Department of Aerospace Engineering

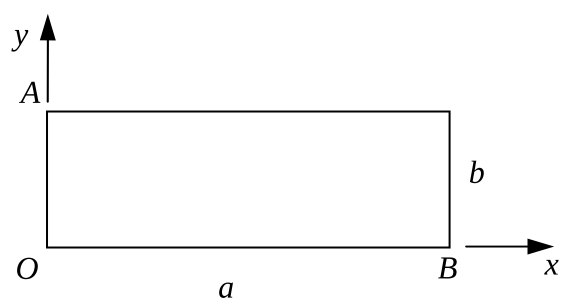
AE530 Aerospace Structures I
MidTerm Exam, Nov. 13th, 2006 – Total Mark 40

Name:__________________________ **ID No.:__________________________**

Question ONE (8 Marks)

As a result of loading, the rectangle shown deforms into a parallelogram in which sides *OA* and *BC* shorten 0.003 mm, and rotate 500 μ radian counterclockwise, while sides *AB* and *OC* elongate 0.005 mm and rotate 800 μ radian clockwise, determine the principal strains and the directions of the principal axes. Take a = 30 mm and b = 10 mm.

Solution:

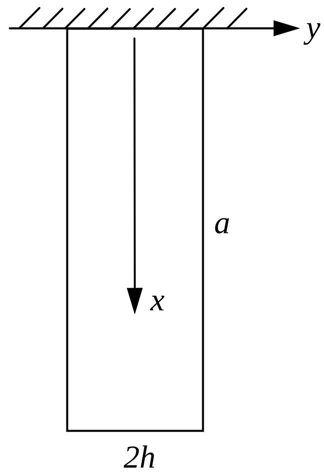


Question TWO (15 Marks)

A uniform bar of rectangular cross section $2h\text{x}b$ and weight per unit volume g hangs in the vertical plane. Its weight results in displacements:

$$u = -\frac{\nu g}{E}xz \quad , \quad v = -\frac{\nu g}{E}yz \quad , \quad w = \frac{g}{2E}\left[(z^2 - a^2) + \nu(x^2 + y^2)\right]$$

Demonstrate whether this solution satisfies the 15 equations of elasticity and the boundary conditions.

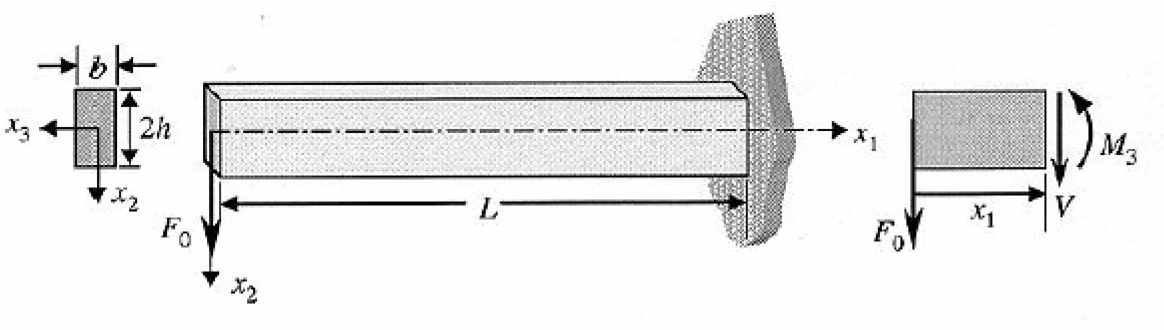


Solution:

Question THREE (12 Marks)

For the shown cantilever beam along x_1 axis, the bending moment at the free end is given by: $M_3 = -F_0 x_1$. The bending stress σ_{11} is given by: $\sigma_{11} = \frac{M_3}{I_3} x_2$ where I_3 is the moment of inertia of the cross-section about the x_3-axis.

Use the equilibrium equations to determine the stresses σ_{22} and σ_{12}.



Solution:

Question FOUR (5 Marks)

Let **r** be the position vector of a typical point, with magnitude *r*. Evaluate $\Delta(r^5)$.

Solution:

Elasticity Equations

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad , \quad i, j = 1, 2, 3 \; or \; x, y, z$$

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2G\varepsilon_{ij} \qquad \text{or} \qquad \varepsilon_{ij} = \frac{1}{E}[(1+\nu)\sigma_{ij} - \nu \sigma_{kk} \delta_{ij}]$$

$$\frac{\partial \sigma_{ij}}{\partial x_j} + X_i = 0$$

Compatibility Equations:

$$\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0$$

Material Constants Relations:

$$E = 2(1+\nu)G$$

$$\lambda = \frac{2G\nu}{1-2\nu} = \frac{E\nu}{2(1+\nu)(1-2\nu)}$$

$$K = \lambda + \frac{2}{3}G = \frac{E}{3(1-2\nu)}$$

$$G = \frac{3KE}{9K - E}$$

Important Relations:

$$\left| \varepsilon_{ij} - \varepsilon \delta_{ij} \right| = 0$$

$$\varepsilon_1, \varepsilon_2 = \frac{(\varepsilon_x + \varepsilon_y)}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\nabla \cdot v = \frac{\partial v_i}{\partial x_i}$$

$$\nabla^2 v = \Delta v = \frac{\partial^2 v_j}{\partial x_i \partial x_i}$$

Good Luck *Dr. Wael Abdelrahman*