

**ON ENHANCED STATISTICAL GRAPHICAL
TESTS FOR DESIGN OF EXPERIMENTS**

BY

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This thesis is dedicated to my cherished parents, siblings, and friends for their unwavering love, steadfast support, and continual encouragement throughout every phase of my journey.

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LIST OF ABBREVIATIONS

DOE	Design of experiments
ANOM	Analysis of means
N	Total number of observations
τ_i	The fixed effect of i^{th} treatment level
CRD	Completely randomized design
LDL	Lower decision limit
UDL	Upper decision limit
ANOMC	Analysis of Means using Covariate
g	Sampling fraction
s	Skewness
kr	kurtosis
ρ	Correlation
n	Sample size
v	Variance ratios
δ	Effect size

t	Number of treatments
$B_2(x_{i.})$	the i^{th} coefficient of kurtosis
$C_{Y_{i.}}, C_{X_{i.}}$	the i^{th} sample coefficients of variation of Y and X
$r_{i.}$	the i^{th} sample correlation
$k_{i.}$	The i^{th} unbiasing constant
B	The slope of the fixed effect model
B^*	The slope between the covariate variables in fixed effect model
MVN	Multivariate normal
MVLN	Multivariate log normal
MVT	Multivariate t
Σ	covariance between variables
H_0	Null hypothesis
H_1	Alternative hypothesis
A_i	The effect of factor A at the i^{th} level
B_j	The effect of factor B at the j^{th} level
$(AB)_{ij}$	the effect of the interaction between A_i and B_j

f	Number of levels for the factor of interest
$k_{ij.}$	The unbiasing constant
$r_{ij.}$	The sample correlation for each (i^{th}, j^{th}) level
$S_{ij.}$	sample standard deviation of the mean estimator for each (i^{th}, j^{th}) level
$C_{Y_{ij.}}, C_{X_{ij.}}$	Sample coefficients of variation of Y and X for each (i^{th}, j^{th}) level
Z_{ij}	The j^{th} observation of covariate for the i^{th} treatment level
X_{ij}	the j^{th} observation of covariate for the i^{th} treatment level
$B_2(x_{i.})$	the i^{th} coefficient of kurtosis

ABSTRACT

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In numerous experiments, our focus is on assessing the statistical significance of means relative to the overall mean of the study variable, which is referred to as Analysis of Means (ANOM). Additional linearly related covariates are occasionally recorded alongside the primary variable data. ANOM is designed to test the independent of treatment means for the primary variable alone, without taking into account the covariates. However, a novel test called Analysis of Means using Covariate (ANOMC) has recently been introduced to address the need to test the independent of treatment means for the primary variable while considering the influence of covariates. In this study, we have introduced and proposed the ANOMC methodology for two distinct scenarios: single fixed factor design and factorial design. In the case of a single fixed factor design, during phase I, ANOMC was developed to address the issue of positively skewed distributions by employing a series of ratio family estimators. However, further investigation is necessary to explore the capabilities of ANOMC in this context. Consequently, during phase II, ANOMC was expanded to incorporate an additional covariate variable using a family of ratio estimators. In the case of a factorial design, the ANOMC test was developed using regression techniques and a selection of ratio estimators. The performance of the proposed methods in both cases is assessed and compared using various informative metrics, including the percentage of type I error and power. Monte Carlo simulation techniques are employed to obtain reliable estimates of these measures. In addition, we have used practical datasets from

mechanical/industrial engineering and biomedical and implemented our proposed techniques to show their application in real world/ life situations.

ملخص الرسالة

الاسم الكامل: هاجر السبيعي

عنوان الرسالة: تحسين الاختبارات الإحصائية الرسومية لتصميم التجارب

التخصص: الإحصاء التطبيقي

تاريخ الدرجة العلمية: ابريل 2024

في العديد من التجارب، ينصب تركيزنا على تقييم الأهمية الإحصائية للوسائل بالنسبة إلى المتوسط العام لمتغير الدراسة، والذي يشار إليه باسم تحليل الوسائل (ANOM). يتم تسجيل متغيرات مشتركة إضافية مرتبطة خطيا في بعض الأحيان جنبا إلى جنب مع بيانات المتغير الأساسي. تم تصميم ANOM لاختبار استقلالية وسائل العلاج للمتغير الأساسي وحده، دون مراعاة المتغيرات المشتركة. ومع ذلك، تم مؤخرا تقديم اختبار جديد يسمى تحليل الوسائل باستخدام المتغير المشترك (ANOMC) لتلبية الحاجة إلى اختبار استقلالية وسائل العلاج للمتغير الأساسي مع مراعاة تأثير المتغيرات المشتركة. في هذه الدراسة، قدمنا واقترحنا منهجية ANOMC لسيناريوين منفصلين: تصميم عامل ثابت واحد وتصميم عاملي. في حالة تصميم عامل ثابت واحد، خلال المرحلة الأولى، تم تطوير ANOMC لمعالجة مسألة التوزيعات المنحرفة بشكل إيجابي من خلال استخدام سلسلة من مقدرات عائلة النسبة. ومع ذلك، من الضروري إجراء مزيد من التحقيقات لاستكشاف قدرات اختبار ANOMC في هذا السياق. ونتيجة لذلك، وخلال المرحلة الثانية، تم توسيع نطاق الشبكة لتشمل متغيرا مشتركا إضافيا باستخدام عائلة من مقدرات النسبة. في حالة التصميم العاملي، تم تطوير اختبار ANOMC باستخدام تقنيات الانحدار ومجموعة مختارة من مقدرات النسبة. يتم تقييم أداء الطرق المقترحة في كلتا الحالتين ومقارنتها باستخدام مقاييس مختلفة، بما في ذلك النسبة المئوية للخطأ من النوع الأول والقوة. يتم استخدام تقنية محاكاة مونت كارلو للحصول على تقديرات موثوقة لهذه القياسات. بالإضافة إلى ذلك، استخدمنا مجموعات بيانات عملية من الهندسة الميكانيكية / الصناعية والطبية الحيوية ونفذنا تقنياتنا المقترحة لإظهار تطبيقها في الواقع.

CHAPTER 1

INTRODUCTION

In this chapter, we will cast some light on the principles of the Design of Experiments (DOE) and specifically examine the statistical technique called Analysis of Means (ANOM) and its underlying concept. The structures of hypothesis testing are also discussed in the following sections. Furthermore, we will provide a summary of the main components that make up the thesis.

1.1 Design of Experiments (DOE)

Envision a cost-effective and impactful methodology known as Design of Experiments (DOE), which empowers you to identify crucial product/process parameters and establish a functional correlation between them and the desired performance measure. This technique allows for fine-tuning the parameter settings to bring the performance measure precisely to the target and minimize any variability around that target. By harnessing the potential of DOE, organizations can greatly enhance product quality while simultaneously lowering production costs [1]. This approach originated in the early 1920s at the Rothamsted Agricultural Station in England under the guidance of Sir R. A. Fisher. Since

its inception, DOE has undergone remarkable advancements, particularly in the design and data analysis aspects. These developments have further enhanced the method's efficacy in improving quality and fostering innovation; it stands as a powerful technique to minimize performance measure variability and achieve targeted levels in a highly effective and efficient manner. As schematically represented in Figure 1.1 [2], a process is a set of connected activities in which inputs are transformed into outputs for a specific purpose [2]. Some of the process's parameters $(X_1, X_2, \dots, X_k)$ are controllable, whereas others $(Z_1, Z_2, \dots, Z_k)$ are uncontrollable (although they may be controllable during experimentation). There is no direct control over the performance measure (response variable) Y ; in the classical cause and effect approach, it is the effect. In the world of products and their quality, DOE involves deliberately manipulating the parameters that are known or believed to influence the performance of a studied process. This methodology significantly departs from the older "scientific" tradition of altering one parameter at a time. Instead, it focuses on simultaneously examining multiple parameters to assess their interaction effects cost-effectively. (for a brief discussion, one may see [3-5]).

1.1.1 Completely Randomized Design (CRD)

Completely Randomized Design (CRD) is a research approach where treatments are randomly assigned to experimental units without deliberate bias or systematic pattern. This methodology involves connecting the fundamental principles of a particular response variable with various necessary levels, while the assignment of factors and their levels is done through a random process [6].

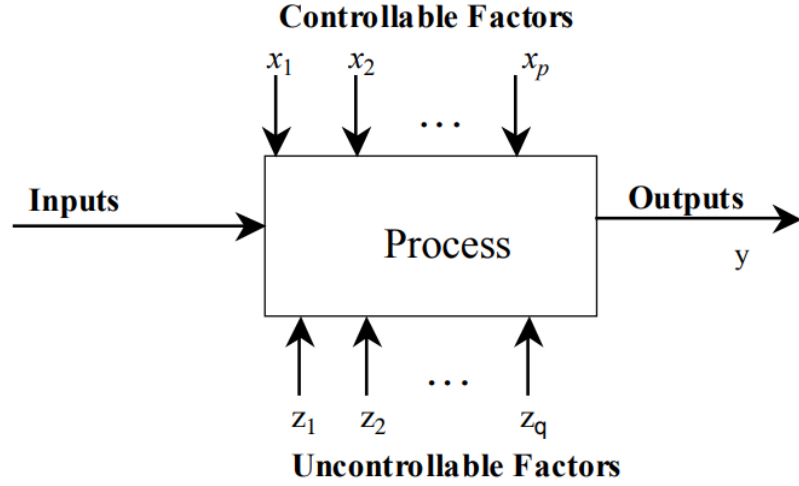


Figure 1. 1: The general DOE framework.

The design strategy of CRD revolves around three essential factors: f , which represents the number of factors; t , which denotes the number of possible levels for each factor; and n , which indicates the number of replications or experimental units. CRD effectively addresses the inherent variability among experimental units by randomly assigning treatments, thereby mitigating potential biases. Consequently, CRD has become an indispensable tool in a wide range of domains, including agriculture, medicine, industrial engineering, and quality control analysis. CRD is especially advantageous in scenarios with limited control over external variables. By harnessing its inherent randomness, CRD effectively neutralizes potential confounding factors. This means that each experimental unit has an equal chance of being assigned any particular treatment, thus establishing a fair and unbiased experimental setup. The random allocation of treatments plays a crucial role in eliminating systematic bias and enhancing the validity of experimental findings.

I. CRD One Factor Design:

Let us consider a single factor completely randomized design (CRD) having t treatments, each with n_i observations while the total number of observations is $N = \sum_{i=1}^t n_i$. The corresponding fixed effects model can be expressed as follows:

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, i = 1, 2, 3, \dots, t, j = 1, 2, 3, \dots, n_i \quad (1.1)$$

where Y_{ij} is the j^{th} observation of response variable for the i^{th} treatment level, $\tau_i = \mu_i - \mu$ is the fixed effect of i^{th} treatment level with respect to the overall mean μ , and we assumed that the random errors $\epsilon_{ij} \stackrel{i.i.d.}{\sim} N(\mu, \sigma^2)$.

- Data Model without Covariate

Table 1.1 illustrates a One-Factor CRD without a covariate. It displays the treatments ($\tau_1, \tau_2, \dots$) and their respective response values (Y_{ij}) for each experimental unit (R_{nj}).

Table 1. 1: General layout for a one-factor CRD design without Covariate.

Terms	τ_1	τ_2	τ_3	...	τ_i	...	τ_t
R_1	Y_{11}	Y_{21}	Y_{31}	...	Y_{i1}	...	Y_{t1}
R_2	Y_{12}	Y_{22}	Y_{32}	...	Y_{i2}	...	Y_{t2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_j	Y_{1j}	Y_{2j}	Y_{3j}	...	Y_{ij}	...	Y_{tj}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_{n_i}	Y_{1n_1}	Y_{2n_2}	Y_{3n_3}	...	Y_{in_i}	...	Y_{tn_t}

- Data Model with Covariate

Table 1.2 represents a general layout for a One-Factor CRD with a covariate (X). The terms ($\tau_1, \tau_2, \dots$) denote the different treatments being studied. The rows R_{nj} correspond to the different experimental units or observations in the study.

Table 1. 2: General layout for a one-factor CRD design with covariate (X).

Terms	τ_1	τ_2	τ_3	...	τ_t	...	τ_t
R_1	$Y_{11}(X_{11})$	$Y_{21}(X_{21})$	$Y_{31}(X_{31})$	...	$Y_{i1}(X_{i1})$	...	$Y_{t1}(X_{t1})$
R_2	$Y_{12}(X_{12})$	$Y_{22}(X_{22})$	$Y_{32}(X_{32})$	...	$Y_{i2}(X_{i2})$	...	$Y_{t2}(X_{t2})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_j	$Y_{1j}(X_{1j})$	$Y_{2j}(X_{2j})$	$Y_{3j}(X_{3j})$	...	$Y_{ij}(X_{ij})$	...	$Y_{tj}(X_{tj})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_{n_i}	$Y_{1n_i}(X_{1n_i})$	$Y_{2n_i}(X_{2n_i})$	$Y_{3n_i}(X_{3n_i})$	...	$Y_{in_i}(X_{in_i})$	...	$Y_{tn_i}(X_{tn_i})$

II. CRD Multi-Factor Design:

Let us consider two fixed factors (say A and B) completely randomized design (CRD) having t and k treatments, each with n_i observations while the total number of observations is $N = ntk$. The corresponding fixed effects model can be expressed as follows:

$$y_{ijl} = \mu + A_i + B_j + (AB)_{ij} + \epsilon_{ijl}, i = 1, 2, \dots, t; j = 1, 2, \dots, k; l = 1, 2, \dots, n, \quad (1.2)$$

where Y_{ij} is the observed response when factor A is at the i^{th} level and factor B at the j^{th} level for the l^{th} replicate, μ is the overall mean effect, A_i is the effect of the i^{th} level of the column factor A, B_j is the effect of the j^{th} level of the row factor B, $(AB)_{ij}$ is the effect of the interaction between A_i and B_j and we assumed that the random errors $\epsilon_{ij} \stackrel{i.i.d.}{\sim} N(\mu, \sigma^2)$. Furthermore, the treatment effects are defined as deviations from the overall mean, so $\sum_{i=1}^a A_i = 0$ and $\sum_{j=1}^b B_j = 0$. Similarly, the interaction effects are fixed and are defined such that $\sum_{i=1}^a (AB)_{ij} = \sum_{j=1}^b (AB)_{ij} = 0$.

- Data Model without covariate

Table 1.3 represents a Two-Factor Factorial CRD. The Table showcases the two factors, Factor A and Factor B, and their respective levels ($a_1, a_2, \dots$) for Factor A, and ($b_1, b_2, \dots$) for Factor B. The subscripts indicate the specific level of each factor.

Table 1. 3: General layout for a two-factor factorial CRD design without covariate.

		Factor A						
		$k = 1, 2, 3, \dots, n$						
		$i = 1, 2, 3, \dots, t$						
		$j = 1, 2, 3, \dots, r$						
Factor B	b_1	a_1	a_2	a_3	...	a_t	...	a_r
		Y_{111}	Y_{211}	Y_{311}		Y_{t11}		Y_{r11}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		Y_{11k}	Y_{21k}	Y_{31k}	...	Y_{t1k}	...	Y_{r1k}
	b_j	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		Y_{11n}	Y_{21n}	Y_{31n}		Y_{t1n}		Y_{r1n}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	Y_{1j1}	Y_{2j1}	Y_{3j1}		Y_{tj1}		Y_{rj1}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		Y_{1jk}	Y_{2jk}	Y_{3jk}	...	Y_{tjk}	...	Y_{rjk}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	Y_{1jn}	Y_{2jn}	Y_{3jn}		Y_{tjn}		Y_{rjn}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	Y_{1r1}	Y_{2r1}	Y_{3r1}		Y_{tr1}		Y_{rr1}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		Y_{1rk}	Y_{2rk}	Y_{3rk}	...	Y_{trk}	...	Y_{rrk}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	Y_{1rn}	Y_{2rn}	Y_{3rn}		Y_{trn}		Y_{rrn}
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$

- **Data Model with covariate**

Table 1.4 represents a Two-Factor Factorial CRD with covariate X . The Table showcases the two factors, Factor A and Factor B, and their respective levels ($a_1, a_2, \dots$) for Factor A, and ($b_1, b_2, \dots$) for Factor B. The subscripts indicate the specific level of each factor.

Table 1. 4: General layout for a two-factor factorial CRD design with covariate (X).

		Factor A						
		$k = 1, 2, 3, \dots, n$						
		$i = 1, 2, 3, \dots, t$						
		$j = 1, 2, 3, \dots, r$						
Factor B	b_1	a_1	a_2	a_3	...	a_t	...	a_r
		$Y_{111}(X_{111})$	$Y_{211}(X_{211})$	$Y_{311}(X_{311})$		$Y_{t11}(X_{t11})$		$Y_{r11}(X_{r11})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$Y_{11k}(X_{11k})$	$Y_{21k}(X_{21k})$	$Y_{31k}(X_{31k})$	...	$Y_{t1k}(X_{t1k})$	...	$Y_{r1k}(X_{r1k})$
	b_j	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$Y_{11n}(X_{11n})$	$Y_{21n}(X_{21n})$	$Y_{31n}(X_{31n})$		$Y_{t1n}(X_{t1n})$		$Y_{r1n}(X_{r1n})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	$Y_{1j1}(X_{1j1})$	$Y_{2j1}(X_{2j1})$	$Y_{3j1}(X_{3j1})$		$Y_{tj1}(X_{tj1})$		$Y_{rj1}(X_{rj1})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$Y_{1jk}(X_{1jk})$	$Y_{2jk}(X_{2jk})$	$Y_{3jk}(X_{3jk})$	...	$Y_{tjk}(X_{tjk})$	...	$Y_{rjk}(X_{rjk})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	$Y_{1jn}(X_{1jn})$	$Y_{2jn}(X_{2jn})$	$Y_{3jn}(X_{3jn})$		$Y_{tjn}(X_{tjn})$		$Y_{rjn}(X_{rjn})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	$Y_{1r1}(X_{1r1})$	$Y_{2r1}(X_{2r1})$	$Y_{3r1}(X_{3r1})$		$Y_{tr1}(X_{tr1})$		$Y_{rr1}(X_{rr1})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
		$Y_{1rk}(X_{1rk})$	$Y_{2rk}(X_{2rk})$	$Y_{3rk}(X_{3rk})$	...	$Y_{trk}(X_{trk})$	...	$Y_{rrk}(X_{rrk})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	b_r	$Y_{1rn}(X_{1rn})$	$Y_{2rn}(X_{2rn})$	$Y_{3rn}(X_{3rn})$		$Y_{trn}(X_{trn})$		$Y_{rrn}(X_{rrn})$
		$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$

1.2 Analysis of Mean (ANOM)

The Analysis of Means (ANOM) is a graphical testing procedure that shares similarities with Shewhart control charts and presents results in a user-friendly manner, making it easier to interpret. It is utilized to compare multiple groups. ANOM offers several advantages over traditional Analysis of Variance (ANOVA). However, introducing ANOVA to individuals with limited statistical knowledge can be challenging. The concept of partitioning the variability in data into potential sources to compare and contrast means may seem conceptually perplexing when transitioning from one and two-sample t-tests. Furthermore, drawing inferences on treatment means using ANOVA involves a two-stage process. Initially, an overall F-test is conducted to determine if at least one treatment mean significantly differs from the others. If this is confirmed, a post-hoc test is subsequently employed to identify the specific means responsible for the observed significance [7]. A regrettable outcome of conducting post hoc tests in ANOVA is that they may not consistently align with the conclusion of the overall F-test. For instance, the overall F-test may indicate rejection, while pairwise comparisons using methods like Tukey or Bonferroni reveal no significant differences. On the other hand, ANOM is a straightforward graphical testing procedure that simultaneously compares each individual mean to the overall mean. The visual comparisons are designed to control the familywise error rate and directly correspond to the overall ANOM testing procedure. The ANOM procedure has gained increasing popularity among practitioners and data scientists due to its incorporation of statistical inference within a visual framework. For example, ANOM can serve as a preliminary test to assess the randomness of sampled data by plotting systematic group means during the early stages of survey sampling problems [8].

Furthermore, ANOM has found applications in various research domains, such as medicine, healthcare, quality management, and environmental sciences, as evidenced by studies conducted in [7-12].

The layout of the ANOM data set is presented in Table 1.5, where the structure of the ANOM test under the same assumptions as the model in Eq (1.1) is used to test the following hypotheses:

Null hypothesis, $H_0: \mu_1 = \mu_2 = \dots = \mu_t$;

Alternative hypothesis, H_1 : at least one of the μ_i is different.

In a balanced design (equal sample size), the overall mean $\bar{Y}_{..}$ is calculated as $\bar{Y}_{..} = \sum_{i=1}^{n_i} \bar{Y}_{i.} / t$. Similarly, the overall mean square error $MSE_{\bar{Y}_{..}}$ is calculated as $MSE_{\bar{Y}_{..}} = \sum_{i=1}^{n_i} S_{\bar{Y}_{i.}} / t$.

Table 1. 5: The framework of the ANOM test.

Terms	τ_1	τ_2	τ_3	...	τ_t	...	τ_t
R_1	Y_{11}	Y_{21}	Y_{31}	...	Y_{t1}	...	Y_{t1}
R_2	Y_{12}	Y_{22}	Y_{32}	...	Y_{t2}	...	Y_{t2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_j	Y_{1j}	Y_{2j}	Y_{3j}	...	Y_{tj}	...	Y_{tj}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_{n_i}	Y_{1n_i}	Y_{2n_i}	Y_{3n_i}	...	Y_{tn_i}	...	Y_{tn_i}
$Y_{i.} = \sum_{j=1}^{n_i} Y_{ij}$	$Y_{1.}$	$Y_{2.}$	$Y_{3.}$	...	$Y_{t.}$	...	$Y_{t.}$
$\bar{Y}_{i.} = \sum_{j=1}^{n_i} Y_{ij} / n_i$	$\bar{Y}_{1.}$	$\bar{Y}_{2.}$	$\bar{Y}_{3.}$	...	$\bar{Y}_{t.}$	...	$\bar{Y}_{t.}$
$S_{Y_{i.}}^2 = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2 / (n_i - 1)$	$S_{Y_{1.}}^2$	$S_{Y_{2.}}^2$	$S_{Y_{3.}}^2$	...	$S_{Y_{t.}}^2$	...	$S_{Y_{t.}}^2$

In a balanced design (equal sample size), the overall mean $\bar{Y}_{..}$ is calculated as $\bar{Y}_{..} = \sum_{i=1}^{n_i} \bar{Y}_{i.} / t$. Similarly, the overall mean square error $MSE_{\bar{Y}_{..}}$ is calculated as $MSE_{\bar{Y}_{..}} = \sum_{i=1}^{n_i} S_{\bar{Y}_{i.}} / t$. Furthermore, for an unbalanced design (unequal sample size), the overall mean ($\bar{Y}_{..}$) and overall mean square error ($MSE_{\bar{Y}_{..}}$) can be calculated using the following expressions:

$$\bar{Y}_{..} = \frac{n_1 \bar{Y}_{1.} + n_2 \bar{Y}_{2.} + \dots + n_t \bar{Y}_{t.}}{N} \quad (1.2)$$

$$MSE_{\bar{Y}_{..}} = \frac{(n_1-1)S_{\bar{Y}_{1.}} + (n_2-1)S_{\bar{Y}_{2.}} + \dots + (n_t-1)S_{\bar{Y}_{t.}}}{N-t} \quad (1.3)$$

Under the balanced design (equal sample size), the lower and upper decision lines are obtained as follows:

$$LDL = \bar{Y}_{..} - h(\alpha, t, n)(MSE_{\bar{M}_{..}})\sqrt{\frac{t-1}{N}} \quad (1.4)$$

$$UDL = \bar{Y}_{..} + h(\alpha, t, n)(MSE_{\bar{M}_{..}})\sqrt{\frac{t-1}{N}} \quad (1.5)$$

Similarly, under the unbalanced design (unequal sample size), the lower and upper decision lines are obtained as follows:

$$LDL = \bar{Y}_{..} - m(\alpha, t)(MSE_{\bar{Y}_{..}})\sqrt{\frac{N-n_i}{Nn_i}} \quad (1.6)$$

$$UDL = \bar{Y}_{..} + m(\alpha, t)(MSE_{\bar{Y}_{..}})\sqrt{\frac{N-n_i}{Nn_i}} \quad (1.7)$$

Where the symbols $h(\alpha, t, n)$ and $m(\alpha, t)$ denote the critical values for balanced and unbalanced design [13].

1.3 Analysis of Means Using Covariate (ANOMC)

In practice, some linearly related covariates are recorded while collecting the information on the main variable. ANOM is developed to test the t independent treatment means of the main variable only without considering the covariates. However, for testing the t independent treatment means of the main variable while keeping the effect of a covariate, a new test called the Analysis of Means using Covariate (ANOMC) was proposed by Mahmood, et al. [14]. This test is designed to provide more accurate and effective analytical results compared to the previous method. Usually, the conventional ANOM test

fails to consider concomitant variables. This can inflate the mean square error, resulting in erroneous conclusions about potential treatment differences. Mahmood, et al. [14] found that the ANOMC test based on the regression estimator [15-19] outperformed the traditional ANOM in making accurate decisions. However, it was also noted that ANOMC exhibited poor performance when applied to datasets with extremely positively skewed distributions. To address this limitation, we need to use robust methods to reduce the limitation and achieve a robust version of the ANOMC method.

Consider a scenario with a single-factor model with a covariate exhibiting a linear relationship. The model includes t treatments, each with t levels, and n_i observations are collected for each level. The total number of observations is denoted as $N = \sum_{i=1}^t n_i$. This fixed effect model expression can be rephrased as:

$$Y_{ij} = \mu + \tau_i + B(X_{ij} - \bar{X}_{..}) + \epsilon_{ij}, i = 1, 2, 3, \dots, t, j = 1, 2, 3, \dots, n_i \quad (1.8)$$

The variables and parameters in Eq (1.8) have similar interpretations as in Eq (1.1), with the following additions:

X_{ij} is the j^{th} observation of covariate for the i^{th} treatment level corresponding to Y_{ij} , B is the slope indicating the relationship between Y_{ij} and X_{ij} , $\bar{X}_{..}$ is the grand mean of X_{ij} observations. In model (1.8), several assumptions are made. Firstly, we assume that the slope $B \neq 0$, indicating a linear relationship between Y_{ij} and X_{ij} . Additionally, the regression coefficients for each treatment level are assumed to be identical. Furthermore, it is assumed that the concomitant variable X_{ij} is not affected by treatment levels. Lastly, the treatment effects sum to zero (i.e., $\sum_{i=1}^t \tau_i = 0$). The continuous covariate is considered as explanatory variable, where r_i is the i^{th} sample correlation, B_i is the i^{th} slope, the

i^{th} regression mean estimator is denoted as $\bar{M}_{i.}$, and the adjusted means are calculated using this estimator is i^{th} unbiasing constant and $S_{\bar{M}_{i.}}$ is the sample standard deviation of i^{th} regression mean estimator. The layout of the ANOMC data set is presented in Table 1.6, where the structure of the ANOMC test under the same assumptions as the model in Eq (1.2) is used to test the following hypotheses:

Null hypothesis, $H_0: adj\mu_1 = adj\mu_2 = \dots = adj\mu_t;$

Alternative hypothesis, H_1 : at least one of the $adj\mu_i$ is different.

In a balanced design (equal sample size), the overall mean $\bar{M}_{..}$ is calculated as $\bar{M}_{..} = \sum_{i=1}^{n_i} \bar{M}_{i.}/t$. Similarly, the overall mean square error $MSE_{\bar{M}_{..}}$ is calculated as $MSE_{\bar{M}_{..}} = \sum_{i=1}^{n_i} S_{\bar{M}_{i.}}/t$.

Furthermore, for an unbalanced design (unequal sample size), the overall mean ($\bar{M}_{..}$) and overall mean square error ($MSE_{\bar{M}_{..}}$) can be calculated using the following expressions:

$$\bar{M}_{..} = \frac{n_1 \bar{M}_{1.} + n_2 \bar{M}_{2.} + \dots + n_t \bar{M}_{t.}}{N} \quad (1.9)$$

$$MSE_{\bar{M}_{..}} = \frac{(n_1-1)S_{\bar{M}_{1.}} + (n_2-1)S_{\bar{M}_{2.}} + \dots + (n_t-1)S_{\bar{M}_{t.}}}{N-t} \quad (1.10)$$

Under the balanced design (equal sample size), the lower and upper decision lines are obtained as follows:

$$LDL = \bar{M}_{..} - h^*(\alpha, t, n)(MSE_{\bar{M}_{..}})\sqrt{\frac{t-1}{N}} \quad (1.11)$$

$$UDL = \bar{M}_{..} + h^*(\alpha, t, n)(MSE_{\bar{M}_{..}})\sqrt{\frac{t-1}{N}} \quad (1.12)$$

Table 1. 6: The framework of the ANOMC test based on regression estimator.

Terms	τ_1	τ_2	τ_3	...	τ_i	...	τ_t
R_1	$Y_{11}(X_{11})$	$Y_{21}(X_{21})$	$Y_{31}(X_{31})$	...	$Y_{i1}(X_{i1})$	...	$Y_{t1}(X_{t1})$
R_2	$Y_{12}(X_{12})$	$Y_{22}(X_{22})$	$Y_{32}(X_{32})$	...	$Y_{i2}(X_{i2})$	...	$Y_{t2}(X_{t2})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_j	$Y_{1j}(X_{1j})$	$Y_{2j}(X_{2j})$	$Y_{3j}(X_{3j})$	...	$Y_{ij}(X_{ij})$	...	$Y_{tj}(X_{tj})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
R_{n_i}	$Y_{1n_i}(X_{1n_i})$	$Y_{2n_i}(X_{2n_i})$	$Y_{3n_i}(X_{3n_i})$	...	$Y_{in_i}(X_{in_i})$	...	$Y_{tn_i}(X_{tn_i})$
$Y_{i.} = \sum_{j=1}^{n_i} Y_{ij}$	$Y_{1.}(X_{1.})$	$Y_{2.}(X_{2.})$	$Y_{3.}(X_{3.})$	...	$Y_{i.}(X_{i.})$	...	$Y_{t.}(X_{t.})$
$X_{i.} = \sum_{j=1}^{n_i} X_{ij}$							
$\bar{Y}_{i.} = \sum_{j=1}^{n_i} Y_{ij}/n_i$	$\bar{Y}_{1.}(\bar{X}_{1.})$	$\bar{Y}_{2.}(\bar{X}_{2.})$	$\bar{Y}_{3.}(\bar{X}_{3.})$	...	$\bar{Y}_{i.}(\bar{X}_{i.})$	...	$\bar{Y}_{t.}(\bar{X}_{t.})$
$\bar{X}_{i.} = \sum_{j=1}^{n_i} X_{ij}/n_i$							
$S_{Y_{i.}}^2 = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2 / (n_i - 1)$	$S_{Y_{1.}}^2 (S_{\bar{X}_{1.}}^2)$	$S_{Y_{2.}}^2 (S_{\bar{X}_{2.}}^2)$	$S_{Y_{3.}}^2 (S_{\bar{X}_{3.}}^2)$	...	$S_{Y_{i.}}^2 (S_{\bar{X}_{i.}}^2)$	...	$S_{Y_{t.}}^2 (S_{\bar{X}_{t.}}^2)$
$S_{\bar{X}_{i.}}^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^2 / (n_i - 1)$							
$S_{X_{i.}Y_{i.}} = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})(Y_{ij} - \bar{Y}_{i.}) / (n_i - 1)$	$S_{X_{1.}Y_{1.}}$	$S_{X_{2.}Y_{2.}}$	$S_{X_{3.}Y_{3.}}$	...	$S_{X_{i.}Y_{i.}}$	...	$S_{X_{t.}Y_{t.}}$
$r_{i.} = S_{X_{i.}Y_{i.}} / \sqrt{S_{Y_{i.}}^2 S_{\bar{X}_{i.}}^2}$	$r_{1.}$	$r_{2.}$	$r_{3.}$	...	$r_{i.}$	...	$r_{t.}$
$b_{i.} = r_{i.}(S_{Y_{i.}}/S_{Y_{i.}})$	$b_{1.}$	$b_{2.}$	$b_{3.}$	...	$b_{i.}$	...	$b_{t.}$
$\bar{M}_{i.} = \bar{Y}_{i.} + b_{i.}(\bar{X}_{i.} - \bar{X}_{i.})$	$\bar{M}_{1.}$	$\bar{M}_{2.}$	$\bar{M}_{3.}$	...	$\bar{M}_{i.}$	...	$\bar{M}_{t.}$
$k_{i.} = \sqrt{(1 - r_{i.}^2)(1 + (1/n_i - 3))}$	$k_{1.}$	$k_{2.}$	$k_{3.}$	...	$k_{i.}$	...	$k_{t.}$
$S_{\bar{M}_{i.}} = k_{i.} b_{i.} / \sqrt{n_i}$	$S_{\bar{M}_{1.}}$	$S_{\bar{M}_{2.}}$	$S_{\bar{M}_{3.}}$	...	$S_{\bar{M}_{i.}}$	...	$S_{\bar{M}_{t.}}$

Similarly, under the unbalanced design (unequal sample size), the lower and upper decision lines are obtained as follows:

$$\text{LDL} = \bar{M}_{..} - m^*(\alpha, t)(\text{MSE}_{\bar{M}_{..}}) \sqrt{\frac{N - n_i}{N n_i}} \quad (1.13)$$

$$\text{UDL} = \bar{M}_{..} + m^*(\alpha, t)(\text{MSE}_{\bar{M}_{..}}) \sqrt{\frac{N - n_i}{N n_i}} \quad (1.14)$$

Where the symbols $h^*(\alpha, t, n)$ and $m^*(\alpha, t)$ denote the critical values for balanced and unbalanced design [13].

1.4 ANOMC: A Balanced Assessment of Its Advantages and Limitations

ANOMC was able to outperform the standard ANOM approach by incorporating a continuous covariate variable during the experiment. This allowed ANOMC to consider the uncontrollable factors that affected the experiment as covariates, providing several advantages:

- I. Increased Statistical Power: ANOMC increases the ability to find meaningful differences between the treatment means, therefore significantly increasing the statistical power of the analysis.
- II. Minimizes Errors: ANOMC actively minimizes the errors and confounding factors in the analysis.
- III. Adjusted means: the adjusted means provided by the ANOMC analysis allow to make more informed and well-grounded inferences about the treatment's effectiveness, accounting for relevant characteristics.

While ANOMC provides several advantages over the standard ANOM approach, it also has some inherent limitations:

- I. Non-Normal Environments: under non-normal environments, especially with highly skewed distributions such as the chi-square or Cauchy distributions, the ANOMC test may increase the error.
- II. Minimum Sample Size Requirement: ANOMC requires a minimum sample size of four or more observations per treatment group. With smaller sample sizes, the analysis becomes infeasible.
- III. Assumption Violations: ANOMC relies on several assumptions mentioned in previous section. Violations of these assumptions can undermine the validity of the analysis. Specifically, when the assumption of homogeneity of variance is violated, the ANOMC test cannot adequately control the type I error rate and may increase the error.

1.5 Comprehensive Literature Review

In the context of DOE perspectives, let us delve into a comprehensive and detailed review of the ANOM test. We will focus on studies that aim to discover an enhanced version of the ANOM test. Before delving into the enhanced versions of ANOM tests, it is important to acknowledge the widely used Analysis of Variance (ANOVA) method, which relies on the F-statistic. ANOVA is a well-established statistical technique employed to differentiate between group means in a one-way fixed effect design [20]. However, it is crucial to note that several other statistical techniques are utilized to compare the means of independent groups.

Researchers have explored various approaches to enhance the ANOM test. These efforts aim to build upon the foundations of ANOM and provide improved tools for data analysis. Some notable alternatives to ANOVA include the James-second-order test [21], which offers enhanced discrimination capabilities; the Welch test [22], which accounts for unequal variances; the Alexander-Govern test [23], which addresses heteroscedasticity, and the Brown-Forsythe test [24], which is robust to violations of the equal variance assumption. These approaches expand the range of options available for researchers conducting mean comparisons among independent groups. The aforementioned tests are designed to enable the assessment of pairwise significance among treatment means. Analysis of Covariate (ANCOVA), an extension of ANOVA, is employed to investigate the presence of a statistically significant difference between the means of three or more independent groups while accounting for one or more covariates [25-27]. This analysis allows for the adjustment of the effect of covariates on the dependent variable, providing a more accurate assessment of group differences.

As previously discussed, the ANOM test is a statistical tool commonly utilized to evaluate differences in treatment means relative to their grand mean. This test was originally introduced by Ott [28] and later reproduced by Ott [29]. The ANOM test finds applications in various fields, including environmental studies [12], medical science [8, 30], nanomaterials [31], tourism [32], and healthcare studies [33]. Schilling [34] proposed an extension of the ANOM test to accommodate mixed effect designs and balanced incomplete block designs. In fact, it is a versatile graphical method that can be effectively utilized to compare rates or proportions, as highlighted by Ramig [35]. These extensions and applications of the ANOM test have enhanced its utility and widened its scope in various experimental settings. Initially, the ANOM test was specifically designed to assess the equality of means. However, Wludyka and Nelson [36] introduced an extension of the ANOM mechanism to address the equality of variances (ANOMV). Bernard and Wludyka [37] and Wludyka and Sa [38] have introduced an approach to enhance the robustness of ANOMV by combining the Fligner and Killeen test and the Levene test. This combined method improves the reliability and stability of ANOMV in assessing the equality of variances across multiple groups. Furthermore, Nelson and Dudewicz [39] and Dudewicz and Nelson [40] proposed an extension of the ANOM test under a heteroscedastic model, known as Heteroscedastic Analysis of Means (HANOM).

A nonparametric version of the ANOM test called ANOMR was introduced by Bakir [41]. Additionally, Chang et al. conducted a comparison between the ANOM and ANOVA tests using parametric bootstrap. Determining exact control limits for the balanced design, where sample sizes are equal, was presented by Nelson [42]. Conversely, for the unbalanced design, where sample sizes are unequal, the precise control limits were

provided by Soong and Hsu [43]. Additionally, the results of the ANOM test were presented in research papers [44-47] for studies with equal sample sizes and in papers [48, 49] for studies with unequal sample sizes. Recently, Mendes and Yiğit conducted a comprehensive analysis that compared the performance of ANOVA-F and ANOM tests when the assumptions of normality and homogeneity of variances were violated [50]. The study aimed to evaluate these tests' type I error rate and power under such circumstances. The ANOM test was analyzed for its statistical properties by Guirguis and Tobias [51] using the Fortran programming language. Pallmann and Hothorn [52] showcased the practical implementation of the ANOM test by utilizing the R programming language. Pallmann and Hothorn [52] presented a novel methodology for the generalized application of ANOM by incorporating the notion of multiple contrast tests (MCTs), particular comparisons to the overall average, and extended applications of MCTs by utilizing a linear model with a covariate. The previous iterations of ANOM were primarily applied to the fixed-effect model. Another investigation into the application of the ANOM test, specifically in the context of the random effect model, was carried out by Jayalath and Ng [53]. Additionally, Jayalath and Ng [54] put forth the ANOM test for designs involving hierarchically nested and split-plot structures. A concise examination of the ANOM test can be located in the reference [55]. Rao [56] proposed an individual measurement control chart utilizing ANOM control limits. A study on the impact of measurement errors on the effectiveness of the ANOM test proposed by Khurshid and Chakraborty [57]. Additionally, Apley [58] presented a Bayesian graphical approach for analyzing the location parameter of the process. Lopez-Mejia and Roldan-Valadez [59] derived the bootstrap confidence intervals for both the ANOM and ANOVA methods. ANOM is designed to assess the

equality of treatment means for a given variable while disregarding the influence of covariates. Nevertheless, Mahmood, et al. [14] presented a recent test called the Analysis of Means using Covariate (ANOMC) to assess the t independent treatment means of the primary variable while accounting for the impact of a covariate. This test aims to deliver more precise and efficient analytical outcomes in contrast to prior approaches. Typically, the traditional ANOM test overlooks covariate variables, leading to an inflation of the mean square error. As a result, misleading conclusions regarding potential disparities in treatment may arise. Mahmood, et al. [14] discovered that the ANOMC test utilizing the regression estimator demonstrated superior performance compared to the conventional ANOM in making precise decisions.

1.6 Objectives of The Thesis

The utilization of covariance in the analysis of means is a crucial aspect in numerous manufacturing experiments aimed at monitoring auxiliary variables that exhibit a linear relationship with the target variable. In this research study, we intend to examine and improve the effectiveness of the ANOM test by incorporating covariance. The specific objectives of our study are as follows:

- I. To formulate a comprehensive review of the Analysis of Means (ANOM) studies and identify potential areas for further contribution by addressing any existing gaps.
- II. To enhance the performance of the ANOMC test approach by incorporating ratio estimators.
- III. To evaluate the effectiveness of the ANOMC test when multiple auxiliary variables are considered.
- IV. Developing and extending the ANOMC test in complex designs such as for factorial

design.

- V. To implement the proposed methodologies to solve real-life case studies.

1.7 Outline of The Thesis

The investigation in this study has been done only under CRD. In Chapter 2, we have introduced the proposed ANOMC test utilizing ratio family estimators named (ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) to reduce the lack of the ANOMC-Reg test when it applies to positively skewed distributions. The performance of the aforementioned tests is evaluated using the percentage of type I errors and the power. The findings reveal that the ANOMC, when compared to ANOMC-Reg, demonstrates relatively superior performance when employing ratio family estimators. The proposed methods apply to problems with mechanical/industrial engineering and medical phenomena.

In Chapter 3, an additional covariate variable is considered in the ANOMC test to find out the performance of the test with two auxiliary variables with a family of ratio estimators. The proposed tests are namely ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10, and will be evaluated in terms of Type I error rate and power. The findings reveal that the proposed tests show different performance under the parameters considered in the study; thus, the choice of an appropriate method is contingent upon the specific parameters selected for the study. Moreover, the proposed tests apply to applications in biomedical and mechanical engineering.

In Chapter 4, we are expanding the scope of the ANOMC test to more complex treatment structures, specifically factorial designs to investigate the effectiveness of factorial

ANOMC tests in comparison to factorial ANOM tests and to enhance the performance of the factorial ANOM test by incorporating a covariate variable. The proposed tests, known as $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$ and $ANOMC - MR_F$ and will be evaluated in terms of Type I error rate and power. The findings reveal that the $ANOMC_F$ tests, when compared to $ANOM_F$, demonstrates relatively superior performance, practically when employing a regression estimator. Additionally, in the application part, a data set taken from the literature is analyzed to show the implementation of the methodology presented in the study. Finally, the conclusion, summaries, and recommendations of the work are presented in Chapter 5.

CHAPTER 2

ON NOVEL COVARIATE BASED COMPUTATIONAL TESTS FOR ANALYZING MEANS

In real-life scenarios, it is often necessary to consider linearly related covariates that are measured alongside the main variable. In such cases, the traditional ANOM approach does not account for the influence of these covariates when assessing mean equivalence. To address this limitation, an analysis of means using covariates (ANOMC) has been proposed recently. The initial implementation of ANOMC, known as ANOMC-Reg, utilizes a regression estimator. However, it has been observed that ANOMC-Reg performs inadequately when dealing with distributions that exhibit extreme positive skewness. Thus, this chapter aims to enhance the performance of the ANOMC test. A series of ratio estimators are introduced in the ANOMC structure and refer to ANOMC-R, ANOMC-

MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests. A simulation study utilizing Monte Carlo simulation methodology is conducted to evaluate and compare the performance of the existing and proposed tests. The tests are assessed based on their percentage of type I error and power under various design parameters.

2.1 Introduction

Historically, statisticians have advocated for the incorporation of covariates as a valuable tool to account for additional variability in a dependent variable. By doing so, the inclusion of covariates aims to enhance the statistical power of an experiment (e.g., [60, 61]). Usually, the conventional ANOM test fails to consider concomitant variables. This can inflate the mean square error, resulting in erroneous conclusions about potential treatment differences. The proposed approach ANOMC based on regression estimator (cf. section 1.3) is designed to provide more accurate and effective analytical results compared to the previous method. However, it was also noted that ANOMC-Reg exhibited poor performance when applied to datasets with extremely positively skewed distributions.

This chapter aims to enhance the performance of the ANOMC test using ratio family estimators, including both classical and modified ratio estimators. The proposed tests are referred to as ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4. The performance of ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 is evaluated in terms of two measures: percentage type I error and power of the test under normal and non-normal environments, different correlation structures, sample sizes, heterogeneous variances, and different number of treatments. The rest of the article is organized as follows: Section 2 consists of

mathematical details of ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests; Section 3 describes the study's parameters design; Section 4 assesses the performance of the proposed and existing methods; Section 5 focuses on applications in mechanical/industrial engineering and medical phenomena; Finally; in Section 6, we provide our concluding remarks for the study.

2.2 Description of Existing and Proposed Methodologies

In this section, we will provide the mathematical details of the ANOMC test based on a regression estimator and then introduce the proposed methods. Our proposal utilizes the ANOMC test based on ratio estimators to test adjusted means when a covariate is present. In Chapter 1, we established the fixed effect model, as represented by Eq (1.8). In this chapter, we utilize this model and present the structure of the test dataset, along with the ANOMC-Reg and ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests in Table 2.1.

2.2.1 The ANOMC test based on the regression estimator

The analysis of means with covariate (ANOMC) based on the regression estimator is referred to by the ANOMC-Reg test. In the ANOMC-Reg test, the continuous covariate is considered as explanatory variable, where r_i is the i^{th} sample correlation, b_i is the i^{th} slope, the i^{th} regression mean estimator is denoted as $\bar{M}_i$, and the adjusted means are calculated using this estimator, k_i is i^{th} unbiasing constant and $S_{\bar{M}_i}$ is the sample standard deviation of i^{th} regression mean estimator. Under the balanced design (equal sample size) and unbalanced design (unequal sample size), the overall mean ($\bar{M}_{..}$) and overall mean square error ($MSE_{\bar{M}_{..}}$) can be calculated using the expressions in Table 2.2. The lower

decision lines (LDL) and upper decision lines (UDL) can be obtained using the expressions in Table 2.3. The symbols $C(\alpha, t, n)$ and $C^*(\alpha, t)$ denote the critical values for balanced and unbalanced design, respectively.

2.2.2 The ANOMC test based on ratio estimators

The ratio estimators can be used as an alternative to the regression estimator by treating it as a ratio variable when testing the mean with a covariate. The ANOMC test based on the ratio estimators is referred to ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 test. In the classical ratio (R) estimator, $\bar{T}_{(1)i}$ is the i^{th} ratio mean estimator, $g = n_i / N$ is sampling fraction, r_i is the i^{th} sample correlation, C_{Y_i} and C_{X_i} are the i^{th} sample coefficients of variation of Y and X , $S_{(1)\bar{T}_i}$ is the sample standard deviation of the i^{th} ratio mean estimator.

On the other hand, the modified ratio (MR) estimators, $\bar{T}_{(2)i}$, $\bar{T}_{(3)i}$, $\bar{T}_{(4)i}$ and $\bar{T}_{(5)i}$ are the i^{th} MR means estimators, $B_2(x_i)$ is the i^{th} coefficient of kurtosis, $S_{(2)\bar{T}_i}$, $S_{(3)\bar{T}_i}$, $S_{(4)\bar{T}_i}$ and $S_{(5)\bar{T}_i}$ are the sample standard deviation of the i^{th} MR means estimators. However, under the balanced design (equal sample size) and unbalanced design (unequal sample size), the overall mean for R and MR estimators ($\bar{T}_{(1)..}$, $\bar{T}_{(2)..}$, $\bar{T}_{(3)..}$, $\bar{T}_{(4)..}$ and $\bar{T}_{(5)..}$) and the overall mean square error ($MSE_{\bar{T}_{(1)..}}$, $MSE_{\bar{T}_{(2)..}}$, $MSE_{\bar{T}_{(3)..}}$, $MSE_{\bar{T}_{(4)..}}$ and $MSE_{\bar{T}_{(5)..}}$) can be calculated using the expressions in Table 2.2. Furthermore, the lower decision lines (LDL) and upper decision lines (UDL) can be obtained using the expressions in Table 2.3. The symbols $V(\alpha, t, n)$, $Z(\alpha, t, n)$, $K(\alpha, t, n)$, $P(\alpha, t, n)$ and $O(\alpha, t, n)$ denote the critical values for balanced design while $V^*(\alpha, t)$, $Z^*(\alpha, t)$, $K^*(\alpha, t)$, $P^*(\alpha, t)$ and $O^*(\alpha, t)$ denote the critical values for unbalanced design.

2.3 Performance Measures and Simulation Procedure

In this section, we will outline the design structure of the study, which will be utilized for conducting a simulation study using the Monte Carlo simulation technique [62].

Table 2. 1: The framework of the ANOMC test based on regression and ratio family estimators and data layout.

Test	Terms	τ_1	τ_2	τ_3	...	τ_i	...	τ_t
ANOMC-Reg	R_1	$Y_{11}(X_{11})$	$Y_{21}(X_{21})$	$Y_{31}(X_{31})$	...	$Y_{i1}(X_{i1})$	...	$Y_{t1}(X_{t1})$
	R_2	$Y_{12}(X_{12})$	$Y_{22}(X_{22})$	$Y_{32}(X_{32})$	...	$Y_{i2}(X_{i2})$	...	$Y_{t2}(X_{t2})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
	R_j	$Y_{1j}(X_{1j})$	$Y_{2j}(X_{2j})$	$Y_{3j}(X_{3j})$	...	$Y_{ij}(X_{ij})$	...	$Y_{tj}(X_{tj})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
	R_{n_i}	$Y_{1n_i}(X_{1n_i})$	$Y_{2n_i}(X_{2n_i})$	$Y_{3n_i}(X_{3n_i})$	...	$Y_{in_i}(X_{in_i})$	...	$Y_{tn_i}(X_{tn_i})$
	$Y_{i.} = \sum_{j=1}^{n_i} Y_{ij}$	$Y_{1.}(X_{1.})$	$Y_{2.}(X_{2.})$	$Y_{3.}(X_{3.})$	...	$Y_{i.}(X_{i.})$	...	$Y_{t.}(X_{t.})$
	$X_{i.} = \sum_{j=1}^{n_i} X_{ij}$							
	$\bar{Y}_{i.} = \sum_{j=1}^{n_i} Y_{ij}/n_i$	$\bar{Y}_{1.}(\bar{X}_{1.})$	$\bar{Y}_{2.}(\bar{X}_{2.})$	$\bar{Y}_{3.}(\bar{X}_{3.})$	...	$\bar{Y}_{i.}(\bar{X}_{i.})$	...	$\bar{Y}_{t.}(\bar{X}_{t.})$
	$\bar{X}_{i.} = \sum_{j=1}^{n_i} X_{ij}/n_i$							
	$S_{Y_{i.}}^2 = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2 / (n_i - 1)$	$S_{Y_{1.}}^2 (S_{X_{1.}}^2)$	$S_{Y_{2.}}^2 (S_{X_{2.}}^2)$	$S_{Y_{3.}}^2 (S_{X_{3.}}^2)$	...	$S_{Y_{i.}}^2 (S_{X_{i.}}^2)$	...	$S_{Y_{t.}}^2 (S_{X_{t.}}^2)$
	$S_{X_{i.}}^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^2 / (n_i - 1)$							
	$S_{X_{i.}Y_{i.}} = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})(Y_{ij} - \bar{Y}_{i.}) / (n_i - 1)$	$S_{X_{1.}Y_{1.}}$	$S_{X_{2.}Y_{2.}}$	$S_{X_{3.}Y_{3.}}$	...	$S_{X_{i.}Y_{i.}}$	...	$S_{X_{t.}Y_{t.}}$
	$r_{i.} = S_{X_{i.}Y_{i.}} / \sqrt{S_{Y_{i.}}^2 S_{X_{i.}}^2}$	$r_{1.}$	$r_{2.}$	$r_{3.}$	...	$r_{i.}$	...	$r_{t.}$
	$b_{i.} = r_{i.} (S_{Y_{i.}} / S_{Y_{i.}})$	$b_{1.}$	$b_{2.}$	$b_{3.}$	...	$b_{i.}$	...	$b_{t.}$
	$\bar{M}_{i.} = \bar{Y}_{i.} + b_{i.}(\bar{X}_{i.} - \bar{X}_{..})$	$\bar{M}_{1.}$	$\bar{M}_{2.}$	$\bar{M}_{3.}$	...	$\bar{M}_{i.}$	...	$\bar{M}_{t.}$
	$k_{i.} = \sqrt{(1 - r_{i.}^2)(1 + (1/n_i - 3))}$	$k_{1.}$	$k_{2.}$	$k_{3.}$	...	$k_{i.}$	...	$k_{t.}$
	$S_{\bar{M}_{i.}} = k_{i.} b_{i.} / \sqrt{n_i}$	$S_{\bar{M}_{1.}}$	$S_{\bar{M}_{2.}}$	$S_{\bar{M}_{3.}}$	...	$S_{\bar{M}_{i.}}$	...	$S_{\bar{M}_{t.}}$
ANOMC-R	$\theta_i = (1 - g) / n_i$	θ_1	θ_2	θ_3	...	θ_i	...	θ_t
	$C_{Y_{i.}}^2 = S_{Y_{i.}}^2 / \bar{Y}_{i.}^2$	$C_{Y_{1.}}^2$	$C_{Y_{2.}}^2$	$C_{Y_{3.}}^2$	...	$C_{Y_{i.}}^2$	...	$C_{Y_{t.}}^2$
	$C_{X_{i.}}^2 = S_{X_{i.}}^2 / \bar{X}_{i.}^2$	$C_{X_{1.}}^2$	$C_{X_{2.}}^2$	$C_{X_{3.}}^2$	...	$C_{X_{i.}}^2$	...	$C_{X_{t.}}^2$
ANOMC-MR1	$\bar{T}_{(1)i.} = \bar{y}_{i.}(\bar{X}_{i.} / \bar{x}_{i.})$	$\bar{T}_{(1)1.}$	$\bar{T}_{(1)2.}$	$\bar{T}_{(1)3.}$	...	$\bar{T}_{(1)i.}$	...	$\bar{T}_{(1)t.}$
	$s_{\bar{T}_{(3)i.}} = \theta_i \bar{Y}_{i.}^2 [C_{Y_{i.}}^2 + C_{X_{i.}}^2 - 2r_{i.} C_{Y_{i.}} C_{X_{i.}}]$	$s_{\bar{T}_{(3)1.}}$	$s_{\bar{T}_{(3)2.}}$	$s_{\bar{T}_{(3)3.}}$	...	$s_{\bar{T}_{(3)i.}}$	...	$s_{\bar{T}_{(3)t.}}$
	$\bar{T}_{(2)i.} = \bar{y}_{i.}(\bar{X}_{i.} + C_{X_{i.}}) / (\bar{x}_{i.} + C_{X_{i.}})$	$\bar{T}_{(2)1.}$	$\bar{T}_{(2)2.}$	$\bar{T}_{(2)3.}$	...	$\bar{T}_{(2)i.}$	...	$\bar{T}_{(2)t.}$
ANOMC-MR2	$\alpha_i = \frac{\bar{X}_{i.}}{\bar{X}_{i.} + C_{X_{i.}}}$	α_1	α_2	α_3	...	α_i	...	α_t
	$s_{\bar{T}_{(2)i.}} = \theta_i \bar{Y}_{i.}^2 [C_{Y_{i.}}^2 + C_{X_{i.}}^2 \alpha_i^2 - 2r_{i.} \alpha_i C_{Y_{i.}} C_{X_{i.}}]$	$s_{\bar{T}_{(2)1.}}$	$s_{\bar{T}_{(2)2.}}$	$s_{\bar{T}_{(2)3.}}$	...	$s_{\bar{T}_{(2)i.}}$	...	$s_{\bar{T}_{(2)t.}}$
	$B_2(x_{i.}) = \frac{n_i(n_i + 1) \sum_{i=1}^t (x_{i.} - \bar{X}_{i.})^4}{(n_i - 1)(n_i - 2)(n_i - 3) S_{X_{i.}}^4} - \frac{3(n_i - 1)^2}{(n_i - 2)(n_i - 3)}$ (See Subramani and Gnanasegaran [63])	$B_2(x_{1.})$	$B_2(x_{2.})$	$B_2(x_{3.})$	...	$B_2(x_{i.})$	...	$B_2(x_{t.})$
ANOMC-MR3	$\bar{T}_{(3)i.} = \bar{y}_{i.}(\bar{X}_{i.} + B_2(x_{i.})) / (\bar{x}_{i.} + B_2(x_{i.}))$	$\bar{T}_{(3)1.}$	$\bar{T}_{(3)2.}$	$\bar{T}_{(3)3.}$	...	$\bar{T}_{(3)i.}$	...	$\bar{T}_{(3)t.}$
	$\delta_i = \frac{\bar{X}_{i.}}{\bar{X}_{i.} + B_2(x_{i.})}$	δ_1	δ_2	δ_3	...	δ_i	...	δ_t
	$s_{\bar{T}_{(3)i.}} = \theta_i \bar{Y}_{i.}^2 [C_{Y_{i.}}^2 + C_{X_{i.}}^2 \delta_i^2 - 2r_{i.} \delta_i C_{Y_{i.}} C_{X_{i.}}]$	$s_{\bar{T}_{(3)1.}}$	$s_{\bar{T}_{(3)2.}}$	$s_{\bar{T}_{(3)3.}}$	...	$s_{\bar{T}_{(3)i.}}$	...	$s_{\bar{T}_{(3)t.}}$
ANOMC-MR4	$\bar{T}_{(4)i.} = \bar{y}_{i.}(\bar{X}_{i.} B_2(x_{i.}) + C_{X_{i.}}) / (\bar{x}_{i.} B_2(x_{i.}) + C_{X_{i.}})$	$\bar{T}_{(4)1.}$	$\bar{T}_{(4)2.}$	$\bar{T}_{(4)3.}$	...	$\bar{T}_{(4)i.}$	...	$\bar{T}_{(4)t.}$
	$\omega_i = \frac{\bar{X}_{i.} B_2(x_{i.})}{\bar{X}_{i.} B_2(x_{i.}) + C_{X_{i.}}}$	ω_1	ω_2	ω_3	...	ω_i	...	ω_t
	$s_{\bar{T}_{(4)i.}} = \theta_i \bar{Y}_{i.}^2 [C_{Y_{i.}}^2 + C_{X_{i.}}^2 \omega_i^2 - 2r_{i.} \omega_i C_{Y_{i.}} C_{X_{i.}}]$	$s_{\bar{T}_{(4)1.}}$	$s_{\bar{T}_{(4)2.}}$	$s_{\bar{T}_{(4)3.}}$	...	$s_{\bar{T}_{(4)i.}}$	...	$s_{\bar{T}_{(4)t.}}$
ANOMC-MR5	$\bar{T}_{(5)i.} = \bar{y}_{i.}(\bar{X}_{i.} C_{X_{i.}} + B_2(x_{i.})) / (\bar{x}_{i.} C_{X_{i.}} + B_2(x_{i.}))$	$\bar{T}_{(5)1.}$	$\bar{T}_{(5)2.}$	$\bar{T}_{(5)3.}$	...	$\bar{T}_{(5)i.}$	...	$\bar{T}_{(5)t.}$
	$\varphi_i = \frac{\bar{X}_{i.} C_{X_{i.}}}{\bar{X}_{i.} C_{X_{i.}} + B_2(x_{i.})}$	φ_1	φ_2	φ_3	...	φ_i	...	φ_t
	$s_{\bar{T}_{(5)i.}} = \theta_i \bar{Y}_{i.}^2 [C_{Y_{i.}}^2 + C_{X_{i.}}^2 \varphi_i^2 - 2r_{i.} \varphi_i C_{Y_{i.}} C_{X_{i.}}]$	$s_{\bar{T}_{(5)1.}}$	$s_{\bar{T}_{(5)2.}}$	$s_{\bar{T}_{(5)3.}}$	...	$s_{\bar{T}_{(5)i.}}$	...	$s_{\bar{T}_{(5)t.}}$

A set of independently and normally distributed random numbers is generated using the Box-Muller transformation. Fleishman mechanism with four specified moments is used to generate non-normal random numbers. For more details about the Fleishman mechanism, see Fleishman [64]. In this study, ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests are compared with respect to type I error (α) and test power ($1 - \beta$). Type I error is the degree of the incorrect rejection of a true null hypothesis ($H_0: adj\mu_1 = adj\mu_2 = \dots = adj\mu_t$), while the power of the test is the degree of correct rejection of the false null hypothesis (H_1 : at least one of the $adj\mu_i$ is different). Following [14], the study examines the performance of the aforementioned tests under different experimental conditions, such as choices of distributions $D = (s, kr)$ with several values of skewness (s) and kurtosis (kr), for instance, the combination of (0,0) represents a normal distribution, (0,3) represents a double exponential distribution, (2,6) represents a Chi Square distribution, and (0,25) represents a Cauchy distribution. Additionally, the study considers several values of correlation (ρ), sample size (n), variance ratios (v), effect size (δ), and number of treatments (t).

Table 2. 2: Overall mean and mean squared error regarding ratio and regression estimators under difference designs.

Balanced design			
$\bar{M}_{..} = \sum_{i=1}^{n_i} \bar{M}_i / t$		$MSE_{\bar{M}_{..}} = \sum_{i=1}^{n_i} S_{\bar{M}_i} / t$	
$\bar{T}_{(1)_{..}} = \sum_{i=1}^{n_i} \bar{T}_{(1)i} / t$		$MSE_{\bar{T}_{(1)_{..}}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(1)i}} / t$	
$\bar{T}_{(2)_{..}} = \sum_{i=1}^{n_i} \bar{T}_{(2)i} / t$		$MSE_{\bar{T}_{(2)_{..}}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(2)i}} / t$	
$\bar{T}_{(3)_{..}} = \sum_{i=1}^{n_i} \bar{T}_{(3)i} / t$		$MSE_{\bar{T}_{(3)_{..}}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(3)i}} / t$	
$\bar{T}_{(4)_{..}} = \sum_{i=1}^{n_i} \bar{T}_{(4)i} / t$		$MSE_{\bar{T}_{(4)_{..}}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(4)i}} / t$	
$\bar{T}_{(5)_{..}} = \sum_{i=1}^{n_i} \bar{T}_{(5)i} / t$		$MSE_{\bar{T}_{(5)_{..}}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(5)i}} / t$	
Unbalanced design			
$\bar{M}_{..} = \frac{n_1 \bar{M}_{1.} + n_2 \bar{M}_{2.} + \dots + n_t \bar{M}_{t.}}{N}$		$MSE_{\bar{M}_{..}} = \frac{(n_1 - 1)S_{\bar{M}_{1.}} + (n_2 - 1)S_{\bar{M}_{2.}} + \dots + (n_t - 1)S_{\bar{M}_{t.}}}{N - t}$	
$\bar{T}_{(1)_{..}} = \frac{n_1 \bar{T}_{(1)1.} + n_2 \bar{T}_{(1)2.} + \dots + n_t \bar{T}_{(1)t.}}{N}$		$MSE_{\bar{T}_{(1)_{..}}} = \frac{(n_1 - 1)S_{\bar{T}_{(1)1.}} + (n_2 - 1)S_{\bar{T}_{(1)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(1)t.}}}{N - t}$	
$\bar{T}_{(2)_{..}} = \frac{n_1 \bar{T}_{(2)1.} + n_2 \bar{T}_{(2)2.} + \dots + n_t \bar{T}_{(2)t.}}{N}$		$MSE_{\bar{T}_{(2)_{..}}} = \frac{(n_1 - 1)S_{\bar{T}_{(2)1.}} + (n_2 - 1)S_{\bar{T}_{(2)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(2)t.}}}{N - t}$	
$\bar{T}_{(3)_{..}} = \frac{n_1 \bar{T}_{(3)1.} + n_2 \bar{T}_{(3)2.} + \dots + n_t \bar{T}_{(3)t.}}{N}$		$MSE_{\bar{T}_{(3)_{..}}} = \frac{(n_1 - 1)S_{\bar{T}_{(3)1.}} + (n_2 - 1)S_{\bar{T}_{(3)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(3)t.}}}{N - t}$	
$\bar{T}_{(4)_{..}} = \frac{n_1 \bar{T}_{(4)1.} + n_2 \bar{T}_{(4)2.} + \dots + n_t \bar{T}_{(4)t.}}{N}$		$MSE_{\bar{T}_{(4)_{..}}} = \frac{(n_1 - 1)S_{\bar{T}_{(4)1.}} + (n_2 - 1)S_{\bar{T}_{(4)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(4)t.}}}{N - t}$	
$\bar{T}_{(5)_{..}} = \frac{n_1 \bar{T}_{(5)1.} + n_2 \bar{T}_{(5)2.} + \dots + n_t \bar{T}_{(5)t.}}{N}$		$MSE_{\bar{T}_{(5)_{..}}} = \frac{(n_1 - 1)S_{\bar{T}_{(5)1.}} + (n_2 - 1)S_{\bar{T}_{(5)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(5)t.}}}{N - t}$	

The findings are evaluated at a significance level of 5% ($\alpha = 5\%$) across different numbers of treatments. The simulation study is conducted using R software version (4.2.3), where simulations are iterated 10,000 times, and each combination of design parameters is outlined in Table 2.5.

Table 2. 3: Overall mean and mean squared error regarding ratio and regression estimators under difference designs.

Balanced design	
$\bar{M}_{..} = \sum_{i=1}^{n_i} \bar{M}_{i.} / t$	$MSE_{\bar{M}_{..}} = \sum_{i=1}^{n_i} S_{\bar{M}_{i.}} / t$
$\bar{T}_{(1).} = \sum_{i=1}^{n_i} \bar{T}_{(1)i.} / t$	$MSE_{\bar{T}_{(1).}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(1)i.}} / t$
$\bar{T}_{(2).} = \sum_{i=1}^{n_i} \bar{T}_{(2)i.} / t$	$MSE_{\bar{T}_{(2).}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(2)i.}} / t$
$\bar{T}_{(3).} = \sum_{i=1}^{n_i} \bar{T}_{(3)i.} / t$	$MSE_{\bar{T}_{(3).}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(3)i.}} / t$
$\bar{T}_{(4).} = \sum_{i=1}^{n_i} \bar{T}_{(4)i.} / t$	$MSE_{\bar{T}_{(4).}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(4)i.}} / t$
$\bar{T}_{(5).} = \sum_{i=1}^{n_i} \bar{T}_{(5)i.} / t$	$MSE_{\bar{T}_{(5).}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(5)i.}} / t$
Unbalanced design	
$\bar{M}_{..} = \frac{n_1 \bar{M}_{1.} + n_2 \bar{M}_{2.} + \dots + n_t \bar{M}_{t.}}{N}$	$MSE_{\bar{M}_{..}} = \frac{(n_1 - 1)S_{\bar{M}_{1.}} + (n_2 - 1)S_{\bar{M}_{2.}} + \dots + (n_t - 1)S_{\bar{M}_{t.}}}{N - t}$
$\bar{T}_{(1).} = \frac{n_1 \bar{T}_{(1)1.} + n_2 \bar{T}_{(1)2.} + \dots + n_t \bar{T}_{(1)t.}}{N}$	$MSE_{\bar{T}_{(1).}} = \frac{(n_1 - 1)S_{\bar{T}_{(1)1.}} + (n_2 - 1)S_{\bar{T}_{(1)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(1)t.}}}{N - t}$
$\bar{T}_{(2).} = \frac{n_1 \bar{T}_{(2)1.} + n_2 \bar{T}_{(2)2.} + \dots + n_t \bar{T}_{(2)t.}}{N}$	$MSE_{\bar{T}_{(2).}} = \frac{(n_1 - 1)S_{\bar{T}_{(2)1.}} + (n_2 - 1)S_{\bar{T}_{(2)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(2)t.}}}{N - t}$
$\bar{T}_{(3).} = \frac{n_1 \bar{T}_{(3)1.} + n_2 \bar{T}_{(3)2.} + \dots + n_t \bar{T}_{(3)t.}}{N}$	$MSE_{\bar{T}_{(3).}} = \frac{(n_1 - 1)S_{\bar{T}_{(3)1.}} + (n_2 - 1)S_{\bar{T}_{(3)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(3)t.}}}{N - t}$
$\bar{T}_{(4).} = \frac{n_1 \bar{T}_{(4)1.} + n_2 \bar{T}_{(4)2.} + \dots + n_t \bar{T}_{(4)t.}}{N}$	$MSE_{\bar{T}_{(4).}} = \frac{(n_1 - 1)S_{\bar{T}_{(4)1.}} + (n_2 - 1)S_{\bar{T}_{(4)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(4)t.}}}{N - t}$
$\bar{T}_{(5).} = \frac{n_1 \bar{T}_{(5)1.} + n_2 \bar{T}_{(5)2.} + \dots + n_t \bar{T}_{(5)t.}}{N}$	$MSE_{\bar{T}_{(5).}} = \frac{(n_1 - 1)S_{\bar{T}_{(5)1.}} + (n_2 - 1)S_{\bar{T}_{(5)2.}} + \dots + (n_t - 1)S_{\bar{T}_{(5)t.}}}{N - t}$

The evaluation of test performance is based on two criteria: tests with a type I error probability close to $\alpha = 0.05$ and high power are considered the best. The empirical estimation of Type I error rates for ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests was calculated by determining the number of null hypotheses (H_0) that erroneously fell outside the interval (either above the upper decision line (UDL) or below the lower decision line (LDL)) in 10,000 simulation trials. The calculated number was then divided by 10,000 to obtain the empirical Type I error rate.

Table 2. 4: Lower and upper decision lines regarding ratio estimators under balanced and unbalanced design.

Balanced design	
$LDL_{\bar{M}_L} = \bar{M}_{..} - C(\alpha, t, n)(MSE_{\bar{M}_{..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{M}_L} = \bar{M}_{..} + C(\alpha, t, n)(MSE_{\bar{M}_{..}}) \sqrt{\frac{t-1}{N}}$
$LDL_{\bar{T}_{(1)L}} = \bar{T}_{(1)..} - V(\alpha, t, n)(MSE_{\bar{T}_{(1)..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{T}_{(1)L}} = \bar{T}_{(1)..} + V(\alpha, t, n)(MSE_{\bar{T}_{(1)..}}) \sqrt{\frac{t-1}{N}}$
$LDL_{\bar{T}_{(2)L}} = \bar{T}_{(2)..} - Z(\alpha, t, n)(MSE_{\bar{T}_{(2)..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{T}_{(2)L}} = \bar{T}_{(2)..} + Z(\alpha, t, n)(MSE_{\bar{T}_{(2)..}}) \sqrt{\frac{t-1}{N}}$
$LDL_{\bar{T}_{(3)L}} = \bar{T}_{(3)..} - K(\alpha, t, n)(MSE_{\bar{T}_{(3)..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{T}_{(3)L}} = \bar{T}_{(3)..} + K(\alpha, t, n)(MSE_{\bar{T}_{(3)..}}) \sqrt{\frac{t-1}{N}}$
$LDL_{\bar{T}_{(4)L}} = \bar{T}_{(4)..} - P(\alpha, t, n)(MSE_{\bar{T}_{(4)..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{T}_{(4)L}} = \bar{T}_{(4)..} + P(\alpha, t, n)(MSE_{\bar{T}_{(4)..}}) \sqrt{\frac{t-1}{N}}$
$LDL_{\bar{T}_{(5)L}} = \bar{T}_{(5)..} - O(\alpha, t, n)(MSE_{\bar{T}_{(5)..}}) \sqrt{\frac{t-1}{N}}$	$UDL_{\bar{T}_{(5)L}} = \bar{T}_{(5)..} + O(\alpha, t, n)(MSE_{\bar{T}_{(5)..}}) \sqrt{\frac{t-1}{N}}$
Unbalanced design	
$LDL_{\bar{M}_L} = \bar{M}_{..} - C^*(\alpha, t)(MSE_{\bar{M}_{..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{M}_L} = \bar{M}_{..} + C^*(\alpha, t)(MSE_{\bar{M}_{..}}) \sqrt{\frac{N-n_i}{Nn_i}}$
$LDL_{\bar{T}_{(1)L}} = \bar{T}_{(1)..} - V^*(\alpha, t)(MSE_{\bar{T}_{(1)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{T}_{(1)L}} = \bar{T}_{(1)..} + V^*(\alpha, t)(MSE_{\bar{T}_{(1)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$
$LDL_{\bar{T}_{(2)L}} = \bar{T}_{(2)..} - Z^*(\alpha, t)(MSE_{\bar{T}_{(2)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{T}_{(2)L}} = \bar{T}_{(2)..} + Z^*(\alpha, t)(MSE_{\bar{T}_{(2)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$
$LDL_{\bar{T}_{(3)L}} = \bar{T}_{(3)..} - K^*(\alpha, t)(MSE_{\bar{T}_{(3)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{T}_{(3)L}} = \bar{T}_{(3)..} + K^*(\alpha, t)(MSE_{\bar{T}_{(3)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$
$LDL_{\bar{T}_{(4)L}} = \bar{T}_{(4)..} - P^*(\alpha, t)(MSE_{\bar{T}_{(4)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{T}_{(4)L}} = \bar{T}_{(4)..} + P^*(\alpha, t)(MSE_{\bar{T}_{(4)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$
$LDL_{\bar{T}_{(5)L}} = \bar{T}_{(5)..} - O^*(\alpha, t)(MSE_{\bar{T}_{(5)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$	$UDL_{\bar{T}_{(5)L}} = \bar{T}_{(5)..} + O^*(\alpha, t)(MSE_{\bar{T}_{(5)..}}) \sqrt{\frac{N-n_i}{Nn_i}}$

To obtain the test power, the following steps illustrate the process:

- 1- The parameter space was specified according to Table 2.5.
- 2- Specific constant values representing the effect sizes ($\delta = 0.25, 0.50, 0.75, 1.00$) were added to the random numbers of one or more treatment groups. This ensured that the means of at least one treatment group fell outside the decision lines.
- 3- The new datasets underwent ANOMC tests, and decision lines were calculated. This process was repeated 10,000 times.

- 4- The count of trials (j) in which the means of any of the treatments exceeded the decision lines was recorded.
- 5- The test's power was obtained by dividing this count by 10,000, representing the total number of repetitions.

Table 2. 5: Critical values against different levels of significance (α).

Balanced Design								
t	ANOMC-Reg		ANOMC-R	ANOMC-MR 1	ANOMC-MR 2	ANOMC-MR 3	ANOMC-MR 4	
	$C(\alpha, t, n)$		$V(\alpha, t, n)$	$Z(\alpha, t, n)$	$K(\alpha, t, n)$	$P(\alpha, t, n)$	$O(\alpha, t, n)$	
	n	α	0.01					
3	n_1		11.2	10	9.99	11	9.98	9.8
	n_2		11.1	12.5	12.36	12.5	12.38	12.16
	n_3		12.65	14.7	14.7	14.5	14.54	14.51
4	n_1		11.5	9.51	9.5	11.2	9.43	9.4
	n_2		11.3	12.05	12.1	12	12	11.7
	n_3		12.85	14.21	14.2	14.1	14.2	14
5	n_1		11.86	9.35	9.4	12.7	9.1	9.1
	n_2		11.7	11.91	11.93	11.73	11.8	11.7
	n_3		12.8	14.19	14.1	14.1	14.2	13.78
t	α		0.05					
	n	α						
3	n_1		7.67	7.39	7.36	7.52	7.38	7.25
	n_2		8.59	9.652	9.55	9.58	9.62	9.54
	n_3		9.95	11.564	11.5	11.43	11.55	11.48
4	n_1		8.04	7.23	7.23	7.4	7.16	7
	n_2		8.96	9.52	9.47	9.4	9.45	9.35
	n_3		10.4	11.46	11.26	11.34	11.4	11.3
5	n_1		8.3	7.21	7.21	7.5	7.1	6.92
	n_2		9.33	9.56	9.52	9.4	9.47	9.32
	n_3		10.8	11.4	11.3	11.34	11.43	11.27
Unbalanced Design								
t	ANOMC-Reg		ANOMC-R	ANOMC-MR1	ANOMC-MR2	ANOMC-MR3	ANOMC-MR4	
	$C^*(\alpha, t)$		$V^*(\alpha, t)$	$Z^*(\alpha, t)$	$K^*(\alpha, t)$	$P^*(\alpha, t)$	$O^*(\alpha, t)$	
	n	α	0.01					
3	n_4		12.6	11.6	11.65	14.8	11.5	11.5
	n_5		12.78	13.1	13.2	13.5	13.13	12.9
	n_6		15.65	16.1	16.05	16.55	16.26	15.6
4	n_4		13.2	10.04	10.28	17.75	9.99	9.9
	n_5		12.6	12.4	12.34	12.64	12.1	12
	n_6		16.8	18.41	18.3	18.42	18.36	18.2
5	n_4		14.3	11.76	11.8	17.86	11.48	11.4
	n_5		15.24	14.75	14.8	15.6	14.6	14.4
	n_6		18.57	19.86	19.86	19.6	19.7	19.7
t	α		0.05					
	n	α						
3	n_4		8.6	8.68	8.69	9.35	8.65	8.54
	n_5		9.36	10.058	10	10.1	10	9.92
	n_6		11.27	12.4	12.19	12.53	12.28	12.1
4	n_4		8.7	7.65	7.65	8.7	7.57	7.47
	n_5		9.5	9.6	9.56	9.6	9.56	9.38
	n_6		13.65	14.75	14.57	14.7	14.7	14.65
5	n_4		9.99	9.26	9.24	10.1	9.22	9
	n_5		11.8	11.85	11.75	11.9	11.8	11.7
	n_6		15.4	16.2	16.1	16	16.16	16.06

2.4 Comparison Performance

To evaluate and compare the performance of ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests, Tables 2.6-2.16 present the empirical type I error rates and power tests for ANOMC tests.

2.4.1 Results of type I error rates

The relative performances for ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests in terms of empirical type I error rates under null case (δ_1) are given in Tables 2.6-2.11.

I. Empirical type I error rate under homogeneous variances

Tables 2.6-2.8 present the results for the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests, considering both balanced and unbalanced designs while assuming the homogeneity of variances. The performance of these tests is evaluated using Bradley's liberal criterion [65], where values falling within the range of 2.5% to 7.5% are considered liberal or conservative. In the case of a balanced design, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests generally exhibit similar levels of type I error rates, except when applied to the Chi-square and Cauchy distributions. Similar patterns are also observed in the unbalanced design, but the ANOMC-Reg test shows higher than acceptable type I error rates, specifically with the Chi-square distribution. Furthermore, it is important to note that in all scenarios, whether the data follows a normal or non-normal distribution, the ANOMC-Reg test displays a high percentage of type I error rates when dealing with the direct pairing of correlations (ρ_2).

Table 2. 6: Choices considered for different design parameters.

Parameter space	Number of treatments (<i>t</i>)		
	3	4	5
Distribution (<i>s</i> , <i>kr</i>)	(0,0), (0,3), (2,6), (0,25)	(0,0), (0,3), (2,6), (0,25)	(0,0), (0,3), (2,6), (0,25)
Correlation (ρ)			
ρ_1	0.5: 0.5: 0.5	0.5: 0.5: 0.5: 0.5	0.5: 0.5: 0.5: 0.5: 0.5
ρ_2	0.5: 0.5: 0.8	0.5: 0.5: 0.8: 0.8	0.5: 0.5: 0.5: 0.8: 0.8
ρ_3	0.8: 0.5: 0.5	0.8: 0.8: 0.5: 0.5	0.8: 0.8: 0.5: 0.5: 0.5
ρ_4	0.8: 0.8: 0.8	0.8: 0.8: 0.8: 0.8	0.8: 0.8: 0.8: 0.8: 0.8
Sample size (<i>n</i>)			
n_1	5: 5: 5	5: 5: 5: 5	5: 5: 5: 5: 5
n_2	10: 10: 10	10: 10: 10: 10	10: 10: 10: 10: 10
n_3	15: 15: 15	15: 15: 15: 15	15: 15: 15: 15: 15
n_4	4: 7: 10	4: 4: 7: 7	4: 6: 8: 10: 15
n_5	5: 10: 15	5: 8: 10: 15	5: 10: 15: 20: 25
n_6	5: 15: 25	10: 20: 30: 40	10: 20: 30: 40: 50
Variance ratios (<i>v</i>)			
v_1	1: 1: 1	1: 1: 1: 1	1: 1: 1: 1: 1
v_2^+	1: 1: 1.05	1: 1: 1: 1.05	1: 1: 1: 1: 1.05
v_2^-	1.05: 1: 1	1.05: 1: 1: 1	1.05: 1: 1: 1: 1
v_3^+	1: 1: 1.1	1: 1: 1: 1.1	1: 1: 1: 1: 1.1
v_3^-	1.1: 1: 1	1.1: 1: 1: 1	1.1: 1: 1: 1: 1
Effect size (δ)			
δ_1	0: 0: 0	0: 0: 0: 0	0: 0: 0: 0: 0
δ_2	0: 0: 1	0: 0: 0: 1	0: 0: 0: 0: 1
δ_3	0: 0.25: 1	0: 0.50: 0.50: 1	0: 0.25: 0.50: 0.75: 1
δ_4	0: 0.50: 1	0: 0.25: 0.75: 1	0: 0: 0.25: 0.75: 1
δ_5	0: 0.75: 1	0: 0: 1: 1	0: 0: 0.25: 0.25: 1
δ_6	0: 1: 1	0: 0.25: 0.50: 1	0: 0: 0: 1: 1
δ_7	0.25: 0: 1	0.25: 0: 0: 1	0.25: 0: 0: 0: 1
δ_8	1: 0: 0.25	1: 0: 0: 0.25	1: 0: 0: 0: 0.25
δ_9	0.50: 0: 1	0.50: 0: 0: 1	0.50: 0: 0: 0: 1
δ_{10}	1: 0: 0.50	1: 0: 0: 0.50	1: 0: 0: 0: 0.50

In general, across both balanced and unbalanced designs, an increase in the number of treatments leads to excessive type I error rates for the ANOMC-Reg test, particularly when considering the Cauchy distribution. It is worth highlighting that unequal correlations, whether direct or indirect, can also impact the percentage of type I error rate, deviating from the specified value of $\alpha = 5\%$.

Table 2. 7: Type I error (%) of ANOMC tests under homogeneous variances when $t = 3$.

		Balanced Design											
n	ρ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	5.09	4.81	10.25*	4.52	4.94	4.44	4.97	3.12	5.04	4.56	6.19	3.05
	ρ_2	5.87	5.13	10.9*	4.82	5.42	4.8	5.28	3.63	5.54	4.61	6.13	3.57
	ρ_3	5.31	5.04	11.35*	4.66	5.35	4.65	5.1	3.1	5.59	4.88	6.32	3.38
	ρ_4	5.08	4.94	10.79*	4.61	4.64	4.09	4.44	3.02	5.02	4.59	5.53	2.97
n_2	ρ_1	5.31	5.08	7.6*	5.77	5.11	4.77	5.11	3.89	5.08	5.12	5.35	4.26
	ρ_2	5.52	5.62	8.54*	5.57	5.23	4.72	5.6	3.8	5.34	5.42	6.45	4.6
	ρ_3	5.4	5.54	8.68*	5.76	5.5	5.02	5.8	4.1	5.89	5.22	6.33	4.59
	ρ_4	5.13	5.26	7.38	5.65	5.11	4.61	4.45	3.84	5.24	5.22	5.52	3.62
n_3	ρ_1	4.89	5.1	6.78	5.58	4.68	4.77	4.98	4.15	5.31	4.65	5.65	4.41
	ρ_2	5.6	5.31	6.94	6.07	5.15	4.84	5.17	4.05	5.8	5.19	6.3	4.21
	ρ_3	5.23	5.59	7.03	6.37	5.27	5.47	5.65	4.64	5.89	5.03	5.87	4.91
	ρ_4	5.08	5.35	6.75	5.31	5.17	4.78	4.44	4.12	5.01	5.04	5.37	3.68
n_1		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	5.04	4.43	4.8	3.52	4.67	4.03	5.98	3.31	5.05	4.13	5.67	3.41
	ρ_2	4.96	4.32	4.87	3.11	5.37	5.36	6.15	3.36	4.82	4.8	5.83	3.33
	ρ_3	4.75	4.07	5.2	3.35	5.1	4.69	6.28	3.54	5.09	4.4	5.8	3.3
	ρ_4	4.77	4.22	5.03	3.16	4.75	4.19	5.22	3.07	4.47	4.47	6.24	3.42
n_2	ρ_1	4.97	4.89	6.04	3.99	4.96	4.79	5.63	3.68	4.79	4.65	5.39	3.79
	ρ_2	4.63	4.78	5.73	3.67	5.09	5.01	5.95	3.99	4.84	4.48	5.65	3.56
	ρ_3	4.95	4.72	5.77	3.99	5.24	4.98	6.22	3.9	4.95	4.66	5.46	3.81
	ρ_4	4.93	4.64	5.6	3.48	4.64	4.4	5.54	3.35	5.1	4.74	5.6	4.23
n_3	ρ_1	4.69	5.19	5.38	4.39	4.57	4.66	5.67	4.05	4.9	4.71	5.08	4.13
	ρ_2	5.27	4.79	5.6	4.27	5.58	4.62	5.86	4.31	4.86	4.72	5.54	4.16
	ρ_3	4.88	5.25	5.56	4.32	5.2	5.26	5.54	4.45	4.89	4.48	6.04	4.27
	ρ_4	4.74	4.95	5.73	4.19	5.5	4.79	5.13	4.12	4.95	4.93	5.57	4.13
Unbalanced Design													
		(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_4		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	5.22	4.52	9.39*	4.7	5.22	4.97	4.7	3.45	4.81	4.71	5.42	3.31
	ρ_2	7	6.76	12.28*	5.99	5.58	5.27	5.81	3.39	5.83	5.23	6.57	3.33
	ρ_3	3.95	3.37	8.18*	4.09	4.34	4.18	4.66	3.52	4.48	4.16	5.22	3.11
	ρ_4	5.17	4.93	10.12*	4.74	5.28	4.48	4.42	3	4.73	4.22	4.95	2.74
n_5	ρ_1	5.13	4.99	8.48*	5.09	5.22	4.53	5.42	3.49	5.06	4.42	5.73	3.74
	ρ_2	7.69*	7.42	11.6*	7.39	5.94	5.18	5.76	3.66	6.02	5.4	6.91	3.78
	ρ_3	3.35	3.48	6.77	3.55	4.31	4.31	4.76	3.87	4.82	4.45	5.66	3.85
	ρ_4	4.93	5.08	8.85*	5.2	4.89	4.76	5.02	3.2	4.9	4.82	5.5	3.81
n_6	ρ_1	5.21	4.93	7.87*	5.58	4.56	4.49	4.11	3.47	5.13	4.11	5.69	3.71
	ρ_2	8.34*	8.37*	11.27*	8.42*	5.79	5.43	5.06	3.62	5.58	5.12	6.03	3.86
	ρ_3	3.17	3.06	5.99	3.33	4.58	4.42	4.47	3.68	5	4.34	5.64	4.16
	ρ_4	5.14	5.36	7.55*	5.21	4.8	4.48	4.73	3.29	5.12	4.67	5.91	3.59
n_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	4.85	4.22	5.03	2.75	4.77	4.41	5.81	3.18	4.8	4.04	5.69	3.58
	ρ_2	5.59	4.06	4.75	2.63	6.13	5.46	6.23	3.64	5.22	4.42	5.74	3.32
	ρ_3	5.77	4.77	5.06	2.59	4.77	4.44	5.4	3.43	4.92	4.32	5.92	3.64
	ρ_4	5.43	4.57	5.3	2.86	4.81	4.61	5.41	3.19	4.85	4.79	5.96	3.37
n_5	ρ_1	4.83	4.24	5.36	3.64	5.1	4.79	5.39	3.25	4.7	4.41	5.55	3.56
	ρ_2	4.72	4.85	5.52	3.33	5.43	5.52	6.34	4.01	4.34	4.76	5.82	3.75
	ρ_3	5.07	4.47	5.91	3.52	5.19	4.58	5.62	3.78	5.02	4.62	5.52	3.45
	ρ_4	5.12	4.6	5.6	3.87	4.93	4.69	5.96	3.52	4.6	4.28	5.77	3.93
n_6	ρ_1	4.65	4.24	5.45	3.37	5.13	4.26	5.29	3.34	5.04	4.25	5.78	3.86
	ρ_2	4.92	4.14	5.29	3.29	5.86	5.34	6.54	4.45	5.4	4.74	5.83	4.1
	ρ_3	4.85	4.37	5.34	3.72	4.52	4.72	5.11	4.52	5.06	4.65	5.83	4.03
	ρ_4	5.37	4.4	4.93	3.27	5.01	4.42	5.18	3.56	4.88	4.75	5.74	3.83

* the percentage type I error lies outside Bradley's liberal criterion.

Table 2. 8: Type I error (%) of ANOMC tests under homogeneous variances when $t = 4$.

		Balanced Design											
n	ρ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	5.17	5.14	12.63*	6.12	4.53	4.67	5.36	3.49	4.72	4.03	6.51	3.47
	ρ_2	5.51	5.63	12.81*	6.92	6.31	5.28	5.96	3.89	6.12	5.1	6.91	4.35
	ρ_3	5.87	5.7	13.09*	6.73	5.61	5.28	6.03	4.15	5.33	5.33	7.3	4.08
	ρ_4	5.24	5.38	12.29*	5.94	4.54	4.55	4.76	4.02	4.59	4.27	6	3.81
n_2	ρ_1	5.22	5.57	8.76*	7.43	4.93	4.76	5.2	4.17	5.16	4.92	6.05	4.06
	ρ_2	6.09	6.27	9.85*	8.94*	5.73	5.92	6.27	4.82	6.26	5.76	6.94	4.94
	ρ_3	6.29	6.83	9.64*	8.64*	5.78	6.03	6.68	5.13	5.68	5.78	6.5	5.09
	ρ_4	5.15	5.24	8.74*	7.54*	4.95	4.63	4.85	4.44	5.11	4.84	5.83	3.99
	ρ_1	4.94	5.49	7.51*	7.21	4.77	4.7	5.14	4.59	5.43	5.11	6.27	4.73
n_3	ρ_2	6.51	6.89	8.69*	7.98*	6.24	5.53	5.87	5.1	6.14	6.41	7.33	5.87
	ρ_3	6.25	6.61	8.21*	8.46*	5.57	5.67	6.5	5.43	6.44	6.22	7.06	5.83
	ρ_4	4.98	5.2	7.24	6.84	4.91	4.72	4.67	4.4	5.06	5.77	5.8	4.49
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	5.11	4.66	5.27	3.63	4.98	4.54	6.64	4.11	5.26	4.76	6.35	4.64
n_1	ρ_2	4.99	4.27	5.52	3.73	5.68	5.7	6.74	4.12	4.83	5.1	6.51	4.41
	ρ_3	5.09	4.15	5.2	3.58	5.94	5.31	7.01	4.45	4.81	4.7	6.66	4.22
	ρ_4	4.71	4.27	5.35	3.82	4.81	4.55	6.25	3.86	4.86	4.89	6.68	4.2
	ρ_1	4.63	4.88	6.08	4.29	5.35	5.13	6.41	4.57	5	4.4	6.29	4.86
	ρ_2	4.9	4.75	6.09	4.66	6.24	5.81	6.9	5.14	5.15	5.16	6.16	4.22
n_2	ρ_3	4.98	5.09	6.46	4.31	6.12	5.79	7.05	5.23	5.2	5.19	6	4.4
	ρ_4	5.38	4.88	5.78	4.48	5.34	4.83	5.33	4.25	5.41	4.69	5.88	4.75
	ρ_1	5.17	5.35	5.48	4.87	5.45	4.97	5.66	4.43	5.01	4.63	6.23	4.37
	ρ_2	5.23	4.72	6.14	4.34	6.12	5.68	6.46	5.28	5.02	4.97	5.96	4.75
	ρ_3	5.32	4.94	5.91	4.73	5.84	6.08	6.76	5.46	5.11	4.84	5.78	4.79
ρ_4	4.9	5.02	5.91	4.15	4.79	4.73	5.41	4.36	5.43	4.78	5.95	4.62	
Unbalanced Design													
		(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_4		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	5.06	5.23	12.46*	5.68	4.9	4.45	5.25	3.9	4.53	4.8	6.43	3.57
	ρ_2	7.79*	7.43	14.77*	8.08*	6.59	6.14	6.47	4.67	6.54	6.18	8.03*	4.51
	ρ_3	3.6	3.85	10.42*	4.44	5.02	5.02	5.33	3.76	4.75	4.7	6.29	4.15
	ρ_4	5.13	4.79	12.15*	5.85	5.25	4.35	4.86	3.74	4.64	4.2	6.06	3.75
n_5	ρ_1	5.1	5.23	9.61*	7	5.11	4.96	5.29	4.6	4.78	4.61	6.37	4.11
	ρ_2	8.76*	7.99*	13.19*	10.45*	6.73	5.99	6.42	5.12	6.41	5.96	7.38	5.01
	ρ_3	4.22	4.65	7.49	5.91	5.34	5.01	5.29	4.36	5.39	5.54	6.29	4.81
	ρ_4	5.08	5.84	9.67*	7.24	4.8	4.61	5.35	3.85	4.45	4.21	5.62	3.77
	ρ_1	4.99	5	6.24	7.1	5.16	5.27	5.19	4.75	5.81	4.99	6.48	5.03
n_6	ρ_2	9.24*	9.3*	9.82*	10.38*	5.66	6.39	6.39	5.66	6.35	6.72	7.44	5.99
	ρ_3	3.9	4.15	6.14	5.44	6.02	5.93	6.74	5.7	6.29	6.67	7.13	5.77
	ρ_4	4.89	4.84	6.43	6.58	5.37	5.23	5.21	4.3	5.27	5.11	5.69	5.1
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	4.54	4.44	3.86	2.79	5.27	4.91	6.49	3.77	5.58	4.93	5.67	4.02
n_4	ρ_2	5.03	4.11	4.02	2.6	7.43	6.4	7.81*	4.56	4.79	4.75	6.61	4.37
	ρ_3	4.82	4.21	4.42	2.57	5.21	4.91	6.99	4	4.88	4.7	6.16	4.4
	ρ_4	5.27	4.06	3.81	2.88	4.84	4.3	5.4	3.46	5.12	5.31	6.25	3.97
	ρ_1	4.97	4.55	5.83	4.3	4.47	5.06	5.91	4.39	5	5.21	6.66	4.27
	ρ_2	4.73	4.39	5.8	3.83	6.23	5.23	7.09	4.68	5.01	4.58	5.99	4.09
n_5	ρ_3	5.41	4.57	5.35	4.14	5.25	5.52	6.39	4.6	4.95	4.81	6.02	4.01
	ρ_4	5.36	4.64	5.62	4.03	4.66	4.73	5.4	3.99	4.93	4.67	6.24	4.56
	ρ_1	4.86	4.74	5.67	4.53	5.1	4.98	5.88	4.79	5.18	4.93	5.57	4.82
	ρ_2	5.22	4.61	5.67	4.7	6.67	6.21	6.94	5.28	5.41	4.67	5.78	4.83
	ρ_3	5.1	4.98	5.71	4.76	6.37	5.97	7.04	5.98	5.03	5	5.53	4.79
ρ_4	5.08	4.61	5.56	4.78	5.3	5.09	5.6	4.65	5.26	5.2	6.09	4.57	

* the percentage type I error lies outside Bradley's liberal criterion

Table 2. 9: Type I error (%) of ANOMC tests under homogeneous variances when $t = 5$.

		Balanced Design											
n	ρ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	5.35	5.84	15.1*	8.17*	5.02	4.71	5.11	4.57	4.64	4.09	7.1	4.08
	ρ_2	5.38	6.49	15*	8.05*	5.58	5.73	6.22	4.96	5.51	5.16	7.04	4.34
	ρ_3	5.78	6.19	14.73*	8.86*	5.91	5.59	6.71	4.86	5.82	5	7.46	4.79
	ρ_4	4.75	5.63	14.8*	7.9*	4.26	4.43	5.37	4.32	4.16	4.07	6.23	4.11
n_2	ρ_1	4.92	5.45	9.8*	8.98*	4.83	5.07	5	4.68	4.51	4.84	6.36	4.71
	ρ_2	5.62	6.64	10.42*	10.27*	6.11	5.83	6.35	5.79	5.88	5.7	6.77	5.28
	ρ_3	5.98	6.99	10.32*	9.87*	6.02	6.28	6.17	5.57	6.5	5.98	7.21	5.56
	ρ_4	5.2	5.27	9.9*	8.62*	5.14	4.75	5.2	4.97	4.7	4.74	5.91	4.38
n_3	ρ_1	4.73	5.88	7.72*	9.08*	5.35	5.17	5.21	5.3	5.39	5.49	6.4	5.52
	ρ_2	6.26	7.45	9.13*	9.86*	6.38	6.34	6.11	5.94	6.51	6.32	7.59*	5.93
	ρ_3	5.83	6.89	9.08*	9.88*	5.98	6.15	5.88	6.19	6.56	6.84	7.41	6.54
	ρ_4	5.15	5.44	7.91*	9.01*	5.06	5.34	5.54	5.34	5.01	4.89	5.94	5.65
n_1		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	4.72	4.3	5.16	3.8	5.02	4.57	7.09	4.67	4.91	4.48	6.95	4.75
	ρ_2	5.1	4.2	5.05	3.66	5.87	5.7	7.79*	5.03	4.69	4.61	7.64*	5.23
	ρ_3	4.96	4.36	5.2	3.75	6.22	5.9	7.87*	4.87	5.08	5.05	6.88	5.2
	ρ_4	5.14	4.48	5.14	3.64	4.7	4.08	7.06	4.5	5.16	5.11	7.14	4.91
n_2	ρ_1	4.91	5.29	6.55	5.09	5.22	5.28	6.4	4.75	4.74	4.99	6.61	4.9
	ρ_2	5.09	4.84	6.43	5.03	6.07	6.63	7.54*	5.47	5.09	4.72	6.66	5.27
	ρ_3	4.88	5.1	6.1	5.24	6.31	6.14	7.85*	5.51	5.22	5.54	6.17	4.62
	ρ_4	4.99	4.69	6.14	4.74	4.64	4.85	6.07	4.69	4.95	5.16	5.98	4.89
n_3	ρ_1	5.09	4.76	5.45	5.32	5.09	4.97	5.91	4.77	4.9	5.37	5.97	5.13
	ρ_2	4.89	4.89	6.21	4.8	6.12	6.04	6.79	6.11	5.32	4.87	6.11	4.96
	ρ_3	5.17	4.6	6.05	5.05	5.92	5.96	6.91	5.74	5.16	5.35	6.78	5.04
	ρ_4	4.68	4.75	6.23	4.89	4.33	4.79	5.58	4.97	4.79	5.19	5.87	4.97
Unbalanced Design													
		(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_4		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	ρ_1	4.73	5.26	11.89*	8.41*	4.66	4.84	4.97	4.8	4.48	4.46	6.16	4.56
	ρ_2	8.35*	8.54*	14.5*	11.44*	6.22	6.18	6.59	5.33	6.29	6.02	7.61*	5.23
	ρ_3	3.68	4.12	9.64*	6.1	5.47	5.47	5.85	5.63	5.21	5.06	6.95	4.97
	ρ_4	5.32	5.45	11.53*	7.76*	4.67	4.35	5.32	4.89	4.54	4.64	6.67	4.38
n_5	ρ_1	5.33	5.26	9.73*	8.68*	4.9	4.86	5.37	4.81	4.8	5.29	6.12	5.05
	ρ_2	8.91*	9.7*	13.22*	11.91*	6.62	6.27	6.01	5.76	6.59	6.4	7.4	5.36
	ρ_3	3.68	4.68	7.28	6.62	5.44	5.61	6.44	5.67	5.77	6.17	7.43	6.01
	ρ_4	5.38	5.65	9.57*	8.72*	4.67	4.8	4.81	4.76	4.62	5.06	5.86	5.1
n_6	ρ_1	5.32	5.16	6.76	7.64*	4.8	5	5.43	5.25	5.28	5.15	5.82	4.93
	ρ_2	8.29*	9.24*	10.21*	11.66*	5.89	5.83	6.23	5.26	6.11	5.94	6.84	5.83
	ρ_3	4.47	4.73	5.79	6.1	6.4	6.49	6.68	6.27	6.33	6.86	6.8	6.54
	ρ_4	4.74	5.16	6.47	7.68*	4.76	4.58	5.27	4.99	4.53	5.27	5.35	5.25
n_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	ρ_1	4.63	4.09	4.4	3.42	5.02	4.53	6.27	4.63	4.98	5.06	6.29	4.92
	ρ_2	5	4.17	4.51	2.98	6.57	6.22	7.74*	5.06	5.18	5.43	6.63	4.96
	ρ_3	5.54	4.48	4.53	3.63	5.51	5.52	6.69	5.39	5.27	4.87	6.75	5.19
	ρ_4	5.15	4.14	4.05	3.15	4.97	4.83	6.26	4.29	4.77	5.34	6.92	5.56
n_5	ρ_1	4.96	4.85	5.59	4.48	4.79	4.43	5.89	4.93	4.88	4.86	5.72	5.14
	ρ_2	5.2	4.72	5.66	4.28	6.5	5.78	7.53	5.45	4.74	4.56	5.85	5.15
	ρ_3	5.07	4.07	5.55	4.56	5.56	6.2	6.71	5.52	4.75	4.83	6.05	4.9
	ρ_4	5.33	4.68	5.78	4.36	5.32	4.94	5.46	4.66	5	4.84	5.76	5.19
n_6	ρ_1	4.82	4.55	5.61	5.7	5.22	4.65	5.48	4.69	5.01	4.83	5.34	4.99
	ρ_2	4.99	4.7	5.74	5.12	6.3	6.39	6.23	6.2	4.84	4.87	6.1	5.09
	ρ_3	5.33	5.25	5.59	5.15	6.24	6.79	6.95	6.25	5.04	4.89	5.42	4.98
	ρ_4	5.39	4.83	5.61	5.29	5.02	5.22	5.29	5.31	4.67	4.84	5.78	5.17

* the percentage type I error lies outside Bradley's liberal criterion

Table 2. 10: Type I error (%) of ANOMC tests under heterogeneous variances when $t = 3$.

n	ρ	v	Balanced Design											
			(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
			ANOMC-Reg				ANOMC-R				ANOMC-MR1			
n_1	ρ_1	v_2^+	6.05	6.41	11.54	7.19	6.28	6.08	6.3	5.56	6.54	5.73	7.87	5.66
		v_2^-	6.03	5.81	11.92	7.14	6.06	5.79	6.15	5.55	5.89	5.81	7.94	5.45
		v_3^+	8.64	9.45	15.8	15.36	9.24	9.52	11.22	12.03	9.94	10.29	13.14	12.58
		v_3^-	8.34	9.66	14.65	15.25	9.54	9.91	10.81	11.99	9.85	9.91	13.47	12.19
	ρ_2	v_2^+	6.06	6.07	13.16	7.76	7.02	6.34	8.04	6.13	6.74	6.07	9.42	5.72
		v_2^-	6.56	6.71	12.52	8.07	6.85	6.31	7.02	6.64	7.34	6.99	8.46	6.44
		v_3^+	10.14	10.45	18.88	18.36	11.05	11.2	14.88	15.46	10.45	10.4	16.29	15.15
		v_3^-	10.7	11.09	16.16	18.83	11.4	11.87	12.56	15.24	12.56	12.48	14.94	16.41
	ρ_3	v_2^+	6.49	6.74	13.06	8.44	6.68	7.07	6.83	6.56	7.16	6.77	8.49	6.66
		v_2^-	6.42	6.11	13.63	7.67	6.33	6.38	8.34	6.01	6.44	6.07	9.13	6.02
		v_3^+	10.59	11.33	15.76	19.4	11.73	12.21	12.94	15.3	12.5	12.17	15.07	16.99
		v_3^-	10.07	10.3	18.78	19	10.94	10.54	15.13	15.01	10.83	11.02	16.64	15.35
	ρ_4	v_2^+	7.06	7.39	13.32	9.99	7.74	7.39	8.2	9.61	8.41	7.91	10.33	10.58
		v_2^-	6.74	6.96	13.7	9.56	7.92	8.36	8.19	8.97	8.03	8.38	9.63	10.36
		v_3^+	13.71	14.58	21.23	27.69	17.63	19.03	20.75	28.42	17.92	19.49	23.25	30.2
		v_3^-	12.32	15.1	21.45	26.95	17.78	18.87	21.05	27.87	17.96	19.8	24.06	30.67
	ρ_1	v_2^+	5.5	5.3	5.94	4.71	6.17	5.34	7.48	5.09	5.84	5.73	7.26	5.33
		v_2^-	5.41	5.47	6.02	4.67	5.97	5.09	7.36	5.4	6.09	5.87	7.24	5.65
		v_3^+	8.04	7.99	9.25	10.2	9.64	9.88	12.22	12.37	9.67	10.22	11.67	12.54
		v_3^-	8.48	8.52	9.53	10.33	10.25	10.08	12.46	11.75	9.01	10.05	12.36	13
	ρ_2	v_2^+	5.85	5.42	6.13	5.18	5.69	6.21	8.67	5.79	6.21	5.47	7.49	5.65
		v_2^-	5.55	5.13	6.34	4.6	6.98	6.91	7.85	6.27	6.16	5.81	7.53	5.64
		v_3^+	7.64	8.17	9.63	9.17	9.85	10.38	15.56	13.9	8.92	9.77	11.86	12.31
		v_3^-	7.79	7.89	9.7	9.4	11.89	11.75	14.55	15.88	9.63	10.07	11.31	11.73
	ρ_3	v_2^+	5.72	5.34	6.37	4.73	6.51	6.74	8.34	6.44	5.84	5.51	6.88	4.93
		v_2^-	5.76	5.62	5.58	4.78	6.07	6.09	8.66	5.8	6	5.86	7.54	5.83
		v_3^+	8.83	8.66	9.04	9.79	11.62	10.87	14.33	16.08	10.07	9.79	11.55	12.09
		v_3^-	8.38	7.88	9.33	10.15	10.26	10.37	15.23	14.75	9.46	9.51	11.88	12.59
	ρ_4	v_2^+	5.64	5.27	5.83	4.81	7.81	8.19	9.58	8.65	6.07	5.76	7.73	5.25
		v_2^-	5.79	4.73	6	5.15	7.87	7.56	9.61	9.39	5.89	5.7	7.07	5.59
		v_3^+	7.69	7.56	9.21	9.87	17.92	18.51	22.47	27.85	8.81	9.6	11.39	11.94
		v_3^-	7.85	7.91	8.93	9.76	17.63	19.02	22.87	28.32	9.71	9.82	11.26	12.53
n_2	ρ_1	v_2^+	8.13	8.84	11.59	12.46	7.86	7.71	8.28	7.95	7.85	8.21	9.78	8.53
		v_2^-	8.11	8.69	11.25	12.74	7.81	7.97	8.25	7.87	8.42	7.93	9.48	9.07
		v_3^+	18.89	20.35	23	32.56	16.73	17.02	18.11	20.59	18.18	18.63	20.81	22.88
		v_3^-	18.45	20.04	23.88	31.92	16.47	17.23	19.01	20.8	17.82	18.64	21.94	22.44
	ρ_2	v_2^+	9.53	9.6	14	13.81	8.42	8.36	10.35	9.18	8.39	8.21	10.93	8.73
		v_2^-	9.36	10.61	12.16	14.78	8.97	9.09	9.12	9.76	9.09	10.11	11.11	10.41
		v_3^+	21.77	24.01	31.49	37.89	20.55	21.32	24.37	26.91	19.75	20.99	27.76	27.2
		v_3^-	22.7	24.94	26.16	38.23	19.98	20.64	21.22	26.34	21.82	23.03	25.02	28.56
	ρ_3	v_2^+	9.69	9.86	12.51	14.99	9.38	9.46	8.62	9.88	9.54	9.79	10.64	10.8
		v_2^-	8.67	9.59	14.56	14.34	8.33	8.44	9.91	9.58	8.37	8.85	11.81	9.37
		v_3^+	22.93	25.12	26.46	37.18	19.23	20.36	21.13	25.86	21.78	22.45	24.44	27.43
		v_3^-	23.09	24.47	31.22	38.88	20.32	20.57	24.62	27.07	19.49	20.98	26.89	28.01
	ρ_4	v_2^+	11.83	13.44	15.89	19.47	12.9	12.34	12.56	15.34	12.92	13.84	15.58	18.1
		v_2^-	12.02	13.48	15.93	19.62	11.69	12.78	12.76	15.13	12.9	12.96	16.05	16.31
		v_3^+	34.87	37.79	41.48	53.69	34.37	38.7	40.3	47.57	38.44	39.72	42.69	49.85
		v_3^-	34.6	38.73	41.92	53.65	36.88	37.34	40.39	47.28	37.55	39.7	42.4	49.93
	ρ_1	v_2^+	7.43	7.98	8.89	8.01	8.21	8.19	8.7	8.06	8.05	7.19	9.13	7.97
		v_2^-	7.91	7.14	8.95	8.79	7.92	7.68	8.97	8.3	7.66	7.8	8.61	8.15
		v_3^+	15.7	16.89	18.71	20.41	16.3	17.27	19.76	21.48	17.39	17.59	19.34	21.11
		v_3^-	16.64	17.28	17.81	20.42	16.19	17.38	19.93	20.73	16.85	17.85	18.7	21.03
	ρ_2	v_2^+	7.84	7.84	8.75	7.79	7.81	7.82	10.64	8.69	7.81	7.6	9.15	8.35
		v_2^-	7.89	7.7	9.23	7.74	9.64	9.06	10.15	10.45	7.61	7.69	9.58	8.01
		v_3^+	16.72	16.29	18.56	21.08	18.17	19.99	24.35	25.67	16.98	17.56	18.9	21.66
		v_3^-	16.66	16.04	18.78	21.22	20.47	20.47	23.54	26.68	17.12	17.21	19.65	21.55
	ρ_3	v_2^+	7.93	7.65	8.74	7.57	9.02	9.04	10.3	10.17	7.23	7.7	9.27	8.06
		v_2^-	7.55	7.49	9.31	8.29	8.44	8.42	11.02	8.93	8	7.44	8.94	8.12
		v_3^+	16.2	17.52	18.83	20.08	21	22.31	23.35	26.52	16.64	17.71	19.58	20.92
		v_3^-	15.9	17.53	19.06	21.07	19.74	19.82	25.11	25.69	16.87	17.22	19.04	21.58
	ρ_4	v_2^+	7.77	7.76	8.76	7.61	12.23	12.64	14.1	15.66	7.23	7.78	9.23	7.98
		v_2^-	7.77	7.38	8.73	7.62	12.36	13.37	14.02	15.32	8.05	7.35	8.79	8.11
		v_3^+	16.52	17.02	18.35	21.4	35.42	37.63	41.34	46.89	16.79	17.73	18.67	21.18
		v_3^-	15.68	17.04	17.94	20.45	35.81	39.21	41.87	46.78	17.41	16.82	19.54	21.39

n	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)		
n_3	ρ_1	v_2^+	ANOMC-Reg				ANOMC-R				ANOMC-MR1					
		v_2^-	9.99	11.22	13.18	15.28	9.02	9.37	9.23	10.58	9.81	9.63	11.31	10.23		
		v_3^+	10.92	11.72	12.32	15.77	8.7	9.29	9.8	10.09	9.91	9.98	10.6	10.87		
		v_3^-	28.49	30.94	32.86	42.17	23.72	24.58	25.03	28.54	25.54	25.71	27.5	31.17		
		v_2^+	28.38	30.3	33	42.67	23.44	25.07	25.2	28.8	24.89	26.07	28.22	30.5		
		v_2^-	11.49	13.05	16.65	17.91	10.22	10.64	12.04	11.45	10.83	10.02	13.7	12.28		
		ρ_2	v_2^-	12.85	13.1	14.12	18.66	11.16	11.29	10.89	12.76	10.98	12.33	12.48	13.47	
			v_3^+	36.14	38.07	42.05	51.05	29.85	31.31	34.26	37.04	30.31	31.94	35.3	38.3	
	v_3^-		34.13	37.51	37.15	48.14	28.47	28.82	29.84	35.3	30.3	31.05	32.46	38.01		
	v_2^+		12.34	13.71	14.83	17.94	10.62	10.96	11	12.73	12.76	11.86	12.69	13.43		
	ρ_3	v_2^-	11.97	12.23	16.3	18.44	10.34	10.59	12.21	11.66	10.25	10.25	13.18	11.73		
		v_3^+	34.4	36.71	37.43	49.67	28.19	29.78	30.11	35.33	30.3	31.86	32.06	38.56		
		v_3^-	34.95	38.64	42.31	50.51	30.78	31.19	34.06	38.23	29.74	31.56	35.07	37.72		
		v_2^+	16.66	17.74	20.65	26.63	16.78	16.77	17.41	21.42	17.29	18.32	19.84	22.7		
	ρ_4	v_2^-	16.84	18.19	19.97	26.86	16.61	17.63	17.35	21.55	17.31	18.25	19.13	21.98		
		v_3^+	55.19	55.89	58.69	67.77	52.84	53.52	56.49	60.37	55.22	56.73	59.03	63.34		
		v_3^-	54.88	57.43	58.74	67.48	52.17	54.13	55.27	60.78	54.7	56.11	59.59	62.28		
	n_3	ρ_1	v_2^+	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
			v_2^-	10.08	9.97	10.88	10.86	9	9.72	10.32	10.19	9.24	9.08	10.19	10.76	
			v_3^+	9.76	9.52	10.86	10.5	9.35	9.29	10.82	10.19	9.01	9.34	10.51	10.06	
			v_3^-	24.69	24.57	26.26	29.64	24.1	23.95	26.58	29.24	24.16	25.04	26.66	28.63	
			v_2^+	23.45	25.1	27.44	29.56	23.66	24.03	26.8	28.68	23.97	25.39	26.56	29.04	
			v_2^-	10	9.5	10.5	10.29	9.94	10.14	12.5	11.23	9.3	9.38	10.73	10.3	
			ρ_2	v_2^-	9.06	9.62	10.88	10.22	10.54	11.64	11.88	12.25	9.11	9.77	10.44	10.86
				v_3^+	24.18	24.63	25.95	29.34	27.79	29.98	33.59	35.5	23.25	24.39	26.88	29.03
		v_3^-		24.7	25.84	25.85	29.85	29.1	30.58	31.68	35.95	24.07	24.45	27.36	29.47	
		v_2^+		9.63	9.7	11.16	10.22	11.1	11.72	12.15	12.26	9.42	9.2	11.06	10.58	
		ρ_3	v_2^-	9.46	9.71	10.18	9.72	10.22	9.83	12.15	11.08	9.17	9.75	10.19	10.36	
			v_3^+	24.76	25.36	26.16	30.08	29.14	29.81	30.77	35.35	23.31	24.84	26.19	29.1	
			v_3^-	24.28	24.97	26.41	29.33	27.95	29.09	33.45	35.6	24.12	24.96	25.41	28.74	
			v_2^+	9.29	9.39	10.37	10.7	16.99	17.23	18.27	21.49	9.76	9.62	10.17	10.1	
ρ_4		v_2^-	9.48	9.75	10.59	10.74	15.86	17.39	18.51	20.69	9.52	9.66	10.91	10.29		
		v_3^+	24.81	24.65	26.19	29.81	53.08	54.85	57.85	60.66	24.28	25.37	26.4	28.93		
		v_3^-	24.62	25.12	25.88	28.75	53.07	53.62	56.48	60.25	24.32	25.12	26.77	29.25		
Unbalanced Design																
n_4		ρ_1	v_2^+	ANOMC-Reg				ANOMC-R				ANOMC-MR1				
			v_2^-	6.31	6.72	11.45	8.87	6.8	6.74	7.63	6.55	6.68	6.57	8.81	6.76	
			v_3^+	6.47	6.3	11.43	6.87	6.42	5.76	6.15	5.54	6.3	6.06	6.85	5.38	
			v_3^-	11.19	11.66	18.32	20.58	12.93	13.52	15.7	18.1	13.33	13.24	18.72	18.04	
			v_2^+	10.12	10.05	15.2	14.12	10.03	10.33	10.13	10.83	10.52	10.07	11.47	11.65	
			v_2^-	9.21	9.2	16.54	11.47	8.09	8.13	10.57	8.54	7.57	7.2	12.12	8.28	
			ρ_2	v_2^-	9.12	8.91	13.79	10.03	7.64	7.25	7.23	6.18	8.02	7.49	8.59	7.03
				v_3^+	15.58	17.43	26.84	28.09	16.01	16.35	22.82	23.37	14.84	16.06	23.61	22.44
		v_3^-		13.97	14.13	18.76	20.12	11.73	13.11	11.99	15.16	12.34	12.64	13.53	15.59	
		v_2^+		5.47	5.22	9.54	7.81	7.3	7.52	8.42	7.88	7.12	7.5	9.41	8.39	
		ρ_3	v_2^-	4.81	4.47	9.48	5.57	5.63	5.26	5.88	5	5.13	5.28	7.15	5.02	
			v_3^+	10.9	12.76	17.63	23.84	15.16	15.86	17.19	20.75	15.52	15.97	19.76	21.84	
			v_3^-	7.49	8.54	14.61	14.52	9.53	10.69	11.27	12.91	9.06	9	12.61	11.88	
	v_2^+		8.06	8.51	13.51	13.71	10.49	9.95	12.57	12.75	9.65	10.48	13.8	14.84		
	ρ_4	v_2^-	7.27	7.61	12.76	9.12	8.42	7.78	7.55	8.61	7.7	8.27	7.85	9.15		
		v_3^+	20.44	22.4	29.76	38.14	27.16	28.53	34.22	39.52	27.78	29.13	36.3	41.25		
		v_3^-	14.89	16.06	20.44	26.74	18.41	19.22	18.2	26.46	17.8	18.6	19.74	27.49		
	n_4	ρ_1	v_2^+	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
			v_2^-	5.87	5.02	7.01	4.51	6.15	6.41	8.62	7.01	6.39	6.5	8.81	6.6	
			v_3^+	6.53	5.67	4.93	4.1	6.49	5.91	6.95	5.21	6.15	5.71	6.88	5.22	
			v_3^-	8.68	8.58	12.9	10.26	12.8	13.14	16.93	17.21	12.49	13.7	18.43	17.36	
			v_2^+	8.78	8.94	6.51	8.6	10.13	9.91	11.12	11.2	10.1	10.48	10.37	11.72	
			v_2^-	5.92	5.42	7.15	4.55	7.98	7.79	10.86	7.76	6.51	6.78	9.18	7.1	
			ρ_2	v_2^-	6.34	5.82	4.72	4.23	8.01	7.32	7.85	6.78	6.31	6.05	6.63	5.42
				v_3^+	8.67	8.38	13.53	10.79	15.11	15.96	23.54	21.5	12.8	13.36	18.07	18.1
		v_3^-		9.51	8.6	6.49	8.34	12.26	12.28	12.86	15.79	10.05	10.28	9.66	11.64	
		v_2^+		5.94	5.02	7.45	4.25	7.23	7.26	9.09	8.01	6.46	6.5	9.49	6.89	
		ρ_3	v_2^-	6.89	5.64	5.05	4.69	5.23	5.23	6.73	4.83	6.07	6.15	6.35	5.58	
			v_3^+	8.42	7.72	12.77	10.47	15.59	16.07	19.42	21.21	12.06	13.34	17.09	17.5	
			v_3^-	9.34	8.78	6.26	8.57	9.18	9.36	11.68	12.96	10.04	10.57	10.1	11.3	
			v_2^+	5.85	4.78	7.32	3.89	10.21	9.88	14.16	12.96	6.79	6.6	8.64	6.78	
ρ_4		v_2^-	6.69	5.89	4.88	4.5	7.84	8.06	7.87	8.5	6.43	5.54	6.93	5.26		
		v_3^+	8.46	8.77	12.99	10.63	27.1	28.83	35.77	39.05	12.31	12.57	17.37	16.38		
		v_3^-	9.45	9.05	6.24	8.47	17.66	18.87	19.33	26.38	9.84	10.19	10.29	11.11		

n	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)		
n_5	ρ_1	v_2^+	7.5	7.89	12.13	11.81	8.16	7.95	9.08	8.59	7.66	8.28	11.15	8.71		
		v_2^-	6.94	6.68	10.73	8.91	6.97	6.59	6.54	6.13	6.73	6.87	7.79	6.68		
		v_3^+	17.25	18.76	24.35	29.92	18.31	18.62	20.77	23.13	18.92	19.06	23.86	25.36		
		v_3^-	13.07	13.21	16.32	19.63	11.99	11.78	11.29	14.28	11.84	12.25	13.86	14.58		
		ρ_2	v_2^+	11.64	12.08	18.36	16.1	9.1	9.21	12.59	9.85	9.02	8.87	13.76	10.64	
			v_2^-	10.44	10.33	13.64	11.75	8.33	7.68	7.57	8.08	8.58	7.88	9.07	7.8	
			v_3^+	25.69	27.45	37.36	41.84	22.12	23.18	28.52	30.86	21.74	22.53	30.19	30.37	
			v_3^-	18.55	19.95	21.4	26.18	14.07	15.1	13.17	17.22	14.93	14.74	15.99	19.42	
		ρ_3	v_2^+	7.34	7.15	10.17	10.74	9.01	9.24	9.24	10.08	8.96	9.27	11.35	11.07	
			v_2^-	5.27	5.15	9.29	7.29	5.84	6.33	6.33	5.89	6.36	6.35	7.38	5.48	
			v_3^+	18.57	19.86	24.13	33.8	21.32	22.36	22.36	28.24	22.27	24.06	27.7	30.3	
			v_3^-	10.17	11.27	16.14	18.87	12.29	12.57	12.57	16.7	11.31	12.32	13.82	15.58	
	ρ_4	v_2^+	11.57	12.32	17.49	19.39	12.74	13.34	16.9	17.02	13.7	14.05	18.6	18.56		
		v_2^-	8.78	9.33	12.14	11.87	9.09	8.98	7.83	10.05	9.34	8.77	8.92	10.46		
		v_3^+	33.93	37.92	44.4	52.64	38.52	42.2	45.69	50.97	41.03	43.56	48	52.07		
		v_3^-	20.89	22.8	24.77	34.32	22.08	23.79	21.86	32.36	23.53	24.88	24.79	35.23		
	ρ_1	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4						
		v_2^+	7.54	7.49	9.84	8.1	7.78	8.31	10.81	8.03	7.5	7.5	11.01	8.36		
		v_2^-	6.87	6.67	5.88	5.91	7.14	6.37	7.91	5.97	6.64	6.59	7.12	5.98		
		v_3^+	15.7	16.37	21.85	21.32	17.9	18.57	23.6	23.1	17.94	18.61	23.47	23.6		
		ρ_2	v_3^-	11.78	11.97	9.72	12.72	12.35	11.62	13.29	13.71	11.97	12.54	11.88	14.18	
			v_2^+	7.31	7.59	10.03	7.66	8.35	8.69	13.01	10.09	7.85	7.94	10.52	8.19	
			v_2^-	7.02	6.92	5.78	5.98	7.81	8.67	8.07	7	6.9	6.47	7.28	5.71	
			v_3^+	15.56	16.61	20.76	20.99	21.17	22.15	28.61	28.56	17.13	18.59	23.15	23.42	
		ρ_3	v_3^-	12.04	11.28	10.45	12.64	15.08	15.17	15.07	17.95	12.03	12.29	11.33	14.16	
			v_2^+	7.56	6.86	10	7.48	8.82	8.79	10.95	10.28	7.28	7.57	10.83	8.34	
			v_2^-	6.95	6.09	6.43	5.69	6.36	6.14	7.11	6.21	6.8	6.8	7.22	6.13	
			v_3^+	15.42	15.56	21	20.56	21.1	23.02	27.4	28.9	17.45	18.52	23.17	23.07	
		ρ_4	v_3^-	11.12	11.17	10.33	12.93	10.9	11.69	13.63	14.51	12.14	12.71	12.2	13.43	
			v_2^+	7	7.04	10.45	7.73	12.9	13.2	18.03	17.3	7.86	7.67	10.57	7.97	
			v_2^-	6.48	6.45	6.3	5.81	9.13	9.01	9.1	10.22	6.74	6.09	7.25	6.46	
			v_3^+	15.6	16.32	20.75	19.92	39.1	41.69	47.56	50.35	17.55	18.23	22.75	22.12	
			v_3^-	11.95	12.17	10.34	13.1	22.54	24.24	24.62	32.42	12.14	12.74	11.47	14	
			ρ_1	ANOMC-Reg				ANOMC-R				ANOMC-MR1				
				v_2^+	9.75	10.26	13.52	14.53	9.4	9.32	11.41	10.35	10.21	9.95	12.55	11.54
				v_2^-	8.09	7.35	9.92	8.66	7	6.19	6.54	5.63	7.47	7.56	7.42	6.37
		v_3^+		26.77	29.03	33.97	40.55	24.67	26.27	28.99	30.81	26.17	27.37	32.29	33.29	
		ρ_2		v_3^-	14.19	14.29	17.28	18.55	11.91	12.53	11.23	13.18	13.13	13.54	13.5	14.95
				v_2^+	15.13	15.96	22.31	21.74	10.53	10.78	14.25	12.41	11.25	11.4	16.63	13.1
				v_2^-	11.65	11.73	14.37	12.88	7.64	7.35	7.12	6.6	8.09	7.63	8.26	7.57
	v_3^+			39.96	42.58	49.06	53.94	30.38	32.68	37.32	39.27	31.81	32.55	38.72	40.15	
	ρ_3	v_3^-		21.71	21.66	21.49	27.75	15.09	14.74	12.84	16.77	16.38	16.33	15.29	19.37	
		v_2^+		8.52	8.76	11.91	12.83	11.11	10.91	12.82	12.78	11.7	11.68	14.46	13.58	
		v_2^-		4.52	4.8	8.23	5.99	6.03	5.95	6.18	5.99	6.1	6.46	6.59	6.25	
		v_3^+		27.65	30.39	33.58	42.33	30.91	31.88	34.13	37.81	33.05	34.58	36.94	39.54	
	ρ_4	v_3^-	10.62	10.97	14.68	16.98	12.09	12.42	11.7	14.32	12.34	12.64	13.65	15.23		
		v_2^+	16.69	17.05	22.83	25.01	17.09	17.66	21.3	22.84	18.17	18.61	24.39	24.45		
		v_2^-	9.23	9.53	12.78	11.19	9.31	8.77	8.1	9.38	9.88	9.69	9.27	10.19		
v_3^+		53.02	55.51	60.6	66.48	55.88	56.87	59.58	63.53	57.93	59.82	62.52	65.87			
v_3^-		24.13	25.57	26.24	35.82	24.22	24.4	21.24	33.19	25.43	26.7	24.31	36.27			
ρ_1		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4						
		v_2^+	8.36	8.22	11.34	9.68	9.59	9.39	12.33	10.7	9.45	9.89	12.68	11.42		
		v_2^-	6.79	6.31	5.89	5.53	7.01	6.68	6.89	5.88	7.01	6.78	7.18	6.28		
	v_3^+	20.88	22.28	28.54	28	25.25	26.06	31.29	31.05	25.95	25.91	31.21	32.16			
	ρ_2	v_3^-	12.14	12.4	10.11	13.35	12.58	12.54	12.6	13.24	12.53	13.48	12	14.58		
		v_2^+	9.02	8.54	11.51	9.07	9.99	10.89	14.96	13.11	9.16	9.66	13.44	10.88		
		v_2^-	7.06	5.95	5.85	5.29	7.82	7.96	7.84	6.81	6.76	6.97	6.52	6.64		
		v_3^+	21.37	23.21	28.6	28.28	29.18	31.14	37.01	39.1	25.4	25.64	31.49	32.12		
	ρ_3	v_3^-	12.46	11.19	9.77	12.68	15.27	15.62	15.1	18.16	12.71	12.38	11.02	14.6		
		v_2^+	7.55	7.66	11.77	9.14	10.86	11.13	13.23	13.27	9.25	10.02	12.87	10.6		
		v_2^-	7.09	6.1	5.82	5.7	6.14	5.9	6.8	5.7	7.09	6.92	6.46	6.49		
		v_3^+	21.45	21.94	27.95	27.71	30.22	31.95	35.15	36.97	25.83	25.68	31.24	31.39		
	ρ_4	v_3^-	12.02	11.49	9.37	12.97	11.19	11.71	12.74	14.31	12.47	12.89	11.62	15.05		
		v_2^+	8.8	8.21	11.91	8.73	17.53	18.18	23.4	23.54	9.83	9.7	13.08	10.48		
		v_2^-	6.47	6.03	5.72	5.71	9.38	9.31	8.88	9.36	6.89	6.98	7	6.28		
		v_3^+	20.57	22.71	27.62	27.66	55.67	56.62	60.51	63.64	26.08	26.63	30.96	32.06		
		v_3^-	11.68	12.43	9.59	12.65	24.24	25.02	23.44	32.39	13.39	12.81	11.41	13.89		

Table 2. 11: Type I error (%) of ANOMC tests under heterogeneous variances when t = 4.

n	ρ	v	Balanced Design												
			(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	
			ANOMC-Reg				ANOMC-R				ANOMC-MR1				
n ₁	ρ ₁	v ₂ ⁺	5.75	6.17	13.75	8.11	6.05	5.74	6.21	5.16	6.05	5.82	7.57	5.56	
		v ₂ ⁻	5.67	6.3	13.53	6.91	6.1	5.89	5.92	5.18	5.73	5.62	7.62	4.92	
		v ₃ ⁺	8.28	8.85	17.18	14.76	9.26	9.03	10.41	11.01	9.09	9.51	12.26	11.58	
		v ₃ ⁻	8.29	8.71	15.43	14.69	9.41	9.38	10.14	10.64	9.43	9.5	12.87	11.36	
	ρ ₂	v ₂ ⁺	6.45	6.28	14.73	9.22	6.82	6.2	7.82	6.52	6.68	5.86	9.35	6.28	
		v ₂ ⁻	7.16	6.6	14.42	8	8.35	8.26	8.04	7.52	7.79	7.69	9.36	7.99	
		v ₃ ⁺	9.59	10.09	18.98	18.7	10.8	11.55	13.47	15.12	10.94	10.69	15.9	14.18	
		v ₃ ⁻	10.92	11.45	18.31	20.15	12.87	13.13	13.3	16.58	13.36	13.34	16.75	17.95	
	ρ ₃	v ₂ ⁺	6.74	7.65	14.39	9.79	7.64	7.66	7.91	7.45	8.15	7.44	9.21	7.75	
		v ₂ ⁻	6.33	6.77	14.19	8.27	6.86	6.72	8.1	6.4	5.87	5.99	9.27	6.04	
		v ₃ ⁺	11.07	12.29	18.32	20.51	12.56	13.5	13.93	16.39	14.09	13.17	16.13	17.79	
		v ₃ ⁻	8.77	10.42	18.96	18.1	10.93	11.51	13.72	15.58	10.26	10.97	15.85	14.81	
	ρ ₄	v ₂ ⁺	6.62	6.71	14.65	10.71	7.57	7.37	7.94	8.54	7.52	6.89	10.02	8.33	
		v ₂ ⁻	6.43	7.02	15.2	10.07	6.96	7.61	8.17	8.34	7.21	7.31	9.32	8.68	
		v ₃ ⁺	12.56	13.56	21.23	25.98	16.73	18.06	19.38	25.45	16.76	18.55	22.33	27.39	
		v ₃ ⁻	11.71	13.8	21.49	26.29	16.7	19.02	19.33	25.94	17.39	18.08	21.9	27.23	
	n ₂	ρ ₁	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
			v ₂ ⁺	5.7	5.09	6.2	5	5.89	5.83	7.6	5.47	6.39	5.88	7.44	6.03
			v ₂ ⁻	5.62	4.82	6.42	4.69	6.01	5.85	7.84	6.01	6.04	6.03	7.67	5.92
			v ₃ ⁺	8.04	7.73	9.19	8.95	9.42	8.88	12.57	11.21	9.6	9.61	12.3	12.13
		ρ ₂	v ₃ ⁻	7.46	8	9.25	8.86	9.17	9.8	12.89	11.14	8.92	9.81	12.42	11.57
			v ₂ ⁺	5.51	5.19	5.96	4.61	6.21	6.76	8.94	6.54	6.35	5.65	7.83	5.88
			v ₂ ⁻	5.49	4.95	6.46	4.61	8.02	7.95	9.9	7.7	5.36	6.13	7.97	5.75
			v ₃ ⁺	8.21	7.1	8.97	8.58	10.2	10.9	15.84	14.71	8.84	10.02	11.81	12.18
ρ ₃		v ₃ ⁻	7.34	7.52	9.01	8.42	14.26	14.04	16.21	17.78	9.77	9.41	11.23	11.25	
		v ₂ ⁺	5.35	4.89	6.47	4.72	8.33	7.82	9.23	7.58	5.99	6.26	8.11	5.9	
		v ₂ ⁻	5.76	5.07	6	4.87	6.85	6.91	9.25	6.55	5.97	5.66	7.94	6.06	
		v ₃ ⁺	8	7.28	8.85	8.52	13.51	14.1	16.64	17.72	9.51	9.31	11.21	11.69	
ρ ₄		v ₃ ⁻	7.61	7.19	8.8	8.62	10.64	11.08	15.76	14.27	9.1	9.62	11.25	10.92	
		v ₂ ⁺	5.46	4.55	6.22	4.27	7.58	7.7	9.26	8.83	5.96	5.85	7.82	6.07	
		v ₂ ⁻	5.37	5.27	6.14	4.46	7.85	7.83	9.31	8.58	6.49	5.62	7.76	5.83	
		v ₃ ⁺	7.75	7.17	8.76	8.57	17.37	17.65	21.74	26.12	9.5	10.1	12.04	12.12	
n ₂		ρ ₁	v ₃ ⁻	7.61	6.92	9.09	8.53	16.31	17.79	21.4	25.73	9.92	10.19	11.68	11.46
			ANOMC-Reg				ANOMC-R				ANOMC-MR1				
			v ₂ ⁺	7.91	8.69	11.89	13.16	7.29	7.46	8.23	7.77	7.86	7.7	9.43	7.91
			v ₂ ⁻	7.44	8.78	11.9	13.21	7.27	6.8	8.1	8.07	7.12	8.12	9.59	8.22
		ρ ₂	v ₃ ⁺	18.94	19.78	23.62	31.91	16.15	16.5	16.65	19.32	16.04	17.12	19.25	20.53
			v ₃ ⁻	18.38	20.28	22.85	31.17	15.94	15.7	17.47	19.65	16.9	17.29	19.27	20.12
			v ₂ ⁺	9.45	11.11	14.77	15.64	8.49	8.35	9.62	8.94	8.31	8.64	10.9	8.75
			v ₂ ⁻	11.61	11.5	14.57	17.55	10.13	10.3	10.44	10.98	10.32	10.82	11.86	12.08
	ρ ₃	v ₃ ⁺	24.41	26.95	31.68	40.65	20.88	21.65	24.9	27.89	20.68	21.71	25.84	28.31	
		v ₃ ⁻	26.09	28.32	30.17	41.22	22.7	23.96	23.38	29.7	24.5	25.28	26.95	31.27	
		v ₂ ⁺	10.51	11.97	14.01	16.8	10.23	10.09	10.65	10.93	10.91	10.82	11.08	11.81	
		v ₂ ⁻	9.19	10.56	15.08	15.49	8.33	8.46	9.34	9.51	7.84	8.39	11.24	9.33	
	ρ ₄	v ₃ ⁺	25.61	28.64	29.22	40.77	22.75	24.65	23.62	29.68	24.22	25.62	26.55	30.81	
		v ₃ ⁻	23.64	27.68	31.56	40.71	21.35	22.31	25.2	27.06	19.99	21.01	25.72	28.48	
		v ₂ ⁺	11.41	12.59	15.93	20.08	11.61	11.99	12.13	13.87	11.63	12.18	13.88	15.14	
		v ₂ ⁻	11.52	12.36	16.32	19.88	11.63	11.81	12.07	14.56	11.89	12.97	13.72	15.12	
	n ₂	ρ ₁	v ₃ ⁺	35.66	38.77	40.86	54.06	36.68	37.87	38.9	46.22	36.58	39	42.1	48.48
			v ₃ ⁻	35.8	39.45	40.96	52.82	35.87	38.44	38.22	45.45	37.97	38.93	42.52	47.62
			ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
			v ₂ ⁺	8.1	7.53	8.91	8.19	7.24	7.7	8.59	7.81	7.68	7.41	8.78	8.34
		ρ ₂	v ₂ ⁻	8.08	6.98	8.74	8.05	7.44	7.19	9.52	7.74	7.65	7.28	8.71	7.93
			v ₃ ⁺	15.92	17.06	18.58	19.17	16.44	16.29	19.6	19.31	16.69	17.28	18.41	20.59
			v ₃ ⁻	16.62	16.8	18.29	20.33	16.51	17.08	18.51	19.68	16.62	18.06	18.41	20.5
			v ₂ ⁺	7.6	7.47	9.33	7.85	8.08	8.4	10.3	8.93	7.44	7.84	9.15	8.18
ρ ₃		v ₂ ⁻	8	7.09	8.79	8.1	10.64	11.2	11.91	11.12	7.04	7.52	8.53	8.17	
		v ₃ ⁺	15.75	16.74	17.82	19.37	21.1	21.69	24.14	27.35	17.03	16.96	18.96	19.62	
		v ₃ ⁻	15.55	16.9	17.94	19.22	23.72	24.82	25.57	30.32	16.6	17.38	18.7	19.32	
		v ₂ ⁺	7.55	7.02	8.46	8.14	10.45	10.44	11.52	11.25	7.27	7.46	8.85	8.37	
ρ ₄		v ₂ ⁻	7.97	7.48	8.66	7.75	8.28	8.42	10.78	9.04	7.58	7.45	9.13	8.18	
		v ₃ ⁺	16.3	16.77	18.44	19.51	23.67	24.77	26.45	29.98	16.84	18.17	19.02	19.9	
		v ₃ ⁻	16.29	16.3	18.35	19.32	20.05	20.47	25.89	26.96	16.97	17.25	18.8	20.34	
		v ₂ ⁺	7.74	7.59	8.73	7.96	11.91	12.59	13.39	14.24	7.49	8.27	8.99	8.09	
ρ ₄		v ₂ ⁻	7.68	7.5	8.6	7.8	11.5	12.24	12.59	14.5	7.24	7.66	9.25	7.95	
		v ₃ ⁺	17.1	15.74	18.45	18.93	36.61	37.66	40.79	45.38	17.45	16.9	18.61	20.6	
		v ₃ ⁻	15.64	17.08	18.32	19.76	36.5	37.47	39.51	45.74	16.71	16.1	19.17	19.69	
		v ₂ ⁺	15.64	17.08	18.32	19.76	36.5	37.47	39.51	45.74	16.71	16.1	19.17	19.69	

	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	
			ANOMC-Reg				ANOMC-R				ANOMC-MR1				
n_3	ρ_1	v_2^+	9.99	11.19	13.37	15.59	8.63	9.02	9.14	9.54	10.1	9.93	11.07	9.91	
		v_2^-	10.52	10.21	12.92	16.08	8.78	8.53	9.72	9.4	9.9	9.53	11.3	10.78	
		v_3^+	29.27	30.5	31.21	41.03	22.71	23.89	23.07	27.46	25.54	26.44	27.39	29.21	
		v_3^-	29.26	29.71	31.45	41.89	23	22.7	23.43	27.24	24.81	26.09	27.95	30.17	
	ρ_2	v_2^+	12.86	12.85	16.82	19.08	10.81	10.14	11.07	11.16	10.7	10.43	12.79	12.02	
		v_2^-	14.27	15.59	16.53	20.63	12.15	13.1	12.88	13.7	14.41	13.64	14.59	15.4	
		v_3^+	39.76	42.64	44.53	55.21	32.78	32.76	34.54	38.35	33.49	34.08	37.65	40.41	
		v_3^-	39.56	41.86	42.12	53.09	32.42	33.08	33.2	38.31	36.02	37.15	37.54	42.03	
	ρ_3	v_2^+	14.12	15.42	16.27	20.77	12.57	12.88	12.33	14.12	14.24	13.46	14.77	15.55	
		v_2^-	12.28	13.34	16.19	18.34	9.84	10.69	11.26	11.71	11.32	10.86	13.49	12.47	
		v_3^+	39.91	41.94	41.98	52.78	32.63	33.74	33.09	38.49	35.81	37.52	37.08	42.88	
		v_3^-	39.46	42.7	44.34	54.71	31.28	32.55	34.92	38.5	33.58	35.08	37.12	41.46	
	ρ_4	v_2^+	16.23	18.19	19.75	25.95	16.34	16.65	16.16	19.1	17.01	18.28	19.11	21.86	
		v_2^-	16.35	17.55	18.27	25.83	15.7	16.5	16.88	19.84	17.25	18.21	18.78	20.85	
		v_3^+	55.33	58.85	59.05	68.31	54.66	55.05	54	60.5	56.47	58.21	59.29	62.69	
		v_3^-	55.39	57.97	58.79	68.14	54.35	55.19	54.72	59.87	57.22	59.58	59.7	63.95	
	n_4	ρ_1	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
			v_2^+	9.52	9.6	9.74	9.49	8.86	9.2	10.51	9.47	8.96	9.46	9.9	9.72
			v_2^-	8.91	8.91	9.95	9.73	9.15	8.58	10.45	9.63	9.15	9.82	10.09	9.87
			v_3^+	24.25	24.63	24.7	27.79	23.6	23.46	26.06	26.48	24.02	24.86	25.29	28.23
		ρ_2	v_3^-	23.11	23.82	24.85	28.04	22.95	23.64	25.39	27.24	24.34	25.48	25.6	28.58
			v_2^+	8.83	8.82	10.08	9.3	9.54	9.6	12.07	11.26	9	9.64	10.56	9.97
			v_2^-	8.41	9.29	10.16	9.22	12.75	13.19	13.69	14.43	8.66	9.11	10.02	9.78
			v_3^+	23.54	23.81	25.3	27.48	29.89	33.14	35	37.86	24.52	23.4	25.59	29.13
		ρ_3	v_3^-	24.05	24.4	25.07	27.97	33.37	34.37	34.95	39.87	24.21	24.56	25.69	27.65
			v_2^+	8.66	8.87	9.93	9.79	12.86	13.44	13.84	14.22	9.1	8.89	10.43	10.35
			v_2^-	8.94	8.82	10.08	9.37	10.61	10.22	13.14	11.43	9.14	9.53	10.54	10.05
			v_3^+	23.99	23.94	24.58	26.91	32.86	34.47	35.73	39.11	24.83	25.15	26.34	27.72
		ρ_4	v_3^-	23.36	24.13	25.13	27.95	30.51	31.64	34.39	36.76	24.05	24.48	26.46	27.65
			v_2^+	8.92	8.99	9.73	9.53	15.47	16.71	17.86	19.37	9.52	9.14	10.19	9.28
			v_2^-	9.12	9.55	10.06	9.59	16.78	16.46	16.96	19.22	9.01	9.14	10.16	9.5
			v_3^+	23.1	23.3	24.73	26.01	54.26	54.83	56.05	60.43	24.51	25.31	25.9	28.49
Unbalanced Design															
n_4		ρ_1	v_3^-	24.03	23.85	24.66	27.32	53.84	55.55	56.65	59.96	23.59	24.46	26.44	28.1
			ANOMC-Reg				ANOMC-R				ANOMC-MR1				
			v_2^+	5.54	5.27	12.84	7.92	5.98	5.71	6.35	5.72	5.98	5.92	8.33	6.01
	v_2^-		6.13	5.91	12.86	7.39	6.12	6.06	5.83	5.01	5.91	5.53	7.5	4.8	
	ρ_2	v_3^+	8	8.42	16.08	15.83	10.19	10.9	12.13	13.57	10.49	11.17	14.21	14.49	
		v_3^-	7.68	8.4	15.36	12.01	8.7	9.11	9.24	10.29	8.69	9.45	11.36	10.08	
		v_2^+	8.14	8.74	16.9	10.71	8.15	8.44	9.03	7.78	7.52	7.02	10.61	7.26	
		v_2^-	9.1	8.76	16.27	10.38	8.41	8.29	8.09	7.21	8.79	8.12	9.9	7.49	
	ρ_3	v_3^+	12.53	13.05	22.53	22.8	13.81	14.11	18.14	19.68	13.03	13.98	20.53	19.34	
		v_3^-	12.02	12.66	19.38	18.85	14.05	13.46	12.47	15.41	14.3	14.23	16.06	16.48	
		v_2^+	4.99	5.11	11.36	7.52	7.74	7.4	7.92	7.63	7.75	7.8	9.28	7.77	
		v_2^-	4.48	4.84	11.42	6.17	5.38	6.02	6.79	5.2	5.72	5.71	7.77	5.05	
	ρ_4	v_3^+	8.73	9.68	16.65	18.35	13.94	15.01	15.7	19.23	15.13	15.18	18.47	21.06	
		v_3^-	6.3	6.64	14.88	11.78	8.88	9.72	10.96	11.69	8.99	8.78	12.55	12.17	
		v_2^+	6.52	7.05	14.03	9.67	8.15	8.37	8.92	10.64	8.35	8.69	11.53	10.98	
		v_2^-	6.22	6.23	14.19	8.13	6.97	7.15	6.94	7.17	6.8	7.12	8.81	8.12	
	ρ_4	v_3^+	13.31	14.61	23.1	27.96	20.48	23.19	24.91	31.48	20.99	22.87	28.5	33.14	
		v_3^-	11.21	11.28	19.47	20.19	15.15	15.99	16.54	22.11	14.78	16.63	18.31	24.49	
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4					
		v_2^+	5.69	4.02	4.42	3.33	6.49	6.42	8.61	5.58	6.13	6.08	8.04	6.05	
ρ_1		v_2^-	5.83	5.01	4.43	3.46	6.36	6.04	7.35	5.22	6.16	5.94	7.03	5.53	
		v_3^+	6.07	5.82	7.61	5.65	10.97	11.23	14.83	14.18	10.48	10.63	13.61	13.63	
		v_3^-	6.82	6.48	5.69	6.23	9.44	8.86	11.45	9.98	9.32	8.42	10.09	10.34	
		v_2^+	4.8	3.88	4.74	3.3	8	7.84	10.56	7.29	6.51	6.02	7.98	6.51	
ρ_2	v_2^-	4.96	4.76	4.23	3.29	9.16	8.5	9.95	7.75	6	5.52	7.56	5.72		
	v_3^+	6.12	5.14	6.87	6.4	13.63	13.68	20.44	19.27	10.74	10.62	13.66	13.81		
	v_3^-	7.26	6.17	5.53	5.98	14.28	13.65	16.09	16.52	8.71	8.84	10.3	10.33		
	v_2^+	5.05	4.52	4.87	2.86	7.5	8.39	9.66	8.16	6.09	6.16	8.06	6.33		
ρ_3	v_2^-	5.99	4.63	4.22	3.06	5.71	5.56	7.78	5.48	5.89	5.74	7.15	5.56		
	v_3^+	6.16	5.19	7.21	5.6	14.63	14.84	18.76	19.56	10.1	10.8	13.13	12.93		
	v_3^-	7.17	7.04	5.78	5.5	9.15	9.59	12.43	12.36	8.95	8.69	10.34	10.26		
	v_2^+	5.24	4.06	4.77	3.11	8.25	8.97	10.86	10.1	6.05	6.01	7.89	6.11		
ρ_4	v_2^-	5.75	4.57	4.45	3.46	7.31	7.4	8.8	7.62	6.17	5.57	6.87	5.4		
	v_3^+	6.31	5.12	7.3	5.2	21.15	22.03	26.69	31.42	10.27	10.05	13.77	13.2		
	v_3^-	7.66	6.49	5.82	5.42	15.47	16.33	18.64	21.92	8.71	8.9	10.17	9.75		

n	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_5	ρ_1	v_2^+	ANOMC-Reg				ANOMC-R				ANOMC-MR1			
			7.71	7.97	12.62	12.16	7.38	6.8	8.4	8.68	7.73	7.81	10.24	8.36
			6.59	7.05	11.6	9.49	6.44	6.43	6.28	5.73	6.63	6.74	7.61	6.02
			18.71	20.62	24.56	33.3	18.1	18.59	21.18	22.79	18.45	19.58	24.36	23.62
		v_3^+	12.15	12.25	17.08	18.41	11.05	11.34	10.8	11.76	10.9	11.2	12.64	11.95
			13.26	12.78	19.73	19.01	9.59	9.52	11.77	11.28	9.69	9.78	13.63	10.53
			11.87	11.12	15.25	14.61	8.67	8.37	8.87	8.22	9.26	8.69	10.41	8.71
			29.29	31.38	39.9	47.79	25.77	25.65	30.73	33.6	24.78	25.61	33.02	33.45
		v_3^-	19.9	20.99	22.89	28.73	16.03	16.25	15.65	18.4	16.74	17.27	18.35	20.19
			7.83	8.65	12.56	14.22	10.62	10.11	11.56	11.29	10.93	10.45	13.11	12.06
			5.19	5.45	10.11	8.16	6.43	6.29	6.94	6.44	6.57	6.42	7.45	5.77
			23.57	24.98	29.32	39.8	25.62	26.11	27.73	32.01	28.3	28.26	30.31	35.23
	ρ_2	v_2^+	11.02	11.17	16.78	18.67	11.46	12.18	12.56	14.62	11.6	11.93	14.38	15.24
			11.41	13.05	17.21	20.39	12.36	12.89	15.05	17.31	13.16	13.06	17.11	17.9
			8.32	8.67	12.99	11.65	8.44	7.93	8.04	8.71	7.9	8.34	8.79	9.5
			36.71	41.18	45.32	56.26	41.59	43.49	45.67	51.62	42.76	45.64	49.17	54.48
		v_3^+	20.33	20.91	25.2	32.49	20.19	21.85	19.39	27.5	21.78	22.89	23.03	29.18
			ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
			7.46	6.82	9.55	7.77	7.69	7.69	9.76	8.32	7.88	7.41	10.88	8.77
			6.46	6.12	6.82	6.14	6.53	6.23	7.73	5.94	6.55	5.82	7.48	6.03
		v_3^-	16.29	17.29	21.77	20.98	17.49	18.23	23.01	23.12	18.73	19.45	23.75	24.38
			11.14	10.65	10.39	12.18	10.48	10.86	12.11	11.95	11.27	11	11.61	12.89
			8.08	6.99	9.75	7.91	9.37	8.9	12.76	10.37	8.19	7.9	9.78	8.53
			6.86	6.17	6.41	5.62	9.57	8.63	9.82	8.27	6.7	6.38	7.66	6.3
		v_3^+	17.19	16.91	20.92	21.44	22.88	25.21	30.82	32.35	19.36	20.12	24.02	23.86
			11.05	10.2	9.93	11.38	16.57	16.58	17.38	18.8	11.27	11.41	12.06	12.82
			7.8	6.94	9.04	7.96	10.81	10.36	12.77	12.1	7.68	7.65	11.41	8.96
			6.8	6.38	6.81	6.19	6.3	6.03	8.1	6.21	6.67	6.08	7.45	6.48
	ρ_3	v_3^+	16.45	17.13	20.78	20.53	26.49	27.08	31.03	32.39	19.37	20.27	23.39	23.93
			11.21	10.66	10.39	11.15	10.96	11.73	13.56	13.82	11.19	11.03	12.36	12.91
			7.21	7.32	9.52	7.46	12.57	13.06	16.99	16.75	7.91	8.1	9.98	8.53
			6.55	5.75	6.52	5.72	8.53	7.8	8.72	8.38	6.37	6.65	7.83	6.34
		v_3^-	16.24	16.06	20.75	20.5	40.27	42.75	47.58	50.72	18.37	19.56	23.05	23.01
			11.32	10.65	10.3	11.65	20.7	22.18	22.56	27.32	10.86	11.15	11.99	13.05
			ANOMC-Reg				ANOMC-R				ANOMC-MR1			
			16.33	18.92	18.73	24.34	14.8	15.26	15.49	17.2	16.17	16.16	18.31	17.77
			8.72	10.1	10.89	11.92	8.34	8.25	7.92	8.08	9.49	8.95	9.56	9.12
		v_3^+	57.82	60.28	61.58	67.13	47.95	49.91	49.66	52.68	50.84	51.82	54.33	54.82
			24.45	24.64	26.14	31.12	19.44	19.18	18.78	21.51	20.41	21.52	21.38	23.62
			27.32	28.75	31.36	37.44	17.85	18.36	20.6	21.03	18.51	18.75	22.75	22.98
			16.71	17.01	17.56	20.62	12.15	11.25	10.73	11.65	12.69	12.51	12.36	12.56
		v_3^-	81.42	82.32	82.85	85.19	66.56	66.83	67.84	69.88	68.44	68.93	69.42	71.83
			38.71	39.32	37.28	47.63	26.7	27.57	26.74	31.85	29.5	30.59	30.9	35.91
			20.41	21.67	22.8	27.74	21.69	22.15	22.94	24.63	23.57	24.49	24.6	26.02
			8.08	7.96	10.75	10.46	9.1	9.47	8.94	9.53	9.61	9.78	10.63	9.94
	ρ_4	v_3^+	68.02	68.72	69.78	73.67	63.71	64.58	64.43	66.6	67.86	67.94	67.5	69.2
			27.41	29.03	29.17	36.45	25.96	27.41	25.65	30.44	27	27.46	28.13	32.55
			33.17	35.24	35.9	44.11	32.82	32.37	34.55	37.98	34.82	35.83	37.59	40.28
			13.86	14.91	16.26	18.43	13.12	13.67	12.65	14.03	14.29	14.68	14.44	16.3
		v_3^-	90.91	89.96	90.71	91.15	88.82	88.66	87.95	87.85	90.72	89.91	89.93	89.1
			47.24	48.79	46.67	57.43	43.77	45.52	42.45	52.44	47.05	48.48	48.03	56.63
			ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
			14.47	14.73	16.94	16.09	15.16	14.69	16.81	16.33	14.9	14.79	17.37	17.04
			7.49	8.57	7.87	8.1	8.3	8.43	8.95	8.54	8.6	7.79	8.38	8.32
			47.79	48.78	50.46	52.35	48.3	47.83	50.47	52.1	48.8	49.58	51.08	52.88
		v_3^+	19.27	20.2	18.35	22.14	19.14	20.56	19.42	21.78	19.94	20.22	18.81	22.58
			14.69	14.64	16.79	16.01	16.96	17.87	21.77	20.22	14.74	15.08	17.43	16.26
			8.22	8.09	8.15	8.65	12.18	12.22	11.53	12.01	8.53	8.4	7.97	7.98
			47.8	49.4	50.82	52.1	65.32	66.7	67.26	68.44	49.89	49.54	50.78	51.84
	ρ_5	v_3^-	19.81	19.95	17.27	22.32	28.61	29.06	27	34.19	20.04	21.17	18.63	22.11
			14.08	14.47	17.35	16.67	22.33	21.91	23.32	24.44	14.93	15.27	16.62	16.04
			9.15	8.25	8.52	8.09	8.79	9.4	9.35	9.31	8.49	8.41	8.39	8.36
			47.94	49.34	49.43	52.27	64.18	65.1	65.08	66.85	49.59	49.18	50.14	52.45
		v_3^+	19.91	19.68	19.1	21.13	26.07	26.44	26.16	29.09	19.93	20.08	20.24	21.72
			15.23	15.15	16.68	16.38	32.04	33.3	36.09	36.87	14.63	14.97	16.72	16.76
			8.4	8.34	8.09	8.11	13.52	14.17	12.92	15.27	7.99	8.39	8.69	8.54
			48.81	48.69	50.29	51.81	89.71	88.97	87.82	87.66	49.09	50.89	51.46	51.92
		v_3^-	19.81	20.26	18.45	21.93	44.37	46.27	44.88	52.46	20.25	19.41	19.14	22.47

Table 2. 12: Type I error (%) of ANOMC tests under heterogeneous variances when t = 5.

Balanced Design														
n	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n ₁	ρ ₁	v ₂ ⁺	ANOMC-Reg				ANOMC-R				ANOMC-MR1			
			5.46	6.18	15.23	9.44	5.55	5.53	6.6	5.85	5.17	5.51	8.03	5.11
			5.75	6.19	15.12	9.58	5.74	5.69	6.67	5.41	5.44	5.51	8.12	5.26
		v ₃ ⁺	8.4	8.74	17.77	15.21	8.53	8.58	9.72	10.55	8.42	8.64	11.87	10.03
			8.15	8.36	17.3	15.28	8.84	8.81	9.66	10.63	8.49	8.71	12.12	10.26
			6.16	6.87	16.32	9.86	6.05	5.84	6.73	6.31	6.08	5.73	8.47	5.51
		v ₂ ⁻	6.86	7.29	17.27	10.42	7.11	6.99	7.66	7.48	6.87	7.04	9.93	6.65
			8.67	9.07	19.24	17.49	8.77	9.91	11.25	12.37	8.26	8.97	13.96	11.5
			10.01	11.35	20.16	19.29	12.25	11.97	13.12	14.69	12.16	11.79	15.93	15.25
		v ₃ ⁻	6.8	7.64	16.32	10.57	7.56	6.91	7.64	7.14	7.3	6.77	9.63	7.54
			6.35	6.69	16.29	9.53	6.45	5.98	6.43	5.87	6.32	5.39	7.82	5.33
			9.68	11.22	19.3	19.11	12.26	12.12	13.04	14.59	12.84	13.39	15.57	15.04
	ρ ₂	v ₂ ⁺	8.6	9.6	19.52	17.53	9.74	9.49	11.27	12.03	8.46	8.9	13.49	11.23
			6.59	7.19	16.45	11.14	6.88	6.84	7.81	8.12	6.6	6.67	9.21	8.56
			6.34	6.86	16.86	11.2	6.46	7.18	8.04	7.79	7.03	6.89	9.4	7.91
		v ₃ ⁺	11.27	13.68	21.99	25.8	15.32	16.86	17.56	23.39	15.34	17.54	20.32	25.01
			12.17	13.22	22.72	24.19	15.76	16.66	17.19	23.69	15.84	16.49	21.13	25.3
			ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
		v ₂ ⁻	5.49	4.66	6.29	4.49	5.75	5.94	8.49	5.89	6.3	5.79	8.32	5.92
			5.36	4.81	5.62	4.53	5.84	6.13	8.27	5.23	6.17	5.72	8.91	6.2
			7.28	7.47	7.9	7.85	8.81	9.06	11.96	10.7	9.62	9.6	12.06	11.38
		v ₃ ⁻	6.8	6.96	7.94	7.8	9.11	9.2	12.21	10.95	9.04	9.63	12.17	11.52
			5.25	4.88	5.91	4.19	6.39	5.8	8.82	6.34	5.89	5.96	8.19	6.48
			5.06	4.43	5.89	4.48	7.68	7.58	10.17	7.69	5.8	5.8	8.28	6.48
	ρ ₃	v ₂ ⁺	7.42	6.65	8.5	7.42	9.36	9.36	12.79	11.83	9.37	9.82	11.89	11.48
			7.18	6.42	8.03	7.61	12.55	13.18	16.22	15.47	9.24	9.2	11.54	11.51
			5.07	4.47	5.66	4.42	8.06	7.35	10.07	7.73	5.98	5.82	7.84	6.46
		v ₃ ⁺	5.56	4.91	6.1	4.38	6.11	6.27	8.43	6.49	6.11	5.9	8.05	6.42
7.26			6.47	8.26	7.82	12.86	12.68	16.38	15.66	9.18	9.2	11.88	11.38	
7.1			6.73	8.01	8.24	8.84	9.16	13.25	11.99	9.68	9.12	12.67	11.36	
v ₂ ⁻		5.3	4.34	5.44	4.23	7.41	7.52	9.64	7.87	6.15	5.84	8.36	6.77	
		5.25	4.93	5.5	4.3	7.28	7.14	9.71	8.09	5.58	6.27	8.49	6.21	
		6.72	6.62	8.09	6.89	16.18	17.47	20.65	23.38	8.99	9.5	12.05	10.98	
v ₃ ⁻		6.81	6.45	8.42	7.06	16.43	16.81	20.49	24.38	9.34	9.66	11.67	11.1	
		ANOMC-Reg				ANOMC-R				ANOMC-MR1				
		v ₂ ⁺	7.44	8.19	12.41	13.64	6.94	6.78	7.39	7.47	7.15	6.95	8.55	7.62
7.24	7.74		11.82	13.19	6.84	7.18	7.37	7.58	6.92	6.91	8.91	7.85		
17.69	18.82		21.94	30.8	15	14.82	15.59	18.08	15.86	16.03	18.2	18.77		
v ₃ ⁺	16.94	19.13	22.33	30.76	14.97	15.42	16.01	17.13	15.03	15.76	18.19	19.58		
	8.42	9.08	13.05	14.79	7.88	7.51	7.88	8.21	7.18	7.01	9.22	7.75		
	9.94	10.67	14.78	16.96	9.01	9.47	10.03	10.81	9.22	10.03	11.1	10.58		
ρ ₄	v ₂ ⁻	20.46	22.8	28.64	36.97	17.05	17.78	19.21	23.14	16.63	17.59	20.86	23.08	
		22.75	25.91	27.27	37.18	21.01	21.47	21.73	26.02	21.89	22.76	24.36	26.09	
		10.22	11.05	14.9	16.95	9.88	9.43	9.87	9.66	9.49	10.18	11.47	10.67	
	v ₃ ⁻	8.56	9.29	13.44	14.8	7.38	7.42	7.81	7.97	7.08	6.84	8.95	7.94	
		23.52	25.72	27.71	39.16	21.26	21.77	21.56	25.46	22.32	22.36	24.06	27.34	
		20.22	22.72	27.58	37.74	17.66	17.6	19.05	22.24	17.45	17.22	21.74	23.37	
	v ₂ ⁺	10.41	11.19	15.61	19.62	10.74	10.63	10.84	13.4	11.12	11.1	12.78	13.94	
		11.34	12.05	15.74	20.16	10.9	11.54	11.59	13.03	10.73	11.1	13.03	13.72	
		34.48	37.61	39.58	52.7	35.03	36.59	36.17	43.48	36.39	37.77	39.88	46.19	
	v ₃ ⁺	34.66	37.96	39.97	52.05	35.59	36.82	36.37	43.28	36.96	38.23	39.73	46.76	
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
		v ₂ ⁻	6.74	6.49	8.81	8.11	7.49	7.49	8.46	7.78	7.82	7.54	8.82	8.37
6.82	7.17		8.7	7.76	7.38	6.84	9.08	7.68	7.36	7.66	8.8	7.7		
16.13	16.32		17.07	17.39	15.87	15.17	17.97	18.58	16.7	16.32	18.38	19.34		
n ₂	ρ ₁	v ₃ ⁺	16.13	15.93	17.24	18.25	15.24	15.95	18.58	18.63	16.33	16.81	18.6	19.4
			7.09	6.87	8.38	8.05	7.02	7.92	9.22	8.16	7.47	6.83	9.07	8.23
			7.34	7.5	8.55	7.47	9.78	10.72	11.63	9.97	6.72	7.48	9.05	8.42
	ρ ₂	v ₃ ⁻	15.7	16.1	17.95	18.36	16.69	17.32	21.45	21.58	16.51	16.75	18.13	19.12
			15.56	14.88	17.52	18.23	22.34	23.02	24.06	25.34	16.44	16.79	17.7	19.24
			7.57	7.03	9.04	7.35	10.15	9.98	11.98	10.5	7.28	7.85	8.9	7.8
	ρ ₃	v ₂ ⁻	7.01	7.08	9.17	7.73	7.53	7.43	9.46	8.01	7.42	7.49	8.82	7.96
			15.36	15.55	17.5	18.49	21.43	21.46	24.67	25.95	16.47	16.5	18.16	18.92
			15.47	15.47	17.72	17.82	17.58	17.35	20.78	21.68	15.86	16.49	18.33	19.02
	ρ ₄	v ₂ ⁺	6.79	7.54	8.55	7.68	10.86	11.39	12.64	13.63	6.72	7.59	9.2	8.37
			6.93	6.77	8.3	7.59	10.53	10.98	13.54	13.31	7.08	7.43	8.46	8.1
			15.54	16.35	16.88	17.68	34.95	36.81	38.43	43.14	16.69	16.23	17.77	18.49
v ₃ ⁺	16.19	15.42	17.55	17.83	34.57	36.43	38.36	43.81	16.01	16.58	18.01	18.49		

			(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	
n	ρ	v	ANOMC-Reg				ANOMC-R				ANOMC-MR1				
			v_2^+	v_2^-	v_3^+	v_3^-	v_2^+	v_2^-	v_3^+	v_3^-	v_2^+	v_2^-	v_3^+	v_3^-	
n_3	ρ_1	v_2^+	9.37	10.06	12.27	15.24	8.69	8.82	9.59	9.65	8.92	9.78	10.49	9.64	
		v_2^-	9.26	10.58	11.99	16.07	8.54	8.88	9.19	9.48	8.98	9.04	10.52	10.21	
		v_3^+	27.38	29.62	31.9	40.97	23.61	23.31	24.11	26.83	24.36	24.47	26.32	28.11	
		v_3^-	27.45	30.02	30.34	40.94	22.82	22.48	23.53	26.94	24.51	25.04	25.86	27.67	
	ρ_2	v_2^+	10.34	12.06	14.61	17.98	9.06	9.66	10.11	10.29	9.04	9.27	11.9	10.6	
		v_2^-	13.55	13.59	16.01	20.25	11.7	12.57	12.22	13.27	12.81	13.9	14.34	13.75	
		v_3^+	34.27	37.05	40.75	50.89	27.34	28.91	30.61	33.51	28.43	30.1	32.59	35.93	
		v_3^-	36.13	39.18	38.94	50.15	29.72	30.45	30.99	35.81	32.45	33.93	35.18	38.36	
	ρ_3	v_2^+	13.34	14.17	15.94	20.6	12.42	12.67	11.87	12.65	12.92	12.96	14	14.55	
		v_2^-	10.58	11.54	14.52	18.25	8.93	9.06	9.82	9.46	9.72	9.7	11.64	10.91	
		v_3^+	36.74	40.68	39.35	49.68	30.82	31.58	31.07	35.4	33.82	33.88	34.92	37.29	
		v_3^-	35.09	37.45	40.68	51.46	28.45	29.07	30.54	34.12	28.73	29.77	33.23	36.44	
	ρ_4	v_2^+	15.65	17.18	19.2	23.66	15.4	15.57	15.71	18.62	15.98	16.21	17.69	19.73	
		v_2^-	15.93	16.94	18.74	25.29	14.7	15.62	15.13	17.8	16.35	16.77	17.87	19.73	
		v_3^+	55.03	58.31	57.19	69.04	54.13	55.92	54.03	60.76	56.24	57.51	59.01	62.92	
		v_3^-	55.33	57.5	58.26	68.55	53.78	55.74	55.26	59.88	56.7	57.35	57.82	63.38	
				ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	n_4	ρ_1	v_2^+	8.54	8.18	9.17	9.25	8	8.35	9.58	9.1	8.82	9.54	9.71	9.82
			v_2^-	8.41	7.96	10.01	9.21	9.22	8.72	9.7	9.38	8.19	8.72	10.49	9.66
			v_3^+	23.01	23.59	23.54	25.49	22.68	22.7	24.76	26.48	23.62	24.33	24.76	26.92
			v_3^-	22.3	23.99	24.6	26.39	22.5	23.07	24.01	25.57	24.19	23.91	24.59	27.28
		ρ_2	v_2^+	8.44	8.57	9.44	9.17	8.53	9.01	10.18	9.75	8.48	8.59	10.4	10.1
			v_2^-	8.5	8.91	10.15	8.41	11.72	12.02	13.35	13.24	8.99	8.73	9.75	9.42
			v_3^+	23.21	23.35	24.56	26.39	26.39	27.34	30.1	31.31	24.46	24.76	24.26	26.63
			v_3^-	23.07	23.51	23.87	26.32	30.99	31.22	32.55	35.26	23.47	24.22	25.06	26.83
		ρ_3	v_2^+	8.64	8.38	9.76	9.41	12.03	12.06	12.32	12.61	8.82	9.22	9.26	9.85
			v_2^-	8.42	7.94	9.59	9.25	8.4	8.47	9.9	9.57	8.72	9.22	10.16	9.56
			v_3^+	23.37	22.87	23.24	26.8	30.5	31.31	33.29	35.52	23.93	24.04	24.34	27.2
			v_3^-	23.24	22.72	24.98	26.78	25.9	27.26	30.73	32.19	24.08	24.75	24.78	27.32
		ρ_4	v_2^+	8.37	8.28	9.63	9.03	14.98	14.66	15.54	17.82	9.11	8.53	10.53	9.62
			v_2^-	8.37	8.06	9.24	9.36	14.84	15.99	16.01	17.9	8.47	8.48	9.75	9.59
			v_3^+	23.27	23.31	24.21	26.22	53.53	52.84	55.03	59.39	23.47	24.5	25.01	27
			v_3^-	22.38	24.16	23.51	25.42	53.43	54.2	55.02	58.71	23.74	23.91	24.53	26.44
Unbalanced Design															
n_3	ρ_1	v_2^+	6.88	7.18	13.26	11.72	6.69	7.66	8.13	8.32	7.23	6.89	9.85	8.25	
		v_2^-	6.13	6.33	12.89	9.17	6.16	6.13	6.52	5.35	5.91	5.62	8.1	5.85	
		v_3^+	15.12	17.21	23.62	29.6	16.06	17.91	19.04	22.09	17.23	17.85	22.61	22.34	
		v_3^-	8.74	10.06	16.5	13.47	9.47	9.81	9.22	9.24	8.65	8.65	11.26	9.78	
	ρ_2	v_2^+	10.08	11.68	18.82	17.72	8.79	8.4	10.24	10.14	8.18	7.83	11.55	9.26	
		v_2^-	10.46	11.05	17.32	13.23	8.58	8.49	8.2	7.21	8.08	8.22	10.46	7.58	
		v_3^+	22.56	25.58	34.27	41.5	20.51	22.14	25.46	28.53	20.72	22.07	28.91	29.61	
		v_3^-	15.47	16.2	21.39	21.66	13.46	12.99	13.44	13.7	13.85	13.41	15.74	14.31	
	ρ_3	v_2^+	6.04	6.29	12.66	11.82	9.8	9.62	9.9	10.71	9.75	9.46	12.3	11.73	
		v_2^-	4.26	4.48	10.37	7.47	5.98	5.84	6.02	5.7	5.89	5.82	7.29	5.3	
		v_3^+	17.21	19.1	25.05	33.06	23.73	24.45	25.79	28.3	23.95	25.35	28.93	30.94	
		v_3^-	6.48	7.41	13.51	11.78	8.82	8.88	8.87	9.47	7.97	8.02	10.78	9.81	
	ρ_4	v_2^+	9.38	10.27	17.08	17.27	11.9	12.82	13.48	15.4	11.29	12.16	15.78	16.87	
		v_2^-	6.91	7.59	13.82	10.71	7.62	7.23	7.32	7.04	7.36	7.54	8.65	7.65	
		v_3^+	31.86	36.44	41.28	51.74	40.9	43.02	43.32	49.78	43.08	44.77	47.18	52.24	
		v_3^-	14.49	15.27	20.9	22.27	16.01	16.78	14.81	20.28	16.56	16.36	18.46	21.68	
				ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	n_4	ρ_1	v_2^+	5.81	5.15	6.43	4.64	6.6	7.24	9.44	8.02	7.93	7.89	9.82	8.75
			v_2^-	6.2	5.47	4.97	4.35	5.89	6.38	7.66	6.17	5.94	5.99	7.87	6.23
			v_3^+	10.4	10.56	13.17	12.53	16.78	17.37	22.36	21.93	18.09	18.98	22.8	23.77
			v_3^-	9.43	7.97	7.09	7.68	9.87	9.11	10.56	9.64	9.5	9.06	10.52	10.55
		ρ_2	v_2^+	5.71	4.75	5.73	4.7	8.24	8.51	11.2	8.61	7.61	7.69	10.02	8.34
			v_2^-	6.58	5.22	4.85	4.64	8.34	7.91	9.5	7.69	5.88	6.36	7.42	6.08
			v_3^+	10.02	10.95	14.02	13.13	19.96	20.53	28.57	28.45	18.18	17.98	22.33	22.05
			v_3^-	8.98	7.95	6.84	7.25	13.46	12.72	14.74	14.35	9.98	9.19	10.37	10.26
		ρ_3	v_2^+	5.59	4.7	6.42	4.68	9.43	9.17	12.33	10.87	7.59	8.01	10.66	8.39
			v_2^-	6.44	5.69	5.39	4.71	5.82	5.3	7.51	5.72	6.05	6.02	7.64	6.29
			v_3^+	9.89	9.8	13.93	12.74	23.62	23.7	27.44	28.8	18.5	17.81	22.48	22.64
			v_3^-	9.59	8.6	7.8	7.32	7.86	8.02	10.24	9.31	9.13	9.57	10.91	10.5
		ρ_4	v_2^+	5.87	4.68	6.24	4.94	12.13	12.36	14.57	15.59	7.3	7.87	10.1	8.9
			v_2^-	6.65	5.63	4.76	4.07	7.13	6.82	8.24	7.32	5.8	6.2	7.27	6.17
			v_3^+	9.85	10.06	13.28	12.23	41.03	41.71	45.54	49.75	17.54	18.68	22	22.56
			v_3^-	9.09	8.84	7.01	7.41	15.65	15.75	17.21	20.34	9.37	9.07	10.91	9.81

n	ρ	v	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
n_5	ρ_1	v_2^+	ANOMC-Reg				ANOMC-R				ANOMC-MR1			
		10.52	11.93	14.94	17.78	10.13	9.9	10.66	11.17	9.72	10.51	12.56	11.77	
		v_2^-	6.63	7.29	11.88	10.48	6.2	6.47	6.67	6.64	6.43	6.61	7.94	6.1
		v_3^+	34.75	36.36	40.72	48.3	29.58	30.34	32.51	35.35	31.99	32.61	36.22	36.8
		v_3^-	13.1	12.88	17.2	16.58	11.4	11.09	11.25	11.08	11.19	11.26	13.66	11.37
		v_2^+	15.67	16.58	22.13	24.79	10.34	10.49	13.6	13.58	10.69	11	14.6	13.03
		v_2^-	12.87	12.41	16.5	15.34	9.19	8.66	8.77	8.34	9.57	9.22	9.78	8.21
		v_3^+	50.82	54.13	56.87	65.97	38.56	39.03	42.15	46.34	39.48	40.74	45.45	48.07
		v_3^-	21.29	22.61	23.39	26.91	15.38	14.93	14.46	16.17	16.42	15.98	17.18	17.37
		v_2^+	11.34	12.26	15.01	18.04	14.04	13.72	15	15.49	14.72	15.37	16.8	17.34
		v_2^-	4.54	5.36	8.98	8.47	6.63	6.81	7.27	6.5	6.76	6.74	7.83	6.61
		v_3^+	39.62	42.65	44.37	54.04	40.06	41.48	41.69	45.9	42.31	43.19	45.59	48.12
	v_3^-	9.81	10.51	14.47	15.51	9.9	10.69	10.52	11.06	10.15	10.19	12.13	11.37	
	ρ_2	v_2^+	18.68	20.59	23.82	29.36	19.31	20.42	21.6	24.64	20.26	21.34	24.02	26.85
		v_2^-	8.88	8.62	13.92	12.45	8.2	8.28	8.51	8.48	8.32	8.61	9.67	8.91
		v_3^+	68.64	69.47	71.75	76.66	68.89	69.58	69.31	71.98	71	71.71	72.27	73.85
		v_3^-	21.49	22.76	24.12	30.39	21.25	20.84	19.31	25.39	22.22	23.01	22.32	26.7
	ρ_3	v_2^+	ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
		9.51	9.26	11.63	10.58	9.8	10.24	12.14	10.59	9.92	10.09	12.02	11.86	
		v_2^-	6.12	6.35	6.77	6.14	6.28	6.49	8.06	6.05	6.14	5.7	7.69	6.52
		v_3^+	27.18	28.94	31.89	32.86	29.65	30.27	34.63	34.7	30.83	30.65	34.01	35.36
		v_3^-	11.2	11.06	10.93	11.26	11.17	10.46	12.42	11.29	10.52	11.02	11.87	11
		v_2^+	9.58	9.67	10.83	10.28	10.39	10.7	13.69	12.44	10.34	9.67	12.19	10.9
		v_2^-	7.05	6.06	7.15	6.56	9.07	8.83	10.03	7.79	5.96	5.84	7.3	6.45
v_3^+		28.1	28.78	31.99	33.31	37.28	37.74	42.86	44.85	30.55	31.77	33.99	35.41	
v_3^-		11.14	11.26	10.37	10.78	15.82	15.89	16.44	16.68	10.75	10.82	11.13	11.58	
v_2^+		9.37	9.34	11.17	10.45	14.26	14.2	16.65	15.74	10.03	10.3	11.82	11.68	
v_2^-		6.52	6.21	6.57	6.26	6.76	6.54	7.48	6.26	6.04	5.62	7.14	6.37	
v_3^+		27.49	28.43	31.08	32.99	40.94	41.71	43.28	45.89	31.26	30.72	32.54	34.91	
ρ_4	v_3^-	11.22	10.94	11.19	11.24	9.77	9.98	11.8	10.61	9.83	10.96	11.16	11.61	
	v_2^+	9.29	9.11	10.83	9.73	19.45	20.21	21.69	24.29	10.06	9.96	12.44	11.66	
	v_2^-	6.53	6.67	6.78	6.28	7.76	8.74	9.07	8.15	6.01	6.44	7.5	6.04	
	v_3^+	28.5	27.84	30.97	32.09	68.6	68.96	70.18	70.85	31.49	31	33.98	35.52	
n_6	ρ_1	v_3^-	11.17	10.68	10.94	11.34	21.33	21.51	21.18	25.5	11.39	10.55	11.7	10.86
		ANOMC-Reg				ANOMC-R				ANOMC-MR1				
		v_2^+	21.03	21.09	22.73	29.17	17.31	16.96	17.37	18.59	17.92	18.29	19.78	20.69
		v_2^-	8.5	9.38	10.98	12.22	7.62	7.86	7.62	7.92	8.23	8.45	8.92	8.3
		v_3^+	72.65	73.35	73.98	77.13	60.39	61	61.2	61.93	62.65	62.29	63.55	65.12
		v_3^-	23.49	24.62	24.43	29	17.88	18.18	17.43	19.54	19.4	20.01	20.01	21.07
		v_2^+	30.36	32.84	34.92	40.41	19.05	18.81	20.87	22.64	19.27	19.4	23.29	23.77
		v_2^-	15.55	16.38	16.47	19.04	10.45	10.17	10.05	10.67	11.14	11.38	11.29	10.68
		v_3^+	90.17	90.8	90.37	91.69	76.48	76.87	76.76	77.6	78.15	79.3	79.02	78.87
		v_3^-	36.06	37.15	34.67	43.54	25.01	25.06	23.69	27.48	26.49	26.57	26.73	28.6
		v_2^+	23.58	25.59	25.64	31.72	23.95	24.59	25.55	27.49	25.57	25.89	27.42	27.75
		v_2^-	6.67	6.99	9.53	10.01	8.01	7.71	8.18	8.42	7.58	7.72	9.11	8.24
	ρ_2	v_3^+	78.76	79.98	80.2	81.77	72.13	72.78	73.32	73.1	74.95	74.86	75.59	76.14
		v_3^-	23.17	24.71	25	30.72	20.58	21.31	20.9	23.34	20.56	21.54	21.8	23.63
		v_2^+	42.19	43.46	44.7	52.99	40.76	41.3	40.84	43.9	42.46	42.6	43.77	46.21
		v_2^-	13.64	13.59	15.24	17.31	12.47	12.8	12.09	13.21	13.03	13.27	12.66	14.15
	ρ_3	v_3^+	97.44	97.35	97.41	96.49	96.33	95.82	95.35	94.53	96.91	96.88	96.12	94.88
		v_3^-	46.42	47.65	44.61	55.64	42.21	44.6	41.65	49.63	44.21	46.63	44.53	52.02
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4				
		v_2^+	17.54	17.91	19.54	19.37	17.02	17.52	19.68	18.61	17.31	17.54	19.18	18.82
	ρ_4	v_2^-	8.02	8.19	8.69	7.84	8.03	7.81	8.57	7.85	7.48	8.1	7.78	7.86
		v_3^+	60.33	61.75	62.49	63.54	60.87	60.61	61.11	63.54	61.32	61.63	62.37	64.09
		v_3^-	19.21	18.94	18.09	20.54	18.41	18.28	18.84	20.09	18.81	18.49	18.33	20.12
		v_2^+	17.07	17.91	19.62	19.77	18.48	19.52	21.22	22.37	17.29	17.62	19.36	19.36
ρ_5	v_2^-	8.44	8.01	7.59	7.78	10.78	10.3	11.34	10.5	7.92	7.86	8.01	7.52	
	v_3^+	61.85	61.98	62.74	64.74	75.96	76.54	76.4	76.1	61.09	61.56	62.19	63.64	
	v_3^-	19.11	18.38	18.82	19.92	25.59	25.19	24.93	27.63	18.25	19.35	17.46	20.05	
	v_2^+	17.26	17.69	19.06	18.82	24.33	24.75	25.72	27.11	17.43	17.71	18.96	19.28	
ρ_6	v_2^-	7.99	7.88	8.53	7.92	8.12	8.21	8.66	8.38	7.41	7.57	7.7	8.12	
	v_3^+	60.93	61.44	61.49	64.82	73.05	72.7	73.34	73.91	61.56	61.86	61.07	62.6	
	v_3^-	19.25	19.35	17.66	19.42	19.98	20.07	21.45	22.15	18.22	19.18	17.7	19.29	
	v_2^+	17.29	18.25	19.89	19.93	39.65	41.15	42.4	44.54	17.45	17.66	18.86	19.74	
ρ_7	v_2^-	8.16	8.14	8.17	8.23	12.47	12.22	12.77	13.37	7.98	7.8	8.24	7.96	
	v_3^+	61.6	62.08	60.71	63.24	96.26	95.76	95.7	94.26	61	61.5	61.84	63.34	
	v_3^-	18.51	18.5	18.32	19.62	42.42	43.42	41.93	49.41	19.91	18.79	17.81	19.82	

II. Empirical type I error rate under heterogeneous variances

In Tables 2.9-2.11, it is observed that the ANOMC-MR2 test demonstrates slightly better percentage type I error rates compared to ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests. However, all the tests exceed the level of significance ($\alpha = 5\%$) due to the impact of heterogeneity. The actual type I error rates for all the tests significantly surpass the specified level when sample sizes and variances are inversely paired. Although improvements are observed at various levels of heterogeneity, it is important to note that increasing the sample size leads to an increase in type I error rates. Under both balanced and unbalanced designs, both tests are affected by variances' heterogeneity. However, the ANOMC-MR2 test shows a relatively lower susceptibility to heterogeneity than the other tests. Regarding the ANOMC-Reg test, it exhibits higher type I error rates (%) when large sample sizes are associated with substantial variances. Additionally, the ANOMC-Reg test also displays higher type I error rates (%) in cases with an association between large sample sizes and small variances. Conversely, the ANOMC-MR1 test demonstrates higher type I error rates (%) when there is an association between small sample sizes and large variances. However, in the specified experimental conditions (cf. Table 2.5), the ANOMC-MR2 test demonstrates a slight superiority in terms of robustness when compared to the remaining tests.

2.4.2 Results of the power test

Tables 2.12-2.17 present the comparative performances of ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests across nine distinct non-null cases (δ_{2-10}) for both balanced design (n_1) and unbalanced design (n_4).

I. Empirical test power estimates under balanced design (n_1)

For the fixed treatment level ($t = 4$) with fixed correlations (ρ_1), the results in scenarios of non-null case δ_6 reveal that under a normal distribution, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests exhibit power rates of 15%, 17.54%, 17.95%, 13.4%, 18.42%, and 18.7%, respectively. Under a double exponential distribution and for a non-null case δ_2 , the corresponding power rates are 25.09%, 26.81%, 27.17%, 19.7%, 26.15%, and 28.32%. Furthermore, when considering a Chi Square distribution, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests exhibit power rates of 27.31%, 27.31%, 22.08%, 26.29%, 18.21%, 25.46%, and 25.25% for non-null case δ_4 , respectively. Finally, in the context of non-null case δ_8 under a Cauchy distribution, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests demonstrate power rates of 38.8%, 30.45%, 31.34%, 24.46%, 30.9%, and 32.63%.

II. Empirical test power estimates under unbalanced design (n_4)

At a fixed treatment level ($t = 5$) and fixed correlations (ρ_3), for the non-null scenario δ_4 , and in the presence of a Cauchy distribution, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests exhibit power values of 61.17%, 49.85%, 55.04%, 28.2, 52.77%, and 47.21%, respectively. Under a double exponential distribution, the corresponding power rates for the non-null case δ_7 are 73.45%, 71.31%, 73.17%, 41.11%, 71.62%, and 62.13%, respectively.

Table 2. 13: Power (%) of ANOMC tests under homogeneous variances under balanced design (n_1) when $t = 3$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	23.47	26.36	34.45	45.54	25.34	28.09	32.52	37.25	26.55	27.99	36.52	38.25
	δ_3	18.8	21.98	30.07	39.46	20.68	22.47	27.03	30.91	21.94	23.15	31.37	32.51
	δ_4	17.71	20.81	28.19	37.28	19.3	21.48	24.51	28.45	20.08	21.89	28.58	29.65
	δ_5	19.16	22.46	29.76	39.03	20	21.44	26.85	29.99	20.96	22.06	30.63	31.92
	δ_6	22.86	26.58	34.59	46.97	23.02	25.56	30.8	34.69	25.44	26.49	34.89	36.15
	δ_7	18.4	21.36	29.91	40.07	20.77	22.69	26.93	30.8	21.75	23.52	29.76	32.42
	δ_8	18.88	21.69	30.24	39.79	21	22.07	27.23	31.21	21.32	23.39	30.29	31.98
	δ_9	17.4	20.63	28.21	37.51	19.83	19.93	25.24	29.53	20.73	21.54	28.47	30.56
	δ_{10}	17.16	19.52	28.73	36.49	18.92	20.84	24.79	28.86	20.16	21.01	28.73	29.06
ρ_2		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	20.05	21.62	27.01	32.1	25.46	26.35	34.22	36.72	25.46	27.34	34.17	39
	δ_3	17.29	17.68	22.2	26.31	21.36	21.25	28.69	29.85	21.95	23.1	28.02	33.15
	δ_4	16.28	17.16	20.5	24.61	19.81	20.02	27.4	27.79	19.66	22.66	26.4	31.27
	δ_5	16.88	18.13	22.84	25.7	19.94	21.48	28.06	29.79	22.06	23.26	28.69	33.31
	δ_6	20.4	21.36	26.54	31.07	23.53	25.46	33.32	34.72	26	28.68	33.89	37.99
	δ_7	16.9	18.04	22.75	26.4	20.35	22.83	28.78	29.85	22.42	23.07	29.7	33.04
	δ_8	17.65	17.84	22.31	26.29	20.71	21.42	29.15	30.49	21.25	23.6	28.15	33.18
	δ_9	15.51	17.23	21.16	24.25	18.85	20.13	27.07	27.76	19.99	21.45	27.81	29.93
	δ_{10}	15.91	16.87	20.89	24.78	19.15	20.95	27.24	28.8	20.71	21.48	26.43	31.11
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	27.53	31.7	43.46	52.77	33.37	34.94	41.92	45.9	32.41	34.24	43.85	46.44
	δ_3	23.38	27.08	38.13	46.1	26.75	29.06	34.72	38.6	25.94	27.63	36.91	38.64
	δ_4	22.47	24.85	36.73	43.69	24.42	27.09	32.27	36.44	23.13	26.57	34.36	36.47
	δ_5	23.44	27.15	37.19	40.15	25.25	27.89	33.06	37.59	25.69	27.27	36.21	38.07
	δ_6	28.24	32.22	42.39	52.7	29.62	32.62	38.65	43	30.91	32.36	41.54	45.23
	δ_7	23.07	26.76	38.21	46.11	26.73	28.08	35.37	39.59	24.91	27.97	38.47	38.72
	δ_8	23.01	26.25	33.65	46.71	25.51	28.06	33.21	37.69	26.88	29.47	35.92	39.48
	δ_9	20.93	25.06	35.49	43.97	23.51	25.61	32.29	35.98	23.78	27.3	35.35	35.65
	δ_{10}	22.05	24.3	32.34	43.72	23.11	25.05	30.59	33.7	24.7	25.96	34.31	36.63
ρ_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	20.24	21.61	25.92	30.33	29.93	31.69	41.29	43.75	25.4	28.51	33.55	38.09
	δ_3	16.2	17.48	21.9	26.86	23.94	26.52	34.91	36.6	21.39	23.29	28.45	33.52
	δ_4	15.29	16.9	19.96	24.87	21.56	23.43	32.42	34.48	19.96	21.71	26.67	32.28
	δ_5	16.04	17.54	21.97	25.82	23.79	26.46	35.3	36.15	21	23.19	29.05	32.85
	δ_6	19.34	20.3	26.35	31.35	28.52	29.88	38.73	41.18	26.06	28.97	34.83	38.65
	δ_7	16.83	18.12	21.83	26.01	24.38	25.78	35.14	37.4	21.45	23.56	28.66	33.59
	δ_8	16.27	18.18	21.66	25.88	25.63	27.37	35.07	37.81	21.82	23.86	28.15	33.21
	δ_9	15.74	16.24	20.82	23.81	22.01	24.02	32.84	32.83	20.46	21.77	26.62	31.14
	δ_{10}	15.7	17.12	20.11	24.36	21.67	24.88	31.67	33.99	20.7	22.37	27.49	31.51
ρ_5		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	27.51	32.24	39.29	52.39	30.86	34.61	38.1	44.5	33.05	36.43	41.2	46.96
	δ_3	22.72	26.95	33.42	46.16	25.71	27.59	31.82	37.71	27.58	29.25	36.52	41.15
	δ_4	21.56	25.25	30.5	44.15	22.68	24.41	28.62	34.9	26.1	27.19	33.7	39.4
	δ_5	23.51	26.93	32.53	46.44	24.19	25.91	30.43	36.68	27.48	29.45	34.82	40.65
	δ_6	27.28	32.72	38.14	52.48	28.49	31.03	34.95	42.88	31.6	35.83	41.31	47.6
	δ_7	23.28	26.52	34.4	46.51	25.63	27.54	32.81	37	27.05	29.61	36.14	39.77
	δ_8	23.22	26.58	37.9	46.52	26.66	29.33	35.04	39.23	26.43	27.49	36.78	39.41
	δ_9	20.87	25.15	33.07	43.33	23.09	26.14	31.1	35.6	24.27	25.8	33.49	37.01
	δ_{10}	21.85	24.42	36.11	43.83	23.4	26.4	31.76	36.2	23.74	25.31	34.37	36.64
ρ_6		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	19.01	21.29	26.5	30.2	31.21	33.48	41.79	44.82	25.87	28	34.19	38.24
	δ_3	16.56	17.81	22.03	25.78	26.94	28.33	35.14	37.95	21.53	23.01	27.94	32.9
	δ_4	14.9	16.8	20.08	24.31	26.01	25.59	31.82	35.32	20.78	22.04	26.87	31.05
	δ_5	17.18	17.5	22.01	25.49	26.18	27.74	34.66	38.43	21.04	24.24	28.92	33.61
	δ_6	19.26	20.78	26.13	31.18	30.16	33.28	40.09	44.68	25.78	28.11	33.5	40.1
	δ_7	16.52	16.95	22.32	26.63	24.63	27.08	34.87	37.91	21.11	23.27	28.29	33.07
	δ_8	16.75	17.95	21.49	25.65	23.78	26.39	34.87	36.26	21.59	23.41	28.52	32.27
	δ_9	15.8	16.03	20.37	24.66	22.71	24.14	31.98	33.98	20.1	22.11	26.57	30.51
	δ_{10}	15.06	17.16	20.19	25.12	22.94	24.14	32.16	35.33	20.36	21.71	26.58	31.32
ρ_7		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	45.26	49.92	56.1	68.24	55.97	59.33	62.42	68.03	57.17	61.3	66.03	70.52
	δ_3	36.34	41.8	49.5	62.61	45.52	49.7	54.19	60.07	48.07	50.84	57.64	63.65
	δ_4	33.09	37.73	46.43	58.89	42.63	45.93	50.36	56.87	44.17	48.05	54.4	59.58
	δ_5	35.9	42.33	49.69	62.35	44.33	48.93	52.24	58.21	47.09	50.16	58.31	63.06
	δ_6	44.25	50.54	56.99	67.72	53.23	55.87	60.65	65.8	56.17	58.66	64.51	69.53
	δ_7	35.37	41.52	48.83	61.63	46.53	49.38	54.1	60.44	49.26	51.82	58.24	62.61
	δ_8	36.99	42.04	50.35	62.15	46.52	48.87	53.91	61.4	47.74	51.81	58.32	64.06
	δ_9	32.86	37.43	46.82	58.88	41.97	44.81	50.11	56.5	44.74	47.37	54.42	59.64
	δ_{10}	34.11	39.09	45.81	59.21	41.95	44.46	50.91	55.71	44.68	47.23	54.35	60.58
ρ_8		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	18.39	20.3	25.49	29.54	55.04	57.78	63.56	66.94	25.73	27.76	31.68	37.53
	δ_3	15.97	16.74	20.93	24.72	46.88	48.18	55	58.89	21.56	23.15	27.97	32.44
	δ_4	15.05	15.75	19.67	23.13	41.54	44.74	51.35	55.68	19.89	21.74	26.43	30.53
	δ_5	15.5	16.5	20.81	25.45	45.12	46.77	54.51	58.51	21.82	22.42	28.44	32.93
	δ_6	18.85	20.47	24.72	30.11	52.28	56.01	62.27	64.53	26.23	27.8	33.97	37.88
	δ_7	15.63	16.48	21.43	24.59	45.46	49.52	55.43	59.35	20.69	22.58	28.77	31.8
	δ_8	15.52	17.07	20.62	25.57	46	48.77	54.86	59.39	21.12	23.37	28.06	32.39
	δ_9	13.8	15.87	19.26	23.58	41.12	44.25	52.44	56.98	19.01	21.91	25.33	30.47
	δ_{10}	14.75	16.27	19.94	22.9	41.65	43.78	52.35	56.16	19.56	20.82	25.88	30.53

In the scenario of a non-null case δ_{10} under a normal distribution, the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests demonstrate power percentages of 21.78%, 28.36%, 27.65%, 17.28%, 27.16%, and 26.29%, respectively. Lastly, considering the non-null case δ_3 within a Chi-Square distribution, the power rates for the ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests are found to be 35.09%, 31.35%, 36.96%, 23.81%, 35.42%, and 34.05%.

In general, when examining the correlation (ρ_1), the ANOMC-MR4 test consistently exhibits a heightened power rate compared to the other ANOMC tests. Conversely, in cases where the data follows a Cauchy distribution and the correlation (ρ_3) is taken into account, the ANOMC-Reg test demonstrates a notably higher power rate. However, ANOMC-MR2 demonstrates the lowest power rate across all correlation (ρ_{1-4}), non-null cases (δ_{2-10}) and sample size (n_1, n_4). Nevertheless, a comprehensive evaluation across several scenarios reveals that the ANOMC-MR1 test shows relatively superior performance compared to the other ANOMC tests. It is also noteworthy that the performance of all ANOMC tests (including ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) improves with an increase in the correlation between the study variable and the concomitant variable.

Table 2. 14: Power (%) of ANOMC tests under homogeneous variances under balanced design (n_1) when $t = 4$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	20.98	25.09	33.11	44.58	24.9	26.81	30.65	34.75	25.66	27.17	33.96	36.54
	δ_3	13.58	15.55	26.01	30.35	16.48	16.85	20.15	22.18	16.66	17.98	23.37	23.33
	δ_4	15.18	17.53	27.31	32.49	18.05	18.36	22.08	23.89	18.3	19.27	26.29	25.34
	δ_5	20.66	24.48	33.54	42.88	24.84	25.26	30.25	33.05	24.94	26.42	34.84	34.3
	δ_6	15	16.74	26.54	32.21	17.54	17.91	21.21	24.83	17.95	17.91	25.1	24.71
	δ_7	17.93	21.67	30.34	39.35	21.12	22.69	26.54	30.89	22.27	23.4	31.76	31.27
	δ_8	18.81	20.76	30.77	38.8	21.1	23.14	26.82	30.45	21.98	23.46	29.69	31.34
	δ_9	17.24	19.86	27.99	37.08	20.57	21.28	25.25	28.94	21.3	21.76	29.17	29.34
	δ_{10}	17.4	20.6	29.05	36.03	21.31	21.65	25.64	28.68	21.19	22.2	28.57	29.56
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	18.04	19.7	24.6	28.75	24.69	26.15	33.75	34.96	26.14	28.32	33.59	37.42
	δ_3	13.76	13.44	16.89	18.12	16.6	16.68	22.68	22.56	17.65	18.65	23.42	25.87
	δ_4	14.56	15.37	18.21	19.81	17.64	18.5	25.46	24.22	19.17	20.48	25.25	27.81
	δ_5	19.75	20.06	24.96	27.61	25.2	25.35	33.7	34.17	26.44	27.32	34.85	37.41
δ_6	13.4	14.04	17.13	19.51	18.42	18.15	24.13	23.88	18.7	19.16	24.55	26.59	
δ_7	16.68	18.1	21.78	24.99	22.72	22.78	30.48	30.09	22.78	24.51	29.45	32.68	
δ_8	16.52	17.46	21.57	24.46	22.29	23.47	30.64	30.9	23.05	24.8	29.88	32.63	
δ_9	16.21	16.63	20.61	23.3	20.77	20.95	29.02	28.92	22.04	23.28	28.05	30.72	
δ_{10}	15.77	16.87	20.73	22.65	20.21	21.5	27.53	28.19	21.76	22.88	28.42	31.57	
ρ_2		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	28.95	34.03	44.25	55.52	35.45	38.01	44.15	48.4	35.94	37.87	46.14	48.22
	δ_3	19.62	21.22	33.11	40.24	22.87	25	29.77	32.58	22.03	23.71	31.96	31.86
	δ_4	20.99	24.87	35.94	41.54	24.26	26.99	31.79	35.26	24.35	25.24	34.46	33.6
	δ_5	28.77	33.17	44.51	53.34	35.38	37.32	42.86	46.28	32.56	34.03	44.27	45.68
	δ_6	19.78	23.17	35.13	42.2	24.36	25.72	30.65	34.22	22.2	23.99	33.59	33.94
	δ_7	24.33	29.53	39.62	51.31	30.62	32.77	37.03	44.37	29.86	31.65	41.19	43.58
	δ_8	27.18	30.98	36.96	49.96	31.32	33.22	37.35	43.51	33.23	34.67	40.97	45.93
	δ_9	22.58	27.09	37.41	46.86	28.4	30.42	34.77	40.68	27.34	29.73	39.62	39.69
	δ_{10}	24.71	28.2	34.94	47.43	29.51	30.97	34.23	39.83	30.68	31.44	38.58	42.1
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	18.21	19.06	23.59	27.5	34.45	37.38	45.3	46.55	25.14	27.62	31.96	36.43
	δ_3	12.85	13.05	15.9	18.12	22.02	22.91	31.77	31.17	17.55	19.35	24.28	25.84
	δ_4	14.15	14.95	17.58	19.98	23.33	24.67	32.13	32.76	19.05	20.86	25.39	27.67
	δ_5	19.43	18.95	24.66	27.49	32.11	34.51	43.55	42.78	26.3	27.65	34.97	36.3
δ_6	14.2	14.93	17.08	19.64	22.18	24.69	32.05	33.3	18.43	19.64	24.43	26.57	
δ_7	16.36	17.74	20.69	24.27	28.88	31	39.17	41.5	22.95	24.82	29.23	32.78	
δ_8	16.35	16.97	20.66	23.99	32.87	33.94	41.08	43.81	22.23	23.81	29.1	31.61	
δ_9	15.79	16.07	19.89	22.47	27.51	28.8	37.19	38.98	21.22	23.07	28.49	31.19	
δ_{10}	15.81	15.94	20.09	22.59	29.88	31.27	37.65	39.76	22.28	22.96	28.17	30.93	
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	31.01	34.65	41.02	55.44	35.59	37.37	41.05	48.02	38.67	40.2	46	51.1
	δ_3	18.83	22.34	29.44	39.86	21.66	23.76	26.59	31.75	23.8	25.53	31.08	33.87
	δ_4	21.27	24.75	31.91	41.98	24.22	25.49	29.5	33.82	27.48	28.65	34.58	37.52
	δ_5	29.19	33.02	40.33	54.09	32.89	35.04	38.79	44.45	37.81	40.29	45.71	49.85
	δ_6	20.86	23.9	30.23	41.26	24.02	25.45	28.92	33.96	26.58	28.18	33.41	36.82
	δ_7	26.93	30.66	37.5	50.38	30.9	33.19	36.59	43.77	33.07	35.23	41.15	44.71
	δ_8	25.68	28.49	40.5	50.2	30.07	32.72	37.43	42.41	29.65	32.12	40.85	43.59
	δ_9	24.69	28.19	36.08	47.9	28.73	31.03	34.14	39.66	29.99	31.07	37.38	41.04
	δ_{10}	22.74	26.67	37.95	47.43	27.37	30.24	35.11	39.64	28.14	29.5	38.92	40.57
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	18.43	19.57	23.19	26.92	36.47	39.25	44.5	49.42	26.14	28.3	32.72	36.97
	δ_3	13.1	13.37	16.27	18.11	23.22	24.31	30.7	32.91	17.26	18.12	22.81	24.93
	δ_4	14.11	14.4	17.49	19.61	26.49	28.5	34.03	36.85	18.57	20.04	25.21	27.47
	δ_5	18.43	19.17	23.96	26	36.81	38.37	45.42	47.93	25.95	28.61	34.09	36.93
δ_6	12.33	13.25	17.44	19.47	26.72	26.62	32.64	36.12	18.56	19.53	24.52	26.8	
δ_7	15.57	17.12	20.32	23.57	32.59	33.58	41.1	43.53	22.85	24.25	28.68	32.66	
δ_8	16.59	17.28	20.72	23.61	29.51	31.43	39.07	40.37	22.59	22.98	28.85	32.35	
δ_9	15.58	16.32	19.56	22.47	30.69	31.17	37.33	39.53	21.28	22.88	28.2	30.84	
δ_{10}	15.34	15.79	20.17	22.95	28.18	29.83	37.37	38.7	20.95	21.86	27.85	30.67	
ρ_4		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	44.31	49.77	56.52	68.11	58.1	60.85	63.39	68.1	59.59	62.59	66.33	69.49
	δ_3	27.19	31.36	40.35	52.69	36.02	38.39	42.88	49.02	37.3	40.07	48.82	51.82
	δ_4	29.02	33.09	43.65	54.3	38.93	41.75	46.89	51.42	40.56	43.28	50.65	54.03
	δ_5	39.36	45.23	54.26	66.23	52.39	55.09	58.99	65	54.06	56.82	63.8	66.45
	δ_6	28.27	33.59	42.08	53.17	38.24	40.99	44.87	50.73	39.77	42.24	49.96	53.32
	δ_7	38.02	42.64	50.77	62.9	49.74	53.54	56.14	61.37	51.66	54.87	60.68	64.98
	δ_8	37.78	44.28	50.92	63.03	50.63	52.68	56.2	62.18	52.03	54.84	60.54	64.31
	δ_9	35.04	39.14	46.96	59.78	45.87	47.87	52.52	58.09	47.77	50.11	56.5	60.04
	δ_{10}	33.51	38.43	48.42	60.15	46.62	48.17	51.89	58.56	48.3	50.88	57.04	60.31
		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	17.19	18.88	22.78	27.07	58.58	59.93	65.21	66.3	25.33	27.35	32	35.72
	δ_3	12.54	13.04	15.21	17.39	35.94	38.47	47.47	48.21	17.24	18.21	22.97	24.71
	δ_4	13.56	13.79	17.45	19.4	38.93	40.62	49.48	51.01	19.11	19.88	25.36	27.19
	δ_5	17.03	19.72	24.09	26.48	52.49	55.01	61.93	63.37	24.97	28.02	34.34	36.64
δ_6	12.71	13.77	16.33	18.43	38.18	40.3	48.43	50.71	17.84	19.56	23.19	26.48	
δ_7	15.38	16.92	21.04	23.3	50.65	53.7	58.28	62.1	23.03	23.9	28.56	32.01	
δ_8	15.4	16.38	19.74	23.23	50.04	52.96	59.1	61.28	22.7	24	28.03	31.66	
δ_9	14.61	16.1	20	21.72	46.48	48.82	54.99	57.41	21.02	21.74	26.76	30.68	
δ_{10}	14.82	15.31	19.28	21.43	45.58	48.25	54.25	57.14	20.9	23	27.03	31.58	

Table 2. 15: Power (%) of ANOMC tests under homogeneous variances under balanced design (n_1) when $t = 5$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	20.03	22.7	33.01	43.03	24.12	25.9	29.44	33.77	24.33	26.91	32.46	34.2
	δ_3	13.74	14.95	25.74	28.78	15.33	16.27	18.75	20.57	15.76	16.4	22.22	20.28
	δ_4	16.02	18.17	28.78	33.98	19.29	19.65	22.94	26.19	19.51	20.14	27.56	26.33
	δ_5	16.13	18.03	28.91	35.57	18.97	20.48	23.86	27.09	20.29	20.48	27.46	27.58
	δ_6	21.05	24.25	33.97	43.75	25.87	27.14	30.27	34.01	25.76	27.46	34.46	35.69
	δ_7	17.99	21.17	30.17	39.27	21.3	22.62	26.23	29.63	22.74	23.27	30.06	30.74
	δ_8	17.38	20.88	30.86	39.53	20.92	22.69	26.01	30.35	22.46	23.06	30.33	31.22
	δ_9	16.82	19.45	29.53	37.17	19.65	21.91	25.39	28.03	21.15	21.83	28.8	29
	δ_{10}	17.44	20.02	30.02	36.9	21.46	21.83	25.23	28.22	20.66	21.6	29.13	28.96
ρ_2		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	16.77	17.75	21.83	23.81	24.25	26.35	32.65	33	25.76	27.61	31.62	35
	δ_3	11.82	11.41	15.44	15.43	16.05	16.56	21.77	20.84	17.89	18.57	22.72	24.03
	δ_4	13.75	14.74	17.88	19.03	19.26	20.58	27.6	26.38	20.96	21.22	26.49	29.61
	δ_5	13.88	14.47	18.32	19.69	19.45	20.19	27.09	26.91	20.67	22.45	26.14	28.85
	δ_6	18.72	19.11	23.5	25.59	25.54	25.99	34.74	34.49	27.1	29.59	35.24	37.09
	δ_7	15.21	16.44	19.45	22.45	22.81	23.21	29.79	28.97	22.39	24.38	29.22	32.17
	δ_8	14.72	15.37	19.3	21.74	21.91	23.06	29.8	30.35	23.79	24.53	29.52	31.26
	δ_9	14.7	15.12	18.24	20.45	22.15	22.54	28.6	27.84	21.94	23.84	28.14	31.2
	δ_{10}	14.58	15.68	19.72	20.72	20.85	22.54	29.28	28.41	22.51	23.07	27.88	31.55
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	24.46	29.67	41.6	52.07	31.26	33.45	38.16	43.13	29.67	32.25	41.94	43.5
	δ_3	17.02	18.94	31.81	36.2	19.54	20.45	24.99	26.84	18.41	18.59	27.64	25.66
	δ_4	20.41	23.25	34.94	40.93	23.72	25.49	30.18	32.53	21.18	22.8	31.48	31.95
	δ_5	19.71	23.75	35.02	44.05	23.95	25.89	30.38	34.62	22.67	24.87	33.05	34.28
	δ_6	26	29.83	42.45	50.57	31.36	33.25	39.06	42.37	28.35	30.35	40.16	39.48
	δ_7	23.08	26.9	37.66	48.51	27.4	29.65	34.12	39.35	26.84	28.3	38.03	38.78
	δ_8	24.25	28.64	36.79	49.21	29.65	31.97	35.18	39.84	30.82	33.05	38.72	41.87
	δ_9	21.29	24.57	36.41	44.94	25.71	26.68	32.06	37.84	25.38	26.78	35.83	37.12
	δ_{10}	24.01	26.06	35.85	45.23	27.56	29.21	33.2	37.83	29.1	30.42	36.2	38.3
ρ_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	16.71	17.33	20.61	23.83	29.75	32.77	40.71	41.96	24.92	27.41	31.41	35.44
	δ_3	11.46	11.56	14.1	15.9	18.84	19.5	26.89	25.07	17.42	18.07	22.74	23.95
	δ_4	13.81	13.86	17.97	18.76	22.24	23.03	31.75	31.45	20.95	21.86	27.31	28.9
	δ_5	13.3	13.52	17.59	18.89	23.78	24.86	33.32	33.65	20.56	21.72	25.79	28.46
	δ_6	17.81	18.35	22.96	25.05	28.43	30.36	39.67	38.52	27.09	28.5	34.58	37.04
	δ_7	15.5	16.03	19.64	21.32	25.67	28.58	37.68	36.96	23.36	24.48	29.62	31.46
	δ_8	15.25	14.97	19.13	22.23	31.11	32.38	38.57	40.17	22.91	25.09	29.02	31.87
	δ_9	14.35	15.01	17.81	20.18	25.51	27.31	34.96	36.21	22.46	22.96	28.03	30.45
	δ_{10}	13.79	14.63	19.07	20.07	28.99	31.02	36.85	37.82	21.38	22.87	26.83	31.43
ρ_5		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	27.65	32.41	40.22	52.03	33.42	35.01	38.12	44.46	34.33	36.13	42.77	45.94
	δ_3	17.07	19.27	28.73	35.36	20.25	20.87	22.79	27.02	22.71	22.3	27.44	29.18
	δ_4	21.73	24.57	33.21	42.69	25.7	25.77	29.04	33.84	27.82	29.03	34.73	36.31
	δ_5	21.94	25.25	33.2	44.17	25.79	27.44	30.39	36.04	26.91	30.25	34.4	38.25
	δ_6	28.19	31.9	41.1	53.04	33.65	35.52	39.64	44.67	37.17	39.08	45.54	47.54
	δ_7	25.06	28.53	36.68	49.03	29.47	30.88	34.82	40.37	31.13	33.14	38.17	41.64
	δ_8	22.43	26.34	37.89	47.91	27.15	29.93	33.08	40.15	25.63	28.42	37.51	39.1
	δ_9	23.7	27	35.39	46.39	28.24	30.03	32.33	37.63	29.02	30.26	37.54	38.97
	δ_{10}	21.11	24.8	36.12	45.91	25.9	26.69	32.16	37.16	25.61	26.97	35.91	36.7
ρ_6		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	15.99	16.45	20.64	24.04	34.56	36.23	41.34	44.41	25.1	26.67	32.36	35.09
	δ_3	10.74	11.36	14.75	14.97	21.71	22.95	28.52	28.05	16.87	18.04	22.68	23.7
	δ_4	13.28	13.38	17.45	18.54	27.84	28.56	34.92	35.14	20.47	21.72	27.77	29.02
	δ_5	13.12	13.77	17.69	18.92	27.71	29.49	35.03	35.63	20.88	22.08	25.61	28.42
	δ_6	16.84	17.55	22.7	24.31	36.82	37.84	45.49	46.93	27.37	29.06	34.68	37.16
	δ_7	14.86	15.38	18.94	21.34	31.47	32.58	39.02	40.42	23.34	24.74	28.15	31.61
	δ_8	14.97	15.27	18.44	21.41	26.54	28.57	36.87	37.62	23.07	24.53	29.09	32.43
	δ_9	14.5	14.69	17.59	19.45	28.83	30.11	36.59	37.08	23.33	23.23	28.4	29.56
	δ_{10}	14.45	15.17	17.15	20.44	24.78	26.8	35.96	35.76	22.74	23.86	28.5	30.37
ρ_7		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	43.97	50.41	56.51	68.46	58.51	59.48	61.73	67.23	60.71	62.66	65.87	69.32
	δ_3	25.74	28.96	40.1	49.93	33.95	36.36	39.54	46.44	35.23	37.6	46.31	48.77
	δ_4	31.8	34.95	45.84	57.44	43.35	45.8	48.46	53.99	44.83	47.75	52.92	57.44
	δ_5	34.53	38.68	47.32	60.01	46.22	49.36	51.71	57.78	48.26	50.34	54.88	60.55
	δ_6	40.94	47.2	56.16	67.82	55.25	58.89	60.52	66.45	58.25	59.97	64.96	68.53
	δ_7	39.57	44.31	51.67	64.74	52.87	55.06	56.17	63.17	53.5	56.52	61.54	65
	δ_8	38.17	44.01	50.91	63.56	52.76	54.17	56.21	63.25	53.26	55.6	61.03	65.62
	δ_9	35.19	39.47	49.78	61.35	48.76	50	52.66	59.52	49.46	52.52	57.34	61.47
	δ_{10}	34.71	40.64	49.62	62.28	48.91	50.9	53.02	60.03	50.85	52.3	57.33	61.13
ρ_8		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	15.5	16.11	20.07	23.25	58.23	60.19	65.19	66.57	25.52	27.17	31.65	33.98
	δ_3	11.14	10.76	14.18	14.97	35.15	37.56	45.1	46.18	17.41	18.13	23.22	22.01
	δ_4	12.73	13.01	16.39	18.64	43.89	45.57	52.32	54.6	21	22.27	25.96	28.67
	δ_5	13.49	13.15	16.4	18.01	47.19	49.22	54.75	56.66	19.6	21.28	26.16	28.31
	δ_6	16.8	17.86	21.01	23.81	57.74	58.72	64.87	65.5	26.46	28.44	34.38	35.79
	δ_7	13.42	15.02	18.26	20.26	52.96	54.78	59.53	62.61	23.1	24.05	27.96	30.09
	δ_8	14.15	15.23	18.6	20.15	53.09	54.44	60.71	61.7	22.84	23.96	28.33	31.66
	δ_9	13.5	13.92	17.47	19.31	49.79	51.28	56.52	58.03	22.03	22.95	27.15	29.97
	δ_{10}	13.25	14.36	16.94	19.82	49.7	51.15	56.39	59.64	21.93	22.67	28.18	29.9

Table 2. 16: Power (%) of ANOMC tests under homogeneous variances under unbalanced design (n_4) when $t = 3$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	40.14	44.47	52.96	61.67	40.44	42.61	47.58	50.24	41.16	43.84	49.72	52.42
	δ_3	29.18	32.72	42.23	51.04	30.41	30.88	36.29	40.19	30.94	32.37	39.64	41.05
	δ_4	23.42	26.14	36.74	41.25	23.08	24.59	29.96	32.22	24.04	24.32	33.15	34.24
	δ_5	21.17	24.52	33.98	41.07	21.21	22.74	28.86	30.28	22.02	22.55	32.04	31.97
	δ_6	24.64	27.37	37.38	44.78	24.03	25.21	31.33	33.41	23.71	25.88	34.97	35.87
	δ_7	35.6	40.28	48.13	58.21	36.24	36.52	42.05	46.26	36.05	38.23	45.62	47.98
	δ_8	20.42	21.27	27.05	37.13	21.13	23	26.35	29.25	35.33	22.27	27.54	30.35
	δ_9	32.58	36.57	46.02	54.84	33.16	34.39	39.06	44.15	33.15	35.1	42.5	44.29
	δ_{10}	19.14	21.61	28.09	36.59	20.21	21.92	25.45	28.48	21.09	21.61	27.51	28.61
ρ_2		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	25.63	28.01	37.64	39.02	39.64	42.11	49.8	50.37	40.2	43.83	49.81	53.78
	δ_3	18.04	19.81	29.92	29.85	29.65	31.48	38.87	39.85	31.03	32.76	40.86	42.68
	δ_4	15.56	16.31	25.9	23.9	23.09	23.74	32.6	32.55	24.39	26.11	34.45	35.75
	δ_5	13.25	14.58	23.78	21.84	20.91	23.42	30.82	30.59	23.38	24.17	33.78	34.2
	δ_6	16.01	17.17	26.77	25.36	23.7	25.08	33.46	32.28	26.54	28.22	37.07	38.68
	δ_7	22.09	24.58	32.82	34.78	34.62	36.43	44.45	45.57	34.86	39.07	44.85	48.88
	δ_8	16.34	16.07	13.54	21.56	20.4	21.65	27.43	29.26	22.08	22.51	23.39	31.12
	δ_9	20.88	22.99	29.79	33.22	32.01	33.58	40.49	42.59	34.47	36.1	42.37	46.7
	δ_{10}	15.45	15.86	14.51	20.75	20.01	21.34	26.8	28.58	20.33	22.61	25.56	30.46
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	53.59	58.9	65.17	72.3	51.17	53.8	57.46	61.88	49.06	51.86	58.17	60.43
	δ_3	40.68	45.7	55.95	61.99	38.08	40.23	46.73	50.68	36.54	38.69	47.66	49.33
	δ_4	32.03	36.28	47.34	54	29.88	32.41	38.67	41.95	28.55	31.32	40.79	42.72
	δ_5	31.39	33.7	45.96	51.44	27.18	29.71	36.78	38.93	27.19	29.25	39.79	40.41
	δ_6	34.36	37.8	47.87	54.88	30.18	32.58	39.53	42.38	29.82	31.29	42.05	42.95
	δ_7	47.9	53.09	59.27	67.78	44.35	46.88	51.81	56.88	43.19	45.21	52.88	55.35
	δ_8	28.37	29.73	34.41	47.47	24.09	26.96	30.73	36.1	43.48	27.41	32.93	37.07
	δ_9	44.98	49.03	57.49	65.22	40.44	43.56	47.83	52.4	38.86	42.9	49.62	53.74
	δ_{10}	27.35	29.8	36.51	47.13	25.41	26.08	31.33	35.33	25.79	26.63	33.85	36.14
ρ_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	25.74	28.71	36.31	38.67	47.36	49.35	56.63	59.64	41.08	43.58	49.33	53.49
	δ_3	18.42	20.16	29.92	28.71	36.04	37.99	46.45	48.38	30.78	32.73	41.01	42.49
	δ_4	14.94	15.69	25.44	23.23	28.01	29.29	40.11	40.51	24.88	26.36	33.94	35.45
	δ_5	14.34	15.26	25.15	21.35	26.03	27.11	37.7	38.62	22.98	24.5	33.22	34.89
	δ_6	16.1	16.48	27.16	25.01	28.31	30.52	39.94	41.62	26.3	27.04	36.34	37.44
	δ_7	22.03	25.13	32.2	35.19	42.03	44.42	51.57	53.48	35.92	37.17	44.28	48.79
	δ_8	16.17	16.13	13.6	21.28	25.7	26.98	32.63	36.17	20.99	23.36	24.82	30.55
	δ_9	20.37	23.3	30.82	33.68	38.01	40.77	48.41	50.18	34.39	36.33	41.56	45.49
	δ_{10}	14.87	15.08	14.88	20.87	24.21	25.8	32.4	35.64	20.83	22.12	25.63	31.91
ρ_5		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	44.13	49.45	55.67	66.77	48.38	50.13	53.38	59.43	49.88	51.68	57.47	60.79
	δ_3	32.96	37.62	43.94	54.63	35.44	38.28	41.91	46.89	37.25	41.32	46.08	50.22
	δ_4	24.59	28.4	34.59	46.31	27.54	29.77	34.07	38.07	30.17	31.95	38.84	40.66
	δ_5	22.91	25.97	31.84	44.59	24.58	27	30.4	36.32	27.89	29.4	36.61	40.86
	δ_6	25.69	29.05	37.22	49.28	26.47	28.29	34.27	39.64	30.62	32.83	40.73	43.27
	δ_7	38.41	43.09	50.34	60.35	41.54	43.8	47.45	53.01	44.03	45.56	51.74	54.39
	δ_8	20.21	22.78	29.88	40.45	24.65	26.39	31.5	35.47	43.78	24.63	32.51	34.68
	δ_9	35.51	40.01	46.28	57.76	38.99	41	44.81	49.64	38.83	41.44	47.66	51.37
	δ_{10}	20.73	22.97	30.85	40.3	24.22	26.36	30.94	34.33	23.46	24.54	32.73	34.18
ρ_6		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	23.42	26.66	36.08	38.51	49.38	50.42	55.56	59.21	41.42	43.13	49.06	53.73
	δ_3	17.62	19.17	29.53	29.07	36.4	37.91	44.56	48.09	30.59	33.06	40.16	42.96
	δ_4	14.2	15.09	26.35	22.18	28.15	30.53	37.3	38.93	24.29	26.17	34.66	35.91
	δ_5	13.43	14.63	25.6	21.48	26.13	27.75	35.29	36.93	22.75	24.26	32.8	33.99
	δ_6	15.66	15.91	26.9	24.2	29.03	31.17	38.83	41.13	25.35	27.89	36.91	38.42
	δ_7	21.66	23.38	31.73	34.25	41.35	44.13	49.34	52.89	36.48	37.81	44.37	48.62
	δ_8	16.06	15.97	13.28	22.12	22.62	24.19	31.56	33.04	20.5	23.01	24.75	31.04
	δ_9	20.27	22.94	28.61	32.44	37.83	40.18	45.84	49.78	33.66	35.84	41.65	47.07
	δ_{10}	14.72	15.74	13.8	20.85	23.03	24.44	31.53	32.91	21.27	22.63	25.72	31.36
ρ_7		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	73.58	76.25	78.82	84.26	80.3	80.78	79.22	82.2	80.34	81.46	81.17	82.97
	δ_3	57.41	62.24	68.33	74.54	65.05	66.66	68.94	73.06	65.52	66.73	70.63	74.26
	δ_4	46.65	50.59	58.76	67.06	52.48	54.5	59.53	63.17	54.75	57.47	62.44	68.4
	δ_5	43.29	48.14	56.78	65.31	48.47	52.08	55.73	61.82	50.41	55.14	60.54	65.36
	δ_6	48.26	53.14	61.12	69.32	54.43	57.43	61.36	66.13	55.52	58.6	64.8	69.92
	δ_7	66.19	70.04	73.85	79.92	72.14	73.29	74.42	77.64	73.81	74.25	76.04	78.6
	δ_8	38.39	42.99	46.68	59.94	44.05	48.93	52.3	59.42	73.98	50.49	55.62	62.14
	δ_9	63.82	65.5	70.14	77.47	68.73	70.19	72.28	75.07	70.4	71.67	73.34	77.29
	δ_{10}	37.53	42.28	47.94	60.28	45.02	47.87	50.93	58.11	46.29	49.07	55.56	60.87
ρ_8		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	23.67	26.41	35.32	38.15	79.08	79.02	79.75	80.36	40.99	43.61	49.66	52.4
	δ_3	17.2	19.24	28.38	28	63.23	65.26	69.79	70.9	30.26	32.17	39.98	42.33
	δ_4	14.18	15.21	25.64	22.34	51.74	54.69	60.92	63.29	24.43	26.06	34.11	34.78
	δ_5	12.84	14.14	24.01	20.88	47.5	51.16	57.55	61.59	22.76	25.69	33.01	33.1
	δ_6	14.49	15.95	27.07	23.47	53.12	57.62	62.19	65.63	25.43	26.94	35.96	37.59
	δ_7	21.11	23.14	31.09	33.85	71.85	73.62	73.66	76.24	35	37.9	44.76	46.85
	δ_8	15.44	16.84	12.97	20.68	44.64	48.39	52.6	58.52	21.26	21.59	23.96	30.54
	δ_9	19.21	22.2	29.24	32.1	68.37	69.34	71.67	74.27	33.63	35.9	42.16	45.96
	δ_{10}	14.21	15.2	13.63	19.98	43.31	47.17	52.34	58.56	21.11	22.07	24.8	30.3

Table 2. 17: Power (%) of ANOMC tests under homogeneous variances under unbalanced design (n_4) when $t = 4$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	25.32	28.92	38.35	48.59	31.18	33.49	37.2	42.92	32.03	33.88	42.39	44.07
	δ_3	12.91	15.11	24.25	29.09	16.57	16.76	20.99	23.1	17.45	17.5	24.45	23.56
	δ_4	14.53	16.67	26.41	29.21	16.8	18.23	22.44	24.23	18.09	19.66	25.98	24.74
	δ_5	19.63	22.32	32.49	38.8	23.87	24.53	29.98	32.97	25.44	26.64	35.64	35.37
	δ_6	14.63	15.89	25.36	30.51	18.1	19.71	23.37	25.2	19.18	20.05	27.64	26.65
	δ_7	22.08	25.5	35.55	45	28.6	30.02	34.26	38.28	28.5	31.36	38.02	38.52
	δ_8	14.92	17.43	24.56	31.3	19.22	20.65	23.31	26.32	19.68	20.68	26.34	27.85
	δ_9	20.49	24.08	33.61	42.31	27.11	28.34	32.04	34.84	26.78	28.32	35.81	36.76
	δ_{10}	14.55	16.68	24.9	28.66	18.76	19.44	22.33	25.55	19.53	20.24	26.24	26.52
ρ_2		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	13.29	14.99	21.58	22.93	32.25	33.41	42.1	41.97	32.93	34.48	39.74	43.64
	δ_3	8.39	8.33	12.62	10.71	17.13	17.44	24.52	22.03	18.06	19.24	25.06	24.77
	δ_4	8.87	8.78	14.55	11.93	18.39	18.97	25.37	24.12	18.67	20.49	27.95	27.64
	δ_5	11.08	11.21	18.8	17.13	25.47	26.4	34.93	33.22	26.36	27.4	35.77	37.34
	δ_6	8.47	8.48	13.64	11.94	18.36	19.81	26.46	25.49	19.11	19.98	26.88	27.59
	δ_7	11.88	13.62	19.35	20.97	28.38	29.57	37.43	39.1	29.08	31.05	36.56	40.29
	δ_8	11.39	10.9	10.46	14.26	18.39	20.79	25.71	25.55	19.64	20.77	23.7	27.41
	δ_9	11.71	12.5	17.97	18.9	26.65	28.16	36.13	35.39	27.75	28.91	35.11	38.14
	δ_{10}	10.88	10.37	10.7	13.66	19.24	19.71	25.42	26.02	19.1	20.89	23.72	26.45
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	39.66	44.68	53.74	64.59	46.17	48.88	53.46	58.54	45.55	48.86	57.18	59.02
	δ_3	21.66	24.55	36.3	40.78	24.62	26.68	32.42	34.34	23.39	25.04	35.02	35.19
	δ_4	24.19	26.68	39.2	42.6	26.87	29.12	34.15	36.69	24.75	26.1	37.4	35.65
	δ_5	33.31	36.18	48.57	54	36.36	38.82	45.71	49.29	34.4	37.31	48.35	48.1
	δ_6	23.39	26.8	38.02	44.05	27.13	28.92	34.14	36.61	26.82	27.43	37.03	38.69
	δ_7	35.07	40.85	50.11	61.22	41.06	43.7	48.9	53.58	41.43	42.66	51.27	54.62
	δ_8	25.94	29.14	33.45	45.24	28.74	29.73	32.94	40.13	29.87	32.47	37.11	41.48
	δ_9	32.89	36.98	49.03	57.21	37.85	40	45.08	50.68	37.37	40.29	49.81	51.66
	δ_{10}	24.16	26.24	33.41	42.39	26.49	28.68	32.45	38.09	28.64	30.54	36.42	39.25
ρ_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	13.2	14.48	21.46	23.21	44.52	47.49	55.1	56.64	31.71	34.44	39.67	43.6
	δ_3	8.12	8.28	12.86	10.74	23.41	24.12	34.81	32.61	17.82	18.44	25.06	25.35
	δ_4	8.01	8.97	13.86	11.53	24.53	26.9	36.67	34.62	18.74	19.83	26.6	27.08
	δ_5	11.01	11.53	18.61	16.43	33.79	35.97	46.72	45.59	26.32	27.14	35.92	36.24
	δ_6	7.97	8.37	13.23	11.81	25.1	27.01	35.96	35.96	19.08	20.82	25.71	27.31
	δ_7	11.58	12.3	18.48	20.04	39.75	42.98	50.22	52.26	28.79	30.05	37.34	39.94
	δ_8	11.4	11.07	10.73	13.45	30.05	31.85	37.1	40.22	19.26	20.59	22.77	27.89
	δ_9	11.74	12.5	16.98	19.06	36.93	39.78	48.52	49.5	27.56	28.5	34.9	37.57
	δ_{10}	10.66	10.49	10.45	13.69	28.81	30.03	36.78	38.53	18.55	20.24	23.23	26.38
ρ_5		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	32.72	37.02	44.03	57.54	43.62	45.35	48.74	54.08	46.24	48.51	52.84	57.41
	δ_3	15.75	18.16	24.89	34.7	21.95	23.42	25.71	30.91	24.6	25.8	30.51	34.28
	δ_4	16.53	18.53	26.1	34.9	23.27	25.04	28.15	31.58	26.3	27.64	33.32	36.02
	δ_5	22.13	25.05	34.27	45.28	31.43	32.57	36.7	43.11	36.84	38.06	44.91	47.18
	δ_6	17.73	20.2	26.82	37.06	23.97	25.66	48.62	34.55	27.86	29.31	33.96	36.53
	δ_7	28.79	33.04	40.95	53.22	38.06	40.45	44.96	50.92	40.75	44.25	47.86	53.22
	δ_8	15.12	18.31	28.09	37.59	24.78	26.92	31.35	36.42	25.02	26.66	34.3	36.72
	δ_9	26.01	29.81	38.49	49.81	35.71	37.81	42.25	46.61	38.15	40.14	46.36	49.29
	δ_{10}	15.99	17.76	28.48	35.12	24.26	26	30.45	35.89	25.56	25.51	31.71	35.59
ρ_6		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	11.96	13.06	20.26	21.96	45.09	47.04	52.81	55.87	31.71	33.37	40.53	43.24
	δ_3	8.03	7.75	12.41	10.74	24.74	24.9	30.61	32.88	17.48	17.87	24.05	25.15
	δ_4	7.69	7.91	13.63	10.95	25.1	26.72	32.96	33.81	18.43	19.47	26.98	27.28
	δ_5	10.4	11.11	18.27	16.7	36.04	37.28	44.23	46.77	25.71	27.11	34.42	35.63
	δ_6	7.92	8.51	13.31	11.16	27.01	27.91	33.75	36.09	19.24	20.34	26.53	27.87
	δ_7	10.58	11.97	18.46	18.38	40.58	43.19	48.26	50.53	28.83	30.81	36.3	39.24
	δ_8	10.64	10.42	10.06	13.87	24.38	26.03	32.95	34.47	19.43	20.77	22.84	27
	δ_9	11.04	10.65	16.82	17.64	37.46	38.98	45.08	47.81	26.17	28.73	34.6	37.04
	δ_{10}	11.06	11.19	10.93	12.36	23.31	25.59	31.63	34.49	18.79	20.14	23.42	26.73
ρ_7		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	53.25	59.53	64.64	75.22	70.92	71.29	72.18	75.55	72.42	72.92	75.4	77.58
	δ_3	26.34	30.05	42.16	50.06	37.56	40.45	44.83	49.98	38.86	41.38	50.21	54.15
	δ_4	27.37	30.33	42.16	51.09	39.81	42.11	48.02	52.14	40.39	45.03	51.86	55.52
	δ_5	39.23	43.26	53.42	63.22	53.75	56.43	60.41	65.13	55.87	59.18	64.63	67.8
	δ_6	28.35	32.96	43	53.78	41.58	44.78	48.74	54.42	44.45	47.09	53.5	57.38
	δ_7	46.17	52.28	60.34	70.24	64.78	65.48	67.24	71.22	65.3	67.97	70.3	72.62
	δ_8	29.4	35.25	40.61	53.91	43.26	47.61	49.25	56.36	44.92	48.82	53.84	59.64
	δ_9	43.55	48.26	56.87	66.53	59.1	61.02	63.01	69.04	60.93	63.61	67.47	69.76
	δ_{10}	28.38	31.78	39.99	51.6	40.85	44.73	47.86	53.45	41.85	45.97	52.49	57.02
ρ_8		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	12.39	12.81	19.87	21.37	70.67	71.34	73.75	74.77	31.12	33.99	38.61	42.23
	δ_3	7.54	7.2	12.58	10.26	38.27	40.28	49.1	49.79	17.13	17.89	24.41	24.37
	δ_4	7.99	7.7	13.34	10.81	39.96	43.13	51.5	52.26	18.21	20	27.02	26.51
	δ_5	9.72	10.37	17.53	15.15	54.29	56.66	62.85	66.21	24.14	27.51	35.09	36.22
	δ_6	8.47	7.91	12.96	11.26	42.45	45.18	52.45	53.96	18.71	20.08	25.27	27.65
	δ_7	11.4	11.45	17.51	19.07	64.51	65.22	69.05	70.76	27.65	30.15	36.67	39.53
	δ_8	11.34	11.04	10.45	13.85	44.19	45.72	52.41	56.31	19.19	19.7	23.33	27.54
	δ_9	10.72	11.53	16.6	17.31	59.45	60.62	65.4	67.75	25.97	28.94	34.16	37.58
	δ_{10}	10.53	9.82	10.38	13.21	41.54	44.09	50.56	53.62	18.07	19.96	22.83	26.16

Table 2. 18: Power (%) of ANOMC tests under homogeneous variances under unbalanced design (n_4) when $t = 5$.

ρ	δ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
ρ_1		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	64.94	68.42	71.75	79.51	60.83	62.29	63.4	67.73	63.6	64.13	67.39	67.78
	δ_3	24.58	27.42	35.48	42.2	22.97	23.97	27.4	29.86	24.73	24.43	31.19	29.48
	δ_4	36.04	39.21	46.77	56.41	34	35.71	39.25	42.96	35.82	37.12	42.64	43.45
	δ_5	47.78	51.72	56.97	66.54	45.77	46.04	47.72	52.33	46.66	47.54	52.19	54.12
	δ_6	44.87	48.98	56.61	65.59	45.73	45.9	47.87	52.71	45.85	47.07	53.27	54.07
	δ_7	62.23	65.31	70.05	76.66	57.84	59.43	60.91	64.36	59.72	60.96	65.54	64.61
	δ_8	20.08	22.58	27.22	34.78	21.33	22.71	23.9	26.92	22.13	22.14	26.51	26.83
	δ_9	58.67	63.3	66.98	74.89	54.93	56.57	57.81	62.37	56.61	57.65	61.59	62.38
	δ_{10}	22.1	24.18	30.34	37.41	24.24	24.63	26.54	29.3	24.39	25.15	30.36	28.8
ρ_2		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	41.86	45.74	52.72	54.04	61.19	62.51	65.45	65.92	64.53	66.14	68.74	69.57
	δ_3	14.97	15.26	22.78	20.15	22.48	23.79	29.67	28.2	28.33	28.41	34.56	34.54
	δ_4	20.86	22.5	30.34	28.43	34.73	34.59	41.51	40.29	37.56	39.77	45.53	47.56
	δ_5	28.68	30.97	38.85	39.23	43.53	44.49	50.69	51.03	47.8	49.45	54.32	55.94
	δ_6	27.16	28.61	36.52	38.46	44.89	44.66	51.37	51.67	48.34	49.58	54.2	56.96
	δ_7	39.97	42.66	48.99	51.9	57.21	58.17	61.85	61.58	62.62	62.8	65.8	66.79
	δ_8	16.89	16.79	14.46	18.66	20.91	21.97	25.03	25.4	21.43	21.98	24.25	27.65
	δ_9	38.18	40.41	46.66	48.47	53.58	55.42	60.58	59.34	58.77	60.43	62.74	64.94
	δ_{10}	16.85	17.27	16.81	19.17	23.71	24.16	28.34	28.28	24.58	25.23	28.84	30.27
ρ_3		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	84.84	85.33	86.41	89.57	78.95	78.67	77.54	79.83	78.68	78.56	79.53	79.42
	δ_3	38.61	40.54	52.19	57.77	32.21	33.64	37.59	39.98	30.42	32.15	40.44	39.6
	δ_4	53.11	57.06	64.42	71.23	45.77	47.25	51.58	55.33	44.17	46.64	53.37	53.38
	δ_5	67.11	71.03	73.93	81.01	59.86	60.23	61.7	66.07	59.26	60.58	65.05	66.21
	δ_6	63.28	66.64	73.52	78.78	57.09	58.45	61.05	64.86	54.14	55.94	61.63	62.69
	δ_7	81.97	83.25	84.27	88.78	74.65	74.66	74.67	77.12	74.91	76.14	77.35	76.94
	δ_8	32.88	34.72	36.97	50.07	29.09	29.59	32.57	36.99	29.2	31.26	34.7	38.09
	δ_9	79.59	81.55	83.14	86.7	70.82	71.25	72.41	74.93	72.56	72.94	74.68	75.55
	δ_{10}	33.84	36.79	42.59	52.83	30.75	31.56	34.74	38.44	30.94	31.64	38.41	40.44
ρ_4		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	43.11	46.16	51.84	54.32	76.02	77.11	78.63	77.27	64.75	65.5	67.62	68.44
	δ_3	15.61	15.54	23.22	19.79	29.67	29.95	37.69	37.04	26.96	27.59	35.21	33.7
	δ_4	20.61	21.91	30.02	29.16	41.96	43.62	51.24	52.3	38.29	40.11	44.99	46.64
	δ_5	28.89	30.64	37.89	39.91	56.82	58.87	63.56	63.97	49.53	50.21	53.99	55.97
	δ_6	26.86	28.11	36.49	36.78	51.16	53.39	60.35	59.76	48.17	48.95	55.5	56.77
	δ_7	40.18	42.33	48.73	51.33	72.67	72.33	75.44	74.45	60.78	64.5	66.12	66.08
	δ_8	17.68	17.11	14.25	18.65	29	30.54	34.9	36.48	21.58	22.79	24.7	27.81
	δ_9	37.73	39.26	46.79	48.57	69.56	69.92	72.91	72.15	60.12	60.46	63.14	64.43
	δ_{10}	17.74	17.37	16.91	19.5	29.96	31.62	36.83	37.29	24.95	25.29	28.85	30.76
ρ_5		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	75.16	75.98	78.76	83.11	73.61	74.34	74.1	75.85	76.08	75.81	77.8	78.48
	δ_3	25.55	29.41	35.09	45.75	28.31	29.66	31.35	34.75	31.6	32.47	36.96	38.58
	δ_4	39.96	44.64	49.13	61.17	42.71	44.15	45.05	49.85	47.44	46.97	52.26	55.04
	δ_5	54.85	58.98	62.91	72.14	55.99	56.6	57.27	61.93	59.53	60.07	62.96	65.28
	δ_6	51.84	56.56	61.25	70.87	55.45	57.24	58.26	63.3	59.73	60.7	63.99	65.85
	δ_7	71.11	73.45	76.77	81.15	69.82	71.31	70.73	73.03	72.35	73.17	74.79	75.55
	δ_8	19.81	22.21	28.45	38.41	24.96	26.24	29.35	32.33	23.81	25.88	30.77	32.65
	δ_9	67.53	69.87	73.75	79.41	66.74	67.85	69.11	71.48	68.87	70.23	71.47	73.47
	δ_{10}	21.78	25.29	31.78	42.07	28.36	29.66	32.21	36	27.65	28.68	34.19	36.31
ρ_6		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	40.66	43.22	49.39	53.6	74.01	74.45	75.2	75.59	64.98	66.29	67.05	69.66
	δ_3	14.94	14.2	23.81	19.37	29.69	31.48	35.42	36.37	26.85	28.45	34.05	34.04
	δ_4	20.18	21.36	29.24	28.2	45.54	45.5	49.84	52.77	36.93	39.48	44.75	47.21
	δ_5	27.78	29.73	37.41	39.25	57.63	58.04	60.24	61.59	47.7	50.14	54.15	55.46
	δ_6	25.73	27.06	36.17	36.88	57.19	58.83	62.05	63.74	48.58	49.97	54.39	56.61
	δ_7	37.92	41.11	47.35	50.67	70.42	71.62	72.75	72.73	62.39	62.13	66.32	66.64
	δ_8	17.08	16.26	13.89	18.62	23.84	24.78	29.74	30.99	21.91	22.2	23.84	27.69
	δ_9	35.99	39.25	45.38	48.18	67.48	66.1	69.61	70.8	58.87	59.47	63.62	63.37
	δ_{10}	17.28	17.34	16.86	19.66	27.16	27.89	33.19	34.28	26.29	25.72	28.93	30.64
ρ_7		ANOMC-Reg				ANOMC-R				ANOMC-MR1			
	δ_2	96.23	95.5	95.57	95.12	96.75	96.24	94.55	93.29	97.36	96.76	95.14	93.87
	δ_3	51.35	54.21	62.32	68.92	54.86	56.55	58.68	61.56	58.18	58.83	62.98	65.88
	δ_4	69.9	72.86	76.93	82.05	73.91	74.49	74.06	76.98	76.12	76.27	79.09	80.19
	δ_5	84.93	87	87.06	89.02	87.44	87.04	85.7	85.76	89.19	89.13	87.29	87.43
	δ_6	80.5	82.46	85.05	89.11	85.59	85.15	83.49	85.86	86.67	86.72	85.87	86.5
	δ_7	94.74	94.33	94.55	94.51	95.77	95.02	92.7	92.03	96.4	95.69	94.01	93.24
	δ_8	43.2	46.43	48.27	63.27	49.67	52.49	53	60.83	51.95	52.95	56.7	60.96
	δ_9	92.97	92.08	93.17	93.43	94.14	92.72	91.69	90.74	94.85	94.02	92.78	91.76
	δ_{10}	45.02	48.2	51.75	65.21	52.74	54.65	56.77	62.24	52.59	56.88	60.29	64.07
ρ_8		ANOMC-MR2				ANOMC-MR3				ANOMC-MR4			
	δ_2	40.63	41.95	50.46	51.99	96.61	95.59	95.11	92.55	65.43	65.5	67.81	69.79
	δ_3	14.63	14.97	22.51	18.43	54.56	56.64	61.91	60.93	26.87	27.63	33.62	33.55
	δ_4	19.26	21.01	29.18	27.97	73.43	73.9	75.58	76.5	38.13	39.81	44.6	45.95
	δ_5	27.27	29.57	36.14	37.88	87.04	86.57	86.21	85.45	48.81	49.81	53.24	55.61
	δ_6	24.42	27.37	34.81	35.97	83.84	84.11	83.7	84.38	48.02	50.02	53.5	55.76
	δ_7	37.74	39.95	47.49	49.37	95.39	94.47	93.6	91.37	61.92	62.39	64.5	66.61
	δ_8	16.79	16.08	14.23	18.15	48.55	51.16	54.78	58.9	21.92	22.95	23.62	27.25
	δ_9	35.48	37.7	44.74	47.35	93.58	92.97	92.14	90.3	57.75	60.25	62.57	65.31
	δ_{10}	16.98	17.46	16.68	18.97	52.11	53.77	58.32	60.4	24.66	26.07	28.1	30.2

2.5 Illustrative Examples

To evaluate the performance of our proposed methods, as well as existing method, that utilize auxiliary information through the analysis of means using covariate (ANOMC), we provide real-life examples that compare the means while considering the auxiliary variable. This illustration is based on real manufacturing industry and medical science data. The ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 methods were implemented for analysis in balanced and unbalanced cases.

2.5.1 An example of the balanced case (equal sample size)

The mechanical manufacturing problem dataset [66] was utilized to implement the ANOMC methods for a balanced case with equal sample sizes. Dutta, et al. [66] conducted an experimental study on the effects of electrical discharge machine (EDM) parameters on machining time and surface roughness for machining Inconel 800. They utilized an (EDM) and investigated the impact of pulse current, pulse-on-time, and pulse-off-time. The EDM process employed a negatively polarized brass wire with a diameter of 0.25mm and a tensile strength of 500N/mm as the electrode. Deionized water was used as dielectric fluid to facilitate insulation and material removal. Samples with dimensions of 25mm × 25mm × 5mm were cut using the Electronic 4-axis CNC sprint cut wire electrical discharge machine. Machining time (in minutes) and surface roughness (in μm) were recorded while varying the pulse current (amp), which was one of the manipulated parameters.

In this example, the focus is on studying two variables: surface roughness (Y) as the study variable and machining time (X) as the concomitant variable. These variables are reported

at different levels of pulse current (*amp*). The relationship between surface roughness and machining time is considered to be linear based on ten experiments, excluding the first and seventh experiments. Hence, 10 observations ($n = 10$) correspond to three levels ($t = 3$) of pulse current, namely $210\mu m$, $220\mu m$, and $230\mu m$. The ANOMC methods (ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) are applied for testing the means of surface roughness considering machining time with a significance level (α) of 5%. Moreover, the estimator's mean ($\bar{M}_i, \bar{T}_{(1)i}, \bar{T}_{(2)i}, \bar{T}_{(3)i}, \bar{T}_{(4)i}$, and $\bar{T}_{(5)i}$) are calculated for each pulse current level, and the critical values are presented in Table 2.4. Furthermore, Figure 2.1 illustrates the graphical depiction of the ANOMC tests performed with a balanced design.

ANOMC-Reg involves calculating the overall mean ($\bar{M}_{..}$) as 3.073646 and the overall mean square error ($MSE_{\bar{M}_{..}}$) as 0.071436. The individual regression means ($\bar{M}_i$) are plotted against the decision interval, with the critical value $C(\alpha, t, n) = 8.59$, the lower decision limit (LDL) set at 2.915205, and the upper decision limit (UDL) at 3.232086. For ANOMC-R, the overall mean ($\bar{T}_{(1)..}$) is determined as 3.084764, and the overall mean square error ($MSE_{\bar{T}_{(1)..}}$) is calculated as 0.312417. Using the critical value $V(\alpha, t, n) = 9.652$, the decision interval is established with the LDL at 2.306178, and the UDL at 3.863351, and it is plotted against the individual modified ratio means ($\bar{T}_{(1)i}$). ANOMC-MR1 involves calculating the overall mean ($\bar{T}_{(2)..}$) as 3.077092 and the overall mean square error ($MSE_{\bar{T}_{(2)..}}$) as 0.296145. The critical value $Z(\alpha, t, n) = 9.55$ is used to determine the decision interval, resulting in the LDL of 2.346857 and the UDL of 3.807328, and it is plotted against the individual modified ratio means ($\bar{T}_{(2)i}$). In ANOMC-MR2, the overall

mean ($\bar{T}_{(3)..}$) is calculated as 3.07224, and the overall mean square error ($MSE_{\bar{T}_{(3)..}$) is determined as 0.017299. The individual modified ratio means ($\bar{T}_{(3)i.}$) are plotted against the decision interval. By utilizing the critical value $K(\alpha, t, n) = 9.58$, the decision interval is established with the LDL at 3.02945 and the UDL at 3.115031. In ANOMC-MR3, the overall mean ($\bar{T}_{(4)..}$) is calculated to be 3.084763, and the overall mean square error ($MSE_{\bar{T}_{(4)..}$) is determined as 0.3101119.

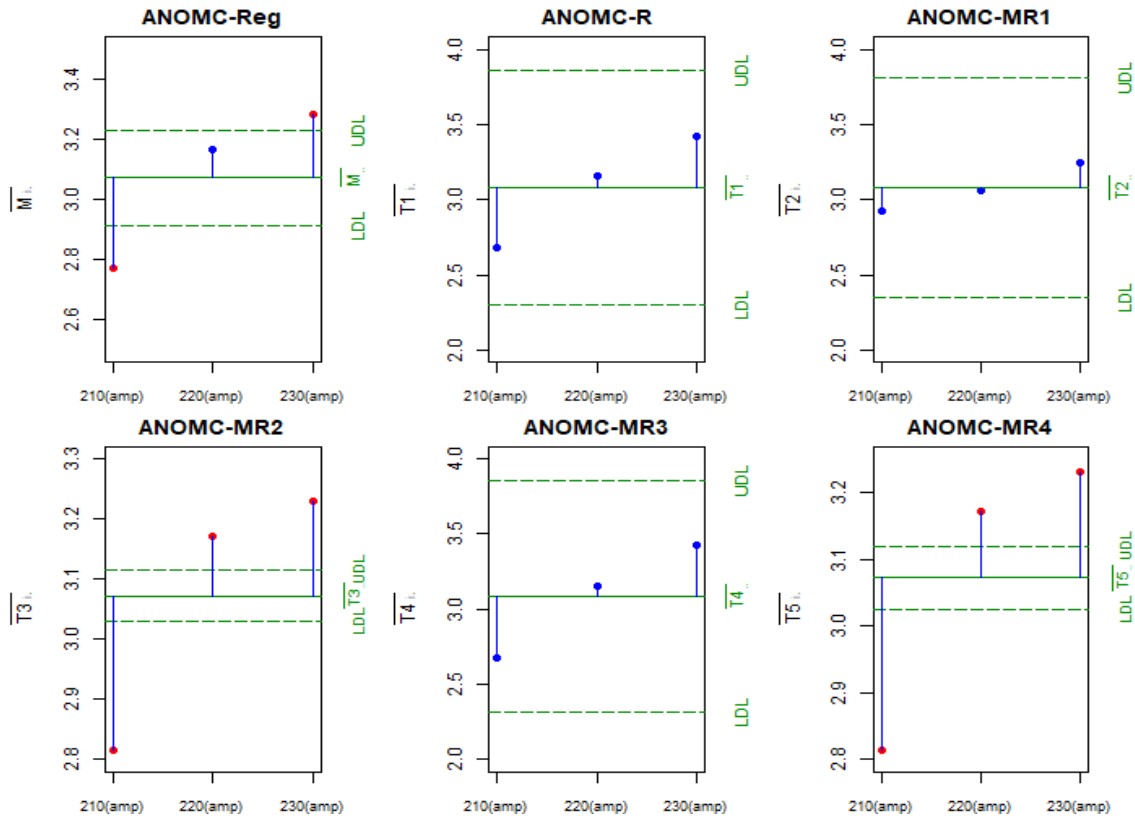


Figure 2. 1: The performance of ANOMC methods for EDM problem.

The critical value $P(\alpha, t, n) = 9.62$ is utilized to establish the decision interval. The individual modified ratio means ($\bar{T}_{(4)i.}$) are plotted against this decision interval, resulting in the lower decision limit (LDL) of 2.314484 and the upper decision limit (UDL) of

3.855042. Finally, in ANOMC-MR4, the overall mean ($\bar{T}_{(5)..\}$) is found to be 3.071949, and the overall mean square error ($MSE_{\bar{T}_{(5)..}}$) is determined as 0.01894093. The critical value $O(\alpha, t, n) = 9.54$ is used to establish the decision interval. The individual modified ratio means ($\bar{T}_{(5)i.}$) are plotted against this decision interval, with the lower decision limit (LDL) set at 3.025294 and the upper decision limit (UDL) set at 3.118605.

2.5.2 An example of an unbalanced case (unequal sample size)

In the field of medical science, a dataset related to sexual desire (libidos) applies ANOMC methods in an unbalanced case with unequal sample sizes. Where the effects of three different dosages of Viagra on both participants' and their partners' libidos are recorded. The dataset [67], consists of three treatment groups corresponding to different Viagra dosages. The first treatment group represents the placebo (no Viagra), including 9 participant and partner libidos samples. The remaining two treatment groups represent low and high dosages of Viagra, with 8 and 13 samples of participant and partner libidos, respectively. In this analysis, the number of participant libidos was considered as the study variable (Y), while the number of partner libidos served as a covariate (X). Figure 2.2 illustrates the graphical depiction of the ANOMC tests performed with an unbalanced design.

The ANOMC methods (ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) are applied for testing the means of participant libidos considering partner libidos served with a significance level (α) of 5%. Moreover, the estimators mean ($\bar{M}_{i.}, \bar{T}_{(1)i.}, \bar{T}_{(2)i.}, \bar{T}_{(3)i.}, \bar{T}_{(4)i.}$ and $\bar{T}_{(5)i.}$) are calculated for each participant's libidos level.

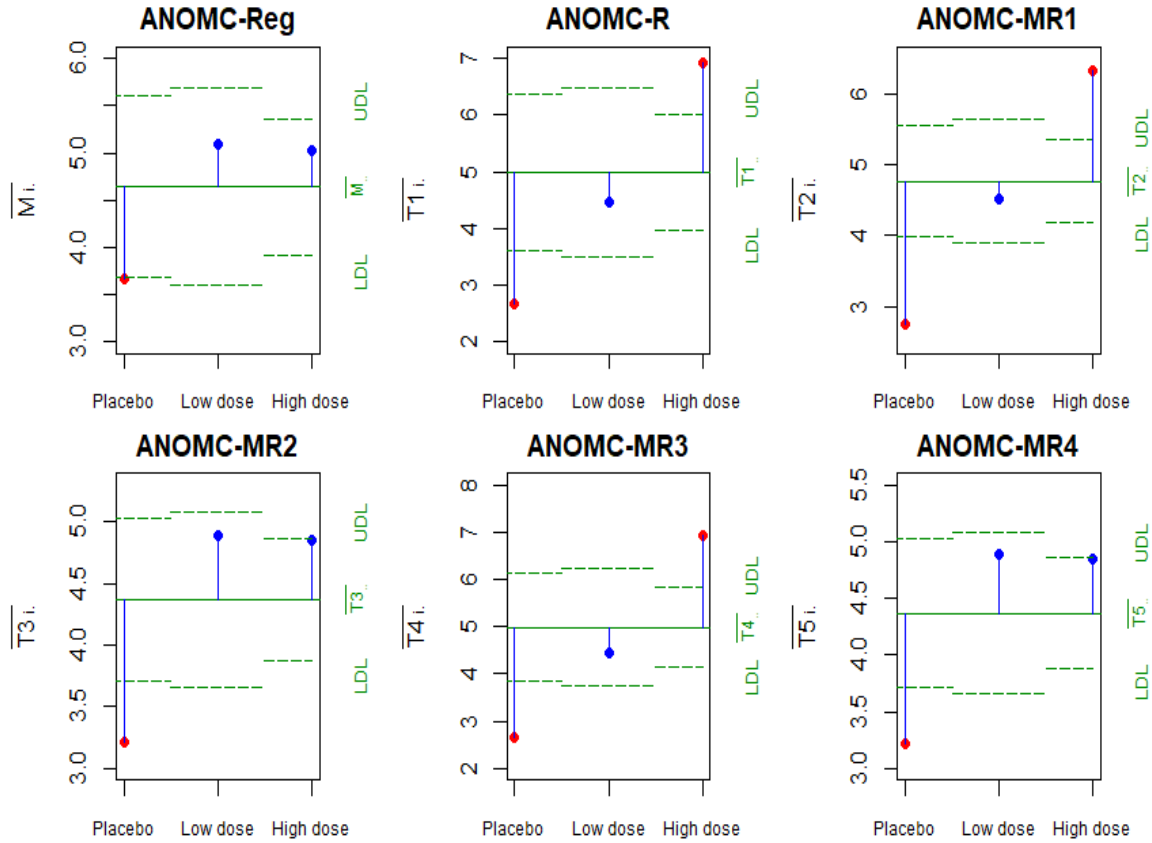


Figure 2. 2: The performance of ANOMC methods for Viagra dosage problem.

ANOMC-Reg involves the computation of the overall mean ($\bar{M}_{..}$) as 4.641178 and the overall mean square error ($MSE_{\bar{M}_{..}}$) as 0.393557. The individual regression means ($\bar{M}_{i.}$) are plotted against the decision interval using the critical value $C^*(\alpha, t) = 8.73$. The lower decision limit (LDL) is set at (3.682991;3.600950;3.923855), and the upper decision limit (UDL) is set at (5.599366;5.681406;5.358502). For ANOMC-R, the overall mean ($\bar{T}_{(1)_{..}}$) is determined as 4.98926, and the overall mean square error ($MSE_{\bar{T}_{(1)_{..}}}$) is calculated as 0.505509. Using the critical value $V^*(\alpha, t) = 9.74$, the decision interval is established with the LDL at (3.616115;3.498546;3.961289) and the UDL at (6.362404;6.479973;6.017230). The individual modified ratio means ($\bar{T}_{(1)i.}$) are plotted against this interval. ANOMC-MR1 involves calculating the overall mean ($\bar{T}_{(2)_{..}}$) as

4.769833 and the overall mean square error ($MSE_{\bar{T}_{(2)}}$) as 0.292333. The critical value $Z^*(\alpha, t) = 9.743$ is used to determine the decision interval, resulting in the LDL of (3.975752; 3.907763; 4.175364) and the UDL of (5.563914; 5.631904; 5.364302). The individual modified ratio means ($\bar{T}_{(2)i}$) are plotted against this interval. In ANOMC-MR2, the overall mean ($\bar{T}_{(3)}$) is calculated as 4.369636, and the overall mean square error ($MSE_{\bar{T}_{(3)}}$) is determined as 0.2433905. The individual modified ratio means ($\bar{T}_{(3)i}$) are plotted against the decision interval, established using the critical value $K^*(\alpha, t) = 9.72$. The LDL is (3.709859; 3.653368; 3.875710), and the UDL is (5.029414; 5.085904; 4.863562). ANOMC-MR3 involves calculating the overall mean ($\bar{T}_{(4)}$) as 4.989049, and the overall mean square error ($MSE_{\bar{T}_{(4)}}$) is determined as 0.419637. The critical value $P^*(\alpha, t) = 9.7$ is utilized to establish the decision interval, resulting in the LDL of (3.853844; 3.756647; 4.139206) and the UDL of (6.124253; 6.221450; 5.838891). The individual modified ratio means ($\bar{T}_{(4)i}$) are plotted against this interval. Finally, in ANOMC-MR4, the overall mean ($\bar{T}_{(5)}$) is found to be 4.368331, and the overall mean square error ($MSE_{\bar{T}_{(5)}}$) is determined as 0.241897. The critical value $O^*(\alpha, t) = 9.67$ is used to establish the decision interval. The individual modified ratio means ($\bar{T}_{(5)i}$) are plotted against this interval, with the LDL set at (3.715975; 3.660120; 3.879961) and the UDL set at (5.020687; 5.076542; 4.856701).

2.6 Summary and Concluding Remarks

The ANOM test compares means within the main variable; however, the ANOMC test represents an improved version of the ANOM test, incorporating the influence of covariates. This study expands upon the ANOMC test, utilizing ratio estimators instead of

the regression estimator. In this context, the extension ANOMC test is denoted as ANOMC-Reg, while the proposed ANOMC is presented as ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4. The ANOMC tests (namely, ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) work under a set of assumptions, such as normality, linearity, and homogeneity. The impact of these assumptions, as well as considerations like sample sizes (equal or unequal), different treatments, and the formulation of null and non-null hypotheses, is meticulously assessed concerning the ANOMC tests. This evaluation compares the tests' performance in terms of the test's percentage type I error and percentage power. At the end of this simulation, the study showed that in case of homogeneity of variances with the null case, the ANOMC-Reg is not as robust as the ANOMC based on ratio family (ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) under both Chi-square and Cauchy distributions. Furthermore, it is observed that when unequal correlations, pairing (direct or indirect) of correlations are considered may lead to a change in the percentage type error rate from pre-specified ($\alpha = 5\%$). In the presence of heterogeneity of variances with the null case, regardless of the distribution shape and sample size combinations, the aforementioned ANOMC tests are affected. Still, the ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4 tests exhibit less susceptibility to the effects compared to the ANOMC-Reg test, particularly the ANOMC-MR2 test, in both balanced and unbalanced design under normal and non-normal distribution. ANOMC-Reg is affected when large sample sizes are associated with more substantial variances. On the other hand, the ANOMC-MR1 test shows higher rates of type I error when there is an association between small sample sizes and large variances.

As anticipated, the power values of the ANOMC tests (ANOMC-Reg, ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4) vary based on the effect size, sample size, and distribution environment. ANOMC, when compared to ANOMC-Reg, demonstrates relatively superior performance when employing ratio family estimators. However, it is important to note that the specific experimental conditions significantly determine the outcomes. Furthermore, the effectiveness of the ANOMC tests improves as the correlation between the study variable and the concomitant variable increases.

CHAPTER 3

GRAPHICAL MEANS TESTING

APPROACH USING TWO

COVARIATES

In real-world applications, it is frequently necessary to account for covariates that exhibit linear relationships with the main variable. However, in such cases, the conventional ANOM approach fails to consider the impact of these covariates when assessing the equivalence of group means. In the previous chapter, we proposed a new ANOMC methods that adjusts for one covariate. Use of the ANOMC test is not sufficient; more investigations should be conducted on its utility in experiments where there are multi-covariates involved. Therefore, in this chapter, we aim to find out the performance of the ANOMC test with two auxiliary variables with a family of ratio estimators. The proposed test is namely ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 and will be evaluated in terms of Type I error rate and the power of the test. In order to evaluate and compare performance, a Monte Carlo simulation was conducted. The tests are

assessed based on their percentage of type I error and power under various design parameters.

3.1 Introduction

Historically, statisticians have placed significant emphasis on utilizing covariates as valuable tools for minimizing unwanted variability in a dependent variable. Nonetheless, further research and exploration are required to comprehensively investigate the role of multiple covariates in reducing errors and obtaining accurate and reliable results, particularly in experimental settings. Recent studies have discussed and focused on the effectiveness of two auxiliary information (covariates) used in control charts, and generally, they show promising results [68-73]. In addition, ratio estimators have been recognized as trustworthy estimators in many diverse areas, such as control charts, and have exhibited outstanding influence [70, 72, 74]. As proven by Hussain, et al. [74], Jayalath and Ng [54], Khan, et al. [75], and Noor-ul-Amin, et al. [76], the aforementioned estimators performed well with the applications of control charts. The application of ratio estimators using two auxiliary variables conducted by Yan and Tian [77], Kadilar and Cingi [78], and Lu [79] indicates that the ratio estimators are powerful in a wide variety of areas, however within the Design of Experiments (DOE) arena, the application of ratio estimators, in particular when involving two auxiliary variables, has in fact been quite Infrequent.

Though ratio estimators have demonstrated their utility in applications of control charts, in the DOE field, their expectable values and effectiveness in ANOMC test, which makes use of auxiliary information, are thus far less known. Therefore, the prime objective of the current study is to evaluate the effectiveness of the ANOMC test with multiple auxiliary

variables using ratio estimators. The proposed tests, referred to as ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10, will be assessed in terms of two measures: the percentage of Type I error and the power of the test. The evaluation will be conducted under different distributions, covariance structures, sample sizes of the treatments, heterogeneous variances, and different number of treatments.

The remaining part of the chapter is organized as follows: Section 2 consists of mathematical details of ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests; Section 3 describes the study's parameters design; Section 4 assesses the performance of the proposed methods; Section 5 focuses on applications in biomedical and mechanical engineering; Finally; In Section 6, we provide our concluding remarks for the study.

3.2 Proposed Approach

In this section, we will introduce the mathematical derivations and details of the ANOMC test, specifically focusing on the inclusion of two auxiliary variables. Our proposal involves utilizing the ANOMC test based on ratio estimators to test adjusted means when two covariates are present.

3.2.1 The fixed-effect model

Let us consider a completely randomized design (CRD) with a single factor and two linearly related covariates X and Z with t treatment levels, each with n_i observations while the total number of observations is $N = \sum_{i=1}^t n_i$. The corresponding fixed effects model can be expressed as follows:

$$Y_{ij} = \mu + \tau_i + B(X_{ij} - \bar{X}_{..}) + B^*(Z_{ij} - \bar{Z}_{..}) + \epsilon_{ij}, i = 1, 2, 3, \dots, t, j = 1, 2, 3, \dots, n_i \quad (3.1)$$

Where Y_{ij} is the j^{th} observation of response variable for the i^{th} treatment level, $\tau_i = \mu_i - \mu$ is the fixed effect of i^{th} treatment level with respect to the overall mean μ , and we assumed that the random errors $\epsilon_{ij} \stackrel{i.i.d.}{\sim} N(\mu, \sigma^2)$, X_{ij} is the j^{th} observation of covariate for the i^{th} treatment level corresponding to Y_{ij} , B is the slope indicating the relationship between Y_{ij} and X_{ij} , $\bar{X}_{..}$ is the grand mean of X_{ij} observations. Z_{ij} is the j^{th} observation of covariate for the i^{th} treatment level corresponding to Y_{ij} , B^* is the slope indicating the relationship between Y_{ij} and Z_{ij} , $\bar{Z}_{..}$ is the grand mean of Z_{ij} observations.

In model (3.1), several assumptions are made. Firstly, we assume that the slope B and B^* is not equal to zero, indicating a linear relationship between Y_{ij} and X_{ij} and between Y_{ij} and Z_{ij} . Additionally, it is assumed that the observations for each treatment level are not identical. Furthermore, it is assumed that the concomitant variables X_{ij} and Z_{ij} are not affected by treatment levels. Moreover, X_{ij} and Z_{ij} are assumed to have the same effect on Y_{ij} Which implies equal slopes for these concomitant variables. Lastly, the treatment effects sum to zero (i.e., $\sum_{i=1}^t \tau_i = 0$). The structure of the test dataset and ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests, is reported in Table 3.1.

3.2.2 The ANOMC test based on ratio estimators

A class of generalized ratio-type estimators is employed, treating it as a ratio variable for the purpose of testing the mean while incorporating the two covariates. The ANOMC test based on the ratio estimators considering two covariates is referred to ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests.

Table 3. 1: The framework of the ANOMC test based on ratio family estimators and data layout.

Test	Terms	τ_1	τ_2	τ_3	...	τ_t	...	τ_t
	R_1	$Y_{11}(X_{11}, Z_{11})$	$Y_{21}(X_{21}, Z_{21})$	$Y_{31}(X_{31})$	...	$Y_{i1}(X_{i1})$	...	$Y_{t1}(X_{t1})$
	R_2	$Y_{12}(X_{12}, Z_{12})$	$Y_{22}(X_{22}, Z_{22})$	$Y_{32}(X_{32})$	...	$Y_{i2}(X_{i2})$	...	$Y_{t2}(X_{t2})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
	R_j	$Y_{1j}(X_{1j}, Z_{1j})$	$Y_{2j}(X_{2j}, Z_{2j})$	$Y_{3j}(X_{3j})$	...	$Y_{ij}(X_{ij})$	...	$Y_{tj}(X_{tj})$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
	R_{n_i}	$Y_{1n_i}(X_{1n_i}, Z_{1n_i})$	$Y_{2n_i}(X_{2n_i}, Z_{2n_i})$	$Y_{3n_i}(X_{3n_i}, Z_{3n_i})$	...	$Y_{in_i}(X_{in_i}, Z_{in_i})$	...	$Y_{tn_i}(X_{tn_i}, Z_{tn_i})$
	$Y_{i.} = \sum_{j=1}^{n_i} Y_{ij}$							
	$X_{i.} = \sum_{j=1}^{n_i} X_{ij}$	$Y_1(X_{1.}, Z_{1.})$	$Y_2(X_{2.}, Z_{2.})$	$Y_3(X_{3.}, Z_{3.})$	...	$Y_i(X_{i.}, Z_{i.})$	...	$Y_t(X_{t.}, Z_{t.})$
	$Z_{i.} = \sum_{j=1}^{n_i} Z_{ij}$							
	$\bar{Y}_{i.} = \sum_{j=1}^{n_i} Y_{ij}/n_i$							
	$\bar{X}_{i.} = \sum_{j=1}^{n_i} X_{ij}/n_i$	$\bar{Y}_1(\bar{X}_1, \bar{Z}_1)$	$\bar{Y}_2(\bar{X}_2, \bar{Z}_2)$	$\bar{Y}_3(\bar{X}_3, \bar{Z}_3)$	...	$\bar{Y}_i(\bar{X}_i, \bar{Z}_i)$	...	$\bar{Y}_t(\bar{X}_t, \bar{Z}_t)$
	$\bar{Z}_{i.} = \sum_{j=1}^{n_i} Z_{ij}/n_i$							
	$S_{Y_{i.}}^2 = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2 / (n_i - 1)$							
	$S_{X_{i.}}^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^2 / (n_i - 1)$	$S_{Y_1}^2(S_{X_1}^2, S_{Z_1}^2)$	$S_{Y_2}^2(S_{X_2}^2, S_{Z_2}^2)$	$S_{Y_3}^2(S_{X_3}^2, S_{Z_3}^2)$	...	$S_{Y_i}^2(S_{X_i}^2, S_{Z_i}^2)$	...	$S_{Y_t}^2(S_{X_t}^2, S_{Z_t}^2)$
	$S_{Z_{i.}}^2 = \sum_{j=1}^{n_i} (Z_{ij} - \bar{Z}_{i.})^2 / (n_i - 1)$							
	$S_{Y_i X_i} = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})(X_{ij} - \bar{X}_{i.}) / (n_i - 1)$	$S_{Y_1 X_1}$	$S_{Y_2 X_2}$	$S_{Y_3 X_3}$	...	$S_{Y_i X_i}$	...	$S_{Y_t X_t}$
	$S_{Y_i Z_i} = \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})(Z_{ij} - \bar{Z}_{i.}) / (n_i - 1)$	$S_{Y_1 Z_1}$	$S_{Y_2 Z_2}$	$S_{Y_3 Z_3}$	...	$S_{Y_i Z_i}$	...	$S_{Y_t Z_t}$
	$S_{X_i Z_i} = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})(Z_{ij} - \bar{Z}_{i.}) / (n_i - 1)$	$S_{X_1 Z_1}$	$S_{X_2 Z_2}$	$S_{X_3 Z_3}$	...	$S_{X_i Z_i}$	...	$S_{X_t Z_t}$
	$\theta_i = (1 - g)/n_i$	θ_1	θ_2	θ_3	...	θ_i	...	θ_t
	$C_{Y_i}^2 = S_{Y_i}^2 / \bar{Y}_i^2$	$C_{Y_1}^2$	$C_{Y_2}^2$	$C_{Y_3}^2$	...	$C_{Y_i}^2$	...	$C_{Y_t}^2$
	$C_{X_i}^2 = S_{X_i}^2 / \bar{X}_i^2$	$C_{X_1}^2$	$C_{X_2}^2$	$C_{X_3}^2$	...	$C_{X_i}^2$	...	$C_{X_t}^2$
	$C_{Z_i}^2 = S_{Z_i}^2 / \bar{Z}_i^2$	$C_{Z_1}^2$	$C_{Z_2}^2$	$C_{Z_3}^2$	...	$C_{Z_i}^2$	...	$C_{Z_t}^2$
	$S_{\bar{Y}_{(k)l}} \approx \theta_l \bar{Y}_l^2 C_{Y_l}^2 \left[1 - \frac{\rho_{yx}^2 + \rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}}{1 - \rho_{xz}^2} \right]$	$S_{\bar{Y}_{(k)1}}$	$S_{\bar{Y}_{(k)2}}$	$S_{\bar{Y}_{(k)3}}$	...	$S_{\bar{Y}_{(k)l}}$	...	$S_{\bar{Y}_{(k)t}}$
Where $K = 6, 7, 8, 9, 10$								
ANOMC-MR5	$R_{(1)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{C_{X_l}(1 - \rho_{xz}^2)}, R_{(2)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{C_{Z_l}(1 - \rho_{xz}^2)}$	$R_{(1)1}, R_{(2)1}$	$R_{(1)2}, R_{(2)2}$	$R_{(1)3}, R_{(2)3}$	...	$R_{(1)l}, R_{(2)l}$	...	$R_{(1)t}, R_{(2)t}$
	$\bar{T}_{(6)l} = \bar{Y}_l \left(\frac{\bar{X}_l}{\bar{X}_l} \right)^{R_{(1)l}} \left(\frac{\bar{Z}_l}{\bar{Z}_l} \right)^{R_{(2)l}}$	$\bar{T}_{(6)1}$	$\bar{T}_{(6)2}$	$\bar{T}_{(6)3}$	...	$\bar{T}_{(6)l}$	...	$\bar{T}_{(6)t}$
ANOMC-MR6	$\xi_{(1x)i} = \frac{\bar{X}_i}{\bar{X}_i + C_{X_i}}, \xi_{(1z)i} = \frac{\bar{Z}_i}{\bar{Z}_i + C_{Z_i}}$	$\xi_{(1x)1}, \xi_{(1z)1}$	$\xi_{(1x)2}, \xi_{(1z)2}$	$\xi_{(1x)3}, \xi_{(1z)3}$	...	$\xi_{(1x)l}, \xi_{(1z)l}$	...	$\xi_{(1x)t}, \xi_{(1z)t}$
	$J_{(1)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(1x)l}C_{X_l}(1 - \rho_{xz}^2)}, J_{(2)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(1z)l}C_{Z_l}(1 - \rho_{xz}^2)}$	$J_{(1)1}, J_{(2)1}$	$J_{(1)2}, J_{(2)2}$	$J_{(1)3}, J_{(2)3}$	...	$J_{(1)l}, J_{(2)l}$	...	$J_{(1)t}, J_{(2)t}$
ANOMC-MR7	$\bar{T}_{(7)l} = \bar{Y}_l \left(\frac{\bar{X}_l + C_{X_l}}{\bar{X}_l + C_{X_l}} \right)^{J_{(1)l}} \left(\frac{\bar{Z}_l + C_{Z_l}}{\bar{Z}_l + C_{Z_l}} \right)^{J_{(2)l}}$	$\bar{T}_{(7)1}$	$\bar{T}_{(7)2}$	$\bar{T}_{(7)3}$	...	$\bar{T}_{(7)l}$	...	$\bar{T}_{(7)t}$
	$B_2(X_{i.}) = \frac{n_i(n_i + 1) \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^4}{(n_i - 1)(n_i - 2)(n_i - 3)S_{X_{i.}}^4} - \frac{3(n_i - 1)^2}{(n_i - 2)(n_i - 3)}$	$B_2(X_1)$	$B_2(X_2)$	$B_2(X_3)$	...	$B_2(X_i)$	...	$B_2(X_t)$
ANOMC-MR8	$B_2(Z_{i.}) = \frac{n_i(n_i + 1) \sum_{j=1}^{n_i} (Z_{ij} - \bar{Z}_{i.})^4}{(n_i - 1)(n_i - 2)(n_i - 3)S_{Z_{i.}}^4} - \frac{3(n_i - 1)^2}{(n_i - 2)(n_i - 3)}$	$B_2(Z_1)$	$B_2(Z_2)$	$B_2(Z_3)$	...	$B_2(Z_i)$	...	$B_2(Z_t)$
	(See [63])							
ANOMC-MR9	$\xi_{(2x)i} = \frac{\bar{X}_i}{\bar{X}_i + B_2(X_{i.})}, \xi_{(2z)i} = \frac{\bar{Z}_i}{\bar{Z}_i + B_2(Z_{i.})}$	$\xi_{(2x)1}, \xi_{(2z)1}$	$\xi_{(2x)2}, \xi_{(2z)2}$	$\xi_{(2x)3}, \xi_{(2z)3}$	...	$\xi_{(2x)l}, \xi_{(2z)l}$	...	$\xi_{(2x)t}, \xi_{(2z)t}$
	$J_{(1)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(2x)l}C_{X_l}(1 - \rho_{xz}^2)}, J_{(2)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(2z)l}C_{Z_l}(1 - \rho_{xz}^2)}$	$J_{(1)1}, J_{(2)1}$	$J_{(1)2}, J_{(2)2}$	$J_{(1)3}, J_{(2)3}$	...	$J_{(1)l}, J_{(2)l}$	...	$J_{(1)t}, J_{(2)t}$
ANOMC-MR10	$\bar{T}_{(8)l} = \bar{Y}_l \left(\frac{\bar{X}_l + B_2(X_{i.})}{\bar{X}_l + B_2(X_{i.})} \right)^{J_{(1)l}} \left(\frac{\bar{Z}_l + B_2(Z_{i.})}{\bar{Z}_l + B_2(Z_{i.})} \right)^{J_{(2)l}}$	$\bar{T}_{(8)1}$	$\bar{T}_{(8)2}$	$\bar{T}_{(8)3}$	...	$\bar{T}_{(8)l}$	...	$\bar{T}_{(8)t}$
	$\xi_{(3x)i} = \frac{\bar{X}_i B_2(X_{i.})}{\bar{X}_i B_2(X_{i.}) + C_{X_i}}, \xi_{(3z)i} = \frac{\bar{Z}_i B_2(Z_{i.})}{\bar{Z}_i B_2(Z_{i.}) + C_{Z_i}}$	$\xi_{(3x)1}, \xi_{(3z)1}$	$\xi_{(3x)2}, \xi_{(3z)2}$	$\xi_{(3x)3}, \xi_{(3z)3}$	...	$\xi_{(3x)l}, \xi_{(3z)l}$	...	$\xi_{(3x)t}, \xi_{(3z)t}$
ANOMC-MR11	$J_{(1)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(3x)l}C_{X_l}(1 - \rho_{xz}^2)}, J_{(2)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(3z)l}C_{Z_l}(1 - \rho_{xz}^2)}$	$J_{(1)1}, J_{(2)1}$	$J_{(1)2}, J_{(2)2}$	$J_{(1)3}, J_{(2)3}$	...	$J_{(1)l}, J_{(2)l}$	...	$J_{(1)t}, J_{(2)t}$
	$\bar{T}_{(9)l} = \bar{Y}_l \left(\frac{\bar{X}_l B_2(X_{i.}) + C_{X_i}}{\bar{X}_l B_2(X_{i.}) + C_{X_i}} \right)^{J_{(1)l}} \left(\frac{\bar{Z}_l B_2(Z_{i.}) + C_{Z_i}}{\bar{Z}_l B_2(Z_{i.}) + C_{Z_i}} \right)^{J_{(2)l}}$	$\bar{T}_{(9)1}$	$\bar{T}_{(9)2}$	$\bar{T}_{(9)3}$	...	$\bar{T}_{(9)l}$	...	$\bar{T}_{(9)t}$
ANOMC-MR12	$\xi_{(4x)i} = \frac{\bar{X}_i C_{X_i}}{\bar{X}_i C_{X_i} + B_2(X_{i.})}, \xi_{(4z)i} = \frac{\bar{Z}_i C_{Z_i}}{\bar{Z}_i C_{Z_i} + B_2(Z_{i.})}$	$\xi_{(4x)1}, \xi_{(4z)1}$	$\xi_{(4x)2}, \xi_{(4z)2}$	$\xi_{(4x)3}, \xi_{(4z)3}$	...	$\xi_{(4x)l}, \xi_{(4z)l}$	...	$\xi_{(4x)t}, \xi_{(4z)t}$
	$J_{(1)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(4x)l}C_{X_l}(1 - \rho_{xz}^2)}, J_{(2)l} = \frac{C_{Y_l}(\rho_{xy} - \rho_{yz}\rho_{xz})}{\xi_{(4z)l}C_{Z_l}(1 - \rho_{xz}^2)}$	$J_{(1)1}, J_{(2)1}$	$J_{(1)2}, J_{(2)2}$	$J_{(1)3}, J_{(2)3}$	...	$J_{(1)l}, J_{(2)l}$	...	$J_{(1)t}, J_{(2)t}$
ANOMC-MR13	$\bar{T}_{(10)l} = \bar{Y}_l \left(\frac{\bar{X}_l C_{X_i} + B_2(X_{i.})}{\bar{X}_l C_{X_i} + B_2(X_{i.})} \right)^{J_{(1)l}} \left(\frac{\bar{Z}_l C_{Z_i} + B_2(Z_{i.})}{\bar{Z}_l C_{Z_i} + B_2(Z_{i.})} \right)^{J_{(2)l}}$	$\bar{T}_{(10)1}$	$\bar{T}_{(10)2}$	$\bar{T}_{(10)3}$	...	$\bar{T}_{(10)l}$	...	$\bar{T}_{(10)t}$

The modified ratio (MR) estimators with two covariates were proposed by Gupta and Shabbir [72], denoted as $(\bar{T}_{(6)i.}, \bar{T}_{(7)i.}, \bar{T}_{(8)i.}, \bar{T}_{(9)i.}$ and $\bar{T}_{(10)i.})$, are also referred to as $(Y_{m(R)}, Y_{m(SD)}, Y_{m(SK)}, Y_{m(US1)}$ and $Y_{m(US2)})$, respectively. These estimators represent the i^{th} MR means estimators. $B_2(x_{i.})$ and $B_2(z_{i.})$ represent the i^{th} coefficient of kurtosis for the variables X and Z , respectively. The sampling fraction is denoted as $g = n_i / N$, $C_{Y_{i.}}, C_{X_{i.}}$ and $C_{Z_{i.}}$ denote the i^{th} sample coefficients of variation for the variables Y , X , and Z , respectively. $S_{(r)\bar{T}_{i.}}$ represents the sample standard deviation of the i^{th} MR means estimators, where $r = 6, 7, 8, 9$, and 10 . Furthermore, ρ_{yx} , ρ_{yz} , and ρ_{xz} represent the correlation coefficients between Y and X , Y and Z , and X and Z , respectively. These correlations can be calculated as $\rho_{yx} = cov(Y, X) / S_{Y_{i.}} S_{X_{i.}}$, $\rho_{yz} = cov(Y, Z) / S_{Y_{i.}} S_{Z_{i.}}$ and $\rho_{xz} = cov(X, Z) / S_{X_{i.}} S_{Z_{i.}}$ where $S_{X_{i.}}, S_{Y_{i.}}$ and $S_{Z_{i.}}$ represent the population standard deviation of the i^{th} X, Y and Z , respectively. It is to be noted that under the balanced design (equal sample size) and unbalanced design (unequal sample size), the overall mean for MR estimators $(\bar{T}_{(6)..<}, \bar{T}_{(7)..<}, \bar{T}_{(8)..<}, \bar{T}_{(9)..<}$ and $\bar{T}_{(10)..<})$ and the overall mean square error $(MSE_{\bar{T}_{(6)..<}}, MSE_{\bar{T}_{(7)..<}}, MSE_{\bar{T}_{(8)..<}}, MSE_{\bar{T}_{(9)..<}}$ and $MSE_{\bar{T}_{(10)..<}})$ can be calculated using the expressions given in Table 3.2. Furthermore, the lower decision lines (LDL) and upper decision lines (UDL) can be obtained using the expressions in Table 3.3.

Table 4 provides a comprehensive list of the critical values for both balanced and unbalanced designs. The critical values for balanced designs are denoted by $V(\alpha, t, n)$, $Z(\alpha, t, n)$, $K(\alpha, t, n)$, $P(\alpha, t, n)$ and $O(\alpha, t, n)$, while the critical values for unbalanced designs are represented by $V^*(\alpha, t)$, $Z^*(\alpha, t)$, $K^*(\alpha, t)$, $P^*(\alpha, t)$ and $O^*(\alpha, t)$.

Table 3. 2: Overall mean and mean squared error regarding ratio estimators under balanced and unbalanced design.

Balanced design	
$\bar{T}_{(6)\cdot} = \sum_{i=1}^{n_i} \bar{T}_{(6)i} / t$	$MSE_{\bar{T}_{(6)\cdot}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(6)i}} / t$
$\bar{T}_{(7)\cdot} = \sum_{i=1}^{n_i} \bar{T}_{(7)i} / t$	$MSE_{\bar{T}_{(7)\cdot}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(7)i}} / t$
$\bar{T}_{(8)\cdot} = \sum_{i=1}^{n_i} \bar{T}_{(8)i} / t$	$MSE_{\bar{T}_{(8)\cdot}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(8)i}} / t$
$\bar{T}_{(9)\cdot} = \sum_{i=1}^{n_i} \bar{T}_{(9)i} / t$	$MSE_{\bar{T}_{(9)\cdot}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(9)i}} / t$
$\bar{T}_{(10)\cdot} = \sum_{i=1}^{n_i} \bar{T}_{(10)i} / t$	$MSE_{\bar{T}_{(10)\cdot}} = \sum_{i=1}^{n_i} S_{\bar{T}_{(10)i}} / t$
Unbalanced design	
$\bar{T}_{(6)\cdot} = \frac{n_1 \bar{T}_{(6)1} + n_2 \bar{T}_{(6)2} + \dots + n_t \bar{T}_{(6)t}}{N}$	$MSE_{\bar{T}_{(6)\cdot}} = \frac{(n_1 - 1)S_{\bar{T}_{(6)1}} + (n_2 - 1)S_{\bar{T}_{(6)2}} + \dots + (n_t - 1)S_{\bar{T}_{(6)t}}}{N - t}$
$\bar{T}_{(7)\cdot} = \frac{n_1 \bar{T}_{(7)1} + n_2 \bar{T}_{(7)2} + \dots + n_t \bar{T}_{(7)t}}{N}$	$MSE_{\bar{T}_{(7)\cdot}} = \frac{(n_1 - 1)S_{\bar{T}_{(7)1}} + (n_2 - 1)S_{\bar{T}_{(7)2}} + \dots + (n_t - 1)S_{\bar{T}_{(7)t}}}{N - t}$
$\bar{T}_{(8)\cdot} = \frac{n_1 \bar{T}_{(8)1} + n_2 \bar{T}_{(8)2} + \dots + n_t \bar{T}_{(8)t}}{N}$	$MSE_{\bar{T}_{(8)\cdot}} = \frac{(n_1 - 1)S_{\bar{T}_{(8)1}} + (n_2 - 1)S_{\bar{T}_{(8)2}} + \dots + (n_t - 1)S_{\bar{T}_{(8)t}}}{N - t}$
$\bar{T}_{(9)\cdot} = \frac{n_1 \bar{T}_{(9)1} + n_2 \bar{T}_{(9)2} + \dots + n_t \bar{T}_{(9)t}}{N}$	$MSE_{\bar{T}_{(9)\cdot}} = \frac{(n_1 - 1)S_{\bar{T}_{(9)1}} + (n_2 - 1)S_{\bar{T}_{(9)2}} + \dots + (n_t - 1)S_{\bar{T}_{(9)t}}}{N - t}$
$\bar{T}_{(10)\cdot} = \frac{n_1 \bar{T}_{(10)1} + n_2 \bar{T}_{(10)2} + \dots + n_t \bar{T}_{(10)t}}{N}$	$MSE_{\bar{T}_{(10)\cdot}} = \frac{(n_1 - 1)S_{\bar{T}_{(10)1}} + (n_2 - 1)S_{\bar{T}_{(10)2}} + \dots + (n_t - 1)S_{\bar{T}_{(10)t}}}{N - t}$

3.3 Performance Measures and Simulation Procedure

In this section, We will highlight the study's design structure, which will be utilized to conduct a simulation study using the Monte Carlo simulation technique [80]. For the simulations, the trivariate samples (Y, X, Z) are generated from multivariate normal (MVN), multivariate t (MVT), and multivariate log normal (MVLN) distributions. The sample mean in the study is assumed to be 5. Consequently, the sample mean vector is $\mu = (5, 5, 5)^T$, while several scenarios of covariance choice are evaluated to exhibit different correlation structures among the main or covariates variables as follows:

Case I: When all covariances are the same (All variables have same correlation):

$$\Sigma_1 = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}, \quad (3.2)$$

Case II: The covariance between X and Z is changed (the covariates are highly correlated):

$$\Sigma_2 = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.8 \\ 0.5 & 0.8 & 1 \end{bmatrix}, \quad (3.3)$$

Case III: The covariance between Y and X is changed (the covariate (X) is highly correlated with the response variable (Y)):

$$\Sigma_3 = \begin{bmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}, \quad (3.4)$$

Case IV: When all covariances are the same (All variables have the same strong correlation):

$$\Sigma_4 = \begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}. \quad (3.5)$$

It is important to note that we did not investigate the scenario where the covariance gets changed in Y and Z for the sake of brevity, as it would yield the same results as Case III.

In this study, ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests are compared with respect to type I error (α) and test power ($1 - \beta$). The degree of incorrect rejection of a true null hypothesis ($H_0: adj\mu_1 = adj\mu_2 = \dots = adj\mu_t$) is known as a type I error, whereas the degree of correct rejection of the false null hypothesis (H_1 : at least one of the $adj \mu_i$ is different) is known as the power of the test. Besides the aforementioned distributions, the study examines the performance of the aforementioned tests under different experimental conditions such as covariance between variables (Σ), sample size (n), variance ratios (v), effect size (δ), and number of treatments (t).

Table 3. 3: Lower and upper decision lines regarding ratio estimators under balanced and unbalanced design

Balanced design			
LDL $\bar{r}_{(6)L} = \bar{T}_{(6).} - V(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(6).}} \right) \sqrt{\frac{t-1}{N}}$		UDL $\bar{r}_{(6)L} = \bar{T}_{(6).} + V(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(6).}} \right) \sqrt{\frac{t-1}{N}}$	
LDL $\bar{r}_{(7)L} = \bar{T}_{(7).} - Z(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(7).}} \right) \sqrt{\frac{t-1}{N}}$		UDL $\bar{r}_{(7)L} = \bar{T}_{(7).} + Z(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(7).}} \right) \sqrt{\frac{t-1}{N}}$	
LDL $\bar{r}_{(8)L} = \bar{T}_{(8).} - K(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(8).}} \right) \sqrt{\frac{t-1}{N}}$		UDL $\bar{r}_{(8)L} = \bar{T}_{(8).} + K(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(8).}} \right) \sqrt{\frac{t-1}{N}}$	
LDL $\bar{r}_{(9)L} = \bar{T}_{(9).} - P(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(9).}} \right) \sqrt{\frac{t-1}{N}}$		UDL $\bar{r}_{(9)L} = \bar{T}_{(9).} + P(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(9).}} \right) \sqrt{\frac{t-1}{N}}$	
LDL $\bar{r}_{(10)L} = \bar{T}_{(10).} - O(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(10).}} \right) \sqrt{\frac{t-1}{N}}$		UDL $\bar{r}_{(10)L} = \bar{T}_{(10).} + O(\alpha, t, n) \left(\text{MSE}_{\bar{T}_{(10).}} \right) \sqrt{\frac{t-1}{N}}$	
Unbalanced design			
LDL $\bar{r}_{(6)L} = \bar{T}_{(6).} - V^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(6).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$		UDL $\bar{r}_{(6)L} = \bar{T}_{(6).} + V^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(6).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$	
LDL $\bar{r}_{(7)L} = \bar{T}_{(7).} - Z^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(7).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$		UDL $\bar{r}_{(7)L} = \bar{T}_{(7).} + Z^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(7).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$	
LDL $\bar{r}_{(8)L} = \bar{T}_{(8).} - K^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(8).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$		UDL $\bar{r}_{(8)L} = \bar{T}_{(8).} + K^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(8).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$	
LDL $\bar{r}_{(9)L} = \bar{T}_{(9).} - P^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(9).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$		UDL $\bar{r}_{(9)L} = \bar{T}_{(9).} + P^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(9).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$	
LDL $\bar{r}_{(10)L} = \bar{T}_{(10).} - O^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(10).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$		UDL $\bar{r}_{(10)L} = \bar{T}_{(10).} + O^*(\alpha, t) \left(\text{MSE}_{\bar{T}_{(10).}} \right) \sqrt{\frac{N-n_i}{Nn_i}}$	

To visually represent the effects of the different covariance choices ($\Sigma_1 - \Sigma_4$) under different distributions, mean, variance, and covariance changes are portrayed in Figure 1. In the case of all variables in the study (Y, X , and Z) following MVN distribution, the mean and variance remain constant at 5 and 1, respectively. However, the covariance may vary and might not remain at 0.5. On the other hand, under MVT distribution, the mean remains consistent at 5, but the variance and covariance will vary and may not remain at 1 and 0.5, respectively. In the case of MVLN distribution, the mean remains the same as before, while the variance and covariance may fluctuate. The findings of ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests are evaluated at a significance level of 5% ($\alpha = 5\%$) across different numbers of treatments.

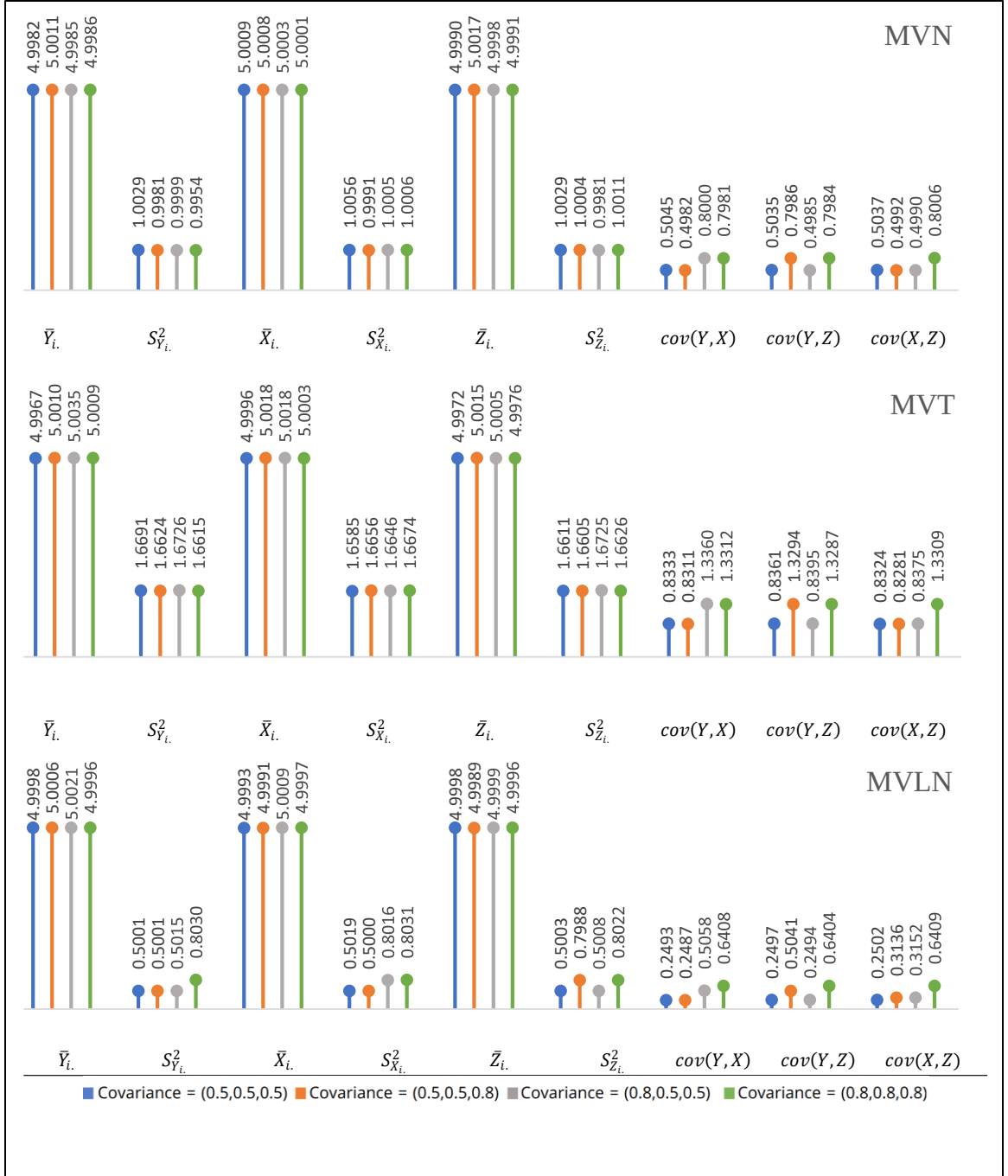


Figure 3. 1: Descriptive properties of the distributional data.

The simulation study is conducted using R software version (4.2.3), where simulations are iterated 10,000 times, and each combination of design parameters is outlined in Table 3.5. The evaluation of test performance is based on two criteria: tests with a type I error

probability close to $\alpha = 0.05$ and high power are considered the best. The empirical estimation of Type I error rates for ANOMC tests was calculated by determining the number of null hypotheses (H_0) that erroneously fell outside the interval (either above the upper decision line (UDL) or below the lower decision line (LDL)) in 10,000 simulation trials. The calculated number was then divided by 10,000 to obtain the empirical Type I error rate.

Table 3. 4: Critical values against different levels of significance (α)

Balanced Design											
		ANOMC-MR6 $V(\alpha, t, n)$		ANOMC-MR7 $Z(\alpha, t, n)$		ANOMC-MR8 $K(\alpha, t, n)$		ANOMC-MR9 $P(\alpha, t, n)$		ANOMC-MR10 $O(\alpha, t, n)$	
n	α	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
3	n_1	20.4	11.8	19.15	11.58	30.5	12.3	20.5	11.58	21.6	11.7
	n_2	17.1	12	16.18	11.54	16.5	11.6	16.3	11.57	16.5	11.55
	n_3	18.2	13.48	17	12.9	17.25	12.9	17.2	12.9	17.22	12.9
4	n_1	19.95	12.38	19.55	11.92	40.33	13.1	19.35	12.1	23	12.28
	n_2	17	12.20	15.7	11.65	16	11.68	16.3	11.66	15.85	11.65
	n_3	18.25	13.65	17.25	13	17.23	13.02	17.38	13	17.25	13
5	n_1	20.65	12.9	20.83	12.62	50.25	13.95	20.7	12.63	24	12.87
	n_2	17	12.4	16.1	11.82	16.35	11.9	16.65	11.84	16.2	11.85
	n_3	18.23	13.8	16.6	13.14	17.1	13.16	17	13.15	17.26	13.17
Unbalanced Design											
t		ANOMC-MR6 $V^*(\alpha, t)$		ANOMC-MR7 $Z^*(\alpha, t)$		ANOMC-MR8 $K^*(\alpha, t)$		ANOMC-MR9 $P^*(\alpha, t)$		ANOMC-MR10 $O^*(\alpha, t)$	
n	α	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%
3	n_1	19.5	12.56	18.9	12.35	42	13.25	19.2	12.33	19.65	12.48
	n_2	19.3	13.55	18.6	13.17	20	13.4	18.65	13.2	18.58	13.2
	n_3	22.5	16.75	21.75	16.22	23.85	16.5	22.15	16.26	22.48	16.3
4	n_1	20.95	12.75	20.85	12.6	60	14.85	20.75	12.57	23.75	12.8
	n_2	18.5	13.18	18.05	12.68	18.75	12.9	18.22	12.65	18.3	12.75
	n_3	24.3	18.46	22.5	17.65	23	17.65	22.85	17.63	22.8	17.63
5	n_1	20.1	13.45	18.92	12.95	40.4	13.72	19.42	12.93	20.1	13.1
	n_2	19.95	15.34	19.35	14.65	19.7	14.85	19.55	14.65	19.36	14.68
	n_3	25.75	20.1	24.5	19.15	24.5	19.18	24.55	19.16	24.35	19.15

To obtain the test power, the following steps illustrate the process:

- 1- The parameter space was specified according to Table 3.5.
- 2- Specific constant values representing the effect sizes ($\delta = 0.25, 0.50, 0.75, 1.00$) were added to the random numbers of one or more treatment groups. This ensured that the means of at least one treatment group fell outside the decision lines.
- 3- The new datasets underwent ANOMC tests, and decision lines were calculated. This process was repeated 10,000 times.

- 4- The count of simulation trials (j) in which the means of any of the treatments exceeded the decision lines was recorded.
- 5- The test's power was determined by dividing this count by 10,000, representing the total number of repetitions.

Table 3. 5: Choices considered for different design parameters.

Parameter space	Number of treatments (t)		
	3	4	5
Distribution	MVN; MVT; MVLN	MVN; MVT; MVLN	MVN; MVT; MVLN
Sample size (n)			
n_1	5: 5: 5	5: 5: 5: 5	5: 5: 5: 5: 5
n_2	10: 10: 10	10: 10: 10: 10	10: 10: 10: 10: 10
n_3	15: 15: 15	15: 15: 15: 15	15: 15: 15: 15: 15
n_4	4: 7: 10	4: 4: 7: 7	4: 6: 8: 10: 15
n_5	5: 10: 15	5: 8: 10: 15	5: 10: 15: 20: 25
n_6	5: 15: 25	10: 20: 30: 40	10: 20: 30: 40: 50
Variance ratios (v)			
v_1	1: 1: 1	1: 1: 1: 1	1: 1: 1: 1: 1
v_2^+	1: 1: 1.05	1: 1: 1: 1.05	1: 1: 1: 1: 1.05
v_2^-	1.05: 1: 1	1.05: 1: 1: 1	1.05: 1: 1: 1: 1
v_3^+	1: 1: 1.1	1: 1: 1: 1.1	1: 1: 1: 1: 1.1
v_3^-	1.1: 1: 1	1.1: 1: 1: 1	1.1: 1: 1: 1: 1
Effect size (δ)			
δ_1	0: 0: 0	0: 0: 0: 0	0: 0: 0: 0: 0
δ_2	0: 0: 1	0: 0: 0: 1	0: 0: 0: 0: 1
δ_3	0: 0.25: 1	0: 0.50: 0.50: 1	0: 0.25: 0.50: 0.75: 1
δ_4	0: 0.50: 1	0: 0.25: 0.75: 1	0: 0: 0.25: 0.75: 1
δ_5	0: 0.75: 1	0: 0: 1: 1	0: 0: 0.25: 0.25: 1
δ_6	0: 1: 1	0: 0.25: 0.50: 1	0: 0: 0: 1: 1
δ_7	0.25: 0: 1	0.25: 0: 0: 1	0.25: 0: 0: 0: 1
δ_8	1: 0: 0.25	1: 0: 0: 0.25	1: 0: 0: 0: 0.25
δ_9	0.50: 0: 1	0.50: 0: 0: 1	0.50: 0: 0: 0: 1
δ_{10}	1: 0: 0.50	1: 0: 0: 0.50	1: 0: 0: 0: 0.50

3.4 Comparison Performance

To evaluate and compare the performance of ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests, Tables 3.6-3.12 present the empirical type I error rates and power tests for ANOMC tests.

3.4.1 Results of type I error rates

The performance of ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests is evaluated using Bradley's liberal criterion [65], where values falling within the range of 2.5% to 7.5% are considered liberal or conservative. The relative performances for ANOMC tests in terms of empirical type I error rates under null case (δ_1) is given in Tables 3.6-3.9.

I. Empirical type I error rate under homogeneous variances

Table 3.6 presents the results for the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests, considering both balanced and unbalanced designs while assuming the homogeneity of variances.

The ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests generally exhibit similar levels of type I error rates under Σ_1 and Σ_2 in covariance ratio. However, ANOMC-MR6 stands out as it effectively controls the type I error under Σ_1 and Σ_2 but shows weaknesses in Σ_3 and Σ_4 . When using the MVT distribution, the test slightly fails to control the type I error. Moreover, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 follow a similar pattern. Under Bradley's liberal criterion, except for Σ_3 with a small number of treatments and small sample size, these tests demonstrate robustness in effectively controlling the type I error. However, as the number of treatments and sample size increase, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 become less effective in controlling the type I error, but they remain strong tests in Σ_1 and Σ_4 . In general, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 still exhibit acceptable rates in Σ_2 , indicating their

overall robustness except in Σ_3 . It is crucial to highlight that under Σ_3 , the results of all the distributions and tests in the study are significantly impacted when the covariance between the response variable (Y) and the auxiliary variable (X or Z) deviates from the specified value of $\alpha = 5\%$. This emphasizes the sensitivity of the results to changes in the covariance parameter. Furthermore, it is noteworthy to mention that the type of design used in the study does not have an impact on the results. The observed effects on the results are primarily driven by the deviation in the covariance between the response variable and the auxiliary variable, rather than the specific design employed in the study.

Table 3. 6: Type I error (%) of ANOMC tests under homogeneous variances.

Distribution		MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	
Test		ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10			
t	n	Σ	Balanced Design														
3	n ₁	Σ ₁	5.22	5.45	5.39	5.22	5.42	5.18	5.19	5.34	4.22	5.23	5.41	5.18	5.28	5.44	4.89
		Σ ₂	5.8	6.37	5.51	3.12	3.43	2.91	3.37	3.77	2.77	3.13	3.41	2.91	3.17	3.61	2.76
		Σ ₃	7.21	8.5*	7.07	16.4*	18.1*	16.8*	15*	16.1*	15.56*	16.4*	18.1*	16.82*	16.2*	17.3*	17.43*
		Σ ₄	17.2*	18.4*	17.3*	6.71	6.7	7.43	6.56	6.94	5.89	6.73	6.7	7.44	6.76	6.95	6.54
	n ₂	Σ ₁	5.27	5.46	4.84	5.05	5.38	4.77	5.06	5.46	4.57	5.02	5.33	4.66	5.09	5.46	4.65
		Σ ₂	6.64	7.23	6.65	2.68	2.53	2.54	2.51	2.57	2.22*	2.4*	2.46*	2.2*	2.45*	2.61	2.27*
		Σ ₃	8.29*	8.82*	8.07*	20*	21.3*	20.2*	19.5*	21.1*	20.04*	19.8*	21.2*	20.5*	19.7*	21.15*	20.29*
		Σ ₄	23.5*	26.7*	24.7*	5.27	6.55	5.96	5.28	6.66	6.09	5.24	6.48	6.27	5.3	6.67	6.19
	n ₃	Σ ₁	4.93	6.11	5.16	4.82	6.13	5.05	4.83	6.21	5.27	4.83	6.13	5.24	4.83	6.17	5.27
		Σ ₂	6.86	8.09*	6.74	2.36*	2.55	2.06*	2.3*	2.6	2.06*	2.35*	2.54	2.06*	2.31*	2.56	2.06*
		Σ ₃	8.17*	9.04*	8.24*	20.9*	23*	21*	21.2*	22.9*	20.86*	20.9*	23.1*	21.01*	21*	22.88*	20.84*
		Σ ₄	27.1*	28.9*	26.9*	5.68	6.08	5.6	5.69	6.28	5.6	5.67	6.09	5.67	5.66	6.25	5.6
	Unbalanced Design																
	n ₄	Σ ₁	4.98	6.26	4.61	4.95	6.23	4.77	5.25	6.09	4.04	5	6.27	5.19	5.06	6.36	4.88
		Σ ₂	5.7	7.12	5.76	2.59	3.02	2.77	3.18	3.59	2.5	2.6	3.06	2.87	2.74	3.24	2.73
		Σ ₃	7.82*	8.73*	7.57*	17.93*	18.5*	18.3*	16.1*	16.55*	15.53*	18.1*	18.6*	18.4*	17.4*	18.08*	17.61*
		Σ ₄	20.3*	21.6*	19.5*	6.94	7.24	6.51	6.84	7.28	5.89	7	7.32	6.93	6.9	7.31	6.45
	n ₅	Σ ₁	5.08	5.63	4.91	5.05	5.45	5.32	5.16	5.56	4.59	5.04	5.35	4.83	5.15	5.64	4.79
		Σ ₂	6.1	6.9	6.49	2.58	2.8	2.27*	2.39*	3.18	2.17*	2.23*	2.8	2.31*	2.29*	2.92	2.29*
		Σ ₃	7.74*	8.73*	7.89*	19.59*	19.8*	19.1*	18.7*	18.84*	18.57*	19.6*	19.8*	19.56*	19.2*	19.23*	19.36*
		Σ ₄	21.9*	23.3*	22.6*	5.64	6.35	6.18	5.58	6.58	5.77	5.56	6.3	6.21	5.78	6.55	5.96
	n ₆	Σ ₁	5.11	5.54	4.96	4.78	5.65	5.06	4.98	5.77	4.63	4.74	5.57	4.93	4.79	5.64	4.87
		Σ ₂	6.95	7.24	6.23	2.33*	2.62	1.98*	2.47*	2.71	1.86*	2.3*	2.55	2.06*	2.35*	2.68	2.05*
		Σ ₃	7.67*	8.55*	7.89*	18.91*	18.8*	19.8*	18.5*	18.36*	18.41*	18.8*	18.7*	19.36*	18.6*	18.66*	19.01*
		Σ ₄	22.7*	24.2*	22.6*	6.09	6.35	6.06	5.97	6.46	5.73	6.07	6.23	6.08	6.14	6.43	5.94
t	n	Σ	Balanced Design														
4	n ₁	Σ ₁	5.26	5.23	4.54	5.33	5.62	5.1	5.2	5.63	3.83	5.16	5.3	5.05	5.24	5.43	4.73
		Σ ₂	5.5	5.81	5.34	3.2	3.54	3.12	3.24	4.03	2.78	3.09	3.38	3.22	3.05	3.57	3.12
		Σ ₃	7.43	7.44	7.29	17.32*	17.3*	18.0*	14.3*	14.22*	14.53*	16.7*	16.8*	18.06*	16.1*	16.25*	16.84*
		Σ ₄	17.2*	18.71*	17.5*	6.79	7.62*	7.81	6.52	7.04	5.95	6.52	7.26	6.95	6.32	7.46	6.5
	n ₂	Σ ₁	4.91	5.18	4.74	4.85	5.37	4.86	4.93	5.59	5.09	4.84	5.36	5.19	4.87	5.59	5.12
		Σ ₂	6.35	6.81	6.57	2.19*	2.35*	2.5	2.18*	2.52	2.31*	2.19*	2.35*	2.35*	2.17*	2.38*	2.32*
		Σ ₃	8.04*	8.89*	7.87*	20.41*	22*	21.4*	20.4*	22.09*	21.08*	20.4*	22*	21.74*	20.5*	22.08*	21.23*
		Σ ₄	27.1*	28.42*	27.5*	5.74	7.14	6.04	5.83	7.22	6.01	5.71	7.09	6.14	5.85	7.19	6.02
	n ₃	Σ ₁	5.01	5.79	4.74	5.22	5.85	4.78	5.21	5.86	4.68	5.22	5.84	4.71	5.24	5.85	4.71
		Σ ₂	6.7	7.15	6.6	2.01*	2.19*	1.88*	1.91*	2.19*	2.11*	2*	2.19*	2.2*	1.94*	2.16*	2.16*
		Σ ₃	8.73*	8.99*	8.5*	22.03*	23.5*	22.5*	22*	23.11*	22.41*	22.1*	23.4*	22.69*	22.1*	23.18*	22.53*
		Σ ₄	28.7*	31.78*	29.2*	5.08	6.31	5.63	5.13	6.34	5.76	5.06	6.32	5.82	5.17	6.34	5.8
	Unbalanced Design																
	n ₄	Σ ₁	5.43	4.73	5.24	5.2	4.75	5.08	5.65	5.16	3.02	5.25	4.75	4.9	5.35	5.09	4.65
		Σ ₂	5.42	5.9	5.39	2.93	3.46	3.07	4.1	4.84	2.5*	2.98	3.45	2.72	3.13	3.73	2.6
		Σ ₃	7.61*	8*	7.87*	16.81*	17.2*	18.5*	12.4*	12.7*	11.8*	16.9*	17.4*	18.32*	16.3*	16.69*	17.18*
Σ ₄		18.1*	19.59*	18*	7.15	6.84	7.21	6.75	7.22	5.1	7.21	6.9	6.82	7.33	7.1	6.52	
n ₅	Σ ₁	5.2	4.48	4.48	4.98	4.84	4.46	4.94	5.05	4.49	5.03	4.87	4.96	5	4.84	4.72	
	Σ ₂	6.11	6.78	5.77	2.56	2.62	2.19*	2.6	3.06	2.23*	2.57	2.61	2.26*	2.53	2.74	2.24*	
	Σ ₃	7.7*	8.7*	7.94	19.22*	21.3*	20*	18.4*	20.26*	19.46*	19.5*	21.5*	21.15*	18.9*	20.53*	20.05*	
	Σ ₄	24.6*	25.14*	25*	5.91	6.42	6.54	5.9	6.93	5.48	5.99	6.46	6.09	6.01	6.76	5.66	
n ₆	Σ ₁	5.2	5.4	5.09	5.06	5.67	5.22	5.06	5.76	5.41	5.09	5.71	5.44	5.09	5.78	5.44	
	Σ ₂	6.93	7.97	6.62	1.83*	2.2*	1.84*	1.86*	2.3*	1.9*	1.83*	2.22*	1.93*	1.84*	2.29*	1.92*	
	Σ ₃	8.31*	8.96*	8.13*	20.72*	21.7*	21.3*	20.8*	21.88*	21.9*	20.8*	21.9*	22.05*	20.9*	21.97*	22.01*	
	Σ ₄	27.7*	29.52*	27.9*	5.29	5.94	5.34	5.42	6.09	5.37	5.31	5.98	5.37	5.45	6.04	5.4	
t	n	Σ	Balanced Design														
5	n ₁	Σ ₁	5.36	4.78	4.59	5.12	4.84	4.82	5.16	5.07	3.58	5.08	4.83	4.92	5.11	5.06	4.63
		Σ ₂	5.6	5.61	4.91	3.24	3.1	3.12	3.83	4	2.46*	3.26	3.12	3.06	3.32	3.41	2.94
		Σ ₃	7.38	7.83	7.72	16.57*	16.5*	17.7*	13.7*	13.82*	12.59*	16.6*	16.6*	17.35*	15.8*	15.72*	15.82*
		Σ ₄	17.9*	19.08*	17.7*	6.89	6.97	7.39	6.63	7.18	6.25	6.89	6.94	7.27	6.77	6.81	6.97
	n ₂	Σ ₁	5.03	5.2	5.16	4.86	5.01	5.1	4.87	5.23	4.92	4.81	4.97	5.01	4.87	5.13	5.06
		Σ ₂	6.74	7.22	6.28	2.7	2.81	2.4*	2.23*	2.84	2.06*	2.23*	2.79	2.13*	2.31*	2.8	2.14*
		Σ ₃	8.29*	9.23*	8.9*	21.3*	23.6*	23.0*	20.9*	23.28*	21.93*	21.1*	23.5*	22.4*	21.2*	23.35*	22.3*
		Σ ₄	28.6*	29.81*	29*	6.05	6.65	6.1	6.02	6.89	6.04	6	6.61	6.2	6.06	6.77	6.19
	n ₃	Σ ₁	5.11	5.31	4.96	5.3	5.28	4.84	5.31	5.38	4.65	5.3	5.26	4.71	5.27	5.26	4.64
		Σ ₂	6.7	7.35	6.45	1.88*	1.93*	1.64*	1.89*	1.89*	1.66*	1.88*	1.93*	1.66*	1.87*	1.85*	1.66*
		Σ ₃	8.93*	9.4*	8.41*	23.34*	25.4*	24.2*	23.3*	25.09*	25.83*	23.3*	25.4*	25.88*	23.2*	25.05*	25.73*
		Σ ₄	31.7*	34.46*	32.1*	5.08	6.4	5.53	5.19	6.44	5.48	5.04	6.37	5.53	5.13	6.44	5.45
	Unbalanced Design																
	n ₄	Σ ₁	4.67	4.95	4.74	4.68	5.19	4.94	4.94	5.36	4.01	4.72	5.25	4.92	4.56	5.28	4.58
		Σ ₂	5.85	6.01	5.33	2.81	2.53	2.26*	3.21	3.44	2.29*	2.45*	2.43*	2.43*	2.49*	2.51	2.34*
		Σ ₃	8.5*	8.39*	8.39*	20.41*	20.3*	21.1*	18.5*	18.27*	17.27*	20.6*	20.5*	21.08*	20.0*	19.5*	19.88*
Σ ₄		24.3*	24.95*	24.3*	7.09	6.38	6.75	6.99	6.76	5.96	7.15	6.41	6.95	7.04	6.61	6.77	
n ₅	Σ ₁	4.67	5.18	4.81	4.7	5.56	5.31	4.62	5.59	4.29	4.71	5.57	4.69	4.75	5.62	4.63	
	Σ ₂	6.63	7.28	6.08	2.2	2.11*	1.69*	2.19*	2.4*	1.88*	2.19*	2.1*	1.96*	2.19*	2.23*	1.92*	
	Σ ₃	8.7*	9.36*	8.66*	22.4*	23.3*	22.8*	21.8*	22.75*	22.09*	22.4*	23.3*	23.53*	22.2*	23.25*	22.86*	
	Σ ₄	28.5*	30.02*	28.8*	5.24	6.2	5.67	5.28	6.42	5.91	5.27	6.2	6.18	5.47	6.35	6.14	
n ₆	Σ ₁	4.94	5.31	4.93	5.02	5.42	5.04	5.06	5.45	4.93	5	5.38	4.93	5.09	5.47	5	
	Σ ₂	6.9	7.18	6.82	1.51*	1.75*	1.78*	1.51*	1.87*	1.69*	1.51*	1.75*	1.68*	1.47*	1.83*	1.71*	
	Σ ₃	8.55*	9.16*	8.33*	22.43*	23.8*	22.8*	22.5*	23.59*	22.77*	22.4*	23.8*	22.91*	22.6*	23.65*	22.86*	
	Σ ₄	31.5*	32.47*	30.8*	5.18	5.92	5.24	5.16	6.02	5.6	5.16	5.91	5.71	5.13	6.1	5.67	

Table 3. 7: Type I error (%) of ANOMC tests under heterogeneous variances when $t = 3$.

Distribution			MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
n	Σ	ν	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
n_1	Σ_1	v_2^+	6.52	6.5	7.2	6.48	6.84	8	5.57	5.67	5.54	6.49	6.85	7.59	6.48	6.83	7.32
		v_2^-	6.47	7.19	7.31	6.33	7.32	7.6	5.46	6.37	5.5	6.34	7.35	7.73	6.36	7.35	7.49
		v_3^+	9.68	10.07	13.91	9.73	10.23	14.82	7.67	8.41	11.16	9.73	10.25	15.43	9.64	10.28	14.86
		v_3^-	9.93	9.2	14.16	10.17	9.27	14.56	8.18	7.72	11.09	10.17	9.32	14.6	9.97	9.19	14.13
	Σ_2	v_2^+	7.09	7.6	7.92	4.14	4.22	4.85	3.86	4.35	3.58	4.33	4.23	4.87	4.32	4.32	4.64
		v_2^-	6.99	7.68	7.95	3.91	4.33	5.03	3.94	3.98	3.66	4.09	4.01	5.15	4.13	4.22	4.92
		v_3^+	10.34	10.46	14.55	6.83	6.97	11.43	6.47	5.99	8.16	7.76	6.95	10.82	7.58	6.88	10.4
		v_3^-	10.02	10.01	13.93	7.12	6.26	10.71	6.01	5.8	8.26	7.28	6.72	11.05	7.09	6.72	10.78
	Σ_3	v_2^+	9.96	10.25	12.41	19.31	20.19	23.84	16.04	16.53	17.47	19.7	20.87	22.6	18.98	20.03	21.74
		v_2^-	10.04	10.95	11.54	19.5	20.63	23.04	15.35	17.06	17.82	19.45	21.38	22.68	18.86	20.75	21.95
		v_3^+	16.17	16.12	24.6	26.95	26.75	36.25	22.11	21.46	29.6	26.79	26.16	36.77	26.06	25.35	35.95
		v_3^-	16.47	15.24	24.51	27.75	26.23	37.18	22.46	21.24	30.02	27.14	26.23	36.9	26.39	25.28	36
	Σ_4	v_2^+	20.33	21.38	20.69	9.58	9.44	10.12	7.75	8.31	8.21	9.55	9.85	10.7	9.55	9.86	10.18
		v_2^-	20.45	22.1	20.64	9.35	10.27	10.65	7.95	8.43	8.48	9.66	9.47	11.15	9.73	9.56	10.73
		v_3^+	27.57	28.25	31.33	17.16	16.61	21.45	14.47	13.87	16.21	17.82	16.95	21.13	17.56	16.5	20.43
		v_3^-	27.05	28.17	30.36	18.09	15.86	20.75	14.16	12.9	16.2	17.52	15.53	21.02	17.07	15.4	20.42
n_2	Σ_1	v_2^+	9.19	8.54	11.71	9.34	8.95	12.56	9.22	8.82	12.24	9.24	8.86	12.47	9.32	8.81	12.47
		v_2^-	8.64	8.57	12.52	8.94	8.72	13.08	8.9	8.73	12.59	8.91	8.64	12.75	8.94	8.81	12.71
		v_3^+	19.32	17.46	35.2	20.34	18.7	38.73	20.06	18.67	37.34	20.18	18.63	37.48	20.3	18.78	37.7
		v_3^-	19.89	16.96	34.97	20.87	17.62	38.07	20.51	17.3	36.31	20.66	17.52	36.59	20.69	17.62	36.65
	Σ_2	v_2^+	10.63	10.55	13.51	5.25	5.12	8.11	5.29	5.4	7.61	5.24	5.23	7.75	5.35	5.36	7.74
		v_2^-	10.53	9.94	13.51	5.26	4.84	7.96	5.42	5.11	7.48	5.46	5.09	7.57	5.55	5.14	7.6
		v_3^+	20.28	17.83	33.87	15.19	12.66	29.86	15.37	12.76	30.69	15.43	12.81	30.92	15.53	12.86	31.09
		v_3^-	20.58	18.14	33.47	15.23	12.63	30.6	15.47	12.96	29.71	15.54	12.89	29.91	15.65	13.01	30.09
	Σ_3	v_2^+	15.78	14.65	22.47	27.44	27.18	35.74	28.05	26.89	34.59	28.31	27.13	34.89	28.29	27.01	34.87
		v_2^-	15.4	14.73	22.43	27.49	27.53	35.93	27.83	26.05	34.02	28.14	26.5	34.33	28.15	26.24	34.37
		v_3^+	35.2	30.32	60.27	47.07	42.68	65.63	46.6	40.96	65.69	46.88	41.23	66.04	46.83	41.15	65.98
		v_3^-	36	30.18	60.2	45.8	40.85	66.36	46.76	41.79	66.4	47.27	42.09	66.76	47.11	42.09	66.69
	Σ_4	v_2^+	33.24	33.41	33.47	15.19	13.49	17.06	14.32	12.81	17.36	14.36	12.73	17.57	14.5	12.88	17.56
		v_2^-	32.11	32.89	33.69	14.82	13.2	17.03	14.55	13.29	16.85	14.57	13.18	17.19	14.66	13.42	17.07
		v_3^+	49.48	45.83	55.84	41.83	33.48	51.82	41.36	32.94	51.13	41.56	32.97	51.6	41.78	33.06	51.56
		v_3^-	50.75	46.23	55.96	41.28	32.94	51.32	41.64	32.13	50.98	41.86	32.15	51.16	42.05	32.5	51.33
n_3	Σ_1	v_2^+	11.37	10.59	17.37	11.41	11.29	18.71	11.46	11.32	18.58	11.4	11.3	18.56	11.45	11.31	18.58
		v_2^-	11.74	10.68	17.17	12.52	11.15	18.67	12.53	11.2	18.07	12.53	11.13	18.05	12.49	11.18	18.07
		v_3^+	30.67	24.1	55.11	33.24	26.37	59.59	33.2	26.38	58.21	33.24	26.38	58.29	33.19	26.38	58.21
		v_3^-	30.82	24.06	54.29	33.35	25.95	58.21	33.24	25.97	58.79	33.34	25.94	58.93	33.21	25.93	58.8
	Σ_2	v_2^+	12.86	12.19	17.97	6.75	6.17	11.75	6.92	6.34	11.5	6.88	6.27	11.49	6.89	6.31	11.51
		v_2^-	13.21	12.51	18.87	7.33	6.23	11.72	7.58	5.58	11.69	7.48	5.49	11.76	7.58	5.55	11.68
		v_3^+	30.14	25.87	52.29	26.6	20.01	53.49	26.21	18.65	52.96	26.14	18.65	52.98	26.23	18.65	52.96
		v_3^-	30.63	24.27	52.05	26.02	19.29	53.33	25.4	19.13	51.88	25.45	19.24	51.95	25.42	19.1	51.89
	Σ_3	v_2^+	20.65	18.57	32.48	33.78	31.08	44.55	32.8	31.05	44.16	33	31.07	44.22	32.85	31.02	44.15
		v_2^-	21.15	18.7	32.4	33.48	31.02	44.1	32.93	31.54	45.39	32.87	31.87	45.38	32.98	31.57	45.37
		v_3^+	53.16	42.6	82.21	61.39	52.87	81.93	59.81	51.17	82.02	60.07	51.26	81.98	59.8	51.21	82.02
		v_3^-	53.23	41.24	81.75	60.43	51.32	81.84	60.67	52.11	81.84	60.92	52.13	81.96	60.68	52.07	81.86
	Σ_4	v_2^+	38.62	38.24	41.96	21.14	17.68	25.43	21.94	17.93	25.19	21.93	17.89	25.27	21.95	17.89	25.17
		v_2^-	38.75	38.16	42.27	21.61	17.72	25.19	21.3	17.46	25.08	21.09	17.13	25.1	21.27	17.38	25.08
		v_3^+	63.54	56.08	70.99	64.09	48.76	74.86	62.69	47.59	74.49	63.05	47.29	74.64	62.68	47.58	74.5
		v_3^-	63.96	54.92	70.5	63.09	47.66	74.37	63.2	48.49	74.61	63.33	48.6	74.74	63.18	48.46	74.61
n_4	Σ_1	v_2^+	8.58	8.93	10.63	8.48	8.88	10.74	8.15	8.51	8.91	8.52	8.91	11.1	8.62	9.07	10.64
		v_2^-	5.9	6.9	6.86	6.14	6.43	7.23	5.95	6.3	5.49	6.18	6.46	6.9	6.08	6.47	6.56
		v_3^+	18.7	16.35	29.69	18.65	15.97	30.85	17.16	14.66	26.91	18.69	16.01	31.47	18.52	15.9	30.82
		v_3^-	9.09	8.59	12.4	9.27	8.77	13.3	8.31	8.3	10.67	9.33	8.79	13.39	9.1	8.65	12.71
	Σ_2	v_2^+	9.09	9.96	11.89	5.17	5.64	7.31	5.54	5.12	5.82	5.4	4.9	6.86	5.54	5.01	6.7
		v_2^-	6.61	7.63	7.56	3.34	3.51	4.03	4.04	4.12	3.28	3.86	3.53	4.17	3.77	3.57	4.04
		v_3^+	18.71	16.77	29.74	14.16	11.66	25.76	12.73	10.49	21.49	14.39	11.27	25.52	14.1	11.23	24.96
		v_3^-	9.95	9.58	13.33	6.01	5.53	9.09	5.39	5.66	7.38	5.74	5.4	9.14	5.68	5.48	8.64
	Σ_3	v_2^+	14.19	14.09	19.45	25.17	24.52	30.43	22.04	21.95	27.89	24.63	24.21	31.65	24.19	23.49	30.93
		v_2^-	9.2	10.09	11.51	21.07	21.36	23.32	17.64	19.04	19.86	20.61	21.61	23.82	19.46	20.55	22.58
		v_3^+	32.26	27.27	53.09	40.31	35.35	58.35	36.71	32.64	54.4	40.12	35.35	57.99	40	35.06	57.79
		v_3^-	14.56	13.76	21.81	27.06	25.98	35	22.93	22.22	28.7	27.11	26.03	34.21	25.84	24.41	32.57
	Σ_4	v_2^+	27.19	27.02	28.8	13.82	12.97	15.79	12.73	12.12	13.71	13.92	12.77	16.06	14.18	13.16	15.89
		v_2^-	20.83	23.86	22.58	8.43	8.23	9.57	7.65	8.09	8.01	8.82	8.77	9.97	8.42	8.45	9.17
		v_3^+	43.04	39.16	48.3	35.77	28.83	43.92	33.46	27.19	40.09	36.56	29.45	44.54	36.79	29.75	44.13
		v_3^-	28.77	27.88	31.26	15.48	13.6	18.16	12.3	12.67	14.57	14.52	14.19	18.39	13.78	13.6	17.2

n_5	Σ_1	v_2^+	12.05	11.25	16.78	11.73	11.19	17.51	11.7	11.2	15.93	11.62	11.07	16.44	12	11.42	16.69
		v_2^-	5.94	6.34	7.58	5.85	6.79	7.85	5.63	6.67	7.13	5.77	6.72	7.6	5.87	6.76	7.53
		v_3^+	29.46	23.9	49.99	30.86	24.59	51.92	30.49	24.33	51.7	30.66	24.47	52.51	31.36	25.04	52.93
		v_3^-	8.83	9.06	14.47	9.26	9.49	15.75	8.98	9.11	13.74	9.23	9.47	14.83	9.1	9.32	14.47
	Σ_2	v_2^+	13.13	12.38	17.29	7.71	7.21	11.92	7.15	6.69	11.12	7.19	6.71	11.56	7.42	6.97	11.79
		v_2^-	6.96	7.88	8.76	2.85	3.41	4.32	2.9	3.46	3.79	2.8	3.17	4.13	2.81	3.27	4.04
		v_3^+	28.73	24.39	48.09	25.44	19.75	48.68	25.4	19.14	47.68	25.63	19.18	48.48	26.02	19.71	48.87
		v_3^-	10.26	10.11	15.68	5.34	5.07	9.69	5.34	5.18	9.59	5.69	5.25	10.15	5.53	5.2	10
	Σ_3	v_2^+	20.24	18.12	29.25	29.5	27.92	39.65	29.74	27.98	38.21	30.11	28.12	38.63	30.4	28.26	39.05
		v_2^-	10.2	10.32	12.84	22.52	22.38	25.02	20.72	21.06	24.39	21.78	22.3	25.8	21.35	21.65	25.26
		v_3^+	49.5	38.88	76.9	54.1	45.23	74.65	53.98	45.27	74.03	54.03	45.13	74.19	54.65	46	74.68
		v_3^-	15.75	15.75	26.52	29.39	27.67	39.23	27.87	26.32	37.26	29.33	27.95	39.29	28.48	27.05	38.44
	Σ_4	v_2^+	34.13	32.28	36.28	20.08	17.54	23.86	20.23	17.02	22.27	19.9	16.51	22.71	20.61	17.5	23.11
		v_2^-	25.03	26.55	25.28	8.54	9.05	10	7.94	8.54	9.08	8.48	8.6	9.78	8.17	8.55	9.54
		v_3^+	56.49	49.15	64.2	58.35	43.85	68.34	58.26	43.51	67.98	58.17	43.26	68.1	59.28	44.54	68.85
		v_3^-	32.44	32.26	36.02	16.25	14.99	21.27	15.26	13.93	19.41	16.24	14.79	21.07	15.97	14.41	20.41
n_6	Σ_1	v_2^+	16.02	13.1	25.27	17.03	13.39	26.71	17.1	13.49	26.02	16.88	13.19	26.57	17.3	13.55	26.61
		v_2^-	6.05	6.16	7.01	6.15	6.52	7.37	5.79	6.38	6.45	6.07	6.41	6.95	5.85	6.46	6.69
		v_3^+	45.84	33.07	73.31	48.23	34.38	75.93	47.87	34.1	75.04	48.1	34.26	75.56	48.57	34.71	75.8
		v_3^-	8.62	8.58	13.49	9	8.55	13.67	8.46	8.11	13.13	8.93	8.49	13.94	8.61	8.34	13.61
	Σ_2	v_2^+	17.15	14.78	24.76	11.84	9.32	20.45	11.76	9.4	19.95	11.76	9.24	20.43	11.97	9.56	20.55
		v_2^-	7.38	7.72	8.53	2.79	2.9	3.69	2.63	2.96	3.45	2.5	2.89	3.8	2.49	2.91	3.73
		v_3^+	42.8	32.91	69.14	43.94	30.14	75.9	43.09	29.7	75.31	43.61	30.05	76.03	43.96	30.35	76.26
		v_3^-	9.78	10.41	13.44	4.95	4.59	8.22	4.45	4.15	7.47	4.71	4.14	8.08	4.59	4.09	7.88
	Σ_3	v_2^+	28.25	21.95	43.53	36.17	31.06	48.87	36.01	30.13	48.56	35.9	29.99	48.84	36.34	30.66	49.3
		v_2^-	9.85	10.18	12.18	21.27	21.03	23.93	20.16	19.88	22.7	21.21	20.93	23.91	20.55	20.41	23.22
		v_3^+	68.44	53.33	92.72	69.44	55.05	88.4	67.85	55.93	87.82	67.58	55.55	87.82	68.35	56.54	88.11
		v_3^-	14.85	13.96	22.82	27.78	25.71	35.3	26.16	24.43	32.8	27.77	25.78	34.63	26.75	24.68	33.67
	Σ_4	v_2^+	39.83	35.04	43.69	30.63	22.61	36.88	31.49	24.89	37.26	31.11	23.96	37.17	31.91	25.2	38
		v_2^-	25.02	26.14	26.52	8.18	8.16	9	7.26	7.2	8.39	7.82	7.33	8.99	7.45	7.06	8.54
		v_3^+	69.9	58.82	77.31	80.43	61.64	88.78	79.89	62.09	88.65	79.92	61.81	88.56	80.44	62.95	89.12
		v_3^-	30.81	30.23	33.26	14.72	12.94	18.3	13.17	11	16.93	14.33	11.52	18.44	13.58	11.13	17.54

II. Empirical type I error rate under heterogeneous variances

In Tables 3.7-3.9, for both balanced and unbalanced designs, it is expected that the rates will be affected due to the heterogeneity of variances. The observations indicate that the ANOMC-MR6 and ANOMC-MR8 tests demonstrate slightly better percentage type I error rates (%) compared to ANOMC-MR7, ANOMC-MR9, and ANOMC-MR10. It is noted that as the number of treatments increases, the rates also increase. Furthermore, under the MVLN distribution, the rates exhibit higher type I error rates, while under the MVN distribution, the impact is less pronounced. This effect becomes more prominent with an increase in the variance ratio (v_3^+ , v_3^-). In general, ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 show similar rates in Σ_1 .

However, the actual type I error rates for all the tests significantly exceed the specified level when sample sizes and variances are inversely paired. Although improvements are

observed at various levels of heterogeneity, it is important to note that increasing the sample size leads to an increase in type I error rates. Regarding the ANOMC-MR6 test, it exhibits higher type I error rates under Σ_4 , suggesting that this particular configuration may result in inflated type I error rates compared to the other cases (Σ_{1-3}). It is worth noting that under Σ_3 when the assumption of variance is satisfied, all the tests (ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10) struggle in controlling Type I error rates. However, when the assumption of variance is violated, ANOMC-MR5 is less affected compared to the other tests.

3.4.2 Results of the power test

Tables 3.10-3.12 display the empirical outcomes regarding the test powers of the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests under the specific experimental conditions examined. These tables provide the rates of the test powers across nine different non-null cases (δ_{2-10}). For brevity, we examine the outcomes for both a balanced design (n_1) and an unbalanced design (n_4).

I. Empirical test power estimates under balanced design (n_1)

For the fixed treatment level ($t = 3$) with Σ_1 , the results in scenarios of non-null case δ_4 reveal that under MVN distribution, the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests exhibit power rates of 22.4%, 23.12%, 20.97%, 23.12% and 22.76%, respectively. Under MVT distribution and for a non-null case δ_6 , the corresponding power rates are 24.27%, 25.42%, 23.22%, 25.43% and 25.07%. Finally, in the context of non-null case δ_8 under MVLN distribution, the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests exhibit power rates of 44.49%, 45.38, 41.66%, 46.45% and 45.63%.

II. Empirical test power estimates under unbalanced design (n_4)

At a fixed treatment level ($t = 5$) and considering Σ_4 , for the non-null scenario δ_3 , and in the presence of MVN distribution, the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests exhibit power values of 77.99%, 82.4%, 79.89%, 82.1% and 82.38%, respectively. Under MVT distribution, the corresponding power rates for the non-null case δ_5 are 83.53%, 91.35%, 89.82%, 91.25% and 91.07%, respectively. Lastly, In the scenario of a non-null case δ_{10} under MVLN distribution, the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests demonstrate power percentages of 70.93%, 68.74%, 63.07%, 68.58% and 67.68%, respectively. The analysis of the power values reveals that there are similarities between the power values under the MVN and MVT distributions when there is no change in covariance (Σ_1) or a change between the auxiliary variables (Σ_2). However, the power of the tests is influenced by differences in sample size. When examining the effect of sample size on test power, it is observed that the power of the tests is impacted by variations in sample size. In general, when conducting experiments under Σ_1 , the ANOMC-MR7 test consistently exhibits higher power rates compared to the other ANOMC tests. On the other hand, in scenarios where the data follows a MVLN distribution, all ANOMC tests demonstrate notably higher power rates. However, a comprehensive evaluation across multiple scenarios reveals that the ANOMC-MR7 test shows relatively superior performance compared to the other ANOMC tests. It is also worth noting that the performance of all ANOMC tests (ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests) improves with a higher covariance between the study variable and the concomitant variables.

Table 3. 8: Type I error (%) of ANOMC tests under heterogeneous variances when $t = 4$.

Distribution			MVN	MVT	MVLN	MVN	MVT	MVLI	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
n	Σ	v	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
n_1	Σ_1	v_2^+	6.17	6.14	6.56	6.54	6.37	6.97	5.81	6.19	5.55	6.15	6.07	6.89	6.11	6.24	6.51
		v_2^-	5.72	5.74	6.32	6.01	6.08	6.79	5.45	5.82	5.57	5.71	5.79	7.1	5.83	5.95	6.64
		v_3^+	8.65	8.21	11.68	9.25	8.93	13.25	8.13	8.31	9.89	8.88	8.58	12.39	8.87	8.6	11.84
		v_3^-	8.29	8.58	12.23	8.91	8.95	13.56	7.66	8.08	9.63	8.5	8.62	12.44	8.3	8.54	11.8
	Σ_2	v_2^+	6.92	6.57	6.79	4.4	3.83	4.63	3.61	4.39	3.5	3.38	3.66	4.17	3.5	3.72	3.97
		v_2^-	6.71	6.73	6.71	3.91	4.18	4.7	3.76	4.27	3.32	3.89	3.7	4.12	3.93	3.97	3.93
		v_3^+	9.23	8.92	12.24	6.42	6.25	9.34	6.48	5.46	7.39	6.74	5.08	8.92	6.69	5.25	8.58
		v_3^-	9.07	8.95	11.97	6.13	6.21	9.14	5.84	5.97	6.69	6.21	5.62	8.37	5.99	5.7	7.98
	Σ_3	v_2^+	9.92	9.38	11.25	19.82	19.75	23.24	16.87	17.01	17.99	19.63	19.45	22.27	18.75	18.39	21.1
		v_2^-	9.06	9.36	10.99	19.4	20.12	21.76	15.73	16.65	17.25	18.99	19.08	21.56	18.01	18.22	20.38
		v_3^+	14.64	13.65	21.69	26.35	24.64	35.13	22.33	20.77	28.78	25.69	24.33	34.51	24.85	23.25	32.89
		v_3^-	14.24	13.55	22	26.08	25.61	35.26	21.34	21.02	27.63	24.93	24.46	32.92	24.12	23.26	31.63
	Σ_4	v_2^+	20.91	21.03	20.96	9.7	9.31	10.22	8.24	8.82	8.19	9.23	8.8	9.8	9.03	8.86	9.35
		v_2^-	20.17	20.57	20.69	9.35	9.06	10.26	8.21	8.2	8.27	8.83	8.56	9.9	8.95	8.56	9.31
		v_3^+	26.18	25.54	28.69	16.49	14.65	19.28	13.26	13.39	15.53	15.25	14.46	18.77	14.87	14.32	17.86
		v_3^-	25.97	25.92	28.47	16.39	15.37	19.06	13.74	13.77	15.13	15.25	14.99	18.29	14.62	14.65	17.52
n_2	Σ_1	v_2^+	7.79	7.93	10.49	7.71	8.23	11.35	7.73	8.4	10.97	7.7	8.2	11.18	7.75	8.4	11.13
		v_2^-	8.3	7.73	10.64	8.76	7.75	11.8	8.77	7.89	11.12	8.74	7.71	11.29	8.8	7.85	11.2
		v_3^+	17.58	16.09	31.75	19.14	17	35.26	19.14	16.98	33.51	19.1	16.97	33.77	19.24	17.06	33.78
		v_3^-	18.13	15.17	32.27	19.12	16.44	35.11	19.08	16.46	34.34	19.06	16.38	34.52	19.16	16.6	34.53
	Σ_2	v_2^+	9.63	9.52	11.69	4.49	4.52	6.22	4.45	4.5	6.51	4.44	4.32	6.59	4.45	4.42	6.6
		v_2^-	9.5	9.11	11.62	4.56	4.1	6.34	4.74	3.99	6.46	4.67	3.93	6.5	4.69	3.95	6.51
		v_3^+	18.57	17.23	31.51	13.62	11.72	28.08	14.2	11.22	27.23	14.23	11.08	27.4	14.28	11.22	27.46
		v_3^-	19.21	16.37	31.06	14.23	11.29	28.15	13.47	11.57	27.03	13.49	11.34	27.26	13.56	11.57	27.24
	Σ_3	v_2^+	14.3	13.86	20.37	27.71	28.07	35.47	28.11	27.14	35.08	28.14	27.4	35.57	28.11	27.24	35.32
		v_2^-	14.87	13.37	21.04	29.34	27.65	35.92	27.28	27.67	35.25	27.57	27.64	35.65	27.43	27.72	35.43
		v_3^+	33.04	28.09	57.69	46.64	41.49	66.1	47.08	41.06	65.45	47.29	41.49	65.79	47.27	41.25	65.67
		v_3^-	33.42	28.04	58.15	46.94	41.84	65.99	46.8	41.26	65.83	47.17	41.6	66.04	47.06	41.36	66.02
	Σ_4	v_2^+	34.62	34.46	35.51	12.87	12.72	15.27	13.11	12.3	15.98	13.07	12.22	16.13	13.2	12.39	16.19
		v_2^-	33.53	33.42	35.83	13.75	11.75	15.43	13.46	12.54	16.18	13.3	12.42	16.3	13.5	12.66	16.3
		v_3^+	50.48	47.07	57.52	39.03	31.47	49.48	39.3	30.8	48.75	39.5	30.61	49.2	39.4	30.84	49.12
		v_3^-	50.59	46.74	56.39	39.47	30.99	49.05	39.81	30.22	48.72	39.8	30.23	48.92	39.96	30.23	49.01
n_3	Σ_1	v_2^+	9.49	9.64	14.52	9.89	10.04	16.27	9.92	10.09	16.48	9.9	10.03	16.62	9.94	10.11	16.58
		v_2^-	10.38	9.21	15.4	10.92	9.51	16.55	10.92	9.61	16.76	10.92	9.52	16.85	10.92	9.62	16.89
		v_3^+	28.06	22.04	51.64	30.36	23.91	56.7	30.2	23.88	56.74	30.38	23.96	56.95	30.33	23.94	56.89
		v_3^-	27.93	21.98	51.53	30.99	23.84	56.47	30.84	23.75	56.43	31	23.86	56.69	31.05	23.8	56.59
	Σ_2	v_2^+	11.29	11.52	16.1	5.38	5.47	9.95	5.94	5.18	9.7	6	5.14	9.78	5.95	5.19	9.82
		v_2^-	11.84	11.08	16.63	6.2	5.06	10.07	6.18	5.17	10.38	6.15	5.19	10.5	6.2	5.18	10.49
		v_3^+	28.45	22.46	48.59	24.03	17.67	49.63	23.3	17.34	50.42	23.4	17.38	50.75	23.44	17.39	50.7
		v_3^-	27.67	23.3	49.72	23.7	17.32	50.91	23.98	17.34	50.85	23.97	17.35	51.11	24.07	17.42	51.02
	Σ_3	v_2^+	18.35	17.55	29.53	34.3	31.61	45.52	34.28	32.14	44.65	34.44	32.33	44.95	34.41	32.23	44.79
		v_2^-	19.3	16.62	30.03	34.86	32.69	44.74	34.15	32.79	44.44	34.39	32.97	44.7	34.27	32.89	44.51
		v_3^+	51.13	39.43	80.67	61.66	52.21	82.54	60.55	52.21	83	60.74	52.47	83.09	60.7	52.32	83.11
		v_3^-	51.02	39.7	81.18	61.85	51.87	82.56	60.73	51.93	82.36	61.01	52.36	82.62	60.87	52.04	82.48
	Σ_4	v_2^+	40.58	39.98	43.15	18.17	15.57	23.34	19.1	14.71	23.83	19.09	14.74	24.09	19.19	14.79	23.98
		v_2^-	41.1	39.03	44.1	19.3	15.49	22.3	19.17	15.98	22.62	19.23	15.82	22.73	19.22	16.04	22.7
		v_3^+	64.44	56.62	71.47	61.43	45.54	74.64	61.8	45.22	74.44	61.97	45.49	74.56	61.91	45.37	74.56
		v_3^-	63.88	56.01	72.4	61.75	45.6	74	61.91	45.63	74.08	62.1	45.64	74.27	62.05	45.85	74.19
n_4	Σ_1	v_2^+	7.09	6.16	8.06	6.78	6	7.5	6.38	5.87	4.66	6.82	6.08	7.63	6.86	6.25	7.2
		v_2^-	5.43	6.07	6.22	5.29	6	6.43	5.08	6.14	3.83	5.33	6.06	5.86	5.33	6.11	5.47
		v_3^+	11.16	10.01	16.55	11.03	9.72	17.24	8.68	8.14	10.13	11.13	9.8	17.28	10.9	9.92	16.25
		v_3^-	8.14	7.58	9.98	8.15	7.61	10.35	7.02	7.75	6.32	8.22	7.71	10.48	8.1	7.78	9.75
	Σ_2	v_2^+	7.61	6.8	7.75	4.48	3.6	4.87	4.6	4.89	3.61	3.89	3.99	4.97	3.94	4.32	4.77
		v_2^-	6.09	6.64	6.81	3.4	3.95	3.9	4.44	4.98	3.31	3.59	3.94	4.04	3.7	4.05	3.87
		v_3^+	11.64	10.65	16.52	7.9	6.72	12.08	6.6	6.27	7.67	8.24	6.48	12.05	8.07	6.58	11.39
		v_3^-	8.66	8.22	10.79	5.74	4.97	7.37	5.19	5.86	5.45	5.59	5.24	7.54	5.48	5.37	7.1
	Σ_3	v_2^+	11.22	10.33	13.07	19.98	20.48	24.69	15.28	15.35	16.37	21.05	20.84	24.54	20.22	19.98	23.31
		v_2^-	8.64	9.37	10.46	18.59	19.59	21.78	13.73	14.79	14.31	19.03	20.11	21.47	17.85	18.83	20.28
		v_3^+	19.87	17.77	32.33	29.89	27.5	42.78	23.56	20.71	31.2	31.72	28.15	42.84	30.65	26.93	41.61
		v_3^-	13.66	12.92	17.5	23.81	23.62	30.48	16.61	16.7	20.93	23.43	22.43	31.22	22.29	21.27	29.19
	Σ_4	v_2^+	23.12	22.29	23.64	10.43	9.65	11.18	8.59	8.55	7.79	10.5	9.93	10.9	10.3	10.11	10.45
		v_2^-	19.73	21.77	20.21	8.22	8.89	9.15	7.88	7.49	6.61	8.65	8.06	9.45	8.66	7.91	8.85
		v_3^+	33.05	30.74	36.37	20.53	17.51	25.85	15.99	13.94	16.95	21.44	18.17	25.24	21.02	18.21	24.1
		v_3^-	24.93	25.17	26.74	13.97	12.58	15.01	10.16	10.49	10.4	12.65	12.47	15.06	12.33	12.5	14.13

n_5	Σ_1	v_2^+	11.37	10.05	15.86	11.64	10.01	17.21	11.52	10.27	16.05	11.71	10.04	16.92	11.64	10.24	16.71
		v_2^-	5.75	6.06	6.19	5.84	5.96	6.76	5.74	6.13	6.5	5.88	6.03	7.05	5.79	5.98	6.74
		v_3^+	29.59	22.9	52.32	31.81	23.87	56.21	31.08	23.72	55.06	32.02	23.95	56.38	32.01	24.26	56.09
		v_3^-	8.4	7.67	11.62	8.67	7.84	12.55	8.33	7.67	11.41	8.7	7.94	12.37	8.45	7.63	12
	Σ_2	v_2^+	12.81	11.47	16.96	7.5	6.46	11.95	7.72	6.78	10.89	7.83	6.58	11.44	7.73	6.75	11.29
		v_2^-	6.88	7.63	7.51	3.11	2.97	3.45	3.02	3.27	3.55	2.97	3.03	3.77	2.87	2.91	3.63
		v_3^+	29.94	23.15	49.57	26.62	19.11	51.19	26.53	19.53	49.45	27.09	19.58	50.98	26.95	19.74	50.63
		v_3^-	9.44	9.13	12.24	5.22	4.98	7.77	4.91	5.14	7.18	4.98	5.11	7.75	4.76	4.92	7.37
	Σ_3	v_2^+	20.63	17.39	30.34	49.06	28.18	40.9	31.63	28.6	40.88	32.28	29.05	41.6	32.1	29.04	41.47
		v_2^-	9.79	10.28	11.14	43.56	22.66	25.14	20.98	21.71	23.77	22.21	23.16	25.61	21.31	22.25	24.62
		v_3^+	51.84	39.61	79.94	68.57	48.4	79.04	57.31	48.4	77.85	58.09	48.64	78.46	58	48.96	78.39
		v_3^-	14.71	13.62	21.91	50.59	27.47	37.49	26.58	25.92	35.22	28.4	27.95	37.63	27.21	26.41	36.18
	Σ_4	v_2^+	36.04	33.13	38.36	20.91	16.14	24.65	20.85	17.8	24.08	20.75	17.15	24.53	21.19	17.82	24.69
		v_2^-	25.74	28.16	27.35	8.09	8.28	9.43	7.76	8.37	8.07	8.1	8.47	9.08	7.88	8.4	8.34
		v_3^+	60.25	51.11	67.63	61.97	46.09	71.73	61.66	46.2	71.6	62.15	46.29	72.3	62.27	46.67	72.3
		v_3^-	33.08	32.18	34.21	14.73	13.1	17.5	13.66	12.56	16.35	14.73	13.19	17.85	14.03	12.48	16.93
n_6	Σ_1	v_2^+	27.65	20.07	46.41	29.53	21.8	50.05	29.75	22.15	51.03	29.58	21.81	50.86	29.79	22.18	51.13
		v_2^-	5.34	5.81	7.18	5.76	5.83	7.61	5.78	5.9	7.51	5.78	5.89	7.57	5.8	5.88	7.55
		v_3^+	74.04	55.96	96.66	77.97	59.43	97.63	78.2	59.74	97.51	78.03	59.54	97.48	78.27	59.79	97.52
		v_3^-	8.96	8.03	15.07	9.63	8.56	16.82	9.54	8.59	16.79	9.69	8.6	17.1	9.51	8.55	16.85
	Σ_2	v_2^+	28.21	20.88	45.04	24.18	15.79	45.82	23.17	16.15	46.06	23.09	15.92	45.99	23.28	16.13	46.16
		v_2^-	7.31	8.18	8.81	2.14	2.46	3.3	2.34	2.49	3.26	2.32	2.46	3.32	2.29	2.45	3.3
		v_3^+	69.97	53.57	94.04	77.68	56.84	98.3	77.83	56.39	98.53	77.75	56.12	98.51	77.91	56.48	98.54
		v_3^-	10.51	9.88	16.23	4.55	3.86	9.86	5.13	3.98	9.74	5.21	4.03	9.85	5.16	3.94	9.81
	Σ_3	v_2^+	46.23	34.81	73.44	53.51	44.04	72.66	54.18	44.2	74.19	53.44	43.36	73.73	54.22	44.19	74.23
		v_2^-	10.28	10.45	13.56	23.9	24.23	28.11	24.11	23.96	27.52	24.63	24.38	28.09	24.2	24.03	27.62
		v_3^+	92.88	79.48	99.9	90.91	79.4	99.35	91.1	79.58	99.33	90.84	79.12	99.37	91.12	79.65	99.34
		v_3^-	18.45	16.44	32.83	33.57	31.78	49.13	33.6	30.22	47.91	34.46	31.09	48.72	33.68	30.22	48.08
	Σ_4	v_2^+	56.43	48.54	64.09	55.9	39.82	65.27	56.91	40.45	66.7	56.19	39.92	66.17	56.97	40.52	66.79
		v_2^-	31.67	31.65	32.99	7.64	7.56	8.84	7.45	7.84	9.04	7.62	8.04	9.38	7.46	7.87	9.13
		v_3^+	90.82	79.79	95.25	98.37	89.75	99.7	98.52	89.31	99.73	98.44	89.09	99.71	98.53	89.32	99.74
		v_3^-	41.03	39	46.15	18.39	15.05	24.64	17.72	14.34	23.68	18.23	14.75	24.42	17.8	14.26	23.74

Table 3. 9: Type I error (%) of ANOMC tests under heterogeneous variances when $t = 5$.

Distribution			MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
n	Σ	v	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
n_1	Σ_1	v_2^+	5.96	5.76	5.87	5.95	5.79	6.33	5.63	5.75	4.53	5.94	5.78	6.33	6.05	5.98	5.85
		v_2^-	5.43	5.58	5.94	5.4	5.37	6.14	5.28	5.96	4.58	5.4	5.38	6.38	5.42	5.56	5.92
		v_3^+	7.96	7.08	10.86	7.96	7.4	11.38	7.25	7.14	7.75	7.99	7.43	10.48	7.72	7.41	9.79
		v_3^-	8.69	6.9	10.64	8.13	7.16	11.32	7.44	7.33	8.27	8.11	7.13	11.09	7.93	7.15	10.51
	Σ_2	v_2^+	6.77	6.26	6.39	4.02	3.68	4.05	3.87	4.69	3.31	3.49	3.7	4.16	3.69	3.88	3.88
		v_2^-	6.65	6.23	6.58	3.98	3.64	3.96	4.24	4.47	3.45	3.93	3.52	4.09	3.98	3.63	3.89
		v_3^+	8.37	7.93	11.2	5.21	4.99	7.92	5.18	5.39	5.89	5.45	4.94	7.39	5.36	4.84	7.01
		v_3^-	8.96	7.8	10.8	6.07	5.13	7.7	5.49	5.49	6.24	5.77	5.32	8.15	5.73	5.4	7.71
	Σ_3	v_2^+	6.77	6.26	10.76	18.52	18.09	21.91	15.61	14.48	15.81	19	17.78	20.76	18.01	16.77	19.41
		v_2^-	6.65	6.23	10.32	18.36	17.72	21.23	14.4	15.21	15.94	18.14	17.9	21.32	16.76	17.05	19.83
		v_3^+	8.37	7.93	20.48	24.15	22.98	32.28	19.35	19.16	24.35	23.72	23.5	31.64	22.54	22.15	29.73
		v_3^-	8.96	7.8	19.91	23.19	23.22	32.14	20.03	19.14	24.67	23.97	23.14	31.24	23.13	22.01	29.58
	Σ_4	v_2^+	21.1	20.33	21.42	9.07	8.71	10.15	7.9	7.91	7.92	8.91	8.03	9.9	8.74	8.08	9.18
		v_2^-	20.22	19.78	20.23	8.32	8.2	9.09	7.98	8.27	7.76	8.56	8.61	9.51	8.69	8.67	8.95
		v_3^+	26.12	25.14	28.34	14.26	12.71	17.08	12.26	11.63	13.08	14.62	13.24	16.51	14.3	12.92	15.75
		v_3^-	26.28	25.51	27.46	14.91	12.79	17.17	12.37	11.68	13.02	14.51	13.2	16.54	13.97	13.22	15.69
n_2	Σ_1	v_2^+	7.63	6.89	9.39	7.81	7.33	10.16	7.71	7.4	9.95	7.72	7.33	10.17	7.69	7.34	10.09
		v_2^-	8.13	7.48	9.86	8.42	7.86	10.61	8.32	7.91	9.41	8.36	7.82	9.62	8.4	7.9	9.64
		v_3^+	16.54	14.43	28.82	17.83	15.47	32.64	17.47	15.29	32.18	17.77	15.37	32.67	17.66	15.29	32.55
		v_3^-	16.49	14.13	29.15	17.92	15.19	32.77	17.75	14.95	31.52	17.86	15.01	31.93	17.95	15.01	31.84
	Σ_2	v_2^+	9.04	8.41	10.34	4.4	3.93	5.53	4.13	4.31	5.72	4.11	4.14	5.87	4.04	4.15	5.83
		v_2^-	9.77	9.24	11.25	4.76	4.23	5.81	4.78	4	5.72	4.76	3.89	5.89	4.78	3.91	5.86
		v_3^+	17.39	15.6	29.77	12.57	10.31	25.21	12.92	10.34	24.76	13.06	10.27	25.26	13.08	10.38	25.16
		v_3^-	17.46	15.51	28.68	12.19	10.1	25.34	12.37	9.88	24.71	12.67	9.95	25.16	12.63	9.95	25.1
	Σ_3	v_2^+	14.1	13.12	18.82	28.71	28.03	35.41	28.12	27.74	34.02	28.47	28.24	34.43	28.45	27.98	34.39
		v_2^-	14.76	13.29	19.04	28.79	27.47	35.88	28.44	27.94	34.72	28.64	28.32	35.39	28.72	28.17	35.2
		v_3^+	31.76	26.77	55.92	46.44	41.19	65.72	46.13	40.11	64.86	46.72	40.76	65.39	46.52	40.53	65.15
		v_3^-	32.03	26.24	55.67	46.8	40.83	66.03	46.26	41.11	65.62	46.42	41.32	65.93	46.61	41.28	65.88
	Σ_4	v_2^+	36.06	35.47	36.92	11.85	11.46	15	12.63	11.71	14.26	12.5	11.5	14.65	12.73	11.72	14.52
		v_2^-	35.28	35.44	37.2	12.95	11.88	14.49	12.23	11.97	14.46	12.2	11.65	15.02	12.34	12.07	14.72
		v_3^+	50.91	47.6	57.83	37.73	29.04	47.17	36.8	29.59	45.99	37.17	29.55	46.73	37.06	29.76	46.5
		v_3^-	50.7	46.48	57.04	37.16	28.37	46.64	36.76	29.39	46.57	37.23	29.3	47.13	37.05	29.51	46.93
n_3	Σ_1	v_2^+	9.45	8.77	13.95	10.1	9.4	15.15	10.06	9.53	14.67	10.08	9.4	14.77	10.01	9.43	14.66
		v_2^-	9.36	8.44	13.58	10.28	8.74	14.78	10.21	8.79	14.72	10.25	8.71	14.78	10.16	8.68	14.69
		v_3^+	26.61	20.61	49.5	29.36	22.6	54.17	29.2	22.42	54.67	29.28	22.56	54.84	29.08	22.34	54.59
		v_3^-	26.57	20.95	50.17	28.99	22.64	55.07	28.97	22.53	53.48	28.9	22.58	53.71	28.87	22.44	53.36
	Σ_2	v_2^+	11.65	10.8	15.32	5.21	4.91	9.1	5.19	4.63	8.69	5.22	4.67	8.77	5.19	4.58	8.68
		v_2^-	11.04	10.33	15.29	5.31	4.4	8.79	5.43	4.68	8.76	5.42	4.54	8.82	5.38	4.59	8.74
		v_3^+	27.3	21.21	47.53	22.93	15.68	48.73	22.47	15.81	48.81	22.47	15.75	48.83	22.42	15.7	48.69
		v_3^-	27.21	21.71	47.35	22.45	16.31	49	21.95	16.43	48.85	21.96	16.41	48.89	21.89	16.31	48.78
	Σ_3	v_2^+	18.42	16.63	29.37	35.22	32.67	46.39	34.63	33.24	45.56	34.69	33.49	45.7	34.59	33.03	45.48
		v_2^-	17.95	16.54	28.68	35.04	33.22	45.12	35.54	32.59	45.07	35.8	32.71	45.24	35.45	32.54	45
		v_3^+	49.44	38.34	80.28	61.14	52.88	82.76	60.29	52.28	82.76	60.15	52.04	82.82	60.22	52.11	82.74
		v_3^-	49.31	37.89	80.31	61.51	52.54	82.96	60.87	51.78	82.73	60.96	51.82	82.99	60.8	51.67	82.7
	Σ_4	v_2^+	42.83	41.07	45.86	17.92	15.23	22.23	18.23	15.33	22.02	18.21	15.21	22.06	18.15	15.21	21.95
		v_2^-	42.03	41.42	45.28	18.29	14.78	21.69	18.11	15.17	21.98	18.2	15.01	22.21	18.05	15.09	21.93
		v_3^+	65.51	57.26	72.78	60.13	44.5	72.8	59.77	43.72	73.22	59.91	43.56	73.24	59.7	43.62	73.1
		v_3^-	64.63	58.33	72.81	59.59	44.39	73.47	60.04	43.76	72.17	60.12	43.52	72.31	59.95	43.66	72.11
n_4	Σ_1	v_2^+	10.95	9.87	14.98	11.07	10.2	15.93	10.58	9.85	13.49	11.13	10.24	16.03	11.01	10.41	15.52
		v_2^-	5.55	5.77	5.37	5.59	5.9	6.01	5.52	6.38	4.77	5.63	5.96	6.24	5.57	5.97	5.81
		v_3^+	29.39	22.09	51.92	31.44	23.1	54.41	28.61	21.71	49.32	31.55	23.14	54.27	31.2	23.16	53.74
		v_3^-	7.78	7.06	9.08	8.03	7.29	9.66	7.32	7.36	8.24	8.1	7.31	10.18	7.79	7.36	9.64
	Σ_2	v_2^+	11.78	10.86	15.84	7.08	6.2	10.77	7.36	6.57	9.58	7.52	6.3	11.37	7.47	6.41	10.94
		v_2^-	6.55	6.53	6.65	3.21	2.81	3.53	3.5	4.34	2.87	3.14	3.42	3.58	3.08	3.48	3.34
		v_3^+	29.07	22.26	48.94	25.42	18.25	50.68	22.16	17.34	43.87	24.76	18.87	49.2	24.39	18.92	48.39
		v_3^-	8.54	8.28	9.89	5.13	4.37	6.09	5.04	5.08	5.53	4.85	4.5	6.48	4.84	4.6	6.27
	Σ_3	v_2^+	19.65	17.39	28.88	31.3	28.76	41.74	28.97	26.69	38.16	31.88	29.29	41.47	31.55	28.91	41.2
		v_2^-	9.53	9.29	10.65	21.65	22.28	24.55	18.59	18.65	20.21	21.71	21.47	24.37	20.6	20.51	22.84
		v_3^+	51.16	38.53	78.87	57.75	48	78.39	55.49	45	75.35	57.77	47.49	77.99	57.81	47.74	77.74
		v_3^-	13.27	11.66	17.15	26.16	25.57	32.01	22.42	21.99	26.28	25.82	25.73	31.65	24.4	24.14	29.8
	Σ_4	v_2^+	35.95	33.32	38.59	19.72	16.86	23.47	18.97	15.69	20.98	20.43	15.81	23.38	20.56	16.66	23.23
		v_2^-	25.65	25.81	26.53	7.79	7.76	8.76	7.34	8.06	7.64	8.05	8.08	9	7.74	7.91	8.34
		v_3^+	60.94	50.19	68.22	61.47	44.86	71.94	57.18	42.98	67.02	60.66	45.33	70.79	60.87	45.8	70.66
		v_3^-	29.63	29.14	30.81	12.77	11.51	14.37	11.24	10.54	12	12.74	11.58	14.46	12.03	11.04	13.58

n_5	Σ_1	v_2^+	17.19	14.18	28.48	18.54	14.87	30.89	18.21	15.04	30.22	18.57	14.85	31.01	18.62	15.22	31.15
		v_2^-	5.41	5.49	5.69	5.66	5.75	5.7	5.67	5.89	5.91	5.66	5.77	6.22	5.7	5.81	6.14
		v_3^+	52.18	36.89	82.53	56.24	39.85	85.49	55.67	39.44	85.46	56.2	39.81	86.03	56.49	40.12	86.04
		v_3^-	6.31	6.75	9.08	6.77	7.04	9.47	6.47	7.05	9.32	6.77	7.06	10	6.71	6.93	9.71
	Σ_2	v_2^+	18.02	15.35	28.25	12.58	9.89	24.66	13.21	10.13	23.61	13.35	9.92	24.46	13.49	10.13	24.59
		v_2^-	6.79	7.46	6.97	2.44	2.34	2.64	2.62	2.6	2.65	2.5	2.3	2.87	2.48	2.32	2.78
		v_3^+	50.31	36.38	78.12	52.89	34.64	85.92	52.02	35.56	86.1	52.81	36.02	86.64	53.06	36.22	86.74
		v_3^-	7.99	8.23	10.18	3.36	3.51	5.15	3.64	3.82	4.79	3.69	3.62	5.08	3.66	3.68	4.94
	Σ_3	v_2^+	31.35	25.2	51.84	42.22	37.41	57.84	43.25	36.97	58.95	43.75	37.31	59.23	44.01	37.62	59.62
		v_2^-	9.74	9.52	10.7	23.69	24.36	25.37	22.79	23.54	24.45	23.81	24.17	25.74	23.47	24.02	25.37
		v_3^+	78.36	60.16	97.34	79.2	65.51	95.18	78.9	65.69	94.85	79.02	65.78	95.05	79.27	66.39	95.04
		v_3^-	12.66	11.91	17.18	27.41	27.18	34.52	26.65	26.66	32.46	28.18	28.14	34.55	27.59	27.41	33.5
	Σ_4	v_2^+	47.95	43.58	53.02	35.93	26.93	44.35	37.02	27.06	42.16	36.88	26.62	42.69	37.74	27.58	43.27
		v_2^-	30.11	31.4	30.91	7.3	7.13	7.31	6.67	6.71	7.21	6.97	6.7	7.81	6.8	6.76	7.42
		v_3^+	80.07	67.49	87.13	89.25	71.16	95.43	88.89	71.43	94.95	89.06	71.54	95.23	89.21	71.9	95.23
		v_3^-	34.08	33.95	36.2	10.67	10.2	13.63	10.3	9.99	12.28	11.06	10.29	13.43	10.65	10.08	12.66
n_6	Σ_1	v_2^+	33.61	24.2	59.28	36.29	26.06	63.5	36.4	26.24	62.74	36.22	26.06	62.53	36.59	26.32	62.91
		v_2^-	5.24	5.13	6.07	5.53	5.56	6.38	5.44	5.55	5.75	5.52	5.53	5.85	5.5	5.56	5.82
		v_3^+	85.08	66.22	99.39	88.45	70.25	99.7	88.52	70.34	99.63	88.45	70.19	99.63	88.6	70.52	99.64
		v_3^-	7.19	6.58	10.62	7.49	6.87	11.34	7.34	6.78	11.85	7.47	6.86	12.05	7.39	6.81	11.98
	Σ_2	v_2^+	33.4	25.65	55.57	30.4	20.04	60.44	30.64	20.21	59.57	30.5	19.94	59.44	30.76	20.22	59.72
		v_2^-	7.21	6.92	8.56	1.78	1.75	2.5	1.92	2.11	2.56	2.01	2.13	2.61	1.95	2.11	2.55
		v_3^+	80.81	62.68	98.29	89.08	68.51	99.91	89.85	68.86	99.78	89.75	68.63	99.78	89.92	68.98	99.78
		v_3^-	9.08	8.89	11.75	3.15	2.84	5.75	3.06	2.89	5.78	3.14	2.9	5.92	3.11	2.86	5.87
	Σ_3	v_2^+	56.57	42.28	83.44	61.62	50.35	82.21	62.25	51.84	82.78	61.87	51.45	82.62	62.39	52.03	82.83
		v_2^-	9.54	9.35	10.91	24.67	24.87	27.25	24.72	23.85	26.65	25.05	24.27	27.07	24.87	23.94	26.77
		v_3^+	97.83	87.52	99.98	96.33	87.73	99.86	96.17	87.46	99.87	96.14	87.26	99.88	96.19	87.46	99.87
		v_3^-	14.9	13.23	24.02	31.82	30.05	43.57	30.24	28.69	41.83	30.83	29.35	42.67	30.36	28.81	42.02
	Σ_4	v_2^+	65.74	56.25	71.77	68.53	49.37	77.26	68.67	50.49	78.86	68.27	49.55	78.29	68.76	50.53	78.93
		v_2^-	33.16	32.91	34.44	6.75	6.66	7.56	5.95	5.91	7.3	6.03	5.96	7.47	5.95	5.88	7.36
		v_3^+	95.91	86.85	98.45	99.72	95.59	99.96	99.84	95.32	99.99	99.83	95.27	99.99	99.84	95.34	99.99
		v_3^-	40.36	38.44	42.97	13.48	11.38	17.21	12.82	11.22	16.44	13.38	11.55	17.06	12.86	11.32	16.57

Table 3. 10: Power (%) of ANOMC tests under homogeneous variances under balanced & unbalanced design n_1 & n_4 when $t = 3$.

Distribution		MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
Σ	δ	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
Σ_1	δ_2	30.36	25.57	54.14	31.63	26.54	51.08	28.57	24.31	48.58	31.62	26.57	53.77	31.08	26.28	52.91
	δ_3	25.63	22.27	44.54	26.76	22.89	45.45	23.94	20.98	41.5	26.74	22.9	46.38	26.4	22.62	45.49
	δ_4	22.4	20.85	39.18	23.12	21.69	41.1	20.97	20.09	35.82	23.12	21.71	40.56	22.76	21.73	39.66
	δ_5	24.1	21.32	40.22	24.54	21.7	40.06	22.44	19.92	37.22	24.53	21.71	42.12	24.17	21.45	41.13
	δ_6	27.28	24.27	46.38	28.89	25.42	43	25.85	23.22	43.63	28.91	25.43	48.72	28.36	25.07	47.81
	δ_7	25.37	23.38	43.89	26.89	23.75	44.36	24.34	22.04	42.13	26.89	23.8	47.45	26.32	23.47	46.64
	δ_8	25.11	22.98	44.49	26.18	23.95	45.38	23.34	21.71	41.66	26.18	23.95	46.45	25.63	23.48	45.63
	δ_9	22.51	20.53	39.88	23.39	21.05	41.08	21.1	19.14	36.99	23.4	21.1	41.82	23.1	20.91	41.04
	δ_{10}	24.22	20.8	38.97	25.06	21.29	41.5	22.49	19.46	37.87	25.07	21.34	42.26	24.58	21.01	41.54
	Σ_2	δ_2	29.36	25.65	50.41	26.91	22.21	44.68	23.86	20.52	45.68	26.49	22.6	50.67	26.07	22.26
δ_3		25.65	22.44	42.75	22.1	18.23	37.51	20.13	17.02	37.01	22.25	18.56	41.96	22	18.45	40.94
δ_4		22.78	21.4	38.59	19.07	17.14	34.19	17.57	16.61	31.34	19.69	17.74	36.12	19.28	17.62	35.46
δ_5		23.24	21.38	39.37	19.47	16.85	32.72	16.55	15.84	30.79	18.68	17.1	35.33	18.47	16.82	35.8
δ_6		27.12	24.67	43.99	23.27	20.1	35.42	20.53	17.8	37.25	23.28	19.95	42.34	22.94	19.81	41.61
δ_7		25.13	22.82	42.65	21.92	19.27	37.47	18.95	16.83	35.74	21.24	18.47	40.78	20.88	18.3	39.76
δ_8		25.13	22.81	42.46	21.45	19.16	38.27	19.32	17.38	36.79	21.93	19.03	41.52	21.41	18.81	40.4
δ_9		22.95	20.94	37.49	18.78	16.3	33.64	17.05	15.21	31.75	18.74	16.59	35.96	18.58	16.61	35.26
δ_{10}		23.81	21.13	37.76	19.99	16.52	34.63	17.05	15.28	32.41	19.2	16.61	36.7	18.86	16.41	35.56
Σ_3		δ_2	49.84	43.28	79.63	57.41	50.84	79.73	54.41	45.77	75.96	58.24	49.58	79.28	57.09	50.23
	δ_3	43.13	36.69	70.7	51.76	45.82	72.52	48.06	43.12	69.93	51.88	46.62	73.62	51.74	46.24	72.27
	δ_4	39.19	34.55	64.69	49.25	44.25	69.28	45.1	40.39	65.07	48.89	43.48	68.95	48.6	44.27	68.69
	δ_5	41	35.27	65.65	51.31	45.86	71.02	45.54	42.37	66.44	49.8	46.06	70.31	50.19	44.96	69.45
	δ_6	47.39	40.92	72.11	57.12	49.64	76.08	52.04	47.12	71.9	55.94	50.86	75.23	56.86	50	75.49
	δ_7	43.85	38.29	70.96	52.9	46.06	72.19	48.16	43	69.84	51.8	46.95	73.48	51.2	45.81	72.06
	δ_8	42.73	37.99	71.55	52.28	46.41	73.57	48.19	42.61	68.33	51.69	46.54	72.17	51.59	45.17	72.41
	δ_9	39.37	34.41	65.28	49.19	43.61	69.04	45.5	40.33	66.54	49.65	44	70.18	48.53	43.81	68.71
	δ_{10}	40.21	35.45	63.72	50.25	44.56	68.77	45.61	41.02	65.77	49.56	45.06	69.53	49.28	43.73	68.94
	Σ_4	δ_2	58.43	52.58	68.47	58.59	48.51	76.72	53.97	44.77	64.61	58.62	48.9	69.72	58	47
δ_3		53.9	47.86	62.2	50.51	42.05	69.23	46.09	38.97	56.02	50.08	42.33	61.11	50.47	42.76	60.56
δ_4		50.7	46.76	57.11	45.51	39.36	63.71	42.21	36.43	50.9	46.51	39.26	55.75	45.48	38.63	54.97
δ_5		52.62	48.29	58.45	47.55	40.44	61.38	42.14	36.79	51.1	46.58	40.1	56.01	45.86	40.22	55.53
δ_6		58.38	52.7	63.67	54.73	46.09	65.56	49.07	42.26	58.08	53.68	46.05	63.25	54.55	45.59	62.29
δ_7		53.08	49.57	61.84	51.17	43.02	67.84	46.45	39.48	56.15	51.1	42.74	60.84	50.2	41.88	60.7
δ_8		52.45	48.77	60.45	50.94	42.76	69.06	47.11	38.24	56.44	51.07	41.72	61.26	50.07	41.78	60.61
δ_9		50.57	46.42	56.86	46.09	38.45	63.56	42.62	36.29	51.18	46.77	39.53	55.99	46.22	37.7	54.71
δ_{10}		51.32	47.01	58.24	46.29	39	64.78	42.31	35.67	51.4	46.55	38.63	56.35	45.3	38.59	54.31
Σ_5		δ_2	66.83	52.84	92.96	68.84	53.99	76.78	64.43	50.3	91.09	68.9	54.03	93.22	68.44	54.06
	δ_3	53.63	42.38	82.67	55.22	43.68	64.99	51.07	40.28	79.61	55.26	43.78	83.76	54.95	43.56	83.25
	δ_4	40.05	33.05	65.85	40.88	33.5	52.38	37.39	31.17	62.03	41.02	33.54	67.3	40.79	33.7	66.78
	δ_5	31.4	26.43	50.28	31.75	26.97	41.98	29.1	24.6	46.07	31.84	27.04	51.18	31.84	27.14	50.73
	δ_6	27.15	23.32	41.04	27.92	23.43	37.62	24.84	21.28	37.55	27.98	23.56	43.15	27.83	23.66	42.49
	δ_7	60.46	47.99	89.09	62.05	48.48	71.41	57.65	44.79	85.85	62.19	48.54	88.92	62.01	48.52	88.61
	δ_8	17.95	16.73	31.16	18.93	17.12	37.35	16.47	15.09	28.57	19	17.23	34.5	18.37	16.65	33.12
	δ_9	54.96	43.16	82.83	56.04	44.09	65.67	51.84	40.33	79.79	56.15	44.2	83.75	55.66	44.04	83.23
	δ_{10}	19.31	18.39	33.89	20.21	18.6	36.95	17.46	16.34	29.91	20.3	18.68	35.48	19.68	18.29	34.45
	Σ_6	δ_2	62.47	50.03	90.35	66.22	50.61	76.9	61.05	46.22	92.42	66.33	50.26	94.75	65.8	50.1
δ_3		50.18	41.15	78.53	51.05	39.11	62.19	46.45	34.9	79.28	51.36	38.11	83.68	50.86	38.23	83.22
δ_4		38.77	32.74	62.95	36.75	28.82	46.04	33.01	25.43	59.48	36.82	28.09	65.58	36.57	28.15	64.94
δ_5		30.38	26.54	48.9	25.87	21.41	34.71	23.32	19.73	40.56	26.17	21.69	45.8	25.97	21.79	45.2
δ_6		27.31	24.38	40.46	21.4	17.79	29.18	18.71	16.43	29.62	21.34	18.38	34.73	21.01	18.46	34.21
δ_7		57.47	45.88	85.16	59.51	44.8	69.8	52.99	41.19	86.98	58.17	45.65	90.2	57.64	45.18	89.83
δ_8		18.49	17.19	30.92	14.28	12.74	29.26	12.8	11.23	22.77	14.63	12.28	27.73	14.34	12.16	26.59
δ_9		52.37	41.65	79.27	52.41	39.4	62.35	46.7	36.05	79.7	51.88	40.46	84.24	51.35	40.1	83.55
δ_{10}		20.2	18.67	32.73	15.07	13.25	28.45	13.35	12.47	24.01	15.63	13.65	29.37	15.11	13.44	28.24
Σ_7		δ_2	87.98	74.51	99.36	85.55	74.73	97.18	83.47	72.86	96.38	85.16	75.14	97.12	85.38	75.19
	δ_3	77.71	63.3	97.06	75.86	65.46	92.32	73.14	62.74	91.12	75.89	65.48	92.51	75.79	65.32	92.46
	δ_4	63.22	51.52	89.26	65.15	56.22	84.35	62.17	54.07	82.06	64.89	56.9	84.48	65.03	56.81	84.36
	δ_5	49.95	41.9	74.87	57.49	49.69	75.23	54.22	47.24	72.41	57.25	50.07	75.42	56.99	50	75.31
	δ_6	44.64	37.99	66.33	54.27	49.6	71.73	51.34	46.43	68.24	55.11	49.78	72.06	54.79	49.45	71.67
	δ_7	83.73	69.55	98.66	81	70.16	95.19	79.18	67.81	94.82	81.49	70.84	95.61	81.43	70.68	95.6
	δ_8	31.68	28.68	55.57	43.99	39.28	63.28	39	35.66	56.84	44.66	40.52	63.48	42.98	38.79	62.24
	δ_9	78.46	65.01	97.58	77.77	67.47	93.46	74.79	64.68	91.68	77.33	67.94	92.96	77.34	67.92	92.97
	δ_{10}	34.68	30.51	58.51	46.79	42.03	65.36	41.03	37.82	60.94	46.85	42.46	66.76	45.3	41.6	65.58
	Σ_8	δ_2	86.43	75.87	93.64	82.98	75.39	95.39	83.21	79.92	97.53	94.66	82.76	98.1	94.56	82.62
δ_3		78.49	67.06	86.07	75.27</											

Table 3. 11: Power (%) of ANOMC tests under homogeneous variances under balanced & unbalanced design n_1 & n_4 when $t = 4$.

Distribution		MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
Σ	δ	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
n_1	δ_2	26.63	22.78	47.47	28.68	24.39	50.93	24.19	20.8	43.5	27.75	23.55	50.51	27.1	23.22	49.15
	δ_3	18.14	16.03	30.98	19.79	17.44	33.64	16.44	15.14	25.97	18.88	16.75	31.38	18.52	16.48	30.18
	δ_4	20.69	18.29	34.36	22.17	19.72	37.87	18.39	17.01	30.44	21.43	18.94	36.56	20.82	18.77	35.25
	δ_5	28.61	24.5	47.6	30.71	26.57	51.89	25.31	22.32	42.86	29.65	25.56	50.4	28.82	25.03	49.04
	δ_6	19.65	17.2	32.98	21.16	18.83	35.68	17.36	15.74	29.57	20.3	18.09	35.43	19.87	17.83	34.38
	δ_7	24.29	20.69	41.62	25.92	22.03	45.73	21.49	18.86	36.91	24.94	21.26	43.63	24.3	20.91	42.31
	δ_8	23.98	20.6	42.43	25.95	22.29	45.86	21.91	18.93	37.33	25.09	21.41	44.21	24.45	20.94	42.95
	δ_9	22.94	20.17	38.71	25.24	21.49	42.69	20.79	18.44	34.64	24.34	20.71	41.27	23.53	20.43	39.91
	δ_{10}	22.86	19.82	39.63	24.46	21.18	42.77	20.46	18.46	35.53	23.62	20.47	42.32	22.84	20.16	40.97
	n_2	δ_2	26.31	22.35	43.5	23.91	20.19	45.45	19.44	16.88	38.01	22.86	18.81	44.78	22.21	18.56
δ_3		18.38	16.29	28.85	14.9	12.69	26.8	12.49	11.4	21.34	14.22	12.37	26	13.95	12.23	25.07
δ_4		20.16	19.1	33.47	17.4	14.89	30.61	14.73	12.51	24.52	17	13.84	29.35	16.71	13.79	28.31
δ_5		28.05	24.77	46.07	24.6	21.34	44.82	19.85	18.4	36.64	23.05	20.61	43.27	22.43	20.28	41.9
δ_6		19.95	17.74	31.79	16.68	14.7	29.88	13.17	12.95	24.48	15.29	14.53	29.63	14.74	14.37	28.48
δ_7		24.21	21.07	40.9	21.58	17.98	40.99	17.77	15.74	32.83	20.36	17.66	39.2	19.81	17.37	37.91
δ_8		24.45	20.53	40.25	21.46	17.58	40.09	17.8	15.05	31.66	20.74	17.04	38.2	20.06	16.75	36.93
δ_9		22.89	19.88	38.02	19.75	16.59	37.09	16.36	14.42	29.82	18.58	16.16	35.26	18.13	15.98	34.04
δ_{10}		23.01	19.5	37.95	19.83	16	37.12	16.6	14.34	29.65	19.26	15.99	35.55	18.68	15.73	34.42
n_3		δ_2	46.12	38.63	74.82	55.36	48.13	77.86	48.11	42.96	71.56	53.53	47.6	76.72	52.25	46.7
	δ_3	31.76	27.6	54.16	45.48	41.24	63.98	38.39	34.41	56.84	44.19	39.61	63.18	42.9	38.22	61.85
	δ_4	35.52	31.95	58.46	49.7	44.12	69.74	42.77	38.45	61.98	48.87	43.45	68.42	47.63	42.31	67.36
	δ_5	48.87	42.21	75.75	62.01	54.94	82.89	54.26	48.35	76.46	60.08	54.14	81.51	58.91	52.83	80.79
	δ_6	34.81	30.28	57.33	47.78	41.82	66.94	40.3	37.29	59.67	45.72	41.93	65.98	44.48	40.9	64.83
	δ_7	41.99	35.15	69.51	52.64	46.2	73.94	46.25	40.05	68.06	51.69	45.02	73.69	50.48	44	72.68
	δ_8	42.09	35.31	69.52	52.71	46.06	74.37	45.33	40.53	67.81	50.64	45.34	73.24	49.82	44.43	72.29
	δ_9	40.21	34.46	66.06	52.98	46.44	72.68	44.57	39.8	66.73	50.13	45.28	72.49	48.84	43.74	71.25
	δ_{10}	39.85	34.45	66.29	51.74	46.07	73.56	46	40.13	67.11	51.35	45.7	72.81	50.37	44.36	71.71
	n_4	δ_2	56.06	49.76	65.02	54.99	44.79	66.05	47.81	40.03	58.67	53.7	44.52	65.55	52.47	43.71
δ_3		45.88	42.05	51.43	38.72	32.81	46.23	32.2	27.85	39.36	37.22	31.54	45.46	36.4	31.08	44.33
δ_4		49.08	46.58	56.02	42.64	37.35	51.49	36.23	31.53	43.17	41.34	35.8	49.79	40.39	35.18	48.33
δ_5		61.93	57.11	68.92	57.65	50.21	67.79	50.79	42.66	58.18	57.47	47.93	65.57	56.21	47.18	64.4
δ_6		47.45	43.19	54.38	42.11	34.71	50.16	35.73	30.41	43.18	40.87	34.11	49.47	40.12	33.55	48.17
δ_7		52.86	47.4	61.03	50.65	41.72	60.04	42.66	36.36	53	48.46	40.42	60.23	47.41	39.99	58.67
δ_8		53.64	46.85	61.63	50.64	40.97	60.8	42.93	36.04	52.83	48.29	40.08	59.44	47.35	39.33	58.21
δ_9		52.27	48.48	59.62	48.76	40.88	57.98	41.8	34.36	49.33	47.6	38.87	56.25	46.48	38.41	54.89
δ_{10}		52.24	47.99	59.92	47.98	39.2	57.52	40.29	34.44	50.75	45.69	38.85	57.38	44.54	38.12	55.94
n_5		δ_2	41.71	33.33	71.4	42.86	33.72	72.79	31.85	26	56.94	43.02	33.87	72.39	41.98	33.25
	δ_3	21.36	18.76	37.13	21.74	18.66	38.67	15.98	14.19	26.03	21.88	18.79	39.05	21.39	18.55	37.81
	δ_4	22.79	19.62	36.71	22.78	20	38.2	16.82	14.6	26.73	23	20.11	38.8	22.41	19.94	37.69
	δ_5	30.35	27.06	49.26	31.3	27.61	49.79	22.01	19.75	35.62	31.47	27.82	51.3	30.62	27.38	49.99
	δ_6	24.82	22.16	43.94	25.06	21.98	44.94	18.52	16.87	30.22	25.14	22.14	44.22	24.6	21.87	42.84
	δ_7	37.48	30.83	67.11	38.44	31.22	68.21	28.84	23.24	52.05	38.57	31.41	67.47	37.61	30.76	65.89
	δ_8	18.31	16.73	31.16	18.55	16.82	32.34	14.06	13.28	21.95	18.72	16.89	31.8	18	16.47	30.56
	δ_9	35.6	29.41	61.57	36.24	29.19	63.43	26.52	21.92	47.49	36.5	29.34	64.04	35.57	28.8	62.47
	δ_{10}	19.02	17.16	32.15	19.92	17.51	33.38	14.44	13.58	23.48	20.1	17.66	34.76	19.2	17.19	33.38
	n_6	δ_2	39.72	33.13	68.14	37.17	29.44	70.09	26.36	20.45	52.81	36.92	28.13	69.89	36.2	27.59
δ_3		21.49	19.62	36.13	16.66	14.43	32.64	12.56	12.11	20.95	17.61	15.46	31.45	17.27	15.32	30.51
δ_4		22.92	20.03	36.3	17.75	14.6	31.89	13.26	11.69	21.66	18.5	15.09	32.55	17.89	14.87	31.76
δ_5		29.67	26.54	47.89	24.32	20.65	43.09	17.86	15.39	30.07	25.12	20.82	44.16	24.42	20.76	42.75
δ_6		25.07	22.75	41.56	20.24	17.33	38.74	14.75	13.08	26.1	20.27	17.24	38.43	19.71	17.01	37.02
δ_7		36.23	30.27	61.92	33.14	26.46	63.54	22.94	19.03	47.35	32.93	26.45	64.4	32.06	25.92	62.82
δ_8		18.15	16.87	29.86	14.51	12.97	27.39	12.06	10.96	18.16	15.25	12.87	27.16	14.88	12.84	26.01
δ_9		34.39	28.71	59.24	30.76	24.17	59.2	21.07	17	41.72	29.94	23.15	58.62	29.01	22.82	56.85
δ_{10}		18.98	17.39	32.16	14.9	13.03	27.94	11.57	10.55	18.18	15.05	12.88	28.1	14.54	12.77	26.66
n_7		δ_2	67.15	55.27	93.79	69.72	59.72	90.71	59.15	49.73	83.4	69.56	60.13	90.26	69.23	59.4
	δ_3	38.24	32.66	63.66	49.09	41.78	67.38	38.57	34.36	57.01	48.66	43.22	68.26	47.68	42.5	67.51
	δ_4	39.72	34.03	63.95	49.96	45.36	70.14	39.47	35.89	58.72	49.52	45.54	70.64	49.01	45.11	69.77
	δ_5	53.33	45.61	78.31	61.42	55.51	82.84	49.58	43.21	71.53	61.85	54.26	82.05	61.07	53.51	81.39
	δ_6	44.29	37.72	71.69	52.07	45.62	73.4	41	37.03	61.5	52.2	46.26	72.97	51.33	45.51	72.1
	δ_7	62.34	51.2	91.24	67.05	56.89	87.93	56.28	46.96	79.54	66.95	58.1	87.37	65.88	56.91	86.7
	δ_8	31.68	28.1	53.15	43.24	38.22	61.91	31.74	29.31	48.57	43.23	38.55	62.4	41.35	37.11	60.72
	δ_9	58.73	49.22	88.15	65.17	55.98	86.34	54.29	45.07	77.73	65.39	55.9	86.73	64.5	54.79	86.05
	δ_{10}	33.87	29.37	56.78	45.75	40.31	66.42	34.11	29.89	51.98	45.58	40.2	65.66	43.95	38.76	64.06
	n_8	δ_2	71.99	63.01	82.13	75.96	62.29	87.06	63.47	49.93	75.78	75.85	60.77	86.88	74.83	60.1
δ_3		50.4	46.77	57.03	43.8	35.73	53.41	34.22	28.62	41						

Table 3. 12: Power (%) of ANOMC tests under homogeneous variances under balanced & unbalanced design n_1 & n_4 when $t = 5$.

Distribution		MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN	MVN	MVT	MVLN
Σ	δ	ANOMC-MR6			ANOMC-MR7			ANOMC-MR8			ANOMC-MR9			ANOMC-MR10		
n_1	δ_2	24.29	20.56	43.66	25.1	21.4	45.83	21.18	18.46	36.36	25.07	21.35	45.01	24.44	20.77	42.94
	δ_3	17.8	15.56	27.78	18.14	15.72	29.25	15.04	13.81	22.99	18.11	15.68	29.84	17.68	15.53	28.41
	δ_4	21.79	18.77	35.97	22.4	19.1	37.88	18.5	16.66	28.86	22.34	19.13	37.41	21.65	18.63	35.65
	δ_5	20.54	17.43	34.79	21.06	18	36.81	17.65	15.19	30.29	21.02	17.88	37.6	20.4	17.4	36.04
	δ_6	28	24.93	47.78	28.8	25.54	50.42	23.89	21.87	40.63	28.77	25.46	50.29	27.69	24.78	48.4
	δ_7	23	19.33	39.48	23.74	19.74	41.54	19.69	17.16	33.9	23.72	19.73	41.79	22.9	19.27	40.21
	δ_8	22.58	19.26	39.03	23.39	19.49	41.49	19.27	16.69	34.02	23.37	19.45	42.13	22.4	19.2	40.49
	δ_9	22.28	19.13	38.5	22.93	19.8	40.37	19.08	17.24	32.39	22.89	19.78	40.63	22.1	19.35	39.15
	δ_{10}	22.56	18.51	37.98	23.2	19.08	40.14	19.06	16.27	31.89	23.19	18.99	39.95	22.15	18.73	38.53
	δ_2	24.06	20.37	39.52	20.83	17.19	39.45	16.55	14.99	32.23	20.04	17.06	40.63	19.4	16.82	39.04
n_2	δ_3	18.16	16.18	29.02	13.69	11.78	24.74	11.6	10.1	18.76	13.69	11.51	24.41	13.25	11.34	23.17
	δ_4	21.73	19.09	33.7	17.21	15.03	30.41	13.97	13.61	24.3	16.76	15.4	31.54	16.33	15.17	30.09
	δ_5	19.94	17.85	33.08	16.68	14.14	31.35	13.83	12.25	25.11	16.82	14.01	32.22	16.39	13.72	30.81
	δ_6	28.06	24.84	46.66	23.39	20.49	43.2	19.92	16.54	34.46	24.4	19.5	43.29	23.3	19.11	41.53
	δ_7	22.45	18.95	37.13	19.45	15.61	35.97	16.45	13.63	28	20.17	15.92	35.1	19.32	15.65	33.72
	δ_8	21.82	19.32	37.57	18.86	15.79	36.39	15.84	14.04	28.2	18.98	16.22	35.89	18.22	15.75	34.46
	δ_9	21.97	19.12	35.27	17.96	15.38	32.37	15.81	14.44	25.96	18.93	16.15	32.41	18.3	15.91	30.87
	δ_{10}	22.69	18.6	35.31	19.11	15.11	33.36	15.22	12.9	26.61	18.16	15.01	33.83	17.62	14.91	32.43
	δ_2	43.17	36.21	72.03	51.64	45.18	78.35	43.92	37.68	67.48	51.11	44.26	74.88	49.57	42.9	73.68
	δ_3	31.26	27.6	49.99	44.19	39.48	67.56	35.95	33.22	53.48	43.29	39.77	62.01	41.9	38.33	60.29
n_3	δ_4	38.09	33.02	62.05	50.45	44.32	75.7	41.43	37.45	62.34	49.57	44.32	70.65	47.77	42.71	68.91
	δ_5	36.46	31.72	61.28	47.04	41.14	72.23	39.83	34.86	59.73	47.29	41.34	67.39	45.88	40.08	65.89
	δ_6	49.09	43.08	76.85	59.55	53.74	86.77	51.77	44.7	76.37	60.47	52.51	83.83	58.64	50.87	82.37
	δ_7	40.35	33.96	67.66	49.69	43.44	75.17	42.2	36.76	64.41	49.13	42.9	71.84	47.56	41.61	70.4
	δ_8	39.89	34.04	67.46	50	42.68	75.29	42.04	37.09	63.81	48.98	43.06	71.67	47.75	41.49	70.15
	δ_9	39.09	33.62	65.67	49.96	44.35	75.8	42.75	36.57	64.89	49.8	43.36	72.54	48.29	41.45	71.33
	δ_{10}	39.47	32.92	65.39	49.78	43.23	75.73	42.49	37.5	64.07	50.07	43.91	72.26	48.36	42.47	70.49
	δ_2	54.04	47.54	62.82	50.22	41.48	61.36	43.27	36.29	53.1	50.77	42.02	61.89	49.38	41.07	60.05
	δ_3	46.24	43.04	51.13	34.95	30.61	42.73	29.32	25.53	34.99	35.73	30.38	42.24	34.74	29.84	40.72
	δ_4	52.96	48.15	58.09	44.2	37.23	51.73	36.38	31.57	43.4	43.53	36.76	52.28	42.12	36.17	50.5
n_4	δ_5	49.24	44.81	56.62	42.88	35.52	52.74	36.53	30.04	43.56	42.56	34.41	52.19	41.58	33.93	50.52
	δ_6	63.38	57.46	70.17	56.19	48.31	66.49	48.21	40.83	56.88	56.26	47.67	65.95	54.65	46.47	64.2
	δ_7	52.35	45.99	59.63	46.72	38.45	56.76	39.41	33.3	47.96	46.32	38.26	57.03	44.79	37.6	55.18
	δ_8	51.75	46.45	59.74	47.05	37.64	57.33	39.28	32.2	48.79	46.11	37.76	57.25	44.78	36.59	55.38
	δ_9	51.9	47.85	59.35	46.03	38.33	55.7	38.68	32.45	46.03	45.24	38.01	55.02	44.05	37.13	53.31
	δ_{10}	52.75	47.47	59.02	46.26	37.69	55	37.78	32.5	46.49	44.81	37.81	55.48	43.51	37.03	53.82
	δ_2	89.94	74	99.72	91.89	76.64	99.8	90.3	74.28	99.78	91.93	76.68	99.84	91.76	76.52	99.84
	δ_3	47.29	36.6	75.34	50.07	38.46	77.31	46.31	36.63	74.29	50.19	38.53	78.01	49.94	38.71	77.54
	δ_4	64.52	49.5	90.78	67.48	52.31	92.53	63.94	49.25	90.22	67.6	52.39	92.42	67.41	52.45	92.18
	δ_5	79.19	62.09	98.41	82.16	64.71	98.62	79.59	62.1	98.12	82.26	64.82	98.66	82	64.82	98.57
n_5	δ_6	71.16	57.61	95.41	74.08	60.35	96.56	69.45	56.46	94.47	74.14	60.42	96.41	73.8	60.37	96.2
	δ_7	88.11	72.32	99.74	90.3	75.46	99.76	88.39	72.57	99.62	90.35	75.55	99.77	90.13	75.35	99.76
	δ_8	17.34	16.15	28.45	18.34	17.36	31.32	16.52	15.74	26.83	18.41	17.41	31.52	17.98	16.92	30.28
	δ_9	86.25	70.22	99.56	88.65	73.26	99.68	86.5	70.77	99.42	88.69	73.3	99.6	88.51	73.05	99.59
	δ_{10}	29.6	24.64	49.1	31.94	25.74	52.87	28.47	23.65	46.64	32.05	25.81	52.43	31.63	25.76	51.36
	δ_2	86.15	70.65	99.44	92.44	75.86	99.96	90.71	73.51	99.93	92.87	76.53	99.98	92.54	76.35	99.98
	δ_3	45.68	36.33	72.47	44.5	32.59	77.01	41.43	30.07	72.94	45.08	32.91	77.46	44.73	32.83	76.76
	δ_4	61.64	47.86	87.63	64.06	46.51	92.49	58.07	43.41	90.61	62.34	46.79	93.03	61.93	46.53	92.7
	δ_5	75.08	58.89	96.83	82.02	62.4	99.23	78.72	58.73	98.71	82.24	62.13	99.2	81.86	61.99	99.13
	δ_6	69.01	56.14	92.51	69.66	53.82	95.68	63.71	49.34	94.29	68.62	53.26	96.25	67.86	52.8	95.96
n_6	δ_7	84.03	68.54	99.33	90.91	73.95	99.91	89.13	70.33	99.85	91.4	73.38	99.92	91.07	73.11	99.92
	δ_8	17.83	16.77	27.69	13.65	12.64	24.14	12.9	11.65	20.32	14.18	12.56	24.1	13.77	12.28	23.26
	δ_9	82.5	67.13	98.93	89.12	71.78	99.8	86.27	68.33	99.7	88.82	71.71	99.87	88.61	71.55	99.87
	δ_{10}	29.94	24.87	46.93	24.97	19.29	44.07	21.17	17.42	38.11	23.78	19.19	43.95	23.51	18.93	42.74
	δ_2	98.83	91	100	97.39	90.5	99.97	96.93	89.38	99.83	97.35	90.29	99.89	97.41	90.48	99.86
	δ_3	72.14	58.34	95.46	75.48	65.38	93.92	73.24	62.85	90.7	75.53	64.71	91.94	75.58	65.12	91.91
	δ_4	87.46	73.24	99.46	86.86	77	98.67	85.93	74.92	97.74	87.47	77.18	98.09	87.84	77.41	98.1
	δ_5	95.65	83.57	99.98	93.15	83.18	99.59	91.76	82.1	99.1	92.72	83.52	99.34	92.76	83.54	99.27
	δ_6	93.24	82.32	99.91	93.77	84.43	99.71	92.42	82.94	99.22	93.68	84.84	99.4	93.75	84.96	99.43
	δ_7	98.52	90.79	100	97.13	89.71	99.88	96.44	88.01	99.84	96.9	89.33	99.91	96.86	89.33	99.87
n_7	δ_8	31.23	27.82	55.19	45.44	41.68	71.48	41.8	37.78	60.01	46.92	42.17	66	45.46	40.77	64.71
	δ_9	98.05	88.73	99.99	96.72	88.55	99.86	95.89	87.46	99.74	96.61	88.76	99.81	96.5	88.74	99.76
	δ_{10}	51.14	42.18	79.68	62.36	54.47	87.09	57.03	49.19	79.06	61.61	53.37	82.72	60.76	52.68	81.99
	δ_2	97.45	90.43	99.63	99.88	96.81	99.99	99.85	95.9	99.98	99.88	96.62	99.99	99.89	96.41	99.99
	δ_3	77.99	68.78	84.31	82.4	66.89	89.96	79.89	64.27	88.83	82.1	66.96</				

3.5 Real-life Applications

To demonstrate the implementation of the proposed methods, we provide real-life examples that compare the means of the response variable while considering the two auxiliary variables. This illustration is based on biomedical and mechanical engineering data. The ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 methods were implemented for analysis in balanced and unbalanced cases.

3.5.1 An example of balanced case (equal sample size)

In the field of biomedical applications, microchannels play a significant role and are frequently created using the laser ablation technique. However, microchannels produced through laser processing often encounter significant challenges, such as the heat-affected zone (HAZ), dimensional inaccuracies, and subpar surface quality. Abdo, et al. [81] conducted a study on pulsed Nd:YAG laser micro-milling to explore the impact of key parameters of the laser process on the precision of dimensions and the quality of the surface of micro-channels fabricated on zirconia dental material. The machining process took into account parameters such as the speed of laser scanning, frequency of pulses, and intensity of pulses, each with four different levels. The experiment considered the following performance criteria: accuracy of channel dimensions (deviation in-depth, deviation in top width, and taper angle), surface roughness and the extent of the heat-affected zone (HAZ). In this particular study, we examined the Nd:YAG laser micro-channeling data using the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 methods. It is noted that the surface roughness is affected by the laser scanning and intensity of pulses; therefore, the investigation focused on the surface roughness (Y) as the

variable of interest. Additionally, we considered laser scanning (X) and intensity of pulses (Z) as concomitant variables.

All variables were analyzed across different levels of frequency of pulses (kHz). The frequency of pulses was divided into four levels ($t = 4$): 5kHz, 8kHz, 11kHz, and 14kHz, and within each frequency level, 16 observations were recorded, resulting in a total of 16 observations ($n_i = 16$) for each treatment group. The ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 are applied for testing the means of surface roughness (Y) considering laser scanning (X), the intensity of pulses (Z) with a significance level (α) of 5%. Moreover, the estimator's mean ($\bar{T}_{(6)i}$, $\bar{T}_{(7)i}$, $\bar{T}_{(8)i}$, $\bar{T}_{(9)i}$ and $\bar{T}_{(10)i}$) is calculated for each frequency of pulse level. Furthermore, Figure 3.2 illustrates the graphical depiction of the ANOMC tests performed with a balanced design. In the ANOMC-MR6 method, the overall mean ($\bar{T}_{(6)..}$) was calculated to be 4.156406. The individual modified ratio means ($\bar{T}_{(6)i}$) were then plotted against the decision interval, with the critical value $V(\alpha, t, n)$ set at 13.93. The lower and upper decision limits (LDL, UDL) were determined to be 4.026005 and 4.286807, respectively. For the ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 methods, the overall means ($\bar{T}_{(7)..}$, $\bar{T}_{(8)..}$, $\bar{T}_{(9)..}$ and $\bar{T}_{(10)..}$) were found to be 4.15642. In the ANOMC-MR7 test, the critical value $Z(\alpha, t, n)$ was used, resulting in lower and upper decision limits (LDL, UDL) of 4.032839 and 4.279973, respectively. Similarly, the ANOMC-MR8 test utilized the critical value $K(\alpha, t, n)$, leading to LDL and UDL of 4.032371 and 4.280441. In the ANOMC-MR9 test, the critical value $P(\alpha, t, n)$ was employed, resulting in LDL and UDL of 4.032839 and 4.279973. Finally, for the ANOMC-MR10 method, the critical value $O(\alpha, t, n)$ was used, and the lower and upper decision

limits (LDL, UDL) were determined as 4.03209 and 4.280722, respectively. The decision lines are plotted against the individual modified ratio means ($\bar{T}_{(r)i.}$) and the overall mean square error ($MSE_{\bar{T}_{(r)}}$) is calculated as 0.043237 where $r = 6,7,8,9,10$. It is worth noting that ANOMC tests did not demonstrate significant performance in assessing the response of accuracy of channel dimensions (deviation in-depth, deviation in top width, and taper angle) and the extent of the heat-affected zone (HAZ).

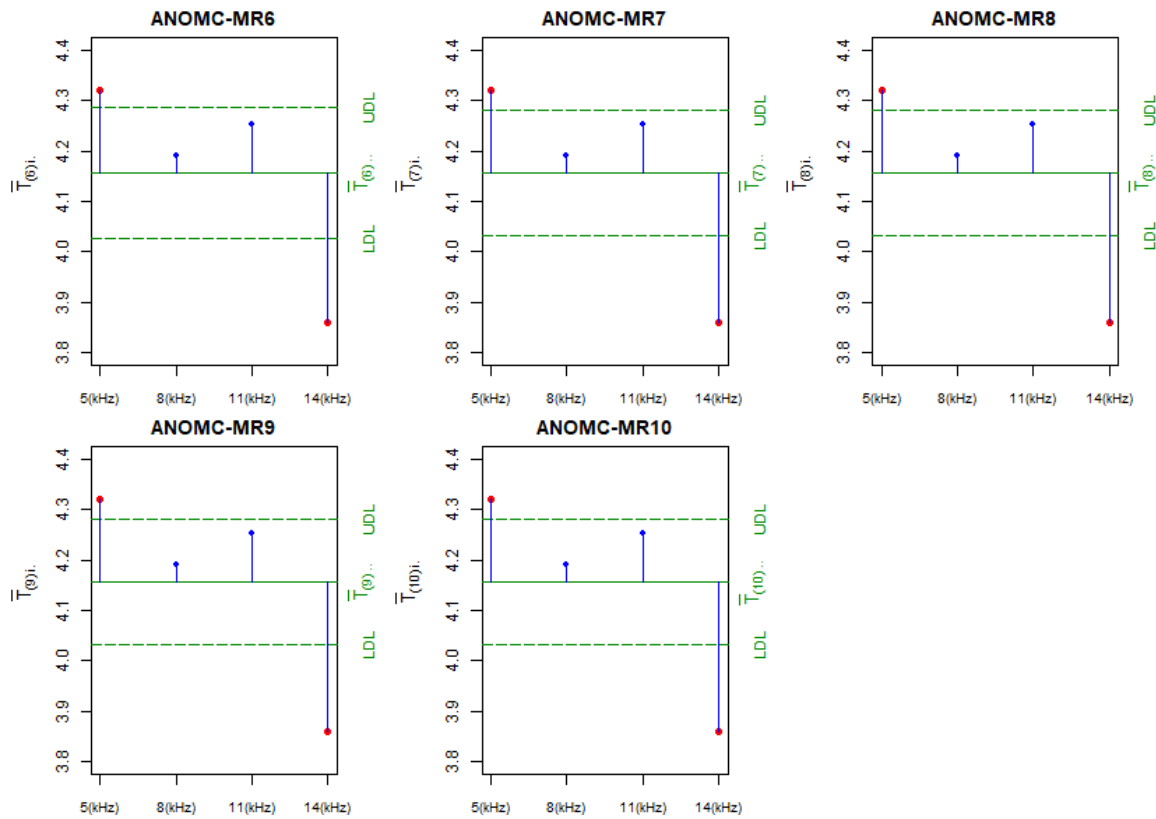


Figure 3. 2: The implementation of ANOMC methods for pulsed Nd: YAG laser problem.

3.5.2 An example of unbalanced case (unequal sample size)

The Carbon-Fiber Reinforced Epoxy Composite (CEC) has gained wide recognition as a structural material in various spheres. The drilled holes are essential in the assembly of

composite material components. In order to accomplish improved surface quality and integrated drilled structures to avoid serious problems, such as damage and thermal expansion, an experimental investigation by Rao, et al. [82] was carried out employing three samples of carbon-fiber-reinforced epoxy composite (CEC). The samples studied were classed as follows: clean CEC, CEC loaded with the disulfide molybdenum (MoS_2), and CEC filled with hexagonal-boron nitride (h-BN). In addition, the investigation examined the impact of particular input parameters, which are the supply, spindle speed, filler concentration, and drill diameter, on the drilling responses of the CEC samples on each of the drilling responses, including delamination factor, surface roughness, thrust force, and temperature.

In this work, we investigated the drilling response of Carbon Fabric data using the ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 approaches. It is recognized that the thrust force is governed by drill diameter and feed; consequently, the main focus of the investigation focused on the thrust force (Y) as the variable of interest. While the main critical elements, such as drill diameter (X) and feed (Z) were deemed concurrent variables under the specimen of MoS_2 reinforced CEC. The investigation was done across multiple levels of spindle speed (rpm), which were divided into three categories: 1000rpm , 3250rpm , and 5500rpm . Each speed level had a different number of observations: 5 observations under 1000rpm , 10 observations under 3250rpm , and 5 observations under 5500rpm .

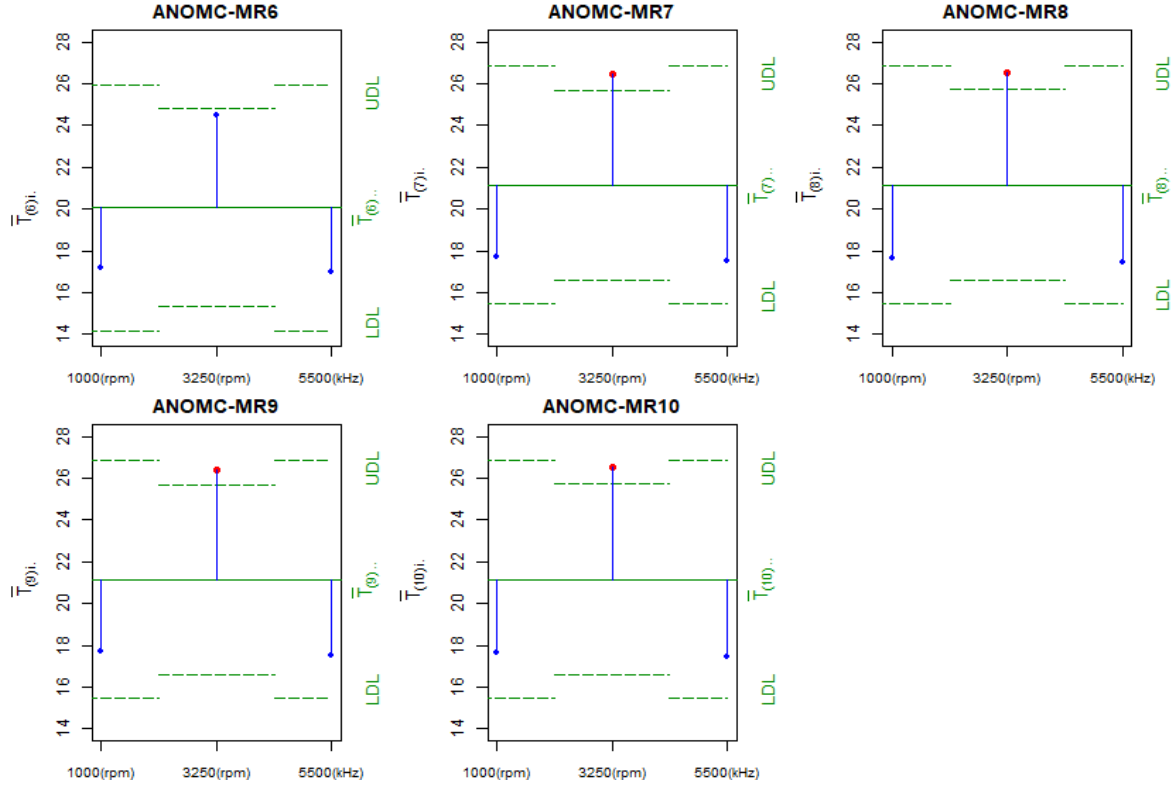


Figure 3. 3: The implementation of ANOMC methods for the Drilling Response of Carbon Fabric problem.

The ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 are applied for testing the means of thrust force (Y) considering drill diameter (X), feed (Z) with a significance level (α) of 5%. Moreover, the estimator's mean ($\bar{T}_{(6)i}$, $\bar{T}_{(7)i}$, $\bar{T}_{(8)i}$, $\bar{T}_{(9)i}$ and $\bar{T}_{(10)i}$) is calculated for each frequency of speed level. Furthermore, Figure 3.3 illustrates the graphical depiction of the ANOMC tests performed with an unbalanced design.

In the ANOMC-MR6 method, the overall mean ($\bar{T}_{(6)..\}$) was determined to be 20.05246. The lower decision limits (LDL) were computed as (14.14062; 15.31244; 14.14062), while the upper decision limits (UDL) were calculated as (25.96430; 24.79247; 25.96430) with

a critical value $V^*(\alpha, t)$ set at 12.25. For the ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 methods, the overall means ($\bar{T}_{(7)..\}, \bar{T}_{(8)..\}, \bar{T}_{(9)..\}$ and $\bar{T}_{(10)..\}$) were obtained as (21.14291; 21.15757; 21.14578; 21.15803), respectively. The ANOMC-MR7 test used a critical value $Z^*(\alpha, t)$ of 11.8, with lower decision limits (LDL) of (15.44824; 16.57702; 15.44824) and upper decision limits (UDL) of (26.83758; 25.70881; 26.83758). In the same way, the critical value $K^*(\alpha, t)$ for the ANOMC-MR8 test is 11.82, and the lower decision limits (LDL) are estimated as (15.45324 16.58393 15.45324) and upper decision limits (UDL) as (26.86189; 25.73120; 26.86189). Furthermore, the critical value $P^*(\alpha, t)$ for the ANOMC-MR9 approach is 11.8, and the lower decision limits (LDL) computed as (15.45111 16.57989 15.45111) and upper decision limits (UDL) as (26.84045 25.71167 26.84045). Finally, the critical value $O^*(\alpha, t)$ for the ANOMC-MR10 approach is 11.78, and the lower decision limits (LDL) are estimated as (15.47301; 16.59988; 15.47301) and upper decision limits (UDL) as (26.84305; 25.71619; 26.84305). The decision lines are plotted against the individual modified ratio means ($\bar{T}_{(r)i.}$) and the overall mean square error ($MSE_{\bar{T}_{(r)}}$) is calculated as 1.730449 given $r = 6, 7, 8, 9, 10$.

3.6 Summary and Concluding Remarks

The ANOM test compares the means of the main variable with respect to different treatments; however, the ANOMC test represents an improved version of the ANOM test, incorporating the influence of covariates. This study introduces and evaluates the effectiveness of the ANOMC test with multiple auxiliary variables using ratio estimators. The proposed ANOMC is presented as ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10. The ANOMC tests (namely, ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10) work under a set

of assumptions, such as normality, linearity, and homogeneity. The impact of these assumptions, as well as considerations like sample sizes (equal or unequal), different treatments, the formulation of null and non-null hypotheses, and the difference scenarios in covariance ratio between the variables in the study, is meticulously assessed concerning the ANOMC tests. This evaluation focuses on comparing the tests' performance in terms of the test's percentage type I error and percentage power. At the conclusion of this simulation, the study revealed that when considering the homogeneity of variances in the null case, the ANOMC-MR6 test demonstrated less robustness compared to the ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 tests across all distributions, particularly in scenarios where the covariance changes in all variables (Σ_4). Additionally, when the covariance changes between the response variable and the auxiliary variable (Σ_3), all ANOMC tests exceeded the pre-specified nominal level of Type I error rate ($\alpha = 5\%$). Indeed, it is worth noting that the type of design and sample size used did not have any impact on the findings mentioned above. In the presence of heterogeneity of variances with the null case, regardless of the distribution shape and sample size combinations, the ANOMC mentioned above tests are affected. Still, the ANOMC-MR6 and ANOMC-MR8 tests exhibit less susceptibility to the effects compared to the ANOMC-MR7, ANOMC-MR9, and ANOMC-MR10 tests, in both balanced and unbalanced designs under all the distributions. Under the MVLN distribution, the rates show higher type I error rates, while under the MVN distribution, the impact is less pronounced. Heterogeneity of variances has a more prominent effect with an increase in the variance's ratio (v_3^+ , v_3^-). Furthermore, all tests significantly exceed the specified level of Type I error rates when sample sizes and variances are inversely paired. As anticipated, the power values of the ANOMC tests

(ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10) vary based on the effect size, sample size, and distribution environment and the important one, the covariance ratio between the variables. The ANOMC-MR7 test consistently and relatively exhibits higher power rates compared to the other ANOMC tests. Moreover, in scenarios where the data follows a MVLN distribution, all ANOMC tests demonstrate notably higher power rates. However, it is important to note that the specific experimental conditions significantly determine the outcomes. Furthermore, the effectiveness of the ANOMC tests improves as the correlation between the study variable and the concomitant variables increases. This implies that when there is a strong relationship between the response variable and the auxiliary variables, the power of the tests increases for all the ANOMC tests. This study is designed to investigate a specific set of parameters, including a limited number of treatments, choices of sample size, covariance between variables, and the limited number of distributions. It is important to note that these limitations constrain the study's scope, but there is potential for future extensions to broaden the range of distributions considered. Furthermore, the current study utilizes the ANOMC method, with a primary focus on examining fixed factor effects. However, an interesting avenue for future research would be to extend the application of the ANOMC approach to more complex experimental designs to enhance the robustness of the method. Finally, we have implemented the ANOMC tests to biomedical and mechanical engineering datasets. The analysis shows that under balanced design, all of the ANOMC tests displayed a significant level of performance. However, when examining the unbalanced design, we observed that ANOMC-MR6 did not demonstrate a statistically significant level of performance.

CHAPTER 4

ANALYSIS OF MEANS UNDER FACTORIAL DESIGN

In order to conduct experiments in fields such as mechanical engineering and industrial engineering, it is important to take into account more than one factor that impacts the experiment. In previous chapters, we focused on ANOMC tests involving only one fixed factor and covariate variables. Now, we are expanding the scope of the test to more complex treatment structures, specifically factorial designs. This chapter aims to examine the effectiveness of factorial ANOMC tests compared to factorial ANOM tests and to improve the performance of the factorial ANOM test by incorporating a covariate variable. The initial implementation of ANOMC is referred to as $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ utilizes regression and ratio estimators to compare it with the traditional factorial ANOM, known as $ANOM_F$.

4.1 Introduction

When we are interested in studying more than one factor, the ANOM procedure is a suitable method for analyzing data from completely randomized designs with factorial treatment structures. ANOM has well-developed techniques for analyzing the fixed effects within the factorial structure. Suppose we are interested in studying the effects of time and temperature on the yield of a chemical process. If we believe that the effect of time is the same across different temperatures, a common but inefficient approach is to study the effects of time and temperature separately. This approach is known as a one-factor-at-a-time design. In this design, you would conduct three separate experiments. First, the temperature is fixed, and the effects of time are studied with three observations. Then, fixing time, one might study two temperatures using three observations for each temperature. With these 15 total observations, we can estimate the average yield at a particular temperature and at a particular time. The uncertainty in the average yield for a fixed temperature would be less than that for a fixed time because more observations were taken (nine versus six). A much more efficient way to study the two factors is to study them together. Taking two observations for each time-temperature combination would require only 12 total observations but would provide estimates for both the average yield at a particular temperature and the average yield at a particular time [83]. Thus, one can obtain better information with fewer observations by studying the two factors together. The simplest experimental designs for studying two or more factors are complete layouts, known as Complete Multifactor design.

In previous chapters, we explored the ANOMC test, which focused on analyzing data with a single fixed factor. Now, we are expanding the scope of the test to more complex

treatment structures, specifically factorial designs. This study aims to investigate the effectiveness of factorial ANOMC tests in comparison to factorial ANOM tests and to enhance the performance of the factorial ANOM test by incorporating a covariate variable.

4.2 Factorial Treatment Structure of ANOM

The ANOM procedure can be used to analyze data from completely randomized designs with factorial treatment structures. The analysis of the fixed effects in the factorial structure is well-developed in ANOM and can be found in Jayalath and Ng [84]. Consider a complete experimental design with two fixed factors, denoted as A and B, within a factorial treatment structure. The model consists of I levels of factor A and J levels of factor B:

$$Y_{ijl} = \mu + A_i + B_j + (AB)_{ij} + \epsilon_{ijl}, \quad i = 1, 2, \dots, t; j = 1, 2, \dots, k; l = 1, 2, \dots, n, \quad (4.1)$$

where Y_{ij} is the observed response when factor A is at the i^{th} level and factor B at the j^{th} level for the l^{th} replicate, μ is the overall mean effect, A_i is the effect of the i^{th} level of the column factor A, B_j is the effect of the j^{th} level of the row factor B, $(AB)_{ij}$ is the effect of the interaction between A_i and B_j and we assumed that the random errors would be normally distributed with mean 5, constant variance and $N = ntk$. Furthermore, the treatment effects are defined as deviations from the overall mean, so $\sum_{i=1}^a A_i = 0$ and $\sum_{j=1}^b B_j = 0$. Similarly, the interaction effects are fixed and are defined such that $\sum_{i=1}^a (AB)_{ij} = \sum_{j=1}^b (AB)_{ij} = 0$. The layout of the ANOM data set is presented in Table 4.1, where the structure of the ANOM test under the same assumptions as the model in Eq (4.1) is used to test the following hypotheses:

Null Hypothesis:

$$H_{0(A)}: A_i = 0 \text{ for all } i; \text{ factor A has no effect.}$$

$H_{0(B)}: B_j = 0 \text{ for all } j; \text{ factor B has no effect.}$

$H_{0(AB)}: AB_{ij} = 0 \text{ for all } (i, j); \text{ there is no AB interaction.}$

Testing the hypothesis $H_{0(AB)}$ is the primary step as subsequent analyses depend on whether an interaction is present.

Alternative Hypothesis:

$H_{1(A)}: \text{Factor A has an effect.}$

$H_{1(B)}: \text{Factor B has an effect.}$

$H_{1(AB)}: \text{There is an interaction effect between factor A and factor B.}$

Table 4. 1: General layout for a Two-Factor Factorial CRD design for ANOM test.

		Factor A					$\bar{Y}_{.j} = \sum_{i=1}^t \bar{Y}_{ij} / t$
		a_1	...	a_i	...	a_t	
		Y_{111}		Y_{i11}		Y_{t11}	
Factor B	$k = 1, 2, 3, \dots, n$	$\vdots$		$\vdots$		$\vdots$	
	$i = 1, 2, 3, \dots, t$	Y_{11k}	...	Y_{ik}	...	Y_{tk}	
	$j = 1, 2, 3, \dots, r$	$\vdots$		$\vdots$		$\vdots$	
	b_1	Y_{11n}		Y_{in}		Y_{tn}	
	$\bar{Y}_{i1.} = \sum_{k=1}^n Y_{ik} / n$	$\bar{Y}_{11.}$	...	$\bar{Y}_{i1.}$	...	$\bar{Y}_{t1.}$	$\bar{Y}_{.1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	Y_{1j1}	Y_{ij1}		Y_{tj1}			
	$\vdots$	$\vdots$		$\vdots$			
	b_j	Y_{1jk}	...	Y_{ijk}	...	Y_{tjk}	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	Y_{1jn}	Y_{ijn}		Y_{tjn}			
	$\bar{Y}_{ij.} = \sum_{k=1}^n Y_{ijk} / n$	$\bar{Y}_{1j.}$	...	$\bar{Y}_{ij.}$	...	$\bar{Y}_{tj.}$	$\bar{Y}_{.j}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	Y_{1r1}	Y_{ir1}		Y_{tr1}			
	$\vdots$	$\vdots$		$\vdots$			
	b_r	Y_{1rk}	...	Y_{irk}	...	Y_{trk}	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	Y_{1rn}	Y_{irn}		Y_{trn}			
	$\bar{Y}_{ir.} = \sum_{k=1}^n Y_{irk} / n$	$\bar{Y}_{1r.}$	...	$\bar{Y}_{ir.}$	...	$\bar{Y}_{tr.}$	$\bar{Y}_{.r}$
	$\bar{Y}_{i.} = \sum_{j=1}^r \bar{Y}_{ij} / r$	$\bar{Y}_{1.}$		$\bar{Y}_{i.}$		$\bar{Y}_{t.}$	$\bar{Y}_{..}$

Assume one of the factors contains only two levels. To assess the interaction effect, we plot the slopes alongside the decision lines for levels of factor A (i.e. $a = 2$) and factor B

at any level (i.e. $b \geq 2$). The slopes are $D_{j(12)} = \bar{Y}_{1j.} - \bar{Y}_{2j.}$, and plot along with the following decision limits where $\bar{D}_{.12} = \sum_{j=1}^r D_{j(12)}$.

$$\begin{aligned} LDL &= \bar{D}_{.12} - S(\alpha, r, 2r(n-1))(\sqrt{MSE_{\bar{Y}_{...}}})\sqrt{\frac{t(r-1)}{rn}} \\ UDL &= \bar{D}_{.12} + S(\alpha, r, 2r(n-1))(\sqrt{MSE_{\bar{Y}_{...}}})\sqrt{\frac{t(r-1)}{rn}} \end{aligned} \quad (4.2)$$

If the observed interaction between the factors is not statistically significant, it is appropriate to proceed with testing the main effects. In this case, we can create plots that show the mean estimator values for i^{th} levels of the factor A and another plot for j^{th} levels of the factor B along with the decision limits in Eq (4.3). Where $\bar{\bar{Y}}_{...} = \sum_{i=1}^t \sum_{j=1}^r \sum_{k=1}^n Y_{ijk.} / tr$ is the grand mean, $MSE_{\bar{Y}_{...}} = \sum_{i=1}^t \sum_{j=1}^r \sum_{k=1}^n \frac{(Y_{ijk.} - \bar{Y}_{ij.})^2}{tr(n-1)}$. The symbols $S(\alpha, f, tr(n-1))$ denote the critical values used in the general form for assessing the interaction between Factor A and Factor B, the effect of Factor A, and the effect of Factor B in the ANOM test. Here, f represents the number of levels for the factor of interest, such as Factor A or Factor B. The critical values can be found in Table 4.2.

In case is the interaction is significant, one can continue to test the $I \times J$ treatment combinations where we can plot the mean estimator for ij^{th} levels of the factor A and factor B along with the following decision limits where in this case $f = I \times J$:

$$\begin{aligned} LDL &= \bar{\bar{Y}}_{...} - S(\alpha, f, tr(n-1))(\sqrt{MSE_{\bar{Y}_{...}}})\sqrt{\frac{f-1}{N}} \\ UDL &= \bar{\bar{Y}}_{...} + S(\alpha, f, tr(n-1))(\sqrt{MSE_{\bar{Y}_{...}}})\sqrt{\frac{f-1}{N}} \end{aligned} \quad (4.3)$$

4.3 Factorial Treatment Structure of ANOM With Covariate (ANOMC)

Suppose we utilize the ANOM procedure to examine completely randomized designs (CRD) with factorial treatment structures, while incorporating the covariate variable X . In

this scenario, we focus on a complete experimental design with two fixed factors A and B along with an auxiliary variable X . The model consists of I levels of factor A, J levels of factor B and the covariate X and considered within the framework of a factorial treatment structure:

$$Y_{ijl} = \mu + A_i + B_j + (AB)_{ij} + \beta(X_{ij.} + \bar{X}_{...}) + \epsilon_{ijl}, i = 1, 2, \dots, t; j = 1, 2, \dots, k; l = 1, 2, \dots, n, \quad (4.4)$$

where Y_{ijl} is the observed response when factor A is at the i^{th} level and factor B at the j^{th} level for the l^{th} replicate, μ is the overall mean effect, A_i is the effect of the i^{th} level of the column factor A, B_j is the effect of the j^{th} level of the row factor B, $(AB)_{ij}$ is the effect of the interaction between A_i and B_j , β is the slope of i^{th} level of factor A and j^{th} level of factor B, $X_{ij.}$ is the covariate observation for the i^{th} and j^{th} treatment levels, $\bar{X}_{...}$ is the overall mean over all $X_{ij.}$ and we assumed that the random errors is normally distributed with mean 5 and constant variance $\sigma^2 = 1$ and $N = ntk$. Furthermore, the treatment effects are defined as deviations from the overall mean, so $\sum_{i=1}^a A_i = 0$ and $\sum_{j=1}^b B_j = 0$. Similarly, the interaction effects are fixed and are defined such that $\sum_{i=1}^a (AB)_{ij} = \sum_{j=1}^b (AB)_{ij} = 0$. In model (4.4), several assumptions are made. Firstly, we assume that the slope $\beta \neq 0$, indicating a linear relationship between Y_{ijl} and X_{ijl} . Additionally, the regression coefficients for each treatment level are assumed to be identical. Furthermore, it is assumed that the concomitant variable X_{ijl} is not affected by treatment levels. The structure of the test dataset along with ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests are reported in Table 4.2. Further, where the structure of the ANOMC test under the same assumptions as the model in Eq (4.4) is used to test the same hypotheses in ANOM test.

4.3.1 The ANOMC test based on the regression estimator

The factorial analysis of means with covariate (ANOMC) based on the regression estimator is referred to by ANOMC – Reg_F test. In the ANOMC – Reg_F test, the continuous covariate is considered as explanatory variable. For each (i^{th}, j^{th}) combination of Factor A and Factor B levels, the test utilizes the sample correlation $r_{ij.}$ and slope $b_{ij.}$ values. The regression mean estimator $\bar{M}_{ij.}$ is then calculated, and adjusted means are derived based on this estimator. The unbiasing constant $k_{ij.}$ and the sample standard deviation of the regression mean estimator $S_{\bar{M}_{ij.}}$ are important components of the test.

Under the complete design (equal sample size), the overall mean $\bar{M}_{...} = \sum_{i=1}^t \sum_{j=1}^r M_{ij.} / tr$ and overall mean square error $MSE_{\bar{M}_{...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{M}_{ij.}}^2 / tr$. The symbols $I(\alpha, f, 2r(n - 1))$ denote the critical values used in the general form for assessing the interaction between Factor A and Factor B, the effect of Factor A, and the effect of Factor B in the ANOMC – Reg_F test. Here, f represents the number of levels for the factor of interest, such as Factor A or Factor B. The critical values can be found in Table 4.2.

The lower decision lines (LDL) and upper decision lines (UDL) can be determined using the expressions provided in Table 4.3. In cases where one factor has only two levels ($a = 2$) and the other factor has at least two levels ($b \geq 2$), the ANOMC – Reg_F procedure test for interaction by comparing the slopes of these lines. The b slopes are $Z_{j(12)} = \bar{M}_{1j.} - \bar{M}_{2j.}$ and plot along with the decision limits where $\bar{Z}_{.12} = \sum_{j=1}^r Z_{j(12)}$. Then, proceed to check the significance of the interaction. This can be done using a similar process as described in section 4.2, utilizing the expression provided in Table 4.4.

Table 4. 2: The framework of the ANOMC test based on regression, ratio and modified ratio estimators and data layout.

	$k = 1, 2, 3, \dots, n$ $i = 1, 2, 3, \dots, t$ $j = 1, 2, 3, \dots, r$	Factor A				$\bar{Y}_{.j} = \sum_{i=1}^t \bar{Y}_{ij} / t$	$\bar{X}_{.j} = \sum_{i=1}^t \bar{X}_{ij} / t$
		a_1	...	a_i	...	a_t	
Factor B	b_1	$Y_{111}(X_{111})$ $\vdots$ $Y_{11k}(X_{11k})$ $\vdots$ $Y_{11n}(X_{11n})$	...	$Y_{i11}(X_{i11})$ $\vdots$ $Y_{i1k}(X_{i1k})$ $\vdots$ $Y_{i1n}(X_{i1n})$	...	$Y_{t11}(X_{t11})$ $\vdots$ $Y_{t1k}(X_{t1k})$ $\vdots$ $Y_{t1n}(X_{t1n})$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	b_j	$Y_{1j1}(X_{1j1})$ $\vdots$ $Y_{1jk}(X_{1jk})$ $\vdots$ $Y_{1jn}(X_{1jn})$	...	$Y_{ij1}(X_{ij1})$ $\vdots$ $Y_{ijk}(X_{ijk})$ $\vdots$ $Y_{ijn}(X_{ijn})$	...	$Y_{tj1}(X_{tj1})$ $\vdots$ $Y_{tjk}(X_{tjk})$ $\vdots$ $Y_{tjn}(X_{tjn})$	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	b_r	$Y_{1r1}(X_{1r1})$ $\vdots$ $Y_{1rk}(X_{1rk})$ $\vdots$ $Y_{1rn}(X_{1rn})$	...	$Y_{ir1}(X_{ir1})$ $\vdots$ $Y_{irk}(X_{irk})$ $\vdots$ $Y_{irn}(X_{irn})$	...	$Y_{tr1}(X_{tr1})$ $\vdots$ $Y_{trk}(X_{trk})$ $\vdots$ $Y_{trn}(X_{trn})$	
	$\bar{Y}_{.j} = \sum_{k=1}^n Y_{ijk} / n$ $\bar{X}_{.j} = \sum_{k=1}^n X_{ijk} / n$	$Y_{1j.}$ $\bar{X}_{1j.}$	...	$Y_{ij.}$ $\bar{X}_{ij.}$	...	$Y_{tj.}$ $\bar{X}_{tj.}$	
	$\bar{Y}_{.j} = \sum_{j=1}^r \bar{Y}_{ij} / r$ $\bar{X}_{.j} = \sum_{j=1}^r \bar{X}_{ij} / r$	$\bar{Y}_{1.}$ $\bar{X}_{1.}$	...	$\bar{Y}_{i.}$ $\bar{X}_{i.}$	...	$\bar{Y}_{t.}$ $\bar{X}_{t.}$	$\bar{Y}_{...}$ $\bar{X}_{...}$
	$S_{Y_{ij.}}^2 = \frac{\sum_{k=1}^n (Y_{ijk} - \bar{Y}_{ij.})^2}{n-1}$ $S_{X_{ij.}}^2 = \frac{\sum_{k=1}^n (X_{ijk} - \bar{X}_{ij.})^2}{n-1}$	$S_{Y_{1j.}}^2$ $S_{X_{1j.}}^2$	...	$S_{Y_{ij.}}^2$ $S_{X_{ij.}}^2$	...	$S_{Y_{tj.}}^2$ $S_{X_{tj.}}^2$	
	$S_{X_{ij}Y_{ij.}} = \sum_{k=1}^n (X_{ijk} - \bar{X}_{ij.})(Y_{ijk} - \bar{Y}_{ij.}) / (n-1)$	$S_{X_{1j}Y_{1j.}}$	...	$S_{X_{ij}Y_{ij.}}$	...	$S_{X_{tj}Y_{tj.}}$	
	$r_{ij.} = S_{X_{ij}Y_{ij.}} / \sqrt{S_{Y_{ij.}}^2 S_{X_{ij.}}^2}$	$r_{1j.}$	...	$r_{ij.}$	...	$r_{tj.}$	
ANOMC-Reg	$b_{ij.} = r_{ij.}(S_{Y_{ij.}}/S_{X_{ij.}})$ $k_{ij.} = \sqrt{(1 - r_{ij.}^2)(1 + (1/n - 3))}$ $\bar{M}_{ij.} = \bar{Y}_{ij.} + b_{ij.}(\bar{X}_{...} - \bar{X}_{ij.})$ $S_{\bar{M}_{ij.}} = k_{ij.} S_{Y_{ij.}} / n$ $\theta_{ij} = (1 - g) / n$	$b_{1j.}$	...	$b_{ij.}$	...	$b_{tj.}$	
	$C_{Y_{ij.}}^2 = S_{Y_{ij.}}^2 / \bar{Y}_{...}^2$ $C_{X_{ij.}}^2 = S_{X_{ij.}}^2 / \bar{X}_{...}^2$	$C_{Y_{1j.}}^2$ $C_{X_{1j.}}^2$	...	$C_{Y_{ij.}}^2$ $C_{X_{ij.}}^2$	...	$C_{Y_{tj.}}^2$ $C_{X_{tj.}}^2$	
ANOMC-R	$\bar{T}_{(1)ij.} = \bar{Y}_{ij.}(\bar{X}_{...} / \bar{X}_{ij.})$ $S_{\bar{T}_{(1)ij.}} = \theta_{ij} \bar{Y}_{ij.}^2 [C_{Y_{ij.}}^2 + C_{X_{ij.}}^2 - 2r_{ij.} C_{Y_{ij.}} C_{X_{ij.}}]$	$\bar{T}_{(1)1j.}$	...	$\bar{T}_{(1)ij.}$	...	$\bar{T}_{(1)tj.}$	
ANOMC-MR1	$\bar{T}_{(2)ij.} = \bar{Y}_{ij.}(\bar{X}_{...} + C_{X_{ij.}}) / (\bar{X}_{ij.} + C_{X_{ij.}})$ $\alpha_{ij} = \frac{\bar{X}_{...}}{\bar{X}_{ij.} + C_{X_{ij.}}}$ $S_{\bar{T}_{(2)ij.}} = \theta_{ij} \bar{Y}_{ij.}^2 [C_{Y_{ij.}}^2 + C_{X_{ij.}}^2 \alpha_{ij}^2 - 2r_{ij.} \alpha_{ij} C_{Y_{ij.}} C_{X_{ij.}}]$	$\bar{T}_{(2)1j.}$	...	$\bar{T}_{(2)ij.}$	...	$\bar{T}_{(2)tj.}$	
	$B_2(x_{ij.}) = \frac{n(n+1) \sum_{i=1}^t (x_{ij.} - \bar{X}_{...})^4}{(n-1)(n-2)(n-3) S_{X_{ij.}}^2} - \frac{3(n-1)^2}{(n-2)(n-3)}$ (See Subramani and Gnanasegaran [63])	$B_2(x_{1j.})$	...	$B_2(x_{ij.})$	...	$B_2(x_{tj.})$	
ANOMC-MR2	$\bar{T}_{(3)ij.} = \bar{Y}_{ij.}(\bar{X}_{...} + B_2(x_{ij.})) / (\bar{X}_{ij.} + B_2(x_{ij.}))$ $\delta_{ij} = \frac{\bar{X}_{...}}{\bar{X}_{ij.} + B_2(x_{ij.})}$ $S_{\bar{T}_{(3)ij.}} = \theta_{ij} \bar{Y}_{ij.}^2 [C_{Y_{ij.}}^2 + C_{X_{ij.}}^2 \delta_{ij}^2 - 2r_{ij.} \delta_{ij} C_{Y_{ij.}} C_{X_{ij.}}]$	$\bar{T}_{(3)1j.}$	...	$\bar{T}_{(3)ij.}$	...	$\bar{T}_{(3)tj.}$	
ANOMC-MR3	$\bar{T}_{(4)ij.} = \bar{Y}_{ij.}(\bar{X}_{...} B_2(x_{ij.}) + C_{X_{ij.}}) / (\bar{X}_{ij.} B_2(x_{ij.}) + C_{X_{ij.}})$ $\omega_{ij} = \frac{\bar{X}_{...} B_2(x_{ij.})}{\bar{X}_{ij.} B_2(x_{ij.}) + C_{X_{ij.}}}$ $S_{\bar{T}_{(4)ij.}} = \theta_{ij} \bar{Y}_{ij.}^2 [C_{Y_{ij.}}^2 + C_{X_{ij.}}^2 \omega_{ij}^2 - 2r_{ij.} \omega_{ij} C_{Y_{ij.}} C_{X_{ij.}}]$	$\bar{T}_{(4)1j.}$	...	$\bar{T}_{(4)ij.}$	...	$\bar{T}_{(4)tj.}$	
ANOMC-MR4	$\bar{T}_{(2)ij.} = \bar{Y}_{ij.}(\bar{X}_{...} C_{X_{ij.}} + B_2(x_{ij.})) / (\bar{X}_{ij.} C_{X_{ij.}} + B_2(x_{ij.}))$ $\varphi_{ij} = \frac{\bar{X}_{...} C_{X_{ij.}}}{\bar{X}_{ij.} C_{X_{ij.}} + B_2(x_{ij.})}$ $S_{\bar{T}_{(2)ij.}} = \theta_{ij} \bar{Y}_{ij.}^2 [C_{Y_{ij.}}^2 + C_{X_{ij.}}^2 \varphi_{ij}^2 - 2r_{ij.} \varphi_{ij} C_{Y_{ij.}} C_{X_{ij.}}]$	$\bar{T}_{(5)1j.}$	...	$\bar{T}_{(5)ij.}$	...	$\bar{T}_{(5)tj.}$	

4.3.2 The ANOMC test based on the ratio estimators

The ratio estimators can be used as an alternative to the regression estimator by treating it as a ratio variable when testing the mean with a covariate. The factorial analysis of means based on the ratio estimators is referred to ANOMC – R_F , ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F test. For each (i^{th}, j^{th}) combination of Factor A and Factor B levels, the classical ratio estimator [4], $\bar{T}_{(1)ij}$ is the ratio mean estimator, $g = n / N$ is sampling fraction and the sample correlation is r_{ij} , further, C_{Yij} and C_{Xij} are the sample coefficients of variation of Y and X , $S_{\bar{T}_{(1)ij}}$ is the sample standard deviation of the classical ratio mean estimator.

On the other hand, for each (i^{th}, j^{th}) combination of Factor A and Factor B levels, the modified ratio (MR) estimators, $\bar{T}_{(2)ij}$, $\bar{T}_{(3)ij}$, $\bar{T}_{(4)ij}$ And $\bar{T}_{(5)ij}$ are the MR means estimators, $B_2(x_{ij})$ Is the coefficient of kurtosis, $S_{\bar{T}_{(2)ij}}$, $S_{\bar{T}_{(3)ij}}$, $S_{\bar{T}_{(4)ij}}$ and $S_{\bar{T}_{(5)ij}}$ are the sample standard deviation of the MR means estimators. However, the overall mean for R and MR estimators ($\bar{T}_{(1)...}$, $\bar{T}_{(2)...}$, $\bar{T}_{(3)...}$, $\bar{T}_{(4)...}$ and $\bar{T}_{(5)...}$) and the overall mean square error ($MSE_{\bar{T}_{(1)...}}$, $MSE_{\bar{T}_{(2)...}}$, $MSE_{\bar{T}_{(3)...}}$, $MSE_{\bar{T}_{(4)...}}$ and $MSE_{\bar{T}_{(5)...}}$) can be calculated as $\bar{T}_{(1)...} = \sum_{i=1}^t \sum_{j=1}^r T_{(1)ij} / tr$, $\bar{T}_{(2)...} = \sum_{i=1}^t \sum_{j=1}^r T_{(2)ij} / tr$, $\bar{T}_{(3)...} = \sum_{i=1}^t \sum_{j=1}^r T_{(3)ij} / tr$, $\bar{T}_{(4)...} = \sum_{i=1}^t \sum_{j=1}^r T_{(4)ij} / tr$ and $\bar{T}_{(5)...} = \sum_{i=1}^t \sum_{j=1}^r T_{(5)ij} / tr$. Furthermore, the overall mean square error can be determined by $MSE_{\bar{T}_{(1)...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{T}_{(1)ij}}^2 / tr$, $MSE_{\bar{T}_{(2)...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{T}_{(2)ij}}^2 / tr$, $MSE_{\bar{T}_{(3)...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{T}_{(3)ij}}^2 / tr$, $MSE_{\bar{T}_{(4)...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{T}_{(4)ij}}^2 / tr$ and $MSE_{\bar{T}_{(5)...}} = \sum_{i=1}^t \sum_{j=1}^r S_{\bar{T}_{(5)ij}}^2 / tr$. Additionally, the lower decision lines (LDL) and upper decision lines (UDL) can be obtained using the expressions in Table 5.3.

The symbols $Z(\alpha, f, 2r(n - 1))$, $M(\alpha, f, 2r(n - 1))$, $K(\alpha, f, 2r(n - 1))$, $E(\alpha, f, 2r(n - 1))$ and $P(\alpha, f, 2r(n - 1))$ denote the critical values used in the general form for assessing the interaction between Factor A and Factor B, the effect of Factor A, and the effect of Factor B in the ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests, respectively. Here, f represents the number of levels for the factor of interest, such as Factor A or Factor B. These critical values can be referenced in Table 4.2.

In cases where one factor has only two levels ($a = 2$) and the other factor has at least two levels ($b \geq 2$), the ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F procedure tests for interactions by comparing the slopes of these lines. The b slopes are $O_{j(12)} = \bar{T}_{(1)1j} - \bar{T}_{(1)2j}$, $Q_{j(12)} = \bar{T}_{(2)1j} - \bar{T}_{(2)2j}$, $H_{j(12)} = \bar{T}_{(3)1j} - \bar{T}_{(3)2j}$, $V_{j(12)} = \bar{T}_{(4)1j} - \bar{T}_{(4)2j}$ and $U_{j(12)} = \bar{T}_{(5)1j} - \bar{T}_{(5)2j}$ and plot along with the decision lines of ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F respectively where $\bar{O}_{.12} = \sum_{j=1}^r O_{j(12)}$, $\bar{Q}_{.12} = \sum_{j=1}^r Q_{j(12)}$, $\bar{H}_{.12} = \sum_{j=1}^r H_{j(12)}$, $\bar{V}_{.12} = \sum_{j=1}^r V_{j(12)}$ and $\bar{U}_{.12} = \sum_{j=1}^r U_{j(12)}$. Then, proceed to check the significance of the interaction. This can be done using a similar process as described in section 4.2, utilizing the expression provided in Table 4.4.

Table 4. 3: Critical values against different levels of significance (α) when $t = 2$.

$\alpha = 0.01$																$\alpha = 0.05$															
Levels of The Factor	2	3	5		2	3	5		2	3	5		2	3	5																
$n = 5$	$S(\alpha, f, 2r(n - 1))$																														
	2.07			$n = 10$	2			$n = 5$	2.13			$n = 10$	2.02																		
	2.02	2.34			1.93	2.37			2.08	2.5			2	2.42																	
	1.98		2.42		1.89		2.5		2.03		2.67		1.99		2.61																
$n = 5$	$I(\alpha, f, 2r(n - 1))$																														
	5.2			$n = 10$	6.2			$n = 5$	5.7			$n = 10$	6.9																		
	5.47	6.2			6.5	8.4			5.67	6.92			6.85	8.24																	
	5.36		7.38		6.44		8.95		5.6		7.68		6.85		9.04																
$n = 5$	$Z(\alpha, f, 2r(n - 1))$																														
	5.97			$n = 10$	7.6			$n = 5$	5.97			$n = 10$	7.68																		
	5.63	6.96			7.22	8.15			5.63	6.96			7.25	8.75																	
	5.33		7.56		6.34		9		5.33		7.56		6.94		9.46																
$n = 5$	$M(\alpha, f, 2r(n - 1))$																														
	5.88			$n = 10$	7.35			$n = 5$	6.02			$n = 10$	7.6																		
	5.36	6.78			6.89	8.54			5.64	7.06			7.13	8.78																	
	5.12		7.37		6.65		9.28		5.34		7.65		6.86		9.45																
$n = 5$	$K(\alpha, f, 2r(n - 1))$																														
	7.08			$n = 10$	8.12			$n = 5$	7.25			$n = 10$	8.32																		
	6.84	8.76			7.76	9.54			7.07	9			7.91	9.75																	
	6.8		9.5		7.42		10.53		7.05		10.86		7.64		10.71																
$n = 5$	$E(\alpha, f, 2r(n - 1))$																														
	5.88			$n = 10$	7.47			$n = 5$	6.02			$n = 10$	7.72																		
	5.4	6.95			6.77	8.62			5.65	7.03			7.27	8.8																	
	5.12		7.28		6.92		9.46		5.34		7.6		6.96		9.5																
$n = 5$	$P(\alpha, f, 2r(n - 1))$																														
	7.2			$n = 10$	8.25			$n = 5$	7.12			$n = 10$	8.32																		
	6.27	8.28			7.81	9.68			6.7	8.58			7.88	9.68																	
	6.68		9.28		7.45		10.23		6.38		9.48		7.59		10.6																

4.4 Performance Measures and Simulation Procedure

In this section, we will outline the design structure of the study, which will be utilized for conducting a simulation study using the Monte Carlo simulation technique [62]. A set of independently and normally distributed random numbers is generated using the Box-Muller transformation. Fleishman mechanism with four specified moments is used to generate non-normal random numbers. For more details about the Flieshman mechanism, see Fleishman [64]. In this study, $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests are compared with respect to type I error (α) and test power ($1 - \beta$). In the context of a factorial design with Factor A and Factor B, there are three main null hypotheses: the null hypothesis for Factor

A, the null hypothesis for Factor B, and the null hypothesis for the interaction between the factors (for more details, see Section 4.3).

Table 4. 4: Lower and upper decision lines regarding regression, ratio and modified ratio estimators.

Interaction between Factor A and Factor B	
$LDL_{\bar{M}_{ij}} = Z_{(12)} - I(\alpha, r, 2r(n-1))(\sqrt{MSE_{\bar{M}_{...}}})\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{\bar{M}_{ij}} = \bar{Z}_{(12)} + I(\alpha, r, 2r(n-1))(\sqrt{MSE_{\bar{M}_{...}}})\sqrt{\frac{t(r-1)}{rn}}$
$LDL_{T_{(1)ij}} = \bar{O}_{(12)} - Z(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(1)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{T_{(1)ij}} = \bar{O}_{(12)} + Z(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(1)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$
$LDL_{T_{(2)ij}} = \bar{Q}_{(12)} - M(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(2)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{T_{(2)ij}} = \bar{Q}_{(12)} + M(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(2)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$
$LDL_{T_{(3)ij}} = \bar{H}_{(12)} - K(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(3)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{T_{(3)ij}} = \bar{H}_{(12)} + K(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(3)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$
$LDL_{T_{(4)ij}} = \bar{V}_{(12)} - E(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(4)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{T_{(4)ij}} = \bar{V}_{(12)} + E(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(4)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$
$LDL_{T_{(5)ij}} = \bar{U}_{(12)} - P(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(5)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$	$UDL_{T_{(5)ij}} = \bar{U}_{(12)} + P(\alpha, r, 2r(n-1))\left(\sqrt{MSE_{\bar{T}_{(5)...}}}\right)\sqrt{\frac{t(r-1)}{rn}}$
Main effects; Factor A or Factor B	
$LDL_{\bar{M}_{ij}} = \bar{M}_{...} - I(\alpha, f, tr(n-1))(\sqrt{MSE_{\bar{M}_{...}}})\sqrt{\frac{f-1}{N}}$	$LDL_{\bar{M}_{ij}} = \bar{M}_{...} + I(\alpha, f, tr(n-1))(\sqrt{MSE_{\bar{M}_{...}}})\sqrt{\frac{f-1}{N}}$
$LDL_{T_{(1)ij}} = \bar{T}_{(1)...} - Z(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(1)...}}}\right)\sqrt{\frac{f-1}{N}}$	$UDL_{T_{(1)ij}} = \bar{T}_{(1)...} + Z(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(1)...}}}\right)\sqrt{\frac{f-1}{N}}$
$LDL_{T_{(2)ij}} = \bar{T}_{(2)...} - M(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(2)...}}}\right)\sqrt{\frac{f-1}{N}}$	$UDL_{T_{(2)ij}} = \bar{T}_{(2)...} + M(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(2)...}}}\right)\sqrt{\frac{f-1}{N}}$
$LDL_{T_{(3)ij}} = \bar{T}_{(3)...} - K(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(3)...}}}\right)\sqrt{\frac{f-1}{N}}$	$UDL_{T_{(3)ij}} = \bar{T}_{(3)...} + K(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(3)...}}}\right)\sqrt{\frac{f-1}{N}}$
$LDL_{T_{(4)ij}} = \bar{T}_{(4)...} - E(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(4)...}}}\right)\sqrt{\frac{f-1}{N}}$	$UDL_{T_{(4)ij}} = \bar{T}_{(4)...} + E(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(4)...}}}\right)\sqrt{\frac{f-1}{N}}$
$LDL_{T_{(5)ij}} = \bar{T}_{(5)...} - P(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(5)...}}}\right)\sqrt{\frac{f-1}{N}}$	$UDL_{T_{(5)ij}} = \bar{T}_{(5)...} + P(\alpha, f, tr(n-1))\left(\sqrt{MSE_{\bar{T}_{(5)...}}}\right)\sqrt{\frac{f-1}{N}}$

The Type I error in a factorial design refers to the probability of erroneously rejecting any of the null hypotheses. In other words, it represents the likelihood of falsely concluding that there is a significant effect or interaction between the factors when, in reality, there is no true effect or interaction. Conversely, the power of a test in a factorial design corresponds to the probability of correctly rejecting the null hypothesis when it is false. It quantifies the test's ability to accurately detect a true effect or interaction between the factors. Following [13], the study examines the ANOM and the ANOMC test under different experimental conditions, such as choices of distributions $D = (s, kr)$ with several

values of skewness (s) and kurtosis (kr), for instance, the combination of (0,0) represents a normal distribution, (0,3) represents a double exponential distribution, (2,6) represents a Chi Square distribution, and (0,25) represents a Cauchy distribution. Additionally, the study considers several values of correlation (ρ), sample size (n), variance ratios (v), effect size (δ), and the number of treatments in factor B, while keeping a fixed number of treatments in factor A ($t = 2$).

The findings are evaluated at a significance level of 5% ($\alpha = 5\%$) across different numbers of treatments. The simulation study is conducted using R software version (4.2.3), where simulations are iterated 10,000 times, and each combination of design parameters is outlined in Table 4.5. The evaluation of test performance is based on two criteria: tests with a type I error probability close to $\alpha = 0.05$ and high power are considered the best. The empirical estimation of Type I error rates for $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests was calculated by determining the number of null hypotheses (H_0) that erroneously fell outside the interval (either above the upper decision line (UDL) or below the lower decision line (LDL)) in 10,000 simulation trials. The calculated number was then divided by 10,000 to obtain the empirical Type I error rate.

To obtain the test power, the following steps illustrate the process:

- 1- The parameter space was specified according to Table 5.5.
- 2- Specific constant values representing the effect sizes ($\delta = 0.25, 0.50, 0.75, 1.00$) were added to the random numbers of one or more treatment groups. This ensured that the means of at least one treatment group fell outside the decision lines.

- 3- The new datasets underwent ANOMC tests, and decision lines were calculated. This process was repeated 10,000 times.
- 4- The count of trials (j) in which the means of any of the treatments exceeded the decision lines was recorded.
- 5- The test's power was obtained by dividing this count by 10,000, representing the total number of repetitions.

Table 4. 5a: Choices considered for different design parameters

Parameter space	Number of treatments in Factor A and B (i,j)		
	(2,2)	(2,3)	(2,5)
Distribution (s,kr)	(0,0), (0,3), (2,6), (0,25)	(0,0), (0,3), (2,6), (0,25)	(0,0), (0,3), (2,6), (0,25)
Correlation (ρ)			
ρ_1	0.5: 0.5: 0.5: 0.5	0.5: 0.5: 0.5: 0.5: 0.5: 0.5	0.5: 0.5: 0.5: 0.5: 0.5: 0.5: 0.5: 0.5: 0.5
ρ_2	0.5: 0.5: 0.8: 0.8	0.5: 0.5: 0.5: 0.8: 0.8: 0.8	0.5: 0.5: 0.5: 0.5: 0.5: 0.8: 0.8: 0.8: 0.8: 0.8
ρ_3	0.8: 0.8: 0.5: 0.5	0.8: 0.8: 0.8: 0.5: 0.5: 0.5	0.8: 0.8: 0.8: 0.8: 0.8: 0.5: 0.5: 0.5: 0.5: 0.5
ρ_4	0.8: 0.8: 0.8: 0.8	0.8: 0.8: 0.8: 0.8: 0.8: 0.8	0.8: 0.8: 0.8: 0.8: 0.8: 0.8: 0.8: 0.8: 0.8: 0.8
Sample size (n)			
n_1	5: 5: 5: 5	5: 5: 5: 5: 5: 5	5: 5: 5: 5: 5: 5: 5: 5: 5: 5
n_2	10: 10: 10: 10	10: 10: 10: 10: 10: 10	10: 10: 10: 10: 10: 10: 10: 10: 10: 10
Variance ratios (v)			
v_1	1: 1: 1: 1	1: 1: 1: 1: 1: 1	1: 1: 1: 1: 1: 1: 1: 1: 1: 1
v_2^+	1: 1: 1.05: 1.05	1: 1: 1: 1: 1.05: 1.05	1: 1: 1: 1: 1: 1: 1.05: 1.05: 1.05: 1.05
v_2^-	1.05: 1.05: 1: 1	1.05: 1.05: 1: 1: 1: 1	1.05: 1.05: 1.05: 1.05: 1: 1: 1: 1: 1: 1
v_3^+	1: 1: 1.1: 1.1	1: 1: 1: 1: 1.1: 1.1	1: 1: 1: 1: 1: 1: 1.1: 1.1: 1.1: 1.1
v_3^-	1.1: 1.1: 1: 1	1.1: 1.1: 1: 1: 1: 1	1.1: 1.1: 1.1: 1.1: 1: 1: 1: 1: 1: 1

Table 4. 5b: Patterns of Effect size (δ) for 2-way factorial Design.

Number of True Null	No. of Pattern	δ	(2,2)	(2,3)	(2,5)	True Effect
3	1	δ_1	0:0:0:0	0,0,0,0,0	0:0:0:0:0:0:0:0	A, B, AB
0	2	δ_2	0:0:1:0	0,0,0,0,1,1	0,0,0,1,1,0,1,1,1,0	None
0	3	δ_3	0:0.25:0.5:1	0,0.25,0.5,1,0,1	0:0.25:0.5:0:0:1:0:1:1:0.5	None
0	4	δ_4	0:0.25:0:0.75	0,0.25,0,0.75,0,1	0:0.25:0:1:0.75:0:0.75:0:1:0.5:0.25	None
0	5	δ_5	1:0:0:0.5	1:0:1:0:0.25:0.5	1:0:1:0.5:0:0:0.25:0.5:0.75:0	None

Note: Pattern 1 for Type I error rate and pattern 2 – 5 for statistical power.

4.5 Comparison Performance

To evaluate and compare the performance of ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests, Tables

4.6-4.9 present the empirical type I error rates, and Figure 4.1-4.6 present the power of ANOM and ANOMC tests.

4.5.1 Results of type I error rates

The relative performances for $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests in terms of empirical type I error rates under null case (δ_1) are given in Tables 4.6-4.9.

I. Empirical type I error rate under homogeneous variances

Tables 4.6 and 4.7 present the results for the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests, assuming the homogeneity of variances. The performance of these tests is evaluated using Bradley's liberal criterion [65], where values falling within the range of 2.5% to 7.5% are considered liberal or conservative. The findings of the results are listed below:

- 1- In the $ANOM_F$ test, regardless of the scenarios considered in the study, similar levels of Type I error rates were observed, and the type of distribution did not have an impact on these rates.
- 2- For the $ANOMC - Reg_F$ test, except for the Chi-square and Cauchy distributions, the test exhibited a percentage around 5% for the Type I error rate. However, with the chi-square and Cauchy distributions, as the sample size increased, the Type I error rates approached the upper value in the Bradley range (7.5%). Furthermore, the Type I error rates increased with an increase in the number of levels in factor B. Notably, in factor A, when the correlation (ρ_1, ρ_4) was equal, and the test was able to control the Type I error rate.

- 3- Regarding the ANOMC – R_F test, except for the Cauchy distribution, the test showed similar levels of Type I error rates around 5% in all scenarios. However, with a small correlation (ρ_1) and a direct pairing of correlations (ρ_2) under the chi-square distribution, the test encountered difficulties and could not effectively control the error.
- 4- Focusing on the ANOMC – $MR1_F$ test, except in the condition of equal small correlation (ρ_1), the test exhibits difficulties in regulating the Type I error rates across all the explored scenarios. Furthermore, in the equal small correlation (ρ_1) scenario, elevating the levels of factor B leads to an increase in the Type I error rates.
- 5- With regards to the ANOMC – $MR2_F$, except for the Cauchy distributions, the test exhibited a percentage around 5% for the Type I error rate.
- 6- Considering the ANOMC – $MR3_F$, apart from the Cauchy distribution, in the scenario of direct pairing of correlations (ρ_2), increasing the levels of factor B results in the Type I error rate climbing above the upper end of the Bradley range (7.5%).
- 7- In the ANOMC – $MR4_F$ test, the Type I error rates varied across different scenarios, ranging from 2.5% to 7.5%. However, with a high correlation (ρ_4), the test exhibited lower Type I error rates around 1%.

II. Empirical type I error rate under heterogeneous variances

In Tables 4.8 and 4.9, as expected, the tests are affected by variances' heterogeneity, particularly in factor A and it is observed that the ANOMC – $MR1_F$ and ANOMC – $MR4_F$ tests demonstrate better percentage type I error rates and relatively lower susceptibility to heterogeneity compared to ANOM_F, ANOMC – Reg_F , ANOMC – R_F , ANOMC – $MR2_F$ and ANOMC – $MR3_F$ tests. However, all the tests exceed the level of significance ($\alpha = 5\%$) due to the impact of heterogeneity. The

actual type I error rates for all the tests significantly surpass the specified level when sample sizes and variances are inversely paired. Although improvements are observed at various levels of heterogeneity, it is important to note that increasing the sample size leads to an increase in type I error rates. Furthermore, increasing the levels of factor B will also increase the type I error rates. In the interaction and factor B in the case of $(i, j) = (2, 2)$, except for Chi-square, Cauchy distribution and ANOMC – MR1_F, all the tests were not affected by the heterogeneous. However, in the specified experimental conditions (cf. Table 4.5), under the high correlation (ρ_4), ANOMC – MR4_F is less effected due to the impact of heterogeneity, on the other hand under the small correlation (ρ_1), ANOMC – MR1_F is less effected due to the impact of heterogeneity. Regarding the ANOMC – Reg_F test, it exhibits higher type I error rates (%) when large sample sizes are associated with substantial variances. Additionally, the ANOMC – Reg_F test also displays higher type I error rates (%) in cases with an association between large sample sizes and small variances. Conversely, the ANOMC – R_F test demonstrates higher type I error rates (%) when there is an association between small sample sizes and large variances in the interaction and factor B rates.

Table 4. 6: Type I error (%) of ANOM and ANOMC tests under homogeneous variances and n_1 .

(s, kr)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
	$t = 2, r = 2$				$t = 2, r = 3$				$t = 2, r = 5$			
ANOM _F												
ρ	Factor A											
ρ_1	4.94	4.72	4.39	4.1	5.2	4.8	5.4	5.7	4.92	4.65	4.5	4.16
ρ_2	5.3	4.6	4.38	3.4	5.2	4	4.3	4.9	5.19	4.65	4.85	4.66
ρ_3	5.48	4.61	4.55	3.95	4.8	4.5	5.4	3.9	4.94	4.67	5.11	4.69
ρ_4	4.88	4.64	4.9	3.69	5.3	4.6	5.7	3.2	5.05	4.93	4.99	4.3
Interaction												
ρ_1	4.99	4.98	4.63	4.11	4.99	4.75	4.91	3.9	5	4.74	5.4	5.54
ρ_2	4.83	4.96	4.52	3.87	5.2	4.71	4.72	3.83	4.78	5.2	5.01	5.11
ρ_3	4.7	4.78	4.38	3.87	4.95	4.76	4.73	3.88	5.17	4.73	4.92	5.16
ρ_4	5.27	4.84	4.68	3.73	4.92	4.64	4.71	3.63	5.35	5.04	4.92	5.01
Factor B												
ρ_1	4.76	4.6	4.32	3.9	5.32	4.86	4.68	3.77	5.15	5.04	5.13	4.95
ρ_2	5.1	4.8	4.65	3.78	4.92	4.77	4.76	4.01	4.9	5.19	5.24	4.98
ρ_3	5.31	4.58	4.72	3.95	5.4	4.44	4.54	3.6	4.77	5.04	5.19	5.21
ρ_4	4.81	4.46	4.48	3.8	5.27	4.68	4.19	4.05	4.86	5.02	5.43	4.78
ANOMC – Reg _F												
Factor A												
ρ_1	4.97	5	9.05*	7.97*	4.7	5.17	10.69*	8.68*	4.73	5.22	12.46*	8.8*
ρ_2	5.93	5.95	10.48*	9.36*	5.46	6.38	11.57*	9.93*	5.53	6.2	12.56*	10.36*
ρ_3	5.74	5.79	9.97*	9.6*	5.86	6.23	11.58*	9.32*	5.3	5.81	13.17*	9.82*
ρ_4	5.24	5.48	9.84*	8.63*	4.67	5.45	10.53*	8.25*	5.18	5.43	12.44*	9.03*
Interaction												
ρ_1	5.37	5.28	10.04*	8.32*	5.39	5.63	12.66*	10.13*	5	5.44	16.98*	12.94*
ρ_2	5.4	5.88	9.67*	8.83*	5.28	5.8	12.26*	10.11*	5.61	6.29	16.97*	13.35*
ρ_3	5.4	5.75	9.94*	8.69*	5.61	5.47	12.52*	10.19*	5.63	6.49	17.4*	12.99*
ρ_4	5.5	5.38	10.04*	8.31*	5.11	5.43	12.84*	9.4*	4.76	5.47	17.17*	13.01*
Factor B												
ρ_1	4.93	5.19	9.38*	8.47*	5.09	5.59	12.53*	9.84*	5.22	5.69	16.67*	12.53*
ρ_2	6.06	5.9	10.24*	9.14*	5.23	5.89	12.96*	9.9*	5.7	6.3	16.81*	13.59*
ρ_3	5.83	5.94	9.74*	8.56*	5.43	5.46	12.61*	10.03*	5.6	6.55	17.35*	13.61*
ρ_4	5.13	5.04	9.94*	8.51*	5.03	5.14	12.51*	9.44*	4.89	5.51	17.37*	12.95*
ANOMC – R _F												
Factor A												
ρ_1	4.74	6.02	7.5	13.09*	4.8	6.33	7.13	14.12*	5.1	6.73	7.35	15*
ρ_2	6.11	7.98*	8.41*	14.91*	5.95	7.71*	8.42*	15.79*	6.35	7.37	8.38*	15.69*
ρ_3	4.44	5.37	5.9	9.99*	4.77	5.64	5.95	11.73*	4.09	5.5	6.74	12.61*
ρ_4	5.09	5.67	6.07	10.43*	4.87	5.26	6.07	11.32*	4.39	5.68	6.91	12.06*
Interaction												
ρ_1	5.22	6.46	7.32	12.96*	5.21	7.3	8.43*	17.92*	5.09	7.64*	9.75*	22.06*
ρ_2	6.08	7.36	7.91*	15.36*	6.37	8.58*	9.57*	18.52*	6.68	9.05*	10.59*	22.47*
ρ_3	4.4	5.27	5.53	10.13*	4.68	5.59	6.64	14.29*	4.21	5.91	8.2*	17.69*
ρ_4	5.35	5.53	5.7	10.25*	4.93	6.09	7.02	13.68*	4.72	6.44	7.9*	17.21*
Factor B												
ρ_1	4.97	6.21	6.8	12.95*	4.89	7.07	8.12*	17.52*	4.85	7.38	9.88*	21.77*
ρ_2	6.25	7.5	8.33*	15.51*	6.04	8.37*	9.6*	18.64*	6.34	9.28*	10.62*	22.99*
ρ_3	4.55	5.55	5.4	9.95*	4.56	5.78	6.09	13.66*	4.14	6.46	7.82*	17.59*
ρ_4	5.16	5.66	5.97	10.8*	4.88	6.08	7.03	13.32*	4.56	6.6	7.65*	17.13*
ANOMC – MR1 _F												
Factor A												
ρ_1	4.94	6.56	6.91	13.37*	4.97	6.59	7.2	14.87*	4.97	7	6.82	15.32*
ρ_2	9.69*	12.02*	12*	18.88*	10.1*	11.76*	11.84*	20.38*	9.84*	11.81*	11.29*	20.93*
ρ_3	8.19*	9.39*	9.69*	16.41*	8.01*	9.51*	8.95*	16.28*	7.73*	9.58*	9.57*	18*
ρ_4	16.3*	17.35*	17.63*	23.96*	15.83*	17.47*	17.22*	23.84*	15.32*	17.39*	16.98*	25.36*
Interaction												
ρ_1	5.31	6.3	7.08	12.86*	5.04	6.87	8.01*	18.76*	5.09	8.46*	8.1*	21.73*
ρ_2	9.4*	11.8*	11.63*	18.92*	10.75*	13.78*	13.85*	23.97*	11.9*	15.73*	14.66*	29.72*
ρ_3	7.77*	9.45*	9.79*	15.34*	8.76*	10.93*	11.41*	20.45*	9.03*	12.2*	12.31*	25.39*
ρ_4	16.77*	17.35*	17.65*	23.48*	19.65*	21.37*	20.44*	29.39*	22.29*	24.4*	23.58*	35.96*
Factor B												
ρ_1	4.97	6.8	6.99	13.37*	4.96	7.06	8.11*	17.93*	4.9	7.8*	7.93*	22.43*
ρ_2	9.5*	12.41*	12*	19.12*	11.03*	13.59*	13.56*	24.06*	11.78*	15.35*	14.64*	29.28*
ρ_3	7.82*	9.63*	9.36*	16.07*	8.86*	11.02*	10.99*	20.03*	8.87*	12.25*	12.07*	25.26*
ρ_4	16.11*	17.13*	17.56*	23.86*	19.01*	21.97*	21.4*	29.18*	21.93*	24.77*	23.86*	35.73*

ANOMC – MR2 _F												
Factor A												
ρ_1	4.77	5.63	6.41	11.28*	4.93	5.31	5.52	11.38*	4.96	5.2	5.2	11.15*
ρ_2	5.92	5.92	6.66	10.93*	5.59	5.7	6.03	11.5*	5.49	5.81	5.43	11.19*
ρ_3	5.06	5.27	6.05	10.6*	4.97	5.47	5.64	10.59*	4.91	5.29	5.49	10.67*
ρ_4	5.05	5.76	6.4	10.89*	5.09	5.23	6.18	10.95*	4.99	5.64	5.48	11.25*
Interaction												
ρ_1	4.84	5.71	6.25	11.03*	4.99	5.2	5.89	12.68*	4.91	5.21	5.53	13.07*
ρ_2	5.24	5.85	6.53	10.97*	5.5	5.85	6.49	12.99*	5.66	6.05	5.24	13.26*
ρ_3	4.92	5.31	6.17	10.81*	5.05	5.43	6.33	13.1*	4.92	5.09	5.01	13.39*
ρ_4	5.2	5.81	6.04	10.83*	5.45	5.49	6.49	12.13*	5.5	5.61	5.76	13.26*
Factor B												
ρ_1	4.95	5.6	5.93	11.07*	5.03	5.41	5.81	13.25*	5.06	5.35	5.15	12.69*
ρ_2	5.59	6.19	6.53	11.12*	5.65	5.79	6.66	12.93*	5.79	6.04	5.57	13.63*
ρ_3	5.14	5.55	6.03	10.98*	5.22	5.34	6.13	12.45*	4.84	5.44	5.22	13.09*
ρ_4	5.21	5.65	5.98	10.88*	5.67	5.55	6.36	12.44*	5.52	5.97	5.49	13.57*
ANOMC – MR3 _F												
Factor A												
ρ_1	4.71	5.93	7.52*	12.83*	4.82	6.25	7.39	14.19*	5.09	6.63	7.78*	15.13*
ρ_2	5.99	7.92*	8.43*	14.73*	6	7.59*	8.57*	15.9*	6.29	7.45	8.76*	15.66*
ρ_3	4.41	5.26	5.94	9.9*	4.77	5.66	6.06	11.87*	4.13	5.65	6.98	12.68*
ρ_4	4.91	5.68	6.05	10.46*	4.73	5.26	6.14	11.29*	4.52	5.81	7.08	12.15*
Interaction												
ρ_1	5.16	6.48	7.5	12.98*	5.13	7.16	8.61*	17.71*	5.2	7.61*	10.23*	22.08*
ρ_2	6.06	7.24	8*	15.3*	6.13	8.29*	9.8*	18.34*	6.69	9.13*	10.94*	22.46*
ρ_3	4.53	5.34	5.61	10.12*	4.65	5.6	6.69	14.02*	4.28	5.99	8.23*	17.67*
ρ_4	5.28	5.47	5.68	9.98*	4.98	6.02	6.88	13.71*	4.71	6.51	7.88*	17.17*
Factor B												
ρ_1	4.8	6.07	6.85	13*	4.75	6.85	8.2*	17.23*	4.83	7.42	10.23*	21.75*
ρ_2	6.2	7.63*	8.72*	15.22*	5.92	8.21*	9.71*	18.48*	6.47	9.23*	11.21*	22.75*
ρ_3	4.72	5.52	5.6	9.97*	4.59	5.68	6.11	13.63*	4.16	6.31	8.24*	17.54*
ρ_4	5.07	5.55	5.9	10.64*	4.7	6.08	6.96	13.26*	4.72	6.68	7.61*	17.15*
ANOMC – MR4 _F												
Factor A												
ρ_1	4.69	5.78	6.81	12.25*	5.07	6.29	6.57	13.45*	5.05	6.56	6.74	13.96*
ρ_2	2.89	4.14	4.24	8.98*	3.04	3.93	4.2	10.27*	3.39	4.23	4.15	10.05*
ρ_3	2.85	3.55	3.91	7.79*	3.03	3.95	3.9	9.07*	3.13	3.81	4.55	10*
ρ_4	1.16*	1.67*	1.71*	4.68	1.16*	1.6*	1.8*	5.23	1.2*	1.66	1.87	5.97
Interaction												
ρ_1	4.91	5.9	6.55	12.02*	4.88	6.44	6.64	15.33*	5.21	6.9	8.2	19.36*
ρ_2	2.96	3.92	3.99	9.58*	2.92	4.2	4.52	11.34*	2.79	4.26	4.54	13.18*
ρ_3	2.81	3.39	3.66	7.77*	2.81	3.88	3.98	10.4*	2.98	3.75	4.92	12.9*
ρ_4	1.14*	1.47*	1.6*	4.6	1.11*	1.68*	1.72*	5.52	1.11*	1.66*	1.85*	6.81
Factor B												
ρ_1	5.05	5.9	6.05	12.01*	5.01	6.4	6.82	15.59*	4.93	6.72	8.21*	19.44*
ρ_2	2.86	3.92	3.91	9.6*	2.98	4.05	4.34	11.01*	2.97	4.36	4.66	13.47*
ρ_3	2.73	3.6	3.63	7.69*	2.75	3.58	3.88	10.25*	2.77	3.59	4.68	13.03*
ρ_4	1.3*	1.51*	1.5*	4.99	1.08*	1.4*	1.76*	5.55	1.08*	1.77*	1.93*	6.87

* the percentage type I error lies outside Bradley's liberal criterion

Table 4. 7: Type I error (%) of ANOM and ANOMC tests under homogeneous variances and n_2 .

ρ	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
	$t = 2, r = 2$				$t = 2, r = 3$				$t = 2, r = 5$			
ANOM _F												
Factor A												
ρ_1	4.49	5.32	4.83	4.37	4.87	4.89	4.6	4.5	4.65	5.58	5.05	4.64
ρ_2	5.45	5.12	5.01	4.81	4.94	4.57	5.05	4.78	4.98	5.25	4.54	4.59
ρ_3	5.02	4.97	5.06	4.57	5.15	5.17	5.52	4.67	5.18	5.13	4.91	4.75
ρ_4	4.88	5.11	5.34	4.67	5.16	4.57	4.64	4.91	4.91	4.85	4.71	4.59
Interaction												
ρ_1	5.13	5.29	4.49	4.35	4.85	4.86	4.92	4.47	5.06	5.52	5.36	4.89
ρ_2	4.82	4.88	4.97	4.75	5.23	4.84	4.55	4.26	4.84	4.98	5.19	5.28
ρ_3	5.14	5.26	4.75	4.46	4.78	4.48	4.92	4.48	4.88	5.06	4.89	4.96
ρ_4	5	4.9	4.81	4.67	4.91	4.74	4.87	4.54	5.12	5	4.76	5.09
Factor B												
ρ_1	5.31	5.08	4.58	4.43	4.88	5.01	4.74	4.46	4.94	4.89	5.15	4.95
ρ_2	5.27	5.28	4.85	4.31	5.03	4.6	4.69	4.5	5.22	5.36	5.19	4.82
ρ_3	5.3	5.35	5.06	4.42	5.12	4.95	4.77	4.05	4.94	5.15	4.54	5.38
ρ_4	5.08	4.94	4.72	4.4	5	4.58	4.74	4.52	4.65	5.08	5.04	5.75
ANOMC – Reg _F												
Factor A												
ρ_1	4.86	5.57	6.22	6.72	4.95	5.53	6.9	7.06	4.74	5.87	7.07	7.92*
ρ_2	5.93	6.36	7.94*	8.22*	5.94	5.77	8.29*	8.21*	6.07	6.16	8.55*	8.96*
ρ_3	5.42	5.95	8.21*	8.35*	5.49	6.74	8.26*	8.26*	5.71	6.25	8.29*	9.01*
ρ_4	4.97	4.92	7.12	6.73	5.11	5.4	6.7	7.48	4.93	5.21	7.25	7.69
Interaction												
ρ_1	4.91	5.39	5.9	6.68	5.06	5.43	7.92*	8.07*	5.05	5.94	9.58*	11.11*
ρ_2	5.17	5.54	7.31	6.74	6.06	5.56	8.51*	8.57*	5.4	6.77	10.76*	12.14*
ρ_3	5.77	5.48	6.91	7.04	5.68	6.03	8.27*	8.71*	5.09	6.39	10.74*	11.23*
ρ_4	4.93	4.92	7.06	6.81	5.26	5.63	7.95*	8*	4.98	5.69	9.5*	11.02*
Factor B												
ρ_1	4.93	5.32	6.54	6.74	5.24	5.6	7.68*	8.01*	5.07	6.16	9.42*	10.79*
ρ_2	5.68	5.85	7.21	7.44	5.96	5.69	8.43*	8.24*	5.87	6.46	10.22*	11.98*
ρ_3	5.33	5.21	7.51*	7.08	5.74	6.14	8.9*	8.44*	5.63	6.23	9.32*	11.66*
ρ_4	5.07	5.11	6.95	6.33	5	5.52	8.22*	8.15*	4.63	5.42	9.15*	10.78*
ANOMC – R _F												
Factor A												
ρ_1	4.76	6.08	6.18	11.1*	4.69	5.84	6.65	10.96*	5.08	6.58	6.88	11.9*
ρ_2	6.2	7.19	7.68*	11.78*	5.82	6.5	7.18	12.12*	5.58	6.4	7.59*	12.75*
ρ_3	4.57	5.13	6.01	8.01*	4.22	5.6	6.05	9.08*	4.51	5.45	6.03	9.62*
ρ_4	4.98	5.47	6.02	8.98*	4.77	5.78	5.73	9.07*	5.05	5.46	5.85	9.42*
Interaction												
ρ_1	4.82	6.25	5.9	10.95*	5.41	6.36	7.63*	14.34*	4.7	6.59	8.49*	17.88*
ρ_2	5.72	6.45	7.95*	11.79*	6.09	7.41	8.58*	15.89*	5.7	8.13*	9.71*	19.83*
ρ_3	5	5.16	5.85	8.29*	4.93	6.04	6.27	11.45*	4.32	5.93	6.7	14.95*
ρ_4	4.79	4.93	5.53	9.02*	5.54	6.11	6.43	11.73*	4.73	6.62	7.29	14.66*
Factor B												
ρ_1	5.01	6.17	6.43	11*	4.8	6.4	7.82*	14.17*	5.14	7.5	8.75*	17.84*
ρ_2	5.86	6.98	7.2	12.37*	5.9	7.12	8.94*	15.66*	6.14	8.15*	9.8*	20.04*
ρ_3	4.67	5.11	5.41	7.9*	4.79	5.72	6.89	11.15*	4.67	6.23	6.64	15.02*
ρ_4	4.93	5.2	5.77	8.39*	5	6.55	6.6	11.75*	4.74	6.44	6.96	14.48*
ANOMC – MR1 _F												
Factor A												
ρ_1	4.29	6.03	6.83	12.17*	4.74	6.24	6.62	12.04*	5.28	6.41	6.61	12.25*
ρ_2	10.28*	11.6*	11.81*	17.41*	10.13*	11.69*	12.25*	18.43*	9.42*	12.02*	12.05*	18.19*
ρ_3	9.42*	10.13*	9.94*	14.66*	9.1*	10.47*	10.78*	15.64*	8.74*	10.22*	10.45*	15.63*
ρ_4	18.42*	19.28*	19.71*	23.71*	18.22*	19.39*	19.14*	23.54*	18.53*	19.89*	19.47*	23.76*
Interaction												
ρ_1	4.82	6.36	6.53	12.15*	4.97	6.47	7.39	14.77*	4.93	7.32	7.52*	18.54*
ρ_2	9.54*	11.38*	11.8*	17.36*	11.51*	13.69*	14.26*	22.94*	12.02*	16.07*	16.33*	27.77*
ρ_3	8.9*	9.75*	10.53*	14.47*	9.93*	12.41*	12.08*	19.33*	10.8*	13.42*	12.91*	24.15*
ρ_4	18.35*	19.61*	19.42*	24.23*	23.99*	25.7*	24.96*	29.84*	27.75*	31.09*	29.08*	38.07*
Factor B												
ρ_1	4.93	6.47	6.76	11.33*	4.93	6.85	7.68*	14.78*	5.03	7.42	7.74*	18.57*
ρ_2	9.66*	11.01*	11.89*	17.41*	11.51*	13.02*	14.66*	23.19*	12.16*	16.07*	16.18*	28.11*
ρ_3	8.93*	10.12*	10.3*	14.74*	10.1*	12.22*	12.22*	19.23*	9.97*	13.69*	12.93*	24.31*
ρ_4	18.49*	19*	19.59*	23.5*	24.28*	25.3*	24.79*	29.63*	28.21*	30.54*	29.32*	37.3*

ANOMC – MR2 _F												
Factor A												
ρ_1	4.74	5.82	6.73	10.73*	4.91	5.91	6.17	10.91*	4.53	6.56	6.65	11.77*
ρ_2	5.25	6.17	6.61	10.46*	4.66	5.34	6.64	10.63*	5.38	6.1	6.27	11.58*
ρ_3	4.88	5.93	6.5	10.72*	5.11	6.22	7.4	11.28*	5.16	6.2	6.43	11.28*
ρ_4	4.93	5.56	6.5	10.51*	5.18	5.33	5.91	10.58*	4.96	5.79	6.37	10.84*
Interaction												
ρ_1	4.94	6.01	6.17	10.5*	5.27	6.09	7.66	13.57*	4.84	6.78	7.87	16.23*
ρ_2	5.04	5.86	6.63	10.25*	4.99	6.07	6.94	12.76*	4.9	6.15	7.31	16.04*
ρ_3	4.89	6	6.15	10.53*	5.05	6.27	7.7	13.55*	5.02	6.5	7.61	16.11*
ρ_4	4.83	5.54	6.73	10.09*	5.14	6.36	7.16	12.69*	4.9	6.48	6.92	15.77*
Factor B												
ρ_1	5.1	5.64	6.34	10.55*	5.32	6.87	7.52	13.44*	5.05	6.47	7.8*	17.1*
ρ_2	5.08	6.16	6.49	9.43*	4.79	6.09	7.02	13.38*	5.38	6.57	7.43	16.05*
ρ_3	4.93	5.82	6.39	10.8*	5.07	6.42	7.92	14.18*	5.13	6.89	7.48	16.05*
ρ_4	5	5.56	6.27	9.85*	5.27	5.92	6.91	13.16*	4.75	6.74	7.11	15.21*
ANOMC – MR3 _F												
Factor A												
ρ_1	4.72	6.07	6.28	11.03*	4.61	5.85	6.79	10.86*	5	6.47	7.02	11.89*
ρ_2	6.19	7.13	7.62*	11.79*	5.83	6.59	7.39	12.16*	5.55	6.56	7.62*	12.7*
ρ_3	4.45	5.14	5.97	8.04*	4.19	5.55	5.89	9.06*	4.53	5.41	6.07	9.64*
ρ_4	4.95	5.34	6.2	8.87*	4.83	5.69	5.82	9.02*	5.05	5.48	5.75	9.44*
Interaction												
ρ_1	4.76	6.14	6.23	11.05*	5.54	6.18	7.83*	14.21*	4.78	6.65	8.73*	17.85*
ρ_2	5.76	6.43	8.19*	11.78*	6.09	7.43	8.76*	15.89*	5.68	8.13*	9.94*	19.72*
ρ_3	4.92	5.06	5.73	8.27*	4.83	6.05	6.36	11.43*	4.36	5.84	6.7	14.77*
ρ_4	4.58	4.75	5.67	9.02*	5.45	6.19	6.47	11.59*	4.77	6.59	7.31	14.64*
Factor B												
ρ_1	4.91	6.2	6.49	10.86*	4.69	6.45	7.99*	14.22*	5.11	7.57*	8.9*	17.76*
ρ_2	5.75	6.97	7.19	12.27*	5.84	7.07	9.06*	15.62*	6.03	8.24*	9.99*	19.91*
ρ_3	4.64	4.99	5.49	7.78*	4.73	5.62	6.84	11.13*	4.56	6.28	6.76	15.04*
ρ_4	4.83	5.03	5.89	8.26*	4.85	6.48	6.57	11.5*	4.72	6.34	6.93	14.56*
ANOMC – MR4 _F												
Factor A												
ρ_1	4.46	5.89	6.21	10.97*	4.69	6.2	6.5	11.35*	4.92	6.13	6.82	12.31*
ρ_2	2.48*	3.33	3.49	7.23	2.55	3.04	3.15	7.7	2.28*	3.01	3.7	7.78*
ρ_3	2.55	2.97	3.48	6.18	2.47*	3.36	3.4	7.14	2.38*	3.23	3.38	7.53*
ρ_4	0.58*	0.81*	1.07*	3.17	0.53*	0.87*	1*	3.2	0.57*	1*	0.94*	3.26
Interaction												
ρ_1	4.53	6.08	5.66	10.96*	5.44	6.56	7.14	14.24*	4.59	6.73	7.75*	17.12*
ρ_2	2.49*	3.24	3.76	7.33	2.26	2.93	3.2	9.14*	1.85*	3.24	3.75	11.4*
ρ_3	2.63	3.16	3.18	6.03	2.43	3.32	4.02	8.13*	2.12*	3.04	3.64	10.69*
ρ_4	0.54*	0.82*	0.98*	2.87	0.38*	0.72*	0.97*	3.59	0.2*	0.6*	0.84*	4.31
Factor B												
ρ_1	5.23	5.93	6.07	11.1	4.69	6.2	7.12	14.14*	4.93	7.21	8.08*	17.13*
ρ_2	2.42*	3.47	3.68	7.5	2.11*	2.76	3.43	8.84*	1.86*	2.95	3.56	11.12*
ρ_3	2.31*	2.73	3.18	5.78	2.43*	3	4.1	8.11*	2.06*	2.9	3.33	10.68*
ρ_4	0.61*	0.9*	0.88*	2.72	0.4*	0.65*	0.99*	3.77	0.4*	0.52*	0.71*	4.07

* the percentage type I error lies outside Bradley's liberal criterion

Table 4. 8: Type I error (%) of ANOM and ANOMC tests under heterogeneous variances and n_1 .

(s, kr)		(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
$t = 2, r = 2$					$t = 2, r = 3$					$t = 2, r = 5$			
ANOM _F													
ρ		Factor A											
ρ_1	v_2^+	7.76	7.64	7.92	9.82	8.3	6.2	7.3	9.5	10.33	9.92	10.23	11.33
	v_2^-	7.85	8.23	8.06	9.28	6.1	5.3	8.5	7	10.53	10.76	11.12	11.83
	v_3^+	17.18	17.66	17.79	23.25	14.2	12.6	12.4	16.1	26.95	26.38	27.36	31.13
	v_3^-	17.06	18.2	16.97	23.02	13.2	14	13.2	17	25.28	26.92	27.55	30.58
ρ_2	v_2^+	7.68	8.34	7.78	9.03	6.95	6.9	6.62	7.55	10.42	9.87	10.66	11.9
	v_2^-	7.96	8.35	7.97	9.59	7.35	6.98	7.31	6.83	9.95	10.34	10.33	11.56
	v_3^+	16.67	18.24	17.92	23.66	13.35	13.1	13.47	16.29	25.53	26.69	26.9	30.74
	v_3^-	16.73	18.01	17.59	23.17	13.11	13.33	12.9	16.54	25.91	25.78	26.4	31.9
ρ_3	v_2^+	7.81	8.31	7.73	8.57	6.78	7.46	6.99	7.53	10.02	9.81	9.9	11.45
	v_2^-	7.72	8.19	8	9.26	7.43	7.19	6.74	7.33	10.48	10.34	10.75	11.67
	v_3^+	16.9	18.39	17.08	23.41	13.22	13	13.53	16.92	26.92	27.02	27.44	30.94
	v_3^-	17.13	18.2	18.17	23.92	13.07	13.28	13.88	16.66	26.88	27.42	26.72	31.65
ρ_4	v_2^+	7.85	7.98	8.11	9.24	6.4	6.8	7.2	9	10.68	10.73	9.93	11.56
	v_2^-	7.85	8.25	7.84	8.5	6.7	6.4	7.3	8.4	10.16	10.31	10.79	11.58
	v_3^+	15.95	18.04	17.48	23.12	13.7	13.3	11.9	16.1	26.14	26.86	26.4	31.36
	v_3^-	17.09	18.21	17.24	22.67	13	14.7	12.9	15.7	26.49	27.33	26.85	31.23
Interaction													
ρ_1	v_2^+	5.05	5	4.67	4.15	5.73	5.5	5.56	5.06	5.77	5.29	5.57	6.13
	v_2^-	5	4.88	4.45	3.91	5.99	5.2	5.19	4.77	5.54	5.5	5.44	5.53
	v_3^+	4.92	4.92	4.33	3.78	7.74	7.26	7.23	7.85	7.28	6.86	6.87	7.77
	v_3^-	5.18	4.93	4.54	3.7	7.62	7.36	7.01	7.93	7.32	7.11	6.81	7.61
ρ_2	v_2^+	5.19	4.76	4.47	3.94	5.89	5.54	5.37	4.8	5.66	5.32	6.11	5.91
	v_2^-	4.77	4.45	4.69	4.15	5.52	5.5	5.45	4.5	5.59	5.66	5.35	5.85
	v_3^+	4.98	5.2	4.48	3.86	7.78	7.98	7.25	7.87	6.58	6.91	6.9	7.48
	v_3^-	4.92	4.71	4.12	3.84	7.92	7.55	7.54	7.8	7.1	7.02	7.11	7.61
ρ_3	v_2^+	4.75	4.62	4.8	3.49	5.04	5.18	5.5	4.77	5.41	6.1	5.72	5.86
	v_2^-	5.2	4.2	5.2	4.07	5.3	5.49	5.52	4.73	5.77	5.57	5.4	5.55
	v_3^+	4.99	4.52	4.32	3.73	7.46	7.71	7.18	7.84	6.78	7.31	7.16	7.51
	v_3^-	4.92	5.25	4.83	3.53	7.18	7.39	7.56	7.75	6.52	6.91	7.17	8.06
ρ_4	v_2^+	5.2	4.36	4.71	4.03	5.77	5.87	5.41	5.25	5.55	5.55	5.56	5.62
	v_2^-	5.11	4.94	4.62	3.88	5.04	5.18	5.61	5.57	5.4	5.18	5.5	5.95
	v_3^+	5.37	4.71	4.75	3.82	7.62	7.77	6.81	7.78	7.01	6.64	7.38	7.63
	v_3^-	5.1	4.47	4.67	3.67	7.82	7.89	7.95	7.71	7.26	7.29	6.94	7.46
Factor B													
ρ_1	v_2^+	4.74	4.65	4.31	3.86	5.75	5.82	5.55	5.05	5.56	5.52	5.57	5.64
	v_2^-	5.16	4.91	4.53	3.93	5.35	5.49	5.66	4.71	5.26	5.42	5.65	5.62
	v_3^+	5.16	4.83	4.76	3.7	8.19	7.44	7.37	7.58	6.94	7.2	7.12	7.91
	v_3^-	5	4.41	4.24	3.63	7.83	7.23	7.13	7.45	7.15	7.19	7.01	8.01
ρ_2	v_2^+	4.53	4.83	4.52	3.65	5.31	5.92	5.22	5.13	5.29	5.57	5.38	6.02
	v_2^-	4.96	4.81	4.55	3.94	5.84	5.62	4.99	5.22	5.59	5.61	5.47	5.23
	v_3^+	5.15	4.86	4.41	3.91	7.73	7.8	7.29	8.13	7.25	6.83	6.86	7.77
	v_3^-	4.85	4.69	4.45	3.64	7.52	7.99	7.43	7.81	7.18	6.87	7.02	7.43
ρ_3	v_2^+	4.88	4.48	4.48	3.71	6	5.37	5.35	4.63	5.37	5.33	5.64	5.66
	v_2^-	4.88	4.4	4.36	3.9	5.71	5.21	5.74	4.76	6.03	5.45	5.32	5.42
	v_3^+	4.42	4.9	4.63	4.06	7.39	7.7	7.62	8.08	7.43	7.3	7	7.65
	v_3^-	4.95	4.68	4.84	3.61	7.5	8.15	7.66	8.33	7.3	6.8	7.24	7.71
ρ_4	v_2^+	5.02	4.66	4.28	3.87	5.58	5.38	5.39	5.1	5.23	5.97	5.48	5.75
	v_2^-	4.56	4.52	4.64	3.88	5.71	5.52	5.39	4.76	5.38	5.5	5.55	5.79
	v_3^+	4.56	4.75	4.58	3.83	7.47	7.67	7.35	7.92	6.71	7	6.98	7.68
	v_3^-	5	5.06	5.01	3.82	7.65	7.66	7.18	8.01	6.98	7.24	6.84	7.39
ANOMC – Reg _F													
Factor A													
ρ_1	v_2^+	7.9	7.93	13.05	15.66	6.33	7.21	12.86	13.37	9.46	9.83	17.83	18.12
	v_2^-	7.79	8.61	13.5	16.01	7.02	7.52	13.43	13.28	9.15	10.86	17.22	18.09
	v_3^+	16.11	18.39	22.93	32.24	12.29	13.88	19.69	24.7	23.96	26.7	32.54	40.8
	v_3^-	16.51	18.08	23.94	33.04	12.23	13.4	19.51	24.31	23.56	27.44	32.48	40.97
ρ_2	v_2^+	9.65	10.49	17.47	19.28	7.78	9.01	16.04	15.66	12.52	13.18	22.31	23.07
	v_2^-	9.68	11.27	14.1	19.32	8.77	8.94	14.43	15.74	12.47	13.37	18.59	22.26
	v_3^+	21.73	25.25	32.3	40.9	16.2	18.5	26.31	31.11	31.31	35.34	43.11	50.95
	v_3^-	21.12	24.02	25.93	39.97	16	17.64	21.17	30.24	32.85	34.25	37.97	49.76
ρ_3	v_2^+	9.83	10.28	13.09	18.84	8.24	8.99	13.54	15.71	11.94	12.98	18.47	22.02
	v_2^-	10.11	11.21	18.13	19.67	8.52	9	16.25	16.04	12.35	13.8	23.09	22.41
	v_3^+	21.31	24.09	25.57	40.24	15.6	17.62	21.25	30.96	32.44	34.77	38.71	49.32
	v_3^-	21.76	25.93	33.26	40.77	16.61	18.61	27.05	31.15	32.94	35.59	42.1	50.49
ρ_4	v_2^+	11.35	12.09	17.34	23.13	8.29	9.67	15.49	17.6	15.3	17.17	23.6	27.22
	v_2^-	11.12	12.49	16.72	22.51	9.43	9.63	16.25	17.68	14.89	16.7	23.65	26.75
	v_3^+	28.04	33.48	37.78	51.03	20.73	23.59	29.54	38.97	43.71	47.92	52.66	64.64
	v_3^-	29.17	33.58	37.14	50.55	21.17	23.3	29.37	38.62	43.97	47.98	52.04	63.04

Interaction													
ρ_1	v_2^+	5.31	5.32	10.09	8.34	5.75	6.52	13.42	11.45	5.22	5.96	17.64	13.89
	v_7^-	4.96	5.57	9.21	8.43	5.37	6.05	12.49	11	5.27	6.2	16.7	13.58
	v_3^+	4.89	5.23	9.61	8.47	7.5	7.56	14.64	14.96	6.43	7.42	19.03	15.33
	v_3^-	5.54	5.45	10.25	8.35	7.29	7.46	15.52	14.81	5.98	7	18.89	15.55
ρ_2	v_7^+	5.4	5.31	9.76	8.63	6.14	6.18	12.79	11.69	6.19	6.53	18.46	14.07
	v_2^-	5.44	5.52	10.22	8.61	6.21	6.66	14.01	11.72	5.99	7.23	17.7	14.51
	v_4^+	5.61	5.86	9.89	8.36	8.63	9.34	16.77	17.89	6.87	8.35	19.95	17.43
	v_3^-	5.83	5.96	10.03	8.92	8.83	9.5	17.07	17.78	7.5	9.07	20	18.37
ρ_3	v_2^+	5.91	5.73	10.13	8.83	6.41	6.37	13.69	12.03	6.08	6.85	18.6	14.87
	v_7^-	5.44	5.51	10.21	8.73	6.18	6.44	13.29	11.65	6.11	7.58	18.46	15.3
	v_4^+	5.6	6.03	10.01	8.89	8.65	9.12	16.78	17.63	7.78	8.36	20.27	18.81
	v_3^-	5.54	5.82	10.69	8.85	8.13	9.37	16.76	17.39	7.24	8.31	20.31	18.24
ρ_4	v_7^+	5.16	5.54	10.02	8.82	6.15	7.11	13.94	12.37	5.53	6.58	17.44	14.19
	v_7^-	5.27	5.36	8.98	8.44	5.95	6.9	13.47	12.12	5.47	6.71	18.57	14.56
	v_3^+	4.9	5.66	9.81	8.04	9.57	10.78	19.15	20.85	8.1	8.8	21.39	19.92
	v_3^-	5.54	5.69	10.25	8.12	9.98	10.79	19.2	20.21	7.9	9	20.54	19.35
Factor B													
ρ_1	v_7^+	5.01	5.19	9.36	8.43	5.67	6.1	13.16	10.88	5.62	5.88	16.96	13.65
	v_2^-	5.82	5.4	9.59	8.64	5.47	6.17	13.18	11.03	5.51	5.97	16.73	13.69
	v_3^+	5.35	5.52	9.66	8.2	7.47	7.77	14.9	14.34	6.73	7.64	18.78	16.11
	v_3^-	5.08	5.15	10.06	8.5	7.25	7.7	15.22	14.57	5.97	7.07	18.95	15.56
ρ_2	v_2^+	5.59	5.44	9.93	8.85	5.97	6.8	13.01	12.02	5.46	6.98	18.01	14.03
	v_2^-	5.43	5.81	10.38	8.79	6.1	7.31	14.01	12.38	5.74	7.21	17.68	13.76
	v_4^+	5.12	5.95	9.71	8.17	8.68	9.18	16.93	18.07	7.12	8.42	19.92	17.81
	v_3^-	5.61	5.79	10.53	8.68	8.8	10.03	16.57	18.03	7.65	8.69	19.86	18.03
ρ_3	v_2^+	5.54	5.83	10.12	8.31	6.46	6.85	14.63	11.85	6.07	7.09	18.06	13.72
	v_7^-	5.56	5.28	10.31	8.4	6.18	6.74	13.46	11.48	5.95	6.9	18.52	14.55
	v_3^+	5.31	6.07	10.43	8.89	8.96	9.92	17.11	17.15	7.67	8.35	20.61	18.54
	v_3^-	5.73	5.52	10.37	8.53	8.31	9.65	16.63	17.2	7.04	8.52	19.9	17.51
ρ_4	v_7^+	5.25	5.56	10.1	8.58	6.42	6.68	13.36	11.78	5.11	6.74	17.57	14.61
	v_2^-	4.92	4.82	9.24	8.81	6.08	6.62	12.94	11.73	5.6	6.48	18.26	14.41
	v_3^+	4.96	5.35	9.89	7.92	9.84	10.67	18.34	20.83	7.85	9.29	21.12	19.54
	v_3^-	5.19	5.22	9.75	8.49	9.88	10.83	18.43	20.07	8.02	9.48	20.92	18.81
ANOMC – R _F													
Factor A													
ρ_1	v_2^+	7.5	9.17	9.98	17.57	6.18	8.14	9.18	16.94	9.41	11.14	12.45	20.88
	v_7^-	7.81	9.66	11.3	17.77	6.56	8.25	9.83	17.31	9.02	11.29	13.25	21.39
	v_4^+	14.81	17.71	19.04	28.84	11.51	13.79	15.07	24.67	21.42	24.33	27.15	36.72
	v_3^-	15.37	17.44	20.49	29.09	11.77	13.12	16.15	24.22	21.07	24.46	27.72	37.06
ρ_2	v_7^+	9.78	11.73	13.85	21.35	7.86	9.97	11.61	19.6	11.01	13.31	16.43	24.73
	v_7^-	9.99	11.67	12.31	21.94	8.41	9.85	11.41	19.55	11.95	14.7	15.36	24.3
	v_3^+	20.79	24.19	27.99	36.9	15.66	18.39	20.88	30.12	28.15	31.56	35.53	44.69
	v_3^-	20.8	23.01	24.97	35.68	15.37	17.89	19.25	29.66	28.7	32.44	34.73	44.66
ρ_3	v_7^+	8.07	9.25	9.17	16.2	6.31	8.16	8.1	15.03	9.73	12.04	12.06	21.69
	v_2^-	7.98	9.36	12.19	16.46	6.9	7.51	9.74	14.97	9.41	11.2	14.74	20.05
	v_4^+	17.39	19.26	19.89	30.59	13.04	14.84	15.6	25.05	26.32	28.97	30.51	40.06
	v_3^-	17.77	21	24.79	31.34	13.08	14.99	19.16	25.43	25.44	28.93	32.38	39.25
ρ_4	v_2^+	10.67	12.41	13.2	21.22	8.17	9.62	10.13	17.99	14.05	16.34	17.2	24.97
	v_7^-	10.91	12.2	14.18	21.35	8.6	9.8	11.72	17.64	13.6	16.64	18.25	24.16
	v_3^+	27.11	30.39	32.92	43.67	19.93	22.49	23.63	33.37	39.47	42.07	46.18	52.4
	v_3^-	27.17	31.27	33.86	43	19.65	21.8	25.14	32.39	38.77	42.48	45.25	51.31
Interaction													
ρ_1	v_2^+	5.19	6.33	7.36	12.85	5.53	7.85	9.06	18.78	5.61	7.96	10.17	22.55
	v_7^-	5.22	6.24	6.96	13.89	5.76	7.8	8.78	18.31	5.37	8.25	9.78	21.52
	v_3^+	4.84	6.43	6.59	12.8	7.38	9.31	10.64	20.52	5.89	9	11.43	23.74
	v_3^-	5.55	6.57	6.9	12.73	7.26	9.42	10.62	21.19	6.63	9.16	11.52	24.35
ρ_2	v_7^+	5.79	7.51	8.42	14.76	6.84	9.2	9.81	20.34	6.77	9.7	11.37	23.93
	v_2^-	6.22	7.55	8.48	15.04	7.07	9.4	10.68	20.24	6.6	10.11	11.13	23.56
	v_3^+	5.65	7.06	8.38	15.62	9.02	11.88	13.5	24.23	8.36	11.47	13.2	26.35
	v_3^-	6.18	7.89	8.93	15.71	9.67	12.43	13.28	25.19	8.14	11.83	12.95	26.27
ρ_3	v_2^+	4.85	5.15	5.44	9.86	4.91	6.45	7.78	15.1	4.5	6.93	8.66	19.52
	v_2^-	4.36	5.27	5.72	9.88	4.88	6.63	6.6	14.18	4.26	6.86	7.87	19.21
	v_4^+	4.57	5.6	6.18	10.44	6.96	8.63	9.89	17.65	6.17	8.35	10.14	20.81
	v_3^-	4.5	5.6	5.66	10.42	7.11	8.87	9.96	19.36	5.7	8.27	8.98	20.93
ρ_4	v_2^+	4.77	5.76	5.89	10.2	5.99	7.2	8.33	15.91	5.31	7.47	8.59	17.72
	v_2^-	5.08	5.52	6.18	10.23	6.32	7.62	8.2	16.42	5.63	8.11	8.56	18.42
	v_3^+	4.83	5.37	6.03	10.9	9.05	11.58	12.92	21.18	6.81	10.22	11.74	22.11
	v_3^-	4.93	6.24	6.51	10.99	9.65	11.6	12.55	22.47	7.62	9.97	10.98	22.97
Factor B													
ρ_1	v_2^+	4.89	6.09	6.71	12.85	5.4	7.7	8.99	18.44	5.32	7.63	10.24	22.77
	v_7^-	5.1	6.38	7.1	13.44	5.84	7.67	9.28	18.4	5.23	8.06	10.13	22.77
	v_3^+	4.89	6.48	6.44	12.7	7.54	9.33	10.51	20.71	6.51	8.9	11.77	24.07
	v_3^-	5.59	6.15	7.15	13.12	7.02	9.51	11.69	20.78	5.99	9.51	11.51	24.37
ρ_2	v_7^+	5.9	7.64	8.28	14.82	6.54	9.9	10.08	20.8	6.55	9.95	11.55	23.83
	v_2^-	6.21	7.73	8.19	14.87	7.22	9.75	10.97	20.05	6.78	10.17	10.68	23.19
	v_4^+	6.22	7.5	8.11	15.02	9.81	11.6	13.63	23.79	8.5	11.04	12.96	27.24
	v_3^-	6.41	7.57	8.74	15.43	9.33	12.21	13.55	25.27	7.93	12.17	14.28	26.68

ρ_3	v_2^+	4.5	5.13	5.97	9.78	4.96	6.58	7.61	14.95	4.31	7.42	8.48	19.16
	v_2^-	4.67	5.04	5.75	10.34	5.31	6.75	7.94	14.44	4.44	6.78	8.19	18.47
	v_3^+	4.51	5.63	5.98	9.99	6.96	8.47	9.92	17.89	6.09	8.11	10.3	21.15
	v_3^-	4.67	5.32	5.47	9.87	7.09	9.36	11.01	19.47	6.02	8.25	10.67	21.01
ρ_4	v_2^+	4.34	5.62	6.02	10.88	6.03	7.25	8.41	15.13	4.99	7.6	8.28	17.72
	v_2^-	4.92	5.57	5.57	10.2	6.38	7.42	8.7	16.65	5.48	7.59	8.94	18.62
	v_3^+	4.98	5.65	5.97	10.69	8.96	11.74	12.98	21.53	7.24	9.76	11.88	22.26
	v_3^-	5.08	5.9	6.45	10.77	10.01	11.29	13.92	22.24	8.01	9.8	12.34	22.77
ANOMC – MR1_F													
Factor A													
ρ_1	v_2^+	4.89	6.55	6.91	13.35	4.96	6.56	7.2	14.92	4.99	6.99	6.8	15.34
	v_2^-	5.06	6.9	6.89	13.21	5.55	6.68	6.65	14.79	5.34	6.81	6.79	15.34
	v_3^+	5.32	6.29	6.97	14.04	5.12	6.79	6.57	14.04	4.83	6.73	7.12	15.63
	v_3^-	5.33	6.2	7.02	13.48	4.99	6.39	7.18	14.26	4.8	6.8	7.38	15.12
ρ_2	v_2^+	9.15	11.91	11.45	19.03	9.17	12.04	11.43	19.73	9.79	12.22	11.6	21.15
	v_2^-	9.07	11.4	11.94	18.9	9.82	11.14	11.68	18.78	10.07	11.76	11.3	20.15
	v_3^+	9.88	11.6	12.28	19.81	10.08	11.98	12.23	19.71	9.82	12.33	11.64	20.84
	v_3^-	9.83	11.5	12.42	18.36	9.27	11.36	11.12	19.38	8.75	11.92	11.78	19.75
ρ_3	v_2^+	7.73	9.07	9.68	15.74	7.23	8.99	9.9	16.19	7.22	9.7	9.5	18.25
	v_2^-	7.52	9.4	9.91	15.61	8.17	9.99	9.34	15.92	8.15	9.5	9.94	18.05
	v_3^+	7.38	9.49	9.37	15.59	7.32	9.08	9.52	15.94	7.79	9.06	9.35	17.89
	v_3^-	8.25	9.55	9.88	16.13	7.91	9.88	9.68	17.41	7.72	10.93	9.78	17.35
ρ_4	v_2^+	15.87	16.85	18.1	24.05	15.71	17.43	16.77	23.82	15.9	17.61	16.41	24.21
	v_2^-	15.73	17.42	17.58	23.42	15.45	17.45	16.74	24.95	15.58	18.04	17.11	24.56
	v_3^+	15.86	17.66	17.75	23.67	14.88	18.03	17.3	24.29	16.04	17.56	17.22	25.03
	v_3^-	15.84	18.62	17.87	23.08	15.71	17.34	16.76	23.76	15.75	17.51	17.46	24.85
Interaction													
ρ_1	v_2^+	5.25	6.21	7.02	12.86	5.03	6.89	7.98	18.71	5.09	8.45	8.11	21.7
	v_2^-	4.89	6.83	7.05	13.4	4.84	7.13	7.9	17.18	5	8.35	7.77	21.65
	v_3^+	5	6.41	6.9	12.95	5.04	7.03	7.92	17.37	5.31	8.05	8.51	21.93
	v_3^-	5.47	6.16	7.15	13.72	5.07	7.23	7.71	17.31	5.16	7.96	8.24	21.49
ρ_2	v_2^+	9.53	11.97	11.86	18.44	10.45	13.52	13.39	24.14	11.35	15.63	14.92	29.96
	v_2^-	9.22	10.85	12.14	18.07	10.41	13.12	13.39	24.28	11.19	15.44	14.88	27.93
	v_3^+	9.81	11.71	12.51	19.97	11.23	13.31	14.62	23.81	11.8	15.59	15.97	30.62
	v_3^-	9.24	11.19	12.01	18.14	10.77	13	13.62	22.94	10.62	15.57	14.68	28.63
ρ_3	v_2^+	7.45	9.09	9.92	15.17	7.44	10.12	10.98	19.95	8.16	12.07	11.47	25.95
	v_2^-	7.83	8.97	10.75	15.22	8.18	11.17	10.96	19.27	8.7	12.47	12.36	25.19
	v_3^+	7.42	9.06	9.67	15.44	7.81	10.68	10.96	19.84	8.55	11.48	11.51	25.53
	v_3^-	7.89	9.58	9.51	16.23	8.47	11.41	11.16	20.82	8.64	12.83	12.45	25.71
ρ_4	v_2^+	16.08	17.35	17.83	23.35	19.07	21.38	21.18	29.55	21.48	25.41	23.02	35.25
	v_2^-	16.49	17.74	17.28	23.52	18.31	21.43	21	30.05	21.62	25.88	23.24	35.66
	v_3^+	15.46	17.39	18.1	23.97	18.36	21.55	21.48	29.83	21.29	24.54	23.83	35.34
	v_3^-	16.06	18.31	17.28	23.77	19.73	21.86	21.2	29.15	21.6	24.95	23.71	35.59
Factor B													
ρ_1	v_2^+	4.89	6.68	7	13.36	4.91	7.02	8.09	17.9	4.93	7.77	7.95	22.37
	v_2^-	4.8	7.26	7.14	13.25	5.2	7.26	7.46	17.66	4.98	7.94	7.76	21.35
	v_3^+	4.69	6.61	6.59	13.69	5.09	7.16	7.29	16.97	5.29	8.17	7.98	21.93
	v_3^-	4.98	6.65	7.24	13.41	5.13	7.51	7.96	17.15	5.25	8.14	8.16	21.68
ρ_2	v_2^+	9.16	11.77	12.16	19.21	10.39	13.84	13.54	23.57	12.1	15.57	15.04	30.5
	v_2^-	9.23	11.18	11.59	18.66	10.63	13.28	13.38	23.27	10.95	15.57	14.45	28.35
	v_3^+	9.87	11.97	12.31	19.41	11.16	13.38	14.84	23.88	12.07	15.7	14.98	30.81
	v_3^-	9.78	11.49	12.13	18.2	10.63	13.35	13.63	23.19	11.2	15.32	14.73	28.39
ρ_3	v_2^+	7.82	8.98	9.55	15.35	7.79	10.69	11.01	19.88	8.34	12.08	12.11	26.38
	v_2^-	8.11	9.49	9.75	16.01	8.78	10.91	10.72	20.04	8.52	12.33	12.85	25.48
	v_3^+	7.21	8.92	9.8	15.47	7.79	10.64	10.59	19.63	8.76	11.64	11.55	24.49
	v_3^-	8.06	10.04	10.02	15.72	8.88	10.98	11.78	20.64	9.21	13.07	12.19	25.32
ρ_4	v_2^+	15.84	17.06	17.14	23.64	19.39	21.35	20.95	29.81	21.35	24.9	23.21	35.33
	v_2^-	15.71	17.84	16.89	23.39	19.29	21.26	20.95	29.72	21.44	25.47	23.43	35.61
	v_3^+	15.97	17.69	17.56	24.21	18.97	22.16	21.9	29.22	20.29	24.46	23.75	35.57
	v_3^-	16.4	17.38	18.06	23.25	20.04	21.52	21.46	29.08	21.37	25.2	23.36	36.09
ANOMC – MR2_F													
Factor A													
ρ_1	v_2^+	6.43	7.73	6.53	14.73	5.96	6.87	5.24	13.49	6.86	7.69	6.27	15.55
	v_2^-	5.71	7.01	10.3	13.9	5.81	6.26	8.28	12.72	6.57	7.42	9.1	14.77
	v_3^+	11.54	13.32	10.25	23.05	9.33	10.31	8.6	19.61	12.81	14.46	13.07	26.48
	v_3^-	10.33	11.69	16.46	21.22	7.53	8.77	12.74	17.7	10.71	13.64	16.79	24.46
ρ_2	v_2^+	7.11	8.34	6.33	14.45	6.21	7.38	6.21	12.89	7.94	7.6	6.84	15.93
	v_2^-	6.07	6.63	10.08	14.18	5.56	6.79	8.5	12.76	6.7	7.69	9.21	14.34
	v_3^+	12.03	13.66	10.79	23.62	9.56	10.11	8.85	18.96	13	15.12	13	26.68
	v_3^-	9.89	11.6	16.36	22.13	7.72	9.12	12.77	18.31	12.14	13.41	16.69	25.2
ρ_3	v_2^+	6.5	7.94	5.93	13.65	5.45	6.83	5.16	12.55	6.84	7.73	6.14	15.76
	v_2^-	5.49	6.44	10.06	14.1	5.35	5.94	7.67	12.14	6.43	7.16	9.14	14.56
	v_3^+	10.75	13.04	10.22	22.47	8.39	10.24	7.98	18.02	12.67	14.21	12.88	25.61
	v_3^-	9.43	11.21	16.27	21.72	7.88	8.69	12.78	17.79	11.48	13.69	16.59	24.86
ρ_4	v_2^+	7.37	7.67	6.36	13.97	6.25	6.68	6	12.71	7.44	8.57	6.49	15.08
	v_2^-	6.39	7.55	9.58	14.06	5.61	6.35	8.47	12.95	6.58	7.91	9.12	14.26
	v_3^+	10.95	13.27	10.24	23.61	9.16	10.66	8.59	19	12.69	14.98	12.68	26.04
	v_3^-	9.46	12.04	16	21.68	7.85	9.84	12.76	18.68	12.27	13.41	17.25	25.65

Interaction													
ρ_1	v_2^+	4.81	5.7	6.24	11.01	5.26	5.57	6	13.13	4.99	5.39	5.56	13.43
	v_2^-	4.88	5.61	6.81	10.93	5.42	5.7	5.57	13.67	5.37	5.57	4.72	12.96
	v_3^+	5.03	5.48	6.29	10.64	6.22	7.28	7	15.61	5.3	5.76	5.44	14.31
	v_3^-	5.19	5.79	6.13	11.06	5.99	6.64	5.69	15.37	5.3	5.82	5.07	14.51
ρ_2	v_2^+	5.26	6.15	5.85	11.18	5.52	6.17	6.31	13.7	6.04	5.99	5.43	13.9
	v_2^-	5.23	5.99	6.58	11.06	5.55	6.7	5.97	13.53	6.05	6.51	5.14	13.27
	v_3^+	5.45	6.01	6.48	11.54	6.47	7.19	6.97	14.76	6.2	6.41	6.11	14.59
	v_3^-	5.19	5.84	6.28	11.56	6.66	7.71	6.39	15.85	6.51	6.7	5.53	14.42
ρ_3	v_2^+	5.23	5.48	6.4	10.44	4.84	5.51	6.02	13.42	5.46	5.37	5.48	13.85
	v_2^-	4.83	4.98	6.4	10.71	4.97	6.08	5.65	12.85	5.34	5.57	4.69	13.25
	v_3^+	4.83	5.22	6.05	10.56	5.56	6.93	6.67	14.31	5.6	5.83	5.86	14.09
	v_3^-	4.94	5.67	6.42	10.53	6.31	7.13	5.92	15.76	5.52	6.21	5.32	14.58
ρ_4	v_2^+	5.55	5.32	6.58	10.35	5.55	6.47	6.49	14.06	5.58	6.38	5.26	13.17
	v_2^-	5.59	6.09	6.02	10.85	5.42	6.44	5.94	14.28	5.47	7.01	5.47	13.7
	v_3^+	5.32	5.85	6.45	10.95	6.34	7.14	6.67	15.23	6.26	6.46	6.35	14.53
	v_3^-	5.29	5.99	6.75	11.02	6.7	7.83	6.38	15.13	6.34	6.6	5.27	14.48
Factor B													
ρ_1	v_2^+	4.96	5.59	5.94	10.96	5.27	5.94	5.86	13.97	5.15	5.53	5.39	13.19
	v_2^-	5.08	5.72	6.63	11.26	5.33	5.31	7.45	12.83	5.12	5.39	5.58	13.04
	v_3^+	5.25	5.7	6.23	10.93	6.37	6.86	6.65	14.77	5.26	6.17	5.55	14.85
	v_3^-	4.82	5.61	5.99	11.32	5.38	5.91	9.09	14.86	4.97	5.59	7.27	14.39
ρ_2	v_2^+	5.29	5.74	6.33	11.1	5.3	6.34	6.55	13.14	6.02	6.31	5.67	13.62
	v_2^-	5.33	5.82	6.41	11.06	5.42	6.19	7.57	13.4	5.77	5.97	6.18	13.34
	v_3^+	5.77	6.07	6.37	11.24	6.73	7.04	7.55	14.82	6.14	6.53	5.91	15.11
	v_3^-	5.54	5.64	6.42	11.11	5.79	7.09	9.28	15.46	5.95	6.25	6.96	14.18
ρ_3	v_2^+	4.99	5.33	5.91	10.28	4.68	6.01	5.97	13.07	5.42	5.65	5.23	13.66
	v_2^-	5.02	5.53	6.38	11.07	4.72	5.56	7.33	12.74	5.02	4.93	5.54	13.05
	v_3^+	4.58	4.99	5.75	10.23	5.73	6.46	6.29	14.71	5.82	5.88	5.76	14.24
	v_3^-	4.93	5.12	6.07	10.85	5.25	6.4	9.36	14.89	5.23	6.06	6.91	14.38
ρ_4	v_2^+	5.88	5.46	6.25	10.21	5.33	6.57	6.62	13.3	5.86	6.27	5.52	13.1
	v_2^-	5.52	5.93	6	10.65	5.14	6.11	7.62	13.82	5.24	6.21	6.18	13.13
	v_3^+	4.99	6.03	6.31	10.94	6.38	7.58	7.05	15.1	6.06	6.32	6.16	14.54
	v_3^-	5.12	6.39	6.85	11.18	5.9	6.84	8.92	14.92	6.35	5.94	6.97	14.44
ANOMC – MR3 _F													
Factor A													
ρ_1	v_2^+	7.47	9.2	10.45	17.4	6.2	8.1	9.42	16.87	9.47	11.37	12.8	21
	v_2^-	7.55	9.61	11.63	17.56	6.63	8.38	10.17	17.24	9.15	11.38	13.59	21.37
	v_3^+	14.68	17.54	19.5	28.65	11.38	13.73	15.49	24.64	21.57	24.5	27.77	36.74
	v_3^-	15.16	17.16	20.73	28.71	11.69	13.18	16.35	24.22	20.99	24.65	28.33	37.14
ρ_2	v_2^+	9.14	10.94	13.62	20.85	7.7	10.14	11.85	18.45	10.78	12.88	16.27	24.54
	v_2^-	10.3	11.97	13.17	22.01	8.56	10.32	11.7	19.77	12.37	15.14	16.15	24.41
	v_3^+	19.48	22.72	27.14	35.79	15.73	17.21	21.11	28.85	26.96	30.55	35.16	44.26
	v_3^-	21.39	23.73	26.01	36	16.25	19.38	20.81	29.88	29.68	33.41	36.21	45.31
ρ_3	v_2^+	8.33	9.68	9.76	16.28	7.11	8.37	8.04	15.36	10.46	12.48	12.62	22.1
	v_2^-	7.5	8.58	12.01	16.02	6.14	7.58	9.65	14.77	9.08	10.83	14.6	19.87
	v_3^+	18.31	20.07	20.77	30.81	13.49	15.59	16.88	25.04	27.47	29.94	31.71	40.43
	v_3^-	16.49	19.64	23.86	30.68	12.71	14.4	18.86	25.25	24.48	28.06	31.94	38.55
ρ_4	v_2^+	10.59	12.23	13.3	20.99	8.46	10.12	10.5	17.58	14.06	16.22	17.3	24.96
	v_2^-	10.73	12.05	14.37	21.15	8.12	9.84	11.57	17.9	13.62	16.54	18.25	24.18
	v_3^+	27.02	30.35	32.85	43.24	19.84	22.24	24.35	33.58	39.48	42.29	46.03	52.33
	v_3^-	27.06	30.91	33.77	42.33	20.1	22.33	24.84	33.61	39.18	42.58	45.06	51.21
Interaction													
ρ_1	v_2^+	5.25	6.46	7.47	12.86	5.5	7.7	9.21	18.63	5.62	8.07	10.59	22.58
	v_2^-	5.1	6.09	7.14	13.76	5.5	7.87	9.03	18.23	5.35	8.38	10.04	21.42
	v_3^+	5	6.37	6.74	12.66	7.42	9.15	10.85	20.48	5.9	8.97	11.83	23.74
	v_3^-	5.48	6.25	7.08	12.71	6.96	9.51	10.83	20.96	6.69	9.14	11.64	24.16
ρ_2	v_2^+	5.76	7.42	8.55	14.47	6.76	8.83	9.75	19.98	6.65	9.53	11.57	23.64
	v_2^-	6.2	7.53	8.84	15.08	6.93	9.33	11.04	20.07	6.77	10.26	11.33	23.74
	v_3^+	5.51	6.71	8.39	15.14	8.51	11.69	13.34	23.48	7.93	11.13	13.2	25.8
	v_3^-	6.44	8.15	9.3	15.62	9.68	12.43	13.84	25.15	8.57	12.17	13.79	26.88
ρ_3	v_2^+	4.81	5.3	5.64	9.82	5.02	6.36	8.06	14.93	4.64	7.12	8.93	19.5
	v_2^-	4.19	5.14	5.79	9.54	4.71	6.44	6.73	13.8	4.3	6.79	7.91	18.91
	v_3^+	4.89	5.9	6.7	10.7	7.15	8.78	10.07	17.8	6.6	8.79	10.82	21.16
	v_3^-	4.4	5.38	5.4	9.96	6.85	8.47	9.59	18.88	5.44	8.04	8.88	20.69
ρ_4	v_2^+	4.71	5.76	5.76	10.15	5.8	7.27	8.26	15.63	5.33	7.57	8.67	17.63
	v_2^-	4.91	5.49	6.02	10.32	6.13	7.32	7.95	16.44	5.62	8.04	8.51	18.32
	v_3^+	4.82	5.5	6.17	10.94	8.93	11.38	12.74	21.1	6.93	10.36	11.86	21.95
	v_3^-	4.74	6.08	6.5	10.9	9.54	11.32	12.32	22.06	7.55	9.79	10.95	22.71
Factor B													
ρ_1	v_2^+	4.83	6.05	6.9	12.87	5.47	7.41	9.06	18.19	5.23	7.65	10.53	22.9
	v_2^-	5.04	6.46	7.3	13.57	5.66	7.53	9.45	18.23	5.38	7.99	10.47	22.65
	v_3^+	4.81	6.41	6.74	12.78	7.3	9.3	11.01	20.58	6.45	8.87	12.05	24.12
	v_3^-	5.6	6.06	7.25	13.08	6.85	9.43	11.74	20.7	6.12	9.42	11.92	24.29
ρ_2	v_2^+	5.61	7.38	8.45	14.75	6.29	9.64	10.14	20.46	6.32	9.7	11.51	23.69
	v_2^-	6.11	7.72	8.55	14.78	7.14	9.6	11.36	19.8	6.9	10.38	11.31	23.61
	v_3^+	5.92	7.29	8.2	14.63	9.36	11.26	13.48	23.24	8.05	10.7	12.91	26.93
	v_3^-	6.44	7.75	9.32	15.42	9.46	12.32	14.07	25.31	8.53	12.48	14.81	27.11

ρ_3	v_2^+	4.65	5.32	5.97	9.88	4.96	6.65	7.77	14.83	4.61	7.63	8.75	19.35
	v_2^-	4.51	5.01	5.76	10.19	5.12	6.7	7.79	14.27	4.34	6.7	8.11	18.12
	v_3^+	4.68	5.69	6.17	10.14	7.34	8.8	10.38	17.81	6.54	8.49	10.94	21.29
	v_3^-	4.57	5.01	5.43	9.52	6.67	8.87	10.64	19.04	5.84	7.97	10.51	20.86
ρ_4	v_2^+	4.32	5.58	6.11	10.73	6.16	7.15	8.34	15.1	5.11	7.74	8.22	17.89
	v_2^-	4.93	5.37	5.67	10.24	6.15	7.18	8.88	16.46	5.46	7.51	9	18.62
	v_3^+	4.97	5.69	6.11	10.58	8.89	11.68	12.78	21.31	7.35	9.88	11.93	22.29
	v_3^-	4.97	5.71	6.48	10.66	9.82	11.2	13.81	22.05	7.96	9.67	12.2	22.83
ANOMC – MR4 _F													
Factor A													
ρ_1	v_2^+	6.79	8.43	8.15	16.27	6.65	8.1	7.22	15.99	8.61	10.15	10.29	20.04
	v_2^-	6.58	8.7	11.13	16.43	6.22	7.36	9.55	16.15	8.03	10	12.24	19.98
	v_3^+	13.7	15.67	14.27	26.46	11.05	13.1	11.43	23.35	18.2	21.27	22.15	33.46
	v_3^-	12.76	15.07	18.43	26.44	9.82	11.91	15.1	22.42	17.62	20.87	24.59	33.96
ρ_2	v_2^+	5.03	5.87	5.8	13.22	4.39	5.55	5.36	12.22	5.84	7.05	8.08	16.56
	v_2^-	5.12	6.35	7.21	13.65	4.41	5.32	6.56	12.51	6.53	8.15	8.78	16.21
	v_3^+	10.55	12.61	13.1	23.96	7.98	9.5	10.15	20.02	15.27	17.98	19.98	31.88
	v_3^-	11.13	12.76	15.22	24.29	8.34	11.14	12.43	20.54	16.45	19.85	22.24	32.63
ρ_3	v_2^+	5.5	6.35	5.11	12.23	4.84	5.76	4.62	12.1	6.47	7.85	7.46	17.47
	v_2^-	4.4	5.13	8.15	12.73	3.98	4.88	6.45	11.81	5.58	6.81	9.79	15.72
	v_3^+	11.37	13.74	11.5	24.32	8.63	10.9	9.37	20.2	17.4	20.31	19.92	33.83
	v_3^-	9.31	11.56	16.18	23.51	7.22	8.82	12.91	20.05	14.91	18.48	22.07	32.09
ρ_4	v_2^+	2.99	3.53	3.51	8.96	2.61	2.68	3	8.54	4.17	5.55	5.17	12.91
	v_2^-	2.67	3.52	4.61	9.67	2.44	3.22	3.78	8.54	3.79	5.3	6.51	12.54
	v_3^+	7.83	10.2	10.02	22.41	5.66	7.2	7.52	16.94	12.96	16.64	18.12	31.89
	v_3^-	7.35	10.03	11.59	22.33	5.4	7.47	8.92	17.02	13.15	16.73	18.84	31.25
Interaction													
ρ_1	v_2^+	4.9	5.8	6.4	11.96	5.72	7.11	6.79	16.31	5.65	7.09	8.68	19.85
	v_2^-	4.98	6.13	6.58	13.04	5.77	7.38	7.16	15.75	5.26	7.61	7.51	18.88
	v_3^+	5.14	5.82	6.24	12.04	7.26	8.68	8.25	17.96	6.03	7.99	9.67	20.45
	v_3^-	5.27	5.91	6.15	11.97	6.44	8.81	7.99	19.13	6.39	8.15	8.7	20.6
ρ_2	v_2^+	2.6	3.92	4.01	8.92	3.15	4.48	4.36	11.85	3.17	4.39	4.79	14.58
	v_2^-	3	4.1	4.38	9.7	3.26	4.61	4.83	12.15	3.47	4.76	4.92	14
	v_3^+	2.91	3.7	4.05	9.38	4.11	5.56	5.85	13.96	3.51	4.87	5.43	15.2
	v_3^-	3.04	4.14	4.71	9.88	4.42	6.13	6.03	15.67	4.1	5.57	5.64	15.74
ρ_3	v_2^+	3.14	3.79	3.8	7.78	3.12	4.25	4.17	11.6	2.82	4.21	4.95	14.64
	v_2^-	2.91	3.19	3.93	7.52	3.2	4.13	3.25	10.49	3	4	4.28	13.56
	v_3^+	3.05	3.65	4.12	8.33	4.08	5.28	5.3	13.73	3.93	5.55	5.73	15.81
	v_3^-	3.02	3.35	3.6	7.91	3.84	5.41	4.41	13.86	4.04	5.26	4.45	15.6
ρ_4	v_2^+	1.21	1.49	1.7	4.47	1.35	1.8	1.66	6.67	1.27	1.87	1.94	7.44
	v_2^-	1.33	1.6	1.8	4.54	1.53	2.02	1.85	6.5	1.23	2.03	1.98	8.25
	v_3^+	1.13	1.62	1.77	4.95	2.17	3.44	2.96	9.52	1.76	2.63	2.82	9.75
	v_3^-	1.05	1.82	1.77	5.1	2.08	3.32	2.77	9.51	1.89	2.45	2.31	9.71
Factor B													
ρ_1	v_2^+	5.02	5.93	5.94	11.82	5.41	7.08	6.93	16.38	5.26	7.07	8.33	20.05
	v_2^-	4.87	6.48	6.41	12.44	5.35	6.7	8.38	16.43	5.15	7.11	8.29	19.37
	v_3^+	4.84	5.82	6.28	11.62	7.17	8.62	8.41	18.07	6.19	7.95	9.43	20.99
	v_3^-	5.33	6.02	6.06	12.47	6.07	7.96	10.35	18.33	6.05	7.99	10.25	20.76
ρ_2	v_2^+	2.92	3.62	4.12	9.02	3.21	4.82	4.35	12.47	3.31	4.54	4.85	13.87
	v_2^-	3.09	4.19	4.16	9.26	3.33	4.46	5.14	11.49	3.18	4.85	4.68	13.7
	v_3^+	2.94	3.49	3.98	9.17	4.17	5.23	5.92	14.17	3.69	4.84	5.41	16.31
	v_3^-	2.98	3.88	4.53	9.46	4.51	6.01	6.52	15.39	3.92	5.42	6.6	15.64
ρ_3	v_2^+	3.17	3.55	3.73	7.61	3.36	4.04	4.49	11.49	2.91	4.23	5.17	14.08
	v_2^-	2.86	3.31	3.86	7.78	2.9	3.6	4.96	10.81	3.13	4.34	4.97	13.49
	v_3^+	2.65	3.64	4.09	8.16	4.41	5.28	5.56	13.77	3.97	5.62	5.88	16.02
	v_3^-	2.92	3.38	3.52	7.75	3.6	5.03	7.09	13.94	3.27	4.89	6.49	15.37
ρ_4	v_2^+	1.16	1.35	1.8	4.53	1.4	1.71	1.91	6.38	1.22	1.96	1.82	7.17
	v_2^-	1.45	1.67	1.44	4.65	1.33	1.75	2.25	6.36	1.09	1.96	2.34	8.1
	v_3^+	1.25	1.57	1.86	5.13	2.3	3.17	2.78	9.31	1.54	2.59	2.73	9.68
	v_3^-	1.28	1.62	1.68	4.96	2.11	2.94	4.04	9.27	1.78	2.03	2.75	9.8

Table 4. 9: Type I error (%) of ANOM and ANOMC tests under heterogeneous variances and n_2 .

(s, kr)		(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)	(0,0)	(0,3)	(2,6)	(0,25)
t = 2, r = 2						t = 2, r = 3				t = 2, r = 5			
ANOM _F													
ρ		Factor A											
ρ ₁	v ₂ ⁺	11.44	12.23	12.16	14.18	9.16	9.99	9.77	10.04	16.05	16.65	16.71	17.75
	v ₂ ⁻	12.27	12.53	12.32	13.89	9.48	9.61	10.38	10.78	16.54	16.09	16.62	17.97
	v ₃ ⁺	31.25	31.74	33.04	38.02	23.51	23.77	23.58	27.72	47.52	48.35	49.12	50.12
	v ₃ ⁻	30.73	31.72	32.64	37.58	23.72	23.56	23.87	27.6	47.85	49.15	48.55	50.74
ρ ₂	v ₂ ⁺	11.07	11.52	12.02	14.2	9.26	9.79	9.68	10.65	17.11	16.14	17.05	18.4
	v ₂ ⁻	11.7	11.78	11.63	13.89	10.13	10.12	9.62	11.13	16.51	16.29	15.92	17.4
	v ₃ ⁺	31.52	32.4	32.94	37.64	23.14	24.44	23.69	26.54	46.64	48.15	47.53	50.42
	v ₃ ⁻	31.38	31.97	33.2	38.27	23.68	24.12	23.52	26.71	47.19	47.23	48.65	51.52
ρ ₃	v ₂ ⁺	11.47	12.4	11.97	13.81	9.48	9.94	9.37	10.27	16.05	16.37	16.21	17.56
	v ₂ ⁻	12.18	12.08	11.93	14.52	9.66	10.21	9.67	10.5	16.32	15.78	16.09	18.28
	v ₃ ⁺	31.41	32.48	32.15	37.47	23.95	24.43	24.8	27.82	47.41	49.04	48.38	51.3
	v ₃ ⁻	30.97	32.29	33.06	37.88	23.61	24.15	23.85	27.37	47.02	48.01	49.09	50.63
ρ ₄	v ₂ ⁺	11.82	11.92	11.94	13.78	9.89	10.12	9.6	10.75	16.25	16.73	16.65	17.63
	v ₂ ⁻	12.31	11.99	11.81	13.68	9.94	9.61	9.32	10.75	16.13	16.67	17.06	17.22
	v ₃ ⁺	31.35	32.3	32.28	37.25	23.13	23.23	24.02	27.04	47.84	48.28	49.47	50.5
	v ₃ ⁻	30.7	33.24	32.34	37.63	23.72	24.12	23.19	27.67	47.19	47.7	47.92	50.5
Interaction													
ρ ₁	v ₂ ⁺	5.22	5.28	4.46	4.39	6.47	6.21	6.47	5.86	6.09	6.34	6.02	6.01
	v ₂ ⁻	4.97	4.82	4.97	4.46	6.48	6.28	6.04	5.88	6.03	6.02	6.59	6.16
	v ₃ ⁺	5.17	5.19	4.66	4.63	10.86	10.71	10.57	11.83	9.42	9.71	9.61	10.16
	v ₃ ⁻	5.21	4.82	4.46	4.75	10.48	11.45	10.61	11.94	9.4	9.4	9.37	10.02
ρ ₂	v ₂ ⁺	5.46	4.95	4.85	4.18	6.19	6.28	6.34	6.16	6.3	5.91	6.18	6.41
	v ₂ ⁻	5.02	5.13	4.48	4.31	6.56	6.22	6.28	5.99	5.72	6.11	6.08	6.14
	v ₃ ⁺	5.24	5.37	4.89	4.19	10.87	10.7	10.45	11.75	9.43	9.67	9.5	10.01
	v ₃ ⁻	4.93	5.12	4.83	4.48	10.57	9.83	10.75	11.15	9.4	9.36	9.69	10.05
ρ ₃	v ₂ ⁺	5.04	5.5	4.83	4.18	6.64	6.14	6.57	6.1	6.23	6.25	6.22	6.3
	v ₂ ⁻	4.9	4.75	5.08	4.62	6.53	6.35	6.45	6.23	6.04	6.36	6.08	6.64
	v ₃ ⁺	4.72	4.95	4.87	4.58	10.99	10.4	10.98	11.6	9.38	9.54	9.68	10.84
	v ₃ ⁻	5.32	5.26	5.25	4.71	10.74	10.14	11.23	11.97	8.38	9.9	10.1	10.58
ρ ₄	v ₂ ⁺	5.14	5.01	4.54	4.38	6.42	5.84	6.22	6.22	6.2	5.67	5.86	5.87
	v ₂ ⁻	5.2	4.67	5.03	4.41	6.35	6.32	6.14	6.35	5.52	6.15	6.04	5.95
	v ₃ ⁺	5.07	5.03	5.24	4.27	10.6	10.7	10.36	12.04	9.1	9.32	9.41	10.07
	v ₃ ⁻	5.36	5.4	5.08	4.65	10.77	10.77	10.69	11.74	9.54	9.29	9.65	10.05
Factor B													
ρ ₁	v ₂ ⁺	5.32	5.02	4.66	4.52	6.33	6.22	6.01	6.5	6.17	6.07	5.98	6.23
	v ₂ ⁻	5.32	5.32	4.72	4.55	6.13	5.84	6.19	5.97	6.17	6.35	5.88	6.04
	v ₃ ⁺	5.05	5.18	4.98	4.43	10.98	10.87	10.91	11.65	9.27	9.79	9.08	10.52
	v ₃ ⁻	5.28	4.88	4.73	4.43	10.79	10.8	10.48	11.83	9.07	9.66	9.33	10.48
ρ ₂	v ₂ ⁺	5.67	5.04	5.04	4.33	6.04	6.44	6.13	6.14	6.13	6.31	5.86	6.48
	v ₂ ⁻	4.95	4.87	4.87	4.58	6.16	6.06	6.26	5.81	5.86	5.96	6.05	6.47
	v ₃ ⁺	5.04	5.29	4.69	4.51	10.1	10.13	10.67	12.13	8.99	9.65	9.91	10.39
	v ₃ ⁻	5	5.28	5.08	4.54	10.38	11.22	11.37	11.9	9.56	9.22	9.58	10.55
ρ ₃	v ₂ ⁺	4.97	5.27	5.18	4.53	6.76	6.5	5.71	6.38	6.35	5.94	6.24	6.19
	v ₂ ⁻	5.15	5.21	5.04	4.42	6.1	6.05	6.11	6.1	5.66	5.55	5.82	6.47
	v ₃ ⁺	5.06	5.09	4.79	4.53	10.73	11.12	10.88	11.88	9.42	9.76	9.53	10.11
	v ₃ ⁻	4.87	5.07	4.79	4.41	10.67	11.2	10.85	11.7	9.35	9.44	9.61	10.22
ρ ₄	v ₂ ⁺	5.28	4.94	4.98	4.38	6.55	6.16	5.93	6.23	5.84	6.06	6	6.48
	v ₂ ⁻	5.38	4.96	4.84	4.44	6.34	6.33	5.59	5.64	6.04	6.42	5.96	6.28
	v ₃ ⁺	5.06	4.97	4.77	4.64	10.19	11.49	10.39	11.7	10.01	9.31	9.48	10.42
	v ₃ ⁻	5.58	5.04	4.74	4.53	10.52	10.98	10.48	11.97	9.85	8.77	9.09	9.89
ANOMC – Reg _F													
Factor A													
ρ ₁	v ₂ ⁺	12.58	14.5	15.82	21.34	9.89	11.49	13.15	16.53	17.65	19.74	22.12	27.38
	v ₂ ⁻	13.58	14.35	16.83	21.39	10.08	11.24	13.48	16.45	18.67	19.79	22.63	27.91
	v ₃ ⁺	36.25	38.13	41.39	50.91	27.68	29.09	30.94	39.31	54.22	56.77	58.16	64.8
	v ₃ ⁻	35.28	38.28	40.42	51.15	27.15	28.42	31.37	40.29	53.91	57.08	58.62	65.63
ρ ₂	v ₂ ⁺	16.66	18.44	22.93	28.2	13.56	14.65	18.75	21.42	24.9	25.57	30.5	35.5
	v ₂ ⁻	16.64	18.65	18.02	28.11	13.63	14.71	15.23	20.95	23.34	26.04	26.57	34.5
	v ₃ ⁺	48.03	49.89	53.69	62.24	35.84	38.42	41.6	49.39	67.98	70.31	70.32	76.7
	v ₃ ⁻	47.2	49.51	50.34	61.39	35.81	37.94	38.52	48.9	66.13	68.26	71.19	76.27
ρ ₃	v ₂ ⁺	16.69	18.48	18.21	27.35	13.51	14.53	14.7	21.2	24.3	26.04	27.22	34.56
	v ₂ ⁻	17.31	19.17	22.91	27.59	13.51	15.32	18.83	21.28	24.27	25.97	30.01	35.41
	v ₃ ⁺	47.37	49.53	49.77	60.96	36.2	37.94	38.72	49.18	67.8	69.67	70.89	76.12
	v ₃ ⁻	46.4	49.7	54.16	62.17	35.39	38.48	41.43	49.03	67.43	70.14	71.09	76.03
ρ ₄	v ₂ ⁺	22.12	24.18	26.13	34.86	16.81	17.97	20.8	26.15	32.66	35.64	38.03	44.83
	v ₂ ⁻	22.59	24.52	25.91	34.86	16.63	18.18	20.28	26.19	32.13	35.5	37.42	44.07
	v ₃ ⁺	63.85	67.03	67.36	74.06	48.55	50.76	53.19	61.98	84.2	85.55	86.12	89.05
	v ₃ ⁻	63.7	66.47	67.59	74.23	48.55	51.95	53.57	63.1	84.36	85.47	86.16	89.12

Interaction													
ρ_1	v_2^+	4.78	5.38	6.03	6.74	7.1	7.17	9.91	11.17	6.13	7.24	10.84	13.29
	v_2^-	4.98	5.25	6.97	6.53	7.1	7.57	10.29	11.18	5.88	7.33	11.48	12.74
	v_3^+	5.57	5.25	6.82	6.84	12.19	13.54	15.56	20.71	9.84	11.44	16.46	19.77
	v_3^-	4.98	5.03	6.96	6.91	12.13	13.35	16.67	20.36	9.81	11.59	15.54	19.88
ρ_2	v_2^+	5.42	5.76	7.76	6.98	7.72	8.9	11.04	13.09	7.02	8.23	12.41	14.66
	v_2^-	5.05	5.54	6.75	7.17	8.08	8.65	11.52	12.77	7.2	8.41	12.41	14.27
	v_3^+	5.11	5.53	7.14	6.84	16.59	17.53	20.32	26.37	12.98	14.91	18.94	24.42
	v_3^-	5.39	5.89	7.12	7.5	15.3	16.77	20.82	25.74	13.25	14.74	20.57	24.83
ρ_3	v_2^+	5.1	6.18	6.84	6.81	8.61	8.75	11.61	13.32	7.38	8.73	12.36	15.36
	v_2^-	5.05	5.18	7	7.23	7.86	8.53	11.43	13.05	7.16	8.1	12.17	15.08
	v_3^+	5.46	5.99	7.06	7.01	15.91	18.17	20.24	26.55	13.31	15.15	19.38	25.34
	v_3^-	5.18	5.62	6.83	6.75	15.6	17.15	20.87	26.12	12.5	14.98	19.63	24.4
ρ_4	v_2^+	5.11	5.3	7.02	6.58	8.59	9.46	12.55	14.84	7.52	8.28	12.57	15
	v_2^-	5.2	4.97	6.44	6.72	8.88	9.72	12.47	14.71	6.96	8.81	13.23	15.34
	v_3^+	4.85	4.96	6.74	6.57	20.29	22.68	25.74	34.57	17.71	19.04	24.24	31.06
	v_3^-	5.23	5.29	7.03	6.66	20.16	22.28	26.29	34.04	17.4	19.61	24.94	31.2
Factor B													
ρ_1	v_2^+	4.9	5.29	6.47	6.81	6.96	7.52	9.32	11.41	6.41	7.24	10.51	12.73
	v_2^-	5.1	5.42	6.78	6.91	7.1	7.15	9.71	11.25	6.39	7.16	11.06	13.11
	v_3^+	5.06	4.96	6.99	6.81	12.63	13.81	16.3	20.49	10	11.27	15.36	19.86
	v_3^-	5.11	5.15	7.06	6.35	11.88	13.18	16.15	20.36	9.54	11.55	15.77	19.61
ρ_2	v_2^+	5.51	5.31	7.21	6.77	7.83	8.72	11.67	12.75	6.87	8.44	12.04	14
	v_2^-	5.4	5.37	6.9	7.64	8.05	8.93	11.51	12.36	7.05	8.36	12.19	15.27
	v_3^+	4.72	5.22	7.16	7.32	15.59	17.05	20.9	27.16	12.78	14.84	19.91	24.66
	v_3^-	5.69	5.93	7.53	7.57	16.04	17.43	20.75	26.71	12.86	14.27	20.04	25.03
ρ_3	v_2^+	5.15	6.04	7.26	6.95	8.53	8.95	11.22	13.36	7.6	8.56	12.08	15.01
	v_2^-	5.04	5.6	7.57	6.84	8.06	8.54	11.45	12.67	7.22	7.95	11.52	15.11
	v_3^+	5.46	5.83	7.51	7.1	16.32	17.54	20.99	27.11	13.35	14.88	20.09	25.09
	v_3^-	5.04	5.26	6.97	6.83	15.46	17.69	20.5	25.94	13.05	15.2	19.59	24.65
ρ_4	v_2^+	5.29	5.27	6.62	6.47	8.49	9.47	12.32	15.55	7.17	9.16	13.07	15.38
	v_2^-	4.96	5.37	6.56	6.7	8.86	9.65	12.45	14.78	7.35	8.34	13.24	15.44
	v_3^+	5.19	5.19	7	6.94	20.48	22.22	25.16	34.56	17.74	19.95	25.91	31.89
	v_3^-	5.09	5.05	6.87	6.88	20.47	22.76	26.72	34.05	17.1	19.09	25.13	31.01
ANOMC – R _F													
Factor A													
ρ_1	v_2^+	10.42	12.54	13.26	18.93	8.58	10.14	11.07	17.06	14.93	16.71	18.14	24.07
	v_2^-	11.56	12.17	14.06	19.27	9.43	10.64	11.68	17.14	14.5	16.62	18.38	24.67
	v_3^+	28.27	29.96	32.74	39.43	21.87	23.6	24.65	31.54	42.62	44.76	47.31	52.06
	v_3^-	28.71	29.38	32.51	38.27	21.2	23.33	25.1	31.51	42.06	45.08	47.26	51.97
ρ_2	v_2^+	15.23	16.87	19	25.09	11.35	13.25	14.9	20.31	19.41	20.97	23.65	29.99
	v_2^-	13.91	16.23	16.54	24.4	11.7	12.88	13.24	19.61	19.4	22.2	23.01	30.93
	v_3^+	39.98	41.5	43.91	49.78	29.54	31.19	34.47	40.46	56.75	57.98	59.62	63.02
	v_3^-	38.27	40.22	40.96	49.06	28.34	31.02	32.08	39.44	55.44	57.21	59.72	63.67
ρ_3	v_2^+	12.32	13.95	13.41	19.63	10.26	11.3	11.05	17.24	18.77	19.46	20.36	25.97
	v_2^-	13.67	14.56	16.48	19.69	10.07	11.58	13.31	16.58	17.92	19.23	21.83	26.71
	v_3^+	35.69	37.46	38.38	44.69	27.02	28.51	28.74	35.42	54.87	55.81	57.84	58.53
	v_3^-	36.74	38.63	41.89	45.86	25.97	28.84	30.95	35.63	53.73	54.6	57.75	57.39
ρ_4	v_2^+	19.78	20.86	22.11	28.09	14.78	15.67	17.33	21.16	28.1	30.24	31.74	35.55
	v_2^-	20.3	21.16	21.74	27.98	14.85	16.32	17.11	20.87	28.3	30.24	30.94	35.35
	v_3^+	57.98	58.75	59.34	60.86	42.23	42.86	44.84	49.27	77.05	76.91	77.47	73.94
	v_3^-	56.58	57.8	59.27	60.88	42.42	43.31	45.6	48.62	77.89	76.46	77.14	74.49
Interaction													
ρ_1	v_2^+	4.78	6.22	5.85	10.8	6.86	7.84	9.14	16.07	5.46	7.65	9.38	19.52
	v_2^-	5.07	6	7.25	10.44	6.76	8.01	9.05	17.09	5.74	8.14	9.42	20.18
	v_3^+	5.45	5.81	6.81	10.23	10.16	13.26	13.51	21.7	7.86	10.8	12.26	23.57
	v_3^-	5.27	5.61	6.57	10.76	10.79	13.23	13.51	21.31	8.01	11.79	12.36	23.63
ρ_2	v_2^+	5.56	6.45	7.91	12.52	7.62	9.97	11.16	18.82	6.96	9.64	11.19	21.29
	v_2^-	6.12	6.72	6.84	12.5	7.84	9.71	11.15	19.46	6.7	9.79	10.91	22.18
	v_3^+	5.42	6.09	6.63	11.62	13.91	16.4	18.29	26.68	11.1	14.64	15.62	27.28
	v_3^-	5.79	7.16	7.36	12.75	13.62	16.35	17.46	26.03	11.03	14.43	15.94	27.58
ρ_3	v_2^+	4.02	5.33	5.44	8.25	6.51	7.87	8.5	14.12	5.41	7.24	8.2	16.18
	v_2^-	4.53	4.89	5.18	8.25	6.55	7.7	8.18	13.75	5.34	7.47	8.44	16.95
	v_3^+	4.56	5.08	5.55	8.02	11.46	13.19	13.72	20.73	8.78	11.39	12.56	22.38
	v_3^-	4.25	5.4	5.55	8.61	12.14	13.3	14.48	21.58	9.22	11.6	12	21.82
ρ_4	v_2^+	4.87	5.66	5.71	8.51	8.04	9.55	10.7	15.69	6.63	8.53	9.64	17.24
	v_2^-	4.96	5.35	5.73	8.86	8.38	9.51	9.82	16.16	6.81	9.07	9.13	17.73
	v_3^+	5.07	5.1	5.59	8.58	17.8	19.69	20.93	28.81	14.54	16.25	17.51	26.03
	v_3^-	4.9	5.3	6	8.84	18	19.78	21.23	28.07	13.8	16.7	17.74	27.16
Factor B													
ρ_1	v_2^+	5.01	6.21	6.4	10.85	6.54	7.99	8.96	16.44	5.73	8.46	9.98	19.59
	v_2^-	5.17	6.13	6.68	10.69	6.34	7.49	9.56	16.53	5.81	7.55	9.76	19.46
	v_3^+	4.99	5.73	6.17	9.8	10.59	12.78	13.35	21.21	8.37	11.04	11.95	23.68
	v_3^-	5.27	5.54	6.8	9.98	10.31	12.66	14.15	22.96	8.17	10.97	12.86	23.34
ρ_2	v_2^+	5.56	6.56	7.54	12.34	7.58	10.03	11.42	18.61	6.98	10.27	10.96	21.01
	v_2^-	5.45	6.46	7.16	12.83	7.95	9.29	10.73	18.75	6.61	9.53	11.02	22.27
	v_3^+	5.17	6.33	7.26	11.91	13.74	16.26	18.4	26.78	11.42	14.64	16.17	27.86
	v_3^-	5.7	7.26	7.46	12.97	13.97	15.98	17.92	26.77	11.05	14.05	16.05	27.89

ρ_3	v_2^+	4.3	5.37	5.79	8.13	6.51	7.66	8.83	14.7	5.57	7.52	7.9	16.24
	v_2^-	4.58	5.28	5.53	8.25	6.52	7.47	8.98	13.13	5.54	7.21	8.29	16.57
	v_3^+	4.09	5.45	5.68	8.09	11.98	13.35	14.36	20.67	9	11.56	13.21	21.99
	v_3^-	4.61	5.2	5.55	8.03	11.38	13.75	15.43	21.71	9.1	11.38	13.98	22.73
ρ_4	v_2^+	4.9	5.5	5.69	8.81	8.28	9.52	9.81	16.19	6.8	8.48	9.85	17.11
	v_2^-	4.7	5.73	5.6	9.22	8.54	9.41	10.49	15.7	6.66	8.65	10.04	17.38
	v_3^+	5.26	5.03	5.72	8.22	18.22	19.39	20.59	28.81	14.39	17.19	17.86	26.23
	v_3^-	5.18	5.43	5.9	9.04	17.92	19.38	21.24	28.21	14.08	16.95	18.64	27.17
ANOMC – MR1 _F													
Factor A													
ρ_1	v_2^+	4.31	5.99	6.82	12.18	4.72	6.25	6.6	12	5.32	6.37	6.56	12.26
	v_2^-	5.32	6.59	6.67	11.36	5.08	6.1	7.1	12.55	4.65	6.61	6.99	12.69
	v_3^+	5.42	6.37	6.56	11.46	4.9	6.55	6.92	12.16	4.92	6.35	6.61	12.27
	v_3^-	5.46	5.93	6.9	11.56	5.18	6.14	6.47	12.04	4.85	5.82	6.77	12.39
ρ_2	v_2^+	10.58	12.15	12.38	17.02	10.36	11.79	12.29	17.21	9.84	11.44	11.88	17.74
	v_2^-	9.73	11.25	11.08	16.94	10.03	11.12	11.54	17.46	9.79	11.36	11.02	18.16
	v_3^+	10.82	11.42	12.11	17.54	10.85	11.75	12.34	17.89	10.4	11.89	12.04	18.2
	v_3^-	9.56	11.44	11.61	16.85	10.33	11.28	11.38	17.72	9.21	11.53	10.94	17.88
ρ_3	v_2^+	8.56	10.1	9.95	13.37	8.73	9.54	10.43	15.33	8.51	9.5	9.84	14.29
	v_2^-	8.81	10.37	10.11	14.84	9.31	10.59	10.22	14.93	9.31	10.32	10.02	15.69
	v_3^+	8.34	9.42	9.86	14.32	9.02	9.95	10.09	14.64	8.47	9.77	9.44	14.95
	v_3^-	8.97	10.73	11.54	15.7	9.72	10.5	10.8	15.44	9.49	10.81	10.29	15.45
ρ_4	v_2^+	18.66	19.59	19.55	23.73	19.16	20.33	19.52	23.12	18.56	19.33	19.36	24.2
	v_2^-	18.95	19.99	19.23	23.63	19.54	19.67	19.26	23.66	18.84	19.3	19.11	23.32
	v_3^+	19.52	19.35	19.58	23.58	18.03	19.21	19.26	23.34	18.48	19.81	19.32	23.24
	v_3^-	18.54	20.07	19.79	23.32	18.79	19.61	19.14	23.58	19	20.55	18.99	24.24
Interaction													
ρ_1	v_2^+	4.79	6.31	6.53	12.19	4.93	6.47	7.32	14.75	4.95	7.3	7.51	18.53
	v_2^-	4.84	6.18	6.6	11.4	5.13	6.82	7.22	15.21	4.82	7.54	8.01	18.68
	v_3^+	5.03	5.93	6.43	11.16	4.79	6.93	7.2	15.02	5.1	7.21	7.36	18.63
	v_3^-	5.21	6.04	7.08	11.2	5.06	7.13	7.06	14.55	4.61	7.46	7.63	19.02
ρ_2	v_2^+	10.32	11.13	12.26	17.4	11.77	13.19	14.82	22.32	13	16.59	16.39	28.14
	v_2^-	9.61	11.13	11.35	16.74	11.42	13.56	14.16	21.89	12.36	15.93	15.43	29.41
	v_3^+	10.63	12.21	11.96	17.28	11.94	14.04	14.43	22.61	13.54	16.2	16.19	28.51
	v_3^-	9.84	11.17	11.17	16.74	11.47	13.38	13.88	21.81	11.49	15.75	14.45	27.02
ρ_3	v_2^+	8.82	9.95	9.64	14.09	9.83	11.2	12	19.04	10.37	12.79	12.89	23.42
	v_2^-	8.78	10.65	10.44	13.98	10.44	12.73	12.22	18.47	11.43	13.83	13.66	25.05
	v_3^+	8.16	9.68	10.06	14.04	9.36	11.09	11.44	18.22	9.76	13.33	12.9	23.08
	v_3^-	9.73	10.55	10.87	15.13	10.45	12.87	12.39	19.13	11.71	14.38	13.98	24.03
ρ_4	v_2^+	18.8	20.08	19.73	23.29	24.13	24.94	25.3	30	28.55	30.38	30.23	37.26
	v_2^-	18.45	19.74	18.71	23.57	24.55	25.22	25.46	29.58	28.64	30.5	28.6	37.27
	v_3^+	18.5	19.55	19.44	22.95	23.39	24.48	24.81	30.85	29.15	30.33	29.64	35.89
	v_3^-	18.91	19.77	19.18	23.5	23.71	24.99	25.15	29.84	28.17	31.03	29.52	37.18
Factor B													
ρ_1	v_2^+	4.94	6.44	6.7	11.4	4.89	6.82	7.73	14.77	5.01	7.44	7.73	18.57
	v_2^-	4.88	6.28	6.67	11.48	4.86	6.26	7.4	15.74	4.86	7.35	7.84	18.71
	v_3^+	5.12	6.06	6.39	11.4	4.95	7.2	7.18	14.76	4.52	7.24	7.43	19.12
	v_3^-	5.2	5.94	6.28	11.28	5.45	6.47	6.97	14.6	4.92	7.56	7.59	18.69
ρ_2	v_2^+	10.59	11.86	12.53	17.24	11.49	13.95	14.6	22.53	12.74	16.54	15.7	27.33
	v_2^-	9.28	11.25	11.69	16.93	10.6	13.21	13.98	22.25	12.21	16.06	14.98	28.34
	v_3^+	10.63	12.04	11.68	17.08	11.68	14.07	15.11	22.51	13.56	16.66	16.34	28.05
	v_3^-	9.48	11.75	11.12	16.75	10.83	13.35	13.95	21.46	12.1	14.68	15.37	27.85
ρ_3	v_2^+	8.46	10.26	8.65	9.77	9.69	11.25	11.67	18.98	9.98	12.72	12.66	22.97
	v_2^-	8.97	10.58	8.5	9.03	10.01	12.13	12.64	18.94	10.53	14.11	13.43	24.02
	v_3^+	8.29	9.49	8.97	10.41	9.31	11.05	11.63	18.14	10.19	13.09	12.41	23.14
	v_3^-	9.21	8.76	9.59	9.87	10.47	12.16	12.5	19.3	10.93	13.59	14.56	24.11
ρ_4	v_2^+	10.46	14.48	18.62	19.48	23.76	25.25	25.05	30.52	28.53	30.81	29.23	37.11
	v_2^-	10.47	15.32	18.32	19.48	24.27	26.02	24.99	29.61	28.54	30.11	28.87	37.48
	v_3^+	10.15	14.91	18.56	20	22.78	25.35	24.8	31.16	28.16	30.16	28.79	36.48
	v_3^-	14.23	19.28	19.59	20.02	23.98	25.14	24.52	30.24	28.51	30.93	29.24	37
ANOMC – MR2 _F													
Factor A													
ρ_1	v_2^+	11.11	12.46	9.87	19.56	8.65	10.17	8.83	17.16	13.25	16.28	14.3	23.69
	v_2^-	9.54	10.77	15.78	17.64	7.56	8.85	12.63	16.59	12.75	14.06	18.73	23.59
	v_3^+	26.94	28.61	26.95	38.51	20.55	22.25	19.67	31.29	37.62	40.06	40.83	49.01
	v_3^-	24.07	26.28	31.68	36.56	18.16	19.64	23.97	29.77	36.28	40.25	43.29	48.37
ρ_2	v_2^+	11.2	12.34	9.84	19.09	8.59	9.96	8.96	16.54	14.39	15.4	14.27	23.75
	v_2^-	9.01	10.6	14.83	16.74	7.95	9.45	12.34	15.6	12.42	13.76	16.73	22.11
	v_3^+	27.47	28.52	26.14	38.01	20.12	21.83	19.4	29.87	36.64	39.87	39.68	48.55
	v_3^-	23.78	26.56	31.36	36.55	18.26	19.96	24.08	29.42	36.33	38.76	42.81	48.93
ρ_3	v_2^+	10.48	12.76	9.74	19.14	9	10.22	8.72	16.76	13.78	15.56	14.35	23.23
	v_2^-	8.98	11.14	15.19	18.3	7.53	9.51	12.4	15.94	12.42	14	17.84	23.69
	v_3^+	26.45	28.95	26.68	38.43	19.95	22.16	19.93	31.32	37.34	40.27	41.06	50.17
	v_3^-	23.89	26.97	32.59	37.63	17.7	20.24	24.39	29.12	35.85	39.46	43.62	48.75
ρ_4	v_2^+	11.39	12.69	9.47	18.92	9.46	10.64	8.65	16.19	13.95	15.89	14.1	22.67
	v_2^-	9.44	10.55	14.67	17.31	7.96	8.78	12.09	16.08	12.53	14.48	18.06	21.92
	v_3^+	26.91	28.77	26	37.79	19.49	21.6	19.12	30.58	37.04	41.16	40.66	48.6
	v_3^-	24.23	27.47	31.25	36.94	17.86	19.53	23.7	28.39	35.68	38.53	41.66	48.46

Interaction													
ρ_1	v_2^+	4.9	6	6.24	10.36	6.48	7.35	8.24	15.2	5.74	7.68	8.36	17.3
	v_2^-	4.97	6.14	6.84	10.7	6.66	7.87	6.67	15.2	6.46	7.43	7.52	18.08
	v_3^+	5.01	6.12	6.33	10.35	9.89	11.43	11.08	20.32	7.76	10.07	10.54	20.91
	v_3^-	5.23	5.4	6.54	10.31	9.77	12.04	9.73	21.14	7.86	10.4	8.19	21.54
ρ_2	v_2^+	5.15	5.71	6.72	9.96	6.28	7.41	8.01	14.88	6	7.43	7.99	16.98
	v_2^-	4.8	6.08	6.21	10.48	6.48	7.65	6.73	14.75	5.69	7.86	6.77	17.59
	v_3^+	4.83	5.66	6.4	9.68	10.03	11.19	10.59	20.1	8.06	10.04	9.62	20.44
	v_3^-	5.05	6.26	6.12	10.82	10.23	11.58	9.06	20.63	8.57	10.73	8.3	20.64
ρ_3	v_2^+	4.54	6.08	6.8	10.17	6.64	7.01	7.69	15.91	5.92	7.17	7.87	16.23
	v_2^-	4.63	6.12	7.43	10.16	6.59	7.99	7.19	15.31	6.1	7.74	7.24	18.56
	v_3^+	4.74	5.78	6.79	10.11	9.78	11.51	10.81	20.15	8.15	10.22	11.31	21.08
	v_3^-	5.25	5.74	6.69	11.02	10.16	11.71	9.72	20.88	7.97	10.68	8.57	21.26
ρ_4	v_2^+	4.95	5.69	6.52	9.98	6.4	7.32	8.17	14.58	5.67	7.38	7.7	16.42
	v_2^-	4.81	5.78	6.39	10.48	6.75	7.69	7.11	14.82	6.08	7.24	6.68	17.39
	v_3^+	5.25	5.54	6.55	9.46	9.07	11.22	10.91	20.33	7.78	9.87	9.94	20.01
	v_3^-	5.24	6.13	6.6	10.32	10.39	11.33	9.46	20.69	8.15	10.62	8.24	21.06
Factor B													
ρ_1	v_2^+	5.1	5.57	6.36	10.52	6.19	7.91	7.94	15.9	5.72	7.2	8.14	18.04
	v_2^-	5.18	6.39	6.36	9.94	5.53	6.53	10.35	15.1	5.47	6.74	9.96	17.94
	v_3^+	5.16	5.52	6.47	10.54	10.15	11.84	11.68	20.63	7.77	10.4	10.5	21.2
	v_3^-	5.03	5.7	6.74	10.52	8.81	10.01	14.83	20.38	6.36	9.16	13.18	20.42
ρ_2	v_2^+	5.27	5.76	6.85	10.35	5.99	7.83	8.01	14.74	6.01	7.61	7.67	17.34
	v_2^-	4.73	5.42	6.32	10.45	5.57	6.93	9.77	15.02	5.04	6.58	8.69	17.26
	v_3^+	4.87	5.81	6.19	9.99	9.07	11.28	10.95	19.99	7.55	9.52	9.46	20.65
	v_3^-	4.79	6	6.25	10.53	7.52	9.95	14.61	19.74	6.87	9.04	12.96	20.14
ρ_3	v_2^+	4.87	5.87	6.75	10.01	6.51	7.6	7.37	16.16	5.73	6.78	8.15	17.51
	v_2^-	5.11	6.09	6.6	10.65	5.56	6.73	10.35	14.91	4.54	6.51	9.63	16.97
	v_3^+	4.85	6.22	6.65	10.21	9.84	11.61	11.23	20.59	7.97	10.57	10.47	20.16
	v_3^-	4.7	6.09	6.6	11.1	7.85	10.71	14.86	19.88	6.71	9.53	13.5	20.95
ρ_4	v_2^+	5.08	5.7	6.85	9.85	6.53	7.54	7.68	14.94	5.87	7.04	7.6	17.39
	v_2^-	5.24	6.13	6.3	10.51	5.6	6.74	9.4	13.8	5.54	6.78	8.67	17.58
	v_3^+	5.16	5.5	6.51	10.64	8.96	11.71	10.01	20.3	7.69	9.79	9.82	19.9
	v_3^-	5.57	5.77	6.65	10.83	7.94	9.91	14.25	19.24	7.23	9.1	13.01	19.83
ANOMC – MR3 _F													
Factor A													
ρ_1	v_2^+	10.49	12.52	13.55	18.85	8.55	10.17	11.22	16.99	14.92	16.58	18.29	23.99
	v_2^-	11.6	12.34	14.24	19.19	9.37	10.46	11.9	16.97	14.42	16.48	18.62	24.67
	v_3^+	28.16	29.94	33.04	39.16	21.77	23.7	24.99	31.67	42.8	44.57	47.7	52.12
	v_3^-	28.32	29.55	32.74	37.98	21.12	23.32	25.42	31.41	42.12	44.92	47.47	51.93
ρ_2	v_2^+	14.41	16.21	18.74	24.35	10.82	12.92	14.65	19.74	18.7	20.51	23.28	29.38
	v_2^-	14.49	16.67	17.35	24.59	12.12	13.26	13.89	19.86	19.99	22.88	23.58	31.28
	v_3^+	38.15	39.98	42.45	48.73	28.37	30.31	34.02	39.8	55.45	56.85	58.58	62.45
	v_3^-	39.73	40.95	42.18	49.76	29.51	31.8	33.13	39.97	56.59	58.29	60.53	63.74
ρ_3	v_2^+	13.05	14.34	13.99	20.28	10.66	11.74	11.62	17.64	19.35	20.04	21.03	26.16
	v_2^-	12.73	13.64	15.89	18.97	9.57	11.18	13.06	16	17.31	18.77	21.4	26.3
	v_3^+	36.96	38.55	39.85	45.32	27.93	29.44	30.04	36.04	55.69	56.58	58.8	59.01
	v_3^-	35.02	37.19	40.63	44.68	24.84	27.85	29.99	34.9	52.65	53.66	57.14	57
ρ_4	v_2^+	19.53	20.85	21.89	27.87	14.81	15.58	17.27	21.14	27.98	30.25	31.47	35.19
	v_2^-	20.01	20.96	21.6	27.74	14.67	16.13	17.01	21.08	28.26	30.14	30.73	35.02
	v_3^+	57.67	58.2	58.72	60.61	41.99	42.69	44.72	49.18	77.1	76.79	77.02	73.93
	v_3^-	56.3	57.61	58.88	60.76	42.53	43.5	45.33	48.34	77.81	76.6	76.8	74.59
Interaction													
ρ_1	v_2^+	4.77	6.1	6.21	10.91	6.53	7.78	9.23	16.05	5.53	7.68	9.69	19.51
	v_2^-	5	6.07	7.35	10.43	6.75	7.86	9.11	16.91	5.63	8.15	9.67	19.97
	v_3^+	5.41	5.73	6.87	10.24	10.1	13.15	13.8	21.63	8.03	10.77	12.42	23.61
	v_3^-	5.1	5.45	6.6	10.74	10.65	13.04	13.76	21.21	7.86	11.69	12.51	23.66
ρ_2	v_2^+	5.32	6.22	7.92	12.31	7.28	9.82	10.98	18.34	6.96	9.41	10.92	20.69
	v_2^-	6.08	6.88	7.2	12.61	7.98	9.8	11.7	19.29	6.86	9.95	11.27	22.31
	v_3^+	4.97	5.78	6.56	11.14	13.19	15.69	17.97	26.11	10.4	13.78	14.99	26.57
	v_3^-	6	7.44	7.83	12.94	13.95	16.57	18.3	26.33	11.61	14.89	16.99	28.27
ρ_3	v_2^+	4.28	5.32	5.6	8.44	6.58	7.93	8.75	14.21	5.54	7.29	8.43	16.3
	v_2^-	4.44	4.69	5.14	8.11	6.41	7.54	8.16	13.5	5.19	7.05	8.23	16.7
	v_3^+	4.95	5.36	5.76	8.38	12	13.6	14.47	21.1	9.52	12.1	13.3	22.71
	v_3^-	3.93	5	5.3	8.34	11.43	12.65	13.99	21.04	8.62	10.76	11.51	21.27
ρ_4	v_2^+	4.9	5.53	5.73	8.49	8.01	9.52	10.61	15.63	6.55	8.32	9.54	17.24
	v_2^-	4.9	5.15	5.72	8.66	8.28	9.4	9.73	16.01	6.83	8.97	8.86	17.58
	v_3^+	5	5.15	5.7	8.64	17.78	19.7	20.79	28.67	14.61	16.24	17.46	26.14
	v_3^-	4.88	5.27	5.86	8.75	17.68	19.48	21.04	27.94	13.65	16.56	17.67	26.99
Factor B													
ρ_1	v_2^+	4.92	6.18	6.47	10.78	6.52	7.9	9.08	16.48	5.93	8.5	10.13	19.34
	v_2^-	5.08	5.99	6.77	10.56	6.28	7.38	9.66	16.36	5.81	7.58	10.02	19.47
	v_3^+	4.98	5.86	6.31	9.75	10.43	12.84	13.75	21.01	8.46	11.05	12.2	23.69
	v_3^-	5.13	5.56	6.8	9.91	10.13	12.58	14.41	22.62	8.06	11.06	13.2	23.27
ρ_2	v_2^+	5.31	6.47	7.55	11.93	7.4	9.75	11.42	18.29	6.87	10.14	10.88	20.68
	v_2^-	5.69	6.62	7.4	13.12	8	9.54	11.13	18.74	6.89	9.8	11.39	22.49
	v_3^+	4.9	6.02	7.13	11.59	13.2	15.59	18.13	26.07	10.73	13.88	15.52	27.42
	v_3^-	6	7.56	7.9	13.15	14.31	16.41	18.81	27.02	11.77	14.7	16.87	28.39

ρ_3	v_2^+	4.31	5.46	6.07	8.18	6.59	7.7	9.18	14.67	5.83	7.58	8.21	16.19
	v_2^-	4.31	5.15	5.38	8.02	6.29	7.3	8.88	12.85	5.33	6.98	8.05	16.37
	v_3^+	4.5	5.72	5.93	8.27	12.46	13.71	14.9	21.12	9.75	12.15	14.01	22.19
	v_3^-	4.3	4.93	5.21	7.76	10.77	13.05	14.82	21.11	8.48	10.71	13.38	21.96
ρ_4	v_2^+	5.01	5.38	5.67	8.79	8.22	9.55	9.78	16.01	6.74	8.42	9.69	17.02
	v_2^-	4.77	5.62	5.57	9.13	8.36	9.33	10.48	15.76	6.59	8.55	9.87	17.31
	v_3^+	5.28	5.11	5.55	8.37	18.35	19.4	20.44	28.41	14.36	17.27	17.92	26.27
	v_3^-	5.06	5.51	5.8	9.01	17.7	19.17	21.31	28.03	13.82	16.79	18.45	27.2
ANOMC – MR4_F													
ρ_1	Factor A												
	v_2^+	10.28	12.17	10.2	18.82	8.93	10.61	9.48	17.16	13.95	15.94	15.83	24.31
	v_2^-	10.73	11.74	15.15	18.26	8.65	9.53	12.34	16.86	13.35	15.52	18.08	24.11
	v_3^+	26.68	28.68	28.35	39.25	21.15	23.19	21.78	31.58	40.04	42.47	43.42	51.25
ρ_2	v_2^+	27.76	27.36	32.12	37.38	19.23	21.41	25.05	30.95	38.81	41.86	45.29	51.3
	v_2^-	7.48	9.08	8.33	15.52	5.5	6.83	7.04	12.73	10.34	11.29	12.27	20.71
	v_3^+	7.05	8.85	10.12	16.38	5.98	7.14	8.28	12.96	10.55	12.99	14.02	21.95
	v_3^-	23.32	24.64	26.55	35.84	16.01	17.97	19.84	28.61	37.08	40.11	41.9	49.82
ρ_3	v_2^+	23.67	26.69	28.07	36.67	17.05	19.02	20.98	28.28	38.97	41.62	44.33	51.59
	v_2^-	8.22	9.59	7.67	15.65	6.34	7.43	6.39	14.32	12.21	13.13	12.74	21.7
	v_3^+	6.27	8.06	10.75	14.39	4.61	5.88	8.78	12.22	9.2	11.77	14.34	21.8
	v_3^-	24.52	27.24	25.99	37.92	18.17	20.49	18.24	29.88	39.98	43.18	43.82	52.12
ρ_4	v_2^+	24.74	24.96	30.5	37.32	14.96	18.39	22.04	28.07	35.48	39.86	44.19	49.54
	v_2^-	4.24	5.32	4.78	11.03	3.01	3.67	3.63	8.82	6.5	9.13	9.29	17.3
	v_3^+	3.51	4.75	5.9	11.75	2.39	3.43	4.51	8.47	6.34	8.41	9.62	17.35
	v_3^-	19.55	22.38	23.4	35.66	11.75	14.52	15.27	26.04	35.27	39.85	42.04	51.5
ρ_1	Interaction												
	v_2^+	4.65	6.03	5.64	10.98	6.92	7.94	7.94	16.03	5.3	7.6	8.37	18.38
	v_2^-	4.93	6.15	6.67	10.97	6.83	7.67	7.47	16.59	5.79	7.97	7.99	19.11
	v_3^+	5.17	6	6.17	10.7	10.43	12.55	11.84	21.74	7.81	10.09	11.03	22.38
ρ_2	v_2^+	5.05	5.74	6.14	11.26	10.47	12.16	10.72	20.79	7.6	10.66	9.69	22.75
	v_2^-	2.34	2.89	3.63	7.49	2.96	4.26	4.43	11.08	2.48	3.64	3.88	11.63
	v_3^+	2.72	3.2	3.09	7.9	3.36	4.41	4.71	11.19	2.24	3.99	3.57	12.55
	v_3^-	2.16	2.84	3.28	6.7	5.31	7.3	7.87	15.8	3.96	5.78	5.52	15.06
ρ_3	v_2^+	2.25	3.64	3.57	8.24	6.36	8.03	7.9	16.05	4.73	6.21	6.04	16.27
	v_2^-	2.31	3.23	3.44	6.28	3.52	4.46	4.45	10.51	2.66	3.96	4.43	11.85
	v_3^+	2.34	2.83	3.11	6.23	3.28	4.09	3.16	9.92	2.54	3.42	3.15	11.62
	v_3^-	2.56	3.18	3.4	6.12	6.13	8.12	6.48	16.42	4.62	6.32	6.56	16.67
ρ_4	v_2^+	2.6	3.1	3.16	6.04	6.39	7.33	5.35	15.73	4.28	5.86	4.25	15.26
	v_2^-	0.67	1.04	1.03	2.79	0.91	1.31	1.57	5.32	0.55	0.94	1.21	5.19
	v_3^+	0.8	0.78	1.05	3.11	1.17	1.38	1.17	5.37	0.64	1.08	0.89	5.37
	v_3^-	0.65	0.83	1.05	2.91	2.64	3.46	3.68	10.69	1.55	2.46	2.55	8.91
ρ_1	Factor B												
	v_2^+	5.2	5.92	6.02	11.08	6.1	7.87	7.92	15.79	5.52	8.05	8.94	18.73
	v_2^-	5.12	6.29	6.6	10.82	6.08	6.62	9.59	16	5.56	7.2	9.8	18.62
	v_3^+	5.07	5.79	6.01	9.85	10.6	12.16	11.97	20.36	8.03	10.47	10.55	22.83
ρ_2	v_2^+	5.06	5.57	6.53	10.38	9.45	10.98	14.34	21.43	7.19	9.69	13.05	21.99
	v_2^-	2.28	3.07	3.36	7.21	2.87	4	4.48	10.65	2.26	3.65	3.87	11.93
	v_3^+	2.61	3.1	3.26	8.12	3.11	3.86	5.05	11.02	2.07	3.57	4.23	12.73
	v_3^-	1.9	2.9	2.89	6.74	5.75	7.07	7.89	15.95	3.95	5.29	5.88	16.23
ρ_3	v_2^+	2.52	3.45	3.66	8.21	5.76	7.28	9.05	16.15	4.39	6.17	7.26	16.71
	v_2^-	2.46	3.19	3.54	6.23	3.41	3.85	4.03	10.44	2.83	3.65	4.22	11.84
	v_3^+	2.29	3.22	3.27	6.05	2.86	3.82	5.92	9.43	1.95	3.35	4.94	11.52
	v_3^-	2.11	3.2	3.63	6.36	6.69	8.14	7.1	16.22	4.69	6.28	6.5	16.32
ρ_4	v_2^+	2.28	2.88	2.91	5.82	4.71	6.54	9.91	15.09	3.48	5.14	8.27	15.37
	v_2^-	0.7	0.82	0.92	2.92	0.89	1.5	1.33	5.34	0.54	0.97	1.32	5.32
	v_3^+	0.58	0.95	0.85	3.01	1.01	1.24	1.78	5.14	0.54	0.81	1.11	5.59
	v_3^-	0.68	0.87	0.97	2.94	2.56	3.43	3.32	10.79	1.48	2.53	2.36	8.94
ρ_1	v_2^+	0.74	0.89	1.03	3.12	2.06	3.1	4.49	10.64	1.2	2.21	3.25	9.04

4.5.2 Results of the power test

Figures 4.1-4.6 present the comparative performances of ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests across four distinct non-null cases (δ_{2-5}) for the sample size (n_1) and (n_2).

For the fixed treatment compensation level $(i, j) = (2, 3)$ with fixed correlations (ρ_1) and small sample size (n_1) , the results in scenarios of non-null case δ_2 reveal that under a normal distribution, the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests in the main interaction exhibit power rates of 17.7%, %, 15.54%, 15.12%, 6.72%, 11.67%, 14.84% and 16.66%, respectively. In factor A, 40.46%, 37.76%, 33.91%, 6.51%, 23.9%, 34.3% and 33.9%, for factor B, 17.24%, 15.5%, 15.12%, 6.76%, 11.3%, 15.04% and 16.66%. Under a double exponential distribution and for a non-null case δ_3 , the corresponding power rates in the main interaction are 21.06%, 19.93%, 20.57%, 9.49%, 13.79%, 20.46% and 21%. In factor A, 20.44%, 20.46%, 19.82%, 8.32%, 15.74%, 19.87% and 21.2%, for factor B, 21.02%, 19.97%, 19.72%, 9.55%, 14.4%, 20.24% and 21.17%. Furthermore, when considering a Chi Square distribution, the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests in the main interaction exhibit power rates of 23.66%, 30.05%, 26.19%, 10.35%, 14.62%, 26.69% and 23.65% for non-null case δ_4 , respectively. In factor A, 28.02%, 32.67%, 29.33%, 8.28%, 18.01%, 29.9% and 27.81%, for factor B, 10.24%, 17.55%, 17.26%, 9.75%, 9.2%, 14.99% and 13.81%. Finally, in the context of non-null case δ_5 under a Cauchy distribution, the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests in the main interaction demonstrate power rates of 25.3%, 35.71%, 32.35%, 18.28%, 23.44%, 32.24% and 30.54%, respectively. In factor A, 25.05%, 33.62%, 29.33%, 15.41%, 22.15%, 29.21% and 28.94%, for factor B, 25.81%, 35.99%, 35.59%, 18.42%, 24.99%, 33.1% and 31.61%.

In general, when examining the correlation (ρ_1), the ANOMC – Reg_F test consistently exhibits a heightened power rate compared to the other tests. Conversely, in cases where the data follows a Cauchy distribution is taken into account, the ANOMC – Reg_F and ANOMC – MR3_F tests demonstrate a notably higher power rate. However, the ANOMC – MR1_F approach exhibits the lowest power rate, particularly in situations with non-null effects (δ_3, δ_4), and sample sizes (n_1, n_2). Additionally, the ANOMC – MR1_F approach demonstrates similar power rates across the different non-null cases (δ_{2-5}) within each correlation ratio (ρ_{1-4}). Nevertheless, a comprehensive evaluation across several scenarios reveals that the ANOMC – Reg_F and ANOMC – MR3_F tests show relatively superior performance compared to the other ANOM and ANOMC tests. It is also noteworthy that the performance of all ANOM and ANOMC tests (including ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F) improves with an increase in the sample size and the levels of factor B, as well as increasing the correlation (ρ_{1-4}) which also leads to an increase in the power of the tests. Further, ANOMC tests show a higher power rate than ANOM tests.

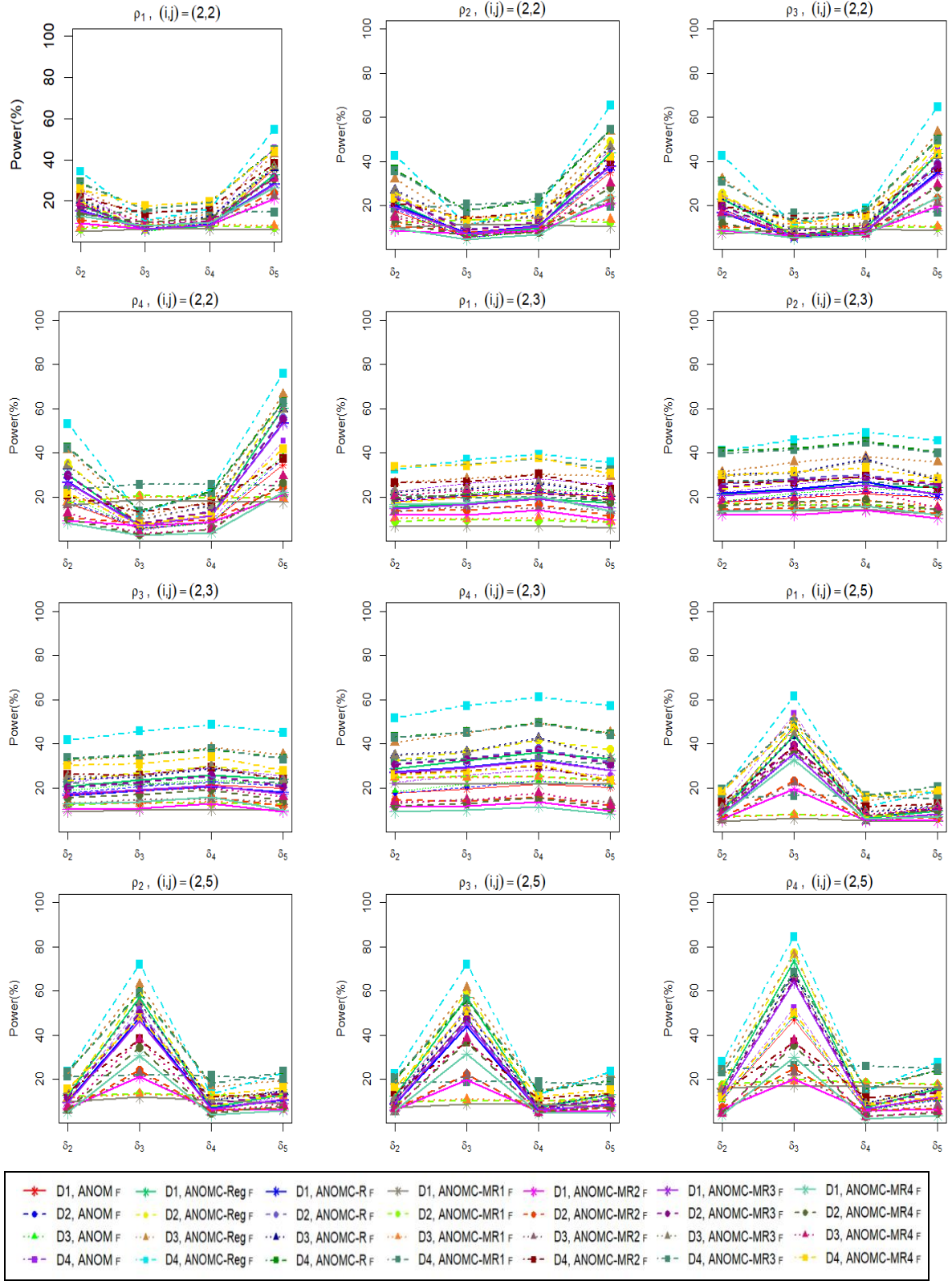


Figure 4. 1: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of the Interaction between Factor A and Factor B under homogeneous variances and sample size n_1).

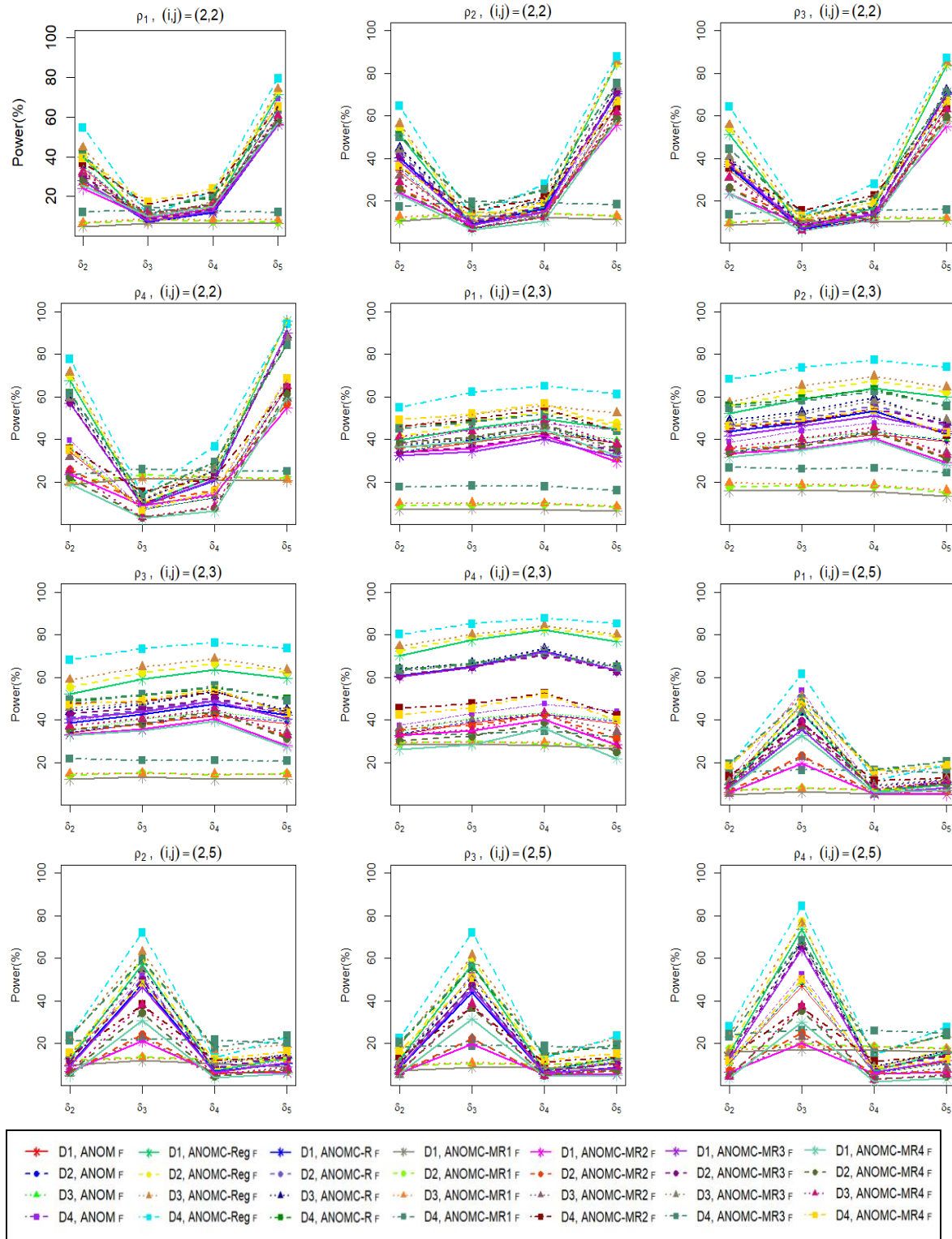


Figure 4. 2: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of the Interaction between Factor A and Factor B under homogeneous variances and sample size n_2).

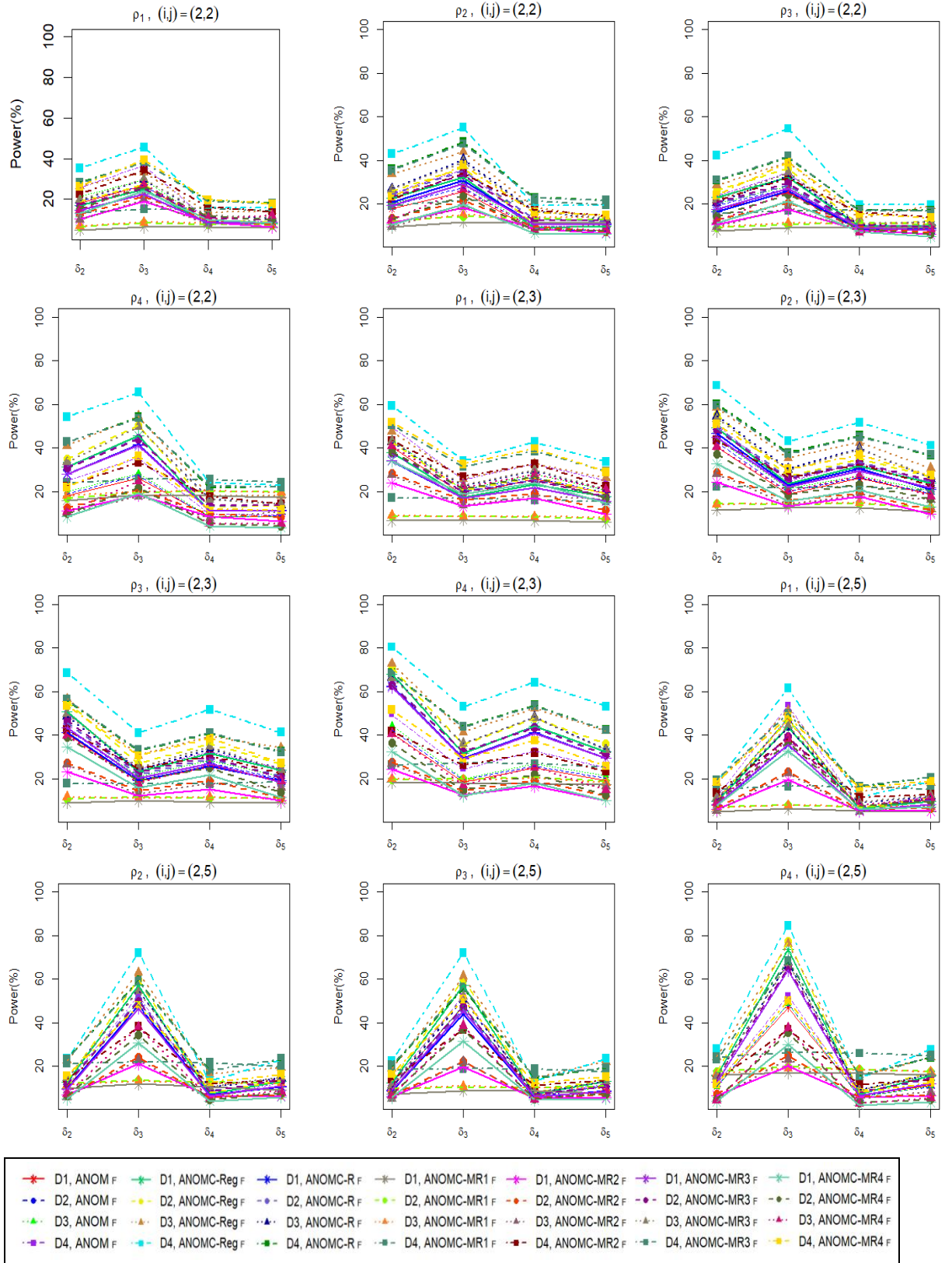


Figure 4. 3: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of Factor A under homogeneous variances and sample size (n_1).

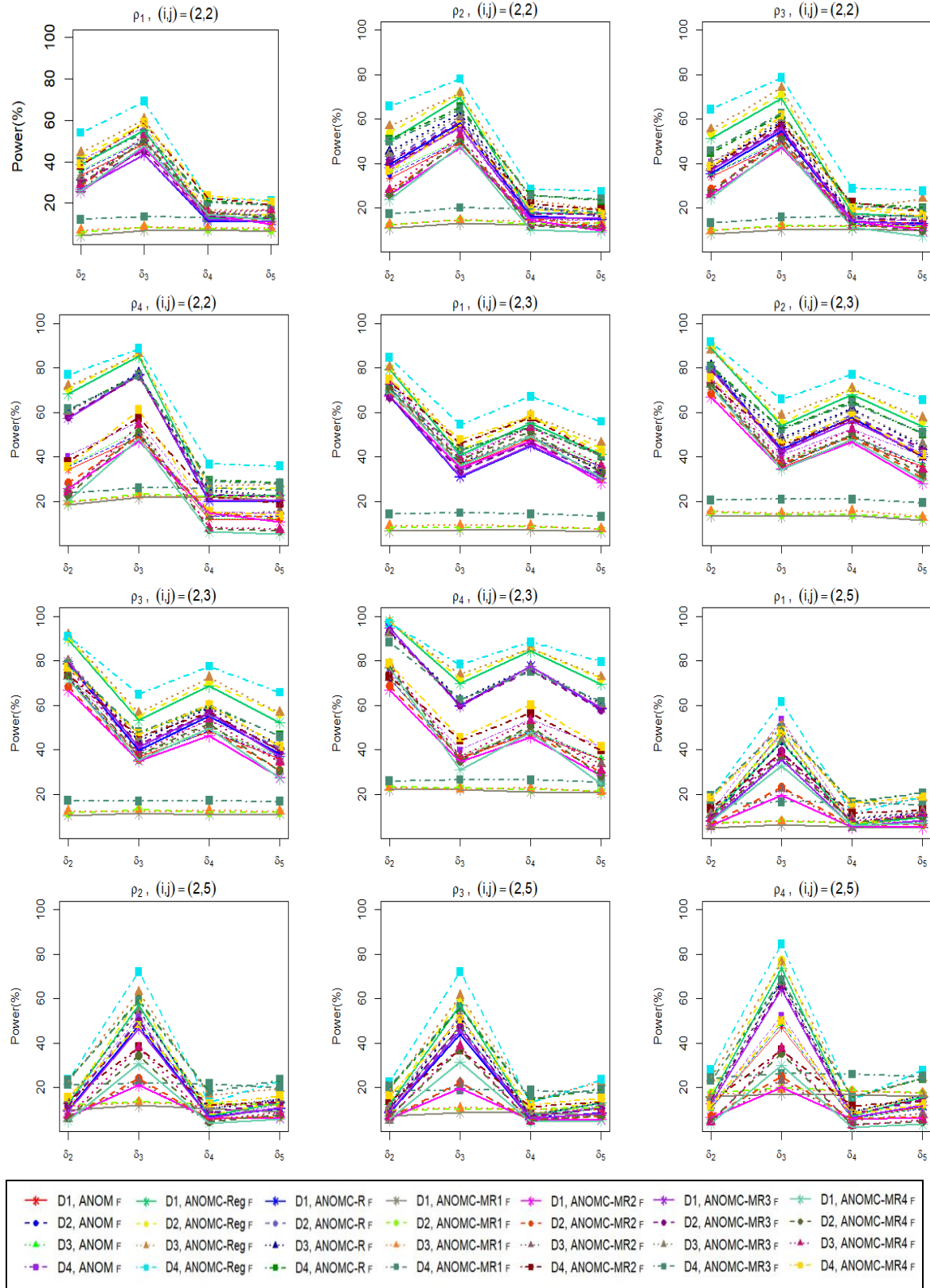


Figure 4. 4: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of Factor A under homogeneous variances and sample size (n_2).

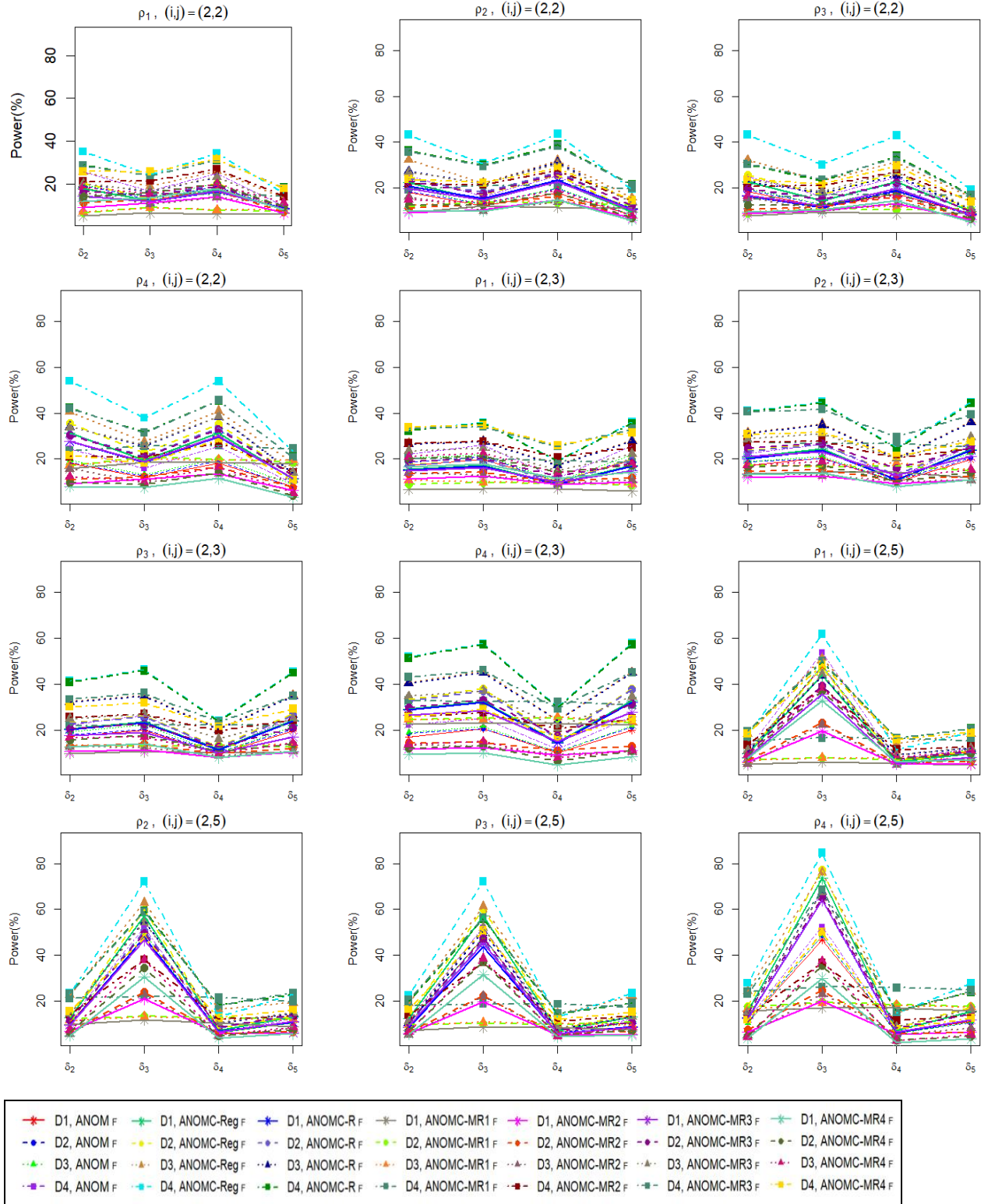


Figure 4.5: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of Factor B under homogeneous variances and sample size (n_1).

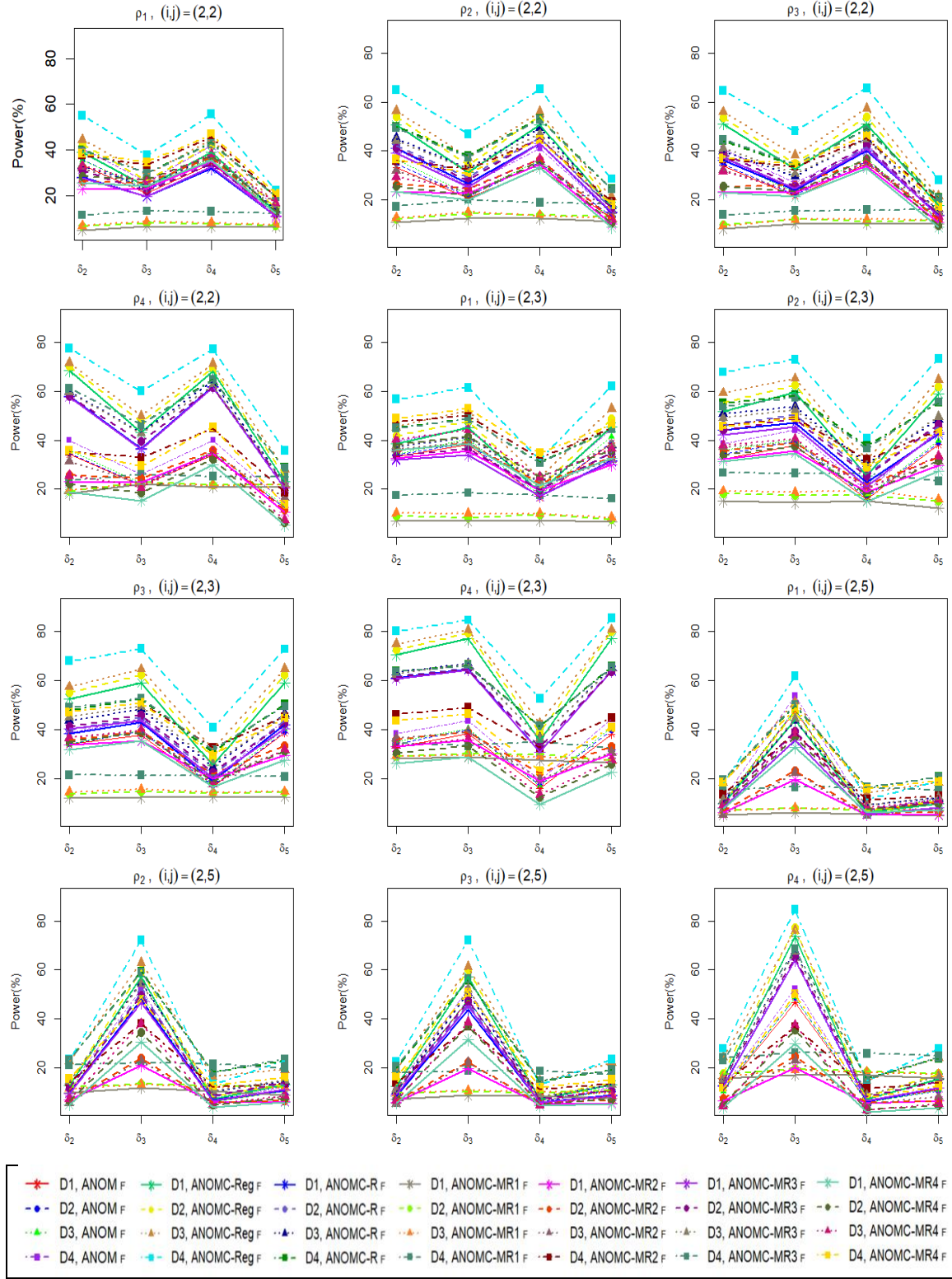


Figure 4. 6: Effect of non-null cases on tests (ANOMC and ANOM) in terms of % power of Factor B under homogeneous variances and sample size (n_2).

4.6 Illustrative Examples

Montgomery [85] presents an illustrative example involving factorial experiments with covariates. The experiment comprises two factors, A and B, with two levels each (-1 and 1), accompanied by a covariate term (X). The dataset associated with this experiment is documented in Table 15.16 [85]. In this work, we investigated Montgomery data using the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests. The investigation was done across two levels of factor A and two levels of factor B, which were divided into four treatments (say A1B1, A1B2, A2B1, A2B2). Each treatment level had 4 observations. The $ANOM_F$ test is applied for testing the response (Y), while the $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests are applied for testing the response (Y) considering the covariate (X) with a significance level (α) of 5%.

4.6.1 Study the interaction effect between factor A and factor B:

In order to conduct the aforementioned tests, we first calculate the interaction slope terms $D_{j(12)}$, $Z_{j(12)}$, $O_{j(12)}$, $Q_{j(12)}$, $H_{j(12)}$, $V_{j(12)}$ and $U_{j(12)}$ for $j = 1, 2$ in $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests, respectively. That yields, $D_{1(12)} = 60.2$, $D_{2(12)} = -15.4$ in $ANOM_F$ test, $Z_{1(12)} = 65.2$, $Z_{2(12)} = -11.98$ in $ANOMC - Reg_F$ test, $O_{1(12)} = 64.6$, $O_{2(12)} = -9.89$ in $ANOMC - R_F$ test, $Q_{1(12)} = -9.95$, $Q_{2(12)} = 2.31$ in $ANOMC - MR1_F$, $H_{1(12)} = 60.49$, $H_{2(12)} = -15.41$ in $ANOMC - MR2_F$, $V_{1(12)} = 64.68$, $V_{2(12)} = -9.89$ in $ANOMC - MR3_F$. Finally, $U_{1(12)} = 64.68$, $U_{2(12)} = -9.89$ in $ANOMC - MR4_F$.

test. These values are plotted along with the appropriate decision limits given in Equations (4.2) and Table 4.4, as shown in Figure 4.7, using the critical values $S(0.05, 2, 12) = 2.19$, $I(0.05, 2, 12) = 5.68$, $Z(0.05, 2, 12) = 5.72$, $M(0.05, 2, 12) = 5.69$, $K(0.05, 2, 12) = 7.85$, $E(0.05, 2, 12) = 5.77$ and $P(0.05, 2, 12) = 6.88$. The mean square error for the estimator's values are as follows: $MSE_{\bar{Y}_{...}} = 27.464$, $MSE_{\bar{M}_{...}} = 2.9359$, $MSE_{\bar{T}_{(1)...}} = 7.1997$, $MSE_{\bar{T}_{(2)...}} = 6.3426$, $MSE_{\bar{T}_{(3)...}} = 7.0301$, $MSE_{\bar{T}_{(4)...}} = 7.3986$ and $MSE_{\bar{T}_{(5)...}} = 7.0153$. Figure 4.7 show that all the slopes estimates fall outside the decision limits indicating a significant interaction effect except under the ANOMC – MR1_F test, which shows an insignificant interaction effect.

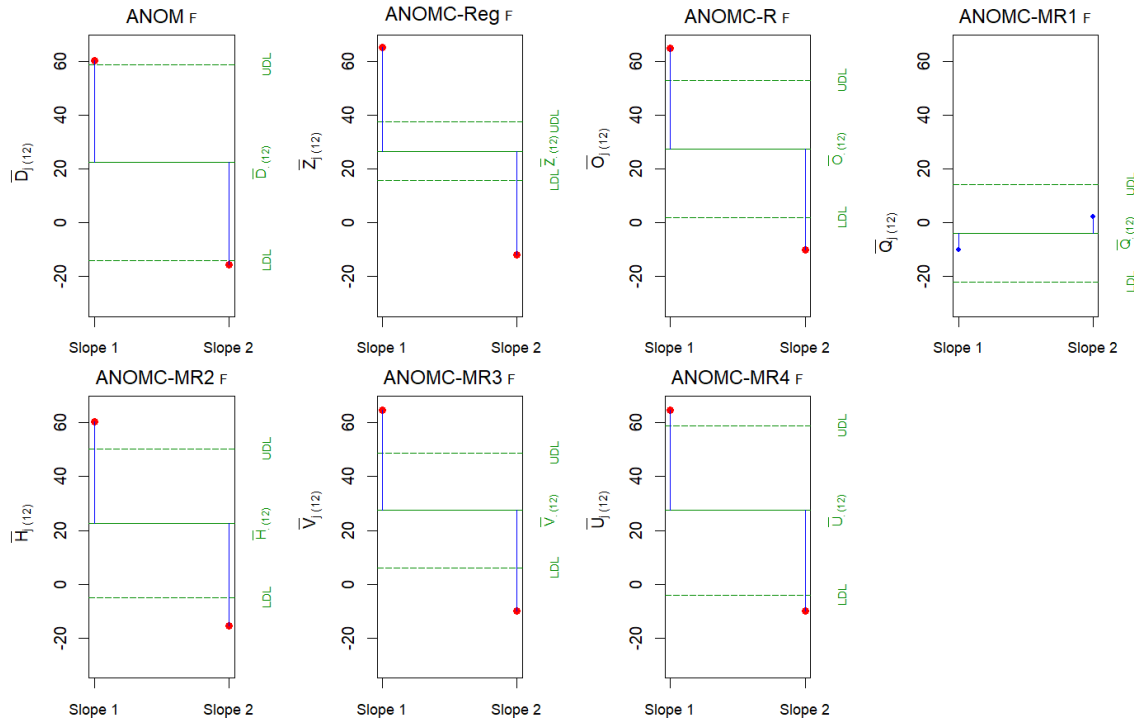


Figure 4. 7: ANOM and ANOMC for testing the interaction.

4.6.2 Study the effect of both factor A and factor B:

Since the interaction is not significant, then we can proceed to test the $I \times J$ treatment combinations where $\bar{Y}_{ij.} = (-23.1300, 50.7900, 37.0775, 35.3775)$ and $\bar{M}_{ij.} = (-31.32550, 46.97387, 33.87910, 34.98804)$, $\bar{T}_{(1)ij.} = (-31.58910, 44.78793, 33.10994, 34.89096)$, $\bar{T}_{(3)ij.} = (-23.41853, 50.78972, 37.07726, 35.37737)$ and $\bar{T}_{(4)ij.} = \bar{T}_{(5)ij.} = (-31.57329, 44.78794, 33.10995, 34.89096)$ as shown in Figure 4.8 and plotted along with the appropriate decision limits given in Eq (4.3) and Table 4.4 using the critical values $S(0.05, 4, 12) = 2.65$, $I(0.05, 4, 12) = 7.4$, $Z(0.05, 4, 12) = 7.05$, $K(0.05, 4, 12) = 13.2$, $E(0.05, 4, 12) = 7.13$ and $P(0.05, 4, 12) = 8.95$.

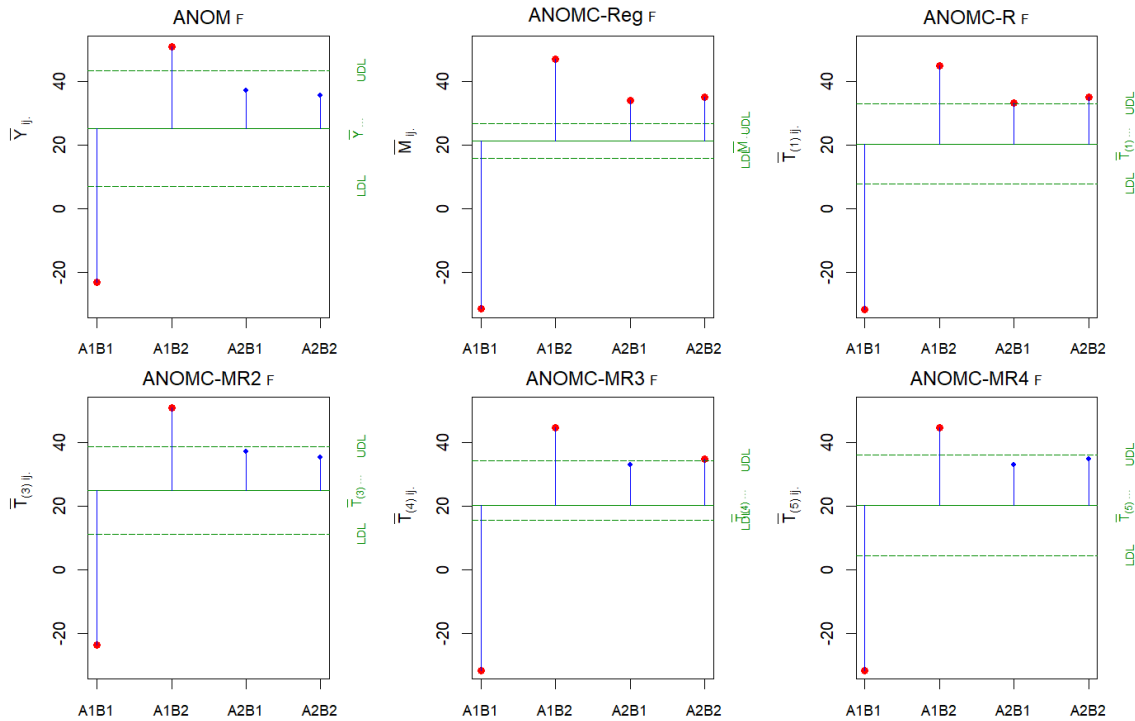


Figure 4. 8: ANOM and ANOMC for testing the Factors.

On the other hand, ANOMC – MR1_F test shows an insignificant interaction effect. Therefore, Figure 4.9 demonstrates the main effects of factors A and B by plotting the

levels of the factors (e.g., plotting the levels of factor A at the different levels of factor B). This shows that factors A and B have no significant effect.

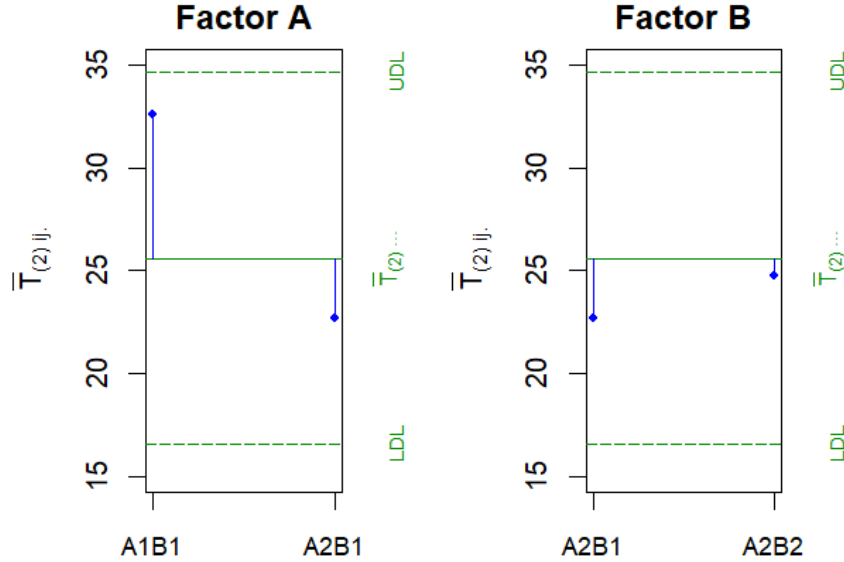


Figure 4. 9: The effect of factor A and factor B in ANOMC – MR1_F test.

Upon applying the proposed methods, namely ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests, all of which demonstrate a significant performance in detecting the interaction effect except ANOMC – MR1_F test. However, it is worth noting that the ANOMC – Reg_F test exhibits a smaller error compared to the other tests.

4.7 Summary and Concluding Remarks

The factorial ANOM test compares the means by considering more than one factor within the main variable; however, the fixed factor ANOMC test represents an improved version of the fixed factor ANOM test by incorporating the influence of covariates. To expand the ANOMC test into more complex designs, this study introduces the factorial ANOMC test

utilizing regression and series of ratio estimators named by ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests and compare the performance with the factorial ANOM test named as ANOM_F test. The ANOMC tests (namely, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F) work under a set of assumptions, such as normality, linearity, and homogeneity. The impact of these assumptions, as well as considerations like sample sizes, different levels of (i^{th}, j^{th}) combination (say treatments) and the formulation of null and non-null hypotheses is meticulously assessed concerning the ANOMC tests. This evaluation focuses on comparing the tests' performance in terms of the test's percentage type I error and percentage power.

At the end of this simulation, the study showed that in the case of homogeneity of variances with the null case, the ANOM_F test consistently maintained similar levels of type I error rates regardless of the specific scenarios considered in the study while the ANOMC tests struggled to maintain control over the error practically in Cauchy distribution. Further, in the ANOMC – MR1_F test, except in the condition of equal small correlation (ρ_1), the test exhibits difficulties in regulating the type I error rates across all the explored scenarios unlike the ANOMC – MR2_F test, exhibited a percentage around 5% for the type I error rate. Apart from the chi-square distribution, ANOMC – Reg_F and ANOMC – R_F test demonstrated a type I error rate of approximately 5%. Furthermore, ANOMC – R_F with direct pairing of correlations (ρ_2), where it struggled to maintain control over the error. In the ANOMC – MR3_F, apart from the Cauchy distribution, in the scenario of direct pairing of correlations (ρ_2). In the ANOMC – MR4_F test, Type I error rates varied across different

scenarios, ranging between 2.5% and 7.5%, but remained low at approximately 1% in cases with high correlations. However, increasing the number of levels in factor B also contributed to elevated error rates. However, the ANOMC – Reg_F test was effective in controlling the Type I error rate when the correlation in factor A was equal.

In the presence of heterogeneity of variances with the null case, regardless of the distribution shape and sample size combinations, the aforementioned tests ANOM and ANOMC are affected. Except Cauchy distribution, ANOMC – MR1_F and ANOMC – MR4_F tests demonstrate better percentage type I error rates and relatively lower susceptibility to heterogeneity compared to ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR2_F and ANOMC – MR3_F tests particularly in factor A. Regarding the ANOMC – Reg_F test, it exhibits higher type I error rates (%) when large sample sizes are associated with substantial variances. It is worth noting that in the interaction and factor B in the case of $(i, j) = (2, 2)$, except for Chi-square, Cauchy distribution and ANOMC – MR1_F, all the tests were not affected by the heterogeneous. under the high correlation (ρ_4), ANOMC – MR4_F is less effected due to the impact of heterogeneity, on the other hand under the small correlation (ρ_1), ANOMC – MR1_F is less effected due to the impact of heterogeneity. Additionally, the ANOMC – Reg_F test also displays higher type I error rates (%) in cases with an association between large sample sizes and small variances. Conversely, the ANOMC – R_F test demonstrates higher type I error rates (%) when there is an association between small sample sizes and large variances in the interaction and factor B rates. For the power of the test, as anticipated, the power values of the tests (ANOM_F, ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F tests) vary based on the effect size, sample size,

and distribution environment. $ANOMC_F$ tests, when compared to $ANOM_F$, demonstrates relatively superior performance, practically when employing regression estimator. However, it is important to note that the specific experimental conditions significantly determine the outcomes. Furthermore, the effectiveness of the $ANOM_F$, $ANOMC - Reg_F$, $ANOMC - R_F$, $ANOMC - MR1_F$, $ANOMC - MR2_F$, $ANOMC - MR3_F$ and $ANOMC - MR4_F$ tests improve as the sample size and the levels of factor B increase.

CHAPTER 5

SUMMARY, CONCLUSIONS AND FUTURE RECOMMENDATIONS

In this chapter, the findings of this thesis, along with future recommendations, are reported. These are given in the following sections:

5.1 Summary and Conclusion

In many experiments, our interest lies in testing the significance of means from the grand mean of the study variable known as Analysis of Means (ANOM). Sometimes, some linearly related covariates are also recorded while collecting the information on the main variable. ANOM is developed to test the t independent treatment means of the main variable only, without considering the covariates. However, for testing the t independent treatment means of the main variable while keeping the effect of covariate, a new test called the analysis of means using covariate (ANOMC) was proposed recently. This new approach capitalizes on the covariate effect to refine the traditional structure and rectify misleading decisions, especially when covariates are present.

In Chapter 2, The proposed test (referred to as ANOMC-Reg) drew from the regression estimator. The purpose of developing the ANOMC test was to address certain limitations associated with the regression estimator, particularly its inadequate performance when handling distributions that display highly skewed, positively skewed data by using a family of ratio estimators named ANOMC-R, ANOMC-MR1, ANOMC-MR2, ANOMC-MR3, and ANOMC-MR4. The results show that the ANOMC, compared to ANOMC-Reg, demonstrates relatively superior performance when employing ratio family estimators. The use of the ANOMC test is insufficient; more investigations should be conducted on its utility in experiments involving multi-covariates.

In Chapter 3, additional covariate variable is considered, proposing new tests refer as ANOMC-MR6, ANOMC-MR7, ANOMC-MR8, ANOMC-MR9, and ANOMC-MR10 using a series of ratio family estimators. ANOMC-MR6 shows less performance compared to other tests. Moreover, in previous chapters, we explored the ANOMC test, which focused on analyzing data with one fixed factor. Now, we are expanding the scope of the test to more complex treatment structures, specifically factorial designs.

In Chapter 4, we investigate the effectiveness of factorial ANOMC tests (referred to as ANOMC – Reg_F, ANOMC – R_F, ANOMC – MR1_F, ANOMC – MR2_F, ANOMC – MR3_F and ANOMC – MR4_F) utilizes regression and ratio estimators to compare it with the traditional factorial ANOM, known as ANOM_F, furthermore, to enhance the performance of the factorial ANOM test by incorporating a covariate variable. Finally, the performance of the proposed tests in this work is evaluated on the basis of well-known performance measures such as type I error and test power. The simulations of this work were done by using the Monte Carlo simulation technique.

5.2 Limitations of The Study

The limitations of our study are given in the following points:

- I. This study is designed to investigate a specific set of parameters, including a limited number of treatments, choices of sample size, covariance between variables, and the limited number of distributions.
- II. Generally, in ANOM and ANOMC two well-known models are used such as fixed effect model and random effect model. This study comprises fixed effect model in ANOM and ANOMC.
- III. This study focuses on single factor and two factor factorial design.

5.3 Future Recommendations

- I. The study in Chapter 4 is based on “one factor at two levels”; one may extend this idea to both factors at more than one level.
- II. The complete dissertation consists of methods that are used to enhance the ANOM and ANOMC under fixed effect model. One may use these methods to improve the ANOM and ANOMC under the random effect model.
- III. Mostly, the present study has covered Completely Random Design (CRD). However, the scope of this study may be extended to cover Randomize Completely Block Design (RCBD) and Latin Square Design (LSD).
- IV. It is potential for future work to extend the study by investigating the proposed methods under 2^k factorial, fractional factorial, nested and split plot designs.

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