

**MIXED MEMORY STRUCTURES:
PROPERTIES AND ENHANCEMENTS**

BY

USMAN IBRAHIM

A Thesis Presented to the
COLLEGE OF GRADUATE AND INTERDISCIPLINARY STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

APPLIED STATISTICS

DECEMBER 2024

This thesis, written by **USMAN IBRAHIM** under the direction of his thesis adviser and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN APPLIED STATISTICS**.

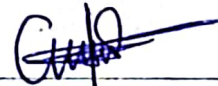
Thesis Committee

 08/12/2024

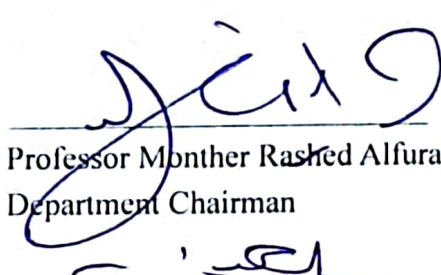
Dr. Nasir Abbas (Adviser)

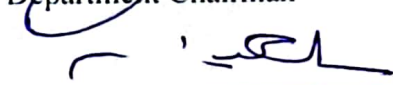
 8/12/2024

Professor Muhammad Riaz (Member)

 08/12/24

Dr. Emmanuel Osita Afuecheta (Member)


Professor Monther Rashed Alfuraidean
Department Chairman


Professor Suliman S. Al-Homidan
Dean of Graduate Studies

Date



©Usman Ibrahim 2024

To my parents

ACKNOWLEDGMENTS

First and foremost, I thank Almighty Allah for His countless blessings, guidance, and strength throughout this journey. Without His mercy, this accomplishment would not have been possible.

A heartfelt thank you to my advisor, Dr. Nasir Abbas, for being a good mentor. Your guidance, patience, and constant support have played a huge role in the completion of this thesis. I'm also grateful to my committee members, Prof. Muhammad Riaz and Dr. Emmanuel Osita Afuecheta, for their insightful feedback and encouragement throughout this process. I am also thankful to the Mathematics Department and King Fahd University of Petroleum and Minerals (KFUPM) for providing me with a great academic environment to grow and learn as well as a generous financial support that made my academic journey as seamless as possible. Many thanks to Dr. Muhammad Alhaji for his unwavering support and kindness toward Nigerians and international students at KFUPM. His guidance, generosity, and availability have been invaluable throughout my academic journey.

I can't express enough gratitude to my family. To my late parents, I know your prayers and sacrifices continue to guide me throughout my life. To my wife, Asmau Sulaiman Umar, thank you for your unwavering love, patience, and understanding for all the time I have spent far away from home. I also want to specially thank my eldest sibling, Alhaji Yahaya Ibrahim, for his constant support throughout my academic endeavors, my brothers and sisters, and the entire extended family for their endless support.

To my friends at KFUM, especially my roommate Muhammad Mustapha Adam whom I relate with in many situations, you made this journey much more enjoyable. Thanks for

being there through the tough and fun times alike. I also appreciate, Hassan Umar Hassan, Ahmad Usman, Ahmad Muhammad Bukar, and Faisal Muhammad Lawan, and the entire KFUPM student community for your support and good times.

A special thanks to all my Gombe State University lecturers, especially Dr. Semiu Oladejo, Dr. Afiz Saliu, and Dr. Aliyu Muhammad Awwal, for being incredible mentors and for always offering valuable advice. And to all my friends from undergrad, especially Abubakar Muhammad Awwal, Adamu Saad Abubakar, and Muhammad Rabi'u Tsoho—thank you for the friendship, the memories, and the support over the years. You all have been a big part of this journey.

To everyone who has been there for me throughout this journey, I am truly thankful. This work is possible only because of your support and contributions.

TABLE OF CONTENTS

ACKNOWLEDGMENTS	v
LIST OF TABLES	x
LIST OF FIGURES.....	xi
LIST OF ABBREVIATIONS.....	xii
THESIS ABSTRACT	xiii
THESIS ABSTRACT (ARABIC).....	xiv
CHAPTER 1	1
INTRODUCTION	1
1.1 Quality and Statistical Process Control.....	1
1.2 Basic Concepts of Control Charts.....	2
1.2.1 Classification of Control Charts by Data Types.....	3
1.2.2 Classification of Control Charts by Structure	4
1.2.3 Phases in Control Charts Application	4
1.3 Aim and Objectives of the Thesis	4
1.3.1 Evaluation of Mixed Memory Charts	5
1.3.2 Development of a New Dispersion Chart	5
1.3.3 Introduction of New Charts for Joint Monitoring.....	5
1.4 Outline of the Thesis	5
CHAPTER 2	7
LITERATURE REVIEW	7
2.1 Control Charts for Monitoring Process Location.....	7
2.2 Control Charts for Monitoring Process Dispersion.....	9
2.3 Joint Monitoring Control Charts	11
CHAPTER 3	15
EVALUATION OF MIXED EWMA-CUSUM CHARTS.....	15
3.1 The Classical Shewhart $\bar{X}$ Chart	15
3.2 The Classical CUSUM Control Chart.....	16
3.3 The Classical EWMA Control Chart.....	17
3.4 The Mixed EWMA-CUSUM Control Chart.....	18
3.5 Performance Measures	19
3.5.1 Zero-state Average Run Length	20

3.5.2 Conditional Expected Delay	20
3.5.3 Steady-state Average Run Length	21
3.5.4 Extra Quadratic Loss.....	21
3.5.5 Absolute Expected Loss	21
3.5.6 Sequential Extra Quadratic Loss.....	22
3.6 Results and Discussion.....	22
3.6.1 Performance Evaluation Under Zero-State Condition	24
3.6.2 Performance Evaluation Under Steady-State Condition.....	28
3.7 Comparative Analysis	35
3.7.1 Zero-State Comparisons.....	36
3.7.2 Steady-State Comparisons	39
3.8 Design of MEC Chart	42
3.9 Real-Time Monitoring of Water pH Levels in Aquatic Ecotoxicology	46
3.9.1 pH Data Description.....	47
3.9.2 Data Assumptions Validation	48
3.9.3 Monitoring pH Level Using Control Charts	49
3.10 Chapter 3 Conclusion.....	51
CHAPTER 4	52
NEW STEP CUSUM CONTROL CHART FOR MONITORING PROCESS VARIABILITY	52
4.1 Existing Methods	52
4.1.1 The parameter transformation	53
4.1.2 EWMA-S ² Chart	54
4.1.3 CUSUM-S ² Chart.....	55
4.2 The proposed Step-CUSUM (SC-S ²) chart for Variance	56
4.3 Performance Evaluation Measures.....	58
4.4 Results and Discussion.....	60
4.4.1 IC and OOC performance of the proposed chart	61
4.4.2 Optimal parameters	64
4.5 Comparisons.....	68
4.5.1 Zero-state comparisons	68
4.5.2 Steady-state comparisons	70
4.5.3 Graphical Comparison using CED Curves	80
4.6. Illustrative Examples.....	83
4.6.1 Example 1: Simulated Data.....	83

4.6.2 Example 2: Combined Cycle Power Plant (CCPP)	84
4.7 Chapter 4 Conclusion.....	87
CHAPTER 5	89
PROPOSED JOINT CAM CHARTS.....	89
5.1 Existing Methods	89
5.1.1 Transformations	90
5.1.2 Max-EWMA and SS-EWMA	92
5.1.3 Max-CUSUM and SS-CUSUM.....	94
5.2 New Charts for Joint Monitoring.....	95
5.2.1 Max-CAM Charts	95
5.2.2 SS-CAM Charts	99
5.3 Results and Discussion.....	100
5.3.1 Performance Evaluation of the Proposed Charts	102
5.4 Comparative Study.....	107
5.4.1 Zero-State Comparisons.....	107
5.4.2 CED Comparisons.....	110
5.5 Real-world Example	116
5.6 Chapter 5 Conclusion.....	119
CHAPTER 6	121
CONCLUSION.....	121
6.1 SUMMARY	121
6.2 RECOMMENDATION.....	122
APPENDICES	123
Appendix A (Distribution of CAM statistic).....	123
Appendix B (Effects of the Design Parameter on the Proposed Charts)	125
REFERENCES.....	132
VITAE.....	139

LIST OF TABLES

Table 3.1: Zero- and steady-state EQL values of MEC control chart.	25
Table 3.2: Zero- and steady-state AEL values of MEC control chart.....	27
Table 3.3: Steady-state ARL profiles of MEC control chart, $kM = 0.125$	29
Table 3.4: Steady-state ARL profiles of MEC control chart, $kM = 0.25$	30
Table 3.5: Steady-state ARL profiles of MEC control chart, $kM = 0.5$	31
Table 3.6: Steady-state ARL profiles of MEC control chart, $kM = 2$	32
Table 3.7: Steady-state ARL profiles of MEC control chart, $kM = 2.5$ & 3.	33
Table 3.8: EWMA and CUSUM overall performance using EQL and AEL metrics	38
Table 3.9: SEQL values of MEC, CUSUM, and EWMA charts.	39
Table 3.10: The MEC chart optimal values of kM and λM	45
Table 3.11: pH levels Data	48
Table 4.1: $An, Bn, Cn, \mu n, \sigma n$, and Zo constants for $n = 3, 4, \dots, 15$	54
Table 4.2: Zero-state ARL values of SC-S ² for.	62
Table 4.3: SC-S ² Optimal combinations ($Kopt, Lopt$) and $ARLopt$	65
Table 4.4: Zero-state minimum ARL of EWMA-S ² , CUSUM-S ² and SC-S ²	69
Table 4.5: ARL comparisons among CUSUM-S ² , EWMA-S ² and SC-S ²	72
Table 4.6: Steady-state ARL comparison among CUSUM-S ² , EWMA-S ² and SC-S ²	73
Table 4.7: Steady-state ARL comparisons for a target shift $\gamma_{target} = 1.5$	74
Table 4.8: SC-S ² , CUSUM-S ² , and EWMA-S ² EQL	78
Table 5.1. (ARL, SDRL, MDR) values of the six proposed charts	104
Table 5.2: Zero-state ARL Comparison of Proposed and Existing Charts.	106

LIST OF FIGURES

Figure 3.1: Steady-state ARL profiles of MEC chart.....	34
Figure 3.2: Effect of kM and λM on EQL performance of MEC chart.	35
Figure 3.3: Steady-state ARL profiles of MEC EQL	41
Figure 3.4: 3D Surface Plot and Contour Plot of kM , λM , and hM	45
Figure 3.5: Application of Shewhart, EWMA, CUSUM and MEC charts pH data	50
Figure 4.1: (KSC, LSC) pairs such that $ARL_0 = 370$	62
Figure 4.2. Three-Dimensional Surface Plot of K_{opt} , L_{opt} , and γ_{target}	66
Figure 4.3. SC-S ² steady-state ARL as a function of K	67
Figure 4.4. Zero-state Log (ARL) of the proposed, EWMA, and CUSUM charts	70
Figure 4.5. CED curves comparison when $\gamma = 0.5$	81
Figure 4.6. CED curves comparison when $\gamma = 0.95$	82
Figure 4.7. CED curves comparison when $\gamma = 1.1$	82
Figure 4.8. CED curves comparison when $\gamma = 1.5$	83
Figure 4.9. Histogram, normal Q-Q plot and ACF plot for the AP data.	86
Figure 4.10. Application of the charts on AP data.....	87
Figure 5.1: IC CED Curves of Proposed and Existing Charts	111
Figure 5.2: OOC CED Curves of Max-CAM and Max-CUSUM Charts.	112
Figure 5.3: OOC CED Curves of SS-CAM and SS-CUSUM Charts.	113
Figure 5.4: OOC CED Curves of Max-CAM and Max-EWMA Charts.	114
Figure 5.5: OOC CED Curves of SS-CAM and SS-EWMA Charts.	115
Figure 5.6: Application of Proposed and Existing Charts on Real Data	119

LIST OF ABBREVIATIONS

AEL	Absolute Expected Loss
AP	Atmospheric Pressure
ARL	Average Run Length
CAM	Conditional Aggregate Mean
CCPP	Combined Cycle Power Plant
CED	Conditional Expected Delay
CUSUM	Cumulative Sum
EQL	Extra Quadratic Loss
EWMA	Exponentially Weighted Moving Average
IC	In-Control
Max	Maximum
MDRL	Median Run Length
MEC	Mixed EWMA-CUSUM
OOC	Out-of-Control
pH	Potential of Hydrogen
SC	Step CUSUM
SDRL	Standard Deviation of Run Length
SEQL	Sequential Extra Quadratic Loss
SS	Sum of Squares

THESIS ABSTRACT

Full Name : Usman Ibrahim
Thesis Title : Mixed Memory Structures: Properties and Enhancements
Major Field : Applied Statistics
Date of Degree : December, 2024

Industrial and service processes are often characterized by some form of variation. To distinguish disruptive variations from natural variations, special statistical tools called control charts are commonly employed. Traditionally, two separate control charts are used to monitor variations in process location and scale parameters. Recently, single charts that simultaneously monitor both parameters have gained significant attention due to their efficiency in detecting joint shifts in process parameters. Despite advancements in control charting methods, there remains a need for improved techniques that enhance accuracy and adaptability in diverse process conditions.

This thesis evaluates the performance of mixed memory-type charts for process monitoring and introduces a new chart for monitoring process dispersion, along with an additional six charts for the simultaneous monitoring of process location and dispersion parameters. The efficiency of the proposed charts was assessed through Monte Carlo simulations and compared with existing alternatives. The results showed that the new charts had an advantage over their counterparts in certain conditions. Several examples using real data from industrial and laboratory processes were provided to demonstrate the construction and performance of the proposed charts in real-world applications.

THESIS ABSTRACT (ARABIC)

الاسم الكامل : عثمان إبراهيم

عنوان الرسالة : هياكل الذاكرة المختلطة: الخصائص والتحسينات

التخصص الرئيسي: الإحصاء التطبيقي

تاريخ الدرجة : ديسمبر 2024

غالبًا ما تتسم العمليات الصناعية والخدمية ببعض أشكال التباين. وللتمييز بين التباينات التخريبية والتباينات الطبيعية، تُستخدم أدوات إحصائية خاصة تُعرف باسم مخططات التحكم. تقليديًا، يتم استخدام مخططين منفصلين للتحكم في مراقبة التباينات في موقع العملية ومعلومات الحجم. مؤخرًا، حظيت المخططات الفردية التي تراقب كلا المعلمتين في وقت واحد باهتمام كبير بسبب كفاءتها في اكتشاف التحولات المشتركة في معلمات العملية. وعلى الرغم من التقدم في أساليب رسم مخططات التحكم، لا تزال هناك حاجة إلى تقنيات محسنة لتعزيز الدقة والقدرة على التكيف في ظروف العمليات المتنوعة.

تعيد هذه الأطروحة تقييم أداء مخططات الذاكرة المختلطة لمراقبة العمليات، كما تقدم مخططًا جديدًا لمراقبة تشتت العملية، إلى جانب ستة مخططات إضافية للمراقبة المتزامنة لمعلمات موقع العملية وتشتتها. تم تقييم كفاءة المخططات المقترحة باستخدام محاكاة مونت كارلو ومقارنتها بالمخططات الحالية. وقد أظهرت النتائج تفوق المخططات الجديدة على نظيراتها في بعض الظروف. ولإثبات بناء وأداء المخططات المقترحة في التطبيقات الواقعية، تم تقديم العديد من الأمثلة باستخدام بيانات حقيقية من العمليات الصناعية والمعملية.

CHAPTER 1

INTRODUCTION

This Chapter introduces the basic concepts of Statistical Process Control (SPC) and its importance in maintaining process quality. It also briefly outlines different types of control charts and their classifications, such as Shewhart-type and memory-type charts. Additionally, the chapter discusses the scope and structure of the thesis and summarizes the topics covered in each chapter.

1.1 Quality and Statistical Process Control

Quality has always been an essential component of all products and services. Statisticians and production managers have different perspectives on quality improvement. To production managers, producing uniform goods defines quality, whereas statisticians recognize that variability is unavoidable. Therefore, from a statistical standpoint, improving quality involves reducing variability in the process.

Excessive variation in processes can lead to inefficiency, thus resulting in poor quality and unnecessary wastage. Nevertheless, irrespective of how excellent the design of a process is, there will always be an inherent variation that emerges due to the accumulation of several minor effects caused by practically inescapable factors. These variations are referred to as chance or common cause variations. A process is deemed in-control (IC) or, more formally, in a state of statistical control when it is subject only to variations caused by common factors. On the other hand, some variations may result from avoidable causes such as human mistakes, improperly adjusted machinery, and out-of-spec raw materials.

These variations are known as special or assigned cause variations. A process is considered to be out-of-control (OOC) when it experiences both sources of variation. To guarantee high-quality manufacturing or service, it is essential to minimize process variations as much as possible.

Defining or measuring quality can be challenging. [1] provided some guidelines to evaluate the key components of quality as follows:

- Performance: Does the product fulfill its intended purpose effectively?
- Reliability: Does the product experience failures? How often?
- Durability: What is the lifespan of the product?
- Serviceability: What is the difficulty level when repairing the product?
- Aesthetics: How visually appealing is the product?
- Features: What capabilities or functionalities does the product provide?
- Perceived Quality: How is the product or its manufacturer regarded?
- Conformance to Standards: Does the product meet the exact specifications set by its designer?

SPC is a powerful collection of techniques that are very effective in improving quality by detecting and help in reducing process variability. Control charts, histograms, flowcharts, Pareto charts, scatter plots, and fishbone diagrams are often used in quality management and process monitoring [1].

1.2 Basic Concepts of Control Charts

Quality control charts, first introduced by [2], are the most commonly used techniques for distinguishing between natural process fluctuations and those triggered by certain factors, including malfunctioning machinery and operator faults. In its most basic form, a control

chart consists of a graph of measurement on some quality characteristic with a horizontal line representing the IC average of the process, and two additional horizontal lines marking the upper and lower boundaries, beyond which the process is considered to be OOC.

Since their inception, control charts have been widely applied for process monitoring in the manufacturing, healthcare, business, and finance sectors. This broad applicability highlights the need to develop new methods and enhance the existing structures to address the unique challenges and requirements of each sector.

The performance of control charts is usually evaluated through the average run length (ARL), which is the average number of samples that must be taken before an OOC is produced.

Control charts can be classified based on their structures or the type of data they monitor. This helps in choosing the most appropriate chart for various process monitoring needs and specific applications.

1.2.1 Classification of Control Charts by Data Types

Control charts can be divided into variable control charts and attribute control charts based on the nature of the data being monitored. Variable control charts are designed to monitor continuous data, which can be measured on a numerical scale (e.g., weight, temperature, dimensions). Some commonly used variable charts include $\bar{X}$ and R, $\bar{X}$ and S, and individual measurement charts.

On the other hand, attribute control charts deal with categorical or discrete data, such as defect counts or binary outcomes (e.g., pass/fail). These charts are usually used in monitoring quality in terms of conformity or the occurrence of defects. The most popular attribute control charts are p , np , c and u charts [1].

1.2.2 Classification of Control Charts by Structure

Control charts can also be categorized in to two based on their structural design according to how historical data are handled. These are memoryless charts and memory-based charts. Memoryless charts, also known as Shewhart charts, have plotting statistics that depend only on the current sample, without considering any previous information. The second category is the memory-based control charts that incorporate current and past information in the computation of their plotting statistics. The most widely used memory-based charts are the Exponentially Weighted Moving Average (EWMA) chart [3] and the Cumulative Sum (CUSUM) chart [4].

1.2.3 Phases in Control Charts Application

The application of control charts involves two phases: Phase I and Phase II. Each phase serves a distinct purpose in process monitoring. In Phase I, historical data is collected and analyzed in a retrospective manner to setup trial limits and establish a process baseline. The goals of this phase is to detect and remove special causes of variation, ensuring the process is stable, and to determine control limits to be used in Phase II. In Phase II, the control charts are used in on-line application to monitor the process and maintain stability. The focus shifts to detecting deviations from the established baseline to ensure product quality and process performance.

1.3 Aim and Objectives of the Thesis

This thesis aims to provide some contributions to statistical process control by developing and evaluating advanced control charting schemes. The works presented in Chapters 3, 4, and 5 address this aim through the following objectives.

1.3.1 Evaluation of Mixed Memory Charts

The first aim of this study is to perform a comprehensive evaluation and comparison of Mixed EWMA-CUSUM (MEC) chart. The main objectives here are:

- To critically analyze the performance of the MEC chart under zero-state and steady-state conditions.
- To identify optimal parameter settings through sensitivity analysis for easy application by practitioners.
- To provide a comprehensive comparison between the MEC chart and other memory-based charts

1.3.2 Development of a New Dispersion Chart

The second aim of the study is to develop a new memory-type chart that is efficient in detecting a wide range of shifts in process dispersion. The main objectives here are:

- To propose a new Step CUSUM (SC-S²) control chart for monitoring process variability.
- To evaluate the performance of the SC-S² chart and compare it with existing counterparts.

1.3.3 Introduction of New Charts for Joint Monitoring

The third aim of the study is to propose new joint schemes for simultaneous monitoring of process location and dispersion parameters. The main objectives here are:

- To design six new control chart schemes for joint monitoring of process mean and variance that are efficient for a wide range of shifts.
- To assess the efficiency of the proposed charts by evaluating performance measures using Monte Carlo simulations.

1.4 Outline of the Thesis

This thesis is divided into six chapters. Chapter 1 introduces the statistical process control tools and why they are important in ensuring quality in manufacturing processes and

services. Chapter 2 provides a comprehensive review of the literature on different monitoring schemes. Chapters 3-5 present the contribution of the thesis which include reevaluation of mixed charts in Chapter 3, introduction of a new control chart for monitoring process variability in Chapter 4, and proposal of six new schemes for the joint monitoring of process mean and variability.

CHAPTER 2

LITERATURE REVIEW

This chapter provides an overview of existing research on control charts in SPC, highlighting studies focused on charts designed to monitor process location, scale, or both simultaneously.

2.1 Control Charts for Monitoring Process Location

Control charts for location are primarily employed in the monitoring process mean (or other location parameters such as median). These include the classical Shewhart $\bar{X}$ – chart, CUSUM, EWMA, and several other modifications of these charts such as the incorporation of run-rules schemes [5-7], a combination of two or more schemes [8, 9], and the design of adaptive charts [10-12]. [13] have discussed and evaluated several extensions of the EWMA chart, including the Generally Weighted Moving Average, Double, Tripple, and Quadruple EWMA charts, and concluded that those new charts have better performance than the classical EWMA in both zero and steady-state conditions.

Another significant improvement attempt that has received considerable attention in the literature is mixing two classical or modified control chart features, for example, the combination of EWMA and CUSUM charts, to form a new compound structure. The mixed EWMA-CUSUM (MEC) control chart scheme by [14] was the first in this category. In the study, the authors used the EWMA statistic as an input in the CUSUM chart to monitor shifts in a process location. They compared several schemes, such as the classical, fast

initial response (FIR), and run rules-based EWMA and CUSUM schemes. Using the zero-state ARL comparisons, they concluded that, in most cases, their newly proposed MEC chart is superior in detecting small shifts in the process mean than other schemes considered for the same purpose. Since then, several modifications and variations of the MEC and its properties under certain conditions have been suggested and investigated. For example, [15] proposed enhancing the MEC scheme by incorporating head starts and run rules features, which improved the performance of the chart, especially for small shifts that occur at the start-up of the process. [16] investigated the MEC performance under different scenarios in a comparative study with other memory schemes, namely, CUSUM and EWMA charts. Their research considered the unknown parameter case and employed six different location estimators to evaluate their robustness using several run length distribution properties as performance measures. They concluded that there is no single scheme or particular estimator that is superior in all the situations in terms of early detection; however, the EWMA chart designed using the median estimator has the best performance on average, and the MEC chart shows a certain level of robustness in the contaminated variance environments. Another study by [17] found that the performance of the MEC chart is severely affected by measurement error; nonetheless, the chart was shown to be more resilient than both EWMA and CUSUM in the case of measurement error.

Another mixed form was also developed by [18] for data following Conway Maxwell Poisson distribution. Their study showed that the new chart has performed better than other memory charts in detecting moderate shifts in count data processes. [19] extended the idea of the MEC chart to monitor the mean of multivariate processes. [20] used the MEC chart

to monitor the mean of an autocorrelated process and found that it performs well in detecting small shifts in autocorrelated data. [21] proposed the median form of the MEC chart that provides protection against outliers in contaminated environments. [22] introduced the mixed modified EWMA-EWMA (MEME) control chart for improved detection of small to moderate shifts in the process location. [23] incorporated auxiliary variables in a combined MEC chart for simultaneous monitoring of location and dispersion. [24] investigated the performance of MEC control charts under two different sampling techniques: median-ranked set sampling (MRSS) and double-ranked set sampling (DRSS) techniques.

2.2 Control Charts for Monitoring Process Dispersion

Most recent studies on memory-based dispersion charts have relied on transforming the sample variance so that the new statistic closely resembles a normally distributed variable. This allows the standard or modified versions of the CUSUM and EWMA frameworks to be applied. Here, we briefly discuss some recent developments in dispersion charts using transformed sample variance and/or sample standard deviation. Building on the earlier work of [25], an improved S^2 -EWMA control chart was introduced by [26]. The previous method used a one-sided chart to identify increases in variability, while the method of [26] uses a two-sided chart to detect increases and decreases in process variability to improve accuracy. In comparison to [25], the S^2 -EWMA chart shows superior overall sensitivity to minor changes in process variability. Also inspired by the work of [27], [28] developed a CUSUM- S^2 control chart for monitoring process variance using a three-parameter logarithmic transformation. The suggested approach enhances variance shift detection over the prior research, particularly in small sample sizes. They showed that the proposed chart

performs better than the EWMA- S^2 chart when there is a decrease in variability. However, the EWMA- S^2 is better suited to identify increases in process variance. In a similar vein, motivated by the prior work of [26], [29] introduced the EWMA-J- S^2 control chart, an extension using Johnson's SB transformation. While the previous method used a three-parameter logarithmic transformation, their new approach enhances the detection of process variability by applying a four-parameter Johnson SB transformation. This modification improves the normality of the transformed data and the detection of shifts in variance, especially when there is a reduction in process variability. The study also presents a comprehensive comparison with the previous EWMA- S^2 chart, showing that both methods have comparable effectiveness, although the EWMA-J- S^2 chart has minor benefits in identifying decreases in variance.

The CS-EWMA control chart, which tracks process variability by combining the features of CUSUM and EWMA charts, was first suggested by [30], utilizing both historical and present data to enhance the detection of increases and decreases in the process dispersion. Their simulation results showed that the new chart performs better than conventional methods such as S^2 -EWMA and CUSUM- S^2 , making it a valuable tool for process change monitoring. The reverse version of CS-EWMA, the Mixed CUSUM-EWMA (MCE) chart was suggested by [31] to improve monitoring changes in process variance. This new chart utilizes features of CUSUM and EWMA charts, making it more successful at detecting minor variation changes. The study demonstrates that the MCE chart performs better than classical charts such as Shewhart, EWMA, and CUSUM. In another study, [32] introduced two new control charts, the floating $T - S^2$ and floating $U - S^2$, for monitoring process variability based on the three- and four-parameter transformations using the progressive

mean technique. The study shows that floating $T - S^2$ works best for small shifts, while the floating $U - S^2$ is better for larger changes. The floating charts showed excellent zero-state performance but performed poorly in the steady-state.

Special sampling techniques are also considered for enhancing the performance of dispersion charts. For example, [33] proposed new CUSUM control charts using ranked set sampling (RSS) and double RSS for monitoring process variability. Other versions of control charts for variability related to the EWMA chart include the AEWMA- S^2 chart by [34], S^2 -HEWMA by [35], S^2 -TEWMA by [36], S^2 -QEWMA by [37], and S^2 -Double GWMA by [38].

2.3 Joint Monitoring Control Charts

Recently, joint monitoring of the process mean and standard deviation has gained significant attention in the literature. This section provides a brief overview of some of these techniques. [39] suggested using two independent charting statistics on a single chart to develop the first control chart that jointly monitors process level and variability.

A different approach involves the use of a single charting statistic for simultaneous monitoring of the mean and dispersion. For example, [40] introduced an enhanced semicircle chart for simultaneously monitoring the mean and standard deviation with a single plotting statistic. [41] also suggested a control chart for variables that combines the standardized mean and standard deviation into a single statistic using the maximum function. [42] proposed a maximum EWMA (Max-EWMA) chart, which combines two independent EWMA statistics: one for the mean, derived from the standardized sample mean, and the other for the standard deviation, based on a transformation of the sample

variance. The study demonstrated that this chart outperforms its counterparts in detecting increases in process variability associated with changes in the process mean. In a similar effort to improve efficiency of the chart, four different EWMA-type charts for joint monitoring have been proposed by [43]: the Max-EWMA chart based on the maximum of the normal transformation of the sample mean and standard deviation; the EWMA-Max chart, in which the maximum of the two statistics is used as an input in an EWMA structure; the EWMA sum of squares (SS-EWMA), which builds on the idea of SC charts by plotting the sum of squares of the sample statistics; and the EWMA-SC chart, which addresses the insensitivity to the small shift problem with the SC chart. An extension on the EWMA-type charts of [43] was presented by [44]. [45] proposed the Max-DEWMA chart as an extension of the Max-EWMA chart, and showed that the proposed chart outperforms the Max-EWMA in identifying changes in the monitored parameters. In a similar fashion, [46] further extended the idea using triple EWMA to develop a Max-TEWMA chart and concluded that the proposed chart has excellent performance, notably in the detection of a wide range of shifts in both mean and variance. [47] used a progressive approach to develop a Max-P chart with impressive zero-state properties. [48] suggest several new approaches to joint control charting, including the Max-EWMAVAR, SS-EWMAVAR, Max-CUSUMVAR, and SS-CUSUMVAR charts, all based on the three-parameter transformation of the sample variance.

In addition to EWMA-type charts, CUSUM-type charts have been widely considered as alternative methods for enhancing the sensitivity to small shifts in process parameters. [49] plotted two independent CUSUM charts, one for controlling the mean and the other for

standard deviation, on a single chart that jointly monitors the parameters. However, it is somewhat difficult to interpret OOC signals using this approach; therefore, various other methods have been proposed. For instance, [50] introduced a new CUSUM chart for simultaneous monitoring, utilizing a single statistic derived from a weighted loss function. Also, [51] developed a version of single CUSUM chart for individual observations. In a manner analogous to Max-EWMA, [52] proposed a Max-CUSUM chart to jointly control the mean and standard deviation of the process and recommend it over the Max-EWMA chart for applications in high-quality manufacturing processes. To enhance the detection capability of Max CUSUM-type charts across a range of joint shifts, [53] developed the sum of squares version of CUSUM chart (SS-CUSUM) and showed that it is efficient in identifying both small and large changes in the parameters of the process. [54] suggested weighted adaptive CUSUM charts using several unbiased estimators of the process parameters, and showed their superiority over several competitors including the Max-EWMA, Max-DEWMA, and Max-CUSUM. In another study, [55] developed a maximum dual CUSUM chart that excels over previous adaptive charts in the detection of a range of minor shifts in the parameters.

Furthermore, by complementing the parametric EWMA and CUSUM approaches, nonparametric charts provide a reliable alternative for situations where the underlying process distribution deviates from normality or not known. Some of the works in this direction include the distribution-free CUSUM-Lepage chart by [56]; the fuzzy scheme approach to joint monitoring by [57]; the one-sided Shewhart-Lepage chart for monitoring either upward or downward joint shifts in the process location and dispersion by [58]; the

fast initial response (FIR) equipped nonparametric EWMA chart by [59]; the EWMA-type distribution-free chart based on the Wilcoxon rank-sum statistics by [60]; two new distribution-free charts based on the van der Waerden (VW) and Ansari-Bradley (AB) tests, and the VW and Mood tests by [61]; a Max-EWMA chart for joint monitoring of location, scale, and parameters of the process by [61]. Finally, [62] provided an extensive review of joint monitoring techniques.

CHAPTER 3

EVALUATION OF MIXED EWMA-CUSUM CHARTS

This Chapter evaluates the performance of the MEC chart under a more reliable steady-state condition in addition to zero-state. This is in contrast to many previous studies focusing only on zero-state performance. To help practitioners in setting up the chart according to different needs, we have identified the optimal parameter settings of the chart through sensitivity analysis. Finally, the chart is applied in monitoring changes in potential of Hydrogen (pH) levels to illustrate its real-world application.

3.1 The Classical Shewhart $\bar{X}$ Chart

The statistic plotted on the Shewhart chart is the sample mean, $\bar{X}_i$, which is computed as

$\bar{X}_i = \frac{\sum_{j=1}^n X_{ij}}{n}$. This statistic is plotted against the lower and upper control limits (LCL_S and UCL_S), defined as follows:

$$\left. \begin{aligned} LCL_S &= \mu_0 - L_S \frac{\sigma_0}{\sqrt{n}} \\ UCL_S &= \mu_0 + L_S \frac{\sigma_0}{\sqrt{n}} \end{aligned} \right\} \quad (3.1)$$

Here, L_S is the control limit coefficient that determines the width of the control limits. A common choice for L_S is 3, which results in the well-known 3-sigma limits. If the sample mean $\bar{X}_i$ falls outside these control limits, it signals that the process may be OOC, meaning a significant shift in the process mean has occurred.

Despite its simplicity and ease of use, the Shewhart chart is not very effective for detecting small to moderate shifts in the process mean. Its primary strength lies in identifying large shifts quickly. For minor shifts, the chart may not signal promptly, leading to potential delays in corrective actions. Therefore, alternative control charts such as the EWMA and CUSUM charts are often preferred for processes where smaller shifts are of concern.

3.2 The Classical CUSUM Control Chart

The CUSUM control chart is a memory-type chart designed to detect small and moderate shifts in the process mean more effectively than the Shewhart chart. Unlike the Shewhart chart, the CUSUM chart incorporates information from all past samples, making it more sensitive to gradual changes in the process. The CUSUM chart can be implemented in two forms: the tabular CUSUM and the V-mask CUSUM. The tabular CUSUM is more commonly used in practice, where separate one-sided CUSUMs are calculated for detecting upward and downward shifts. The upward and downward CUSUMs are defined as:

$$\left. \begin{aligned} C_i^- &= \max[0, C_{i-1}^- + (\mu_0 - K_C) - \bar{X}_i] \\ C_i^+ &= \max[0, C_{i-1}^+ + \bar{X}_i - (\mu_0 + K_C)] \end{aligned} \right\} \quad (3.2)$$

where $K_C = k_C \frac{\sigma_0}{\sqrt{n}}$ is the sensitivity parameter of CUSUM chart. If either C_i^- or C_i^+ exceeds the decision interval $H_C = h_C \frac{\sigma_0}{\sqrt{n}}$, it indicates that the process mean has shifted. The strength of the CUSUM chart lies in its ability to detect small, sustained shifts in the process mean. As it accumulates information over time, it quickly picks up on trends that

may go unnoticed by the Shewhart chart. However, its sensitivity to small shifts comes with the trade-off of being slower in detecting large shifts compared to the Shewhart chart.

3.3 The Classical EWMA Control Chart

The EWMA control chart also addresses some limitations of the Shewhart chart by placing more weight on recent data points while still incorporating historical information. This characteristic makes the EWMA chart particularly effective for detecting small shifts in the process mean. The EWMA statistic is calculated using the formula:

$$EWMA_i = \lambda_E \bar{X}_i + (1 - \lambda_E)EWMA_{i-1} \quad (3.3)$$

Where $EWMA_i$ represents the current EWMA statistic and λ_E is the sensitivity parameter that determines the weight assigned to the most recent sample. The value of λ_E lies between 0 and 1, with larger values placing more emphasis on recent data. When λ_E equals 1, the EWMA chart becomes equivalent to the Shewhart chart.

The control limits for the EWMA chart are established based on the expected value and asymptotic variance of the EWMA statistic. The limits are given by:

$$\left. \begin{aligned} LCL_E &= \mu_0 - L_E \frac{\sigma_0}{\sqrt{n}} \sqrt{\frac{\lambda_E}{(2 - \lambda_E)}} \\ UCL_E &= \mu_0 + L_E \frac{\sigma_0}{\sqrt{n}} \sqrt{\frac{\lambda_E}{(2 - \lambda_E)}} \end{aligned} \right\} \quad (3.4)$$

Here, L_E determines the width of the control limits. A well-chosen L_E value balances the sensitivity and specificity of the chart, minimizing false alarms while effectively detecting shifts.

One of the primary advantages of the EWMA chart is its ability to quickly signal small shifts in the process mean. By continuously updating the statistic with each new sample, the EWMA chart is more responsive to changes compared to the Shewhart chart. This responsiveness is particularly valuable in industries where maintaining a consistent process is crucial, as it allows for timely interventions before significant deviations occur (see [63]). In practice, practitioners can optimize the parameters of the EWMA chart, such as λ_E and L_E , based on the specific characteristics of the process and the shifts of interest. [64] provides guidance on selecting optimal values for these parameters, emphasizing that lower λ_E values are preferable for detecting minor deviations, while higher values are better suited for larger shifts.

3.4 The Mixed EWMA-CUSUM Control Chart

The MEC control chart combines the strengths of both EWMA and CUSUM control charts to enhance the detection of small shifts in the process mean. Instead of relying solely on raw observations, the MEC chart utilizes the EWMA statistic as input into the CUSUM framework, thus allowing for a more responsive monitoring system. The MEC chart calculates two monitoring statistics: the upper MEC statistic (MEC_i^+) and the lower MEC statistic (MEC_i^-). These statistics are defined as follows:

$$\left. \begin{aligned} MEC_i^- &= \max[0, MEC_{i-1}^- + (\mu_0 - K_M) - EWMA_i] \\ MEC_i^+ &= \max[0, MEC_{i-1}^+ + EWMA_i - (\mu_0 + K_M)] \end{aligned} \right\} \quad (3.5)$$

In the above equation, $EWMA_i$ is derived from the current sample mean using the EWMA formula, and K_M is a reference parameter that is set to a predetermined value based on the

expected shift. The initial values for both MEC statistics are zero (i.e., $MEC_0^- = MEC_0^+ = 0$).

To assess whether the process is OOC, the MEC statistics are compared against a control limit (H_M). An upward shift in the process average is indicated when MEC_i^+ exceeds H_M , while a downward deviation is signaled if MEC_i^- exceeds H_M . The design parameters of the MEC chart include the sensitivity parameter from the EWMA chart (denoted by λ_M for MEC chart) and the reference values K_M and H_M . These parameters are determined in a manner analogous to the CUSUM chart, where K_M and H_M are expressed as follows:

$$\left. \begin{aligned} K_M &= k_M \frac{\sigma_0}{\sqrt{n}} \sqrt{\frac{\lambda_M}{2 - \lambda_E}} \\ H_M &= h_M \frac{\sigma_0}{\sqrt{n}} \sqrt{\frac{\lambda_M}{2 - \lambda_E}} \end{aligned} \right\}. \quad (3.6)$$

Where k_M and h_M serve similar purposes as in the traditional CUSUM scheme. The value of h_M is identified by fixing λ_M and k_M , and searching for the corresponding h_M that achieves the desired IC ARL (ARL_0). The unique structure of MEC chart allows it to efficiently detect both small and moderate shifts in process mean, making it a valuable tool in the field of SPC. The EWMA control chart exemplifies the principle that informed decision-making relies on understanding both current and historical data, enabling effective process monitoring and improvement.

3.5 Performance Measures

Performance measures are crucial metrics that allow for the assessment of the efficiency of control charts and enable comparisons between competing schemes. Historically,

traditional hypothesis testing measures of type I and type II errors served as performance measures for control charts until [65] introduced the concept of ARL. The run length (RL) is defined as the number of units or samples inspected until a signal is generated, which may indicate either a true OOC condition or a false alarm. Two important metrics in this context are ARL_0 , which represents the IC ARL, and ARL_1 , which denotes the OOC ARL. Ideally, the goal is to maximize ARL_0 when the process is in a state of control and minimize ARL_1 when the process is OOC. The evaluation of performance typically relies on the characteristics of the RL distribution. The following outlines key performance measures utilized in the analysis of control charts.

3.5.1 Zero-state Average Run Length

The zero-state ARL is the expected value of the RL when the computation begins at the start of the monitoring period (i.e., at time $i = 1$). It is expressed as $ARL_{\text{zero-state}} = \mathbb{E}(RL)$. This measure helps assess the performance of the chart under initial conditions.

3.5.2 Conditional Expected Delay

Conditional Expected Delay (CED) quantifies the average number of samples required to detect a change after it has occurred, given that the process was in control prior to the change and there were no previous false alarms. CED can be mathematically expressed as $CED_{\tau} = \mathbb{E}[(RL - \tau + 1) | (RL \geq \tau)]$, where τ denotes the unknown point at which the change occurs. This measure is particularly useful for evaluating the responsiveness of the control chart, as demonstrated by [66] in their recent study comparing the performance of several control charts.

3.5.3 Steady-state Average Run Length

The steady-state ARL ($ARL_{\text{steady-state}}$) is similar to its zero-state counterpart, but the computation begins when the process has been operating for a while without prior false signals and is in a stable condition.

It is defined as $ARL_{\text{steady-state}} = \lim_{\tau \rightarrow \infty} \mathbb{E}[(RL - \tau + 1) | (RL \geq \tau)]$. The steady-state ARL represents the limit of CED as τ approaches infinity. Recent studies, such as [67], have integrated this metric into their evaluations to provide a more comprehensive performance assessment.

3.5.4 Extra Quadratic Loss

Extra Quadratic Loss (EQL) is an important performance measure that accounts for the uncertainty in the magnitude of shifts in process parameters. Given that the exact shift magnitude is rarely known in advance, practitioners often seek a chart that exhibits good overall performance across a range of shifts. EQL is defined as the weighted expected ARL that considers shifts from a minimum to a maximum range (i.e., $\delta_{\max} - \delta_{\min}$) and uses the square of the shift as the weight. Mathematically, it is represented as $EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta$, where $ARL(\delta)$ denotes the OOC ARL for a shift of size δ .

3.5.5 Absolute Expected Loss

Similar to EQL, Absolute Expected Loss (AEL) is another overall performance measure that provides an expected weighted ARL using the absolute value of the shift as the weight function. It is expressed mathematically as $AEL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} |\delta| ARL(\delta) d\delta$. This measure is particularly useful for evaluating the performance of control charts across a

range of potential shifts. The choice of using the absolute value of delta instead of delta squared in the weight function is significant. In EQL, the weight is proportional to δ^2 , which assigns disproportionately low weight to shifts smaller than 1, as the square of a value less than 1 approaches zero. Conversely, shifts larger than 1 receive amplified weight due to squaring. AEL addresses this limitation by using the absolute value of delta, ensuring that all shifts, regardless of their size, contribute appropriately to the overall performance assessment of the control chart.

3.5.6 Sequential Extra Quadratic Loss

Sequential EQL (SEQL) refers to an EQL calculated progressively over a series of shifts. The evaluation starts with the smallest shift and incorporates one additional shift at each step until the final shift is reached. The final SEQL value corresponds to the EQL for the entire range of shifts. This measure aids in assessing the overall performance of a control chart across specific shift ranges.

These performance measures provide a comprehensive framework for evaluating the effectiveness of these charts across various operational scenarios. In the subsequent Section, we will present the results of our comparative study, discussing the performance metrics of the MEC chart in detail and illustrating its advantages in monitoring process shifts.

3.6 Results and Discussion

In this Section, we evaluate the performance of MEC control chart using key metrics such as the ARL, EQL, and AEL under both zero-state and steady-state conditions. The analysis encompasses small, moderate, and large parameter values to study their effects on the

detection performance of chart. We explore a range of k_M values from 0.125 to 3 and λ_M values from 0.01 to 1. The special case of λ_M equal to 1 simplifies the MEC chart to the classical CUSUM chart, while $\lambda_M = 1$ and $k_M = h_m$ corresponds to the classical Shewhart chart. A Monte Carlo simulation approach is employed to estimate the performance of the MEC control chart by generating random samples and calculating the ARL for different shifts in the process mean. The simulation uses 10^6 iterations to compute the ARL, and a search is performed to determine the decision interval h_M that yields ARL_0 approximately equal to 370.

The following algorithm outline the procedure used to compute the MEC ARL using simulation:

1. Specify τ (where $\tau = 1$ corresponds to the zero-state condition).
2. Generate random samples based on the following:
 - For sample number i less than τ , X_i is normally distributed with mean μ_0 and variance σ_0^2 .
 - For sample number i greater than or equal to τ , X_i is normally distributed with mean μ and variance σ_0^2 .
3. Compute the plotting statistics using the relevant formulas (equations 5 and 7).
4. Compare the plotting statistics with the control limit (from equation 8) and denote the first i for which the plotting statistics exceed the limit as r .
5. If r is less than τ , discard the value; otherwise, save it.
6. Repeat steps 2 to 5 a total of 10^6 times.

7. Compute the ARL by taking the mean of r , conditional on r being greater than or equal to τ , and subtract $\tau + 1$ from the result.

The performance of MEC control chart across various parameter settings is discussed in the following sections.

3.6.1 Performance Evaluation Under Zero-State Condition

The EQL metric assesses the overall performance of a control chart over the entire range of shifts. Smaller EQL values indicate better detection ability of shifts. There are certain parameter combinations for which the MEC chart with $ARL_0 = 370$ does not exist, as indicated by the (*) symbol in Tables 3.1 and 3.2.

Table 3.1: Zero- and steady-state EQL values of MEC control chart.

Zero-state									
λ_M	k_M								
	0.125	0.25	0.5	0.75	1	1.5	2	2.5	3
0.01	267.9132	239.0157	33.9723	146.9892	112.1147	61.0010	*	*	*
0.025	209.4027	187.3235	151.2752	122.9308	99.6575	63.9854	41.0289	*	*
0.05	165.0904	146.0910	117.8222	96.9490	80.4818	56.3164	38.9433	*	*
0.1	126.4136	109.5690	87.4637	72.5641	61.1478	45.2213	33.5855	28.4067	*
0.175	100.8031	85.3668	66.5829	55.6337	47.0487	35.3674	29.0487	24.0390	*
0.25	86.8803	72.2484	55.9796	46.1856	39.9217	31.1321	25.0454	21.6455	*
0.375	73.1363	59.3328	45.1806	36.8806	31.9678	26.4421	21.1334	19.9595	*
0.5	64.4493	50.5329	37.9541	31.6312	28.8741	22.2003	19.7598	19.6970	*
0.75	53.1480	40.6349	30.6360	25.6712	21.8358	19.5835	20.2607	21.3659	*
1	46.3701	33.9723	24.9039	20.5922	19.5808	21.1758	24.1427	26.2089	26.7919
Steady-state									
0.01	226.6431	207.9915	173.4082	139.9925	109.3752	60.3999	*	*	*
0.025	174.1998	161.1018	137.7157	116.1950	96.3152	63.3561	40.8457	*	*
0.05	135.6649	125.1107	107.2736	91.6542	77.8678	55.3115	38.8427	*	*
0.1	103.1400	94.1180	79.8700	68.5422	59.1317	44.3433	33.4553	27.4662	*
0.175	81.9707	73.5404	61.3297	52.7330	45.7726	35.3198	28.2183	23.9503	*
0.25	70.5654	62.3648	51.4180	43.9943	38.6101	30.8111	24.7808	21.9134	*
0.375	59.3849	51.3294	51.4180	35.3760	31.2606	25.9396	21.2907	20.2013	*
0.5	52.4251	44.2388	35.2568	30.3935	27.7506	22.1311	19.8507	19.7933	*
0.75	43.4774	35.3843	28.3184	24.3274	21.3034	19.4754	20.2276	21.3276	*
1	37.5910	29.8393	22.8436	19.8058	19.1996	21.0432	24.0654	26.1351	26.7304

From Table 3.1, it is evident that some values of the parameters (k_M and λ_M) lead to extremely large EQL values, indicating poor overall performance. However, for a fixed k_M , the EQL drastically decreases as λ_M increases.

For example, when $k_M = 0.125$, the EQL reduces by approximately 82% when λ_M changes from 0.01 to 1. The same trend can be observed when λ_M is fixed, and k_M is increased. For instance, fixing λ_M at 0.01 results in the EQL dropping from 267.91 at $k_M = 0.125$ to 61.00 at $k_M = 1.5$.

The overall performance improves when both k_M and λ_M are set at larger values. The best EQL from Table 3.1 is 19.581 at $k_M = 1$ and $\lambda_M = 1$, which corresponds to the classical CUSUM chart.

The best EQL for the MEC chart is 19.583 at $k_M = 1.5$ and $\lambda_M = 0.75$. Both values are nearly identical, indicating a close competition between the MEC and CUSUM charts.

The AEL, like EQL, measures the overall performance of control charts. However, because EQL gives too much weight to large shifts by squaring the shift magnitude, AEL provides a more balanced perspective by using the absolute value of the shift. The AEL results are given in Table 3.2.

Table 3.2: Zero- and steady-state AEL values of MEC control chart.

Zero-state									
λ_M	k_M								
	0.125	0.25	0.5	0.75	1	1.5	2	2.5	3
0.01	74.4776	66.5821	11.1971	41.5935	32.2195	18.5923	*	*	*
0.025	58.6200	52.5198	42.6172	34.8969	28.5967	19.0688	13.0602	*	*
0.05	46.6453	41.3156	33.4684	27.7676	23.3146	16.8296	12.3487	*	*
0.1	36.2516	31.4355	25.1927	21.0907	18.0333	13.8163	10.8731	9.5031	*
0.175	29.4025	24.8954	19.5840	16.5400	14.3074	11.3880	9.7938	8.6683	*
0.25	25.6954	21.3522	16.6977	14.0673	12.4561	10.3441	9.0540	8.3895	*
0.375	22.0348	17.8768	13.8325	11.7329	10.5538	9.4576	8.5748	8.5828	*
0.5	19.7191	15.5816	12.0256	10.4395	9.8548	8.8625	8.7988	9.2916	*
0.75	16.7048	12.9346	10.1645	9.2225	8.7823	9.2958	10.6953	11.8799	*
1	14.8033	11.1971	9.0145	8.5724	9.1750	11.9529	14.9576	16.5904	16.9782
Steady-state									
0.01	62.9188	57.8840	48.5690	39.5837	31.3821	18.3877	*	*	*
0.025	48.6697	45.0998	38.7545	32.9421	27.5988	18.8198	12.9357	*	*
0.05	38.2144	35.3063	30.4304	26.1970	22.4927	16.5051	12.2087	*	*
0.1	29.4287	26.9038	22.9707	19.9033	17.3947	13.5332	10.7632	9.2436	*
0.175	23.7538	21.3576	17.9721	15.6569	13.8609	11.2622	9.5550	8.5594	*
0.25	20.7180	18.3585	15.3159	13.3692	12.0284	10.1964	8.9177	8.3574	*
0.375	17.7557	15.4127	13.3159	11.2035	10.2781	9.2773	8.5325	8.5658	*
0.5	15.9199	13.5476	11.1357	10.0029	9.5167	8.7734	8.7578	9.2554	*
0.75	13.5621	11.2384	9.4220	8.7896	8.5712	9.2170	10.6494	11.8312	*
1	11.9974	9.8082	8.3396	8.2579	8.9965	11.8740	14.8999	16.5407	16.9332

As expected, the AEL values are much smaller than their corresponding EQL values. For small values of k_M and λ_M , the AEL is quite large, indicating poor overall performance. In particular, the row for $\lambda_M = 1$ in Table 3.2 corresponds to the AEL values for the classical CUSUM chart, which shows the smallest values across most columns except for $k_M \geq 1$. This suggests that the MEC chart is inferior to the classical CUSUM chart when $k_M < 1$.

Nevertheless, the minimum AEL in Table 3.2 is 8.389, corresponding to the MEC chart with $k_M = 2.5$ and $\lambda_M = 0.25$. In general, moderate to large values of both k_M and λ_M produce smaller AEL values, indicating better overall performance in these regions.

3.6.2 Performance Evaluation Under Steady-State Condition

In this Section, we reevaluate the MEC chart properties under steady-state conditions, focusing primarily on its ARL characteristics. Steady-state ARLs are calculated after the process has been running for some time and has stabilized, which allows for a more reliable comparison between different control chart designs.

For this analysis, the change point is set at $\tau = 201$. Various other values of τ , ranging from $\tau = 51$ to $\tau = 501$, were considered. The ARL values of MEC chart appears to be most stable at $\tau = 501$, but for computational efficiency, we have set $\tau = 201$, as the results are nearly identical to those obtained at $\tau = 501$.

Table 3.3: Steady-state ARL profiles of MEC control chart. Zero-state $ARL_0 \cong 370$, $k_M = 0.125$ and $\tau = 201$.

δ	λ_M									
	0.01	0.025	0.05	0.1	0.175	0.25	0.375	0.5	0.75	1
0	250.75	261.49	268.45	277.10	287.36	295.32	305.23	313.14	325.92	334.80
0.25	87.10	76.65	70.09	65.80	63.73	62.93	62.32	62.30	62.62	62.10
0.5	57.37	47.30	40.23	34.84	31.88	30.44	29.06	28.18	26.96	26.10
0.75	45.50	36.66	30.23	25.08	22.05	20.58	19.17	18.29	17.08	16.14
1	38.78	30.82	25.02	20.27	17.38	15.89	14.52	13.68	12.54	11.70
1.25	34.29	27.06	21.75	17.36	14.59	13.16	11.83	11.02	9.97	9.19
1.5	31.05	24.36	19.44	15.34	12.74	11.38	10.09	9.32	8.31	7.59
1.75	28.61	22.33	17.73	13.87	11.40	10.10	8.86	8.11	7.16	6.49
2	26.61	20.71	16.37	12.73	10.39	9.15	7.96	7.23	6.31	5.67
2.25	25.00	19.40	15.29	11.82	9.59	8.40	7.25	6.54	5.65	5.06
2.5	23.63	18.30	14.38	11.08	8.94	7.79	6.68	6.00	5.14	4.57
2.75	22.47	17.37	13.61	10.45	8.39	7.30	6.22	5.56	4.73	4.17
3	21.46	16.56	12.95	9.91	7.93	6.88	5.84	5.20	4.39	3.85
3.25	20.57	15.86	12.38	9.44	7.54	6.51	5.51	4.89	4.10	3.58
3.5	19.79	15.22	11.87	9.04	7.19	6.20	5.22	4.62	3.85	3.35
3.75	19.08	14.67	11.43	8.68	6.89	5.93	4.98	4.39	3.64	3.14
4	18.46	14.17	11.02	8.35	6.62	5.68	4.76	4.19	3.45	2.97
4.25	17.88	13.72	10.66	8.07	6.38	5.47	4.57	4.01	3.30	2.82
4.5	17.36	13.31	10.33	7.80	6.16	5.27	4.39	3.85	3.15	2.69
4.75	16.87	12.91	10.01	7.57	5.96	5.09	4.24	3.70	3.02	2.57
5	16.42	12.57	9.74	7.35	5.78	4.93	4.10	3.57	2.90	2.47
5.25	16.01	12.24	9.48	7.14	5.61	4.79	3.97	3.45	2.79	2.38
5.5	15.63	11.94	9.24	6.95	5.46	4.65	3.86	3.35	2.69	2.29
5.75	15.26	11.67	9.01	6.78	5.32	4.52	3.74	3.26	2.62	2.19
6	14.94	11.41	8.82	6.62	5.18	4.40	3.63	3.17	2.55	2.11

From Table 3.3, starting with $k_M = 0.125$, it becomes evident that for small values of λ_M , the ARL_0 is as low as 250, indicating a higher likelihood of false alarms for smaller values of k_M . As λ_M approaches 1, the ARL_0 steadily increases, reaching a peak of 334.8 at $\lambda_M = 1$. For ARL_1 , the performance of MEC chart improves as λ_M increases. From Table 3.3, it is clear that $\lambda_M = 1$ produces the smallest ARL values for all shifts, particularly at lower k_M . However, this trend changes as k_M increases beyond 0.25. For $k_M = 0.5$ and larger, moderate values of λ_M (around 0.1) produce smaller ARL values.

Table 3.4. Steady-state ARL profiles of MEC control chart with zero-state $ARL_0 \cong 370$,
 $k_M = 0.25$ and $\tau = 201$.

δ	λ_M									
	0.01	0.025	0.05	0.1	0.175	0.25	0.375	0.5	0.75	1
0	268.61	283.01	293.71	306.03	318.57	326.74	335.12	340	348.69	354.59
0.25	83.88	74.2	68.1	64.41	63.14	63.16	63.97	65.54	70.52	77.53
0.5	54.05	44.7	37.87	32.52	29.47	27.95	26.64	25.89	25.34	25.66
0.75	42.51	34.38	28.21	23.1	19.97	18.39	16.9	15.98	14.87	14.27
1	36.09	28.81	23.26	18.58	15.61	14.05	12.58	11.69	10.56	9.85
1.25	31.81	25.19	20.16	15.88	13.08	11.6	10.18	9.31	8.23	7.53
1.5	28.74	22.66	18.01	14.02	11.41	10.01	8.65	7.83	6.78	6.11
1.75	26.41	20.74	16.4	12.67	10.21	8.89	7.59	6.8	5.81	5.16
2	24.55	19.23	15.14	11.63	9.3	8.04	6.81	6.05	5.1	4.49
2.25	23.04	17.99	14.12	10.79	8.59	7.39	6.21	5.48	4.56	3.97
2.5	21.76	16.96	13.28	10.11	8	6.86	5.73	5.03	4.15	3.58
2.75	20.68	16.09	12.56	9.54	7.52	6.42	5.34	4.66	3.81	3.26
3	19.72	15.34	11.95	9.04	7.11	6.05	5.02	4.36	3.54	3.01
3.25	18.91	14.67	11.42	8.61	6.76	5.74	4.73	4.1	3.31	2.8
3.5	18.16	14.08	10.95	8.25	6.45	5.47	4.5	3.89	3.11	2.62
3.75	17.52	13.56	10.54	7.91	6.18	5.23	4.29	3.7	2.94	2.46
4	16.93	13.1	10.16	7.62	5.94	5.02	4.11	3.53	2.8	2.32
4.25	16.39	12.68	9.82	7.36	5.72	4.83	3.94	3.39	2.68	2.2
4.5	15.91	12.29	9.51	7.12	5.52	4.65	3.79	3.26	2.58	2.09
4.75	15.46	11.94	9.24	6.9	5.34	4.5	3.66	3.13	2.48	2
5	15.04	11.61	8.97	6.7	5.18	4.36	3.54	3.01	2.38	1.93
5.25	14.66	11.31	8.73	6.52	5.04	4.24	3.44	2.91	2.28	1.87
5.5	14.31	11.03	8.51	6.34	4.9	4.12	3.35	2.82	2.19	1.82
5.75	13.98	10.77	8.31	6.18	4.78	4.01	3.26	2.75	2.1	1.79
6	13.67	10.52	8.11	6.04	4.65	3.9	3.17	2.69	2.03	1.75

Table 3.5. Steady-state ARL profiles of MEC control chart with zero-state $ARL_0 \cong 370$, $k_M = 0.5$ and $\tau = 201$.

δ	λ_M									
	0.01	0.025	0.05	0.1	0.175	0.25	0.375	0.5	0.75	1
0	297.21	310.34	324.02	334.74	343.85	348.69	348.69	356.22	361.08	363.04
0.25	78.55	70.63	66.2	64.8	66.51	69.44	69.44	82.45	98.09	118.38
0.5	48.11	40.32	34.25	29.44	26.9	25.98	25.98	26.21	28.78	33.61
0.75	37.1	30.44	24.98	20.28	17.35	15.96	15.96	14.28	14.22	15.15
1	31.13	25.27	20.4	16.1	13.31	11.87	11.87	9.87	9.22	9.18
1.25	27.25	21.98	17.59	13.64	11.05	9.68	9.68	7.64	6.84	6.52
1.5	24.49	19.69	15.64	12.01	9.59	8.29	8.29	6.32	5.48	5.06
1.75	22.4	17.95	14.19	10.82	8.56	7.34	7.34	5.44	4.6	4.15
2	20.74	16.6	13.08	9.91	7.78	6.63	6.63	4.82	4	3.53
2.25	19.41	15.49	12.18	9.19	7.17	6.08	6.08	4.35	3.56	3.09
2.5	18.29	14.58	11.44	8.6	6.68	5.64	5.64	3.99	3.22	2.75
2.75	17.34	13.81	10.81	8.1	6.27	5.28	5.28	3.7	2.95	2.49
3	16.52	13.14	10.27	7.68	5.93	4.97	4.97	3.45	2.73	2.29
3.25	15.81	12.57	9.8	7.32	5.63	4.72	4.72	3.25	2.55	2.12
3.5	15.17	12.05	9.4	6.99	5.38	4.49	4.49	3.07	2.4	1.99
3.75	14.61	11.6	9.03	6.71	5.15	4.3	4.3	2.93	2.26	1.89
4	14.1	11.19	8.7	6.46	4.94	4.12	4.12	2.81	2.14	1.79
4.25	13.63	10.82	8.4	6.23	4.76	3.96	3.96	2.7	2.05	1.71
4.5	13.22	10.48	8.14	6.03	4.6	3.82	3.82	2.61	1.98	1.62
4.75	12.84	10.17	7.9	5.84	4.46	3.7	3.7	2.51	1.92	1.54
5	12.49	9.89	7.67	5.67	4.32	3.6	3.6	2.42	1.88	1.46
5.25	12.16	9.63	7.46	5.51	4.2	3.5	3.5	2.32	1.85	1.37
5.5	11.86	9.39	7.27	5.37	4.07	3.4	3.4	2.22	1.81	1.29
5.75	11.58	9.16	7.1	5.24	3.96	3.3	3.3	2.14	1.78	1.22
6	11.32	8.96	6.93	5.11	3.85	3.2	3.2	2.06	1.74	1.16

It is also seen from Tables 3.4 and 3.5 that as both k_M and λ_M increase, the ARL_0 continues to improve, moving closer to the desired ARL_0 value of 370. This indicates that under steady-state conditions, the MEC chart exhibits a lower false alarm rate with larger design parameter values.

From Tables 3.6 and 3.7, we observe that larger values of k_M (ranging from 1.5 to 2.5) tend to perform better with smaller values of λ_M . For instance, with $k_M = 2$ and $\lambda_M = 0.025$ (as shown in Table 3.6), only 26.89 samples are required on average to detect a shift of 0.5

standard deviations, whereas about 93 samples are needed for the same shift when $\lambda_M = 0.75$.

Additionally, from Figure 3.1, it is noted that for larger values of k_M , different λ_M values exhibit nearly identical performance when detecting large shifts ($\delta \geq 3$).

Table 3.6: Steady-state ARL profiles of MEC control chart. Zero-state $ARL_0 \cong 370$, $k_M = 2$ and $\tau = 201$.

δ	λ_M									
	0.01	0.025	0.05	0.1	0.175	0.25	0.375	0.5	0.75	1
0	346.84	353.78	360.44	363.11	364.58	367.04	367.99	367.72	368.36	346.84
0.25	64.28	69.81	83.92	103.9	122.07	149.57	174.99	225.33	272.77	64.28
0.5	26.89	25.87	26.98	30.92	36.12	46.4	58.81	93.01	142.97	26.89
0.75	16.82	15.4	14.52	14.87	16.1	19.42	24.13	40.16	70.82	16.82
1	12.32	11.09	9.93	9.42	9.54	10.52	12.3	19.52	36.25	12.32
1.25	9.77	8.77	7.66	6.92	6.68	6.84	7.5	10.8	19.65	9.77
1.5	8.13	7.32	6.3	5.55	5.18	5.03	5.22	6.75	11.42	8.13
1.75	6.99	6.31	5.4	4.68	4.29	4	3.97	4.64	7.13	6.99
2	6.15	5.58	4.76	4.07	3.69	3.34	3.21	3.47	4.8	6.15
2.25	5.5	5.02	4.28	3.63	3.26	2.9	2.72	2.75	3.45	5.5
2.5	4.98	4.58	3.9	3.3	2.93	2.57	2.37	2.29	2.65	4.98
2.75	4.55	4.22	3.59	3.03	2.68	2.32	2.11	1.97	2.12	4.55
3	4.21	3.93	3.34	2.81	2.47	2.12	1.91	1.73	1.79	4.21
3.25	3.91	3.67	3.13	2.63	2.3	1.96	1.75	1.55	1.55	3.91
3.5	3.66	3.46	2.95	2.47	2.16	1.83	1.62	1.41	1.38	3.66
3.75	3.44	3.27	2.8	2.33	2.03	1.71	1.5	1.3	1.26	3.44
4	3.25	3.11	2.66	2.21	1.92	1.61	1.41	1.21	1.17	3.25
4.25	3.08	2.96	2.54	2.1	1.83	1.52	1.32	1.15	1.11	3.08
4.5	2.93	2.83	2.42	2.01	1.74	1.43	1.24	1.1	1.07	2.93
4.75	2.8	2.7	2.32	1.92	1.67	1.35	1.18	1.06	1.04	2.8
5	2.68	2.6	2.23	1.85	1.59	1.28	1.13	1.04	1.02	2.68
5.25	2.57	2.49	2.15	1.78	1.52	1.22	1.09	1.02	1.01	2.57
5.5	2.48	2.4	2.07	1.72	1.45	1.16	1.06	1.01	1	2.48
5.75	2.39	2.32	2	1.66	1.39	1.12	1.03	1	1	2.39
6	2.31	2.24	1.94	1.6	1.32	1.08	1.02	1	1	2.31

Table 3.7. Steady-state ARL profiles of MEC control chart with zero-state $ARL_0 \cong 370$, $k_M = 2.5$ & 3 and $\tau = 201$.

δ	λ_M							
	0.1	0.175	0.25	0.375	0.5	0.75	1	1 (k=3)
0	360.9	363.72	365.42	366.41	369.26	368.23	368.92	369.49
0.25	86.74	109.96	130.92	162.55	191.36	239.64	279.09	280.09
0.5	27.31	32.69	39.45	52.55	68.34	106.35	153.19	154.86
0.75	14.31	15.28	17.22	21.89	28.42	48.13	79.48	80.98
1	9.52	9.37	9.84	11.57	14.31	24	42.49	43.69
1.25	7.12	6.68	6.69	7.27	8.46	13.29	23.95	24.88
1.5	5.71	5.2	5.04	5.17	5.67	8.13	14.24	14.94
1.75	4.79	4.28	4.06	3.99	4.18	5.43	8.92	9.44
2	4.14	3.65	3.41	3.26	3.29	3.92	5.92	6.28
2.25	3.66	3.2	2.96	2.77	2.72	3	4.15	4.4
2.5	3.29	2.86	2.63	2.42	2.33	2.42	3.06	3.24
2.75	3	2.6	2.37	2.16	2.05	2.03	2.37	2.48
3	2.76	2.38	2.17	1.96	1.84	1.76	1.92	2
3.25	2.56	2.21	2.01	1.8	1.67	1.55	1.62	1.67
3.5	2.39	2.06	1.87	1.67	1.54	1.4	1.41	1.44
3.75	2.26	1.94	1.76	1.56	1.42	1.29	1.28	1.29
4	2.13	1.84	1.66	1.46	1.33	1.2	1.18	1.19
4.25	2.03	1.75	1.58	1.38	1.25	1.13	1.11	1.11
4.5	1.94	1.67	1.5	1.3	1.18	1.09	1.07	1.07
4.75	1.85	1.6	1.42	1.23	1.13	1.05	1.04	1.04
5	1.78	1.53	1.36	1.18	1.09	1.03	1.02	1.02
5.25	1.72	1.47	1.3	1.13	1.06	1.02	1.01	1.01
5.5	1.66	1.41	1.24	1.09	1.04	1.01	1	1
5.75	1.6	1.35	1.19	1.06	1.02	1	1	1
6	1.55	1.3	1.15	1.04	1.01	1	1	1

Steady-State EQL Performance: As in the previous Section, the overall performance of the MEC chart is analyzed under steady-state conditions using the EQL metric. The lower rows of Table 3.1 present the steady-state EQL values of MEC chart. The same trends observed under the zero-state condition apply here as well.

From Table 3.5, it is clear that nearly all EQL values below 20 occur when $k_M \geq 1$ and $\lambda_M \geq 0.5$. In general, larger values of the MEC design parameters are preferred when the goal is to optimize overall performance.

For a graphical representation, Figure 3.2 illustrates the effect of the parameters k_M and λ_M on the overall performance, as measured by EQL. Clearly, increasing the values of both parameters up to certain limits has a positive impact on the performance of MEC chart.

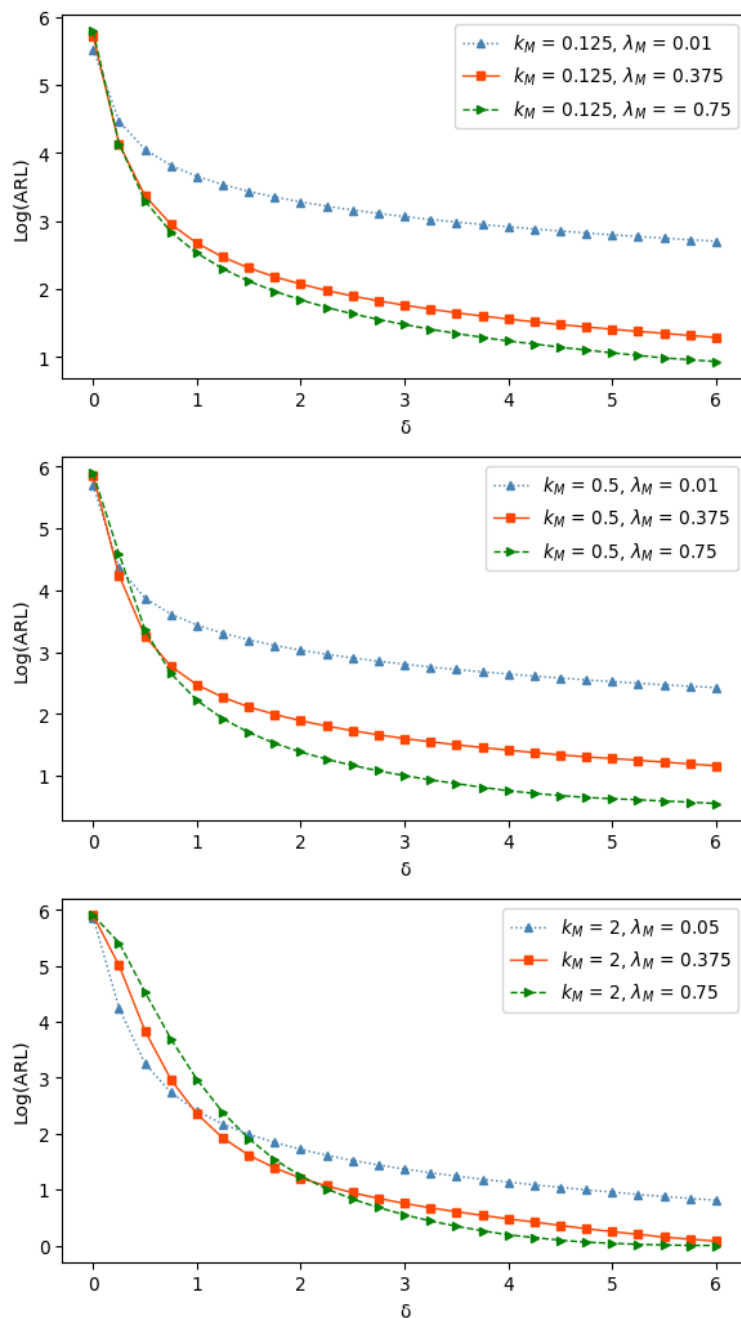


Figure 3.1: Steady-state ARL profiles (in logarithm scale) of nine designs of MEC control chart for shifts $\delta = 0$ to $\delta = 6$. Zero-state $ARL_0 \cong 370$.

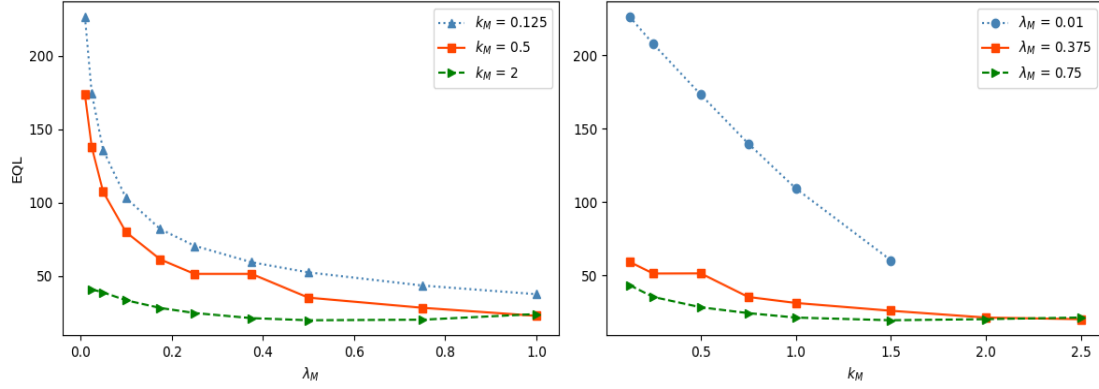


Figure 3.2: Effect of k_M and λ_M on EQL performance of MEC chart.

Steady-State AEL Performance: The steady-state AEL values of MEC chart are also provided in Table 3.2. As seen in the zero-state scenario, the largest AEL values occur when both k_M and λ_M are at their smallest values. By fixing λ_M , we also observe that AEL steadily decreases as k_M increases. For instance, the AEL value decreases from 62.9188 at $k_M = 0.125$ and $\lambda_M = 0.01$ to 18.3877 at $k_M = 1.5$ for the same value of λ_M .

Among all parameter combinations considered, the lowest AEL value of 8.3574 occurs when $k_M = 2.5$ and $\lambda_M = 0.25$. This value is slightly lower than the 8.389 observed under the zero-state condition.

3.7 Comparative Analysis

This Section presents a detailed comparison between the MEC chart and two other memory-type control charts: the EWMA and CUSUM charts. Since the MEC chart combines features from both EWMA and CUSUM charts, we use the same parameter values for the comparisons. Various methods exist for comparing control charts, including

direct comparisons of performance metrics such as the ARL or ensuring equivalent variance properties for the competing charts (for details, see [68]).

However, to simplify the comparison process, we focus on the CED profiles and overall performance using the EQL and AEL metrics.

For a meaningful comparison, the performance of the MEC chart is evaluated based on both zero-state and steady-state conditions. In zero-state comparisons, the process starts under IC conditions, whereas in steady-state comparisons, we assume the process has been operating for some time, allowing for a deeper understanding of long-term performance. The metrics we examine include the EQL, AEL, and SEQL values for different process shifts. By comparing these metrics, we assess how well the charts detect both small and large shifts in the process mean, helping practitioners choose the best control chart depending on the situation.

3.7.1 Zero-State Comparisons

In this subsection, we compare the performance of the MEC, EWMA, and CUSUM charts based on zero-state conditions, where the process is assumed to start under IC conditions. We first focus on the EQL values, followed by the AEL metrics.

EQL Comparisons: Table 3.1 presents the zero-state EQL values for the MEC chart. The values less than 20 are highlighted in bold, and it is observed that the chart generally performs better for $k_M \geq 1$ and $\lambda_M \geq 0.375$. The minimum EQL value is attained when both k_M and λ_M are equal to 1, which suggests that the CUSUM chart with $k_C = 1$ performs slightly better than the MEC chart designs considered here. From Table 3.8, the lowest EQL value of EWMA chart is 19.8530 at $\lambda_E = 0.5$, which is slightly higher than the EQL

of MEC chart at $k_M = 1.5$ and $\lambda_M = 0.75$. This indicates the superiority of MEC chart over the EWMA chart in this context. In summary, the order of performance based on EQL is: CUSUM, MEC, and EWMA, with CUSUM leading the way.

AEL Comparisons: Next, we turn to the AEL metric, which provides a different perspective on chart performance due to its unique weighing method. As with EQL, larger k_M values tend to result in smaller AEL values. However, unlike EQL, moderate values of λ_M seem to offer better performance. Table 3.2 presents the AEL values for the MEC chart. The minimum AEL value of 8.3895 occurs at $\lambda_M = 0.25$ and $k_M = 2.5$, with nearly all values close to 8 occurring at $k_M \geq 1.5$. For the EWMA chart, the minimum AEL value is also at $\lambda_E = 0.25$ and is slightly lower than the AEL of MEC chart (see Table 3.8). The minimum AEL value of CUSUM chart is 8.5750 at $k_C = 0.75$. Consequently, the order of performance based on AEL is: EWMA, MEC, and CUSUM. Thus, when comparing the charts based on EQL and AEL, the MEC chart consistently ranks second in performance, with CUSUM leading in EQL and EWMA performing best in AEL.

SEQL Comparisons: Finally, we examine the SEQL values to understand how the charts perform across varying shifts. Table 3.9 presents these values. For shifts up to $\delta = 1$, the EWMA chart exhibits the best performance, with the MEC chart closely trailing. For shifts ranging from $\delta = 1.25$ to $\delta = 4$, the MEC chart generally performs better, although CUSUM and EWMA occasionally outperform it in specific scenarios. For $\delta > 4$, the CUSUM chart dominates, although the MEC chart remains competitive.

Table 3.8. EWMA and CUSUM overall performance under both zero- and steady-state conditions using EQL and AEL metrics

EWMA					CUSUM				
	Zero-state		Steady-state			Zero-state		Steady-state	
λ_E	EQL	AEL	EQL	AEL	k_C	EQL	AEL	EQL	AEL
0.01	47.9397	15.1407	47.4117	37.591	0.125	46.3217	14.7884	37.591	11.9974
0.025	38.7397	12.4472	38.7579	29.8393	0.25	33.9741	11.1987	29.8393	9.8082
0.05	32.9309	10.7152	32.5849	22.8436	0.5	24.9044	9.0156	22.8436	8.3396
0.1	28.2043	9.4534	27.2624	19.8058	0.75	20.5995	8.575	19.8058	8.2579
0.175	23.6592	8.6006	23.66	19.1996	1	19.5742	9.1734	19.1996	8.9965
0.25	21.3753	8.376	21.6975	21.0432	1.5	21.1803	11.9557	21.0432	11.874
0.375	19.9223	8.6902	20.1824	24.0654	2	24.1269	14.9412	24.0654	14.8999
0.5	19.853	9.491	19.9149	26.1351	2.5	26.2144	16.5988	26.1351	16.5407
0.75	21.7631	12.2247	21.7323	26.7304	3	26.798	16.9801	26.7304	16.9332
1	26.8056	16.9915	26.7367	37.591					

In conclusion, the zero-state comparisons show that while no single chart consistently outperforms the others across all metrics, the MEC chart offers a good balance, particularly for moderate shifts. The choice of control chart should depend on the specific process shift magnitude and the focus of practitioner, whether it is minimizing EQL, AEL, or SEQL.

Table 3.9. Zero- and steady state minimum Sequential EQL values of MEC, CUSUM, and EWMA control charts.

δ	Zero-state			Steady-state		
	MEC	CUSUM	EWMA	MEC	EWMA	CUSUM
0.5	5.5647	6.2194	5.5400	5.266	5.353	5.266
0.75	6.7069	7.2131	6.6027	6.360	6.406	6.426
1	7.6552	8.2460	7.6290	7.263	7.421	7.263
1.25	8.3791	9.2967	8.3851	8.149	8.171	8.149
1.5	9.1485	9.7872	9.1474	8.866	8.927	9.071
1.75	9.9295	10.3471	9.9481	9.590	9.721	9.622
2	10.5276	10.9627	10.5447	10.164	10.329	10.164
2.25	11.1700	11.6237	11.1679	10.753	10.946	10.753
2.5	11.8490	12.3216	11.8292	11.383	11.604	11.383
2.75	12.4167	12.7993	12.4523	12.045	12.222	12.045
3	13.0280	13.2779	13.0377	12.660	12.791	12.66
3.25	13.6814	13.8046	13.6659	13.148	13.399	13.148
3.5	14.2968	14.3701	14.3345	13.672	14.044	13.672
3.75	14.7891	14.9605	14.9078	14.224	14.676	14.224
4	15.2977	15.4036	15.3932	14.795	15.187	14.795
4.25	15.7686	15.7951	15.8862	15.366	15.726	15.366
4.5	16.2193	16.2103	16.3915	15.783	16.285	15.783
4.75	16.6664	16.6579	16.9116	16.231	16.865	16.231
5	17.1523	17.1443	17.4465	16.721	17.472	16.721
5.25	17.6799	17.6724	17.9929	17.257	18.026	17.257
5.5	18.2552	18.2480	18.5533	17.846	18.598	17.846
5.75	18.8856	18.8787	19.1702	18.495	19.227	18.495
6	19.5808	19.5742	19.8530	19.200	19.915	19.2

3.7.2 Steady-State Comparisons

In this subsection, we analyze and compare the performance of the MEC, EWMA, and CUSUM charts under steady-state conditions. In the steady-state, the process is allowed to reach its natural variability, simulating the long-term operation of a process.

EQL Comparisons: In Table 3.3, we presented the steady-state EQL values for the MEC chart. The performance trends are consistent with the zero-state results, where larger k_M values (greater than or equal to 1) result in better performance. The smallest EQL value is observed when $k_M = 1$ and $\lambda_M = 0.75$, with a value of 17.3321. This is comparable to the CUSUM chart, whose best steady-state EQL value is 17.2503 at $k_C = 1$. For the EWMA

chart, the minimum steady-state EQL value is 18.1540 at $\lambda_E = 0.25$ (see Table 3.8). Therefore, as with the zero-state comparison, the CUSUM chart performs slightly better than the MEC and EWMA charts in terms of EQL, with the MEC chart ranking in second place. As shown in Figure 3.3, the MEC, EWMA, and CUSUM charts exhibit nearly identical performance across all shifts. This comparison is made under the condition that each chart is designed to achieve the minimum EQL value within its respective scheme.

AEL Comparisons: Moving to the AEL metric, the results in steady-state show a slightly different trend. Table 3.4 shows that the MEC chart achieves its minimum AEL value of 8.2345 at $k_M = 2.0$ and $\lambda_M = 0.5$. This value is very close to the minimum steady-state AEL value of the EWMA chart, which is 8.1748 at $\lambda_E = 0.25$ (see Table 3.8). On the other hand, the CUSUM chart has a minimum steady-state AEL value of 8.4532 at $k_C = 1.25$. Thus, based on AEL, the order of performance in the steady-state scenario mirrors that of the zero-state comparison: EWMA, MEC, and CUSUM.

SEQL Comparisons: Next, we examine the SEQL values, which capture the dynamic behavior of the charts for various shifts over time. Table 3.9 presents the SEQL values for the different charts. As observed in the zero-state comparison, for small shifts ($\delta \leq 1.25$), the EWMA chart maintains its superiority, with the MEC chart being a close contender. For moderate shifts, $1.5 \leq \delta \leq 4$, the MEC chart displays its strength by achieving lower SEQL values than both the EWMA and CUSUM charts. For $\delta > 4$, the CUSUM chart dominates the SEQL metric, though the MEC chart still performs reasonably well.

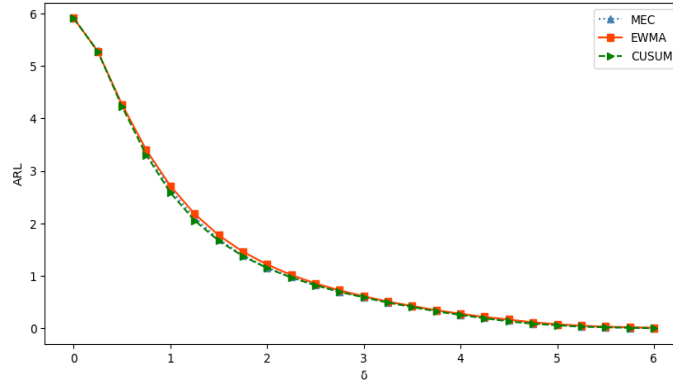


Figure 3.3: Steady-state ARL profiles (in logarithm scale) of MEC, EWMA, and CUSUM charts having minimum EQL values.

The steady-state comparisons reinforce the zero-state findings, with the MEC chart performing best for moderate shifts, while the CUSUM chart excels in detecting larger shifts, and the EWMA chart remains optimal for small shifts. The MEC chart provides a balanced alternative to the other two, making it a viable choice across different process shift scenarios.

- In terms of EQL, the CUSUM chart is marginally superior to the MEC and EWMA charts.
- Based on AEL, the EWMA chart outperforms both the MEC and CUSUM charts.
- For SEQL, the MEC chart shows excellent performance for moderate shifts, while the EWMA chart is best for small shifts and the CUSUM chart for large shifts.

Practitioners should select the control chart that aligns with their specific process monitoring needs, considering the expected magnitude of shifts and their focus on minimizing EQL, AEL, or SEQL.

3.8 Design of MEC Chart

In some cases, the practitioner may want to optimize the control chart to detect a specific shift with the minimum ARL. To this end, we provide the steady-state optimal values of the pair (k_M, λ_M) for each anticipated shift size (δ_{Target}) and their corresponding control limit h_M coefficient in Table 3.10. For a specified δ_{Target} , the unique combination (k_M, λ_M, h_M) gives the minimum ARL_1 (ARL^*). It is observed that for all the different shift magnitudes presented here, except for $\delta_{\text{Target}} = 0.25$, the MEC chart is optimized at $\lambda_M = 0.75$ with different values of k_M depending on the size of the shift. For $\delta_{\text{Target}} \geq 3.5$, the optimal setting is $k_M = 2.5$ and $\lambda_M = 0.75$.

Using the information from Table 3.10 and the 3D plot in Figure 3.4, we summarize the relationship among the parameters, magnitude of shifts, and the optimal ARL as follows:

- For almost all $\delta_{\text{Target}} \geq 0.5$, λ_M remains constant at 0.75, suggesting this value provides a reliable optimization parameter across a wide range of shifts. This simplifies the control chart configuration since λ_M does not need to be adjusted frequently. The exception is $\delta_{\text{Target}} = 0.25$, where $\lambda_M = 0.5$, indicating that smaller shifts may require a different value for λ_M to optimize the ARL^* .
- k_M increases with increasing δ_{Target} , but shows a nonlinear relationship:
 - At low target shifts (e.g., $\delta_{\text{Target}} = 0.25$ to $\delta_{\text{Target}} = 0.75$), k_M changes from 0.125 to 0.5.
 - For medium target shifts (e.g., $\delta_{\text{Target}} = 1$ to $\delta_{\text{Target}} = 2.5$), k_M ranges from 0.5 to 1.5.

- For higher target shifts ($\delta_{\text{Target}} \geq 2.75$), k_M reaches 2 or higher.
- This gradual increase suggests that as the magnitude of the shift increases, a larger value of k_M is necessary to detect it efficiently with minimal ARL.
- The control limit h_M decreases as δ_{Target} increases:
 - At $\delta_{\text{Target}} = 0.25$, $h_M = 24.72$, indicating that a larger control limit is required for smaller shifts.
 - At $\delta_{\text{Target}} = 4$, $h_M = 0.505$, indicating a much smaller control limit for larger shifts.
- This trend shows that larger shifts require tighter control limits to detect OOC situations quickly.
- ARL* decreases dramatically with increasing δ_{Target} :
 - For $\delta_{\text{Target}} = 0.25$, ARL* is 62.30, meaning it takes more time to detect small shifts.
 - For larger shifts, such as $\delta_{\text{Target}} = 4$, ARL* drops to 1.20, meaning the process can detect large shifts quickly.

For $\delta_{\text{Target}} = 0.25$, both λ_M and k_M take lower values (0.5 and 0.125, respectively), indicating a unique optimization for small shifts. This is in contrast to larger shifts, where k_M and λ_M stabilize at higher values.

At higher shift magnitudes ($\delta_{\text{Target}} \geq 3.5$), the combination of $k_M = 2.5$ and $\lambda_M = 0.75$ remains constant, while h_M reduces significantly. This suggests the process stabilizes at a

particular combination for large shifts, optimizing detection without further parameter changes.

The relationship between k_M and h_M is nonlinear, as seen in the 3D plot in Figure 3.4. It can be observed that while k_M increases steadily with δ_{Target} , the corresponding decrease in h_M is much steeper, especially at higher shift values. Figure 3.4 can be used as a guide to determine the value of the control limit h_M for any pair given optimal pairs k_M and λ_M .

Table 3.10. The MEC chart optimal values of k_M and λ_M and corresponding limit h_M for target shifts. $\tau = 201$.

δ	k_M	λ_M	h_M	ARL*
0.25	0.125	0.5	24.72	62.30
0.5	0.25	0.75	11.47	25.34
0.75	0.5	0.75	6.815	14.22
1	0.5	0.75	6.815	9.22
1.25	0.75	0.75	4.650	6.50
1.5	1	0.75	3.360	4.92
1.75	1	0.75	3.360	3.92
2	1.5	0.75	1.870	3.18
2.25	1.5	0.75	1.870	2.65
2.5	1.5	0.75	1.870	2.28
2.75	2	0.75	1.068	1.97
3	2	0.75	1.068	1.73
3.25	2	0.75	1.068	1.55
3.5	2.5	0.75	0.504	1.4
3.75	2.5	0.75	0.504	1.29
4	2.5	0.75	0.505	1.2

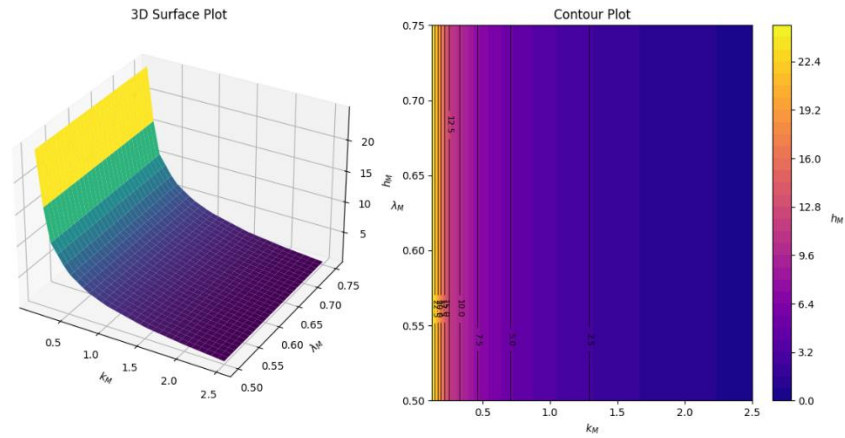


Figure 3.4: 3D Surface Plot and Contour Plot of k_M , λ_M , and h_M

The above analysis indicates that practitioners must carefully balance these parameters based on the anticipated shift magnitude to ensure efficient detection. Understanding the interaction between k_M , λ_M , and h_M is crucial for minimizing ARL.

For smaller shifts, lower values of k_M and λ_M require larger control limits h_M , while for more significant shifts, higher k_M with a stable λ_M and smaller h_M achieve the desired ARL.

3.9 Real-Time Monitoring of Water pH Levels in Aquatic Ecotoxicology

This Section illustrates the practical application of the MEC chart, alongside the CUSUM and EWMA charts, for monitoring water pH levels in an aquatic ecotoxicology laboratory.

The goal is to assess the performance of these control charts in detecting changes in water pH, a critical factor in ensuring optimal living conditions for aquatic organisms in the lab.

The laboratory under study maintains a controlled environment for mysids, small crustaceans used in water toxicity assessments. Consistent monitoring of water pH is crucial to maintaining the health of these organisms and ensuring the accuracy of ecotoxicological experiments. In this application, we aim to detect any deviations from the target pH level of 8.2, which could indicate a potential issue in the water quality management system.

To achieve this, the MEC chart is applied alongside traditional control charts like the CUSUM, EWMA, and Shewhart charts. A fair comparison is ensured by setting the parameters of all charts such that their ARL_0 is 370. The control charts are then used to analyze the pH data, which includes both an IC period and an OOC period caused by scheduled maintenance in the laboratory, leading to a known disruption in water

circulation. Further details about the data, assumptions, and control chart setup are provided in the following subsections.

3.9.1 pH Data Description

The ecotoxicology laboratory is designed to maintain a controlled setting for investigating the harmful effects of chemicals on aquatic life, particularly focusing on mysids. These small crustaceans, measuring approximately 1 cm in length, inhabit both fresh and saltwater environments and are frequently used as indicators in water toxicity assessments. The laboratory houses mysids in tanks filled with either fresh or saltwater, and the water undergoes continuous circulation and filtration for 22 hours daily to ensure cleanliness and prevent algae growth. The laboratory supervisor provides food for the mysids twice daily. The facility maintains specific water conditions critical for mysid health, including a temperature of 23.8°C (75°F), illumination at 941.775 LUX, a pH level of 8.2, an ammonia (NH₃) concentration of 0.1 mg/L, and salinity ranging from 20 to 22 parts per thousand (ppt). These controlled conditions are essential for safeguarding the health of the mysids and ensuring reliable experimental outcomes. For more detailed information about the lab setting, refer to [69]. The data collected for this study consists of daily pH measurements taken over a period of 60 days, from April 1 to May 30, 2016, which are reproduced in Table 3.11.

Table 3.11: pH levels Data

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
pH value	8.2	8.2	8.2	8.19	8.25	8.19	8.19	8.34	8.12	8.29	8.25	8.12	8.19	8.3	8.25
Days	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
pH value	8.2	8.2	8.2	8.15	8.25	8.29	8.3	8.3	8.28	8.27	8.19	8.28	8.3	8.4	8.3
Days	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
pH value	8.11	8.35	8.22	8.24	8.29	8.3	8.15	8.14	8.24	8.11	8.13	8.11	8.15	8.14	8.2
Days	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
pH value	8.36	8.26	8.11	8.32	8.11	8.33	8.42	8.32	8.41	8.39	8.37	8.31	8.29	8.26	8.27

3.9.2 Data Assumptions Validation

Before applying the control charts, it is important to validate the underlying assumptions concerning the pH data, particularly focusing on its normality and independence. The normality assumption is foundational in SPC, as many control chart methodologies rely on the data being normally distributed. To assess this, we employed the Anderson-Darling normality test (see [70]). The test yielded a p-value of 0.1191, indicating that there is no significant departure from normality. In addition to normality, we also evaluated the independence of observations, which is essential for ensuring that the control chart signals are reliable and not influenced by autocorrelation. The Durbin-Watson test (see [71]) was utilized for this purpose, yielding a p-value of 0.902. This high p-value indicates a lack of significant autocorrelation among the pH measurements, confirming that the observations are independent. Thus, the independence assumption holds, allowing us to proceed with the application of the control charts without the risk of misleading results due to correlated data.

3.9.3 Monitoring pH Level Using Control Charts

We apply the three control charts discussed in this research, along with the Shewhart chart, to the pH data provided in Table 3.11. According to [69], the IC mean and standard deviation of the water pH are 8.2 and 0.1, respectively. The control charts are established using these known values, with the following charting constants for each chart:

- **Shewhart Chart:** Control limit coefficient $L_S = 3$
- **EWMA Chart:** Sensitivity parameter $\lambda_E = 0.02$, asymptotic control limit coefficient $L_E = 2.1349$
- **CUSUM Chart:** Sensitivity parameter $k_C = 0.125$ and control limit coefficient $h_C = 12.0829$
- **MEC Chart:** Sensitivity parameters $k_M = 0.125$, $\lambda_M = 0.5$, and control limit coefficient $h_M = 24.72$

These charting parameters and constants are selected for each chart so that the $ARL_0 = 370$, allowing for a fair comparison among the four charts.

The IC region (light green in Figure 3.5) encompasses the initial 50 days, while the OOC region (light red) spans from day 51 to 60. During the final 10-day period (days 51-60), the laboratory underwent annual maintenance, necessitating the temporary disconnection of an air supply pipe. This disruption affected water circulation, leading to comparatively elevated pH levels.

The Shewhart chart, despite its simplicity and ease of use, demonstrates limited sensitivity to small shifts in process parameters. This characteristic may render it less effective in detecting early signs of instability, especially in sensitive laboratory environments where

maintaining a specific pH level is crucial for the health of mysids. The EWMA chart, known for its sensitivity to shifts, signaled four false alarms starting from day 35, indicating an increased responsiveness to minor fluctuations in pH levels. However, this heightened sensitivity came at the expense of generating misleading signals, as evidenced by the fact that the first valid OOC signal from the EWMA chart occurred on day 53. In contrast, both the CUSUM and MEC charts issued their first valid OOC signal on day 54, which shows their capacity to accurately detect the shift without any false alarms. This is particularly significant, as it highlights the advantages of using these charts in scenarios where maintaining a high level of accuracy in monitoring is critical.

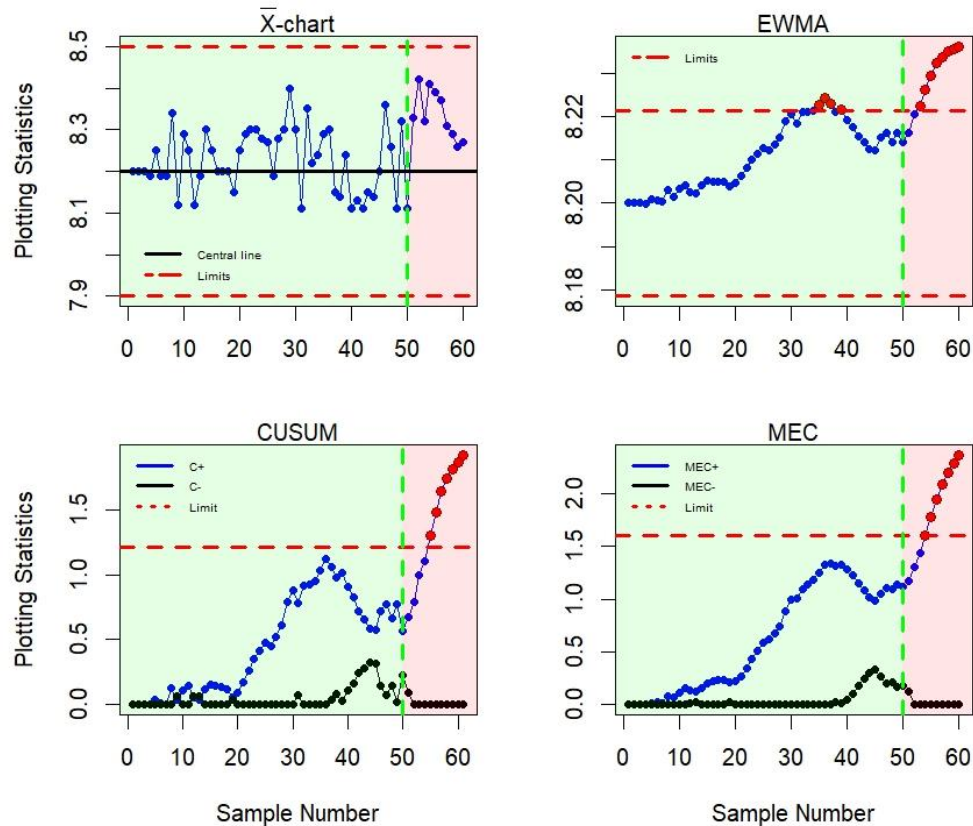


Figure 3.5: Application of Shewhart, EWMA, CUSUM, and MEC control charts on the pH data.

Overall, the performance comparison reveals that while the EWMA chart excels in early detection of OOC conditions, it is less effective in maintaining IC performance due to the numerous false alarms generated. The MEC and CUSUM charts exhibited a more balanced performance, effectively detecting shifts in pH levels with no false alarms, thereby providing reliable monitoring.

3.10 Chapter 3 Conclusion

The MEC control chart is a memory control chart that effectively identifies small to moderate changes in process parameters. The efficiency of this chart has been evaluated in previous research using properties of zero-state run length distribution. Nevertheless, it is essential to note that better performance based purely on these indicators does not provide solid evidence to support the superiority of charts, as it relies on certain assumptions that are deemed less realistic. This study comprehensively reevaluates the performance of the MEC chart in both zero-state and steady-state conditions. Our findings clearly indicate that the MEC chart exhibits strong performance across a broad spectrum of shift magnitudes, as shown by good overall performance. Optimal parameters for specific target shifts were provided as a guide for quality practitioners.

Finally, monitoring the pH level of water in the ecotoxicology lab has shown that the MEC control chart has practical application in areas such as a laboratory or an industry where the regulation of process parameters is important. This monitoring chart can be used in many areas to improve the quality and stability of processes.

CHAPTER 4

NEW STEP CUSUM CONTROL CHART FOR MONITORING PROCESS VARIABILITY

Recently, [72] introduced a Step-CUSUM control chart for monitoring process mean, aimed at enhancing the detection of various shifts. Unlike the traditional CUSUM chart, which targets a specific shift, this updated version accumulates all deviations following a change, making it more effective in identifying a wider range of shifts in the process mean. The main aim of this Chapter is to extend the work of [72] by developing a new version of the Step-CUSUM control chart for monitoring dispersion, utilizing the three-parameter transformation of the sample variance as outlined by [28]. Additionally, the performance of the proposed chart will be evaluated under both zero- and steady-state conditions. We will also identify the optimal design parameters for detecting specific shifts in process dispersion and will compare the performance of the new chart with that of the EWMA-S² and CUSUM-S² control charts.

4.1 Existing Methods

In this Section, we discuss the three-parameter transformation popularly used to transform sample variance in dispersion charts. We also discuss some of the existing control charts for monitoring process variability.

Let X be the quality characteristic to be monitored. Assume X_{ij} to be i th observation of the j th sample of size n from a normal distribution with an IC mean and variance μ_0 and σ_0^2

respectively, i.e. $X_{ij} \sim N(\mu_0, \sigma_0^2)$. It is important to note that this research focuses on monitoring changes in the scale of the process. Therefore, any OOC condition indicates a shift in the process standard deviation from the nominal value σ_0 to a new value σ_1 . If $\sigma_1 > \sigma_0$, this signals an increase in the process variability, which can lead to a wider dispersion of the output. On the other hand, $\sigma_1 < \sigma_0$ reflects a reduction in variability, which may result in a more uniform output of the process. The magnitude of the shift is indicated by $\gamma = \sigma_1/\sigma_0$.

4.1.1 The parameter transformation

To present the parameter transformation, we first define the sample variance as follows:

$$S_j^2 = \frac{\sum_{i=1}^n (X_{ij} - \bar{X}_j)^2}{n-1}, \quad (4.1)$$

where $\bar{X}_j = \frac{\sum_{i=1}^n X_{ij}}{n}$ is the sample mean for the j th sample.

The dispersion charts presented in this Section are based on the three-parameter logarithmic transformation of sample variance in equation (4.1), represented by:

$$T_j = a + b \ln(S_j^2 + c) \quad (4.2)$$

Where a , b , and c are positive constants defined in [26] as $b = B(n)$, $c = C(n)\sigma$, and $a = A(n) - 2B(n)\ln(\sigma)$. Here, $A(n)$, $B(n)$ and $C(n)$ are functions of the sample size n with their values for various n reproduced from [26] and presented in Table 4.1. Although the sampling distribution of S_j^2 is not normal, he showed that the constants a , b , and c can be selected in a way that the resulting distribution of the transformed variable in equation (4.2) is approximately normal, with mean and standard deviation as functions of sample size n , and independent of σ , i.e. $E(T_j) = \mu_n$ and $\sigma(T_j) = \sigma_n$. Values

of μ_n and σ_n are obtained from [26] and presented in Table 4.1 to make the research self-contained.

Table 4.1: $A(n), B(n), C(n), \mu_n, \sigma_n$, and Z_o constants for $n = 3, 4, \dots, 15$

n	$A(n)$	$B(n)$	$C(n)$	μ_n	σ_n	Z_o
3	-0.6627	1.8136	0.6777	0.02472	0.9165	0.276
4	-0.7882	2.1089	0.6261	0.01266	0.9502	0.237
5	-0.8969	2.3647	0.5979	0.00748	0.967	0.211
6	-0.9940	2.5941	0.5801	0.00485	0.9765	0.193
7	-1.0827	2.8042	0.5678	0.00335	0.9825	0.178
8	-1.1647	2.9992	0.5588	0.00243	0.9864	0.167
9	-1.2413	3.182	0.5519	0.00182	0.9892	0.157
10	-1.3135	3.3548	0.5465	0.00141	0.9912	0.149
11	-1.3820	3.5189	0.5421	0.00112	0.9927	0.142
12	-1.4473	3.6757	0.5384	0.0009	0.9938	0.136
13	-1.5097	3.826	0.5354	0.00074	0.9947	0.131
14	-1.5697	3.9705	0.5327	0.00062	0.9955	0.126
15	-1.6275	4.11	0.5305	0.00052	0.996	0.122

4.1.2 EWMA-S² Chart

[26] proposed an S²-EWMA, which is also written as EWMA-S², control chart to monitor process dispersion. This chart is based on a three-parameter logarithmic transformation in equation (4.2). As mentioned, the constants in equation (4.2) ensure that the transformed variable (T_j) approximately follows a normal distribution. Then, the classical EWMA chart is employed with T_j as the input variable. The plotting statistic for the S²-EWMA chart is defined as:

$$EWMA_j = \lambda_E T_j + (1 - \lambda_E) EWMA_{j-1} \quad (4.3)$$

where λ_E is the smoothing parameter with $(0 < \lambda_E \leq 1)$, and $EWMA_0$ is initialized with a value derived from the parameters. $EWMA_0$ values, denoted as Z_0 , for different sample sizes are also available in Table 4.1.

The lower and upper control limits for the S^2 -EWMA chart are given, respectively by:

$$\left. \begin{aligned} LCL &= \mu_n - L_E \sigma_n \sqrt{\frac{\lambda_E}{2 - \lambda_E}} \\ UCL &= \mu_n + L_E \sigma_n \sqrt{\frac{\lambda_E}{2 - \lambda_E}} \end{aligned} \right\}, \quad (4.4)$$

where μ_n and σ_n are the mean and standard deviation of the transformed statistic (T_j), respectively, and L_E is the width of the control limits, usually set to provide a specific ARL_0 .

If the plotting statistic in equation (4.3) is plotted outside the limits in equation (4.4), an OOC situation is declared.

4.1.3 CUSUM- S^2 Chart

[28] introduced the CUSUM- S^2 control chart for monitoring process dispersion. This chart uses two statistics, $(CUSUM_j^+)$ and $(CUSUM_j^-)$, to track upward and downward shifts in the process variance, respectively:

$$\left. \begin{aligned} CUSUM_j^+ &= \max[0, (T_j - \mu_n) - K_C + CUSUM_{j-1}^+] \\ CUSUM_j^- &= \max[0, -(T_j - \mu_n) - K_C + CUSUM_{j-1}^-] \end{aligned} \right\}, \quad (4.5)$$

where the reference value K_C is a constant that determines the sensitivity of the chart and μ_n is the mean of T_j . The chart signals an OOC state if either statistic exceeds a predefined threshold H_C , also referred to as the upper control limit. The decision limits for the

CUSUM-S² chart are: $CUSUM_j^+ \geq H_C$ or $CUSUM_j^- \geq H_C$. H_C is chosen to achieve the desired ARL_0 for a fixed K_C .

4.2 The proposed Step-CUSUM (SC-S²) chart for Variance

Inspired by [72], we propose a modified version of the CUSUM chart, called SC-S², for monitoring changes in process variance. The CUSUM-S² chart defined in Section 4.1 accumulates deviations that are greater than K , which is what should be done until a shift is detected. Step-CUSUM is modified so that it accumulates, without loss, all the deviations after a change is detected. Our proposed chart uses CUSUM statistics given in equation (4.5) to detect change points in the process variance, then initializes two count variables that keep track of how many samples are taken since an upward or downward deviation occurred.

To present the structure of the proposed chart mathematically, we first standardize the transformed variable T_j defined in equation (4.2) as follows:

$$Z_j = \frac{T_j - \mu_n}{\sigma_n} \quad (4.6)$$

Now, the standardized variable Z_j is approximately standard normal with a mean $\mu = 0$ and standard deviation $\sigma = 1$, i.e. $Z_j \sim N(0,1)$. Note that the standardization here will naturally lead to a CUSUM for monitoring changes in dispersion parameter and also have a reference parameter that is independent of the scale [73].

Similar to equation (4.5), we define two statistics C_j^+ and C_j^- using the standardized variable Z_j , to respectively detect upward and downward shifts in the process variance.

$$\left. \begin{aligned} C_j^+ &= \max[0, Z_j - K_{SC} + C_{j-1}^+] \\ C_j^- &= \max[0, -Z_j - K_{SC} + C_{j-1}^-] \end{aligned} \right\} \quad (4.7)$$

The CUSUM statistics in equation (4.7) guide the proposed chart to determine a change point. We define two statistics D_j^+ and D_j^- that, respectively, count the number of samples investigated since last $C_j^+ = 0$ and $C_j^- = 0$, as follows:

$$\begin{aligned} D_j^+ &= \begin{cases} 1, & C_{j-1}^+ = 0 \\ 1 + D_{j-1}^+, & C_{j-1}^+ > 0 \end{cases} \\ D_j^- &= \begin{cases} 1, & C_{j-1}^- = 0 \\ 1 + D_{j-1}^-, & C_{j-1}^- > 0 \end{cases} \end{aligned} \quad (4.8)$$

The definitions of the discrete variables in equation (4.8) ensure that the count restarts at 1 for D_j^+ or D_j^- every time CUSUM goes back to zero for C_{j-1}^+ or C_{j-1}^- respectively. The proposed SC-S² chart is based on these variables with the following two statistics to monitor increases/decreases in the process variance:

$$\begin{aligned} SC_j^+ &= \frac{1}{D_j^+} Z_j + \left(1 - \frac{1}{D_j^+}\right) SC_{j-1}^+ \\ SC_j^- &= \frac{1}{D_j^-} Z_j + \left(1 - \frac{1}{D_j^-}\right) SC_{j-1}^- \end{aligned} \quad (4.9)$$

The plotting statistics of the proposed chart in equation (4.9) in a way resemble the EWMA plotting statistic with the standardized variable Z_j as the input variable and reciprocals $\frac{1}{D_j^+}$ and $\frac{1}{D_j^-}$ as the weights for the upper SC_j^+ and lower SC_j^- respectively.

The SC-S² chart statistics represent the cumulative average of the transformed variable Z_j from the point when the CUSUM statistic was last reset to zero. Therefore, in the IC state, the expected values of the SC-S² statistics are described by $E[SC_j^+] = E[SC_j^-] = 0$, and the variances are: $\sigma^2(SC_j^+) = \frac{1}{D_j^+}$; $\sigma^2(SC_j^-) = \frac{1}{D_j^-}$. Using this information, the upper and lower control limits of the proposed chart are constructed as follows:

$$\left. \begin{aligned} UCL_j^{SC} &= \frac{L_{SC}}{\sqrt{D_j^+}} \\ LCL_j^{SC} &= -\frac{L_{SC}}{\sqrt{D_j^-}} \end{aligned} \right\}, \quad (4.10)$$

where the constant L_{SC} serves as the control limit multiplier for the proposed chart. The process is considered IC as long as the two plotting statistics in equation (4.9) are within the upper and lower limits provided by equation (4.10). However, $SC_j^+ > UCL_j^{SC}$ indicates an upward shift in the process variance while $SC_j^- < LCL_j^{SC}$ is a signal that the process variance has shifted downward. Both cases mean that the process is OOC and needs special attention.

4.3 Performance Evaluation Measures

This Section discusses the measures used in evaluating the efficiency of the proposed chart and making comparisons with existing charts. The measures are based on the properties of run length (RL), which is defined as the number of subgroups that must be taken before an OOC signal is given by the chart. If the computation of the RL starts at the beginning of the monitoring process (i.e., change-point $\tau = 1$) it is referred to as a zero-state condition, while steady-state means that the computation is done after a very long time ($\tau \rightarrow \infty$) and the process has attained a stable condition without any prior signal.

There are two types of measures based on the method of evaluation. The first category involves individual performance measures, which assess the control charts' ability to detect specific changes in process parameters. The most commonly used measure of individual performance is ARL, which represents the average of run lengths. There are two types: IC ARL (ARL_0) when the process is under normal conditions, and OOC ARL (ARL_1) when

there is a shift from the normal condition. The goal is to have a large ARL_0 to reduce false alarms and a small ARL_1 for quick detection of actual changes. Given the skewed nature of run length distributions, other properties of the RL distribution, such as median run length (MRL) and standard deviation run length (SDRL) are often reported alongside ARL [74]. In some situations, control charts, especially for monitoring dispersion, can be ARL-biased, a phenomenon used to describe the case when $ARL_1 > ARL_0$, meaning that the chart takes more time to detect the shift than to trigger a false alarm. Many methods are introduced to design ARL-unbiased control charts [75]. The conditional expected delay (CED) curve is another measure of individual performance for evaluating the efficiency of a control chart for different change point values (τ). CED_τ is the expected RL at τ on condition that no false alarm was generated before the change point τ [76].

The second category, overall performance measures, assesses how effectively a control chart performs across a range of shifts. These measures offer a summary of the chart's ability to detect shifts, regardless of their magnitude or direction. In this study, extra quadratic loss (EQL) is considered for evaluating overall performance. EQL represents the weighted mean of ARL over a range of shifts, with the square of the shifts used as weights. EQL is the weighted mean of the ARL over a range of shifts, using the square of the shifts as the weights [77]. The mathematical expressions of CED and EQL are as follows:

$$\left. \begin{aligned} CED_\tau &= E(RL - \tau + 1 | RL \geq \tau) \\ EQL &= \frac{1}{\gamma_{max} - \gamma_{min}} \int_{\gamma_{min}}^{\gamma_{max}} (\gamma^q)^2 ARL(\gamma) d\gamma \end{aligned} \right\}, \quad (4.11)$$

where γ_{max} and γ_{min} are the maximum and minimum shifts in process variability, and $\gamma^q = \gamma$ for upward shifts and $\gamma^q = 1/\gamma$ for downward shifts.

4.4 Results and Discussion

This Section discusses the performance of the proposed scheme and draws comparisons with some competing charts (CUSUM-S² and EWMA-S²) in terms of zero- and steady-state ARL, as well as the overall performance metric, EQL. Many techniques, including Markov chain approximation [78] and Monte Carlo simulation [79], are used in estimating the ARL of control charts. In this study, we employ the Monte Carlo simulation which involves conducting repeated random sampling (10^6 iterations). For each iteration, the rules of RL specific to the chart are applied. Finally, the results from these simulations are then averaged to obtain an estimated ARL. More details on ARL computation using Monte Carlo simulation can also be found in [38]. The following algorithm outlines the steps for estimating the ARL of the proposed chart through Monte Carlo simulations:

1. Specify the sample size n and select the reference parameter K_{SC} , limit constant L_{SC} , and change point τ ($\tau = 1$ correspond to zero-state).
2. Draw n random observations from:
 - i. $X_{ij} \sim N(\mu_0 = 0, \gamma = \sigma_1/\sigma_0)$, if sample number $j \geq \tau$.
 - ii. $X_{ij} \sim N(\mu_0 = 0, \sigma_0 = 1)$, otherwise.
3. Use the formula in equation (4.1) to estimate the sample variance, S_j^2 .
4. Transform S_j^2 to T_j using equation (4.2) and normalize T_j using its mean and standard deviation provided in Table 4.1.
5. Calculate the CUSUM statistics C_j^+ and C_j^- from equation (4.7).

6. Compute the discrete variables D_j^+ and D_j^- according to the conditions given by equation (4.8).
7. Calculate the plotting statistics SC_j^+ and SC_j^- from equation (4.9)
8. Use equation (4.8) to Compute the control limits UCL_j^{SC} and UCL_j^{SC} .
9. Compare the plotting statistics in step #7 with the control limits in step #8, if either $SC_j^+ > UCL_j^{SC}$ or $SC_j^- < UCL_j^{SC}$, save the index $(j - \tau + 1)$ as run length, otherwise, go to step #2
10. Repeat step #2 to step #9 m times (say, 10^6) to record m run lengths.
11. Estimate the ARL_0 by taking the average of the m run lengths.
12. If the estimated ARL_0 is not approximately equal to the target ARL_0 , adjust the value of L_{SC} in step #1 and repeat step #2 to step #11.
13. To compute ARL_1 , draw the random observations in step#2 using $X_{ij} \sim N(\mu_0 = 0, \gamma > 1)$ for upward shifts, and $X_{ij} \sim N(\mu_0 = 0, \gamma < 1)$ for downward shifts.

4.4.1 IC and OOC performance of the proposed chart

The proposed chart monitors both increases and decreases in process dispersion, so our analysis included various OOC scenarios for both cases. Without loss of generality, we assume the IC process standard deviation is 1. Therefore, $\sigma_1 > 1$ ($\sigma_1 < 1$) corresponds to an increase (decrease) in the process standard deviation. We have chosen all the parameters of the charts such that ARL_0 is approximately 370. Figure 4.1 visualizes the relationship between different choices of the parameter K_{SC} and the corresponding control limit L for sample sizes $n \in (3, 5, 7, 9)$ such that $ARL_0 \cong 370$.

Table 4.2: Zero-state ARL values of SC-S² for γ from 0.5 to 2, $n = 3, 5, 7$, and 9, $K_{SC} = (0.01, 0.1, 0.5)$. Zero-state $ARL_0 \cong 370$.

K_{SC}												
	$n = 3$			$n = 5$			$n = 7$			$n = 9$		
γ	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5	0.01	0.1	0.5
0.5	8.61	10.99	12.65	4.4	5.41	6.09	3.06	3.7	4.14	2.41	2.87	3.2
0.6	12.64	16.27	18.94	6.43	7.97	8.96	4.44	5.42	6.06	3.45	4.17	4.66
0.7	20.86	27.16	35.71	10.58	13.27	14.96	7.27	8.98	10.04	5.62	6.88	7.7
0.8	41.7	55.7	116.39	21.34	27.23	34.89	14.65	18.42	21.29	11.28	14.08	15.84
0.9	114.1	167.2	563.77	66.63	88.62	178.64	47.08	61.15	100.76	36.54	47.18	69.47
0.95	209.1	325.67	645.63	161.17	223.51	417.25	127	171.03	298.97	104.58	139.3	235.02
1	369.86	370.19	369.72	369.64	370.37	370.71	370.24	370.12	370.46	369.91	369.73	370.01
1.05	209.53	225.33	184.98	141.03	172.76	160.8	109.7	140.23	143.78	90.61	118.56	129.68
1.1	87	113.83	100.49	53.76	70.88	72.36	39.96	52.6	57.19	32.15	42.07	46.91
1.2	30.07	40.87	39.96	18.14	23.48	24.44	13.38	17.01	18.01	10.75	13.52	14.45
1.3	15.94	21.31	21.54	9.66	12.26	12.84	7.2	8.92	9.5	5.82	7.15	7.7
1.4	10.34	13.51	13.95	6.34	7.86	8.34	4.76	5.77	6.21	3.88	4.65	5.05
1.5	7.48	9.62	10.1	4.65	5.66	6.06	3.52	4.2	4.54	2.9	3.41	3.7
1.6	5.84	7.36	7.81	3.67	4.39	4.71	2.8	3.28	3.55	2.32	2.69	2.91
1.7	4.77	5.92	6.34	3.03	3.58	3.85	2.34	2.71	2.91	1.96	2.23	2.41
1.8	4.04	4.95	5.32	2.6	3.03	3.26	2.03	2.31	2.48	1.71	1.92	2.06
1.9	3.52	4.25	4.58	2.29	2.64	2.83	1.8	2.03	2.17	1.54	1.71	1.82
2	3.13	3.74	4.03	2.06	2.34	2.51	1.64	1.82	1.95	1.41	1.55	1.64

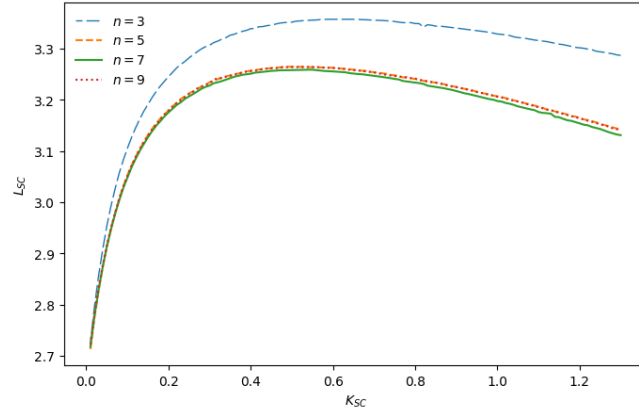


Figure 4.1: (K_{SC}, L_{SC}) pairs such that $ARL_0 = 370$.

It can be observed from Table 4.2, as with many other charts, that the zero-state ARLs of the proposed chart have a direct relationship with the magnitude of the shift (γ) in the process scale. Specifically, for small shifts, whether in the positive (increase) or negative (decrease) direction, the ARLs tend to be large, while for larger deviations, they are smaller. This indicates that larger shifts are detected more quickly than smaller ones. It should be noted here that by small deviations we mean γ values close to unity, and vice-versa for large deviations.

Sample size plays a key role in the performance of the proposed chart as seen in Table 4.2. As the sample size increases, the ARL values drop, meaning the chart detects shifts faster for larger sample sizes. For instance, when $\gamma = 0.5$ and $K_{SC} = 0.01$, the ARL is 8.61 for $n = 3$, but it falls by about 72% to 2.41 for $n = 9$. This is also true for upward shifts, for example, when $\gamma = 1.5$ and $K_{SC} = 0.01$, the ARL is 7.48 for $n = 3$, but drops to 2.90 for $n = 9$, which is a reduction of more than 60% in the ARL. A similar pattern is observed across different K_{SC} values and shift sizes.

It can be seen that the choice of the parameter K_{SC} also directly affects the performance of the proposed chart. For example, when $n = 5$ and $\gamma = 1.1$, the ARLs for $K_{SC} = 0.01, 0.1$, and 0.5 , respectively are 53.76, 70.88, and 72.36, indicating smaller values of K_{SC} almost always results in better performance as indicated by smaller ARLs. A similar trend across all sample sizes is observed, meaning that smaller K_{SC} values consistently lead to faster detection of both upward and downward shifts in the variance. Larger K_{SC} values slow down the chart's response, making it less sensitive to shifts in the process. Therefore, in zero-state, we consider $K_{SC} = 0.01$ to be the optimal parameter value for detecting any

shift in the process variance. Note that using a large value of K_{SC} for a small sample size could render the chart to become ARL-biased (i.e., $ARL_1 > ARL_0$) at $\gamma = 0.95$.

4.4.2 Optimal parameters

It is often useful to the quality practitioner to know the optimal combinations of the parameters of the chart to be used so that the chart can be designed to have maximum sensitivity at a specified shift while keeping the desired ARL_0 unchanged. Following [28], the optimal values (K_{opt}) of the proposed chart for specific shifts (γ_{target}) in the process variability are presented in Table 4.3, for $\gamma \in (0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.05, 1.1, 1.15, 1.2, 1.25, 1.3, 1.35, 1.4, 1.45, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3, 3.5, 4, 4.5, 5)$ along with their corresponding limit coefficients (L_{opt}) and the minimum possible steady-state ARL (ARL_{opt}). The optimal values satisfy the following two conditions:

- (a) When the process is IC, that is $\gamma = 1$, the couples (K_{opt}, L_{opt}) yield zero-state

$$ARL_0 \approx 370;$$

- (b) When the process is OOC, that is $\gamma = \gamma_{target}$, the couples (K_{opt}, L_{opt}) yield the minimum possible steady-state ARL_{opt} .

All steady-state ARLs are obtained at $\tau = 501$. Since similar ARL values are observed for $\tau > 501$, we assume that all charts in this study are stable at $\tau = 501$.

Table 4.3. SC-S² Optimal combinations (K_{opt}, L_{opt}) and the corresponding minimum steady-state ARL (ARL_{opt}) for $\gamma = 0.5, 0.55, \dots, 5, n = 3, 5, 7$, and 9. Zero-state $ARL_0 \cong 370$.

	n = 3			n = 5			n = 7			n = 9		
γ_{target}	L_{opt}	K_{opt}	ARL_{opt}	L_{opt}	K_{opt}	ARL_{opt}	L_{opt}	K_{opt}	ARL_{opt}	L_{opt}	K_{opt}	ARL_{opt}
0.5	3.307	0.3	11.29	3.2641	0.54	5.64	3.1941	1.02	3.82	3.1616	1.21	2.93
0.55	3.2752	0.24	13.43	3.2629	0.46	6.72	3.2279	0.83	4.58	3.1813	1.12	3.51
0.6	3.2541	0.21	16.33	3.2516	0.37	8.2	3.2449	0.72	5.61	3.2302	0.87	4.32
0.65	3.2456	0.2	20.42	3.246	0.34	10.29	3.2558	0.44	7.07	3.2583	0.65	5.45
0.7	3.2046	0.16	26.48	3.2294	0.29	13.37	3.2453	0.36	9.2	3.2639	0.48	7.12
0.75	3.1625	0.13	36.09	3.1793	0.2	18.25	3.2116	0.26	12.57	3.2515	0.37	9.73
0.8	3.1265	0.11	52.23	3.1635	0.18	26.64	3.183	0.21	18.38	3.2329	0.3	14.25
0.85	3.0825	0.09	83.5	3.104	0.13	43.15	3.1496	0.17	29.87	3.1871	0.21	23.25
0.9	2.9924	0.06	151.4	3.0528	0.1	83.37	3.1005	0.13	58.52	3.1523	0.17	45.81
0.95	2.954	0.05	301.96	3.0065	0.08	212.02	3.0482	0.1	162.79	3.0514	0.1	132.46
1.05	3.342	0.82	180.99	3.2294	0.29	155.39	3.2023	0.24	132.61	3.1792	0.2	113.89
1.1	3.3462	0.45	98.07	3.2216	0.27	67.14	3.2069	0.25	50.94	3.2159	0.26	41.31
1.15	3.3434	0.43	58.67	3.248	0.35	36.44	3.2369	0.33	27.01	3.2454	0.34	21.73
1.2	3.3452	0.44	38.63	3.2573	0.4	23.15	3.2541	0.42	17.05	3.2588	0.42	13.7
1.25	3.348	0.46	27.53	3.2644	0.48	16.26	3.2581	0.49	11.96	3.2644	0.53	9.63
1.3	3.351	0.48	20.82	3.2637	0.55	12.21	3.2524	0.64	8.99	3.2588	0.64	7.24
1.35	3.3575	0.58	16.4	3.2635	0.57	9.61	3.2515	0.65	7.11	3.2458	0.76	5.72
1.4	3.3576	0.6	13.35	3.253	0.71	7.84	3.2371	0.77	5.81	3.2261	0.89	4.7
1.45	3.3576	0.61	11.2	3.2375	0.82	6.58	3.2095	0.94	4.88	3.2233	0.91	3.97
1.5	3.342	0.82	9.56	3.2338	0.85	5.64	3.213	0.92	4.2	3.1906	1.08	3.42
1.75	3.342	0.82	5.39	3.1442	1.29	3.24	3.1603	1.17	2.47	3.1445	1.28	2.05
2	3.3219	1.05	3.74	3.1645	1.2	2.32	3.1402	1.26	1.81	3.1641	1.2	1.55
2.25	3.3027	1.19	2.91	3.162	1.21	1.86	3.1349	1.28	1.49	3.1479	1.27	1.3
2.5	3.2941	1.25	2.42	3.1416	1.3	1.59	3.1372	1.27	1.31	3.1408	1.3	1.17
2.75	3.3027	1.19	2.11	3.1562	1.24	1.43	3.1764	1.1	1.21	3.2281	0.88	1.1
3	3.2929	1.26	1.89	3.1645	1.2	1.32	3.1764	1.1	1.14	3.1919	1.07	1.06
3.5	3.3009	1.21	1.62	3.1909	1.08	1.19	3.2253	0.85	1.07	3.2217	0.92	1.02
4	3.3151	1.11	1.46	3.2205	0.93	1.12	3.1902	1.04	1.03	3.2644	0.55	1.01
4.5	3.287	1.3	1.35	3.2358	0.83	1.08	3.2547	0.62	1.02	3.2628	0.58	1
5	3.3064	1.17	1.28	3.2102	0.98	1.05	3.259	0.55	1.01	3.2588	0.42	1

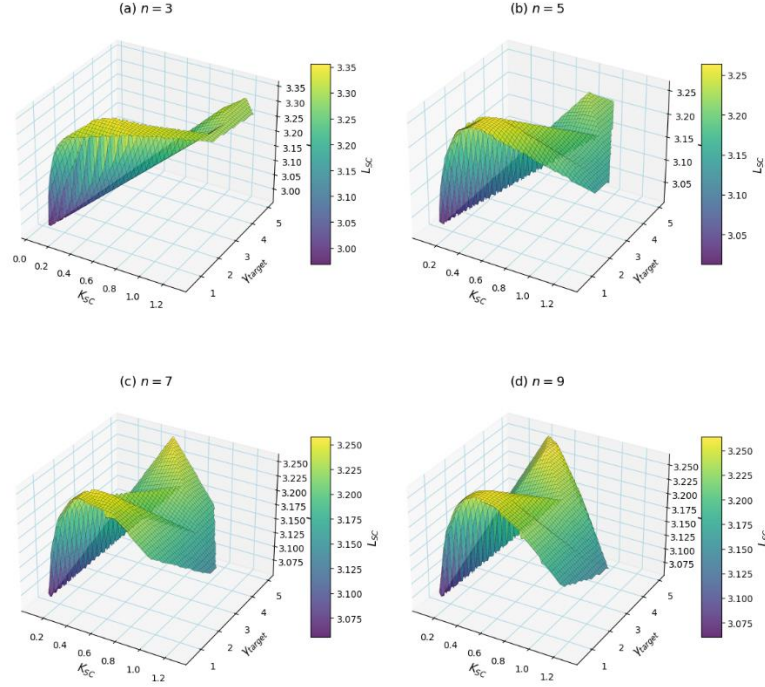


Figure 4.2. Three-Dimensional Surface Plot of K_{opt} , L_{opt} , and γ_{target}

It is observed from Table 4.3, and also from the three-dimensional surface plot in Figure 4.2, that although there is no direct relationship between γ_{target} and the optimal parameter K_{opt} , general pattern is observed pertaining the shift magnitude and sample size. For larger values of shifts, for example, $\gamma = 0.5$ and $\gamma = 1.5$, the K_{opt} values tend to be higher. For example, at $\gamma = 0.5$, K_{opt} values range from 0.3 for $n = 3$ to 1.21 for $n = 9$. Also, for $\gamma = 1.5$, K_{opt} values go even higher, $K_{opt} = 0.82$ for $n = 3$ and 1.08 for $n = 9$. This particular trend suggests that shifts in either direction, but larger in magnitude, require higher K_{SC} values to attain an acceptable level so that the chart can detect those shifts as quickly as possible. In contrast, lower K_{SC} values are required for smaller shifts like $\gamma = 0.95$. For $\gamma = 0.95$, K_{SC} values are 0.05 for $n = 3$ and 0.1 for $n = 9$. The reason for that is that the chart is more sensitive to small shifts, therefore, it requires smaller K_{SC} to detect them.

With an increase in sample size, the K_{opt} values for the same shift size increase. At $\gamma = 0.5$, for instance, the K_{opt} value starts as low as 0.3 for $n = 3$ but increases to 1.21 for $n = 9$. This implies that for increasing sample sizes, generally higher K_{SC} values are required for optimized chart performance. Similarly, for $\gamma = 1.5$, K_{opt} increases from 0.82 for $n = 3$ to 1.08 for $n = 9$. This steady increase in K_{SC} with sample size shows that when more observations are considered, the K_{SC} value should be big enough to protect the chart from being overly sensitive and generating too many false alarms. It should be noted here that these optimal combinations are recommended for efficient steady-state performance. In the case of zero-state, however, it is shown that a small value of the parameter K_{SC} is optimal for all the shifts in the process variability. Figure 3 highlights the effect of the parameter K_{SC} on the steady-state ARL for some selected shifts and different sample sizes.

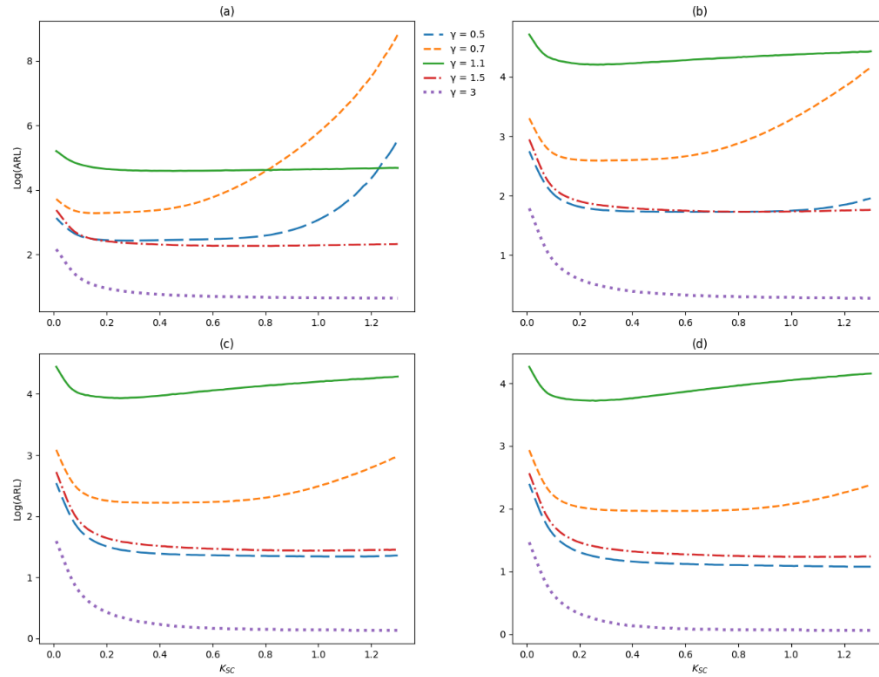


Figure 4.3. SC-S² steady-state ARL as a function of K_{SC} for different shifts γ and sample size: (a) $n = 3$, $n = 5$, $n = 7$, and $n = 9$. $ARL_0 = 370$.

4.5 Comparisons

In this Section, the performance of the proposed chart in zero- and steady-state conditions is compared with some existing charts for dispersion, specifically, EWMA-S² and CUSUM-S² based on the three-parameter transformation. The ARL and CED are used as performance criteria for specific shifts, while the EQL metric indicates overall performance across a range of shifts.

4.5.1 Zero-state comparisons

Under zero-state, the ARL values of the proposed chart are generated using $K_{SC} = 0.01$ as the optimal parameter for all the shifts, while the minimum possible ARL for each shift is used in the case of EWMA-S² and CUSUM-S² charts. All the zero-state ARL results are reported in Table 4.4. It is seen that for small to moderate shifts in the negative direction, the SC-S² chart consistently outperforms both EWMA-S² and CUSUM-S² in detecting changes, as evidenced by lower ARL values. For instance, at $\gamma = 0.5$ and $n = 3$, SC-S² has an ARL of 8.61, while CUSUM-S² has 10.8 and EWMA-S² has 12.3. This trend continues across larger sample sizes for almost all shifts $\gamma \leq 0.95$.

In the case of larger shifts in the positive direction, like $\gamma \geq 1.5$, SC-S² still exhibits superior performance for all $n \geq 5$. However, the proposed chart is inferior to EWMA-S² in detecting positive changes in the variance when the sample size is small. Overall, SC-S² demonstrates faster detection ability across both small and large shifts compared to EWMA-S² and CUSUM-S², especially for larger sample sizes, making it the most effective choice among the charts for monitoring process variability. The last row of Table 4.4 labeled 'EQL' summarizes overall chart performance over a range of shifts. The lower EQL values the better the chart performance. For example, at $n = 3$, SC-S² has an EQL of

103.75 and thus is more efficient overall than EWMA-S² (and thus CUSUM-S², which has an EQL of 114.04). The case is the same for $n = 5, 7$, and 9 . The EQL metric confirms that the SC-S² chart consistently provides better results across a range of shifts, hence it is the most effective chart in terms of overall performance in monitoring process changes. Figure 4.4 provides a graphical comparison of 3 different designs of the proposed chart, EWMA-S² and CUSUM-S² control charts in terms of ARL in the logarithmic scale.

Table 4.4. Zero-state minimum ARL profiles of EWMA-S², CUSUM-S², and SC-S² for $\gamma = 0.5, 0.6, \dots, 2$. $ARL_0 \cong 370$.

	EWMA-S ²				CUSUM-S ²				SC-S ²			
γ	n = 3	n = 5	n = 7	n = 9	n = 3	n = 5	n = 7	n = 9	n = 3	n = 5	n = 7	n = 9
0.50	12.3	6.2	4.2	3.2	10.8	5.6	3.8	2.9	8.61	4.4	3.06	2.41
0.60	17.7	9.0	6.1	4.7	15.4	8.3	5.7	4.4	12.64	6.43	4.44	3.45
0.70	28.4	14.6	10.0	7.7	24.1	13.4	9.4	7.3	20.86	10.58	7.27	5.62
0.80	54.2	28.1	19.5	15.1	44	25.4	18.2	14.3	41.7	21.34	14.65	11.28
0.90	155.2	80.8	56.7	44.5	108.9	68.4	51.1	41.3	114.1	66.63	47.08	36.54
0.95	323.5	202.1	150.1	120.5	216.9	154.8	122.9	103.1	209.1	161.17	127	104.58
1.05	153.5	121.2	99.3	84.1	183.3	145.7	117.5	99.5	209.53	141.03	109.7	90.61
1.10	65.1	44.9	35.0	29.1	98.6	64.6	49.2	40.3	87	53.76	39.96	32.15
1.20	21.8	15.3	12.5	10.8	39.5	24.3	18	14.5	30.07	18.14	13.38	10.75
1.30	11.7	8.8	7.4	6.6	21.7	13.1	9.6	7.7	15.94	9.66	7.2	5.82
1.40	7.9	6.2	5.4	4.8	14.1	8.3	6.1	4.9	10.34	6.34	4.76	3.88
1.50	5.9	4.8	4.3	3.9	10.2	5.9	4.3	3.5	7.48	4.65	3.52	2.9
1.60	4.8	4.0	3.6	3.2	7.8	4.5	3.3	2.7	5.84	3.67	2.8	2.32
1.70	4.1	3.5	3.1	2.9	6.2	3.6	2.7	2.2	4.77	3.03	2.34	1.96
1.80	3.6	3.1	2.8	2.6	5.2	3.0	2.3	1.9	4.04	2.6	2.03	1.71
1.90	3.2	2.8	2.5	2.4	4.4	2.6	2.0	1.7	3.52	2.29	1.8	1.54
2.00	2.9	2.5	2.3	2.2	3.8	2.3	1.8	1.5	3.13	2.06	1.64	1.41
EQL	124.45	72.53	53.60	43.50	114.04	70.22	52.18	42.15	103.75	61.82	45.19	36.07

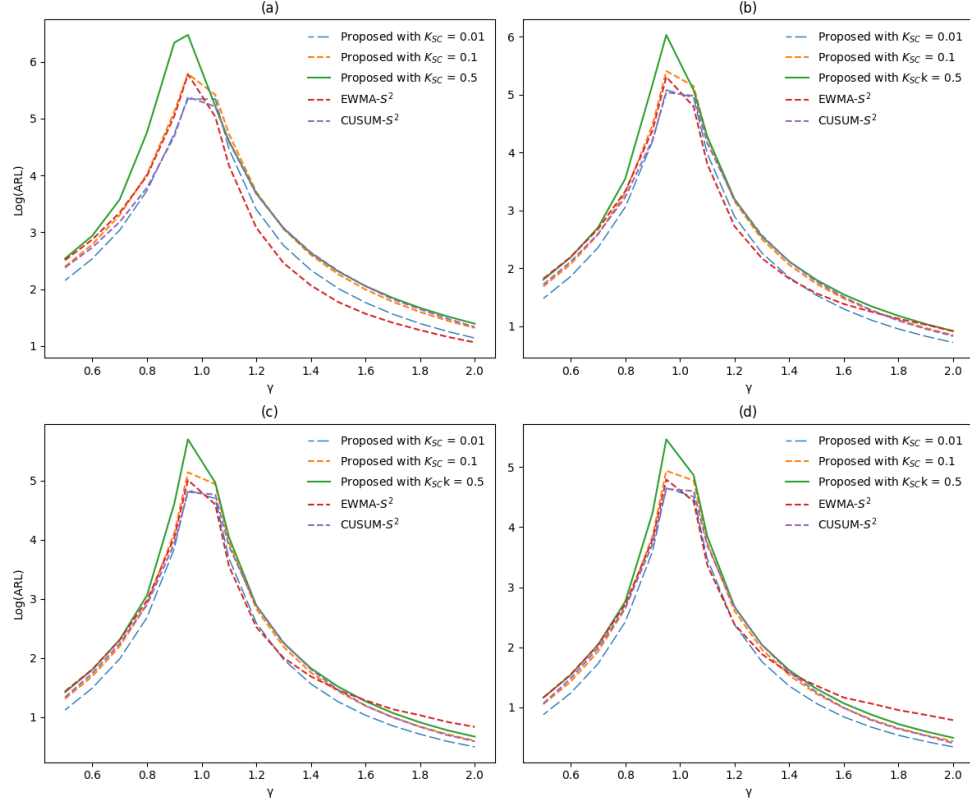


Figure 4.4. Zero-state Log (ARL) of the proposed, EWMA, and CUSUM charts for shifts $\gamma \in (0.5, 2)$ and sample size: (a) $n = 3$, $n = 5$, $n = 7$, and $n = 9$. $ARL_0 \cong 370$.

4.5.2 Steady-state comparisons

This Section compares the performance of the proposed chart and the competing charts in the steady-state environment using CED and its limit, the steady-state ARL, as well as the overall performance metric, EQL. The steady-state ARL comparisons are done by optimizing all the competing charts for a specific shift γ_{target} and then evaluating their performance on a range of shifts that includes the target shift and other shifts in the process variance. For the steady-state, the shift is introduced after sample number 500 ($\tau = 500$). To make the results comparable, the zero-state ARL_0 is set at approximately 370 for all the charts. For a concise discussion, three cases are considered here and the results are reported

in Tables 4.5 – 4.7 with dedicated columns “min”, the dominance zone, indicating for each case which chart has the least ARL_1 . In all 3 Tables, (C) denotes that CUSUM-S² has the least ARL_1 , while (E) is for EWMA-S², and (S) stands for the proposed SC-S². Tables 4.5 and 4.6 are for the cases when the charts are optimized for a downward shift, specifically $\gamma_{target} = 0.5$ and 0.95 , respectively. Table 4.7 presents the case when the chart is optimized for an upward shift of $\gamma_{target} = 1.5$. Finally, the EQL comparison is presented in Table 4.8.

Table 4.5. Steady-state ARL comparisons among CUSUM-S², EWMA-S², and the proposed chart when the charts are optimized for a target shift $\gamma_{target} = 0.5$. Zero-state $ARL_0 \cong 370$.

	n = 3				n = 5				n = 7				n = 9			
γ	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min
0.5	9.7	10.28	11.29	C	5.26	5.5	5.64	C	3.66	3.85	3.82	C	2.83	3.02	2.93	C
0.55	11.6	12.57	13.51	C	6.38	6.67	6.73	C	4.46	4.7	4.6	C	3.46	3.7	3.52	C
0.6	14.43	16.1	16.49	C	8.14	8.51	8.24	C	5.75	6.04	5.81	C	4.5	4.81	4.47	S
0.65	19.06	21.97	20.87	C	11.13	11.57	10.45	S	8.04	8.3	7.98	S	6.36	6.74	6.18	S
0.7	27.28	32.38	27.55	C	16.9	17.14	13.95	S	12.6	12.52	12.39	S	10.12	10.41	9.65	S
0.75	43.7	52.39	38.69	S	21.13	28.19	20.42	S	22.87	21.06	22.07	E	18.86	18.05	17.42	S
0.8	79.91	94.06	61	S	59.12	51.75	35.15	S	48.2	39.95	45.33	E	40.71	35.31	36.37	E
0.85	163.55	185.48	117.93	S	132.01	107.32	74.56	S	111.25	85.23	104.55	E	97.14	76.65	84.75	E
0.9	340.93	381.57	275.5	S	299.73	241.54	191.39	S	262	199.39	251.33	E	233.56	182.49	208.19	E
0.95	508.79	569.82	489.24	S	511.09	467.88	430.55	S	481.03	421.76	480.98	E	453.31	399.81	431.67	E
1	362.09	374.17	363.84	C	365.86	366.55	363.95	S	367.25	367.79	368.25	C	368.08	366.86	368.53	E
1.05	182.93	176.89	187.58	E	162.54	154.49	158	E	154.88	147.84	154.13	E	152.14	144.43	150.45	E
1.1	96.73	91.23	99.6	E	76.24	69.99	71.14	E	67.58	61.82	67	E	63.04	57.97	61.99	E
1.15	57.19	53.93	59.28	E	40.78	37.69	38.06	E	34.34	31.5	34.22	E	30.65	28.33	30.29	E
1.2	37.55	35.89	39.2	E	25.06	23.53	23.57	E	20.1	18.8	20.11	E	17.45	16.38	17.26	E
1.25	26.87	25.97	28.24	E	17.13	16.33	16.33	S	13.26	12.68	13.32	E	11.19	10.77	11.15	E
1.3	20.46	19.95	21.58	E	12.59	12.22	12.25	E	9.59	9.28	9.54	E	7.9	7.78	7.91	E
1.35	16.31	16.09	17.2	E	9.8	9.69	9.67	S	7.33	7.28	7.32	E	6.01	5.99	6.02	E
1.4	13.58	13.46	14.2	E	7.97	8	7.95	S	5.92	5.95	5.88	S	4.8	4.86	4.82	C
1.45	11.56	11.53	12.03	E	6.72	6.82	6.71	S	4.95	5.02	4.91	S	3.99	4.08	4.01	C
1.5	10.07	10.11	10.4	C	5.81	5.93	5.79	S	4.25	4.38	4.21	S	3.43	3.54	3.42	S
1.75	6.22	6.34	6.1	S	3.55	3.72	3.46	S	2.58	2.73	2.5	S	2.08	2.2	2.06	S
2	4.64	4.74	4.35	S	2.65	2.82	2.51	S	1.93	2.09	1.84	S	1.57	1.68	1.55	S
2.25	3.77	3.87	3.45	S	2.17	2.33	2.02	S	1.6	1.74	1.52	S	1.33	1.42	1.31	S
2.5	3.24	3.33	2.89	S	1.87	2.03	1.73	S	1.4	1.52	1.33	S	1.19	1.26	1.18	S
2.75	2.87	2.96	2.53	S	1.67	1.82	1.54	S	1.27	1.38	1.22	S	1.12	1.17	1.1	S
3	2.61	2.68	2.27	S	1.52	1.67	1.41	S	1.19	1.28	1.15	S	1.07	1.11	1.06	S
3.5	2.24	2.31	1.93	S	1.33	1.46	1.25	S	1.09	1.15	1.07	S	1.03	1.04	1.02	S
4	2	2.07	1.73	S	1.22	1.33	1.16	S	1.05	1.09	1.04	S	1.01	1.02	1.01	S
4.5	1.82	1.9	1.59	S	1.15	1.24	1.11	S	1.03	1.05	1.02	S	1	1.01	1	C, S
5	1.69	1.77	1.48	S	1.11	1.17	1.07	S	1.01	1.03	1.01	C,S	1	1	1	C,E,S

Table 4.6. Steady-state ARL comparisons among CUSUM-S², EWMA-S², and the proposed chart when the charts are optimized for a target shift $\gamma_{target} = 0.95$. Zero-state $ARL_0 \cong 370$.

γ	n = 3				n = 5				n = 7				n = 9			
	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min
0.5	14.62	11.03	16.69	E	9.56	7.1	8.48	E	7.51	4.67	5.85	E	6.1	4.81	4.92	E
0.55	16.25	12.6	18.91	E	10.59	7.97	9.6	E	8.31	5.27	6.62	E	6.74	5.34	5.54	E
0.6	18.32	14.72	21.93	E	11.92	9.12	11.12	E	9.34	6.08	7.64	E	7.57	6.05	6.36	E
0.65	21.05	17.63	26.02	E	13.7	10.71	13.21	E	10.72	7.23	9.09	E	8.68	7	7.5	E
0.7	24.8	22.14	32.01	E	16.13	13.04	16.26	E	12.59	8.93	11.2	E	10.21	8.35	9.19	E
0.75	30.09	29.48	41.29	E	13.7	16.64	20.96	C	15.32	11.75	14.45	E	12.43	10.43	11.75	E
0.8	38.5	42.86	56.81	C	25.12	22.84	29.04	E	19.55	16.89	19.98	E	15.87	13.81	16.11	E
0.85	52.92	71.04	86.58	C	34.66	35.58	44.8	C	26.99	28.36	31.09	C	22.04	20.39	24.69	E
0.9	83.2	141.75	152.3	C	55.5	68.93	84.12	C	43.21	60.93	58.72	C	35.64	37.34	46.39	C
0.95	165.94	327.26	301.96	C	123.29	188.09	212.02	C	99.1	182.88	162.79	C	85.08	108.55	132.46	C
1	286.84	419.78	445.38	C	305.48	384.25	385.87	C	312.32	367.08	374.14	C	322.78	368.8	374.05	C
1.05	156.71	198.38	312.52	C	118.32	143.52	184.76	C	96.66	124.88	141.49	C	83.34	94.65	119.01	C
1.1	83	94.7	143.65	C	56.18	59.38	76.55	C	44.11	47.5	54.93	C	36.52	36.16	44.62	E
1.15	54.5	55.62	83.76	C	36.14	33.91	43.17	E	28.17	25.39	30.53	E	23.08	20.97	24.83	E
1.2	40.53	37.75	56.94	E	26.6	23.19	29.09	E	20.86	16.64	20.42	E	16.97	14.69	16.77	E
1.25	32.31	28.18	42.9	E	21.13	17.57	21.63	E	16.59	12.28	15.17	E	13.48	11.35	12.56	E
1.3	26.92	22.45	34.21	E	17.59	14.17	17.19	E	13.81	9.74	12.02	E	11.24	9.31	10.03	E
1.35	23.07	18.62	28.51	E	15.13	11.91	14.27	E	11.87	8.07	10	E	9.69	7.9	8.36	E
1.4	20.28	15.91	24.38	E	13.28	10.29	12.2	E	10.45	6.94	8.54	E	8.52	6.9	7.19	E
1.45	18.1	13.93	21.36	E	11.89	9.1	10.66	E	9.37	6.09	7.49	E	7.65	6.15	6.33	E
1.5	16.38	12.4	19.06	E	10.78	8.17	9.5	E	8.52	5.46	6.69	E	6.94	5.56	5.68	E
1.75	11.33	8.18	12.52	E	7.51	5.54	6.28	E	5.98	3.7	4.45	E	4.9	3.88	3.84	S
2	8.85	6.29	9.53	E	5.92	4.34	4.83	E	4.74	2.91	3.45	E	3.91	3.1	3	S
2.25	7.37	5.19	7.85	E	4.98	3.63	4.02	E	4.01	2.46	2.9	E	3.32	2.64	2.54	S
2.5	6.4	4.49	6.75	E	4.36	3.18	3.49	E	3.53	2.18	2.53	E	2.94	2.34	2.24	S
2.75	5.72	3.99	5.96	E	3.92	2.86	3.12	E	3.19	1.97	2.29	E	2.67	2.13	2.04	S
3	5.2	3.63	5.41	E	3.59	2.63	2.85	E	2.93	1.83	2.11	E	2.46	1.97	1.88	S
3.5	4.48	3.13	4.62	E	3.13	2.3	2.48	E	2.58	1.61	1.87	E	2.17	1.75	1.69	S
4	4	2.8	4.12	E	2.82	2.08	2.24	E	2.35	1.47	1.71	E	1.97	1.61	1.56	S
4.5	3.65	2.57	3.74	E	2.61	1.93	2.08	E	2.17	1.36	1.6	E	1.84	1.5	1.47	S
5	3.4	2.39	3.47	E	2.44	1.81	1.95	E	2.03	1.28	1.52	E	1.76	1.41	1.4	S

Table 4.7. Steady-state ARL comparisons among CUSUM-S², EWMA-S², and SC-S² when both charts are optimized for a target shift $\gamma_{target} = 1.5$. Zero-state $ARL_0 = 370$.

	n = 3				n = 5				n = 7				n = 9			
γ	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min	CUSUM-S ²	EWMA-S ²	SC-S ²	min
0.5	16.98	11.03	13.51	E	6.53	7.1	5.66	S	3.9	4.67	3.83	S	2.9	4.81	2.95	C
0.55	26.72	12.6	19.08	E	9.19	7.97	6.86	S	5.08	5.27	4.58	S	3.65	5.34	3.52	S
0.6	49.34	14.72	30.25	E	14.45	9.12	8.79	S	7.19	6.08	5.69	S	4.96	6.05	4.36	S
0.65	105.52	17.63	54.02	E	26.01	10.71	12.3	E	11.47	7.23	7.52	E	7.5	7	5.75	S
0.7	247.2	22.14	106.89	E	52.85	13.04	19.48	E	21.03	8.93	11	E	12.99	8.35	8.47	E
0.75	597.18	29.48	227.63	E	26.01	16.64	35.24	E	43.43	11.75	18.63	E	26.11	10.43	14.41	E
0.8	1379.1	42.86	505.29	E	261.43	22.84	72.93	E	96.39	16.89	37.12	E	58.51	13.81	29.1	E
0.85	2382.17	71.04	1025.45	E	565.54	35.58	166.34	E	218.79	28.36	85.36	E	137.65	20.39	67.86	E
0.9	1831.19	141.75	1268.32	E	960.47	68.93	379.39	E	455.89	60.93	213.96	E	311.74	37.34	174.01	E
0.95	823.84	327.26	761.35	E	790.96	188.09	580.7	E	617.65	182.88	446.13	E	521.95	108.55	397.51	E
1	367.28	419.78	361.79	S	367.05	384.25	368.03	C	366.82	367.08	367.25	C	369.24	368.8	368.19	S
1.05	182.07	198.38	180.99	S	163.16	143.52	161.8	E	157.43	124.88	151.77	E	152.92	94.65	147.5	E
1.1	101.5	94.7	99.95	E	80.9	59.38	76.88	E	71.43	47.5	64.86	E	65.71	36.16	59.24	E
1.15	62.02	55.62	61.23	E	45.01	33.91	41.92	E	37.23	25.39	32.94	E	32.58	20.97	28.7	E
1.2	41.13	37.75	40.44	E	27.76	23.19	25.74	E	21.97	16.64	19.4	E	18.49	14.69	16.45	E
1.25	29.21	28.18	28.68	E	18.72	17.57	17.35	S	14.32	12.28	12.89	E	11.82	11.35	10.69	S
1.3	21.83	22.45	21.37	S	13.48	14.17	12.72	S	10.13	9.74	9.34	S	8.24	9.31	7.66	S
1.35	17.02	18.62	16.69	S	10.28	11.91	9.8	S	7.63	8.07	7.24	S	6.17	7.9	5.88	S
1.4	13.78	15.91	13.46	S	8.17	10.29	7.91	S	6.05	6.94	5.83	S	4.89	6.9	4.75	S
1.45	11.48	13.93	11.22	S	6.73	9.1	6.6	S	4.98	6.09	4.9	S	4.03	6.15	3.97	S
1.5	9.78	12.4	9.56	S	5.71	8.17	5.64	S	4.22	5.46	4.2	S	3.42	5.56	3.42	S
1.75	5.57	8.18	5.39	S	3.24	5.54	3.3	C	2.45	3.7	2.52	C	2.03	3.88	2.08	C
2	3.94	6.29	3.79	S	2.34	4.34	2.39	C	1.81	2.91	1.86	C	1.53	3.1	1.57	C
2.25	3.13	5.19	2.96	S	1.89	3.63	1.92	C	1.5	2.46	1.53	C	1.29	2.64	1.32	C
2.5	2.63	4.49	2.48	S	1.63	3.18	1.64	C	1.32	2.18	1.34	C	1.17	2.34	1.18	C
2.75	2.31	3.99	2.16	S	1.46	2.86	1.47	C	1.21	1.97	1.22	C	1.1	2.13	1.11	C
3	2.08	3.63	1.93	S	1.35	2.63	1.35	S	1.14	1.83	1.14	C,S	1.06	1.97	1.06	C,S
3.5	1.77	3.13	1.65	S	1.21	2.3	1.2	S	1.07	1.61	1.07	C,S	1.02	1.75	1.02	C,S
4	1.58	2.8	1.48	S	1.13	2.08	1.13	C,S	1.03	1.47	1.03	S,C	1.01	1.61	1.01	C,S
4.5	1.46	2.57	1.38	S	1.09	1.93	1.09	C,S	1.02	1.36	1.02	C,S	1	1.5	1	C,S
5	1.37	2.39	1.3	S	1.06	1.81	1.06	C,S	1.01	1.28	1.01	C,S	1	1.41	1	C,S

Steady-state ARL Comparison

Based on the results from Tables 4.5 – 4.7, we categorize our comparison discussion into 3 cases:

Case 1: $\gamma_{target} = 0.5$ (Table 4.5)

It is seen that when the charts are optimized for a large shift in the negative direction, there is a mixed performance among the competing charts. CUSUM-S² dominates at $\gamma = 0.5$ and values close to 0.5, for all the different sample sizes ($n = 3, 5, 7$, and 9). Note that this is the shift all the charts are optimized to detect the fastest. For example, when $n = 3$ and $\gamma = 0.5$, the ARL value for CUSUM-S² is 9.7 as compared to 10.28 and 11.29 for EWMA-S² and the proposed chart respectively. However, for smaller shifts ($0.75 \leq \gamma \leq 0.95$), the proposed chart outperforms both CUSUM-S² and EWMA-S² for sample sizes $n = 3$ and 5 , while for $n = 7$ and 9 , the EWMA-S² chart takes the lead. All the charts are ARL-biased at $\gamma = 0.95$, and the biasedness is more pronounced when the sample size is small. The proposed chart is the least biased for $n = 3$ and 5 , while the EWMA-S² chart is the least when $n = 7$ and 9 . For minor upward shifts ($1.05 \leq \gamma \leq 1.2$), EWMA-S² dominates the competition, consistently showing the least ARL values. However, the proposed showed improved performance at higher shifts like $\gamma = 2$ and above, uniformly outperforming its competitors. At large shifts, the performance of all the charts converge especially when the sample size is large. [72] noted that in monitoring process location, no chart surpasses CUSUM in terms of steady-state ARL at a specific target when the chart is optimized for that target. This holds for process dispersion monitoring as well, in most cases, however, there are exceptions.

Case 2: $\gamma_{target} = 0.95$ (Table 4.6)

It is observed again that CUSUM-S² beats other charts for shifts $\gamma = \gamma_{target}$ and other changes around it. For instance, for shifts in the range $0.8 \leq \gamma \leq 1.15$, the chart consistently shows the least ARL values compared to the other charts across different sample sizes, except for larger sample sizes. It can be seen that the improved ARL_1 performance of the CUSUM-S² chart in this case comes with a cost of a significant reduction in the desired ARL_0 especially when $n = 3$, which results in a higher false alarm rate. In contrast, the other charts, especially EWMA-S², have shown remarkable increases in the ARL_0 values for different sample sizes. Take for instance when $n = 3$, the ARL_0 values are 286.84, 419.78, and 445.38 respectively for CUSUM-S², EWMA-S², and the proposed chart. It is also clear that for almost all the shifts not close to the target shift EWMA-S² outperforms the other charts under investigation. This may be due to the fact that, although all the charts are optimized for a shift $\gamma_{target} = 0.95$, the optimal parameter of the EWMA-S² for this shift is also optimal for all other shifts $\gamma \geq 0.95$, and hence, in this case, the EWMA-S² chart may have a competitive advantage over the charts. In this case, the proposed chart is not the best choice as it is uniformly dominated by the other competing charts for all the shifts and sample sizes, except in a few instances where n is large.

Case 3: $\gamma_{target} = 1.5$ (Table 4.7)

The major issue to point out in this case is how the charts behave in detecting downward shifts when they are optimized for an upward shift. As mentioned, the optimal parameter for EWMA-S² is the same for all shifts $\gamma \geq 0.95$ so the chart is not affected by this issue.

However, for CUSUM-S² and the proposed chart, the case is different, as they are significantly ARL-biased for most of the negative shifts. The reason for this poor performance is that both charts require very small values of their respective reference parameters to efficiently detect small negative shifts, and moderate to large values of the parameters for the positive shifts. However, it can be seen that for upward shifts the proposed chart gains dominance in faster detection of shifts at $\gamma \geq 1.3$. The proposed chart and CUSUM-S² have similar performance at larger shifts, while EWMA-S² lags behind them.

Table 4.8. SC-S², CUSUM-S², and EWMA-S² EQL values computed from steady-state ARLs using optimal parameters

SC-S ²												
	n = 3			n = 5			n = 7			n = 9		
γ_{target}	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL
0.5	128.57	28.12	156.69	86.54	18.34	104.88	101.63	15.82	117.45	85.06	14.75	99.81
0.6	107.61	30.70	138.31	66.75	19.06	85.81	68.22	15.65	83.87	57.82	14.49	72.32
0.7	99.93	33.51	133.45	60.08	19.91	80.00	43.91	16.24	60.14	37.57	14.43	52.00
0.8	95.87	38.91	134.78	54.34	22.62	76.96	38.86	18.15	57.01	31.97	15.06	47.03
0.9	99.71	54.20	153.91	54.72	28.58	83.29	38.89	21.33	60.23	30.51	17.30	47.81
0.95	102.34	59.95	162.28	56.67	32.07	88.74	40.20	23.92	64.12	32.58	21.27	53.85
1.05	569.03	25.18	594.21	60.08	19.91	80.00	39.53	17.53	57.07	30.52	16.48	47.00
1.1	185.88	26.28	212.16	58.78	20.23	79.01	39.74	17.36	57.09	31.16	15.44	46.60
1.2	181.28	26.36	207.64	69.84	18.85	88.69	46.89	15.94	62.83	35.36	14.52	49.88
1.3	201.81	26.10	227.91	87.96	18.30	106.26	61.41	15.63	77.04	44.51	14.35	58.86
1.4	283.77	25.60	309.37	115.14	18.17	133.31	72.70	15.66	88.37	59.02	14.50	73.52
1.5	569.03	25.18	594.21	147.13	18.13	165.27	88.83	15.74	104.56	73.40	14.64	88.04
2	1518.36	25.30	1543.66	289.83	18.31	308.14	141.73	16.01	157.74	84.23	14.75	98.98
CUSUM-S ²												
	n = 3			n = 5			n = 7			n = 9		
γ_{target}	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL
0.5	147.81	30.59	178.40	118.12	19.28	137.40	104.74	16.07	120.81	92.93	14.84	107.77
0.6	108.79	32.87	141.66	79.62	20.51	100.12	68.82	16.45	85.28	61.50	14.83	76.33
0.7	85.04	36.04	121.08	57.90	22.77	80.67	48.84	17.39	66.23	41.93	15.15	57.08
0.8	71.85	40.84	112.69	46.36	26.43	72.79	36.98	20.17	57.15	31.08	16.62	47.70
0.9	67.71	49.53	117.24	42.74	32.21	74.96	33.54	25.72	59.25	27.51	21.75	49.26
0.95	68.43	54.44	122.87	44.55	37.82	82.37	35.88	30.94	66.82	29.49	25.98	55.47
1.05	488.52	27.15	515.67	44.08	33.80	77.88	34.47	28.48	62.95	28.74	24.84	53.58
1.1	74.36	39.39	113.75	43.26	29.27	72.53	33.72	23.67	57.40	27.57	20.18	47.76
1.2	117.12	32.28	149.40	55.61	23.18	78.79	42.71	18.27	60.98	34.91	15.72	50.62
1.3	214.12	28.93	243.05	94.86	19.93	114.79	64.78	16.53	81.31	51.88	14.88	66.76
1.4	401.19	27.40	428.58	192.14	18.63	210.77	111.76	16.03	127.79	81.61	14.81	96.41
1.5	1138.26	26.39	1164.65	346.05	18.36	364.41	172.25	15.98	188.24	119.81	14.89	134.70
2	50743.99	25.94	50769.93	1818.08	18.53	1836.60	324.58	16.11	340.70	158.30	15.00	173.30
EWMA-S ²												
	n = 3			n = 5			n = 7			n = 9		
γ_{target}	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL	EQL ⁻	EQL ⁺	EQL
0.50	167.61	31.14	198.76	106.22	20.17	126.39	86.98	16.40	103.39	78.78	14.88	93.66
0.60	116.58	33.65	150.22	78.96	21.36	100.33	62.88	16.96	79.84	53.48	15.05	68.53
0.70	100.09	35.55	135.64	50.83	25.73	76.57	47.48	18.12	65.61	38.76	15.69	54.45
0.80	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	31.18	17.23	48.41
0.90	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.04	20.36	48.40
0.95	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.05	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.10	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.20	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.30	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.40	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
1.50	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34
2.00	85.06	40.27	125.33	46.85	28.82	75.67	38.01	20.36	58.37	28.03	21.31	49.34

Finally, the three charts are compared in terms of overall (rather than individual shifts) performance using the EQL metric. The computation of the EQL is done using the optimal parameters of the charts and generating steady-state ARLs for a range of shifts from 0.5 to 5. Table 4.8 presents the EQL values of the proposed, CUSUM-S², and EWMA-S² charts. EQL⁻ is the EQL computed only for downward shifts, while EQL⁺ is the EQL computed for only upward shifts, and when combined they form the overall EQL. For Example, for CUSUM-S² at $\gamma_{target} = 0.5$, the EQL⁻, EQL⁺ and EQL values are 147.81, 30.59, and 178.40 respectively when $n = 3$. This means that when CUSUM-S² is optimized for a shift of 0.5, the EQL for: $0.5 \leq \gamma \leq 0.95$ is 147.81, $1.05 \leq \gamma \leq 5$ is 30.59, and the overall EQL is 178.40. The following points summarized the comparisons in terms of EQL among the competing charts.

- Proposed VS CUSUM-S²: The proposed chart dominates CUSUM-S² for upward shifts, especially for larger sample size, as indicated by consistently lower EQL⁺ values. It also outperforms CUSUM-S² when the charts are optimized for larger shifts in either direction. For example, when $n = 3$ and $\gamma_{target} = 0.5$, the overall EQL of the proposed chart is 156.69 while that of CUSUM-S² is 178.40, and at $\gamma_{target} = 1.3$, the values are 227.91 and 243.05 for the proposed chart and CUSUM-S² respectively.
- Proposed VS EWMA-S²: It is also seen here that the proposed chart outperforms EWMA-S² when we only consider the upward shifts in the process dispersion. However, the overall performance of the proposed chart deteriorates in the case of a reduction in the process dispersion. For instance, at $\gamma_{target} = 1.2$, when $n = 3$,

EQL^- values are 181.28 and 85.06, while, EQL^+ are 26.36 and 40.27, respectively for the proposed and EWMA- S^2 .

In summary, we see that the proposed chart performs the best in overall performance for increases in the variability of the process, and trails behind the other charts in the case of a decrease in the process variability. However, in most real applications, practitioners are more concerned with an increase in the process variance rather than a decrease, as the former will result in lower uniformity of final output, while the latter might be an indication of improved process performance. Therefore, an upper one-sided version of the proposed chart can be developed to monitor increases in the process variance. It is expected that this version may outperform both CUSUM- S^2 and EWMA- S^2 charts in terms of individual and overall performance.

4.5.3 Graphical Comparison using CED Curves

Figures 4.5-4.8 CED compare the performance of the competing charts for selected shift values ($\gamma = 0.5, 0.95, 1.1, \text{ and } 1.5$) and sample sizes $n = 3, 5, 7, \text{ and } 9$. These shifts represent both downward and upward small to moderate changes in the process standard deviation, along with a range of small to large sample sizes. The results are obtained by optimizing each chart for the specific shift, with the assumption that the zero-state $ARL_0 \approx 370$. Change points (τ) ranging from 1 to 501 are taken into account, but it's important to note that the plots do not include change points where the CED values for all three charts remain constant. It is clear that for small to moderate reduction in the process dispersion CUSUM- S^2 is optimal across different sample sizes (Figures 4.5 and 4.6). However, for a small increase ($\gamma = 1.1$), EWMA- S^2 shows a better performance than the other two charts, especially for larger sample sizes. The proposed chart excels at detecting larger positive

shifts in the process variability. These scenarios further highlight that no single chart always performs the best, hence the need for an informed selection of the chart to be used based on the nature of the process and the anticipated shift. In many practical scenarios, the exact shift that will occur is often unknown. As discussed in earlier sections, the proposed chart offers broader protection for a range of shifts rather than targeting a specific shift.

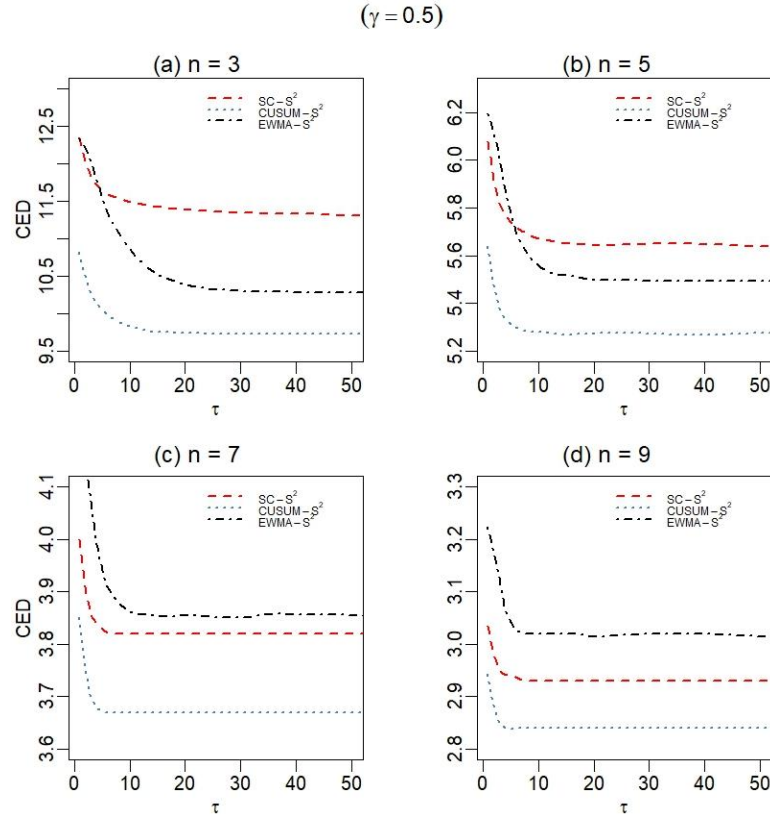


Figure 4.5. CED curves comparison of the proposed, CUSUM-S², and EWMA-S² charts when $\gamma = 0.5$ for different sample sizes.

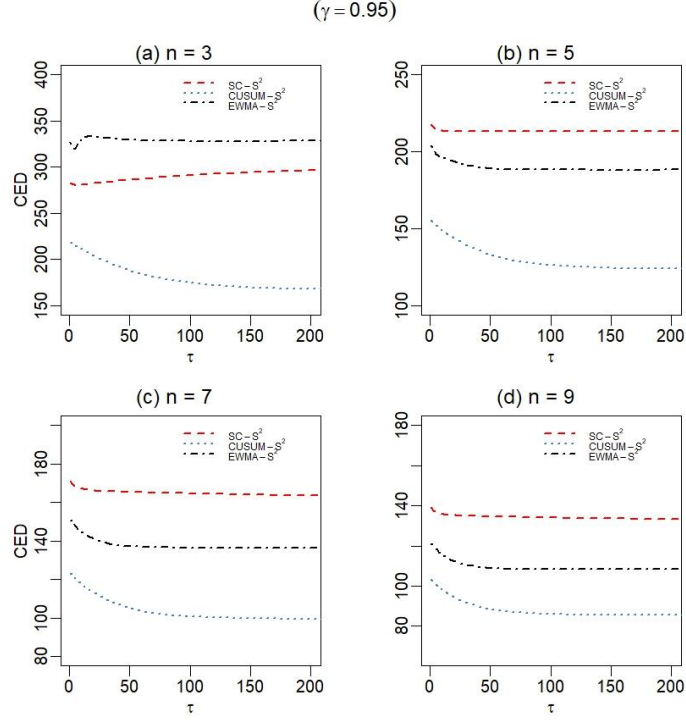


Figure 4.6. CED curves comparison of the proposed, CUSUM-S², and EWMA-S² charts when $\gamma = 0.95$ for different sample sizes.

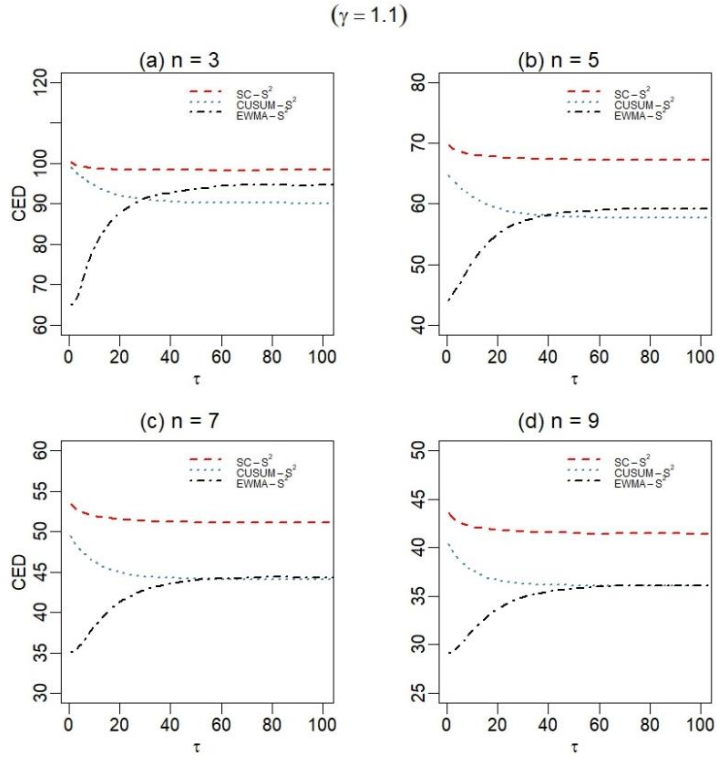


Figure 4.7. CED curves comparison of the proposed, CUSUM-S², and EWMA-S² charts when $\gamma = 1.1$ for different sample sizes.

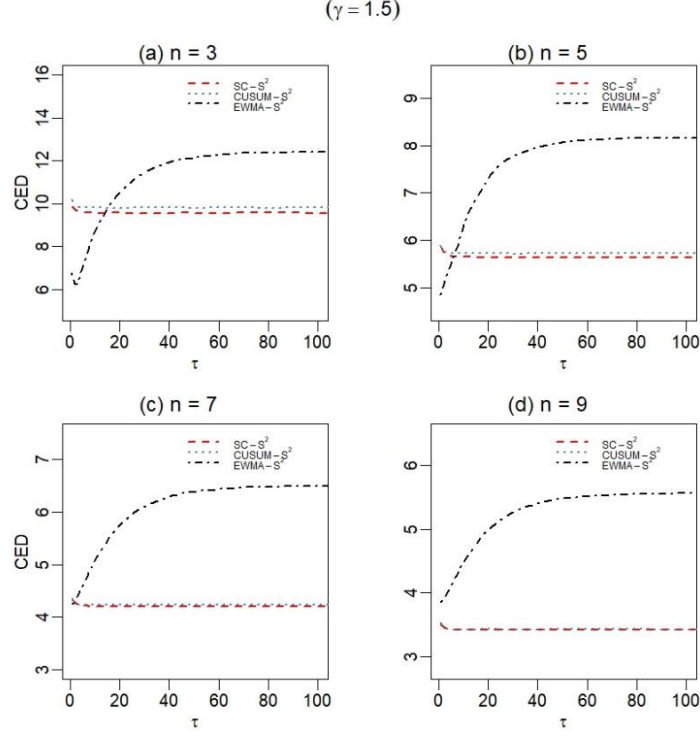


Figure 4.8. CED curves comparison of the proposed, CUSUM- S^2 , and EWMA- S^2 charts when $\gamma = 1.5$ for different sample sizes.

4.6. Illustrative Examples

Consistent with other studies on control charts, this Section presents a practical example using both simulated and real-life data to show the implementation and applicability of the proposed chart. In the following subsections, two examples are demonstrated, the first with simulated dataset and the second with real dataset. To provide performance comparisons, in both cases, the proposed chart, CUSUM- S^2 , and EWMA- S^2 are applied to the data to detect departures from target values.

4.6.1 Example 1: Simulated Data

In this example, 25 independent samples each of size 5 were drawn from a process following Normal distribution with mean $\mu_0 = 10$ and standard deviation $\sigma_0 = 2$ i.e.,

$X_{ij} \sim N(\mu_0 = 10, \sigma_0 = 2)$ for $i = 1, 2, \dots, 5$ and $j = 1, 2, \dots, 25$, as a realization of the IC state. Then 15 additional samples were drawn from the Normal distribution with a mean $\mu_0 = 10$ and standard deviation $\sigma_1 = 1.6$, indicating a downward shift of $\gamma = \sigma_1/\sigma_0 = 0.8$ in the process standard deviation. Accordingly, the 3 charts are constructed so that each chart is optimal for the target shift of 0.8 with $ARL_o \approx 370$. For the proposed chart, the parameter combination $(K_{SC}, L_{SC}) = (0.18, 3.1635)$ is used, while for the CUSUM-S² and EWMA-S² charts, the combinations are $(K_C, H_C) = (0.24, 7.7933)$ and $(\lambda_E, L_E) = (0.08, 2.634)$ respectively. Figure 4.10 displays the plots of the three charts, showing how effectively they detect the introduced shift. The IC region is plotted in the light green area, while the OOC region is highlighted in the light red area, and any OOC signal is indicated by a red marker point. It is seen that both the proposed chart and EWMA-S² signal the first OOC situation at sample number 34. However, while the proposed chart detects all subsequent shifts, giving a total of 7 signals, EWMA-S² misses some afterward, detecting only 4 OOC samples.

In contrast, CUSUM-S² signals slightly later at sample 36 and detects all subsequent shifts. This example highlights the importance of proper selection of control chart depending on the need and nature of the process to be monitored, as it is clear that different charts perform differently in terms of quick detection of shifts and number of shifts detected.

4.6.2 Example 2: Combined Cycle Power Plant (CCPP)

This example follows a study originally carried out by [80], which focuses on predicting the electrical power output of a Combined Cycle Power Plant (CCPP), which generates electricity by combining gas and steam turbines. The gas turbine burns natural gas,

producing both electricity and hot exhaust gases, which are then used to create steam that drives a steam turbine for additional power generation, resulting in high efficiency. The study used data collected over six years, comprising 9,568 hourly data points recorded from 674 different days on four key variables affecting power output: ambient temperature (AT), atmospheric pressure (AP), relative humidity (RH), and vacuum (V). Machine learning models were used to predict power output based on these variables.

Studies like [81] have since applied this dataset for control chart applications. In this study, the AP is chosen as the critical variable of interest to be monitored. According to [80], AP is one of the significant factors affecting the overall power output of the plant. Specifically, lower AP could decrease the efficiency of the gas turbine which in turn adversely impacts the power generation. For illustration purposes, the first 600 AP values are assumed to be the IC data, and 30 samples each of size 5 are drawn from the data representing the IC samples. The mean standard and standard deviation of the data are found to be $\mu_0 = 1013.546$ and $\sigma_0 = 5.8222$ respectively. The normality assumption is investigated and found to be satisfied through the Anderson-Darling test with test statistic $A = 0.3481$ and p-value: 0.4765. Furthermore, for testing the independence assumption, we found that the first-order autocorrelation is insignificant with the Ljung-Box test P-value = 0.7782 (the same is true for higher orders). Figure 4.9 displays the graphical tests of the normality assumption using histogram and normal probability plots, as well as the ACF plot which tests the autocorrelation among the AP observations at various lags of time. All the plots further confirm that the normality and independence assumptions are satisfied by the AP data.

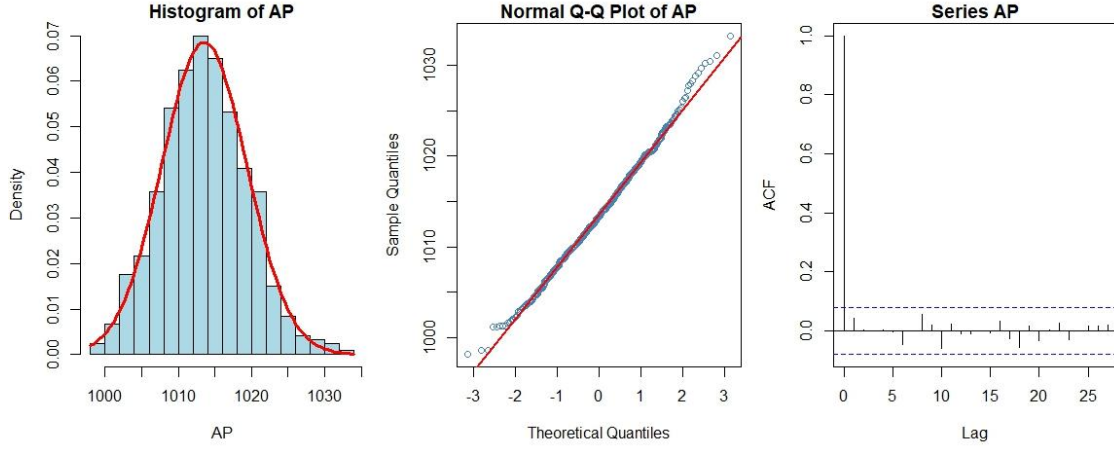


Figure 4.9. Histogram, normal Q-Q plot and ACF plot for the AP data.

Next, assuming $ARL_0 \approx 370$, the proposed, CUSUM-S², and EWMA-S² control charts are applied to the AP data using the 30 samples drawn in the IC state and additional 20 samples from a normal distribution with mean $\mu_0 = 1013.546$ and standard deviation $\sigma_1 = 8.7333$, meaning that a shift of $\gamma = 8.7333/5.8222 \approx 1.5$ has been introduced to the process dispersion. The following parameters are used in setting up the charts so that each chart is optimal for the targeted shift (1.5):

The proposed chart: $(K_{SC}, L_{SC}) = (0.85, 3.2338)$; CUSUM-S²: $(K_C, H_C) = (0.98, 2.2409)$; EWMA-S²: $(\lambda_E, L_E) = (0.05, 2.514)$.

Figure 4.10 shows that the proposed chart detects the first OOC condition at sample 37, following an increase in variability. It successfully identifies all subsequent shifts, except for sample 42. EWMA-S² signals its first OOC slightly later, at sample 39. CUSUM-S², however, signals even later, at sample 43, detecting only 4 true shifts and generating 2 false alarms. Overall, the proposed chart identifies more OOC samples than both EWMA-S² and CUSUM-S² and notably, it produces no false alarms.

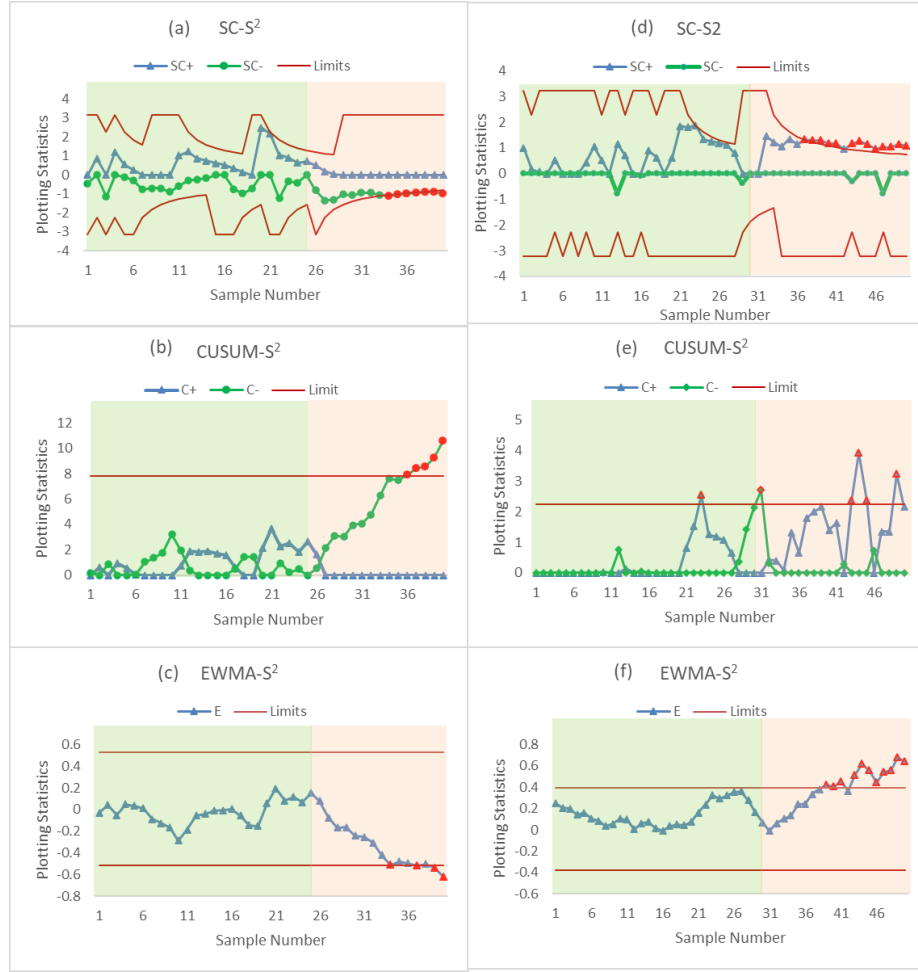


Figure 4.10. The SC-S², CUSUM-S², and EWMA-S² charts for the simulated data (a, b, c), Example 1, and SC-S², CUSUM-S², and EWMA-S² charts for the AP data (d, e, f), Example 2.

4.7 Chapter 4 Conclusion

This Chapter has introduced a new control chart for monitoring changes in the process dispersion based on the popular three-parameter transformance of the sample variance. The efficiency of the proposed chart was evaluated in both zero- and steady-state conditions using ARL and EQL measures for individual and overall performance respectively. Through extensive simulations study, the optimal parameter choices of the proposed chart

have been provided for easy implementation by quality practitioners. The proposed chart has been compared with existing CUSUM-S² and EWMA-S² control charts that have been developed for the same purpose. The comparison shows that the proposed chart dominates its competitors in the zero-state setting and continues to deliver strong results in the steady-state, particularly when larger positive shifts occur or when the sample size is large.

Two examples, using both simulated and real data, demonstrate the applicability of the proposed chart in real-life scenarios. The first example uses simulated data to show the basic behavior of the chart in comparison to other charts, while the second applies it to real atmospheric pressure (AP) data from a Combined Cycle Power Plant (CCPP), highlighting its practical utility in the energy sector.

CHAPTER 5

PROPOSED JOINT CAM CHARTS

Recently, [72] introduced a new Step CUSUM (SC) control chart that utilizes a CUSUM structure within a cumulative aggregate mean approach to effectively detect a wide range of shifts in the process mean. His findings demonstrate that this new chart outperforms the traditional CUSUM chart in identifying shifts that differ from the values for which the charts are optimized. Taking inspiration from the works of [43, 52, 72], we propose six new charts for the joint monitoring of a normal process location and dispersion parameters based on the cumulative aggregate mean. The charts were developed using two popular methods: the maximum method and the sum of squares method. In each approach, we incorporated three different variance transformations: chi-square transformation, three-parameter transformation, and four-parameter transformation of the sample variance. Using Monte Carlo simulations, the effectiveness of the proposed charts was assessed under both zero- and steady-state conditions by examining the various features of their run-length distributions. These new charts respond to changes in the process mean and/or standard deviation, demonstrating good performance in identifying a range of shifts in the process parameters.

5.1 Existing Methods

This Section discusses different transformations commonly employed to convert sample variance into an approximately normal distribution. Then, we briefly review CUSUM and

EWMA charts for simultaneous monitoring based on the maximum and sum of squares methods.

Let $X_{1,t}, X_{2,t}, \dots, X_{n,t}$ be n independent sequence of random observations from a process $\{X_t; t \geq 1\}$ following a normal distribution with a stable known mean μ_0 and standard deviation σ_0 , where X_t is a measurement from a critical-to-quality variable taken at time t . The process $\{X_t; t \geq 1\}$ is assumed to be IC for $t \leq \tau$, where τ denotes the change-point at which the process becomes out-of-control (OOC) due to a shift in the mean and/or standard deviation. Mathematically, when the process is IC ($t \leq \tau$), $X_t \sim N(\mu_0, \sigma_0)$, whereas for OOC ($t \geq \tau$), $X_t \sim N(\mu_1 = \mu_0 + \delta\sigma_0, \sigma_1 = \theta\sigma_0)$. It should be noted here that IC state corresponds to when $\delta = 0$ and $\theta = 1$. Moreover, we considered the following OOC scenarios: $(N - D)$, $(U - N)$, $(U - D)$, and $(U - U)$, which represent a negative shift only in the standard deviation, a positive shift only in the mean, a positive shift in the mean and a negative shift in the standard deviation, and positive shifts in both the mean and standard deviation, respectively. When the shift is introduced at the beginning of monitoring, that is, when $\tau = 1$, the condition is known as zero-state, whereas steady-state refers to the condition where the shift is applied after the process has been operating for some time without any prior signals, that is, when $\tau \rightarrow \infty$.

5.1.1 Transformations

To jointly control for the process mean and dispersion, we first define sample mean $\bar{X}_t = (X_{1,t} + X_{2,t} + \dots + X_{n,t})/n$, and sample variance $S_t^2 = \sum_{i=1}^n (X_{i,t} - \bar{X}_t)^2 / (n - 1)$. Then, $\bar{X}_t$ and S_t^2 are independent (Basu's theorem) and have the following distributions: $\bar{X}_t \sim N(\mu_0, \sigma_0/\sqrt{n})$, and $(n - 1)S_t^2/\sigma_0^2 \sim \chi_{n-1}^2$.

To transform the mean and variance estimators $(\bar{X}_t, S_t^2)$ to follow the same distribution, [52] use the following transformations:

$$V_{1,t} = \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}}, \quad (5.1)$$

$$V_{2,t} = \varphi^{-1} \left[\omega_{\chi_{n-1}^2} \left(\frac{(n-1)S_t^2}{\sigma_0^2} \right) \right], \quad (5.2)$$

where $\varphi^{-1}[\cdot]$ represents the inverse of the cumulative distribution function (CDF) of the standard normal distribution and $\omega_{\chi_{n-1}^2}(\cdot)$ represents the CDF of a Chi-squared random variable having $(n-1)$ degrees of freedom. It can be shown that under IC process, both $V_{1,t}$ and $V_{2,t}$ follow a standard normal distribution, $N(0,1)$. However, a shift in the process parameters is reflected by a shift in the mean and/or variance of the transformed variables, according to the nature of the deviation. Therefore, the transformed variables can be employed to monitor shifts in the process parameters at the same time. To further enhance the detection ability of the joint charts, [48] suggested using a three-parameter logarithm transformation of S_t^2 before the normal standardization as follows:

$$T_{1,t} = a + b \ln(S_t^2 + c) \quad (5.3)$$

[26] defined the constants a, b , and c as follows: $b = B(n)$, $c = C(n)\sigma_0$, and $a = A(n) - 2B(n)\ln(\sigma_0)$, and showed that the distribution of $T_{1,t}$ is approximately normal with mean $E[T_{1,t}] = \mu_n$ and standard deviation $\sigma[T_{1,t}] = \sigma_n$. Where μ_n and σ_n depend only on the sample size n . Using this information, we define another variable for controlling the variance as follows:

$$V_{3,t} = \frac{T_{1,t} - \mu_n}{\sigma_n} \quad (5.4)$$

It is widely recognized that when the process is in IC state, the distribution of $V_{3,t}$ is also $N(0,1)$, and when the process is OOC, it is also reflected in the mean and/or variance of $V_{3,t}$, depending on the nature of the contamination.

In an effort to make the distribution of S_t^2 more symmetric, [29] proposed using a four-parameter Johnson transformation on S_t^2 before using it in controlling the dispersion parameter. The transformation is defined as follows:

$$T_{2,t} = a + b \left(\frac{S_t^2 - c}{c + d - S_t^2} \right) \quad (5.5)$$

The rationality of using this transformation and the distribution of $T_{2,t}$, as well as $E[T_{2,t}]$, $\sigma[T_{2,t}]$, and the constants a, b, c , and d are given in [29]. Based on $T_{2,t}$ we define another variable $V_{4,t}$ as follows:

$$V_{4,t} = \frac{T_{2,t} - E[T_{2,t}]}{\sigma[T_{2,t}]} \quad (5.6)$$

Similar to $V_{3,t}$, $V_{4,t} \sim N(0,1)$ if the process $\{X_t\}$ is under IC state, and reflect shifts in the OOC situations. In the following subsections, the transformed variables, $V_{1,t}, V_{2,t}, V_{3,t}$, and $V_{4,t}$ are used to present the existing charts and in the construction of new charts.

5.1.2 Max-EWMA and SS-EWMA

[43] proposed using the two transformed variables in equations (5.1) and (5.2) in an EWMA structure and combining them using a max function to construct a single Max-EWMA (denoted Max-EWMA- χ^2 in this study) chart that jointly detects shifts in the process mean and variability. The two independent EWMA statistics are given as:

$$\left. \begin{aligned} E_{1,t} &= \lambda V_{1,t} + (1 - \lambda)V_{1,t-1} \\ E_{2,t} &= \lambda V_{2,t} + (1 - \lambda)V_{2,t-1} \end{aligned} \right\} \quad (5.7)$$

Where $0 < \lambda \leq 1$ is the sensitivity parameter of the EWMA chart that controls its sensitivity to process shifts and the values $E_{1,0}$ and $E_{2,0}$ are set to zero. Since the two statistics in equation (5.7) have the same distribution, they are merged into a single statistic, $M_{1,t} = \max(|E_{1,t}|, |E_{2,t}|)$ with the following upper control limit (UCL): $UCL = \sqrt{\lambda/(2 - \lambda)} (1.128379 + 0.602810L_1)$. L_1 controls the width of the limit.

In a similar way, [48] proposed another version of the EWMA chart by replacing the variable controlling the dispersion parameter in equation (5.7) with the transformation defined in equation (4) to form a new chart called EWMAVAR (denoted EWMA-3T in this study).

In another study, [43] proposed combining the EWMA statistics defined in equation (5.7) by the method of sum of squares to construct a new chart called SS-EWMA (SS-EWMA- χ^2) to monitor both the process mean and variability simultaneously using a single charting statistic, $SS_{1,t} = E_{1,t}^2 + E_{2,t}^2$, having a mean, $E(SS_{1,t}) = 2\sigma_E^2$, and variance, $\sigma^2(SS_{1,t}) = 4\sigma_E^4$. Where $\sigma_E^2 = \sigma_{E_{1,t}}^2 = \sigma_{E_{2,t}}^2$. The steady-state UCL of the SS-EWMA- χ^2 chart is given as $UCL = (2\lambda/(2 - \lambda))(1 + L_2)$. L_2 is the coefficient that controls the width of the limit. The SS-EWMAVAR (SS-EWMA-3T) of [48] is formed in a similar fashion by replacing $V_{2,t}$ in equation (5.7) with $V_{3,t}$ from equation (5.4).

5.1.3 Max-CUSUM and SS-CUSUM

Similar to the EWMA-type single charts for joint monitoring, CUSUM structure was also explored to enhance detection of especially minor irregular shifts in the process mean and/or variance parameter(s). [52] combined two independent two-sided CUSUM statistics, first two for tracking upward/downward shifts in the mean and the other two for variability, to jointly monitor the process parameters. These statistics are calculated based on $V_{1,t}$ and $V_{2,t}$ as follows:

$$\left. \begin{aligned} C_{1,t}^+ &= \max(0, V_{1,t} - k_1 + C_{1,t-1}^+) \\ C_{1,t}^- &= \max(0, -V_{1,t} - k_1 + C_{1,t-1}^-) \end{aligned} \right\}, \quad (5.9)$$

$$\left. \begin{aligned} C_{2,t}^+ &= \max(0, V_{2,t} - k_1 + C_{2,t-1}^+) \\ C_{2,t}^- &= \max(0, -V_{2,t} - k_1 + C_{2,t-1}^-) \end{aligned} \right\} \quad (5.10)$$

These values are combined into a single plotting statistic for the Max-CUSUM (Max-CUSUM- χ^2) chart, $M_{2,t} = \max(C_{1,t}^+, C_{1,t}^-, C_{2,t}^+, C_{2,t}^-)$. The initial values of the statistics in equations (5.9) and (5.10) are all equal to zero, and k_1 is reference parameter of the CUSUM chart that controls its sensitivity to shifts in the process parameter(s). An OOC signal is given when $M_{2,t} > h_1$, where h_1 is the threshold limit. In a related manner [48] created the Max-CUSUMVAR (Max-CUSUM-3T) by substituting $V_{2,t}$ in equation (5.8) with $V_{3,t}$ from equation (5.4).

Further enhancement was achieved by [53] with the SS-CUSUM chart, where the sum of squares of the maximum CUSUM statistics from equations (5.9) and (5.10) was calculated as follows:

$$C_{1,t} = \max(C_{1,t}^+, C_{1,t}^-), C_{2,t} = \max(C_{2,t}^+, C_{2,t}^-), SS_{2,t} = C_{1,t}^2 + C_{2,t}^2.$$

The SS-CUSUM (SS-CUSUM- χ^2) chart is then constructed by plotting $SS_{2,t}$ against a specified threshold, h_2 .

Building on these developments, [48] created a new chart, known as the SS-CUSUMVAR (SS-CUSUM-3T), by modifying this approach: substituting $V_{2,t}$ in equation (5.8) with $V_{3,t}$ from equation (5.4) to incorporate the three-parameter transformation, enabling more refined joint monitoring of process variability alongside mean shifts.

The four-parameter Johnson transformation in equation (5.5) has not, to our knowledge, been applied in prior studies for joint monitoring, and therefore it can be considered an additional contribution of the current work.

5.2 New Charts for Joint Monitoring

Building on the structure of a recent chart by [72], we introduce six new memory-based control charts designed to simultaneously monitor shifts in both the process location (mean) and dispersion (variance/standard deviation) parameters. These charts employ the normal standardization of the sample mean from equation (5.1) and incorporate different standardized sample variance transformations outlined in equations (5.2), (5.4), and (5.6). Since these charts rely on cumulative aggregate mean (CAM) for monitoring, they are referred to as CAM charts. Three of the charts are constructed using the maximum of the chart's plotting statistics, while the other three use the sum of squares of these statistics.

5.2.1 Max-CAM Charts

The Max-CAM charts make up the first three types of charts within the CAM structure. These charts are built by taking the maximum of the statistics that track changes in the

process mean and variance. Using the standardized transformations outlined in equations (5.1) and (5.2), the Max-CAM- χ^2 chart is set up as follows:

First, two independent two-sided CUSUM statistics are employed to identify change-points in the process location and dispersion, as represented in equation (5.11) for the mean and equation (5.12) for the variance:

$$\left. \begin{aligned} CUSUM_{1,t}^+ &= \max(0, V_{1,t} - k_2 + CUSUM_{1,t-1}^+) \\ CUSUM_{1,t}^- &= \max(0, -V_{1,t} - k_2 + CUSUM_{1,t-1}^-) \end{aligned} \right\}, \quad (5.11)$$

$$\left. \begin{aligned} CUSUM_{2,t}^+ &= \max(0, V_{2,t} - k_2 + CUSUM_{2,t-1}^+) \\ CUSUM_{2,t}^- &= \max(0, -V_{2,t} - k_2 + CUSUM_{2,t-1}^-) \end{aligned} \right\} \quad (5.12)$$

Where the reference parameter k_2 is used to adjust the sensitivity of the chart. The initial values are set as $CUSUM_{1,0}^+ = CUSUM_{1,0}^- = CUSUM_{2,0}^+ = CUSUM_{2,0}^- = 0$.

Next, we introduce two pairs of discrete variables, $D_{1,t}^+$ and $D_{1,t}^-$, along with $D_{2,t}^+$ and $D_{2,t}^-$, which respectively count the number of samples analyzed since the last occurrence of $CUSUM_{1,t}^+ = 0$, $CUSUM_{1,t}^- = 0$, $CUSUM_{2,t}^+ = 0$, and $CUSUM_{2,t}^- = 0$, as defined below:

$$\left. \begin{aligned} D_{1,t}^+ &= \begin{cases} 1, & \text{if } CUSUM_{1,t-1}^+ = 0 \\ 1 + D_{1,t-1}^+, & \text{if } CUSUM_{1,t-1}^+ > 0 \end{cases} \\ D_{1,t}^- &= \begin{cases} 1, & \text{if } CUSUM_{1,t-1}^- = 0 \\ 1 + D_{1,t-1}^-, & \text{if } CUSUM_{1,t-1}^- > 0 \end{cases} \end{aligned} \right\} \quad (5.13)$$

$$\left. \begin{aligned} D_{2,t}^+ &= \begin{cases} 1, & \text{if } CUSUM_{2,t-1}^+ = 0 \\ 1 + D_{2,t-1}^+, & \text{if } CUSUM_{2,t-1}^+ > 0 \end{cases} \\ D_{2,t}^- &= \begin{cases} 1, & \text{if } CUSUM_{2,t-1}^- = 0 \\ 1 + D_{2,t-1}^-, & \text{if } CUSUM_{2,t-1}^- > 0 \end{cases} \end{aligned} \right\} \quad (5.14)$$

In equations (5.13) and (5.14), the definitions of the discrete variables ensure that the count for $D_{1,t}^+$, $D_{1,t}^-$, $D_{2,t}^+$, $D_{2,t}^-$ restarts at 1 whenever the respective previous CUSUM statistic $CUSUM_{1,t-1}^+$, $CUSUM_{1,t-1}^-$, $CUSUM_{2,t-1}^+$, or $CUSUM_{2,t-1}^-$ resets to zero. This resetting mechanism effectively eliminates the drawback in detecting delayed shifts often faced by progressive mean charts. The proposed Max-CAM- χ^2 chart uses these variables along with the following statistics to monitor shifts in the process mean (equation (5.15)) and variance (equation (5.16)):

$$\left. \begin{aligned} CAM_{1,t}^+ &= \frac{1}{D_{1,t}^+} V_{1,t} + \left(1 - \frac{1}{D_{1,t}^+}\right) CAM_{1,t-1}^+ \\ CAM_{1,t}^- &= \frac{1}{D_{1,t}^-} V_{1,t} + \left(1 - \frac{1}{D_{1,t}^-}\right) CAM_{1,t-1}^- \end{aligned} \right\}, \quad (5.15)$$

$$\left. \begin{aligned} CAM_{2,t}^+ &= \frac{1}{D_{2,t}^+} V_{2,t} + \left(1 - \frac{1}{D_{2,t}^+}\right) CAM_{2,t-1}^+ \\ CAM_{2,t}^- &= \frac{1}{D_{2,t}^-} V_{2,t} + \left(1 - \frac{1}{D_{2,t}^-}\right) CAM_{2,t-1}^- \end{aligned} \right\} \quad (5.16)$$

Since the CAM statistics in equations (5.15) and (5.16) represent the cumulative average of the transformed variables $V_{1,t}$ and $V_{2,t}$ from the point when the CUSUM statistics were last reset to zero, the mean and variance values of the statistics in the IC state are (proofs provided in Appendix A) $E(CAM_{1,t}^+) = E(CAM_{1,t}^-) = E(CAM_{2,t}^+) = E(CAM_{2,t}^-) = 0$, and $\sigma^2(CAM_{1,t}^+) = 1/D_{1,t}^+$, $\sigma^2(CAM_{1,t}^-) = 1/D_{1,t}^-$, $\sigma^2(CAM_{2,t}^+) = 1/D_{2,t}^+$, and $\sigma^2(CAM_{2,t}^-) = 1/D_{2,t}^-$. The CAM statistics are incorporated into the Max statistic, which takes the maximum of the absolute values of scaled cumulative averages and serves as the plotting statistic for the proposed charting scheme. The independence of $CAM_{1,t}^+$, $CAM_{1,t}^-$, $CAM_{2,t}^+$, and $CAM_{2,t}^-$ is assumed.

and $CAM_{1,t}^+$ is implied by the fact that $V_{2,t}$ and $V_{2,t}$ are independent. Mathematically, the Max statistic is defined as:

$$\text{MaxCAM}_t = \max \left(\begin{array}{l} |CAM_{1,t}^+ \times \sqrt{D_{1,t}^+}|, |CAM_{1,t}^- \times \sqrt{D_{1,t}^-}|, \\ |CAM_{2,t}^+ \times \sqrt{D_{2,t}^+}|, |CAM_{2,t}^- \times \sqrt{D_{2,t}^-}| \end{array} \right) \quad (5.16)$$

To construct the proposed Max-CAM- χ^2 chart, MaxCAM_t is calculated at each time point $t = 1, 2, \dots$ and plotted to monitor process stability. MaxCAM_t over time reveals whether the process remains stable:

- A large MaxCAM_t value indicates that the process mean has likely shifted from the IC or target value and/or that variability has changed.
- A smaller MaxCAM_t suggests that both the mean and variability are near their target levels.

Since MaxCAM_t is non-negative, it is compared only to a specified positive threshold, say h_2 . Values of MaxCAM_t above h_2 signal a potential process shift, while values below it indicate the process is likely stable.

The other two charts for monitoring both the process mean and variance simultaneously, Max-CAM-3T and Max-CAM-4T, are based on three- and four-parameter transformations of the sample variance. They are constructed in a similar manner by substituting $V_{2,t}$ in equations (5.12) and (5.16) with $V_{3,t}$ from equation (5.4) for Max-CAM-3T chart, and $V_{4,t}$ from equation (5.6) for Max-CAM-4T chart. In this way, the study examined different transformations using the new charting structure to determine the most effective charts for various conditions.

5.2.2 SS-CAM Charts

Building on [53], we present three new charts based on the sum of squares of the maximum statistics, which track changes in the process mean and variance as outlined in equations (5.15) and (5.16). These charts have similar properties to the Max-CAM charts, but with the added benefit of being easier to use in practical applications.

Starting with SS-CAM- χ^2 , we define two variables that take the maximum of the scaled CAM statistics from equations (5.15) and (5.16) as follows:

$$CAM_{1,t} = \max \left(\left| CAM_{1,t}^+ \times \sqrt{D_{1,t}^+} \right|, \left| CAM_{1,t}^- \times \sqrt{D_{1,t}^-} \right| \right) \quad (5.17)$$

$$CAM_{2,t} = \max \left(\left| CAM_{2,t}^+ \times \sqrt{D_{2,t}^+} \right|, \left| CAM_{2,t}^- \times \sqrt{D_{2,t}^-} \right| \right) \quad (5.18)$$

Using to these statistics, the new chart is constructed by computing $SSCAM_t = CAM_{1,t}^2 + CAM_{2,t}^2$ and plotting it against a predetermined threshold, say h_3 . An OOC scenario is declared at time t if $SSCAM_t > h_3$.

Similar to the Max-CAM charts, the SS-CAM-3T and SS-CAM-4T charts for joint monitoring are based on three- and four-parameter transformations. These charts follow a similar construction, where $V_{2,t}$ in equations (5.12) and (5.16) is replaced by $V_{3,t}$ from equation (5.4) for the SS-CAM-3T chart, and $V_{4,t}$ from equation (5.6) for the SS-CAM-4T chart. This approach allowed the study to explore different transformations using the new charting structure, aiming to identify the most suitable charts for various conditions.

5.3 Results and Discussion

The performance of control charts is commonly assessed by examining their run-length (RL) distribution, where RL represents the number of observations or subgroups investigated before the chart signals a shift. This study uses ARL—the expected count of samples plotted until an OOC signal is generated—to evaluate the performance of the proposed charts and compare them to other existing options. When the process is IC, the ARL is represented as ARL_0 , while for OOC situations, it is indicated as ARL_1 . To provide a comprehensive assessment of chart efficiency, we have also reported the median run length (MDRL) and the standard deviation of run length (SDRL). The median is a more robust measure when the RL distribution is skewed, and SDRL reflects the stability of the RL—the lower the SDRL, the more consistent the chart's performance. Another important individual performance measure is the conditional expected delay (CED) curve, which assesses a control chart's efficiency across different change points τ . Specifically, CED_τ represents the expected run length at τ given that no alarms were triggered prior to this change point.

Moreover, overall performance measures through extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) are used in evaluating the effectiveness of the charts in a range of shifts. Refer to [82] for more discussion on the overall performance measures. This study incorporated the EQL metric estimated using the following formular:

$$EQL = \frac{1}{M} \sum_{i=1}^M (\delta_i^2 + \theta_i^2) ARL(\delta_i \theta_i) \quad (5.19)$$

Where M is the number of OOC observations considered and $ARL(\delta_i, \theta_i)$ is the i th ARL_1 value for the joints shift δ_i, θ_i .

The literature offers several ways to calculate the run length (RL) for control charts, such as using integral equations, Markov Chains, and Monte Carlo simulations. In this study, we use Monte Carlo simulations to assess RL characteristics, because the other methods don't work for all chart types, especially memory-type control charts, where the nature of the plotting statistics can make other methods less practical. For consistency, we have fixed the ARL_0 at approximately 250 (a popular choice in simultaneous monitoring) and examined the OOC behavior of the charts through ARL_1 under different shift scenarios: $(N - D)$, $(U - N)$, $(U - D)$, and $(U - U)$.

The following algorithm is used in evaluating the RL characteristics of the proposed charts:

1. Fix the sample size n and desired ARL_0 , and select the reference parameter k , limit threshold h , and change point τ (with $\tau = 1$ representing the zero-state).
2. At each time t , draw n random observations according to the following conditions:
 - i. $X_t \sim N(\mu_0, \sigma_0)$, if $t > \tau$.
 - ii. $X_t \sim N(\mu_1 = \mu_0 + \delta\sigma_0, \sigma_1 = \theta\sigma_0)$, otherwise.
3. Estimate $\bar{X}_t$ and S_t^2 and apply transformations according to the chart in question.
4. Calculate the CUSUM statistics $CUSUM_{1,t}^+$, $CUSUM_{1,t}^-$, $CUSUM_{2,t}^+$ and $CUSUM_{2,t}^-$ based on the transformations applied in step #3.
5. Compute the discrete variables $D_{1,t}^+$, $D_{1,t}^-$, $D_{2,t}^+$, and $D_{2,t}^-$ using equations (5.13) and (5.14).

6. Calculate the CAM plotting statistics $CAM_{1,t}^+$, $CAM_{1,t}^-$, $CAM_{2,t}^+$, and $CAM_{2,t}^-$ using the calculated statistics in step #4 and step #5.
7. Combine the statistics in step #6 into a single statistic using either the $\max CAM_t$ defined in equation (5.16) for Max-CAM charts, or $SSCAM_t$ for SS-CAM charts.
8. Plot the single statistic in step #7 against time t . If, statistic is greater than the specified threshold h , record $(t - \tau + 1)$ as run length, otherwise, go to step #2
9. Repeat step #2 to step #8 m times (say, 10^6) to record m run lengths.
10. Estimate the ARL_0 by taking the average of the m run lengths.
11. If the estimated ARL_0 is not approximately equal to the target ARL_0 , adjust the limit h in step #1 and repeat step #2 to step #10 until the desired ARL_0 is achieved.
12. To compute ARL_1 , draw the random observations in step#2 after introducing a shift in the mean ($\delta \neq 0$), or variance ($\theta \neq 1$), or both.

5.3.1 Performance Evaluation of the Proposed Charts

In this study, we assessed the ARL of the proposed chart with a sample size of $n = 5$, varying sample sizes can be explored to understand their impact. In this study, the reference parameter k was set to 0.5. However, the impact of using different k values (0.25, 0.75, and 1) on the performance of the proposed charts was also analyzed, with the results provided in Tables B1–B3 in Appendix B. Table 5.1 contains the ARL values of the proposed charts along with SDRL and MDRL for a range of small, moderate, and large shifts in the process mean ($\delta \in (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 1, 1.25, 1.5, 2, 3)$) and variance ($\theta \in (0.25, 0.5, 0.75, 0.9, 1, 1.1, 1.25, 1.5, 2)$). The results show that the propose charts are highly sensitive to changes in the process parameters. For example, the ARL for the Max-

CAM-3T chart reduced from 250, for IC state, to 3.53 when a shift $\theta = 0.25$ is introduced in the process variance. Comparable reductions are observed across the other charts, with the Max-CAM- χ^2 having the least ARL of 2.31.

All charts yield relatively high ARL values when only the process variance experiences a small shift in the variance ($\theta = 0.9$). However, the ARLs decrease significantly when the shift is accompanied by even a small change in the process mean. For instance, with a joint shift of $(\delta, \theta) = (0.2, 0.9)$ the ARL for the SS-CAM-4T chart decreases by approximately 80%, from 250 to as low as 47.88.

In terms of the MDRL, all the charts show exhibit almost similar behavior across different shift combinations. In almost all the cases, $ARL > MDRL$ indicating that the RL distribution of the chart is positively skewed. For example, when $\delta = 0.2$ and $\theta = 1.25$, the Max-CAM-3T has $ARL = 13.33$ and $MDRL = 11$, and SS-CAM-3T has 12.39 and 10 respectively

It's also seen that the SDRL values drop as the magnitude of the shifts increases, which shows that the charts tend to perform more consistently when detecting moderate to large shifts in the parameters of the process.

Overall, none of the proposed charts consistently outperforms the others across all scenarios. However, the charts that use the three-parameter transformation (Max-CAM-3T and SS-CAM-3T) appear to be more efficient than others when detecting moderate changes in the process variability. Moreover, for $\delta \geq 0.5$ and θ is moderate to large, the performance of all six charts is nearly identical.

Table 5.1. (ARL, SDRL, MDR) values of the six proposed charts for different shifts in the process mean (δ) and variance (θ).

$$ARL_0 \cong 250, k = 0.5$$

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.53,0.51,4)	(3.53,0.51,4)	(3.53,0.51,4)	(3.53,0.51,4)	(3.53,0.51,4)	(3.53,0.51,4)	(3.52,0.50,4)	(2.86,0.39,3)	(2.01,0.15,2)	(1.59,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.61,1.66,6)	(6.61,1.65,6)	(6.61,1.65,6)	(6.60,1.65,6)	(6.58,1.63,6)	(6.39,1.49,6)	(4.55,1.09,5)	(2.88,0.74,3)	(2.04,0.50,2)	(1.55,0.51,2)	(1.02,0.13,1)	(1.00,0.00,1)
	0.75	(23.44,11.87,21)	(23.37,11.78,21)	(22.12,10.55,20)	(17.92,7.47,17)	(13.01,5.13,12)	(9.33,3.65,9)	(4.71,1.79,5)	(2.91,1.06,3)	(2.06,0.72,2)	(1.57,0.57,2)	(1.08,0.27,1)	(1.00,0.00,1)
	0.90	(205.66,192.73,147)	(144.25,131.71,104)	(52.83,41.35,41)	(23.27,14.12,20)	(13.71,7.09,13)	(9.31,4.53,9)	(4.67,2.10,4)	(2.92,1.23,3)	(2.07,0.83,2)	(1.60,0.62,2)	(1.12,0.33,1)	(1.00,0.01,1)
	1.00	(250.60,245.84,175)	(135.21,128.38,96)	(46.97,38.76,36)	(21.81,14.62,19)	(13.11,7.60,12)	(9.01,4.87,8)	(4.62,2.27,4)	(2.92,1.34,3)	(2.09,0.90,2)	(1.62,0.66,2)	(1.15,0.36,1)	(1.00,0.02,1)
	1.10	(61.77,56.79,45)	(50.06,44.76,37)	(30.25,24.49,24)	(18.04,12.70,15)	(11.86,7.49,10)	(8.47,5.00,8)	(4.52,2.40,4)	(2.91,1.43,3)	(2.10,0.96,2)	(1.64,0.70,2)	(1.17,0.39,1)	(1.00,0.04,1)
	1.25	(15.95,12.11,13)	(15.19,11.45,12)	(13.33,9.74,11)	(10.91,7.61,9)	(8.70,5.78,7)	(6.94,4.40,6)	(4.21,2.43,4)	(2.84,1.51,3)	(2.10,1.03,2)	(1.66,0.75,2)	(1.21,0.42,1)	(1.00,0.06,1)
	1.50	(5.67,3.81,5)	(5.60,3.76,5)	(5.40,3.61,5)	(5.10,3.37,4)	(4.71,3.07,4)	(4.29,2.75,4)	(3.30,2.00,3)	(2.54,1.43,2)	(2.01,1.05,2)	(1.65,0.80,1)	(1.25,0.47,1)	(1.01,0.11,1)
	1.75	(3.27,2.06,3)	(3.25,2.04,3)	(3.20,2.01,3)	(3.12,1.94,3)	(3.01,1.86,3)	(2.87,1.76,2)	(2.51,1.48,2)	(2.14,1.20,2)	(1.83,0.96,2)	(1.59,0.77,1)	(1.26,0.50,1)	(1.03,0.16,1)
2.00	(2.32,1.37,2)	(2.31,1.36,2)	(2.29,1.35,2)	(2.26,1.32,2)	(2.22,1.29,2)	(2.16,1.25,2)	(2.00,1.12,2)	(1.82,0.98,2)	(1.64,0.84,1)	(1.48,0.71,1)	(1.25,0.49,1)	(1.04,0.19,1)	
Max-4T	0.25	(3.01,0.34,3)	(3.01,0.34,3)	(3.01,0.34,3)	(3.01,0.34,3)	(3.01,0.34,3)	(3.01,0.34,3)	(3.01,0.34,3)	(2.81,0.40,3)	(2.01,0.15,2)	(1.60,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.04,1.77,6)	(6.04,1.77,6)	(6.04,1.77,6)	(6.04,1.77,6)	(6.02,1.74,6)	(5.88,1.61,6)	(4.41,1.06,4)	(2.87,0.73,3)	(2.04,0.50,2)	(1.56,0.51,2)	(1.02,0.13,1)	(1.00,0.00,1)
	0.75	(22.77,12.14,20)	(22.70,12.07,20)	(21.51,10.82,20)	(17.49,7.62,17)	(12.80,5.15,12)	(9.25,3.63,9)	(4.71,1.78,5)	(2.92,1.06,3)	(2.06,0.72,2)	(1.57,0.57,2)	(1.08,0.27,1)	(1.00,0.00,1)
	0.90	(185.10,173.42,132)	(134.31,122.71,97)	(51.78,40.53,40)	(23.16,14.03,20)	(13.71,7.08,13)	(9.31,4.53,9)	(4.68,2.10,4)	(2.93,1.23,3)	(2.08,0.83,2)	(1.60,0.62,2)	(1.12,0.33,1)	(1.00,0.01,1)
	1.00	(249.61,245.03,174)	(135.55,128.80,96)	(47.18,38.99,36)	(21.88,14.67,19)	(13.15,7.63,12)	(9.03,4.88,8)	(4.63,2.28,4)	(2.93,1.34,3)	(2.09,0.90,2)	(1.62,0.66,2)	(1.15,0.36,1)	(1.00,0.02,1)
	1.10	(64.85,59.89,47)	(52.00,46.65,38)	(30.91,25.13,24)	(18.24,12.87,15)	(11.95,7.55,10)	(8.50,5.01,8)	(4.53,2.41,4)	(2.92,1.43,3)	(2.10,0.96,2)	(1.64,0.70,2)	(1.17,0.39,1)	(1.00,0.04,1)
	1.25	(16.41,12.50,13)	(15.64,11.79,13)	(13.65,9.98,11)	(11.11,7.76,9)	(8.82,5.89,8)	(7.01,4.47,6)	(4.22,2.45,4)	(2.85,1.52,3)	(2.10,1.03,2)	(1.66,0.75,2)	(1.21,0.43,1)	(1.00,0.07,1)
	1.50	(5.75,3.90,5)	(5.68,3.86,5)	(5.47,3.68,5)	(5.16,3.45,4)	(4.75,3.13,4)	(4.32,2.80,4)	(3.31,2.02,3)	(2.54,1.44,2)	(2.01,1.06,2)	(1.65,0.80,1)	(1.25,0.47,1)	(1.01,0.11,1)
	1.75	(3.28,2.10,3)	(3.26,2.09,3)	(3.20,2.05,3)	(3.12,1.99,3)	(3.01,1.90,3)	(2.88,1.79,2)	(2.50,1.50,2)	(2.13,1.21,2)	(1.82,0.97,2)	(1.58,0.77,1)	(1.26,0.49,1)	(1.03,0.16,1)
2.00	(2.31,1.39,2)	(2.30,1.38,2)	(2.28,1.37,2)	(2.25,1.34,2)	(2.20,1.31,2)	(2.15,1.26,2)	(1.99,1.13,2)	(1.80,0.98,2)	(1.63,0.84,1)	(1.47,0.71,1)	(1.24,0.49,1)	(1.04,0.19,1)	
Max- χ^2	0.25	(2.31,0.60,2)	(2.31,0.60,2)	(2.31,0.60,2)	(2.31,0.60,2)	(2.31,0.60,2)	(2.31,0.60,2)	(2.31,0.60,2)	(2.25,0.55,2)	(1.95,0.26,2)	(1.62,0.49,2)	(1.00,0.01,1)	(1.00,0.00,1)
	0.50	(5.75,2.03,6)	(5.75,2.03,6)	(5.74,2.03,6)	(5.74,2.03,6)	(5.73,2.00,6)	(5.60,1.86,5)	(4.29,1.16,4)	(2.87,0.74,3)	(2.06,0.51,2)	(1.58,0.50,2)	(1.02,0.14,1)	(1.00,0.00,1)
	0.75	(22.89,12.80,21)	(22.81,12.74,21)	(21.63,11.42,20)	(17.54,8.00,17)	(12.85,5.33,12)	(9.31,3.72,9)	(4.77,1.80,5)	(2.96,1.07,3)	(2.09,0.73,2)	(1.59,0.57,2)	(1.09,0.28,1)	(1.00,0.00,1)
	0.90	(170.25,160.43,121)	(128.72,118.56,93)	(51.84,40.80,40)	(23.40,14.20,21)	(13.90,7.18,13)	(9.46,4.58,9)	(4.76,2.12,4)	(2.97,1.24,3)	(2.11,0.84,2)	(1.62,0.63,2)	(1.13,0.34,1)	(1.00,0.01,1)
	1.00	(250.60,246.77,175)	(138.52,131.63,98)	(48.34,39.82,37)	(22.31,14.87,19)	(13.41,7.74,12)	(9.21,4.94,8)	(4.71,2.30,4)	(2.97,1.35,3)	(2.12,0.91,2)	(1.64,0.67,2)	(1.16,0.37,1)	(1.00,0.02,1)
	1.10	(70.81,65.44,51)	(56.15,50.44,41)	(32.54,26.43,25)	(18.89,13.28,16)	(12.29,7.71,11)	(8.71,5.10,8)	(4.62,2.45,4)	(2.96,1.45,3)	(2.13,0.97,2)	(1.66,0.71,2)	(1.18,0.40,1)	(1.00,0.04,1)
	1.25	(17.48,13.23,14)	(16.60,12.44,14)	(14.38,10.43,12)	(11.64,8.08,10)	(9.18,6.08,8)	(7.23,4.58,6)	(4.32,2.49,4)	(2.90,1.54,3)	(2.13,1.05,2)	(1.68,0.76,2)	(1.21,0.43,1)	(1.00,0.07,1)
	1.50	(6.01,4.05,5)	(5.93,3.99,5)	(5.72,3.83,5)	(5.37,3.56,5)	(4.94,3.24,4)	(4.49,2.89,4)	(3.41,2.08,3)	(2.60,1.48,2)	(2.05,1.08,2)	(1.68,0.81,1)	(1.26,0.48,1)	(1.01,0.12,1)
	1.75	(3.40,2.17,3)	(3.39,2.16,3)	(3.33,2.12,3)	(3.23,2.05,3)	(3.12,1.95,3)	(2.98,1.85,3)	(2.58,1.54,2)	(2.19,1.24,2)	(1.86,0.99,2)	(1.60,0.79,1)	(1.27,0.51,1)	(1.03,0.16,1)
2.00	(2.38,1.43,2)	(2.37,1.42,2)	(2.35,1.41,2)	(2.32,1.38,2)	(2.27,1.34,2)	(2.21,1.30,2)	(2.04,1.16,2)	(1.85,1.01,2)	(1.66,0.86,1)	(1.50,0.72,1)	(1.26,0.50,1)	(1.04,0.20,1)	
SS-3T	0.25	(4.42,0.50,4)	(4.39,0.50,4)	(4.26,0.45,4)	(4.07,0.33,4)	(3.85,0.40,4)	(3.47,0.50,3)	(2.90,0.31,3)	(2.08,0.27,2)	(1.99,0.12,2)	(1.52,0.50,2)	(1.00,0.01,1)	(1.00,0.00,1)
	0.50	(8.22,1.88,8)	(8.09,1.87,8)	(7.60,1.78,7)	(6.83,1.59,7)	(6.01,1.37,6)	(5.24,1.19,5)	(3.70,0.84,4)	(2.69,0.62,3)	(2.07,0.44,2)	(1.69,0.47,2)	(1.05,0.22,1)	(1.00,0.00,1)
	0.75	(27.38,12.95,25)	(25.80,12.40,24)	(20.87,9.99,19)	(15.16,6.55,14)	(11.17,4.37,11)	(8.52,3.15,8)	(4.87,1.68,5)	(3.18,1.05,3)	(2.30,0.73,2)	(1.78,0.58,2)	(1.17,0.38,1)	(1.00,0.01,1)
	0.90	(208.13,193.96,149)	(130.96,118.31,95)	(49.04,37.57,38)	(23.15,13.55,20)	(14.31,7.05,13)	(10.00,4.58,9)	(5.22,2.18,5)	(3.31,1.30,3)	(2.36,0.88,2)	(1.82,0.67,2)	(1.23,0.42,1)	(1.00,0.03,1)
	1.00	(249.99,243.50,175)	(131.41,123.19,94)	(46.90,37.59,36)	(22.66,14.56,20)	(14.04,7.75,13)	(9.82,5.03,9)	(5.16,2.39,5)	(3.30,1.42,3)	(2.36,0.96,2)	(1.82,0.71,2)	(1.25,0.44,1)	(1.00,0.04,1)

Chart	θ	δ											
		0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
SS-4T	1.10	(58.02,51.97,42)	(46.34,40.21,34)	(28.32,22.13,22)	(17.57,11.88,15)	(11.96,7.21,11)	(8.78,4.92,8)	(4.88,2.45,4)	(3.20,1.49,3)	(2.33,1.01,2)	(1.81,0.75,2)	(1.26,0.46,1)	(1.00,0.06,1)
	1.25	(15.25,11.07,13)	(14.40,10.34,12)	(12.39,8.63,10)	(10.13,6.71,9)	(8.18,5.14,7)	(6.66,3.99,6)	(4.24,2.31,4)	(2.96,1.49,3)	(2.23,1.05,2)	(1.77,0.78,2)	(1.28,0.48,1)	(1.01,0.09,1)
	1.50	(5.60,3.57,5)	(5.52,3.51,5)	(5.28,3.33,5)	(4.93,3.07,4)	(4.52,2.78,4)	(4.11,2.47,4)	(3.18,1.81,3)	(2.49,1.33,2)	(2.01,1.00,2)	(1.68,0.78,2)	(1.28,0.49,1)	(1.02,0.14,1)
	1.75	(3.28,1.97,3)	(3.26,1.95,3)	(3.20,1.90,3)	(3.10,1.84,3)	(2.97,1.74,3)	(2.83,1.64,2)	(2.44,1.37,2)	(2.09,1.11,2)	(1.79,0.90,2)	(1.56,0.74,1)	(1.26,0.49,1)	(1.03,0.17,1)
	2.00	(2.34,1.32,2)	(2.33,1.32,2)	(2.31,1.30,2)	(2.27,1.27,2)	(2.22,1.24,2)	(2.16,1.19,2)	(1.98,1.06,2)	(1.79,0.92,2)	(1.61,0.79,1)	(1.46,0.67,1)	(1.23,0.47,1)	(1.04,0.19,1)
	0.25	(3.67,0.52,4)	(3.65,0.53,4)	(3.55,0.53,4)	(3.37,0.49,3)	(3.19,0.40,3)	(3.05,0.29,3)	(2.48,0.50,2)	(2.02,0.13,2)	(1.93,0.25,2)	(1.32,0.47,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.51,2.01,7)	(7.39,1.99,7)	(6.98,1.88,7)	(6.33,1.67,6)	(5.63,1.44,5)	(4.96,1.24,5)	(3.57,0.86,4)	(2.62,0.63,3)	(2.04,0.44,2)	(1.66,0.48,2)	(1.05,0.22,1)	(1.00,0.00,1)
	0.75	(26.72,13.26,24)	(25.18,12.67,23)	(20.42,10.14,18)	(14.90,6.65,14)	(11.03,4.43,10)	(8.43,3.20,8)	(4.84,1.70,5)	(3.17,1.06,3)	(2.30,0.74,2)	(1.78,0.58,2)	(1.18,0.38,1)	(1.00,0.01,1)
	0.90	(192.43,179.34,137)	(124.16,112.01,90)	(47.88,36.68,37)	(22.95,13.45,20)	(14.24,7.05,13)	(9.97,4.59,9)	(5.23,2.19,5)	(3.32,1.30,3)	(2.37,0.88,2)	(1.82,0.67,2)	(1.23,0.42,1)	(1.00,0.03,1)
	1.00	(250.37,243.81,175)	(131.65,123.32,94)	(47.14,37.79,36)	(22.75,14.60,20)	(14.08,7.75,13)	(9.86,5.04,9)	(5.18,2.39,5)	(3.31,1.42,3)	(2.37,0.96,2)	(1.83,0.72,2)	(1.25,0.44,1)	(1.00,0.04,1)
SS- χ^2	1.10	(60.73,54.73,44)	(48.06,41.86,36)	(29.04,22.70,23)	(17.84,12.05,15)	(12.12,7.30,11)	(8.85,4.95,8)	(4.92,2.47,5)	(3.22,1.49,3)	(2.34,1.02,2)	(1.82,0.75,2)	(1.27,0.46,1)	(1.00,0.06,1)
	1.25	(15.67,11.38,13)	(14.79,10.67,12)	(12.69,8.86,11)	(10.33,6.87,9)	(8.32,5.24,7)	(6.75,4.05,6)	(4.28,2.33,4)	(2.99,1.50,3)	(2.24,1.05,2)	(1.78,0.79,2)	(1.28,0.48,1)	(1.01,0.09,1)
	1.50	(5.66,3.66,5)	(5.58,3.59,5)	(5.33,3.41,5)	(4.98,3.15,4)	(4.56,2.84,4)	(4.14,2.52,4)	(3.20,1.83,3)	(2.51,1.35,2)	(2.02,1.02,2)	(1.69,0.79,2)	(1.28,0.50,1)	(1.02,0.14,1)
	1.75	(3.28,2.01,3)	(3.25,2.00,3)	(3.19,1.95,3)	(3.10,1.87,3)	(2.97,1.78,3)	(2.82,1.67,2)	(2.44,1.39,2)	(2.09,1.13,2)	(1.80,0.91,2)	(1.57,0.74,1)	(1.26,0.49,1)	(1.03,0.17,1)
	2.00	(2.32,1.34,2)	(2.31,1.34,2)	(2.29,1.32,2)	(2.25,1.29,2)	(2.20,1.25,2)	(2.14,1.21,2)	(1.96,1.08,2)	(1.78,0.93,2)	(1.60,0.79,1)	(1.45,0.67,1)	(1.23,0.47,1)	(1.04,0.19,1)
	0.25	(2.84,0.67,3)	(2.82,0.67,3)	(2.78,0.65,3)	(2.69,0.63,3)	(2.58,0.60,3)	(2.46,0.57,2)	(2.12,0.44,2)	(1.89,0.35,2)	(1.63,0.48,2)	(1.20,0.40,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.12,2.29,7)	(7.01,2.26,7)	(6.64,2.13,6)	(6.07,1.88,6)	(5.43,1.61,5)	(4.81,1.39,5)	(3.50,0.94,3)	(2.60,0.67,3)	(2.03,0.48,2)	(1.66,0.49,2)	(1.06,0.23,1)	(1.00,0.00,1)
	0.75	(26.94,13.95,25)	(25.38,13.31,23)	(20.55,10.59,19)	(15.01,6.89,14)	(11.10,4.58,11)	(8.49,3.30,8)	(4.89,1.74,5)	(3.21,1.08,3)	(2.32,0.75,2)	(1.80,0.59,2)	(1.19,0.39,1)	(1.00,0.01,1)
	0.90	(182.85,171.39,131)	(120.31,109.09,87)	(47.74,36.73,37)	(23.10,13.55,20)	(14.39,7.15,13)	(10.11,4.67,9)	(5.29,2.22,5)	(3.36,1.32,3)	(2.40,0.90,2)	(1.84,0.67,2)	(1.24,0.43,1)	(1.00,0.03,1)
	1.00	(249.96,243.54,175)	(133.03,124.81,95)	(47.81,38.30,37)	(23.09,14.77,20)	(14.31,7.85,13)	(10.02,5.11,9)	(5.26,2.43,5)	(3.36,1.44,3)	(2.40,0.97,2)	(1.85,0.72,2)	(1.26,0.45,1)	(1.00,0.05,1)
SS- χ^2	1.10	(65.26,58.91,48)	(51.08,44.54,38)	(30.26,23.66,24)	(18.37,12.37,16)	(12.39,7.40,11)	(9.05,5.03,8)	(5.00,2.50,5)	(3.27,1.51,3)	(2.37,1.03,2)	(1.84,0.76,2)	(1.28,0.47,1)	(1.00,0.07,1)
	1.25	(16.49,11.92,14)	(15.56,11.14,13)	(13.26,9.18,11)	(10.73,7.08,9)	(8.58,5.36,7)	(6.94,4.13,6)	(4.37,2.37,4)	(3.04,1.52,3)	(2.28,1.07,2)	(1.81,0.80,2)	(1.29,0.49,1)	(1.01,0.10,1)
	1.50	(5.91,3.78,5)	(5.81,3.71,5)	(5.54,3.51,5)	(5.15,3.22,4)	(4.72,2.90,4)	(4.27,2.58,4)	(3.28,1.87,3)	(2.56,1.37,2)	(2.06,1.03,2)	(1.71,0.80,2)	(1.29,0.50,1)	(1.02,0.14,1)
	1.75	(3.39,2.06,3)	(3.37,2.05,3)	(3.30,2.00,3)	(3.19,1.92,3)	(3.06,1.83,3)	(2.91,1.71,3)	(2.50,1.42,2)	(2.13,1.15,2)	(1.82,0.93,2)	(1.59,0.76,1)	(1.27,0.50,1)	(1.03,0.17,1)
	2.00	(2.39,1.38,2)	(2.38,1.37,2)	(2.35,1.35,2)	(2.31,1.33,2)	(2.26,1.29,2)	(2.20,1.24,2)	(2.01,1.10,2)	(1.81,0.95,2)	(1.63,0.81,1)	(1.47,0.69,1)	(1.25,0.48,1)	(1.04,0.19,1)

Table 5.1 (Continued)

Table 5.2: Zero-state ARL Comparison of Proposed and Existing Charts.

Chart	δ	θ	Method					
			Max			SS		
			Transformation			Transformation		
			χ^2	3T	4T	χ^2	3T	4T
CAM	0.00	1.00	(250.60,246.77,175)	(250.60,245.84,175)	(249.61,245.03,174)	(249.96,243.54,175)	(249.99,243.50,175)	(250.37,243.81,175)
	0.20	1.00	(48.34,39.82,37)	(46.97,38.76,36)	(47.18,38.99,36)	(47.81,38.30,37)	(46.90,37.59,36)	(47.14,37.79,36)
	1.50	1.00	(1.64,0.67,2)	(1.62,0.66,2)	(1.62,0.66,2)	(1.85,0.72,2)	(1.82,0.71,2)	(1.83,0.72,2)
	0.00	0.75	(22.89,12.80,21)	(23.44,11.87,21)	(22.77,12.14,20)	(26.94,13.95,25)	(27.38,12.95,25)	(26.72,13.26,24)
	0.00	1.50	(6.01,4.05,5)	(5.67,3.81,5)	(5.75,3.90,5)	(5.91,3.78,5)	(5.60,3.57,5)	(5.66,3.66,5)
	0.20	0.75	(21.63,11.42,20)	(22.12,10.55,20)	(21.51,10.82,20)	(20.55,10.59,19)	(20.87,9.99,19)	(20.42,10.14,18)
	1.50	1.50	(1.68,0.81,1)	(1.65,0.80,1)	(1.65,0.80,1)	(1.71,0.80,2)	(1.68,0.78,2)	(1.69,0.79,2)
		EQL	17.10	16.80	16.74	17.38	17.14	17.11
CUSUM	0.00	1.00	(250.31, 243.71, 176)	(250.12, 243.41, 175)	(249.94, 242.97, 175)	(249.87, 242.96, 175)	(250.10, 243.69, 175)	(250.01, 243.00, 176)
	0.20	1.00	(45.46, 38.34, 34)	(44.97, 37.93, 34)	(45.06, 38.01, 34)	(45.24, 38.14, 34)	(44.80, 37.69, 33)	(44.87, 37.71, 34)
	1.50	1.00	(2.33, 0.54, 2)	(2.32, 0.54, 2)	(2.32, 0.54, 2)	(2.38, 0.56, 2)	(2.37, 0.55, 2)	(2.37, 0.55, 2)
	0.00	0.75	(21.34, 13.54, 18)	(22.09, 13.14, 19)	(21.34, 13.10, 18)	(22.15, 13.90, 19)	(22.91, 13.51, 19)	(22.12, 13.48, 19)
	0.00	1.50	(6.92, 3.64, 6)	(6.68, 3.49, 6)	(6.72, 3.58, 6)	(6.63, 3.48, 6)	(6.41, 3.35, 6)	(6.46, 3.44, 6)
	0.20	0.75	(19.83, 11.90, 17)	(20.50, 11.56, 18)	(19.83, 11.55, 17)	(19.27, 11.54, 16)	(19.81, 11.20, 17)	(19.23, 11.16, 16)
	1.50	1.50	(2.38, 0.76, 2)	(2.37, 0.75, 2)	(2.36, 0.76, 2)	(2.31, 0.71, 2)	(2.29, 0.71, 2)	(2.29, 0.71, 2)
		EQL	17.51	17.46	17.35	17.36	17.31	17.21
EWMA	0.00	1.00	(249.78, 245.44, 174)	(250.14, 245.46, 175)	(250.67, 246.45, 175)	(250.00, 245.31, 175)	(250.5, 245.98, 175)	(250.28, 246.01, 175)
	0.20	1.00	(48.13, 42.51, 35)	(46.33, 40.78, 34)	(46.91, 41.32, 34)	(48.25, 42.70, 35)	(46.8, 41.29, 34)	(47.22, 41.59, 35)
	1.50	1.00	(2.15, 0.51, 2)	(2.13, 0.51, 2)	(2.13, 0.51, 2)	(2.23, 0.53, 2)	(2.22, 0.52, 2)	(2.22, 0.52, 2)
	0.00	0.75	(24.81, 18.78, 19)	(28.68, 21.44, 22)	(25.6, 19.02, 20)	(28.83, 22.39, 22)	(34.69, 26.96, 27)	(30.14, 23.12, 23)
	0.00	1.50	(6.52, 3.80, 6)	(6.21, 3.56, 5)	(6.3, 3.7, 5)	(6.32, 3.67, 5)	(6.06, 3.47, 5)	(6.12, 3.57, 5)
	0.20	0.75	(22.73, 16.55, 18)	(25.52, 18.26, 20)	(23.24, 16.59, 19)	(18.95, 12.94, 15)	(20.29, 13.6, 16)	(19.1, 12.8, 16)
	1.50	1.50	(2.17, 0.75, 2)	(2.14, 0.74, 2)	(2.14, 0.74, 2)	(2.11, 0.70, 2)	(2.09, 0.69, 2)	(2.1, 0.7, 2)
		EQL	18.19	18.37	17.99	18.13	18.44	

5.4 Comparative Study

This Section evaluates the performance of the proposed charts against several existing charts for joint monitoring, specifically the Max-CUSUM and Max-EWMA charts, which are based on the parameter transformations discussed earlier. The ARL and CED are used as performance criteria for specific shifts, while the EQL metric provides an overall assessment of performance across a range of joint shifts in both the process mean and variability. To ensure a fair comparison, following [48], the design parameters for the competing charts are set as follows: For all proposed charts and CUSUM charts, $k = 0.5$; for all EWMA charts, $\lambda = 0.2$. Additionally, ARL_0 is fixed at 250, and the sample size is set to $n = 5$.

5.4.1 Zero-State Comparisons

Table 5.2 presents a detailed (ARL, SDRL, MDRL) comparison among the proposed and CUSUM and EWMA charts, based on two methods (SS and Max) and three transformations (3T, 4T, and Chi-square). Various shifts have been considered to compare the charts' effectiveness in detecting changes in both the mean and variability of the process. In particular, we have considered the following scenarios: small and moderate shifts in the process location only ($\delta = 0.2, 0.5, \theta = 1$); small and moderate shifts in the process scale only ($\delta = 0, \theta = 0.75, 1.5$); and hybrid and large shifts in the process parameters ($\delta = 0.2, 1.5, \theta = 0.75, 1.5$).

From Table 5.2, the following observations can be summarized:

- Max-CAM charts vs Max-CUSUM charts: The proposed Max-CAM charts show a slight advantage over the Max-CUSUM charts in identifying moderate changes in

the process mean and/or variability. For instance, when $\delta = 1.5$ and $\theta = 1.0$, the ARL for the Max-CAM-3T is reduced by 99.35%, reaching 1.62, compared to the 99.07% reduction for the Max-CUSUM-3T, which has an ARL of 2.32. This is the case when $\delta = 1.5$ and $\theta = 1.0$. However, for minor changes such as $\delta = 0.2$ and $\theta = 0.75$, the Max-CUSUM charts slightly outperform the proposed Max-CAM charts, though the difference remain minimal.

- Max-CAM charts vs Max-EWMA charts: In most cases, the Max-CAM charts consistently dominate the Max-EWMA charts in terms of lower ARL values for both small and moderate changes in the mean and/or variance. The only exemption is when $\delta = 0.2$ and $\theta = 1$, where both charts show similar performance. Though the differences are mostly minor, there are certain cases that show a clear advantage. For example, when $\delta = 0$ and $\theta = 0.75$, the ARL for the Max-CAM-3T is 23.44 while that of Max-EWMA-3T is 28.68, marking about 20% difference in ARL reduction.
- SS-CAM charts vs. SS-CUSUM charts: Similar to the Max-CAM the SS-CAM charts demonstrate a slight advantage over SS-CUSUM charts in detecting moderate shifts in the process mean and variability. For instance, under the χ^2 transformation with $\delta = 1.5$ and $\theta = 1.5$, the ARL for SS-CAM decreases by 99.32%, reaching 1.71, while SS-CUSUM shows a 99.08% reduction, with an ARL of 2.31. This advantage is evident for $\delta=1.5$ and $\theta=1.5$. However, for smaller shifts, such as $\delta = 0.2$ and $\theta = 1$, the SS-CUSUM charts perform slightly better than SS-CAM, though the differences are marginal.

- SS-CAM charts vs SS-EWMA charts: The SS-CAM charts uniformly outperform the SS-EWMA charts for moderate changes in the process mean and/or variance. However, for small shifts the SS-EWMA charts take the lead. For example, when $\delta = 0.2$ and $\theta = 0.75$, the ARL for the SS-EWMA- χ^2 is 18.95 while that of SS-CAM- χ^2 is 20.55, indicating a 92.42% and 91.78% ARL reduction for SS-EWMA- χ^2 and SS-CAM- χ^2 respectively.

In a nutshell, the proposed Max-CAM and SS-CAM charts perform well, often better than the Max-CUSUM and SS-CUSUM charts, especially when there are moderate changes in the process mean and/or variability. These CAM-based charts detect shifts faster in most situations, as indicated by low ARL values when larger shifts occur. The Max-CAM charts mostly dominate the Max-EWMA charts, except for very small shifts, where both perform well with the Max-EWMA charts having slight edge. Similarly, the SS-CAM charts generally perform better than SS-EWMA for moderate shifts, though SS-EWMA has a slight advantage with smaller shifts. Overall, CAM-based charts provide a balanced approach to detecting shifts in process parameters, making them especially useful for keeping processes stable when moderate shifts are important.

Because the exact shift in process parameters is usually not known beforehand, we have also considered the overall performance of the charts across a range of shifts. In addition to the ARL values, the EQL metric is also reported in Table 2. As expected, all the proposed charts have lower EQL values compared to their CUSUM and EWMA counterparts. This is an indication of the superiority of our proposals in detecting shifts efficiently. The lower EQL values demonstrate the ability of the charts to maintain a quicker and more consistent

response across a wide range of shifts rather than specific shifts. The inter proposal comparison between the Max and SS-based CAM charts reveals that the Max CAM charts are more efficient in general than their SS CAM counterparts. However, for the specific shifts investigated in Table 5s.2, a mixed performance is observed with the two methods outperforming each other for different shifts.

5.4.2 CED Comparisons

The steady-state metrics are usually more relevant than the zero-state because they better reflects real-life scenarios. The zero-state condition assumes OOC at the start, however, in practice the process stays IC for some time before a change in the process parameters occurs, which aligns with the steady-state assumption. To this end, we have analyzed the IC and OOC CED curves of the proposed charts using the same OOC shifts considered in the zero-state, and compared them with other competing charts at various change-points $\tau \in [1,50]$. The results are presented as plots in Figures 5.1 – 5.5.

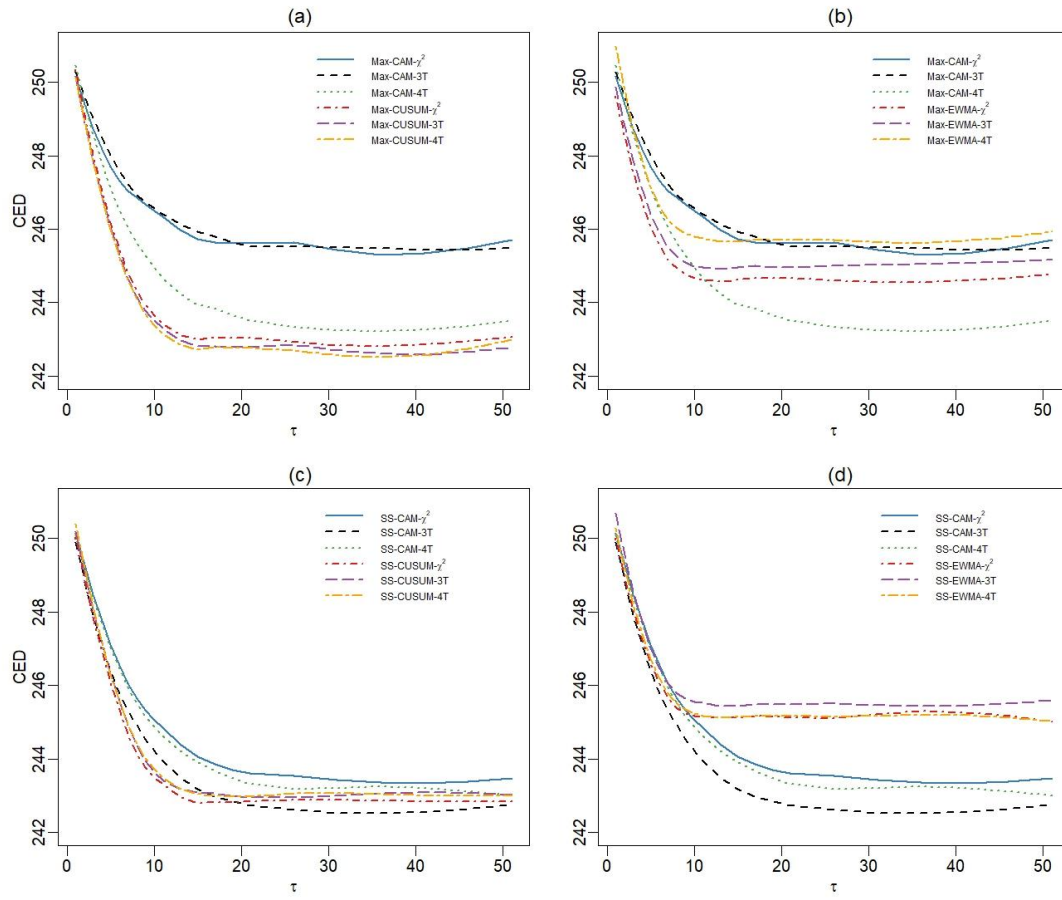


Figure 5.1: IC CED Curves of Proposed and Existing Charts

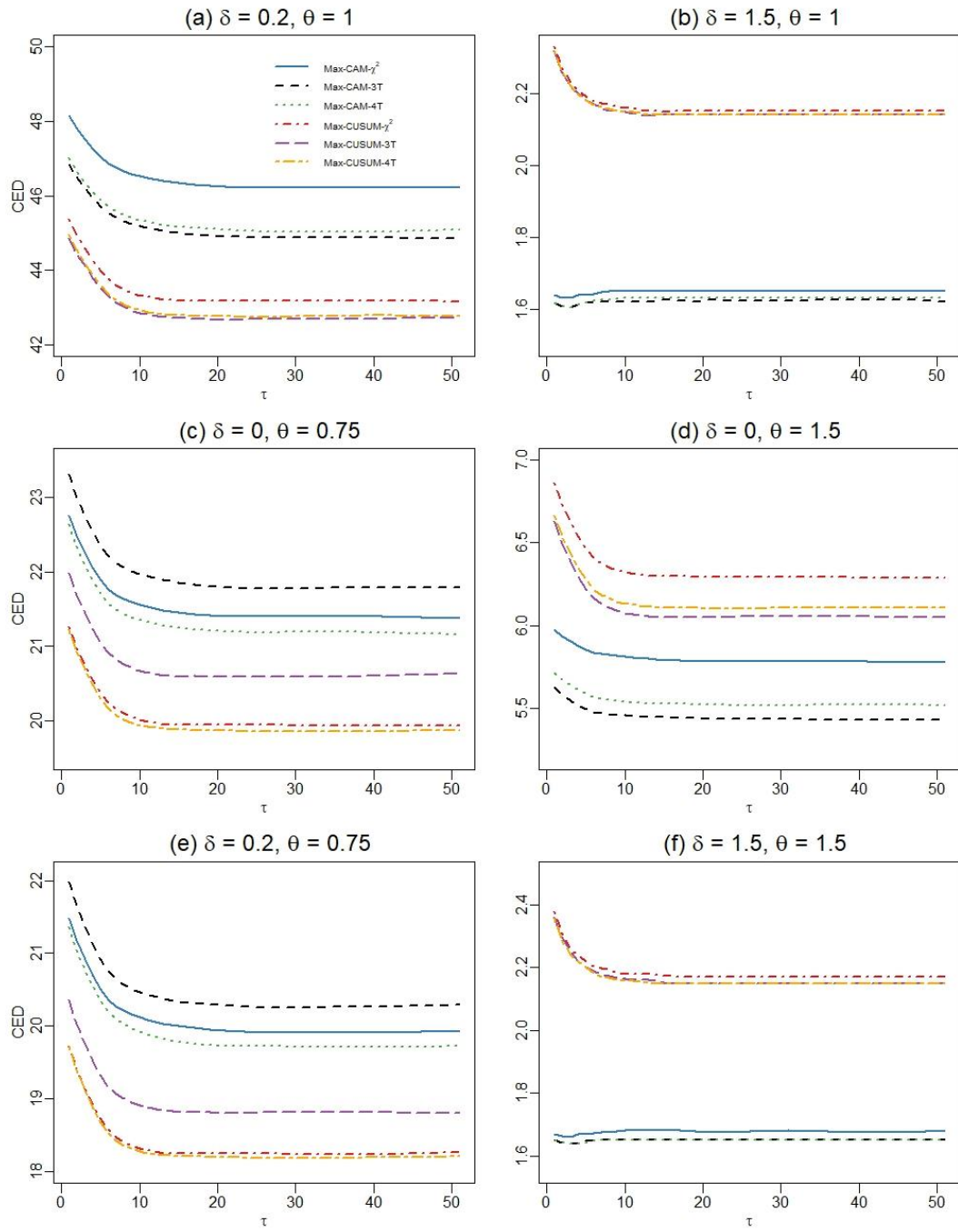


Figure 5.2: OOC CED Curves of Max-CAM and Max-CUSUM Charts.

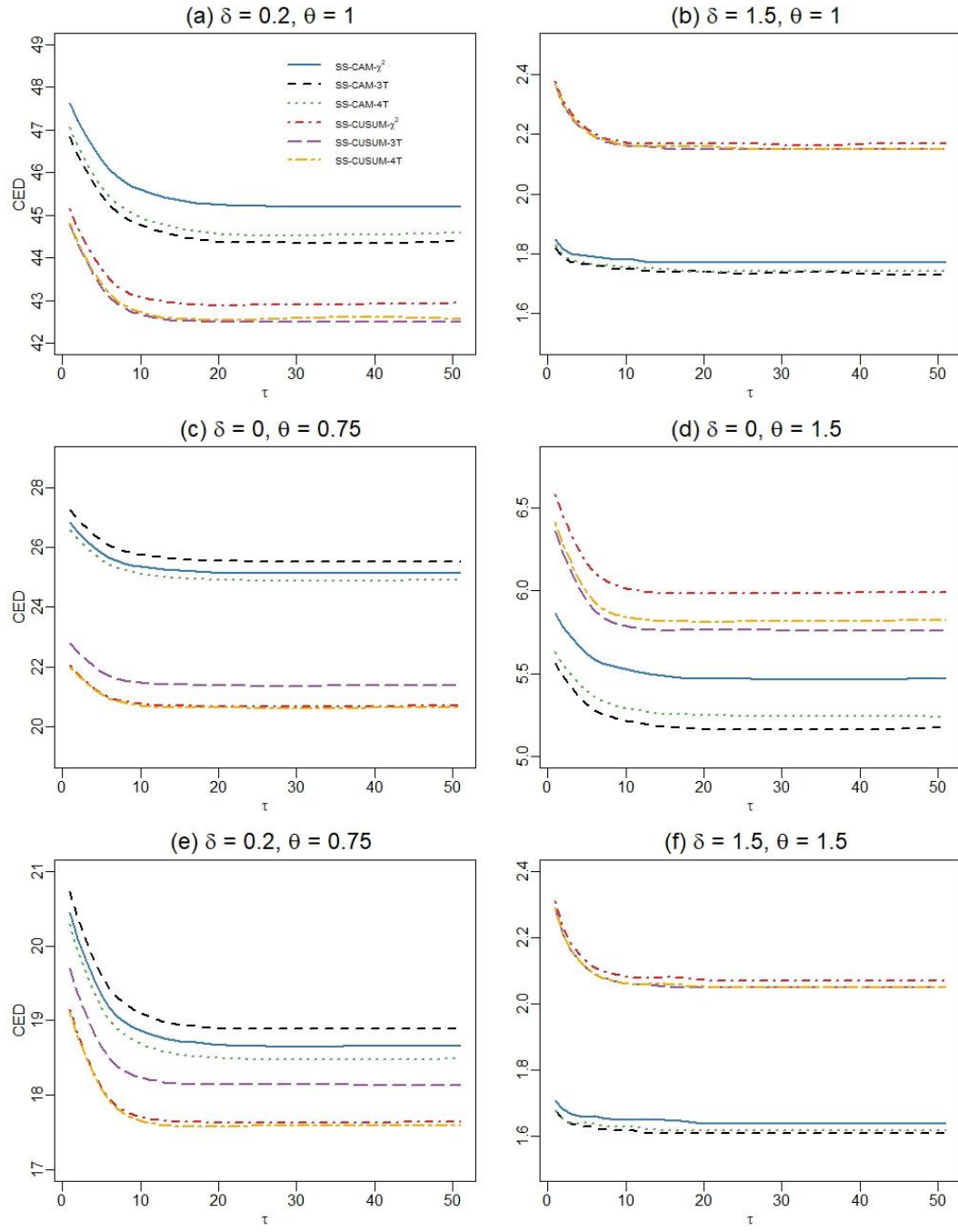


Figure 5.3: OOC CED Curves of SS-CAM and SS-CUSUM Charts.

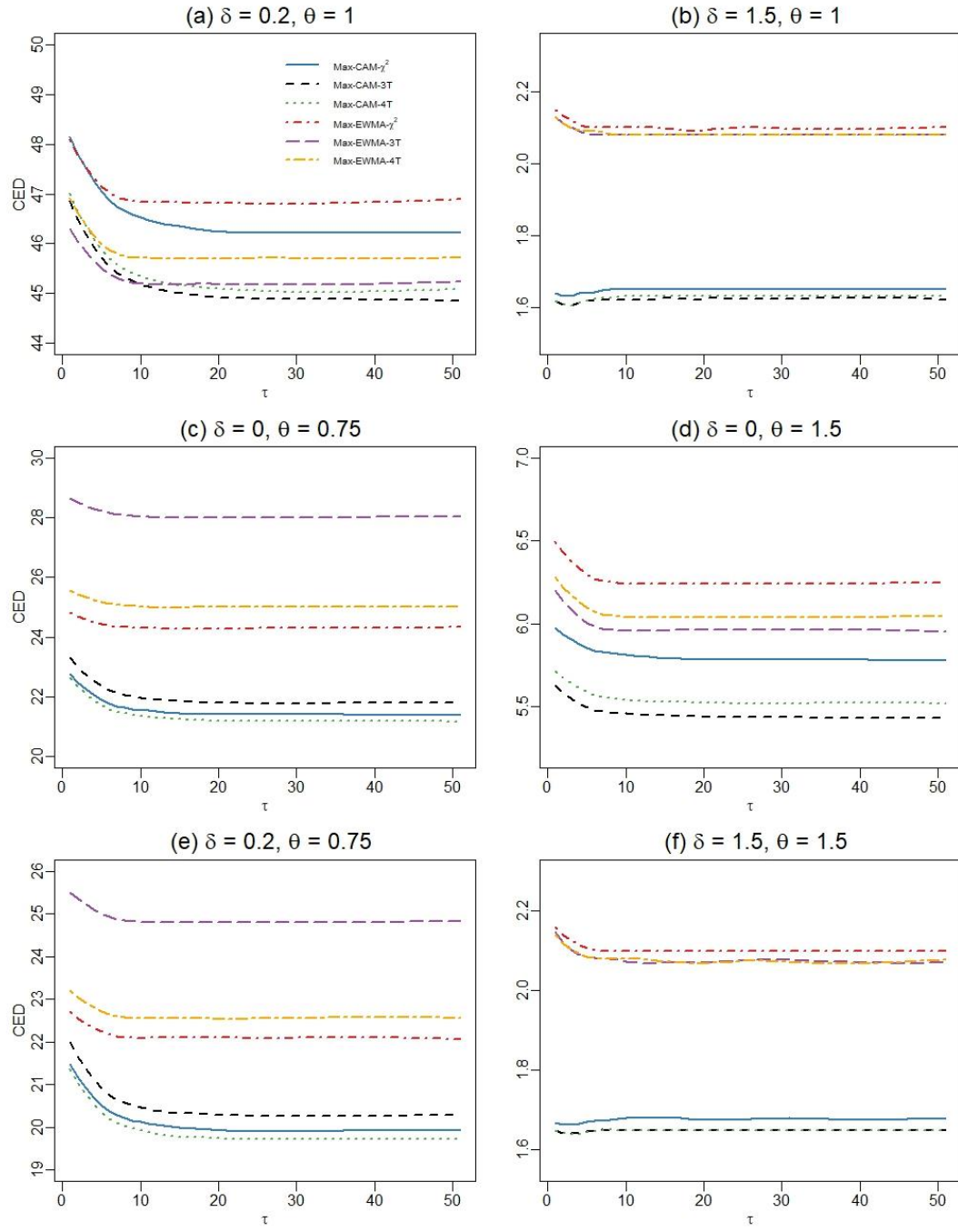


Figure 5.4: OOC CED Curves of Max-CAM and Max-EWMA Charts.

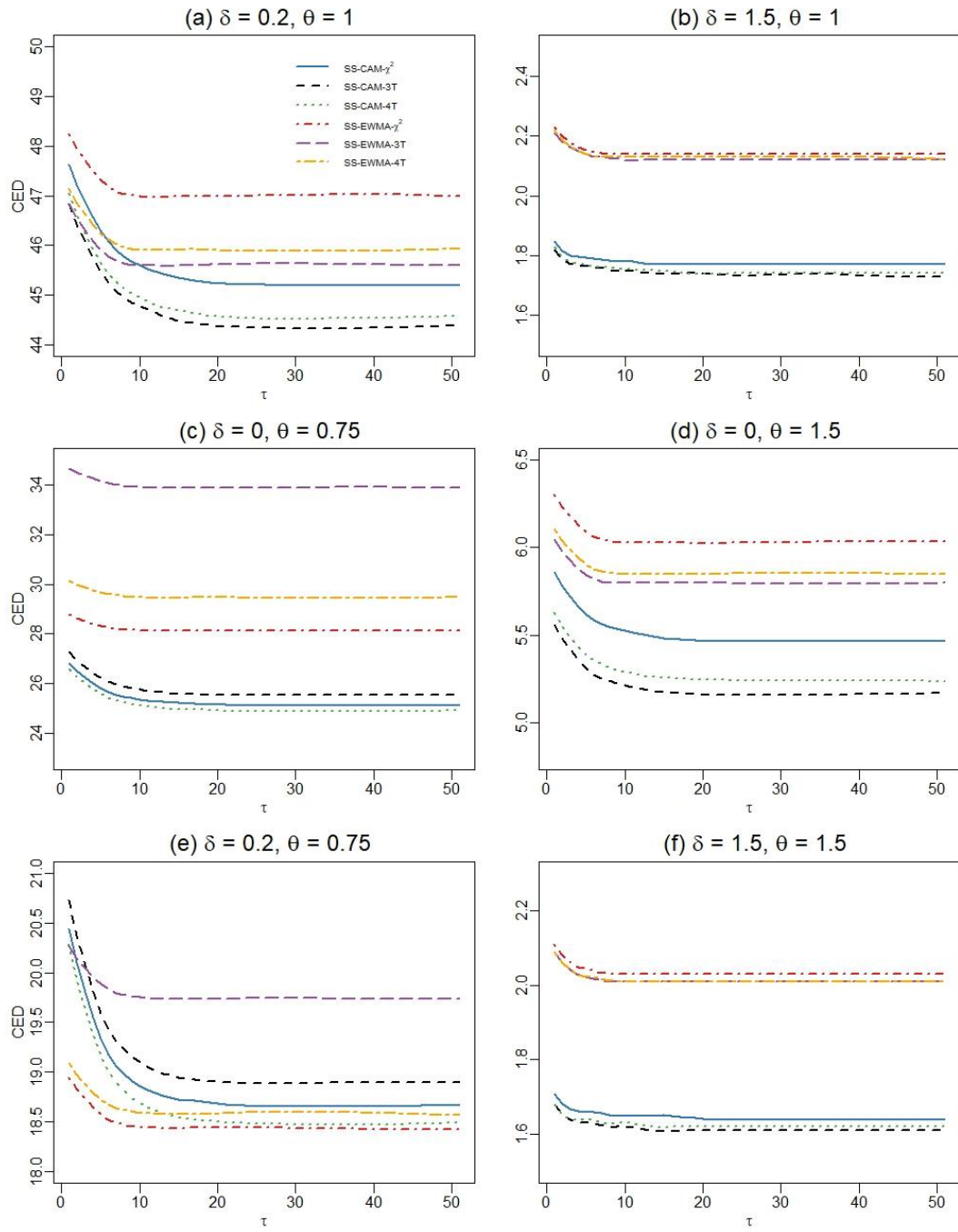


Figure 5.5: OOC CED Curves of SS-CAM and SS-EWMA Charts.

In the IC state (Figure 5.1), all the charts show a reduction in ARL_0 , indicating an increased false alarm rate in the steady-state. The CUSUM charts are the most affected, followed by the proposed charts, while the EWMA charts are the least impacted. Overall, charts based on the sum of squares method are more affected than their Max counterparts.

Figures 5.2 and 5.3 compare the OOC CED curves of the proposed CAM charts with the CUSUM charts for various small and moderate shifts, while Figures 5.4 and 5.5 show the comparison with the EWMA charts. In the OOC state, all charts exhibit a slight reduction in ARL_1 as τ increases, reflecting good steady-state performance.

The proposed charts perform better than CUSUM charts for moderate shifts. Specifically, when $\delta = 1.5$ and/or $\theta = 1.5$, the CUSUM charts are less effective compared to the proposed charts. However, for smaller shifts, such as $\delta = 0.2$ and $\theta = 0.75$, the CUSUM charts perform better. The CED comparisons with EWMA charts in Figures 4 and 5 show that the proposed charts generally outperform EWMA charts, with consistently lower CED values, except in cases of small shifts.

5.5 Real-world Example

As a common practice of studies on control charts, this section includes a practical industrial example using real-life data to illustrate the implementation and applicability of the proposed charts. The example is based on a study by [80] on predicting the electrical power output of a certain Combined Cycle Power Plant (CCPP). The plant combines gas and steam turbines to improve power generation. The data is taken in six-year time period with 9,568 hourly records covering four critical variables: ambient temperature (AT),

atmospheric pressure (AP), relative humidity (RH), and vacuum (V). The study used Machine learning models to predict power output based on these variables.

In this example, the AP variable is used as the important characteristic to be monitored. For brevity, we have considered the first 600 AP measurements as IC data and estimated the IC process parameters as follows: $\mu_0 = 1013.55$ and $\sigma_0 = 5.82$. The normality and independence assumptions were validated through the Anderson-Darling test (p-value = 0.4765) and the Ljung-Box test (p-value = 0.7782), respectively.

To set up the charts, 20 phase I samples, each of size 5, were taken from the IC data, and an additional 10 phase II samples in the first case (Figure 5.6a) were generated under an OOC condition with a joint minor shift of $\delta = 0.2$ in the mean and $\theta = 0.75$ in the variance of the process. In the second case (Figure 5.6b) 30 additional samples were generated with the moderate joint shifts $\delta = 1.25$ and $\theta = 1.25$. For CAM and CUSUM charts, the reference parameters are $k_1 = k_2 = 0.5$, and thresholds are $h_1 = 5.05$ and $h_1 = 3.453$, respectively. For EWMA chart, $\lambda = 0.2$ and limit coefficient $L_1 = 3.036$. Note that the control limits for all of the charts are selected to achieve $ARL_0 \approx 250$. Since the performance of the charts under different transformations and methods are comparable, we have set up all charts using the popular transformation, $V_{2,t}$ given in equation (2) under the Max method.

As shown in Figure 5.6, all control charts confirmed the process was IC for the first 20 samples, corresponding to the Phase I dataset, indicating no false alarms. For small shifts in the process parameters (Figure 5.6a), the EWMA and CAM charts were the first to signal OOC conditions at sample 45, while the CUSUM chart issued its signal at sample 48. In

total, both the CUSUM and EWMA charts identified only two OOC samples, whereas the proposed CAM chart detected six samples, which demonstrated its superiority in this scenario. However, when moderate shifts were introduced in the process parameters (Figure 5.6b), all charts issued their first alarms at sample 23 and detected the same eight OOC conditions.

The two cases showed that the proposed charts outperform their alternatives in detecting moderate changes in process parameters and remain competitive for small changes.

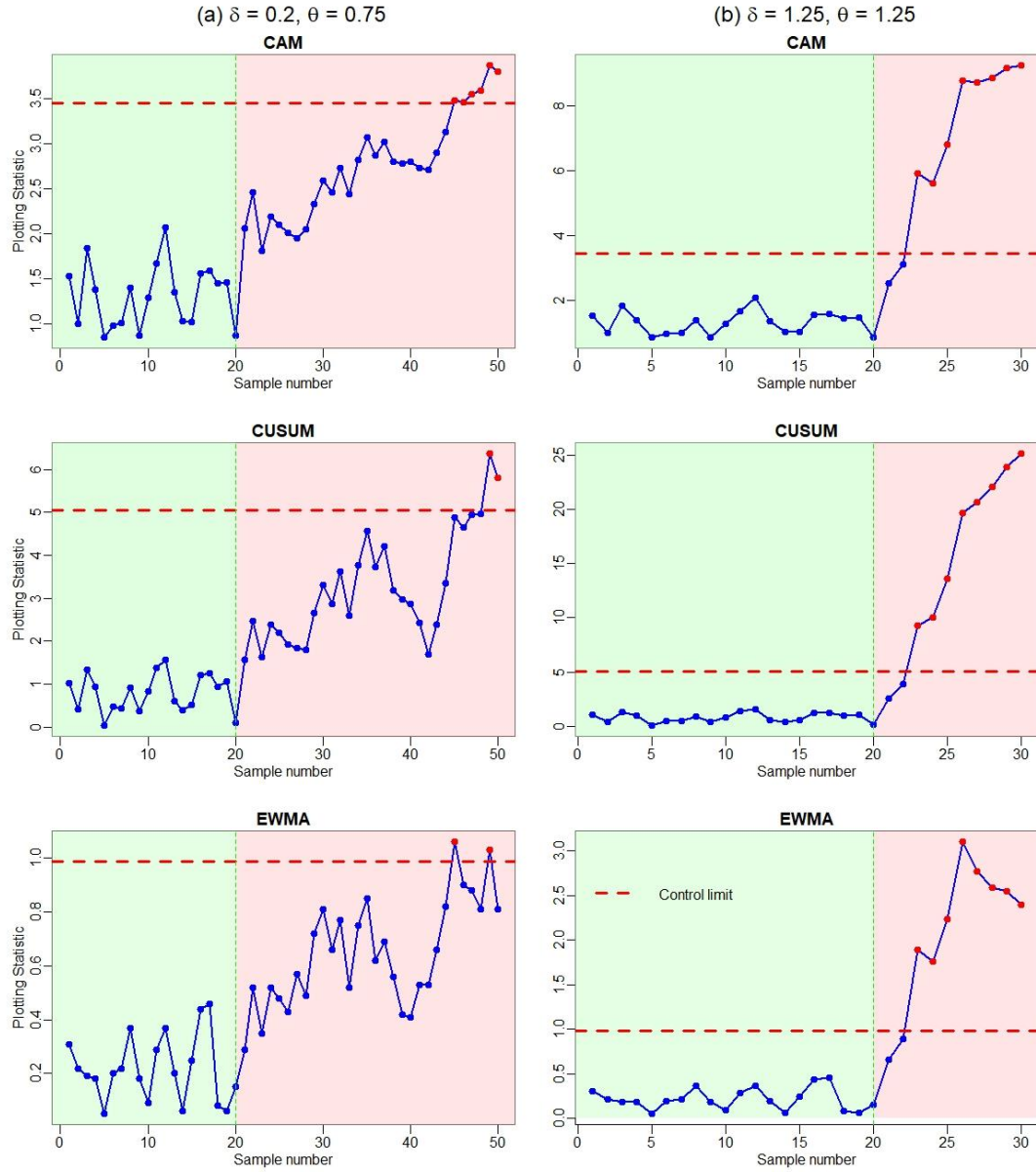


Figure 5.6: Application of Proposed and Existing Charts on Real Data

5.6 Chapter 5 Conclusion

In this chapter, six new memory-based control charts were proposed for the joint monitoring of process location (mean) and dispersion (variance) parameters. These charts, referred to as CAM charts, were developed using two methods—the maximum method and

the sum of squares method—and incorporated three different variance transformations: chi-square transformation, three-parameter transformation, and four-parameter transformation. The Monte Carlo simulation method was employed to evaluate the run-length characteristics of the proposed charts. The charts' specific and overall performance under both zero- and steady-state conditions was assessed using average run length, conditional expected delay curves, and extra quadratic loss metrics. The findings showed that the CAM charts are highly sensitive to all types of shifts in process parameters. Comparative analysis showed that the CAM charts often outperformed existing alternatives, such as joint CUSUM and EWMA schemes, especially for moderate shifts in the process mean and/or variance. Additionally, the proposed charts have lower EQL values than the CUSUM and EWMA charts, indicating they have better overall performance.

To demonstrate the applicability of the proposed charts, an example using real data from a combined cycle power plant (CCPP) was provided to monitor joint shifts in a critical variable of interest and compare the results with some competing schemes. The example showed that the proposed charts were better than their counterparts in detecting moderate changes in the parameters.

CHAPTER 6

CONCLUSION

6.1 SUMMARY

Statistical Process Control (SPC) is the primary technique employed to attain process stability and ensure the production and delivery of high-quality goods and services. Control charts are highly effective in identifying assignable causes and are simple to implement. This thesis has explored various charting schemes in Statistical Process Control, specifically memory-type parametric control charts for monitoring process parameters.

Chapters 1 and 2 presented the background of the study and comprehensively reviewed scholarly contributions on process monitoring using control charts. Chapter 3 reevaluated the performance of the MEC chart in zero-state and steady-state scenarios. The results through EQL and AEL measures showed that the MEC chart has good performance across a range of shift. The chapter also provides optimal parameters for specific target shifts were provided as a guide for quality practitioners. A practical example was used to show the construction of the chart.

In Chapter 4, a new control chart was proposed for monitoring shifts in the process variance using a three-parameter transformation of the sample variance. The performance of the proposed chart was assessed in zero- and steady-state scenarios through ARL and EQL metrics. The Chapter also provided optimal parameters of the proposed chart to aid practitioner in setting up the proposed chart. The comparison of the proposed chart with some existing alternatives showed that the proposed chart outperformed other in the zero-

state and has comparable performance in the steady-state. Two examples using real and simulated data that are included to show the practical implementation of the proposed chart. In Chapter 5, six new control charts were developed for monitoring of process mean and variance simultaneously using a single chart. The charts were constructed using two methods—the maximum method and the sum of squares method—and incorporated three different variance transformations: chi-square transformation, three-parameter transformation, and four-parameter transformation. The chapter presented the performance evaluation of the proposed charts and compared them with selected alternatives. The new charts were found to outperform their competitors in detecting moderate shifts in either one or both parameters. An example in industrial setting was provided to demonstrate the applicability of the proposed charts.

6.2 RECOMMENDATION

In Chapter 4, the three-parameter transformation of the sample variance was employed, further research could explore the four-parameter transformation to potentially improve the performance of the proposed chart. Additionally, the effectiveness of other versions of the proposed chart, such as a one-sided chart or one using untransformed sample variance, could be assessed and compared with other charts monitoring process dispersion.

Additionally, future studies may investigate the effects of sample size and parameter estimations on the proposed charts in Chapter 5. Also, this study assumes normal distribution, other non-normal or distribution-free methods could be explored.

APPENDICES

Appendix A (Distribution of CAM statistic)

The distribution of CAM_t statistic conditional on $CUSUM_{t-1}$, is normal.

Proof: The CAM_t statistic for the negative CUSUM case is defined as:

$$CAM_t^- = \frac{1}{D_t^-} V_t + \left(1 - \frac{1}{D_t^-}\right) CAM_{t-1}^-.$$

where D_t^- is the number of consecutive samples for which $CUSUM_{t-1}^- > 0$. Since V_t represents one the standardized transformations in equations (5.2), (5.4), and (5.6), then V_t observations are independent and standard normal:

$$V_t \sim N(0,1).$$

The CAM statistic is a linear combination of normally distributed random variables. By the properties of linear combinations of normal variables, any such combination of independent normal random variables will also follow a normal distribution.

Thus, since CAM_t^- is a weighted average of V_t , it follows a normal distribution. Therefore, given $CUSUM_{t-1}^-$, CAM_t^- follows a normal distribution:

$$CAM_t^- |_{CUSUM_{t-1}^-} \sim N(\mu_{CAM^-}, \sigma_{CAM^-}^2).$$

By similar reasoning, the CAM_t statistic for the positive CUSUM is also normal.

Lemma A.1: Conditional Mean of CAM_t^- given $CUSUM_{t-1}^-$ is 0.

Proof: We begin by representing the CAM statistic as the average of the previous D_t^- values of V starting from when the last CUSUM was zero:

$$CAM_t^- |_{CUSUM_{t-1}^-} = \frac{1}{D_t^-} \sum_{m=t-D_t^-+1}^t V_m.$$

Next, we apply the expectation operator as f:

$$\mathbb{E}(CAM_t^- |_{CUSUM_{t-1}^-}) = \frac{1}{D_t^-} \sum_{m=t-D_t^-+1}^t \mathbb{E}(V_m).$$

Substituting $\mathbb{E}(V_t) = 0$, yields:

$$\mathbb{E}(CAM_t^- |_{CUSUM_{t-1}^-}) = \frac{1}{D_t^-} \sum_{m=t-D_t^-+1}^t (0).$$

$$\mathbb{E}(CAM_t^- |_{CUSUM_{t-1}^-}) = 0.$$

Lemma A.2: For a given $CUSUM_{t-1}^-$, the conditional variance of CAM_t^- is $\frac{1}{D_t^-}$.

Proof: By representing the CAM statistic as the average of the previous D_t^- values of V starting from when the last CUSUM was zero and applying the variance, we have:

$$\mathbb{V}(CAM_t^- |_{CUSUM_{t-1}^-}) = \frac{1}{(D_t^-)^2} \mathbb{V}\left(\sum_{m=t-D_t^-+1}^t V_m\right).$$

The variance of a sum of independent random variables V_m is the sum of their variances.

Thus:

$$\mathbb{V}(CAM_t^- | CUSUM_{t-1}^-) = \frac{1}{(D_{(j)}^-)^2} \sum_{m=t-D_t^-+1}^t \mathbb{V}(V_m) = \frac{1}{(D_t^-)^2} \sum_{m=t-D_t^-+1}^t (1). \quad (1).$$

The summation from $t - D_t^- + 1$ to t is performed D_t^- times. Therefore,

$$\begin{aligned} \mathbb{V}(CAM_t^- | CUSUM_{t-1}^-) &= \frac{1}{(D_t^-)^2} [D_t^-(1)] \\ &= \frac{1}{D_t^-}. \end{aligned}$$

Lemma A.3: By following the same approach, the distribution of CAM_t^+ statistic conditional on the positive $CUSUM_{t-1}^+$, is given as:

$$CAM_t^+ | CUSUM_{t-1}^+ \sim N\left(0, \frac{1}{D_t^+}\right).$$

Appendix B (Effects of the Design Parameter on the Proposed Charts)

Table B1. (ARL, SDRL, MDR) values of the six proposed CAM charts for different shifts in the process mean (δ) and variance (θ). $ARL_0 \cong 250$ and $k_2 = 0.25$.

Chart	θ	δ											
		0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.36,0.48,3)	(2.79,0.43,3)	(2.00,0.15,2)	(1.50,0.50,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.41,1.64,6)	(6.41,1.64,6)	(6.41,1.64,6)	(6.41,1.63,6)	(6.38,1.60,6)	(6.19,1.47,6)	(4.40,1.07,4)	(2.80,0.73,3)	(1.99,0.50,2)	(1.50,0.51,2)	(1.01,0.11,1)	(1.00,0.00,1)
	0.75	(21.96,9.68,21)	(21.89,9.62,21)	(20.90,8.92,20)	(17.27,6.93,16)	(12.66,5.01,12)	(9.09,3.60,9)	(4.59,1.77,4)	(2.84,1.04,3)	(2.00,0.71,2)	(1.53,0.56,1)	(1.07,0.25,1)	(1.00,0.00,1)
	0.90	(112.50,83.53,91)	(86.20,59.01,73)	(41.40,23.53,38)	(21.89,11.67,20)	(13.42,6.86,12)	(9.12,4.50,8)	(4.56,2.08,4)	(2.85,1.22,3)	(2.02,0.82,2)	(1.56,0.61,1)	(1.11,0.31,1)	(1.00,0.01,1)
	1.00	(250.43,244.34,176)	(111.51,95.75,86)	(40.40,27.11,35)	(20.89,12.70,19)	(12.90,7.41,12)	(8.87,4.87,8)	(4.51,2.26,4)	(2.85,1.32,3)	(2.04,0.89,2)	(1.58,0.65,2)	(1.13,0.34,1)	(1.00,0.02,1)
	1.10	(60.04,51.71,46)	(48.52,39.98,38)	(29.39,21.69,24)	(17.86,12.09,15)	(11.76,7.46,10)	(8.36,5.02,7)	(4.42,2.40,4)	(2.84,1.41,3)	(2.05,0.94,2)	(1.60,0.69,1)	(1.16,0.37,1)	(1.00,0.03,1)
	1.25	(16.11,12.17,13)	(15.43,11.61,13)	(13.49,9.97,11)	(11.01,7.87,9)	(8.69,5.95,7)	(6.87,4.50,6)	(4.12,2.43,4)	(2.78,1.49,2)	(2.05,1.01,2)	(1.63,0.74,1)	(1.19,0.41,1)	(1.00,0.06,1)
	1.50	(5.61,3.91,5)	(5.55,3.86,5)	(5.34,3.69,4)	(5.03,3.44,4)	(4.65,3.13,4)	(4.22,2.79,4)	(3.23,2.00,3)	(2.48,1.42,2)	(1.97,1.04,2)	(1.62,0.78,1)	(1.23,0.46,1)	(1.01,0.11,1)
	1.75	(3.21,2.07,3)	(3.19,2.05,3)	(3.14,2.02,3)	(3.05,1.95,3)	(2.95,1.86,2)	(2.82,1.76,2)	(2.45,1.47,2)	(2.10,1.19,2)	(1.79,0.95,2)	(1.56,0.76,1)	(1.25,0.48,1)	(1.02,0.15,1)
2.00	(2.27,1.36,2)	(2.26,1.35,2)	(2.24,1.33,2)	(2.21,1.31,2)	(2.17,1.27,2)	(2.12,1.24,2)	(1.96,1.10,2)	(1.78,0.96,2)	(1.61,0.82,1)	(1.46,0.69,1)	(1.23,0.48,1)	(1.03,0.18,1)	
Max-4T	0.25	(2.93,0.39,3)	(2.93,0.39,3)	(2.93,0.39,3)	(2.93,0.39,3)	(2.93,0.39,3)	(2.93,0.39,3)	(2.93,0.38,3)	(2.70,0.46,3)	(2.00,0.14,2)	(1.51,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.86,1.75,6)	(5.87,1.75,6)	(5.86,1.75,6)	(5.86,1.75,6)	(5.84,1.72,6)	(5.70,1.59,6)	(4.26,1.04,4)	(2.79,0.72,3)	(1.99,0.50,2)	(1.51,0.51,2)	(1.01,0.11,1)	(1.00,0.00,1)
	0.75	(21.44,10.05,20)	(21.36,10.03,20)	(20.41,9.24,19)	(16.88,7.09,16)	(12.45,5.04,12)	(9.01,3.59,9)	(4.58,1.76,4)	(2.84,1.04,3)	(2.01,0.71,2)	(1.53,0.56,2)	(1.07,0.25,1)	(1.00,0.00,1)
	0.90	(110.46,83.70,90)	(84.97,59.30,72)	(41.05,23.58,37)	(21.81,11.68,20)	(13.40,6.85,12)	(9.12,4.49,8)	(4.57,2.08,4)	(2.85,1.22,3)	(2.03,0.82,2)	(1.56,0.61,2)	(1.11,0.31,1)	(1.00,0.01,1)
	1.00	(250.05,244.08,176)	(111.70,96.11,86)	(40.57,27.20,35)	(20.92,12.73,19)	(12.92,7.43,12)	(8.89,4.88,8)	(4.52,2.26,4)	(2.86,1.32,3)	(2.04,0.89,2)	(1.58,0.65,2)	(1.13,0.35,1)	(1.00,0.02,1)
	1.10	(62.24,53.38,48)	(49.84,40.81,40)	(29.86,21.93,25)	(18.05,12.20,15)	(11.84,7.51,10)	(8.40,5.05,7)	(4.44,2.40,4)	(2.84,1.42,3)	(2.05,0.95,2)	(1.60,0.69,1)	(1.16,0.37,1)	(1.00,0.03,1)
	1.25	(16.52,12.43,14)	(15.81,11.86,13)	(13.77,10.18,11)	(11.20,8.01,9)	(8.80,6.03,7)	(6.93,4.55,6)	(4.13,2.45,4)	(2.78,1.50,2)	(2.05,1.02,2)	(1.63,0.74,1)	(1.19,0.41,1)	(1.00,0.06,1)
	1.50	(5.69,3.99,5)	(5.61,3.93,5)	(5.41,3.77,5)	(5.09,3.52,4)	(4.70,3.19,4)	(4.26,2.84,4)	(3.24,2.02,3)	(2.48,1.43,2)	(1.97,1.04,2)	(1.62,0.78,1)	(1.23,0.46,1)	(1.01,0.11,1)
	1.75	(3.22,2.11,3)	(3.20,2.09,3)	(3.15,2.06,3)	(3.06,1.99,3)	(2.95,1.90,2)	(2.82,1.79,2)	(2.45,1.49,2)	(2.09,1.20,2)	(1.79,0.95,2)	(1.55,0.76,1)	(1.24,0.48,1)	(1.02,0.15,1)
2.00	(2.27,1.38,2)	(2.26,1.37,2)	(2.24,1.35,2)	(2.20,1.33,2)	(2.16,1.29,2)	(2.11,1.25,2)	(1.95,1.12,2)	(1.77,0.97,2)	(1.60,0.82,1)	(1.45,0.69,1)	(1.23,0.47,1)	(1.03,0.18,1)	
Max- χ^2	0.25	(2.24,0.59,2)	(2.24,0.59,2)	(2.24,0.59,2)	(2.24,0.59,2)	(2.24,0.59,2)	(2.24,0.59,2)	(2.24,0.59,2)	(2.17,0.53,2)	(1.93,0.28,2)	(1.51,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.57,2.00,5)	(5.57,2.00,5)	(5.57,2.00,5)	(5.57,2.00,5)	(5.55,1.97,5)	(5.42,1.83,5)	(4.13,1.14,4)	(2.78,0.73,3)	(2.00,0.50,2)	(1.53,0.51,2)	(1.01,0.12,1)	(1.00,0.00,1)
	0.75	(21.55,10.67,20)	(21.45,10.62,20)	(20.49,9.77,19)	(16.90,7.46,16)	(12.46,5.22,12)	(9.03,3.67,9)	(4.63,1.78,4)	(2.87,1.05,3)	(2.03,0.72,2)	(1.55,0.56,2)	(1.07,0.26,1)	(1.00,0.00,1)
	0.90	(110.31,85.58,89)	(85.07,60.81,72)	(41.28,24.05,37)	(22.00,11.84,20)	(13.54,6.93,13)	(9.23,4.54,9)	(4.62,2.10,4)	(2.88,1.23,3)	(2.05,0.83,2)	(1.58,0.62,2)	(1.11,0.32,1)	(1.00,0.01,1)
	1.00	(250.06,245.18,175)	(113.46,97.56,87)	(41.12,27.48,36)	(21.24,12.87,19)	(13.12,7.50,12)	(9.01,4.92,8)	(4.58,2.28,4)	(2.89,1.34,3)	(2.07,0.89,2)	(1.60,0.66,2)	(1.14,0.35,1)	(1.00,0.02,1)
	1.10	(66.06,56.30,51)	(52.53,42.74,42)	(30.97,22.55,26)	(18.53,12.43,16)	(12.12,7.63,11)	(8.57,5.12,8)	(4.50,2.43,4)	(2.88,1.43,3)	(2.08,0.95,2)	(1.62,0.69,2)	(1.16,0.38,1)	(1.00,0.03,1)
	1.25	(17.41,12.93,14)	(16.59,12.27,14)	(14.44,10.54,12)	(11.65,8.26,10)	(9.10,6.20,8)	(7.14,4.66,6)	(4.23,2.50,4)	(2.82,1.52,3)	(2.08,1.03,2)	(1.64,0.75,2)	(1.20,0.42,1)	(1.00,0.06,1)
	1.50	(5.93,4.12,5)	(5.85,4.05,5)	(5.63,3.89,5)	(5.28,3.62,4)	(4.86,3.28,4)	(4.40,2.92,4)	(3.32,2.07,3)	(2.53,1.46,2)	(2.00,1.06,2)	(1.64,0.80,1)	(1.24,0.47,1)	(1.01,0.11,1)
	1.75	(3.33,2.17,3)	(3.31,2.16,3)	(3.25,2.12,3)	(3.16,2.04,3)	(3.04,1.95,3)	(2.91,1.84,2)	(2.52,1.53,2)	(2.14,1.23,2)	(1.82,0.97,2)	(1.57,0.77,1)	(1.25,0.49,1)	(1.02,0.16,1)
2.00	(2.33,1.41,2)	(2.32,1.41,2)	(2.30,1.39,2)	(2.26,1.36,2)	(2.22,1.33,2)	(2.16,1.28,2)	(1.99,1.15,2)	(1.81,0.99,2)	(1.63,0.84,1)	(1.47,0.71,1)	(1.24,0.48,1)	(1.04,0.19,1)	
SS-3T	0.25	(4.42,0.50,4)	(4.36,0.49,4)	(4.22,0.42,4)	(4.07,0.32,4)	(3.86,0.40,4)	(3.47,0.50,3)	(2.90,0.31,3)	(2.08,0.27,2)	(1.99,0.12,2)	(1.53,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.38,0.49,3)	(3.36,0.48,3)	(2.79,0.43,3)	(2.00,0.15,2)	(1.50,0.50,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(8.19,1.87,8)	(8.01,1.83,8)	(7.51,1.71,7)	(6.81,1.55,7)	(6.03,1.37,6)	(5.26,1.19,5)	(3.71,0.84,4)	(2.69,0.62,3)	(2.07,0.44,2)	(1.69,0.47,2)	(1.05,0.22,1)	(1.00,0.00,1)
	0.75	(26.29,10.67,25)	(24.31,9.86,23)	(19.61,7.75,19)	(14.88,5.69,14)	(11.21,4.18,11)	(8.58,3.12,8)	(4.89,1.69,5)	(3.19,1.05,3)	(2.30,0.73,2)	(1.79,0.58,2)	(1.18,0.38,1)	(1.00,0.01,1)
	0.90	(112.59,80.14,93)	(77.60,51.41,66)	(38.56,20.78,35)	(22.16,10.95,21)	(14.36,6.74,13)	(10.10,4.55,9)	(5.25,2.19,5)	(3.32,1.30,3)	(2.37,0.88,2)	(1.82,0.67,2)	(1.23,0.42,1)	(1.00,0.03,1)
	1.00	(249.74,237.18,178)	(106.90,87.51,83)	(41.51,26.09,37)	(22.50,12.79,20)	(14.31,7.64,13)	(10.00,5.08,9)	(5.20,2.41,5)	(3.31,1.43,3)	(2.36,0.96,2)	(1.82,0.72,2)	(1.25,0.44,1)	(1.00,0.04,1)
	1.10	(57.33,46.18,46)	(45.32,34.65,37)	(28.21,19.29,24)	(18.04,11.33,16)	(12.35,7.29,11)	(9.00,5.04,8)	(4.93,2.49,5)	(3.22,1.50,3)	(2.33,1.02,2)	(1.81,0.75,2)	(1.26,0.46,1)	(1.00,0.06,1)
	1.25	(16.18,11.35,14)	(15.28,10.66,13)	(13.10,8.94,11)	(10.65,7.01,9)	(8.52,5.39,7)	(6.88,4.18,6)	(4.30,2.37,4)	(2.99,1.51,3)	(2.24,1.05,2)	(1.78,0.79,2)	(1.28,0.48,1)	(1.01,0.09,1)
	1.50	(5.80,3.80,5)	(5.72,3.73,5)	(5.46,3.54,5)	(5.08,3.26,4)	(4.65,2.93,4)	(4.20,2.59,4)	(3.22,1.87,3)	(2.51,1.36,2)	(2.02,1.02,2)	(1.68,0.79,2)	(1.28,0.49,1)	(1.02,0.14,1)
	1.75	(3.33,2.04,3)	(3.32,2.03,3)	(3.25,1.98,3)	(3.15,1.91,3)	(3.01,1.81,3)	(2.86,1.70,2)	(2.46,1.40,2)	(2.10,1.13,2)	(1.80,0.91,2)	(1.57,0.74,1)	(1.26,0.49,1)	(1.03,0.17,1)
2.00	(2.36,1.36,2)	(2.35,1.35,2)	(2.33,1.33,2)	(2.29,1.30,2)	(2.23,1.26,2)	(2.17,1.22,2)	(1.99,1.08,2)	(1.79,0.93,2)	(1.61,0.80,1)	(1.46,0.67,1)	(1.23,0.47,1)	(1.04,0.19,1)	
SS-4T	0.25	(3.67,0.53,4)	(3.63,0.53,4)	(3.52,0.52,4)	(3.36,0.49,3)	(3.19,0.40,3)	(3.05,0.29,3)	(2.47,0.50,2)	(2.02,0.13,2)	(1.93,0.25,2)	(1.32,0.47,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.47,1.99,7)	(7.32,1.95,7)	(6.91,1.82,7)	(6.32,1.64,6)	(5.65,1.44,6)	(4.98,1.25,5)	(3.57,0.87,4)	(2.62,0.63,3)	(2.04,0.44,2)	(1.66,0.48,2)	(1.05,0.22,1)	(1.00,0.00,1)
	0.75	(25.69,11.04,24)	(23.80,10.20,22)	(19.27,7.97,18)	(14.65,5.83,14)	(11.06,4.25,11)	(8.49,3.16,8)	(4.86,1.70,5)	(3.17,1.06,3)	(2.29,0.74,2)	(1.78,0.58,2)	(1.18,0.38,1)	(1.00,0.01,1)
	0.90	(111.49,80.85,91)	(76.71,51.57,65)	(38.26,20.92,35)	(22.04,11.00,20)	(14.30,6.76,13)	(10.06,4.56,9)	(5.24,2.19,5)	(3.32,1.30,3)	(2.36,0.88,2)	(1.82,0.67,2)	(1.23,0.42,1)	(1.00,0.03,1)
	1.00	(249.68,237.17,177)	(106.84,87.51,83)	(41.55,26.11,37)	(22.54,12.80,20)	(14.32,7.64,13)	(10.01,5.09,9)	(5.21,2.42,5)	(3.31,1.43,3)	(2.37,0.96,2)	(1.83,0.72,2)	(1.25,0.44,1)	(1.00,0.04,1)
	1.10	(58.95,47.36,47)	(46.34,35.28,38)	(28.65,19.50,24)	(18.22,11.41,16)	(12.45,7.32,11)	(9.06,5.06,8)	(4.96,2.50,5)	(3.23,1.51,3)	(2.34,1.02,2)	(1.82,0.75,2)	(1.27,0.46,1)	(1.00,0.06,1)
	1.25	(16.54,11.58,14)	(15.59,10.84,13)	(13.35,9.08,11)	(10.83,7.12,9)	(8.65,5.47,8)	(6.95,4.22,6)	(4.34,2.39,4)	(3.00,1.52,3)	(2.25,1.06,2)	(1.79,0.79,2)	(1.28,0.48,1)	(1.01,0.09,1)
	1.50	(5.85,3.88,5)	(5.76,3.81,5)	(5.50,3.60,5)	(5.12,3.32,4)	(4.68,2.98,4)	(4.23,2.63,4)	(3.24,1.89,3)	(2.52,1.37,2)	(2.02,1.02,2)	(1.69,0.79,2)	(1.28,0.50,1)	(1.02,0.14,1)
	1.75	(3.32,2.08,3)	(3.29,2.06,3)	(3.23,2.01,3)	(3.13,1.94,3)	(3.00,1.84,3)	(2.85,1.72,2)	(2.46,1.42,2)	(2.09,1.14,2)	(1.80,0.92,2)	(1.57,0.75,1)	(1.26,0.49,1)	(1.03,0.17,1)
2.00	(2.33,1.37,2)	(2.32,1.36,2)	(2.30,1.35,2)	(2.26,1.32,2)	(2.21,1.28,2)	(2.15,1.23,2)	(1.97,1.09,2)	(1.78,0.94,2)	(1.61,0.80,1)	(1.45,0.67,1)	(1.23,0.47,1)	(1.04,0.19,1)	
SS- χ^2	0.25	(2.83,0.67,3)	(2.81,0.66,3)	(2.76,0.65,3)	(2.68,0.62,3)	(2.57,0.60,3)	(2.45,0.57,2)	(2.12,0.44,2)	(1.88,0.35,2)	(1.62,0.49,2)	(1.19,0.39,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.07,2.27,7)	(6.94,2.22,7)	(6.57,2.07,6)	(6.04,1.85,6)	(5.43,1.62,5)	(4.81,1.39,5)	(3.49,0.95,3)	(2.59,0.67,3)	(2.02,0.48,2)	(1.65,0.49,2)	(1.05,0.23,1)	(1.00,0.00,1)
	0.75	(25.88,11.66,24)	(23.96,10.72,23)	(19.37,8.35,18)	(14.72,6.06,14)	(11.11,4.40,11)	(8.53,3.26,8)	(4.89,1.74,5)	(3.20,1.08,3)	(2.31,0.75,2)	(1.80,0.59,2)	(1.19,0.39,1)	(1.00,0.01,1)
	0.90	(112.63,82.95,92)	(77.21,52.64,65)	(38.46,21.24,35)	(22.22,11.20,21)	(14.43,6.87,13)	(10.17,4.64,10)	(5.30,2.23,5)	(3.36,1.32,3)	(2.39,0.89,2)	(1.84,0.67,2)	(1.24,0.43,1)	(1.00,0.03,1)
	1.00	(250.23,238.27,178)	(107.44,87.87,84)	(41.87,26.27,37)	(22.81,12.93,21)	(14.51,7.73,13)	(10.15,5.16,9)	(5.28,2.44,5)	(3.35,1.44,3)	(2.39,0.97,2)	(1.84,0.72,2)	(1.26,0.45,1)	(1.00,0.05,1)
	1.10	(61.91,49.43,50)	(48.20,36.51,40)	(29.48,19.86,25)	(18.65,11.57,16)	(12.70,7.42,11)	(9.22,5.12,8)	(5.04,2.53,5)	(3.28,1.52,3)	(2.37,1.03,2)	(1.84,0.76,2)	(1.28,0.47,1)	(1.00,0.07,1)
	1.25	(17.29,11.94,15)	(16.27,11.18,14)	(13.87,9.33,12)	(11.18,7.28,10)	(8.88,5.56,8)	(7.12,4.29,6)	(4.42,2.43,4)	(3.05,1.54,3)	(2.28,1.07,2)	(1.80,0.80,2)	(1.29,0.49,1)	(1.01,0.10,1)
	1.50	(6.08,3.98,5)	(5.98,3.91,5)	(5.71,3.71,5)	(5.30,3.40,5)	(4.83,3.06,4)	(4.35,2.69,4)	(3.31,1.93,3)	(2.56,1.39,2)	(2.05,1.04,2)	(1.71,0.80,2)	(1.29,0.50,1)	(1.02,0.14,1)
	1.75	(3.43,2.14,3)	(3.41,2.12,3)	(3.33,2.07,3)	(3.23,1.99,3)	(3.09,1.88,3)	(2.93,1.76,3)	(2.51,1.45,2)	(2.13,1.16,2)	(1.83,0.93,2)	(1.59,0.76,1)	(1.27,0.50,1)	(1.03,0.17,1)
2.00	(2.40,1.41,2)	(2.39,1.40,2)	(2.37,1.38,2)	(2.32,1.35,2)	(2.27,1.31,2)	(2.21,1.26,2)	(2.01,1.12,2)	(1.81,0.96,2)	(1.63,0.82,1)	(1.47,0.69,1)	(1.24,0.48,1)	(1.04,0.20,1)	

Table B2. (ARL, SDRL, MDR) values of the six proposed CAM charts for different shifts in the process mean (δ) and variance (θ). $ARL_0 \cong 250$ and $k_2 = 0.75$.

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.48,0.50,3)	(2.84,0.40,3)	(2.01,0.15,2)	(1.57,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.55,1.66,6)	(6.56,1.66,6)	(6.56,1.66,6)	(6.56,1.66,6)	(6.52,1.63,6)	(6.33,1.49,6)	(4.51,1.08,4)	(2.86,0.74,3)	(2.02,0.50,2)	(1.54,0.51,2)	(1.02,0.13,1)	(1.00,0.00,1)
	0.75	(33.28,25.11,26)	(33.15,25.01,25)	(30.66,22.53,24)	(22.10,14.21,18)	(13.93,7.11,12)	(9.38,4.01,9)	(4.66,1.78,4)	(2.89,1.05,3)	(2.04,0.72,2)	(1.56,0.57,2)	(1.08,0.27,1)	(1.00,0.00,1)
	0.90	(355.36,347.78,249)	(237.68,230.90,167)	(82.52,76.00,59)	(29.90,23.87,23)	(14.92,9.68,13)	(9.42,5.09,9)	(4.62,2.08,4)	(2.90,1.22,3)	(2.06,0.83,2)	(1.59,0.62,2)	(1.12,0.32,1)	(1.00,0.01,1)
	1.00	(249.89,246.50,174)	(153.66,149.70,108)	(59.39,54.56,43)	(25.61,20.81,19)	(13.95,9.63,12)	(9.07,5.38,8)	(4.56,2.25,4)	(2.90,1.32,3)	(2.07,0.89,2)	(1.61,0.66,2)	(1.14,0.35,1)	(1.00,0.02,1)
	1.10	(64.32,61.08,46)	(53.16,49.75,38)	(33.16,29.40,24)	(19.39,15.60,15)	(12.20,8.67,10)	(8.46,5.35,7)	(4.46,2.37,4)	(2.88,1.41,3)	(2.09,0.95,2)	(1.63,0.70,2)	(1.17,0.38,1)	(1.00,0.04,1)
	1.25	(16.21,13.32,12)	(15.42,12.54,12)	(13.37,10.52,11)	(10.87,8.12,9)	(8.60,6.01,7)	(6.82,4.46,6)	(4.14,2.39,4)	(2.82,1.49,3)	(2.09,1.02,2)	(1.65,0.75,2)	(1.20,0.42,1)	(1.00,0.06,1)
	1.50	(5.54,3.75,5)	(5.46,3.69,5)	(5.28,3.53,4)	(4.97,3.28,4)	(4.60,2.99,4)	(4.20,2.67,4)	(3.25,1.94,3)	(2.51,1.41,2)	(2.00,1.04,2)	(1.65,0.79,1)	(1.24,0.47,1)	(1.01,0.11,1)
	1.75	(3.22,2.01,3)	(3.20,1.99,3)	(3.15,1.95,3)	(3.07,1.89,3)	(2.96,1.81,3)	(2.83,1.71,2)	(2.48,1.44,2)	(2.13,1.18,2)	(1.82,0.95,2)	(1.58,0.77,1)	(1.26,0.49,1)	(1.03,0.16,1)
2.00	(2.30,1.34,2)	(2.29,1.33,2)	(2.27,1.32,2)	(2.24,1.29,2)	(2.20,1.26,2)	(2.14,1.22,2)	(1.98,1.10,2)	(1.81,0.96,2)	(1.63,0.83,1)	(1.48,0.70,1)	(1.24,0.49,1)	(1.04,0.19,1)	
Max-4T	0.25	(2.99,0.35,3)	(2.99,0.35,3)	(2.99,0.35,3)	(2.99,0.35,3)	(2.99,0.35,3)	(2.99,0.35,3)	(2.99,0.34,3)	(2.79,0.41,3)	(2.01,0.14,2)	(1.58,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.99,1.77,6)	(5.99,1.77,6)	(5.99,1.77,6)	(5.99,1.77,6)	(5.97,1.74,6)	(5.82,1.60,6)	(4.37,1.05,4)	(2.86,0.73,3)	(2.03,0.50,2)	(1.54,0.51,2)	(1.02,0.13,1)	(1.00,0.00,1)
	0.75	(31.02,23.67,24)	(30.82,23.46,24)	(28.73,21.32,22)	(21.20,13.77,17)	(13.63,7.03,12)	(9.30,3.99,9)	(4.67,1.77,4)	(2.90,1.05,3)	(2.05,0.72,2)	(1.56,0.57,2)	(1.08,0.27,1)	(1.00,0.00,1)
	0.90	(285.63,278.91,200)	(205.36,198.98,144)	(78.89,72.47,57)	(29.61,23.58,23)	(14.90,9.67,13)	(9.42,5.08,9)	(4.63,2.08,4)	(2.91,1.22,3)	(2.07,0.83,2)	(1.59,0.62,2)	(1.12,0.33,1)	(1.00,0.01,1)
	1.00	(250.56,247.63,174)	(154.48,150.20,109)	(59.85,55.07,43)	(25.70,20.88,20)	(14.00,9.66,12)	(9.10,5.39,8)	(4.58,2.26,4)	(2.90,1.32,3)	(2.08,0.89,2)	(1.61,0.66,2)	(1.15,0.36,1)	(1.00,0.02,1)
	1.10	(68.12,64.91,48)	(55.75,52.37,40)	(34.17,30.40,25)	(19.70,15.90,15)	(12.31,8.77,10)	(8.50,5.37,7)	(4.47,2.38,4)	(2.89,1.41,3)	(2.09,0.96,2)	(1.63,0.70,2)	(1.17,0.39,1)	(1.00,0.04,1)
	1.25	(16.82,13.92,13)	(15.95,13.06,12)	(13.80,10.93,11)	(11.11,8.33,9)	(8.73,6.13,7)	(6.89,4.52,6)	(4.15,2.40,4)	(2.82,1.49,3)	(2.09,1.02,2)	(1.65,0.75,2)	(1.20,0.42,1)	(1.00,0.06,1)
	1.50	(5.63,3.86,5)	(5.56,3.80,5)	(5.36,3.63,5)	(5.04,3.37,4)	(4.65,3.06,4)	(4.24,2.73,4)	(3.26,1.97,3)	(2.51,1.42,2)	(2.00,1.05,2)	(1.64,0.79,1)	(1.24,0.47,1)	(1.01,0.11,1)
	1.75	(3.23,2.05,3)	(3.22,2.04,3)	(3.17,2.00,3)	(3.08,1.93,3)	(2.97,1.85,3)	(2.84,1.75,2)	(2.47,1.47,2)	(2.12,1.19,2)	(1.81,0.96,2)	(1.57,0.77,1)	(1.25,0.49,1)	(1.03,0.16,1)
2.00	(2.29,1.36,2)	(2.28,1.36,2)	(2.26,1.34,2)	(2.23,1.32,2)	(2.19,1.28,2)	(2.13,1.24,2)	(1.97,1.12,2)	(1.79,0.97,2)	(1.62,0.83,1)	(1.47,0.70,1)	(1.24,0.48,1)	(1.04,0.19,1)	
Max- χ^2	0.25	(2.30,0.60,2)	(2.30,0.60,2)	(2.30,0.60,2)	(2.30,0.60,2)	(2.30,0.60,2)	(2.30,0.60,2)	(2.30,0.60,2)	(2.24,0.55,2)	(1.95,0.26,2)	(1.61,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.71,2.03,6)	(5.71,2.03,6)	(5.71,2.03,6)	(5.71,2.03,6)	(5.69,2.00,6)	(5.56,1.86,5)	(4.27,1.15,4)	(2.86,0.74,3)	(2.05,0.50,2)	(1.57,0.50,2)	(1.02,0.14,1)	(1.00,0.00,1)
	0.75	(30.58,23.87,23)	(30.45,23.79,23)	(28.49,21.67,22)	(21.20,14.12,17)	(13.69,7.19,12)	(9.37,4.07,9)	(4.74,1.79,5)	(2.95,1.06,3)	(2.08,0.73,2)	(1.59,0.57,2)	(1.09,0.28,1)	(1.00,0.00,1)
	0.90	(232.08,226.91,163)	(180.42,174.94,127)	(77.59,71.46,56)	(29.98,23.90,23)	(15.13,9.75,13)	(9.59,5.14,9)	(4.72,2.11,4)	(2.96,1.24,3)	(2.10,0.84,2)	(1.61,0.63,2)	(1.13,0.34,1)	(1.00,0.01,1)
	1.00	(250.39,247.14,175)	(158.64,154.79,111)	(61.94,56.96,45)	(26.51,21.51,20)	(14.33,9.82,12)	(9.30,5.46,8)	(4.67,2.29,4)	(2.96,1.34,3)	(2.11,0.90,2)	(1.63,0.67,2)	(1.15,0.37,1)	(1.00,0.02,1)
	1.10	(76.14,72.81,54)	(61.49,57.88,44)	(36.64,32.61,27)	(20.67,16.69,16)	(12.76,9.06,11)	(8.74,5.50,8)	(4.57,2.42,4)	(2.94,1.43,3)	(2.12,0.97,2)	(1.65,0.70,2)	(1.18,0.39,1)	(1.00,0.04,1)
	1.25	(18.16,15.04,14)	(17.19,14.09,13)	(14.72,11.67,12)	(11.74,8.81,10)	(9.13,6.39,8)	(7.16,4.68,6)	(4.27,2.46,4)	(2.88,1.52,3)	(2.13,1.04,2)	(1.68,0.76,2)	(1.21,0.43,1)	(1.00,0.07,1)
	1.50	(5.92,4.03,5)	(5.84,3.96,5)	(5.62,3.78,5)	(5.29,3.51,5)	(4.86,3.18,4)	(4.41,2.82,4)	(3.37,2.03,3)	(2.58,1.45,2)	(2.04,1.07,2)	(1.67,0.81,1)	(1.25,0.48,1)	(1.01,0.12,1)
	1.75	(3.36,2.13,3)	(3.34,2.11,3)	(3.29,2.07,3)	(3.20,2.00,3)	(3.08,1.91,3)	(2.94,1.80,3)	(2.55,1.51,2)	(2.17,1.23,2)	(1.85,0.98,2)	(1.60,0.79,1)	(1.27,0.50,1)	(1.03,0.16,1)
2.00	(2.36,1.41,2)	(2.35,1.40,2)	(2.34,1.39,2)	(2.30,1.36,2)	(2.25,1.32,2)	(2.20,1.28,2)	(2.03,1.15,2)	(1.84,1.00,2)	(1.65,0.85,1)	(1.49,0.72,1)	(1.25,0.50,1)	(1.04,0.19,1)	
SS-3T	0.25	(4.25,0.44,4)	(4.24,0.43,4)	(4.19,0.41,4)	(4.05,0.39,4)	(3.72,0.48,4)	(3.31,0.47,3)	(2.81,0.40,3)	(2.04,0.19,2)	(1.97,0.18,2)	(1.39,0.49,1)	(1.00,0.00,1)	(1.00,0.00,1)

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.49,0.51,3)	(3.48,0.50,3)	(2.84,0.40,3)	(2.01,0.15,2)	(1.57,0.49,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.96,1.86,8)	(7.88,1.86,8)	(7.56,1.85,7)	(6.86,1.76,7)	(5.93,1.50,6)	(5.08,1.22,5)	(3.57,0.82,4)	(2.60,0.61,3)	(2.02,0.43,2)	(1.63,0.49,2)	(1.04,0.19,1)	(1.00,0.00,1)
	0.75	(39.56,29.75,30)	(37.27,27.84,29)	(29.71,21.67,23)	(19.25,12.54,16)	(12.21,6.42,11)	(8.62,3.73,8)	(4.74,1.70,5)	(3.09,1.04,3)	(2.23,0.72,2)	(1.73,0.58,2)	(1.15,0.36,1)	(1.00,0.01,1)
	0.90	(367.57,359.96,258)	(226.59,219.62,159)	(78.83,72.10,57)	(29.90,23.47,23)	(15.42,9.71,13)	(9.96,5.15,9)	(5.05,2.16,5)	(3.21,1.27,3)	(2.29,0.87,2)	(1.77,0.66,2)	(1.20,0.40,1)	(1.00,0.02,1)
	1.00	(250.03,246.03,175)	(152.84,148.31,107)	(59.71,54.36,43)	(26.30,20.97,20)	(14.62,9.76,12)	(9.66,5.50,9)	(4.97,2.35,5)	(3.19,1.39,3)	(2.29,0.94,2)	(1.77,0.70,2)	(1.22,0.43,1)	(1.00,0.04,1)
	1.10	(60.63,56.85,43)	(49.62,45.79,36)	(31.10,27.10,23)	(18.62,14.62,14)	(12.06,8.33,10)	(8.57,5.24,7)	(4.70,2.41,4)	(3.09,1.45,3)	(2.26,0.99,2)	(1.76,0.74,2)	(1.24,0.44,1)	(1.00,0.06,1)
	1.25	(15.16,12.10,12)	(14.34,11.36,11)	(12.31,9.43,10)	(9.98,7.24,8)	(7.98,5.42,7)	(6.42,4.07,6)	(4.08,2.26,4)	(2.87,1.45,3)	(2.17,1.02,2)	(1.73,0.77,2)	(1.25,0.46,1)	(1.01,0.08,1)
	1.50	(5.33,3.49,5)	(5.25,3.41,4)	(5.03,3.25,4)	(4.71,2.99,4)	(4.33,2.70,4)	(3.94,2.39,3)	(3.06,1.74,3)	(2.41,1.29,2)	(1.96,0.98,2)	(1.64,0.76,1)	(1.26,0.48,1)	(1.02,0.13,1)
	1.75	(3.15,1.89,3)	(3.13,1.87,3)	(3.07,1.83,3)	(2.97,1.76,3)	(2.86,1.68,2)	(2.73,1.58,2)	(2.36,1.31,2)	(2.03,1.08,2)	(1.75,0.88,2)	(1.53,0.72,1)	(1.24,0.47,1)	(1.03,0.16,1)
2.00	(2.27,1.27,2)	(2.26,1.27,2)	(2.24,1.25,2)	(2.20,1.23,2)	(2.15,1.19,2)	(2.10,1.15,2)	(1.92,1.03,2)	(1.75,0.90,2)	(1.58,0.77,1)	(1.43,0.65,1)	(1.22,0.46,1)	(1.03,0.18,1)	
SS-4T	0.25	(3.53,0.53,4)	(3.51,0.53,3)	(3.46,0.52,3)	(3.32,0.48,3)	(3.14,0.37,3)	(2.99,0.31,3)	(2.35,0.48,2)	(2.01,0.10,2)	(1.87,0.33,2)	(1.22,0.41,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.26,1.97,7)	(7.19,1.97,7)	(6.91,1.94,7)	(6.33,1.81,6)	(5.54,1.53,5)	(4.81,1.26,5)	(3.44,0.85,3)	(2.54,0.62,2)	(1.99,0.45,2)	(1.60,0.50,2)	(1.04,0.18,1)	(1.00,0.00,1)
	0.75	(36.98,27.94,28)	(34.87,26.27,27)	(28.01,20.53,22)	(18.51,12.11,15)	(11.95,6.36,11)	(8.50,3.73,8)	(4.70,1.72,4)	(3.08,1.05,3)	(2.23,0.73,2)	(1.73,0.58,2)	(1.15,0.36,1)	(1.00,0.01,1)
	0.90	(304.60,297.79,213)	(198.69,192.02,140)	(74.44,67.77,54)	(29.26,22.92,22)	(15.30,9.63,13)	(9.93,5.16,9)	(5.05,2.17,5)	(3.21,1.28,3)	(2.29,0.87,2)	(1.77,0.66,2)	(1.20,0.41,1)	(1.00,0.02,1)
	1.00	(250.26,246.22,175)	(152.86,148.38,107)	(59.93,54.52,43)	(26.43,21.11,20)	(14.64,9.77,12)	(9.70,5.52,9)	(4.99,2.35,5)	(3.20,1.39,3)	(2.29,0.94,2)	(1.78,0.70,2)	(1.23,0.43,1)	(1.00,0.04,1)
	1.10	(64.24,60.57,46)	(52.00,48.17,37)	(32.08,28.04,24)	(19.00,14.96,15)	(12.25,8.47,10)	(8.67,5.30,8)	(4.73,2.42,4)	(3.11,1.45,3)	(2.27,0.99,2)	(1.77,0.74,2)	(1.24,0.45,1)	(1.00,0.06,1)
	1.25	(15.73,12.67,12)	(14.82,11.79,12)	(12.69,9.79,10)	(10.23,7.47,8)	(8.13,5.55,7)	(6.53,4.16,6)	(4.12,2.28,4)	(2.89,1.46,3)	(2.18,1.03,2)	(1.74,0.77,2)	(1.26,0.46,1)	(1.01,0.08,1)
	1.50	(5.40,3.58,5)	(5.32,3.52,5)	(5.10,3.33,4)	(4.76,3.06,4)	(4.37,2.76,4)	(3.97,2.45,3)	(3.08,1.78,3)	(2.43,1.30,2)	(1.97,0.99,2)	(1.65,0.77,1)	(1.26,0.48,1)	(1.02,0.13,1)
	1.75	(3.15,1.93,3)	(3.13,1.91,3)	(3.07,1.87,3)	(2.98,1.80,3)	(2.86,1.71,2)	(2.73,1.61,2)	(2.37,1.34,2)	(2.03,1.09,2)	(1.75,0.89,2)	(1.54,0.72,1)	(1.25,0.48,1)	(1.03,0.16,1)
2.00	(2.25,1.30,2)	(2.24,1.29,2)	(2.22,1.27,2)	(2.19,1.25,2)	(2.14,1.21,2)	(2.08,1.17,2)	(1.91,1.04,2)	(1.74,0.90,2)	(1.57,0.77,1)	(1.43,0.65,1)	(1.22,0.46,1)	(1.03,0.18,1)	
SS- χ^2	0.25	(2.75,0.66,3)	(2.74,0.66,3)	(2.71,0.65,3)	(2.64,0.64,3)	(2.52,0.60,3)	(2.39,0.56,2)	(2.07,0.42,2)	(1.85,0.37,2)	(1.56,0.50,2)	(1.14,0.35,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.90,2.26,7)	(6.83,2.25,7)	(6.58,2.20,6)	(6.05,2.02,6)	(5.35,1.70,5)	(4.67,1.40,5)	(3.39,0.93,3)	(2.52,0.66,2)	(1.98,0.48,2)	(1.60,0.50,2)	(1.04,0.20,1)	(1.00,0.00,1)
	0.75	(36.69,28.26,28)	(34.61,26.53,27)	(27.96,20.88,22)	(18.56,12.39,15)	(12.02,6.50,11)	(8.58,3.83,8)	(4.76,1.76,5)	(3.12,1.07,3)	(2.26,0.74,2)	(1.75,0.59,2)	(1.16,0.37,1)	(1.00,0.01,1)
	0.90	(260.25,254.03,183)	(179.82,173.79,127)	(72.62,66.31,52)	(29.33,22.99,22)	(15.45,9.70,13)	(10.07,5.22,9)	(5.13,2.20,5)	(3.26,1.30,3)	(2.33,0.88,2)	(1.79,0.67,2)	(1.22,0.42,1)	(1.00,0.02,1)
	1.00	(249.86,245.77,174)	(155.47,150.72,109)	(61.33,55.90,44)	(26.98,21.51,21)	(14.94,9.93,13)	(9.89,5.60,9)	(5.08,2.39,5)	(3.25,1.41,3)	(2.33,0.95,2)	(1.80,0.71,2)	(1.24,0.44,1)	(1.00,0.04,1)
	1.10	(70.55,66.72,50)	(56.35,52.39,40)	(33.98,29.80,25)	(19.79,15.58,15)	(12.62,8.70,11)	(8.90,5.41,8)	(4.83,2.46,4)	(3.17,1.48,3)	(2.31,1.01,2)	(1.80,0.75,2)	(1.26,0.45,1)	(1.00,0.06,1)
	1.25	(16.78,13.54,13)	(15.78,12.57,12)	(13.37,10.32,11)	(10.72,7.82,9)	(8.43,5.74,7)	(6.73,4.26,6)	(4.22,2.32,4)	(2.94,1.49,3)	(2.22,1.04,2)	(1.76,0.78,2)	(1.27,0.47,1)	(1.01,0.09,1)
	1.50	(5.65,3.71,5)	(5.56,3.63,5)	(5.31,3.45,5)	(4.95,3.17,4)	(4.53,2.84,4)	(4.11,2.52,4)	(3.16,1.81,3)	(2.48,1.33,2)	(2.01,1.01,2)	(1.67,0.78,2)	(1.27,0.49,1)	(1.02,0.13,1)
	1.75	(3.27,1.98,3)	(3.24,1.97,3)	(3.18,1.92,3)	(3.08,1.85,3)	(2.96,1.76,3)	(2.81,1.65,2)	(2.43,1.37,2)	(2.08,1.11,2)	(1.79,0.90,2)	(1.56,0.73,1)	(1.26,0.49,1)	(1.03,0.17,1)
2.00	(2.32,1.33,2)	(2.31,1.33,2)	(2.29,1.31,2)	(2.25,1.28,2)	(2.20,1.24,2)	(2.14,1.20,2)	(1.96,1.07,2)	(1.77,0.92,2)	(1.60,0.79,1)	(1.45,0.67,1)	(1.23,0.47,1)	(1.04,0.19,1)	

Table B3. (ARL, SDRL, MDR) values of the six proposed CAM charts for different shifts in the process mean (δ) and variance (θ). $ARL_0 \cong 250$ and $k_2 = 1$.

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.39,0.49,3)	(2.80,0.42,3)	(2.00,0.14,2)	(1.52,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.57,1.93,6)	(6.57,1.93,6)	(6.58,1.94,6)	(6.57,1.93,6)	(6.53,1.87,6)	(6.29,1.63,6)	(4.43,1.08,4)	(2.81,0.73,3)	(1.99,0.50,2)	(1.51,0.51,2)	(1.01,0.11,1)	(1.00,0.00,1)
	0.75	(65.07,59.29,47)	(64.42,58.51,46)	(58.39,52.55,42)	(38.29,32.72,28)	(19.28,14.33,15)	(10.69,6.38,9)	(4.60,1.84,4)	(2.84,1.04,3)	(2.01,0.71,2)	(1.54,0.56,2)	(1.07,0.25,1)	(1.00,0.00,1)
	0.90	(542.59,538.71,377)	(350.96,347.32,244)	(123.86,119.65,87)	(42.71,38.66,31)	(18.78,15.02,14)	(10.47,7.07,9)	(4.58,2.18,4)	(2.85,1.21,3)	(2.03,0.82,2)	(1.57,0.61,2)	(1.11,0.31,1)	(1.00,0.01,1)
	1.00	(250.38,247.38,175)	(168.39,165.46,118)	(73.03,69.83,52)	(31.85,28.60,23)	(16.28,13.13,12)	(9.84,6.91,8)	(4.52,2.36,4)	(2.85,1.31,3)	(2.05,0.88,2)	(1.59,0.65,2)	(1.13,0.35,1)	(1.00,0.02,1)
	1.10	(66.98,64.59,47)	(56.44,53.90,40)	(36.76,34.13,26)	(21.72,19.01,16)	(13.34,10.67,10)	(8.90,6.40,7)	(4.42,2.48,4)	(2.84,1.40,3)	(2.06,0.94,2)	(1.61,0.69,2)	(1.16,0.37,1)	(1.00,0.03,1)
	1.25	(16.97,14.78,13)	(16.11,13.91,12)	(13.91,11.72,10)	(11.24,9.08,9)	(8.80,6.70,7)	(6.88,4.88,6)	(4.08,2.44,4)	(2.77,1.47,2)	(2.05,1.01,2)	(1.63,0.73,1)	(1.19,0.41,1)	(1.00,0.06,1)
	1.50	(5.49,3.90,5)	(5.41,3.83,4)	(5.21,3.65,4)	(4.90,3.37,4)	(4.53,3.04,4)	(4.13,2.70,4)	(3.18,1.92,3)	(2.47,1.38,2)	(1.97,1.02,2)	(1.63,0.78,1)	(1.23,0.46,1)	(1.01,0.11,1)
	1.75	(3.16,1.99,3)	(3.14,1.97,3)	(3.09,1.94,3)	(3.01,1.87,3)	(2.91,1.79,3)	(2.78,1.69,2)	(2.43,1.42,2)	(2.09,1.16,2)	(1.79,0.93,2)	(1.56,0.75,1)	(1.25,0.48,1)	(1.02,0.15,1)
2.00	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.39,0.49,3)	(2.80,0.42,3)	(2.00,0.14,2)	(1.52,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)	
Max-4T	0.25	(2.94,0.38,3)	(2.94,0.38,3)	(2.94,0.38,3)	(2.94,0.38,3)	(2.94,0.38,3)	(2.94,0.38,3)	(2.94,0.37,3)	(2.72,0.45,3)	(2.00,0.14,2)	(1.52,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.98,1.97,6)	(5.98,1.97,6)	(5.98,1.98,6)	(5.97,1.97,6)	(5.95,1.92,6)	(5.78,1.71,6)	(4.29,1.05,4)	(2.80,0.72,3)	(2.00,0.50,2)	(1.51,0.51,2)	(1.01,0.12,1)	(1.00,0.00,1)
	0.75	(52.59,47.48,38)	(52.25,47.08,38)	(48.41,43.22,35)	(34.02,28.84,25)	(18.37,13.53,14)	(10.51,6.24,9)	(4.61,1.83,4)	(2.85,1.04,3)	(2.02,0.71,2)	(1.54,0.56,2)	(1.07,0.26,1)	(1.00,0.00,1)
	0.90	(394.89,390.95,275)	(283.62,279.27,198)	(115.35,111.26,81)	(41.91,37.90,30)	(18.65,14.87,14)	(10.48,7.07,9)	(4.59,2.18,4)	(2.86,1.21,3)	(2.03,0.82,2)	(1.57,0.61,2)	(1.11,0.31,1)	(1.00,0.01,1)
	1.00	(250.39,247.90,174)	(168.90,166.25,118)	(73.44,70.16,52)	(31.94,28.66,23)	(16.32,13.15,12)	(9.86,6.92,8)	(4.54,2.36,4)	(2.86,1.31,3)	(2.05,0.88,2)	(1.59,0.65,2)	(1.14,0.35,1)	(1.00,0.02,1)
	1.10	(71.07,68.93,50)	(59.38,56.97,42)	(37.97,35.33,27)	(22.15,19.44,16)	(13.47,10.80,10)	(8.94,6.42,7)	(4.43,2.48,4)	(2.84,1.40,3)	(2.06,0.94,2)	(1.61,0.69,2)	(1.16,0.38,1)	(1.00,0.03,1)
	1.25	(17.74,15.56,13)	(16.78,14.62,12)	(14.39,12.21,11)	(11.50,9.34,9)	(8.95,6.85,7)	(6.96,4.97,6)	(4.11,2.46,4)	(2.77,1.48,2)	(2.06,1.01,2)	(1.63,0.74,1)	(1.19,0.41,1)	(1.00,0.06,1)
	1.50	(5.59,4.02,5)	(5.51,3.95,5)	(5.30,3.76,4)	(4.98,3.47,4)	(4.59,3.13,4)	(4.17,2.77,4)	(3.20,1.96,3)	(2.47,1.40,2)	(1.97,1.03,2)	(1.62,0.78,1)	(1.23,0.46,1)	(1.01,0.11,1)
	1.75	(3.18,2.04,3)	(3.15,2.02,3)	(3.10,1.98,3)	(3.02,1.92,3)	(2.92,1.82,2)	(2.79,1.72,2)	(2.43,1.44,2)	(2.09,1.17,2)	(1.79,0.94,2)	(1.55,0.75,1)	(1.25,0.48,1)	(1.02,0.15,1)
2.00	(2.25,1.34,2)	(2.25,1.34,2)	(2.23,1.32,2)	(2.20,1.30,2)	(2.15,1.26,2)	(2.10,1.22,2)	(1.94,1.09,2)	(1.77,0.95,2)	(1.60,0.82,1)	(1.45,0.69,1)	(1.23,0.48,1)	(1.03,0.18,1)	
Max- χ^2	0.25	(2.27,0.59,2)	(2.27,0.59,2)	(2.27,0.59,2)	(2.27,0.59,2)	(2.27,0.59,2)	(2.27,0.59,2)	(2.27,0.59,2)	(2.21,0.54,2)	(1.94,0.27,2)	(1.56,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(5.71,2.21,5)	(5.71,2.22,5)	(5.71,2.22,5)	(5.70,2.21,5)	(5.68,2.17,5)	(5.53,1.96,5)	(4.19,1.15,4)	(2.82,0.73,3)	(2.03,0.50,2)	(1.55,0.51,2)	(1.02,0.13,1)	(1.00,0.00,1)
	0.75	(47.84,43.42,35)	(47.60,43.10,34)	(44.75,40.24,32)	(32.76,28.02,24)	(18.26,13.57,14)	(10.59,6.30,9)	(4.69,1.85,4)	(2.90,1.05,3)	(2.05,0.72,2)	(1.57,0.57,2)	(1.08,0.27,1)	(1.00,0.00,1)
	0.90	(283.43,280.02,198)	(227.02,223.80,158)	(109.37,105.39,77)	(42.36,38.31,31)	(19.05,15.20,14)	(10.70,7.19,9)	(4.68,2.21,4)	(2.91,1.22,3)	(2.07,0.83,2)	(1.60,0.62,2)	(1.12,0.33,1)	(1.00,0.01,1)
	1.00	(250.43,248.13,174)	(173.45,171.01,121)	(77.07,73.73,54)	(33.35,29.98,24)	(16.89,13.61,13)	(10.14,7.09,8)	(4.64,2.40,4)	(2.91,1.33,3)	(2.09,0.90,2)	(1.61,0.66,2)	(1.15,0.36,1)	(1.00,0.02,1)
	1.10	(80.51,78.00,57)	(66.61,64.14,47)	(41.36,38.64,30)	(23.58,20.74,17)	(14.11,11.34,11)	(9.25,6.64,8)	(4.54,2.53,4)	(2.90,1.42,3)	(2.10,0.95,2)	(1.64,0.70,2)	(1.17,0.39,1)	(1.00,0.04,1)
	1.25	(19.43,17.12,14)	(18.36,16.06,14)	(15.61,13.33,12)	(12.32,10.04,9)	(9.45,7.26,7)	(7.28,5.18,6)	(4.23,2.53,4)	(2.84,1.51,3)	(2.10,1.03,2)	(1.66,0.75,2)	(1.21,0.42,1)	(1.00,0.06,1)
	1.50	(5.90,4.24,5)	(5.83,4.18,5)	(5.59,3.96,5)	(5.24,3.64,4)	(4.81,3.27,4)	(4.35,2.88,4)	(3.31,2.02,3)	(2.54,1.43,2)	(2.01,1.05,2)	(1.65,0.80,1)	(1.25,0.47,1)	(1.01,0.12,1)
	1.75	(3.31,2.12,3)	(3.29,2.10,3)	(3.23,2.06,3)	(3.15,1.98,3)	(3.03,1.89,3)	(2.89,1.79,2)	(2.51,1.49,2)	(2.14,1.21,2)	(1.83,0.97,2)	(1.58,0.77,1)	(1.26,0.50,1)	(1.03,0.16,1)
2.00	(2.33,1.38,2)	(2.32,1.38,2)	(2.30,1.36,2)	(2.27,1.34,2)	(2.23,1.30,2)	(2.17,1.26,2)	(2.00,1.13,2)	(1.82,0.98,2)	(1.64,0.84,1)	(1.48,0.71,1)	(1.24,0.49,1)	(1.04,0.19,1)	
SS-3T	0.25	(4.12,0.33,4)	(4.11,0.33,4)	(4.09,0.32,4)	(4.01,0.36,4)	(3.74,0.50,4)	(3.28,0.46,3)	(2.65,0.48,3)	(2.02,0.12,2)	(1.92,0.27,2)	(1.25,0.43,1)	(1.00,0.00,1)	(1.00,0.00,1)

		δ											
Chart	θ	0.00	0.10	0.20	0.30	0.40	0.50	0.75	1.00	1.25	1.5	2.00	3.00
Max-3T	0.25	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.40,0.50,3)	(3.39,0.49,3)	(2.80,0.42,3)	(2.00,0.14,2)	(1.52,0.50,2)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.77,2.09,7)	(7.71,2.09,7)	(7.52,2.09,7)	(7.06,2.08,7)	(6.20,1.92,6)	(5.19,1.55,5)	(3.47,0.86,3)	(2.52,0.61,2)	(1.96,0.44,2)	(1.56,0.50,2)	(1.02,0.15,1)	(1.00,0.00,1)
	0.75	(83.52,76.60,60)	(77.42,70.86,56)	(58.35,52.57,42)	(33.22,28.26,25)	(17.02,12.54,13)	(9.96,5.91,8)	(4.66,1.82,4)	(2.99,1.04,3)	(2.16,0.72,2)	(1.67,0.58,2)	(1.12,0.33,1)	(1.00,0.00,1)
	0.90	(560.60,554.38,390)	(346.86,342.07,242)	(122.98,118.66,86)	(43.58,39.16,32)	(19.41,15.31,15)	(11.00,7.24,9)	(4.92,2.25,5)	(3.09,1.26,3)	(2.21,0.85,2)	(1.71,0.65,2)	(1.17,0.38,1)	(1.00,0.02,1)
	1.00	(250.35,247.44,174)	(167.98,164.71,117)	(73.96,70.37,52)	(32.62,29.06,24)	(16.87,13.41,13)	(10.31,7.07,8)	(4.83,2.44,4)	(3.07,1.37,3)	(2.21,0.92,2)	(1.71,0.69,2)	(1.20,0.40,1)	(1.00,0.03,1)
	1.10	(63.33,60.75,45)	(53.06,50.48,38)	(34.49,31.73,25)	(20.78,17.96,15)	(13.06,10.27,10)	(8.86,6.23,7)	(4.57,2.49,4)	(2.98,1.43,3)	(2.18,0.97,2)	(1.71,0.72,2)	(1.21,0.42,1)	(1.00,0.05,1)
	1.25	(15.67,13.42,12)	(14.84,12.60,11)	(12.71,10.52,10)	(10.26,8.13,8)	(8.10,6.05,6)	(6.43,4.48,5)	(3.96,2.31,3)	(2.77,1.43,3)	(2.10,1.00,2)	(1.68,0.75,2)	(1.23,0.44,1)	(1.01,0.08,1)
	1.50	(5.19,3.59,4)	(5.12,3.53,4)	(4.90,3.34,4)	(4.57,3.07,4)	(4.20,2.76,4)	(3.83,2.44,3)	(2.96,1.74,3)	(2.34,1.27,2)	(1.90,0.95,2)	(1.60,0.74,1)	(1.24,0.46,1)	(1.01,0.12,1)
	1.75	(3.03,1.86,3)	(3.01,1.84,3)	(2.96,1.80,3)	(2.87,1.73,2)	(2.76,1.65,2)	(2.63,1.55,2)	(2.29,1.29,2)	(1.97,1.05,2)	(1.71,0.85,2)	(1.50,0.70,1)	(1.22,0.46,1)	(1.02,0.15,1)
	2.00	(2.19,1.24,2)	(2.18,1.23,2)	(2.16,1.22,2)	(2.13,1.19,2)	(2.09,1.16,2)	(2.03,1.12,2)	(1.87,1.00,2)	(1.70,0.87,1)	(1.54,0.74,1)	(1.40,0.63,1)	(1.20,0.44,1)	(1.03,0.17,1)
SS-4T	0.25	(3.37,0.49,3)	(3.36,0.49,3)	(3.32,0.48,3)	(3.26,0.45,3)	(3.13,0.40,3)	(2.92,0.40,3)	(2.23,0.42,2)	(2.00,0.08,2)	(1.77,0.42,2)	(1.13,0.33,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(7.07,2.14,7)	(7.02,2.14,7)	(6.85,2.13,7)	(6.45,2.09,6)	(5.73,1.90,5)	(4.87,1.54,5)	(3.34,0.88,3)	(2.46,0.61,2)	(1.93,0.46,2)	(1.53,0.50,2)	(1.02,0.15,1)	(1.00,0.00,1)
	0.75	(66.95,60.84,48)	(62.60,56.63,45)	(48.87,43.54,36)	(29.71,24.99,22)	(16.12,11.83,13)	(9.68,5.74,8)	(4.62,1.81,4)	(2.98,1.04,3)	(2.15,0.72,2)	(1.67,0.58,2)	(1.12,0.33,1)	(1.00,0.00,1)
	0.90	(421.97,417.17,294)	(283.26,279.59,198)	(111.77,107.61,79)	(41.86,37.57,30)	(19.08,15.04,15)	(10.90,7.18,9)	(4.91,2.26,5)	(3.10,1.26,3)	(2.21,0.85,2)	(1.71,0.65,2)	(1.17,0.38,1)	(1.00,0.02,1)
	1.00	(249.45,246.82,173)	(168.01,164.91,117)	(73.95,70.56,52)	(32.74,29.12,24)	(16.92,13.42,13)	(10.34,7.10,8)	(4.85,2.44,4)	(3.08,1.37,3)	(2.22,0.92,2)	(1.72,0.69,2)	(1.20,0.41,1)	(1.00,0.03,1)
	1.10	(67.19,64.66,47)	(55.84,53.21,40)	(35.80,33.01,26)	(21.30,18.43,16)	(13.30,10.49,10)	(8.99,6.34,7)	(4.60,2.50,4)	(3.00,1.44,3)	(2.19,0.97,2)	(1.72,0.72,2)	(1.22,0.42,1)	(1.00,0.03,1)
	1.25	(16.37,14.13,12)	(15.44,13.20,12)	(13.17,10.99,10)	(10.56,8.42,8)	(8.28,6.23,7)	(6.55,4.57,5)	(4.01,2.35,4)	(2.79,1.44,3)	(2.11,1.00,2)	(1.69,0.75,2)	(1.23,0.45,1)	(1.01,0.08,1)
	1.50	(5.28,3.71,4)	(5.20,3.63,4)	(4.97,3.45,4)	(4.64,3.16,4)	(4.27,2.84,4)	(3.87,2.50,3)	(2.99,1.78,3)	(2.36,1.29,2)	(1.91,0.96,2)	(1.61,0.75,1)	(1.24,0.46,1)	(1.01,0.12,1)
	1.75	(3.05,1.91,3)	(3.03,1.89,3)	(2.97,1.84,3)	(2.88,1.77,2)	(2.77,1.69,2)	(2.64,1.59,2)	(2.29,1.32,2)	(1.98,1.07,2)	(1.71,0.86,2)	(1.50,0.70,1)	(1.23,0.46,1)	(1.02,0.15,1)
	2.00	(2.18,1.26,2)	(2.17,1.26,2)	(2.16,1.24,2)	(2.12,1.22,2)	(2.08,1.18,2)	(2.02,1.14,2)	(1.86,1.01,2)	(1.70,0.88,1)	(1.54,0.75,1)	(1.40,0.64,1)	(1.20,0.44,1)	(1.03,0.17,1)
SS- χ^2	0.25	(2.66,0.65,3)	(2.65,0.65,3)	(2.63,0.65,3)	(2.57,0.64,3)	(2.47,0.61,2)	(2.33,0.56,2)	(2.01,0.40,2)	(1.80,0.41,2)	(1.47,0.50,1)	(1.09,0.29,1)	(1.00,0.00,1)	(1.00,0.00,1)
	0.50	(6.72,2.40,6)	(6.68,2.40,6)	(6.53,2.38,6)	(6.16,2.30,6)	(5.51,2.06,5)	(4.73,1.66,5)	(3.30,0.96,3)	(2.45,0.66,2)	(1.92,0.49,2)	(1.54,0.50,2)	(1.03,0.17,1)	(1.00,0.00,1)
	0.75	(60.88,55.53,44)	(57.24,52.05,41)	(45.72,40.92,33)	(28.86,24.41,21)	(16.01,11.82,12)	(9.74,5.80,8)	(4.69,1.86,4)	(3.02,1.07,3)	(2.19,0.74,2)	(1.70,0.58,2)	(1.14,0.34,1)	(1.00,0.01,1)
	0.90	(321.99,318.10,225)	(235.39,231.57,164)	(105.51,101.53,74)	(41.66,37.44,30)	(19.26,15.17,15)	(11.08,7.28,9)	(5.00,2.29,5)	(3.15,1.28,3)	(2.26,0.87,2)	(1.74,0.65,2)	(1.19,0.39,1)	(1.00,0.02,1)
	1.00	(249.66,246.98,174)	(171.28,168.25,120)	(76.60,73.20,54)	(33.85,30.12,25)	(17.42,13.86,13)	(10.59,7.25,9)	(4.96,2.49,5)	(3.14,1.39,3)	(2.26,0.94,2)	(1.75,0.70,2)	(1.21,0.42,1)	(1.00,0.04,1)
	1.10	(74.92,72.20,53)	(61.77,58.99,44)	(38.60,35.73,28)	(22.51,19.55,17)	(13.83,10.91,11)	(9.28,6.51,8)	(4.71,2.55,4)	(3.07,1.46,3)	(2.23,0.99,2)	(1.75,0.73,2)	(1.23,0.44,1)	(1.00,0.05,1)
	1.25	(17.75,15.37,13)	(16.70,14.37,12)	(14.10,11.80,11)	(11.16,8.96,9)	(8.67,6.51,7)	(6.81,4.75,6)	(4.12,2.39,4)	(2.85,1.47,3)	(2.15,1.02,2)	(1.72,0.76,2)	(1.25,0.45,1)	(1.01,0.08,1)
	1.50	(5.54,3.88,5)	(5.46,3.81,5)	(5.21,3.60,4)	(4.85,3.30,4)	(4.44,2.94,4)	(4.02,2.59,3)	(3.08,1.82,3)	(2.41,1.31,2)	(1.95,0.98,2)	(1.63,0.76,1)	(1.25,0.47,1)	(1.02,0.12,1)
	1.75	(3.16,1.96,3)	(3.14,1.95,3)	(3.08,1.90,3)	(2.98,1.83,3)	(2.86,1.74,2)	(2.73,1.63,2)	(2.36,1.35,2)	(2.02,1.09,2)	(1.74,0.88,2)	(1.53,0.72,1)	(1.24,0.47,1)	(1.03,0.16,1)
	2.00	(2.25,1.30,2)	(2.25,1.29,2)	(2.22,1.28,2)	(2.18,1.25,2)	(2.14,1.21,2)	(2.08,1.17,2)	(1.91,1.04,2)	(1.73,0.90,2)	(1.57,0.77,1)	(1.42,0.65,1)	(1.21,0.45,1)	(1.03,0.18,1)

REFERENCES

- [1] D. C. Montgomery, *Introduction to statistical quality control*. John wiley & sons, 2019.
- [2] W. A. Shewhart, "Quality control charts," *The Bell System Technical Journal*, vol. 5, no. 4, pp. 593-603, 1926, doi: 10.1002/j.1538-7305.1926.tb00125.x.
- [3] S. W. Roberts, "Control Chart Tests Based on Geometric Moving Averages," *Technometrics*, vol. 1, no. 3, pp. 239-250, 1959/08/01 1959, doi: 10.1080/00401706.1959.10489860.
- [4] E. S. Page, "Control charts for the mean of a normal population," *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 16, no. 1, pp. 131-135, 1954.
- [5] R. Mehmood, M. H. Lee, S. Hussain, and M. Riaz, "On efficient construction and evaluation of runs rules-based control chart for known and unknown parameters under different distributions," *Quality and Reliability Engineering International*, vol. 35, no. 2, pp. 582-599, 2019, doi: <https://doi.org/10.1002/qre.2423>.
- [6] N. Abbas, M. Riaz, and R. J. M. M. Does, "Enhancing the performance of EWMA charts," *Quality and Reliability Engineering International*, vol. 27, no. 6, pp. 821-833, 2011, doi: <https://doi.org/10.1002/qre.1175>.
- [7] C. A. Acosta-Mejia, "Two Sets of Runs Rules for the Chart," *Quality Engineering*, vol. 19, no. 2, pp. 129-136, 2007/04/04 2007, doi: 10.1080/17513470701263641.
- [8] M. Klein, "Composite Shewhart-EWMA statistical control schemes," *IIE Transactions*, vol. 28, no. 6, pp. 475-481, 1996/06/01 1996, doi: 10.1080/07408179608966294.
- [9] J. M. Lucas, "Combined Shewhart-CUSUM Quality Control Schemes," *Journal of Quality Technology*, vol. 14, no. 2, pp. 51-59, 1982/04/01 1982, doi: 10.1080/00224065.1982.11978790.
- [10] A. Haq, "Adaptive EWMA and CUSUM charts with dynamic probability control limits," *Communications in Statistics: Simulation and Computation*, Article 2024, doi: 10.1080/03610918.2024.2417819.
- [11] W. Ugaz, I. Sánchez, and A. M. Alonso, "Adaptive EWMA control charts with time-varying smoothing parameter," *International Journal of Advanced Manufacturing Technology*, Article vol. 93, no. 9-12, pp. 3847-3858, 2017, doi: 10.1007/s00170-017-0792-1.
- [12] Z. Wu, J. Jiao, M. Yang, Y. Liu, and Z. Wang, "An enhanced adaptive CUSUM control chart," *IIE Transactions (Institute of Industrial Engineers)*, Article vol. 41, no. 7, pp. 642-653, 2009, doi: 10.1080/07408170802712582.
- [13] V. Alevizakos, K. Chatterjee, and C. Koukouvinos, "On the performance and comparison of various memory-type control charts," *Communications in Statistics: Simulation and Computation*, 2024, doi: 10.1080/03610918.2024.2310692.
- [14] N. Abbas, M. Riaz, and R. J. M. M. Does, "Mixed exponentially weighted moving average-cumulative sum charts for process monitoring," *Quality and Reliability Engineering International*, vol. 29, no. 3, pp. 345-356, 2013/4// 2013, doi: 10.1002/qre.1385.

- [15] J. O. Ajadi, M. Riaz, and K. Al-Ghamdi, "On increasing the sensitivity of mixed EWMA–CUSUM control charts for location parameter," *Journal of Applied Statistics*, vol. 43, no. 7, pp. 1262-1278, 2016/5// 2016, doi: 10.1080/02664763.2015.1094453.
- [16] H. Z. Nazir, N. Abbas, M. Riaz, and R. J. M. M. Does, "A comparative study of memory-type control charts under normal and contaminated normal environments," *Quality and Reliability Engineering International*, vol. 32, no. 4, pp. 1347-1356, 2016/6// 2016, doi: 10.1002/qre.1835.
- [17] M. Noor-Ul-Amin, "Impact of measurement error on mixed EWMA-CUSUM control chart," *Scientia Iranica*, vol. 29, no. 4, pp. 2134-2148, 2022/7// 2022, doi: 10.24200/sci.2020.53453.3244.
- [18] G. S. Rao, M. Aslam, U. Rasheed, and C.-H. Jun, "Mixed EWMA–CUSUM chart for COM-Poisson distribution," *Journal of Statistics and Management Systems*, vol. 23, no. 3, pp. 511-527, 2020/4// 2020, doi: 10.1080/09720510.2019.1639947.
- [19] J. O. Ajadi and M. Riaz, "Mixed multivariate EWMA-CUSUM control charts for an improved process monitoring," *Communications in Statistics - Theory and Methods*, vol. 46, no. 14, pp. 6980-6993, 2017/7// 2017, doi: 10.1080/03610926.2016.1139132.
- [20] D. Tyagi and V. Yadav, "Modified Mixed Exponentially Weighted Moving Average-Cumulative Sum Control Charts for Autocorrelated Process," *Journal of Reliability and Statistical Studies*, vol. 14, no. 2, pp. 471-490, 2021/7// 2021, doi: 10.13052/jrss0974-8024.1425.
- [21] S. Hussain, X. Wang, S. Ahmad, and M. Riaz, "On a class of mixed EWMA-CUSUM median control charts for process monitoring," *Quality and Reliability Engineering International*, vol. 36, no. 3, pp. 910-946, 2020/4// 2020, doi: 10.1002/qre.2608.
- [22] K. Talordphop, S. Sukparungsee, and Y. Areepong, "On designing new mixed modified exponentially weighted moving average - exponentially weighted moving average control chart," *Results in Engineering*, vol. 18, 2023/6// 2023, doi: 10.1016/j.rineng.2023.101152.
- [23] S. M. Anwar, M. Aslam, B. Zaman, and M. Riaz, "Mixed memory control chart based on auxiliary information for simultaneously monitoring of process parameters: An application in glass field," *Computers & Industrial Engineering*, vol. 156, pp. 107284-107284, 2021, doi: 10.1016/j.cie.2021.107284.
- [24] A. Mohammadkhani and A. Amiri, "Developing mixed EWMA-CUSUM and CUSUM-EWMA control charts based on MRSS and DRSS procedures," *Scientia Iranica*, vol. 29, no. 5 E, pp. 2756-2771, 2022/9// 2022, doi: 10.24200/sci.2020.55632.4328.
- [25] S. V. Crowder and M. D. Hamilton, "An EWMA for monitoring a process standard deviation," *Journal of Quality Technology*, vol. 24, no. 1, pp. 12-21, 1992, doi: 10.1080/00224065.1992.11979369.
- [26] P. Castagliola, "A new S2-EWMA control chart for monitoring the process variance," *Quality and Reliability Engineering International*, vol. 21, no. 8, pp. 781-794, 2005/12// 2005, doi: 10.1002/qre.686.

- [27] T. C. Chang and F. F. Gan, "Cumulative sum control chart for monitoring process variance," *Journal of Quality Technology*, vol. 27, no. 2, pp. 109-119, 1995, doi: 10.1080/00224065.1995.11979574.
- [28] P. Castagliola, G. Celano, and S. Fichera, "A new CUSUM-S2 control chart for monitoring the process variance," *Journal of Quality in Maintenance Engineering*, vol. 15, no. 4, pp. 344-357, 2009/9// 2009, doi: 10.1108/13552510910997724.
- [29] P. Castagliola, G. Celano, and S. Fichera, "A Johnson's type transformation EWMA-S 2 control chart," *Int. J. Quality Engineering and Technology*, vol. 1, pp. 253-275, 2010, doi: 10.1504/IJQET.2010.034611.
- [30] N. Abbas, M. Riaz, and R. J. M. M. Does, "CS-EWMA chart for monitoring process dispersion," *Quality and Reliability Engineering International*, vol. 29, no. 5, pp. 653-663, 2013/7// 2013, doi: 10.1002/qre.1414.
- [31] B. Zaman, N. Abbas, M. Riaz, and M. H. Lee, "Mixed CUSUM-EWMA chart for monitoring process dispersion," *International Journal of Advanced Manufacturing Technology*, vol. 86, no. 9-12, pp. 3025-3039, 2016/10// 2016, doi: 10.1007/s00170-016-8411-0.
- [32] N. Abbas, M. Riaz, and R. J. M. M. Does, "Memory-type control charts for monitoring the process dispersion," *Quality and Reliability Engineering International*, vol. 30, no. 5, pp. 623-632, 2014, doi: 10.1002/qre.1514.
- [33] W. Munir and A. Haq, "New cumulative sum control charts for monitoring process variability," *Journal of Statistical Computation and Simulation*, vol. 87, no. 15, pp. 2882-2899, 2017/10// 2017, doi: 10.1080/00949655.2017.1346102.
- [34] B. Zaman, "Adaptive CUSUM control charts for efficient monitoring of process dispersion," *Quality and Reliability Engineering International*, vol. 38, no. 5, pp. 2273-2302, 2022/7// 2022, doi: 10.1002/qre.3075.
- [35] S. Tariq, M. Noor-ul-Amin, M. Aslam, and M. Hanif, "Design of hybrid EWMA-S 2 control chart," *Journal of Industrial and Production Engineering*, vol. 36, no. 8, pp. 554-562, 2019, doi: 10.1080/21681015.2019.1702111.
- [36] K. Chatterjee, C. Koukouvinos, and A. Lappa, "A new S 2-TEWMA control chart for monitoring process dispersion," *Quality and Reliability Engineering International*, vol. 37, no. 4, pp. 1334-1354, 2021, doi: <https://doi.org/10.1002/qre.2798>.
- [37] V. Alevizakos, K. Chatterjee, and C. Koukouvinos, "The quadruple exponentially weighted moving average control chart," *Quality Technology & Quantitative Management*, vol. 19, no. 1, pp. 50-73, 2022, doi: 10.1080/16843703.2021.1989141.
- [38] V. Alevizakos, K. Chatterjee, C. Koukouvinos, and A. Lappa, "A double generally weighted moving average control chart for monitoring the process variability," *Journal of Applied Statistics*, vol. 50, no. 10, pp. 2079-2107, 2023/7// 2023, doi: 10.1080/02664763.2022.2064977.
- [39] E. M. White and R. Schroeder, "A Simultaneous Control Chart," *Journal of Quality Technology*, vol. 19, no. 1, pp. 1-10, 1987/01/01 1987, doi: 10.1080/00224065.1987.11979028.
- [40] M. T. Chao and S. W. Cheng, "Semicircle Control Chart for Variables Data," *Quality Engineering*, vol. 8, no. 3, pp. 441-446, 1996/03/01 1996, doi: 10.1080/08982119608904646.

- [41] G. Chen and S. W. Cheng, "MAX CHART: COMBINING X-BAR CHART AND S CHART," *Statistica Sinica*, vol. 8, no. 1, pp. 263-271, 1998. [Online]. Available: <http://www.jstor.org/stable/24306354>.
- [42] G. Chen, S. W. Cheng, and H. Xie, "Monitoring Process Mean and Variability with One EWMA Chart," *Journal of Quality Technology*, vol. 33, no. 2, pp. 223-233, 2001/04/01 2001, doi: 10.1080/00224065.2001.11980069.
- [43] H. Xie, "Contributions to Qualimetry," Ph.D. Thesis, University of Manitoba, 1999.
- [44] G. Chen, S. W. Cheng, and H. Xie, "A New EWMA Control Chart for Monitoring Both Location and Dispersion," *Quality Technology & Quantitative Management*, vol. 1, no. 2, pp. 217-231, 2004/01/01 2004, doi: 10.1080/16843703.2004.11673074.
- [45] M. B. C. Khoo, S. Y. Teh, and Z. Wu, "Monitoring Process Mean and Variability with One Double EWMA Chart," *Communications in Statistics - Theory and Methods*, vol. 39, no. 20, pp. 3678-3694, 2010/10/14 2010, doi: 10.1080/03610920903324866.
- [46] K. Chatterjee, C. Koukouvinos, and A. Lappa, "Monitoring process mean and variability with one triple EWMA chart," *Communications in Statistics - Simulation and Computation*, vol. 53, no. 2, pp. 611-641, 2024/02/01 2024, doi: 10.1080/03610918.2022.2025835.
- [47] R. F. Zafar, T. Mahmood, N. Abbas, M. Riaz, and Z. Hussain, "A progressive approach to joint monitoring of process parameters," *Computers & Industrial Engineering*, vol. 115, pp. 253-268, 2018/01/01/ 2018, doi: <https://doi.org/10.1016/j.cie.2017.11.015>.
- [48] J. O. Ajadi and M. Riaz, "NEW MEMORY-TYPE CONTROL CHARTS FOR MONITORING PROCESS MEAN AND DISPERSION," *Scientia Iranica*, vol. 24, pp. 3423-3438, 2017, doi: 10.24200/sci.2017.4483.
- [49] D. M. Hawkins, "CUMULATIVE SUM CONTROL CHARTING: AN UNDERUTILIZED SPC TOOL," *Quality Engineering*, vol. 5, no. 3, pp. 463-477, 1993/01/01 1993, doi: 10.1080/08982119308918986.
- [50] Z. Wu and Y. Tian, "Weighted-loss-function CUSUM chart for monitoring mean and variance of a production process," *International Journal of Production Research*, vol. 43, no. 14, pp. 3027-3044, 2005/07/15 2005, doi: 10.1080/00207540500057639.
- [51] Z. Wu and Q. Wang, "A single CUSUM chart using a single observation to monitor a variable," *International Journal of Production Research*, vol. 45, no. 3, pp. 719-741, 2007/02/01 2007, doi: 10.1080/00207540600792267.
- [52] S. W. Cheng and K. Thaga, "The Max-CUSUM Chart," in *Frontiers in Statistical Quality Control 9*, H.-J. Lenz, P.-T. Wilrich, and W. Schmid Eds. Heidelberg: Physica-Verlag HD, 2010, pp. 85-98.
- [53] K. Thaga, "SS-CUSUM Chart," vol. 24, no. 1, pp. 117-128, 2009, doi: 10.1515/EQC.2009.117.
- [54] A. Haq and F. Razzaq, "Maximum weighted adaptive CUSUM charts for simultaneous monitoring of process mean and variance," *Journal of Statistical Computation and Simulation*, vol. 90, no. 16, pp. 2949-2974, 2020/11/01 2020, doi: 10.1080/00949655.2020.1793154.

- [55] A. Haq and Q. Ali, "A maximum dual CUSUM chart for joint monitoring of process mean and variance," *Quality Technology & Quantitative Management*, vol. 21, no. 3, pp. 287-308, 2024/05/03 2024, doi: 10.1080/16843703.2023.2193769.
- [56] S. Chowdhury, A. Mukherjee, and S. Chakraborti, "Distribution-free Phase II CUSUM Control Chart for Joint Monitoring of Location and Scale," *Quality and Reliability Engineering International*, vol. 31, no. 1, pp. 135-151, 2015, doi: <https://doi.org/10.1002/qre.1677>.
- [57] Z. L. Chong, A. Mukherjee, and M. B. C. Khoo, "Distribution-free Shewhart-Lepage type premier control schemes for simultaneous monitoring of location and scale," *Computers & Industrial Engineering*, vol. 104, pp. 201-215, 2017/02/01/ 2017, doi: <https://doi.org/10.1016/j.cie.2016.12.004>.
- [58] Z. L. Chong, A. Mukherjee, and M. B. C. Khoo, "Some distribution-free Lepage-type schemes for simultaneous monitoring of one-sided shifts in location and scale," *Computers & Industrial Engineering*, vol. 115, pp. 653-669, 2018/01/01/ 2018, doi: <https://doi.org/10.1016/j.cie.2017.11.029>.
- [59] Z. Song, A. Mukherjee, and J. Zhang, "An efficient approach of designing distribution-free exponentially weighted moving average schemes with dynamic fast initial response for joint monitoring of location and scale," *Journal of Statistical Computation and Simulation*, vol. 90, no. 13, pp. 2329-2353, 2020/09/01 2020, doi: 10.1080/00949655.2020.1776711.
- [60] T. Zhang, Z. He, X. Zhao, and L. Qu, "Joint monitoring of post-sales online review processes based on a distribution-free EWMA scheme," *Computers & Industrial Engineering*, vol. 158, p. 107372, 2021/08/01/ 2021, doi: <https://doi.org/10.1016/j.cie.2021.107372>.
- [61] S. R. Shinde, S. B. Mahadik, and A. C. Rakitzis, "New Lepage-type control charts for joint monitoring of location and scale," *Computers & Industrial Engineering*, vol. 197, p. 110614, 2024/11/01/ 2024, doi: <https://doi.org/10.1016/j.cie.2024.110614>.
- [62] Z. Jalilibal, A. Amiri, and M. B. C. Khoo, "A literature review on joint control schemes in statistical process monitoring," *Quality and Reliability Engineering International*, vol. 38, no. 6, pp. 3270-3289, 2022, doi: <https://doi.org/10.1002/qre.3114>.
- [63] M. Mansouri, M. F. Harkat, M. Nounou, and H. Nounou, "Midpoint-radii principal component analysis -based EWMA and application to air quality monitoring network," *Chemometrics Intellig. Lab. Syst.*, vol. 175, pp. 55-64, 2018/04/15/ 2018, doi: <https://doi.org/10.1016/j.chemolab.2018.01.016>.
- [64] S. V. Crowder, "Design of exponentially weighted moving average schemes," *Journal of Quality technology*, vol. 21, no. 3, pp. 155-162, 1989.
- [65] E. S. Page, "Control Charts for the Mean of a Normal Population," *Journal of the Royal Statistical Society: Series B (Methodological)*, vol. 16, no. 1, pp. 131-135, 1954/01/01 1954, doi: <https://doi.org/10.1111/j.2517-6161.1954.tb00154.x>.
- [66] X. Qin, J. Zhang, A. Mukherjee, D. Yu, and L. Liu, "Comparisons of some new and existing EWMA schemes for binomial count processes with applications to monitoring hospital No-Shows," *Quality Technology & Quantitative Management*, pp. 1-23, 2024, doi: <https://doi.org/10.1080/16843703.2024.2411120>.

- [67] Y. Tan, A. Mukherjee, and J. Zhang, "Two Bayesian approaches of monitoring mean of Gaussian process using Bayes factor," *Qual. Reliab. Eng. Int.*, vol. 40, no. 6, pp. 3320-3339, 2024/10/01 2024, doi: <https://doi.org/10.1002/qre.3567>.
- [68] V. Alevizakos, K. Chatterjee, and C. Koukouvinos, "On the performance and comparison of various memory-type control charts," *Communications in Statistics - Simulation and Computation*, pp. 1-21, 2024, doi: <https://doi.org/10.1080/03610918.2024.2310692>.
- [69] F. Marini, "Culture Of Mysid Shrimp And Bivalve Trochphores (veligers)," (in English), *Reefs*, 2003, doi: <https://reefs.com/magazine/the-breeder-s-net-culture-of-mysid-shrimp-and-bivalve-trochphores-veligers>.
- [70] G. Marsaglia and J. Marsaglia, "Evaluating the Anderson-Darling Distribution," *Journal of Statistical Software*, vol. 9, no. 2, pp. 1 - 5, 02/25 2004, doi: <https://doi.org/10.18637/jss.v009.i02>.
- [71] M. Verbeek, *A Guide to Modern Econometrics, 5th Edition*. Hoboken, NJ, USA: Wiley (in English), 2017.
- [72] N. Abbas, "On efficient change point detection using a step cumulative sum control chart," *Quality Engineering*, 2023, doi: 10.1080/08982112.2023.2193896.
- [73] D. C. Montgomery, *Introduction to Statistical Quality Control*. Wiley, 2020.
- [74] Z. Rasheed, H. Zhang, M. Khan, and S. M. Anwar, "Modified exponentially weighted moving average control chart for monitoring process dispersion," *Communications in Statistics: Simulation and Computation*, 2024, doi: 10.1080/03610918.2024.2358137.
- [75] S.-F. Yang and B. C. Arnold, "Monitoring Process Variance Using an ARL-unbiased EWMA-p Control Chart," *Quality and Reliability Engineering International*, vol. 32, no. 3, pp. 1227-1235, 2016/4// 2016, doi: <https://doi.org/10.1002/qre.1829>.
- [76] N. Khan, M. Aslam, and M. Albassam, "Design of Control Charts Using Repetitive Sampling: A Comparative Study of Conditional Expected Delay," *Journal of Reliability and Statistical Studies*, pp. 223-240, 2024/8// 2024, doi: 10.13052/jrss0974-8024.1719.
- [77] S. M. Anwar, M. Aslam, B. Zaman, and M. Riaz, "Mixed memory control chart based on auxiliary information for simultaneously monitoring of process parameters: An application in glass field," *Computers & Industrial Engineering*, vol. 156, p. 107284, 2021/06/01/ 2021, doi: 10.1016/j.cie.2021.107284.
- [78] D. Brook and D. A. Evans, "An Approach to the Probability Distribution of Cusum Run Length," *Biometrika*, vol. 59, no. 3, pp. 539-549, 1972, doi: 10.2307/2334805.
- [79] M. C. Fu and J.-Q. Hu, "Efficient Design and Sensitivity Analysis of Control Charts Using Monte Carlo Simulation," *Management Science*, vol. 45, no. 3, pp. 395-413, 1999/3// 1999, doi: 10.1287/mnsc.45.3.395.
- [80] P. Tüfekci, "Prediction of full load electrical power output of a base load operated combined cycle power plant using machine learning methods," *International Journal of Electrical Power & Energy Systems*, vol. 60, pp. 126-140, 2014, doi: 10.1016/j.ijepes.2014.02.027.
- [81] M. A. Raza, F. Tariq, A. A. Zaagan, G. M. Engmann, A. M. Mahnashi, and M. Z. Meetei, "A nonparametric mixed exponentially weighted moving average-moving average control chart with an application to gas turbines," *PLOS ONE*, vol. 19, no.

- 8, p. e0307559, 2024/8// 2024. [Online]. Available:
<https://doi.org/10.1371/journal.pone.0307559>.
- [82] Z. Wu, M. Yang, M. B. C. Khoo, and F.-J. Yu, "Optimization designs and performance comparison of two CUSUM schemes for monitoring process shifts in mean and variance," *European Journal of Operational Research*, vol. 205, no. 1, pp. 136-150, 2010/08/16/ 2010, doi: <https://doi.org/10.1016/j.ejor.2009.12.005>.

VITAE

Name : Usman Ibrahim

Nationality : Nigerian

Date of Birth : 10/06/1994

Email : uibrahim474@gmail.com

Address : 1 Abacha Road, Gombe, Gombe State, Nigeria. Postal Code 760251.

Academic Background : BSc. Statistics, Gombe State University, Gombe, 2019

Research :

- New Step-CUSUM Chart for Monitoring Dispersion: An Application in Combined Cycle Power Plan. (Research article under review)
- Smart Monitoring Solutions: MEC Chart for Real-Time Water pH Regulation in Aquatic Ecotoxicology. (Research article under review)
- Joint Monitoring of Process Parameters Using Conditional Aggregate Mean. (Research article under review)
- Term Paper 2022: Community Mobility and Economic Condition in the Era of Post-COVID
- Undergraduate Project: Time Series Analysis on temperature in Gombe metropolis (2019).