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NUMERICAL MODELS FOR BOUNDARY-LAYER ANALYSIS ON ROUGH SURFACES

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Abstract: The paper presents a brief overview of the two most commonly used approaches, namely the equivalent sand-grain approach and the discrete-element approach, used to study boundary-layer characteristics of a rough surface. The salient features as well as some of the recent improvements and the limitations of these approaches are highlighted. Some results are presented at the end to show a comparison with known experimental data.
Keywords: Roughness, equivalent sand-grain roughness, discrete-element, boundary-layer analysis, aircraft icing.

1. INTRODUCTION

Ice accretion and insect contamination are one of the major causes of aircraft performance degradation since their presence leads to large penalties in lift and drag (see Fig. 1). As a consequence, there is a growing concern among the aeronautical engineering community to better understand the process of ice accretion. This concern is largely due to a number of icing related accidents in recent years. In order to improve flight safety, a better understanding of the effect of ice accretion on the aerodynamic performance of wings is required. The prediction of boundary-layer characteristics on rough surfaces [1] has been the focus of research for many years. In an effort to better predict the performance penalties associated with rough surfaces, the main focus of a joint research underway at École Polytechnique de Montréal under the *Bombardier Aeronautical* chair, is to develop reliable icing and anti-icing simulation tools such as *CANICE* [2]. The validity of these tools, specifically the roughness models associated with ice accretion and insect contamination, is being established through the experimental effort at The Queen's University of Belfast, N. Ireland, UK.

In order to improve the prediction of boundary-layer characteristics on rough surfaces, a comparative study on the effect of different turbulence models on skin-friction and heat-transfer prediction using discrete-element^5 and sand-grain approach is underway. The goal of this study is to determine the strength and weakness of each approach and the associated turbulence model and then make a final recommendation as to which of the combinations maybe used in ice accretion prediction codes. This paper briefly discusses the models that are being used for boundary-layer analysis of a rough surface.

2. NUMERICAL MODELS

Although the development of a numerical model based on the Navier-Stokes (NS) equations would provide greater understanding of the complex flow structure around a roughness element or a distribution of such elements, it does not offer any significant advantage over the methods based on empirical or semi-empirical correlations and BL equations in terms of determining the skin-friction and heat-transfer characteristics of the flow. The research effort was, therefore, devoted to the development of numerical models based on the latter methods. In literature, two approaches have generally been used to study the BL characteristics of a rough wall. These include: (1) the equivalent sand-grain approach, and (2) the discrete-element approach.

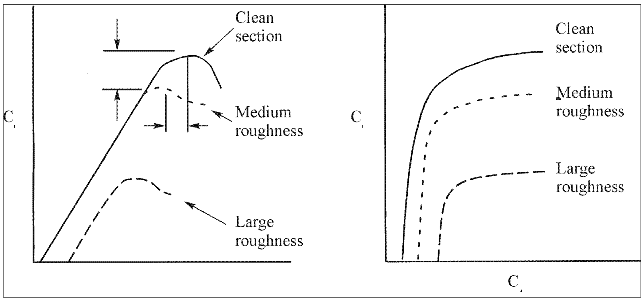

Figure 1 Influence of roughness on airfoil lift and drag

2.1 THE EQUIVALENT SAND-GRAIN APPROACH

The study of the effects of surface roughness on fluid flow and heat transfer had its origin with the classic works of Nikuradse [3] in 1933 and Schlichting [4] in 1936. The parameter commonly used to characterize a rough surface is the well-known equivalent sand-grain height h_S. This parameter is determined by comparing the skin friction and velocity profiles for a particular surface with the results of Nikuradse [3] on fluid flow in channels with rough walls. With the use of the measured friction on the smooth and rough parts of the wall and the associated wall laws, Schlichting reduced the three-dimensional roughness to an equivalent sand-grain roughness of height h_S that would cause the same increase in the drag. He then associated to each of the surface studied, a geometrical coefficient

$$a = h / h_S \qquad (1)$$

defined as the ratio of the mean height h of the roughness element to the equivalent sand-grain height h_s established from correlations of the geometrical characteristics (height, spacing density, and shape) of the roughness elements. Since then, several correlation have been tested with more or less success. In addition to the empirical correlations, semi-empirical methods have also been proposed which use empirical correlations in addition to solving the associated integral and local equations. These include the work of Dvorak [5], Dirling [6], Grabow and White [7], Rotta [8], Cebeci and Chang [9], Chan [10], Van Driest [11], Krogstadt [12], Wilcox *et al.* [13, 14] and Patel and Sheuer [15].

2.2 LIMITATIONS OF THE EQUIVALENT SAND-GRAIN APPROACH

- The correlations are not universal.
- Additional parameters are necessary to characterize the geometry of the surface such as the spacing between the rough elements and roughness density. This presents a difficult task for complex flows and different types of surfaces.
- The Reynolds analogy, linking the drag and the heat transfer, gives good results for a flat plate and for small roughness. It presents the inconvenience of not being valid near the separation point and is not valid for average and big rough elements. What is more, there is no physical basis for the correlation between the height h_s of the equivalent sand-grain and the heat transfer [16—18]. Indeed, the increase in the wall shear stress is more rapid than the increase of the heat flux, in the presence of roughness. Moreover the wall shear stress is "seen" by the wall as a form drag which has no equivalent in the energy equation.

2.3 THE DISCRETE-ELEMENT APPROACH

Schlichting [4] generalized the work of Nikuradse [3] to other types of three-dimensional roughness. These roughness elements consisted of simple geometrical figures such as spheres, cones, hemispherical elements, see Fig. 2, of different height and distributed uniformly on a portion of the wall of interest. To adequately describe a rough surface at least three measures are required: height, spacing density, and shape. He proposed that the decomposition of skin friction consisted of two contributions, one due to the friction of the smooth wall between the roughness elements and the other due to the pressure drag on roughness elements. Thus, the Discrete-Element approach takes into account the physical characteristics as well as the local effects due to the presence of roughness elements on the boundary layer, in the form of supplementary coefficients and terms in the equations of motion. The equations of mass, momentum and energy are derived using mass, force and energy balance on an elementary control volume which surrounds enough roughness elements, see Fig. 3. The presence of a roughness element forces the fluid flow to go above and around the element and, thus, results in a blocking effect [19, 20] which is accounted for in the equations by a blocking factor ***b***, see Fig. 4.

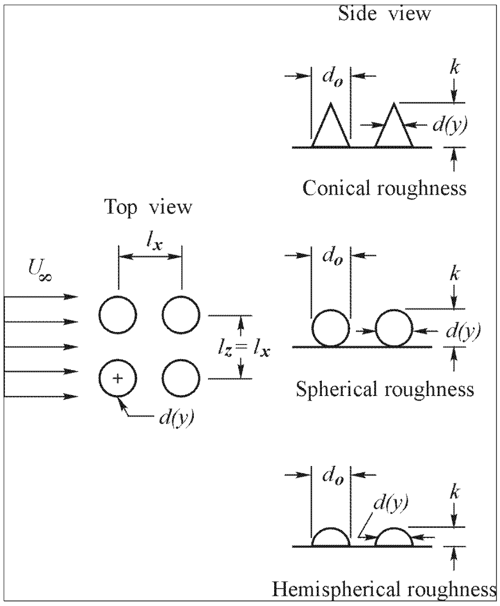

Figure 2 Typical roughness configurations

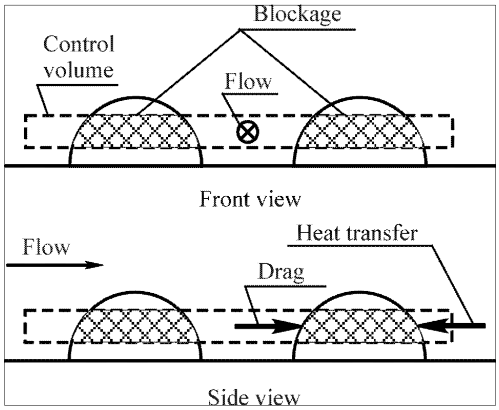

Figure 3 Schematic of a rough surface [17]

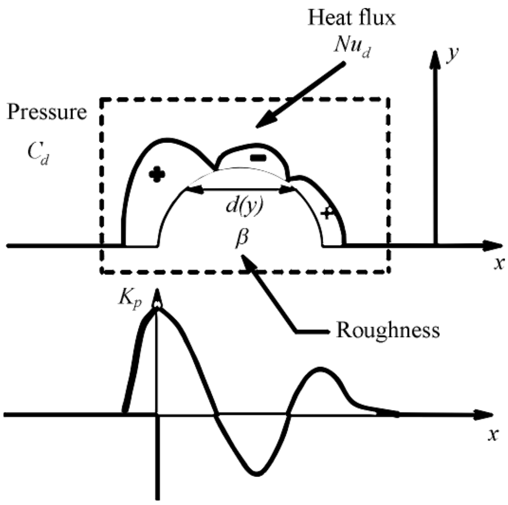

Figure 4 The blockage effect

The flow exerts pressure and viscous stresses on the roughness elements, which translate into a supplementary drag term in the momentum equation, including the pressure difference ΔP between the downstream and upstream faces of the roughness element and the effect of proximity of other roughness elements. If the fluid temperature is different from the wall temperature T_w, a heat flux between the elements and the fluid is modeled by an additional term in the energy equation involving the local Nusselt number Nu_d. The latter increases with the regime of the flow and the gap between the wall's temperature and the fluid's temperature. Similar to the skin-friction coefficient C_f, the Stanton number St is calculated directly from the local or global equations.

Different models utilizing the Discrete-Element concept have been investigated. The most common amongst these models are those of Finson [19, 20], Lin and Bywater [21], Cristoph [22], Taylor *et al.* [23], and Coleman *et al.* [17, 24—27], Zukauskas [28], and Carrau [29]. Improvements to these models using volume-averaging has been demonstrated by Crapiste [30] and Gray [31].

2.4 Advantages and Disadvantages of the Discrete-Element Approach

The Discrete-Element approach fills many of the gaps left
by the Equivalent Sand-Grain approach such as:

- The physical geometry of the rough surface is accounted for into the formulation of the governing equations and includes height, shape and density distribution, etc.
- Solution of global equations in which the local effect on the boundary layer due to roughness elements are taken into account. The drawback is that one must assume a specific geometry for the roughness elements. The method is not valid for sphere or any type of joint roughness elements.
- Schlichting [4] showed the existance of “dead zones” downstream of joint sphere roughness elements. For joint spheres, the flow “sees” an apparent wall located at a distance $y_0 = d_0/5$, where d_0 is the diameter of the spheres.
- The problem of effective wall location and ill-defined boundary conditions are also eliminated.
- The method uses empirical data only through the geometry of the roughness which appears in the definition of the blocking coefficient \beta, the drag coefficient C_d and the local Nusselt number Nu_d.
- The approach works for 2D and 3D roughness, requiring a change for the definition of the drag coefficient.

The models presented above distinguish themselves by:

- The blocking coefficient: two different blocking coefficients, one for the horizontal direction and one for the normal (to the wall) direction. If the same blocking coefficient is used for both directions, then it represents an average blocking in the horizontal direction.
- The different location of the blocking coefficient in the convective terms, diffusive terms and pressure gradient.
- The modeling of drag coefficient C_d, the local Nusselt number that appear in the sink terms in the momentum and energy equations.
- The turbulence model used; they vary from simple algebraic models to two-equation type models.
- Some models consider the influence of the roughness only in the equations describing the mean flow, while other models consider the influence on the turbulent field and include them in the turbulence model.

3. Choice of a Numerical Model

In the above sections, a number of mathematical models have been presented which can be used to study the flow of a fluid over a rough wall. The emphasis of this work has basically been on two methods: the Equivalent Sand-Grain method and the Discrete-Element method. The inherent advantage of using the Discrete-Element method and the use of volume-averaging leads to additional *dissipation* terms due to the presence of roughness elements and justifies the inclusion of the effects of the roughness in the turbulence modeling. For numerical purposes a Discrete-Element as well as an Equivalent Sand-Grain numerical model have been developed. Currently, the models use (a zero-order turbulence model) the Cebeci-Smith’s turbulence model, modified by Cebeci-Chang to take into account the effects of the roughness [9]. Other model's such as the Van Driest [11] mixing length model with van Driest damping used for smooth wall are also being investigated.

4. Preliminary Results

The results from the two numerical models are presented along with the comparison with the experimental results and results by other investigators [17]. The comparison shown consists of turbulent velocity (Fig. 5) and temperature (Fig. 6) profiles at a station sufficiently downstream of local transition as well as the skin-friction coefficient distribution (Fig. 7). The roughness geometry chosen for the comparison consists of hemispherical elements with d_o = 3 mm, k = 1.5 mm and $l_x = l_y$ = 4 mm. Other input conditions include: U_e = 27 m/s, T_e = 300 K and T_w = 273 K. The nomenclature used in the figures is listed below for the convenience of the reader.

Nomenclature

T = local temperature, K
T_e = free-stream temperature, K
T_w = wall temperature, K
U = local velocity, m/s
U_e = free-stream velocity, m/s

y = vertical distance from the wall, m
d = boundary-layer thickness, m
d_T = thermal boundary-layer thickness, m
dp/dx = pressure gradient

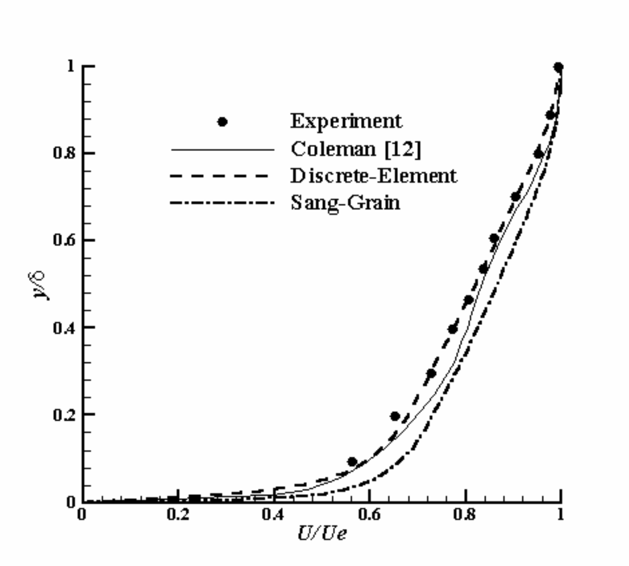

Figure 5 Velocity profile for U_e = 27 m/s

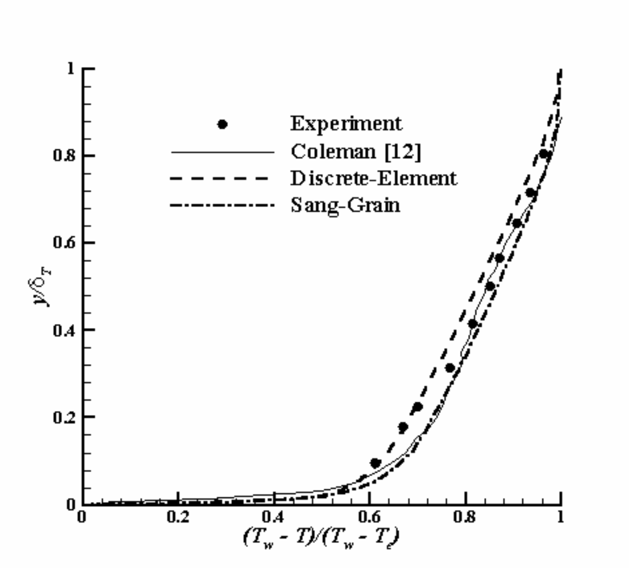

Figure 6 Temperature profile for U_e = 27 m/s

Figures 5 and 6 show a comparison of turbulent velocity and temperature profiles, respectively, at some location downstream of transition for $dp/dx \approx 0$. The profiles in the laminar region exhibit Blasius profiles. As evident from the two methods, the discrete-element approach predicts better results for velocity profile while the sand-grain approach is seen to perform better in the thermal profile. This effect is seen in skin-friction (Fig. 7) and heat-transfer coefficient prediction as well and, therefore, shows the inherent advantages of both the methods. Work is presently underway to study the effect of different turbulence models and pressure gradient and determine their influence on predictions. Details of the analysis and experimental results can be found in Refs. [32] through [36].

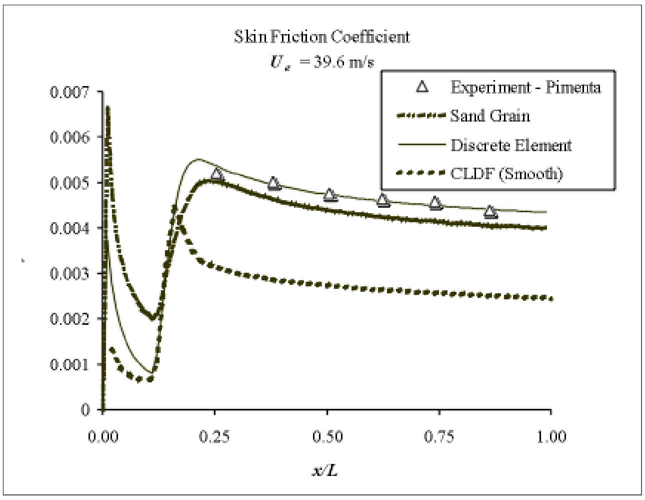

Figure 7 Skin friction coefficient distribution for U_e = 39.6 m/s

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