

# Failure Forecasting of Aircraft Air-Conditioning/Cooling Pack with Field Data

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DOI: 10.2514/1.26561

This paper presents methods for modeling the failure of air-conditioning/cooling packs for a particular type of aircraft with field data. In many regards, field data are highly desirable for more accurate failure prediction by aircraft operators, because the data implicitly account for all actual usage and environmental stresses. It is not always possible to accurately anticipate or simulate these stresses in a laboratory or even in a field test. Field data, in a larger extent, are also important to the manufacturer, because the data identify product deficiencies and areas of improvement. In this study, the failure of the aircraft air-conditioning/cooling pack under a customer-use environment is first modeled at the component level by using the Weibull distribution and its extensions. These include the two-parameter Weibull model, three-parameter Weibull model, mixture model, and phased bi-Weibull model. The number of failures over time is estimated by a renewal process. The failure of the air-conditioning/cooling pack at the system level is then modeled by using the power law process model. The failure trend is tested by the Laplace test. The results give an insight into the reliability and quality of the air-conditioning/cooling pack under actual operating conditions. The models presented here can be used by aircraft operators for assessing system and component failures and customizing the maintenance programs recommended by the manufacturer.

## Nomenclature

|          |   |                                      |
|----------|---|--------------------------------------|
| $c$      | = | number of cycles                     |
| $D$      | = | common beta hypothesis statistic     |
| $F(t)$   | = | cumulative distribution function     |
| $K$      | = | total number of systems              |
| $M(t)$   | = | renewal function                     |
| $N(t)$   | = | total number of failures by time $t$ |
| $p$      | = | Weibull mixture parameter            |
| $R(t)$   | = | reliability function                 |
| $S$      | = | observation starting time            |
| $T$      | = | observation ending time              |
| $t$      | = | flight hour                          |
| $u(t)$   | = | intensity function                   |
| $\alpha$ | = | power law parameter                  |
| $\beta$  | = | Weibull shape parameter              |
| $\gamma$ | = | location parameter                   |
| $\eta$   | = | Weibull scale parameter              |
| $\theta$ | = | power law parameter                  |

## Subscripts

|     |   |                |
|-----|---|----------------|
| $i$ | = | failure number |
| $q$ | = | system number  |

## Introduction

THE maintenance planning document (MPD) prepared by the manufacturer is the main document that is used by aircraft operators to develop their maintenance programs for a particular type of aircraft. The MPD sets minimum maintenance requirements for the aircraft. Each operator should customize the MPD based upon its

own operating and environmental conditions, maintenance capabilities, and practices and rules of the local regulatory authority. However, most of the operators usually use the inspection or replacement intervals, mean time between failures, etc., as recommended by the manufacturer in their maintenance programs, as long as they do not conflict with the local regulations. Manufacturer recommendations are based on the test data. Even the most faithful and rigorous testing will fail to precisely simulate all field conditions. It is quite likely that there would be variations between the field reliability and the manufacturer's reliability test results. Hence, two airlines may need different maintenance programs for the same aircraft model, depending on their field conditions. Field data are capable of capturing the operating and environmental stresses associated with the actual usage conditions. This enables the operators to develop appropriate inspection or replacement programs and spare-part plans that are based on their own operating and environmental conditions, which will decrease the flight delays and cancellations due to unexpected failures. Analysis of failure data for the fielded systems is also very important to the manufacturers, because the information received from the field gives a true measure of product performance and points out the areas that need design changes to refine the product. However, there are limited numbers of studies on the fielded systems, because in-service failure data are often more difficult to obtain.

The objective of this study is to assess the reliability characteristics of an aircraft air-conditioning/cooling pack that is subjected to an airline-use environment. The data analyzed here were obtained from a local aviation company in Saudi Arabia using the air-conditioning/cooling packs installed in a very popular aircraft. The aircraft is used by most of the airlines around the world. The way airlines maintain and support their fleets is rather sensitive information. To respect the sentiments of the airline and of the aircraft and component manufacturers, their names are not disclosed.

In the present work, distributions based on the Weibull family are applied to the air-conditioning/cooling pack failure data to determine models for the various modes of failure at the component level. The renewal function, which is described by an integral equation involving the failure function, is used to predict the expected number of failures and the number of components needed over a certain time period. The results are compared with those recommended by the manufacturer. The failure of the air-conditioning/cooling pack is then analyzed at the system level. In general, a distribution (such as the Weibull distribution) cannot be used to estimate the failure

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pattern of a repairable system. At the system level, a distribution applies to the very first failure. To address the failure characteristic issues of a repairable system, a process is often used instead of a distribution. In this study, the power law model is used to model the air-conditioning/cooling pack failures over time at the system level. The results give an insight into the reliability and quality of air-cooling packs under actual operating conditions and into the determination of possible inspection plans.

The rest of this paper is organized as follows. A brief description of the air-conditioning/cooling pack and the data used in this study are presented in the next section, with explanations for failures and suspensions. In the third section, the Weibull models are described and failures at the component level are investigated. In the fourth section, the air-conditioning/cooling pack failures at the system level are modeled by the power law model. Finally, the paper is concluded in the fifth section.

### Air-Conditioning/Cooling Pack Characteristics and Failure Data

The air-conditioning/cooling pack is an important system that is used to feed the cabin with cool air at a certain temperature. Air cycle machine refrigeration is the predominant means of air-cooling for commercial and military aircraft of all types. The aircraft air cycle machine-cooling system uses the high-pressure bleed air extracted from the engine compressor or auxiliary power unit. The system consists of the following main components, as shown in Fig. 1: air-conditioning/bleed-air control panel, flow control shutoff valve, heat exchangers (two), air cycle machine, ram air system, low-limit system, and water separator. After the bleed air goes through a flow control shutoff valve, it enters the primary heat exchanger. As the bleed air goes through the heat exchanger, ram air removes heat. The cooled bleed air then enters the compressor section of the air cycle machine and is compressed. The compressed air then goes to a secondary heat exchanger and back to the turbine section of the air cycle machine. The bleed air is rapidly expanded and goes through a water separator. The water separator collects and removes moisture from the air before it goes into the distribution system.

The data were collected from maintenance records of the company over a period of about five years. The aircraft and the air-conditioning/cooling packs were commissioned in 2000 and were new as of the beginning of the data collection. There are three aircraft, with registration numbers N737A, N738A, and N739A. Each aircraft has two air-conditioning/cooling packs, right and left. For convenience, aircraft were named in serial order from A to C. Air-conditioning/cooling packs were numbered as one to the left and two to the right. Thus, for example, B2 refers to the right air-conditioning/cooling pack in aircraft N738A. The data have the following entries: routine/nonroutine work, aircraft registration number, description of reported fault, Air Transport Association chapter, part number, date,

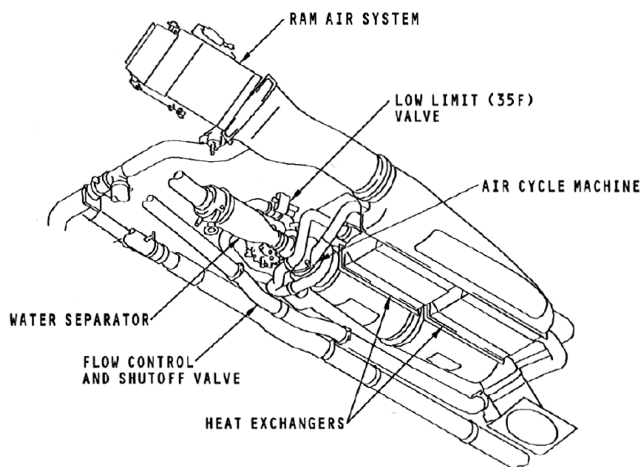


Fig. 1 Air-conditioning/cooling pack.

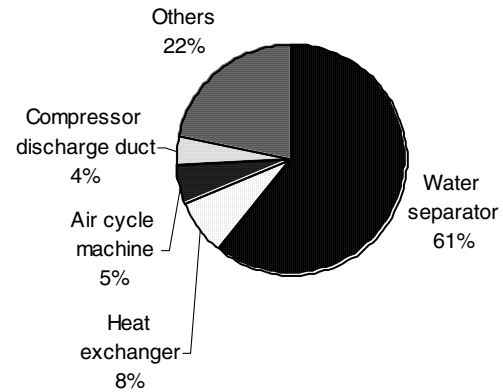


Fig. 2 Breakdown of air-conditioning/cooling pack component failures.

corrective action, aircraft total time, aircraft total cycles, total man hours, and engine parameters. Table 1 shows the summarized failure data related to aircraft A. There are similar failure data records for each aircraft. In this study, a failure is considered as degradation below a defined level of limit set by the manufacturer's specifications. Maintenance records for the air-conditioning/cooling pack were reviewed in detail. This enabled the determination of whether a field removal was a confirmed failure or a "no-fault-found," thus eliminating false removals in the data. A total of 232 confirmed failures were observed for all aircraft. Few items have failed sufficiently often to allow an analysis and modeling at the component level. Figure 2 indicates a breakdown of failures over various components. The water separator is the component with the most observed failures (141 failures). The heat exchanger, which has failed 19 times, follows the water separator. The air cycle machine has failed 11 times. Other components have failed less than 10 times.

Maintenance task cards indicate that the air-conditioning/cooling system is maintained under an "on-condition" maintenance program that requires inspections of certain components at fixed intervals. There is no "hard-time" maintenance process that involves periodic replacement or overhaul schedule for the components. In the status column of Table 1, F means that component was replaced as a result of failure in service or did not pass periodic inspection; S refers to suspension. For this study, suspensions represent the components currently running that have not failed to date.

The data were analyzed at the system level, which means considering a system failure to be when the pack is not operating properly and is serviced, and the particular components that have failed are ignored. In addition, an analysis at the component level of the water separator history was undertaken. The water separator is a cylindrical chamber with a taper at the upstream end. The coalescer bag in the water separator collects water mist from the air. It also helps to prevent damage from ice or debris that may be present in the air. As dirt and contamination collect on the coalescer bag, the airflow rate through the bag decreases. The water separator is subjected to periodic visual inspection at every 500 flight hours, as recommended by the manufacturer. When the water separator indication disk is in the red range, the coalescer bag is replaced; otherwise, it continues in service. Maintenance records reveal that only two water separator bags were removed during these periodic checks and these water separators are handled as failures.

The heat exchangers remove the heat from air going to the compressor and turbine section of the air cycle machine. Ram air is used to remove the heat. This component is subjected to a restoring maintenance task at every 3000 flight hours, as recommended by the manufacturer. The restoring task requires the cleaning of primary and secondary heat exchangers.

There are no periodic maintenance tasks for the other components. The air cycle machine and the compressor discharge duct are repaired when they fail. Others are replaced when they fail. Hence, it is quite safe to assume that the failure data represent in-service failures (in other words, unscheduled maintenance actions), which are the main concern of all airlines.

**Table 1 Air-conditioning/cooling system failure data for aircraft A**

| Right air-cooling pack      |        |                      | Left air-cooling pack       |        |                      |
|-----------------------------|--------|----------------------|-----------------------------|--------|----------------------|
| Failure time (flight hours) | Status | Component            | Failure time (flight hours) | Status | Component            |
| 90.88                       | F      | Water separator      | 63.04                       | F      | Water separator      |
| 111.93                      | F      | Water separator      | 112.18                      | F      | Low limit switch     |
| 168.01                      | F      | Water separator      | 167.72                      | F      | Water separator      |
| 333.52                      | F      | Heat exchanger       | 269.27                      | F      | Water separator      |
| 604.25                      | F      | Water separator      | 604.25                      | F      | Water separator      |
| 688.13                      | F      | Water separator      | 688.13                      | F      | Shutoff value        |
| 842.17                      | F      | Compressor discharge | 794.00                      | F      | Water separator      |
| 948.72                      | F      | Water separator      | 1016.60                     | F      | Water separator      |
| 1106.93                     | F      | Water separator      | 1393.58                     | F      | Water separator      |
| 1405.68                     | F      | Water separator      | 1568.24                     | F      | Water separator      |
| 1445.22                     | F      | Air cycle machine    | 1603.43                     | F      | Panel switch         |
| 1568.24                     | F      | Water separator      | 1649.14                     | F      | Heat exchanger       |
| 1603.43                     | F      | Water separator      | 1688.26                     | F      | Water separator      |
| 1649.14                     | F      | Heat exchanger       | 1851.84                     | F      | Compressor discharge |
| 1796.84                     | F      | Water separator      | 2042.74                     | F      | Air cycle machine    |
| 1804.21                     | F      | Water separator      | 2360.54                     | F      | Water separator      |
| 2010.74                     | F      | Water separator      | 2378.01                     | F      | Shutoff value        |
| 2231.63                     | F      | Ram inlet actuator   | 2459.37                     | F      | Water separator      |
| 2378.01                     | F      | Water separator      | 2476.17                     | F      | Water separator      |
| 2459.37                     | F      | Water separator      | 2607.41                     | F      | Water separator      |
| 2481.57                     | F      | Panel switch         | 2680.23                     | F      | Water separator      |
| 2607.41                     | F      | Water separator      | 2691.87                     | F      | Water separator      |
| 2676.58                     | F      | Water separator      | 2765.43                     | F      | Ram air              |
| 2731.53                     | F      | Shutoff value        | 2812.93                     | F      | Water separator      |
| 2765.43                     | F      | Water separator      | 2874.60                     | F      | Compressor discharge |
| 2814.10                     | F      | Heat exchanger       | 3082.42                     | F      | Water separator      |
| 2855.58                     | F      | Water separator      | 3212.05                     | F      | Heat exchanger       |
| 2990.27                     | F      | Compressor discharge | 3368.90                     | F      | Water separator      |
| 3046.54                     | F      | Water separator      | 3717.82                     | F      | Water separator      |
| 3212.05                     | F      | Heat exchanger       | 3901.38                     | F      | Water separator      |
| 3368.90                     | F      | Water separator      | 4014.72                     | F      | Air cycle machine    |
| 3405.62                     | F      | Air cycle machine    | 4098.58                     | F      | Water separator      |
| 3513.53                     | F      | Water separator      | 4213.40                     | F      | Panel switch         |
| 3888.26                     | F      | Water separator      | 4357.17                     | F      | Water separator      |
| 4098.58                     | F      | Water separator      | 4810.06                     | S      | -                    |
| 4423.90                     | F      | Low limit value      |                             |        |                      |
| 4578.89                     | F      | Water separator      |                             |        |                      |
| 4736.80                     | F      | Water separator      |                             |        |                      |
| 4810.06                     | S      | -                    |                             |        |                      |

The times to failure are measured in terms of flying hours. The data also include the failure times in terms of the number of cycles (landings). A relation between these two parameters can be written as  $t \propto c$  [1]. However, in the present study, flight hour is used as indicator of product life, because the periodic inspection recommended by the manufacturer is based on this criterion.

**Air-Conditioning/Cooling Pack Failures at the Component Level**

**Weibull Models**

There are several models to identify the failure characteristics of components. The Weibull model is one of the most commonly used models for this purpose. Brief descriptions of the Weibull models that have been explored in this study are given next [2–4].

*Two-Parameter Weibull Model*

This is the simplest model considered. The reliability function is given by

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right] \quad \eta > 0, \quad \beta > 0, \quad t \geq 0 \quad (1)$$

A complimentary function  $F(t)$  to the reliability function can be defined as

$$F(t) = 1 - R(t) \quad F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right] \quad (2)$$

where  $F(t)$  indicates the probability that a failure occurs before time  $t$ ;  $\beta$  indicates whether the failure rate is increasing, constant, or decreasing; and  $\eta$  represents the characteristic life of the part.

In this model and in the following models, the reliability function given is defined for  $t > 0$ . Similarly, the shape and scale parameters are taken to the positive; that is,  $\beta > 0$  and  $\eta > 0$ .

*Three-Parameter Weibull Model*

This model is the same as the two-parameter Weibull model, but it contains a delay or location parameter with reliability function.

$$R(t) = \exp\left[-\left(\frac{t - \gamma}{\eta}\right)^\beta\right] \quad (3)$$

*Mixture Model*

Many failure analysis problems include mixes of various failure modes. A mixed population occurs when there are two or more subpopulations in the analysis. The model assumes that each population has its own failure mode and is not subjected to failure modes of other subpopulations. The reliability function for the whole population can be expressed as [5]

$$R(t) = p \exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right] + (1 - p) \exp\left[-\left(\frac{t}{\eta_2}\right)^{\beta_2}\right] \quad (4)$$

where  $0 \leq p \leq 1$

**Table 2** Parameter estimates for modeling water separator coalescer bag failures

| Model                   | $\beta_1$    | $\beta_2$    | $\eta_1$      | $\eta_2$       | $p$   | $t_0$         | $\gamma$ | $lof$        |
|-------------------------|--------------|--------------|---------------|----------------|-------|---------------|----------|--------------|
| Two-parameter Weibull   | 1.227        | -            | 220.754       | -              | -     | -             | -        | 0.024        |
| Three-parameter Weibull | 1.062        | -            | 216.379       | -              | -     | -             | 6.445    | 0.012        |
| Mixture model           | 10.648       | 1.176        | 78.678        | 227.400        | 0.069 | -             | -        | 0.018        |
| <b>Phase bi-Weibull</b> | <b>1.989</b> | <b>1.070</b> | <b>91.856</b> | <b>213.347</b> | -     | <b>31.050</b> | -        | <b>0.011</b> |

*Phased Bi-Weibull Model*

This model is composed of different Weibull distributions for different time periods. In effect, the time axis is split into two distinct phases. Each phase is associated with a different Weibull distribution. The reliability function with parameters  $\beta_1, \beta_2, \eta_1,$  and  $\eta_2$  is [6]

$$R(t) = \exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right] \quad 0 \leq t < t_0 \quad (5)$$

$$R(t) = \exp\left\{-\left[\left(\frac{t}{\eta_1}\right)^{\beta_1} + \left(\frac{t-t_0}{\eta_2}\right)^{\beta_2}\right]\right\} \quad t_0 \leq t < \infty \quad (6)$$

**Model Fitting and Analysis**

Time between failures is readily obtained from the data for the water separator and then ordered in rank. It is considered that the failure data for water separators could be amalgamated across the three aircraft, resulting in a total of 141 failures. Distribution models based on the Weibull family can be fitted to failure data by rank regression or maximum likelihood methods. In addition to selecting a model to be fitted to the data, along with a fitting method, a criterion used to measure the fit of a model and then determine the one of best fit is required. This objective function should be a measure that can be used to discriminate between models and values of parameters. In this study, the method is used for minimizing a sum of squared deviations that is based on the distance of the data points from the cumulative probability of failure curve, as measured on a linear scale. The sum of squared deviations is referred to as a *lack of fit* between the data and the fitted model and is denoted as *lof* in Table 2. The algorithm can also use the minimum sum of absolute deviations or the minimum of the maximum deviation as an objective function. The results are similar for all objective functions and, consequently, the results for the latter two are not included here.

The results are presented in Table 2 for the water separator bag (component) failures, with accompanying graphs. The graphs for the model of best fit and, for comparison, that of the two-parameter Weibull model are included. The parameter estimates of each model with respective *lof* are given, and the model of best fit is highlighted in bold type in the table.

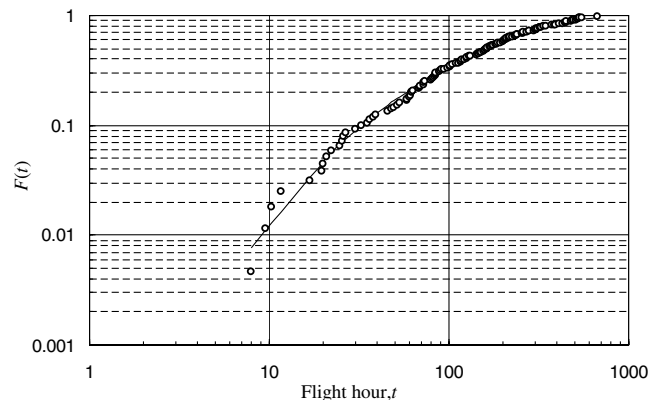
Table 2 shows that the failure of the water separator bag seems to be best fitted by the phased bi-Weibull model, and comparison with a two-parameter Weibull model is shown in Figs. 3 and 4. The model indicates that the coalescer bag initially has an increasing failure rate (shape parameter 1.989) for a short period (31.05 h), followed by a constant failure rate (shape parameter 1.07) for the rest of the time. The two-phase failures indicate that either two failure mechanisms are at play or that there is a single one that undergoes a sharp change in a short time. The former is the likely explanation. A total of 14 water separators have failed in the first 31 flying hours. An increasing failure rate indicates that the water separators have failed because of wear-out causes in this early period. After that, change failures (which are usually externally induced failures) predominate. It is possible that during early life, wear-out failures may occur because of poor storage or long shelf life, which results in deterioration of the component. In maintenance records, the causes of the water separator bag failures were given in 126 occurrences. Fifteen failure causes were not recorded. Seven failure causes were recorded as “damaged bag,” and all remaining causes were recorded as “dirt contamination.” Hence, dirt contamination is identified as the most

frequently occurring failure type for the water separator. It is noticed that all of the seven water separators that failed due to damage in the bag are among the 14 bags that failed in the first (wear-out) period, and 114 failures due to dirt contamination have occurred in the constant failure rate period. These findings support the model with two failure mechanisms. However, in general, the model suggests that the water separator coalescer bags mainly exhibit a high and constant failure rate. A constant failure rate is indicative of random failures. A random failure pattern of water separator bags can be attributed to application of loads to this component in excess of its design strength at a constant average rate, which likely results from the climatic conditions in the region. It is also interesting to note that the three-parameter Weibull model is also a very reasonable fit and is a simpler model than the phased bi-Weibull. This model also suggests that the water separator bag exhibits constant failure rate.

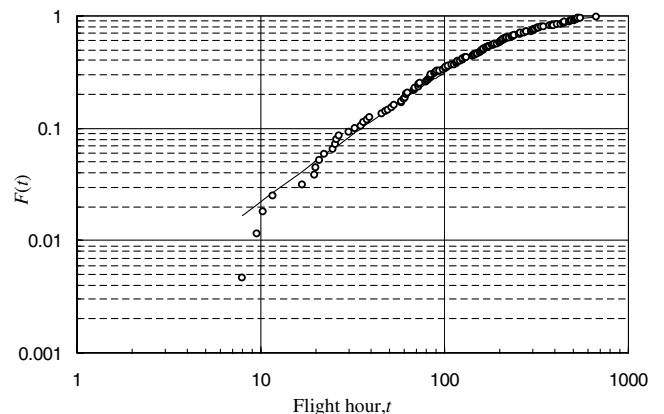
**Modeling of Component Failures over Time**

Because it is reasonable to model the water separator coalescer bag replacements as a renewal process, a useful characterization of the process is the renewal function, which is described by an integral equation involving the cumulative failure function.

$$M(t) = F(t) + \int_0^t M(t-x) dx \quad (7)$$



**Fig. 3** Plot of cumulative failure probability of the water separator with the phase bi-Weibull model.



**Fig. 4** Plot of cumulative failure probability of the water separator with the two-parameter Weibull model.

Equation (7) is difficult to solve analytically, but it can be approximated or evaluated numerically with the method proposed by Blischke and Murthy [7].

The renewal function gives the expected number of failures (or the number of components needed) for an operating period of  $t$  hours. The estimated number of water separator coalescer bag failures per year for the entire fleet (approximately 1000 flying hours per aircraft) is 32. The observed number of water separator bag failures over a 5-year period is close to the estimated number.

Mean time to failure of the water separator coalescer bag is estimated as 217.63 flight hours. Because the component exhibits a random failure pattern, a hard-time maintenance program has no benefit. The appropriate response to a constant failure rate is to develop an appropriate inspection and monitoring program and to employ on-condition maintenance. The manufacturer recommends a monitoring program with a visual inspection of the component at every 500 flying hours, in an attempt to catch the next failure and to reduce in-service failures. However, the result indicates that an inspection interval for this component is likely to be appropriate at about 200 flying hours, which is quite different from the time interval recommended by the manufacturer.

In addition to this reactive measure, some proactive measures may also be taken to alter and decrease the failure rate of water separators through changes in the component design or operational context. A component is designed with an inherent failure probability that is usually known as a designed-in failure rate. The designed-in failure rate is based on the overlap between the distributions of strength and expected operational loads of the component. The relative separation of mean values of load and strength is the static margin. The standard deviation of the load is known as the loading roughness. The safety margin and loading roughness allow the manufacturer to analyze the way in which load and strength distributions overlap and to generate a probability of failure. This overlap represents the design-in unreliability of the component. Failures in this area are usually change failures. On the other hand, there are always some uncertainties in load and stress distributions. Therefore, any difference between the actual and expected operation load distribution drastically affects the field failure rate and reliability of the component. It is clear that the water separator experiences a field failure rate higher than the manufacturer's designed-in failure rate. The primary cause of the failures is contamination. The region in which the airline operates is known for its harsh weather conditions and sand storms. Local air traffic control regulations prohibit the airline from flying above a certain altitude, which allows the aircraft to avoid bad weather conditions. Furthermore, most of the airline's destinations are in remote sandy areas. The runways at these areas are surrounded by deserts and often filled with dust that finds its way to air-cooling system through engine bleed air. Hence, the water separator is subjected to contaminating loads at an almost constant rate, which exceeds the expected operation loads by the manufacturer. In other words, high loading roughness due to the wide spread of the load distribution results in failure of the large population of water separators. Under these conditions, the option to reduce the failure rate may be to curtail the load distribution by some device, such as water spray injection, to reduce the quantity of dirt entering the water separator as it is employed in some air freighters.

Another factor that contributes to the constant failure rate of the water separators may be the replacement of some units before failure.

Maintenance work forms indicate that only one of the water separators (either right or left) has been replaced as a corrective action in response to the 126 reported faults. On the other hand, in 13 cases, maintainers replaced both of the water separators. It is likely that only one of the water separators actually failed in these cases, but maintainers used the opportunity to also replace the unfailed one, in an attempt to prevent a future failure. Therefore, these failures may be considered as human-induced failures.

### Air-Conditioning/Cooling Pack Failures at the System Level

The underlying assumption in all of the models used for the water separator component failures is that the times between failures are independent. This is the assumption of a renewal process, whereby the system is like new after a failure. This is plausible for individual components, such as the water separator coalescer bags, which are replaced rather than repaired. However, at the system level (which is repaired, not replaced), this assumption is seldom true. To address the reliability characteristics of repairable systems, a process is often used instead of distribution. The power law model is one of the most common processes applied to analyze the repairable systems in the field. The model approximates the cumulative number of failures for a system under minimal repair, using a power function of the form [8]

$$N(t) = \theta r^\alpha \tag{8}$$

The power law model can be viewed as an extension of the Weibull distribution. The Weibull distribution governs the first system failure, and the power law model governs each succeeding system failure. The derivative of power function given in Eq. (9) is the intensity function, which is usually referred to as the rate of occurrence of failures (ROCOF). This function gives the expected number of failures per unit time.

$$u(t) = \theta \alpha t^{\alpha-1} \tag{9}$$

The parameters can be determined using least-squares fitting or by maximum likelihood estimation. If  $\alpha < 1$ , failure intensity is decreasing; if  $\alpha > 1$ , failure intensity is increasing; and if  $\alpha = 1$ , the failure intensity is constant. The latter is the special case that represents a homogenous Poisson process, because there is no change in the intensity function.

In this study, the failure of each air-conditioning/cooling pack system is modeled by the power law model. Maximum likelihood estimation is used to determine parameters. The general maximum likelihood estimates for  $\theta$  and  $\alpha$  can be obtained from [9]

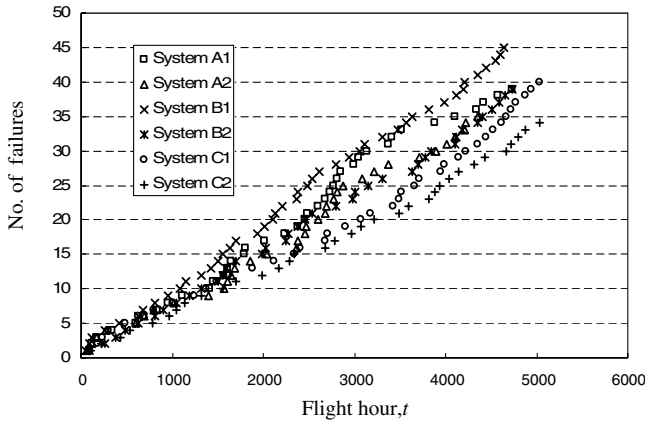
$$\theta = \frac{\sum_{q=1}^K N_q}{\sum_{q=1}^K (T_q^\alpha - S_q^\alpha)} \tag{10}$$

$$\alpha = \frac{\sum_{q=1}^K N_q}{A \sum_{q=1}^K [T_q^\alpha \ln(T_q) - S_q^\alpha \ln(S_q)] - \sum_{q=1}^K \sum_{i=1}^{N_q} \ln(n_{iq})} \tag{11}$$

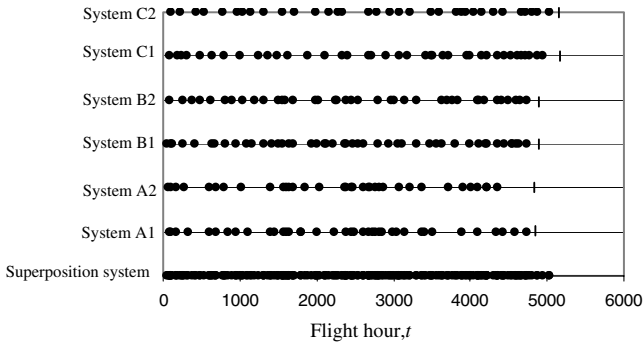
The estimated parameter values are presented in Table 3. Goodness of fit is tested by the Cram'er-von Mises method. The Laplace test is used to test for trend. Cumulative failures over time for each system are shown in Fig. 5. Values of  $\alpha$  lie between 0.9326 and 1.1054,

**Table 3** Parameter estimates for modeling air-conditioning/cooling system failures

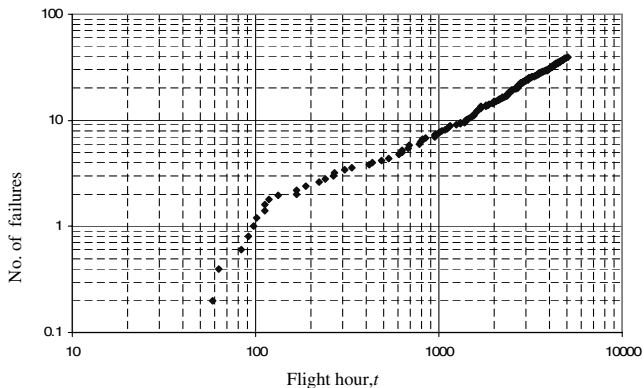
| System    | Failures | $\alpha$ | $\theta$ | Cram'er-von Mises |        |         | Laplace trend |        |          |
|-----------|----------|----------|----------|-------------------|--------|---------|---------------|--------|----------|
|           |          |          |          | Test              | Upper  | Lower   | Test          | Upper  | Trend    |
| System A1 | 34       | 0.9326   | 0.0129   | 0.0801            | 0.1722 | -1.6449 | -0.5113       | 1.6449 | No trend |
| System A2 | 38       | 0.9666   | 0.0108   | 0.0726            | 0.1723 | -1.6449 | -0.4719       | 1.6449 | No trend |
| System B1 | 45       | 0.9701   | 0.0121   | 0.0189            | 0.1725 | -1.6449 | -0.1104       | 1.6449 | No trend |
| System B2 | 39       | 1.0980   | 0.0035   | 0.0178            | 0.1723 | -1.6449 | 0.5308        | 1.6449 | No trend |
| System C1 | 41       | 1.0971   | 0.0034   | 0.1321            | 0.1723 | -1.6449 | 1.1497        | 1.6449 | No trend |
| System C2 | 35       | 1.1054   | 0.0027   | 0.0351            | 0.1721 | -1.6449 | 0.7381        | 1.6449 | No trend |



**Fig. 5** Cumulative failures over time for each air-conditioning/cooling system.



**Fig. 6** Operation of each air-conditioning/cooling system and superposition system.



**Fig. 7** Cumulative failures over time for the superposition system.

which indicates that the ROCOF is almost constant for each system. The Laplace tests show no trend in failure data. Systems C1 and C2 have slightly higher failure rates (1.0971 and 1.1054, respectively) than other systems. Aircraft C is the aircraft with the highest flight hours (5,100.2 h) to date. It may indicate that wear-out of some components, due to aging, is likely to come into play around that time.

To determine if the failure data for the systems should be combined into a single superposition (equivalent) system that can be used to estimate the reliability metrics of interest, a common beta hypothesis (CBH) test [10] is employed. The CBH test compares the ROCOF of each system by comparing the  $\beta$  of each system to determine if the interarrival rate of failures across the systems is fairly consistent. In other words, the CBH test tests the hypothesis  $H_0$ , such that  $\beta_1 = \beta_2 = \beta_3 = \dots = \beta_K$ .

The likelihood ratio procedure is used for the CBH test of the systems. The calculated statistic for the systems under investigation is found as  $D = 1.211$ . Using the chi-squared tables with  $K - 1 =$  five degrees of freedom, the critical values at the 2.5 and 97.5

percentiles are 1.064 and 7.779, respectively. Because  $1.064 < D < 7.779$ , the hypothesis is accepted that  $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6$  at the 5% significance level.

Operation of each system and superimposed system is shown in Fig. 6. Estimated parameter values for superimposed systems are  $\alpha = 1.063$  with a 95% confidence interval of 0.8886–1.1498 and  $\theta = 0.0066$  with a 95% confidence interval of 0.0057–0.0075;  $\alpha = 1.063$  indicates that the rate of occurrence of failures is constant. The cumulative mean time between failures (MTBF) is calculated as 127.90 flight hours.

The fitted function can be used directly to estimate the expected number of failures over specified time intervals. The estimated number of air-conditioning/cooling pack failures per year (approximately 1000 flight hours per aircraft) is 48. In other words, there will be 48 unscheduled maintenance tasks per year for all aircraft, due to air-conditioning/cooling pack failures.

Figure 7 indicates a plot of  $N(t)$  against  $t$  for the superimposed system. It is interesting to note that there is a change in the failure pattern around the 10th failure. For this reason, a phased power law model that is composed of two power models for different time periods is developed as follows:

$$N(t) = \theta_1(t)^{\alpha_1} \quad 0 \leq t \leq t_1 \quad (12)$$

$$N(t) = \theta_1(t)^{\alpha_1} + \theta_2(t - t_1)^{\alpha_2} \quad t_1 < t < \infty \quad (13)$$

Up to  $t = 126$  h, a power law model with  $\alpha_1 = 2.904$  and  $\theta_1 = 6.89 \times 10^{-6}$  fits very well and indicates an increasing ROCOF. If, after 126 h, the time origin is reset to zero, it is found that a power law model with  $\alpha_2 = 1.054$  and  $\theta_2 = 0.0038$  is appropriate, which implies no evidence of trend. Thus, an occurrence around  $t = 126$  h has clearly changed the ROCOF. Although there is no information available to identify that occurrence, it may be attributed to the elimination of an initial fault around that time.

### Conclusions

Failure of the aircraft air-conditioning/cooling packs for a particular type of aircraft with field data is investigated at the component level and the system level. Distributions based on the Weibull family are used to model the failures at the component level. Failures at the system level are modeled by the power law model for each system and superposition system, and failure trend is tested by the Laplace test. Expected numbers of failures and MTBF are estimated for the component and superposition system. The results indicate that the water separator is the component with the most observed failures. This component mostly exhibits a random failure pattern with a high rate, except for a short period in early life, in which increasing failure rate predominates. Dirt contamination is identified as the most frequently occurring failure type for the water separator. The rate of occurrence of failures for the system indicates no trend and is almost constant. The constant failure rate at the component level and constant ROCOF at the system level are indicative of externally induced failures, which are likely due to weather conditions in the region. Results also point out that the failures occur at a higher rate than that estimated by the manufacturer. Thus, a revision in monitoring and inspection program recommended by the manufacturer and a means to decrease the contaminating load acting on the system are likely needed.

For the assessment of component and system reliability, field data are highly desirable for aircraft operators, because the data inherently capture the operational and environmental stresses associated with actual usage conditions, which are not always possible to accurately simulate in the tests conducted by the manufacturer. As with all field reliability studies, there are limitations associated with the data. The main disadvantage of the field data is incomplete or lost information, because for most consumer goods, maintenance records are kept haphazardly. However, this is less of a problem for airlines, which usually operate with strict data reporting requirements. Therefore, methods presented in this study can be used to assess the failure characteristics of the systems or components and to customize the

maintenance program in accordance with the airline's unique operating and environmental conditions.

### Acknowledgments

The authors are grateful to the King Fahd University of Petroleum and Minerals for supporting this research and to the local aviation company for supplying the data.

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