Charge Dependent Effects
On Nuclear Forces

by

Al-Aithan Thamer Abdul-Muhsen

A Thesis Presented to the
FACULTY OF THE COLLEGE OF GRADUATE STUDIES
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

NUCLEAR PHYSICS

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Charge dependent effects on nuclear forces

Abdul-Muhsen, Al-Aithan Thamer, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1987
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AL-AITAN THAMIR ABDUL-MUHSEN

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This thesis, written by THAMER A. AL-AITHAN
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MASTERS OF SCIENCES IN PHYSICS

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Date July 26, 1987
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Dedicated to my family
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إن دراسة عدم الاعتماد على الشحنة وتناول الشحنة عن طريق
بعد التشذب عندما تكون قوة الدفع الراوي لتفاعل وحدات القوة
النوية (النيترن والبروتون) مفروشة فإنها تعطي دالة على تأثير
الشحنة على القوة النووية. لقد تم في هذا البحث إعتبار جميع
المساهمات المحتملة المعروفة لتفسير الفروقات التجريبية من
جهة أخرى تم تقسيم هذه المشكلة إلى جزئين، الأول هو التأثير
الكهربائي المباشر والثاني التأثير الكهرومغناطيسي غير المباشر. تم التركيز في هذا البحث على التأثير في حالة تناول
الشحنة والنتائج من فرق الكتلة بين الكوارد العالي والكوارد
الواطي والذي يجب اختتام جزيء بابون مع جزيء أثير من طريق الكهرومغناطيسي. ولقد حسب هذا التأثير من خلال حساب
تأثير الكهرومغناطيسي على شحات الدمج لوحدات القوة
النوية عند تبادل جزيء البابون ومن ثم مايجب من تأثير على
بعد التشذب.

لقد تم التوصل إلى أهمية مساهمة تبادل جزيئين في القوة
النوية وكذلك تحديد شحات دمج الوحدة النووية مع جزيء كما
لتكون مفيدة جدا ومحددة بالقيمة التجريبية لتناول الشحنة.
The study of charge independence and charge symmetry on the $1s_0$ N-N scattering lengths provides a test of charge dependent effects of nuclear forces. All possible known contributions to the scattering lengths are considered to account for the experimental discrepancies. This problem was broken into two parts, the first considering the direct electromagnetic and the second the indirect electromagnetic effects. The effect on the charge symmetry breaking correction due to the isospin breaking part of the QCD Hamiltonian, determined by the up-down current quark mass difference, on the difference of scattering lengths for charge symmetry is determined. This is done by first calculating the effect of QCD on the pion-nucleon coupling constants via isospin mixing, giving rise to CSB one pion exchange (OPEP) and two pion exchange (TPEP) potentials. The importance of CSB-TPEP is emphasized. The eta-nucleon coupling constant appears to be constrained by experimental values of CS to be very small.
CHAPTER ONE

INTRODUCTION

The strong nuclear force is one of the three forces in nature, the other two being the gravitational and electroweak forces. For a long time it has been realized that the strong nuclear force, as represented by the nucleon-nucleon (N-N) interaction, is charge independent. But its charge independence is broken by certain small effects which violate the isospin symmetry of this interaction. In this thesis, the problem of the charge dependence of the nuclear force is studied in some detail.

The problem of the charge independence (CI) and charge symmetry (CS) of nuclear forces has a distinguished history in modern physics. CI and CS can be expressed as isospin invariance of the strong nuclear force. Isospin symmetry is one of the first internal symmetries which are independent of space-time symmetries. This problem, at the same time, provides an interplay between nuclear physics and particle physics
(ref.9). The explanation of discrepancies between observed parameters "which are indicators of the strength of the nuclear force" and those predicted by the CI and CS of nuclear forces in terms of effects which break the isospin symmetry of nuclear forces continues to be studied in the literature. In the past it was attempted to solve the problems by considering QED, but later QCD was also considered to explain the small observed charge dependence of nuclear forces. The isospin symmetry of the strong interaction between nucleons may be broken by electromagnetic forces either directly or indirectly or by effects which do not appear to be of electromagnetic origin but nevertheless break isospin symmetry, e.g. the up-down quark mass difference in QCD. The breaking of CI and CS are useful guides in understanding the symmetry breaking of the nuclear, or strong, interaction.

As is well known the $^1S_0$ pp, nn and pn scattering lengths provide a test of the charge symmetry (CS) and charge independence (CI) of nuclear forces. CS, implying vanishing of the difference $\delta a_{\text{CSB}} = (a_{\text{pp}} - a_{\text{nn}})$, holds within the experimental error ($\delta a_{\text{CSB}} = 0.55 \pm 0.46 \text{ fm}$). CI, which would imply
the vanishing of \( \Delta a = \left[ \left( \frac{a_{pp} + a_{nn}}{2} \right) - a_{np} \right] \), is
broken by the measured\(^1\) \( \Delta a = (5.5 \pm 0.3 \text{ fm}) \).

The understanding of \( \delta a \) and \( \Delta a \) in terms of charge dependent effects has a long history (ref.2). The study of charge independence breaking (CIB) and charge symmetry breaking (CSB) effects have been reconsidered (ref.3) more carefully with the progress and development of particle physics. CIB, as exemplified by the above value of \( \Delta a \), is almost solved by taking into account the \( \pi^\pm - \pi^0 \) mass difference and electromagnetic (e.m.) corrections to the pion-nucleon coupling constants in the one pion exchange potential (OPEP) and the two pion exchange potential (TPEP), and by the photon-pion exchange potential. The other possible contributions to CIB, such as short range effects due to six quark contributions, exchange of heavy \( I = 1 \) mesons etc., have been found to be very small.

The situation with respect to CSB is not so clear. Apart from many small effects such as \( \rho^0 - \omega^0 \) particle mixing, \( \pi^0 - \eta, \eta' \) particle mixings, and \( \gamma \pi \) boson exchange, an important contribution to CSB arises from electromagnetic corrections to the \( \pi^0 pp \) and \( \pi^0 nn \) coupling constants.
In this thesis, we will concentrate on charge symmetry breaking (CSB) due to QCD, its effect on the coupling constants, and also on the scattering lengths. These effects are due to the up-down current (not constituent) quark mass difference,

\[ H_{\text{QCD}} = \frac{1}{2} (m_u - m_d)(\bar{u}u - \bar{d}d). \]

Many authors (refs. 2-4) found that the one boson exchange potential, OBEP, due to QCD gives a very small contribution to the scattering length. On the other hand, the two-pion exchange potential TPEP-QCD contributions were not considered and some argued that they are small compared to OBEP (ref. 5). However, in our work we will show the importance of TPEP-QCD.

The approach we wish to follow is to estimate the effect of \( \Delta m_q \) on the pion-nucleon coupling constants, where \( \Delta m_q \) appears as the current quark mass difference in the basic Lagrangian or Hamiltonian:

\[ H = H_{\text{QED}} + H_{\Delta m_q}. \]  

Here \( H_{\text{QED}} \) is the usual electromagnetic current-current Hamiltonian due to an internal photon loop while

\[ H_{\Delta m_q} = \frac{1}{2} \Delta m_q (\bar{u}u - \bar{d}d). \]
The effect of $H_{\text{QED}}$ on the pion-nucleon coupling constants has previously been calculated by various authors (see ref. 2 for a review). What about the effect of $\Delta m_q$ on the pion-nucleon coupling constants? One way such an effect can be taken into account is by observing that $H_{\Delta m_q}$ can give rise to $\eta - \pi^0$ mixing.

The effects due to such a mixing on the one meson exchange potential and that of the latter on the scattering lengths have been calculated in ref. 7. However, if one looks on the third and fourth lines of table 4.1, one observes the important fact that there is an almost complete cancellation between $\Delta a_{\text{OPEP}}^{\text{cc}}$ and $\Delta a_{\text{TPEP}}^{\text{cc}}$, i.e. to say when one considers only the effect of $H_{\text{QED}}$ on the $\pi$-$N$ coupling constants there is an almost complete cancellation from contributions arising from the one pion exchange and two pion exchange potentials. On the other hand, the calculation of ref. 7 in effect takes into account modifications due to $\eta - \pi^0$ mixing in OPEP only. In view of the above remarks it is important that the effect of $\Delta m_q$ on the $\eta - \pi^0$ mixing should also be studied in TPEP. Thus it should be more consistent to add the effect of (1.2) on the
π-N coupling constants that of $H_{\text{QED}}$ (which has previously been calculated) and then study the combined effect of (1.1) on $\Lambda^{\text{OPEP}}_{cc}$ and $\Lambda^{\text{TPEP}}_{cc}$ in order to see whether the cancellation between the two still persists.

For a complete study of this problem, we will firstly review the theory. In chapter two, the definition of isospin, scattering length theory, and effective range theory are considered in some detail. In chapter three we will review the experimental observations of scattering lengths and effective ranges for $^1S_0$ pp, nn and np scattering. In chapter four we will consider the charge dependent effects which has been previously calculated. In chapter five we will consider computation of various effects. Finally we will consider corrections due to QCD and then present our conclusions and discuss our results.
CHAPTER TWO

THEORETICAL REVIEW

In this chapter the theory and definitions which are needed to explain CI (charge independence) and CS (charge symmetry) and their breaking are explained in some detail. The relation between the scattering length and the nucleon-nucleon potential is also derived. Contributions from different charge dependent effects leading to CIB and CSB will also be considered.
2.1-ISO SPIN

Isospin is introduced to incorporate the CI of nuclear forces. It is considered to be a quantum number in an internal space called isospin space. The neutron state \( |n> \) and proton state \( |p> \) are regarded to be two different states of the nucleon, so that

\[
X_p = X_+ = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = |p>
\tag{2.1}
\]

\[
X_n = X_- = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = |n>
\tag{2.2}
\]

define the fundamental (doublet) representation of the SU\(_1\) (2) group generated by the isospin operator \( I = (1/2)\mathbf{I} \)

where \( I=(I_1,I_2,I_3) \) is a vector in isospin space and \( \mathbf{I} \) is the usual Pauli isospin matrix with components
\[ \begin{align*}
\tau_1 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \\
\tau_2 &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \\
\tau_3 &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}
\end{align*} \quad (2.3) \]

These matrices obey the following commutation relations

\[ [\tau_i, \tau_j] = \tau_i \tau_j - \tau_j \tau_i = 2i\varepsilon_{ijk}\tau_k \quad (2.4) \]

Where

\[ \varepsilon_{ijk} = \begin{cases} +1 & \text{even permutation of indices} \\ -1 & \text{odd permutation of indices} \\ 0 & \text{if any two of indices are equal} \end{cases} \quad (2.5) \]

In other words I behaves similarly to the spin angular momentum so that we can use the eigenvalues of \( I^2 \) and \( I_3 \), where

\[ I^2 = I_1^2 + I_2^2 + I_3^2 \quad (2.6) \]

to label the states. Thus for the state \( I^3 \), called the isospin wave function,

\[ I^2 I^3 = I(I+1) I^3 \quad (2.7) \]
\( I_3 X_3 = I_3 X_3 \)  \(  \)  \( (2.8) \)

\( \) 

I and \( I_3 \) are the eigenvalues of the isospin magnitude of \( I^2 \) and 3-rd component of \( I \) respectively. The eigenvalues of \( I^2 \) and \( I_3 \) for a nucleon are 

\( I = \frac{1}{2} \), \( I_3 = \pm \frac{1}{2} \)

i.e.

\( I^2 X_N = \frac{1}{2} \left( \frac{1}{2} + 1 \right) X_N \), \( N = n, p \)  \(  \)  \( (2.9) \)

\( I_3 X_n = -\left( \frac{1}{2} \right) X_n \)  \(  \)  \( (2.10) \)

\( I_3 X_p = \left( \frac{1}{2} \right) X_p \)  \(  \)  \( (2.11) \)

Note that the choice of sign of the eigenvalues of the 3-rd component of \( I \) may be used reversely i.e.

\( I_3 X_n = \left( \frac{1}{2} \right) X_n \)  \(  \)  \( (2.12) \)

\( I_3 X_p = -\left( \frac{1}{2} \right) X_p \)
From Eqs(2.10), and (2.11) by definition, $I_3$ distinguishes between the neutron and proton. The electric charge operator $Q$ is related to $I_3$ by,

$$Q = e \left( \frac{1}{2} + I_3 \right),$$  \hspace{1cm} (2.13)

where $e$ is the charge of the proton. This implies

$$Q \left| p > \right> = e \left( \frac{1}{2} + 1/2 \right) \left| p > \right> = e \left| p > \right>$$ \hspace{1cm} (2.14)

$$Q \left| n > \right> = e \left( \frac{1}{2} - 1/2 \right) \left| n > \right> = 0$$

which gives the correct charge for each nucleon type.

The state of a system of nucleons is labelled by $I$ and $I_3$, as well as other quantum numbers, i.e. it is expressed as

$$\left| \alpha, I, I_3 > \right>$$ \hspace{1cm} (2.15)

so that the neutron and proton states are represented by

$$\left| p > \right> = \left| \alpha, 1/2, 1/2 > \right>$$ \hspace{1cm} (2.16)

$$\left| n > \right> = \left| \alpha, 1/2, -1/2 > \right>,$$

where $\alpha$ stands for all the other quantum numbers which are necessary to distinguish one state from others. Similar to the angular momentum vector, $I_3$ can take $(2I+1)$ values given by:-

-I, -I+1, ...., I-1, I.

Out of the components $I_i$ $(i=1,2,3)$ we can also form
raising and lowering operators:

\[ I_\pm = I_1 \pm iI_2, \quad (2.17) \]

which for the case of the nucleon take the form

\[ I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad (2.18) \]

\[ I_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \]

The effect of these operators on the nucleon states, defined by Eq (2.12), is given by

\[ I_+ X_p = X_n \]

\[ I_- X_p = 0 \quad (2.19) \]

\[ I_+ X_n = 0 \]

\[ I_- X_n = X_p \]

These operators change the neutron to proton or vice versa.
2.2-ISOSPIN INVARIANCE AND ITS BREAKING

Charge independence in a hadronic interaction means that the force between nucleons is independent of their electric charge. In other words the hadronic interaction is independent of the direction in isospin space. This leads to the fact that the hadronic interaction is invariant under rotations in isospin space, in complete analogy with ordinary angular momentum.

The unitary transformation corresponding to a rotation in isospin space can be expressed by the following unitary operator:

\[ U_I(\alpha) = \exp(iI.\alpha), \quad (2.20) \]

where \( \alpha \) is the angle of rotation, \( I \) is the isospin operator. Isospin invariance means;

\[ [H_h, U_I(\alpha)] = 0, \quad (2.21) \]

where \( H_h \) is the hadronic interaction Hamiltonian, responsible for example for the N-N interaction. For an infinitesimal rotation

\[ U_I(\alpha) = 1 + i I . \alpha \quad (2.22) \]
\[ [H_h, I] = 0 \] \hspace{1cm} (2.23)

Thus \( I \) is a conserved quantum number in hadronic interactions, and so is \( I_3 \), i.e.

\[
\Delta I^2 = I_f^2 - I_i^2 = 0 \hspace{1cm} (2.24)
\]
\[
\Delta I_3 = I_{3f} - I_{3i} = 0,
\]

where \( f \) stands for the final state and \( i \) for the initial state.

Including the electromagnetic interaction we can write the complete Hamiltonian in the form

\[ H = H_h + H_{em} \hspace{1cm} (2.25) \]

where \( H_h \) is invariant under an isospin rotation, but \( H_{em} \) breaks the isospin invariance i.e.

\[ [H, I] = [H_{em}, I] \neq 0, \hspace{1cm} (2.26) \]

since as is clear from eqn.(2.3), \( H_{em} \) involving electric charge has selected a particular direction in isospin space. For example, the Coulomb potential between two nucleons which can be written as:
\[ v_{ij}^c = \frac{e^2}{r} \left( \frac{1}{2} + I_{13}^i \right) \left( \frac{1}{2} + I_{13}^j \right) \]  

(2.27)

breaks isospin symmetry, i.e.

\[
[ V_c^i, I ] \neq 0,
\]

(2.28)

for the p-p interaction, but note that \( I_3 \) is still conserved. Another example is the mass Hamiltonian, \( H_m \), if we take \( m_n \neq m_p \). The Kinetic energy term is

\[
\text{K.E.} = - \frac{A}{\Xi} \sum_{i=1}^{V_i} \left[ \frac{B_{i}^+(i)}{2m_n} + \frac{B_{i}^-(i)}{2m_p} \right]
\]

(2.29)

where K.E. stands for kinetic energy, \( A \) is the number of nucleons, \( m_p \) is the mass of the proton, \( m_n \) is the mass of the neutron, and \( B_{\pm} = \frac{1}{2} (1 \mp I_3) \) are projection operators with the properties:

- \( B_+ \) projects out neutron
- \( B_- \) projects out proton

Then by rearranging eqn.(2.29) we get

\[
\text{K.E.} = - \frac{A}{\Xi} \sum_{i=1}^{V_i} \left[ \frac{1}{2} - I_{13}^i \right] + \left[ \frac{1}{2} + I_{13}^i \right]
\]

(2.30)
and thus

$$K.E. = - \sum_{i=1}^{A} \frac{v_i^2}{2M} \left[ 1 - \frac{\Delta M}{M} I_3^i \right] + O\left( \frac{\Delta M}{M} \right)^2, \quad (2.31)$$

where $M = 1/2 \ (m_n + m_p)$ and $\Delta M = (m_n - m_p)$. Thus it is clear from eqn. (2.31) that K.E. breaks isospin symmetry since $[K.E., I] \neq 0$, because $I_3$ does not commute with $I_1$ or $I_2$. If, however $\Delta M$ is neglected, i.e. $\Delta M = 0$, then it does not break isospin symmetry. From the above two terms i.e. $V_{ij}^c$ and K.E., and previous work, we can see that $I_3$ is always conserved but not $I^2$, and isospin symmetry is broken by the e.m. force and the small mass difference between $n$ and $p$, and $\Delta M$ may arise due to e.m. interactions between nucleons.

We now discuss the effect of CI on isospin invariance for the two nucleon system. Since each nucleon has $I = 1/2$, the two nucleon system can be in $I = 0$ or $I = 1$ states:

$$I = 0 : \left\{ \frac{1}{\sqrt{2}} \right\} \{np> - |pn> \}, \quad I_3 = 0 \quad (2.32)$$

$$I = 1 : \quad |pp> \quad I_3 = +1.$$
\[
\left( \frac{1}{\sqrt{2}} \right) \{ |np> + |pn> \} \quad I_3=0 \quad (2.33)
\]

\[
|nn> \quad I_3=-1.
\]

where eqn.(2.32) is antisymmetric under interchange of the two nucleons and eqn.(2.33) is symmetric under interchange of the two nucleons.

Thus if we consider the two nucleon system in a state of relative orbital angular momentum \( L=0 \) (so that the spatial wave function is symmetric), the generalized Pauli principle (namely, that the total wave function involving isospin, spin and space parts must be antisymmetric for a two fermion system) requires that the \( I=0 \) two nucleon state must be in a spin triplet \( S=1 \) symmetric state, and the \( I=1 \) two nucleon state must be in a spin singlet \( S=0 \) antisymmetric state.

Hence isospin invariance or CI requires that the \( ^1S_0 \) N-N interaction must be the same for the three members of the \( I=1 \) triplet listed above, i.e.

\[
\nu_{pp}(^1S_0) = \nu_{nn}(^1S_0) = \nu_{np}(^1S_0) \quad . \quad (2.34)
\]

This can be tested by measuring the \( ^1S_0 \) phase shift \( \delta \) at very low energies as the above relation implies that:
\[ a_{pp} = a_{nn} = a_{np}, \quad (2.35) \]

where the \( l_S^0 \) scattering length \( a \) is defined by

\[ a = - \lim_{k \to 0} \frac{\sin \delta_0}{k} = - \lim_{k \to 0} \frac{\delta_0}{k}, \quad (2.36) \]

More details about the scattering length are given in section 2.5.

We now discuss charge symmetry, CS, which is the operation of reversing the sign of the 3rd component of isospin. The operator performing this reflection is

\[ R = \exp(i \pi I_2) \quad (2.37) \]

where \( I_2 \) is the generator of rotations about the second axis in isospin space. Its effect on states is to transform them into states with the opposite sign of \( I_3 \). Thus if we apply the above CS operator to \( I=1, |NN> \) states, we have
\( R \ket{\text{NN}\, I_3 = +1} = \ket{\text{NN}\, I_3 = -1} \)

(2.38)

\( R \ket{\text{NN}\, I_3 = 0} = \ket{\text{NN}\, I_3 = 0} \)

(2.39)

implying under \( R \)

\( \ket{pp} \leftrightarrow \ket{nn} \)

Hence CS invariance of the \( ^1S_0 \) NN interaction requires that

\[ V_{pp}(^1S_0) = V_{nn}(^1S_0) \]

or in terms of the \( ^1S_0 \) scattering length

\[ a_{pp} = a_{nn} \]

Thus CS invariance is less restrictive than CI invariance.
From $SU_3^I(2)$ symmetry, relations between coupling constants of different N-N interactions can be found.

Consider the Feynman diagram of Fig.1 which gives rise to the N-N interaction through pion exchange:

![Feynman diagram](image)

**Fig.1**

Feynman diagram for exchanged pion.

From Fig.1, at each vertex the $SU_3^I(2)$ invariant interaction is

$$H_{\text{int}} = g \bar{N} \Gamma \cdot \Gamma N,$$

where $g$ is the strength of the vertex and
\[ \bar{N} = (\bar{p} \quad \bar{n}) , \quad N = \begin{bmatrix} [p] \\ [n] \end{bmatrix} . \] (2.41)

Here \( \bar{p}, \bar{n} \) and \((n, p)\) are creation (destruction) operators for nucleon states and \( \pi \), which mediates the nuclear force between nucleons, is an isospin vector representing three pion states \( \pi^+, \pi^0 \) as follows:

\[ \pi^+ |0> = \frac{(\pi_1 \pm i \pi_2)}{\sqrt{2}} > \] (2.42)

\[ \pi^0 |0> = |\pi^0> , \]

where \( |0> \) is the vacuum state, and \( \pi^+, \pi^0 \) are operators which create \( \pi^+ \) and \( \pi^0 \) mesons respectively. Since \( \pi \) is an isospin vector, and the only isospin vector which acts on nucleon states available to us is \( \tau \), we can form an isospin scalar to have \( SU_\tau(2) \) invariance for the pion-nucleon vector as follows:

\[ \tau \cdot \pi = \tau_1 \pi_1 + \tau_2 \pi_2 + \tau_3 \pi_3 \]

giving us the expression (2.40). We can now write

\[ H_{int} = g \bar{N} [\sqrt{2} \tau_+ \pi^+ + \sqrt{2} \tau_- \pi^- + \tau_3 \pi^0 ] N \] (2.43)
$$H_{\text{int}} = g \left[ \sqrt{2} \, p_n \pi^+ + \sqrt{2} \, n_p \pi^- + p_p \pi^0 - n_n \pi^0 \right], \quad (2.44)$$

where the matrix form of the isospin operator $\tau$ is as follows:

$$\tau_+ = \left( \frac{1}{2} \right) (\tau_1 + i \tau_2), \quad \tau_- = \left( \frac{1}{2} \right) (\tau_1 - i \tau_2)$$

$$\tau_+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \tau_- = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (2.45)$$

Then isospin invariance or CI gives \{c.f. Eq.(2.44)\}

$$g_{np \pi^-} = g_{pn \pi^+} = \sqrt{2} \, g_{pp \pi^0} = -\sqrt{2} \, g_{nn \pi^0} \quad (2.46)$$

To find relations between coupling constants of other pseudoscalar $0^-$ mesons one must consider SU(3) symmetry. The matrix elements of an octet operator are given by

$$\langle 8, k | 0_i | 8, j \rangle = i \, f_{ijk} \, F + d_{ijk} \, D \quad (2.47)$$

where $0_i$ is the octet operator, and $|8,j\rangle$ is the baryon octet state. If the octect operator $0_i$ is taken to be $0^-$
pseudoscalar mesons, then the coupling constants are given by

\[ g_{ijk} = 2g \left[ i f_{ijk} F + d_{ijk} D \right], \]  \hspace{1cm} (2.48)

where \( 2g \) is a normalizing factor so that

\[ g_{pp\pi'} = g[F+D] \]  \hspace{1cm} (2.49)

\( F \) and \( D \) are the anti-symmetric and symmetric reduced matrix elements respectively. For the case of

\[ g_{pp\pi'} \], we normalize

\[ g_{pp\pi'} = g, \]  \hspace{1cm} so that

\[ F + D = 1 \]  \hspace{1cm} (2.50)

The \( \frac{D}{F} \) ratio can be determined experimentally.

We are interested to find \( g_{\eta NN} \) and \( g_{\eta' NN} \) for our later work [sec.6.2]. In order to do so one has to use Eq. (2.48), we have to relate the physical states \( |\eta\rangle \) and \( |\eta'\rangle \) to the SU(3) singlet and octet states \( |\eta_1\rangle \) and \( |\eta_8\rangle \) or to \( |\eta_{NS}\rangle \) and \( |\eta_S\rangle \), defined as follows (in terms of \( q \bar{q} \)):
\[ |\eta_{s}\rangle = \left(\frac{1}{\sqrt{6}}\right) |\bar{u}u+\bar{d}d-\bar{s}s\rangle \quad (2.51) \]

\[ |\eta_{1}\rangle = \left(\frac{1}{\sqrt{3}}\right) |\bar{u}u+\bar{d}d+\bar{s}s\rangle \quad (2.52) \]

\[ |\eta_{ns}\rangle = \left(\frac{1}{\sqrt{2}}\right) |\bar{u}u+\bar{d}d\rangle \quad (2.53) \]

\[ |\eta_{s}\rangle = |\bar{s}s\rangle \quad . \quad (2.54) \]

In terms of the above states, \(|\eta\rangle\) and \(|\eta'\rangle\) are given by

\[ |\eta\rangle = \cos \phi \ |\eta_{ns}\rangle - \sin \phi \ |\eta_{n}\rangle \quad (2.55) \]

\[ |\eta'\rangle = \sin \phi \ |\eta_{ns}\rangle + \cos \phi \ |\eta_{n}\rangle \quad (2.56) \]

where \(\phi\) is the mixing angle, and \(|\eta\rangle\) and \(|\eta'\rangle\) are also related to \(|\eta_{1}\rangle\) and \(|\eta_{s}\rangle\) with the mixing angle \(\theta\), as follows:

\[ |\eta\rangle = -\sin \theta \ |\eta_{1}\rangle + \cos \theta \ |\eta_{s}\rangle \quad (2.57) \]

\[ |\eta'\rangle = \cos \theta \ |\eta_{1}\rangle + \sin \theta \ |\eta_{s}\rangle \quad . \quad (2.58) \]

The angles \(\theta\) and \(\phi\) are related by

\[ \phi = \theta + \tan^{-1}\sqrt{2} \quad . \quad (2.59) \]

Note also that \(\tan^{-1}\sqrt{2}\) in (2.59) is called the ideal
mixing angle.

Since p contains no strange quark, the quark line rule implies that it is not coupled to $\eta_s$ so that

$$ g_{\eta pp} = \cos \phi \ g_{\eta ns pp} \quad (2.60) $$

$$ g_{\eta' pp} = \sin \phi \ g_{\eta ns pp} \quad (2.61) $$

Now using Eq. (2.48), we obtain

$$ g_{\eta ns pp} = g_{(3F-D)} = g_{\eta nn} \quad (2.62) $$

so that Eqs. (2.60) and (2.61) give

$$ g_{\eta pp} = g_{(3F-D)} \cos \phi = g_{\eta nn} \quad (2.63) $$

and

$$ g_{\eta' pp} = g_{(3F-D)} \sin \phi = g_{\eta' nn} \quad (2.64) $$

and thus

$$ \frac{g_{\eta' pp}}{g_{\eta pp}} = \tan \phi = \frac{g_{\eta' nn}}{g_{\eta nn}} \quad (2.65) $$
2.4-CHARGE INDEPENDENT ONE PION EXCHANGE POTENTIAL

One can derive the one pion exchange potential OPEP, which is relevant for the long range part of the N-N potential, from considering the following diagram:

![Diagram of one pion exchange potential](image)

Fig. 2

Feynman diagram for exchanged pion with isospin and strength at each vertex.

As is well known, the above diagram gives the N-N potential

\[
V(r) = -\left(\frac{g_{\pi NN}}{2m}\right)^2 \left[\frac{\exp(-\nu r)}{r}\right] \left[-\frac{4}{3} \vec{S}_1 \cdot \vec{S}_2 + \frac{4}{3} [\vec{S}_1 \cdot \vec{S}_2 - 3 \frac{(\vec{S}_1 \cdot \vec{r}) (\vec{S}_2 \cdot \vec{r})}{r^2}]\right] \\
\times \left[1 + \frac{3}{(\nu r)} + \frac{3}{(\nu r)^2}\right].
\]  

(2.66)

In isospin space, using isospin invariance, this
potential can be written as

\[ \bar{V}(r) = \vec{\tau}_1 \cdot \vec{\tau}_2 \, V(r), \]  

(2.67)

where \((1/2) \vec{\tau}_1\) and \((1/2) \vec{\tau}_2\) refer to the isospin of nucleon 1 and nucleon 2.

In Eq.(2.66), \(m\) is the common nucleon mass, \(\vec{s}_1 = (1/2) \vec{d}_1\) is the spin of nucleon 1 and \(\vec{s}_2 = (1/2) \vec{d}_2\) is the spin of nucleon 2. For the two nucleon system I=1 (\(\vec{\tau}_1 \cdot \vec{\tau}_2 = 1\)) or I=0 (\(\vec{\tau}_1 \cdot \vec{\tau}_2 = -3\)). For a system of two nucleons in a state of relative angular momentum \(L = 0\), the generalised Pauli principle requires that I=1, S=0 or I=0, S=1 where \(S\) is the total spin of the two nucleon system. Thus the CI potential for the I=1 two nucleon system, namely pp, nn, and \((1/\sqrt{2}) \, \mid np+pn\rangle\) in the \(1s_0\) state is obtained from Eqs.(2.66) and (2.67)

\[ V(r) = - \left[ \frac{g_r \mu}{2m} \right]^2 \left[ \frac{\exp(-\mu r)}{r} \right] \]  

(2.68)

which is also known as the Yukawa potential. Here \(\mu\) is the pion mass. The strength is given by \(g^2_r = \frac{g^2}{4\pi}\). For convenience we shall often use \(f^2 = (\frac{\mu}{2m})^2 \frac{g^2}{r}\).
2.5 EFFECTIVE RANGE THEORY

For the scattering of two nucleons at very low energies only the S-wave is important. This can be seen easily from the semi-classical approximation

\[ L = Pr , \quad (2.69) \]

where \( L \) is the angular momentum, \( P \) is the linear momentum, and \( r \) is the impact parameter of scattering approximated to be the radius of the nucleon (or the nuclear force range) which is approximately 1.5 fm.

The linear momentum is given by

\[ P = \sqrt{2m_N E_N} , \quad (2.70) \]

where \( E_N \) is the total energy, and \( m_N \) the average nucleon mass. Making the replacement \( L^2 \rightarrow \ell(\ell+1)\hbar^2 \) where \( \ell \) is the angular momentum quantum number, Eq.(2.69) implies that

\[ \frac{1}{2} \left[ \frac{\ell(\ell + 1) \hbar^2}{L} \right] \frac{1}{2m_N E_N} = r = 1.5 \text{ fm} \quad (2.71) \]
For $l = 1$, this gives $E_N = 19$ MeV or $E_{cm}$ (center of mass energy for N-N interaction) = 9 MeV. Thus we can conclude that for very low energy $l = 0$, i.e. only $S$- wave is important.

Then the radial Schrodinger equation for $L=0$ is

\[
\left[ \frac{d^2}{dr^2} + k^2 \right] X(r) = \frac{2\mu}{\hbar^2} V(r) X(r), \quad (2.72)
\]

where

\[ k^2 = \left(2\mu \frac{E^2}{\hbar^2}\right), \quad E \text{ is the center of mass energy of the system, and } \mu = M/2 \text{ is the reduced nucleon mass. The modified radial function is } X(r) = r R(r), \text{ where } R(r) \text{ is the radial wave function of the system. The asymptotic form of } X(r) \text{ i.e. as } r \to \infty, \text{ is given by}
\]

\[ X(r) \approx C \left(\frac{1}{k}\right) \sin{(kr + \delta_0)} , \quad (2.73)\]

where $C$ is a constant, and $\delta_0$ is the $S$-wave phase shift caused by a non-zero potential at small $r$. For $k \to 0$ writing the wave function as $X_0$, we have

\[
\frac{d^2}{dr^2} X_0(r) = \frac{2\mu}{\hbar^2} V(r) X_0(r), \quad (2.74)
\]
where from Eq. (2.73)

\[
X_0(r) \xrightarrow{r \to \infty} C \left( \frac{1}{k} \right) \left[ \sin(kr) \cdot \cos(\delta_0) + \cos(kr) \cdot \sin(\delta_0) \right].
\]  

(2.75)

Now for \( k \to 0 \), \( \delta_0 \to 0 \) [but not \( \frac{\delta_0}{k} \)] and

\[
\sin(kr) \approx kr, \quad \cos(kr) \approx 1.
\]  

(2.76)

Thus

\[
X_0(r) \xrightarrow{r \to \infty} C \left( \frac{1}{k} \right) \left[ kr \cot \delta_0 + \sin \delta_0 \right],
\]  

(2.77)

\[
X_0 \xrightarrow{k \to 0} \left( \frac{\sin \delta_0}{k} \right) \left[ kr \cot \delta_0 + 1 \right].
\]  

(2.78)

Define

\[
a = \lim_{k \to 0} \frac{\sin \delta_0}{k} = \lim_{k \to 0} \frac{\delta_0}{k},
\]  

(2.79)

where \( a \) is called the scattering length. Note that the magnitude of \( a \) depends on the strength of the N-N potential since \( \delta_0 \) depends on the strength of the N-N potential.

Thus finally
\[ X_0(r) = C(-a) \left[ -\frac{r}{a} + 1 \right], \quad r \to \infty \]  

(2.80)

We can choose \( C = -\frac{1}{a} \), so that

\[ X_0(r) \xrightarrow{r \to \infty} \left( 1 - \frac{r}{a} \right) \equiv \phi_0(r) \]  

(2.81)

where now \( \phi_0 \) is the asymptotic solution of (2.74) i.e.

\[ \frac{d^2}{dr^2} \phi_0(r) = 0 \]  

(2.82)

since for \( r \to \infty \), \( V(r) \to 0 \).

Now multiply Eq. (2.72) by \( X_0(r) \) and (2.74) by \( X(r) \) and subtract to get

\[ \left[ X_0 \frac{d^2 X}{dr^2} - X \frac{d^2 X_0}{dr^2} \right] = -k^2 X(r) X_0(r) \]  

(2.83)

i.e.

\[ \frac{d}{dr} \left[ X_0 \frac{dX}{dr} - X \frac{dX_0}{dr} \right] = -k^2 X(r) X_0(r) \]  

Since \( X(0) = X_0(0) = 0 \), integration from 0 to \( r' \) gives
\[
\left[ x \frac{dx}{dr} - X \frac{dx_0}{dr} \right] \bigg|_{r=r'} = -k^2 \int_0^{r'} X(r) X_0(r) \, dr \quad (2.84)
\]

Similarly the same steps can be carried out for the asymptotic solutions \( \phi(r) \) and \( \phi_0(r) \) where

\[
X(r) \underset{r \to \infty}{\longrightarrow} \phi(r) \quad (2.85)
\]

and

\[
\frac{d^2 \phi}{dr^2} + k^2 \phi = 0 \quad (2.86)
\]

\[
\frac{d^2 \phi_0}{dr^2} = 0 \quad (2.87)
\]

Thus we get

\[
\left[ \phi \frac{d\phi}{dr} - \phi_0 \frac{d\phi_0}{dr} \right] \bigg|_{r=r'} - \left[ \phi_0 \frac{d\phi}{dr} - \phi \frac{d\phi_0}{dr} \right] \bigg|_{r=0} = -k^2 \int_0^{r'} \phi \phi_0 \, dr \quad . \quad (2.88)
\]

Note that \( \phi(0) \) and \( \phi_0(0) \neq 0 \) and from Eqs(2.73) and
(2.85) with \( C = -\frac{1}{a} \),

\[
\phi_0(0) = \phi(0) = 1, \quad \frac{d\phi_0}{dr} = -\frac{1}{a}, \quad (2.89)
\]

\[
\phi(r) = -\frac{1}{ka} \sin(kr + \delta_0) \quad .
\]

Let \( r \to \infty \), and subtract (2.88) from (2.84) and use Eq(2.85) to get

\[
\left[ \phi_0 \frac{d\phi}{dr} - \phi \frac{d\phi_0}{dr} \right] \bigg|_{r=0}^{r=\infty} = -k^2 \int_{0}^{\infty} (\phi_0 \phi - XX_0) \, dr \quad (2.90)
\]

Then using Eqs(2.89), it is easy to see that we have

\[
\left[ 1(-\frac{1}{a}) \cos \delta_0 - \left( -\frac{\sin \delta_0}{ka} \right) (-\frac{1}{a}) \right] = k^2 \int_{0}^{\infty} (\phi_0 \phi - XX_0) \, dr, \quad (2.91)
\]

or

\[
k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 r_0(k) \quad (2.92)
\]
where
\[
  r_0(k) = \left[ \frac{-ka}{\sin \delta_0} \right] \cdot 2 \int_0^\infty (\phi \phi_0 - XX_0) \, dr.
\]  

(2.93)

This is an exact result.

If \( k \) is small then to leading order in \( k \),
\[
  \frac{\sin \delta_0}{k} = -a,
\]

and we can write Eq.(2.93) as
\[
  r_0 = 2 \int_0^\infty (\phi \phi_0 - XX_0) \, dr.
\]

(2.94)

where \( r_0 \) is now independent of \( k \). Eq.(2.92) then becomes

\[
  k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 r_0.
\]

(2.95)

\( r_0 \) is called the effective range and is a characteristic of the potential \( V(r) \), since \( X_0(r) \) is determined by this potential.

Thus we see that low energy nucleon-nucleon data can be analyzed in terms of just two parameters, the scattering length \( a \) and the effective range \( r_0 \).
2.6 CHANGE IN SCATTERING LENGTH FOR A CHANGE IN POTENTIAL

Consider the radial Schrödinger equation for \( L=0 \) for two different potentials \( V_1 \) and \( V_2 \), but for \( k \to 0 \), i.e.

\[
\frac{d^2 X_1}{dr^2} = \frac{2\mu}{\hbar^2} V_1(r) X_1(r) \quad ,
\]

\[
(2.96)
\]

\[
\frac{d^2 X_2}{dr^2} = \frac{2\mu}{\hbar^2} V_2(r) X_2(r) \quad .
\]

\[
(2.97)
\]

Now multiply Eq.(2.96) by \( X_2 \) and Eq.(2.97) by \( X_1 \) and subtract, to get

\[
\frac{d}{dr} \left[ X_1 \frac{dX_2}{dr} - X_2 \frac{dX_1}{dr} \right] = -\frac{2\mu}{\hbar^2} (V_2 - V_1) X_1 X_2 \quad .
\]

\[
(2.98)
\]

Integrating both sides of (2.98) over \( r \) from \( 0 \to r' \) and noting that \( X_1(0) = X_2(0) = 0 \), we get

\[
\left[ X_1 \frac{dX_2}{dr} - X_2 \frac{dX_1}{dr} \right] \bigg|_r = -\frac{2\mu}{\hbar^2} \int_{r=r'}^{\infty} (V_2 - V_1) X_1(r) X_2(r) dr \quad .
\]

\[
(2.99)
\]
Consider now the asymptotic form of the Schrodinger equation as $k \to 0$

\[
\frac{d^2 \phi_1}{dr^2} = 0 ,
\]

\[
\frac{d^2 \phi_2}{dr^2} = 0 .
\]

(2.100)

This is because $V_1$ and $V_2$ vanish as $r \to \infty$. Then we have

\[
X_1(r) \quad \longrightarrow \quad \phi_1 = (1 - \frac{r}{a_1}) ,
\]

(2.101)

\[
X_2(r) \quad \longrightarrow \quad \phi_2 = (1 - \frac{r}{a_2}) .
\]

Now multiply the first of Eqs(2.100) by $\phi_2$ and the second by $\phi_1$, subtract and integrate from 0 to $r'$ to get

\[
\left[ \phi_1 \frac{d\phi_2}{dr} - \phi_2 \frac{d\phi_1}{dr} \right]_{r=r'} - \left[ \phi_1 \frac{d\phi_2}{dr} - \phi_2 \frac{d\phi_1}{dr} \right]_{r=0} = 0
\]

(2.102)
Now from Eqs(2.101)

$$\phi_1(0) = \phi_2(0) = 1,$$

$$\frac{d\phi_1(0)}{dr} = -\frac{1}{a_1},$$

$$\frac{d\phi_2(0)}{dr} = -\frac{1}{a_2}.$$  \hspace{1cm} (2.103)

Using (2.103) in (2.102), we obtain

$$\left[ \phi_1 \frac{d\phi_2}{dr} - \phi_2 \frac{d\phi_1}{dr} \right]_{r=r'} = -\frac{1}{a_1} - \frac{1}{a_1}.$$  \hspace{1cm} (2.104)

Let now $r \to \infty$, subtract Eq(2.104) from (2.99) and make use of

$$X_1 \xrightarrow{r \to \infty} \phi_1$$

$$X_2 \xrightarrow{r \to \infty} \phi_2$$

\hspace{1cm} (2.105)

to obtain

37
\[ \left[ -\frac{1}{a_1} - \frac{1}{a_1} \right] = \frac{2\mu}{\hbar^2} \int_0^\infty (V_2 - V_1) X_1(r) X_2(r) \, dr \quad (2.106a) \]

or

\[ \left[ \frac{\Delta a}{a_1 a_2} \right] = \frac{2\mu}{\hbar^2} \int_0^\infty \Delta V X_1(r) X_2(r) \, dr \quad . \quad (2.106b) \]

This is an exact result and relates the change in scattering length due to a change in the potential.
CHAPTER THREE

EXPERIMENTAL PARAMETERS

The experimental values of the N-N scattering lengths gave the first suggestion for charge independence. In the case of n-p scattering there is no difficulty in collecting the data needed to find the scattering length, but the bound state effect must be removed. This can be achieved by scattering at an energy $E < 10$ MeV which is then dominated by scattering in the $S=0$ state, with np scattering in the $S=1$ "deuteron" state being energetically unallowed.

In p-p scattering there is a difficulty of removing the effect of the electromagnetic interaction, and this is model dependent in some aspects. For the third case, n-n scattering, it is very difficult to collect data because of the difficulty of getting neutron targets and high intensity neutron beams. We will consider each one of these three cases separately.
3.1- n-p SCATTERING

The n-p scattering data are the most accurate low energy information which exist. As stated earlier, the energy must be below 10 MeV (otherwise contributions from the deuteron must be removed, which is a very difficult process) but there are no long range effects which need to be corrected for. There are other effects which can be overcome [see ref.2 and references therein]. The accepted value for \( a_{np} \) is [see ref.1],

\[
a_{np} = -23.748 \pm 0.010 \text{ fm}.
\] (3.1)

The effective range is also given by (ref.1)

\[
r_{np} = 2.75 \pm 0.05 \text{ fm}.
\] (3.2)

3.2- p-p SCATTERING

The collected data on p-p scattering is very precise because of the charge of the proton, since it is much easier to detect charged particles than uncharged ones.
On the other hand, it is easier to accelerate protons and one can also have pure targets of protons i.e. liquid hydrogen. The experimental values for $a_{pp}$ and $r_{pp}$ are given by [e.g. ref. 2]

\[
a_{pp}^E = -7.8098 \pm 0.0023 \text{ fm} .
\]

\[
r_{pp}^E = 2.767 \pm 0.010 \text{ fm} .
\]

where $E$ stands for the scattering length uncorrected due to long range interactions like the Coulomb interaction and vacuum polarization. The difficulties with p-p scattering are the corrections due to the Coulomb interaction and vacuum polarization. The scattering length corrected for the Coulomb interaction is given by the somewhat model-dependent formula

\[
\frac{1}{a_{pp}} = \frac{1}{a_{pp}^c} + \frac{1}{D} \left[ \ln\left(\frac{D}{r_{pp}}\right) - 0.33 \right]
\]

where $a_{pp}^c$ is the scattering length in the presence of Coulomb interaction, and

\[
D = \frac{1}{Me^2} = 28.82 \text{ fm} .
\]

Eq. (3.5) still has to be corrected for vacuum polarization. For more detail see (ref. 2).
The accepted values of $a_{pp}$ and $r_{pp}$ are (from ref.1),

$$a_{pp} = -17.9 \pm 0.002 \text{ fm},$$

$$r_{pp} = 2.82 \pm 0.01 \text{ fm}.$$ \hspace{1cm} (3.7)

In (ref.1) the authors did not combine their values with errors, but we will take them to be as indicated above [c.f.ref.6].

3.3- n-n SCATTERING

There are difficulties in collecting data for n-n scattering. The main difficulty is having a pure neutron target, while the other one is to have a high intensity beam of neutrons. On account of these difficulties, and others, direct n-n scattering is very difficult to carry out.

Because of such difficulties, indirect experiments have been used to measure the n-n scattering length and effective range. Usually three-body final state interactions occur in indirect n-n scattering, where two of the final particles are neutrons. The spectrum of
the third particle is most often measured, where the number of events near the upper end of its kinematical limit is the measure of the low energy n-n interaction. The best third particle is one that does not interact with neutrons, i.e. the interaction of the third particle with n is negligible, otherwise one will face the three body problem which is complicated. If the third particle interacts significantly with neutrons, then the scattering length and effective range depend on having a valid theory of three body interactions. The first suggested indirect reaction is

$$\pi + d \longrightarrow \gamma + n + n$$  \hspace{1cm} (3.8)

where the $\gamma$-n interaction is negligible. The second indirect reaction is

$$n + d \longrightarrow p + n + n$$  \hspace{1cm} (3.9)

In this reaction the n-p interaction is as strong as n-n and a 3-body analysis must be made. From (ref.1) the scattering length and effective range are given by

$$a_{nn} = -18.45 \pm 0.46 \text{ fm}$$

$$r_{nn} = 2.83 \pm 0.16 \text{ fm}$$  \hspace{1cm} (3.10)

and
\[ a_{nn} = -16.73 \pm 0.47 \text{ fm}, \quad \]
\[ \text{from } d(n,p)2n, \quad \]
\[ r_{nn} = 2.68 \pm 0.16 \text{ fm}, \quad \]
\[ (3.11) \]

The preferred values are those determined by [see ref.1]. the \( d(\pi^-,\gamma)2n \) reaction i.e. the values in Eq.(3.10) [For more details see ref.2] .

From the above experimental data, we see that

\[ a_{pp} - a_{nn} = 0.55 \pm 0.46 \text{ fm} \quad (3.12) \]

\[ \Delta a = \left( \frac{a_{pp} + a_{nn}}{2} \right) - a_{np} \]

\[ = 5.5 \pm 0.3 \text{ fm} \quad . \quad (3.13) \]

As discussed in chapter two, CS implying vanishing of \( \delta a = a_{pp} - a_{nn} \), holds within the experimental error but more precise data is needed to test it. The vanishing of \( \Delta a \) implies CI but the above experimental value of \( \Delta a \) shows that CI is broken. In the next chapter we discuss the various charge dependent effects on NN interactions in order to explain the above value of \( \Delta a \).
<table>
<thead>
<tr>
<th>TERMS</th>
<th>EXP.(fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_{np}</td>
<td>-23.748±0.01</td>
</tr>
<tr>
<td>a_{nn}</td>
<td>-18.45 ±0.46</td>
</tr>
<tr>
<td>a_{pp}</td>
<td>17.9 ±0.002</td>
</tr>
<tr>
<td>r_{np}</td>
<td>2.75 ±0.05</td>
</tr>
<tr>
<td>r_{nn}</td>
<td>2.83 ±0.16</td>
</tr>
<tr>
<td>r_{pp}</td>
<td>2.82 ±0.001</td>
</tr>
</tbody>
</table>

**TABLE 3.1**

Experimental data (ref.1) for N-N scattering length a and effective range r are given (in fm). The error in a_{pp} is not stated in (ref.1), but was taken from (ref.6).
CHAPTER FOUR

CHARGE DEPENDENT EFFECTS

In this chapter the charge dependent effects on the $I=1, ^1S_0$ scattering lengths are considered. The measured values of the scattering lengths $a$ and the effective ranges $r$, are given in table 3.1. Charge symmetry, as we saw earlier, implying vanishing of the difference $a_{pp} - a_{nn}$, holds within the experimental errors since $\delta a = 0.55 \pm 0.46$ fm. Charge independence, which would imply the vanishing of the quantity $\Delta a = \left[\frac{a_{pp} + a_{nn}}{2}\right] - a_{np}$, is broken by the measured $\Delta a = 5.5 \pm 0.3$ fm. Note that we used $\bar{a} = \frac{1}{2} \left[a_{pp} + a_{nn}\right]$ in charge independence (CI) to remove the effect of charge symmetry (CS) from CI. The understanding of this discrepancy in terms of direct electromagnetic effects (other than Coulomb) and charge dependence corrections to the meson masses and coupling constants has a long history summarized in ref.2.
Briefly, charge dependent effects may be broken into two types, direct and indirect electromagnetic effects. The direct e.m. effects, apart from the Coulomb effect which has already been taken care of in quoting the values in table 3.1, include (a) vacuum polarization, (b) the neutron-proton mass difference, and (c) magnetic effects due to the proton-neutron magnetic moment difference and finite size effects due to the charge and magnetic moment distributions of the proton and neutron. The indirect e.m. effects include (a) the \( \pi^+ - \pi^0 \) meson mass difference in the one pion exchange potential (OPEP), (b) the \( \pi^+ - \pi^0 \) meson mass difference in the two pion exchange potential (TPEP), (c) charge dependence of \( \pi-N \) coupling constants, (d) \( \pi\gamma \) exchange potential, and (e) up-down quark mass difference. The direct effects result in a small contribution to \( a \), as follows:

\[
\begin{align*}
\Delta a_{pp} \text{ (magn. + finite size)} & \sim -0.06 \text{ fm.} \\
\Delta a_{nn} \text{ (magn. + finite size)} & \sim +0.02 \text{ fm.} \\
\Delta a_{np} \text{ (magn. + finite size)} & \sim +0.20 \text{ fm.}
\end{align*}
\]

The effect of vacuum polarization and the \( n-p \) mass difference are still smaller. Out of the indirect e.m. effects, the most important is due to the \( \pi^+ - \pi^0 \) mass difference.
Now we will consider the indirect effects in some detail. The perturbation expression for $\Delta a$ is given by Eq. (2.106), i.e.

$$\Delta a = (\bar{a} - a_{np}) = \bar{a} a_{np} m \int_0^\infty (\bar{V} - V_{np}) X X_{np} \, dr,$$

(4.2)

where $\bar{a}$ stands for the average scattering length [in the case of CI of $a_{pp}$ and $a_{nn}$ this is $\bar{a} = \frac{1}{2} (a_{nn} + a_{pp})$], $a_{np}$ stands for the n-p scattering length. The $\bar{V}$ potential stands for $\bar{V} = \frac{1}{2} (V_{pp} + V_{nn})$, and $V_{np}$ denotes the n-p potential.

The wave functions $X$ and $X_{np}$ are the zero energy $^1S_0$ wave functions associated with $\bar{V}$ and $V_{np}$ respectively, and normalized so that

$$X(r) \quad \xrightarrow{r \to \infty} \quad (1 - \frac{r}{a}).$$

(4.3)

$m$ is the average nucleon mass. The large factor $\bar{a} a_{np} m$ on the right hand side of Eq. (4.2) makes $\Delta a$ very sensitive to even a small change in the potential difference $\Delta V = \bar{V} - V_{np}$.

It is convenient to use experimental values for
and $a_{np}$ such that the perturbation potential $\Delta V$ is to
result in the experimental values of $\Delta a$ (see ref. 3).
Now, after fixing the value of the factor $\bar{a}_{np}$ m in
Eq. (4.2), $\Delta a$ is additive since $\Delta V$ is additive. The
distinction between $X(r)$ and $X_{np}(r)$ is not important,
since consideration of this difference will be of second
order in $\Delta V$. We will use $|X_{\pi}(r)|^2$ instead of $XX_{np}$. $X_{\pi}(r)$
is generated by using the Reid soft core potential (ref.
15). The details of computing the wave function are
considered in chapter 5. As stated earlier $\Delta a$ is
additive, allowing us to evaluate the various indirect
effects listed earlier separately.

4.1 ELECTROMAGNETIC CORRECTIONS TO COUPLINGS

In this section we will consider the electromagnetic
e.m.) correction to the pion-nucleon coupling
constants. This correction is of the order $\alpha^2$, where $\alpha$
is the fine structure constant $\alpha = 1/137$. This
correction causes the charged pion couplings to differ
from the neutral pion couplings, i.e.
\[ f_0 = f \left[ 1 - \frac{\delta_0}{2} \right] \]  
\[ f_0 = f \left[ 1 + \frac{\delta_0}{2} \right] \]  
\[ \frac{\delta_0}{2} \]

\[ f_C = \left( \frac{1}{2} \right) \left[ f_{\pi^+} + f_{\pi^-} \right] = f \left( 1 + \delta_C \right) \]  
\[ \frac{\delta_C}{2} \]

\( \delta_0 \) and \( \delta_C \) are the correction to the couplings when \( \alpha^2 \) is considered in calculating the coupling constants from the Feynman diagram, where \( f \) is the coupling when all the couplings are the same, i.e. the e.m. second order corrections to couplings are neglected. Also \( f_{\pi^+} = f_{\pi^-} \). These corrections have been evaluated by Morrison (ref.10) as

\[ \delta_0 = 0.0023 \quad , \quad \delta_C = -0.0058 \]  

In the next section we consider Feynman diagrams to see the effect of these corrections on OPEP and TPEP.
4.2 POTENTIALS

1 - ONE PION MASS DIFFERENCE EFFECT

The effect of the $\pi^+ - \pi^0$ mass difference on OPEP as given in Eq.(2.98), can be derived from Figs.3(a,b,c) and is given by:

$$\Delta V_{CI} = \overline{V} - V_{np},$$

where from Fig.3(a)

$$\overline{V} \equiv \frac{1}{2} (V_{pp} + V_{nn}) = C(0),$$

and from Fig.3(b)&(c),

$$V_{np} = -C(0) + \sqrt{2} \cdot \sqrt{2} C(1),$$

so that

$$\Delta V_{CI} = 2 \left[ C(0) - C(1) \right]$$

where

$$C(0) = -g^2 \left( \mu_\pi/2m \right)^2 \{ \exp(-\mu_\pi r)/r \}$$

and

$$C(1) = -g^2 \left( \mu_\pi/2m \right)^2 \{ \exp(-\mu_\pi r)/r \},$$

as taken from Eq(2.106). Thus
FIGURE 3

Charge dependence of OPEP and TPEP. Graphs (a) to (c) are the three OPEP cases, graphs (d) to (h) represent the uncrossed TPEP diagrams, and graphs (i) to (m) represent the crossed TPEP diagrams. The isospin relative coefficients are given at each vertex. These graphs were taken from (ref.12, Fig.1).

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\[ \Delta V_{CI} , \Delta \mu = g^2 \left( \mu_0 / \mu_c \right)^2 \{ \exp(-\mu_0 r)/r \} \left[ 1 - \left( \mu_c / \mu_0 \right)^2 \right] \times \{ \exp(-\Delta \mu r)/r \} \]  \hspace{1cm} (4.9) \]

where

\[ \mu_0 = \text{mass of } \pi^0 \]

\[ \mu_c = \text{mass of } \pi^\pm \]

\[ f^2 = g^2 (\mu_0 / 2m)^2 \]

\[ m = \text{average nucleon mass} \]

\[ \Delta \mu = \mu_c - \mu_0 \]

2 - COUPLING CONSTANT EFFECTS IN OPEP

The OPEP due to the e.m. correction can be found by using equ.(4.4) and (4.5) and replacing in Fig.3(a,b,c) the factors 1, -1 and \( \sqrt{2} \) respectively by \( (1-\delta_0/2) \), \( -(1+\delta_0/2) \) and \( (1+\delta_c) \), and neglecting terms of second order in the \( \delta \)'s. Thus \( V_{np} \) becomes, from Fig.3(c),

\[ V_{np} = -C(0) + \sqrt{2} (1+\delta_c) \sqrt{2} (1+\delta_c) C(1). \]  \hspace{1cm} (4.10a) \]

Note that there will be no correction in \( C(0) \) due to \( \delta_c \) but there will be one due to \( \delta_0 \) but it is of the order of \( (\delta_0^2) \) which is very small. Thus finally
\[ \Delta V^{OPEP}_{CI, \mu+cc} = 2 \{ C(0) - C(1) \} - 4 \delta_c C(1) + O(\delta_c^2) + O(\delta^2), \]  

(4.10b)

where the effect of \( \delta^o \) is of higher order as can be seen from

\[ V_{np} = - \left(1 - \frac{\delta^o}{2}\right) \left(1 - \frac{\delta^o}{2}\right) C(0) + 2 \left(1 + \delta_c\right)^2 C(1) \]  

(4.11a)

\[ \bar{V} = \frac{1}{2} \left[ \left(1 + \frac{\delta^o}{2}\right)^2 + \left(1 - \frac{\delta^o}{2}\right)^2 \right] C(0) = \frac{1}{2} \left[ 2 + \frac{\delta^2}{2} \right] C(0) \]  

(4.11b)

so that

\[ \Delta V^{OPEP}_{CI} = 2 \ C(0) + O(\delta^2). \]  

(4.11c)

This implies for OPEP [c.f. Eq. (4.7)] that

\[ \Delta V^{CI}_{cc} = 4 \ \delta_c f^2 \left[ \exp(-\mu r)/r \right] \]  

(4.12)

where "cc" denotes correction due to coupling constants.

3 - TWO PION MASS DIFFERENCE EFFECT

In the case of TPEP, Figs. 3[d,...m] are used to write down the potentials, and we denote the amplitude of the uncrossed diagrams \{d,...,h\} by A, and those for the crossed diagrams \{i,...,m\} by B. Each amplitude is assigned two indices, which denote the kind of exchanged \( \pi^- \)-meson (i.e. 0 & 1 for \( \pi^0 \) & \( \pi^+ \) respectively). In
this way, we obtain for the effect of the \( \tau^\pm - \tau^0 \) mass difference on TPEP:

\[
V = A(00) + B(00) + 4B(11),
\]

(4.13)

\[
V_{np} = A(00) + 4A(11) - 4A(01) + B(00) + 4B(01)
\]

implying

\[
\Delta V_{\text{TPEP}}^{\text{CI}, \Delta \mu} = 4\left[ \{ B(11) - A(11) \} - \{ B(01) - A(01) \} \right]
\]

(4.14)

4 - COUPLING CONSTANT EFFECT IN TPEP

The coupling constant's correction to the TPEP \( \Delta V_{\text{CI}, cc} \) can be obtained from Fig.3(d,----m) by replacing the factors 1, -1 and \( \sqrt{2} \) in the appropriate Feynman diagrams respectively by \( 1 - \frac{\delta_0}{2} \), \( -(1 + \frac{\delta_0}{2}) \) and \( 1 + \delta_c \) and neglecting terms of second order in the \( \delta \)'s. This term is given by

\[
\Delta V_{\text{CI}, cc}^{\text{TPEP}} = 8 \delta_c \left[ B(11) - A(11) \right], \quad (4.15)
\]

For more details of the derivation of Eqs (4.14) and (4.15) and the complete form of these potentials see Appendix A.
The last CI term is the $\pi\gamma$ exchange potential and its value has been evaluated to be $\Delta a_{\pi\gamma} = 1.1$ fm (ref. 3).

4.3-CHARGE SYMMETRY BREAKING TERM DUE TO ELECTROMAGNETIC CORRECTION

The charge symmetry breaking potential is given by $\Delta V_{CS} = V_{pp} - V_{nn}$. This expression is to be evaluated for the OPEP and TPEP due to the e.m. correction of the coupling constants.

In the case of OPEP there is no $\pi^\pm - \pi^0$ mass difference correction to $\Delta V_{CS}$ and thus we consider the effect due to the e.m. correction of the pion-nucleon coupling constants. Using equ.(4.4) and Fig.3(a), we have,

$$V_{pp} = (1 - \frac{\delta_0}{2})^2 \ C(0)$$

(4.16)

$$V_{nn} = (1 + \frac{\delta_0}{2})^2 \ C(0)$$

so that
\[ \Delta V_{\text{CSB}}^{\text{OPEP}} = \left[ (1 - \delta_0 + \frac{\delta_0^2}{4}) - (1 + \delta_0 + \frac{\delta_0^2}{4}) \right] C(0) \]
\[ = -2 \delta_0 C(0) \quad (4.17) \]

[neglecting second order term in \( \delta_0^2 \)]

and thus

\[ \Delta V_{\text{CSB}}^{\text{OPEP}} = 2\delta_0 f^2 \left[ \exp(-\mu_0 r)/r \right]. \quad (4.18) \]

In the case of TPEP due to the e.m. correction, using Fig.3{d,...m} and Eq.(4.4) we get:

\[ \Delta V_{\text{CSB}}^{\text{TPEP}} = -4 \delta_0 \left[ A(00) + B(00) \right]. \quad (4.19) \]

For the derivation of (4.19) see Appendix B.

The contributions to \( \Delta a \) from various potentials are summarised in table 4.1, and the shapes of the integrands are shown in Figs. 4 and 5.
<table>
<thead>
<tr>
<th>TERMS</th>
<th>$\Delta a$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a_{OPEP}^{\Lambda \mu}$</td>
<td>$3.6 \pm 0.15$</td>
</tr>
<tr>
<td>$\Delta a_{TPEP}^{\Lambda \mu}$</td>
<td>$0.88 \pm 0.1$</td>
</tr>
<tr>
<td>$\Delta a_{OPEP}^{cc}$</td>
<td>$-1.6 \pm 0.2$</td>
</tr>
<tr>
<td>$\Delta a_{TPEP}^{cc}$</td>
<td>$+1.6 \pm 0.2$</td>
</tr>
<tr>
<td>$\Delta a_{\pi \gamma}$</td>
<td>$1.1 \pm 0.5$</td>
</tr>
<tr>
<td>$\Delta a_{tot}$</td>
<td>$5.6 \pm 0.5$</td>
</tr>
</tbody>
</table>

**TABLE 4.1**

Contribution to $\Delta a$ from different potentials discussed in this chapter. The errors were taken from (ref.3). For more details of how these errors are evaluated see (ref.3).
FIGURE 4

Integrands of Eqn.(4.1), for the indicated potentials.
FIGURE 5
Integrands of Eqn. (4.1), for the indicated potentials.
CHAPTER FIVE

COMPUTATION

In this chapter we will consider the computational techniques used to get the results of this thesis. The first section is an introduction for this chapter which gives an overview and reasons for considering this chapter. In the second section, we will consider evaluation of the wave function. In section three, the ways of calculating the OPEP and OBEP contributions are considered. In section four, TPEP will be considered in more detail. The final section will contain numerical values of the physical constants used in this thesis.
5.1 - INTRODUCTION

In our work we needed evaluation of quantities which were very sensitive to small changes in potentials, for example the CIB and CSB TPEP expressions, so accurate programmes were needed which could do this job. The most difficult potential - in these calculations, the TPEP, was calculated but unfortunately it was not exactly the same as what Ericson and Miller (ref.3) computed. During the work, it was found many times that it was needed - for self training and confidence - to numerically compute work which had previously been done. Unfortunately, some times it was not easy to reproduce such results. The missing point in the above statement was that there was almost not any written programme included with the results stated. For this reason it is better to include some of the programmes used in this thesis where it is neccessary to include them. This will be done in Appendix C.
5.2 - N-N WAVE FUNCTION

We need the N-N wave function $u(r)$ in the perturbative expression Eq.(4.1) so that one can evaluate the different contributions to CI and CS. To find such a wave function one has to solve the Schrodinger equation i.e.

$$\frac{d^2}{dr^2} u(r) = \frac{2\mu'}{\hbar^2} V(r)u(r) \quad (5.1)$$

where $\mu'$ is the reduced nucleon mass and we have taken $k \to 0$ and $L=0$ as in the case for very low energy scattering.

Since $V(r)$ is not well known, and even the approximate potential is not known in terms of integrable expressions, we can not find an analytic solution for the wave function. One way to get the wave function is by solving eq(5.1) numerically. In this section we will use the same notation as in ref.(15). For N-N scattering $2\mu' = m = \text{average nucleon mass}$. To have the same notation as in ref.15, divide Eq(5.1) by the pion-mass and let $x=\mu r$, then we will get
\[
\frac{d^2}{dx^2} u(r) = V'(r)u(r)
\]  
(5.2)

where

\[
V'(r) = \left( \frac{m}{\mu^2 h^2} \right) V(r)
\]

\[
\mu = \left( \frac{m_0 c}{h} \right) \approx 0.7 \text{ fm.} \quad \text{(we use natural units)}
\]

A $^1S_0$ I=1 phenomenological N-N potential, called the Reid soft-core potential, is given by ref.(15). Its form is;

\[
V(x) = -h \frac{\exp(-x)}{x} - 1650.6 \frac{\exp(-4x)}{x} + 6484.2 \frac{\exp(-7x)}{x}, \quad (5.3)
\]

where \( h = 10.463 \text{ MeV} \), and the unit of the potential is in MeV.

The asymptotic solution of the wave function is

\[
u(r) \underset{k \rightarrow 0}{\longrightarrow} \left(1 - \frac{r}{a}\right) . \quad (5.4)
\]

We used the programme listed at the end of this thesis to find the solution of the second order differential equation. The boundary conditions needed to solve this differential equation are

\[
u(0) = 0 ,
\]
\[ u'(0) = \frac{du(0)}{dr} \neq 0 \quad (5.5) \]

and the asymptotic solution (5.4). We used an arbitrary value for \( u'(0) \) at \( r=0 \) and then normalized the wavefunction by matching it to the asymptotic solution (5.4). Integration was done from \( x = 0.035 \) to 3.22, with a step size of \( \Delta x = 0.35 \) and error tolerance of 0.0001, implying 0.01 percent is the maximum error tolerated. From \( x = 3.22 \) to 10.0, we used (5.4) to find \( u(r) \) since \( V(r) \) is negligible beyond \( x = \mu r = 3.22 \), i.e. for \( r \geq 4.6 \) fm. Actually we followed the same method used by ref.3 as it appears to be from Fig.1 of ref.3. Then we matched the asymptotic solution with the small \( r \) solution at \( R=4.6 \) fm to get the shape in Fig.6. The final comment to mention here is that we used

\[ a_{\text{exp.}} = -18.2 \text{ fm} = \frac{1}{2} \{ a_{pp} + a_{nn} \}, \]

in (5.4), since there is no difference in using any of the experimental scattering lengths in evaluating Eq.(5.4), and this is due to the insensitivity of the wave function to the scattering length.
FIGURE 6

The N-N wave function $u(r)$ and the asymptotic solution $u_a(r)$ at very low energy.
5.3-EVALUATION OF ONE BOSON EXCHANGE POTENTIAL (OBEP)

CONTRIBUTION

Since we know each expression of OBEP [i.e. Eqn.'s (4.9, 12, 18, and 6.33)], there is no difficulty in evaluating these expressions. There is one restriction in evaluating these terms, i.e. the potentials must be evaluated at the same radial points as the wave function. This solution will make one loose error control. For this reason I have checked the error for some similar known functions and found that for the interval length \( \Delta r = 0.05 \) fm the maximum error in a case was 7.0 percent. This restriction is not limited to OBEP only, but extended to TPEP also.
5.4-EVALUATION OF TPEP

The most difficult and time consuming potential was the valuation of TPEP. The expressions of TPEP are given in ref. (12), and each potential is formed out of more than two terms and those terms are either two or three dimensional integrals. Since, as we mentioned in chapter four, the perturbative expression [Eq(4.1)] being additive allows us to evaluate each term separately. Thus we need to evaluate TPEP first and then put it in Eq(4.1) to find $\Delta a_{\text{TPEP}}$. In ref. (12), they advised to use appendix B of their paper to evaluate the different terms. I used their advice but, unfortunately, I got $\Delta V_{\text{TPEP}}$ about 20 times larger than the actual potential difference in the case of CS. I think the reason behind this is that a lot of error is introduced during computation, specially that we have multi-dimensional integrals, and also we have a large number of terms to be added or subtracted many times until we get the last required value of $\Delta V_{\text{TPEP}}$.

The second way I used to evaluate these terms is by using a programme that controls error. The subroutine QB01A from the Harwell Subroutine Library ref. (17),
which controls the error during integration, was used to evaluate those potentials. Note that in using QB01A, I used the transformation suggested by ref.(12), in their appendix B, but I did not use 21 points at each dimension. In this case I also faced difficulties, the first one was that the $\Delta V_{TPEP}^{CS}$ potential difference changes sign at some values of $r$, while it should not, the second difficulty was the long time needed to run each programme (not CPU time). When the error tolerance is specified by $10^{-5}$, it took about 24 hours to run the programme because of the three dimensional integrations and because of the small error tolerance. The last method I used was to have an error tolerance of $10^{-4}$ and then I got the shape in Fig.4.

Still there is some problem in the advice of ref.(12) with respect to the limits of integration. When we followed the advice of ref.(12) and changed variables to expand the domain of integration and to get the required result, we got a wrong shape. The important point for integration, i.e. to get the correct result, is by unifying the limits before changing the variables and to make the change of limit according to the change of variables. Table (5.1), shows the change of limits. Normally the limits of the integrations can be written
from 0 to 1, but during computation we cannot use 0 or 1, otherwise there will be singularities. To scan all the domain of integration and not to lose any important contribution to the potentials, we chose very close values to the limits, i.e. the variables before change have the following values, [c.f.App.A and B].

\[ \beta_L = \xi_L = \xi^{-1}_L = 10^{-12} \]

\[ \beta_u = \xi_u = \xi^{-1}_u = 0.99 \]

where \( L \) stands for the lower limit and \( u \) for the upper limit.
TABLE 5.1

This table shows the transformation of variables and limits for the expressions listed in appendices A and B and according to appendix B of ref.12.

<table>
<thead>
<tr>
<th>TERM</th>
<th>VARIABLE</th>
<th>UPPER LIMIT</th>
<th>LOWER LIMIT</th>
<th>COMMENTS</th>
</tr>
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<td>all terms</td>
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<td>$10^{-12}$</td>
<td>no change of variables</td>
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<td>X's</td>
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<td>variables did change</td>
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<td>Y's</td>
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<td>0.01</td>
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<td>W</td>
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<td>0.996655</td>
<td>$10^{-4}$</td>
<td>variables did change</td>
</tr>
<tr>
<td></td>
<td>$w = \zeta^\frac{1}{3}$</td>
<td>0.99</td>
<td>$10^{-4}$</td>
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<tr>
<td>Z</td>
<td>$u = \zeta^\frac{1}{5}$</td>
<td>0.996655</td>
<td>0.01</td>
<td>variables did change</td>
</tr>
<tr>
<td></td>
<td>$w = \zeta^\frac{1}{3}$</td>
<td>0.99</td>
<td>$10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 5.2

This table contains all the constants used in this thesis. Note that natural units are used in this table, i.e. $\hbar = c = 1$, except for $\hbar c$.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>VALUE</th>
<th>NAME</th>
</tr>
</thead>
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<tr>
<td>$\mu^+$</td>
<td>139.6 MeV/c</td>
<td>charged pion mass</td>
</tr>
<tr>
<td>$\mu^0$</td>
<td>135.0 MeV/c</td>
<td>neutral pion mass</td>
</tr>
<tr>
<td>$m$</td>
<td>938.906 MeV/c</td>
<td>average nucleon mass</td>
</tr>
<tr>
<td>$\eta$</td>
<td>548.8 MeV/c</td>
<td>$\eta$-particle mass</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>958.0 MeV/c</td>
<td>$\eta'$-particle mass</td>
</tr>
<tr>
<td>$f^2$</td>
<td>0.08</td>
<td>unit less coupling constant</td>
</tr>
<tr>
<td>$hc$</td>
<td>197.33 MeV fm</td>
<td>in natural unit they equal to unity</td>
</tr>
<tr>
<td>$K$</td>
<td>493 MeV/c</td>
<td>$K$-meson mass</td>
</tr>
<tr>
<td>$m_u$</td>
<td>360 MeV/c</td>
<td>up, down and strange constituent quark mass</td>
</tr>
<tr>
<td>$m_d$</td>
<td>360 MeV/c</td>
<td></td>
</tr>
<tr>
<td>$m_s$</td>
<td>510 MeV/c</td>
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</tr>
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<td>$m_u$</td>
<td>5 MeV/c</td>
<td>up and down current quark mass</td>
</tr>
<tr>
<td>$m_d$</td>
<td>10 MeV/c</td>
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CHAPTER SIX

CHARGE SYMMETRY BREAKING CORRECTIONS TO NN SCATTERING LENGTH DUE TO

UP-DOWN CURRENT QUARK MASS DIFFERENCE

In this chapter, the charge symmetry breaking (CSB) correction due to the isospin breaking part of the QCD Hamiltonian, determined by the up-down current quark mass difference \( H_{QCD} = (1/2) (m_u - m_d)(\bar{u}u - \bar{d}d) \), on \((\delta a)_{CBS} = a_{pp} - a_{nn}\) is estimated. This is done by first calculating the effect of \( H_{QCD} \) on the pion-nucleon coupling constants via \( \pi^0 - \eta, \eta' \) mixing, giving rise to CSB one pion exchange (OPEP) and two pion exchange (TPEP) potentials. The importance of CSB-TPEP is emphasized. The \( \eta \)-nucleon coupling constant appears to be constrained by \((\delta a)_{CSB}^{\exp}\) to be very small.
6.1 - INTRODUCTION

As seen in chapter 4, $(\Delta a)_{\text{CIB}}$ is almost entirely explained by the various corrections listed in Table-4.1, the most important of these corrections is the $\pi^+ - \pi^0$ mass difference. The situation with respect to CSB effects is, however, not so clear as it is in CI. Apart from CSB, OPEP and TPEP potentials (ref.3) generated by the e.m. corrections to the $p\pi^0$ and $n\pi^0$ coupling constants, there is an additional effect due to the difference between the masses of the up and down quarks. Although such a mass difference is regarded as of non-electromagnetic origin and is believed to arise from QCD and the Higgs mechanism, it does violate isospin symmetry. Various authors (refs. 4,5,6 & 7) have estimated the effect of such a mass difference on the scattering lengths and have generally found it to be small. In ref.4 the effect of $\Delta m_q = (m_u - m_d)$ has been investigated on the pion nucleon coupling constants within the framework of the MIT bag model. The effect of $\Delta m_q$ on an effective nucleon-nucleon potential was studied in ref.5 in a quark cluster model. A related investigation has been done in ref.6 where the so called color magnetic correction to the short range nucleon-nucleon potential arising from the one gluon exchange quark-quark
potential has been calculated. The isospin violating effect on the scattering lengths in such a calculation also arises from $\Delta m_q$. A more recent calculation has studied CSB in OPEP arising from the small difference in the charge RMS radii for u- and d-quark distributions in a nucleon due to $\Delta m_q$ (ref.7). A common feature of all these calculations is that they need the up-down constituent quark mass difference rather than the current quark mass difference. On the other hand it is for the latter that a reliable estimate exists: $\Delta m_q \approx 5$ MeV (with current quark masses being $m_u \approx 5$ MeV, $m_d \approx 10$ MeV). Thus in the above mentioned calculations one has to rely on the expectation that the mass difference $\Delta m_q \approx 5$ MeV is not lost in the renormalization of the current quark masses to those of the constituent quarks (notice that the common constituent quark mass is 360 MeV). The approach we wish to follow is to estimate the effect of $\Delta m_q$ on the pion-nucleon coupling constants where $\Delta m_q$ appears as the current quark mass difference in the basic Lagrangian or Hamiltonian:

$$H = H_{QED} + H_{QCD}.$$  \hspace{1cm} (6.1)

Here $H_{QED}$ is the usual electromagnetic current-current Hamiltonian due to an internal photon loop which we considered in chapter 4, while
\[ H_{QCD}(\Delta I=1) = -\left(\frac{1}{2}\right)\Delta m_q (\bar{u}u-\bar{d}d) = -\Delta m_q S_3 \]  (6.2)

(Here \(\Delta m_q\) is the current quark mass difference by definition and \(S_3 = (\frac{1}{2})(\bar{u}u-\bar{d}d) = \bar{q}(\frac{\lambda_3}{2}) q\). What about the effect of \(H_{QCD}\) on the \(\pi-N\) coupling constants? One way such an effect can be taken into account is by observing that \(H_{QCD}\) can give rise to \(\eta, \eta' - \pi^0\) mixings. The effect due to such mixings on the meson exchange potential and that of the latter on the scattering lengths has been calculated previously (refs.8 and 9). However, these calculations in effect take into account the modification due to \(\eta, \eta' - \pi^0\) mixings on the one meson exchange potential only. On the other hand if one considers the effect of \(H_{QED}\) on the pion-nucleon coupling constants, one observes the importance of this effect on TPEP, namely an almost complete cancellation between \(\Delta a_{cc}^{TPEP}\) and \(\Delta a_{cc}^{OPEP}\) for CIB and \(\delta a_{cc}^{TPEP} \approx 3\delta a_{cc}^{TPEP}\) for CSB. In view of this, it is important that the effect of \(H_{QCD}\) on \(\eta - \pi^0\) and \(\eta' - \pi^0\) mixings should also be studied in TPEP. Thus it should be more consistent to add the effect of (6.2) on the pion-nucleon coupling constants to that of \(H_{QED}\) (which has previously been calculated) and then study the combined effect.
of (6.1) on $\delta a_{cc}^{\text{TPEP}}$ and $\delta a_{cc}^{\text{OPEP}}$ for CSB. [It is for this reason that we do not include the contribution of $H_{\text{QED}}^{\eta, \eta'-\eta^0}$ mixings for the calculation of their effect on the pion-nucleon coupling constants]. We indeed also find for $H_{\text{QCD}}$ that $\delta a_{cc}^{\text{TPEP}} = 3\delta a_{cc}^{\text{OPEP}}$. Our results are summarized in table 6.1 and in sec.3 we discuss the implications of these results and summarize our conclusions.
6.2 - CSB POTENTIAL WITH QCD CORRECTIONS TO PION-NUCLEON COUPLING CONSTANTS

We write the $\pi^0\text{pp}$ and $\pi^0\text{nn}$ coupling constants as

$$g_{\pi^0\text{pp}} = [1 - (\frac{1}{2})(\delta_{\text{QCD}})] g_\pi$$

$$g_{\pi^0\text{nn}} = [1 + (\frac{1}{2})(\delta_{\text{QCD}})] g_\pi .$$

Note that the definition of Eqs.(6.3) are similar to Eqs.(4.4). In the above equation, $g_\pi$ and $f$ are given by

$$\frac{g_\pi^2}{4\pi} = 14.3$$

and

$$f^2 = (\frac{m_\pi}{2m_N})^2 \frac{g_\pi^2}{4\pi} = 0.08.$$  

$\delta_{\text{QCD}}$ is the correction due to $H_{\text{QCD}}$ and is discussed below. As remarked earlier $H_{\text{QCD}}$ leads to $\eta, \eta' - \pi^0$ mixings as shown in Fig.7, which in turn give rise to corrections to the pion-nucleon coupling constants. Before considering the QCD correction to the coupling constants, let us first consider the total Hamiltonian in terms of the quark masses, i.e.

$$H = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s,$$

where $m_u$, $m_d$ and $m_s$ are the masses of the up, down and strange quark respectively.
FIGURE 7

Feynman diagram for the $\eta-\pi$ mixing that results from $H_{QCD}$. 
Rearranging some of the terms of Eq.(6.4), we get:

\[ H = \frac{(m_u^+m_d)}{2}(\bar{u}u+\bar{d}d) + \frac{(m_u^-m_d)}{2}(\bar{u}u+\bar{d}d) + m_s \bar{s}s, \]  

(6.5)

or

\[ H = \left( \frac{1}{3} \right)(m_u^+m_d+m_s)(\bar{u}u+\bar{d}d+\bar{s}s) + \left( \frac{1}{3} \right)(m_u^-m_d)(\bar{u}u+\bar{d}d-2\bar{s}s) \]

\[ + \left( \frac{1}{2} \right)(m_u^-m_d)(\bar{u}u-\bar{d}d) \]

which leads to

\[ H = \bar{m}(\bar{u}u+\bar{d}d+\bar{s}s) + \left( \frac{2}{\sqrt{3}} \right)(m_u^-m_d)(\bar{u}u+\bar{d}d-2\bar{s}s) \]

\[ + \left( \frac{1}{2} \right)(m_u^-m_d)(\bar{u}u-\bar{d}d) \]  

(6.6)

where \( \bar{m} = \frac{(m_u^+m_d+m_s)}{3} \) and \( \bar{m} = \frac{(m_u^+m_d)}{2} = m_{ns} \).

Now put

\[ S_3 = \bar{q}(\frac{\lambda_3}{2}) q , \]

(6.7)

and

\[ S_8 = \bar{q}(\frac{\lambda_8}{2}) q , \]

where \( \bar{q} = (\bar{u} \; \bar{d} \; \bar{s}) , \quad q = \begin{bmatrix} u \\ d \\ s \end{bmatrix} \)  

(6.8)

and \( \lambda_3 \), \( \lambda_8 \) are two of generators of SU(3) group. Thus finally we can write (6.6) as

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\[ H = \bar{m}(\bar{u}u + \bar{d}d + \bar{s}s) + \left( \frac{2}{\sqrt{3}} \right)(\bar{m}_s - m_s) S_8 + (m_u - m_d) S_3 \]  \quad (6.9)

The first term of Eq. (6.9) is SU(3) symmetry invariant, and \((m_u - m_d)\) is the SU(2) isospin breaking term, while \((m_s - m_s)\) is the SU(3) breaking term. If we assume that \(m_u = m_d = m_s\), then SU(3) will not be broken, and if we assume \(m_u = m_d\), then SU(2) is not broken.

Now in the quark model, the Gell-Mann-Okubo mass splittings are given by

\[ H_{ns} = \left( \frac{2}{\sqrt{3}} \right)(\bar{m}_s - m_s) S_8 \]  \quad (6.10)

Our \(H^{AI=1}_{QCD}\) is \((m_u - m_d) S_3\). Since \(S_3\) and \(S_8\) belong to the same octet, we have

\[ <\pi^0 | S_3 | \eta_1 > = d_{330} s_d = \left( \frac{\sqrt{2}}{3} \right) s_d \]  \quad (6.11)

\[ <\pi^0 | S_3 | \eta_8 > = d_{338} s_d = \left( \frac{1}{\sqrt{3}} \right) s_d \]  

The reduced matrix element \(s_d\) is now determined by the Gell-Mann-Okubo mass splitting:
\[ \Delta m_k^2 = \frac{2}{\sqrt{3}}(\bar{m}_s - m_s) \frac{d_{4+15}}{\sqrt{2}} \frac{4-i5.8}{\sqrt{2}} s_d , \]  
\[ \Delta m_\pi^2 = \frac{2}{\sqrt{3}}(\bar{m}_s - m_s) \frac{d_{338}}{s_d} \]  
\[ \text{and from the equations we get} \]
\[ s_d = - \frac{(m_k^2 - m_\pi^2)}{(m_{ns} - m_s)} . \]

From the Feynman diagram shown in Fig. 7, using the last term of the Hamiltonian eqn. (6.9), we find the corrections to the pion-nucleon coupling constants as follows:

\[ \left( \frac{\delta g}{g} \right)_\pi = \left( \frac{\delta g}{g} \right)_\eta \]

\[ = - \frac{\eta_{\pi,pp}}{g_\pi} \]

\[ = - \frac{\eta_{\pi,nn}}{g_\pi} . \]  

where

\[ \eta_{\pi} = -(m_d - m_u) \left< \pi^0 | S_3 | \eta \right> \]

\[ \eta_{\pi,pp} = -(m_d - m_u) \left< \pi^0 | S_3 | \eta \right> . \]
The physical states $\eta$ and $\eta'$ are linear combinations of $\eta_8$ (member of SU(3) octet) and $\eta_1$ (SU(3) singlet) and are given by Eqs(2.57) and (2.58). Using Eqs(2.57), (2.58), (6.11) and (6.17) we get

\[ g_{\eta \pi} = (m_u - m_d) \left\{ \cos \theta \, d_{338} - \sin \theta \, d_{330} \right\} s_d \]

\[ = (m_u - m_d) \left\{ \cos \theta - \sqrt{2} \sin \theta \right\} \frac{1}{\sqrt{3}} s_d . \quad (6.18) \]

Similarly for $g_{\eta' \pi}$ we find

\[ g_{\eta' \pi} = (m_u - m_d) \left\{ \sin \theta + \sqrt{2} \cos \theta \right\} \frac{1}{\sqrt{3}} s_d . \quad (6.19) \]

Using Eqs (6.18) and (6.19), we get from (6.16)

\[ \left( \frac{\delta g}{g} \right)_\pi = \varepsilon \left[ 1 + \frac{(m_{\eta}^2 - m_{\pi}^2)}{(m_{\eta'}^2 - m_{\pi}^2)} \right] \tan(\phi) \times \]

\[ \times \left( \frac{g_{\eta' \pi p p}}{g_{\eta \pi p p}} \right) \times 3 \cos(\theta) \left( \frac{g_{\eta p p}}{g_{\pi}} \right) , \quad (6.20) \]

$\phi$ is the mixing angle between $|\eta_S \rangle = |\bar{s}s\rangle$

and $|\eta_{NS} \rangle = (\frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle :$

\[ \cos(\phi) = \left( \frac{1}{\sqrt{3}} \right) \left[ \cos(\theta) - \sqrt{2} \sin(\theta) \right] \quad (6.21) \]

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\[
\sin(\phi) = \left(\frac{1}{\sqrt{3}}\right) \left[ \sin(\theta) + \sqrt{2} \cos(\theta) \right] 
\]

and

\[
\epsilon = \left(\frac{\sqrt{3}}{4}\right) \left[ \frac{m_d - m_u}{m_s - m} \right] \{ 1 + \frac{\Lambda}{3} \} \tag{6.22a}
\]

\[
\Delta = \frac{(4m_K^2 - m_\pi^2 - 3m_\eta^2)}{(m_\eta^2 - m_\pi^2)} . \tag{6.22b}
\]

The parameter \(\epsilon\) is evaluated by using the numerical values (ref.11)

\[
\left[ \frac{m_d - m_u}{m_s - m} \right] = \left(\frac{1}{43.5}\right) \text{ and } \Delta = 0.21 \text{ giving } \epsilon = 1.07 \times 10^{-2} .
\]

essentially determines the QCD correction to the pion-nucleon coupling constants. The quark-line rule, namely that the \(|\eta_s> = |\bar{\eta}s>\) component of \(|\eta>\) and \(|\eta'>\) is not coupled to nucleons and by using Eqs(2.57) and (2.58), gives (ref.9)

\[
\frac{g_{\eta'}}{g_\eta} = \frac{g_{\eta'}_{pp}}{g_{\eta_{pp}}} = \tan(\phi) \tag{6.23}
\]

Using Eqs(6.3), (6.20-22) and (6.23), we have finally

\[
\delta_{QCD} = -2 \epsilon \left[ 1 + \frac{(m_\eta^2 - m_\pi^2)}{(m_\eta^2 - m_\eta')^2} \tan^2(\phi) \right] \times 3 \cos(\phi) \left( \frac{g_\eta}{g_\pi} \right) . \tag{6.24}
\]

Using the convention of ref.12 for TPEP, we find CSB
corrections to OPEP and TPEP due to $\delta_0$ and $\delta_{QCD}$ \{ as derived in appendix B, with $\delta_0$ replaced by $\delta_0 + \delta_{QCD}$ \} as follows:

$$\Delta V_{CSB} = \Delta V_{OPEP}^{CSB} + \Delta V_{TPEP}^{CSB}$$

$$= \left[ f^2 \frac{\exp(-m_\pi r)}{r} - 2 \{ A(00) + B(00) \} \right] 2 \delta$$

(6.25)

where

$$\delta = \delta_0 + \delta_{QCD}.$$  \hspace{1cm} (6.26)

Note that in ref.(3) $[A(11) + B(11)]$ has been used instead of $[A(00)+B(00)]$ but the difference between the two expressions which is due to the $\pi^\pm - \pi^0$ mass difference effect will be of higher order in Eq(6.25).

As we see from Eq.(6.24), $\delta_{QCD}$ depends on the mixing angle $\phi$ or $\theta$ and the ratio $\frac{q_\eta}{q_\pi}$. Using again that the $|\eta_5>$ coupling to nucleons is zero and SU(3) symmetry, one obtains (see 2.3)

$$\frac{q_\eta}{q_\pi} = \cos(\phi) \ [3F-D]$$

(6.27)

with $F+D = 1$, where as discussed in references 8 and 9, one may have $1.7 \leq \frac{D}{F} \leq 3$. Recently values of $-\theta[\phi]$ larger \{smaller\} than the standard value (ref.13) $-\theta = 13^\circ$ \{\$= 42^\circ\}$
have been obtained (ref.14) from two photon decays of \( \eta \) and \( \eta' \). For the numerical results in Table 6.1 we thus employ \( 13^\circ \leq \theta \leq 25^\circ \) and \( 1.7 \leq \frac{D}{F} \leq 3 \). In obtaining these results, we have used as in ref.3 the Reid soft core potential (ref.15) to generate the wave function \( u(r) \) in the integral of Eq.(4.1).

Before we discuss the implications of these results in the next section, we give the CSB corrections for the \( \eta \) and \( \eta' \) exchange potentials due to \( H_{QCD} \). Let us define

\[
\begin{align*}
g_{\eta pp} & = g_\eta (1 + \frac{1}{2} \delta_\eta^{QCD}) \\
g_{\eta' pp} & = g_{\eta'} (1 + \frac{1}{2} \delta_{\eta'}^{QCD}) \\
g_{\eta nn} & = g_\eta (1 - \frac{1}{2} \delta_\eta^{QCD}) \\
g_{\eta' nn} & = g_{\eta'} (1 - \frac{1}{2} \delta_{\eta'}^{QCD})
\end{align*}
\]

where \( \delta_{QCD} \) can be obtained from the first and second part of Eq.(6.24) by interchanging \( \eta, \eta' \) by \( \pi \) so that

\[
\delta_\eta^{QCD} = -(2\sqrt{3} \varepsilon) \cos(\phi) \frac{g_\pi}{g_\eta} = \delta(1) \frac{g_\pi^2}{g_\eta^2}
\]

(6.30)
\[ \delta_{QCD}^{\eta'} = \delta_{QCD}^{(2)} \left( \frac{g_\pi^2}{g_\eta^2} \right) \]

where

\[ \delta_{QCD}^{(1)} = -\varepsilon \ 2\sqrt{3} \ \cos(\phi) \ \left( \frac{g_\eta}{g_\pi} \right) \]  \hspace{1cm} (6.31a)

\[ \delta_{QCD}^{(2)} = \left[ \frac{(m_\eta^2 - m_\pi^2)}{m_\eta^2 - m_\pi^2} \right] \tan^2(\phi) \ \delta_{QCD}^{(1)} \]  \hspace{1cm} (6.31b)

so that

\[ \delta_{QCD} = \delta_{QCD}^{(1)} + \delta_{QCD}^{(2)} \] \hspace{1cm} (6.32)

Using Eqs. (6.24), and (6.29-33) we obtain for CSB in OPEP and One \( \eta \) or \( \eta' \) Exchange Potential (O\( \eta, \eta' \)EP) the following expression:

\[ \Delta V_{CSB}^{OPEP}(QCD) = f^2 \left\{ \left( \frac{\exp(-m_\pi r)}{r} \right) 2 \delta_{QCD}^{(1)} - \left( \frac{m_\eta}{m_\pi} \right)^2 \left( \frac{\exp(-m_\eta r)}{2} \right) 2 \delta_{QCD}^{(1)} \right. \]

\[ \left. - \left( \frac{m_\eta'}{m_\pi} \right)^2 \left[ \frac{\exp(-m_\eta' r)}{r} \right] 2 \delta_{QCD}^{(2)} \right\} \] \hspace{1cm} (6.33)

In Fig. 9, we display the integrand of integral (4.1) for OPEP and OBEPEP, the integrals of which give the corresponding
contributions to $\delta a$ in Table 6.1. As is seen from Table 6.1 and as was also found in refs. 8 and 9, there is a cancellation between CSB contributions in OPEP and $0\eta\eta'\text{EP}$. This is brought about by the relation (6.31) almost independent of values of $-\theta$ and $\frac{g_\eta}{g_\pi}$, although the cancellation tends to be more complete for $-\theta = 13^\circ$ for a given value of D/F.
6.3 - DISCUSSION AND CONCLUSIONS:

From Table 6.1, where we also give the QED-CSB contribution (ref.3) \( \delta \), to \( \delta a \) for both OPEP and TPEP for comparison, one observes the following:

\[(i) \quad (\delta a)^{\text{TPEP}}_{\text{QED}} = 3 (\delta a)^{\text{OPEP}}_{\text{QED}}, \quad (6.34)\]

with

\[(\delta a)^{\text{QED}}_{\text{QED}} = 1.2 \text{ fm} = (\delta a)^{\text{OPEP}}_{\text{QED}} + (\delta a)^{\text{TPEP}}_{\text{QED}}\]

\[(ii) \quad (\delta a)^{\text{TPEP}}_{\text{QCD}} = 3 (\delta a)^{\text{OPEP}}_{\text{QCD}}, \quad (6.35)\]

the sign for both of these expressions is opposite to the corresponding QED expressions.

\[(iii) \text{ There is strong cancellation between } (\delta a)^{\text{OPEP}}_{\text{QCD}} \text{ and } (\delta a)^{\text{Oe}, \eta', \text{EP}}_{\text{QCD}} \text{ essentially because of the relation } (6.31) \text{ which shows that}\]

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\[ \delta_{\text{QCD}} (\pi) = \delta_{\text{QCD}}^{(1)} (\eta) + \delta_{\text{QCD}}^{(2)} (\eta') \]

(iv) Unless there is strong cancellation between \((\delta a)^{\text{TPEP}}\) and the corresponding CSB contributions from two boson \([\pi^0, (\eta, \eta')]\) and \([((\eta, \eta'), (\eta, \eta'))\) potential, \((\delta a)^{\text{TPEP}}_{\text{QCD}}\) is too large to be consistent with the small experimental value for \((\delta a)^{\text{CSB}}\) except when \(D/F = 2.8\) implying a very small value for
\[ \left( \frac{g_\pi^2}{4\pi} \right) = 0.02. \]

We now discuss the last point in some detail. Using again the convention of ref.12 so that denoting the \((\eta, \pi^0)\) and \((\eta, \eta')\) etc. exchange contributions from uncrossed and crossed diagrams as shown in Fig.8 can be written as follows:

\[
\overline{A}(O, \eta) = \left( \frac{g_\pi^2}{g_\eta^2} \right) A(O, \eta)
\]

\[
\overline{A}(\eta, \eta) = \left( -\frac{g_\pi^2}{g_\eta^2} \right) A(\eta, \eta), \quad \text{(6.36)}
\]

and similarly for \(B\) and \(\eta'\). Thus we can write

\[
\delta V_{\text{TPEP}}^{\text{CSB}} = -2 \left[ A(O, O) + B(O, O) \right] 2 \delta_{\text{QCD}}
\]

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+2 \left[ A(O, \eta) + B(O, \eta) \right] \left[ - \left( \frac{\eta}{2} \right)^2 \delta^{(1)}_{\text{QCD}} + \delta^{(1)}_{\text{QCD}} \right] \\
+2 \left[ A(\eta, \eta) + B(\eta, \eta) \right] \left[ 2 \left( \frac{\eta}{2} \right)^2 \delta^{(1)}_{\text{QCD}} + \right] \\
\quad + \eta \to \eta' \quad \text{and} \quad \delta^{(1)}_{\text{QCD}} \to \delta^{(2)}_{\text{QCD}} \\
(6.37)
FIGURE 8

Feynman diagrams of two boson exchange $\pi^0 - \eta$. Graphs (a) to (e) represent uncrossed TBEP diagrams, and graphs (f) to (j) represent the crossed TBEP diagrams. Note that we can write other possible diagrams of TBEP by considering $\eta \rightarrow \eta'$ and $\pi^0 \rightarrow \eta'$. 
For OBEP in Eq.(6.33), assuming that $\pi, \eta$ and $\eta'$ dependent terms contribute equally to $\delta a$ the cancellation is due to (6.32). This assumption for OBEP is supported by the results in Table 6.1. Making, therefore, a similar assumption that $A$ and $B$ do not depend very much on the $\pi, \eta$ and $\eta'$ masses, we obtain from Eq.(6.37), on using Eq.(6.32),

$$
\Delta V_{\text{CSB}}^{\text{TPEP}} = -4 \frac{g_\eta^2}{g_\pi^2} \delta_{\text{QCD}} + [\{2(\frac{g_{\eta'}^2}{g_\pi^2}) - 2(\frac{g_\eta^2}{g_\pi^2})\}] \delta_{\text{QCD}} + 2\delta_{\text{QCD}}^{(1)} + 2\delta_{\text{QCD}}^{(2)}
$$

$$
+ 2 \left[ \left(\frac{g_{\eta'}^2}{g_\pi^2}\right) + \left(\frac{g_\eta^2}{g_\pi^2}\right) \right] \left[\delta_{\text{QCD}}^{(1)} + \delta_{\text{QCD}}^{(2)} \right]
$$

$$
+ 2 \frac{g_{\eta'}^2}{g_\pi^2} \delta_{\text{QCD}}^{(1)} + \frac{g_{\eta'}^2}{g_\pi^2} \delta_{\text{QCD}}^{(2)}
$$

$$
= -2 \left[1 - \left(\frac{g_{\eta'}^2}{g_\pi^2}\right)\right] \delta_{\text{QCD}}^{(1)} - 2\left[1 - \left(\frac{g_\eta^2}{g_\pi^2}\right)\right] \delta_{\text{QCD}}^{(2)}
$$

(6.38)

Thus we see that for values of $\left(\frac{g_{\eta}}{g_\pi}\right)$ and $\left(\frac{g_{\eta'}}{g_\pi}\right)$ in Table 6.1 we do not see a severe cancellation for TPEP similar to the one for OBEP. If this conjecture is true, then one can conclude that the small experimental value for $(\delta a)_{\text{CSB}}$ strongly constrains the $\eta$-nucleon coupling constant to be very small ($<0.02$). In any case the QCD-CSB contribution to TPEP is

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important and study of CSB effect due to
H_{QCD} through π^0-η, η' mixings appears to strongly constrain
the η-nucleon coupling constant to be very small.

The QCD correction to coupling constants contributing to CI
is given by considering Fig.3 (a→c) and using Eq.(6.3):

\[ V_{np} = -(1+\frac{\delta_{QCD}}{2})(1-\frac{\delta_{QCD}}{2}) C(0) + 2 C(1) \]  \hspace{1cm} (6.39a)

\[ V = \frac{1}{2}[(1+\frac{\delta_{QCD}}{2})^2 C(0) + (1-\frac{\delta_{QCD}}{2})^2 C(0)] = \frac{1}{2}[V_{nn}+V_{pp}] \]  \hspace{1cm} (6.39b)

Thus

\[ \Delta V_{CI} = -2 C(0) + 2 C(1) - \frac{\delta_{QCD}^2}{4} C(0) \]  \hspace{1cm} (6.39c)

neglecting higher order term \( \delta_{QCD}^2 \ll 1 \), we have

\[ \Delta V_{CI,CC}^{QCD} = 0 \]
TABLE 6.1

QED and QCD CSB contributions to the scattering lengths:

\[ \delta a = a_{pp} - a_{nn} \]

<table>
<thead>
<tr>
<th>Type of correction</th>
<th>$\theta^\circ$</th>
<th>$\phi^\circ$</th>
<th>$D/F$</th>
<th>$\frac{g_{ppn'}}{g_{pp\eta}}$</th>
<th>$\frac{g_{pp\eta}}{g_{pp\pi^0}}$</th>
<th>OPEP $\delta a$(fm)</th>
<th>TPEP $\delta a$(fm)</th>
<th>$\eta'\eta$ EP $\delta a$(fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED Ref(3)</td>
<td>~</td>
<td>~</td>
<td>~</td>
<td>~</td>
<td>~</td>
<td>0.283</td>
<td>0.927</td>
<td>~</td>
</tr>
<tr>
<td>QED</td>
<td>-13</td>
<td>42</td>
<td>1.7</td>
<td>0.900</td>
<td>0.358</td>
<td>-1.621</td>
<td>-5.304</td>
<td>1.659</td>
</tr>
<tr>
<td></td>
<td>-19</td>
<td>36</td>
<td>-</td>
<td>0.727</td>
<td>0.390</td>
<td>-1.748</td>
<td>-5.718</td>
<td>1.890</td>
</tr>
<tr>
<td></td>
<td>-25</td>
<td>30</td>
<td>-</td>
<td>0.577</td>
<td>0.477</td>
<td>-1.862</td>
<td>-6.089</td>
<td>2.101</td>
</tr>
<tr>
<td>QCD</td>
<td>-13</td>
<td>42</td>
<td>2.0</td>
<td>0.900</td>
<td>0.248</td>
<td>-1.123</td>
<td>-3.674</td>
<td>1.149</td>
</tr>
<tr>
<td></td>
<td>-19</td>
<td>36</td>
<td>-</td>
<td>0.727</td>
<td>0.270</td>
<td>-1.210</td>
<td>-3.959</td>
<td>1.309</td>
</tr>
<tr>
<td></td>
<td>-25</td>
<td>30</td>
<td>-</td>
<td>0.577</td>
<td>0.289</td>
<td>-1.290</td>
<td>-4.220</td>
<td>1.456</td>
</tr>
<tr>
<td>QCD</td>
<td>-13</td>
<td>42</td>
<td>2.8</td>
<td>0.900</td>
<td>0.039</td>
<td>-0.177</td>
<td>-0.578</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>-19</td>
<td>36</td>
<td>-</td>
<td>0.727</td>
<td>0.043</td>
<td>-0.193</td>
<td>-0.630</td>
<td>0.209</td>
</tr>
<tr>
<td></td>
<td>-25</td>
<td>30</td>
<td>-</td>
<td>0.577</td>
<td>0.046</td>
<td>-0.205</td>
<td>-0.672</td>
<td>0.323</td>
</tr>
</tbody>
</table>
FIGURE 9

Integrands of equ.(4.1) for indicated potential with $\phi = 42^0$ and $\frac{D}{F} = 2$ in QCD. The integrand of OPEP - $\Lambda\mu$ is shown for comparison.
REFERENCES:


Riazuddin and Fayyazuddin, to be published.


APPENDIX A

In this appendix, we will derive different terms of the TPEP. Referring to Fig.2, we will be using the same notation as in ref(12) where each potential is denoted by \( V(\text{PPII}_3) \) where PP stands for two pions, I=1,0 for the isospin and \( I_3=+,0,- \) for the third component of isospin. The uncrossed diagrams are denoted by A and crossed diagrams by B. The two amplitudes A or B are assigned two numbers 0 or 1 where 0 stands for \( \pi^0 \) and 1 for \( \pi^\pm \). The relative coupling constants are shown at each vertex of Fig.3, as given in chapter 4.

From Fig.3 (d),

\[
V_A(\text{PPL}^+) = A(00) \tag{1A}
\]

\[
V_A(\text{PPL}^0) = A(00) + (\sqrt{2})^4 A(11) - (\sqrt{2})^2 A(01) - (\sqrt{2})^2 A(10). \tag{2A}
\]

From Fig.3 (i) and (j), we get

\[
V_B(\text{PPL}^+) = B(00) + (\sqrt{2})^4 B(11), \tag{3A}
\]

\[
V_B(\text{PPL}^0) = B(00) + (\sqrt{2}) B(01) + (\sqrt{2})^2 B(10), \tag{4A}
\]

Note that \( V(\text{PPL}^-) = V(\text{PPL}^+) \) in (1A) and (3A).

Al
Thus we see that the complete \( V(PP1^+) \) satisfies

\[
V(PP1^+) = V_A + V_B
\]

\[
V(PP1^+) = A(00) + B(00) + 4B(11)
\]  \hspace{1cm} (5A)

and also \( V(PP10) \) satisfies

\[
V(PP10) = A(00) + 4A(11) - 4A(10) + B(00) + 4B(10)
\]  \hspace{1cm} (6A)

Note that \( A(10) = A(01) \) and \( B(10) = B(01) \) and

\[
\Delta V_{TPEP} = V(PP1^+) - V(PP10)
\]

\[
= 4 \left\{ [B(11) - A(11)] - [B(10) - A(10)] \right\}
\]  \hspace{1cm} (7A)

To derive eqn. (4.15) for TPEP due to the corrections to the coupling constants and using eqn. (4.15), we can see that;

\[
V_A(PP1^+) = A(00) \hspace{1cm} \text{(unchanged)}
\]

\[
V_A(PP1^+) = A(00) + \left\{ \sqrt{2 \left( \frac{1+\delta_c}{2} \right)} \right\}^4 A(11) - 4A(01)
\]  \hspace{1cm} (8A)
\[ V_B(PP1+) = B(00) + \left( \sqrt{2(1 + \delta_c)} \right)^4 B(11) \]  

\[ V_B(PP10) = B(00) + 4B(10), \]  

so that

\[ \Delta V_{TPEP}^{cc} + \Delta V_{TPEP}^{\Delta m} = V_A(PP1+) + V_B(PP1+) - V_A(PP10) - V_B(PP10), \]

\[ = A(00) + B(00) + 4(1 + 2\delta_c + \delta_c^2) B(11) - A(00) - 4(1 + 2\delta_c + \delta_c^2) A(11) + 4A(01) - B(00) - 4B(010) \]  

Neglecting higher order terms since \( \delta_c^2 \ll 1 \).

\[ \Delta V_{TPEP}^{cc} + \Delta V_{TPEP}^{\Delta m} = 4\{ B(11) - A(11)\} - \{B(01) - A(01)\} + 8 \delta_c \{ B(11) - A(11) \} \]

from (13A),

A3
\[ \Delta V_{\text{TPEP}}^{\text{CC}} = 8 \delta_{\text{C}} \{B(11) - A(11)\} \]  

(14A)

where \( A \) and \( B \) are given by

\[ A(\alpha) = \left\{ \frac{G_{\pi}^2}{(4\pi^2)} \right\} \{X_A(\alpha) + Y_A(\alpha) + W_A(\alpha) + Z_A(\alpha) + Z_B(\alpha)\}, \]  

(15A)

\[ B(\alpha) = \left\{ \frac{G_{\pi}^2}{(4\pi^2)} \right\} \{X_B(\alpha) + X_C(\alpha)\}, \]  

(16A)

\( J=1,2,3,4 \) represent central, tensor and spin-spin terms.

\( X, Y, W \) and \( Z \) are integrals whose form are given in eqn.'s[19A,---27A] where

\[ G_{\pi}^2 = 14.3 \] (is the N-N coupling constant)

Then

\[ A(\alpha) = \{A(1) + \Omega_{\text{SO}} A(2) + \Omega_{\text{T}} A(3) + \Omega_{\text{SS}} A(4)\}(\alpha) \]  

\[ B(\alpha) = \{B(1) + \Omega_{\text{SO}} B(2) + \Omega_{\text{T}} B(3) + \Omega_{\text{SS}} A(B)\}(\alpha) \]  

(17A)

where \( A(1) \), \( B(1) \) arise from the central potential

A(2), B(2) " " the spin orbit potential

A(3), B(3) " " the tensor potential

A(4), B(4) " " the spin - spin potential

A4
Note that $\Omega_{s_0}$ and $\Omega_T$ are equal to zero in our case and $\alpha$ stands for the mass of the pion. In case of A(00) or B(00), the $\mu^0$ -mass is used instead of $\mu$. In A(01) or B(01), the $M$ -mass is used where $M^2 = (\mu_0^2 + \mu_c^2)$. In A(11) or B(11), the $\mu^c$ -mass is used. For more details see (ref.12).

The final forms of A and B which are used in this thesis after making appropriate change of variables suggested by Appendix B in ref(12), are as follows:

$$X_B(1) = 6\pi^3 \left( \frac{u}{x} \right) \int_0^1 \int_0^1 du \, d\beta \left[ \frac{u^5}{(1-u^3)(1-\beta^2)} \right] \times \exp \left[ -\frac{2}{\sqrt{(1-u^3)(1-\beta^2)}} \right] \times \left[ 1 + \left( \frac{m}{u} \right)^2 \frac{u^6}{(1-u^3)} \right]^{1/2} \mu r \right] , \quad (19A)$$

$$X_C(1) = -12\pi^3 \left( \frac{1}{x} \right) (m/\mu) \int_0^1 \int_0^1 du \, d\beta \left[ u^{11}/(1-u^3)^{2.5} (1-\beta^2)^{1.5} \right] \times \left\{ (m/\mu)^2 + [u^6/(1-u^3)] \right\}^{-1/2} \times \exp \left[ -2(1-u^3)(1-\beta^2)^{1/2} \right] \left[ 1 + \left( \frac{m}{u} \right)^2 \frac{u^6}{(1-u^3)} \right]^{1/2} \mu r \} , \quad (20A)$$

$$Y_A(1) = -12\pi^3 \left( \frac{1}{x} \right) \mu \int_0^1 \int_0^1 du \, d\beta \left[ u^2/(1-u^6)(1-\beta^2) \right] \times \left\{ (1-x)^2 \right\}$$

A5
\[
\exp\left\{-\frac{2}{\left(1-u^6(1-\beta^2)\right)^{1/2}}\right\} (1+\frac{m}{\mu})^2 \left\{u^6/(1-u^6)\right\}^{1/2} \mu r, \quad (21A)
\]

\[
Z_B(1) = 72\pi^3 (1/x) \mu \int \int \int du \, d\beta \, dw \left\{u^2 w^2 / \left\{(1-u^6)(1-\beta^2)\right\}\right\} \times
\]

\[
\exp\left\{-\frac{2}{\left(1-u^6(1-\beta^2)\right)^{1/2}}\right\} (1+\frac{m}{\mu})^2 \left\{u^6/(1-u^6)\right\}^{1/2} \mu r, \quad (22A)
\]

A = A(1) + \Omega_{so} A(2) + \Omega_{T} A(3) + \Omega_{ss} A(4), \quad (23A)

Including the \(\Omega_{ss}\) term with \(\Omega_{ss} = -3\) we then find that

\[
\{B(1)-A(1)\} = \left\{\frac{G\pi^4}{(4\pi^4)} \right\} \left\{2X_B(1) + X_C(1) - Y_A(1) - 3X_B(4) \right\} \]

\[
+ 3 \left\{W_A(4) + Z_A(4) - Z_B(1) \right\}, \quad (24A)
\]

Note that \(X_A(1) = -X_B(1)\) (see ref(12)).

\[
X_B(4) = 4\pi^3 (1/x) (\mu/m)^2 \mu \int \int \int du \, d\beta \, \left\{1+\frac{m}{\mu}\right\}^2 \left\{u^6/(1-u^3)\right\} \times \left\{u^5/\left\{(1-u^3)(1-\beta^2)\right\}^2\right\}
\]

\[
\exp\left\{-\frac{2}{\left(1-u^3(1-\beta^2)\right)^{1/2}}\right\} (1+\frac{m}{\mu})^2 \left\{u^6/(1-u^3)\right\}^{1/2} \mu r, \quad (25A)
\]

A6
\[
W_A(4) = 12\pi^3 \left(1/x\right) (\mu/m)^2 \mu \int_0^1 \int_0^1 \int_0^1 \frac{du \, d\beta \, dw}{w^4 \{\left(1-u^3\right)(1-\beta^2)\}^2} \left\{ u^5 / w^4 \{\left(1-u^3\right)(1-\beta^2)\}^2 \right\} \\
\times \{1+(m/\mu)^2 [u^6/(1-u^3)]\} \times \exp\left\{ -2/\{\left(1-u^3\right)(1-\beta^2)\}^{1/2} \right\} (1+(m/\mu)^2 [u^6/(w^6(1-u^3))]^{1/2} \mu r \right\},
\]

(26A)

\[
Z_A(4) = -24\pi^3 \left(1/x\right) (\mu/m)^2 \mu \int_0^1 \int_0^1 \int_0^1 \frac{du \, d\beta \, dw}{w^4 \{\left(1-u^6\right)(1-\beta^2)\}^2} \left\{ u^8 / w^4 \{\left(1-u^6\right)(1-\beta^2)\}^2 \right\} \\
\times \{1+(m/\mu)^2 [u^6/(1-u^6)]\} \times \exp\left\{ -2/\{\left(1-u^6\right)(1-\beta^2)\}^{1/2} \right\} (1+(m/\mu)^2 [u^6/(w^6(1-u^6))]^{1/2} \mu r \right\},
\]

(27A)
To derive the correction due to $H_{\text{QED}}$ in TPEP, we use the same notation as in Appendix A. Using Eqn. (4.4) for writing the potentials in TPEP and Fig. 2, and noting that

\[ V_{pp}(\text{PP}1^+) \neq V_{nn}(\text{PP}1^-) \]

(1B)

\[ V_{pp}(\text{PP}10) \neq V_{nn}(\text{PP}10) \]

Eqn. (1B) holds because of relation (4.4), i.e. the couplings are different in n-n and p-p pairs.

so that

\[ V_{pp}(\text{pp}1^+) = (1-\delta_g/2)^2 A(00) + (1-\delta_g/2)^2 B(00) + 4B(11) \]

(2B)

\[ V_{pp}(\text{pp}10) = (1+\delta_g/2)^2 A(00) + 4A(11) - 4A(10) + (1-\delta_g/2)^2 B(00) + 4B(11) \]

(3B)

\[ V_{nn}(\text{nn}1^-) = (1+\delta_g/2)^2 A(00) + (1+\delta_g/2)^2 B(00) + 4B(11) \]

(4B)
\[ V_{nn}(pp10) = (1+\delta \delta /2)^2 A(00) + 4A(11) - 4A(10) \]

\[ + (1+\delta /2)^2 B(00) + 4B(11) , \]  \hspace{1cm} (5B)

such that

\[ \Delta V^{\text{TPEP}}_{\text{CSB}} = \{V_{pp}(pp1+) + V_{pp}(PP10)\} - \{V_{nn}(pp1-) + V_{nn}(pp10)\}, \]  \hspace{1cm} (6B)

Neglecting higher order terms in Eqn. (6B), since \( \delta \delta^2 \ll 1 \), we get,

\[ \Delta V^{\text{TPEP}}_{\text{CSB}} = -4\delta \delta \{ A(00) + B(00) \} , \]  \hspace{1cm} (7B)
APPENDIX C

This subroutine is for solving second order differential equation, and was taken from SSP library.

```
C SUBROUTINE RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C
DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)
DOUBLE PRECISION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1),
X,H

DO 1 I=1,NDIM
1  AUX(8,I)=.06666667*DERY(I)
  X=PRMT(1)
  XEND=PRMT(2)
  H=PRMT(3)
  PRMT(5)=0.
  CALL FCT(X,Y,DERY)

C ERROR TEST
  IF(H*(XEND-X))38,37
3    2
37   2
C  PREPARATIONS FOR RUNGE-KUTTA METHOD
2    A(1)=.5
2    A(2)=.2928932
2    A(3)=1.707107
2    A(4)=.1666667
2    B(1)=2.
2    B(2)=1.
2    B(3)=1.
2    B(4)=2.
2    C(1)=.5
2    C(2)=.2928932
2    C(3)=1.707107
2    C(4)=.5

C  PREPARATIONS OF FIRST RUNGE-KUTTA STEP
3    DO 1 I=1,NDIM
3      AUX(1,I)=Y(I)
3      AUX(2,I)=DERY(I)
3      AUX(3,I)=0.
3      AUX(6,I)=0.
3      IREC=0
3      H=H+H
```
IHLF=-1
ISTEP=0
IEND=0

C
C START OF A RUNGE-KUTTA STEP
4 IF((X+H-XEND)^2)*H)7,6,5
5 H=XEND-X
6 IEND=1
C
C RECORDING OF INITIAL VALUES OF THIS STEP
7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)
   IF(PRMT(5))40,8,40
8 ITEST=0
9 ISTEP=ISTEP+1
C
C START OF INNERMOST RUNGE-KUTTA LOOP
J=1
10 AJ=A(J)
BJ=B(J)
CJ=C(J)
DO 11 I=1,NDIM
   R1=H*DERY(I)
   R2=AJ*(R1-BJ*AUX(6,I))
   Y(I)=Y(I)+R2
   R2=R2+R2+R2
11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
   IF(J-4)12,15,15
12 J=J+1
   IF(J-3)13,14,13
13 X=X+.5*H
14 CALL FCT(X,Y,DERY)
   GOTO 10
C
C END OF INNERMOST RUNGE-KUTTA LOOP
C
C TEST OF ACCURACY
15 IF(ISTEST)16,16,20
C
C IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
16 DO 17 I=1,NDIM
17 AUX(4,I)=Y(I)
   ITEST=1
   ISTEP=ISTEP+ISTEP-2
18 IHLF=IHLF+1
   X=X-H
   H=.5*H
   DO 19 I=1,NDIM
   Y(I)=AUX(1,I)
   DERY(I)=AUX(2,I)
19  AUX(6,I)=AUX(3,I)
   GOTO 9

C    IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
20  IMOD=ISTEP/2
    IF(ISTEP-IMOD-IMOD)21,23,21
21  CALL FCT(X,Y,DERY)
    DO 22 I=1,NDIM
       AUX(5,I)=Y(I)
22  AUX(7,I)=DERY(I)
   GOTO 9

C    COMPUTATION OF TEST VALUE DELT
23  DELT=0.
    DO 24 I=1,NDIM
24  DELT=DELT+AUX(8,I)*DABS(AUX(4,I)-Y(I))
    IF(DELT-PRMT(4))28,28,25

C    ERROR IS TOO GREAT
25  IF(IHLF-10)26,36,36
26  DO 27 I=1,NDIM
27  AUX(4,I)=AUX(5,I)
    ISTEP=ISTEP+ISTEP-4
    X=X-H
    IEND=0
   GOTO 18

C    RESULT VALUES ARE GOOD
28  CALL FCT(X,Y,DERY)
    DO 29 I=1,NDIM
       AUX(1,I)=Y(I)
       AUX(2,I)=DERY(I)
       AUX(3,I)=AUX(6,I)
       Y(I)=AUX(5,I)
29  DERY(I)=AUX(7,I)
    CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)
    IF(PRMT(5))40,30,40
30  DO 31 I=1,NDIM
    Y(I)=AUX(1,I)
31  DERY(I)=AUX(2,I)
    IREC=IHLF
    IF(IEND)32,32,39

C    INCREMENT GETS DOUBLED
32  IHLF=IHLF-1
    ISTEP=ISTEP/2
    H=H+H
    IF(IHLF)4,33,33
33  IMOD=ISTEP/2
    IF(ISTEP-IMOD-IMOD)4,34,4
34  IF(DELT-.02*PRMT(4))35,35,4

C3
35 IHLF=IHLF-1  
   ISTEP=ISTEP/2  
   H=H+H  
   GOTO 4

C
C RETURNS TO CALLING PROGRAM
36 IHLF=11  
   CALL FCT(X,Y,DERY)  
   GOTO 39
37 IHLF=12  
   GOTO 39
38 IHLF=13  
39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
40 RETURN
END

C******************************************************************************

DOUBLE PRECISION PRMT(5),AUX(8,2),DERY(2),Y(2),X,C
EXTERNAL FCT,OUTP
C=18.2  
PRMT(1)=0.02  
PRMT(2)=4.6*0.7  
PRMT(3)=0.05*0.7  
PRMT(4)=0.0001  
Y(1)=1.0/(C*0.7)  
Y(2)=1.0+(10.0/(C*0.7))  
Y(1)=0.1  
Y(2)=0.000  
DERY(1)=0.5  
DERY(2)=0.5
NDIM=2
CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
STOP
END
SUBROUTINE FCT(X,Y,DERY)
DOUBLE PRECISION Y(2),DERY(2),H,C,V,X,C1,C2,C5
H=10.463  
C1=1650.6  
C2=6484.2  
C=938.26/((197.4**2)*(0.7**2))  
DERY(1)=-C*((H*DEXP(-X)+C1*DEXP(-4.0*X)-C2*DEXP(-7.0*X))/(X))*Y(2)*2  
DERY(2)=Y(1)
RETURN
END
SUBROUTINE OUTP(X,Y,DERY,IREC,NDIM,PRMT)
DOUBLE PRECISION Y(2),DERY(2),PRMT(5),X,V,C1,C2,H
WRITE(9,2)X,Y(2)
FORMAT (1X,2F15.7)
RETURN
END

C4
This programme is for multi-dimentional integration and was used with the subroutine QB01A taken from HARWEL SUBRUTINE LIBRARY.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION MAXCO(6),ITYPE(6),ICOUNT(6),R(100),U(100),X1(100)
COMMON R,U,I,MMAX
ITYPE(1)=0
ITYPE(2)=0
ITYPE(3)=0

C
L=48
N=95

C SET THE NO. OF INTEGRATIONS
MMAX=2
C SET THE ERROR REQUIRED
ERROR=0.0001

2 FORMAT(1X,2F15.7)
DO 10 I=1,N
10 READ(9,2)X1(I),U(I)
DO 20 I=54,N
R(I)=X1(I)/.7
CALL QB01A(ERROR,MAXCO,ITYPE,MMAX,F,ICOUNT)

20 WRITE(7,2)R(I),F
STOP
END

C
C
C
C SUBROUTINE LIMITS(M,A,B,G,X,F,FA)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R(100),U(100),FA(6),X(6),A1(6),B1(6)
COMMON R,U,I,MMAX
A1(1)=0.000464
A1(2)=0.1D -9
B1(1)=.9999996
B1(2)=.999999
A=A1(M)
B=B1(M)
G=0.0
FA(M)=1.0
IF(M.EQ.MMAX+1) THEN

C EVALUATE THE FUNCTION TO BE INTEGRATED
C F=X(2)*X(3)**2*EXP(-X(1)*X(2)*X(3)*R(I))
CP=(14.4**2)*6.0/(3.14159)**139.6

C5
C = -150.0
C2 = 45.2551
C3 = 0.70745
C4 = 2.0/C2
C5 = 1.0
C6 = -2.0*C2
C X B IS MUTL. BY 2.0, SINCE B(1)-A(1) =XB+XC-XA-YA-ZB
C BUT XB=-XA. C3=(NUCLEON MASS/M)**2, C4=M/CHARGED PION
C
T1 = DLLOG(2.0*((X1)**5)/((1.0-X1)**3)*(1.0-X2)**2)) +
((-2.0/(DSQRT(1.0-X1)**3)*DSQRT(1.0-X2)**2)) * DSQRT(C1 +
(X1)**6.0)*C2/(1.0-X1)**3)*R(1)*C3)
IF(T1.LE.C) THEN
   XB = 0.0
ELSE
   XB = DEXP(T1)
END IF

C
C
C
T2 = DLLOG(((X1)**11)/(((1.0-X1)**3)**2.5)*(1.0-X2)**2*(1.0-X2)**3)*
(1.0/(DSQRT(C1+C2*(X1)**6)/(1.0-X1)**3))) +
((-2.0/(DSQRT(1.0-X1)**3)*DSQRT(1.0-X2)**2)) * DSQRT(C1 +
(X1)**6.0)*C2/(1.0-X1)**3)*R(1)*C3)
IF(T2.LE.C) THEN
   XC = 0.0
ELSE
   XC = DEXP(T2)
END IF

C
C
C
T4 = DLLOG(((X1)**2)/((1.0-X1)**6)*(1.0-X2)**2)) +
((-2.0/(DSQRT(1.0-X1)**6)*DSQRT(1.0-X2)**2)) * DSQRT(C1 +
(X1)**6.0)*C2/(1.0-X1)**6)*R(1)*C3)
IF(T4.LE.C) THEN
   YA1 = 0.0
ELSE
   YA1 = DEXP(T4)
END IF

C
C
C
T5 = DLLOG((X1)**5*(C1+C2*(X1)**6)/(1.0-X1)**3)/((1.0-X1)**3)*
(1.0-X2)**2)*2) +
((-2.0/(DSQRT(1.0-X1)**3)*DSQRT(1.0-X2)**2)) * DSQRT(C1 +
(X1)**6.0)*C2/(1.0-X1)**3)*R(1)*C3)
IF(T5.LE.C) THEN
   XB4 = 0.0
ELSE
   XB4 = DEXP(T5)
END IF

C
C
C
C6
F=CP*((XB-XB4*C4)/(R(I)*C3)+C6*XC)
XB+2.0*YA1-XB4*C4)/(R(I)*C3)+C6*XC)
END IF
RETURN
END