

# CAVITY-MEDIATED LASER COOLING

BY

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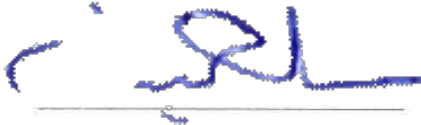
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*Dedicated to*  
*My Beloved Mother.*

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## LIST OF ABBREVIATIONS

|             |   |                                          |
|-------------|---|------------------------------------------|
| <b>CMLC</b> | : | Cavity Mediated Laser Cooling            |
| <b>QTT</b>  | : | Quantum Trajectory Theory                |
| <b>QTM</b>  | : | Quantum Trajectory Method                |
| <b>QMWF</b> | : | Quantum Monte-Carlo Wave Function Method |
| <b>TLS</b>  | : | Two-level system                         |

# ABSTRACT

Full Name : Faiz Khan

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This thesis explores cavity-mediated laser cooling (CMLC) of a single two-level atom, a laser cooling method that surpasses traditional laser cooling techniques by confining the atom strongly within an optical cavity. This method enables unprecedented control over an atom's motion and temperature, achieving sub-recoil level cooling. The dynamics of a two-level system in the so-called Lamb-Dicke regime are studied using both the quantum Monte-Carlo wave function method and the Lindblad master equation. Our results demonstrate the existence of three resonant atom-cavity detuning values for which the stationary state phonon number of the system exhibits a sharp minimum. For these detuning values, the atom can be cooled below the sub-recoil limit, reaching its ground state.

We also analyzed the cooling rate and phonon number distribution for the cavity-mediated laser cooling. These results provide critical insight into the efficiency and fundamental limits of cavity-mediated laser cooling in the Lamb-Dicke regime over a wide range of parameters.

## ملخص الرسالة

الاسم الكامل: فايز خان

عنوان الرسالة: التبريد بالليزر بواسطة التجايف

المجال الرئيسي: [الفيزياء]

تاريخ الحصول على الدرجة: [ديسمبر 2024]

لذرة مفردة ذات مستويين، وهي تقنية تبريد (CMLC) تتناول هذه الرسالة دراسة التبريد بالليزر بواسطة التجايف بالليزر تتفوق على الأساليب التقليدية من خلال حصر الذرة بشكل قوي داخل تجويف بصري. تتيح هذه الطريقة تحكمًا غير مسبوق في حركة الذرة ودرجة حرارتها، حيث تحقق تبريدًا إلى مستويات أقل من حدود الارتداد.

تمت دراسة ديناميكيات نظام المستويين في ما يعرف بنظام لامب-ديك باستخدام كل من طريقة دالة موننت كارلو الموجية الكمومية ومعادلة ليندبلاد الرئيسية. أظهرت النتائج وجود ثلاث قيم مميزة لتحويلات الرنين بين الذرة والتجويف، حيث يُظهر العدد الثابت للفونونات في النظام حدًا أدنى حادًا. عند هذه القيم، يمكن تبريد الذرة إلى ما دون حد الارتداد، مما يجعلها تصل إلى حالتها الأساسية.

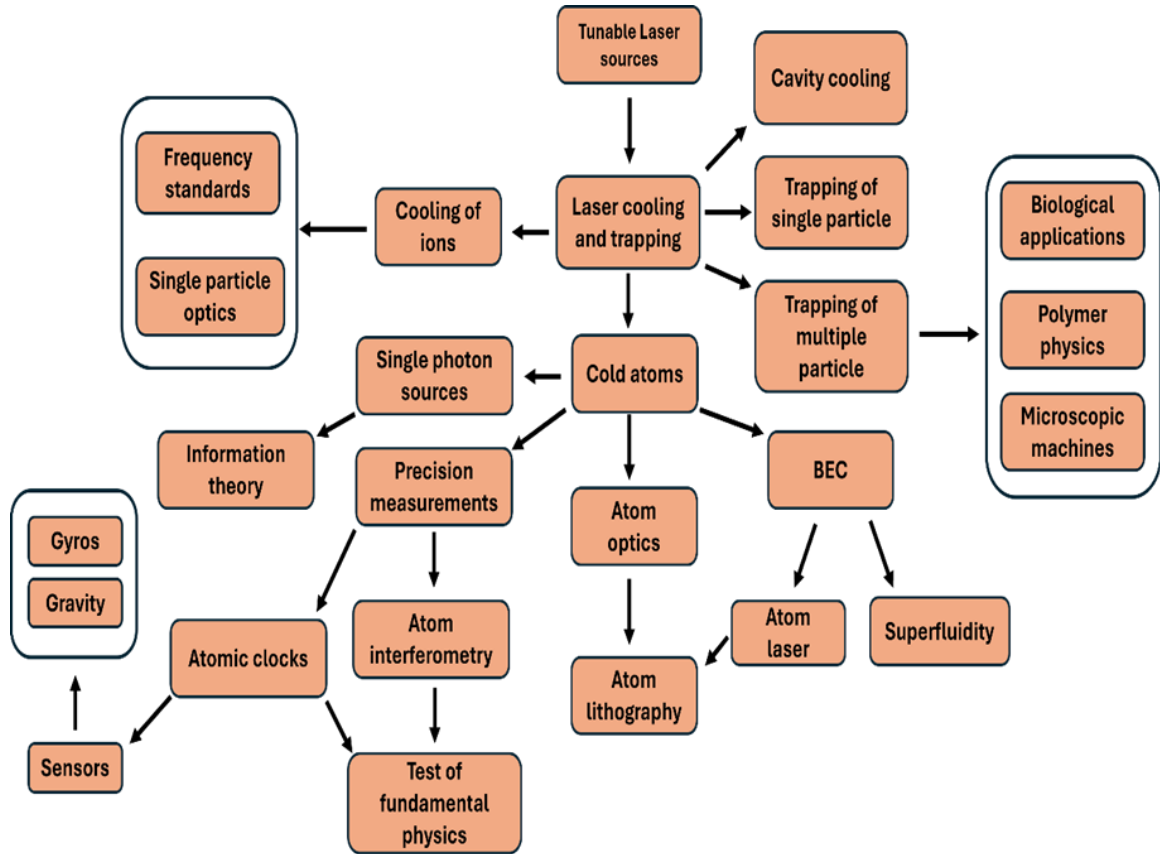
كما قمنا بتحليل معدل التبريد وتوزيع عدد الفونونات في تقنية التبريد بالليزر بواسطة التجايف. توفر هذه النتائج رؤى مهمة حول كفاءة وحدود التبريد بالليزر بواسطة التجايف في نظام لامب-ديك ضمن نطاق واسع من المتغيرات.

# CHAPTER 1

## INTRODUCTION

The study of light-matter interaction has long been a foundational area of research in physics, sparking developments across numerous research fields. Within this domain, one prominent area of research involves the use of light to control and manipulate matter particles, especially through cooling techniques that reduce the thermal motion of particles [1][2]. Lowering the temperature of an ensemble of particles not only enables greater control over their behavior but also opens pathways to observe quantum phenomena. The ability to probe and manipulate particles at extremely low temperatures has proven invaluable in fields like quantum information science, spectroscopy, and quantum computing [3]. With the development of theory of quantum mechanics in the early 1900s, scientists devised methods to examine the quantum interactions between light and matter, setting the stage for the development of techniques to cool atoms to lower temperatures. For effective interaction at the quantum level, it is ideal for the particle's speed to approach zero, since lower velocities and hence lower temperatures reduce thermal disturbances that would otherwise prevent quantum effects from manifestation [4]. However, traditional cooling methods-such as air cooling, water cooling, and evaporative cooling-were inadequate for reaching the extremely low temperatures required to access the quantum regime [5]. These methods failed to sufficiently reduce particle motion, leaving scientists in search of alternative techniques. In the early 20th century, Planck's discovery that

radiation is 'quantized' led to the concept of "light quanta" (photons), each carrying a discrete packet of energy. These photons, by virtue of their momentum, exert radiation pressure on particles. This interaction suggested that light could potentially be used to influence and even control the movement of particles. Building on this concept, Schawlow and Hänsch proposed in 1975 [6] that a laser with precisely controlled intensity could exert significant force on an atom, making it a reliable tool for cooling. Around the same time, Wineland and Dehmelt suggested a similar approach for cooling trapped ions using lasers [7]. In 1978, Ashkin extended these ideas by introducing a method to cool and manipulate neutral atoms using laser light [8]. These methods laid the groundwork for Doppler cooling, a laser cooling technique based on the Doppler shift principle, where the laser frequency is tuned to interact with moving particles in a way that reduces their speed. Initially, laser cooling experiments focused on sodium atoms because of their near-ideal two-level system structure, which simplified the process of driving cooling transitions with the lasers available at the time. As laser technology advanced and more tunable lasers were available, researchers expanded their focus to include heavier atoms like rubidium (Rb)



**Figure 1-1 Emergence of new research fields and applications after development of tunable laser sources [9].**

and cesium (Cs), enabling them to explore more complex systems [10]. Over time, laser cooling techniques improved, achieving temperatures in the microkelvin range and finding applications across various fields, leading to the emergence of new research areas (as illustrated in Figure 1-1).

The fundamental principle of Doppler laser cooling is based on the conservation of energy and momentum during photon absorption and emission processes. Although this mechanism seems relatively simple, further investigations revealed unexpected complexities. For instance, some experiments demonstrated temperatures below the Doppler limit, the theoretically predicted minimum temperature achieved with Doppler cooling, indicating the presence of additional cooling mechanisms. This unexpected result

sparked new research, leading to the discovery that factors such as the multi-level structure of atoms and spatial variations in light polarization contributed significantly to the cooling process. These insights led to the development of sub-Doppler cooling techniques, such as *polarization gradient cooling*, which further reduced temperatures to around  $10^{-5}$  K [11]. Despite its effectiveness, sub-Doppler cooling encountered an intrinsic temperature limit called the *recoil limit*, where atoms could not be cooled beyond recoil energy gained from recoil of a photon. This recoil limit generally corresponds to temperatures between  $10^{-7}$  and  $10^{-6}$  K. Further advancements led to more sub-recoil cooling methods, including sideband cooling and cavity cooling, which enabled even lower temperatures by allowing atoms to scatter photons until they nearly reached zero velocity[12][13]. These advanced methods achieved three-dimensional cooling to temperatures below  $10^{-6}$  K.

The main objective of cooling techniques is to enhance the phase space density of particles, defined as  $n\lambda^3$ , where  $n$  is the particle density and  $\lambda$  is the thermal de Broglie wavelength. A higher phase space density, characterized by a tightly confined spatial and velocity distribution, is essential for many atomic physics experiments. Additionally, the phase space density determines whether an atomic ensemble behaves as a classical or quantum gas. When  $n\lambda^3$  surpasses the critical value of 2.62, the system enters the quantum degenerate regime, where quantum effects dominate, where Bose-Einstein condensation and other quantum effects like superfluidity can emerge, allowing neighboring bosonic atoms' wavefunctions to overlap [14]. Although laser cooling can significantly enhance phase space density, combining it with trapping techniques amplifies its effectiveness. The first simultaneous demonstration of cooling and trapping, achieved in 1987 using combined light and magnetic fields, led to the creation of the magneto-optical trap (MOT) [15]. This

device became essential in cold atom research, trapping up to  $10^8$  atoms at temperatures between 10 and  $100\mu\text{K}$  and reaching densities of around  $10^{12}/\text{cm}^3$ . However, despite increasing the phase space density by about  $10^{12}$  times compared to a standard atomic beam, the MOT did not reach the quantum degenerate regime due to limitations posed by near-resonant light, which restricted both the density and temperature of the trapped particles. Understanding these constraints has led researchers to explore alternative laser cooling techniques, each with its own set of challenges. Most traditional laser cooling methods rely on cycles of optical pumping and spontaneous emission. While spontaneous emission helps reduce the atom's entropy by emitting photons in random directions, it also introduces a fundamental limit to cooling [2]. Each emitted photon imparts a small momentum kick to the atom, setting a lower bound on the achievable temperature. These inherent limitations have prompted the scientific community to seek new approaches for cooling atoms to even lower temperatures.

In recent years, a promising solution has emerged in the form of cavity-mediated laser cooling. This approach replaces spontaneous emission as the primary dissipative mechanism with the escape of photons from an optical cavity [16]. By coupling a single atom to a high-finesse cavity, the process of cooling is no longer limited by the spontaneous emission rate  $\gamma$  but instead by the cavity decay rate  $\kappa$ . The cavity decay rate  $\kappa$  can be precisely controlled, unlike  $\gamma$ , which is determined by the atomic structure and difficult to alter. This flexibility allows researchers to make  $\kappa$  sufficiently small, enhancing the cooling effect and pushing the system to much lower temperatures than previously achievable. Cavity cooling also has the advantage of applicability to particles with complex internal level structures, including atoms and molecules that would otherwise be challenging to

cool using traditional methods. The first demonstration of cavity cooling for a single atom was carried out by Maunz, Puppe, Schuster, Syassen, Pinkse and Rempe, who showed that cavity cooling not only extends the storage time of atoms but also improves localization within the cavity [17]. Since then, several research groups, including those led by Rempe [18] and Vuletić [19], have conducted systematic studies on this technique. Recent advancements in atom-cavity experiments have expanded the achievable range of parameters, leading to innovations such as optical ring cavities [20], atom chips [21], atomic conveyor belts [22], and ion traps [23].

The technique of CMLC enhances the effectiveness of laser cooling and extends its reach to particle types and quantum systems that were previously inaccessible. By achieving high control over temperature and particle motion, cavity-mediated cooling has become an invaluable tool for exploring quantum mechanics at new depths and creating conditions conducive to studying complex quantum phenomena. By integrating high-finesse optical cavities with laser cooling, scientists have developed a method that transcends conventional limitations. In this setup, an atom or particle inside the cavity interacts with a highly controlled optical field, which cools it and stabilizes its position within the cavity. The optical cavity traps light in a defined space, enhancing the interaction between the particle and the light field and creating an exceptionally effective cooling environment [24].

An extraordinary feature of cavity-mediated cooling is its ability to localize particles with exceptional precision. In conventional cooling methods, random spontaneous emissions lead to momentum kicks in unpredictable directions, limiting localization. In contrast, cavity cooling confines particles to specific regions within the cavity, as photon emission

occurs in well-defined modes aligned with the cavity's structure. This increased precision has paved the way for achieving quantum control over particles, which is essential for applications in quantum computing and quantum communication [17]. Another advantage of cavity-mediated cooling is its applicability to systems with complex or multilevel internal structures, such as molecules and large atomic clusters. Traditional laser cooling techniques are typically effective only for simple two-level systems or atoms with straightforward electronic structures. However, the cavity environment allows for controlled photon exchange that can be fine-tuned to accommodate more complicated systems [25]. This makes it possible to cool particles that were previously too challenging to address with laser cooling alone.

Furthermore, the sub-recoil temperatures achievable with cavity cooling are highly beneficial in experiments requiring ultra-precise measurements or interference setups where thermal motion would otherwise introduce noise. By reaching temperatures below the recoil limit, cavity cooling brings particles into a regime where quantum effects—such as wave-particle duality and coherence can be observed and harnessed more clearly [17]. By incorporating cavity-mediated cooling into experimental setups, researchers have significantly broadened the horizons of quantum research. This technique enables the study of quantum phenomena with unprecedented precision and control, opening new avenues in fields like quantum computing, communication, and fundamental physics.

In Chapter 2, we lay out the foundation of our theoretical framework by deriving the Jaynes-Cummings Hamiltonian, which models light-matter interaction starting from fundamental principles. This chapter will define the interaction framework we rely on throughout the thesis, allowing us to explore how atoms interact with quantized fields.

Special attention is given to how this Hamiltonian changes when an atom is placed inside a cavity, setting the stage for understanding cavity-mediated cooling.

In Chapter 3, we dive into laser cooling techniques, examining the fundamental forces at play when radiation interacts with atomic systems. We describe methods like Doppler cooling and Sisyphus cooling, analyzing their efficacy in reducing atomic motion and cooling down particles to near absolute zero. By discussing these techniques in a semiclassical framework, we provide a clear picture of how cooling forces are generated, particularly through radiation pressure and dipole interactions. The challenges and limits of these methods, such as the Doppler limit and recoil energy considerations, will also be addressed.

Chapter 4 introduces the theory of open quantum systems, essential for handling real-world quantum systems that interact with external environments. We explore the master equation approach to model these interactions, but the focus shifts towards Quantum Trajectory Theory (QTT) as a more effective tool for simulating open quantum systems. By demonstrating QTT's effectiveness in resolving dissipation in quantum systems, we show its advantage over conventional density matrix approaches, especially for systems with complex, time-dependent dynamics. Examples are provided to illustrate QTT's applicability and its potential in tracking single quantum events.

In Chapter 5, we apply the theoretical tools developed in the previous chapters to study cooling mechanisms for a single particle confined within an optical cavity. Starting with the derivation of the system's Hamiltonian, we treat the setup as an open quantum system and employ both QTT and master equation methods to simulate cooling dynamics.

In Chapter 6 we discuss the results of our simulations by analyzing how cooling is achieved in the cavity by examining phonon and photon modes, atom-cavity detuning, and various cooling rates, providing both numerical and analytical insights. By comparing the simulations to the theoretical predictions, we highlight the effectiveness of cavity-mediated cooling and explore how adjusting cavity parameters impacts the cooling process.

Finally, Chapter 7 concludes by summarizing the primary findings and discussing the broader implications of this research in the fields of quantum control and low-temperature physics. We will also outline potential avenues for future work, particularly in optimizing cavity-based cooling techniques and extending these findings to more complex multi-level systems or molecular cooling applications.

## CHAPTER 2

### Quantum Mechanics of Light-Matter Interaction

In this chapter, I will describe the dynamics of light-matter interactions using the Jaynes-Cummings model. This model is fundamental in quantum optics for understanding how a two-level atom interacts with a single mode of a quantized electromagnetic field, typically within an optical cavity. The Jaynes-Cummings Hamiltonian comprises three terms: one representing the energy levels of the atom,  $H_{atom}$ , one for the quantized field mode (photons)  $H_{cavity}$ , and one for the interaction between the atom and the field,  $H_{atom-cavity}$ :

$$H_{jc} = H_{atom} + H_{cavity} + H_{atom-cavity} \quad (2.1)$$

#### 2.1 Hamiltonian for the quantized cavity field

A detailed discussion of this section can be found in [26]. We start the derivation of cavity Hamiltonian by quantizing the free radiation field. The main difference between the classical theory of the radiation field and its quantum counterpart is that, in quantum theory, classical quantities are replaced by operators that obey specific commutation relations. For example, the classical harmonic oscillator is replaced by a quantum harmonic oscillator, and the classical position and momentum variables become operators satisfying the Heisenberg uncertainty principle.

Consider a radiation field confined to a cavity. The Maxwell equation for electric field,  $\mathbf{E}$  and magnetic field  $\mathbf{B}$  may be given by,

$$\begin{aligned}
\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J} \\
\nabla \cdot \mathbf{B} &= 0 \\
\varepsilon_0 \nabla \cdot \mathbf{E} &= \sigma.
\end{aligned} \tag{2.2}$$

Where  $\mathbf{J}$  is current density,  $\sigma$  is charge density,  $\mu_0$  is the vacuum permeability, and  $\varepsilon_0$  is the vacuum permittivity. Letters written in bold are assumed to be vectors. In terms of vector and scalar potential  $\mathbf{A}$  and  $\varphi$ , the electric and magnetic field may be written as,

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \tag{2.3}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{2.4}$$

With these definitions of fields, the Maxwell's equations become,

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} + \frac{1}{c^2} \frac{\partial}{\partial t} \nabla\varphi + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \mu_0 \mathbf{J}$$

$$\varepsilon_0 \left( \nabla^2 \varphi + \nabla \cdot \frac{\partial \mathbf{A}}{\partial t} \right) = -\sigma \tag{2.5}$$

These two equations are known as *Maxwell's Field Equations*. Under the so-called *Coulomb gauge*, electric and magnetic fields of equation (2.4) remain invariant if we replace  $\{ \mathbf{A}, \varphi \}$  by  $\{ \mathbf{A}', \varphi' \}$  related by the gauge transformation,  $\mathbf{A} = \mathbf{A}' - \nabla\Theta$  and  $\varphi = \varphi' + \frac{\partial\Theta}{\partial t}$ . Where gauge function  $\Theta$  is an arbitrary function of position  $\mathbf{r}$  and time  $t$  [26]. So, we have freedom to choose this gauge which imposes the condition,  $\nabla \cdot \mathbf{A} = 0$ . Then equations (2.5) become,

$$-\nabla^2 A + \frac{1}{c^2} \frac{\partial}{\partial t} \nabla \varphi + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = \mu_0 J \quad (2.6)$$

$$\varepsilon_0 (\nabla^2 \varphi) = -\sigma \quad (2.7)$$

These equations can be simplified further if we use the Helmholtz' theorem [27], which states that a vector field can always be written as a vector sum of its transverse component with zero divergence and longitudinal component with zero curl. Using this theorem for electric and magnetic field we write,

$$\mathbf{E} = \mathbf{E}_T + \mathbf{E}_L$$

$$\mathbf{J} = \mathbf{J}_T + \mathbf{J}_L$$

Using these equations along with the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ , equation (2.7) become [26],

$$-\nabla^2 A + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = \mu_0 J_T \quad (2.8)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} (\nabla \varphi) = \mu_0 J_L \quad (2.9)$$

In addition, we also have [26],

$$\begin{aligned} \mathbf{E}_T &= -\frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{E}_L &= -\nabla \varphi \end{aligned}$$

Thus, we see that transverse field components are in general described vector potential  $\mathbf{A}$  and longitudinal components by scalar potential  $\varphi$ . In a region free of charges  $\mathbf{J}_T = 0$ , we can write equation (2.7) as,

$$-\nabla^2 \mathbf{A} + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0 \quad (2.10)$$

To solve this equation inside the cavity, we apply boundary conditions appropriate for the cavity walls. For a cavity of length  $L$  with periodic boundary conditions, the allowed wave vectors  $\mathbf{k}$  are quantized:

$$\mathbf{k} = \frac{2\pi}{L} \mathbf{n} \text{ with } \mathbf{n} = (n_x, n_y, n_z) \text{ and } n_x, n_y, n_z = 0, \pm 1, \pm 2, \dots$$

The vector potential can be expanded as a sum over these allowed modes:

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\lambda=1,2} \mathbf{e}_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) \quad (2.11)$$

For each  $A_{\mathbf{k}\lambda}(\mathbf{r}, t)$ , there are two independent polarization directions that are orthogonal both to wave vector  $\mathbf{k}$  and to each other. These two polarization states correspond to  $\lambda=1$  and  $\lambda=2$ . The vector potential  $\mathbf{A}(\mathbf{r}, t)$  can be decomposed as,

$$A_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) = A_{\mathbf{k}\lambda}(t)e^{i\mathbf{k}\cdot\mathbf{r}} + A_{\mathbf{k}\lambda}^*(t)e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (2.12)$$

In free space, electromagnetic waves are transverse, meaning the electric and magnetic fields are perpendicular to the direction of propagation. Unit polarization vectors along the transverse directions,  $\mathbf{e}_{\mathbf{k}\lambda}$ , satisfy,

$$\begin{aligned} \mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{k} &= 0 \\ \mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{e}_{\mathbf{k}\lambda'} &= \delta_{\lambda,\lambda'} \end{aligned} \quad (2.13)$$

The modal components of vector potential are thus, independent of each other and both satisfy the field equation,

$$\ddot{A}_{\mathbf{k}\lambda}(t) + \omega_k^2 A_{\mathbf{k}\lambda}(t) = 0 \quad (2.14)$$

Where  $\omega_k = c |\mathbf{k}|$  is angular frequency of  $k^{th}$  mode. The general solution of equation (2.14) is:

$$A_{\mathbf{k}\lambda}(t) = A_{\mathbf{k}\lambda} e^{-i\omega_k t} \quad (2.15)$$

and equation (2.11) gives the complete modal components of vector potential,

$$A_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) = A_{\mathbf{k}\lambda} e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} + A_{\mathbf{k}\lambda}^* e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}} \quad (2.16)$$

Inserting this expression in equation (2.9), we get the complete expression for vector potential  $\mathbf{A}(\mathbf{r}, t)$ . From where the expressions for electric and magnetic fields are easily calculated using equation (2.4),

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\mathbf{k}, \lambda} \mathbf{e}_{\mathbf{k}\lambda} E_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) \quad (2.17)$$

Where,

$$E_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) = i\omega_k (A_{\mathbf{k}\lambda}(t) e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} - A_{\mathbf{k}\lambda}^*(t) e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}}) \quad (2.18)$$

and the magnetic field,

$$\mathbf{B}(\mathbf{r}, t) = \sum_{\mathbf{k}, \lambda} \frac{1}{k} \mathbf{k} \times \mathbf{e}_{\mathbf{k}\lambda} B_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) \quad (2.19)$$

With,

$$B_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) = ik (A_{\mathbf{k}\lambda}(t) e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} - A_{\mathbf{k}\lambda}^*(t) e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}}) \quad (2.20)$$

The total energy of the radiation field is given by [28],

$$H = \frac{1}{2} \int_V dV \left[ \epsilon_0 |\mathbf{E}(\mathbf{r}, t)|^2 + \frac{1}{\mu_0} |\mathbf{B}(\mathbf{r}, t)|^2 \right] \quad (2.21)$$

Substituting the expressions for  $\mathbf{E}$  and  $\mathbf{B}$  in this equation and inducing the Dirac-delta identity,

$$\int_V dV e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} dV = V\delta_{\mathbf{k},\mathbf{k}'}$$

and integrating over the cavity volume  $V$ , we find:

$$H = \sum_{\mathbf{k},\lambda} \varepsilon_0 V \omega_k^2 [A_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda}^* + A_{\mathbf{k}\lambda}^* A_{\mathbf{k}\lambda}] \quad (2.22)$$

Comparing this Hamiltonian with Hamiltonian of a harmonic oscillator,

$$H = \frac{1}{2} \hbar \omega (a_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^* + a_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda}) \quad (2.23)$$

and identifying,

$$\begin{aligned} A_{\mathbf{k}\lambda} &\rightarrow \sqrt{\frac{\hbar}{2\varepsilon_0 V \omega_k}} a_{\mathbf{k}\lambda} \\ A_{\mathbf{k}\lambda}^* &\rightarrow \sqrt{\frac{\hbar}{2\varepsilon_0 V \omega_k}} a_{\mathbf{k}\lambda}^\dagger \end{aligned} \quad (2.24)$$

where  $a_{\mathbf{k}\lambda}$  is annihilation operator,  $a_{\mathbf{k}\lambda}^\dagger$  is creation operator for the mode  $k\lambda$ . These operators satisfy the canonical commutation relations:

$$[a_{\mathbf{k}\lambda}, a_{\mathbf{k}'\lambda'}^\dagger] = \delta_{\mathbf{k}\mathbf{k}'} \delta_{\lambda\lambda'} \quad (2.25)$$

Attention must be paid to the fact that the modal components  $A_{\mathbf{k}\lambda}$ ,  $A_{\mathbf{k}\lambda}^*$  are no longer classical vector potentials instead they are now quantum operators given by,

$$A(\mathbf{r}, t) = \sum_{\mathbf{k}, \lambda} \mathbf{e}_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda}(t)(\mathbf{r}, t)$$

With,

$$A_{\mathbf{k}\lambda}(t)(\mathbf{r}, t) = \sqrt{\frac{\hbar}{2\varepsilon_0 V \omega_k}} (a_{\mathbf{k}\lambda} e^{-i\omega_k t + i\mathbf{k} \cdot \mathbf{r}} + a_{\mathbf{k}}^\dagger e^{i\omega_k t - i\mathbf{k} \cdot \mathbf{r}})$$

With these substitutions we have,

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= i \sum_{\mathbf{k}, \lambda} \mathbf{e}_{\mathbf{k}\lambda} \sqrt{\frac{\hbar \omega_k}{2\varepsilon_0 V}} (a_{\mathbf{k}\lambda} e^{-i\omega_k t + i\mathbf{k} \cdot \mathbf{r}} - a_{\mathbf{k}\lambda}^\dagger e^{i\omega_k t - i\mathbf{k} \cdot \mathbf{r}}) \\ \mathbf{B}(\mathbf{r}, t) &= i \sum_{\mathbf{k}, \lambda} \mathbf{k} \times \mathbf{e}_{\mathbf{k}\lambda} \sqrt{\frac{\hbar}{2\varepsilon_0 V \omega_k}} (a_{\mathbf{k}\lambda} e^{-i\omega_k t + i\mathbf{k} \cdot \mathbf{r}} - a_{\mathbf{k}\lambda}^\dagger e^{i\omega_k t - i\mathbf{k} \cdot \mathbf{r}}) \end{aligned} \quad (2.26)$$

Using the commutation relation of  $a_{\mathbf{k}\lambda}^\dagger$  and  $a_{\mathbf{k}\lambda}$  in equation (2.23), we get,

$$H = \sum_{\mathbf{k}, \lambda} \hbar \omega_k \left( a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + \frac{1}{2} \right) \quad (2.27)$$

This Hamiltonian is completely analogous to the Hamiltonian of quantum harmonic oscillator having a discrete equally spaced energy level spectrum. For a single mode plane wave in x-direction, we can drop the labels,  $\lambda$  and  $\mathbf{k}$  from  $a_{\mathbf{k}\lambda}$  and  $a_{\mathbf{k}\lambda}^\dagger$ . We can write,

$$\mathbf{E}(x, t) = i \left( \frac{\hbar \omega}{2\varepsilon_0 V} \right)^{\frac{1}{2}} \mathbf{e}_x [a e^{ik \cdot x - i\omega t} - a^\dagger e^{-ik \cdot x + i\omega t}] \quad (2.28)$$

In almost all practical examples of quantum optics, the fields vary negligibly over the dimensions of atom. Because the wavelength  $\lambda$ , is usually of the order of few thousand Ås

while the dimensions of the atom are of the order of Å. For example, the wavelength of visible light ranges from 3500-8000 Å while radius of Hydrogen atom is 0.53 Å and that of Cs atom is 2.62 Å. Thus, we can approximate,

$$\frac{\lambda}{2\pi} = \frac{1}{|\mathbf{k}|} \gg |\mathbf{r}_{\text{atom}}|,$$

$$|\mathbf{k}| \cdot |\mathbf{r}_{\text{atom}}| \ll 1 \quad (2.29)$$

Under this approximation, we can write,

$$e^{\pm i\mathbf{k} \cdot \mathbf{r}} \approx 1 \pm i\mathbf{k} \cdot \mathbf{r} \approx 1 \quad (2.30)$$

So that electric field in equation (2.27) becomes,

$$E(x, t) \approx E(t) = i \left( \frac{\hbar\omega}{2\varepsilon_0 V} \right)^{\frac{1}{2}} [ae^{-i\omega t} - a^\dagger e^{i\omega t}] \quad (2.31)$$

The approximation in the equation (2.29) is called *dipole approximation*. This field loses its time dependence in the Schrodinger's picture and becomes,

$$E = i \left( \frac{\hbar\omega}{2\varepsilon_0 V} \right)^{\frac{1}{2}} [a - a^\dagger] \quad (2.32)$$

To  $\frac{\pi}{2}$  to get rid of the  $i$  we redefine,  $ae^{i\frac{\pi}{2}} \rightarrow a$  and hence  $a^\dagger e^{-i\frac{\pi}{2}} \rightarrow a^\dagger$  so that,

$$i[a - a^\dagger] = ae^{i\frac{\pi}{2}} - a^\dagger e^{i\frac{\pi}{2}} \rightarrow a - a^\dagger e^{i\pi} = a + a^\dagger$$

Thus, electric field of equation (2.32) becomes,

$$E = \left( \frac{\hbar\omega}{2\varepsilon_0 V} \right)^{\frac{1}{2}} [a + a^\dagger] \quad (2.33)$$

Moreover, the expression for the Hamiltonian of the radiation field for a single mode takes the form,

$$H_{field} = \hbar\omega \left( a^\dagger a + \frac{1}{2} \right) \quad (2.34)$$

The zero point energy term  $\frac{1}{2}\hbar\omega$  is a constant. If we take the vacuum state i.e. ground state with no photons as the reference point for energy, we can drop the zero-point energy term.

We get,

$$H_{field} = \hbar\omega(a^\dagger a) \quad (2.35)$$

This is the Hamiltonian for quantized cavity field.

## 2.2 Atomic Hamiltonian

A detailed discussion of this section can be found in [26][28]. An atom is considered a two-level atom when the energy difference between the two levels of our interest is smaller than the energy-gaps of all other levels. In this case the dynamics of atom will effectively be described by these two levels alone and the influence of other energy levels can be neglected. Consider a two-level atom with quantized atomic levels. Let  $|i\rangle$  be an energy eigenket of the Hamiltonian with eigenvalue  $\hbar\omega_i$ , then,

$$H|i\rangle = \hbar\omega_i|i\rangle \quad (2.36)$$

The completeness relation of the eigenkets ensures that,

$$\sum_i |i\rangle\langle i| = 1 \quad (2.37)$$

The orthonormality relationship gives,

$$\langle i|H|j\rangle = \hbar\omega_i\delta_{ij} \quad (2.38)$$

where  $\delta_{ij}$  is the Kronecker delta. The Hamiltonian in the basis of energy eigenkets is then given as,

$$H = \sum_i \hbar\omega_i |i\rangle\langle i| \quad (2.39)$$

The operator  $|i\rangle\langle j|$  is a transition operator of state  $|i\rangle$  which collapses a state  $|k\rangle$  on the state  $|i\rangle$ ,

$$|i\rangle\langle j|k\rangle = |i\rangle\delta_{jk} \quad (2.40)$$

Thus, for an atom having two energy eigenkets, ground state  $|g\rangle$  and excited state  $|e\rangle$ , the atomic raising and lowering operator, also called atomic transition operators, are given by,

$$\sigma^+ = |e\rangle\langle g| \text{ and } \sigma^- = |g\rangle\langle e| \quad (2.41)$$

If we apply  $\sigma^+$  on the ground state of the atom, it shifts it to an excited state and  $\sigma^-$  applied on excited state shifts it to the ground state,

$$\sigma^+|g\rangle = |e\rangle \text{ and } \sigma^-|e\rangle = |g\rangle \quad (2.42)$$

Also, it can be easily verified that,

$$(\sigma^\pm)^2 = 0 \quad (2.43)$$

Some other properties of these transition operators are given as follows,

$$\sigma^- \sigma^+ = |g\rangle\langle e| e \langle g| = |g\rangle\langle g| \quad (2.44)$$

$$\sigma^+ \sigma^- = |e\rangle\langle g| g \langle e| = |e\rangle\langle e|$$

Operators,  $\sigma^+$  and  $\sigma^-$  do not commute with each other. Instead, they have the following anti-commutator relationship,

$$\sigma^+ \sigma^- + \sigma^- \sigma^+ = 1 \quad (2.45)$$

Since the atom in our problem is a two-level atom with transition frequency  $\omega_0$  that means the difference between the two energy levels is  $\hbar\omega_0 = E_e - E_g = \hbar\omega_e - \hbar\omega_g$ . Here  $\omega_i$  represents the eigenfrequency corresponding to the basis state  $|i\rangle$  with  $i = g, e$ . We define the ground state  $|g\rangle$  to be at zero energy and make it our reference point then the Hamiltonian in terms of energy eigenstates can be written as,

$$H_{atom} = \hbar\omega_e |e\rangle\langle e| - 0. |g\rangle\langle g|$$

$$H_{atom} = \hbar\omega_e |e\rangle\langle e|$$

Using equation (2.44), Hamiltonian can be rewritten in the form,

$$H_{atom} = \hbar\omega_0 \sigma^+ \sigma^- \quad (2.46)$$

We have neglected the zero-point energy of the atom by taking the reference of energy from the middle of the energy gap. This is the Hamiltonian for a two-level atom which forms the basis for understanding how atoms interact with light.

### 2.3 Atom- Cavity Interaction Hamiltonian

A detailed discussion of this section can be found in [28]. Building upon the previous sections where we discussed the Hamiltonians for the quantized electromagnetic field and a two-level atom, we now focus on the interaction between the atom and the quantized light field within an optical cavity. When a light beam falls upon the atom, its infinite number of modes interact with the atom. However, by using a cavity we can filter out unwanted modes except for a single mode. Then quantized electric field operator in the cavity expressed as [28],

$$\mathbf{E} = \left( \frac{\hbar \omega_c}{2 \epsilon_0 V} \right)^{\frac{1}{2}} [a + a^\dagger] \sin(kz) \hat{\mathbf{e}} \quad (2.47)$$

Where  $\hat{\mathbf{e}}$  is a unit vector in the direction of polarization of electric field and the axis of cavity is taken in the z-direction. The term  $\sin(kz)$  appears because the cavity imposes the boundary conditions on the field which modify the field from a plane wave to a standing wave. The interaction of the radiation with the atom is due to electric field of the radiation inducing a dipole moment in the atom. The dipole moment vector  $\mathbf{d}$  then interacts with the electric field of the radiation - a term needs to be added to the Hamiltonian, which is given [28],

$$H_{atom-cavity} = -\mathbf{d} \cdot \mathbf{E} = -\sqrt{\frac{\hbar \omega_c}{2 \epsilon_0 V}} d \sin(kz) (a + a^\dagger) \quad (2.48)$$

Here, we have taken the direction of dipole moment of atom along the direction of electric field polarization and perpendicular to the direction of cavity axis so that,

$$\mathbf{d} = d\hat{\mathbf{e}} \quad (2.49)$$

Because of parity conservation, only off diagonal elements contribute to dipole operator measurement [28] and  $\langle e|d|e\rangle = 0 = \langle g|d|g\rangle$ . Hence, In terms of atomic raising and lowering operators:

$$d = |g\rangle\langle g|d|e\rangle\langle e| + |e\rangle\langle e|d|g\rangle\langle g| \quad (2.50)$$

$$d_0|g\rangle\langle e| + d_0|e\rangle\langle g| = d_0(\sigma^- + \sigma^+) \quad (2.51)$$

where  $d_0 = \langle g|d|e\rangle = \langle e|d|g\rangle$  is a real constant. Thus, the interaction of light with the dipole of the atom adds the term,

$$H_{atom-cavity} = \hbar G(\sigma^+ + \sigma^-)(a + a^\dagger) \quad (2.52)$$

With,

$$G = -\sqrt{\frac{\omega_k}{2\hbar\epsilon_0 V}} d \sin(kz) \quad (2.53)$$

In the free field case, operators  $a$  and  $a^\dagger$  evolve in time as [28],

$$a(t) = a(0)e^{-i\omega t}, \quad a^\dagger(t) = a^\dagger(0)e^{i\omega t}$$

Similarly, for the atomic lowering and raising operators we have,

$$\sigma^\pm(t) = \sigma^\pm(0)e^{\pm i\omega_0 t}$$

Thus, the time evolution of the products of these operators in interaction Hamiltonian (2.52) yield,

$$\begin{aligned} \sigma^+ a &\sim e^{i(\omega_0 - \omega)t} \\ \sigma^- a^\dagger &\sim e^{-i(\omega_0 - \omega)t} \\ \sigma^+ a^\dagger &\sim e^{i(\omega + \omega_0)t} \\ \sigma^- a &\sim e^{-i(\omega + \omega_0)t} \end{aligned}$$

The term  $\sigma^+ a^\dagger$  represents photon emission as the atom makes a transition to excited state, while  $\sigma^- a$  corresponds to photon absorption when the atom moves from an excited state back to the ground state. When integrating the time-dependent Schrödinger equation, similar to the perturbative approach, these two terms yield exponential terms involving  $\omega_0 + \omega$  in contrast to  $\omega_0 - \omega$  for the terms,  $\sigma^- a^\dagger$  and  $\sigma^+ a$ . We will discard terms that do not conserve energy, effectively applying the rotating wave approximation (RWA). Consequently, within this approximation, our Hamiltonian simplifies accordingly.

$$H_{atom-cavity} = \hbar G(\sigma^+ a + \sigma^- a^\dagger) \quad (2.53)$$

This Hamiltonian encapsulates the interaction between radiation field and a two-level atom. Combining the atomic Hamiltonian  $H_{atom}$  the cavity field Hamiltonian  $H_{field}$ , and the atom-field interaction Hamiltonian  $H_{int}$ , the total Hamiltonian for the atom-cavity system is:

$$H_{JC} = \hbar\omega a^\dagger a + \hbar\omega_0 \sigma^+ \sigma^- + \hbar g(\sigma^+ a + \sigma^- a^\dagger) \quad (2.54)$$

This is the so-called Jaynes-Cummings Hamiltonian, a fundamental equation in quantum optics that describes the coherent interaction between a two-level atom and a single quantized mode of the electromagnetic field inside a cavity. We will return back to deriving the remaining terms in the total Hamiltonian of equation (2.2) when we will discuss the cavity cooling of single particle. Where we will treat the atom-cavity system in a strong coupling regime.

## CHAPTER 3

### Techniques of Laser Cooling

In this chapter, we provide a review of some basic methods of laser cooling, focusing on a semiclassical approach that describes the cooling process using friction-like forces. These forces, responsible for slowing down atoms, are primarily of two types: the radiation pressure force and the dipole (or gradient) force due to the radiation field. When an atom of mass  $m$  absorbs a photon, the photon's energy  $h\nu$  is transferred to the atom's internal energy states, elevating it to an excited state. Simultaneously, the atom experiences a recoil due to the conservation of momentum, resulting in a change in velocity  $\Delta v = \hbar k/m$  in the direction of the incoming photon. After a short period, the atom returns to its ground state by emitting a photon through spontaneous emission. This emission is isotropic, meaning the photon is emitted in a random direction, imparting another recoil to the atom. However, because the directions of spontaneous emissions are random, their net contribution to the atom's momentum averages out to zero over many cycles. Repeated absorptions of photons by atom reduce its speed significantly. The cumulative effect of repeated absorption and emission cycles is a net force that opposes the atom's motion when illuminated by a laser beam counter-propagating with respect to the atom's trajectory. This process effectively reduces the atom's velocity in the direction opposite to the laser beam. As this 'slowed atom' atom undergoes Brownian motion, its collision with other faster-moving atoms, gradually reduces the velocity of the other atoms in atomic ensemble. When these cycles

are repeated many times, it results in cooling of the entire ensemble of atoms. The cycle of cooling is illustrated in Figure 3-1.

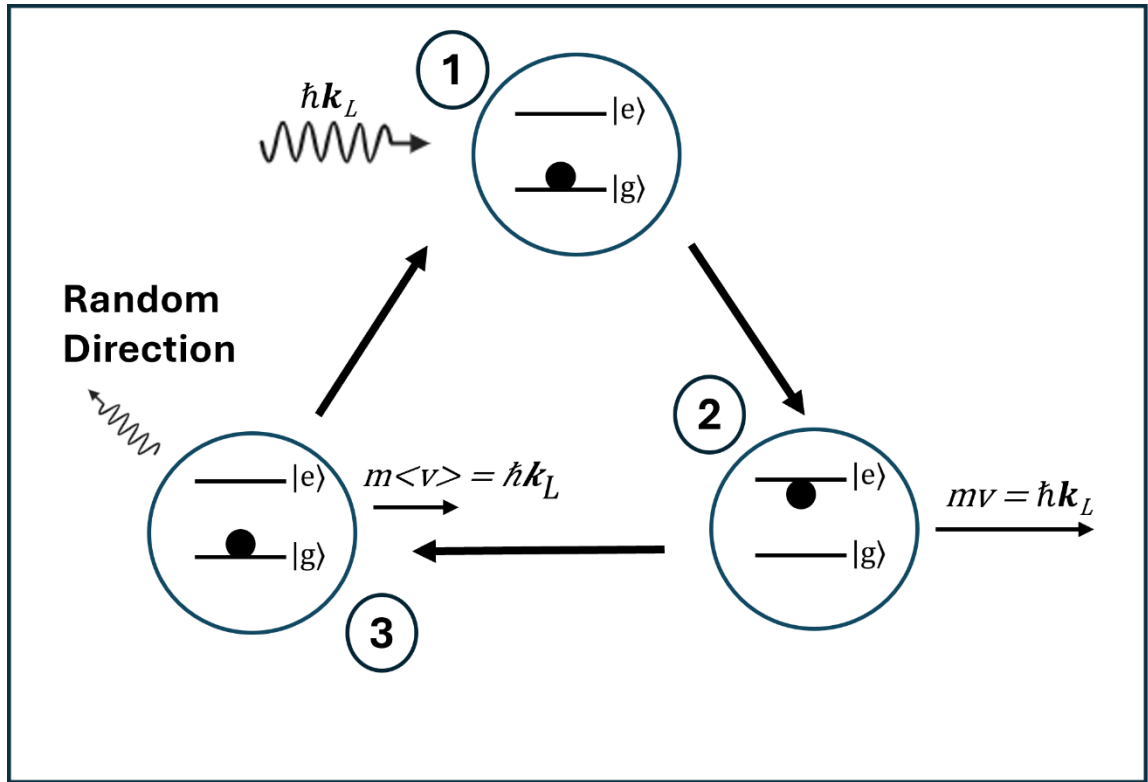


Figure 3-1 A two-level atom in its ground state gets kicked by photon with momentum  $\hbar k$ , resulting in an increase in its velocity by  $\hbar k/m$  in the direction of photon propagation. The atom releases this energy by spontaneously emitting a photon. Since the photon emission happens in a random direction, there is no net change in the atom's average velocity from spontaneous emission step. The atom then returns to its ground state, ready to repeat the cycle.

In 1933, Otto Frisch observed the slowing down of a beam of collimated Sodium atoms when it was illuminated transversely by light from a sodium lamp. Due to the limited intensity and broad spectral linewidth of the sodium lamp, only about one-third of the atoms were excited to the higher energy state, resulting in a relatively small change in speeds of atoms compared to typical speeds of Sodium atom ( $\approx 570$  m/s) at room temperature [2]. The development of narrow-bandwidth lasers in 1960s significantly improved the excitation rate by providing higher intensities and precise frequency control.

This advancement allowed for more efficient momentum transfer between light and atoms, paving the way for sophisticated laser cooling methods.

### **3.1 Description of Laser Cooling Using Semiclassical Approach**

In the semiclassical description of laser cooling, the external motion of atom is treated classically while their internal states are treated quantum mechanically. This approach allows us to use classical force concepts to describe how atoms are slowed down by light. The forces acting on an atom affect both its internal and external degrees of freedom. Internal degrees of freedom refer to the atom's quantum states related to its electronic configuration, such as energy levels, spin orientations, and charge distributions. These degrees of freedom determine how the atom interacts with external fields, leading to transitions between different energy states when subjected to laser radiation or electromagnetic fields. Changes in the internal states involve quantum processes like absorption and emission of photons. External degrees of freedom involve the atom's center-of-mass motion, including its position and linear momentum. In the semiclassical approximation, these are treated classically, meaning the atom moves along well-defined trajectories influenced by forces exerted by the light field [29].

While these two sets of degrees of freedom are handled differently, they are *interdependent*. For example, when an atom absorbs or emits a photon, its internal state changes, and it experiences a recoil that alters its external motion. Conversely, the atom's velocity affects the frequency of the light in its rest frame due to the Doppler effect, influencing the internal transitions. Additionally, the spatial variation of the laser's polarization and intensity can affect both the internal state transitions and the forces acting

on the atom, especially when the laser field is not uniform across the interaction region [10].

Consider a two-level atom with a ground state  $|g\rangle$  and an excited state  $|e\rangle$ . The ground state is stable with an effectively infinite lifetime, while the excited state is unstable, characterized by a finite lifetime  $\tau=1/\gamma$ , where  $\gamma$  is the spontaneous decay rate. Typically,  $\tau$  is of the order of a few nanoseconds. When this atom is illuminated by a laser of frequency  $\omega_L$ , it can absorb photons and make transition to the excited state if the laser frequency is near resonance with the atomic transition frequency ( $\omega_L \approx \omega_0$ ). The interaction with the laser field results in forces acting on the atom due to the momentum exchange during absorption and emission processes.

### 3.1.1 Analyzing the Forces

To analyze the forces acting on the atom using the semiclassical approach, we need to consider the conditions under which this approximation is valid:

1. *Large Photon Number:* The laser field is intense enough that its quantum fluctuations are negligible, allowing it to be treated as a classical electromagnetic wave.
2. *Atomic Motion:* The de Broglie wavelength of the atom is much smaller than the wavelength of the laser light, justifying a classical treatment of the atomic motion.
3. *Timescale Separation:* The internal dynamics (electronic transitions) occur on much faster timescales than the external motion (center-of-mass movement), enabling us to separate the two processes.

Under these conditions, we can derive expressions for the forces acting on the atom. There are two types of forces, [1], [29]:

- **Radiation Pressure Force:** This force arises from the net momentum transfer due to the absorption of photons from the laser beam and subsequent spontaneous emission,
- **Dipole (Gradient) Force:** This force results from the interaction of the induced atomic dipole moment with spatial variations in the laser field's intensity or phase. It can either attract or repel the atom from regions of higher intensity, depending on the detuning between the laser frequency and the atomic transition.

### 3.1.2 Validity of Semiclassical Treatment of Laser Cooling

Recall that in the semiclassical model, the atom is considered as a classical particle with a well-defined position  $\mathbf{r}$  and momentum  $\mathbf{p}$ . However, quantum mechanically an atom is considered as a wave packet with extensions in position and momentum  $\Delta\mathbf{r}$  and  $\Delta\mathbf{p}$ . As long as  $\Delta\mathbf{r}$  and  $\Delta\mathbf{p}$  are small in comparison of characteristic parameters of the given problem, an atom can be treated as a point like particle. There are following conditions for this approach to be valid in our case [30][31],

- 1- The uncertainty in position of an atom is small when compared to the wavelength of the laser field.

$$\Delta\mathbf{r} \ll \lambda$$

- 2- The uncertainty in the doppler shift  $k\Delta\mathbf{v}$  is much smaller than the width of the resonance frequency of the atom:

$$k\Delta\mathbf{v} \ll \gamma$$

These two conditions, together with the Heisenberg's Uncertainty relation,

$$m \Delta r \Delta v \geq \frac{\hbar}{2} \quad (3.1)$$

result in:

$$E_r = \frac{\hbar^2 k^2}{2m} \ll \hbar \gamma \quad (3.2)$$

$E_r$  is called the recoil energy. The condition in equation (3.2) is called the *broadline limit* or *recoil limit*. This limit ensures that the recoil energy  $E_r$  must be much smaller than the width of the energy of excited state of the atom. If the condition in equation (3.2) is fulfilled, the internal variable vary on a much smaller timescale than the external variables [30].

### 3.2 Description of Forces

When the conditions of semiclassical treatment are fulfilled, we can derive a formula for the total force acting on an atom. Suppose an atom in the vicinity of position  $\mathbf{r}$  with momentum  $\mathbf{p}$ , moves in a quasi-resonant radiation field of frequency  $\omega_L$ . By quasi-resonant radiation field, we mean that the frequency of atomic transitions,  $\omega_0$  differ from that of the radiation field frequency  $\omega_L$  by an amount  $\Delta = \omega_L - \omega_0$  such that,  $|\Delta| \ll \omega_0, \omega_L$ . In this limit, radiation pressure  $\mathbf{f}_{\text{RP}}$  is given by [29],

$$\mathbf{f}_{\text{RP}} = \hbar \mathbf{k} \gamma \frac{\frac{\Omega^2(\mathbf{r})}{4}}{\Delta^2 + \frac{\gamma^2}{4} + \frac{\Omega^2(\mathbf{r})}{2}} \quad (3.5)$$

Where  $\Omega(\mathbf{r}) = \frac{d \cdot \mathbf{E}}{\hbar}$  is the Rabi frequency,  $\mathbf{k}$  is the wave vector of the laser field,  $\gamma$  is spontaneous decay rate of the atom. This is the expression for the radiation pressure force acting on an atom at rest in a radiation field with velocity  $\mathbf{v}$ . The dipole force is given by [29],

$$\mathbf{f}_{\text{dip}} = -\frac{\hbar \Delta}{4} \frac{\nabla \Omega(\mathbf{r})^2}{\Delta^2 + \frac{\gamma^2}{4} + \frac{\Omega(\mathbf{r})^2}{2}} \quad (3.6)$$

With these expressions of the forces acting on the atom in a radiation field, we delve into the theories of some of the laser cooling techniques.

### 3.3 Doppler cooling

The temperature of an assembly of particles is related to their average velocity. The temperature of classical non-interacting particles is proportional to the square of their rms velocity. Thus, if we say we want to cool down an ensemble of particles, what we mean is that we want to reduce the rms velocity of the particles. A scheme to exploit radiation fields to cool down atoms was first proposed by Hansch and Schawlow in 1975 [6]. In their experiment, they illuminated an atom with laser light which was slightly tuned below (red detuned) the atomic transition line. Figure 3-2 shows a schematic of such a process. When an atom moves toward a beam of radiation which is red detuned, it perceives the light Doppler shifted towards its resonance. But if the atom is moving away from a beam of laser

light, it sees the light beam shifted more away from its resonance. Thus, the atom absorbs

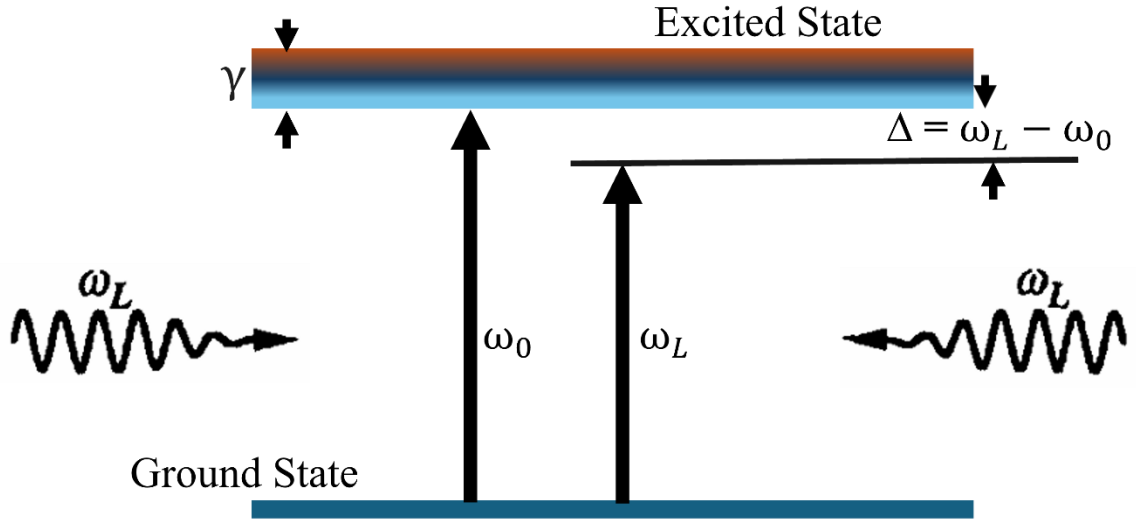


Figure 3-2 Laser light is detuned by an amount  $\Delta = \omega_L - \omega_0$  from the atomic resonance frequency  $\omega_0$ . Due to Doppler shift, atom perceives the incoming laser light frequency equal to  $\omega_0$ .

Photons from the beam towards which it is moving. To increase the efficiency of the cooling, we can collimate laser beams propagating in opposite direction and keep atomic samples in between the beams. The process is referred to as *Doppler Cooling* because the Doppler effect is the central concept in this scheme.

The process of cooling is purely quantum mechanical because of the discrete nature of the photon momentum. The radiation pressure force on the atom because of one laser beam when it was at rest is given by equation (3.5). When this atom moves in the laser field with velocity  $\mathbf{v}$ , it perceives frequency of the laser field Doppler shifted by an amount  $\mathbf{k} \cdot \mathbf{v}$ . In this case force on the atom modifies to,

$$\mathbf{f}(\mathbf{v}) = -\hbar \frac{\gamma}{2} \left[ \frac{\frac{\Omega^2(\mathbf{r})}{4}}{(\omega_L - \omega_0 + \mathbf{k} \cdot \mathbf{v})^2 + \frac{\gamma^2}{4} + \frac{\Omega^2(\mathbf{r})}{2}} \mathbf{k} \right]$$

Similarly, the radiation pressure force on the atom moving in field of two-counterpropagating laser beams will be given by [30],

$$\begin{aligned} \mathbf{f}(\mathbf{v}) = & -\hbar \frac{\gamma}{2} \left[ \frac{\frac{\Omega^2(\mathbf{r})}{4}}{(\omega_L - \omega_0 + \mathbf{k} \cdot \mathbf{v})^2 + \frac{\gamma^2}{4} + \frac{\Omega^2(\mathbf{r})}{2}} \mathbf{k} \right] \\ & -\hbar \frac{\gamma}{2} \left[ \frac{\frac{\Omega^2(\mathbf{r})}{4}}{(\omega_L - \omega_0 - \mathbf{k} \cdot \mathbf{v})^2 + \frac{\gamma^2}{4} + \frac{\Omega^2(\mathbf{r})}{2}} \mathbf{k} \right] \end{aligned} \quad (3.6)$$

For small  $\mathbf{k} \cdot \mathbf{v}/(\omega_L - \omega_0)$ , this force can be Taylor expanded around  $\mathbf{v}=0$ ,

$$\mathbf{f}(\mathbf{v}) = \mathbf{f}_0 - \alpha \mathbf{v} + \dots \quad (3.7)$$

Where the coefficient  $\alpha$  referred to as *friction* or *viscosity coefficient*, is positive constant for red detuned radiation field. The expression for  $\alpha$  is given as [30],

$$\alpha = \hbar k^2 s \frac{2\gamma|\Delta|}{\Delta^2 + \frac{\gamma^2}{4}} \quad (3.8)$$

Where we have assumed,  $s = \frac{\frac{\Omega^2(\mathbf{r})}{2}}{\Delta^2 + \frac{\gamma^2}{4}} \ll 1$ . To get the maximum rate of cooling, we want to

maximize  $\alpha$ . The value of  $\alpha$  is at a maximum when, when  $\Delta = -\gamma/2$  and  $s = 1$  i.e.  $\Omega = \gamma$ .

Which gives,

$$\alpha_{\max} = \frac{\hbar k^2}{4} \quad (3.9)$$

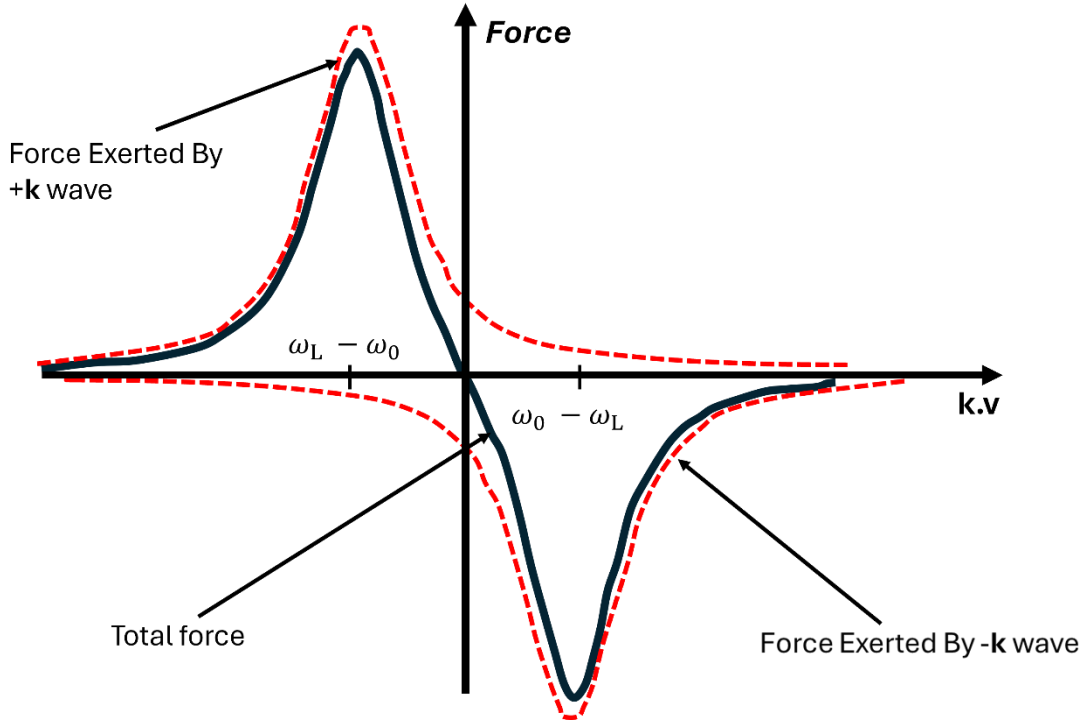


Figure 3-3 Profile of the total force on a particle in optical molasses [9].

When analyzed independently, the force profiles of equation (3.6) take the shape of Lorentzian curves with maxima at  $k \cdot v = \omega_0 \pm \omega_L$ , signifying that the detuning is exactly offset by the Doppler shift. Considering both lasers together and examining Equation (3.6), it becomes clear that the constant term  $f_0$  in the series expansion of the force from one laser is canceled out by the corresponding term from the other laser. Close to  $v = 0$ , the net force is linear and acts in the direction opposite to the velocity. Extending this to three dimensions is straightforward: three pairs of laser beams are aligned along three orthogonal axes to create what is known as optical molasses. The optical molasses refers to a situation where the counterpropagating laser beams along all three axis form a force field which behaves like viscous fluid for the atom.

Equation (3.7) shows that the motion of particles in a highly directed radiation field is very similar to the particle motion in a viscous medium. Thus, the particle, which is subject to slightly red detuned laser light, after many cycles of absorption and emission of the photons, shows a narrow spread of velocity, leading to an exponential decay of its velocity toward zero. However, the random nature of absorption and spontaneous emission processes imposes a lower limit on the narrowing of the atomic velocity distribution. This limit is called *Doppler limit* and is given by [32],

$$k_B T_D = \frac{\hbar \gamma}{2} \quad (3.10)$$

This minimum temperature  $T_D$  represents the limit for Doppler cooling, constrained by the recoil energy imparted from each spontaneous emission event. Physically, this limit highlights that, while Doppler cooling is effective in reducing atomic velocities, the random nature of photon emission prevents it from achieving absolute zero.

### 3.4 Laser Cooling Beyond Doppler Limit: Sisyphus Cooling

In 1988, a group of physicists led by William D. Phillips working on laser cooling at the National Institute of Standards and Technology (NIST), USA, found that the temperature achieved by laser cooling of sodium atoms was substantially lower than the theoretical prediction for the Doppler cooling. Later in year 1988, a new mechanism of laser cooling was proposed which was based on combined effects of several factors such that pumping of sublevels of the ground state, AC Stark shifts and the polarization gradient. The concept behind this new cooling mechanisms is based on the interaction between laser beam electric fields and an atom's polarization. This polarization, influenced by the ground-state sublevel

and the laser polarization, generates a dipole that interacts with the laser field. As a result, a shift in the energy levels of the ground-state sublevels occurs, which is referred to as the light shift, or ac Stark shift. When the laser intensity or polarization changes across space, these shifts become dependent on position, leading to a force known as the polarization gradient force, proportional to the gradient of the atomic energy [4][32].

Additionally, optical pumping between different atomic sublevels occurs when the laser's intensity or polarization is spatially non-uniform. This means the population distribution among the sublevels varies with position. For an atom moving slowly in a spatially periodic light field, the average forces over time might cancel out. However, because optical pumping requires a finite time—determined by the laser intensity—it's possible to adjust the laser's frequency and spatial gradients so that the atom loses kinetic energy while climbing potential energy "hills" before being optically pumped into a different sublevel where it experiences less influence from the potential. This cyclic process leads to a net loss of kinetic energy, analogous to the myth of Sisyphus perpetually pushing a boulder uphill, hence the name *Sisyphus cooling*.

### 3.4.1 Qualitative Discussion of the Mechanism of Sisyphus Cooling

Two laser beams of same frequency  $\omega_L$  travelling opposite to one another give rise to two configurations of polarization [11]

- 1- In the first configuration known as the *lin  $\perp$  lin configuration*, two plane waves with linear polarizations that are perpendicular to each other are combined while traveling in opposite directions. The polarization alternates between  $\sigma^-$  and  $\sigma^+$  states, with intermediate regions exhibiting  $\pi$  polarization states. As a result, there

is a gradient of ellipticity in the polarization when we move along direction of propagation of light (as shown in Figure 3-4(a)). When light has an elliptical polarization, the electric field vector traces out an ellipse over time. The degree of ellipticity quantifies how stretched or circular this ellipse is. A gradient of ellipticity occurs when this property changes gradually along a certain direction, such as the propagation axis or a transverse axis. This means that as we move along the specified direction, the shape of the polarization ellipse evolves continuously [11].

- 2- In the second configuration called the  $\sigma^+ \text{-} \sigma^-$  configuration, two circularly polarized waves travel in opposite directions. In this configuration, a continuous rotation of the polarization is observed (shown in Figure 3-4(b)) Unlike the lin  $\perp$  lin setup, the polarization in this configuration remains linear at every point, and its extremity forms a helix with a pitch  $\lambda$ .

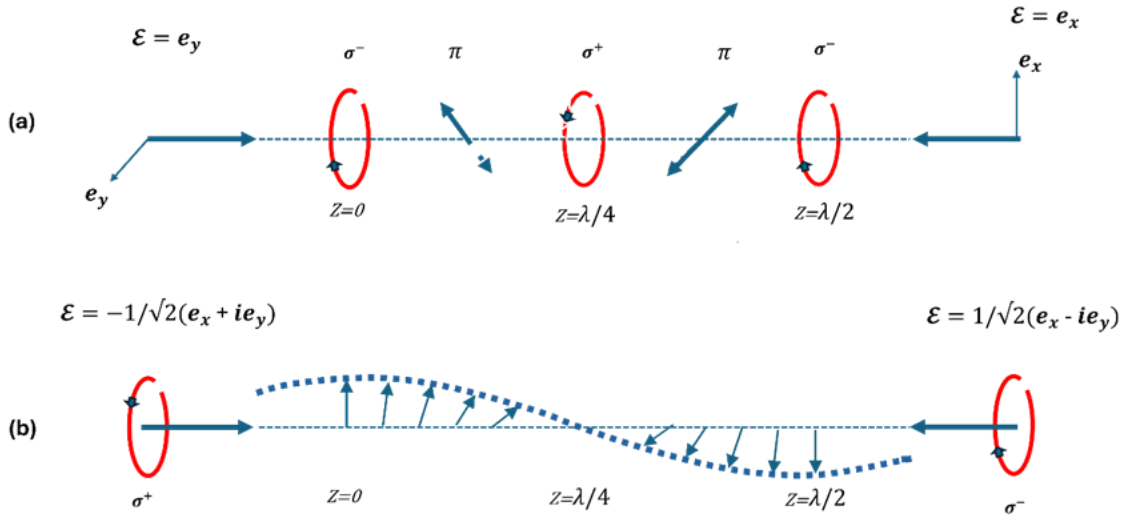


Figure 3-4 Two orthogonally polarized beams superpose to give rise to lin  $\perp$  lin polarization configuration. Polarization switches from  $\sigma^-$  to  $\sigma^+$  every quarter of the wavelength. (b) Two circularly polarized beams give rise to rotation of linear polarization with period  $\lambda/2$ . Figure is adopted from [9].

Now consider a two-level atom with the ground state total angular momentum  $J = 1/2$  and excited state total angular momentum  $J = 3/2$  moving through such a standing wave formed

by the  $\text{lin} \perp \text{lin}$  configuration. The resulting polarization pattern varies depending on the relative phase of the laser beams, and this pattern changes with position, as illustrated in Figure 3-4. This spatial variation in polarization creates a periodic modulation in the light shift (AC Stark shift) experienced by the states in the lower energy level. The interaction strength between the atom and the light depends on the magnetic quantum numbers ( $M_J$ ) in both the lower and upper states of the atom. To better understand this concept, we will consider a location in the standing wave where the light is  $\sigma^+$ -polarized. In this case, the transition from  $M_J = +1/2$  in the lower state to  $M_J = +3/2$  in the upper state is more probable than the transition from  $M_J = -1/2$  to  $M_J = +1/2$ . This can be understood from the Clebsch Gordan coefficients, which indicate that the square of the transition amplitude is  $2/3$  for the first transition and  $1/3$  for the second. When the laser light is red-detuned (its frequency is lower than the atomic transition frequency), both ground-state sublevels experience energy shifts, but the amount of shift varies between them due to their different interaction strengths with the light field. As the atom moves through regions of varying polarization—from  $\sigma^-$  to  $\sigma^+$  over a distance of  $\Delta z = \lambda/4$  the light shifts of the ground-state sublevels oscillate periodically along the standing wave [32][33].

As the atom moves through this periodic potential region, it alternates between regions of higher and lower kinetic and potential energy. While the atom accelerates and decelerates depending on its position within the standing wave, its total energy remains unchanged as long as it remains in the same quantum state. To cool an atom, there must be a way to dissipate energy. This is achieved through absorption and spontaneous emission. When an atom absorbs light at the top of a potential energy hill and then spontaneously decays, it is more likely to end up at the bottom of a valley than back at the top. This means that the

kinetic energy the atom uses to climb the hill is lost through the photon emitted during decay, and the atom ends up moving more slowly at the bottom of the valley (see Figure 3.5). In the Sisyphus cooling technique. Let's take a closer look at how atoms transfer between the  $M_J$  states of the lower energy level at a point where the light is  $\sigma^+$ -polarized (as shown in Figure 3-3).

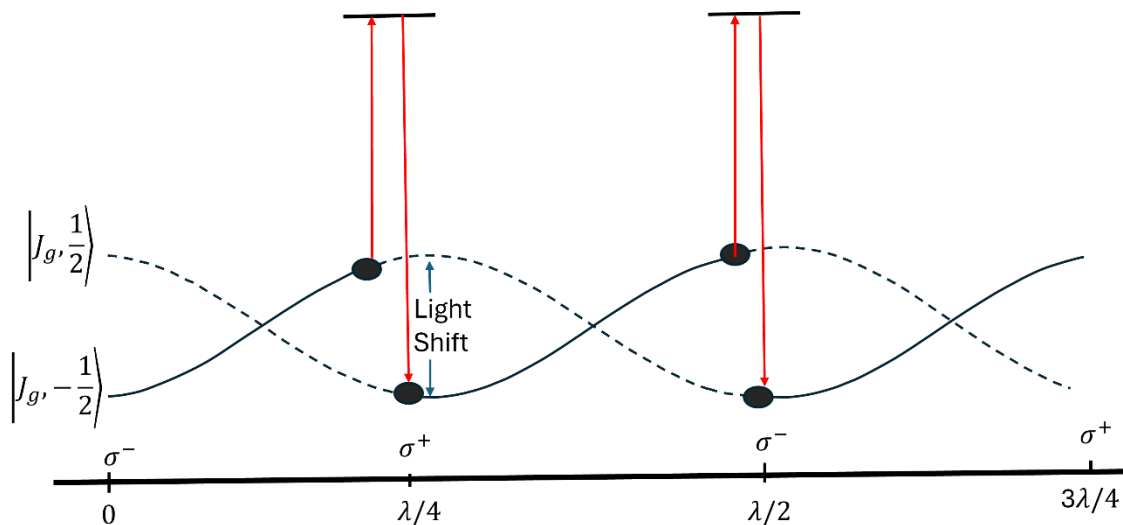


Figure 3-5 The Sisyphus cooling mechanism: Atom moves in polarized field region, climbs up the potential hill at the expense of its kinetic energy from there it makes transition to lower state losing energy equal to light shift in each of these cycles. Figure is adopted from[9].

For example, when an atom is in the  $M_J = -1/2$  state and absorbs  $\sigma^+$ -polarized light, it transitions to an excited state and can decay back to any of the two ground-state sublevel. If it decays to  $M_J = +1/2$ , it finds itself at a lower potential energy due to the position-dependent light shifts. This optical pumping acts as a one-way mechanism transferring population from higher to lower potential energy states, thereby continuously reducing the atom's kinetic energy. During each of these transitions, the atom loses an energy amount approximately equal to  $U_0$ , which corresponds to the height of the hills relative to the valleys. The treatment shows that atoms moving in a standing wave typically have kinetic energy on the order of  $U_0$ . This suggests that Sisyphus cooling continues until the atoms

can no longer climb the potential hills, effectively trapping them in the valleys, much like what occurs in an optical lattice. This simple model predicts that the temperature is related to the intensity and frequency detuning of the light by the relationship [32]

$$k_B T \propto U_0 \propto \frac{I}{|\Delta|} \quad (3.12)$$

Where  $\Delta = \omega_L - \omega_0$ , is the detuning between atomic transition frequency and frequency of the laser and  $I$  is the intensity of the laser. However, the temperature does not decrease indefinitely with decreasing laser intensity or detuning. The cooling limit is reached when the energy lost by the atom during one optical pumping cycle equals the recoil energy  $E_r$  gained from spontaneous emission. At this point, no further net energy is lost, and the lowest attainable temperature is on the order of a few times the recoil temperature,  $T_r \approx E_r/k_B$ .

The recoil limit temperature,  $T_r$ , is defined by the following equation:

$$k_B T_r = \frac{\hbar^2 k^2}{M} = \frac{h^2}{M \lambda^2} \quad (3.13)$$

where  $k$  is the wave number,  $M$  is the atomic mass,  $\lambda$  is the wavelength of light,  $h$  is Planck's constant, and  $k_B$  is Boltzmann's constant. For sodium atoms, the recoil limit temperature is about  $2.4\mu$  K, which is significantly lower than the Doppler cooling limit of  $240\mu$  K. In typical experiments, optical molasses can achieve temperatures close to the recoil limit, but not exactly at it.

### 3.5 Sideband Cooling

Laser sideband cooling is another method to achieve temperatures below the Doppler limit. Unlike Doppler cooling, which reduces the thermal motion by exploiting the Doppler shift, sideband cooling targets the quantized vibrational states (phonons) of a particle confined in a harmonic potential. Sideband cooling is particularly effective for systems where the particle's motion is tightly confined, such as ions in Paul traps or atoms in optical lattices [34].

In sideband cooling, we consider an atom that is strongly confined in the trapping potential such that the trap or phonon frequency  $\nu$  is larger than the natural linewidth  $\gamma$  of the electronic transitions. When this condition ( $\nu > \gamma$ ) is fulfilled, the sidebands of the atomic transitions get spectrally resolved. The method employs a cooling laser that is tuned to the red sideband frequency, detuned from the electronic transition frequency  $\omega_0$  by the trap frequency  $\nu$ :

$$\omega_L = \omega_0 - \nu$$

Tuning the laser frequency to lower sideband,  $\omega_L = \omega_0 - \nu$ , the particle absorbs the photon at this frequency and goes into the vibrational state  $m' = m - 1$  in the excited state. Where  $m'$  and  $m$  are labels of excited and ground state vibrational quantum number. This excited state subsequently decays back to the ground electronic state. Assuming that particle is in the so-called Lamb-Dicke regime ( $\eta \ll 1$ ), then most probable spontaneous transition is one where the vibrational quantum number remains unchanged, resulting in the particle, on average, returning to the ground state with a lower vibrational level than it

initially occupied [32]. The rate of red sideband transitions depends on the overlap between the initial and final motional states, quantified by the Lamb-Dicke parameter  $\eta$ , which is defined as [35]:

$$\eta = \sqrt{\frac{\hbar k^2}{2M\nu}}$$

Here,  $k$  is the wave number of the cooling laser, and  $M$  is the mass of the particle. In the Lamb-Dicke regime ( $\eta \ll 1$ ), the particle's motion is so tightly confined that the probability of multiphoton transitions is negligible, allowing efficient single-phonon sideband cooling [34]. The process of sideband cooling is explained in more detail below

**Cooling Cycle:** The cooling cycle consists of the following steps:

- Absorption: The particle absorbs a red-detuned photon, transitioning from  $|g, n\rangle$  to  $|e, n - 1\rangle$ .
- Emission: The excited state  $|e, n - 1\rangle$  spontaneously decays back to  $|g, n - 1\rangle$ , emitting a photon without changing the phonon number.

Each cycle results in a net loss of one phonon, systematically cooling the particle toward its motional ground state. By repeatedly applying this cycle, the particle can be cooled to the lowest vibrational state, achieving temperatures below the Doppler limit [34].

In sideband cooling, the steady-state phonon number is fundamentally limited by the linewidth  $\gamma$  of the electronic transition. This limitation arises because the finite linewidth introduces a residual heating rate due to spontaneous emission, setting a lower bound on

the achievable temperature and thus restricting the method's effectiveness. Theoretically, minimum achievable temperature,  $T_s$  using sideband cooling is given by [36],

$$\begin{aligned} T_s &\simeq \hbar\nu/[k_B \ln (\langle m \rangle^{-1})] \\ &= \hbar\nu/[k_B \ln (\nu^2/C_s\gamma^2)] \end{aligned}$$

Where  $k_B$  is Boltzmann constant,  $C_s$  is a constant on the order of 1 which depends upon the selection rules of absorption and emission,  $\langle m \rangle$  is the average of minimum achievable phonon number given by,

$$\langle m \rangle = C_s \left( \frac{\gamma}{\nu} \right)^2$$

To overcome this limit, several approaches have been developed. The use of optical resonators, or cavities, are some of the most prominent solutions [37][38] . In cavity-mediated laser cooling, particles are strongly coupled to high-finesse optical cavities. In this regime, the cavity modifies the spontaneous emission properties of the particles. Instead of emitting photons spontaneously into free space, the particles preferentially emit photons into the cavity mode. The photons then escape from the cavity mirrors, effectively replacing spontaneous emission with cavity-enhanced emission. Horak et al. [37] proposed cavity-induced atom cooling in the strong coupling regime, showing that the cavity can significantly alter the atom's emission properties and cooling behavior. Vuletić & Chu [38] proposed cooling by coherent scattering into a cavity mode, offering a method to surpass conventional cooling limits. We will return to this topic in more detail in Chapter 5 where we will analyze the cooling dynamics of an atom by coupling it strongly to a cavity.

## CHAPTER 4

### Theoretical Background

#### 4.1 Theory of Open Quantum Systems

Schrödinger's equation is central to describing the time evolution and behavior of systems that are isolated from any external interaction. In such cases, the system is completely independent from the surrounding environment, which is why we refer to it as a closed or isolated system. The state of such a system is represented by a normalized state vector. By solving the Schrödinger equation, we can determine the system's state at any point in time, provided initial conditions are known. However, in many real-world scenarios, systems do not remain completely isolated—they interact with their surroundings. We call these open quantum systems. They exhibit behavior that goes beyond what the Schrödinger's equation alone can describe. Since open systems cannot be described using normalized state vectors, we must adopt a different formalism: the theory of open quantum systems. The theory of open quantum systems deals with issues like dissipation and dephasing by considering that all real systems of interest are, in fact, open systems interacting with their environments. In this framework, any open quantum system is viewed as part of a larger composite system, consisting of the system of interest (S) and a reservoir (R) to which it is coupled, collectively referred to as System-Reservoir (SR).

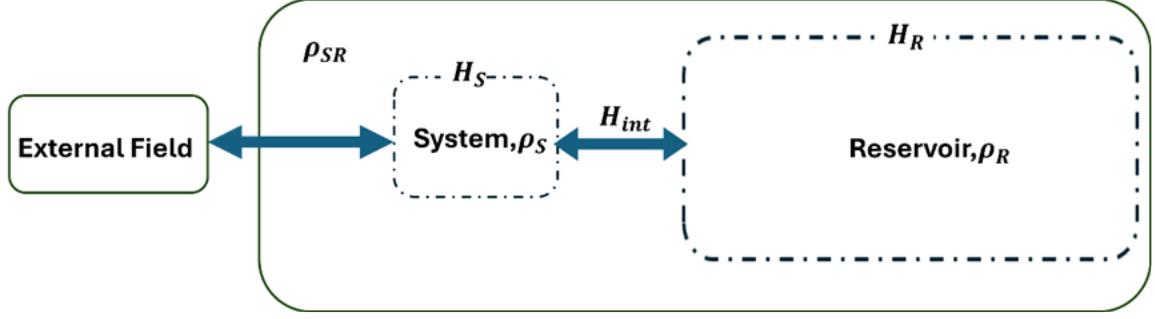


Figure 4-1 Diagram represents an open quantum system interacting with reservoir and external field. Figure is adopted from [9].

An example of an open quantum system could be a group of excited atoms interacting with their surroundings. An illustration of such a system can be seen in Figure 4-1. Similar systems also find application in fields such as cavity quantum electrodynamics and quantum Brownian motion.

In open quantum systems, we symbolize the Hilbert space of the system as  $\mathbb{H}_S$ , the Hilbert space of the reservoir as  $\mathbb{H}_R$ , and the combined Hilbert space for the entire system as  $\mathbb{H}_{SR}$ . The composite system, denoted as  $SR$ , is described by the total Hamiltonian [9]:

$$H_{SR} = H_S \otimes 1_R + 1_S \otimes H_R + H_{int} \quad (4.1)$$

Here:

- $H_S$  represents the Hamiltonian of the system we are interested in,
- $H_R$  denotes the Hamiltonian of the surrounding reservoir,
- $H_{int}$  is the interaction Hamiltonian that describes the coupling between the system and the reservoir,
- $1_i$  is the identity operator acting on the Hilbert space  $H_i$  (where  $i = S, R$ ).

Open quantum systems can be categorized based on the commutation relation between the system's Hamiltonian  $H_S$  and the interaction Hamiltonian  $H_{\text{int}}$ . Two cases are commonly identified [9]:

- Quantum non-demolition (QND) systems, where  $[H_S, H_{\text{int}}] = 0$ , causing decoherence without dissipation.
- Quantum dissipative systems, where  $[H_S, H_{\text{int}}] \neq 0$ , leading to both decoherence and dissipation.

In our study, we focus on the case where  $[H_S, H_{\text{int}}] \neq 0$ , corresponding to dissipative dynamics.

At the initial time, we assume that the system is described by a density operator  $\rho_S(0)$ , which acts on the system's Hilbert space  $H_S$  of dimension  $d_S$ . Similarly, the reservoir is described by a density operator  $\rho_R(0)$ , acting on the reservoir's Hilbert space  $H_R$ , which may have a very large, potentially infinite dimension  $d_R$ . The combination of the system and reservoir form a composite system, described initially by the density operator  $\rho_{SR}(0)$ , acting on the combined Hilbert space  $\mathbb{H}_{SR} = \mathbb{H}_S \otimes \mathbb{H}_R$  with dimension  $d_{SR} = d_S \times d_R$ . Since the composite system is isolated, its evolution follows unitary dynamics, expressed as [33]:

$$\rho_{SR}(t) = U(t)\rho_{SR}(0)U^\dagger(t) = e^{-\frac{iH_{SR}t}{\hbar}}\rho_{SR}(0)e^{\frac{iH_{SR}t}{\hbar}} \quad (4.2)$$

This equation describes the time evolution of the composite system in terms of its unitary transformation. The state of the system can also be described using the Liouville-von Neumann equation, which is given by:

$$i\hbar \frac{d\rho_{SR}(t)}{dt} = [H_{SR}, \rho_{SR}(t)] \quad (4.3)$$

However, due to the large dimensionality of  $H_R$  (and, as a result,  $H_{SR}$ ), the density operator  $\rho_{SR}(t)$ , which represents the combined system and reservoir, becomes very complex, with a huge number of components. This makes exact computations highly expensive, if not impossible.

While we could theoretically determine the evolution of the system  $S$  from the overall system dynamics, in practice, we are often more interested in the behavior of the system  $S$  rather than that of the reservoir. Thus, our focus is to derive an equation that describes the reduced dynamics of the system, accounting for its interaction with the environment. To do this, we introduce the reduced density operator  $\rho_S(t)$ , which is obtained by taking the partial trace of  $\rho_{SR}(t)$  over the reservoir degrees of freedom:

$$\rho_S(t) = \text{Tr}_R[\rho_{SR}(t)] \quad (4.4)$$

The partial trace operation  $\text{Tr}_R(\dots)$  ensures that, for any physical observable  $O_S$  associated with the system  $S$ , and for the observable  $O_{SR} = O_S \otimes 1_R$  of the combined system  $SR$ , the measurement statistics remain consistent regardless of whether we measure  $O_{SR}$  on the combined system  $SR$  in the state  $\rho_{SR}$ , or measure  $O_S$  on  $S$  in the state  $\rho_S = \text{Tr}_R(\rho_{SR})$ . For example, the expectation value of the physical observable  $O_S$  is given by:

$$\langle O_S \rangle = \text{Tr}_S(O_S \rho_S) = \text{Tr}_{SR}[(O_S \otimes 1_R) \rho_{SR}] \quad (4.5)$$

Since  $\rho_S$  is a positive operator ( $\langle \phi_S | \rho_S | \phi_S \rangle \geq 0$ ) and has a unit trace ( $\text{Tr}_S(\rho_S) = 1$ ) [33], it can be appropriately interpreted as a density operator. To derive the equation governing the evolution of  $\rho_S$ , we take the partial trace of the total system's evolution equation (Eq.

(4.3)) over the reservoir states. With appropriate assumptions, this leads to a differential equation that describes the evolution of  $\rho_S$ , commonly referred to as a master equation.

In the next two sections we will explore the methods of master equation and quantum trajectory theory for studying the dynamics of open quantum systems.

## 4.2 Description of Open Quantum Systems: Master Equation

### Approach

The Master equation approach for the density operator provides a powerful framework to analyze the behavior of quantum systems that are weakly coupled to their surrounding environment. Initially, it was developed in quantum optics to explore the interaction between light and matter [40]. However, equations are now widely used to model various quantum phenomena. Examples include noisy models in the context of quantum information processing, it is used to obtain transient (short-lived or temporary) emission and absorption spectra of optically active materials, and it can help describe experiments involving electronic and nuclear spins [41].

To obtain the master equation, we take partial trace of the equation (4.3) over all degrees of freedom of the environment,

$$\frac{d\rho_S(t)}{dt} = \frac{1}{i\hbar} \text{Tr}_R([H_{SR}, \rho_{SR}(t)]) \quad (4.6)$$

A more physical insightful form of this equation can be derived if we impose Born-Markov approximations,

$$\frac{d\rho_S(t)}{dt} = \frac{-i}{\hbar} [H_S, \rho_S] + \sum_{k=1}^{N^2-1} \gamma_k \left( A_k \rho_S A_k^\dagger - \frac{1}{2} \{A_k^\dagger A_k, \rho_S\} \right) \quad (4.7)$$

Where  $\mathcal{L}$  is the so-called *Liouvillian superoperator*. Superoperators are those operators which map a set of operators into itself.  $N$  is the dimension of Hilbert space. The operators  $\{A_k\}$  are called Lindblad operators [33], are the dissipation operators for the system. Equation 4.7 is called *Lindblad Master Equation*. The superoperator  $\mathcal{L}$  is generator of a quantum dynamical semi-group, which can be physically understood as describing the time evolution of an open quantum system, with time being the only parameter involved. Unlike in a full group, where every element has an inverse, a semi-group lacks this property. This means that the evolution between two times,  $t_1$  and  $t_2$  generally cannot be reversed, reflecting the irreversible nature of open quantum systems. In other words, for many cases, once the system evolves from  $t_1$  to  $t_2$  it cannot return to its previous state, capturing the inherent dissipation and loss of information to the environment. However, this irreversibility depends on the specific choice of  $A_k$ , as not all such choices lead to irreversible dynamics. This possibility of irreversible dynamics is a key feature of open quantum systems, distinguishing them from closed quantum systems where evolution can be inverted through unitary transformations [33] [9].

Regrouping terms in equation 4.7 as below can sometimes be useful and physically insightful,

$$\frac{d\rho_S(t)}{dt} = \mathcal{L}_{\text{unitary}}[\rho_S(t)] + \mathcal{L}_{\text{relax}}[\rho_S(t)], \quad (4.8)$$

where,

$$\begin{aligned}\mathcal{L}_{\text{unitary}}[\rho_S(t)] &= \frac{-i}{\hbar} [H_S, \rho_S] \\ \mathcal{L}_{\text{relax}}[\rho_S(t)] &= \sum_k \gamma_k A_k \rho_S A_k^\dagger - \frac{1}{2} \sum_k \gamma_k \{A_k^\dagger A_k, \rho_S\}\end{aligned}\tag{4.9}$$

This grouping into two primary contributions is physically motivated. The first term,  $\mathcal{L}_{\text{unitary}}[\rho_S(t)]$ , represents the *coherent and reversible* evolution of the system under an effective Hermitian Hamiltonian. The second term,  $\mathcal{L}_{\text{relax}}[\rho_S(t)]$  typically describes the *irreversible and non-unitary* dynamics of the system due to its interaction with the surrounding environment (the reservoir). While this part often accounts for dissipative processes such as decoherence and energy loss, it is important to note that irreversibility is not guaranteed in all cases, as certain specific conditions can lead to exceptions. The operator responsible for this dissipation is known as the *dissipator*, or the *Lindbladian*, which has a specific form that captures the system-reservoir coupling.

Operators  $A_k$  are dimensionless and  $\gamma_k$  have units of  $1/\text{time}$  to ensure that  $\rho_S$  retains its interpretation as a density operator. These  $A_k$  operators form a countable set of positive, bounded operators known as Lindblad operators. The number of these operators (whether it is one, several, or infinite) and their actions depend on the nature of the problem being studied. They describe how the system is influenced by an implicit, hidden measurement performed by the reservoir. Thus, they can project the system's state onto a well-defined state in specific scenarios. For instance, when the  $A_k$  operators are proportional to projectors (e.g., in quantum jump processes corresponding to measurement-like events),

they collapse the wavefunction to a particular state. However,  $A_k$  operators are not always projectors; in more general cases, they describe dissipative or decoherent processes where the system evolves non-unitarily without a strict projection onto a single state. For this reason, the  $A_k$  operators are sometimes referred to as collapse operators, as they cause a collapse of the wavefunction due to the hypothetical measurement by the reservoir. However, these operators are more often called jump operators, a term we will use going forward [9][33].

Jump operators describe transformations of a system's states due to specific interactions, such as decay processes or measurements. They typically correspond to an irreversible, instantaneous transformation of the system's wavefunction. Point should be noted that while some jump operators can correspond to projective measurements, this is not generally the case. Physically, in the context of quantum optics, they represent the system's contribution to the system-reservoir interaction through the various decay channels that feed into the quantized field [9]. Consider an atom that has a ground state  $|g\rangle$  and an excited state  $|e\rangle$ . This atom is coupled to a quantized electromagnetic field, which is initially in a vacuum (no-photon) state. In this case, the coupling with the vacuum field is described by a dissipator in the form of equation (4.9), with a single jump operator:  $A = |g\rangle\langle e|$ , and  $\gamma$  representing the spontaneous emission rate. That the anticommutator in equation (4.9) arises during the derivation of the master equation, which comes from analyzing the evolution of the atom + field wavefunction component within the initial zero-photon state of the field, and it represents the coupling between the atom and the electromagnetic field, accounting for the interaction terms that lead to the dissipation and decoherence of the system. The presence of the anticommutator is a direct consequence of the quantum nature

of the interaction, particularly in the context of quantum optics and cavity-QED systems. In the two-level atom model, this anticommutator accounts for the decay of the excited state population and the coherences between the ground and excited states. Additionally, the "sandwich term" originates from tracing over the field's degrees of freedom to construct the reduced density operator, involving field states different from the initial vacuum state—specifically, reservoir states containing one fluorescence photon [42].

While deriving the master equation, the Born-Markov approximations have been assumed. The Born approximation assumes that the reservoir is only minimally affected by its interaction with the system. This is valid when the dimension of the reservoir's Hilbert space  $H_R$  is much larger than that of the system's Hilbert space  $H_S$  (i.e.,  $d_R \gg d_S$ ). In such cases, the system's influence on the reservoir can be considered negligible. This is typically true when dealing with large reservoirs like the quantized electromagnetic field. Additionally, if the reservoir is in a stationary state where  $[H_R, \rho_R] = 0$ , meaning  $\rho_R(t) = \rho_R(t_0)$ , the Born approximation can be applied also. In this approximation, the total density operator  $\rho_{SR}(t)$  can be factorized as  $\rho_S(t) \otimes \rho_R$ , neglecting any correlations between the system and the reservoir. This simplification implies that while the reservoir interacts with the system, it does not significantly alter the system's evolution [33].

The Markov approximation assumes that the evolution of the reduced density operator  $\rho_S$  can be described using a first-order differential equation with respect to time, meaning that the system's evolution is memoryless. In this approximation, the density operator at time  $t$  only depends on its state at an earlier time  $t_0$ , and not on its state at multiple past times. While real reservoirs often retain some memory of past interactions, in the Markov

approximation, these memory effects are neglected. For this approximation to hold, there must be a characteristic time  $\tau_B$ , representing the time over which reservoir correlations (due to past system interactions) decay. The Markov approximation is valid as long as this characteristic time is sufficiently short [33].

### 4.3 Quantum Trajectory Theory

Quantum trajectory techniques emerged in the field of quantum optics in the early 1990s [42] to simulate dissipative dynamics numerically. These techniques are applicable to any system where the density operator's time evolution is governed by a Lindblad-form master equation. The approach involves expressing the master equation as a stochastic average over individual trajectories, which can be evolved over time as pure states. This allows for bypassing the need to propagate a full density matrix, which is often computationally demanding, and instead utilizes stochastic sampling. The key advantage here is that for a Hilbert space of dimension  $N_H$ , propagating a full density matrix requires tracking an object of size  $N_H^2$ , whereas the stochastic sampling approach only requires state vectors of size  $N_H$ . However, this approach requires a sufficiently large number of samples to achieve low statistical errors. Ideally it should be fewer samples than the size of the Hilbert space to remain efficient. The term "quantum trajectories" was coined by Carmichael, while other groups referred to this approach as the "quantum jump" or "Monte Carlo wavefunction" method. Notably, this term should not be confused with "quantum Monte Carlo" techniques, as the Monte Carlo approach here is classical, focusing on expectation values obtained through stochastic sampling rather than coherent amplitude summation [43].

### 4.3.1 Steps of Quantum Trajectory Method

The quantum trajectory method involves the following steps,

- 1- The wave function of system at time  $t$ ,  $|\psi(t)\rangle$  evolves under a non-Hermitian

Hamiltonian  $H = H_S - \frac{i\hbar}{2} \sum_k \gamma_k A_k^\dagger A_k$  for a sufficiently small time step  $\delta t$ . In this

case, the wavefunction at time  $t + \delta t$  is given by,

$$|\psi(t + \delta t)\rangle = \left(1 + \frac{1}{i\hbar} H \delta t\right) |\psi(t)\rangle \quad (4.10)$$

the resulting wave function is not normalized since the Hamiltonian  $H$ , is non-Hermitian that means,  $H \neq H^\dagger$ . To preserve the probability, the wave function  $|\psi(t + \delta t)\rangle$  must be normalized, however. The norm square of the wave function after time  $t + \delta t$  is given by,

$$\begin{aligned} \langle \psi(t + \delta t) | \psi(t + \delta t) \rangle &= \left\langle \psi(t) \left| \left(1 + \frac{i\delta t}{\hbar} H^\dagger\right) \left(1 - \frac{i\delta t}{\hbar} H\right) \right| \psi(t) \right\rangle \\ &= 1 - P_{\text{jump}}(t) + \mathcal{O}(\delta t^2) \end{aligned} \quad (4.11)$$

Assuming that the time step  $\delta t$  is small enough, we can neglect second or higher order terms in the norm.

- 2- At same time  $t + \delta t$ , a uniformly distributed random number  $r_n \in [0,1]$  is compared with the jump probability [42],

$$P_{\text{jump}}(t) = \frac{\delta t}{\hbar} i \langle \psi(t) | H - H^\dagger | \psi(t) \rangle = \sum_k \delta p_k \quad (4.12)$$

where  $\delta p_k$  is the jump probability associated with  $k^{\text{th}}$  jump channel.

If  $r_n \geq P_{\text{jump}}(t)$ , the wave function keeps evolving under the effective Hamiltonian as,

$$|\psi(t + \delta t)\rangle = \left(1 - \frac{iH\delta t}{\hbar}\right) |\psi(t)\rangle \quad (4.13)$$

Or if  $r_n < P_{\text{jump}}(t)$ , a jump takes place. Then the wave function is given by,

$$|\psi(t + \delta t)\rangle = A_k |\psi(t)\rangle$$

Where  $A_k$  is a jump operator. If there are  $N_c$  jump channels with jump probabilities  $P_1, P_2, P_3 \dots P_{N_c}$  and associated jump operators  $A_1, A_2, A_3 \dots A_{N_c}$  then we apply jump operator  $A_1$  if  $r_n$  falls between 0 and  $P_1$ , operator  $A_2$  if  $r_n$  falls between  $P_1$  and  $P_1 + P_2$ , and so on. In this case no collapse occurs when  $r_n > P_1 + P_2 + P_3 + \dots P_{N_c}$  [44].

In both cases, the wave function is not normalized. The normalized wave function at time  $t + \delta t$  is,

$$|\psi(t + \delta t)\rangle \rightarrow \frac{|\psi(t+\delta t)\rangle}{\sqrt{\langle \psi(t + \delta t) | \psi(t + \delta t) \rangle}} = \frac{|\psi(t+\delta t)\rangle}{\sqrt{1 - P_{\text{jump}}(t)}} \quad (4.14)$$

After normalizing the wave function the cycle is started again. The term "quantum jump" is intentionally left general, as it refers to a conceptual process where the wavefunction is collapsed onto the eigenstate corresponding to a specific measurement outcome. The role of the jump operator will be explored further in examples that will be discussed later. Figure 4-2 depicts a flowchart of the quantum trajectory theory. Thus, starting with a normalized wavefunction  $|\psi(t)\rangle$ , the wavefunction can evolve in two possible ways over a sufficiently

small timestep  $\delta t$  : it follows a non-Hermitian evolution with probability  $1 - P_{\text{jump}}(t)$ , or it undergoes quantum jump evolution with probability  $P_{\text{jump}}(t)$ .

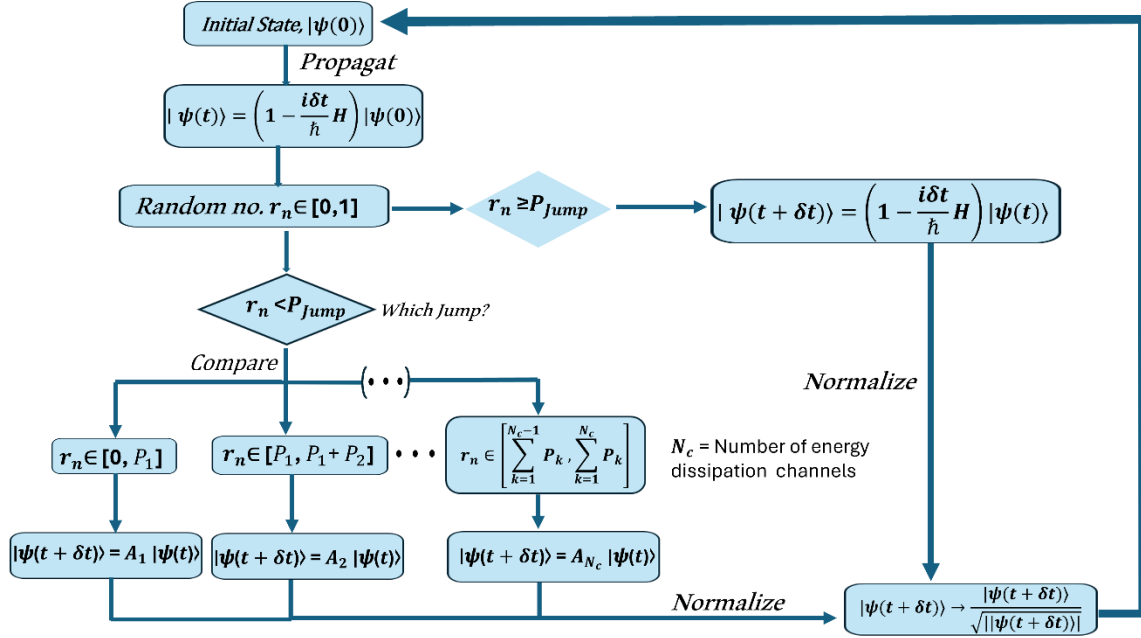


Figure 4-2 Flowchart of decision-making process of Quantum Trajectory Method.

Once we know how to evolve a Monte Carlo Wavefunction (MCWF) in time, we can calculate numerical values of observables. One of the primary goals of the MCWF approach is to enable the calculation of average values of physical observables. Using a Monte Carlo-type approach, the desired observable is computed for each realization, and the results are averaged over many realizations. Suppose  $N$  independent MCWF realizations have been propagated up to time  $t$  and we are interested in an observable  $A$ , represented by the Hermitian operator  $A$ . The expectation value for  $A$  across all wavefunctions (or trajectories) is then given by:

$$\langle A \rangle_{(N)}(t) = \frac{1}{N} \sum_{i=1}^N \langle \psi_i(t) | A | \psi_i(t) \rangle \quad (4.15)$$

Using a master equation approach, we would obtain the exact reduced density operator  $\rho_s$  and calculate the expectation value of any observable by computing  $\text{Tr}(\rho_s A)$ . The equation (4.15) approximates  $\text{Tr}(\rho_s A)$ , with the accuracy increasing as the number of realizations  $N$  grows [42].

In statistics, the error associated with the mean of a sample distribution is estimated by [42]:

$$\delta A_{(N)}(t) = \frac{\Delta A_{(N)}(t)}{\sqrt{N}}$$

with  $(\Delta A_{(N)}(t))^2$  being the sample variance at time  $t$  for the  $N$  expectation values:

$$(\Delta A_{(N)}(t))^2 = \frac{1}{N} \left( \sum_{i=1}^N \langle \psi_i(t) | A | \psi_i(t) \rangle^2 \right) - (\langle A \rangle_{(N)}(t))^2 \quad (4.16)$$

Thus, expectation values of physical observables come with an error margin of half-width  $\Delta A_{(N)}/\sqrt{N}$ . For large  $N$ , the fluctuations in the stochastic results average out by the law of large numbers of statistics and the MCWF average,  $\langle A \rangle_{(N)}(t)$  approaches  $\langle A \rangle(t) = \text{Tr}[\rho_s(t)A]$ .

We define the signal-to-noise ratio as  $\langle A \rangle / \Delta A_{(N)}$ , which indicates the quality of the computed average value for  $A$ . A good signal-to-noise ratio requires [42]:

$$\sqrt{N} \gg \frac{\Delta A_{(N)}(t)}{\langle A \rangle(t)} \quad (4.17)$$

This implies that the larger  $N$  is, the more reliable the results. However, beyond a certain  $N$ , additional realizations yield diminishing improvements, raising questions about the

requirements imposed by equation (4.17). To clarify, we categorize operators into "local" and "global" types. For example, the average kinetic energy is a global operator, while the population of a specific state  $|i\rangle$  is a local operator. Local operators often exhibit greater fluctuations in expectation values than global operators [42]. For local operators involving  $N_{\text{states}}$  electronic states, we get [42]:

$$\langle A \rangle(t) \sim \frac{1}{N_{\text{states}}}, (\Delta A)^2(t) \sim \frac{1}{N_{\text{states}}}, \text{ when } N_{\text{states}} \gg 1$$

Substituting these expectations into equation (4.17), we see that:

$$N \gg N_{\text{states}}$$

We recall that a larger number of realizations improves the signal-to-noise ratio. However, we now see that at least  $N_{\text{states}}$  simulations are needed, meaning the MCWF approach requires a complexity of  $O(N_{\text{states}}^2)$  for local operators.

For global operators, we expect [42]:

$$\Delta A(t) \sim \langle A \rangle(t)$$

Substituting this into equation (4.17) suggests:

$$\sqrt{N} \gg 1$$

Thus, for global operators, the method yields good results as soon as  $N$  is significantly greater than one. As noted by Mølmer, achieving 10% accuracy in the expectation value of such an operator requires around 100 realizations. Hence, if the number of realizations

needed for a desired accuracy is greater than the number of electronic states, the MCWF approach is more efficient than using a density matrix method [40], [42].

### 4.3.2 Examples of the Quantum Trajectory Method

#### 1- Two-level atom decaying spontaneously:

We begin by considering a two-level atom with damping assuming that the thermal photon number  $\bar{n}$  is zero. This system can be described by the following master equation [44]:

$$\dot{\rho} = -\frac{i}{2}\omega_a[\sigma_z, \rho] + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \quad (4.18)$$

where  $\gamma$  represents the rate of spontaneous emission and  $\omega_0$  is the frequency of atomic transition. In this equation,  $\sqrt{\gamma}\sigma_-$  and  $\sqrt{\gamma}\sigma_+$  are jump operators and  $\sigma_-$ ,  $\sigma_+$  are atomic lowering and raising operators respectively, corresponding to the radiative transitions in the two-level atom.

Generally, the wavefunction for a two-level system is given by,

$$|\psi(t)\rangle = C_g|g\rangle + C_e|e\rangle \quad (4.19)$$

The Hamiltonian for the system including the dissipation term is given by,

$$H_{\text{eff}} = \frac{\hbar\omega_a}{2}\sigma_z - i\frac{\hbar\gamma}{2}\sigma_+\sigma_- \quad (4.20)$$

The jump probability is given by,

$$P_{\text{jump}}(t) = \gamma \delta t |C_e|^2 \quad (4.21)$$

Having Hamiltonian, wavefunction and the probability of jump in hand, we can proceed with the steps of QTM as described in section 4.3.1. The simulation results are shown in

the figures below. Here, a transition from the excited state to the ground state results in photon emission to the environment. If we start with an atom in the excited state, the probability of being in the excited state remains at 1 until a quantum jump occurs, at which point the excited state probability becomes 0. After the jump occurs, the atom collapses to ground state, and the non-unitary Schrodinger's equation keeps the system in that state forever. No evolution happens between jumps; however, between jumps, the excited state amplitude decays due to the non-Hermitian term  $-i\hbar\gamma$  [44]. Figure 4-3 shows the simulation of the two-level atom decaying spontaneously using both master equation method and quantum trajectory method. The effect of increasing the number of trajectories is that the results using QTM converge rapidly to that of the master equation method as shown by the blue curve (100 trajectory) and crayon curve (1000 trajectory).

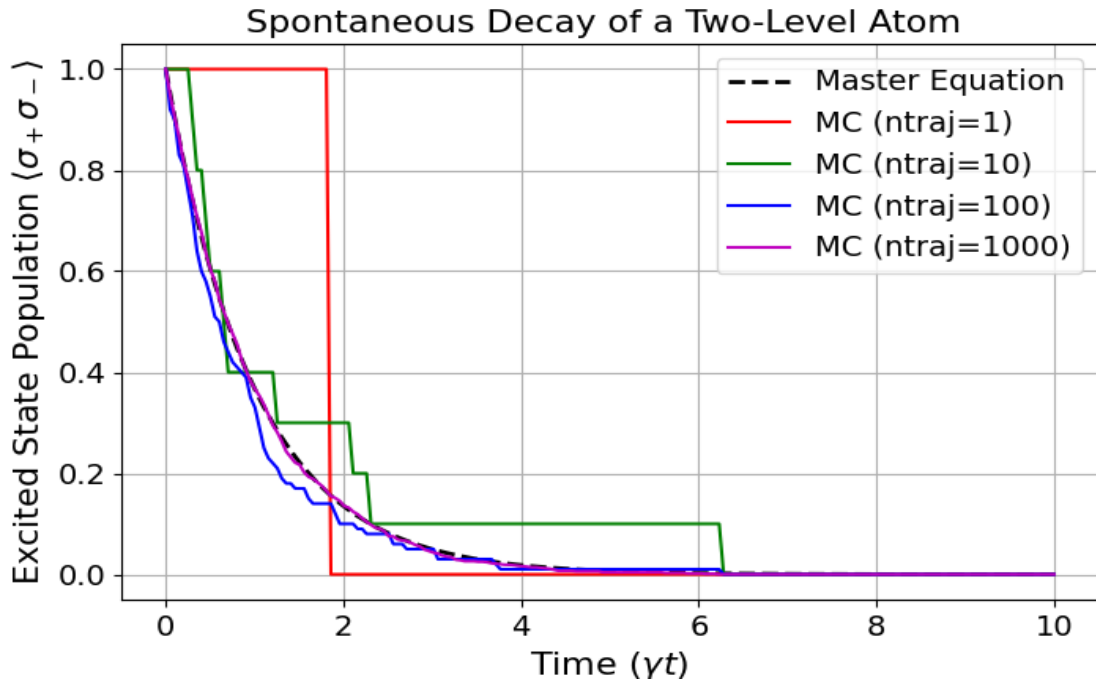


Figure 4-3 Decay of a two-level atom using both master equation method and quantum trajectory method for  $\gamma = 1$  and  $\gamma dt = 0.05$ . Figure shows that the solution using QTM (dashed black curve) becomes equivalent to master equation method when no. of trajectories is large. The python code for this figure is listed in appendix A program-1.

## 2- Cavity in Fock state decaying spontaneously:

Let us consider the example of the cavity in a bosonic Fock state at time  $t=0$ , given by,  $|\psi(0)\rangle = |n\rangle$ , where  $n$  is number of photons in the Fock state. A Fock state is a state with well-defined particle number that can be written as product of creation operators operating on vacuum state. The effective non-Hermitian Hamiltonian for this system including the dissipation term will be given by [44],

$$H_{\text{eff}} = \hbar\omega a^\dagger a - i\hbar\kappa a^\dagger a \quad (4.22)$$

Where  $\omega$  is the resonant angular frequency of cavity and  $a^\dagger, a$  are creation and annihilation operators for the cavity,  $\kappa$  is the spontaneous decay rate of the cavity. The Lindblad master equation for this system with the jump operator  $a$  will be given by,

$$\dot{\rho} = -i\omega[a^\dagger a, \rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \quad (4.23)$$

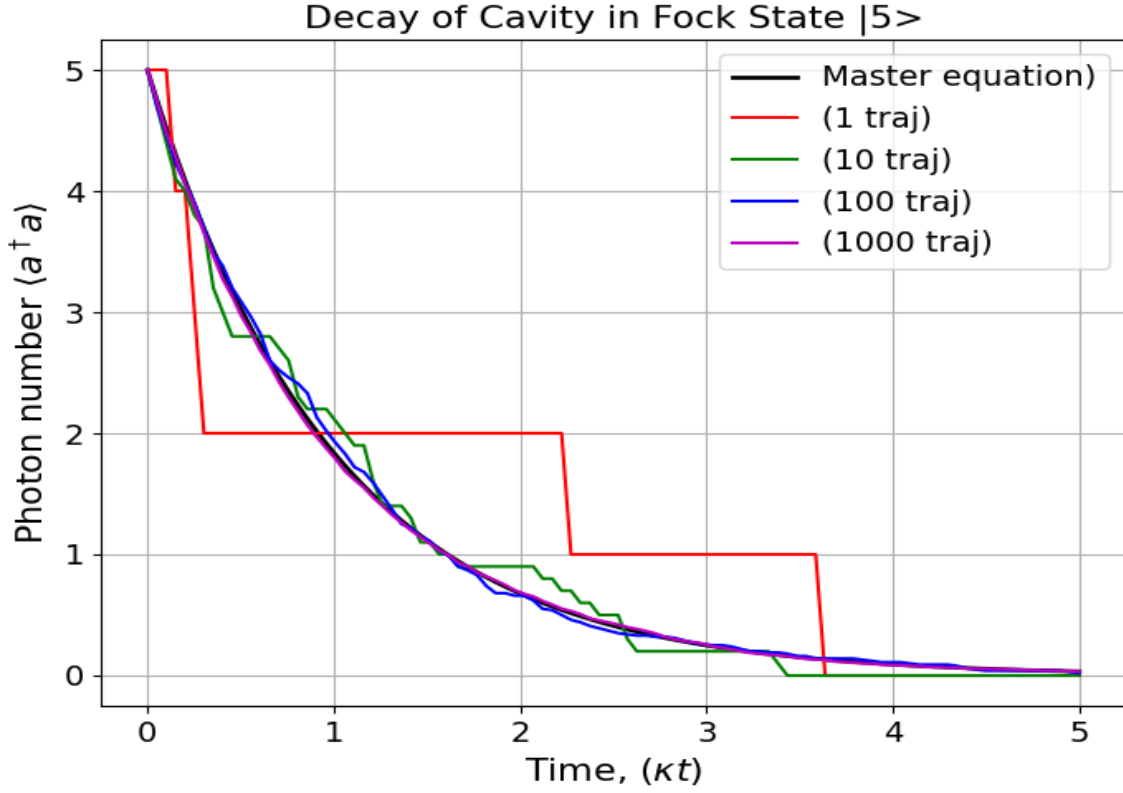


Figure 4-4 Decay of cavity initially in the Fock state  $|5\rangle$ . Simulation parameters are:  $\kappa = 1$  and  $\kappa dt = 0.05$ . The python code to generate this figure is listed in appendix A program-2.

Using these preliminaries one can proceed with steps of QTT to simulate the problem. The Figure 4-4 shows the quantum trajectory simulation of a cavity prepared in Fock state  $|\psi(t)\rangle = |5\rangle$ . The red steps in the plot represent abrupt "jumps" in the system's state, which correspond to quantum jumps caused by photon emission events. After each jump, the system's wavefunction is renormalized, which resets the cycle of evolution. The black curve represents the result obtained from the master equation. Taking many trajectories to simulate the results, converges the results of quantum trajectory method to that of master equation method. Green, blue and magenta curves in Figure 4-4 show the evolution of the same cavity Fock state with 10, 100 and 1000 trajectories respectively. Compared to single trajectory (red curve), the green, blue and magenta curves become increasingly smoother

because it averages over more and more realizations of the decay process. In this case, the individual quantum jumps become less pronounced, and the overall result begins to approach the smooth black curve obtained from the master equation.

We can also take a cavity prepared in the superposition state instead of pure Fock state given by,

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|2\rangle + |0\rangle)$$

The effective Hamiltonian is still same as equation (4.28),

$$H_{\text{eff}} = \hbar\omega a^\dagger a - i\hbar\kappa a^\dagger a \quad (4.24)$$

The Lindblad form of master equation for this system will be given by,

$$\dot{\rho} = -i\omega[a^\dagger a, \rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

The collapse operator is:  $c = \sqrt{\kappa}a$ . Figure 4-5 shows the simulation of the given system for 10 and 100 trajectories respectively. With a small number of trajectories, the decay curve exhibits significant fluctuations due to stochastic quantum jumps. Each trajectory corresponds to a unique quantum evolution of the system, so the average over a few realizations is noisy (red curve) and does not accurately approximate the smooth master equation solution (black curve). As the number of trajectories increases (100 trajectories), the blue curve closely matches the black curve. The fluctuations are smoothed out because averaging over a large number of trajectories effectively reproduces the ensemble average predicted by the master equation.

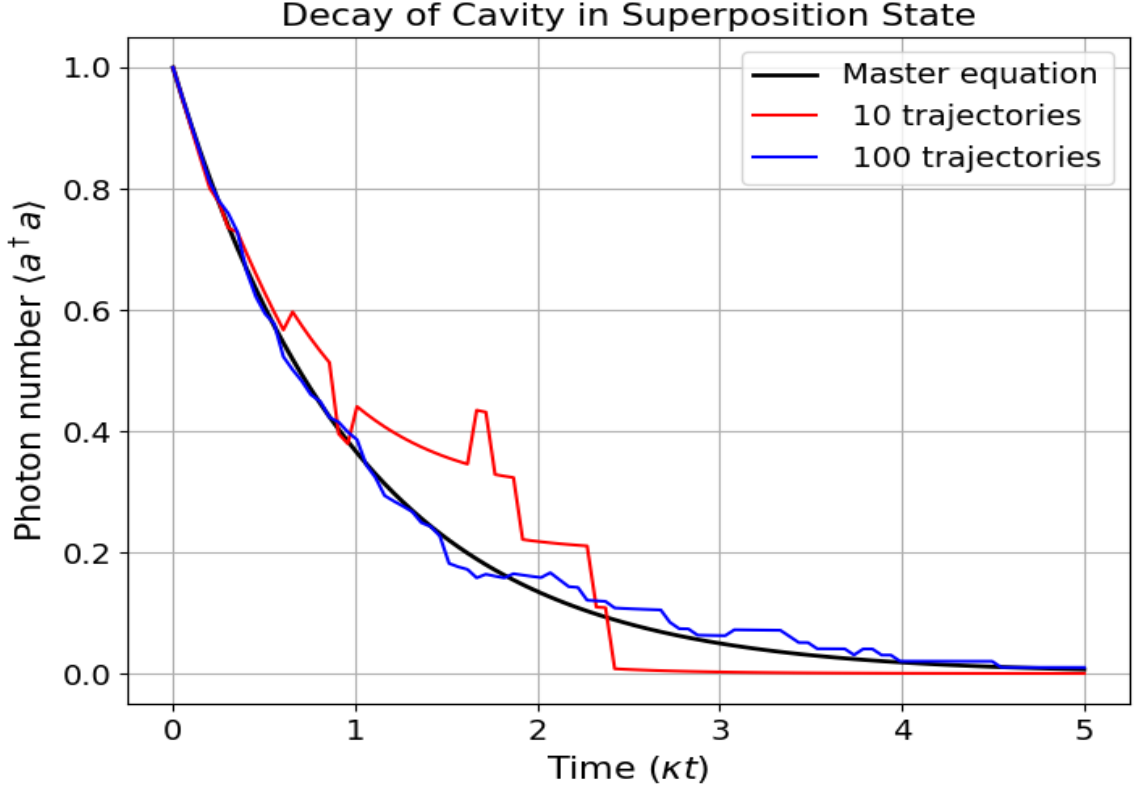


Figure 4-5 Decay of the cavity initially in superposition state with 10 and 100 trajectories. Simulation parameters are:  $\kappa = 1$  and  $\kappa dt = 0.05$ . The python code for this figure is listed in appendix A program-3.

### 3- Resonance Fluorescence from Two-Level Atom

Till now we have considered rather trivial examples to illustrate the evolution of an open system using the quantum trajectory method. Now we are in a comfortable position to tackle some more advanced problems. We consider a two-level atom being driven by a laser. The effective Hamiltonian for such a system including the dipole interaction term and the decay term within the *rotating wave approximation* is given by [45],

$$H_{\text{eff}} = \frac{1}{2} \hbar \omega_a \sigma_z + \hbar \frac{\Omega}{2} (\sigma_+ + \sigma_-) - i \frac{\hbar \gamma}{2} \sigma_+ \sigma_- \quad (4.25)$$

Where  $\Omega = -\frac{\mathbf{d}\cdot\mathbf{E}}{\hbar}$  is Rabi frequency of the laser with  $\mathbf{E}$  being the electric field of the laser and  $\mathbf{d}$ , dipole moment of the atom. In equation (4.34),  $\omega_a$  is the frequency of atomic transition,  $\sigma_+$ ,  $\sigma_-$  are atomic raising and lowering operators respectively,  $\gamma$  is the spontaneous emission rate and  $\sigma_z$  is a Pauli's spin-z matrix. The master equation for such a system in Lindblad form is given by,

$$\dot{\rho} = -i\frac{\omega_a}{2}[\sigma_z, \rho] - i\frac{\Omega}{2}[(\sigma_+ + \sigma_-), \rho] + \frac{\gamma}{2}(2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_-) \quad (4.26)$$

The jump operator is recognized as,  $A = \sqrt{\gamma}\sigma_-$ .

For our discussion, we assume that initially the state of the atom is in a superposition of ground state  $|g\rangle$  and excited state  $|e\rangle$ ,

$$|\psi(0)\rangle = c_1(0)|g\rangle + c_2(0)|e\rangle \text{ with } |c_1|^2 + |c_2|^2 = 1$$

We know that a single fluorescing atom evolves into a steady state [45]. In the quantum trajectory approach, we assume that the conditioned wavefunction's evolution follows a steady-state stochastic process. This stochastic process is relatively simple, as it obeys the collapse rule. After each jump (photon emission), the atom ends up in its lower state, meaning that the evolution between jumps starts from the same initial condition. However, unlike spontaneous emission, in the presence of a driving field, the atom does not stay in the lower state after a collapse. Instead, it evolves to a new state  $|\psi(\delta t)\rangle = c_1(\delta t)|0\rangle + c_2(\delta t)|1\rangle$  where  $c_2(\delta t) \neq 0$  and  $\delta t$  represents the time since the last collapse. This

evolution gives the atom a continuous probability of undergoing another collapse and emitting another photon.

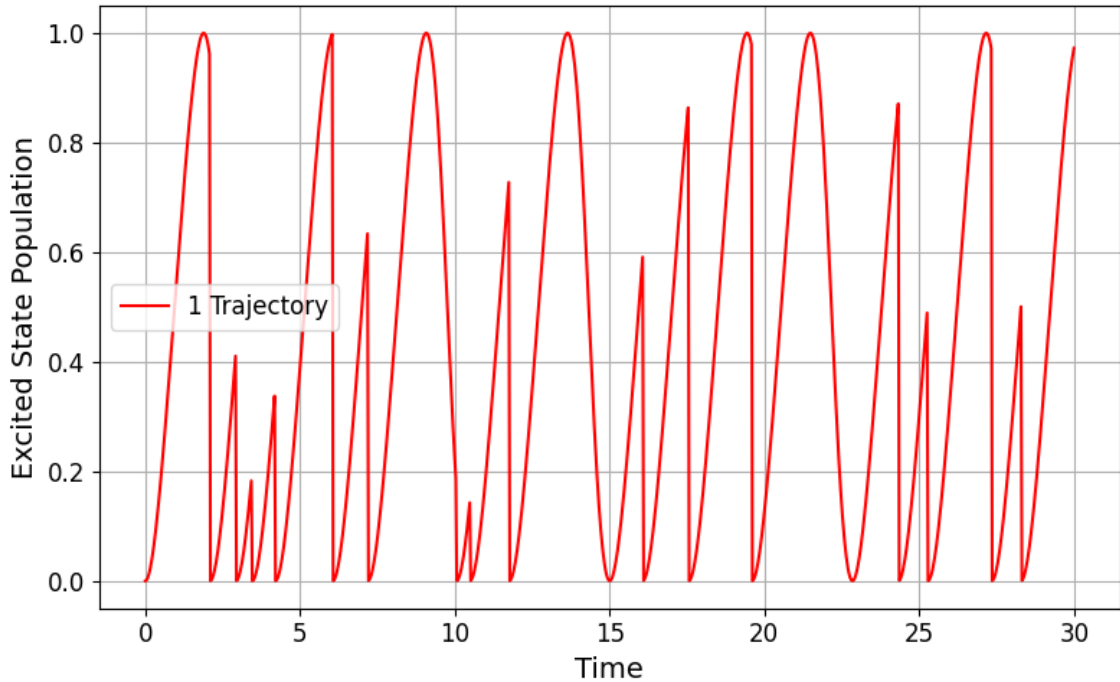


Figure 4-6 Single trajectory for resonance fluorescence of strongly driven atom ( $\Omega = 4\gamma$ ). Simulation parameters are:  $\Omega = 4\gamma$  and  $\gamma dt = 0.06$ . The python code for this figure is listed in appendix A program-4.

The population of the excited state exhibits *Rabi oscillations*, oscillating between 0 and 1 at the Rabi frequency. As shown in Figure 4-7. These oscillations are oscillations of excited state population of a two-level atom when a laser drives this atom. The frequency of these oscillations is called Rabi frequency. When the spontaneous emissions from the atom are considered with rate  $\gamma$ , these oscillations are periodically interrupted by quantum jumps, which occur at random intervals. Each quantum jump collapses the wavefunction onto the ground state, effectively resetting the system. **Error! Reference source not found.** i

illustrates the comparison between the excited-state population dynamics of a quantum

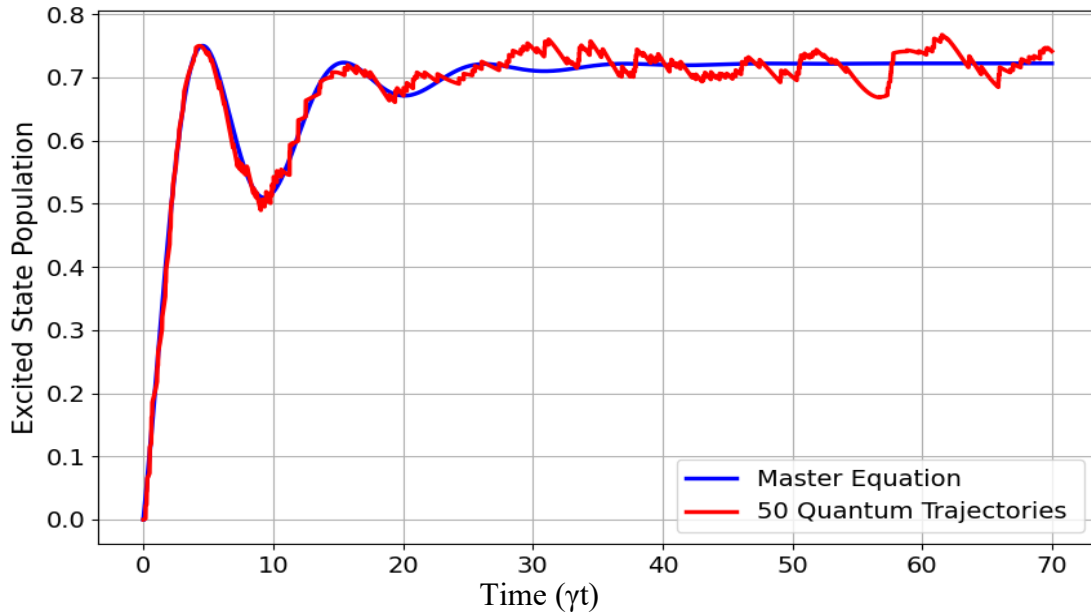


Figure 4-7 Effect of increasing no. of trajectories on excited state population of two-level atom. Simulation parameters are:  $\Omega = 2\gamma$  and  $\gamma dt = 0.06$ . The quantum trajectory method with 50 trajectories is compared with master equation method. The python code for this figure is listed in appendix A program-4.

system over time, obtained using the deterministic master equation (blue) and the stochastic quantum trajectory method with 50 trajectories (red). Both methods demonstrate consistent results, particularly in the steady-state regime, where the master equation produces a smooth population curve, while the trajectory method exhibits fluctuations due to the stochastic nature of the approach. These fluctuations arise from the finite number of trajectories and would reduce as the number of trajectories increases. This comparison highlights the quantum trajectory method offers insights into individual quantum realizations.

## 4.4 Simulating Open Quantum System Using QuTip Library of Python

QuTiP (Quantum Toolbox in Python) is an open-source software library designed for the numerical simulation of open quantum systems. It is a versatile tool widely used in quantum physics and quantum optics research, providing robust methods to model quantum systems, especially when they interact with an environment. This interaction with the environment typically results in dissipation, decoherence, and other non-unitary processes, which are critical for understanding real-world quantum systems. This interaction is described using theoretical framework such as the Lindblad master equation and quantum trajectory methods. QuTiP enables researchers to simulate these processes efficiently, offering a range of solvers and pre-built tools for numerical analysis.

QuTiP's main advantage lies in its ability to describe the evolution of density matrices for quantum systems under the influence of dissipative dynamics. The library provides a straightforward interface for defining Hamiltonians, collapse operators, and system observables. Users can define custom Hamiltonians, including those representing light-matter interactions, vibrational modes, and even complex many-body systems.

For simulating open quantum systems, QuTiP's master equation solvers are particularly useful. The Lindblad master equation, can be directly implemented in QuTiP. The user specifies the system Hamiltonian and collapse operators that describe various forms of dissipation, such as spontaneous emission or dephasing. QuTiP then calculates the system's time evolution, producing results in the form of density matrices or expectation values of observables. Another strength of QuTiP is its support for quantum trajectory simulations.

This approach is particularly advantageous for studying measurement-induced collapse, photon counting statistics, and individual quantum events. QuTiP's implementation of quantum trajectory methods is both flexible and efficient, allowing us to gain insights into specific stochastic processes within their systems.

QuTiP also supports the modeling of complex quantum systems such as those encountered in cavity quantum electrodynamics, trapped ions, and superconducting qubits. Simulating cavity-mediated laser cooling, QuTiP can model atom-cavity interaction, including the coupling of phonon modes to external fields. We define the system's Hamiltonian, add collapse operators for cavity decay and atomic transitions, and solve for the steady-state density matrix. From this, observables like the phonon number distribution or cooling rates can be extracted. The library includes pre-defined tools for visualizing results, such as plotting Wigner functions, expectation values, or eigenstate populations. Its integration with Python's ecosystem of libraries like NumPy, SciPy, and Matplotlib further enhances its usefulness, making it a comprehensive solution for open quantum system simulations. Some of the main functions of QuTiP that I have used for simulation purposes are listed below.

### **1. Master Equation Solver**

The “mesolve” function in QuTiP is designed to solve the Lindblad master equation, which governs the time evolution of a quantum system's density matrix. Key features of this function are given below.

1      Input Parameters:

- **H (Hamiltonian):** Specifies the system's dynamics. It can be time-independent or time dependent, representing the coherent evolution of the system.
- **rho0 (Initial Density Matrix):** Describes the initial state of the quantum system.
- **tlist (Time Points):** Specifies the time intervals over which the simulation is performed.
- **c\_ops (Collapse Operators):** Represent the dissipative processes, such as spontaneous emission, dephasing, or cavity loss.
- **e\_ops (Expectation Operators):** Define the observables to be calculated during the simulation, such as population, coherence, or phonon numbers.

## 2 Outputs:

- Returns a **Result** object containing the time evolution of the density matrix or expectation values of observables.

## 2. Steady State Solver

The “steadystate” function computes the steady-state solution of an open quantum system by solving the Lindblad master equation in the long-time limit. It finds the density matrix where the system's evolution has equilibrated.

### 1 Input Parameters:

- **H (Hamiltonian):** The system Hamiltonian driving coherent dynamics.
- **c\_ops (Collapse Operators):** Include all dissipative processes affecting the system.

## 2 Outputs:

- Returns the steady-state density matrix, which satisfies  $\mathcal{L}(\rho) = 0$ , where  $\mathcal{L}$  is the Liouvillian superoperator.

## 3. Monte-Carlo Solver

The “mcsolve” function is a quantum trajectory solver that simulates the evolution of a system by averaging over multiple quantum trajectories. Unlike mesolve, which directly solves the master equation for the density matrix, mcsolve provides individual realizations of quantum states.

Key Features:

### 1 Input Parameters:

- **H (Hamiltonian):** Governs coherent dynamics.
- **psi0 (Initial State):** Specifies the initial pure state of the system.
- **tlist (Time Points):** Time intervals for the simulation.
- **c\_ops (Collapse Operators):** Represent the dissipative processes causing quantum jumps.
- **ntraj (Number of Trajectories):** Specifies how many trajectories to simulate for statistical averaging.

### 2 Outputs:

- Returns a Result object containing the quantum states for each trajectory or averaged expectation values over trajectories.

In summary, QuTiP is an indispensable tool for simulating and understanding open quantum systems. Its ability to model non-unitary dynamics, support for both master equation and quantum trajectory methods, and ease of use make it highly suitable for addressing the problem of CMLC. However, despite its versatility, the computational speed of QuTiP proved to be a significant slow for our system. Specifically, this limitation proved to be an obstacle in our ability to perform Monte Carlo Wave Function (MCWF) simulations effectively, as the time required to handle the computational demands exceeded practical constraints.

## CHAPTER 5

### Cavity-Mediated Laser Cooling of a two-level atom

When a particle is trapped inside a harmonic potential, depending upon the strength of the trap, the dynamics of a particle differs from that of a free particle. When cooling trapped atoms or ions, we typically distinguish between two different regimes based on the relationship between the trap's ground-state size and the wavelength  $\lambda$  of the cooling laser. The ground-state size of the particle refers to the characteristic spatial spread,  $r$  of the ground state wave function and can be determined by taking root-mean-square (RMS) of the particle's position expectation value. When the ground-state size of the particles motion  $r$  is much larger than the laser wavelength ( $r \gg \lambda$ ), the effect of the trapping potential on the cooling process is minimal. Because in this case, the spatial oscillations of the particle appear relatively unconfined with respect to the laser's periodic electromagnetic field. As a result, the spatial localization provided by the trapping potential does not strongly influence the cooling process because the laser's effect averages out over the large motion of the particle. In this scenario, standard cooling methods designed for free atoms—such as Doppler cooling—are effective and can be applied without significant modifications [46].

On the other hand, when the wavelength  $\lambda$  is much larger than ground state size of the trap ( $\lambda \gg r$ ), the situation changes drastically. This regime is known as the Lamb-Dicke limit (LDL), where the atom's motion within the trap becomes quantized and starts to play a major role in the cooling process. In the Lamb-Dicke regime, the particle is confined to a

region smaller than the laser wavelength, which reduces the impact of photon recoil during absorption and emission [46]. This tight confinement necessitates specialized cooling techniques that take the atom's quantized motion into account, leading to unique cooling behaviors distinct from those observed in free-space cooling. In the Lamb-Dicke limit, when the cavity is blue detuned from the atomic transition frequency, the spontaneous emissions from atom into cavity mode dominate over the spontaneous emissions in the free space, so every absorption-emission cycle accompanies loss of amount of energy corresponding to the detuning frequency. When the absorption-emission process is repeated many times, then the atom is cooled down to ground state of its trap [47].

In my thesis, we will analyze the cooling dynamics of a two-level atom which has been precooled and is trapped using a harmonic trap inside a cavity. A relevant experimental setup is shown in Figure 5-1. When the atom inside the cavity decays spontaneously, the emitted photon applies a backward recoil momentum  $p_r = \hbar k_L$  on the atom. Where  $k_L$  is the wave vector of the laser field. This recoil momentum increases the energy of the atom of mass  $M$  by an amount  $E_r = \hbar^2 k_L^2 / 2M$ . The energy of a quantum of vibrational motion is represented by  $\hbar \nu$  where  $\nu$  is the trap or phonon frequency. So, when the atom gains energy from the recoil of the photon, it gains energy equal to an amount  $m\hbar \nu$  and goes to  $m$ th vibrational state. Where  $m=0,1,2,3,\dots$  is the number of quanta of vibrational energy of the atom. Thus if  $E_r > \hbar \nu$ , the atom can gain energy more than energy of one quantum of

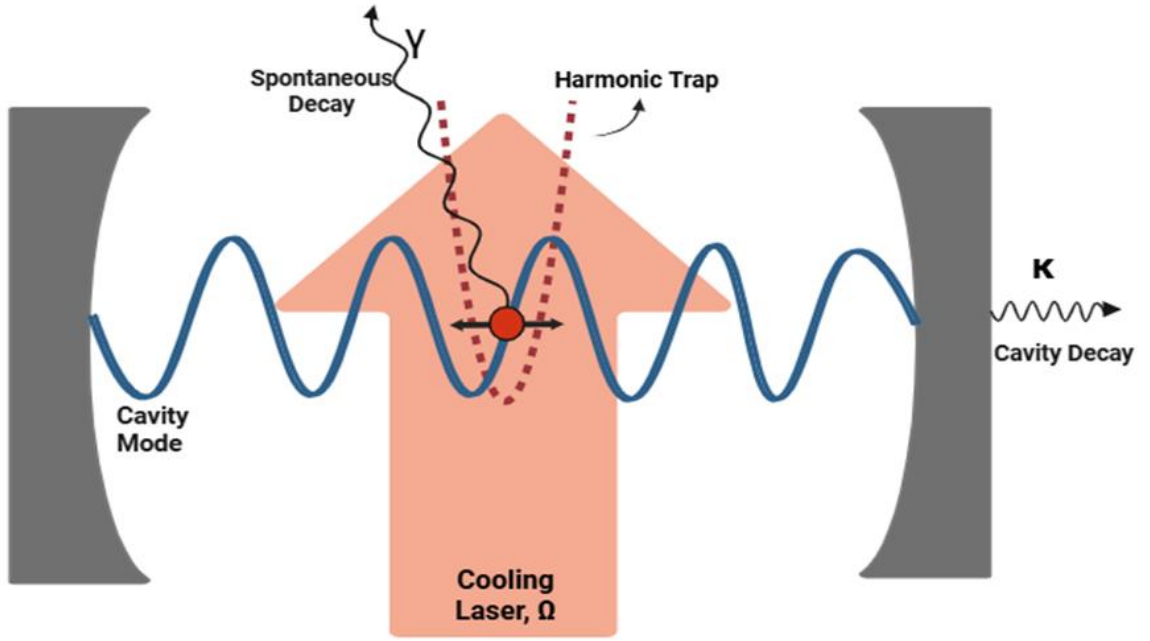


Figure 5-1 Experimental setup showing an atom trapped in cavity by an external trap.

vibration i.e. phonon and can make transitions in which the phonon number changes by more than one. However, if  $E_r \ll \hbar\nu$ , then higher-order transitions where  $m > 1$  are suppressed, and the particle primarily moves between adjacent levels. This leads us to define a key parameter - the Lamb-Dicke parameter  $\eta$ , which characterizes the interaction between the particle's internal states and its external vibrational motion, and it is given by,

$$\eta = \sqrt{\frac{\hbar k_L^2}{2M\nu}} = \sqrt{\frac{E_r}{\hbar\nu}} \quad (5.1)$$

Any particle trapped inside a cavity is said to be in Lamb-Dicke regime (LDL) if  $\eta \ll 1$ . To understand this limit, we examine the particle's root mean square (RMS) position [46] [3]:

$$\langle r \rangle_{\text{rms}} = \sqrt{\langle m | (r)^2 | m \rangle}$$

$$\begin{aligned}
&= \sqrt{\left\langle m \left| \left( \frac{\hbar}{2Mv} \right) (b + b^\dagger)^2 \right| m \right\rangle} \\
&= \sqrt{\left( \frac{\hbar}{2Mv} \right) (2m + 1)} \tag{5.2}
\end{aligned}$$

Where  $b^\dagger$ ,  $b$  are phononic creation and annihilation operators respectively. In the Lamb-Dicke regime, the particle's oscillations are so tightly confined that its RMS position  $\langle r \rangle_{\text{rms}}$  is much smaller than the wavelength of the laser light. This restriction on motion suppresses transitions between vibrational levels that would change the quantum number by more than one. To quantify [46],

$$\text{Or,} \quad \langle r \rangle_{\text{rms}} \ll \frac{1}{k_L}$$

$$\text{Or,} \quad k_L \langle r \rangle_{\text{rms}} \ll 1$$

$$\text{Or,} \quad \sqrt{k_L^2} \sqrt{\langle m | (r)^2 | m \rangle} \ll 1$$

$$\text{Or,} \quad \sqrt{\eta^2 (2m + 1)} \ll 1 \tag{5.3}$$

When a particle is in the ground state  $m = 0$ , then  $\eta \ll 1$  is the typical value of  $\eta$ .

Having all the prerequisites in hand, we are ready to tackle the problem of CMLC using the quantum trajectory method (QTM). As already described in the previous chapter, to solve a system using QMWF, first of all we must set up the Hamiltonian for the system

and recognize the decay or jump channels i.e. jump operators. In the literature the problem of CMLC of a single two-level atom has already been solved using approximate analytical approaches [48]. The analytical approach exploits the master equation formalism to set up rate equations for the steady state phonon number.

## 5.1 The Hamiltonian for Atom-Cavity-Phonon System

A discussion of this section can be found in [48] [49]. A relevant experimental setup for our problem is shown in the **Error! Reference source not found..** The total Hamiltonian for the system depicted in this figure consists of following terms,

$$H_{\text{total}} = H_{\text{trap}} + H_{\text{atom}} + H_{\text{cavity}} + H_{\text{atom-laser}} + H_{\text{atom-cavity}} \quad (5.4)$$

Here,  $H_{\text{vibrational}}$  is the Hamiltonian describing atomic vibrations and  $H_{\text{electronic}}$  is the Hamiltonian that captures the dynamics of the electronic transition between the excited and ground state of the atom. Moreover,  $H_{\text{cavity}}$  is the Hamiltonian that describes the cavity field,  $H_{\text{atom-laser}}$  the Hamiltonian that describes interactions between the atom and incoming laser field and  $H_{\text{interaction}}$  is the interaction Hamiltonian due to interaction among cavity photons, electronic and the vibrational states of the atom.

We discussed the Jaynes-Cummins Hamiltonian for interaction of light with a two-level atom in chapter 2. There we made an approximation that the light is interacting with an atom at ‘rest’. Hence, we ignored the external degrees of freedom of the atom. For an atom trapped inside a harmonic trap, the motion of the atom can be quantified as that of a simple harmonic oscillator. In what follows, we will consider that the atom is trapped such that it can only vibrate along the axis of the cavity. Then, the Hamiltonian for the

motion of the atom inside the trap is given by,

$$H_{\text{trap}} = \frac{p_z^2}{2M} + \frac{1}{2} M v^2 z^2 \quad (5.5)$$

Where  $p_z$  is the momentum,  $M$  is the mass of the atom,  $v$  is the velocity, and the cavity axis is assumed to be the z-axis. In the quantum mechanical treatment of a harmonic oscillator, we can replace momentum and position operator by creation and annihilation operator:

$$b = \sqrt{\frac{Mv}{2\hbar}} \left( z - \frac{i}{Mv} p_z \right)$$

$$b^\dagger = \sqrt{\frac{Mv}{2\hbar}} \left( z + \frac{i}{Mv} p_z \right) \quad (5.6)$$

With,

$$[b, b^\dagger] = 1$$

Substituting these operators in equation (5.5), the atom's vibrational Hamiltonian becomes,

$$H_{\text{trap}} = \left( b^\dagger b + \frac{1}{2} \right) \hbar v$$

The constant term  $\frac{1}{2} \hbar v$  is the zero point energy term. It does not contribute to the time-evolution of the operators. Also, since this term has no operators, it only introduces a global phase factor  $e^{-\frac{i}{2}vt}$  to the wavefunction which does not impact the observable quantities. Hence, without loss of generality, we can drop the ground state energy term from the above Hamiltonian,

$$H_{\text{vib}} = \hbar \nu b^\dagger b \quad (5.7)$$

As shown in section chapter 2, equation (2.46) and (2.35), the Hamiltonian for electronic states of the atom is given by,

$$H_{\text{electronic}} = \hbar \omega_a \sigma^+ \sigma^- \quad (5.8)$$

and the Hamiltonian for the cavity field is given by,

$$H_{\text{cavity}} = \hbar \omega_c a^\dagger a \quad (5.9)$$

In what follows the atom-laser interaction will be described using a semiclassical model, where the atom is treated quantum mechanically while the laser field is represented as a classical electromagnetic wave. The Hamiltonian for this interaction is given by:

$$H_{\text{atom-laser}} = \mathbf{d} \cdot \mathbf{E}_L = (d\sigma^+ + d\sigma^-)(E_0 e^{i\omega_L t} + E_0 e^{-i\omega_L t}) \quad (5.10)$$

where we have used the expression from Equation (2.51). In this expression  $d$  is the dipole moment of the atom.  $E_0 e^{i\omega_L t}$  and  $E_0 e^{-i\omega_L t}$  represent the oscillating electric field of the laser with amplitude  $E_0$  and frequency  $\omega_L$ . To maximize the interaction between atom and laser field, we can tune our laser to the frequency of electronic transition of the atom:

$$\omega_L = \omega_a$$

This resonance condition enhances the probability of energy exchange between the laser field and the atom, making the interaction more prominent.

From chapter 2, equation (2.52) the cavity-atom interaction Hamiltonian term is given by,

$$H_{\text{int}} = \hbar G(z)(\sigma^+ a + \sigma^- a^\dagger)$$

With coupling,

$$G(x) = g \sin(kz)$$

With,  $g = -\sqrt{\frac{\omega_L}{2\hbar\epsilon_0 V}}d$ . In the Lamb-Dicke regime, the amplitude of oscillations of the atom around the center of the trap is small. So, we can approximate  $\sin(kz) \approx kz$ . Moreover, the quantity  $kz$  cannot be treated as just a number, instead  $z$  is now the position operator and can be represented in terms of creation and annihilation operators,  $b$  and  $b^\dagger$ , giving,

$$kz = \eta(b + b^\dagger) \quad (5.11)$$

The interaction Hamiltonian in equation (5.10) is then given by,

$$H_{\text{int}} = \hbar\eta g(b + b^\dagger)(\sigma^+ a + \sigma^- a^\dagger) \quad (5.12)$$

Collecting all the terms, the total Hamiltonian for the system become,

$$\begin{aligned} H_{\text{total}} = & \hbar\omega_a \sigma^+ \sigma^- + \hbar\nu b^\dagger b + \hbar\omega_c a^\dagger a \\ & + (d_0 \sigma^- + d_0^* \sigma^+) (E_0 e^{-i\omega_a t} + E_0^* e^{i\omega_a t}) \\ & + \eta g(b + b^\dagger)(a\sigma^+ + a^\dagger \sigma^-) \end{aligned}$$

In practical situations, the optical frequencies fulfill  $\omega_a, \omega_c \gg \Delta, \nu, g$ . Where  $\Delta = \omega_c - \omega_a$  is the cavity-atom detuning. Identifying the time evolution operator for this Hamiltonian as  $U_0(t, 0) \equiv e^{-iH_0 t/\hbar}$  and  $H_0 = \hbar\omega_a \sigma^+ \sigma^- + \hbar\omega_a a^\dagger a$ , We get the total Hamiltonian in the interaction picture as,

$$H_I = e^{\frac{iH_0 t}{\hbar}} (H_{\text{total}} - H_0) e^{-\frac{iH_0 t}{\hbar}}$$

Or more explicitly, it can be rewritten as the equation below,

$$\begin{aligned}
H_I = & \hbar v b^\dagger b + \hbar(\omega_c - \omega_a) a^\dagger a + d_0 E_0 \sigma^- e^{-2i\omega_a t} + d_0 E_0^* \sigma^- \\
& + d_0^* E_0 \sigma^+ + d_0^* E_0^* \sigma^+ e^{2i\omega_a t} + \eta g b a \sigma^+ + \eta g b a^\dagger \sigma^- \\
& + \eta g b^\dagger a \sigma^+ + \eta g b^\dagger a^\dagger \sigma^-
\end{aligned} \tag{5.13}$$

The optical frequencies,  $\omega_c$  and  $\omega_a$  are much larger than frequencies  $v$ ,  $g$  and  $\Delta = (\omega_c - \omega_a)$ . In the rotating-wave approximation (RWA), we can neglect fast oscillating terms containing  $2\omega_a$ . Then interaction Hamiltonian can also be written as,

$$\begin{aligned}
H_I = & \hbar v b^\dagger b + \hbar \Delta a^\dagger a + \frac{1}{2} \hbar \Omega (\sigma^- + \sigma^+) \\
& + \hbar \eta g (b + b^\dagger) (\sigma^+ a + \sigma^- a^\dagger)
\end{aligned} \tag{5.14}$$

Where  $\Omega = \frac{d \cdot E}{\hbar}$ , is Rabi frequency. This is the expression of Hamiltonian which encapsulates the dynamics of a particle confined strongly to a cavity. We will solve the Schoedinger equation for this Hamiltonian to gain insight into the physics of CMLC of single atom.

## 5.2 Master Equation Method for Cooling a Single Atom Coupled to a Cavity

A master equation analysis with approximations for cooling of a single particle mediated by a cavity has previously been performed by Kim and Beige [50]. The master equation for the system in Figure 5-1 in Lindblad form is given by,

$$\begin{aligned}
\frac{d\rho}{dt} = & -\frac{i}{\hbar} [H_I, \rho] + \frac{1}{2} \kappa (2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \\
& + \frac{1}{2} \gamma (2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-)
\end{aligned} \tag{5.17}$$

The vibrations of the atom around its center of mass position are characterized by the number of phonons in the cavity. We want to ultimately reduce the number of phonons in the system. The atom is cooled to its ground state when there are no more phonons left in the system. That means, we are interested how the expectation value of the phononic number operator  $m = \langle b^\dagger b \rangle$  evolves in time. For the Lindblad equation (5.17), the expectation value of any operator  $O$  is given by,

$$\begin{aligned} \langle \dot{O} \rangle = & -\frac{i}{\hbar} \langle [O, H_1] \rangle + \frac{1}{2} \kappa \langle 2a^\dagger O a - O a^\dagger a - a^\dagger a O \rangle \\ & + \frac{1}{2} \gamma \langle 2\sigma^+ O \sigma^- - O \sigma^+ \sigma^- - \sigma^+ \sigma^- O \rangle \end{aligned} \quad (5.18)$$

For the zeroth and first-order approximations in the interaction strength  $\eta$ , the mean phonon number  $m$  remains constant, showing no time-dependent evolution. However, extending the analysis to second order in  $\eta$  reveals that the mean phonon number evolves over a much longer timescale [48]. This extended timescale is crucial for implementing the adiabatic elimination of faster-evolving variables. By ensuring that the mean phonon number changes significantly more slowly than other dynamical variables, the system's behavior can be analyzed more effectively within this approximation. We assume that the atom-phonon-photon interaction constant  $\eta g$  is considerably smaller than the atom-cavity detuning  $\Delta$ , cavity decay rate  $\kappa$ , or phonon frequency  $\nu$ . That is,  $\eta g \ll \Delta, \kappa$ , or  $\nu$ , enables a quasi-steady-state approximation, allowing for adiabatic elimination of fast-evolving coherences. This framework facilitates a deeper understanding of the cooling dynamics, as it isolates the slow evolution of the mean phonon number from other rapidly fluctuating variables within the system.

Using (5.18) equation for mean phonon number  $m$ , a rate equation for phonon number  $m$  comes out to be [48]

$$\dot{m} = -R_c m + C \quad (5.19)$$

Where  $R_c$  is the effective cooling rate given by,

$$R_c = \frac{2\eta^2 g^2 \Omega^2}{\gamma^2 + 2\Omega^2} (R_+ - R_-) \quad (5.20)$$

with:

$$R_- = \frac{(\gamma_0 \gamma_1 \gamma_2 + \gamma_{-1} \zeta_+^2)(\gamma_2^2 + \zeta_+^2) + 4\Omega^2(\gamma_0 \gamma_2^2 + \gamma_4 \zeta_+^2)}{(\gamma_0^2 + \zeta_+^2)((\gamma_1^2 + \zeta_+^2)(\gamma_2^2 + \zeta_+^2) + 8\Omega^2(\gamma_1 \gamma_2 - \zeta_+^2) + 16\Omega^4)}$$

$$R_+ = \frac{(\gamma_0 \gamma_1 \gamma_2 + \gamma_{-1} \zeta_-^2)(\gamma_2^2 + \zeta_-^2) + 4\Omega^2(\gamma_0 \gamma_2^2 + \gamma_4 \zeta_-^2)}{(\gamma_0^2 + \zeta_-^2)((\gamma_1^2 + \zeta_-^2)(\gamma_2^2 + \zeta_-^2) + 8\Omega^2(\gamma_1 \gamma_2 - \zeta_-^2) + 16\Omega^4)}$$

and,

$$C = \frac{2\eta^2 g^2 \Omega^2}{\gamma^2 + 2\Omega^2} \left( \frac{\gamma_1}{\gamma_1^2 + \zeta_+^2} + \frac{(\gamma_0 \gamma_1 \gamma_2 + \gamma_{-1} \zeta_+^2)(\gamma_2^2 + \zeta_+^2) + 4\Omega^2(\gamma_0 \gamma_2^2 + \gamma_4 \zeta_+^2)}{(\gamma_0^2 + \zeta_+^2)[(\gamma_1^2 + \zeta_+^2)(\gamma_2^2 + \zeta_+^2) + 8\Omega^2(\gamma_1 \gamma_2 - \zeta_+^2) + 16\Omega^4]} \right) \quad (5.21)$$

Where  $\gamma_n = \gamma_0, \gamma_1, \gamma_2, \gamma_4$ ,  $\zeta_+$  and  $\zeta_-$  are defined as,

$$\gamma_n = \kappa + n \gamma \quad (5.22)$$

$$\zeta_{\pm} \equiv 2(\Delta \pm v) \quad (5.23)$$

When a steady state has been established in the system, in equation (5.19) we can put  $\dot{m} = 0$  to get the steady state phonon number  $m_{ss}$ .

$$m_{ss} = \frac{C}{R_c} \quad (5.24)$$

Having equation (5.24) in hand, we can look for the optimized values of parameters which give the lowest  $m_{ss}$ . In the following section we will do a detailed analysis of the dependence of  $m_{ss}$  on different parameters.

## CHAPTER 6

### Results and Discussion

#### 6.1 Dependence of $m_{ss}$ on Atom-Cavity Detuning

We used the master equation method to simulate the problem of cavity mediated laser cooling. Plotting  $m_{ss}$  against the parameters of the system e.g. spontaneous decay rate of atom,  $\gamma$ , frequency of trap  $\nu$ , cavity decay rate  $\kappa$ , atom-cavity detuning  $\Delta = (\omega_c - \omega_a)$  and laser rabi frequency,  $\Omega$  is one way to find out for which values of these parameters would  $m_{ss}$  be at a minimum. When the atom is driven weakly (this means  $\Omega$  is small), we can drop all 2<sup>nd</sup> or higher order terms of  $\Omega$  from the expression of  $m_{ss}$ . In this case the expression for  $m_{ss}$  is similar to the one found by using the laser sideband cooling except that the atom spontaneous emission rate  $\gamma$  is replaced by the cavity decay rate  $\kappa$  [50],

$$m_{ss} = \frac{\kappa^2 + 4(\Delta - \nu)^2}{16\delta\nu}$$

This expression clearly shows that  $m_{ss}$  is at a minimum when atom-cavity  $\Delta$  detuning is equal to the frequency  $\nu$  of the vibrational mode of the atom. Moreover  $m_{ss}$  is at a maximum when  $\Delta = -\nu$ . Thus, we see that for a weakly driven atom, only one heating and one cooling resonance appears. Simulations that are shown in Figure 6-1 confirm the presence of these cooling and heating resonances.

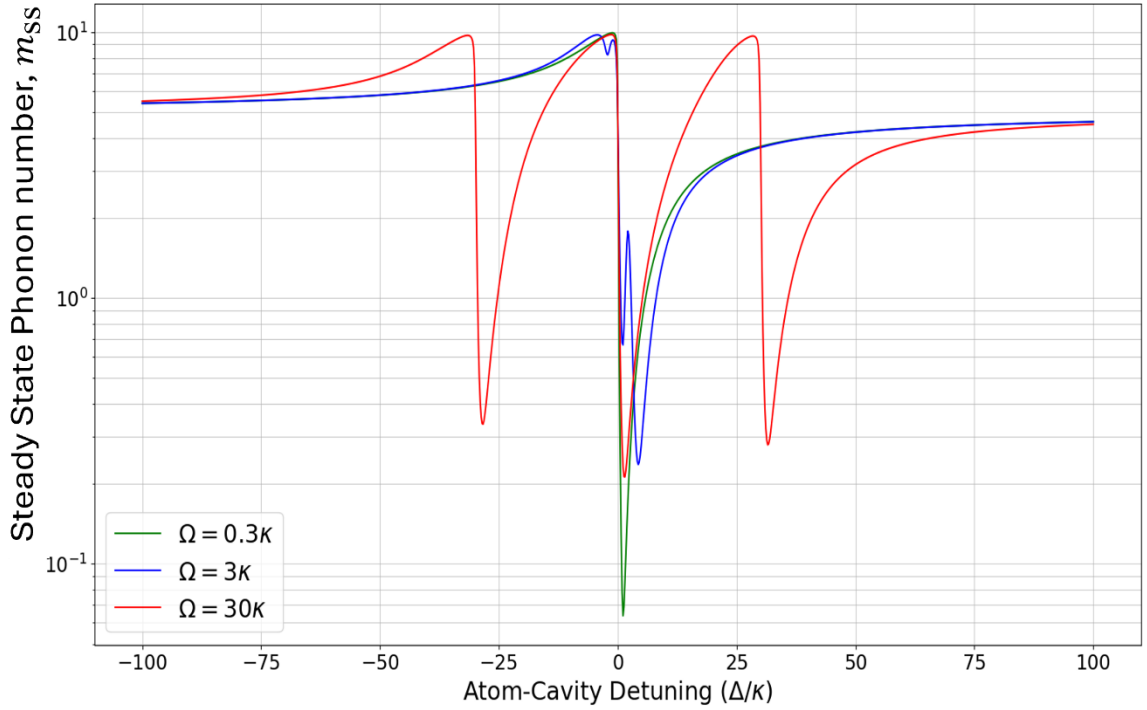


Figure 6-1 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against detuning  $\Delta$  for  $\mathbf{v} = \boldsymbol{\kappa} = \boldsymbol{\gamma}$  and  $\Omega = 0.3\kappa$  (Green Curve),  $\Omega = 3\kappa$  (Blue Curve),  $\Omega = 30\kappa$  (Red Curve). The python code for this figure is listed in appendix A program-5.

The situation changes drastically when the strength of the drive i.e. the Rabi frequency is increased. In the limit,  $\Omega, \zeta_{\pm} \gg \kappa, \gamma$ , rate equation for the phonon number becomes,

$$\dot{m} = \eta^2 g^2 \left( \frac{\gamma_{-1} \zeta_+^2 + 4\gamma_4 \Omega^2}{\zeta_+^4 - 8\Omega^2 \zeta_+^2 + 16\Omega^4} \right) (1 + m) - \eta^2 g^2 \left( \frac{\gamma_{-1} \zeta_-^2 + 4\gamma_4 \Omega^2}{\zeta_-^4 - 8\Omega^2 \zeta_-^2 + 16\Omega^4} \right) m$$

Thus, the expression for  $m_{ss}$  is given by

$$m_{ss} = \frac{-\left( \frac{\gamma_{-1} \zeta_+^2 + 4\gamma_4 \Omega^2}{\zeta_+^4 - 8\Omega^2 \zeta_+^2 + 16\Omega^4} \right)}{\left( \frac{\gamma_{-1} \zeta_+^2 + 4\gamma_4 \Omega^2}{\zeta_+^4 - 8\Omega^2 \zeta_+^2 + 16\Omega^4} \right) - \left( \frac{\gamma_{-1} \zeta_-^2 + 4\gamma_4 \Omega^2}{\zeta_-^4 - 8\Omega^2 \zeta_-^2 + 16\Omega^4} \right)} \quad (5.25)$$

This expression can be simplified further to get,

$$m_{ss} = \frac{-(\gamma_{-1}\zeta_+^2 + 4\gamma_4\Omega^2)(\zeta_-^4 - 8\Omega^2\zeta_-^2 + 16\Omega^4)}{(\gamma_{-1}\zeta_+^2 + 4\gamma_4\Omega^2)(\zeta_-^4 - 8\Omega^2\zeta_-^2 + 16\Omega^4) - (\gamma_{-1}\zeta_-^2 + 4\gamma_4\Omega^2)(\zeta_+^4 - 8\Omega^2\zeta_+^2 + 16\Omega^4)} \quad (5.26)$$

This expression clearly shows that when  $\zeta_-^2 = 4\Omega^2$ ,

$$m_{ss} = 0$$

That means that we get minimum phonon number in the system when we have,

$$(2\Delta + 2v)^2 = 4\Omega^2$$

Or

$$\Delta = v \pm \Omega \quad (5.27)$$

This shows the existence of two additional resonance conditions that dependent on  $\Omega$ , in addition to  $\Delta = v$ . The blue curve in Figure 6-1 illustrates the stationary phonon number  $m_{ss}$  as a function of detuning  $\Delta$  over a wide parameter range when atom is being driven with laser rabi frequency  $\Omega = 0.3\kappa$ . The predictions hold true even for relatively large spontaneous decay rates  $\gamma$ . In this analysis, the parameters  $\kappa$  and  $\gamma$  are chosen to be comparable in magnitude to the phonon frequency  $v$  and the atom-cavity detuning  $\Delta$ .

Returning to equation (5.26), we can see that when  $\zeta_+^2 = 4\Omega^2$ , i.e.  $\Delta = -v \pm \Omega$ ,  $m_{ss}$  is a maximum. Thus, we conclude that for these values of detuning, extreme heating occurs.

Red curve in Figure 6-1 highlights the presence of both cooling and heating resonances.

When  $\Omega$  is significantly smaller than  $v$ , these resonances start to overlap, leading to

identical values for cooling and heating. In this particular scenario, cooling is restricted to  $\Delta = \nu$ , whereas strong heating is observed at  $\Delta = -\nu$ .

## 6.2 Explanation of the Cooling Mechanism

Cavity-mediated cooling can be understood using the dressed state picture of the atom-cavity system. The dressed state picture refers to the eigenstates of the combined system of the atom and the interacting laser or cavity field. These states arise when the bare atomic and field states are coupled, forming a new basis due to interaction. The dressed state picture provides an intuitive understanding of how the interaction between the atom, the cavity field, and an external laser leads to the reduction of the vibrational energy, effectively cooling it to its vibrational ground state. Let us assume that initially the atom is in ground state  $|g\rangle$  and vibrational (phonon) states  $|m\rangle$  and cavity is initially in a Fock state with  $n$  photons, denoted as  $|n\rangle$ . The cooling laser drives transitions between the atomic ground and excited states. The laser frequency  $\omega_L$  is resonant with the atomic transition frequency  $\omega_a$ , i.e.,  $\omega_L = \omega_a$ . Simultaneously, the atom interacts with the cavity field, which is blue detuned from the atomic resonance by an amount equal to the vibrational frequency  $\nu$ , so  $\omega_c = \omega_a + \nu$ . This specific detuning is crucial for the cooling process because it ensures that the emission of photons into the cavity is accompanied by a reduction in the atom's vibrational energy. The interaction between the atom and the cavity field leads to the formation of dressed states:

$$|\pm, n, m\rangle = \frac{1}{\sqrt{2}}(|e, n, m\rangle \pm |g, n+1, m\rangle)$$

The coupling laser enables following transitions,

- $|g, n, m\rangle \xrightarrow{\text{Laser Excitation}} |e, n, m\rangle.$
- $|e, n, m\rangle \xrightarrow{\text{De-Excitation in cavity mode}} |g, n + 1, m - 1\rangle.$
- $|g, n + 1, m - 1\rangle \xrightarrow{\text{cavity decay}} |g, n, m - 1\rangle.$

In the case of an atom trapped but the harmonic trap such that  $\nu > \gamma$ , driven weakly ( $\Omega < \gamma$ ) the energy levels of its ground and excited state get resolved as shown in Figure 6-2. The difference between split energy level is  $\hbar\nu$ . When the laser drives the atom at its resonance frequency  $\omega_a$ , it absorbs photons at frequency  $\omega_a$ , but emits photons into cavity mode which in our case is tuned at frequency  $\omega_c = \omega_a + \nu$ , effectively reducing the energy of the system by an amount  $\hbar\nu$  (see Figure 6-3). This corresponds to cooling resonance  $\Delta = \omega_c - \omega_a = \nu$  shown by dip in the green curve in Figure 6-1. Similarly, when atom emits photons at frequency  $\Delta = \omega_c - \omega_a = -\nu$ , it increases the energy of the system by an amount  $\hbar\nu$ , effectively heating the system shown by peak in the green curve in Figure 6-1. In the case of strongly driven atom ( $\Omega \gg \gamma$ ), the ground and excited state energy levels of atom further split up in two parts by an amount  $\hbar\Omega$ . In this case atom absorbs the photon at  $\omega_a$  but it can emit photons at either of the three resonances  $\omega_c = \omega_a + \nu$ ,  $\omega_c = \omega_a + \Omega + \nu$  and  $\omega_c = \omega_a - \Omega + \nu$ , effectively reducing the energy of the system again by an amount  $\hbar\nu$ . These three transitions correspond to three cooling resonances of Figure 6-1 (red curve). Similarly, when atom emits at frequency  $\omega_c = \omega_a - \nu$ ,  $\omega_c = \omega_a + \Omega - \nu$  and  $\omega_c = \omega_a - \Omega - \nu$ , it effectively increases the energy of the system by an amount of  $\hbar\nu$  hence heating the system. These heating resonances correspond to three peaks in the red curve in Figure 6-1. By repeating this cycle of laser excitation followed by emission into

the cavity with a loss of one phonon, the atom progressively reduces its vibrational energy, eventually reaching the ground state  $m = 0$ .

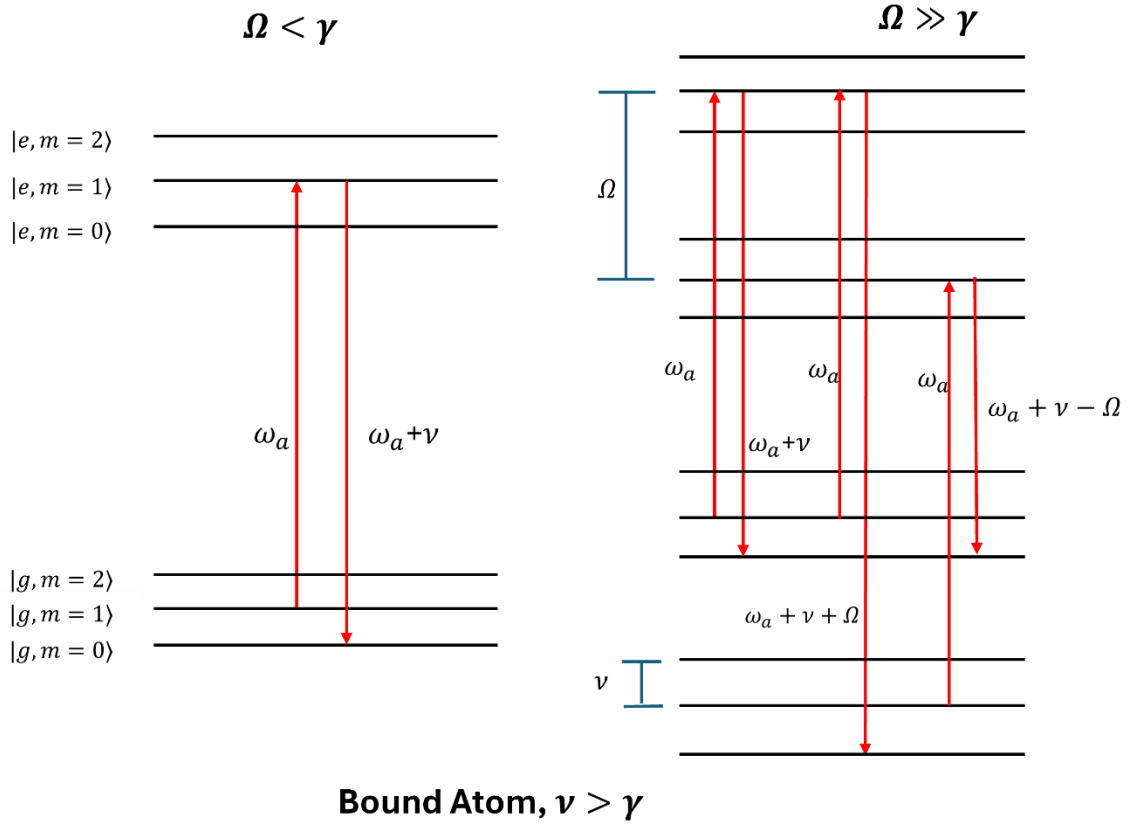


Figure 6-2 Energy level diagram of a two-level atom trapped inside a harmonic potential when it is being driven weakly ( $\Omega < \gamma$ ) (left). The ground and excited states split up further when atom is driven strongly ( $\Omega \gg \gamma$ ) (right).

### 6.3 Dependence of $m_{ss}$ on Parameters of the System

Next, we turn our attention to the dependence of  $m_{ss}$  on the laser Rabi frequency  $\Omega$ . Figure 6-3 shows the steady-state phonon number as a function of the Rabi frequency ( $\Omega$ ) for three different detuning values ( $\Delta$ ). For small values of Rabi frequency  $\Omega \approx 1\kappa$  that is when strength of the laser field is mild,  $m_{ss}$  for the two cooling resonances has values close to

zero. Which shows efficient cooling. For cooling resonance  $\Delta = \text{resonance}$ ,  $m_{ss}$  has value close to 1.

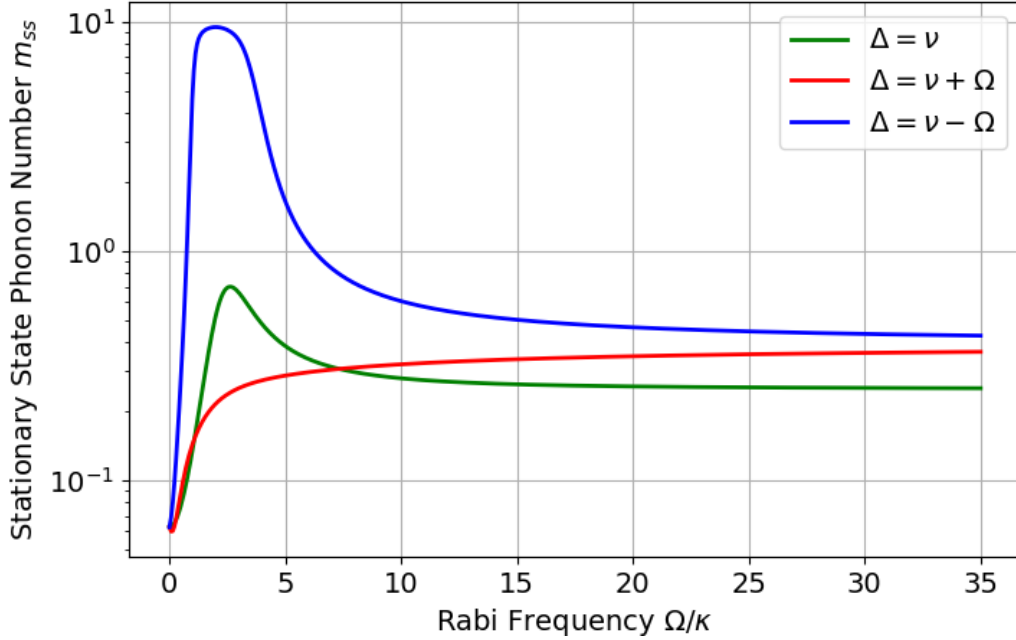


Figure 6-3 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against laser rabi freq.  $\Omega$  for  $\nu = \kappa = \gamma$ , for all three cooling resonances. The python code for this figure is listed in appendix A program-6.

This is evidence of the fact that the driving laser field works as a coupling agent between the atom and the cavity. Thus, when the atom is being driven weakly with respect to  $\kappa$  around its resonant detuning, it shows behavior of free space cooling predominantly emitting photons via spontaneous emission of its electronic state. From this discussion one might infer that further increasing the strength of laser would return larger values of  $m_{ss}$ . However this is not entirely true. In fact, after a certain value of  $\Omega \approx 3\kappa$ , the stationary state phonon number  $m_{ss}$ , starts to decrease and then settles down to a constant value. Thus, increasing the strength of the laser beyond  $\Omega \approx 10\kappa$ , does have much of an effect on the value of  $m_{ss}$ .

Now we will take a closer look at how the phonon frequency  $\nu$  affects the cooling process. To do this, we examine two cases: one with a relatively small value of the Rabi

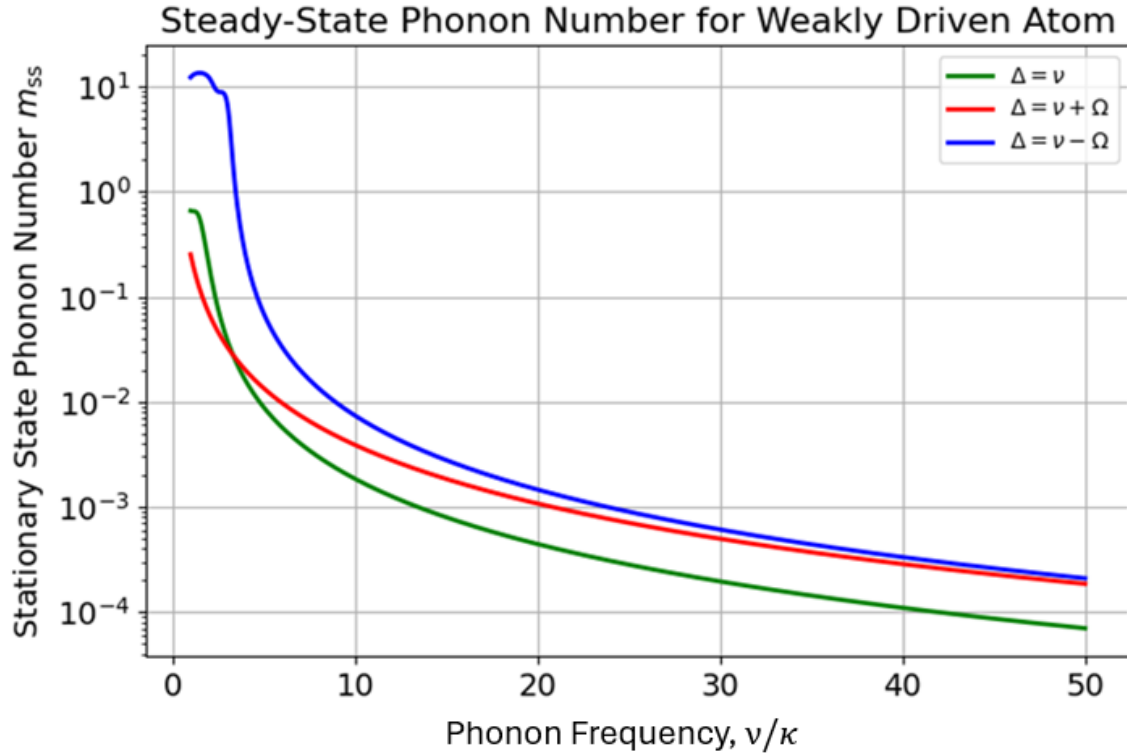


Figure 6-4 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against phonon freq.  $\nu$  for  $\kappa = \gamma$  and  $\Omega = 3\kappa$  for all three cooling resonances. The python code for this figure is listed in appendix A program-7.

frequency  $\Omega=3\kappa$  (Figure 6-5) and another with a larger value,  $\Omega=30\kappa$  (Figure 6-6) while keeping other parameters consistent with previous simulations. Additionally, both figures show that when the detuning  $\Delta = \nu - \Omega$  is used, it results in a higher stationary phonon number, which makes it less practical than using  $\Delta = \nu + \Omega$ ,  $\Delta = \nu$ . For lower vibrational frequency,  $\nu$  the smallest stationary phonon number is achieved when the detuning is set to  $\Delta = \nu + \Omega$ . This behavior can be understood in terms of dressed atom picture referring to Figure 6-5. When the phonon frequency  $\nu$  is small, the spacing between the energy levels of the dressed atom is relatively narrow. In this scenario, the heating and cooling resonances are situated close to each other in frequency space. This

overlap reduces the effectiveness of cooling, as the system cannot be efficiently tuned to preferentially drive cooling transitions. Consequently, this inefficiency leads to a higher steady-state phonon number in the system. Conversely, when  $\nu$  is large, the spacing between the dressed atom's energy levels increases significantly. This larger separation allows for precise tuning of the atom-cavity detuning to selectively enhance cooling transitions while minimizing heating effects. As a result, the system achieves more efficient cooling and a lower steady-state phonon number.

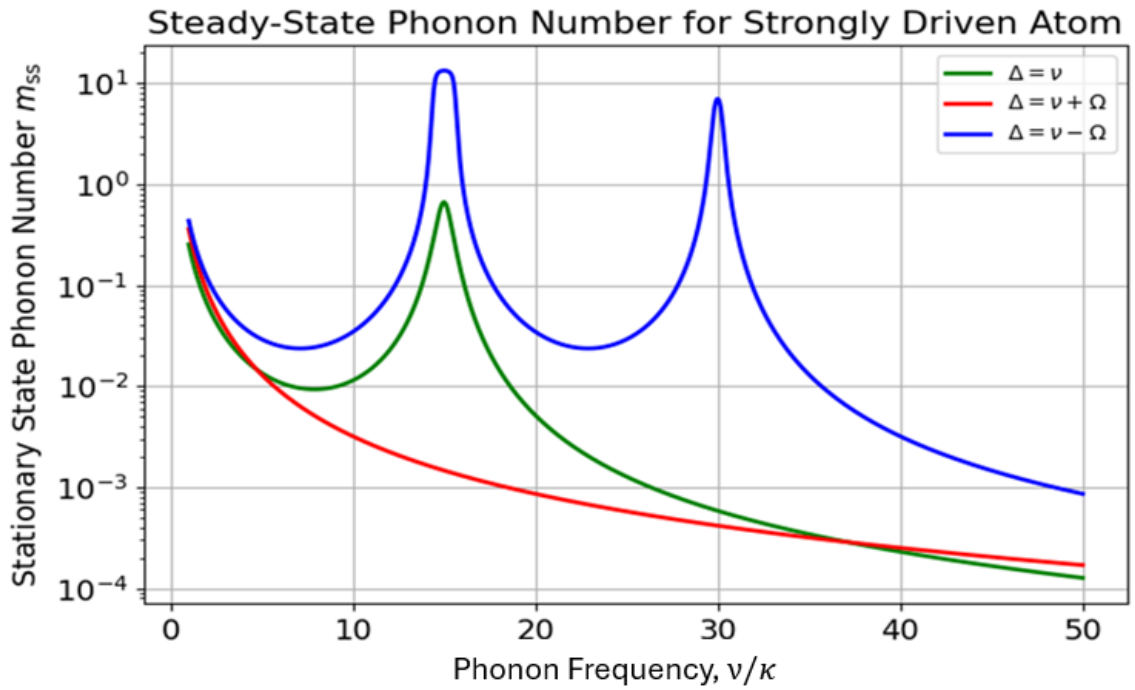


Figure 6-6 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against phonon freq.  $\nu$  for  $\kappa = \gamma$  and  $\Omega = 30\kappa$  for all three cooling resonances. The python code for this figure is listed in appendix A program-7.

Lastly, let's look at how the stationary phonon number  $m_{ss}$  depend on the spontaneous cavity decay rate  $\kappa$ . Similar to the previous section, we examine two scenarios: one with  $\Omega = 3\gamma$  (Figure 6-7) and another with  $\Omega = 30\gamma$  (Figure 6-8). Across a broad range of parameters, we observe that setting the detuning to  $\Delta = \nu + \Omega$  results in the lowest  $m_{ss}$ . This effect is especially noticeable when the cavity decay rate  $\kappa$  is relatively high. When  $\kappa$

becomes too large, however, the cooling transitions become overdamped. This overdamping significantly reduces the efficiency of the cooling, causing the  $m_{ss}$  to increase rapidly. However, after a certain value of  $\kappa$ ,  $m_{ss}$  becomes a constant.

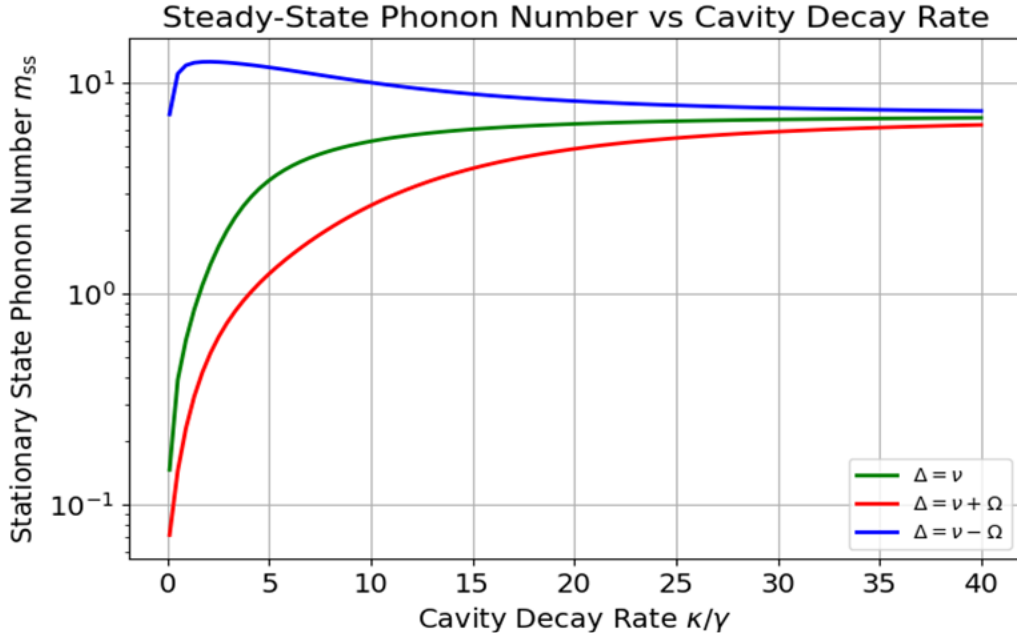


Figure 6-7 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against phonon freq.  $\kappa$  for  $\gamma = \nu$ ,  $\Omega = 3\gamma$ . The python code for this figure is listed in appendix A program-8.

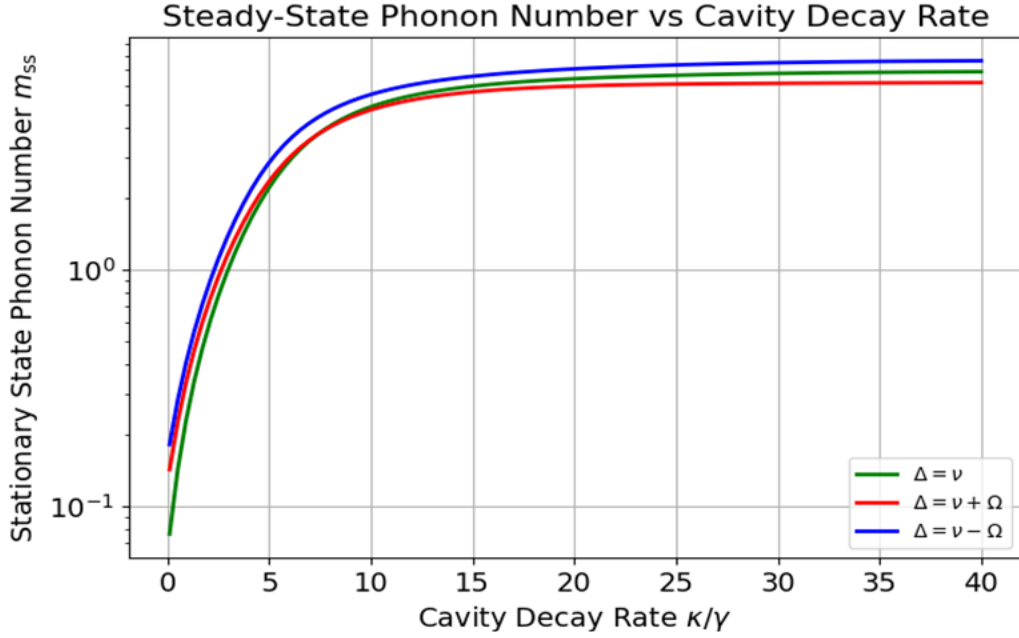


Figure 6-8 Stationary-state phonon number  $m_{ss}$  plotted on a logarithmic scale against phonon freq.  $\kappa$  for  $\gamma = \nu$ , and  $\Omega = 30\gamma$  for all three cooling resonances. The python code for this figure is listed in appendix A program-8.

## 6.4 Dependence of Cooling Rate on System parameters

The cooling rate of the system works as a measure of how quickly the atom dissipates its vibrational energy to reach its lowest quantum state. To evaluate the efficiency of cooling for the setup depicted in Figure 5-1, we compare the stationary state phonon numbers  $m_{ss}$  and the effective cooling rates  $R_c$  at the three resonances:  $\Delta = \nu$  and  $\Delta = \nu \pm \Omega$ . As previously discussed, the phonon number  $m$  and the stationary phonon number  $m_{ss}$  are equal at zeroth order, i.e., when  $\eta = 0$ . In this case, there is no coupling between the atom and the cavity, the initial phonon number of the system remains unchanged indefinitely. Referring to equation (5.19), this behavior is captured mathematically:

$$\dot{m} = -R_c m + C$$

the steady-state behavior of  $m(t)$  can be analytically described by:

$$m(t) = \frac{C}{R_c} (1 - e^{-R_c t}) + m_0 e^{-R_c t}$$

Or,

$$m(t) = [m_0 - m_{ss}]e^{-R_c t} + m_{ss} \quad (5.28)$$

where  $m_{ss} = \frac{C}{R_c}$  and  $m_0$  is number of phonons in the system at  $t = 0$ . This equation holds

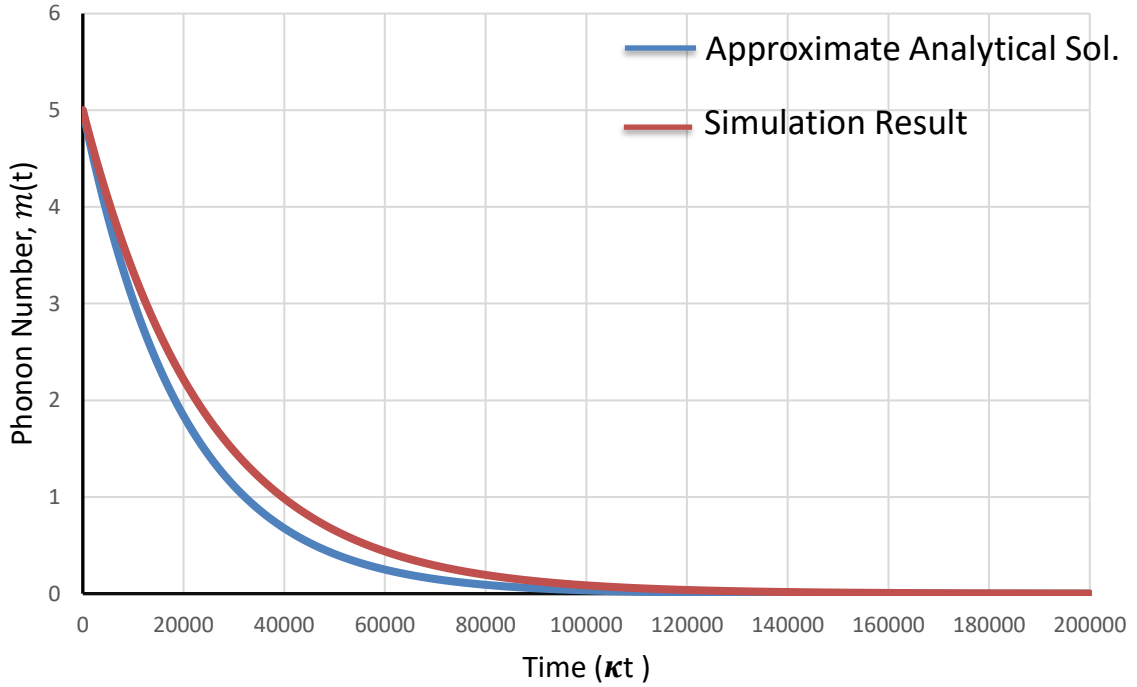


Figure 6-9 Comparison of phonon number evolution over time using the approximate analytical solution (blue curve) and simulation result from the master equation method (red curve). The python code for this figure is listed in appendix A program-9.

true irrespective of the relative magnitude of  $\nu$  with respect to  $\gamma$ , making it applicable in both the strong and weak confinement regimes. However, its validity is limited to a transition period of approximately  $1/R_c$ . Thus when  $m(t)$  is plotted against time  $t$ , allowing the system to reach its steady-state, and then fitting the curve with the exponential decay, we can get the cooling rate  $R_c$  by fitting the curves using equation (5.28). Figure 6-9

shows a plot of phonon number against time. The red curve shows the simulation result using master equation method and blue curve shows the result using the approximate analytical method (see equation 5.19). Upon fitting the curves, we get the cooling rate  $R_c = 5 \times 10^{-5}/\kappa t$  for the master equation simulated graph. The reason for such small value of cooling rate is that the phonon number evolves on the scale of  $\eta^2 g^2$  as evident from equation (5.20). Since the value of  $\eta$  must be much less than 1 to stay in Lamb-Dicke

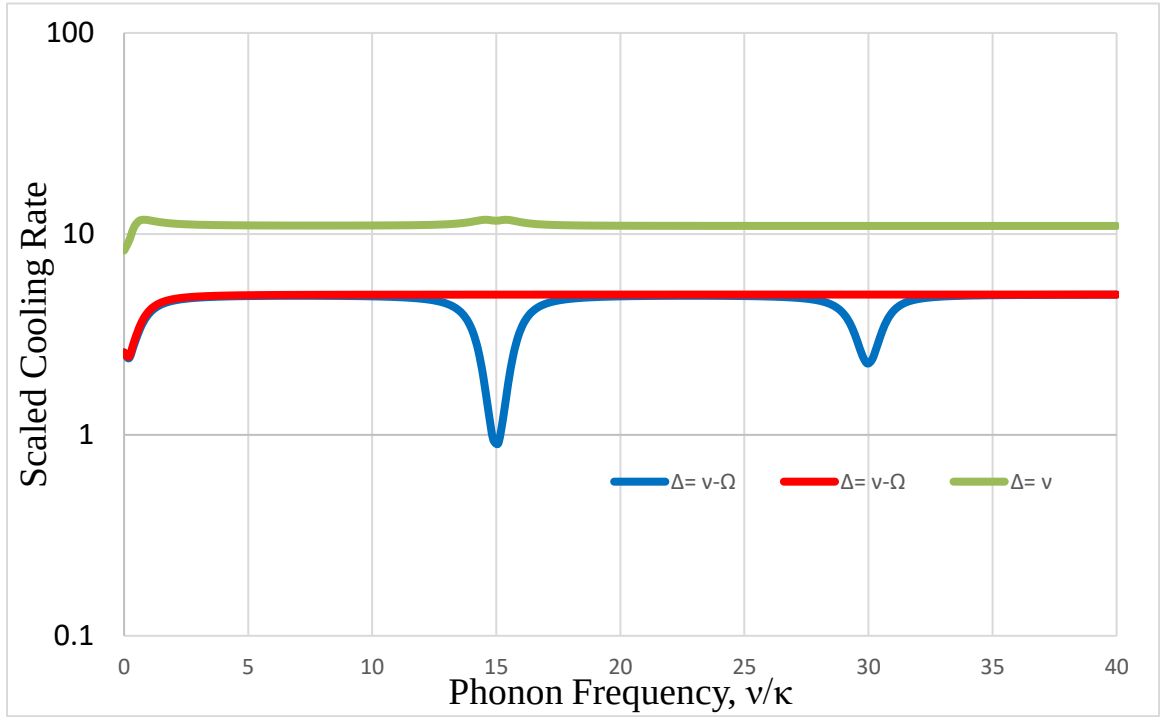


Figure 6-10 Scaled cooling plotted against phonon frequency,  $\nu$ . Other parameters as set to,  $\kappa = \gamma$  and  $\Omega = 30\gamma$ . The python code for this figure is listed in appendix A program-10.

Regime ( $\eta=0.01$  for my simulations), the phonon number evolves very slowly resulting in such small value of  $R_c$ .

Next, we turn our attention to the effect of system parameters on cooling rate,  $R_c$ . Figure **6-10** shows the scaled cooling rate  $\frac{R_c \gamma}{\eta^2 g^2}$  plotted against phonon frequency for the tree detuning cases,  $\Delta = \nu \pm \Omega$  and  $\Delta = \nu$  when the system is driven strongly ( $\Omega = 30$ ).

The alignment between the analytical and numerical solutions across these parameter choices is promising. Figure shows that the cooling rate for all three resonances remains almost constant for a wide range of phonon frequencies however cooling rates are slightly higher when the driving laser is tuned to relatively small values. When  $\nu$  is set close to  $\gamma$  or  $\kappa$ , we observe a noticeable boost in the cooling rate for the resonance  $\Delta = \nu$  as evident in Figure **6-10**. This behavior suggests that matching  $\nu$  with the spontaneous emission or cavity decay rates provides an optimal environment for dissipating vibrational energy. Additionally, smaller values of  $\nu$  generally yield higher cooling rates, especially in the presence of a strong driving field. Thus, adjusting  $\nu$  according to experimental conditions could enhance cooling efficiency. The cooling rate dips significantly when phonon frequencies approach a resonance condition, aligning with our expectation that resonances affect the energy exchange rate. These dips suggest points of reduced cooling efficiency

due to heating effects or sub-optimal energy transfer, underscoring the importance of precise parameter selection.

## 6.5 The Distribution of Phonons in the System

The phonon number distribution provides insights into the behavior of the phonon population in the mechanical mode over time. Starting from an initial state characterized by a specific number of phonons, in a Fock state  $|m\rangle$  for phonons,  $|n\rangle$  for photons, and the atom in the ground state  $|g\rangle$ , the dynamics evolve under both coherent and dissipative processes. The evolution of the phonon number distribution  $P(m)$ , which represents the probability of finding  $m$  phonons in the system at time  $t$ , is influenced by the coupling between the phonon, photon, and atomic subsystems. This interaction is governed by the system Hamiltonian and is affected by cavity damping and atomic decay processes, modeled by Lindblad terms in the master equation.

To study this distribution, we focus on the reduced density matrix for the phonon subsystem,  $\rho_{\text{phonon}}(t)$ , obtained by tracing out the photon and atomic degrees of freedom from the total density matrix  $\rho(t)$ . Physically, this process isolates the phonon population dynamics, revealing how energy dissipates from the phonon mode due to interactions mediated by the cavity field and atomic transitions. Given the initial state of the system,

$$|\psi(0)\rangle = |m\rangle_{\text{phonon}} \otimes |n\rangle_{\text{photon}} \otimes |g\rangle_{\text{atom}} \quad (5.29)$$

the dynamics of the system are governed by master equation,

$$\begin{aligned}\frac{d\rho}{dt} = & -\frac{i}{\hbar} [H_1, \rho] + \frac{1}{2} \kappa (2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \\ & + \frac{1}{2} \gamma (2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-)\end{aligned}\quad (5.30)$$

Taking out the partial trace of full density matrix  $\rho(t)$  at later time over the atomic and photonic degrees of freedom gives the phononic dynamics of the system,

$$\rho_{\text{phonon}}(t) = \text{Tr}_{\text{photon, atom}} (\rho(t)) \quad (5.31)$$

From this reduced density matrix, the phonon number distribution  $P_m$  can be found as,

$$P_m = \langle n | \rho_{\text{phonon}}(t) | n \rangle \quad (5.32)$$

Alternatively,  $P_m = \text{Tr} \left( (|m\rangle\langle m| \otimes I_{\text{photon}} \otimes I_{\text{atom}}) \rho(t) \right)$ . Using these preliminaries, phonon number distribution at steady state is show in Figure **6-11**. The average number of phonons at steady state came out to be  $\langle m \rangle \approx 0.202$ .

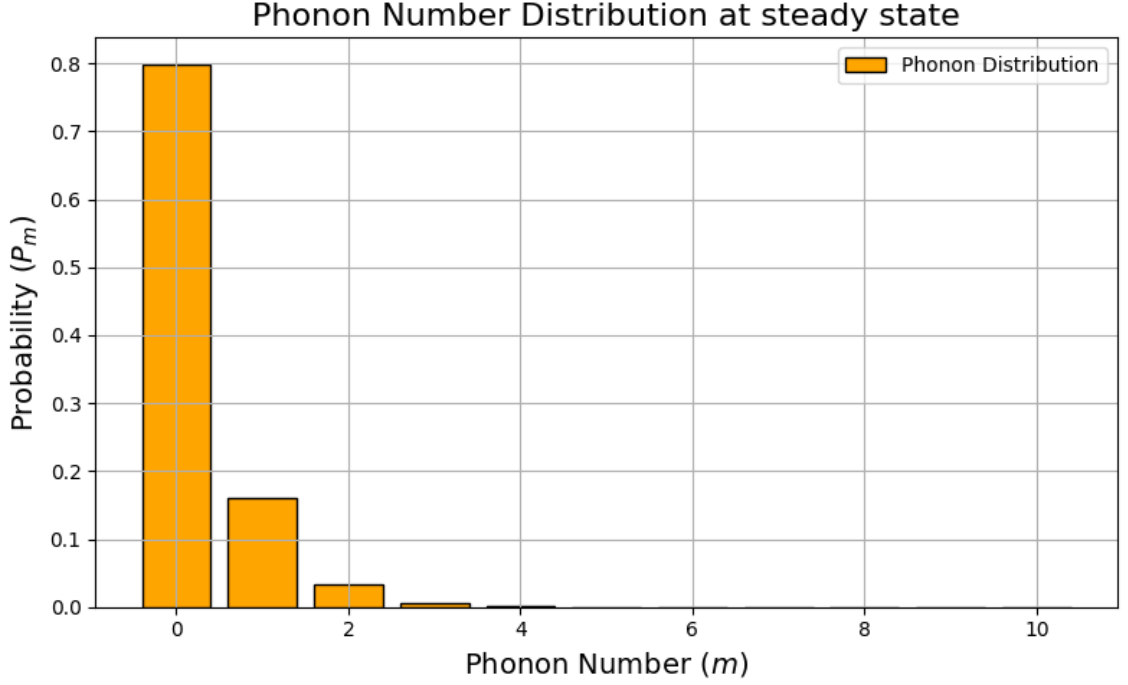


Figure 6-11 Distribution of phonon number,  $m$  at steady state. At steady state the probability of having zero phonons in the system is much higher than having any other number of phonons. The python code for this figure is listed in appendix A program-11.

The laser cooling gives a “thermal distribution” of occupation number thus approximate expression for the steady-state temperature is given by [3], [31] :

$$T_s \approx \frac{\hbar\nu}{k_B \ln(\langle m \rangle^{-1})} \quad (5.33)$$

Using this formula, we can calculate the temperature of the system at steady state. Putting the values of Boltzmann constant  $k_B = 1.38 \times 10^{-23} J/K$ ,  $\hbar/2\pi = 6.626 \times 10^{-34} J.s$  and typical trap frequency  $\nu = 1 MHz$  in the above formula we get  $T_s \approx 30 \mu K$ . Although the minimum temperature achieved by this method depends solely on the specific values of the system temperature. We can still guess its value by taking typical values of parameters. If we take  $\nu = \Delta = \gamma = \kappa = \Omega = g = 1 MHz$  and  $\eta = 0.01$ , theoretically predicted approximate minimum temperature of the system  $T_t$  can be calculated using

equations (5.20), (5.21) and (5.33). Which comes out to be equal to be  $T_t \approx 7.87 \mu K$ . This value is close to the minimum temperature ( $\approx 10 \mu K$ ) achieved by M. Hosseini *et al.* at MIT [51] for an assembly of atoms by method of cavity-mediated cooling.

## CHAPTER 7

### Conclusion

The main goal of this thesis was to study cavity-mediated laser cooling. This thesis has focused on cavity-mediated laser cooling as a powerful technique for achieving ultra-low temperatures in atomic systems. By exploiting the interactions between atoms, a cavity field, and an external laser, cavity-mediated cooling addresses limitations in traditional laser cooling methods like Doppler and Sisyphus cooling, which are restricted by the Doppler limit and recoil limit respectively. Our work highlights how the cavity plays an essential role in enhancing cooling efficiency, enabling temperature reduction beyond what is achievable with free-space cooling. This is done by utilizing the controlled escape of photons from the cavity.

In the initial chapters, we “derived” the theoretical foundation for cavity-mediated interactions, starting from the Jaynes-Cummings Hamiltonian to describe the atom-cavity-laser system. This model provided the basis for our understanding of how coupling an atom to a high-finesse cavity modifies the cooling dynamics. That is, effectively it is replacing the spontaneous emission process with cavity-mediated photon decay. Through this approach, we demonstrated how cavity parameters—such as detuning, photon coupling strength, and decay rates—can be optimized to significantly lower the vibrational energy of the atom, facilitating sub-Doppler cooling.

The discussions in Chapter 4 provided a theoretical foundation for understanding open quantum systems. Using the master equation approach, the chapter effectively modeled the evolution of systems interacting with external environments, focusing on dissipation and decoherence. The introduction of Quantum Trajectory Theory (QTT) offered a stochastic simulation method that efficiently captures individual quantum events, making it superior for complex and time-dependent dynamics. Examples highlighted how QTT accurately tracks the dynamics of open quantum systems, providing insights into their interaction with reservoirs and external fields under various conditions. This framework is crucial for studying real-world quantum systems and lays the groundwork for the applications explored in subsequent chapters.

The results of our simulations in chapter 5 demonstrated the effectiveness of cavity-mediated cooling in achieving a stationary phonon distribution with a low steady-state phonon number, signifying successful energy dissipation through the cavity. We explored various parameter regimes, revealing that cooling rates are highly dependent on both phonon frequencies and atom-cavity detuning, with certain resonances yielding optimal cooling conditions.

Future research can build on this work by exploring multi-atom configurations within cavities or extending the model to more complex, multi-level atomic systems. Additionally, investigating other geometries such as ring cavities or integrating atom chips could further enhance the practical applications of cavity-mediated cooling, opening new possibilities for quantum technology development and ultra-cold atomic physics.

# Appendix A

## List of Programs

### Program-1. Spontaneous decay of two-level atom:

```
import numpy as np
import matplotlib.pyplot as plt
from qutip import (basis, mesolve, mcsolve, sigmam)
gamma = 1.0
e = basis(2, 0)
g = basis(2, 1)
psi0 = e
H = 0 * sigmam()
c_ops = [np.sqrt(gamma) * sigmam()]
tlist = np.linspace(0, 10, 200)
me_result = mesolve(
    H=H,
    rho0=psi0,
    tlist=tlist,
    c_ops=c_ops,
    e_ops=[sigmam().dag() * sigmam()]
)
pop_exc_me = me_result.expect[0]
traj_list = [1, 10, 100, 1000]
pop_exc_mc_list = []
for ntraj in traj_list:
    mc_result = mcsolve(
        H=H,
        state=psi0,
        tlist=tlist,
        c_ops=c_ops,
        e_ops=[sigmam().dag() * sigmam()],
        ntraj=ntraj
    )
    pop_exc_mc_list.append(mc_result.expect[0])
plt.rcParams.update({
    "font.size": 14,
    "axes.labelsize": 16,
    "axes.titlesize": 16,
    "legend.fontsize": 14,
    "xtick.labelsize": 14,
    "ytick.labelsize": 14
})
plt.figure(figsize=(8, 5))
plt.plot(gamma * tlist, pop_exc_me, 'k--', lw=2, label="Master Equation")
colors = ["r", "g", "b", "m"]
for i, ntraj in enumerate(traj_list):
    plt.plot(
```

```

        gamma * tlist,
        pop_exc_mc_list[i],
        color=colors[i],
        label=f"MC (ntraj={ntraj})"
    )
plt.xlabel(r"Time ($\gamma t$)")
plt.ylabel(r"Excited State Population $\langle \sigma_+ \sigma_- \rangle$")
plt.title("Spontaneous Decay of a Two-Level Atom ")
plt.legend(loc="best")
plt.grid(True)
plt.savefig("atomdecay_noH.png")
plt.show()

```

## Program-2. Spontaneous decay of cavity in Fock state:

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import *
kappa = 1.0
N = 10
n0 = 5
tmax = 5.0
numt = 100
tlist = np.linspace(0, tmax, numt)
a = destroy(N)
psi0 = fock(N, n0)
c_ops = [np.sqrt(kappa) * a]
H = 0 * a
me_result = mesolve(
    H=H,
    rho0=psi0,
    tlist=tlist,
    c_ops=c_ops,
    e_ops=[a.dag() * a],
    options={"progress_bar": False}
)
n_me = me_result.expect[0]
traj_list = [1, 10, 100, 1000]
n_mc_list = []
for ntraj in traj_list:
    mc_result = mcsolve(
        H=H,
        state=psi0,
        tlist=tlist,
        c_ops=c_ops,
        e_ops=[a.dag() * a],
        ntraj=ntraj,
        options={"progress_bar": False}
    )
    n_mc_list.append(mc_result.expect[0])
plt.rcParams.update({
    "font.size": 14,
    "axes.labelsize": 16,

```

```

        "axes.titlesize": 16,
        "legend.fontsize": 14,
        "xtick.labelsize": 14,
        "ytick.labelsize": 14
    })
    plt.figure(figsize=(8,6))
    plt.plot(tlist, n_me, 'k', lw=2, label="Master equation")
    colors = ["r", "g", "b", "m"]
    for i, ntraj in enumerate(traj_list):
        plt.plot(tlist, n_mc_list[i], color=colors[i], label=f"({ntraj} traj)")
    plt.xlabel("Time, ( $\kappa t$ )")
    plt.ylabel(r"Photon number  $\langle a^\dagger a \rangle$ ")
    plt.title("Decay of Cavity in Fock State  $|5\rangle$ ")
    plt.legend(loc="best")
    plt.grid(True)
    plt.savefig("cavitydecay.png")
    plt.show()

```

### Program-3. Spontaneous decay of cavity in superposition state

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import *
kappa = 1.0
N = 10
tmax = 5.0
numt = 100
tlist = np.linspace(0, tmax, numt)
a = destroy(N)
psi0 = (fock(N, 0) + fock(N, 2)).unit()
c_ops = [np.sqrt(kappa) * a]
H = 0 * a
me_result = mesolve(
    H=H,
    rho0=psi0,
    tlist=tlist,
    c_ops=c_ops,
    e_ops=[a.dag() * a],
    options={"progress_bar": False}
)
n_me = me_result.expect[0]
traj_list = [10, 100]
n_mc_list = []
for ntraj in traj_list:
    mc_result = mcsolve(
        H=H,
        state=psi0,
        tlist=tlist,
        c_ops=c_ops,
        e_ops=[a.dag() * a],
        ntraj=ntraj,
        options={"progress_bar": False}
    )
    n_mc_list.append(mc_result.expect[0])

```

```

plt.rcParams.update({
    "font.size": 14,
    "axes.labelsize": 16,
    "axes.titlesize": 16,
    "legend.fontsize": 14,
    "xtick.labelsize": 14,
    "ytick.labelsize": 14
})

plt.figure(figsize=(8,6))
plt.plot(tlist, n_me, 'k', lw=2, label="Master equation")
colors = ["r", "b"]
for i, ntraj in enumerate(traj_list):
    plt.plot(
        tlist,
        n_mc_list[i],
        color=colors[i],
        label=f"{ntraj} trajectories"
    )
plt.xlabel(r"Time $(\kappa t)$")
plt.ylabel(r"Photon number $\langle a^\dagger a \rangle$")
plt.title("Decay of Cavity in Superposition State")
plt.legend(loc="best")
plt.grid(True)

plt.savefig("cavitydecay_10_100.png")
plt.show()

```

#### Program-4. Resonance fluorescence from two-level atom:

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import basis, mesolve, mcsolve, sigmam, sigmap, sigmaz
gamma = 1.0
Omega = 3.0
g = basis(2, 1)
e = basis(2, 0)
psi0 = g
H = (Omega / 2) * (sigmap() + sigmam())
c_ops = [np.sqrt(gamma) * sigmam()]
tlist = np.linspace(0, 60, 1000)
me_result = mesolve(H, psi0, tlist, c_ops, e_ops=[sigmap() * sigmam()])
pop_me = me_result.expect[0]
ntraj = 50
mc_result = mcsolve(H, psi0, tlist, c_ops, e_ops=[sigmap() * sigmam()], ntraj=ntraj)
pop_mc = mc_result.expect[0]
plt.figure(figsize=(8,5))
plt.plot(tlist, pop_me, 'b', lw=2, label="Master Equation")
plt.plot(tlist, pop_mc, 'r-', label=f"(ntraj={ntraj})")
plt.xlabel("Time", fontsize=14)

```

```

plt.ylabel("Excited State Population", fontsize=14)
plt.title("Resonant Driving & Spontaneous Emission", fontsize=16)
plt.legend(fontsize=12)
plt.grid(True)
plt.tick_params(axis='both', labelsize=12)
plt.tight_layout()
plt.savefig("ResoFloure.png")
plt.show()

```

### Program-5. Stationary state phonon number vs atom-cavity detuning:

```

import numpy as np
import matplotlib.pyplot as plt
import csv
from qutip import destroy, sigmaz, basis, tensor, qeye, sigmap, sigmam, steadystate, expect, ptrace
N = 2
M = 25
g = 0.001
gamma = 1
kappa = 1
nu = 1
pi = np.pi
sm = sigmam()
sp = sigmap()
sz = sigmaz()
I_atom = qeye(2)
c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
delta_vals = np.linspace(-100, 100, 1000)
Omega_vals = [0.3 * kappa, 3 * kappa, 30 * kappa]
phonon_steady_states_all = []
csv_data = [["Detuning (Delta)", "Omega = 0.3kappa", "Omega = 3kappa", "Omega = 30kappa"]]
H_phonon = nu * tensor(I_atom, I_photon, bd * b)
c_ops = [
    np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
    np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
]
for Omega in Omega_vals:
    phonon_steady_states = []
    H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
    H_interaction = g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    for delta in delta_vals:
        H_photon = delta * tensor(I_atom, cd * c, I_phonon)
        H = H_phonon + H_photon + H_atom + H_interaction
        rho_ss = steadystate(H, c_ops)
        rho_phonon = ptrace(rho_ss, 2)
        phonon_number = expect(bd * b, rho_phonon)
        phonon_steady_states.append(phonon_number)
    phonon_steady_states_all.append(phonon_steady_states)

```

```

plt.figure(figsize=(8, 5))
colors = ['green', 'blue', 'red']
labels = [r'$\Omega = 0.3\kappa$', r'$\Omega = 3\kappa$', r'$\Omega = 30\kappa$']
for i, Omega in enumerate(Omega_vals):
    plt.plot(delta_vals, phonon_steady_states_all[i], color=colors[i], label=labels[i])
plt.yscale('log')
plt.xlabel(r'Atom-Cavity Detuning ($\Delta / \kappa$)', fontsize=22)
plt.ylabel(r'Steady-State Phonon Number, $m$', fontsize=22)
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
plt.legend(fontsize=22)
plt.grid(True, which="both", linewidth=0.5)
plt.tight_layout()
plt.show()

```

### Program-6. Stationary state phonon number versus Laser Rabi Frequency:

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import destroy, sigmaz, basis, tensor, qeye, sigmap, sigmam, steadystate, expect
N = 2
M = 11
g = 0.01
gamma = 1
kappa = 1
nu = 1
pi = np.pi
sm = sigmam()
sp = sigmap()
sz = sigmaz()
I_atom = qeye(2)
c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
Omega_vals = np.linspace(0.0001, 35, 500)
phonon_steady_states_0 = []
phonon_steady_states_plus = []
phonon_steady_states_minus = []
H_phonon = nu * tensor(I_atom, I_photon, bd * b)
H_interaction = g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
c_ops = [
    np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
    np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
]
phonon_number_op = tensor(I_atom, I_photon, bd * b)
for Omega in Omega_vals:
    H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
    H_photon_0 = nu * tensor(I_atom, cd * c, I_phonon)

```

```

H_0 = H_phonon + H_photon_0 + H_atom + H_interaction
rho_ss_0 = steadystate(H_0, c_ops)
phonon_number_0 = expect(phonon_number_op, rho_ss_0)
phonon_steady_states_0.append(phonon_number_0)
H_photon_plus = (nu + Omega) * tensor(I_atom, cd * c, I_phonon)
H_plus = H_phonon + H_photon_plus + H_atom + H_interaction
rho_ss_plus = steadystate(H_plus, c_ops)
Mss_plus = expect(phonon_number_op, rho_ss_plus)
phonon_steady_states_plus.append(Mss_plus)
H_photon_minus = (nu - Omega) * tensor(I_atom, cd * c, I_phonon)
H_minus = H_phonon + H_photon_minus + H_atom + H_interaction
rho_ss_minus = steadystate(H_minus, c_ops)
Mss_minus = expect(phonon_number_op, rho_ss_minus)
phonon_steady_states_minus.append(Mss_minus)
plt.figure(figsize=(8, 5))
plt.plot(Omega_vals, phonon_steady_states_0, 'g-', label=r'$\Delta = \nu$', linewidth=2)
plt.plot(Omega_vals, phonon_steady_states_plus, 'r', label=r'$\Delta = \nu + \Omega$', linewidth=2)
plt.plot(Omega_vals, phonon_steady_states_minus, 'b', label=r'$\Delta = \nu - \Omega$', linewidth=2)
plt.xlabel(r'Rabi Frequency $\Omega / \kappa$', fontsize=14)
plt.ylabel(r'Stationary State Phonon Number $m_{ss}$', fontsize=14)
plt.legend(fontsize=14)
plt.grid(True)
plt.yscale('log')
plt.tick_params(axis='both', which='major', labels=14)
plt.savefig("MssVsOmega.png")
plt.show()

```

## Program-7. Stationary state phonon number versus Phonon Frequency

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import destroy, sigmaz, basis, tensor, qeye, sigmap, sigmam, steadystate, expect
N = 2
M = 15
g = 0.001
gamma = 1
kappa = 1
Omega = 30
pi = np.pi
sm = sigmam()
sp = sigmap()
sz = sigmaz()
I_atom = qeye(2)
c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
nu_vals = np.linspace(1, 50, 1000)
phonon_steady_states_0 = []
phonon_steady_states_plus = []

```

```

phonon_steady_states_minus = []
c_ops = [
    np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
    np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
]
phonon_number_op = tensor(I_atom, I_photon, bd * b)
for nu in nu_vals:
    H_phonon = nu * tensor(I_atom, I_photon, bd * b)
    H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
    H_photon_0 = nu * tensor(I_atom, cd * c, I_phonon)
    H_0 = H_phonon + H_photon_0 + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_0 = steadystate(H_0, c_ops)
    phonon_number_0 = expect(phonon_number_op, rho_ss_0)
    phonon_steady_states_0.append(phonon_number_0)
    H_photon_plus = (nu + Omega) * tensor(I_atom, cd * c, I_phonon)
    H_plus = H_phonon + H_photon_plus + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_plus = steadystate(H_plus, c_ops)
    Mss_plus = expect(phonon_number_op, rho_ss_plus)
    phonon_steady_states_plus.append(Mss_plus)
    H_photon_minus = (nu - Omega) * tensor(I_atom, cd * c, I_phonon)
    H_minus = H_phonon + H_photon_minus + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_minus = steadystate(H_minus, c_ops)
    Mss_minus = expect(phonon_number_op, rho_ss_minus)
    phonon_steady_states_minus.append(Mss_minus)

plt.figure(figsize=(10, 7))
plt.plot(nu_vals, phonon_steady_states_0, 'g-', label=r'$\Delta = \nu$', linewidth=4)
plt.plot(nu_vals, phonon_steady_states_plus, 'b', label=r'$\Delta = \nu + \Omega$', linewidth=4)
plt.plot(nu_vals, phonon_steady_states_minus, 'r', label=r'$\Delta = \nu - \Omega$', linewidth=4)
plt.xlabel(r'Phonon Frequency $\nu / \kappa$', fontsize=20)
plt.ylabel(r'Stationary State Phonon Number (m)', fontsize=20)
plt.legend(fontsize=18, loc='best')
plt.yscale('log')
plt.grid(True, which="both", linestyle="--", linewidth=0.5)
plt.tick_params(axis='both', which='major', labels=16)
plt.tight_layout()
plt.savefig("MssVsNu.png")
plt.show()

```

## Program-8. Stationary state phonon number versus Cavity Decay Rate

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import destroy, sigmaz, basis, tensor, qeye, sigmap, sigmam, steadystate, expect
N = 2
M = 15
g = 0.01
gamma = 1
Omega = 30
pi = np.pi
sm = sigmam()
sp = sigmap()
sz = sigmaz()
I_atom = qeye(2)

```

```

c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
nu = 1
kappa_vals = np.linspace(0.1, 40, 100) * gamma
phonon_steady_states_0 = []
phonon_steady_states_plus = []
phonon_steady_states_minus = []
phonon_number_op = tensor(I_atom, I_photon, bd * b)

for kappa in kappa_vals:
    c_ops = [
        np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
        np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
    ]
    H_phonon = nu * tensor(I_atom, I_photon, bd * b)
    H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
    H_photon_0 = nu * tensor(I_atom, cd * c, I_phonon)
    H_0 = H_phonon + H_photon_0 + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_0 = steadystate(H_0, c_ops)
    phonon_number_0 = expect(phonon_number_op, rho_ss_0)
    phonon_steady_states_0.append(phonon_number_0)
    H_photon_plus = (nu + Omega) * tensor(I_atom, cd * c, I_phonon)
    H_plus = H_phonon + H_photon_plus + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_plus = steadystate(H_plus, c_ops)
    Mss_plus = expect(phonon_number_op, rho_ss_plus)
    phonon_steady_states_plus.append(Mss_plus)

    H_photon_minus = (nu - Omega) * tensor(I_atom, cd * c, I_phonon)
    H_minus = H_phonon + H_photon_minus + H_atom + g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    rho_ss_minus = steadystate(H_minus, c_ops)
    Mss_minus = expect(phonon_number_op, rho_ss_minus)
    phonon_steady_states_minus.append(Mss_minus)

plt.figure(figsize=(8, 5))
plt.plot(kappa_vals / gamma, phonon_steady_states_0, 'g-', label=r'$\Delta = \nu$', linewidth=2)
plt.plot(kappa_vals / gamma, phonon_steady_states_plus, 'r', label=r'$\Delta = \nu + \Omega$', linewidth=2)
plt.plot(kappa_vals / gamma, phonon_steady_states_minus, 'b', label=r'$\Delta = \nu - \Omega$', linewidth=2)
plt.xlabel(r'Cavity Decay Rate $\kappa / \gamma$', fontsize=14)
plt.ylabel(r'Stationary State Phonon Number $m_{\mathrm{ss}}$', fontsize=14)
plt.title('Steady-State Phonon Number vs Cavity Decay Rate', fontsize=16)
plt.legend()
plt.grid(True)
plt.yscale('log')
plt.tick_params(axis='both', which='major', labelsize=14)
plt.savefig("MssVsKappa.png")

```

## Program-9. Phonon number vs time

```
import numpy as np
```

```

import matplotlib.pyplot as plt
from qutip import destroy, basis, tensor, qeye, sigmap, sigmam, mesolve, Options
import pandas as pd
N = 2
M = 6
g = 1
gamma = 1
kappa = 1
Omega = 30
eta = 0.01
sm = sigmam()
sp = sigmap()
I_atom = qeye(2)
c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
c_ops = [
    np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
    np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
]
t_max = 150000
num_points = 10000
tlist = np.linspace(0, t_max, num_points)
options = Options(nsteps=10000, atol=1e-3, rtol=1e-2)
n_phonons = 5
phonon_number_op = tensor(I_atom, I_photon, bd * b)
nu = 20
delta = nu
H_phonon = nu * tensor(I_atom, I_photon, bd * b)
H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
H_photon = delta * tensor(I_atom, cd * c, I_phonon)
H_interaction = g * eta * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
H = H_phonon + H_photon + H_atom + H_interaction
initial_state = tensor(basis(2, 0), basis(N, 0), basis(M, n_phonons))
result = mesolve(H, initial_state, tlist, c_ops, [phonon_number_op], options=options)
phonon_numbers = result.expect[0]
data = {
    'Time ': tlist,
    'Phonon Number': phonon_numbers
}
df = pd.DataFrame(data)
df.to_csv('PhononNumberVsTime.csv', index=False)
plt.figure(figsize=(8, 5))
plt.plot(tlist, phonon_numbers, label='Phonon Number', color='blue', linewidth=2)
plt.xlabel('Time ', fontsize=14)
plt.ylabel(r'Phonon Number  $\langle b^\dagger b \rangle$ ', fontsize=14)
plt.title('Phonon Number vs Time', fontsize=16)
plt.grid(True)
plt.legend()
plt.savefig("PhononNumberVsTime.png")
plt.show()

```

### Program-10. Scaled Cooling rate versus Phonon frequency

```
import numpy as np
import matplotlib.pyplot as plt
from qutip import (destroy, basis, tensor, qeye, sigmap, sigmam, mesolve, Options)
from scipy.optimize import curve_fit
from multiprocessing import Pool
import pandas as pd
N = 2
M = 6
g = 1
gamma = 1
kappa = 1
Omega = 30
eta = 0.01
t_max = 100000
num_points = 20000
tlist = np.linspace(0, t_max, num_points)
n_phonons = 5
nu_vals = np.linspace(0, 40, 200)
scaling_factor = gamma / ((eta * g) ** 2)
def cooling_model(t, m0, gamma_c, n_ss):
    return m0 * np.exp(-gamma_c * t) + n_ss
def compute_cooling_rate(args):
    nu, delta_type = args
    sm = sigmam()
    sp = sigmap()
    I_atom = qeye(2)
    c = destroy(N)
    cd = c.dag()
    I_photon = qeye(N)
    b = destroy(M)
    bd = b.dag()
    I_phonon = qeye(M)
    c_ops = [
        np.sqrt(gamma) * tensor(sm, I_photon, I_phonon),
        np.sqrt(kappa) * tensor(I_atom, c, I_phonon)
    ]
    if delta_type == 'minus':
        delta = nu - Omega
    elif delta_type == 'plus':
        delta = nu + Omega
    elif delta_type == 'zero':
        delta = nu
    else:
        raise ValueError("Invalid delta_type.")
    H_phonon = nu * tensor(I_atom, I_photon, bd * b)
    H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
    H_photon = delta * tensor(I_atom, cd * c, I_phonon)
    H_interaction = g * eta * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
    H = H_phonon + H_photon + H_atom + H_interaction
    initial_state = tensor(basis(2, 0), basis(N, 0), basis(M, n_phonons))
    options = Options(nsteps=10000)
    try:
```

```

    result = mesolve(H, initial_state, tlist, c_ops, [tensor(I_atom, I_photon, bd * b)], options=options)
    phonon_number = result.expect[0]
    popt, _ = curve_fit(cooling_model, tlist, phonon_number, p0=(n_phonons, 0.0005, phonon_number[-
1]))
    _, gamma_c_fit, _ = popt
    modified_cooling_rate = gamma_c_fit * scaling_factor
    return 1 / modified_cooling_rate if modified_cooling_rate != 0 else np.nan
except Exception as e:
    print(f"Error for nu={nu}, delta_type={delta_type}: {e}")
    return np.nan
if __name__ == "__main__":
    delta_types = ['minus', 'plus', 'zero']
    results = {}
    for delta_type in delta_types:
        args_list = [(nu, delta_type) for nu in nu_vals]
        with Pool() as pool:
            modified_cooling_rates = pool.map(compute_cooling_rate, args_list)
        results[delta_type] = modified_cooling_rates
    data = {
        'nu': nu_vals,
        'Rc_minus': results['minus'],
        'Rc_plus': results['plus'],
        'Rc_zero': results['zero']
    }
    df = pd.DataFrame(data)
    df.to_csv('cooling_rates_vs_nu1.csv', index=False)
    plt.figure(figsize=(8, 5))
    colors = {'minus': 'r', 'plus': 'b', 'zero': 'g'}
    labels = {'minus': r'$\Delta = \nu - \Omega$', 'plus': r'$\Delta = \nu + \Omega$', 'zero': r'$\Delta = \nu$'}
    for delta_type in delta_types:
        modified_cooling_rates = results[delta_type]
        plt.plot(nu_vals, modified_cooling_rates, '-', color=colors[delta_type], label=labels[delta_type])
    plt.xlabel(r'$\nu / \gamma$', fontsize=14)
    plt.ylabel(r'$\frac{R_c}{\gamma} \{(\eta g)^2\}$', fontsize=14)
    plt.title('Scaled Cooling Rate vs. Phonon Frequency')
    plt.yscale('log')
    plt.ylim(1e-4, 10)
    plt.legend()
    plt.grid(True)
    plt.savefig("CrVsNuFinal007.png")
    plt.show()

```

## Program-11. Phonon Distribution

```

import numpy as np
import matplotlib.pyplot as plt
from qutip import destroy, sigmaz, tensor, qeye, sigmap, sigmam, steadystate, ptrace
# System parameters
N = 2
M = 11
g = 0.001
gamma = 1

```

```

kappa = 1
Omega = 30
nu = 1 # Phonon frequency
delta = nu # Set detuning to delta = nu
# Operators
sm = sigmam()
sp = sigmap()
sz = sigmaz()
I_atom = qeye(2)
c = destroy(N)
cd = c.dag()
I_photon = qeye(N)
b = destroy(M)
bd = b.dag()
I_phonon = qeye(M)
# Collapse operators
c_ops = [
    np.sqrt(gamma) * tensor(sm, I_photon, I_phonon), # Atomic decay
    np.sqrt(kappa) * tensor(I_atom, c, I_phonon) # Cavity decay
]
# Hamiltonian
H_phonon = nu * tensor(I_atom, I_photon, bd * b)
H_photon = delta * tensor(I_atom, cd * c, I_phonon)
H_atom = 0.5 * Omega * tensor(sp + sm, I_photon, I_phonon)
H_interaction = g * (tensor(sp, c, b + bd) + tensor(sm, cd, b + bd))
H = H_phonon + H_photon + H_atom + H_interaction
# Compute steady-state density matrix
rho_ss = steadystate(H, c_ops)
# Partial trace over atom and photon modes to get the phonon density matrix
rho_phonon = ptrace(rho_ss, 2)

# Debugging: Check the shape of rho_phonon
print("rho_phonon shape:", rho_phonon.shape)
# Ensure rho_phonon is converted to a NumPy array
rho_phonon_np = rho_phonon.full()
# Extract the diagonal (phonon number distribution)
phonon_distribution = np.real(np.diag(rho_phonon_np))
phonon_distribution = np.copy(phonon_distribution)
# Normalize the distribution to sum to 1
phonon_distribution /= np.sum(phonon_distribution)
# Prepare phonon number values
n_values = np.arange(len(phonon_distribution))
import csv
with open('phonon_distribution_steady_state.csv', mode='w', newline='') as csvfile:
    csv_writer = csv.writer(csvfile)
    csv_writer.writerow(['Phonon Number (m)', 'Probability (P_m)'])
    for n, prob in zip(n_values, phonon_distribution):
        csv_writer.writerow([n, prob])
print("Phonon distribution has been saved to 'phonon_distribution_steady_state.csv'.")
# Plot the phonon number distribution
plt.figure(figsize=(8, 5))
plt.bar(n_values, phonon_distribution, color='orange', edgecolor='black', label="Phonon Distribution")
plt.xlabel(r'Phonon Number ($m$)', fontsize=14)
plt.ylabel(r'Probability ($P_m$)', fontsize=14)
plt.title(r'Phonon Number Distribution at steady state', fontsize=16)
plt.grid(True)

```

```
plt.legend()  
plt.tight_layout()  
plt.savefig("Pnnnnnn1515.png")  
plt.show()
```

## References

- [1] S. Chu, “Nobel Lecture: The manipulation of neutral particles,” *Rev Mod Phys*, vol. 70, no. 3, p. 685, Jul. 1998, doi: 10.1103/RevModPhys.70.685.
- [2] H. J. Metcalf and P. Van der Straten, “Laser Cooling and Trapping,” Springer, pp. 57–58, 1999.
- [3] D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E. King, and D. M. Meekhof, “Experimental issues in coherent quantum-state manipulation of trapped atomic ions,” *J Res Natl Inst Stand Technol*, vol. 103, no. 3, pp. 259–328, Oct. 1997, doi: 10.6028/jres.103.019.
- [4] N. C. T. Claude and W. D. Phillips, “New Mechanisms for Laser Cooling,” *Phys Today*, vol. 43, no. 10, pp. 33–40, Oct. 1990, doi: 10.1063/1.881239.
- [5] W. D. Phillips, “Nobel Lecture: Laser cooling and trapping of neutral atoms,” *Rev Mod Phys*, vol. 70, no. 3, p. 721, Jul. 1998, doi: 10.1103/RevModPhys.70.721.
- [6] T. W. Hänsch and A. L. Schawlow, “Cooling of gases by laser radiation,” *Opt Commun*, vol. 13, no. 1, pp. 68–69, 1975, doi: 10.1016/0030-4018(75)90159-5.
- [7] W. M. Itano and D. J. Wineland, “Laser cooling of ions stored in harmonic and Penning traps,” *Phys Rev A (Coll Park)*, vol. 25, no. 1, p. 35, Jan. 1982, doi: 10.1103/PhysRevA.25.35.

- [8] A. Ashkin, “Trapping of Atoms by Resonance Radiation Pressure,” *Phys Rev Lett*, vol. 40, no. 12, p. 729, Mar. 1978, doi: 10.1103/PhysRevLett.40.729.
- [9] R. Chrétien, “Laser cooling of atoms: Monte-Carlo wavefunction simulations,” Master’s Thesis, Faculté des Sciences Appliquées Université de Liège, June. 2014.
- [10] W. D. Phillips and H. Metcalf, “Laser deceleration of an atomic beam,” *Phys Rev Lett*, vol. 48, no. 9, pp. 596–599, 1982, doi: 10.1103/PHYSREVLETT.48.596.
- [11] J. Dalibard and C. cohen-tannoudji, “Laser cooling below the Doppler limit by polarization gradients: simple theoretical models,” *Journal of The Optical Society of America B-optical Physics*, vol. 6, no. 11, p. 2023, Nov. 1989, doi: 10.1364/JOSAB.6.002023.
- [12] D. R. Leibbrandt, J. Labaziewicz, V. Vuletić, and I. L. Chuang, “Cavity sideband cooling of a single trapped ion,” *Phys Rev Lett*, vol. 103, no. 10, Aug. 2009, doi: 10.1103/PHYSREVLETT.103.103001.
- [13] A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. cohen-tannoudji, “Laser cooling below the one-photon recoil energy by velocity-selective coherent population trapping: theoretical analysis,” *Journal of The Optical Society of America B-optical Physics*, vol. 6, no. 11, p. 2112, Nov. 1989, doi: 10.1364/JOSAB.6.002112.

- [14] E. A. Cornell and C. E. Wieman, "Nobel Lecture: Bose-Einstein condensation in a dilute gas, the first 70 years and some recent experiments," *Rev. Mod. Phys.*, vol. 74, no. 3, pp. 875–893, Aug. 2002.
- [15] E. L. Raab, M. Prentiss, A. Cable, S. Chu, and D. E. Pritchard, "Trapping of Neutral Sodium Atoms with Radiation Pressure," *Phys Rev Lett*, vol. 59, no. 23, p. 2631, Dec. 1987, doi: 10.1103/PhysRevLett.59.2631.
- [16] S. Zippilli and G. Morigi, "Mechanical effects of optical resonators on driven trapped atoms: Ground-state cooling in a high-finesse cavity," *Phys Rev A*, vol. 72, no. 5, Nov. 2005, doi: 10.1103/PHYSREVA.72.053408.
- [17] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, "Cavity cooling of a single atom," *Nature* 2004 428:6978, vol. 428, no. 6978, pp. 50–52, Mar. 2004, doi: 10.1038/nature02387.
- [18] P. W. H. Pinkse, T. Fischer, P. Maunz, and G. Rempe, "Trapping an atom with single photons," *Nature* 2000 404:6776, vol. 404, no. 6776, pp. 365–368, Mar. 2000, doi: 10.1038/35006006.
- [19] M. H. Schleier-Smith, I. D. Leroux, H. Zhang, M. A. Van Camp, and V. Vuletić, "Optomechanical cavity cooling of an atomic ensemble," *Phys Rev Lett*, vol. 107, no. 14, p. 143005, Sep. 2011, doi: 10.1103/PHYSREVLETT.107.143005/FIGURES/4/THUMBNAIL.
- [20] T. Elsässer, B. Nagorny, and A. Hemmerich, "Optical bistability and collective behavior of atoms trapped in a high-Q ring cavity," *Phys Rev A*,

vol. 69, no. 3, p. 033403, Mar. 2004, doi:  
10.1103/PHYSREVA.69.033403/FIGURES/10/THUMBNAIL.

- [21] M. Trupke *et al.*, “Microfabricated high-finesse optical cavity with open access and small volume,” *Appl Phys Lett*, vol. 87, no. 21, pp. 1–3, Nov. 2005, doi: 10.1063/1.2132066.
- [22] M. Khudaverdyan *et al.*, “Controlled insertion and retrieval of atoms coupled to a high-finesse optical resonator,” *New J Phys*, vol. 10, no. 7, p. 073023, Jul. 2008, doi: 10.1088/1367-2630/10/7/073023.
- [23] P. F. Herskind, A. Dantan, J. P. Marler, M. Albert, and M. Drewsen, “Realization of collective strong coupling with ion Coulomb crystals in an optical cavity,” *Nature Physics 2009 5:7*, vol. 5, no. 7, pp. 494–498, Jun. 2009, doi: 10.1038/nphys1302.
- [24] H. J. Kimble, “Strong interactions of single atoms and photons in cavity QED,” *Physica Scripta T*, vol. 76, no. T76, pp. 127–137, Jan. 1998, doi: 10.1238/PHYSICA.TOPICAL.076A00127/XML.
- [25] S. Nußmann, K. Murr, M. Hijkema, B. Weber, A. Kuhn, and G. Rempe, “Vacuum-stimulated cooling of single atoms in three dimensions,” *Nature Physics 2005 1:2*, vol. 1, no. 2, pp. 122–125, Oct. 2005, doi: 10.1038/nphys120.
- [26] R. Loudon, “The Quantum Theory of Light,” 3rd ed., Oxford University Press, p. 126-133 and 162-165, 2023.

- [27] G. B. Arfken, H. J. Weber, and F. E. Harris, “Mathematical Methods for Physicists: A Comprehensive Guide,” 7th ed., pp. 97–100, Jan. 2012.
- [28] C. Gerry and P. Knight, “Introductory Quantum Optics,” Cambridge University Press, pp. 11–15, 90–95, Oct. 2004.
- [29] C. Cohen-Tannoudji, “Atomic Motion in Laser Light,” Course 1, College de France and Department de Physique, Ecole Normale Supérieure, Paris, France, Elsevier Science Publishers, pp. 26–29, Nov. 1990.
- [30] Jean Dalibard and Yvan Castin, "Laser Cooling from the Semi-Classical to the Quantum Regime," Proceedings of the International School of Physics "Enrico Fermi", Course CXX, Frontiers in Laser Spectroscopy, 23 June - 3 July 1992, p. 445-476, edited by T.W. Hänsch and M. Inguscio (NorthHolland, 1994).
- [31] S. Stenholm, “The semiclassical theory of laser cooling,” *Rev Mod Phys*, vol. 58, no. 3, pp. 699–739, 1986, doi: 10.1103/RevModPhys.58.699.
- [32] C. J. Foot, “Atomic Physics,” Oxford University Press, pp. 190–191 and 203–208, 2004.
- [33] H.-P. Breuer and F. Petruccione, “The Theory of Open Quantum Systems,” Oxford University Press, pp. 110–126 and 131-132, 2002.
- [34] F. Diedrich, J. C. Bergquist, W. M. Itano, and D. J. Wineland, “Laser Cooling to the Zero-Point Energy of Motion,” *Phys Rev Lett*, vol. 62, no. 4, p. 403, Jan. 1989, doi: 10.1103/PhysRevLett.62.403.

- [35] R. Blatt, F. Schmidt-Kaler, J. Eschner, and G. Morigi, “Laser cooling of trapped ions,” *JOSA B*, Vol. 20, Issue 5, pp. 1003-1015, vol. 20, no. 5, pp. 1003–1015, May 2003, doi: 10.1364/JOSAB.20.001003.
- [36] D. J. Wineland, W. M. Itano, J. C. Bergquist, and R. G. Hulet, “Laser-cooling limits and single-ion spectroscopy,” *Phys Rev A (Coll Park)*, vol. 36, no. 5, pp. 2220–2232, 1987, doi: 10.1103/PhysRevA.36.2220.
- [37] P. Horak, G. Hechenblaikner, K. M. Gheri, H. Stecher, and H. Ritsch, “Cavity-Induced Atom Cooling in the Strong Coupling Regime,” *Phys Rev Lett*, vol. 79, no. 25, p. 4974, Dec. 1997, doi: 10.1103/PhysRevLett.79.4974.
- [38] V. Vuletić and S. Chu, “Laser Cooling of Atoms, Ions, or Molecules by Coherent Scattering,” *Phys Rev Lett*, vol. 84, no. 17, p. 3787, Apr. 2000, doi: 10.1103/PhysRevLett.84.3787.
- [39] H. Carmichael, “An Open Systems Approach to Quantum Optics,” vol. 18, Springer, pp. 130–132, 1993 doi: 10.1007/978-3-540-47620-7.
- [40] M. A. Nielsen and I. L. Chuang, “Quantum Computation and Quantum Information: 10th Anniversary Edition,” *Quantum Computation and Quantum Information*, Dec. 2010, doi: 10.1017/CBO9780511976667.
- [41] K. Mølmer, Y. Castin, and J. Dalibard, “Monte Carlo wave-function method in quantum optics,” *JOSA B*, Vol. 10, Issue 3, pp. 524-538, vol. 10, no. 3, pp. 524–538, Mar. 1993, doi: 10.1364/JOSAB.10.000524.

- [42] A. J. Daley, “Quantum trajectories and open many-body quantum systems,” *Adv Phys*, vol. 63, no. 2, pp. 77–149, May 2014, doi: 10.1080/00018732.2014.933502.
- [43] P. Rice, “An introduction to quantum optics: An open systems approach,” *An Introduction to Quantum Optics: An open systems approach*, pp. 9-1-9-17, Jan. 2020.
- [44] H. Carmichael, “An Open Systems Approach to Quantum Optics,” vol. 18, 1993, doi: 10.1007/978-3-540-47620-7.
- [45] G. Nemova, “Field Guide to Laser Cooling Methods,” *Field Guide to Laser Cooling Methods*, Nov. 2019, doi: 10.1117/3.2538938.
- [46] J. I. Cirac, M. Lewenstein, and P. Zoller, “Laser cooling a trapped atom in a cavity: Bad-cavity limit,” *Phys Rev A*, vol. 51, no. 2, p. 1650, Feb. 1995, doi: 10.1103/PhysRevA.51.1650.
- [47] O. Kim and A. Beige, “Mollow triplet for cavity-mediated laser cooling,” *Phys Rev A*, vol. 88, no. 5, p. 053417, Nov. 2013, doi: 10.1103/PHYSREVA.88.053417/FIGURES/7/THUMBNAIL.
- [48] T. Blake, A. Kurcz, and A. Beige, “Comparing cavity and ordinary laser cooling within the Lamb–Dicke regime,” *J Mod Opt*, vol. 58, no. 15, pp. 1317–1328, Sep. 2010, doi: 10.1080/09500340.2010.543957.

- [49] T. Blake, A. Kurcz, and A. Beige, “Rate-equation approach to cavity-mediated laser cooling,” *Phys Rev A*, vol. 86, no. 1, p. 013419, Jul. 2012, doi: 10.1103/PHYSREVA.86.013419/FIGURES/10/THUMBNAIL.
- [50] M. Hosseini, Y. Duan, K. M. Beck, Y. T. Chen, and V. Vuletić, “Cavity Cooling of Many Atoms,” *Phys Rev Lett*, vol. 118, no. 18, May 2017, doi: 10.1103/PHYSREVLETT.118.183601/FIGURES/3/THUMBNAIL.

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