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Compressible Flow

(AE 520)

Term Project Report

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ABSTRACT

The topic will be a research about flow inside ducts with friction. I will look for knowledge of compressible flow in case of flow with friction exists. And I am also going to discuss how to calculate the exit Mach number and the friction factor by using some numerical methods and explain how such type of problems could be solved not for air only but generally for all different types of fluids and how to calculate flow properties like pressure ,density and temperature.

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1. Background information

Flow of an ideal gas through a constant-area adiabatic duct with wall friction is referred to as Fanno flow. An ideal flow used to study the flow of fluids in long pipes; the flow obeys the same simplifying assumptions as Rayleigh flow except that the assumption there is no friction is replaced by the requirement the flow be adiabatic.

In effect, this is similar to a Moody-type pipe flow but with large changes in kinetic energy, enthalpy, and pressure in the flow. The conservation equations for the Fanno flow combined with the definition of friction factor, equation of state, the Mach number equation, and the definitions of the stagnation temperature and pressure, leads to the following equations; a complete analysis is given in Anderson [2003]:

$$\frac{4f}{D}L^* = \left(\frac{1-M^2}{\gamma M^2}\right) + \frac{\gamma+1}{2\gamma}\ln\left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right] \qquad \textbf{(1)}$$

$$\frac{T}{T^*} = \frac{(\gamma+1)}{2+(\gamma-1)M^2} \qquad \textbf{(2)}$$

$$\frac{p}{p^*} = \frac{1}{M}\left[\frac{(\gamma+1)}{2+(\gamma-1)M^2}\right]^{1/2} \qquad \textbf{(3)}$$

$$\frac{p_0}{p_0{}^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{(\gamma+1)}\right]^{(\gamma+1)/[2(\gamma-1)]} \qquad \textbf{(4)}$$

In Equation (1) through Equation (4) sonic flow (M = 1) has been used as a reference condition, where the flow properties are denoted by T*, p*, and p0*. L* is defined as the length of the duct necessary to change the Mach number of the flow from M to unity and *f* is an average friction factor. Equations (1) through Equation (4) were used to generate Figure 1.1 for $\gamma = 1.4$. These equations are also normally tabulated in standard compressible flow texts in “Fanno Flow Tables, We Consider a duct of given cross-sectional area and variable length. If the inlet, mass flow rate, and average friction factor are fixed, there is a maximum length of the duct that can transmit the flow. Since the Mach number is unity at the duct exit in that case, the length is designated L* and the flow is said to be friction-chocked**.** In other words, friction always derives the Mach number toward unity, decelerating a supersonic flow and accelerating a subsonic flow. From Equation (1) we can see that at any point in the duct (say point 1), the variable *f* L*/D depends only on the Mach number at that point (M1) and γ. Since the diameter (D) is constant and *f* is assumed constant, then at some other point (2) a distance L (L < L*) downstream from point 1, we have:

$$\left(\frac{4f}{D}L^*\right)_2 = \left(\frac{4f}{D}L^*\right)_1 - \left(\frac{4f}{D}L\right) \qquad \textbf{(5)}$$

From Equation (5), we can determine M_2. If in a given situation M_2 was fixed, then Equation (5) can be rearranged to determine the length of duct required (L) for Mach number M_1 to change to Mach number M_2.

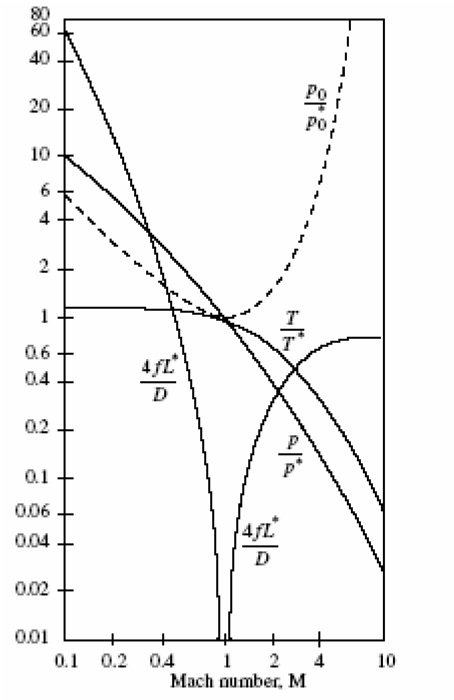

FIGURE 1.1 Fanno flow property variations with Mach number for $\gamma = 1.4$.

Since there is friction in the duct, Fanno flow is irreversible. For subsonic Fanno flows, the equation variables change along the pipe (or duct) as indicated below in Figure 1.2:

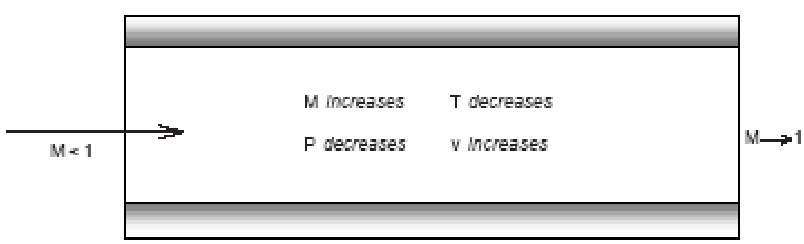

FIGURE 1.2 Fanno flow duct

Fanno flow moves toward the Mach 1 state as the compressible fluid propagates down a frictional pipe. Given an initial Mach number, M, the Sonic Length Formula can be used to calculate the length of pipe downstream through which the fluid must flow to reach sonic velocity:

$$S = (f\,L^*/D) = (1-M^2)/(\gamma\, M^2) + [(\gamma+1)/2\gamma]\ln[(\gamma+1)M^2/(2+(\gamma-1)M^2)] \qquad (6)$$

Where:
L^* = length of pipe the fluid must flow through to reach sonic velocity
D = pipe diameter
f = Darcy friction factor
M = initial Mach number
γ = specific heat ratio

2. Duct Addition

Duct addition is a recurring circumstance that is involved in many Fanno flow situations. In duct addition, information on the initial state, the friction factor, and the duct diameter and length are given; the final state after the flow traverses the duct length is desired.

Conditions at the duct inlet (M_1, P_1, T_1, f, D, and L) are known. The duct length required to achieve Mach 1 is L_1*, and for state 2 the duct length required to achieve Mach 1 is L_2*. The duct location of the sonic point (M = 1) and the conditions at the sonic point in a given Fanno flow are unique. For both the inlet and the outlet the sonic location and the conditions at the sonic location must be the same. Figure 1.3 on the following page illustrates the problem.

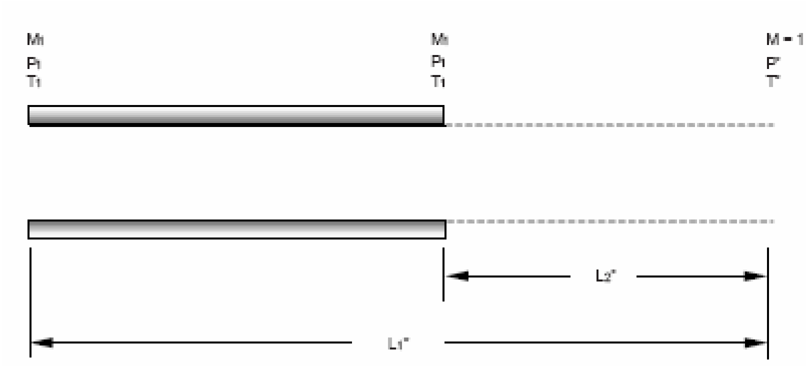

FIGURE 2.1 Schematic of Duct Addition Problem

L_1* can be computed directly by substituting the known inlet Mach number (M1) into the Sonic Length Formula (equation 1). L_2* can then be computed by subtracting the actual duct length from L1*:

$$(fL_2^*/D) = (fL_1^*/D) - (f\,L/D)_{actual} \qquad (7)$$

Since the Sonic Length Formula is non-analytic with respect to the Mach number, the outlet Mach number (M_2) cannot be directly calculated. A numerical method is required to determine this value. This method is called Newton's Method and it will be fully explained next.

3. Newton's Method

As noted above, the sonic length function is a non-analytic function with respect to the Mach number. Thus, to determine the outlet Mach number PIPE-FLO Compressible employs Newton's method. A brief description of this method follows.
NOTE: For convenience, S will be used to refer to the dimensionless quantity (fL/D).

1- We start with an initial guess for the outlet Mach number, M_2^0.
2- From this initial value of M_2^0, compute $(S_2^*)^0$.
3 - We Compute the value of the derivative dS*/dM evaluated at M_2^0.
4 -Find the equation of the line that is tangent to the sonic length function at M_2^0 and $(S_2^*)^0$:

$$S^* = (M - M_2^0)(dS^*/dM)|M_2^0 + (S_2^*)^0 \tag{8}$$

5- Find the intersection of the tangent line with horizontal line $S^* = S_2^*$.
$S_2^* = (M - M_2^0)(dS^*/dM)|M_2^0 + (S_2^*)^0$
Solving for M,

$$M = [S_2^* - (S_2^*)^0]/(dS^*/dM)|M_2) + M_2^0 \tag{9}$$

The value of M calculated in equation (9) is the improved guess for M_2, M_{21}.
6- Repeat steps 2 through 5, substituting the improved guess M_{21} for the initial guess. When $(S_2^*)^n$ is within an acceptable tolerance of the target S_2^* (calculated with equation 15), stop. At this point, $(M_2)^n$ is a good approximation to the theoretical M_2 we are trying to calculate.

4. Calculating the Friction Factor

The pipeline friction factor is a function of the Reynolds number. The Reynolds number is a dimensionless parameter which describes the characteristics of the fluid flowing in the piping system.

Reynolds developed the following relationship:

$$Re = W/(d\mu) \qquad (10)$$

Where:

W = flow rate
d = pipe diameter
μ = fluid dynamic viscosity

It was determined that for Newtonian fluids with a Reynolds number below a specific value, the fluid particles move in slip streams or laminar layers. Above a critical value of Reynolds number, the motion of the fluid particles becomes random or turbulent.

For engineering calculations, the upper practical limit of laminar flow has been set at a Reynolds number of Re = 2100. Above the laminar flow region, the flow starts to become turbulent. As the Reynolds number of the system increases, the flow becomes more turbulent, until the motion of the fluid particles is completely turbulent. The range between laminar flow and fully turbulent flow is referred to as the transition region. Because most compressible gas flow in pipes is turbulent, the program considers only turbulent flow.

A scientist called Nikuradse performed a series of experiments in order to develop a relationship between the friction factor and Reynolds number in pipes with turbulent flow.

The value of material surface roughness was arrived at by coating the interior of a smooth pipe with uniform grains of sand. The results of his experimentation presented some valuable relationships. He made the following discoveries:

1- At high Reynolds numbers the pipe friction factor becomes constant.

2- For rough pipes the ratio of surface material roughness to pipe diameter, or relative roughness, is more important than the Reynolds number for determining the friction factor.

Another scientist called Colebrook experimented with commercial pipes of various materials and roughness and developed the following equation for pipes in the transition region to the complete turbulence zone:

$$1/(f^{1/2}) = -0.869\ln[(e/D)/3.7 + 2.523/(Ref^{1/2})] \qquad (11)$$

Since this relationship has the friction factor term on both sides of the equation, it must be solved by iteration.

Iterative equations are easily solved by computer, but they take longer to solve than a straightforward relationship. For this reason, PIPE-FLO Compressible uses an equation that provides a direct calculation of the friction factor and is within 1% of the Colebrook equation:

$$f = 1.325/[\ln(e/(3.7D) + 5.74/Re0.9)]^2 \qquad (12)$$

Equation (12) gives accurate values of the friction factor and can be solved quickly without performing iterative calculations.

5. Compressible Flow with Friction Example

Now we give this problem to explain how to deal with this type of problems using previous techniques:
Air enters a 0.01-m-diameter duct (f = 0.05) with Ma = 0.05. The pressure and temperature at the duct inlet are 1.5 MPa and 400 K. What are the (a) Mach number, (b) pressure, and (c) temperature in the duct 50 m from the entrance?
At the duct entrance, with f = 0.05, D = 0.01 m, and Ma = 0.05, we obtain

$$\left(\frac{\bar{f} L^*}{D}\right)_1 = \left[\frac{1-Ma^2}{k\,Ma^2} + \frac{k+1}{2k}\ln\frac{(k+1)\,Ma^2}{2+(k-1)\,Ma^2}\right]_1$$

$$\left(\frac{\bar{f} L^*}{D}\right)_1 = \left[\frac{1-0.05^2}{1.4(0.05)^2} + \frac{2.4}{2.8}\ln\frac{(2.4)\,0.05^2}{2+(0.4)\,0.05^2}\right]_1 = 280$$

Then, at the duct exit we obtain

$$\left(\frac{\bar{f} L^*}{D}\right)_2 = \left(\frac{\bar{f} L^*}{D}\right)_1 - \frac{\bar{f}\,\Delta L}{D} = 280 - \frac{(0.05)\,50}{0.01} = 30$$

We can now write for the duct exit that:

$$\left(\frac{\bar{f} L^*}{D}\right)_2 = 30 = \left[\frac{1-Ma^2}{k\,Ma^2} + \frac{k+1}{2k}\ln\frac{(k+1)\,Ma^2}{2+(k-1)\,Ma^2}\right]_2$$

Or

$$30 = \frac{1-Ma_2^2}{1.4\,Ma_2^2} + \frac{2.4}{2.8}\ln\frac{2.4\,Ma_2^2}{2+0.4\,Ma_2^2}$$

The solution of the second of these equations gives answer (a) $Ma2$ = 0.145.
Writing the following expression for pressure ratios yields for (b),

$$P_2 = P_1 \frac{P_2}{P_2^*}\frac{P_2^*}{P_1^*}\frac{P_1^*}{P_1}$$

$$P_2 = (1.5)\frac{1}{Ma_2}\left[\frac{(k+1)}{2+(k-1)\,Ma_2^2}\right]^{1/2}(1)\frac{Ma_1}{1}\left[\frac{2+(k-1)\,Ma_1^2}{k+1}\right]^{1/2}$$

$$P_2 = (1.5)\frac{1}{0.145}\left[\frac{2.4}{2+(0.4)0.145^2}\right]^{-1/2}(1)\frac{0.05}{1}\left[\frac{2+(0.4)0.05^2}{2.4}\right]^{-1/2} = 0.516$$

Application of the temperature ratios yields answer (c),

$$T_2 = T_1\frac{T_1^*}{T_1}\frac{T_2^*}{T_1^*}\frac{T_2}{T_2^*} = 400\frac{2+(k-1)Ma_1^2}{2+(k-1)Ma_2^2} = 400\frac{2+(0.4)0.05^2}{2+(0.4)0.145^2} = 399$$

It is noted that in both of the previous expressions, P_2^* / P_1^* *and* T_2/T_1^* equal 1 as the sonic reference conditions are constant between two points.
This example demonstrates how Mach number changes in adiabatic frictional flow in a duct. When the flow at the inlet to the duct is subsonic, the Mach number increases as the duct gets longer. When the inlet flow is supersonic, the Mach number decreases as the duct gets longer. A plot of the specific entropy of the fluid as a function of the duct Mach number (and therefore length) is presented in Figure 5.1 for both subsonic and supersonic flow.

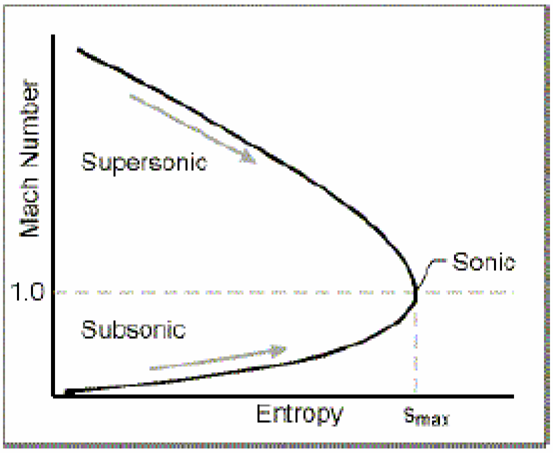

FIGURE 5.1 Entropy VS Mach number

These results clearly illustrate that the Mach number in the duct approaches unity as the length of the duct is increased. Once the sonic condition exists at the duct exit, the flow becomes choked. This figure also demonstrates that the flow can never proceed from subsonic to supersonic (or supersonic to subsonic) flow, as this would result in a violation of the second law of thermodynamics.

6. Conclusion

From the previous discussion, we got the knowledge of compressible flow in case of flow with friction exists. And also we discussed how to calculate the exit Mach number and the friction factor by using some numerical methods and were able to explain how such type of problems could be solved not for air only but generally for all different types of fluids and how to calculate flow properties like pressure ,density and temperature.

From this research lots of benefit I earned by learning more about the concept and application of Fanno flow and I got an idea how properties could change inside a duct having friction and it became easily to design ducts according to some requirements as next step of this research.