A WAVE DISPERSION MODEL FOR HEALTH MONITORING OF PLATES WITH PIEZOELECTRIC COUPLING IN AEROSPACE APPLICATIONS

Ahmed Z. El-Garni and Wael G. Abdelrahman

King Fahd University of Petroleum and Minerals

Dhahran, Saudi Arabia

ABSTRACT

Plates with piezoelectric coupling have been used extensively for active vibration damping of aerospace structures. Ensuring the existence of the correct amount of the piezoelectric constituent over time and health monitoring are essential in the reliability analysis of these plates. NDT techniques involving wave propagation in solid media proved to be effective inspection tools for aerospace structures. In this paper, a recently developed continuum mixture model for studying guided wave propagation in bi-laminated periodic composites of piezoelectric materials is used to investigate the effect of piezoelectric coupling on the wave dispersion characteristics of such plates. The theory leads to the simple governing coupled equations for the actual composite which retain the integrity of the propagation process in each constituent but allow them to coexist under analytically derived interaction parameters. As a consequence of the analysis, effective mixture properties of the composite are obtained in the zero-frequency limit. The analysis for the cases with and without the piezoelectric constituent can be coupled with experimentally obtained dispersion curves to give an accurate estimate of the percentage of the piezoelectric patch. The procedure lends itself to modifications that allow it to be

used as an effective tool in quality control of piezoelectric coupled plates manufacturing or for health monitoring of such structures.

INTRODUCTION

There has been an increased interest in determining the effective bulk properties of piezoelectric composite materials. See for example Benveniste, 1993, Bisegna et al., 1996, and Dunn et al., 1993. This has been perhaps prompted by the fact that, since the late seventies, piezoelectric composites have been used in the manufacturing of high tech components such as ultrasonic transducers and actuators.

In some idealized situations, and for simple systems, exact solutions might be obtained. On the other hand, for simple deterministic geometries, limited success has been realized in calculating some of the properties; this is based primarily on solving appropriate boundary value problems. For the most situations, however, properties are calculated or estimated by using bounds schemes or by using various theories of mixture depending upon available information about the variability in constituent properties, geometrical arrangements and interactions. See, for example, Nayfeh, 1995 for detailed references.

In the case of dynamic applications and, in particular those involving wave propagation, the applicability of the effective modules theories is somewhat restrictive. Specially, these theories are incapable of reproducing the dispersion and pronounced alteration (spreading and attenuation) of propagation pulses in these composites. The necessity to simulate such effects on the mechanical, thermal and electromagnetic response of composites has led to development and applications of several theories reflecting the influence of the microstructure. Wang, 2002, derived theoretical expressions for the dispersive characteristics and mode shapes of the transverse displacement and electric potential of the piezoelectric layers and studied the limits of the wave velocity as the wave number increases. Later, Qian et al., 2004, obtained the dispersion relations for wave propagation in periodic piezoelectric composite layers.

In this paper, a continuum mixture model that was developed by Nayfeh et al., 1999 for the study of guided wave propagation in piezoelectric plates is adapted to investigate the effect of the piezoelectric content on the coupling characteristics, dispersion properties and the zero-frequency limit of the wave speed. This provides a means of estimating the piezoelectric content of composite plates by measuring these characteristics. The model employs the use of four coupled simple equations for the propagation process in each constituent subject to some interactions. The derived system of equations is readily adaptable to the study of harmonic excitation in the system. The utility and range of applicability of this simple theory was established by comparison with an exact treatment. See Nayfeh et al., 1999.

FORMULATION OF THE PROBLEM

FIELD EQUATIONS

Details of the micromechanical model can be found in Nayfeh et al., 1999. A partial development of the development of this model is provided here. Consider the propagation of waves in the direction parallel to the interface of a periodic array of bi-laminated composite as shown in figure 1a. From symmetry all field variables are independent of the y-coordinate. Thus, the problem reduces to strictly two-dimensional one. For longitudinal wave propagation along the x-direction, the wave motion is further restricted to yield symmetric u_x and antisymmetric u_z displacement components. Insuring such symmetries also leads to the vanishing of the shear stress and the transverse displacements (mechanical and electric) at the center of each layer. These symmetry conditions allow the isolation of a repeating unit cell of the composite as shown in figure 1b. For each constituent of this cell a local transverse z-coordinate with the origin located at the layer's center is assigned. Symmetry and the applicable continuity conditions are shown on figure 1c.

With respect to this geometrical arrangement, the relevant piezoelectric field equations for each constituent consist of the momentum equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u_x}{\partial t^2}, \qquad (1a)$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2}, \qquad (1b)$$

the charge equation of electrostatics

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0, \qquad (2)$$

the strain-mechanical displacement relations

$$S_x = \frac{\partial u_x}{\partial x}, \qquad S_z = \frac{\partial u_z}{\partial z}, \qquad \gamma_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}, \qquad (3)$$

the electric field-electric potential relations

$$E_x = -\frac{\partial \phi}{\partial x}, \qquad E_z = -\frac{\partial \phi}{\partial z}, \qquad (4)$$

and the coupled constitutive relations

$$\sigma_{ij} = c^E_{ijkl} S_{kl} - e_{kij} E_k, \qquad D_i = e_{ikl} S_{kl} + \varepsilon^S_{ik} E_k. \qquad (5a, b)$$

The elastic, piezoelectric and dielectric constants for the orthotropic piezoelectric system are given in the expanded matrix forms

$$C_{pq}^{E}=\begin{bmatrix} C_{11}^{E} & C_{12}^{E} & C_{13}^{E} & 0 & 0 & 0 \\ C_{12}^{E} & C_{22}^{E} & C_{23}^{E} & 0 & 0 & 0 \\ C_{13}^{E} & C_{23}^{E} & C_{33}^{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^{E} \end{bmatrix}$$

$$e_{ip}=\begin{bmatrix} e_{11} & e_{12} & e_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{26} \\ 0 & 0 & 0 & e_{35} & 0 \end{bmatrix}, \qquad \varepsilon_{ij}^{S}=\begin{bmatrix} \varepsilon_{11}^{S} & 0 & 0 \\ 0 & \varepsilon_{22}^{S} & 0 \\ 0 & 0 & \varepsilon_{33}^{S} \end{bmatrix},$$

which reflect renaming c_{ijkl} as C_{pq} such that p and q = 1,2, …6 are replaced by 11, 22, 33, 23 or 32, 31 or 13, and 12 or 21, respectively. For simplicity of the notation, we shall thereafter suppress the superscripts E and S in the equations (6) and (8). According to the above relations, the constitutive equations (5) take the expanded form

$$\sigma_x = C_{11}\frac{\partial u_x}{\partial x} + C_{13}\frac{\partial u_z}{\partial z} + e_{11}\frac{\partial \phi}{\partial x}, \tag{9a}$$

$$\sigma_z = C_{13}\frac{\partial u_x}{\partial x} + C_{33}\frac{\partial u_z}{\partial z} + e_{13}\frac{\partial \phi}{\partial x}, \tag{9b}$$

$$\sigma_{xz} = C_{55}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) + e_{35}\frac{\partial \phi}{\partial z}, \tag{9c}$$

$$D_x = e_{11}\frac{\partial u_x}{\partial x} + e_{13}\frac{\partial u_z}{\partial z} - \varepsilon_{11}\frac{\partial \phi}{\partial x}, \tag{9d}$$

$$D_z = e_{35}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) - \varepsilon_{33}\frac{\partial \phi}{\partial z}. \tag{9e}$$

which, once again, hold for each layer.

SYMMETRY AND CONTINUITY CONDITIONS

The above field equations are supplemented with the symmetry conditions

$$u_z^{(1)}(x,0,t)=0\,,\qquad \sigma_{xz}^{(1)}(x,0,t)=0\,,\qquad D_z^{(1)}(x,0,t)=0\,,\tag{10a}$$

$$u_z^{(2)}(x,0,t)=0\,,\qquad \sigma_{xz}^{(2)}(x,0,t)=0\,,\qquad D_z^{(2)}(x,0,t)=0\,,\tag{10b}$$

that hold at the center of layers 1 and 2, respectively. The continuity conditions at the interface defined by $z_1=h_1$ and $z_2=-h_2$

$$u_z^{(1)}(x,h_1,t)=u_z^{(2)}(x,-h_2,t)\,,\qquad \sigma_{xz}^{(1)}(x,h_1,t)=\sigma_{xz}^{(2)}(x,-h_2,t)\,,\tag{11a}$$

$$u_x^{(1)}(x,h_1,t)=u_x^{(2)}(x,-h_2,t)\,,\qquad \sigma_{z}^{(1)}(x,h_1,t)=\sigma_{z}^{(2)}(x,-h_2,t)\,,\tag{11b}$$

$$\phi^{(1)}(x,h_1,t)=\phi^{(2)}(x,-h_2,t)\,,\qquad D_z^{(1)}(x,h_1,t)=D_z^{(2)}(x,-h_2,t)\,.\tag{11c}$$

where superscripts (1) and (2) and subscripts 1 and 2 refer to layers 1 and 2, respectively.

ACROSS-THICKNESS AVERAGING

Eliminating the z-dependence by performing the across-thickness integration:

$$\overline{(\quad)}^{(1)}=\frac{1}{h_1}\int_0^{h_1}(\quad)^{(1)}dz_1\,,\qquad \overline{(\quad)}^{(2)}=\frac{1}{h_2}\int_{-h_2}^{0}(\quad)^{(2)}dz_2\,.\tag{12}$$

If the symmetry and continuity conditions on $\sigma_{xz}^{(\alpha)}$, α =1,2 are taken into account, then applying the averaging to the equation of motion (1a), and the charge equation of electrostatics (2) leads to

$$n_1\frac{\partial\overline{\sigma}_x^{(1)}}{\partial x}=-\frac{1}{h}\sigma_{xz}^{*}+n_1\rho_1\frac{\partial^2\overline{u}_x^{(1)}}{\partial t^2}\,,\tag{13a}$$

$$n_2\frac{\partial\overline{\sigma}_x^{(2)}}{\partial x}=\frac{1}{h}\sigma_{xz}^{*}+n_2\rho_2\frac{\partial^2\overline{u}_x^{(2)}}{\partial t^2}\,.\tag{13b}$$

$$n_1\frac{\partial\overline{D}_x^{(1)}}{\partial x}+\frac{1}{h}D_z^{*}=0\,,\tag{14a}$$

$$n_2 \frac{\partial \overline{D}_x^{(2)}}{\partial x} - \frac{1}{h} D_z^* = 0 . \tag{14b}$$

Here

$$\sigma_{xz}^* = \sigma_{xz}^{(1)}(x, h_1, t) = \sigma_{xz}^{(2)}(x, -h_2, t) , \qquad D_z^* = D_z^{(1)}(x, h_1, t) = D_z^{(2)}(x, -h_2, t) , \tag{15}$$

define certain interface interaction terms and

$$n_1 = \frac{h_1}{h} , \qquad n_2 = \frac{h_2}{h} , \tag{16}$$

with $h = h_1 + h_2$ define the volume fractions for layers 1 and 2, respectively.

Similarly, the constitutive relations (9a) and (9d) for σ_x and D_x are averaged to yield

$$n_1 \overline{\sigma}_x^{(1)} = n_1 C_{11}^{(1)} \frac{\partial \overline{u}_x^{(1)}}{\partial x} + \frac{1}{h} C_{13}^{(1)} u_z^* + n_1 e_{11}^{(1)} \frac{\partial \overline{\phi}^{(1)}}{\partial x} , \tag{17a}$$

$$n_2 \overline{\sigma}_x^{(2)} = n_2 C_{11}^{(2)} \frac{\partial \overline{u}_x^{(2)}}{\partial x} - \frac{1}{h} C_{13}^{(2)} u_z^* + n_2 e_{11}^{(2)} \frac{\partial \overline{\phi}^{(2)}}{\partial x} , \tag{17b}$$

$$n_1 \overline{D}_x^{(1)} = n_1 e_{11}^{(1)} \frac{\partial \overline{u}_x^{(1)}}{\partial x} + \frac{1}{h} e_{13}^{(1)} u_z^* - n_1 \varepsilon_{11}^{(1)} \frac{\partial \overline{\phi}^{(1)}}{\partial x} , \tag{17c}$$

$$n_2 \overline{D}_x^{(2)} = n_2 e_{11}^{(2)} \frac{\partial \overline{u}_x^{(2)}}{\partial x} - \frac{1}{h} e_{13}^{(2)} u_z^* - n_2 \varepsilon_{11}^{(2)} \frac{\partial \overline{\phi}^{(2)}}{\partial x} , \tag{17d}$$

where

$$u_z^* = u_z^{(1)}(x, h_1, t) = u_z^{(2)}(x, -h_2, t) , \tag{18}$$

also defines an interface interaction term.

EVALUATION OF THE INTERACTION TERMS

APPROXIMATIONS

To solve for the interaction terms, we assume that $\sigma_{xz}^{(1)}(x,z_1,t)$, $u_z^{(1)}(x,z_1,t)$ and $D_z^{(1)}(x,z_1,t)$ vary linearly with z_1 according to

$$\sigma_{xz}^{(1)}(x,z_1,t)=A(x,t)z_1,\quad u_z^{(1)}(x,z_1,t)=B(x,t)z_1,\quad D_z^{(1)}(x,z_1,t)=C(x,t)z_1, \qquad \text{(19a,b,c)}$$

and that $\sigma_{xz}^{(2)}(x,z_2,t)$, $u_z^{(2)}(x,z_2,t)$ and $D_z^{(2)}(x,z_2,t)$ vary linearly with z_2 according to

$$\sigma_{xz}^{(2)}(x,z_2,t)=-\frac{h_1}{h_2}A(x,t)z_2, \qquad \text{(20a)}$$

$$u_z^{(2)}(x,z_2,t)=-\frac{h_1}{h_2}B(x,t)z_2, \qquad \text{(20b)}$$

$$D_z^{(2)}(x,z_2,t)=-\frac{h_1}{h_2}C(x,t)z_2. \qquad \text{(20c)}$$

These relations are intentionally chosen which automatically satisfy the individual symmetry and interface conditions as required by equations (10) and (11).

Subject to these approximations, we now proceed to average the remaining constitutive relations (9c) and (9e) for each constituent. The procedure is summarized as follows. First, we substitute the approximate expressions for $\sigma_{xz}^{(\alpha)}$ and $D_z^{(\alpha)}$, α=1, 2, into equations (9c) and (9e). Second, we multiply the resulting equations by z_1 and z_2 for α =1 and α =2, respectively. Third, we average in accordance with equations (12); to finally arrive at

$$\frac{h_1}{3}\sigma_{xz}^{*}=C_{55}^{(1)}(u_x^{*}-\bar{u}_x^{(1)})+e_{35}^{(1)}(\phi^{*}-\bar{\phi}^{(1)}), \qquad \text{(21a)}$$

$$-\frac{h_1Q}{3}\sigma_{xz}^{*}=C_{55}^{(2)}(u_x^{*}-\bar{u}_x^{(2)})+e_{35}^{(2)}(\phi^{*}-\bar{\phi}^{(2)}), \qquad \text{(21b)}$$

$$\frac{h_1}{3}D_z^{*}=e_{35}^{(1)}(u_x^{*}-\bar{u}_x^{(1)})-\varepsilon_{33}^{(1)}(\phi^{*}-\bar{\phi}^{(1)}), \qquad \text{(21c)}$$

$$-\frac{h_1 Q}{3} D_z^* = e_{35}^{(2)} (u_x^* - \bar{u}_x^{(2)}) - \varepsilon_{33}^{(2)} (\phi^* - \bar{\phi}^{(2)}) . \qquad (21d)$$

with

$$Q = \frac{n_2}{n_1} . \qquad (22)$$

INTERFACE TERMS u_x^* AND ϕ^*

By eliminating σ_{xz}^* and D_z^*, the four equations (21) can be solved for the interface terms u_x^* and ϕ^* as

$$u_x^* = \frac{1}{T_1} (T_2 \bar{u}_x^{(1)} + T_3 \bar{u}_x^{(2)} - T_4 \bar{\phi}^{(1)} - T_5 \bar{\phi}^{(2)}) , \qquad (23a)$$

$$\phi^* = P_1 \bar{u}_x^{(1)} + P_2 \bar{u}_x^{(2)} - P_3 \bar{\phi}^{(1)} - P_4 \bar{\phi}^{(2)} , \qquad (23b)$$

where

$$T_1 = (e_{35}^{(1)} Q + e_{35}^{(2)}) R_1 + C_{55}^{(1)} Q + C_{55}^{(2)} , \qquad T_2 = (e_{35}^{(1)} Q + e_{35}^{(2)}) R_2 + C_{55}^{(1)} Q ,$$

$$T_3 = (e_{35}^{(1)} Q + e_{35}^{(2)}) R_3 + C_{55}^{(2)} , \qquad T_4 = (e_{35}^{(1)} Q + e_{35}^{(2)}) R_4 - e_{35}^{(1)} Q ,$$

$$T_5 = (e_{35}^{(1)} Q + e_{35}^{(2)}) R_5 - e_{35}^{(2)} ,$$

and

$$P_1 = R_1 \frac{T_2}{T_1} - R_2 , \qquad P_2 = R_1 \frac{T_3}{T_1} - R_3 , \qquad P_3 = R_1 \frac{T_4}{T_1} - R_4 , \qquad P_4 = R_1 \frac{T_5}{T_1} - R_5 .$$

with

$$R_1 = \frac{e_{35}^{(1)} Q + e_{35}^{(2)}}{\varepsilon_{33}^{(1)} Q + \varepsilon_{33}^{(2)}} , \qquad R_2 = \frac{e_{35}^{(1)} Q}{\varepsilon_{33}^{(1)} Q + \varepsilon_{33}^{(2)}} , \qquad R_3 = \frac{e_{35}^{(2)}}{\varepsilon_{33}^{(1)} Q + \varepsilon_{33}^{(2)}} ,$$

$$R_4 = \frac{\varepsilon_{33}^{(1)} Q}{\varepsilon_{33}^{(1)} Q + \varepsilon_{33}^{(2)}} , \qquad R_5 = \frac{\varepsilon_{33}^{(2)}}{\varepsilon_{33}^{(1)} Q + \varepsilon_{33}^{(2)}} .$$

INTERACTION TERMS σ_{xz}^* *AND* D_z^*

Substitution of the expressions for u_x^* and ϕ^* from equation (23) back into equations (21) finally results in

$$\sigma_{xz}^* = F_1(\bar{u}_x^{(1)} - \bar{u}_x^{(2)}) + F_2(\bar{\phi}^{(1)} - \bar{\phi}^{(2)}) , \tag{24a}$$

$$D_z^* = G_1(\bar{u}_x^{(1)} - \bar{u}_x^{(2)}) + G_2(\bar{\phi}^{(1)} - \bar{\phi}^{(2)}) , \tag{24b}$$

where

$$F_1 = \frac{3}{h_1}\left[\frac{T_2}{T_1}(C_{55}^{(1)} + e_{35}^{(1)}R_1) - (C_{55}^{(1)} + e_{35}^{(1)}R_2)\right],$$

$$F_2 = \frac{3}{h_1}\left[-\frac{T_4}{T_1}(C_{55}^{(1)} + e_{35}^{(1)}R_1) - (e_{35}^{(1)} - e_{35}^{(1)}R_4)\right],$$

$$G_1 = \frac{3}{h_1}\left[\frac{T_2}{T_1}(e_{35}^{(1)} - \varepsilon_{33}^{(1)}R_1) - (e_{35}^{(1)} - \varepsilon_{33}^{(1)}R_2)\right],$$

$$G_2 = \frac{3}{h_1}\left[-\frac{T_4}{T_1}(e_{35}^{(1)} - \varepsilon_{33}^{(1)}R_1) - (-\varepsilon_{33}^{(1)} + \varepsilon_{33}^{(1)}R_4)\right].$$

INTERACTION TERM u_z^*

The equation relating the final interaction term u_z^* to $\bar{u}_x^{(1)}$, $\bar{u}_x^{(2)}$, $\bar{\phi}^{(1)}$ and $\bar{\phi}^{(2)}$ must be obtained from the only remaining interface condition (11), namely $\sigma_z^{(1)} = \sigma_z^{(2)}$ at $z_1 = h_1$. To this end, substituting for $u_z^{(1)}$, $u_z^{(2)}$ their respective approximations from equations (19b) and (20b) and in turn into the expressions (9b) for σ_z leads to

$$\sigma_z^{(1)}(x, h_1, t) = C_{13}^{(1)}\frac{\partial u_x^*}{\partial x} + \frac{C_{33}^{(1)}}{h_1}u_z^* + e_{13}^{(1)}\frac{\partial \phi^*}{\partial x} , \tag{25a}$$

$$\sigma_z^{(2)}(x, -h_2, t) = C_{13}^{(2)}\frac{\partial u_x^*}{\partial x} - \frac{C_{33}^{(2)}}{h_2}u_z^* + e_{13}^{(2)}\frac{\partial \phi^*}{\partial x} . \tag{25b}$$

Invoking the continuity relation on σ_z and eliminating u_x^* and ϕ^* as per equation (21) finally leads to

$$\frac{1}{h}u_z^* = (E_1 \frac{\partial \bar{u}_x^{(1)}}{\partial x} - E_2 \frac{\partial \bar{u}_x^{(2)}}{\partial x}) + (E_3 \frac{\partial \bar{\phi}^{(1)}}{\partial x} - E_4 \frac{\partial \bar{\phi}^{(2)}}{\partial x}), \tag{26}$$

where

$$E_1 = \frac{1}{E}\left[\frac{T_2}{T_1}(C_{13}^{(2)} - C_{13}^{(1)}) + (e_{13}^{(2)} - e_{13}^{(1)})P_1\right], \quad E_2 = -\frac{1}{E}\left[\frac{T_3}{T_1}(C_{13}^{(2)} - C_{13}^{(1)}) + (e_{13}^{(2)} - e_{13}^{(1)})P_2\right],$$

$$E_3 = -\frac{1}{E}\left[\frac{T_4}{T_1}(C_{13}^{(2)} - C_{13}^{(1)}) + (e_{13}^{(2)} - e_{13}^{(1)})P_3\right], \quad E_4 = \frac{1}{E}\left[\frac{T_5}{T_1}(C_{13}^{(2)} - C_{13}^{(1)}) + (e_{13}^{(2)} - e_{13}^{(1)})P_4\right],$$

and

$$E = \frac{C_{33}^{(1)}}{n_1} + \frac{C_{33}^{(2)}}{n_2}.$$

MIXTURE EQUATIONS OF MOTION

So far, we have solved for all interaction terms u_z^*, σ_{xz}^*, and D_z^* in terms of the average field variables $\bar{u}_x^{(1)}$, $\bar{u}_x^{(2)}$, $\bar{\phi}^{(1)}$ and $\bar{\phi}^{(2)}$. Equations (13), (14), and (17), together with equations (24) and (26) define the general quasi-one-dimensional model for the piezoelectric system. These equations are now combined into the following system of four coupled partial differential equations

$$n_1\rho_1 \frac{\partial^2 \bar{u}_x^{(1)}}{\partial t^2} - a_{11}\frac{\partial^2 \bar{u}_x^{(1)}}{\partial x^2} - a_{12}\frac{\partial^2 \bar{u}_x^{(2)}}{\partial x^2} - a_{13}\frac{\partial^2 \bar{\phi}^{(1)}}{\partial x^2} - a_{14}\frac{\partial^2 \bar{\phi}^{(2)}}{\partial x^2} = \frac{1}{h}[F_1(\bar{u}_x^{(1)} - \bar{u}_x^{(2)}) + F_2(\bar{\phi}^{(1)} - \bar{\phi}^{(2)})], \tag{27a}$$

$$n_2\rho_2 \frac{\partial^2 \bar{u}_x^{(2)}}{\partial t^2} - a_{21}\frac{\partial^2 \bar{u}_x^{(1)}}{\partial x^2} - a_{22}\frac{\partial^2 \bar{u}_x^{(2)}}{\partial x^2} - a_{23}\frac{\partial^2 \bar{\phi}^{(1)}}{\partial x^2} - a_{24}\frac{\partial^2 \bar{\phi}^{(2)}}{\partial x^2} = \frac{1}{h}[F_1(\bar{u}_x^{(2)} - \bar{u}_x^{(1)}) + F_2(\bar{\phi}^{(2)} - \bar{\phi}^{(1)})], \tag{27b}$$

$$a_{31}\frac{\partial^2\overline{u}_x^{(1)}}{\partial x^2}+a_{32}\frac{\partial^2\overline{u}_x^{(2)}}{\partial x^2}-a_{33}\frac{\partial^2\overline{\phi}^{(1)}}{\partial x^2}-a_{34}\frac{\partial^2\overline{\phi}^{(2)}}{\partial x^2}=\frac{1}{h}[G_1(\overline{u}_x^{(2)}-\overline{u}_x^{(1)})+G_2(\overline{\phi}^{(2)}-\overline{\phi}^{(1)})],$$

(27c)

$$a_{41}\frac{\partial^2\overline{u}_x^{(1)}}{\partial x^2}+a_{42}\frac{\partial^2\overline{u}_x^{(2)}}{\partial x^2}-a_{43}\frac{\partial^2\overline{\phi}^{(1)}}{\partial x^2}-a_{44}\frac{\partial^2\overline{\phi}^{(2)}}{\partial x^2}=\frac{1}{h}[G_1(\overline{u}_x^{(1)}-\overline{u}_x^{(2)})+G_2(\overline{\phi}^{(1)}-\overline{\phi}^{(2)})].$$

(27d)

Here

$$a_{11}=C_{11}^{(1)}n_1+C_{13}^{(1)}E_1,\quad a_{12}=-C_{13}^{(1)}E_2,\qquad a_{13}=e_{11}^{(1)}n_1+C_{13}^{(1)}E_3,\quad a_{14}=-C_{13}^{(1)}E_4,$$

$$a_{21}=-C_{13}^{(2)}E_1,\quad a_{22}=C_{11}^{(2)}n_2+C_{13}^{(2)}E_2,\qquad a_{23}=-C_{13}^{(2)}E_3,\qquad a_{24}=e_{11}^{(2)}n_2+C_{13}^{(2)}E_4,$$

$$a_{31}=e_{11}^{(1)}n_1+e_{13}^{(1)}E_1,\quad a_{32}=-e_{13}^{(1)}E_2,\qquad a_{33}=\varepsilon_{11}^{(1)}n_1-e_{13}^{(1)}E_3,\quad a_{34}=e_{13}^{(1)}E_4,$$

$$a_{41}=-e_{13}^{(2)}E_1,\quad a_{42}=e_{11}^{(2)}n_2+e_{13}^{(2)}E_2,\qquad a_{43}=e_{13}^{(2)}E_3,\qquad a_{44}=\varepsilon_{11}^{(2)}n_2-e_{13}^{(2)}E_4.$$

The four coupled equations (27) comprise a mixture system for the laminated piezoelectric composite. In general, these equations retain the integrity of the propagation process in the individual constituents and allow them to coexist under the derived interaction parameters u_z^*, σ_{xz}^*, and D_z^*. In particular, information as to the distribution of the field variables in the individual constituent is readily obtainable.

As a further observation, assuming that the left hand sides of equations (27) remain finite while letting $h\to 0$ dictates that $\overline{u}_x^{(1)}$ approaches $\overline{u}_x^{(2)}$ and $\overline{\phi}^{(1)}$ approaches $\overline{\phi}^{(2)}$. For this limiting case, $\overline{u}_x^{(1)}\equiv\overline{u}_x^{(2)}$ and $\overline{\phi}^{(1)}\equiv\overline{\phi}^{(2)}$, and the sum of equation (27a) and (27b), together with the sum of equation (27c) and (27d) yield the two coupled equations

$$\rho_c\frac{\partial^2\overline{u}_x}{\partial t^2}-A_{11}\frac{\partial^2\overline{u}_x}{\partial x^2}-A_{12}\frac{\partial^2\overline{\phi}}{\partial x^2}=0,\qquad -A_{21}\frac{\partial^2\overline{u}_x}{\partial x^2}+A_{22}\frac{\partial^2\overline{\phi}}{\partial x^2}=0,\qquad (28a, b)$$

where $\bar{u}_x$ stands for either $\bar{u}_x^{(1)}$ or $\bar{u}_x^{(2)}$. And a similar remark holds for $\bar{\phi}$. Here

$A_{11} = a_{11} + a_{12} + a_{21} + a_{22}$, $\quad A_{12} = a_{13} + a_{14} + a_{23} + a_{24}$,

$A_{21} = a_{31} + a_{32} + a_{41} + a_{42}$, $\quad A_{22} = a_{33} + a_{34} + a_{43} + a_{44}$, $\quad \rho_c = \rho_1 n_1 + \rho_2 n_2$.

Inspection of equations (28) shows that A_{11}, $A_{12} = A_{21}$ and A_{22} define effective $C_{11}^{(e)}$, $e_{11}^{(e)}$ and $\varepsilon_{11}^{(e)}$ for the composite respectively. These quantities are further reduced and written in terms of the individual constituents properties and volume fractions as

$$C_{11}^{(e)} = A_{11} = n_1 C_{11}^{(1)} + n_2 C_{11}^{(2)} - \frac{n_1 n_2 (C_{13}^{(1)} - C_{13}^{(2)})^2}{n_1 C_{33}^{(2)} + n_2 C_{33}^{(1)}} \tag{29a}$$

$$e_{11}^{(e)} = A_{12} = A_{21} = n_1 e_{11}^{(1)} + n_2 e_{11}^{(2)} - \frac{n_1 n_2 (C_{13}^{(1)} - C_{13}^{(2)})(e_{13}^{(1)} - e_{13}^{(2)})}{n_1 C_{33}^{(2)} + n_2 C_{33}^{(1)}} \tag{29b}$$

$$\varepsilon_{11}^{(e)} = A_{22} = n_1 \varepsilon_{11}^{(1)} + n_2 \varepsilon_{11}^{(2)} + \frac{n_1 n_2 (e_{13}^{(1)} - e_{13}^{(2)})^2}{n_1 C_{33}^{(2)} + n_2 C_{33}^{(1)}} \tag{29c}$$

ILLUSTRATION

As an illustration for the effect of the piezoelectric constituent on the composite dynamic characteristics, we study the three arrangements, with PZT65/35 being the piezoelectric constituents in two of them, and PZT5 in the third. In all cases we use h_1= 0.25 mm and h_2 = 0.25 mm, which result in the volume fractions n_1= 0.5 and n_2= 0.5. The material properties used in the calculations are presented in Table 1.

To quantify the influence of piezoelectric coupling, Figures 2-4 depict the variation of the effective properties $C_{11}^{(e)}$, $e_{11}^{(e)}$ and $\varepsilon_{11}^{(e)}$ with the PZT65/35 volume fraction. These figures show characteristics similar to those predicted by Auld et al., 1991. By correctly selecting the frequency of the propagating wave, and measuring the corresponding value of effective properties, these figure provide a means for predicting the piezoelectric content in the composite. If applied over longer periods of time, it can provide useful information regarding the deterioration of the material properties with time.

CONCLUSION

In this work, a previously developed model for the wave propagation in piezoelectric composite plates is utilized to study the effective properties of several piezoelectric arrangements. As a consequence of the analysis, the variations of the effective stiffness, piezoelectric coupling, and permittivity with the piezoelectric volume fraction of the composite are obtained. The work can be used to estimate the piezoelectric volume fraction based on measurements of these effective properties.

ACKNOWLEDGMENTS

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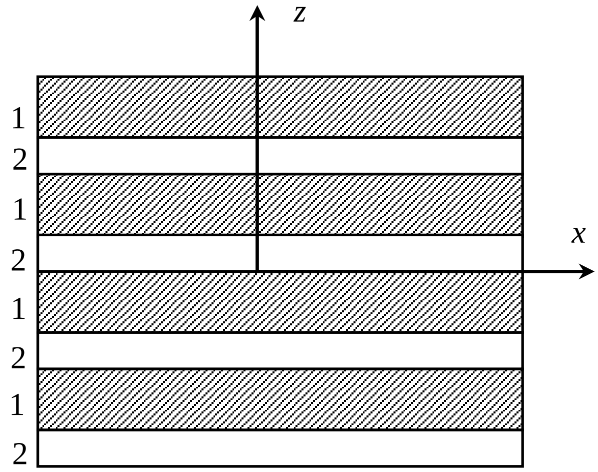

Figure 1: (a) The geometry of the composite model.

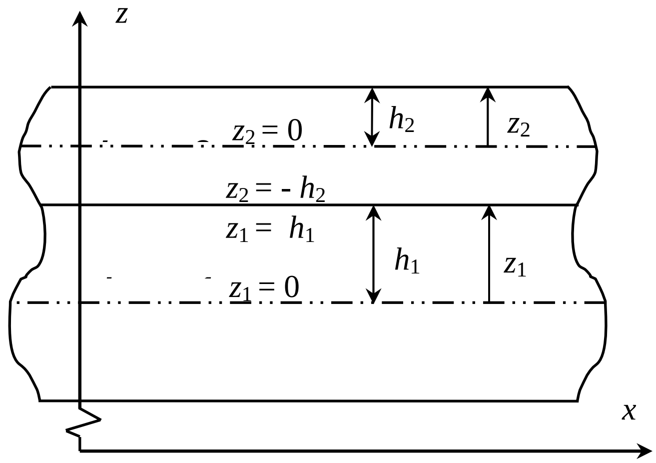

Figure 1: (b) Unit cell geometry and coordinate system.

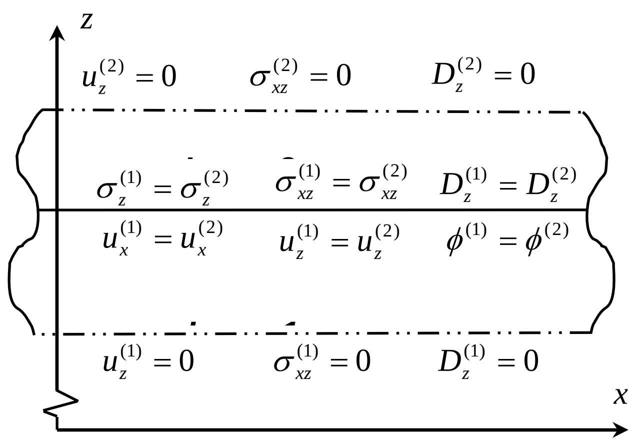

Figure 1: (c) Unit cell symmetry and continuity conditions.

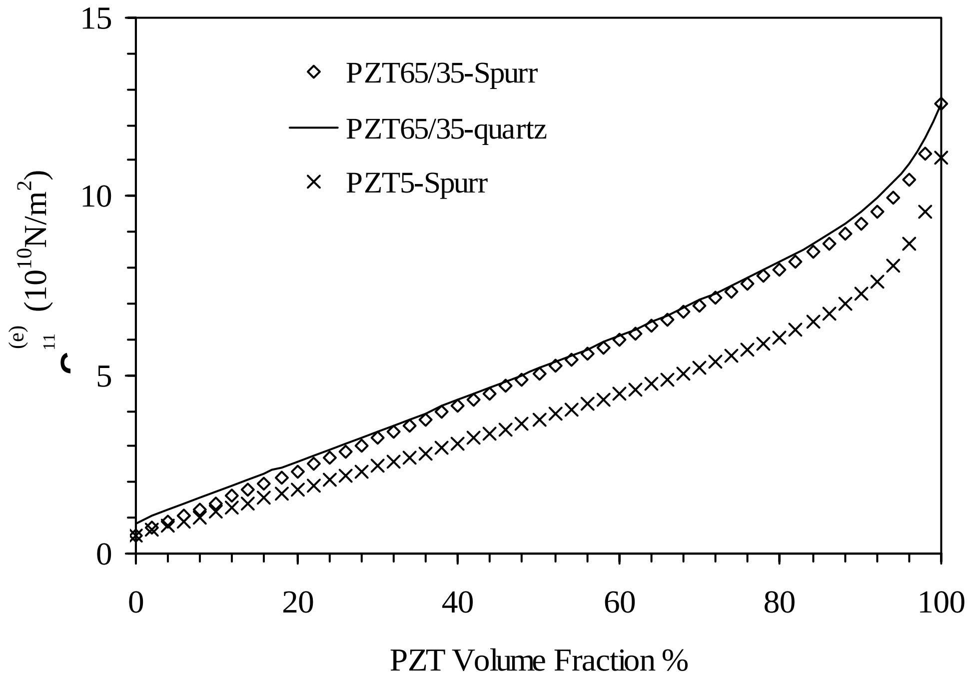

Figure 2. Variation of effective stiffness with the piezoelectric volume fraction.

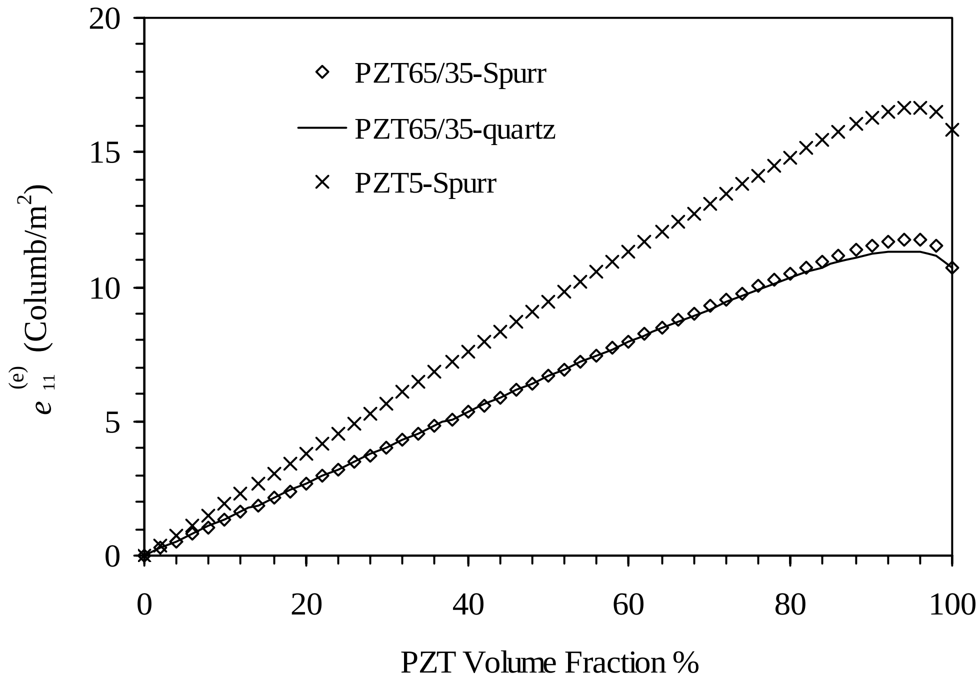

Figure 3. Variation of effective piezoelectric coupling with the piezoelectric volume fraction.

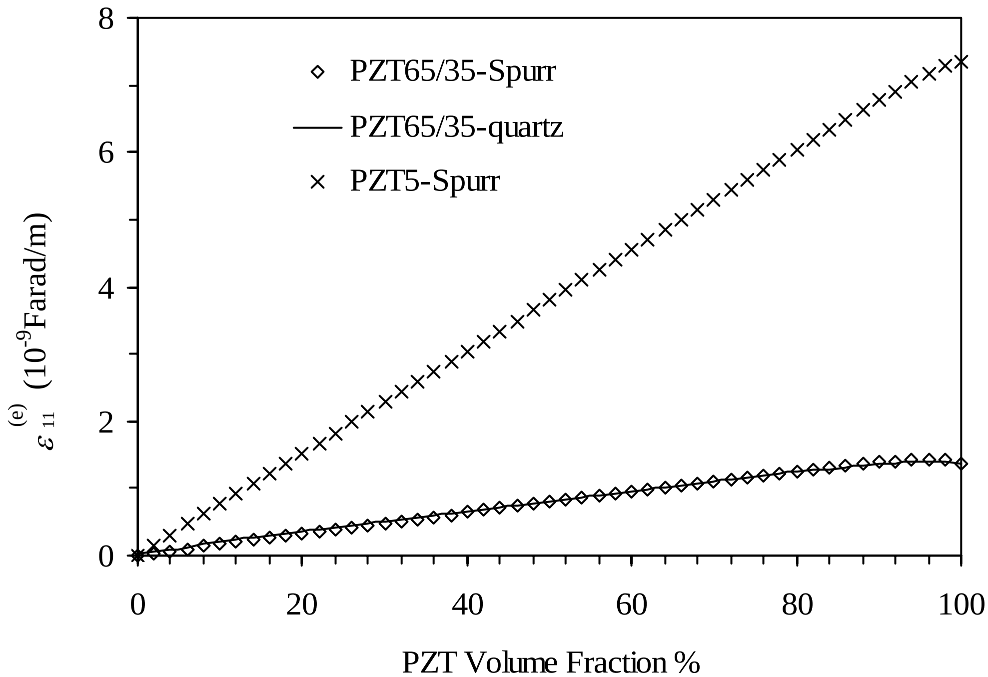

Figure 4. Variation of effective permittivity with the piezoelectric volume fraction.

Table I. The Material Properties of Selected Materials.

Units of C_{pq} and e_{ip} are 10^9 N/m^2 and $coulomb/m^2$, respectively. ε_{ij} is given non-dimensional as $\varepsilon^s / \varepsilon_o^S$ where $\varepsilon_o^S = 8.85410{*}10^{-12} = 10^{-9}/36\pi$ *farad/m*.

Materials Constants	*PZT 65/35*	*PZT5*	*Spurr*	*Quartz*
ρ(g/cm^3)	7.500	7.75	1.100	2.651
C_{11}	159.4	121	5.3	10.72
C_{12}	73.9	75.2	3.1	0.699
C_{13}	73.9	75.2	3.1	1.191
C_{22}	159.4	121	5.3	8.674
C_{23}	73.9	75.4	3.1	1.191
C_{33}	159.4	121	5.3	10.72
C_{44}	42.8	22.9	1.1	5.794
C_{55}	38.9	22.9	1.1	5.794
C_{66}	38.9	22.9	1.1	3.988
e_{11}	10.7	15.8		
e_{12}	-6.13	-5.4		
e_{13}	-6.13	-5.4		
e_{26}	8.39	10.6		
e_{35}	8.39	10.6		
ε_{11}	153.3	830	1.0	4.6
ε_{22}	639.3	635	1.0	4.5
ε_{33}	639.3	635	1.0	4.5

Figure 1(a). Composite model geometry.

Figure 1(b). Unit cell geometry and coordinate system.

Figure 1(c). Symmetry and continuity conditions.

Figure 2. Variation of effective stiffness with the piezoelectric volume fraction.

Figure 3. Variation of effective piezoelectric coupling with the piezoelectric volume fraction.

Figure 4. Variation of effective permittivity with the piezoelectric volume fraction.