KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Computer Graphics
(ICS – 535)

Term Project Progress Report

Superellipse Fitting to Partial Data

SUBMITTED TO

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BY

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Abstract:

The fitting of primitive models to image data is a basic task in computer graphics and computer vision, allowing reduction and simplification of data. One of the most commonly used models is the ellipse and superellipse. Superellipse is formed by incorporating additional parameter into the equation of ellipse. It can be used to represent in a compact form a large variety of shapes. However, fitting them to data is difficult and computationally expensive. Moreover, when partial data is available the parameter estimates become unreliable. This project discusses the least square fitting (LSF) method for fitting ellipses and superellipses to scattered or partial data. Furthermore, this project attempts to improve the process of fitting to partial data by combining gradient and curvature information with the LSF method. The main objective is accomplished by fitting ellipse or superellipse to real data. This helps in the representation of complex data in a simpler way. Moreover, tests are performed based on the gradient and curvature information to observe the robustness of the fit.

Keywords: Ellipse, Superellipse, Curve Fitting, Algebraic distance, Least Square Fitting, Generalized Eigenvalue Problem.

1. Introduction

Image feature extraction is an important part of computer vision system. The accuracy and reliability for representing features can have a profound impact on the higher level processes build on top [1]. Curves are common feature types which are used to represent various conics. Ellipse and Superellipse are an interesting abstraction of curves. The following equation represents a superellipse with origin as the center:

$$\left(\frac{x}{a}\right)^{2/\varepsilon} + \left(\frac{y}{b}\right)^{2/\varepsilon} = 1 \qquad (1)$$

if ε=1, it represents an ellipse. The advantage of superellipse is that it provides a fairly compact parameterization that still enables a wide variety of shapes to be represented. Unfortunately the super ellipse's disadvantage is the highly non-linear nature of ε. In particular, fitting is complicated and problematic since iterative methods are required. This report concentrates on Superellipse curve fitting. This is done using the least square solution. This is achieved by finding the set of model parameters that minimize the sum of squares of the *distances* between the model curve and given pixel data, this technique is discussed in detail in the following subsections. The notion of distance can be interpreted in various ways, Rosin and West [7] investigate several examples. The Euclidean distance between a pixel and the point on the curve closest to it is probably the most suitable measure. Unfortunately, finding this distance is rarely used. A simpler measure is the algebraic distance given by

$$Q_0(x,y) = \left(\frac{x}{a}\right)^{2/\varepsilon} + \left(\frac{y}{b}\right)^{2/\varepsilon} - 1. \qquad (2)$$

The best fit superellipse is determined by finding the parameters which minimize the objective function $Q_0^2(x,y)$. One of the main problems with the algebraic distance is that it results in different distance estimates for different parts of superellipse curves depending on the local curvature of the curve [1]. Many distance measures has been suggested to compensate the inaccuracy but none of them are totally satisfactory [2]. The inaccuracies of the distance approximations are not such a serious problem when they are adopted for fitting superellipse with relatively complete pixel samples. The literature on ellipse or superellipse fitting is divided into two broad techniques [2],

- Clustering (Such as Hough based methods).
- Least Square Fitting.

Realistic applications frequently require fitting based on pixel data covering part of superellipses. In such cases, a conventional algebraic distance measure often leads to gross misestimating. Fig. 1 shows an example of such error. The project proposes a modification scheme based on introducing higher order derivatives information and demonstrate experimentally that it produces superellipse fitting partial data.

The rest of the paper is organized as follows. Section 2 briefs the literature in this area, section 3 discusses the proposed strategy. Section 4 discusses the experimental steps followed by a conclusion and a description of the future work.

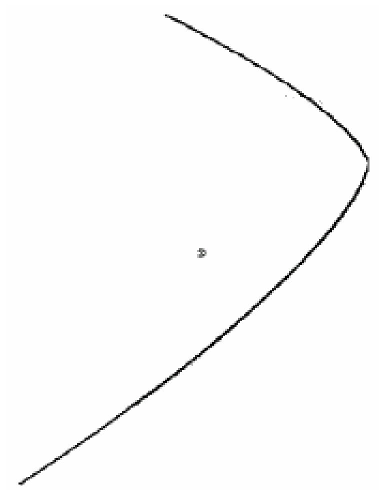

Fig. 1. Gross misestimation of centre.

2. Least Square Fitting Strategy

Least square techniques center on finding the set of parameters that minimize some distance measure between the data points and the ellipse. There are several approaches depending on the constraints applied on the least squares. To illustrate this constraints let us consider the a general conic as:

$$F(\overline{a},\overline{x}) = \overline{a}.\overline{x} = ax^2 + bxy + cy^2 + dx + ey + f = 0\,, \qquad (3)$$

where $\overline{a} = [a\,b\,c\,d\,e\,f]^T$ and $\overline{x} = [x^2\; xy\; y^2\; x\; y\; 1]^T$. $F(\overline{a};\overline{x}_i)$ is called the "algebraic distance" of a point *(x,y)* to the conic $F(\overline{a};\overline{x}_i)$=0. The fitting of general conic may be approached by minimizing the sum of squared algebraic distance of the curve to the N data points.

$$= \min\left\{\sum_{i=1}^{N}\left(F(\overline{a},\overline{x})\right)^2\right\} \qquad (4)$$

In order to avoid the trivial solutions $\overline{a} = 0_6$, the parameter $\overline{a}$ is constraint in many ways. The constraints are as follows:

- Many authors suggested that $\|a\|^2 = 1$
- Rosin [6] and Gander [5] imposed the following $a + c = 1$
- Bookstein [4] proposed $a^2 + \frac{1}{2}b^2 + c^2 = 1$

The above constraints are either linear, of the form $C.\bar{a}=1$ or quadratic $\bar{a}^T C\bar{a}=1$ where C is a 6x6 *constraint matrix*. To fit a superellipse efficiently, it is required to apply the following constraint:

$$Minimize \sum_i F(\bar{a},\bar{x}_i)^2 subject\ to\ b^2-4ac<0 \qquad (5)$$

The results using the least square fitting (i.e., using eqn (5)) on the partial data is shown in Fig. 2 (a) and 2 (b).

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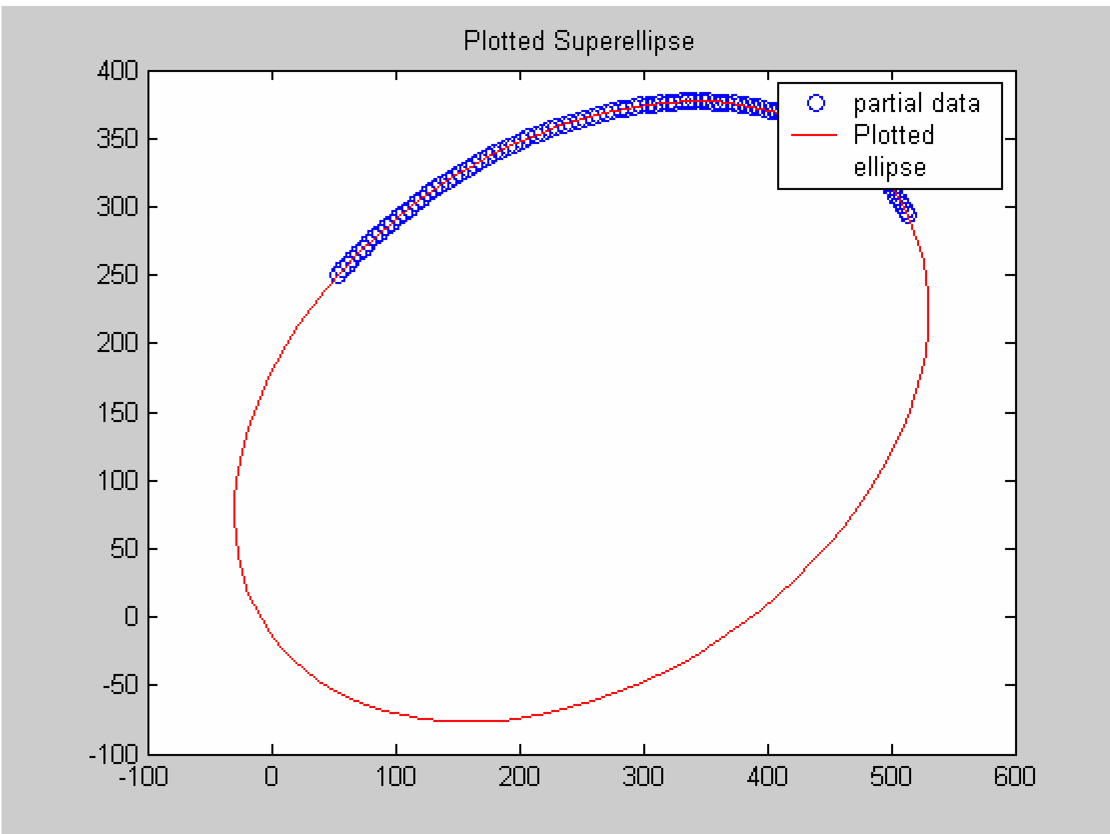


Figure 2 (a) Ellipse drawn with centre (250,150), major axis=300, minor axis= 200

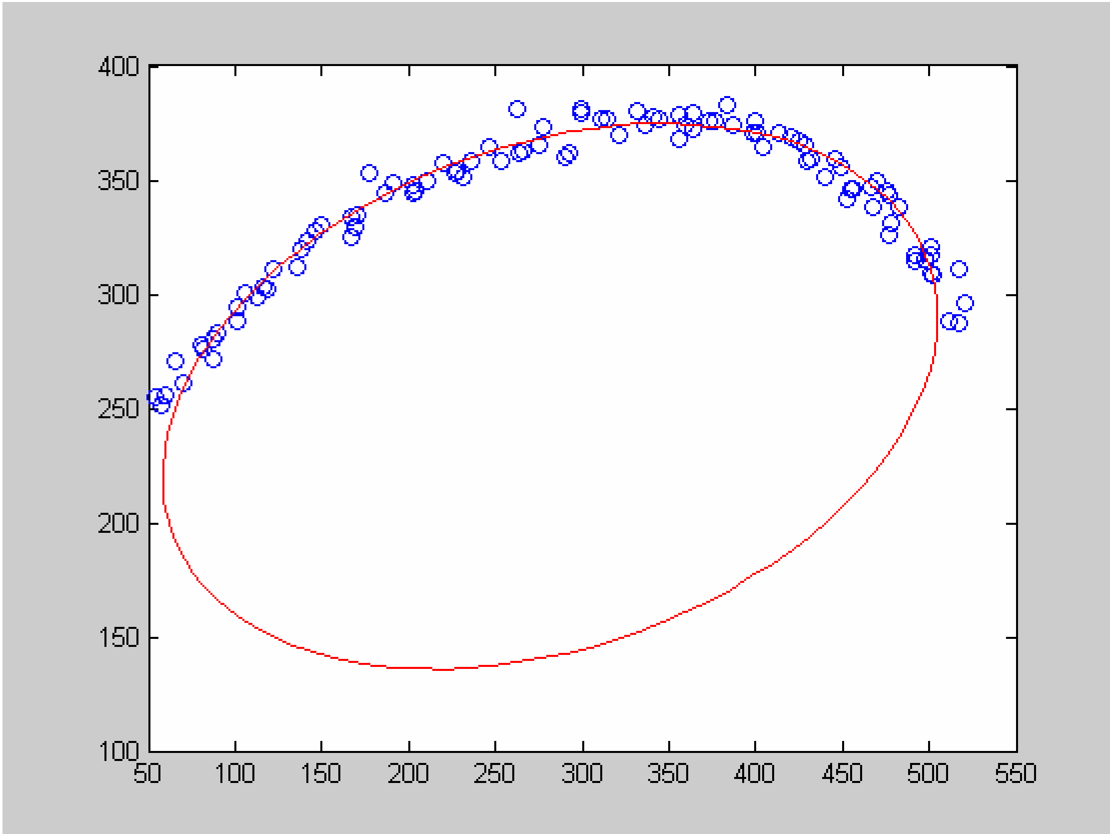


Figure 2 (b) Ellipse drawn with Least Square Fitting having centre (250,150), major axis=300, minor axis= 200 with scattered data

3. Proposed Scheme

3.1 Rationale

A problem with fitting superellipse to partial data using the conventional algebraic distance is that the error landscape does not provide enough information to guide the optimization process. The conventional algebraic distance measure treats pixels as individual data points and relations between pixels are not exploited. However, incorporating these relations is potentially very useful for curve fitting. In particular, the local gradient and curvature features of a superellipse are the functions of its parameters. Therefore, examining the gradient and curvature features of a point on a curve should reveal information about the true parameters. In order to utilize such information, reasonable gradient and curvature distance measures need to be defined. The next section devises these distance measures.

3.2 Modified Objective Function

For generality, the function if first expressed in polar form, i.e.,

$$Q_0(x,y)=\left[\frac{(x-x_c)\cos\theta-(y-y_c)\sin\theta}{a}\right]^{2/\varepsilon}+\left[\frac{(x-x_c)\sin\theta+(y-y_c)\cos\theta}{b}\right]^{2/\varepsilon}-1 \quad (6)$$

where (x_c,y_c) are the coordinates of center of the superellipse and θ is the degree of rotation. To simplify the expression of the following derivation, define the one-to-one transformation $T(x_c,y_c,a,b,\theta,\varepsilon)$ from the original space to the new {X,Y} space:

$$X=\left[\frac{(x-x_c)\cos\theta-(y-y_c)\sin\theta}{a}\right]^{1/\varepsilon},$$

$$Y=\left[\frac{(x-x_c)\sin\theta+(y-y_c)\cos\theta}{b}\right]^{1/\varepsilon}, \quad (7)$$

Notice that transformation T does not only shift, rotate, and rescale superellipses but also transform them into circles in the XY domain. As a result the following relationship between an algebraic distance to a unit circle in the XY domain and the $Q_0(x,y)$ in the *xy* domain.

$$Q_0^{'}(X,Y)=X^2+Y^2-1=Q_0(x,y). \quad (8)$$

Since the inverse mapping $T^{-1}(x_c,y_c,a,b,\theta,\varepsilon)$ exists and is one-to-one as well, the *xy* and XY notations are interchangeable. The conventional algebraic distance is calculated by evaluating $Q_0(u,v)$ where *(u,v)* is the coordinates of the point in question. If point *(u,v)* is on the superellipse described by $Q_0(x,y)=0$, then $Q_0(u,v)=0$, in other words the algebraic distance from *(u,v)* to $Q_0(x,y)=0$ is 0. Otherwise, $Q_0(u,v)\neq 0.$. Hence, curve fitting can be achieved by minimizing objective function $Q_0^2(x,y)$.

The main work is to extend the concept of algebraic distance to incorporate a measure of local shape of the curve in terms of gradient and curvature. In order to achieve this first look at the unit circle in XY plane. For any point on the unit circle, (U,V), we have

$$U^2+V^2=1 \quad (9)$$

and (U,V) can be expressed in parametric form as

$$U = \cos t,\ V = \sin t. \tag{10}$$

By differentiating both sides of Eqn (9) with respect to ‘t’ we have.

$$\frac{d(U^2 + V^2)}{dt} = 2UU_t^{'} + 2VV_t^{'} = 0. \tag{11}$$

Similarly,

$$\frac{d(UU_t^{'} + VV_t^{'})}{dt} = \left[UU_t^{"} + (U_t^{'})^2 + VV_t^{"} + (V_t^{'})^2\right] = 0 \tag{12}$$

Notice that from Eqn. (10) we get $(U_t^{'})^2 + (V_t^{'})^2 = 1$,

giving $$\left(UU_t^{"} + VV_t^{"} + 1\right) = 0. \tag{13}$$

Let $Q_1(x,y) = 2XX_t^{'} + 2YY_t^{'}$ and $Q_2(x,y) = XX_t^{"} + YY_t^{"} + 1$. Thus in a similar manner to the algebraic distance at any point, *(u,v)*, on the superellipse $Q_0(u,v) = Q_1(u,v) = Q_2(u,v) = 0$. In general, for any point off the ellipse $Q_1(x,y) \neq 0$ and $Q_2(x,y) \neq 0$**,** although depending on the values of $X_t^{'}, Y_t^{'}, X_t^{"}, and\, Y_t^{"}$ this may not hold. In contrast, Q_0 is guaranteed to be nonzero for all points off the superellipse. Q_1 *and* Q_2 is introduced in the objective function for minimization. Now we define a new objective function in XY domain

$$F(x,y) = (1 - w_1 - w_2)Q_0^2(x,y) + w_1 Q_1^2(x,y) + w_2 Q_2^2(x,y)$$
$$(w_1, w_2 \geq 0\, and\, w_1 + w_2 \leq 1) \tag{14}$$

so that the curve fitting problem is now to minimize $F(x,y)$, which involves simultaneous minimizing $Q_0^2(x,y), Q_1^2(x,y),\ and\ Q_2^2(x,y)$. The purpose of the objective function is to measure how well the gradient and curvature of a given data set fits a theoretical superellipse model.

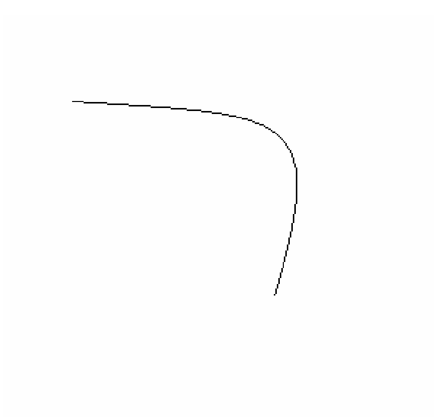
It should be noted that the reduction of *F(x,y)* does not necessarily lead to reduction of a given distance in the *xy* domain. However an exact fit in XY domain does guarantee an exact superellipse fit in the *xy* domain.

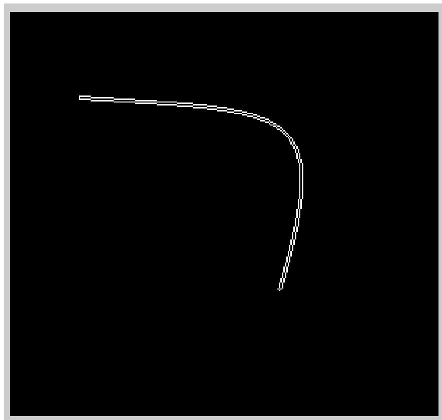
4. Experimental Test

Experiments were carried out for different values of major and minor axis along with some added noise. Test are done with different variations of w1 and w2. If w1=w2=0 then the superellipse is plotted with the standard algebraic method. Variations for different values of w1 and w2 are plotted to get a better plot. However the main goals of this project is to fit superellipse based on the objective function in Eqn. (14) and show how to fit a superellipse on a real data. The results of the fits are overlaid on the data, and plotted both when the standard algebraic distance is used and when gradient information is incorporated. By using the gradient information the fits are better than that fitted using algebraic distance.

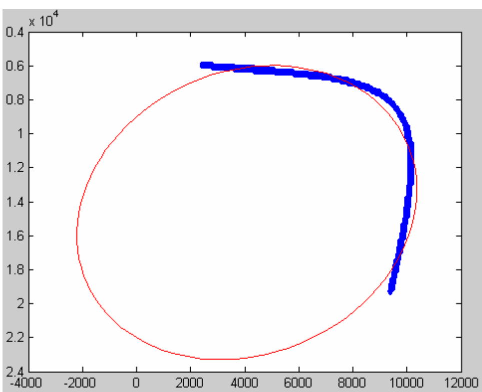
4.1 Fitting to Partial Data

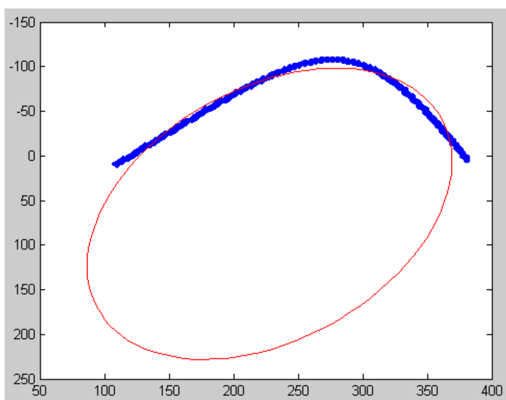
Example of fitting to data are shown in Figure(). It is shown that superellipse plotted using least square gives a large deviation, but by using the gradient and curvature information the result shows a better plot.





(a) (b)





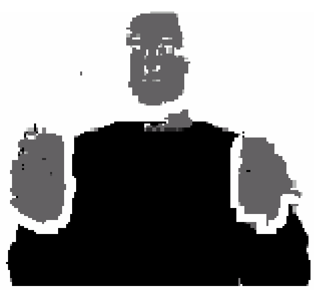
(c) (d)

Figure 3. (a) Original Image (b) Edged detected image (c) Ellipse fitted using Least square method (d) Ellipse fitted using gradient and curvature information.

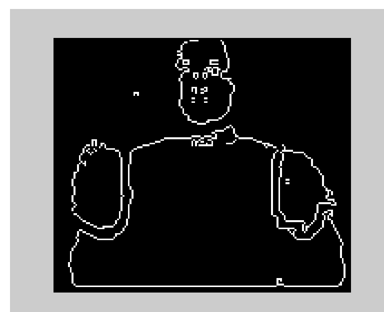
4.2 Fitting to Real Data

Ellipse and superellipse fitting is employed on real data, which is used as an intermediate result for further processing. It can be used in many applications such as human face detection, Storing information about different poses of a person for human detection. The plot shows the results for fitting superellipse on real images using the modified objective function in Eqn(14). The result for some of the real data is shown in Figure 4 . The result shows that the incorporation of gradient and curvature information, gives better results than fitting using least square. To fit a superellipse to real data, the following steps needs to be followed:

1. Read the given real image.
2. Find the edged image of the given input.
3. Extract regions by finding the 8-connected components from the edged image.
4. Draw a superellipse using the modified function in equation (14).



(a)



(b)

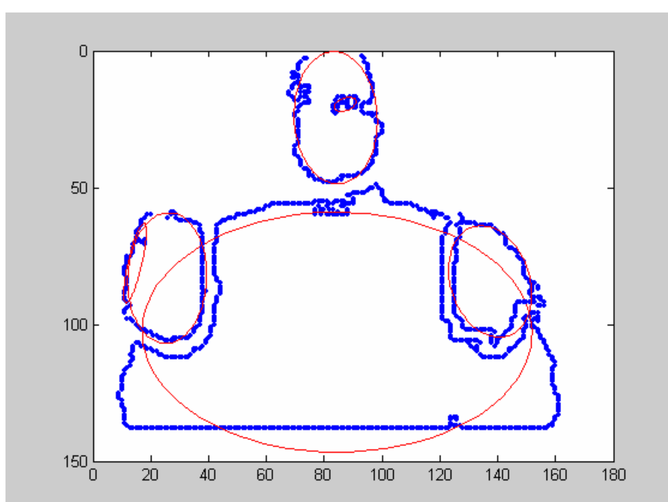
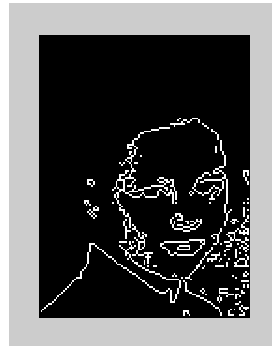
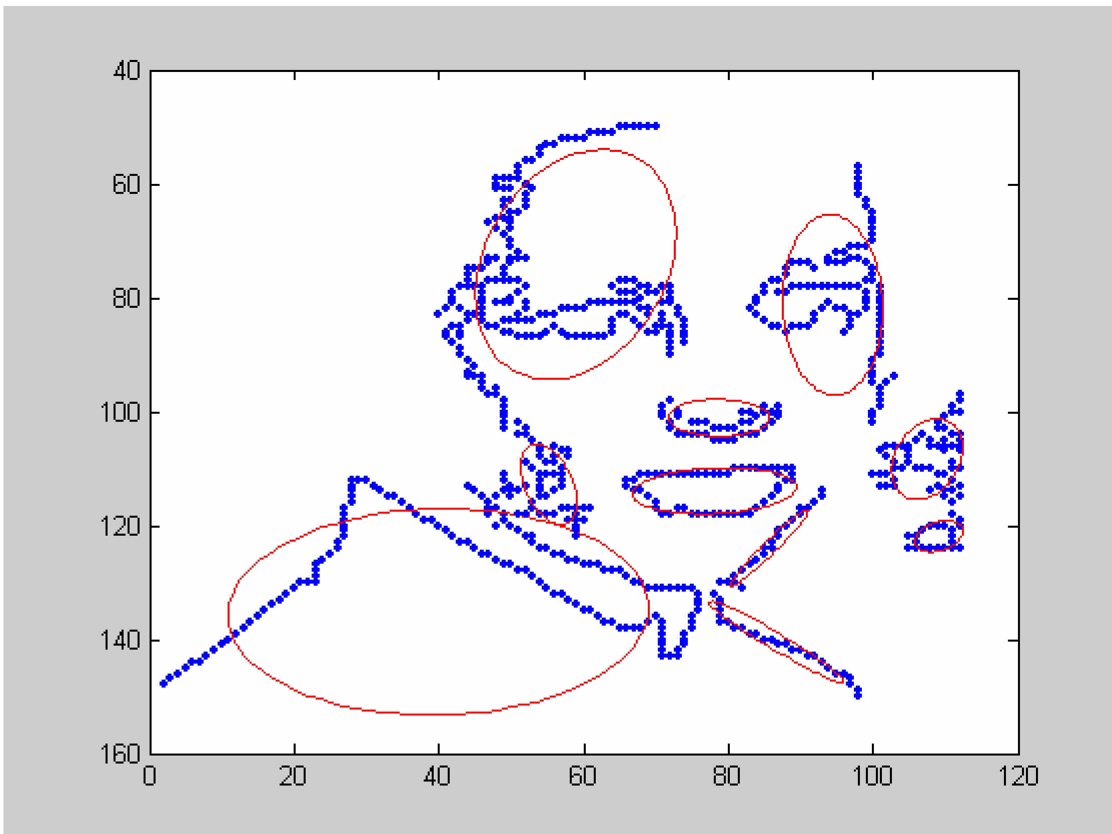


Figure 4(a) Original Image (b) Edged image (c) Fitted superellipse using modified function.

(a)



(b)



(c)

Figure 5(a) Original Image (b) Edged image (c) Fitted superellipse using modified function.

4.3 Application to Classification

A final test of the fitting technique is given in the classification example. Figure () shows a variety of beans to be distinguished. In this case we can distinguish a large set of beans of different classes and classify them based on their fits.

Similarly superellipse fitting can be used in many applications, such as tracking of poses of a human being and some are explained in section 5.

5. Applications

1 Superellipses are used to model parts of objects (such as toys..) in a coarse but very compact way.
2 Human Detection – as an intermediate path
3 Real Time Face Tracker using ellipse fitting [8]
In the case of real time face tracker using ellipse fitting, each face candidate blob is fitted by an ellipse, and its major and minor axes are computed. The direction of the major axis determines the planar rotation angle of a face. The length of the minor axis determines the size of the face template. This method makes the face detection fast and detects the 2D rotation angle.
4 Face Detection using ellipse fitting. [9]

6. Conclusion

Reliably fitting superellipse to partial data is both difficult and computationally expensive, requiring iterative methods to minimize some objective function. This report described least square method to fit data and which is extended to fit using the gradient information. Finally, it is also shown how to apply the gradient method for real data.

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