Adaptive Control Schemes for a class of Hammerstein-Wiener nonlinear systems

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Abstract

In this work we develop two different adaptive control schemes for a class of nonlinear systems. The class of systems belongs to the Hammerstein-Wiener nonlinear systems. The techniques developed are presented and an example is given in illustration. Using an approximate inverse of the nonlinear plant model, the overall system is forced to track the desired reference signal.

Keywords: *nonlinear systems, adaptive control precompensation, Hammerstein-Wiener systems.*

1. Introduction

The rapidly advancing technology has always been the prime mover for the development of more sophisticated and accurate controllers. With systems becoming increasingly complex and nonlinear, it becomes almost a necessity to design self-adjusting systems which will keep the numerous control parameters at optimal or near optimal [1]. Adaptive control systems have evolved as an attempt to avoid degradation of the dynamic performance of a control system when environmental variations occur [2]. The Adaptive control problem has attracted the attention of many researchers in the field for a long time [1]. In the adaptive identification or control, a set of parameters are adjusted so that the output of the given plant and that of a model approach each other asymtotically.

Al-Naemi and Phillips [3] investigated the control of nonlinear plant by a conventional adaptive controller based on the identification of time varying parameters of a proposed affine model. Qin [3] presented the solution of the adaptive control of nonlinear systems with unknown (certain or uncertain) parameters, using H_∞ approach. Ding and Xie [4] presented the inverse operator method of nonlinear systems with a nonlinearity and established the adaptive control algorithm of this type of nonlinear systems.

Fernando and Sesay [5] proposed a Hammerstein type decision feedback equalizer (HDFE) that compensates for both the nonlinear distortion and the linear dispersion of a Wiener type system. Kang *et al* [6] derived an adaptive algorithm for Wiener system, adjusting the parameters of the precompensator, structured by a Hammerstein model by using the stochastic gradient method. Lim *et al* [7] discussed the linearization of a nonlinear system by connecting a nonlinear Volterra prefilter tandemly with the nonlinear system and by adaptively adjusting the coefficients of the prefilter.

In this paper we develop an adaptive controllers for a class of Hammerstein-Wiener nonlinear systems. The adaptive schemes are based on the identification of the system parameters and precompensating the nonlinear system by the approximate inverse of the plant.

This paper is organized as follows. In Section 2, the Bai's system for which we are developing the adaptive controler is presented. In Section 3 the proposed technique using the approximate inverse method is given. In Section 4 the proposed technique using the estimate of the plant noise is presented and Section 5 includes illustrative example and the simulation results for the developed technique.

2. The Hammerstein-Wiener Model

A block diagram of the well known Hammerstein-Wiener Model where two static nonlinear elements N_1 and N_2 surround a linear block L is shown in Fig. 1.

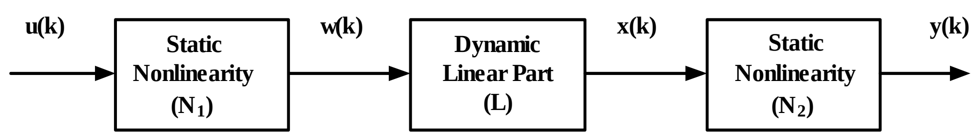

Fig. 1. Hammerstein-Wiener Model

A representation for a class of this type of nonlinear systems is given by Bai [8], where a discrete-time nonlinear dynamic system considered is given by:

$$y(k) = \sum_{i=1}^{p} a_i \{ \sum_{l=1}^{q} d_l g_l [\, y(k-i)] \} + \sum_{j=1}^{n} b_j \{ \sum_{t=1}^{m} c_t\, f_t [\, u(k-j)] \} + e(k) \qquad (1)$$

In (1), $y(k)$, $u(k)$ and $e(k)$ are the system output, input and noise at time k respectively. The $g_l(\cdot)s$ and $f_t(\cdot)s$ are nonlinear functions and

$$a = (a_1,.......,a_p)' \quad b = (b_1,.......,b_n)' \quad c = (c_1,.......,c_m)' \; and \; d = (d_1,.......,d_q)'$$

denote the system parameter vectors.

In the model (1), the purpose of identification is to estimate the unknown parameter vectors *a*, *b*, *c* and *d*, from the observed input-output measurements. The functions $f_t^{'}s(t = 1....m)$ and $g_l^{'}s(l = 1....q)$ are assumed to be *a priori known smooth* nonlinear functions and the orders *q*,*n*,*p* and *m* are assumed to be known as well.

Identification of the Bai's system

Bai in [8] proposed the system identification scheme for the system in (1).

Define:

$$\theta = (a_1 d_1,..,a_1 d_q,...,a_p d_1,...,a_p d_q, b_1 c_1,...,b_1 c_{m,},...,b_n c_1,...,b_n c_m)^T \qquad (2)$$

Let,

$$\theta_{ad} = ad^T = \begin{pmatrix} a_1 d_1, a_1 d_2 ....., a_1 d_p \\ a_2 d_1, a_2 d_2 ....., a_2 d_q \\ .... \\ .... \\ a_p d_1, a_p d_2 ....., a_p d_q \end{pmatrix} \qquad (3)$$

and

$$\theta_{bc} = bc^T = \begin{pmatrix} b_1 c_1, b_1 c_2 ....., b_1 c_m \\ b_2 c_1, b_2 c_2 ....., b_2 c_m \\ .... \\ .... \\ b_n c_1, b_n c_2 ....., b_n c_m \end{pmatrix} \qquad (4)$$

For a given data set of input and output $\{u(k), y(k)\}_{k=1}^N$

Step 1: Calculate the least square estimate of the vector in (2).

$$\hat{\theta}(N) = \hat{\theta}_{ls}(N) = \left(\Phi_N^T \Phi_N\right)^{-1} \Phi^T Y_N \qquad (5)$$

Step 2: Obtain $\hat{\theta}_{ad}(N)$ and $\hat{\theta}_{ab}(N)$ from (5), such that

$$\hat{\theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T \qquad \hat{\theta}_{ab}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i \upsilon_i$$

Where $\xi_i^{'} s\,(i = 1,2,...., p)$ and $\zeta_i^{'} s\,(i = 1,2,...., q)$ $\mu_i^{'} s \quad (i = 1,2,....,n), \upsilon_i^{'} s \quad (i = 1,2,....,m)$, , are *p*,*q*, *n*, *m*-dimensional orthonormal vectors, and $\sigma_i^{'} s(i = 1,2,......,\min(n,m))$, $\delta_i^{'} s(i = 1,2,......,\min(p,q))$ are the nonzero singular values of $\hat{\theta}_{ad}$ and $\hat{\theta}_{ab}$, respectively .

Which gives, using the SVD

$$\hat{b} = s_\mu \mu_1, \hat{c} = s_\mu \sigma_1 \upsilon_1 \;,\; \hat{a}(N) = s_\xi \xi_1, \; and \;\; \hat{d}(N) = s_\zeta \zeta_1 \;,$$

Where s_{ξ} and s_{ξ} denotes the sign of the first non-zero elements of μ_1 and ξ_1 , respectively. We use a recursive version of the above scheme by doing the estimation of $\hat{\theta}$ at every single iteration and applying the Singular Valued Decomposition (SVD) also.

3. An approximate inverse based adaptive control

Here we develop an adaptive controller for the Bai's system (1). It is desired that the output of the overall system follow a desired signal. The desired signal will be injected at the precompensator side and using an approximate inverse of the system, the system is forced to track the desired signal. The block diagram for the adaptive control scheme is as shown in Fig. 2 and the flow chart for a second order plant in Fig. 3.

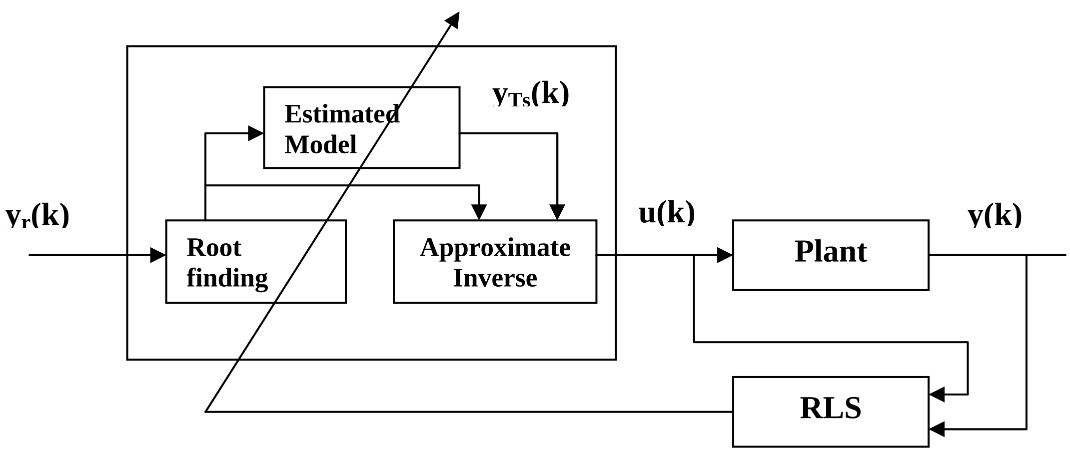

Fig 2. Block Diagram for the Precompensator technique

Consider the Hammerstein-Wiener model (1) and consider the inverse problem of finding *u(k)* to achieve a given $y_r(k)$. Given the estimates $\hat{a}, \hat{b}, \hat{c}\ and\ \hat{d}$ at the precompensator, the description is given by:

$$y_r(k) = \sum_{i=1}^{p} \hat{a}_i \{ \sum_{l=1}^{q} \hat{d}_l g_l [y_r(k-i)]\} + \sum_{j=1}^{n} \hat{b}_j \{ \sum_{t=1}^{m} \hat{c}_t f_t [u(k-j)]\} \qquad (6)$$

The terms with the regressors of $y_r(k)$ are known and will be written one side such that a function in terms of the unknowns *u(k-j)* for *j=1...n* is formed. This is given by:

$$y_r(k) - \sum_{i=1}^{p} \hat{a}_i \{ \sum_{l=1}^{q} \hat{d}_l \, g_l [y_r(k-i)] \} = \sum_{j=1}^{n} \hat{b}_j \{ \sum_{t=1}^{m} \hat{c}_t \, f_t [u(k-j)] \} \qquad (7)$$

This is now solved for *u(k)*. Since the solution amounts to the problem of finding the roots of (7), we need to find among the roots the root that will result in a value closest to the given $y_r(k)$ signal. To do this, the roots are substituted into the model with the actual parameters:

$$y_{T_s}(k) = \sum_{i=1}^{p} a_i \{ \sum_{l=1}^{q} d_l g_l [y_{T_s}(k-i)] \} + \sum_{j=1}^{n} b_j \{ \sum_{t=1}^{m} c_t \, f_t [u(k-j)] \} \qquad (8)$$

Now a root $u_s(k-j)$ is selected such that it gives closest value of the output $y_{Ts}(k)$ to the injected signal $y_r(k)$. The pair of parameters are updated using the recursive least squares (RLS) technique.

The RLS parameter estimation scheme we use a modified version of the scheme give in [8]. The scheme in [8] is a batch estimation scheme. We modify it to be a recursive scheme by solving the SVD problem at every step.

The RLS identification algorithm is given by [10]:

$$\begin{aligned}
&\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\,\varepsilon(k) \\
&K(k) = P(k)\,\varphi(k) \\
&\varepsilon = y(k) - \varphi^T(k)\,\hat{\theta}(k-1) \\
&P(k) = P(k-1) - \frac{P(k-1)\,\varphi(k)\,\varphi^T(k)\,P(k-1)}{\left[1 + \varphi^T(k)\,P(k-1)\,\varphi(k)\right]}
\end{aligned}$$

Where $P(k) = (\Phi^T \Phi)^{-1}$ is the variance of $\hat{\theta}(k)$, Φ is the regressor vector $\varphi^T(k)\,\hat{\theta}(k-1)$ is one step ahead predicted output and ε is the prediction error to be minimized. Each step $\hat{\theta}(k)$ is splitted as shown in (3)-(4) and the SVD is used.

The adaptive control scheme

The flow chart Fig. 3 describes the adaptive control scheme.

i. The desired output, $y_r(k)$ is injected into the inverse system, the precompensator block with some initialized estimates of the parameters.

ii. Now the problem at the precompensator side is a root finding problem. Calculate the roots, the input to the actual system.

iii. Substitute the obtained values in the true system and obtain the output $y_{Ts}(k)$ for each of the '*s*' roots.

iv. Compare $y_{Ts}(k)$ to the injected signal $y_r(k)$. A root is selected such that it gives the output value $y_{Ts}(k)$ closest to $y_r(k)$.

v. Using the input-output pair *u(k)* and *y(k)*, use the RLS method to update the parameters.

vi. The procedure is carried for *N* data points and the formulation is obtained.

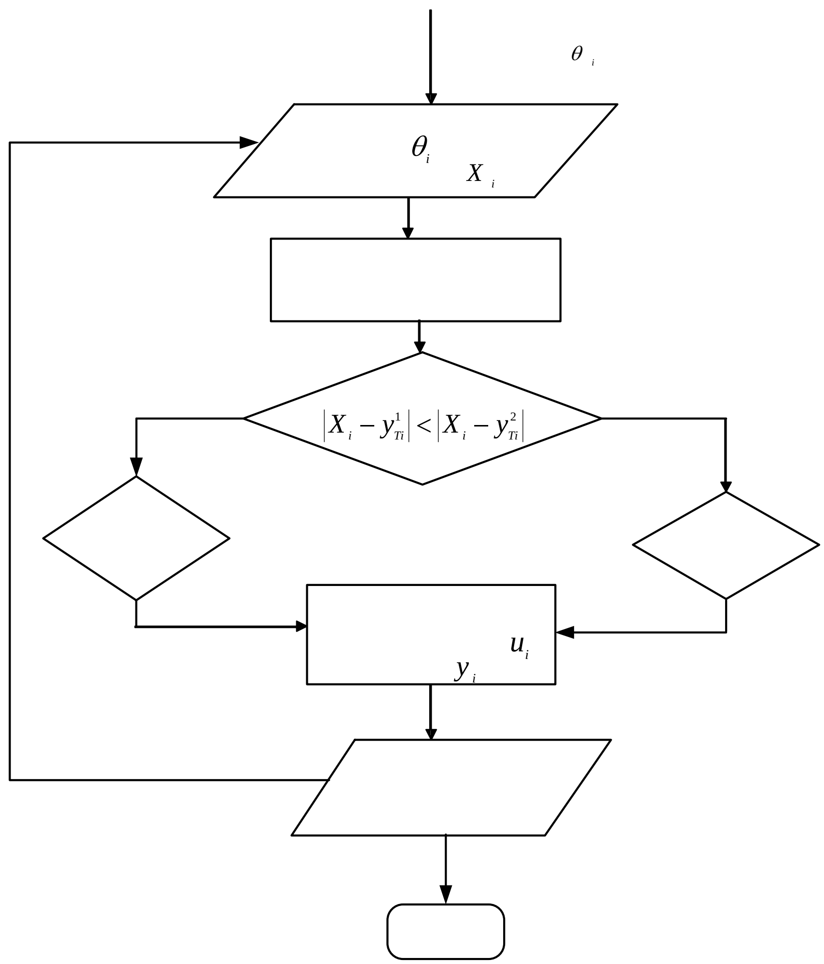

Fig. 3. Flow chart for the Precompensator technique

4. Noise estimation based Adaptive Control scheme

The elimination of noise and the extraction of signal are important issues in adaptive control techniques and is a subject of much scientific research in recent years. Literature is available discussing about some particular type of noise, such as a step noise and a sinusoidal noise (11)-(12) with practical examples.

Our scheme mentioned before, approximate inverse based adaptive control scheme is enhanced here by using the feedback loop for the estimate of the plant noise. This scheme can be summarized as follows:

i. The plant *P* is identified using the online identification scheme, RLS.
ii. The compensator block is updated based upon the information from the online identification for the plant with noise *e(k)*.
iii. An estimate of the plant noise is obtained using the actual plant output *y(k)* and an identified plant output $\hat{y}(k)$.
iv. The compensator block is provided with the information of the plant noise using the estimate of the noise and thus the reference signal $y_r(k)$ is tracked.

Fig. 4 shows the block diagram for the technique used. Where $\hat{P}$ is the estimated plant model. A particular type of noise signal, sinusoidal noise is used for simulation.

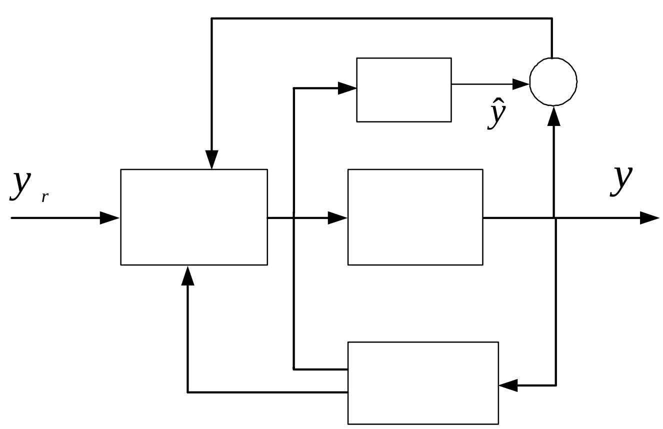

Fig. 4. Block diagram of the adaptive compensation technique with the feedback of the estimated plant noise

5. An Example

Consider a Hammerstein-Wiener model with 7 parameters.

$$y(k) = a_1\{d_1 \cos\ y(k-1) + d_2 \sin(\ y(k-1))\} + a_2\{d_1 \cos\ y(k-2) + d_2 \sin(\ y(k-2))\} + b_1\{c_1 u(k-1) + c_2 u^2(k-1)\} + e(k) \quad (8)$$

Simulations are carried for sinusoidal noise case. The results are plotted each time for the parameter convergence, the reference signal, the plant output, the control signal and the mismatch performance. Results are compared for the case while not using the noise estimates in the FB loop, scheme 1.

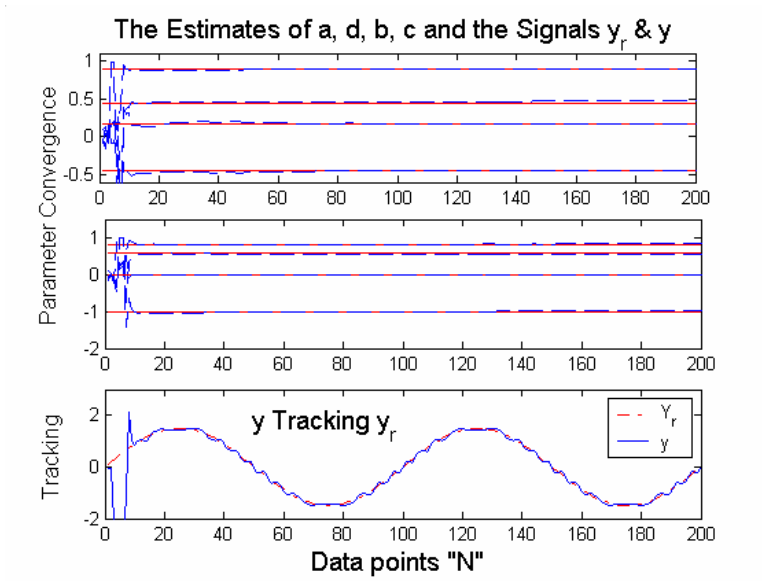

Fig. 5. Parameters and tracking: model with out the estimated noise in the feedback , in case of a sinusoidal noise

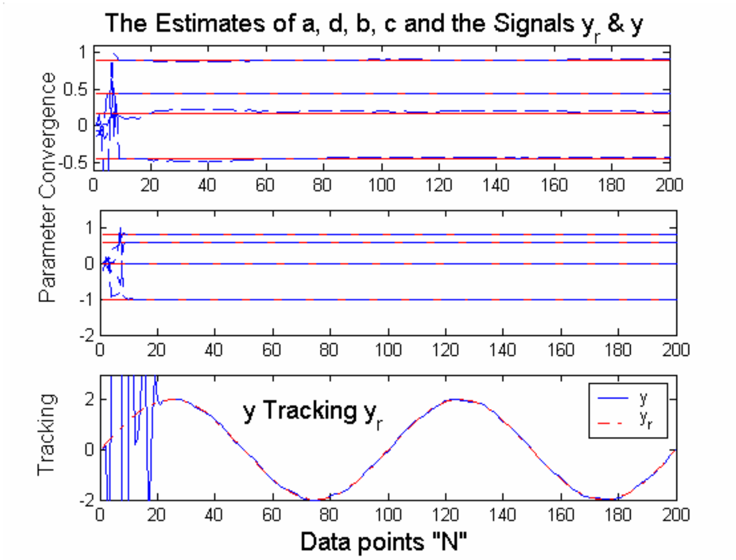

Fig. 6. Parameters and tracking: model with the estimated noise in the feedback, in case of a sinusoidal noise

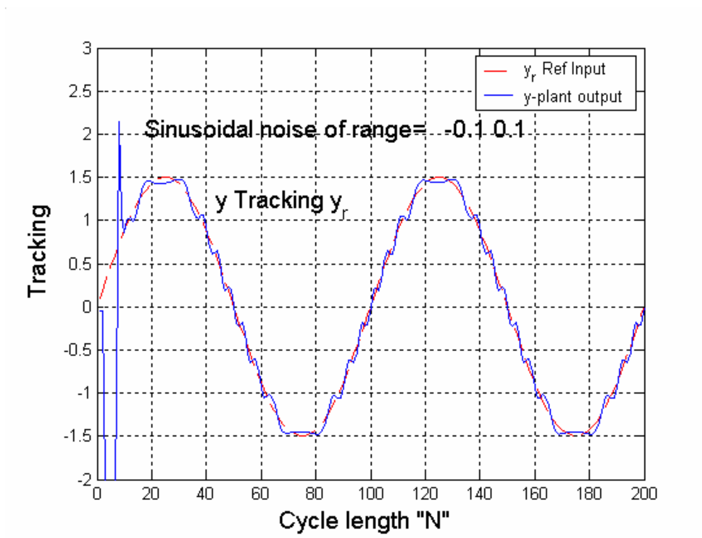

Fig. 7. Tracking: model with out the estimated noise in the feedback

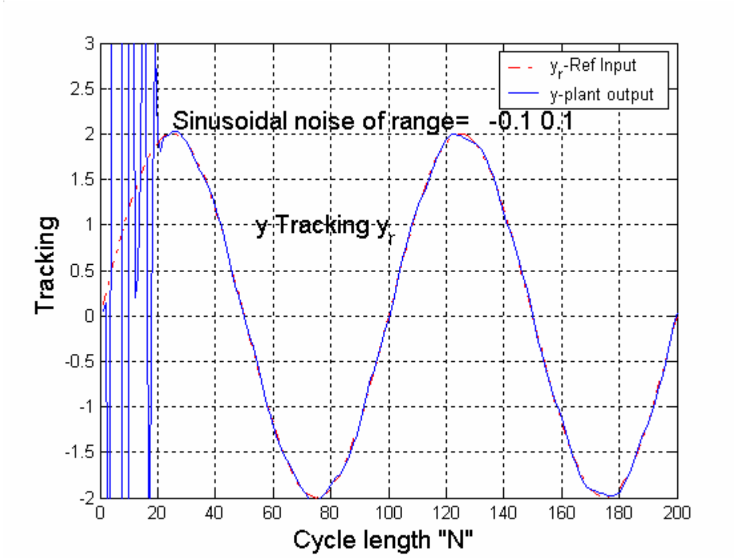

Fig. 8. Tracking: model with the estimated noise in the feedback

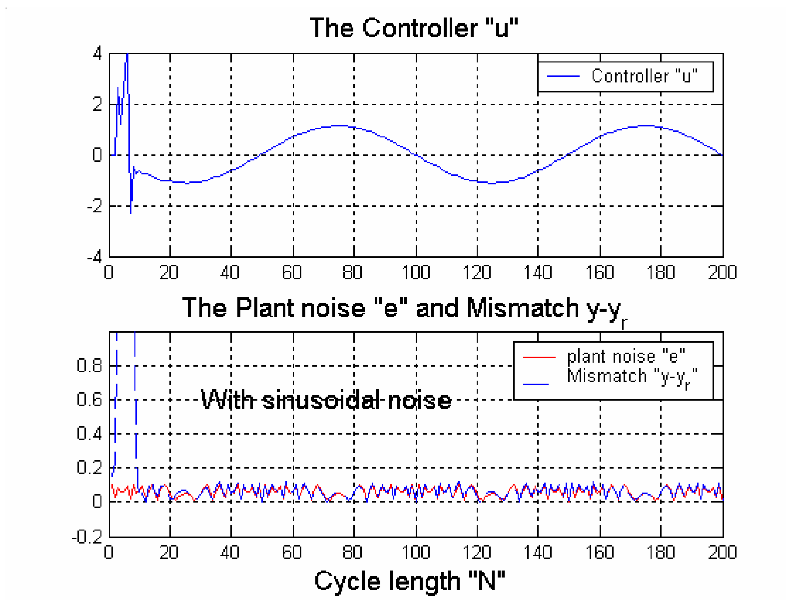

Fig. 9. Controller u and Mismatch performance with absolute values plot: model with out the estimated noise in the feedback, in case of a sinusoidal noise

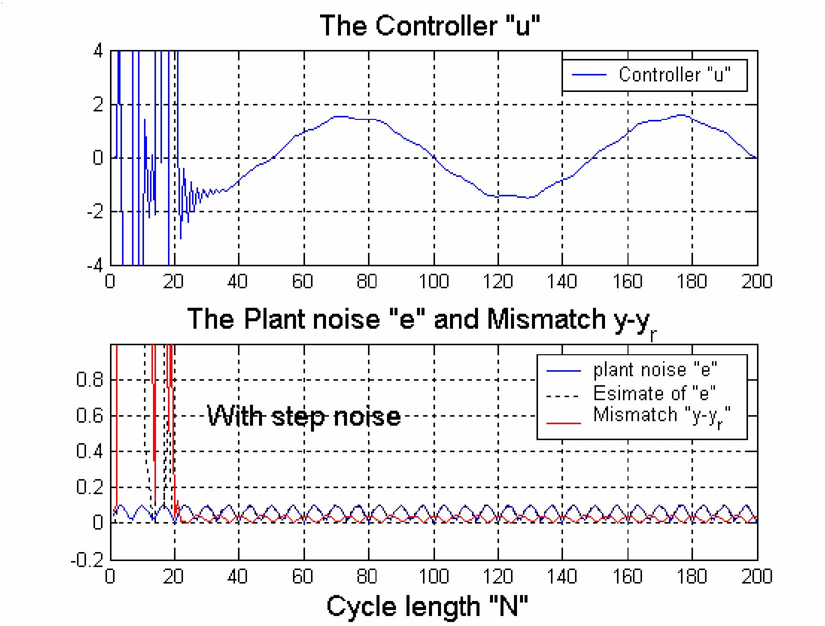

Fig. 10. Controller u and Mismatch performance with absolute values plot: model with the estimated noise in the feedback, in case of a sinusoidal noise

The plots in Figs. 5, 7 and 9 show the performance for scheme 1 with sinusoidal noise for the plant in the example above. While Figs. 6, 8 and 10 show the performance for the scheme with FB of estimated noise (Section 4), with sinusoidal noise. Comparing the results obtained, it can be inferred that an improved performance, hence better tracking is obtained in scheme 2 (Section 4), while we use the information of the estimated noise in the FB loop. The effect of the above mentioned noises still remain with the plant output in scheme 1 while it is almost all removed in scheme 2 because of the added information of the plant noise. In case of the random noise, this scheme does not perform so well and the tracking is not so perfect.

6. Conclusions and Observations

We have developed adaptive controllers for a class of nonlinear systems. Simulation for the tracking of the desired signal is carried for the same class of nonlinear systems, which shows that the output follows the desired reference signal precisely. Using an approximate inverse of the system, the system is forced to track the desired signal. The method of using the estimated noise in the feedback loop is implemented and thus obtaining the plant output noise cancellation.

Acknowledgments

The authors would like to express their appreciation for the support provided by King Fahd University of Petroleum and Minerals.

7. References.

[1] K. J. Astrom and B. Wittenmark. *Adaptive Control* Addison-Wesley Publishing Company, Inc.,1995.

[2] K. S. Narendra and A. M. Annaswamy. *Stable Adaptive Systems*. Prentice-Hall International Editions, 1989.

[3] M. Al-Naemi and S. M. Philips, Time varying approach to adaptive control of nonlinear systems, Proceedings of the American Control Conference, 2: 1218-1219, June 1997.

[4] F. Ding and X. M. Xie. Adaptive control of a class of nonlinear systems. *Second IEEE Conference on Control Applications*, 2:817–818, September 1993.

[5] X. N. Fernando and A. B. Sesay. A Hammerstein type equalizer for the wiener type fiber-wireless channel. *IEEE Pacific Rim Conference on Communications, Computers and signal Processing*, 2:546–549, 2001.

[6] H. W. Kang Y. S. Cho and D. H. Youn. Adaptive precompensation of wiener systems. *IEEE Transactions on Signal Processing*, 46(10):2825– 2829, October 1998.

[7] Y. H. Lim Y. S. Cho, W. Cha and D. H. Youn. An adaptive nonlinear prefilter for compensation of distortion in nonlinear systems. *IEEE Transactions on Signal Processing*, 46(6):1726–1730, June 1998.

[8] Er-Wei Bai. An Optimal Two Stage Identification *Algorithm for Hammerstein-Wiener Nonlinear Systems*. Proceeding of the American Control Conference, 1998.

[9] Y. Yang B. Qin and Z. Han. Adaptive control ofnonlinear systems with unkown parameters. *IEEE International Conference on Man, Machine and Cybernetics*, 2:809–814, 1996.

[10] T. Soderstrom and P. Stoica. *System Identification*. Prentice-Hall, 1989.

[11] M. Bodson J. S. Jensen and S. C. Douglas. Active noise control for periodic disturbances. *Proceedings of the American Control Conference*, pages 2616– 2620, June 1998.

[12] R. Sankar and N. Demir. Spectral estimation and sinusoidal noise cancellation using structural signal processoing. *IEEE Proceedings of Southeastcon*, 1:218– 220, April 1991.