Reliable Assessment of Deteriorating Structures

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ABSTRACT

A precondition for determining the condition of a concrete structure is to know what to look for. This implies that the assessment engineer must have a good knowledge of the deterioration mechanisms, and must know which parameters govern these mechanisms.

Depending on the situation, together with the depth of technical knowledge and the level of experience, the assessment engineer can select different inspection and testing strategies. A selective approach can be valuable in determining the cause of damage and the means to remedy the defects, and generates usually the least amount of data. A random inspection and testing approach can be valuable to determine the size or extent of damage when very large structures or when many similar structural components are deteriorating, and testing all parts would become insurmountable, but it does accumulate relatively much information causing additional difficulties when deciphering the accumulated data.

Previously, the inherent variability and randomness in material parameters of structural concrete has been coarsely treated in assessment engineering. Nevertheless, the large variability has constantly given cause for concern when only a very limited number or a small size of specimen was available. Therefore, this has been an area where much confidence has had to be placed on the subjective experience of the assessment engineer.

Today, means are available to take this inherent variability and the often limited number of test results into account, and combine this in a scientifically rational way with the subjective competence and experience of the assessment expert. The modern theories of safety and reliability provide now these tools which will revolutionise future inspection and testing, and in particular the interpretations made from information obtained. A much more reliable decision basis can now be provided, and service life forecasting with corresponding cost implications is a fully operational tool on the doorstep to the next millennium.

KEYWORDS

Concrete structures; inspection; testing; assessment; reliability updating; probabilistic approach; decision theory.

INTRODUCTION

Major structures are expected to fulfil two basic requirements

1. They must have a long service life to ensure that the investment is spent in a rational way.

2. They must be designed and maintained such that the probability of failure both with respect to serviceability and collapse is acceptable.

It is a well-known fact that concrete structures are subjected to a number of different destructive mechanisms which during the lifetime of the structure may lead to a situation where the structure is not able to meet its design requirements. This problem can be solved by designing robust structures where the influence of the destructive mechanisms is negligible. Alternatively, the required safety can be obtained by performing regular inspections and measurements on the basis of which the reliability of the structure can be updated. Already at the design stage the engineer must choose an optimal plan for the inspection and maintenance of the given structure. This requires a careful and realistic assessment of the interrelation between design, choice of material, deterioration processes and future maintenance. The assessment can be performed on the basis of an evaluation of the expected costs related to the considered structure throughout its lifetime.

An evaluation of the expected costs related to a given structure must be performed taking into account the probability that the structure enters an unwanted state. For example, the higher risk of deterioration, the more need for care in selection of materials, geometrical form and correct execution, and for an adequate safe and economic maintenance scheme.

The natural degeneration will require an assessment of the structure carried out at regular intervals to reveal which mechanisms are threatening the structure, and to identify which parameters are governing the type and rate of deterioration. The assessment of the state of a structure can be performed by the use of a wide range of different inspection and measurement methods. It is the responsibility of the engineer planning the inspections and measurements to select a set of methods which give the optimal amount of information regarding the state of the structure at the lowest cost.

DECISION FRAMEWORK FOR PLANNING OF ASSESSMENT AND MAINTENANCE

A fundamental challenge for the engineer is to identify a design and an assessment plan as well as a maintenance strategy which minimise the overall life cycle costs and at the same time ensure that the safety is kept within the limits specified by legislation and being acceptable to society.

Two decision situations can be distinguished for practical reasons, namely design of new structures and maintenance of existing structures.

For new structures the design parameters such as choice of material, member dimensions, type of joints and specific detailing are determined on the basis of their influence on the design costs and future maintenance costs. Optimal design parameters may hence minimise the overall costs, including design costs, expected costs of maintenance and repair and expected costs of failure, i.e. the costs related to the structure not being able to fulfil its design requirements.

For existing structures the optimal inspection, repair and strengthening actions may be identified, based on evaluations of their influence on the immediate repair or strengthening costs, the expected future maintenance costs and expected failure.

Based on the above economic considerations, there are no principal differences between the situations when a new structure is to be designed and an existing structure is to be maintained, because the design parameters and the repair or strengthening parameters can be treated alike. The only difference is the reliability of the data available as factual information can be gained from testing the existing structure. This is not possible at the design stage of new structures.

The Rationale of Assessment Engineering

Structural maintenance planning usually involves one or more assessment analyses and actions, followed by decisions on requalification, rehabilitation and sometimes even replacement of the structure. Due to the close interrelation between the use of the structure, the actual and the future state and safety of the structure, decisions regarding requalification and rehabilitation cannot be carried out, if a strategy for the future maintenance has not been decided upon.

As the available information regarding e.g. loading, material properties and deterioration processes in general is incomplete or uncertain, maintenance decisions will be based on uncertain information. In normal structural design such uncertainties are treated by the chosen safety format. Such an approach is not feasible in predictions of deterioration and planning of maintenance. In the latter case rational decisions are made based on optimal service life costs. If the uncertainties are significant as e.g. predicting the deterioration in concrete structures, the decision problem may conveniently be treated within the framework of modern structural reliability methods and the economic decision theory.

In an assessment situation the decision problem is typically to choose if and how additional information about the state of the structure is to be collected. This information may concern the state of deterioration, the "as built" geometry, material characteristics, concrete cover, etc.

Information can be collected in different ways according to the wanted accuracy and relevance and thus be more or less costly. Depending on the gained knowledge, a requalification or a rehabilitation such as do nothing, strengthen and/or repair must then be chosen, again at different costs depending on the state of the structure. The overall safety and the residual service life are estimated with corresponding benefits and costs.

Rational planning of assessment and maintenance actions is seen to be an advanced and multi-disciplinary task requiring a close collaboration between engineers with different backgrounds including advanced structural mechanics, structural durability, reliability and decision theory and steel and concrete materials technology.

The Decision Process of The Assessment Engineer

In practical decision problems such as reassessment, inspection and maintenance planning for structures, the number of alternative actions can be extremely large. Therefore, a framework for the systematic analysis of the actions and their corresponding consequences is necessary. A framework suitable for this purpose is decision analysis.

The reassessment decision problem may conveniently be represented by a decision tree as illustrated in Figure 1.

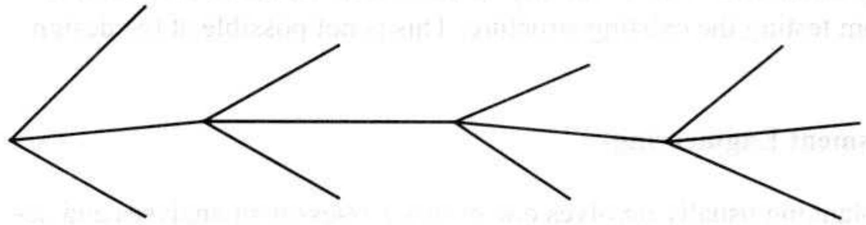

Figure 1: Decision tree.

Because different methods for collecting information have different costs and yield information of different accuracy and relevance, the owner of a structure, which must be reassessed, is typically faced with the problem to choose if and how to collect additional information about the state of the structure. The information may concern the state of deterioration, as built, geometry, materials characteristics etc.

At the first level of the decision tree shown in Figure 1 the decision related to the planning of tests and assessment is made, i.e. the number and type of inspections and tests are determined. The variables describing the inspection and testing plan are denoted $\mathbf{e}$ and the set of available decisions is denoted Ω_e. An inspection and testing plan must contain all the information necessary to carry out the assessment such that the purpose can be fulfilled, i.e. the following information should be given:

- The type of inspections and tests
- The number of each type of tests
- The conditions under which the inspections and tests should be performed
- The order of the tests
- The location of the tests
- The time at which the tests should be carried out

At the second level of the decision tree observations of the inspections and tests are obtained. It is important to take into account that the information gained by the additional inspections and tests are unknown at the time where it is decided to collect it. These observations are, therefore, modelled as stochastic variables, $\mathbf{S}$, with the admissible range Ω_s.

Depending on the state of knowledge after having collected the information, a requalification action such as do nothing, strengthening and/or repair must be chosen. Different requalification actions have different costs and yield different effects on the state of the structure.

At the third level in the decision tree it is decided which action to take. This decision is made by the owner of the structure and is denoted $\mathbf{a}$. The set of possible actions is denoted Ω_a.

At the fourth level in the decision tree a realisation is observed. Typically this realisation is related to observations of some critical event, for example cracking, delamination or spalling of the concrete cover. The uncertainties related to the loads (environment) and resistances (material) are modelled by the stochastic variables $\mathbf{Z}$ with the admissible range Ω_z. The limit state function

$$g(\mathbf{e}, \mathbf{s}, \mathbf{a}, \mathbf{z}) = 0 \qquad (1)$$

is used to model the critical event, identified or selected by the assessment engineer as the relevant limit state. The structure can be in a safe region where all requirements are fulfilled or in some other state where it is not able to fulfil one or more of the required performance criteria. Each of these states can be associated with a given cost.

On the basis of the decision tree the cost associated with each outcome of the test, **s**, and nature, **z**, can be determined for a given testing plan, **e**, and repair and maintenance strategy, **a**. For more detailed information on Bayesian decision analysis see e.g. Raiffa and Schlaifer (1961), Benjamin and Cornell (1970) and Ang and Tang (1984). For examples of practical applications see e.g. Engelund et al. (1998b) and Kroon (1994).

Planning of inspections and planning of maintenance according to decision analysis is relatively time-consuming. However, the principles involved give an excellent overview of the factors which must be taken into account by rational planning of inspections, tests and maintenance, i.e.

- The cost of the inspection and maintenance strategy being chosen
- The accuracy of the information obtained on the basis of the inspections and tests
- The relevancy of the information obtained by the inspections and tests
- The effect of the maintenance strategy

These factors must be taken into account in a rational manner by the planning of inspections, tests and maintenance.

If a model for the future deterioration of a given structure exists this model can be used in conjunction with planning of inspections and tests. For a given measurement accuracy the model can be used to determine the relevancy of different types of information. Hence, the prediction of deterioration is an integral aspect of the inspection planning and planning of maintenance.

DETERIORATION PROCESSES

As mentioned above the prediction of the future deterioration of a given structure is an integral part of experimental planning. In some cases detailed models for the prediction of deterioration exists. However, this is by no means the general case. Often little or no information on the basis of which the deterioration of a structure can be predicted is available. Further, it is difficult to formulate general models because different structural materials have individual durability characteristics. Furthermore, deterioration processes are to a large extent depending on the choice of structural detailing, workmanship and general quality control during production and execution.

The importance of designing structural details to facilitate a high quality in the execution process cannot be stressed too much. Poorly designed structural details have far too often resulted in specifications which could not be executed with a satisfactory quality. Examples are welded connections which are virtually impossible for the welder to access, and detailing in reinforced concrete structures where proper compacting of the concrete is not feasible.

For concrete structures, deterioration due to corrosion of the reinforcement is initiated by depassivation of the reinforcement. This is typically due to penetration of chlorides or to carbonation of the concrete. After depassivation has occurred, the deterioration is governed by the rate of corrosion which in part depends on the size of the depassivated zone and the resulting area of the anode and cathode. For these deterioration processes the important parameters are, therefore, the thickness of the concrete cover, the environmental impact and the ability of the concrete to transport chloride and the ability of carbon dioxide to penetrate the concrete; in short the importance is the robustness of the structure against premature deterioration.

A comparison to similar behaviour for steel structures can be valuable. In steel the dominating deterioration processes are fatigue and corrosion. Fatigue is caused by cyclic stress changes, resulting in formation of slip-bands in the crystalling material structure and finally in fatigue cracks. The factors, governing the fatigue life of steel structures, are the so-called local stress risers and the material characteristics which can be estimated by laboratory tests. Stress risers are typically caused by poor detailing which results in extreme stress concentrations in e.g. welded joints. However, stress risers can also come from imperfections in the welding such as slag inclusions and undercuts. Local stress risers can even come from corrosion formed "pit"-like imperfections in the surface.

Corrosion of steel structures is typically avoided by appropriate surface protection or by dehumidification of the air to which the steel is exposed, when possible. Due to interaction between corrosion and fatigue it is important to avoid corrosion in locations which are sensitive to fatigue. Fatigue of cables in cable-supported structures must be avoided in order not to reduce the strength of the cable drastically. Furthermore, it is recognised that even the slightest degree of corrosion in a cable may eventually lead to fatigue. Therefore, it is of utmost importance to design cables to be insensitive to fatigue and to protect them against corrosion. Finally, the design of the bridge must allow for the possibility of cables being exchanged, if they should deteriorate during the life of the structure. These are elements which determine the robustness of steel structures against premature deterioration.

Predicting Deterioration

On the basis of considerations such as the above the critical deterioration mechanisms in a given structure must be identified. Having identified the relevant deterioration mechanisms a model for the future deterioration must be formulated using the available prior knowledge or an existing model which represents the problem must be chosen. Alternatively, a hypothesis concerning the degradation of the structure can be formulated.

Having identified the model or having formulated a hypothesis concerning the degradation of the structure a set of experiments must now be performed. The purpose of these experiments is naturally to estimate the unknown model parameters and/or to accept or reject the hypothesis concerning the future degradation.

1. *The observation of an assumed constant* such as the gravitational constant.

2. *The observation of an outcome of one or more stochastic variables.* This could e.g. be the compressive strength of concrete.

3. *The observation of an event which depends on one or more variables.* Such an event could be an observation of signs of corrosion. For signs of corrosion to occur e.g. chloride must penetrate to the reinforcement and dissolve the protective layer. This event depends on the amount of chloride on the surface of the structure, the permeability of the concrete with respect to chloride ingress and the amount of chloride necessary to initiate corrosion. Evidently, this event depends on a number of uncertain variables.

4. *Comparative experiments.* Suppose we wish to reduce the chloride ingress in a concrete structure by some treatment of the surface of the structure. To assess the effect of the treatment we would have to posses two identical structures exposed to exactly the same environment. However, from observations it is known that the rate of ingress as well as the environment exhibit a substantial random variation between different structures. The only thing we can do is to apply the treatment to several structures and to observe the effect and to compare this with results obtained on the basis of structures where no treatment was performed. This type of experiment is commonly known as a comparative experiment.

The first type of experiment is in general of little relevance for civil engineers.

The second type of experiment is often used to determine the mean value and standard deviation of material properties and characteristic values of the material properties. This is usually done by the use of traditional statistical methods for parameter estimation such as the method of moments, maximum likelihood estimation or Bayesian parameter estimation. For more detailed information on the statistical treatment of such test data see e.g. Ross (1987) or Box and Tiao (1973) for detailed information on Bayesian analysis.

The third type of experiments can be used for model building, i.e. to determine the relation between some measurable input parameters and some output parameters by linear or non-linear regression. Further, this type of experiment can be used to update the reliability of a given structure by the use of Bayes formula. Let E denote a given event and let O be the observation which has been made. The probability of the event, E, occurring given that the observation, O, has been made, i.e. $P(E|O)$, is given by

$$P(E|O) = \frac{P(E \cap O)}{P(O)} \tag{2}$$

The fourth type of experiments can be used to assess the effect of a given maintenance strategy and/or repair method.

MAINTENANCE STRATEGIES

Service life of a structure is dependent on the design as well as of the owner or the management who make the decisions for operation, management and maintenance. Therefore, it is essential that these key players have all the technical and economical facts present. Quality cannot be ensured only by codes, standards and specifications.

A main problem is that the solutions for design and construction are neither black nor white. There is always a grey zone. Quality control following the ISO 9000-system does not function to perfection in the design and construction process of major projects, each to be considered a "prototype", because it is basically aiming at the producing industry which operates within repetitive production. Only main principles can be taken into account in structural design and construction. The main task for the experienced structural engineer is to manage this grey zone to the benefit of the quality of the structure.

Maintenance strategies are often carried out without sufficient knowledge about the effect of the strategy or sufficient knowledge about the future behaviour of the structure given maintenance has been carried out. To make rational decisions concerning the choice of maintenance of a given structure the following information should be available.

- Purpose of the repair method
- Description of the method
- Efficiency of the repair method
- Control of the repair method
- Costs associated with the repair method

At first, a description of the problem, the given repair methods aims to solve, should be given together with a description of how the given repair method solves the problem, i.e. a description of the effect of the repair is given. If for example the repair method is surface coating, the problem this method aims to solve can be chloride ingress and the repair method aims at solving the problem by preventing chlorides to penetrate through the surface coating and cover.

The description of the method is simply to give information about the materials used for the repair and to describe how the repair is performed.

When a given maintenance method has been implemented there may still be a risk of deterioration of the structure. If we again consider the surface coating, the efficiency of the method clearly depends on the amount of chloride present in the concrete at the time when the coating is applied. Further, to ensure the efficiency of the method it is important that a sufficient amount of coating is applied at all surfaces. An important aspect of the description of the efficiency of a given repair method is to describe how the future degradation can be predicted once the repair has been performed. This implies that relevant expressions for the progress of degradation shall be identified. If existing models can be used for the prediction of the degradation the effect of the repair method on the model variables shall be identified.

In some cases it may be necessary to perform an inspection of the repair to ensure that a given repair method solves the given problem. For example if a coating is applied it may be necessary to perform measurements of the chloride concentration in the structure to check that the coating prevents chloride penetration. If control is necessary this should be stated and the amount of control should also be identified. In some cases also the models for a number of variables depends on the amount of control, i.e. an increased amount of control will usually lead to a reduction of the uncertainty related to a given material property.

For a given structure and a given destructive mechanism, usually a number of different repair methods can be applied. To determine the optimal repair strategy, the cost associated with a given repair strategy must be identified. The cost can be given in terms of the cost of materials and the number of man-hours necessary to perform the repair, depending on the size of the structure, e.g. in square meters.

Experience has shown that the costs for repairing damaged structures are much higher than the costs for carrying out some protective measures, while the structure is still visually undamaged. The costs for late repair may be much higher than the costs for an early preventive maintenance. Add to this the often extra costs for inconvenience to the people using the structure and to the owner, because the magnitude of such repairs are usually not foreseen in the operation budget.

PREDICTING THE EXPECTED SERVICE LIFE COSTS

The costs related to a given experimental plan and a given maintenance strategy can be determined as

$$C(\mathbf{e},\mathbf{S},\mathbf{a},\mathbf{Z}) = C_e(\mathbf{e},\mathbf{S}) + C_a(\mathbf{a}) + C_f(\mathbf{e},\mathbf{S},\mathbf{a},\mathbf{Z})\mathbf{I}_f \qquad (3)$$

where $C_e(\mathbf{e},\mathbf{S})$ denotes the cost of the tests and measurements as a function of the plan, **e**, and the outcomes of the tests and measurements, **S**, $C_a(\mathbf{a})$ is the cost of the chosen maintenance action, **a**, and $C_f(\mathbf{e},\mathbf{S},\mathbf{a},\mathbf{Z})$ is the cost of failure depending on the plan, **e**, the observations, **S**, the action, **a**, and the state of nature, **Z**. Finally, the indicator function, $\mathbf{I}_f$ is defined such that it is equal to zero if and only if the structure is in a safe state, i.e. it is able to fulfil all the performance requirements and equal to one if and only if the structure is in an adverse state where it is not able to fulfil one or more of its performance criteria.

The expected cost is naturally determined taking into account the real rate of interest and the time at a given test or measurement is performed as well as the time when a given maintenance action is made. In this manner different strategies can be compared on the basis of their "present time" value.

PROBABILITY OF CHLORIDE-INDUCED CORROSION: EXAMPLE

As mentioned previously in this paper an inspection and testing plan shall focus on the relevant information. The relevancy of a given information can be assessed on the basis of a probabilistic analysis of the given problem using the available information. This analysis will reveal the most important factors, i.e. the factors whose uncertainty gives the largest contribution to the probability of the considered event occurring.

Consider for example a structure in a marine environment. It is possible that at some time corrosion will be initiated due to chloride ingress. The probability of corrosion can be determined using well-known probabilistic methods such as FORM/SORM-analysis (First Order Reliability Method/Second Order Reliability Method), see e.g. Madsen Krenk and Lind (1986).

The limit state function, $g(\mathbf{x},t)$, defined such that it is less than or equal to zero if and only if corrosion is initiated, can be defined as:

$$g(\mathbf{x},t) = c_{cr} - c(d,t) \tag{4}$$

where $\mathbf{x}$ is a vector of stochastic variables, t denotes the exposure time, c_{cr} is the critical chloride concentration and $c(d,t)$ is the chloride concentration at the exposure time t at the depth of the reinforcement, d (the cover thickness).

The chloride concentration around the reinforcement at a given exposure time can e.g. be determined on the basis of the diffusion equation:

$$c(d,t) = c_s\left[1 - erf\left(\frac{d}{2\sqrt{Dt}}\right)\right] \tag{5}$$

where c_s denotes the surface chloride concentration and D is the diffusion coefficient. For a more detailed description of the problem of chloride-initiated corrosion, see Engelund et al. (1998a) or Hoffman and Weyers (1996).

In Table 1 below the distribution functions and the distribution parameters are given for all variables in the problem.

Variable	Distribution	Mean value	Standard deviation	Unit
D	Log-Normal	15.0	5.0	[mm^2/year]
c_s	Log-Normal	1.0	0.30	[%]
d	Normal	50.0	10.0	[mm]
c_{cr}	Log-Normal	0.10	0.025	[%]

Table 1: Stochastic variables.

The chloride concentrations given in Table 1 are given in % relative to the weight of dry concrete.

In Figure 2 the probability of initiation of corrosion is given as a function of the exposure time, t.

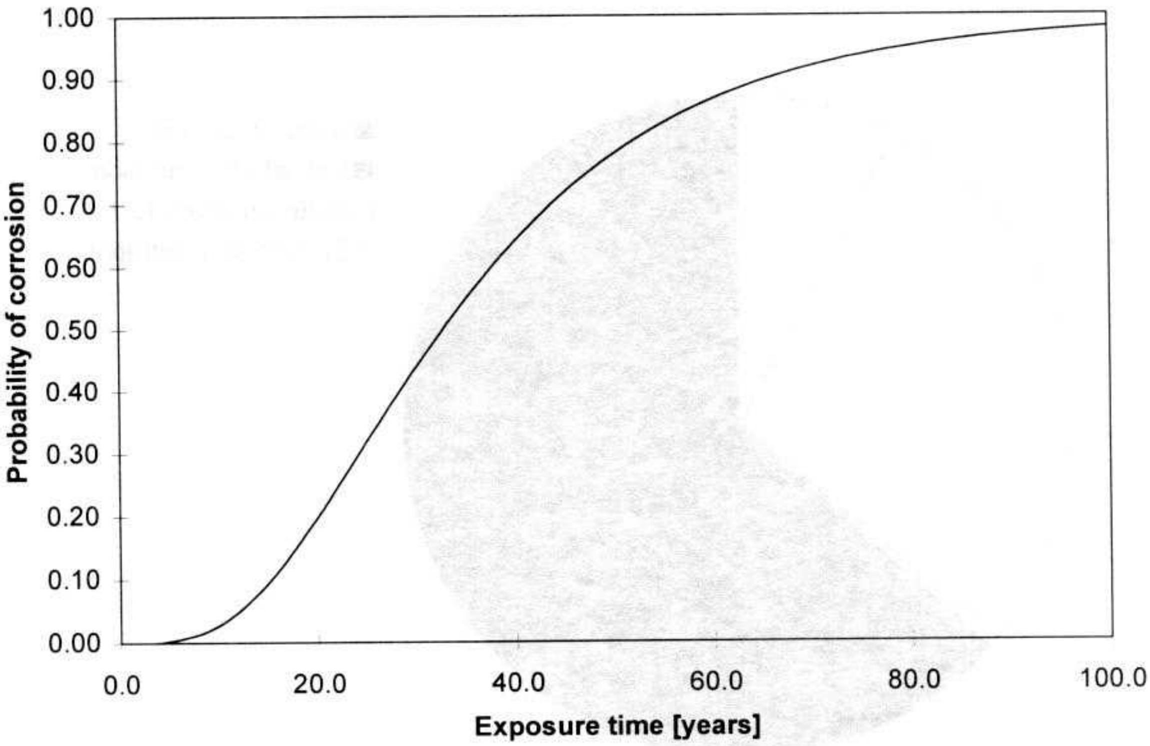

Figure 2: Probability of corrosion.

In Figure 3 the relative importance of the individual variables is shown, i.e. it is shown how much the individual parameters contributes to the total probability of corrosion initiation. The evaluation of the importance of the variables has been performed for t=20 years. However, the results are insensitive toward a change of the exposure time. In Figure 3 it is seen that the major contribution to the failure probability originates from the variable describing the uncertainty related to the cover thickness. This implies that if the uncertainty related to the cover thickness can be reduced by performing additional measurements of the cover thickness such measurements are the most relevant, i.e. have the highest benefit-cost ratio.

In Figure 4 the elasticity of the reliability index with respect to changes in the mean values of the stochastic variables are shown. The reliability index, β, is defined by

$$\beta = -\Phi^{-1}\left(P_f\right) \tag{6}$$

where P_f is the probability of corrosion and Φ is the standard Normal distribution function. Obviously, the reliability index increases with decreasing failure probability. The elasticities given in Figure 4 indicate the change in % of the reliability index for an increase of one % of the mean value of the stochastic variables. Again it is seen that the cover thickness is the most important variable. The reliability index will increase 4.5 % if the cover thickness is increases by 1.0 %. The mean value of the diffusion coefficient is the second most important mean value. By increasing the mean value of the diffusion coefficient by one % the reliability index is reduced by about 2 %. This investigation also leads to the conclusion that the cover thickness is the most important variable and that an investigation of the mean cover thickness is important. Similarly, enduring a reliable large cover in new designs is the most effective parameter to delay corrosion initiation.

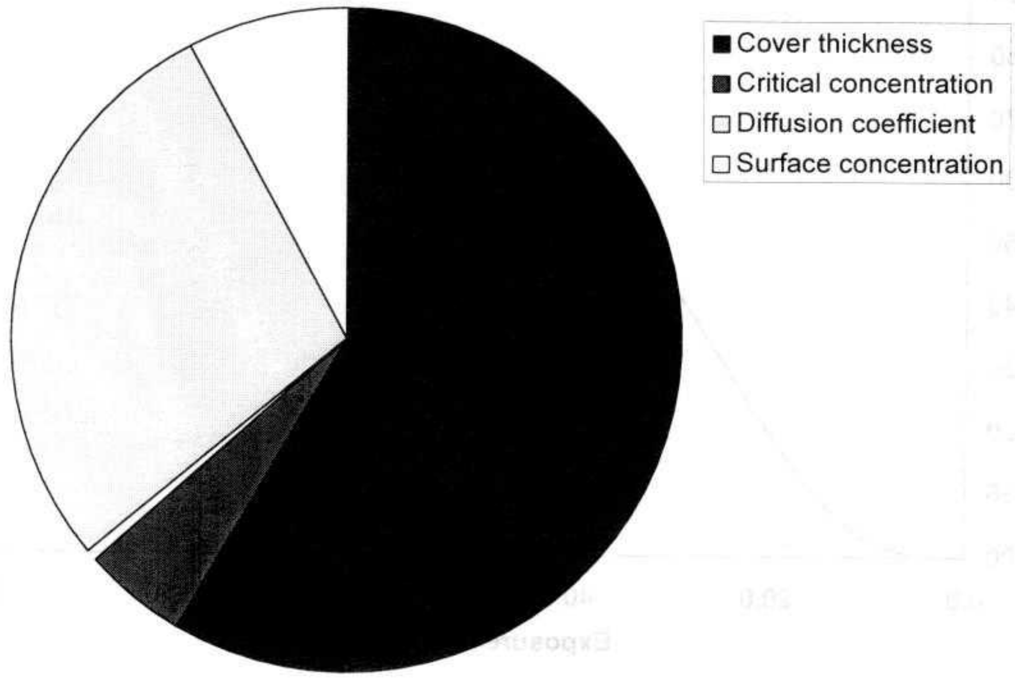

Figure 3: Relative importance of the stochastic variables.

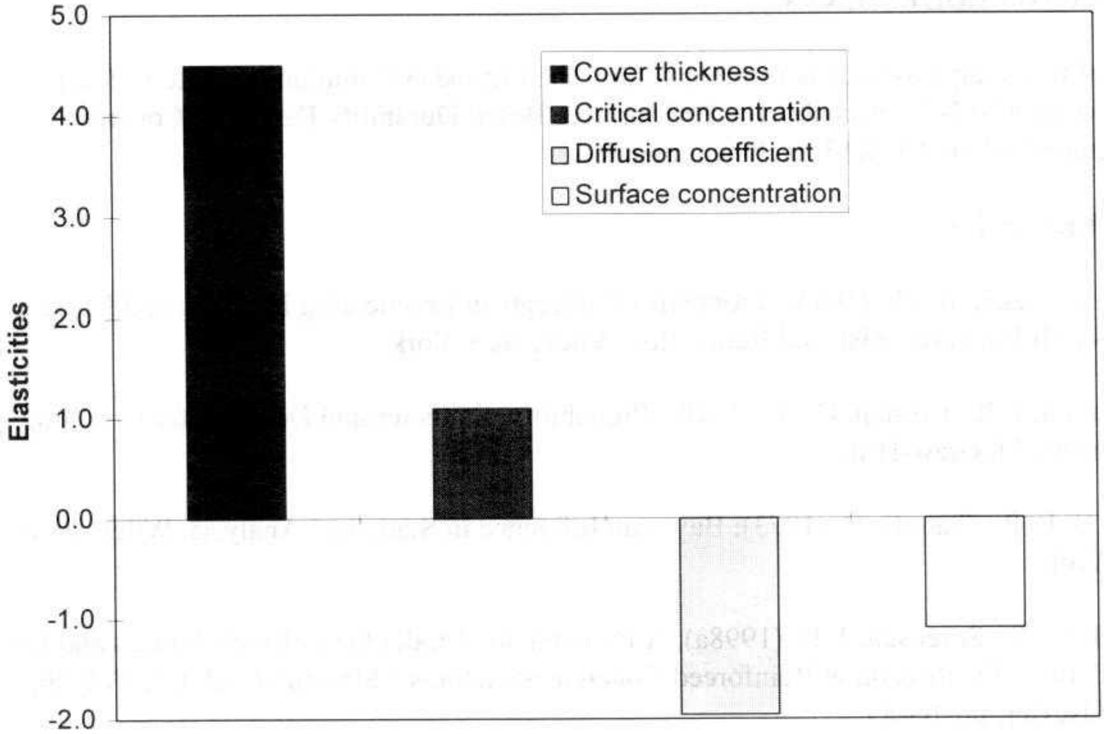

Figure 4: Elasticity of the reliability index with respect to the mean values.

CONCLUSIONS

Reliable assessment of structures must be performed on the basis of a model describing the deterioration of the considered structure together with observations and measurements from the considered structure.

An optimal experimental plan and an optimal plan for maintenance can be determined using decision analysis. Decision analysis allows the decision maker to arrange the large number of potential inspections, tests and methods for maintenance in a rational manner and it allows the decision maker to take into account the uncertainty related to the prediction of the performance of the considered structure. By decision analysis the cost of inspections, tests and maintenance is taken into account as well as the accuracy and relevancy of the information obtained by the inspections and tests.

In order to determine optimal inspection and testing plans and optimal plans for maintenance it is necessary to predict the future behaviour of the considered structure. Hence, a model for predicting the rate of deterioration must exist or most be formulated on the basis of the available information. Such models can only be formulated on the basis of a large number of observations from a large number of different structures. Therefore, it is important that all measurements and observations are reported in a format suitable for future analysis and that the analysis of the results is performed using statistical methods. This also allows the uncertainty related to the prediction of the performance to be quantified.

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