

**Modeling large-scale Microalgae Growth in
Open Raceway Ponds under Saudi Arabia
Climate Conditions**

BY

HUSSAIN ALI MOHAMMED ALSUBAIE

A Thesis Presented to the
COLLEGE OF GRADUATE AND INTERDISCIPLINARY STUDIES
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

CHEMICAL ENGINEERING

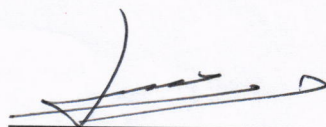
December 2024

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

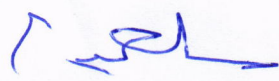
DHAHRAN- 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by **Hussain Ali Mohamed Al Subaie** under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for **the degree of Master of Science in chemical Engineering.**

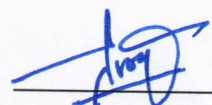


Dr. Hassan M. Baaqeel
Department Chairman

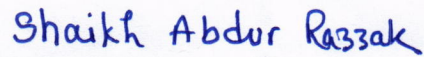


Prof. Suliman Saleh Al-Homidan Dean
of Graduate Studies

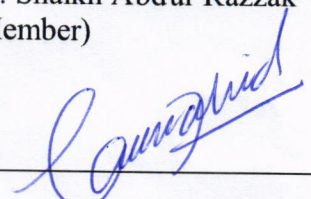
Date



Dr. Wasif Farooq
(Advisor)



Dr. Shaikh Abdur Razzak
(Member)



Dr. Umer Zahid
(Member)



© Hussain AlSubaie

2024

I dedicate this thesis to my family, whose encouragement, love, and guidance helped a lot in this master's degree level and the improvement of my life generally.

ACKNOWLEDGMENTS

At this work, I appreciate my advisor, Dr. Wasif Farooq, for his great effort and impact on this work. His thoughts and expertise served a lot in improving the outcomes of this thesis. I thank the committee members, Dr. Shaikh Abdur Razzak and Dr. Umer Zahid, for their efforts and useful feedback for improving this research. Also, I am grateful to Dr. Yousef from the Ministry of Environment, Water, and Agriculture and Dr. Claudio Grunewald, Algae Program Director at KAUST, for providing the experimental data to validate our model. Finally, my deep gratitude is directed to my family for their continuous support and encouragement.

TABLE OF CONTENTS

ACKNOWLEDGMENTS	V
TABLE OF CONTENTS	VI
LIST OF TABLES.....	VIII
LIST OF FIGURES.....	IX
LIST OF ABBREVIATIONS.....	XI
ABSTRACT	XVIII
ملخص لرسالة	XX
CHAPTER 1: INTRODUCTION	1
1.1. Background	1
1.2. Why mathematical modeling in algae cultivation for Saudi Arabia.	4
1.3. Novelty of the research.....	4
1.4. Objectives of this research	5
1.5. Summary of this work	5
CHAPTER 2: LITERATURE REVIEW	6
2.1. Mathematical kinetic models for microalgae growth based on a singular substrate	6
2.1.1. Light as a growth factor in Mathematical modeling	6
2.1.2. Modelling the Effect of Temperature on algae growth	19
2.1.3. Mathematical modelling of algae growth based on impact of Nutrients	28
2.1.4. Modelling the impact of pH on Algae Growth	38
2.1.5. Modelling the impact of Salinity on Algae Growth	45
2.1. Modelling microalgae growth based on multiple substrates	46
2.2.1. Mathematical growth Models based on non-interactive effect of substrate.....	47
2.2.2. Mathematical growth models based on the Interactive effect of substrates	48
2.2.3. Mathematical growth model based on Additive effect of substrate	49

CHAPTER 3: METHODOLOGY	57
3.1. Energy balance (Thermal model for ORP).....	57
3.1.1. Exchange of heat between ORP system and surrounding	57
3.2. Biological model for algae growth in ORP	62
3.3. Solution of first order differential equations for thermal and biological model	64
3.4. Strain parameters and ORP geometry dimensions were used in this study.....	64
3.5. Weather conditions for thermal and biological model	66
 CHAPTER 4: RESULTS AND DISCUSSION.....	 67
4.1. Impact of weather conditions on the pond temperature	67
4.1.1. Impact of Global Horizontal Irradiance (GHI) on the predicted pond temperature	67
4.1.2. Impact of Relative Humidity (RH) on predicted pond temperature	68
4.1.3. Impact of Wind Speed on the Pond Temperature	69
4.1.4. Impact of Air temperature on the pond temperature	71
4.1.5. The degree of impact of each weather condition on the pond temperature.....	73
4.2. impact of weather conditions (GHI, wind speed, RH, and air temperature) on predicted evaporation rate ..	74
4.3. Biomass Growth rate and impact of GHI, PAR, and ORP temperature on growth rate of <i>Arthrospira maxima</i>	
.....	78
4.3.1. Predicted productivity of algae biomass during summer season	78
4.3.2. Predicted productivity of algae biomass during fall season	81
4.3.3. Predicted productivity of algae biomass during winter season	83
4.3.4. Predicted productivity of algae biomass during spring season	85
4.4. Model validation.....	88
4.4.1. Thermal part of model validation	88
4.4.2. Biological part of the model validation	91
4.5. Impact of ORP depth and harvesting concentration on the biomass productivity	94
 CHAPTER 5: CONCLUSION	 98
 REFERENCES.....	 100
 VITAE	 111

LIST OF TABLES

Table 1: Mathematical models for microalgae growth depending on the light intensity as a single factor affecting algae growth	13
Table 2: Kinetic models of microalgae growth depending on the environmental temperature.....	24
Table 3: Kinetic models of microalgae growth depending on the nitrogen and phosphorus nutrients	33
Table 4: Kinetic models of microalgae growth depending on pH	42
Table 5: Kinetic models of microalgae growth depend on the multiple limitations of substrates.	51
Table 6: The geometry dimensions for ORP and <i>Arthrospira maxima</i> parameters implemented in thermal and biological model.....	65

LIST OF FIGURES

Figure 1: A conventional Open Raceway Pond for microalgae cultivation.....	3
Figure 2: Why mathematical modeling in algae cultivation for Saudi Arabia.....	4
Figure 3: PI curve illustrating the impact of light intensity on algae growth in three distinct regions: light-limited, light-saturated, and light-inhibited	7
Figure 4: The diagram of growth rate (s^{-1}) with temperature (K) with including of three cardinal temperatures with the optimal growth rate	23
Figure 5: The trend of increase in salinity in three different regions for potential microalgae cultivation in Australia.....	46
Figure 6: Impact of GHI on predicted pond temperature for an ORP situated in Jeddah.....	68
Figure 7: Impact of RH on predicted pond temperature for an ORP situated in Jeddah	69
Figure 8: Impact of wind speed on pond temperature for an ORP situated in Jeddah.....	71
Figure 9: Impact of air temperature on pond temperature for an ORP situated in Jeddah	72
Figure 10: Impact of wind speed and RH on predicted Evaporation rate per day (kg/day) for an ORP situated in Jeddah.....	76
Figure 11: Impact of GHI and air temperature on pond Evaporation rate per day (kg/day) for an ORP situated in Jeddah.....	77
Figure 12: Average air and predicted ORP temperature throughout one year of running model for an ORP situated in Jeddah.....	77
Figure 13: Predicted daily evaporation rate and salinity throughout one year of running model for an ORP situated in Jeddah.....	78
Figure 14: Biomass Growth rate in the interval between June and Aug 2022	80
Figure 15: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of <i>Arthrospira maxima</i> in the interval between Jun and Aug 2022	80
Figure 16: Biomass Growth rate in the interval between Sep and Nov 2022	82
Figure 17: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of <i>Arthrospira maxima</i> in the interval between Sep and Nov 2022.....	82
Figure 18: Biomass Growth rate in the interval between Dec 2022 and Feb 2023.....	84
Figure 19: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of <i>Arthrospira maxima</i> in the interval between Dec 2022 and Feb 2023.....	85
Figure 20: Biomass Growth rate in the interval between Mar to May 2023	87
Figure 21: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of <i>Arthrospira maxima</i> in the interval between Mar to May 2023	88
Figure 22: Comparison between temperature results from the thermal part of the model and experimental data of temperature for ORP from a local facility.....	90
Figure 23: Regression of comparison between temperature results from the thermal part of the model and experimental data of temperature for ORP from a local facility	90
Figure 24: Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Inner Mongolia, China	92
Figure 25: Regression of comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Inner Mongolia, China ..	92
Figure 26: Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Italy	93
Figure 27: Regression of Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Italy	93

Figure 28: Comparison of the productivity based on the depth of ORP at 600g/m³; (a) 0.2, (b) 0.25 and (c) 0.3 m	96
Figure 29: Comparison of productivity based on two harvesting concentrations of 25 m³ ORP volume; (a) at 600 g/m³ and (b) at 1100 g/m³	97

LIST OF ABBREVIATIONS

μ_{opt} : maximum growth rate (d^{-1})	
μ : specific growth rate (d^{-1})	T_{opt} : temperature where $\mu = \mu_{max}$ ($^{\circ}C$)
μ_{max} : maximum specific growth rate (d^{-1})	T_{min} : lowest temperature where growth rate is zero for temperature less than T_{min} , ($^{\circ}C$)
t: time, (h or d)	T_{Max} : highest temperature where growth rate is zero for temperature greater than T_{min} , ($^{\circ}C$)
I : light intensity($\mu mol m^{-2} s^{-1}$)	β : dimensionless constant for model
I_0 : incident light intensity at the pond surface($\mu mol m^{-2} s^{-1}$)	u : constant of scaling
I_{av} : average light intensity in a pond($\mu mol m^{-2} s^{-1}$)	R : constant of gas (8.314 J/mole K)
λ : wavelength (m)	$\Delta H_A^{\ddagger}$: activation enthalpy(J/mole)
I_S : light saturation intensity($\mu mol m^{-2} s^{-1}$)	ΔC_p : change of heat capacity (J/K mol-amino acid-residue)
n,m: exponent	n :amino acid residues number
I_k : the representer of cell affinity to light ($\mu mol m^{-2} s^{-1}$)	ΔH^* : change of enthalpy at temperature T_H^* (J/mol amino acid residue)
a: constant	T_H^* : convergence temperature for protein unfolding enthalpy (K)
b: constant($(\mu E m^{-2} s^{-1})^{-1}$)	ΔS^* : change of entropy at temperature T_S^* (J/Kelvin)
c: constant($\mu E m^{-2} s^{-1}$)	T_S^* : convergence temperature for protein unfolding entropy(K)
d: constant	s: empirical constant ($^{\circ}C^{-2}$)

I_{opt} : optimum light intensity for growth ($\mu mol m^{-2} s^{-1}$)	A_1 : specific growth rate at reference temperature (T_o)(d^{-1})
β : sharpness coefficient	A_2 : specific deactivation rate at reference temperature (T_o) (d^{-1})
Z: bioreactor depth(m)	E_1 : Activation energy for growth (kcal/ mol)
X_A : biomass concentration(g/L)	E_2 : activation energy for deactivation (kcal/ mol)
σ : light absorption parameter (L/g m)	γ : dimensionless coefficient
PAR: photosynthetic active radiation (%)	P : photosynthetic rate($\frac{\mu mole CO_2}{m^2 s^{-1}}$)
I_{GHI} : global horizontal irradiance ($\mu mol m^{-2} s^{-1}$)	P_{opt} : maximum photosynthetic rate($\frac{\mu mole CO_2}{m^2 s^{-1}}$)
k_e : extinction coefficient(m^{-1})	k_B : Boltzmann's constant (J/K)
K_I : light saturation coefficient ($\mu E m^{-2} d^{-1}$)	h: Planck's constant (J second)
V: volume of liquid in a bioreactor(m^3)	$\Delta S_{T_o}^\ddagger$: activation entropy at reference temperature (T_o)(KJ/ mole K)
I_a : absorbed energy by bioreactor (mole/ d)	$\Delta C_p^\ddagger$: Heat capacity difference between the enzyme bound to the substrate for the reaction, and the enzyme bound to the transition state for the reaction (KJ/ (mole K))
I_{max} : rate of maintenance (mole/ (kg d) $^{-1}$)	$\Delta G^\ddagger$: change in Gibbs free energy between the reactants the transition state (KJ/mole)
V_F : volume faction of illuminated region in bioreactor	$\Delta H_{T_o}^\ddagger$: activation enthalpy at reference temperature (T_o)(KJ/ mole)
R: constant that has similar meaning to growth yield (kg/ mole)	X_c :microalgae cell number per volume of medium (10^9 cell/ L)
$K_{i,L}$: parameter that represents Photoinhibition ($\mu E m^{-2} d^{-1}$)	$[CO_2^*]$: equilibrium CO_2 concentration in the culture medium(mole/L)

α : Initial slope of the light response curve(d^{-1})	$K_L a$: coefficient of gas-liquid mass transfer (h^{-1})
T : temperutuer ($^{\circ}C$)	p_{CO_2} : partial pressure of CO_2 in the air (atm)
A : the regression coefficient constant	H_{CO_2} : Henry' s constant (atm L/ mole)
B : additional constant to adjust the model based on experimental result (1/K)	$[CO_2^*]$: concentration of CO_2 in culture medium (mole/L)
S : substrate concentration (Nitrogen and Phosphorus) (mg/L)	$[H^+]$: hydrogen ions concentration in the culture medium (mole/L)
k_S : saturation constant or half saturation constant of substrate (mg/L))	k_1 and k_2 are the chemical equilibriums constants (mole/ L)
$K_{i,S}$ and k_i :constant of inhibition of substrate (mg/L)	pH_{max} : maximum pH for microalgae growth
μ_{m1} : maximum specific growth rate when $S > 0$ (d^{-1})	pH_{mim} : minimum pH for microalgae growth
μ_{m2} : maximum specific growth rate for $S = 0$ (d^{-1})	pH_{opt} : optimal pH where the maximum growth rate of microalgae can be.
μ_{m3} : specific growth rate for inhibition (d^{-1})	λf : constant
Q : cell quota that is defined as the mass of internal limiting nutrients (Nitrogen, phosphorus as an example) per total mass of cell (%)	k_A and k_B : model constants (mole/L)
Q_o : minimal cell quota that is defined as the minimal internal mass ratio of limiting nutrients that are required for microalgae growth (%)	pH_{dep} : model constant.
K_Q and K_P : half saturated constant (%)	C_A and C_B : pre-exponential factor
Q_{max} : Maximal cell quota that is defined as the maximal amount of limiting nutrients that can	D_A and D_B : activation energies

be accumulated in a cell before growth of microalgae cell stops (%)	
$[TIC]$: total concentration of inorganic carbon (mole/L)	k_C : model constant
K_{CL} :saturation constant for carbon (mole/ 10^9 cell)	A_4 : factor of degradation(h^{-1})
$Q_{N,0}$: minimum quota for nitrogen (moleNmoleC $^{-1}$)	S_N : concentration of nitrogen(g/L)
$Q_{P,0}$: minimum quota for phosphorus (molePmoleC $^{-1}$)	$S_{glucose}$: concentration of glucose(g/L)
$Q_{N,max}$: quota of nitrogen at maximum growth rate(moleNmoleC $^{-1}$)	$K_{glucose}$: saturation constant of glucose(g/L)
$Q_{P,max}$: quota of phosphorus at maximum growth rate (molePmoleC $^{-1}$)	$S_{Nitrate}$: concentration of nitrate (g/L)
Q_N : quota of nitrogen at time t (moleNmoleC $^{-1}$)	$K_{Nitrate}$: saturation constant of nitrate (g/L)
Q_P : quota of phosphorus at time t (molePmoleC $^{-1}$)	S_{CO_2} : concentration of CO $_2$ (g/L)
$Q_{N,min}$: minimum quota of nitrogen ($10^{-9}\mu mole \frac{N}{cell}$)	K_{CO_2} : saturation constant of CO $_2$ (g/L)
$Q_{P,min}$: minimum quota of phosphorus ($10^{-9}\mu mole \frac{N}{cell}$)	S_I : light intensity ($\mu mole m^{-2}s^{-1}$)
X_{chl} : chl a content of biomass (g chl/ g dw)	K_{IXi} : inhibition constant of light intensity ($\mu Em^{-2}s^{-1}$)
α_1 : chl an Optical cross section($m^2(g chl)^{-1}$)	θ :Arrhenius temperature parameter for growth
$P_{m,chl}$: light-saturated photosynthesis rate (gC (gchl d) $^{-1}$)	μ_A : specific growth rate of autotrophic(d $^{-1}$)
C_s : sodium acetate concentration (g/L)	ϕ : quantum efficiency (g C/ mole photons)

C_x : cell concentration(g/L)	Q_{cm} : maximum cell quota of carbon (g C/ (g dw)
C_p : product concentration (mg/L)	Q_c : carbon subsistence quota (g C/g dw)
C_{xm} : achievable maximum cell concentration (g/L)	$Q_c(t)$: cell quota of carbon at time t (g C/ g dw)
C_{pm} : achievable maximum product concentration (mg/L)	v_{cm} : maximum carbon uptake rate (g C/ g dw d)
K_{XI} : constant of light saturation ($\mu mole\ m^{-2}s^{-1}$)	$C(t)$: organic carbon concentration in a culture medium at time t (g C/m ³)
K_{Xi} : cell growth inhibition constant of sodium acetate (g/L)	v_{ch} : half saturation constant (g C/ m ³)
K_s : saturation constant of sodium acetate (g/L)	A : acetate concentration(g/L)
K_{xs} : saturation constant of substrate(g/L)	$K_{s,A}$: saturation constant of acetate (g/L)
K_{xN} : saturation constant of nitrogen(g/L)	$K_{i,A}$: inhibition constant of acetate (g/L)
K_{NXi} : inhibition constant of nitrogen (g/L)	A_3 : factor of growth (h ⁻¹)
K_{XI} : saturation constant of light intensity ($\mu Em^{-2}s^{-1}$)	α_k : attenuation rate(m^2g^{-1})
ρ : density of the medium ($\frac{kg}{m^3}$)	Re : Reynolds number
V : ORP volume(m^3)	L : equal to the ratio of the ORP area over its perimeter (m)
C_p : the medium's heat capacity ($\frac{J}{kg.K}$)	V : the velocity of the wind (m/s)
T_R : temperature for the ORP (K)	ϑ_a :the kinematic viscosity($\frac{m^2}{s}$)
Q_n : heat lost or gained by the ORP system(w)	Pr : Prandtl number
Q_s : heat gain by ORP from the direct irradiance(w)	α_a : the air thermal diffusivity($\frac{m^2}{s}$)

f_a : the fraction of the sunlight that photosynthesis of algae converts to chemical energy	Nu : Nusselt number
GHI: the global horizontal irradiance($\frac{W}{m^2}$)	Len and w :the length and width of ORP(m)
α_{cult} : the absorptivity of ORP culture	$Q_{evaporation}$: the latent heat of evaporation(w)
$A_{surface}$: the surface area of ORP(m^2)	m_e : the amount of water evaporation($\frac{kg}{m^2.s}$)
Q_{air} : heat exchanger regarding radiation of the air(w)	L_w : the latent heat of evaporation($\frac{J}{kg}$)
$\varepsilon_w \varepsilon_a$: the emissivity of the reactor culture and the air, respectively	Sch : Schmidt number
σ : the Stephan- Boltzmann constant($\frac{W}{m^2 K^4}$)	$D_{w,a}$: the water mass diffusion coefficient on the air($\frac{m^2}{s}$)
T_{surr} : the temperature of the surroundings (k)	Sh : Sherwood number
Q_{ground} : heat transfer between the bottom of ORP and the ground (w)	RH: the relative humidity (%)
K_{con} : concrete thermal conductivity($\frac{W}{m.K}$)	M_W : molecular weight (g/mole)
L_{ref} : the reference depth of the conductivity(m)	R : constant for an ideal gas (Pa m ³ /K.mole)
Q_{rad} : the loss of heat from the pond is due to radiation that comes from the pond(w)	P_w, P_a : vapor pressure at T_{pond} (°C)and T_{surr} (°C) (Pa)
Q_{con} : the heat transfer between the surrounding air and the top surface of ORP(w)	$Q_{water\ inflow}$: heat exchange regarding water flow to the ORP (w)
h_{con} : the coefficient for convection heat flux ($\frac{W}{K.m^2}$)	ϕ_L : an efficient parameter for culture light

c_x : the algae concentration in the reactor($\frac{g}{m^3}$)	I_a and I_s : mean light intensity and saturated light intensity, respectively($\frac{w}{m^2}$)
Z: the ORP depth in(m)	
φ_T : an efficient parameter for the culture temperature	PAR : the rate of photosynthetic spectrum photon incidence ($\frac{w}{m^2}$)
ϕ_{photon} : the number of photons that are required to produce the algae biomass, g/moles	D : the algae biomass lost without light (g/sec)
μ_{dark} : the decay rate of biomass(h^{-1})	τ : biomass losing average via the dark period (%)
Δt : The time interval of the dark (h^{-1})	

ABSTRACT

Full Name : Hussain Ali Mohammed Alsubaie

Thesis Title : Modeling large-scale Microalgae Growth in Open Raceway Ponds
under Saudi Arabia Climate Conditions

Major Field : Chemical Engineering

Date of Degree : December 2024

The growth of microalgae is widely explored around the world due to its various applications. One of the exciting applications of microalgae is aquafeed for fish and shrimp. Saudi Arabia relies on exports for aquafeed. Exploring the potential of microalgae cultivation for its potential use in Saudi Arabia is essential. Therefore, understanding the microalgae growth rate in an Open Raceway Pond (ORP) is essential. One of the best ways to understand the dynamic growth rate of microalgae under real weather conditions that influence the growth rate of microalgae in ORP is to build a time-dynamic growth model. This thesis presents a mathematical model for microalgae “*Arthrospira maxima*” growth using biological, chemical, and physical variables that affect algae growth. The current work will explore the temperature profile for ORP over time by applying energy balance to the ORP system using the thermal part of the model. The temperature profile from a thermal ORP model is then incorporated into the biological model to find the time-resolved growth rate of microalgae in an ORP. The actual weather conditions (wind speed, air temperature, relative humidity, and global horizontal irradiance) are used to build the model and generate the results. The weather data was selected for a specified location in Jeddah, Saudi Arabia. The impact of each weather condition on the ORP temperature profile and biomass productivity is studied in this work. The thermal and biological mathematical model was validated by comparing its results with experimental results from the literature and actual data from a large-scale cultivation facility in Saudi Arabia. The comparison of model prediction with actual data shows a good correlation. Moreover, weather conditions significantly affect the productivity of the microalgae. Furthermore, the harvesting sequence also greatly impacts productivity—frequent harvesting results in more biomass but at the expense of more energy. The model predicted the maximum, average, and minimum productivity of algae as 43.24, 22.51, and 5.32g/m² /day throughout the year. The

developed mathematical growth model for algae can be used with different locations in Saudi Arabia to get a complete idea of the optimum location for algae cultivation and associated production costs throughout the year to find the best production season and location.

Keywords: Microalgae growth, Biological model, Open Raceway Pond, Aquafeed

ملخص لرسالة

الاسم الكامل: حسين علي محمد السبيعي

عنوان الرسالة: نمذجة نمو الطحالب الدقيقة في بركة مفتوحة كبيرة الحجم تحت تأثير الظروف المناخية في المملكة العربية السعودية

التخصص: الهندسة الكيميائية

تاريخ الدرجة العلمية: ديسمبر 2024

بسبب التطبيقات المختلفة لاستخدام الطحالب الدقيقة، استكشاف نمو الطحالب مطلوب في انحاء العالم. واحدة من استخدامات الطحالب الدقيقة هيا استخدامها كمصدر غذائي للأسماك والربيان (الشرم). بما ان المملكة العربية السعودية مهتمة في في تربية الاحياء المائية والطحالب مصدر غذاء لهاذه المخلوقات الأحيائية فإن اختبار إمكانية مقدرة الطحالب الدقيقة لنمو في المملكة العربية السعودية مهم. لذلك فهم نمو الطحالب الدقيقة في البركة المستخدمة في تنمية الطحالب مهم. واحدة من الطرق المستخدمة لمعرفة إمكانية نمو الطحالب الدقيقة في البركة المفتوحة تحت تأثير معاملات الجو في المملكة العربية السعودية هي بواسطة استخدام النموذج المحاكى لنمو مع الوقت. هذه الاطروحة تعرض نموذج رياضي لمعدل نمو الطحالب " من نوع اسبورلينا ماكسما" بي واسطة استخدام متغيرات احيائية, كيميائية, وفيزيائية التي تؤثر على نمو الطحالب. العمل الحالي سوف يقوم بحساب تغير الحرارة للبركة مع الوقت بواسطة تطبيق الجزء الحراري لنموذج. الحرارة الموجودة لي البركة المستخدمة لزراعة الطحالب بواسطة تطبيق الجزء الحراري من النموذج تستخدم في النموذج الأحيائي لإيجاد معدل نمو الطحالب في بركة الزراعة. البيانات الحقيقية لطقس (سرعة الرياح, حرارة الهواء, نسبة الرطوبة, والاشعاع الحراري الشمسي) تستخدم لبناء النموذج واستخراج النتائج. بيانات الطقس المختارة في هذه الدراسة هي لعدة في المملكة العربية السعودية. تأثير كل واحد من معاملات الطقس على التغير في درجة حرارة بركة الزراعة وعلى إنتاج الطحالب تم دراستها في هذه الرسالة. النموذج الرياضي الحراري والأحيائي تم التحقق من صحته بواسطة مقارنة نتائج النموذج مع نتائج تجريبية موجودة في المراجع او مأخوذة من منشأة موجودة في جدة, المملكة العربية السعودية. كانت مقارنة الترابط بين نتائج النموذج والنتائج التجريبية جيدة. أيضا, أوضاع الجو كان لها أثر عالي على إنتاج الطحالب الدقيقة. أيضا, التغير في تركيز الحصاد او دورة الحصاد ليس يؤثر فقط على إنتاجية الطحالب بل تؤثر على تكلفة الإنتاج لطحالب. النموذج استنتج اعلي, متوسط, واقل معدل إنتاج لطحالب ممكن وكانت كتالي 43.24, 22.52, و 5.32 غرام/متر مربع/يوم. خلال سنة كاملة. النموذج الرياضي المطور الأحيائي لي حساب معدل نمو الطحالب ممكن استخدامه في مناطق مختلفة في المملكة العربية السعودية لمعرفة أفضل مكان ممكن لزراعة الطحالب ومعرفة سعر الإنتاجية لطحالب خلال السنة لمعرفة أفضل فصل من السنة لإنتاج الطحالب وأفضل موقع لإنتاج الطحالب.

CHAPTER 1: INTRODUCTION

1.1. Background

Microalgae are a kind of unicellular organisms that can utilize carbon dioxide through photosynthetic processes to produce chemical energy (organic food) (Abdur Razzak et al., 2024). The growth of microalgae using light and CO₂ is termed as phototrophic. Microalgae can also grow in dark or light-limited conditions using organic carbon sources; in this case, microalgae utilize the organic compounds instead of producing organic energy by supporting light sources; this kind of microalgae growth is called heterotrophic growth. In the regime of mixotrophic growth, both carbon dioxide (inorganic source) with light and organic carbon simultaneously contribute to the metabolism of microalgae growth (Perez-Garcia et al., 2011). In recent years, the interest in microalgae cultivation has increased among researchers due to various features of microalgae. Microalgae can grow rapidly in a relatively short period, and it is flexible enough to adjust their biochemical compositions by manipulating the cultivation conditions. Moreover, the production of microalgae biomass does not consume much water; it can grow in seawater, and wastewater is environmentally friendly and utilizes land unsuitable for crop cultivation (Dolganyuk et al., 2020).

The wastewater produced from food, electronics, oil, gas, and various industries is hazardous to dispose its directly to the environment. WW includes organic and inorganic contaminants (solid particles) that are highly toxic to organisms and eutrophication of aquatic environments (Abdelfattah et al., 2023). Therefore, WW treatment is needed before discharging or reusing it. The disadvantages of implementing conventional wastewater. Wastewater treatment processes emit greenhouse gas emissions, excessive solid landfilling, and high energy consumption to improve sustainable solutions for wastewater. Wastewater treatment and disposal, alternative environmental, environment-friendly, less energy consumption, and economical methods are needed. One of the choices is to use microalgae for wastewater treatment. The pollutants of WW can be removed through the metabolism of microalgae, which can grow in highly saline water.

Approximately 1.83 kg of carbon dioxide can be fixed to produce 1 kg of microalgae biomass (Al-Jabri et al., 2021). Currently, microalgae are used in industrial-scale farmed fish feed and also for food and drug compound production such as omega-3 polyunsaturated fatty acid, food colorings including astaxanthin, phycoerythrin, and β -carotene. Many microalgae species contain proteins, vitamins, antioxidants, and low carbohydrate composition, so they are classified as compounds of healthy food (Rozenberg et al., 2024). Generally, commercial microalgae production has been implemented in outdoor ponds regarding considerations of scale and cost (Hirayama et al., 2022). The cultivation system of microalgae is widely classified into closed photobioreactors (tubular, vertical-column, flat-panel) and open ponds (Tan et al., 2020). Closed photobioreactors' main advantages are the greater ability to control cultivation conditions, greater biomass production, less loss of water due to evaporation, and fewer contaminants. In contrast, a closed photobioreactor is more expensive than an open pond in terms of operating and building costs. Open pond systems have high production capacity, less design, building, and operating effort, and low energy consumption. However, they are highly affected by local climate conditions and exposure to contamination. Additionally, low carbon dioxide transfer, uncontrolled light intensity, and variation of temperature are presented in open ponds (Prado et al., 2023). The widely used method for microalgae cultivation is open pond culture since less maintenance is required and cost-efficient (Avinash et al., 2020). Open ponds are termed as Open raceway ponds with rotating peddle for keep the culture suspended.

The growth rate of microalgae can be influenced not only by the cultivation system (open or closed photobioreactor) but also by many environmental factors such as pH, temperature, light intensity, carbon dioxide, dissolved oxygen concentration, and salinity (Chowdury et al., 2020). Metabolic and physiological changes of microalgae happen due to environmental stresses such as starvation due to limited nutrients, ultraviolet radiation, and temperature variation. When stress are very high or compounded, microalgae growth will stop and lead to death (Elisabeth et al., 2021). Effects of cultural conditions on algae growth can be tested using experiments. But, it takes too long and is costly (Yuan et al., 2020). Modeling of microalgae growth is an efficient method that connects the and incorporates the influence of each factor on algae growth and main output results parameters (biomass growth rate, lipid production, etc.). Therefore, the effect of each change on the environmental conditions of the cultivation culture (temperature, pH, etc.) can be noticed without doing any experiment. The mathematical kinetic growth model can specify microalgae

cultivation's most influential factors. Ultimately, optimization of cultivation conditions for optimal biomass production, algal biofuel, and other contents of algae can be estimated with little effort (Darvehei et al., 2018; Yuan et al., 2020; Lee et al., 2015).



Figure 1: A conventional Open Raceway Pond for microalgae cultivation (Ebadi et al., 2020)

1.2. Why mathematical modeling in algae cultivation for Saudi Arabia.

Figure 2 represented in the below illustrates the role of mathematical modeling in algae cultivation for the future of sustainable aquaculture in Saudi Arabia.

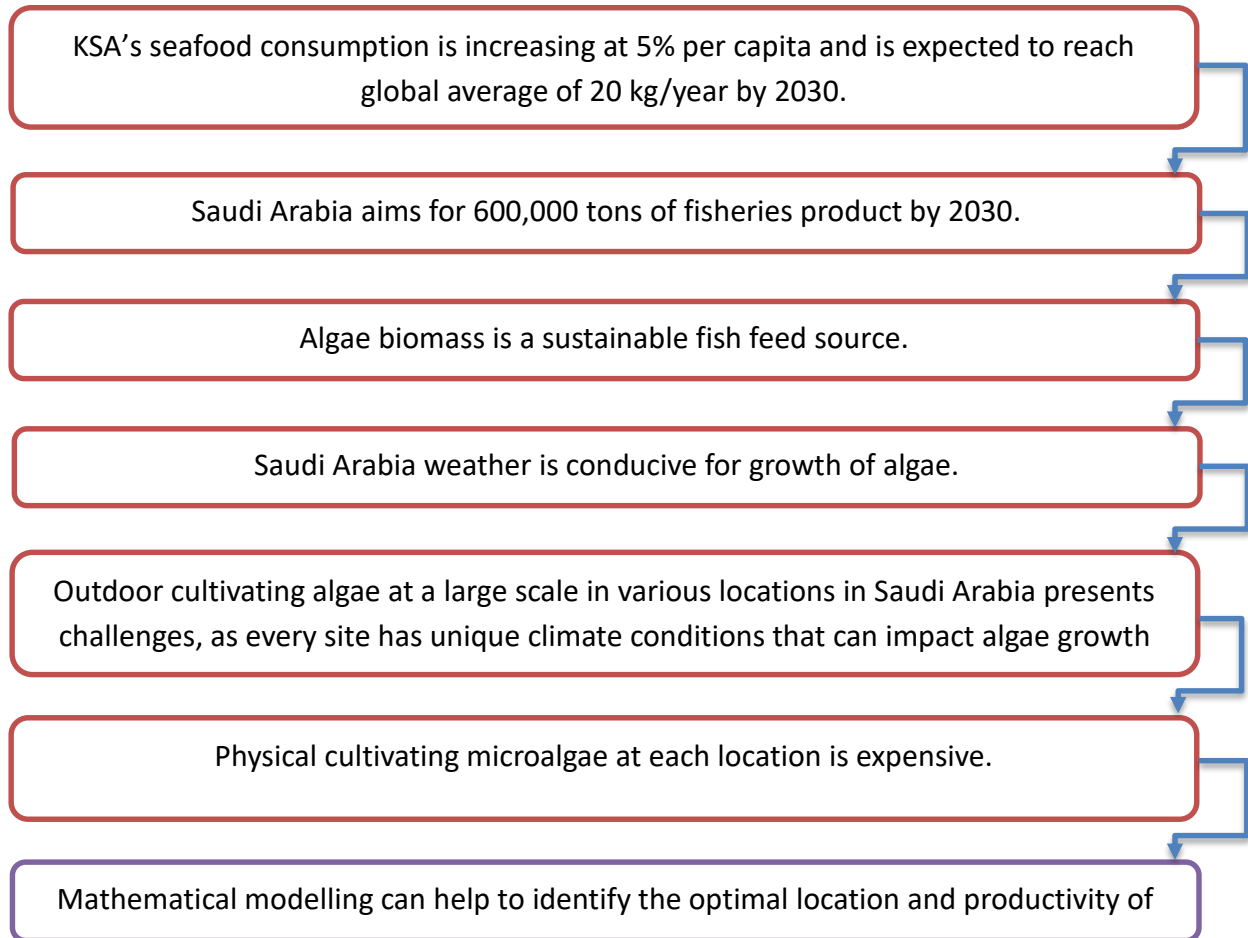


Figure 2: Why mathematical modeling in algae cultivation for Saudi Arabia

1.3. Novelty of the research

This work includes a novel approach to predict the microalgae growth and productivity under Saudi Arabia climate conditions.

1.4. Objectives of this research

The specific objectives for this research are as follows:

- Predict the microalgae growth in open raceway pond using weather conditions prevailing in Saudi Arabia by thermal and biological mathematical modelling
- Test the impact of the harvesting cycle and depth of ponds on algae productivity
- Validation of the model results with experimental results
- Provide a tool to find the optimal location of microalgae cultivation in Saudi Arabia

1.5. Summary of this work

The present work implemented the thermal and biological model for the ORP bioreactor to estimate annual production for a specified microalgae strain (*Arthrospira maxima*). The thermal and biological model was used to find the temperature and microalgae biomass concentration profile for ORP. The local weather conditions (ambient temperature, relative humidity, and global horizontal irradiance (GHI), dimensions of ORP, initial pond temperature, initial biomass concentration inside a pond, and harvested concentration were used as input information in the thermal and biological model based on the weather conditions from Jeddah, Saudi Arabia. The model estimated the annual microalgae production, pond temperature, evaporation rate, and changes in the pond's salinity. To see the effect of ORP dimensions on the yearly production of microalgae, the model was run based on fixing weather conditions and varying ORP dimensions. The model was run based on different harvesting concentrations to obtain the impact of harvesting concentration on the microalgae productivity. Finally, this model can be run in different places in Saudi Arabia with different microalgae species to estimate the biomass productivity potential at specified locations.

CHAPTER 2: LITERATURE REVIEW

Mathematical modeling of microalgae can be categorized into two parts. The first part will include the mathematical growth rate models depending on the single substrate effect of limiting factors such as light intensity, temperature, nitrogen and phosphorus nutrients, pH, and salinity. The second part will describe the growth models based on multiple limiting factors: non-interactive, interactive, and additive mathematical models. The mathematical model will describe the kinetic growth of microalgae in terms of a specific growth rate (μ) related to the biomass concentration of microalgae by the following relation.

$$\mu = \frac{\ln(X_f) - \ln(X_i)}{t_f - t_i} \quad (2-1)$$

X_f and X_i are the final and initial biomass concentration and $t_f - t_i$ is the time interval of the period (Vo et al., 2019).

2.1. Mathematical kinetic models for microalgae growth based on a singular substrate

To avoid the complexity of living cell models, one single factor is assumed to have a critical impact on the cellular growth rate. In general, two kinds of singular substrate models are available in the literature: the extracellular model, which follows the form of classic Monod, and intracellular elements, mainly represented by the Droop model form (Mesut Bekirogullari et al., 2020).

2.1.1. Light as a growth factor in Mathematical modeling

One of the widely used cultivation methods for microalgae is photoautotrophic culture, where light energy is utilized by microalgae cells to transform carbon dioxide into an organic substance (Ye et al., 2023). The growth rate of marine microalgae *Nannochloropsis sp.* was tested under different light intensities and confirmed that the specific growth rate of *Nannochloropsis sp.* decreases when *Nannochloropsis sp.* cells are exposed to light intensities lower or higher than optimum light intensity (Chang et al., 2016). Therefore, the light source is critical to microalgae's mathematical

kinetic growth model (Darvehei et al., 2018). The PI expresses the relationship that connects the photosynthetic process and light intensity in individual cells (P presents the photosynthetic rate, and I express light intensity). The PI curve mainly consists of three central regions. The first range is illustrated by the proportional relation between photosynthetic rate and light intensity, and photosynthesis in this region is limited due to light intensity limitations. The capacity of algae for capturing photons in photosynthesis has a specified limit, and any further increase in light intensity will not influence the rate of photosynthesis; this phenomenon is presented in the second region of the PI curve. The last segment in the PI curve shows the influence of light inhibition because high incident light intensity deactivates photosynthetic systems as shown in Figure 2) (Béchet et al., 2013).

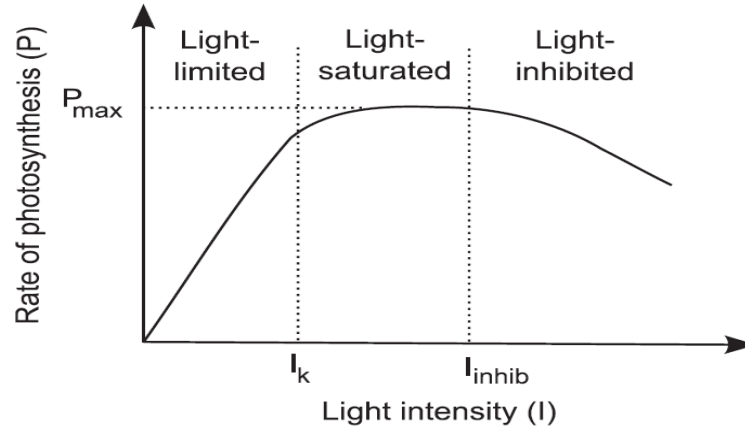


Figure 3: PI curve illustrating the impact of light intensity on algae growth in three distinct regions: light-limited, light-saturated, and light-inhibited (Béchet et al., 2013)

Light attenuation occurs when light penetrates a turbid medium like an algae culture. The intensity of light decreases as it goes deeper into the medium. The Beer-Lambert law fits the attenuation phenomenon. This law explains how light intensity exponentially decreases with the depth of the medium, as described in equation 2-2 (Benson & Rusch, 2006).

$$I(z) = I_o \exp(-\sigma X_A Z) \quad (2-2)$$

Where $I_a(z)$ is the light intensity amount at distance z from the surface of the medium (pond), I_o is incident light intensity at the surface, X_A is microalgae cell concentration, and the σ is the

biomass light absorption (extinction) coefficient for the entire PAR range (400–700 nm). There are two main conditions for applying the Beer-Lambert. The first condition is that the broth's optical properties are not affected by the direction of light, which happens in a well-mixed medium. Another condition is that the light does not scatter by algae cells (Marsullo et al., 2015; Huesemann et al., 2016). The movement of algae inside the culture plays a critical role in the photosynthetic rate because the algae on the surface are exposed to absorb more light intensity than algae deeper in broth (Béchet et al., 2013). The calculation of average light intensity (I_a) for medium (bioreactor) strongly depends on the geometry of the algae growth bioreactor, so for growth on the open pond, the relation for I_a is

$$I_a = \frac{1}{z} \int_0^z I_o \exp(-k_e z) dz \quad (2-3)$$

I_o is incident light on the surface, which can be found by $I_o = PAR \times I_{GHI}$. where PAR (photosynthetic active radiation) is the amount of light that algae use in the photosynthetic process. It is equal to 45 % of total coming light intensity, I_{GHI} is global horizontal irradiance, z is the depth of the bioreactor (pond), and k_e is named the extinction coefficient. It is a function of algae concentration in a pond, as mentioned in Equation 2-4 (Marsullo et al., 2015).

$$K_e = K_{e1} + K_{e2}X_A \quad (2-4)$$

The photosynthetic process can be illustrated in many forms, such as the rate of electron transfer, oxygen release from algae, and carbon dioxide consumption by algae (Darvehei et al., 2018). Many models have been proposed to describe specific growth rates in light intensity, but in this part, only four models will be discussed in detail. Table 1 includes those models with their parameters.

2.1.1.1. Modelling algae growth on incident light intensity as a single factor

Baly and Tamiya propose a model for algae growth by replacing substrate concentration in the Monod model with light intensity, as given in the following equation.

$$\mu = \mu_{max} \frac{I}{I_s + I} \quad (2-5)$$

Where μ is the specific growth rate, μ_{max} is the maximum specific growth rate, I_s is light saturation irradiance, and I is light irradiance (Darvehei et al., 2018).

In real situations, a specific growth rate is inhibited at a high substrate concentration, which often reduces the specific growth rate. The Monod model does not account for the inhibition under high external substrate concentration (Chen & Johns, 1994). So, equation 2-5 (Monod model when light irradiance is substrate) does not describe the third region in the PI curve (Figure 2). This limitation of Baly and Tamiya's Model cannot describe microalgae growth at low substrate concentrations (for instance, light irradiance), which is a lag phase (Figure.2).

a. Models based on average light intensity

Algae cells has a light attenuation effect in growth medium. Using incident light intensity has no more extended meaning since algae receive different amounts of light from the broth. Instead of using the model with incident light intensity, models based on average light intensity were implemented under light attenuation conditions. Average light intensity depends on the length of the medium, the concentration of algae in the broth, the extinction coefficient, the geometry of the cultivation system, and incident light intensity on the surface of the culture medium, as mentioned in equation 2-2 (Lee et al., 2015)—the following two models used average light intensity approach.

b. Hyperbolic growth model without photoinhibition

This Model has the same construction as the Monod model but incorporates an exponential n that has been inserted into this Model to adjust growth kinetic for the lag phase at a low value of average light intensity inside the growth culture. The expression for this Model is

$$\mu = \mu_{max} \frac{I_{av}^n}{I_{av}^n + I_k^n} \quad (2-6)$$

Where I_{av} is the average light intensity of broth, I_k is a light constant that constitutes the cell's (microalgae) affinity to light, which is equivalent to the saturation constant (equation 2-5).

$\mu = \frac{1}{2}\mu_{max}$, when $I_{av} = I_k$, but growth becomes limited by light at I_{av} above the value of I_k until there is no more growth at a saturated point (Molina Grima et al., 1993).

c. Hyperbolic Model with photoinhibition

When the photoinhibition happens, both parameters n and I_k on the hyperbolic model without photoinhibition are modified as a function of incident light intensity on the reactor surface (I_o). The hyperbolic Model that involves photoinhibition is expressed.

$$\mu = \frac{\mu_{max} I_{av}^{(a+bI_o)}}{(c+dI_o)^{(a+bI_o)} + I_{av}^{(a+bI_o)}} \quad (2-7)$$

The new parameters a,b,c, and d were found by correlating 16 experimental data points for *H. pluvialis* microalgae to equation 2-7 using non-linear regression, and the coefficient of this regression (r^2) is equal to 0.9778. The values of μ_{max} , a,b,c, and d are 0.11 h^{-1} , 2.32, -0.0008 ($\mu\text{E m}^{-2}\text{s}^{-1}$) $^{-1}$, $98.7 \mu\text{E m}^{-2}\text{s}^{-1}$, and 0.034, respectively. (García-Malea et al., 2006).

2.1.1.2 Eilers and Peeters Model

This model accounts for the photoinhibition at a certain light intensity level, as shown in Figure 3 (Lee et al., 2015). The model developed by Eilers and Peeters is based on photosynthesis dynamics. The central assumption used to create this model is that the photosynthetic unit (PSU) can follow three different states: state of resting, activated state by only one photon, and deactivated state by a higher amount of photoss. Photoinhibition is described by the third state, which is a speedy sequence of photons deactivation of PSU. When one photon hits PSU in interval Δt , the transition from one state to another happens, and the photosynthetic process occurs. The number of hitting photons per unit Δt is directly proportional to light intensity. The Eilers and Peeters model was expressed by replacing probability (p) in the Poisson model with one hitting photon in Δt (Peeters & Eilers, n.d.; Eilers & Peeters, 1988). The formula of Eilers and Peeters is illustrated as follows:

$$\mu = 2 \mu_{max} (1 + \beta) \frac{x}{x^2 + 2\beta x + 1} \quad (2-8)$$

$$x = \frac{I}{I_{opt}} \quad (2-9)$$

where I is the light energy, I_{opt} is the optimum light energy for growth rate, and β is a sharpness coefficient ranging from -1 to ∞ (Talbot et al., 1991).

2.1.1.3. Light spectrum as an input variable for algae growth

Both light intensity and light spectrum are important parameters for algae growth. Microalgae showed better growth in specific wavelengths than others. Therefore, researchers investigated this impact. Active wavelengths in light for photosynthesis are known as photosynthetically active radiation (PAR), and it is in the range between 400 - 700nm (Darvehei et al., 2018). Based on two assumptions, the energy of light intensity is affected by this range of wavelengths, and light intensity is attenuated in culture. Blanken et al. produced a formula that predicts the amount of light intensity inside culture at a certain depth (Z):

$$I(Z) = \sum_{\lambda=400}^{\lambda=700} I_{\lambda}(0) e^{(-a_{\lambda,x} C_x Z)} \Delta\lambda \quad (2-10)$$

Where $I_{\lambda}(0)$ is light intensity at the culture surface for wavelength λ , $a_{\lambda,x}$ is the light absorption parameter at wavelength λ , and C_x is the concentration of biomass (Blanken et al., 2016).

Ritchie(Ritchie, 2010)'s proposed a different way for light intensity calculation from wavelength based on the assumption that not all wavelengths in the range between 400 - 700 nm contribute the total energy of light intensity (Ritchie, 2010). The source of energy for the photosynthetic process is mainly coming from two light sources: blue (425 - 500 nm) and red (640 -700 nm), and the reason behind this is that algae have chlorophyll a and b which respond to absorb wavelengths of light at blue and red spectra mainly for photosynthesis. Therefore, the estimate of used light intensity in the photosynthetic process is expressed as follows.

$$I_{PUR} \approx 0.668 \times \int_{425}^{500} I_{\lambda} d\lambda + 0.819 \int_{640}^{700} I_{\lambda} d\lambda \quad (2-11)$$

Where 0.668 and 0.819 are the average quantum efficiency for blue and red light, respectively. The maximum sunlight irradiance in the ground of the earth reaches about $2220 \mu\text{molm}^{-2}\text{s}^{-1}$, but the irradiance used for photosynthesis is about $767 \mu\text{molm}^{-2}\text{s}^{-1}$ based on equation 2-11 (Ritchie, 2010).

Additionally, ultraviolet irradiation negatively affects algae growth through photosynthesis inhibition or cell killing (Darvehei et al., 2018).

Table 1: Mathematical models for microalgae growth depending on the light intensity as a single factor affecting algae growth

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light	$\mu = \mu_{max} \frac{I}{I_S + I}$	T1-1	Indoor	<i>Haematococcus pluvialis</i>	(Jeon et al., 2005).
Limitation				Simulated daylight	
				Cell concentration 0.215 gL^{-1}	
				$P_m = 43.5 \text{ mg g}^{-1} \text{ h}^{-1}$	
				$I_S = 177.2 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$	
				Simulated daylight	
				Cell concentration 0.123 gL^{-1}	
				$P_m = 51.7 \text{ mg g}^{-1} \text{ h}^{-1}$	
				$I_S = 254.4 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$	
				Red light	
				Cell concentration 0.123 gL^{-1}	
				$P_m = 59.7 \text{ mg g}^{-1} \text{ h}^{-1}$	
				$I_S = 236.9 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$	

Table 1: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light	$\mu = \mu_{max} \frac{I}{I_S + I}$	T1-1	Indoor	Greenlight	(Jeon et al., 2005).
Limitation				Cell concentration 0.123 gL^{-1} $P_m = 33.6 \text{ mg g}^{-1} \text{ h}^{-1}$ $I_S = 152.9 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$	
			Outdoor	<i>Chrysophyceae</i> $\mu_{max} = 0.0465 \text{ day}^{-1}$ $I_S = 115 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$ <i>Minor species</i> $\mu_{max} = 0.212 \text{ day}^{-1}$ $I_S = 134 \text{ } \mu\text{mol m}^{-2} \text{ s}^{-1}$	(Darvehei et al., 2018).

Table 1: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light Limitation	$\mu = \mu_{max} \left[1 - e^{-\frac{I}{I_K}} \right]$	T1-2	Indoor	<i>Phaeodactylum tricornutum</i> At 27.5 °C $\mu_{max} = 0.006317 \text{ h}^{-1}$ $I_K = 67.92 \mu\text{mol m}^{-2} \text{d}^{-1}$	(Ova Ozcan & Ovez, 2020)
	$\mu = \mu_{max} \frac{I}{(K_I^m + I^m)^{1/m}}$	T1-3	Indoor	<i>Chlorella pyrenoidosa</i> At 25 °C $\mu_{max} = 2.48 \text{d}^{-1} K_I = 5.7 \mu\text{Em}^{-2} \text{d}^{-1} m = 3$	(Bannister, 1979)

Table 1: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light Limitation in present of light attenuation by broth	$\mu = \mu_{max} \frac{I_{av}^n}{I_{av}^n + I_k^n}$	T1-4	Indoor	<i>Isochrysis galbana</i> $\mu_{max} = 0.0426 h^{-1}$ $I_k = 10.92 W m^{-2}$ $n = 5.13$	(Molina Grima et al., 1993).
	$\mu =$ $A \left[\frac{I_a}{X \cdot V} - I_{max}(1 - V_F) \right]$	T1-5	Indoor	<i>Chlorella pyrenoidosa</i> $A = 0.8 kg mol^{-1}$ $X = 0.01905 kg m^{-3}$ $V = 0.00075 m^3$ $I_{max} = 0.13 mol kg^{-1} d^{-1}$ $1 - V_F$ <i>is volume fraction of dark region</i>	(Lee et al., 2015)

Table 1: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light Limitation with photoinhibition	$\mu =$	T1-6	Indoor	<i>Nannochloris eucaryotum</i>	(Concas et al., 2016).
	$\mu_{max} \frac{I}{k_I + I + \frac{I^2}{K_{i,L}}} \quad (6)$			$k_I = 60 \mu Em^{-2} s^{-1}$	
				$K_{i,L} = 60 \mu Em^{-2} s^{-1}$	
	$\mu = 2 \mu_{max} (1 + \beta) \frac{x}{x^2 + 2\beta x + 1}$	T1-7	Indoor	<i>Pophyridium cruentum</i>	(Dermoun et al., 1992)
	$x = \frac{I}{I_{opt}}$	T1-8		$\beta = 0.5$	
				At T = 5°C	
				$\mu_{max} = 0.06 \text{ day}^{-1}$	
				$I_{opt} = 60 \mu mole \text{ photon } m^{-2} s^{-1}$	
				At T = 20°C	
				$\mu_{max} = 1.37 \text{ day}^{-1}$	
				$I_{opt} = 492 \mu mole \text{ photon } m^{-2} s^{-1}$	
				At T = 35°C	
				$\mu_{max} = 1.2 \text{ day}^{-1}$	
				I_{opt}	
				$= 318 \mu mole \text{ photon } m^{-2} s^{-1}$	

Table 1: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Light Limitation with photoinhibition	$\mu = \mu_{max} \frac{I}{I_{opt}} e^{(1-\frac{I}{I_{opt}})}$	T1-9	Indoor	<i>Chlorococcum littorale</i> $\mu_{max} = 0.134 h^{-1}$ I_{opt} $= 505 \mu mole photon m^{-2} s^{-1}$	(Kurano & Miyachi, 2005)
	$\mu = \mu_{max} \frac{I}{I + \frac{\mu_{max}}{\alpha} \left(\frac{I}{I_{opt}} - 1 \right)^2}$	T1-10	Outdoor	<i>Nannochloropsis oceanica</i> $\mu_{max} = 1.85 day^{-1}$ $I_{opt} = 2.03 \times 10^8 \mu E m^{-2} s^{-1}$ $\alpha = 0.12$	(Bernard & Rémond, 2012)

2.1.2. Modelling the Effect of Temperature on algae growth

One of the parameters that plays an important role in a microorganism's growth is temperature (Ming-Ju Chen, Kreuter, 1996). The temperature factor can affect the photosynthetic process in algae since photosystem II (PSII), which responds to derive photosynthesis, is influenced by temperature. Certain conditions, such as low temperature, increases the opportunity for loss of PSII activity (Torzillo et al., 1998). Much research has been done to investigate how temperature affects the growth rate. Many growth models have been introduced to describe relationships between kinetic growth and temperature in microorganisms (Darvehei et al., 2018).

a. The Model of Hinshelwood

Arrhenius equation was used to develop the Hinshelwood model for bacterial growth and algae growth description depending on temperature. This model has two parts, representing microalgae's growth rate and death. The following expression explains the Hinshelwood model.

$$\mu = Ae^{-\frac{E_1}{RT}} - Be^{-\frac{E_2}{RT}} \quad (2-12)$$

where μ is the growth rate, A and B are constants that define all process intensities, E_1 and E_2 are growth and death activation energies, R is the universal gas constant, and T is the growth temperature (Serra-Maia et al., 2016). Hinshelwood Model is valid at a high value of temperature (35 °C), but as the temperature decreases, the specific growth of algae in the Hinshelwood model will not arrive at zero unless at zero kelvin temperature, which is not a realistic condition (Serra-Maia et al., 2016 ; Darvehei et al., 2018). Therefore, a model that describes the growth rate at low temperatures is needed.

b. Modifying of Ratkowsky's Model

Ratkowsky et al (Ratkowsky et al., 1983) showed that Arrhenius's law cannot fit a plot of the logarithm of bacterial growth versus reciprocal absolute temperature (Ratkowsky et al., 1983). In place of Arrhenius's law, the following model (equation 2-13) was proposed.

$$\sqrt{\mu} = A (T - T_o) \quad (2-13)$$

where $\sqrt{\mu}$ is the square root of the growth rate constant, A is the regression coefficient constant of the square root of growth rate versus temperature in degrees Kelvin, and T_o is the temperature where growth rate is zero. The validity range of this model is from low temperature to optimum temperature. However, at temperatures higher than the optimum temperature, the growth rate starts to decrease, and this model does not predict this drop in temperature (Ratkowsky et al., 1982). Another model was proposed to cover the whole temperature range by adding an exponential part to fix the limitation of the previous Model at high temperatures.

$$\sqrt{\mu} = A (T - T_o)(1 - e^{B(T-T_{Max})}) \quad (2-14)$$

Where B is further constant to adjust the Model to fit data at a temperature greater than the optimal temperature and T_{Max} is the highest temperature where the growth rate is zero. The equation 2-14 reduces to equation 2-13 when the value of temperature is much less than T_{Max} (Ratkowsky et al., 1983).

c. Mathematical model based on Cardinal temperature

The Model of Lobry was constructed based on an empirical model that contains three cardinal temperatures (T_{min} , T_{opt} , T_{max}), so the Lobry model is called the cardinal temperature model (CTM). This model can predict the growth rate of microalgae when the temperature is located between maximum and minimum temperatures and the highest growth is reached at optimal temperatures. The growth rate is zero for any temperature higher than T_{max} or smaller than T_{min} . The three cardinal temperatures are characteristics of microalgae strain (Noll et al., 2020). The Lobry model is presented in the following equation (J. Lobry et al., 1991).

$$\mu = \mu_{opt} \left[1 - \frac{(T-T_{opt})^2}{(T-T_{opt})^2 + T(T_{max}+T_{min}-T) - T_{max} \cdot T_{min}} \right] \quad (2-15)$$

The weakness of Lobay's model was that it could not express the curvature of the suboptimal temperature in the growth rate trend. Rosso et al. introduced a suboptimal curve range of temperature in the growth model called the cardinal temperature model with inflection (CTMI; equation 2-16) (Noll et al., 2020).

$$\mu = \mu_{opt} \left[\frac{(T-T_{max})(T-T_{min})^2}{(T_{opt}-T_{min})[(T_{opt}-T_{min}) \cdot (T-T_{opt}) - (T_{opt}-T_{max}) \cdot (T_{opt}+T_{min}-2 \cdot T)]} \right] \quad (2-16)$$

CTMI was tested for microalgae growth culture and fitted with experimental data. Its limitation is that the values of T_{max} and T_{opt} must be close to each other to get growth values in the range between T_{max} and T_{min} . Therefore, the CTMI model will predict negative values of specific growth rates when the mean of T_{max} and T_{min} is less than T_{opt} (Darvehei et al., 2018).

d. Mathematical modelling of short-term temperature

The model developed by Blanchard et al is used to describe the effect of the short-term temperature on the photosynthetic capacity of benthic phytoplankton (Noll et al., 2020; Grimaud et al., 2017). Blanchard modeled that photosynthetic capacity rapidly increases while increasing in temperature until the optimum temperature. When the temperature is the temperature is excess optimum; the photosynthetic capacity comes down rapidly. The three cardinal temperatures (T_{min} , T_{opt} , T_{max}), which are biological features of microorganisms, are used to predict parameters in Blanchard's model with initial guesses easily. Grimaud et al (Grimaud et al., 2017) replaced photosynthetic capacity in Blanchard's Model with growth rate, as Equation 2-17 illustrates (Grimaud et al., 2017).

$$\mu = \mu_{opt} \left(\frac{T_{max}-T}{T_{max}-T_{opt}} \right)^{\beta} e^{-\beta \left(\frac{T_{opt}-T}{T_{max}-T_{opt}} \right)} \quad (2-17)$$

where μ_{opt} is optimal (maximal) specific growth rate (s⁻¹) at optimal temperature (T_{opt}), degrees Kelvin, β is a dimensionless constant of Blanchard, and T_{max} is maximum temperature, degrees Kelvin, where there is a growth.

e. The Model of Corkrey

The biological processes that happen in enzymes of the living cells are highly controlled by temperature, from individual cells to complex populations, in an extending range of temperatures between far below freezing and higher than the boiling point of water. Some organisms prefer to grow in cold conditions, whereas others in hot conditions are more comfortable to growing. Corkrey built his model by developing a thermodynamic model. The model of Corkrey fit 230 growth rate data of various strains from unicellular and multicellular organisms. The temperature domain of fitting experimental data with this model was in the range of 271.2 K to 395.3K. The main assumption for the structure of this model is that the growth rate is controlled by a single-enzyme-catalyzed reaction, which is limiting under all conditions. The numerator of Corkrey's model is explained by an Arrhenius model that depends on temperature and describes the reaction rate. The varying catalytic activity of the enzyme is caused by the effects of temperature on the protein conformation that is accounted on the denominator of Corkrey's Model. Therefore, the predicted rate of growth is changing.

$$\mu = \frac{T e^{(c - \frac{\Delta H_A^\ddagger}{RT})}}{1 + e^{(-n - \frac{\Delta H^* - T\Delta S^* + \Delta C_p \left(T - T_H^* - T \log \left(\frac{T}{T_S^*} \right) \right)}{RT}})} \quad (2-18)$$

c is a constant of scaling, R is constant of gas (8.314 J/mole K), $\Delta H_A^\ddagger$ is the activation enthalpy (J/mole), T is temperature in degrees kelvin, ΔC_p is a change of heat capacity (J/K mol-amino acid-residue) upon denaturation of the putative enzyme, n is the amino acid residues number, ΔH^* is the change of enthalpy (J/mol amino acid residue) at temperature T_H^* , degrees Kelvin, the convergence temperature for protein unfolding enthalpy, ΔS^* is the change of entropy the convergence temperature for protein unfolding enthalpy, ΔS^* is the change of entropy (J/Kelvin) at temperature T_S^* , degrees Kelvin, the convergence temperature for protein unfolding entropy (Corkrey et al., 2014).

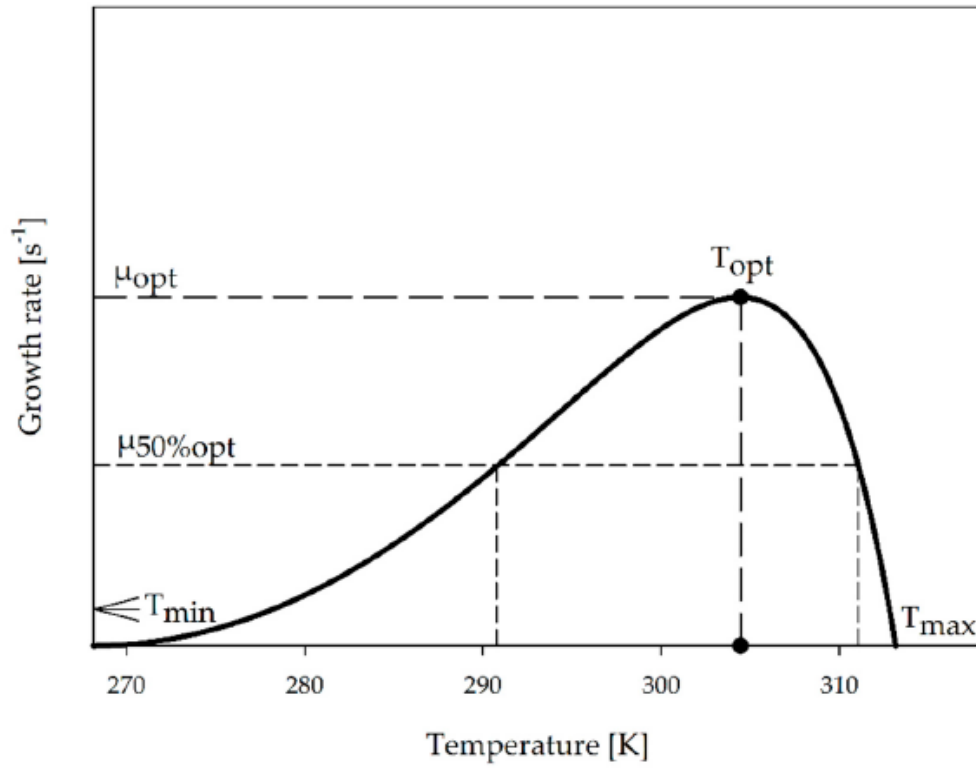


Figure 4: The diagram of growth rate (s^{-1}) with temperature (K) with including of three cardinal temperatures with the optimal growth rate (Noll et al., 2020)

Each species of microalgae has its own maximum and minimum temperature where it can grow. The maximum growth of microalgae happens when the temperature of cultivation medium is equal to the optimal growth temperature of microalgae strain (see Figure 4).

Table 2: Kinetic models of microalgae growth depending on the environmental temperature

Kinetic Model	Eq#	System	Parameters	Reference
$\sqrt{\mu} = A (T - T_o)$	T2-1			(Ratkowsky et al., 1982)
$\sqrt{\mu} = A (T - T_o)(1 - e^{B(T-T_{Max})})$	T2-2			(Ratkowsky et al., 1983)
$\mu = \mu_{opt} e^{-c(T-T_{opt})^2}$	T2-3	Indoor	<i>Phaeodactylum tricornutum</i> $T_{opt} = 20\text{ }^{\circ}\text{C}$ $c = 0.007^{\circ}\text{C}^{-2}$	(James& Boriah, 2010)
$\mu = A_1 e^{\left(\frac{E_1}{RT} \frac{T-T_o}{T}\right)} - A_2 e^{\left(\frac{E_2}{RT} \frac{T-T_o}{T}\right)}$	T2-4		<i>Phaeodactylum tricornutum</i> $A_1 = 0.26h^{-1}$ $A_2 = 0.18h^{-1}$ $E_1 = 0.28 \frac{kcal}{mol}$ $E_2 = 0.39 \frac{kcal}{mol}$ $T_o = 293\text{ }k$	(Bitaubé Pérez et al., 2008)

Table 2: Continued

Kinetic Model	Eq#	System	Parameters	Reference
$\mu = \frac{k_B T}{h} e^{\frac{-\Delta G^\ddagger}{RT}}$ $\Delta G^\ddagger = \Delta H_{T_o}^\ddagger + \Delta C_p^\ddagger (T - T_o) - T \left[\Delta S_{T_o}^\ddagger + \Delta C_p^\ddagger \ln \left(\frac{T}{T_o} \right) \right]$	T2-5			(L. A. Schipper et al., 2014)
$\mu = \frac{T e^{(c - \frac{\Delta H_A^\ddagger}{RT})}}{1 + e^{(-n - \frac{\Delta H^* - T \Delta S^* + \Delta C_p \left(T - T_H^* - T \log \left(\frac{T}{T_S^*} \right) \right)}{RT}})}$	T2-6	Indoor	<i>Chlorella pyrenoidosa</i> $\Delta H_A^\ddagger = 77.4 \text{ Jmol}^{-1}$ ΔC_p $= 63.1 \frac{J}{K \text{ mol} - \text{amino acid}^{-1}}$ $n = 692.3$ $\Delta H^* \Delta S^* T_H^* T_S^*$ are universal parameters.	(Corkrey et al., 2014)

Table 2: Continued

Kinetic Model	Eq#	System	Parameters	Reference
$\mu = \mu_{max} \left[\frac{(T - T_{max})(T - T_{min})^2}{(T_{opt} - T_{min})(C(T) - D(T))} \right]$ $C(T) = (T_{opt} - T_{min}) \cdot (T - T_{op})$ $D(T) = (T_{opt} - T_{max}) \cdot (T_{opt} + T_{min} - 2 \cdot T)$	T2-7	Outdoor	<i>Nannochloropsis oceanica</i> $T_{min} = -0.2^\circ\text{C}$ $T_{min} = 26.7^\circ\text{C}$ $T_{min} = 33.3^\circ\text{C}$	(Bernard & Rémond, 2012)
$\mu = \mu_{opt} \left(\frac{T_{max} - T}{T_{max} - T_{opt}} \right)^\beta e^{-\beta \left(\frac{T_{opt} - T}{T_{max} - T_{opt}} \right)}$	T2-8	Outdoor	<i>Benthic Phytoplankton</i> $\mu_{opt} = 5.81 \mu\text{g}/\mu\text{g Chl } a \text{ h}$ $T_{opt} = 25.3^\circ\text{C}$ $T_{max} = 38.2^\circ\text{C}; \beta = 1.76$	(Blanchard et al., 1996)
$\mu = \begin{cases} \log(2) 0.851 (1.066^T) e^{-\frac{ T - T_{opt} ^3}{T_{low}}} & T \leq T_{opt} \\ \log(2) 0.851 (1.066^T) e^{-\frac{ T - T_{opt} ^3}{T_{high}}} & T > T_{opt} \end{cases}$	T2-9			(Moisan et al., 2002)

Table 2: Continued

Kinetic Model	Eq#	System	Parameters	Reference
$\mu = \begin{cases} \mu_{opt} e^{-2.3 \left(\frac{T-T_{opt}}{T_{low}-T_{opt}} \right)^2}, & T < T_{opt} \\ \mu_{opt} e^{-2.3 \left(\frac{T-T_{opt}}{T_{upp}-T_{opt}} \right)^2}, & T \geq T_{opt} \end{cases}$	T2-10	Indoor	<i>Fragilaria crotonensis</i> $\mu_{opt} = 0.58 day^{-1}$ $T_{opt} = 25^{\circ}C$ <i>Stauroastrum pingue</i> $\mu_{opt} = 0.77 day^{-1}$ $T_{opt} = 27^{\circ}C$ <i>Synechocystis minima</i> $\mu_{opt} = 1.32 day^{-1}$ $T_{opt} = 32^{\circ}C$	(Dauta et al., 1990)
$P = P_{opt} \frac{2 \cdot e^{\left(\frac{E1}{RT_{opt}} - \frac{E1}{RT} \right)}}{1 + \left(e^{\left(\frac{E1}{RT_{opt}} - \frac{E1}{RT} \right)} \right)^2}$	T2-11	Outdoor	<i>Tidestromia oblongifolia</i> $P_{opt} = 54 \frac{\mu mole CO_2}{m^2 s^{-1}}$ $E1 = 66.2 \frac{KJ}{mole}$ $T_{opt} = 41.9^{\circ}C$	(Alexandrov & Yamagata, 2007)

2.1.3. Mathematical modelling of algae growth based on impact of Nutrients

The growth of microalgae is highly influenced by nitrogen and phosphorus, the two major macro-nutrient sources (Gao et al., 2023). Nitrogen is necessary to produce ammonium and nitrate salts used by microalgae cells for growth and metabolism. Additionally, nitrogen is the highest quoted factor that increases the productivity of lipids in many microalgae species (Chin et al., 2023). Inorganic phosphorus such as K_2HPO_4 and $Na_5P_3O_{10}$ is the main source of phosphorous for microalgae growth, but some researchers have proved that the growth of microalgae is affected by organic phosphorous such as ATP and G6P (Xing et al., 2023). Gao et al (Gao et al., 2023) showed the influence of different nitrogen-to-phosphorus concentration ratios in *Scenedesmus obliquus* microalgae, and results illustrated that there is an optimum concentration ratio of nitrogen to phosphorus for *S. obliquus* growth (Gao et al., 2023). This section will present some famous growth models that depend on nutrients (Nitrogen and Phosphorus).

a. Model based on a single substrate (Monod Model)

The Monod growth model was constructed as an empirical model dependent on experimental work. It can be used to explain any microbial growth rate. The main difference between the Monod model and the classical model like those introduced by Gompertz (1825), Verhulst (1845,1847), and Richards (1959) is proposing the principle of limiting nutrients (J. R. Lobry et al., 1992). The main assumption in this model is that the growth rate of microbes stops when the amount of limiting nutrients is exhausted. Another assumption of Monod's model is that only one substrate manages the growth rate, so any changes in other substrates are negligible (Mesut Bekirogullari et al., 2020). The Monod model fits the results of the experimental work done by Tamiya and Baly for microalgae *Chlorella ellipsoidea*. The experiment used light irradiance as a substrate (Darvehei et al., 2018).

$$\mu = \mu_{max} \frac{S}{k_S + S} \quad (2-19)$$

where μ is the specific growth rate, μ_{max} is the maximum specific growth rate, k_S is the saturation constant, and S is substrate concentration (Nitrogen and Phosphorus). When substrate

concentration is much higher than k_s , the maximum specific growth rate is equal to the specific growth rate. In another case, k_s this is called the half-saturation constant because $\mu = \frac{1}{2}\mu_{max}$ when $k_s = S$ (Darvehei et al., 2018; Bekirogullari et al., 2020). Some of the substrate goes for the maintenance of cells during growth, which is not accounted in the Monod model (Muloiwa et al., 2020).

b. Model Haldane-Andrews

The advantage of the Haldane model is the ability to describe growth rates at low and high substrate concentrations (lag and death phase, see Figure 2). Meanwhile, the Monod model only describes microbial (microalgae) growth rate in algae growth's exponential and stationary phases (see Figure 2). The formula of the Haldane model in terms of substrate concentration is

$$\mu = \mu_{max} \frac{S}{k_s + S + \frac{S^2}{K_{i,S}}} \quad (2-20)$$

Where μ is the specific growth rate, μ_{max} is the maximum specific growth rate, S is the substrate concentration, k_s is half saturation constant, and $K_{i,S}$ is the inhibition constant (Muloiwa et al., 2020). The maximum specific growth rate can be achieved in the Haldane model by implementing optimal initial concentration of substrate (Nitrogen and Phosphorus). Equation 2-21 is used to calculate the maximum growth rate that can be achieved by using optimal substrate concentration (equation 2-22) (Mesut Bekirogullari et al., 2020).

$$\mu_{max} = \frac{\mu}{1 + 2 \sqrt{\frac{k_s}{K_{i,S}}}} \quad (2-21)$$

$$S = \sqrt{K_{i,S} k_s} \quad (2-22)$$

c. Droop's Model

Droop realized that the specific growth rate of *Monochrysis Lutheri* versus the external concentration of vitamin B₁₂ did not fit the hyperbola plot mentioned by the model of Monod (Mesut Bekirogullari et al., 2020). So, Droop proposed (Droop, 1968)

that the total concentration of limiting nutrients inside the cell, known as cell quota, controls the cell's growth kinetics, as Droop proposed. The expression for the Model of Droop is as follows.

$$\mu = \mu_{max} \left(1 - \frac{Q_o}{Q}\right) \quad (2-23)$$

Where μ is the specific growth rate, μ_{max} is the maximum specific growth rate, Q (cell quota) is the mass of internal limiting nutrients (Nitrogen, phosphorus as an example) per total mass of biomass cell, and Q_o (minimal cell quota) is the minimal internal mass ratio of limiting nutrients that are required for growth (Quinn et al., 2011; Benavides et al., 2015). Monod and Droop models were compared using data from different phytoplankton species collected from a eutrophic lake in Germany. The Droop model was fitting more precisely (Sommer, 1991). The main issue with the Droop model is that the specific growth rate μ is equal to μ_{max} at the infinity value of cell quota (Q), which is impossible in real situations (Lee et al., 2015).

d. The Caperon–Meyer model

The Caperon-Meyer model is a model that describes growth rate by using the cell quota principle and the structure formula of the Caperon-Meyer model is the same as the equation of Michaelis–Menten enzyme kinetics (Molina-Miras et al., 2022). The basic difference between the Droop model and the Caperon–Meyer model is that the saturation kinetics constant was included in the Caperon–Meyer model which makes the specific growth rate bounded in the upper limit (Mesut Bekirogullari et al., 2020). The following formula is the Caperon–Meyer model (Molina-Miras et al., 2022).

$$\mu = \mu_{max} \left(\frac{Q - Q_o}{K_Q + (Q - Q_o)} \right) \quad (2-24)$$

K_Q is a half-saturated constant with the same unit as Q . Based on equation 2-24, the growth rate will be zero when Q equals Q_o . The cell quota does not go to infinity for the maximum specific growth rate in the Caperon–Meyer model due to the hyperbolic behavior of this Model, and the maximum rate of growth is when $Q - Q_o$ is much greater than K_Q , as mentioned in equation 2-24. The number of parameters in the Droop model is two, less than that in the Caperon–Meyer model. For this reason, the Droop model is preferred (Mesut Bekirogullari et al., 2020).

e. Model of Martinez-Sancho

The Monod model cannot describe the growth rate in case that the nutrients are exhausted in a growth medium. Therefore, Martínez Sancho et al (Das et al., 2023). modified the Monod model by accounting for the maximum specific growth rate (μ_{m2}) in the nonexistence of external nutrients. Martínez Sancho et al. proposed the following relation (Das et al., 2023).

$$\mu = \frac{\mu_{m1}S + k_s\mu_{m2}}{k_s + S} \quad (2-25)$$

Where μ_{m1} is the maximum specific growth rate when $S > 0$ and μ_{m2} is the maximum specific growth rate for $S = 0$ (when S is finished), k_s was identified as a substrate concentration when half of the sum of μ_{m1} and μ_{m2} is equal to the specific growth rate(μ) (Mesut Bekirogullari et al., 2020). Martínez Sancho et al. made another modification by putting all three nutrition cases of growth in one Model: the first case is growth under exhaustion of nutrients, the second case is limited growth under low concentration of nutrients, and the third case is inhabitation of growth under high concentration of nutrients. The following expression includes the effect of a high concentration of nutrients on other nutrition cases of growth (Martínez Sancho et al., 1997).

$$\mu = \frac{\mu_{m1}k_iS + k_s k_i \mu_{m2} + \mu_{m3}S^2}{k_s k_i + k_i S + S^2} \quad (2-26)$$

Where k_i is constant of inhibition, μ_{m1} is the maximum specific growth rate, μ_{m2} is a specific growth rate when the nutrients are non-existent, and μ_{m3} is a specific growth rate for inhibition (Darvehei et al., 2018). The main disadvantage of the Martinez-Sancho Model is the requirement to find more kinetics parameters compared to those in the Monod model (Mesut Bekirogullari et al., 2020).

Table 3: Kinetic models of microalgae growth depending on the nitrogen and phosphorus nutrients

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Model based on external concentration of nutrient	$\mu = \mu_{max} \frac{S}{k_S + S}$	T3-1	Indoor	<i>Scenedesmus sp. LX1.</i> N: $\mu_{N,max} = 1.78 \times 10^6 \frac{Cells}{mL \ day}$ $k_{S,N} = 11.8 \frac{mg}{L}$ P: $\mu_{P,max} = 1.02 \times 10^6 \frac{Cells}{mL \ day}$ $k_{S,P} = 0.28 \frac{mg}{L}$	(Xin et al., 2010).
			Indoor	<i>Gracilaria folifera</i> N: $\mu_{N,max} = 12.8 \pm 1.4 \ day^{-1}$ $k_{S,N} = 0.2 \pm 0.1 \mu MNH_4^+$	(DeBoer et al., 1978).

Table 3: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Model based on external concentration of nutrient	$\mu = \mu_{max} \frac{S}{k_s + S}$	T3-1	Indoor	<i>Chlorella sp.</i> P: $\mu_{P,max} = 0.23 \text{ day}^{-1}$ $k_{S,P} = 3.01 \frac{mg}{L}$	(Wang et al., 2014)
	$\mu = \mu_{max} \frac{S}{k_s + S + \frac{S^2}{K_{i,S}}}$	T3-2	Indoor	<i>Chlorella sp</i> N: $k_{S,N} = 1.78 \frac{mg}{L}, k_{i,N} = 364 \frac{mg}{L}$	(Lee & Zhang, 2016).
Model based on external concentration of nutrient	$\mu = \frac{\mu_{m1}S + k_s\mu_{m2}}{k_s + S}$	T3-3	Indoor	<i>Scenedesmus obliquus</i> P: $k_s = 0.2\mu M$ $\mu_{m1} = 0.0466h^{-1}, \mu_{m2} = 0.0256h^{-1}$ Application range P: $0 - 10.1 \frac{mg}{L}$ as $PO_4\text{-P}$	(Das et al., 2023).

Table 3: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Model based on external concentration of nutrient	$\mu = \frac{\mu_{m1}k_iS + k_s k_i \mu_{m2} + \mu_{m3}S^2}{k_s k_i + k_i S + S^2}$	T3-4	Indoor	<i>Scenedesmus obliquus</i> P: $k_s = 0.25\mu M$, $k_i 955.72\mu M$ $\mu_{m1} = 0.0471h^{-1}$ $\mu_{m2} = 0.0350h^{-1}$ $\mu_{m3} = 0.0272h^{-1}$ Application range P: $0 - 10.1 \frac{mg}{L}$ as $PO_4\text{-P}$	(Das et al., 2023).
Model based on internal concentration of nutrient	$\mu = \mu_{max} \left(\frac{Q - Q_o}{K_Q + (Q - Q_o)} \right)$	T3-5		<i>Aureocumbra lagunensis</i> N: $Q_{o,N} = 0.043 mol N mol C^{-1}$ $\mu_{max} = 0.697 day^{-1}$ $k_{Q,N} = 0.0083 mol N mol C^{-1}$ <i>S. quadricauda</i> P: $Q_{o,P} = 0.25 \times 10^{-8} \mu mol cell^{-1}$ $\mu_{max} = 0.0002 min^{-1}$ $k_{Q,P} = 0.25 \times 10^{-8} \mu mol cell^{-1}$	(Mesut Bekirogullari et al., 2020). (Yao et al., 2011).

Table 3: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Model based on internal concentration of nutrient	$\mu = \mu_{max} \left(1 - \frac{Q_o}{Q}\right)$	T3-6	Outdoor	<i>Nanno- chlorosis oculata</i> N: $Q_{o,N} = 0.01 gg^{-1}$ $\mu_{max} = 2.5 \times 10^{-2} h^{-1}$	(Quinn et al., 2011b).
			Indoor	N: $Q_{o,N} = 0.04 mgN mgC^{-1}$ $\mu_{max} = 1.645 day^{-1}$	(Benavides et al., 2015).
			Indoor	<i>Nitzschia sp.</i> P: $Q_{o,P} = 0.045 Pmol cell^{-1}$ $\mu_{max} = 0.48 day^{-1}$	(Yamamoto et al., 2012).
				<i>Skeletonema costatum</i> P: $Q_{o,P} = 0.0033 Pmol cell^{-1}$ $\mu_{max} = 1.25 day^{-1}$	
				<i>Gymnodinium catenatum</i> P: $Q_{o,P} = 1.83 Pmol cell^{-1}$ $\mu_{max} = 0.37 day^{-1}$	

Table 3: Continued

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Model based on internal concentration of nutrient	$\mu = \mu_{max} \left(\frac{1 - \frac{Q_o}{Q}}{1 - \frac{Q_o}{Q_{max}}} \right)$	T3-7	Indoor	<i>Isochrysis affinis galbana</i> $\mu_{max} = 1.5 \text{ day}^{-1}$ N: $Q_{o,N} = 0.065 \text{ mol N mol C}^{-1}$ $Q_{max,N} = 0.14 \text{ mol N mol C}^{-1}$ P: $Q_{o,P} = 0.0009 \text{ mol P mol C}^{-1}$ $Q_{max,P} = 0.006 \text{ mol P mol C}^{-1}$	(Bougaran et al., 2010).
	$\mu = \mu_{max} \left(\frac{(1+K_P)(Q-Q_o)}{(Q-Q_o)+K_P(Q_{max}-Q_o)} \right)$	T3-8		<i>Alexandrium minutum</i> $\mu_{max} = 0.55 \text{ day}^{-1}$ P: $Q_{o,P} = 3.7 \text{ pg P cell}^{-1}$ $Q_{max,P} = 32.6 \text{ pg P cell}^{-1}$ $K_P = 4.24$	(Chapelle et al., 2010).

2.1.4. Modelling the impact of pH on Algae Growth

The cultivation of microalgae can be optimized by regulating the growth medium's pH. The biochemical composition of microalgae is affected by pH. Moreover, the high pH can reduce unwanted biological contaminants in the growth medium, but the growth of microalgae is inhibited at a high pH value (Qiu et al., 2017). Dissolved carbon dioxide (CO₂) concentration and alkalinity of the medium determine the pH (Marsullo et al., 2015). Each microalgae strain has its optimal pH range for optimal growth and intracellular lipid composition (Yu et al., 2022; Sakarika & Kornaros, 2016). For instance, the growth rate of *Scenedesmus sp.* was tested under different pH, and the maximum growth rate was in the pH range between 7.5 – and 8.5, whereas the growth rate was the lowest at pH 9. (Mohamed et al., 2019). Qiu et al (Qiu et al., 2017). showed the best growth rate for *Chlorella sorokiniana* DOE1412 was at low pH 5.8 and 6 (Qiu et al., 2017). The highest biomass growth rate and total lipid content were 0.541 d⁻¹ and 53.43% g g_{DW}⁻¹ for *Chlorella vulgaris* at pH 7.5, with sulfur as a limiting nutrient (Sakarika & Kornaros, 2016). According to the previous discussion, studying the growth kinetic model of microalgae depending on pH is required to understand how pH influences the growth rate of microalgae further. In the following subsection, some common models of growth kinetic based on pH will be discussed.

a. Modelling the impact of pH due to changes in inorganic carbon

The inorganic carbon dissolved in media comes from carbon dioxide (CO₂), carbonate (CO₃²⁻), and bicarbonate (HCO₃⁻). This Model was built to predict a specific growth rate based on the total concentration of inorganic carbon (*TIC*). Equation 2-27 represents the predicted specific growth rate.

$$\mu = \mu_{max} \frac{[TIC]}{K_{CL} X + [TIC]} \quad (2-27)$$

K_{CL} is the carbon saturation constant, X is the microalgae cell number per volume of medium, and $[TIC]$ is the total inorganic carbon concentration in moles. Equation 2-28 is used to find TIC.

$$\frac{dTIC}{dt} = K_L a ([CO_2^*] - [CO_2]) \quad (2-28)$$

$$[CO_2^*] = \frac{p_{CO_2}}{H_{CO_2}} \quad (2-29)$$

Where $[CO_2^*]$ is the equilibrium CO_2 concentration in the culture medium, $K_L a$ is the coefficient of gas-liquid mass transfer, p_{CO_2} is the partial pressure of CO_2 in the air, and H_{CO_2} is Henry's constant for the solubility of CO_2 in the culture medium. The molar concentration of carbon dioxide ($[CO_2]$) can be found by implementing equation 2-30.

$$[CO_2] = \frac{[TIC]}{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}} \quad (2-30)$$

$$[H^+] = 10^{-pH} \quad (2-31)$$

Where k_1 and k_2 are the chemical equilibrium constants between HCO_3^- and CO_2 and CO_3^{2-} and HCO_3^- respectively, and $[H^+]$ is the concentration of hydrogen ions in the culture medium (Filali et al., 2011).

b. Models considering impact of different pH on growth

Two models of kinetic growth that depend on pH with other parameters of pH (pH_{min} , pH_{max} , and pH_{opt}) are presented. Ifrim et al. illustrated the following model, which includes pH_{min} and pH_{max} (Ifrim et al., 2015).

$$\mu = \mu_{max} \frac{pH - pH_{max}}{pH_{min} - pH} e^{\frac{pH - pH_{max}}{pH_{min} - pH}} \quad (2-32)$$

The cardinal temperature model with an inflection discussed previously in the subsection of the temperature section can be used for pH as the substrate for the Model of microalgae growth rate by replacing temperature with pH, as presented in the work of Ippoliti et al. (Ippoliti et al., 2016).

$$\mu = \mu_{max} \left[\frac{(pH - pH_{max})(pH - pH_{min})^2}{(pH_{opt} - pH_{min})[(pH_{opt} - pH_{min}) \cdot (pH - pH_{opt}) - (pH_{opt} - pH_{max}) \cdot (pH_{opt} + pH_{min} - 2 \cdot pH)]} \right] \quad (2-33)$$

Where pH_{max} is the maximum pH at which the microalgae can still grow, pH_{min} is the minimum pH at which the microalgae can still grow, and pH_{opt} is the optimal pH at which the maximum growth rate of microalgae can be achieved.

c. Model using CO₂ as a factor contributing toward pH

This model was proposed based on the assumption that carbon dioxide is the only compound influencing pH. To describe the carbon effect, the specific growth rate of microalga was described as a function of pH instead of the concentration of carbon dioxide (Jayaraman & Rhinehart, 2015).

$$\mu = \mu_{max} \frac{1}{1 + e^{\lambda(pH - pH_{opt})}} \quad (2-34)$$

where λ is a constant and pH_{opt} is the optima pH for algae growth. Jayaraman and Rhinehart compared their Model with experimental data collected from the University of New Mexico State for the *Galdieria sulphuraria* strain. This Model almost fits the experiment results.

d. Modelling the impact of pH on enzyme activity

Bailey and Ollis modeled the influence of pH on enzymes' reaction rate (Darvehei et al., 2018). They assumed that enzymes can be one of three catalytic classifications: active class, acid inactive, or basic inactive. The model formula of Bailey and Ollis is presented in the following equation.

$$\mu = \mu_{max} \frac{1}{1 + \frac{[H^+]}{k_1} + \frac{k_2}{[H^+]}} \quad (2-35)$$

Where k_1 and k_2 are model constants and $[H^+]$ is defined in equation 2-31. One experiment was done for *Phaeodactylum tricornutum* microalgae, and the parameter constants of fitting the results of this experiment with the Model of Bailey and Ollis were $\mu_{\mu max} = 0.064h^{-1}$, $k_1 = 2.3 \times 10^{-6}mol L^{-1}$, and $k_2 = 1.2 \times 10^{-10}mol L^{-1}$ with regression square $r^2 = 0.92$ (Bitaubé Pérez et al., 2008).

Table 4: Kinetic models of microalgae growth depending on pH

Kinetic Model	Eq#	System	Parameters	Reference
$\mu = \mu_{max} \frac{[TIC]}{K_{CL} X + [TIC]} \quad (1)$	T4-1	Indoor	<i>Chlorella vulgaris</i> at 25 °C $\mu_{max} = 0.08h^{-1}$ K_{CL} $= 1.28 \times 10^{-5} mole \ 10^9 cell^{-1}$	(Filali et al., 2011).
$\mu = \mu_{max} \frac{pH - pH_{max}}{pH_{min} - pH} e^{\frac{pH - pH_{max}}{pH_{min} - pH}}$	T4-2			(Ifrim et al., 2015).
$\mu = \mu_{max} \left[\frac{(pH - pH_{max})(pH - pH_{min})^2}{(pH_{opt} - pH_{min})(A(pH) - B(pH))} \right] \quad (3)$ $A(pH) = (pH_{opt} - pH_{min}) \cdot (pH - pH_{opt})$ $B(pH) = (pH_{opt} - pH_{max}) \cdot (pH_{opt} + pH_{min} - 2 \cdot pH)$	T4-3	Indoor and Out door	<i>Isochrysis galbana</i> $pH_{min} = 2.24$ $pH_{opt} = 7.34$ $pH_{max} = 10$	(Ippoliti et al., 2016).

Table 4: Continued

Kinetic Model	Eq#	System	Parameters	Reference
$\mu = \mu_{max} \left[\frac{(pH - pH_{max})(pH - pH_{min})}{C(pH) - D(pH)} \right]$ $C(pH) = (pH - pH_{max})(pH - pH_{min})$ $D(pH) = (pH - pH_{opt})^2$	T4-4	Indoor	<i>C. vulgaris</i> $pH_{min} = 6.7$ $pH_{opt} = 7.5$ $pH_{max} = 11$	(Amini et al., 2018).
$\mu = \mu_{max} \frac{1}{1 + e^{\lambda(pH - pH_{opt})}}$	T4-5		<i>Galdieria sulphuraria</i> $pH_{opt} = 2.5$	(Jayaraman & Rhinehart, 2015)
$\mu = \mu_{\mu max} \frac{1}{1 + \frac{[H^+]}{k_1} + \frac{k_2}{[H^+]}}$	T4-6	Indoor	<i>Phaeodactylum tricornutum</i> . $k_1 = 2.3 \times 10^{-6} mol L^{-1}$ $k_2 = 1.2 \times 10^{-10} mol L^{-1}$	(Bitaubé Pérez et al., 2008)
$\mu = \mu_{\mu max} (1 - pH - pH_{opt} \times pH_{dep})$ pH_{opt} is where the growth rate is maximum. pH_{dep} is the model constant.	T4-7	Outdoor	$pH_{opt} = 8.5$ $pH_{dep} = 0.3$	(Rarrek et al., 2016)

Table 4: Continued

Kinetic Model	Eq#	System	Parameters	Reference
$P = P_{\mu max} \left[C_1 e^{-\frac{D_1}{pH}} - C_2 e^{-\frac{D_2}{pH}} \right]$	T4-8	Indoor	<i>Scendesmus almeriensi</i>	(Darvehei et al., 2018)
			$C_1 = 2.5$	
			$C_2 = 533$	
			$D_1 = 6.45$	
			$D_2 = 69.2$	
		Outdoor	<i>Scendesmus almeriensis</i>	(Fernández et al., 2016)
			$C_1 = 2.4098; C_2 = 533.009$	
			$D_1 = 6.2684; D_2 = 68.8062$	
$\mu = \mu_{\mu max} \frac{1}{k_3 + \frac{[H^+]}{k_1} + \frac{k_2}{[H^+]}}$	T4-9	Indoor	<i>Chlamydomonas reinhardtii</i>	(Darvehei et al., 2018)
			For nitrate	
			$\mu_{\mu max} = 1.452 h^{-1}$	
			$k_1 = 0.401 \times 10^7 moleL^{-1}$	
			$k_2 = 0.865 \times 10^7 moleL^{-1}$	
			$k_3 = 0.7602$	

2.1.5. Modelling the impact of Salinity on Algae Growth

Fresh water is limited at many places around the globe. Therefore, seawater is an available choice to produce the algae biomass for animal consumption. Salinity not only affects the microalgae growth but also influences the biochemical contents such as lipids and fatty acids (Pugkaew et al., 2019). Using fresh water with nutrients as a culture media for microalgae increases the production of microalgae, but it is a costly option. Therefore, using saline water instead of fresh water is the alternative solution. The productivity of microalgae increases with a rise in light intensity, but the evaporation rate increases when light intensity increases, especially on the open raceway pond, which increases the salinity. Therefore, the production rate of microalgae will be affected (Darvehei et al., 2018). Ishika et al (Ishika, Moheimani, & Bahri, 2017) illustrated the increase in salinity based on evaporation and rainfall rate of three potential places for microalgae cultivation in Western Australia (Geraldton, Perth, and Karratha) (Ishika, Moheimani, & Bahri, 2017)(Figure 4). Five different microalgae (*Chlorella vulgaris*, *Choricystis minor*, *Neochloris* sp., *Pseudococ comyxa* simplex, and *Scenedesmus* sp) were tested with three different saline media: freshwater, seawater, and a mixture of the same volume ratio of freshwater and seawater medium. The salinity of seawater is 40 g/l of sea salt in deionized water. All the microalgae grew on both media, freshwater, and seawater, except *Choricystis minor*, which did not respond to grow in seawater medium. In this work, the nitrate and phosphate consumption rate was measured for *Chlorella vulgaris* in the three mentioned media above. The rate of nitrate consumption was increased with the increase in salinity of culture media. Whereas, the lowest and the highest consumption rates of phosphate were in a mixture of freshwater and seawater (1:1 by volume) and freshwater culture medium, respectively (Luangpipat & Chisti, 2017). The optimal salinity for fucoxanthin production from saline microalgae (*Chrysotila carterae*, *Chaetoceros muelleri*, *Pheodactylum tricornutum*, *Tisochrysis lutea* (T.Iso), *Navicula* sp., and *Amphora* sp) are 35, 45, 45, 45, 75, and 85 % respectively (Ishika, Moheimani, Bahri, et al., 2017). Mohseni et al (Mohseni et al., 2021) studied the rate of organic matter removal from three different saline water culture media (5, 10, and 15 g/l) by using three strains of microalgae for each mentioned medium. (*S. abundans*, *C. vulgaris*, and *N. salina*) . The

highest rate of removal of organic matter was for *S. abundans* in a 5 g/l concentration of saline medium. (Mohseni et al., 2021). Microalgae can be affected by saline culture in three causes : (1) changes in ratio of internal cellular ions, (2) gain or loss of ions that affects ions stress, and (3) resulting osmotic stress. The level of salinity can put microalgae cells under stress, which leads to a decrease in the growth rate of microalgae cells and the accumulation of lipids. No mathematical model has been proposed to include the salinity effect parameter in the growth rate of microalgae (Darvehei et al., 2018).

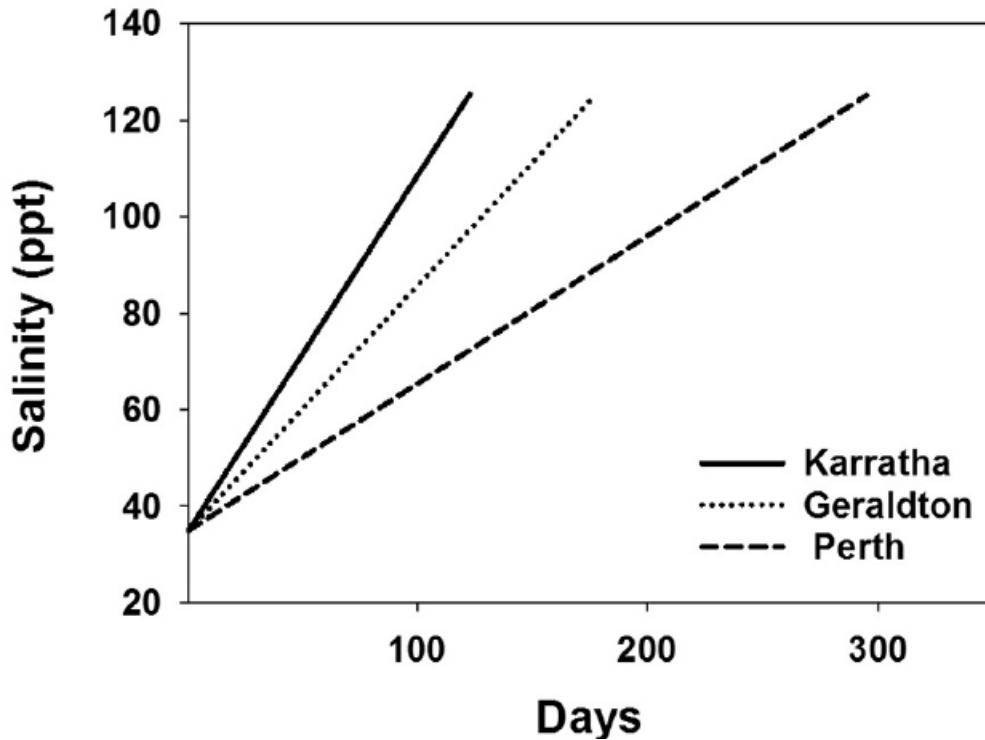


Figure 5: The trend of increase in salinity in three different regions for potential microalgae cultivation in Australia (Ishika, Moheimani, Bahri, et al., 2017)

2.1. Modelling microalgae growth based on multiple substrates

Since the growth rate of microalgae is determined by many factors: pH, nutrients, light intensity, temperature of the medium, and salinity. The mathematical models that simultaneously include multiple factors called co-limitation. The kinetic growth model that provides co-limitation factors (N concentration, P concentration, light intensity, temperature, and CO₂ concentration, for

instance) predicts more realistic results of microalgae growth rate (Ye et al., 2023). Therefore, the cultivation system can be scaled up by optimization factors for commercialization and industrial purposes (M. Bekirogullari et al., 2018). The advantages of single substrate models are that they are easy to build and have fewer model parameters. The predictability of single substrate models is good, but they are limited to cultivation systems on a laboratory scale. In contrast, the co-limitation factors models have more parameters, which require a more complicated method for fitting experimental data and or optimization techniques to calibrate a model depending on experimental results (Mesut Bekirogullari et al., 2020). The next sections will explain the three main frameworks of mathematical modeling of microalgae using multiple substrate kinetics growth based on the interactions of environmental factors (temperature and light intensity), nutrients, and substrate.

2.2.1. Mathematical growth Models based on non-interactive effect of substrate

The transitions between nitrogen (N) and phosphorus (P) levels occur seasonally in seawaters and eutrophic lakes where the phytoplankton grow. For instance, in seawater, the amount of phosphorus is sufficient whereas the nitrogen quantity is limited. The mixing process between seawater and freshwater supplies low phosphorus levels relative to nitrogen levels (Society, 1978). When limited in the culture medium, the two macronutrients, nitrogen and phosphorus, control the growth rate of phytoplankton (Klausmeier et al., 2004). The results from Droop did showed that the growth rate of *Monochrysis lutheri* under two limiting nutrients (phosphorous and vitamin B₁₂) was not following multiple substrate models, but a single nutrient with lower concentration governed the growth rate (Society, 1978). Therefore, Liebig's law was proposed to explain the growth rate that depends on the most limiting nutrient (Klausmeier et al., 2004). The general formula of the most limiting nutrient model is presented as follows:

$$\mu = \mu_{max} \min \cdot [f(x_1), f(x_2), f(x_3) \cdots f(x_i)] \quad (2-36)$$

Where μ_{max} is the maximum growth rate at the most limited factor, and $f(x_i)$ is the model related to the most limited substrate or nutrients such as N, P, CO₂, temperature, and light (Lee et al., 2015).

Bougaran et al. (2010) illustrated a non-interactive model of kinetic growth based on a quota of either nitrogen or phosphorus.

$$\mu = \mu_{max} \min \cdot \left[\frac{1 - \frac{q_{N,0}}{q_N}}{1 - \frac{q_{N,0}}{q_{N,max}}}, \frac{1 - \frac{q_{P,0}}{q_P}}{1 - \frac{q_{P,0}}{q_{P,max}}} \right] \quad (2-37)$$

Where $q_{N,0}$ and $q_{P,0}$ are the minimum quota for nitrogen (moleN.moleC⁻¹) and phosphorus (moleP.moleC⁻¹) respectively for growth, q_N and q_P are the quota for nitrogen and phosphorus, respectively, at any time, and $q_{N,max}$ and $q_{P,max}$ are the quota for nitrogen and phosphorus for maximum growth rate. Equation 2-37 will become the function of either nitrogen or phosphorus quota (equations 2-38 and 2-39) when the limited factor is either nitrogen or phosphorus.

$$\mu = \mu_{max} \left(\frac{1 - \frac{q_{N,0}}{q_N}}{1 - \frac{q_{N,0}}{q_{N,max}}} \right) \quad (2-38)$$

$$\mu = \mu_{max} \left(\frac{1 - \frac{q_{P,0}}{q_P}}{1 - \frac{q_{P,0}}{q_{P,max}}} \right) \quad (2-39)$$

A non-interactive model's prediction accuracy may be good for a biological system with small interactions between cellular subsystems. In contrast, for a more complex cellular system, the non-interactive model is restrictive since a microorganism's physiological status is simultaneously affected by each of all nutrient sources (Mesut Bekirogullari et al., 2020).

2.2.2. Mathematical growth models based on the Interactive effect of substrates

In the interactive model, all the growth factors contribute to the microbial growth rate simultaneously. Thus, if there are two limiting factors, both will control the kinetic growth, and the co-limitation influences of these two factors will be multiplicative. The general formula of the Interactive or multiplicative model is mentioned in the next equation.

$$\mu = \mu_{max} f(x_1) \cdot f(x_2) \cdot f(x_3) \cdots f(x_i) \quad (2-40)$$

where μ_{max} is the maximum growth rate affected by all contributing limiting factors such as temperature, N, P, light intensity, and CO₂. $f(x_i)$ is the relation of contributing limiting factors (Lee et al., 2015). Wu et al (Wu et al., 2013). used the Interactive model with influence of three factors (light, nitrogen, and phosphorus) to model the growth rate of *Scenedesmus sp. LX1* in open pond (Wu et al., 2013).

$$\mu = \mu_{max} \frac{S_N}{K_N + S_N} \left(1 - \frac{q_o}{q_p}\right) \frac{I}{I_{opt}} e^{\left(1 - \frac{I}{I_{opt}}\right)} \quad (2-41)$$

where S_N is the concentration of nitrogen (mg/L), K_N is the half-saturation constant for nitrogen (mg/L), q_o is the minimal phosphorus quota required for growth (%), phosphorus quota at a specific time t (%), I is light intensity (Ix), and I_{opt} is the light intensity where the growth rate is maximum.

The Model that was applied by Beran and Kargi (Beran & Kargi, 2005) includes both non-interactive and interactive formulas for estimating the kinetic growth of algae in wastewater ponds.

$$\mu = \mu_{algae} \text{Min}(f_1(N), f_2(I)) \cdot f_3(pH) \cdot f_4(T) \quad (2-42)$$

where μ_{algae} is algae's maximum growth rate (d⁻¹), the non-interactive part is $\text{Min}(f_1(N), f_2(I))$, which includes the most limiting factor (either N or I). The second term of equation 2-42 contains an interactive part that presents the contributions of influence of all factors which are $f_3(pH)$ and $f_4(T)$.

2.2.3. Mathematical growth model based on Additive effect of substrate

The sum of the specific growth rate of each limiting substrate individually is called the additive model (Mesut Bekirogullari et al., 2020). Adesanya et al (Adesanya et al., 2014). modeled the

growth rate of *Chlorella vulgaris* under mixotrophic condition culture (autotrophic and heterotrophic conditions) (Adesanya et al., 2014). The modeling formula that was used by Adesanya et al. is given in eq.2-43

$$\mu_m = \mu_A + \mu_H \quad (2-43)$$

Where μ_m , μ_A , and μ_H is the specific growth rate (d^{-1}) of mixotrophic, autotrophic, and heterotrophic conditions. The following two equations (equations 2-45 and 2-46) illustrate the formula of μ_A and μ_H .

$$\mu_A = \mu_{A,m} \left[1 - e^{\frac{-\alpha I \varphi}{p_m}} \right] \quad (2-44)$$

Where $\mu_{A,m}$ is the maximum specific growth rate under autotrophic conditions, p_m represents the regime of light-saturated where light intensity does not affect the photosynthesis, I is the light intensity average in the culture medium (mole photons/ m^2), α is chlorophyll a optical cross section (m^2/g chl), and φ is quantum efficiency (g C/ mole photons). The average light intensity in a Z depth medium is $I = \frac{I_o}{\alpha B X Z} (1 - e^{(-\alpha B X Z)})$ and the following mathematical relation describes the growth rate under heterotrophic conditions (μ_H).

$$\mu_H = \frac{q_{cm} - Q_c(t)}{q_{cm} - q_c} \left[\frac{v_{cm} C(t)}{C(t) + v_{ch}} \right] \quad (2-45)$$

Where q_{cm} is the maximum cell quota of carbon (g C/ (g dw)), q_c is carbon subsistence quota (g C/g dw), $Q_c(t)$ is cell quota of carbon at time t (g C/ (g dw)), v_{cm} is maximum carbon uptake rate (g C/ (g dw d)), $C(t)$ is organic carbon concentration in a culture medium at time t (g C/ m^3) and v_{ch} is half saturation constant (g C/ m^3).

Table 5: Kinetic models of microalgae growth depend on the multiple limitations of substrates.

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Non-interactive Model	$\mu = \mu_{max} \min \cdot \left[\frac{1 - \frac{q_{N,0}}{q_N}}{1 - \frac{q_{N,0}}{q_{N,max}}}, \frac{1 - \frac{q_{P,0}}{q_P}}{1 - \frac{q_{P,0}}{q_{P,max}}} \right]$	T5-1	Indoor.	<i>Selenastrum minutum</i> $\mu_{max} = 1.68d^{-1}$ $q_{N,0} = 0.06moleNmoelC^{-1}$ $q_{N,max} = 0.21moleNmoelC^{-1}$ $q_{N,0} = 0.0018molePmoelC^{-1}$ $q_{N,max} = 0.015molePmoelC^{-1}$ <i>Isochrysis affinis galbana</i> $\mu_{max} = 1.5d^{-1}$ $q_{N,0} = 0.065moleNmoelC^{-1}$ $q_{N,max} = 0.14moleNmoelC^{-1}$ $q_{N,0} = 0.0009molePmoelC^{-1}$ $q_{N,max} = 0.006molePmoelC^{-1}$	(Bougaran et al., 2010)
	$\mu = \mu_{max} \min \cdot \left[1 - \frac{Q_{N,min}}{Q_N}, 1 - \frac{Q_{P,min}}{Q_P} \right]$	T5-2	Indoor	Phytoplankton $\mu_{max} = 1.35d^{-1}$ $q_{N,min} = 45.4 (10^{-9}\mu mole \frac{N}{cell})$ $q_{P,min} = 1.64 (10^{-9}\mu mole \frac{P}{cell})$	(Klausmeier et al., 2004)

Table 5: Continued.

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Non-interactive Model	$\mu = \mu_{max} \min \cdot \left[1 - \frac{Q_{N,min}}{Q_N}, X P_m \left(1 - e^{\frac{-\alpha I \varphi}{P_m}} \right) \right]$ $I = \frac{I_o}{\alpha X Z} (1 - e^{(-\alpha X Z)})$	T5-3	Indoor	<i>Pseudochlorococcum sp</i> $\mu_{max} = 3.26 d^{-1}$ $Q_{N,min} = 0.0278 gN(gdw)^{-1}$ $\alpha = 4.82 m^2(g chl)^{-1}$ $P_m = 90.1 gC(g chl d)^{-1}$ $\varphi =$ $0.0984 molC(mole photon)^{-1}$ $Z = 0.03 m$	(Packer et al., 2011)
Multiplicative models	$\mu = \mu_{max} \left(\frac{C_s}{K_s + C_s + \frac{C_s^2}{K_{Xi}}} \right) \left(\frac{I}{K_{XI} + I} \right) \left(1 - \frac{C_x}{C_{Xm}} \right) \left(1 - \frac{C_P}{C_{Pm}} \right)$	T5-4	Indoor	<i>Haematococcus pluvialis</i> $\mu_{max} = 0.5258 d^{-1}$ $C_s = 0.025 g/L$ $K_{Xi} = 53.26 g/L$ $K_s = 0.0211 g/L$ $K_{XI} = 53.26 g/L$ $C_{Xm} = 2.9161 g/L$ $C_{Pm} = 55.6192 g/L$	(Zhao & Su, 2020).

Table 5: Continued.

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Multiplicative models	$\mu = \mu_{max} \left(\frac{S}{K_S + S + \frac{S^2}{K_{Si}}} \right) \left(\frac{N}{K_N + N + \frac{N^2}{K_{Ni}}} \right) \left(\frac{I}{K_I + I + \frac{I^2}{K_{Ii}}} \right) f(T)$ $f(T) = A e^{\left[\frac{Ea}{R} \right] \left(\frac{1}{T} - \frac{1}{T_o} \right)} - B A e^{\left[\frac{Eb}{R} \right] \left(\frac{1}{T} - \frac{1}{T_o} \right)}$	T5-5	Indoor	<i>Chlamydomonas reinhardtii</i> $\mu_{max} = 0.275 d^{-1}$ $K_S = 0.0624 g/L$ $K_{Si} = 12.40 g/L$ $K_N = 0.0812 g/L$ $K_{Ni} = 0.6250 g/L$ $K_I = 45.50 \mu Em^{-2} s^{-1}$ $K_{Ii} = 297.75 \mu Em^{-2} s^{-1}$ $A = 0.6175$ $B = 0.1101$ $Ea = 25.9243 \text{ kcal/mole}$ $Eb = 48.0151 \text{ kcal/mole}$	(M. Bekirogullari et al., 2018).
	$\mu = \mu_{max} \left(\frac{S_N}{K_N + S_N} \right) \left(1 - \frac{Q_{min}}{q_p} \right) \left(\frac{I}{I_{opt}} e^{\left(1 - \frac{I}{I_{opt}} \right)} \right)$	T5-6	Indoor	<i>Scenedesmus sp. LX1</i> $\mu_{max} = 0.79 d^{-1}$ $K_N = 9.5 \text{ mg/L}$ $Q_{min} = 0.019 \%$ $I_{opt} = 5277 \text{ IX}$	(Wu et al., 2013)

Table 5: Continued.

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Multiplicative models	$\mu = \mu_{max} \left(\frac{S_{glucose}}{K_{glucose} + S_{glucose}} \right) \cdot \left(\frac{S_{Nitrate}}{K_{Nitrate} + S_{Nitrate}} \right)$	T5-7	Indoor	<i>Chlorella vulgaris</i>	(Ye et al., 2023).
	$\left(\frac{S_{CO_2}}{K_{CO_2} + S_{CO_2}} \right) \cdot \left(\frac{S_I}{K_I + S_I} \right)$			$\mu_{max} = 1.87 h^{-1}$	
				$K_{glucose} = 0.797 g/L$	
				$K_{Nitrate} = 0.042 g/L$	
				$K_{CO_2} = 0.0316 g/L$	
				$K_I = 31.29 \mu mole\ m^{-2} s^{-1}$	
	$\mu = \mu_{max} \left(\frac{S_N}{K_N + S_N} \right) \left(\frac{S_{CO_2}}{K_{CO_2} + S_{CO_2}} \right) (\theta^{T-20}) \left(\tanh \left(\frac{I_{ave}}{I_k} \right) \right)$	T5-8	Indoor	<i>Chlorella sp</i>	(Lee & Zhang, 2016)
				$\mu_{max} = 0.7\ d^{-1}$	
				$K_N = 0.1 mg/L$	
	$I = \frac{I_o}{\alpha X d} (1 - e^{(-\alpha X d)})$	T5-9		$K_{CO_2} = 3.6 mg/L$	
				$\theta = 1.35$	
				$I_k =$	
				$16.98 \mu mole\ photon\ m^{-2} s^{-1}$	
				$\alpha = 0.2 m^2 g^{-1}$	

Table 5: Continued.

Classification	Kinetic Model	Eq#	System	Parameters	Reference
Additive models	$\mu = \mu_A \left[1 - e^{\frac{-\alpha I \varphi}{p_m}} \right] + \frac{q_{cm} - Q_c(t)}{q_{cm} - q_c} \left[\frac{v_{cm} C(t)}{C(t) + v_{ch}} \right]$ $I = \frac{I_o}{\alpha X d} (1 - e^{(-\alpha X d)})$	T5-10	Indoor	<i>C.vulgaris</i> $\alpha = 25.4804 \text{ m}^2 \text{ gchl}^{-1}$ $\varphi =$ $0.4345 \text{ g C (mole photon)}^{-1}$ $P_m = 93.94 \text{ g C (g dw d)}^{-1}$ $q_{cm} = 0.4571 \text{ g C (g dw)}^{-1}$ $q_c = 0.4143 \text{ g C (g dw)}^{-1}$ $v_{cm} = 9.789 \text{ g C (g dw d)}^{-1}$ $v_{ch} = 3.15 \text{ g C m}^{-3}$ $d = 0.03 \text{ m}$ $I_o = 6 \text{ mole photons m}^{-2} \text{ d}^{-1}$	(Adesanya et al., 2014)
	$\mu = \mu_{max} \cdot \left[W_H \cdot \frac{A}{K_{S,A} + A + \frac{A^2}{K_{i,A}}} + W_I \cdot \frac{I}{K_{S,I} + I + \frac{I^2}{K_{i,I}}} \right]$ $W_H = \frac{A/K_{S,A}}{A/K_{S,A} + I/K_{S,I}}; W_I = \frac{I/K_{S,I}}{A/K_{S,A} + I/K_{S,I}}$	T5-11	Indoor	<i>C.reinhardtii</i> $\mu_{max} = 0.106 \text{ h}^{-1}$ $K_{S,A} = 1.789 \text{ g C/L}$ $K_{i,A} = 0.109 \text{ g C/L}$ $K_{S,I} = 1.4 \text{ } \mu\text{mole m}^{-2} \text{ s}^{-1}$ $K_{i,I} = 186.52 \text{ } \mu\text{mole m}^{-2} \text{ s}^{-1}$	(Figueroa-Torres et al., 2017).

The extensive literature review showed that the best mathematical growth model integrates the dynamic effect of variables that influence algae growth support. Adoption of such models with good predictability can not only estimate the actual potential of a location for growth and productivity of a specific strain of algae but can also help estimate the resource requirement. However, the success of these models depends on a few inherent characteristics of algae strain, such as saturation light intensity and optimal growth temperature. Besides algae characterization, weather conditions such as incident light intensity, humidity, air temperature and velocity, and culture composition are important factors.

This thesis attempts to explore the potential of algae growth in prevailing weather conditions in Saudi Arabia for microalgae *Arthrospira maxima*, a potential strain of algae for the production of aqua feed in an open raceway pond. Various factors, such as pond temperature, growth rate, the effect of harvesting frequency, and culture concentration before harvesting on biomass productivity, are evaluated.

CHAPTER 3: Methodology

The mathematical model used in this study includes two main parts: the thermal and biological parts. The purpose of implementing the thermal part of the model is to estimate the ORP temperature. In this part of the model, weather conditions are used to calculate the time-resolved temperature. By connecting the results of the predicted temperature in the thermal part of the model with local light intensity, the biological part of the model can estimate the biomass concentration of microalgae in a bioreactor (ORP). The following subsections describe details of thermal and biological model calculations.

3.1. Energy balance (Thermal model for ORP)

In this part of the thesis of the materials and methods section, all kinds of heat gain or loss from the system (ORP), assumptions, and parameters used in heat transfer are presented. The central governing equation for thermal balance for ORP used in this work is

$$\rho c_p V \frac{dT_R}{dt} = \sum Q_n \quad (3-1)$$

where ρ is the density of the medium, which is assumed to be the density of water ($1000 \frac{kg}{m^3}$).

V is ORP volume. (m^3). C_p is the medium's heat capacity, which does not change with time ($4184 \frac{J}{kg.K}$). $\frac{dT_R}{dt}$ is the temperature variation for the ORP with time (K/s). $\sum Q_n$ is the summation of all kinds of heat lost or gained by the ORP system (w) (Endres et al., 2016).

3.1.1. Exchange of heat between ORP system and surrounding

Heat can be exchange between the ORP system and its surroundings though seven different ways. In the following parts of this chapter, all these ways are explained in detail with their governing equations.

a. Heat transfers from directly and diffuses radiation

The surface area of ORP is parallel to the earth's surface, so direct and diffuse irradiance can be combined into global horizontal irradiance (GHI). The equation that is used to do the calculation for heat gain by ORP from the direct irradiance is

$$Q_s = (1 - f_a) GHI \alpha_{cult} A_{surface} \quad (3-2)$$

where f_a is the fraction of the sunlight is that photosynthesis of algae converts to chemical energy ($f_a = 0.025$). GHI is the global horizontal irradiance ($\frac{W}{m^2}$). α_{cult} is the absorptivity for ORP culture($\alpha_{cult} = 0.82$). $A_{surface}$ is the surface area of ORP (m^2).

b. Heat transfer from the air radiation

The relation that covers the heat exchanging regarding radiation of the air is

$$Q_{air} = \varepsilon_w \varepsilon_a \sigma T_{surr}^4 A_{surface} \quad (3-3)$$

where

$\varepsilon_w \varepsilon_a$ is the emissivity of the reactor culture and the air, respectively ($\varepsilon_w = 0.97$, $\varepsilon_a = 0.85$). σ is the Stephan- Boltzmann constant ($\sigma = 5.76 \times 10^{-8} \frac{W}{m^2 K^4}$). T_{surr} is the temperature of the surroundings (K).

c. Heat flux by ground conduction

The heat transfer between the bottom of ORP and the ground is by using this equation.

$$Q_{ground} = - K_{con} \frac{T_P - 290}{L_{ref}} A_s \quad (3-4)$$

where

K_{con} is concrete thermal conductivity ($K_{con} = 1.4 \frac{W}{m.K}$). L_{ref} is the reference depth of the conductivity $L_{ref} = 3.66 \text{ m}$.

d. Heat transfers due to radiation of pond

By implementing the Stefan-Boltzmann relation, the loss of heat from the pond is due to radiation that comes from the pond.

$$Q_{rad} = \varepsilon_w \sigma T_{pond}^4 A_{surface} \quad (3-5)$$

where T_{pond} is the temperature of ORP.

e. Heat flux due to convection of heat at the pond's surface

The general equation that governs the heat transfer between the surrounding air and the top surface of ORP is

$$Q_{con} = h_{con} (T_{amb} - T_{pond}) A_{surface} \quad (3-6)$$

where $T_{amb} = T_{air}$ h_{con} is the coefficient for convection heat flux ($\frac{W}{K.m^2}$).

The steps for calculating the coefficient of heat transfer convection are presented below.

1. Calculation of the Reynolds number by using the following equation

$$Re = \frac{LV}{\vartheta_a} \quad (3-7)$$

Where L is equal to the ratio of the ORP area over its perimeter (m).
V is the velocity of the wind (m/s).

ϑ_a is the kinematic viscosity

$$(\vartheta_a = 0.00001679 \frac{m^2}{s})$$

2. Calculation of Prandtl number by using the presented equation

$$Pr = \frac{\nu_a}{\alpha_a} \quad (3-8)$$

Where α_a is the air thermal diffusivity ($\alpha_a = 0.000002 \frac{m^2}{s}$).

3. Calculation of the Nusselt number based on the value of Reynold

$$Nu = 0.628 (Re)^{0.5} (Pr)^{1/3} \quad \text{for } Re < 300,000 \quad (3-9)$$

$$Nu = 0.035 (Re)^{0.8} (Pr)^{1/3} \quad \text{for } Re > 500,000 \quad (3-10)$$

Moreover, the Nu number's value is equal to the average of the above two values when the value of Re is between the two above values.

4. To calculate the value of h_{con} , use the following equation.

$$Nu = \frac{h_{con} L}{\gamma_a} \quad (3-11)$$

where γ_a is air thermal conductivity ($\gamma_a = 0.025 \frac{W}{m.K}$).

$$L = \frac{A_s}{2 \times Len + 2 \times w} \quad (3-12)$$

where Len and w are the length and width of ORP (m).

f. Heat loss from ORP due to evaporation

Some water in the ORP is evaporating to the atmosphere, so the latent heat of this amount of water is lost from the system, and it is calculated by the presented equation.

$$Q_{evaporation} = -m_e L_w A_{surface} \quad (3-13)$$

where m_e is the amount of water evaporation

L_w is the latent heat of evaporation $L_w = 2260000 \frac{J}{kg}$.

There are steps to calculate the value of $m_e (\frac{kg}{m^2.s})$.

The first step is calculating the K (m/s), the mass transfer coefficient.

1. Calculation of the Reynolds number by using the following equation

$$Re = \frac{LV}{\nu_a} \quad (3-14)$$

2. Calculation of Schmidt number (Sch) by using the presented equation

$$Sch = \frac{\nu_a}{D_{w,a}} \quad (3-15)$$

Where $D_{w,a}$ it is the water mass diffusion coefficient on the air ($D_{w,a} = 0.000025 \frac{m^2}{s}$).

3. Calculation of the Sherwood number based on the value of Reynold

$$Sh = 0.628 (Re)^{0.5} (Sch)^{1/3} \quad \text{for } Re < 300,000 \quad (3-16)$$

$$Sh = 0.035 (Re)^{0.8} (Sch)^{1/3} \quad \text{for } Re > 500,000 \quad (3-17)$$

and the Nu number's value is equal to the average of the above two values when the value of Re is between the two above values of Re.

4. To calculate the value of K is, simply by using the following equation.

$$Sh = \frac{KL}{D_{w,a}} \quad (3-18)$$

The second step is to find m_e that is equal to

$$m_e = K \left(\frac{P_w}{T_{pond}} - \frac{RHP_a}{T_{surr}} \right) \frac{M_w}{R} \quad (3-19)$$

Where RH, M_w , R , P_w , P_a are the relative humidity (%), molecular weight (kg/mole), constant for an ideal gas ($\text{Pa m}^3/\text{K.mole}$) and vapor pressure (Pa) at T_{pond} ($^{\circ}\text{C}$) and T_{surr} ($^{\circ}\text{C}$) respectively.

The equation is used to find the values of P_w and P_a .

$$P_i = 3385.5 e^{(-8.0929 + 0.97608(T_i + 42.607 + 273)^{0.5})}. \quad (3-20)$$

g. Flux of heat for ORP by the water makeup

The level of water in the ORP remains constant, so the water that is pumping to the ORP substitutes the amount of water that is missing due to evaporation. The following equation is to find the heat exchanging regarding water flow to the ORP.

$$Q_{water\ inflow} = C_{p,water} (T_{surr} - T_{pond}) \quad (3-21)$$

$C_{p,water}$ is the heat capacity of water added to ORP ($C_{p,water} = 4184 \frac{J}{kg.K}$).

3.2. Biological model for algae growth in ORP

The second part of the model contains governing equation for the biomass growth rate for algae in ORP. The following equation contains two main parts that determine the time-resolved growth rates depending on the light intensity and temperature of the cultivation culture.

$$\frac{dc_x}{dt} = \frac{\varphi_L(t)\varphi_T(t)PAR(t)\phi_{photon} A_{surface}}{V} + \frac{D(t)}{V} \quad (3-22)$$

where c_x is the algae concentration in the reactor ($\frac{g}{m^3}$).

$\varphi_L(t)$ is an efficient parameter for culture light, so

$$\varphi_L(t) = \frac{I_a}{I_s} e^{\left(1 - \frac{I_a}{I_s}\right)} \quad (3-23)$$

where I_a and I_s are mean light intensity and saturated light intensity, respectively ($\frac{W}{m^2}$).

The average intensity can be calculated by calling this relation

$$I_a = GHI \cdot 0.458 \cdot \frac{1 - e^{-OD_{750}Z}}{OD_{750}Z} \quad (3-24)$$

Z is the ORP depth in the meter unit.

$\varphi_T(t)$ is an efficiency parameter for the culture temperature,

$$\varphi_T(t) = \frac{(T_{pond} - T_{max})(T_{pond} - T_{min})}{(T_{optimal} - T_{min})[g(T_{pond}) - f(T_{pond})]} \quad (3-25)$$

where T_{max} and T_{min} are the maximum and minimum temperatures of the specific strain that can be loaded. T_{pond} is the optimal temperature for algae growth.

$$g(T_{pond}) = (T_{optimal} - T_{max})(T_{pond} - T_{optimal}) \quad (3-26)$$

$$f(T_{pond}) = (T_{optimal} - T_{max})(T_{optimal} + T_{min} - 2T_{pond}) \quad (3-27)$$

$PAR(t)$ is the rate of photosynthetic spectrum photon incidence

$$PAR(t) \text{ is equal to } 0.458 \times 4.57 \times 10^{-6} \times \text{GHI} \quad (3-28)$$

Φ_{photon} is the number of photons that are required to produce the algae biomass, and

$$\Phi_{photon} = \frac{8 \text{ g biomass}}{12 \text{ moles of photons}} \quad (3-29)$$

$D(t)$ is the algae biomass lost without light.

$$D(t) = C_x \frac{\mu_{dark}}{3600} V \varphi_T(t) \quad \text{for GHI} < 5 \quad (3-30)$$

$$\text{And } D(t) = 0 \quad \text{for GHI} > 5 \quad (3-31)$$

$$\mu_{dark} = \frac{\ln(1-\tau)}{\Delta t} \quad (3-32)$$

μ_{dark} is the decay rate of biomass, τ biomass losing average via the dark period, and Δt is the time interval of the dark (hour).

3.3. Solution of first order differential equations for thermal and biological model

As discussed in the previous section, thermal energy (equation 3-1) was used to find the culture temperature. The thermal model results along with input of local light intensity were used to estimate the biomass growth rate of microalgae by using equations 3-22. Equations 3-1 and 3-22 were solved numerically by implementing the explicit Euler method in MS Excel software (Atkinson et al., 2011).

$$\frac{y_{i+1}-y_i}{h} = f(x_i, y_i) \quad (3-33)$$

3.4. Strain parameters and ORP geometry dimensions were used in this study

The following table includes all parameters for *Arthrospira maxima* and dimensions for ORP implemented in this study.

Table 6: The geometry dimensions for ORP and *Arthrospira maxima* parameters implemented in thermal and biological model.

Inputs	Values	Units	Parameter specification	References
Minimum Temperature	18	°C	Strain parameter	(Kumar et al., 2011)
Maximum Temperature	38	°C	Strain parameter	(Kumar et al., 2011)
Optimal Temperature	35	°C	Strain parameter	(Furmaniak et al., 2017)
Saturated Light Intensity	540	$\frac{\mu mol}{m^2 s}$	Strain parameter	(Li et al., 2022)
Dark Respiration Rate	4.5	%	Strain Parameter	(Greene et al., 2021a)
Initial Concentration	200	$\frac{g}{m^3}$	Model parameter	Experimental data
Initial ORP temperature	25	°C	Model parameter	Experimental data
Harvested Concentration	600	$\frac{g}{m^3}$	Model parameter	Experimental data
Length	25	m	Geometry parameter	Experimental data
Width	4	m	Geometry parameter	Experimental data
Depth	0.25	m	Geometry parameter	Experimental data
The time interval	60	Minutes	Model parameter	Experimental data
Optical Density	0.8	$\frac{g}{L m}$	Strain parameter	Experimental data

3.5. Weather conditions for thermal and biological model

The thermal model was run based on the parameters of ORP geometry, equations that govern the heat transfer between ORP and surroundings, and the weather conditions (air temperature, wind speed, related humidity, and global horizontal irradiance) that were collected from June 2022 till May 2023 in Jeddah. These weather conditions were taken each one hour during the above-mentioned period. The only weather condition used for the biological part of the model is GHI.

CHAPTER 4: Results and Discussion

4.1. Impact of weather conditions on the pond temperature

This part of the results and discussion chapter includes mainly the influence of four weather conditions (mentioned in the previous chapter) on the average predicted ORP temperature for each day throughout the whole year (from June 2022 to May 2023) (Figures 5, 6, 7, and 8). Additionally, this section of Chapter 4 contains the degree of impact of each weather condition on the average predicted ORP temperature.

4.1.1. Impact of Global Horizontal Irradiance (GHI) on the predicted pond temperature

ORP can gain heat from direct and diffuse radiation (GHI). This heat gained by ORP only influences the pond temperature during daytime, whereas the impact of sun radiation on the pond temperature disappears after sunset. Therefore, the presented readings of maximum, average, and minimum GHI in Figure 5 are only for daytime for each day of the running model. The highest readings of average GHI were distributed in the summer, fall, and spring seasons, but the highest predicted average pond temperatures were in the range between mid of Aug and Sep 2022. Analyzing the impacts of other weather conditions can give a clear explanation for the previous result. In the duration between Nov 2022 and Mar 2023, the graph trend of average predicted ORP temperature was decreasing due to a decrease in maximum and average GHI. The lowest measurements for maximum and average GHI were in Jan 2023, but the predicted pond temperature was not the lowest at this period. The minimum GHI in the last days of Aug 2022 was high, and the average predicted pond temperature was high. In contrast, the recordings of minimum GHI in Dec 2022, Mar, and Apr 2023 were high, and the predicted average pond temperature was low compared to other periods of the year. As a result, minimum GHI had little impact on the ORP temperature.

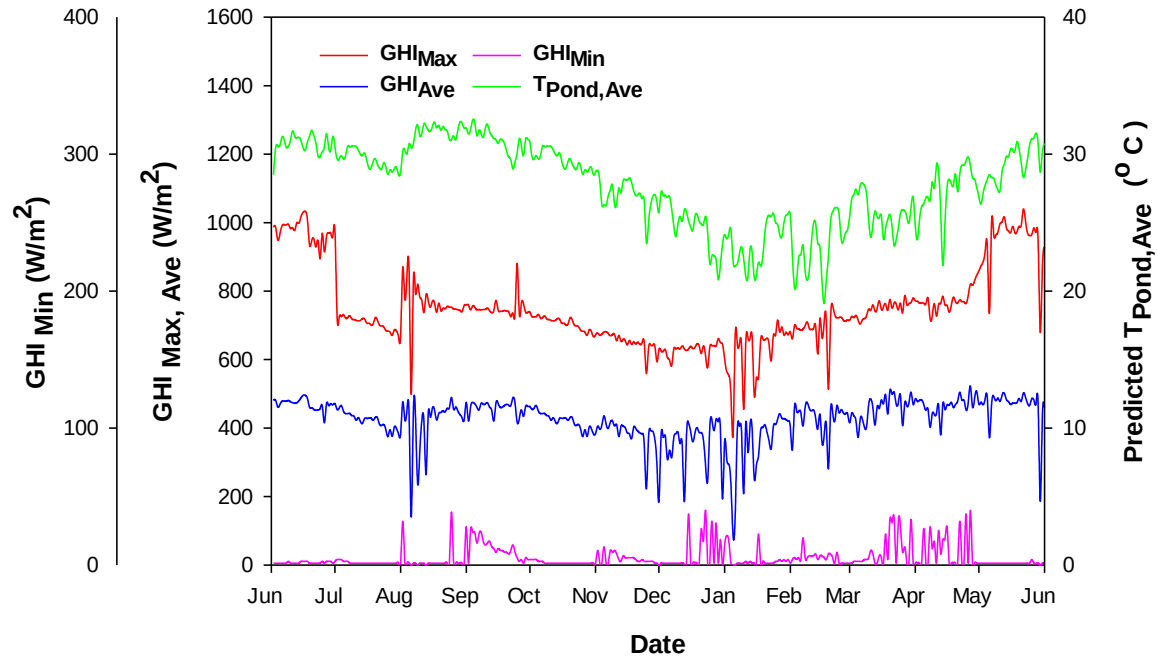


Figure 6: Impact of GHI on predicted pond temperature for an ORP situated in Jeddah

4.1.2. Impact of Relative Humidity (RH) on predicted pond temperature

Relative humidity (RH) influences pond temperature by one way of heat transfer. RH can influence the ORP temperature indirectly. From Equation 3-19, the evaporation rate of water in the pond decreases with an increase in RH, and water evaporation leads to heat loss from the pond, equation 3-13. Therefore, as RH increases, the heat loss of the pond decreases, so the pond temperature rises. The maximum RH was in mid-July and Oct 2022 (Figure 6). In the interval between Aug and Sep 2022, the average RH readings were in the middle compared to all readings of the whole year, but the average predicted pond temperatures were the highest. The explanation for this result is that other weather conditions increased the predicted pond temperature. The graph trend of maximum, average, and minimum RH were the same, so the predicted average ORP temperature had

the same influence by these three RHs. RH alternated throughout the year. The winter season had high measurements of RH, but the predicted pond temperature was lowest in this season. GHI measurements were lowest in the winter season, as presented in Figure 5. Another reason is low-temperature readings for ambient air.

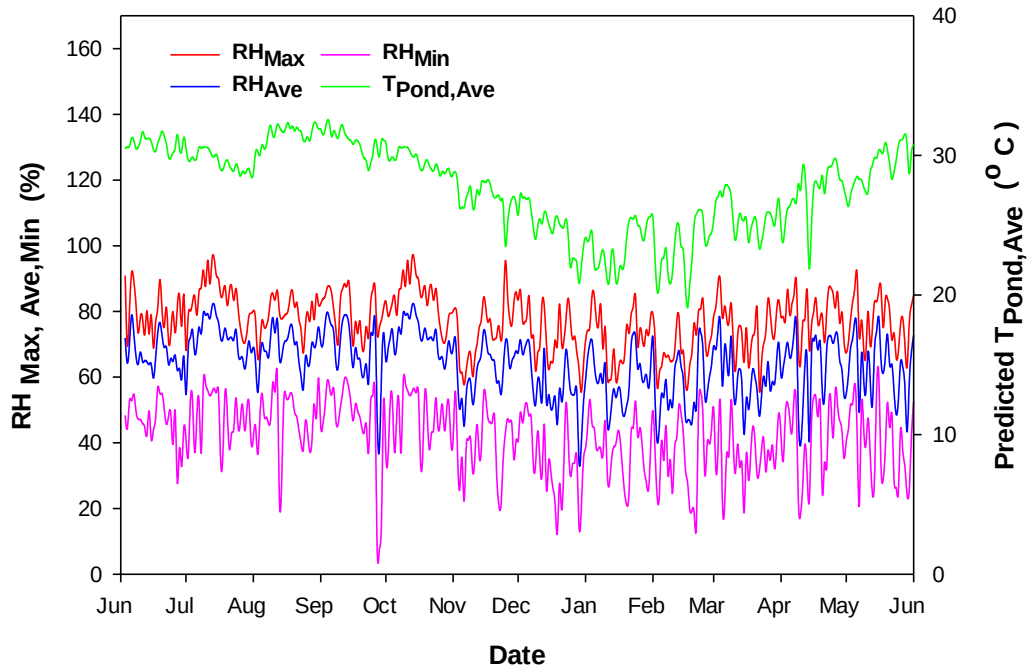


Figure 7: Impact of RH on predicted pond temperature for an ORP situated in Jeddah

4.1.3. Impact of Wind Speed on the Pond Temperature

Wind speed can influence the heat that ORP can gain or lose due to heat convection between the top surface of the pond and ambient air. Heat is lost from ORP due to water evaporation, and wind speed impacts the evaporation rate. The predicted pond temperature can be increased, decreased, or not changed by the influence of wind speed. When the heat is lost by evaporation due to wind speed is close to the heat gained by convection of heat,

the overall change in pond temperature is almost negligible. Pond temperature increases due to wind speed in case of less evaporation and heat gained by convection. ORP temperature decreases due to high wind speed since evaporation rate and heat loss by convection can be high. The fluctuation maximum wind speed was higher than the average and minimum wind speed. The graph trend for maximum, average, and minimum wind speed was the same throughout the year. The maximum measurement of average wind speed was in the last of Jul 2022. At this time, the average predicted pond temperature decreased to the lowest in Jul 2022. The other highest measurements of average wind speed were distributed in summer, winter, and spring seasons, but the average predicted pond temperatures did not always increase for these highest average wind speeds; they were alternating depending on the seasons. The main factors that determine lost or gained heat by ORP through heat convection are pond and ambient temperature, as mentioned in equations 3-6. Wind speed can determine the amount of heat transfer between ORP and its surroundings. Most periods of the whole year almost included small fluctuations in average wind speed, so the amount of evaporation rate or heat transfer between ORP and surrounding (controlling pond temperature) according to wind speed was almost stable for most periods of the running model.

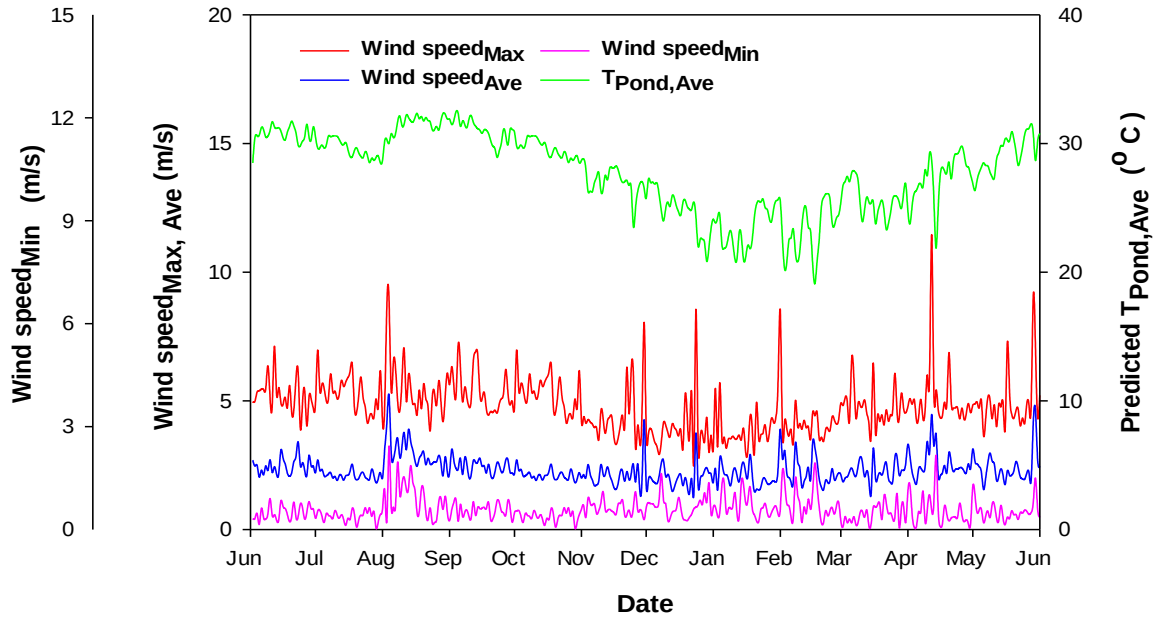


Figure 8: Impact of wind speed on pond temperature for an ORP situated in Jeddah

4.1.4. Impact of Air temperature on the pond temperature

Heat transfer methods by air radiation, surface pond convection, evaporation, and water makeup are controlled by air temperature (from the materials and methods chapter). So, the influence of air temperature in ORP pond temperature is more significant than other weather conditions (wind speed, relative humidity, and global horizontal irradiance), and the reason for this result is all weather conditions except air temperature govern heat transfer to or from the pond by one kind only (as discussed in previous sections). The graph trend was almost the same for average pond temperature and maximum, average, and minimum air temperature. The explanation for this result is that air temperature governs more ways of transferring heat from or to ORP, as discussed previously. The average air temperature fluctuated in a small range between 29 and 31 °C from June to Aug 2022. During this period, the average pond temperature was in the range between 27 and 31 °C.

In the next three months (Aug, Set, and Oct 2022), average air temperature oscillated between 30 and 34 °C. At this interval, not only the average air temperature but also the average pond temperature was one of the highest for a whole year of cultivation. The average air temperature started to decrease from Nov 2022 till the end of Dec 2022, and it was the same for the average pond temperature. On Jan, Feb, and Mar 2023, the average air and pond temperature were stable and oscillated in the range between 23 and 26 °C and 19 and 27 °C, respectively. From the beginning of Apr till the end of May, both temperatures of average air and ORP increased in the form of oscillation.

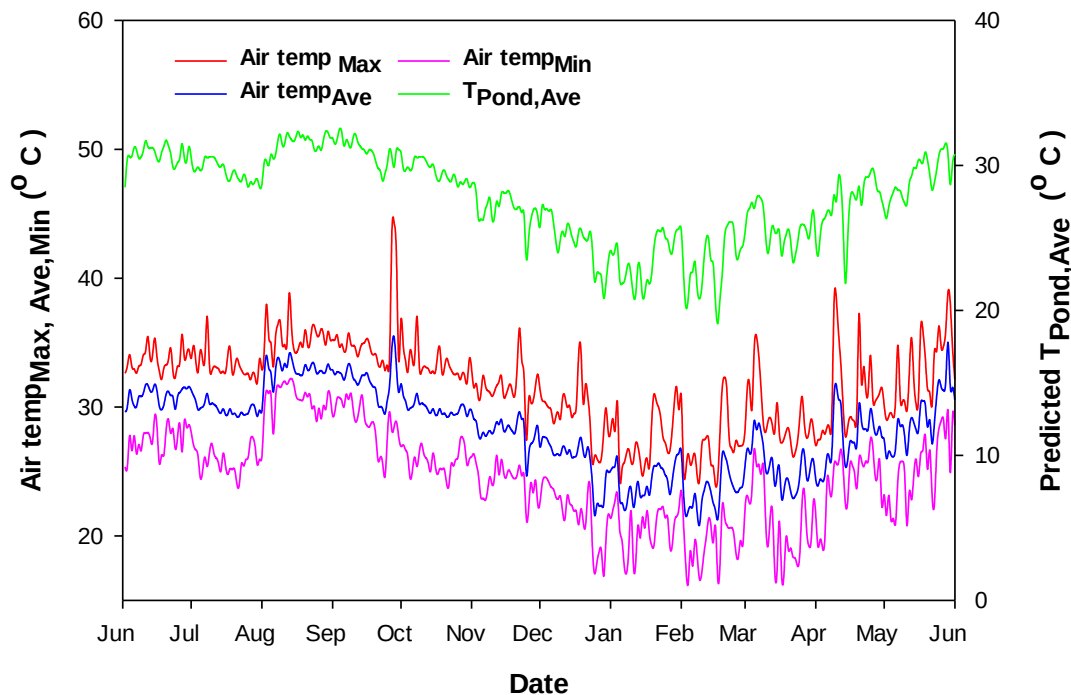


Figure 9: Impact of air temperature on pond temperature for an ORP situated in Jeddah

4.1.5. The degree of impact of each weather condition on the pond temperature

By comparing the impact of GHI, RH, and wind speed with the impact of air temperature on pond temperature, the graph trend for air temperature was closer to the average pond temperature graph trend. Therefore, the highest impact among the four weather conditions on pond temperature was air temperature. The reason for this result was discussed in detail in the previous section (see Figures 5, 6, 7, and 8 for more clarity). There was a similarity between the maximum and average GHI and average pond temperature graph trend, but the graph trend for average pond temperature and GHI was not close to the graph trend for air and pond temperature. From Figures 6 and 7, the distribution of RH and wind speed during the whole year period was equal. So, the impact of these two weather conditions (wind speed and RH) on pond temperature was the lowest. The highest predicted average pond temperature was in the Aug and Sep 2022 period. During this period, the average air temperature and GHI were high but not the highest. The explanation for the predicted average pond temperature was maximum during the Aug and Sep 2022 period; GHI and air temperature were stable at high readings. The RH and wind speed almost fluctuated with constant frequency for most of the periods of the year. So, their impact on pond temperatures was almost the same throughout most periods of the year. The lowest measurements for GHI were in the winter season (Dec 2022, Jan 2023), whereas, from the end of Dec 2022 till the end of Mar 2023, the lowest readings of air temperature were. The lowest predicted average pond temperatures were between the end of Dec 2022 and the end of Feb 2023.

4.2. impact of weather conditions (GHI, wind speed, RH, and air temperature) on predicted evaporation rate

Equation 3-19 showed that the evaporation rate increases as wind speed increases and RH decreases. From Figure.9, average RH oscillated with higher amplitude compared to average wind speed during most of the periods. Therefore, the influence of wind speed on the predicted evaporation rate for most of the year was more stable than that of RH. Average RH decreased sometimes to 38%, and it reached close to 80%. So, this big fluctuation of RH throughout the running periods impacted the fluctuation amplitude of the predicted evaporation rate. The periods that included high predicted evaporation rates had either high wind speed and low RH, low wind speed and RH, or high wind and intermediate RH. The lowest predicted evaporation rates were in Dec 2022 and Jan 2023, whereas the highest predicted evaporation rate was at the end of May 2023. For further investigation of the results of the predicted evaporation rate, the impact of average GHI and air temperature must be analyzed. Figure.10 includes the graph trend for the average predicted evaporation rate, air temperature, and GHI throughout the running period. GHI influences the predicted evaporation rate indirectly. The ORP temperature increases by increasing GHI; therefore, the evaporation rate increases. By comparing the graph trends of predicted evaporation rate and average GHI, the periods where the predicted evaporation rates were highest and the average readings of GHI were the lowest. The results were supposed to be reversed. The reason for this result was the impact of GHI on the predicted evaporation rate was small compared with other weather conditions. Air temperature can influence the average predicted evaporation rate by influencing ORP temperature. Heat gained or lost by ORP is affected by air temperature, as mentioned in the materials and methods chapter, so air temperature can increase, decrease, or does not change ORP temperature; the last case happened when heat gained by one method is equivalent to heat lost by another method. By comparing the maximum peaks in the average predicted evaporation rate and air temperature graphs (Figure 10), the predicted evaporation rate increased as the average air temperature increased. From Figure 10, the predicted evaporation rate did not decrease from Oct 2022 till the end of May 2023, whereas the average air temperature decreased till

the end of Feb 2023 and started to increase till the end of May 2023. The answer for this result is either ORP temperature was higher than or close to air temperature or the impact of other weather conditions (RH, wind speed, and GHI) was high on the evaporation rate at these periods. The analysis of all-weather conditions together leads to clarification of the results that were unpredicted by using only one weather condition to analyze them (using Figures 9 and 10 together). Using the graph trends of average predicted pond temperature and air temperature (Figure.11) helps for more investigation about these unexpected results for predicted evaporation rate in running periods. In Jun and Jul 2022, the average air and pond temperatures were almost the same, so the heat transfer by differences between these two temperatures had no impact on the evaporation rate. From Figure.9, most of the evaporation came from RH and wind speed factors. From Aug to the end of Sep 2022, there was a difference in the average pond and air temperatures, and the average air temperature was higher in this period. Therefore, the graph trends of the average predicted evaporation rate and air temperature were almost similar. From Oct 2022 till the end of May 2023, the difference in average air and predicted pond temperatures was almost zero except for some days. In this period, the predicted evaporation rates were oscillating with constant amplitude except for some days. From Oct 2022 till the end of May 2023, RH and wind speed controlled the predicted evaporation rate. During the days when the maximum peaks of predicted evaporation rate happened, the difference between an average pond and air temperature and wind speed was high, and RH was low. The highest predicted evaporation rate was at the end of May 2023, when the difference in average air and predicted pond temperature and wind speed were most of the highest readings, and RH was most of the lowest readings. GHI had the lowest impact factor in the predicted evaporation rate since the maximum peaks of the predicted evaporation rate were on days when average GHI readings were low on the same days. Figure 12 shows the predicted evaporation rate daily with the corresponding predicted salinity. Increasing the predicted evaporation rate led to an increase in predicted salinity, and the same happened in the reverse case. The amplitude range of predicted salinity was between 41 and 44 g NaCl/L. The impact of weather conditions on predicted salinity is the same as the evaporation rate, so the previous results and discussion on the influence of weather conditions on predicted evaporation rate are the same for predicted salinity.

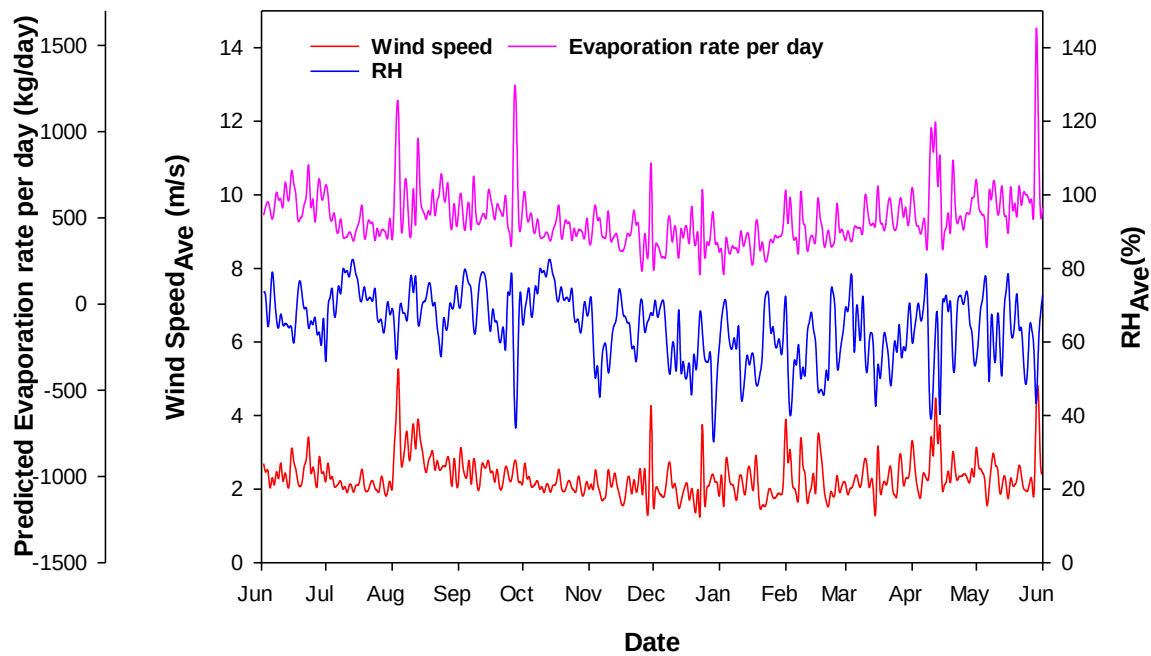


Figure 10: Impact of wind speed and RH on predicted Evaporation rate per day (kg/day) for an ORP situated in Jeddah

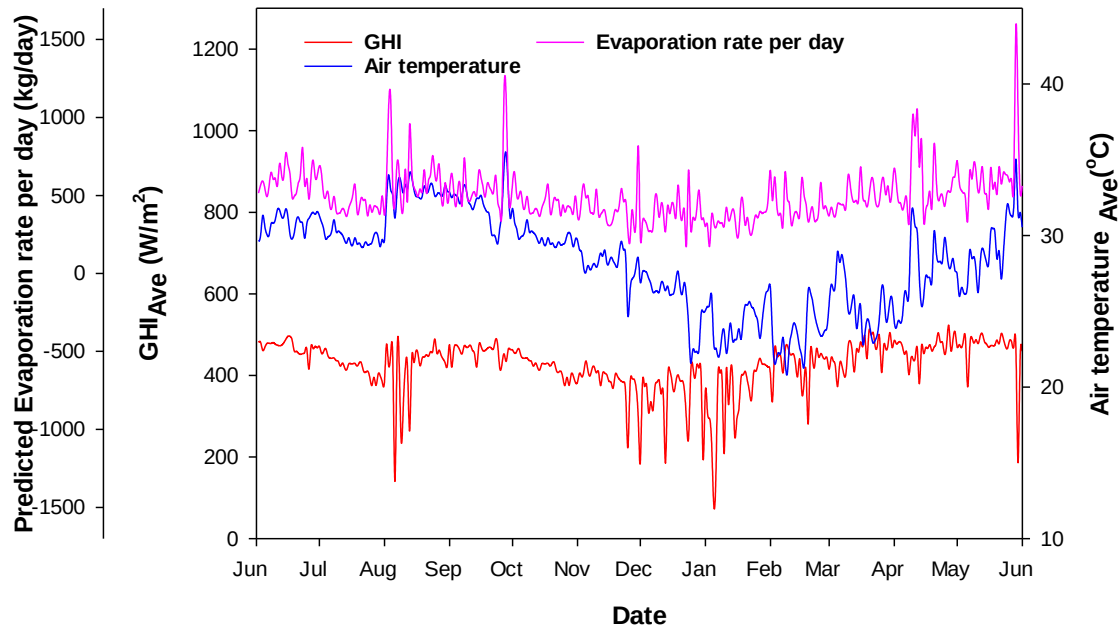


Figure 11: Impact of GHI and air temperature on pond Evaporation rate per day (kg/day) for an ORP situated in Jeddah

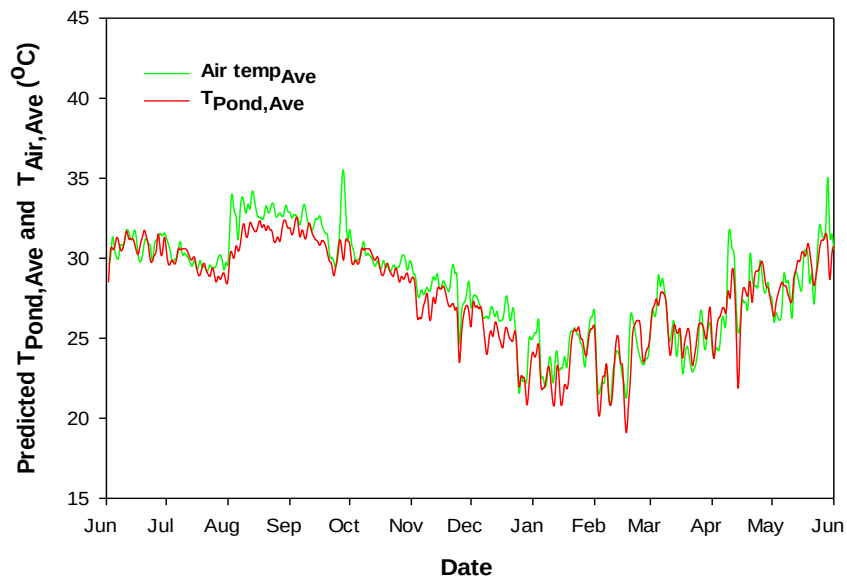


Figure 12: Average air and predicted ORP temperature throughout one year of running model for an ORP situated in Jeddah

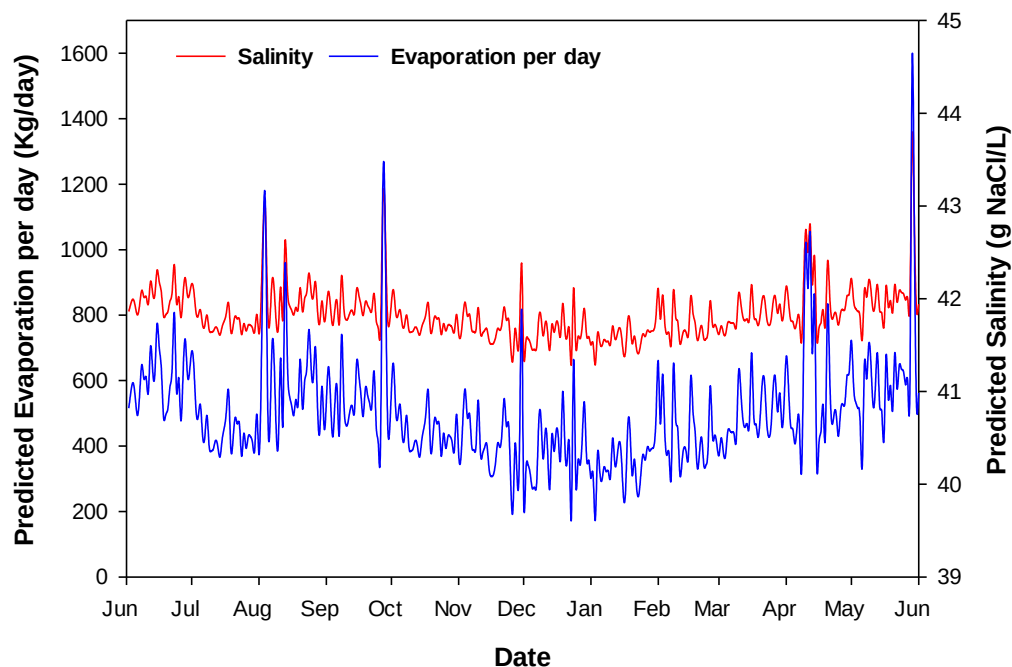


Figure 13: Predicted daily evaporation rate and salinity throughout one year of running model for an ORP situated in Jeddah

4.3. Biomass Growth rate and impact of GHI, PAR, and ORP temperature on growth rate of *Arthrospira maxima*

4.3.1. Predicted productivity of algae biomass during summer season

The initial and maximum biomass harvested concentration was set to be 200 and 600 g/m³, respectively, in the biological part of the model, so the predicted biomass concentration increased from 200 g/m³ to a little above 600 g/m³ for each repeated periodic cycle (Figure.13). The increase in predicted biomass concentration was sharp in each repeated cycle for this period. In Jul, the time needed to achieve harvested concentration for repeated cycles was higher compared to other repeated periods in Jun and Aug. The maximum,

average, and minimum productivity were 43.23, 31.41, and 21.1 g/m² /day, respectively; the number of harvestings in this period was 27 times. The explanation of the above results needs to analyze the factors that influenced the growth rate (concentration of biomass per time) throughout the cultivation period. From equations 3-22 in the materials and methods chapter, the three factors governing the biomass growth rate are culture light (ϕ_L) and temperature (ϕ_T) efficiency and the rate of photosynthetic spectrum photon incidence (*PAR*). Culture temperature efficiency depends on the maximum and minimum temperature at which microalgae can still grow and the optimal temperature for optimal algae growth. It was calculated by using equation 3-25. Culture light efficiency depends on GHI and saturated light intensity, which provide maximum growth of algae. To study the impact of these factors, Figure.14 showed the graph trend for average GHI, predicted pond temperature, culture light efficiency, and culture temperature efficiency for this period. Average daily culture light efficiency was stable at around 0.32 most of the period, but it reached a maximum of about 0.47 on day seven of Aug. GHI can increase culture light efficiency if the GHI of the pond is close to saturated light intensity (algae parameter) (see equation 3-23). Therefore, decreasing GHI may cause an increase in culture light efficiency, so an increase in the growth rate of algae due to culture light efficiency as happened on day seven of Aug. The graph trend for average predicted pond temperature and culture temperature efficiency were the same. This was expected. The average culture temperature efficiency fluctuated in the range between 0.5 to around 0.82 in this interval. The maximum predicted culture temperature efficiency was between the eighth of Aug and the end of Aug. It was clear from Figure 14 that the maximum productivity in the period between Jun and Aug was in the second half of Aug since most of the highest average readings of GHI, and highest average culture temperature were in this period of cultivation, whereas average culture light intensity in this period was close to other periods of algae cultivation in range between Jun and Aug. So, the impact of GHI on biomass growth was almost stable for most of this interval.

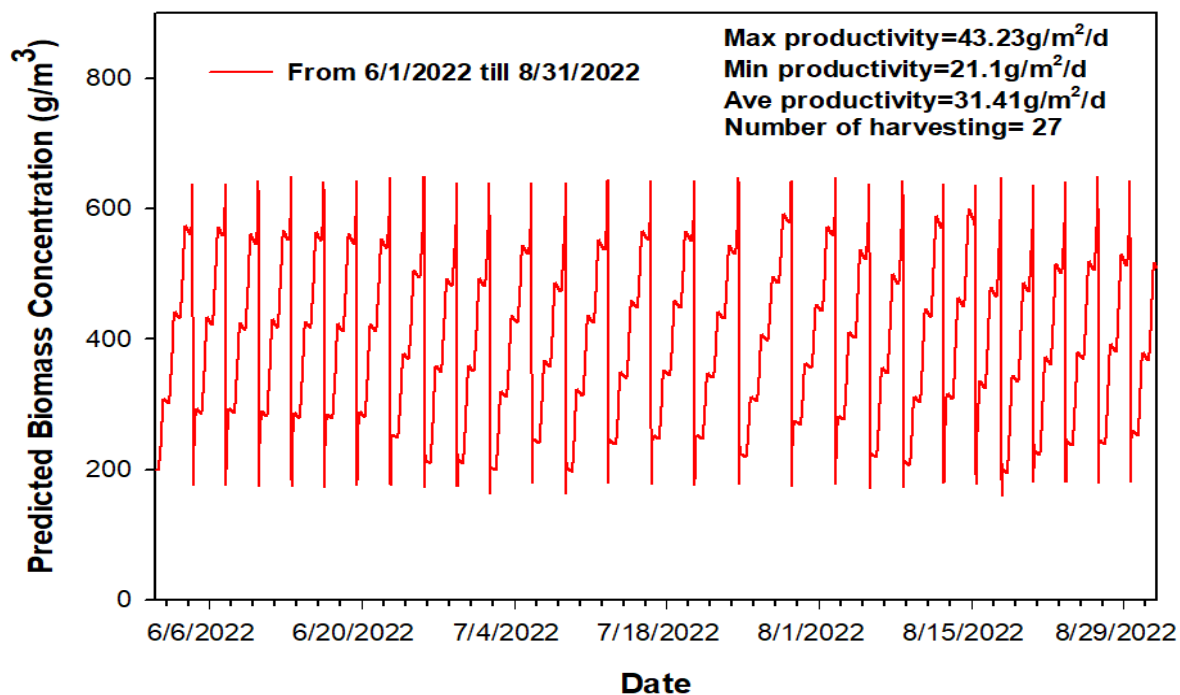


Figure 14: Biomass Growth rate in the interval between June and Aug 2022

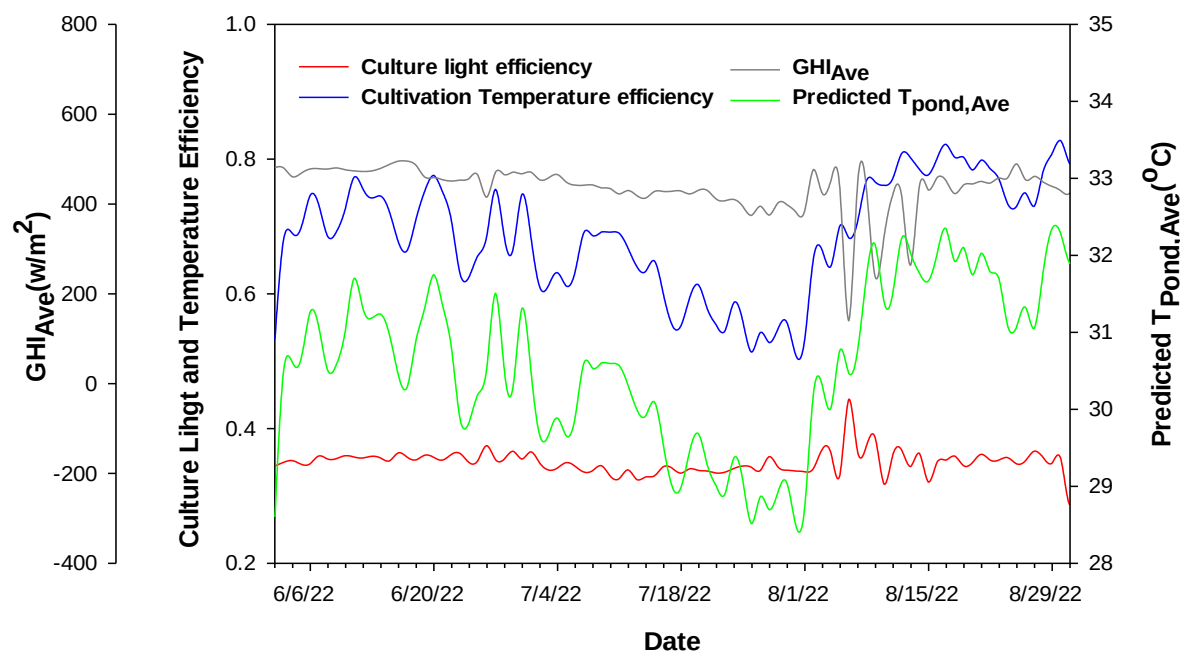


Figure 15: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of *Arthrospira maxima* in the interval between Jun and Aug 2022

4.3.2. Predicted productivity of algae biomass during fall season

The time for biomass to reach the harvested concentration was maximum for repeated cycles located in the first twenty days of Sep; then, it started to increase till the end of Nov, generally. In the last ten days of September, some of the complete cycles were higher or equal to those in October. The maximum, average, and minimum productivity and harvesting numbers were lower in this interval compared to the Jun to Aug interval. The average GHI and culture light efficiency had the same graph trend. Average culture light efficiency was stable around 0.35 throughout the interval, except for the lowest value, which was on the 26th of Nov. On this day of cultivation, not only was the average culture light efficiency the lowest, but the average predicted pond temperature, GHI, and culture efficiency temperature also remained low. Since the average culture light efficiency and GHI were almost stable for the whole of this interval, the impact of these two parameters on microalgae cultivation was almost steady. By comparing the results presented in Figure 15 with impact parameters in Figure 16, the maximum productivity was on the sixth of Sep. Since the culture temperature efficiency was the highest on this day. Culture temperature efficiency generally decreased in oscillation mode from September to the end of November. Therefore, productivity decreased throughout the interval. The average productivity for this period was in October since the mean average culture temperature efficiency was in the same month. The minimum productivity was on the twenty-sixth of Nov, whereas all the growth rate impact parameters were the lowest. The main Parameter that influenced the biomass growth rate was culture temperature efficiency for this interval (Figures 15 and 16).

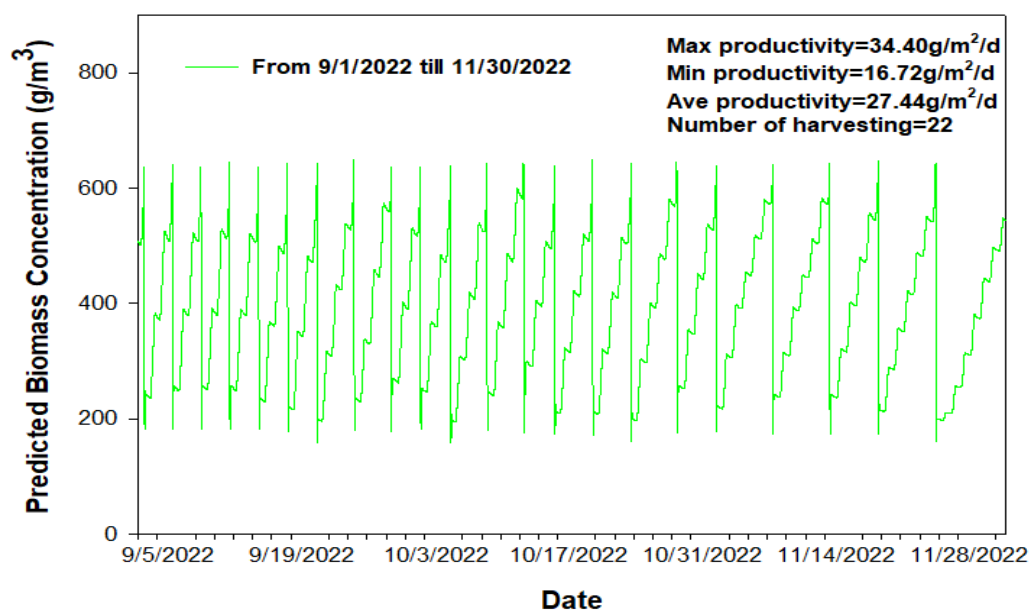


Figure 16: Biomass Growth rate in the interval between Sep and Nov 2022

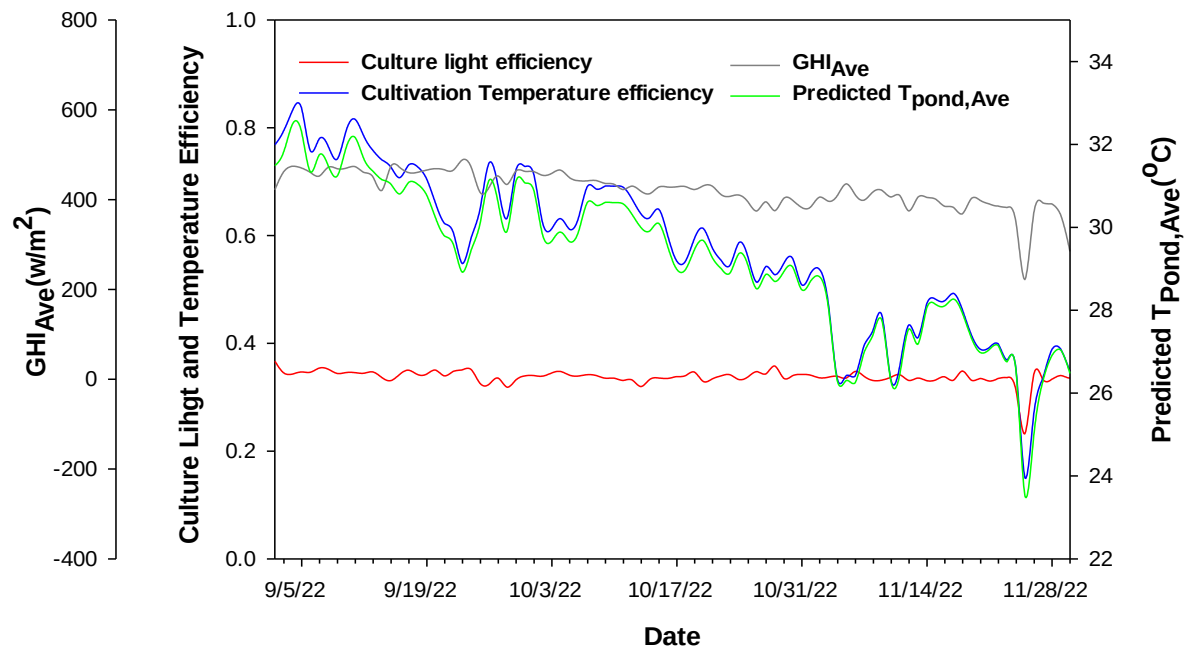


Figure 17: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of *Arthrospira maxima* in the interval between Sep and Nov 2022

4.3.3. Predicted productivity of algae biomass during winter season

From Figure 17, the time to complete the cycle increases from the beginning of the interval till the second of Jan. Then, it alternated till the end of the interval. The lowest complete cycle time was between twenty seconds of Jan till the second Feb in the interval between twenty seconds to the end of Mar. The lowest productivity in this period (highest complete cycle time) was between the third and the second of January. In the first ten days of December, the maximum productivity was at this interval. The total number of harvestings was 8 in this period. This result reflects that the impact factors in this period were far from the optimal parameters of biomass growth rate compared to previous periods. Average GHI was oscillating between around 150 and 450 w/m^2 throughout this period. In the two previous periods, the average GHI was almost stable except in some intervals and higher. Therefore, the GHI reduced the biomass growth rate in this period since it decreased the PAR, which impacted the growth rate of biomass, but it impacted slightly. Since, from equation 3-28, the impact of PAR on biomass growth rate is small. The average culture light efficiency in this period was stable and almost equivalent to the two previous periods. As a result, the contribution of average culture light efficiency for microalgae growth was almost similar to the previous and this period of cultivation. The productivity of microalgae biomass was lower in this interval compared to the last two intervals, and average GHI and culture light efficiency were similar in this period and the last two periods. The decrease in biomass productivity came from a high drop in culture temperature efficiency. The average predicted pond temperature reduced during the winter season (this period), and as the difference between culture temperature and optimal temperature (where biomass maximum growth rate happened) increases, the culture temperature efficiency decreases (see equation 3.25). Generally, the average culture temperature efficiency throughout this period was less than the last two periods. The maximum productivity in this period was in intervals, including the maximum average culture temperature efficiency. The average culture temperature efficiency during the first twenty days of Dec oscillated; it decreased till the twenty-fifth of Dec, whereas from the twenty-fifth of Dec till the twentieth of Jan,

the culture temperature efficiency fluctuated with the maximum peak around 0.24. After that, Culture temperature efficiency increased and oscillated with the maximum peak around 0.3 till the third of Feb. Then, until the end of February, the amplitude of oscillation of culture temperature efficiency increased to between 0.15 and 0.34 for maximum and minimum limits, respectively. By comparing the results of Figures 17 and 18, the time needed for the complete cycle and average culture temperature efficiency were consistent.

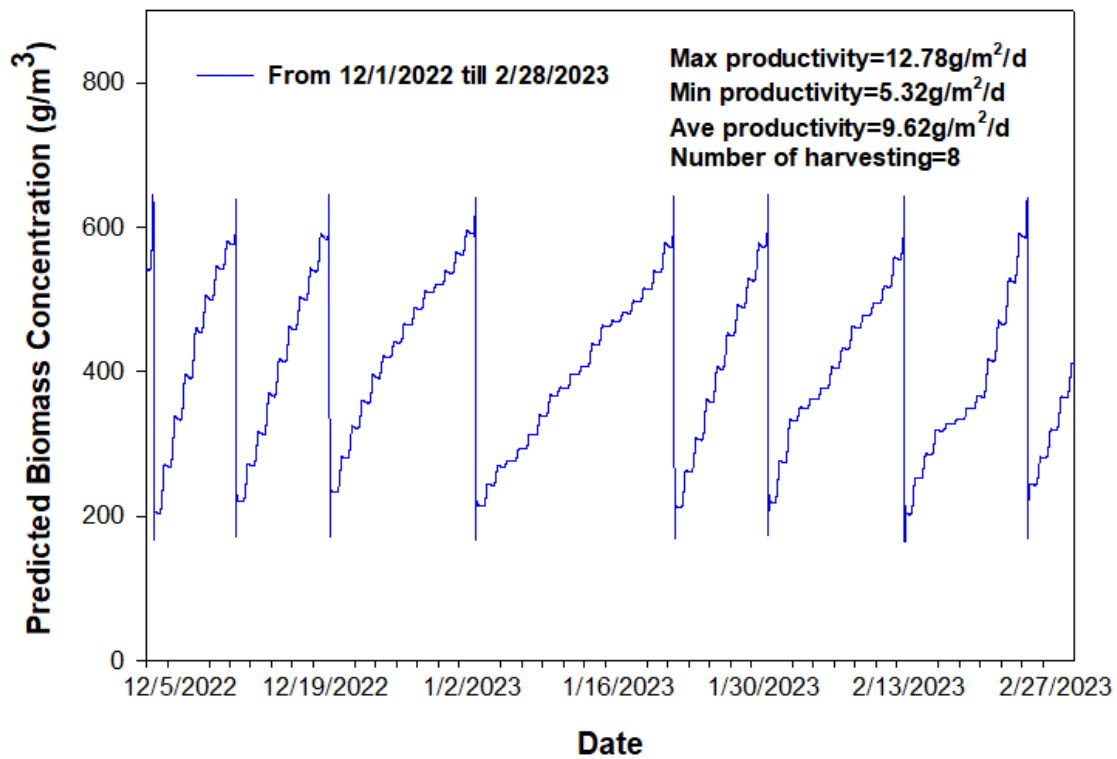


Figure 18: Biomass Growth rate in the interval between Dec 2022 and Feb 2023

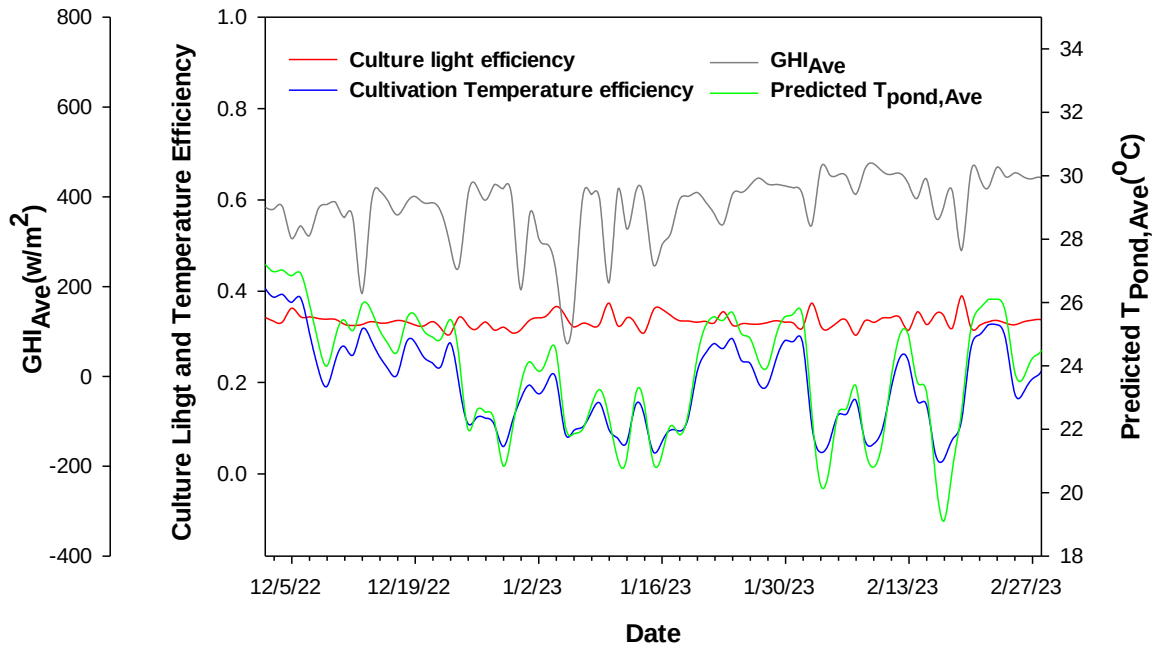


Figure 19: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of *Arthrospira maxima* in the interval between Dec 2022 and Feb 2023

4.3.4. Predicted productivity of algae biomass during spring season

In general, the productivity for cycles completed increased during this period (from March to May). From Figure.19, productivity was stable in Mar, except it decreased in the range between the twentieth to the sixth of Mar, and this was the lowest productivity interval with 12.48 g/m²/d for microalgae cultivation between Mar and May. The maximum productivity from the Mar to May period and the period between Sep to Nov were almost the same; they were 34.4 and 33.5 g/m²/d for the period between Sep and Nov 2022 and the period between Mar and May 2023, respectively. Also, these two periods were the closest in average and minimum productivity, as well as the number of harvestings. The

average GHI in this period oscillated with short amplitude. The oscillation of average GHI for most of this interval was steady at around 450 w/m^2 . Since GHI impacts culture light efficiency, as discussed previously. Therefore, average culture light efficiency was also stable for most of this period. Based on stable values for average GHI and culture light efficiency throughout this period, the impact of these two factors on the productivity of each cycle of this period was constant. The average culture temperature efficiency was around 0.4 in the first eight days of Mar. Then, it decreased and fluctuated between 0.2 and 0.4 till the seventh of Apr. The amplitude of fluctuation increased in the interval between the seventh and nineteenth of Apr. After that, till the end of May, the average culture temperature efficiency oscillated in the range between 0.35 and 0.8. The lowest average culture temperature efficiency was on the fourteenth day of April during the whole period, but the productivity was not the lowest in the interval, including day fourteen. The reason for this result was that most of the average culture temperature efficiencies for the interval that had the lowest productivity were less compared to those on an interval that had the lowest average culture efficiency. Since the maximum average culture temperature efficiency was in the last ten days of May, the highest productivity was in this range of this cultivation period (from Mar to May).

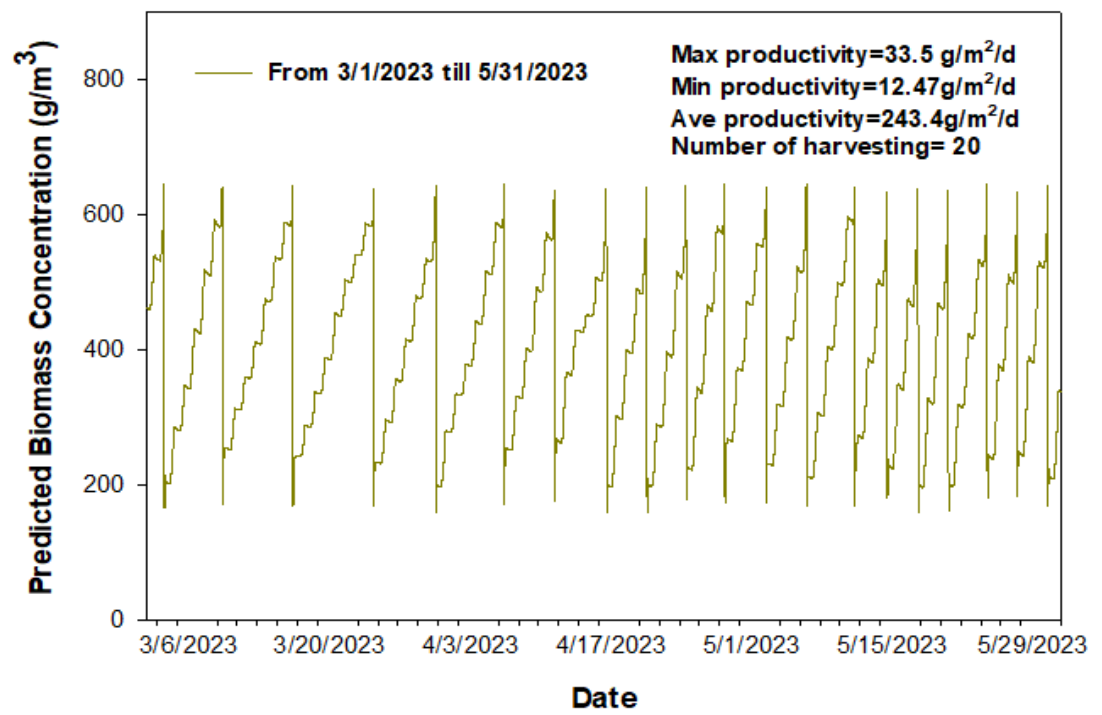


Figure 20: Biomass Growth rate in the interval between Mar to May 2023

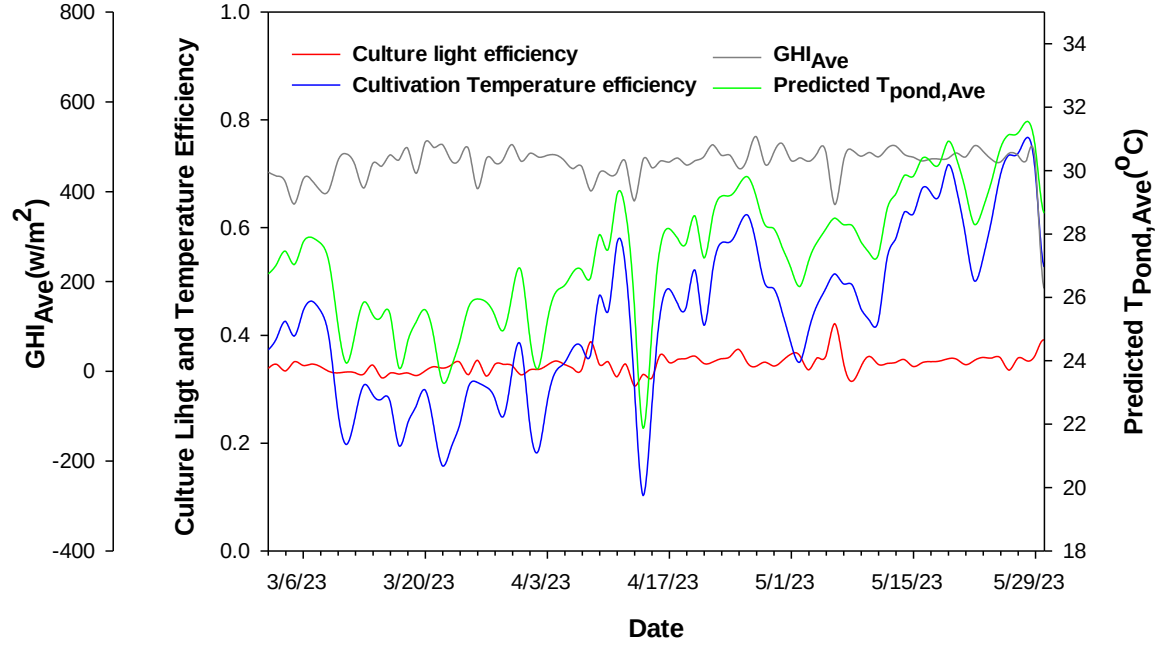


Figure 21: Impact of GHI, ORP temperature, and culture light and temperature efficiency on the growth rate of *Arthrospira maxima* in the interval between Mar to May 2023

4.4. Model validation

The mathematical model used in this study includes two main divisions: thermal and biological. The purpose and results of these two divisions were discussed in detail previously. The main subject of this section is to show the performance of the model. The first part of this section is to test the validation of the thermal division of the model, and the second part is to check the efficiency of the biological section of the model.

4.4.1. Thermal part of model validation

By using the thermal part of the model, the maximum predicted ORP temperature was 36.8°C. K. Schipper et al. simulated the maximum temperature for the culture of ORP

located in Saudi Arabia (Jeddah), and it was 33.4 °C (K. Schipper et al., 2021). The thermal model that was used in this study was run by using weather conditions of a specific location in KAUST (Jeddah). The difference in the results of maximum ORP culture temperatures simulated in this study and K. Schipper et al.'s work could be due to some differences in the assumptions following. In place of comparing one result reported in the literature. Experimental results from the local facility for ORP temperature were compared with the thermal part of the used model in this research as presented in Figure 21 for one completed year. The trend of experimental and model temperature results was almost fitted. The difference in the results between the experiment and model in the winter and spring seasons was less related to the fall season. The explanation for these results is that the model used the main assumption that the temperature of makeup water is the same as the instantaneous temperature of the ambient air; this is correct in a few situations since the heat capacity of water allows water to save energy for a long time and if the fluctuation in surroundings temperature is high, the assumption assumed air temperature is equal to supplier tank temperature is not valid in this case. In the winter and spring seasons, the fluctuation in air temperature was less compared to the fall season as shown in Figure 21. The comparison between the results of the model and experiment for the summer season is not included in Figure.21 but the expectation of diversity between experiment and model results would be close to the fall season since the fluctuation of air temperature was almost similar for these two seasons (see Figure 8). The actual makeup water temperature that impacted ORP temperature was hotter and colder than the air temperature in the fall season, for this reason, previous results (Figure 21) happened. The regression plot for the thermal validation part of the model is shown in Figure.22 illustrates the degree of accuracy of the thermal part of the model, $R^2 = 0.996$. The average error of the thermal part of the model for ORP was -0.346 ± 2.12 °C. The same thermal model used in this study was implemented to estimate the temperature of ORP used to cultivate *D. intermedius* at Georgia Tech (Jun-Jul), *N. oceanica* at Georgia Tech (Mar-Jun), and *C. vulgaris* at Arizona Center for Algal Technology and Innovation (Jan-Feb) and the average error for the thermal model was -1.02 ± 2.13 °C (Greene et al., 2021b). The average error range of the thermal model used in this study was lower than that of the one reported in the literature (Greene et al. work).

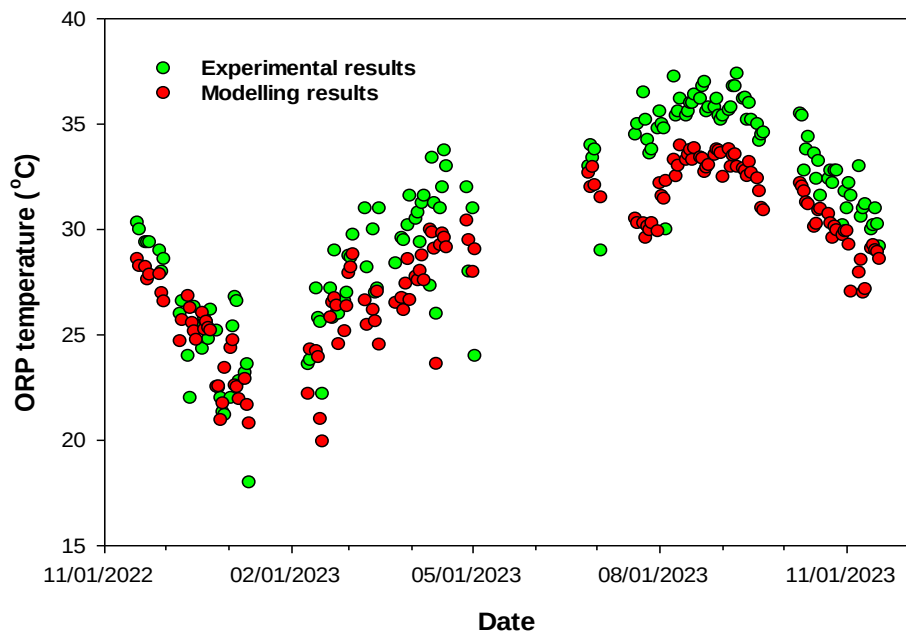


Figure 22: Comparison between temperature results from the thermal part of the model and experimental data of temperature for ORP from a local facility

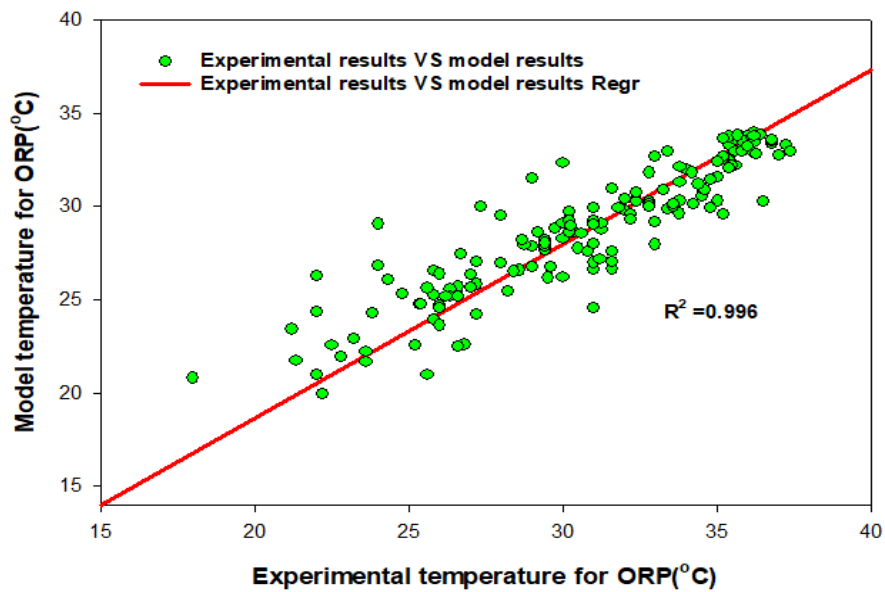


Figure 23: Regression of comparison between temperature results from the thermal part of the model and experimental data of temperature for ORP from a local facility

4.4.2. Biological part of the model validation

The strain type of *Spirulina* (*Arthrospira maxima*) was cultivated in ORP in Inner Mongolia, China, and the cultivation was outdoor (Zhu et al., 2020). This experimental work of *Spirulina* cultivation in China reported data that was used to run and validate the biological part of the model implemented in this study. These data are biomass concentration readings of microalgae, harvested time, period of running an experiment, dimensions of ORP used, GHI readings for the weather during the period of the running experiment, and temperature measurements of ORP during microalgae cultivation. Figure 23 illustrates the performance evaluation of the biological part of the model by comparing the experimental work data collected from *Spirulina* cultivation in China, Zhu et al. work, with model results for the same microalgae (*Spirulina*). The microalgae were cultivated for eight days and harvested every four days, so the model was set to harvest based on the number of days (every four days), not the concentration of microalgae (as done in the previous discussion). The regression plot for Zhu et al. experimental work and model results of biomass concentration is illustrated in Figure. 24 shows the efficiency of the biological part of the model. The value of R^2 was 0.998, so the model was well fitting the experimental results of *Spirulina* cultivation in China for eight days. The average error of comparing the model results with Zhu et al. experimental results is 3.95 % with a standard deviation of ± 3.04 . The same model was used to estimate the biomass concentration of *C. vulgaris* that was cultivated in ORP located at Arizona Center for Algal Technology and Innovation; the average error of comparison between the model and experimental results was 5.29% with a standard deviation of $\pm 6.7\%$ (Greene et al., 2021b). The cultivation of *Spirulina* in commercial scale ORP (300 m²) was monitored for one year located in central Europe (Kurpan et al., 2024). The model used in this research was implemented based on the given conditions of *Spirulina* cultivation in Kurpan et al. work. Figures 25 and 26 show the comparison between the results of the model and Kurpan et al. experiment work and regression for this comparison, respectively.

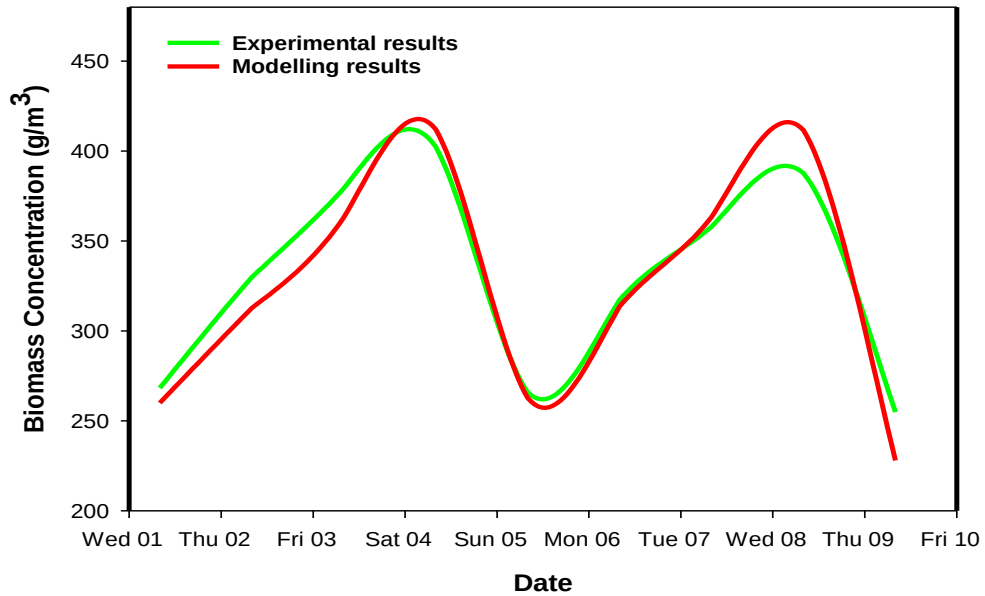


Figure 24: Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Inner Mongolia, China

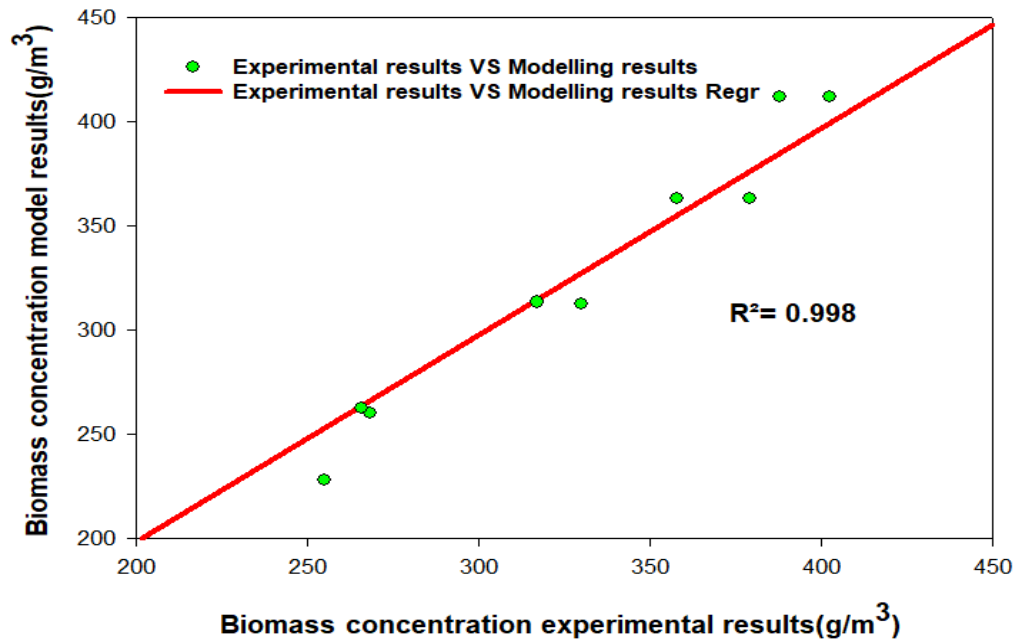


Figure 25: Regression of comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Inner Mongolia, China

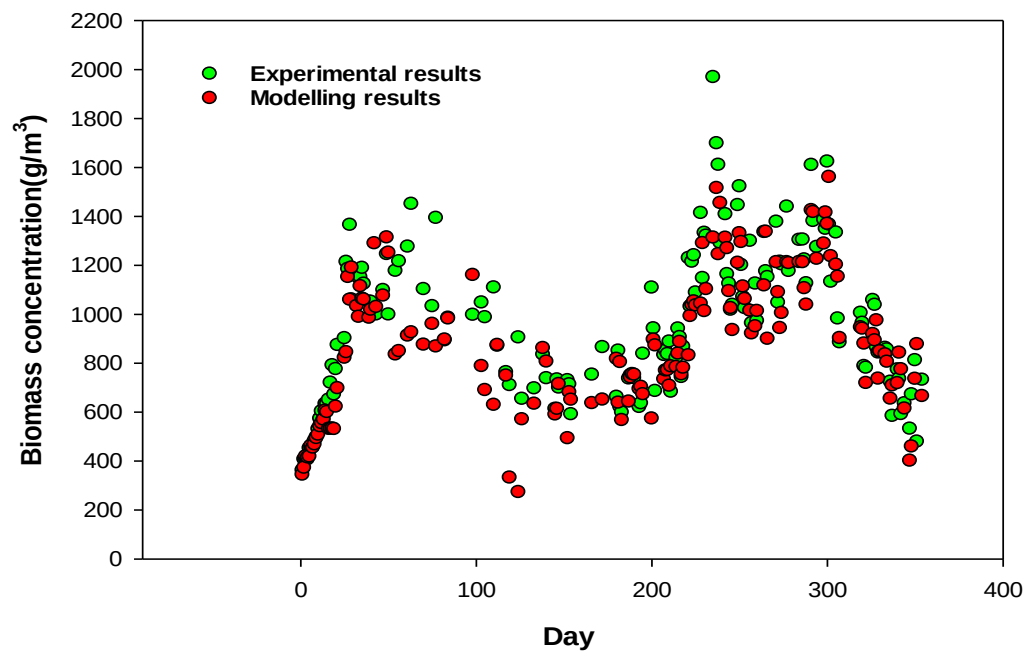


Figure 26: Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Italy

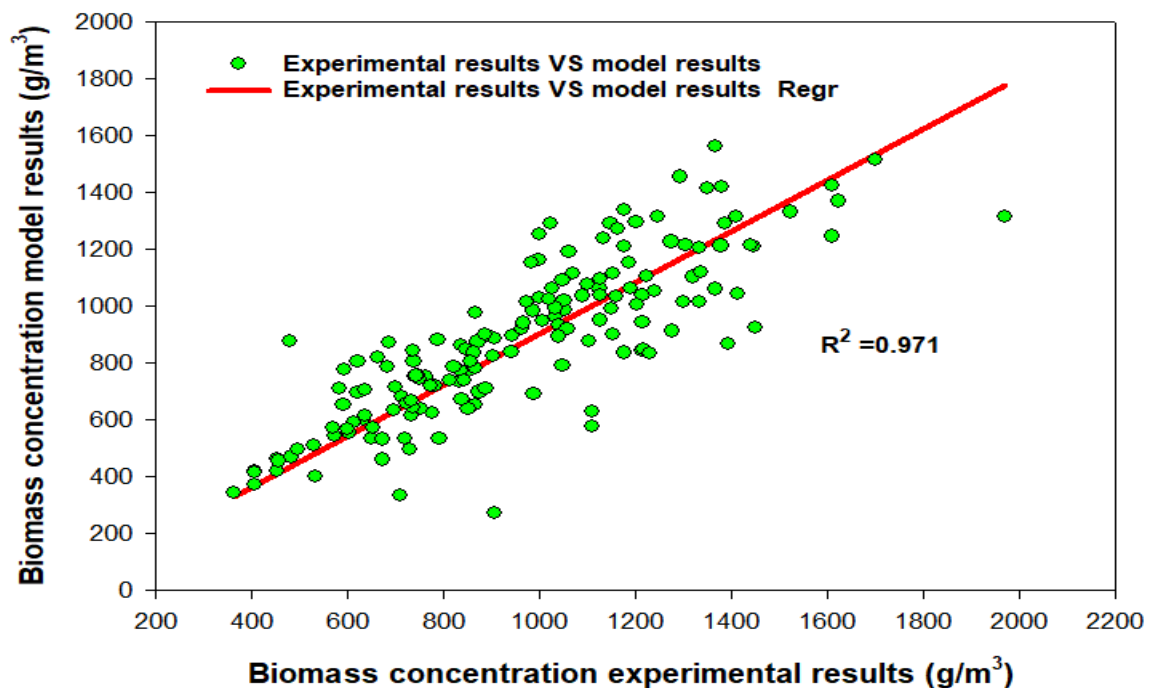
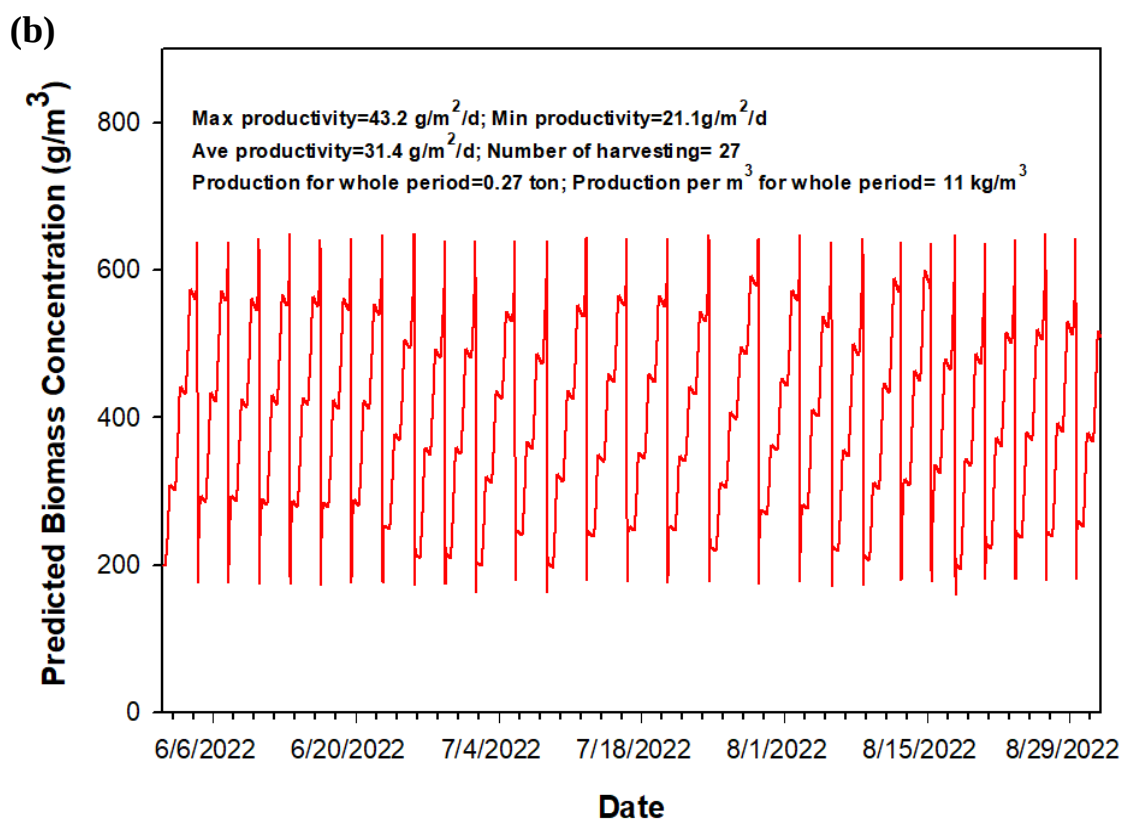
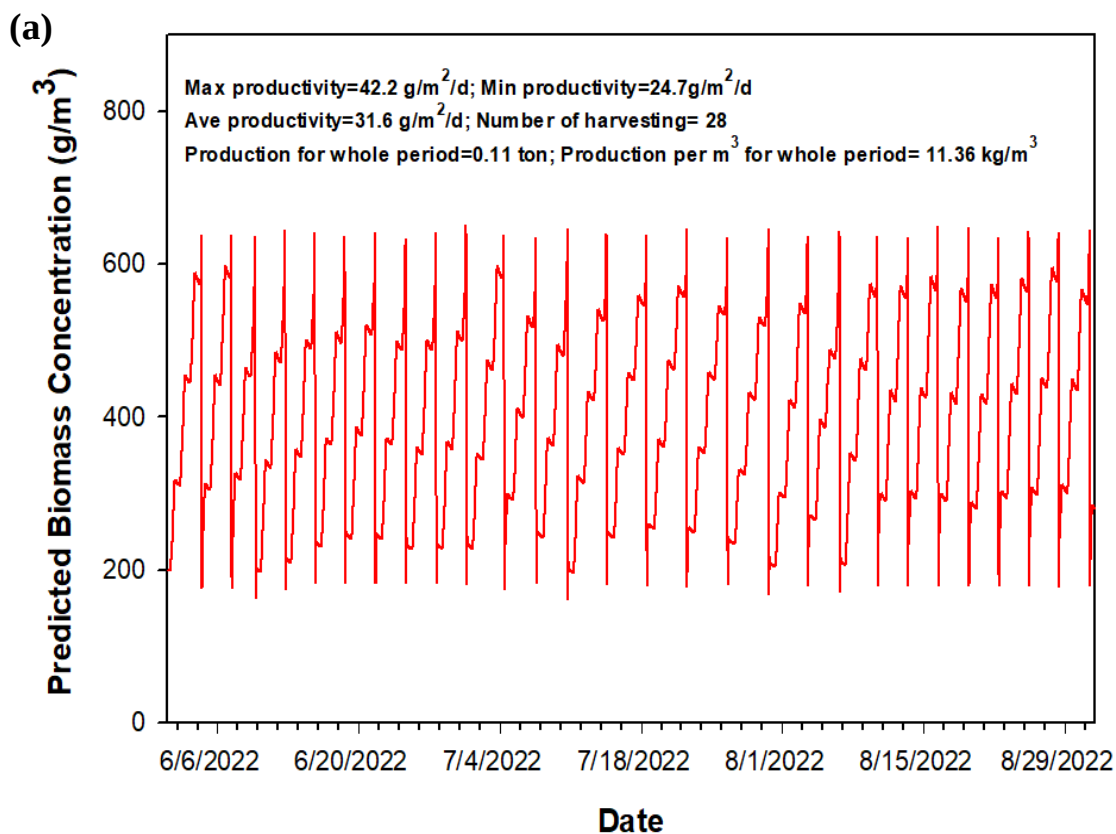


Figure 27: Regression of Comparison between biomass concentration results of the model and experimental data of biomass concentration from experimental work in Italy

4.5. Impact of ORP depth and harvesting concentration on the biomass productivity

The model was run based on three different sizes of ORP (20, 25, and 30 m³), each size had the same surface area but different depth (z) (0.2, 0.25, and 0.3 m) at fixed harvesting concentration (600g/m³) to study the impact of ORP depth on the biomass productivity. The impact of ORP depth on productivity was only studied for three months (June, July, and August). The biomass production per unit volume decreased as the depth of ORP increased from 0.2 to 0.3 m. The biomass production per unit volume (kg/m³) decreased by 23 and 34.8% when the depth of ORP increased from 0.2 to 0.25 and 0.3m, respectively. As the harvestings number increases the production of biomass increases, so the number of harvestings for 0.2, 0.25, and 0.3m of ORP depth was 35, 27, and 23 times, respectively (see Figure 31). The governing equation of biomass growth rate (equation 3-22) can explain the previous result. The two main parameters in equation 3-22 that are influenced by the depth of ORP at fixed surface area are culture light efficiency (ϕ_L) and the ratio of surface area to volume of ORP ($A_{surface}/V$). In this section, the surface area is fixed, and depth is changed, so as the depth decreases the ratio between $A_{surface}$ and V increases, finally the biomass growth rate (productivity) increases. As the depth of ORP increases, the culture light efficiency decreases, therefore the biomass growth rate reduces.

The productivity of biomass was tested at two different cases of harvesting concentrations, 600 and 1100g/m³, in interval between June and August. The average productivity for both cases was the same. Also, the biomass production per unit volume was similar, 11 and 10.86 kg/m³ for 600 and 1100g/m³ of harvesting concentration, respectively. In contrast, the harvesting number for 600g/m³ was 27 whereas it was 12 for 1100g/m³ (see Figure 32). From these results, harvesting concentration does not impact biomass productivity but influences the harvesting number, and harvesting number can impact the operation cost for large-scale cultivation, as a result, total cost increases as the harvesting number increases.



(c)

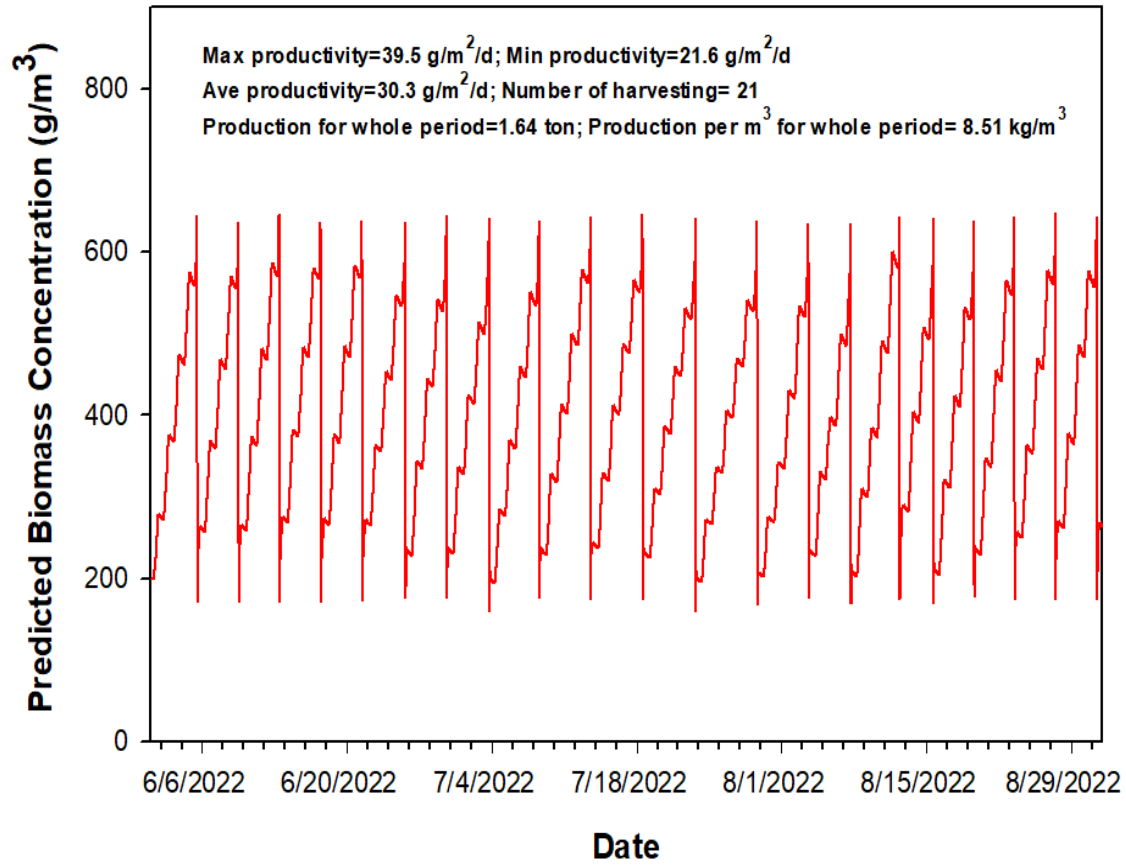


Figure 28: Comparison of the productivity based on the depth of ORP at 600g/m³; (a) 0.2, (b) 0.25 and (c) 0.3 m

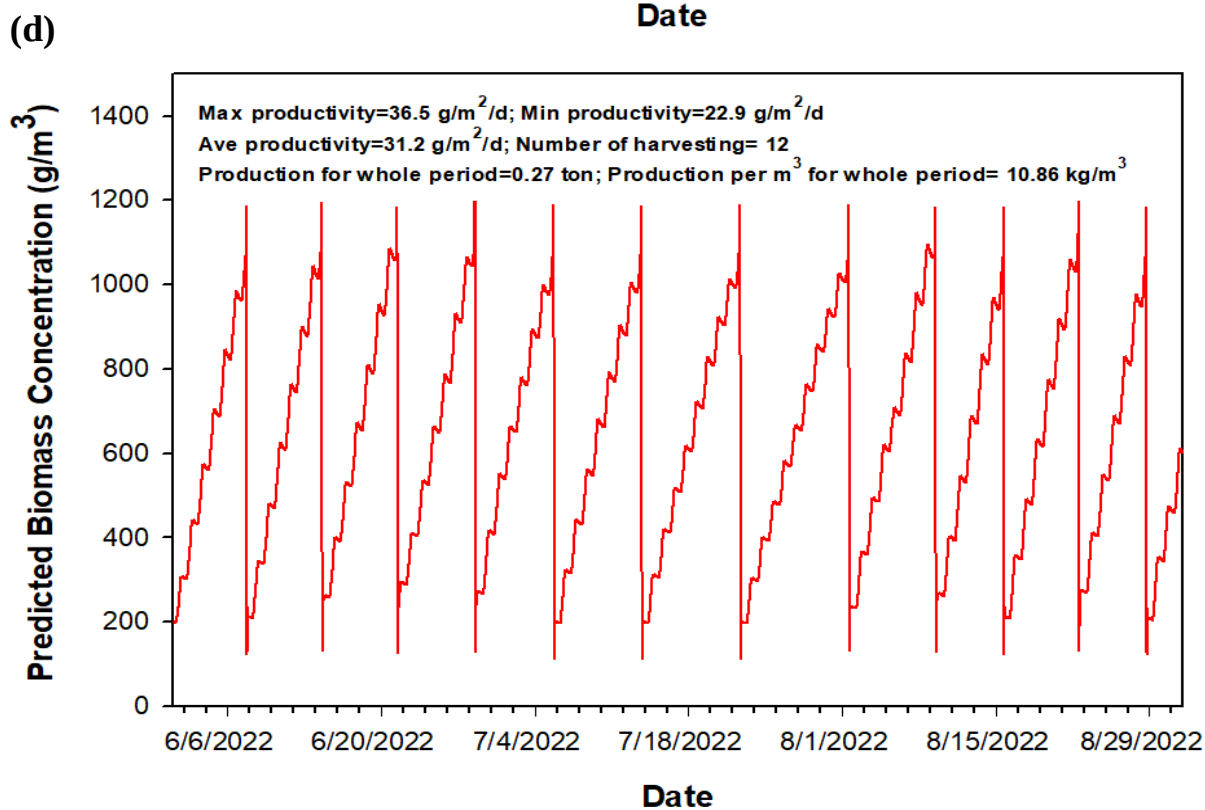
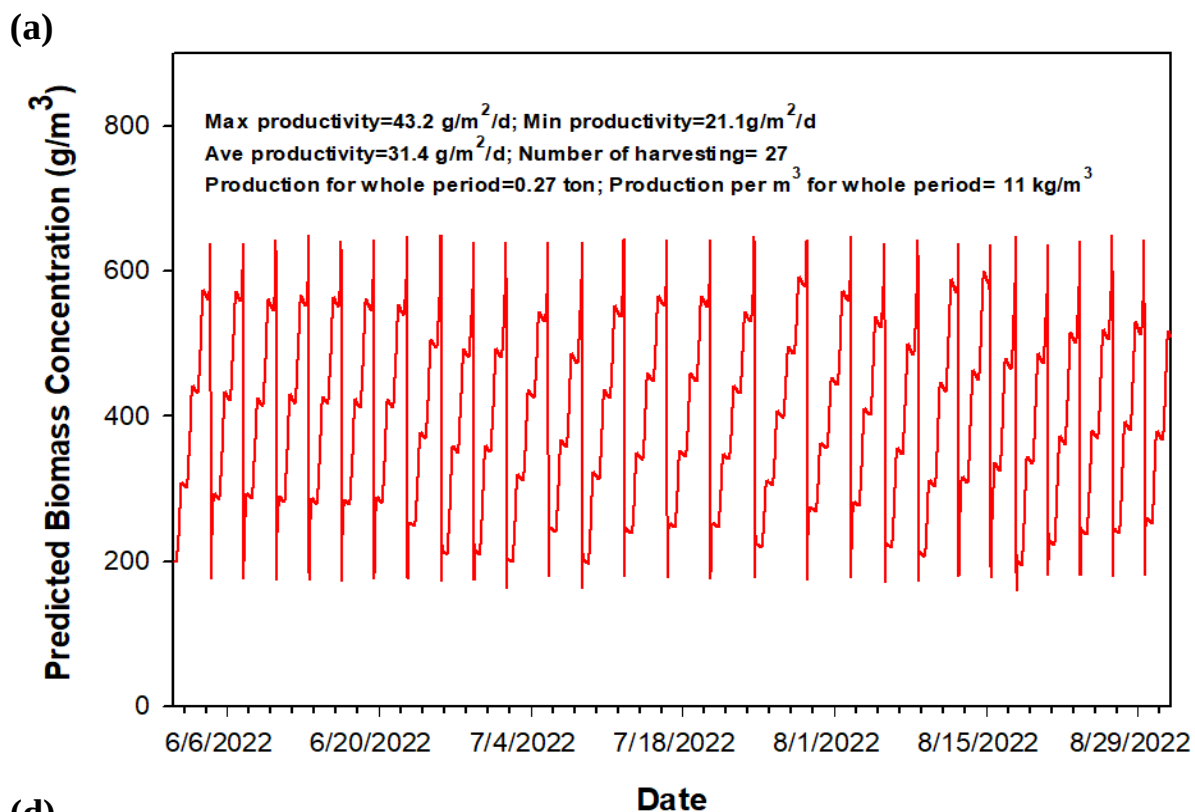


Figure 29: Comparison of productivity based on two harvesting concentrations of 25 m³ ORP volume; (a) at 600 g/m³ and (b) at 1100 g/m³

CHAPTER 5: Conclusion

The model develops to predict the ORP temperature, evaporation rate and microalgae growth predicted the variables with good accuracy. The average error of the thermal part with actual and predicted value was -0.346 ± 2.12 °C and -63.64 ± 130.81 g/m³ by using experimental results from the local facility of 10 m³ ORP volume. The reason for the relatively high average error in the biological part of the model was that the harvesting cycle was not consentient. In general, the performance of the model in predicting temperature and biomass concentration for the ORP was in acceptable range with relatively low input parameters (strain characteristics and ORP dimensions). The application of this model using the data based on potential strains and locations can provide an initial assessment of the optimal location to grow microalgae in Saudi Arabia with high productivity. Moreover, the ability of the model to simulate the growth rate of algae by using historical weather data allows us to predict the biomass growth rate for coming years of similar weather conditions. However, the current model does not include some of the environmental conditions, such as rainfall, that may influence the pH of the cultivation medium and medium contamination. To treat these limitations in the current model, new functions that include the impact of these weather events can be introduced to simulate the realistic growth rate of microalgae under the impact of these weather conditions. The prediction of dark respiration can be modified by simulating dark respiration as a function of the growth phase of algae. By using this model to predict the evaporation rate, places with low evaporation rates can be avoided since a low evaporation rate leads to a rise in ORP temperature and the growth rate of microalgae is impacted negatively by the high temperature of the cultivation medium. The second advantage of evaporation rate prediction is to optimize the cost of a big-scale cultivation system as its affect the salinity of medium and need additional make-up water to maintain the optimal salinity. Therefore, building large ORP systems in areas with weather conditions that enhance evaporation rates leads to high costs regarding water loss from the ORP system. The cost estimation for large-scale cultivation plants can be reduced by optimizing the input of the model (location, strain kind, and ORP parameters). The outputs of the model, such as productivity, pond

temperature (if cooling or heating is needed), evaporation rate (water loss), wastewater treatment, energy consumption, amount of nutrients consumed, harvesting cycles, cleaning periods, and dimensions of ORP are used as inputs on economic analysis for commercial production.

References

- Abdelfattah, A., Ali, S. S., Ramadan, H., El-Aswar, E. I., Eltawab, R., Ho, S. H., Elsamahy, T., Li, S., El-Sheekh, M. M., Schagerl, M., Kornaros, M., & Sun, J. (2023). Microalgae-based wastewater treatment: Mechanisms, challenges, recent advances, and future prospects. *Environmental Science and Ecotechnology*, 13, 100205. <https://doi.org/10.1016/j.esse.2022.100205>
- Abdur Razzak, S., Bahar, K., Islam, K. M. O., Haniffa, A. K., Faruque, M. O., Hossain, S. M. Z., & Hossain, M. M. (2024). Microalgae cultivation in photobioreactors: sustainable solutions for a greener future. *Green Chemical Engineering*, 5(4), 418–439. <https://doi.org/10.1016/j.gce.2023.10.004>
- Adesanya, V. O., Davey, M. P., Scott, S. A., & Smith, A. G. (2014). Bioresource Technology Kinetic modelling of growth and storage molecule production in microalgae under mixotrophic and autotrophic conditions. *BIORESOURCETECHNOLOGY*, 157, 293–304. <https://doi.org/10.1016/j.biortech.2014.01.032>
- Al-Jabri, H., Das, P., Khan, S., Thaher, M., & Abdulquadir, M. (2021). Treatment of wastewaters by microalgae and the potential applications of the produced biomass—a review. *Water (Switzerland)*, 13(1). <https://doi.org/10.3390/w13010027>
- Alexandrov, G. A., & Yamagata, Y. (2007). A peaked function for modeling temperature dependence of plant productivity. *Ecological Modelling*, 200(1–2), 189–192. <https://doi.org/10.1016/j.ecolmodel.2006.07.012>
- Amini, H., Wang, L., Hashemisohi, A., Shahbazi, A., Bikdash, M., KC, D., & Yuan, W. (2018). An integrated growth kinetics and computational fluid dynamics model for the analysis of algal productivity in open raceway ponds. *Computers and Electronics in Agriculture*, 145(January), 363–372. <https://doi.org/10.1016/j.compag.2018.01.010>
- Atkinson, K. E., Han, W., & Stewart, D. (2011). Numerical Solution of Ordinary Differential Equations. *Numerical Solution of Ordinary Differential Equations*, 1–252. <https://doi.org/10.1002/9781118164495>
- Avinash, A., Sasikumar, P., & Pugazhendhi, A. (2020). Analysis of the limiting factors for large scale microalgal cultivation: A promising future for renewable and sustainable biofuel industry. *Renewable and Sustainable Energy Reviews*, 134(September), 110250. <https://doi.org/10.1016/j.rser.2020.110250>
- Bannister, T. T. (1979). Quantitative description of steady state, nutrient-saturated algal growth, including adaptation. *Limnology and Oceanography*, 24(1), 76–96. <https://doi.org/10.4319/lo.1979.24.1.0076>

- Béchet, Q., Shilton, A., & Guieysse, B. (2013). Modeling the effects of light and temperature on algae growth: State of the art and critical assessment for productivity prediction during outdoor cultivation. In *Biotechnology Advances* (Vol. 31, Issue 8, pp. 1648–1663). <https://doi.org/10.1016/j.biotechadv.2013.08.014>
- Bekirogullari, M., Pittman, J. K., & Theodoropoulos, C. (2018). Multi-factor kinetic modelling of microalgal biomass cultivation for optimised lipid production. *Bioresource Technology*, 269(May), 417–425. <https://doi.org/10.1016/j.biortech.2018.07.121>
- Bekirogullari, Mesut, Figueroa-Torres, G. M., Pittman, J. K., & Theodoropoulos, C. (2020). Models of microalgal cultivation for added-value products - A review. *Biotechnology Advances*, 44(July), 107609. <https://doi.org/10.1016/j.biotechadv.2020.107609>
- Benavides, M., Telen, D., Lauwers, J., Logist, F., Van Impe, J., & Wouwer, A. Vande. (2015). Parameter identification of the droop model using optimal experiment design. *IFAC-PapersOnLine*, 28(1), 586–591. <https://doi.org/10.1016/j.ifacol.2015.05.094>
- Benson, B. C., & Rusch, K. A. (2006). Investigation of the light dynamics and their impact on algal growth rate in a hydraulically integrated serial turbidostat algal reactor (HISTAR). *Aquacultural Engineering*, 35(2), 122–134. <https://doi.org/10.1016/j.aquaeng.2005.09.005>
- Beran, B., & Kargi, F. (2005). A dynamic mathematical model for wastewater stabilization ponds. *Ecological Modelling*, 181(1), 39–57. <https://doi.org/10.1016/j.ecolmodel.2004.06.022>
- Bernard, O., & Rémond, B. (2012). Validation of a simple model accounting for light and temperature effect on microalgal growth. *Bioresource Technology*, 123, 520–527. <https://doi.org/10.1016/j.biortech.2012.07.022>
- Bitaubé Pérez, E., Caro Pina, I., & Pérez Rodríguez, L. (2008). Kinetic model for growth of *Phaeodactylum tricornutum* in intensive culture photobioreactor. *Biochemical Engineering Journal*, 40(3), 520–525. <https://doi.org/10.1016/j.bej.2008.02.007>
- Blanchard, G. F., Guarini, J. M., Richard, P., Gros, P., & Mornet, F. (1996). Quantifying the short-term temperature effect on light- saturated photosynthesis of intertidal microphytobenthos. *Marine Ecology Progress Series*, 134(1–3), 309–313. <https://doi.org/10.3354/meps134309>
- Blanken, W., Postma, P. R., de Winter, L., Wijffels, R. H., & Janssen, M. (2016). Predicting microalgae growth. *Algal Research*, 14, 28–38. <https://doi.org/10.1016/j.algal.2015.12.020>
- Bougaran, G., Bernard, O., & Sciandra, A. (2010). Modeling continuous cultures of

- microalgae colimited by nitrogen and phosphorus. *Journal of Theoretical Biology*, 265(3), 443–454. <https://doi.org/10.1016/j.jtbi.2010.04.018>
- Chang, H. X., Huang, Y., Fu, Q., Liao, Q., & Zhu, X. (2016). Kinetic characteristics and modeling of microalgae *Chlorella vulgaris* growth and CO₂ biofixation considering the coupled effects of light intensity and dissolved inorganic carbon. *Bioresource Technology*, 206, 231–238. <https://doi.org/10.1016/j.biortech.2016.01.087>
- Chapelle, A., Labry, C., Sourisseau, M., Lebreton, C., Youenou, A., & Crassous, M. P. (2010). *Alexandrium minutum* growth controlled by phosphorus. An applied model. *Journal of Marine Systems*, 83(3–4), 181–191. <https://doi.org/10.1016/j.jmarsys.2010.05.012>
- Chen, F., & Johns, M. R. (1994). Substrate inhibition of *Chlamydomonas reinhardtii* by acetate in heterotrophic culture. *Process Biochemistry*, 29(4), 245–252. [https://doi.org/10.1016/0032-9592\(94\)80064-2](https://doi.org/10.1016/0032-9592(94)80064-2)
- Chin, G. J. W. L., Andrew, A. R., Abdul-Sani, E. R., Yong, W. T. L., Misson, M., & Anton, A. (2023). The effects of light intensity and nitrogen concentration to enhance lipid production in four tropical microalgae. *Biocatalysis and Agricultural Biotechnology*, 48(February), 102660. <https://doi.org/10.1016/j.bcab.2023.102660>
- Chowdury, K. H., Nahar, N., & Deb, U. K. (2020). The Growth Factors Involved in Microalgae Cultivation for Biofuel Production: A Review. *Computational Water, Energy, and Environmental Engineering*, 09(04), 185–215. <https://doi.org/10.4236/cweee.2020.94012>
- Concas, A., Pisu, M., & Cao, G. (2016). A novel mathematical model to simulate the size-structured growth of microalgae strains dividing by multiple fission. *Chemical Engineering Journal*, 287, 252–268. <https://doi.org/10.1016/j.cej.2015.11.021>
- Corkrey, R., McMeekin, T. A., Bowman, J. P., Ratkowsky, D. A., Olley, J., & Ross, T. (2014). Protein thermodynamics can be predicted directly from biological growth rates. *PLoS ONE*, 9(5). <https://doi.org/10.1371/journal.pone.0096100>
- Darvehei, P., Bahri, P. A., & Moheimani, N. R. (2018). Model development for the growth of microalgae: A review. In *Renewable and Sustainable Energy Reviews* (Vol. 97, pp. 233–258). Elsevier Ltd. <https://doi.org/10.1016/j.rser.2018.08.027>
- Das, P., Halder, G., & Bal, M. (2023). A critical review on remediation of microplastics using microalgae from aqueous system. *Science of the Total Environment*, 898(July), 166425. <https://doi.org/10.1016/j.scitotenv.2023.166425>
- Dauta, A., Devaux, J., Piquemal, F., & Boumnick, L. (1990). Growth rate of four freshwater algae in relation to light and temperature. *Hydrobiologia*, 207(1), 221–226. <https://doi.org/10.1007/BF00041459>

- DeBoer, J. A., Guigli, H. J., Israel, T. L., & D'Elia, C. F. (1978). Nutritional Studies of Two Red Algae. I. Growth Rate As a Function of Nitrogen Source and Concentration. *Journal of Phycology*, 14(3), 261–266. <https://doi.org/10.1111/j.1529-8817.1978.tb00296.x>
- Dermoun, D., Chaumont, D., Thebault, J. M., & Dauta, A. (1992). Modelling of growth of *Porphyridium cruentum* in connection with two interdependent factors: Light and temperature. *Bioresource Technology*, 42(2), 113–117. [https://doi.org/10.1016/0960-8524\(92\)90069-A](https://doi.org/10.1016/0960-8524(92)90069-A)
- Dolganyuk, V., Belova, D., Babich, O., Prosekov, A., Ivanova, S., Katserov, D., Patyukov, N., & Sukhikh, S. (2020). Microalgae: A promising source of valuable bioproducts. *Biomolecules*, 10(8), 1–24. <https://doi.org/10.3390/biom10081153>
- Droop, M. R. (1968). Vitamin B12 and Marine Ecology. IV. The Kinetics of Uptake, Growth and Inhibition in *Monochrysis Lutheri*. *Journal of the Marine Biological Association of the United Kingdom*, 48(3), 689–733. <https://doi.org/10.1017/S0025315400019238>
- Ebadi, A. G., Toughani, M., Najafi, A., & Babaee, M. (2020). A brief overview on current environmental issues in Iran. *Cent. Asian. J. Environ. Sci. Technol. Innov*, 1(1), 1–11. <https://doi.org/10.22034/CAJESTI.2020.01.08>
- Eilers, P. H. C., & Peeters, J. C. H. (1988). A model for the relationship between light intensity and the rate of photosynthesis in phytoplankton. *Ecological Modelling*, 42(3–4), 199–215. [https://doi.org/10.1016/0304-3800\(88\)90057-9](https://doi.org/10.1016/0304-3800(88)90057-9)
- Elisabeth, B., Rayen, F., & Behnam, T. (2021). Microalgae culture quality indicators: a review. *Critical Reviews in Biotechnology*, 41(4), 457–473. <https://doi.org/10.1080/07388551.2020.1854672>
- Endres, C. H., Roth, A., & Brück, T. B. (2016). Thermal Reactor Model for Large-Scale Algae Cultivation in Vertical Flat Panel Photobioreactors. *Environmental Science and Technology*, 50(7), 3920–3927. <https://doi.org/10.1021/acs.est.5b05414>
- Fernández, I., Acien, F. G., Guzmán, J. L., Berenguel, M., & Mendoza, J. L. (2016). Dynamic model of an industrial raceway reactor for microalgae production. *Algal Research*, 17, 67–78. <https://doi.org/10.1016/j.algal.2016.04.021>
- Figuerola-Torres, G. M., Pittman, J. K., & Theodoropoulos, C. (2017). Kinetic modelling of starch and lipid formation during mixotrophic, nutrient-limited microalgal growth. *Bioresource Technology*, 241, 868–878. <https://doi.org/10.1016/j.biortech.2017.05.177>
- Filali, R., Tebbani, S., Dumur, D., Isambert, A., Pareau, D., & Lopes, F. (2011). Growth modeling of the green microalga *Chlorella vulgaris* in an air-lift photobioreactor. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 44(1 PART 1), 10603–10608.

<https://doi.org/10.3182/20110828-6-IT-1002.01955>

- Furmaniak, M. A., Misztak, A. E., Franczuk, M. D., Wilmotte, A., Waleron, M., & Waleron, K. F. (2017). Edible cyanobacterial genus *Arthrospira*: Actual state of the art in cultivation methods, genetics, and application in medicine. *Frontiers in Microbiology*, 8(DEC), 1–21. <https://doi.org/10.3389/fmicb.2017.02541>
- Gao, L., Ding, W., Xi, J., Gao, S., Zhou, X., Chen, Y., Song, K., Mao, X., Tu, R., & Jiang, G. (2023). Effects of different nitrogen/phosphorus ratios on the growth and metabolism of microalgae *Scenedesmus obliquus* cultured in the mixed wastewater from primary settling tank and sludge thickener. *Process Safety and Environmental Protection*, 170(December 2022), 824–833. <https://doi.org/10.1016/j.psep.2022.12.059>
- García-Malea, M. C., Acién, F. G., Fernández, J. M., Cerón, M. C., & Molina, E. (2006). Continuous production of green cells of *Haematococcus pluvialis*: Modeling of the irradiance effect. *Enzyme and Microbial Technology*, 38(7), 981–989. <https://doi.org/10.1016/j.enzmictec.2005.08.031>
- Greene, J. M., Quiroz, D., Compton, S., Lammers, P. J., & Quinn, J. C. (2021a). A validated thermal and biological model for predicting algal productivity in large scale outdoor cultivation systems. *Algal Research*, 54. <https://doi.org/10.1016/j.algal.2021.102224>
- Greene, J. M., Quiroz, D., Compton, S., Lammers, P. J., & Quinn, J. C. (2021b). A validated thermal and biological model for predicting algal productivity in large scale outdoor cultivation systems. *Algal Research*, 54(February). <https://doi.org/10.1016/j.algal.2021.102224>
- Grimaud, G. M., Mairet, F., Sciandra, A., & Bernard, O. (2017). Modeling the temperature effect on the specific growth rate of phytoplankton: a review. *Reviews in Environmental Science and Biotechnology*, 16(4), 625–645. <https://doi.org/10.1007/s11157-017-9443-0>
- Hirayama, A., Sueyoshi, M. N., Nakano, T., Ota, Y., Kurita, H., Tasaki, M., Kuroiwa, Y., Kato, T., Serizawa, S., Kojima, K., Al-Maamari, R. S., Hasegawa, T., Thomas-Hall, S. R., & Schenk, P. M. (2022). Development of large-scale microalgae production in the Middle East. *Bioresource Technology*, 343(September 2021), 126036. <https://doi.org/10.1016/j.biortech.2021.126036>
- Huesemann, M., Crowe, B., Waller, P., Chavis, A., Hobbs, S., Edmundson, S., & Wigmosta, M. (2016). A validated model to predict microalgae growth in outdoor pond cultures subjected to fluctuating light intensities and water temperatures. *Algal Research*, 13, 195–206. <https://doi.org/10.1016/j.algal.2015.11.008>
- Ifrim, G. A., Barbu, M., Ceanga, E., & Caraman, S. (2015). Modeling and control of a multipurpose biotechnological plant. Photobioreactor modeling. *2015 19th*

International Conference on System Theory, Control and Computing, ICSTCC 2015 - Joint Conference SINTES 19, SACCS 15, SIMSIS 19, October 2015, 55–60.
<https://doi.org/10.1109/ICSTCC.2015.7321269>

Ippoliti, D., Gómez, C., del Mar Morales-Amaral, M., Pistocchi, R., Fernández-Sevilla, J. M., & Acién, F. G. (2016). Modeling of photosynthesis and respiration rate for *Isochrysis galbana* (T-Iso) and its influence on the production of this strain. *Bioresource Technology*, 203, 71–79.
<https://doi.org/10.1016/j.biortech.2015.12.050>

Ishika, T., Moheimani, N. R., & Bahri, P. A. (2017). Sustainable saline microalgae co-cultivation for biofuel production: A critical review. *Renewable and Sustainable Energy Reviews*, 78(May), 356–368. <https://doi.org/10.1016/j.rser.2017.04.110>

Ishika, T., Moheimani, N. R., Bahri, P. A., Laird, D. W., Blair, S., & Parlevliet, D. (2017). Halo-adapted microalgae for fucoxanthin production: Effect of incremental increase in salinity. *Algal Research*, 28(November), 66–73.
<https://doi.org/10.1016/j.algal.2017.10.002>

James, S. C., & Boriah, V. (2010). Modeling algae growth in an open-channel raceway. *Journal of Computational Biology*, 17(7), 895–906.
<https://doi.org/10.1089/cmb.2009.0078>

Jayaraman, S. K., & Rhinehart, R. R. (2015). Modeling and optimization of algae growth. *Industrial and Engineering Chemistry Research*, 54(33), 8063–8071.
<https://doi.org/10.1021/acs.iecr.5b01635>

Jeon, Y. C., Cho, C. W., & Yun, Y. S. (2005). Measurement of microalgal photosynthetic activity depending on light intensity and quality. *Biochemical Engineering Journal*, 27(2), 127–131. <https://doi.org/10.1016/j.bej.2005.08.017>

Klausmeier, C. A., Litchman, E., & Levin, S. A. (2004). Phytoplankton growth and stoichiometry under multiple nutrient limitation. *Limnology and Oceanography*, 49(4 II), 1463–1470. https://doi.org/10.4319/lo.2004.49.4_part_2.1463

Kumar, M., Kulshreshtha, J., & Singh, G. P. (2011). Growth and biopigment accumulation of cyanobacterium. *Brazilian Journal of Microbiology*, 42, 1128–1135.

Kurpan, D., Idà, A., Körner, F. G., Bombelli, P., da Silva Aguiar, J. P., Marinho, L. M., do Valle, A. F., Acién, F. G., Trasatti, S. P., & Schievano, A. (2024). Long-term evaluation of productivity and harvesting efficiency of an industrial *Spirulina* (*Arthrospira platensis*) production facility. *Bioresource Technology Reports*, 25(November 2023).
<https://doi.org/10.1016/j.biteb.2023.101741>

Lee, E., Jalalizadeh, M., & Zhang, Q. (2015). *Growth kinetic models for microalgae cultivation : A review*. 12, 497–512. <https://doi.org/10.1016/j.algal.2015.10.004>

- Lee, E., & Zhang, Q. (2016). Integrated co-limitation kinetic model for microalgae growth in anaerobically digested municipal sludge centrate. *Algal Research*, 18, 15–24. <https://doi.org/10.1016/j.algal.2016.05.019>
- Li, W., Liu, M., Zhang, X., & Yang, J. (2022). The influences of a variety of light parameters on biomass production and energy efficiency of *Spirulina*. *2022 Joint Conference - 11th International Conference on Energy Efficiency in Domestic Appliances and Lighting and 17th International Symposium on the Science and Technology of Lighting, EEDAL/LS:17 2022*, 1–6.
- Lobry, J. R., Flandrois, J. P., Carret, G., & Pave, A. (1992). Monod's bacterial growth model revisited. *Bulletin of Mathematical Biology*, 54(1), 117–122. <https://doi.org/10.1007/BF02458623>
- Lobry, J., Rosso, L., & Flandrois, J. (1991). A fortran subroutine for the determination of parameter confidence limits in non-linear models. In *Binary* (Vol. 3, pp. 86–93).
- Luangpipat, T., & Chisti, Y. (2017). Biomass and oil production by *Chlorella vulgaris* and four other microalgae — Effects of salinity and other factors. *Journal of Biotechnology*, 257, 47–57. <https://doi.org/10.1016/j.jbiotec.2016.11.029>
- Marsullo, M., Mian, A., Ensinas, A. V., Manente, G., Lazzaretto, A., & Marechal, F. (2015). Dynamic modeling of the microalgae cultivation phase for energy production in open raceway ponds and flat panel photobioreactors. *Frontiers in Energy Research*, 3(SEP), 1–18. <https://doi.org/10.3389/fenrg.2015.00041>
- Martínez Sancho, M. E., Jiménez Castillo, J. M., & El Yousfi, F. (1997). Influence of phosphorus concentration on the growth kinetics and stoichiometry of the microalga *Scenedesmus obliquus*. *Process Biochemistry*, 32(8), 657–664. [https://doi.org/https://doi.org/10.1016/S0032-9592\(97\)00017-4](https://doi.org/https://doi.org/10.1016/S0032-9592(97)00017-4)
- Ming-Ju Chen, Kreuter, J. Y.-T. K. (1996). Nanoparticles and microparticles for drug and vaccine delivery. *Journal of Anatomy*, 189 (Pt 3(ii)), 503–505. <https://doi.org/10.1002/bit>
- Mohamed, R. M. S. R., Apandi, N., Miswan, M. S., Gani, P., Al-Gheethi, A. A. S., Kassim, A. H. M., & Fitriani, N. (2019). Effect of pH and light intensity on the growth and biomass productivity of microalgae *scenedesmus* sp. *Ecology, Environment and Conservation*, 25(May), S1–S5.
- Mohseni, A., Fan, L., & Roddick, F. A. (2021). Impact of microalgae species and solution salinity on algal treatment of wastewater reverse osmosis concentrate. *Chemosphere*, 285(June), 131487. <https://doi.org/10.1016/j.chemosphere.2021.131487>
- Moisan, J. R., Moisan, T. A., & Abbott, M. R. (2002). Modelling the effect of temperature on the maximum growth rates of phytoplankton populations. *Ecological Modelling*,

153(3), 197–215. [https://doi.org/10.1016/S0304-3800\(02\)00008-X](https://doi.org/10.1016/S0304-3800(02)00008-X)

Molina-Miras, A., Bueso-Sánchez, A., Cerón-García, M. del C., Sánchez-Mirón, A., Contreras-Gómez, A., & García-Camacho, F. (2022). Effect of Nitrogen, Phosphorous, and Light Colimitation on Amphidinol Production and Growth in the Marine Dinoflagellate Microalga *Amphidinium carterae*. *Toxins*, 14(9). <https://doi.org/10.3390/toxins14090594>

Molina Grima, E., Sánchez Pérez, J. A., Garcia Camacho, F., Garcia Sánchez, J. L., & López Alonso, D. (1993). n-3 PUFA productivity in chemostat cultures of microalgae. *Applied Microbiology and Biotechnology*, 38(5), 599–605. <https://doi.org/10.1007/BF00182796>

Muloiwa, M., Nyende-Byakika, S., & Dinka, M. (2020). Comparison of unstructured kinetic bacterial growth models. *South African Journal of Chemical Engineering*, 33(May 2019), 141–150. <https://doi.org/10.1016/j.sajce.2020.07.006>

Noll, P., Lilge, L., Hausmann, R., & Henkel, M. (2020). Modeling and exploiting microbial temperature response. *Processes*, 8(1). <https://doi.org/10.3390/pr8010121>

Ova Ozcan, D., & Ovez, B. (2020). Evaluation of the interaction of temperature and light intensity on the growth of *Phaeodactylum tricornutum*: Kinetic modeling and optimization. *Biochemical Engineering Journal*, 154(March 2019), 107456. <https://doi.org/10.1016/j.bej.2019.107456>

Packer, A., Li, Y., Andersen, T., Hu, Q., Kuang, Y., & Sommerfeld, M. (2011). Growth and neutral lipid synthesis in green microalgae: A mathematical model. *Bioresource Technology*, 102(1), 111–117. <https://doi.org/10.1016/j.biortech.2010.06.029>

Peeters, J. C. H., & Eilers, P. (n.d.). *The relationship between light intensity and photosynthesis - a simple mathematical model*. 134–136.

Perez-Garcia, O., Escalante, F. M. E., de-Bashan, L. E., & Bashan, Y. (2011). Heterotrophic cultures of microalgae: Metabolism and potential products. *Water Research*, 45(1), 11–36. <https://doi.org/10.1016/j.watres.2010.08.037>

Prado, L. de O., Bolzani, H. R., Souza, H. H. de S., Ruas, G., & da Silva, G. H. R. (2023). Microalgal cultivation in open and closed systems under a tropical climate: A life cycle comparison. *Journal of Cleaner Production*, 422(April), 138631. <https://doi.org/10.1016/j.jclepro.2023.138631>

Pugkaew, W., Meetam, M., Yokthongwattana, K., Leeratsuwan, N., & Pokethitiyook, P. (2019). Effects of salinity changes on growth, photosynthetic activity, biochemical composition, and lipid productivity of marine microalga *Tetraselmis suecica*. *Journal of Applied Phycology*, 31(2), 969–979. <https://doi.org/10.1007/s10811-018-1619-7>

- Qiu, R., Gao, S., Lopez, P. A., & Ogden, K. L. (2017). Effects of pH on cell growth, lipid production and CO₂ addition of microalgae *Chlorella sorokiniana*. *Algal Research*, 28(November), 192–199. <https://doi.org/10.1016/j.algal.2017.11.004>
- Quinn, J., de Winter, L., & Bradley, T. (2011a). Microalgae bulk growth model with application to industrial scale systems. *Bioresource Technology*, 102(8), 5083–5092. <https://doi.org/10.1016/j.biortech.2011.01.019>
- Quinn, J., de Winter, L., & Bradley, T. (2011b). Microalgae bulk growth model with application to industrial scale systems. *Bioresource Technology*, 102(8), 5083–5092. <https://doi.org/10.1016/j.biortech.2011.01.019>
- Rarrek, A., Mostertz, M., Kistenmacher, H., Rehfeldt, S., & Klein, H. (2016). Simulation and optimization of large open algae ponds. *Chemical Engineering Research and Design*, 114(1983), 220–235. <https://doi.org/10.1016/j.cherd.2016.08.018>
- Ratkowsky, D. A., Lowry, R. K., McMeekin, T. A., Stokes, A. N., & Chandler, R. E. (1983). Model for bacterial culture growth rate throughout the entire biokinetic temperature range. *Journal of Bacteriology*, 154(3), 1222–1226. <https://doi.org/10.1128/jb.154.3.1222-1226.1983>
- Ratkowsky, D. A., Olley, J., McMeekin, T. A., & Ball, A. (1982). Relationship between temperature and growth rate of bacterial cultures. *Journal of Bacteriology*, 149(1), 1–5. <https://doi.org/10.1128/jb.149.1.1-5.1982>
- Ritchie, R. J. (2010). Modelling photosynthetic photon flux density and maximum potential gross photosynthesis. *Photosynthetica*, 48(4), 596–609. <https://doi.org/10.1007/s11099-010-0077-5>
- Rozenberg, J. M., Sorokin, B. A., Mukhambetova, A. N., Emelianova, A. A., Kuzmin, V. V., Belogurova-Ovchinnikova, O. Y., & Kuzmin, D. V. (2024). Recent advances and fundamentals of microalgae cultivation technology. *Biotechnology Journal*, 19(3). <https://doi.org/10.1002/biot.202300725>
- Sakarika, M., & Kornaros, M. (2016). Effect of pH on growth and lipid accumulation kinetics of the microalga *Chlorella vulgaris* grown heterotrophically under sulfur limitation. *Bioresource Technology*, 219, 694–701. <https://doi.org/10.1016/j.biortech.2016.08.033>
- Schipper, K., Al-Jabri, H. M. S. J., Wijffels, R. H., & Barbosa, M. J. (2021). Techno-economics of algae production in the Arabian Peninsula. *Bioresource Technology*, 331(March). <https://doi.org/10.1016/j.biortech.2021.125043>
- Schipper, L. A., Hobbs, J. K., Rutledge, S., & Arcus, V. L. (2014). Thermodynamic theory explains the temperature optima of soil microbial processes and high Q₁₀ values at low temperatures. *Global Change Biology*, 20(11), 3578–3586. <https://doi.org/10.1111/gcb.12596>

- Serra-Maia, R., Bernard, O., Gonçalves, A., Bensalem, S., & Lopes, F. (2016). Influence of temperature on *Chlorella vulgaris* growth and mortality rates in a photobioreactor. *Algal Research*, 18, 352–359. <https://doi.org/10.1016/j.algal.2016.06.016>
- Society, A. (1978). Effects of N : P Atomic Ratios and Nitrate Limitation on Algal Growth , Cell Composition , and Nitrate Uptake. *Limnology and Oceanography*, 23(1), 10–25. <http://www.jstor.org/stable/2835687>
- Sommer, U. (1991). A Comparison of the Droop and the Monod Models of Nutrient Limited Growth Applied to Natural Populations of Phytoplankton. *Functional Ecology*, 5(4), 535. <https://doi.org/10.2307/2389636>
- Talbot, P., Thébault, J. M., Dauta, A., & de la Noüe, J. (1991). A comparative study and mathematical modeling of temperature, light and growth of three microalgae potentially useful for wastewater treatment. *Water Research*, 25(4), 465–472. [https://doi.org/10.1016/0043-1354\(91\)90083-3](https://doi.org/10.1016/0043-1354(91)90083-3)
- Tan, J. Sen, Lee, S. Y., Chew, K. W., Lam, M. K., Lim, J. W., Ho, S. H., & Show, P. L. (2020). A review on microalgae cultivation and harvesting, and their biomass extraction processing using ionic liquids. *Bioengineered*, 11(1), 116–129. <https://doi.org/10.1080/21655979.2020.1711626>
- Torzillo, G., Bernardini, P., & Masojídek, J. (1998). On-line monitoring of chlorophyll fluorescence to assess the extent of photoinhibition of photosynthesis induced by high oxygen concentration and low temperature and its effect on the productivity of outdoor cultures of *Spirulina platensis* (Cyanobacteria). *Journal of Phycology*, 34(3), 504–510. <https://doi.org/10.1046/j.1529-8817.1998.340504.x>
- Vo, H. N. P., Ngo, H. H., Guo, W., Liu, Y., Chang, S. W., Nguyen, D. D., Nguyen, P. D., Bui, X. T., & Ren, J. (2019). Identification of the pollutants' removal and mechanism by microalgae in saline wastewater. *Bioresource Technology*, 275(October 2018), 44–52. <https://doi.org/10.1016/j.biortech.2018.12.026>
- Wang, M., Kuo-Dahab, W. C., Dolan, S., & Park, C. (2014). Kinetics of nutrient removal and expression of extracellular polymeric substances of the microalgae, *Chlorella* sp. and *Micractinium* sp., in wastewater treatment. *Bioresource Technology*, 154, 131–137. <https://doi.org/10.1016/j.biortech.2013.12.047>
- Wu, Y. H., Li, X., Yu, Y., Hu, H. Y., Zhang, T. Y., & Li, F. M. (2013). An integrated microalgal growth model and its application to optimize the biomass production of *Scenedesmus* sp. LX1 in open pond under the nutrient level of domestic secondary effluent. *Bioresource Technology*, 144, 445–451. <https://doi.org/10.1016/j.biortech.2013.06.065>
- Xin, L., Hong-ying, H., Ke, G., & Ying-xue, S. (2010). Effects of different nitrogen and phosphorus concentrations on the growth, nutrient uptake, and lipid accumulation of a freshwater microalga *Scenedesmus* sp. *Bioresource Technology*, 101(14),

5494–5500. <https://doi.org/10.1016/j.biortech.2010.02.016>

Xing, D., Li, X., Wang, Y., Deng, S., Jin, C., Zhao, Y., & Guo, L. (2023). The comprehensive impact of phosphorus sources on microalgae biochemical metabolism and phosphorus transformation. *Journal of Water Process Engineering*, 51(January), 103477. <https://doi.org/10.1016/j.jwpe.2022.103477>

Yamamoto, T., Suzuki, M., Kim, K., & Asaoka, S. (2012). Growth and uptake kinetics of phosphate by benthic microalga *Nitzschia* sp. isolated from Hiroshima Bay, Japan. *Phycological Research*, 60(3), 223–228. <https://doi.org/10.1111/j.1440-1835.2012.00653.x>

Yao, B., Xi, B., Hu, C., Huo, S., Su, J., & Liu, H. (2011). A model and experimental study of phosphate uptake kinetics in algae: Considering surface adsorption and P-stress. *Journal of Environmental Sciences*, 23(2), 189–198. [https://doi.org/10.1016/S1001-0742\(10\)60392-0](https://doi.org/10.1016/S1001-0742(10)60392-0)

Ye, Y., Ma, S., Peng, H., Huang, Y., Zeng, W., Xia, A., Zhu, X., & Liao, Q. (2023). Insight into the comprehensive effect of carbon dioxide, light intensity and glucose on heterotrophic-assisted phototrophic microalgae biofilm growth: A multifactorial kinetic model. *Journal of Environmental Management*, 325(PB), 116582. <https://doi.org/10.1016/j.jenvman.2022.116582>

Yu, H., Kim, J., Rhee, C., Shin, J., Shin, S. G., & Lee, C. (2022). Effects of Different pH Control Strategies on Microalgae Cultivation and Nutrient Removal from Anaerobic Digestion Effluent. *Microorganisms*, 10(2). <https://doi.org/10.3390/microorganisms10020357>

Yuan, S., Hu, J., Liu, Z., Hong, Y., & Wang, X. (2020). Modeling microalgae growth in continuous culture: Parameters analysis and temperature dependence. *Energy*, 195, 116961. <https://doi.org/10.1016/j.energy.2020.116961>

Zhao, B., & Su, Y. (2020). Macro assessment of microalgae-based CO₂ sequestration: Environmental and energy effects. *Algal Research*, 51(August), 102066. <https://doi.org/10.1016/j.algal.2020.102066>

Zhu, B., Shen, H., Li, Y., Liu, Q., Jin, G., Han, J., Zhao, Y., & Pan, K. (2020). Large-Scale Cultivation of *Spirulina* for Biological CO₂ Mitigation in Open Raceway Ponds Using Purified CO₂ From a Coal Chemical Flue Gas. *Frontiers in Bioengineering and Biotechnology*, 7(January), 1–8. <https://doi.org/10.3389/fbioe.2019.00441>

Vitae

Name : Hussain Ali Mohammed AlSubaie

Nationality : Saudi Arabia

Date of Birth : 5/16/1991

Email : kfu211522236@gmail.com

Address : FAAB5229 Baldat Al Halilah 179, Al Halilah,
JAWATHA

Academic Background:

- **Bachelor degree in Chemical Engineering**, King Faisal University, KSA,
August 2015
- **Master degree in Chemical Engineering**, King Fahd University of Petroleum &
Minerals, KSA, December 2024

Work Experience:

- Summer Trainee at Saudi Aramco at Oil and Gas Reservoir Management
Department Specially in Logging.
June-July 2015
- Working as Laboratory Technician in KFUPM (Chemical Engineering
department)
August ,2016 - currently

Computer Literacy:

- Programming and Simulation Software (ASPEN, MATLAB)
- Microsoft Packages (Excel, Word, and Power point)

Senior Project:

Production of Di-Methyl ether(C_2H_6O) from Dehydration of Methanol. The main parts of Senior Project are General Information about project, Material and Energy Balance, Equipment Design and Cost Estimation

Publications:

- Khairuddin, N.F.M., Farooq, W., **AlSubaie, H.A.** et al. Nitrate influence on Scenedesmus Obliquus growth and organic removal from produced water. Int. J. Environ. Sci. Technol. (2024). <https://doi.org/10.1007/s13762-024-06267-3>

Manuscript in Progressing:

- Microalgae-driven treatment of Produced water from Oil and Gas exploration; A comparison between Real and Synthetic produced water
- Modeling large-scale Microalgae Growth in Open Raceway Ponds under Saudi Arabia Climate Conditions