A Web-Based Automatic Outline Capturing of Images

BY

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This thesis, written by **Muhammad Faisal Abdul Razzak** under the direction of his thesis advisor and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of

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عمادة الدراسات العليا
DEANSHIP OF GRADUATE STUDIES

Dedicated to

My Parents

and

Sisters

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In the name of Allah, Most Gracious, Most Merciful

Alhamdulillah , All praise is due to Allah, the Lord of the Worlds. The Beneficent, the Merciful. Master of the Day of Judgment. Thee do we worship and Thee aid we seek.

Peace and blessing of Allah be upon last Prophet Muhammad (sallallaahu 'alaihi wa sallam).

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THESIS ABSTRACT

Name: Muhammad Faisal Abdul Razzak

Title: A Web-Based Automatic Outline Capturing of Images

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This thesis introduces a system which will automatically capture the outline of the images. It is a World Wide Web based system for converting bitmap images which specifies each individual pixel in the image to outline which specifies the image as a collection of mathematically-specified curves.

Designing of curves has been one of the significant problems of Computer Graphics. There are number of applications where finding a mathematical curve description of the desired shape is beneficial. Font designing, Capturing Hand drawn images on computer screens, Data Visualization and Cartooning are main motivation towards curve designing.

An efficient and modular design approach has been taken to automate the process of capturing outline of images. The aim is to find the minimal number of significant points to fit the curve keeping in mind to optimize the closeness of fit between original digitized curve and our parametric curve.

The system is also deployed over the World Wide Web to present an easy-to-use interface to the user. The design of the interface system is such that the user feels a sense of control over the system by varying some parameters and seeing the results online. Moreover, in addition to the standard images available, a user can upload his own image for testing purposes.

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خلاصة الرسالة

الاســـم : محمد فيصل عبدالرزاق

العنـــوان: التحديد التلقائي للرسم الكفافي من خلال الشبكة العنكبوتيه

الدرجـــة: الماجستير في العلوم

التخصص الرئيسي: علوم الحاسب الآلي

تاريخ التخرج: كانون الأول "ديسمبر" ٢٠٠١

يقدم هذا البحث نظاماً يقوم بتحديد الخط الكفافي للرسم بصورة تلقائية. فهذا النظام مبنياً على الشبكة العنكبوتية لأجل تحويل صور الخارطة النقطية **bitmap images** وذلك بتحديد كل نقطة ضوئية **pixel** موجوده على الرسم ومن ثم استخلاص الخط الكفافي في شكل مجموعة من المنحنيات المحدده رياضياً.

إن تصميم هذه المنحنيات تمثل أحد أبرز المشكلات في مجال الرسم الحاسوبي **Computer Graphics**. فهناك العديد من التطبيقات تستفيد من ايجاد وصف المنحنى الرياضي للشكل المطلوب مثل تصميم الخط، التعرف على الرسم اليدوي على شاشة الحاسب، **Data Visualization** والرسومات الكرتونية **Cartooning** ، فهذه التطبيقات كانت الحافز الرئيسي باتجاه تصميم هذه المنحنيات.

تم طرح طريقة فعالة وبتصميم معياري لآلية عملية التقاط الخط الكفافي للرسم. فالهدف هو الحصول على أقل عدد من النقاط البارزه والتي تناسب المنحنى المكوّن للرسم مع المحافظه على أكبر قدر من التقارب ين المنحنى الأصلي والمنحنى القياسي الذي نريد ايجاده.

تم أيضاً نشر هذا النظام على الشبكة العنكبوتيه ليقدّم للمستخدم بطريقة سلسة وسهلة. فواجهة البرنامج تتيح للمستخدم حرية التحكم على النظام وتغيير القياسات ورؤية النتيجة مباشرةً. إلى جانب الصور المتوفرة في النظام، يمكن للمستخدم أن يقوم بإنزال صورة من عنده لغرض الاختبار.

جامعـــة الملك فهـــد للبترول والمعـــادن

كانون اول "ديسمبر" ٢٠٠١

Chapter 1

Introduction

Designing of curves has been one of the significant problems of Computer Graphics. There are number of applications where finding a mathematical curve description of the desired shape is beneficial. Moreover the curves, which are robust and easy to control and compute, are of more interest. Font designing, Capturing Hand drawn images on computer screens, Data Visualization and Cartooning are main motivation towards curve designing. In curve designing, the cubic functions are the most powerful tools as they can define space curves and curves with inflections. The ideas such as end point interpolation, detection of characteristic points, least square method, recursive subdivision and parameterization can be used for curve fitting. There is a fair amount of literature on this topic particularly in [1, 2, 3]. The readers can refer to [4, 5, 6, 7, 8, 9, 10, 11] as well.

This thesis deals with an algorithm to eliminate the human interaction in obtaining the outline of original digital image and the system's appearance over the web. In the traditional approaches [12], initially, a digitized image is obtained either by scanning or from some electronic device. From this digitized image, boundary or contour of the image is obtained. Then corner points of the image are determined from contour. These corner points can be obtained by some interactive method or by some automated corner detection algorithm [13, 14, 15]. Optimal curve fitting is done by segmenting the contour outline at the corner points. Normally, the curve fitting methods are based on Bézier cubics [6].

1.1 Methodology

The methodology, in this thesis, mainly differs to the traditional approaches in various ways and follows mainly the work of [1, 3]. Since, some times corners are not detected precisely and some times only corner points are not sufficient to fit the curve which represent the original image, some more points are needed to achieve a best fit. These points are called the break points and are used along with corner points to achieve the best fit by using the least square method. The subdivision methodology is used to conquest the desired solution. Another major difference lies in the curve model for the description of design curve. The outline capturing technique, instead of traditional Bézier cubics, is based upon a cubic model which has attracting features to control the curve segments.

1.2 Advantages of a Web based System

The growth of information technology and the World Wide Web motivates us to make the system appear over the Web. The advantages of a web based system are:

- a system will be widely accessible on the Internet
- share the knowledge with the whole Internet Community
- good and easy-to-use user interface using HTML technology
- no additional software is required at the client side except Web browser
- no special hardware platform is required - any client computer capable of running a web browser will suffice
- user can test their own images and get a quick and faster response

The client/server model is used to achieve our goal. The Matlab Web Server, Apache and Tomcat Web Server along with HTML, JSP and JavaBeans technology are used to implement the system.

1.3 Organization of chapters

This thesis is organized as follows. Chapter 2 provides some background information on cubic curves and curve fitting. Also some web terminologies are described and explained. The corner detection algorithm is discussed in chapter 3. The details of fitting parametric cubic model are given in chapter 4. The web application structure and implementation is discussed in chapter 5. Chapter 6 shows some comparison and the results obtained. Chapter 7 ends with a conclusion and future work.

Chapter 2

Background

This chapter gives a brief introduction of parametric cubic curves and discusses the significance of parametric representation of curves along with the past approaches for curve fitting. A more detailed presentation of this material can be found in [11] and in [1, 3]. Also the work described in this thesis is concerned with the design of a web-based interface and presentation system. It is therefore appropriate to describe also the context in which the system will exist - the World Wide Web, as well as the major tools such as Common Gateway Interface. Java programming language etc.

2.1 Parametric Representation of Curves

Polylines are first-degree, piecewise approximation to curves. If the curves being approximated are not piecewise linear then a large number of endpoint coordinates must be created and stored to achieve reasonable accuracy. This is expensive in terms of both computation and space. Higher degree functions can be used to approximate the desired curves. The higher-degree approximations can be based on one of three methods. (The discussion is restricted to only 2-D curves.)

- Explicit Functions
- Implicit Equations
- Parametric Representation

2.1.1 Explicit Functions

We can express y as explicit function of x (e.g. $y = f(x)$). The difficulties with this approach are (1) it is impossible to get multiple values of y for a single value of x, so curves such as circle and ellipse must be represented by multiple curve segments; (2) such curves are not rotationally invariant and may require breaking a curve segment into many segments; (3) describing curves with vertical tangents is difficult, because a slope of infinity is difficult to represent.

2.1.2 Implicit Equations

We can choose to model curves as solutions to implicit equations of the form $f(x, y) = 0$; The difficulties with this approach are: (1) the given equation may have more solutions than required, for example, in modeling a circle, we might want to use $x^2 + y^2 = 1$, which is fine. But how do we model one-half of a circle? We must add constraints such as $x \geq 0$ which cannot be contained within the implicit equation; (2) if two implicit defined curve segments are joined together, it may be difficult to determine whether their tangent directions agree at join point.

2.1.3 Parametric Representation

The parametric form overcomes the problems caused by functional and implicit forms. The points on a curve are represented as ordered set of values: $p_i = [x_i, y_i]$. There are corresponding parametric functions that may be used to represent arbitrary curves; these are of the form $Q(t) = [x(t), y(t)]$. The parameter t takes the values from a specified range; conventionally from 0 to 1. A curve represented in this way can be thought of as the projection of the curve in 3-D, t as the third dimension, perpendicular to x and y plane. A unit circle can be represented in parametric form $Q(t) = [cos(t), sin(t)]$. Parametric form of curve allows multiple values of y for single or more values of x. Parametric curves replace the use of geometric slopes (which may be infinite) with parametric tangent vectors (which are never infinite). A parametric curve is approximated by *piecewise polynomial curves.* Cubic polynomials are most often used because lower-degree polynomials give little flexibility in controlling the shape of the curve, and higher degree polynomials require more computation and can introduce unwanted wiggles. No lower-degree polynomials allow a curve segment to interpolate (pass through) two specified end points with specified derivates at each end point.

2.2 Overview of Curves

This section describes various forms of parametric cubic curves and the past approaches of curve fitting [11],[16] and [1]. A detailed description and comparison of these curves can be found in [11].

2.2.1 Spline

The term spline goes back to the long flexible strips of metal used by draftsmen to lay out the surfaces of ships, cars, aircrafts etc. Weight attached to splines called "ducks", were used to pull the spline in various directions. These metal splines had second-order continuity. Figure (2.1) shows draftmen's spline. The mathematical equivalent of these strips, the natural cubic spline, is a C^0, C^1, and C^2 continuous cubic polynomial that interpolates the control points.

A huge amount of work has been done towards the construction of splines in the last two decades. Various families of splines for different objectives have been discovered.

Figure 2.1: Draftmen's Spline

Curve fitting by Spline

The use of splines for approximation of functions appears in [17]. This method applied to single-valued functions. Reinsch [17] used cubic splines to fit the curve. Grossman [18] described a parametric curve fitting technique. He noted that ordinary least-square fitting method does not take into account the ordering of points, and can not handle the multi-valued functions. His approach is to treat the data points parametrically but without piecewise representation. In order to fit complex curves, higher-degree polynomials were required. Almost the same technique is the basis of curve fitting systems in [2] and [1]. Generalized cubic splines [7, 19, 20] provide a reasonable methodology for curve fitting and curve designing. Rational cubic splines [21-68] are also good candidates for curve fitting.

2.2.2 B-Spline

B-splines consist of curve segments whose polynomial coefficients depend on just a few control points. This behavior is called *local control*. Thus, moving a control point affects only a small part of the curve. This local behavior is due to the fact that each vertex is associated with a unique basis function. The B-spline basis allows the order of basis function and hence the degree of the resulting curve to be changed without changing the number of defining polygon vertices.

Letting $P(t)$ be the position vectors along the curve as a function of the parameter t, a B-spline curve is given by

$$P(t) = \sum_{i=1}^{n+2} B_i N_{i,k}(t) \quad t_{min} \leq t < t_{max}; \quad 2 \leq k \leq n+1 \tag{2.1}$$

where B_i are the control points(position vectors) of the $n+1$ defining vertices and $N_{i,k}$ are the normalized B-spline basis functions. The i^{th} normalized B-spline basis function of order k (degree $k-1$) is defined by the Cox-deBoor recursion formulas.

$$N_{i,1} = \begin{cases} 1 & \text{if} \quad x_i \leq t < x_{i+1}; \\ 0 & otherwise. \end{cases}$$

and

$$N_{i,k}(t) = \frac{(t - x_i)N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t)N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}} \tag{2.2}$$

In above equations, x_i's are *knots* , satisfying the relation $x_i \leq x_{i+1}$. The parameter t varies from t_{min} to t_{max} along the cure $P(t)$. Since the denominators in the recursive calculations can have a value 0, the convention 0/0=0 is assumed.

Formally a B-spline curve is defined as a polynomial spline function of order k (degree $k-1$). $P(t)$ and its derivatives of order $1, 2, \ldots, k-2$ are all continuous over the entire curve. Thus, for example, a fourth-order B-spline curve is a piecewise cubic curve (C^2 continuity).

From equation (2.2) it is clear that the choice of knot vector (x_i) has a significance influence on the B-spline basis function $N_{i,k}(t)$ and hence on the resulting B-spline curve. Fundamentally three types of knot vectors are used: *uniform, open-uniform (or open) and non-uniform.*

Curve fitting by B-spline

If data points and knot values are known then control points can be computed and thereby B-spline curve can be fitted. This is know as inversion method. Wu, Abel and Greenburg [69] used B-spline for surface representation. The user of the system has to choose knots from a digitized curve then by inversion method B-spline curve is approximated. Yamaguchi [70] developed a method of an interactive curve fitting using B-spline. A least square cubic B-spline curve fitting technique is described by Dierchx [71]. A curve fitting technique based on quadratic B-splines is described by Yang [72]. Vercken [73] has also described a system based on B-spline. Some evolutionary methods, using B-splines, can be seen in [74, 75, 76, 77, 78]. These methods are based upon knot selection.

2.2.3 Hermite

Hermite curves are defined by two points P_1 and P_4 and two tangent vectors R_1 and R_4. Figure (2.2) shows a series of Hermite parametric cubic curves.

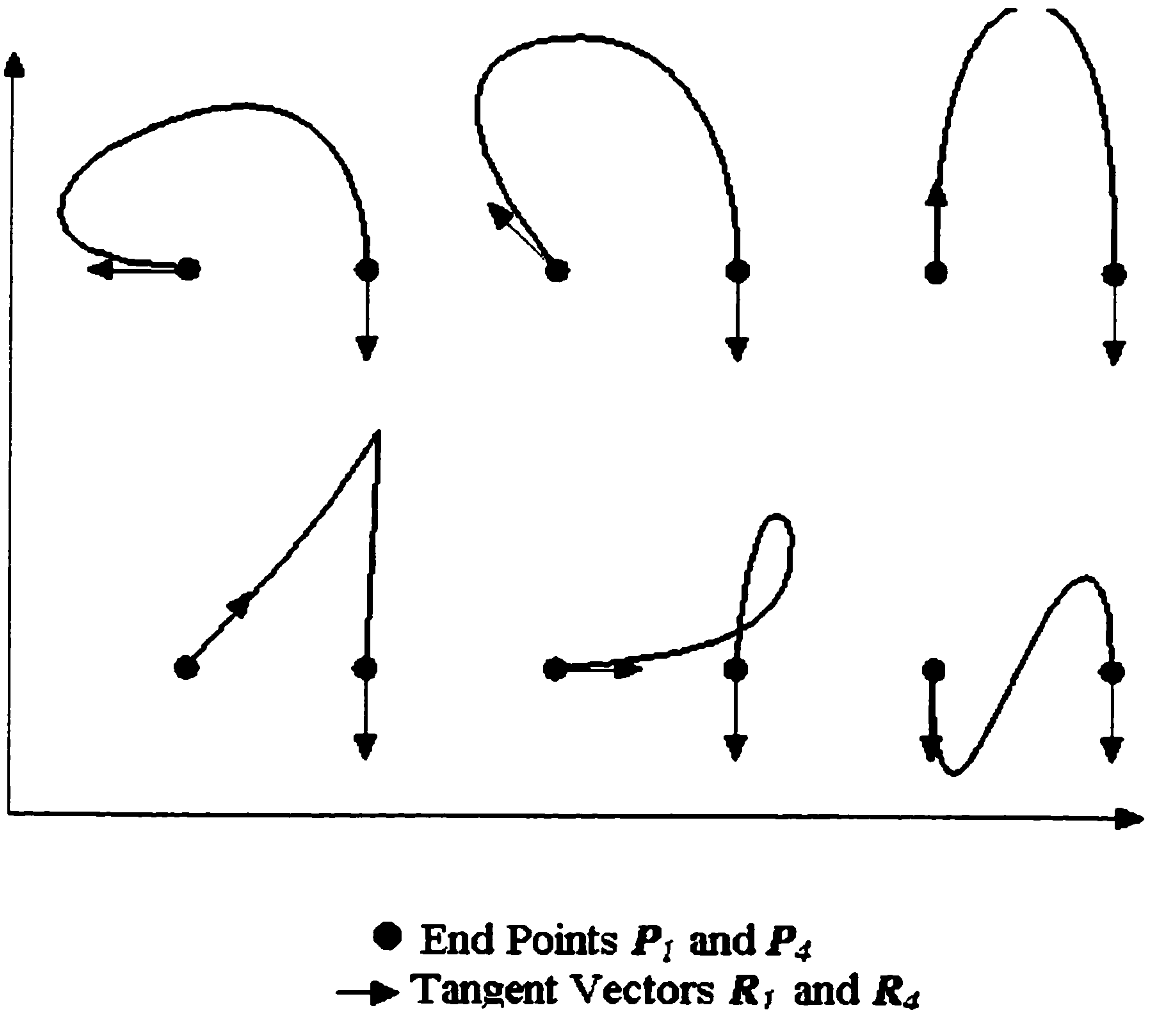

● End Points P_1 and P_4
→ Tangent Vectors R_1 and R_4

Figure 2.2: Hermite Curves

To find *Hermite basis matrix* M_H, which relates the Hermite geometry vector G_H to the polynomial coefficients, we write four equations, one for each of the constraints (P_1, P_4, R_1, R_4), in the four unknown polynomial coefficients, and then solve for the unknowns.

Defining G_{H_x} , the x component of the Hermite geometry matrix, as

$$G_{H_x} = \begin{bmatrix} P_{1_x} & P_{2_x} & P_{3_x} & P_{4_x} \end{bmatrix} \tag{2.3}$$

We express the curve as:

$$\begin{aligned} x(t) &= a_x t^3 + b_x t^2 + c_x t + d_x && (2.4) \\ &= G_{H_x}.M_H.T \\ x(t) &= G_{H_x}.M_H \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} && (2.5) \end{aligned}$$

The constraints on $x(0)$ and $x(1)$ are found by substituting $t = 0$ and $t = 1$ in equation (2.5)

$$x(0) = P_{1_x} \;=\; G_{H_x}.M_H \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{2.6}$$

$$x(1) = P_{4_x} \;=\; G_{H_x}.M_H \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{2.7}$$

To get $x'(t)$ we have to differentiate equation (2.5)

$$x'(t) \;=\; G_{H_x}.M_H \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \tag{2.8}$$

Now we can find $x'(0)$ and $x'(1)$

$$x'(0) = R_{1_x} \;=\; G_{H_x}.M_H \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T \tag{2.9}$$

$$x'(1) = R_{4_x} \;=\; G_{H_x}.M_H \begin{bmatrix} 3 & 2 & 1 & 0 \end{bmatrix}^T \tag{2.10}$$

Now equation(2.3) can be written as follows

$$G_{H_x} \;=\; G_{H_x}.M_H \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \tag{2.11}$$

M_H is the inverse of 4x4 matrix in equation (2.11)

$$M_H = \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{2.12}$$

M_H can now be substituted in (2.5) to find $x(t)$. Similarly $y(t)$ and $z(t)$ can be found out. So we can write for Hermite polynomial $Q(t)$

$$Q(t) = \begin{bmatrix} x(t) & y(t) & z(t) \end{bmatrix} = G_H.M_H.T \tag{2.13}$$

where

$$G_H = \begin{bmatrix} P_1 & P_4 & R_1 & R_4 \end{bmatrix}$$

Hermite blending function B_H can be written as

$$B_H = M_H.T \tag{2.14}$$

Now $Q(t)$ can be written as follows

$$Q(t) = G_H.B_H$$

$$Q(t) = (2t^3 - 3t^2 + 1)P_1 + (-2t^3 + 3t^2)P_4 + (t^3 - 2t^2 + t)R_1 + (t^3 - t^2)R_4 \quad (2.15)$$

These basis functions are shown in Figure(2.3)

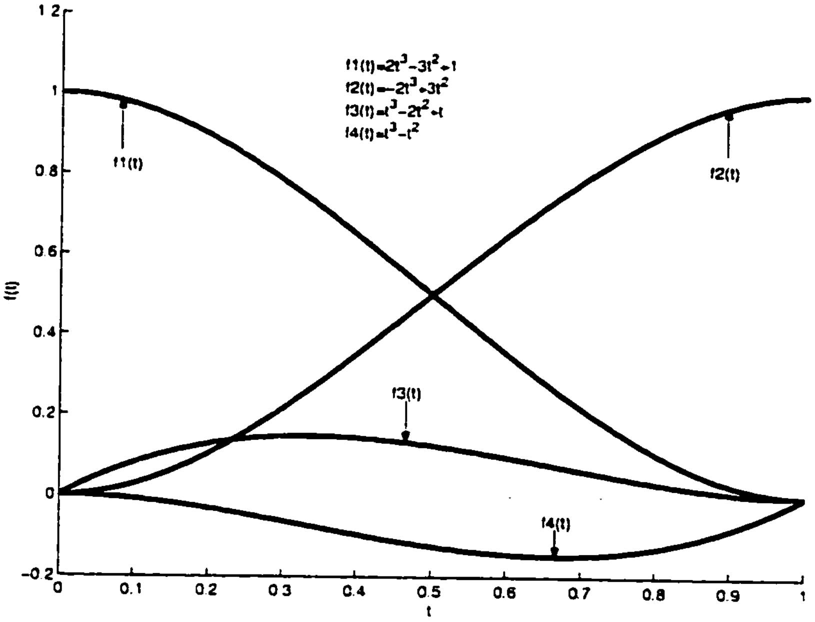


Figure 2.3: Hermite Basis Functions

Curve fitting by Hermite

Ichida [79] described curve fitting by one pass method. His method produces curve of C^1 continuity. Chong [80] described an automatic curve fitting algorithm, produces C^1 continuity. It is worth to mention here that the curve fitting model used in this thesis is a generalized form of Hermite cubics [81, 82]. The detailed description of this technique is given in chapter 4.

2.2.4 Bézier

Bézier curves are developed by Pierre Bézier [83] and [84] for use in designing automobiles at Renault.

The Bézier form of cubic polynomial curve segment has four control points P_0, P_1, P_2 and P_3. Two intermediate points P_1 and P_2 are not on the curve. The Bézier curve interpolates the two end control points P_0 and P_3 and approximates the two intermediate points P_1 and P_2. Some typical Bézier curves and their control polygon are shown in Figure(2.4)

The starting and end tangent vectors R_0 and R_3 are determined by P_0P_1 and P_2P_3 as follows:

$$R_0 = Q'(0) = 3(P_1 - P_0) \tag{2.16}$$

$$R_3 = Q'(1) = 3(P_3 - P_2) \tag{2.17}$$

The *Bézier geometry matrix* G_B is defined as

$$G_B = \begin{bmatrix} P_0 & P_1 & P_2 & P_3 \end{bmatrix} \tag{2.18}$$

The matrix M_{HB} defines the relation between *Hermite geometry matrix* G_H and

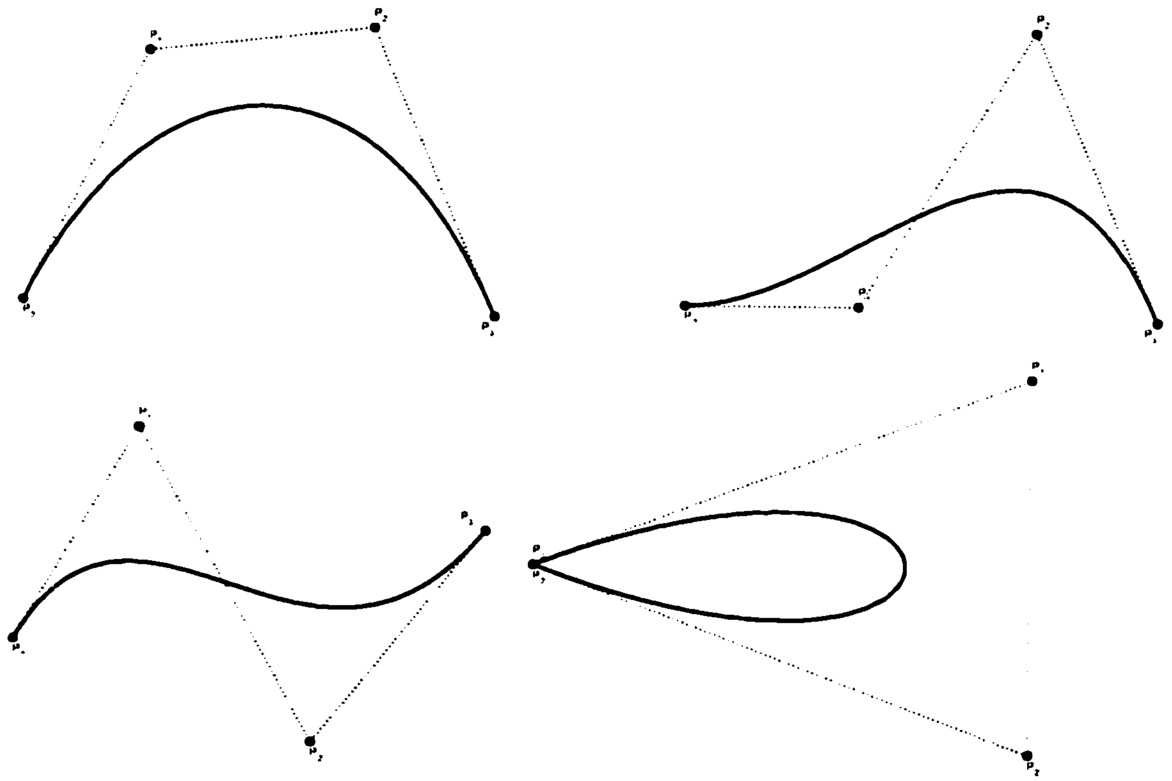


Figure 2.4: Bézier Curves and their Control Polygons

Bézier geometry matrix G_B:

$$\begin{aligned} G_H &= G_B.M_{HB} \\ &= \begin{bmatrix} P_0 & P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3 \end{bmatrix} \end{aligned} \tag{2.19}$$

To find *Bézier basis matrix* M_B, we use equation (2.13) for the Hermite form

$$\begin{aligned} Q(t) &= G_H.M_H.T \\ &= (G_B.M_{HB}).M_H.T \\ &= G_B.(M_{HB}.M_H).T \end{aligned} \tag{2.20}$$

we define

$$M_B = M_{HB}.M_H = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{2.21}$$

Now we can write equation (2.20) as follows

$$Q(t) = G_B.M_B.T \tag{2.22}$$

Substituting the G_B, M_B and T yields

$$Q(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3 \tag{2.23}$$

Bernstein polynomials $B_0(t), B_1(t), B_2(t)$ and $B_3(t)$ can be defined as follows

$$\begin{aligned} B_0(t) &= (1-t)^3 \\ B_1(t) &= 3t(1-t)^2 \\ B_2(t) &= 3t^2(1-t) \\ B_3(t) &= 3t^3 \end{aligned} \tag{2.24}$$

Figure (2.5) shows Bézier basis functions.

General form of Bézier Bernstein polynomials

A Bézier curve of degree n is defined in terms of *Bernstein polynomials* as follows:

$$Q(t) = \sum_{i=0}^{n} P_i B_i^n(t) \tag{2.25}$$

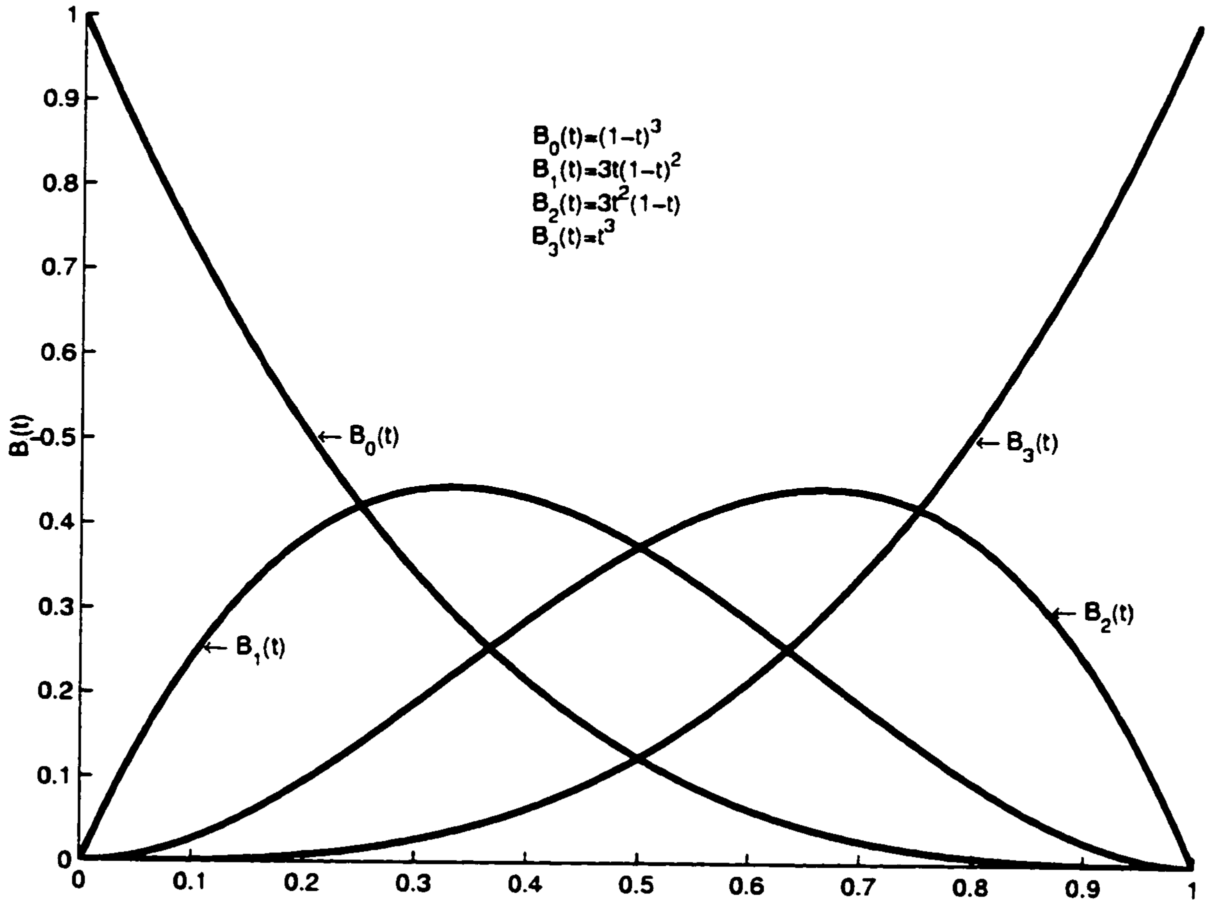


Figure 2.5: Bézier Basis Functions

$B_i^n(t)$ are *Bernstein polynomials*

$$B_i^n(t) = \left(\frac{n!}{i!(n-i)!}\right) t^i(1-t)^{n-i}, \quad i = 0, \ldots, n \tag{2.26}$$

P_i are the control points, and the sequence $P_0, P_2, \ldots, P_n$ form the control polygon of the curve segment.

Properties of Bézier Curve

Some of the important properties of Bézier curves are following:

Affine invariance: Bézier curves are invariant under affine maps. It means that we can rotate, translate, or scale the set of control points, and the curve associated with them will retain its relationship with the control points.

Convex hull: For $t \in [0, 1]$, the curve lies within the convex hull of control polygon. A simple consequence of the convex hull property is that a planar control polygon always generates a planar curve.

Endpoint interpolation: The Bézier curve passes through first and last control points. This can be easily verified by substituting $t = 0$ and $t = 1$ in Equation (2.23). This property ensures that the curve passes through significant points of each segment.

Derivative of Bézier Curve: Derivatives of Bézier curve are used to enforce continuities. The k^{th} derivative of Bézier curve is given by

$$\frac{d^kQ(t)}{dt^k} = \frac{n!}{(n-k)!}\sum_{i=0}^{n-k} \Delta^k P_i B_i^{n-k}(t) \tag{2.27}$$

where

$$\begin{aligned} \Delta^1 P_i &= \Delta P_i = P_{i+1} - P_i \\ \Delta^k P_i &= \Delta^k P_{i+1} - \Delta^{k-1} P_i \end{aligned}$$

2.3 Web Terminologies

The work described in this thesis is also concerned with the design of a web-based interface and presentation system. It is therefore appropriate to briefly mention the World Wide Web, as well as the major tools that are used in realizing the system - hypertext transfer protocol (HTTP), hypertext markup language (HTML), the Common Gateway Interface (CGI), and the Java programming language. For brevity, the readers can refer to [85, 86, 87, 88] or the electronic references at the end of this thesis.

2.3.1 The World Wide Web

The World Wide Web (also known as WWW or simply the Web) is a distributed hypermedia information system. It allows hypermedia information to be located on a network of computers called Web servers around the world, which are connected through the Internet. User friendly tools called Web clients or Web browsers make the Web easily accessible to anybody who has access to a personal computer and a modem. For that reason it has experienced tremendous growth over the past three years and has become the most used portion of the Internet. The World Wide Web uses a client-server model. Web clients are programs that send simple request to Web servers. Web servers reply to these requests by either sending documents or error codes.

The World Wide Web integrates several Internet protocols and defines its own protocol for hypertext transfer called Hypertext Transfer Protocol (HTTP). To define the location of objects on the Web, it uses Uniform Resource Locators (URLs). The standard format for hypertext documents on the World Wide Web is the Hypertext Markup Language (HTML). To enable building gateways to databases and other applications the Common Gateway Interface (CGI) was defined. For more interactivity on the Web JavaScript and Java allow the integration of executable content into Web pages. These components of the World Wide Web are introduced in the next sections.

2.3.2 Hypertext Transfer Protocol (HTTP)

The Hypertext Transfer Protocol (HTTP) is an application-level protocol that provides a fast and flexible mechanism to retrieve units of information distributed on the Web. It is based on the Internet protocol TCP/IP and HTTP servers usually listen on port TCP 80. The name Hypertext Transfer Protocol name might be misleading as information units transported by HTTP may not only be hypertext, but every kind of data such as images or sounds. Like most protocols on the Internet HTTP is a simple client-server protocol. In a typical transaction an HTTP-client opens a connection to a server and sends a request. The server replies to that request and the connection is closed again. Every request stands on its own and no state can be maintained. Thus, HTTP is a connectionless protocol.

2.3.3 Hypertext Markup Language (HTML)

Documents on the World Wide Web may be of arbitrary format, but the format generic to the Web is the Hypertext Markup Language (HTML). The HyperText Markup Language can be said to be the language of the World Wide Web. It allows text data objects to embed simple formatting information and references to other objects. It is a markup language used to describe and encapsulate the content of web documents. HTML hypertext documents are portable from one platform to another, as they are plain (ASCII) text files formatted according to a specification.

There is no standard for HTML. The World Wide Web Consortium (W3C) defines recommendations for HTML and the current recommendation is HTML 4.0 [89].

2.3.4 Common Gateway Interface (CGI)

Documents delivered by Web servers are static. They are files that do not change. As the Web evolved, its users started to look for dynamic documents created on the fly on the server. For that reason, the Common Gateway Interface (CGI) was introduced. CGI defines a standard for interfacing external programs with Web servers. These programs are often called CGI scripts, although they are not necessarily scripts and can be written in arbitrary programming languages. CGI scripts are addressed like normal documents by the use of URLs. They output an HTTP response in real time, which is then sent back to the client by the Web server. By this means, dynamic Web objects of arbitrary type can be created.

A CGI program is an executable program that can be run independently, but it is most often started by the client browser when posting a form. The purpose of the CGI program is then to receive the posted data and act upon it. The range of possible applications of the combination of html forms and CGI scripts is wide. Databases can be made accessible through the World Wide Web. Simulations can be run based on user input and their results presented to the user. Companies can provide credit card product order services. In short, the Web becomes an interactive medium instead of a static information presentation system.

2.3.5 Java

In some cases not even the power of the CGI is sufficient. In our system a user needs to upload the image file which is not currently supported by the Matlab server and the Matlab programs which acts as a CGI. To implement the upload feature, JSP and JavaBeans technologies are chosen which are nothing but Java.

Java is a new object-oriented programming language developed at Sun Microsystems. It is designed to be particularly useful as a programming language for the World Wide Web as it is made compact and secure. In the context of the web it is used for creating portable mini-applications, called applets, which are downloaded to a client as the result of a special tag in the source code for the currently viewed web page. The code of the applet is stored on the server until a browser fetches it, much in the same way as a graphic or a picture would be fetched. It is brought into the web browser where the browser then starts executing the code.

2.3.6 JSP and JavaBeans

$JavaServerPages^{TM}$ (JSP^{TM}) technology allows web developers and designers to rapidly develop and easily maintain, information-rich, dynamic web pages that leverage existing business systems. As part of the $Java^{TM}$ family, JSP technology enables rapid development of web-based applications that are platform independent. JavaServer Pages technology separates the user interface from content generation enabling designers to change the overall page layout without altering the underlying dynamic content.

JavaServer Pages (JSP) is a web-scripting technology similar to Microsoft Active Server Pages (ASP). It is a presentation layer technology and allows mixing static HTML content with server-side scripting to produce dynamic output. JSP uses Java as its default scripting language and all Java capabilities and APIs can be used. JSP uses JavaBeans to implement the business and data logic for an application. JSP provides tags and scripting platform for exposing the content generated or returned by the beans in HTML pages. Web designers generally develop JSP pages that uses beans for presentation logic. Java developers/programmers create JavaBeans to implement business logic. By separating the page logic from its design and display and supporting a reusable component-based design, JSP technology makes it faster and easier than ever to build web-based applications. Figure 2.6 shows how JSP and JavaBeans work on the Internet/Intranet.

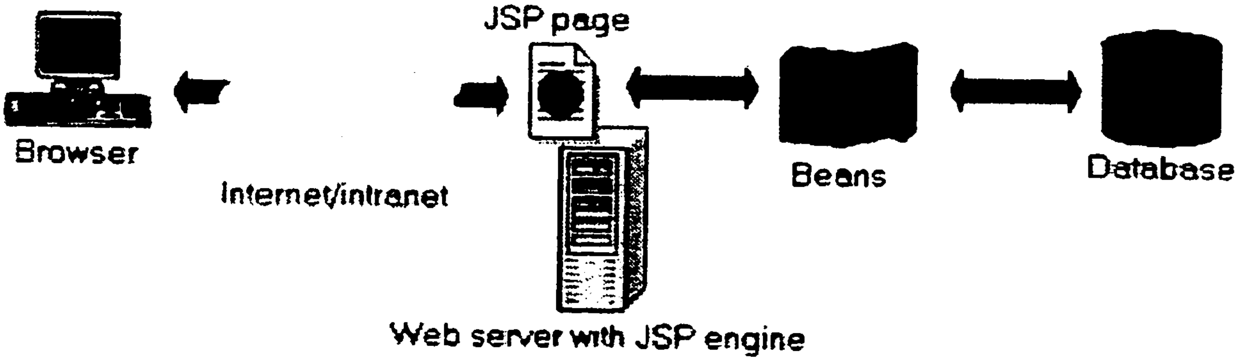


Figure 2.6: JSP and JavaBeans

2.3.7 Servers and Clients

There is a wide range of different Web servers available. Netcrafts Web server survey [90], in which more than 3 million servers were surveyed, shows that three server products hold more than 80 percent share of the market. The shareware Web server Apache holds more than 50 percent, the commercial products Microsoft Internet Information Server and Netscape Enterprise Server hold about 20 and 10 percent respectively. Microsoft's and Netscape's Web servers come with a range of tools, which help with information management and server administration, while Apache does not provide such tools. The Apache Web Server is chosen due to the simplicity in configuration and integration with the Matlab server.

There is a large number of World Wide Web clients or Web browsers, as they are often called, available. They differ in their support of the various features the World Wide Web offers. For example some support extensions made to HTTP and HTML, while others do not. The most common Web browsers are the Netscape Navigator and Microsoft Internet Explorer.

Chapter 3

Corner Detection

3.1 Introduction

Corners in digital images give important clues for shape representation and analysis. Since dominant information regarding shape is usually available at the corners, they provide important features for object recognition, shape representation and image interpretation. Corners are the robust features in the sense that they provide important information regarding objects under translation, rotation and scale change. If the corner points are identified properly, a shape can be represented in an efficient and compact way with sufficient accuracy in many shape analysis problem.

Corner detection schemes can be broadly divided into two categories based on their applications:

- binary (suitable for binary images) and

- gray level (suitable for gray level images)

Corner detection approaches for binary images usually involve segmenting the image into regions and extracting boundaries from those regions that contain them. The techniques for gray level images can be categorized into two classes: (a) Template-based and (b) gradient-based. The template-based technique utilizes correlation between a subimage and a template of a given angle. A corner point is selected by finding the maximum of the correlation output. Gradient-based techniques require computing curvature of an edge that passes through a neighborhood in a gray level image.

Corner detection is related to detection of high curvature points in planar curves. Various corner detection algorithms have been developed. Frequently cited approaches of corner detection are discussed and compared by [13] and [15]. For images a corner detection algorithm, based on the property of corners that the change of image intensity should be high in all directions is described by [91].

What is a corner? The notion of corner seems to be intuitively clear but no generally accepted mathematical definition exists, at least for digital curves. Different approaches give different but conceptually related computational definitions to a visual phenomenon. Since curvature measure is used to detect corner, therefore in this thesis some threshold value of angle is set to declare a point as corner point. If the corner points are detected precisely then the computation will be minimized in the later stages.

A curve consists of sequence of points $p_i = (x_i, y_i), i = 1, 2, \ldots, n$. For each point its cornerness (measure of corner strength) is determined. Points whose cornerness is above predefined threshold are declared as corner points. When processing a point p_i, the algorithm considers a number of subsequent points (p_k^+) and previous points (p_k^-) in the sequence, as candidates for potential corner points.

3.2 Corner Detection Algorithm

The proposed algorithm in [15] is used in this thesis where corner point is defined as a point where triangle of specified angle can be inscribed within specified distance from its neighbor points. The number of neighbor points to be checked are also predefined. It is a two pass algorithm. In the first pass the algorithm scans the sequence and selects candidate corner points. The second pass is post-processing to remove superfluous candidates.

First Pass: For each point p_i it is checked if triangle of specified size and angle is inscribed or not. Following three conditions are used.

$$d_{min}^2 \leq |p - p_k^+|^2 \leq d_{max}^2 \tag{3.1}$$

$$d_{min}^2 \leq |p - p_k^-|^2 \leq d_{max}^2 \tag{3.2}$$

$$\alpha \leq \alpha_{max} \tag{3.3}$$

where

p is the point under consideration for corner point.

p_k^+ is the k^{th} clockwise neighbor of p.

p_k^- is the k^{th} anti-clockwise neighbor of p.

Taking

$a = |p - p_k^+|$, the distance between p and p_k^+

$b = |p - p_k^-|$, the distance between p and p_k^-

$c = |p_k^+ - p_k^-|$, the distance between p_k^+ and p_k^-

The angle α can be computed by using cosine law

$$a^2 + b^2 - c^2 - 2ab\,cos\alpha = 0 \tag{3.4}$$

$$\alpha = cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \tag{3.5}$$

All the three conditions described in equations (3.1), (3.2) and (3.3) are necessary for the first pass. Now each point p may have zero, one or more than one alpha values. Among all alpha values, minimum value is taken as the alpha value of that point p.

Second Pass: Second pass removes some superfluous points. A candidate corner point p from the first pass is discarded if it has a sharper valid neighbor p_v:α(p) > α(p$_v$). A candidate point p_v is a valid neighbor of p if $|p - p_v|^2 \leq d^2_{max}$. As alternative definitions, one can use $|p - p_v|^2 \leq d^2_{min}$ or the points adjacent to p.

d_{min}, d_{max} and α_{max} are the parameters of the algorithm. Small values of d_{min} responds to fine corners. The upper limit d_{max} is necessary to avoid false sharp triangles formed by distant points in highly varying curves. α_{max} is the angle limit that determines the minimum sharpness accepted as high curvature.

The procedure of detecting corner points is given in the Flow chart of Figure (3.1).

Demonstration of the corner detection algorithm is shown from Figures (3.2) to (3.12). The pictures are selected from different categories to show the results with the default values of d_{min}, d_{max} and α_{max}. Alphabets of english and japanese language, flower, plane, pound sign and an arabic word 'Lillah' are the selected images. Also the arabic word 'Lillah' has been shown with different values of α_{max} to indicate the effect of changing α_{max}. Although the algorithm works fine and detects corner correctly in most of the images but it may not find all of the corners at the right position such as at the tail in the figure of plane (figure 3.11). But the method employed in this thesis is such that it will take care of these points and fit the curve correctly by inserting some break points in the later stage.

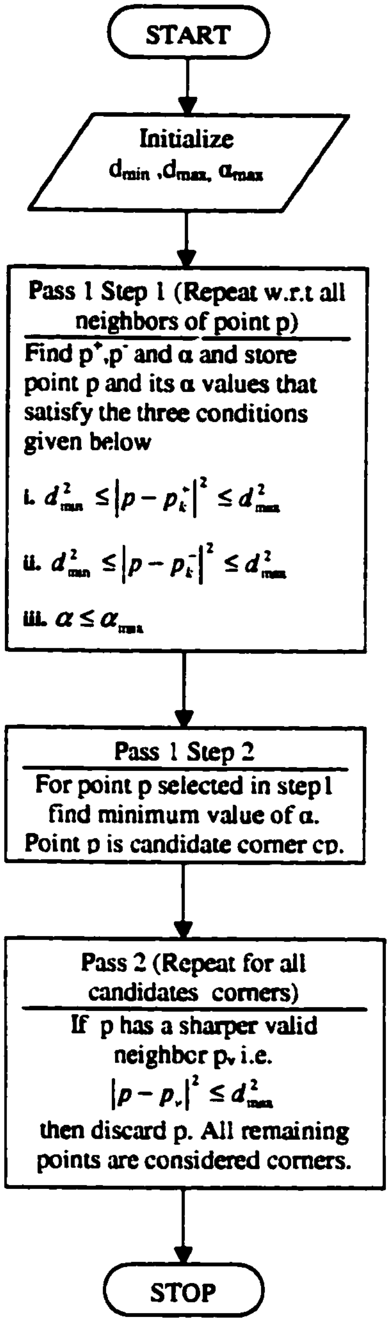

START
Initialize
d_{min} ,d_{max}, α_{max}
Pass 1 Step 1 (Repeat w.r.t all neighbors of point p)
Find p^+,p^- and α and store point p and its α values that satisfy the three conditions given below
i. $d_{min}^2 \le |p - p_k^+|^2 \le d_{max}^2$
ii. $d_{min}^2 \le |p - p_k^-|^2 \le d_{max}^2$
iii. $\alpha \le \alpha_{max}$
Pass 1 Step 2
For point p selected in step1 find minimum value of α. Point p is candidate corner cp.
Pass 2 (Repeat for all candidates corners)
If p has a sharper valid neighbcr p_v i.e.
$|p - p_v|^2 \le d_{max}^2$
then discard p. All remaining points are considered corners.
STOP

Figure 3.1: Flow Chart of Corner Detection Algorithm

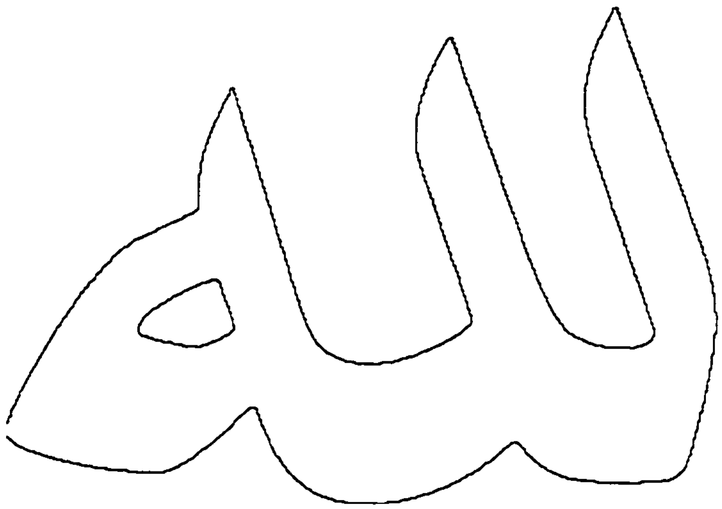


Figure 3.2: Contour of Image

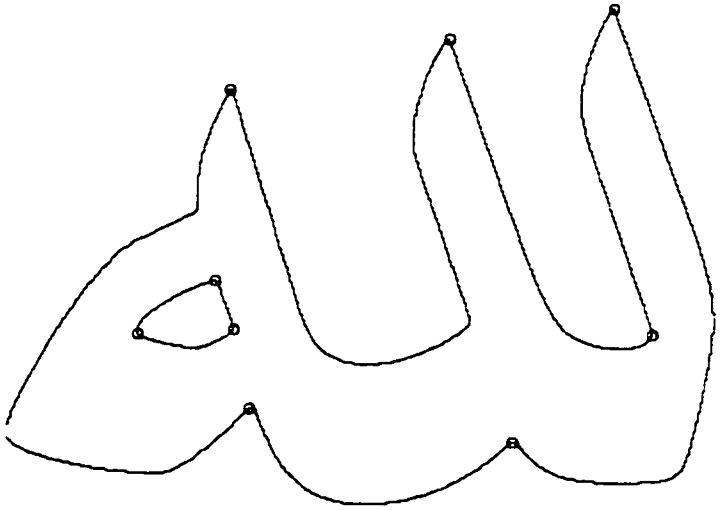


Figure 3.3: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 120$

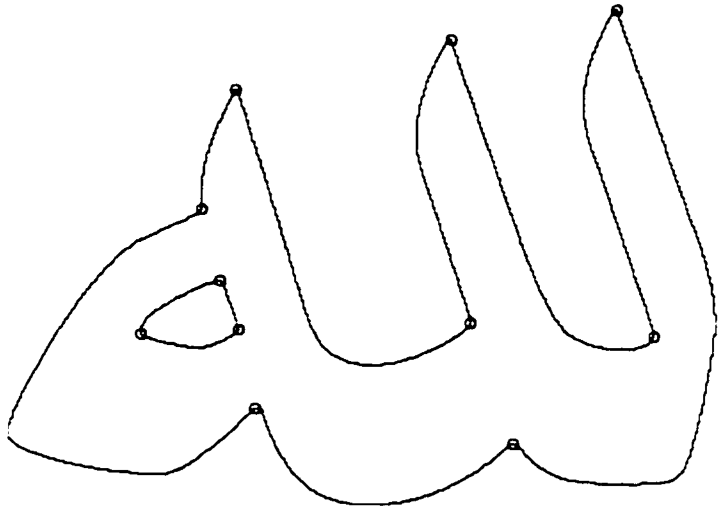


Figure 3.4: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 130$

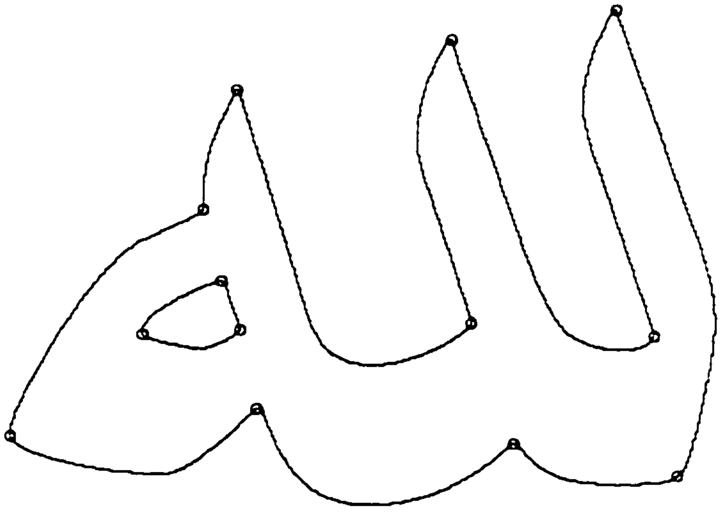


Figure 3.5: Corner candidates after Pass2. $d_{min} = 5$, $d_{max} = 8$, $\alpha_{max} = 150$

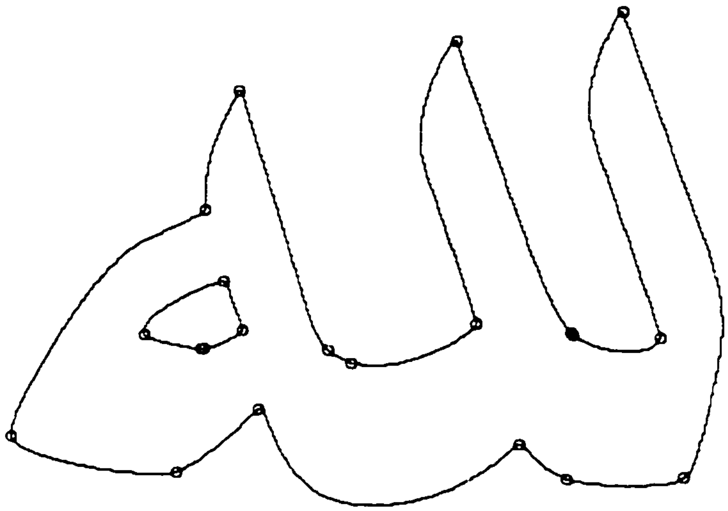


Figure 3.6: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 160$

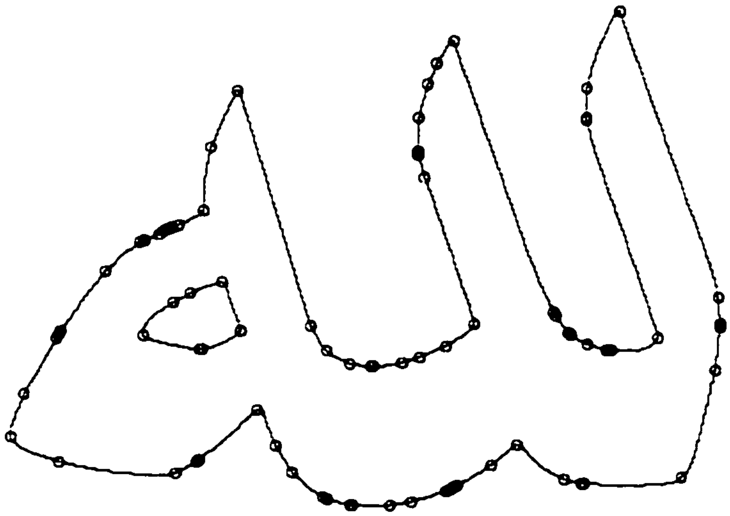


Figure 3.7: Corner candidates after Pass2. $d_{min} = 5$, $d_{max} = 8$, $\alpha_{max} = 170$

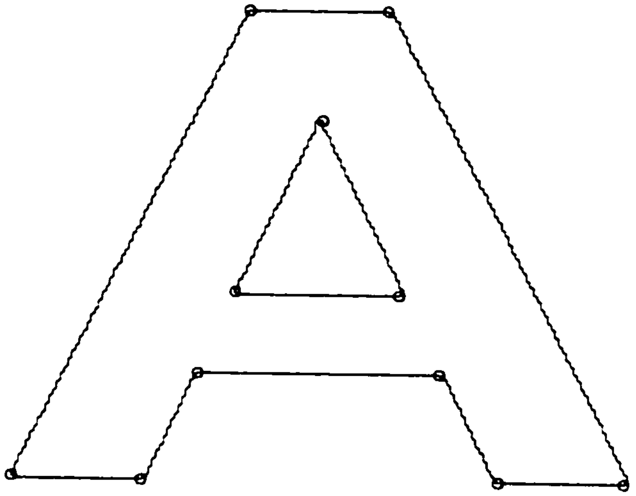


Figure 3.8: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 150$

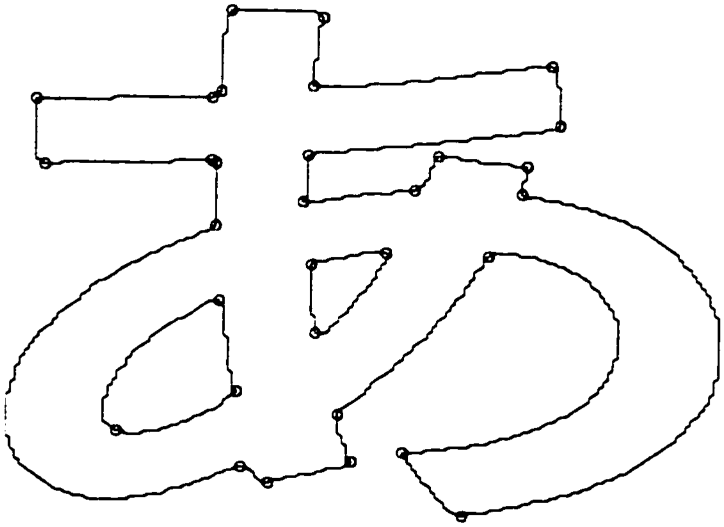


Figure 3.9: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 150$

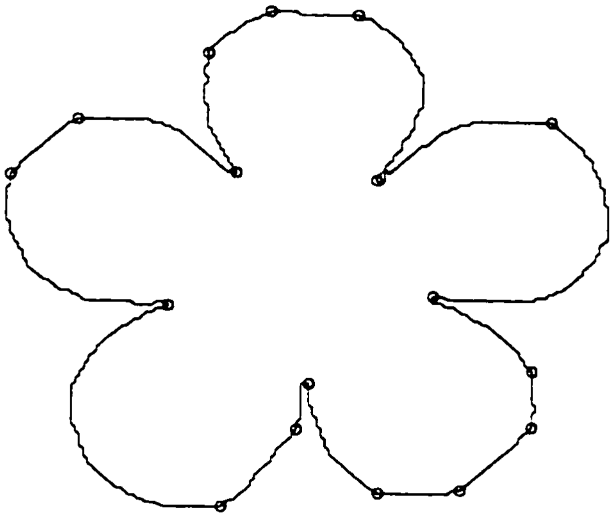


Figure 3.10: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 150$

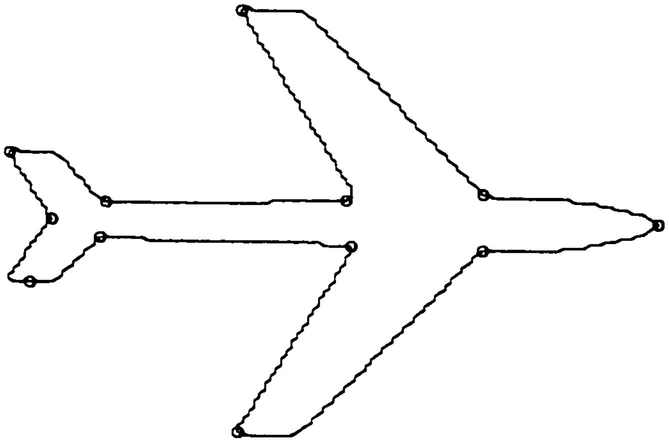


Figure 3.11: Corner candidates after Pass2. $d_{min} = 5$, $d_{max} = 8$, $\alpha_{max} = 150$

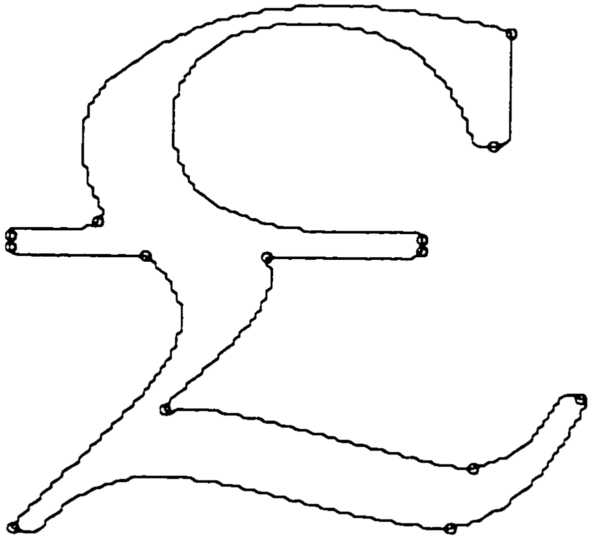


Figure 3.12: Corner candidates after Pass2. $d_{min} = 7$, $d_{max} = 9$, $\alpha_{max} = 150$

Chapter 4

Work Flow

4.1 Introduction

This chapter gives the details about the work flow of the system. Computer generated images are used in many applications, especially in font designing, data visualization etc. They also help in designing automobiles, ships, mechanical parts etc. The designer uses the iterative process to produce the physical design of the shape and it must be then conveyed to computer via user interface constrained to some parameters. To design a curve, many representation schemes exist nowadays. In traditional systems, the designer communicates the initial shape of the curve by specifying some control points interactively (with a mouse etc.). The computer then generates a curve whose shape depends on the control points. A curve may pass through (interpolate) control points, or may not, depending on the mathematical formulation defining the curve's shape. The shape of the curve depends on the placement of control points and the mathematical description that relates the curve [1,7,19,20-68]. Human interaction in designing the shape of the curve by moving some control points is not feasible as the users may not be familiar with the mathematics of the system. Automation needs to be done at various steps which we will see later in this chapter.

The work done in this thesis differs in various ways to the traditional approaches and the work done recently by Schneider [1] and Murtaza [3]. The main difference lies in the mathematical formulation of the curve which is a Hermite cubic or generalized Hermite cubic spline [81, 82] as compared to Bézier cubics used in previous work. There exists a continuity as well which was not present earlier. Segment breaking at the worst point error is also done using different approach discussed later in this chapter. However, the effect of noise filtering and reparameterization is not discussed in this thesis. Thus a modular and iterative procedure is developed to achieve the goal and the goal is to convert a bitmap image (which specifies each individual pixel in the image) to outline (which specifies the image as a collection of mathematically-specified curves). Specifically, the input is a bitmap image and the output is the outline of an image as shown in Figure (4.1).

The black box in Figure (4.1) consists of several steps. The output of one step is an input for another and all of them are automated to get the desired output. Figure 4.2 shows the steps involved in getting a good outline of bitmap image.

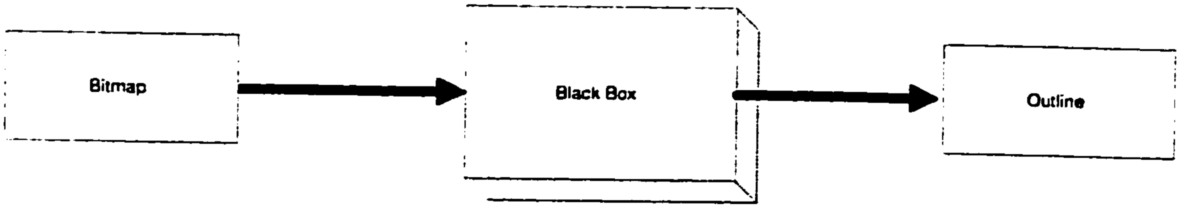


Figure 4.1: System showing input and output

4.2 Getting Digitized Image

Digitized image can be obtained directly from some electronic device or by scanning an image. The quality of digitized scanned image depends on various factors such as image on paper, scanner and attributes set during scanning. The quality of digitized image obtained directly from electronic device depends on the resolution of device, source of image, type of image etc. Some of the digitized images are shown in Figures (4.3) and (4.4).

4.3 Boundary Extraction

The next step is to find all the cyclical outlines (i.e., closed curves) in the bitmap image. The resulting list is called a pixel outline list which consists of the pixel coordinates of each edge on the outline. For example, the pixel outline list for an 'i' has two elements: one for the dot, and one for the stem. The pixel outline list for an 'o' also has two elements: one for the outside of the shape, and one for the

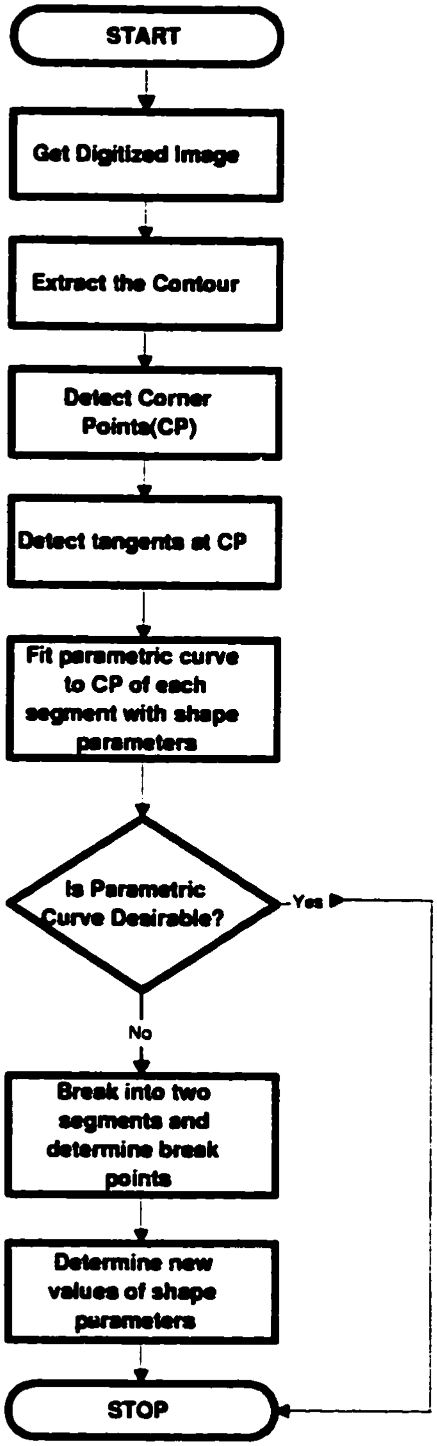

START
Get Digitized Image
Extract the Contour
Detect Corner Points(CP)
Detect tangents at CP
Fit parametric curve to CP of each segment with shape parameters
Is Parametric Curve Desirable?
Yes
No
Break into two segments and determine break points
Determine new values of shape parameters
STOP

Figure 4.2: Flow chart of the system

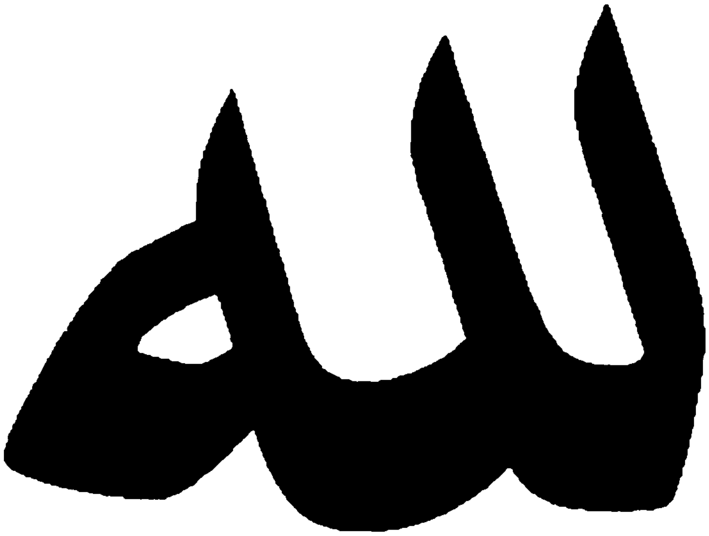


Figure 4.3: Digitized Image of ‘Lillah’ character

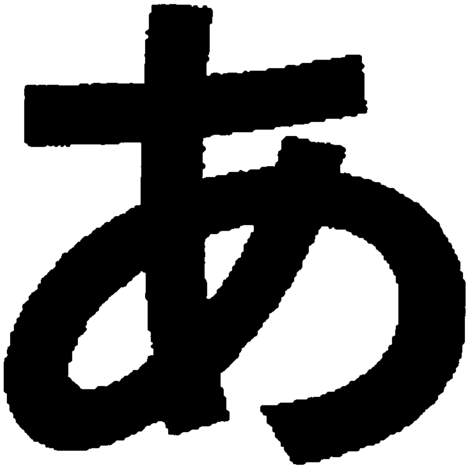


Figure 4.4: Digitized Image of ’Kanji’ character

inside.

Boundary of digitized image is extracted by using some boundary detection algorithm. There are numerous algorithms for detecting boundary [92]. The algorithm in [93] is used in this thesis. The input to this algorithm is a bitmap file. The algorithm returns number of pieces and for each piece number of boundary points and values of these boundary points $p_i = (x_i, y_i), i = 1, \ldots, N$. Figures 4.5 and 4.6 show detected boundary of the images of Figures 4.3 and 4.4 respectively. One of the text file obtained using the algorithm in [93] for figure 4.3 can be seen in appendix A.1.

Figure #	# of Pieces	# of Boundary Points
4.5	2	1522+116=1638
4.6	3	870+102+67=1039

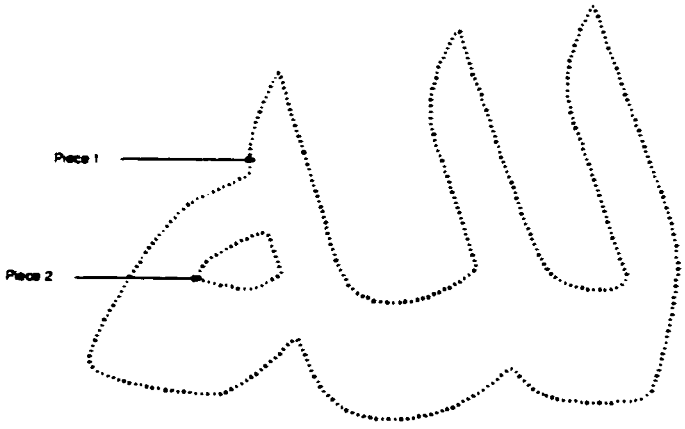


Figure 4.5: Detected Boundary consists of Two Pieces

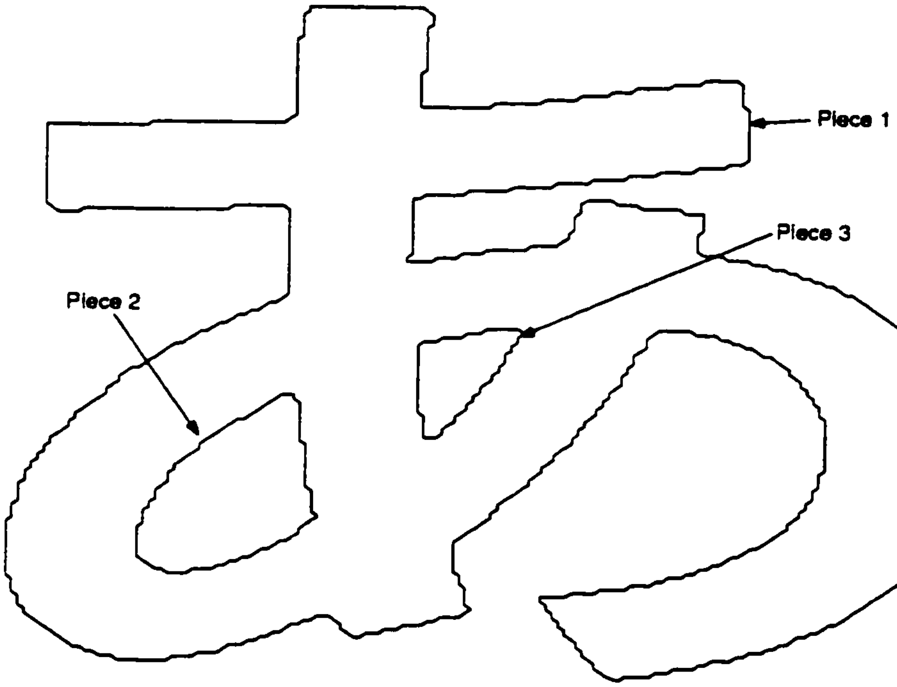


Figure 4.6: Detected Boundary consists of Three Pieces

4.4 Detecting Corner Points

The final goal is to fit continuous curves to the discrete bitmap image. We look for points where the outline makes such a sharp turn that a single curve cannot possibly fit well. These points are sometimes called characteristic points or corner points or significant points. If the corner points are detected precisely then the computation will be minimized in the later stages. The detail of this step is mentioned in chapter 3.

4.5 Cubic Interpolant

We divide the whole set of contour points into groups called segments such that each segment lies between the two consecutive corner points. The parametric representation of curves is then used to fit the curve piecewisely. Each segment of the overall curve is given by functions x and y which are cubic polynomials in the parameter t. Cubic polynomials are most often used because lower-degree polynomials give too little flexibility in controlling the shape of the curve, and higher-degree polynomials can introduce unwanted wiggles and also require more computation. No lower-degree representation allows a curve segment to interpolate (pass through) two specified endpoints with specified derivatives at each endpoint. Given a cubic polynomial with its four coefficients, four knowns are used to solve for the unknown coefficients. The four knowns might be the two endpoints and the derivatives at the endpoints.

Let $\mathbf{F}_i,\mathbf{F}_{i+1},i \in Z$ be the two end characteristic points given at the distinct knots $t_i,t_{i+1},i \in Z$ with interval spacing $h_i = t_{i+1} - t_i > 0$. Also let $\mathbf{D}_i,\mathbf{D}_{i+1},i \in Z$ denote the first derivative values defined at the knots. Then the generalized form of the cubic is defined by

$$\mathbf{P}|_{(t_i,t_{i+1})}(t) = (1-t)^3\mathbf{F}_i + 3t(1-t)^2\mathbf{V}_i + 3t^2(1-t)\mathbf{W}_i + t^3\mathbf{F}_{i+1}, \tag{4.1}$$

where

$$\mathbf{V}_i = \mathbf{F}_i + v_i h_i \mathbf{D}_i/3, \qquad \mathbf{W}_i = \mathbf{F}_{i+1} - w_i h_i \mathbf{D}_{i+1}/3 \tag{4.2}$$

The interpolation conditions are as follows:

$$\left.\begin{array}{ll} \mathbf{P}(t_i) = \mathbf{F}_i, & \mathbf{P}(t_{i+1}) = \mathbf{F}_{i+1} \text{ and} \\ \mathbf{P}^{(1)}(t_i) = v_i\mathbf{D}_i, & \mathbf{P}^{(1)}(t_{i+1}) = w_i\mathbf{D}_{i+1}, i \in Z. \end{array}\right\} \tag{4.3}$$

Equation (4.1) can be rewritten as

$$\mathbf{P}|_{(t_i,t_{i+1})}(t) = R_{0,i}(t)\mathbf{F}_i + R_{1,i}(t)\mathbf{V}_i + R_{2,i}(t)\mathbf{W}_i + R_{3,i}(t)\mathbf{F}_{i+1}, \tag{4.4}$$

where

$$\left.\begin{aligned} R_{0,i}(t) &= (1-t)^3, \\ R_{1,i}(t) &= 3t(1-t)^2, \\ R_{2,i}(t) &= 3t^2(1-t), \\ R_{3,i}(t) &= t^3 \end{aligned}\right\} \tag{4.5}$$

The functions $R_{j,i}$, $j = 0, 1, 2, 3$ are Bernstein Bézier like basis functions, such that

$$\sum_{j=0}^{3} R_{j,i}(t) = 1. \tag{4.6}$$

From the Bernstein-Bézier theory it follows that the curve segment $\mathbf{P}|_{[t_i,t_{i+1}]}$ lies in the convex hull of the control points $\{\mathbf{F}_i, \mathbf{V}_i, \mathbf{W}_i, \mathbf{F}_{i+1}\}$ and is variation diminishing with respect to the control polygon joining these points.

We observe the following properties defined by equation (4.1) and (4.2):

- The curve always passes through $\mathbf{F}_i$ and $\mathbf{F}_{i+1}$
- If $v \rightarrow 0$, then the curve exhibits the biased tension behavior to the left and is pulled towards the control point $\mathbf{F}_i$
- If $w \rightarrow 0$, then the curve exhibits the biased tension behavior to the right and is pulled towards the control point $\mathbf{F}_{i+1}$

- If $v, w \to 0$, then the curve exhibits the internal tension behavior and approaches to the linear interpolant

$$\mathbf{P}(t) = (1 - t)\mathbf{F}_i + t\mathbf{F}_{i+1}$$

- For $0 < v, w \leq 1$, the cubic ensures the convex hull property and hence the curve segment is more flexible than the traditional cubic Bézier curve.

4.5.1 Parameterization

There are number of parameterizations techniques in the literature [94, 95] such as uniform parameterization, linear or chord length parameterization, parabolic parameterization and cubic parameterization. Chord length parameterization scheme can be adapted to continuous sets of points and this scheme is used to estimate the parametric value t associated with each point $p_i = (x_i, y_i)$.

$$t_i = \begin{cases} 0 & \text{if } \ i = 1; \\ \frac{|p_1p_2| + |p_2p_3| + \cdots + |p_{i-1}p_i|}{|p_1p_2| + |p_2p_3| + \cdots + |p_{n-1}p_n|} & \text{if } \ 2 \leq i \leq n - 1; \\ 1 & \text{if } \ i = n \end{cases}$$

It should be noted that t_i is in normalized form and varies from 0 to 1, and hence h_i in our case is always equal to 1.

4.5.2 Estimation of TangentVectors

We define a distance based choice for tangents vectors $\mathbf{D}_i$'s at $\mathbf{F}_i$'s as follows:

For open curves:

$$\left.\begin{aligned} \mathbf{D}_0 &= 2(\mathbf{F}_1 - \mathbf{F}_0) - (\mathbf{F}_2 - \mathbf{F}_0)/2, \\ \mathbf{D}_n &= 2(\mathbf{F}_n - \mathbf{F}_{n-1}) - (\mathbf{F}_n - \mathbf{F}_{n-2})/2, \\ \mathbf{D}_i &= a_i(\mathbf{F}_i - \mathbf{F}_{i-1}) + (1 - a_i)(\mathbf{F}_{i+1} - \mathbf{F}_i), i = 1, .., n-1. \end{aligned}\right\} \tag{4.7}$$

For close curves:

$$\left.\begin{aligned} \mathbf{F}_{-1} = \mathbf{F}_{n-1}, \mathbf{F}_{n+1} = \mathbf{F}_1, \\ \mathbf{D}_i = a_i(\mathbf{F}_i - \mathbf{F}_{i-1}) + (1 - a_i)(\mathbf{F}_{i+1} - \mathbf{F}_i), i = 0, .., n. \end{aligned}\right\} \tag{4.8}$$

where

$$a_i = \frac{|\mathbf{F}_{i+1} - \mathbf{F}_i|}{|\mathbf{F}_{i+1} - \mathbf{F}_i| + |\mathbf{F}_i - \mathbf{F}_{i-1}|}, i = 0, .., n. \tag{4.9}$$

Also the tangents $\mathbf{D}_i$ and $\mathbf{D}_{i+1}$ in equation (4.1) can be computed by using Least Square method. The number of cases that are discussed in the next section uses both the techniques to calculate the tangents.

4.5.3 Optimal Design Curve

Now we have five cases to consider and they are discussed below one by one.

Case 1: $v_i = w_i = 1$ and tangents are estimated as described in §4.5.2.

It is a C^1 Hermite Spline curve and can be treated as a default design curve. This is a simplest case to consider and requires less computation initially. Since v_i,w_i and h_i are equal to 1, we have:

$$\mathbf{V}_i = \mathbf{F}_i + \mathbf{D}_i/3 \quad \text{and} \quad \mathbf{W}_i = \mathbf{F}_{i+1} - \mathbf{D}_{i+1}/3.$$

The results of this case for the four images are shown in chapter 6.

Case 2: $v_i = w_i$ and tangents are estimated as described in §4.5.2.

In this case the shape parameters are treated as equal and we have to solve the equation for only one variable using Least Square method. Suppose, for $i = 0, 1, 2, ..., n-1$, the data segments

$$\mathbf{P}_{i,j} = (x_{i,j}, y_{i,j}), \qquad j = 1, 2, ..., m_i \tag{4.10}$$

are given as the ordered sets of the universal set of the data points. Then the squared sums S_i's of distances between $\mathbf{P}_{i,j}$'s and their corresponding parametric points $\mathbf{P}(t_j)$'s on the curve are computed as:

$$S_i = \sum_{j=1}^{m_i} [\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}]^2, \qquad i = 0, 1, 2, ..., n-1 \tag{4.11}$$

where the parameterization over u's is in accordance with the chord length parameterization.

For the best fitting of the curve to the given data, we have to find out the parameter v_i so that sums S_i's are minimal and this can be done by using least square approximation. The minimum of S_i's occur if partial derivative of S_i's with respect to v_i's becomes zero. We have

$$\frac{\partial S_i}{\partial v_i} = 2\sum_{j=1}^{m_i} \frac{\partial \mathbf{P}_i(u_{i,j})}{\partial v_i} . [\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, \qquad i = 0, 1, 2, ..., n-1,$$

Since

$$\frac{\partial \mathbf{P}_i(u_{i,j})}{\partial v_i} = \frac{h_i}{3}(R_{1,i}(u_{i,j})\mathbf{D}_i - R_{2,i}(u_{i,j})\mathbf{D}_{i+1}),$$

therefore

$$\frac{\partial S_i}{\partial v_i} = \frac{2h_i}{3}\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{D}_i - R_{2,i}(u_{i,j})\mathbf{D}_{i+1}).[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, i = 0, 1, 2, ..., n-1, \tag{4.12}$$

Equation (4.12) can be simplified as

$$\begin{aligned}&\mathbf{D}_i.\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{P}_i(u_{i,j}) - \mathbf{D}_{i+1}.\sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{P}_i(u_{i,j}) = \\ &\quad \mathbf{D}_i.\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{P}_{i,j} - \mathbf{D}_{i+1}.\sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{P}_{i,j}, \qquad i = 0, 1, 2, ...n-1.\end{aligned} \tag{4.13}$$

This is equivalent to (by replacing $\mathbf{P}_i(u_{i,j})$ from equation (4.1)):

$$\begin{aligned}&\mathbf{D}_i.\mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{D}_i.\mathbf{F}_{i+1}(a_{2,i} + a_{3,i}) - \mathbf{D}_i.\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{P}_{i,j} - \mathbf{D}_{i+1}.\mathbf{F}_i(a_{2,i} + a_{4,i}) - \\ &\quad \mathbf{D}_{i+1}.\mathbf{F}_{i+1}(a_{5,i} + a_{6,i}) + \mathbf{D}_{i+1}.\sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{P}_{i,j} = \\ &\quad \frac{h_i}{3}v_i(-|\mathbf{D}_i|^2 a_{1,i} + 2\mathbf{D}_i.\mathbf{D}_{i+1}a_{2,i} - |\mathbf{D}_{i+1}|^2 a_{5,i}), \qquad i = 0, 1, 2, ...n-1.\end{aligned} \tag{4.14}$$

where

$$\left.\begin{aligned}
a_{0,i} &= \sum_{j=1}^{m_i} R_{0,i}(u_{i,j})R_{1,i}(u_{i,j}),\\
a_{1,i} &= \sum_{j=1}^{m_i} R_{1,i}^2(u_{i,j}),\\
a_{2,i} &= \sum_{j=1}^{m_i} R_{1,i}(u_{i,j})R_{2,i}(u_{i,j}),\\
a_{3,i} &= \sum_{j=1}^{m_i} R_{1,i}(u_{i,j})R_{3,i}(u_{i,j}),\\
a_{4,i} &= \sum_{j=1}^{m_i} R_{0,i}(u_{i,j})R_{2,i}(u_{i,j}),\\
a_{5,i} &= \sum_{j=1}^{m_i} R_{2,i}^2(u_{i,j}),\\
a_{6,i} &= \sum_{j=1}^{m_i} R_{2,i}(u_{i,j})R_{3,i}(u_{i,j}),
\end{aligned}\right\} \qquad (4.15)$$

Equation (4.14) leads to the solution:

$$v_i = \frac{3(A_{2,i} - A_{3,i} + A_{4,i})}{h_i.A_{1,i}}, \qquad i = 0, 1, 2, ...n - 1. \qquad (4.16)$$

where

$$\left.\begin{aligned}
A_{1,i} &= |\mathbf{D}_i|^2 a_{1,i} - 2\mathbf{D}_i.\mathbf{D}_{i+1} a_{2,i} + |\mathbf{D}_{i+1}|^2 a_{5,i},\\
A_{2,i} &= \mathbf{D}_i . \sum_{j=1}^{m_i} R_{1,i}(u_{i,j})\mathbf{P}_{i,j} - \mathbf{D}_{i+1} . \sum_{j=1}^{m_i} R_{2,i}(u_{i,j})\mathbf{P}_{i,j},\\
A_{3,i} &= \mathbf{D}_i.\mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{D}_i.\mathbf{F}_{i+1}(a_{2,i} + a_{3,i}),\\
A_{4,i} &= \mathbf{D}_{i+1}.\mathbf{F}_i(a_{2,i} + a_{4,i}) + \mathbf{D}_{i+1}.\mathbf{F}_{i+1}(a_{5,i} + a_{6,i}).
\end{aligned}\right\} \qquad (4.17)$$

Thus the curve fitted using the above values of v_i's will be a good candidate of best fit.

Case 3: $v_i \neq w_i$ and tangents are estimated as described in §4.5.2.

In this case the shape parameters are not treated equal and we have to solve the equation for two variables using Least Square method. Suppose, for $i = 0, 1, 2, ..., n-1$, the data segments

$$\mathbf{P}_{i,j} = (x_{i,j}, y_{i,j}), \qquad j = 1, 2, ..., m_i \tag{4.18}$$

are given as the ordered sets of the universal set of the data points. Then the squared sums S_i's of distances between $\mathbf{P}_{i,j}$'s and their corresponding parametric points $\mathbf{P}(t_j)$'s on the curve are computed as:

$$S_i = \sum_{j=1}^{m_i} [\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}]^2, \qquad i = 0, 1, 2, ..., n-1 \tag{4.19}$$

where the parameterization over u's is in accordance with the chord length parameterization.

For the best fitting of the curve to the given data, we have to find out the parameters v_i's and w_i's so that sums S_i's are minimal and this can be done by using least square approximation. The minimum of S_i's occur if partial derivative of S_i's with respect to v_i's and w_i's becomes zero. We have

$$\frac{\partial S_i}{\partial v_i} = 2\sum_{j=1}^{m_i} \frac{\partial \mathbf{P}_i(u_{i,j})}{\partial v_i}.[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, \qquad i = 0, 1, 2, ..., n-1,$$

and

$$\frac{\partial S_i}{\partial w_i} = 2\sum_{j=1}^{m_i} \frac{\partial \mathbf{P}_i(u_{i,j})}{\partial w_i}.[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, \qquad i = 0, 1, 2, ..., n-1,$$

which implies

$$\frac{\partial S_i}{\partial v_i} = \frac{2h_i}{3}\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{D}_i).[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, i = 0, 1, 2, ..., n-1, \qquad (4.20)$$

and

$$\frac{\partial S_i}{\partial w_i} = -\frac{2h_i}{3}\sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{D}_{i+1}).[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, i = 0, 1, 2, ..., n-1, \qquad (4.21)$$

Replacing $\mathbf{P}_i(u_{i,j})$ from equation (4.1) and simplifying equation (4.20) and (4.21), we get

$$\mathbf{D}_i.\mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{D}_i.\mathbf{F}_{i+1}(a_{2,i} + a_{3,i}) - \mathbf{D}_i.\sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{P}_{i,j} =$$
$$\frac{h_i}{3}(-v_i|\mathbf{D}_i|^2 a_{1,i} + w_i\mathbf{D}_i.\mathbf{D}_{i+1}a_{2,i}), \qquad i = 0, 1, 2, ...n - 1. \tag{4.22}$$

and

$$\mathbf{D}_{i+1}.\mathbf{F}_i(a_{2,i} + a_{4,i}) + \mathbf{D}_{i+1}.\mathbf{F}_{i+1}(a_{5,i} + a_{6,i}) - \mathbf{D}_{i+1}.\sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{P}_{i,j} =$$
$$\frac{h_i}{3}(-v_i\mathbf{D}_i.\mathbf{D}_{i+1}a_{2,i} + w_i|\mathbf{D}_{i+1}|^2 a_{5,i}), \qquad i = 0, 1, 2, ...n - 1. \tag{4.23}$$

where $a_{0,i}, ..., a_{6,i}$ are same as in equation (4.15) described in case 2.

Solving equation (4.22) and equation (4.23), we get

$$v_i = \frac{3}{h_i}\left[\frac{A_{3,i}B_{2,i} - A_{2,i}B_{3,i}}{A_{1,i}B_{2,i} - A_{2,i}B_{1,i}}\right] \tag{4.24}$$

and

$$w_i = \frac{3}{h_i}\left[\frac{A_{3,i}B_{1,i} - A_{1,i}B_{3,i}}{A_{2,i}B_{1,i} - A_{1,i}B_{2,i}}\right] \tag{4.25}$$

where

$$\left.\begin{aligned} A_{1,i} &= -|\mathbf{D}_i|^2 a_{1,i}, \\ A_{2,i} &= \mathbf{D}_i.\mathbf{D}_{i+1} a_{2,i}, \\ A_{3,i} &= \mathbf{D}_i.\mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{D}_i.\mathbf{F}_{i+1}(a_{2,i} + a_{3,i}) - \mathbf{D}_i.\sum_{j=1}^{m_i} R_{1,i}(u_{i,j})\mathbf{P}_{i,j}, \end{aligned}\right\} \quad (4.26)$$

and

$$\left.\begin{aligned} B_{1,i} &= -\mathbf{D}_i.\mathbf{D}_{i+1} a_{2,i}, \\ B_{2,i} &= |\mathbf{D}_{i+1}|^2 a_{5,i}, \\ B_{3,i} &= \mathbf{D}_{i+1}.\mathbf{F}_i(a_{2,i} + a_{4,i}) + \mathbf{D}_{i+1}.\mathbf{F}_{i+1}(a_{5,i} + a_{6,i}) - \mathbf{D}_{i+1}.\sum_{j=1}^{m_i} R_{2,i}(u_{i,j})\mathbf{P}_{i,j}, \end{aligned}\right\} \quad (4.27)$$

Thus the curve fitted using the above values of v_i's and w_i's will be a good candidate of best fit.

Case 4: $v_i = w_i = 1$ and tangents are calculated using Least Square method. In this case the shape parameters v_i's and w_i's are treated as constant for each segment and the tangents are approximated using Least Square method. Suppose, for $i = 0, 1, 2, ..., n-1$, the data segments

$$\mathbf{P}_{i,j} = (x_{i,j}, y_{i,j}), \qquad j = 1, 2, ..., m_i \quad (4.28)$$

are given as the ordered sets of the universal set of the data points. Then the squared sums S_i's of distances between $\mathbf{P}_{i,j}$'s and their corresponding parametric points $\mathbf{P}(t_j)$'s on the curve are computed as:

$$S_i = \sum_{j=1}^{m_i} [\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}]^2, \qquad i = 0, 1, 2, ..., n-1 \tag{4.29}$$

where the parameterization over u's is in accordance with the chord length parameterization.

We have to find out the tangents $\mathbf{D}_i$'s and $\mathbf{D}_{i+1}$'s so that sums S_i's are minimal and this can be done by using least square approximation. The minimum of S_i's occur if partial derivative of S_i's with respect to $\mathbf{D}_i$'s and $\mathbf{D}_{i+1}$'s becomes zero. We have

$$\frac{\partial S_i}{\partial \mathbf{D}_i} = 2\sum_{j=1}^{m_i} \frac{\partial \mathbf{P}_i(u_{i,j})}{\partial \mathbf{D}_i}.[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, \qquad i = 0, 1, 2, ..., n-1,$$

and

$$\frac{\partial S_i}{\partial \mathbf{D}_{i+1}} = 2\sum_{j=1}^{m_i} \frac{\partial \mathbf{P}_i(u_{i,j})}{\partial \mathbf{D}_{i+1}}.[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, \qquad i = 0, 1, 2, ..., n-1,$$

Since

$$\frac{\partial \mathbf{P}_i(u_{i,j})}{\partial \mathbf{D}_i} = \frac{h_i}{3}(R_{1,i}(u_{i,j}),$$

and

$$\frac{\partial \mathbf{P}_i(u_{i,j})}{\partial \mathbf{D}_{i+1}} = -\frac{h_i}{3}(R_{2,i}(u_{i,j}),$$

therefore

$$\frac{\partial S_i}{\partial \mathbf{D}_i} = \frac{2h_i}{3}\sum_{j=1}^{m_i} R_{1,i}(u_{i,j}).[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, i = 0, 1, 2, ..., n-1, \tag{4.30}$$

and

$$\frac{\partial S_i}{\partial \mathbf{D}_{i+1}} = -\frac{2h_i}{3}\sum_{j=1}^{m_i} R_{2,i}(u_{i,j}).[\mathbf{P}_i(u_{i,j}) - \mathbf{P}_{i,j}] = 0, i = 0, 1, 2, ..., n-1, \tag{4.31}$$

Replacing $\mathbf{P}_i(u_{i,j})$ from equation (4.1) and simplifying equation (4.30) and (4.31), we get

$$\begin{aligned}\mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{F}_{i+1}(a_{2,i} + a_{3,i}) - \sum_{j=1}^{m_i}(R_{1,i}(u_{i,j})\mathbf{P}_{i,j} = \\ \frac{h_i}{3}(-\mathbf{D}_i a_{1,i} + \mathbf{D}_{i+1}a_{2,i}), \qquad i = 0, 1, 2, ...n-1.\end{aligned} \tag{4.32}$$

and

$$\begin{aligned}&\mathbf{F}_i(a_{2,i} + a_{4,i}) + \mathbf{F}_{i+1}(a_{5,i} + a_{6,i}) - \sum_{j=1}^{m_i}(R_{2,i}(u_{i,j})\mathbf{P}_{i,j} = \\ &\frac{h_i}{3}(-\mathbf{D}_i a_{2,i} + \mathbf{D}_{i+1} a_{5,i}), \qquad i = 0, 1, 2, ...n - 1.\end{aligned} \tag{4.33}$$

where $a_{0,i}, ..., a_{6,i}$ are same as in equation (4.15) described in case 2.

Solving equation (4.32) and equation (4.33), we get

$$\mathbf{D}_i = \frac{3}{h_i(a_2^2 - a_1 a_5)}[a_5 \mathbf{A}_{1,i} - a_2 \mathbf{A}_{2,i}] \tag{4.34}$$

and

$$\mathbf{D}_{i+1} = \frac{3}{h_i(a_2^2 - a_1 a_5)}[a_2 \mathbf{A}_{1,i} - a_1 \mathbf{A}_{2,i}] \tag{4.35}$$

where

$$\left.\begin{aligned} A_{1,i} &= \mathbf{F}_i(a_{0,i} + a_{1,i}) + \mathbf{F}_{i+1}(a_{2,i} + a_{3,i}) - \sum_{j=1}^{m_i} R_{1,i}(u_{i,j})\mathbf{P}_{i,j}, \\ A_{2,i} &= \mathbf{F}_i(a_{2,i} + a_{4,i}) + \mathbf{F}_{i+1}(a_{5,i} + a_{6,i}) - \sum_{j=1}^{m_i} R_{2,i}(u_{i,j})\mathbf{P}_{i,j} \end{aligned}\right\} \tag{4.36}$$

Thus the curve fitted using the above values of $\mathbf{D}_i$'s and $\mathbf{D}_{i+1}$'s will be a good candidate of best fit.

Case 5: v_i's, w_i's and $\mathbf{D}_i$'s are calculated using Least Square method.

In this case the shape parameters v_i's and w_i's and the tangents are approximated using Least Square method. Here we combined the results of case 3 and 4. So we use equations (4.24) and (4.25) to calculate the best values of shape parameters and equations (4.34) and (4.35) to approximate the tangents at the characteristic points. The results at the end shows that this case gives good results in terms of less number of break points as compared to other cases.

4.5.4 Breaking Segments

A fitted Bezeir curve to a segment may not satisfy the threshold tolerance limit. The curve is then to be subdivided in two at the point of worst error - the point where the fitted spline is farthest from the digitized curve. In the literature [9, 10] it is computed as the squared distance between each point p_i of digitized curve and its corresponding point $p(t_i)$ of parametric curve. A different approach is taken in this thesis and each segment of the curve is broken at the maximum difference of x or y coordinates. The new break point will be considered as a significant point and the curve is again fitted between these characteristic points. The distance 'd' (in terms of pixels) between original curve points $\mathbf{P}_{i,j}$ and their corresponding points

$\mathbf{P}(t_{i,j})$ on the parametric curve is given by

$$d = \max(|\mathbf{P}_{x_{i,j}} - \mathbf{P}_x(t_{i,j})|, |\mathbf{P}_{y_{i,j}} - \mathbf{P}_y(t_{i,j})|)$$

If d exceeds predefined error tolerance limit then the segment is broken into two segments at the point of maximum distance and the point corresponding to maximum distance is added to list of significant points. Number of segments and number of Significant points are increased by one.

The process is repeated for each segment until all the segments of all the pieces meet the threshold tolerance limit.

4.6 Conclusion

In this chapter the details of the system has been described. The first few basic steps of the system were mentioned briefly. The curve fitting step is the main step of the system and discussed in detail. The generalized Hermite cubic is used to fit the curve. The two corner points for each segment are taken as the two end control points of the curve whereas the least square method is used to get the optimal shape parameter values.

Chapter 5

Web Application

The recent explosion in the popularity of the world wide web and its associated hypertext markup Language (HTML) and hypertext transfer protocol (HTTP) presents an exciting new opportunity to provide widely distributed access to sophisticated software applications [96].

A system or application appearance over the World Wide Web is nowadays common and sometimes essential and economical. Our aim is to implement and test the curve fitting approaches that are discussed in chapter 4 providing end user with an easy-to-use interface. Another reason for making the system Web based is to share the knowledge amongst the whole Internet community.

Initially two options were available for deploying the system over the Web. Either to use Matlab Web Server with some httpd server or to use Microsoft IIS or Tomcat Web Server in combination with ASP, JSP and JavaBeans. All the cases mentioned in chapter 4 were implemented on Matlab, so the Matlab Web Server option along with Apache Web Server is used. Since the Matlab Web Server does not support the uploading of files, the combination of JSP, JavaBeans and Tomcat Web Server to implement the upload feature is used. Here the Matlab Web Server Environment, it's components and how to build the applications in it are briefly discussed. The following information is extracted from [97].

5.1 MATLAB Web Server Environment

The *MATLAB*® Web Server enables to create MATLAB applications that use the capabilities of the World Wide Web to send data to MATLAB for computation and to display the results in a Web browser. The MATLAB Web Server depends upon TCP/IP networking for transmission of data between the client system and MATLAB. The required networking software and hardware must be installed on the system prior to using the MATLAB Web Server. In the simplest configuration, a Web browser runs on client workstation, while MATLAB, the MATLAB Web Server (matlabserver), and the Web server daemon (httpd) run on another machine.

In a more complex network, the Web server daemon can run on a machine apart from the others.

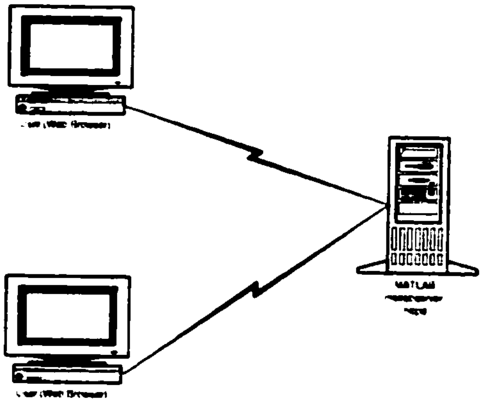


Figure 5.1: Simple Configuration

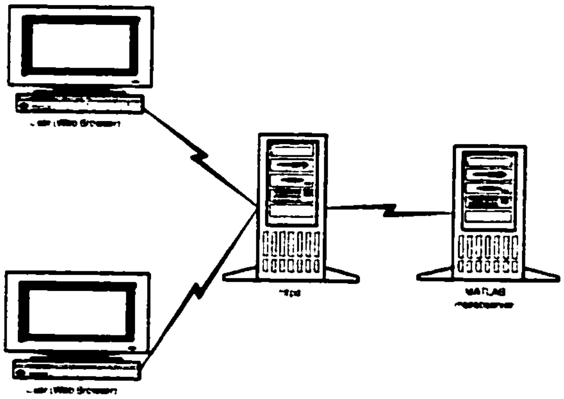


Figure 5.2: Complex Configuration

5.2 Building MATLAB Web Server Applications

MATLAB Web Server applications are a combination of M-files (Matlab Files), Hypertext Markup Language (HTML), and graphics.

The process of creating a MATLAB Web Server application involves the creation of:

1. An HTML input document for data submission to MATLAB.
2. An HTML output document for display of MATLAB's computations.
3. A MATLAB M-file to process input data and compute results. The task of this file is to:
   - receives the data entered in the HTML input form.
   - analyzes the data and generates any requested graphics.
   - places the output data into a MATLAB structure.
   - calls Matlab function 'htmlrep' to place the output data into an HTML output document template.
4. List the application name and associated configuration data in the configuration file matweb.conf.

5.3 MATLAB Web Server Components

The MATLAB Web Server consists of a set of programs that enable MATLAB programmers to create MATLAB applications and access them on the Web:

- matlabserver: Manages the communication between the Web application and MATLAB. matlabserver is a multithreaded TCP/IP server. It runs the MATLAB program (M-file) specified in a hidden field named mlmfile contained in the HTML document. matlabserver invokes matweb.m, which in turn runs the M-file. matlabserver can be configured to listen on any legal TCP/IP port by editing the matlabserver.conf file on Windows NT or running webconf on Solaris/ Linux.

- matweb: A TCP/IP client of matlabserver. This program uses the Common Gateway Interface (CGI) to extract data from HTML documents and transfer it to matlabserver.

- matweb.m: Calls the M-file that we want the Web application to run.

- matweb.conf: A configuration file that matweb needs for connecting to matlabserver. Applications must be listed in matweb.conf.

The following figure shows how Matlab operates over the Web.

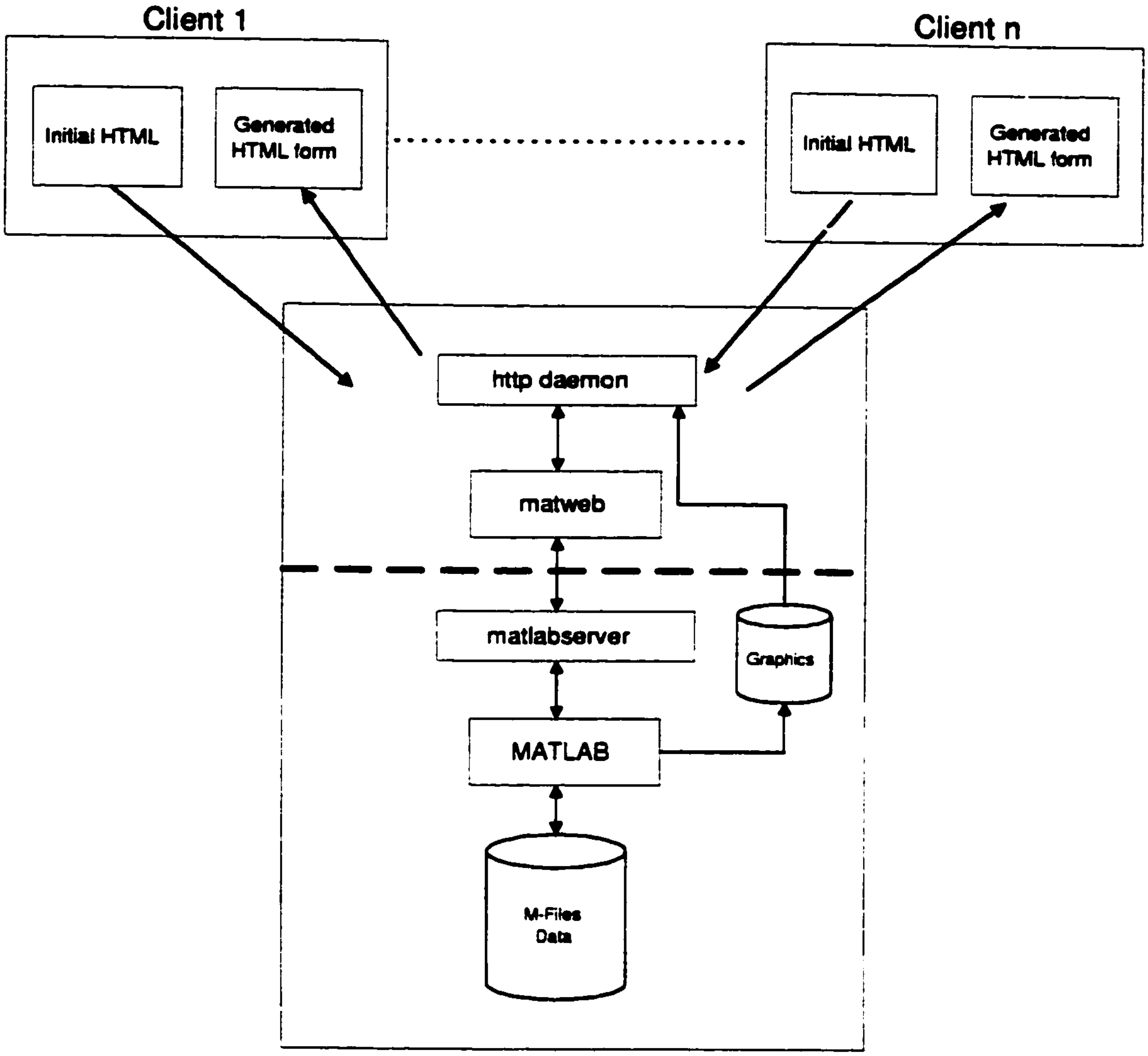


Figure 5.3: Matlab on the Web

5.4 Structure and Implementation

The structure of our system is similar to the one depicted in Figure 5.1. The Apache Web Server is used as the httpd daemon and the web system works as shown in Figure 5.3. Five cases that were discussed in chapter 4 were implemented and made available for testing over the Internet. Several different gray scale bitmap images, set of arabic and english alphabets are provided for testing. In addition to this a user can upload his own bitmap image for testing with any of the five cases. Figures 5.5 and 5.6 shows the set of arabic and english characters. Figure 5.7 shows the miscellaneous bitmap images page. Also a user uploaded images page have the same structure and appearance. A user has to select an image by clicking on the radio button next to the images and then select two more things from the list of values. The first one is the threshold value in pixels which specify how much error can be tolerated as described in chapter 4. The default value of threshold is 3 pixels. Another one is the selection of any one case from the five available cases discussed in chapter 4. By default, case 5 will be selected and the user has to press the 'Go' button to start the whole process described in chapter 4. A flow chart showing the client and server flow is depicted in figure (5.4).

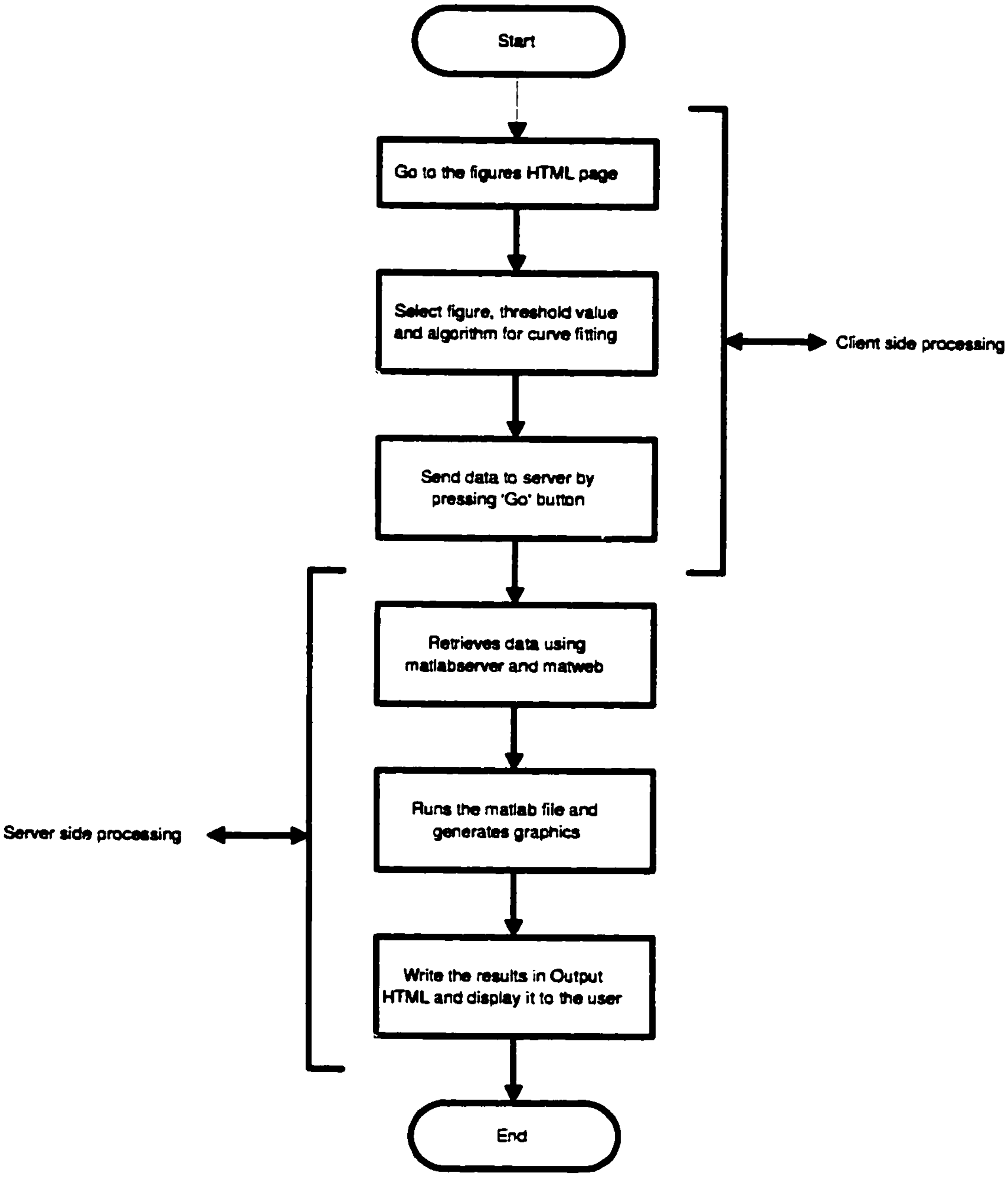

Start
Go to the figures HTML page
Select figure, threshold value and algorithm for curve fitting
Send data to server by pressing 'Go' button
Retrieves data using matlabserver and matweb
Runs the matlab file and generates graphics
Write the results in Output HTML and display it to the user
End
Client side processing
Server side processing

Figure 5.4: Flow chart of Matlab Web Application

Figure 5.8 shows the final result screen that appears after the processing. It shows some statistics such as total number of points, total number of corner and break points. It also shows the digitized image, outline of the curve and the fitted curve with corner and break points.

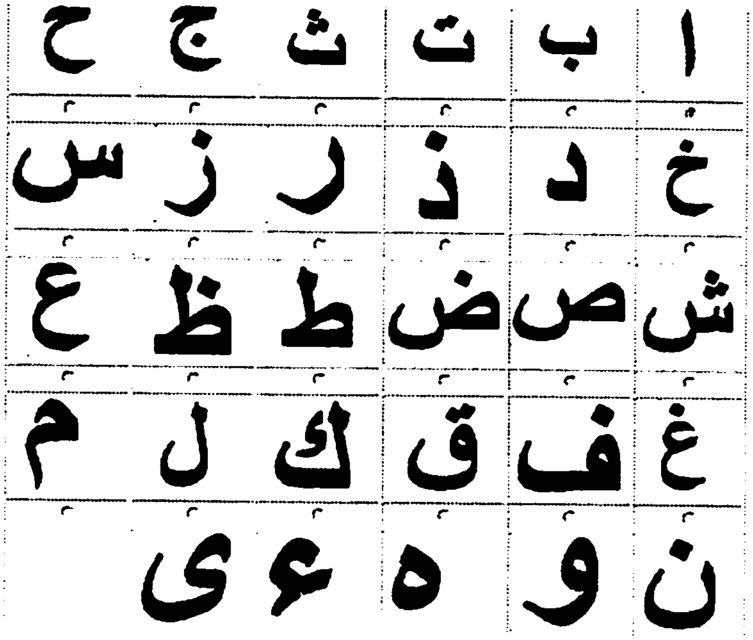


Figure 5.5: Set of Arabic characters

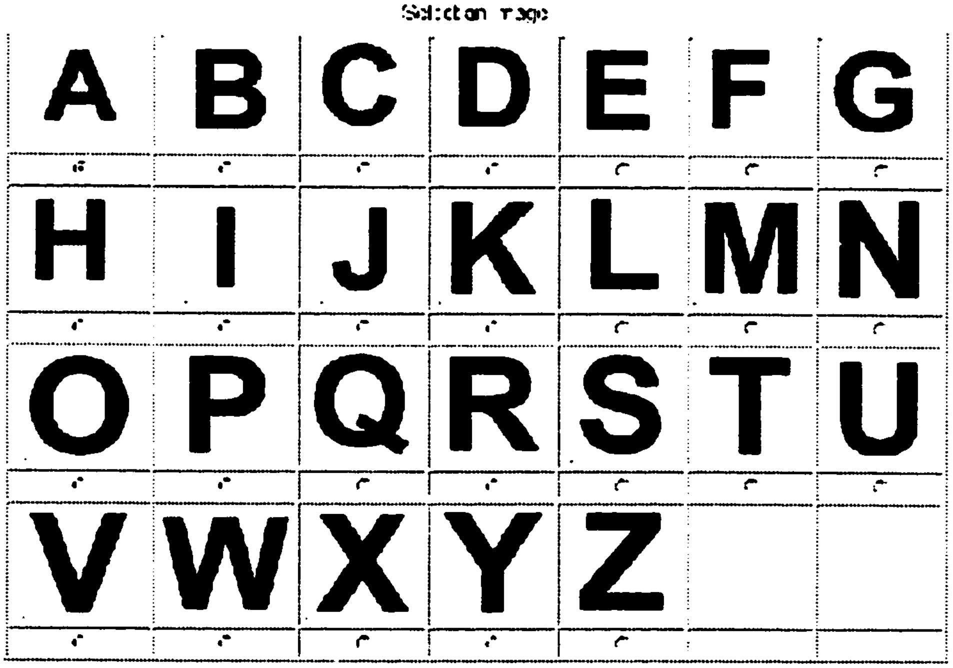
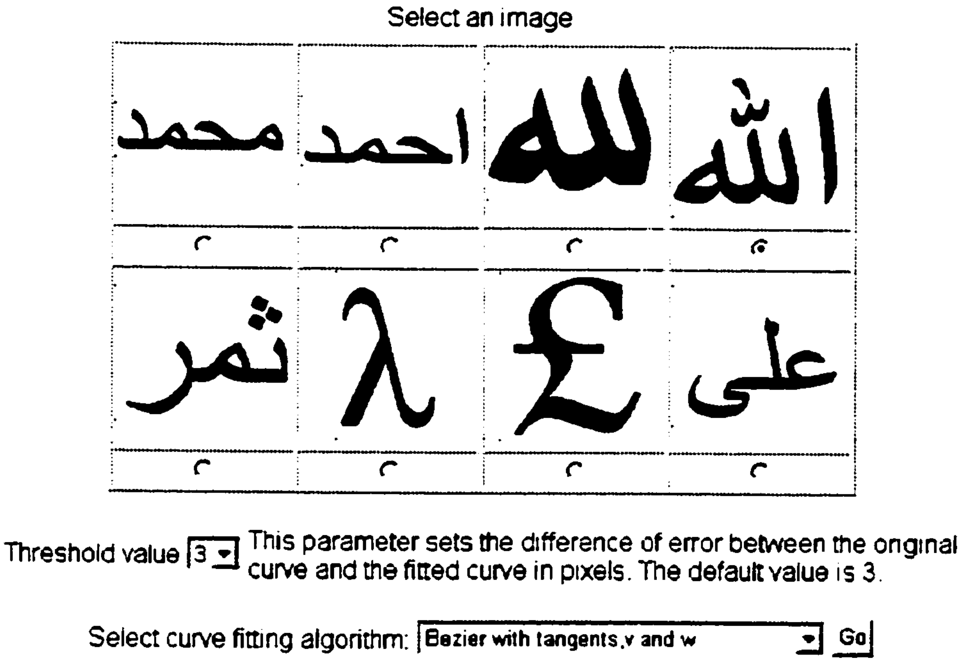


Figure 5.6: Set of English characters



Select an image

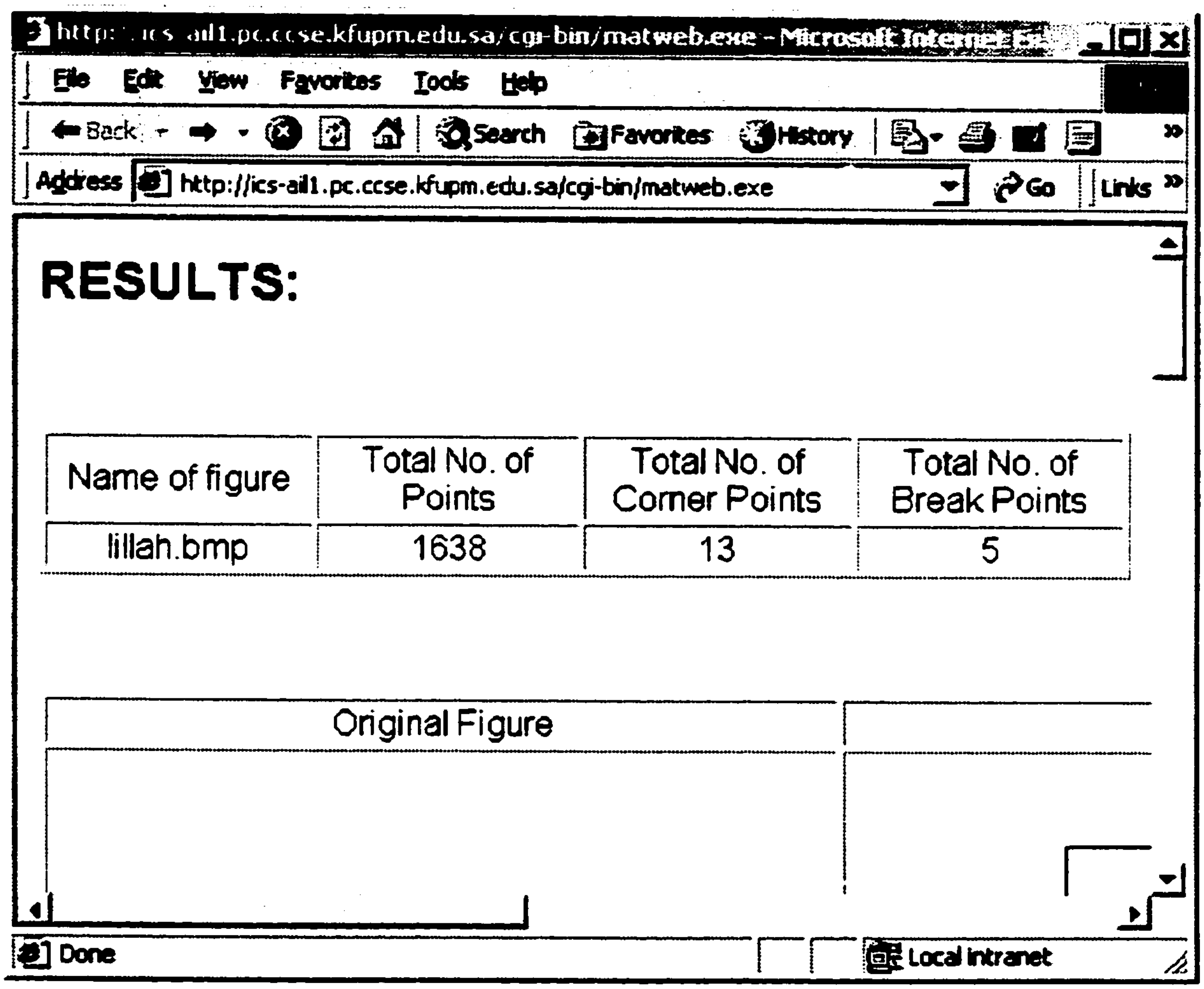
○ ○ ○ ●

○ ○ ○ ○

Threshold value 3 ▾ This parameter sets the difference of error between the original curve and the fitted curve in pixels. The default value is 3.

Select curve fitting algorithm: Bezier with tangents,v and w ▾ Go

Figure 5.7: Misc. figures page


http://ics-ail1.pc.ccse.kfupm.edu.sa/cgi-bin/matweb.exe - Microsoft Internet Ex
File Edit View Favorites Tools Help
Back Search Favorites History
Address http://ics-ail1.pc.ccse.kfupm.edu.sa/cgi-bin/matweb.exe
Go Links
Done
Local intranet

RESULTS:

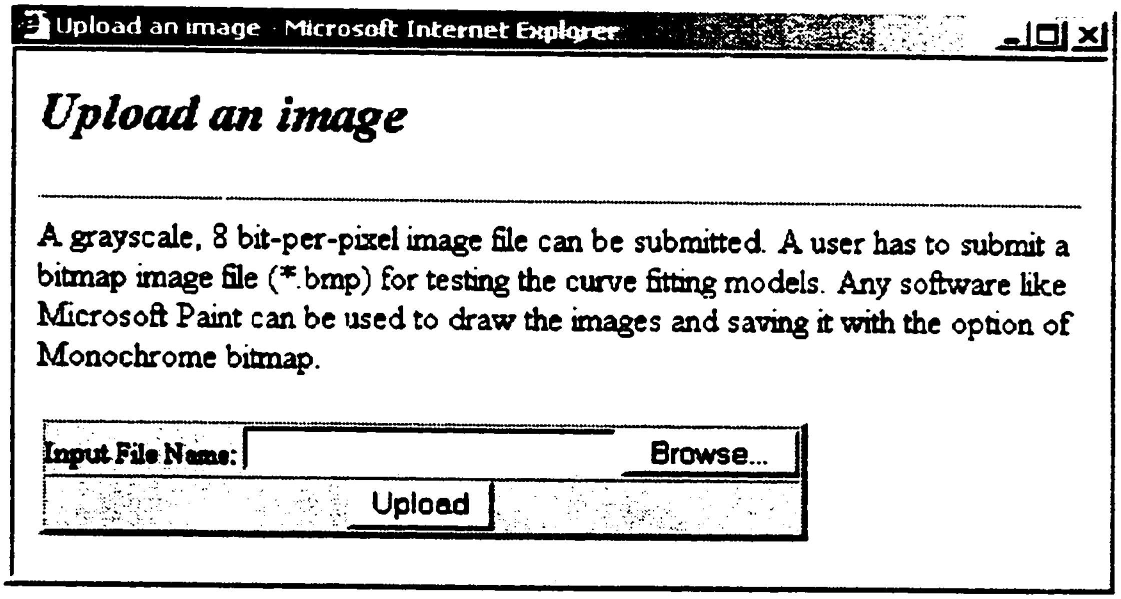
Name of figure	Total No. of Points	Total No. of Corner Points	Total No. of Break Points
lillah.bmp	1638	13	5

Original Figure	

Figure 5.8: Final result screen

5.4.1 Uploading Feature

In addition to providing some standard images and sets of alphabets, a user can upload his own bitmap image. Since Matlab Web Server does not support the uploading feature, the Tomcat Web Server is used as it supports JSP and JavaBeans technology. By using JSP and JavaBean technology, a user bitmap image file is uploaded and then one html file is also updated according to it, so that a user can see and test his image. Although the uploading feature uses different Web Server, this is completely transparent to the novice user. All he has to do is to select the file and then upload it by clicking a button. A sample screen is shown in Figure 5.9.

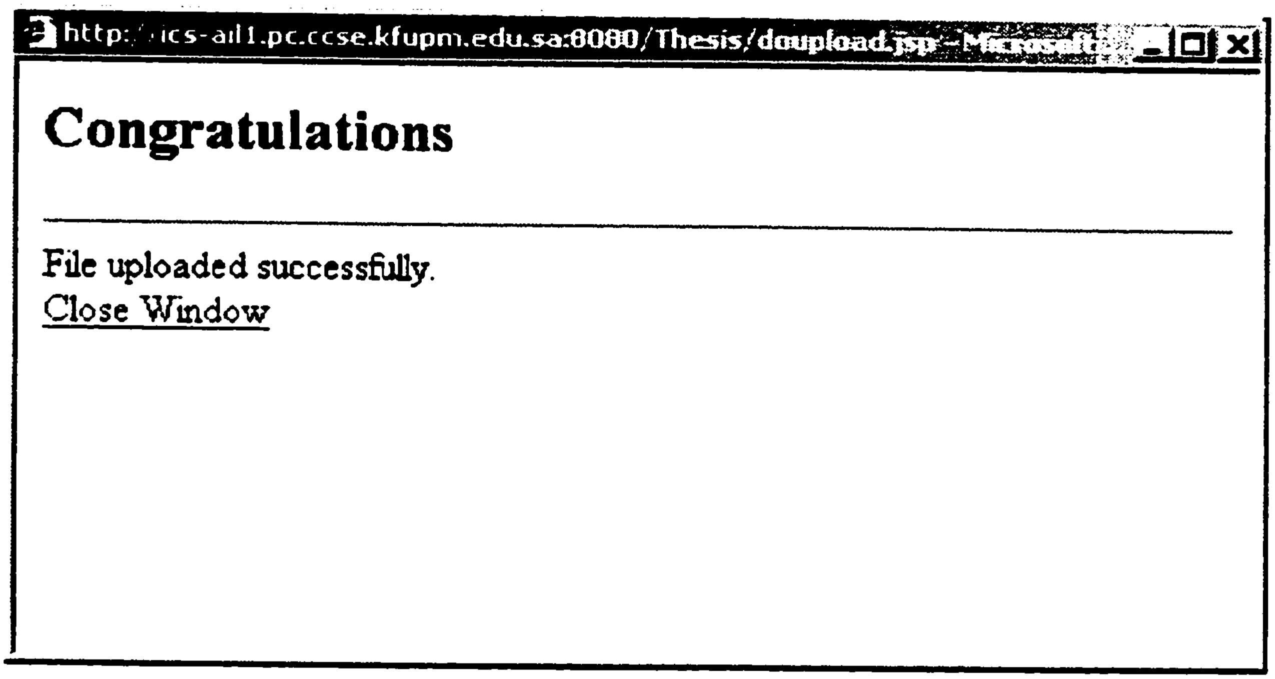


Upload an image

A grayscale, 8 bit-per-pixel image file can be submitted. A user has to submit a bitmap image file (*.bmp) for testing the curve fitting models. Any software like Microsoft Paint can be used to draw the images and saving it with the option of Monochrome bitmap.

Figure 5.9: Screen shot of an upload page

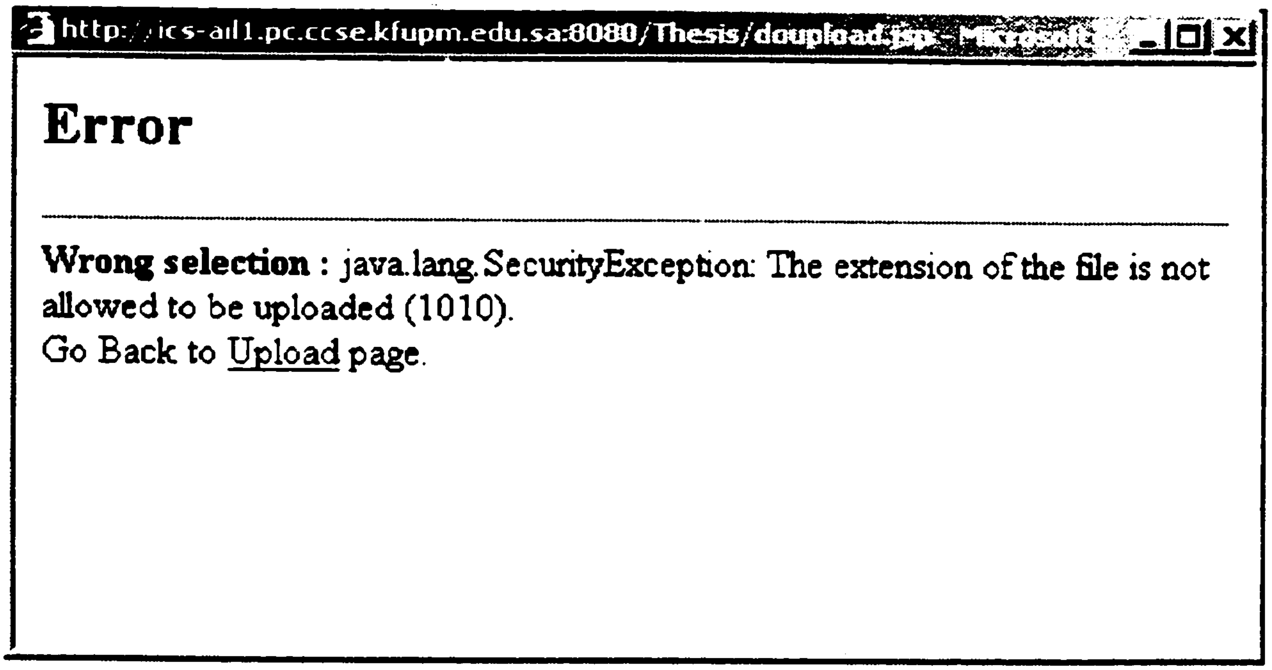
Currently only the monochrome bitmaps files are supported for testing. If the user tries to upload a file other than the bitmap then he will get an error message. Figure 5.11 shows the error screen that a user will see and he will be asked to repeat the process. If an image is successfully uploaded then a screen shown in figure 5.10 will be displayed.


http://ics-ail1.pc.ccse.kfupm.edu.sa:8080/Thesis/doupload.jsp - Microsoft

Congratulations

File uploaded successfully.
Close Window

Figure 5.10: Screen shot of an upload success page


http://ics-ai11.pc.ccse.kfupm.edu.sa:8080/Thesis/doupload.jsp - Microsoft

Error

Wrong selection : java.lang.SecurityException: The extension of the file is not allowed to be uploaded (1010).
Go Back to Upload page.

Figure 5.11: Screen shot of an upload failure page

Chapter 6

Comparison and Results

In this chapter the results of the five cases that were discussed in chapter 4 are shown. Also the comparison is done for the five cases with varying threshold values. In the end a comparison with a previous work is also done. Four images are being used for the purpose. One is an arabic word 'Lillah' (Figure 6.1) and the other is the 'Kanji' (Figure 6.2) character from Japanese language. The other two are the images of an aeroplane (Figure 6.3) and a flower (Figure 6.4). They are selected to emphasize that not only the approach is useful for font designing, it is appropriate for any other image (currently only monochrome bitmap images are used). The results are shown in the order from worst to excellent case in terms of the number of break points required to fit the curve. Also the images obtained with Murtaza's method [3] are also shown.

Figures 6.5-6.18 show the final outlines for 'Lillah' (Figure 6.1) image. For all the cases shown, two values, 1 and 3 are used for the threshold. The results of case 1 are depicted in Figures 6.5 and 6.6. Figures 6.7 and 6.8 show the results for case 2 whereas Figures 6.9 and 6.10 displays the results for case 3. It can be seen that we are getting the best fit with minimum number of significant points as we move along from case 1 to case 3. The final outlines obtained using case 4 are shown in Figures 6.11 and 6.12 whereas Figures 6.13 and 6.14 are the final results for case 5. Case 5 can be regarded as the best in terms of number of significant points required to fit the curve. Figures 6.15 and 6.16 are the final outlines obtained using Murtaza's method [3] when no filtering and reparameterization are used. Finally the results for the case with filtering but without reparameterization are shown in Figures 6.17 and 6.18.

Figures 6.19-6.32 show the final outlines for 'Kanji' (Figure 6.2) image. For all the cases shown, two values, 1 and 3 are used for the threshold. The results of case 1 are depicted in Figures 6.19 and 6.20. Figures 6.21 and 6.22 show the results for case 2 whereas Figures 6.23 and 6.24 displays the results for case 3. It can be seen that we are getting the best fit with minimum number of significant points as we move along from case 1 to case 3. The final outlines obtained using case 4 are shown in Figures 6.25 and 6.26 whereas Figures 6.27 and 6.28 are the final results for case 5. Case 5 can be regarded as the best in terms of number of significant points required to fit the curve. Figures 6.29 and 6.30 are the final outlines obtained using Murtaza's method [3] when no filtering and reparameterization are used. Finally the results for the case with filtering but without reparameterization are shown in Figures 6.31 and 6.32.

Figures 6.33-6.46 show the final outlines for aeroplane (Figure 6.3) image. For all the cases shown, two values, 1 and 3 are used for the threshold. The results of case 1 are depicted in Figures 6.33 and 6.34. Figures 6.35 and 6.36 show the results for case 2 whereas Figures 6.37 and 6.38 displays the results for case 3. It can be seen that we are getting the best fit with minimum number of significant points as we move along from case 1 to case 3. The final outlines obtained using case 4 are shown in Figures 6.39 and 6.40 whereas Figures 6.41 and 6.42 are the final results for case 5. Case 5 can be regarded as the best in terms of number of significant points required to fit the curve. Figures 6.43 and 6.44 are the final outlines obtained using Murtaza's method [3] when no filtering and reparameterization are used. Finally the results for the case with filtering but without reparameterization are shown in Figures 6.45 and 6.46.

Figures 6.47-6.60 show the final outlines for flower (Figure 6.4) image. For all the cases shown, two values, 1 and 3 are used for the threshold. The results of case 1 are depicted in Figures 6.47 and 6.48. Figures 6.49 and 6.50 show the results for case 2 whereas Figures 6.51 and 6.52 displays the results for case 3. It can be seen that we are getting the best fit with minimum number of significant points as we move along from case 1 to case 3. The final outlines obtained using case 4 are shown in Figures 6.53 and 6.54 whereas Figures 6.55 and 6.56 are the final results for case 5. Case

5 can be regarded as the best in terms of number of significant points required to fit the curve. Figures 6.57 and 6.58 are the final outlines obtained using Murtaza's method [3] when no filtering and reparameterization are used. Finally the results for the case with filtering but without reparameterization are shown in Figures 6.59 and 6.60.

Some statistics in tabular form are shown from Tables 6.1 to 6.9. Table 6.1 shows the number of pieces and data points in each of the four figures. Tables 6.2 to 6.9 shows the comparison of different cases. Tables 6.11 to 6.14 shows the shape parameters and tangent values for all the five cases.

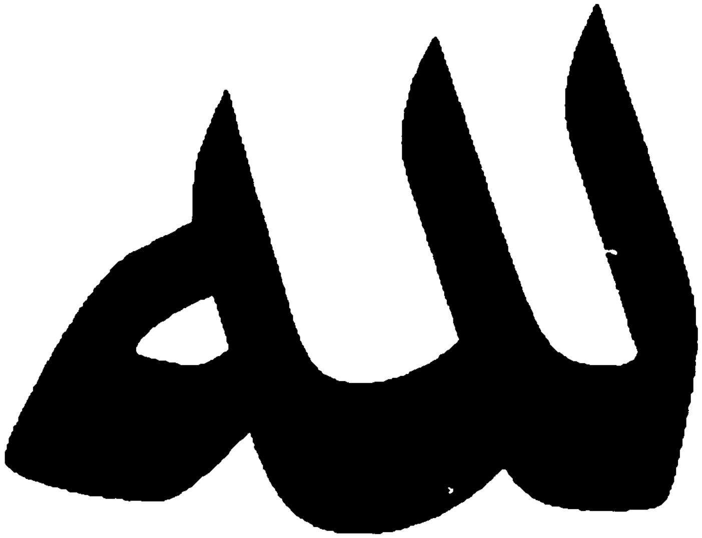


Figure 6.1: Digitized Image of 'Lillah' character

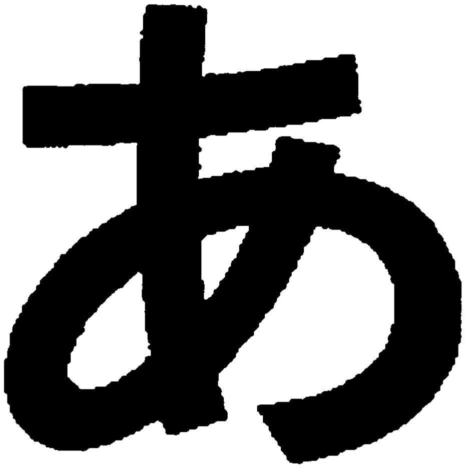


Figure 6.2: Digitized Image of 'Kanji' character

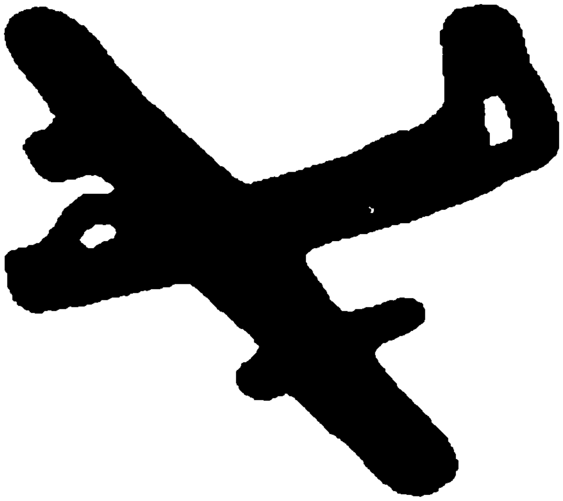


Figure 6.3: Digitized Image of an aeroplane

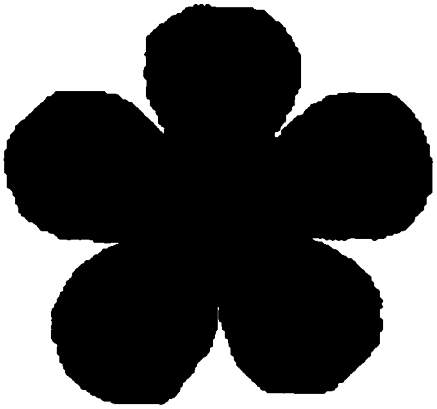


Figure 6.4: Digitized Image of flower

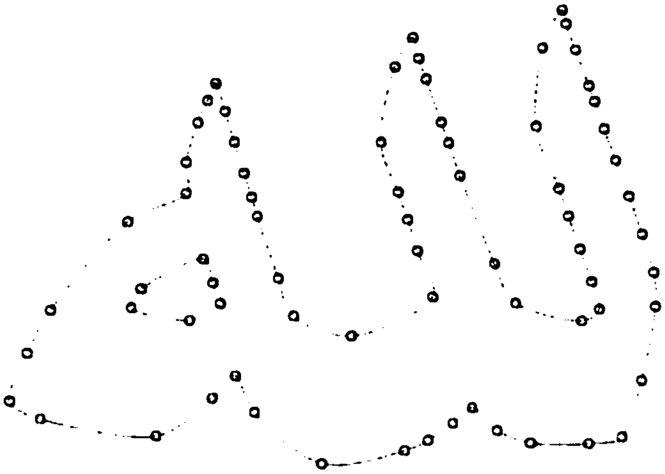


Figure 6.5: Fitted curve with corner and break points, threshold=3 (case 1).

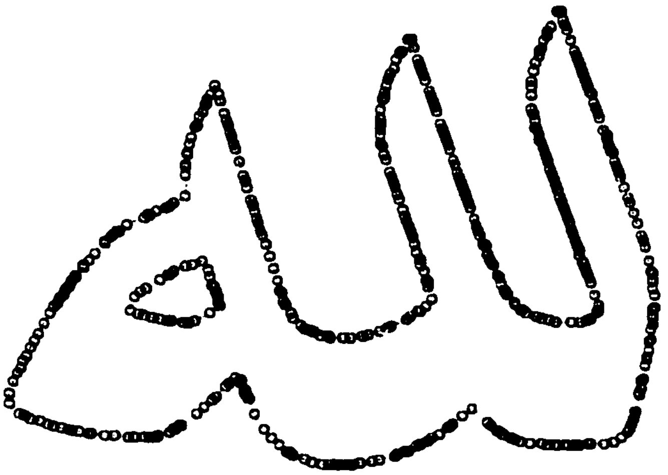


Figure 6.6: Fitted curve with corner and break points, threshold=1 (case 1).

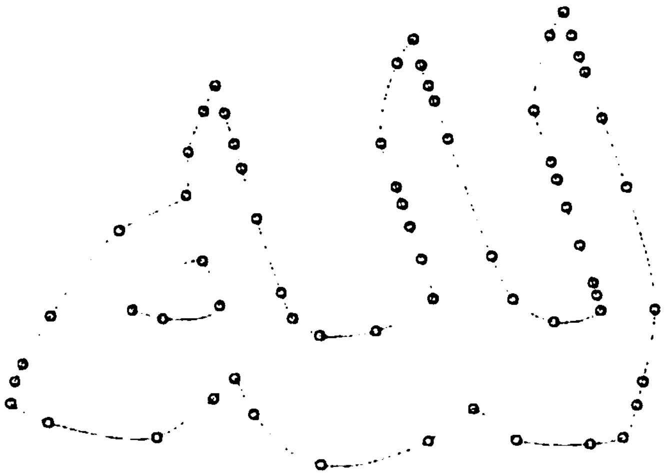


Figure 6.7: Fitted curve with corner and break points, threshold=3 (case 2).

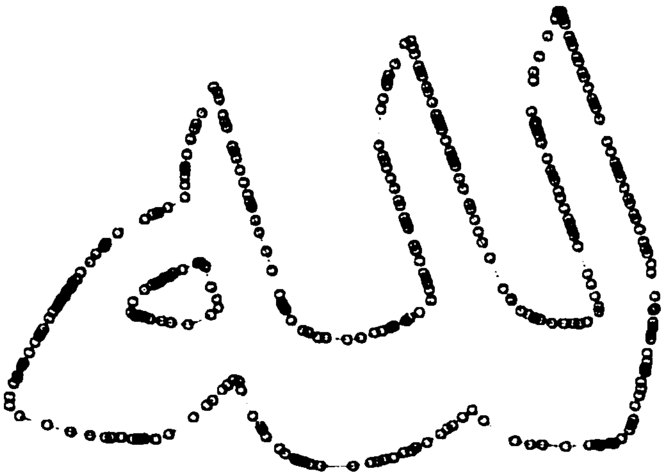


Figure 6.8: Fitted curve with corner and break points, threshold=1 (case 2).

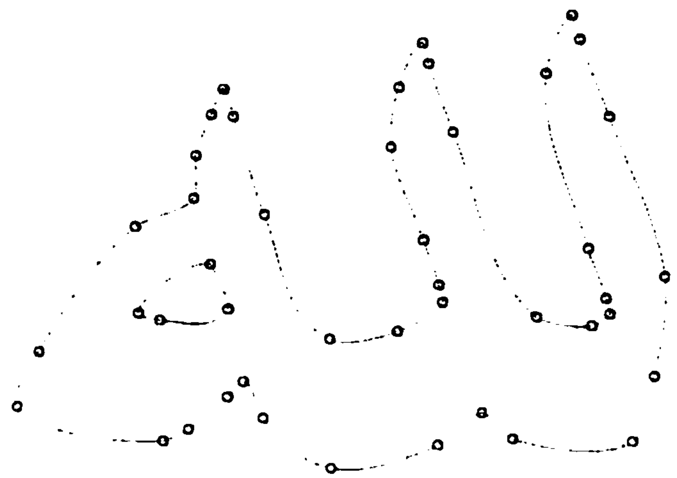


Figure 6.9: Fitted curve with corner and break points, threshold=3 (case 3).

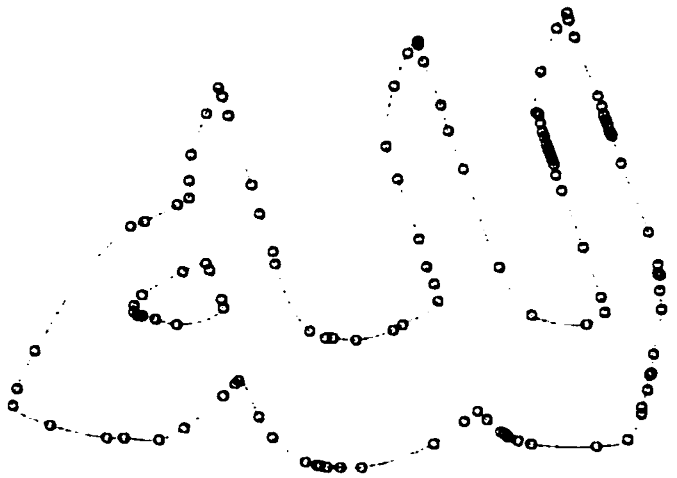


Figure 6.10: Fitted curve with corner and break points, threshold=1 (case 3).

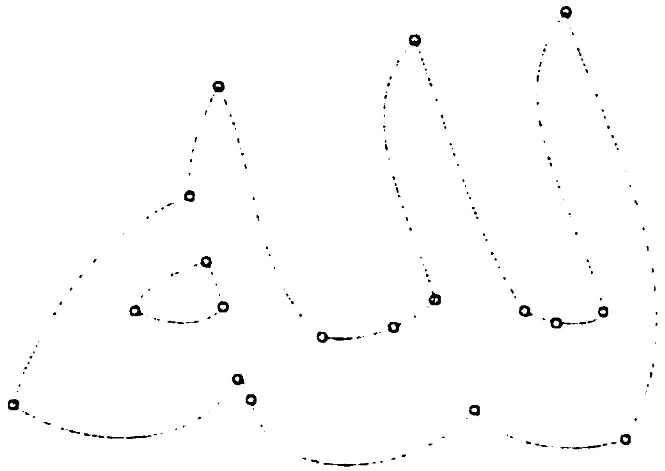


Figure 6.11: Fitted curve with corner and break points, threshold=3 (case 4).

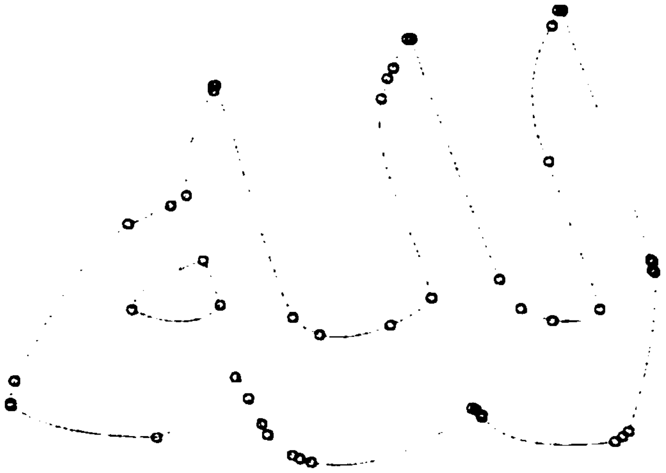


Figure 6.12: Fitted curve with corner and break points, threshold=1 (case 4).

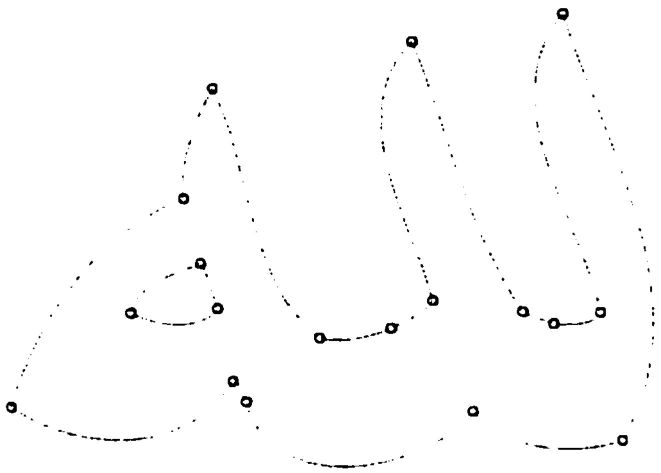


Figure 6.13: Fitted curve with corner and break points, threshold=3 (case 5).

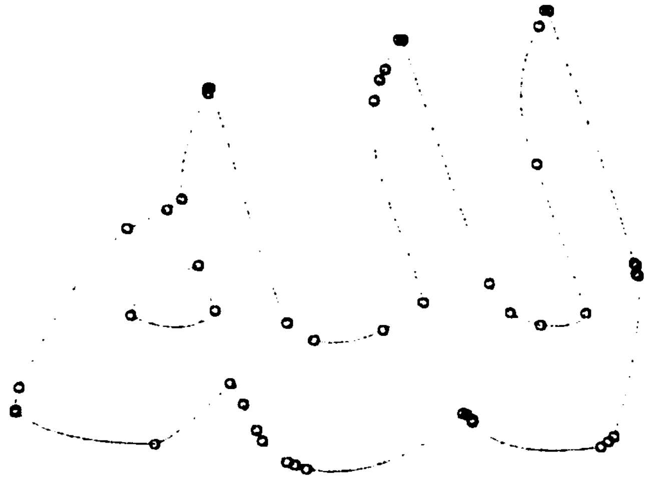


Figure 6.14: Fitted curve with corner and break points, threshold=1 (case 5).

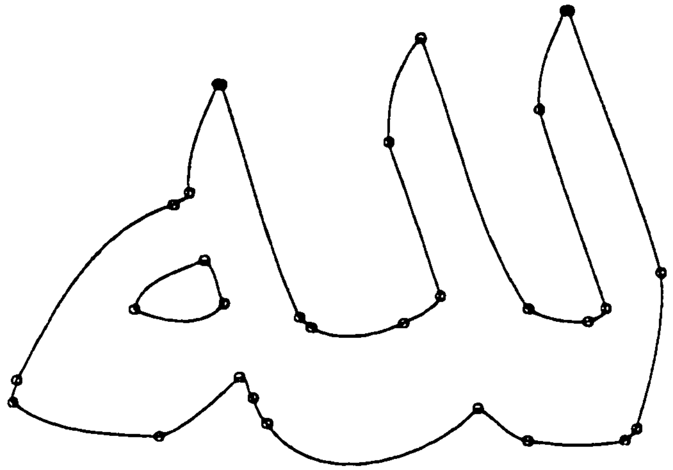


Figure 6.15: Final Outline without filtering and reparameterization, threshold=3

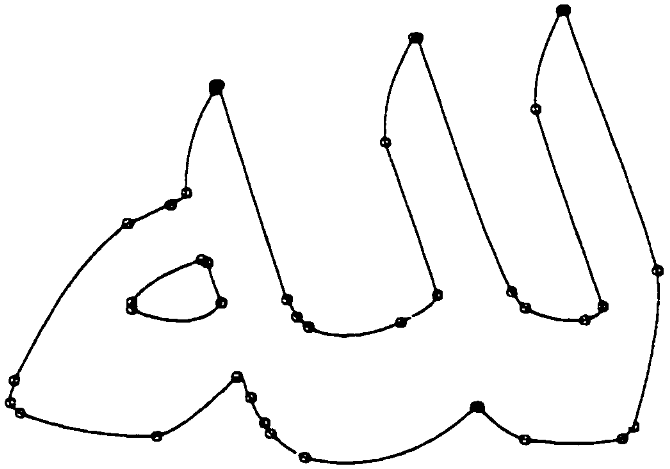


Figure 6.16: Final Outline without filtering and reparameterization, threshold=1

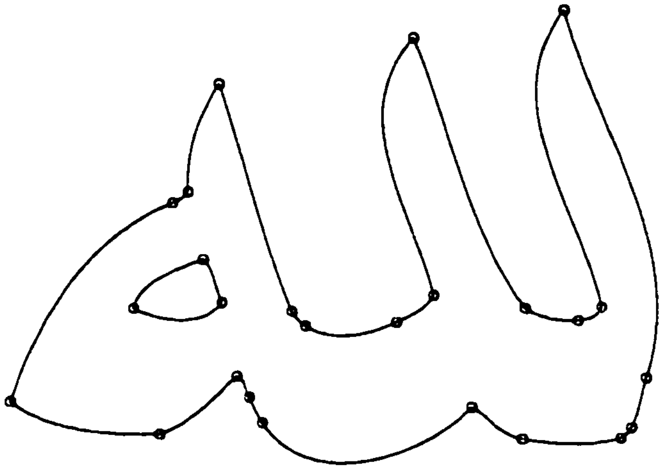


Figure 6.17: Final Outline without reparameterization, threshold=3

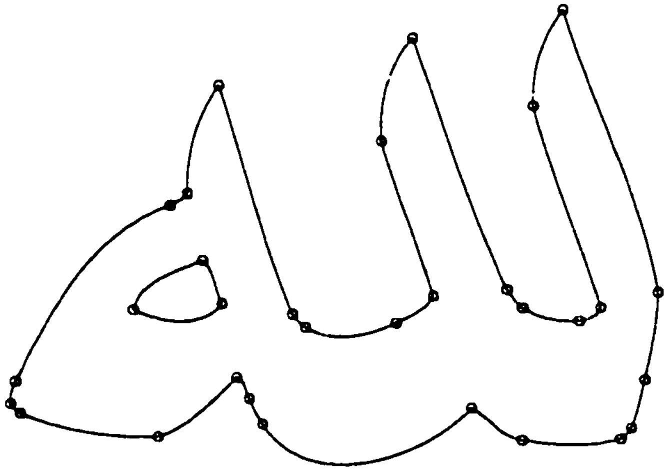


Figure 6.18: Final Outline without reparameterization, threshold=1

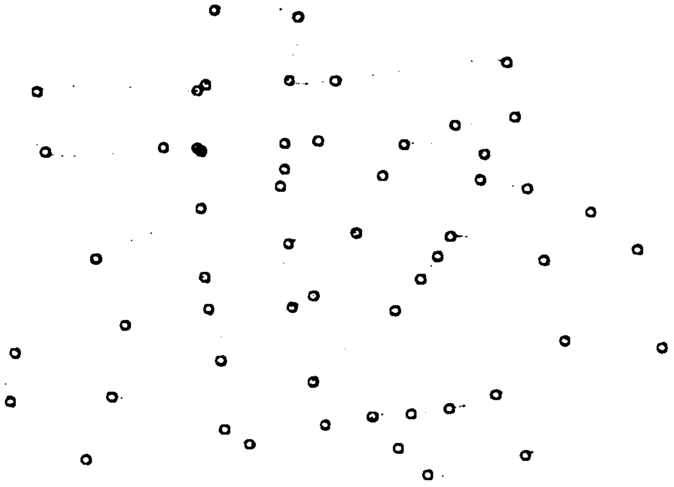


Figure 6.19: Fitted curve with corner and break points, threshold=3 (case 1).

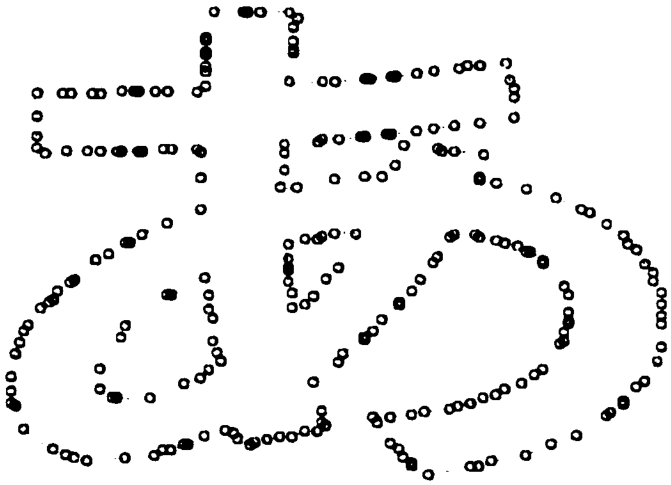


Figure 6.20: Fitted curve with corner and break points, threshold=1 (case 1).

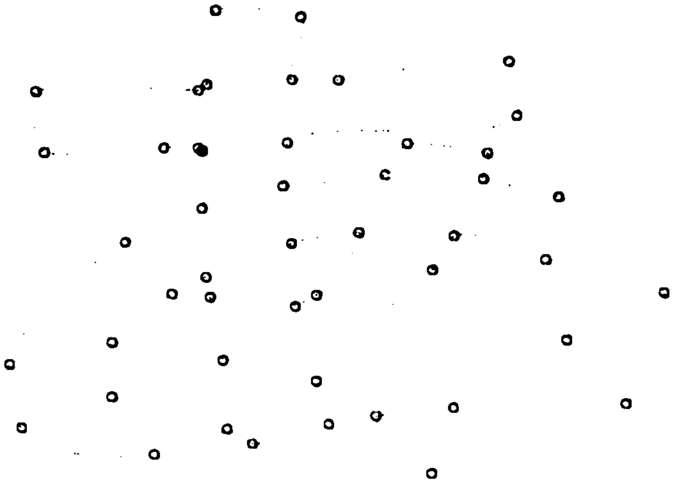


Figure 6.21: Fitted curve with corner and break points, threshold=3 (case 2).

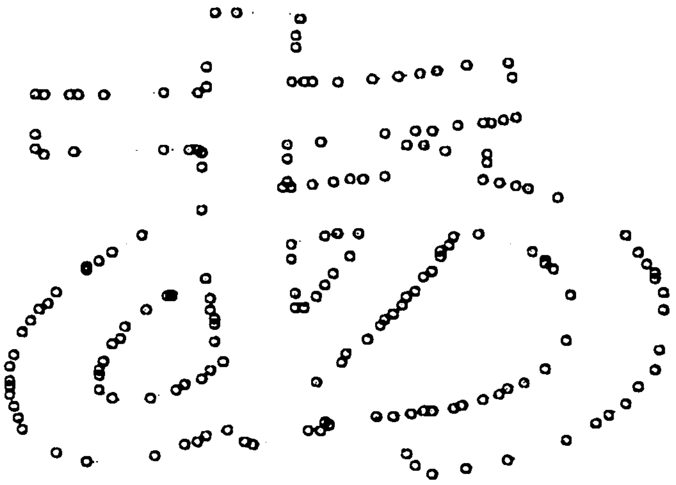


Figure 6.22: Fitted curve with corner and break points, threshold=1 (case 2).

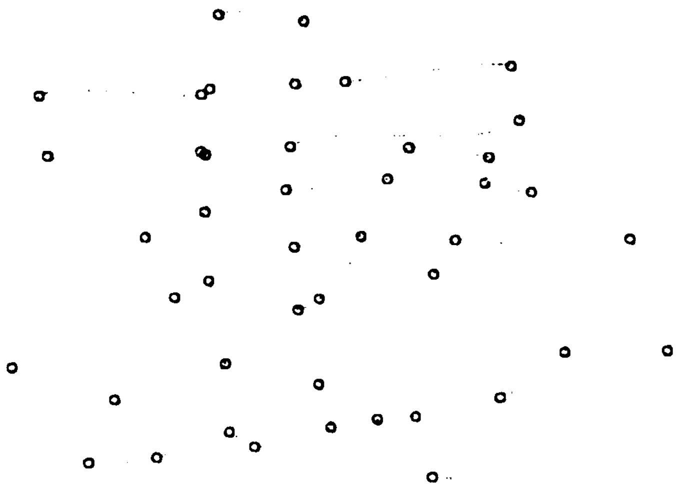


Figure 6.23: Fitted curve with corner and break points, threshold=3 (case 3).

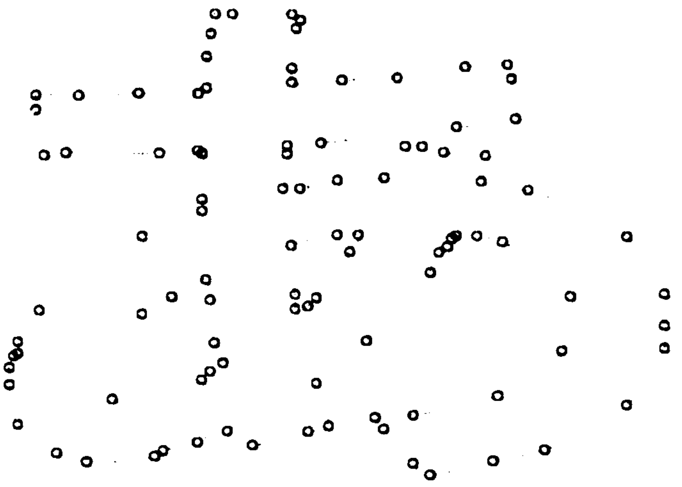


Figure 6.24: Fitted curve with corner and break points, threshold=1 (case 3).

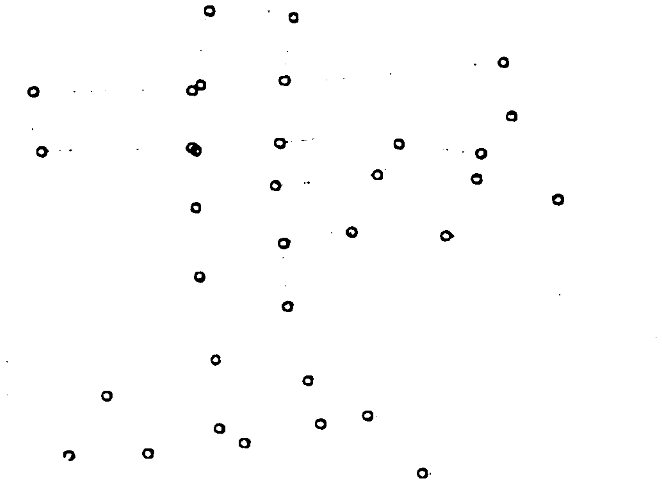


Figure 6.25: Fitted curve with corner and break points, threshold=3 (case 4).

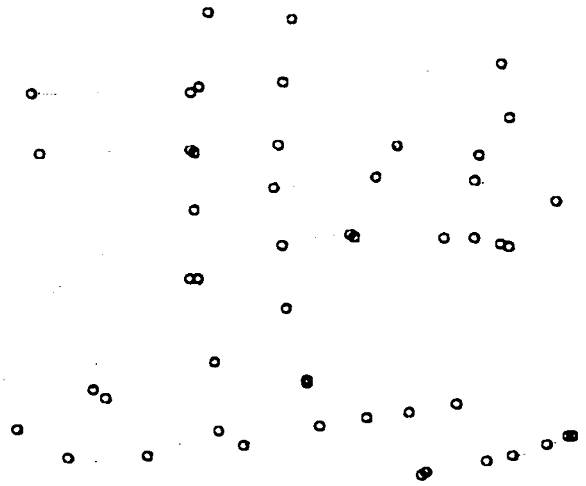


Figure 6.26: Fitted curve with corner and break points, threshold=1 (case 4).

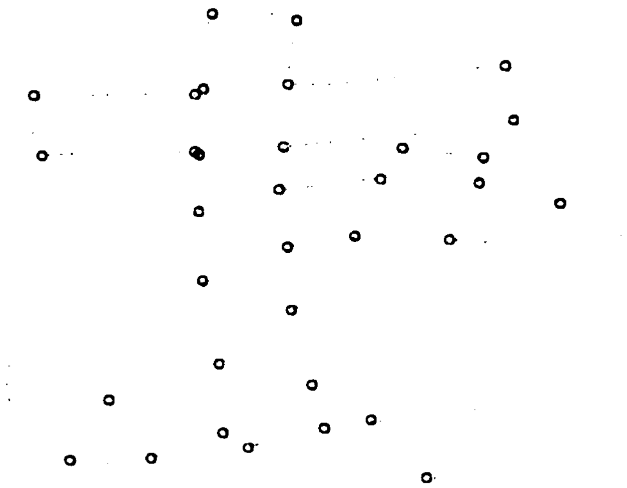


Figure 6.27: Fitted curve with corner and break points, threshold=3 (case 5).

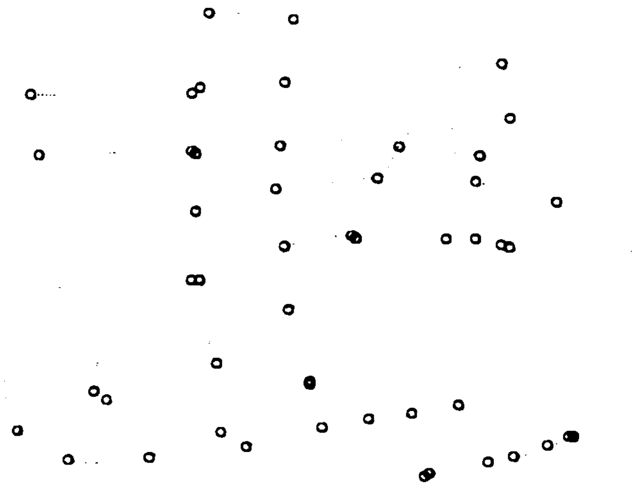


Figure 6.28: Fitted curve with corner and break points, threshold=1 (case 5).

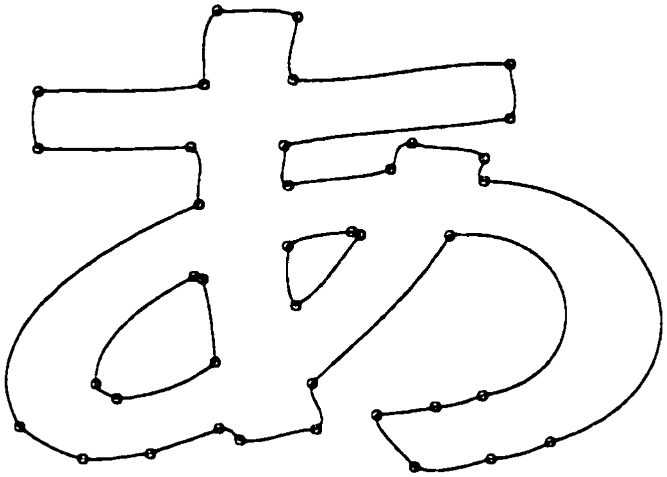


Figure 6.29: Final Outline without filtering and reparameterization, threshold=3

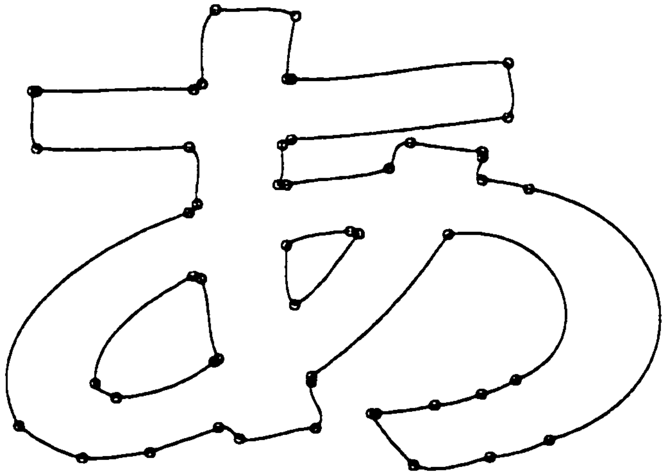


Figure 6.30: Final Outline without filtering and reparameterization, threshold=1

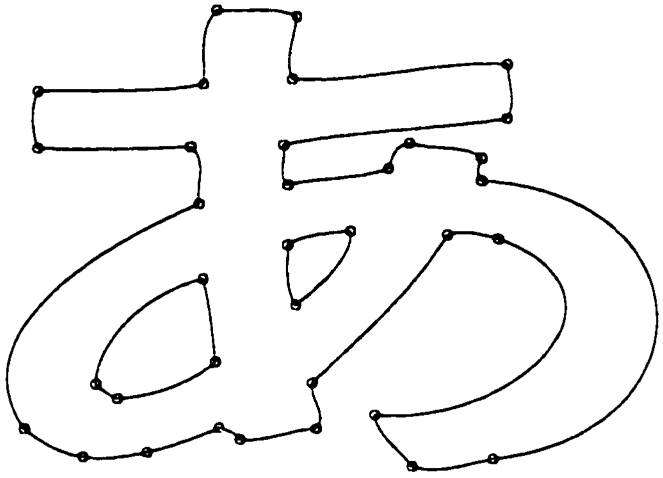


Figure 6.31: Final Outline without reparameterization, threshold=3

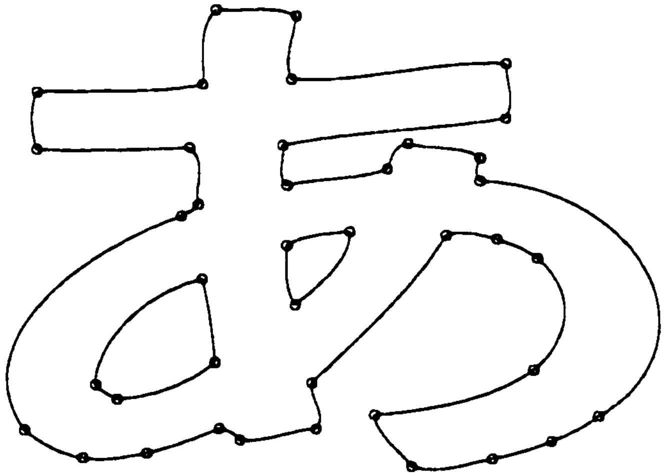


Figure 6.32: Final Outline without reparameterization, threshold=1

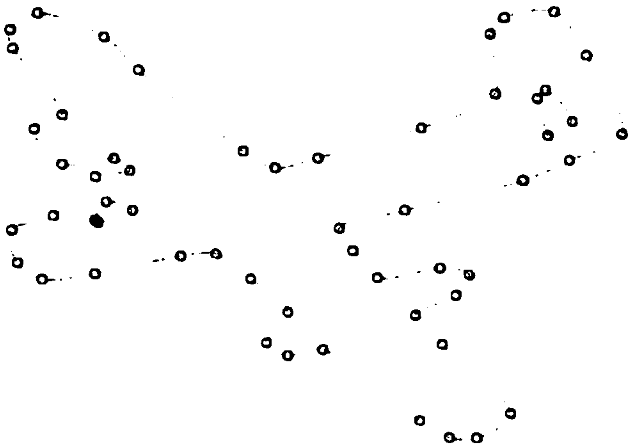


Figure 6.33: Fitted curve with corner and break points, threshold=3 (case 1).

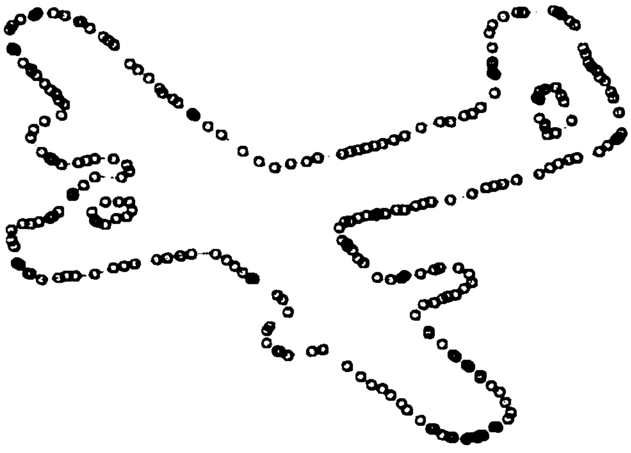


Figure 6.34: Fitted curve with corner and break points, threshold=1 (case 1).

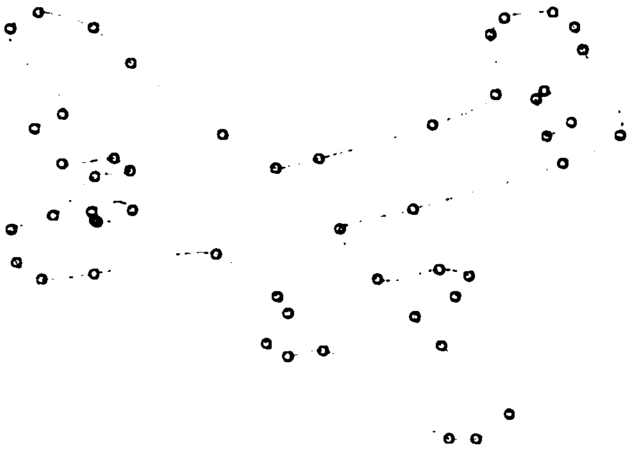


Figure 6.35: Fitted curve with corner and break points, threshold=3 (case 2).

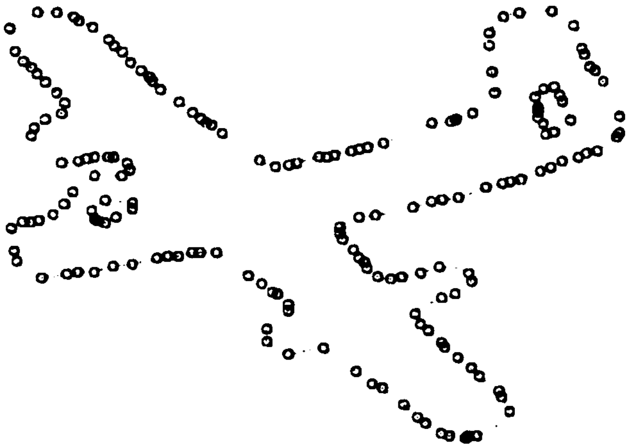


Figure 6.36: Fitted curve with corner and break points, threshold=1 (case 2).

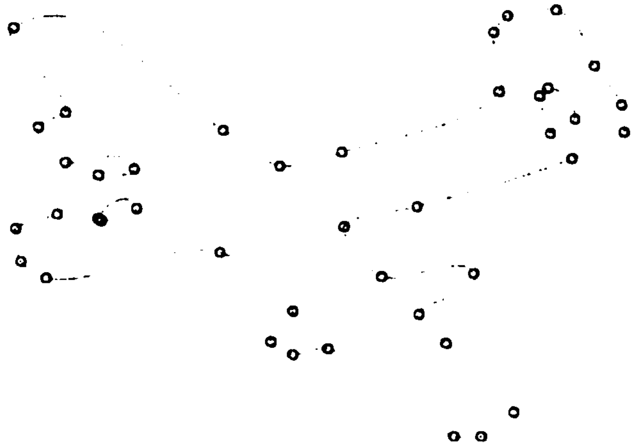


Figure 6.37: Fitted curve with corner and break points, threshold=3 (case 3).

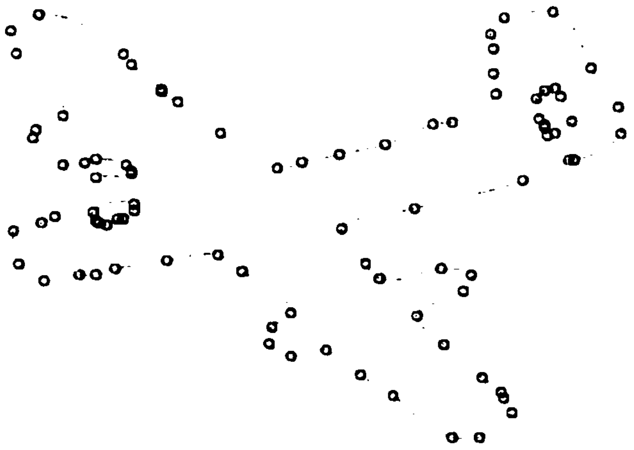


Figure 6.38: Fitted curve with corner and break points, threshold=1 (case 3).

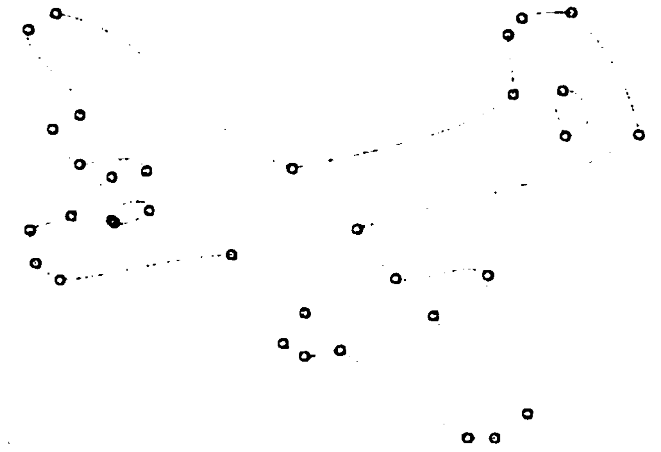


Figure 6.39: Fitted curve with corner and break points, threshold=3 (case 4).

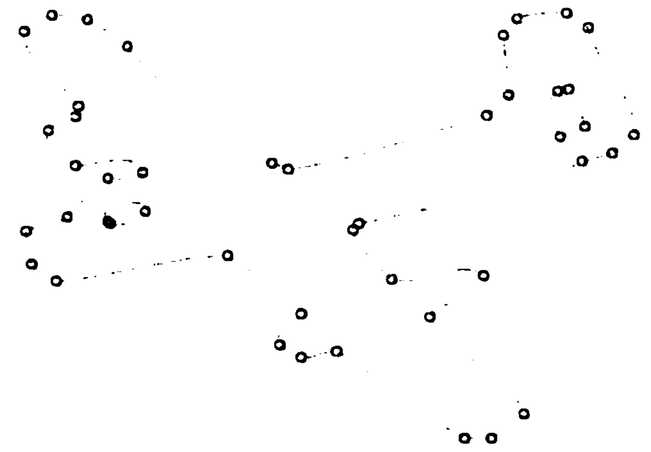


Figure 6.40: Fitted curve with corner and break points, threshold=1 (case 4).

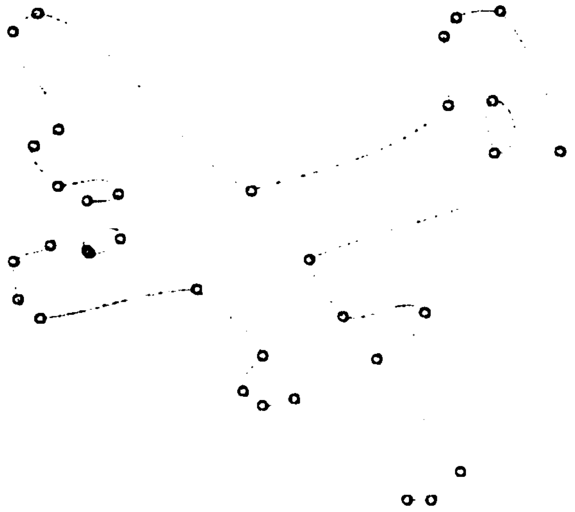


Figure 6.41: Fitted curve with corner and break points, threshold=3 (case 5).

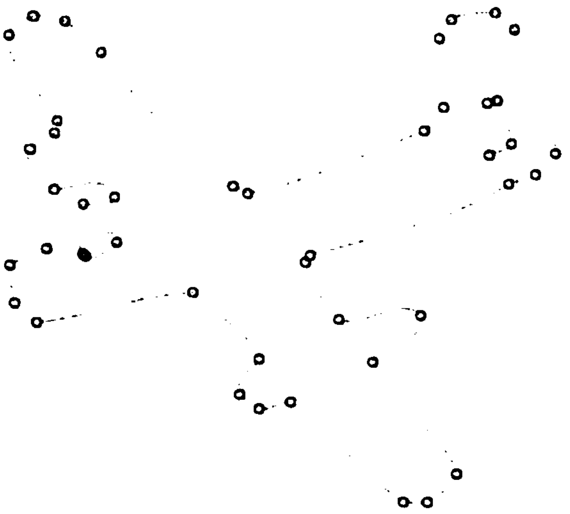


Figure 6.42: Fitted curve with corner and break points, threshold=1 (case 5).

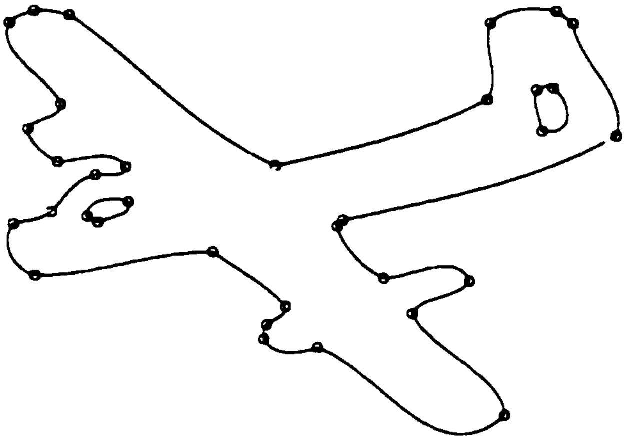


Figure 6.43: Final Outline without filtering and reparameterization, threshold=3

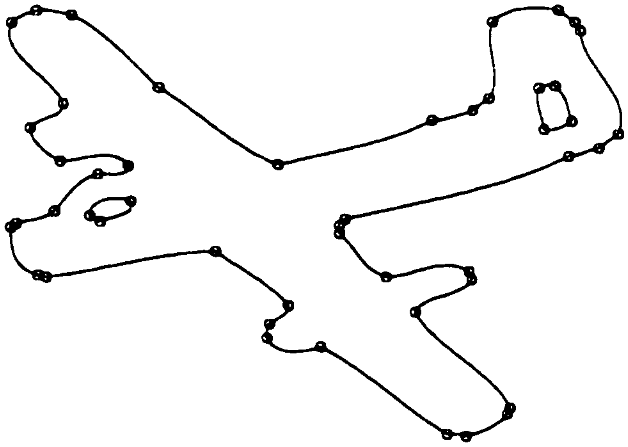


Figure 6.44: Final Outline without filtering and reparameterization, threshold=1

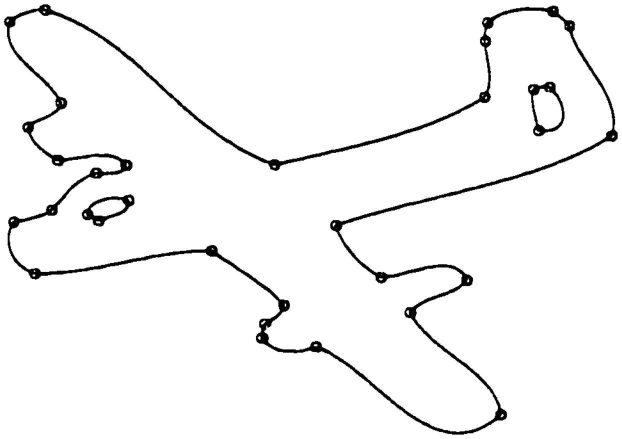


Figure 6.45: Final Outline without reparameterization, threshold=3

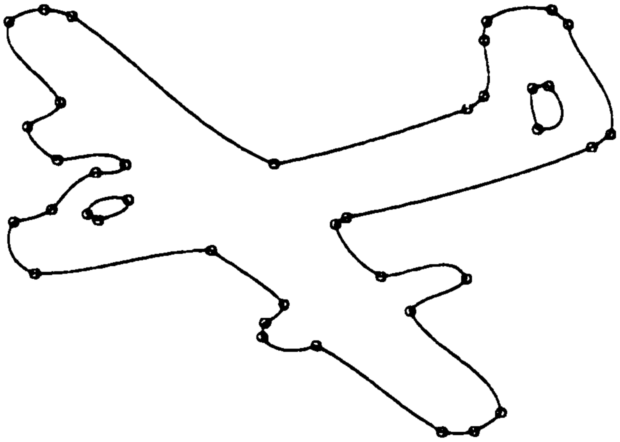


Figure 6.46: Final Outline without reparameterization, threshold=1

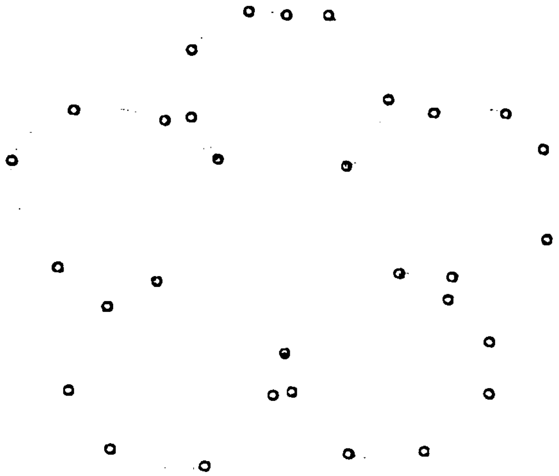


Figure 6.47: Fitted curve with corner and break points, threshold=3 (case 1).

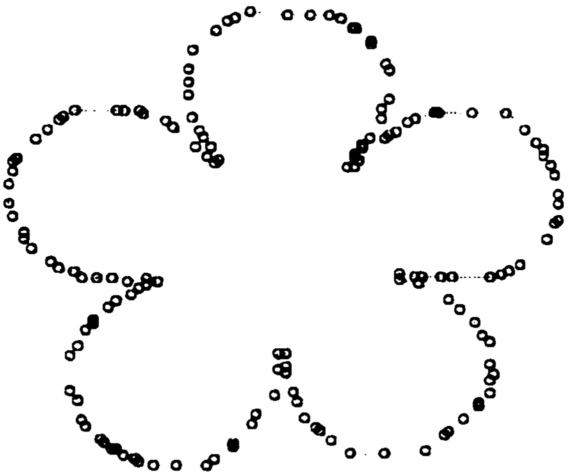


Figure 6.48: Fitted curve with corner and break points, threshold=1 (case 1).

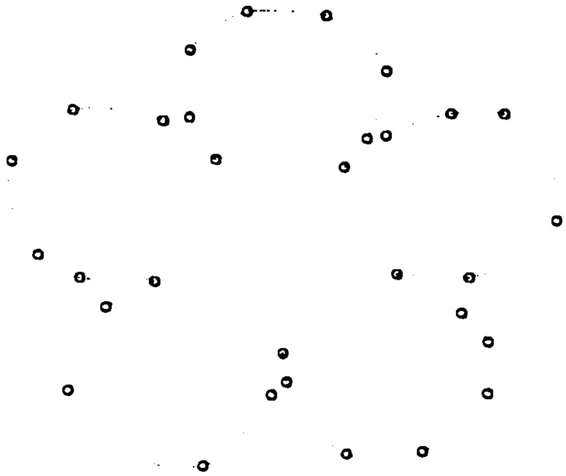


Figure 6.49: Fitted curve with corner and break points, threshold=3 (case 2).

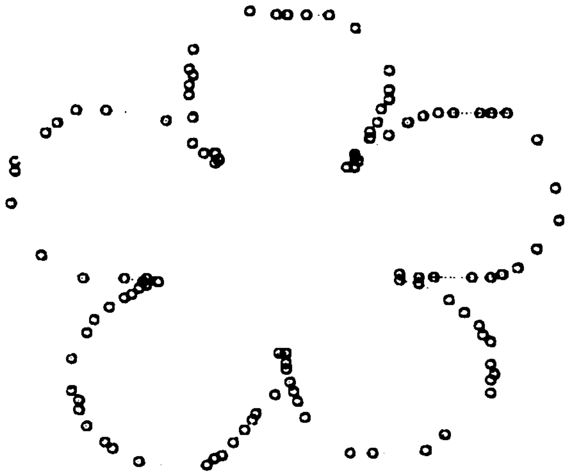


Figure 6.50: Fitted curve with corner and break points, threshold=1 (case 2).

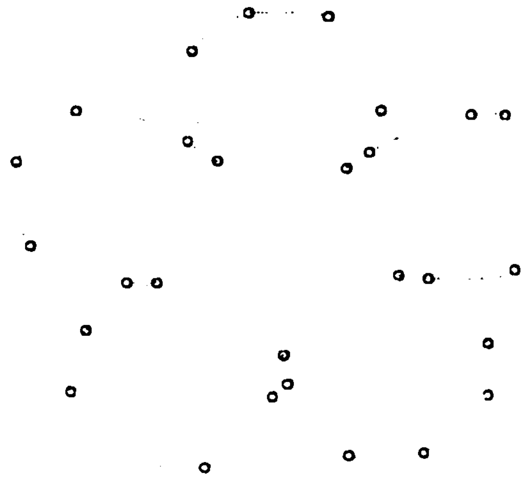


Figure 6.51: Fitted curve with corner and break points, threshold=3 (case 3).

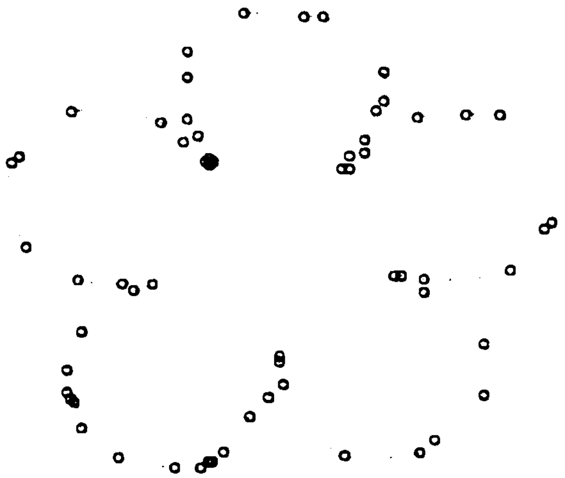


Figure 6.52: Fitted curve with corner and break points, threshold=1 (case 3).

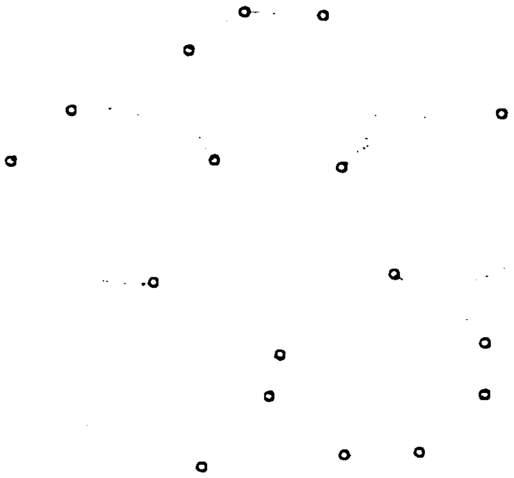


Figure 6.53: Fitted curve with corner and break points, threshold=3 (case 4).

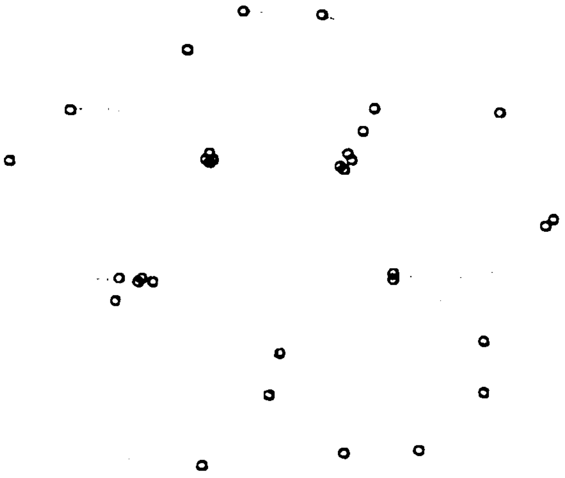


Figure 6.54: Fitted curve with corner and break points, threshold=1 (case 4).

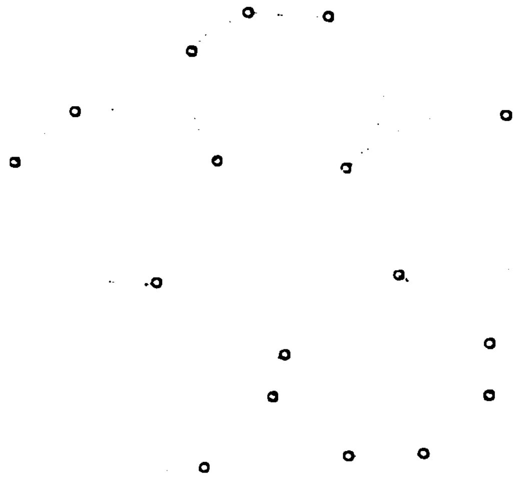


Figure 6.55: Fitted curve with corner and break points, threshold=3 (case 5).

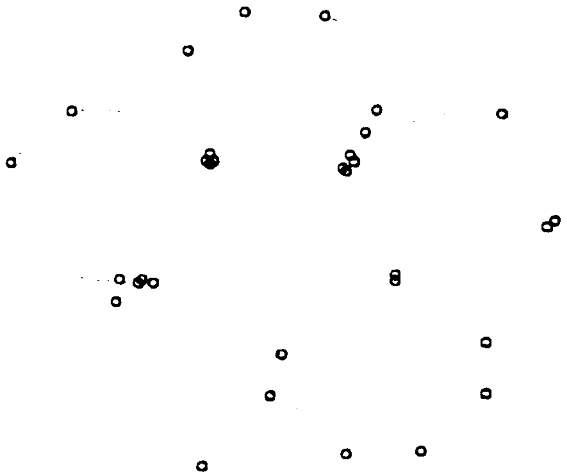


Figure 6.56: Fitted curve with corner and break points, threshold=1 (case 5).

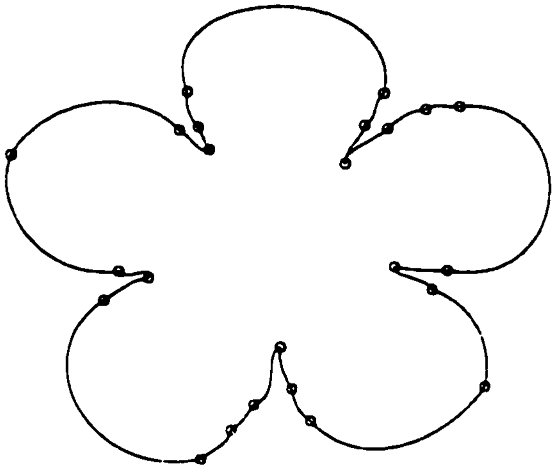


Figure 6.57: Final Outline without filtering and reparameterization, threshold=3

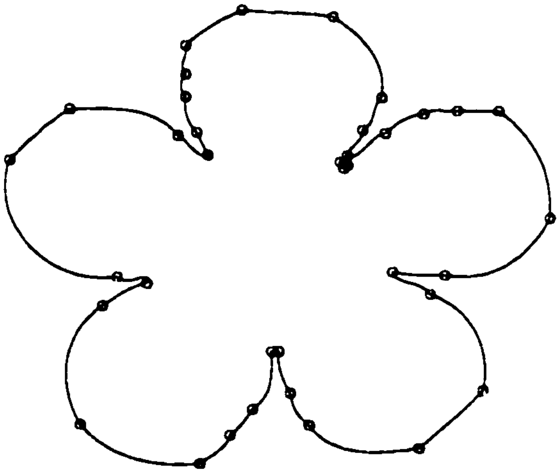


Figure 6.58: Final Outline without filtering and reparameterization, threshold=1

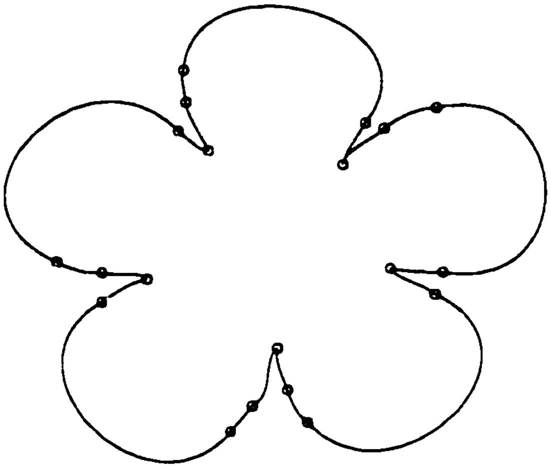


Figure 6.59: Final Outline without reparameterization, threshold=3

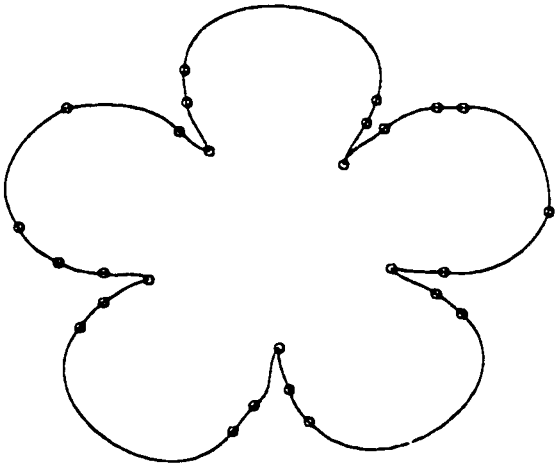


Figure 6.60: Final Outline without reparameterization, threshold=1

Figure #	# of Pieces	# of Boundary Points
6.1	2	1522+116=1638
6.2	3	870+102+67=1039
6.3	3	913+38+55=1006
6.4	1	587

Table 6.1: Statistics of images

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	10+3	55+3	71
case 2	10+3	51+1	65
case 3	10+3	30+1	44
case 4	10+3	5+0	18
case 5	10+3	5+0	18
without filtering and reparameterization [3].	9+3	19+0	31
with filtering and without reparameterization [3].	9+3	13+0	25

Table 6.2: Cases comparison for 'Lillah' image, threshold=3

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	10+3	593+33	639
case 2	10+3	269+21	303
case 3	10+3	110+10	133
case 4	10+3	39+1	53
case 5	10+3	39+1	53
without filtering and reparameterization [3].	9+3	27+2	41
with filtering and without reparameterization [3].	9+3	20+0	32

Table 6.3: Cases comparison for 'Lillah' image, threshold=1

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	25+3+3	23+2+1	57
case 2	25+3+3	13+3+1	48
case 3	25+3+3	12+1+1	45
case 4	25+3+3	3+0+0	34
case 5	25+3+3	3+0+0	34
without filtering and reparameterization [3].	23+3+3	6+1+1	37
with filtering and without reparameterization [3].	23+3+3	5+0+0	34

Table 6.4: Cases comparison for 'Kanji' image, threshold=3

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	25+3+3	235+16+13	295
case 2	25+3+3	126+20+9	186
case 3	25+3+3	60+6+5	102
case 4	25+3+3	19+2+1	53
case 5	25+3+3	19+2+1	53
without filtering and reparameterization [3].	23+3+3	19+3+1	52
with filtering and without reparameterization [3].	23+3+3	9+1+0	39

Table 6.5: Cases comparison for 'Kanji' image, threshold=1

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	28+3+2	20+1+2	56
case 2	28+3+2	16+1+2	52
case 3	28+3+2	7+0+2	42
case 4	28+3+2	1+0+0	34
case 5	28+3+2	1+0+0	34
without filtering and reparameterization [3].	23+3+3	5+0+0	34
with filtering and without reparameterization [3].	23+3+3	1+0+0	30

Table 6.6: Cases comparison for plane image, threshold=3

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	28+3+2	256+7+12	308
case 2	28+3+2	130+5+10	178
case 3	28+3+2	41+5+7	86
case 4	28+3+2	10+0+2	45
case 5	28+3+2	10+0+2	45
without filtering and reparameterization [3].	23+3+3	16+0+1	46
with filtering and without reparameterization [3].	23+3+3	10+0+0	39

Table 6.7: Cases comparison for plane image, threshold=1

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	17	14	31
case 2	17	14	31
case 3	17	11	28
case 4	17	0	17
case 5	17	0	17
without filtering and reparameterization [3].	6	17	23
with filtering and without reparameterization [3].	6	14	20

Table 6.8: Cases comparison for flower image, threshold=3

	# of Corner Points	# of Break Points	Total # of Significant Points
case 1	17	169	186
case 2	17	94	111
case 3	17	46	63
case 4	17	17	34
case 5	17	17	34
without filtering and reparameterization [3].	6	27	33
with filtering and without reparameterization [3].	6	19	25

Table 6.9: Cases comparison for flower image, threshold=1

S.No.	D_x	D_y
1	-18.4626	23.708
2	-11.136	1.68873
3	-17.5601	-15.9436
4	-40.3653	-10.2558
5	-26.3287	9.7598
6	-6.22969	17.0346
7	10.5441	26.4559
8	20.0743	33.0743
9	33.1195	29.1315
10	10.2893	17.657
11	2.5856	20.1547
12	5.36947	16.6947
13	4.43607	11.3082
14	4.39117	-0.17051
15	5	-16.9459
16	5	-18
17	4.43801	-15.752
18	3.43917	-12.3175
19	4.85981	-16.8119
20	9.15029	-27.368
21	17.3024	-17.3488
22	34.8027	2.60756
23	10.5154	25.5188
24	-6.19647	21.5894
25	-5	16.9459
26	-6.42278	20.624
27	-2.51514	35.0796
28	8.3969	25.1418
29	5.34826	-0.6501
30	3.49441	-12
31	5.30075	-16.2274
32	5.30075	-16.2274
33	4.77664	-14.7182
34	9.2307	-27.6152
35	13.2426	-31.9706
36	20.6231	-17.5608
37	15.0852	2.86204
38	3.68625	10.6788
39	-4.90575	17.3586
40	-6	19
41	-5.45691	17.3707
42	-7.14477	22.1279
43	-5.22732	40.5151
44	7.59246	30.2544
45	4.03533	-0.367517
46	3.02831	-10.3994
47	5.83333	-17.5
48	4.2	-12.6
49	3.72281	-11.5298
50	5.46907	-16.9381
51	6.46154	-19.3846
52	7	-21.4895
53	6.49691	-22
54	3.33781	-20.9351
55	-1.51922	-27.2428
56	-8.67459	-37.418
57	-14.6466	-13.7498
58	-21.7833	-2.52822
59	-21.9396	4.34021
60	-15	10
61	-11.2677	0.296313
62	-11.3519	-9.45063
63	-12.4499	-7.7998
64	-19.2769	-6.46948
65	-39.0769	10.1442
66	21.3177	3.54277
67	4.02683	11.1973
68	-4.45971	12.9194
69	-12.7816	4.77737
70	-11.6855	-12.7333
71	4.86052	-9.87308

Table 6.10: Tangent values for 'Lillah' image (case 1).

S.No.	v	D_x	D_y
1	0.6215	-18.4626	23.708
2	-0.0215	-10.5883	1.5869
3	0.6348	-16.7192	-15.3111
4	1.2517	-40.4148	-10.5823
5	0.6829	-29.3449	10.1612
6	0.5819	-5.8676	12.2507
7	0.1193	3.0996	11.3506
8	0.2336	6.5598	14.6076
9	1.2824	21.1196	35.4586
10	0.6944	35	32.6219
11	0.5306	13.9211	23.4799
12	1.0006	4.48065	24.5028
13	0.5178	6.7795	18.5075
14	0.1195	5.5092	-0.2139
15	0.8773	5	-16.9459
16	0.4289	4.438	-15.752
17	0.4301	5.3046	-18.8922
18	1.1503	10.0054	-34.6151
19	0.63	7.8516	-22.4062
20	0.605	9.8742	-12.5786
21	0.7995	19.5667	-5.1755
22	0.9922	29	10.3087
23	0.4925	8.2359	21.373
24	0.8424	-6	20.824
25	0.7628	-4.8114	15.4342
26	0.2431	-3.4343	11.3028
27	0.3659	-4.4228	14.2684
28	1.6271	-2.3188	32.8116
29	0.574	8	22.3402
30	-0.0938	5.9621	-0.775
31	0.7834	4	-13.3469
32	0.2263	3.4286	-10.2857
33	0.15	4.165	-12.7862
34	1.7594	10.8936	-33.194
35	0.6197	14.3074	-36.8516
36	0.9972	16.2514	-18.6984
37	1.3467	22.4909	-3.0607
38	0.3504	5.005	8.4612
39	-0.0306	-2	7.8825
40	0.3436	-3.1987	10.5961
41	0.9111	-7	22
42	0.8005	-5.8413	18.5239
43	0.2653	-3.7676	12.3027
44	0.4414	-4.5	15
45	1.3859	-1.9979	35.7664
46	0.5827	7.2593	21.7778
47	-0.1619	5.4458	-0.5061
48	0.7949	4	-12.9298
49	0.1649	3.4286	-10.2857
50	0.2848	4.5	-13.5
51	0.5607	10.6143	-32.2465
52	1.4101	13.7339	-51.3747
53	0.6323	1.7479	-52.6994
54	0.6752	-3.7569	-21.065
55	1.2465	-4.6569	-16.0711
56	0.5839	-12.3691	-10.9463
57	1.0177	-23.6061	-2.1126
58	0.6505	-28.8414	11.1586
59	0.3082	-22.4879	-0.053
60	1.2346	-34.0254	-17.2773
61	1.2736	-43.8943	9.8772
62	0.3278	9.323	17.5432
63	1.7848	-19.3373	5.6945
64	0.2738	2.0688	-11.3085
65	1.6773	20.9085	-0.4421

Table 6.11: Tangent & shape parameter (v) values for 'Lillah' image (case 2).

S. No.	v	w	D_x	D_y
1	0.698353	1.94471	-18.4626	23.708
2	0.672609	-0.17392	-8.68222	1.23336
3	1.59595	1.11002	-11.6464	-12.0387
4	0.640459	0.326515	-15.4404	-11.1836
5	2.63962	1.75417	-22.9357	-2.8334
6	0.806852	0.317219	-15.1679	28.0444
7	1.81093	1.86182	21.5606	43.3943
8	0.639846	1.16362	35.3525	32.7757
9	0.628886	0.380151	13.1942	21.6361
10	1.00067	0.99976	4.48065	24.5028
11	0.5152	0.672943	6.77945	18.5075
12	1.36447	0.0606299	5.50923	-0.213925
13	2.17796	0.747035	7.42731	-25.0473
14	1.17353	2.13096	23.6761	-63.3968
15	0.859512	0.661014	34.6925	-18.6769
16	0.757827	1.68284	28.3663	11.6337
17	0.507297	-0.00703357	4.57112	11.8399
18	1.69266	0.518342	-3.63601	14.3627
19	1.43485	1.1422	-10.921	35.0876
20	0.887761	0.915105	-4.0551	42.2879
21	0.572011	2.70994	8.41289	30.0355
22	0.441366	-0.140856	5.71489	-0.537142
23	2.0803	0.54379	5.27257	-18.3632
24	1.6231	2.51829	21.2651	-57.8633
25	0.576594	1.27645	31.1573	-25.128
26	0.784132	0.865922	14.4369	3.56614
27	0.804303	-0.0134002	2.91797	8.10582
28	2.04918	0.486117	-3.63038	13.6582
29	2.29478	2.12559	-11.9303	45.4546
30	0.398861	2.2157	3.9481	51.5865
31	0.2368	-0.121551	6.57143	-0.285714
32	2.06102	0.621343	6.5835	-21.2808
33	1.45786	1.2557	19.5846	-60.7185
34	0.838425	1.10478	7.71793	-71.092
35	0.717735	0.775623	-8.57239	-46.092
36	1.45963	1.38163	-30.6683	-22.6587
37	0.620531	0.674384	-27.8993	11.298
38	0.798676	0.321786	-18.9658	0.594818
39	1.41071	1.16926	-34.0254	-17.2773
40	1.24825	1.56227	-43.8943	9.87716
41	0.445517	0.181201	10.153	17.7294
42	1.60753	2.83696	-19.3373	5.69454
43	1.03551	-0.00894506	1.43137	-8.98366
44	2.15041	2.25427	16.9267	-1.2836

Table 6.12: Tangent & shape parameters (v,w) values for 'Lillah' image (case 3).

S. No.	D_{x1}	D_{y1}	D_{x2}	D_{y2}
1	-9.18167	10.0901	-11.7315	-3.20473
2	-72.0141	-136.029	-93.2907	75.7112
3	37.0497	145.831	142.641	61.2406
4	-6.2534	71.347	24.1931	50.7751
5	63.782	-135.085	113.954	-100.745
6	38.3418	-9.91867	25.6115	13.0955
7	21.522	11.397	10.201	19.6185
8	-33.0239	141.746	105.744	129.587
9	56.6044	-156.22	84.6548	-139.624
10	13.8382	-9.7964	11.3133	-3.91049
11	26.4554	-1.19687	8.45233	23.6315
12	-33.0393	165.876	109.348	157.321
13	66.7496	-234.117	-123.845	-243.59
14	-74.4781	-30.8481	-44.9859	68.1507
15	-83.801	-114.652	-34.7117	164.072
16	-8.69017	25.2321	-11.9742	18.9057
17	-40.6759	-14.4403	-4.89211	-43.6354
18	40.8642	-24.2447	30.2858	40.6132

Table 6.13: Tangent values for 'Lillah' image (case 4).

S. No.	v	w	D_{x1}	D_{y1}	D_{x2}	D_{y2}
1	1	1	-9.18167	10.0901	-11.7315	-3.20473
2	1	1	-72.0141	-136.029	-93.2907	75.7112
3	1	1	37.0497	145.831	142.641	61.2406
4	1	1	-6.2534	71.347	24.1931	50.7751
5	1	1	63.782	-135.085	113.954	-100.745
6	1	1	38.3418	-9.91867	25.6115	13.0955
7	1	1	21.522	11.397	10.201	19.6185
8	1	1	-33.0239	141.746	105.744	129.587
9	1	1	56.6044	-156.22	84.6548	-139.624
10	1	1	13.8382	-9.7964	11.3133	-3.91049
11	1	1	26.4554	-1.19687	8.45233	23.6315
12	1	1	-33.0393	165.876	109.348	157.321
13	1	1	66.7496	-234.117	-123.845	-243.59
14	1	1	-74.4781	-30.8481	-44.9859	68.1507
15	1	1	-83.801	-114.652	-34.7117	164.072
16	1	1	-8.69017	25.2321	-11.9742	18.9057
17	1	1	-40.6759	-14.4403	-4.89211	-43.6354
18	1	1	40.8642	-24.2447	30.2858	40.6132

Table 6.14: Tangent & shape parameters (v,w) values for 'Lillah' image (case 5).

6.1 Comparison with Previous Work

The work presented in this thesis is motivated by [3] and is an alternate approach. In addition to some small variations, the overall methodology mainly differs at a major step of curve fitting technique. A comparative study with one of the image is presented in the next paragraph.

The mathematical model of traditional Bézier was used in [3] whereas a generalized form of Hermite cubic is used in this thesis. Along with the shape parameters, it is believed that one can have a more control over the shape of the curve. Also there exists a C^1 continuity in fitting the curve piecewisely which was not present in [3]. It is believed that this will be quite useful especially in the case of smooth curves. Another difference lies in breaking the segment when the error between the digitized and fitted curve is larger than some threshold value. In [3], distance formula is used to calculate the error whereas the x and y pixel differences in both the horizontal and vertical directions is used in this thesis. Although in a high computing environment it does not make much difference but calculating the square values has been saved. Also no filtering and reparameterization is done to get the optimal curve fit. As a comparison for the 'Lillah' (Figure 6.1) image, there were 30 total number of significant points without filtering and 25 with filtering when the threshold value was 3. In the best case here it is fitted with 18 total number of significant points when the threshold value is 3. The detailed comparison for this figure along with three more images can be seen in Tables 6.2 to 6.9. Moreover the application has been deployed over the Web and potential users can test their images with ease and varying some parameters.

Chapter 7

Conclusion and Future Work

An algorithm for the approximation of boundary of digital images has been presented which can be particularly used in font designing method. In addition to the detection of corner points, a strategy to detect a set of significant points is also explained to optimize the outline. A more flexible class of cubic functions is used to fit the curve. It eliminates the human interaction in obtaining the outline of digitized image. Because of a modular design, the system is flexible and adaptable for future enhancements. Such a work is still in progress and some more elegant results can be achieved by using different parametric technique, segment subdivision and cubic model. A World Wide Web based interface has been created for automatic outline capturing of images. An easy-to-use interface is developed whereby users can test the cases discussed in this thesis. In addition to the images available, users can upload their own images. It is a simple interface and more features will be added in the future.

7.1 Future Work

- **Web Application Enhancement**: Several enhancements to the described system will be feasible within short. In addition to the uploading of images a user can be given a free hand to draw any kind of shape, save it and then submit to test the algorithm. Also a user can vary more parameters such as d_{min}, d_{max} and α_{max} of corner detection algorithms and can select edge detection method from various available methods.

- **Distributed/Parallel processing**: The nature of the system makes it suitable for distributed/parallel processing. It can be applied to each piece of the curve and corner detection, breaking the segments,fitting curve to segments can be done separately for each piece.

- **Going for three dimensional and Open curves**: It will be interesting to investigate and extend the method so that it can work for 3D images. Currently the method is applicable for only closed curves. It can be changed with slight ease to tackle with the Open curves and it might be necessary when the Web application is enhanced, so that user can draw any type of image.

- **Higher Order or Rational cubic curves**: The generalized Hermite cubic is used in this thesis. It is possible to use curves of degree four or five. Although using higher order curves will be expensive in terms of computation, but it can be worth to investigate if higher order curves can reduce the number of segments required to produce the outline. Also a curve can be fitted using rational cubics.

- **Colored Images**: Another possibility is to test the colored images. The main change might occur in detecting the edges and an additional field of color must be saved in the file.

Appendix A

Extracted Boundary File

A.1 Lillah.txt

The original data file consist of three columns but to save space it is depicted here in twelve columns. The first column corresponds to the x coordinate of the image and the second column corresponds to the y coordinate of the image. The third column is the index of these x and y coordinates. The same sequence is repeated in the next nine columns.

```
2       -----   Total number of segments
1522    -----   Number of data points in first segment
116     -----   Number of data points in second segment
```

164	25	1	163	25	2	162	25	3	161	25	4
160	26	5	159	26	6	158	26	7	157	27	8
156	27	9	155	27	10	154	28	11	153	28	12
152	29	13	151	29	14	150	30	15	149	31	16
148	31	17	147	32	18	146	33	19	145	34	20
144	35	21	143	36	22	142	37	23	141	38	24
140	39	25	139	40	26	138	41	27	138	42	28
137	43	29	136	44	30	136	45	31	135	46	32
135	47	33	134	48	34	133	49	35	133	50	36
132	51	37	132	52	38	131	53	39	131	54	40
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124	71	57	124	72	58	124	73	59	123	74	60
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118	71	65	117	70	66	116	69	67	115	68	68
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