A Variational finite element solution for the Navier-Stokes equations in two dimensions

Abu Nasser Khondaker

Civil Engineering

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Abstract

Literature review reveals a number of works devoted to the solution of the Navier-Stokes equations using the finite element method. However, in all these cases a Galerkin formulation was used and/or some form of linearization was involved. In the present work, a finite element formulation is presented using an appropriate variational form preserving full non-linearity. This variational functional, previously used by Guymon and Scott for a constant velocity field in connection with the diffusion-convection equation, is also found to be viable for a variable velocity field in the nonlinear equations. The solution algorithm is developed using the 'vorticity' and 'stream function' formulation of the Navier-Stokes equations. The dependent varibles (vorticity and stream function) are approximated over each triangular element using linear interpolation polynomials. The developed algorithm is embodied in a computer code. The application of the developed code to some numerical examples produced comparable results displaying the efficiency and the versatility of the method. Literature review amply demonstrates that mathematical and numberical difficulties increase with the increase in Reynolds number. The proposed method was also found to be limited to low Reynolds numbers.