# Failure Distribution Modeling For Planned Replacement Of Aircraft Auxiliary Power Unit Oil Pumps

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## Abstract

This paper presents an application of Weibull method for forecasting the failure distribution of aircraft auxiliary power unit (APU) oil pumps for planned replacement. The Weibull method is extremely useful for maintenance planning. Using Weibull failure forecasting, a maintenance planner can make quantitative trades between scheduled and unscheduled maintenance or non-destructive inspection and replacement. The method also helps for determining the age at which an operating part in an aircraft system should be replaced with a new part. In this study, the cumulative failure distribution of auxiliary power unit oil pumps of Boeing 737-200/300 aircraft is modeled by using simple Weibull and mixture Weibull techniques. It is demonstrated that the mixture Weibull model provides more accurate prediction for failure of the APU oil pumps. Then, the optimal replacement ages of the pumps based on the estimated Weibull parameters are calculated for various cost ratios.

## 1. Introduction

Determining the age at which an operating part in an aircraft should be replaced with a new part has always been a problem. The age for such a planned replacement should depend on the time-to-failure distribution of the part, and also on the relative costs of an in-service failure and a planned replacement (i.e. replacing an unfailed part).

There are two conditions required to make planned replacement potentially worthwhile[1]. The first is that the planned replacement of a part must cost less than an unexpected or unscheduled replacement. The second condition is that the failure characteristics of the part must display wearout, i.e. failure rate must increase with age. This can be better understood by examining the mortality characteristics of parts which is shown in Figure 1[2]. The descending curve indicates burn-in characteristic in which the failure rate decreases over time. The horizontal curve represents constant random characteristic which indicates that failure rate remains constant over time. Therefore, planned replacement has no advantage in these cases. The rising curve indicates wearout, i.e. increasing failure rate with time. Such units with age-related failure rate may be candidates for planned replacement.

In this study, the failure times of APU oil pumps are modeled using Weibull distribution and its extension. These include the standard distribution without delay parameter and a combination of Weibull distributions (mixture model).The parameters of the Weibull models are estimated by rank regression method. Then based on the best model, optimal replacement intervals for the pumps are proposed for various cost ratios.

The data used in this study are discussed in the next section.The modeling of failures using simple Weibull and mixture Weibull models is discussed in Section 3 and Section 4 respectively. The results obtained from both models are compared in Section 5. In Section 6 optimal replacement age of the pumps is studied.Conclusions of the study are summarized in Section 6.

## 2. The Data

The data analyzed here are obtained from an aviation company in Turkey for 38 oil pumps installed in APU of Boeing 737-200/300 aircraft. There is no planned replacement program for the pumps. The data are collected from maintenance records of the company over a period of 3 years. Beyond the visual checks, APU faults are found out through warnings in the cockpit. There are four types of warning indications: LOW OIL QUANTITY, LOW OIL PRESSURE, HIGH OIL TEMP and OVER SPEED. These warnings are the result of failures of various components in the APU. In order to determine the faults resulting from in-service failure of oil pumps the "non-routine work cards" for APU (ATA chapter 49) are reviewed and the faults resulting from in-service failure of oil pumps are separated and examined in detail. The first time-to- failures for the 32 pumps are obtained from these records. There is no "no fault found (NFF)" condition. Examination of other maintenance records indicates that the oil pumps are subjected to B checks at every 3,000 flight hours in which cleaning, functional checks, etc. are carried out. Furthermore, apart from these periodic checks, pumps also undergo a special operational check at 1,000 flight hours since they are supplied by a new manufacturer. The records reveal that 6 of the pumps are removed during these periodic operation checks. These pumps are handled as suspensions. Thus for this study suspensions or suspended pumps represent the pumps which do not fail in service but are removed during periodic checks. Table 1 summarizes the failure data for the pumps.

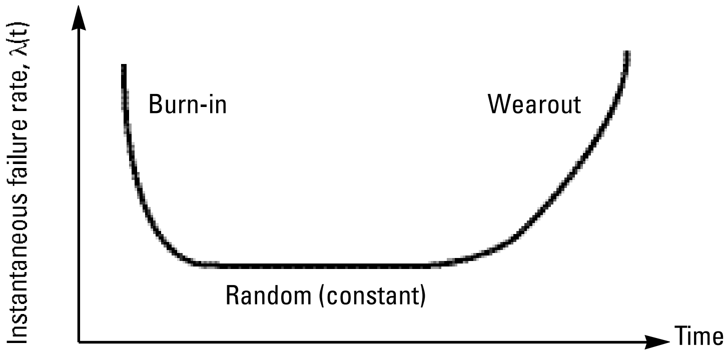


*Figure 1. Three types of morality characteristics.*

| Order | Flight | Failure Type |
|---|---|---|
| 1 | 082 | Low Pressure |
| 2 | 396 | Visable leakage |
| 3 | 763 | Low pressure |
| 4 | 846 | Low oil quantity |
| 5 | 900 | Suspension (at 1,000 hours check) |
| 6 | 935 | Suspension (at 1,000 hours check) |
| 7 | 950 | Low pressure |
| 8 | 958 | Suspension (at 1,000 hours check) |
| 9 | 989 | Suspension (at 1,000 hours check) |
| 10 | 997 | Suspension (at 1,000 hours check) |
| 11 | 1334 | Vissable leakage |
| 12 | 1800 | Low pressure |
| 13 | 1882 | Low pressure |
| 14 | 1947 | Low pressure |
| 15 | 2548 | Low pressure |
| 16 | 2912 | Low pressure |
| 17 | 2957 | Suspension (at 3,000 hours check) |
| 18 | 3200 | Low pressure |
| 19 | 3654 | Low pressure |
| 20 | 3800 | Low pressure |
| 21 | 3830 | Low pressure |
| 22 | 3921 | Low pressure |
| 23 | 3972 | Low oil quantity |
| 24 | 4185 | Low pressure |
| 25 | 4550 | Visable leakage |
| 26 | 4840 | Low oil quantity |
| 27 | 4863 | Low pressure |
| 28 | 4911 | Low pressure |
| 29 | 4918 | Low pressure |
| 30 | 4962 | Low pressure |
| 31 | 4984 | Low pressure |
| 32 | 5443 | Low pressure |
| 33 | 5782 | Low pressure |
| 34 | 5890 | Low oil quantity |
| 35 | 5978 | Low pressure |
| 36 | 5980 | Low pressure |
| 37 | 6810 | Low pressure |
| 38 | 7968 | Low pressure |

*Table 1. APU oil pumps failure data*

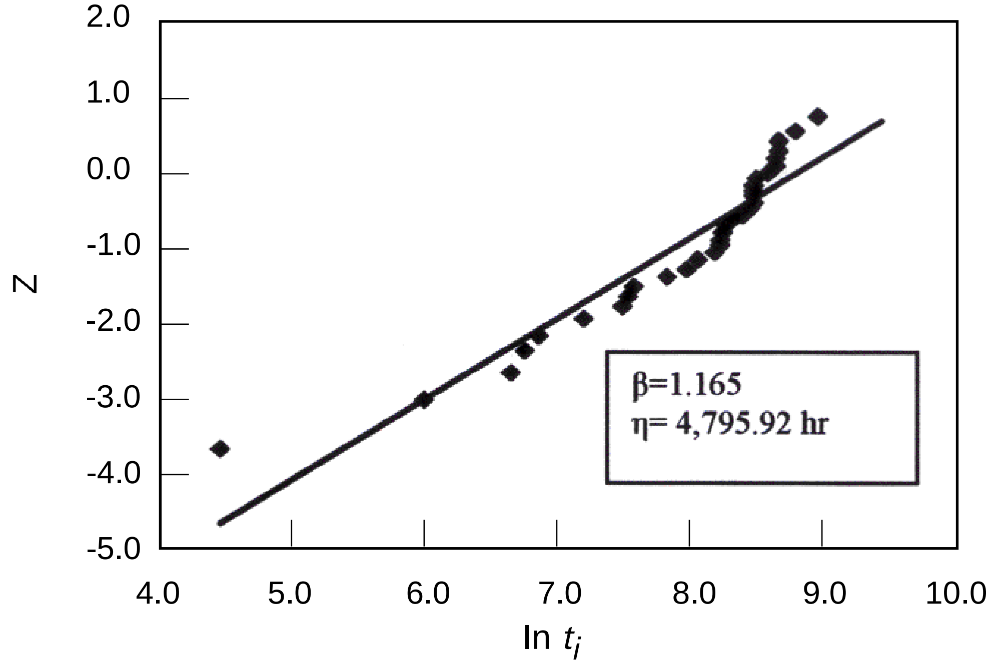


*Fig 2. The Weibull plot for the failure data of APU oil pumps.*

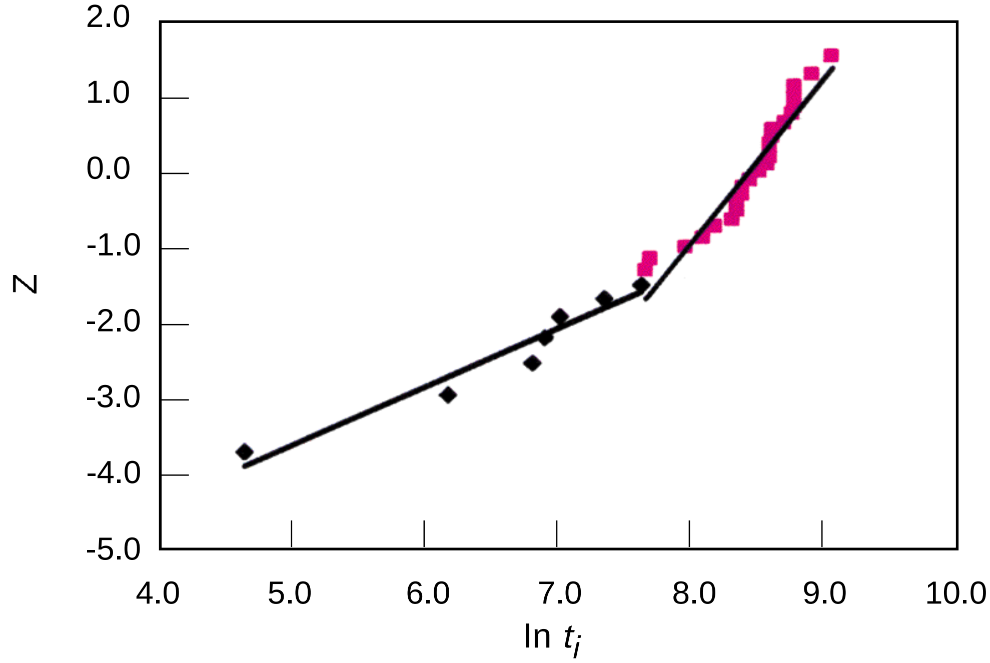


*Fig 3. Plot of APU failure data to identify the subpopulations in the data.*

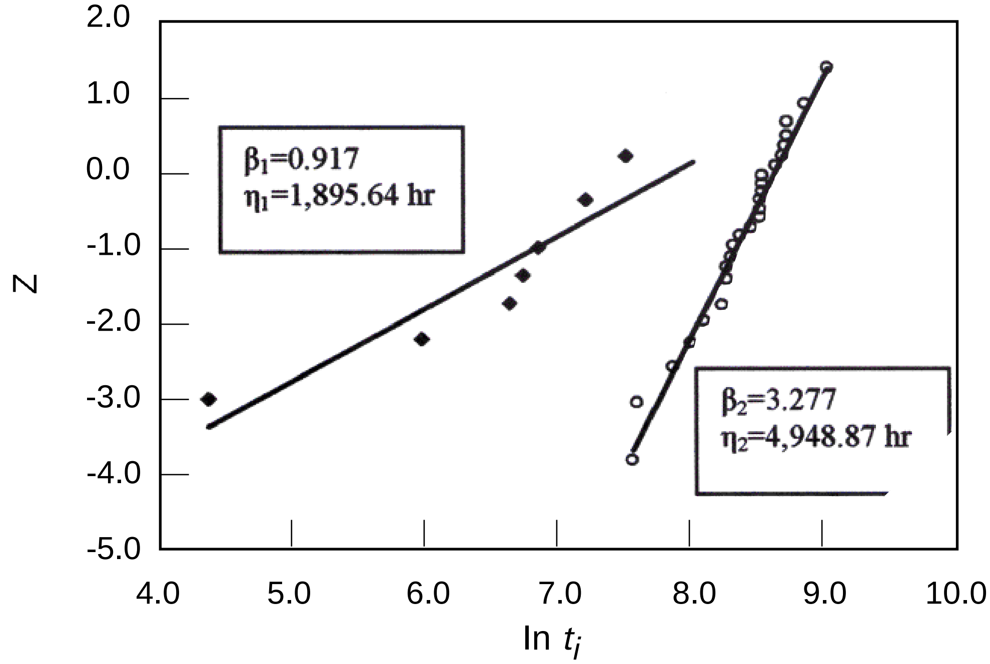


*Fig 4. Weibull plots for each of the two subpopulations drawn seperately.*

## 3. The Simple two-parameter Weibull Model

One of the prerequisites for a worthwhile planned replacement program is that the failure distribution should have a wearout characteristic, or that the failure rate increases with age. There are several models to identify the failure characteristics of parts. The Weibull model is one of the most commonly used model for this purpose.

The primary advantage of the Weibull model is the ability to provide reasonably accurate failure analysis and failure forecast with extremely small samples. This means that it is possible to use data as the first failures emerge and decide appropriate action before more failure data is generated.

The Weibull distribution can be characterized by an instantaneous failure rate function λ(t) of the form [3]

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad \eta > 0,\ \beta > 0,\ t \geq 0 \tag{1}$$

The reliability function *R(t)* which indicates the probability of surviving beyond a given time *t* can be derived from this failure rate function as follows :

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{2}$$

The probability density function (pdf) which describes the shape of the failure

distribution can be given by

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

A complimentary function *F(t)* to the reliability function can be defined as:

$$F(t) = 1 - R(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{3}$$

*F(t)* is known as cumulative distribution function and indicates the probability that a failure occurs before time t.

η is referred to as the shape parameter. It indicates whether the failure rate is increasing, constant, or decreasing. Practically, β<1 indicates that the part has a decreasing failure rate and implies infant mortality. This can be caused by a variety of factors, including design flaws, misassembly, and poor quality control. β=1 indicates a constant failure rate and implies random failures. In this case, one can suspect random events such maintenance errors, human errors, foreign object damage (FOD). β>1 indicates an increasing failure rate. The most common causes of failures in this range are corrosion, erosion and fatigue cracking. η is known as scale parameter. It represents the characteristic life of the part. The value of η is equal to the number of cycles (flight hours, landings, etc) at which 63.2% of the parts have failed[4].

Various approaches are used in fitting the Weibull model to the failure data. In this paper, the cumulative distribution function *F(t)* is transformed as follows[4-6] so that it appears in the form of a straight line.

$$\ln[1 - F(t)] = -\left(\frac{t}{\eta}\right)^{\beta}$$

$$\ln\left\{\ln\left[\frac{1}{1 - F(t)}\right]\right\} = \beta\ln(t) - \beta\ln(\eta) \tag{4}$$

From Eq.(4) it can be seen that β corresponds to the slope and (-β ln η) is the intercept. The cumulative distribution function *F(t)*, can be substituted by its estimate $F(t_i)$ using mean rank formula[3] for the time to failure data which is organized in ascending order.

$$F(t_i) = \frac{i_{t_i}}{n+1} \qquad 0 \leq i \leq n \tag{5}$$

where *n* is the sample size. By performing linear regression analysis using Eq.(4), the parameters β and η can be determined.

The Eqs.(1-4) are for two-parameter Weibull model. If the fit to data is not good enough, then three-parameter Weibull model can be employed. This can be done by estimating a minimum lifetime $t_0$ and transforming the data by letting $t_i = t - t_0$. However, certain criteria related to shape of the plot, sample size, and improvement in correlation coefficient should be met before implementing the three-parameter Weibull model.

Since pump data have suspended units, adjustments must be made to the cumulative probabilities. In this study rank adjustment method is used to account for suspension times occurring prior to ith failure. The method requires calculation of the rank increment as follow[7];

$$\textit{Rank increment} = \frac{(n+1) - i_{t_{i-1}}}{1 + (n - \text{number of preceding units})} \tag{6}$$

The rank increment is recomputed for the next failure following a suspended unit. Its adjusted rank then becomes

$$i_{t_1} = i_{t_{i-1}} + \text{rank increment} \tag{7}$$

The rank increment then remains the same until the next suspension takes place. $F(t_i)$ can be calculated by using Eq.(5).The method described above is applied to the APU oil pump data for fitting the Weibull distribution. The results are given in Table 2. In addition, Weibull plot of failure data is given in Figure 2 with the best fitted line. Resulting Weibull parameters of the oil pumps indicate that shape parameter $\beta$ is 1.165 and scale parameter $\eta$ is 4795.92 hours. The obtained shape parameter indicates that the pumps have a slow wearout failure pattern.

Although resulting index of fit, R=0.946 indicates a strong linear fit to data, it can be observed from Figure 2 that a shallow slope is followed by a steep slope. This obvious change in slope points out that there might be more than one failure mode, a mixture of modes which is analyzed in the next section.

## 4. Mixture Weibull Model

Many failure analysis problems include mixes of various failure modes. There are several methods for modeling mixes of failure modes such as mixed population model and competing risk model. Engineering physical analysis is also helpful to classify the failure data into different modes. In this study, a mixed population model proposed by Kececioglu[8] is used. A mixed population occurs when there are two or more subpopulations in the analysis. The model assumes that each population has its own failure mode and is not subjected to failure modes of other subpopulations. For two subpopulations mixture model the cumulative failure distribution $F(t)m$ can be expressed as[8]:

$$F(t)_m = \left(\frac{n_1}{n}\right) F(t)_1 + \left(\frac{n_2}{n}\right) F(t)_2 \qquad (8)$$

where $n_1$ and $F(t)_1$ are size and cumulative failure distribution of first subpopulation, and $n_2$ and $F(t)_2$ are size and cumulative failure distribution of second subpopulation. When cumulative failure distributions in Eq.(8) are modeled by two parameter Weibull distribution, it gives rise to following:

$$F(t)_m = \left(\frac{n_1}{n}\right)\left(1 - \exp\left(\frac{t}{\eta_1}\right)^{\beta_1}\right) + \left(\frac{n_2}{n}\right)\left(1 - \exp\left(\frac{t}{\eta_2}\right)^{\beta_2}\right) \qquad (9)$$

where 1 and 2 refer to the two subpopulations being considered. Reliability and probability density functions can be expressed accordingly. To compute $F(t)_m$ using Eq.(9), it is necessary to find the subpopulation sizes $n_1$ and $n_2$ and the four Weibull parameters. In order to separate the mixed population into its constituent subpopulations a combined analytical-graphical method is used. For this purpose Figure 2 is replotted to determine if the points appear to fall into distinct groupings by visual inspection. Then the best straight line which represents each subpopulation is drawn. For the case under study two subpopulations are found as shown in Figure 3. Points falling closest to the lower line are in Subpopulation 1, and points falling closest to the higher line are in Subpopulation 2. It should be noted that this plot serves only to separate the data into subpopulations; the Weibull distribution parameters are not determined from this plot.

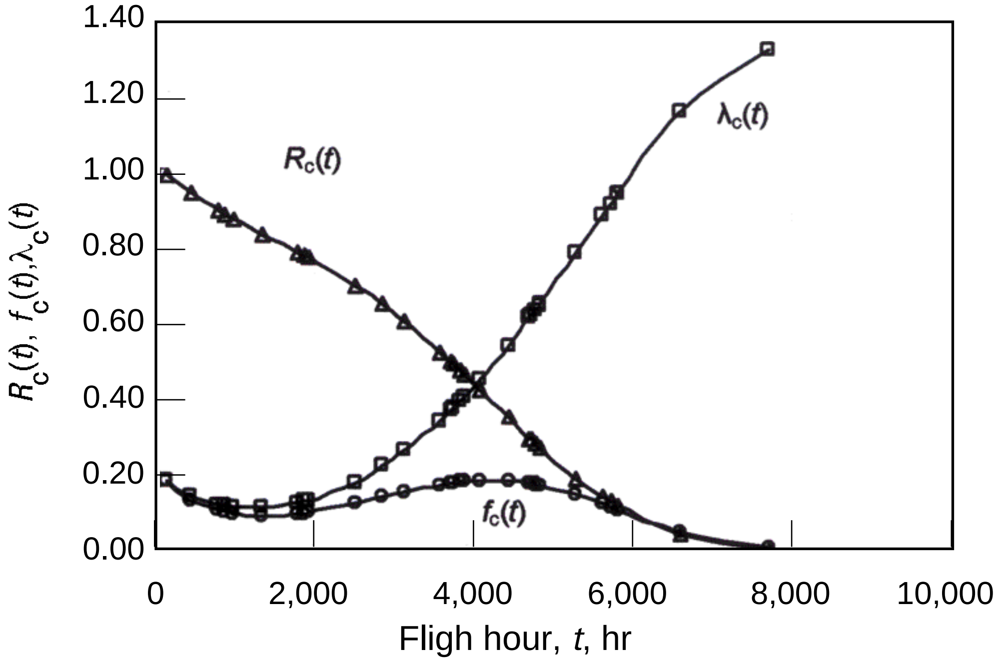


*Fig 5. Relationship between $R_c(t)$, $f_c(t)$, and $\lambda_c(t)$ and flight hour, t.*

Having identified the two subpopulations, all Weibull parameters are determined for each subpopulation by treating the separated, constituent subpopulations individually by using the method described in Section 3. Table 3 indicates the results for Subpopulation 1 and Table 4 indicates the results for Subpopulation 2 yielding the following Weibull parameters:

$\beta_1$=0.917 $\eta_1$=1,895.64 hr

$\beta_2$=3.277 $\eta_2$=4,948.87 hr

Resulting indexes of fits are 0.9238 and 0.9893 for the first and second subpopulations respectively. The Weibull plots of the two subpopulations are drawn separately in Figure 4. Looking at the results of the two-subpopulation analysis it appears that the first 7 failures, comprising the first subpopulation, are due to early and/or random causes ($\beta$=0.917).Since the subpopulation size is limited, it is not attempted to separate this subpopulation into further groups. The last 25 failures, comprising the second population, are due to wear out causes ($\beta$=3.277). Calculated values of $R(t)$, $f(t)$, and $\lambda(t)$ for each flight time are given in Table 5 and plotted in Figure 5.

## 5. Comparison of Results

Table 6 and Figure 6 show the comparison between the cumulative failure distribution estimated by Eq.(5) and calculated failure distributions using simple Weibull model and mixture Weibull model for the APU oil pumps. The performances of the two models may be evaluated by using Kolmogorov-Smirnov goodness-of-fit test which compares the observed failure probability with the corresponding predicted failure probability calculated by each model. This has been done in Table 6 where the values of *D* are listed as obtained from

$$D = |F(t)_p - F(t)_c|$$

The maximum values of *D* obtained are first compared with the allowable critical value of *D*, $D_{cr}$, to see if $D_{max}<D_{cr}$ at a chosen level of significance.If this condition is satisfied, the model with minimum $D_{max}$ is preferred. At the 5% significant level, allowable critical value of *D* is given by[8]

$$D_{cr} = \frac{1.36}{\sqrt{38}} = 0.2206$$

$D$max from both models are below the critical value. However, as the $D$max value for the two-subpopulation Weibull model is smaller than for simple Weibull model, the two-subpopulation model is preferred.

In order to support the results obtained from mixture Weibull model, the failure data on non routine work cards of the oil pumps are also examined in detail. It is found out that the first 7 pump failures are resulted from following causes: Misalignment/misplacement (5 pumps) and contamination (2 pumps). These causes are characteristics causes of early failures and change failures. On the other hand the rest of the pump failures, except two failures, are resulted from deterioration or wear in seals, drive shafts, bearings, and gears which represent aging and wear out failures. Two pumps failures are due to multiple-causes. Thus causes of the oil pump failures support the results obtained from analytical-graphical method.

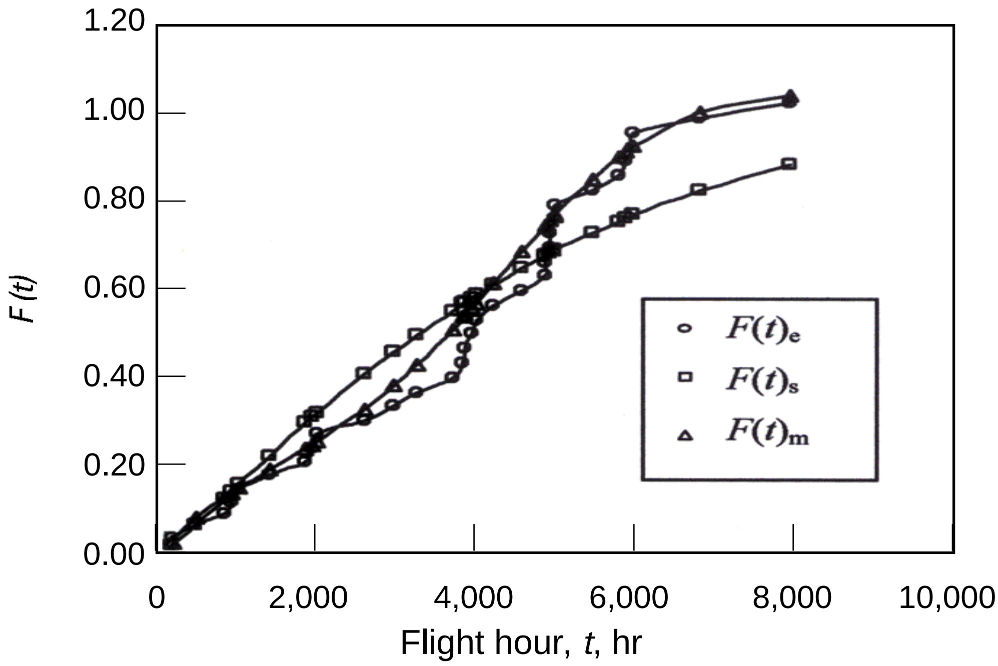


*Fig 6. Comparison of estimated and calculated cumulative failure distribution as obtained from Table 6*

## 6. Optimal Replacement Interval

In previous section, it was found out that the failure rate of pumps increases with time. Thus, a planned replacement of the pumps may be worthwhile. At this point, the problem is to determine the age at which the oil pumps should be replaced with a new one.

The desirable replacement will depend on the criterion to be used to evaluate the decision. Two criteria are available. The first is to choose the replacement age so that the risk of failure is always less than some maximum acceptable amount. Whenever the probability of failure reaches this maximum acceptable risk, the part is replaced. The second criterion is to choose the replacement age $(t_p)$ so as to minimize the expected costs per period of operation. The choice between these two criteria depends upon the circumferences at hand. If a regulatory body, such as a civil aviation authority, requires that the risk of failure be less than some specified amount, then there is little choice. This is generally true when an in-service failure would degrade flight safety and endanger human life. However, with the current emphasis on extensive and thorough redundancy in vital systems for both military and commercial aircraft, situations dominated by flight safety considerations rarely occur. Ordinarily the failure of an aircraft part causes nothing more than a delay, an unfulfilled mission, or an expedited repair action. Hence, the second criterion of minimum expected cost was used in this study to determine optimal replacement age.

The total cost $(C_T)$ of any replacement can be defined[1,3] as the sum of cost of planned replacements $(C_p)$ and cost of replacements at failures (in-service failures) $(C_f)$.

$$C_T = C_p\,[1 - F(t_p)] + C_f F(t_p) \tag{10}$$

where $F(t_p)$ is the percentage of failed components at time $t_p$. Then, the total cost per unit time $J(t_p)$ can be written as follows[1]:

$$J(t_p) = \frac{C_p\,[1 - F(t_p)] + C_f F(t_p)}{\int_0^{t_p} [1 - F(t)]\, dt} \tag{11}$$

Cumulative failure function $F(t)_m$ obtained from mixure Weibull model is substituted in this equation. The value of time $t_p$ that minimizes the total cost per period of operation is the optimal replacement age.

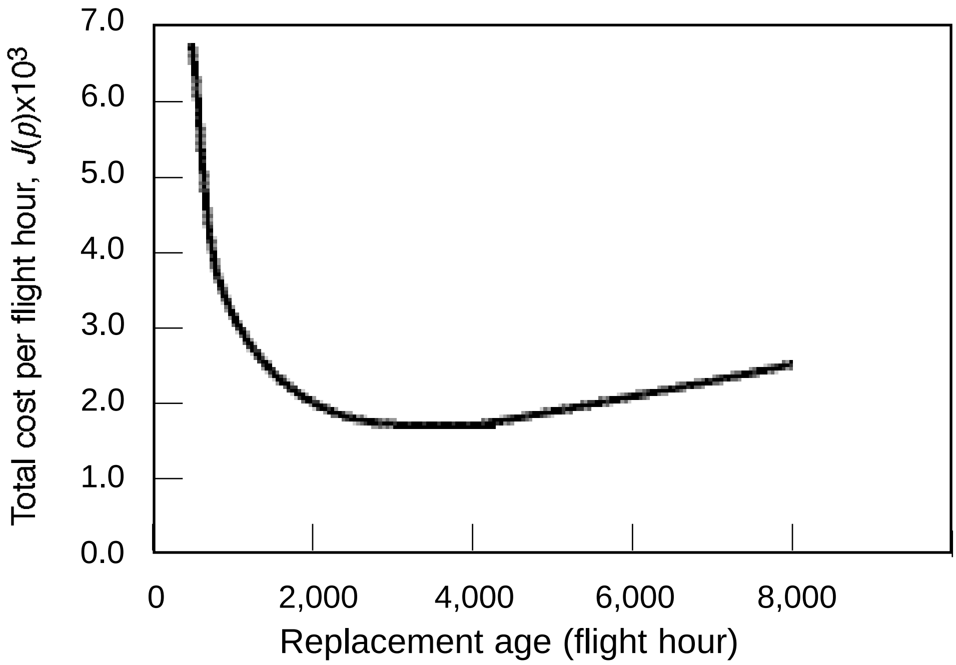


*Fig 7. Graph of the cost per operating hour versus replacement age for $C_f/C_p$=10.*

| Cost Ratio $C_f/C_p$ | Optimal replacement age (in flight hours) |
|---|---|
| 2 | 4,400 |
| 5 | 3,100 |
| 10 | 2,560 |
| 15 | 2,400 |
| 40 | 2,100 |

*Table 7. Optimal replacement ages for oil pumps in flight hour for different $C_f/C_p$ ratios*

In this study optimal replacement ages are found for different values of $C_f/C_p$ ratios by assuming $C_f$=1.0 unit since there is no cost data. The results are indicated in Table 7. Figure 7 indicates the cost per flight hour versus replacement age for $C_f/C_p$=10. As it can be seen the cost per unit time falls initially quite steeply as the replacement age increases. There is then a flat region around the optimal replacement age, and finally the cost per unit time rises again slightly. The lowest point on the graph is at the optimal replacement age which is 2,560 hours. There is an interval around this value where the cost does not vary much. In the absence of flight safety considerations, estimates for the cost of an in-service failure and cost of a planned replacement are required to optimize the replacement interval for the pumps. These estimates are equal in importance to the determination of the underlying failure distribution. If the relative cost of in-service failure is overestimated the replacement age will be too short, and the real cost will be raised because of a waste of useful pump life. If the cost is underestimated, the replacement age will be too high, and the real cost will again be higher than the necessary because of too many emergency actions. However, usually there are reasons to expect that it costs much more to repair an in-service failure than to make a planned replacement under the most general circumstances of both military and commercial operations.

## 7. Conclusion

The Weibull analysis is an effective tool for failure forecasts and predictions of the various aircraft parts and systems, and extremely useful for maintenance planning, particularly reliability centered maintenance. In this case, the failure characteristics of aircraft APU oil pumps have been analyzed by using simple two parameter and mixture Weibull models. The predictive performance of the models is compared using Kolmogorov-Smirnov goodness-of-fit test. It is found that the mixture model with smaller $D_{max}$ value provides more accurate prediction for the failure distribution of the pumps.The resulting parameters indicate that the oil pumps have an approximately random failure rate (shape parameter 0.917) initially, followed by a wearout pattern (shape parameter 3.277).The physics of the failure of the pumps supports the validity of this failure pattern obtained from the analysis. Then based on the predicted failure distribution, the optimal replacement age of the pumps is calculated for various cost ratios.

## Acknowledgement

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## References

1. Kamins,M and McCall J.J.,"Rules for Planned Replacement of Aircraft and Missile Parts," Memorandum RM-2810-PR, Rand Corporation, 1961.
2. Smith,A.M., "Reliability-Centered Maintenance", McGraw Hill, New York, NY, 1993.
3. Ebeling,C.E.,"An Introduction to Reliability and Maintainability Engineering," McGraw-Hill, New York, NY, 1997.
4. Al-Garni,A.Z., Sahin,A.Z., Al-Ghamdi,A.S and Al-Kaabi, S.A.,"Reliability Analysis of Aeroplane Brakes," Quality and Reliability Engineering International, Vol.15,pp. 143-150, 1999.
5. Sheikh,A.K, Al-Garni,A.Z and Badar,M.A.,"Reliability Analysis of Aeroplane Tyres," International Journal of Quality and Reliability Management, Vol.13,pp.28-38, 1996.
6. Al-Garni, A.Z., Sahin. and Al-Farayedhi, A.A., " A Reliability Study of Fokker F-27 Airplane Brakes," Reliability Engineering and System Safety, Vol.56, pp.143-150, 1997.
7. Newton,D.,and Bromley,R., "Practical Reliability Engineering",4th Ed. John Wiley & Sons,New York, N.Y., 2002
8. Kececioglu,D.,"Reliability Engineering Handbook", Prentice-Hall, Englewood Cliffs, N.J., 1991.

## Nomenclature

*$C_T$ = total cost*

*$C_f$ = cost of in-service failure*

*$C_p$ = cost of planned replacement*

*F(t) = cumulative distribution function*

*$J(t_p)$ = total cost per unit time*

*$\lambda$(t) = instantaneous failure rate function*

*n = number of observations*

*R(t) = reliability function*

*t = operation time in flight hours*

*$t_p$ = optimum replacement age in flight hours*

*$\beta$,$\eta$. = Weibull parameters*

EDITORS NOTE:

**This article details the formula and manual methods of performing Weibull analysis. However there are available a number of Weibull analysis software packages that make the analysis much easier. See the Special Maintenance Application Software Survey in the October 2005 issue of the maintenance journal for products such as RelCode (www.albanyint.com.au)**

| Adjusted rank, $i_{ti}$ | $t_i$,hr | ln $t_i$ | $F(t_i)$ | Z=1/(1-$F(t_i)$) | Ln (Ln Z) | Predicted Ln (Ln Z) |
|---|---|---|---|---|---|---|
| 1 | 82 | 4.40672 | 0.02564 | 1.02632 | -3.65060 | -4.74165 |
| 2 | 396 | 5.98141 | 0.05128 | 1.05405 | -2.94421 | -2.90655 |
| 3 | 763 | 6.63726 | 0.07692 | 1.08333 | -2.52519 | -2.14225 |
| 4 | 846 | 6.74052 | 0.10256 | 1.11429 | -2.22365 | -2.02192 |
| 5.061 | 950 | 6.85646 | 0.12977 | 1.14912 | -1.97330 | -1.88680 |

*Table 2. Results of two parameter Weibull analysis for APU oil pump failures.*
*(Only the first 5 results of 32 results are shown)*

| $i_{ti}$ | $t_i$,hr | ln $t$ | $F(t_i)$ | Z=1/(1-$F(t_i)$) | Ln (Ln Z) |
|---|---|---|---|---|---|
| 1.000 | 82 | 4.40672 | 0.07692 | 1.08333 | -2.52519 |
| 2.000 | 396 | 5.98141 | 0.15385 | 1.18182 | -1.78944 |
| 3.000 | 763 | 6.63726 | 0.23077 | 1.30000 | -1.33802 |
| 4.000 | 846 | 6.74052 | 0.30769 | 1.44444 | -1.00042 |
| 5.286 | 950 | 6.85646 | 0.40662 | 1.68525 | -0.65026 |

*Table 3. Failure data analysis for Subpopulation 1.*
*(Only the first 5 results of 7 results are shown)*

| $i_{ti}$ | $t_i$,hr | ln $t$ | $F(t_i)$ | Z=1/(1-$F(t_i)$) | Ln (Ln Z) |
|---|---|---|---|---|---|
| 1.000 | 1882 | 7.54009 | 0.03704 | 1.03846 | -3.27703 |
| 2.000 | 1947 | 7.57405 | 0.07407 | 1.08000 | -2.56446 |
| 3.000 | 2548 | 7.84306 | 0.11111 | 1.12500 | -2.13891 |
| 4.000 | 2912 | 7.97660 | 0.14815 | 1.17391 | -1.83044 |
| 5.045 | 3200 | 8.07091 | 0.18685 | 1.22979 | -1.57580 |

*Table 4. Failure data analysis for Subpopulation 2.*
*(Only the first 5 results of 25 results are shown)*

| Flight Hours $t_i$ | Calculated Reliability $R(t)_c$ | Calculated pdf $f(t)_c$ x$10^3$ | Calculated Failure rate $\lambda(t)_c$ x$10^3$ |
|---|---|---|---|
| 82 | 0.98276 | 0.18747 | 0.19076 |
| 396 | 0.93297 | 0.13857 | 0.14853 |
| 763 | 0.88731 | 0.11313 | 0.12750 |
| 846 | 0.87807 | 0.10945 | 0.12465 |
| 950 | 0.86689 | 0.10568 | 0.12190 |

*Table 5. Calculated values of reliability, $R(t)_c$, probability density function, $f(t)_c$, and Failure rate, $\lambda(t)_c$.*
*(Only the first 5 results results are shown)*

** IF YOU WISH TO SEE THE FULL TABLE OF RESULTS FOR TABLES 2 TO 6 CONTACT:*
*Len Bradshaw mail@maintenancejournal.com*

| Flight Hours $t_i$ | Estimated $F(t)_e$ [Eq.(5)] | Calculated $F(t)_s$ [Eq.(3)] | Calculated $F(t)_m$ [Eq.(9)] | $D_1$ Simple model | $D_2$ Mixture model |
|---|---|---|---|---|---|
| 82 | 0.02564 | 0.00870 | 0.01724 | 0.01694 | 0.00840 |
| 396 | 0.05128 | 0.05324 | 0.06703 | 0.00196 | 0.01575 |
| 763 | 0.07692 | 0.11083 | 0.11269 | 0.03391 | 0.03577 |
| 846 | 0.10256 | 0.12408 | 0.12193 | 0.02152 | 0.01936 |
| 950 | 0.12977 | 0.14071 | 0.13311 | 0.01094 | 0.00334 |
| 1334 | 0.16179 | 0.20165 | 0.17203 | 0.03986 | 0.01024 |
| 1800 | 0.18977 | 0.27333 | 0.21854 | 0.08356 | 0.02877 |
| 1882 | 0.21977 | 0.28559 | 0.22704 | 0.06582 | 0.00727 |
| 1947 | 0.24977 | 0.29521 | 0.23390 | 0.04544 | 0.01587 |
| 2548 | 0.27977 | 0.38037 | 0.30416 | 0.10060 | 0.02439 |
| 2912 | 0.30977 | 0.42833 | 0.35444 | 0.11856 | 0.04467 |
| 3200 | 0.34115 | 0.46428 | 0.39884 | 0.12313 | 0.05768 |
| 3654 | 0.37254 | 0.51735 | 0.47654 | 0.14481 | 0.10400 |
| 3800 | 0.40392 | 0.53349 | 0.50319 | 0.12957 | 0.09927 |
| 3830 | 0.43531 | 0.53676 | 0.50874 | 0.10145 | 0.07344 |
| 3921 | 0.46669 | 0.54654 | 0.52573 | 0.07985 | 0.05904 |
| 3972 | 0.49808 | 0.55195 | 0.53534 | 0.05387 | 0.03726 |
| 4185 | 0.52946 | 0.57396 | 0.57592 | 0.04450 | 0.04646 |
| 4550 | 0.56085 | 0.60957 | 0.64590 | 0.04873 | 0.08506 |
| 4840 | 0.59223 | 0.63604 | 0.70021 | 0.04381 | **0.10798** |
| 4863 | 0.62362 | 0.63807 | 0.70441 | 0.01446 | 0.08080 |
| 4911 | 0.65500 | 0.64228 | 0.71313 | 0.04349 | 0.05813 |
| 4918 | 0.68638 | 0.64289 | 0.71440 | 0.07106 | 0.02801 |
| 4962 | 0.71777 | 0.64671 | 0.72230 | 0.10055 | 0.00453 |
| 4984 | 0.74915 | 0.64860 | 0.72623 | 0.09438 | 0.02293 |
| 5443 | 0.78054 | 0.68616 | 0.80269 | 0.09438 | 0.02215 |
| 5782 | 0.81192 | 0.71159 | 0.85098 | 0.10033 | 0.03906 |
| 5890 | 0.84331 | 0.71930 | 0.86469 | 0.12400 | 0.02138 |
| 5978 | 0.87469 | 0.72545 | 0.87524 | 0.14924 | 0.00054 |
| 5980 | 0.90608 | 0.72559 | 0.87547 | **0.18049** | 0.03061 |
| 6810 | 0.93746 | 0.77788 | 0.94780 | 0.15958 | 0.01034 |
| 7968 | 0.96885 | 0.83578 | 0.98658 | 0.13306 | 0.01774 |

*Table 6. Calculated values of the estimated cumulative failure distribution, $F(t)_e$, cumulative failure distributions from simple Weibull model, $F(t)_s$, and mixtureWeibull model, $F(t)_m$, and Kolmogorov-Simirnov goodness-of-fit test values, $D_1$ and $D_2$ for each model (Maximum values are shown in bold).*