

AN INVESTIGATION OF STATISTICAL CONTROL CHARTING SCHEMES FOR TRACKING DRIFTED PROCESSES

BY

HUDA HAKEM ALSHAMMARI

A Thesis Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

APPLIED STATISTICS

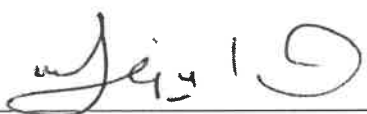
January 2024

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN- 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by **HUDA HAKEM ALSHAMMARI** under the direction of his thesis advisor and approved by her thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN APPLIED STATISTICS**.



Prof. Monther Rashed Alfuraidean

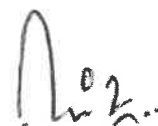
(Department Chairman)



Prof. Suliman Saleh Al-Homidan

(Dean of Graduate Studies)

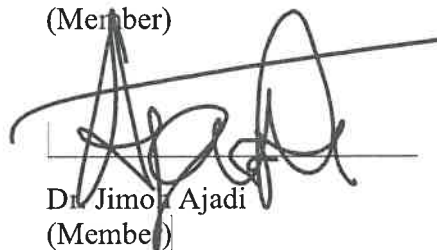
Date



Prof. Muhammad Riaz
(Advisor)



Dr. Nasir Abbas
(Member)



Dr. Jimon Ajadi
(Member)



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I dedicate my thesis work to my family. A special feeling of gratitude to my loving parents, Hakem and Foz Alshammari and my sisters & brothers.

Also, I dedicate this thesis to my lovely husband Mohammad Alshammari, and my kids Albtool, Tariq, Mubarak, and Lean. I am truly thankful for having you all in my life.

|

ACKNOWLEDGMENTS

I am grateful for the guidance and strength provided by Allah for me throughout this journey. I was able to overcome challenges and complete this task successfully because of His boundless mercy. May His blessings continue to guide me in all my endeavors and may the peace of God be upon His beloved prophet.

I would like to express my gratitude to Northern Border University. The support and resources provided by the university have significantly enhanced my learning experience. Also, I am grateful for the great research environment and facilities that King Fahd University of Petroleum and Minerals (KFUPM) has provided for me. My sincere gratitude goes out to the Deanship of Scientific Research (DSR) of KFUPM for providing the necessary funding for me to attend the IEEM 2023 conference in Singapore. Additionally, I want to express my sincere gratitude to everyone at the Mathematics Department of KFUPM. For my academic success, the department's dedication to offering resources and assistance outside of the classroom has also been a major factor. A special thanks to Prof. Monther Alfuraidan for the endless care and support he provided. Thank you for believing in me.

I would like to express my deep gratitude to my advisor, Dr. Muhammad Riaz. His guidance and expertise have been instrumental in shaping my research. Thank you for your invaluable guidance, support, kindness, and patience throughout my academic journey.

I would like to express my sincere gratitude to Dr. Tahir Mahmood. I feel incredibly fortunate to have had the opportunity to work with a mentor who is so dedicated and competent. I'm very grateful that you always pushed me to do my best. Your constant

encouragement and belief in my abilities have been truly motivating. I am grateful for the countless hours you have dedicated to reviewing my work, providing insightful feedback, and helping me overcome challenges. Your mentorship has not only enriched my academic experience but also inspired me to become a better researcher. Thank you for everything you have done for me.

Additionally, I would like to express my gratitude to all committee members and my teachers for supporting and advising me. I would like to mention, especially, Dr. Nasir Abbas, Dr. Jimoh Ajadi, and Dr. Emmanuel Osita Afuecheta. I am grateful for the time and effort you have invested in me. I will always value the lessons I have learned from you.

I would like to express my gratitude to my parents, siblings, husband, and children . It would not have been possible for me to finish my studies without their tremendous support and understanding over the past few years. Their unwavering belief in me and constant encouragement were invaluable throughout this academic journey. I am truly grateful for their love and support, which has fueled my determination to succeed.

Finally, I am also grateful to my friends, especially Alaa Alseaad, who was a constant source of support and encouragement throughout my journey. She has stood by me through thick and thin, and I am truly blessed to have such amazing individuals by my side.

HUDA ALSHAMMARI

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LIST OF ABBREVIATIONS

μ	:	Location Parameter
μ_0	:	In-Control Location Parameter
μ_1	:	Out-of-Control Location Parameter
N	:	Normal Distribution
n	:	Sample Size
L	:	Control limit Coefficient
L_1	:	Control Limit Coefficient for Lower Control Limit
L_2	:	Control Limit Coefficient for Upper Control Limit
h_1	:	Decision Limit for Lower Control Limit
h_2	:	Decision Limit for Upper Control Limit
t	:	Time Point
τ	:	Change Point
σ	:	Standard Deviation
σ_0	:	In-Control Standard Deviation
σ_1	:	Out-Of-Control Standard Deviation

$\tilde{X}$	:	Sample Median
$\bar{X}$	:	Sample Mean
X	:	Independent Random Variable
δ	:	Amount of Shift
ARL_0	:	In-Control Average Run Length
ARL_1	:	Out-Of-Control Average Run Length
ARL	:	Average Run Length.
CL	:	Central Limit
$CUSUM$	:	Cumulative Sum
$\bar{X} - CUSUM$	:	Cumulative Sum of Mean
$\tilde{X} - CUSUM$	:	Cumulative Sum of Median
$\bar{X} - EWMA$	:	Exponentially Weighted Moving Average of Mean
$\tilde{X} - EWMA$	:	Exponentially Weighted Moving Average of Median
$EWMA$	:	Exponentially Weighted Moving Average.
FAR	:	False Alarm Rate
IC	:	In-Control
LCL	:	Lower Control Limit

<i>MDRL</i> :	The Median of Run Length
<i>OOC</i> :	Out-Of-Control
<i>RL</i> :	Run Length
<i>SDRL</i> :	The Standard Deviation of Run Length
<i>SPC</i> :	Statistical Process Control
<i>UCL</i> :	Upper Control Limit.

|

ABSTRACT

Full Name : [Huda Hakem Alshammari]

Thesis Title : [An Investigation of Statistical Control Schemes for Tracking Drifted Processes]

Major Field : [Applied Statistics]

Date of Degree : [January 2024]

Manufacturing procedures today are very capable of producing items that are in compliance. However, over time, non-conforming goods are produced as a result of unstable manufacturing processes caused by the machining parts' wear and tear. For instance, the needle of the machine is crucial for properly stitching clothing items when they are manufactured. When the needle tip is broken, the stitching mistakes get worse over time and result in clothes that do not meet the desired quality standards. To address this issue, manufacturers need to regularly inspect and replace worn-out machine parts to ensure consistent product quality. Such overtime changes are referred to as drift, but a sudden, temporary change is referred to as a shift. Statistical process control (SPC) is a crucial technique used to manage and control processes in such situations. Location control charts are one of the main tools of SPC. Shewhart, as memoryless control chart, with exponentially weighted moving average (EWMA) and Cumulative Sum (CUSUM) as memory charts, are typically used to track the processes' shifted mean. The purpose of this thesis is to demonstrate the performance of location charts under linear and quadratic drifts because these charts are not frequently mentioned in drifted setups. A simulation setup is designed to assess charts' zero-state and steady-state run length performance under normal

and non-normal setups. Moreover, the proposed charts are implemented on a real-life dataset to highlight the stated proposal's importance

ملخص الرسالة

الاسم الكامل: هدى بنت حاكم الشمري

عنوان الرسالة: التحقيق في مخططات الرقابة الإحصائية لتعقب العمليات المنجرفة

التخصص: إحصاء تطبيقي

تاريخ الدرجة العلمية: يناير 2024

إجراءات التصنيع اليوم قادرة جداً على إنتاج مواد متماثلة تطابق المعايير التي يحددها المصنعون. ومع ذلك، فإن هناك احتمالية لظهور سلع غير مطابقة للمعايير الموضوعية والتي قد تُنتج بشكل غير مباشر (بطيء) بمرور الوقت نتيجة لعمليات التصنيع غير المستقرة الناجمة عن التآكل والتلف في الأجزاء الميكانيكية للألات. على سبيل المثال، فإن إبرة الآلة تعد أداة بالغة الأهمية لخياطة أصناف الملابس على نحو سليم ومطابق للمعايير. ولكن عندما يتعرض طرف هذه الأداة للكسر أو التلف بسبب الاستخدام المستمر فإنه يخلف أثراً سلبية تنعكس على عملية التصنيع والانتاج. ستصبح أخطاء الخياطة أسوأ مع مرور الوقت وتؤدي إلى إنتاج قطع لا تفي بمعايير الجودة المطلوبة. ولمعالجة هذه المسألة، يتعين على المصنعين أن يفحصوا ويستبدلوا بانتظام قطع الآلات البالية لضمان اتساق جودة المنتج. ينبغي الإشارة إلى أن التغيير في قراءات عملية ما والذي يحدث باستمرار بمرور الوقت يعرف بمصطلح الانجراف، بينما التغيير المؤقت والمفاجئ في القراءات يُشار إليه بمصطلح التحول. في حالات مماثلة، تعد المراقبة الإحصائية للعمليات تقنية بالغة الأهمية لإدارة ومراقبة العمليات. وعادة ما تستخدم مخططات التحكم في المركز، باعتبارها أهم أدوات المراقبة الإحصائية للعمليات. ويستخدم مخطط شوارت ومخطط المتوسط المتحرك المرجح أسياً ومخطط المجموع التراكمي لمتتبع "المتوسط المحوّل" للعمليات. الغرض من هذه البحث هو فحص أداء مخططات التحكم بالمركز في إطار انجرافات خطية وتربيعية. لأن الدراسات السابقة لم تتطرق لمناقشة أداء هذا النوع من المخططات في ظل وجود الانجراف. قامت هذه الدراسة على نظام المحاكاة وقد صُمّم هذا النظام لتقييم أداء المخططات في حالة المدى الصفري وفي حالة المدى الثابت للمراقبة في إطارات التوزيع الطبيعي وغير الطبيعي. وعلاوة على ذلك، تم تطبيق نتائج الدراسة على مجموعة بيانات حقيقية من الواقع لإبراز أهمية المخططات المقترحة.

CHAPTER 1

INTRODUCTION

This chapter provides an introduction to statistical process control (SPC), particularly control charts like the Shewhart as a memoryless control chart, with exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) as memory charts. This chapter will cover drift concepts in addition to a literature review. At the conclusion, the thesis's outlines are also provided.

1.1 Statistical Process Control

Today, in an increasingly competitive marketplace, we use a wide range of goods and services from a variety of sources, including smart phones, clothing, healthcare, food, transportation, sports, and consultation, not only to meet our needs but also to enjoy the quality and low price of the goods or services. In order to live up to our expectations, these goods and services must comply with specified requirements. Thus, manufacturers and suppliers face intense pressure to maintain a high standard of quality. Due to damage to machining components, production processes deteriorate with time, resulting in non-conforming goods. As a result, manufacturing delays and repair expenses may rise. These problems can be reduced, and high-quality product manufacturing can be ensured by using statistical process control. Statistical process control, or SPC, is not a recent idea in manufacturing fields. The control chart and the notion that a process could be subject to statistical control were developed at Bell Laboratories by Walter A. Shewhart (1929). SPC

seeks to examine process variances using a collection of well-known toolkits such as histograms, cause-and-effect diagrams, check sheets, Pareto charts, scatter diagrams, stratification, and the control chart, which is the most well-known SPC tool.

1.2 Location Control Charts

Control charts are one of the most essential instruments of statistical process control utilized for process monitoring in order to give quality and stability to specified areas. Control charts are a type of graphical tool that can help you monitor and improve a process or certain system. The three decision lines that make up a control chart are the center line (CL), which represents the objective of a process; the lower line (LCL), which stands for lower control limit; and the upper line (UCL), which stands for upper control limit. Making a determination regarding process behavior is made simpler by plotting the monitoring statistics against the aforementioned decision lines. Numerous forms of statistical control charts, each with a specific purpose, can be used to carry out this monitoring. For instance, location charts keep track of the process mean and determine if a change over time has occurred using the sample mean or median. On the other hand, sample variance and sample range are typically used to track the distribution of observations around the average in order to measure process dispersion.

The two primary types of statistical control charts are memory-less charts, which are just based on current data and do not account for the process's previous performance. They work well for spotting significant changes in process variables like the mean or standard deviation. The Shewhart chart, created by Walter A. Shewhart in the 1920s, is the most typical instance of a memory-less chart.

Memory charts are constructed using both past and current data, with a preference for more recent observations. They are more sensitive than memory-less charts, making them useful for spotting slight and moderate changes in the process parameters. The Cumulative Sum (CUSUM) chart and the Exponentially Weighted Moving Average (EWMA) chart, both of which were created in the 1950s and 1960s, are the most typical types of memory charts.

1.2.1 Shewhart Chart

Shewhart charts are a particular kind of control chart that are used to track and enhance the effectiveness and efficiency of process parameters (i.e., location or dispersion) over time. Shewhart charts have been widely used as a tool in sectors like manufacturing, healthcare, and finance. The Shewhart chart is good at identifying significant or brief changes in the process because it is based on the most recent sample data. The upper control limit (UCL), lower control limit (LCL), and central limit (CL) are the three decision lines that make up the Shewhart control chart's structure. A statistic (θ) where $\theta \sim N(\mu_0, \sigma_0^2)$ will be plotted against the aforementioned limits, which are defined as

$$\mu_0 \pm L \frac{\sigma_0}{\sqrt{n}}$$

Where L is charting constants, which are used to determine the width of the limits against the pre-specified IC average run length (ARL_0). Shewhart The central line represents the process target $CL = \mu_0$. Generally, 3σ limits are most frequently used in practice when the distribution of the statistic follows a standard normal distribution. Additionally, the Shewhart chart indicates an *OOC* signal when θ exceed LCL to UCL . Otherwise, the chart indicates the *IC* state of the process.

1.2.2 EWMA Chart

This chart is based on the Exponentially Weighted Moving Average statistical method, the exponentially weighted moving average (EWMA) chart gets its name from it. The decision-making process in the chart is based on an exponentially decreasing function of previous measurements. EWMA control chart is a memory-type structure due to the fact that both historical and contemporary data are used. The EWMA is used to monitor small or moderate shifts in the process parameters. In order to determine the performance of the EWMA chart, two parameters must be carefully selected against a fixed IC average run length ARL_0 . The first parameter of the EWMA chart is the weight allocated to the most recent subgroup mean, known as λ where λ generally must be within $0 < \lambda \leq 1$. L , also known as the control limit coefficient, is another parameter that has to be adjusted since it aids in determining the chart's control limits. The EWMA statistic (Z_i) for a statistic (θ) is defined for each sample number i as

$$Z_i = \lambda\theta + (1 - \lambda)Z_{i-1}$$

The Z_i initial value is set to zero (i.e., $Z_i = \mu_0$). The following are the asymptotic upper and lower bounds for the *EWMA* charts.

$$\mu_0 \pm L \sqrt{\frac{\lambda}{2 - \lambda} \sigma_0^2}$$

In a process, if Z_i exceeds the limits, a signal is generated by the EWMA chart; otherwise, the process is stable. More details can be found in Chapter 3.

1.2.3 CUSUM Chart

The *CUSUM* chart is a memory-type structure because it includes both the most recent and prior values of the process's data. Furthermore, the *CUSUM* chart plots the cumulative sums of the deviations of the sample values from a target value. It is used to monitor small or medium changes in the process parameters. The essential parameters for creating the *CUSUM* chart are a reference value (k) and a decision limit (h). Setting the reference value (k) to be half the drift to be detected and the decision limit (h) to the threshold for declaring the process out of control. The *CUSUM* plotting statistic is defined as

$$C_i^+ = \max [0, (\theta - \mu_0) - k + C_{i-1}^+]$$

$$C_i^- = \max [0, -(\theta - \mu_0) + k + C_{i-1}^-]$$

Where $C_0^+ = C_0^- = 0$. The *CUSUM* limits are defined as follows.

$$UCL = H = h\sigma_0 ; LCL = -H = -h\sigma_0 ; CL = 0$$

For more information regarding *CUSUM* chart see chapter 4.

1.3 Shifts vs Linear and Quadratic Drifts

The study aims to study shift, linear drift, and quadratic drift. The shift is characterized as a sudden increase or decrease in a process's performance. This shift can occur due to various factors, such as technological advancements, changes in market demand, or modifications in production methods.

The conventional shift in the mean is defined as $\mu_1 = \mu_0 + \delta\sigma_0$. Where μ_1 is the shifted mean, and δ is the amount of change times σ_0 in the process mean. Traditionally, δ stays constant across time. In steady-state monitoring, the shift is introduced after a

predetermined point, but in zero-state monitoring, the shift is introduced from the initial point of the process. Therefore, we have considered steady-state shift beyond the 50th point in this study.

However, the terms "drifts" and "trends" refers to the gradual change in the measurement readings over time either linearly or nonlinearly. The difference between a small drift and a large drift lies in the magnitude of this change. A large drift indicates a significant and perceptible change over time, whereas a small drift indicates a gradual and relatively minor change in the measurement readings. The drifts seemed to occur more frequently in practice when cutting tools started to gradually decay from everyday usage or when production machinery started to deteriorate. For instance, in the production of clothing, the machine's needle is crucial to properly sewing the shirts. The sewing defects get worse over time if the needle tip is destroyed. Additionally, over time during the manufacturing process, the turning process of the outside diameter (OD) of steel rods is also impacted by the frequent use of the turning die (die lifetime) Fahmy and Elsayed (2006b).

Linear drift is a constant rate of deviation in a measurement's readings over time, indicating a consistent deviation from the true value per unit of time. The linear drifted mean is defined as $\mu_1 = \mu_0 + \omega t \sigma_0$. Where μ_1 is the drifted mean, ω is the amount of change times σ_0 in the process mean, and t represents the time point. For example, if a pressure gauge consistently reads 2 psi higher than the actual pressure every month, it exhibits linear drift. On the other hand, quadratic drift refers to a situation where the drift in a measurement's readings changes at a non-constant rate over time. This means that the measurements deviate from the true value by an amount that varies with time. the quadratic drifted mean is defined as $\mu_1 = \mu_0 + \rho t^2 \sigma_0$. Where μ_1 is the drifted mean, ρ is the amount of change

times σ_0 in the process mean, and t^2 represents the time point. For example, if a temperature sensor initially reads 1 degree Celsius higher than the actual temperature, but the discrepancy increases by an additional 0.5 degrees Celsius every month, it exhibits quadratic drift. The investigation introduces drift at the 50th point in steady-state drift monitoring, while zero-state drift monitoring starts at the process's starting point.

1.4 Overview of Control Chart Studies Based on Linear and Quadratic

Drifts

Numerous studies have been conducted in the past to study drifts using various control charts. For example, under linear drift, Bissell (1984a) expanded the Markov chain methodology proposed by Brook and Evans (1972) and examined the run-length characteristics of the Shewhart and CUSUM charts. A method for determining the trend from the CUSUM chart was studied by Bissell (1984b). Davis and Woodall (1988) evaluated the performance of the $\bar{X}$ chart using various trend rules to determine the drift on the process mean. Gan (1992), and Gan (1996) developed numerical algorithms. Moreover, tables and instructions are provided to identify the proper EWMA and CUSUM chart parameters of a particular *ARL*. Reynolds and Stoumbos (2001) proposed a EWMA chart to monitor drifts in location and scale parameters and compared their proposed method with the combined Shewhart $\bar{X}$ chart with the Shewhart MR chart. Along with the implementation of the VSI feature, the combined charts with the EWMA chart will provide faster detection of drift rates. Davis and Krehbiel (2002) investigated the performance of a $\bar{X}$ control chart with run-rules in the presence of a linear trend in the process mean. The

findings show that a run-rules based control chart is better than $\bar{X}$ control chart in a situation where a linear trend in the process mean is introduced. Gültekin et al. (2002) studied the performance of cross-correlation charts with a various control chart on processes with mean under linear drift. The proposed chart outperforms the CUSUM and of $\bar{X}$ trend control charts. Fahmy and Elsayed (2006a) developed a chi-square chart that aims to detect linear trends in the process mean and compare its performance with Shewhart, CUSUM, EWMA, and generalized likelihood ratio (GLR) control charts. Shu et al. (2008) investigated the performance of the weighted CUSUM chart under linear drifts. Zou et al. (2009) evaluated the performance of various control charts such as GLR, EWMA, CUSUM and generalized EWMA (GEWMA) in terms of detecting linear drifts. Their findings revealed that *GLR* based control chart outperforms the other charts under consideration. Assareh et al. (2011) modeled and detected the change point in a Bayesian framework, assuming that the underlying change is a linear trend in the odds ratio of a Bernoulli process. In a bivariate environment with linear trend disruption, Atashgar and Noorossana (2011) proposed a model based on artificial neural networks. Amiri and Allahyari (2012) comprehensively reviewed charts based on change point models. Xu et al. (2013) proposed GLR based control chart to examine the linear mean drift. Kazemi et al. (2014) were focused on the issue of identifying the transition point in various processes when the quality characteristics gradually depart from a *IC* state using fuzzy statistical clustering (FSC) to estimate the drift time in different processes. The suggested method is superior in identifying the drift time across a wide range of trend rates. Recently, Yi and Qiu (2022) suggested a new approach to drift detection. The new approach makes use of the general

framework to create a CUSUM chart based on the GLR statistic in order to detect linear drift.

Furthermore, the studies that have examined process drifts along with the EWMA chart are few, to the best of our knowledge. Aerne et al. (1991) assessed the performance of the Shewhart chart, the CUSUM chart, and the EWMA chart under linear drift in the process mean. Gan (1991) evaluated the performance of the EWMA chart in the presence of linear drift. Fahmy and Elsayed (2006a) compared the performance of the chi-square chart, which aims to detect linear trends in the process mean, with Shewhart, CUSUM, EWMA, and generalized likelihood ratio (GLR) control charts. Fahmy and Elsayed (2006a) examined the performance of the maximum likelihood estimator (MLE) based CUSUM and EWMA charts under linear drift. Su et al. (2011) suggested an adaptive EWMA (AEWMA) chart for the detection of linear drift in the process mean.

Additionally, there are, at most, a few studies that have looked into process drifts in association with the CUSUM chart. For example, in his 1984 study, Bissell looked into a method for reading a trend from a *CUSUM* chart Bissell (1984a). Also, Bissell (1984b) expanded the Brook and Evans (1972) Markov chain methodology by examining the run-length characteristics of the Shewhart and CUSUM charts. Gan (1992), and Gan (1996) developed numerical algorithms. Moreover, tables and instructions are provided to identify the proper EWMA and CUSUM chart parameters for a particular *ARL*. Aerne et al. (1991) evaluated how the Shewhart chart, CUSUM chart, and EWMA chart performed when there was linear drift in the process mean. Gültekin et al. (2002) evaluated the effectiveness of cross-correlation charts with various control charts on processes with a mean subject to linear drift. The proposed chart outperforms both the CUSUM and the X trend control

charts. Fahmy and Elsayed (2006a) investigated the performance of the CUSUM and EWMA charts based on the maximum likelihood estimator (*MLE*) under linear drift. , Xu et al. (2013) suggested using a control chart based on a *GLR* to assess the linear mean drift. Recently, a fresh method for drift detection was proposed by Yi and Qiu (2022). In the new method, a CUSUM chart based on the *GLR* statistic is produced using the general framework.

1.5 Objectives of the Study

Drifts and shifts in the process's mean can cause increased product quality variability, more waste, and decreased productivity. Therefore, monitoring the process plays a key role in ensuring the success and effectiveness of any project or system. Using control charts makes it easier to regularly monitor and evaluate performance and outcomes to identify any potential deviations or issues. ultimately improving the quality. The specific objectives of our study are listed below.

- I. Developing memory-less control charts for different types of process drifts under various distributional environments.
- II. Designing memory-type monitoring schemes (Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA)) targeting small to moderate amounts of drifts.
- III. Evaluating the proposed monitoring structures using run length characteristics, such as *ARL*.
- IV. Implementing the proposed methodologies on practical datasets as case studies.

1.6 Outlines of the Thesis

In Chapter 2, we introduce Shewhart charts based on sample mean ($\bar{X}$) and sample median ($\tilde{X}$). Providing a description of the control chart structure for location monitoring in the presence of linear and quadratic drifts. In two monitoring structures (i.e., zero- and steady-state), the performance of the Shewhart charts is assessed using run length (RL) properties. Considering that the evaluation will be based on various subgroup sizes ($n=5, 7, \text{ and } 10$) in different distributional environments. In the end, real-world data is used to demonstrate the implementation of Shewhart charts and to validate their effectiveness in detecting process variations.

In Chapter 3, exponentially weighted moving average chart is based on the sample mean ($\bar{X}$) and sample median ($\tilde{X}$). The goal is to evaluate how well the charts perform when subjected to shifts, linear drifts, and quadratic drifts in non-normal environments such as the student t distribution and gamma distribution. Additionally, different subgroup sizes are taken into account while discussing the performance of the charts, as well as zero-state and steady-state scenarios. In this chapter, we also compare the Shewhart and EWMA charts' abilities to identify and react to process drift under identical circumstances, revealing which chart is more suited for spotting process changes. A real example has been explained of the practical implementation of the charts.

In chapter 4, we propose a cumulative sum control chart (CUSUM). The charts used the sample mean ($\bar{X}$) and sample median ($\tilde{X}$) to track any change in the process mean. The charts are subjected to shifts, linear drifts, and quadratic drifts in non-normal situations. In terms of run length properties, zero- and steady-state monitoring were used to evaluate

the performance. A comparative analysis of CUSUM with Shewhart charts and EWMA charts under drifts is provided. Also, we provide a brief comparison of control charts. Demonstrating how the CUSUM charts (i.e., $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$), the EWMA control charts (i.e., the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts), and Shewhart control charts (i.e., the $\bar{X}$ and $\tilde{X}$ charts) perform under different environments, and the performance analysis is discussed in terms of run length properties. Further, real-life applications related to air quality and maintenance is also included to highlight the importance of the proposed charts.

In Chapter 5, we present the findings along with limitations and recommendations for future research.

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CHAPTER 2

Shewhart Location Charts Under Drifts

In this current era, manufacturing processes are highly capable of producing conforming products. However, due to wear and tear on the machining parts, the manufacturing process does not remain stable and produces non-conforming products with the passage of time. For example, in garment manufacturing, the machine's needle plays a significant role in adequately sewing the shirts. When the needle tip is damaged, the flaws in sewing increase with time. In literature, such gradual changes over time after a specific stable process are known as drift, while a sudden temporary change is known as a shift. Generally, location control charts, (i.e., $\bar{X}$ and $\tilde{X}$ charts) are well-known for monitoring the shifted mean of the process. However, these charts are not widely discussed under the drifted setup; therefore, this chapter is intended to show the performance of $\bar{X}$ and $\tilde{X}$ charts under linear and quadratic drifts. A simulation setup is designed to assess the charts' zero and steady state run length performance, and both charts are also discussed in non-normal environments. The findings of zero-state and steady-state monitoring for shifts and drifts revealed that the $\bar{X}$ chart outperforms $\tilde{X}$ chart under normal and gamma distributions, while $\tilde{X}$ chart performed well under student's t distribution. Both charts are much quicker at detecting quadratic drifts than linear drifts or shifts, and their performance increases with the increase in subgroup size n . Moreover, the proposed charts are implemented on a real-life dataset to highlight the stated proposal's importance.

2.1 Introduction

Statistical process control (SPC) is a well-known toolkit that contains several data analytic tools to study process variations. Usually, two types of variations exist in a process: the common cause of variations and the assignable cause of variations. The common cause of variations is the inherited part of the process, which cannot be controlled and remains in the process. However, the assignable cause of variations is the noise for any process, which can be timely identified, eliminated, or controlled to ensure the conforming yield of the process. Therefore, a control chart, one of the SPC tools, is mostly used for real-time detection of the common cause of variations in the process. The general structure of a control chart depends on three decision lines. First, a central line (CL) represents the target of a process, a lower line characterizes the lower control limit (LCL), and the upper line refers to the upper control limit (UCL). Plotting the monitoring statistics against the above-stated decision lines helps to draw a conclusion about the process behavior. A process is declared in-control (IC) when the monitoring statistic falls within decision limits, i.e., LCL and UCL ; otherwise, the process is stated out-of-control (OOC). A chart is known to be a memory-type chart when the plotting statistic is based on current and past information, such as cumulative sum (CUSUM) by Page (1954) and exponentially weighted moving average (EWMA) by Roberts (1959) control charts. However, when the plotting statistic depends only on the current information, it is known to be a memoryless control chart, such as the Shewhart chart proposed by Shewhart (1930). The memory-type structures are mainly designed to detect small or moderate deviations in the process, while memory-less charts are effective in detecting large variations in the process.

Monitoring a process means constantly detecting deviations from the intended goal and ensuring stability. Hence, it can be done by conducting various statistical control charts, each serving a particular purpose. For instance, location charts monitor the process mean or average and check for a deviation over time using a sample mean or sample median. For process dispersion, sample variance and sample range are usually used to monitor the spread of the observations (Montgomery, 2020). This chapter focuses on the Shewhart-type location charts, where the sample mean and the median are used as the plotting statistics. In literature, the Shewhart structure based on the sample mean is known as the $\bar{X}$ -bar or $\bar{X}$ chart, while the sample median, is termed by median or the $\tilde{X}$ chart. One may see recent related studies on $\bar{X}$ and $\tilde{X}$ charts in Yaqub et al. (2016), Tran et al. (2017), Aslam et al. (2019), and references therein.

The majority of research involving $\bar{X}$ and $\tilde{X}$ charts is created with the assumption that quality characteristics are regularly distributed with μ_0 and σ_0 . A shift in the mean is expressed in a process as $\mu_1 = \mu_0 + \delta\sigma_0$, whereas a linearly drifted mean is $\mu_1 = \mu_0 + \delta t\sigma_0$, and a quadratically drifted mean is defined as $\mu_1 = \mu_0 + \delta t^2\sigma_0$. Here, t stands for a time point, μ_1 is the shifted mean, and δ is the amount of change times σ_0 . Further information about the conventional shift in the mean, linear drifts, and quadratic drift is provided in section 1.3. In the literature, several studies (discussed in Section 1.4) are available for monitoring the drifts, such as Davis and Krehbiel (2002) which investigated the performance of a $\bar{X}$ control chart with a run-rules in the presence of a linear trend in the process mean. The findings show that run-rules based control chart is better than a $\bar{X}$ control chart in a situation where a linear trend in the process mean is introduced. Davis and Woodall (1988) evaluated the performance of $\bar{X}$ chart using various trend rules to

determine the drift on the process mean. Aerne et al. (1991) evaluated the performance of Shewhart, CUSUM, and EWMA charts under linear drift in the mean. From the literature, it is seen that many studies are designed to see the effect of shifts and linear drifts in the data using a $\bar{X}$ chart. However, the $\tilde{X}$ chart is not discussed under drifts, to the best of our knowledge. Furthermore, location-based studies or change-point studies are discussed under the linear change over time; however, to the best of our knowledge, quadratic change over time is not considered. Hence, this chapter is designed to assess the performance of $\bar{X}$ and $\tilde{X}$ charts under shifts, linear drifts, and quadratic drifts. In addition, the performance of $\bar{X}$ and $\tilde{X}$ charts is also discussed under non-normal environments, considering different subgroup sizes, and in terms of zero-state and steady-state.

The rest of the chapter is structured as follows: The control charts structure for the location monitoring is presented in Section 2. The study's simulation design is given in Section 3. In Section 4, performance evaluation and comparative analysis are reported. In Section 5, real-world data is used to demonstrate the implementation of charts, and finally, remarks and recommendations for future research work are provided in Section 6.

2.2 Control Charts for Location monitoring

The popular Shewhart structure for location monitoring, based on the sample mean, is called the X-bar or $\bar{X}$ chart, and the sample median is called the median or $\tilde{X}$ chart. This section is designed to discuss the general structure of the $\bar{X}$ and $\tilde{X}$ control charts.

2.2.1 The $\bar{X}$ Chart

Let $X_{t,i}; i = 1, 2, \dots, n; t = 1, 2, \dots$ be a normally distributed t^{th} subgroup of size n , i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$, where μ_0 and σ_0 are the IC mean and standard deviation, respectively. The term t represents the time, and this linear drifted model transforms into

a regular shifted model when $t = 1$. However, if t is replaced with t^2 , the resulting model is named a quadratic drifted model. Moreover, δ is the amount of the standardized mean shift. If $\delta = 0$, the process is considered as *IC*; otherwise, the process is declared as *OO*.

The t^{th} plotting statistic, which is the sample mean, is obtained as follows:

$$\bar{X}_t = \frac{\sum_{i=1}^n X_i}{n} \quad (1)$$

and the control limits of the $\bar{X}$ chart is given as follows:

$$LCL = \mu_0 - L_1 \frac{\sigma_0}{\sqrt{n}} \quad (2)$$

$$CL = \mu_0 \quad (3)$$

$$UCL = \mu_0 + L_2 \frac{\sigma_0}{\sqrt{n}} \quad (4)$$

Where L_1 and L_2 are charting constants, which are used to determine the width of the limits against the pre-specified IC average run length (ARL_0). The values of the L_1 and L_2 against the fixed $ARL_0 = 370$ are presented in Table 1.1. In a process, the $\bar{X}$ chart provides a signal if $\bar{X}_t$ falls outside decision limits, i.e., LCL and UCL ; otherwise, the process remained stable.

2.2.2 The $\tilde{X}$ Chart

Let $X_{t,i}$; $i = 1, 2, \dots, n$; $t = 1, 2, \dots$ be a normally distributed t^{th} subgroup of size n , i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$. The t^{th} plotting statistic of the $\tilde{X}$ chart, which is the sample median, is obtained as follows:

$$\tilde{X}_t = \begin{cases} X_{t(\frac{n+1}{2})}, & \text{if } n \text{ is odd} \\ \frac{X_{t(\frac{n}{2})} + X_{t(\frac{n}{2}+1)}}{2}, & \text{if } n \text{ is even} \end{cases} \quad (5)$$

and the control limits of the $\tilde{X}$ chart is obtained as follows:

$$LCL = \mu_0 - L_1 \sigma_0 \sqrt{\frac{\pi}{2n}} \quad (6)$$

$$CL = \mu_0 \quad (7)$$

$$UCL = \mu_0 + L_2 \sigma_0 \sqrt{\frac{\pi}{2n}} \quad (8)$$

Similar to the $\bar{X}$ chart, the L_1 and L_2 are different charting constants for $\tilde{X}$ chart, which are used to determine the width of the limits against the pre-specified IC average run length (ARL_0). The values of the L_1 and L_2 against the fixed $ARL_0 = 370$ are presented in Table 1.1. The process remained stable if $\tilde{X}_t$ fell inside the decision limits, i.e., LCL and UCL ; otherwise, the process remained unstable.

2.3 Design of Simulation Study

For the simulation setting, subgroups of different sample sizes, i.e., $n = 5, 7$, and 10 , are generated from the normal distribution ($N(0,1)$), student's t distribution with 4 degrees of freedom (t_4), and gamma distribution ($Gamma(1,1)$). Where the normal distribution is considered to show symmetric behavior, the t distribution is used to show the symmetric behavior with large probabilities on tails (light-tailed distribution), and the Gamma distribution is considered to cover a heavy-tailed, skewed environment.

The performance of the control chart is generally assessed using run length (RL). Therefore, different characteristics of the RL are considered to be performance measures,

such as average run length (ARL) to present an average of RL . The ARL is one of the most common performance measures, which measures the average number of samples until an OOC signal occurs. In performance evaluation, ARL is categorized as ARL_0 (average run length in IC state) and ARL_1 (average run length in OOC state). On the fixed $ARL_0 = 370$, a control chart is considered efficient and has a higher ability to detect the drift, if it has a smaller value of ARL_1 as compared to other charts under consideration. Further, for brevity, the findings of the study are interpreted in terms of the ARL_1 .

In practice, the shifts or drifts appear either at the starting point of the process or after a certain period of time. In SPC literature, if the shifts or drifts appear at the starting point of the process, then the process is known to be a zero-state process, while when shifts or drifts appear after a certain period of time, the process is referred to as a steady-state process. In this chapter, the performance of the $\bar{X}$ and $\tilde{X}$ control charts is assessed in both zero-state and steady-state processes. Further, the number of shifts or drifts is represented by δ and is considered as 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4. It is to be noted that for the steady-state process, we have considered shifts or drifts after the 50th point.

Table 1.1: Charting constant of Shewhart charts at $ARL_0 = 370$

Chart	Distribution	$n = 5$		$n = 7$		$n = 10$	
		L_1	L_2	L_1	L_2	L_1	L_2
$\bar{X}$	Normal	2.99	2.99	2.99	2.99	2.99	2.99
	student's t	3.75	3.74	3.63	3.60	3.51	3.51
	Gamma	1.88	4.20	2.04	4.02	2.18	3.85
$\tilde{X}$	Normal	3.03	3.03	3.02	3.01	3.00	3.01
	student's t	3.43	3.43	3.32	3.30	3.19	3.19
	Gamma	2.04	4.21	2.34	3.84	2.76	3.34

2.4 Results and Discussion

In this section, at fixed $ARL_0 = 370$, we present the performance of the $\bar{X}$ and $\tilde{X}$ control charts under shift, linear drift, and quadratic drift. The zero-state performance of the $\bar{X}$ and $\tilde{X}$ control charts in terms of ARL is presented in Tables 1.2 with respect to $n = 5, 7$, and 10, respectively. Similarly, the steady-state performance of the $\bar{X}$ and $\tilde{X}$ control charts is presented in Tables 1.3 with respect to $n = 5, 7$, and 10, respectively. Moreover, the superiority of the chart is determined by the minimum ARL_1 value as compared to another chart under consideration.

2.4.1 Zero-State Run Length Performance

This section is designed to show the performance assessment of the $\bar{X}$ and $\tilde{X}$ charts in zero-state monitoring. The performance is evaluated in terms of ARL_1 value against different values of δ , i.e., 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4. The performance assessment of the $\bar{X}$ and $\tilde{X}$ charts in zero-state monitoring under normal, student's t and gamma

distributions are presented in Table 1.2 with respect to subgroup size $n = 5, 7$, and 10. The findings of the results are explained as follows:

- Under the normal distribution, the performance of the $\bar{X}$ chart is better than $\tilde{X}$ chart under shifts and drifts. For example, at $n = 5$ and $0.1\sigma_0$ shift (Table 1.2), 73 units decrease in the ARL_1 value of $\bar{X}$ chart is observed while in the $\tilde{X}$ chart, reduction by 66 units in ARL_1 values have been seen. At $n = 7$ and $0.1\sigma_0$ linear drift, $\bar{X}$ chart has 8.93 ARL_1 value while $\tilde{X}$ chart has 10.41 ARL_1 value. Similarly, $\bar{X}$ chart has $ARL_1 = 3.50$ and $\tilde{X}$ chart has $ARL_1 = 3.73$ for $0.1\sigma_0$ quadratic drift at $n = 10$.
- Under the student's t distribution, $\tilde{X}$ chart has better performance in detecting shifts as compared to $\bar{X}$ chart under shifts and drifts. For example, at $n = 5$ and $0.1\sigma_0$ shift (Table 1.2) may cause 26 units reduction in the ARL_1 values for $\bar{X}$ chart. While, in the $\tilde{X}$ chart, the same amount of change may cause a reduction of 67 units in ARL_1 values. At $n = 7$ and $0.1\sigma_0$ linear drift, $\bar{X}$ chart has 8.57 ARL_1 value while $\tilde{X}$ chart has 7.41 ARL_1 value. Similarly, $\bar{X}$ chart has $ARL_1 = 3.22$ and $\tilde{X}$ chart has $ARL_1 = 2.99$ for $0.1\sigma_0$ quadratic drift at $n = 10$.
- Under the gamma distribution, both charts revealed biased ARL's, where the ARL_1 values exceed the ARL_0 value, when shifts are introduced $0.01 \leq \delta \leq 0.1$. This bias is due to the skewness of the distribution and similar behavior is also reported by Huang and Yang (2015) and Riaz (2008a). Moreover, under linear and quadratic drifts, such biased behavior of ARL's not observed. However, under biased and unbiased ARL's, the $\bar{X}$ chart has better detection ability to monitor the shifts and drifts. For example, under biased ARL's, at $n = 5$ and $0.1\sigma_0$ shift (Table 1.2), ARL_1 exceed by almost 148 units for the $\bar{X}$ chart, while 189 units for the $\tilde{X}$

chart. At $n = 7$ and $0.1\sigma_0$ linear drift, $\bar{X}$ chart has 12.27 ARL_1 value while $\tilde{X}$ chart has 14.15 ARL_1 value. Similarly, $\bar{X}$ chart has $ARL_1 = 3.90$ and $\tilde{X}$ chart has $ARL_1 = 4.01$ for $0.1\sigma_0$ quadratic drift (Table 1.2) at $n = 10$.

- From the above discussion points, it is also revealed that both charts have high detection ability when a process has a quadratic drift, while the detection ability of linear drift comes next.
- As the subgroup size n increases, the detection ability of the charts increases. For example, under the normal distribution at $\delta = 0.5$ shift, the ARL_1 values of the $\bar{X}$ chart is reported as 33.31, 21.37, and 12.80 for $n = 5, 7$, and 10, respectively (Table 1.2). For the $\tilde{X}$ chart, at linear drift $\delta = 1$ and under the t distribution, ARL_1 values are observed equal to 4.33, 3.64, and 2.98 for $n = 5, 7$, and 10, respectively.

Table 1.2: Zero-state ARL profile of Shewhart Charts at different subgroup sizes.

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.92	370.08	370.42	370.44	370.30	370.80	370.26	370.15	370.54
		0.01	369.42	368.72	367.90	53.21	47.40	42.05	10.67	9.95	9.22
		0.025	363.65	362.44	358.27	28.10	24.92	21.91	7.24	6.73	6.25
		0.05	348.52	339.63	329.19	16.96	14.98	13.12	5.39	5.02	4.65
		0.1	297.25	273.89	244.52	10.12	8.93	7.79	4.03	3.75	3.50
		0.25	133.15	101.88	73.06	5.06	4.46	3.90	2.78	2.62	2.42
		0.5	33.31	21.37	12.80	3.02	2.67	2.34	2.04	1.96	1.92
		1	4.48	2.76	1.77	1.83	1.65	1.44	1.78	1.64	1.43
		2	1.08	1.01	1.00	1.07	1.01	1.00	1.07	1.01	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	student's t	0	370.70	370.63	370.63	370.10	370.46	370.36	370.55	370.91	370.15
		0.01	378.45	368.45	368.45	61.35	50.52	42.67	10.62	9.64	8.72
		0.025	378.10	366.46	366.46	31.05	25.55	21.34	7.09	6.46	5.85
		0.05	371.08	353.98	353.98	18.04	14.87	12.42	5.24	4.78	4.35
		0.1	344.07	312.08	312.08	10.33	8.57	7.17	3.89	3.58	3.22
		0.25	216.38	147.15	147.15	4.93	4.12	3.48	2.74	2.44	2.14
		0.5	61.37	26.45	26.45	2.85	2.41	2.05	1.99	1.96	1.90
		1	4.11	1.82	1.82	1.77	1.45	1.16	1.75	1.45	1.15
		2	1.01	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	Gamma	0	370.94	370.15	370.87	370.95	370.66	370.18	370.19	370.72	370.20
		0.01	417.98	415.23	398.58	79.70	67.42	57.23	13.05	11.80	10.66
		0.025	481.31	460.42	435.77	42.00	35.20	29.59	8.72	7.90	7.14
		0.05	543.68	502.94	457.76	25.09	20.96	17.53	6.42	5.83	5.29
		0.1	518.78	460.24	396.24	14.72	12.27	10.24	4.72	4.34	3.90
		0.25	301.11	234.18	167.85	7.13	5.95	4.96	3.15	2.90	2.76
		0.5	124.21	77.37	42.72	4.08	3.42	2.87	2.44	2.06	1.98
		1	23.73	10.67	4.44	2.39	1.97	1.77	1.96	1.90	1.77
		2	1.83	1.08	1.00	1.45	1.07	1.00	1.45	1.08	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
$\tilde{X}$	Normal	0	370.80	370.42	370.89	370.75	370.11	370.88	370.71	370.05	370.50
		0.01	358.91	366.25	364.75	60.83	54.53	47.52	11.65	10.87	9.89
		0.025	356.41	361.40	357.25	32.36	28.88	24.88	7.88	7.35	6.70
		0.05	343.00	347.12	337.24	19.60	17.44	14.95	5.86	5.48	4.99
		0.1	304.92	294.86	270.94	11.71	10.41	8.89	4.38	4.09	3.73
		0.25	165.45	138.85	101.15	5.85	5.21	4.44	2.98	2.81	2.61
		0.5	51.23	36.29	21.01	3.48	3.09	2.64	2.22	2.06	1.96
		1	7.81	4.94	2.68	2.09	1.87	1.64	1.88	1.80	1.62
		2	1.30	1.10	1.01	1.24	1.09	1.01	1.24	1.09	1.01
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	student's t	0	370.61	370.87	370.76	370.83	370.55	370.60	370.56	370.53	370.12
		0.01	371.55	378.87	364.86	50.80	42.27	34.14	9.88	8.95	7.97
		0.025	365.33	370.78	352.91	25.96	21.53	17.40	6.63	6.03	5.38
		0.05	353.22	345.14	307.89	15.26	12.71	10.23	4.92	4.48	4.01
		0.1	303.96	265.20	199.15	8.90	7.41	6.01	3.67	3.35	2.99
		0.25	134.90	84.00	39.83	4.33	3.64	2.98	2.55	2.25	2.02
		0.5	26.93	12.13	4.56	2.54	2.16	1.81	1.96	1.91	1.78
		1	2.27	1.37	1.05	1.56	1.26	1.05	1.56	1.26	1.05
		2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	Gamma	0	370.55	370.55	370.84	370.89	370.38	370.31	370.56	370.59	370.95
		0.01	454.43	443.79	430.00	91.88	78.53	61.65	14.05	12.70	11.09
		0.025	588.02	539.99	491.55	48.62	40.96	31.85	9.35	8.46	7.40
		0.05	647.89	603.79	522.90	29.00	24.27	18.85	6.88	6.23	5.46
		0.1	559.21	521.72	430.69	16.96	14.15	10.96	5.04	4.61	4.01
		0.25	361.61	295.05	198.09	8.14	6.77	5.27	3.46	2.99	2.83
		0.5	177.45	117.90	56.41	4.61	3.85	3.03	2.69	2.26	1.98
		1	43.13	20.30	6.19	2.66	2.21	1.84	1.98	1.95	1.84
		2	3.37	1.37	1.00	1.70	1.27	1.00	1.70	1.28	1.00
	4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

2.4.2 Steady-State Run Length Performance

The purpose of this section is to demonstrate the performance evaluation of $\bar{X}$ and $\tilde{X}$ charts in steady-state monitoring. The evaluation of the performance is based on ARL_1 values against the δ values, containing 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4. The chart with the lowest ARL_1 values will be declared as the best. The steady-state performance assessment of $\bar{X}$ and $\tilde{X}$ charts for

- Under the normal distribution, the performance of the $\bar{X}$ chart is better than the $\tilde{X}$ chart under drifts, but the $\tilde{X}$ chart is better under shift. As an example, at $n = 5$ and $0.025\sigma_0$ shift (Table 1.3), 5 unit decrease in the ARL_1 value of $\bar{X}$ chart is observed, while in the $\tilde{X}$ chart, a reduction of 6 units in ARL_1 values were observed. At $n = 7$ and $0.025\sigma_0$ linear drift, $\bar{X}$ chart has 68.58 ARL_1 value while $\tilde{X}$ chart has 71.99 ARL_1 value. Similarly, $\bar{X}$ chart has $ARL_1 = 52.27$ and $\tilde{X}$ chart has $ARL_1 = 52.66$ for $0.025\sigma_0$ quadratic drift at $n = 10$.
- Under the student's t chart has better performance in detecting drifts as compared to $\bar{X}$ chart under shifts. For instance, at $n = 5$ and $0.025\sigma_0$ shift (Table 1.3) may cause 3 units reduction in the ARL_1 values for $\bar{X}$ chart. While in the $\tilde{X}$ chart, the same amount of change may cause an increase of 2 units in ARL_1 values. At $n = 7$ and $0.025\sigma_0$ linear drift, the $\bar{X}$ chart has 69.19 ARL_1 value while the $\tilde{X}$ chart has 65.75 ARL_1 value. Similarly, the $\bar{X}$ chart has $ARL_1 = 64.99$ and $\tilde{X}$ chart has $ARL_1 = 51.62$ for $0.025\sigma_0$ quadratic drift at $n = 10$.
- Under the gamma distribution, when shifts of 0.01 to 0.1 are introduced, both charts show biased ARLs. Whereas these biased ARL behaviors are not seen for linear

and quadratic drifts. The $\bar{X}$ chart can better detect shifts and drifts under biased and unbiased ARLs, though. For instance, under biased ARL's, at $n = 5$ and $0.025\sigma_0$ shift, ARL_1 exceed by almost 90 units for the $\bar{X}$ chart, while 211 units for the $\tilde{X}$ chart. At $n = 7$ and $0.025\sigma_0$ linear drift, $\bar{X}$ chart has 77.56 ARL_1 value while $\tilde{X}$ chart has 81.82 ARL_1 value. Also, $\bar{X}$ chart has $ARL_1 = 53.07$ and $\tilde{X}$ chart has $ARL_1 = 53.34$ for $0.025\sigma_0$ quadratic drift at $n = 10$. Observing that the charts' ability is very close in terms of detecting the quadratic drift.

- The explanation above makes it abundantly evident that both charts are highly sensitive to the presence of quadratic drift in a process. The steady-state performance of the charts is quite similar to one another.
- The ability of the charts to detect the drifts increases as subgroup size (n) increases. For instance, under the gamma distribution at $\delta = 0.5$ shift, the ARL_1 values of the $\bar{X}$ chart are reported as 154.93, 113.28, and 83.39 for $n = 5, 7$, and 10, respectively (Table 1.3). For the $\tilde{X}$ chart, at linear drift $\delta = 1$ and under the t distribution, ARL_1 values are observed equal to 48.04, 47.96, and 47.82 for $n = 5, 7$, and 10, respectively.

Table 1.3: Steady-state ARL profile of Shewhart charts at different subgroup sizes.

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.10	370.71	370.31	370.20	370.61	370.75	370.13	370.58	370.70
		0.01	371.26	370.22	368.22	93.27	88.35	83.62	56.23	55.53	54.81
		0.025	365.80	363.64	358.85	71.39	68.58	66.01	53.14	52.75	52.27
		0.05	350.00	344.20	332.58	61.62	59.96	58.31	51.52	51.25	50.88
		0.1	305.50	286.31	261.07	55.72	54.56	53.60	50.36	50.10	49.96
		0.25	162.26	135.77	111.29	51.21	50.71	50.25	49.26	49.15	48.96
		0.5	75.91	65.48	58.09	49.51	49.18	48.88	48.62	48.53	48.49
		1	50.78	49.24	48.39	48.45	48.27	48.10	48.41	48.26	48.10
		2	47.79	47.75	47.66	47.75	47.76	47.72	47.76	47.65	47.67
		4	47.78	47.68	47.71	47.74	47.69	47.70	47.60	47.72	47.70
	student's t	0	370.51	370.60	370.69	370.61	370.47	370.87	370.41	370.17	370.27
		0.01	370.20	361.46	373.80	98.76	91.47	83.25	101.30	91.71	83.23
		0.025	367.40	359.68	365.08	73.21	69.19	65.11	74.28	69.47	64.99
		0.05	360.23	349.07	350.71	62.12	59.83	57.47	62.86	60.01	57.49
		0.1	336.04	310.53	292.23	55.59	54.33	52.94	56.08	54.46	52.92
		0.25	224.71	170.27	126.93	51.03	50.47	49.74	51.29	50.56	49.82
		0.5	97.04	69.05	56.27	49.25	48.99	48.57	49.48	48.97	48.51
		1	50.20	48.35	47.94	48.35	48.15	47.78	48.51	48.19	47.80
		2	47.71	47.64	47.74	47.66	47.73	47.71	47.84	47.78	47.63
		4	47.71	47.65	47.72	47.71	47.74	47.66	47.81	47.80	47.66
	Gamma	0	370.67	370.62	370.84	370.79	370.05	370.92	370.78	370.46	370.41
		0.01	405.57	399.70	399.23	116.47	105.57	96.31	58.26	56.79	56.13
		0.025	460.75	438.44	428.05	83.55	77.56	72.49	54.45	53.44	53.07
		0.05	514.86	473.21	443.81	68.84	65.01	62.11	52.45	51.63	51.47
		0.1	501.05	439.52	389.83	59.71	57.47	55.76	51.01	50.45	50.28
		0.25	309.36	247.33	191.89	53.08	52.01	51.20	49.61	49.12	49.24
		0.5	154.93	113.28	83.39	50.37	49.75	49.36	49.00	48.36	48.57
		1	67.52	55.98	50.63	48.90	48.48	48.39	48.55	48.27	48.40
		2	48.42	47.76	47.72	48.11	47.69	47.73	48.11	47.55	47.70
		4	47.70	47.69	47.70	47.75	47.64	47.70	47.70	47.53	47.69
$\bar{X}$	Normal	0	370.39	370.27	370.61	370.47	370.89	370.81	370.22	370.15	370.91
		0.01	366.99	381.66	358.79	100.92	94.63	87.97	56.98	56.09	55.59
		0.025	364.43	379.28	354.14	75.48	71.99	68.30	53.71	52.97	52.66
		0.05	353.32	363.97	333.07	64.13	61.99	59.59	51.85	51.40	51.26
		0.1	322.45	320.28	275.11	57.17	55.92	54.44	50.62	50.16	50.16
		0.25	196.44	175.45	132.16	52.00	51.33	50.64	49.44	49.10	49.14
		0.5	93.70	80.33	64.44	49.96	49.55	49.01	48.76	48.50	48.61
		1	53.85	51.44	49.03	48.70	48.43	48.10	48.49	48.28	48.26
		2	47.88	47.88	47.70	47.94	47.80	47.60	47.88	47.66	47.72
		4	47.65	47.80	47.56	47.78	47.66	47.64	47.67	47.53	47.66
	student's t	0	370.92	370.21	370.73	370.95	370.19	370.17	370.89	370.65	370.94
		0.01	375.12	364.34	365.18	90.89	84.03	76.47	55.21	54.69	53.92
		0.025	372.75	358.61	351.55	69.38	65.79	61.84	52.42	52.08	51.62
		0.05	362.12	335.91	315.28	60.08	57.99	55.69	50.92	50.72	50.39
		0.1	316.18	273.20	221.64	54.57	53.36	51.98	49.81	49.78	49.52
		0.25	167.71	117.85	81.90	50.63	50.09	49.29	48.87	48.80	48.60
		0.5	71.12	57.06	50.83	49.04	48.77	48.34	48.32	48.53	48.42
		1	48.89	47.94	47.71	48.23	48.01	47.64	48.04	47.96	47.82
		2	47.75	47.59	47.69	47.68	47.71	47.59	47.61	47.77	47.81
		4	47.79	47.68	47.68	47.66	47.79	47.62	47.63	47.73	47.74
	Gamma	0	370.91	370.81	370.68	370.98	370.84	370.28	370.30	370.77	370.93
		0.01	464.71	432.96	421.95	128.26	114.15	100.86	59.09	57.79	56.50
		0.025	581.62	508.67	474.39	89.69	81.92	74.81	54.98	54.18	53.34
		0.05	615.76	563.46	507.04	72.48	67.61	63.45	52.81	52.34	51.63
		0.1	541.66	487.32	426.68	61.75	58.98	56.43	51.21	50.81	50.40
		0.25	365.71	295.72	220.77	54.02	52.63	51.38	49.90	49.41	49.33
		0.5	201.05	146.71	96.24	50.93	50.14	49.48	49.17	48.76	48.60
		1	84.71	64.13	52.35	49.16	48.65	48.49	48.63	48.56	48.51
		2	49.85	48.00	47.80	48.35	47.87	47.74	48.32	47.94	47.70
		4	47.74	47.65	47.76	47.78	47.68	47.69	47.68	47.66	47.81

In general, the findings from steady-state and zero-state monitoring for shifts and drifts are stated as follows.

- When it comes to linear drift and quadratic drift, under normal and gamma distributions, The $\bar{X}$ chart has a better ability to detect drifts under biased and unbiased ARLs, while $\tilde{X}$ chart performed well under the student's t distribution.
- Both charts can detect quadratic drifts more quickly than linear drifts or shifts.
- as n increases, the ARL_1 value decreases for both charts.

2.5 Illustrative Examples

In this section, we demonstrate the implementation of the proposed charts using two real datasets related to air-quality parameters (Nitrogen Oxides) and process temperature.

2.5.1 A Case study on Nitrogen Oxides Concentration level

The air quality dataset contains average hourly responses from five metal oxide chemistry sensors built into a multi-sensor air quality chemistry instrument. The device was installed in a field in a heavily polluted area at road level in an Italian town. That dataset contains the hourly averaged CO concentration amounts, Non- Metanic Hydrocarbons, Benzene, Total Nitrogen Oxides (NO_x), and Nitrogen Dioxide (NO_2). Among the other variables, NO_x is the most dangerous to human health, including harm to the respiratory system, increased susceptibility, chronic lung disease, irritation of the eyes and skin, and mucous membranes. Therefore, we have considered Nitrogen Oxides (NO_x) concentration as the variable of interest for monitoring.

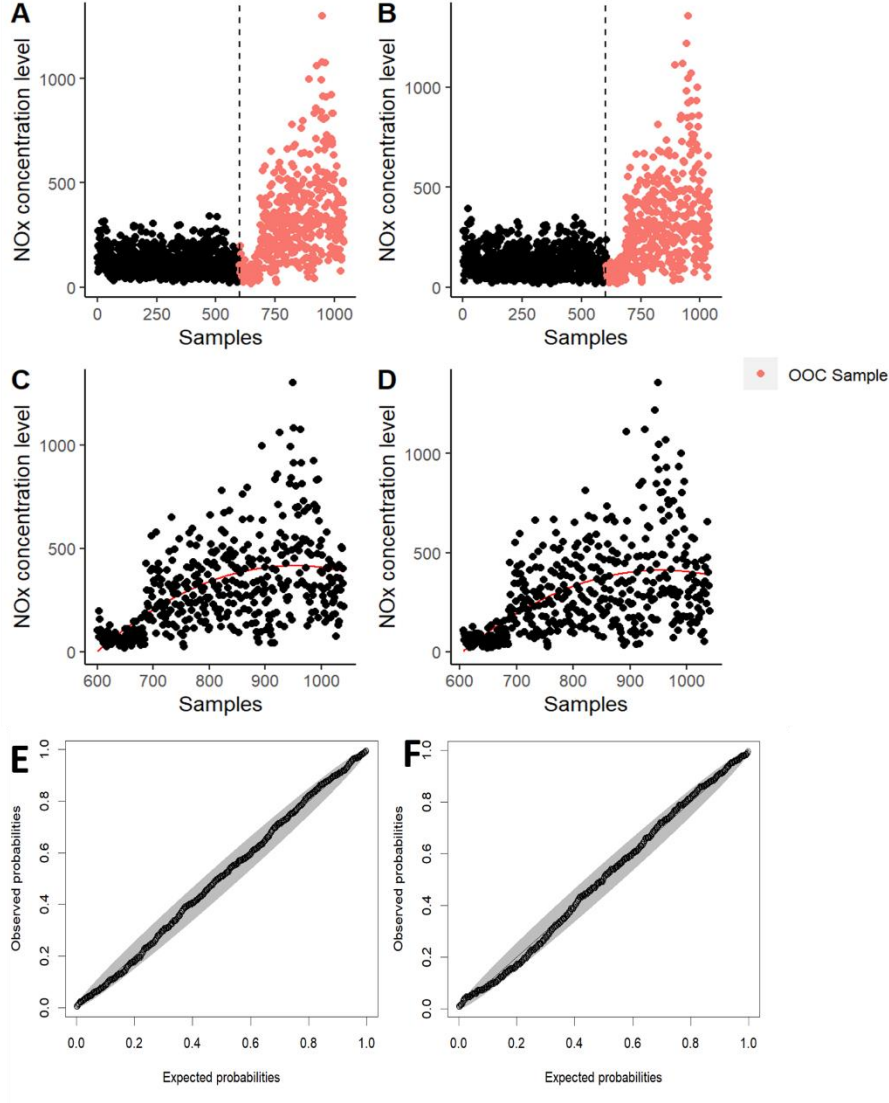


Figure 1.1: Scatter plot for the *IC* and *OOC* data of the NOx concentration level; (A) $\bar{X}$ values, (B) $\tilde{X}$ values, (C) for *OOC* $\bar{X}$ with the quadratic trend, and (D) for *OOC* $\tilde{X}$ with the quadratic trend, (E) PP-plot for the *IC* NOx concentration level $\bar{X}$ values, and (F) PP-plot for the *IC* NOx concentration level $\tilde{X}$ values.

The dataset was taken from The UCI Machine Learning Repository (Vito, 2016) and contained 5200 data points from March 2004 to April 2005. We have considered NO_x concentration from March to July 2004 as the in-control (*IC*) data, while the remaining data is regarded as out-of-control (*OOC*). In the preprocessing step, the unexpected NO_x

concentration value (i.e., -200) is deleted from the data. Further, data is divided into subgroups having size 5 each, and for each subgroup, the mean ($\bar{X}$) and median ($\tilde{X}$) are calculated. To check whether the OOC data drifted linearly or quadratically, we applied the regression and revealed that the data drifted quadratically, with the fitted regression model defined as $\text{NOx} = -2618 + 6.377t - 0.003351t^2$. The IC and OOC data are plotted in Figure 1.1(A and B), while Figure 1(B-C) shows the quadratic trend in OOC $\bar{X}$ and $\tilde{X}$, respectively.

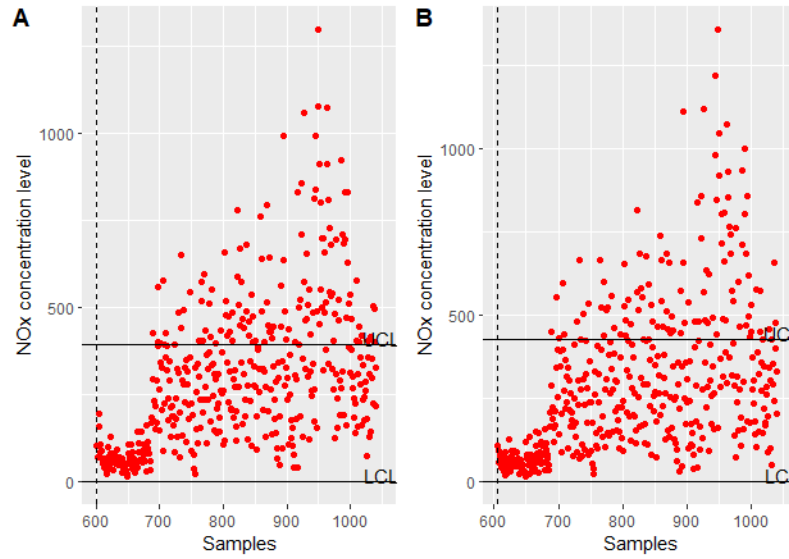


Figure 1.2: For NOx concentration level; (A) $\bar{X}$ chart, and (B) $\tilde{X}$ chart.

The Shapiro-Wilk test and the PP-plots for $\bar{X}$ and $\tilde{X}$ values are presented in Figure 1.1(E and F), which reveals that both variables follow the Gamma distribution. The IC $\bar{X}$ follows gamma distribution with shape 4.09 and rate 0.032, while IC $\tilde{X}$ follows Gamma distribution with shape 3.079 and rate 0.024. The mean and standard deviation of the $\bar{X}$ chart belongs to IC data are estimated as $\bar{X} = 128.84$ and $\sigma_{\bar{X}} = 62.99$, while the mean ($\tilde{X}$) and standard deviation ($\sigma_{\tilde{X}}$) of the $\tilde{X}$ chart belongs to IC data are estimated as 126.59 and 71.43. Using

the Gamma distribution, $ARL_0 = 370$ and the above mentioned mean, and variances, the LCL and UCL are set to be 0 and 355.36 for $\bar{X}$ chart. Moreover, for the $\tilde{X}$ chart, LCL and UCL are defined as 0 and 387.86. In process control, charts that present more signal are generally considered better. Considering the importance of striking a balance between detecting changes and avoiding false alarms. The $\bar{X}$ chart for NOx concentration level is presented in Figure 1.2 (A) and $\tilde{X}$ chart in Figure 1.2 (B) were the focus of performance after the change point for both charts. The $\bar{X}$ chart showed 149 signals, while the $\tilde{X}$ chart captured 130 signals. Furthermore, since the $\tau = 607$, the $\bar{X}$ chart was able to detect the drift at sample 688, while the $\tilde{X}$ chart detected the drift at sample 689. Hence, it is revealed that the $\bar{X}$ chart has better detection ability in detecting quadratic drift, which is aligned with the results obtained from the simulation study.

2.5.2 A Case Study on Maintenance Data

This subsection presents another illustration of the proposed charts using a real predictive maintenance dataset ("AI4I 2020 Predictive Maintenance Dataset," 2020). The dataset was obtained in 2020, consists of 10,000 samples in total, and indicates many variables that may affect the process, such as air temperature, rotational speed, and machine failure. During routine operations, predictive maintenance monitors the performance of the equipment and looks for any anomalies that might point to a potential failure. The process temperature is one of the most crucial factors affecting preventive maintenance. In fact, it is frequently used as an important marker of equipment health. Therefore, in this illustration, we have considered process temperature as the variable of interest. We have only considered the first 5200 observations for the illustration, using break-point procedure given in R package "segmented", data is further divided into in-control (IC) data consisting

of first 1875 samples while remaining are considered as out of control (OOC) data. In the preprocessing step, the data is divided into subgroups of size 5 each, and for each subgroup, the mean ($\bar{X}$) and median ($\tilde{X}$) are calculated.

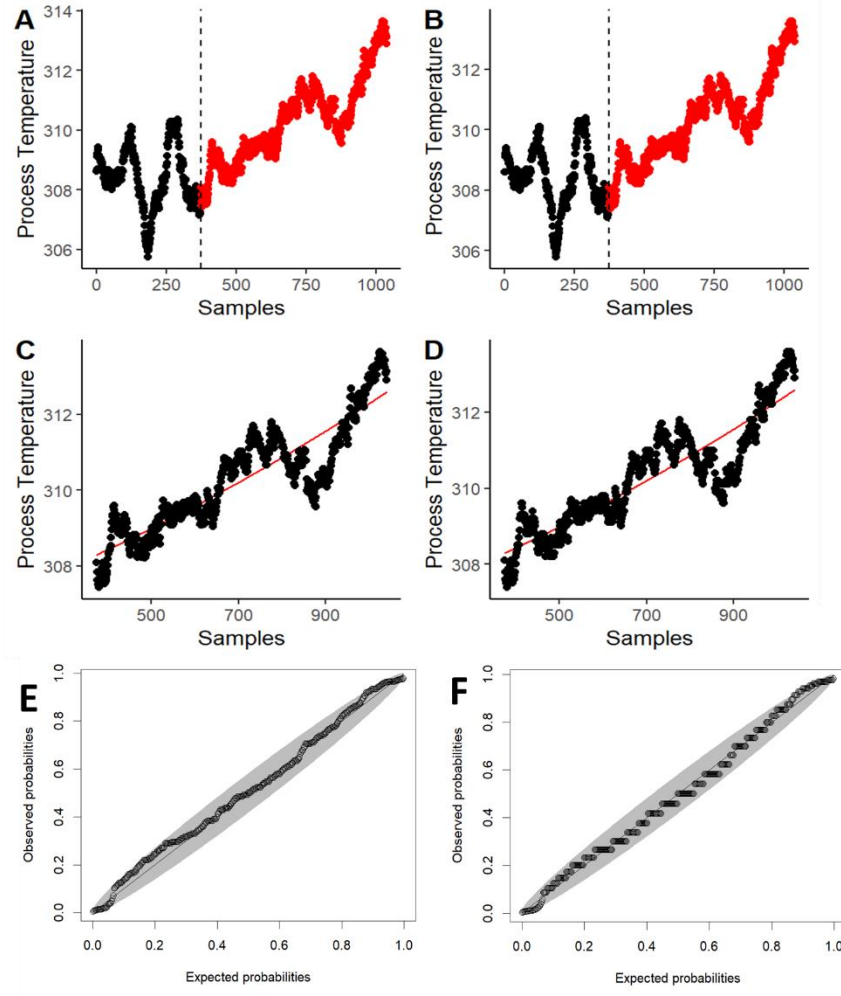


Figure 1.3: Scatter plot for the *IC* and *OOC* data for the temperature of maintenance dataset; (A) $\bar{X}$ values, (B) $\tilde{X}$ values, (C) for *OOC* $\bar{X}$ with the quadratic trend, and (D) for *OOC* $\tilde{X}$ with the quadratic trend, (E) PP-plot for the *IC* process temperature $\bar{X}$ values, and (F) PP-plot for the *IC* process temperature $\tilde{X}$ values.

To check whether the *OOC* data drifted linearly or quadratically, we applied the regression and revealed that the data drifted quadratically, with the fitted regression model defined as

Temperature= $350.70 + 0.006457 t + 0.00000177 t^2$ The IC and OOC data are plotted in Figure 1.3(A and B), while Figure 1.3(C and D) shows the quadratic trend in OOC $\bar{X}$ and $\tilde{X}$ charts , respectively.

The Shapiro-Wilk test and the PP-plots for $\bar{X}$ and $\tilde{X}$ values are presented in Figure 1.3(E and F), which reveals that both variables follow the normal distribution. The mean and standard deviation of the $\bar{X}$ chart belongs to IC data are estimated as 308.39 0.98, while mean (and standard deviation of the $\tilde{X}$ chart belongs to IC data are estimated as 308.39 and 0.99.

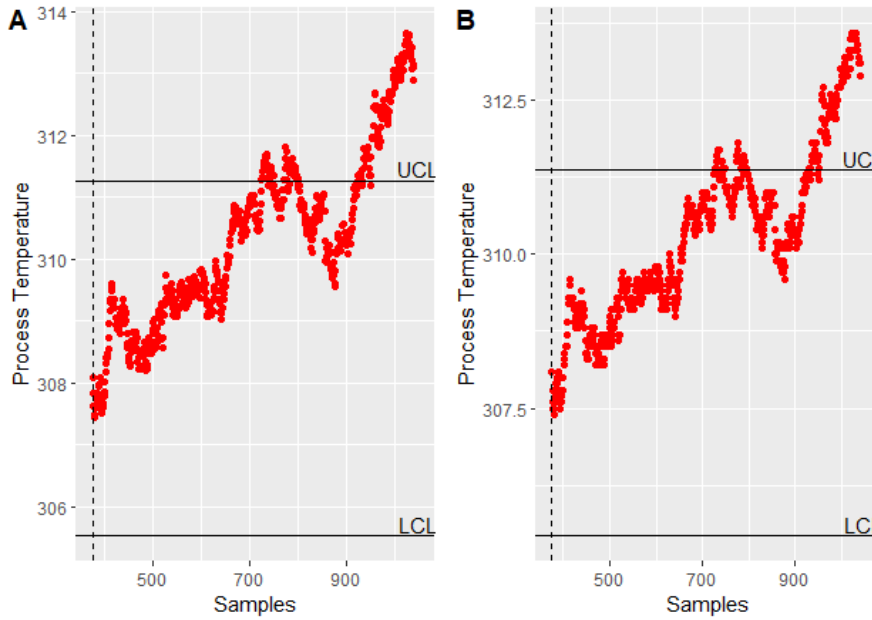


Figure 1.4: For the temperature of the maintenance dataset; (A) $\bar{X}$ chart, and (B) $\tilde{X}$ chart

At $ARL_0 = 370$, and using the above mentioned mean, and variances, the LCL and UCL are set to be 305.52 and 311.26 for $\bar{X}$ chart. Moreover, for the $\tilde{X}$ chart, LCL, and UCL are defined as 305.42 and 311.36. The $\bar{X}$ chart for the temperature of the maintenance dataset is presented in Figure 1.4 (A) and $\tilde{X}$ chart is plotted in Figure 1.4 (B). The focus here was

on the performance of the charts after the change point. The $\bar{X}$ chart showed 152 signals, while the $\tilde{X}$ chart captured 137 signals. Moreover, since $\tau = 375$, the $\bar{X}$ chart was able to detect the drift at sample number 725, while the $\tilde{X}$ chart detected the drift at sample number 726. Hence, it is revealed that the $\bar{X}$ chart has better detection ability in detecting quadratic drift, which is aligned with the results obtained from the simulation study.

2.6 Summary and Recommendations

This chapter is designed to assess the performance of well-known Shewhart based location charts, i.e., the $\bar{X}$ and $\tilde{X}$ charts. The charts were evaluated in two monitoring states, zero and steady states, with various distribution types and sample sizes. Firstly, regarding the subgroup size, it was found that increasing the subgroup size resulted in higher detection abilities of the $\bar{X}$ and $\tilde{X}$ charts. Secondly, in terms of distributional environments under zero-state and steady state-monitoring, the performance evaluation of the $\bar{X}$ chart provided the lowest values of ARL_1 , which makes it the best in detecting the shifts or drifts in normal and Gamma distribution. Conversely, the performance evaluation of the $\tilde{X}$ chart showed that it has higher detection power under student's t distribution. Furthermore, regarding the drifts type, the detection ability of both charts in detecting the quadratic drifts is higher than the linear ones. A special case was observed in the shift scenario under the gamma distribution, that the ARL_1 values exceed ARL_0 in both $\bar{X}$ chart and $\tilde{X}$ chart $0.01 \leq \delta \leq 0.1$. However, such ARL bias does not report under linear, and quadratic drifts. Furthermore, this chapter was limited to assessing the effects of drifts on Shewhart based location charts, which can be extended by seeing the effect of drifts on memory-type (i.e., EWMA and CUSUM charts) location charts.

CHAPTER 3

EWMA Based Location Charts Under Drifts.

Stability is necessary in the environmental devices since inconsistent actions and practices can lead to misleading predictions. Since drift affects the bulk of the environmental sensors, statistical process control (SPC) tools, such as control charts, assist administrators in addressing such problems. Drift is a term used to describe a slow, overtime, deviation in the process parameter(s) caused by equipment wear and tear. This chapter aims to assess the performance of an exponentially weighted moving average control chart using the mean and median as plotting statistics. The run length metric under zero- and steady-states is used to evaluate the performance of the chart against linear and quadratic drifts. The findings reveal that the charts based on sample median are better at identifying shifts and linear drifts under the student's t distribution in zero-state monitoring. On the other hand, control charts based on sample mean are better at detecting shifts and drifts under normal and gamma distributions. This is true for both zero- and steady-states as well as quadratic drift under the t distribution. The control charts based on both estimators identify quadratic drift more quickly than linear drift. However, after comparing the performance of Shewhart charts and EWMA charts, the results showed that under zero-state monitoring, the EWMA chart based on sample mean detects linear drift under all the distributions better than the corresponding Shewhart chart for small drifts. Under steady-state monitoring, the median based EWMA chart detects quadratic drift better than the corresponding Shewhart chart in normal and t

distributions. The use of the suggested charts is demonstrated, along with the importance of the stated idea, using an actual case study related to environmental engineering.

3.1 Introduction

A common statistical methodology for measuring, monitoring, and managing quality throughout a process is known as statistical process control (SPC). Manufacturing companies frequently use primary SPC tools, such as control charts, with an emphasis on improvement for increased productivity, decreased waste, and early issue detection. Control charts display the process data in the order of occurrence. A central line (CL), an upper control limit (UCL), and a lower control limit (LCL) are the decision parameters of a typical control chart. The process mean is represented by the center line, cf. Montgomery (2020). The control limits signify the variation in the process. Control charts aim to differentiate between two types of process variation: (i) special cause variation; which originates from outside sources and indicates that the process is out of statistical control, and (ii) common cause variation; which is inherent to the process and will always be present. By plotting the quality characteristic against the control limits and observing the results over time, it can be determined whether the process is in control (IC) or out of control (OOC). Control charts are classified into two types based on the parameter of interest to be investigated. Control charts that focus on the centrality parameter are known as location charts, which is our interest here, and charts that focus on the variability parameter are known as dispersion charts. Control charts are divided into two types based on the magnitude of process disturbances. To detect large disturbances, memoryless-type charts are used. Memory-type charts, on the other hand, are used to detect minor or moderate disturbances in a process. Our focus will be on memory charts, whose primary structure is the exponentially weighted moving average (EWMA) control chart by Roberts (1959).

Monitoring variables that make use of the entire history of an output is done using an EWMA chart. Unlike other control charts, which consider every data point separately, the EWMA chart does not. It weights observations in a geometrically decreasing order, favoring the most recent observations while giving little weight to the oldest observations. To build an EWMA chart, two parameters need to be carefully selected. The first parameter of the EWMA chart is the weight assigned to the most recent subgroup mean, known as λ where λ generally must be within $0 < \lambda \leq 1$. The other parameter that needs to be set is L , also known as the control limit coefficient, which helps in establishing the control limits of the chart. Noticing that the limit structure of EWMA charts is not symmetric because the EWMA statistic is a weighted average that gives more weight to recent measurements. As a result, the control limits for an EWMA chart are narrower than those for a Shewhart control chart. This allows the EWMA chart to be more sensitive to small changes in the process. By giving more weight to recent measurements, the EWMA chart can quickly detect shifts or trends in the data.

The $\bar{X}$ and $\tilde{X}$ statistics, which are used to track process location, will be used to analyze the EWMA structure in this chapter cf. (Kumar & Jaiswal, 2020) and (Saha et al., 2022) for more details on median based control charts. The Shewhart chart is designed to focus on detecting large shifts in the process mean. On the other hand, since the EWMA chart incorporates information from previous observations and assigns more weight to recent data, it is more sensitive to smaller shifts in the process mean. Furthermore, the EWMA chart has been studied by a number of authors; recent related studies and references can be found in Karoon et al. (2023) and Naveed et al. (2023).

This thesis investigates various drift types. Drift refers to changes in processes that occur gradually over time, either linearly or nonlinearly, see Saha et al. (2022) for more details. For instance, the size of the tool changes in relation to its previous size, which is a typical example of

tool wear. It is possible to take the necessary action once the drift's cause has been identified. The drift should be removed whenever it is economically feasible, for example, by making automatic compensating adjustments to account for tool wear. This chapter discusses shift, linear drift, and quadratic drift. The shift in process mean is known as $\mu_1 = \mu_0 + \delta\sigma_0$. The linear drift and the quadratic drift are defined as $\mu_1 = \mu_0 + \omega t\sigma_0$ and $\mu_1 = \mu_0 + \rho t^2\sigma_0$, respectively, where μ_1 is the shifted mean, and ω and ρ are the amount of change times σ_0 in the process mean. where t represents the time point.

The control chart's performance was discussed in the SPC literature from different viewpoints. To name but a few (see Section 1.4); Aerne et al. (1991) assessed the performance of the Shewhart chart, the CUSUM chart, and the EWMA chart under linear drift in the process mean. Gan (1991) evaluated the performance of the EWMA chart in the presence of linear drift. Fahmy and Elsayed (2006a) compared the performance of the chi-square chart, which aims to detect linear trends in the process mean, with Shewhart, CUSUM, EWMA, and generalized likelihood ratio (GLR) control charts.

From the literature, it can be observed that many studies use the $\bar{X}$ chart to examine the impact of shifts and linear drifts in the data. To the best of our knowledge, $\tilde{X}$ charts are not discussed under drifts. Also, quadratic change over time is not taken into account, although location-based studies are covered under the heading of linear change over time. Therefore, this study's goal is to evaluate how well an exponentially weighted moving average chart based on sample mean ($\bar{X}$) and sample median ($\tilde{X}$) performs when subjected to shifts, linear drifts, and quadratic drifts in non-normal environments. Additionally, different subgroup sizes are taken into account while discussing the performance of the charts, as well as zero-state and steady-state scenarios. This chapter will evaluate the Shewhart and EWMA charts' performance in detecting and responding to process drift

under similar conditions, providing insights into which chart is more suitable for detecting process variations.

The rest of the manuscript is structured as follows: The control chart structure for the location monitoring is presented in Section 2. The design of the proposed charts is given in Section 3. In Section 4, the performance evaluation of the charts is reported. In Section 5, a real example has been explained of the practical implementation of the proposed charts. In Section 6, a comparative analysis of EWMA and Shewhart charts under drifts is provided, and finally, remarks and recommendations for future research work are provided in Section 7

3.2 Control Charts for Location monitoring

This section is designed to discuss the general structure of an exponentially weighted moving average based on sample mean ($\bar{X} - EWMA$) charts and an exponentially weighted moving average based on sample median ($\tilde{X} - EWMA$) control charts.

3.2.1 The $\bar{X} - EWMA$ Chart

Let $X_{t,1}, \dots, X_{t,n}$, $i = 1, 2, \dots, n$; $t = 1, 2, \dots$ be independent random variables follow normal distribution with n sample size, i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$ where μ_0 and σ_0 are the in-control mean and standard deviation, respectively. t represents the time point, and δ is the amount of the standardized mean shift. If $\delta = 0$, the process is in-control *IC*; otherwise, the process is out-of-control *OOC*. The plotting statistic of $\bar{X} - EWMA$ is given as follows:

$$Z_t = \lambda \bar{X}_t + (1 - \lambda) Z_{t-1} \quad (1)$$

where λ is the smoothing parameter set to be equal to 0.2, and $\bar{X}_t$ defined as

$$\bar{X}_t = \frac{\sum_{i=1}^n X_i}{n} \quad (2)$$

The plotting statistic is plotted against the control limits, which are given as follows:

$$LCL = \mu_0 - L_1 \sqrt{\frac{\lambda}{2-\lambda} \frac{\sigma_0^2}{n}} \quad (3)$$

$$UCL = \mu_0 + L_2 \sqrt{\frac{\lambda}{2-\lambda} \frac{\sigma_0^2}{n}} \quad (4)$$

Where L_1 and L_2 are charting constants set against $ARL_0 = 370$ and reported in Table 2.1. If the LCL and UCL of a process's $\bar{X} - EWMA$ chart (i.e., Z_t) are exceeded, the process becomes unstable.

3.2.2 The $\tilde{X} - EWMA$ Chart

Let $X_{t,i}; i = 1, 2, \dots, n; t = 1, 2, \dots$ be a normally distributed t^{th} subgroup of size n , i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$. The t^{th} plotting statistic of the $\tilde{X} - EWMA$ chart is defined as follows:

$$Z_t = \lambda \tilde{X}_t + (1 - \lambda) Z_{t-1} \quad (5)$$

where λ is the smoothing parameter set to be 0.2, and $\tilde{X}_t$ defined as

$$\tilde{X}_t = \begin{cases} X_{t(\frac{n+1}{2})}, & \text{if } n \text{ is odd} \\ \frac{X_{t(\frac{n}{2})} + X_{t(\frac{n}{2}+1)}}{2}, & \text{if } n \text{ is even} \end{cases} \quad (6)$$

The control limits are as follows,

$$LCL = \mu_0 - L_1 \sqrt{\frac{\lambda}{2-\lambda} \sigma_0^2 \sqrt{\left(\frac{\pi}{2n}\right)}} \quad (7)$$

$$UCL = \mu_0 + L_2 \sqrt{\frac{\lambda}{2-\lambda} \sigma_0^2 \sqrt{(\pi/2n)}} \quad (8)$$

Where L_1 and L_2 are charting constants for the $\tilde{X} - EWMA$ chart, determining the limits against ARL_0 . See Maritz and Jarrett (1978) for more information regarding the variance of sample median. Table 2.1 consists of the values of L_1 and L_2 against the fixed $ARL_0=370$. In a process, if Z_t exceeds the LCL and UCL , a signal is generated by the $\tilde{X} - EWMA$ chart; otherwise, the process is stable.

3.2.3 Control Charting Algorithm

Here, the basic procedure described below is to find the appropriate values for the charting coefficients and for the designed $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts, respectively, using a variety of distributions.

Step 1: In this process, set the values for the designed charts with a pre-fixed $ARL_0=370$, such as subgroup size ($n = 5, 7$, and 10), smoothing parameter $\lambda = 0.2$, the amount of change $\delta = 0$, and the change point $\tau = 0$ or 50 .

Step 2: A randomly distributed subgroup with a normal distribution and a size of n is generated using ($\mu = 0, \sigma = 1$). Similar generating was applied for the student's t distribution with four degrees of freedom and the gamma distribution with ($\alpha = 1$ and $\beta = 1$).

Step 3: The sample mean $\bar{X}$ and sample median $\tilde{X}$ are calculated in order to compute the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ plotting statistics.

Step 4: For both $\bar{X} - EWMA$ and $\tilde{X} - EWMA$, the charting coefficients (L_1 and L_2) can be set to any arbitrary value, after which the control limits of the charts are calculated.

To calculate the ARL_0 value, 100,000 iterations in the R language program are used to execute steps 1 through 4. If the obtained ARL_0 does not match the pre-specified value of 370, the L_1 and L_2 values are modified, and the process is repeated until the desired value of ARL_0 is obtained. When L_1 and L_2 are selected appropriately, the ARL_1 values against δ values (i.e., 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4) are then added. More details on evaluating the run length properties of a control chart through Monte Carlo simulations can be found in Bayer et al. (2019).

Table 2.1: Charting constants of EWMA chart against fixed $ARL_0=370$.

Chart	Distribution	$n = 5$		$n = 7$		$n = 10$	
		L_1	L_2	L_1	L_2	L_1	L_2
$\bar{X}$	Normal	2.86	2.86	2.86	2.86	2.86	2.86
	student's t	3.03	3.03	2.999	2.999	2.97	2.97
	Gamma	2.4	3.359	2.462	3.28	2.526	3.21
$\tilde{X}$	Normal	2.86	2.86	2.456	2.456	1.99	1.99
	student's t	2.95	2.95	2.458	2.458	1.954	1.954
	Gamma	2.255	3.56	2.11	2.76	1.873	1.994

3.3 Design of Simulation Study

In the context of control charts, run length (RL) is mostly used to evaluate the efficiency of the chart in detecting disturbances in statistical process control. The average run length (ARL), standard deviation of run length ($SDRL$), and median of run length ($MDRL$) are the three common performance measures used to assess a chart. Average run length ARL is the most widespread performance metric used to assess a control chart's ability to detect disruption at a particular time point. It is measured by the number of samples until an OOC signal occurs. In performance evaluation, there are two categories of ARL . ARL_0 is the average run length in the IC state, and

ARL_1 is the average run length in the *OO*C state. At a fixed ARL_0 , the control chart with a smaller value of ARL_1 is considered to be more efficient. We expect a smaller value of these measures when the process is in *OO*C state by fixing them at a particular level when the process is in *IC* state, e.g., $ARL_0 = 370$.

Monitoring states refer to the various stages of a process where performance and any changes are continuously monitored and evaluated. These states include zero-state, in which the change manifests at the process's beginning, and steady-state, in which the change manifests after a certain amount of time. The performance of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ control charts are assessed in this study using both zero-state and steady-state, with steady-state taking into account changes beyond the 50th point.

The environment for the simulation study is based on steady-monitoring and zero states, as well as distributional environments like the normal distribution with a mean of zero and a variance of one ($N(0, 1)$), the student's t distribution with four degrees of freedom (t_4), and the gamma distribution ($gamma(1, 1)$). The subgroup size influences the strength of the detection ability of the control chart. In this chapter, three different subgroups size were chosen to be $n = 5, 7$, and 10 . The performance of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts will be evaluated in the presence of regular shift, linear drift, and quadratic drift. The comparison between the chart's performance will be provided using the ARL as a performance measure.

3.4 Results and Discussion

Here, we investigate the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ control charts' performance under shifts and drifts. The performance of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts is based on zero-state run length and is summarized in Table 2.2. The performance of the charts under a steady-state run

length is shown in Table 2.3. When shift, linear drift, and quadratic drift occur, the various distributions are taken into account in the tables, revealing ARL values at fixed $ARL_0 = 370$. Analyzing each of these characteristics using the three subgrouping sizes (i.e., $n = 5, 7, \text{ and } 10$)

3.4.1 Zero-State Run Length Performance

This section demonstrates the evaluation of the charts' performance in zero-state monitoring. The performance of the charts is assessed using ARL_1 values against various values of δ . In Table 2.2, the performance evaluations of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ for zero-state monitoring with respect to subgroup sizes $n = 5, 7$, and 10 are shown for normal, t , and gamma distributions. The following is an explanation of the findings:

- Under normal distribution, since the $\bar{X} - EWMA$ chart has the lowest ARL_1 value, it is more sensitive to detecting small changes in the process mean, making its performance superior in spotting shifts and drifts. Additionally, this advantage allows the $\bar{X} - EWMA$ chart to provide more accurate and timely insights into any deviations from the target. In Table 2.2, at $n = 5$ and $0.025\sigma_0$ shift, the ARL_1 value of $\bar{X} - EWMA$ chart is decreasing by 31.24 units, while in the $\tilde{X} - EWMA$ chart, a reduction of 22.66 units in ARL_1 values was observed. At $n = 7$ and $0.25\sigma_0$ linear drift, the ARL_1 value of $\bar{X} - EWMA$ chart equals 4.17 while the ARL_1 value of $\tilde{X} - EWMA$ chart equals 4.65. In the case of quadratic drift at $n = 10$, the $\bar{X} - EWMA$ chart has $ARL_1 = 2.01$ and the $\tilde{X} - EWMA$ chart has $ARL_1 = 2.09$ for $0.5\sigma_0$.
- Under the student's t distribution, the $\tilde{X} - EWMA$ charts have lower ARL_1 values. It performs better than the $\bar{X} - EWMA$ chart at identifying shifts and linear drifts. In Table 2.2 at $n = 5$ and $0.025\sigma_0$ in shift case, for example, the ARL_1 value of $\bar{X} - EWMA$ chart

is decreasing by 42.85 units, while in the $\tilde{X} - EWMA$ chart, there was a 50.37 unit decrease in ARL_1 values. The ARL_1 value of the $\bar{X} - EWMA$ chart has decreased by 361.45 units for $0.25\sigma_0$ in the linear drift case with $n = 7$, while the ARL_1 value of the $\tilde{X} - EWMA$ chart has decreased by 361.51 units. In the case of quadratic drift at $n = 10$, the $\bar{X} - EWMA$ chart has $ARL_1 = 1.994$ and the $\tilde{X} - EWMA$ chart has $ARL_1 = 1.990$ for $0.5\sigma_0$, which indicate that $\bar{X} - EWMA$ chart performance is slightly better in detecting quadratic drift.

- Under gamma distribution, the performance of the $\bar{X} - EWMA$ is better than $\tilde{X} - EWMA$ in the presence of drifts, even if biased ARLs appear. Besides, the bias indicates that the shift in ARL_1 values is noticeable when the lambda values are 0.01 and 0.025. This suggests that the shift case has a stronger effect on the ARL_1 values compared to other scenarios. In Table 2.2, at $n = 5$ when $\delta = 0.025$ in the shift case, the ARL_1 values have increased by 25 units in the $\bar{X} - EWMA$ chart while the ARL_1 value of the $\tilde{X} - EWMA$ chart has increased by 53 units. At $n = 7$ when $\delta = 0.25$ in the linear drift case, the $\bar{X} - EWMA$ chart has $ARL_1 = 4.51$ but the $\tilde{X} - EWMA$ chart has $ARL_1 = 4.68$. At $n = 10$ when $\delta = 0.5$ in the case of quadratic drift, the $\bar{X} - EWMA$ chart has $ARL_1 = 2.02$ and the $\tilde{X} - EWMA$ chart has $ARL_1 = 2.00$.
- The subgroup size affects the control chart's ability to detect the drifts. In the t distribution $\bar{X} - EWMA$ chart, as an example, the ARL_1 values at $0.1\sigma_0$ are 112.4, 80.2, and 54.2, respectively.

3.4.2 Steady-State Run Length Performance

In this section, the discussion will be about the evaluation of the performance of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ control charts in steady-state monitoring. The ARL_1 values are provided to

compare the performance of the charts. The distributions, such as normal, t, and gamma distributions, were evaluated for steady-state monitoring with respect to subgroup sizes $n = 5, 7$, and 10. The findings shown in Table 2.3 are explained in the following manner:

- Under normal distribution, the $\bar{X} - EWMA$ chart is more reliable and effective in detecting various types of shifts and drifts compared to the $\tilde{X} - EWMA$ charts. At $n = 5$ and $0.025\sigma_0$ (shift), the ARL_1 value of the $\bar{X} - EWMA$ chart reported a reduction of 26 units, while the reduction in the $\tilde{X} - EWMA$ chart was observed as 18. At $n = 7$ and $0.25\sigma_0$ (linear drift), the ARL_1 value of $\bar{X} - EWMA$ chart has reduced by 308 units while ARL_1 value of $\tilde{X} - EWMA$ chart has reduced by 306. At $n = 10$ and $0.5\sigma_0$ (quadratic drift), $\bar{X} - EWMA$ chart ability in detection has $ARL_1 = 48.99$ and $\tilde{X} - EWMA$ chart has $ARL_1 = 49.22$.
- Under the student's t distribution, the $\tilde{X} - EWMA$ has a high ability to detect shifts and linear drifts since it shows the lowest ARL_1 value. At $n = 5$ and $0.025\sigma_0$ in shift case, the ARL_1 value of $\bar{X} - EWMA$ chart is decreasing by 35.47 units while the reduction units in ARL_1 values in the $\tilde{X} - EWMA$ chart is 43.73 units. At $n = 7$ and $0.25\sigma_0$ in the linear drift case, the ARL_1 value of $\bar{X} - EWMA$ chart has reduced by 319.78 units while ARL_1 value of $\tilde{X} - EWMA$ chart has reduced by 319.86. At $n = 10$ and $0.5\sigma_0$ in the case of quadratic drift, $\bar{X} - EWMA$ chart has $ARL_1 = 48.87$ and $\tilde{X} - EWMA$ chart has $ARL_1 = 48.88$. which means that $\bar{X} - EWMA$ chart is slightly better at detecting quadratic drift.
- Under gamma distribution, although there is a bias in the ARL_1 values in the shift scenario when $\delta = 0.01$ and 0.025 , the performance of the $\bar{X} - EWMA$ is better than $\tilde{X} - EWMA$ in spotting shifts and drifts. At $n = 7$ and $0.25\sigma_0$ in the linear

drift case, the $0.025\sigma_0$ change may increase the ARL_1 values of the $\bar{X} - EWMA$ chart by 23.79 units but in $\tilde{X} - EWMA$ chart, on the other hand, the $0.025\sigma_0$ change may increase the ARL_1 values by 48.14 units. At $n = 7$ and $0.25\sigma_0$ in the linear drift case, the $\bar{X} - EWMA$ chart $ARL_1 = 57.61$ but $\tilde{X} - EWMA$ chart has $ARL_1 = 58.47$. At $n = 10$ and $0.5\sigma_0$ in the case of quadratic drift, $\bar{X} - EWMA$ chart has $ARL_1 = 48.99$ and $\tilde{X} - EWMA$ chart has $ARL_1 = 49.07$.

- The subgroup size affects the control chart's ability to detect the drifts. In the gamma distribution $\tilde{X} - EWMA$ chart, as an example, the ARL_1 values at $0.1\sigma_0$ (linear drift) are 9.30, 8.14, and 6.82 at $n=5, 7$, and 10, respectively.

Table 2.2: Zero-state ARL profile of EWMA charts at different subgroup sizes.

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.63	370.60	370.74	370.70	370.90	370.03	370.99	370.98	370.71
		0.01	366.43	364.39	359.38	33.06	29.62	26.47	9.49	8.92	8.36
		0.025	339.39	326.47	311.15	18.40	16.53	14.75	6.79	6.39	5.99
		0.05	266.88	239.56	208.62	11.90	10.73	9.63	5.29	4.98	4.68
		0.1	141.72	111.92	84.21	7.81	7.07	6.36	4.14	3.91	3.69
		0.25	29.05	20.95	14.88	4.59	4.17	3.77	3.00	2.90	2.78
		0.5	8.15	6.27	4.85	3.12	2.86	2.61	2.32	2.11	2.01
		1	3.15	2.62	2.20	2.12	2.00	1.95	1.99	1.98	1.95
		2	1.62	1.30	1.06	1.62	1.30	1.06	1.62	1.30	1.06
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.57	370.04	370.38	370.94	370.43	370.03	370.39	370.84	370.66
		0.01	361.35	359.01	356.76	28.30	25.02	22.06	8.54	8.00	7.47
		0.025	327.72	313.37	287.46	15.56	13.84	12.27	6.12	5.75	5.37
		0.05	245.78	209.09	166.40	10.04	8.98	8.02	4.79	4.50	4.19
		0.1	112.49	80.28	54.21	6.60	5.93	5.32	3.78	3.55	3.27
		0.25	18.13	12.51	8.84	3.90	3.53	3.18	2.85	2.66	2.37
		0.5	5.28	4.13	3.27	2.71	2.42	2.14	2.02	2.00	1.99
		1	2.30	1.99	1.71	1.98	1.91	1.69	1.97	1.91	1.70
		2	1.09	1.01	1.00	1.09	1.01	1.00	1.09	1.01	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Gamma	0	370.14	370.08	370.34	371.04	370.68	370.78	370.96	370.16	370.70
		0.01	391.40	382.85	383.38	38.83	34.02	29.58	10.17	9.45	8.76
		0.025	395.84	379.42	361.87	21.27	18.65	16.30	7.23	6.74	6.26
		0.05	348.87	308.85	263.20	13.55	11.92	10.50	5.62	5.26	4.87
		0.1	210.79	163.37	118.64	8.74	7.74	6.86	4.41	4.08	3.83
		0.25	49.46	32.23	20.81	5.04	4.51	4.03	3.10	2.96	2.89
		0.5	11.74	8.17	5.86	3.42	3.05	2.79	2.61	2.26	2.02
		1	3.84	3.03	2.44	2.32	2.03	1.97	1.99	1.99	1.97
$\bar{X}$	Normal	2	1.88	1.62	1.16	1.87	1.62	1.16	1.87	1.61	1.16
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.22	370.43	370.91	370.48	370.72	370.86	370.90	370.87	370.37
		0.01	367.55	367.68	363.83	37.10	33.66	29.39	10.16	9.59	8.88
		0.025	347.56	342.39	327.23	20.66	18.72	16.36	7.26	6.86	6.36
		0.05	290.70	274.61	239.15	13.33	12.12	10.63	5.65	5.35	4.96
		0.1	176.48	147.82	109.96	8.71	7.94	7.00	4.42	4.18	3.89
		0.25	41.53	30.91	20.35	5.09	4.65	4.13	3.17	3.01	2.89
		0.5	11.09	8.52	6.15	3.45	3.17	2.84	2.59	2.36	2.10
		1	3.91	3.25	2.58	2.36	2.14	2.00	2.00	1.99	1.98
	Gamma	2	1.92	1.67	1.28	1.85	1.66	1.27	1.85	1.66	1.27
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.80	370.34	370.57	370.94	370.10	370.59	370.93	370.88	370.17
		0.01	363.01	360.34	348.05	26.72	23.64	20.35	8.34	7.80	7.17
		0.025	320.43	298.56	260.83	14.83	13.19	11.40	5.98	5.60	5.16
		0.05	220.84	181.13	133.36	9.62	8.59	7.49	4.67	4.38	4.03
		0.1	91.95	64.37	40.59	6.34	5.70	5.00	3.69	3.45	3.13
		0.25	15.35	10.78	7.26	3.76	3.40	3.01	2.78	2.56	2.21
		0.5	4.81	3.79	2.90	2.61	2.32	2.05	2.01	2.00	1.99
		1	2.18	1.89	1.48	1.95	1.84	1.48	1.95	1.84	1.48
$\bar{X}$	Normal	2	1.05	1.00	1.00	1.05	1.00	1.00	1.05	1.00	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.83	370.12	370.99	370.65	370.72	370.86	370.73	370.39	370.34
		0.01	401.15	398.51	394.40	42.38	36.66	30.34	10.55	9.75	8.81
		0.025	423.10	406.89	378.37	22.96	19.96	16.53	7.48	6.92	6.30
		0.05	392.68	347.39	283.00	14.53	12.64	10.56	5.81	5.38	4.84
		0.1	253.02	193.62	129.59	9.30	8.14	6.83	4.58	4.17	3.78
		0.25	64.63	40.83	22.05	5.32	4.68	3.96	3.24	2.97	2.85
		0.5	14.79	9.49	5.77	3.60	3.15	2.72	2.75	2.35	2.00
	Gamma	1	4.35	3.21	2.31	2.51	2.07	1.95	2.00	1.99	1.95
		2	1.97	1.70	1.05	1.93	1.71	1.05	1.93	1.71	1.06
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 2.3: Steady-state ARL profile of EWMA charts at different subgroup sizes.

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.61	370.23	370.53	370.32	370.13	370.57	370.70	370.09	370.82
		0.01	367.55	367.19	363.32	76.07	73.13	70.35	55.59	55.04	54.57
		0.025	343.66	332.36	319.60	63.28	61.66	60.08	53.19	52.86	52.47
		0.05	279.95	256.06	228.85	57.69	56.62	55.61	51.87	51.67	51.36
		0.1	171.42	144.75	120.26	54.08	53.43	52.85	50.86	50.64	50.47
		0.25	72.43	65.38	60.08	51.22	50.91	50.56	49.85	49.73	49.67
		0.5	54.32	52.64	51.46	49.97	49.74	49.55	49.33	49.21	48.99
		1	50.03	49.54	49.16	49.15	48.92	48.88	48.95	48.90	48.84
		2	48.67	48.50	48.28	48.68	48.49	48.31	48.69	48.49	48.30
		4	48.15	48.24	48.12	48.11	48.18	48.13	48.18	48.12	48.16
	t	0	370.35	370.03	373.56	370.88	370.83	371.99	370.89	370.71	370.83
		0.01	360.11	363.41	358.71	71.91	68.95	66.40	54.39	54.18	53.72
		0.025	334.53	320.17	300.03	60.70	59.18	57.87	52.49	52.04	51.89
		0.05	261.02	227.99	192.38	55.81	54.99	54.18	51.23	51.03	50.83
		0.1	144.47	117.06	94.31	52.89	52.30	51.79	50.40	50.27	50.01
		0.25	62.66	57.96	54.73	50.45	50.22	49.97	49.51	49.42	49.28
		0.5	51.63	50.71	50.00	49.44	49.29	49.03	48.86	48.92	48.87
		1	49.12	48.86	48.63	48.73	48.69	48.57	48.75	48.76	48.54
		2	48.14	48.09	48.07	48.19	48.13	48.03	48.14	48.12	48.04
		4	47.97	48.05	48.07	47.93	48.01	48.00	47.97	47.97	48.07
	Gamma	0	370.05	370.10	370.57	370.41	370.55	370.59	370.64	370.53	370.00
		0.01	389.33	382.47	381.31	81.35	76.74	73.09	56.23	55.48	54.92
		0.025	393.79	375.59	361.35	65.81	63.58	61.55	53.55	53.13	52.79
		0.05	353.09	319.29	280.29	59.10	57.61	56.43	52.18	51.81	51.47
		0.1	233.16	189.97	151.45	54.91	53.97	53.25	51.16	50.80	50.58
		0.25	90.74	75.39	65.30	51.68	51.16	50.72	50.00	49.85	49.77
		0.5	57.48	54.30	52.29	50.20	49.91	49.66	49.50	49.26	49.08
		1	50.61	49.89	49.36	49.34	49.04	48.93	49.01	48.92	48.94
$\bar{X}$	Normal	2	48.88	48.60	48.36	48.88	48.64	48.36	48.77	48.64	48.33
		4	48.12	48.10	48.11	48.16	48.10	48.10	48.18	48.14	48.14
		0	370.21	370.05	370.29	370.50	370.77	370.16	370.40	370.72	370.65
		0.01	366.12	368.23	364.96	79.59	76.66	72.99	56.05	55.64	55.05
		0.025	351.36	344.92	334.46	65.22	63.63	61.49	53.50	53.29	52.86
		0.05	301.07	284.46	253.52	58.91	57.92	56.51	52.18	51.93	51.68
		0.1	200.60	175.52	143.27	54.83	54.17	53.41	51.13	50.90	50.64
		0.25	83.46	74.01	64.99	51.67	51.31	50.85	50.05	49.91	49.83
	t	0.5	56.91	54.69	52.58	50.28	50.02	49.77	49.55	49.32	49.22
		1	50.63	50.10	49.49	49.37	49.16	48.94	48.97	48.93	48.93
		2	48.85	48.69	48.43	48.76	48.67	48.48	48.77	48.69	48.44
		4	48.17	48.14	48.11	48.15	48.12	48.12	48.19	48.18	48.18
		0	370.40	370.03	370.08	370.79	370.40	370.70	370.82	370.94	370.96
		0.01	365.20	359.88	351.61	70.59	68.00	65.06	54.45	54.07	53.54
		0.025	326.27	308.09	276.05	60.12	58.77	57.09	52.42	52.18	51.77
		0.05	239.73	206.32	163.04	55.59	54.74	53.73	51.26	51.09	50.74
		0.1	127.16	103.38	82.38	52.69	52.22	51.57	50.45	50.20	49.99
	Gamma	0.25	60.46	56.55	53.51	50.47	50.14	49.78	49.57	49.42	49.19
		0.5	51.36	50.45	49.72	49.46	49.27	49.02	49.00	49.00	48.88
		1	49.14	48.90	48.55	48.82	48.74	48.52	48.84	48.74	48.56
		2	48.25	48.14	48.12	48.23	48.07	48.15	48.27	48.14	48.07
		4	48.14	48.13	48.06	48.09	48.13	48.14	48.08	48.10	48.15
		0	370.61	370.26	370.88	370.74	370.07	370.58	370.95	370.29	370.50
		0.01	397.80	397.91	390.36	84.23	79.51	73.75	56.52	55.89	54.97
		0.025	417.19	407.63	380.68	67.32	64.77	61.84	53.80	53.38	52.77
		0.05	392.37	358.45	295.72	60.00	58.47	56.62	52.37	52.06	51.55
		0.1	269.56	221.00	161.19	55.42	54.49	53.36	51.26	50.94	50.59
		0.25	104.60	83.98	67.04	51.99	51.47	50.78	50.15	49.96	49.72
		0.5	60.23	55.80	52.53	50.35	50.03	49.71	49.61	49.42	49.07
		1	51.04	50.18	49.43	49.41	49.16	48.92	49.04	48.99	48.85
		2	48.97	48.80	48.43	48.92	48.70	48.41	48.88	48.79	48.40
		4	48.19	48.14	48.07	48.16	48.16	48.13	48.15	48.16	48.15

In summary, based on the findings of the shifts and drifts from zero-state and steady-state monitoring, the detection ability becomes more precise as the size of the subgroup increases. Moreover, the $\tilde{X} - EWMA$ chart may be more suitable for detecting shifts and linear drift under the student's t distribution, while the $\bar{X} - EWMA$ chart excels in identifying shifts and drifts in the normal and gamma distributions and quadratic drift under the student's t distribution. Also, compared to linear drifts or shifts, both charts are faster at detecting quadratic drifts.

3.4.3 Comparative Analysis Under Drifts

In this section, we show the performance assessment of the EWMA charts and Shewhart charts in zero-state monitoring and steady-state monitoring. The outcomes of the Shewhart control charts (i.e., $\bar{X}$ and $\tilde{X}$ charts) and the EWMA control charts (i.e., $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts) are provided in Table 2.4. Therefore, we compare how well they performed in identifying drifts for various distributions. The performance of the charts is assessed by comparing the ARL_1 values against the δ values, which are 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4 at $n = 10$; since increasing the subgroup size has a positive impact on improving the detection ability. The best chart will be determined by its lowest ARL_1 values.

Considering the zero-state monitoring first, the findings are explained as follows:

- Under normal distribution, when $0.01 \leq \delta \leq 0.25$ (linear drift), $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts provide lower ARL_1 values compared to Shewhart $\bar{X}$ and $\tilde{X}$ charts y. which determines the superiority of $\bar{X} - EWMA$ chart and $\tilde{X} - EWMA$ chart in detecting the drifts. For instance, at $\delta = 0.05$, the ARL_1 values for the $\bar{X} - EWMA$ and $\bar{X}$ charts are reported as 9.63 and 13.12, respectively, while the ARL_1 values for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are 10.63 and 14.95, respectively. However, when $\delta = 0.5$ and above, Shewhart charts have better performance in identifying the drifts because they are designed to detect

a large number of shifts and drifts. When $\delta = 0.01$ and 0.025 (quadratic drift), $\bar{X} - EWMA$ chart better than the $\bar{X}$ chart in detection of the drift. The ARL_1 value of the $EWMA_{\bar{X}}$ chart is equal to 5.99 and for $\bar{X}$ chart is reported as 6.25 at 0.025 . On the other hand, the $\tilde{X} - EWMA$ chart is better than the $\tilde{X}$ chart in detecting the quadratic drift when $\delta = 0.01, 0.025$, and 0.05 . For example, the ARL_1 value of the $\tilde{X} - EWMA$ chart is reported as 4.96 , whereas for $\tilde{X}$ chart is 4.99 at $\delta = 0.05$. However, when $\delta \geq 0.1$, Shewhart charts have better performance in identifying the quadratic drifts.

- Under the Student's t distribution, when $0.01 \leq \delta \leq 0.25$ (linear drift), the $\bar{X} - EWMA$ chart provides lower ARL_1 values compared to the $\bar{X}$ chart. For example, at $\delta = 0.025$, the ARL_1 values for the $\bar{X} - EWMA$ chart equal 3.18 , but for the $\bar{X}$ chart is reported as 3.48 . However, when $\delta \geq 0.5$, Shewhart charts have a better performance in identifying the linear drifts. meanwhile, the $\tilde{X} - EWMA$ chart is better than the $\tilde{X}$ chart in detecting the linear drift when $0.01 \leq \delta \leq 0.1$. However, when $\delta \geq 0.25$, the $\tilde{X}$ chart outperform the $\tilde{X} - EWMA$ chart in identifying the linear drifts. When $0.01 \leq \delta \leq 0.05$ (quadratic drift), $\bar{X} - EWMA$ chart effective more than $\bar{X}$ chart in detection the drift. For example, at $\delta = 0.05$, the ARL_1 value of $\bar{X} - EWMA$ chart is equal to 4.19 and for $\bar{X}$ chart is reported as 4.35 . However, $\tilde{X} - EWMA$ chart outperform the $\tilde{X}$ chart in detecting the quadratic drift when $\delta = 0.01$ and 0.025 . However, when $\delta \geq 0.05$, $\tilde{X}$ chart outperform $\tilde{X} - EWMA$ chart in identifying the quadratic drifts.
- Under gamma distribution, when $0.01 \leq \delta \leq 0.5$ (linear drift), the $EWMA_{\bar{X}}$ and $\tilde{X} - EWMA$ chart's ARL_1 values are lower than those of the $\bar{X}$ and $\tilde{X}$ chart demonstrating a better ability to detect a drift. For instance, at $0.5\sigma_0$, the ARL_1 values for the $\bar{X} - EWMA$

and $\bar{X}$ charts are reported as 2.78 and 2.87, respectively. On the other hand, the ARL_1 values for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are showing as 2.72 and 3.03, respectively. When $0.01 \leq \delta \leq 0.1$ (quadratic drift), the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ chart's ARL_1 values are lower than those of the $\bar{X}$ and $\tilde{X}$ chart demonstrating a better ability to detect a drift. For instance, at $\delta = 0.1$, the ARL_1 values for the $\bar{X} - EWMA$ and $\bar{X}$ charts are reported as 3.83 and 3.90, respectively. On the other hand, the ARL_1 value for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are showing as 3.78 and 4.01, respectively.

Secondly, let us consider steady-state monitoring. The findings are interpreted as follows:

- Under normal distribution, when $0.01 \leq \delta \leq 0.1$ (linear drift), $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts provide lower ARL_1 values compared to Shewhart $\bar{X}$ and $\tilde{X}$ charts correspondingly. which determines the superiority of $\bar{X} - EWMA$ chart and $\tilde{X} - EWMA$ chart in detecting the drifts. For instance, at $\delta = 0.1$, the ARL_1 values for $\bar{X} - EWMA$ and $\bar{X}$ charts reported as 52.85 and 53.60 respectively, while the ARL_1 values for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are 53.41 and 54.44 respectively. However, when $\delta \geq 0.25$, Shewhart charts have the better performance in identifying the drifts. Only at $\delta = 0.01$ (quadratic drift), $\bar{X} - EWMA$ chart and $\tilde{X} - EWMA$ charts are better than $\bar{X}$ chart and $\tilde{X}$ chart in detection the drift otherwise $\bar{X}$ and $\tilde{X}$ charts are the best. The ARL_1 value of $\bar{X} - EWMA$ chart is equal to 54.57 and for $\bar{X}$ chart is reported as 54.81. On the other hand, the ARL_1 value of the $\tilde{X} - EWMA$ chart is reported as 55.05 whereas for $\tilde{X}$ chart is reported as 55.59.

Table 2.4: ARL profile of EWMA charts and Shewhart charts when $n = 10$

State	Distribution	δ	Linear drift				Quadratic drift			
			$EWMA_{\bar{X}}$	$\bar{X}$	$EWMA_{\bar{X}}$	$\bar{X}$	$EWMA_{\bar{X}}$	$\bar{X}$	$EWMA_{\bar{X}}$	$\bar{X}$
Zero-state monitoring	Normal	0	370.03	370.80	370.86	370.88	370.71	370.54	370.37	370.50
		0.01	26.47	42.05	29.39	47.52	8.36	9.22	8.88	9.89
		0.025	14.75	21.91	16.36	24.88	5.99	6.25	6.36	6.70
		0.05	9.63	13.12	10.63	14.95	4.68	4.65	4.96	4.99
		0.1	6.36	7.79	7.00	8.89	3.69	3.50	3.89	3.73
		0.25	3.77	3.90	4.13	4.44	2.78	2.42	2.89	2.61
		0.5	2.61	2.34	2.84	2.64	2.01	1.92	2.09	1.96
		1	1.95	1.44	2.00	1.64	1.95	1.43	1.98	1.62
	t	2	1.06	1.00	1.27	1.01	1.06	1.00	1.27	1.01
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0	370.03	370.36	370.59	370.60	370.66	370.15	370.17	370.12
		0.01	22.06	42.67	20.34	34.14	7.47	8.72	7.17	7.97
		0.025	12.27	21.34	11.40	17.40	5.37	5.85	5.16	5.38
		0.05	8.02	12.42	7.49	10.23	4.19	4.35	4.03	4.01
		0.1	5.32	7.17	5.00	6.01	3.27	3.22	3.13	2.99
		0.25	3.18	3.48	3.01	2.98	2.37	2.14	2.21	2.02
	Gamma	0.5	2.14	2.05	2.05	1.81	1.99	1.90	1.99	1.78
		1	1.69	1.16	1.48	1.05	1.70	1.15	1.48	1.05
		2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		0	370.78	370.18	370.86	370.31	370.70	370.20	370.34	370.95
		0.01	29.57	57.23	30.34	61.65	8.76	10.66	8.81	11.09
		0.025	16.30	29.59	16.53	31.85	6.26	7.14	6.30	7.40
		0.05	10.50	17.53	10.56	18.85	4.87	5.29	4.84	5.46
Steady-state monitoring	Normal	0.1	6.86	10.24	6.83	10.96	3.83	3.90	3.78	4.01
		0.25	4.03	4.96	3.96	5.27	2.88	2.76	2.85	2.83
		0.5	2.78	2.87	2.72	3.03	2.02	1.98	2.00	1.98
		1	1.97	1.77	1.95	1.84	1.97	1.77	1.95	1.84
		2	1.16	1.00	1.05	1.00	1.16	1.00	1.06	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.57	370.75	370.16	370.81	370.82	370.70	370.65	370.91
		0.01	70.35	83.62	72.99	87.97	54.57	54.81	55.05	55.59
		0.025	60.08	66.01	61.49	68.30	52.47	52.27	52.86	52.66
		0.05	55.61	58.31	56.51	59.59	51.36	50.88	51.68	51.26
		0.1	52.85	53.60	53.41	54.44	50.47	49.96	50.64	50.16
		0.25	50.56	50.25	50.85	50.64	49.67	48.96	49.83	49.14
		0.5	49.55	48.88	49.77	49.01	48.99	48.49	49.22	48.61
		1	48.88	48.10	48.94	48.10	48.84	48.10	48.93	48.26
	Gamma	2	48.31	47.72	48.48	47.60	48.30	47.67	48.44	47.72
		4	48.13	47.70	48.12	47.64	48.16	47.70	48.18	47.66
		0	371.99	370.87	370.70	370.17	370.83	370.27	370.96	370.94
		0.01	66.40	83.25	65.06	76.47	53.72	83.23	53.54	53.92
		0.025	57.87	65.11	57.09	61.84	51.89	64.99	51.77	51.62
		0.05	54.18	57.47	53.73	55.69	50.83	57.49	50.74	50.39
		0.1	51.79	52.94	51.57	51.98	50.01	52.92	49.99	49.52
		0.25	49.97	49.74	49.78	49.29	49.28	49.82	49.19	48.60
	Gamma	0.5	49.03	48.57	49.02	48.34	48.87	48.51	48.88	48.42
		1	48.57	47.78	48.52	47.64	48.54	47.80	48.56	47.82
		2	48.03	47.71	48.15	47.59	48.04	47.63	48.07	47.81
		4	48.00	47.66	48.14	47.62	48.07	47.66	48.15	47.74
		0	370.59	370.92	370.58	370.28	370.00	371.83	370.50	370.93
		0.01	73.09	96.31	73.75	100.86	54.92	56.18	54.97	56.50
		0.025	61.55	72.49	61.84	74.81	52.79	53.10	52.77	53.34
		0.05	56.43	62.11	56.62	63.45	51.47	51.45	51.55	51.63
		0.1	53.25	55.76	53.36	56.43	50.58	50.33	50.59	50.40
		0.25	50.72	51.20	50.78	51.38	49.77	49.26	49.72	49.33
		0.5	49.66	49.36	49.71	49.48	49.08	48.53	49.07	48.60
		1	48.93	48.39	48.92	48.49	48.94	48.34	48.85	48.51
		2	48.36	47.73	48.41	47.74	48.33	47.74	48.40	47.70
		4	48.10	47.70	48.13	47.69	48.14	47.72	48.15	47.81

- Under Student's t distribution, when $0.01 \leq \delta \leq 0.1$ (linear drift), $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts provide lower ARL_1 values compared to Shewhart $\bar{X}$ and $\tilde{X}$ charts. For example, at $\delta = 0.1$, the ARL_1 values for $\bar{X} - EWMA$ chart equal 51.79 but for the $\bar{X}$ chart is reported as 52.94. however, the ARL_1 value of the $\tilde{X} - EWMA$ chart is reported as 51.57 whereas for $\tilde{X}$ chart is reported as 51.98. When $0.01 \leq \delta \leq 0.25$ (quadratic drift), $\bar{X} - EWMA$ chart effective more than $\bar{X}$ chart in detection the drift. For example, at $\delta = 0.25$, the ARL_1 value of $\bar{X} - EWMA$ chart is equal to 49.28 and for $\bar{X}$ chart is reported as 49.82. However, $\tilde{X} - EWMA$ chart outperform the $\tilde{X}$ chart only at $\delta = 0.01$ otherwise $\tilde{X}$ chart outperform $\tilde{X} - EWMA$ chart in identifying the quadratic drifts.
- Under gamma distribution, when $0.01 \leq \delta \leq 0.25$ (linear drift), the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ chart's ARL_1 values are lower than those of the $\bar{X}$ and $\tilde{X}$ chart, demonstrating a better ability to detect a drift. For instance, at $0.25\sigma_0$, the ARL_1 values for the $\bar{X} - EWMA$ and $\bar{X}$ charts are reported as 50.72 and 51.20, respectively. On the other hand, the ARL_1 values for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are showing as 50.78 and 51.38 respectively. When $0.01 \leq \delta \leq 0.025$ (quadratic drift), the $\bar{X} - EWMA$ chart's ARL_1 values are lower than $\bar{X}$ chart values. For instance, at $\delta = 0.025$, the ARL_1 values for $\bar{X} - EWMA$ and $\bar{X}$ charts are reported as 52.79 and 53.10, respectively. On the other hand, when $0.01 \leq \delta \leq 0.05$ (quadratic drift), the ARL_1 values for $\tilde{X} - EWMA$ and $\tilde{X}$ charts are showing as 51.55 and 51.63, respectively, at $\delta = 0.05$.

In conclusion, the effectiveness of both charts depends on the precise distribution and drift seen. The results from zero-state monitoring reveal that the $\bar{X} - EWMA$ chart is more effective at detecting the linear drift under all the distributions when $0.01 \leq \delta \leq 0.25$. While $0.01 \leq \delta \leq 0.05$, the $\bar{X} - EWMA$ chart performs better at spotting the quadratic drift for the Student's t

distribution and gamma distribution. According to results from steady-state monitoring, the $\bar{X} - EWMA$ chart performs better at spotting linear drift beneath all the distributions when $0.01 \leq \delta \leq 0.1$. However, the $\tilde{X} - EWMA$ chart is more effective at spotting quadratic drift under the normal and Student's t distribution only at $\delta = 0.01$.

3.5 Nitrogen Oxides Concentration level illustration

Air quality is crucial for human and environmental well-being, as poor air quality can cause respiratory problems, allergies, and chronic diseases. It also negatively impacts ecosystems, affecting plant and animal species. The sources of NO_x emissions measured in the USA in 2023 are shown in Figure 2.1. Road vehicles, boats, airplanes, farm vehicles, and other mobile sources are by far the largest contributors. Therefore, monitoring and improving air quality are essential for a sustainable future.

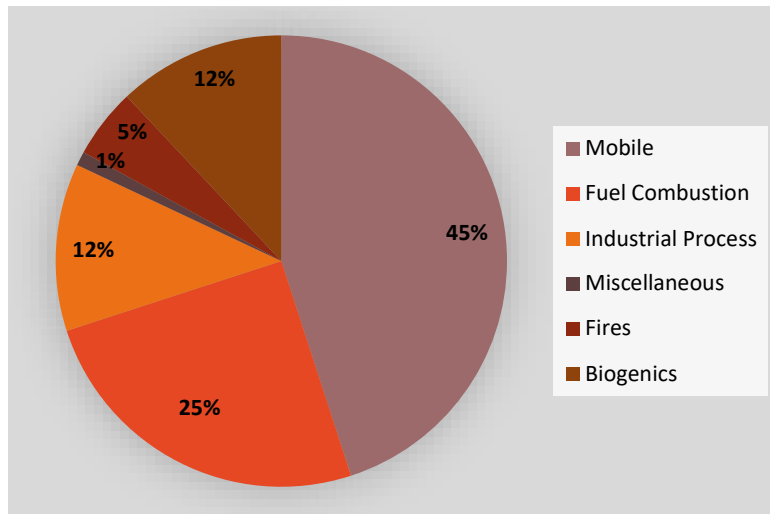


Figure 2.1: Sources of NO_x emissions in the USA – 2023 Source: EPA (2023)

A dataset of air quality was collected from five metal oxide chemistry sensors in a heavily polluted area in an Italian town. The data set takes account of hourly averages of CO concentration, non-Metonic hydrocarbons, benzene, total nitrogen oxides (NO_x), and nitrogen dioxide (NO₂). Based

on the American Lung Association (2023), NO_x is the most dangerous factor for human health, causing respiratory tract damage, increased vulnerability to infections, asthma, chronic lung disease, irritation, lung tissue damage, inflammation, and worsening cough and wheezing.

The concentration of nitrogen oxide (NO_x) has thus been chosen as the variable of interest for monitoring. The dataset was taken from the UCI Machine Learning Repository Vito (2016). There are 5200 data points recorded from March 2004 to April 2005. During preprocessing, in-control NO_x concentrations missing values were taken out, and the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ for each subgroup of 5 were calculated. The R package “segmented” is used again here to locate the change-point Figure 2.2 (A and B). Moreover, the fitted model indicated that the *OOB* data drifted quadratically, as demonstrated by the regression analysis, where $NOx = -2618 + 6.377t - 0.003351t^2$ Figure 2.2 (C and D). The PP-plots suggest that the variables may not be normally distributed, but rather follow a gamma distribution Figure 2(E and F).

To construct the charts, the mean as well as the standard deviation of the $\bar{X} - EWMA$ belongs to IC data are estimated to be $\bar{X} = 131.8625$ and $\sigma_{\bar{X}} = 59.4421$, respectively, whereas the mean and standard deviation of the $\tilde{X} - EWMA$ belongs to IC data are set to be 129.03938 and 69.9816. Therefore, at $ARL_0 = 370$, the *LCL* and *UCL* of the $\bar{X} - EWMA$ chart are set to 86.29027 and 199.23036, respectively. Likewise, the *LCL* and *UCL* for the $\tilde{X} - EWMA$ chart are 75.38676 and 210.6846, respectively.

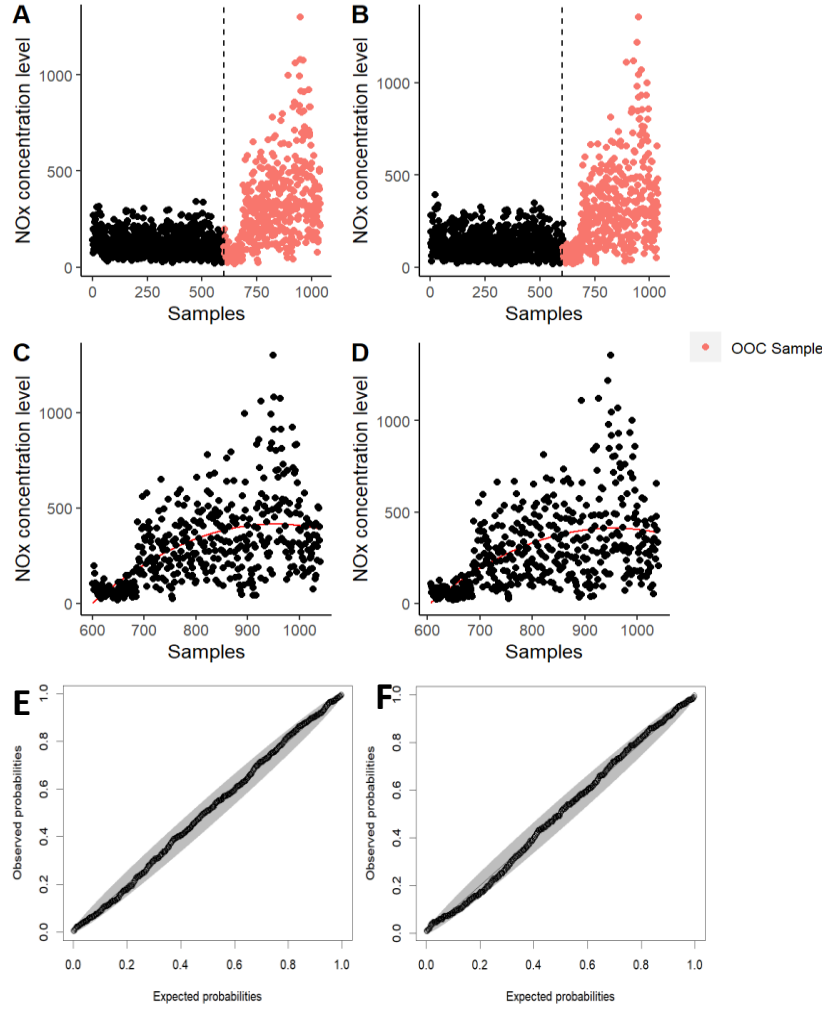


Figure 2.2: Scatter plot for the *IC* and *OOC* data of the NOx concentration level; (A) $\bar{X}$ values, (B) $\tilde{X}$ values, (C) for OOC $\bar{X}$ with the quadratic trend, and (D) for OOC $\tilde{X}$ with the quadratic trend, (E) PP-plot for the IC NOx concentration level $\bar{X}$ values, and (F) PP-plot for the IC NOx concentration level $\tilde{X}$ values.

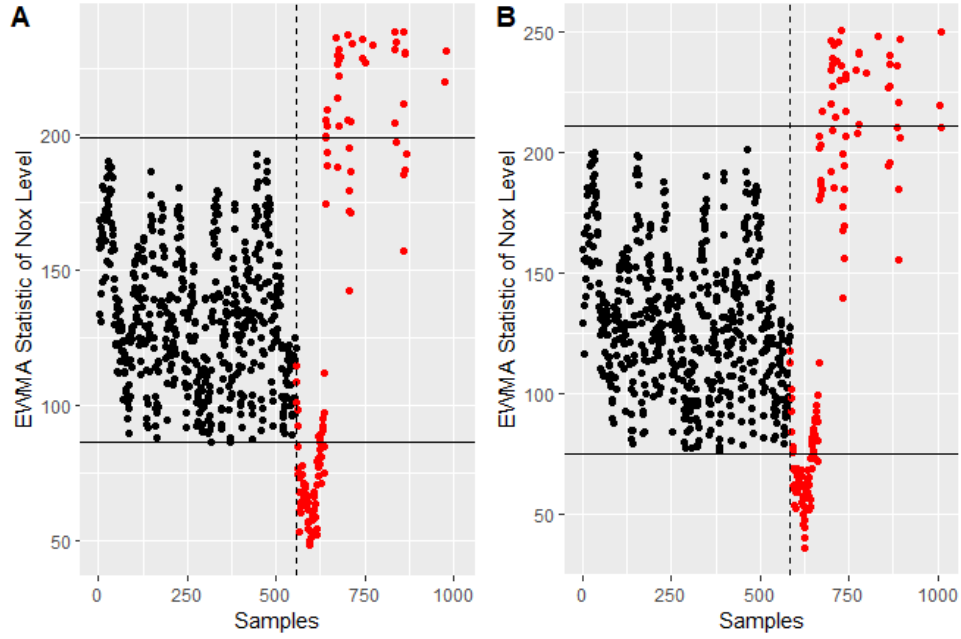


Figure 2.3: (A) the $\bar{X} - EWMA$ chart and (B) the $\tilde{X} - EWMA$ charts for NOx concentration level.

In terms of the ability for identify quadratic drift, the $\bar{X} - EWMA$ chart in Figure 2.3(A) reported 455 signals, and the $\tilde{X} - EWMA$ chart in Figure 2.3(B) captured 401 signals. Furthermore, based on phase 1 data and since $\tau = 556$, the $\bar{X} - EWMA$ chart was able to detect the drift at sample number 561. So, the delay amount is 5. On the other hand, with $\tau = 584$, the $\tilde{X} - EWMA$ chart was able to detect the drift at sample number 593 with the delay amount equal to 9. As a result, the $\bar{X} - EWMA$ chart has a higher detection ability in detecting quadratic drift.

3.6 Summary and Recommendations

In this chapter, we studied the detection behavior of Shewhart and EWMA control charts in the presence of linear and quadratic drifts. Under the two monitoring states (i.e., zero- and steady-states) to ensure the stability and reliability and consistency of the process and gain a comprehensive understanding of its behavior. The performance of the EWMA

control chart using the sample mean and median as location estimators is evaluated into different distributions and subgrouping sizes. It is worth mentioning that the $\bar{X} - EWMA$ chart is more effective at detecting shifts and drifts under normal and gamma distributions in both monitoring states as well as quadratic drift under the t distribution in zero-state monitoring, while the $\tilde{X} - EWMA$ chart is better at identifying shifts and linear drifts under the student's t distribution in zero-state monitoring. However, in the steady-state monitoring case, the $\tilde{X} - EWMA$ chart is better at identifying shifts, linear and quadratic drifts under the student's t distribution. This difference in performance can be attributed to the different statistical properties of the distributions they are designed for. The proposed charts are deployed in real-life scenario related to air quality. A comparative analysis of detection performance has been made between the EWMA charts and the Shewhart charts, where both charts provide insightful information about process drifts. This study assessed drifts' effects on EWMA location-based charts but could extend to other memory-type dispersion charts. The discussion in this chapter only covers linear and quadratic drift, but it can be expanded to cover other types of drift.

CHAPTER 4

CUSUM Based Location Charts Under Drifts.

Nowadays, manufacturers face intense pressure to maintain a high standard of quality. Due to the damage to machining components, manufacturing processes are deteriorating with time, leading to unsatisfactory products. Generally, statistical process control tools like control charts aid in spotting patterns and trends that indicate a change in the process. This study examines the performance of cumulative sum control charts using the sample mean and median as plotting statistics. Run length measurements will be used to assess the performance after the charts have experienced linear and quadratic drifts in non-normal setups under zero- and steady-state conditions. The study revealed that CUSUM charts exhibit superior performance in zero-state monitoring compared to steady-state monitoring. However, the CUSUM chart for the mean is appropriate for normal and gamma distributions and has a greater ability for drift detection under biased and unbiased ARLs. If the distribution is the student's t , the CUSUM chart for the median performs well in spotting the drift. When compared to linear drifts, both are quicker to identify quadratic drifts. The study examined distributions with various subgroup sizes, which revealed that the subgroup size selection impacted the performance of the control chart. The study also compares the detection abilities of Shewhart, EWMA, and CUSUM charts. With Shewhart charts, the CUSUM charts are more effective at detecting drifts between 0.01 and 0.5. Meanwhile, with EWMA charts, the performance of the CUSUM charts is superior in terms

of spotting drifts larger than 0.25. To illustrate the significance of the stated proposal, the suggested charts will also be applied to real-world case studies.

4.1 Introduction

Statistical process control (SPC) is a technique for monitoring and controlling a process or manufacturing method using seven popular instruments. SPC can assist in identifying issues and avoiding the development of faults in a process. Walter A. Shewhart created SPC in the 1920s, and W. Edwards Deming popularized it in Japan following World War II. The core SPC tools, like control charts, are extensively used in industrial operations. Control charts are graphs that display the changes in a process variable over time. Control charts can establish whether a process is under statistical control and identify common and unique causes of variation Montgomery (2020). Common cause variation is always present in processes, and special cause variation comes from outside sources, implying that the process is not under statistical control. Control charts are separated into two groups based on the parameter that needs to be examined.

In contrast to dispersion charts, which focus on the variability parameter, location charts, the topic of this article, are control charts that focus on the centrality parameter. In addition, there are two main kinds of control charts, depending on how severe the process disturbances are. Memoryless charts like Shewhart charts detect large, abrupt changes in processes. In contrast, memory charts like cumulative sum and exponentially weighted moving average control charts identify smaller changes that may not be easily detected. Both types provide distinct insights into process variations, assisting in informed process improvement decision-making. The study will primarily focus on memory charts for

location monitoring, specifically on the cumulative sum (CUSUM) control chart that was developed by Page (1954).

CUSUM charts improve the ability to detect small changes, and the plotting statistic includes both the most recent and prior values of the process's data Koshti (2011). Furthermore, the CUSUM chart plots the cumulative sums of the sample values' deviations from a target value. The CUSUM chart requires two parameters: a reference value (k) and a decision limit (h). Setting the reference value (k) to half of the detected drift and the decision limit (h) to the threshold for declaring the process out-of-control (OOC). The CUSUM structure will be examined using sample mean ($\bar{X}$) and sample median ($\tilde{X}$) statistics, which are typically used to track the location of a process. When compared to the traditional Shewhart chart, the CUSUM chart is more sensitive to smaller shifts and drifts in the process mean. Furthermore, many authors have looked into the CUSUM chart; recent relevant works and references can be found in Erem and Mahmood (2023) and Abbas (2023).

Drift is a data pattern that denotes a slow, linear, or nonlinear change in the process mean over time. The most common causes include things like tool wear, chemical concentration, temperature, etc. that have an impact on the process Reynolds and Stoumbos (2001). For instance, a classic sign of tool wear is when the size of the tool varies in comparison to its former size. Once the reason for the drift has been determined, the required action can be taken. Shift, linear drift, and quadratic drift are all considered in this study. The shift is characterized as a sudden increase or decrease in a process's performance. This shift can occur due to various factors, such as changes in market demand, or modifications in production methods. The conventional shift in the mean is

defined as $\mu_1 = \mu_0 + \delta\sigma_0$. Where μ_1 is the shifted mean, and δ is the amount of change times σ_0 in the process mean. The terms "drifts" and "trends" refers to the gradual change in the measurement readings over time either linearly or nonlinearly. The formulas of the linear drift and the quadratic drift are $\mu_1 = \mu_0 + \omega t\sigma_0$ and $\mu_1 = \mu_0 + \rho t^2\sigma_0$, respectively, where μ_1 is the drifted mean, and (ω, ρ) are the amount of change times σ_0 in the process mean, and t represents the time point. (see 1.3)

The performance of the control chart was investigated from a variety of perspectives in the *SPC* literature. For instance, Bissell examined a technique for interpreting a trend from a CUSUM chart Bissell (1984a). Also, Bissell (1984b) expanded the Brook and Evans (1972) Markov chain methodology by examining the run-length characteristics of the Shewhart and CUSUM charts. Aerne et al. (1991) evaluated how the Shewhart chart, CUSUM chart, and EWMA chart performed when there was linear drift in the process mean. Recent proposals by Yi and Qiu (2022) for a new drift detection technique. Utilizing the general framework, the new method generates a CUSUM chart based on the *GLR* statistics. It is clear from the studies mentioned above that a lot of studies employ $\bar{X}$ chart to investigate the effects of shifts and linear drifts in the data. As far as the authors are aware, drifts are not considered in relation to $\tilde{X}$ charts. Additionally, while location-based research is addressed under the subject of linear drift over time, quadratic drift over time is not considered (See section 1.4 for more details).

The purpose of this study is to assess the performance of a cumulative sum (CUSUM) control chart based on the sample mean ($\bar{X}$) and sample median ($\tilde{X}$) under shifts, linear drifts, and quadratic drifts in normal and non-normal situations. As well as considering various subgroup sizes when analyzing the performance of the charts under zero-state and

steady-state scenarios to shed light on which chart is more suited for spotting process fluctuations. The following would be how the rest of the content is arranged: Section 2 provides a description of the control chart structure for location monitoring. Section 3 contains a design for the suggested charts. Section 4 reports the results of the charts' performance assessment. two real-life examples of how the suggested charts can be used in practice is described in Section 5. In Section 6, a comparative analysis under drifts is provided. In Section 7, comments and suggestions for future research are given.

4.2 Control Charts for Location Monitoring

This section is designed to discuss the general structure of cumulative sum control charts based on sample mean ($\bar{X} - CUSUM$) charts and cumulative sum control charts based on sample median ($\tilde{X} - CUSUM$) control charts.

4.2.1 $\bar{X} - CUSUM$ chart

$X_{t,1}, \dots, X_{t,n}$, $i = 1, 2, \dots, n$; $t = 1, 2, \dots$ are independent, randomly distributed variables that follow normal distribution with an n-sample size, i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$ where μ_0 and σ_0 are the in-control mean and standard deviation, respectively. Additionally, δ represents the magnitude of the standardized mean shift, and t stands for time; if $t = 1$, that indicates a consistent shift in the process mean. The process is under control if $\delta = 0$; otherwise, it is out-of-control.

The plotting statistic of $\bar{X} - CUSUM$ is given as follows:

$$C_i^+ = \text{Max} [0, (\bar{X}_t - \mu_0) - k + C_{i-1}^-] \quad (1)$$

$$C_i^- = \text{Max} [0, -(\bar{X}_t - \mu_0) - k + C_{i-1}^-] \quad (2)$$

with $C_0^+ = C_0^- = 0$ and k is the reference value, also known as the sensitive parameter. The value of k is set to be equal to half of the target's distance ($k = 0.5$). The $\bar{X}_t$ defined as

$$\bar{X}_t = \frac{\sum_{i=1}^n X_i}{n} \quad (3)$$

The control limits, which are provided as follows, are plotted against the plotting of $\bar{X} - \text{CUSUM}$.

$$UCL = h_1 \sigma_0 / \sqrt{n} \quad (4)$$

$$LCL = h_2 \sigma_0 / \sqrt{n} \quad (5)$$

Where the central line is $CL = \mu_0$ and h_1 and h_2 are charting constants calculated with respect to $ARL_0 = 370$ and listed in Table 3.1. The process's stability is dependent on maintaining the LCL and UCL values. Thus, $\bar{X} - \text{CUSUM}$ chart will signify any change if C_i^+ or C_i^- exceeds the limits, and the process is said to be unstable.

4.2.2 $\tilde{X} - \text{CUSUM}$ chart

Let $X_{t,i}; i = 1, 2, \dots, n; t = 1, 2, \dots$ be a normally distributed t^{th} subgroup of size n , i.e., $X_t \sim N(\mu_0 + \delta t \sigma_0, \sigma_0)$. The t^{th} plotting statistic of the $\tilde{X} - \text{CUSUM}$ chart is stated as follows:

$$C_i^+ = \text{Max} [0, (\tilde{X}_t - \mu_0) - k + C_{i-1}^+] \quad (6)$$

$$C_i^- = \text{Max} [0, -(\tilde{X}_t - \mu_0) - k + C_{i-1}^-] \quad (7)$$

with $C_0^+ = C_0^- = 0$ and k is the reference value and it chosen to be 0.5, and $\tilde{X}_t$ expressed as

$$\tilde{X}_t = \begin{cases} X_{t,(\frac{n+1}{2})}, & \text{if } n \text{ is odd} \\ \frac{X_{t,(\frac{n}{2})} + X_{t,(\frac{n}{2}+1)}}{2}, & \text{if } n \text{ is even} \end{cases}, \quad (8)$$

where the t^{th} ordered subgroup represented by $X_{t,(1)}, \dots, X_{t,(n)}$. See Maritz and Jarrett (1978) for details about the variance of the sample median. The control limits as follows:

$$LCL = h_1 \sigma_0 \sqrt{\left(\frac{\pi}{2n}\right)} \quad (9)$$

$$UCL = h_2 \sigma_0 \sqrt{\left(\frac{\pi}{2n}\right)} \quad (10)$$

Where h_1 and h_2 are charting constants for the $\tilde{X} - CUSUM$ chart. Table 3.1 consists of the values of the decision limits against the fixed $ARL_0=370$. A signal will be generated by the $\tilde{X} - CUSUM$ chart if C_i^+ or C_i^- exceeds the LCL and UCL ; else, the process is said to be in-control (IC).

4.2.3 Algorithm for Control Charting Constants

The R language program is employed to determine the appropriate values for the charting coefficients. For the said purpose, the following steps were used:

Step 1: Set the values for the designed charts at a fixed $ARL_0=370$ throughout this procedure. The reference value is set to be $k = 0.5$, the amount of change $\delta = 0$, and change point $\tau = 0$ and subgroup size ($n = 5, 7$, and 10).

Step 2: Generate a subgroup of size n with a normal distribution using ($\mu = 0$, $\sigma = 1$).

However, when a subgroup is generated from the student's t distribution, then four

degrees of freedom is used, while for the gamma distributed subgroup, $\alpha = 1$ and $\beta = 1$ are used.

Step 3: Calculate the sample mean and sample median (i.e., $\bar{X}$ and $\tilde{X}$) to compute the $\bar{X} - CUSUM$ given in Eqs. (1-2) and $\tilde{X} - CUSUM$ provided in Eqs. (6-7) plotting statistics. The starting values for both plotting statistics are considered equal to zero.

Step 4: Set any arbitrary values for the charting coefficients (h_1 and h_2). Following that, the charts' control limits are determined.

Steps 1 through 4 are carried out in the R language software using 1,00,000 iterations to calculate the ARL_0 value. The procedure is repeated until the intended value of ARL_0 is obtained. When h_1 and h_2 are properly chosen, the ARL_1 performance of the proposed charts will be evaluated against δ values where $\delta = 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2$, and 4.

4.3 Design of the Simulation Study

Run length (RL) is a regularly used technique in statistical process control to study, track, and evaluate the effectiveness of the chart in identifying changes in a process. By precisely measuring the run length, valuable insights can be gained, enabling better decision-making of processes. In addition, the standard performance metric is employed to assess a chart based on run length (RL), including average run length (ARL). The ARL measure is the average number of observations before a specific change occurs. This measure is used to monitor and detect any deviations or abnormalities in a process at a specific moment. ARL

is divided into two categories in performance evaluation: (i) ARL_0 is the average run length in the *IC* state, and (ii) ARL_1 is the average run length in the *OOC* state. The control chart with a lower value of ARL_1 is thought to be more effective when the $ARL_0 = 370$. Thus, the performance of the CUSUM charts will be assessed using the *ARL* as a performance metric.

In this study, the term "monitoring states" was used several times, and it refers to the two phases of continuously monitoring and assessing process behavior. These states include zero-state, where the change appears at the start of the operation, and steady-state, where the change appears after a specified amount of time; see Abbas (2023) and Knoth (2006) for more details. In this study, both zero-state and steady-state are used to evaluate the performance of the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ control charts, with steady-state accounting for changes after the 50th point.

This study is designed by simulating various scenarios in which steady-state, zero-state, and different distributional settings are all incorporated into the simulation. Such as the normal distribution with a mean of zero and a variance of one (i.e., $N(0, 1)$), the student's *t* distribution with four degrees of freedom (i.e., t_4), and the gamma distribution (i.e., $gamma(1, 1)$). The size of the subgroups considered $n = 5, 7$, and 10 . The study tries to capture any potential changes in the detection power of the chart during monitoring. Also, it will be determined how well the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts perform when there is a regular shift, linear drift, and quadratic drift.

Table 3.1: Charting constants of the CUSUM chart against fixed $ARL_0=370$.

Chart	Distribution	$n = 5$		$n = 7$		$n = 10$	
		h_1	h_2	h_1	h_2	h_1	h_2
$\bar{X}$	Normal	2.24	2.24	1.865	1.865	1.5	1.5
	student's t	2.95	2.95	2.49	2.49	2.01	2.01
	Gamma	0.914	3.59	0.773	2.998	0.623	2.42
$\tilde{X}$	Normal	6.08	6.08	5.26	5.26	4.05	4.05
	student's t	5.7	5.7	4.62	4.62	3.26	3.26
	Gamma	1.35	9.34	1.23	7.6	0.985	5.44

4.4 Results and Discussion

In this section, we examine how well the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ control charts perform when subjected to shifts and drifts. Table 3.2 provides a summary of the performance of the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts based on zero-state monitoring. While the performance of the charts under a steady-state run length in the presence of shift, linear drift, and quadratic drift is summarized in Table 3.3. The tables, which take into consideration the various distributions, show the ARL values at fixed, $ARL_0 = 370$.

4.4.1 Zero-state run length performance

In this section, the discussion will be about the evaluation of the performance of $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ under zero-state monitoring. The evaluation will be done by comparing the ARL_1 values to various values of δ . A lower ARL_1 value indicates a chart's sensitivity, enabling quicker identification of drifts and prompting administrators to intervene and make necessary adjustments, thereby improving the quality of the process. The performance evaluations of $CUSUM$ charts under zero-state monitoring with respect

to subgroup sizes $n=5, 7$, and 10 are shown in Table 3.2 for normal, student's t , and gamma distributions. The findings are explained as follows:

- Under normal distribution, the $\bar{X} - CUSUM$ chart performance is superior in spotting shifts and drifts. In Table 3.2, at $n = 5$ and $0.025\sigma_0$ change (shift), the ARL_1 value of the $\bar{X} - CUSUM$ chart is decreasing by 12.94 units, while in the $\tilde{X} - CUSUM$ chart, a reduction of 8.66 units in ARL_1 values was observed. At $n = 7$ and $0.25\sigma_0$ change (linear drift), the ARL_1 value of $\bar{X} - CUSUM$ chart equals 4.03 while the ARL_1 value of $\tilde{X} - CUSUM$ chart equals 4.54. In the case of quadratic drift at $n = 10$, the $\bar{X} - CUSUM$ chart has $ARL_1 = 1.93$ and the $\tilde{X} - CUSUM$ chart has $ARL_1 = 1.98$ for $0.5\sigma_0$ change.
- Under the student's t distribution, the $\tilde{X} - CUSUM$ charts have lower ARL_1 values. It outperforms the $\bar{X} - CUSUM$ chart at detecting drifts. In Table 3.2 at $n = 5$ and $0.025\sigma_0$ change (shift), for example, the ARL_1 value of $\bar{X} - CUSUM$ chart is decreasing by 9.91 units, while in the $\tilde{X} - CUSUM$ chart, there was a 15.08 unit decrease in ARL_1 values. The ARL_1 value of the $\bar{X} - CUSUM$ chart equals 6.56 for $0.1\sigma_0$ change in the linear drift case with $n = 7$, while the ARL_1 value of the $\tilde{X} - CUSUM$ chart is 6.14. Moreover, in the case of quadratic drift at $n = 10$, the $\bar{X} - CUSUM$ chart has $ARL_1 = 1.92$ and the $\tilde{X} - CUSUM$ chart has $ARL_1 = 1.80$ for $0.5\sigma_0$ change.

Table 3.2: Zero-state ARL profile of CUSUM charts when $n = 5, 7$, and 10 .

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.16	370.68	370.87	370.83	370.96	370.97	370.50	370.40	370.43
		0.01	367.22	368.70	369.42	40.59	38.51	36.51	9.67	9.17	8.71
		0.025	357.08	357.84	357.26	21.46	20.17	18.96	6.73	6.35	5.99
		0.05	324.10	319.88	319.02	13.21	12.30	11.47	5.13	4.82	4.52
		0.1	232.08	223.13	214.01	8.20	7.56	6.94	3.93	3.68	3.44
		0.25	63.27	55.49	47.97	4.46	4.03	3.64	2.82	2.65	2.42
		0.5	11.55	9.18	7.11	2.87	2.57	2.28	2.08	1.98	1.93
		1	2.73	2.09	1.60	1.91	1.71	1.47	1.87	1.71	1.47
		2	1.13	1.02	1.00	1.13	1.02	1.00	1.13	1.02	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.94	370.65	370.14	370.63	370.97	370.82	370.03	370.70	370.42
		0.01	367.57	370.76	368.53	36.87	35.02	32.88	8.99	8.50	8.01
		0.025	361.03	364.54	358.40	19.00	17.77	16.56	6.27	5.90	5.52
		0.05	337.12	336.46	329.00	11.58	10.72	9.88	4.81	4.51	4.18
		0.1	256.48	247.31	228.87	7.18	6.56	5.97	3.74	3.47	3.15
		0.25	53.85	45.11	35.86	3.96	3.55	3.16	2.75	2.45	2.14
		0.5	6.70	5.11	3.77	2.61	2.28	2.00	2.00	1.97	1.92
		1	2.06	1.57	1.18	1.84	1.53	1.18	1.84	1.53	1.18
		2	1.02	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Gamma	0	370.81	370.60	370.06	370.13	370.98	370.58	370.86	370.35	370.51
		0.01	405.60	404.32	397.65	50.11	47.10	44.14	10.76	10.14	9.55
		0.025	441.49	442.73	427.93	25.99	24.22	22.47	7.50	7.02	6.56
		0.05	460.32	451.62	428.00	15.80	14.59	13.41	5.73	5.36	4.98
		0.1	381.11	359.91	330.64	9.74	8.88	8.06	4.44	4.08	3.79
		0.25	136.17	117.18	97.85	5.30	4.75	4.24	3.07	2.90	2.76
		0.5	23.36	17.81	13.27	3.44	3.04	2.69	2.51	2.10	1.98
		1	4.05	3.03	2.21	2.26	1.96	1.81	1.98	1.94	1.81
		2	1.65	1.16	1.00	1.65	1.16	1.00	1.65	1.16	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\bar{X}$	Normal	0	370.72	370.98	370.09	370.40	370.16	370.77	370.02	370.47	370.41
		0.01	369.94	366.98	366.68	43.08	40.95	38.43	10.28	9.78	9.15
		0.025	362.06	358.71	357.52	23.14	21.76	20.12	7.21	6.81	6.32
		0.05	329.24	325.93	322.91	14.41	13.43	12.26	5.53	5.20	4.80
		0.1	245.92	236.50	224.34	9.02	8.33	7.50	4.27	3.98	3.66
		0.25	74.46	65.67	55.81	5.00	4.54	4.00	3.02	2.85	2.63
		0.5	15.10	12.17	9.00	3.27	2.94	2.55	2.34	2.11	1.98
		1	3.67	2.86	2.05	2.16	1.94	1.69	1.96	1.89	1.69
		2	1.49	1.17	1.01	1.47	1.17	1.01	1.47	1.17	1.01
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t	0	370.31	370.75	370.81	370.66	370.54	370.49	370.24	370.82	370.83
		0.01	364.88	369.49	371.30	34.46	32.36	29.67	8.65	8.15	7.57
		0.025	355.23	355.40	351.03	17.87	16.58	15.06	6.02	5.64	5.19
		0.05	316.32	310.28	296.84	10.91	10.03	9.00	4.62	4.30	3.91
		0.1	215.43	197.69	170.57	6.76	6.14	5.42	3.57	3.27	2.96
		0.25	39.66	31.49	22.45	3.71	3.30	2.85	2.58	2.26	2.02
		0.5	5.71	4.26	2.87	2.42	2.10	1.82	1.98	1.94	1.80
		1	1.79	1.34	1.05	1.68	1.32	1.05	1.68	1.32	1.05
		2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Gamma	0	370.56	370.70	370.32	370.62	370.21	370.90	370.34	370.95	370.48
		0.01	424.26	431.56	415.87	53.56	51.66	48.20	11.19	10.60	9.87
		0.025	487.66	502.88	478.40	27.58	26.27	24.30	7.82	7.34	6.77
		0.05	524.19	525.53	496.31	16.78	15.77	14.39	6.00	5.60	5.13
		0.1	433.26	413.58	381.00	10.36	9.56	8.57	4.64	4.31	3.86
		0.25	172.05	158.51	131.81	5.67	5.11	4.45	3.28	2.96	2.82
		0.5	30.00	26.03	19.16	3.69	3.28	2.79	2.73	2.30	1.99
		1	4.75	3.60	2.44	2.50	2.07	1.86	1.99	1.97	1.86
		2	1.86	1.41	1.00	1.84	1.41	1.00	1.84	1.41	1.00
		4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

- Under gamma distribution, the performance of the $\bar{X} - CUSUM$ is more effective in detecting and adapting to drifts in the data. It is able to accurately identify changes in the process, even when there are variations in the average run length ARL due to bias in both charts. The bias appears when $0.01 \leq \delta \leq 0.1$; authors such as Riaz (2008b) and Huang and Yang (2015) mentioned related appearances. In Table 3.2, at $n = 5$ when $\delta = 0.025$ (shift), the ARL_1 values have increased by 70.68 units in the $\bar{X} - CUSUM$ chart while the ARL_1 value of the $\tilde{X} - CUSUM$ chart has increased by 117.1 units. At $n = 7$ when $\delta = 0.25$ (linear drift), the $\tilde{X} - CUSUM$ chart has $ARL_1 = 5.11$ but the $\bar{X} - CUSUM$ chart has $ARL_1 = 4.75$. At $n = 10$ when $\delta = 0.5$ (quadratic drift), the $\bar{X} - CUSUM$ chart has $ARL_1 = 1.98$ and the $\tilde{X} - CUSUM$ chart has $ARL_1 = 1.99$.
- The subgroup size influences the ability of the control chart to identify drifts. An accurate picture of the process is given by the larger subgroup size, which makes it easier to spot minor drifts. Therefore, the size of the subgroup should be decided in light of the unique requirements and characteristics of the process being observed. In the gamma distribution $\bar{X} - CUSUM$ chart, as an example, the ARL_1 values at $0.5 \sigma_0$ (shift scenario) are 23.36, 17.81, and 13.27 in $n = 5, 7, \text{ and } 10$, respectively.

4.4.2 Steady-state run length performance

The discussion in this section will spotlight how $CUSUM$ control charts performed during steady-state monitoring. The ARL_1 values are given so that the performance of the charts can be compared in Table 3.3. The distributions, including the normal, student's t, and

gamma distributions, were assessed with regard to subgroup sizes $n = 5, 7$, and 10 . Thus, the information shown in Table 3.3 is interpreted as follows:

- Under normal distribution, the $\bar{X} - CUSUM$ chart is more efficient in spotting different kinds of shifts and drifts compared to the other chart. In Table 3.3, at $n = 5$ and $0.025\sigma_0$ change (shift), the ARL_1 value of the $\bar{X} - CUSUM$ chart indicated a reduction of 11 units, but the reduction in the $\tilde{X} - CUSUM$ chart was actually 9.47 units. At $n = 7$ and $0.25\sigma_0$ change (linear drift), the ARL_1 value of $\bar{X} - CUSUM$ chart has reduced by 320.06 units while ARL_1 value of $\tilde{X} - CUSUM$ chart has reduced by 319.45. At $n = 10$ and $0.5\sigma_0$ change (quadratic drift), the $\bar{X} - CUSUM$ chart has $ARL_1 = 48.57$ and the $\tilde{X} - CUSUM$ chart has $ARL_1 = 48.62$.
- Under the student's t distribution, the $\tilde{X} - CUSUM$ has a high ability to detect shifts and drifts, due to the fact that it displays the lowest value of ARL_1 . In Table 3.3 at $n = 5$ and $0.025\sigma_0$ change (shift), the decrease units in ARL_1 value of $\bar{X} - CUSUM$ chart is 9.21 units while the ARL_1 values in the $\tilde{X} - CUSUM$ chart is reducing by 13.5 units. At $n = 7$ and $0.25\sigma_0$ change (linear drift), the ARL_1 value of $\bar{X} - CUSUM$ chart has reduced by 320.07 units while ARL_1 value of $\tilde{X} - CUSUM$ chart has reduced by 321.12. At $n = 10$ and $0.5\sigma_0$ change (quadratic drift), the $\bar{X} - CUSUM$ chart has $ARL_1 = 48.59$ and $\tilde{X} - CUSUM$ chart has $ARL_1 = 48.41$.

Table 3.3: Steady-state ARL profile of CUSUM charts when $n = 5, 7$, and 10 .

Chart	Distribution	δ	Shift			Linear drift			Quadratic drift		
			$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$	$n = 5$	$n = 7$	$n = 10$
$\bar{X}$	Normal	0	370.03	370.64	370.89	370.77	370.45	370.87	370.61	370.67	370.48
		0.01	366.26	370.35	367.61	82.43	80.59	78.86	55.41	54.83	54.40
		0.025	358.24	360.30	358.27	65.66	64.51	63.55	52.80	52.46	52.16
		0.05	332.12	326.64	323.27	58.51	57.64	56.87	51.43	51.12	50.83
		0.1	249.75	242.34	232.21	54.16	53.53	53.00	50.42	50.09	49.85
		0.25	102.24	95.07	88.74	50.82	50.39	50.02	49.39	49.22	48.99
		0.5	56.88	54.83	53.03	49.40	49.15	48.87	48.77	48.61	48.57
		1	49.28	48.75	48.21	48.67	48.40	48.12	48.64	48.37	48.12
		2	47.93	47.76	47.79	47.92	47.81	47.79	47.91	47.74	47.74
		4	47.80	47.72	47.71	47.82	47.74	47.74	47.83	47.76	47.80
	t	0	370.97	370.91	370.98	370.49	370.11	370.89	370.64	370.78	370.90
		0.01	365.87	371.35	368.27	79.00	77.44	75.62	54.74	54.31	53.82
		0.025	361.76	362.31	361.06	63.50	62.44	61.24	52.27	52.12	51.67
		0.05	341.17	340.34	331.89	56.99	56.24	55.51	51.05	50.77	50.48
		0.1	272.14	262.95	247.40	53.12	52.61	52.11	50.20	49.93	49.65
		0.25	93.57	86.38	78.08	50.29	50.04	49.60	49.21	49.03	48.68
		0.5	52.71	51.29	50.12	49.17	48.88	48.63	48.66	48.62	48.59
		1	48.59	48.21	47.85	48.45	48.20	47.83	48.44	48.25	47.82
		2	47.80	47.82	47.65	47.76	47.75	47.75	47.74	47.75	47.66
		4	47.78	47.72	47.73	47.77	47.78	47.75	47.76	47.73	47.77
	Gamma	0	370.53	370.67	370.72	370.82	370.87	370.10	370.19	370.79	370.15
		0.01	400.00	400.52	394.65	90.51	88.05	85.37	56.34	55.77	55.25
		0.025	431.74	429.85	419.66	69.75	67.97	66.47	53.51	53.07	52.56
		0.05	448.26	445.16	421.28	60.70	59.65	58.56	51.93	51.54	51.21
		0.1	380.85	362.08	335.83	55.47	54.64	53.98	50.81	50.43	50.14
		0.25	165.27	148.61	131.92	51.55	51.02	50.60	49.61	49.36	49.28
		0.5	67.11	62.31	58.34	49.93	49.50	49.17	49.12	48.73	48.52
		1	50.40	49.49	48.78	48.89	48.61	48.43	48.66	48.61	48.45
		2	48.32	47.86	47.72	48.37	47.94	47.75	48.36	47.90	47.72
		4	47.82	47.77	47.73	47.80	47.80	47.74	47.80	47.75	47.75
$\bar{X}$	Normal	0	370.70	370.73	370.26	370.73	370.41	370.82	370.23	370.36	370.50
		0.01	369.56	367.24	369.37	84.86	82.84	80.35	55.98	55.42	54.87
		0.025	360.89	359.33	357.83	67.21	66.07	64.45	53.33	52.93	52.38
		0.05	334.25	331.98	327.86	59.62	58.74	57.56	51.83	51.49	51.12
		0.1	261.17	254.54	243.51	54.90	54.22	53.48	50.75	50.41	50.08
		0.25	112.01	104.29	95.46	51.44	51.00	50.43	49.63	49.50	49.11
		0.5	60.04	57.44	54.64	49.82	49.56	49.10	49.13	48.83	48.62
		1	50.16	49.40	48.68	48.90	48.62	48.41	48.78	48.56	48.34
		2	48.29	47.95	47.82	48.28	47.99	47.75	48.28	47.96	47.69
		4	47.91	47.87	47.72	47.88	47.84	47.77	47.93	47.82	47.80
	t	0	370.19	370.20	370.44	370.50	370.85	370.91	370.66	370.52	370.83
		0.01	365.29	364.42	370.11	77.00	75.07	72.84	54.50	53.93	53.49
		0.025	356.69	354.46	352.48	62.55	61.33	60.01	52.18	51.75	51.40
		0.05	325.31	315.34	305.47	56.40	55.60	54.72	50.93	50.63	50.32
		0.1	234.59	219.39	196.89	52.79	52.31	51.63	49.99	49.69	49.42
		0.25	81.20	73.94	66.53	50.11	49.73	49.36	49.16	48.81	48.68
		0.5	51.76	50.50	49.38	49.01	48.64	48.45	48.60	48.59	48.41
		1	48.38	48.05	47.74	48.36	47.98	47.79	48.33	47.92	47.79
		2	47.70	47.76	47.73	47.78	47.76	47.74	47.78	47.74	47.74
		4	47.73	47.71	47.75	47.76	47.76	47.79	47.84	47.73	47.77
	Gamma	0	370.94	370.03	370.70	370.76	370.78	370.17	370.40	370.39	370.53
		0.01	415.03	423.32	412.48	93.72	91.84	88.88	56.79	56.20	55.42
		0.025	473.19	487.11	463.52	71.12	69.81	67.99	53.79	53.27	52.68
		0.05	504.30	503.91	477.12	61.69	60.71	59.50	52.24	51.82	51.29
		0.1	429.19	411.17	379.55	56.00	55.22	54.33	51.04	50.70	50.24
		0.25	197.28	186.06	162.53	52.00	51.34	50.68	49.81	49.46	49.32
		0.5	72.84	69.46	63.61	50.18	49.70	49.30	49.32	48.91	48.57
		1	51.04	49.99	48.94	49.16	48.69	48.45	48.73	48.59	48.44
		2	48.55	48.08	47.66	48.51	48.04	47.67	48.50	48.12	47.67
		4	47.88	47.69	47.71	47.83	47.77	47.72	47.84	47.74	47.67

- Under gamma distribution, at $0.01 \leq \delta \leq 0.1$, both charts show biased behavior under shifting scenarios, but similar biases are not observed for drift scenarios. Despite biases, the $\bar{X} - CUSUM$ chart still demonstrates a good performance in detecting shifts and drifts. In Table 3.3, at $n = 5$ (shift), the $0.1\sigma_0$ change may increase the ARL_1 values of the $\bar{X} - CUSUM$ chart by 10.32 units, but in the $\tilde{X} - CUSUM$ chart, the same change may raise the ARL_1 values by 58.25 units. At $n = 7$ and $0.25\sigma_0$ change (linear drift), the $\bar{X} - CUSUM$ chart $ARL_1 = 51.02$ but $\tilde{X} - CUSUM$ chart has $ARL_1 = 51.34$. At $n = 10$ and $0.5\sigma_0$ change (quadratic drift), $\bar{X} - CUSUM$ chart has $ARL_1 = 48.52$ and $\tilde{X} - CUSUM$ chart has ARL_1 equal to 48.57.
- The subgroup size contributes to the drift detection of the control chart. In the normal distribution, the $\tilde{X} - CUSUM$ chart, as an example, the ARL_1 values at $0.1\sigma_0$ change (quadratic drift) are 50.75, 50.41, and 50.08 in $n = 5, 7$, and 10, respectively.

Overall, according to the results of zero-state and steady-state monitoring for shifts and drifts, the $\bar{X} - CUSUM$ chart and $\tilde{X} - CUSUM$ chart performs better when zero-state monitoring is taken into consideration. When there is quadratic drift in the process, the $\bar{X} - CUSUM$ chart and the $\tilde{X} - CUSUM$ chart performs even better. The choice between the two depends on the process's underlying distribution, with the $\bar{X} - CUSUM$ chart suitable for normal and gamma distributions, with a higher ability to detect drifts under biased and unbiased ARLs when it comes to linear drift and quadratic drift. Whereas the $\tilde{X} - CUSUM$ chart excels under the student's t distribution. However, both charts are faster at spotting quadratic drifts than linear ones. Additionally, for both charts, the value

of ARL_1 drops as n rises. In other words, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts become more sensitive to detecting drifts as the sample size increases.

4.4.3 Comparative Analysis Under Drifts.

In this section, we demonstrate how the CUSUM charts, EWMA charts, and Shewhart charts performed in zero- and steady-state monitoring. Figures 3.1 and 3.2 provide the results of the CUSUM charts (i.e., $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$), the EWMA control charts (i.e., the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts), and Shewhart control charts (i.e., the $\bar{X}$ and $\tilde{X}$ charts). The ARL_1 values are compared to the δ values, which are predefined thresholds such as 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1, 2, and 4. The results are taken at $n = 10$ since increasing the subgroup size significantly affects the ability to detect abnormalities. Ultimately, the chart with the lowest ARL_1 value is considered the best in terms of performance evaluation. Figures 3.1 and 3.2 characterize the ARL curves of the control charts based on the sample mean and sample median. In Figure 3.1, plots A, B, and C represent normal, student's t , and gamma distributions, respectively, under zero-state monitoring, while plots D, E, and F represent the same distributions but under steady-state monitoring in the presence of linear and quadratic drifts monitoring. On the other hand, the ARL Curves of the control charts, based on the sample median in Figure 3.2 represented by plots A, B, and C represent normal, student's t , and gamma distributions, respectively, under zero-state monitoring. In contrast, plots D, E, and F, represent the same distributions under steady-state monitoring in the presence of linear and quadratic drifts.

The results are explained as follows, starting with the zero-state monitoring:

- Under normal distribution (cf. Figures 3.1 (A) and 3.2 (A)), in terms of detecting the linear drift, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts provide

lower ARL_1 values when $\delta \geq 0.25$ compared to the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values when δ is between 0.01 and 0.1. But the superiority goes to the Shewhart $\bar{X}$ and $\tilde{X}$ charts when $\delta \geq 1$, since Shewhart charts are constructed to detect a high number of shifts and drifts. For instance, at $\delta = 0.25$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 3.64 and 4, respectively, while the ARL_1 values for the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 3.77 and 4.13, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 3.90 and 4.44, respectively. In terms of identifying the quadratic drift, $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ chart have a better performance when $\delta \geq 0.025$ compared to $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values only when $\delta = 0.01$. However, when $\delta \geq 0.25$, Shewhart charts perform better at spotting quadratic drifts. For example, at $\delta = 0.25$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 2.42 and 2.63, respectively, while the ARL_1 values for the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 2.78 and 2.89, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 3.90 and 2.61, respectively.

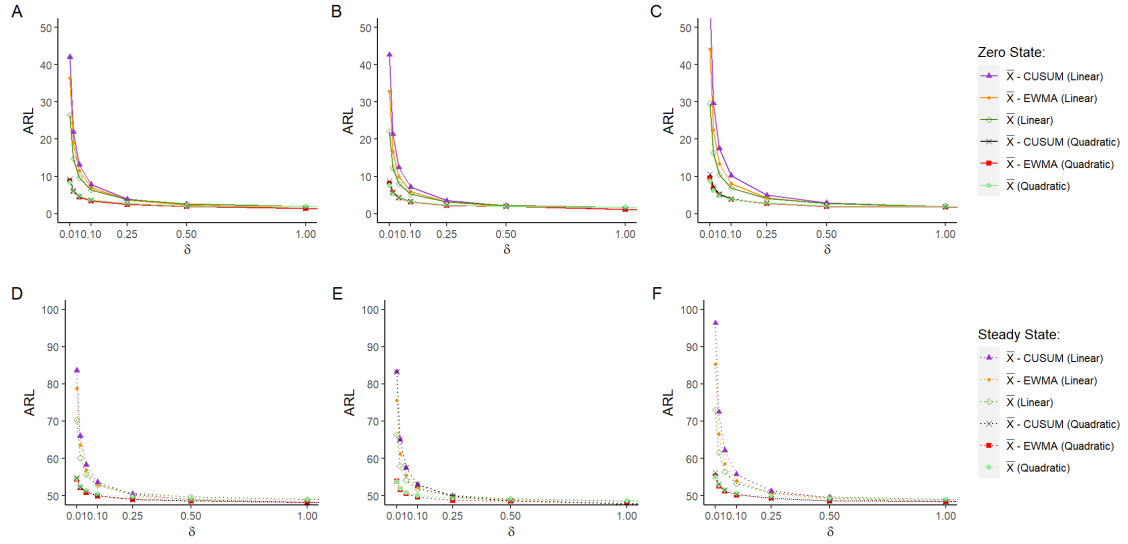


Figure 3.1: ARL Curves of the control charts based on sample mean in presence of linear and quadratic drifts. The first plot represents zero-state: (A) Normal distribution, (B) Student's t distribution, and (C) gamma distribution. The last plot represents steady-state: (D) Normal distribution, (E) t distribution, and (F) gamma distribution.

- Under the student's t distribution (cf. Figures 3.1 (B) and 3.2 (B)), in terms of detecting linear drift, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts provide lower ARL_1 values when $\delta \geq 0.25$ compared to the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values when δ is between 0.01 and 0.1. Generally, the superiority goes to Shewhart $\bar{X}$ chart when $\delta \geq 1$, and for the $\tilde{X}$ chart only when $\delta = 0.5$. For example, at $\delta = 0.25$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts equal 3.16 and 2.85, respectively, but for the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are reported as 3.18 and 3.01, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 3.48 and 2.98, respectively. In terms of identifying the quadratic drift, $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts perform better when $\delta \geq 0.25$ compared to $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1

values only when $\delta = 0.01$ and 0.025 . However, when $\delta \geq 0.5$, Shewhart charts perform better at spotting quadratic drifts. For example, at $\delta = 0.1$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 3.15 and 2.96, respectively, while the ARL_1 values for $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 3.27 and 3.13, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 3.22 and 2.99, respectively. It is worth noting that all of the statistics that are based on the sample median outperform the ones that are based on the sample mean.

- Under gamma distribution (cf. Figures 3.1 (C) and 3.2 (C)), in terms of detecting linear drift, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts exhibit an improvement in drift detection when $\delta \geq 0.5$ compared to the $\bar{X} - EWMA$ chart that performs better when δ is between 0.01 and 0.1 and $\tilde{X} - EWMA$ chart when δ is between 0.01 and 0.5 and for Shewhart $\bar{X}$ and $\tilde{X}$ charts when $\delta \geq 1$. For instance, at $\delta = 0.25$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 3.64 and 4, respectively, while the ARL_1 values for the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 3.77 and 4.13, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 3.90 and 4.44, respectively. In terms of identifying the quadratic drift, $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ chart have a better performance when $\delta \geq 0.1$ compared to $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values only when $\delta = 0.01, 0.025$ and 0.05 . However, when $\delta \geq 1$, Shewhart charts perform better at spotting quadratic drifts. For example, at $\delta = 0.25$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 2.76 and 2.82, respectively, while the ARL_1 values for $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 2.88 and 2.85,

respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$ charts equal 2.76 and 2.83, respectively.

However, at $\delta = 0.25$ and 0.5 , the ARL_1 values for the $\bar{X} - CUSUM$ is reporting similar behavior to Shewhart control charts under specific drifts values.

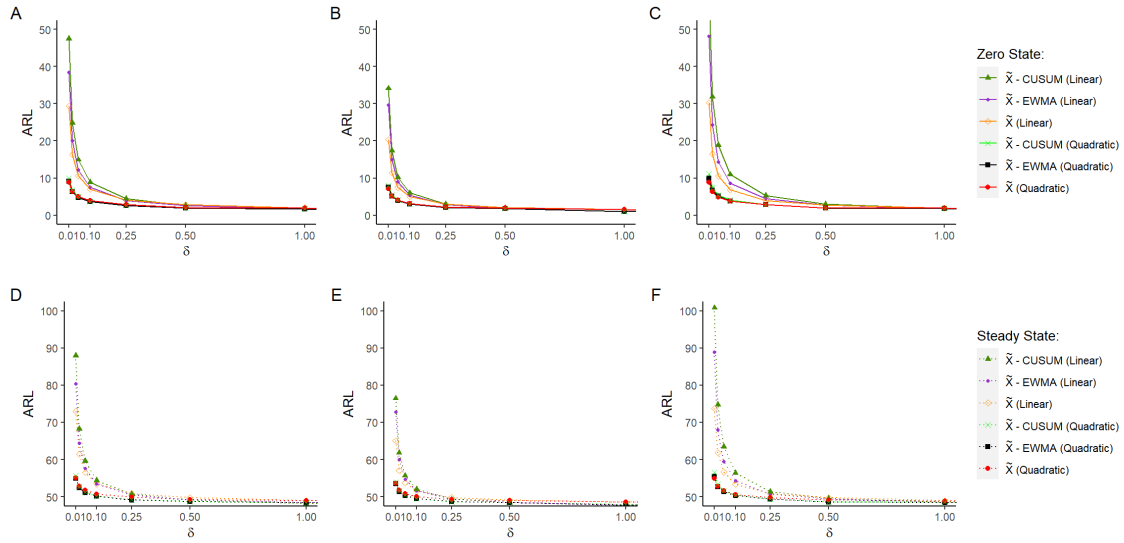


Figure 3.2: ARL Curves of the control charts based on sample median in presence of linear and quadratic drifts. The first plot represents zero-state: (A) Normal distribution, (B) Student's t distribution, and (C) gamma distribution. The last plot represents steady-state: (D) Normal distribution, (E) t distribution, and (F) gamma distribution.

Now that steady-state monitoring is taken into account, the following conclusions are explained:

- Under normal distribution (cf. Figures 3.1 (D) and 3.2 (D)), $\bar{X} - CUSUM$ chart and $\tilde{X} - CUSUM$ provides lower ARL_1 values compared to Shewhart $\bar{X}$ and $\tilde{X}$ charts with the same drift range (i.e., $0.01 \leq \delta \leq 0.5$). Therefore, the superiority goes to the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ chart in detecting the drifts. Meanwhile, $\bar{X} - CUSUM$ chart and $\tilde{X} - CUSUM$ provides lower ARL_1 values compared to $\bar{X} -$

$EWMA$ and $\tilde{X} - EWMA$ charts when $\delta \geq 0.25$. In order to detect the quadratic drift, $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ chart better than to $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts the $\bar{X}$ and $\tilde{X}$ charts. For example, at 0.1σ , the ARL_1 value of the $\bar{X} - CUSUM$ chart and for $\bar{X}$ chart are reported as 49.85 and 49.96. On the other hand, the ARL_1 value of the $\tilde{X} - CUSUM$ chart is reported as 50.08, whereas the $\tilde{X}$ chart is 50.16 at $\delta = 0.01$. For example, at 0.05σ , the values of the $\bar{X} - CUSUM$ chart and $\tilde{X} - CUSUM$ charts are 50.83 and 51.12, respectively, while the ARL_1 values of the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts equal 51.36 and 51.68, respectively. However, Shewhart charts, which are constructed to detect a high magnitude drift, perform better in spotting the drifts when δ becomes higher than $0.5\sigma_0$ both drift types.

- Under the student's t distribution (cf. Figures 3.1 (E) and 3.2 (E)), in terms of detecting linear drift, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts provide lower ARL_1 values when $\delta = 0.25$ compared to the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values when δ is between 0.01 and 0.1. Generally, the superiority goes to the Shewhart $\bar{X}$ chart when $\delta \geq 1$ and for the $\tilde{X}$ chart only when $\delta \geq 0.5$. In terms of identifying the quadratic drift, $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts perform better when $\delta \geq 0.025$ compared $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts that provide lower ARL_1 values only when $\delta = 0.01$. However, when $\delta \geq 0.5$, Shewhart charts perform better at spotting quadratic drifts. For example, at $\delta = 0.1$, the ARL_1 values for the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are reported as 49.65 and 49.42, respectively, while the ARL_1 values for $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 50.01 and 49.99, respectively. The ARL_1 values for the $\bar{X}$ and $\tilde{X}$

charts equal 52.92 and 49.52, respectively. Under the steady-state monitoring scenario, the superiority goes to the $\tilde{X} - CUSUM$ chart since it provides lower ARL_1 values.

- Under gamma distribution (cf. Figures 3.1 (F) and 3.2 (F)), in terms of detecting linear drift, the $CUSUM$ charts show improved drift detection when $\delta \geq 0.25$ compared to the $EWMA$ charts, which perform better when $\delta \leq 0.1$, and the Shewhart charts. The $CUSUM$ charts outperform the other charts when quadratic drifts are present at $\delta > 0.01$. However, superiority goes to the $\bar{X} - CUSUM$ chart in detecting the drifts. For example, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts have ARL_1 values equal to 49.28 and 49.32, respectively, when $\delta = 0.25$. While the ARL_1 values for the $\bar{X} - EWMA$ and $\tilde{X} - EWMA$ charts are 49.77 and 49.72, respectively, and for the $\bar{X}$ and $\tilde{X}$ chart the ARL_1 values equal 49.26 and 49.33, respectively.

The figures clearly indicate that the charts' performance improved in terms of identifying the quadratic drift. Some charts differ in their ability to locate drifts in different ranges of δ , so the choice of which chart to use depends on factors such as the desired detection speed and the type of drift being monitored. Additionally, it is important to consider the distributional settings and whether a zero-state scenario is applicable.

Based on the results from zero- and steady-state, it can be concluded that $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts outperform Shewhart charts when δ is between 0.01 and 0.5 and exceed $EWMA$ charts when $\delta \geq 0.25$. The $\bar{X} - CUSUM$ is also better suited to identifying drifts in normal and gamma distributions. While the $\tilde{X} - CUSUM$ chart is effective at

locating drifts within the student's environment. The slight linear drifts (i.e., $0.01 \leq \delta \leq 0.1$) are detected by EWMA charts before the quadratic one under all distributional settings. The charts using the sample median are faster at detecting quadratic drifts compared to the ones using the sample mean under the student's t distribution. The drifts will be identified quickly under the zero-state scenario (Figure 3.1). The charts detect quadratic drift faster than the linear ones (Figure 3.2). The choice of which chart to use depends on the specific characteristics of the drift being monitored.

4.5 Illustrations

In this section, two real-life examples are used to show the implementation of the proposed charts.

4.5.1 A Case study on Nitrogen Oxides Concentration level

A dataset of air quality in an Italian town was taken from the UCI Machine Learning Repository (Vito, 2016). This data was collected from five metal oxide chemistry sensors, including CO concentration, non-Metonic hydrocarbons, benzene, total nitrogen oxides (NOx), and nitrogen dioxide. The dataset was recorded for a year from March 2004 to April 2005. NOx is the variable of interest for monitoring because it is the most dangerous factor for human and environmental health, which causes respiratory tract damage, increased infection susceptibility, asthma, chronic lung illness, irritation, and lung tissue destruction. In the preliminary computation stage, missing values of NOx levels have been eliminated. The total number of observations was 7718, however we eliminated the last three to consider only 7715. Based on this, we grouped them into groups of five subsamples, yielding a total of 1543 groupings. Using the break-point procedure provided by the

"segmented" R package, there were 607 observations under control, while the rest were not

Figure 3.3 (A and B). Then, for each subgroup of 5, the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ were determined. From the regression analysis, the fitted model of the NOx is defined as $NOx = -2618 + 6.377t - 0.003351t^2$, which indicated that the OOC data drifted quadratically in Figure 3.3 (C and D) . Furthermore, The Shapiro-Wilk test with the PP-plots imply that the variables might not have a normal distribution but instead adhere to a gamma distribution Figure 3.3 (E and F).

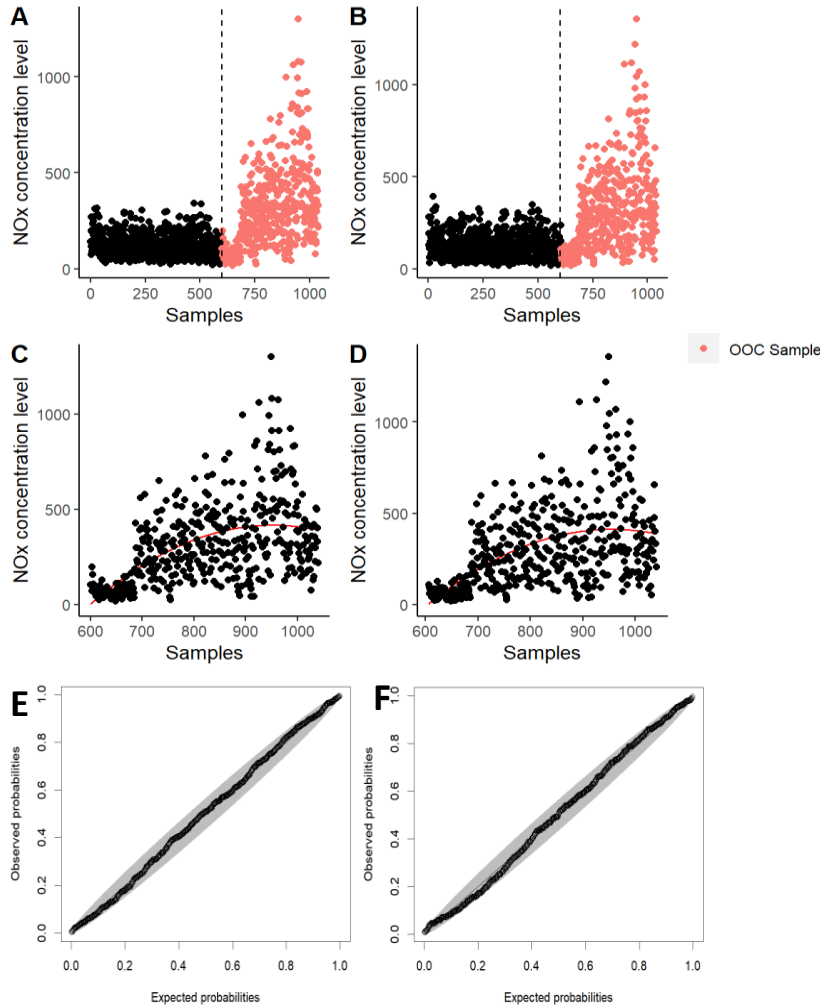


Figure 3.3: Scatter plot for the IC and OOC data of the NOx concentration level; (A) $\bar{X}$ values, (B) $\tilde{X}$ values, (C) for OOC $\bar{X}$ with the quadratic trend, and (D) for OOC $\tilde{X}$ with the quadratic trend, (E)

PP-plot for the IC NO_x concentration level $\bar{X}$ values, and (F) PP-plot for the IC NO_x concentration level $\tilde{X}$ values.

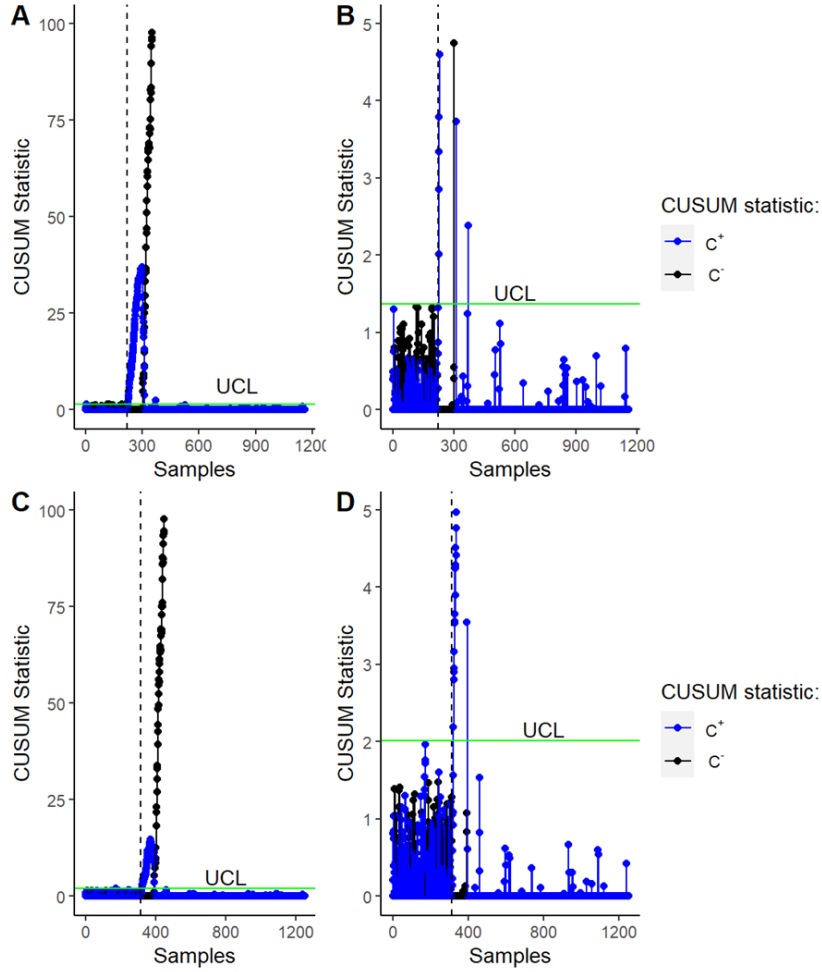


Figure 3.4: (A) the $\bar{X} - CUSUM$ chart, (B) the zoomed $\bar{X} - CUSUM$ chart, (C) the $\tilde{X} - CUSUM$ chart and (D) the zoomed $\tilde{X} - CUSUM$ charts for NO_x level.

The process's baseline performance was determined by analyzing the data. This required computing the data points' mean and standard deviation in order to set control limits. For the $\bar{X} - CUSUM$ chart, the IC mean and the standard deviation are determined to be 128.76 and 63.50, respectively. Moreover, the control limits LCL and UCL of the chart are set to 0 and 1.371, respectively, and the reference value equals $k = 0.5$. In terms of being able to

detect quadratic drift, $\tau = 222$, the $\bar{X} - CUSUM$ chart Figure 3.4(A) was able to detect the drift at sample number 223. Likewise, the IC mean and standard deviation of the $\tilde{X} - CUSUM$ are set to be 126.54 and 71.92, respectively. Also, the LCL and UCL of the chart are set to 0 and 1.997, respectively. In terms of the ability to identify quadratic drift, $\tau = 354$, the $\tilde{X} - CUSUM$ chart Figure 3.4(C) was able to detect the drift at sample number 362. As a result, the $\bar{X} - CUSUM$ detects quadratic drift more effectively compared to $\tilde{X} - CUSUM$, which matches the findings of our study.

study.

4.5.2 A Case Study on Maintenance Data

Another example of the suggested charts is shown using actual predictive maintenance data. ("AI4I 2020 Predictive Maintenance Dataset," 2020). The dataset was collected in 2020, has a total of 10,000 samples, and shows a variety of factors that might influence the operation, including air temperature, rotating speed, and equipment failure. Predictive maintenance keeps an eye on the equipment's performance during ordinary operations and searches for any anomalies that could indicate a probable breakdown. One of the most important elements impacting preventative maintenance is the process temperature. In actuality, it is routinely employed as a crucial indicator of machinery health. Because of this, the variable of importance in this example is process temperature. For the illustration, we have only taken into account the 5200 observations. Data is separated into subgroups of size 5 for the preprocessing stage. Using the break-point procedure provided by the "segmented" R package, we have further divided the data into "in-control" IC data, which consists of the first 375 samples, and "out-of-control" OOC data, which includes the remaining observations Figure 3.5 (A and B). The mean and median are determined for

each subgroup. We used regression to determine if the *OOC* data drifted quadratically or linearly, and the results showed that the data drifted quadratically in Figure 5 (C and D).

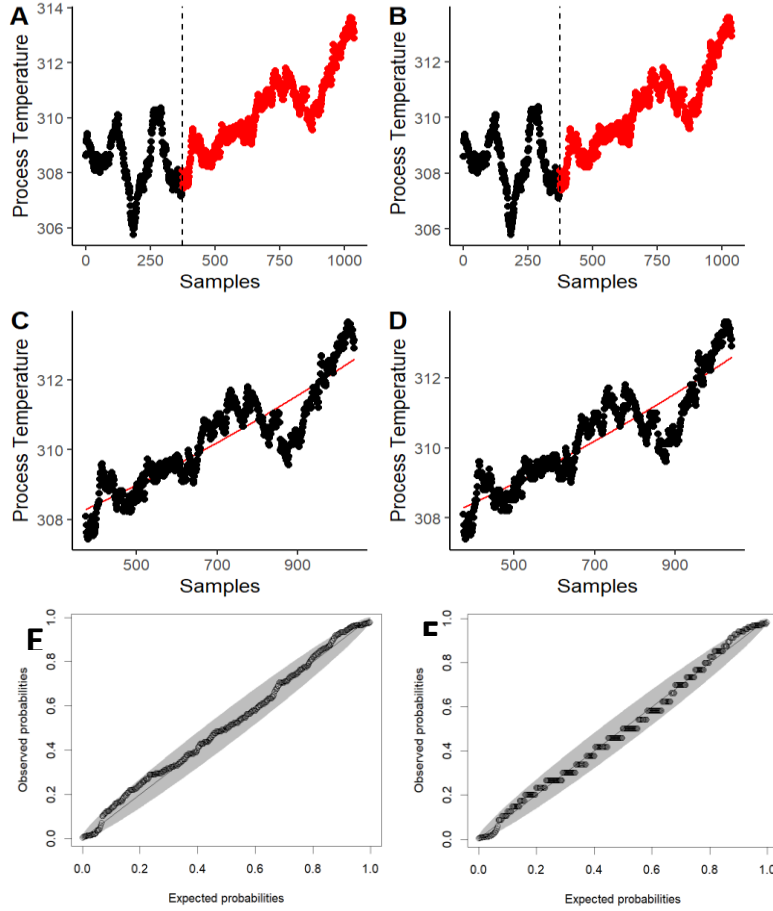


Figure 3.5: Scatter plot for the *IC* and *OOC* data for the temperature of maintenance dataset; (A) $\bar{X}$ values, (B) $\tilde{X}$ values, (C) for OOC $\bar{X}$ with the quadratic trend, and (D) for OOC $\tilde{X}$ with the quadratic trend, (E) PP-plot for the IC process temperature $\bar{X}$ values, and (F) PP-plot for the IC process temperature $\tilde{X}$ values.

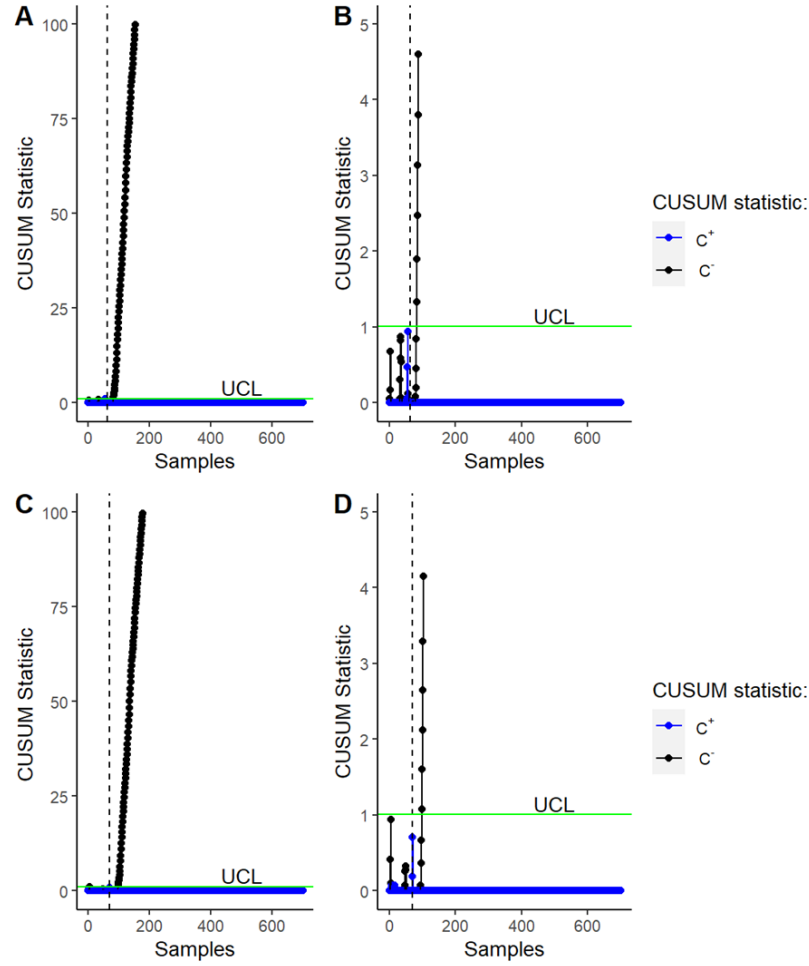


Figure 3.6: (A) the $\bar{X} - CUSUM$ chart, (B) Zoomed the $\bar{X} - CUSUM$ chart, (C) the $\tilde{X} - CUSUM$ charts and (D) Zoomed the $\tilde{X} - CUSUM$ charts of the temperature.

Furthermore, The Shapiro-Wilk test with the pp-plots implies that the variables have a normal distribution Figure 3.5 (E and F). The fitted regression model was Temperature = $350.70 + 0.006457 t + 0.00000177 t^2$. By examining the data, the baseline performance of the process was established. To set control limits, this required calculating the mean and standard deviation of the data points. The IC mean and the standard deviation for $\bar{X} - CUSUM$ chart are found to be $\bar{X} = 297.67$ and $\sigma_{\bar{X}} = 0.879$. Aside from that, the chart's control limits LCL and UCL are set at 0 and 1.001, respectively, and the reference

value is set at $k = 0.5$. Regarding the ability to identify the drift, $\tau = 56$, the $\bar{X} - CUSUM$ chart Figure 3.6(A) was able to detect the drift at sample number 83. Similarly, the $\tilde{X} - CUSUM$ IC mean, and standard deviation are set to 297.67 and 0.878, respectively. The chart's LCL and UCL are further set to 0 and 1.0017, respectively. Since $\tau = 70$, the $\tilde{X} - CUSUM$ chart Figure 3.6(C) was able to detect the drift at sample number 98. Consequently, the $\bar{X} - CUSUM$ chart identifies quadratic drift faster than $\tilde{X} - CUSUM$, which is consistent with the results of our investigation.

4.6 Summary and Recommendations

In this chapter, we aim to study the CUSUM chart's detection behavior in the presence of linear drift and quadratic drift. The performance of the $\bar{X} - CUSUM$ chart and $\tilde{X} - CUSUM$ chart as location charts is being assessed, taking into consideration that $\bar{X}$ and $\tilde{X}$ are the plotting statistics. The performance of charts is evaluated into various distributions and subgrouping sizes under two monitoring states (i.e., zero and steady-states). The study reveals that the size of the subgroup can significantly influence the control chart's performance. For instance, a smaller subgroup size (i.e., $n = 5$) may make the control chart less sensitive to small process drifts, suggesting the need to balance accuracy and sensitivity when selecting the appropriate subgroup size for monitoring a process. The CUSUM charts perform better when zero-state monitoring is considered. They are more effective when quadratic drift is present. Depending on the underlying distribution, the $\bar{X} - CUSUM$ chart is appropriate for normal and gamma distributions. It has a high detection ability for detecting linear and quadratic drift under biased and unbiased ARLs. Meanwhile, $\tilde{X} - CUSUM$ excels under the student's t distribution and is faster at spotting quadratic

drifts. A comparable analysis of detection performance has been conducted between the CUSUM charts, EWMA charts, and Shewhart charts. The comparative analysis (section 5.1) revealed that the Shewhart charts are more effective in detecting larger drifts in the process as they are designed for. However, based on the findings from zero- and steady-state, it can be concluded that the $\bar{X} - CUSUM$ and $\tilde{X} - CUSUM$ charts are superior to the Shewhart charts for δ between 0.01 and 0.5 and the EWMA charts for $\delta \geq 0.25$. The $\bar{X} - CUSUM$ is better suited for identifying drifts in normal and gamma distributions, while the $\tilde{X} - CUSUM$ chart is superior in the student's t distribution. The sample median charts are faster at detecting quadratic drifts than the sample mean under the student's t distribution. Moreover, EWMA charts have a higher sensitivity to detecting quite minor (i.e., between 0.01 and 0.1) in the process mean, making them more effective for early detection. EWMA charts detect slight linear drifts before the quadratic one under all distributional settings. Therefore, depending on the desired level of sensitivity and the magnitude of potential drifts, either chart can be chosen for monitoring process drifts effectively. The proposed charts are deployed in real-life scenarios related to air quality and maintenance data. Additionally, the CUSUM performance is comparable to the EWMA and Shewhart control charts in terms of the number of signals produced. This study assessed drifts' effects on the CUSUM chart with respect to location parameters but could extend to other memory-type dispersion charts. The discussion in this paper only covers linear and quadratic drift, but it could be expanded to cover other types of drift.

CHAPTER 5

Conclusion and Future Recommendations

In this chapter, the findings of this thesis, along with future recommendations, are reported.

5.1 Summary and Discussion

In this thesis, we studied the detection behavior of Shewhart, EWMA, and CUSUM control charts in the presence of linear and quadratic drifts. Under the two monitoring states (i.e., zero- and steady-states), the performance of the control chart using the sample mean and median as location statistics is evaluated into different distributions and subgrouping sizes.

First of all, the results show that the performance of the control chart may be considerably influenced by the size of the subgroup. To illustrate the need to balance accuracy and sensitivity when choosing the right subgroup size for monitoring a process, consider how a lower subgroup size (i.e., $n=5$) may make the control chart less sensitive to tiny process deviations.

For Shewhart control charts, in terms of distributional settings for two states of monitoring, the $\bar{X}$ chart's performance evaluation yielded the lowest values of ARL_1 , making it the best at spotting changes in the normal and gamma distributions. The performance assessment of the $\tilde{X}$ chart, on the other hand, revealed that it had greater

detection power for the t distribution. Additionally, in terms of the drift type, the $\bar{X}$ charts have a stronger detection power for shifts, linear drifts, and quadratic drifts than the $\tilde{X}$ chart does. The shift scenario under the gamma distribution showed a particular situation where the ARL_1 values exceeded ARL_0 in both $\bar{X}$ chart and $\tilde{X}$ chart (i.e., $0.01 \leq \delta \leq 0.1$). However, with linear and quadratic drifts, such ARL bias does not report. Similar results are also shown for the distribution type, subgroup size, and shifts or drifts type under zero-state monitoring.

Moreover, it is also important to note that the $\bar{X} - EWMA$ chart is superior in spotting shifts and drifts under normal and gamma distributions in both monitoring states, as well as quadratic drift under the t distribution in zero-state monitoring. However, in the steady-state monitoring case, the $\tilde{X} - EWMA$ chart is better at identifying shifts and linear quadratic drifts under the t distribution.

Furthermore, depending on the underlying distribution, the $\bar{X} - CUSUM$ chart is appropriate for normal and gamma distributions. It has a high detection ability for detecting linear and quadratic drift under biased and unbiased ARLs. Meanwhile, $\tilde{X} - CUSUM$ excels under the t distribution and is faster at spotting quadratic drifts. The CUSUM charts have good performance in detecting linear drifts in zero-state monitoring and even better performance when quadratic drift is present. The suggested charts are used in real-life scenarios involving air quality and maintenance data. In terms of the quantity of signals produced, the CUSUM performance is being compared to that of the EWMA and Shewhart control charts.

The CUSUM charts, EWMA charts, and Shewhart charts have all been subjected to comparable analyses of detection performance. The comparison study showed that the Shewhart charts are superior at spotting greater drifts in the process, which is what they were intended for. However, for δ between 0.01 and 0.5, it can be concluded that the CUSUM charts are preferable to Shewhart charts. Also, for $\delta \geq 0.25$. The CUSUM charts are superior to the EWMA charts. The $\bar{X} - CUSUM$ is better suited for identifying drifts in normal and gamma distributions, while the $\tilde{X} - CUSUM$ chart is superior in the t distribution. The charts based on sample median are faster at detecting quadratic drifts than the charts based on sample mean under the t distribution. Moreover, EWMA charts have a higher sensitivity to detecting quite minor (i.e., between 0.01 and 0.1) in the process mean, making them more effective for early detection. EWMA charts detect slight linear drifts before the quadratic one under all distributional settings. Therefore, depending on the desired level of sensitivity and the magnitude of potential drifts, either chart can be chosen for monitoring process drifts effectively.

5.2 Limitations

The limitations of our study are given in the following points:

- i. This study evaluated the impact of drifts on control location-based charts.
- ii. The objective of this study is to investigate the normal, t, and gamma distributional frameworks for quality characteristics.
- iii. In this thesis, only linear and quadratic drift are discussed.

5.3 Future Recommendations

The following points provide future recommendations on our proposed charts:

- i. This study's scope may include additional control dispersion-based charts. Since the location version was the study's main focus.
- ii. The scope of this research, which examines the normal, t, and gamma distributional frameworks, may be expanded to include other distributions.
- iii. Additional forms of drift might be included in the study. |

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Vitae

Name :Huda Hakem Alshammari]

Address :Olaya, Al Khobar, 34448, Saudi Arabia]

Email :St.Hudaalshammari@gmail.com]

Education

- July 12, 2010

Bachelor's degree in mathematics from University of Dammam.

- November 2023

Master's degree from in applied statistics from King Fahad university of petroleum and minerals.

Presentations

- December 20,2021 KFUPM

Seminar - Comparative Analysis on Location Charts. In King Fahad
University of Petroleum and Minerals (KFUPM).

- December 19,2023 IEEM2023

Oral Presentation- On Surveillance Methods for Drifted Processes. In IEEE
International Conference on Industrial Engineering and Engineering
Management (IEEM).