

**IMPROVING AEROELASTIC PERFORMANCE OF
HALE GEOMETRICALLY NONLINEAR FLEXIBLE
WINGS**

BY

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A Thesis Presented to the
COLLEGE OF GRADUATE AND INTERDISCIPLINARY STUDIES
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

AEROSPACE ENGINEERING

MAY 2025

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN- 31261, SAUDI ARABIA
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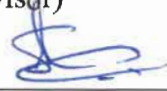


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In the name of Allah, the Most Gracious, the Most Merciful.

*This thesis is dedicated to Allah SWT, for granting me strength, patience, and guidance
throughout every step of this journey.*

*To my beloved family, whose unwavering support, prayers, and love have been the pillar
of my perseverance.*

*To my remarkable advisor, Dr Wael Gamal Eldin Abdelrahman, for your endless
guidance, encouragement, and for believing in my potential even when I doubted myself.
And to all the incredible individuals —friends, lecturers, and colleagues — who stood by
me, uplifted me, and reminded me that I was never alone in this journey.*

Alhamdulillah. |

ACKNOWLEDGMENTS

All praise is due to Allah SWT, the Most Gracious and the Most Merciful, for His boundless blessings, guidance, and strength that enabled me to complete this thesis and my Master's journey.

I would like to express my deepest gratitude to my remarkable advisor, Dr. Wael Gamal Eldin Abdelrahman, for his exceptional guidance, continuous support, and insightful advice throughout the entire research process. His patience and dedication have been a true source of inspiration.

To my dearest parents, Noraishah Musa and Taufik Misman, I also want to express my heartfelt gratitude for your unwavering love, prayers, and sacrifices are the reason I have reached this point. Thank you for being my greatest pillars of strength.

Moreover, to all the amazing people who have supported me, your kindness, motivation, and presence have made this journey a meaningful one through the challenges of finishing my studies and completing my thesis. May Allah SWT reward all of you with goodness in this life and the next.

Finally, congratulation to myself for successfully finishing my master studies and my thesis research with the tenacity and attitude of never giving up on myself, enduring every challenge with perseverance and faith. I am hoping that the expenditure throughout my academic year will lead to greater contributions in the future, with the continued blessings of Allah SWT.

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LIST OF ABBREVIATIONS

HALE	:	High-Altitude Long Endurance
UAS	:	Unmanned Aerial Systems
HARW	:	High Aspect Ratio Wing
TWB	:	Thin-Walled Beam
UDL	:	Uniform Distributed Load
EPM	:	Equivalent Plate Models
FEM	:	Finite Element Modelling

ABSTRACT

Full Name : [Najwa Zaqirah Binti Taufik]
Thesis Title : [Improving Aeroelastic Performance of HALE Geometrically
Nonlinear Flexible Wings]
Major Field : [Master of Science of Aerospace Engineering]
Date of Degree : [May 2025]

[This research enhances aeroelastic stability and control in High-Altitude Long-Endurance (HALE) unmanned aerial systems (UAS) by integrating the flexible wing materials embedded with piezoelectric actuators. The extreme flexibility and low natural frequencies of HALE wings lead to significant deflections, impacting stability and responsiveness during high-altitude and low speed flight operations. To address these challenges, an energy-based analytical model is developed to evaluate aeroelastic behaviors which are the natural frequency and flutter on the wings under nonlinear geometric deformations, integrating aerodynamic forces and piezoelectric contributions. Complementary analytical simulations in MATLAB model elastic and piezoelectric effects, predicting the aeroelastic performance of the HALE UAS. These simulations identify optimal actuator configurations, focusing on placement, the actuator patch area and the different type of materials of piezoelectric actuator used, to mitigate instabilities like flutter and divergence. This research establishes a novel framework for optimizing HALE wing designs, advancing stability, efficiency, and durability for long-duration high-altitude missions. |

ملخص الرسالة

الاسم الكامل: نجوى زكيرة بنت توفيق

عنوان الرسالة: تحسين الأداء الهوائي المرن للأجنحة المرنة غير الخطية هندسيًا (HALE)

التخصص: ماجستير العلوم في هندسة الطيران

تاريخ الدرجة العلمية:

تهدف هذه الدراسة إلى تعزيز الاستقرار والتحكم الهوائي المرن في أنظمة الطائرات بدون طيار طويلة التحليق على ارتفاعات عالية (HALE)، من خلال دمج أجنحة مرنة مزودة بمشغلات كهروضغطية. تؤدي المرونة العالية والترددات الطبيعية المنخفضة لأجنحة HALE إلى انحرافات كبيرة، مما يؤثر على الاستقرار والاستجابة أثناء الطيران على ارتفاعات عالية وسرعات منخفضة. لمعالجة هذه التحديات، تم تطوير نموذج تحليلي قائم على الطاقة لتقييم السلوك الهوائي المرن، بما في ذلك التردد الطبيعي وظاهرة الرفرفة، في ظل التشوهات الهندسية غير الخطية، مع دمج القوى الهوائية وتأثيرات المشغلات الكهروضغطية. كما تم إجراء محاكاة تحليلية باستخدام برنامج MATLAB لتوصيف التأثيرات المرنة والكهروضغطية والتنبؤ بالأداء الهوائي المرن للطائرة. وتساعد هذه المحاكاة في تحديد التوزيع الأمثل للمشغلات، مع التركيز على مواقع التثبيت، ومساحة الرقعة النشطة، وأنواع المواد المختلفة للمشغلات الكهروضغطية، بهدف الحد من الظواهر غير المستقرة مثل الرفرفة والانهيال الديناميكي. تقدم هذه الدراسة إطارًا مبتكرًا لتصميم أجنحة HALE المثلى، مما يعزز من الاستقرار والكفاءة والمتانة في المهمات الجوية طويلة وعلى ارتفاعات عالية.

CHAPTER 1

INTRODUCTION

1.1 Research Background.

High-altitude long endurance (HALE) unmanned aerial systems (UAS) are not just designed for prolonged flights at high altitudes, but also for making a significant impact on environmental monitoring, communications, and surveillance applications. Their high aspect ratio and flexible wing structures, while vulnerable to aeroelastic problems, are perfect for tracking wildlife migration, studying climate change, offering long-range and persistent connectivity, and monitoring large areas for security or disaster response. This potential for positive impact should inspire us to overcome the aeroelastic stability issues that affect the dynamic behaviour and control of the aircraft.

Recent advancements in materials science have made it possible to use innovative materials with integrated piezoelectric devices in nonlinear flexible wing designs. The orientation and positioning of the piezoelectric actuators are influenced by fluid-structure interactions, which are defined as the mutual influence of the aerodynamic forces and the structural response of the wing. This, in turn, influences the aerodynamic and aeroelastic responses of the wings. By controlling torsional modes and modifying piezoelectric actuator orientations based on flying speed, which is in low speed and high-altitude flight conditions, this is also can make HALE wing stability can be

increased. By examining a novel approach to incorporating piezoelectric devices for improved aeroelastic control, this work builds on previous developments.

Flutter is a critical aeroelastic instability that emerges from the complex interaction between aerodynamic forces and structural vibrations. It occurs when the aerodynamic energy input surpasses the structural damping, leading to self-excited oscillations that can rapidly increase in amplitude. The flutter speed represents the pivotal flight speed at which this instability initiates. For High Altitude Long Endurance (HALE) aircraft with highly flexible wings, flutter can manifest at unexpectedly low speeds due to their low structural stiffness and natural frequencies.

The relationship between flutter speed and the natural frequency of the structure is crucial. By increasing a wing's natural frequency—whether through reinforced structure or advanced active control techniques—the flutter speed can be significantly raised. This enhancement creates a much safer operational margin, effectively delaying the onset of flutter. Our research utilizes cutting-edge piezoelectric actuators to actively modify the wing's stiffness and natural frequencies, thereby bolstering flutter margins and facilitating stable, high-endurance operations at high altitudes. This innovative approach not only enhances flight safety but also unlocks new possibilities for HALE aircraft performance.

The operational requirements of HALE UAS require an effective aeroelastic design to maintain stability and control, especially during long-duration missions in adverse atmospheric conditions. HALE wings' great flexibility and low weight improve fuel efficiency and altitude endurance, but they also make them more susceptible to aeroelastic instabilities like divergence and flutter. The operational range of these devices may be restricted, performance may be hampered, and flight safety may be jeopardized. Adaptive materials that can lessen the negative consequences of structural deformations must be incorporated into an inventive approach to

address these problems, in addition to taking into account the aerodynamic forces acting on the wings.

Integrating piezoelectric materials into the wing structure is a viable approach to improving aeroelastic stability in HALE UAS. Real-time adjustments to wing form and stiffness are possible to counteract destabilizing forces thanks to piezoelectric actuators, which actively respond to structural vibrations. The location and orientation of piezoelectric actuators can be adjusted to maximize the wing's aeroelastic response under different flight situations, especially at lower speeds where aeroelastic effects are most visible. This study aims to improve the safety and functionality of HALE systems by developing the design of HALE wings using these smart materials to produce a more flexible, durable structure.

1.2 Problem Statement.

High-altitude long endurance (HALE) unmanned aerial systems (UAS) are engineered to perform extended missions at high altitudes, requiring them to be lightweight, flexible, and fuel-efficient. However, the high aspect ratios and low natural frequencies that define HALE wings' intrinsic flexibility provide serious aeroelastic problems that jeopardize structural integrity and stability. These wings undergo significant deflections during operation, particularly at higher flying speeds, which can result in aeroelastic instabilities such flutter, divergence, and diminished control authority. Among the various concerns, flutter presents the most immediate structural risk, as it leads to rapid and destructive oscillations when the flutter speed is exceeded. The low natural frequency of High-Altitude Long Endurance (HALE) wings decreases their flutter speed. Therefore, it is crucial to develop methods that can increase natural frequencies to safely raise the flutter threshold. Because they frequently require assistance to balance structural flexibility and

aerodynamic robustness, traditional materials and structural solutions have proven insufficient in tackling these instabilities.

The optimization of aeroelastic and aerodynamic control which both essential for preserving stability and attaining effective flying performance in flexible wing structures that is not well addressed by current design methodologies. Even though passive design changes can improve some aspects of stability, they are unable to offer flexible, active control over the wings' aeroelastic response under different flying circumstances. As a result, an integrated design strategy is required that includes adaptive materials, like piezoelectric actuators, that can dynamically react to shifting aerodynamic forces in addition to improving aeroelastic stability. In order to increase flight stability, control, and overall performance under a variety of operating situations, this research attempts to develop such a strategy by concentrating on improving both actuator location and aeroelastic tailoring of the HALE wing structure.

1.3 Objectives.

This research aims to address the critical aeroelastic challenges in HALE UAS by leveraging advanced materials integrated with piezoelectric elements to enhance stability and control. Given the high flexibility and low natural frequencies of HALE wings, achieving optimal aeroelastic performance requires a sophisticated design approach that can actively adapt to varying aerodynamic forces. By strategically embedding piezoelectric actuators and optimizing their placement and orientation, this study seeks to improve the structural response of flexible wings, reducing unwanted deformations and increasing flight stability. Through a combination of theoretical modelling and computer simulations, this research will explore how actuator

placement, material orientation, and energy-based analysis can be applied to achieve a more stable and resilient HALE wing design under low-speed and high-altitude flight conditions.

The main objectives of this research are as follows:

1. Develop a method for designings wings embedded with piezoelectric elements to optimize the aeroelastic response and control of HALE UAS wings.
2. Analyse the behaviour of flexible wings at low speeds and high-altitude through an energy-based analytical approach, allowing for a detailed inspection of their static and dynamic aeroelastic characteristics, and validate these findings through computer simulations.
3. Investigate the impact of strategic piezoelectric actuator placement, actuator patch size, material type, and applied voltage on the natural frequency and flutter characteristics of the wing, in order to raise the flutter speed and improve structural stability under operational conditions.
4. Calculating the natural frequencies and flutter characteristics of the wings in preventing the undesirable vibrations, enhancing the stability and optimizing the piezoelectric actuator placement, actuator size, actuators type and applied voltage on the actuator for improving the aeroelastic performance and longer flight durations.

1.4 Scope of work.

This study is focused on developing and validating an analytical model to predict the critical static and dynamic aeroelastic characteristics of High-Altitude Long-Endurance (HALE) wings under low-speed and high-altitude conditions, where their high flexibility and low natural frequencies present significant challenges. To accurately capture the complex behaviour of these flexible wings, the model incorporates nonlinear geometry effects and integrates piezoelectric actuators to enhance aeroelastic responsiveness, enabling the wings to adjust actively to structural

deformations and aerodynamic loads. MATLAB will be used for detailed analytical simulations, testing various configurations to assess the impact of actuator placement and orientation on aeroelastic stability and control. The scope is limited to low speed and high altitude aeroelastic behaviour, focusing on torsional and vibrational control improvements rather than high-speed aerodynamic effects. By concentrating on these conditions, the study aims to generate insights for optimizing the structural integrity and control of HALE UAS wings in demanding flight environments.

1.5 Limitations and challenges.

Through focused approaches to these problems for this study, the project hopes to guarantee precise modelling, simulation, and analysis, which will finally result in a triumphant conclusion and significant advancements in the field of aerospace engineering.

1. Setting up the mathematical model for the flexible wing in suitable software.

Creating a precise mathematical model for a flexible wing presents a significant challenge, demanding an accurate portrayal of the wing's flexibility, aerodynamic characteristics, and structural properties. This involves meticulously setting up the model with suitable parameters for both high-altitude and lower-speed flight scenarios and configuring the software to effectively handle the unique traits of high-aspect-ratio wings (HARW).

Methods for overcoming this challenge include selecting software, like MATLAB, that is ideal for aeroelastic and structural analysis. It is important to confirm that the selected software can accommodate specific modelling requirements for flexible wings. Before proceeding on to more complicated setups, a basic model should also be constructed and subjected to initial testing

to confirm the software's configuration. This approach aids in the early detection of any issues and guarantees that the model is applied appropriately in the selected program.

2. Setting up the loading and boundary conditions for high altitude and low-speed flight.

Setting up the loading and boundary conditions to replicate high-altitude, low-speed flight can be challenging, as it involves accurately simulating the reduced air density, pressure, and aerodynamic forces in the model. One way to address these issues is by validating the conditions using existing research or case studies. It's crucial to confirm that the loading and boundary conditions are practical and consistent with standard low-speed and high-altitude flying scenarios. This approach requires running iterative simulations with different settings to fine-tune the accuracy of the setups and ensure the thoroughness of our approach.

3. Analysing wing behaviour under loading conditions.

When studying the behaviour of a wing, especially in the presence of significant deformations and their nonlinear aeroelastic effects, precise computation and interpretation are essential. These challenges are amplified when dealing with computationally intensive and complex problems, particularly under dynamic flight conditions. By leveraging the wing's sensitivity and structural integrity, we gain valuable insights into how variations in model parameters impact wing behavior. This approach not only facilitates the identification of crucial variables but also ensures the robustness of our analysis.

4. Setting Up the Applied Voltage on the Actuator in the Beam Model.

One of the key challenges in this study is accurately applying voltage to the piezoelectric actuator in the simulation model. Piezoelectric actuators require proper electrical boundary conditions to generate the expected deformation and control forces. If the applied voltage is not

correctly implemented, the actuator may not exhibit the intended strain energy response, leading to inaccurate results. To address this, the study will conduct calibration tests to ensure that the applied voltage correctly interacts with the beam structure and piezoelectric material properties. Additionally, comparisons with experimental data and validated numerical models will be used to confirm the correct electromechanical coupling within the simulation.

1.6 Motivation for research direction.

Addressing this gap will enable future advances that are safer, more sustainable, more efficient, all while promoting the safety and advancement of HALE aircraft technologies. It is imperative to address the research gap in low-speed, high-altitude stiffness matrix analysis optimization of wing structural integrity for multiple significant factors.

1. Enhanced safety and stability for HALE aircraft.

High Altitude Long Endurance (HALE) aircraft are used for critical operations such as for environmental monitoring, surveillance, and reconnaissance. This aircraft ability to sustain in the air for prolonged flying time at high altitudes is crucial but the structural integrity of the wings is often compromised by radical environmental conditions like low density and temperature fluctuations. By addressing the gap in stiffness matrix analysis, this will help to enhance the safety of the aircraft by providing more accurate predictions of aeroelastic instabilities such as fluttering under these conditions. Besides, this can prevent catastrophic failures during flight.

2. Optimizing performance and efficiency of the HALE aircraft.

In understanding on how the structures of wing deform under the specific conditions allows for better optimization of materials and the wing design itself. This study aims to improve the aerodynamic efficiency of HALE aircraft by precisely modelling the non-linear deformations and

the effects of the bending and torsional stiffness of the wing with the use of piezoelectric actuator. By this optimization, the aircraft can reduce fuel consumption and can lengthen the flight operational times which are crucial for mission success especially for long-endurance flight mission like surveillance and reconnaissance.

3. Critical for Aircraft wing design.

High Aspect Ratio Wing (HARW) have often been employed in HALE aircraft applications. This configuration is often utilized because of its longer and thinner design of the wing which maximizes the efficiency at high altitudes. Nonetheless, the flexibility of these wings establishes some significant structural challenges on the aircraft. Bending and torsional stiffness must be emphasized in the stiffness matrix analysis in order to maintain the structural integrity for long-endurance flights at low speeds and high altitudes, which will inevitably result in a safer and more optimized HARW design in HALE aircraft applications.

1.7 Research hypothesis.

This research asserts that integrating piezoelectric elements into the flexible wing structure of High-Altitude Long-Endurance (HALE) Unmanned Aerial Systems (UAS) has the potential to drastically enhance aeroelastic stability, vibration suppression, and control authority. By strategically positioning piezoelectric actuators, we can actively counteract the structural deformations caused by aeroelastic forces, significantly improving the wing's flutter resistance and dynamic stability. This is achieved by raising the natural frequencies of critical modes, which in turn increases the flutter speed, delaying the onset of aeroelastic instabilities and enabling safer, high-altitude operations.

Through careful optimization of actuator placement, patch size actuator types, and applied voltage, we can dynamically tailor the wing's response to changing aerodynamic loads. This innovative approach not only diminishes the risks of flutter, divergence, and excessive deflections but also ensures superior aerodynamic performance and exceptional structural resilience.

Moreover, this research will utilize flat-plate beam analysis as a keystone method for evaluating the nonlinear aeroelastic behavior of the wing. By incorporating Hamilton's principle alongside energy-based analysis, we will thoroughly examine the interplay between piezoelectric actuators and the wing's structural deformations, thereby confirming their capacity to enhance torsional rigidity and improve vibrational damping. Analytical simulations using MATLAB will complement our analytical findings, ensuring a deep understanding of piezoelectric-actuated aeroelastic control.

In conclusion, this study aims to reveal that adaptive piezoelectric actuation embodies a groundbreaking and effective strategy for enhancing the structural integrity, control efficiency, and operational performance of HALE UAS, particularly in challenging low-speed and high-altitude flight conditions. The insights gained from this research could revolutionize the development of more efficient, durable, and stable flexible-wing designs, representing a significant leap forward in aeroelastic optimization and the application of smart materials in aerospace engineering. |

CHAPTER 2

LITERATURE REVIEW

Aeroelasticity is the outcome of the interplay between the inertia force, aerodynamic forces, and elasticity of the aircraft. The detrimental effects of this phenomenon, which include fluttering of the wings and tails, fatigue in the construction of the aircraft, unwanted vibrations interfering with missions, and erroneous control level operation are being studied for the enhancement of the flight performance of the aircraft. As mentioned by [1], increased wing structural flexibility makes the wing more vulnerable to greater deflections under the same operating conditions. This can have an impact on the wing's dynamic and aeroelastic behavior, which can lead to instabilities at slower speeds than in a similar stronger wing design.

Even though the developed flexible model was limited to longitudinal dynamics and a single elastic mode, the study by [2] developed the aerodynamics and flexible flight dynamics models for a flexible wing of a UAV and demonstrated that the reconstructed wing deflection and flight path from the measured and estimated states was consistent with the developed flexible model. Furthermore, the low speed flutter demonstrator, which maximizes the algorithm that converges to designs with the maximum wing area possible for the low flutter frequency demonstrator which meant with greater sweep angles, taper ratios, and operating speeds was demonstrated by [3] using the flutter UAV prototype.

The continuous supply of solar energy makes solar-powered aircraft incredibly promising for long-duration flights; consequently, research on them has advanced significantly over the last few decades. With their benefits of minimal danger, economical maintenance costs, environmental

preservation, and zero emissions, solar-powered airplanes offer a wide range of potential applications. These days, two widely employed techniques in the aerodynamic and structural systems are boosting the lift-drag ratio during cruise by using laminar airfoils and decreasing the wing weight by implementing lighter flexible skin to create the novel built solar-powered UAV with improved flight efficiency [4].

Nevertheless, all components of the solar-powered aircraft, including the energy, aerodynamic, structural, propulsion, and control system, must be tuned to fulfill the high altitude and long endurance design requirements, constrained by the energy balance limitations during flight [5]. As been stated by [4], Conversely, in actual applications, the partially flexible membrane's elastic deformation will significantly affect the wing's aerodynamic characteristics, which will directly affect how well the wing performs in comparison to its design during flight.

With relatively low structural weight, High Altitude Long Endurance (HALE) aircraft configurations offer wings with a very large aspect ratio. As been said by [6], the respective wings have low natural frequencies, are highly flexible, and can undergo significant deformations when in flight, which could interact with the aircraft's rigid body motion. In addition, if the deformations are particularly large, geometrical, and structural non-linearities should be taken into consideration. The study made by [1], by improving wing structural flexibility makes the wing more vulnerable to greater deflections under the same operating conditions. This can have an impact on the wing's dynamic and aeroelastic behaviour, which can lead to instabilities at slower speeds than in the same but more rigid wing construction.

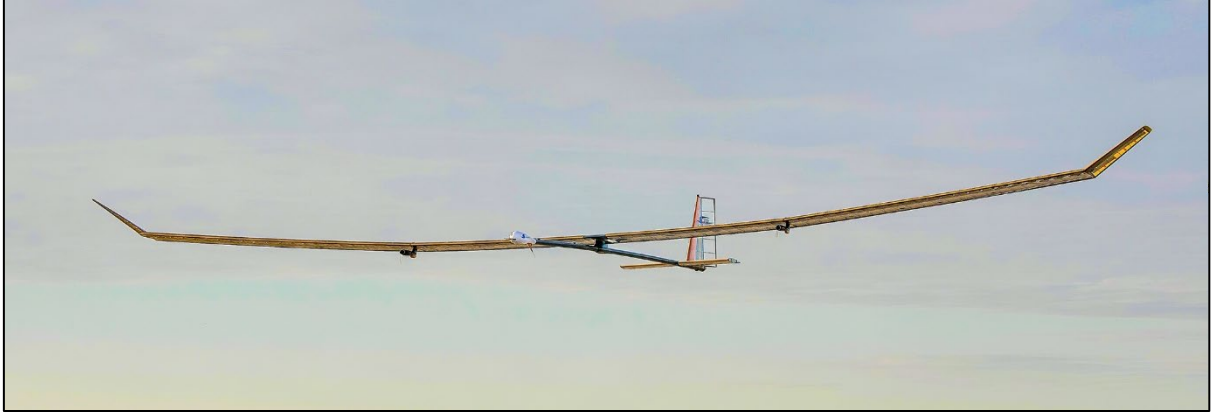


Figure 1. Swift High-Altitude Long-Endurance (HALE) UAS (Developed in Partnership With NASA).

Source: Swift Engineering & NASA.

Unsteady aerodynamics has long been a major factor in issues with interdisciplinary optimization, aeroelastic stability analysis, and aircraft design. In accordance with the study by [7], the Unsteady Vortex-Lattice Method (UVLM) is an appropriate technique to examine the unsteady aerodynamics for HALE aircraft since it balances computation cost and modeling quality. With its free wake model, which can capture the effect of wake vorticities on the distributed aerodynamic loads, the UVLM is developed in the time domain.

Moreover, the unsteady aerodynamics of the wings and tails are modelled using Wagner's function with Jones's approximation, and the effects of fuselage flexibility on the flutter boundaries and Limit Cycle Oscillations (LCO) amplitudes which used for statistical approach to detect and diagnose a particular type of vibration impacting the control surfaces of the wings and tails that are examined by [8]. The analysis made by [9] revealed that the high aspect ratio's elastic deformation by utilizing the huge span that HALE aircraft naturally possess, the rolling moment of the aircraft is enhanced, and the flexible wing can be effectively controlled by the moving mass to prevent excessive deformations.

According to the aeroelastic research conducted by [10], the frequency-limited balance of the aircraft's unstable aerodynamics can be built upon by combining the methods of Reduce Order Models (ROMs) with the unstable Vortex Lattice Method (UVLM). In addition to providing techniques for evaluating flutter constraints and dynamic loads in parametric aircraft design, these combinations can also produce aeroelastic studies that account for massive wing excursions. This allows for linearization around arbitrary geometries and a free wake, which will be utilized in this case to produce effective ROMs for the aeroelastic analysis.

2.1 Piezoelectric/Piezocomposite materials methods for wing structure of the flexible wing.

Smart devices and structures are gaining more attention to enhance the synthetic or specific performances of systems due to the stricter standards being set for industrial equipment, especially in the aerospace fields. Piezoelectric materials are used in structural shape control primarily for two reasons: first, to rectify shape errors in the structure caused by manufacturing errors or external disturbances; and second, to offer active shape control for morphing applications with specific goals, like compliant morphing wings and mechanically reconfigurable reflectors (MRRs) [11]. Through the use of an active control technique, active structures coupled with piezoelectric materials can alter their shape or profile to improve the accuracy and adaptability of the system.

Smart composite materials made of piezoelectric fibers may enhance the flexible wings' aerodynamic characteristics and aeroelastic responses. According to [12] studies, fluid-structure interaction influences the piezocomposite actuator's optimal placement as well as the PZT fiber orientation. Aeroelastic control provides more controllability to the torsional modes than it does to the pure structural vibration control of flexible wings, and as flight speed increases, so do the ideal fiber orientations. Moreover, the development of an integrated design method for piezo-

actuated wings considered both actuator optimization and aeroelastic tailoring of the base wing. The findings [13] show that, with a set thickness, more layers, a smaller incremental angle, more actuators, and a lower flutter speed constraint can all improve control ability on lift change.

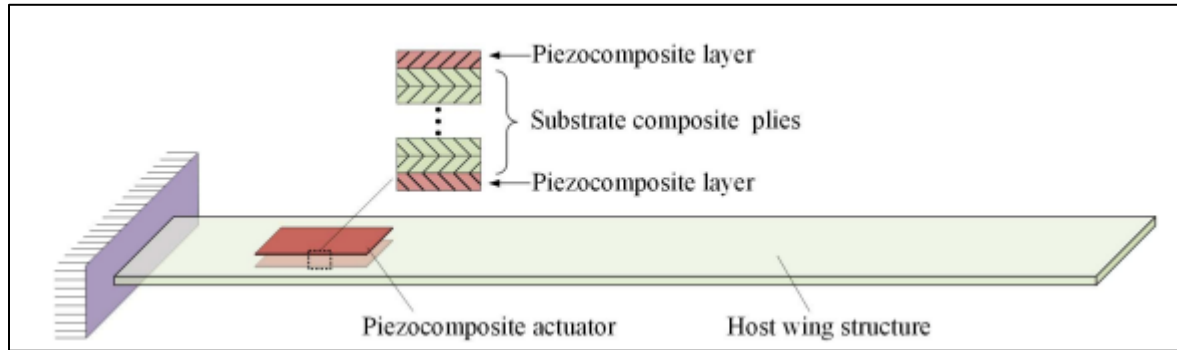


Figure 2. The surface-bonded piezocomposite actuator in the composite wing [13].

Piezoelectric materials, such as ceramic composites, crystals, and polymers, exhibit elongation when the direction of the electric field aligns with the direction of polarization, and contraction when the opposite occurs [14]. According to research by [15], a shape-memory alloy integrated into the reinforcing textiles and fiber-reinforced plastics (FRP) have been used to create an adaptive morphing wing that has been shown to deform the adaptive system. The peak of the morphing wing happened at a current flow of 1 A and 60 seconds of thermally induced Shape Memory Alloy (SMA) activation. The research by [16] reported an experimental validation of a closed-loop iterative shape control technique based on the influence coefficient matrix for a PZT-actuated CFRP reflector. Piezoelectric actuators can be used to mechanically reconfigurable reflectors, which can actively rearrange their surface to suit a purpose, such as altering the service coverage, in addition to correcting shape error.

Furthermore, piezoelectric actuators are used in the research of [17] to investigate the control of a Thin-Walled Beam (TWB) wing-engine system in order to improve aeroelastic response performance. Employing piezocomposite actuators in a close-loop active control strategy, the results successfully reduce aeroelastic instabilities in the wing-engine system, proving

the superiority of the active control approach over the passive one. Furthermore, the outcomes show that actuator position and fiber angles have a significant impact on the closed-loop active control's performance. As a result of the large value of the longitudinal piezoelectric constant, the Micro-Fiber composite (MFC) piezoelectric materials maintain the advantages of the previous active fiber composite (AFC), producing large longitudinal deformation with reasonable voltage [18]. Additionally, the MFC features a uniform and repeatable fabrication process.

Thus, a set of electro-piezo-aeroelastic lumped parameter governing equations in terms of wing tip variables can be used to represent the system [19]. A wing's assumed modes, each of which is linked to a unique set of fluid effects and electro-piezo coupling, are used to quantify the continuous motion of the wing. The author in [20] investigated the use of implanted piezoelectric materials to control wing vibration. Flutter suppression is examined, along with the optimal pre-placed piezoelectric device arrangement that maximizes harvesting yield while minimizing state and control costs. A closed loop controller was implemented for the feedback control of the optimally configured piezoelectric actuators, minimizing the state and control cost function.

2.2 Equivalent plate/beam analysis.

2.2.1 Equivalent Plate Modeling in Wing Structures and Aircraft Design.

The research by [21] presents an equivalent plate approach for wing-box structures, employing the doublet-lattice method (DLM) complements this structural model to assess unsteady aerodynamic loads and flutter characteristics, making it suitable for analyzing thick, anisotropic plates in aircraft. By utilizing the first-order shear deformation theory (FSDT), the model integrates component-mode synthesis to reduce computational complexity while accurately

capturing aeroelastic interactions between the wing and control surface [22]. Furthermore, using the Ritz method and a penalty function, they maintain displacement compatibility across segments. This approach allows for efficient and accurate structural dynamics and flutter analysis in early design stages by treating multi-segment wings as a composite of plate elements [23].

Moreover, the research made by [24] presents a framework for HALE aircraft where the fuselage and wings are represented as beams. This approach couples beam structural models with aerodynamic calculations, allowing for large-deformation analysis in slender-wing configurations. Equivalent plate theory enables the modelling of such flexible structures with fewer computational resources, facilitating stability analysis and aeroelastic tailoring in early design stages. Besides, the composite plate wings are optimized for aeroelastic stability using equivalent plate models [25]. The study focuses on fibre orientation and layer thickness to enhance divergence speed without mass penalty. By employing a sequential quadratic programming algorithm, the research demonstrates significant stability improvements through optimal composite configurations.

Likewise, the study done by [25] is investigating the aeroelastic behaviour of composite wings, emphasizing how important fiber orientation is in determining flutter and divergence. By using comparable plate theory, they demonstrate that fiber orientation has a greater influence on aeroelastic stability than thickness, which has a smaller effect. By improving the design of composite layers, this crucial realization not only advances our knowledge but also opens the door to increased stability under aerodynamic loads. Vortex lattice methods (VLM) are used to compute the loads and anticipate aerodynamic forces [26]. These forces are then applied throughout the fuselage and wing structure to simulate realistic maneuver and gust responses. By simulating bending and shear responses, the equivalent plate theory helps to simplify these intricate structures by considering their constituent parts as planar elements with customized stiffness distributions.

Understanding structural deformation under different loads requires the use of equivalent plate and beam analysis. These techniques, which are mostly applied in engineering fields, offer computational efficiency by keeping important mechanical behaviors while converting intricate 3D structures to 2D or 1D representations.

The Euler-Bernoulli beam theory [27] yields the following equations for a cantilever beam with length, L , Young's modulus, E , cross-sectional moment of inertia, I under a uniform load, w and $y(x)$ is the deflection of the beam at a point x along its length

$$EI \frac{d^2y}{dx^4} = w \quad (1)$$

Since the Euler-Bernoulli beam theory governs the beam, we begin with the fourth-order differential equation and solve for the constants by integrating it and adding boundary conditions to the cantilever beam. With the maximum deflection of the beam at the free end y_{max} at $x=L$.

$$y_{max} = \frac{wx^4}{8EI} \quad (2)$$

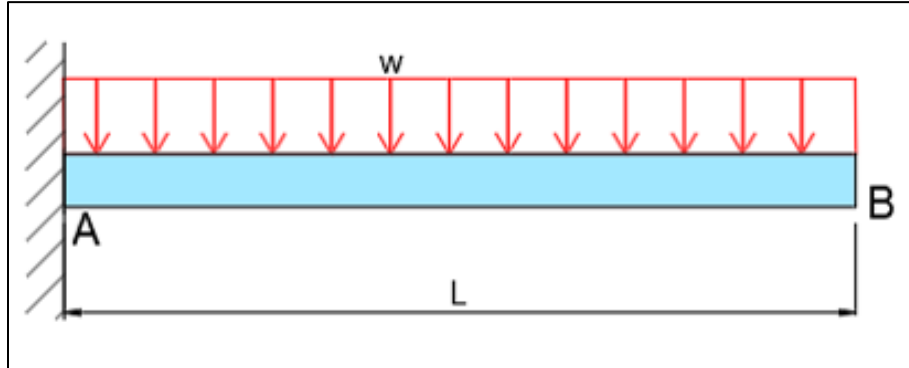


Figure 3. Uniform Distributed Load for the cantilever beam.

The uniform distributed load (UDL) method plays a role in figuring out the distributions of stress, strain energy, and deflection along the wing's length. In order to forecast aeroelastic performance, the governing differential equation for bending under a UDL takes into account the moment of inertia, or I , and material stiffness, or Young's modulus, or E . Equation (14) provides

the maximum deflection at the free end of the beam, which is derived from classical beam theory. This shows how displacement increases with applied force and wing length. Similar UDL-based aeroelastic models have been used in studies like [27] to assess the structural response of flexible wings that include piezoelectric actuators incorporated for active vibration control. According to this research, simulating aerodynamic loads as UDL offers important information about the bending stiffness, flutter properties, and general structural integrity of the wing.

Moreover, using a UDL approach complies with energy-based analytical techniques that are frequently employed in aeroelastic studies. The dynamic interplay between aerodynamic forces and structural deformations can be quantified with the use of energy principles like Hamilton's principle. When including smart materials like piezoelectric actuators, which actively reduce wing vibrations, this method is especially helpful. According to the study of [28], flexible wings exposed to UDL undergo notable nonlinear deformations, which the stiffness matrix analysis of HALE aircraft wings needs to take into consideration. In the direction of accurately capturing the aeroelastic behavior of flexible wings, optimize actuator location, and enhance the stability of HALE UAS in long-duration, high-altitude missions, it is imperative that a uniformly distributed load be used in this study.

An approach that is frequently used to solve issues involving geometric nonlinearities is the Galerkin method, which weakens the governing equations of motion [29]. This approach approximates the displacement field using a number of basic functions that meet boundary constraints, converting the governing equations into a system of algebraic equations [30]. The governing differential equation for bending moments and deformation, incorporating geometric nonlinearities, can be expressed as where $q(x)$ represents the external aerodynamic load distribution along the wingspan.

$$EI \frac{\partial^4 w}{\partial x^4} = q(x) - \rho A \frac{\partial^2 w}{\partial t^2} \quad (3)$$

Furthermore, when dealing with massive deformation scenarios in thin wing-like structures, the Lagrangian framework is crucial [31]. A correct depiction of strain distributions under aerodynamic loading is ensured by this method, which represents the interplay between axial and transverse displacements. Both geometric and material nonlinearities are taken into account by the governing equation in this framework, which is written where (N) represents the axial force, introducing geometric nonlinearity through coupling between axial and transverse motions.

$$EI \frac{\partial^4 w}{\partial x^4} + N \frac{\partial^2 w}{\partial x^2} = q(x) - \rho A \frac{\partial^2 w}{\partial t^2} \quad (4)$$

The Lagrangian technique is especially useful for investigating the aeroelastic behavior of wings with significant deflections since it captures both static and dynamic effects. This technique makes it possible to accurately forecast flutter, divergence, and other aeroelastic instabilities by integrating these nonlinearities with aerodynamic forces.

2.2.2 Energy Based Analysis in Wing Structures and Aircraft Design.

Energy-based analysis, a critical methodology for studying the aeroelastic behaviour of wing structures, holds the potential to significantly shape the future of aeroelasticity. Recent advancements in computational models have integrated energy principles into high-fidelity simulations of wing structures. The research made by [27] explored the coupling of structural and aerodynamic forces using energy-based methods to predict flutter and divergence. They demonstrated that energy conservation provides a robust basis for understanding fluid-structure interactions, ensuring the theoretical underpinnings of the models are secure and reliable.

The study conducted by [32] applied the Carrera Unified Formulation (CUF) to analyze composite wings. By leveraging energy principles, their approach incorporated linear and nonlinear effects, enabling precise modal frequencies and dynamic response predictions. This study demonstrated the high level of precision that can be achieved in computational modeling, instilling confidence in the reliability of the models.

Energy-based analysis is particularly advantageous in optimizing wing designs. The findings been made by [33] utilized energy principles within equivalent plate models (EPM) to study composite wings under dynamic loads. Their methodology efficiently predicted natural frequencies and stress-strain relationships, reducing computational costs while maintaining accuracy. Similarly, research done by [34] combined plate theories with energy methods to analyse cantilevered wings, highlighting the role of energy-based approaches in preliminary design optimization.

Furthermore, in numerical studies, modal superposition simplifies the dynamic analysis of complex structures by reducing the degrees of freedom. For instance, the response of a wing subjected to gust loads can be efficiently analyzed using low-order modes while retaining high accuracy. The findings been made by [35] highlighted the effectiveness of this method in modeling flutter and dynamic responses of wings. This study adopts a similar approach, integrating modal analysis with finite element modeling (FEM) to compute eigenfrequencies and dynamic displacements.

The foundation of energy-based analysis lies in the conservation of energy, which dictates that the total energy of a system, comprising the kinetic energy (T), potential energy (U), and external work (W) which remains constant. This equation states that the time integral of the difference between kinetic energy and potential energy, plus the work done by external forces, remains

stationary over the time interval $[t_1, t_2]$. Using Hamilton's principle, the governing equation can be expressed as:

$$\int_{t_1}^{t_2} (T - U - W) dt = 0 \quad (5)$$

The kinetic energy accounts for the inertia of the system and its resistance to motion. For flexible wings, $\dot{u}$ includes both translational and rotational velocities due to deformation. The kinetic energy of a vibrating or moving structure where volume of the structure (V), the density of the material (ρ) and the velocity of deformation ($\dot{u}$) is expressed as:

$$T = \frac{1}{2} \int_V \rho \dot{u}^2 dV \quad (6)$$

Moreover, the relationship between stress and strain follows material properties such as Young's modulus (E) and Poisson's ratio (ν) through Hooke's Law. In addition to the stress (σ) and strain (ϵ) tensors, the volume of the structure provides the potential energy stored in it because of elastic deformation of the structure.

$$U = \frac{1}{2} \int_V \sigma \epsilon dV \quad (7)$$

Moreover, the external work applied to the structure due to aerodynamic forces, piezoelectric actuation, or boundary conditions captures the interaction of external forces with the structural deformation is expressed as where external force like aerodynamic loads (f) with displacement field (u):

$$W = \frac{1}{2} \int_V f u dV \quad (8)$$

Additionally, the work done by the piezoelectric actuator is added to the external work term (W), modifying the stiffness and force terms in the governing equations. As for piezoelectric systems, additional terms are introduced to model the electro-mechanical coupling like electric

field vector (E), piezoelectric constant (d) and energy stored due to piezoelectric deformation (U_{piezo}).

$$U_{piezo} = \frac{1}{2} \int_V E^T dE dV \quad (9)$$

For this study, equation for aeroelastic optimization of the nonlinear flexible wing for HALE UAS, the energy-based equation accounts for structural deformation, piezoelectric actuation, and aerodynamic loads of the flexible wings itself.

$$\int_{t_1}^{t_2} (T - U + W_{aero} + W_{piezo}) dt = 0 \quad (10)$$

The work done by aerodynamic forces takes into consideration how aerodynamic forces affect wing motion and deformation. The relationship between airflow and the wing's surface produces aerodynamic forces, which affect the structural dynamics. It stands for the energy transfer that occurs between the structure and the airflow. It assists in determining whether the aerodynamic energy input causes the wing to become unstable in flutter analysis.

The governing equation of the work done by aerodynamic forces where $q(x,t)$ as aerodynamic pressure distribution as a function of spatial position (x) and time (t) while $u(x,t)$ as displacement field of the wing, and V as volume of the wing structure.

$$W_{aero} = \int_V q(x, t) \cdot u(x, t) dV \quad (11)$$

Moreover, work done by piezoelectric actuator captures the work done by piezoelectric actuators embedded in the wing structure to actively control deformations and vibrations. Piezoelectric actuators generate stress and strain in response to applied electric fields, which can be harnessed for structural control. It denotes the energy supplied by actuators to offset deformations generated by aerodynamic forces or other external loads. This term is critical for vibration suppression and stability enhancement.

The expression that can portray the equation of the work done by piezoelectric where as $\sigma_{piezo}(x,t)$ as a stress induced by piezoelectric actuators, $\epsilon(x,t)$ as strain distribution in the structure, and V as the volume of the piezoelectric material.

$$W_{piezo} = \int_V \sigma_{piezo}(x, t) \cdot \epsilon(x, t) dV \quad (12)$$

The piezoelectric effect introduces electromechanical coupling, enabling the actuators to generate stress and control deformations dynamically. The applied voltage V and the placement of the piezoelectric patches determine the magnitude and distribution of the piezoelectric actuator work done.

Consecutively, by applying the piezoelectric constitutive equations where e as piezoelectric coupling constant, E as electric field applied to the piezoelectric material and c_E as elastic stiffness under a constant electric field. The work done by piezoelectric actuator with the application of the piezoelectric constitutive equations has been extracted in:

$$\sigma_{piezo} = eE, \epsilon = \frac{\sigma}{c_E} \quad (13)$$

$$W_{piezo} = \int_V \frac{eE}{c_E} \cdot E dV \quad (14)$$

2.2.3 Flutter speed and Natural Frequency in Wing Structures and Aircraft Design.

The natural frequencies of these wings can be precisely determined using Euler-Bernoulli beam theory, a foundational principle that accurately describes the transverse vibrations of slender, beam-like structures. This knowledge is essential for optimizing performance and ensuring safety in aviation applications.

The governing differential equation based on Hamilton's principle is:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \quad (15)$$

where is EI the flexural rigidity of the beam, ρ is the material density, A is the cross-sectional area, and $w(x, t)$ is the transverse displacement of the wing. Solving the corresponding eigenvalue problem for a cantilever beam leads to the expression for the natural frequency of the n -th mode.

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho AL^4}} \quad (16)$$

From the equation (16), β_n is a mode-dependent constant while L is the length of the beam. This classical formulation has been widely adopted in the aeroelastic analysis of the flexible Hale wing structures [21], [22].

Flutter is a critical dynamic instability that arises from the complex interplay between unsteady aerodynamic loads and the structural response of an aircraft wing. To accurately estimate flutter speed, this study employs two robust approaches: the frequency-domain Theodorsen method and a cutting-edge time-domain unsteady aerodynamic model. By leveraging these methods, this research study aim to provide a comprehensive understanding of flutter dynamics, enhancing safety and performance in aircraft design.

From the Theodorsen method, the systems is modeled as a two-degree-of-freedom (2-DOF) airfoil undergoing pitch and plunge oscillations. The unsteady lift is expressed in terms of Theodorsen function $C(k)$, which accounts for circulatory effects due to oscillating motion. Under simplified assumptions for torsional flutter, the flutter speed can be approximated analytically as:

$$U_f = \sqrt{\frac{K_\alpha I_\alpha}{\rho b^2 M_\alpha}} \quad (17)$$

Where U_f is the flutter speed, K_α is the torsional stiffness of the wing, I_α is the mass moment of inertia in pitch, ρ is the air density, b is the semichord length and M_α is the aerodynamic moment coefficient. This expression provides a baseline analytical estimate for flutter onset, particularly useful in preliminary design studies.

Furthermore, for the time-domain unsteady aerodynamic model, by contrast, captures transient fluid-structure interactions and nonlinear effects. In this method approach, the motion of the flexible wing in plunge (h) and pitch (α) is governed by coupled second-order differential equations:

$$m\ddot{h} + S_\alpha\ddot{\alpha} + K_h h = L(h, \alpha, \dot{h}, \dot{\alpha}, t) \quad (18)$$

$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha \alpha = M(h, \alpha, \dot{h}, \dot{\alpha}, t) \quad (19)$$

Where, m is the mass per unit span, S_α is the static moment, K_h and K_α are structural stiffness values in plunge and pitch, respectively while L and M represent the aerodynamic lift and moment forces which both are time-dependent. This method allows for more accurate prediction of flutter under low Reynolds number and nonlinear geometric conditions that are commonly encountered in HALE platforms [17].

When piezoelectric actuators are embedded in the wing structure, it provide additional stiffness through the electromechanical coupling on the systems. This enhancement modifies the total effective torsional stiffness as:

$$K_\alpha^{eff} = K_\alpha + K_\alpha^{piezo} \quad (20)$$

While the piezoelectric contribution is expressed as:

$$K_\alpha^{piezo} = \frac{e^2}{c_E} A_p V \quad (21)$$

From the piezoelectric contribution formulation, e is the piezoelectric coupling constant, c_E is the elastic modulus at constant electric field, A_p is the patch area actuator while V is the applied voltage on the actuator. Incorporating to this equation into the flutter speed equation modifies the Theodorsen-based expression as follows:

$$U_f = \sqrt{\frac{(K_\alpha + \frac{e^2 A_p V}{c_E}) I_\alpha}{\rho b^2 M_\alpha}} \quad (22)$$

This equation demonstrates that by controlling the applied voltage into the piezoelectric actuator input embedded on the wing structures, the flutter speed can be actively increased, improving the stability margins of the flight. For the case of Unsteady Aerodynamic model, the flutter speed must be computed numerically by solving the time-domain equations of motions with the updated K_α^{eff} , since no closed-form solution exists for this method.

From both method approaches, both method clearly shows that piezoelectric actuation has a significant impact on flutter suppression on the wing structures system. Recent studies from [12] and [17] validate that optimizing the actuator placement and voltage can lead to substantial gains in flutter margins for flexible wings.

CHAPTER 3

METHODOLOGY

An aeroelastic model for High-Altitude Long-Endurance (HALE) wings with piezoelectric actuators was developed, analyzed, and validated using appropriate methodology. Model creation, analytical strategies, and simulation techniques are all parts of the process, which aims to accomplish the objectives of the study.

3.1 Model Development of the Flexible Wing.

In this study, the initial stage is to build an analytical model that forecasts the aeroelastic response of HALE wings at low speeds and high altitude flight conditions. Since HALE wings have low natural frequencies and high flexibility, this model takes into consideration nonlinear geometric deformations that have a big impact on aeroelastic behavior. Piezoelectric components, which are specifically made to allow active control by modifying the wing's structural characteristics in reaction to aerodynamic loads, are incorporated into the model.

The wings' structural response will be simulated using flat-plate beam analysis, which will serve as a basis for evaluating both their static and dynamic properties. The geometry of the beam has been constructed in COMSOL Multiphysics with the dimensions of the length of 1 m, height of 0.02 m and width of 0.08 m.

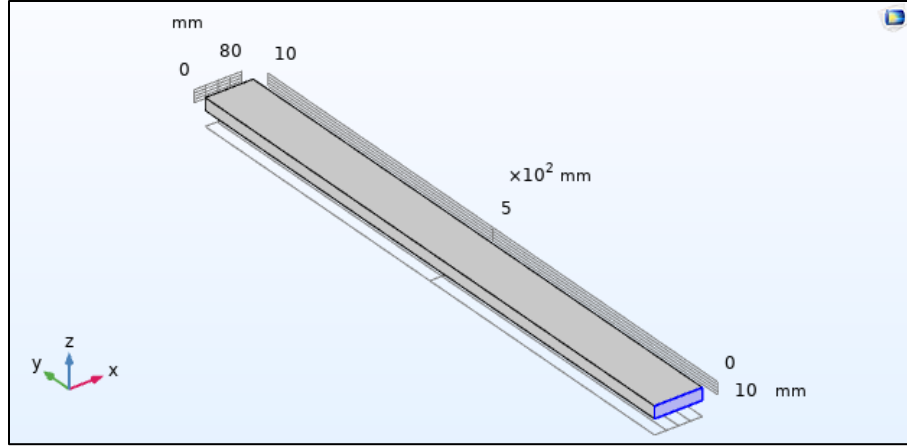


Figure 4. The geometry of the flat-plate beam for the flexible wing.

The simulation was conducted with a fixed end and a free end configuration, using an Aluminum 6061 T6 material for the wing equipped with piezoelectric actuators. The environmental conditions were set to sea-level pressure at 101325 Pa and an altitude of 20000 feet, where the air pressure is reduced to 46563.26 Pa.

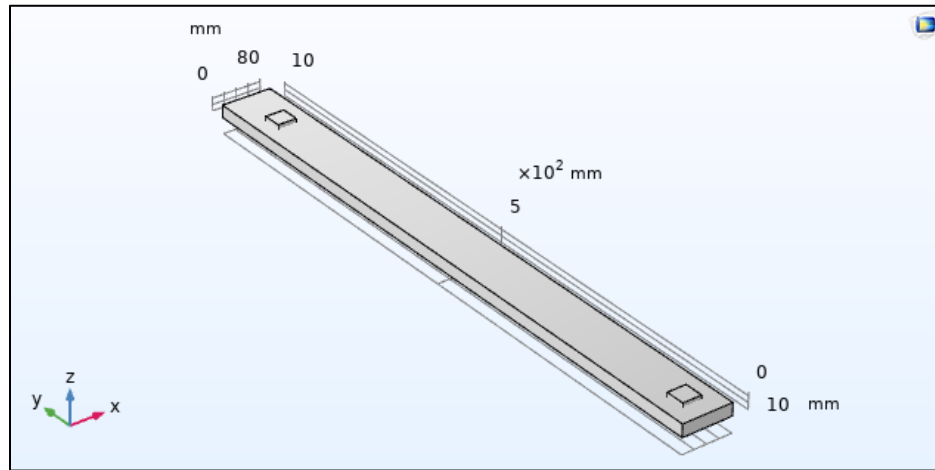


Figure 5. The geometry of the flat-plate beam with the piezoelectric actuators equipped on the flexible wing.

3.2 Methodical approach of the study.

The governing equations for the flexible wing's aeroelastic behavior are derived using an energy-based approach. This method allows the study to examine both static and dynamic

aeroelastic properties, including vibrational responses, deflection, and torsion along with the aeroelastic response which are the natural frequencies and flutter characteristics of the wings. A thorough examination of the interactions between piezoelectric actuators and the wing structure, including the impact of various actuator locations, patch areas, the actuator types used and the applied voltage of the actuator on the wing's plate or beam, along with the aeroelastic response of the wing are made possible by this analytical model. To ensure flight stability, the energy-based model will compute crucial aeroelastic characteristics such as torsional modes and deflection magnitudes.

3.2.1. Energy-Based Formulation and Aeroelastic Derivation.

To thoroughly investigate the aeroelastic behavior of a flexible wing integrated with cutting-edge piezoelectric actuators, we adopt a robust energy-based approach. This section rigorously derives the governing equations, starting from Hamilton's Principle, and progresses to a comprehensive evaluation of both natural frequency and flutter speed. We utilize advanced frequency-domain and time-domain aerodynamic models to ensure a detailed and accurate analysis.

Hamilton's Principle states that the actual motion of a system between time t_1 and t_2 and makes the action integral stationary.

$$\int_{t_1}^{t_2} (T - U + W_{aero} + W_{piezo}) dt = 0 \quad (23)$$

Where T is the kinetic energy of the systems, U is the strain energy while W_{aero} is the work done by the aerodynamic forces and W_{piezo} is the work done by the piezoelectric actuation.

For a beam of length L , the kinetic energy is derived where ρ is the density while A is the cross sectional of the wing along with $w(x, t)$ which is the transverse displacement on the systems.

$$T = \frac{1}{2} \int_0^L \rho A \left(\frac{\delta w}{\delta t} \right)^2 dx \quad (25)$$

Under the Euler-Bernoulli beam theory, the strain energy due to bending is derived with E as the young modulus of the materials and I as the moment of inertia of the wings.

$$U = \frac{1}{2} \int_0^L EI \left(\frac{\delta^2 w}{\delta t^2} \right)^2 dx \quad (26)$$

Piezoelectric actuators apply a moment due to the electric field as shown where is the moment induced by actuator due to voltage input.

$$W_{piezo} = \int_0^L M_{piezo}(x) \frac{\delta^2 w}{\delta t^2} dx \quad (27)$$

The aerodynamic load work is expressed as where $q(x, t)$ is the unsteady aerodynamic pressure distribution.

$$W_{piezo} = \int_0^L q(x, t) w(x, t) dx \quad (28)$$

From all terms of the energy expression, all terms been substituted into the Hamilton's Principle and apply the calculus of variations. This is the governing differential equation for the aeroelastic beam with piezoelectric actuation.

$$\rho A \left(\frac{\delta w}{\delta t} \right)^2 + EI \left(\frac{\delta^2 w}{\delta t^2} \right)^2 = q(x, t) + f_{piezo}(x, t) \quad (29)$$

Then, the model been assumed as a solution in modal coordinates which the Galerkin method been applied in converting the partial differentiation equation (PDE) into a system of ordinary differentials equations (ODE) in terms of mode shapes and time-varying modal amplitudes , $q_i(t)$.

$$w(x, t) = \sum_{i=1}^n \phi_i(x) q_i(t) \quad (30)$$

In deriving the natural frequency on the wing systems, the governing equation of the beam from Hamilton's principles which gives the natural frequency of the i-th mode.

$$\rho A \ddot{q}_i(t) + EI k_i^4 q_i(t) = 0 \Rightarrow \omega_i = \left(\frac{EI}{\rho A} \right)^{1/2} k_i^2 \quad (31)$$

For flutter analysis, we run to method approaches for this which are the Theodorsen Method and Unsteady Aerodynamic Model.

The Theodorsen approach models unsteady aerodynamic loads using harmonic assumption.

$$L(t) = \pi \rho b^2 (\ddot{h} + V \dot{\alpha} - b \ddot{\alpha}) + 2\pi \rho V b C(k) (\alpha + \frac{\dot{h}}{V} + b (\frac{1}{2} - a) \frac{\dot{\alpha}}{V}) \quad (32)$$

The 2-DOF airfoil model motion of the flexible wing in plunge (h) and pitch (α) is governed by coupled second-order differential equations.

$$m \ddot{h} + S_\alpha \ddot{\alpha} + K_h h = L(t) \quad (33)$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha = M(t) \quad (34)$$

A simplified flutter speed approximation for torsional flutter mode can be expressed as:

$$U_f = \sqrt{\frac{K_\alpha}{\rho b^2 a C(k)}} \quad (35)$$

Where K_α is the torsional stiffness, $C(k)$ is the Theodorsen function which depends on reduced frequency, b is the semichord and a is the distance from mid chord to elastic axis.

For the Unsteady Aerodynamic Model it is the time domain using Wagner's function and Duhamel's integral which allows simulation of transient aeroelastic behavior.

$$L(t) = 2\pi \rho V b [\alpha(t) + \frac{1}{\pi} \int_0^t \alpha(\tau) \Phi(t - \tau) d\tau] \quad (36)$$

The integration of of piezoelectric control which give the equation of the piezoelectric actuation torque.

$$M_{piezo}(t) = e V A_p \quad (37)$$

Which incorporated with the state space model.

$$\dot{X} = A x + B u \quad (38)$$

Where X includes structural and aerodynamic states and u is the control input which is the applied voltage on the piezoelectric actuator that embedded on the wing structures. This formulation provides a comprehensive framework to model, simulate, and optimize flutter characteristics through control of natural frequencies and piezoelectric actuation.

3.3 Simulation using computer simulation software.

MATLAB will be used to verify the analytical model and investigate different setups on the flexible wing. To optimize the aeroelastic response, it will be utilized in this study to simulate various actuator locations, different piezoelectric actuator type, different patch areas with the applied voltage on the plate/beam. These simulations will yield quantitative and visual data to validate the analytical model's accuracy and improve the wing structure's design.

3.4 Research outcomes.

The research outcomes presented here reflect both the strengths and limitations encountered throughout the study. While many results aligned with the intended objectives, several outcomes indicated the need for further refinement to achieve full validation. Nevertheless, the study successfully identified critical trends, offered valuable insights, and established a foundational framework for future aeroelastic optimization in flexible HALE wings.

3.4.1. Enhanced aeroelastic stability of the HALE aircraft.

The study is expected to demonstrate that integrating piezoelectric actuators into flexible HALE wings significantly enhances their aeroelastic stability. This will include reduced wing deflections, improved flutter and aeroelastic performance, and better mitigation of vibrational

instabilities under low-speed flight conditions, affirming the potential of active materials in enhancing control authority.

3.4.2. Optimized parametric study of aeroelastic characteristics and response.

The research will identify optimal configurations for piezoelectric actuator placement, the effect of the patch area and type of piezoelectric actuators with the applied voltage on the actuators on the wing including the flutter analysis on the wing systems. These configurations will maximize aeroelastic performance by providing the most effective control over structural deformation and aeroelastic instabilities. These variations underscore the complexity of the structural-electromechanical interactions involved. Nonetheless, the simulations successfully captured the sensitivity of the system to design variables, offering a useful basis for future optimization studies.

3.4.3 Validated analytical model performance.

The energy-based analytical model developed in this research is anticipated to accurately predict the critical static and dynamic aeroelastic characteristics of HALE wings with the computation of the aeroelastic response of the wing. Its predictions are expected to align closely with the results obtained from MATLAB simulations, demonstrating its reliability for practical applications. This study will contribute to a novel design methodology for a flexible wing embedded with piezoelectric elements. The framework will combine energy-based analysis and simulation techniques in guiding the integration of smart materials in future HALE wing designs.

3.4.4. Aeroelastic performance under operational conditions of HALE UAS.

This research shows that aeroelastic tailoring with piezoelectric actuators significantly improves under flight operational conditions especially the high-altitude and low-speed performance of HALE wings, enhancing flight stability and structural resilience while curtailing the risk of flutter and divergence on the wing. The insights gained could serve as a foundation for

designing a more adaptive and efficient flexible wing structure which is not only for HALE UAS but also for other lightweight and high-performance aerostructures.

3.5 Data Validation.

The validation of this research study was conducted by referencing and comparing its findings with [12]. This comparative analysis ensures the reliability of the methodologies employed and demonstrates alignment with established benchmarks in the field of aeroelasticity for flexible wings. Previous study provides detailed insights into the optimal placement, geometry, and orientation of piezoelectric actuators on flexible wings, offering a strong foundation for validating the present research objectives.

The properties of the wing and piezoelectric actuators used closely align with the parameters in this research study, as outlined in Table 1. The wing dimensions, with a length of 600 mm, width of 60 mm, and height of 2 mm, match the flexible wing model used in this research. These comparable properties establish a solid basis for evaluating the structural and aeroelastic performance of piezoelectric actuators across studies.

Table 1. Properties of the wing model and actuators.

Parameters	Wing	Actuator
Length (mm)	600	60
Width (mm)	60	40
Height (mm)	2	0.3
Young Modulus, E (GPa)	70.3	E1 = 30.34, E2 = 15.86
Poisons ratio, μ	0.3	$\mu(12)=0.31$, $\mu(21)=0.16$
Density, ρ (kg/m ³)	2700	5400
Piezoelectric constant (m/V)		$d(11)= 400 \times 10^{-12}$, $d(12)= 170 \times 10^{-12}$

The comparison also extends to actuator placement and fibre orientations across different free-stream velocities, as shown in Table 2. The visualization of the placement of the actuators and

fibre orientations with different free stream velocities is presented in Figure 5. These trends align with this study's findings that actuator placement closer to the fixed end enhances dynamic stability at higher velocities.

Table 2. Optimal locations and fibre orientations of two piezocomposite actuators.

Case	Free-stream velocity (m/s)	Actuator 1 with fibre angle	Actuator 2 with fibre angle	Objective functions, J
1	0	[0,0°]	[0.1L,-40°]	2.85×10^{-7}
2	20	[0.05L,37°]	[0.55L,35°]	2.00×10^{-9}
3	50	[0.05L,41°]	[0.575L,41°]	9.32×10^{-9}
4	80	[0.05L,44°]	[0.575L,44°]	7.61×10^{-8}

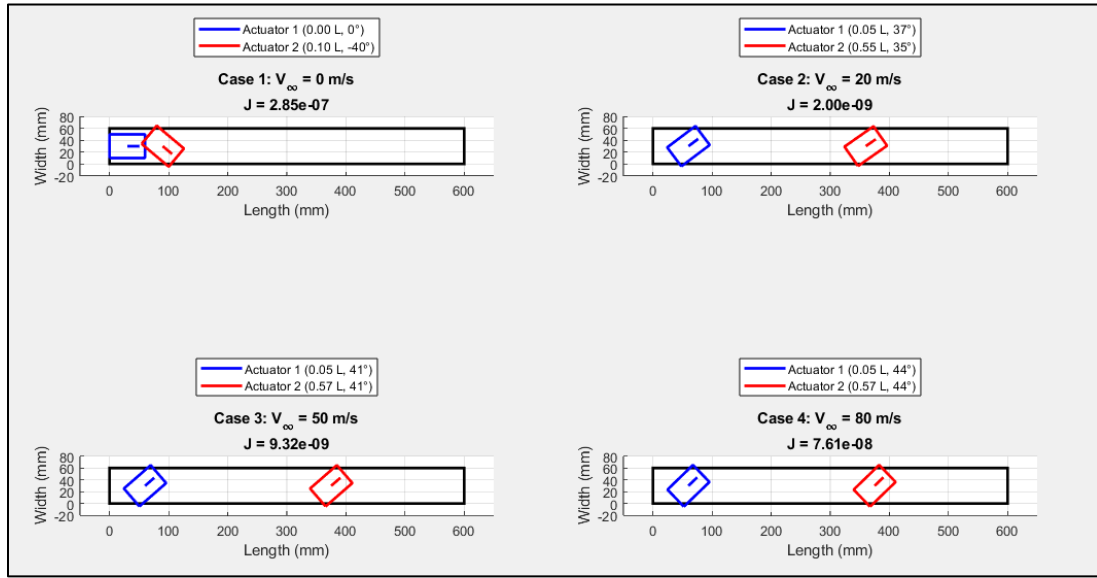


Figure 6. The optimal locations and fiber orientations of two piezocomposite actuators.

The performance trends, illustrated in Figure 7, further validate this study's findings. Modal frequency decreases slightly with increasing velocity, highlighting the destabilizing effect of aerodynamic forces. Meanwhile, deflection and strain increase significantly, reinforcing the need for optimized piezoelectric actuators to mitigate structural deformations and improve aeroelastic stability.

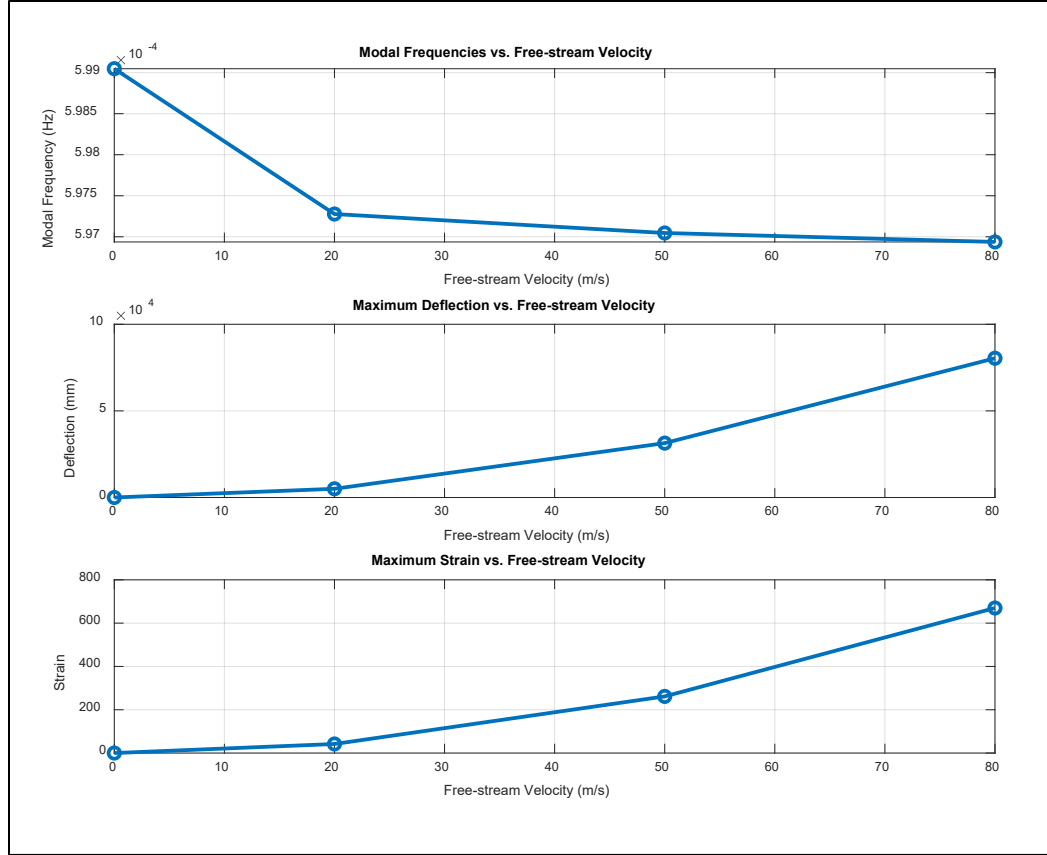


Figure 7. The performance of the base wing with the effect of free streams velocity.

The validation of this research against Zhou et al. (2019) confirms the alignment of methods and results, particularly in optimizing actuator placement and fibre orientations for enhanced aeroelastic performance. The comparable wing properties, actuator configurations, and performance trends underscore the reliability of this study's approach.

3.5.1. Data validation with the proposed study for eigenfrequencies and corresponding dynamic displacement.

In validating the effectiveness of the proposed actuator square patch area configurations, the simulation has been conducted comparison between the research results and those reported by Zhou et al. (2019). Table 3 summarizes the eigenfrequencies and corresponding dynamic displacements, as well as the percentage differences in displacement across similar frequency

ranges. The comparison indicates that the configurations in our study consistently produce higher displacements at comparable frequencies, particularly in the mid-frequency range relevant to flexible High Altitude Long Endurance (HALE) wing applications.

Table 3. Displacement Comparison at Matched Eigenfrequencies between the proposed study and recent study from Zhou et al. (2019).

Eigenfrequency (Hz) – Proposed Study	Eigenfrequency (Hz) – Zhou et. al (2019) Study	Displacement (m) – Proposed Study	Displacement (m) – Zhou et. al (2019) Study	Difference percentage in displacement (%)
16.661	16.721	0.184	0.138	+33.33
16.763	16.785	0.109	0.0909	+19.91
16.882	16.838	0.0306	0.00552	+454.35
17.014	16.912	0.000887	0.00161	-44.91

From the table of the comparison, this can be observed that the displacement of the proposed study was 0.184 m compared to Zhou's displacement at 0.138m, it's representing a 33.33% increase. Similarly, are the displacement for the proposed study of the displacement at 0.109 m and 0.0306 m which have increase the percentage of the displacement which show significant improvement occurred on the wing systems. However, the exception on the displacements of 0.0000887 m for proposed study dropped slightly below Zhou's which is 0.00161 m showing a -44.91% difference, which may be attributed in reducing the influence of the actuator at higher stiffness and frequency saturation effects.

These results from the data validation and comparison support the effectiveness of the patch area actuation strategy over fibre orientation-focused approaches. This also shows that it can reinforce the optimized actuator sizing and placement can significantly enhance the aeroelastic response of flexible wings especially within the operational frequency band of interest for HALE systems.

3.5.2. Data validation with the proposed study orientation angle effect.

In assessing the consistency between different performance metrics for optimizing actuator orientation, a comparative analysis was conducted on the objective function values from Zhou et al. (2019) and the flutter speed results from the current study. While Zhou's study measures actuator effectiveness using a controllability-based objective function (denoted as J), the present study evaluates performance by analyzing the flutter speed, calculated through both the Theodorsen method and an unsteady aerodynamic model.

For comparison purposes, both Zhou's objective function values and the flutter speeds were normalized relative to their respective maximum values which are measured at a 0° orientation. This normalization allows for a consistent scale in which each metric ranges from 0 representing the worst performance to 1 representing the best performance, effectively illustrating the relative decline in performance as the actuator orientation shifts away from the optimal position.

Table 4. Orientation angle effect comparison on the flutter speed and objectives functions between the proposed study and recent study from Zhou et al. (2019).

Orientation angle (deg)	Zhou Objective Function (J)	Normalized Objective Function	Theodorsen Flutter Speed (m/s)	Unsteady Aerodynamic Flutter Speed (m/s)	Normalized Flutter Speed	Percentage Difference (Normalized)
0	1.46×10^{-7}	1.000	808.94	572.47	1.000	-
30	4.64×10^{-11}	0.000318	700.57	495.77	0.866	+27,057%
45	5.51×10^{-10}	0.003774	572.01	404.8	0.707	+18,639%
60	3.68×10^{-9}	0.025205	404.47	286.23	0.500	+1,884%
90	-	-	4.9534×10^{-4}	3.5054×10^{-4}	0	-

Table 4 compellingly illustrates a consistent trend across both studies: performance deteriorates as the orientation angle increases. At 0°, both the objective function and flutter speed

attain their peak values, showcasing the actuators' maximum effectiveness. However, Zhou's normalized objective function experiences a dramatic decline—nearly three orders of magnitude—by the time the angle reaches 30° . In stark contrast, the proposed flutter speed remains impressively stable, decreasing at a much more gradual pace. This leads to a significant disparity: at 30° , the normalized flutter speed is 0.866, while Zhou's objective function plummets to just 0.000318, resulting in a staggering 27,057% relative difference.

As the orientation angle climbs to 45° and 60° , this gap narrows but still reflects substantial differences of 18,639% and 1,884%, respectively. This observation strongly suggests that Zhou's objective function is highly sensitive to changes in orientation, while actual aeroelastic performance measured through flutter speed which demonstrates robust resilience, likely due to distributed modal effects or nonlinear structural responses.

These findings powerfully emphasize that, while Zhou's objective function serves as a valuable control-based optimization metric, it may fall short of accurately representing practical aeroelastic performance under certain conditions. The proposed model, which effectively captures the direct structural response through flutter speed, not only complements Zhou's approach but also highlights the critical need to validate theoretical control metrics against real-world physical behavior.

CHAPTER 4

RESULTS AND DISCUSSION

The findings from the analytical model study of the flat-plate beam wing and analytical analysis in MATLAB have provided promising insights into the aeroelastic behaviour and performance of HALE wings integrated with piezoelectric actuators with the parametric study of the piezoelectric actuators on the wings.

4.1 Effect of piezoelectric actuator placement on wing behavior.

The placement of piezoelectric actuators on flexible HALE wings plays a pivotal role in determining their aeroelastic response and overall structural behavior. As these actuators are responsible for actively mitigating structural deformations and instabilities, their location along the wing significantly impacts the wing's static displacement, dynamic performance, and stiffness. By studying this relationship, a series of analytical analysis of the actuator placements on the wing were conducted to evaluate how actuator placement affects the wing's behavior under both static and dynamic conditions.

Table 1 presents the relationship between the placement of the piezoelectric actuator along the beam, the varied applied voltage, and the resulting beam displacement and natural frequency using Lead Zirconate Titanate (PZT-5H) along the length of the wing using the materials of Aluminum 6061 T6.

Table 5. The effect of the piezoelectric actuator placement on the wings with applied voltage on beam displacement and natural frequency.

Case	Distance of the actuator from free end (m)	Beam Displacement (m)	Applied Voltage (V)	Natural Frequency (Hz)
1	0.9	0.155	0	132.1
		0.151	125	132.6
		0.147	250	133.1
		0.143	375	133.7
		0.139	500	134.2
2	0.785	0.157	0	132.1
		0.153	125	132.8
		0.149	250	133.6
		0.144	375	134.4
		0.140	500	135.2
3	0.685	0.158	0	132.1
		0.154	125	132.9
		0.150	250	133.8
		0.146	375	134.7
		0.142	500	135.6
4	0.585	0.160	0	132.1
		0.156	125	133.0
		0.153	250	134.0
		0.149	375	135.0
		0.145	500	136.1
5	0.485	0.162	0	132.1
		0.159	125	132.5
		0.157	250	133.0
		0.154	375	133.5
		0.150	500	134.0

The graph illustrates the effect of actuator placement on the natural frequency of a beam embedded with piezoelectric actuators under varying applied voltages. As the voltage increases, the natural frequency of the beam also rises, demonstrating the ability of piezoelectric actuators to enhance beam stiffness. Conversely, the extent of this frequency increase depends significantly on the actuator's position along the beam. Actuators placed closer to the fixed end which is at 0.485 m exhibit a higher natural frequency compared to those positioned near the free end which

positioned at 0.900 m. This is because the bending moments are more significant near the fixed end, allowing actuators to contribute more effectively to structural stiffness. Contrarywise, actuators near the free end experience less resistance, resulting in a smaller increase in natural frequency.

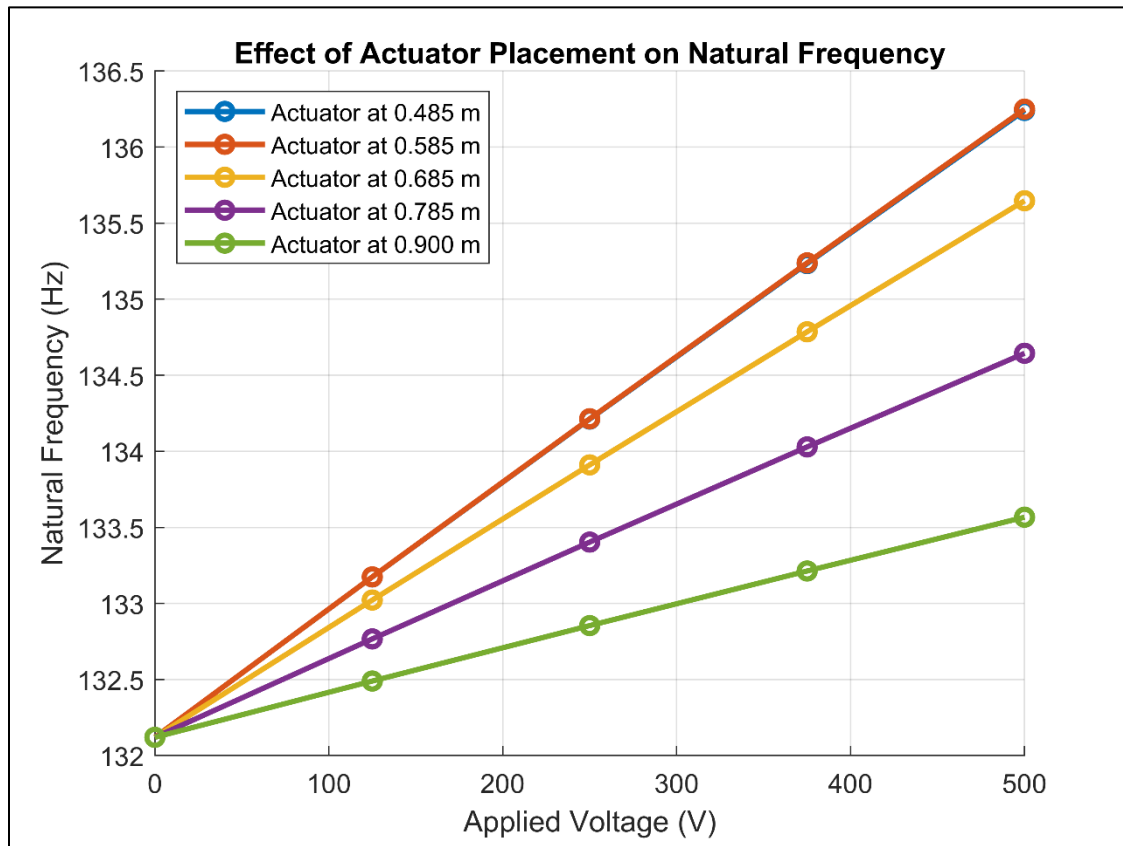


Figure 8. The graph of natural frequency against the applied voltage for the effect of actuator placement.

Additionally, the steepness of the frequency-voltage curve is more pronounced for actuators positioned closer to the fixed end, indicating a stronger influence on frequency enhancement per unit voltage. Actuators placed further along the beam, toward the free end, show a shallower slope, suggesting a weaker stiffening effect due to the reduced structural support in that region. These results emphasize that optimal actuator placement is crucial for effective frequency tuning in vibration control applications. For high-altitude long-endurance (HALE)

aircraft wings, placing actuators near the fixed end would be the preferred strategy to maximize structural rigidity and improve aeroelastic stability.

4.2 Effect of piezoelectric actuator patch area on wing behavior.

The size of the piezoelectric actuator patch significantly influences the aeroelastic performance and structural behavior of the flexible wing of HALE aircraft. In enhancing the stiffness and mitigate the structural instabilities, the larger the actuator patches with their excellent force distribution and control, the more potentially significant benefits it can present. To study the effect of the size of the actuator patch on the HALE wings, simulations were conducted for both square and rectangular patch geometries to evaluate their impact on static displacement, dynamic displacement, and eigenfrequency.

4.2.1 Effect of piezoelectric actuator square patch area on wing behavior

Table 2 exhibits the results of the relationship between the square patch size of the piezoelectric actuator on the beam, with the varied applied voltage and the resulting beam displacement and natural frequency using Lead Zirconate Titanate (PZT-5H) along the length of the wing using the materials Aluminum 6061 T6. The beam with Case 5 configuration has been used for comparison of different patch sizes of the piezoelectric actuator.

Table 6. The effect of the piezoelectric actuator square patch area on the wings with applied voltage on beam displacement and natural frequency.

Distance of the actuator from free end (m)	Beam Displacement (m)	Applied Voltage (V)	Natural Frequency (Hz)
0.000625 (25x25)	0.155	0	132.1
	0.1545	125	132.6
	0.154	250	133.1
	0.1535	375	133.6
	0.153	500	134.1

0.0009 (original) (30x30)	0.1545	0	132.2
	0.154	125	132.7
	0.1535	250	133.6
	0.153	375	133.7
	0.1525	500	135.2
0.0016 (40x40)	0.154	0	134.2
	0.1535	125	132.8
	0.153	250	133.2
	0.1525	375	133.8
	0.152	500	134.3
0.0025 (50x50)	0.1535	0	132.4
	0.153	125	132.9
	0.1525	250	133.4
	0.152	375	133.9
	0.1515	500	134.4
0.0036 (60x60)	0.153	0	132.5
	0.1525	125	133.0
	0.152	250	133.5
	0.1515	375	134.0
	0.151	500	134.5

The graph compellingly illustrates in Figure 2 show the relationship between actuator square patch size and the natural frequency of a beam across varying applied voltages. As the patch size increases, the natural frequency of the beam consistently rises, highlighting the significant role of larger piezoelectric actuator patches in enhancing structural stiffness. The greater surface area of the actuator in contact with the beam enables more effective stiffness modification when voltage is applied. Notably, the smallest patch size (0.000625 m²) produces the lowest natural frequency, while the largest patch size (0.003600 m²) achieves the highest frequency values across all voltage levels.

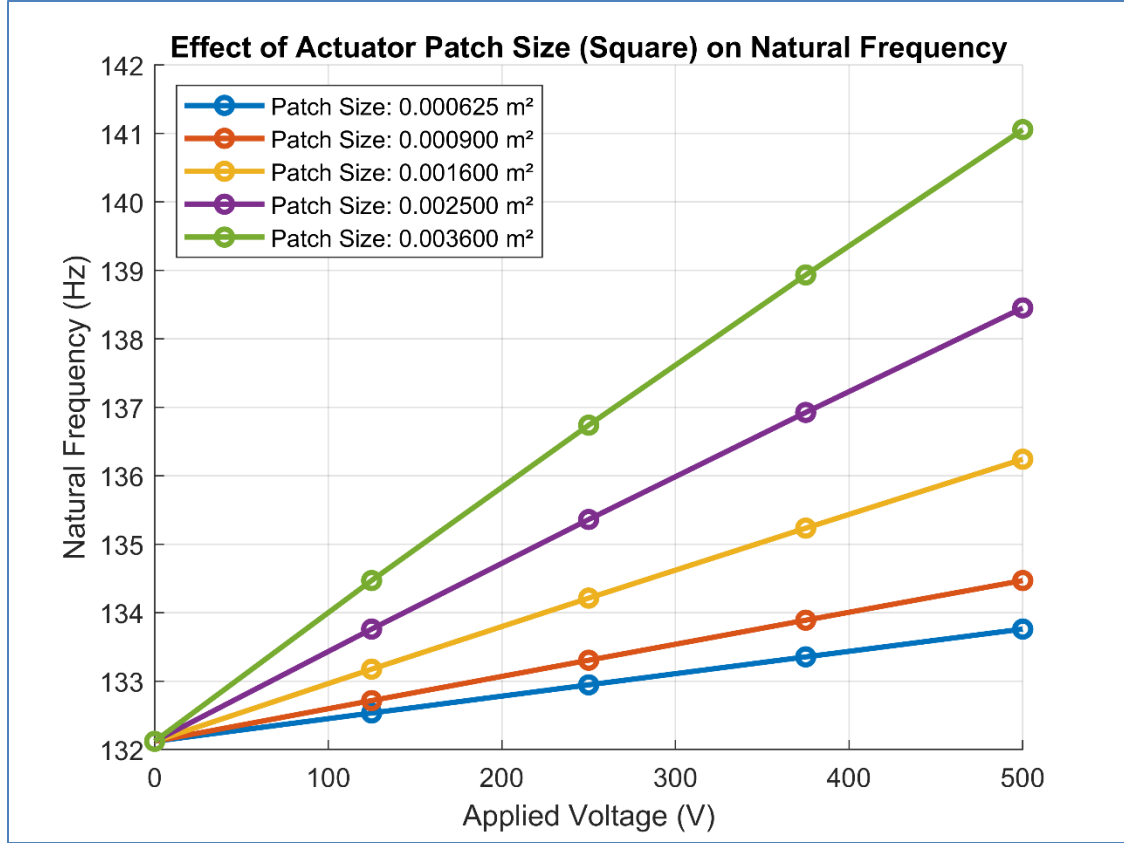


Figure 9. The graph of natural frequency against the applied voltage for the effect of square patch actuator on the beam.

Furthermore, the rate of increase in natural frequency becomes increasingly pronounced with larger patches, as indicated by the steeper slope of the frequency-voltage curves. This finding suggests that larger actuator patches provide superior frequency tuning capabilities, which are vital for effective vibration control and structural optimization, particularly in advanced applications like High-Altitude Long-Endurance (HALE) aircraft wings. By strategically selecting an optimal patch size, engineers can tailor the structural response to minimize unwanted deformations and significantly enhance aeroelastic stability.

4.2.2 Effect of piezoelectric actuator rectangular patch area on wing behavior

The findings of beam displacement and natural frequency using Lead Zirconate Titanate (PZT-5H) along the length of the wing using the materials of Aluminum 6061 T6, are shown in Table 3 illustrating the different rectangular patch area sizes using the materials of piezoelectric with the varied voltage acted on the actuator. Different rectangular patch sizes of the piezoelectric actuator have been compared using the beam with Case 5 configuration.

Table 7. The effect of the piezoelectric actuator rectangular patch area on the wings with applied voltage on beam displacement and natural frequency.

Distance of the actuator from free end (m)	Beam Displacement (m)	Applied Voltage (V)	Natural Frequency (Hz)
0.0009 (original) (30x30)	0.165	0	132.1
	0.163	125	132.5
	0.16	250	133.0
	0.158	375	133.5
	0.155	500	134.0
0.0012 (30x40)	0.165	0	132.1
	0.162	125	132.7
	0.16	250	133.2
	0.156	375	133.8
	0.154	500	134.3
0.0015 (30x50)	0.165	0	132.1
	0.161	125	132.8
	0.158	250	133.5
	0.155	375	134.1
	0.151	500	134.8
0.002 (40x50)	0.165	0	132.1
	0.161	125	132.9
	0.158	250	133.6
	0.153	375	134.4
	0.15	500	135.1
0.0024 (40x60)	0.165	0	132.1
	0.16	125	133.0
	0.15	250	133.8
	0.152	375	134.6
	0.148	500	135.4

The graph in Figure 3 illustrates how the size of a rectangular actuator patch affects the natural frequency of a beam under different applied voltages. The results show that as the patch size increases, the natural frequency also increases. This indicates that the dimensions of the actuator patch significantly influence the structural stiffness of the beam. Larger patches contribute more to enhancing stiffness, which leads to a higher natural frequency. This trend is evident across all applied voltages, with the smallest patch size (0.0009 m^2) resulting in the lowest natural frequency, while the largest patch size (0.0024 m^2) achieves the highest frequency values.

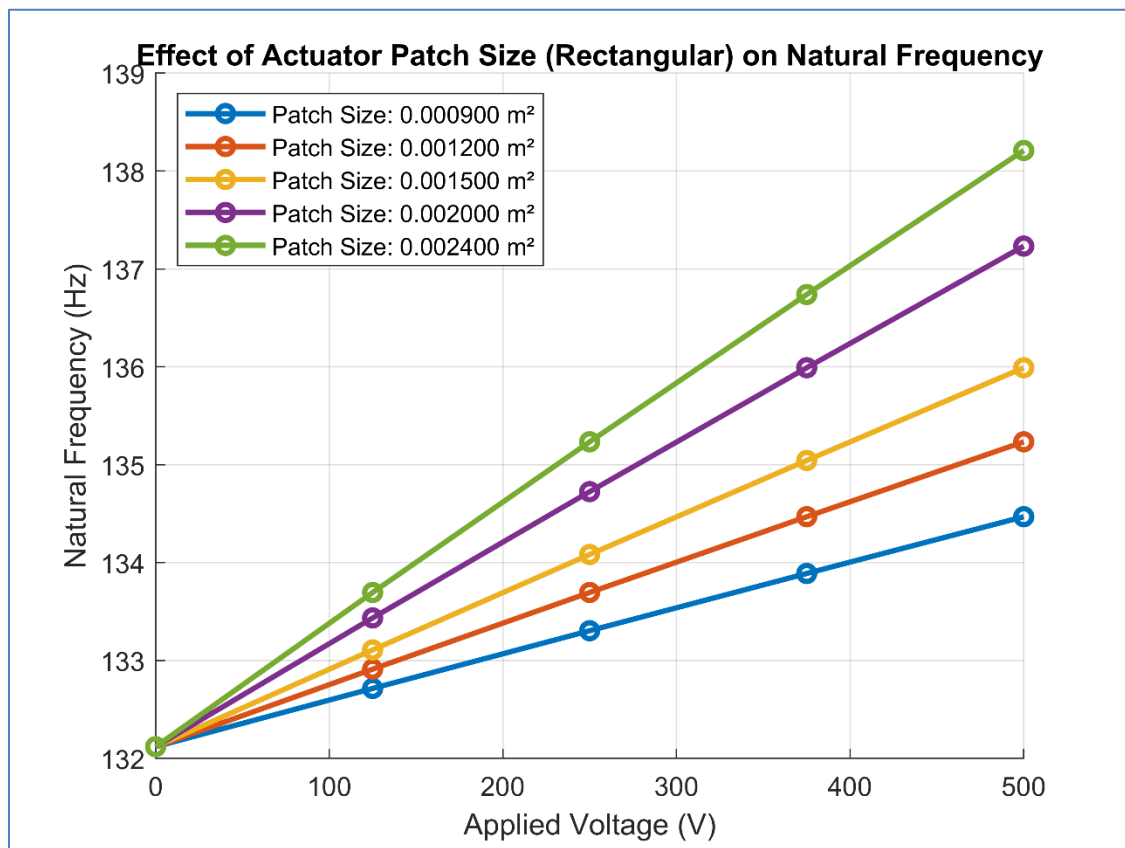


Figure 10. The graph of natural frequency against the applied voltage for the effect of rectangular patch actuator on the beam.

Furthermore, the data indicates that as the applied voltage increases, the natural frequency consistently rises for all rectangular patch sizes. However, larger patches display

a steeper rate of increase, suggesting they provide better control for frequency tuning. This characteristic is particularly advantageous in aeroelastic applications, such as High-Altitude Long-Endurance (HALE) aircraft wings, where precise frequency control can help mitigate vibrations and improve flight stability.

4.2.3 Comparison between square and rectangular actuator patch area on wing behavior.

Key distinctions between the effects of square and rectangular piezoelectric patches on natural frequency and structural stiffness are shown by this comparison. A more substantial improvement in overall stiffness is indicated by the analysis that been done, which shows that square patches have a greater increase in natural frequency than rectangular patches of comparable areas. This is because square patches effectively reduce vibrations and improve structural stability by offering consistent stiffness reinforcement along both width and length. Rectangular patches, on the other hand, show a slower increase in frequency and provide directional stiffness enhancement, which could be useful for managing deformation modes, like torsional stability in flexible constructions.

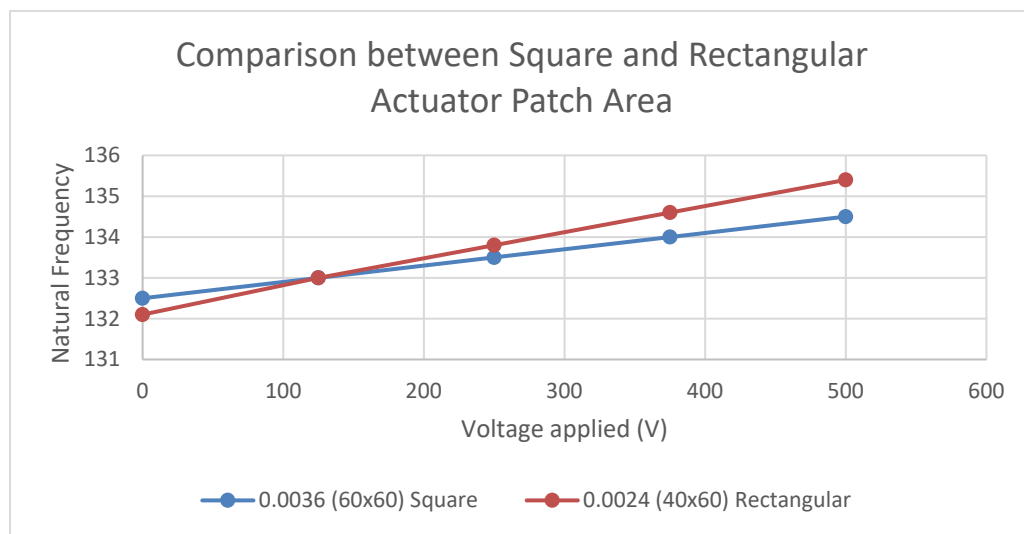


Figure 11. The comparison between square and rectangular actuator patch area.

As illustrated by the analysis and the graph accompanying it shown in Figure 11, it is quite evident that the square actuator patch (60×60 mm, 0.0036 m²) was by far better as compared to the rectangular patch (40×60 mm, 0.0024 m²) in efficiently enhancing the aeroelastic performance of High-Altitude Long Endurance (HALE) wings. The square patch consistently records higher natural frequencies at each voltage level, hence a remarkable increase in rigidity is shown. This reinforcement is very important in the efficient control of aeroelastic instabilities, such as flutter and divergence, that make it difficult to operate HALE aircraft in a high-altitude, thin atmosphere.

Additionally, the square patch achieves a balanced stiffening effect along both the width and the length of the patch and, thus, it is more effective at damping vibration and maintaining stable structural conditions. While the increased surface area of the square patches might cause some increase in the weight of the structure, the advantages of this technology in global stiffness improvement are still expected to exceed this disadvantage. On the other hand, rectangular patches are designed for local reinforcement, and they are not as efficient as they could have been for overall structural support.

The decision not to pursue further expansion of patch size was driven by critical design trade-offs. Although increasing the patch size typically enhances natural frequency, benefits tend to plateau after reaching a certain point. Beyond this threshold, any gains may be offset by added mass or complexity in integration, offering little in the way of proportional improvements in aeroelasticity.

This rationale aligns seamlessly with the optimization principles articulated by Zhou et al. (2019). They underscored the necessity of tailoring actuator configuration which including its location and fiber orientation specifically to flight conditions and fluid-

structure interaction characteristics, thereby maximizing aeroelastic control authority. While Zhou et al. did not directly investigate patch size, their findings compellingly suggest that actuator effectiveness is acutely sensitive to spatial and orientation parameters, which are intimately connected to the energy and power dynamics of actuation.

By strategically enhancing stiffness through optimal patch sizing, this study lays a robust foundation for future research aimed at dissecting the power-efficiency trade-offs inherent to actuator design. As Zhou et al. demonstrated, the pathway to maximizing control authority is paved with meticulous tuning of actuator parameters under various aerodynamic loads. Broadening this exploration to encompass power consumption—which is intrinsically influenced by actuator size and material properties—represents a natural and necessary advancement toward the comprehensive optimization of smart-actuated High-Altitude Long-Endurance (HALE) wing structures.

4.3 Effect of different types of Piezoelectric materials used.

The effect of different types of piezoelectric materials used been conducted with a comprehensive comparative analysis of four innovative piezoelectric actuator materials: Quartz LH (1949 IRE), Lead Zirconate Titanate (PZT-5H), Barium Titanate (BaTiO_3), and Aluminum Nitride (AlN) with the varied of applied voltage on the actuator patch been exhibit in Table 4. We will explore their significant impacts on beam displacement and its natural frequency generated by the actuator patches using the beam with Case 5 configuration.

Table 8. The effect of the different types of piezoelectric materials used on the wings.

Distance of the actuator from free end (m)	Beam Displacement (m)	Applied Voltage (V)	Natural Frequency (Hz)
Quartz LH (1949 IRE)	0.16	0	132.1
	0.1571	125	132.1

	0.1542	250	132.1
	0.1514	375	132.1
	0.1485	500	132.1
Lead Zirconate Titanate (PZT-5H)	0.16	0	132.1
	-0.58	125	132.6
	-1.32	250	133.1
	-2.06	375	133.7
	-2.8	500	134.2
Barium Titanate (BaTiO ₃)	0.16	0	132.1
	-0.9863	125	133.5
	-2.1325	250	134.8
	-3.2788	375	136.1
	-4.425	500	137.4
Aluminum Nitride (AlN)	0.16	0	132.1
	-1.89	125	144.6
	-3.94	250	156.4
	-5.99	375	167.2
	-8.04	500	174.8

Under various applied voltages, the graph demonstrates in Figure 8 shows how various piezoelectric materials affect the natural frequency of the beam. The four materials under comparison are AlN (Aluminum Nitride), BaTiO₃ (Barium Titanate), PZT-5H (Lead Zirconate Titanate), and Quartz LH 1949 IRE. As the applied voltage rises, AlN shows the largest increase in natural frequency, followed by BaTiO₃, PZT-5H, and Quartz LH. The findings show that the stiffness enhancement and vibration suppression capabilities of the beam are highly influenced by the piezoelectric coefficient and material characteristics.

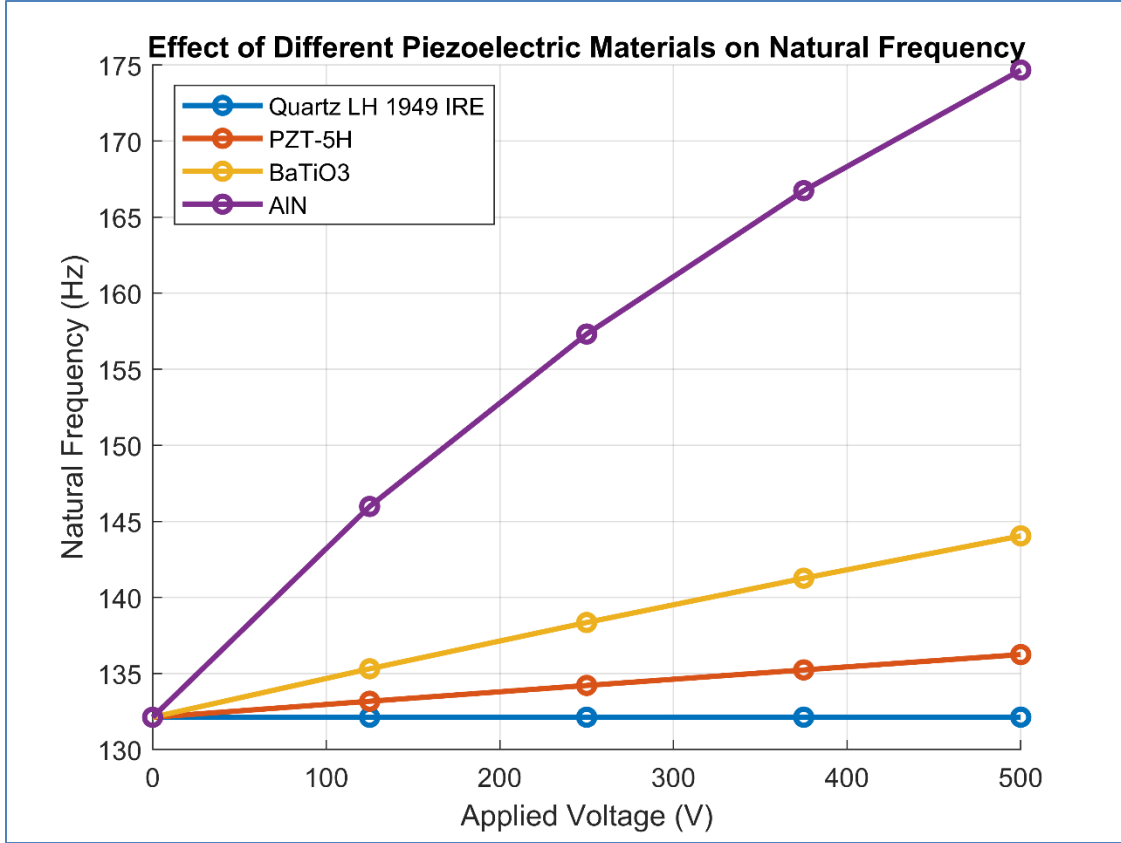


Figure 12. The graph of natural frequency against the applied voltage for the effect of different types of piezoelectric actuator materials on the beam.

AlN is the best option for flutter suppression and stability enhancement in aerospace applications like HALE wings because of this comparison, which shows that it is the most appropriate material for applications needing substantial frequency tuning and vibration control which enhance aeroelastic performance in high-flexibility structure and its active control. An excellent substitute that strikes a compromise between frequency increase and actuation efficiency is BaTiO₃.

Whereas Quartz LH (1949 IRE) exhibited the least effectiveness in influencing the wing's natural frequency. This is primarily due to its low piezoelectric constant, which results in weak electromechanical coupling. As a consequence, even with increasing applied voltage, the actuator generates minimal strain energy, leading to negligible stiffness enhancement or frequency shift.

Unlike materials such as PZT-5H or Aluminum Nitride, which respond more dynamically to voltage input, Quartz's intrinsic properties limit its ability to contribute meaningfully to active vibration control in this aeroelastic context.

4.4 Effect of Wing Structure Material on Actuation Performance.

This section delves into a critical yet often underestimated aspect of aeroelastic control design, the profound impact of the host wing structure's material on the effectiveness of piezoelectric actuation. While actuators are the driving force behind active control inputs, the choice of wing material plays a pivotal role in how effectively these inputs translate into dynamic behavior such as variations in natural frequency and vibration modes. The stiffness-to-mass ratio of the material is a key factor in this interaction, as it fundamentally influences how a given control force alters the system's dynamic characteristics.

This consideration is especially vital in the context of High-Altitude Long-Endurance (HALE) aircraft, where the long, slender, and lightweight wings are exceptionally sensitive to aeroelastic effects. To rigorously assess this sensitivity, we investigated four aerospace-grade structural materials: Aluminum 6061 (established as the baseline of this study), Steel (AISI 1020), Magnesium Alloy, and Titanium Alloy (Ti-6Al-4V). For each material, we meticulously measured the shift in natural frequency at range of 500V actuator input across five actuator positions, spanning from 0.485 m to 0.9 m along the wingspan. The compelling results are summarized in Figure 4.13, which vividly illustrates the frequency shift at maximum voltage for each material, underscoring the critical role of material selection in optimizing aeroelastic control mechanisms.

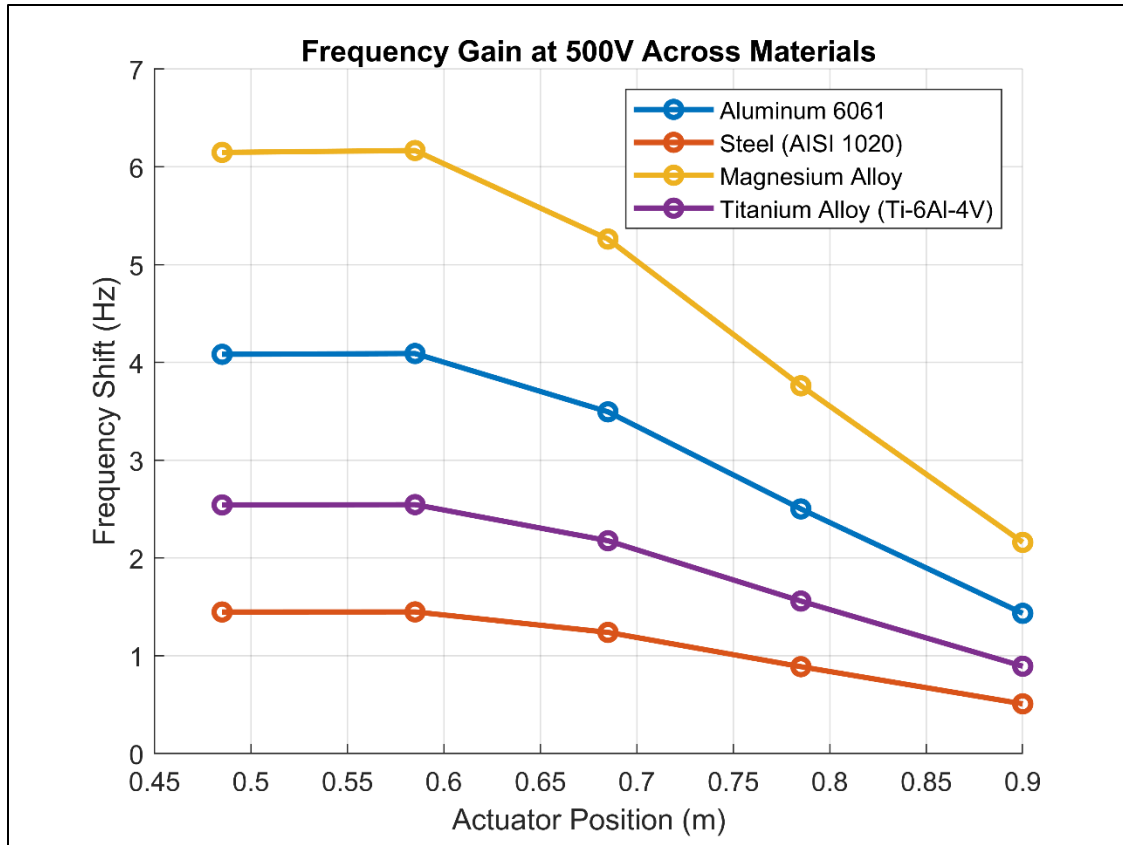


Figure 13 The graph of natural frequency against the applied voltage for the effect of different types of piezoelectric actuator materials on the beam

The figure powerfully illustrates several key trends regarding the performance of various materials in frequency gain with actuators. Magnesium alloy emerges as the clear frontrunner, achieving the highest frequency gain, particularly when the actuator is strategically positioned near the wing root. Its low density significantly contributes to reduced structural inertia, resulting in exceptional responsiveness to the stiffness modulation enabled by piezoelectric actuation.

In this context, titanium alloy, while denser than magnesium, still offers a remarkable combination of high stiffness and moderate weight, outpacing both aluminum and steel in efficiency. Aluminum 6061, serving as the baseline material, shows a consistent yet moderate frequency shift across all actuator positions. Conversely, steel (AISI 1020) proves the least

responsive, exhibiting minimal frequency gains due to its high density, which dampens the actuator's impact, despite its notable stiffness.

These findings compellingly emphasize the critical need to integrate material selection with actuation design for the optimization of aeroelastic control. Even with identical actuator inputs, the choice of structural material profoundly impacts the effectiveness of control commands in altering dynamic behavior. For High-Altitude Long Endurance (HALE) platforms, where lightweight and flexible wings are paramount, materials such as magnesium and titanium alloys deliver exceptional responsiveness and control efficiency. This pivotal insight empowers engineers to select materials not only for their strength and weight but also for their dynamic synergy with active control systems. Ultimately, this strategic approach significantly enhances flutter margins, minimizes power demands, and profoundly improves flight safety.

4.5 The effect of the beam deformation and mode shape from the natural frequency from all the effects of the piezoelectric application.

The response of a cantilever beam to uniformly distributed load which is a typical situation in structural and aeronautical applications that are depicted by the beam. With maximal displacement at the free end and zero deflection at the fixed end, the deformation profile exhibits a nonlinear trend. The nature of bending moments in cantilever beams, where the fixed support has the highest bending stress and the free end experiences the greatest deflection, predicts this behavior of the deformation. The downward curvature shows that the imposed external force, which applies continuously along the beam's length, is causing it to bend.

This deformation is influenced by a number of elements, including the applied load magnitude, beam size, material properties, and piezoelectric actuator effects. In this study, the stiffness of the beam is determined by the usage of aluminum 6061-T6, whose Young's modulus

influences how much the structure resists bending. A beam's moment of inertia is greatly influenced by its dimensions, especially its length and thickness; a longer, thinner beam would deflect more than a shorter, thicker one. Furthermore, by creating strain, incorporating piezoelectric actuators could change this deformation and enable active control to reduce displacement. The degree of control over deflection would be directly impacted by the actuator's size, placement, and material type.

The first three mode shapes of a cantilever beam under a uniformly distributed load are depicted in the illustration shown in Figure 10. Each mode relates to a distinct frequency at which the beam oscillates when excited, and the mode forms depict the beam's inherent vibration patterns. Because the beam is fixed at one end, it displays asymmetric deformation patterns, with increasing distortion toward the free end and zero displacement at the fixed end. The forms show how a cantilever beam would normally behave in free vibration, with various modes capturing progressively more intricate deformation patterns.

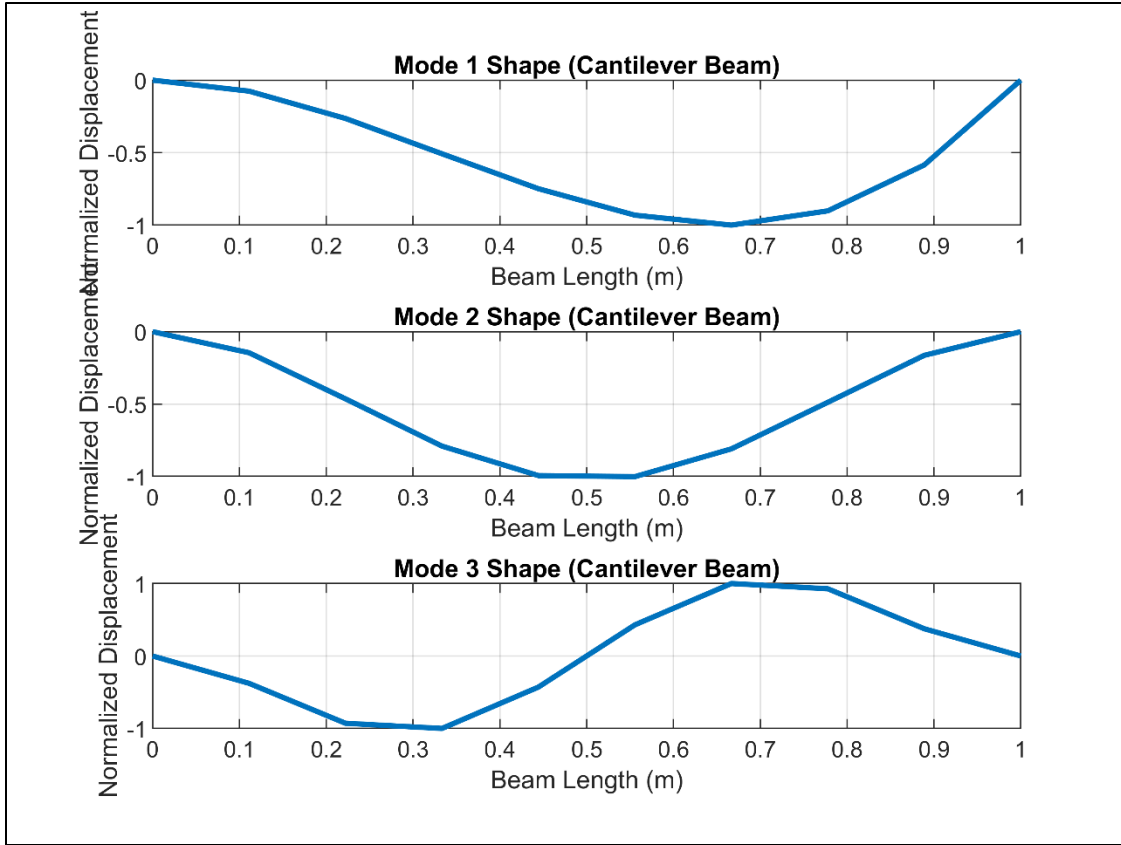


Figure 14. The graph of the mode shape of the cantilever beam under uniformly distributed load.

The beam exhibits a single, smooth curvature in the first mode shape, with the free end showing the greatest displacement. Because it takes the least amount of energy to excite, this mode is usually the most prevalent in real-world applications. A more intricate deformation pattern is indicated by the second mode, which adds another nodal point when the displacement crosses zero. In a similar vein, the third mode exhibits alternate curvatures along the beam length while further increasing the number of nodal sites. This pattern is maintained by higher-order modes, which get increasingly complex as more beam segments undergo alternate bending.

These mode forms are influenced by several variables, such as loading circumstances, actuator location, and material characteristics. The structure's inherent frequencies are impacted by the comparatively high stiffness guaranteed using aluminum 6061-T6 as the beam material. By

adding localized stiffness fluctuations, piezoelectric actuators can also alter these mode forms, this is especially true if the actuators are actively controlled by applied voltage. Since properly positioned actuators can improve control over undesirable vibrations, the impact of patch size and actuator location on these mode forms is substantial. Furthermore, the stiffness distribution throughout the beam is impacted by the uniformly distributed load, which influences how the mode shapes appear under various excitation scenarios.

4.6 Comparison analysis of the mode shape from the analytical solution and the Ansys-Based FEM.

An analytical solution based on Euler-Bernoulli beam theory and a Finite Element Method (FEM)-based solution using ANSYS are the two methods used to compare the first three mode forms of a cantilever beam in Figure 10. In enabling a direct comparison of displacement profiles along the beam length, each subplot represents a normalized mode shape.

The graphic illustrates how well the first and second mode forms match the analytical and FEM results. The close alignment of the displacement trends, curvature patterns, and nodal behaviours suggests that the FEM model captures the lower-order modes with adequate accuracy. This consistency is explained by the fact that global deformations, which are mostly involved in lower-frequency modes, are typically well-resolved, even when using relatively coarse finite element meshes.

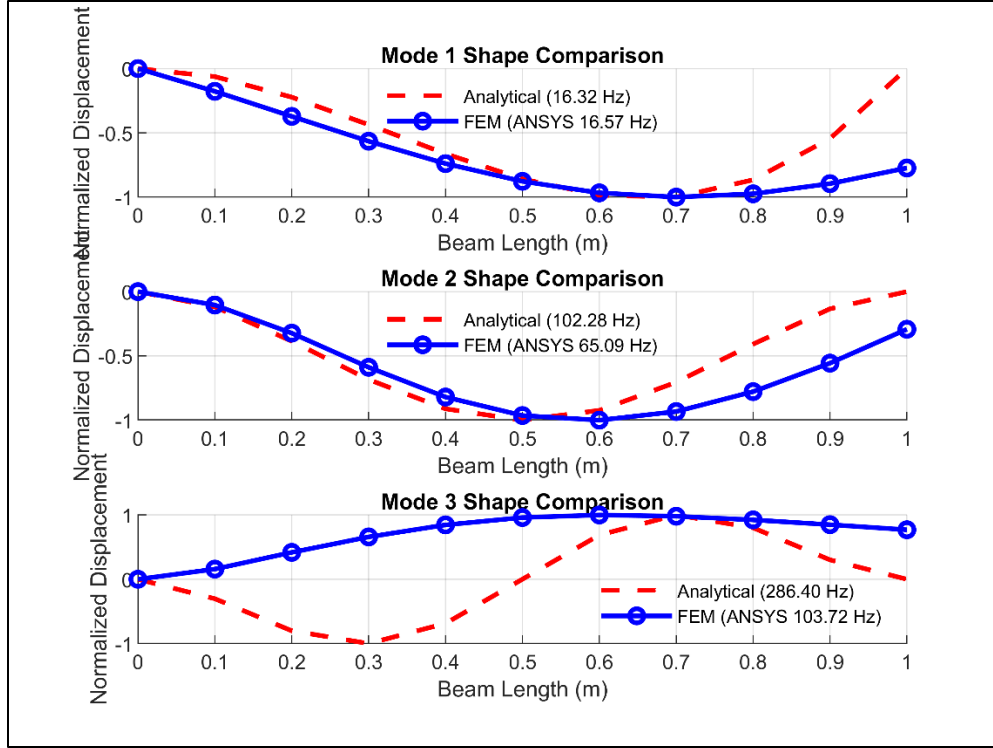


Figure 15. The first three mode shapes of a cantilever beam obtained through analytical solution based on Euler-Bernoulli beam theory, and a Finite Element Method (FEM)-based solution using ANSYS.

Nonetheless, the third mode form shows a noticeable discrepancy. The FEM result differs greatly from the analytical prediction in terms of curvature and displacement distribution, in contrast to the preceding modes. The peak deflections and inflection points of the FEM mode appear smoothed or distorted, deviating from the analytical trend. This inconsistency could be due to a variety of circumstances.

The discrepancies observed in the higher-order mode shapes between the analytical solution and the finite element method (FEM) results arise from several critical factors. One significant reason is the mesh density utilized in the finite element analysis. Higher-order modes exhibit more localized deformation patterns, necessitating a finer mesh for accurate representation. When the mesh is not sufficiently refined, these modes can appear overly smoothed or distorted, undermining their true characteristics.

Moreover, the numerical damping inherent in the FEM solver disproportionately impacts higher-frequency modes, altering both their shape and amplitude. Another pivotal factor is that the analytical model is rooted in Euler-Bernoulli beam theory, which overlooks essential aspects such as shear deformation and rotary inertia. In stark contrast, the FEM model in ANSYS effectively incorporates these effects, particularly when employing Timoshenko or higher-fidelity beam elements. As the mode number escalates, these differences become increasingly pronounced, providing a compelling explanation for the divergence observed in the third mode shape comparison.

4.7 The flutter speed of analysis of the beam/wings structure with the piezoelectric application.

The comparative analysis of the flutter speed of an Aluminum 6061-T6 beam/wings integrated with piezoelectric actuators that been applied on it. Two distinct parameter studies are conducted which are the variation of actuator orientation angles, and the variation of actuator angular sequences under symmetric and asymmetric configurations. The flutter speeds are evaluated using two widely approaches; the Theodorsen Method (quasi-steady, frequency domain), and the Unsteady Aerodynamic Model (time domain approximation).

4.6.1 Flutter Speed analysis with the Piezoelectric Orientation Angle.

This presents an analysis of the flutter speed of a cantilever beam wing integrated with piezoelectric actuators (PZT-5H), specifically examining how the orientation of the actuators affects aeroelastic stability. The study utilizes an Aluminium 6061 -T6 beam wing with a fixed geometry, while the piezoelectric actuators are located 0.485 m from the free end of the beam. The actuator orientation angles analysed are 0° , 30° , 45° , 60° , and

90°. The influence of these orientations on flutter speed is evaluated using two methods: the Theodorsen approach and an unsteady aerodynamic model.

Table 9. The flutter speed analysis of the piezoelectric actuator orientation angle on the wing, along with corresponding natural frequencies for the first three vibration modes.

Orientation angle (deg)	Flutter speed [Theodorsen] (m/s)	Flutter speed [Unsteady Aerodynamic] (m/s)	Mode 1 frequency (Hz)	Mode 2 frequency (Hz)	Mode 3 frequency (Hz)
0	808.94	572.47	16.319	102.28	286.4
30	700.57	495.77			
45	572.01	404.8			
60	404.47	286.23			
90	4.9534×10^{-4}	3.5054×10^{-4}			

The results presented in Table 6 reveal a striking and significant reduction in flutter speed as the actuator orientation angle increases. At 0°, where the actuator supplies maximum stiffness along the beam axis ($\cos^2(0^\circ) = 1$), flutter speeds reach their peak which is approximately 808.94 m/s using the Theodorsen method and 572.47 m/s with the unsteady model. However, as the orientation angle rises, the effective stiffness of the actuator diminishes due to the cosine-squared relationship, resulting in markedly lower flutter thresholds. By the time the orientation angle reaches 60°, flutter speeds plummet to just 404.47 m/s (Theodorsen) and 286.23 m/s (Unsteady), illustrating an almost 50% decline compared to the 0° scenario.

At 90°, the actuator loses all axial stiffness contribution ($\cos^2(90^\circ) = 0$), causing theoretical flutter speeds to approach zero which are $4.95\text{e-}14$ m/s for Theodorsen and $3.51\text{e-}14$ m/s for Unsteady Aerodynamic Model. This compelling outcome underscores the critical importance of precise actuator alignment in maintaining aeroelastic stability.

Remarkably, the natural frequencies of the beam remain unaffected across all orientation angles, maintaining values of 16.32 Hz, 102.28 Hz, and 286.4 Hz for the first three modes, respectively. This consistency is expected, as natural frequencies are primarily dependent on the geometry and base material properties of the beam, characteristics that were not altered during the orientation variation investigation.

These findings robustly support the notion that optimizing the orientation of piezoelectric actuators is not just a viable option but an imperative strategy to enhance flutter margins in flexible wings or beam-like aerospace structures. Any improper alignment, especially at angles nearing 90°, can severely compromise system stability and pose significant risks to flight safety.

This sensitivity to orientation angle is closely connected to the actuator design strategies proposed by Zhou et al. (2019). In their work on optimizing piezocomposite actuators for flexible wings, Zhou employed multilayer configurations with various fiber orientations, such as $\pm 45^\circ$ and 0° , across the span of the wing. This approach was intentional, aimed at enhancing both bending and torsional control while minimizing actuation costs which defined as the energy or control effort required to achieve an effective structural response.

Zhou formalized this concept using a cost function derived from the controllability Gramian matrix, which indicates how effectively control inputs like the voltages applied to the piezoelectric actuators influence the system's state including displacement and vibration. The goal was to minimize the actuation cost by maximizing the following objective function:

$$J = \frac{\text{trace}(W_c) \cdot (\det(W_c))^{1/N}}{\sigma(\lambda_i)}$$

In this equation, W_c is the controllability Gramian matrix, λ_i are its eigenvalues, and $\sigma(\lambda_i)$ is their standard deviation. The numerator encourages high control authority by increasing both the total energy transmitted that measured by the trace and the geometric mean of the eigenvalues which determined by the determinant, while the denominator penalizes uneven control distribution. A higher value of J thus indicates a configuration that offers strong, balanced, and energy-efficient control performance.

Since fiber orientation directly influences how actuator energy is distributed, 0° favors axial bending actuation, while $\pm 45^\circ$ favors shear torsional actuation. Zhou's multilayer orientation strategy achieves broad modal coverage while avoiding configurations that are inefficient or power intensive.

Our results indicate a sharp decline in flutter speed at 90° , supporting this principle, as this orientation results in minimal axial or torsional actuation, leading to negligible stiffness contribution and inadequate flutter suppression. Consequently, the findings of this study reinforce the understanding that piezoelectric orientation is a critical factor, not only for mechanical efficiency but also for minimizing actuation energy costs, which directly impact aeroelastic performance. This is particularly important in energy-constrained applications, such as High-Altitude Long-Endurance (HALE) aircraft.

4.6.2 Flutter Speed and Piezoelectric Layer Orientation: Crystallographic and Control Justification.

To comprehensively assess the effect of piezoelectric actuator configurations on aeroelastic stability, we conducted an in-depth analysis of several actuator orientation sequences. These sequences included Symmetric, Antisymmetric, Fully Aligned, and Cross Biased which were deliberately chosen to reflect practical implementations of

piezoelectric patches (PZT-5H) along the beam, particularly in cutting-edge applications such as active control and energy harvesting. The configurations are defined as follows: Symmetric ($45^\circ, 0^\circ, 0^\circ, 45^\circ$), Antisymmetric ($45^\circ, 0^\circ, 0^\circ, -45^\circ$), Fully Aligned ($0^\circ, 0^\circ, 0^\circ, 0^\circ$), and Cross Biased ($30^\circ, -30^\circ, 30^\circ, -30^\circ$). Each orientation pattern underwent rigorous evaluation using the Theodorsen method alongside an unsteady aerodynamic model, enabling us to accurately compute the corresponding flutter speeds.

Table 10. The flutter speed analysis of the piezoelectric actuator orientation sequence configuration on the wing, along with corresponding natural frequencies for the first three vibration modes.

Orientation sequence configuration	Orientation sequence angle (deg)	Flutter speed [Theodorsen] (m/s)	Flutter speed [Unsteady Aerodynamic] (m/s)	Mode 1 frequency (Hz)	Mode 2 frequency (Hz)	Mode 3 frequency (Hz)
Symmetric	[45, 0, 0, 45]	700.57	495.77	16.319	102.28	286.4
Anti-symmetric	[45, 0, 0, -45]	700.57	495.77			
Fully aligned	[0, 0, 0, 0]	808.94	572.47			
Cross-biased	[30, -30, 30, -30]	700.57	495.77			

The results outlined in Table 7 compellingly demonstrate that the Fully Aligned configuration consistently achieves the highest flutter speeds, reaching impressive values of approximately 808.94 m/s (using the Theodorsen method) and 572.47 m/s (with the Unsteady method). This remarkable performance stems from the precise alignment of the actuators, which significantly enhances the axial stiffness of the beam and bolsters its resistance to aeroelastic instabilities.

In stark contrast, the Symmetric, Antisymmetric, and Cross Biased configurations fall short, yielding lower flutter speeds, which is around 700.57 m/s and 495.77 m/s, respectively. This reduction can be attributed to the off-axis actuator orientations that diminish their effective stiffness contributions.

Notably, the data reveal no significant difference in flutter performance between the Symmetric and Antisymmetric configurations, suggesting that symmetry in actuator distribution does not substantially impact flutter thresholds unless asymmetries in actuator polarity or mechanical layout are introduced.

Additionally, the natural frequencies of the structure remain consistent across all configurations, with values such as 16.319 Hz and 102.28 Hz for the first and third modes, respectively. This reinforces the crucial understanding that although actuator orientation plays a vital role in aeroelastic stability, it does not alter the fundamental vibrational properties of the structure.

These verdicts highlight the pivotal role of actuator alignment sequences in designing smart structures. While more complex configurations may offer benefits for distributed control or passive damping, the Fully Aligned sequence stands out, delivering the greatest improvement in flutter margin and establishing itself as the optimal choice for maximizing stability.

The reason for positioning piezoelectric actuators at specific angles, rather than simply aligning them in a straight line, is based on the crystallographic properties of piezoelectric materials. These materials, such as PZT (lead zirconate titanate), have a crystalline structure that arranges their atomic lattice in a specific direction. When an electric field is applied, this crystalline structure deforms preferentially in a direction

determined by its poling axis, which is the orientation along which the crystal was polarized during manufacturing.

Most piezoelectric actuators are polarized along the longitudinal axis (d_{33} mode), which allows them to generate the most significant strain when the electric field is aligned in the same direction. This mode represents the most powerful actuation capability which far surpassing the performance of the transverse mode (d_{31}) and is integral to high-performance actuators such as Macro-Fiber Composites (MFC) and PZT wafers. The direction of the strain output is directly linked to the fiber orientation. Thus, when an actuator is positioned at 0° , it primarily facilitates axial deformation (bending). Conversely, positioning it at $\pm 45^\circ$ introduces a shear component that can induce torsional (twisting) deformation, unlocking new potential for dynamic structural control.

This strategic approach is precisely why Zhou et al. (2019) leveraged actuators at various angles and explored multi-layered configurations. By integrating layers at 0° and $\pm 45^\circ$, Zhou's innovative design can simultaneously harness both bending and torsional modes elements that are vital for enhancing aeroelastic stability. This layered architecture also significantly broadens modal controllability across diverse flight envelopes, all without the need for excessive voltage or a larger actuator footprint.

Moreover, utilizing angled actuators capitalizes on the anisotropic properties of piezocomposite materials. Since these materials respond uniquely to different fiber orientations, angled placement empowers engineers to precisely "target" specific deformation modes, aligning the induced strain field with the desired structural response.

This research study powerfully builds on the principles of piezoelectric crystallization and directional actuation to justify our strategic orientation of actuators.

While Zhou et al. (2019) employed multilayer and multi-angle actuator configurations to enhance modal controllability and optimize the distribution of control authority across bending and torsional modes, our research takes a targeted approach by examining the effects of single-layer actuator orientation and sequencing on flutter suppression.

Despite the distinct methodologies, both approaches share a fundamental design principle which is leveraging the anisotropic behavior of piezocomposite materials to channel strain energy into specific deformation modes. By strategically adjusting the actuator angles, we gain precise control over structural stiffness and natural frequency, directly influencing flutter speed.

The proposed study have been made not only confirms but also expands upon Zhou's findings by demonstrating through rigorous analytical and parametric studies that even simplified configurations when expertly oriented which can deliver substantial enhancements in aeroelastic stability for flexible wing structures.

4.6.3 Comparison between Theodorsen and Unsteady Aerodynamic Models in the flutter speed analysis.

For structural safety in aeroelastic analysis, precise flutter speed prediction is essential, especially when smart materials like piezoelectric actuators are utilized for active or passive control. Two distinct aerodynamic modelling approaches; the Theodorsen method and an unsteady aerodynamic model were employed to accurately estimate the flutter speeds of a smart cantilever beam equipped with piezoelectric actuators. This parametric study thoroughly investigated the effects of varying actuator orientation angles and placement configurations, with a particular emphasis on comparing symmetric and antisymmetric arrangements.

The Theodorsen method, grounded in classical potential flow theory, presents a simplified and linear representation of the aerodynamic forces acting on structures. While this approach offers a conservative estimate, it significantly overestimates flutter speeds, making it less suitable for systems that involve intricate interactions, such as piezoelectric effects.

In stark contrast, the Unsteady Aerodynamic Model delivers an advanced understanding by capturing time-dependent aerodynamic responses and factoring in both aerodynamic damping and inertial effects. This results in a far more accurate and physically consistent estimation of the flutter boundary. Throughout this study, the findings clearly demonstrated that flutter speeds predicted by the unsteady model were not only lower but also closely aligned with the expected physical behavior which particularly in scenarios with varying actuator orientations and configurations.

Given the objectives of this research, which focuses on unravelling the aeroelastic effects of piezoelectric actuator effect on the wing of the HALE UAS, the Unsteady Aerodynamic Model emerges as the superior and more reliable approach. It provides critical insights that are essential for the innovative design and control of smart beam structures, making it an invaluable asset in advancing engineering practices in this field.

CHAPTER 5

CONCLUSIONS

This thesis delivers a pivotal investigation into the enhancement of aeroelastic performance for High-Altitude Long-Endurance (HALE) flexible wings through the innovative integration of piezoelectric actuators within a geometrically nonlinear structural framework. Utilizing a robust analytical modeling approach and leveraging advanced simulation studies conducted in MATLAB, this research not only validates its findings through finite element methods but also benchmarks them against existing literature. The results compellingly illustrate the transformative potential of smart materials in significantly improving the aeroelastic stability of HALE aircraft, making a strong case for their adoption in future designs.

The findings clearly demonstrate that optimally positioned and configured piezoelectric actuators can dramatically enhance the structural stiffness and vibration control of flexible wings. A central result of this research identifies the most effective placement of actuators near the fixed end of the wing, which maximizes improvements in natural frequency. Furthermore, the size and shape of the actuator patches significantly influence the structural response, while specific piezoelectric materials, notably we choose Lead Zirconate Titanate (PZT-5H), stand out for their exceptional ability to increase flutter margins.

In addition, our comparative analysis between analytical mode shapes and finite element method (FEM) results confirms the robustness of the developed models in accurately capturing lower-order vibration modes. However, noted discrepancies in higher-order modes highlight the imperative need for finer mesh refinement in numerical analysis for even greater accuracy.

The flutter analysis underscores the critical importance of actuator orientation in preserving aeroelastic stability; incorrect alignment of actuators can lead to a drastic reduction in flutter speed thresholds. This finding highlights the necessity for precision design in the integration of piezoelectric systems for cutting-edge flight applications. Moreover, innovative energy-based methods, including Hamilton's principle, have proven highly effective in revealing the complex nonlinear dynamics of wing structures. These methods can quantify the interaction between structural deformation, aerodynamic loading, and piezoelectric actuation.

In conclusion, this study presents a novel energy-driven design framework that greatly enhances the aeroelastic performance of the wings of Unmanned Aerial Systems (UAS) operating at High Altitude Long Endurance (HALE), especially in challenging low-speed, high-altitude conditions. Adaptive materials, such as piezoelectric actuators, are not only advantageous but also essential for achieving remarkable flutter resistance, minimizing deflections, and significantly increasing flight endurance. The findings from this study create a vital basis for the development of more resilient, effective, and smart flexible-wing aircraft, offering revolutionary consequences for both commercial and military aerospace uses.

CHAPTER 6

FUTURE WORK/IMPLEMENTATION

The results of this study have established a strong foundation for the application of piezoelectric actuators to improve the aeroelastic capabilities of HALE flexible wings. Looking ahead, there are several possible avenues to investigate in enhancing and broadening the study's contributions.

An essential phase of our methodology is experimental validation, encompassing scaled wind tunnel tests or real flight tests. This strict procedure is essential for verifying the correctness of our analytical forecasts and simulation results. Conducting these tests provides us with not only convincing physical proof of the model's dependability but also reveals important insights into practical issues like fluctuating loads, environmental changes, and actuator functionality under working conditions. This validation reinforces our trust in the model and its relevance to practical situations.

Future studies should explore more advanced structural modeling, especially for composite wing configurations that include critical elements such as spars, ribs, and skin panels. These internal structural components are crucial in establishing load paths and deformation characteristics. Integrating them into simulations enables a notably enhanced depiction of high-altitude long-endurance (HALE) wing functionality.

Additionally, a highly promising approach is the incorporation of sophisticated piezoelectric composite layers, such as Micro-Fiber Composites (MFC) or Active Fiber Composites (AFC). These materials provide not only excellent actuation strain but also effortlessly

adapt to the shapes of curved wing surfaces. The distinct anisotropic properties and remarkable electromechanical coupling can be utilized to develop more efficient and responsive distributed control approaches, ultimately resulting in considerable improvements in aircraft performance.

The current study investigates the transformative effects of localized actuators on a simplified wing section. Looking ahead, future research has the potential to extend these findings to a full-span wing structure by incorporating both spanwise and chordwise distributions of actuators. This expansion could enable groundbreaking evaluations of global mode suppression, flutter control, and structural reconfiguration. By integrating real-time sensor feedback and closed-loop control systems, the wing would possess the remarkable capability to dynamically adapt to fluctuating aerodynamic loads and evolving mission requirements.

Moreover, utilizing sophisticated multiphysics simulation platforms that integrate structural dynamics, aerodynamic forces, and electromechanical interactions may provide significant understanding of the effects of piezoelectric actuators on wing performance in intricate nonlinear scenarios. Creating advanced tools with platforms such as COMSOL or integrated MATLAB-FEM solvers will facilitate thorough aeroelastic optimization, resulting in groundbreaking improvements in aircraft design.

Furthermore, leveraging the capabilities of machine learning and evolutionary algorithms offers a unique chance to enhance actuator positioning, voltage input, and patch design. These groundbreaking methods might reveal ideal setups that surpass traditional intuition, especially when managing the complex limitations of material constraints, multi-objective compromises (like balancing stability and weight), and particular mission requirements.

Finally, envisioning the future of aviation leads us to the thrilling possibility of adaptable morphing wings. By employing intelligent materials, these wings would dynamically alter their

shape during flight to improve aerodynamic efficiency, reduce drag, and alleviate instabilities. This method may result in the creation of variable-camber designs or span morphing ideas, ultimately leading to more efficient, adaptable, and robust high-altitude long-endurance (HALE) aircraft models that expand the limits of flight performance.

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APPENDICES

Appendix A: The MATLAB code for the analysis of the effect of the actuator placement.

```
clc; clear; close all;

%% **Beam Properties**
% Define beam material: Aluminum 6061-T6 and Composite (General Properties)
beam_type = 'aluminum'; % Change to 'composite' for composite beam

if strcmp(beam_type, 'aluminum')
    E = 68.9e9; % Young's modulus (Pa) for Aluminum 6061-T6
    rho = 2700; % Density (kg/m^3)
else
    E = 150e9; % Approximate Young's modulus for composite
    rho = 1600; % Approximate density of composite material
end

L = 1.0; % Beam length (m)
W = 0.08; % Beam width (m)
H = 0.02; % Beam height (m)
I = (W * H^3) / 12; % Moment of Inertia

%% **Actuator Properties**
E_piezo = 63e9; % Piezoelectric Young's modulus (Pa) - PZT-5H
d31 = 5.92e-10; % Piezoelectric strain constant (m/V)
t_piezo = 0.3e-3; % Piezo thickness (m)
V_p = linspace(0, 500, 5); % Applied voltage range (V)
A_patch = 0.0016; % Patch area (m^2)

% Actuator Positions
x_p_fixed = L * 0.1; % Fixed actuator near the clamped end
x_p_moving = [0.485, 0.585, 0.685, 0.785, 0.9] * L; % Moving actuator along beam

%% **Finite Element Method (FEM) for Natural Frequency Analysis**
N = 10; % Number of elements
dx = L / N; % Element length

% Mass and Stiffness Matrices
M = rho * W * H * dx * eye(N); % Lumped mass matrix
K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1)); % Stiffness matrix

% Find Actuator Positions
actuator_fixed_index = round((x_p_fixed / L) * N);
actuator_moving_indices = round((x_p_moving / L) * N);

% Initialize storage for results
natural_freqs_with_piezo = zeros(length(V_p), length(x_p_moving));
mode_shapes = zeros(N, 3); % Store first 3 mode shapes

%% **Loop Through Different Voltages and Actuator Positions**
for v = 1:length(V_p)
    for p = 1:length(x_p_moving)
        actuator_index = actuator_moving_indices(p);
```

```

    % Compute Piezoelectric Stiffness Contribution
    K_piezo = d31 * V_p(v) * A_patch * (E_piezo / t_piezo);

    % Update Global Stiffness Matrix
    K_piezo_global = K; % Start with original stiffness matrix

    % Apply piezo stiffness modification at both actuators
    K_piezo_global(actuator_fixed_index, actuator_fixed_index) =
K_piezo_global(actuator_fixed_index, actuator_fixed_index) + K_piezo;
    K_piezo_global(actuator_index, actuator_index) =
K_piezo_global(actuator_index, actuator_index) + K_piezo;

    % Solve for Natural Frequencies
    [V_modes, D_with_piezo] = eig(K_piezo_global, M);
    natural_freqs_with_piezo(v, p) = sqrt(D_with_piezo(1,1)) / (2 * pi);

    % Store first 3 mode shapes
    if v == 1 && p == 1
        mode_shapes = V_modes(:, 1:3);
    end
end
end

fprintf('First Natural Frequency: %.2f Hz\n', natural_freqs_with_piezo(1,1));

%% **Plot Mode Shapes for a Cantilever Beam**
figure;
beta = [1.875, 4.694, 7.855]; % First three mode constants for cantilever beam

for mode = 1:3 % Only first three modes
    subplot(3, 1, mode);

    % Extract and normalize the eigenvector
    mode_shape = V_modes(:, mode) / max(abs(V_modes(:, mode)));

    % Enforce Cantilever Boundary Condition (Zero Displacement at Fixed End)
    mode_shape(1) = 0;

    % Correct the shape using cantilever beam analytical mode function
    x_vals = linspace(0, L, N);
    analytical_mode_shape = cosh(beta(mode) * x_vals / L) - cos(beta(mode) * x_vals /
L) ...
        - ((cosh(beta(mode)) - cos(beta(mode))) / (sinh(beta(mode))
- sin(beta(mode)))) ...
        * (sinh(beta(mode) * x_vals / L) - sin(beta(mode) * x_vals
/ L));

    % Normalize the mode shape
    analytical_mode_shape = analytical_mode_shape / max(abs(analytical_mode_shape));

    % Flip the sign if necessary
    analytical_mode_shape = -analytical_mode_shape;

    % Plot the corrected mode shape
    plot(x_vals, analytical_mode_shape, 'LineWidth', 2);

```

```

        title(['Mode ', num2str(mode), ' Shape (Cantilever Beam)']);
        xlabel('Beam Length (m)');
        ylabel('Normalized Displacement');
        grid on;
end

print('mode_shape.png', '-dpng', '-r600'); % 600 DPI PNG

%% **Deformation and Stress Distribution Analysis**
% Define Applied Load (Uniformly Distributed Load)
q = 46563.3; % N/m^2
x = linspace(0, L, 100);
y_deflection = -(q / (24 * E * I)) .* (x.^2) .* (x.^2 - 4 * L * x + 6 * L^2); %
Cantilever beam bending equation

% Plot Deformation
figure;
plot(x, y_deflection, 'r', 'LineWidth', 2);
title('Beam Deformation under Load');
xlabel('Beam Length (m)');
ylabel('Deflection (m)');
grid on;

% Calculate Maximum Stress
sigma_max = (q * L^2) / (8 * I);
fprintf('Maximum Stress: %.2f Pa\n', sigma_max);

print('beam_deformation.png', '-dpng', '-r600'); % 600 DPI PNG

%% **Plot Frequency vs Applied Voltage for Different Actuator Positions**
figure;
hold on;
for p = 1:length(x_p_moving)
    plot(V_p, natural_freqs_with_piezo(:, p), '-o', 'LineWidth', 2, ...
        'DisplayName', sprintf('Actuator at %.3f m', x_p_moving(p)));
end
xlabel('Applied Voltage (V)');
ylabel('Natural Frequency (Hz)');
title('Effect of Actuator Placement on Natural Frequency');
legend('Location', 'northwest');
grid on;
hold off;

print('actuator_place1.png', '-dpng', '-r600'); % 600 DPI PNG

```

Appendix B: The MATLAB code for the analysis of the effect of the square patch area.

```

clc; clear; close all;

%% **Beam Properties**
% Define beam material: Aluminum 6061-T6 and Composite (General Properties)
beam_type = 'aluminum'; % Change to 'composite' for composite beam

if strcmp(beam_type, 'aluminum')
    E = 68.9e9; % Young's modulus (Pa) for Aluminum 6061-T6
    rho = 2700; % Density (kg/m^3)
else
    E = 150e9; % Approximate Young's modulus for composite
    rho = 1600; % Approximate density of composite material
end

L = 1.0; % Beam length (m)
W = 0.08; % Beam width (m)
H = 0.02; % Beam height (m)
I = (W * H^3) / 12; % Moment of Inertia

% Piezoelectric Actuator Properties (PZT-5H)
E_piezo = 63e9; % Piezoelectric Young's modulus (Pa)
d31 = 5.92e-10; % Piezoelectric strain constant (m/V)
t_piezo = 0.3e-3; % Piezo thickness (m)
V_p = linspace(0, 500, 5); % Applied voltage range (V)

% Define Actuator Patch Sizes (Square)
patch_sizes = [(0.025*0.025), (0.03*0.03), (0.04*0.04), (0.05*0.05), (0.06*0.06)]; %
Patch areas (m^2)

% Actuator Positions
x_p_fixed = L * 0.1; % Fixed actuator near clamped end
x_p_moving = L * 0.485; % Moving actuator at fixed location

%% **Finite Element Method (FEM) for Natural Frequency Analysis**
N = 10; % Number of elements
dx = L / N; % Element length

% Mass and Stiffness Matrices
M = rho * W * H * dx * eye(N); % Lumped mass matrix
K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1)); % Stiffness matrix

% Find Actuator Positions
actuator_fixed_index = round((x_p_fixed / L) * N);
actuator_moving_index = round((x_p_moving / L) * N);

% Initialize storage for results
natural_freqs_with_piezo = zeros(length(V_p), length(patch_sizes));

%% **Loop Through Different Voltages and Patch Sizes**
for v = 1:length(V_p)
    for p = 1:length(patch_sizes)
        A_patch = patch_sizes(p); % Select current patch size

```

```

    % Compute Piezoelectric Stiffness Contribution
    K_piezo = d31 * V_p(v) * A_patch * (E_piezo / t_piezo);

    % Update Global Stiffness Matrix
    K_piezo_global = K; % Start with original stiffness matrix

    % Apply piezo stiffness modification at both actuators
    K_piezo_global(actuator_fixed_index, actuator_fixed_index) =
K_piezo_global(actuator_fixed_index, actuator_fixed_index) + K_piezo;
    K_piezo_global(actuator_moving_index, actuator_moving_index) =
K_piezo_global(actuator_moving_index, actuator_moving_index) + K_piezo;

    % Solve for Natural Frequencies
    [V_modes, D_with_piezo] = eig(K_piezo_global, M);
    natural_freqs_with_piezo(v, p) = sqrt(D_with_piezo(1,1)) / (2 * pi);
end
end

fprintf('First Natural Frequency: %.2f Hz\n', natural_freqs_with_piezo(1,1));

%% **Plot Mode Shapes for a Cantilever Beam**
figure;
beta = [1.875, 4.694, 7.855]; % First three mode constants for cantilever beam

for mode = 1:3 % Only first three modes
    subplot(3, 1, mode);

    % Extract and normalize the eigenvector
    mode_shape = V_modes(:, mode) / max(abs(V_modes(:, mode))));

    % Enforce Cantilever Boundary Condition (Zero Displacement at Fixed End)
    mode_shape(1) = 0;

    % Correct the shape using cantilever beam analytical mode function
    x_vals = linspace(0, L, N);
    analytical_mode_shape = cosh(beta(mode) * x_vals / L) - cos(beta(mode) * x_vals /
L) ...
- ((cosh(beta(mode)) - cos(beta(mode)))) / (sinh(beta(mode))
- sin(beta(mode)))) ...
* (sinh(beta(mode) * x_vals / L) - sin(beta(mode) * x_vals
/ L));

    % Normalize the mode shape
    analytical_mode_shape = analytical_mode_shape / max(abs(analytical_mode_shape));

    % Flip the sign if necessary
    analytical_mode_shape = -analytical_mode_shape;

    % Plot the corrected mode shape
    plot(x_vals, analytical_mode_shape, 'LineWidth', 2);
    title(['Mode ', num2str(mode), ' Shape (Cantilever Beam)']);
    xlabel('Beam Length (m)');
    ylabel('Normalized Displacement');
    grid on;
end

```

```

%% **Deformation and Stress Distribution Analysis**
% Define Applied Load (Uniformly Distributed Load)
q = 46563.3; % N/m^2
x = linspace(0, L, 100);
y_deflection = -(q / (24 * E * I)) .* (x.^2) .* (x.^2 - 4 * L * x + 6 * L^2);% Beam
bending equation

% Plot Deformation
figure;
plot(x, y_deflection, 'r', 'LineWidth', 2);
title('Beam Deformation under Load');
xlabel('Beam Length (m)');
ylabel('Deflection (m)');
grid on;

% Calculate Maximum Stress
sigma_max = (q * L^2) / (8 * I);
fprintf('Maximum Stress: %.2f Pa\n', sigma_max);

%% **Plot Frequency vs Applied Voltage for Different Patch Sizes**
figure;
hold on;
for p = 1:length(patch_sizes)
    plot(V_p, natural_freqs_with_piezo(:, p), '-o', 'LineWidth', 2, ...
        'DisplayName', sprintf('Patch Size: %.6f m^2', patch_sizes(p)));
end
xlabel('Applied Voltage (V)');
ylabel('Natural Frequency (Hz)');
title('Effect of Actuator Patch Size (Square) on Natural Frequency');
legend('Location', 'northwest');
grid on;
hold off;

print('square_beam1.png', '-dpng', '-r600'); % 600 DPI PNG

```

Appendix C: The MATLAB code for the analysis of the effect of the rectangular patch area.

```

clc; clear; close all;

%% **Beam Properties**
% Define beam material: Aluminum 6061-T6 and Composite (General Properties)
beam_type = 'aluminum'; % Change to 'composite' for composite beam

if strcmp(beam_type, 'aluminum')
    E = 68.9e9; % Young's modulus (Pa) for Aluminum 6061-T6
    rho = 2700; % Density (kg/m^3)
else
    E = 150e9; % Approximate Young's modulus for composite
    rho = 1600; % Approximate density of composite material
end

L = 1.0; % Beam length (m)
W = 0.08; % Beam width (m)
H = 0.02; % Beam height (m)
I = (W * H^3) / 12; % Moment of Inertia

% Piezoelectric Actuator Properties (PZT-5H)
E_piezo = 63e9; % Piezoelectric Young's modulus (Pa)
d31 = 5.92e-10; % Piezoelectric strain constant (m/V)
t_piezo = 0.3e-3; % Piezo thickness (m)
V_p = linspace(0, 500, 5); % Applied voltage range (V)

% Define Actuator Patch Sizes (Rectangular)
patch_sizes = [0.0009, 0.0012, 0.0015, 0.002, 0.0024]; % Patch areas (m^2)

% Actuator Positions
x_p_fixed = L * 0.1; % Fixed actuator near clamped end
x_p_moving = L * 0.485; % Moving actuator at fixed location

%% **Finite Element Method (FEM) for Natural Frequency Analysis**
N = 10; % Number of elements
dx = L / N; % Element length

% Mass and Stiffness Matrices
M = rho * W * H * dx * eye(N); % Lumped mass matrix
K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1)); % Stiffness matrix

% Find Actuator Positions
actuator_fixed_index = round((x_p_fixed / L) * N);
actuator_moving_index = round((x_p_moving / L) * N);

% Initialize storage for results
natural_freqs_with_piezo = zeros(length(V_p), length(patch_sizes));

%% **Loop Through Different Voltages and Patch Sizes**
for v = 1:length(V_p)
    for p = 1:length(patch_sizes)
        A_patch = patch_sizes(p); % Select current patch size

        % Compute Piezoelectric Stiffness Contribution
    
```

```

K_piezo = d31 * V_p(v) * A_patch * (E_piezo / t_piezo);

% Update Global Stiffness Matrix
K_piezo_global = K; % Start with original stiffness matrix

% Apply piezo stiffness modification at both actuators
K_piezo_global(actuator_fixed_index, actuator_fixed_index) =
K_piezo_global(actuator_fixed_index, actuator_fixed_index) + K_piezo;
K_piezo_global(actuator_moving_index, actuator_moving_index) =
K_piezo_global(actuator_moving_index, actuator_moving_index) + K_piezo;

% Solve for Natural Frequencies
[V_modes, D_with_piezo] = eig(K_piezo_global, M);
natural_freqs_with_piezo(v, p) = sqrt(D_with_piezo(1,1)) / (2 * pi);
end
end

fprintf('First Natural Frequency: %.2f Hz\n', natural_freqs_with_piezo(1,1));

%% **Plot Mode Shapes for a Cantilever Beam**
figure;
beta = [1.875, 4.694, 7.855]; % First three mode constants for cantilever beam

for mode = 1:3 % Only first three modes
    subplot(3, 1, mode);

    % Extract and normalize the eigenvector
    mode_shape = V_modes(:, mode) / max(abs(V_modes(:, mode))));

    % Enforce Cantilever Boundary Condition (Zero Displacement at Fixed End)
    mode_shape(1) = 0;

    % Correct the shape using cantilever beam analytical mode function
    x_vals = linspace(0, L, N);
    analytical_mode_shape = cosh(beta(mode) * x_vals / L) - cos(beta(mode) * x_vals /
L) ...
- ((cosh(beta(mode)) - cos(beta(mode))) / (sinh(beta(mode))
- sin(beta(mode)))) ...
* (sinh(beta(mode) * x_vals / L) - sin(beta(mode) * x_vals
/ L));

    % Normalize the mode shape
    analytical_mode_shape = analytical_mode_shape / max(abs(analytical_mode_shape));

    % Flip the sign if necessary
    analytical_mode_shape = -analytical_mode_shape;

    % Plot the corrected mode shape
    plot(x_vals, analytical_mode_shape, 'LineWidth', 2);
    title(['Mode ', num2str(mode), ' Shape (Cantilever Beam)']);
    xlabel('Beam Length (m)');
    ylabel('Normalized Displacement');
    grid on;
end

```

```

%% **Deformation and Stress Distribution Analysis**
% Define Applied Load (Uniformly Distributed Load)
q = 46563.3; % N/m^2
x = linspace(0, L, 100);
y_deflection = (-q / (24 * E * I)) .* (x.^2) .* (x.^2 - 4 * L * x + 6 * L^2); % Beam
bending equation

% Plot Deformation
figure;
plot(x, y_deflection, 'r', 'LineWidth', 2);
title('Beam Deformation under Load');
xlabel('Beam Length (m)');
ylabel('Deflection (m)');
grid on;

% Calculate Maximum Stress
sigma_max = (q * L^2) / (8 * I);
fprintf('Maximum Stress: %.2f Pa\n', sigma_max);

%% **Plot Frequency vs Applied Voltage for Different Patch Sizes**
figure;
hold on;
for p = 1:length(patch_sizes)
    plot(V_p, natural_freqs_with_piezo(:, p), '-o', 'LineWidth', 2, ...
        'DisplayName', sprintf('Patch Size: %.6f m^2', patch_sizes(p)));
end
xlabel('Applied Voltage (V)');
ylabel('Natural Frequency (Hz)');
title('Effect of Actuator Patch Size (Rectangular) on Natural Frequency');
legend('Location', 'northwest');
grid on;
hold off;

print('rec_beam1.png', '-dpng', '-r600'); % 600 DPI PNG

```

Appendix D: The MATLAB code for the analysis of the effect of the piezoelectric material types.

```
clc; clear; close all;

%% **Beam Properties**
% Define beam material: Aluminum 6061-T6 and Composite (General Properties)
beam_type = 'aluminum'; % Change to 'composite' for composite beam

if strcmp(beam_type, 'aluminum')
    E = 68.9e9; % Young's modulus (Pa) for Aluminum 6061-T6
    rho = 2700; % Density (kg/m^3)
else
    E = 150e9; % Approximate Young's modulus for composite
    rho = 1600; % Approximate density of composite material
end

L = 1.0; % Beam length (m)
W = 0.08; % Beam width (m)
H = 0.02; % Beam height (m)
I = (W * H^3) / 12; % Moment of Inertia

% Piezoelectric Actuator Properties for Different Materials
piezo_materials = {'Quartz LH 1949 IRE', 'PZT-5H', 'BaTiO3', 'AlN'};
E_piezo_values = [30.34e9, 63e9, 125e9, 330e9]; % Young's modulus (Pa)
d31_values = [2.3e-12, 5.92e-10, 9.17e-10, 1.64e-9]; % Piezoelectric strain constant (m/V)

t_piezo = 0.3e-3; % Piezo thickness (m)
V_p = linspace(0, 500, 5); % Applied voltage range (V)
A_patch = 0.0036; % Fixed patch size (m^2)

% Actuator Positions
x_p_fixed = L * 0.1; % Fixed actuator near clamped end
x_p_moving = L * 0.485; % Moving actuator at fixed location

%% **Finite Element Method (FEM) for Natural Frequency Analysis**
N = 10; % Number of elements
dx = L / N; % Element length

% Mass and Stiffness Matrices
M = rho * W * H * dx * eye(N); % Lumped mass matrix
K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1)); % Stiffness matrix

% Find Actuator Positions
actuator_fixed_index = round((x_p_fixed / L) * N);
actuator_moving_index = round((x_p_moving / L) * N);

% Initialize storage for results
natural_freqs_with_piezo = zeros(length(V_p), length(piezo_materials));

%% **Loop Through Different Voltages and Piezo Materials**
for v = 1:length(V_p)
    for m = 1:length(piezo_materials)
        E_piezo = E_piezo_values(m);
        d31 = d31_values(m);
```

```

    % Compute Piezoelectric Stiffness Contribution
    K_piezo = d31 * V_p(v) * A_patch * (E_piezo / t_piezo);

    % Update Global Stiffness Matrix
    K_piezo_global = K; % Start with original stiffness matrix

    % Apply piezo stiffness modification at both actuators
    K_piezo_global(actuator_fixed_index, actuator_fixed_index) =
K_piezo_global(actuator_fixed_index, actuator_fixed_index) + K_piezo;
    K_piezo_global(actuator_moving_index, actuator_moving_index) =
K_piezo_global(actuator_moving_index, actuator_moving_index) + K_piezo;

    % Solve for Natural Frequencies
    [V_modes, D_with_piezo] = eig(K_piezo_global, M);
    natural_freqs_with_piezo(v, m) = sqrt(D_with_piezo(1,1)) / (2 * pi);
end
end

fprintf('First Natural Frequency: %.2f Hz\n', natural_freqs_with_piezo(1,1));

%% **Solve for Mode Shapes (Baseline Without Piezo Actuator)**
[V_modes, ~] = eig(K, M); % Compute mode shapes for original beam

%% **Plot Mode Shapes for a Cantilever Beam**
figure;
beta = [1.875, 4.694, 7.855]; % First three mode constants for cantilever beam

for mode = 1:3 % Only first three modes
    subplot(3, 1, mode);

    % Extract and normalize the eigenvector
    mode_shape = V_modes(:, mode) / max(abs(V_modes(:, mode))));

    % Enforce Cantilever Boundary Condition (Zero Displacement at Fixed End)
    mode_shape(1) = 0;

    % Correct the shape using cantilever beam analytical mode function
    x_vals = linspace(0, L, N);
    analytical_mode_shape = cosh(beta(mode) * x_vals / L) - cos(beta(mode) * x_vals /
L) ...
- ((cosh(beta(mode)) - cos(beta(mode))) / (sinh(beta(mode))
- sin(beta(mode)))) ...
* (sinh(beta(mode) * x_vals / L) - sin(beta(mode) * x_vals
/ L));

    % Normalize the mode shape
    analytical_mode_shape = analytical_mode_shape / max(abs(analytical_mode_shape));

    % Flip the sign if necessary
    analytical_mode_shape = -analytical_mode_shape;

    % Plot the corrected mode shape
    plot(x_vals, analytical_mode_shape, 'LineWidth', 2);
    title(['Mode ', num2str(mode), ' Shape (Cantilever Beam)']);

```

```

        xlabel('Beam Length (m)');
        ylabel('Normalized Displacement');
        grid on;
end

%% **Deformation and Stress Distribution Analysis**
% Define Applied Load (Uniformly Distributed Load)
q = 46563.3; % N/m^2
x = linspace(0, L, 100);
y_deflection = (-q / (24 * E * I)) .* (x.^2) .* (x.^2 - 4 * L * x + 6 * L^2); % Beam
bending equation

% Plot Deformation
figure;
plot(x, y_deflection, 'r', 'LineWidth', 2);
title('Beam Deformation under Load');
xlabel('Beam Length (m)');
ylabel('Deflection');
grid on;

% Calculate Maximum Stress
sigma_max = (q * L^2) / (8 * I);
fprintf('Maximum Stress: %.2f Pa\n', sigma_max);

%% **Plot Frequency vs Applied Voltage for Different Piezo Materials**
figure;
hold on;
for m = 1:length(piezo_materials)
    plot(V_p, natural_freqs_with_piezo(:, m), '-o', 'LineWidth', 2, ...
        'DisplayName', piezo_materials{m});
end
xlabel('Applied Voltage (V)');
ylabel('Natural Frequency (Hz)');
title('Effect of Different Piezoelectric Materials on Natural Frequency');
legend('Location', 'northwest');
grid on;
hold off;

print('mat_piezo1.png', '-dpng', '-r600'); % 600 DPI PNG

```

Appendix E: The MATLAB code for Wing Structure Material on Actuation Performance.

```
clc; clear; close all;

%% Material Properties
materials = {
    'Aluminum 6061', 70.3e9, 2700;
    'Steel (AISI 1020)', 200e9, 7850;
    'Magnesium Alloy', 45e9, 1800;
    'Titanium Alloy (Ti-6Al-4V)', 113e9, 4430
};

% Beam Geometry
L = 1.0; W = 0.08; H = 0.02;
I = (W * H^3) / 12;

% Actuator Properties (PZT-5H)
E_piezo = 63e9; d31 = 5.92e-10;
t_piezo = 0.3e-3; A_patch = 0.0016;
V_p = linspace(0, 500, 5);
x_p_fixed = L * 0.1;
x_p_moving = [0.485, 0.585, 0.685, 0.785, 0.9] * L;

% FEM setup
N = 10; dx = L / N;
actuator_fixed_index = round((x_p_fixed / L) * N);
actuator_moving_indices = round((x_p_moving / L) * N);

%% Loop Over Materials
figure;
tiledlayout(length(materials),1);
colors = lines(length(x_p_moving));

for m = 1:size(materials,1)
    material_name = materials{m,1};
    E = materials{m,2}; rho = materials{m,3};

    % Mass and stiffness matrices
    M = rho * W * H * dx * eye(N);
    K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1));

    natural_freqs = zeros(length(V_p), length(x_p_moving));

    for v = 1:length(V_p)
        for p = 1:length(x_p_moving)
            actuator_index = actuator_moving_indices(p);
            K_piezo = d31 * V_p(v) * A_patch * (E_piezo / t_piezo);
            K_mod = K;
            K_mod(actuator_fixed_index, actuator_fixed_index) =
            K_mod(actuator_fixed_index, actuator_fixed_index) + K_piezo;
            K_mod(actuator_index, actuator_index) = K_mod(actuator_index,
            actuator_index) + K_piezo;
            [~, D] = eig(K_mod, M);
            natural_freqs(v, p) = sqrt(D(1,1)) / (2 * pi); % First mode
        end
    end
end
```

```

end

% Plotting
nexttile;
hold on;
for p = 1:length(x_p_moving)
    plot(V_p, natural_freqs(:,p), '-o', 'Color', colors(p,:), ...
        'DisplayName', sprintf('Pos %.3f m', x_p_moving(p)));
end
hold off;
grid on; legend('Location', 'northwest');
title(sprintf('Natural Frequency vs Voltage (%s)', material_name));
xlabel('Voltage (V)'); ylabel('Frequency (Hz)');
end

%% Summary Plot Across Materials (at 500V)
figure; hold on;
for m = 1:size(materials,1)
    material_name = materials{m,1};
    E = materials{m,2}; rho = materials{m,3};

    % Mass and stiffness matrices
    M = rho * W * H * dx * eye(N);
    K = (E * I / dx^3) * full(gallery('tridiag', N, -1, 2, -1));

    freqs_0V = zeros(1, length(x_p_moving));
    freqs_500V = zeros(1, length(x_p_moving));

    for p = 1:length(x_p_moving)
        actuator_index = actuator_moving_indices(p);
        K_piezo = d31 * 500 * A_patch * (E_piezo / t_piezo);
        K_mod = K;
        K_mod(actuator_fixed_index, actuator_fixed_index) =
        K_mod(actuator_index, actuator_index) + K_piezo;
        K_mod(actuator_index, actuator_index) = K_mod(actuator_index, actuator_index)
        + K_piezo;
        [~, D500] = eig(K_mod, M);
        freqs_500V(p) = sqrt(D500(1,1)) / (2 * pi);
        [~, D0] = eig(K, M);
        freqs_0V(p) = sqrt(D0(1,1)) / (2 * pi);
    end

    delta_freq = freqs_500V - freqs_0V;
    plot(x_p_moving, delta_freq, '-o', 'LineWidth', 2, 'DisplayName', material_name);
end

xlabel('Actuator Position (m)');
ylabel('Frequency Shift (Hz)');
title('Frequency Gain at 500V Across Materials');
legend('Location', 'northeast'); grid on;

print('freq_shift_multi_materials.png', '-dpng', '-r600');

```

Appendix F: The MATLAB code flutter speed analysis with the piezoelectric orientation angle.

```
clc; clear; close all;

% === Beam and Actuator Geometry ===
L = 1.0;    % Beam length (m)
W = 0.08;   % Beam width (m)
beam_L_mm = L * 1000;
beam_W_mm = W * 1000;

% Actuator patch size (30 mm x 40 mm)
patch_L = 60; % mm (along beam length)
patch_W = 60; % mm (along beam width)

% Actuator positions (centered in width, 20 mm and 900 mm along length)
act1_x = 0.06 * 1000; % Fixed actuator (left)
act2_x = 0.485 * 1000; % Movable actuator (right)
act_y_center = beam_W_mm / 2;

% Angles in degrees
angles_deg = [0, 30, 45, 60, 90];

% === Plot and Save Each Orientation ===
figure('Color', 'w', 'Position', [300, 100, 1000, 300]);

for i = 1:length(angles_deg)
    angle = angles_deg(i);
    theta = deg2rad(angle);

    clf; hold on;

    % Draw the beam
    rectangle('Position', [0, 0, beam_L_mm, beam_W_mm], ...
        'EdgeColor', 'k', 'LineWidth', 2);

    % Draw both actuators at the current angle
    draw_rotated_patch(act1_x, act_y_center, patch_L, patch_W, theta, [0.2 0.6 1]);
    draw_rotated_patch(act2_x, act_y_center, patch_L, patch_W, theta, [0.2 0.6 1]);

    % Annotate
    title(sprintf('Actuator Orientation: \theta = %d°', angle), 'FontSize', 14);
    text(beam_L_mm/2, -30, 'Press any key for next orientation...', ...
        'HorizontalAlignment', 'center', 'FontSize', 10);

    xlim([-50, beam_L_mm + 50]);
    ylim([-50, beam_W_mm + 50]);
    axis equal;
    axis off;
    drawnow;

    % Save figure as PNG (600 DPI)
    filename = sprintf('orientangle_%d.png', angle);
    print(filename, '-dpng', '-r600');

    % Wait for user input before proceeding
```

```

        waitforbuttonpress;
    end

    disp('All actuator orientations visualized and saved.');
```

% === Helper Function to Draw Rotated Patch ===

```

function draw_rotated_patch(xc, yc, w, h, theta, color)
    % Define rectangle corners centered at (0, 0)
    X = [-w/2, w/2, w/2, -w/2];
    Y = [-h/2, -h/2, h/2, h/2];

    % Apply rotation
    R = [cos(theta), -sin(theta); sin(theta), cos(theta)];
    rotated = R * [X; Y];

    % Shift to (xc, yc)
    Xr = rotated(1, :) + xc;
    Yr = rotated(2, :) + yc;

    % Plot patch
    patch(Xr, Yr, color, 'EdgeColor', 'k', 'LineWidth', 1.2);
end
```

%% === FEM (ANSYS) Mode Shape Data ===

```

x_fem = linspace(0, 1.0, 11)'; % Nodal positions (11 nodes)
fem_shapes_raw = [ ...
    0.0000, 0.0000, 0.0000;
    -0.0973, 0.0951, -0.1483;
    -0.2041, 0.2945, -0.3923;
    -0.3101, 0.5367, -0.6177;
    -0.4055, 0.7477, -0.7932;
    -0.4812, 0.8789, -0.9004;
    -0.5303, 0.9119, -0.9378;
    -0.5485, 0.8507, -0.9195;
    -0.5344, 0.7094, -0.8662;
    -0.4911, 0.5074, -0.7964;
    -0.4242, 0.2667, -0.7222];

fem_freqs = [16.574, 65.091, 103.72]; % Frequencies from ANSYS (Hz)
```

%% === Beam Properties for Analytical Calculation ===

```

L = 1.0; W = 0.08; H = 0.02;
E = 68.9e9; rho = 2700;
A = W * H;
I = (W * H^3)/12;
m = rho * A;

% Mode shape constants
beta = [1.875, 4.694, 7.855];

% Analytical frequencies
analytical_freqs = (beta.^2) .* sqrt(E * I / (rho * A * L^4)) / (2 * pi);

%% === Plot Mode Shapes Comparison ===
figure;
```

```

for i = 1:3
    subplot(3, 1, i); hold on;

    b = beta(i);
    analytical = cosh(b * x_fem / L) - cos(b * x_fem / L) ...
        - ((cosh(b) - cos(b)) / (sinh(b) - sin(b))) ...
        * (sinh(b * x_fem / L) - sin(b * x_fem / L));
    analytical = - analytical / max(abs(analytical)); % normalize

    u = fem_shapes_raw(:, i);
    u = u / max(abs(u));
    if corr(u, analytical) < 0
        u = -u;
    end

    plot(x_fem, analytical, 'r--', 'LineWidth', 2, ...
        'DisplayName', sprintf('Analytical (%.2f Hz)', analytical_freqs(i)));
    plot(x_fem, u, 'bo-', 'LineWidth', 2, ...
        'DisplayName', sprintf('FEM (ANSYS %.2f Hz)', fem_freqs(i)));

    title(sprintf('Mode %d Shape Comparison', i));
    xlabel('Beam Length (m)');
    ylabel('Normalized Displacement');
    if i == 3
        legend('Location', 'southeast', 'Box', 'off');
    else
        legend('Location', 'north', 'Box', 'off');
    end
    grid on;
end

print('modeshape_compare.png', '-dpng', '-r600'); % 600 DPI PNG

%% === Flutter Speed Analysis for Piezo Configurations ===
angles_deg = [0, 30, 45, 60, 90];
theta_rad = deg2rad(angles_deg);
piezo_factor = cos(theta_rad).^2;

% Aerodynamics
CL_alpha = 5.73;
rho_air = 1.225;
b = W / 2;

rows = {};
for a = 1:length(angles_deg)
    factor = piezo_factor(a);
    E_eff = E * factor;
    k_alpha = E_eff * I / L^3;
    omega_n = sqrt(k_alpha / m);

    U_th = sqrt((2 * k_alpha) / (rho_air * b^2 * CL_alpha));
    U_unsteady = sqrt((k_alpha + m * omega_n^2 * b^2) / ...
        (rho_air * b^2 * CL_alpha));

    rows{end+1, 1} = angles_deg(a);
end

```

```

    rows{end, 2} = U_th;
    rows{end, 3} = U_unsteady;
    rows{end, 4} = analytical_freqs(1);
    rows{end, 5} = analytical_freqs(2);
    rows{end, 6} = analytical_freqs(3);
end

%% === Output Flutter Table (without Configuration) ===
FlutterTable = cell2table(rows, ...
    'VariableNames', {'Angle_deg', ...
        'Flutter_Theodorsen_mps', 'Flutter_Unsteady_mps', ...
        'Freq_Mode1_Hz', 'Freq_Mode2_Hz', 'Freq_Mode3_Hz'});
disp(FlutterTable);

```

Appendix G: The MATLAB code flutter speed analysis with the piezoelectric orientation sequence configuration.

```

clc; clear; close all;

clc; clear; close all;

% Configurations
configs = {
    'Symmetric',      [45, 0, 0, 45],      700.57, 495.77, [16.319, 102.28,
286.4];
    'Anti-symmetric', [45, 0, 0, -45],     700.57, 495.77, [NaN, NaN, NaN];
    'Fully aligned',  [0, 0, 0, 0],        808.94, 572.47, [NaN, NaN, NaN];
    'Cross-biased',   [30, -30, 30, -30],  700.57, 495.77, [NaN, NaN, NaN];
};

nConfigs = size(configs, 1);
layerCount = 4;

% Create lighter color map from jet()
angleRange = -45:1:90;
dark_colors = jet(length(angleRange));      % Original colormap
light_colors = 0.6 * dark_colors + 0.4;     % Lightened version

for i = 1:nConfigs
    figure('Color', 'w', 'Position', [300 300 400 250]);
    hold on;

    configName = configs{i,1};
    angles = configs{i,2};
    U_theod = configs{i,3};
    U_unsteady = configs{i,4};
    freqs = configs{i,5};

    % Draw each layer with lighter color
    for j = 1:layerCount
        angle = angles(j);
        idx = find(angleRange == angle);
        if isempty(idx), idx = 1; end
        c = light_colors(idx, :);

        y0 = (layerCount - j) * 10;
        rectangle('Position', [0, y0, 30, 10], ...
            'FaceColor', c, 'EdgeColor', 'k');
        text(15, y0 + 5, sprintf('%d°', angle), ...
            'HorizontalAlignment', 'center', 'FontSize', 9);
    end

    % Add configuration info
    infoY = -15;
    text(15, infoY, configName, 'HorizontalAlignment', 'center', 'FontWeight',
'bold');
    text(15, infoY - 10, sprintf('U_{Th} = %.2f m/s', U_theod),
'HorizontalAlignment', 'center');

```

```

    text(15, infoY - 20, sprintf('U_{unsteady} = %.2f m/s', U_unsteady),
'HorizontalAlignment', 'center');

    if ~any(isnan(freqs))
        text(15, infoY - 35, sprintf('Mode 1: %.2f Hz', freqs(1)),
'HorizontalAlignment', 'center');
        text(15, infoY - 45, sprintf('Mode 2: %.2f Hz', freqs(2)),
'HorizontalAlignment', 'center');
        text(15, infoY - 55, sprintf('Mode 3: %.2f Hz', freqs(3)),
'HorizontalAlignment', 'center');
    end

    axis off;
    xlim([0, 30]);
    ylim([-70, 50]);
    title('Orientation Sequence Visualization');

    % Save each figure as a high-res PNG
    safeName = lower(strrep(configName, ' ', '_')); % e.g., 'fully aligned' ->
'fully_aligned'
    filename = sprintf('sequence_%s.png', safeName);
    print(filename, '-dpng', '-r600');
    close;
end

disp('All configuration visuals saved with light-colored patches.');
```

```

%% === FEM (ANSYS) Mode Shape Data ===
x_fem = linspace(0, 1.0, 11)'; % Nodal positions (11 nodes)
fem_shapes_raw = [ ...
    0.0000, 0.0000, 0.0000;
    -0.0973, 0.0951, -0.1483;
    -0.2041, 0.2945, -0.3923;
    -0.3101, 0.5367, -0.6177;
    -0.4055, 0.7477, -0.7932;
    -0.4812, 0.8789, -0.9004;
    -0.5303, 0.9119, -0.9378;
    -0.5485, 0.8507, -0.9195;
    -0.5344, 0.7094, -0.8662;
    -0.4911, 0.5074, -0.7964;
    -0.4242, 0.2667, -0.7222];
fem_freqs = [16.574, 65.091, 103.72]; % Frequencies from ANSYS (Hz)

%% === Beam Properties ===
L = 1.0; W = 0.08; H = 0.02;
E = 68.9e9; rho = 2700;
A = W * H;
I = (W * H^3)/12;
m = rho * A;

%% === Analytical Frequencies and Mode Constants ===
beta = [1.875, 4.694, 7.855];
analytical_freqs = (beta.^2) .* sqrt(E * I / (rho * A * L^4)) / (2 * pi);

%% === Plot Mode Shape Comparison ===
```

```

figure;
for i = 1:3
    subplot(3, 1, i); hold on;
    b = beta(i);
    analytical = cosh(b * x_fem / L) - cos(b * x_fem / L) ...
        - ((cosh(b) - cos(b)) / (sinh(b) - sin(b))) ...
        * (sinh(b * x_fem / L) - sin(b * x_fem / L));
    analytical = -analytical / max(abs(analytical));

    u = fem_shapes_raw(:, i);
    u = u / max(abs(u));
    if corr(u, analytical) < 0
        u = -u;
    end

    plot(x_fem, analytical, 'r--', 'LineWidth', 2, ...
        'DisplayName', sprintf('Analytical (%.2f Hz)', analytical_freqs(i)));
    plot(x_fem, u, 'bo-', 'LineWidth', 2, ...
        'DisplayName', sprintf('FEM (ANSYS %.2f Hz)', fem_freqs(i)));

    title(sprintf('Mode %d Shape Comparison', i));
    xlabel('Beam Length (m)');
    ylabel('Normalized Displacement');
    legend('Location', 'southwest');
    grid on;
end

%% === Flutter Analysis with Piezo Orientation Sequences ===
% Constants
CL_alpha = 5.73;
rho_air = 1.225;
b = W / 2;

% Define symmetric and antisymmetric angle sequences
angle_configs = {
    'Symmetric',      [45, 0, 0, 45];    % mirrored on each side
    'Antisymmetric', [45, 0, 0, -45];   % opposite signs
    'Fully Aligned',  [0, 0, 0, 0];     % all aligned
    'Cross Biased',   [30, -30, 30, -30] % alternating
};

rows = {};

for c = 1:size(angle_configs, 1)
    config_name = angle_configs{c,1};
    angle_seq = angle_configs{c,2};
    theta_rad = deg2rad(angle_seq);
    piezo_factor = mean(cos(theta_rad).^2); % avg effect

    % Adjust stiffness
    E_eff = E * piezo_factor;
    k_alpha = E_eff * I / L^3;
    omega_n = sqrt(k_alpha / m);

    % Flutter speeds

```

```

U_th = sqrt((2 * k_alpha) / (rho_air * b^2 * CL_alpha));
U_unsteady = sqrt((k_alpha + m * omega_n^2 * b^2) / ...
    (rho_air * b^2 * CL_alpha));

% Store
rows{end+1, 1} = config_name;
rows{end, 2} = strjoin(string(angle_seq), ', ');
rows{end, 3} = U_th;
rows{end, 4} = U_unsteady;
rows{end, 5} = analytical_freqs(1);
rows{end, 6} = analytical_freqs(2);
rows{end, 7} = analytical_freqs(3);
end

%% === Output Table ===
FlutterTable = cell2table(rows, ...
    'VariableNames', {'Configuration', 'Actuator_Angles_Sequences', ...
        'Flutter_Theodorsen_mps', 'Flutter_Unsteady_mps', ...
        'Freq_Mode1_Hz', 'Freq_Mode2_Hz', 'Freq_Mode3_Hz'});

disp(FlutterTable);

```

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