Failure Data Analysis for Aircraft Maintenance Planning

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Abstract
This paper presents an application of Weibull method for forecasting the failure rate of Boeing 737 Auxiliary power unit (APU) oil pumps. The Weibull method is extremely useful for maintenance planning. Using Weibull failure forecasting, a maintenance planner can make quantitative trades between scheduled and unscheduled maintenance or non-destructive inspection and replacement. The method also helps for determining the age at which an operating part in an aircraft system should be replaced with a new part. In this study, the failure rate of APU oil pump of Boeing 737 aircraft is modeled by using the Weibull technique. The results were in close agreement with the real data indicating the validity of the Weibull model in predicting failure rate of failure for APU oil pumps. In addition, the optimum replacement age of the pumps is also calculated for various cost ratios.

I. Introduction

Determining the age at which an operating part in an aircraft should be replaced with a new part has always been a problem. The age for such a planned replacement should depend on the time-to-failure distribution of the part, and also on the relative costs of an in-service failure and a planned replacement (i.e. replacing an unfailed part).

There are two conditions required to make planned replacement potentially worthwhile. The first is that the planned replacement of a part must cost less than an unexpected or unscheduled replacement. The second condition is that the failure characteristics of the part must display wearout, i.e. failure rate must increase with age. This can be better understood by examining the mortality characteristics of parts (Fig.1)[5,6].The descending curve indicates burn-in characteristic in which the failure rate decreases over time. The horizontal curve represents constant random characteristic which indicates that failure rate remains constant over time. Therefore, planned replacement has no advantage in these cases. The rising curve indicates wearout, i.e. increasing failure rate with time. Such units with age-related failure rate may be candidates for planned replacement.

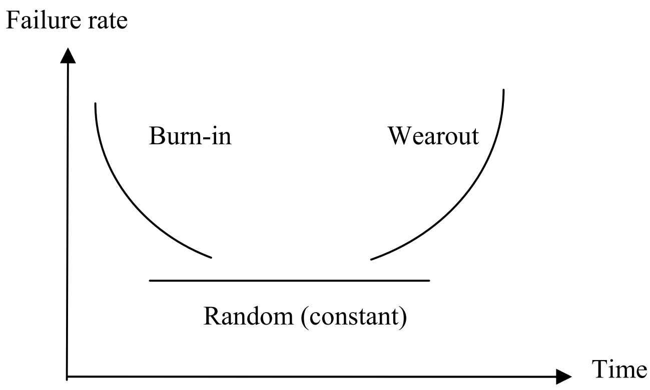

Fig.1. Three types of mortality characteristics

II. The Weibull Failure Distribution Model

One of the prerequisites for a worthwhile planned replacement program is that the failure distribution should have a wearout characteristic, or that the failure rate increases with age. There are several models to identify the failure characteristics of parts. The Weibull model is one of the most commonly used model for this purpose. The primary advantage of the Weibull model is the ability to provide reasonably accurate failure analysis and failure forecast with extremely small samples. This means that it is possible to use data as the first failure emerges and decide appropriate action before more failure data is generated.

The Weibull distribution can be characterized by a failure rate function $\lambda(t)$ of the form [4]

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \quad \eta > 0, \beta > 0, t \geq 0 \tag{1}$$

The reliability function *R(t)* which indicates the probability of surviving beyond a given time *t* can be derived from this failure rate function as follows :

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{2}$$

A complimentary function $F(t)$ to the reliability function can be defined as:

$F(t) = 1 - R(t)$

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{3}$$

$F(t)$ is known as cumulative distribution function and indicates the probability that a failure occurs before time *t*.

β is known as the shape parameter. It indicates whether the failure rate is increasing, constant, or decreasing. Practically, $\beta<1$ indicates that the part has a decreasing failure rate and implies infant mortality. This can be caused by a variety of factors, including design flaws, misassembly, and poor quality control. $\beta=1$ indicates a constant failure rate and implies random failures. In this case, one can suspect random events such as maintenance errors, human errors, foreign object damage (FOD). $\beta>1$ indicates an increasing failure rate. The most common causes of failures in this range are corrosion, erosion and fatigue cracking.

η is referred to as scale parameter. The value of η is equal to the number of cycles (flight hours, landings, etc) at which 63.2% of the parts have failed.

III. Fitting the Weibull Model to Failure Data

Various approaches are used in fitting the Weibull model to the failure data. In this paper, the cumulative distribution function *F(t)* is transformed as follows so that it appears in the familiar form of a straight line $y=mx+b$ [1,3].

$$\ln\left[1-F(t)\right]=-\left(\frac{t}{\eta}\right)^{\beta}$$

$$\ln\left\{\ln\left[\frac{1}{1-F(t)}\right]\right\}=\beta\ln(t)-\beta\ln(\eta) \qquad (4)$$

From equation (4) it can be seen that β corresponds to the slope and ($-\beta \ln \eta$) is the intercept. The cumulative distribution function *F(t)*, can be substituted by its estimate $F(t_i)$ using median rank formula for the time to failure data which is organized in ascending order.

$$F(t_i)=\frac{i}{N+1} \qquad 0\leq i \leq N \qquad (5)$$

By performing linear regression analysis using equation (4), the parameters β and η can be determined.

The equations (1-4) are for two-parameter Weibull model. If the fit to data is not good enough, then three-parameter Weibull model can be employed. This can be done by estimating a minimum lifetime, or repair time t_0 and transforming the data by letting $t_i=t-t_0$. However, certain criteria related to shape of the plot, sample size, and improvement in correlation coefficient should be met before implementing the three-parameter Weibull model.

IV. Analysis of Failure Data of APU Pumps

In this part, a group of data obtained from a Turkish airline will be analyzed. Data represent time to failure of 38 APU hydraulic pumps for Boeing 737 aircraft over a period of 3 years. The ages at removal of the pumps are measured in terms of flying hours. The data do not include any planned removals. An Excel program is used for the analysis. The result of the analysis is shown in Table 1. The resulting index of fit, R=0.9771 indicates a strong linear fit to data, thus supporting the hypothesis that the data came from a Weibull distribution. In addition, $\ln\{\ln[1-(1-F(t))]\}$ versus $\ln t$ is plotted in Fig. 2. An assessment of Weibull parameters of the oil pumps indicates that:

- Shape parameter is greater than one (β>1) which reflects an increasing failure rate of pumps.
- Scale parameter η is about 3867 hours which indicates that about 63 percent of the pumps has failed up to that time.

i	t_i,hr	ln t	$F(t_i)=i/(N+1)$	$Z=1/(1-F(t_i))$	Ln (Ln Z)	Predicted Ln (Ln Z)
1	182	5.204007	0.025641	1.026316	-3.65060	-3.861429
2	396	5.981414	0.051282	1.054054	-2.94421	-2.879261
3	763	6.637258	0.076923	1.083333	-2.52519	-2.050675
4	846	6.740519	0.102564	1.114286	-2.22365	-1.920215
5	900	6.802395	0.128205	1.147059	-1.98631	-1.842043
6	935	6.840547	0.153846	1.181818	-1.78944	-1.793842
7	950	6.856462	0.179487	1.218750	-1.62037	-1.773735
8	958	6.864848	0.205128	1.258065	-1.47153	-1.763140
9	989	6.896694	0.230769	1.300000	-1.33802	-1.722905
10	997	6.904751	0.256410	1.344828	-1.21650	-1.712727
11	1334	7.195937	0.282051	1.392857	-1.10456	-1.344845
12	1800	7.495542	0.307692	1.444444	-1.00042	-0.966328
13	1882	7.540090	0.333333	1.500000	-0.90272	-0.910046
14	1947	7.574045	0.358974	1.560000	-0.81039	-0.867148
15	2548	7.843064	0.384615	1.625000	-0.72256	-0.527272
16	2912	7.976595	0.410256	1.695652	-0.63853	-0.358570
17	2957	7.991931	0.435897	1.772727	-0.55771	-0.339196
18	3200	8.070906	0.461538	1.857143	-0.47959	-0.239419
19	3654	8.203578	0.487179	1.950000	-0.40372	-0.071803
20	3800	8.242756	0.512821	2.052632	-0.32972	-0.022305
21	3830	8.250620	0.538462	2.166667	-0.25723	-0.012370
22	3921	8.274102	0.564103	2.294118	-0.18591	0.017297
23	3972	8.287025	0.589744	2.437500	-0.11544	0.033623
24	4185	8.339262	0.615385	2.600000	-0.04551	0.099619
25	4550	8.422883	0.641026	2.785714	0.02421	0.205264
26	4840	8.484670	0.666667	3.000000	0.09405	0.283326
27	4863	8.489411	0.692308	3.250000	0.16437	0.289315
28	4911	8.499233	0.717949	3.545455	0.23560	0.301725
29	4918	8.500657	0.743590	3.900000	0.30820	0.303524
30	4962	8.509564	0.769231	4.333333	0.38277	0.314777
31	4984	8.513988	0.794872	4.875000	0.46003	0.320366
32	5443	8.602086	0.820513	5.571429	0.54096	0.431668
33	5782	8.662505	0.846154	6.500000	0.62690	0.508001
34	5890	8.681011	0.871795	7.800000	0.71985	0.531381
35	5978	8.695841	0.897436	9.750000	0.82298	0.550118
36	5980	8.696176	0.923077	13.000000	0.94194	0.550540
37	6810	8.826147	0.948718	19.500000	1.08870	0.714745
38	7968	8.983189	0.974359	39.000000	1.29844	0.913149

Regression Statistics	
R (index of fit)	0.977193102
R Square	0.954906358
Standard Error	0.247951678
Observations	38

Table 1. Failure data analysis of APU oil pumps

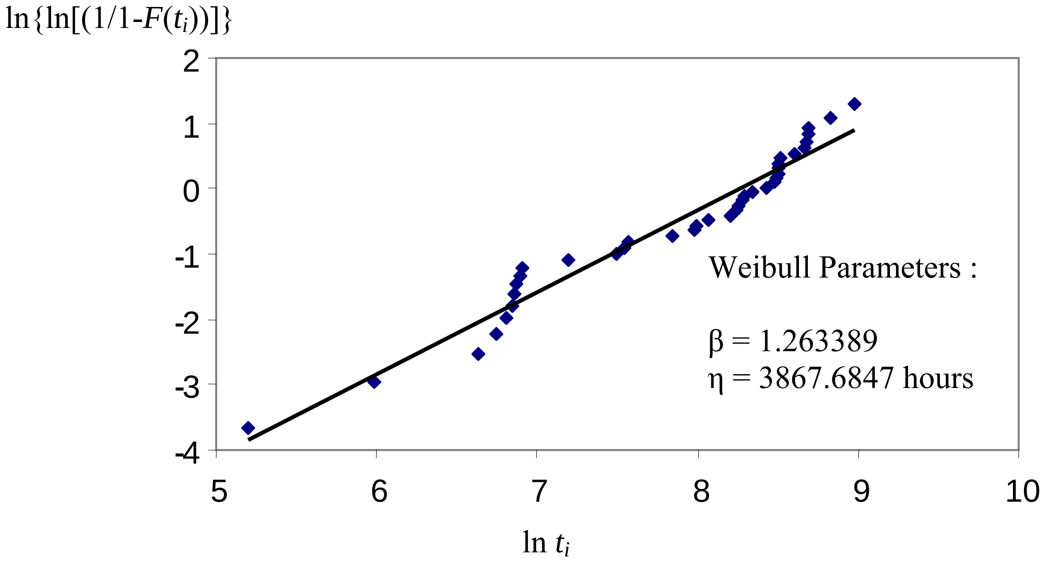

Fig.2. The Weibull plot for the failure data in hours of APU oil pumps

V. Model Adequacy

Model adequacy is important for examining whether the fitted model is in agreement with the observed data. Fig.3 indicates the comparison between the actual and predicted cumulative distribution of failures of pumps using the Weibull model. It can be seen that the results are in close agreement with the actual data.

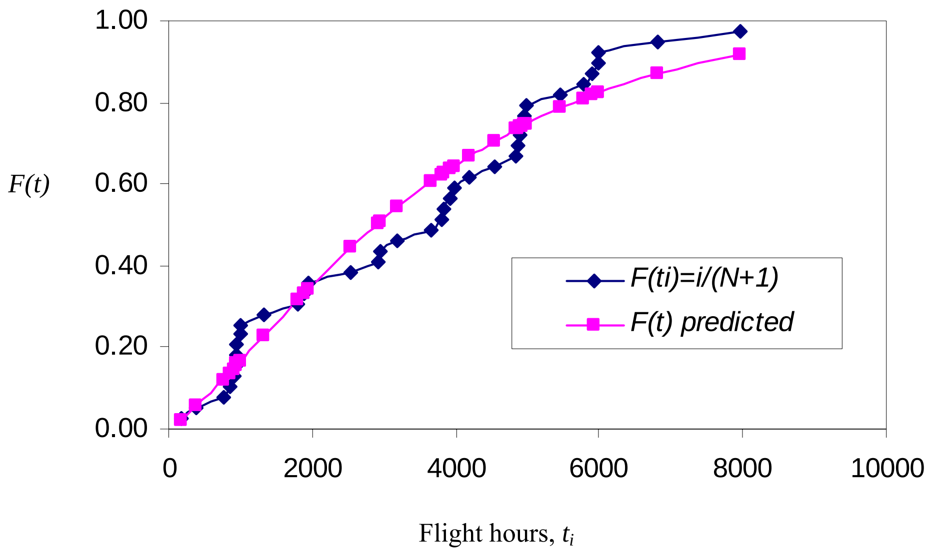

Fig.3. Comparison of actual and estimated cumulative failures

VI. Optimal Replacement Interval

In previous section, it was found out that the failure rate of pumps increases with time. Thus, a planned replacement of the pumps may be worthwhile. At this point, the problem is to determine the age at which the oil pumps should be replaced with a new one.

The desirable replacement will depend on the criterion to be used to evaluate the decision. Two criteria are available. The first is to choose the replacement age so that the risk of failure is always less than some maximum acceptable amount. Whenever the probability of failure reaches this maximum acceptable risk, the part is replaced. The second criterion is to choose the replacement age so as to minimize the expected costs per period of operation. The choice between these two criteria depends upon the circumferences at hand. The second criterion of minimum expected cost was used in this study to determine optimum replacement age.

The total cost (C_t) of any replacement can be defined as the sum of cost of planned replacements (C_p) and cost of replacements at failures (in-service failures) (C_f) [5].

$$C_t = C_p[1-F(t_p)] + C_f F(t_p)$$

Where $F(tp)$ is the percent of failed components at time t_p. Then, the total cost per unit time $J(t_p)$ can be written as follows:

$$J(t_p) = \frac{C_p\left[1 - F(t_p)\right] + C_f F(t_p)}{\int_0^{t_p} \left[1 - F(t)\right] dt}$$

The value of time t_p that minimizes the total cost per period of operation is the optimum replacement age.

In this study optimum replacement ages for different values of C_f / C_p ratios are found by using WinSMITH probability software. The results are indicated in Table 2.

β	Cost ratio C_f / C_p											
	6	7	8	10	15	20	25	30	35	40	45	50
1.2633	3547	2977	2597	2090	1457	1140	950	760	697	633	570	507

Table 2. Optimum replacement age in flight hours for different C_f/C_p ratios

The table indicates that the planned replacement has no advantage if C_f/C_p is less than six. In this case, the pumps should be allowed to operate to failure. This may be due to relatively small value of β. However, usually there are reasons to expect that it costs much more to repair an in-service failure than to make a planned replacement under the most general circumstances of both military and commercial operations.

The variation of total cost with time for $C_f / C_p = 10$ is shown in Fig.4. The graph points out that there is a minimum at about 2090 hours.

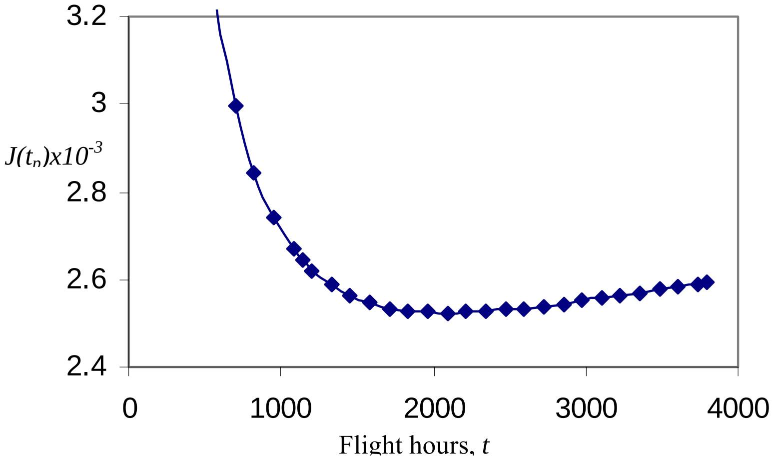

Fig.4. Variation of total cost over time for C_f/C_p=10

VII. Conclusion

The Weibull analysis is an effective tool for failure forecasts and predictions of the various aircraft parts and systems, and extremely useful for maintenance planning, particularly reliability centered maintenance. In this case, the failure characteristics of APU oil pumps have been analyzed by using Excel program. The good straight line fit to the transformed data supports the validity of the Weibull model. The resulting parameters indicate that the oil pumps have an increasing failure rate which makes a planned replacement policy worthwhile. Thus, optimum replacement age in flight hours are calculated for various C_f/C_p ratios.

If the cost for an unplanned failure is very high compared to a planned replacement, then beta greater than 1 is easy to handle on a predictive replacement basis. However, if the cost of an unplanned failure is approximately equal to a planned replacement then it is advised to run the component to failure. If the failures modes are due to chance failure (β=1) or infant mortality (β<1), then the component should run to failure for any ratio of costs.

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Nomenclature

$\lambda(t)$ = Failure rate function

$R(t)$ = Reliability function

$F(t)$ = Cumulative distribution function

t = Operation time in flight hours

β,η = Weibull parameters

N = Number of observations

C_t = Total cost

C_f = Cost of in-service failure

C_p = Cost of planned replacement

$J(t_p)$ = Total cost per unit time