

**On the Thermal Bending Behavior of Porous Cross-Ply
Laminated Plates Using an Improved FSDT**

BY

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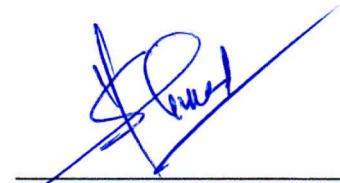
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Dear family,
For your encouragement, love, and support; I dedicate this thesis work to you.

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Praise be to Allah, a praise that befits his majesty, who gave me the health, the patience, and the will to complete this work. I extend my sincere thanks and appreciation to King Fahd University of Petroleum and Minerals, where I did both my bachelor's and master's degrees, for providing me with this valuable opportunity. I would like to thank the Deanship of Graduate Studies for their generous support. I'm grateful to the Civil and Environmental Engineering Department, with all its staff and professors, who never hesitated to serve and assist students.

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LIST OF ABBREVIATIONS

ALFC	:	Advanced Laminated Fibrous Composite
FSDT	:	First-Order Shear Deformation Theory
GDE	:	Governing Differential Equation
CFRP	:	Carbon Fiber-Reinforced Polymer
ESL	:	Equivalent Single Layer
CLPT	:	Classical Laminated Plate Theory
HSDT	:	Higher-Order Shear Deformation Theory
TSDT	:	Third-Order Shear Deformation Theory
I-FSDT	:	Improved First-Order Shear Deformation Theory
CBT	:	Classical Laminated Beam Theory
TBT	:	Timoshenko's Beam Theory
TrSDT	:	Trigonometric Shear Deformation Theory
I-TBT	:	Improved Timoshenko's Beam Theory

ABSTRACT

Full Name : Muayad Abdo Ali Rajeh

Thesis Title : On the Thermal Bending Behavior of Porous Cross-Ply Laminated
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Advanced laminated fibrous composites (ALFCs) are increasingly being used for their enhanced properties (e.g., high specific stiffness and strength, low thermal expansion coefficient, and resistance to both corrosion and temperature.), that can meet the specifications of advanced applications of engineering that require tailored properties. These enhancements in properties were made possible due to the recent development in materials and manufacturing technologies, such as maximizing the directional properties of the laminate fibers. These laminated composites are excellent in withstanding axial and bending loads and therefore are being used as structural plates. During the manufacturing stage of these laminated plates, porosities are introduced either purposefully (e.g., for sound and thermal insulation), or by error. These voids under loading will affect the plate's mechanical behavior and are better accounted for during the analysis. This thesis aims to investigate the flexural behavior of porous cross-ply laminated plates under nonlinear thermal load along the thickness of the plate. Innovatively, three porosity distribution models will be considered. Also, a novel, simple, and efficient improved first-order shear

deformation theory (FSDT) with four variables as opposed to five in conventional FSDT, will be used to conduct the thermal flexural analysis. The plate's top and bottom boundary conditions are satisfied and the need for a shear correction factor is eliminated. The plate is simply supported on all its edges. The governing differential equations (GDEs) and the boundary conditions were obtained using the virtual work principle. The analytical solutions are derived by Navier's procedure. The present theory validity is attested by comparison with the following theories' results: classical, first-order, higher-order, and exact elasticity theories results. The effects of porosity volume fractions and porosity distribution models on the laminated plate flexural response are investigated.

ملخص الرسالة

الاسم الكامل: مؤيد عبده علي راجح

عنوان الرسالة: السلوك الانتنائي الحراري للصفائح ذات الطبقات المتقاطعة المسامية باستخدام نظرية FSDT محسنة

التخصص: هندسة مدنية وبيئية

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تُستخدم المركبات الليفية الطبقيّة المتقدمة (ALFCs) بشكل متزايد لخصائصها المحسنة (على سبيل المثال، الصلابة والقوة النوعية العالية، ومعامل التمدد الحراري المنخفض، والمقاومة للتآكل ودرجة الحرارة)، والتي يمكن أن تلبّي مواصفات التطبيقات المتقدمة للهندسة التي تستلزم خصائص مفصلة. أصبحت هذه التحسينات في الخصائص ممكنة بسبب التطور الحديث في علم المواد وتقنيات التصنيع، مثل تعظيم الخصائص الاتجاهية للألياف المكونة للطبقات. تتميز هذه المركبات الطبقيّة بقدرتها على تحمل الأحمال المحورية والانحناء، وبالتالي يتم استخدامها كصفائح هيكلية. أثناء مرحلة تصنيع هذه الصفائح الطبقيّة، تتكون مسامات عن طريق الخطأ. تؤثر هذه الفراغات تحت التحميل على السلوك الميكانيكي للصفحة ومن الأفضل أخذها بعين الاعتبار أثناء التحليل. تهدف هذه الأطروحة إلى التحقيق في السلوك الانتنائي للصفائح ذات الطبقات المتقاطعة المسامية تحت الحمل الحراري غير الخطي على طول سمك الصفحة. بشكل مبتكر، سيتم النظر في ثلاثة نماذج لتوزيع المسامات. كما سيتم استخدام نظرية محسنة لنظرية الإزاحة بقوى القص ذات الرتبة الأولى (FSDT) جديدة وبسيطة وفعالة ذات أربع متغيرات بدلاً من خمسة في نظرية FSDT التقليدية، لإجراء تحليل الانحناء الحراري. تم استيفاء الشروط الحدودية العلوية والسفلية للصفحة وتم إزالة الحاجة إلى عامل تصحيح القص. الصفحة مدعمة بركائز مفصلية على جميع حوافها. تم الحصول على المعادلات التفاضلية الحاكمة (GDEs) والشروط الحدودية باستخدام مبدأ العمل الافتراضي. تم اشتقاق الحلول التحليلية من خلال إجراء "نافير". تم التحقق من صحة النظرية الحالية من خلال المقارنة بنتائج النظريات التالية: الكلاسيكية، ذات الرتبة الأولى

التقليدية، ذات الرتب العليا، ونظرية المرونة الدقيقة. تم التحقيق في تأثيرات كلا من حجم المسامات ونماذج توزيع المسامات على الاستجابة الانحنائية للصفحة الطبقية.

CHAPTER 1

INTRODUCTION

1.1 Background

Throughout history, materials have been employed in different applications to facilitate people's lives. These materials are subjected to various environmental factors and load conditions during their lifespan. Composite materials, which are made of at least two distinct materials that are not soluble when combined and have preferred properties over the use of any of its constituents individually, have historical records of their employment [1], [2]. Some prominent examples include clay walls reinforced with bamboo and glued wood laminates in ancient Egypt.

Composite materials usually consist of reinforcement materials (in the form of fibers or particles) embedded in a matrix medium [1]. Composite materials could be classified into fibrous composites (fibers in a matrix), particulate composites (macro-scale particles in a matrix), laminated composites (two or more layers attached together), or a combination of the previous three types. This thesis will study the effect of thermal loads on the flexural response of porous plates made with a special type of advanced laminated fibrous composites (ALFCs) called cross-ply laminate as will be explained in the following segments. The analysis will be conducted using an improved, simple version of first-order shear deformation theory (FSDT) for laminated plates.

In the following subsections, more elaboration on some fundamental background of the topic will be discussed. The discussion will contain a brief introduction to ALFCs and their

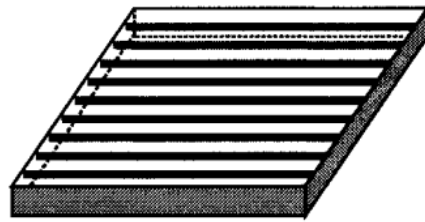
types, with more emphasis on unidirectional fibrous lamina and laminates, laminate notation, and laminate types.

1.1.1 Advanced Laminated Fibrous Composites

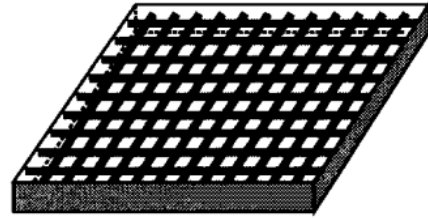
In the early 20th century, ALFCs were introduced when boats and airplanes were made of glass fibers embedded in resin matrices [1], [2]. From the 1970s onwards, new fibers (e.g., carbon, and aramids) were developed along with matrices of ceramics and some metals, increasing the applicability of advanced composites. These composites had superior properties compared to conventional materials, such as higher strength, stiffness, corrosion resistance, wear resistance, and temperature resistance, in addition to having lighter weight. Another advantage is that these composites' properties could be tailored for a specific application.

ALFCs are made with at least two materials on the macroscopic level, one material in the form of fibers serving as reinforcement that are embedded in the other base material called matrix [1]–[3]. Fibers mainly carry the load, while the matrix holds the fibers, transfers the load between them, and shields them from environmental effects. The fibers of a material have superior properties than the bulk form made of the same material [1]. The first reason is that the fibers' small units are processed in such a way that their directional properties are maximized and aligned in the fibers' direction. The second reason is that the fibers are processed on a small scale compared to the bulk form; fibers have a diameter in micrometers scale, reducing the inherited defects in manufacturing. The fibers could be continuous or discontinuous. Continuous fibrous composites have superior properties than discontinuous ones. Fibers in continuous fibrous composites could be unidirectional, bidirectional, or woven. Figure 1 depicts the types of fibrous composites. Unidirectional

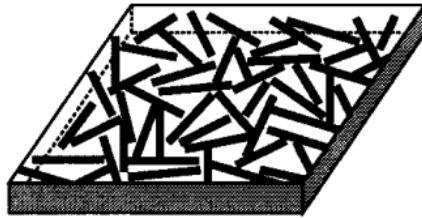
fibrous composites have superior strength than the other types in the fibers' direction. Henceforth, the focus will be on unidirectional fiber composites only, unless otherwise mentioned.



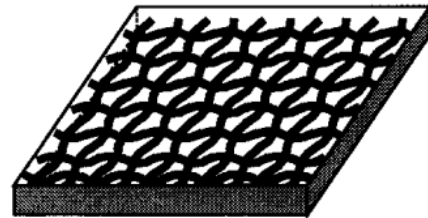
(a) Unidirectional



(b) Bi-directional



(c) Discontinuous fiber



(d) Woven

Figure 1: Types of fibers used in fibrous composites [1].

The building block of ALFCs is called lamina. A lamina contains numerous fibers that are aligned at a given direction and inserted in a matrix base [1], [4]. Usually, the fibers and matrix are better bonded through a chemical compound. Figure 2 (a) shows a lamina with its constituents.

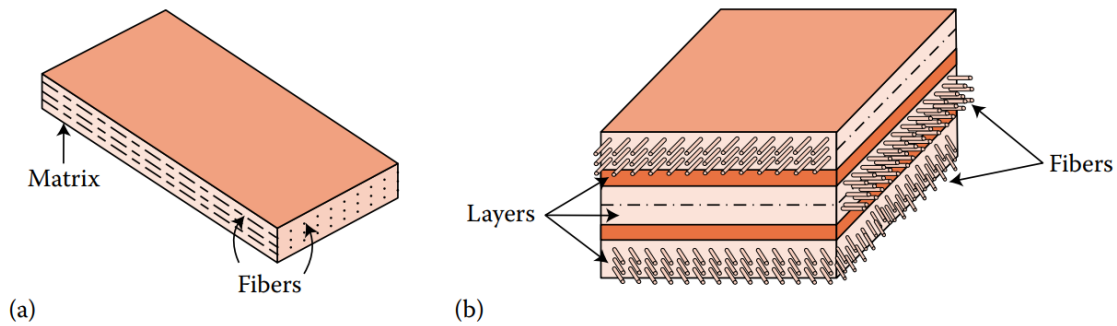


Figure 2: (a) Constituents of a lamina, (b) A laminate made of 3 laminae [4].

Laminated composite members, like beams and plates, are made by stacking the plies, or laminae, at different orientations to attain the required properties for the member [1]. Both the direction of fibers and the sequence of the stacked layers for a given composite material are manipulated to get the desired properties for a certain application.

Laminates are stacks of laminae sequenced and oriented to reach the desired strength, thickness, and stiffness [1], [2]. Figure 2 (b) shows a 3-ply laminate made by stacking 3 laminae at different fibers directions. For example, laminates with the fibers of some laminae directed at angles (30 or 45) degrees are good at handling shear forces. So, depending on the application, the designer can utilize different stacking sequences using different materials. These stacked plies are mostly bonded together using the lamina's matrix material. The manufacturing of laminates will always contain defects, like voids between the laminae, varied thickness, improper orientation, and more, that are better accounted for during analysis [1].

1.1.2 Laminate Notation and Types

Since the lamination scheme is important in the characterization of a laminate, it is crucial to understand the notation used in naming different types of laminate. Laminates are named after the angles sequence of laminae separated by a slash sign, as they are stacked in the direction of the third global axis z [2]. In the following discussion, the z -axis will be considered directed upward unless otherwise mentioned. For example, the laminate in Figure 3 is named (60/90/-45/0) as the z -axis is assumed to be directed upward, or (0/-45/90/60) if the positive z -axis was taken downward. Also, the coding does imply that all laminae have the same thickness and material.

0
-45
90
60

Figure 3: Lamination scheme example 1.

In the case where several plies have the same angle, a subscript demonstrating the number of these plies is denoted. For example, a laminate of the code (0/60/90₂/0) resembles a laminate as shown in Figure 4.

0
90
90
60
0

Figure 4: Lamination scheme example 2.

If the same sequence of laminae is repeated multiple times for a laminate, the number of repetitions is written as a subscript. For example, a laminate with the laminae stacking (0/60/0/60/0/60) could be written for short as (0/60)₃. In the case of symmetric laminates, the line of symmetry is the midplane, the laminae will have the same thickness, angle orientation, and material. A subscript s is used to denote the symmetry of a laminate. For an even number of plies, the midplane will be at the interface of the two middle laminae, for example (0/45/60)_s is shown in Figure 5.

0
45
60
60
45
0

Figure 5: Lamination scheme example 3.

For symmetric laminate with an odd number of layers, the midplane will be at the middle of the center lamina and will be designated by a bar on it, for instance $(0/90/\overline{60})_s$ is depicted in Figure 6.

0
90
60
90
0

Figure 6: Lamination scheme example 4.

Now, after the notation of laminate naming is explained, laminate types will be introduced. Single-layer laminates are those where all layers have the same orientation, material, and thickness [1], [2]. Symmetric laminates are those having the upper laminate half as a mirror image of the other half. Cross-ply laminates, or specially orthotropic, are those made of plies oriented at 0 and 90° only [1]. A laminate could be both symmetric and cross-ply as an example $(0/90/0)_s$. Angle-ply laminates are those having layers of similar material and thickness, oriented at angles $+\theta$ and $-\theta$. An example would be $(-40/40)_2$. Antisymmetric laminates are those having angle orientations that are negative of each other when going the same distance above or below the midplane of the laminate but having the same material and thickness, as in $(45/-60/-30/30/60/-45)$. Since 0 is considered $-\theta$ of 90, antisymmetric laminates could also be cross-ply at the same time as in $(0/90/0/90)$.

1.2 Thesis Objectives

This thesis aims to formulate the governing differential equations (GDEs) of the thermal bending behavior of porous cross-ply laminated plates. Then, solving the GDEs

analytically. After that, some numerical examples of solved problems will be provided. A list of specific thesis objectives is as follows:

- To come up with an efficient improved version of FSDT for plates and beams.
- To investigate the thermal bending behavior of porous cross-ply laminated plates and beams.
- To study the effect of porosity volume fraction (ξ) and three porosity distribution models on the response.

1.3 Significance of the Research

ALFCs market share has increased globally to reach more than \$20,000,000,000 in 2014 and is forecasted to increase even more [5]. This high demand is attributed to them having high specific strengths and stiffnesses, high corrosion and temperature resistances, and lighter than isotropic materials counterparts. All these features made various engineering branches interested in using these materials for their applications. Many publications were done considering different mechanical and thermal loadings applied on cross-ply laminated plates utilizing conventional analytical and numerical methods. However, publications on the thermal bending behavior of porous plates considering multiple porosity models are deficient. Therefore, this thesis is an attempt to fill the gap via an improved efficient FSDT that does not require a shear correction factor, with less computational complexity and effort compared to higher-order theories.

1.4 Thesis organization

This thesis is organized into five chapters, which are further organized into sections, subsections, and subsubsections. Chapter 1 introduces composite materials' history, ALFCs, laminate notation (naming) and types. Such an introduction is important to familiarize readers with the structure to be analyzed. Then, a problem statement and detailed thesis objectives are stated. In Chapter 2, the literature on the topic is presented. It starts with ALFC's applications in different fields and their main manufacturing processes. Then, important concepts required for the problem-solving methodology are expressed. These include structural theories for laminated plates, theoretical formulation with the consideration of thermal effects, and finally porosity effects and considerations. As the analysis of plates could be a bit complex to tackle straightaway, chapter 3 presents a full journal paper on the flexural behavior of porous anti-symmetric cross-ply laminated beams under mechanical and linear thermal loadings. In the mechanics of beams, unlike plates, the rate of change of load, deformation, stress, etc., along y is considered zero. So, the work on beams was essential as it paved the way and built the confidence to properly analyze the bending response of porous plates. Chapter 4 covers a full journal paper, on porous cross-ply laminated plates under nonlinear thermal loading, with its introduction, literature survey, theoretical formulation considering both thermal and porosity effects, results, and discussions. Chapter 5 offers a summary of the important points drawn from this study, in addition to suggestions for what can be done after it.

CHAPTER 2

LITERATURE REVIEW

2.1 Advanced Laminated Fibrous Composites Applications

ALFCs are used in various areas for numerous applications, and this is not the place to mention them all. However, some of the major ones will be mentioned.

In the civil engineering field, high-strength lightweight materials have always been of great interest. Couple that with the corrosion-resistant capabilities of ALFCs, this is why these composites have great potential in solving civil engineering problems. Bridge structural components span long distances and usually endure harsh weather conditions, that is why polymer reinforced with glass fibers are used in their construction [6]. Another example is carbon fiber-reinforced polymer (CFRP) wraps used for concrete elements strengthening. A new type of ALFCs called fiber-reinforced inorganic polymer composites have been found to increase concrete's fire resistance.

In the Aerospace field, ALFCs are extensively used mainly for their high specific strength and stiffness. Furthermore, ALFC's lightness helps reduce fuel consumption. Different fiber materials are used to manufacture various parts of the aircraft as Figure 7 shows [7]. In automobiles, ALFCs are also used for their weight reduction and corrosion resistance advantages. Figure 8 shows the CFRP weight reduction capability of a car's body weight by 30% [8]. ALFCs are also used to manufacture wind turbine components [3]. Glass and carbon fibers are both used in turbine blades increasing their energy-generating efficiency and decreasing energy cost.

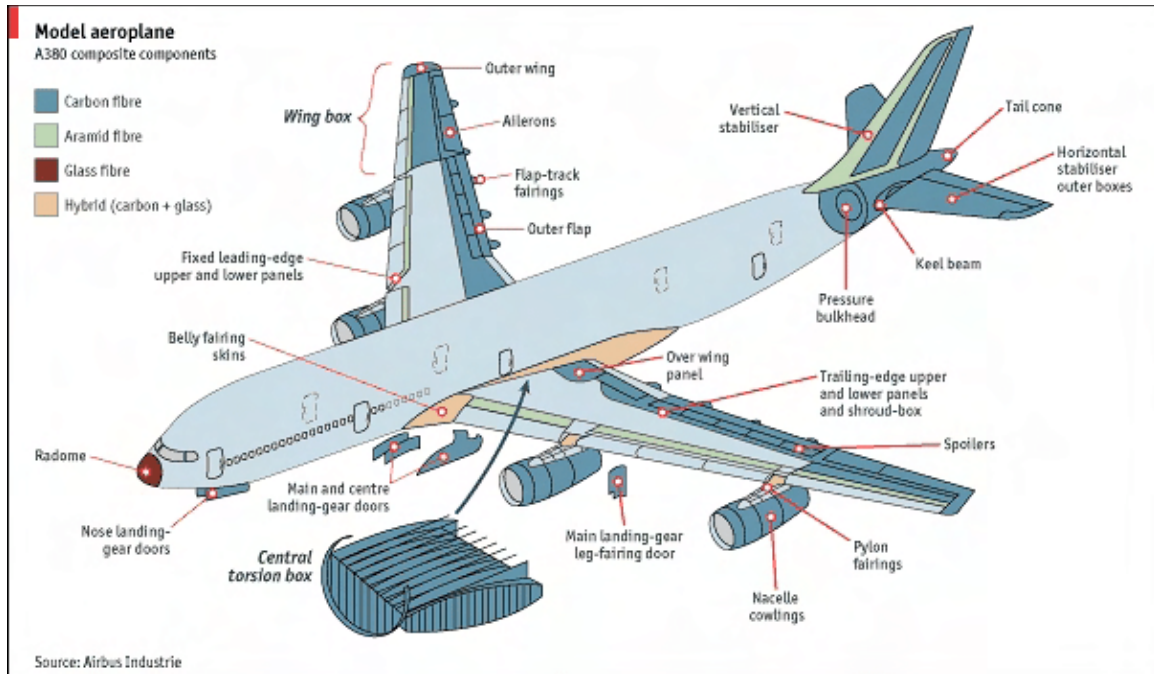


Figure 7: ALFC's usage in Airbus 380 aircraft [7].

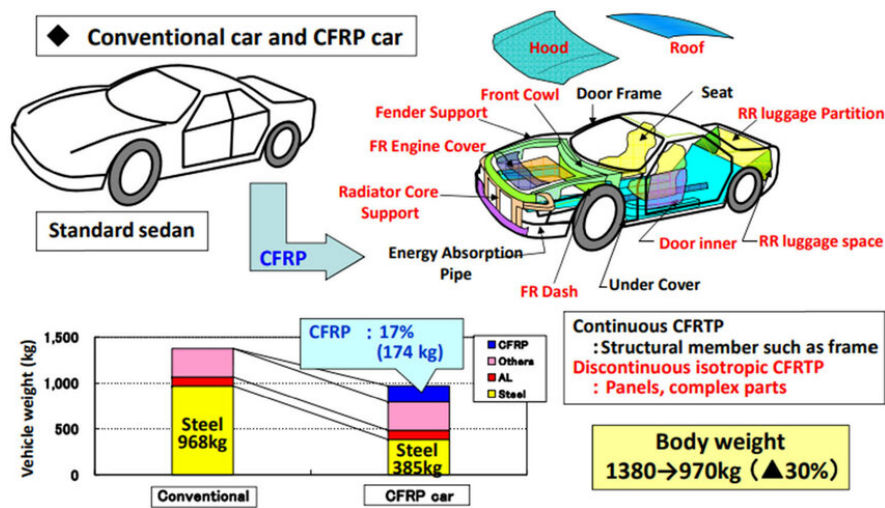


Figure 8: CFRP potential in reducing cars body weight by 30% [8].

2.2 Manufacturing of ALFCs

Many ALFC manufacturing techniques are used depending on the required ALFC's properties and the intended application. Some of the main procedures will be mentioned

and explained here. These techniques are hand lay-up, spray-up, vacuum bagging, resin transfer molding, vacuum-assisted resin transfer molding, compression molding, pultrusion, and filament winding.

The most commonly used open-mold process is called the hand lay-up, or wet lay-up, technique [6], [9]. In this relatively easier manual manufacturing process, dry fiber reinforcements are placed in a mold that is pre-coated with a non-stick material. Then, the resin is brushed or poured on the fibers. Rollers are used to level the resin and remove air bubbles. The process of adding fiber layers continues until the desired thickness is reached. The composite is then cured at room temperature. Another similar but more cost-effective process is called the spray-up technique, where random discontinuous fibers are sprayed along with resin material.

The vacuum bagging technique is an improved variation of the hand lay-up process used in high-performance applications [6]. After the desired thickness is achieved from the hand lay-up process, a vacuum bag is used to enclose the composite, as shown in Figure 9, and a vacuum is operated [6]. This process removes entrapped air, better distributes the resin, and properly impregnates the fibers.

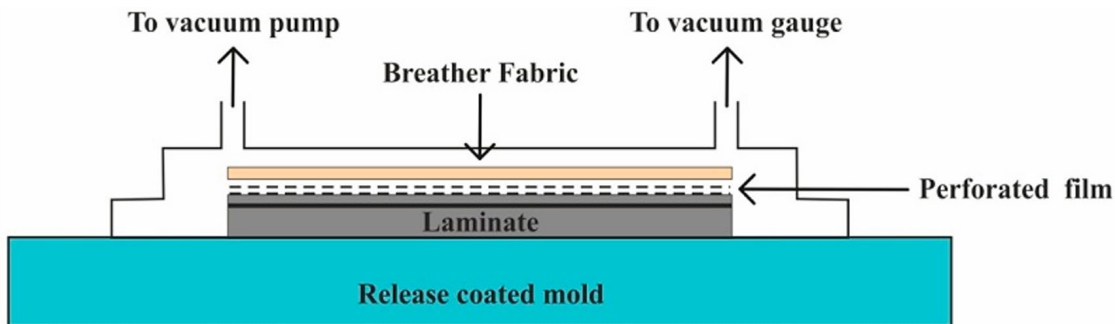


Figure 9: Vacuum bagging technique [6].

In resin transfer molding, fiber preforms in dry condition are arranged in a closed mold where heated resin is injected under pressure as Figure 10 shows [6], [9]. This process is good in applications with high dimension intolerance, such as automotive or aerospace, to manufacture medium-volume components. However, this technique is more expensive and time-consuming compared to earlier mentioned processes. Another version of this technique is when vacuum bagging is also used. Vacuum-assisted resin transfer molding technique is when the resin is pulled by the vacuum instead of pressure injection. This variation is better in removing air voids and decreasing the required tools' cost when compared to resin transfer molding.

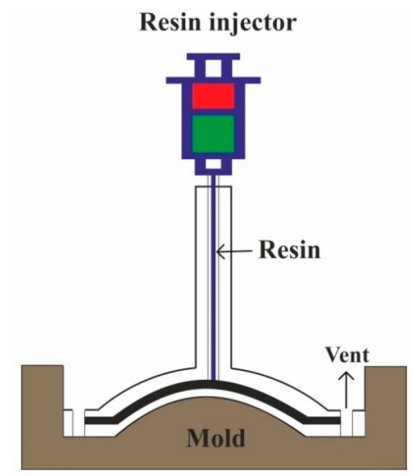


Figure 10: Resin transfer molding technique [6].

In the compression molding technique, prepreg fiber reinforcements are pressed to shape with a hydraulic press under heat as illustrated in Figure 11 [6], [9]. This process is used in the automotive industry for its automation and high productivity.

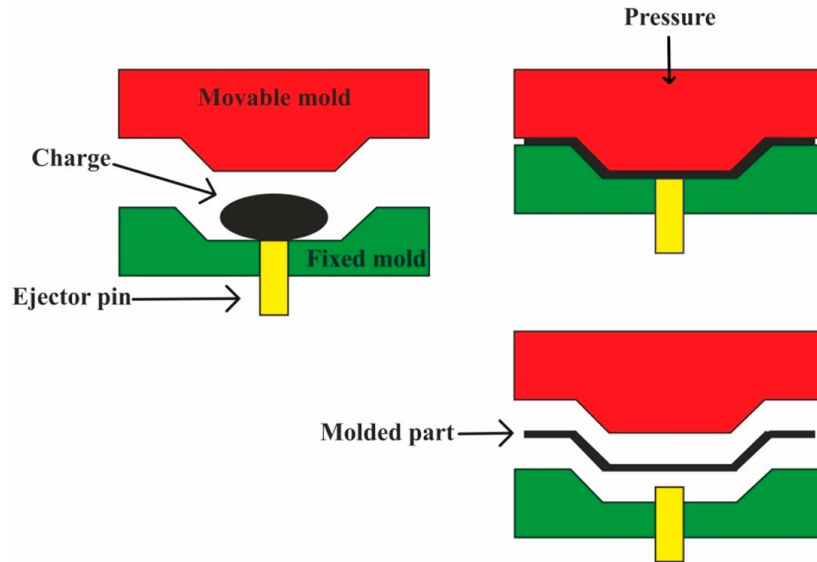


Figure 11: Compression molding technique [6].

Pultrusion is a continuous process where fibers are impregnated in a resin bath, heated, and cut to the desired length in an automated production line as Figure 12 shows [6], [9]. This process is ideal for the production of uniform cross-section elements.

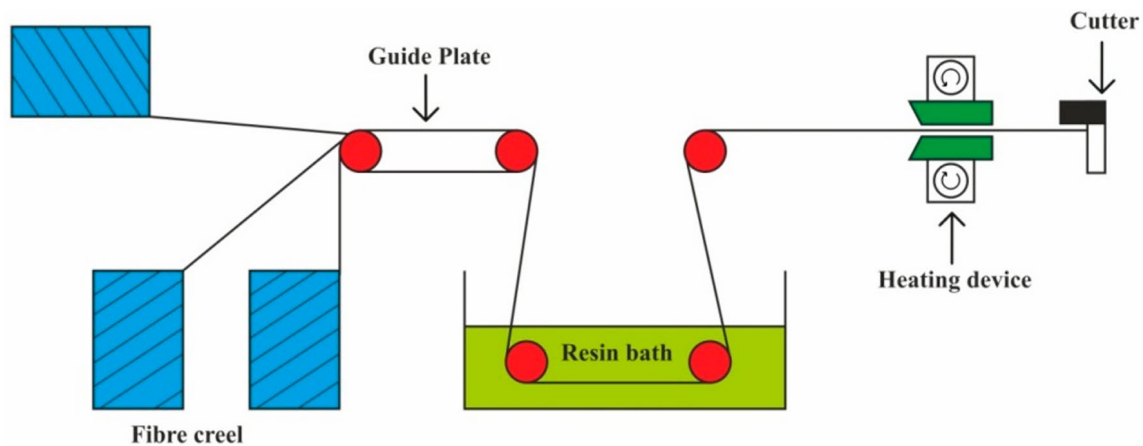


Figure 12: Pultrusion technique [6].

The filament winding technique is an automated process to manufacture cylindrical parts like pipes [6]. Fiber reinforcements are pulled through a resin bath and then winded around a mandrel until the desired shape is achieved as shown in Figure 13. After that, it's either left to be cured or heat is applied to speed up the process.

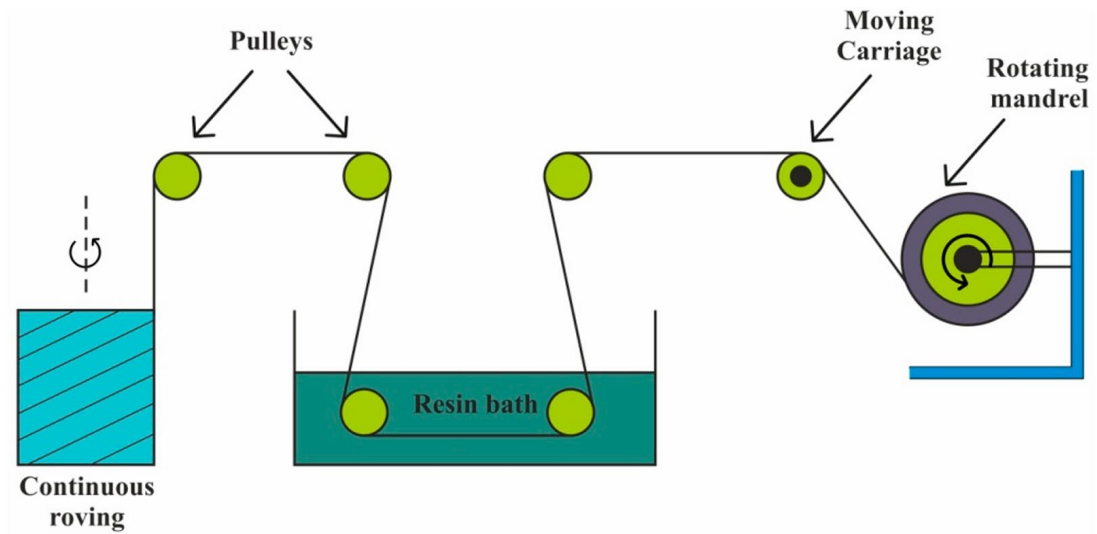


Figure 13: Filament winding technique [6].

2.3 Structural Theories for Laminated Plates

Structural theories of laminated composite plates are generally grouped into equivalent single layer 2D theories (ESL), 3-D elasticity theories, and multiple model methods for both 2-D and 3-D as shown below in Figure 14 [1].

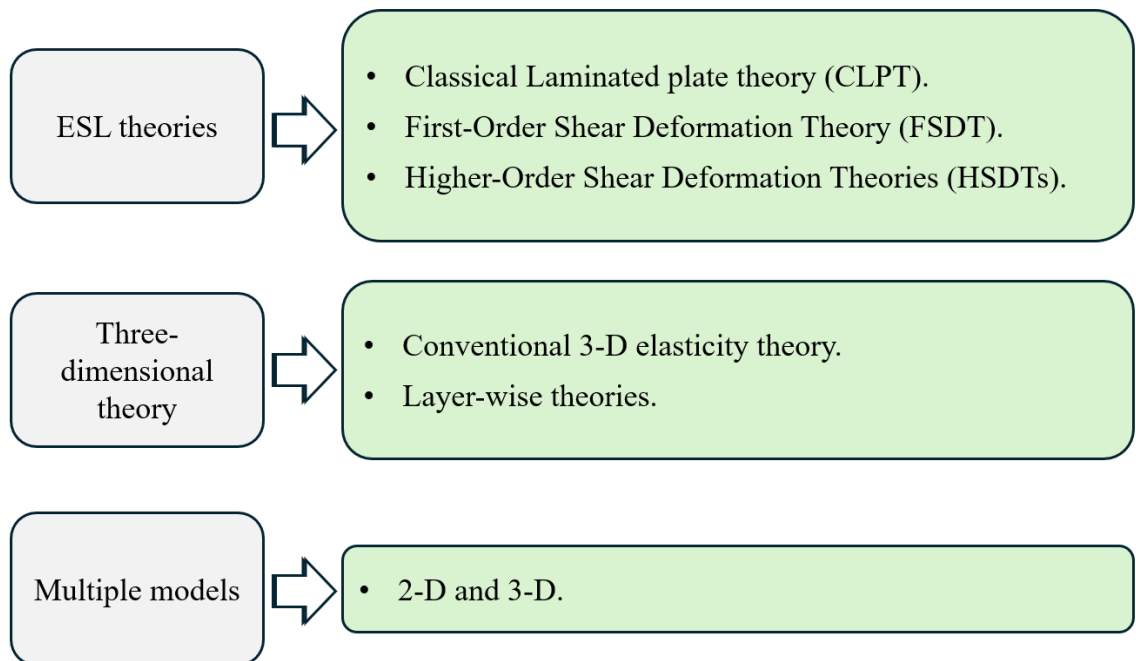


Figure 14: Structural theories for laminated plates.

In the 3-D elasticity, each lamina is analyzed separately as a solid 3-D model [1]. In the ESL method, the heterogeneous laminae of the structural member are assumed as a statically ESL, reducing the problem from 2-D to 3-D via certain assumptions. These assumptions are either regarding deformation kinematics or the state of the stress along the laminate thickness. ESL includes classical laminated plate theory and (first-order, second-order, third-order, parabolic, trigonometric, hyperbolic, and other higher-order) shear deformation laminated plate theories [1], [10].

Two approaches are used to come up with laminated plate theories, either to assume the form of displacement or stress fields [1]. The common one is where the displacement field form is assumed to be made of both unknown functions and height coordinate combinations. Then, both strain-displacement and stress-strain (constitutive) relations are used to find internal forces through the integration of stresses. Finally, GDEs are derived via virtual displacements principle or the dynamic version of it in case the structure behavior is time dependent. The other approach employs the assumption of a stress field with unknown functions. The displacement field is then derived from Hook's law. The remaining procedure steps are similar to the first approach with the exception that the virtual forces principle is now utilized to derive the GDEs.

In this section, the focus will be on some selected ESL theories which are derived through the first approach. Those interested in stress field theories could refer to the book by Panc [11].

2.3.1 The Classical Laminated Plate Theory (CLPT)

CLPT is an extended version of classical (Kirchhoff) plate theory but for laminated plates [1], [2]. This theory assumes the following:

- 1) Mid-surface perpendicular lines that were straight before deformation will continue to be straight after deformation.
- 2) Transverse normal is inextensible.
- 3) Every lamina is homogenous, orthotropic, and elastic.
- 4) Laminae do not slip at their interfaces.
- 5) Thin laminate under plane stress condition.
- 6) Displacements are small compared to laminate thickness.

These assumptions amount to $\sigma_z = \tau_{xz} = \tau_{yz} = \gamma_{xz} = \gamma_{yz} = \varepsilon_z = 0$. Also, the displacement in the thickness direction does not depend on the thickness coordinate itself.

The CLPT displacement field is as follows:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0(x, y)}{\partial x} \\ v(x, y, z) &= v_0(x, y) - z \frac{\partial w_0(x, y)}{\partial y} \\ w(x, y, z) &= w_0(x, y) \end{aligned} \tag{1}$$

Where $u, v, \text{ and } w$ are the displacement in $x, y, \text{ and } z$ directions, respectively. And $u_0, v_0, \text{ and } w_0$ are the mid-surface deformations along the $x, y, \text{ and } z$ directions, respectively. In this theory, if the mid-surface deformations are determined, then the displacement of any point on the laminate can be found. It can be deduced that the planar strains (i.e., $\varepsilon_x, \varepsilon_y, \text{ and } \gamma_{xy}$) will vary linearly along the laminate thickness.

The effects of transverse shear deformation are developed more profoundly in fibrous laminates than in isotropic plates, making the choice of CLPT not preferred for laminated plates except for thin ones [10], [12], [13].

2.3.2 The FSDT-Order Shear Deformation Theory (FSDT)

FSDT for laminated plates has the same assumptions as in CLPT, except for the first assumption (i.e., after deformation, a transverse normal does not remain perpendicular) [1], [10], [12], [13]. This relaxation introduces transverse shear strains into FSDT and, therefore, τ_{xz} , τ_{yz} , γ_{xz} , and γ_{yz} are not neglected. The FSDT displacement field is as follows:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \phi_x(x, y) \\ v(x, y, z) &= v_0(x, y) - z \phi_y(x, y) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (2)$$

where $\phi_x = \frac{\partial u}{\partial z}$, and $\phi_y = \frac{\partial v}{\partial z}$, (i.e., ϕ_x and ϕ_y are the transverse normal rotations about the y and x axes, respectively). So, mid-surface deformations and transverse normal rotations are the unknowns that need to be found in this method. As a plate gets thinner, the rotations ϕ_y and ϕ_x reach the transverse deflection slopes (i.e., $\phi_x = -\frac{\partial w_0(x, y)}{\partial x}$ and $\phi_y = -\frac{\partial w_0(x, y)}{\partial y}$), and therefore getting CLPT displacement field.

In FSDT, the planar strains will also vary linearly along the laminate thickness; however, transverse shear strains (i.e., γ_{xz} and γ_{yz}) are constant [1], [10]. Transverse shear stresses will be constant as well, following the strains. In laminated plates, the actual state of transverse shear stresses is quadratic at least along the thickness. This discrepancy is accounted for by the introduction of a shear correction factor in the calculation of transverse

shear force resultants. This factor is used to satisfy the plate's top and bottom boundaries. The accuracy of FSDT results depends heavily on the choice of shear correction factor. The shear factor depends on laminate characteristics, i.e., lamination sequence and lamina properties, and this problem-dependency of the shear correction factor is the main drawback in this theory.

2.3.3 The Third-Order Shear Deformation Theory (TSDT)

TSDT for laminated plates has the same assumptions as CLPT and FSDT, except for the relaxation of the transverse normal straightness and normality. Many TSDT theories were developed with slight variation, here Reddy's TSDT will be presented [1].

$$\begin{aligned} u &= u_0 + z \phi_x + z^2 \theta_x + z^3 \lambda_x \\ v &= v_0 + z \phi_y + z^2 \theta_y + z^3 \lambda_y \\ w &= w_0 \end{aligned} \quad (3)$$

These nine unknowns are dependent and could be reduced by imposing certain boundary conditions, such as traction-free conditions at the upper and lower faces. Reducing the displacement field unknowns into five, as shown below:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z \phi_x(x, y) - \frac{4 z^3}{3 h^2} \left(\phi_x(x, y) + \frac{\partial w_0(x, y)}{\partial x} \right) \\ v(x, y, z) &= v_0(x, y) + z \phi_y(x, y) - \frac{4 z^3}{3 h^2} \left(\phi_y(x, y) + \frac{\partial w_0(x, y)}{\partial y} \right) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (4)$$

The displacement field includes functions of the third order in thickness coordinate, and therefore, transverse shear strains and stresses would have quadratic variation, eliminating the need for the shear correction factor used in FSDT.

Figure 15 shows a plate's edge transverse normal deformation kinematics for the discussed ESL theories.

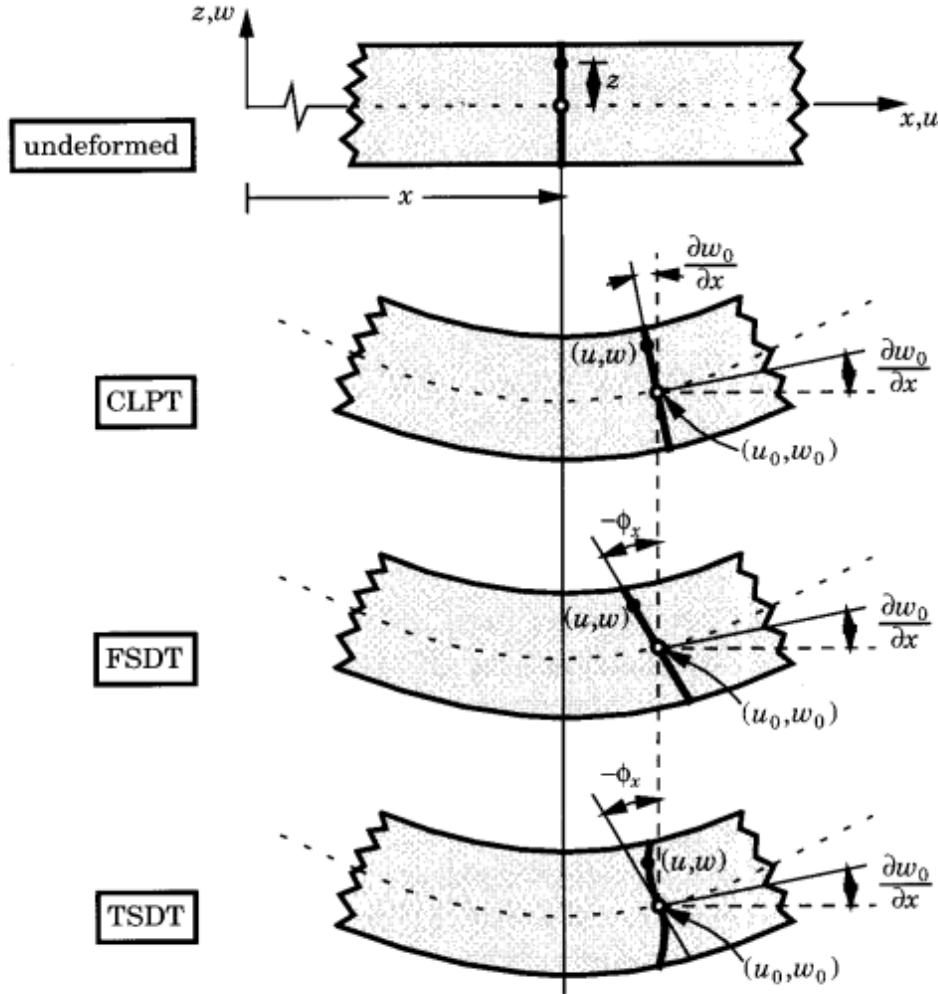


Figure 15: Transverse normal deformation according to CLPT, FSDT, and TSDT [1].

CLPT and FSDT are the simplest ESL theories; however, they could reasonably predict the kinematic behavior of most thin laminates [1]. More complex and higher-order theories are used to describe kinematics in a better way and more accurately represent interlaminar stresses, and generally, shear correction factors are not needed. These higher-order theories

contain stress resultants that require greater computational effort, and their physical interpretation is hard for just a marginal improvement in accuracy, therefore, in practice only used in necessary applications.

2.4 Theoretical Formulation

The discussion of this section is on the assumption that the lamina is a continuum, and it follows Hook's law, behaving like a linear elastic material. Although a lamina is heterogeneous in nature, it could be assumed homogeneous from a macroscopic understanding, with its properties being the weighted average of its component's materials [1].

The following subsections present the general steps through which the GDEs could be derived. Starting with the kinematics, followed by the constitutive equations, and finally the virtual work principle.

2.4.1 Kinematics

The conventional FSDT, as shown in equation (2), has five unknown variables and requires a shear correction factor. In the proposed improved FSDT (I-FSDT), the transverse displacement w is split into a bending part w_b and a shear part w_s reducing the number of unknowns to four u_0, v_0, w_b , and w_s , as expressed below.

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w_b(x, y)}{\partial x} \\ v(x, y, z) &= v_0(x, y) - z \frac{\partial w_b(x, y)}{\partial y} \\ w(x, y, z) &= w_b(x, y) + w_s(x, y) \end{aligned} \tag{5}$$

Where $u, v, \text{ and } w$ are the displacement of a point in $x, y, \text{ and } z$ directions, respectively.

And $u_0, \text{ and } v_0$ are the middle plane deformations along $x, \text{ and } y$, respectively. Also, the proposed theory eliminates the need for a shear correction factor.

2.4.2 Constitutive Equations

a) Strain-Displacement Relations

The strain tensor components for small deformations are found through equation (6).

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (i, j = 1, 2, 3) \quad (6)$$

Where $x_1, x_2, \text{ and } x_3$ are $x, y, \text{ and } z$, respectively. Also $u_1, u_2, \text{ and } u_3$ are $u, v, \text{ and } w$, respectively. After the partial derivation of the displacement field is taken, the strain-displacement relations are expressed in equation (7).

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0(x, y)}{\partial x} - z \frac{\partial^2 w_b(x, y)}{\partial x^2} \\ \frac{\partial v_0(x, y)}{\partial y} - z \frac{\partial^2 w_b(x, y)}{\partial y^2} \\ \frac{\partial u_0(x, y)}{\partial y} + \frac{\partial v_0(x, y)}{\partial x} - 2z \frac{\partial^2 w_b(x, y)}{\partial x \partial y} \\ \frac{\partial w_s(x, y)}{\partial y} \\ \frac{\partial w_s(x, y)}{\partial x} \end{Bmatrix} \quad (7)$$

However, a shear distribution function $g(z)$ is introduced to improve FSDT. So, the shear strains become:

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix}, \quad \text{where } g(z) = 1.35 - 5.4 \frac{z^2}{h^2} \quad (8)$$

b) Stress-Strain Relations

Although, the transverse shear stresses τ_{yz} and τ_{xz} are small compared to σ_x , σ_y , and τ_{xy} but are considered in theories of shear deformation. However, normal transverse stress σ_z , is usually neglected [1]. Since laminates are mostly thin in thickness, therefore, they undergo a plane state of stress. Figure 16 shows the local coordinate system of a lamina. Notice that the local direction x is in the fibers' direction, y is in the lamina plane but perpendicular to the fibers, and z is perpendicular to the lamina plane.

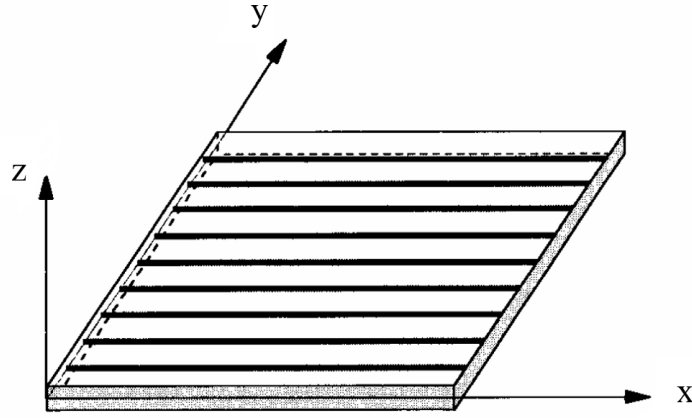


Figure 16: Modified version of lamina coordinate system from [1].

For a single lamina, the local stress-strain relationships including the temperature effect are stated as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x - \alpha_x T \\ \epsilon_y - \alpha_y T \\ \gamma_{xy} - \alpha_{xy} T \end{Bmatrix} \quad (9a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (9b)$$

Q_{ij} are the reduced stiffnesses and are linked to the lamina engineering constants as follows:

$$Q_{44} = \frac{E_1}{1 - v_{12}v_{21}}, \quad Q_{12} = \frac{v_{12} E_2}{1 - v_{12}v_{21}}, \quad Q_{22} = \frac{E_2}{1 - v_{12}v_{21}} \quad (10a)$$

$$Q_{44} = G_{23}, \quad Q_{55} = G_{13}, \quad Q_{66} = G_{12} \quad (10b)$$

where E_i, G_{ij} , and v_{ij} , are Young's modulus, shear modulus, and Poisson's ratio, respectively, and alphas " α "s are the coefficients of thermal expansion.

The applied thermal load T , along the plate, is defined as:

$$T(x, y, z) = T_1(x, y) + \frac{z}{h} T_2(x, y) + \frac{f(z)}{h} T_3(x, y) \quad (11a)$$

where

$$f(z) = \frac{h}{\pi} \sin \left(\frac{\pi z}{h} \right) \quad (11b)$$

As discussed before, a laminate will be stacked with laminae of different orientations depending on the application. Therefore, it is important to derive the stress-strain relations transformed from the local to the global coordinate system. The transformed constitutive equations are as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \epsilon_x - \alpha_x T \\ \epsilon_y - \alpha_y T \\ \gamma_{xy} - \alpha_{xy} T \end{Bmatrix}_k \quad (12a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}_k \quad (12b)$$

$\bar{Q}_{ij}$ are the transformed reduced stiffnesses shown in equation (13), as stated by Reddy [1].

$$\bar{Q}_{11} = Q_{11} \cos^4(\theta) + Q_{22} \sin^4(\theta) + 2(Q_{12} + 2Q_{66}) \cos^2(\theta) \sin^2(\theta)$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) \cos^2(\theta) \sin^2(\theta) + Q_{12}(\cos^4(\theta) + \sin^4(\theta))$$

$$\bar{Q}_{22} = Q_{11} \sin^4(\theta) + Q_{22} \cos^4(\theta) + 2(Q_{12} + 2Q_{66}) \cos^2(\theta) \sin^2(\theta) \quad (13a)$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) \cos^3(\theta) \sin(\theta) - (Q_{22} - Q_{12} - 2Q_{66}) \sin^3(\theta) \cos(\theta)$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3(\theta) \cos(\theta) - (Q_{22} - Q_{12} - 2Q_{66}) \cos^3(\theta) \sin(\theta)$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \cos^2(\theta) \sin^2(\theta) + Q_{66}(\cos^4(\theta) + \sin^4(\theta))$$

$$\bar{Q}_{44} = Q_{44} \cos^2(\theta) + Q_{55} \sin^2(\theta)$$

$$\bar{Q}_{45} = (Q_{55} - Q_{44}) \sin(\theta) \cos(\theta) \quad (13b)$$

$$\bar{Q}_{55} = Q_{55} \cos^2(\theta) + Q_{44} \sin^2(\theta)$$

$\alpha_x^{(k)}$, $\alpha_y^{(k)}$, and $\alpha_{xy}^{(k)}$ are the transformed coefficients of thermal expansion, defined by

Reddy [1] as equation (14) shows.

$$\begin{aligned}
\alpha_x^{(k)} &= \alpha_x \cos^2(\theta) + \alpha_y \sin^2(\theta) \\
\alpha_{xy}^{(k)} &= (\alpha_x - \alpha_y) \sin(\theta) \cos(\theta) \\
\alpha_y^{(k)} &= \alpha_y \cos^2(\theta) + \alpha_x \sin^2(\theta)
\end{aligned} \tag{14}$$

Where θ is the angle of transformation, measured positive counterclockwise going from global to local axes.

2.4.3 Virtual Work Principle

The virtual work principle, expressed in equation (15), is used to derive the GDEs.

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \int_0^a (\sigma_x \delta\epsilon_x + \sigma_y \delta\epsilon_y + \tau_{xy} \delta\gamma_{xy} + \tau_{yz} \delta\gamma_{yz} + \tau_{xz} \delta\gamma_{xz}) dx dy dz = 0 \tag{15}$$

Where total virtual work is found through volume integration of stresses and virtual displacements product. GDEs are then found through integration by parts and collection of unknown coefficients.

2.5 Porosity Effects and Considerations

In laminated composite members, porosities and micro-voids are usually present due to manufacturing defects or intentionally incorporated to either help reduce the member's mass or to increase sound and thermal insulation [14]. However, after reaching a certain level, these micro-voids could significantly affect the member's mechanical behavior as they reduce its effective properties [14], [15]. There are two important voids' characteristics to consider, their volume fraction and distribution. The volume fraction of porosity (ξ) is

defined as the ratio of the voids' volume over the entire member's volume. So, when $\xi = 0$ it means that the member is perfectly manufactured with no intended voids. For porosity distribution, three models will be considered for each lamina, as depicted in Figure 17.

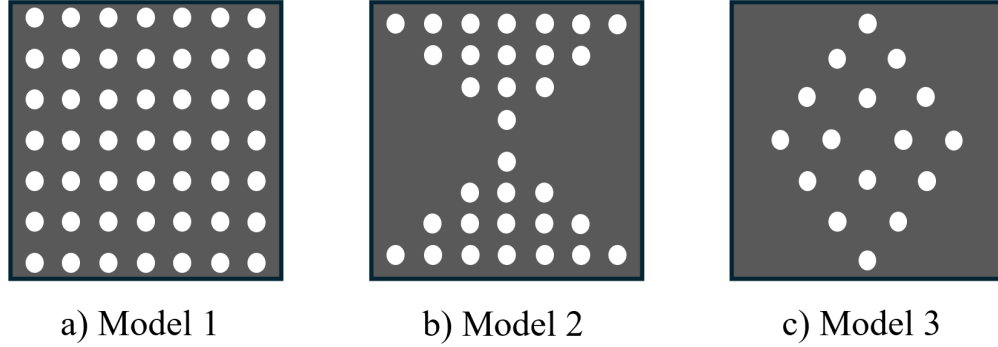


Figure 17: Visual illustration of the three porosity distribution models.

The first model has the voids uniformly distributed through the thickness of the lamina. In the second model, the voids are condensed in the top and bottom portions of the lamina as they decrease linearly going to the center. In contrast, the voids in the third model are centered as their presence increases linearly moving either from the top or bottom of the lamina to its center. The lamina properties that will be affected by porosity are Young's moduli, shear moduli, Poisson's ratios, and the coefficients of thermal expansion. A material property (P) will then be affected by the porosity characteristics as shown in equation (5).

$$\text{Model 1: } P(\lambda) = P(1 - \xi)$$

$$\text{Model 2: } P(\lambda) = P\left(1 - \xi \frac{2|z|}{h}\right) \quad (16)$$

$$\text{Model 3: } P(\lambda) = P\left(1 - \frac{\xi}{2}\left(1 - \frac{2|z|}{h}\right)\right)$$

CHAPTER 3

Journal Article I

Mechanical and Thermal Flexural Response of Porous Cross-Ply Laminated Beams Via an Improved Timoshenko Beam Theory

3.1 Abstract

This article studies the flexural response of porous cross-ply laminated beams under sinusoidal mechanical and thermal loads. The sinusoidal thermal load is linearly varying in the transverse direction. For the first time, this work takes into consideration three porosity distribution models. Moreover, the analysis is performed via an improved Timoshenko beam theory that eliminates the need for a shear correction factor and introduces a parabolic distribution for the transverse shear strain. The beams are simply supported. The virtual work principle is utilized to derive the governing equations. The governing equations are then solved analytically via Navier's rule for the simple support condition. The present work's validity is checked by comparison with the results of Bernoulli-Euler's classical, Timoshenko's first-order, Sayyad-Ghugal's trigonometric, Reddy's higher-order, and exact elasticity beam theories. The proposed theory is found to provide the closest results to those of the elasticity theory in comparison with the other theories. Parametric studies are conducted to measure the influence of porosity parameters on the response. Porosity distribution model 1 seems to have the greatest effect on the displacement results for both the thermal and mechanical load cases, followed by porosity model 2. In the mechanical load case, the lateral and axial displacements increase as the porosity volume fraction increases. However, the displacements decrease for the thermal load case as the porosity volume fraction increases.

Keywords: Cross-ply laminated beam; flexural analysis; porosity distribution models; improved Timoshenko beam theory; transverse linearly varying thermal load; sinusoidal mechanical and thermal loads.

3.2 Introduction

Laminated composite structures are increasingly used for their enhanced properties, weight reduction, and ability to be tailored for a given application. Entrapped air bubbles could accumulate during their production, affecting the laminates' mechanical behavior under load. Therefore, it's crucial to consider the porosities effect during analysis.

Numerous theories have been used for the analysis of laminated beams. The classical laminated beam theory (CBT), an extension of the Bernoulli-Euler beam theory, ignores the effect of transverse shear deformation [16]. However, transverse shear deformation has profound effects in laminated beams, rendering the analysis via CBT unfavorable [17]. Timoshenko's beam theory (TBT), also known as first-order shear deformation theory, considers a constant transverse shear stress across a beam's height. Although TBT provides better results than CBT, it requires a shear correction factor and doesn't satisfy the beam's top and bottom boundary conditions. Subsequently, multiple higher-order shear deformation theories (HSDTs) were proposed to solve the deficiencies in CBT and TBT [1], [18]–[28].

Several research studies were conducted on laminated beam analyses under thermal and mechanical loadings. Khedeir and Reddy [20] conducted a bending analysis of cross-ply laminated beams via CBT, TBT, and HSDTs. Kapuria *et al.* [22] introduced a higher-order zigzag theory for simply supported laminated beam bending analysis under a thermal

loading field. Beam's boundary conditions were satisfied, and a shear correction factor was not used. The virtual work principle was utilized to derive governing equations. Navier's method was used to obtain the analytical solution. Karama *et al.* [23] proposed a novel multilayered model with transverse shear stress continuity for both static and dynamic response of laminated beams. The principle of virtual work was used to derive governing equations and boundary conditions. The results were compared with finite element software. Vidal and Polit [29] performed a mechanical bending analysis of simply supported laminated beams under sinusoidal mechanical load via a refined finite element method with three nodes. No shear correction factor was used, as the beam's top and bottom boundary conditions were satisfied. In another study, Vidal and Polit [30] used a layer-refined sine-distribution-based finite element approach that includes a transverse normal strain effect for laminated beams' mechanical and thermo-mechanical static response. Introducing a cosine function in the transverse shear strain calculation eliminated the need for a shear correction factor. A study by Sayyad and Ghugal [17] examined the influence of including a transverse normal deformation on the flexural analysis of simply supported laminated beams subjected to sinusoidal and uniformly distributed mechanical loads. The analysis was conducted using a trigonometric shear deformation theory (TrSDT) that doesn't require a shear correction factor. Both boundary conditions and governing equations were found via the virtual work principle. Murugesan and Rajamohan [31] studied laminated beams combined with thermo-mechanical interlaminar stresses using finite element commercial software. Different boundary conditions and lamination sequences were considered. Mantari and Canales [32] analyzed laminated beams under sinusoidal, uniformly distributed, linearly varying, and concentrated mechanical loads

using a unified high-order shear deformation theory with the stretching effect. Kulkarni and Ghugal [33] analyzed the thermo-mechanical flexural response of single, antisymmetric, and symmetric laminated beams via sinusoidal shear deformation theory. The beams are all simply supported. Ghalami-Choobar *et al.* [34] used the unified zig-zag theory to perform static analysis of laminated composite beams under thermal and mechanical. Fang *et al.* [35] studied the effects of porosity parameters on beam flexural analysis and free vibration via a quasi-3D method. The effects of normal deformation, shear deformation, and thickness stretching were considered. Srinivasan and Singh [36] developed an exact unified state space theory for the flexural analysis of cross-ply laminated beams under mechanical load with various boundary conditions.

Since entrapped air voids are common during the fabrication of advanced composites, several studies were conducted to include their effect on composite structures' analysis results. Yüksel and Akbaş [14] examined the buckling behavior of laminated composite plates having porosity. The conventional first-order shear theory was used for the analysis. Navier's method was used to provide a simple support analytical solution. The effects of porosity volume, models, and lamination sequence on the response were studied. Kumar and Kumar [37] examined the effect of evenly distributed porosity on laminated plates' flexural response via a third-order shear theory. Chen *et al.* [38] investigated the buckling and bending of porous functionally graded (FG) beams under uniformly distributed and concentrated loads using TBT. Hamilton's principle was employed to derive the governing equations. Transverse bending was obtained through the Ritz rule. Various boundary conditions were considered.

The aforementioned literature survey detects knowledge gaps in consideration of different porosity distribution models for the analysis of porous laminated composite beams' static and dynamic responses, with little consideration of the porosity effect on the flexural behavior of laminated composite beams. This article aims to examine the sinusoidal mechanical and thermal loads flexural response of porous cross-ply laminated beams via an improved TBT. The thermal load will vary linearly in thickness direction. The proposed theory discards the use of a shear correction factor. The present method's findings will be compared with CBT, conventional TBT, TrSDT, HSDT, and exact elasticity theory. The effects of porosity volume fraction variation and three porosity distribution models on the flexural response will be checked.

3.3 Theoretical Formulation

Consider a simply supported laminated beam of rectangular shape with n number of orthotropic laminae, length a , width b (where $b \ll a$), and total thickness h as shown in Figure 18. The x - y plane goes through the middle of the beam. The beam is subjected to a thermal load $T(x, z)$ and transverse mechanical load q .

The porosity is distributed through each lamina. Three distribution models are considered, as Figure 17 depicts. In model 1, the micro-voids are evenly distributed along the lamina's thickness; in model 2, these voids are concentrated in the lamina's top and bottom surfaces. In model 3, the voids are more condensed in the middle portion of the lamina. Porosities will reduce the effective properties (P) of a lamina, such as Young's moduli (E), shear moduli (G), Poisson's ratios (ν), and the coefficients of thermal expansion (α). The mathematical expressions of the porosity models are presented in Table 1.

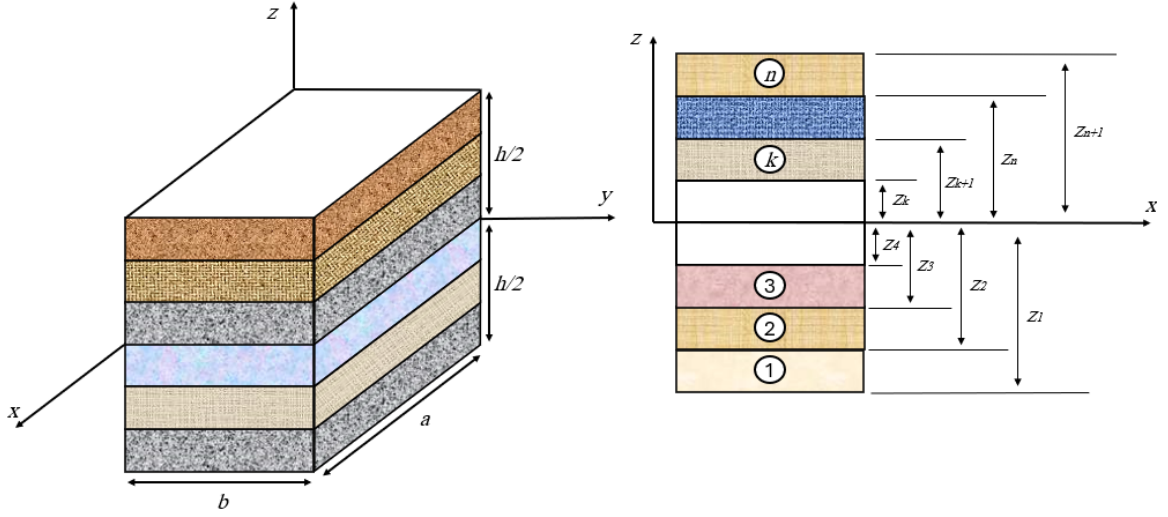


Figure 18: Geometry and coordinate system of the beam.

Table 1: Material effective properties expression under porosity influence.

Porosity distribution models	The material property (P)
Model 1	$P(\xi) = P(1 - \xi)$
Model 2	$P(\xi) = P \left(1 - \xi \frac{2 z }{h} \right)$
Model 3	$P(\xi) = P \left(1 - \frac{\xi}{2} \left(1 - \frac{2 z }{h} \right) \right)$

where ξ represents the porosity volume fraction. $\xi = 0$ means a porous-free beam.

3.3.1 Kinematics

The beam's behavior will be described using an improved Timoshenko's beam theory (I-TBT). In I-TBT, the transverse deflection w is partitioned into a bending component w_b and a shear component w_s .

$$u(x, z) = u_0(x) - z \frac{\partial w_b(x)}{\partial x} \quad (17)$$

$$w(x, z) = w_b(x) + w_s(x)$$

The unknowns are u_0 , w_b , and w_s . u and w are the displacement of a point in the x and z directions, respectively. And u_0 is the middle plane deformations along x .

The normal strain according to assumptions is:

$$\{\epsilon_x\} = \{\epsilon_x^0\} + z\{\kappa_x^b\} \quad (18a)$$

where

$$\{\epsilon_x^0\} = \left\{ \frac{\partial u_0}{\partial x} \right\}; \quad \{\kappa_x^b\} = \left\{ -\frac{\partial^2 w_b}{\partial x^2} \right\} \quad (18b)$$

The transverse shear strain is:

$$\{\gamma_{xz}^s\} = \left\{ \frac{\partial w_s}{\partial x} \right\} \quad (19a)$$

However, a shear distribution function $g(z)$ is introduced to improve TBT's transverse shear distribution along the thickness and eliminate the need for a shear correction factor.

So, the shear strain becomes:

$$\{\gamma_{xz}\} = g(z)\{\gamma_{xz}^s\} \quad (19b)$$

where

$$g(z) = 1.35 - 5.4 \frac{z^2}{h^2} \quad (19c)$$

3.3.2 Constitutive Equations

The local stress-strain relationships for a single lamina in plane stress condition, including temperature and porosity effects, are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11}(\xi) & Q_{12}(\xi) & 0 \\ Q_{12}(\xi) & Q_{22}(\xi) & 0 \\ 0 & 0 & Q_{66}(\xi) \end{bmatrix} \begin{Bmatrix} \epsilon_x - \alpha_x(\xi) T \\ \epsilon_y - \alpha_y(\xi) T \\ \gamma_{xy} - \alpha_{xy}(\xi) T \end{Bmatrix} \quad (20a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44}(\xi) & 0 \\ 0 & Q_{55}(\xi) \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (20b)$$

$Q_{ij}(\xi)$ are the reduced stiffnesses and are linked to the lamina engineering constants as equation (21) states.

$$Q_{11}(\xi) = \frac{E_1(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)}, \quad Q_{12}(\xi) = \frac{\nu_{12} E_2(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)},$$

$$Q_{22}(\xi) = \frac{E_2(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)} \quad (21a)$$

$$Q_{44}(\xi) = G_{23}(\xi), \quad Q_{55}(\xi) = G_{13}(\xi), \quad Q_{66}(\xi) = G_{12}(\xi) \quad (21b)$$

The thermal load T , linearly varying along the beam, is defined as:

$$T(x, z) = \frac{2z}{h} T_1(x) \quad (22)$$

The constitutive relation of the beam's k th lamina transformed from the local to the global (x, y, z) coordinate system is expressed below.

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11}(\xi) & 0 \\ 0 & \bar{Q}_{55}(\xi) \end{bmatrix}_k \begin{Bmatrix} \epsilon_x - \alpha_x(\xi) T \\ \gamma_{xz} \end{Bmatrix}_k \quad (23)$$

where $\bar{Q}_{ij}(\xi)$ are the transformed reduced stiffnesses, calculated as:

$$\bar{Q}_{11}(\xi) = Q_{11}(\xi) \cos^4(\theta) + Q_{22}(\xi) \sin^4(\theta) + 2(Q_{12}(\xi) + 2Q_{66}(\xi)) \cos^2(\theta) \sin^2(\theta)$$

$$\bar{Q}_{55}(\xi) = Q_{55}(\xi) \cos^2(\theta) + Q_{44}(\xi) \sin^2(\theta) \quad (24)$$

and $\alpha_x^{(k)}(\xi)$ is the transformed coefficient of thermal expansion, defined as:

$$\alpha_x^{(k)}(\xi) = \alpha_x(\xi) \cos^2(\theta) + \alpha_y(\xi) \sin^2(\theta) \quad (25)$$

where θ is the angle of transformation, positive measured counterclockwise going from global to local axes.

3.3.3 Governing Equations

By integrating (23) along the thickness, stress resultants are found as follows:

$$\begin{Bmatrix} N_x \\ M_x^b \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \kappa_x^b \end{Bmatrix} - \begin{Bmatrix} N_x^T \\ M_x^{bT} \end{Bmatrix} \quad (26a)$$

$$\{S_{xz}^s\} = [A_{55}^s] \{\gamma_{xz}^s\} \quad (26b)$$

where these stiffnesses are defined as:

$$(A_{11}, B_{11}, D_{11}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}(\xi) (1, z, z^2) dz \quad (27a)$$

$$(A_{55}^s) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \bar{Q}_{55}^{(k)}(\xi) g^2(z) dz \quad (27b)$$

The laminated beam's governing equations are derived using virtual work principle, as shown below.

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^a (\sigma_x \delta \epsilon_x + \tau_{xz} \delta \gamma_{xz}) dx dz - \int_0^a q \delta w dx = 0 \quad (28)$$

Substituting (18-23) into (28), and performing the integration along z :

$$\int_0^a \left(N_x \frac{\partial \delta u_0}{\partial x} - M_x^b \frac{\partial^2 \delta w_b}{\partial x^2} + S_{xz}^s \frac{\partial \delta w_s}{\partial x} - q \delta w \right) dx = 0 \quad (29)$$

Performing integration by parts and collecting the unknowns $\delta u_0, \delta w_b, \delta w_s$ coefficients, the governing equations become:

$$\begin{aligned}\delta u_0: \frac{\partial N_x}{\partial x} &= 0; \\ \delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + q &= 0; \\ \delta w_s: \frac{\partial S_{xz}^s}{\partial x} + q &= 0;\end{aligned}\tag{30}$$

Substituting (26) into (30), the governing equations in terms of displacements are:

$$\delta u_0: A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_b}{\partial x^3} - \frac{\partial N_x^T}{\partial x} = 0;\tag{31a}$$

$$\delta w_b: B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - \frac{\partial^2 M_x^{bT}}{\partial x^2} + q = 0;\tag{31b}$$

$$\delta w_s: A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + q = 0;\tag{31c}$$

3.4 Analytical Solution

Navier's method derives the analytical solution for a simply supported anti-symmetric cross-ply laminated beam. The boundary conditions are expressed as:

$$w_b = w_s = N_x = M_x^b = 0, \quad \text{at } x = 0, x = a\tag{32}$$

The loads are written in Fourier series expansion as:

$$\begin{Bmatrix} T_1(x) \\ q(x) \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} T_{1m} \\ q_m \end{Bmatrix} \sin(\mu x)\tag{33a}$$

where T_{1m} , and q_m are stated as:

$$\begin{Bmatrix} T_{1m} \\ q_m \end{Bmatrix} = \frac{2}{a} \int_0^a \begin{Bmatrix} T_1(x) \\ q(x) \end{Bmatrix} \sin(\mu x) dx \quad (33b)$$

and $\mu = \frac{m\pi}{a}$.

Considering a single sinusoidal thermal and mechanical loading, then $T_{1m} = T_0$ and $q_m = q_0$ when (33b) is integrated for $m = 1$. T_0 , and q_0 are the loads' maximum intensity.

The unknowns u_0 , w_b , and w_s solution form is expressed as:

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\mu x) \\ W_{bm} \sin(\mu x) \\ W_{sm} \sin(\mu x) \end{Bmatrix} \quad (34)$$

where the parameters U_m, W_{bm}, W_{sm} are arbitrary. Substituting (33-34) into (31), the following algebraic operator equation is formed:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{bmatrix} \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad (35)$$

where

$$\begin{aligned} K_{11} &= -A_{11} \mu^2 \\ K_{12} &= B_{11} \mu^3 \\ K_{13} &= 0 \\ K_{22} &= -D_{11} \mu^4 \\ K_{23} &= 0 \\ K_{33} &= -A_{55}^s \mu^2 \end{aligned} \quad (36)$$

and

$$\begin{aligned}
F_1 &= 2 P_{11} T_{1m} \mu \\
F_2 &= -2 F_{11} T_{1m} \mu^2 - q_m \\
F_2 &= -q_m
\end{aligned} \tag{37}$$

where

$$(P_{11}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \alpha_x^{(k)}(\xi) \bar{Q}_{11}^{(k)}(\xi) \left(\frac{z}{h} \right) dz \tag{38a}$$

$$(F_{11}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \alpha_x^{(k)}(\xi) \bar{Q}_{11}^{(k)}(\xi) \left(\frac{z^2}{h} \right) dz \tag{38b}$$

3.5 Numerical Results and Discussion

This section is partitioned into two parts. The first part is concerned with the validation of the I-TBT model's accuracy and studies the porous laminated beam bending response under a transverse mechanical load. The second part examines the thermal bending response of the porous laminated beam. In both parts, the influences of porosity parameters' variation are investigated. Consider a porous simply supported anti-symmetric cross-ply laminated beam of the lamination scheme $(0^\circ/90^\circ)$. The perfect porous-free lamina material properties are taken as [17], [39]:

$$\frac{E_1}{E_2} = 25, \quad G_{12} = G_{13} = 0.5 E_2, \quad G_{23} = 0.2 E_2, \quad \nu_{12} = 0.25, \quad \frac{\alpha_2}{\alpha_1} = 1125 \tag{39}$$

3.5.1 Mechanical Flexural Behavior of Porous Laminated Beams

For the $(0^\circ/90^\circ)$ laminated beam, the mechanical load dimensionless displacements and stresses are stated in (x, z) coordinate fashion as shown below.

$$\bar{u} = \frac{E_2 b}{q_0 h} u\left(0, \frac{z}{h}\right), \bar{w} = \frac{100E_2}{q_0 h s^4} w\left(\frac{a}{2}, \frac{z}{h}\right), \bar{\sigma}_x = \frac{b}{q_0} \sigma_x\left(\frac{a}{2}, \frac{z}{h}\right), \bar{\tau}_{xz} = \frac{b}{q_0} \tau_{xz}\left(0, \frac{z}{h}\right) \quad (40)$$

where $s = \frac{a}{h}$.

The numerical results of the present theory (I-TBT) for the flexural response of a two-lamina anti-symmetric cross-ply laminated beam for both perfect and porous beams under transverse sinusoidal mechanical load are demonstrated in Table 2. The perfect beam's results are compared with Bernoulli-Euler's CBT, Timoshenko's TBT, Sayyad-Ghugal's TrSDT, Reddy's HSDT, and exact elasticity theory [17].

In the case of the porous-free beam, the present I-TBT theory predicted very close results with those of exact, HSDT, and TrSDT theories in the calculation of the transverse shear stress obtained through constitutive relations $\bar{\tau}_{xz}^{CR}(0)$. For the lateral deflection $\bar{w}(0)$ prediction, I-TBT is consistent with TBT, TrSDT, HSDT, and exact theory results. The maximum transverse shear stress was calculated using the 3-D equilibrium equations $\bar{\tau}_{xz}^{EE}(max)$, all theories predict similar results. The axial stress $\bar{\sigma}_x(-\frac{h}{2})$ and axial displacement $\bar{u}(-\frac{h}{2})$ are both underestimated by CBT, TBT, and I-TBT. Whereas TrSDT and HSDT overestimate them.

For the porous beam, Table 2 reveals that $\bar{u}(-\frac{h}{2})$ and $\bar{w}(0)$ are affected the most, in decreasing order, by porosity distribution models 1, 2, and then 3. As the porosity volume fraction increases, both $\bar{u}(-\frac{h}{2})$ and $\bar{w}(0)$ increase regardless of the distribution model.

Table 2: Dimensionless displacements and stresses for two layers ($0^\circ/90^\circ$) anti-symmetric cross-ply laminated composite perfect and porous beams under transverse sinusoidal mechanical load ($T = 0$).

s	Model	Theory		$\bar{u}(-h/2)$	$\bar{w}(0)$	$\bar{\sigma}_x(-h/2)$	$\bar{\tau}_{xz}^{CR}(0)$	$\bar{\tau}_{xz}^{EE}(*)$
4	Exact	Elasticity (From Ref. [17])		1.5288	4.7080	-30.0190	2.7212	-
	CBT	Bernoulli-Euler (From Ref. [17])		1.4176	2.6188	-27.9049	-	2.9468
	TBT	Timoshenko (From Ref. [17])		1.4176	4.4281	-27.9049	1.8189	2.9468
	HSDT	Reddy (From Ref. [17])		1.7108	4.4511	-33.5921	2.4769	2.9768
	TrSDT	Sayyad-Ghugal (From Ref. [17])		1.7183	4.3955	-33.8252	2.5185	2.9885
	I-TBT	Present	Perfect	1.4176	4.4803	-27.9049	2.5263	2.9503
		Present (Model-1)	$\xi = 0.1$	1.5759	4.9795	-27.9049	2.5263	2.9503
			$\xi = 0.2$	1.7736	5.6033	-27.9049	2.5263	2.9503
			$\xi = 0.3$	2.0278	6.4052	-27.9049	2.5263	2.9503
			$\xi = 0.4$	2.3665	7.4741	-27.9049	2.5263	2.9503
			$\xi = 0.5$	2.8406	8.9704	-27.9049	2.5263	2.9503
	I-TBT	Present (Model-2)	$\xi = 0.1$	1.5326	4.7105	-27.1380	2.6078	2.9713
			$\xi = 0.2$	1.6683	4.9722	-26.2473	2.6947	2.9964
			$\xi = 0.3$	1.8310	5.2736	-25.1970	2.7876	3.0253
			$\xi = 0.4$	2.0299	5.6264	-23.9352	2.8872	3.0604
			$\xi = 0.5$	2.2786	6.0485	-22.3839	2.9941	3.1022
	I-TBT	Present (Model-3)	$\xi = 0.1$	1.4361	4.6002	-28.2676	2.4854	2.9398
			$\xi = 0.2$	1.4551	4.7281	-28.6421	2.4415	2.9286
			$\xi = 0.3$	1.4747	4.8647	-29.0291	2.3942	2.9182
			$\xi = 0.4$	1.4951	5.0110	-29.4295	2.3432	2.9071
			$\xi = 0.5$	1.5162	5.1683	-29.8444	2.2879	2.8951
10	Exact	Elasticity (From Ref. [17])		22.4760	2.9611	-176.530	7.2678	-
	CBT	Bernoulli-Euler (From Ref. [17])		22.1505	2.6188	-174.4052	-	7.3670
	TBT	Timoshenko (From Ref. [17])		22.1505	2.9083	-174.4052	4.5473	7.3670
	HSDT	Reddy (From Ref. [17])		22.9424	2.9225	-180.1890	6.2973	7.3780
	TrSDT	Sayyad-Ghugal (From Ref. [17])		22.9353	2.9113	-180.5434	6.4254	7.3845
	I-TBT	Present	Perfect	22.1505	2.9167	-174.4050	6.3157	7.3757
		Present (Model-1)	$\xi = 0.1$	24.6233	3.2421	-174.4050	6.3157	7.3757
			$\xi = 0.2$	27.7131	3.6488	-174.4050	6.3157	7.3757
			$\xi = 0.3$	31.6840	4.1714	-174.4050	6.3157	7.3757
			$\xi = 0.4$	36.9766	4.8681	-174.4050	6.3157	7.3757
			$\xi = 0.5$	44.3842	5.8432	-174.4050	6.3157	7.3757
	I-TBT	Present (Model-2)	$\xi = 0.1$	23.9466	3.0965	-169.6120	6.5194	7.4283
			$\xi = 0.2$	26.0669	3.3044	-164.0460	6.7367	7.4909
			$\xi = 0.3$	28.6094	3.5482	-157.4810	6.9690	7.5632
			$\xi = 0.4$	31.7165	3.8395	-149.5950	7.2179	7.6510
			$\xi = 0.5$	35.6028	4.1953	-139.8990	7.4852	7.7554
	I-TBT	Present (Model-3)	$\xi = 0.1$	22.4384	2.9810	-176.6720	6.2135	7.3494
			$\xi = 0.2$	22.7356	3.0491	-179.0130	6.1037	7.3214
			$\xi = 0.3$	23.0428	3.1213	-181.4320	5.9856	7.2955
			$\xi = 0.4$	23.3607	3.1982	-183.9340	5.8580	7.2678
			$\xi = 0.5$	23.6901	3.2802	-186.5280	5.7199	7.2378

* Designates the maximum $\bar{\tau}_{xz}^{EE}$ magnitude.

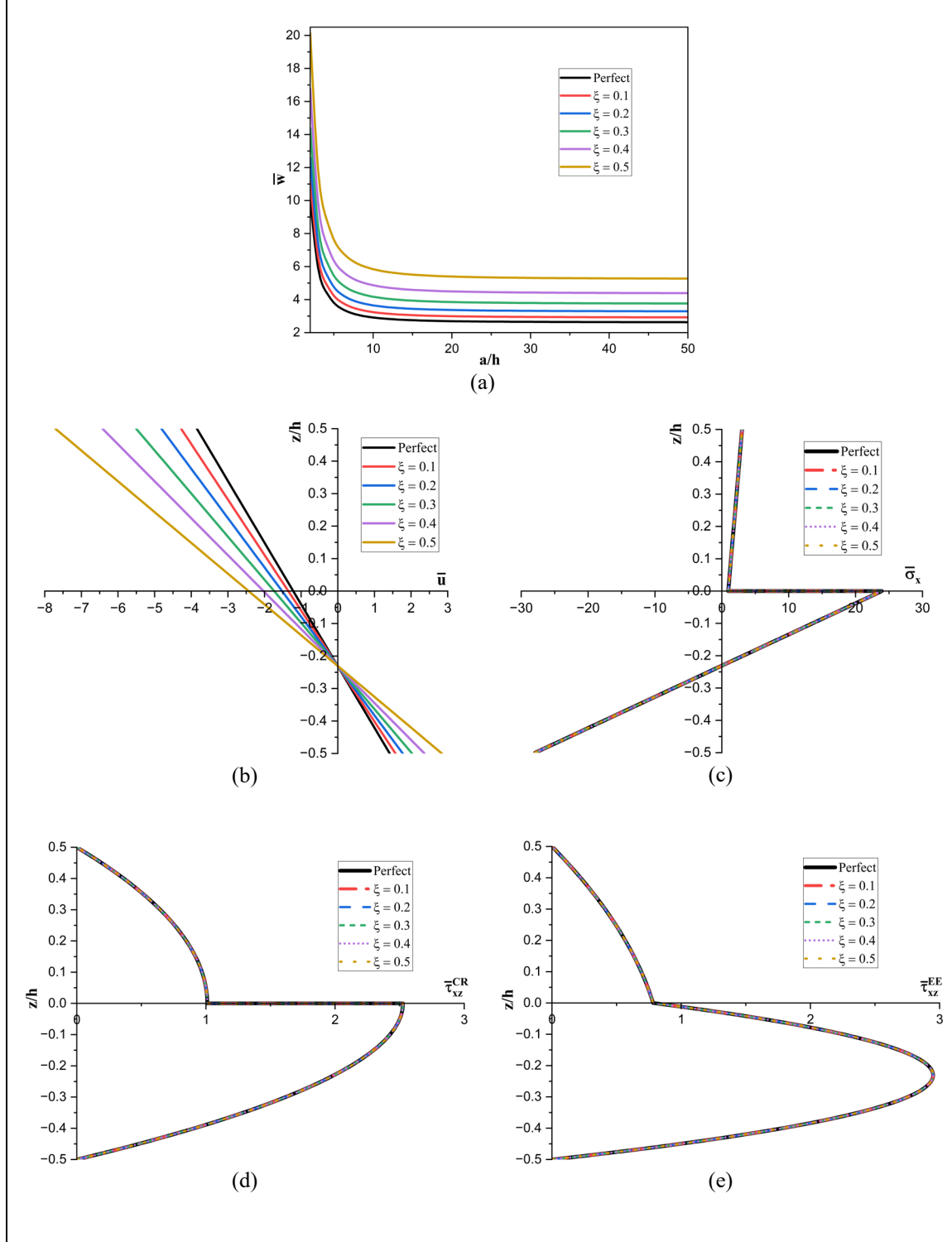


Figure 19: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under transverse sinusoidal mechanical load according to porosity distribution model 1 with $\frac{a}{h} = 4$: (a) lateral deflection $\bar{w}$ vs. a/h , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) out-of-plane shear stress $\bar{\tau}_{xz}^{CR}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

Furthermore, $\bar{\sigma}_x(-\frac{h}{2})$, $\bar{\tau}_{xz}^{CR}(0)$, and $\bar{\tau}_{xz}^{EE}(max)$ are affected the most by porosity model 2, followed by model 3, while model 1 doesn't seem to affect them. As ξ increases, the magnitude of $\bar{\sigma}_x(-\frac{h}{2})$ decreases in porosity model 2, however, it increases in model 3. On the contrary, as ξ increases both transverse shear stresses increase for porosity model 2 and decrease for model 3.

In Figure 19, I-TBT is used to draw normalized results variation considering ξ values from 0.0 to 0.5 for porosity model 1. Similarly, the normalized results for porosity models 2 and 3 are shown in Figure 20 and Figure 21, respectively.

Figure 19 (a)-(b) show that at any z location, as ξ increases the values of $\bar{w}$, and $\bar{u}$ increase too. In Figure 19 (a), as the aspect ratio enlarges for the perfect beam, the I-TBT deflection reaches the CBT results. Figure 19 (c)-(e) illustrations confirm that model 1 porosity doesn't have an effect on $\bar{\sigma}_x$, $\bar{\tau}_{xz}^{CR}$, and $\bar{\tau}_{xz}^{EE}$ under the mechanically induced response.

For porosity distribution model 2, Figure 20 (b) shows that at any z location, except from $\frac{z}{h} = (-1.5 \text{ to } -2.2)$, as porosity volume fraction increases the magnitude of $\bar{u}$ increase as well. Figure 20 (c) depicts that as the beam moves from $\xi = 0.0$ to 0.5 the maximum value for $\bar{\sigma}_x$ shifts from $z = -h/2$ to $z = 0$ in the (0°) lamina. Figure 20 (d) and (e) illustrate that as ξ increases, the transverse shear stresses' maximum value increases as well.

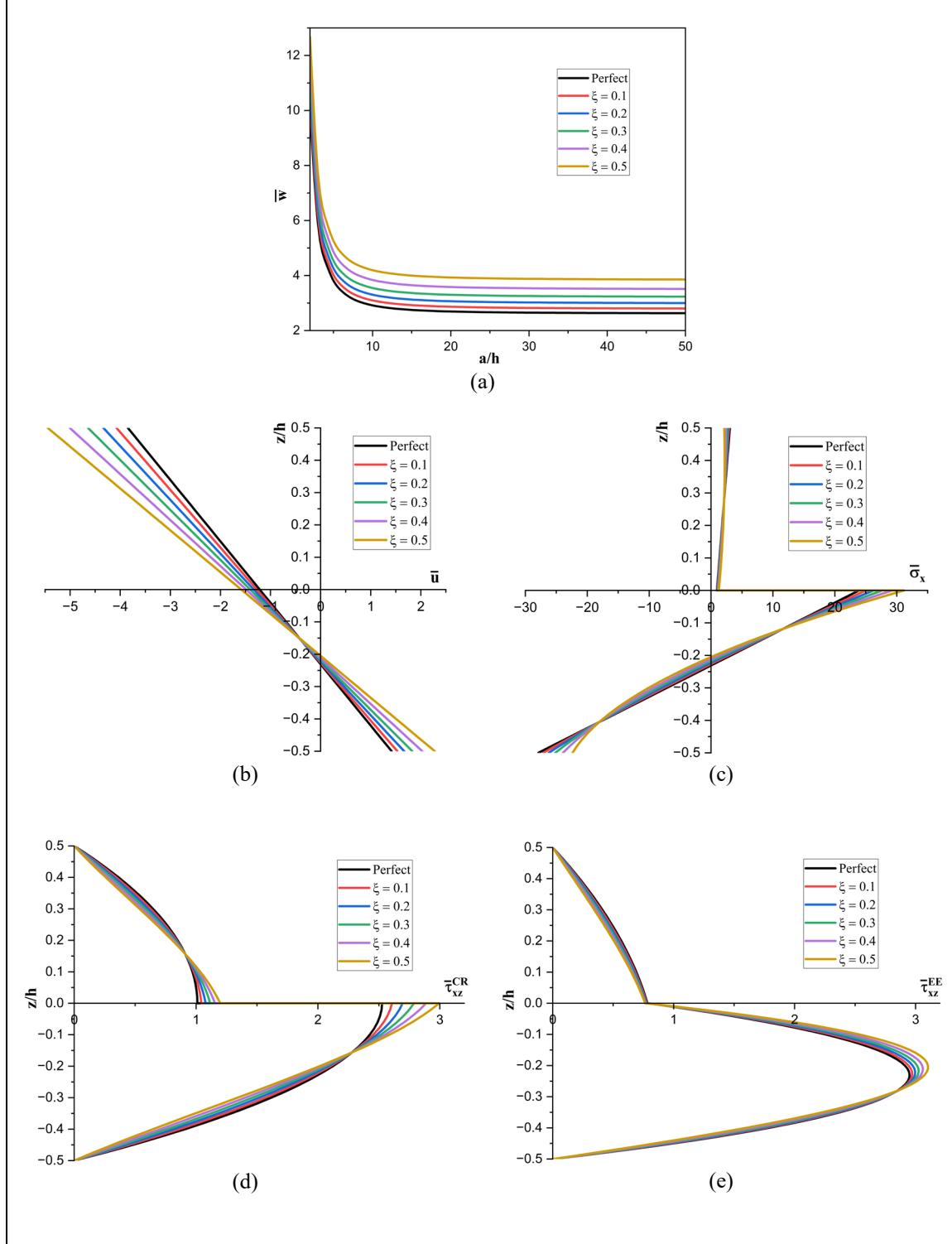


Figure 20: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under transverse sinusoidal mechanical load according to porosity distribution model 2 with $\frac{a}{h} = 4$: (a) lateral deflection $\bar{w}$ vs. a/h , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) out-of-plane shear stress $\bar{\tau}_{xz}^{CR}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

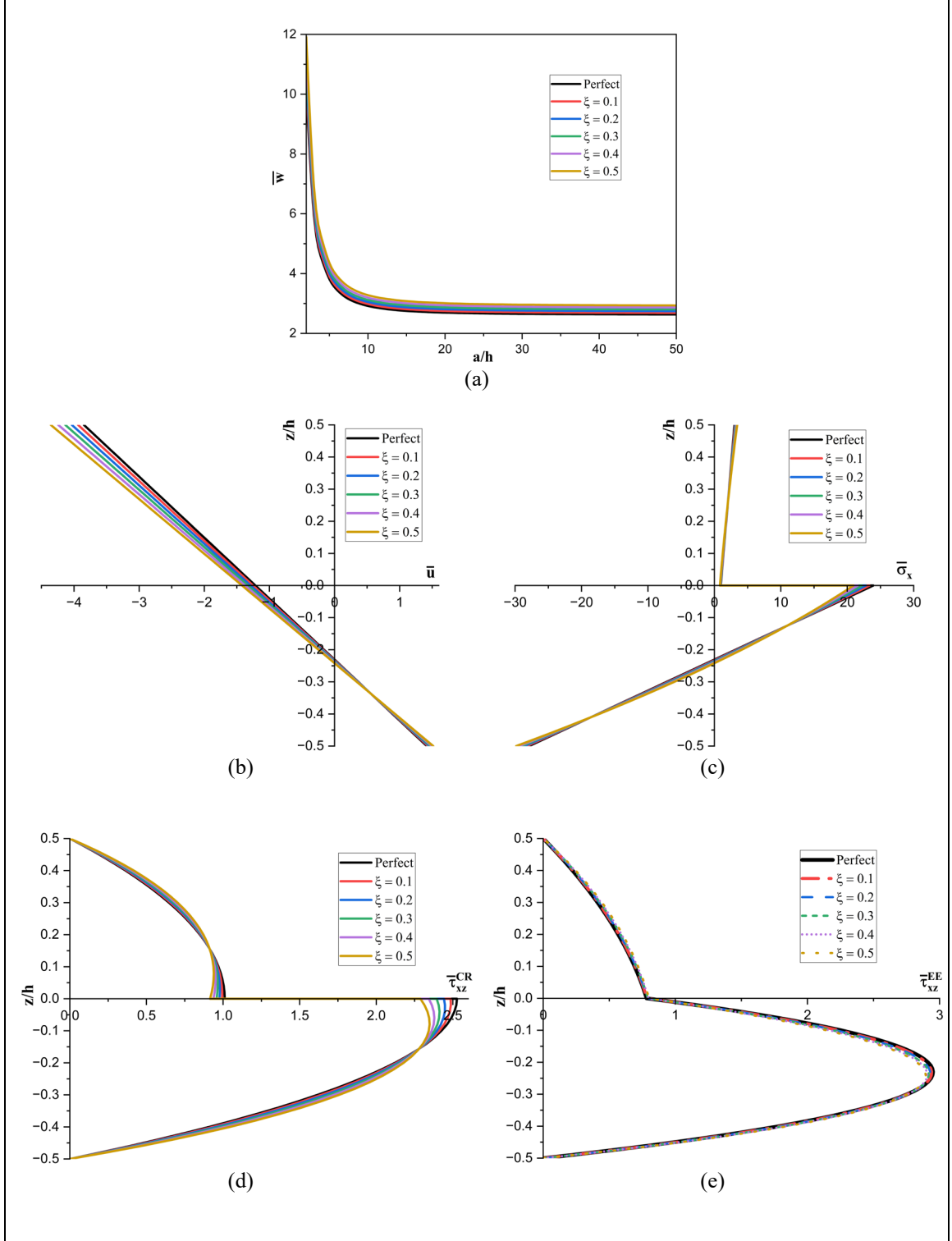


Figure 21: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under transverse sinusoidal mechanical load according to porosity distribution model 3 with $\frac{a}{h} = 4$: (a) lateral deflection $\bar{w}$ vs. a/h , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) out-of-plane shear stress $\bar{\tau}_{xz}^{CR}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

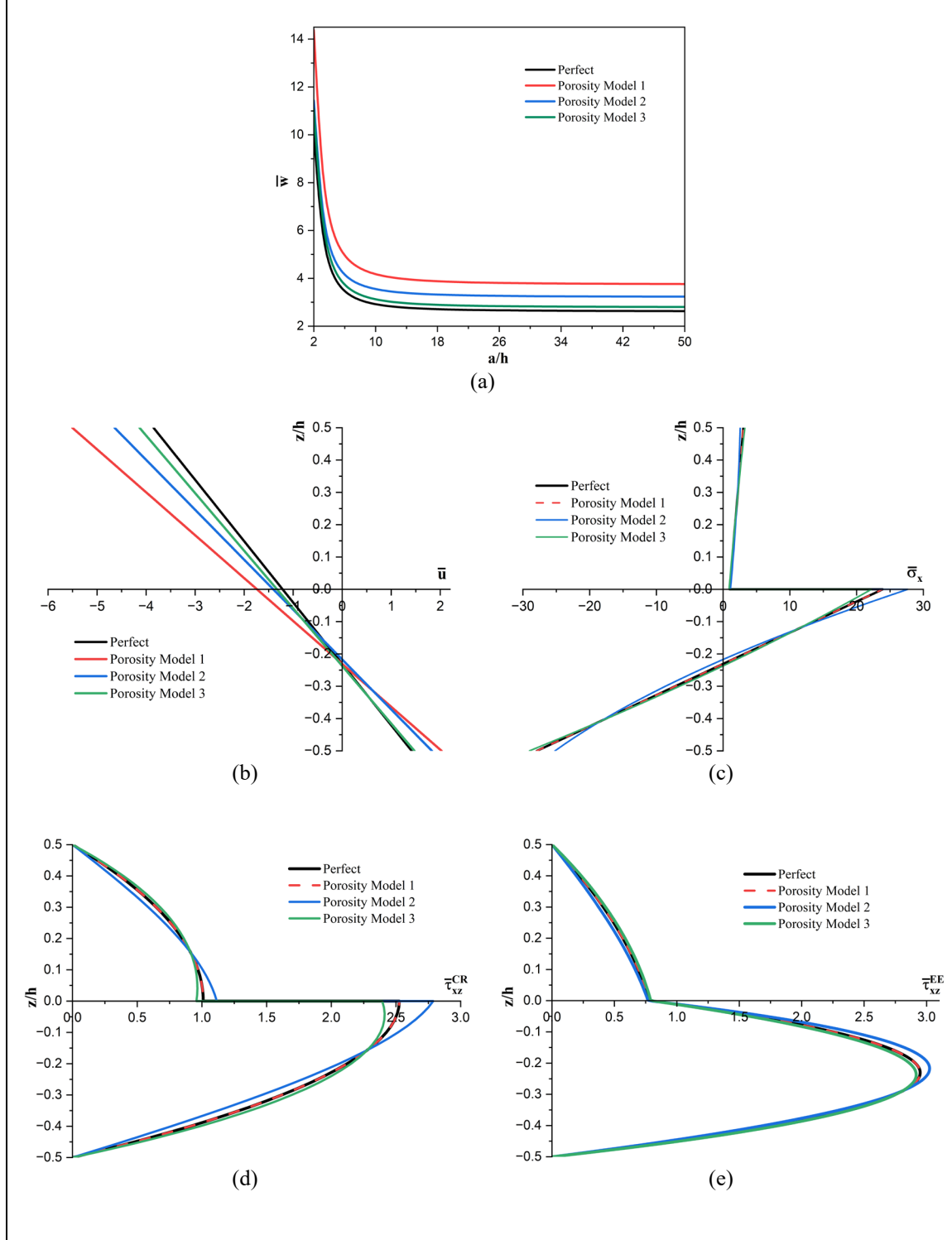


Figure 22: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under transverse sinusoidal mechanical load comparing all porosity models at $\xi = 0.3$ with $\frac{a}{h} = 4$: (a) lateral deflection $\bar{w}$ vs. a/h , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) out-of-plane shear stress $\bar{\tau}_{xz}^{CR}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

Figure 21 (a)-(e) show that porosity model 3 does affect the beam's mechanically induced flexural response, but rather minimally. Nonetheless, generally as ξ increases $\bar{w}$, and $\bar{u}$ increase also. Figure 21 (d) demonstrates that the porosity according to model 3 primarily affects the $\bar{\tau}_{xz}^{CR}$ at the laminae interface, at which as porosity volume fraction increases the transverse shear stresses decrease.

Figure 22 graphs compare the effects of porosity models at $\xi = 0.2$ on the $(0^\circ/90^\circ)$ laminated beam's dimensionless stresses and displacements. In Figure 22 (a) and (b), porosity model 1 seems to have a greater effect on the transverse and axial displacements, followed by model 2. However, when it comes to the stresses, model 1 doesn't affect them, with the highest impact due to porosity by model 2, then model 3, as Figure 22 (c)-(e) shows. For the axial stress, the porosity model 2 tends to have the maximum magnitude at $z = 0$, while for model 3 the maximum is at $z = -h/2$, as portrayed by Figure 22 (c). Figure 22 (d) shows that at $(\frac{z}{h} = 0)$ for a constant ξ , $\bar{\tau}_{xz}^{CR}$ increases by porosity model 2 and decreases by model 3.

3.5.2 Thermal Flexural Behavior of Porous Laminated Beams

For the $(0^\circ/90^\circ)$ laminated beam, the sinusoidal linearly varying thermal load dimensionless displacements and stresses are stated in (x, z) coordinates:

$$\begin{aligned}\bar{u} &= \frac{u}{\alpha_1 T_0 a} \left(0, \frac{z}{h}\right), \bar{w} = \frac{h w}{\alpha_1 T_0 a^2} \left(\frac{a}{2}, 0\right), \\ \bar{\sigma}_x &= \frac{\sigma_x}{E_2 \alpha_1 T_0} \left(\frac{a}{2}, \frac{z}{h}\right), \bar{\tau}_{xz} = \frac{\tau_{xz}}{E_2 \alpha_1 T_0} \left(0, \frac{z}{h}\right)\end{aligned}\quad (41)$$

τ_{xz} is calculated by the equilibrium equations.

Table 3: Dimensionless displacements and stresses for $(0^\circ/90^\circ)$ laminated perfect and porous beams with $\frac{a}{h} = 4$ under linearly varying along the thickness sinusoidal thermal load ($q = 0$).

Porosity Model and Volume Fraction		$\bar{u} (h/2)$	$\bar{w}$	$\bar{\sigma}_x (-h/2)$	$\bar{\tau}_{xz}^{EE} (0)$
CBT*	Perfect, $\xi = 0$	101.4844	41.2721	2188.1140	135.6737
TBT*		101.4938	41.2759	2188.1140	135.6736
I-TBT		101.494	41.276	2193.60	136.01
(I-TBT) Porosity Model 1	$\xi = 0.1$	91.345	37.148	1775.97	110.12
	$\xi = 0.2$	81.195	33.021	1402.64	86.97
	$\xi = 0.3$	71.046	28.893	1073.49	66.56
	$\xi = 0.4$	60.896	24.766	788.43	48.89
	$\xi = 0.5$	50.747	20.638	547.37	33.94
(I-TBT) Porosity Model 2	$\xi = 0.1$	92.068	37.621	1830.02	120.14
	$\xi = 0.2$	82.966	34.084	1501.42	105.31
	$\xi = 0.3$	74.231	30.681	1207.25	91.54
	$\xi = 0.4$	65.916	27.432	946.82	78.85
	$\xi = 0.5$	58.087	24.367	719.30	67.26
(I-TBT) Porosity Model 3	$\xi = 0.1$	101.244	41.080	2164.66	130.64
	$\xi = 0.2$	101.073	40.911	2136.47	125.37
	$\xi = 0.3$	100.989	40.773	2109.08	120.22
	$\xi = 0.4$	101.001	40.670	2082.54	115.18
	$\xi = 0.5$	101.120	40.604	2056.91	110.25

*From Ref. [33]

Table 3 provides I-TBT's numerical solution for the flexural behavior of a two-layer cross-ply laminated beam with $\frac{a}{h} = 4$, considering all porosity models with $\xi = (0.1 - 0.5)$, subjected to the linear sinusoidal thermal load expressed in (6) and (17) with $q = 0$. The porosity distribution model 1 has the greatest effect on the results, followed by model 2. As ξ increases in porosity model 1 and model 2, the magnitude of $\bar{u}(h/2)$, $\bar{w}$, $\bar{\sigma}_x(-h/2)$, and $\bar{\tau}_{xz}^{EE} (0)$ decrease significantly. Porosity model 3, on the other hand, has marginal

effects on $\bar{u}(h/2)$, $\bar{w}$, and $\bar{\sigma}_x(-h/2)$. But, in model 3 as the porosity volume fraction increases $\bar{w}$, $\bar{\sigma}_x(-h/2)$, and $\bar{\tau}_{xz}^{EE}(0)$ decrease.

Figure 23 demonstrates the normalized results for porosity model 1. Similarly, Figure 24 and Figure 25 depict the normalized solution for models 2 and 3, respectively.

As ξ increases for porosity model 1 the magnitudes of $\bar{u}$, $\bar{\sigma}_x$, and $\bar{\tau}_{xz}^{EE}$ considerably decrease at any given z location, as manifested in Figure 23 (a)-(c). This could be attributed to the thermal insulation capabilities of porosity air pockets. Although Figure 23 (b) displays that $\bar{\sigma}_x$ discontinuity at the laminae border decreases as the volume fraction of porosity model 1 increases, the maximum axial stress's magnitude is still at the (0°) lamina side of the border ($z = 0$). $\bar{\tau}_{xz}^{EE}$ on the other hand, has its maximum value at ($\frac{z}{h} = -0.28$) for all ξ as Figure 23 (c) reveals.

For porosity model 2 also, as ξ increases the magnitudes of $\bar{u}$, $\bar{\sigma}_x$, and $\bar{\tau}_{xz}^{EE}$ decrease, as shown in Figure 24 (a)-(c). In Figure 24 (b), the axial stress displays more curvy nonlinear behavior transversely as the porosity volume fraction increases. Moreover, the maximum value is reached at $z = 0$ in the (0°) ply side. As ξ increases, the maximum transverse shear stress's location elevates a little moving from $z = -0.28h$ to $z = -0.26h$ illustrated in Figure 24 (c).

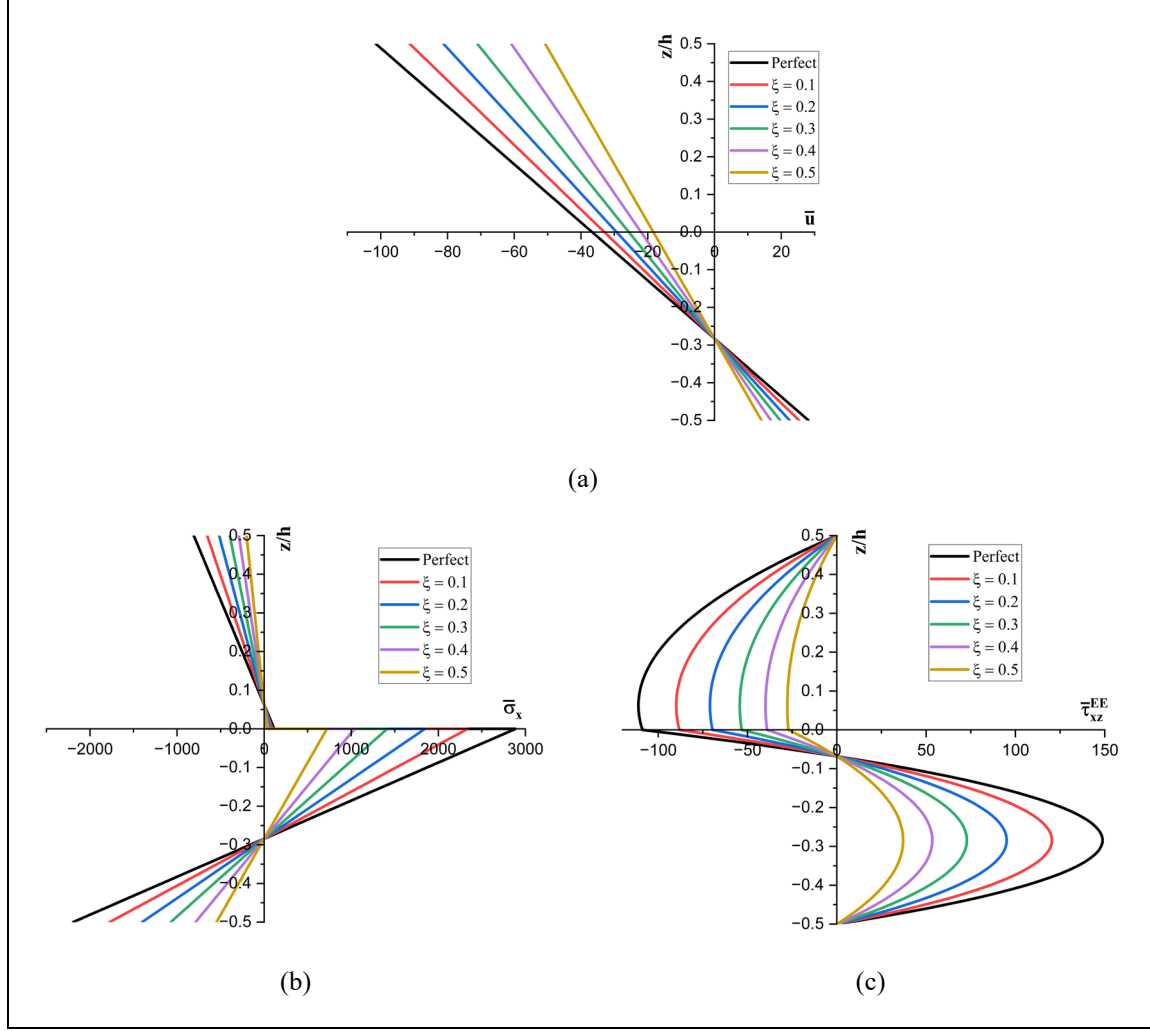


Figure 23: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under linearly varying sinusoidal thermal load according to porosity distribution model 1 with $\frac{a}{h} = 5$: (a) axial displacement $\bar{u}$ vs. z/h , (b) axial stress $\bar{\sigma}_x$ vs. z/h , (c) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

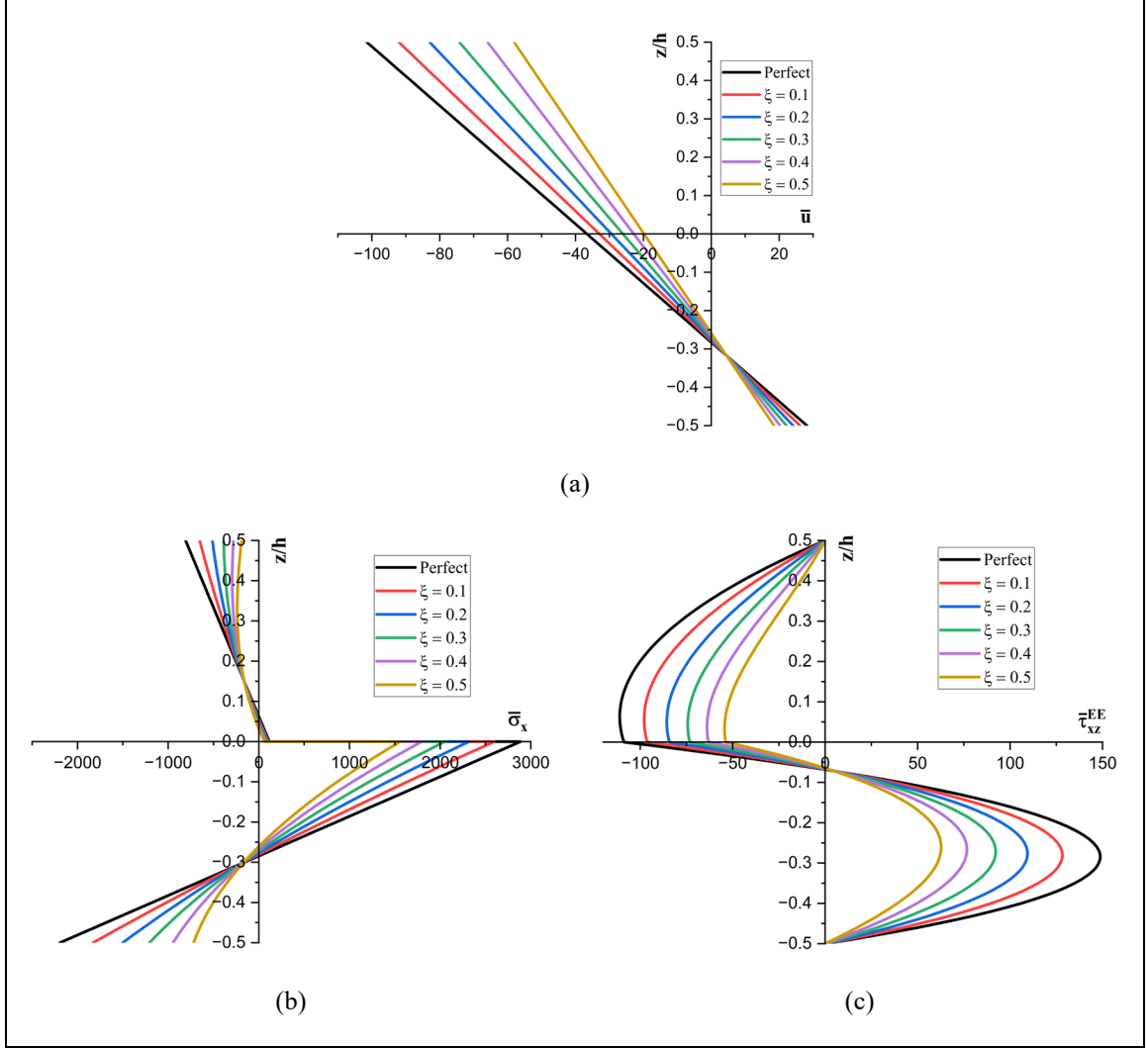


Figure 24: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under linearly varying sinusoidal thermal load according to porosity distribution model 2 with $\frac{a}{h} = 5$: (a) axial displacement $\bar{u}$ vs. z/h , (b) axial stress $\bar{\sigma}_x$ vs. z/h , (c) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

Figure 25 (a) shows that the effect porosity model 3 has on the axial displacement is marginal. As the porosity volume fraction increases, both the axial stress and the transverse shear stress decrease as shown by Figure 25 (b) and (c). While as ξ increases $\bar{\sigma}_x$ decreases more at $z = 0$ compared to $z = -h/2$, the maximum axial stress is at the interface in the (0°) side. $\bar{\tau}_{xz}^{EE}$ has the highest value at $z = -0.29h$ for all ξ values.

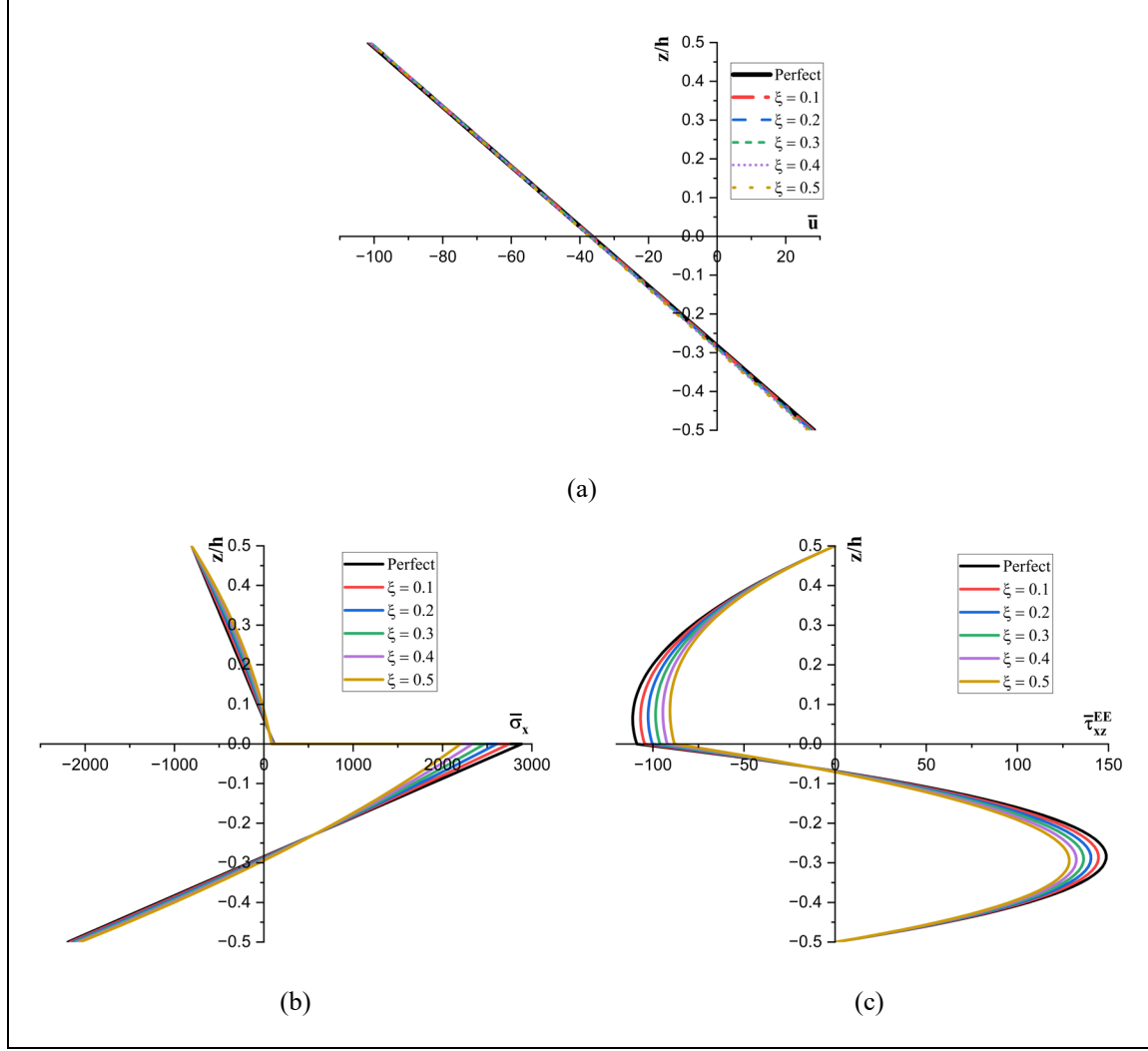


Figure 25: Variation of dimensionless stresses and displacements for the (0°/90°) porous laminated beam under linearly varying sinusoidal thermal load according to porosity distribution model 3 with $\frac{a}{h} = 5$: (a) axial displacement $\bar{u}$ vs. z/h , (b) axial stress $\bar{\sigma}_x$ vs. z/h , (c) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

Figure 26 compares the porosity models effects, at $\xi = 0.2$, on the cross-ply laminated beam's nondimensional stresses and displacements under the linear thermal load via the present I-TBT theory. The beam's deflection, as depicted in Figure 26 (a), decreases as ξ increases for all models, with model 3 causing the least effect. For a constant ξ , Figure 26 (a)-(d) indicate that the flexural response is affected the most by porosity model 1, followed by model 2. The axial displacement is insignificantly affected by porosity model 3.

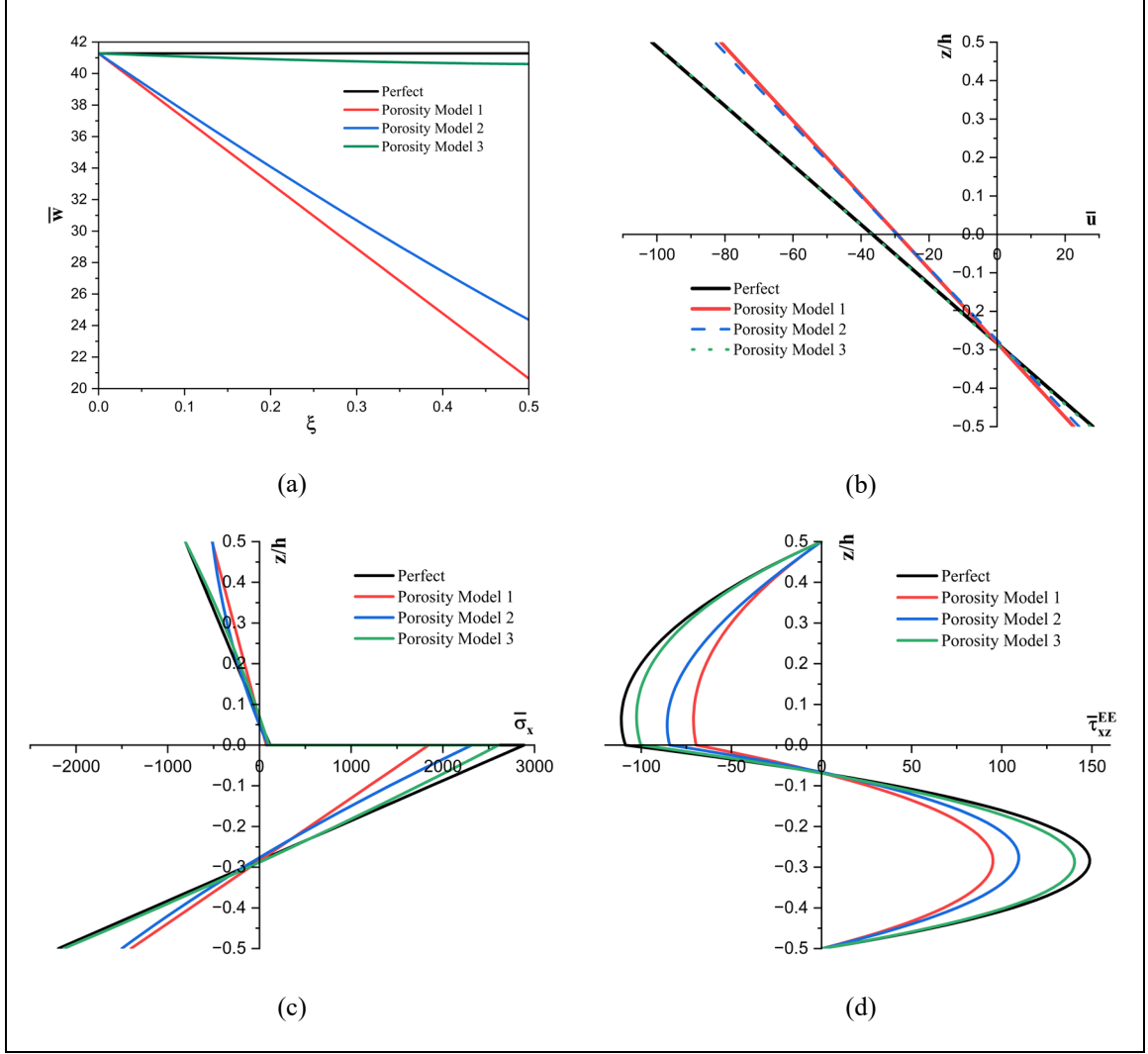


Figure 26: Variation of dimensionless stresses and displacements for the $(0^\circ/90^\circ)$ porous laminated beam under linearly varying sinusoidal thermal load comparing all porosity models with $\frac{a}{h} = 5$: (a) transverse deflection $\bar{w}$ vs. ξ , (b) axial displacement $\bar{u}$ vs. z/h at $\xi = 0.2$, (c) axial stress $\bar{\sigma}_x$ vs. z/h at $\xi = 0.2$, (d) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h at $\xi = 0.2$.

CHAPTER 4

Journal Article II

On the Thermal Bending Behavior of Porous Cross-Ply Laminated Plates Using an Improved First-Order Shear Deformation Theory

4.1 Abstract

This article investigates the stresses and displacements of porous anti-symmetric cross-ply laminated plates under nonlinear thermal load applied along the thickness of the plate. Innovatively, this study considers three porosity distribution models and utilizes an improved first-order shear deformation theory (FSDT) with four variables, as opposed to five in conventional FSDT, to conduct the thermal flexural analysis of the plate. Furthermore, the present theory has a parabolic shear distribution across the thickness, so the need for a shear correction factor is removed. The plate's top and bottom boundary conditions are satisfied. The plate is simply supported on all its edges. The governing differential equations (GDEs) and the boundary conditions were obtained using the virtual work principle. Navier's procedure was utilized to derive the analytical solutions. The present theory's validity is attested by the comparison with classical, first-order, higher-order, quasi-3D, and exact elasticity plate theories' results. The influence of certain parameters, such as porosity volume fraction and porosity distribution models, on the plate's flexural response are investigated. The results show that the thermally induced laminated plate's response is most affected by porosity model 1. While the effects of porosity model 3 are fairly negligible in comparison, except for planar normal stresses and transverse shear stresses. Moreover, the effect of porosity becomes more significant as the porosity volume fraction increases.

Keywords: Anti-symmetric cross-ply laminated plate; flexural response; porosity distribution models; improved first-order shear deformation theory; nonlinear thermal load.

4.2 Introduction

Advanced laminated composites are increasingly being used for their enhanced properties (e.g., high specific stiffness and strength, low thermal expansion coefficient, and resistance to both corrosion and temperature.), which can meet the specifications of advanced engineering applications that usually require tailored properties. Advanced laminated composites are those made with at least two materials on the macroscopic level, one material in the form of fibers serving as reinforcement that are embedded in the other base material called matrix [1], [2]. Fibers mainly carry the load, while the matrix holds the fibers, transfers the load between them, and shields them from environmental effects. The enhancements in properties were made possible due to the advancement in manufacturing technologies, such as maximizing the directional properties of fibers. During the manufacturing stage of these composites, porosities are introduced either purposefully (e.g., for sound and thermal insulation), or by error as a manufacturing defect [2], [14]. These voids under loading will affect the composite's mechanical behavior and are better accounted for during the analysis.

Many analysis theories have been used to study plates. The classical laminated plate theory (CLPT) is based on the Kirchhoff-Love plate theory, where transverse shear strains and stresses are taken to be zero [10], [40]. Laminated plates' transverse shear deformation has profound effects, making the choice of CLPT not preferred. Yet, this method could still provide satisfactory results for thin plates [12], [13]. The FSDT relaxes the assumption that after deformation a transverse normal remains perpendicular, which introduces the effect

of shear deformation [13], [41]. In FSDT, a shear correction factor is required, and the plate's top and bottom shear boundary conditions are not satisfied. This shear factor depends on laminate characteristics, and this dependency is the main drawback in FSDT. Basset [42] first stated the possibility of power series expansion of displacement field in thickness coordinate. Since then, numerous higher-order shear deformation theories (HSDTs) have been developed to model structures behavior [11], [18], [19], [21], [23], [25], [27]–[28], [40], [43]–[55]. HSDTs were proposed to tackle CLPT and FSDT limitations and better represent laminates' kinematics and interlaminar stresses. However, higher-order theories contain stress resultants that require greater computational effort, and sometimes their physical interpretation is complex [1]. So, these theories ought to only be used once it's necessary.

Multiple studies were done on laminated plate analyses under thermal loading. Wu and Tauchert [56] analyzed laminated plate stresses under thermal load using CLPT. Using finite-element formulation, Reddy and Hsu [57] investigated the bending behavior of laminated plates under thermal and mechanical loads. Khdeir and Reddy [58] found the cross-ply laminated plates' deflections and stresses under thermal loading by refining plate theories to find an exact analytical solution. Both Levy and state space methods were used to solve the GDEs. Kant and Swaminathan [59] performed the static analysis of fibrous laminated composite plates analytically based on a refined HSDT. Reddy [1], in his book, extended FSDT to find stresses due to thermal loading applied to laminated plates. The plate bending behavior of cross-ply symmetric and antisymmetric laminates under thermal and mechanical loadings was studied and solved analytically by Zenkour [60] using a unified shear deformation theory. The effects of aspect ratio, width-to-length ratio, and

stacking sequence variations were Examined. Ghugal and Kulkarni [61], [62] used a trigonometric shear deformation theory to study flexural analysis of cross-ply laminated symmetric and antisymmetric plates under nonlinear thermal loading, and nonlinear thermo-mechanical load that is uniformly distributed. Another study by Ghugal and Kulkarni [63] considered thermomechanical loading, both linear and nonlinear, on symmetrical cross-ply laminates. The flexural response was evaluated using trigonometric shear deformation theory. In their studies, the GDEs are derived via the virtual work principle. Sayyad and Ghugal [64] studied the flexural response of cross-ply laminated plates using a trigonometric shear deformation theory. Sayyad *et al.* [65] performed thermo-elastic analysis for laminated plates under linear thermal loading distributed sinusoidally via four variable plate theory. Moreover, laminated cross-ply plates stress analysis under linear variation of thermal load along the thickness was investigated by Sayyad *et al.* [66] using an exponential shear deformation theory. No shear correction factor was required in all of Sayyad's three studies mentioned above. Han *et al.* [67] included the effect of transverse normal to improve FSDT using the theorem of mixed variation. Then, they analyzed the thermo-mechanical effect on both laminated and sandwich plates. Joshan *et al.* [68] proposed a four-variable displacement field with an inverse hyperbolic function to study the hygro-thermo-mechanical behavior of laminated plates. Gandhe *et al.* [69] proposed a trigonometric shear deformation theory with three variables to study an isotropic plate's critical sinusoidal thermal flexural response. No shear correction factor was required as the transversed shear stresses were directly obtained from the constitutive relationships. Yüksel and Akbaş [70] analyzed temperature increment effects on cross-ply laminated plates free vibration using FSDT. Belbachir *et al.* [40]

delved into antisymmetric cross-ply laminate plate flexural behavior under nonlinear thermal loads analyzed by improved four-variable plate theory. Majeed [71] analyzed the buckling of cross-ply laminated plates under thermal via a new high-order displacement field. The GDEs were derived and solved analytically by Hamilton's principle and Navier's method, respectively.

Due to the inevitability of porosity presence during most advanced composites' manufacturing stages, several researchers investigated their effects on plates' behavior. Jabbari *et al.* [72] studied the voids' effects on the buckling of circular porous plates that include piezoelectric actuators and subjected to thermal loading using FSDT. Cong *et al.* [73] investigated buckling and post-buckling of porous functionally graded (FG) plates under nonlinear thermal, mechanical, and thermo-mechanical loadings via the HSDT developed by Reddy. Yüksel and Akbaş [14] studied porous fibrous laminated plates buckling behavior using FSDT. Porosity parameters' variations were investigated. Bennai *et al.* [74] Investigated the free vibration of porous FG plates using high-order hyperbolic shear deformation theory where thickness stretching was considered. Vinh *et al.* [15] used enhanced FSDT and finite-element method to perform buckling and static bending analyses of porous bi-directional FG plates. Transverse shear stresses are zero at the top and bottom boundaries with parabolic distribution, and therefore, no shear correction factor was necessary. Kumar and Kumar [37] used third-order shear deformation theory to study the flexural behavior of laminated plates with uniformly distributed porosity.

According to the literature survey above, publications on the thermal response of porous laminated plates with multiple porosity models are deficient. This research aims to analyze the nonlinear thermal bending behavior of porous cross-ply laminated plates, considering

three porosity distribution models, via an improved and efficient FSDT that does not require a shear correction factor with less computational complexity than higher-order theories. The proposed theory has four unknowns instead of five in ordinary FSDT. The virtual work principle will be used to obtain the GDEs. The analytical solutions are derived by Navier's procedure. The present theory will be validated by comparison with CLPT, ordinary FSDT, HSDT, quasi-3D, and exact theories. The impacts of porosity distribution models and porosity volume fraction on the porous laminated plate thermal flexural response are investigated.

4.3 Theoretical Formulation

Consider an all-edges simply supported cross-ply laminated plate of rectangular shape with n number of orthotropic laminae, length a , width b , and total thickness h as shown in Figure 27. The x - y plane goes through the mid-plane of the plate. The plate is subjected to a thermal load $T(x, y, z)$. For the porosity distribution along each lamina, three models are considered, as Figure 17 illustrates. In model 1, the micro-voids are evenly distributed along the lamina's thickness; in model 2, these voids are concentrated in the lamina's top and bottom surfaces. In model 3, the voids are more condensed in the middle portion of the lamina.

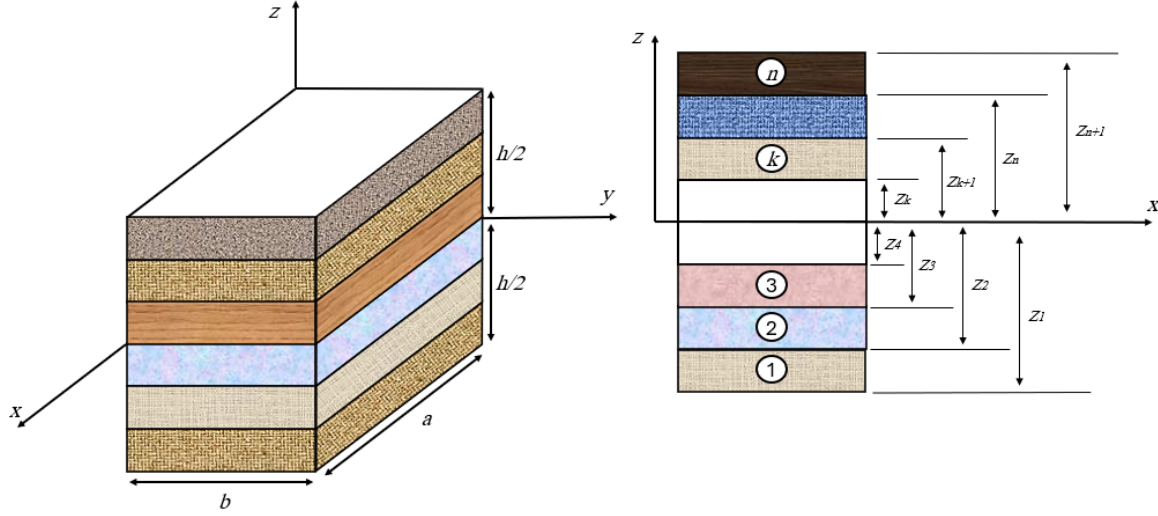


Figure 27: Geometry and coordinate system of the plate.

These porosities will affect the lamina's effective properties (P), such as Young's moduli, shear moduli, Poisson's ratios, and the coefficients of thermal expansion. The porosity distribution models are stated in Table 1.

4.3.1 Kinematics

The plate's behavior will be described using an improved FSDT, where the transverse displacement w will be split into a bending component w_b and a shear component w_s , reducing the number of unknown variables to four instead of five in simple FSDT, as expressed below.

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w_b(x, y)}{\partial x} \\ v(x, y, z) &= v_0(x, y) - z \frac{\partial w_b(x, y)}{\partial y} \\ w(x, y, z) &= w_b(x, y) + w_s(x, y) \end{aligned} \quad (42)$$

The four unknowns are u_0 , v_0 , w_b , and w_s . u , v , and w are the displacement of a point in x , y , and z directions, respectively. And u_0 , and v_0 are the middle plane deformations along x , and y , respectively.

The normal strains derived from the displacement field are as follows:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} \quad (43a)$$

where

$$\begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}; \quad \begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_b}{\partial x^2} \\ -\frac{\partial^2 w_b}{\partial y^2} \\ -2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} \quad (43b)$$

The shear strains are derived as follows:

$$\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x} \end{Bmatrix} \quad (44a)$$

However, a shear distribution function $g(z)$ is introduced to improve FSDT. So, the shear strains become:

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad (44b)$$

where

$$g(z) = 1.35 - 5.4 \frac{z^2}{h^2} \quad (44c)$$

4.3.2 Constitutive Equations

The local stress-strain relationships for a single lamina, including temperature and porosity effects, are stated as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11}(\xi) & Q_{12}(\xi) & 0 \\ Q_{12}(\xi) & Q_{22}(\xi) & 0 \\ 0 & 0 & Q_{66}(\xi) \end{bmatrix} \begin{Bmatrix} \epsilon_x - \alpha_x(\xi) T \\ \epsilon_y - \alpha_y(\xi) T \\ \gamma_{xy} - \alpha_{xy}(\xi) T \end{Bmatrix} \quad (45a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44}(\xi) & 0 \\ 0 & Q_{55}(\xi) \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (45b)$$

$Q_{ij}(\xi)$ are the reduced stiffnesses and are linked to the lamina engineering constants as in equation (46).

$$Q_{11}(\xi) = \frac{E_1(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)}, \quad Q_{12}(\xi) = \frac{\nu_{12} E_2(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)},$$

$$Q_{22}(\xi) = \frac{E_2(\xi)}{1 - \nu_{12}(\xi) \nu_{21}(\xi)} \quad (46a)$$

$$Q_{44}(\xi) = G_{23}(\xi), \quad Q_{55}(\xi) = G_{13}(\xi), \quad Q_{66}(\xi) = G_{12}(\xi) \quad (46b)$$

where E_i, G_{ij} , and ν_{ij} , are Young's modulus, shear modulus, and Poisson's ratio, respectively, and alphas " α "s are the coefficients of thermal expansion.

The applied thermal load T , along the plate, is defined as:

$$T(x, y, z) = T_1(x, y) + \frac{z}{h} T_2(x, y) + \frac{f(z)}{h} T_3(x, y) \quad (47a)$$

where

$$f(z) = \frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right) \quad (47b)$$

The plate will be stacked with laminae of different orientations. Therefore, it is important to derive the constitutive relation of the k th lamina transformed from the local to the global (x, y, z) coordinate system. The transformed constitutive equations are shown as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11}(\xi) & \bar{Q}_{12}(\xi) & \bar{Q}_{16}(\xi) \\ \bar{Q}_{12}(\xi) & \bar{Q}_{22}(\xi) & \bar{Q}_{26}(\xi) \\ \bar{Q}_{16}(\xi) & \bar{Q}_{26}(\xi) & \bar{Q}_{66}(\xi) \end{bmatrix}_k \begin{Bmatrix} \epsilon_x - \alpha_x(\xi) T \\ \epsilon_y - \alpha_y(\xi) T \\ \gamma_{xy} - \alpha_{xy}(\xi) T \end{Bmatrix}_k \quad (48a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{44}(\xi) & \bar{Q}_{45}(\xi) \\ \bar{Q}_{45}(\xi) & \bar{Q}_{55}(\xi) \end{bmatrix}_k \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}_k \quad (48b)$$

$\bar{Q}_{ij}(\xi)$ are the transformed reduced stiffnesses, calculated as:

$$\begin{aligned} \bar{Q}_{11}(\xi) &= Q_{11}(\xi)\cos^4(\theta) + Q_{22}(\xi)\sin^4(\theta) + 2(Q_{12}(\xi) + 2Q_{66}(\xi))\cos^2(\theta)\sin^2(\theta) \\ \bar{Q}_{12}(\xi) &= (Q_{11}(\xi) + Q_{22}(\xi) - 4Q_{66}(\xi))\cos^2(\theta)\sin^2(\theta) + Q_{12}(\xi)(\cos^4(\theta) + \sin^4(\theta)) \\ \bar{Q}_{22}(\xi) &= Q_{11}(\xi)\sin^4(\theta) + Q_{22}(\xi)\cos^4(\theta) + 2(Q_{12}(\xi) + 2Q_{66}(\xi))\cos^2(\theta)\sin^2(\theta) \\ \bar{Q}_{16}(\xi) &= (Q_{11}(\xi) - Q_{12}(\xi) - 2Q_{66}(\xi))\cos^3(\theta)\sin(\theta) - (Q_{22}(\xi) - Q_{12}(\xi) \\ &\quad - 2Q_{66}(\xi))\sin^3(\theta)\cos(\theta) \\ \bar{Q}_{26}(\xi) &= (Q_{11}(\xi) - Q_{12}(\xi) - 2Q_{66}(\xi))\sin^3(\theta)\cos(\theta) \\ &\quad - (Q_{22}(\xi) - Q_{12}(\xi) - 2Q_{66}(\xi))\cos^3(\theta)\sin(\theta) \\ \bar{Q}_{66}(\xi) &= (Q_{11}(\xi) + Q_{22}(\xi) - 2Q_{12}(\xi) - 2Q_{66}(\xi))\cos^2(\theta)\sin^2(\theta) \\ &\quad + Q_{66}(\xi)(\cos^4(\theta) + \sin^4(\theta)) \end{aligned} \quad (49a)$$

$$\begin{aligned}
\bar{Q}_{44}(\xi) &= Q_{44}(\xi) \cos^2(\theta) + Q_{55}(\xi) \sin^2(\theta) \\
\bar{Q}_{45}(\xi) &= (Q_{55}(\xi) - Q_{44}(\xi)) \sin(\theta) \cos(\theta) \\
\bar{Q}_{55}(\xi) &= Q_{55}(\xi) \cos^2(\theta) + Q_{44}(\xi) \sin^2(\theta)
\end{aligned} \tag{49b}$$

$\alpha_x^{(k)}(\xi)$, $\alpha_y^{(k)}(\xi)$, and $\alpha_{xy}^{(k)}(\xi)$ are the transformed coefficients of thermal expansion, defined as:

$$\begin{aligned}
\alpha_x^{(k)}(\xi) &= \alpha_x(\xi) \cos^2(\theta) + \alpha_y(\xi) \sin^2(\theta) \\
\alpha_{xy}^{(k)}(\xi) &= (\alpha_x(\xi) - \alpha_y(\xi)) \sin(\theta) \cos(\theta) \\
\alpha_y^{(k)}(\xi) &= \alpha_y(\xi) \cos^2(\theta) + \alpha_x(\xi) \sin^2(\theta)
\end{aligned} \tag{50}$$

where θ is the angle of transformation, positive measured counterclockwise going from global to local axes.

4.3.3 Governing Differential Equations

By integrating (48) along the height, moments and stress resultants are found as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} - \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^{bT} \\ M_y^{bT} \\ M_{xy}^{bT} \end{Bmatrix} \tag{51a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \tag{51b}$$

where these stiffnesses are defined as:

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)}(\xi) (1, z, z^2) dz, \quad (i, j = 1, 2, 6) \quad (52a)$$

$$(A_{ij}^s) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)}(\xi) g^2(z) dz, \quad (i, j = 4, 5) \quad (52b)$$

The GDEs are found by the virtual work principle as shown below [75]:

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^b \int_0^a (\sigma_x \delta \epsilon_x + \sigma_y \delta \epsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}) dx dy dz = 0 \quad (53)$$

Substituting (43-48) into (53), and performing the integration along z :

$$\begin{aligned} \int_0^b \int_0^a \left(N_x \frac{\partial \delta u_0}{\partial x} - M_x^b \frac{\partial^2 \delta w_b}{\partial x^2} + N_y \frac{\partial \delta v_0}{\partial y} - M_y^b \frac{\partial^2 \delta w_b}{\partial y^2} + N_{xy} \left(\frac{\partial \delta v_0}{\partial x} + \frac{\partial \delta u_0}{\partial y} \right) \right. \\ \left. - 2M_{xy}^b \frac{\partial^2 \delta w_b}{\partial x \partial y} + S_{yz}^s \frac{\partial \delta w_s}{\partial y} + S_{xz}^s \frac{\partial \delta w_s}{\partial x} \right) dx dy = 0 \end{aligned} \quad (54)$$

Then, performing integration by parts along with collecting $\delta u_0, \delta v_0, \delta w_b, \delta w_s$ coefficients, the GDEs become:

$$\begin{aligned} \delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0; \\ \delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} &= 0; \\ \delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} &= 0; \\ \delta w_s: \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} &= 0; \end{aligned} \quad (55)$$

Substituting (51) into (55), the GDEs are written in terms of displacements as follows:

$$\begin{aligned}
& A_{11} \frac{\partial^2 u_0}{\partial x^2} + 2A_{16} \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + A_{16} \frac{\partial^2 v_0}{\partial x^2} + A_{26} \frac{\partial^2 v_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} \\
& - B_{11} \frac{\partial^3 w_b}{\partial x^3} - B_{26} \frac{\partial^3 w_b}{\partial y^3} - 3B_{16} \frac{\partial^3 w_b}{\partial x^2 \partial y} - (B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \partial y^2} - \frac{\partial N_x^T}{\partial x} \\
& - \frac{\partial N_{xy}^T}{\partial y} = 0;
\end{aligned} \tag{56a}$$

$$\begin{aligned}
& A_{16} \frac{\partial^2 u_0}{\partial x^2} + A_{26} \frac{\partial^2 u_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} + A_{22} \frac{\partial^2 v_0}{\partial y^2} + 2A_{26} \frac{\partial^2 v_0}{\partial x \partial y} + A_{66} \frac{\partial^2 v_0}{\partial x^2} \\
& - B_{16} \frac{\partial^3 w_b}{\partial x^3} - B_{22} \frac{\partial^3 w_b}{\partial y^3} - 3B_{26} \frac{\partial^3 w_b}{\partial x \partial y^2} - (B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x^2 \partial y} - \frac{\partial N_y^T}{\partial y} \\
& - \frac{\partial N_{xy}^T}{\partial x} = 0;
\end{aligned} \tag{56b}$$

$$\begin{aligned}
& B_{11} \frac{\partial^3 u_0}{\partial x^3} + B_{26} \frac{\partial^3 u_0}{\partial y^3} + 3B_{16} \frac{\partial^3 u_0}{\partial x^2 \partial y} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + B_{16} \frac{\partial^3 v_0}{\partial x^3} \\
& + B_{22} \frac{\partial^3 v_0}{\partial y^3} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} + 3B_{26} \frac{\partial^3 v_0}{\partial x \partial y^2} - D_{11} \frac{\partial^4 w_b}{\partial x^4} \\
& - D_{22} \frac{\partial^4 w_b}{\partial y^4} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - 4D_{16} \frac{\partial^4 w_b}{\partial x^3 \partial y} \\
& - 4D_{26} \frac{\partial^4 w_b}{\partial x \partial y^3} - \frac{\partial^2 M_x^{bT}}{\partial x^2} - \frac{\partial^2 M_y^{bT}}{\partial y^2} - 2 \frac{\partial^2 M_{xy}^{bT}}{\partial x \partial y} = 0;
\end{aligned} \tag{56c}$$

$$A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + A_{44}^s \frac{\partial^2 w_s}{\partial y^2} + 2A_{45}^s \frac{\partial^2 w_s}{\partial x \partial y} = 0; \tag{56d}$$

4.4 Analytical Solution

For a simply supported anti-symmetric cross-ply laminated plate, the following is correct:

$$A_{16} = A_{26} = D_{16} = D_{26} = B_{12} = B_{16} = B_{26} = B_{66} = A_{45}^s = 0 \quad (57)$$

The boundary conditions are expressed as:

$$v_0 = w_b = w_s = N_x = M_x^b = 0, \quad \text{at } x = 0, a \quad (58a)$$

$$u_0 = w_b = w_s = N_y = M_y^b = 0, \quad \text{at } y = 0, b \quad (58b)$$

The thermal loads are written in Fourier series expansion as:

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} T_{1mn} \\ T_{2mn} \\ T_{3mn} \end{Bmatrix} \sin(\mu x) \sin(\lambda y) \quad (59a)$$

where T_{1mn} , T_{2mn} , and T_{3mn} are stated as:

$$\begin{Bmatrix} T_{1mn} \\ T_{2mn} \\ T_{3mn} \end{Bmatrix} = \frac{4}{a b} \int_0^b \int_0^a \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} \sin(\mu x) \sin(\lambda y) dx dy \quad (59b)$$

and $\mu = \frac{m \pi}{a}$, and $\lambda = \frac{n \pi}{b}$.

Considering a single sinusoidal thermal loading. So, $T_{1mn} = T_{2mn} = T_{3mn} = T_0$ when

(59b) is integrated for $m = n = 1$. T_0 is the thermal load maximum intensity.

The unknowns u_0 , v_0 , w_b , and w_s solution form is expressed as:

$$\begin{pmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{pmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{pmatrix} U_{mn} \cos(\mu x) \sin(\lambda y) \\ V_{mn} \sin(\mu x) \cos(\lambda y) \\ W_{bmn} \sin(\mu x) \sin(\lambda y) \\ W_{smn} \sin(\mu x) \sin(\lambda y) \end{pmatrix} \quad (60)$$

where the parameters $U_{mn}, V_{mn}, W_{bmn}, W_{smn}$ are arbitrary. Substituting (59-60) into (56), an algebraic operator equation is formed, written in a matrix form as:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} \begin{pmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} \quad (61)$$

where

$$\begin{aligned} K_{11} &= -A_{11} \mu^2 - A_{66} \lambda^2 \\ K_{12} &= -(A_{12} + A_{66}) \mu \lambda \\ K_{13} &= B_{11} \mu^3 \\ K_{14} &= 0 \\ K_{22} &= -A_{66} \mu^2 - A_{22} \lambda^2 \\ K_{23} &= B_{22} \lambda^3 \\ K_{24} &= 0 \\ K_{33} &= -D_{11} \mu^4 - D_{22} \lambda^4 - 2(D_{12} + 2D_{66}) \mu^2 \lambda^2 \\ K_{34} &= 0 \\ K_{44} &= -A_{55}^s \mu^2 - A_{44}^s \lambda^2 \end{aligned} \quad (62)$$

and

$$\begin{aligned}
F_1 &= (L_{11} + L_{21}) T_{1mn} \mu + (P_{11} + P_{21}) T_{2mn} \mu + (R_{11} + R_{21}) T_{3mn} \mu \\
F_2 &= (L_{12} + L_{22}) T_{1mn} \lambda + (P_{12} + P_{22}) T_{2mn} \lambda + (R_{12} + R_{22}) T_{3mn} \lambda \quad (63) \\
F_3 &= -(S_{11} + S_{21}) T_{1mn} \mu^2 - (F_{11} + F_{21}) T_{2mn} \mu^2 - (U_{11} + U_{21}) T_{3mn} \mu^2 \\
&\quad - (S_{12} + S_{22}) T_{1mn} \lambda^2 - (F_{12} + F_{22}) T_{2mn} \lambda^2 - (U_{12} + U_{22}) T_{3mn} \lambda^2 \\
F_4 &= 0
\end{aligned}$$

where

$$(L_{ij}, P_{ij}, R_{ij}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \alpha_i^{(k)}(\xi) \bar{Q}_{ij}^{(k)}(\xi) \left(1, \frac{z}{h}, \frac{f(z)}{h}\right) dz, \quad (i, j = 1, 2) \quad (64a)$$

$$(S_{ij}, F_{ij}, U_{ij}) = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \alpha_i^{(k)}(\xi) \bar{Q}_{ij}^{(k)}(\xi) \left(z, \frac{z^2}{h}, \frac{z f(z)}{h}\right) dz, \quad (i, j = 1, 2) \quad (64b)$$

Also, $\alpha_1^{(k)}(\xi)$ and $\alpha_2^{(k)}(\xi)$ are the same as $\alpha_x^{(k)}(\xi)$ and $\alpha_y^{(k)}(\xi)$, respectively, mentioned in (50).

4.5 Numerical Results and Discussion

This section is partitioned into two parts. The first part is concerned with the validation of the proposed model's accuracy. The second one studies the thermal response and the influence of multiple factors. Consider a porous simply supported anti-symmetric cross-ply laminated plate with the lamination sequence $(0^\circ/90^\circ)$. The perfect lamina material properties are taken as:

$$\frac{E_1}{E_2} = 25, \quad G_{12} = G_{13} = 0.5 E_2, \quad G_{23} = 0.2 E_2, \quad \nu_{12} = 0.25, \quad \frac{\alpha_2}{\alpha_1} = 3 \quad (65)$$

4.5.1 Mathematical Model Validation

For the validation of the proposed theory, a transversely applied mechanical load $q(x, y)$ is considered. It is expanded as $q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin(\mu x) \sin(\lambda y)$, where $Q_{mn} = \frac{4}{ab} \int_0^b \int_0^a q(x, y) \sin(\mu x) \sin(\lambda y) dx dy = q_0$ for sinusoidal load. The K_{ij} in (62) are the same, but the F_i in (63) are $\{0, 0, -q_0, -q_0\}$. In this case also, $T = 0$. For the $(0^\circ/90^\circ)$ laminated plate, the mechanical load dimensionless displacements and stresses are stated as follows:

$$\begin{aligned} \bar{u} &= \frac{E_2}{q_0 h s^3} u\left(0, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{w} = \frac{100E_2}{q_0 h s^4} w\left(\frac{a}{2}, \frac{b}{2}, 0\right), \quad \bar{\sigma}_x = \frac{1}{q_0 s^2} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right), \\ \bar{\sigma}_y &= \frac{1}{q_0 s^2} \sigma_y\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{\tau}_{xy} = \frac{1}{q_0 s^2} \tau_{xy}\left(0, 0, \frac{z}{h}\right), \\ \bar{\tau}_{xz} &= \frac{1}{q_0 s} \tau_{xz}\left(0, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{\tau}_{yz} = \frac{1}{q_0 s} \tau_{yz}\left(\frac{a}{2}, 0, \frac{z}{h}\right) \end{aligned} \quad (66)$$

where $s = \frac{a}{h}$.

The numerical results of the present I-FSDT theory for the flexural response of a two-layer anti-symmetric cross-ply laminated square plate, for both perfect and porous plates, under sinusoidal mechanical load are illustrated in Table 4. The perfect plate's results are also compared with the following theories' results presented by Frih *et al.* [76]: Kirchhof's CLPT, Mindlin's 1951 FSDT, Reddy's 1984 HSDT, Frih *et al.* quasi-3D theory, and elasticity exact theory.

For the perfect plate, the present theory has very similar predictions to those attained by Reddy's 1984 HSDT, quasi-3D, and elasticity exact theories for the results of the lateral displacement $\bar{w}(0)$, and the transverse shear stresses obtained through the constitutive relations, $\bar{\tau}_{xz}^{CR}(0)$ and $\bar{\tau}_{yz}^{CR}(0)$. Meanwhile, Mindlin's FSDT tends to underestimate them. For the transverse shear stresses obtained by the 3-D equilibrium equations $\bar{\tau}_{xz}^{EE}(0)$ and $\bar{\tau}_{yz}^{EE}(0)$, I-FSDT's results are matching to those found by CLPT and FSDT, and in good agreement with other theories. The I-FSDT also predicts identical values for $\bar{\tau}_{xz}$, and $\bar{\tau}_{yz}$ for a square plate. The values of $\bar{\sigma}_x(-\frac{h}{2})$ are underestimated by CLPT, FSDT, and I-FSDT, whereas Reddy's HSDT and the quasi-3D theory overestimate them. For the porous plate, the results are obtained using the present I-FSDT theory considering porosity distribution model 1, as visualized in Figure 17 with the mathematical expression stated in Table 1, for ξ values from 0.1 to 0.5. Model 1 porosity affects the values of $\bar{w}(0)$ and $\bar{u}(-\frac{h}{2})$ the most. As the porosity volume fraction increases $\bar{\sigma}_y(-\frac{h}{2})$ decreases, while $\bar{u}(-\frac{h}{2})$, $\bar{w}(0)$, $\bar{\sigma}_x(-\frac{h}{2})$, $\bar{\tau}_{xy}(-\frac{h}{2})$, $\bar{\tau}_{xz}^{EE}(0)$, and $\bar{\tau}_{yz}^{EE}(0)$ all increase for the chosen z location. The porosity in model 1 doesn't seem to affect $\bar{\tau}_{xz}^{CR}$ and $\bar{\tau}_{yz}^{CR}$. Figure 28 graphs compare the effects of the three porosity models on the nondimensional results at $\xi = 0.3$ using the I-FSDT. Figure 28 (e), (f) show that the transverse shear stresses predicted by I-FSDT have parabolic variation, contrary to conventional Mindlin's 1951 FSDT where the transverse shear stresses are constant across the thickness.

Table 4: Dimensionless displacements and stresses for two layers ($0^\circ/90^\circ$) anti-symmetric cross-ply laminated composite perfect and porous square plates under sinusoidal mechanical load.

s	Model	Theory		$\bar{u}$ $(-\frac{h}{2})$	$\bar{w}$ (0)	$-\bar{\sigma}_x$ $(-\frac{h}{2})$	$-\bar{\sigma}_y$ $(-\frac{h}{2})$	$\bar{\tau}_{xy}$ $(-\frac{h}{2})$	$\bar{\tau}_{xz}^{CR}$ (0)	$\bar{\tau}_{xz}^{EE}$ (0)	$\bar{\tau}_{yz}^{CR}$ (0)	$\bar{\tau}_{yz}^{EE}$ (0)
4	Exact	Elasticity [76].		-	2.0670	0.8410	0.1090	0.0591	0.120	-	0.135	-
	Quasi-3D	Frih <i>et al.</i> [76].		0.0111	1.9494	0.9039	0.0965	0.0562	0.127	0.112	0.127	0.112
	CLPT	Kirchhoff [76].		0.0088	1.0636	0.7157	0.0843	0.0525	-	0.122	-	0.122
	FSDT	Mindlin [76].		0.0088	1.9682	0.7157	0.0843	0.0525	0.091	0.122	0.091	0.122
	HSDT	Reddy [76].		0.0113	1.9985	0.9060	0.0891	0.0577	0.125	0.110	0.125	0.110
	I-FSDT	Present	Perfect	0.00884	1.9943	0.7157	0.0843	0.0525	0.126	0.122	0.126	0.122
			$\xi = 0.1^*$	0.00986	2.2181	0.7160	0.0837	0.0526	0.126	0.1221	0.126	0.1221
			$\xi = 0.2^*$	0.01113	2.4978	0.7163	0.0831	0.0527	0.126	0.1222	0.126	0.1222
			$\xi = 0.3^*$	0.01277	2.8573	0.7165	0.0825	0.0528	0.126	0.1222	0.126	0.1222
			$\xi = 0.4^*$	0.01495	3.3367	0.7168	0.0819	0.0529	0.126	0.1223	0.126	0.1223
			$\xi = 0.5^*$	0.01800	4.0078	0.7171	0.0812	0.0530	0.126	0.1224	0.126	0.1224
10	Exact	Elasticity [76].		-	1.2250	0.7302	0.0886	0.0535	0.121	-	0.125	-
	Quasi-3D	Frih <i>et al.</i> [76].		0.0092	1.2100	0.7463	0.0872	0.0530	0.130	0.120	0.130	0.120
	CLPT	Kirchhoff [76].		0.0088	1.0636	0.7157	0.0843	0.0525	-	0.122	-	0.122
	FSDT	Mindlin [76].		0.0088	1.2083	0.7157	0.0843	0.0525	0.091	0.122	0.091	0.122
	HSDT	Reddy [76].		0.0092	1.2161	0.7468	0.0851	0.0533	0.127	0.120	0.127	0.120
	I-FSDT	Present	Perfect	0.00884	1.2125	0.7157	0.0843	0.0525	0.126	0.122	0.126	0.122
			$\xi = 0.1^*$	0.00986	1.3494	0.7160	0.0837	0.0526	0.126	0.1221	0.126	0.1221
			$\xi = 0.2^*$	0.01113	1.5205	0.7163	0.0831	0.0527	0.126	0.1222	0.126	0.1222
			$\xi = 0.3^*$	0.01277	1.7405	0.7165	0.0825	0.0528	0.126	0.1222	0.126	0.1222
			$\xi = 0.4^*$	0.01495	2.0337	0.7168	0.0819	0.0529	0.126	0.1223	0.126	0.1223
			$\xi = 0.5^*$	0.01800	2.4442	0.7171	0.0812	0.0530	0.126	0.1224	0.126	0.1224

*Porosity Distribution Model-1

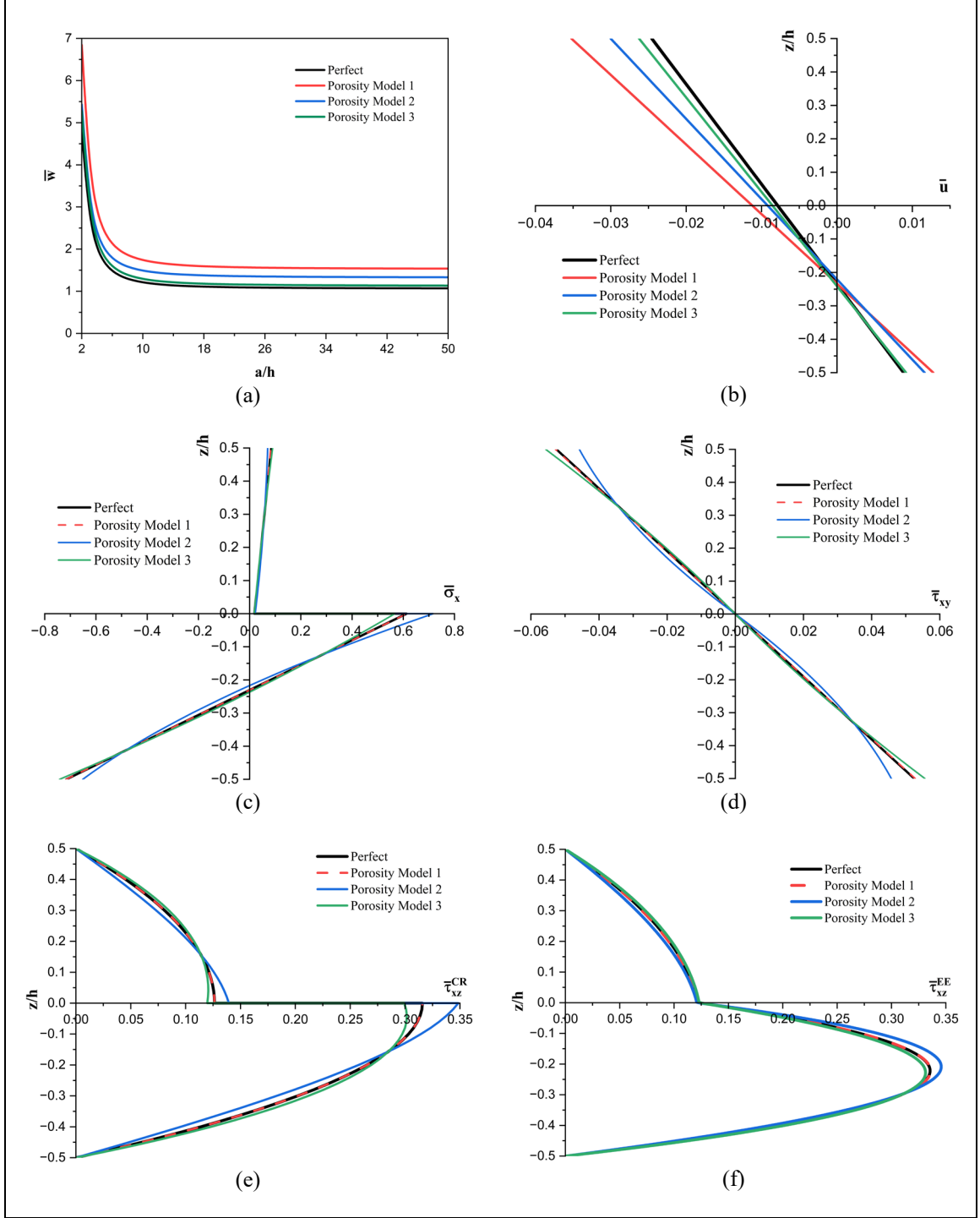


Figure 28: Variation of normalized stresses and displacements considering all porosity models at $\xi = 0.3$ using the I-FSDT for the square ($0^\circ/90^\circ$) laminated plate under transverse mechanical sinusoidal load: (a) lateral displacement $\bar{w}$ vs. a/h , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) planar shear stress $\bar{\tau}_{xy}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{CR}$ vs. z/h , (f) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

4.5.2 Thermal Bending Behavior of Porous Cross-Ply Laminated Plates via I-FSDT

For the $(0^\circ/90^\circ)$ laminated square plate, the nonlinear thermal load dimensionless displacements and stresses are stated as follows:

$$\begin{aligned}\bar{u} &= \frac{1}{\alpha_1 T_0 a} u\left(0, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{v} = \frac{1}{\alpha_1 T_0 a} v\left(\frac{a}{2}, 0, \frac{z}{h}\right), \quad \bar{w} = \frac{10h}{\alpha_1 T_0 a^2} w\left(\frac{a}{2}, \frac{b}{2}, 0\right), \\ \bar{\sigma}_x &= \frac{\sigma_x}{E_2 \alpha_1 T_0} \left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{\sigma}_y = \frac{\sigma_y}{E_2 \alpha_1 T_0} \left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right), \\ \bar{\tau}_{xy} &= \frac{\tau_{xy}}{E_2 \alpha_1 T_0} \left(0, 0, \frac{z}{h}\right), \quad \bar{\tau}_{xz} = \frac{\tau_{xz}}{E_2 \alpha_1 T_0} \left(0, \frac{b}{2}, \frac{z}{h}\right), \quad \bar{\tau}_{yz} = \frac{\tau_{yz}}{E_2 \alpha_1 T_0} \left(\frac{a}{2}, 0, \frac{z}{h}\right)\end{aligned}\tag{67}$$

The transverse shear stresses are attained only through the 3-D equilibrium equations as they satisfy continuity conditions.

Table 5 has the numerical results obtained by I-FSDT for the flexural response of a two-layer anti-symmetric cross-ply laminated square plate with $\frac{a}{h} = 5$, for both perfect and porous plates considering all mentioned porosity models at various ξ values, subjected to nonlinear thermal load as stated by (47) and (59) with $T_1 = 0$. Close inspection of Table 5 reveals that the thermally induced response is most affected by porosity model 1, due to the presence of voids throughout the lamina's thickness heavily reducing its rigidity compared to the other porosity distribution models. The values of $\bar{u}(-\frac{h}{2})$, $\bar{v}(-\frac{h}{2})$, $\bar{w}(0)$, and $\bar{\tau}_{xy}(-\frac{h}{2})$ are affected the most by porosity model 1 followed by model 2, while the effects of porosity model 3 are somewhat negligible in comparison.

Table 5: Dimensionless displacements and stresses for two layers ($0^\circ/90^\circ$) anti-symmetric cross-ply laminated composite perfect and porous square plates with $\frac{a}{h} = 5$ under nonlinear thermal load ($T_1 = 0$).

Porosity Model and Volume Fraction		$\bar{u}$ $(-\frac{h}{2})$	$\bar{v}$ $(-\frac{h}{2})$	$\bar{w}$ (0)	$\bar{\sigma}_x$ $(-\frac{h}{2})$	$\bar{\sigma}_y$ $(+\frac{h}{2})$	$\bar{\tau}_{xy}$ $(-\frac{h}{2})$	$\bar{\tau}_{xz}^{EE}$ (0)	$\bar{\tau}_{yz}^{EE}$ (0)
Perfect	$\xi = 0$	0.2926	0.3325	1.9899	-2.1765	2.1765	0.9820	-0.1010	-0.1010
Porosity Model 1	$\xi = 0.1$	0.2630	0.2990	1.7888	-1.7629	1.7629	0.7945	-0.0818	-0.0818
	$\xi = 0.2$	0.2334	0.2656	1.5882	-1.3930	1.3930	0.6270	-0.0647	-0.0647
	$\xi = 0.3$	0.2039	0.2322	1.3881	-1.0666	1.0666	0.4795	-0.0495	-0.0495
	$\xi = 0.4$	0.1745	0.1988	1.1884	-0.7837	0.7837	0.3519	-0.0182	-0.0182
	$\xi = 0.5$	0.1452	0.1655	0.9892	-0.5443	0.5443	0.2441	-0.0253	-0.0235
Porosity Model 2	$\xi = 0.1$	0.2704	0.3008	1.8180	-2.2894	2.2894	0.8074	-0.0979	-0.0979
	$\xi = 0.2$	0.2484	0.2695	1.6487	-2.3410	2.3410	0.6509	-0.0945	-0.0945
	$\xi = 0.3$	0.2268	0.2390	1.4827	-2.3306	2.3306	0.5122	-0.0908	-0.0908
	$\xi = 0.4$	0.2057	0.2093	1.3209	-2.2569	2.2569	0.3911	-0.0867	-0.0867
	$\xi = 0.5$	0.1852	0.1807	1.1646	-2.1172	2.1172	0.2873	-0.0822	-0.0822
Porosity Model 3	$\xi = 0.1$	0.2890	0.3318	1.9760	-1.8892	1.8892	0.9751	-0.0925	-0.0925
	$\xi = 0.2$	0.2854	0.3312	1.9627	-1.6059	1.6059	0.9685	-0.0843	-0.0843
	$\xi = 0.3$	0.2818	0.3307	1.9498	-1.3269	1.3269	0.9622	-0.0764	-0.0764
	$\xi = 0.4$	0.2784	0.3303	1.9376	-1.0523	1.0523	0.9561	-0.0688	-0.0688
	$\xi = 0.5$	0.2749	0.3301	1.9259	-0.7822	0.7822	0.9504	-0.0615	-0.0615

However, $\bar{\sigma}_x(-\frac{h}{2})$, $\bar{\sigma}_y(+\frac{h}{2})$, $\bar{\tau}_{xz}^{EE}(0)$ and $\bar{\tau}_{yz}^{EE}(0)$ are sensitive the most to model 1 followed by model 3, whereas model 2 has a smaller effect compared to them. For all porosity models, as the porosity volume fraction increases the dimensionless results decrease, except for porosity model 2 when calculating $\bar{\sigma}_x(-\frac{h}{2})$ and $\bar{\sigma}_y(+\frac{h}{2})$ as they increase till $\xi = 0.2$ then decrease.

In Figure 29, I-FSDT is used to draw nondimensional results variation for porosity model 1 using multiple ξ values. A similar thing is done for porosity models 2 and 3 in Figure 30 and Figure 31, respectively. Figure 29 shows that for the whole plate's height, as ξ in porosity model 1 increases the values of $\bar{w}$, $\bar{u}$, $\bar{\sigma}_x$, $\bar{\tau}_{xy}$, and $\bar{\tau}_{xz}^{EE}$ significantly decrease. In

Figure 29 (a) as the plate moves from a square to a more rectangular shape, the effect of porosity becomes even more apparent. Figure 30 (a) also confirms that for porosity model 2, the effect of ξ on $\bar{w}$ increase as b/a values move from 1 to 5. Figure 30 (d) illustrates that $\bar{\tau}_{xy}$ is linearly varying across the thickness for the perfect porous-free plate. However, as the porosity volume fraction increases, it exhibits more of a slightly curvy nonlinear variation across the thickness. In Figure 30 (c), $\bar{\sigma}_x$ has a discontinuity jump at the plies interface that decreases as the volume fraction of porosity increases. Also, $\bar{\sigma}_x$ fluctuates across the thickness, however more rapidly in the bottom (0°) lamina and reaching a maximum magnitude at $(\frac{z}{h} = -0.5)$. $\bar{\tau}_{xz}^{EE}$ have a similar pattern, however reaching maximum magnitude at $(\frac{z}{h} = -0.34)$, as shown in Figure 30 (e). Figure 31 (a) illustrates that porosity model 3 has minimal effects on $\bar{w}$, but the effect of porosity volume fraction becomes more noticeable as $\frac{b}{a}$ changes from (1 to 5). The axial displacement and the planar shear stress are marginally affected by model 3 ξ variation, as depicted by Figure 31 (b) and (d). $\bar{\tau}_{xz}^{EE}$ in porosity model 3 decreases along the thickness as ξ increases, except near the laminae interface from the (0°) lamina side. Figure 32 graphs compare the effects of porosity models at $\xi = 0.2$ on the ($0^\circ/90^\circ$) laminated square plate's normalized stresses and displacements resulting from the nonlinear thermal load obtained by I-FSDT. In Figure 32 (a), both porosity models 1 and 2 effect is present through the range of $\frac{b}{a}$ (1 to 5), while for model 3 the porosity effect becomes noticeable as the plate moves from a square to a more rectangular shape. Figure 32 (b) and (d) show that $\bar{u}$ and $\bar{\tau}_{xy}$ are affected the most by models 1 and 2. Figure 32 (c) reveals that the maximum $\bar{\sigma}_x$ magnitude is for model 2 at $(\frac{z}{h} = -0.5)$. Also, the discontinuity at the plies interface for $\bar{\sigma}_x$ is larger for model 3.

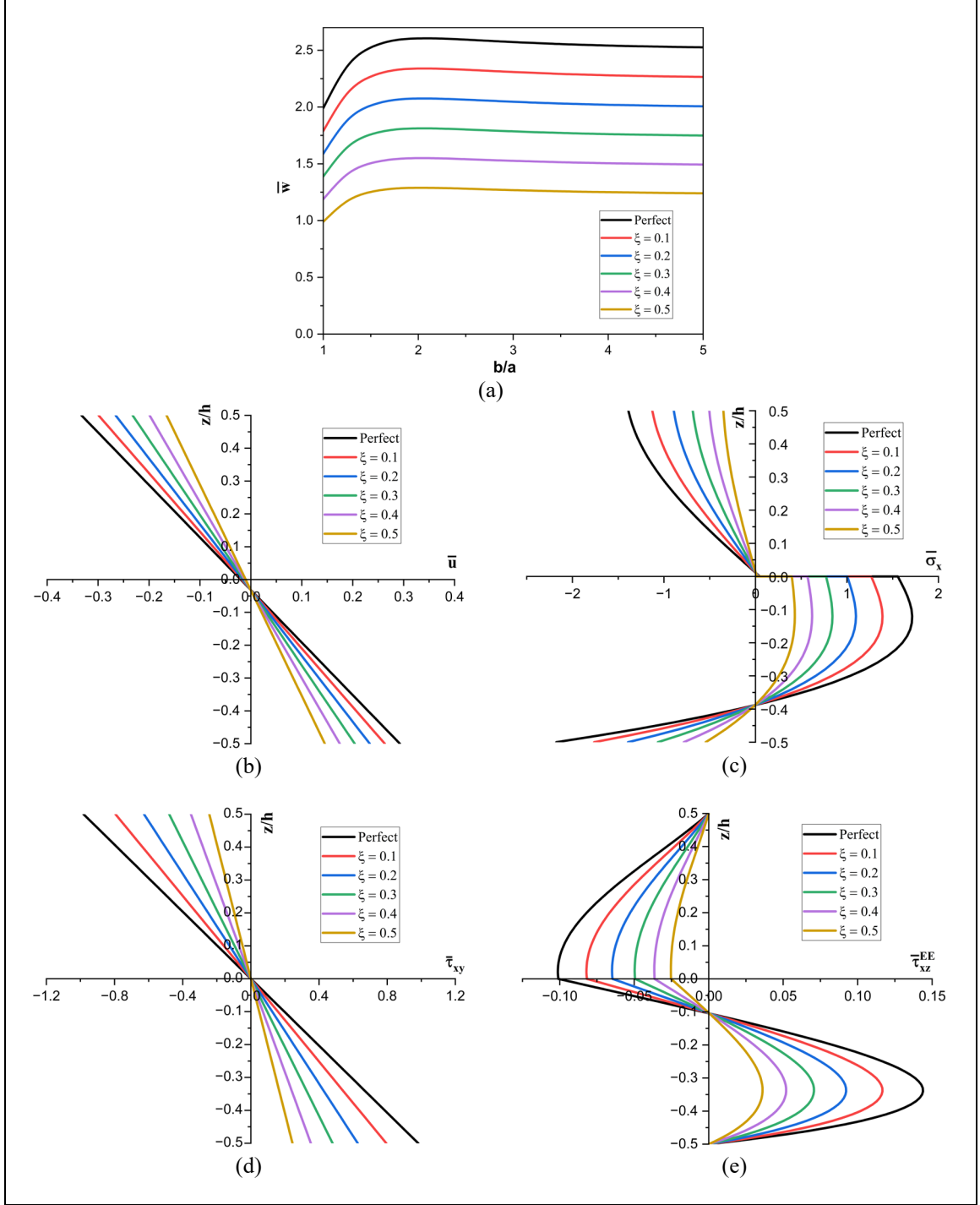


Figure 29: Variation of normalized stresses and displacements via I-FSDT for porosity model 1 using multiple ξ values for the square ($0^\circ/90^\circ$) laminated plate with $\frac{a}{h} = 5$ under nonlinear thermal load ($T_1 = 0$): (a) lateral displacement $\bar{w}$ vs. b/a , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) planar shear stress $\bar{\tau}_{xy}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

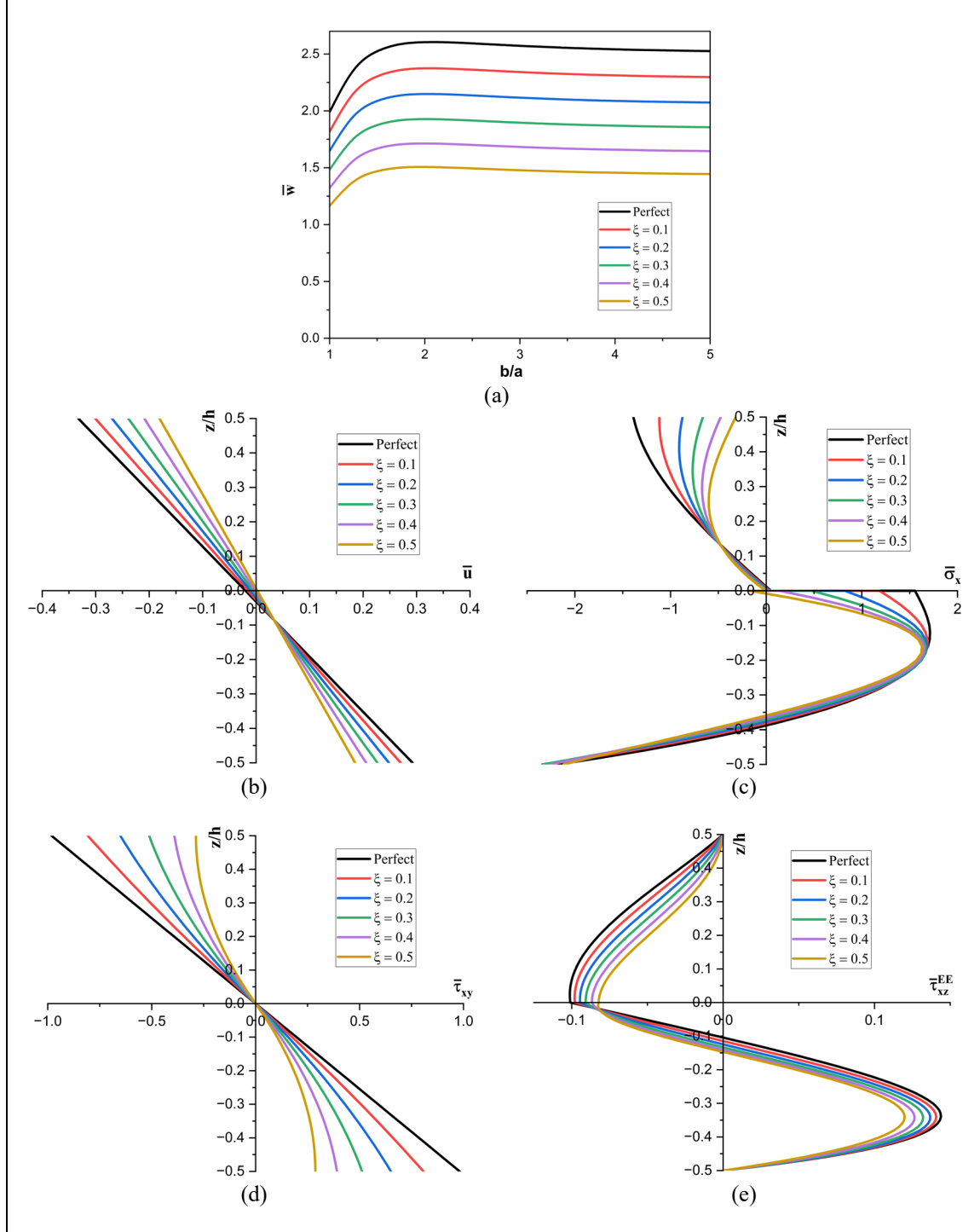


Figure 30: Variation of normalized stresses and displacements via I-FSDT for porosity model 2 using multiple ξ values for the square ($0^\circ/90^\circ$) laminated plate with $\frac{a}{h} = 5$ under nonlinear thermal load ($T_1 = 0$): (a) lateral displacement $\bar{w}$ vs. b/a , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) planar shear stress $\bar{\tau}_{xy}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

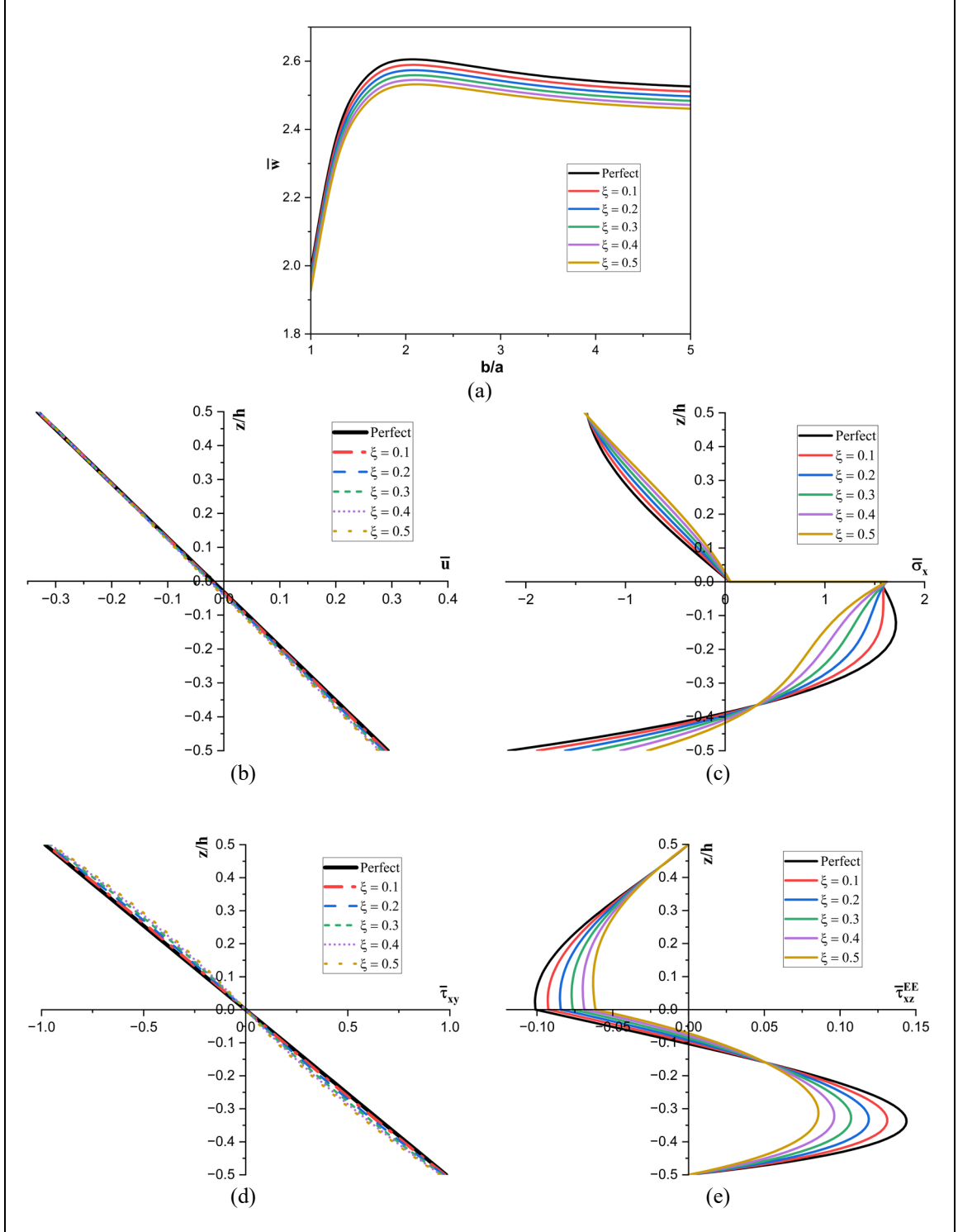


Figure 31: Variation of normalized stresses and displacements via I-FSDT for porosity model 3 using multiple ξ values for the square (0°/90°) laminated plate with $\frac{a}{h} = 5$ under nonlinear thermal load ($T_1 = 0$): (a) lateral displacement $\bar{w}$ vs. b/a , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) planar shear stress $\bar{\tau}_{xy}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

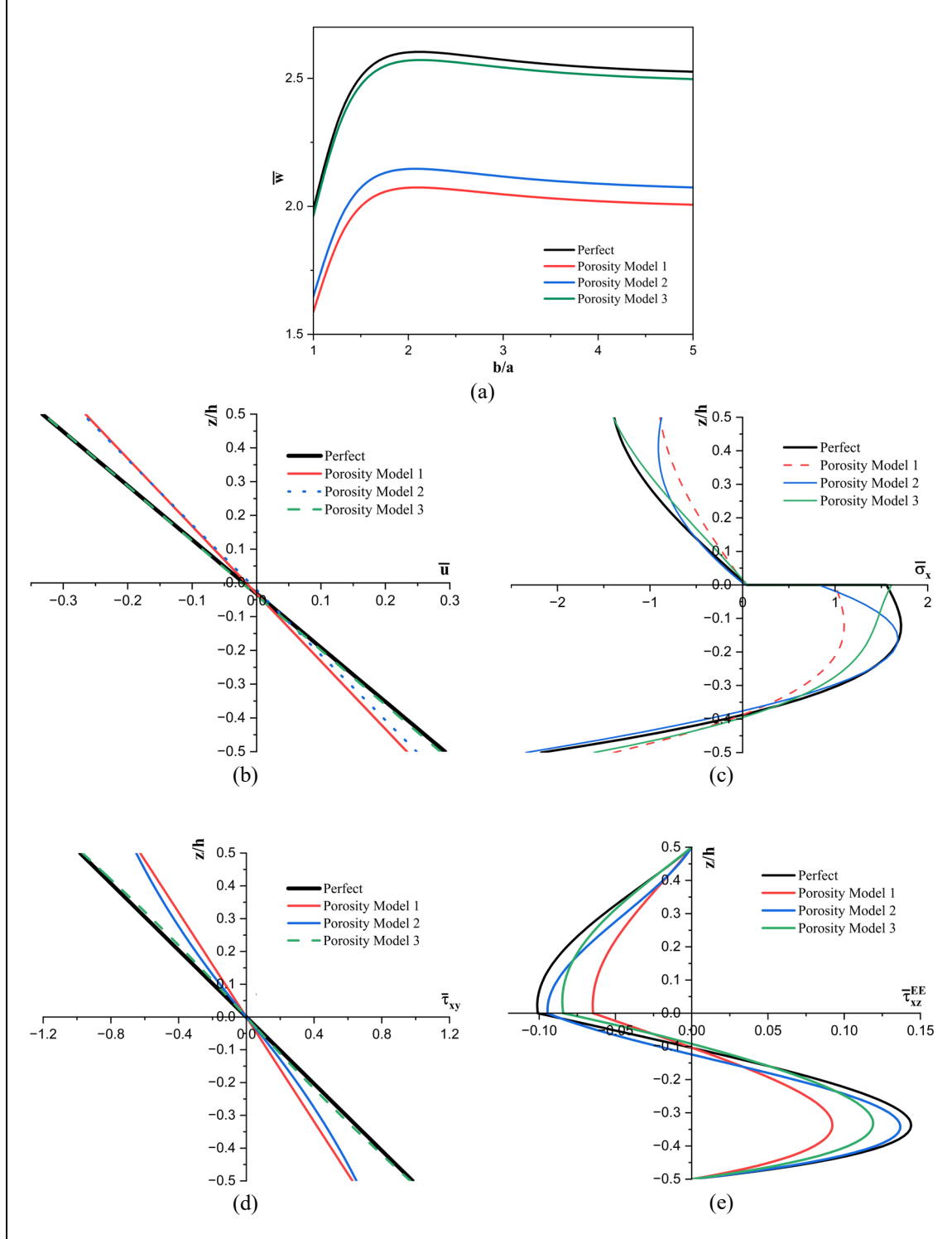


Figure 32: Variation of normalized stresses and displacements considering all porosity models at $\xi = 0.2$ using the I-FSDT for the square ($0^\circ/90^\circ$) laminated plate with $\frac{a}{h} = 5$ under nonlinear thermal load ($T_1 = 0$): (a) lateral displacement $\bar{w}$ vs. b/a , (b) axial displacement $\bar{u}$ vs. z/h , (c) axial stress $\bar{\sigma}_x$ vs. z/h , (d) planar shear stress $\bar{\tau}_{xy}$ vs. z/h , (e) out-of-plane shear stress $\bar{\tau}_{xz}^{EE}$ vs. z/h .

CHAPTER 5

Conclusions and Recommendations

5.1 Conclusions

This work was an attempt to come up with an improved first-order shear deformation theory (I-FSDT) to study the thermal-induced bending behavior of a simply supported ($0^\circ/90^\circ$) porous anti-symmetric cross-ply laminated plate. First, the improvement on the first-order shear deformation theory is done and tested on beams, resulting in the I-FSDT beam's version of the proposed theory called improved Timoshenko's beam theory (I-TBT). Then the I-FSDT was developed for plates. The conclusion is written in two parts, one for beams and the second for plates.

5.1.1 I-TBT for the Analysis of Porous Laminated Beams

An improved Timoshenko's beam theory is developed to conduct mechanical and thermal flexural analysis of simply supported porous cross-ply laminated beam. The theory introduces a parabolic variation of the transverse shear stress and therefore doesn't require a shear correction factor. The governing equations were determined by the principle of virtual work, after which Navier's procedure was used to find the analytical solution for the simple support condition. In the mechanical load case, I-TBT's results were compared with those of Bernoulli-Euler's CBT, Timoshenko's TBT, Sayyad-Ghugal's TrSDT, Reddy's HSDT, and exact elasticity theory as reported by Sayyad and Ghugal [17]. I-TBT is found to provide the closest results to the exact elasticity theory in comparison with the other theories. The effects of porosity parameters' variations were studied, and the following points were noticed:

- As the porosity volume fraction increases, $\bar{u}$ and $\bar{w}$ both increase the most by porosity distribution model 1, then 2.
- The highest influence on the normalized stresses is done by porosity model 2, followed by 3. While porosity model 1 doesn't seem to have an effect.
- As ξ increases for porosity model 2, the maximum magnitude for $\bar{\sigma}_x$ shifts from $z = -h/2$ to the plies interface in the (0°) lamina side. Whereas for model 3 it is always reached at $z = -h/2$.
- The maximum values of the transverse shear stresses $\bar{\tau}_{xz}^{CR}$ and $\bar{\tau}_{xz}^{EE}$ increase in porosity model 2 as ξ increases. In contrast, $\bar{\tau}_{xz}^{CR}$ and $\bar{\tau}_{xz}^{EE}$ maximum will decrease as ξ increases in model 3.
- Although the presence of porosity does have an impact on stresses, the impact on the displacements seems to be much more pronounced.

For the linearly varying sinusoidal thermal load case, the porosity characteristics' impacts were investigated, the findings are summarized below.

- All the dimensionless stresses and displacements are affected the most by porosity model 1, then model 2.
- All the dimensionless stresses and displacements decrease as the porosity volume fraction increases.
- Porosity model 3 has negligible effects on the normalized displacements.
- For all porosity models and ξ values, the extreme magnitude of the axial stress is reached at $z = 0$ in the (0°) ply region.

- As ξ value increases from 0.1 to 0.5, the maximum transverse shear stress's position raises slightly moving from $z = -0.28h$ to $z = -0.26h$, while for porosity models 1 and 3 the maximum is always at $\frac{z}{h} = -0.28$ and $\frac{z}{h} = -0.29$, respectively.

5.1.2 I-FSDT for the Analysis of Porous Laminated Plates

An improved first-order shear deformation theory (I-FSDT) is developed to study the nonlinear thermal-induced flexural behavior of a simply supported ($0^\circ/90^\circ$) porous anti-symmetric cross-ply laminated plate. The proposed theory has four variables instead of five in conventional FSDT and eliminates the need for a shear correction factor. First, I-FSDT's results of a simply supported ($0^\circ/90^\circ$) perfect plate under sinusoidal mechanical load were validated with Kirchhof's CLPT, Mindlin's 1951 FSDT, Reddy's 1984 HSDT, Frih *et al.* quasi-3D theory, and elasticity exact theory as reported by Frih *et al.* [76]. I-FSDT is shown to predict excellent results, closer to those by HSDT, quasi-3D, and elasticity theories. The transverse shear stresses' figures by I-FSDT have parabolic variation, contrary to conventional FSDT where the transverse shear stresses are constant across the thickness. Then, I-FSDT was used to predict the flexural response of porous anti-symmetric cross-ply laminated plates under nonlinear thermal load. Three porosity distribution models with multiple porosity volume fractions effects on the thermally induced behavior were studied. Several observations were recognized:

- The thermal flexural response is most affected by porosity model 1, followed by model 2, while the effects of porosity model 3 are somewhat negligible in comparison except for planar normal stresses and transverse shear stresses.

- The effects of porosity models 1 and 2 on the deflection, $\bar{w}$, are clear through the range of $\frac{b}{a} = (1 \text{ to } 5)$, while for model 3, the effects become more visible as the plate moves from a square to a more rectangular shape.
- When ξ increases $\bar{w}$ decreases for all porosity models.
- As ξ increases in porosity models 2 and 3, $\bar{\tau}_{xy}$ exhibits a slight curvy nonlinear variation across the plate's thickness as opposed to a linear variation for the perfect plate.
- $\bar{\sigma}_x$ in porosity model 2 would reach its maximum magnitude at $(\frac{z}{h} = -0.5)$, while $\bar{\tau}_{xz}^{EE}$ would reach it at $(\frac{z}{h} = -0.34)$.
- The values of $\bar{u}(-\frac{h}{2})$, $\bar{v}(-\frac{h}{2})$, $\bar{w}(0)$, and $\bar{\tau}_{xy}(-\frac{h}{2})$ are affected the most by porosity model 1 followed by model 2, while the effects of porosity model 3 are somewhat negligible in comparison. However, $\bar{\sigma}_x(-\frac{h}{2})$, $\bar{\sigma}_y(+\frac{h}{2})$, $\bar{\tau}_{xz}^{EE}(0)$ and $\bar{\tau}_{yz}^{EE}(0)$ are affected the most by model 1 and then model 3, whereas model 2 has a smaller effect in comparison.

5.2 Recommendations

This work examined the thermal flexural response of simply supported porous anti-symmetric cross-ply laminated plates using I-FSDT and beams using I-TBT. In the future, the effect of thickness stretching could be added to the present theory. Future studies could also focus on:

- The effect of different types of plates' foundations on the response.
- Considering other types of boundary conditions.

- Studying the hygrothermal static response of laminated plates.
- Analyzing laminated plates' dynamic response.

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