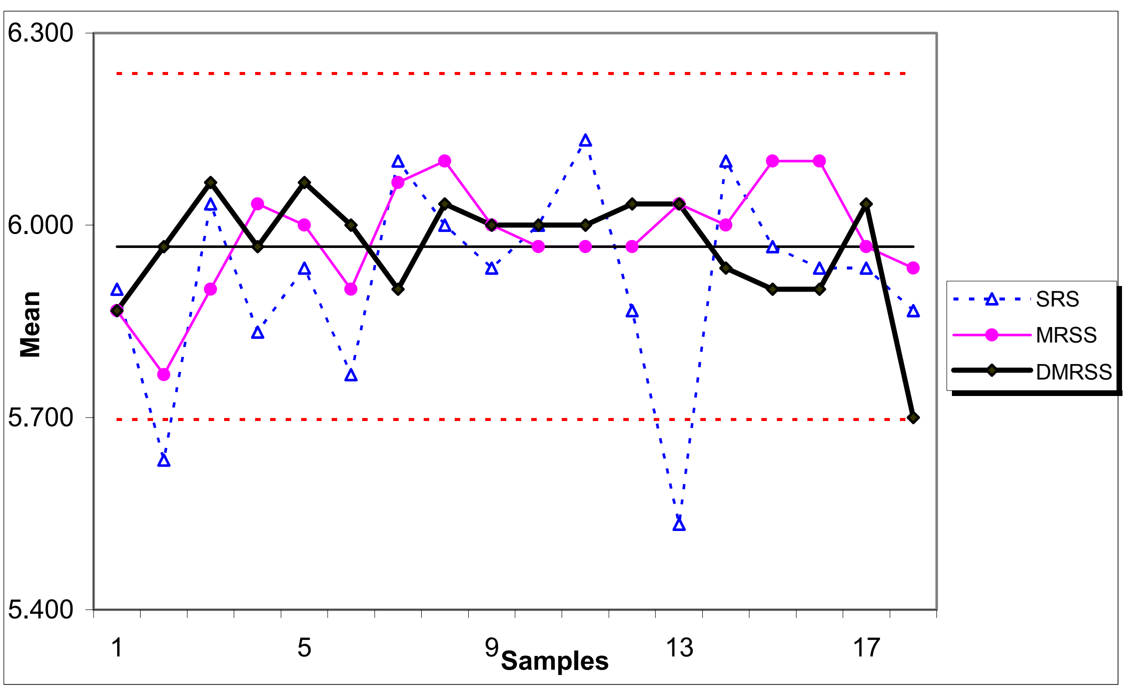

Figure 6.15: Control chart for mean using SRS, MRSS and DMRSS with $n = 3$

6.3.6 Control Charts Using EDRSS Data

We use the EDRSS data obtained from applying ERSS method on the regular RSS data in appendix A to construct quality control charts. Figures 6.16 and 6.17 show the control charts for means when n = 3 using the combination of methods SRS, RSS, EDRSS and SRS, MRSS, EDRSS respectively. In Figures in 6.18 and 6.19, we construct the control charts for means when n = 4 for SRS, RSS, EDRSS and SRS, ERSS, EDRSS data respectively. Also using SRS, ERSS, EDRSS data, control charts for range when n = 4 is given Figure 6.20.

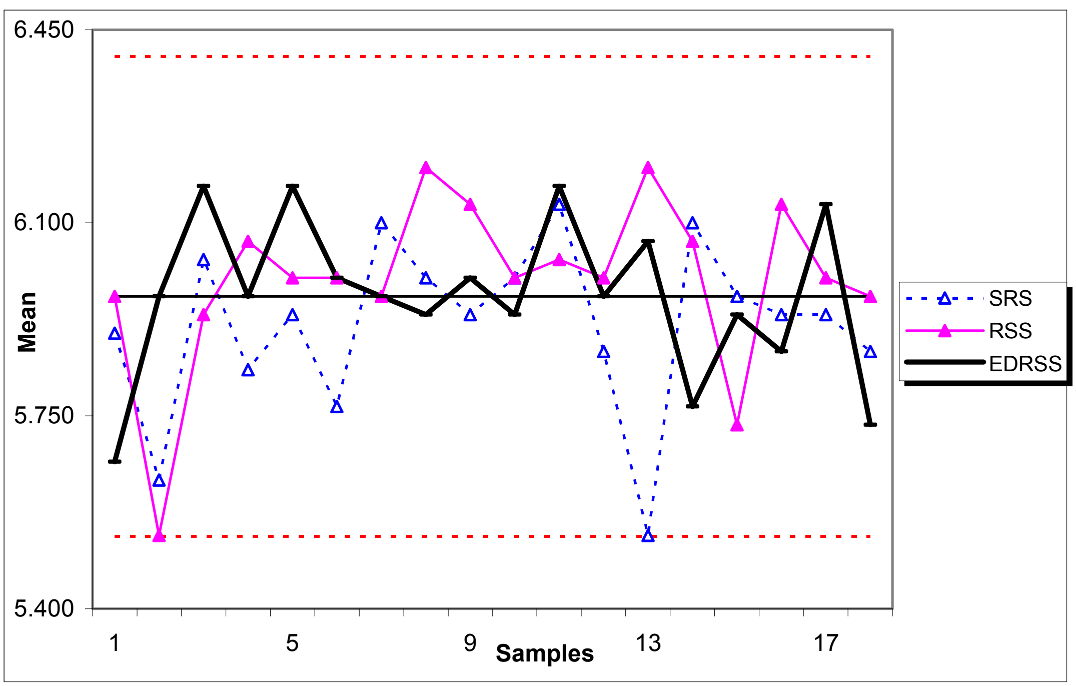

Figure 6.16: Control chart for mean using SRS, RSS and EDRSS with n = 3

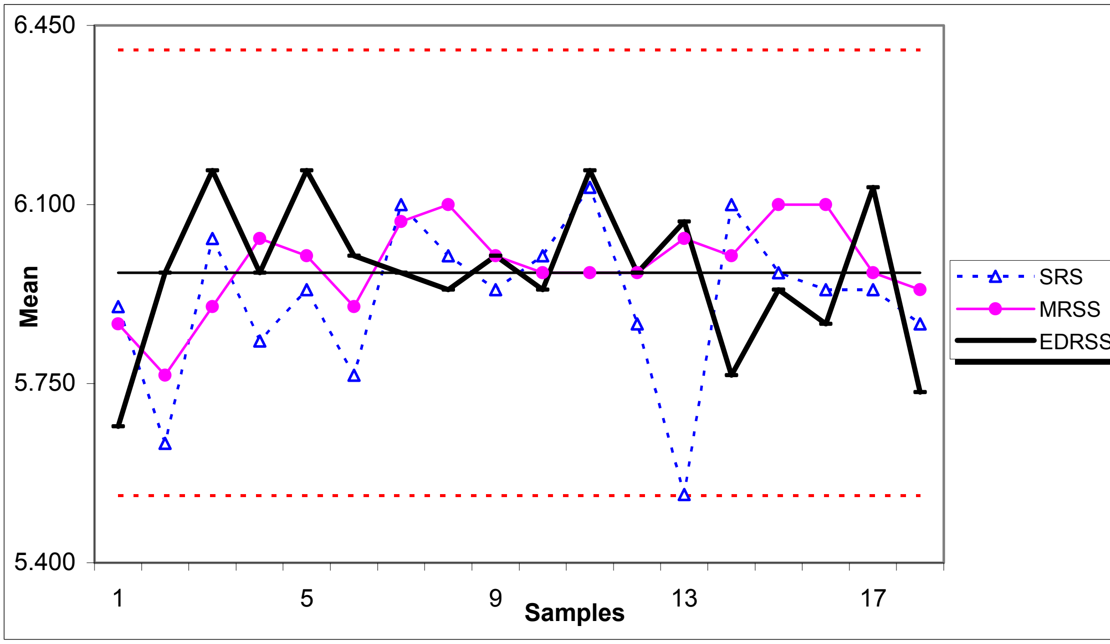

Figure 6.17: Control chart for mean using SRS, MRSS and EDRSS with $n = 3$

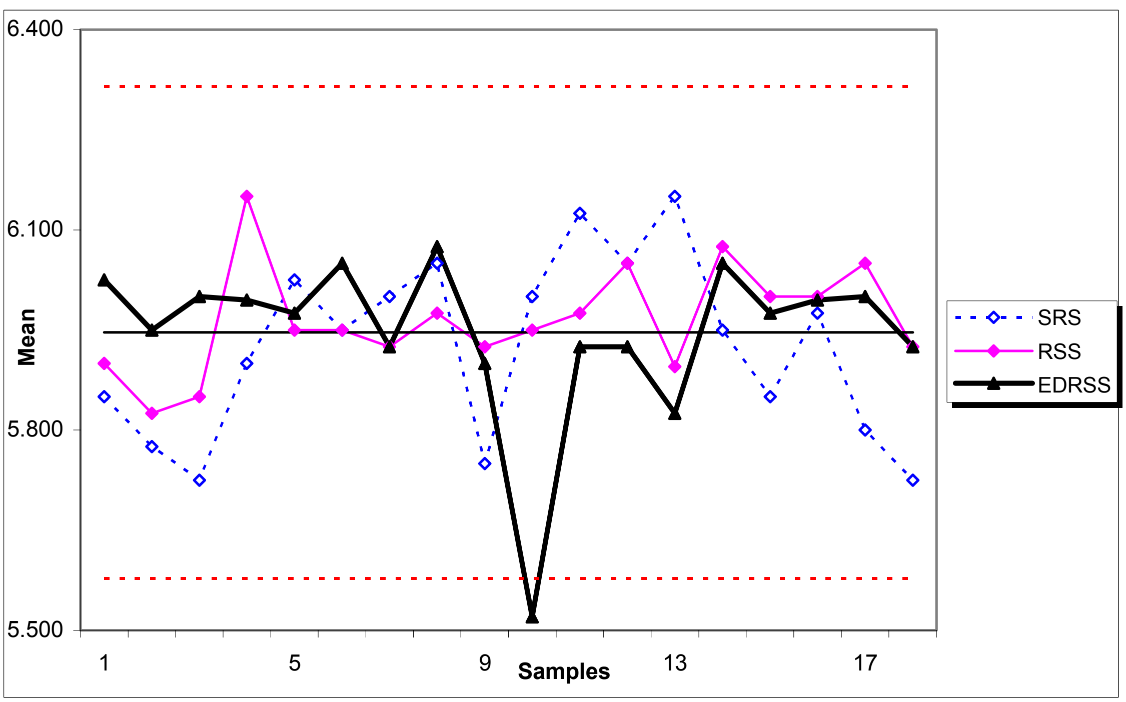

Figure 6.18: Control chart for mean using SRS, RSS and EDRSS with $n = 4$

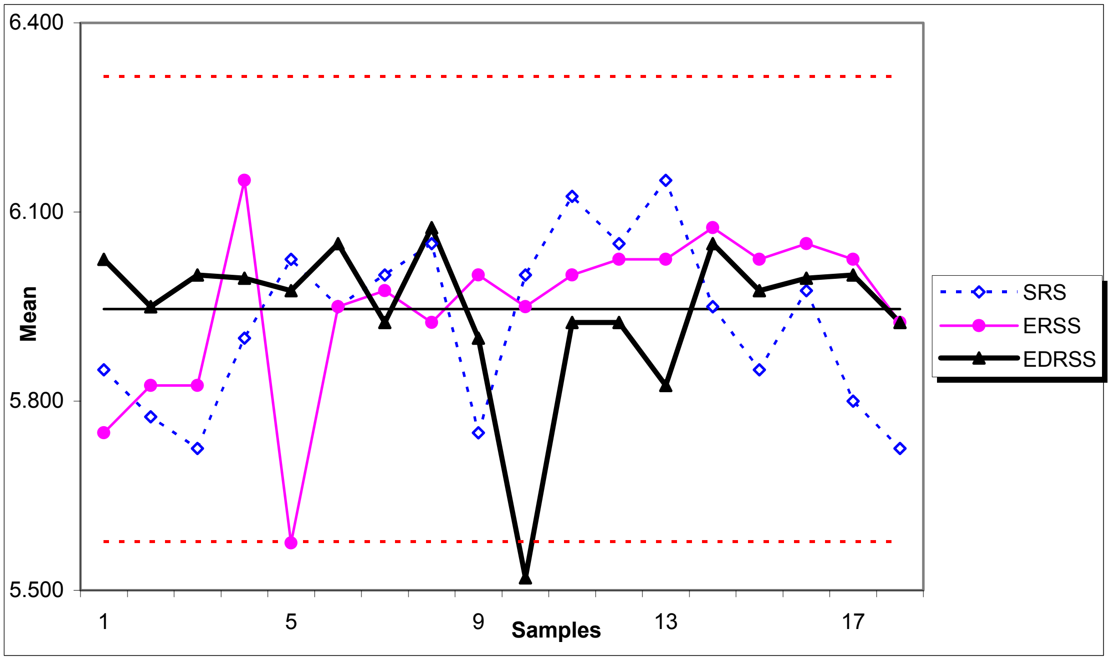

Figure 6.19: Control chart for mean using SRS, ERSS and EDRSS with $n = 4$

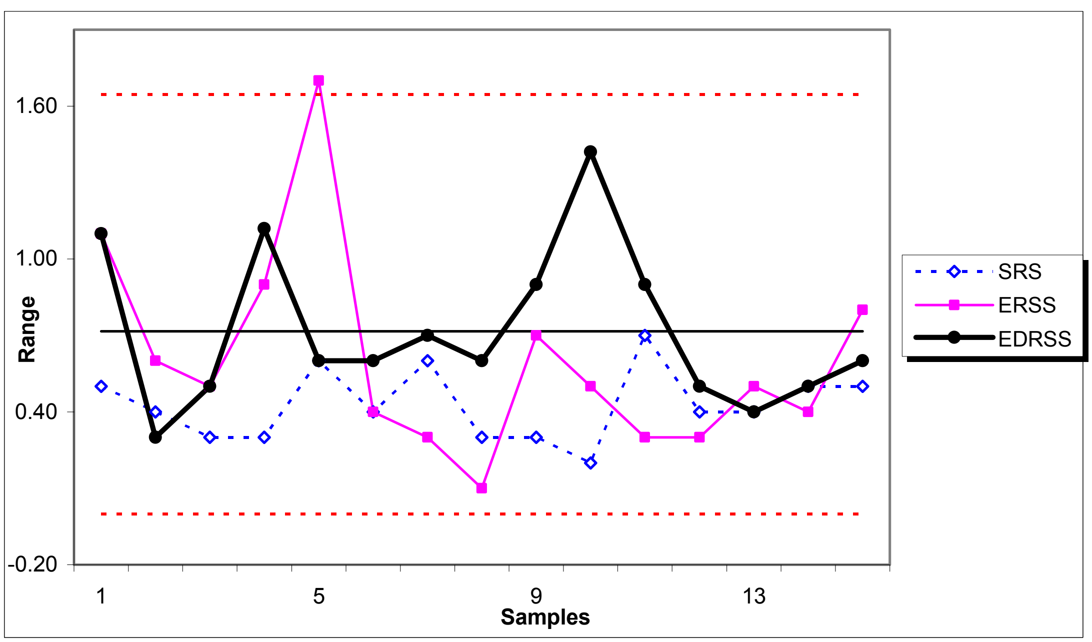

Figure 6.20: Control chart for range using SRS, ERSS and EDRSS with $n = 4$

6.3.7 Control Charts Using DRSS, MDRSS, DMRSS and EDRSS Data

Finally, we use the sets of data obtained using DRSS, MDRSS, DMRSS and EDRSS with sample sizes n = 3 and 4 for the case of perfect ranking to construct the control charts. Figures 6.21 shows the control chart for mean based on the sampling techniques namely: DRSS, MDRSS, DMRSS, and EDRSS data for n = 3 while Figure 6.22 gives the mean chart for DRSS, MDRSS, and EDRSS data when the set size n = 4.

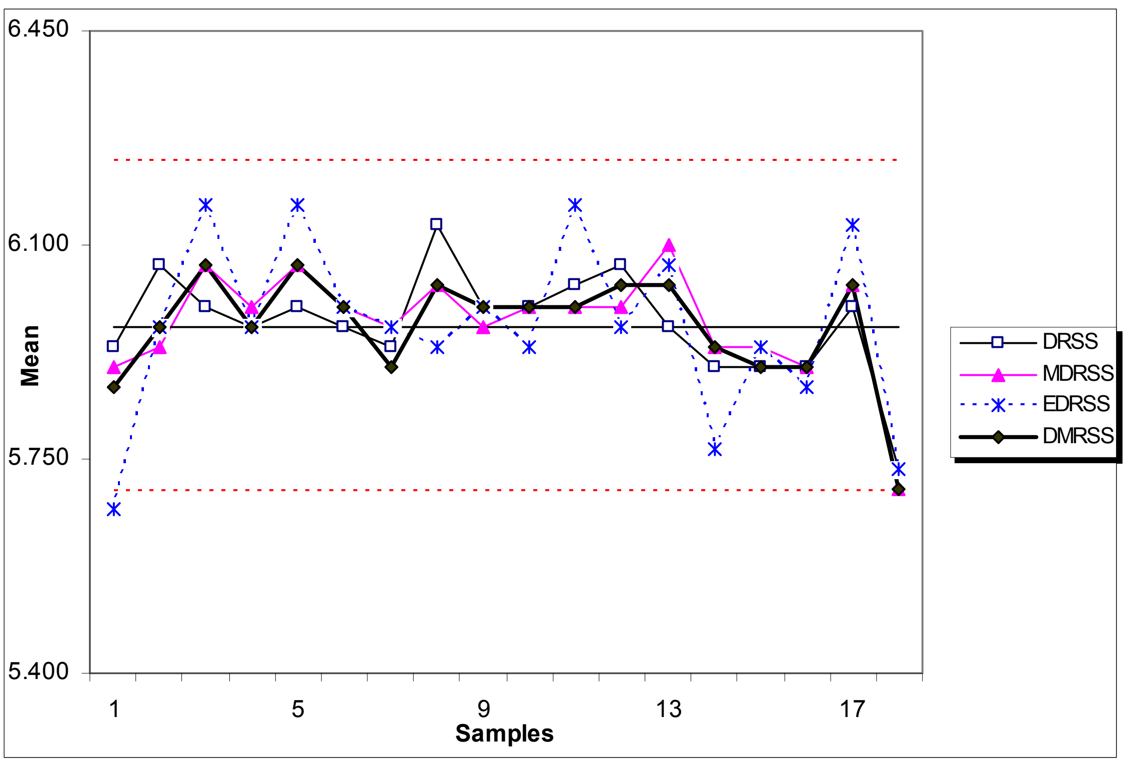

Figure 6.21: Control chart for mean using DRSS, MDRSS, EDRSS and DMRSS with n = 3

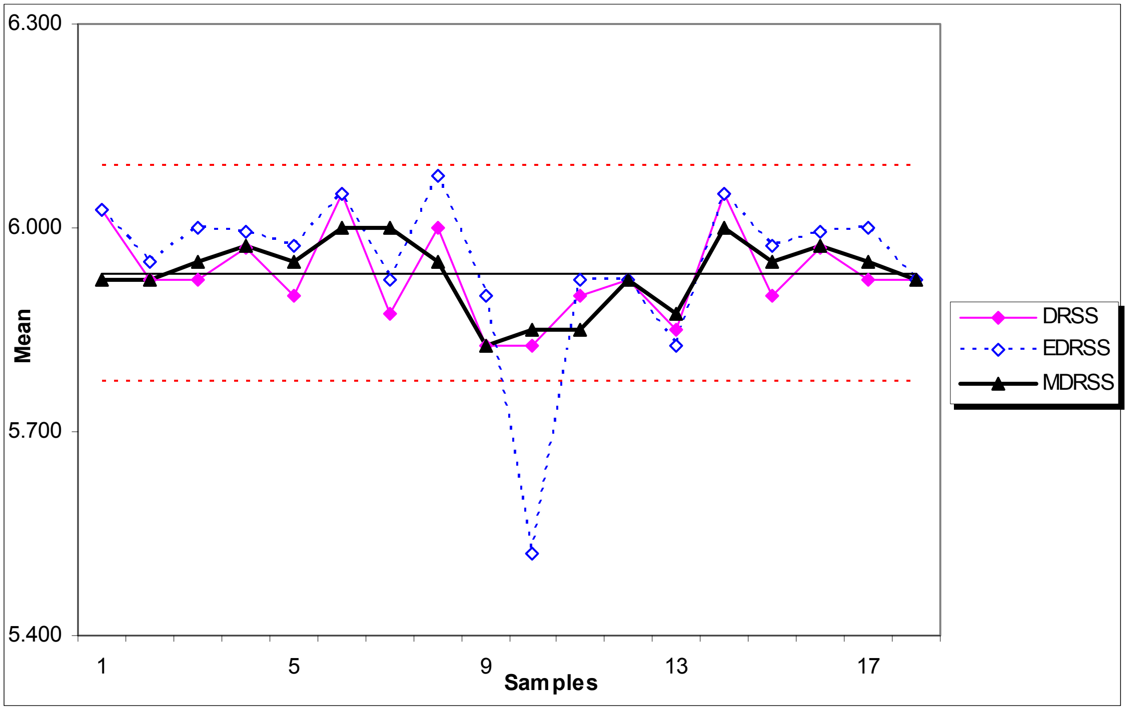

Figure 6.22: Control chart for mean using DRSS, MDRSS and EDRSS with $n = 4$

6.3.8 Comments on the Control Charts

Having used SRS, RSS, MRSS, ERSS, DRSS, MDRSS, DMRSS and EDRSS to implement quality control charts using real data, Figures 6.1-6.22, we make the following observations based on our data set used in this application. It should be noted that these comments cannot be generalized to other data set.

1. The RSS, MRSS and ERSS methods appear to be doing better in estimating the population mean as compared to the traditional SRS method. This is evident through Figures 6.1-6.4 where it can be observed that the means estimated by RSS, MRSS or ERSS method have less variability as compared with the points on SRS mean chart.

2. On the control of process standard deviation, the ERSS method seems to be better in estimating the range than the SRS as we can see that the control chart for range using ERSS have less variability as compared to SRS, Figure 6.5.
3. Using DRSS method produces a very effective control charts for mean which is not only better than the SRS method but also the RSS and ERSS methods. Although, it appears to be just as good as MRSS. See Figures 6.6-6.9.
4. The MDRSS method through Figures 6.10-6.13 demonstrates its superiority in estimating the population mean better than the SRS, RSS, MRSS and ERSS methods for same process.
5. Using DMRSS method to estimate the population mean clearly shows through Figures 6.14-6.15 to be doing a better job than the SRS, RSS and MRSS.
6. The use of EDRSS method in estimating the population mean looks like is performing better than the SRS, RSS methods and possibly as good as the MRSS for same process. This can be seen in Figures 6.16-6.19.
7. Figure 6.20 indicates that the EDRSS method may be better in estimating the range as compared to the SRS and ERSS methods. In other words, the EDRSS method may detect increase in standard deviation faster than the SRS and ERSS methods.
8. Finally, the use of DMRSS produced the most efficient control charts for mean as can be seen in Figures 6.21 and 6.22. This is followed by MDRSS, then DRSS and hence EDRSS.

Chapter 7

CONCLUSION AND RECOMMENDATIONS

7.1 Summary

The ranked set sampling have proven to be very effective where measurements of quality characteristics are difficult or costly but could readily be ranked with respect to the characteristic of interest by visual inspections. In this thesis, we gave the following main contributions:

- The introduction of three new sampling techniques namely: median double ranked set sampling (MDRSS), double median ranked set sampling (DMRSS) and extreme double ranked set sampling (EDRSS).
- Construction of variable control charts using double ranked set sampling (DRSS) and new sampling techniques, MDRSS, DMRSS and EDRSS.
- The development of statistical quality control chart for the range, R chart, using extreme ranked set sampling (ERSS) and extreme double ranked set sampling (EDRSS).
- Control Chart for monitoring the process mean and variance using ranked set sampling (RSS), median ranked set sampling (MRSS), ERSS, DRSS, MDRSS, DMRSS and EDRSS.

7.2 Conclusion

The comparison of the newly developed control charts with the classical charts based on simple random sampling reveled that the all the new charts are considerably more efficient than classical control charts, while some are doing better than others. The results from our work suggest the following.

- The suggested sampling techniques namely: MDRSS, DMRSS and EDRSS are doing better job in estimating the population mean than SRS, RSS and ERSS if the underlying distribution is symmetric with DMRSS dominating all the other methods.
- The new methods still dominates the SRS in terms of population mean estimation even if the underlying distribution is not symmetric and are doing as good as the RSS, MRSS and ERSS.
- On quality control, all the newly developed control charts for mean dominates the classical mean chart using SRS. There is a general reduction in the average run length (ARL) values of the new charts as the process starts to go out of statistical control.
- To increase the efficiency of estimating the population, we suggest the use of MDRSS, DMRSS and EDRSS methods instead of DRSS and MRSS and ERSS instead of RSS. Clearly, all the suggested methods are prone to less error in ranking and could easily be applied in real life. In terms of ARL reduction, the DMRSS is doing a better job than all other methods if the process starts going out of control.

- ❖ The control charts for range developed using ERSS and EDRSS signifies that the new methods considerably more efficient in detecting shifts in process standard deviation as compared to their SRS counterpart as the process begins to go out of statistical control.
- ❖ If there is a shift in the process mean or process mean and standard deviation, then the control charts for mean is suggested as it will be quicker to detect such a shift than the corresponding *R* chart. While a shift in only the process standard deviation will be more properly handle by the *R* chart than the mean chart.

7.3 Recommendations

We recommend the following for future works.

- ◊ The Imperfect cases of DRSS, MDRSS, DMRSS and EDRSS should be investigated and more modifications of RSS that will be easy to apply in practical situations be studied.
- ◊ The MDRSS, DMRSS and/or EDRSS are recommended for the achievement of smaller ARL and hence increase in the efficiency of the estimators.
- ◊ Further studies should be carried out on how to reduce the risk of false alarm rate when the process in under control for RSS based charts.
- ◊ The use of RSS and its modifications in the construction of control charts for mean and range could be extended to other types control charts like the control charts for attributes

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Vita

- Mu'azu Ramat Abujiya
- Born in Patigi, Kwara State of Nigeria.
- Received Bachelor of Science Degree in Mathematical Science at Bayero University Kano, Nigeria in August 1998.
- Joined King Fahd University of Petroleum and Minerals, Dhahran Saudi Arabia in September 2000 as a graduate student on Research Assistantship.
- Received Master of Science Degree in Mathematics at KFUPM in May 2003.