

BAYESIAN HIERARCHICAL MODELING AND EXTREME
VALUE THEORY FOR PREDICTING EXTREME AIR
TEMPERATURE EVENTS

BY

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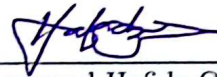
DEANSHIP OF GRADUATE STUDIES

This thesis, written by **BUNJA KOMMA** under the direction of his thesis adviser and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN APPLIED STATISTICS**.

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Dedication

To my beloved uncle, Ba Sainey Camara, thank you for walking this journey with me through every step of the way. Your endless support, quiet sacrifices, and unshakable faith in me have meant more than words can say. This thesis is for you, with all my love and deepest gratitude.

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THESIS ABSTRACT

NAME: BUNJA KOMMA

TITLE OF STUDY: Bayesian Hierarchical Modeling and Extreme Value Theory for Predicting Extreme Air Temperature Events

MAJOR FIELD: APPLIED STATISTICS

DATE OF DEGREE: MAY, 2025

This thesis employs Bayesian Hierarchical Modeling and Extreme Value Theory (EVT) to analyze and predict extreme air temperature events in The Gambia, a region increasingly vulnerable to climate change. Using daily maximum temperature data from ten meteorological stations spanning 2005 to 2022, the research applies the Generalized Extreme Value (GEV) distribution via the Block Maxima (BM) method and the generalized Pareto (GP) distribution via the Peak Over Threshold (POT) approach to model extreme behavior. The results reveal that all the stations exhibit a Weibull-type distribution (shape parameter $\xi < 0$), indicating bounded upper tails for extreme temperatures. A Bayesian framework with non-informative priors and Markov Chain Monte Carlo (MCMC)

methods is used for parameter estimation, ensuring robust and flexible inference. The study identifies significant regional variability, with Basse recording the highest return levels (for example, 45.32°C for a 1000-year return period) and Yundum the lowest (43.68°C), highlighting localized climate dynamics. Nonstationary GEV models incorporating linear and quadratic trends for location and scale parameters further enhance predictive accuracy, validated through likelihood ratio tests and diagnostic plots. The findings underscore the importance of tailored adaptation strategies to mitigate the impacts of extreme temperatures on agriculture, health, and infrastructure in The Gambia. This research bridges a critical gap in regional climate resilience planning.

Keywords: Extreme Value Theory, Bayesian Hierarchical modeling, Generalized Extreme Value Distribution, Generalized Pareto Distribution, Climate change, The Gambia.

ملخص الرسالة

الاسم: بونجا كوما

عنوان الدراسة: نمذجة بيز الهرمية و نظرية القيمة القصوى للتنبؤ بأحداث درجات الحرارة الجوية القصوى

التخصص: الاحصاء التطبيقي

تاريخ الدرجة العلمية: أبريل 2025

تستخدم هذه البحث النمذجة الهرمية لبيز ونظرية القيم المتطرفة لتحليل وتوقع ظواهر درجات الحرارة القصوى في غامبيا، وهي منطقة معرضة بشكل متزايد لتغير المناخ. باستخدام بيانات درجات الحرارة القصوى اليومية من عشر محطات أرصاد جوية تمتد من عام 2005 إلى عام 2022، يطبق البحث توزيع القيم القصوى المعممة باستخدام طريقة الحدود القصوى الكتلية، وتوزيع باريتو المعمم باستخدام نهج تجاوز الذروة لعتبة نمذجة السلوك المتطرف. تكشف النتائج أن جميع المحطات تُظهر توزيعًا من نوع ويبل، مما يشير إلى ذيول عليا محدودة لدرجات الحرارة القصوى. يُستخدم إطار بيز مع مُسبقات غير معلوماتية وأساليب سلسلة ماركوف مونت كارلو لتقدير المعاملات، مما يضمن استدلالًا قويًا ومرنًا. حددت الدراسة تباينًا إقليميًا كبيرًا، حيث سجلت باس أعلى مستويات العائد (32.45 درجة مئوية)، بينما سجلت ياندوم أدنى درجة حرارة (68.43 درجة مئوية)، مما يُبرز ديناميكيات المناخ المحلية. وتُعزز نماذج القيم المتطرفة المعممة غير الثابتة، التي تتضمن اتجاهات خطية وتربيعية في معلمات الموقع والحجم، دقة التنبؤات، التي تم التحقق من صحتها من خلال اختبارات نسبة الاحتمالية والرسوم البيانية التشخيصية. وتؤكد النتائج على أهمية استراتيجيات التكيف المُصممة خصيصًا للتخفيف من آثار درجات الحرارة القصوى على الزراعة والصحة والبنية التحتية في غامبيا. ويسد هذا البحث فجوةً حرجيةً في تخطيط المرونة المناخية الإقليمية.

CHAPTER 1

EXTREME VALUE THEORY AND BAYESIAN MODELING

1.1 Introduction

Human activities like burning fossil fuels, industrial processes, deforestation, and agricultural practices, have largely fueled the increase in global temperatures [1]. Furthermore, the increasing emission of greenhouse gases (GHG), primarily from the industrialized world, has aggravated this trend, leading to increased global temperatures and alterations in other climatic variables, such as rainfall and relative humidity [2]. As a result, extreme temperature events have become more frequent and intense in recent years [1], adversely impacting various aspects of our lives. These has led to droughts, landslides, and floods disrupting agricultural production causing hunger, loss of human and animal lives, and different categories of plants [3]. Recently, The Gambia has experienced highly extreme temperatures, particularly in its rural areas, predominantly occurring from February to June. The significant rise in extreme

temperatures in The Gambia prompts an exploration into the underlying factors and potential consequences of these climatic shifts.

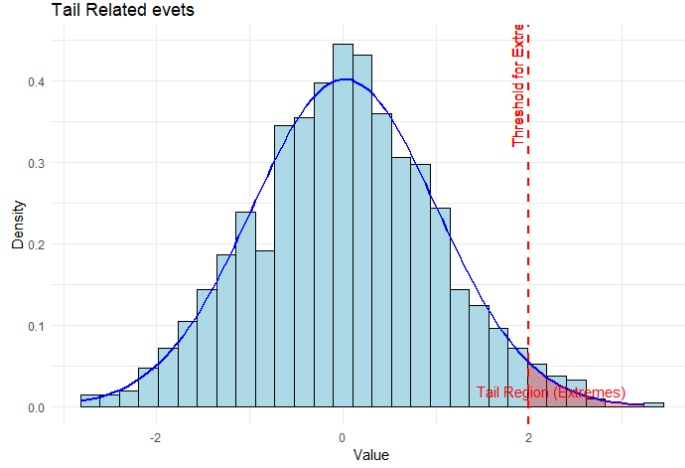


Figure 1.1: **Tail region/ extremes**

Research on extreme climatological events has become a central area of scientific focus, highlighting the increasing awareness of their impact on environmental patterns and resilience of societies [4], [5]. These extremes related events as in Figure 1.1 have a huge impact on diverse sectors of human society and its environment. Moreover, extreme temperatures and climatic disturbances disrupt habitat integrity and imperil biodiversity [6]. In exploring methods to model extreme climatological events, various statistical approaches have been employed, each with its own merits and limitations. Recently, EVT has gained prominence as a statistical tool for modeling of extreme weather events, owing to its effectiveness in capturing rare events and tail distributions [7], [8]. EVT is a statistical field that examines the behavior of extreme events in the limit. Initially developed to model and evaluate risks associated with extreme events [9], EVT has since found application across various disciplines, spanning meteorology [10], hydrology [11], ecology [12], finance [13] and many others.

In practice, modeling extreme values from a set of independent observations generally follows one of two primary approaches. The first method, known as Block Maxima (BM) involves segmenting data into blocks of length n and identifying the maximum values in each block, which is especially useful for large n values. The GEV distribution provides a model for the distribution of these block maxima. However, if other extreme values are available, modeling only the block maxima can lead to inefficiencies by disregarding additional useful data. The second method is the Peaks Over Threshold (POT) approach, which applies GP distribution to values that exceed a predefined high threshold. When modeling extreme events, the POT method has the advantage of including a greater number of observations in the analysis. EVT has recently found application in various research endeavors focusing on extreme climatic events.

In this thesis, we concentrate on using EVT to characterize instances of extreme temperature. The justification for this study lies in the dearth of research on extreme temperatures in The Gambia, despite extensive work on modeling climatic extremes. Given that economic activities, particularly agricultural production, such as crop farming, are heavily dependent on climate conditions, it is essential to study how extreme temperature events could affect this region. By utilizing EVT, we aim to shed light on the nature and behavior of extreme temperature events in The Gambia.

This study focuses on extreme temperature in The Gambia, a West Africa country with a predominantly tropical climate. The Gambia experiences abundant sunshine, consistently high temperatures, and notable humidity levels throughout the year (World Bank, 2021) [14]. However, seasonal variations in cloud cover influence the amount

of sunshine received, with the country averaging around 6-8 (six to eight) hours of sunlight per day (National Meteorological Service, 2020) [15]. The Gambia's climate is significantly shaped by the Atlantic Ocean and prevailing wind systems, which contribute to temperature fluctuations (Jallow, C., et al. 2019).

Over the years, an increasing trend in daily air temperature has been observed across the country, with the western coastal regions experiencing a more pronounced rise compared to inland areas (IPCC,2021) [16]. The hottest periods are typically recorded from March to May and during the late rainy season from September to October (Njie et al., 2020). Research on extreme temperature is crucial for understanding climate patterns and potential risks associated with extreme heat events. Policymakers, climate researchers, and disaster risk management professionals can utilize this knowledge to develop effective strategies, policies, and public awareness initiatives aimed at mitigating the impact of extreme temperatures on communities and the environment (UNDP, 2022) [17].

The estimation of distribution parameters in this study is conducted using a Bayesian framework with a non-informative prior,utilizing MCMC methods. The theoretical foundation Bayesian analysis for extreme value modeling is well-established and has been extensively discussed in scholarly works, such as those by Coles et al. [18], [19]. This approach is particularly valuable, as it simplifies the estimation process by circumventing the need for complex mathematical calculations, which are often required in traditional estimation methods. A key advantage of the Bayesian MCMC approach lies in its ability to produce reliable parameter estimations through repeated iterations, allowing for convergence over time. This method offers flexibility by in-

incorporating specific initial values into the iterative process, which can enhance the efficiency and robustness of the estimation procedure.

The influence of Bayesian MCMC techniques extends across a wide range of statistical methodologies, demonstrating significant potential for application in various research fields. The theoretical justification for applying a non-informative prior distribution is well supported in the literature [20], [21], and there is extensive research available that highlights both the theoretical foundations and practical applications of MCMC methods. This framework provides a robust and flexible approach for parameter estimation in extreme value analysis, making it highly suitable for this study.

1.2 Problem statement

Throughout the 21st century, our planet has grappled with a myriad of obstacles stemming from the effects of global warming and environmental decline. Projections indicate a considerable decline in rainfall and a notable global temperature increase, ranging from 1.4°C to 5.8°C by the century's end. These alarming trends are primarily ascribed to the escalating concentrations of greenhouse gases in the atmosphere [22] and [23]. Extreme temperature events are emerging as significant challenges for African economies, especially those reliant on rain-fed agriculture, as highlighted by the IPCC (2007). Disasters caused by extreme weather are getting worse every day and becoming a threat to the food security of households across the world, more especially on the continent of Africa [24]. With a substantial portion of the African population already residing in drought-prone areas, the effects of climate change are

anticipated to worsen, potentially leading to a heightened risk of hunger for millions by 2080 [25].

These climatic shifts pose unprecedented challenges to regions like The Gambia, where erratic weather patterns are increasingly evident. Despite their critical importance, there is a lack of a comprehensive understanding of the patterns, trends, and underlying factors driving these extreme events across different regions of the country. This knowledge gap hinders effective decision making, planning, and adaptation strategies aimed at mitigating the adverse impacts of extreme temperatures on human health, agriculture, water resources, infrastructure, and socio-economic well-being. As such, there is a pressing need for comprehensive research and analytical frameworks to understand and predict extreme temperature occurrences in The Gambia.

1.3 Objectives of the study

In this thesis, our main objective is to use extreme value theory to model The Gambia's extreme maximum daily air temperature. To achieve this objective, we:

- design extreme value techniques capable of reliably modeling the changes in air temperature over time in The Gambia, based on the EVT with at least 90% fit effectiveness of describing maximum variability of temperature.
- construct Bayesian hierarchical models that offer accurate inferences and flexibility while also predicting the timing of extreme events, such as return levels for temperature.

- demonstrate the application of these models and conduct Monte Carlo simulations to evaluate the uncertainty surrounding the parameters of the most effective model.

1.4 Extreme value distribution

The Extremal Types Theorem [2], [26], which distinguishes between three different kinds of extreme behavior, is the foundation of extreme value theory. Given the sequence: $W = w_1, w_2, \dots, w_i$, where W_i is the daily maximum air temperature of IID and follow a common distribution function F such that:

$$F(W) = P_r\{W_i \leq w\}, \quad (1.1)$$

The maximum value of the n th block is given by $M_n = \max(w_1, w_2, \dots, w_i)$, which is also relevant. The maximum order statistic M_n of the n th block's cumulative distribution function is taken into consideration such that;

$$\begin{aligned}
P\{M_n \leq w\} &= P\{W_1 \leq w, W_2 \leq w, \dots, W_i \leq w\} \\
&= P[W_1 \leq w] \cdot P[W_2 \leq w] \cdot \dots \cdot P[W_i \leq w] \\
&= \prod_{i=1}^n P[W_i \leq w] \\
&= [P(W \leq w)]^n \\
&= F^n(w)
\end{aligned}$$

It has been noted by [27] and [28], and F^n provides the distribution function $\forall w < \sup\{w; F(w) < 1\}$, $F^n(w) \rightarrow 0$ as $\eta \rightarrow \infty$. Allowing a linear re-scaling prevents the degenerate limit [27].

$$M_\eta^* = \frac{M_\eta - b_\eta}{a_\eta}, \quad (1.2)$$

here $b_\eta = \mu$ and $a_\eta = \frac{\sigma}{\sqrt{\eta}}$, σ represent the population standard deviation and η is sample size. We have to define $a_\eta > 0$ and b_η such that;

$$P_r\left\{\frac{M_\eta - b_\eta}{a_\eta} \leq w\right\} = F(b_\eta w + a_\eta)^n \rightarrow G(w), \quad (1.3)$$

which shows $G(w)$ as a non-degenerate distribution. As stated by [29] the three theorems for non-degenerate G , has been proven and which were initially stated by [26], [29] and later on [19] put forward that for the non-degenerate G to exist, such

that , as $\eta \rightarrow \infty$

$$P_r\left\{\frac{M_\eta - b_\eta}{a_\eta} \leq w\right\} \rightarrow G(w), \quad (1.4)$$

For any non-degenerate distribution G , we can classify G into one of the standard types of distributions based on its form. According to extreme value theory, these limiting distributions for extreme values fall into one of three distinct types:

Type I

$$G(w) = \exp\{-\exp(-w)\}, -\infty < w < \infty \quad (1.5)$$

This distribution is applicable when the extreme values are unbounded in both directions. It is widely used to model extremes in cases where the data does not have a natural boundary.

Type II

$$G(w) = \begin{cases} 0 & ; w \leq 0 \\ \exp(-w^{-\alpha}) & ; w > 0, \alpha > 0 \end{cases} \quad (1.6)$$

This distribution is used when extremes are bounded from below that is for modeling heavy-tailed phenomena. Above, α is a shape parameter that controls the tail behavior of the distribution.

Type III

$$G(w) = \begin{cases} \exp(-w^{-\alpha}) & ; w < 0, \alpha > 0 \\ 1 & ; w \geq 0 \end{cases} \quad (1.7)$$

This distribution is suitable when extremes are bounded from above, as it models

light-tailed behavior by restricting values to remain below a certain threshold.

The distributions found in equation (1.5), equation (1.6), and equation (1.7) referred to as the Gumbel, Fréchet, and Weibull types are known as the "extreme value distributions", respectively. Each type is associated with specific tail behaviors and plays a critical role in extreme value theory by characterizing the limiting behavior of maximum values in large samples. These distributions are combined to form the GEV distribution as applied in the analysis of extreme values.

1.4.1 Techniques for modeling EVT

Block Maxima (BM) and Peak Over Threshold (POT) Method

BM approach involves selecting extreme values by dividing the time series data into specific blocks and extracting the maximum value within each block. For daily time series data, these blocks are often chosen empirically, with common intervals including weekly (7 days per block), monthly (30 days per block), quarterly (60 days per block), or yearly (365 or 366 days per block) selections, as demonstrated in [30]. Potential high threshold observations could be wasted if they share the same block as other observations with higher values, according to a potential problem with the block maxima strategy mentioned in [31]. The peak-over-threshold is thought to be the better option in this context.

Figure 1.2 (a) illustrates that, in the BM method, a maximum value is chosen for each of the intervals between 1 and 4, the value x_1, x_2, x_3 and x_4 have been selected based on the BM method and hence, each value is from a unique block. As discussed below

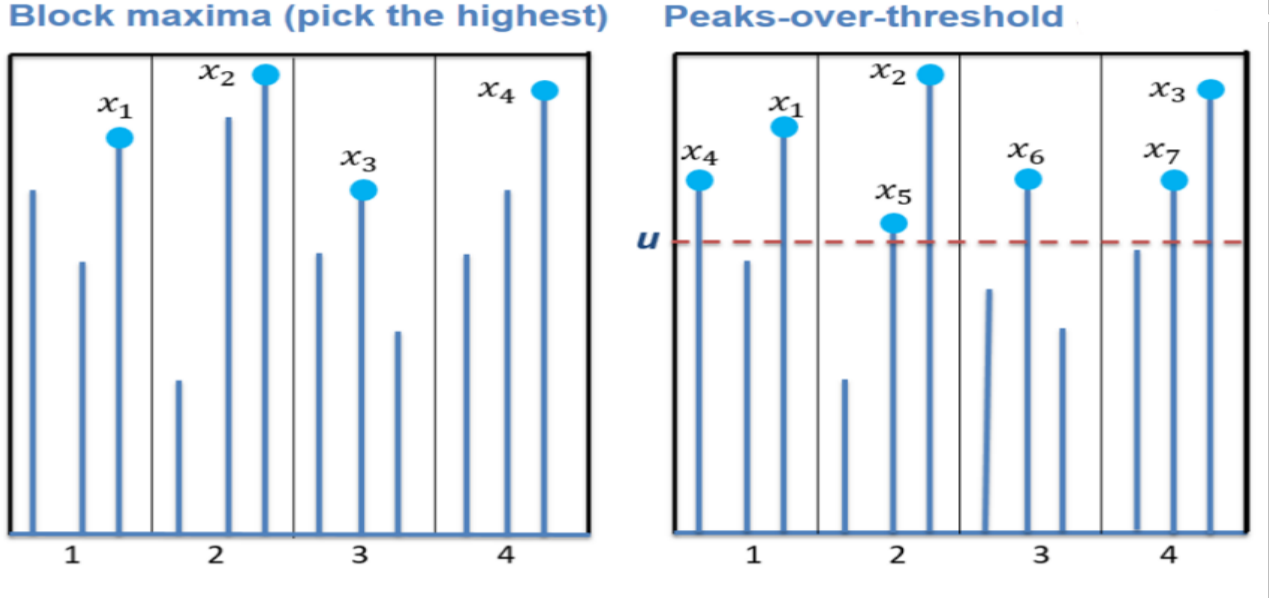


Figure 1.2: **Diagram for BM and POT**

in chapter 3, Figure 1.2(b) POT displays a selection of observations that are all above the threshold u , the value $x_4, x_1, x_2, x_5, x_6, x_7$ and x_3 have been selected by method of the POT and hence, each selected value is a value above the threshold u .

1.5 Area of the study and Data description

The Gambia, located on the west coast of Africa, is the smallest country on the Africa mainland, covering a total land area of $11,300km^2$. With an estimated population of 2.4 million as of 2024 from The Gambia Bureau of Statistics (GBoS), it sits at a latitude of $13^\circ 28'N$ and a longitude of $16^\circ 34'W$. Senegal borders the nation on three sides, and its western boundary is a slender stretch of the Atlantic Ocean. The Gambia experiences a tropical Savannah climate characterized by high temperatures, abundant sunshine, dry conditions, and moderate winds throughout the year. There are two primary seasons in the climate: the dry season, which runs from November to

May, and the rainy season, which runs from June to October. The dry season spans approximately 6 to 7 months with little to no rainfall, during which dry, dusty winds from the Sahara Desert dominate the weather patterns [32]. In this period, temperatures generally range between 18 and 30°C, while in the rainy season, temperatures rise slightly, from 23 to 33°C. Annual rainfall varies across the region, averaging about 700 millimeters in the northern areas and up to 1000 millimeters in the south and southeast.

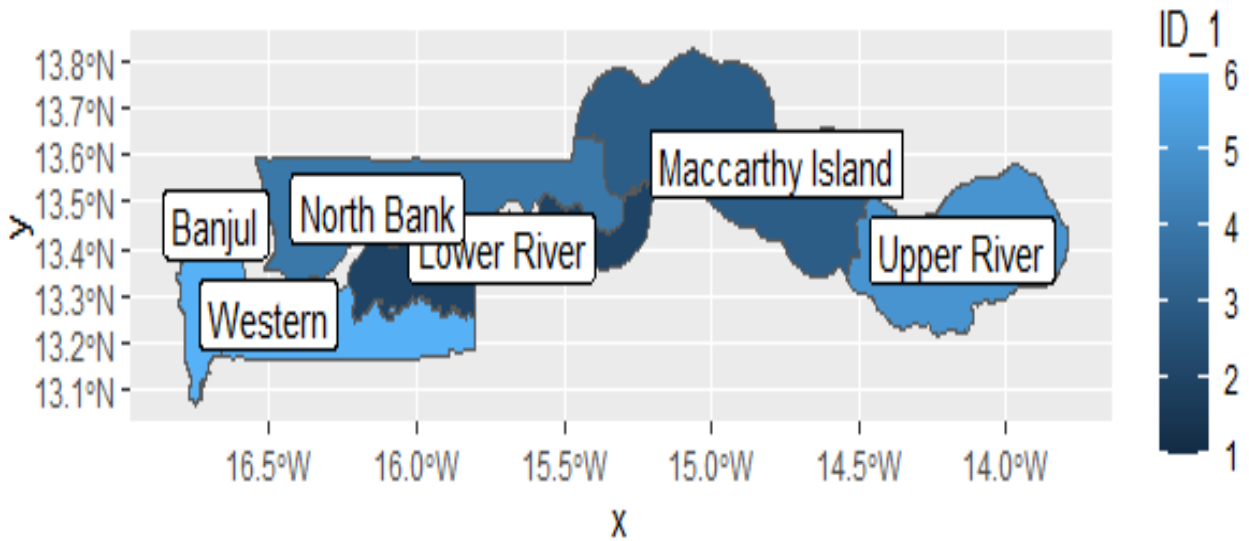


Figure 1.3: **A map of The Gambia showing the main regions and the locations of weather stations within them**

The daily maximum air temperature data were collected from Ten (10) weather stations dispersed throughout the country recorded by Department of Water Resource (DWR) in The Gambia from January 2005 to December 2022. As a result of The Gambia's geographical position, those stations are distributed at random, with varying temperature patterns. This study relies on time series secondary data consisting of daily maximum air temperature readings, recorded in degree Celsius. The data

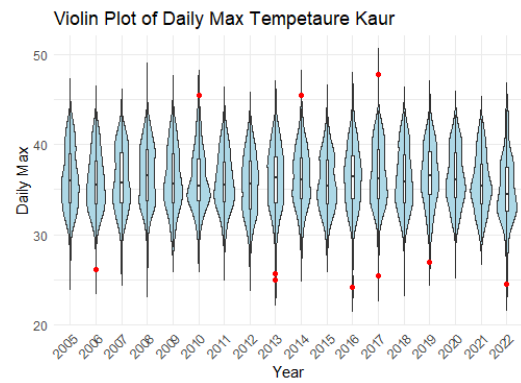
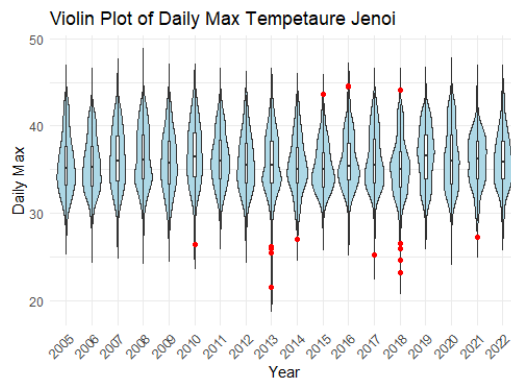
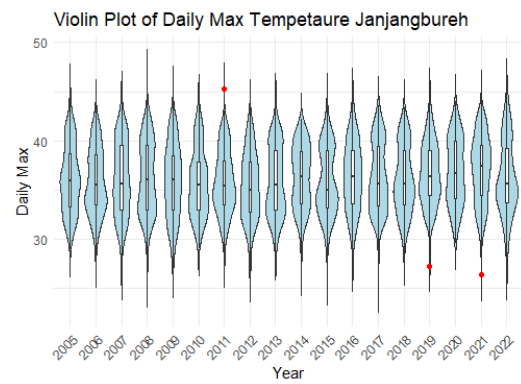
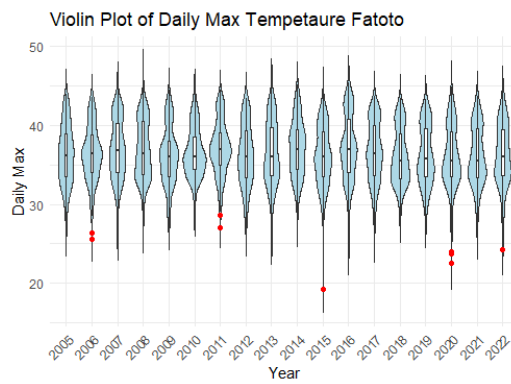
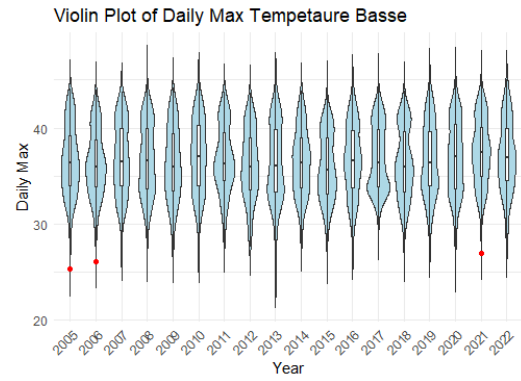
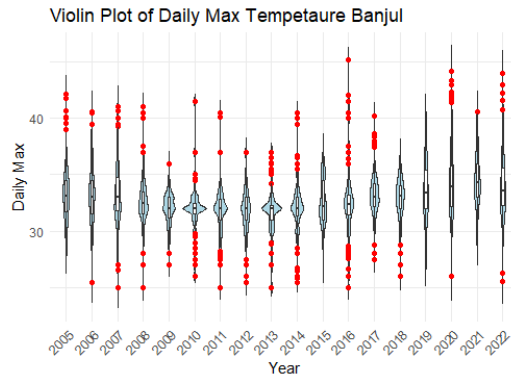
were sourced from the Gambia Water Resources database and represent daily average maximum temperatures. The analysis includes meteorological observations from all major weather stations across The Gambia: Banjul, Basse, Yundum, Kerewan, Fatoto, Janjangbureh, Kaur, Kerewan, Sibanor, Sapu and Jenoï from 2005 – 2022. The year 2005 to 2022 was considered because it is the longest available period that is provided by the Gambian meteorological service.

The dataset contains 6574 observations and one climatic variable for each station. Based on the Table 1.1, Basse recorded the highest 36.61°C followed closely by Fatoto 36.48°C and Sapu 36.28°C . Banjul 32.84°C and Yundum 33.25°C had the lowest mean temperatures, indicating relatively cooler conditions. In terms of maximum recorded temperatures, Kaur 47.70°C and Sapu 47.00°C experience the highest extremes, while Sibanor 45.00°C and Yundum 44.40°C recorded the lowest. This reflects distinct climatic patterns across the regions, with notable differences in both mean and extreme temperatures.

Table 1.1: Summary Statistics for Daily Maximum Data (Rounded to 2 Decimal Places)

Station	Mean	Min	Max	Median	SD	Skewness	Kurtosis
Banjul	32.84	25.00	45.20	32.50	2.30	0.69	1.82
Basse	36.61	24.50	45.50	36.50	3.65	-0.01	-0.73
Fatoto	36.48	19.20	46.50	36.20	3.74	-0.01	-0.44
Janjangbureh	36.12	25.40	46.20	35.90	3.47	0.07	-0.74
Jenoi	35.95	21.50	46.00	35.60	3.29	0.13	-0.33
Kaur	36.07	24.20	47.70	35.90	3.36	0.07	-0.44
Kerewan	35.54	24.70	45.70	35.40	3.16	0.17	-0.23
Sapu	36.28	22.10	47.00	36.20	3.37	-0.04	-0.52
Sibanor	35.27	23.50	45.00	35.00	3.26	0.15	-0.15
Yundum	33.25	21.60	44.40	33.00	2.67	0.48	0.93

We employed Violin plots to the daily maximum air temperature for the ten (10) stations in The Gambia against the year to look for any underlying patterns in the data. These can be seen in Figure (1.4).



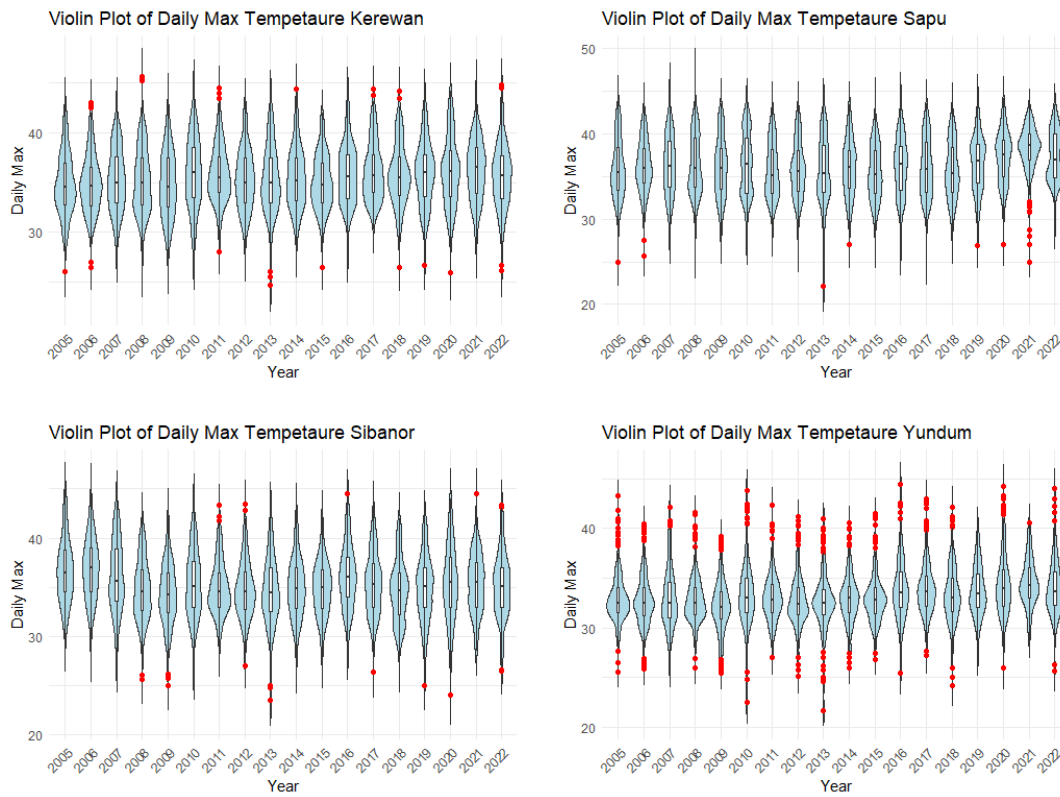


Figure 1.4: Violin plots of Daily maximum air temperatures for the ten stations from 2005 to 2022.

In general, the variation seems to show some form of non-stationarity for all stations with the exception of a few stations in The Gambia. Since environmental factors and processes frequently show this behavior, it is not surprising. This could be explained by seasonal variations in climatic patterns or perhaps by long-term trends brought on by climate change [33]. We used the Mann-Kendall and Cox-Stuart trend test to check for trends and non-stationarity in the daily air temperature maxima, to validate we used the 5% significance level.

1.6 Test for Trend

1.6.1 Mann-Kendall (MK) Test:

The Mann-Kendall (Mk) test is a non-parametric statistical method commonly used to identify trends in datasets, especially when the data do not follow a normal distribution. This test is highly useful for analyzing trends in datasets with missing values, making it versatile for real-world applications. The MK test works by assessing whether there is an increasing or decreasing trend over time, without assuming any specific data distribution. The null hypothesis (H_0) states no trend presence in the data while the alternative (H_1) states there is trend test is performed to show the probability of occurring by chance as indicate by P-value.If the P-value is less than 0.05, a trend is statistically significant at 5% level of significance otherwise it will not be statistically significant when P-value is greater than 5% significance level.

1.6.2 Cox-Stuart (CS) Test:

To further investigate the trend in the daily maximum air temperature, a Cox-Stuart test was conducted. It is also non-parametric method used to detect trends in time series data. In this test, the null hypothesis (H_0) states that there is no trend in the daily maximum air temperature data while alternative (H_1) there is a trend in daily maximum air temperature dataset, indicating an upward or downward pattern overtime. The CS test provides a straightforward way to asses trends without making assumption about the distribution of the dataset, which makes it important for identifying trends in environmental or observational datasets. By performing this test, we gain additional confirmation of wether a significant trend exist in the daily maximum air temperature.

Table 1.2: Mann-Kendall Test Results

Station	Tau	p-value
Banjul	0.132	$< 2.22 \times 10^{-16}$
Basse	0.0102	0.21809
Fatoto	-0.0378	4.6136×10^{-06}
Janjangbureh	0.0329	7.1049×10^{-05}
Jenoi	-0.00636	0.44246
Kaur	-0.0235	0.0044445
Kerewan	0.0566	$< 2.22 \times 10^{-16}$
Sapu	0.0589	$< 2.22 \times 10^{-16}$
Sibanor	-0.0576	3.5129×10^{-12}
Yundum	0.128	$< 2.22 \times 10^{-16}$

Table 1.3: Cox-Stuart Test Results

Station	Statistic	p-value
Banjul	2013	$< 2.2 \times 10^{-16}$
Basse	1643	0.1797
Fatoto	1518	0.0008212
Janjangbureh	1868	$< 2.2 \times 10^{-16}$
Jenoi	1521	0.005805
Kaur	1629	0.8195
Kerewan	1864	$< 2.2 \times 10^{-16}$
Sapu	1792	7.007×10^{-09}
Sibanor	1495	0.0001359
Yundum	2004	$< 2.2 \times 10^{-16}$

CHAPTER 2

GENERALIZED EXTREME VALUE (GEV)

2.1 GEV Distribution

The GEV distribution is a classic choice for modeling EVT, especially when using the BM approach. In this method, the GEV distribution is applied to a data set comprising the maximum values from defined blocks of time within a time series, as outlined in [34]. The GEV distribution effectively combines the types of EVT shown in equation (1.5), equation (1.6), and equation (1.7) into a unified parametric form. [35] demonstrates how these 3 types based on equation (1.3) can be merged to form the GEV distribution, represented by a single parametric family. Given the CDF of the GEV distribution in equation (2.1).

$$G(w) = \exp \left\{ - \left[1 + \xi \left(\frac{w - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}, 1 + \xi \left(\frac{w - \mu}{\sigma} \right) > 0 \quad (2.1)$$

where this form is defined for values satisfying $1 + \xi(w - \mu)/\sigma > 0$. Here, w represent the extreme values in blocks, $\xi \in \Re$, $\sigma > 0$, and $\mu \in \Re$ are the shape parameter that governs the behavior of the tailed distribution, the scale parameter and the location parameter, respectively. The distribution is denoted as $GEV(\mu, \sigma, \xi)$, where the parameter values of μ, σ , and ξ cover each of the three types of extreme value distributions, enabling flexibility to model different extremal behaviors.

The Generalize Extreme Value (GEV) distribution can behave in three different forms depending on the shape (ξ) parameter;

2.1.1 Shape parameter $\xi = 0$:

If the shape ξ parameter in the aforementioned distribution is set to zero, the distribution simplifies to the Gumbel distribution, which is a special case of the GEV distribution. The Gumbel distribution, sometimes referred to as the Laplace distribution and Log-Weibll distribution, is classified as a light-tailed distributions. It is defined over the entire real line and is widely used for fitting the distribution of maximum of a samples of values from various distributions. The CDF and PDF of Gumbel distribution are presented in equation (2.2) and equation (2.3) respectively.

$$G(w) = \exp\left\{-\exp\left(-\frac{w-\mu}{\sigma}\right)\right\}, -\infty < w < \infty. \quad (2.2)$$

$$g(w) = \frac{1}{\sigma} \exp\left\{-\left(\frac{w-\mu}{\sigma} + \exp\left(-\frac{w-\mu}{\sigma}\right)\right)\right\} \quad (2.3)$$

In these equations, μ and σ represents the location and scale parameters. This distribution is useful in modeling extreme values because it provides insights into the behavior of maximum values in large datasets across various domains.

2.1.2 Shape parameter $\xi < 0$:

When the shape parameter ξ in the Generalized Extreme Value (GEV) distribution is less than zero, it results in the Weibull distribution, commonly applied when analyzing minimum values instead of maximums. In this context, the Weibull distribution has an additional parameter compared to the traditional two-parameter Weibull distribution. Since our focus is on maximum values, we employ the reversed Weibull distribution, which provides an upper bound rather than a lower bound. The reversed Weibull can be seen as a reflection of the standard Weibull distribution; while both distributions share similar shapes, they are skewed in opposite directions. The Cumulative distribution function (CDF) of the Weibull depends on the number of parameters it has. The standard Weibull distribution, with zero shift has two parameters (σ, ξ) , whereas with a non-zero shift, it requires three parameters (μ, σ, ξ) . The probability density function (PDF) for the Weibull distribution is shown in equation (2.4), while equation (2.5) and equation (2.6) provide the CDF for two-parameter and three-parameters, respectively.

$$g(w) = \frac{\xi}{\sigma} \left(\frac{w}{\sigma}\right)^{\xi-1} \exp \left\{ -\left(\frac{w}{\sigma}\right)^{\xi} \right\} \quad (2.4)$$

$$G(w) = 1 - \exp \left\{ -\left(\frac{w}{\sigma}\right)^\xi \right\} \quad (2.5)$$

$$G(w) = 1 - \exp \left\{ -\left(\frac{w-\mu}{\sigma}\right)^{-\xi} \right\} \quad (2.6)$$

Here, μ represents the location parameter, σ is the scale parameter, and ξ is the shape parameter. The flexibility of the Weibull distribution in capturing different types of extreme values, along with its parameter variations, makes it well-suited for diverse applications in modeling extreme events.

2.1.3 Shape parameter $\xi > 0$:

When the shape parameter ξ of the GEV distribution is positive, the resulting distribution is the heavy-tailed Fréchet distribution. This distribution is often associated with the Pareto, Student's t, and Cauchy distributions, which are all derived from the Fréchet. As the inverse of the Weibull distribution and a special case within the GEV family, the Fréchet distribution is the only limiting cumulative distribution function (CDF) for maxima defined on the non-negative real numbers that is well-behaved in this regard. It is particularly useful for characterizing random variables with large values, making it valuable in modeling the statistical behavior of material properties in various engineering applications. The probability density function (PDF) for the Fréchet distribution is shown in equation (2.7). For a two-parameter Fréchet distribution, its CDF is given in equation (2.8) while equation (2.9) provides the CDF for a three-parameter version.

$$g(w) = \frac{\xi}{\sigma} \left(\frac{\sigma}{w-\mu} \right)^{\xi-1} \exp \left\{ - \left(\frac{\sigma}{w-\mu} \right)^{\xi} \right\} \quad (2.7)$$

$$G(w) = \exp \left\{ - \left(\frac{\sigma}{w} \right)^{\xi} \right\} \quad (2.8)$$

$$G(w) = \exp \left\{ - \left(\frac{\sigma}{w-\mu} \right)^{\xi} \right\} \quad (2.9)$$

Various estimation methods have been developed for fitting the GEV distribution, including least squares estimation [36], Bayes estimation [37], direct estimation also on order statistics and records [38], maximum likelihood estimation [39], and the method of moments [40]. Each of these approaches offers unique advantages and is suited to specific characteristics of extreme value data.

2.2 MDA (Maximum Domain of Attraction)

In EVT, the concept of MDA describes how the distribution of the maximum values from a set of random variables that are independent and identically distributed (i.i.d.) approaches a specific limiting distribution. This limiting distribution belongs to one of the three EVDs: Fréchet, Gumbel, and Weibull. The particular domain of attraction for each of these distributions is determined by the tail behavior of the original, underlying distribution. In the following, we summarize the MDA for various common distributions, including the Gumbel, Fréchet, Weibull, Exponential, and Normal distributions.

- Gumbel distribution

This distribution models limit of maxima for distributions with **light tails** (exponential decay). Distributions that converge to the Gumbel distribution are said to belong to the Gumbel (MDA). A distribution $F(w)$ is in the MDA of Gumbel distribution when there is a sequence of constant $a_\eta > 0$ and b_η that meet the following condition:

$$\begin{aligned}
\lim_{\eta \rightarrow \infty} P\left(\frac{\text{Max}(W_1, W_2, \dots, W_\eta) - b_\eta}{a_\eta} \leq w\right) &= \lim_{\eta \rightarrow \infty} \frac{1 - F(\eta + w)}{1 - F(\eta)} \\
&= \frac{1 - [\exp(-\exp(-\eta - w))]}{1 - \exp(-\exp(-\eta))} \\
&= \frac{\ln(1) - \ln(\exp(-\exp(-\eta - w)))}{\ln(1) - \ln(\exp(-\exp(-\eta)))} \\
&= \frac{\exp(-\eta) * \exp(-w)}{\exp(-\eta)} \\
&= \exp(-w)
\end{aligned}$$

where $\exp(-e^{-w})$ is the CDF of Gumbel distribution. The following distributions belong to the Gumbel MDA, Normal distribution $F(w) \sim \mathcal{N}(\mu, \sigma^2)$, Exponential distribution $F(w) \sim \exp(\lambda)$ and Log-Normal distribution [41].

- Fréchet distribution

The Fréchet distribution describes the behavior of maxima for distribution with **heavy tails**, where extreme values occur with a non-negligible probability. A distribution $F(w)$ is considered to be in MDA of Fréchet distribution if the tail

decays polynomially. The CDF of the Fréchet distribution

$$F(w) = \begin{cases} 0 & ; w \leq 0 \\ \exp(-w^{-\alpha}) & ; w > 0 \end{cases}$$

where $\alpha > 0$ is the shape parameter that determines the thickness of the tail.

A distribution $F(w)$ belongs to the MDA of the Fréchet distribution if:

$$\begin{aligned} \lim_{\eta \rightarrow \infty} \frac{1 - F(\eta * w)}{1 - F(\eta)} &= \frac{1 - [\exp(-(\eta * w))^{-\alpha}]}{1 - \exp(-\eta)^{-\alpha}} \\ &= \frac{\ln(1) - \ln(\exp(-(\eta * w))^{-\alpha})}{\ln(1) - \ln(\exp(-\eta)^{-\alpha})} \\ &= \frac{(\eta * w)^{-\alpha}}{(\eta)^{-\alpha}} \\ &= w^{-\alpha} \end{aligned}$$

which characterizes the heavy tail. The following distribution belongs to the Fréchet MDA: Pareto distribution $F(W) \sim \text{Pareto}(\alpha)$, Cauchy distribution, and Weibull distribution with shape parameter $k < 1$ [42].

- Pareto distribution

This distribution CDF is $F(w) = 1 - (\frac{k}{w})^\alpha$, here $k > 0$ shows scale parameter, and $\alpha > 0$ represent shape parameter, which also indicates heaviness of the tail (larger α will results in a less heavy tail). The limiting distribution of Pareto $F(w)$ is shown below:

$$\begin{aligned}
\lim_{\eta \rightarrow \infty} \frac{1 - F(\eta * w)}{1 - F(\eta)} &= \frac{1 - [1 - (\frac{k}{\eta * w})^\alpha]}{1 - [1 - (\frac{k}{\eta})^\alpha]} \\
&= \frac{(k/\eta * w)^\alpha}{(k/\eta)^\alpha} \\
&= \frac{1}{w^\alpha} \\
&= w^{-\alpha}
\end{aligned}$$

so, the Pareto CDF belongs to $F(w)$ for Fréchet domain of attraction.

- Weibull distribution

Above distribution is used to model data with finite upper bounds. Distributions that base bounded support converge to the Weibull distribution, and these belong to the Weibull MDA. Below is the CDF of Weibull distribution

$$G(w) = \begin{cases} \exp(-w^{-\alpha}) & ; w < 0 \\ 1 & ; w \geq 0 \end{cases}$$

where $\alpha > 0$ determines the tail behavior. The Weibull distribution is characterized by bounded support, meaning that the random variable has finite upper bound. A distribution $F(w)$ is in the MDA of the Weibull distribution if:

$$\lim_{w \rightarrow w^*} \frac{F(w - w^*)}{F(w^*)} = t^\alpha$$

where w^* represent the upper limit of the support of F [43]. And the following distribution belongs to Weibull MDA; Beta distribution $F(w) \sim Beta(\alpha, \beta)$ and Weibull distribution with shape parameter $k > 1$.

- Exponential distribution

The exponential distribution $F(w) \sim Exp(\lambda)$ models the time between events in a Poisson process and is known distribution for waiting time. It has a **light tail** that decays exponentially, the exponential CDF $F(w) = 1 - \exp(-w)$ maximum domain of attraction is given as:

$$\begin{aligned}
 \lim_{\eta \rightarrow \infty} \frac{1 - F(\eta + w)}{1 - F(\eta)} &= \frac{1 - [1 - \exp(-\eta - w)]}{1 - [-\exp(-\eta)]} \\
 &= \frac{\exp(-\eta - w)}{\exp(-\eta)} \\
 &= \frac{\exp(-\eta) * \exp(-w)}{\exp(-\eta)} \\
 &= \exp(-w)
 \end{aligned}$$

we can conclude that the exponential CDF $F(w)$ belongs to the Gumbel domain of attraction. The maximum of a sample from an exponential distribution converges to the Gumbel distribution as sample size increases [44]. The tail behavior of the exponential distribution decays rapidly as;

$$\lim_{w \rightarrow \infty} \frac{1 - F(w)}{\exp(-\lambda w)} = 1$$

showing that the tail follows an exponential decay.

- Normal distribution

The Normal distribution $F(w) \sim \mathcal{N}(\mu, \sigma^2)$, has **light tails**, meaning rare of extreme value value. As the sample size increases the maxima of a large sample from a normal distribution converge to the Gumbel distribution [45]. The tail of the normal distribution decays exponentially as:

$$\lim_{w \rightarrow \infty} \frac{1 - F(w)}{\exp(-w^2/2)} = 1$$

Distributions are grouped by the Maximum Domain of Attraction that relies on how their tails behave. Gumbel distribution captures light-tailed behavior ,while Fréchet and Weibull distributions handles heavy-tailed and bounding distributions respectively. Each domain provides a limiting form for the maxima of independent and identically distributed samples.

2.3 MML (Method of Maximum Likelihood) of GEV

In our study, we employed the MML to fit the data for each distribution, as it provides a robust and reliable approach for estimating the parameters of the statistical models. This method ensures that the fitted distributions align closely with the observed data, thereby enhancing the accuracy and validity of our analysis.

1. Probability Density Function (PDF)

The PDF is derived by differentiating the CDF:

$$g(w) = \frac{d}{dw}G(w) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{w - \mu}{\sigma} \right) \right]^{-1/\xi - 1} \exp \left\{ - \left[1 + \xi \left(\frac{w - \mu}{\sigma} \right) \right]^{-1/\xi} \right\},$$

with the constraint:

$$1 + \xi \left(\frac{w - \mu}{\sigma} \right) > 0.$$

For the special case $\xi = 0$ (Gumbel case), the PDF simplifies to:

$$g(w) = \frac{1}{\sigma} \exp \left(-\frac{w - \mu}{\sigma} \right) \exp \left[-\exp \left(-\frac{w - \mu}{\sigma} \right) \right].$$

2. Likelihood Function

Given a sample of block maxima $w_1, w_2, \dots, w_n$, the likelihood function is:

$$L(\mu, \sigma, \xi) = \prod_{i=1}^n g(w_i) = \prod_{i=1}^n \frac{1}{\sigma} \left[1 + \xi \left(\frac{w_i - \mu}{\sigma} \right) \right]^{-1/\xi - 1} \exp \left\{ - \left[1 + \xi \left(\frac{w_i - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}.$$

3. Log-Likelihood Function

For computational stability, the log-likelihood is maximized:

$$\ell(\mu, \sigma, \xi) = \log L(\mu, \sigma, \xi) = \sum_{i=1}^n \log g(w_i).$$

Substituting the PDF gives:

$$\ell(\mu, \sigma, \xi) = -n \log \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \log \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right) - \sum_{i=1}^n \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)^{-1/\xi}.$$

In the special case when $\xi = 0$:

$$\ell(\mu, \sigma, 0) = -n \log \sigma - \sum_{i=1}^n \left(\frac{w_i - \mu}{\sigma}\right) - \sum_{i=1}^n \exp\left(-\frac{w_i - \mu}{\sigma}\right).$$

4. Maximizing the Log-Likelihood

The maximum likelihood estimates $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\xi}$ are found by solving:

$$\frac{\partial \ell}{\partial \mu} = 0, \quad \frac{\partial \ell}{\partial \sigma} = 0, \quad \frac{\partial \ell}{\partial \xi} = 0.$$

Partial derivatives:

For μ :

$$\frac{\partial \ell}{\partial \mu} = \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \frac{-\xi/\sigma}{1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)} + \frac{1}{\sigma} \sum_{i=1}^n \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)^{-1/\xi - 1}.$$

For σ :

$$\frac{\partial \ell}{\partial \sigma} = -\frac{n}{\sigma} + \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \frac{-\xi(w_i - \mu)}{\sigma^2 \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)} + \frac{\xi}{\sigma^2} \sum_{i=1}^n (w_i - \mu) \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)^{-1/\xi - 1}.$$

For ξ :

$$\frac{\partial \ell}{\partial \xi} = \sum_{i=1}^n \left[\frac{1}{\xi^2} \log \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right) - \left(1 + \frac{1}{\xi}\right) \frac{(w_i - \mu)/\sigma}{1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)} \right] -$$

$$\sum_{i=1}^n \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)^{-1/\xi} \left[\frac{1}{\xi^2} \log \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right) - \frac{(w_i - \mu)/\sigma}{\xi \left(1 + \xi \left(\frac{w_i - \mu}{\sigma}\right)\right)} \right].$$

2.4 Model Checking Techniques for GEV distribution

2.4.1 Probability Plot

Probability plots are graphical tool for comparing the empirical distribution of the data with the theoretical GEV distribution [19]. We follow the following steps for constructing a probability plot:

- Sort the temperature data in ascending order : $w_{(1)} \leq w_{(2)} \leq w_{(3)} \leq \dots \leq w_{(m)}$
- compute the empirical CDF for each data point: $G(w_{(i)}) = \frac{i}{m+1}$
- compute the theoretical CDF values using the GEV distribution: $G(w_{(i)}, \hat{\mu}, \hat{\sigma}, \hat{\xi})$

$$G(w_{(i)}) = \exp \left\{ - \left[1 + \hat{\xi} \left(\frac{w_{(i)} - \hat{\mu}}{\hat{\sigma}} \right) \right]^{-\frac{1}{\hat{\xi}}} \right\}$$

- plot the empirical CDF against the theoretical CDF
- $$\left\{ \left(G(w_{(i)}, \frac{i}{m+1}) : i = 1, 2, 3, \dots, m \right) \right\}$$

A good fit is indicated by points lying close to the 45 – *degree* line. Deviations, particularly in the tails, suggest that the GEV model may not be appropriate.

2.4.2 Quantile-Quantile plot

Quantile plots focus on comparing the quantiles of observed data with those of the fitted GEV distribution [41]. The steps to follow:

- compute the empirical quantiles of data
- compute the theoretical quantiles using the GEV quantile function:

$$G^{-1}\left(\frac{i}{m+1}\right) = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}} \left\{ 1 - \left[-\log\left(\frac{i}{m+1}\right) \right]^{-\hat{\xi}} \right\}$$

- plot the empirical quantiles against the theoretical quantiles.

$$\left\{ \left(G^{-1}\left(\frac{i}{m+1}\right), w_{(i)} \right) : i = 1, 2, 3, \dots, m \right\}$$

A good fit is indicated by points aligning closely with the 45 – *degree* line. Discrepancies in the upper quantiles suggest issues with the model’s tail behavior.

2.4.3 Return level plot

A return level plot is constructed by graphing the return level w_k against the transformed return period $y_k = -\log(1 - k)$ on a logarithmic scale [19]. The return level w_k is defined as:

$$w_k = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}} \left\{ 1 - [-\log(1 - k)]^{-\hat{\xi}} \right\}$$

The transformed return period $y_k = -\log(1 - k)$ is plotted on the horizontal axis(log scale), while the return level w_k is plotted on the vertical axis. This transformation compresses the tail of the distribution, allowing for a clearer visualization of extreme

events associated with a long return period [41]. The return level plot exhibits a linear relationship when $\xi = 0$, which is the baseline for assessing the affect of the shape parameter ξ . Deviations from the linear relationship indicate the influence of $\xi \neq 0$ either $\xi < 0$ or $\xi > 0$ [43]. The fitted model is represented by the point location $\{(\log y_k, w_k) : 0 < k < 1\}$. The closeness of these points to the observed data provides a measure of goodness fit of the model.

2.4.4 Density plot

Density plots compare the empirical density of the data with the theoretical density of the fitted GEV distribution. The following steps should be considered:

- Estimate the empirical density using histograms
- compute the theoretical GEV density

$$G(w) = \exp \left\{ - \left[1 + \hat{\xi} \left(\frac{w - \hat{\mu}}{\hat{\sigma}} \right) \right]^{-\frac{1}{\hat{\xi}}} \right\}$$

- plot the empirical density and the theoretical density on the same graph.

A good fit is indicated by the empirical density closely following the theoretical density. Discrepancies, especially in the tails, suggest that the model may not adequately capture extreme event behavior [41].

2.5 The GEV Models Based on Time-Varying Linear and Quadratic Trends

To better understand and analyze the trends in the data, as shown in Figure 1.4, we utilized equation (2.5) as noted as Model (I) to investigate how the location and scale parameters varied over time. In this analysis, we kept the shape parameter fixed at a constant value across all levels to simplify the modeling process. The location and scale parameters were tested for potential linear and quadratic variations to capture any underlying trends, using the models described below.

The reference model, denoted as model M_I , is defined such that

$$M_I: = GEV(\mu, \sigma, \xi) : \mu, \sigma, \text{ and } \xi \text{ all constants.}$$

$$M_{II} : = GEV(a, \sigma, \xi, b) : \mu = a + b \times t, \sigma = \text{constant} \text{ and } \xi = \text{constant}$$

$$M_{III} : = GEV(a, \sigma, \xi, b, c) : \mu = a + b \times t + c \times t^2, \sigma = \text{constant}, \text{ and } \xi = \text{constant}$$

$$M_{IV}: = GEV(\mu, a, \xi, b) : \mu = \text{constant}, \sigma = \exp(a + b \times t), \text{ and } \xi = \text{constant}$$

$$M_V: = GEV(\mu, a, \xi, b, c) : \mu = \text{constant}, \sigma = \exp(a + b \times t + c \times t^2), \text{ and } \xi = \text{constant}$$

$$M_{VI} : = GEV(a, c, \xi, b, d) : \mu = a + b \times t, \sigma = \exp(c + d \times t), \text{ and } \xi = \text{constant}$$

$$M_{VII} : = GEV(a, c, \xi, b, d, e) : \mu = a + b \times t + e \times t^2, \sigma = \exp(c + d \times t), \text{ and } \xi = \text{constant}$$

$$M_{VIII} : = GEV(a, c, \xi, b, d, e) : \mu = a + b \times t, \sigma = \exp(c + d \times t + e \times t^2) \text{ and } \xi = \text{constant}$$

$$M_X: = GEV(a, c, \xi, b, d, e, f) : \mu = a + b \times t + e \times t^2, \sigma = \exp(c + d \times t + f \times t^2), \text{ and } \xi = \text{constant}$$

2.5.1 Comparing Nested models

We used the (LRT) Likelihood Ratio Test, which is a statistical test used to compare the fit of two models; a full model (more complex) and a reduced model (simpler, nested within the full model). To evaluate whether a more complex model significantly improves the fit to our temperature data compared to a simpler one. The LRT compares the null hypothesis, with parameters restricted to a subset, and the alternative hypothesis, with parameters in the full space. In our study, the negative log-likelihood is computed for two models of the GEV distribution that are minimized to find the best-fitting parameters. The MLEs with null and alternative are $\hat{H}_0$ and $\hat{H}_1$, respectively. The test statistic $\tau = 2[\ell(H_0) - \ell(H_1)]$ asymptotically follows the χ^2 distribution with degree of freedom k which is the difference in the number of parameters between H_1 and H_0 , such that $T = 2[(-\ell(\hat{H}_0)) - (-\ell(\hat{H}_1))] < \chi_{k, 0.95}^2$ at the significant level of 5%, the simpler model would be preferred; otherwise, we will reject the simpler model indicating that the full space significantly improves the fit.

2.6 Return Period (RP) and Return Levels (RL) for Max Air Temperature

The RP K is the reciprocal of the probability that the event exceeds the return level w_K in any given time period:

$$P(W > w_K) = K^{-1}$$

$$P(W \leq w_K) = 1 - K^{-1}$$

For a random variable Y_K , the return period K signifies the probability that Y_K is exceeded in any given year. The RL associated with this return period is the value that is expected on average to be surpassed once in every K years. In this context, the return level W_K represents the magnitude of the extreme temperature expected to be exceeded, on average, only once in K years. This relationship can be expressed mathematically as $W_K = 1 - K^{-1}$ [3]. The cumulative distribution function (CDF) $G(w)$ of the GEV and GP distributions gives the probability $P(W \leq w_K)$. Therefore, we set:

$$G(w_K) = 1 - K^{-1}$$

When the optimal model for the data is identified, the next step is to estimate the return levels of the maximum air temperature. The concept of RL, denoted as W_K , is crucial to understanding extreme events. Specifically, the $K - year$ return level represents the value that is expected to be exceeded, on average, once every K years. This value provides an estimate of the magnitude of an extreme event that occurs with a return period of K years-the time span, expressed in years, over which such an extreme is expected to occur.

2.6.1 Estimating RL for GEV distribution

The GEV distribution has the following CDF:

$$G(w) = \exp \left\{ - \left[1 + \xi \left(\frac{w - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}, 1 + \xi \left(\frac{w - \mu}{\sigma} \right) > 0$$

1. setting $G(w_K) = 1 - K^{-1}$, we have:

$$\exp \left\{ - \left[1 + \xi \left(\frac{w_K - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} = 1 - K^{-1}.$$

2. Taking the natural logarithm on both sides:

$$- \left[1 + \xi \left(\frac{w_K - \mu}{\sigma} \right) \right]^{-1/\xi} = \log (1 - K^{-1}).$$

3. Multiply both sides by -1 :

$$\left[1 + \xi \left(\frac{w_K - \mu}{\sigma} \right) \right]^{-1/\xi} = -\log (1 - K^{-1}).$$

4. Taking both sides to the power of $-\xi$:

$$1 + \xi \left(\frac{w_K - \mu}{\sigma} \right) = [-\log (1 - K^{-1})]^{-\xi}.$$

5. making w_K the subject:

$$\xi \left(\frac{w_K - \mu}{\sigma} \right) = [-\log (1 - K^{-1})]^{-\xi} - 1,$$

$$\frac{w_K - \mu}{\sigma} = \frac{[-\log(1 - K^{-1})]^{-\xi} - 1}{\xi},$$

$$w_K = \mu + \frac{\sigma}{\xi} \left([-\log(1 - K^{-1})]^{-\xi} - 1 \right), \quad \xi \neq 0.$$

If $\xi = 0$ (Gumbel case), the formula simplifies to:

$$w_K = \mu - \sigma \log(-\log(1 - K^{-1})).$$

2.7 Results

2.7.1 Estimation and Fitting of the Generalized Extreme Value (GEV) Distribution (M_I)

The Table 2.1 provides maximum likelihood estimates for model (M_I) of the GEV distribution parameters across 10 stations, along with their associated standard errors and 95% CI (confidence intervals). Paying attention to the behavior of the shape parameter, ξ , it is evident that it is consistently negative for all stations from Banjul to Yundum. This indicates that the extreme values observed follow a Weibull distribution due to the fact that $0 > \xi$ and the CI for ξ are significantly far from 0, characterized by a bounded upper tail. The value of ξ differs between stations, ranging from -0.136 at the Banjul station to -0.339 at Basse station. Stations with values close to zero, such as Banjul station, exhibit less truncation in their extreme values, suggesting that larger extremes are slightly more likely to occur. Conversely, stations with more negative values, such as Basse station, experience stronger truncation, indicating a tighter

upper limit on the extreme values. These estimates can be seen, as the standard errors are small and the confidence intervals narrow. This reliability in our data strengthens the conclusion that extreme events for all stations are naturally limited in magnitude.

Table 2.1: MML Estimates of GEV parameters for 10 stations with the Standard Errors (SE in brackets) and 95% Confidence Interval (CI).

Station Name	Parameter	Estimate	SE	95% CI
Banjul	μ	31.9029	(8.00×10^{-4})	(31.90156, 31.90424)
	σ	0.758128	(7.73×10^{-5})	(0.757975, 0.758281)
	ξ	-0.1363627	(2.39×10^{-5})	(-0.136411, -0.136315)
Basse	μ	35.41981	(2.56×10^{-3})	(35.41478, 35.42484)
	σ	1.306668	(1.03×10^{-4})	(1.306462, 1.306874)
	ξ	-0.3398513	(8.26×10^{-5})	(-0.340013, -0.339689)
Fatoto	μ	35.23162	(2.59×10^{-3})	(35.22651, 35.23673)
	σ	1.328617	(9.46×10^{-5})	(1.328432, 1.328802)
	ξ	-0.3226042	(5.69×10^{-5})	(-0.322716, -0.322492)
Janjangbureh	μ	34.90867	(2.16×10^{-3})	(34.90443, 34.91291)
	σ	1.232688	(9.72×10^{-5})	(1.232495, 1.232881)
	ξ	-0.2895524	(6.11×10^{-5})	(-0.289675, -0.28943)
Jenoi	μ	34.7581	(1.91×10^{-3})	(34.75433, 34.76187)
	σ	1.168958	(9.17×10^{-5})	(1.168776, 1.16914)
	ξ	-0.2594	(6.25×10^{-5})	(-0.259525, -0.259275)
Kaur	μ	34.83129	(1.87×10^{-3})	(34.82759, 34.83499)
	σ	1.19254	(8.53×10^{-5})	(1.192372, 1.192708)
	ξ	-0.2507694	(1.75×10^{-5})	(-0.250803, -0.250736)
Kerewan	μ	34.35688	(1.72×10^{-3})	(34.35344, 34.36032)
	σ	1.119151	(8.96×10^{-5})	(1.118973, 1.119329)
	ξ	-0.2336275	(5.56×10^{-5})	(-0.233738, -0.233517)
Sapu	μ	35.07349	(1.93×10^{-3})	(35.0697, 35.07728)
	σ	1.21344	(8.50×10^{-5})	(1.21327, 1.21361)
	ξ	-0.2789088	(1.32×10^{-5})	(-0.278935, -0.278883)
Sibanor	μ	34.06415	(1.86×10^{-3})	(34.06046, 34.06784)
	σ	1.158364	(8.87×10^{-5})	(1.158187, 1.158541)
	ξ	-0.2419897	(5.50×10^{-5})	(-0.242098, -0.241881)
Yundum	μ	32.19165	(1.12×10^{-3})	(32.18945, 32.19385)
	σ	0.9229715	(7.94×10^{-5})	(0.922814, 0.923129)
	ξ	-0.1664535	(3.22×10^{-5})	(-0.166517, -0.16639)

2.7.2 Graphical Model Validation for each Station

The following figures present the diagnostic plots for the fitted GEV model, applied to the maximum air temperature data from all meteorological stations. These plots provide a visual assessment of the model's performance and its suitability in capturing the behavior of extreme temperature values. The return level plot, density plot, Q-Q plot and P-P plots for maximum temperatures recorded at Banjul, Basse, Fatoto, Janjangbureh, Jenoi, Kaur, Kerewan, Sapu, Sibanor and Yundum stations show a linear pattern in the plotted points, given the idea that the GEV offers a good fit to the observed data.

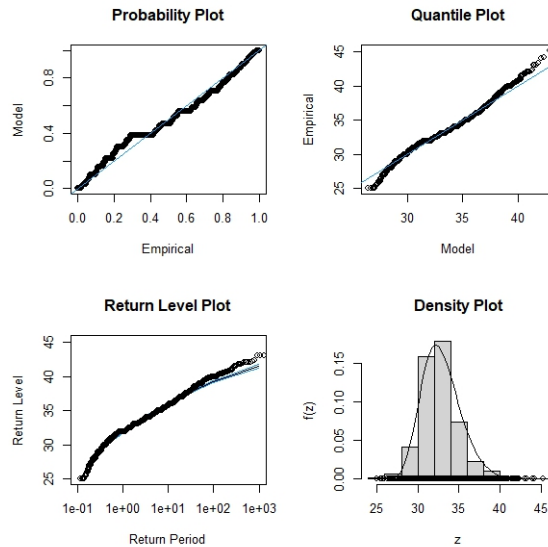


Figure 2.1: GEV fits plots Banjul temperature data.

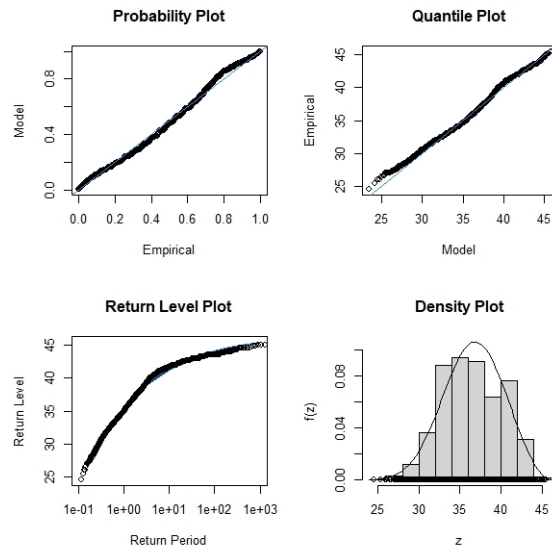


Figure 2.2: GEV fits plots Basse temperature data.

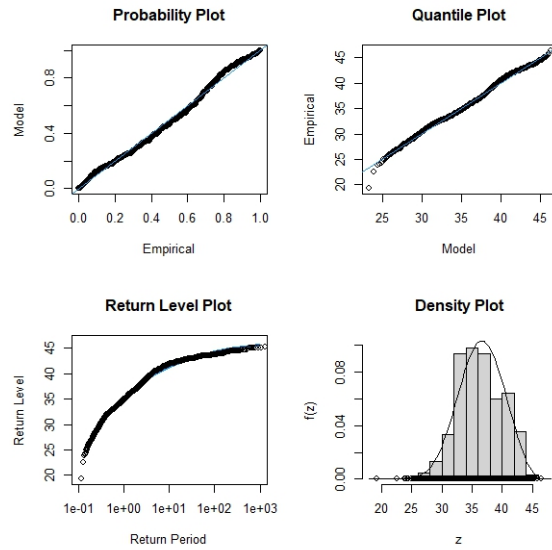


Figure 2.3: GEV fits plots Fatoto temperature data.

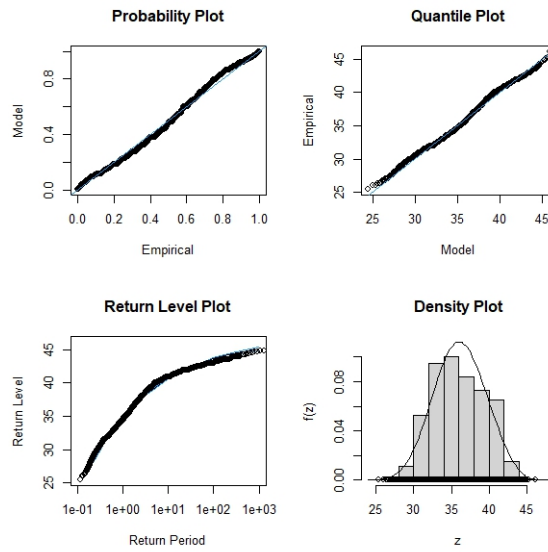


Figure 2.4: GEV fits plots Janjangbureh temperature data.

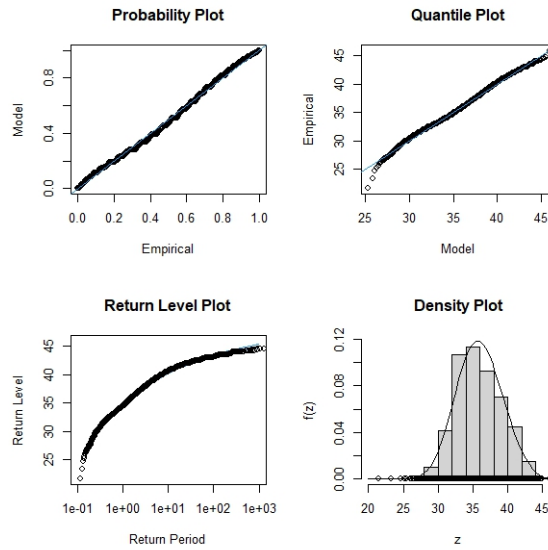


Figure 2.5: GEV fits plots Jenoi temperature data.

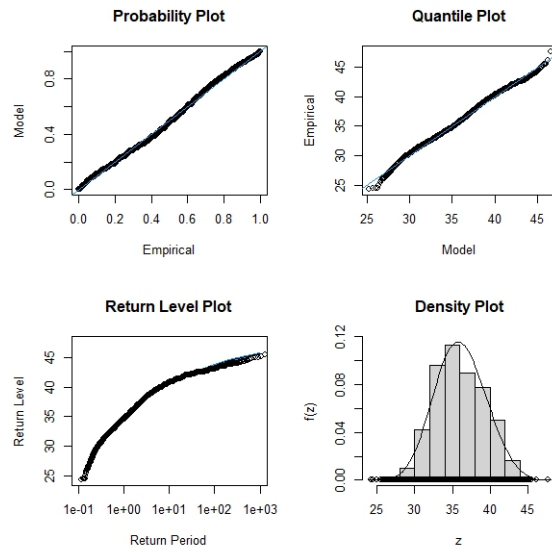


Figure 2.6: GEV fits plots Kaur temperature data.

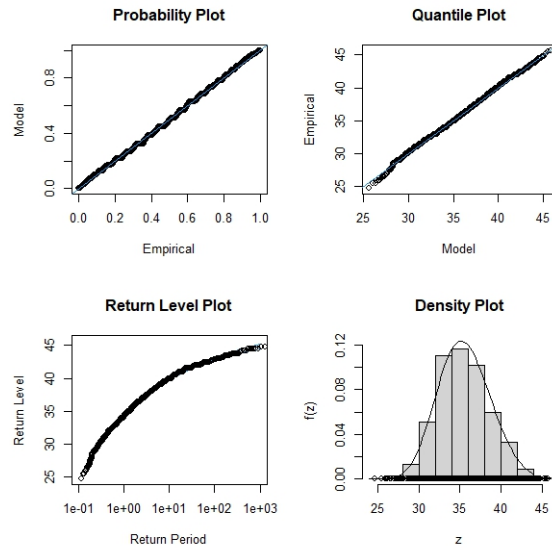


Figure 2.7: GEV fits plots Kerewan temperature data.

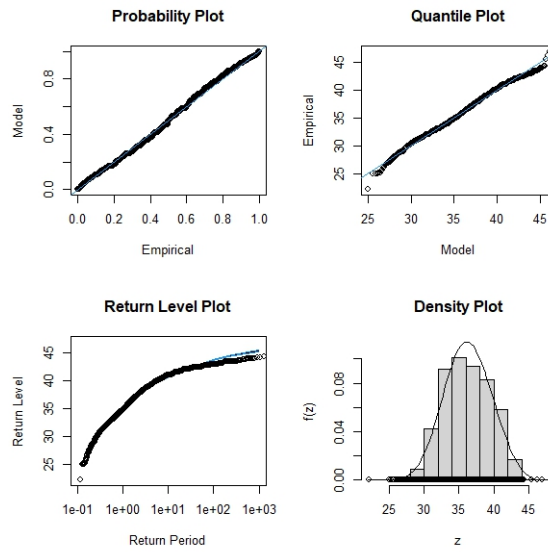


Figure 2.8: GEV fits plots Sapu temperature data.

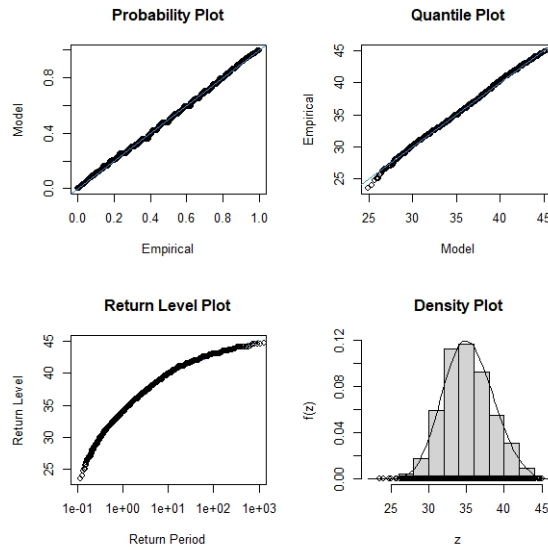


Figure 2.9: GEV fits plots Sibamor temperature data.

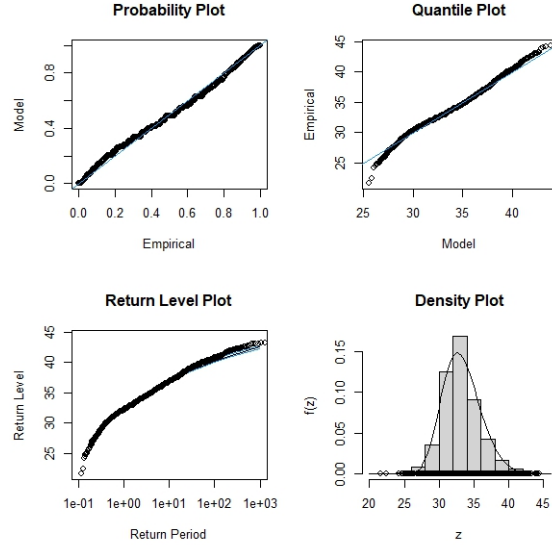


Figure 2.10: GEV fits plots Yundum temperature data.

Table 2.2: Return Levels (RL) for Different Stations

Return Period	Banjul	Basse	Fatoto	Janjangbureh	Jenoi	Kaur	Kerewan	Sapu	Sibanor	Yundum
2	32.17	35.87	35.69	35.34	35.17	35.25	34.75	35.50	34.47	32.52
5	32.93	36.96	36.81	36.41	36.21	36.32	35.77	36.56	35.52	33.42
10	33.37	37.48	37.36	36.95	36.75	36.88	36.32	37.10	36.07	33.92
30	33.96	38.05	37.97	37.57	37.39	37.55	36.97	37.73	36.74	34.58
50	34.19	38.24	38.18	37.79	37.62	37.79	37.22	37.96	36.99	34.84
100	34.49	38.46	38.42	38.04	37.89	38.08	37.51	38.22	37.28	35.16
200	34.76	38.63	38.60	38.25	38.12	38.33	37.76	38.43	37.52	35.44
500	35.08	38.79	38.79	38.46	38.36	38.59	38.03	38.66	37.79	35.76
1000	35.29	38.89	38.91	38.59	38.51	38.75	38.19	38.79	37.95	35.98

2.7.3 RL analysis based on Table 2.2

The results in Table 2.2 presents the RP's and their corresponding RL's for maximum air temperatures across various stations in The Gambia, based on the GEV model M_I . The analysis reveals that returns levels consistently increase with increasing re-

turn periods, indicating that more extreme temperature events are expected as return periods grow longer. This trend is consistent across all stations, reflecting the general behavior of extreme temperature distributions.

Among the stations, Basse stands out with the highest return levels across most return periods. For example, the 10 – *year* return level at Basse is 37.48°C , the highest compared to other stations. For the same stations, the 100 – *year* return level at Basse reaches 38.46°C , exceeding the temperatures observed at other locations. These findings highlight Basse’s susceptibility to extreme-temperature events.

In contrast, Yundum consistently exhibits the lowest return levels across all return periods. For example, the 10 – *year* return level at Yundum is 33.92°C , significantly lower than the temperatures at Basse. The 100 – *year* return level at Yundum is 35.16°C , making it the station with the least extreme temperature events. This indicates that Yundum experiences relatively stable temperature patterns compared to other regions.

The projections for longer return periods, such as 500 and 1000 years, provide insight into the potential for unprecedented temperature extremes. At Basse, the 100 – *year* return level reaches 38.90°C , suggesting the likelihood of severe temperature events in the distant future. On the other hand, Yundum’s 1000 – *year* return level is 35.98°C , remaining the lowest among the stations analyzed. The results suggest significant regional variability in temperature extremes. While stations such as Basse and Fatoto appear more prone to frequent and intense temperature extremes, stations like Yundum and Kerewan exhibit more moderate and stable patterns. This variability is crucial to understanding the regional climate dynamics in The Gambia and highlights

the need for localized strategies to address the impacts of extreme temperatures.

2.7.4 Fitted Non-Stationary GEV model Parameters for Daily Maximum Temperature

Table 2.3: Banjul station

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 31.9029 (8.00 \times 10^{-4})$, $\hat{\sigma} = 0.758128 (7.73 \times 10^{-5})$, $\hat{\xi} = -0.1363627 (2.39 \times 10^{-5})$
II	$\hat{a} = 31.20474 (3.16 \times 10^{-3})$, $\hat{\sigma} = 0.7463654 (7.68 \times 10^{-5})$, $\hat{\xi} = -0.1415838 (2.21 \times 10^{-5})$, $\hat{b} = 1.438473 (9.67 \times 10^{-3})$
III	$\hat{a} = 32.37276 (6.26 \times 10^{-3})$, $\hat{\sigma} = 0.7293919 (7.51 \times 10^{-5})$, $\hat{\xi} = -0.1459579 (1.00 \times 10^{-5})$, $\hat{b} = -5.280689 (1.29 \times 10^{-1})$, $\hat{c} = 6.668114 (1.19 \times 10^{-1})$
IV	$\hat{\mu} = 31.87065 (8.08 \times 10^{-4})$, $\hat{a} = 0.6503882 (2.43 \times 10^{-4})$, $\hat{\xi} = -0.1460841 (2.26 \times 10^{-5})$, $\hat{b} = 0.2195337 (7.39 \times 10^{-4})$
V	$\hat{\mu} = 31.76617 (8.38 \times 10^{-4})$, $\hat{a} = 0.8610039 (6.09 \times 10^{-4})$, $\hat{\xi} = -0.1453555 (6.47 \times 10^{-6})$, $\hat{b} = -1.154367 (1.30 \times 10^{-2})$, $\hat{c} = 1.429457 (1.30 \times 10^{-2})$
VI	$\hat{a} = 31.26002 (3.13 \times 10^{-3})$, $\hat{c} = 0.6827025 (2.61 \times 10^{-4})$, $\hat{\xi} = -0.1429074 (2.24 \times 10^{-5})$, $\hat{b} = 1.329448 (1.02 \times 10^{-2})$, $\hat{d} = 0.1254382 (7.50 \times 10^{-4})$
VII	$\hat{a} = 32.39239 (5.53 \times 10^{-3})$, $\hat{d} = 0.6453344 (2.35 \times 10^{-4})$, $\hat{\xi} = -0.1489339 (1.04 \times 10^{-5})$, $\hat{b} = -5.435731 (1.23 \times 10^{-1})$, $\hat{c} = 0.165153 (6.65 \times 10^{-4})$, $\hat{e} = 6.854821 (7.19 \times 10^{-1})$
VIII	$\hat{a} = 31.12788 (3.77 \times 10^{-3})$, $\hat{c} = 0.8961158 (6.54 \times 10^{-4})$, $\hat{\xi} = -0.1440933 (6.68 \times 10^{-6})$, $\hat{b} = 1.400875 (1.26 \times 10^{-2})$, $\hat{d} = -1.207525 (1.34 \times 10^{-2})$, $\hat{e} = 1.363659 (1.30 \times 10^{-2})$
X	$\hat{a} = 32.45936 (6.86 \times 10^{-3})$, $\hat{d} = 0.8536316 (6.01 \times 10^{-4})$, $\hat{\xi} = -0.1372397 (5.52 \times 10^{-6})$, $\hat{b} = -6.06388 (1.43 \times 10^{-1})$, $\hat{c} = -1.211853 (1.31 \times 10^{-2})$, $\hat{e} = 7.591014 (1.41 \times 10^{-1})$, $\hat{f} = 1.41298 (1.28 \times 10^{-2})$

Model	Neg.log-likelihood
Model I	14718.84
Model II	14613.50
Model III	14434.35
Model IV	14687.78
Model V	14605.48
Model VI	14603.06
Model VII	14414.15
Model VIII	14529.19
Model X	14330.85

Table 2.4: **Neg.log-likelihood values for each model Banjul.**

BANJUL models for maximum air temperature data

Consider the pair of models M_I vs M_{II} presented in Table 2.3 where M_I is designated as the reference model. Using the critical value of $\chi^2_1, 0.05 = 3.841459$, the test statistic is computed as $\tau = 2[14718.84 - 14613.50] = 210.68$. Since the computed statistic τ exceeds the critical value ($\tau > \chi^2_1, 0.05$), we conclude that M_{II} provides a statistically significant improvement in fit compared to the basic GEV model M_I . This result underscores the importance of including a linear trend for μ parameter, as the model M_{II} demonstrates a better capacity to describe the data. Thus, adopting the model M_{II} is justified and adds substantial value in capturing the observed variability.

The second pair of models under consideration is M_I vs M_{III} , with the critical value

given as $\chi^2_{2,0.05} = 5.991465$ and the computed test statistic $\tau = 568.98$. Since the test statistic is greater than the critical value ($\tau > \chi^2_{2,0.05}$), this shows that the quadratic trend in the parameter μ of M_{III} offers a statistically improved fit compared to the reference GEV model M_I . The third pair under consideration is M_I vs M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 62.12$. Since the computed τ is greater than critical value ($\tau > \chi^2_{1,0.05}$), we can conclude that the linear trend for σ parameter in M_{IV} statically improved the fit compared to the basic GEV model.

The fourth pair under consideration is M_I vs M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 226.72$. Since the computed τ is greater than critical value ($\tau > \chi^2_{2,0.05}$), we can conclude that the quadratic trend for σ parameter in M_V statistically improved the fit compared to the basic GEV model.

The fifth pair under consideration is M_I vs M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and computed statistic $\tau = 231.56$. Since the τ statistic exceeds critical value ($\tau > \chi^2_{2,0.05}$) at 5% the level of significant. The result shows that the linear trend in both the parameter μ and the parameter σ in M_{VI} statistically improved the fit compared to the basic GEV as the reference model.

The other pair under consideration are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 609.38$ and $\tau = 379.3$, respectively. Since the computed τ statistic of M_{VII} and M_{VIII} greater than the critical value ($\tau > \chi^2_{3,0.05}$), and for model M_{VII} quadratic trend for μ parameter and linear trend for σ parameter, in model M_{VIII} where the location parameter μ follows Linear trend and scale parameter σ follows quadratic trend, the result show a noticeable improvement in fit over the basic GEV model.

For the pair M_I vs M_X , with the critical value $\chi^2_{4,0.05} = 9.487729$ and computed statistic $\tau = 775.98$. Since the τ statistic is greater than the critical ($\tau > \chi^2_{4,0.05}$), we can conclude the quadratic trend in model M_X for both μ parameter and σ parameter significantly improved model fit compared to the reference model GEV M_I .

The best nonstationary model for the Banjul station is M_{III} for the daily maximum temperature and location parameter has a linear and quadratic trend shown in below (2.1).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.1459579 \left(\frac{w_i - (32.37276 - 5.280689 * t + 6.668114 * t^2)}{0.7293919} \right) \right]^{\frac{1}{0.1459579}} \right\} \quad (2.1)$$

where w_i is the daily temperature max and t_i is the time in years.

Table 2.5: **Basse station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 35.41981 (2.56 \times 10^{-3})$, $\hat{\sigma} = 1.306668 (1.03 \times 10^{-4})$, $\hat{\xi} = -0.3398513 (8.26 \times 10^{-5})$
II	$\hat{a} = 35.31288 (7.78 \times 10^{-3})$, $\hat{\sigma} = 1.306771 (1.03 \times 10^{-4})$, $\hat{\xi} = -0.3405107 (8.30 \times 10^{-5})$, $\hat{b} = 0.2164998 (2.12 \times 10^{-2})$
III	$\hat{a} = 35.68156 (1.73 \times 10^{-2})$, $\hat{\sigma} = 1.306237 (1.03 \times 10^{-4})$, $\hat{\xi} = -0.3417134 (8.38 \times 10^{-5})$, $\hat{b} = -1.955612 (3.54 \times 10^{-1})$, $\hat{c} = 2.161996 (3.30 \times 10^{-1})$
IV	$\hat{\mu} = 35.42063 (2.57 \times 10^{-3})$, $\hat{a} = 1.299261 (2.37 \times 10^{-4})$, $\hat{\xi} = -0.3405806 (8.36 \times 10^{-5})$, $\hat{b} = 0.01551057 (5.81 \times 10^{-4})$
V	$\hat{\mu} = 35.42011 (2.57 \times 10^{-3})$, $\hat{a} = 1.327624 (5.53 \times 10^{-4})$, $\hat{\xi} = -0.3423348 (8.41 \times 10^{-5})$, $\hat{b} = -0.1454756 (1.06 \times 10^{-2})$, $\hat{c} = 0.1589292 (9.71 \times 10^{-3})$
VI	$\hat{a} = 35.31172 (9.22 \times 10^{-3})$, $\hat{c} = 1.307173 (2.77 \times 10^{-4})$, $\hat{\xi} = -0.340492 (8.34 \times 10^{-5})$, $\hat{b} = 0.2187526 (2.66 \times 10^{-2})$, $\hat{d} = -0.0008227465 (7.26 \times 10^{-4})$
VII	$\hat{a} = 35.67791 (1.83 \times 10^{-2})$, $\hat{d} = 1.307769 (2.79 \times 10^{-4})$, $\hat{\xi} = -0.3416625 (8.41 \times 10^{-5})$, $\hat{b} = -1.949237 (3.57 \times 10^{-1})$, $\hat{c} = -0.003119652 (7.28 \times 10^{-4})$, $\hat{e} = 2.163013 (3.30 \times 10^{-1})$
VIII	$\hat{a} = 35.31233 (9.39 \times 10^{-3})$, $\hat{c} = 1.334908 (5.89 \times 10^{-4})$, $\hat{\xi} = -0.3422072 (8.40 \times 10^{-5})$, $\hat{b} = 0.2163308 (2.72 \times 10^{-2})$, $\hat{d} = -0.1589037 (1.07 \times 10^{-2})$, $\hat{e} = 0.1564328 (9.71 \times 10^{-3})$
X	$\hat{a} = 35.67152 (1.97 \times 10^{-2})$, $\hat{d} = 1.311099 (6.13 \times 10^{-4})$, $\hat{\xi} = -0.3416854 (8.41 \times 10^{-5})$, $\hat{b} = -1.909835 (4.06 \times 10^{-1})$, $\hat{c} = -0.02268753 (1.22 \times 10^{-2})$, $\hat{e} = 2.123274 (3.79 \times 10^{-1})$, $\hat{f} = 0.01938614 (1.12 \times 10^{-2})$

Model	Neg.log-likelihood
Model I	17740.04
Model II	17738.93
Model III	17731.83
Model IV	17739.83
Model V	17738.53
Model VI	17738.93
Model VII	17731.82
Model VIII	17737.67
Model X	52 17731.80

Table 2.6: **Neg.log-likelihood values for each model Basse.**

BASSE models for maximum air temperature data

The pair of model under consideration is M_I vs M_{II} from Table 2.5 has $\chi^2_{1,0.05} = 3.841459$, test statistic is computed as $\tau = 2[17740.04 - 17738.93] = 2.22$. Since critical value is greater than τ statistic ($\tau < \chi^2_{1,0.05}$), this follows that the Linear trend for μ parameter of M_{II} does not offer statistically improve in fit compared to the reference GEV model M_I .

Second pair of models being considered is M_I vs M_{III} , with the critical value given as $\chi^2_{2,0.05} = 5.991465$ and the computed test statistic $\tau = 16.42$. Since critical value is less than τ statistic ($\tau > \chi^2_{2,0.05}$), it follows that the quadratic trend for parameter μ of M_{III} offers a statistically improved fit compared to the reference GEV model M_I .

Third pair under consideration is M_I vs M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 0.42$. Since the computed τ statistic is less than critical ($\tau < \chi^2_{1,0.05}$), we can conclude that the linear trend for the parameter σ in M_{IV} does not statistically improve the fit compared to the basic GEV model. In this case, the stationary model M_I is considered.

The fourth pair of models is M_I vs M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 3.02$. Since the critical value is more than τ statistic ($\tau < \chi^2_{2,0.05}$), we can conclude that the quadratic trend for the parameter σ in M_V does not statistically improve fit compared to the basic GEV model. The fifth pair of models is M_I vs M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and computed statistic $\tau = 2.22$. Since the critical value exceeds the test statistic ($\tau < \chi^2_{2,0.05}$) at 5% the level of significant. The result shows that the linear trend in both parameter μ and parameter σ in M_{VI} does not statistically improve in fit over the basic GEV as the reference model, so M_{VI} is not considered.

The other pair of models are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 16.44$ and $\tau = 4.74$, respectively. Since the test statistic of M_{VII} is greater than the critical value ($\tau > \chi^2_{3,0.05}$), in model M_{VII} where parameter μ follows a quadratic trend and the σ follows a linear trend, the results show a noticeable improvement in fit compared to the basic GEV model. For model M_{VIII} the linear trend for μ parameter and σ follows quadratic trend, with the test statistic computed of M_{VIII} is less than the critical value ($\tau < \chi^2_{3,0.05}$), which show there is no statistical improvement in fit over the basic M_I model.

For the pair of models M_I vs M_X , with the critical value $\chi^2_{4,0.05} = 9.487729$ and computed statistic $\tau = 16.48$. Since the τ statistic is greater than the critical ($\tau > \chi^2_{4,0.05}$), showing quadratic trend in model M_X for both μ parameter and σ parameter significantly improved the model fit compared to the reference model GEV M_I .

Among the various non-stationary models considered for analyzing the maximum temperature data for Basse, two models emerged as the most competitive based on their deviance statistics. The primary model (2.2), incorporates a quadratic trend in the location parameter, making it the best-fitting model. As an alternative, Model(2.3) was identified, which shows quadratic trend in μ and linear trend in σ parameter. The preferred model, denoted as M_{III} , is defined as follows:

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.3417134 \left(\frac{w_i - (35.68156 - 1.955612 * t + 2.161996 * t^2)}{1.306237} \right) \right]^{\frac{1}{0.3417134}} \right\} \quad (2.2)$$

where w_i is the daily temperature max and t_i is the time in years. The second model (2.3) non-stationary GEV is:

$$G(w_i, a, \xi, b, c, e, d) = \exp \left\{ - \left[1 - 0.3416625 \left(\frac{w_i - (35.67791 - 1.949237 * t + 2.163013 * t^2)}{-0.003119652 + 1.307769 * t} \right) \right]^{\frac{1}{0.3416625}} \right\} \quad (2.3)$$

Model	Neg.log-likelihood
Model I	17929.20
Model II	17918.33
Model III	17913.58
Model IV	17929.20
Model V	17928.51
Model VI	17917.20
Model VII	17912.51
Model VIII	17916.25
Model X	17912.44

Table 2.8: Neg.log-likelihood values for each model Fatoto.

Table 2.7: **Fatoto Station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 35.23162 (2.59 \times 10^{-3}), \hat{\sigma} = 1.328617 (9.46 \times 10^{-5}), \hat{\xi} = -0.3226042 (5.69 \times 10^{-5})$
II	$\hat{a} = 35.59508 (8.56 \times 10^{-3}), \hat{\sigma} = 1.327013 (9.65 \times 10^{-5}), \hat{\xi} = -0.3235356 (6.76 \times 10^{-5}),$ $\hat{b} = -0.718294 (2.37 \times 10^{-2})$
III	$\hat{a} = 35.29153 (1.82 \times 10^{-2}), \hat{\sigma} = 1.326588 (9.64 \times 10^{-5}), \hat{\xi} = -0.3241535 (6.62 \times 10^{-5}),$ $\hat{b} = 1.116253 (3.76 \times 10^{-1}), \hat{c} = -1.835671 (3.53 \times 10^{-1})$
IV	$\hat{\mu} = 35.23163 (2.59 \times 10^{-3}), \hat{a} = 1.328593 (2.94 \times 10^{-4}), \hat{\xi} = -0.3226018 (6.03 \times 10^{-5}),$ $\hat{b} = 4.47 \times 10^{-5} (6.92 \times 10^{-4})$
V	$\hat{\mu} = 35.22999 (2.58 \times 10^{-3}), \hat{a} = 1.310799 (5.20 \times 10^{-4}), \hat{\xi} = -0.3232533 (5.88 \times 10^{-5}),$ $\hat{b} = 0.1156637 (1.04 \times 10^{-2}), \hat{c} = -0.1189346 (1.02 \times 10^{-2})$
VI	$\hat{a} = 35.6302 (8.99 \times 10^{-3}), \hat{c} = 1.305993 (2.90 \times 10^{-4}), \hat{\xi} = -0.3225293 (6.41 \times 10^{-5}),$ $\hat{b} = -0.7922661 (2.61 \times 10^{-2}), \hat{d} = 0.04083747 (7.40 \times 10^{-4})$
VII	$\hat{a} = 35.33365 (1.86 \times 10^{-2}), \hat{d} = 1.306153 (2.89 \times 10^{-4}), \hat{\xi} = -0.3230234 (6.25 \times 10^{-5}),$ $\hat{b} = 1.021547 (3.76 \times 10^{-1}), \hat{c} = 0.03956515 (7.33 \times 10^{-4}),$ $\hat{e} = -1.826493 (3.54 \times 10^{-1})$
VIII	$\hat{a} = 35.62972 (8.87 \times 10^{-3}), \hat{c} = 1.284308 (5.29 \times 10^{-4}), \hat{\xi} = -0.3233805 (6.24 \times 10^{-5}),$ $\hat{b} = -0.7937148 (2.56 \times 10^{-2}), \hat{d} = 0.1781511 (1.05 \times 10^{-2}), \hat{e} = -0.1396428 (1.01 \times 10^{-2})$
X	$\hat{a} = 35.3474 (1.98 \times 10^{-2}), \hat{d} = 1.299789 (5.80 \times 10^{-4}), \hat{\xi} = -0.3230526 (6.22 \times 10^{-5}),$ $\hat{b} = 0.9385534 (4.24 \times 10^{-1}), \hat{c} = 0.07879692 (1.19 \times 10^{-2}), \hat{e} = -1.743082 (4.03 \times 10^{-1}),$ $\hat{f} = -0.03972647 (1.14 \times 10^{-2})$

FATOTO models for maximum air temperature data

The pair of models M_I vs M_{II} presented in Table 2.7 where M_I is designated as the reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 21.74$. Since the computed statistic τ exceeds the critical value ($\tau > \chi^2_{1,0.05}$), we conclude that M_{II} provides a statistically significant improvement in fit compared to the basic GEV model M_I . This result underscores the importance of including a linear trend for μ parameter, as the M_{II} demonstrates a better capacity to describe the data. Thus, adopting the model M_{II} is justified and adds substantial value in capturing the observed variability.

The second pair of models under consideration is M_I vs M_{III} , with the critical value given as $\chi^2_{2, 0.05} = 5.991465$ and the computed test statistic $\tau = 31.24$. Since the τ is greater than the critical value ($\tau > \chi^2_{2, 0.05}$), this demonstrates that the quadratic trend for μ parameter of M_{III} offers a statistically improved fit compared to the reference GEV model M_I . The third pair of models is M_I vs M_{IV} , with $\chi^2_{1, 0.05} = 3.841459$ and $\tau = 0.0$ exactly the same as the basic model. Since the computed τ statistic is less than the critical value ($\tau < \chi^2_{1, 0.05}$), we can conclude that the linear trend for σ parameter in M_{IV} does not statically improve the fit compared to the basic GEV model. In this case, the stationary model M_I is considered.

The fourth pair of models is M_I vs M_V , with $\chi^2_{2, 0.05} = 5.991465$ and $\tau = 1.38$. Since the computed τ statistic is less than the critical value ($\tau < \chi^2_{2, 0.05}$), we can conclude that the quadratic trend for σ parameter in M_V does not statistically improve the fit compared to the basic GEV model.

The fifth pair of models under consideration M_I vs M_{VI} , with $\chi^2_{2, 0.05} = 5.991465$ and the computed statistic $\tau = 24.0$. Since the test statistic exceeds the critical value ($\tau > \chi^2_{2, 0.05}$) at 5% the level of significance. The result shows that the linear trend in both the μ parameter and the σ parameter in M_{VI} statistically improved the fit compared to the basic GEV as the reference model.

The other pair of model are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3, 0.05} = 7.814728$, test statistic $\tau = 33.38$ and $\tau = 25.9$, respectively. Since the computed test statistic of M_{VII} and M_{VIII} greater than the critical value ($\tau > \chi^2_{3, 0.05}$), and for model M_{VII} quadratic trend for μ parameter and linear trend for scale parameter σ , for model M_{VIII} the linear trend for μ parameter μ and

quadratic trend for σ parameter, we can say that there is statistical improvement in the fit compared to the basic GEV model.

For the last pair M_I vs M_X , with the critical value $\chi^2_{4, 0.05} = 9.487729$ and computed statistic $\tau = 33.52$. Since the test statistic is greater than the critical ($\tau > \chi^2_{4, 0.05}$), we can conclude the quadratic trend in model M_X for both location parameter μ and scale parameter σ significantly improved the model fit compared to the reference model GEV M_I .

The various non-stationary models considered for analyzing the maximum temperature data for Fatoto data, two models emerged as the most competitive based on their deviance statistics and the test results. The primary model, known as Model(2.4), incorporates a quadratic trend in μ parameter, making it the best-fitting model. As an other choice for consideration, Model(2.5) was identified, which quadratic trend in μ and linear trend for σ parameter. The preferred model, denoted as M_{III} , is defined as follows:

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.3241535 \left(\frac{w_i - (35.29153 + 1.116253 * t - 1.835671 * t^2)}{1.326588} \right) \right]^{\frac{1}{0.3241535}} \right\} \quad (2.4)$$

where w_i is the daily temperature max and t_i is the time. The second choice of model (2.5) non-stationary GEV model is:

$$G(w_i, a, \xi, b, c, e, d) = \exp \left\{ - \left[1 - 0.3230234 \left(\frac{w_i - (35.33365 + 1.021547 * t - 1.826493 * t^2)}{0.03956515 + 0.04083747 * t} \right) \right]^{\frac{1}{0.3230234}} \right\} \quad (2.5)$$

Table 2.9: Janjangbureh Station

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 34.90867 (2.16 \times 10^{-3}), \hat{\sigma} = 1.232688 (9.72 \times 10^{-5}), \hat{\xi} = -0.2895524 (6.11 \times 10^{-5})$
II	$\hat{a} = 34.64278 (7.43 \times 10^{-3}), \hat{\sigma} = 1.230216 (9.61 \times 10^{-5}), \hat{\xi} = -0.2866258 (5.47 \times 10^{-5}),$ $\hat{b} = 0.5241543 (2.04 \times 10^{-2})$
III	$\hat{a} = 35.03905 (1.63 \times 10^{-2}), \hat{\sigma} = 1.230031 (9.57 \times 10^{-5}), \hat{\xi} = -0.2887419 (5.16 \times 10^{-5}),$ $\hat{b} = -1.815535 (3.31 \times 10^{-1}), \hat{c} = 2.334648 (3.10 \times 10^{-1})$
IV	$\hat{\mu} = 34.90913 (2.17 \times 10^{-3}), \hat{a} = 1.234209 (2.54 \times 10^{-4}), \hat{\xi} = -0.2896121 (6.19 \times 10^{-5}),$ $\hat{b} = -0.003031347 (6.23 \times 10^{-4})$
V	$\hat{\mu} = 34.9085 (2.17 \times 10^{-3}), \hat{a} = 1.285478 (6.07 \times 10^{-4}), \hat{\xi} = -0.2930401 (5.98 \times 10^{-5}),$ $\hat{b} = -0.2968407 (1.19 \times 10^{-2}), \hat{c} = 0.2913123 (1.11 \times 10^{-2})$
VI	$\hat{a} = 34.60788 (8.05 \times 10^{-3}), \hat{c} = 1.250763 (2.88 \times 10^{-4}), \hat{\xi} = -0.2883707 (6.07 \times 10^{-5}),$ $\hat{b} = 0.6003567 (2.28 \times 10^{-2}), \hat{d} = -0.0399041 (7.16 \times 10^{-4})$
VII	$\hat{a} = 35.00938 (1.72 \times 10^{-2}), \hat{d} = 1.247311 (2.91 \times 10^{-4}), \hat{\xi} = -0.2904558 (5.77 \times 10^{-5}),$ $\hat{b} = -1.721269 (3.40 \times 10^{-1}), \hat{c} = -0.03329509 (7.18 \times 10^{-4}), \hat{e} = 2.29178 (3.10 \times 10^{-1})$
VIII	$\hat{a} = 34.59222 (8.38 \times 10^{-3}), \hat{c} = 1.304871 (6.48 \times 10^{-4}), \hat{\xi} = -0.2916681 (5.82 \times 10^{-5}),$ $\hat{b} = 0.6290827 (2.40 \times 10^{-2}), \hat{d} = -0.3484547 (1.21 \times 10^{-2}), \hat{e} = 0.3046279 (1.11 \times 10^{-2})$
X	$\hat{a} = 34.95671 (1.85 \times 10^{-2}), \hat{d} = 1.283859 (6.55 \times 10^{-4}), \hat{\xi} = -0.2913171 (5.81 \times 10^{-5}),$ $\hat{b} = -1.443108 (3.68 \times 10^{-1}), \hat{c} = -0.24617 (1.27 \times 10^{-2}), \hat{e} = 2.037242 (3.35 \times 10^{-1}),$ $\hat{f} = 0.2100651 (1.17 \times 10^{-2})$

Model	Neg.log-likelihood
Model I	17432.61
Model II	17425.85
Model III	17417.05
Model IV	17432.60
Model V	17428.73
Model VI	17424.74
Model VII	17416.28
Model VIII	17420.50
Model X	17414.37

Table 2.10: **Neg.log-likelihood values for each model Janjangbureh.**

JANJANGBUREH models for maximum air temperature data

The pair of models M_I vs M_{II} presented in Table 2.9 where M_I is designated as the reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 13.52$. Since the computed statistic τ exceeds the critical value ($\tau > \chi^2_{1,0.05}$), we conclude that M_{II} provides a statistically significant improvement in fit compared to the basic GEV model M_I . This result underscores the importance of including a linear trend for μ parameter, as the model M_{II} demonstrates a better capacity to describe the data. Thus, adopting the model M_{II} is justified and adds substantial value in capturing the observed variability. The second pair of models under consideration is M_I vs M_{III} , with the critical value given as $\chi^2_{2,0.05} = 5.991465$ and

the computed test statistic $\tau = 31.12$. Since test statistic is higher than the critical value ($\tau > \chi^2_{2,0.05}$), which means that the quadratic trend in the position parameter μ of M_{III} offers a statistically better fit than the reference GEV model M_I .

The next model comparison is M_I vs M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 0.02$. As the calculated τ statistic is inferior to the critical value ($\tau < \chi^2_{1,0.05}$), we can deduce that the linear trend for σ parameter in M_{IV} does not statistically enhance the fit relative to the fundamental GEV model. The stationary model M_I is being studied in this instance.

The fourth pair of models is M_I vs M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 7.76$. Since the computed τ statistic is greater than the critical value ($\tau > \chi^2_{2,0.05}$), we can conclude that the quadratic trend for σ parameter in M_V statistically improves the fit compared to the basic GEV model. The fifth pair of models is M_I vs M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and the computed statistic $\tau = 15.74$. Since the τ statistic exceeds the critical value ($\tau > \chi^2_{2,0.05}$) at 5% the level of significance. The result shows that the linear trend for both the parameter μ and the parameter σ in M_{VI} statistically improved the fit compared to the basic GEV as the reference model.

The other pair of model are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 32.66$ and $\tau = 24.22$, respectively. Since the computed test statistic of M_{VII} and M_{VIII} greater than the critical value ($\tau > \chi^2_{3,0.05}$), and for model M_{VII} quadratic trend for μ parameter and linear trend in σ parameter, for model M_{VIII} the linear trend in μ parameter and quadratic trend for σ parameter, we can deduce that there is statistical improvement in the fit over the basic GEV model.

For the model pair M_I vs M_X , with the critical value $\chi^2_{4,0.05} = 9.487729$ and computed statistic $\tau = 36.48$. Since the test statistic is greater than the critical ($\tau > \chi^2_{4,0.05}$), we can say that the quadratic trend in model M_X for both the parameter μ and the parameter σ significantly improved the model fit compared to the reference model GEV M_I .

The best non-stationary model for Janjangbureh is M_{III} for the daily maximum temperature and has a linear and quadratic trend in parameter μ shown in model (2.6).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.2887419 \left(\frac{w_i - (35.03905 - 1.815535 * t + 1.835671 * t^2)}{2.334648} \right) \right]^{\frac{1}{0.2887419}} \right\} \quad (2.6)$$

where w_i is the daily temperature max and t_i is the time.

Table 2.11: Jenoi Station

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 34.7581 (1.91 \times 10^{-3}), \hat{\sigma} = 1.168958 (9.17 \times 10^{-5}), \hat{\xi} = -0.2594 (6.25 \times 10^{-5})$
II	$\hat{a} = 34.83195 (6.69 \times 10^{-3}), \hat{\sigma} = 1.169064 (9.18 \times 10^{-5}), \hat{\xi} = -0.2597395 (6.28 \times 10^{-5}),$ $\hat{b} = -0.146485 (1.89 \times 10^{-2})$
III	$\hat{a} = 34.91733 (1.45 \times 10^{-2}), \hat{\sigma} = 1.168978 (9.19 \times 10^{-5}), \hat{\xi} = -0.2597515 (6.30 \times 10^{-5}),$ $\hat{b} = -0.6600944 (3.02 \times 10^{-1}), \hat{c} = 0.5145155 (2.84 \times 10^{-1})$
IV	$\hat{\mu} = 34.7583 (1.91 \times 10^{-3}), \hat{a} = 1.191633 (2.78 \times 10^{-4}), \hat{\xi} = -0.2599011 (6.33 \times 10^{-5}),$ $\hat{b} = -0.04511188 (7.34 \times 10^{-4})$
V	$\hat{\mu} = 34.75825 (1.91 \times 10^{-3}), \hat{a} = 1.189101 (6.25 \times 10^{-4}), \hat{\xi} = -0.2598234 (6.36 \times 10^{-5}),$ $\hat{b} = -0.03024366 (1.27 \times 10^{-2}), \hat{c} = -0.01485211 (1.20 \times 10^{-2})$
VI	$\hat{a} = 34.80903 (7.02 \times 10^{-3}), \hat{c} = 1.189334 (2.86 \times 10^{-4}), \hat{\xi} = -0.2598795 (6.33 \times 10^{-5}),$ $\hat{b} = -0.09994633 (1.99 \times 10^{-2}), \hat{d} = -0.04063685 (7.73 \times 10^{-4})$
VII	$\hat{a} = 34.89988 (1.50 \times 10^{-2}), \hat{d} = 1.189662 (2.86 \times 10^{-4}), \hat{\xi} = -0.2598943 (6.35 \times 10^{-5}),$ $\hat{b} = -0.6406518 (3.05 \times 10^{-1}), \hat{c} = -0.04147715 (7.74 \times 10^{-4}), \hat{e} = 0.5388392 (2.84 \times 10^{-1})$
VIII	$\hat{a} = 34.80875 (7.01 \times 10^{-3}), \hat{c} = 1.187186 (6.30 \times 10^{-4}), \hat{\xi} = -0.2598142 (6.37 \times 10^{-5}),$ $\hat{b} = -0.09944749 (1.99 \times 10^{-2}), \hat{d} = -0.02796984 (1.27 \times 10^{-2}), \hat{e} =$ $-0.01267525 (1.20 \times 10^{-2})$
X	$\hat{a} = 34.90248 (1.50 \times 10^{-2}), \hat{d} = 1.184668 (6.30 \times 10^{-4}), \hat{\xi} = -0.2597889 (6.37 \times 10^{-5}),$ $\hat{b} = -0.6598543 (3.09 \times 10^{-1}), \hat{c} = -0.01183224 (1.30 \times 10^{-2}), \hat{e} = 0.5592701 (2.89 \times 10^{-1}),$ $\hat{f} = -0.02969228 (1.22 \times 10^{-2})$

Model	Neg.log-likelihood
Model I	17100.74
Model II	17100.17
Model III	17099.71
Model IV	17099.36
Model V	17099.35
Model VI	17099.10
Model VII	17098.59
Model VIII	17099.10
Model X	63 17098.56

Table 2.12: Neg.log-likelihood values extracted for each model Jenoi.

JENOI models for maximum air temperature data

We found that the model pair M_I vs. M_{II} in 2.11 has $\chi^2_{1,0.05} = 3.841459$, and the test statistic is $\tau = 2[17100.74 - 17100.17] = 1.14$. The test statistic is less than the important value ($\tau < \chi^2_{1,0.05}$), which means that the linear trend for position parameter μ of M_{II} does not provide a statistically significant improvement in fit when compared to the reference GEV model M_I . The second set of models being looked at is M_I vs. M_{III} . The critical number is $\chi^2_{2,0.05} = 5.91465$, and the test statistic is $\tau = 2.06$. The test statistic is less than the important value ($\tau < \chi^2_{2,0.05}$), which means that the quadratic trend for the position parameter μ of M_{III} does not provide a statistically better fit than the reference GEV model M_I .

The third comparison between models M_I and M_{IV} shows that the test statistic, $\tau = 2.76$, is lower than the critical value $\chi^2_{1,0.05} = 3.841459$. This suggests that adding a linear trend to the scale parameter σ in model M_{IV} does not result in a meaningful improvement over the simpler stationary model M_I . Therefore, there's no strong statistical reason to prefer M_{IV} over M_I for this dataset. Similarly, when comparing M_I to M_V , where the scale parameter includes a quadratic trend, the test statistic $\tau = 2.78$ again falls short of the threshold $\chi^2_{2,0.05} = 5.991465$. This indicates that even with the added complexity, model M_V doesn't significantly outperform the basic GEV model. In both cases, the simpler, stationary structure of model M_I remains the more appropriate choice for modeling the data.

The fifth model comparison involves M_I and M_{VI} , where the critical value at the 5% significance level is $\chi^2_{2,0.05} = 5.991465$, and the calculated test statistic is $\tau = 3.28$. Since the test statistic is below the threshold ($\tau < \chi^2_{2,0.05}$), this indicates that including

a linear trend for both the location (μ) and scale (σ) parameters of model M_{VI} does not lead to a better fit compared to the simpler GEV model M_I . As a result, model M_{VI} is not selected for further consideration. Another set of comparisons includes model M_I versus M_{VII} and M_{VIII} . Both of these models are evaluated using the same critical value of $\chi^2_{3,0.05} = 7.814728$, with test statistics of $\tau = 4.3$ and $\tau = 3.28$, respectively. For M_{VII} , which incorporates a quadratic trend for μ parameter and a linear trend in the σ parameter, the test statistic remains below the critical threshold. This suggests that M_{VII} does not provide a statistically better fit than the baseline model M_I . Similarly, M_{VIII} adds a linear trend for μ parameter and a quadratic trend for the σ parameter, but its test statistic also falls short of the critical value. Thus, no significant improvement is observed in this case either, and the basic GEV model M_I continues to be the preferred choice.

Lastly, the comparison between M_I and M_X shows a test statistic of $\tau = 4.36$, while the critical value is $\chi^2_{4,0.05} = 9.487729$. Since the test statistic does not exceed the threshold, the quadratic trends applied to both the location and scale parameters in model M_X do not offer any substantial improvement in model fit. Therefore, even the more complex M_X model does not outperform the simpler reference model M_I .

For the daily temperature max of the Jenoi station, the best model is the basic GEV model, M_I and given by:

$$G(w_i, \mu, \sigma, \xi) = \exp \left\{ - \left[1 - 0.2594 \left(\frac{w_i - 34.7581}{1.168958} \right) \right]^{\frac{1}{0.2594}} \right\} \quad (2.7)$$

where w_i is the daily maximum temperature.

Table 2.13: **Kaur Station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 34.83129 (1.87 \times 10^{-3})$, $\hat{\sigma} = 1.19254 (8.53 \times 10^{-5})$, $\hat{\xi} = -0.2507694 (1.75 \times 10^{-5})$
II	$\hat{a} = 35.03598 (6.72 \times 10^{-3})$, $\hat{\sigma} = 1.191259 (8.51 \times 10^{-5})$, $\hat{\xi} = -0.249105 (1.68 \times 10^{-5})$, $\hat{b} = -0.4133297 (1.99 \times 10^{-2})$
III	$\hat{a} = 34.67824 (1.48 \times 10^{-2})$, $\hat{\sigma} = 1.190428 (8.54 \times 10^{-5})$, $\hat{\xi} = -0.2507043 (1.85 \times 10^{-5})$, $\hat{b} = 1.74926 (3.16 \times 10^{-1})$, $\hat{c} = -2.157445 (2.95 \times 10^{-1})$
IV	$\hat{\mu} = 34.83201 (1.89 \times 10^{-3})$, $\hat{a} = 1.190139 (2.63 \times 10^{-4})$, $\hat{\xi} = -0.2509776 (1.91 \times 10^{-5})$, $\hat{b} = 0.004887304 (7.37 \times 10^{-4})$
V	$\hat{\mu} = 34.83241 (1.89 \times 10^{-3})$, $\hat{a} = 1.200957 (5.44 \times 10^{-4})$, $\hat{\xi} = -0.2500971 (2.07 \times 10^{-5})$, $\hat{b} = -0.06466083 (1.22 \times 10^{-2})$, $\hat{c} = 0.07051405 (1.17 \times 10^{-2})$
VI	$\hat{a} = 35.04131 (6.77 \times 10^{-3})$, $\hat{c} = 1.184159 (2.67 \times 10^{-4})$, $\hat{\xi} = -0.249746 (1.87 \times 10^{-5})$, $\hat{b} = -0.421716 (2.02 \times 10^{-2})$, $\hat{d} = 0.01458552 (7.70 \times 10^{-4})$
VII	$\hat{a} = 34.67918 (1.55 \times 10^{-2})$, $\hat{d} = 1.189945 (2.71 \times 10^{-4})$, $\hat{\xi} = -0.2507443 (1.98 \times 10^{-5})$, $\hat{b} = 1.745936 (3.25 \times 10^{-1})$, $\hat{c} = 0.0009929917 (7.84 \times 10^{-4})$, $\hat{e} = -2.155059 (2.99 \times 10^{-1})$
VIII	$\hat{a} = 35.03933 (6.85 \times 10^{-3})$, $\hat{c} = 1.188407 (5.63 \times 10^{-4})$, $\hat{\xi} = -0.2494192 (2.04 \times 10^{-5})$, $\hat{b} = -0.4175306 (2.05 \times 10^{-2})$, $\hat{d} = -0.0122282 (1.26 \times 10^{-2})$, $\hat{e} = 0.02700429 (1.19 \times 10^{-2})$
X	$\hat{a} = 34.66995 (1.60 \times 10^{-2})$, $\hat{d} = 1.198427 (5.68 \times 10^{-4})$, $\hat{\xi} = -0.2501795 (2.08 \times 10^{-5})$, $\hat{b} = 1.778872 (3.32 \times 10^{-1})$, $\hat{c} = -0.05185898 (1.22 \times 10^{-2})$, $\hat{e} = -2.179759 (3.05 \times 10^{-1})$, $\hat{f} = 0.05322897 (1.16 \times 10^{-2})$

Model	Neg.log-likelihood
model I	17255.96
model II	17251.67
model III	17243.81
model IV	17255.94
model V	17255.73
model VI	17251.53
model VII	17243.81
model VIII	17251.50
model X	17243.68

Table 2.14: **Neg.log-likelihood values for each model Kaur.**

KAUR models for maximum air temperature data

The first model comparison considers M_I versus M_{II} . In this case, the test statistic is computed as $\tau = 8.58$, while the critical value at the 5% level of significance is $\chi^2_{1,0.05} = 3.841459$. Since the τ statistic exceeds the threshold ($\tau > \chi^2_{1,0.05}$), it suggests that introducing a linear trend in the location parameter μ of M_{II} leads to a statistically significant improvement over the stationary GEV model M_I . In the second comparison, we evaluate M_I against M_{III} . The test statistic here is $\tau = 24.3$, which is well above the critical value $\chi^2_{2,0.05} = 5.991465$. This result indicates that incorporating a quadratic trend in the location parameter μ significantly enhances the model fit, making M_{III} a better-performing alternative to the baseline model M_I .

The third pair involves M_I and M_{IV} , with a test statistic of $\tau = 0.04$ and a critical value $\chi_{1,0.05}^2 = 3.841459$. Since the test statistic is much smaller than the critical value, it suggests that adding a linear trend for scale parameter σ does not offer any meaningful improvement in model performance. Therefore, model M_I remains the more suitable choice in this comparison. Next, we compare M_I with M_V . The test statistic of $\tau = 0.46$ again falls below the critical value $\chi_{2,0.05}^2 = 5.991465$, indicating that the quadratic trend for scale parameter σ of M_V does not contribute a statistically significant enhancement to model fit. As such, model M_V is not considered preferable over the simpler model M_I .

In contrast, the comparison between M_I and M_{VI} yields a test statistic of $\tau = 8.86$, which exceeds the critical value $\chi_{2,0.05}^2 = 5.991465$. This result supports the conclusion that including linear trends in both the location μ and scale σ parameters provides a statistically meaningful improvement in model fit. Hence, model M_{VI} is considered a valuable alternative to M_I . Further, M_I is compared against models M_{VII} and M_{VIII} . Both comparisons use the same critical value $\chi_{3,0.05}^2 = 7.814728$, while their respective test statistics are $\tau = 24.3$ for M_{VII} and $\tau = 8.92$ for M_{VIII} . In both cases, the test statistics are greater than the critical threshold, which implies that the combination of a quadratic trend for μ location parameter and a linear trend for σ scale parameter M_{VII} , as well as a linear trend for the location and a quadratic trend for σ parameter in M_{VIII} , lead to statistically significant improvements over the base model M_I .

Finally, the model M_X is compared with M_I . The test statistic is calculated as $\tau = 24.56$, which surpasses the critical value $\chi_{4,0.05}^2 = 9.487729$. This finding confirms

that the use of quadratic trends in both the location and scale parameters in model M_X leads to a substantial enhancement in model performance relative to the reference model M_I .

The best model for Kaur station is M_{III} for the daily temperature max and has linear and quadratic trend for μ parameter shown in model (2.8).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.2507043 \left(\frac{w_i - (34.67824 + 1.74926 * t - 2.157445 * t^2)}{1.190428} \right) \right]^{\frac{1}{0.2507043}} \right\} \quad (2.8)$$

where w_i is the daily temperature max and t_i is the time in years.

Model	Neg.log-likelihood
model I	16852.4
model II	16828.76
model III	16827.66
model IV	16852.34
model V	16851.32
model VI	16828.62
model VII	16827.49
model VIII	16827.69
model X	16826.33

Table 2.16: **Neg.log-likelihood values for each model Kerewan.**

Table 2.15: **Kerewan Station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 34.35688 (1.72 \times 10^{-3}), \hat{\sigma} = 1.119151 (8.96 \times 10^{-5}), \hat{\xi} = -0.2336275 (5.56 \times 10^{-5})$
II	$\hat{a} = 33.90586 (6.06 \times 10^{-3}), \hat{\sigma} = 1.114736 (8.93 \times 10^{-5}), \hat{\xi} = -0.2322385 (5.39 \times 10^{-5}),$ $\hat{b} = 0.9062309 (1.73 \times 10^{-2})$
III	$\hat{a} = 33.77802 (1.35 \times 10^{-2}), \hat{\sigma} = 1.114302 (8.94 \times 10^{-5}), \hat{\xi} = -0.2318073 (5.41 \times 10^{-5}),$ $\hat{b} = 1.66676 (2.80 \times 10^{-1}), \hat{c} = -0.7587121 (2.61 \times 10^{-1})$
IV	$\hat{\mu} = 34.3558 (1.73 \times 10^{-3}), \hat{a} = 1.114518 (2.66 \times 10^{-4}), \hat{\xi} = -0.2339125 (5.60 \times 10^{-5}),$ $\hat{b} = 0.009621163 (7.63 \times 10^{-4})$
V	$\hat{\mu} = 34.35743 (1.73 \times 10^{-3}), \hat{a} = 1.141592 (6.39 \times 10^{-4}), \hat{\xi} = -0.23368 (5.66 \times 10^{-5}),$ $\hat{b} = -0.1508433 (1.36 \times 10^{-2}), \hat{c} = 0.1582632 (1.24 \times 10^{-2})$
VI	$\hat{a} = 33.90194 (6.14 \times 10^{-3}), \hat{c} = 1.122105 (2.84 \times 10^{-4}), \hat{\xi} = -0.2324323 (5.44 \times 10^{-5}),$ $\hat{b} = 0.9147677 (1.76 \times 10^{-2}), \hat{d} = -0.01458465 (7.60 \times 10^{-4})$
VII	$\hat{a} = 33.77063 (1.38 \times 10^{-2}), \hat{d} = 1.122508 (2.84 \times 10^{-4}), \hat{\xi} = -0.2320004 (5.46 \times 10^{-5}),$ $\hat{b} = 1.690283 (2.83 \times 10^{-1}), \hat{c} = -0.01627307 (7.62 \times 10^{-4}), \hat{e} = -0.7708075 (2.62 \times 10^{-1})$
VIII	$\hat{a} = 33.89883 (6.25 \times 10^{-3}), \hat{c} = 1.147736 (6.48 \times 10^{-4}), \hat{\xi} = -0.2321431 (5.50 \times 10^{-5}),$ $\hat{b} = 0.9231344 (1.80 \times 10^{-2}), \hat{d} = -0.1674205 (1.34 \times 10^{-2}), \hat{e} = 0.1511449 (1.24 \times 10^{-2})$
X	$\hat{a} = 33.75172 (1.43 \times 10^{-2}), \hat{d} = 1.151461 (6.59 \times 10^{-4}), \hat{\xi} = -0.2319918 (5.50 \times 10^{-5}),$ $\hat{b} = 1.784113 (2.92 \times 10^{-1}), \hat{c} = -0.1873449 (1.35 \times 10^{-2}), \hat{e} = -0.8546393 (2.69 \times 10^{-1}),$ $\hat{f} = 0.1689635 (1.24 \times 10^{-2})$

KEREWAN models for maximum air temperature data

Consider the pair of models M_I vs M_{II} presented in Table 2.15 where M_I is designated as a reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 47.28$. Since the computed statistic τ exceeds the critical value ($\tau > \chi^2_{1,0.05}$), we conclude that M_{II} provides a statistically better in fit compared to the basic GEV model M_I . This result underscores the importance of including linear trend for the position μ parameter, as the model M_{II} demonstrates a better capacity to describe the data. Thus, adopting the model M_{II} is justified and adds substantial value in capturing the observed variability.

The second pair of models under consideration is M_I vs M_{III} , with the critical value given as $\chi^2_{2,0.05} = 5.991465$ and the computed test statistic $\tau = 49.48$. Since the test statistic is greater than the critical value ($\tau > \chi^2_{2,0.05}$), which shows that the quadratic trend for the μ parameter of M_{III} offers a statistically improved fit compared to the stationary GEV model M_I .

The third comparison involves models M_I and M_{IV} , where the calculated test statistic is $\tau = 0.12$, which is well below the critical value of $\chi^2_{1,0.05} = 3.841459$. This outcome suggests that including linear trend for scale parameter σ within model M_{IV} does not result in a significant improvement over the standard GEV model. As a result, the stationary model M_I remains the more appropriate choice for representing the data in this scenario. In the fourth comparison, model M_I is evaluated against M_V , where the test statistic $\tau = 2.16$ is also below the critical threshold $\chi^2_{2,0.05} = 5.991465$. This indicates that the addition of a quadratic component to the scale parameter σ in M_V does not lead to a meaningful enhancement in model performance. Hence, model M_V is considered statistically unnecessary in this case.

The fifth evaluation compares M_I with model M_{VI} , resulting in a test statistic of $\tau = 47.56$, which clearly exceeds the critical value $\chi^2_{2,0.05} = 5.991465$. This substantial difference provides strong evidence that incorporating linear trends for both the position parameter μ and the scale parameter σ leads to a significant improvement in model fit. Therefore, M_{VI} stands out as a more suitable model than M_I .

Further comparisons involve M_I versus M_{VII} and M_{VIII} . Both models are assessed using the same threshold, $\chi^2_{3,0.05} = 7.814728$, with respective test statistics of $\tau = 49.82$ and $\tau = 49.42$. Since both values far exceed the critical value, we conclude that these

models offer substantial improvements over the baseline model M_I . Specifically, M_{VII} captures a quadratic trend for the position parameter μ along with a linear trend for the σ parameter, while M_{VIII} models a linear pattern in μ and a quadratic trend in σ . In both cases, these enhancements contribute to significantly better model performance.

Finally, the comparison between M_I and M_X reveals a test statistic of $\tau = 52.14$, which surpasses the critical value $\chi^2_{4,0.05} = 9.487729$. This clearly demonstrates that incorporating quadratic trends in both the location and scale parameters within model M_X leads to a notable improvement in capturing the underlying behavior of the temperature extremes when compared to the stationary GEV model M_I .

The best model for the Kerewan station is M_{III} as the primary for the daily temperature max and has linear and quadratic trend for the position μ parameter shown in model (2.9).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.2318073 \left(\frac{w_i - (33.77802 + 1.66676 * t - 0.7587121 * t^2)}{1.114302} \right) \right]^{\frac{1}{0.2318073}} \right\} \quad (2.9)$$

where w_i is the daily temperature max and t_i is the time in years. The second choice of model has a linear trend for the μ parameter of the model M_{II} , which is shown below as model(2.10):

$$G(w_i, a, \sigma, \xi, b) = \exp \left\{ - \left[1 - 0.2322385 \left(\frac{w_i - (33.90586 + 0.9062309 * t)}{1.114736} \right) \right]^{\frac{1}{0.2322385}} \right\} \quad (2.10)$$

Table 2.17: **Sapu Station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 35.07349 (1.93 \times 10^{-3}), \hat{\sigma} = 1.21344 (8.50 \times 10^{-5}), \hat{\xi} = -0.2789088 (1.32 \times 10^{-5})$
II	$\hat{a} = 34.68248 (7.52 \times 10^{-3}), \hat{\sigma} = 1.207863 (8.49 \times 10^{-5}), \hat{\xi} = -0.2713536 (1.44 \times 10^{-5}),$ $\hat{b} = 0.7629896 (2.14 \times 10^{-2})$
III	$\hat{a} = 35.57382 (1.55 \times 10^{-2}), \hat{\sigma} = 1.199346 (8.54 \times 10^{-5}), \hat{\xi} = -0.2711091 (1.59 \times 10^{-5}),$ $\hat{b} = -4.612398 (3.20 \times 10^{-1}), \hat{c} = 5.419494 (3.03 \times 10^{-1})$
IV	$\hat{\mu} = 35.1037 (2.03 \times 10^{-3}), \hat{a} = 1.260453 (2.79 \times 10^{-4}), \hat{\xi} = -0.2882309 (2.32 \times 10^{-5}),$ $\hat{b} = -0.08920032 (6.73 \times 10^{-4})$
V	$\hat{\mu} = 35.10242 (2.07 \times 10^{-3}), \hat{a} = 1.264681 (6.61 \times 10^{-4}), \hat{\xi} = -0.288412 (2.39 \times 10^{-5}),$ $\hat{b} = -0.1134544 (1.32 \times 10^{-2}), \hat{c} = 0.02411971 (1.24 \times 10^{-2})$
VI	$\hat{a} = 34.54725 (8.67 \times 10^{-3}), \hat{c} = 1.297735 (3.43 \times 10^{-4}), \hat{\xi} = -0.2861854 (2.35 \times 10^{-5}),$ $\hat{b} = 1.084634 (2.41 \times 10^{-2}), \hat{d} = -0.1713307 (8.76 \times 10^{-4})$
VII	$\hat{a} = 35.51859 (1.73 \times 10^{-2}), \hat{d} = 1.289851 (3.26 \times 10^{-4}), \hat{\xi} = -0.2868487 (2.66 \times 10^{-5}),$ $\hat{b} = -4.471336 (3.31 \times 10^{-1}), \hat{c} = -0.1727841 (8.19 \times 10^{-4}), \hat{e} = 5.460155 (2.99 \times 10^{-1})$
VIII	$\hat{a} = 34.54809 (8.60 \times 10^{-3}), \hat{c} = 1.285402 (7.14 \times 10^{-4}), \hat{\xi} = -0.2856301 (2.41 \times 10^{-5}),$ $\hat{b} = 1.090369 (2.40 \times 10^{-2}), \hat{d} = -0.1000521 (1.34 \times 10^{-2}), \hat{e} = -0.0713329 (1.26 \times 10^{-2})$
X	$\hat{a} = 35.55423 (1.66 \times 10^{-2}), \hat{d} = 1.236847 (6.28 \times 10^{-4}), \hat{\xi} = -0.2853841 (2.51 \times 10^{-5}),$ $\hat{b} = -4.73028 (3.28 \times 10^{-1}), \hat{c} = 0.1503889 (1.26 \times 10^{-2}), \hat{e} = 5.733637 (2.98 \times 10^{-1}),$ $\hat{f} = -0.328288 (1.22 \times 10^{-2})$

Model	Neg.log-likelihood
model I	17277.48
model II	17263.86
model III	17216.04
model IV	17271.79
model V	17271.77
model VI	17247.47
model VII	17198.29
model VIII	17247.27
model X	17193.90

Table 2.18: **Neg.log-likelihood values extracted from models Sapu.**

SAPU models for maximum air temperature data

The pair of models M_I vs M_{II} presented in Table 2.17 where M_I is designated as the reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 27.24$. Since the computed statistic τ exceeds the critical value ($\tau > \chi^2_{1,0.05}$), we conclude that M_{II} provides a statistically better in fit compared to the basic GEV model M_I . This result underscores the importance of including linear trend for the position μ parameter, as the M_{II} model demonstrates a better capacity to describe the data. Thus, adopting the model M_{II} is justified and adds substantial value in capturing the observed variability.

The second pair of models under consideration is M_I vs M_{III} , with the critical value

given as $\chi^2_{2,0.05} = 5.991465$ and the computed test statistic $\tau = 122.88$. Since the τ statistic is greater than the critical value ($\tau > \chi^2_{2,0.05}$), which shows that the quadratic trend for the μ parameter of M_{III} offers a statistically improved fit compared to the reference GEV model M_I .

The third pair of models is M_I vs M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 11.38$. Since the computed τ statistic is greater than the critical value ($\tau > \chi^2_{1,0.05}$), we can conclude that the linear trend for scale parameter σ in M_{IV} statically improved the fit compared to the basic GEV model. The fourth pair of models is M_I vs M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 11.42$. Since the computed test statistic is greater than the critical value ($\tau > \chi^2_{2,0.05}$), we can conclude that the quadratic trend for scale parameter σ in M_V statistically improved the fit compared to the basic GEV model.

The fifth pair of models is M_I vs M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and the computed statistic $\tau = 60.02$. Since the test statistic exceeds the critical value ($\tau > \chi^2_{2,0.05}$) at 5% the level of significance. The result shows that the linear trend for both the position parameter μ and σ scale parameter in M_{VI} statistically improved the fit compared to the basic GEV as the reference model.

The other pair of model are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 158.38$ and $\tau = 60.42$, respectively. Since the computed test statistic of M_{VII} and M_{VIII} greater than the critical value ($\tau > \chi^2_{3,0.05}$), and for model M_{VII} quadratic trend for location parameter μ and linear trend in σ parameter, for model M_{VIII} the linear trend in μ parameter and quadratic trend for scale parameter σ , we can conclude that there is improvement in the fit compare to basic GEV M_I .

For the pair of model under consideration M_I vs M_X , with the critical value $\chi_4^2, 0.05 = 9.487729$ and computed statistic $\tau = 167.16$. Since the τ statistic is greater than the critical ($\tau > \chi_4^2, 0.05$), we can conclude the quadratic trend in model M_X for both the position parameter μ and σ scale parameter significantly improved the model fit compared to the reference model GEV M_I .

The best non-stationary model for Sapu station is M_{VII} for the daily maximum temperature and has a linear trend about scale parameter and a quadratic trend for location parameter shown in model (2.11).

$$G(w_i, a, \xi, b, c, e, d) = \exp \left\{ - \left[1 - 0.2868487 \left(\frac{w_i - (35.51859 - 4.471336 * t + 5.460155 * t^2)}{-0.1727841 + 1.289851 * t} \right) \right]^{\frac{1}{0.2868487}} \right\} \quad (2.11)$$

where w_i is the daily temperature max and t_i is the time in years.

Table 2.19: Sibamor Station

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 34.06415 (1.86 \times 10^{-3})$, $\hat{\sigma} = 1.158364 (8.87 \times 10^{-5})$, $\hat{\xi} = -0.2419897 (5.50 \times 10^{-5})$
II	$\hat{a} = 34.61497 (6.53 \times 10^{-3})$, $\hat{\sigma} = 1.154764 (8.90 \times 10^{-5})$, $\hat{\xi} = -0.2444157 (5.59 \times 10^{-5})$, $\hat{b} = -1.08337 (1.86 \times 10^{-2})$
III	$\hat{a} = 35.44754 (1.43 \times 10^{-2})$, $\hat{\sigma} = 1.151552 (8.88 \times 10^{-5})$, $\hat{\xi} = -0.2497342 (5.52 \times 10^{-5})$, $\hat{b} = -5.97444 (2.97 \times 10^{-1})$, $\hat{c} = 4.88698 (2.78 \times 10^{-1})$
IV	$\hat{\mu} = 34.05743 (1.87 \times 10^{-3})$, $\hat{a} = 1.202963 (2.71 \times 10^{-4})$, $\hat{\xi} = -0.2465849 (5.63 \times 10^{-5})$, $\hat{b} = -0.08519895 (6.56 \times 10^{-4})$
V	$\hat{\mu} = 34.03994 (1.87 \times 10^{-3})$, $\hat{a} = 1.314073 (5.51 \times 10^{-4})$, $\hat{\xi} = -0.2517307 (5.54 \times 10^{-5})$, $\hat{b} = -0.7950389 (1.14 \times 10^{-2})$, $\hat{c} = 0.7300054 (1.14 \times 10^{-2})$
VI	$\hat{a} = 34.59485 (7.07 \times 10^{-3})$, $\hat{c} = 1.166848 (2.72 \times 10^{-4})$, $\hat{\xi} = -0.2443959 (5.57 \times 10^{-5})$, $\hat{b} = -1.043189 (2.06 \times 10^{-2})$, $\hat{d} = -0.02422527 (7.29 \times 10^{-4})$
VII	$\hat{a} = 35.46697 (1.49 \times 10^{-2})$, $\hat{d} = 1.142192 (2.74 \times 10^{-4})$, $\hat{\xi} = -0.2499428 (5.55 \times 10^{-5})$, $\hat{b} = -6.054046 (3.09 \times 10^{-1})$, $\hat{c} = 0.01876811 (7.48 \times 10^{-4})$, $\hat{e} = 4.949104 (2.86 \times 10^{-1})$
VIII	$\hat{a} = 34.51526 (7.70 \times 10^{-3})$, $\hat{c} = 1.265703 (6.03 \times 10^{-4})$, $\hat{\xi} = -0.2497279 (5.57 \times 10^{-5})$, $\hat{b} = -0.9234121 (2.29 \times 10^{-2})$, $\hat{d} = -0.6279294 (1.23 \times 10^{-2})$, $\hat{e} = 0.6119855 (1.18 \times 10^{-2})$
X	$\hat{a} = 35.35688 (1.69 \times 10^{-2})$, $\hat{d} = 1.201426 (6.19 \times 10^{-4})$, $\hat{\xi} = -0.2485166 (5.50 \times 10^{-5})$, $\hat{b} = -5.561499 (3.42 \times 10^{-1})$, $\hat{c} = -0.3491347 (1.33 \times 10^{-2})$, $\hat{e} = 4.534715 (3.12 \times 10^{-1})$, $\hat{f} = 0.3692866 (1.26 \times 10^{-2})$

Model	Neg.log-likelihood
model I	17073.38
model II	17042.04
model III	16999.48
model IV	17067.93
model V	17044.01
model VI	17041.64
model VII	16999.24
model VIII	17025.65
model X	16993.74

Table 2.20: Neg.log-likelihood values for each model Sibamor.

SIBANOR models for maximum air temperature data

The pair of models M_I vs M_{II} presented in Table 2.19 where M_I is designated as the reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 62.68$. Since the computed statistic τ exceeds the critical value (> 3.841459), we conclude that M_{II} provides a statistically better fit compared to the basic GEV M_I . This result emphasizes how crucial it is to include a linear trend for the location parameter because the M_{II} shows a greater ability to describe the data. The second pair of models under consideration is M_I vs M_{III} , with the critical value given as $\chi^2_{2,0.05} = 5.991465$ and the computed test statistic $\tau = 147.80$. As the τ exceeds the critical value ($\tau > 5.991465$), it suggests that a quadratic trend of M_{III} location parameter gives statistically better fit than a stationary M_I -based GEV model.

The third pair of models is M_I vs M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 10.90$. In comparison to the basic GEV model, the fit was statistically improved by the linear trend for scale (σ) parameter in M_{IV} , as the computed τ statistic is more than the critical value ($\tau > 3.841459$). The fourth pair of models is M_I vs M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 58.74$. We can infer that the quadratic trend in the σ parameter in M_V statistically enhanced the fit when compared to the basic GEV model because the calculated test statistic is higher than the critical value ($\tau > 5.991465$). The fifth pair of models is M_I vs M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and the computed statistic $\tau = 63.48$. The test statistic is considered significant at the 5 percent level since it surpasses the critical value ($\tau > 5.991465$). The result shows that the linear trend for both the μ location parameter and the scale parameter σ in M_{VI} statistically improved

the fit compared to the basic GEV as the reference model. The other pair of model are M_I vs M_{VII} and M_I vs M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 148.28$ and $\tau = 95.46$, respectively. Since the computed test statistic of M_{VII} and M_{VIII} greater than the critical value ($\tau > \chi^2_{3,0.05}$), and given the quadratic trend for location parameter μ and the linear trend about scale parameter σ in model M_{VII} and the quadratic trend for position parameter μ and scale parameter σ in model M_{VIII} , we can say that the fit is statistically better than that of the basic GEV model. For the model pair M_I vs M_X , with the critical value $\chi^2_{4,0.05} = 9.487729$ and test statistic $\tau = 159.28$. As the test statistic exceeds the crucial ($\tau > \chi^2_{4,0.05}$), model M_X quadratic trend for location and scale parameters significantly improves model fit compared to GEV M_I .

The best non-stationary model for Sibamor is M_{III} for the daily maximum temperature and has a linear and quadratic trend for location parameter shown in model (2.12).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.2497342 \left(\frac{w_i - (35.44754 - 5.97444 * t + 4.88698 * t^2)}{1.151552} \right) \right]^{\frac{1}{0.2497342}} \right\} \quad (2.12)$$

where w_i is the daily temperature max and t_i is the time in years.

YUNDUM models for maximum air temperature data

Table 2.21 displays the M_I vs. M_{II} model pair, with M_I serving as the reference model. Using the critical value of $\chi^2_{1,0.05} = 3.841459$, the test statistic is computed as $\tau = 201.96$. Given that the calculated statistic τ is greater than the critical value

Table 2.21: **Yundum Station**

Model	Parameter Estimates with Standard Errors
I	$\hat{\mu} = 32.19165 (1.12 \times 10^{-3}), \hat{\sigma} = 0.9229715 (7.94 \times 10^{-5}), \hat{\xi} = -0.1664535 (3.22 \times 10^{-5})$
II	$\hat{a} = 31.41415 (4.16 \times 10^{-3}), \hat{\sigma} = 0.9086632 (7.89 \times 10^{-5}), \hat{\xi} = -0.1668366 (3.12 \times 10^{-5}),$ $\hat{b} = 1.59021 (1.23 \times 10^{-2})$
III	$\hat{a} = 31.90956 (8.99 \times 10^{-3}), \hat{\sigma} = 0.9042598 (7.91 \times 10^{-5}), \hat{\xi} = -0.1658601 (3.15 \times 10^{-5}),$ $\hat{b} = -1.34782 (1.89 \times 10^{-1}), \hat{c} = 2.927572 (1.76 \times 10^{-1})$
IV	$\hat{\mu} = 32.17172 (1.16 \times 10^{-3}), \hat{a} = 0.8754628 (2.79 \times 10^{-4}), \hat{\xi} = -0.1700939 (3.25 \times 10^{-5}),$ $\hat{b} = 0.09845768 (8.78 \times 10^{-4})$
V	$\hat{\mu} = 32.17018 (1.17 \times 10^{-3}), \hat{a} = 0.8867496 (6.34 \times 10^{-4}), \hat{\xi} = -0.1706186 (3.30 \times 10^{-5}),$ $\hat{b} = 0.03032982 (1.37 \times 10^{-2}), \hat{c} = 0.06944456 (1.33 \times 10^{-2})$
VI	$\hat{a} = 31.42405 (4.14 \times 10^{-3}), \hat{c} = 0.8885451 (2.88 \times 10^{-4}), \hat{\xi} = -0.1667089 (3.10 \times 10^{-5}),$ $\hat{b} = 1.570222 (1.25 \times 10^{-2}), \hat{d} = 0.04003647 (8.39 \times 10^{-4})$
VII	$\hat{a} = 31.91761 (8.70 \times 10^{-3}), \hat{d} = 0.8784625 (2.87 \times 10^{-4}), \hat{\xi} = -0.1657725 (3.13 \times 10^{-5}),$ $\hat{b} = -1.391303 (1.87 \times 10^{-1}), \hat{c} = 0.05124053 (8.34 \times 10^{-4}), \hat{e} = 2.968717 (1.77 \times 10^{-1})$
VIII	$\hat{a} = 31.4264 (4.15 \times 10^{-3}), \hat{c} = 0.8798368 (6.63 \times 10^{-4}), \hat{\xi} = -0.1662447 (3.22 \times 10^{-5}),$ $\hat{b} = 1.567677 (1.25 \times 10^{-2}), \hat{d} = 0.09119214 (1.39 \times 10^{-2}), \hat{e} = -0.05160097 (1.32 \times 10^{-2})$
X	$\hat{a} = 31.9189 (8.59 \times 10^{-3}), \hat{d} = 0.8657229 (6.37 \times 10^{-4}), \hat{\xi} = -0.1654855 (3.16 \times 10^{-5}),$ $\hat{b} = -1.393036 (1.85 \times 10^{-1}), \hat{c} = 0.1272423 (1.34 \times 10^{-2}), \hat{e} = 2.966237 (1.75 \times 10^{-1}),$ $\hat{f} = -0.07656553 (1.28 \times 10^{-2})$

($\tau > \chi_{1,0.05}^2$), we deduce that M_{II} offers a statistically superior fit to the fundamental GEV model MI. This result emphasizes how crucial it is to include a linear trend in the μ parameter because model M_{II} more accurately describes the data. The second pair of models under consideration is M_I vs. M_{III} , with the critical value given as $\chi_{2,0.05}^2 = 5.991465$ and the computed test statistic $\tau = 250$. The quadratic trend in the μ parameter of M_{III} provides a statistically better fit than the reference GEV model MI because the test τ is greater than the critical ($\tau > \chi_{2,0.05}^2$).

Model	Neg.log-likelihood
model I	15735.39
model II	15634.41
model III	15610.39
model IV	15729.96
model V	15729.78
model VI	15633.45
model VII	15608.81
model VIII	15633.35
model X	15608.58

Table 2.22: **Neg.log-likelihood values for each model Yundum.**

The third pair of models is M_I vs. M_{IV} , with $\chi^2_{1,0.05} = 3.841459$ and $\tau = 10.86$. We may infer that the linear trend in the scale parameter σ in M_{IV} statistically improved the fit in comparison to the basic GEV model since the calculated test statistic is higher than the critical value ($\tau > \chi^2_{1,0.05}$). The fourth pair of models is M_I vs. M_V , with $\chi^2_{2,0.05} = 5.991465$ and $\tau = 11.22$. Since the computed τ is greater than the critical value ($\tau > \chi^2_{2,0.05}$), we can conclude that, in comparison to the basic GEV model, the fit was statistically improved by the quadratic trend for scale parameter σ in M_V .

The fifth pair of models is M_I vs. M_{VI} , with $\chi^2_{2,0.05} = 5.991465$ and calculated test statistic $\tau = 203.88$. Since $\tau > \chi^2_{2,0.05}$, the test statistic is greater than the critical value at the 5% significance level. The result shows that the linear trend in both the location parameter μ and the scale parameter σ in M_{VI} statistically improved the fit compared to the basic GEV as the reference model. The other pair of models are M_I vs. M_{VII} and M_I vs. M_{VIII} , both have the same critical value $\chi^2_{3,0.05} = 7.814728$, test statistic $\tau = 253.16$ and $\tau = 204.08$, respectively. Given that the calculated test statistics for both M_{VII} and M_{VIII} are greater than the critical value ($\tau > \chi^2_{3,0.05}$),

and that the model M_{VII} exhibits a quadratic trend in the location parameter μ and a linear trend in the scale parameter σ , while the model M_{VIII} exhibits a linear trend in the location parameter μ and a quadratic trend in the scale parameter σ , we can conclude that the fit is statistically better than the basic GEV model.

Finally, for the model pair M_I vs. M_X , with the critical value $\chi^2_{4,0.05} = 9.487729$ and computed statistic $\tau = 253.62$. For both location parameter μ and scale parameter σ , the quadratic trend in model M_X greatly enhanced the model fit in comparison to the reference model GEV MI, as the test statistic is greater than the critical ($\tau > \chi^2_{4,0.05}$). The best model for Yundum station is M_{III} for the daily maximum temperature and has a linear and quadratic trend for location parameter shown in model (2.13).

$$G(w_i, a, \sigma, \xi, b, c) = \exp \left\{ - \left[1 - 0.1658601 \left(\frac{w_i - (31.90956 - 1.34782 * t + 2.927572 * t^2)}{0.9042598} \right) \right]^{\frac{1}{0.1658601}} \right\} \quad (2.13)$$

where w_i is the daily temperature max and t_i is the time in years.

CHAPTER 3

GENERALIZED PARETO (GP)

3.1 Generalized Pareto (GP) Distribution

3.1.1 POT (Peaks Over Threshold method)

POT is one of the techniques used to examine distribution of extreme events, which takes into account the maximum variables that exceed a threshold that has been established [46]. It also focuses on all values in the sample that go beyond a chosen upper threshold k . POT method is a means to avoid and uses the data more effectively because in the BM approach, the second most significant value in a particular block may be greater than the most significant value in the next block, which is typically not accounted for. All sample values that are above a predetermined higher threshold k are taken into account by the POT method. How much can k be exceeded for instance, if the temperature in The Gambia is above a high threshold, let's say k ? In order to respond to such a question, we require the excess distribution, which, in the POT methodology, is nearly equivalent to GP.

3.1.2 GP (Generalized Pareto) Distribution

The GP distribution is generated from time series data by means of the POT modeling approach, which is commonly used to examine the distribution of values that exceed a specified high threshold, k . This method provides insights into the tail behavior of dataset [19]. Consider a series of independently distributed random variables that are identically distributed (i.i.d.), $w_1, w_2, w_3, w_4, \dots, w_n$ with the CDF $F(w)$. Our focus is on the tail behavior of F , specifically for values that go beyond a high threshold K . The conditional probability of w given that $w > k$ is then defined as follows:

$$F_k(y) = P_r\{w - k \leq y | w > k\} = \frac{F(k + y) - F(k)}{1 - F(k)}$$

such that $y > 0$, for threshold k , for excesses $y = w - k$. If one knew the parent distribution F , one would also know the threshold exceedance distribution [47].

Asymptotic Distribution of Exceedances

As described by [46], for a wide class of distribution F , the distribution of exceedances $F_k(y)$ approaches a GP distribution as the threshold k move closer to the upper limit of the distribution's support, represented by $x_F = \sup\{x : F(w) < 1\}$. It can be expressed as:

$$\lim_{k \rightarrow x_F} F_k(y) = G(y, \sigma, \xi)$$

where $G(y, \sigma, \xi)$ is the Cumulative function of the GP distribution with scale parameter σ and shape parameter ξ . Fitting the GP distribution to all values over a predetermined threshold k is the framework for the POT [48]. The GP is employed to

model the data that exceeds the threshold. Its CDF and PDF are shown in equation (3.1) and (3.2), respectively.

$$G(y, \sigma, \xi) = \begin{cases} 1 - \{1 + \xi(\frac{y}{\sigma})\}^{-\frac{1}{\xi}}; & \xi \neq 0 \\ 1 - \exp(-\frac{y}{\sigma}); & \xi = 0 \end{cases} \quad (3.1)$$

$$g(y; \xi, \sigma) = \begin{cases} \frac{1}{\sigma} (1 + \xi \frac{y}{\sigma})^{-\frac{1}{\xi}-1}, & \text{if } \xi \neq 0, \\ \frac{1}{\sigma} e^{-\frac{y}{\sigma}}, & \text{if } \xi = 0, \end{cases} \quad (3.2)$$

where $0 \leq y$, if $\xi \geq 0$, or $0 \leq y \leq -\frac{\sigma}{\xi}$ if $\xi < 0$ and $0 < \sigma$ [19]. The generalized Pareto distribution's (GP) tail behavior is defined by the shape parameter ξ , which transforms equation (3.1) for GP into three basic forms. A distribution with a heavy tail indicates a higher probability of extreme events when $\xi > 0$ is leading to Pareto distribution; a distribution with a light tail indicates a lower probability of extreme events when $\xi < 0$ is leading to beta distribution. Furthermore, the GP shows an exponential tail for $\xi \rightarrow 0$, which indicates a constant hazard rate.

3.1.3 MML of GP Distribution

We use MML estimation for parameter estimation in the GP distribution. Consider a random sample $y_1, y_2, \dots, y_m$ representing m threshold excesses above a given threshold k . The log-likelihood function for GP distribution corresponding to these excesses

can be expressed as follows:

$$L(\xi, \sigma; y_1, \dots, y_m) = \prod_{i=1}^m g(y_i; \xi, \sigma).$$

Substituting the PDF from equation (3.2) $g(y; \xi, \sigma)$, we have:

$$L(\xi, \sigma; y_1, \dots, y_m) = \prod_{i=1}^m \frac{1}{\sigma} \left(1 + \xi \frac{y_i}{\sigma}\right)^{-\frac{1}{\xi}-1},$$

for $1 + \xi \frac{y_i}{\sigma} > 0$.

we obtained the log-likelihood by taking the natural log of the above function:

$$\ell(\xi, \sigma) = \log L(\xi, \sigma; y_1, \dots, y_m) = \sum_{i=1}^m \left[-\log \sigma - \left(1 + \frac{1}{\xi}\right) \log \left(1 + \xi \frac{y_i}{\sigma}\right) \right].$$

when $\xi \neq 0$

$$\ell(\xi, \sigma) = -m \log \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^m \log \left(1 + \xi \frac{y_i}{\sigma}\right). \quad (3.3)$$

when $\xi = 0$

$$\ell(\sigma) = -m \log \sigma - \frac{1}{\sigma} \sum_{i=1}^m (y_i). \quad (3.4)$$

3.1.4 Estimating Return Level RL for GP distribution

The cumulative distribution function (CDF) of the generalized Pareto (GP) distribution (3.1):

$$G(w; \sigma, \xi) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-\frac{1}{\xi}}, & \text{if } \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\sigma}\right), & \text{if } \xi = 0, \end{cases}$$

where:

- $y > 0$ and $y = w - \lambda$
- w is the random variable (exceedance),
- λ is the threshold value,

The return period K is related to the exceedance probability by the equation:

$$K = \frac{1}{1 - G(w_K; \sigma, \xi)}.$$

From the above equation:

$$1 - G(w_K; \sigma, \xi) = K^{-1}.$$

Substituting the GP CDF

Using the CDF of the GP distribution, we substitute $G(w_K; \sigma, \xi)$:

For $\xi \neq 0$:

Substitute the first case of the GP CDF:

$$1 - \left[1 - \left(1 + \xi \frac{w_K - \lambda}{\sigma} \right)^{-\frac{1}{\xi}} \right] = K^{-1}.$$

Simplify:

$$\left(1 + \xi \frac{w_K - \lambda}{\sigma} \right)^{-\frac{1}{\xi}} = 1 - K^{-1}.$$

For $\xi = 0$:

Substitute the second case of the GP CDF:

$$1 - \left[1 - \exp \left(-\frac{w_K - \lambda}{\sigma} \right) \right] = K^{-1}.$$

Simplify:

$$\exp \left(-\frac{w_K - \lambda}{\sigma} \right) = K^{-1}.$$

Solving for w_K

For $\xi \neq 0$:

Raise both sides to the power of $-\xi$:

$$1 + \xi \frac{w_K - \lambda}{\sigma} = (1 - K^{-1})^{-\xi}.$$

Rearrange to solve for w_K :

$$w_K = \lambda + \sigma \frac{(1 - K^{-1})^{-\xi} - 1}{\xi}.$$

For $\xi = 0$:

Taking the natural logarithm of both sides:

$$-\frac{w_K - \lambda}{\sigma} = \log(K^{-1}).$$

Rearrange to solve for w_K :

$$w_K = \lambda - \sigma \log(K^{-1}).$$

Final RL Formula for the GP distribution

The return level w_K for the generalized Pareto (GP) distribution is:

$$w_K = \begin{cases} \lambda + \sigma \frac{(1 - K^{-1})^{-\xi} - 1}{\xi}, & \text{if } \xi \neq 0, \\ \lambda - \sigma \log(K^{-1}), & \text{if } \xi = 0. \end{cases}$$

This formula provides the level expected to be exceeded, on average, once every K years, with λ representing the threshold for exceedances.

3.2 Model Checking Techniques for GP distribution

3.2.1 Probability plot

To create a P-P plot under GP distribution, we start by:

- assuming a threshold k
- identifying the threshold excesses $y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(m)}$, and an estimated model $\hat{G}$
- the plot is made by pairing $\frac{i}{m+1}$ with $\hat{G}(y_i)$ for each i from 1 to k .

The function $\hat{G}(y)$ is calculated as $1 - \left(1 + \frac{\hat{\xi}y}{\hat{\sigma}}\right)^{-\frac{1}{\hat{\xi}}}$, provided $\xi \neq 0$. This process is based on the method described by [48].

3.2.2 Quqntile-Quantile plot

Using equation (3.3) and [3.4], the Q-Q plot is constructed given $\xi \neq 0$ or $\xi = 0$

$$\left(\hat{G}^{-1} \left(\frac{i}{m+1} \right), y_{(i)} \right), \quad i = 1, \dots, m,$$

where the inverse of $\hat{G}$ is given as

$$\hat{G}^{-1}(y) = k + \frac{\hat{\sigma}}{\hat{\xi}} \left[y^{-\hat{\xi}} - 1 \right].$$

3.3 Choosing an Appropriate Threshold

3.3.1 Parameter Stability (PS) plot

The *PS* is a widely utilized graphical method for selecting an appropriate threshold k when analyzing extreme values. The underlying concept is that if values exceeding a high threshold k_o follow a G Pareto distribution with parameter $\xi(shape)$ and $\delta_{K_o}(scale)$, then choosing any higher threshold $k > k_o$, will still result in exceedances that follow a G Pareto distribution. In this case, the shape parameter remains constant as $\xi_k = \xi$, and the scale parameter adjusts to

$$\delta_k = \delta_{k_o} + \xi(k - k_o)$$

[48]. To simplify parameterization, a new scale parameter, $\delta^* = \delta_k - \xi k$, is introduced. This re-parameterization ensures that δ^* is independent of k , provided k_o is sufficiently high to satisfy the assumptions of the GP distribution. The parameter stability plot is constructed by plotting $\{(k, \delta^*); k < w_{max}\}$ and $\{(k, \xi k); k < w_{max}\}$, where w_{max} is the maximum observed value in the data set. For an appropriately chosen threshold k_o , the estimates of both δ^* and ξk should remain stable (that is, constant) for all $k > k_o$ [48].

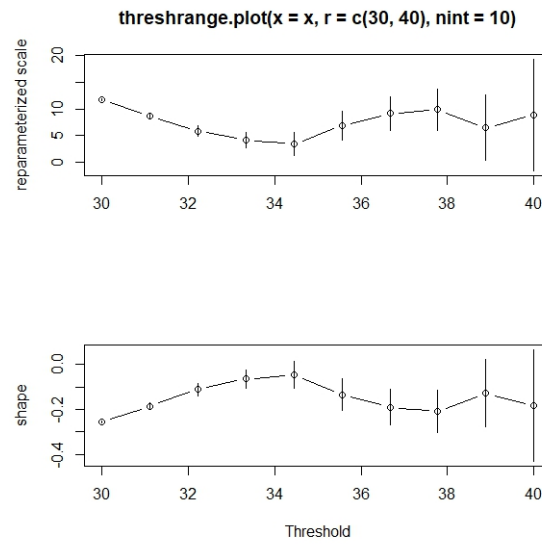


Figure 3.1: PS plots for Banjul station maximum air temperature.

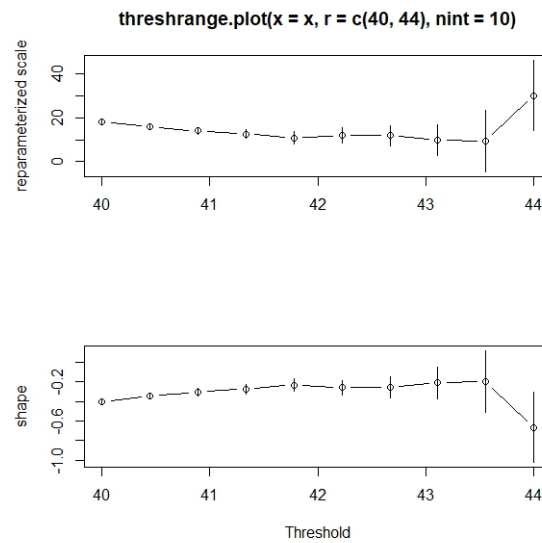


Figure 3.2: PS plots for Basse station maximum air temperature.

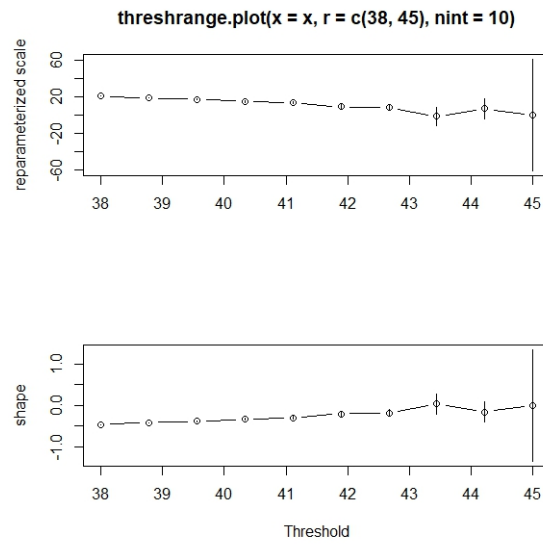


Figure 3.3: PS plots for Fatoto station maximum air temperature.

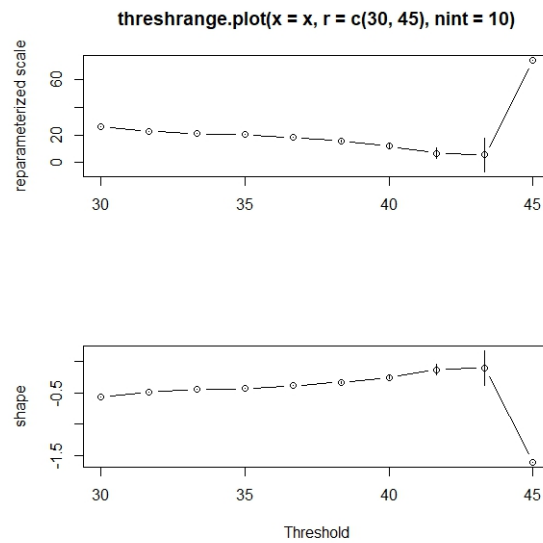


Figure 3.4: PS plots for Janjangbureh station maximum air temperature.

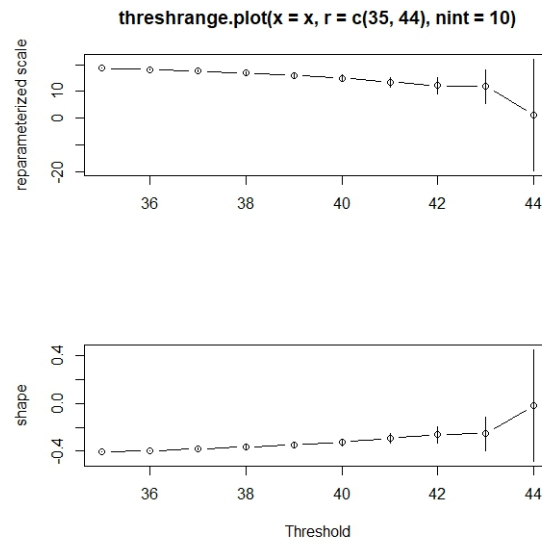


Figure 3.5: PS plots for Jenoi station maximum air temperature.

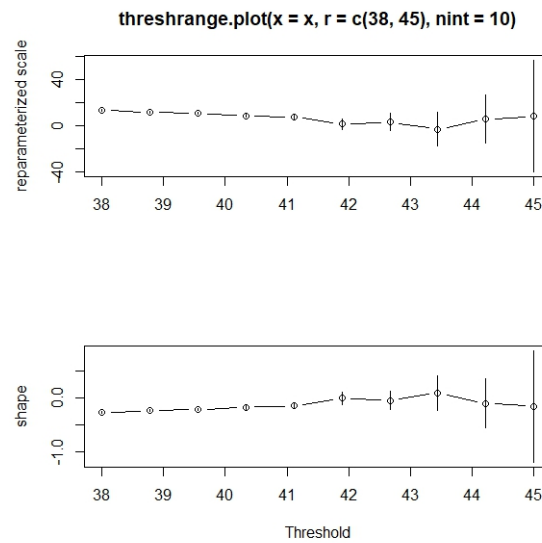


Figure 3.6: PS plots for Kaur station maximum air temperature.

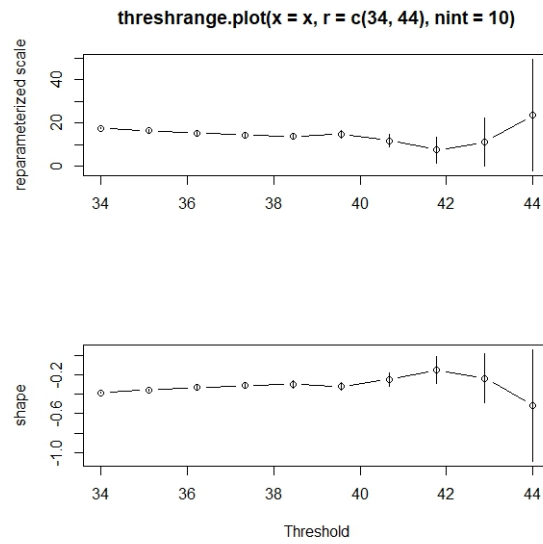


Figure 3.7: PS plots for Kerewan station maximum air temperature.

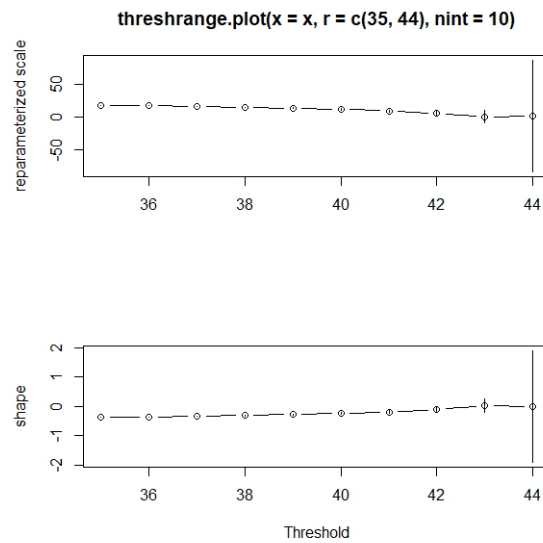


Figure 3.8: PS plots for Sapu station maximum air temperature.

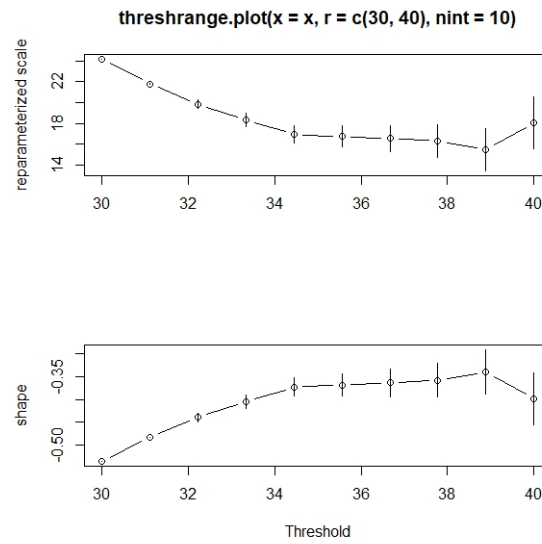


Figure 3.9: PS plots for Sibonor station maximum air temperature.

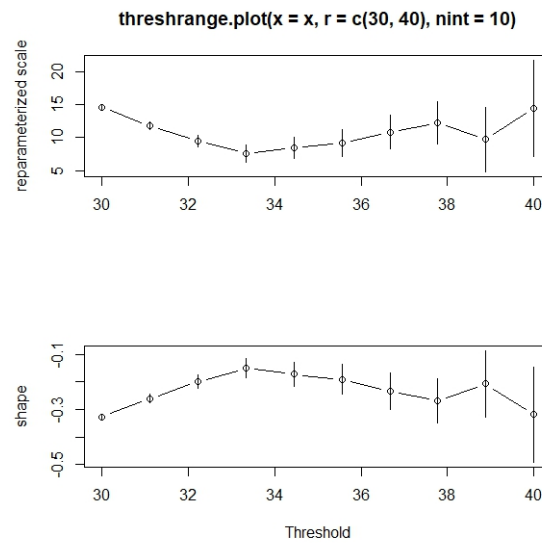


Figure 3.10: PS plots for Yundum station maximum air temperature.

3.3.2 Rules of Thumb

An alternative approach for selecting a threshold involves choosing one of the sample points from the dataset. This method is essentially linked to estimating the $k - th$ upper-order statistic, denoted as $W_{(1)}, \dots, W_{(n)}$. A commonly employed practice is to use the 90% quantile as a threshold. However, this approach has been critiqued for its theoretical limitations [49]. Other practical rules for determining the threshold are based on the dataset size n . For example, the square root rule suggests using $k = \sqrt{n}$, while a more defined option is $k = n^{\frac{2}{3}}/\log(\log(n))$. These rules provide useful guidelines for threshold selection in extreme value analysis, ensuring a balance between sufficient data retention and accurate modeling.

3.3.3 Mean Residual Life (MRL)Plot

This approach considers the expected value of the standardized residuals, which are assumed to be iid GP distribution. For the iid random variable Y following a GPD with parameters δ and ξ , the expected value of Y is given by:

$$E(Y) = \frac{\delta}{1 - \xi}$$

, provided $\xi < 1$. Consider a scenario in which the G Pareto distribution is a valid model for exceedances over specific threshold k_o from a series of observations $W_1, W_2, \dots, W_n$, with any given observation denoted as W . Using the above relation-

ship, the expected value of the excesses of W over the threshold k_o is:

$$E(W - k_o | W > k_o) = \frac{\delta_{k_o}}{1 - \xi},$$

where δ_{k_o} represents the scale parameter of the GP distribution for exceedances above k_o , also $\xi < 1$ [50]. An important property of the GP distribution is that if it is valid for exceedances of k_o , it remains valid for exceedances over any higher threshold k , where $k > k_o$. For all such thresholds, the expected value of excesses over k is given by:

$$E(W - k | W > k) = \frac{\delta_k}{1 - \xi} = \frac{\delta_{k_o} + \xi(k - k_o)}{1 - \xi},$$

where $\delta_k = \delta_{k_o} + \xi(k - k_o)$. This shows that for all thresholds $k > k_o$, the mean excess function $E(W - k | W > k)$ is a linear function of k . The mean excess value $E(W - k | W > k)$ is essentially the mean of the threshold exceedances W , and can be estimated using the sample mean of these exceedances [48]. The procedure involves plotting the mean excess values as a function of the threshold k . Specifically, the locus of points is given by:

$$\left(k, \frac{1}{n_k} \sum_{i=1}^{n_k} (w_i - k) \right), \quad k < w_{\max},$$

where $w_{(1)}, w_{(2)}, \dots, w_{(n_k)}$ are the n_k observations exceeding the threshold k , and $w_{\max}$ represents the largest observed value in the series W [48].

Interpreting MRL plots can be challenging in practice. The goal is to identify the lowest threshold where the plot appears roughly linear, while also considering

the 95% confidence interval, which strikes a good balance between including enough data for reliability and focusing on extremes. This approach helps to ensure that the chosen threshold is statistically sound and suitable for further analysis. Also known as a mean excess plot, is often used in extreme value analysis to help identify a threshold for generalized Pareto (GP) distribution in peak-over-threshold models, which can be seen in Figures 3.11 to 3.20.

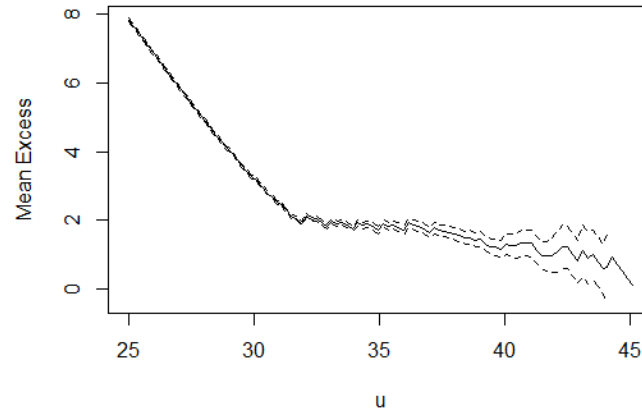


Figure 3.11: **MRL plot for Banjul station maximum air temperature.**

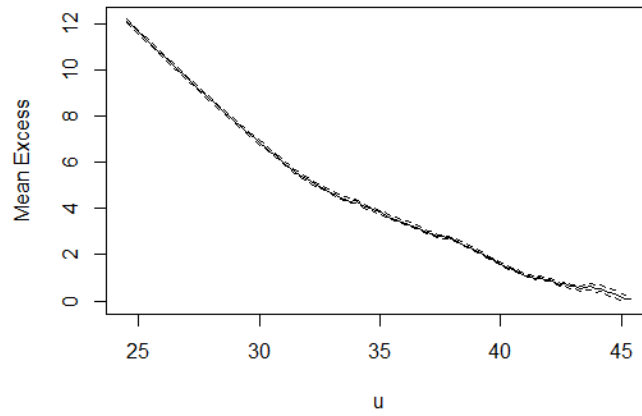


Figure 3.12: MRL plot for Basse station maximum air temperature.

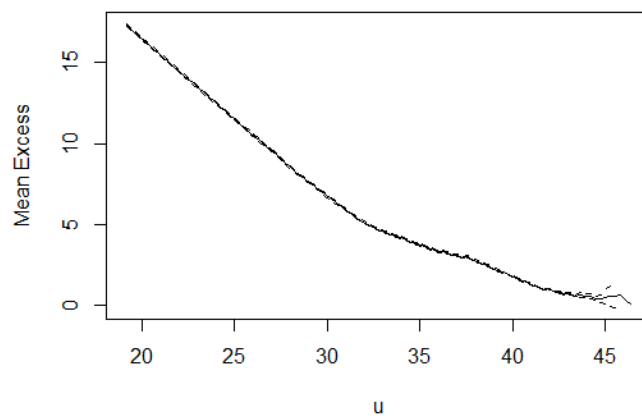


Figure 3.13: MRL plot for Fatoto station maximum air temperature.

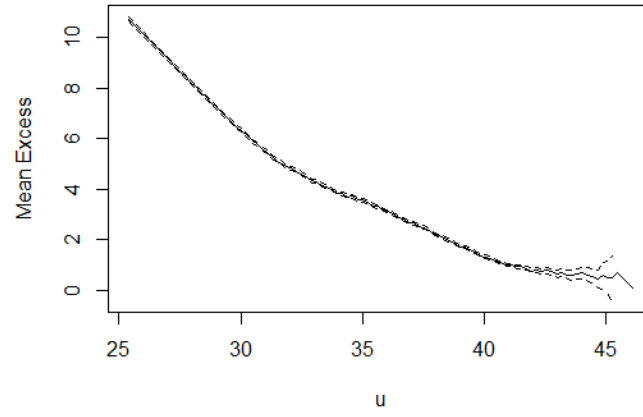


Figure 3.14: MRL plot for Janjangbureh station maximum air temperature.

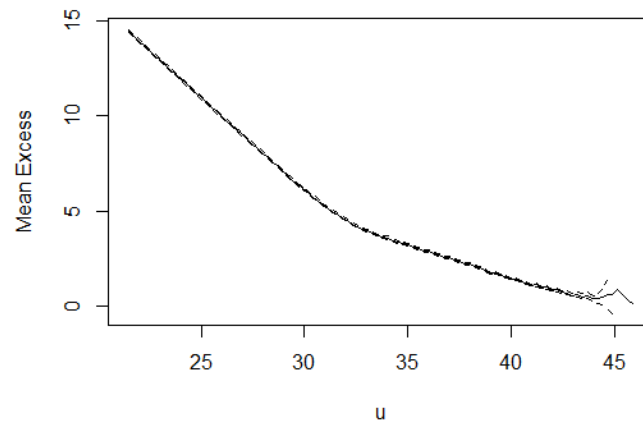


Figure 3.15: MRL plot for Jenoi station maximum air temperature.

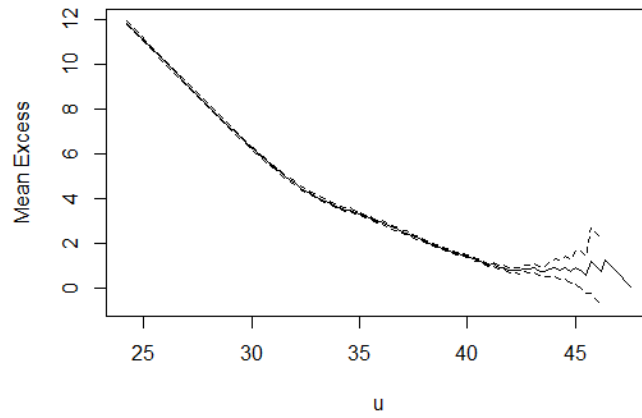


Figure 3.16: MRL plot for Kaur station maximum air temperature.

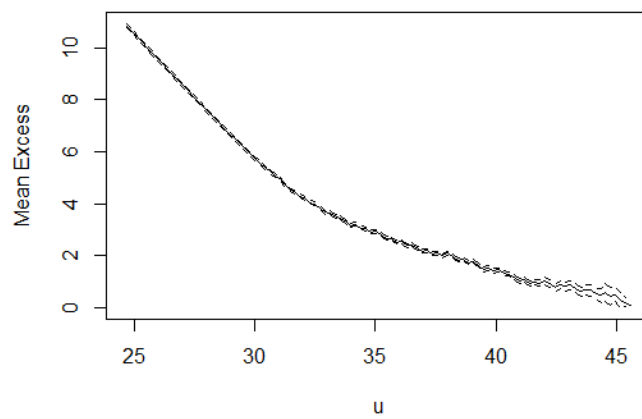


Figure 3.17: MRL plot for Kerewan station maximum air temperature.

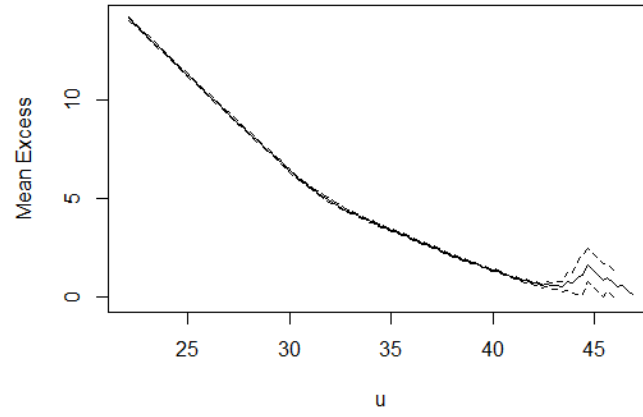


Figure 3.18: MRL plot for Sapu station maximum air temperature.

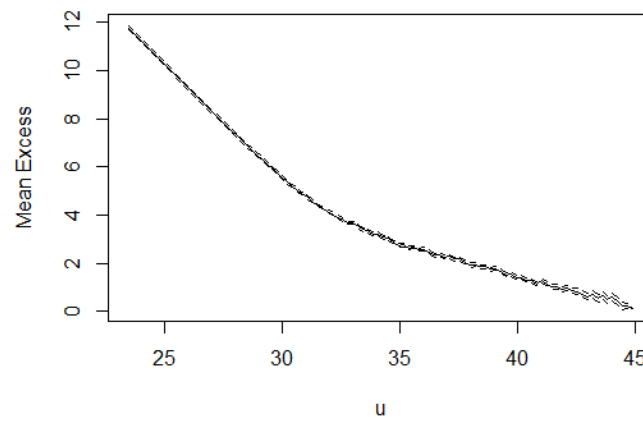


Figure 3.19: MRL plot for Sibamor station maximum air temperature.

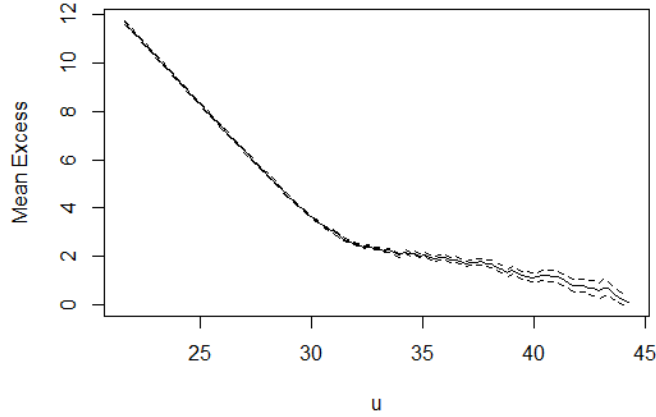


Figure 3.20: **MRL plot for Yundum station maximum air temperature.**

Base on the Figure 3.1 to 3.10 and Figure 3.11 to 3.20 above, from both MRL and PS plots we can conclude that the best choice of the threshold is approximately $k = 33^{\circ}C$ for Banjul station, $k = 41.5^{\circ}C$ for Basse station, $k = 42.5^{\circ}C$ for Fatoto station, $k = 40^{\circ}C$ for Janjangbureh station, $k = 39^{\circ}C$ for Jenoi station, $k = 40^{\circ}C$ for Kaur station, $k = 38.5^{\circ}C$ for Kerewan station, $k = 38^{\circ}C$ for Sapu station, $k = 34^{\circ}C$ for Sibanor station and $k = 33^{\circ}C$ for Yundum station to validate the assumptions of the extreme value approach.

3.4 Declustering

The POT method relies on the assumption that exceedances are mutually independent. However, in temperature data, threshold exceedances often occur in clusters, where an extremely warm day is likely to be followed by another [48]. To address this issue and ensure the applicability of the POT method, a process known as declustering is commonly employed. This technique filters out dependent observations,

yielding a set of threshold exceedances that can be treated as approximately independent. Declustering is carried out as follows: One(1), a threshold is chosen and clusters of exceedances are defined as consecutive occurrences where the threshold is surpassed. In two (2), a minimum separation, often referred to as the run length k , is then specified between the clusters. A cluster is considered complete if the separation between two exceedances exceeds the predefined run length k . Three(3), from each cluster the maximum exceedance is identified, as these maxima are assumed to be independent. These independent cluster maxima can then be used to fit the GP distribution, enabling a more robust statistical analysis [19], [51].

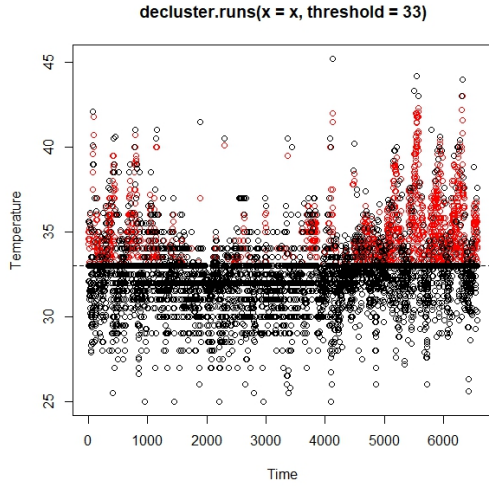


Figure 3.21: Maximum temperature for Banjul, showing declustered values above the $33^{\circ}C$ threshold

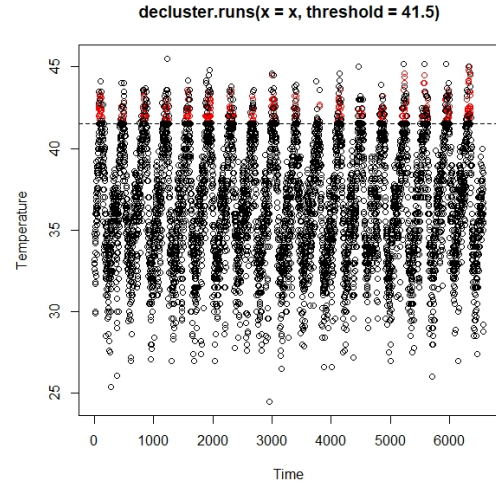


Figure 3.22: Maximum temperature for Basse, showing declustered values above the $41.5^{\circ}C$ threshold

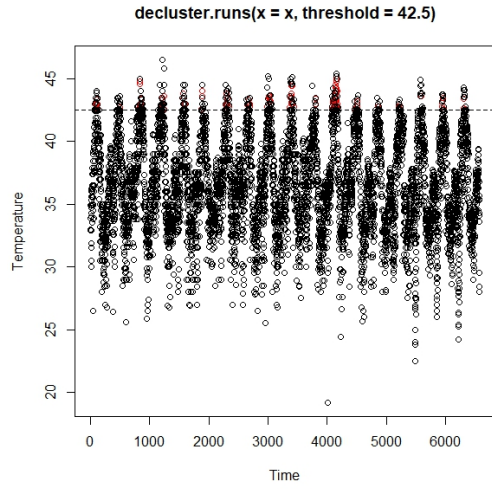


Figure 3.23: Maximum temperature for Fatoto, showing declustered values above the 42.5°C threshold

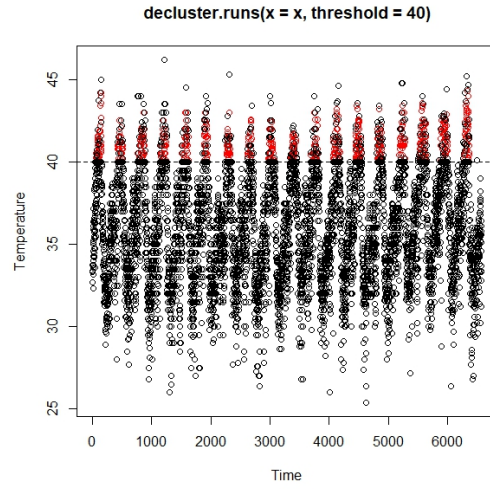


Figure 3.24: Maximum temperature for Janjangbureh, showing declustered values above the 40°C threshold

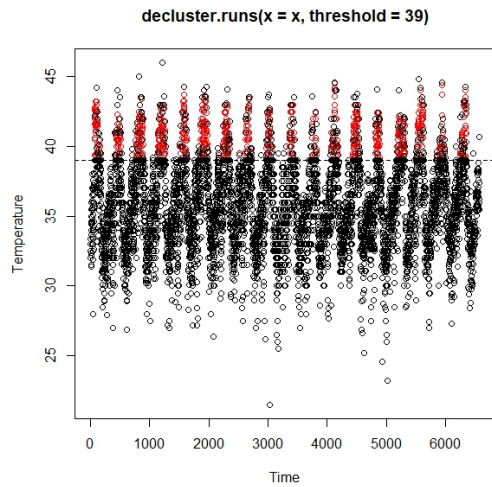


Figure 3.25: Maximum temperature for Jenoi, showing declustered values above the 39°C threshold

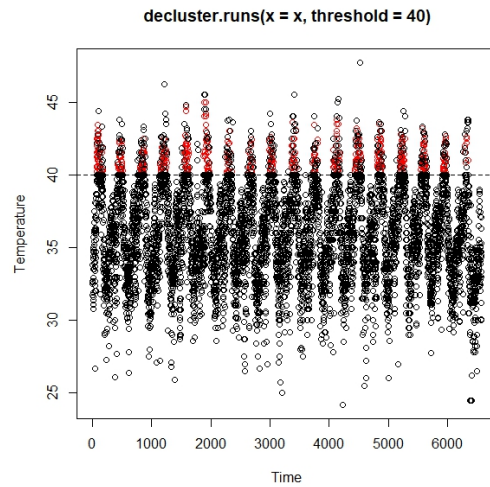


Figure 3.26: Maximum temperature for Kaur, showing declustered values above the 40°C threshold

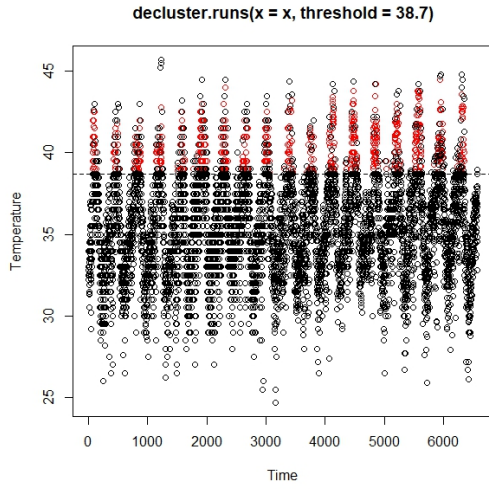


Figure 3.27: Maximum temperature for Kerewan, showing declustered values above the 38.7°C threshold

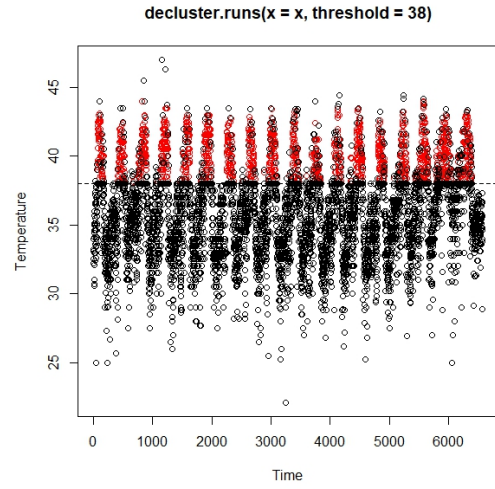


Figure 3.28: Maximum temperature for Sapu, showing declustered values above the 38°C threshold

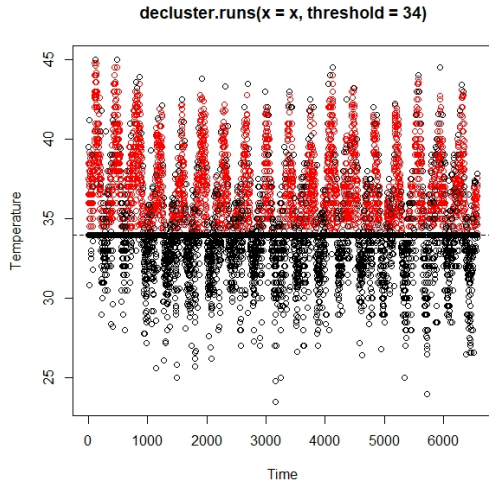


Figure 3.29: Maximum temperature for Sibanor, showing declustered values above the 34°C threshold

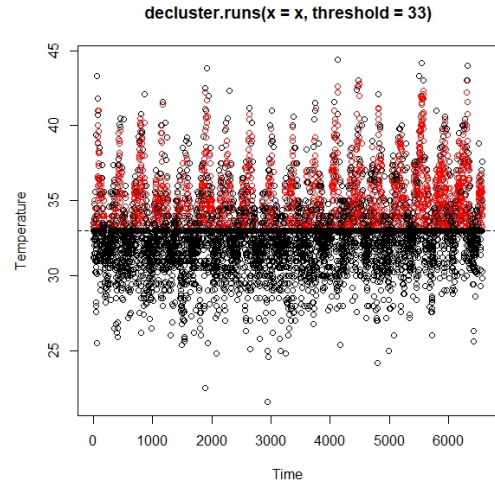


Figure 3.30: Maximum temperature for Yundum, showing declustered values above the 33°C threshold

3.5 Estimation and Fitting of the GP Distribution

The GP distribution parameters are shown in Table 3.1. The results showed the maximum air temperature data for Banjul, Basse, Fatoto, Janjangbureh, Jenoi, Kaur, Kerewan, Sapu, Sibanor and Yundum can be seen the shape parameter ξ is negative for all ten stations and be modeled by well known family of Weibull distribution. Moreover, all the ten station confidence intervals for the shape parameter do not contain 0 (zero).

Table 3.1: **GP Distribution Parameter Estimates for Maximum Air Temperature**

N	Station	Threshold	Parameter	Estimates	SE	CI
1	Banjul	33	Scale (δ_k)	2.361	0.062	(2.240, 2.483)
			Shape (ξ)	-0.134	0.017	(-0.167, -0.101)
2	Basse	41.5	Scale (δ_k)	1.342	0.059	(1.227, 1.457)
			Shape (ξ)	-0.310	0.024	(-0.358, -0.262)
3	Fatoto	42.5	Scale (δ_k)	0.999	0.061	(0.878, 1.121)
			Shape (ξ)	-0.211	0.032	(-0.273, -0.148)
4	Janjangbureh	40	Scale (δ_k)	1.688	0.054	(1.581, 1.794)
			Shape (ξ)	-0.259	0.013	(-0.285, -0.233)
5	Jenoi	39	Scale (δ_k)	2.476	0.070	(2.338, 2.615)
			Shape (ξ)	-0.349	0.011	(-0.370, -0.327)
6	Kaur	40	Scale (δ_k)	1.775	0.062	(1.653, 1.896)
			Shape (ξ)	-0.216	0.013	(-0.241, -0.190)
7	Kerewan	38.5	Scale (δ_k)	2.533	0.080	(2.374, 2.690)
			Shape (ξ)	-0.340	0.015	(-0.369, -0.310)
8	Sapu	38	Scale (δ_k)	2.762	0.053	(2.656, 2.866)
			Shape (ξ)	-0.304	0.006	(-0.314, -0.292)
9	Sibanor	34	Scale (δ_k)	4.589	0.076	(4.440, 4.737)
			Shape (ξ)	-0.411	0.008	(-0.427, -0.394)
10	Yundum	33	Scale (δ_k)	2.901	0.066	(2.771, 3.029)
			Shape (ξ)	-0.206	0.015	(-0.235, -0.178)

3.5.1 Graphical Model Validation for each Station under GP distribution

The following figures present diagnostic plots for the fitted GP distribution, applied to the maximum air temperature data from all meteorological stations. These plots provide a visual assessment of the model's performance and its suitability in capturing the behavior of extreme temperature values. The return level plot, density plot, Q-Q plot and P-P plots for maximum temperatures recorded at Banjul, Basse, Fatoto, Janjangbureh, Jeno, Kaur, Kerewan, Sapu, Sibanor and Yundum stations show a linear pattern in the plotted points, give the idea that the GP distribution offers a best fit to the observed temperature data.

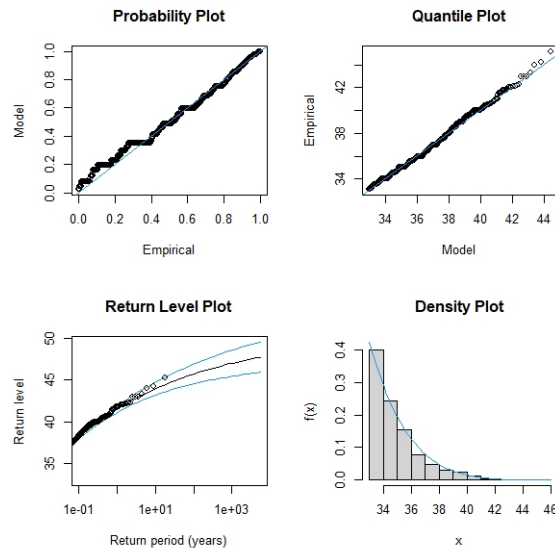


Figure 3.31: GP fits plots Banjul temperature data

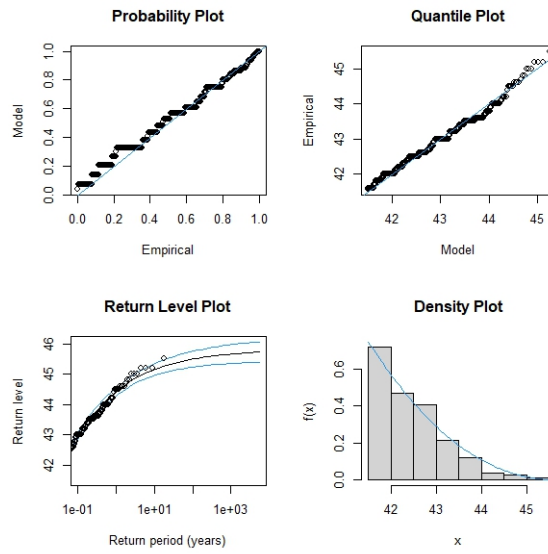


Figure 3.32: GP fits plots Basse temperature data

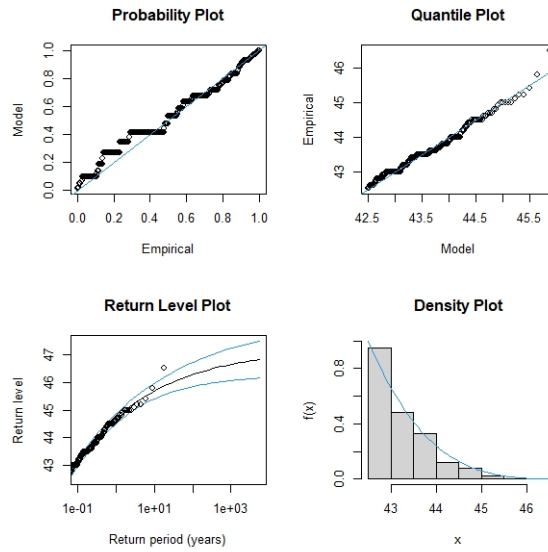


Figure 3.33: GP fits plots Fatoto temperature data

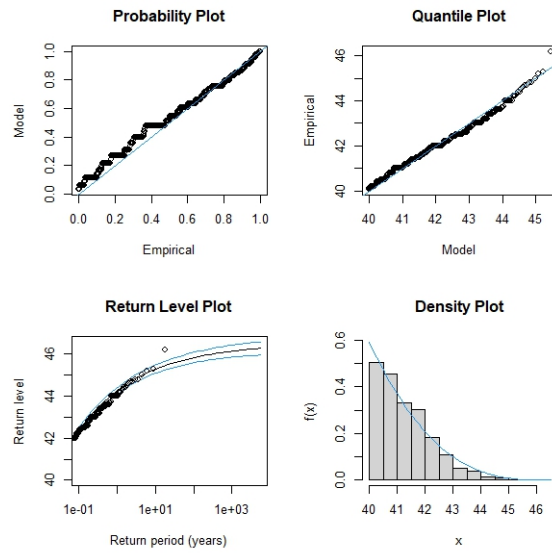


Figure 3.34: GP fits plots Janjangbureh temperature data

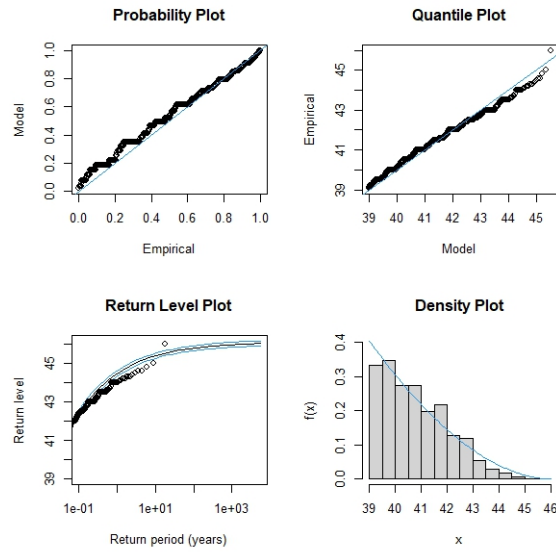


Figure 3.35: GP fits plots Jenoi temperature data

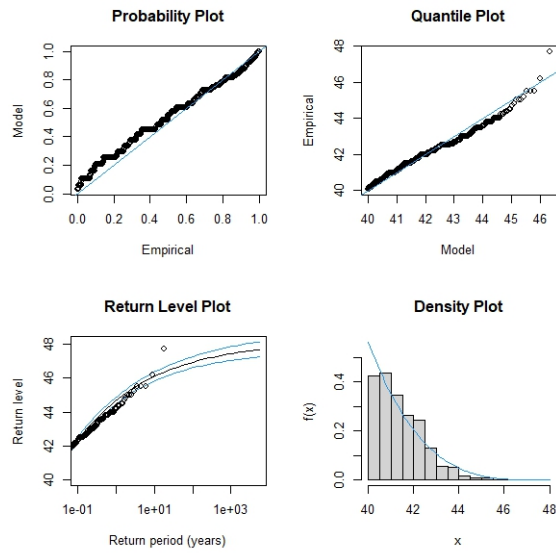


Figure 3.36: GP fits plots Kaur temperature data

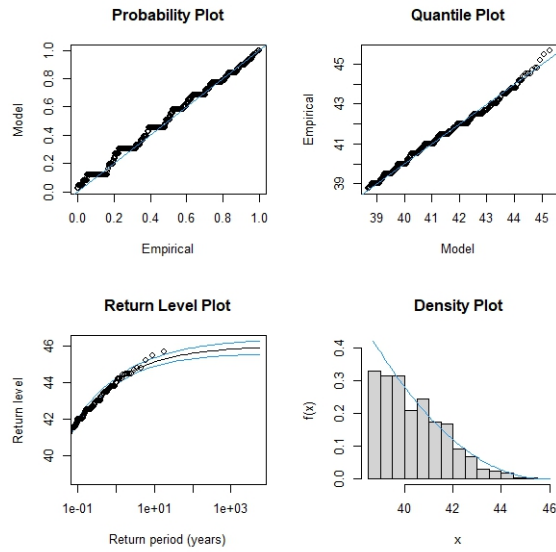


Figure 3.37: GP fits plots Kerewan temperature data.

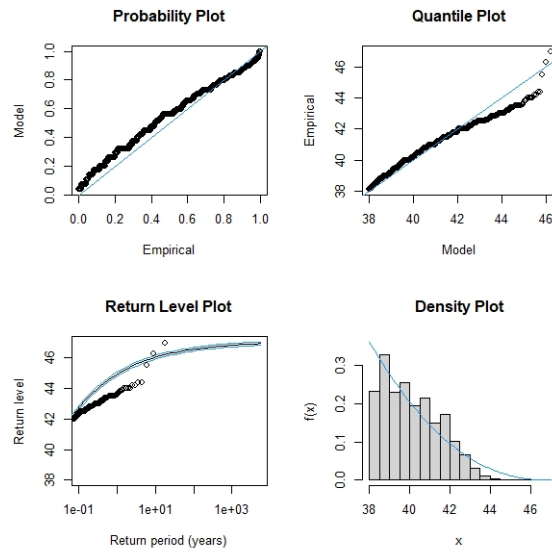


Figure 3.38: GP fits plots Sapu temperature data

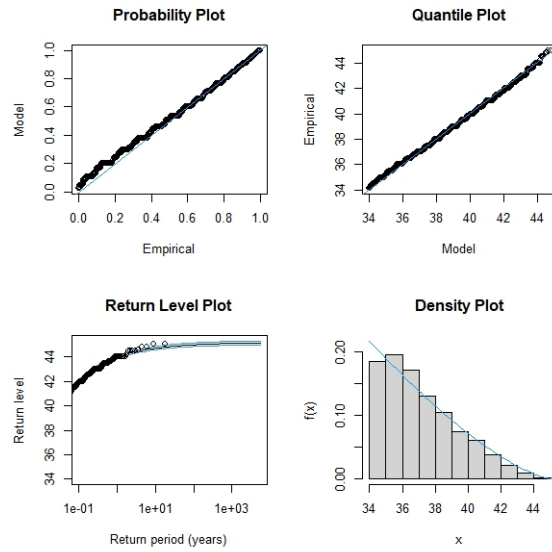


Figure 3.39: GP fits plots Sibamor temperature data

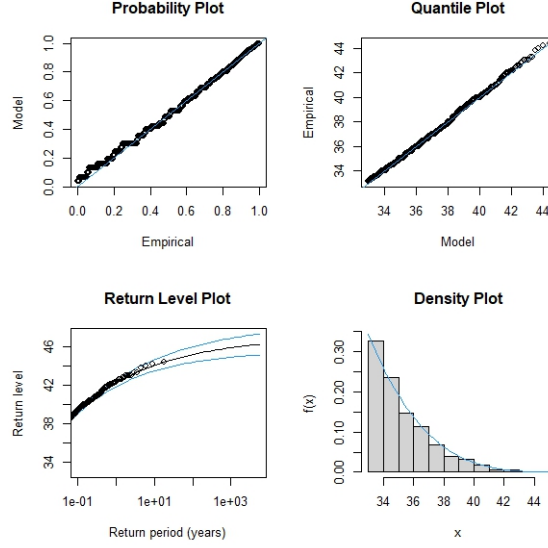


Figure 3.40: GP fits plots Yundum temperature data

3.5.2 Analysis of the Return levels (RLs) for GP distribution

The results in Table 3.2 present the return periods (RP) and their corresponding RL for maximum air temperatures across various stations in The Gambia, based on the GP distribution. The analysis demonstrates a consistent trend in which return levels increase with increasing return periods, indicating that more extreme temperature events are expected as the return periods grow longer. This trend aligns with the expected behavior of extreme temperature distributions.

Among the stations, **Basse** stands out with the highest return levels across most return periods. For example, the 10-year return level at Basse is 43.67°C , the highest compared to other stations. Similarly, the 100-year return level at Basse reaches 44.79°C , indicating that this station is particularly prone to extreme temperature events. These findings emphasize Basse's vulnerability to intense and prolonged heat conditions.

On the other hand, **Yundum** consistently exhibits the lowest return levels across all return periods. For instance, the 10-*year* return level in Yundum is 38.22°C , which is significantly lower than the corresponding value at Basse. Additionally, the 100-*year* return level at Yundum is 41.62°C , reinforcing its position as the station experiences the least extreme temperature events. This greatly enhances Yundum's relatively stable temperature patterns compared to other regions in The Gambia.

The projections for longer return periods, such as 500 and 1000 years, provide insight into the potential for unprecedented temperature extremes. At **Basse**, the 1000-*year* return level reaches 45.32°C , reflecting the likelihood of severe temperature events in the distant future. In contrast, **Yundum's** 1000-*year* return level is 43.68°C , remaining the lowest among the stations analyzed. The analysis also reveals significant regional variability in temperature extremes. While stations like **Basse** and **Fatoto** appear more susceptible to frequent and intense temperature extremes, locations such as **Yundum**, **Banjul** and **Kerewan** exhibit more moderate and stable patterns. For example, Kerewan has a 100-*year* return level of 44.38°C , which is relatively lower than Basse but higher than Yundum. This variability underscores the regional climate dynamics within The Gambia and highlights the need for localized adaptation strategies to mitigate the impacts of extreme temperature.

Table 3.2: **Return Period and Return Levels for Various Stations**

Station	2	5	10	30	50	100	200	500	1000
Banjul	33.84	36.21	37.59	39.42	40.17	41.10	41.95	42.95	43.63
Basse	41.97	43.11	43.67	44.31	44.54	44.79	44.99	45.20	45.32
Fatoto	42.85	43.79	44.29	44.92	45.16	45.44	45.69	45.96	46.14
Janjangbureh	40.59	42.10	42.88	43.80	44.14	44.54	44.86	45.21	45.42
Jenoi	39.85	41.89	42.86	43.92	44.28	44.67	44.98	45.29	45.46
Kaur	40.63	42.27	43.16	44.26	44.68	45.18	45.60	46.08	46.37
Kerewan	39.52	41.52	42.50	43.58	43.96	44.38	44.72	45.06	45.26
Sapu	38.96	41.33	42.50	43.84	44.31	44.85	45.28	45.72	45.98
Sibanor	35.56	39.14	40.74	42.39	42.92	43.48	43.90	44.30	44.52
Yundum	34.02	36.74	38.22	40.07	40.77	41.62	42.34	43.16	43.68

CHAPTER 4

BAYESIAN HIERARCHICAL MODELING (BHM)

4.1 Introduction to BHM

Bayesian hierarchical modeling is a potential tool for statistical modeling and parameter estimation in complex systems, especially those with multilevel or nested structures. Because it models data at multiple degrees, this philosophical approach allows for the exchange of information across classifications while rationalizing for uncertainty at each level. This method proposes a logical way to determine both individual-level and group-level variances, making it very vital when working with hierarchical data, such as repeated observations over time or across different physical locations [21]. Bayes' theorem, which serves as a foundation for the Bayesian hierarchical models which amalgamates prior information with observed data to contemporaneous beliefs about the factors of concern [52]. Imagine a situation where we want to model the air temperature data in dissimilar localities over time in The Gambia. Let W_{ij} represent

the temperature observation for the $i - th$ time point (yearly) at the $j - th$ location (station). The temperature observations may be assumed to be distributed normally with a location-specific mean μ_j and a common variance σ^2 by the observation model, the first level of the hierarchical model. This is expressed as:

$$W_{ij} \sim \mathcal{N}(\mu_j, \sigma^2)$$

In the following stage, location-specific means μ_j are modeled as coming from a common distribution that captures the overall variation in temperatures across different locations. For example, μ_j can be assumed to follow another distribution of normal with a global mean μ_o and variance τ^2 [53].

$$\mu_j \sim \mathcal{N}(\mu_o, \tau^2)$$

The parameters μ_o , σ^2 , and τ^2 themselves require a prior distribution, known as hyperprior, which are often chosen based on prior information or assumption. For instance, the global mean μ_o might be given a prior distribution such as $\mu_o \sim \mathcal{N}(\eta, \xi^2)$, where η and ξ^2 are hyper-parameters that store our prior beliefs about the global mean temperature.

The Bayesian hierarchical model's goal is to compute the posterior distribution of all parameters given the observed data $W = \{W_{ij}\}$. The joint posterior distribution for the parameter $\theta = (\mu_o, \mu_j, \sigma^2, \tau^2)$, which is proportional to the product of the likelihood and the prior distributions as:

$$P(\theta/W) \propto P(W/\theta) \times P(\theta)$$

$$P(\mu_o, \mu_j, \sigma^2, \tau^2/W_{ij}) \propto P(W_{ij}/\mu_j, \sigma^2)P(\mu_j/\mu_o, \tau^2)P(\mu_o)P(\tau^2)P(\sigma^2)$$

here, $P(W/\theta)$ is the likelihood function and $P(\theta)$ is the prior distribution.

Because the posterior distribution is often complex and analytically intractable, computational methods such as (MCMC) Markov Chain Monte Carlo are employed to approximate it [53]. This enables us to integrate the data and prior knowledge in a logical way and generate probabilistic claims about the parameters and predictions.

4.2 Assessment of parameter stability

To our knowledge, only a small number of research have used Bayesian hierarchical models for climate data in conjunction with extensive Monte Carlo simulations designed especially for assessing parameter uncertainty in EVT. The robustness and sensitivity of the suggested model parameters will be thoroughly evaluated by this work using these simulations, leading to a more sophisticated comprehension of uncertainty in severe temperature predictions.

4.2.1 Gibbs Sampler

For example, the Gibbs sampler is a method for drawing samples from multivariate distributions by leveraging the ability to sample from conditional distributions. Consider a target density $\pi(\omega)$, typically representing a posterior density, where

$\omega = (\omega_1, \dots, \omega_d)'$. Assume that we can sample from the full conditional distributions $\pi(\omega_i | \omega_1, \dots, \omega_{i-1}, \omega_{i+1}, \dots, \omega_d) = \pi(\omega_i | \omega_{-i}) = \pi_i(\omega_i)$, $i = 1, \dots, d$, where ω_{-i} represents all components of ω except for ω_i . The Gibbs sampling algorithm can then proceed as follows:

1. For the iteration start counter with $k = 1$ and initialize the chain with an initial state $\omega^{(0)} = (\omega_1^{(0)}, \dots, \omega_d^{(0)})'$;
2. Generate another state $\omega^{(k)}$ from $\omega^{(k-1)}$ by sequentially sampling each component:

$$\begin{aligned}\omega_1^{(k)} &\sim \pi(\omega_1 | \omega_2^{(k-1)}, \dots, \omega_d^{(k-1)}), \\ \omega_2^{(k)} &\sim \pi(\omega_2 | \omega_1^{(k)}, \omega_3^{(k-1)}, \dots, \omega_d^{(k-1)}), \\ &\vdots \\ \omega_d^{(k)} &\sim \pi(\omega_d | \omega_1^{(k)}, \dots, \omega_{d-1}^{(k)}); \end{aligned}$$

3. Increase the counter k by 1, and repeat step 2.

In this approach, each sample relies only on the last sample obtained, and not on earlier values or the iteration count k . The Gibbs sampler is especially useful when sampling directly from the full conditionals is feasible. However, if sampling from the conditionals is challenging, it can be combined with Metropolis–Hastings steps to handle cases where the full conditional distributions are not straightforward to sample from.

4.2.2 Metropolis–Hastings sampling

Assume that the target density is given by $\pi(\omega)$. We have an arbitrary transition kernel $p(\omega', \omega)$, which can be easily simulated to iteratively sample successive values.

The algorithm proceeds as follows:

1. begin by setting the iteration counter to $k = 1$ and initialize the chain at the starting value $\omega^{(0)}$.
2. Use the kernel $p(\omega^{(k-1)}, \omega')$ to generate a proposed value ω' .
3. Determine the acceptance probability $A(\omega^{(k-1)}, \omega')$ associated with the proposed transition, which is given by:

$$A(\omega, \omega') = \min \left(1, \frac{\pi(\omega')L(\omega'|x)p(\omega', \omega)}{\pi(\omega)L(\omega|x)p(\omega, \omega')} \right).$$

4. With probability $A(\omega^{(k-1)}, \omega')$, set $\omega^{(k)} = \omega'$; otherwise, let $\omega^{(k)} = \omega^{(k-1)}$.
5. Increment k by 1 and repeat from step 2.

In this approach, a new value is drawn at each step from the proposal distribution. If accepted, the chain advances; if not, it remains at the current position. Acceptance probability is influenced by the alignment between the target density and the proposal distribution. Frequently used options for the proposal distribution include symmetric kernels, where $p(\omega, \omega') = p(\omega', \omega)$, and random walk kernels. In the random walk approach, the proposed value ω' at iteration k is given by $\omega' = \omega + \epsilon_k$, where ϵ_k represents independently and identically distributed (IID) random variables.

4.3 Bayesian MCMC on GEV Distribution

In Bayesian analysis, prior information about a parameter θ can be formally represented using a probability density function, denoted as $P(\theta)$. This informative prior is chosen based on existing knowledge, historical data, or expert opinions, providing a reasonable starting point for inference. When new data w becomes available, the likelihood function, $p(\theta|w)$, quantifies how well different values of θ explain the observed data [18].

By applying Baye's theorem, the prior distribution and the likelihood defined above are combined to obtained the posterior distribution of θ , which updates our understanding of the parameter in light of both prior knowledge and observed data. Informative priors are particularly useful when relevant background information is avialable, ensuring that parameter estimates are stable and meaningful even with limited data [54]. Bayesian inference, coupled with (MCMC) Markov Chain Monte Carlo methods, enables efficient sampling from the posterior distribution, facilitating parameter estimation in complex models, which is the case for the GEV distribution [55].

From the pdf of the GEV distribution below;

$$f(w \mid \mu, \sigma, \xi) = \frac{1}{\sigma} \left(1 + \xi \frac{w - \mu}{\sigma} \right)^{-\frac{1}{\xi} - 1} \exp \left(- \left(1 + \xi \frac{w - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right), \quad (4.1)$$

and its likelihood function;

$$p(w \mid \mu, \sigma, \xi) = \left[\prod_{i=1}^n f(w_i \mid \mu, \sigma, \xi) \right]$$

4.4 Selection of the prior distributions

To estimate the parameters of the GEV distribution, we employ a MCMC approach. The selection of prior distributions is guided by both the existing literature and the key characteristics observed in the temperature dataset [56]. Each parameter is assigned an independent prior distribution to reflect its inherent properties while maintaining model flexibility [57].

For the scale parameter $\log \sigma$, we assume a **Normal prior**, which ensures positivity and aligns with the empirical variability seen in the data. The location parameter (μ) and the shape parameter (ξ) are assigned **Normal priors**, reflecting their potential to take both positive and negative values. These choices allow for a balance between incorporating prior knowledge and letting the data influence the parameter estimates. To obtain the posterior distribution, we maximize the log-posterior function, which combines prior knowledge with the likelihood of observed data. Since analytical solutions are often intractable, we implement the **Metropolis-Hastings algorithm** to generate posterior samples and explore the parameter space efficiently. Evaluating the marginal likelihood helps assess the overall model fit and compare different prior assumptions. This structured approach ensures that our Bayesian inference framework provides reliable and interpretable estimates for extreme temperature modeling.

4.4.1 When location parameter μ follows a normal prior:

$$\mu \sim N(\mu_0, \tau_\mu^2), \quad (4.2)$$

$$p(\mu) = \frac{1}{\sqrt{2\pi\tau_\mu^2}} \exp \left\{ -\frac{(\mu - \mu_0)^2}{2\tau_\mu^2} \right\}$$

where:

- μ_0 is the prior mean,
- τ_μ^2 is the prior variance.

And Compute the Posterior distribution Using Bayes' Theorem, which states:

$$p(\mu \mid w, \sigma, \xi) \propto p(w \mid \mu, \sigma, \xi)p(\mu). \quad (4.3)$$

Expanding the terms:

- $p(w \mid \mu, \sigma, \xi)$ is the likelihood of the GEV distribution.
- $p(\mu)$ is the prior from equation [4.2].

Thus, the posterior distribution is as follows.

$$p(\mu \mid w, \sigma, \xi) \propto \left[\prod_{i=1}^n f(w_i \mid \mu, \sigma, \xi) \right] \times N(\mu_0, \tau_\mu^2).$$

$$p(\mu, \sigma, \xi \mid w) \propto \prod_{i=1}^n \frac{1}{\sigma} \left(1 + \xi \frac{w_i - \mu}{\sigma} \right)^{-\frac{1}{\xi}-1} \exp \left\{ - \left(1 + \xi \frac{w_i - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right\} \cdot \exp \left\{ -\frac{(\mu - \mu_0)^2}{2\tau_\mu^2} \right\}.$$

Taking the log-posterior:

$$\log p(\mu \mid w, \sigma, \xi) = \sum_{i=1}^n \log f(w_i \mid \mu, \sigma, \xi) - \frac{1}{2\tau_\mu^2}(\mu - \mu_0)^2 + K(\text{constant}),$$

$$\begin{aligned} \log p(\mu \mid \sigma, \xi, w) = K(\text{constant}) + \sum_{i=1}^n \left[-\log \sigma - \left(\frac{1}{\xi} + 1 \right) \log \left(1 + \xi \frac{w_i - \mu}{\sigma} \right) \right. \\ \left. - \left(1 + \xi \frac{w_i - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right] - \frac{1}{2\tau_\mu^2}(\mu - \mu_0)^2. \end{aligned} \quad (4.4)$$

4.4.2 when the log (scale) parameter follows a normal prior

we say $\log \sigma$

$$\log \sigma \sim N(\mu_\sigma, \tau_\sigma^2), \quad (4.5)$$

$$\log(\sigma) \sim \mathcal{N}(\mu_\sigma, \tau_\sigma^2),$$

which implies that the prior for σ is:

$$p(\sigma) = \frac{1}{\sigma \sqrt{2\pi\tau_\sigma^2}} \exp \left(-\frac{(\log \sigma - \mu_\sigma)^2}{2\tau_\sigma^2} \right).$$

where:

- μ_σ is the prior mean,

- τ_σ^2 is the prior variance.

Bayes' Theorem defines the posterior distribution as:

$$p(\mu, \sigma, \xi \mid w) \propto L(w \mid \mu, \sigma, \xi)p(\sigma). \quad (4.6)$$

- $p(w \mid \mu, \sigma, \xi)$ is the likelihood of the GEV distribution.
- $p(\sigma)$ is the prior from equation [4.5].

Thus, the posterior distribution is as follows.

Taking the logarithm:

$$\log p(\mu, \sigma, \xi \mid w) = \log L(\mu, \sigma, \xi) + \log p(\sigma) + K(\text{constant}).$$

$$\log p(\mu, \sigma, \xi \mid w) = -n \log \sigma - \sum_{i=1}^n \left(1 + \xi \frac{w_i - \mu}{\sigma}\right)^{-1/\xi} - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \log \left(1 + \xi \frac{w_i - \mu}{\sigma}\right)$$

$$- \frac{(\log \sigma - \mu_\sigma)^2}{2\tau_\sigma^2} - \log \sigma + K(\text{constant}).$$

$$\begin{aligned} \log p(\sigma \mid w, \mu, \xi) &= -(n+1) \log \sigma - \sum_{i=1}^n \left(1 + \xi \frac{w_i - \mu}{\sigma}\right)^{-1/\xi} \\ &- \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \log \left(1 + \xi \frac{w_i - \mu}{\sigma}\right) - \frac{(\log \sigma - \mu_\sigma)^2}{2\tau_\sigma^2} + K(\text{constant}). \end{aligned}$$

4.4.3 When the shape parameter ξ follows a normal prior:

$$\xi \sim N(\xi_0, \varphi^2) \quad (4.7)$$

which implies the prior density:

$$p(\xi) = \frac{1}{\sqrt{2\pi\varphi^2}} \exp\left(-\frac{(\xi - \xi_0)^2}{2\varphi^2}\right).$$

Here, ξ_0 is the prior mean and φ^2 is the prior variance.

By Bayes' theorem:

$$p(\xi \mid w, \mu, \sigma) \propto p(w \mid \mu, \sigma, \xi)p(\xi). \quad (4.8)$$

Expanding the terms:

- $p(w \mid \mu, \sigma, \xi)$ is the likelihood from the GEV distribution [4.1].
- $p(\xi)$ is the prior, which follows a normal distribution.

Thus, the posterior distribution is given as:

$$p(\xi \mid w, \mu, \sigma) \propto \left[\prod_{i=1}^n f(w_i \mid \mu, \sigma, \xi) \right] \times \exp\left(-\frac{(\xi - \xi_0)^2}{2\varphi^2}\right).$$

logarithm of the posterior:

$$\log p(\xi \mid w, \mu, \sigma) = \sum_{i=1}^n \log f(w_i \mid \mu, \sigma, \xi) - \frac{(\xi - \xi_0)^2}{2\varphi^2} + K(\text{constant}).$$

$$\sum_{i=1}^n \left[-\log \sigma - \left(1 + \xi \frac{w_i - \mu}{\sigma} \right)^{-1/\xi} - \left(1 + \frac{1}{\xi} \right) \log \left(1 + \xi \frac{w_i - \mu}{\sigma} \right) \right].$$

$$\begin{aligned} \log p(\xi \mid w, \mu, \sigma) = & -n \log \sigma - \sum_{i=1}^n \left(1 + \xi \frac{w_i - \mu}{\sigma} \right)^{-1/\xi} - \left(1 + \frac{1}{\xi} \right) \sum_{i=1}^n \log \left(1 + \xi \frac{w_i - \mu}{\sigma} \right) \\ & - \frac{(\xi - \xi_0)^2}{2\varphi^2} + K(\text{constant}). \end{aligned} \tag{4.9}$$

4.5 Bayesian Credible Interval

We use credible intervals to quantify the uncertainty associated with parameter estimation by specifying a range within which the true parameter value is likely to lie, such as the parameters of the GEV distribution given the observed data. Like chalk and cheese, frequentist confidence intervals, which rely on hypothetical repeated sampling, credible intervals are derived from the posterior distribution and provide a more intuitive probabilistic interpretation of parameter estimates [21]. A credible interval for the GEV parameter (μ, σ, ξ) is in an interval $[a, b]$ such that:

$$P(a \leq (\mu, \sigma, \xi) \leq b \mid w) = 1 - \alpha$$

where $[a, b]$ denotes the interval and $1 - \alpha$ corresponds to the posterior probability [58]. The Bayesian credible interval provides a direct probabilistic statement about the parameter itself. This distinction highlights a key advantage of Bayesian inference, particularly in cases where incorporating prior knowledge is beneficial or when data

availability is limited [20].

The derivation of credible intervals relies on the posterior distribution of the parameter of interest. The most common approach is the equal-tailed (quantile-based) credible interval, which is constructed by taking the *2.5th* and *97.5th* percentiles of the posterior distribution, ensuring that 95% of the posterior probability mass is contained within the interval [59]. Another widely used approach is the HPD interval (highest posterior density), which specifies the shortest interval containing 95% of the posterior probability mass. Unlike the equal-tailed interval, which may include less probable values at the tails of the distribution, the HPD interval is particularly useful for skewed or multimodal posterior distributions, ensuring that the interval encompasses the most credible parameter values [58].

We employed credible intervals to assess the uncertainty about the estimation of the GEV parameters (μ, σ, ξ) . Using MCMC sampling, the posterior distributions of these parameters are obtained and the credible intervals 95% are computed to determine the range within which they are most likely to fall. This Bayesian approach ensures a more probabilistic and robust understanding of parameter uncertainty, making it well-suited for extreme value modeling [19].

4.6 Results

The parameters of the GEV distribution were estimated using Bayesian Markov Chain Monte Carlo MCMC techniques, specifically employing the Metropolis-Hastings algorithm. Within the Bayesian framework, the algorithm outline in Subsection [4.2.2] was utilized to obtain parameter estimates. The analysis was carried out on daily air temperature data from ten stations, applying the algorithm in each case. A total of 30,000 iterations were performed for each station to ensure robust parameter estimation.

To assess the convergence of the Markov Chain to correct distribution, the same algorithm was executed using predefined starting points. The convergence was observed within the first 15,000 iterations, confirming the effectiveness of the approach. Furthermore, the proposal distribution developed for the Metropolis-Hastings algorithm was found to perform well. An informative prior distribution was used in the Bayesian MCMC method, ensuring that all initial values led to accurate parameter estimates. It is important to note that the same initial values were applied uniformly across all stations in this study.

The results of the estimation of the GEV parameters for the ten stations, which span from Banjul to Yundum, are presented in Figures [4.1 to 4.10]. These figures illustrate the estimated posterior densities and trace plots for the location (μ), scale (σ) and shape (ξ) parameter derived from the Bayesian MCMC approach. In addition, Table [4.1] provides a summary of the posterior means and credible intervals by Bayesian MCMC for the parameter estimates obtained for the ten stations using the GEV

distribution. The posterior means from the Bayesian MCMC estimation are closely aligned with the MML estimates of maximum likelihood of the GEV distribution, demonstrating the reliability of the Bayesian approach in this analysis.

4.6.1 Analysis of the Return levels (RLs) under Bayesian MCMC

In Table [4.2] the return level estimates for different periods (2, 5, 10, 30, 50, 100, 200, 500, 1000)*years* indicate a clear increasing trend in the extreme temperature in all stations from Banjul down to Yundum. As the return period extends to 100, 500 and 1000 years, the projected temperatures rise significantly, suggesting a future with more frequent and intense heat events. In particular, Fatoto and Janjangbureh exhibit the highest return levels, with Fatoto reaching 45.72°C at the 1000-year return period, making them particularly vulnerable to extreme temperatures. These findings highlight the need for proactive policy measures, particularly in agriculture, the energy sector and public health, to mitigate the impact of rising temperatures. Stations like Banjul and Yundum show comparatively lower increases, but the overall trend suggests that no location is entirely exempt from the effects of climate change. As extreme temperatures become more common, adaptation strategies, such as improved infrastructure and heat resilience planning, will be crucial in ensuring long-term sustainability and safety for communities.

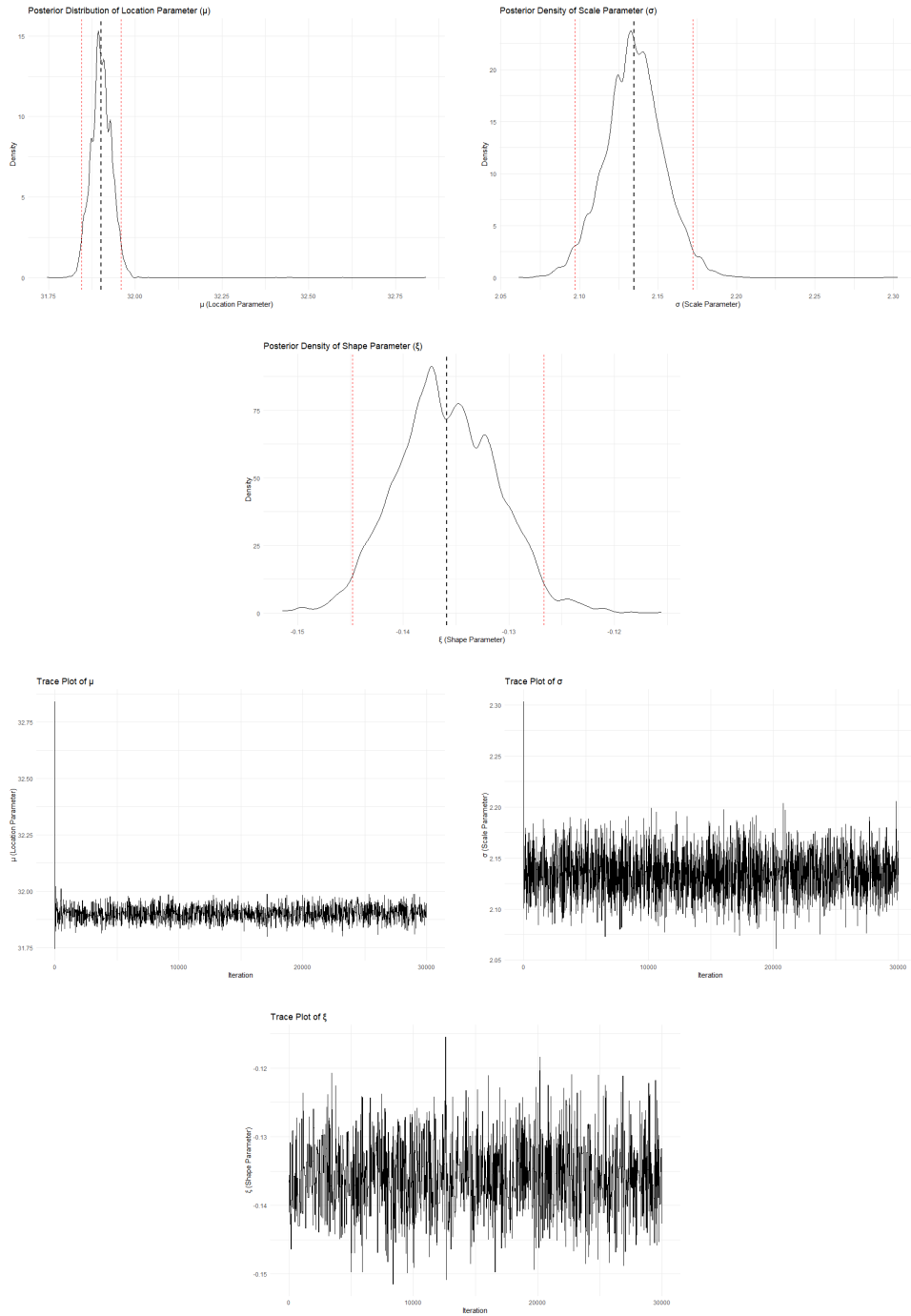


Figure 4.1: Visual summary of GEV parameter for Banjul using trace and posterior plots

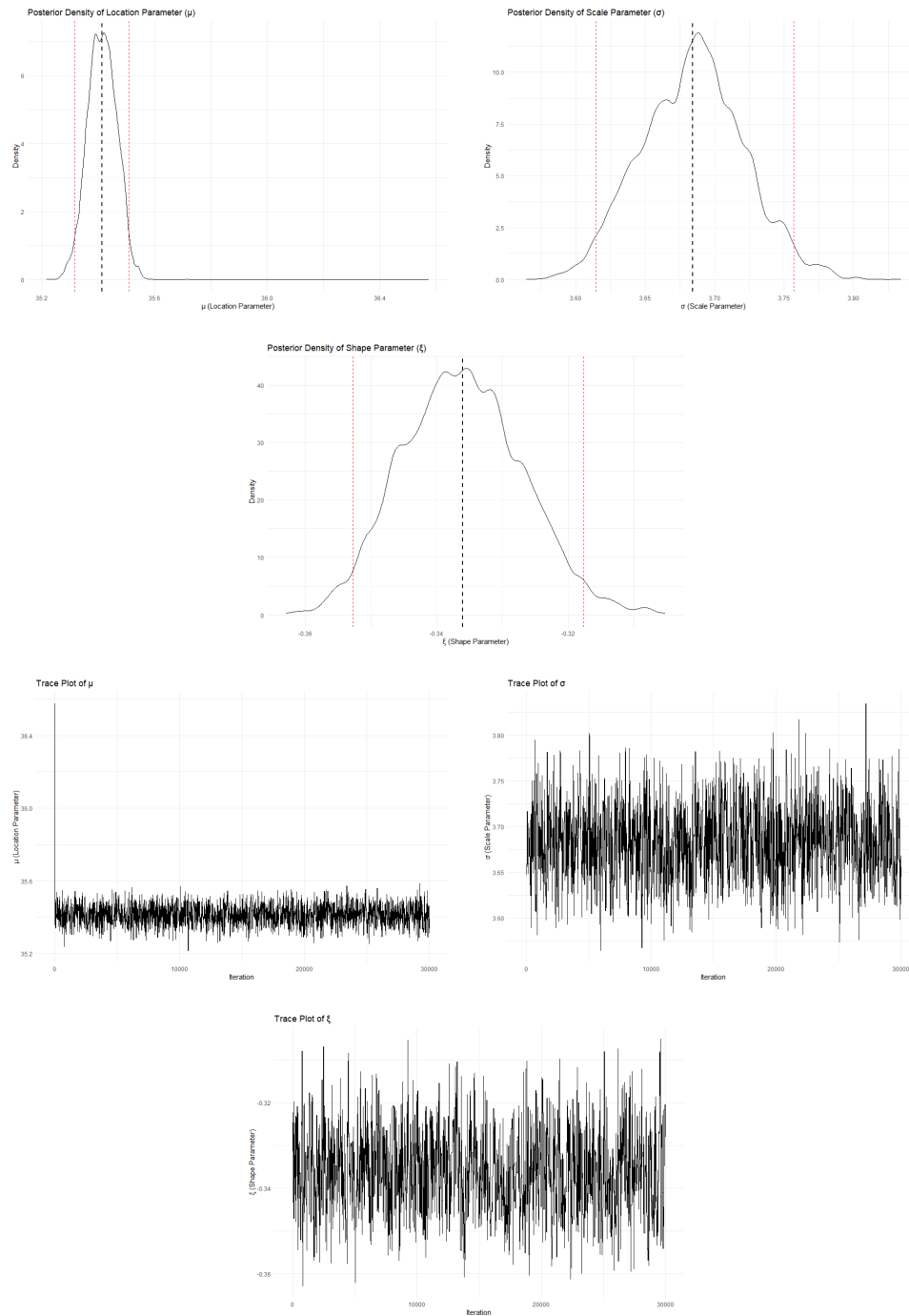


Figure 4.2: Visual summary of GEV parameter for Basse using trace and posterior plots

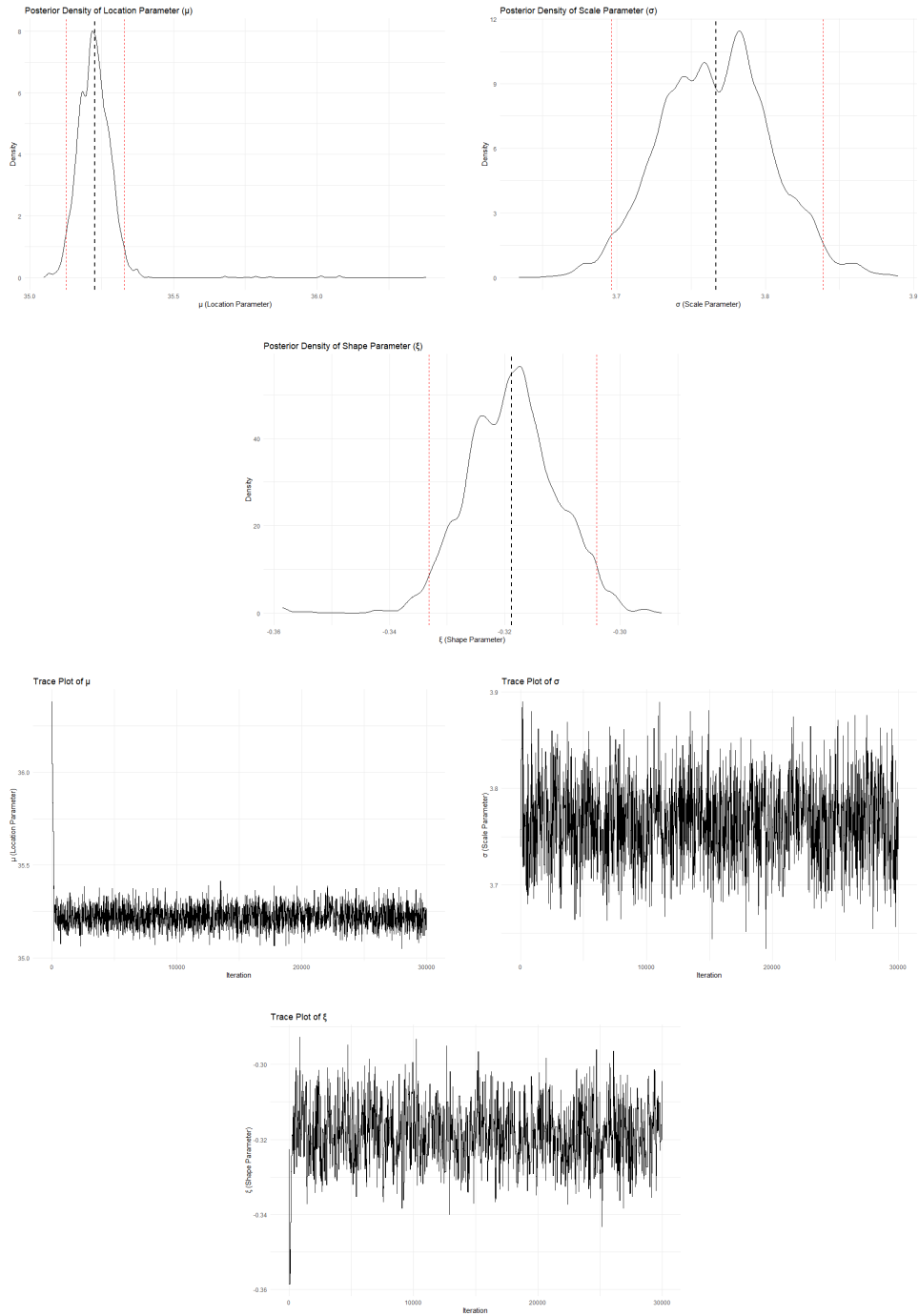


Figure 4.3: Visual summary of GEV parameter for Fatoto using trace and posterior plots

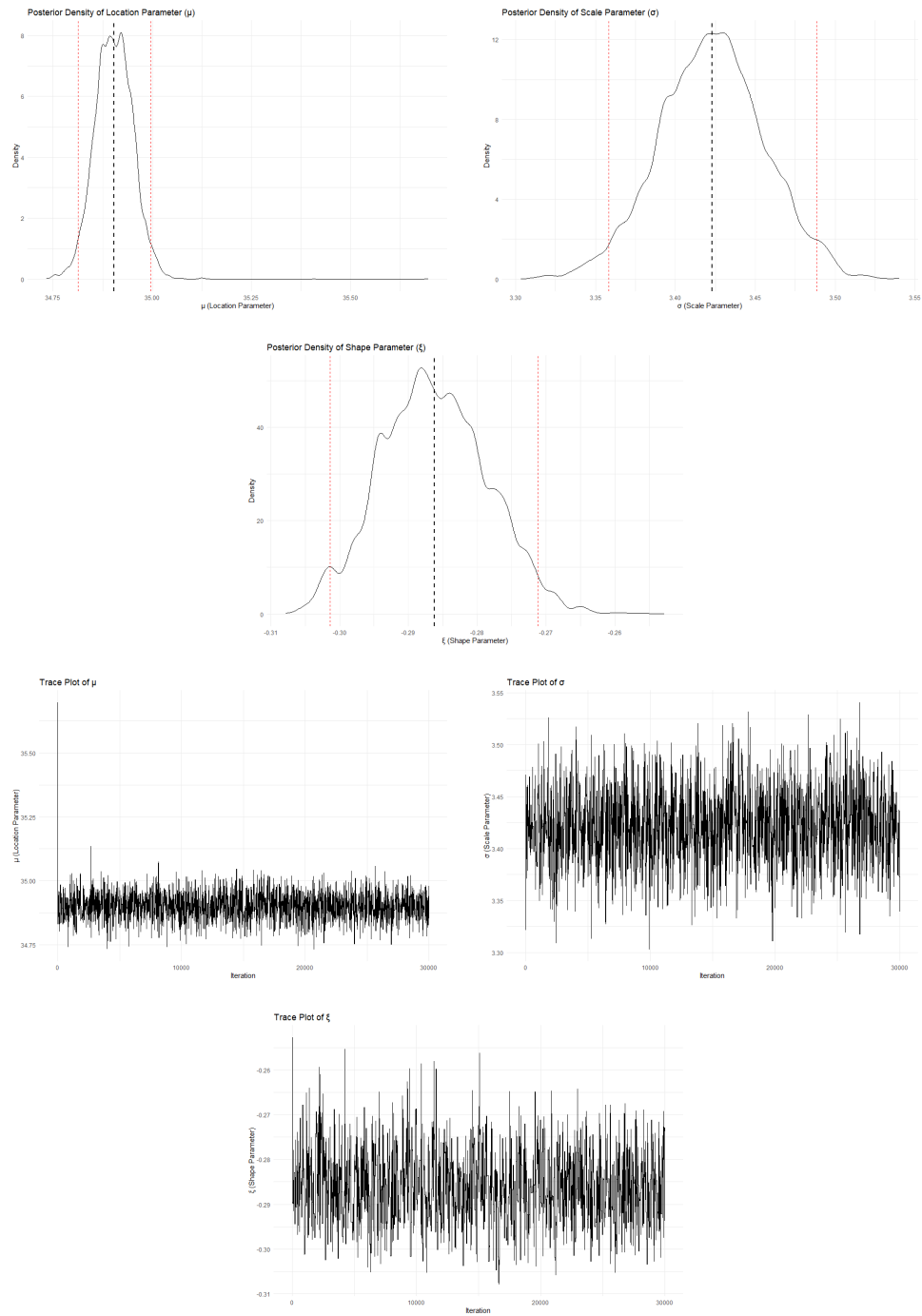


Figure 4.4: Visual summary of GEV parameter for Janjangbureh using trace and posterior plots

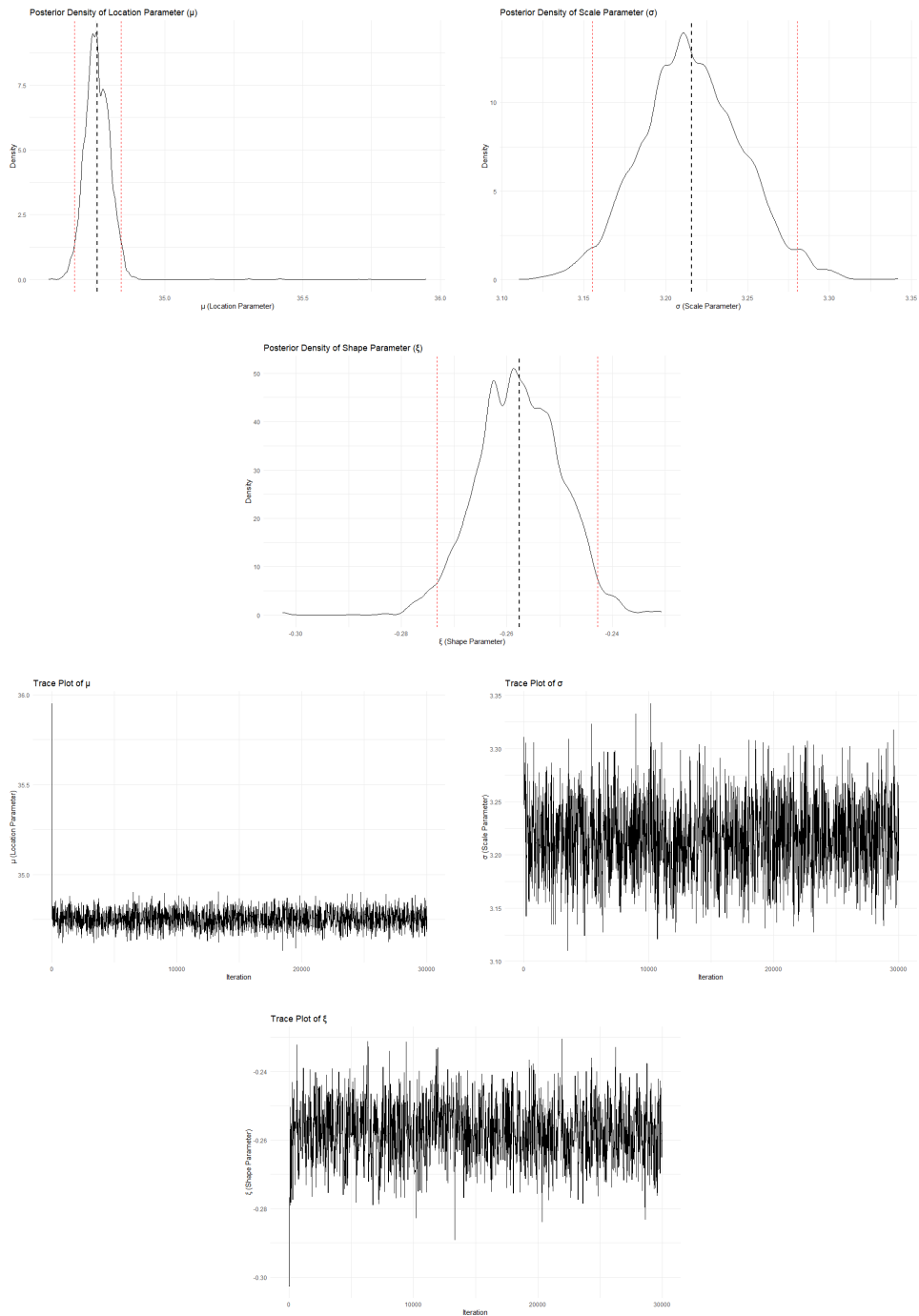


Figure 4.5: Visual summary of GEV parameter for Jenoi using trace and posterior plots

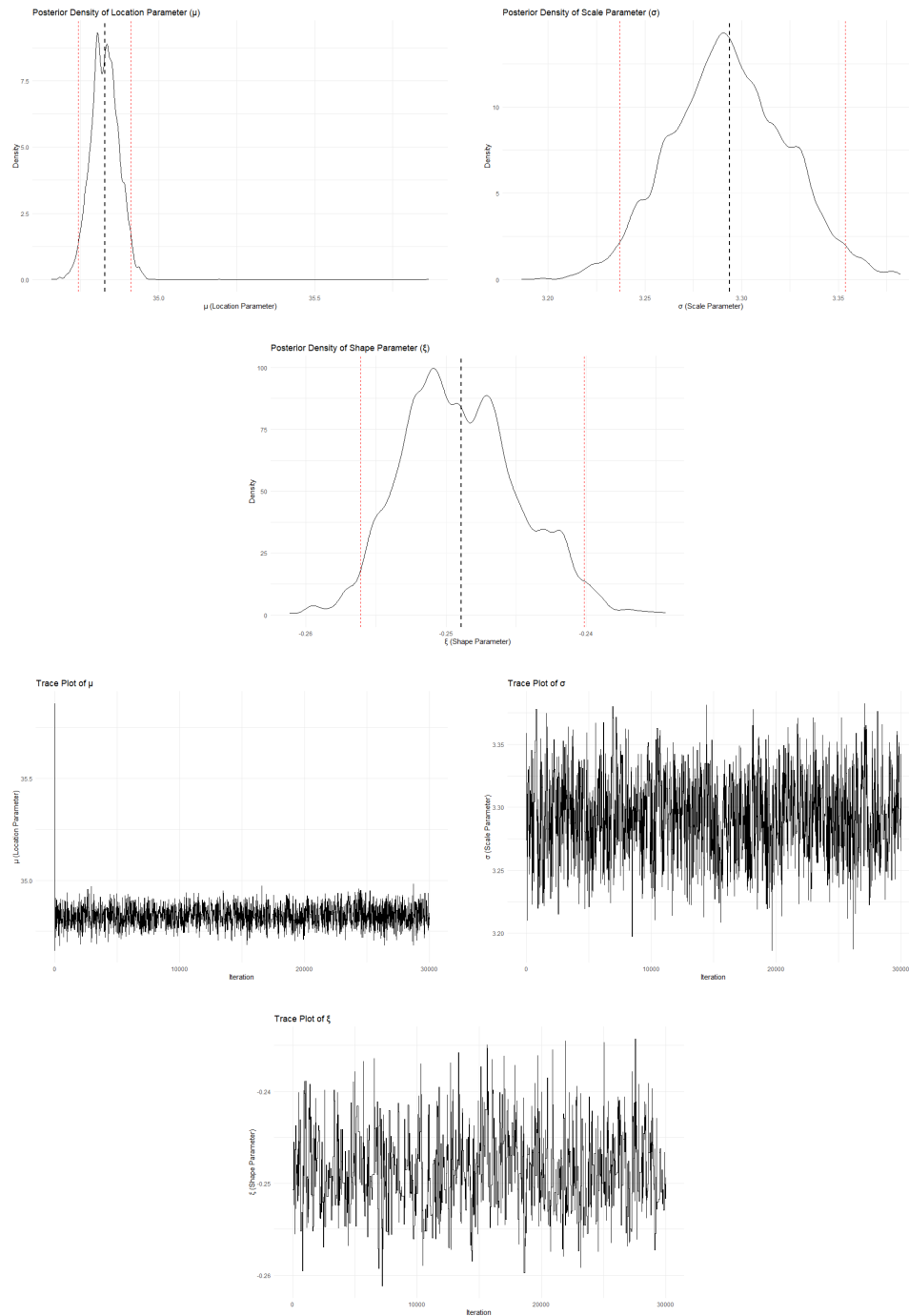


Figure 4.6: Visual summary of GEV parameter for Kaur using trace and posterior plots

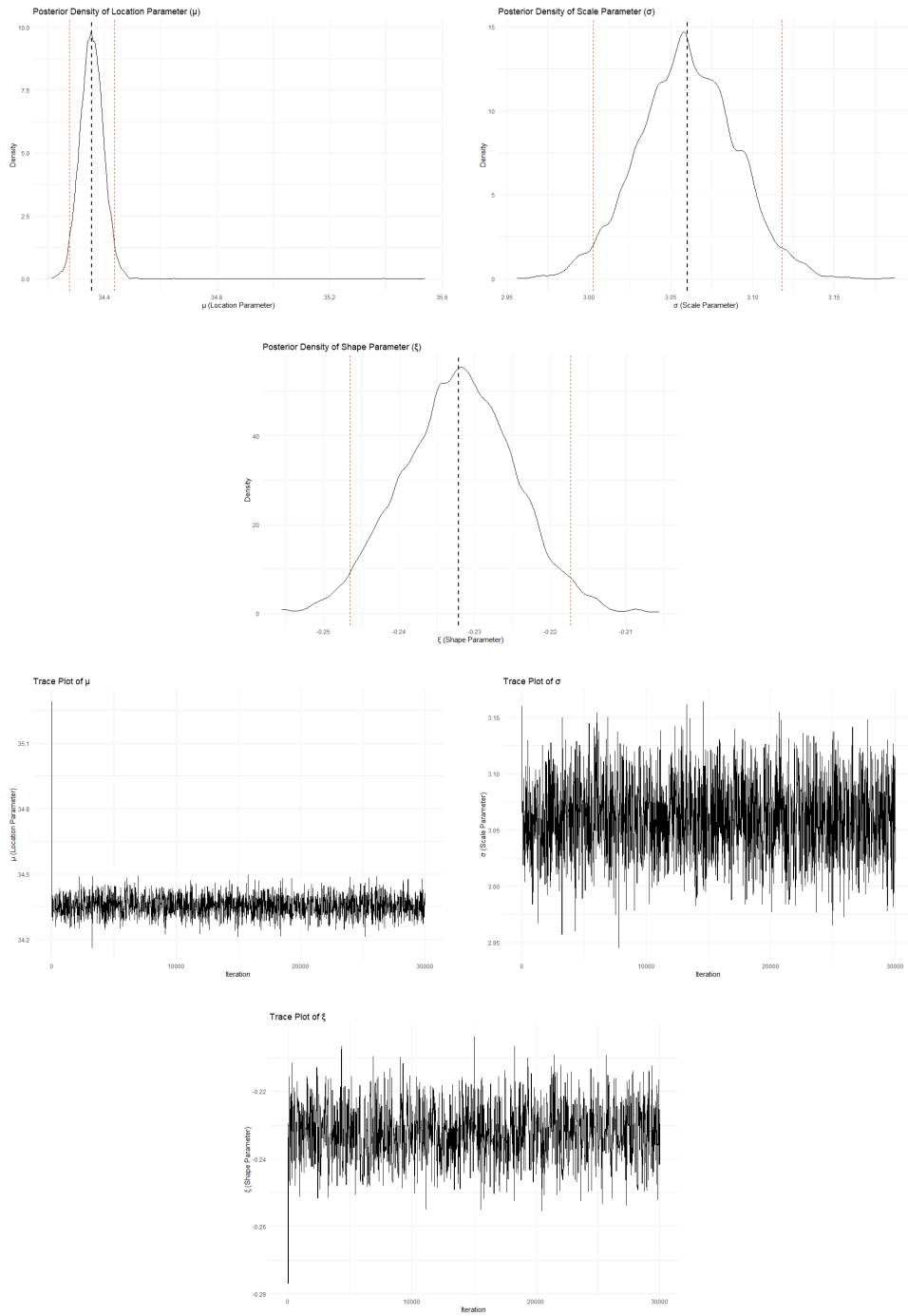


Figure 4.7: Visual summary of GEV parameter for Kerewan using trace and posterior plots

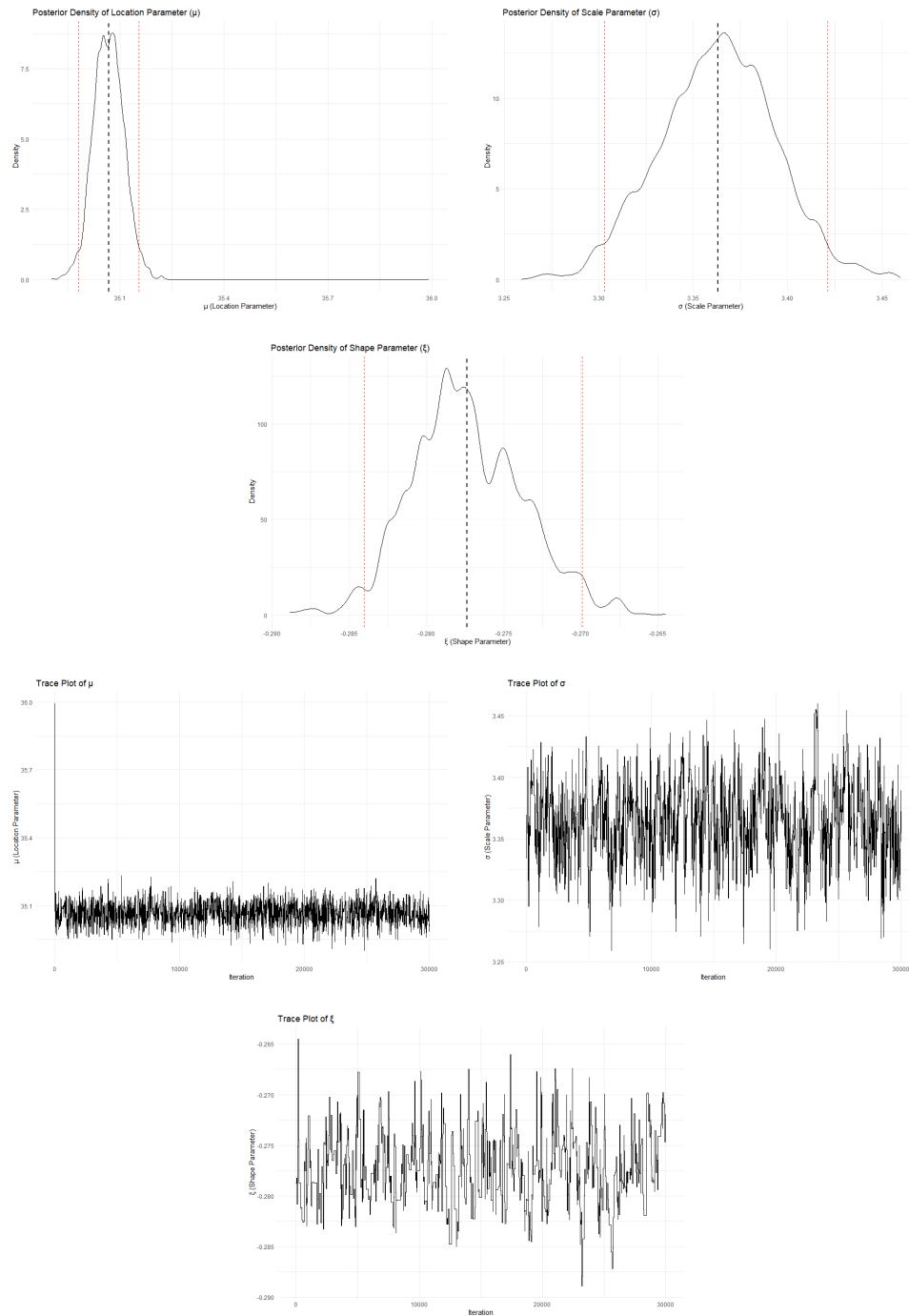


Figure 4.8: Visual summary of GEV parameter for Sapu using trace and posterior plots

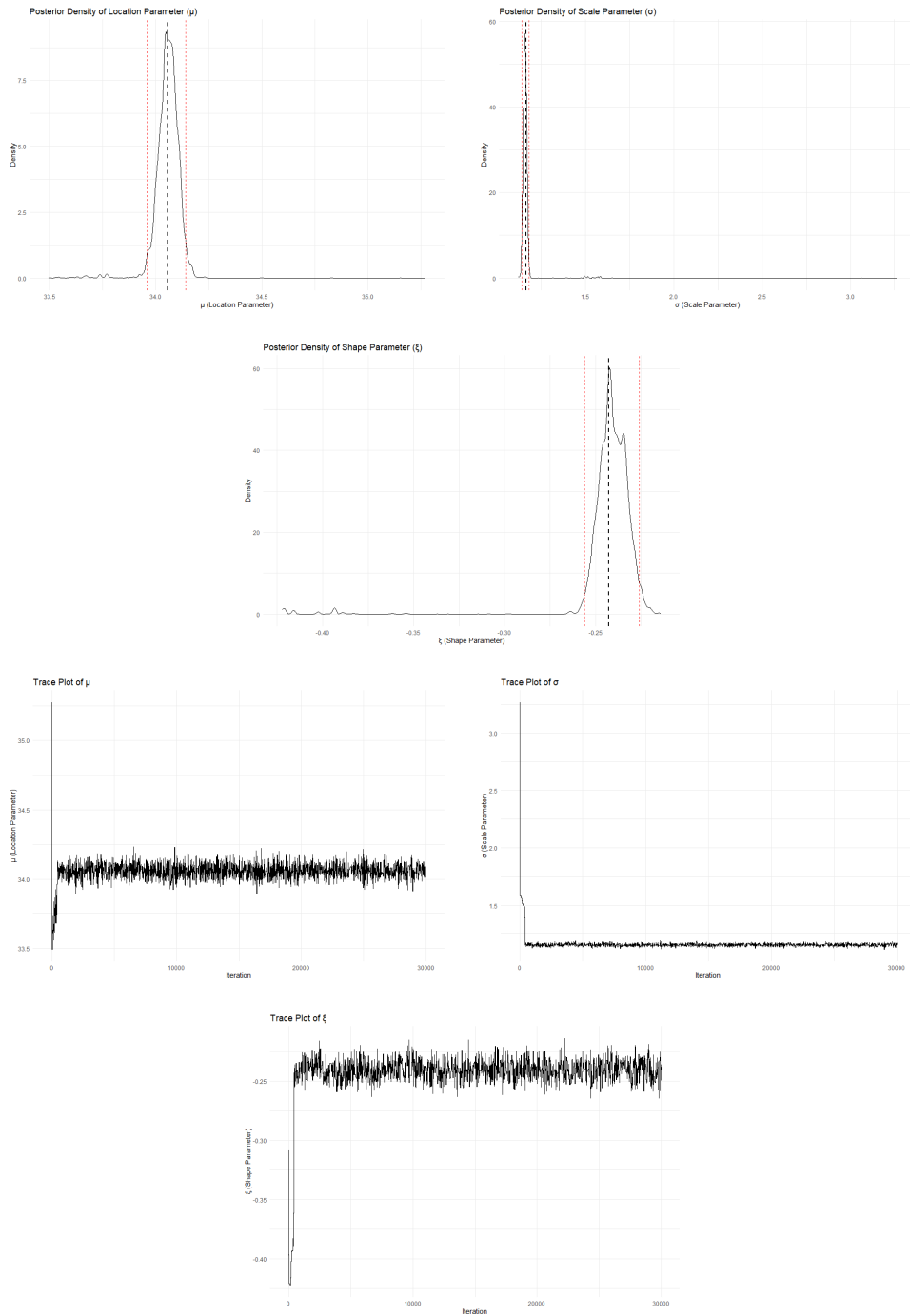


Figure 4.9: Visual summary of GEV parameter for Sibamor using trace and posterior plots

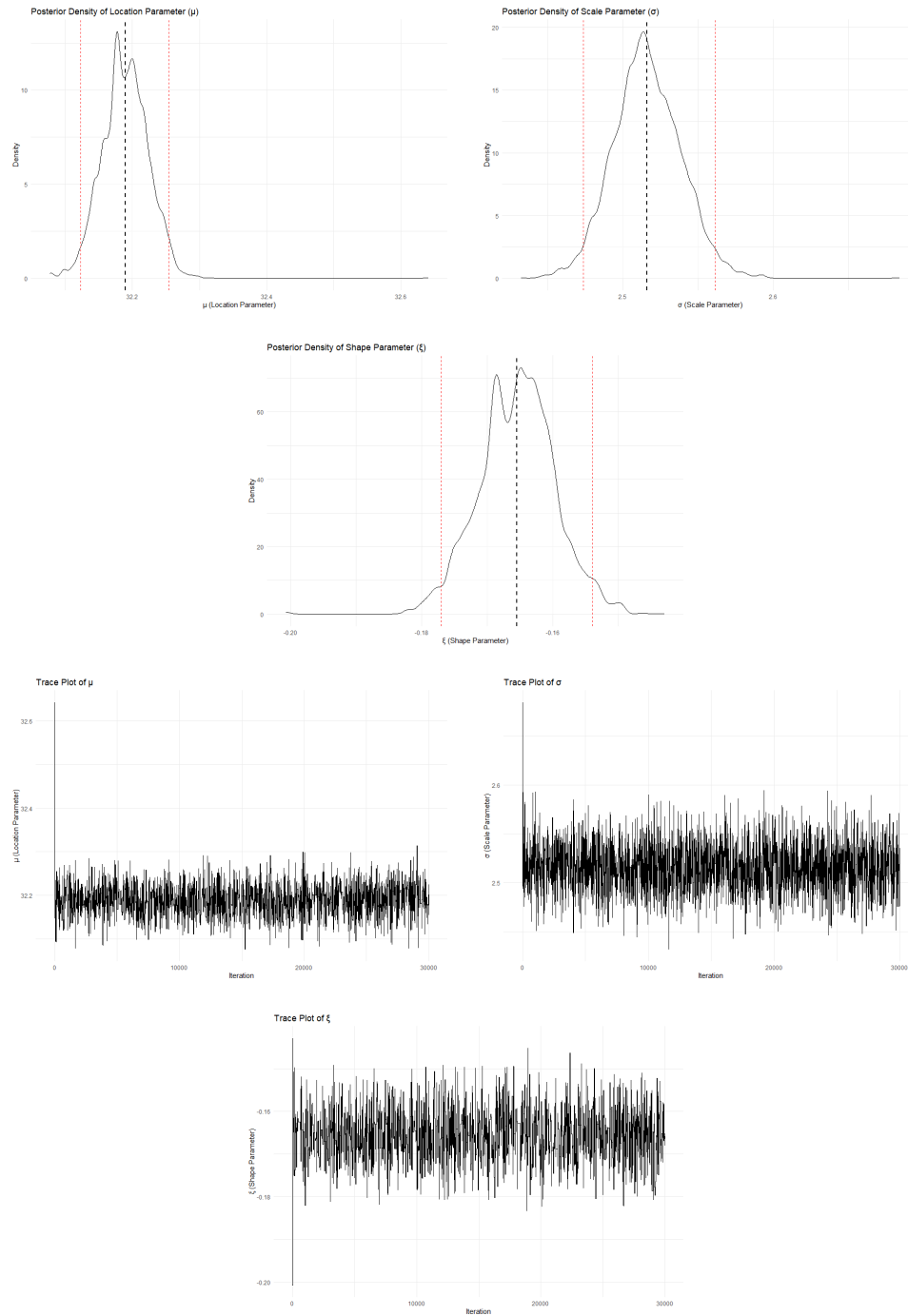


Figure 4.10: Visual summary of GEV parameter for Yundum using trace and posterior plots.

Table 4.2: **Return Period and Return Levels for Various Stations**

Station	2	5	10	30	50	100	200	500	1000
Banjul	32.66	34.79	36.04	37.69	38.36	39.20	39.96	40.86	41.46
Basse	36.68	39.75	41.23	42.86	43.42	44.04	44.52	45.01	45.30
Fatoto	36.52	39.71	41.27	43.02	43.62	44.30	44.84	45.40	45.72
Janjangbureh	36.09	39.07	40.58	42.32	42.94	43.65	44.23	44.84	45.20
Jenoi	35.87	38.75	40.24	42.01	42.67	43.42	44.04	44.72	45.13
Kaur	35.98	38.95	40.50	42.35	43.04	43.84	44.51	45.23	45.68
Kerewan	35.42	38.23	39.71	41.52	42.21	43.00	43.68	44.42	44.88
Sapu	36.23	39.19	40.69	42.44	43.08	43.80	44.40	45.02	45.40
Sibanor	35.17	38.07	39.59	41.43	42.11	42.91	43.59	44.32	44.78
Yundum	33.08	35.53	36.91	38.70	39.41	40.28	41.05	41.94	42.53

Station	Parameter	Estimate	Credible Interval
Banjul	μ	31.902	(31.040, 31.949)
	σ	2.135	(2.097, 2.172)
	ξ	-0.136	(-0.145, -0.127)
Basse	μ	35.414	(35.317,35.509)
	σ	3.684	(3.615,3.757)
	ξ	-0.336	(-0.353,-0.318)
Fatoto	μ	35.226	(35.126,35.330)
	σ	3.767	(3.696,3.839)
	ξ	-0.319	(-0.333,-0.304)
Janjangbureh	μ	34.903	(34.813,34.996)
	σ	3.423	(3.358,3.489)
	ξ	-0.286	(-0.301,-0.271)
Jenoi	μ	34.755	(34.672,34.841)
	σ	3.216	(3.155,3.280)
	ξ	-0.258	(-0.273,-0.243)
Kaur	μ	34.827	(34.742,34.911)
	σ	3.293	(3.237,3.353)
	ξ	-0.249	(-0.256,-0.240)
Kerewan	μ	34.354	(34.277,34.431)
	σ	3.061	(3.003,3.120)
	ξ	-0.232	(-0.247,-0.217)
Sapu	μ	35.067	(34.982,35.154)
	σ	3.363	(3.303,3.421)
	ξ	-0.277	(-0.284,-0.270)
Sibanor	μ	34.061	(33.959,34.144)
	σ	1.163	(1.139,1.179)
	ξ	-0.241	(-0.254,-0.226)
Yundum	μ	32.189	(32.123,32.255)
	σ	2.516	(2.474,2.561)
	ξ	-0.165	(-0.177,-0.154)

Table 4.1: **Posterior means and credible intervals for ten stations by Bayesian MCMC**

CHAPTER 5

CONCLUSIONS

5.1 Conclusion

This study has explored various statistical methodologies for modeling extreme temperature events, focusing on Extreme Value Theory (EVT), Generalized Extreme Value (GEV) distribution, Generalized Pareto (GP) distribution, and Bayesian Hierarchical Modeling. Through a rigorous analysis of these frameworks, we have provided a comprehensive understanding of how extreme temperatures can be statistically modeled, predicted, and analyzed in the context of climate extremes.

(Chapter 1) EVT provides a solid foundation for analyzing rare and extreme temperature events by focusing on the distributional properties of maxima and threshold exceedances. EVT is particularly relevant for climate studies, as it allows for better characterization of extreme occurrences, which can have profound impacts on infrastructure, human health, and agriculture. The theoretical underpinnings of EVT were explored, emphasizing its two primary approaches: the Block Maxima (BM) method and the Peaks Over Threshold (POT) method. These approaches set the stage for

the application of the GEV and GP distributions, respectively, which were further explored in subsequent chapters.

(Chapter 2) The GEV distribution play a crucial role in modeling block maxima, enabling the estimation of return levels for extreme events over different time horizons. By fitting the GEV distribution to the dataset, we were able to obtain the location μ , scale σ , and shape ξ key parameters which define the characteristics of extreme temperatures in different stations.

A crucial outcome of this analysis was the estimation of return levels, which quantify the expected magnitude of extreme temperatures for given return periods (2, 5, 10, 30, 50, 100, 200, 500, 1000) years. These return levels are particularly valuable for policymakers and engineers in designing resilient infrastructure and formulating climate adaptation strategies. The study revealed spatial variability in return levels across stations, emphasizing the need for location-specific modeling when assessing climate risks.

(Chapter 3)The GP distribution, applied through the Peak Over Threshold (POT) approach, provided an alternative method for modeling extreme temperature exceedances. By selecting an appropriate threshold, we were able to fit the GP distribution and estimate its parameters, thus characterizing the tail behavior of the temperature data.

Return level estimates derived from the GP model further reinforced the findings from the GEV analysis, demonstrating consistency in the magnitude of extreme temperature events. However, the GP model showed greater sensitivity to the threshold selection process, highlighting the importance of careful parameter tuning in extreme value

modeling. A Key comparison between the GEV and GP approaches revealed their complementary strengths. While the GEV model provides a comprehensive summary of block maxima, the GP model allows for finer granularity in analyzing exceedances beyond a certain threshold. Together, these models offer a robust statistical toolkit for understanding extreme climate events.

(Chapter 4) The Bayesian framework was employed to refine parameter estimation by incorporating prior knowledge into the analysis. The Bayesian Hierarchical Model (BHM) provided a probabilistic approach to estimating extreme temperature distribution, accounting for uncertainty in a more structured manner compared to classical frequentist methods.

The Bayesian approach proved particularly useful in handling spatial and temporal dependencies among stations, allowing for more flexible modeling of extreme-temperature events. Using Markov Chain Monte Carlo (MCMC) sampling, the study obtained posterior distributions for the GEV parameters, enabling robust inference even in the presence of limited data.

The integration of EVT, GEV, GP, and Bayesian modeling in this study provides a comprehensive framework for analyzing extreme temperatures. These methodologies offer several practical applications:

The return level estimates help quantify future temperature extremes, aiding in risk assessment for critical sectors such as agriculture, water management, and public health. Engineers and urban planners can use return period estimates to design resilient buildings, transportation networks, and energy systems that withstand extreme temperatures.

The probabilistic insights from Bayesian modeling support evidence-based policymaking for climate adaptation and mitigation strategies.

Although the study provides valuable information on extreme temperature modeling, some limitations should be acknowledged. Climate change introduces non-stationary in extreme temperature trends. Future research should explore non-stationary GP models that account for changing climatic conditions over time. The dependence of the GP model on threshold selection can introduce variability in the results. Future work could explore automated threshold selection techniques to improve robustness. This work has demonstrated the power of statistical modeling in understanding and predicting extreme temperature events. By leveraging EVT, GEV, GP, and Bayesian approaches, we have developed a robust framework for assessing climate extremes and informing decision-making processes.

The findings underscore the importance of extreme value analysis in climate resilience planning and highlight avenues for further research to refine and expand these methodologies. As climate variability and extremes continue to pose challenges worldwide, the statistical techniques explored in this study will remain indispensable tools for advancing scientific knowledge and policy interventions in climate risk management.

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VITAE

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Objective

Statistician with a passion for learning, developing skills in statistics, and contributing to collaborative environments.

EDUCATION

Master of Science: Applied Statistics

King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia
January 2023 - May 2025

Bachelor of Science: Mathematics (Distinction)

University of The Gambia, Kanifing, The Gambia
September 2017 - June 2021

Diploma : Information and Technology

Future Starts Now Gambia, Latrikunda Sabiji, The Gambia
January 2020 - March 2021

Higher Teachers Certificate (HTC): Mathematics and Science

The Gambia College, Brikama, The Gambia
September 2015 - 2017

- College Project: Comparative performance Evaluation of boys and girls in Mathematics at National Assessment Test (NAT) in Brikama Methodist 2012-2014

ACADEMIC AWARD

Third Best Graduating Student 2021 Graduating Class (Bronze Medalist)

University of The Gambia

SKILLS

- Competent in R programming, PYTHON, MATLAB, MINITAB, LATEX (Overleaf) and MAPLE
- Proficient in Microsoft Office (Excel, Word, PowerPoint) and Stata
- Strong love for teaching, classroom teaching, and data interpretation

WORK HISTORY

Recitation Teaching Assistant

King Fahd University of Petroleum and Minerals, Dammam

January 2023 - to date

- Graded quizzes, homework, and exams designated by my instructor

- Took attendance in every class
- Conducted problem-solving class for each section weekly
- Proctored exams

Enumerator

National Disaster Management Agency (NDMA), Banjul

July 2021 - September 2022

- Traveled door-to-door in assigned communities and requested participation in the survey interview, clearly explaining process and purpose
- Recognized and gave an account of problems in obtaining data
- Reviewed questionnaire submissions for accuracy and completeness at end of each day, flagging incomplete or problematic surveys for further review
- Executed follow-up visits to houses to resolve survey issues or obtain additional information, expressing appreciation for participation and regret for inconvenience

Enumerator

Central Bank of The Gambia, Banjul

October 2019 - December 2019

- Traveled to Banjul International Airport in assigned and requested participation in the survey interview, clearly explaining process and purpose
- Distributed questionnaires to the Tourist at the departure hall
- Recognized and gave an account of problems in obtaining data
- Reviewed questionnaire submissions for accuracy and completeness at end of each day

Senior Teacher

Kombo Kerewan Senior Secondary School, Banjul

September 2017 - June 2019

- Devising lesson plans based on curriculum guidelines where appropriate
- Leading classes by giving lectures, demonstrating tasks, assigning tasks, and encouraging discussions
- Encouraging student participation and performance
- Supervising students while they complete tasks independently, answering student questions
- Setting homework or exercises for students to practice their skills after class, developing and marking examinations to measure students' progress

- Assessing and reporting on students’ academic achievements and behavior
- Documenting and reporting on students’ progress

ACADEMIC LEADERSHIP

Mathematics Tutor, University of The Gambia September 2020 - June 2021

Sports Minister, University of The Gambia Science Students’ Association UTG-SSA November 2020 - November 2021

Adviser, Mathematics Club Gambia College School of Education September 2016 - September 2017

References

Available upon request