

EXPLORING FERTILITY DYNAMICS

IN THE MENA REGION:

DISTRIBUTION, DETERMINANTS,

AND TEMPORAL TRENDS

BY

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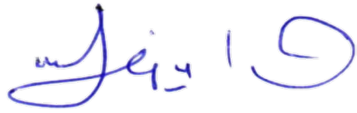
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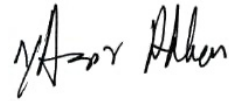
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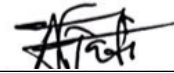
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Dedication

I dedicate this thesis to my unwavering pillars of strength, my parents, whose boundless love, encouragement, and sacrifices have fueled every step of my educational voyage. Your unwavering support has been my anchor, and this achievement is as much yours as it is mine.

To my person and dearest friend, Haifaa Al-Akel, your friendship has been a guiding light in the darkest of times. Your belief in me, your relentless encouragement, and the moments of shared laughter made this journey not just bearable, but enriching. Thank you for being my safe place and savior.

In profound memory and solidarity with my hometown, Gaza, a place marked by resilience and courage, I dedicate this work to the souls lost and those enduring the hardships of the ongoing war.

May this humble contribution be a testament to the strength derived from familial bonds, enduring friendships, and the shared aspiration for a better, more harmonious world.

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LIST OF ABBREVIATIONS

MENA	:	Middle East and North Africa
TFR	:	Total Fertility Rate
GEV	:	Generalized Extreme Value
ARIMA	:	Autoregressive Integrated Moving Average
P-value	:	Probability Value
ACF	:	Autocorrelation Function
PACF	:	Partial Autocorrelation Function
VIF	:	Variance Inflation Factor
KS Test	:	Kolmogorov–Smirnov test
AIC	:	Akaike Information Criterion
BIC	:	Bayesian Information Criterion
ME	:	Mean Error
RMSE	:	Root Mean Squared Error
MAE	:	Mean Absolute Error
MPE	:	Mean Percentage Error
MAPE	:	Mean Absolute Percentage Error
MASE	:	Mean Absolute Scaled Error

ABSTRACT

Full Name : Dena Alhaloul
Thesis Title : Exploring Fertility Dynamics in The MENA Region:
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This thesis delves into fertility dynamics in the Middle East and North Africa (MENA), exploring distribution through spatial analysis and utilizing statistical tests. Regression and ARIMA time series models investigate temporal trends, while determinants like urbanization and female empowerment are studied. The study identifies the Generalized Extreme Value (GEV) distribution as applicable to Total Fertility Rates (TFR) across years. Spatial analysis reveals geographical variations, prompting exploration of factors influencing reproductive behaviors. Despite challenges, statistical models highlight the role of urbanization and female empowerment in shaping demographic patterns. Advocating an interdisciplinary approach, the study offers policymakers insights through spatial analysis, statistical modeling, and determinant exploration. Contributions to demography discourse and actionable insights for evidence-based policies address MENA's unique challenges. The study lays the groundwork for future research, emphasizing sub-regional variations and global trends' impact on reproductive behaviors. Ongoing scientific exploration aligns with a commitment to understanding fertility dynamics for sustainable development.

ملخص الرسالة

الاسم الكامل: دينا الهلول

عنوان الرسالة: استكشاف ديناميات الخصوبة في منطقة الشرق الأوسط وشمال أفريقيا: التوزيع، والعوامل المؤثرة، والاتجاهات الزمنية

التخصص: الإحصاء التطبيقي

تاريخ الدرجة العلمية: يناير ٢٠٢٤

تتناول هذه الرسالة ديناميات الخصوبة في الشرق الأوسط وشمال أفريقيا، حيث تستكشف التوزيع من خلال تحليل المكان واستخدام الاختبارات الإحصائية. تقوم النماذج الزمنية للاسترجاع و ARIMA بدراسة الاتجاهات الزمنية، في حين يتم دراسة عوامل مثل التحضر وتمكين المرأة. تحدد الدراسة توزيع القيم المطلقة الكبرى (GEV) كقابل لتطبيقه على معدلات الخصوبة الكلية (TFR) عبر السنوات. يكشف التحليل المكاني عن اختلافات جغرافية، مما يحفز على استكشاف العوامل التي تؤثر في السلوكيات التكاثرية. على الرغم من التحديات، تبرز النماذج الإحصائية الدور الذي يلعبه التحضر وتمكين المرأة في تشكيل الأنماط الديموغرافية. بدعم منهج متعدد التخصصات، تقدم الدراسة رؤى لصناع السياسات من خلال التحليل المكاني والنمذجة الإحصائية واستكشاف العوامل المحددة. تقديم مساهمات لحوار الديموغرافيا وتقديم رؤى قابلة للتنفيذ لسياسات مستندة إلى الأدلة تتعامل مع تحديات وفرص الشرق الأوسط وشمال أفريقيا الفريدة. تضع الدراسة الأساس للبحث المستقبلي، مع التأكيد على التغيرات الجغرافية الفرعية وتأثيرات الاتجاهات العالمية على السلوكيات التكاثرية. تتسق الاستكشافات العلمية المستمرة مع التفاني في فهم ديناميات الخصوبة لتحقيق التنمية المستدامة.

CHAPTER 1

INTRODUCTION

This chapter lays the foundation for the exploration into fertility dynamics in the Middle East and North Africa (MENA) region. Beginning with an overview of the region's geographical and cultural significance, it navigates through the historical tapestry, acknowledging the diverse landscapes and the interplay of ancient civilizations with contemporary challenges. The heart of the study, fertility dynamics, is introduced, emphasizing its multifaceted nature, the concept of demographic transition, and the tension between tradition and modernity in the MENA context. The chapter then unveils three pivotal research questions that guide the inquiry into the distribution, temporal trends, and determinants of fertility. It delineates the geographical and thematic scope of the study, discusses inherent limitations, and concludes by highlighting the study's empirical, methodological, and practical contributions.

1.1 Background and Context:

1.1.1 The MENA Region

The Middle East and North Africa (MENA) region, a geographic and cultural crossroads between Asia, Europe, and Africa, has long captured the world's attention. Encompassing a diverse array of nations, each with its unique history, traditions, and socio-political landscape, the MENA region stands at the intersection of ancient civilizations and modern complexities.

Spanning from the Atlantic Ocean to the Arabian Sea, the MENA region is characterized by a rich tapestry of landscapes, ranging from the vast deserts of the Arabian Peninsula to the fertile valleys along the Nile River. This geographic diversity contributes to the region's economic activities, cultural practices, and challenges.

With a history dating back millennia, the MENA region has been a cradle of civilization, witnessing the rise and fall of empires, the birth of major religions, and the development of advanced sciences and arts. The remnants of ancient civilizations, such as Mesopotamia and Egypt, coexist with the modern metropolises that have emerged in recent decades.

Culturally, the MENA region is a mosaic of languages, religions, and traditions. The Islamic civilization, in particular, has left an indelible mark on the region, influencing art, architecture, philosophy, and governance. Beyond Islam, there exists a rich tapestry of Christian, Jewish, and other religious communities, each contributing to the region's cultural diversity.

The MENA region has been a focal point of global geopolitics due to its strategic location and significant natural resources, particularly oil. The complex interplay of regional powers, historical conflicts, and socio-political movements has shaped the contemporary landscape, influencing governance structures, diplomatic relations, and societal norms.

While the MENA region boasts tremendous cultural and historical wealth, it also faces a myriad of challenges, including political instability, economic disparities, and social tensions. At the same time, there are opportunities for growth, innovation, and positive transformation as nations in the region navigate the complexities of the modern world.

This introduction merely scratches the surface of the multifaceted MENA region, setting the stage for a deeper exploration into its intricate tapestry of cultures, histories, and

contemporary dynamics. As we delve into the exploration of fertility dynamics within this diverse landscape, we embark on a journey to better understand the forces shaping the demographic landscape of the Middle East and North Africa.

1.1.2 Fertility Dynamics

Fertility dynamics, the study of patterns and changes in human reproduction within a population, constitutes a vital dimension of demographic research. It delves into the intricate interplay of biological, social, economic, and cultural factors that influence the decisions individuals and couples make regarding family size, timing of childbirth, and overall reproductive behavior.

Fertility is a fundamental component of population dynamics, playing a pivotal role in shaping the size, structure, and composition of societies. The choices individuals make regarding when and how many children to have have far-reaching consequences for communities, nations, and the global demographic landscape.

At its core, fertility dynamics reflect both biological and behavioral aspects of human reproduction. Biological factors include age, health, and physiological changes affecting fertility rates. Behavioral factors encompass cultural norms, societal expectations, educational attainment, and the evolving roles of women in the workforce.

The study of fertility dynamics often involves analyzing temporal trends, exploring how fertility rates change over time. Many societies undergo a demographic transition, characterized by a shift from high to low fertility and mortality rates. Understanding these transitions is essential for predicting population growth, aging trends, and the associated social and economic implications.

Fertility dynamics exhibit significant variations across regions and countries. While some regions experience sustained high fertility rates, others have entered a stage of demographic transition marked by declining birth rates. These regional differences are influenced by cultural practices, economic development, healthcare infrastructure, and government policies.

Fertility dynamics profoundly impact social and economic structures. High fertility rates may strain resources and social services, affecting issues such as education, healthcare, and employment opportunities. Conversely, low fertility rates can contribute to aging populations and pose challenges to workforce sustainability and economic growth.

Governments and policymakers closely monitor fertility dynamics as they formulate strategies to address population-related challenges. Family planning programs, healthcare initiatives, and policies supporting work-life balance are often shaped by insights derived from fertility research.

As we embark on an exploration of fertility dynamics in the MENA region, it becomes crucial to unravel the unique factors influencing reproductive behaviors in this context. The intersection of tradition and modernity, socio-economic dynamics, and cultural influences contributes to a distinctive fertility landscape worthy of careful examination and analysis. Through this study, we aim to contribute to the broader understanding of how fertility dynamics shape the demographic future of the Middle East and North Africa.

1.1.3 Fertility Dynamics of MENA Region

The Middle East and North Africa (MENA) region, characterized by its rich history, diverse cultures, and intricate geopolitical landscape, is undergoing a fascinating demographic transformation. Fertility dynamics, a critical aspect of this transformation, encapsulate the

complex interplay of cultural traditions, socio-economic changes, and evolving gender roles that collectively shape the reproductive behaviors of individuals and couples across the region.

Fertility in the MENA region is deeply embedded in cultural and societal norms that have been passed down through generations. Family and kinship ties hold profound significance, influencing decisions related to marriage, childbearing, and the overall structure of the family unit. Understanding the cultural nuances surrounding fertility is paramount to comprehending the unique dynamics at play.

The MENA region, characterized by a juxtaposition of tradition and modernity, experiences the dynamic tension between age-old customs and the pressures of a rapidly changing world. The impact of these influences on fertility choices is profound, as individuals navigate between societal expectations and emerging opportunities in education, career, and personal fulfillment.

Rapid economic development in some parts of the MENA region has brought about shifts in lifestyle and employment patterns. As urbanization and globalization reshape societal structures, individuals are confronted with new challenges and opportunities that influence decisions regarding family size, timing of childbirth, and overall reproductive preferences. Education, particularly for women, has emerged as a powerful catalyst in shaping fertility dynamics. Increasing educational attainment among women in the region has been associated with delayed marriages and childbearing, as well as a reevaluation of traditional gender roles. These shifts contribute to the ongoing transformation of family structures.

Access to healthcare, including reproductive health services and family planning initiatives, plays a pivotal role in fertility dynamics. Government policies and healthcare infrastructure

influence the availability and utilization of family planning resources, impacting the choices individuals make in managing their reproductive lives.

The MENA region is not homogeneous in its fertility patterns. Variations exist across countries and communities, reflecting diverse social, economic, and cultural contexts. Exploring these differences is essential for a nuanced understanding of the factors influencing fertility dynamics in specific sub-regions.

As we embark on an exploration of fertility dynamics in the MENA region, this study seeks to unravel the intricate tapestry of influences that shape reproductive behaviors. By shedding light on the complexities inherent in the intersection of tradition and modernity, we aim to contribute to a deeper understanding of the demographic landscape and its implications for the future of the Middle East and North Africa.

1.2 Research Questions

Research Question 1: What is the distribution of fertility rates in the MENA region?

1.1. What is the distribution of fertility rate of MENA for each year from 1960 to 2021?

1.2. How does fertility vary among different countries within the MENA region?

Research Question 2: What is the temporal trend of fertility rates in MENA, and can it be characterized by a statistical model?

2.1. Can we model the temporal trend using a statistical model, such as a time series analysis or a demographic model?

2.2. How well does the chosen statistical model fit the observed fertility data, and what are the implications for future fertility trends in MENA?

Research Question 3: What are the most significant determinants of fertility in MENA?

3.1. How does education level influence fertility rates in the MENA region?

- 3.2. What is the impact of women's workforce participation on fertility decisions?
- 3.3. To what extent do economic factors, such as income and employment opportunities, influence fertility rates?
- 3.4 How can we regress the determinants to the fertility of MENA in the face of potential violation to some or all of the assumptions of linear regression?
- 3.5 Does the model of fertility determinants in MENA differ from the rest of the world?

1.3 Scope of the Study

This research endeavors to provide a comprehensive analysis of fertility dynamics within the Middle East and North Africa (MENA) region, as delineated by the countries listed by the World Bank. The scope of the study encompasses an exploration of the factors influencing reproductive behaviors, including cultural norms, socio-economic conditions, educational attainment, and healthcare access. By focusing on the countries identified within the MENA region, the study aims to capture the unique and diverse demographic landscape of this geographically and culturally rich area.

The research will employ a multifaceted approach to investigate temporal trends in fertility, examining both historical patterns and contemporary shifts. Additionally, the study will consider the influence of urbanization, economic development, and changing gender roles on fertility dynamics.

1.4 Limitations of the Study

While every effort will be made to conduct a thorough and insightful analysis, it is essential to acknowledge certain limitations inherent in this study:

- **Data Availability and Reliability:** The study relies on the availability and reliability of demographic data from various sources. Discrepancies or gaps in the data may limit the precision of the analysis.
- **Cultural Heterogeneity:** The MENA region is characterized by a diverse array of cultures and subcultures. While the study aims to capture general trends, variations within individual countries and communities may not be fully represented.
- **Governmental Policies:** The impact of governmental policies on fertility dynamics is complex and multifaceted. Changes in policy effectiveness over time and differences in implementation across countries may pose challenges to a comprehensive assessment.
- **Social and Economic Factors:** Socio-economic conditions, though integral to fertility dynamics, are influenced by a multitude of factors beyond the scope of this study. While these factors will be considered, the analysis may not encompass all relevant variables.
- **Regional Instabilities:** The MENA region has experienced political and social instabilities that may affect demographic trends. The study will strive to account for these factors, but their unpredictable nature introduces a level of uncertainty.

By recognizing these limitations, the study aims to maintain transparency and ensure a nuanced interpretation of the findings. Despite these constraints, the research seeks to contribute valuable insights into the intricate dynamics of fertility within the MENA region.

1.5 Significance and contribution of the study

This research holds significant importance for several reasons, contributing to the understanding of fertility dynamics in the Middle East and North Africa (MENA) region and beyond:

- **Empirical understanding:** The distribution analysis provides empirical insights into the variations of fertility rates over time and among different countries in the MENA region. This contributes to the empirical foundation of demographic studies specific to the region.
- **Modeling for precision:** The application of statistical models not only aids in characterizing temporal trends but also enhances precision in predicting future fertility rates. The study's contribution lies in refining existing models or proposing new ones that suit the distinctive features of fertility dynamics in the MENA region.
- **Identification of determinants:** The study identifies and analyzes key determinants of fertility, offering a nuanced understanding of the factors influencing reproductive decisions. This information is valuable for policymakers, researchers, and practitioners seeking targeted interventions to address demographic challenges.
- **Assumption evaluation in regression modeling:** By addressing potential violations of linear regression assumptions in the regression analysis, the study contributes to methodological rigor. This consideration enhances the reliability and validity of the findings, providing a methodological contribution to the broader field of demographic research.
- **Comparative global insight:** The comparative analysis of fertility determinants in MENA against global models contributes to a broader understanding of whether the

factors influencing fertility in the region differ significantly from those observed worldwide. This comparative insight adds a valuable dimension to the global discourse on population dynamics.

In summary, this study not only aims to answer specific research questions related to fertility dynamics in the MENA region but also provides a foundation for informed policymaking, contributes methodologically to demographic research, and offers valuable insights into the intricate interplay of cultural, social, and economic factors shaping reproductive behaviors in the MENA context.

CHAPTER 2

LITERATURE REVIEW

This literature review navigates the multifaceted terrain of fertility dynamics in the MENA region, shedding light on statistical distributions, methodological applications, estimation models, and the myriad factors influencing fertility patterns.

2.1 Statistical Distribution of Fertility

Understanding the distribution of fertility is fundamental to unraveling the intricacies of population dynamics. This section delves into the statistical underpinnings of fertility patterns, examining the tools and methodologies employed to characterize the variability and central tendencies of fertility rates across the MENA region. Findings in the literature collectively contribute to understanding the statistical distribution of fertility rates by exploring regional variations, employing analytical methods, and providing recommendations for sample sizes to enhance the reliability of fertility estimates.

Pezzulo, et al. (2021) offered an extensive descriptive examination of the geographic variations in fertility rates within countries. The study identifies clusters of both high and low fertility that transcend national borders, shedding light on characteristic patterns and behaviors within each cluster. The study found that *TFR* exhibited a range from below replacement (2.1 children per woman) in 36 out of the 932 subnational regions, predominantly situated in India, Myanmar, Colombia, and Armenia, to rates of 8 and above in 14 subnational regions located in sub-Saharan Africa and Afghanistan. Regions

characterized by high-fertility clusters were generally linked to low prevalence of women with secondary or higher education, limited contraceptive usage, and elevated unmet needs for family planning, with some exceptions.

Argote (2007) employed the bootstrap method to generate the statistical distribution of the global fertility rate (GFR). For this study, data from the demography and health national survey of Bolivia in 1998 were utilized, as it represented the most recent fertility survey available until February 2005 and featured a stratified design. The Kolmogorov-Smirnov test applied to the statistical distribution of the global fertility rate did not provide sufficient evidence to reject the normality of the resampling distribution. While the global fertility rate serves as a biased estimator, its standardized bias is lower than the coefficient of variation, indicating consistency. The confidence intervals exhibit consistency and display a convergent trend. Despite the high probability that the GFR confidence interval, assuming normality, encompasses the parameter value, the bootstrap method facilitates more precise estimations. This method proves valuable for assessing sampling error and the bias inherent in estimations.

Mishra, Singh, Singh, & Singh (2018) published a study that aims to determine an appropriate sample size for obtaining reliable fertility estimates using open-birth intervals in India. The analysis utilizes data from the National Family Health Survey (NFHS-3) for the states of Uttar Pradesh, Maharashtra, and Kerala. Treating the NFHS-3 data for these states as the population and the total fertility rate (TFR) values as parameters, samples of varying sizes are systematically selected through a simple random sampling procedure. The analysis of *TFR* distribution from these samples reveals a normal distribution pattern. A sample size of 700 is anticipated to yield a reliable *TFR* estimate when information on the

date of the last birth is considered for all three states. For Maharashtra, a sample size of 1000 or more is recommended, specifically if information on births in the last year alone is considered, while for Uttar Pradesh and Kerala, sample sizes of 1200 and 1000, respectively, are suggested for *TFR* estimation.

Pasupuleti and Chattopadhyay (2013) introduced a straightforward analytical framework, employing a series of analytical models, to ascertain the probability distributions of the number of children and maternal age at different-order births. The framework utilizes data on age-specific fertility rates categorized by birth order. Application of this framework includes the examination of cohort birth order-specific fertility schedules in India and its diverse regions, as well as the period birth order-specific fertility schedules of various countries such as the USA, Russia, and the Netherlands. Noteworthy findings from this analysis include: (1) the prevalence of primary infertility and higher maternal age at various-order births in North-east India compared to other regions in the country, and (2) a projection that over 30% of women in Hungary and Austria are anticipated to remain childless in the near future based on the persistent age-specific fertility rates by birth order in these countries.

Following the confirmation that no literature delves into the statistical distribution of the MENA region and that there exists no overarching Total Fertility Rate (TFR) distribution suitable for application in the MENA context, an examination of various prevalent statistical distributions was undertaken. Among these, the General Extreme Values (GEV) distribution emerged as a significant contender, known for its widespread applications in statistics albeit with limited use. This prompted a more in-depth exploration of the GEV distribution through additional research.

2.2 Applications of Extreme Value Theory in Demography

Extreme Value Theory (EVT) emerges as a powerful lens through which demographers scrutinize the tails of fertility distributions, capturing rare and extreme events. This section explores the applications of EVT in the realm of demography, unveiling its potential to explain outliers and understand the tail behavior of fertility rates. Additionally, the application of the Generalized Extreme Value (GEV) distribution, a cornerstone of EVT, is dissected to comprehend its role in modeling extreme fertility events within the MENA context.

According to Medford and Vaupel (2019), in the field of demography, Extreme Value Theory (EVT) methods have traditionally seen limited use. This may appear somewhat unexpected because population studies often involve situations where the focus is on extreme events and infrequent values rather than averages or high-frequency occurrences. These situations include instances such as the lowest and highest birth rates, the most significant expected migration levels, and maximum lifespans. However, the limited adoption of EVT tools is not entirely surprising, as these models demand a more meticulous and effort-intensive implementation. Given their reliance on asymptotic principles, it is crucial to exercise caution when regarding them as precise results for finite data samples. Furthermore, in practical applications, these models can be seen as inefficient in their utilization of data, leading to criticism on these grounds.

Up to the present time, the primary applications of EVT within the field of demography have primarily revolved around the exploration of maximum lifespans and lifespan limits. This research has encompassed work by scholars like Gumbel (1937, 1958), Aarssen and Haan (1994), Watts, et al. (2006), Hanayama and Sibuya (2015), Gbari, et al. (2017) and

Rootzén and Zholud (2017). Recently, notable applications have emerged, such as Medford (2017) modeling and projection of the Best Practice Life Expectancy (BPLE) line introduced by Oeppen and Vaupel (2002) using extreme value distributions. Basellini, Lenart, Canudas-Romo, and Lenart (2018) also found that these distributions were the best-fitting models when applied to hazard rates at adult ages. Additionally, Canudas-Romo, Mazzuco, and Zanotto (2018) employed EVT models to fit age-specific human mortality data.

From the literature, it can be observed that there are very few sources that discussed the statistical distribution of fertility rates of the MENA region in particular. Hence, there is an opportunity for contribution. One of the objectives of this thesis is to identify the most suitable distribution of fertility in the MENA region. This exploration involves considering various common statistical distributions, including the GEV distribution.

Following the identification of the statistical distribution of fertility in the MENA region, it is imperative to establish an estimation model for MENA fertility. This model will facilitate the estimation of the Total Fertility Rate (TFR) for future periods.

2.3 Estimation Models of Fertility

Navigating the intricacies of fertility estimation models is imperative to grasp the mechanisms that drive population growth or decline. This section critically examines diverse estimation models employed in fertility research, shedding light on their strengths, limitations, and applications. From classical models to innovative approaches, the goal is to discern the efficacy of these models in capturing the nuances of fertility trends over time. Several papers in the literature have delved into the discussion and proposition of models for estimating fertility and analyzing the distribution of fertility. In this context, Gaire and

Aryaf (2015) presented an Inverse Gaussian probability model aimed at capturing the distribution pattern of age-specific fertility rates among Nepalese mothers. The research reveals that the average age for childbearing is 26.12 years, with the initiation of childbearing observed at 15.82 years. The estimated total fertility rate (*TFR*) is computed at 2.60, closely aligning with the observed value of 2.6 per woman. The coefficient of determination and chi-square values indicate that the proposed model provides a robust fit to the data from Nepal Demographic and Health Surveys, highlighting its suitability for describing the fertility dynamics in the region.

Islam, et al. (2014) endeavored to enhance Bongaarts' model by incorporating all fetal wastage, aiming to estimate a more accurate *TFR*. The research investigated the impact of proximate variables on fertility level changes in Bangladesh from 1975 to 2007. Data for the study were derived from Bangladesh Contraceptive Prevalence Surveys, Bangladesh Fertility Surveys, and Bangladesh Demographic and Health Surveys. Proximate determinants play a crucial role in shaping fertility levels within a society, and Bongaarts' model is a widely recognized approach for examining these determinants concerning the total fertility rate (*TFR*). However, Bongaarts' original model considers only induced abortion as a proximate variable for estimating fertility determinants, overlooking other significant factors like miscarriage and stillbirth. The revised model, which incorporates all fetal wastage, yielded a *TFR* estimate more closely aligned with the observed *TFR* ($R^2 = 0.98$) compared to Bongaarts' original model ($R^2 = 0.84$). Furthermore, the contribution of abortion alone (Bongaarts' model) to *TFR* change was approximately 1 birth, while considering all fetal wastage (revised Bongaarts' model) resulted in an approximate change of 1.40 births per woman. The study findings indicate a declining trend in all proximate

indices over time, suggesting that the revised Bongaarts' model provides a more accurate estimation of *TFR* compared to the original Bongaarts' model.

Alkema, et al. (2011) presented a Bayesian projection model designed to generate country-specific forecasts of the total fertility rate (*TFR*) for all nations. This model dissects the *TFR* evolution into three stages: pre-transition high fertility, the fertility transition, and post-transition low fertility. Building upon the deterministic projection methodology of the United Nations Population Division, assuming eventual fertility decline below replacement level, this model captures the *TFR* decline as the sum of two logistic functions contingent on the current *TFR* level and a stochastic term. A Bayesian hierarchical model is employed to project future *TFR*, considering both the country's *TFR* history and the global pattern. The estimation is derived from United Nations estimates of historical *TFR* in all countries using a Markov chain Monte Carlo algorithm. The post-transition low fertility phase is modeled using an autoregressive approach, where long-term *TFR* projections converge to and oscillate around replacement level. The model's performance is assessed through out-of-sample projections for the periods since 1980 and 1995, demonstrating effective calibration.

Peristera and Kostaki (2007) introduced a flexible model designed to describe both traditional and contemporary fertility patterns, with empirical evaluations conducted on various fertility schedules to assess its adequacy. This is because the age-specific fertility pattern, traditionally uniform across human populations over time, has recently exhibited deviations in developed countries. Current datasets from the United Kingdom, Ireland, and the US reveal anomalies characterized by a surge in fertility rates among younger women. Moreover, in nations with distorted fertility, the first-birth pattern displays a pronounced

hump at younger ages, surpassing that of the overall fertility pattern. This heterogeneity observed in recent fertility distributions in European countries and the US may be linked to factors such as marital status, religion, educational level, and variations in social and economic conditions. In the US, ethnic differences in the timing and number of births could contribute to this heterogeneity. Existing models struggle to capture this new fertility pattern, emphasizing the need for more suitable representations.

Singh, Singh, and Chaurasia (2021) presented a simple model that relies on the first and second moments of the age distribution among currently married women of reproductive age, offering a reasonably dependable estimate of *TFR* in India. The proposed model is:

$TFR = 38.717(CV) - 10.381$ where (CV) is the coefficient of variation. Information concerning the age of currently married women in their reproductive years is consistently gathered and managed at the community level by healthcare service providers. Consequently, the proposed methodology in this paper can be applied to gauge *TFR* at the grassroots level. This application will aid in assessing and overseeing fertility transitions at the local level, enhancing the connection with the community.

Kaloyanov (2011) investigated the fertility patterns among women in Bulgaria by involving an analysis of age-specific fertility rate distributions. An employment of all four statistical distribution parameters: the arithmetic mean, standard deviation, coefficient of asymmetry, and coefficient of excess kurtosis was used to achieve this analysis. Schmertmann and Hauer (2019) explored a contemporary statistical method as an alternative to the traditional deterministic approach for demographic estimation. They formulated a Bayesian model to represent the statistical distribution of *TFR* based on the population's age-sex composition. In this model, the unknown demographic variables in the conventional estimation are

treated as parameters with prior probability distributions. They put this model into practice through two case studies: the first focusing on a small indigenous population in the Amazon region of Brazil characterized by exceptionally high fertility rates, and the second encompassing 159 counties within the state of Georgia, USA. The statistical approach provides valuable insights into the origins of errors in indirect estimation and their relative magnitudes.

The above papers collectively contribute to the understanding of fertility dynamics, offering diverse models and methodologies for fertility estimation and analysis.

2.4 Factors affecting fertility of MENA

This section explores the determinants shaping fertility rates. From economic shifts and cultural norms to healthcare policies and environmental considerations, we unravel the multifaceted influences that define the fertility landscape in the MENA region. By synthesizing existing research, we seek to provide a comprehensive understanding of the factors steering fertility dynamics. The recent literature presents a comprehensive exploration of factors influencing fertility across the Middle East and North Africa (MENA) region.

Awad (2014) investigated the factors contributing to the recent decline in fertility across Gulf Cooperation Council (GCC) countries, focusing on the United Arab Emirates (UAE) as a case study. Utilizing data from the 2008 UAE Household Expenditure Survey, the research employs a Poisson fertility count model to analyze determinants of fertility. Contrary to expectations, economic factors related to the costs and benefits associated with raising children do not emerge as significant determinants, attributed to the substantial social insurance provided by the UAE Government. Furthermore, the study finds that labor

market participation, by either gender, does not exert a pivotal influence on fertility in the UAE. Key factors driving the fertility decline include delayed marriages or first births and increased female education levels. Additional contributors encompass marriages between UAE national males and foreign females, along with extended childbirth intervals. Conversely, larger household residences and the presence of domestic workers exhibit a positive correlation with fertility rates.

Lari and Al-Rakeb (2019) investigated the socioeconomic factors influencing fertility rates and preferences among Qatari women and their policy implications. Utilizing data from a 2018 fertility survey involving 607 Qatari households, collected through computer-assisted personal interviews, a multidimensional model was employed for analysis. Logistic and Poisson regression techniques were applied to interpret the findings. The results reveal an average fertility rate of 3.2 among Qatari women, with those in the 20–29 age group exhibiting the highest fertility rate. Notably, women's educational attainment and employment status did not emerge as significant factors affecting age-specific fertility rates. Family planning method usage was more prevalent among older women with higher parity. The paper concludes by providing family-friendly policy recommendations for addressing the declining fertility rate among Qatari women, aligning with sustainable development goals related to national reproductive health.

Islam, Dorvlo, and Al-Qasmi (2011) utilized recent national survey data and Bongaarts' framework of the proximate determinants of fertility to explore the factors contributing to the substantial fertility decline in Oman. The fertility rate in Oman has witnessed a significant decline, plummeting from a high of 8.6 births per woman in 1988 to 4.8 births per woman in 2000. This remarkable decrease, constituting a 44% reduction in total fertility

over twelve years, occurred without the implementation of an official national family planning program. The findings highlight that a reduction in the age-specific proportions of married women, coupled with an increase in contraceptive usage, stands out as the primary mechanisms driving the decline in fertility. Factors such as women's education and employment appear to play a role in encouraging couples to postpone marriage, adopt modern family planning methods, and subsequently reduce marital fertility.

Al-Kandari (2007) investigated the impact of certain sociocultural variables on the fertility of Kuwaiti women. A questionnaire was distributed to a randomly selected sample of 7749 married women, aged 15-78 years, from 10 primary health care clinics in Kuwait. The recorded fertility rate was 3.58 live births per woman. Fertility exhibited higher rates among Sunni Muslim women, those of Bedouin ethnicity, and those in consanguineous marriages ($P < 0.001$). Notably, there was a significant negative correlation between fertility and the respondents' educational level, occupation, age at marriage, socioeconomic status, and type of marriage (consanguineous or not). Conversely, a positive correlation was observed between fertility and the respondents' age and family income.

Sabermahani, Goudarzi, and Nasiri (2017) examined the most significant factors affecting the fertility rate in Iran, considering the impact of various time periods on population changes. This descriptive-analytic study utilized a combination of cross-sectional and time series data (panel data) extracted from 1966 to 2013, covering Iran's 24 provinces. Data sources included statistical yearbooks from the Statistical Center of Iran and the Organization of Civil Registration. The findings revealed that variables such as marriage, women's education level, unemployment, population policies, Sunni population, economic policies, and annual household expenses influenced the fertility rate. According to the

research results, marriage and women's education level had the most positive and most negative effects on the fertility rate, respectively. Unemployment, family planning policies, cash subsidy policies, and total annual household expenses had inverse effects on the fertility rate. Policies related to cash subsidies and the Sunni population showed positive effects on the fertility rate.

Aboulghasem Pourreza et al (2021) published almost the only study that addressed the fertility rates in MENA. The aim of their study is to find the most critical factors affecting the fertility rate decline in MENA region. They identified these factors in five different categories which are health care-related, economic, cultural, social, and political. The health care factors are mostly related to the use of contraception. The cultural factors are mainly concern with the postponement of marriage or childbirth and the impact of religion. Economic factors included costs of child-raising, inflation, and rates of women's employment. Social factors are concerned with level of education and women's employment. Finally, political factors included mainly governments' support for family planning. This study addressed only the factors that affect the decline of fertility rates in MENA. Also, some of the factors were overlapping like the factors in social and cultural categories.

Taher (2020) published another relevant paper that investigated the factors affecting the Tunisian fertility rates over the period from 1960 to 2015 using multivariant regression model. The result has concluded that the main reason for the decrease in fertility rates in Tunisia is the development of the education system after the independence. The paper has also confirmed that education, economic growth, urbanization, political stability,

expanding women's rights in term of education and employment are factors affecting the decline of fertility rates in Tunisia.

Buyukkececi and Engelhardt (2021) compared the relationship between fertility and development in Western countries with the MENA region, where fertility rates are higher than in other regions with identical levels of development. The findings show that fertility and development were positively correlated for only a short period in Western countries and that the relationship became negative again in recent years. Results also show that there is no significant correlation between fertility and development in MENA countries. These findings indicate that the well-acknowledged theories of fertility and development are not applicable in every context.

All major long-term global population forecasts concur on the ongoing decrease in fertility rates, leading to a diminishing growth rate of population size that is anticipated to approach zero and eventually become negative as fertility rates fall further below death rates (Arenberg, et al., 2022). Nkaly (2021) investigated the impact of environmental pollution and socio-economic aspects on fertility rate in MENA and two other regions from 1970 to 2019. The study showed that the total fertility rate of MENA is negatively affected by environmental pollution (such as CO₂ emissions) while it has positive effect on Southeast Asian Nations. This indicates that factors can have different effects on different regions. However, it can be observed from the literature that there are very few sources that discussed the fertility rates of the MENA region in specific.

These studies collectively contribute to a nuanced understanding of fertility dynamics in the MENA region, highlighting the interplay of economic, sociocultural, and environmental factors. However, it can be observed that there is a gap in the study of the determinates

affecting fertility rate in MENA as the only study that addressed this topic only focused on the factors affecting the fertility rate negatively.

After reviewing the literature, this research embarks on a comprehensive exploration of fertility dynamics in the MENA region, addressing three pivotal questions. The next chapter examines the spatial nuances of fertility rates, unraveling the distribution patterns of MENA region. Moving forward, the fourth chapter meticulously analyzes the temporal trends of fertility, employing sophisticated statistical models to characterize the evolving nature of reproductive behaviors over time. Finally, the fifth chapter delves into the determinants of fertility, spotlighting the key factors that significantly shape the reproductive landscape in the MENA region. Together, these chapters weave a thorough narrative, advancing our understanding of fertility complexities from geographical, temporal, and causal perspectives.

CHAPTER 3

DISTRIBUTION OF FERTILITY RATES IN MENA

Unlike biology where fertility is defined as the ability to conceive a child, in demography, fertility represents the output or product of reproduction (Frank, 2022). Fertility in demography is measured by fertility rate which is the number of children born alive to women proportional to the average annual population of women (Fertility Rate, 2021). Fertility rate is important because it helps to estimate how a population is growing, shrinking, or stabilizing over time. It can also help predict other demographic shifts, such as the future distributions within a population (Moran, 2020).

While the field of demography provides various fertility metrics, the most commonly utilized indicator today is the Total Fertility Rate (*TFR*). The *TFR* represents the average number of births a woman would have if she were to go through her reproductive years (typically ages 15 to 49) and give birth at the rates observed during a specific year or period (Bongaarts & Feeney, 1998). This metric is hypothetical in nature since it is not based on the actual experiences of any specific group of women and does not guarantee that these precise rates will be encountered in reality. The Total Fertility Rate (*TFR*) offers the advantage of assessing present fertility, providing current and relevant insights into fertility levels and trends. Additionally, the *TFR*'s popularity is partly due to its straightforward interpretation, which sets it apart from other metrics.

Fertility rates of countries in the Middle East and North Africa (MENA) as a group of countries is interesting. These countries have characteristics in common such as religion,

language, culture, political stability, etc. Islam is certainly the main religion in almost all the MENA territories. (Middle East-North Africa Overview, 2009). Arabic is the most spoken language in MENA region. Economically, most of the countries in MENA region are economically stable due to vast reserves of oil and natural gas. The MENA region has 60% of the world's oil reserves according to the Oil and Gas Journal (2009). On the other hand, politically, the region is the least peaceful in the world according to the Global Peace Index rankings (Global Peace Index 2022: Measuring Peace in a Complex World, 2022). The geographical scope of the MENA region includes 21 countries according to the World Bank (Middle East and North Africa (MENA) Regional Update, 2021). These countries in alphabetical order are: Algeria, Bahrain, Djibouti, Egypt, Iraq, Iran, Israel, Jordan, Kuwait, Lebanon, Libya, Malta, Morocco, Oman, Qatar, Saudi Arabia, Syria, Tunisia, United Arab Emirates, West Bank and Gaza, and Yemen.

As shown in Figure 3-1 below, the average Total Fertility Rate of MENA region has been declining from 7 to 2.7 during the period 1960 to 2021 (Fertility rate, total (births per woman) - Middle East & North Africa, 2023).

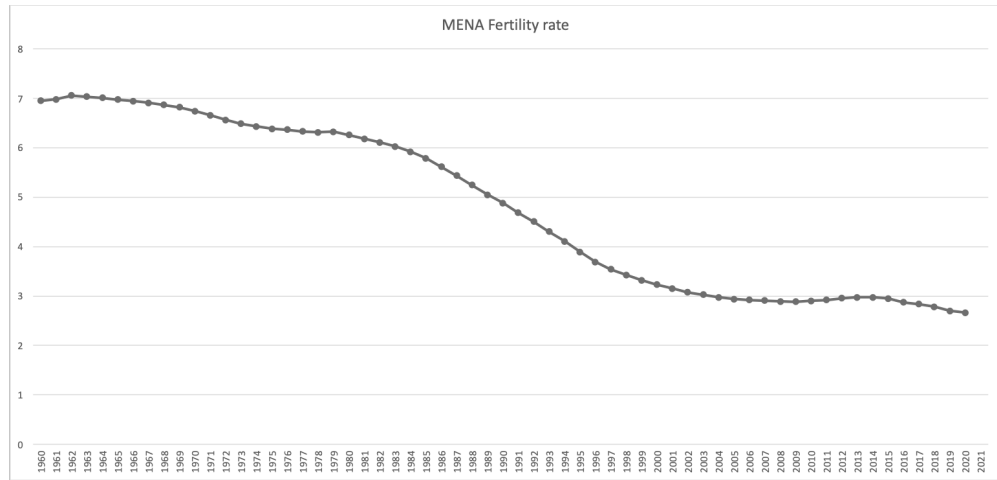


Figure 3-1 MENA Average TFR 1960-2021

Analyzing the statistical distribution of fertility rates in the Middle East and North Africa (MENA) region is important for a variety of reasons:

- **Demographic Trends:** Fertility rates are a key demographic indicator that provides insights into population dynamics. Analyzing fertility rate distributions can help researchers and policymakers understand trends in population growth, aging, and the overall demographic structure of the region.
- **Policy Planning:** Fertility rates influence a wide range of policies, including those related to healthcare, education, social services, and labor markets. Understanding the distribution of fertility rates is essential for effective policy planning and resource allocation.
- **Economic Impact:** Fertility rates have a significant impact on economic development. High fertility rates can strain resources and limit economic growth, while low fertility rates can lead to population aging and workforce challenges. Analyzing fertility rate distributions helps in assessing the economic implications and planning labor force.
- **Healthcare Services:** High fertility rates can place demands on healthcare systems, particularly in terms of maternal and child health. By analyzing fertility rate distributions, healthcare providers can anticipate and address the healthcare needs of the population.
- **Family Planning:** Understanding the distribution of fertility rates is crucial for family planning programs. It helps in identifying regions or communities where family planning services are most needed and tailoring interventions accordingly.

In summary, analyzing the statistical distribution of fertility rates in the MENA region is crucial for informed decision-making, policy formulation, and addressing various socio-economic and demographic challenges. It provides valuable insights into the region's population dynamics and helps in planning for the future.

3.1 Methodology

3.1.1 Data Collection

To accomplish the objective of this study, the first phase is data collection. The research will utilize publicly available demographic databases and reports from the World Bank. The assembled data will include data on fertility rates for MENA region and for countries in MENA region separately. Next, extrapolation will be used to fill the missing data. Some of the countries like the West Bank and Gaza has recorded fertility starting 1990 only while other countries in MENA have recorded fertility rates since 1960. Extrapolation will be used to fill the missing fertility rates for the West Bank and Gaza from 1960 to 1990. Extrapolation involves extending or projecting existing trends or patterns beyond the range of observed data. In this case, the known data from Yemen are used to make predictions about *TFR* in West Bank and Gaza for a period (1960-1990) that is outside the range of directly observed data.

Quadratic regression extrapolation is a statistical method used to model and predict the relationship between variables when a quadratic (second-degree) polynomial equation is deemed to be a good fit for the data. The fitted regression equation is then used for extrapolation, where different values of the year is substituted to predict the corresponding *TFR*. Here's an overview of the method:

- **Quadratic Regression Model:** Fit a quadratic regression model to the available data of Yemen. A quadratic regression equation has the form: $y = ax^2 + bx + c$. In this equation, y represents the dependent variable (*TFR*), x is the independent variable (time in this case), and a , b , and c are coefficients.
- **Curve Fitting:** The quadratic regression model aims to find the coefficients that best fit the observed data points. The resulting quadratic equation represents the curve that best fits the trend in the available data.
- **Extrapolation:** Once the quadratic equation is established, it will be used to predict values beyond the observed range. The quadratic equation will be applied to estimate *TFR* values for West Bank and Gaza for the period 1960-1990.

3.1.2 Data Analysis

The second phase is data analysis where descriptive statistics will be used to summarize and visualize fertility rate data, including measures of central tendency and dispersion. To identify the distribution of fertility rate in MENA, first, summary statistics of the *TFR* in MENA of each year from 1960 to 2021 will be calculated. Summary statistics, including the mean, median, variance, kurtosis, and skewness, play a fundamental role in understanding the statistical distribution of a dataset. They provide a concise and informative snapshot of the dataset's central tendency, spread, shape, and the outlier presence. Here's how each of these summary statistics contributes to understanding the distribution (Casella & Berger, 2002):

- **Central Tendency (Mean and Median):** The mean is a measure of central tendency. It represents the arithmetic average of the dataset. It provides information about where the "center" of the data is located. If the mean is significantly different

from the median, it may indicate skewness or the presence of outliers in the data. The median is another measure of central tendency, specifically the middle value when the data is ordered. It is robust to outliers and resistant to extreme values, making it a good indicator of the dataset's center even when outliers are present. Comparing the mean and median can help detect skewness or asymmetry in the distribution.

- **Variance (Spread/Dispersion):** Variance measures the spread or dispersion of data points around the mean. A higher variance indicates greater data spread, while a lower variance suggests that data points are closer to the mean. Variance helps understand the variability or consistency in the dataset.
- **Skewness (Asymmetry):** Skewness measures the asymmetry of the data distribution. Positive skewness indicates a right-skewed distribution, where the tail on the right (larger values) of the median is longer. Negative skewness, on the contrary, indicates a left-skewed distribution, where the tail on the left (smaller values) is longer. Skewness helps identify the direction and degree of skew in the data.
- **Kurtosis (Peakedness or Tailedness):** Kurtosis quantifies the shape of the distribution by measuring how the tails of the distribution compare to the tails of a normal distribution. More specifically, Excess Kurtosis will be measured. Excessive kurtosis indicates that the distribution of event's outcomes exhibits numerous occurrences of outlier results, leading to wider tails on the bell-shaped distribution curve. Positive kurtosis (leptokurtic) indicates heavier tails than a normal distribution, while negative kurtosis (platykurtic) indicates lighter tails.

Excess kurtosis helps identify whether the data distribution is more or less peaked and tailed than a normal distribution.

In addition to the summary statistics, the data will be visualized using histograms and smoothed empirical density polygons. While summary statistics provide a quantitative summary of the data, visualizations in the form of histograms and smoothed empirical density polygons offer a complementary and often essential way to gain a deeper understanding of the data's distribution, detect patterns, and communicate findings effectively. Together, they provide a more comprehensive view of the dataset, assisting in data exploration, hypothesis testing, and model selection. A histogram is a bar chart that describes a frequency distribution while a smoothed empirical density polygon visually represents this probability as the area under a section of a curve (Kaplan, Gabrosek, Curtiss, & Malone, 2014).

After the summary statistics and visualization of the distribution, statistical tests for common distributions are used to confirm the distribution of the fertility rate of MENA countries for each year. First, each year is tested for unimodal distribution to check the need for bimodal or multi-modal distributions. The Dip test will be used to test for Unimodality. Introduced by Hartigan and Hartigan (1985), the The Dip Test for Unimodality is a statistical test used to assess whether a given sample is unimodal (i.e., has a single mode) or multimodal (has more than one mode). The test is based on the depth function, which quantifies how far a point is from the modes of the distribution.

- Dip Statistic: The Dip Test is based on the dip statistic, which is a measure of the maximum difference between the empirical distribution function (EDF) and a

unimodal distribution function. The dip statistic is calculated by comparing the EDF with its best unimodal fit.

- The null hypothesis of the Dip Test is that the data are unimodal. The test involves comparing the observed dip statistic with those obtained from many random samples drawn from a unimodal distribution. If the observed dip is significantly larger than those from the simulated samples, then the null hypothesis of unimodality is rejected.
- *P-Value*: The *p-value* of the Dip Test indicates the probability of obtaining a dip statistic as extreme as the observed one under the assumption of unimodality. A small *p-value* suggests that the data might not be unimodal. A rejection of the null hypothesis suggests that there is evidence against unimodality, indicating a multimodal distribution.

The Dip Test is nonparametric and distribution-free, making it applicable to a wide range of data types. It is particularly useful for detecting deviations from unimodality in an exploratory data analysis context.

If unimodality is confirmed, the data will be tested for common symmetric and skewed distributions like Normal, Log-normal, Gamma, Weibull and Generalized Extreme Values distributions. Each year will be compared to common distributions to be able to find the distribution that fit all the years. For consistency and comparison purposes, all distributions will be tested using Kolmogorov-Smirnov test.

According to Feller (1971), The Kolmogorov-Smirnov (KS) test is a non-parametric test used to assess whether a sample comes from a specific probability distribution. The test is

based on the cumulative distribution function (CDF) of the theoretical distribution and the empirical distribution function (EDF) of the sample.

- *Null Hypothesis (H_0):* The null hypothesis of the KS test assumes that the sample is drawn from a specific continuous distribution.
- *Alternative Hypothesis (H_1):* The alternative hypothesis is that the sample is not drawn from the specified distribution.
- *Test Statistic (D):* The KS test statistic (D) is calculated as the maximum absolute difference between the empirical distribution function (EDF) of the sample and the cumulative distribution function (CDF) of the theoretical distribution. The formula is given by: $D = \max|F_n(x) - F(x)|$ where $F_n(x)$ is the empirical distribution function, $F(x)$ is the cumulative distribution function of the theoretical distribution, and $\max$ denotes the maximum absolute difference over all data points.
- *Critical Value:* The critical value for the test depends on the chosen significance level (commonly denoted as α), which determines the probability of rejecting the null hypothesis when it is true.
- *Decision Rule:* If the test statistic D is greater than the critical value, the null hypothesis is rejected. If D is less than or equal to the critical value, there is insufficient evidence to reject the null hypothesis.

The KS test is applicable to a wide range of continuous distributions, and it is particularly useful when the distribution is unknown or not assumed to follow a specific parametric form. It is important to note that the KS test is sensitive to differences in location, scale, and shape of distributions, making it a versatile tool for distributional comparisons. However, it may lack power in certain situations, especially when sample sizes are small.

According to Kotz and Nadarajah (2002), Extreme value theory constitutes a distinct statistical branch focusing on rare and extreme events. Its foundation rests on the extremal types theorem, also known as the three types theorem, which asserts that there are just three fundamental distribution types necessary to model the maximum or minimum values among a set of random observations originating from the same distribution. To put it differently, if you generate N datasets from an identical source distribution and form a new dataset comprising the maximum values from these N datasets, this resultant dataset can be accurately represented by one of the three primary models - namely, the Gumbel, Fréchet-type, and Weibull-type distributions. These models, in the Generalized Extreme Value family of distribution, find extensive application in fields such as risk management, finance, insurance, economics, hydrology, material sciences, telecommunications, and various other industries that grapple with extreme events.

The Generalized Extreme Value (GEV) distribution, originating from extreme value theory, amalgamates the Gumbel, Fréchet-type, and Weibull-type distribution families, which are also referred to as the Type I, II, and III extreme value distributions. These are families of continuous probability distributions. According to the extreme value theorem, the GEV distribution represents the sole possible marginal distribution for the properly standardized maximum values obtained from a set of independent and identically distributed random variables. It is worth noting that this requires the presence of a specific marginal distribution and a regularity condition at the distribution's tail. Nevertheless, the GEV distribution is frequently employed as an approximation when modeling the maximum values of lengthy (though finite) sequences of random variables.

The Generalized Extreme Value (GEV) distribution is a versatile three-parameter model that unifies the Gumbel, Fréchet-type, and Weibull-type distributions for modeling extreme values. It has the following PDF:

$$f(x) = \begin{cases} \frac{1}{\sigma} \exp\left(- (1 + kz)^{-1/k}\right) (1 + kz)^{-1-1/k} & k \neq 0 \\ \frac{1}{\sigma} \exp(-z - \exp(-z)) & k = 0 \end{cases}$$

where $z = (x-\mu)/\sigma$, and k , σ , μ are the shape, scale, and location parameters respectively. The shape and location can take on any real value however the scale must be positive ($\sigma > 0$). The range of definition of the GEV distribution depends on shape k :

$$\begin{aligned} 1 + \kappa \frac{(x - \mu)}{\sigma} &> 0 & \text{for } k \neq 0 \\ -\infty < x < +\infty && \text{for } k = 0 \end{aligned}$$

3.2 Results

3.2.1 Data Collection

For the first phase, the MENA *TFR* data was collected from the World Bank. Figure 3-2 shows the *TFR* of MENA countries separately from 1960 to 2021. In 2021, the three highest fertility rates were reported in Yemen (3.9), West Bank and Gaza (3.6), and Iraq (3.6) while the lowest fertility rates were reported in UAE (1.5), Iran (1.7), and Qatar (1.8). In Table 3-1, the highlighted values represent the extrapolated values for West Bank and Gaza, as the *TFR* for this country is only available starting from 1990. The missing values (NA) in the original dataset have been filled in by assigning values that form a quadratic progression between the nearest available *TFR*. In the original dataset, there are missing values between years 1960 and 1989, and then the sequence of values is provided starting from 1990. The extrapolation fills in the missing values with a quadratic progression, creating a continuous

sequence. The transformed dataset shows a smooth transition from the last available value before the missing section to the first available value after the missing section.

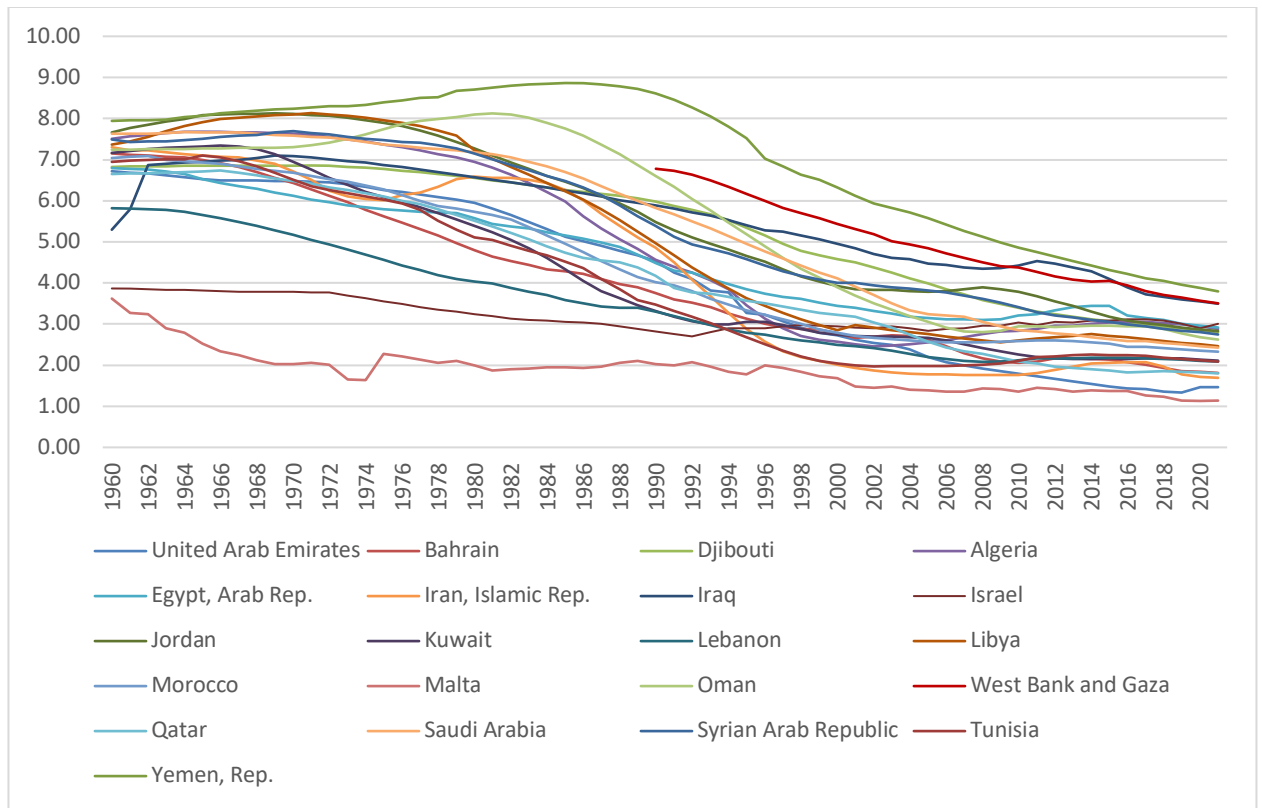


Figure 3-2 TFR of MENA Countries

3-1 MENA Average TFR and TFR of Each Country

	1973	1972	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960	Year
	6.48	6.56	6.66	6.74	6.82	6.87	6.91	6.94	6.97	7.01	7.03	7.05	6.98	6.95	Average
	6.40	6.45	6.47	6.49	6.49	6.50	6.49	6.49	6.52	6.57	6.62	6.66	6.68	6.72	UAE
	5.96	6.13	6.28	6.43	6.56	6.70	6.83	6.92	7.00	7.06	7.07	7.10	7.12	7.15	Bahrain
	6.82	6.85	6.86	6.86	6.85	6.85	6.85	6.85	6.85	6.85	6.84	6.84	6.83	6.83	Djibouti
	7.49	7.55	7.60	7.64	7.65	7.66	7.67	7.67	7.68	7.67	7.65	7.61	7.56	7.50	Algeria
	5.89	5.97	6.03	6.12	6.20	6.29	6.36	6.43	6.52	6.64	6.72	6.76	6.78	6.79	Egypt
	6.11	6.25	6.51	6.71	6.90	6.97	7.05	7.08	7.11	7.13	7.18	7.22	7.23	7.30	Iran
	6.96	7.01	7.06	7.08	7.10	7.05	7.00	6.97	6.95	6.93	6.90	6.87	5.80	5.30	Iraq
	3.70	3.77	3.77	3.78	3.78	3.79	3.79	3.80	3.81	3.83	3.84	3.85	3.86	3.87	Israel
	8.02	8.06	8.09	8.11	8.13	8.12	8.11	8.09	8.08	7.99	7.92	7.85	7.77	7.67	Jordan
	6.38	6.56	6.76	6.95	7.14	7.26	7.32	7.34	7.33	7.31	7.30	7.25	7.21	7.16	Kuwait
	4.81	4.93	5.04	5.17	5.28	5.38	5.49	5.57	5.65	5.72	5.78	5.80	5.81	5.82	Lebanon
	8.07	8.10	8.13	8.10	8.08	8.05	8.02	7.99	7.91	7.82	7.70	7.55	7.45	7.37	Libya
	6.47	6.53	6.61	6.68	6.72	6.78	6.84	6.90	6.93	6.97	7.00	7.09	7.07	7.04	Morocco
	1.65	2.01	2.06	2.03	2.02	2.12	2.24	2.33	2.52	2.79	2.90	3.24	3.27	3.62	Malta
	7.50	7.41	7.35	7.31	7.29	7.29	7.28	7.28	7.27	7.26	7.26	7.25	7.25	7.25	Oman
	7.77	7.75	7.73	7.70	7.67	7.63	7.59	7.54	7.48	7.42	7.35	7.28	7.20	7.20	Palestine
	6.27	6.33	6.41	6.48	6.55	6.61	6.67	6.74	6.72	6.69	6.69	6.67	6.66	6.65	Qatar
	7.48	7.54	7.56	7.58	7.60	7.63	7.66	7.66	7.66	7.67	7.65	7.64	7.63	7.63	Saudi
	7.56	7.61	7.65	7.69	7.67	7.60	7.58	7.55	7.51	7.47	7.44	7.44	7.43	7.49	Syria
	6.19	6.26	6.36	6.52	6.69	6.84	6.96	7.05	7.10	7.02	7.01	6.99	6.97	6.94	Tunisia
	8.30	8.30	8.26	8.24	8.22	8.20	8.17	8.13	8.07	8.04	7.98	7.96	7.96	7.94	Yemen

1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
4.88	5.05	5.24	5.43	5.62	5.78	5.92	6.03	6.11	6.18	6.26	6.32	6.31	6.33	6.37	6.39	6.43
4.54	4.67	4.79	4.90	5.01	5.13	5.31	5.48	5.65	5.81	5.95	6.03	6.09	6.16	6.21	6.26	6.33
3.76	3.90	3.97	4.09	4.22	4.28	4.32	4.44	4.54	4.64	4.80	4.97	5.15	5.31	5.46	5.62	5.79
5.98	6.06	6.12	6.17	6.22	6.26	6.33	6.39	6.44	6.49	6.55	6.61	6.65	6.69	6.73	6.77	6.80
4.56	4.83	5.06	5.33	5.63	5.98	6.22	6.44	6.64	6.81	6.95	7.06	7.14	7.23	7.31	7.37	7.44
4.48	4.69	4.87	4.99	5.07	5.16	5.24	5.33	5.37	5.44	5.58	5.70	5.72	5.74	5.77	5.80	5.84
4.86	5.11	5.39	5.69	6.01	6.26	6.44	6.51	6.55	6.56	6.58	6.53	6.33	6.20	6.14	6.01	6.04
5.88	5.94	6.02	6.10	6.18	6.25	6.33	6.39	6.45	6.51	6.57	6.63	6.69	6.75	6.81	6.88	6.93
2.82	2.89	2.95	3.01	3.03	3.06	3.08	3.11	3.13	3.19	3.24	3.30	3.35	3.41	3.48	3.55	3.63
5.48	5.73	5.93	6.12	6.32	6.46	6.61	6.77	6.93	7.10	7.27	7.43	7.58	7.71	7.81	7.89	7.96
3.32	3.47	3.63	3.80	4.05	4.32	4.61	4.85	5.05	5.23	5.39	5.54	5.70	5.85	5.98	6.09	6.22
3.30	3.39	3.40	3.43	3.50	3.59	3.70	3.79	3.89	3.98	4.03	4.09	4.20	4.31	4.42	4.56	4.69
4.97	5.26	5.53	5.79	6.02	6.24	6.44	6.63	6.83	7.02	7.22	7.58	7.71	7.82	7.90	7.96	8.02
4.02	4.14	4.33	4.54	4.75	4.96	5.15	5.35	5.55	5.65	5.73	5.80	5.87	6.00	6.14	6.27	6.37
2.02	2.10	2.06	1.97	1.93	1.95	1.95	1.92	1.90	1.87	1.99	2.10	2.05	2.14	2.21	2.27	1.64
6.61	6.87	7.14	7.37	7.58	7.76	7.90	8.02	8.10	8.13	8.10	8.04	7.99	7.94	7.86	7.75	7.62
6.78	6.89	6.90	6.97	7.47	7.53	7.58	7.62	7.66	7.69	7.72	7.74	7.76	7.77	7.78	7.78	7.78
4.18	4.37	4.49	4.54	4.61	4.73	4.88	5.06	5.21	5.38	5.51	5.66	5.79	5.91	6.00	6.10	6.18
5.83	6.00	6.17	6.36	6.55	6.70	6.84	6.95	7.05	7.13	7.19	7.23	7.26	7.30	7.33	7.37	7.43
5.38	5.63	5.89	6.13	6.33	6.48	6.61	6.74	6.88	7.01	7.16	7.27	7.35	7.41	7.44	7.47	7.51
3.47	3.59	3.85	4.10	4.36	4.52	4.67	4.79	4.91	5.05	5.11	5.29	5.51	5.77	5.94	6.03	6.11
8.61	8.71	8.79	8.83	8.86	8.86	8.85	8.83	8.79	8.75	8.71	8.67	8.52	8.50	8.44	8.40	8.34

2007	2006	2005	2004	2003	2002	2001	2000	1999	1998	1997	1996	1995	1994	1993	1992	1991
2.91	2.92	2.94	2.98	3.02	3.07	3.15	3.23	3.32	3.43	3.54	3.69	3.89	4.11	4.30	4.50	4.69
1.99	2.07	2.20	2.38	2.49	2.54	2.62	2.73	2.79	2.91	3.06	3.23	3.27	3.77	3.82	4.09	4.25
2.29	2.46	2.62	2.71	2.73	2.69	2.66	2.78	2.87	2.89	2.94	3.01	3.13	3.26	3.42	3.52	3.59
3.72	3.86	3.99	4.11	4.25	4.38	4.50	4.58	4.67	4.78	4.96	5.15	5.34	5.53	5.66	5.77	5.88
2.68	2.60	2.56	2.51	2.48	2.46	2.51	2.57	2.62	2.71	2.90	3.14	3.46	3.85	4.06	4.26	4.39
3.12	3.11	3.15	3.18	3.26	3.32	3.39	3.44	3.52	3.62	3.65	3.73	3.84	3.97	4.10	4.25	4.30
1.77	1.77	1.78	1.80	1.82	1.87	1.94	2.02	2.10	2.20	2.33	2.57	2.89	3.27	3.68	4.08	4.51
4.37	4.44	4.48	4.58	4.62	4.71	4.84	4.95	5.06	5.15	5.24	5.29	5.40	5.53	5.64	5.72	5.81
2.90	2.88	2.84	2.90	2.95	2.89	2.92	2.95	2.97	2.98	2.94	2.90	2.90	2.90	2.80	2.70	2.76
3.85	3.80	3.78	3.80	3.84	3.83	3.85	3.92	4.04	4.16	4.34	4.51	4.66	4.81	4.96	5.11	5.29
2.52	2.60	2.66	2.69	2.69	2.69	2.72	2.74	2.79	2.88	2.97	3.06	3.05	3.00	2.99	3.07	3.18
2.11	2.16	2.20	2.27	2.35	2.41	2.46	2.50	2.55	2.60	2.66	2.74	2.78	2.87	2.97	3.08	3.19
2.65	2.70	2.77	2.81	2.86	2.91	2.97	2.85	2.97	3.11	3.27	3.44	3.64	3.86	4.11	4.38	4.67
2.55	2.56	2.57	2.60	2.63	2.67	2.72	2.80	2.87	3.01	3.12	3.22	3.33	3.47	3.61	3.81	3.94
1.35	1.36	1.38	1.40	1.48	1.45	1.48	1.68	1.73	1.84	1.93	1.99	1.77	1.84	1.96	2.07	1.99
2.81	2.92	3.05	3.20	3.35	3.50	3.69	3.89	4.13	4.34	4.62	4.90	5.19	5.47	5.76	6.05	6.34
4.61	4.72	4.84	4.94	5.02	5.19	5.32	5.44	5.58	5.70	5.82	6.00	6.18	6.34	6.50	6.63	6.72
2.34	2.43	2.58	2.75	2.91	3.04	3.18	3.23	3.27	3.34	3.43	3.50	3.57	3.66	3.74	3.77	3.88
3.18	3.21	3.24	3.34	3.50	3.71	3.91	4.12	4.25	4.42	4.59	4.77	4.95	5.14	5.32	5.49	5.66
3.70	3.76	3.81	3.86	3.90	3.95	4.01	4.00	4.08	4.18	4.28	4.43	4.57	4.72	4.83	4.95	5.12
1.99	1.98	1.98	1.98	1.98	1.97	2.00	2.05	2.10	2.22	2.36	2.51	2.67	2.86	3.02	3.18	3.32
5.27	5.43	5.58	5.71	5.82	5.94	6.11	6.32	6.51	6.64	6.83	7.02	7.53	7.80	8.05	8.27	8.46

2021	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008
2.63	2.66	2.70	2.78	2.84	2.87	2.95	2.97	2.97	2.95	2.92	2.90	2.88	2.89
1.46	1.46	1.33	1.36	1.42	1.44	1.49	1.55	1.60	1.67	1.72	1.79	1.85	1.92
1.81	1.83	1.86	1.93	2.01	2.07	2.13	2.16	2.16	2.16	2.09	2.05	2.09	2.17
2.80	2.85	2.89	2.93	2.97	3.01	3.07	3.11	3.18	3.24	3.31	3.39	3.49	3.59
2.89	2.94	2.99	3.02	3.05	3.05	3.04	3.00	2.96	2.96	2.88	2.84	2.81	2.75
2.92	2.96	3.00	3.10	3.15	3.20	3.44	3.44	3.42	3.34	3.25	3.21	3.11	3.10
1.69	1.71	1.77	1.97	2.07	2.07	2.05	2.04	1.96	1.89	1.80	1.77	1.77	1.77
3.50	3.55	3.60	3.66	3.73	3.90	4.09	4.28	4.38	4.48	4.54	4.43	4.36	4.34
3.00	2.90	3.01	3.09	3.11	3.11	3.09	3.08	3.03	3.05	2.98	3.03	2.96	2.96
2.83	2.87	2.92	2.97	3.03	3.08	3.18	3.31	3.44	3.56	3.68	3.78	3.84	3.89
2.11	2.14	2.16	2.17	2.16	2.16	2.16	2.16	2.15	2.17	2.20	2.26	2.34	2.42
2.09	2.10	2.13	2.15	2.17	2.18	2.18	2.18	2.17	2.17	2.16	2.13	2.09	2.08
2.46	2.51	2.54	2.58	2.63	2.67	2.71	2.75	2.72	2.68	2.65	2.60	2.56	2.60
2.33	2.35	2.38	2.42	2.45	2.45	2.53	2.55	2.59	2.60	2.61	2.59	2.58	2.55
1.14	1.13	1.14	1.23	1.26	1.37	1.37	1.38	1.36	1.42	1.45	1.36	1.42	1.43
2.62	2.69	2.78	2.88	2.94	2.95	2.96	2.96	2.95	2.93	2.94	2.94	2.84	2.81
3.50	3.57	3.64	3.71	3.81	3.94	4.05	4.04	4.08	4.16	4.27	4.38	4.41	4.50
1.80	1.82	1.83	1.85	1.84	1.83	1.86	1.90	1.93	1.97	2.04	2.09	2.18	2.28
2.43	2.47	2.50	2.55	2.58	2.59	2.64	2.69	2.74	2.78	2.81	2.85	2.95	3.06
2.75	2.80	2.84	2.89	2.94	2.99	3.05	3.10	3.17	3.22	3.28	3.40	3.51	3.61
2.09	2.11	2.15	2.17	2.22	2.24	2.25	2.25	2.24	2.22	2.19	2.11	2.05	2.01
3.80	3.89	3.96	4.04	4.11	4.21	4.32	4.43	4.53	4.63	4.74	4.86	4.99	5.13

3.2.2 Data Analysis

For the second phase, summary statistics are summarized in Table 3-2.

- **Central Tendency (Mean and Median):** For MENA *TFR*, the mean and median values of fertility rates generally decrease over the years, indicating a general downward trend in fertility. The mean values range from 6.70145 in 1960 to 2.47681 in 2021, while the median values decrease from 7.096 in 1960 to 2.462 in 2021.
- **Variance (Spread/Dispersion):** For MENA *TFR* in Table 3-2, the variance generally decreases over time. This suggests that fertility rates became less variable or more consistent across the years. That is, together with the mean and median *TFR*, total fertility is consistently decreasing across the years.
- **Skewness (Asymmetry):** The negative skewness values throughout the years indicate a left-skewed distribution. This means that the distribution is tilted to the left, and there may be a tail of lower fertility rates. For MENA *TFR* in Table 3-2, the skewness values started negative but increase over time towards symmetry.
- **Excess Kurtosis (Peakedness or Tailedness):** The positive excess kurtosis values imply that the distribution has heavier tails than a normal distribution. This means there may be more outliers or extreme values in the dataset than those in the normal distribution.

Overall, these summary statistics indicate a consistent and notable decline in fertility rates over the years. The data initially had a left-skewed and leptokurtic distribution, suggesting that the distribution was skewed to the left with heavier tails. However, the skewness and

kurtosis values changed over time, indicating a trend towards a more symmetrical and less heavy-tailed distribution.

3-2 Summary Statistics of MENA Average TFR

Year	Mean	Median	Variance	Skewness	Kurtosis
1960	6.70145	7.096	1.398924	-1.510634	1.203526
1961	6.717	7.0975	1.48116	-1.666591	1.900775
1962	6.78125	7.092	1.473107	-1.82878	2.459905
1963	6.77155	7.0385	1.638971	-1.862077	2.69342
1964	6.7713	7.038	1.728376	-1.819263	2.613312
1965	6.75815	7.048	1.891896	-1.817682	2.712682
1966	6.7423	7.011	2.014634	-1.807109	2.755652
1967	6.7183	6.976	2.08227	-1.765803	2.667792
1968	6.6833	6.913	2.156103	-1.731612	2.620642
1969	6.64425	6.873	2.229365	-1.673721	2.49552
1970	6.59725	6.782	2.240649	-1.573314	2.218039
1971	6.5436	6.6855	2.248706	-1.450784	1.87747
1972	6.4816	6.5485	2.288771	-1.361198	1.664278
1973	6.40115	6.4325	2.497962	-1.391586	1.834998
1974	6.34285	6.352	2.542224	-1.302613	1.559033
1975	6.32115	6.2645	2.304398	-0.9921218	0.5636214
1976	6.2686	6.1735	2.381467	-0.9395611	0.4369969
1977	6.20685	6.179	2.462255	-0.8766802	0.3141956
1978	6.1331	6.2125	2.535449	-0.825549	0.2106705
1979	6.0762	6.2805	2.588664	-0.7051206	-0.0099739
1980	5.9811	6.2505	2.636249	-0.6488725	-0.0289797
1981	5.8874	6.1515	2.689095	-0.5858032	-0.0168156
1982	5.7928	6.047	2.682494	-0.4713995	-0.1225436
1983	5.68885	5.934	2.657912	-0.354168	-0.1905326
1984	5.5734	5.7625	2.627896	-0.2243659	-0.2549846
1985	5.4467	5.5675	2.598226	-0.1026287	-0.2610852
1986	5.3102	5.3475	2.554948	0.01740649	-0.2073045
1987	5.16325	5.158	2.476894	0.1630443	-0.1333586
1988	5.0176	4.965	2.373382	0.32241	-0.0158934
1989	4.86625	4.7555	2.282055	0.4659788	0.1449278
1990	4.801619	4.556	2.315042	0.4453412	-0.0668245
1991	4.631	4.386	2.272204	0.557349	-0.02614085
1992	4.487667	4.253	2.162208	0.6544165	0.006393016
1993	4.332714	4.059	2.078479	0.7027873	-0.000729743
1994	4.185095	3.846	1.974954	0.7157817	-0.014534
1995	4.005286	3.573	1.897606	0.7503564	-0.0755433
1996	3.861	3.439	1.655701	0.7363266	-0.361282
1997	3.725667	3.265	1.592662	0.7431834	-0.3328266
1998	3.60319	3.109	1.53021	0.7411363	-0.3032297
1999	3.498048	2.97	1.4942	0.7480384	-0.2430621
2000	3.406714	2.95	1.411835	0.7427126	-0.2501465
2001	3.323286	2.971	1.35533	0.6583823	-0.3037501

2002	3.24281	2.911	1.26752	0.6572747	-0.2846647
2003	3.186143	2.906	1.171976	0.6831115	-0.2104475
2004	3.119381	2.808	1.138583	0.6962019	-0.1779394
2005	3.050143	2.765	1.091827	0.7011479	-0.2179794
2006	2.990571	2.699	1.048888	0.6728668	-0.3335299
2007	2.940857	2.677	1.001972	0.6288629	-0.4663583
2008	2.903	2.753	0.944894	0.6225888	-0.5648566
2009	2.865857	2.813	0.9054508	0.5864396	-0.6436752
2010	2.850286	2.843	0.8894643	0.5117121	-0.7219211
2011	2.838048	2.814	0.8356917	0.5413538	-0.6775388
2012	2.822143	2.78	0.7831522	0.4623515	-0.7061501
2013	2.797762	2.737	0.7412743	0.3717755	-0.7292037
2014	2.778524	2.754	0.697704	0.3180608	-0.7600487
2015	2.745048	2.711	0.6586461	0.2446815	-0.8079182
2016	2.691	2.673	0.5974973	0.1886777	-0.7819776
2017	2.649952	2.626	0.5611857	0.05682597	-0.764014
2018	2.602714	2.581	0.5483589	0.02287502	-0.7789112
2019	2.544048	2.539	0.5439731	-0.0111066	-0.7654084
2020	2.50681	2.507	0.5039713	0.03144227	-0.7629187
2021	2.47681	2.462	0.4796474	0.01951871	-0.8273185

Both histograms and smoothed empirical density polygons in Figures 3-3 and 3-4 below are consistent with the findings of the summary statistics. The data initially had a left-skewed and leptokurtic distribution. However, the skewness decreased over time, indicating a trend towards a more symmetrical and less heavy-tailed distribution.



Figure 3-3 Histograms of MENA TFRs 1960-2021

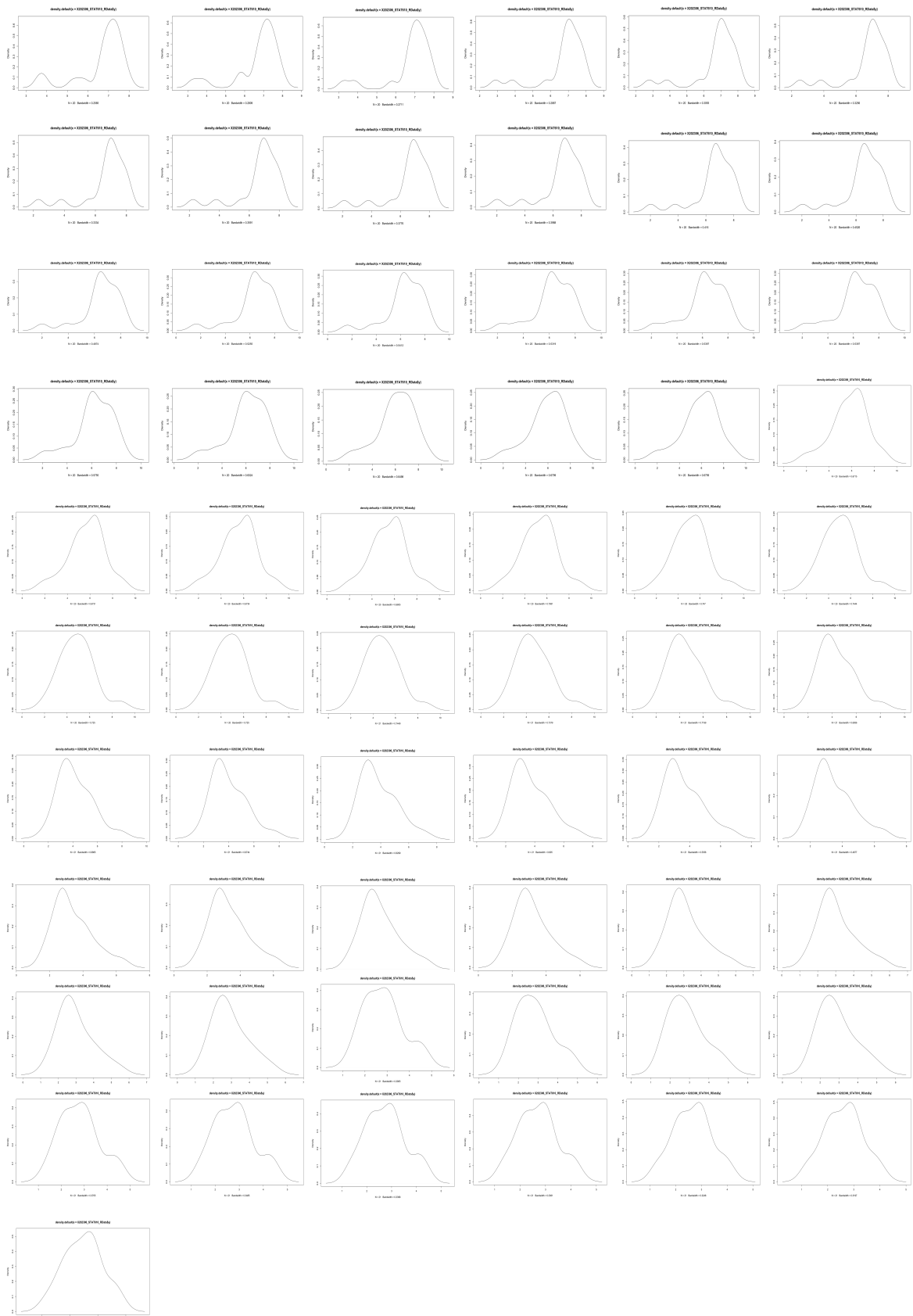


Figure 3-4 Empirical Density Polygons of MENA TFR 1960-2021

After that, statistical tests for common distributions are used to confirm the distribution of the fertility rate of MENA countries for each year. Because some of the early years showed more than one peak in the histograms, unimodal distribution was tested to test if the data has more than one peak for each year. For all the years, the MENA *TFRs* indicate evidence for unimodality, meaning for each year there is a single peak in MENA *TFRs*. After confirming unimodality, each year is tested for Normal, Log-normal, Gamma, Weibull, and Generalized Extreme Values distributions. Table 3-3 provides distribution of fertility results for MENA countries for each year. In particular, the table reports the p -value results of the Kolmogorov-Smirnov test for each distribution.

TABLE 3-3 Test Results of Statistical Distributions by Year

Year	Unimodal	Normal	Log-normal	Gamma	Weibull	GEV
1960	0.05623762	2.2E-16	2.117e-07	2.2e-16	0.2652	0.555746
1961	0.05947614	2.2E-16	9.01e-08	2.2e-16	0.257	0.5816381
1962	0.05286458	2.2E-16	9.065e-08	2.2e-16	0.1738	0.4585547
1963	0.06302993	2.2E-16	9.406e-08	2.2e-16	0.1901	0.6538091
1964	0.0559794	2.2E-16	9.886E-07	2.2e-16	0.2358	0.6484015
1965	0.05908257	2.2E-16	1.054e-07	2.2e-16	0.2634	0.6959013
1966	0.05307692	2.2E-16	1.239e-07	2.2e-16	0.3224	0.7073712
1967	0.05795455	2.2E-16	1.759e-07	2.2e-16	0.358	0.6472909
1968	0.06402439	2.2E-16	2.214e-07	2.2e-16	0.3462	0.6472909
1969	0.06160377	2.2E-16	2.804e-07	2.2e-16	0.2751	0.6472909
1970	0.05695122	2.2E-16	3.645e-07	2.2e-16	0.2798	0.6472909
1971	0.05974843	2.2E-16	4.954e-07	2.2e-16	0.4187	0.6472909
1972	0.07166031	2.2E-16	6.515e-07	2.2e-16	0.5219	0.6472909
1973	0.08958904	2.2E-16	9.084e-07	2.2e-16	0.441	0.6472909
1974	0.07771377	2.2E-16	1.254e-06	2.2e-16	0.4912	0.6472909
1975	0.07982871	2.2E-16	1.781e-06	2.2e-16	0.8045	0.7675814
1976	0.07978723	2.2E-16	2.679e-06	2.2e-16	0.7906	0.7890223
1977	0.06831797	2.2E-16	3.729e-06	2.2e-16	0.6865	0.8536626
1978	0.05480978	2.2E-16	5.28e-06	2.2e-16	0.9032	0.9686826
1979	0.06667505	2.2E-16	7.398e-06	2.2e-16	0.9727	0.9969183
1980	0.0698505	2.2E-16	8.985e-06	2.2e-16	0.8337	0.7516102
1981	0.07068008	2.2e-16	1.055e-05	2.2e-16	0.7843	0.7516102
1982	0.07749511	2.2e-16	1.482e-05	2.2e-16	0.7041	0.7922199
1983	0.08488445	2.2e-16	2.081e-05	2.2e-16	0.6172	0.744482
1984	0.08490391	2.2e-16	2.837e-05	2.2e-16	0.7093	0.7730827
1985	0.08490807	2.2e-16	4.407e-05	2.2e-16	0.8253	0.8427628
1986	0.07082909	2.2e-16	5.564e-05	2.2e-16	0.892	0.8874961
1987	0.05162399	2.2e-16	6.319e-05	2.2e-16	0.8546	0.8917877
1988	0.05389192	2.2e-16	8.824e-05	2.2e-16	0.8159	0.9058166
1989	0.05486402	2.2e-16	9.592e-05	2.2e-16	0.8787	0.9675097
1990	0.05366326	2.2e-16	7.255e-05	2.2e-16	0.9954	0.9998956
1991	0.05200851	2.2e-16	0.0001241	2.2e-16	0.952	0.9999379
1992	0.05691222	2.2e-16	0.0002099	2.2e-16	0.7356	0.9834967
1993	0.06445631	2.2e-16	0.000116	2.2e-16	0.507	0.860981
1994	0.06841046	2.2e-16	8.294e-05	2.2e-16	0.5109	0.8643758
1995	0.06710792	2.2e-16	0.0002547	2.2e-16	0.5612	0.8836177
1996	0.06397275	2.2e-16	0.0007594	2.2e-16	0.5097	0.8745191
1997	0.06227106	2.2e-16	0.002469	2.2e-16	0.6446	0.8705711
1998	0.06743003	2.2e-16	0.006519	2.2e-16	0.5528	0.8649029
1999	0.07174553	2.2e-16	0.01805	2.2e-16	0.5262	0.800655
2000	0.06801553	2.2e-16	0.01938	2.2e-16	0.6007	0.894848

2001	0.05152602	2.2e-16	0.03048	2.2e-16	0.8642	0.9854993
2002	0.05303189	2.2e-16	0.03038	2.2e-16	0.8077	0.9880227
2003	0.05388422	2.2e-16	0.04238	2.2e-16	0.6934	0.9614802
2004	0.05593004	2.2e-16	0.0474	2.2e-16	0.7336	0.9734006
2005	0.05114286	2.2e-16	0.04192	2.2e-16	0.7455	0.9246093
2006	0.06111614	2.2e-16	0.03615	2.2e-16	0.7583	0.9849897
2007	0.05533049	2.2e-16	0.03058	2.2e-16	0.8246	0.9849897
2008	0.0553869	2.2e-16	0.02619	2.2e-16	0.7938	0.9951075
2009	0.0639881	2.2e-16	0.022	2.2e-16	0.8453	0.9879843
2010	0.06215068	2.2e-16	0.01853	2.2e-16	0.9498	0.9385305
2011	0.0793148	2.2e-16	0.01587	2.2e-16	0.8397	0.8397437
2012	0.07315205	2.2e-16	0.01347	2.2e-16	0.8559	0.8597458
2013	0.07285004	2.2e-16	0.0114	2.2e-16	0.9108	0.9224177
2014	0.0752381	2.2e-16	0.009595	2.2e-16	0.9113	0.9428724
2015	0.07742037	2.2e-16	0.007943	2.2e-16	0.9056	0.960634
2016	0.08251794	2.2e-16	0.006451	2.2e-16	0.8915	0.8959226
2017	0.0750601	2.2e-16	0.005227	2.2e-16	0.9111	0.8537701
2018	0.07536232	2.2e-16	0.004496	2.2e-16	0.9153	0.8812671
2019	0.06501548	2.2e-16	0.00374	2.2e-16	0.8503	0.8592054
2020	0.06255567	2.2e-16	0.003102	2.2e-16	0.8553	0.8878882
2021	0.06537713	2.2e-16	0.002453	2.2e-16	0.9921	0.9944589

Table 3-3 presents the p -values from the Diptest and the Kolmogorov-Smirnov (KS) test for normal, log-normal, gamma, Weibull, and Generalized Extreme Value (GEV) distributions across various years.

- For the normal distribution, the p -values are consistently extremely low, indicating a significant deviation from normality.
- Log-normal and gamma distributions also show consistently low p -values, suggesting non-conformance with these distributions.
- For Weibull distribution, there is not enough evidence to reject the null hypothesis of the KS test.
- The GEV distribution exhibits higher p -values, indicating a relatively better fit compared to other distributions.

In summary, the GEV distribution shows a comparatively better fit. The variations across years highlight the dynamic nature of the dataset's conformity to different statistical distributions.

The Generalized Extreme Value (GEV) distribution is considered a better fit than the Weibull distribution based on the provided data because, for the Total Fertility Rates (*TFR*) in the Middle East and North Africa (MENA) region, the p -values from the Kolmogorov-Smirnov (KS) goodness-of-fit test consistently favor the GEV distribution over the Weibull distribution.

In each year from 1981 to 2021, the p -value associated with the GEV distribution is higher than that of the Weibull distribution. The p -value indicates the probability of observing a discrepancy between the empirical data and the assumed distribution as extreme as what is observed, under the null hypothesis that the data follows the specified distribution.

Higher p -values (closer to 1) suggest better agreement between the data and the distribution being tested. Therefore, the consistently higher p -values for the GEV distribution imply that, according to the KS test, the *TFR* data in MENA more closely aligns with the GEV distribution compared to the Weibull distribution across the considered years.

Table 3-4 summarizes the GEV parameters for MENA region for each year:

- Location Parameter (Central Tendency): Changes in the location parameter may suggest shifts in the central tendency of fertility rates in the MENA region. Increasing values might indicate a rise in average fertility rates, while decreasing values could signify a decline.
- Scale Parameter (Variability): Changes in the scale parameter reflect variability in fertility rates. Increasing scale values suggest higher spread in fertility rates, while decreasing values indicate more stability.
- Shape Parameter (Tail Behavior): Positive shape values indicate unbounded upper tails, suggesting the possibility of extreme fertility rates. Negative shape values suggest bounded upper tails, indicating that extreme values are less likely.
- Kolmogorov-Smirnov (KS) Test Results: Lower p -values in the KS test may indicate years where the estimated GEV distribution does not fit the observed fertility data well. Higher p -values suggest better agreement between the estimated distribution and the actual data.

Overall Trends:

- 1960s-1970s: Potential increases in central tendency (location) and variability (scale) during this period. Negative shape values might suggest a bounded upper tail more like the Weibull-type GEV.

- 1980s: Notable decrease in the scale parameter, indicating potential stabilization in fertility rates. The shape parameter becomes less negative, suggesting a transition from a more bounded upper tail.
- 1990s-2000s: Fluctuations in the location and scale parameters. Slightly positive (near zero) shape values in the 1990s may indicate a return to unbounded upper tails more like the Gumbel distribution or the Frechet-type GEV.
- 2010s-2021: Decrease in location and scale parameters. The shape parameter remains negative but less so, indicating a more bounded upper tail.
- Implications: Periods of increased variability may signify economic, social, or political factors influencing fertility rates. Bounded upper tails may suggest policies or societal changes that limit extreme fertility values. High location values (nearing 7 or 7 life offsprings per woman) and negative shape (bounded upper tails) in the early 1960s to lower location value (nearing 2 or 2 children per woman) with negative shape (bounded upper tail) may indicate the gradual change from natural fertility moving towards more active fertility limitations. Stabilization in the 1980s could be linked to socio-economic developments or family planning initiatives.

TABLE 3-4 Parameters of GEV Distribution by Year

Year	Location	Location SE	Scale	Scale SE	Shape	Shape SE	Statistics	P-value
1960	6.8086	0.2789	1.1294	0.2750	-0.9117	0.1838	0.1697	0.5557
1961	6.8240	0.2829	1.1390	0.2781	-1.1020	0.1765	0.1662	0.5816
1962	6.8862	0.2889	1.0768	0.2818	-1.0780	0.1694	0.1833	0.4586
1963	6.8582	0.2912	1.1228	0.2911	-0.9912	0.1577	0.1567	0.6538
1964	6.8456	0.3075	1.1924	0.2983	-1.1000	0.1438	0.1574	0.6484
1965	6.8228	0.3142	1.2552	0.3017	-1.0070	0.1342	0.1511	0.6959
1966	6.8081	0.3251	1.3189	0.3152	-1.0050	0.1255	0.1496	0.7074
1967	6.7828	0.3356	1.3832	0.3187	-1.0450	0.1162	0.1575	0.6473
1968	6.6833	0.3476	1.5447	0.3284	-0.9953	0.0955	0.1575	0.6473
1969	6.6132	0.3518	1.5770	0.3363	-0.9835	0.0841	0.1575	0.6473
1970	6.5663	0.3526	1.5856	0.3463	-0.9455	0.0710	0.1575	0.6473
1971	6.4717	0.3869	1.6014	0.3849	-0.8862	0.2078	0.1575	0.6473
1972	6.4816	0.3894	1.6110	0.3667	-0.8195	0.1928	0.1575	0.6473
1973	6.2702	0.4037	1.6721	0.3745	-0.8085	0.1865	0.1575	0.6473
1974	6.1752	0.4074	1.6888	0.3637	-0.7641	0.1746	0.1575	0.6473
1975	6.0974	0.3929	1.6265	0.3309	-0.6781	0.1626	0.1415	0.7676
1976	6.0188	0.3993	1.6517	0.3293	-0.6494	0.1577	0.1384	0.7890
1977	5.9238	0.4054	1.6765	0.3252	-0.6129	0.1521	0.1287	0.8537
1978	5.8280	0.0060	1.6996	0.0017	-0.4699	0.000002	0.1031	0.9687
1979	5.7143	0.4134	1.7058	0.3132	-0.5259	0.1413	0.0835	0.9969
1980	5.5869	0.4158	1.7138	0.3082	-0.4908	0.1377	0.1237	0.8516
1981	5.4626	0.4184	1.7224	0.3044	-0.4587	0.1350	0.1176	0.7526
1982	5.3358	0.4147	1.7045	0.2965	-0.4190	0.1330	0.1380	0.7922
1983	5.2018	0.4084	1.6763	0.2876	-0.3791	0.1308	0.1446	0.7445
1984	5.0580	0.4005	1.6407	0.279	-0.3392	0.1298	0.1407	0.7731
1985	4.9065	0.3920	1.6033	0.271	-0.3021	0.1293	0.1304	0.8428
1986	4.7497	0.3819	1.5602	0.2622	-0.2666	0.1287	0.1229	0.8875
1987	4.5849	0.3674	1.4966	0.2521	-0.2267	0.1302	0.1222	0.8918
1988	4.4256	0.3492	1.4173	0.2404	-0.1837	0.1340	0.1195	0.9058
1989	4.2652	0.3324	1.3443	0.2302	-0.1456	0.1382	0.1036	0.9675
1990	4.1943	0.3272	1.3498	0.2285	-0.1453	0.1427	0.0665	0.9999
1991	4.0095	0.3144	1.2888	0.2218	-0.1073	0.1514	0.0649	0.9999
1992	3.8591	0.2966	1.2046	0.2128	-0.0633	0.1639	0.0941	0.9835
1993	3.7072	0.2844	1.1545	0.2049	-0.0417	0.1655	0.1246	0.8610
1994	3.5750	0.2752	1.1206	0.1977	-0.0389	0.1625	0.1240	0.8644
1995	3.3943	0.2630	1.0671	0.191	-0.0099	0.1682	0.1208	0.8836
1996	3.2513	0.2373	0.9351	0.1806	0.0691	0.2035	0.1223	0.8745
1997	3.1263	0.2333	0.9151	0.1782	0.0724	0.2082	0.1230	0.8706
1998	3.0221	0.2312	0.9084	0.1754	0.0573	0.2054	0.1239	0.8649
1999	2.9321	0.2299	0.9096	0.1727	0.0401	0.1975	0.0665	0.9999

2000	2.8598	0.2246	0.8901	0.168	0.0323	0.1950	0.1188	0.8948
2001	2.8130	0.2303	0.9262	0.1675	-0.0337	0.1779	0.0995	0.9855
2002	2.7524	0.2234	0.9008	0.1617	-0.0404	0.1747	0.0912	0.9880
2003	2.7136	0.2135	0.8617	0.1544	-0.0356	0.1729	0.1033	0.9615
2004	2.6563	0.2098	0.8513	0.1508	-0.0398	0.1670	0.0990	0.9734
2005	2.5930	0.2041	0.8263	0.1474	-0.0303	0.1702	0.1129	0.9246
2006	2.5400	0.2007	0.8088	0.1458	-0.0271	0.1763	0.0932	0.9850
2007	2.4996	0.1983	0.7941	0.1449	-0.0297	0.1838	0.0930	0.9854
2008	2.4634	0.1915	0.7552	0.1426	-0.0023	0.2009	0.0845	0.9951
2009	2.4377	0.1904	0.7470	0.1419	-0.0123	0.2060	0.0913	0.9880
2010	2.4407	0.1956	0.7700	0.1443	-0.0583	0.2025	0.1097	0.9385
2011	2.4355	0.1878	0.7360	0.1393	-0.0410	0.2066	0.1279	0.8397
2012	2.4491	0.1883	0.7438	0.138	-0.0931	0.1982	0.1248	0.8597
2013	2.4531	0.1893	0.7555	0.1371	-0.1479	0.1879	0.1134	0.9224
2014	2.4532	0.1870	0.7479	0.1353	-0.1753	0.1865	0.1086	0.9429
2015	2.4413	0.1859	0.7453	0.1347	-0.2122	0.1860	0.1036	0.9606
2016	2.4100	0.1791	0.7232	0.1293	-0.2372	0.1781	0.1186	0.8959
2017	2.3955	0.1779	0.7264	0.1283	-0.2916	0.1665	0.1257	0.8538
2018	2.3555	0.1767	0.7235	0.1278	-0.3046	0.1644	0.1212	0.8813
2019	2.3025	0.1771	0.7262	0.1282	-0.3181	0.1628	0.1249	0.8592
2020	2.2689	0.1693	0.6924	0.1222	-0.3014	0.1649	0.1200	0.8879
2021	2.2484	0.1663	0.6782	0.1212	-0.3130	0.1706	0.0853	0.9945

CHAPTER 4

STATISTICAL MODELING OF FERTILITY IN MENA

In the context of the Middle East and North Africa (MENA) region, where demographic transitions are rapid, statistical modeling serves as an essential tool to analyze and predict these transitions. Statistical modeling offers a systematic and structured approach to unravel the complexities embedded in fertility dynamics.

This chapter embarks on a journey to explore and analyze fertility patterns in the MENA region through the lens of statistical modeling. The overarching goal is to provide a nuanced understanding of the fertility trends over time in MENA.

In the broader context of demographic research, statistical modeling acts as a compass, guiding researchers through the intricate terrain of fertility dynamics. This chapter thus serves as a gateway to unraveling the complex tapestry of fertility rates in the MENA region, showcasing the pivotal role statistical modeling plays in enhancing our understanding of demographic phenomena.

4.1 Methodology

The initial phase of the chapter involves employing regression analysis, a cornerstone of statistical modeling, to establish relationships between fertility rates and time. However, the assumptions underpinning regression analysis are not always universally met, and deviations from these assumptions may compromise the reliability of the results.

Recognizing the limitations and potential pitfalls of regression models, the approach is fortified with a robust examination of the assumptions, ensuring the validity and integrity

of the findings. Should the assumptions be violated or if the data exhibit temporal dependencies, necessitating a more dynamic analysis, the approach will pivot towards time series analysis. Time series methods enable to capture the temporal structure inherent in fertility rate data, providing a more nuanced understanding of how these rates evolve over time.

4.1.1 Regression Modeling

In essence, regression analysis serves as a powerful tool in the statistical modeling toolkit, offering a systematic and data-driven approach to understanding the intricacies of fertility dynamics in the MENA region.

Using the average *TFR* data that was collected from World Bank, MENA average *TFR* will be regressed overtime to understand the trends and dynamics. In the model the dependent variable (*y*) is the *TFR* of the countries of MENA and the predictor (*x*) is the year of that *TFR* for each country. The model will be as:

$$TFR = \beta_0 + \beta_1 \times \text{year} + \epsilon$$

Here, β_0 is the intercept, β_1 is the year coefficient, and ϵ represents the error term.

After conducting the regression model, the assumptions of the regression model will be checked. Assumptions of Regression Modeling:

- **Linearity:** The relationship between the dependent variable and independent variables is linear. Testing for linearity in regression involves assessing whether the relationship between the dependent and independent variables is adequately captured by a linear model. One common way to check for linearity is by examining residual plots where a scatterplot of residuals is created against the fitted values. In a linear relationship, it is expected to see a random scatter of points around the horizontal axis with no clear

pattern. A non-linear relationship might be indicated by a systematic pattern or curvature in the residuals. While visual inspection is often informative, statistical tests can also be performed to test for linearity. One common approach is the RESET (Regression Specification Error Test) test is used to examine whether there are omitted nonlinearities in a regression model, which might indicate a lack of fit. The null hypothesis of the RESET test is that the model is correctly specified, meaning it includes all relevant linear terms. A small *p-value* may suggest a lack of linearity. If the test indicates a lack of linearity in the regression model, several strategies may be considered to address this issue such as transforming variables, using splines, etc.

- **Normality of Residuals:** The residuals should follow a normal distribution. This assumption is essential for hypothesis testing and constructing confidence intervals. Normality will be tested visually using histogram and QQ plot. In the QQ plot, points should approximately follow a straight line. Departures from the line may indicate departures from normality. The Shapiro-Wilk test will be also used to test for normality. The Shapiro-Wilk test is a formal statistical test for normality. The null hypothesis is that the data follows a normal distribution. A low *p-value* suggests rejection of the normality assumption. Transformation can be considered on the dependent variable to fix the departure from normality.
- **Homoscedasticity:** Residuals should exhibit constant variance across all levels of the independent variables. Visually, homoscedasticity can be assessed from the residuals vs. fitted values plot. The goal is a random scatter of points around the horizontal line at $y = 0$. If there is a funnel shape or any pattern observed, it may suggest heteroscedasticity. The Breusch-Pagan test will also be used to confirm

homoscedasticity. The Breusch-Pagan test is a diagnostic test designed to detect heteroscedasticity in the residuals of a regression model. The test involves estimating a regression model and then testing whether the variance of the residuals is related to the independent variables. The steps involve fitting the original regression model and obtaining the residuals. Then, a secondary regression model is estimated, where the dependent variable is the squared residuals from the original model, and the independent variables are the same as in the original model. The test statistic is based on the idea that, under the null hypothesis of homoscedasticity, the coefficients in this secondary model should not be significantly different from zero $H_0: \beta_1 = \beta_2 = \dots = 0$. The test statistic follows a chi-square distribution with degrees of freedom equals to the number of the independent variables in the regression model, and if the *p-value* is below a chosen significance level, it suggests rejecting the null hypothesis of homoscedasticity in favor of the alternative hypothesis of heteroscedasticity.

- **Independence of Errors:** Residuals (the differences between observed and predicted values) should be independent of each other. The Durbin-Watson test will be used to test for independence of errors. The Durbin-Watson test statistic ranges from 0 to 4. A value around 2 indicates no autocorrelation (independence of errors). Values significantly different from 2 may suggest autocorrelation.

4.1.2 Time Series Modeling

Time series is defined as a sequence of observations measured at regular time intervals (Box, George, Jenkins, & Reinsel, 2008). The MENA average *TFR* data is considered a time series data. A major assumption of many regression methods is uncorrelated residuals. This assumption makes the structure of time series data challenging for researchers. Within

a single time series, it becomes evident that data points taken in close temporal proximity exhibit greater similarity than those farther apart, leading to correlated residuals, a phenomenon known as "autocorrelation." Models that overlook autocorrelation may produce accurate parameter estimates but incorrect standard errors.

Autoregressive Integrated Moving Average (ARIMA) model is a type of model that accounts for autocorrelation and it is applied using a methodology developed by George Box and Gwilym Jenkins in 1970. The application of ARIMA model requires the formal characterization of the inherent pattern in a time series to be able to forecast future behavior of the time series using this pattern. It can be defined as:

$$y_t^{(d)} = c + \varepsilon_t + \phi_1 y_{t-1}^{(d)} + \phi_2 y_{t-2}^{(d)} + \cdots + \phi_p y_{t-p}^{(d)} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

The MENA average *TFR* data meets the data requirements of ARIMA model as it has a univariate time series with at least 50 observations. The interval over which the count is taken is the same over time (1 year). Data is presented in a vertical vector (column of data). An essential requirement for ARIMA models is that the dataset in question should exhibit stationarity, indicating that it maintains a consistent mean and variance across time. If the dataset is not stationary from the outset, stationarity can be achieved through a technique known as "differencing."

The ARIMA model decomposes the time series pattern into three elements:

- The autoregressive component denoted as "p," which characterizes the relationship between observations due to their proximity in time.
- The differencing component, marked as "d," used to render the time series stationary

- The moving average component, referred to as "q," which explains external disturbances or "shocks" to the system.

The Box-Jenkins methodology involves a sequential process consisting of the following three stages:

1. **Identification:** In this initial step, a class of straightforward ARIMA models is chosen based on data visualizations, autocorrelations, partial autocorrelations, and other relevant information. This entails determining suitable values for p, d, and q.
2. **Estimation:** Following the selection of the model, the parameters (phis and thetas) for the chosen model are estimated using techniques like maximum likelihood estimation, back casting, and others outlined in the Box-Jenkins (1976) methodology.
3. **Diagnostic Checking:** The model obtained in the previous step is subjected to a diagnostic assessment to identify any deficiencies. This evaluation involves examining the autocorrelations of the residual series, which represents the series of residual or error values using Ljung-Box test.

After applying these steps, the final model can be used to forecast the average *TFR* of future years.

4.2 Results

4.2.1 Regression Modeling

Table 4-1 shows the summary of the model. The model is:

$$TFR = 175.314234 - 0.08574 \text{ year}$$

The negative coefficient for "year" implies that, on average, the average *TFR* in the MENA region decreases by 0.0857 units each year. This suggests a declining trend in fertility rates over the years.

The highly significant *p-values* indicate that the relationship between "year" and *TFR* is unlikely to be due to random chance.

The *R-squared* value of 0.5826 indicates that the model explains a substantial portion of the variability in *TFR*. However, there are other factors not included in the model that contribute to the remaining variability.

The F-statistic's low *p-value* supports the overall significance of the model.

In summary, the regression suggests a statistically significant negative relationship between the year and average *TFR* in the MENA region, indicating a declining trend in fertility rates over time.

After conducting the regression model, the assumptions of the regression model are checked:

- **Linearity:** Figure 4-1 shows the Residuals vs. Fitted Values plot. The scatter of points around the horizontal axis is not completely random. There is curvature in the residuals which might indicate the violation of the linearity assumption. To confirm further, statistical test will be used. The RESET (Regression Specification Error Test) test is used to examine whether there are omitted nonlinearities in a regression model, which might indicate a lack of fit. The null hypothesis of the RESET test is that the model is correctly specified, meaning it includes all relevant linear terms. 4-2 shows that the *p-value* is very close to zero ($1.309e-12$), which is well below common significance levels like

0.05. Therefore, the null hypothesis is rejected and there is evidence of omitted nonlinearities in the regression model. This suggests that the model may not adequately capture the relationship between the *TFR* and the year, indicating the need for further investigation or refinement of the model.

- **Normality of Residuals:** Figures 4-2 and 4-3 represent the histogram and Q-Q plot of residuals. Both graphs show departure from normality. The Shapiro-Wilk test result in Table 4-2 confirms that. The low p -value ($5.768\text{e-}11$, which is close to zero) suggests strong evidence against the null hypothesis. The Shapiro-Wilk test indicates that the residuals of your regression model are not normally distributed. This violation of normality might suggest that the assumption of normally distributed errors is not met.
- **Homoscedasticity:** from the residuals vs. fitted values plot (Figure 4-1), it can be observed that the residuals follow a pattern which might indicate heteroscedasticity. The result of Breusch-Pagan test confirms that. The low p -value ($4.284\text{e-}09$, which is close to zero) suggests strong evidence against the null hypothesis. Since the p -value is very small, the null hypothesis is rejected. The studentized Breusch-Pagan test indicates that there is evidence of heteroscedasticity in the residuals of the regression model. Heteroscedasticity means that the variance of the residuals is not constant across all levels of the independent variable (year).
- **Independence of Errors:** The p -value of 0 of Durbin Watson test (Table 4-2) suggests strong evidence against the null hypothesis. Therefore, there is evidence of first-order autocorrelation in the residuals of the regression model.

Since all regression assumptions are violated, it can have several consequences, including biased parameter estimates, incorrect standard errors, unreliable hypothesis tests, and unreliable predictions. It may be challenging to obtain reliable results from the current model, that is why alternative models should be considered.

TABLE 4-1 Regression Model Summary

Model	TFR ~ year	
Multiple <i>R</i>-squared	0.5826	
Adjusted <i>R</i>-squared	0.5822	
F-static	1814	
<i>P</i>-value	<2.2e-16	
Coefficients	(Intercept)	Year
Estimate	175.314234	-0.085740
Std. Error	4.006918	0.002013
Pr(> t)	<2e-16	<2e-16

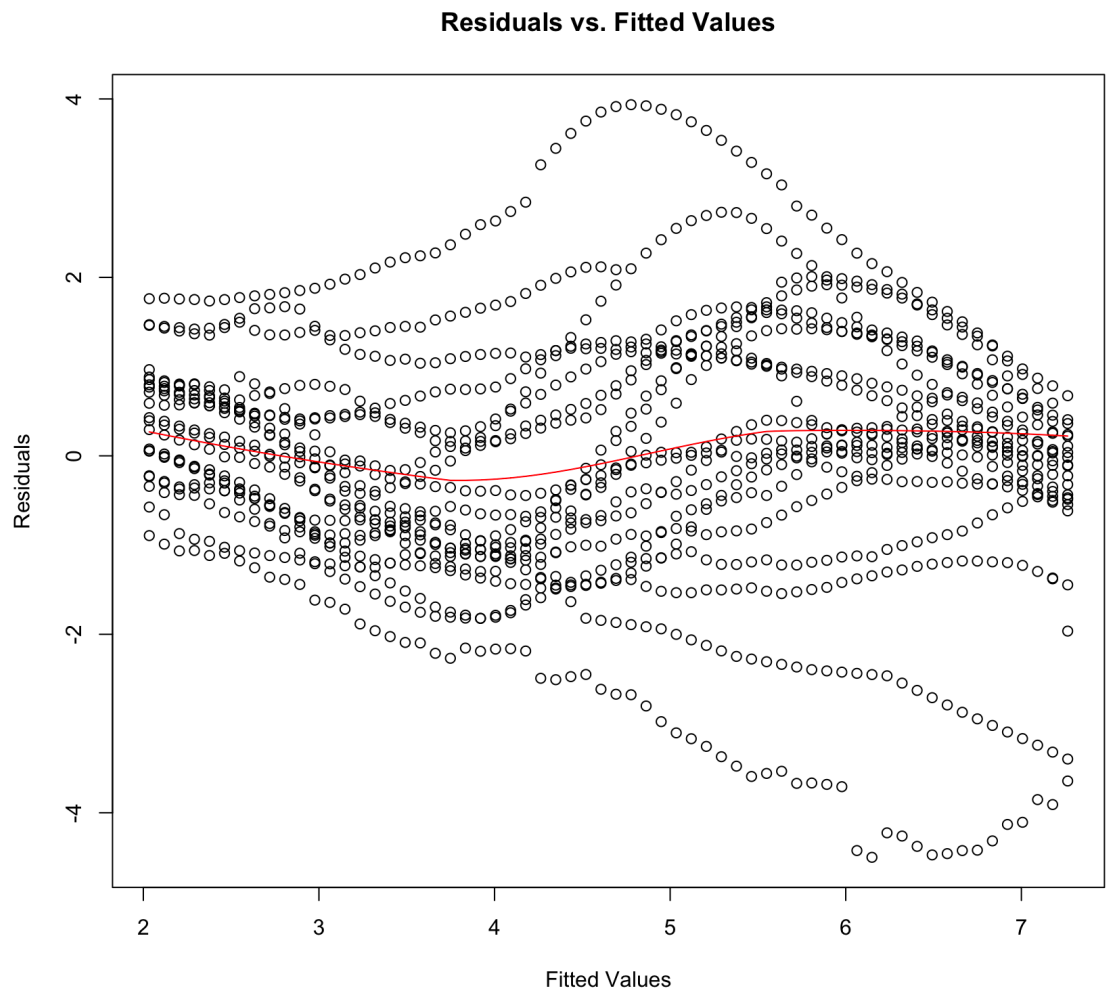


Figure 4-1 Residuals vs. Fitted Values Plot

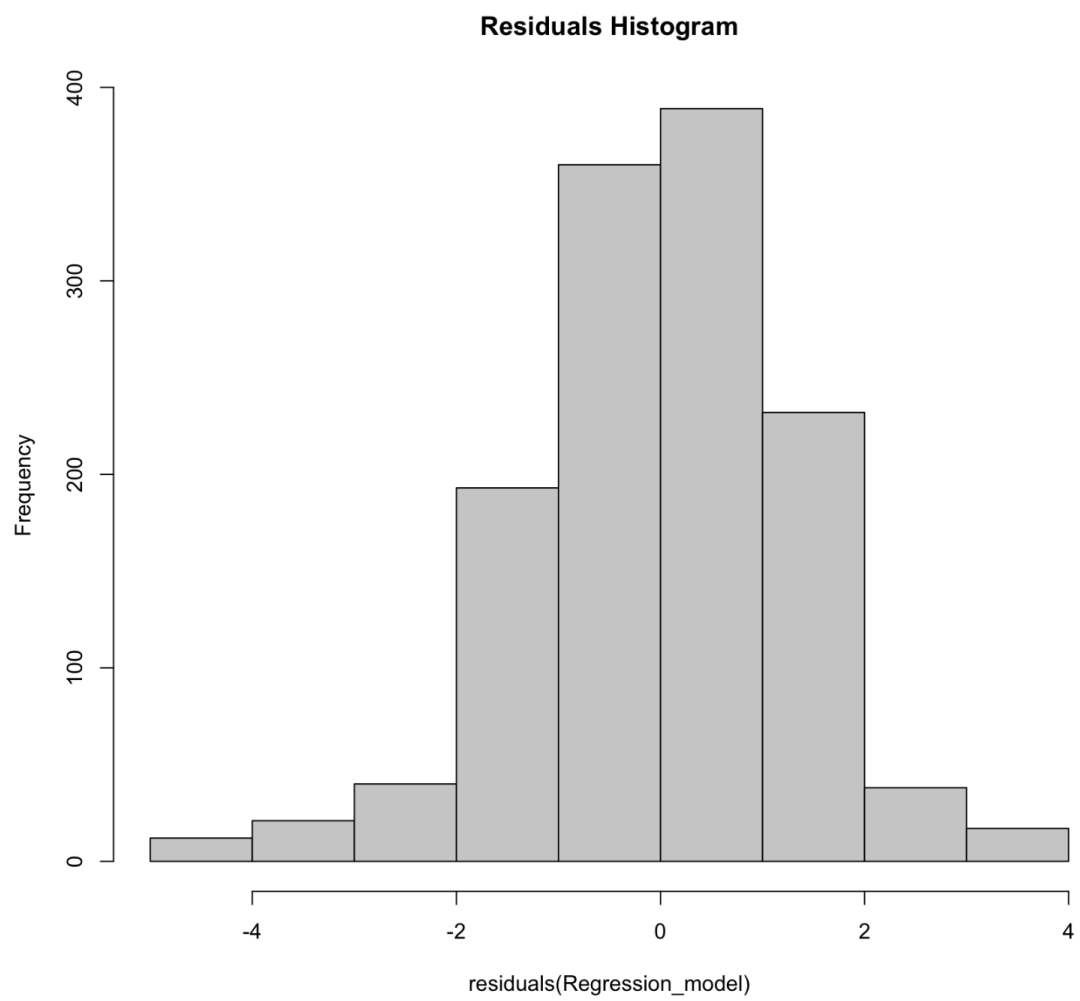


Figure 4-2 Histogram of Residuals

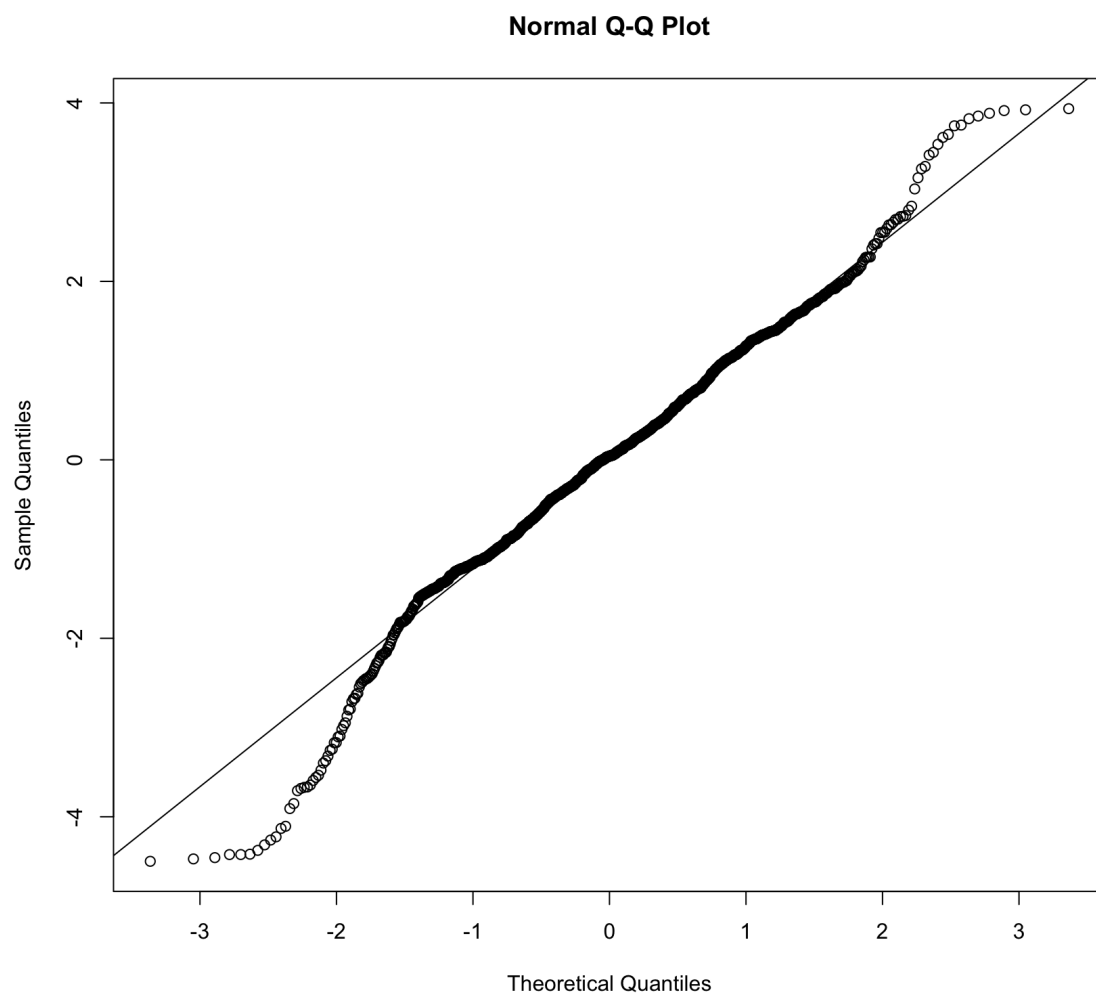


Figure 4-3 Q-Q Plot

TABLE 4-2 Test Results of Regression Model Assumptions

Assumption	Test	<i>P</i>-value
Linearity	RESET test	1.309e-12
Normality of residuals	Shapiro-Wilk normality test	5.768e-11
Homoscedasticity	studentized Breusch-Pagan test	4.284e-09
Independence of errors	Durbin Watson Test	0

4.2.2 Time Series Modeling

Applying the Box-Jenkins Methodology to MENA average *TFR* data:

Step 1: Model Identification

Using R Programming, the MENA average *TFR* will be defined as time series object starting on 1960 and ending on 2021 with frequency = 1. Next, the original series is plotted (Figure 4-4). Based on the average *TFR* Data plot, there is significant downward trend from 1960 until 2021. Since there is a trend component in the series, the series is not stationary. Next, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) for the MENA average *TFR* are plotted (Figure 4-5). Based on Autocorrelation function (ACF) for the MENA average *TFR*, the ACF is slowly declining as the number of lags increase suggesting that the MENA average *TFR* is not stationary. Also, based on Partial Autocorrelation function (PACF) of the MENA average *TFR*, there is one significant spike (exceeding the standard error (SE) band). This is a signal that the first differencing can be performed to turn the MENA average *TFR* into stationary series.

Figure 4-6 shows the series after the first difference is performed to the MENA average *TFR* data. Based on the ACF of the first difference, the rate of decline is much faster which the changes from positive autocorrelation at lag 11. Meanwhile, for PACF shows significant spike at lag one (exceeded the SE bands). Although, the trend has declined, the time series is still not considered stationary after the first differencing. Accordingly, the data needs to be differenced one more time to achieve stationary. Figure 4-7 shows the ACF and PACF after second differencing and it is clear now that the stationary is accomplished.

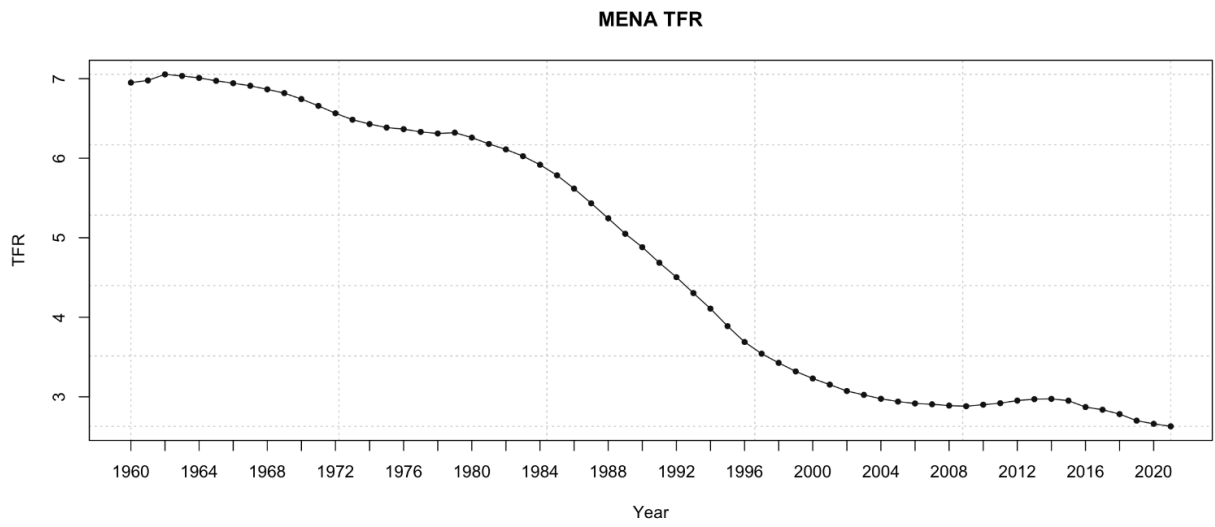


Figure 4-4 Plot of MENA TFR 1960 - 2021

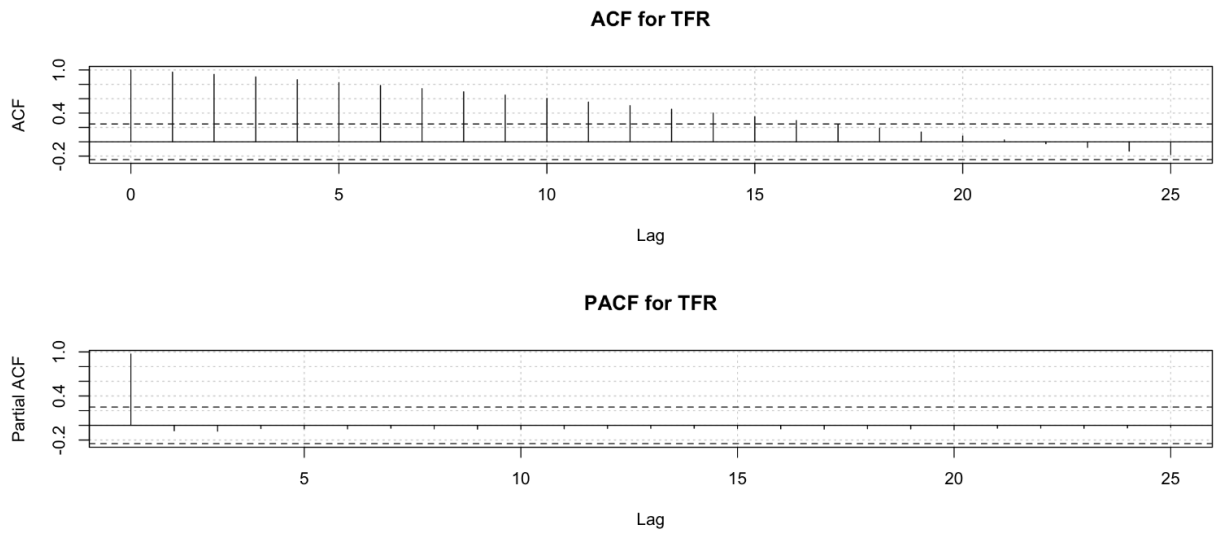


Figure 4-5 ACF and PACF Plots

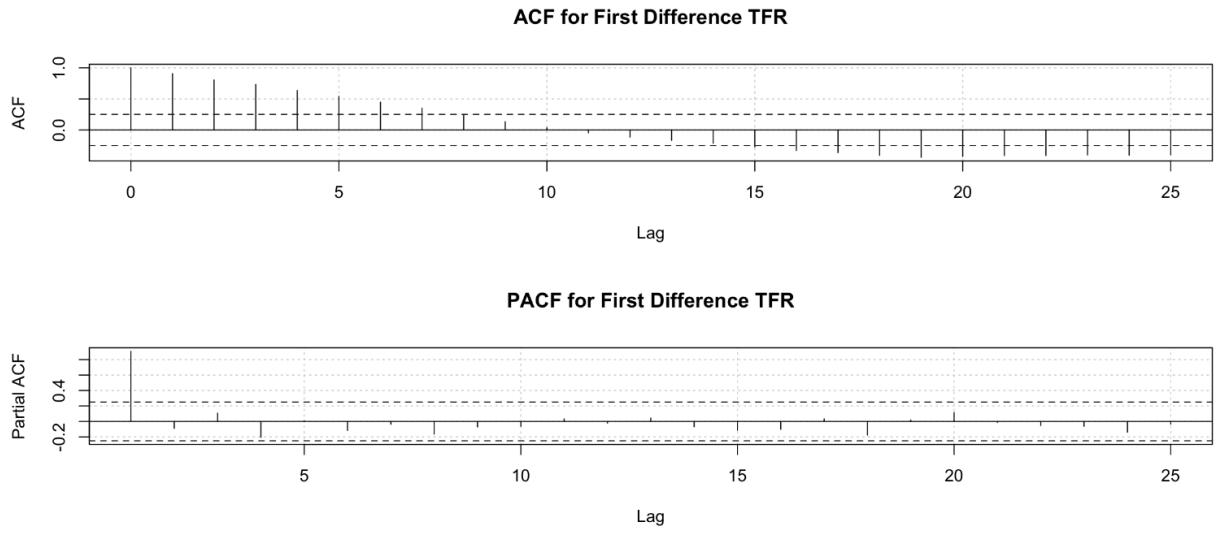


Figure 4-6 First Difference of ACF and PACF

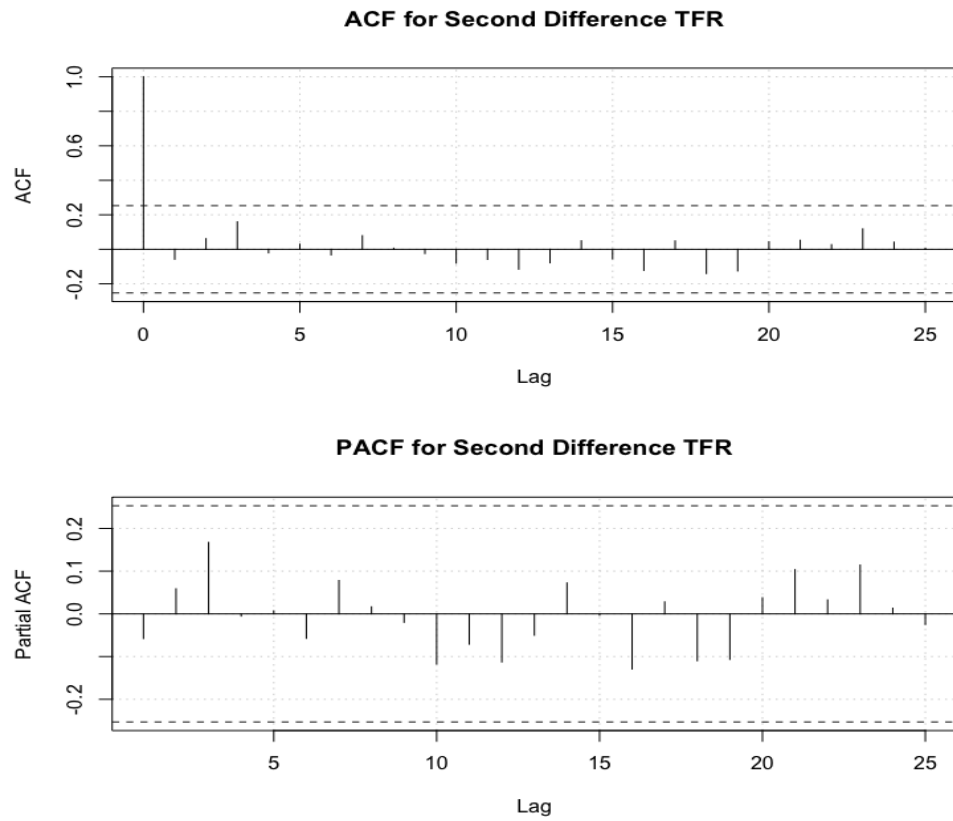


Figure 4-7 Second Difference of ACF and PACF

Stage 2: Model Validation and Estimation

Based on the ACF and PACF after differencing the original MENA average *TFR* data, the order of ARIMA model can be identified. In the context of an ARIMA model, the notation (0,2,0) specifies the orders of the components within the model.

- Autoregressive Order (p): The first number (0 in this case) refers to the autoregressive order, denoted as 'p.' This component represents the number of lagged observations included in the model. In an (0,2,0) model, there are no autoregressive terms, meaning the current value of the time series is not linearly dependent on any past values.
- Integrated Order (d): The second number (2 in this case) is the integrated order, denoted as 'd.' This component represents the number of times the time series needs to be differenced to achieve stationarity. In an (0,2,0) model, the time series has been differenced twice.
- Moving Average Order (q): The third number (0 in this case) is the moving average order, denoted as 'q.' This component represents the number of lagged forecast errors included in the model. In an (0,2,0) model, there are no moving average terms.

Thus, it can be concluded that the suitable model based on ACF and PACF is ARIMA (0, 2, 0). So, an ARIMA (0,2,0) model means that the time series has been differenced twice to achieve stationarity, and the model does not include autoregressive or moving average terms. The notation (0,2,0) suggests a simple differencing approach without considering the influence of past observations or forecast errors.

Below is the summary of the ARIMA (0,2,0) model:

Coefficients:

- ar1: The coefficient for the first autoregressive term is approximately 1.9587.
- ar2: The coefficient for the second autoregressive term is approximately -0.9618.
- intercept: The intercept term is approximately 4.5667.

Standard Errors (s.e.):

Standard errors are provided for each coefficient estimate:

- 0.0265 for ar1.
- 0.0271 for ar2.
- 0.9067 for the intercept.

Variance (σ^2):

The estimated variance (σ^2) of the residuals is approximately 0.0006792.

Log Likelihood and AIC:

Log likelihood is 132.69, and the AIC (Akaike Information Criterion) is -257.39. A lower AIC suggests a better-fitting model, so in this case, the model is evaluated favorably.

Training Set Error Measures:

- ME (Mean Error): Approximately -0.0022
- RMSE (Root Mean Squared Error): Approximately 0.0261
- MAE (Mean Absolute Error): Approximately 0.0197
- MPE (Mean Percentage Error): Approximately -0.0546
- MAPE (Mean Absolute Percentage Error): Approximately 0.4582
- MASE (Mean Absolute Scaled Error): Approximately 0.2534
- ACF1 (Autocorrelation of Residuals at Lag 1): Approximately -0.0881

These error measures provide insights into how well the model fits the training data. Overall, a lower RMSE, MAE, MPE, MAPE, and MASE, and a close-to-zero ACF1 indicate a better fit. The negative sign in front of ME, MPE, and ACF1 suggests a slight underestimation in the model.

To obtain more conclusive evidence, several alternatives will be considered. For example, ARIMA (1, 1, 0), ARIMA (1, 1, 1) and ARIMA (3, 1, 0) can be chosen as competing alternatives. Table 4-3 shows the summaries of these models.

In addition to the competitive models, ARIMA model has been fit to each country in MENA separately (Table 4-4). ARIMA (0,2,0) is the most repeated best model for countries in MENA which shows consistency between member countries of MENA and MENA as a whole.

TABLE 4-3 Time Series ARIMA Competing Models

Model	ARIMA (0,2,0)	ARIMA(1,1,0)	ARIMA(1,1,1)	ARIMA(3,1,0)
Coefficients	NA	ar1=0.9526	ar1=0.9553, ma1= -0.0337	ar1=0.9085, ar2=0.1532, ar3=-0.1107
AIC	-261.07	-262.99	-261.07	-259.63
ME	-0.001051859	-0.003982673	-0.003944315	-0.003944798
RMSE	0.02660994	0.02639741	0.02638337	0.02624779
MAE	0.02009083	0.02010184	0.02024068	0.01982156
MPE	0.02256638	-0.04937207	-0.04702531	-0.05032577
MAPE	0.479026	0.4702958	0.4731494	0.4634338
MASE	0.2589237	0.2590657	0.260855	0.2554536
ACF1	-0.06787385	-0.0384383	-0.004001433	-0.01102234

TABLE 4-4 ARIMA Model for Each Country in MENA

Country	ARIMA
Malta	(0,1,0)
West Bank and Gaza	(0,2,0)
Djibouti	(0,2,0)
Algeria	(0,2,0)
Jordan	(0,2,0)
Morocco	(0,2,0)
Saudi Arabia	(0,2,0)
Syria	(0,2,0)
Tunisia	(0,2,0)
Israel	(0,2,1)
Libya	(0,2,1)
Bahrain	(0,2,2)
Egypt	(1,1,0)
Qatar	(1,1,1)
Iraq	(1,2,0)
Oman	(1,2,0)
Yemen	(1,2,0)
UAE	(1,2,1)
Lebanon	(1,2,1)
Iran	(2,1,0)
Kuwait	(3,2,0)
India	(3,1,0)
Germany	(0,2,1)
USA	(0,2,2)
Brazil	(0,2,1)
Nigeria	(0,2,2)
Australia	(1,2,1)

Step 3: Model diagnostic

Based on the AIC criterion and the magnitude of ACF1, ARIMA(1, 1, 0) appears to be a slightly better fit than ARIMA(0, 2, 0) for the provided data. Both models have similar performance in terms of error measures. The differences in ME, RMSE, MAE, MPE, MAPE, and MASE are relatively small. The ACF1 value provides information about the autocorrelation of the residuals at lag 1. Both models have negative ACF1 values, indicating some level of underestimation in both cases. However, the magnitude is higher in ARIMA(0, 2, 0), suggesting a potentially better fit. ARIMA (0,2,0) provides a better trade-off between model complexity and goodness of fit.

In addition, based on Ljung-Box test, which resulted in p -value of 0.5841, there is not enough evidence to reject the null hypothesis indicated that the residuals are white noise. Figure 4-8 shows the forecasted average *TFR* using ARIMA (0,2,0) and the actual average *TFR* from 2016-2020.

To allow for a quick assessment of how well the forecasting model performed for each year, Table 4-5 compares the actual average *TFR* with the forecasted average *TFR* for the years 2016 to 2020, along with the difference between the two values.

- Actual Average *TFR*: This column represents the observed or real average *TFR* for each respective year.
- Forecasted Average *TFR*: This column shows the average *TFR* values predicted by the forecasting model for the corresponding years.
- Difference: This column calculates the variance between the actual and forecasted average *TFR*. It indicates how much the forecasting model deviated from the actual values.

A negative difference indicates that the forecasted values are lower than the actual values.

For all years, the forecasted *TFR* is lower than the actual *TFR*.

For example, in 2016, the actual *TFR* was 2.8712, but the forecasted *TFR* was 2.9289. The difference is -0.0578, indicating that the model underestimated the *TFR* by approximately 0.058. The magnitude of the underestimation increases from 2016 to 2020.

Since the differences are consistently small and within an acceptable range, it can suggest a reasonably accurate model.

The increasing discrepancies over the years might indicate that the forecasting model may not be capturing some of the more complex or dynamic aspects of fertility trends as the forecast horizon extends. It's not uncommon for forecasting accuracy to decrease as the projection period becomes more distant from the observed data, as there is greater uncertainty about future conditions and events that could influence the outcome. It might be worth exploring whether the model needs adjustments or if additional factors could enhance its predictive performance.

TABLE 4-5 Difference Between Actual and Forecasted Average TFR 2016 - 2020

Year	Actual Average TFR	Forecasted Average TFR	Difference
2016	2.8712	2.9289	-0.0578
2017	2.8380	2.9061	-0.0681
2018	2.7830	2.8833	-0.1002
2019	2.7004	2.8605	-0.1601
2020	2.6605	2.8376	-0.1771

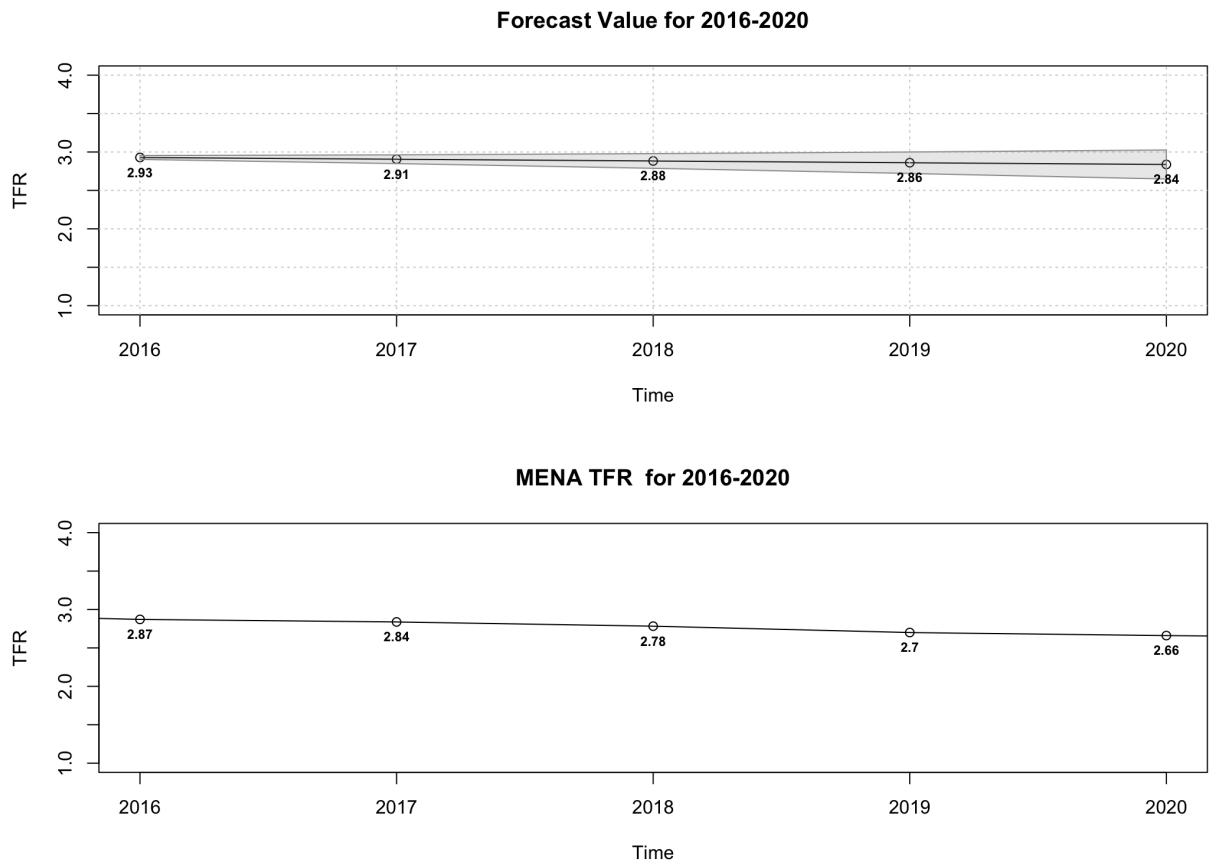


Figure 4-8 Forecasted Vs. Actual MENA Average TFR 2010-2020

Model Application and forecasting

The model ARIMA (0, 2, 0) is used to generate 5 years ahead (2022-2026). Figures 4-9, and 4-10 represent the plot of the forecasted years with the current average *TFR*, and a focus on the forecasted years respectively.

The forecast values for 2022-2026 are 2.6, 2.57, 2.53, 2.5 and 2.47. These forecasts are valuable for understanding and predicting fertility trends in the MENA region, aiding policymakers, researchers, and demographers in making informed decisions based on robust statistical analysis.

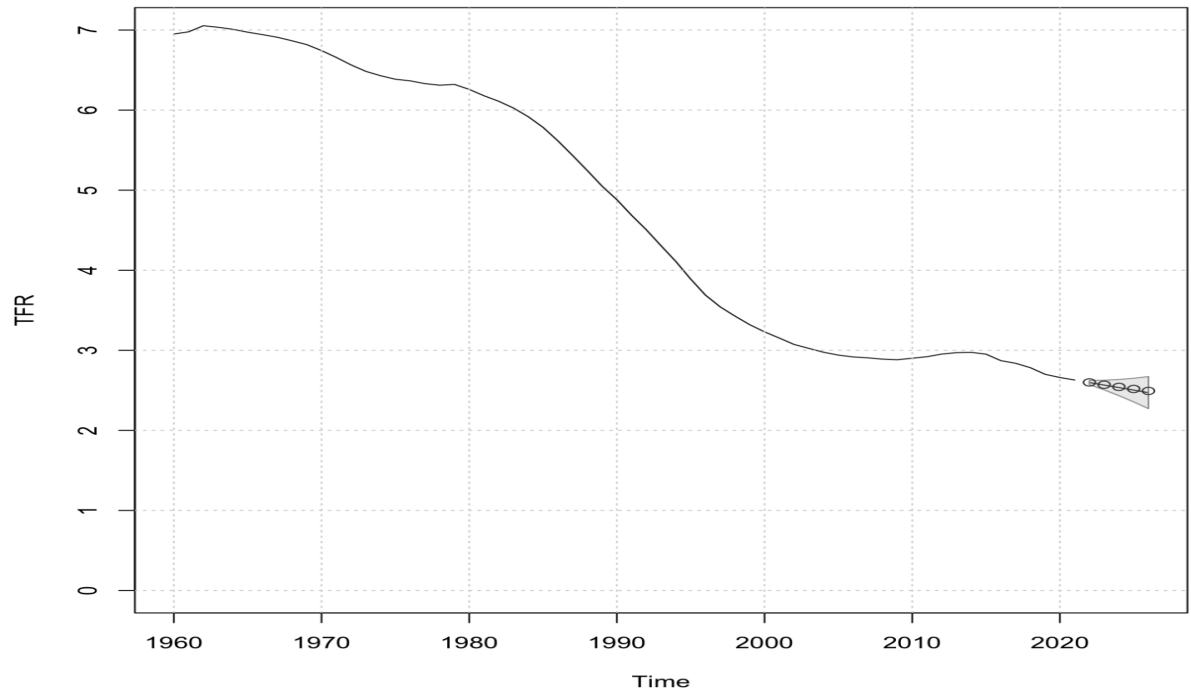


Figure 4-9 MENA TFR 1960-2026

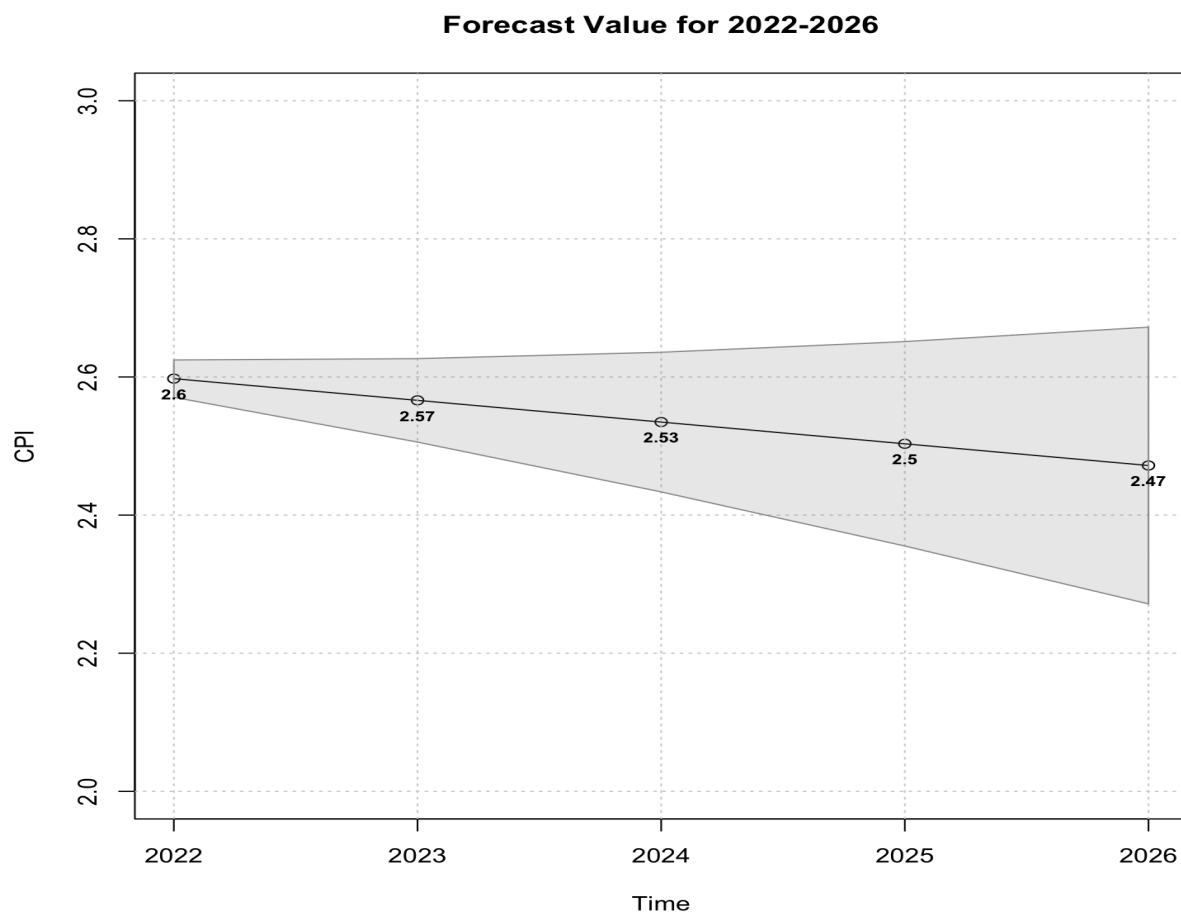


Figure 4-10 Forecasted MENA TFR 2022-2026

Benchmarking

Comparing fertility dynamics between the MENA region (Middle East and North Africa) and another region, such as Europe, can provide several valuable insights and benefits such as understanding how fertility patterns differ or resemble between regions, recognizing patterns, trends, and challenges that may have global implications, especially in the context of population growth, demographic transition, and sustainable development. In addition, benchmarking provides an opportunity to assess the robustness and generalizability of research methodologies and models. Different regions may have unique characteristics that impact the applicability of certain models and approaches. Table 4-6 shows that the best ARIMA model fitting the average *TFR* of Europe is (0,2,1). Table 4-7 shows the summary of the model. Figure 4-11 and 4-12 show the forecasted values of Europe *TFR* while Figure 4-13 shows the average *TFR* of both MENA and Europe from 1960 to 2021. When comparing the summary of Europe model to the summary of ARIMA (0,2,0) of MENA, it can be observed that:

- The model for the MENA region has a better fit compared to the model for Europe as the AIC of MENA is -261.07 while for Europe it is -246.42.
- The smaller RMSE and MAE suggest that the MENA model provides more accurate predictions for average *TFR* in the MENA region compared to the model for Europe.
- The negative ma_1 coefficient indicates a negative impact of the moving average component on the average *TFR* in Europe. This implies that past errors in prediction are influencing current average *TFR* values negatively.

- Both models have negative ACF1 values, indicating that past residuals are influencing current residuals negatively, but MENA model has a more negative ACF1 which suggests better model performance.

Considering the lower AIC, smaller RMSE and MAE, and a more negative ACF1, the ARIMA model for the MENA region appears to be a better fit for predicting *TFR* compared to the model for Europe. The negative coefficient in Europe model suggests that there might be a stronger influence of past errors on current predictions in the European context.

In summary, the decline in the average *TFR* in both the MENA region and Europe reflects significant changes in fertility behavior over the past six decades. Analyzing the specific factors contributing to these trends can provide valuable insights for policymakers and researchers in addressing demographic challenges and opportunities in each region which will be discussed in the next chapter.

TABLE 4-6 Europe ARIMA Competing Models

Model	AIC
ARIMA(0,2,1)	-246.2074
ARIMA(2,2,2)	-243.5081
ARIMA(0,2,0)	-224.5493
ARIMA(1,2,0)	-237.0416
ARIMA(1,2,1)	-244.0095
ARIMA(0,2,2)	-244.0125
ARIMA(1,2,2)	-242.3813

TABLE 4-7 Europe ARIMA (0,2,1) Summary

Model	ARIMA (0,2,1)
Coefficients	ma1 = -0.6918
AIC	-246.42
ME	-0.000122656
RMSE	0.02937969
MAE	0.02280414
MPE	0.05238161
MAPE	1.243296
MASE	0.7528384
ACF1	-0.01869559

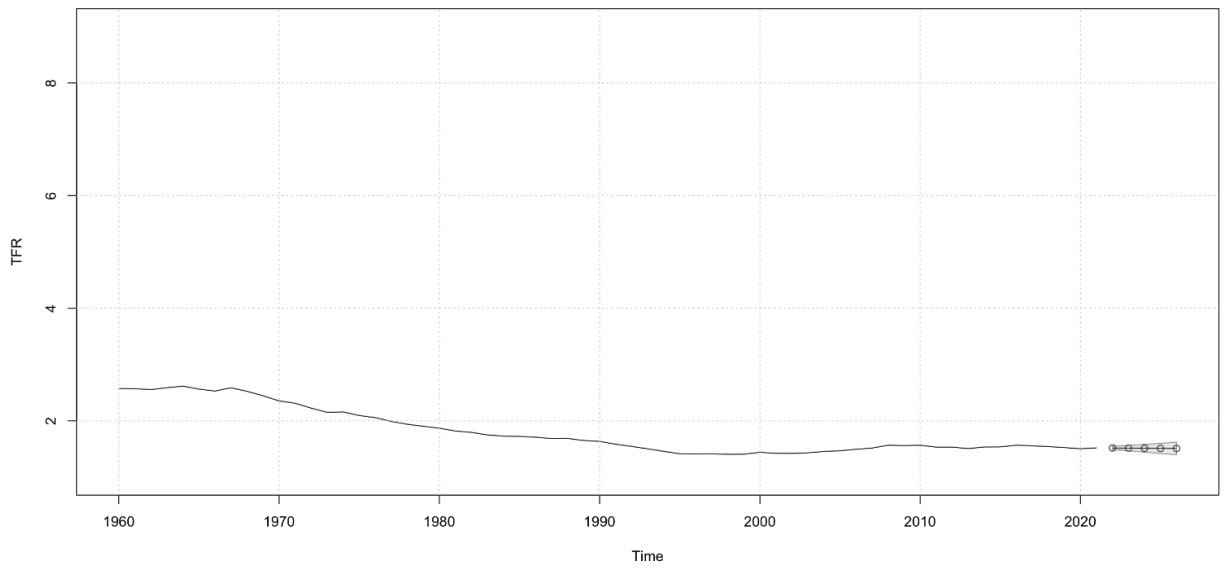


Figure 4-11 Europe TFR 1960-2026

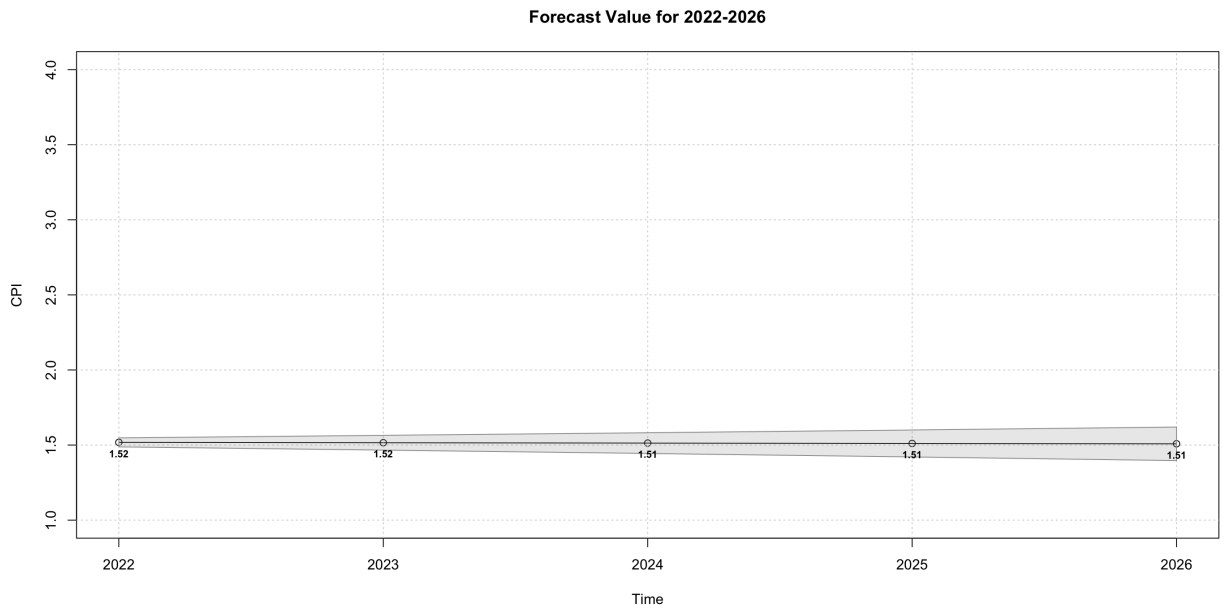


Figure 4-12 Forecasted Europe TFR 2022-2026

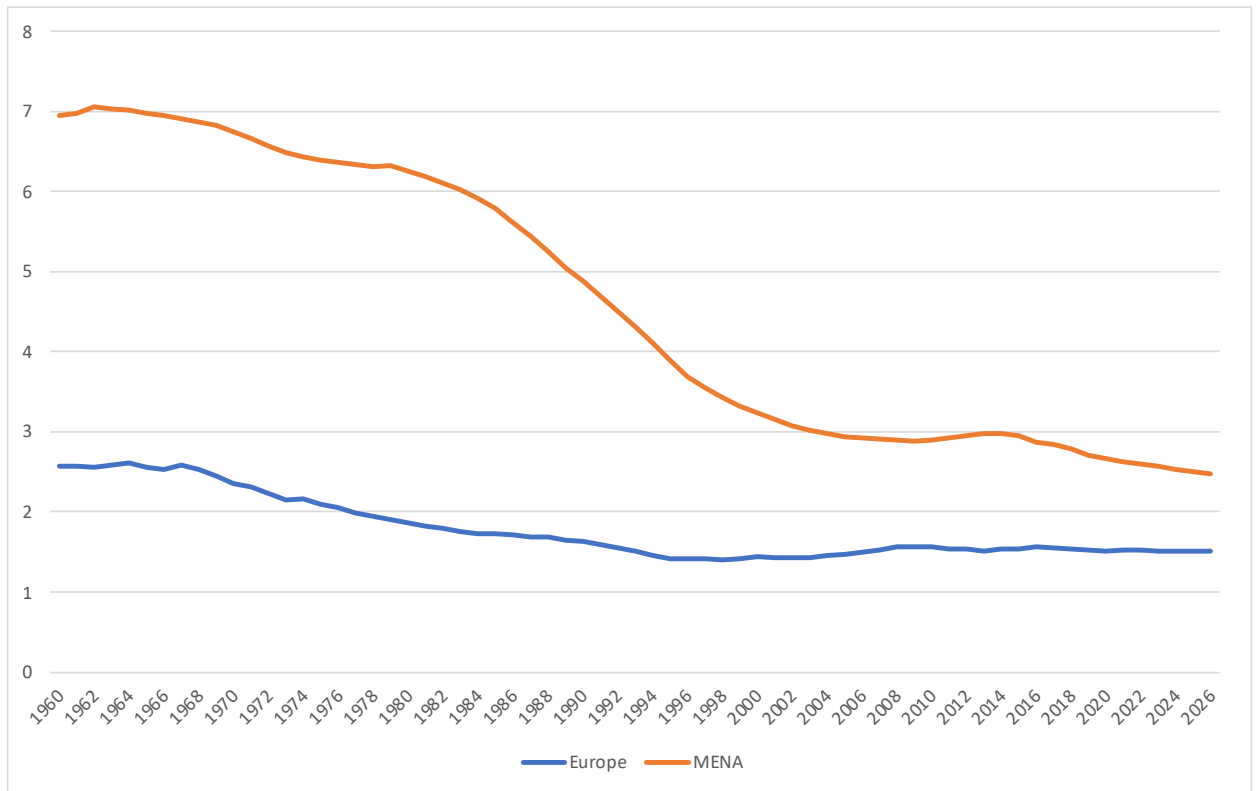


Figure 4-12 TFR of MENA Compared to Europe from 1960-2021

CHAPTER 5

DETERMINANTS OF FERTILITY IN MENA

The dynamics of fertility rates hold profound implications for demographic patterns, economic development, and social structures. In the context of the Middle East and North Africa (MENA) region, understanding the determinants of fertility is a crucial endeavor given the unique blend of socio-cultural, economic, and political factors that shape reproductive behaviors.

MENA, encompassing diverse countries with varying levels of economic development and cultural landscapes, presents an intriguing case for fertility rate studies. This region has witnessed significant socio-economic transformations over the past few decades, leading to shifts in family structures, educational attainment, and workforce participation. Consequently, fertility rates have experienced notable fluctuations, influencing population growth trajectories and demographic transitions.

This chapter seeks to unravel the complex web of factors influencing fertility rates in MENA, offering a comprehensive analysis that goes beyond traditional demographic models. By examining the interplay of socio-cultural norms, economic conditions, healthcare access, and the empowerment of women, the aim is to provide a nuanced understanding of the drivers of fertility within the region.

Understanding the determinants of fertility rates in MENA is not only essential for academic research but also carries significant policy implications. Policymakers can utilize the insights gained from this study to formulate targeted interventions aimed at addressing

specific challenges and opportunities related to population dynamics in the region. Furthermore, the findings may contribute to the broader discourse on demographic transitions, providing valuable insights for global discussions on sustainable development and population growth.

Through a multifaceted exploration of the determinants of fertility rates in MENA, this thesis aims to contribute to the existing literature and offer a nuanced perspective that considers the intricate interplay of factors shaping reproductive behaviors in this diverse and dynamic region.

5.1 Methodology

To accomplish the objective of this study, first, a systematic review to identify the factors that affect fertility rate in MENA will be conducted. After that, a regression analysis is conducted for all the factors in the presence of each other. If regression does not provide conclusive results, time series modeling will be used.

5.1.1 Systematic Review

All research articles published in the English language between the years 2015 and 2023 will be examined. After identifying the main factors, an indicator for each of the identified factors is selected. The selection of the indicators is based on the relevance and the availability of the data. All the data used are extracted from the World Bank. (World Development Indicators, 2022).

5.1.2 Regression Analysis

A multiple regression model is employed to estimate the relationship between the average *TFR* and the selected determinants. The model is specified as follows:

$$TFR_i = \beta_0 + \beta_1 \cdot \text{Determinant1}_i + \beta_2 \cdot \text{Determinant2}_i + \beta_3 \cdot \text{Determinant3}_i + \beta_4 \cdot \text{Determinant4}_i + \epsilon_i$$

The study tests several hypotheses regarding the impact of different identified factors on the average *TFR* in addition to the interactions between these factors.

The regression analysis assumes linearity, independence, homoscedasticity, and normality of errors. Diagnostic tests will be conducted to assess these assumptions. In addition to the assumptions, multicollinearity will be checked as well with the criteria below:

- Variance Inflation Factor (*VIF*) values around 1 indicate low correlation between the predictor and the other variables.
- *VIF* values between 1 and 5 are generally considered moderate.
- *VIF* values above 5 may indicate high correlation and potential multicollinearity.

5.1.3 Transfer Functions

If the assumptions of traditional regression are not met, and there are issues like non-linearity, autocorrelation, or heteroscedasticity, time series transfer functions can be a valuable alternative for analyzing the impact of determinants on fertility rates. A time series transfer function is a model that captures how changes in one time series influence changes in another over time. Transfer functions, in the context of time series analysis, allow for the exploration of the dynamic relationships between a dependent time series (e.g., fertility rates) and one or more independent time series (e.g., determinants). The transfer function equation generally takes the form:

$$\gamma_{t,T} = \sum_{i=1}^m \delta_i \left(\frac{t}{T} \right) \gamma_{t-i,T} + \sum_{j=0}^n \omega_j \left(\frac{t}{T} \right) X_{t-j,T} + \epsilon_t, \quad t = 1, \dots, T,$$

where:

- Y_t is the dependent variable (fertility rates).
- T is the number of observations and ε_t is i.i.d. $(0, \sigma^2)$. The error and the input series are assumed to be independent.
- The functions $\omega_j(u)$ $j = 0, 1, 2, \dots, n$, and $\delta_i(u)$, $i = 1, 2, \dots, n$, are supported on the interval $(0, 1)$ and connected to the underlying series by an appropriate rescaling of time, $u = t/T$.

Identifying an appropriate transfer function involves examining the autocorrelation and partial autocorrelation functions of both the dependent and independent variables. Significant lags in these functions help identify the appropriate lag structure for the transfer function. Transfer functions can handle situations where traditional regression assumptions may not hold, providing more flexibility in capturing the underlying dynamics.

To evaluate the accuracy of a Transfer Function model in predicting average *TFR* in MENA region, the outlined methodology below will be used:

- a) **Data Splitting:** The dataset will be divided into training (1960-2015) and testing (2016-2020) sets, encompassing 90% and 10% of the available data, respectively. The training dataset will be used to build the best transfer function model.
- b) **Transfer Function Models:** Transfer Function models will be applied to the entire set of predictors collectively and individually. The objective is to identify the model with the best fit for average *TFR* prediction.
- c) **Model Evaluation:** The accuracy of each model will be assessed by comparing the AIC and errors. The model demonstrating the highest accuracy during the evaluation phase will be selected.

- d) **Validation:** The selected model will be used to predict average *TFR* for the validation period (2016-2020).
- e) **Comparison and Analysis:** Forecasted average *TFR* values will be compared to actual data for the validation period, and the accuracy of the overall forecasting will be analyzed.
- f) **Forecasting:** The selected model will be used to forecast the average MENA *TFR*

This methodology ensures a rigorous assessment of the Transfer Function model's performance in predicting average *TFR* in the MENA region. The approach of evaluating models collectively and individually provides a nuanced understanding of predictor variable contributions to the accuracy of average *TFR* forecasts.

5.2 Results

5.2.1 Systematic Review

The systematic review has identified key determinants influencing fertility rates in the MENA region as follows:

- **Urbanization:** A negative correlation has been observed between urbanization and fertility rates in MENA. The transition from rural to urban areas is associated with increased access to family planning services, education, and employment opportunities for women, leading to a decline in fertility rates.
- **Economic Growth:** Economic growth shows a positive correlation with fertility rates in MENA. Rising incomes enable families to better support their children, contributing to an increase in fertility rates. However, as economic growth continues, families may opt for fewer children, prioritizing their overall quality of life.

- **Women's Labor Force Participation:** Women's involvement in the labor force is negatively correlated with fertility rates in MENA. Increased participation often results in delayed marriage and childbearing, contributing to a decline in fertility rates.
- **Female Education:** Education exhibits a negative correlation with fertility rates in MENA. Higher levels of education among women are associated with delayed marriage and childbearing, leading to a reduction in fertility rates.

Other factors influencing fertility rates in MENA encompass aspects such as access to healthcare, cultural and religious values, government policies related to family planning, and the availability of contraception. However, this study will primarily focus on the aforementioned key factors.

For each identified determinant, an indicator from the World Bank (World Development Indicators, 2022) has been selected. Fertility rate, measured by the total fertility rate (*TFR*), serves as the metric estimating the average number of children a woman would give birth to during her reproductive years, following the age-specific fertility rates of a given year and assuming she lives until the end of her childbearing period.

- **Urbanization (Figure 5-1)** is measured by the percentage of people living in urban areas to the whole population.
- **Economic growth (Figure 5-2)** is measured by Gross Domestic Product (GDP) at purchaser's prices which is a measure of the total value of goods and services produced within an economy over a specific time period. It is calculated as the sum of the gross value added by all resident producers in the economy, inclusive of any

product taxes, and subtracting any subsidies that are not included in the value of the final products. Data are in constant 2015 prices, expressed in U.S. dollars.

- Women labor force participation (Figure 5-3) is measured as the proportion of women who are currently employed or seeking employment, out of the total number of individuals aged 15 and above who are considered to be part of the labor force. The labor force refers to individuals who are engaged in the production of goods and services during a given time frame.
- Female education (Figure 5-4) is measured as the percentage of female students in the secondary school. Secondary school is not as advanced as college degrees or basic as primary school. It provides a good insight especially for underdeveloped countries where college education is not as common. In addition, data on percentage of females who pursued bachelor's degrees is not available.

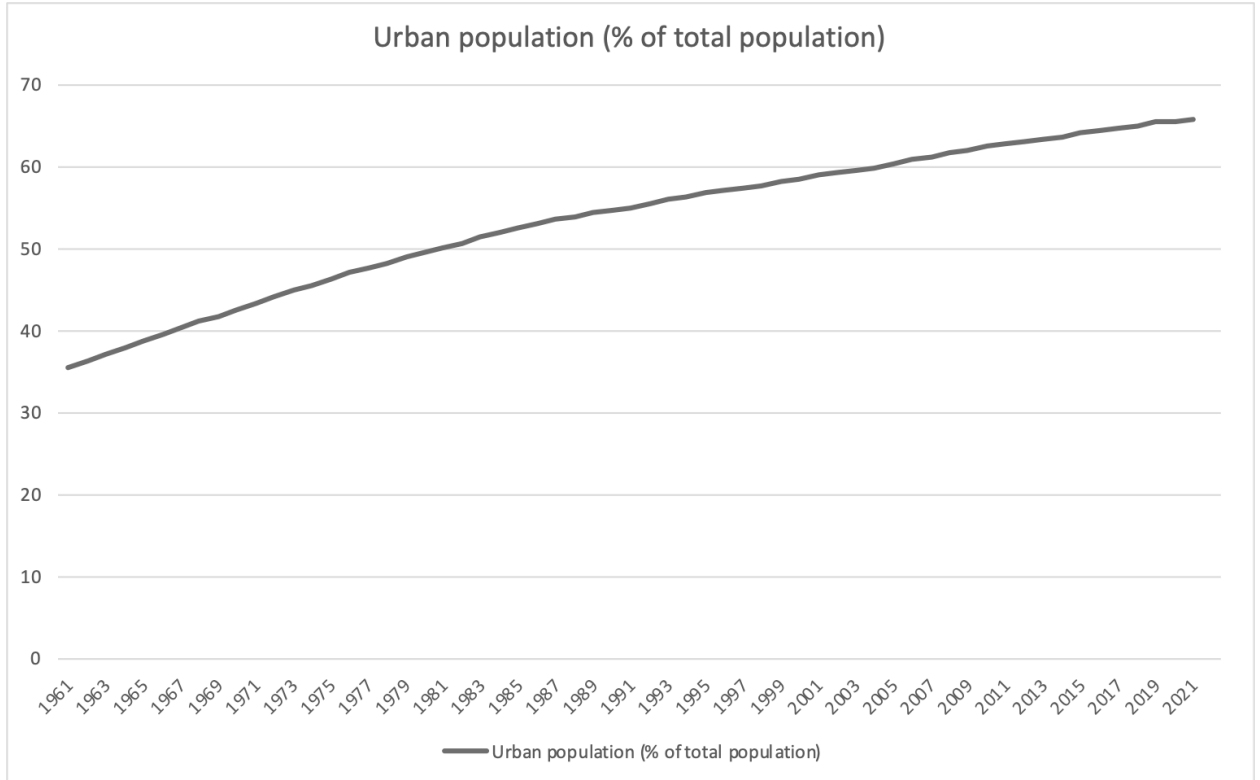


Figure 5-1 MENA Urban Population (% of Total Population)

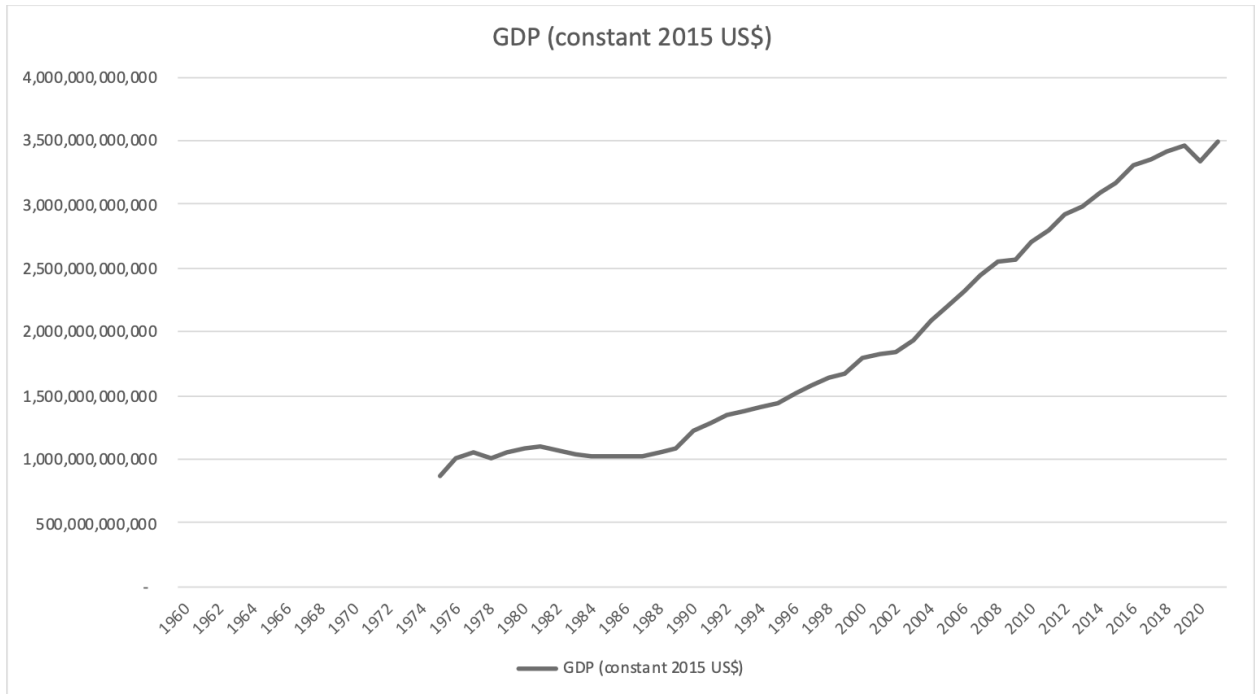


Figure 5-2 MENA Economic Growth 1975 – 2020



Figure 5-3 MENA Women Labor Force Participation

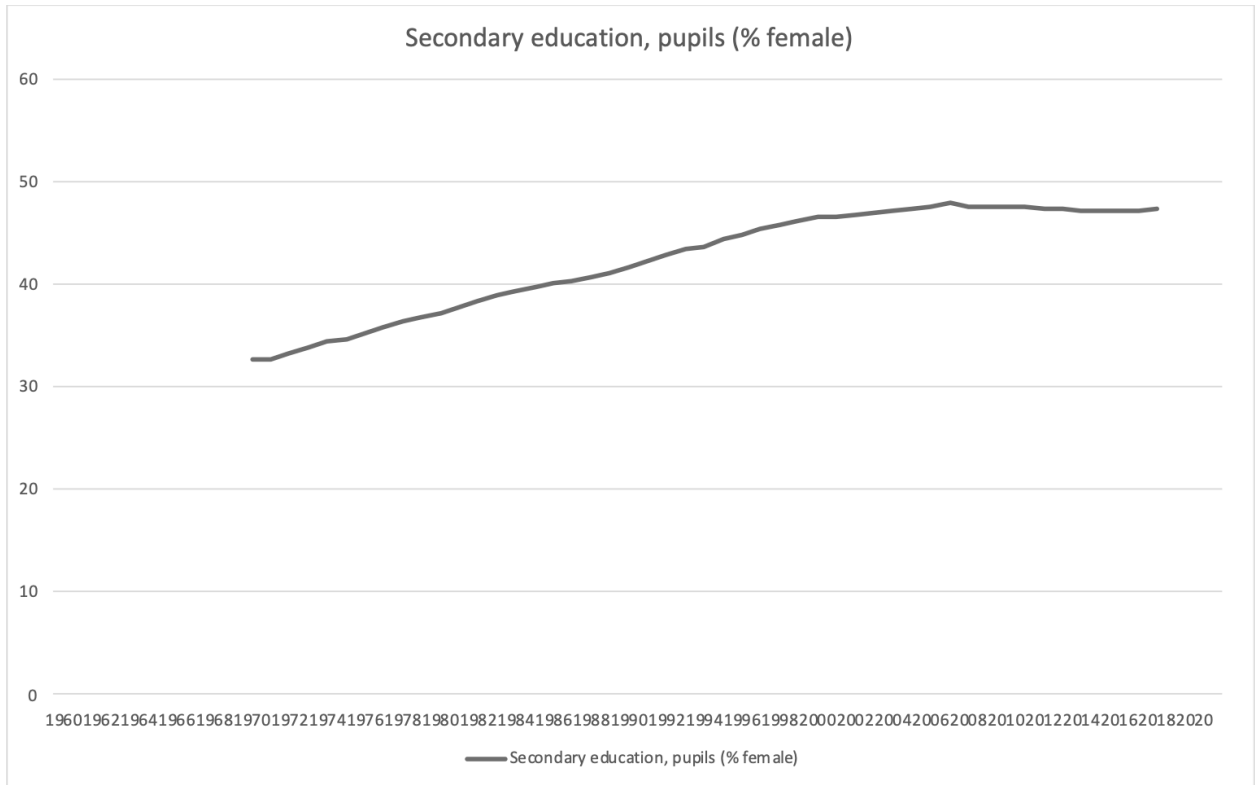


Figure 5-4 Female secondary education %

5.2.2 Regression Analysis

After identifying the determinants and their appropriate measure, multiple regression model will be fit with the below formula:

$$TFR_i = \beta_0 + \beta_1 \cdot \text{Urbanization}_i + \beta_2 \cdot \text{Economic Growth} + \beta_3 \cdot \text{Female Labor Force}_i + \beta_4 \cdot \text{Female Education}_i + \epsilon_i$$

Table 5-1 shows the summary of the model:

- The estimated intercept is 3.276e+00. It represents the expected value of average *TFR* when all predictor variables are zero.
- Urbanization (4.022e-01): For every one-unit increase in urbanization, the average *TFR* is expected to increase by 4.022e-01 units, holding other variables constant.
- GDP (-1.550e-12): The coefficient is extremely small and negative. For practical purposes, it suggests that GDP does not have a substantial linear relationship with the average *TFR* in this model.
- Female Labor (-1.484e-01): For every one-unit increase in the percentage of female labor force participation, the average *TFR* is expected to decrease by 1.484e-01 units, holding other variables constant.
- Female Education (-3.848e-01): For every one-unit increase in the female education variable, the *TFR* is expected to decrease by 3.848e-01 units, holding other variables constant.
- All coefficients are statistically significant ($p\text{-value} < 0.05$), indicating that each predictor contributes significantly to explaining the variability in the average *TFR*.

- Residual Standard Error: The standard deviation of the residuals is 0.1845. It represents the typical difference between the observed and predicted values of the average *TFR*.
- *R*-squared (Multiple): 98.86% of the variability in the average *TFR* is explained by the model.
- Adjusted *R*-squared takes into account the number of predictors and is 98.78%.

In summary, Urbanization, female labor force participation, and female education are significant predictors of the average *TFR*. GDP, in this model, does not show a substantial linear relationship with the average *TFR*. The model has a high level of explanatory power, as evidenced by the high *R*-squared value.

Next, the assumptions of regression model are checked:

- **Linearity:** Figure 5-5 shows the Residuals vs. Fitted Values plot. The scatter of points around the horizontal axis is not completely random. There is curvature in the residuals which might indicate the violation of the linearity assumption. To confirm further, statistical test will be used. The RESET (Regression Specification Error Test) test is used to examine whether there are omitted nonlinearities in a regression model, which might indicate a lack of fit. The null hypothesis of the RESET test (Table 5-2) is that the model is correctly specified, meaning it includes all relevant linear terms. The *p*-value is extremely small, indicating strong evidence against the null hypothesis. Since the RESET test suggests a specification error, there might be a need to revisit the model and consider including additional variables, transforming variables, or exploring more complex model specifications to improve the model's fit.

- **Normality of Residuals:** Figures 5-6 represents the Q-Q plot of residuals. The graph does not show severe departure from normality. The Shapiro-Wilk test result (Table 5-2) confirms that. With a p -value of 0.3309, we fail to reject the null hypothesis of normality. This suggests that the residuals of model1 are not significantly different from a normal distribution. Therefore, the residuals are assumed to follow a normal distribution based on this test.
- **Homoscedasticity:** from the residuals vs. fitted values plot (Figure 5-5), it can be observed that the residuals follow a pattern which might indicate heteroscedasticity. The result of Breusch-Pagan test (Table 5-2) confirms that. With a small p -value (0.002191), there is evidence to reject the null hypothesis of homoscedasticity. This suggests the presence of heteroscedasticity in the residuals of model1. In practical terms, it means that the variance of the residuals is not constant across all levels of the independent variables.
- **Independence of Errors:** The p -value of 0 of the Durbin Watson test (Table 5-2) suggests strong evidence against the null hypothesis. Therefore, there is evidence of first-order autocorrelation in the residuals of the regression model.

In summary, the diagnostic of model 1 reveals potential issues with linearity, homoscedasticity, and independence of errors in the model. Addressing these concerns may involve adjusting model specifications, exploring additional variables, or transforming existing ones to enhance the model's overall fit.

TABLE 5-1 Model 1 Summary

Model	TFR ~ Urbanization + GDP + FemaleLabor + FemaleEducation				
Multiple <i>R</i>-squared	0.9886				
Adjusted <i>R</i>-squared	0.9878				
F-static	1215				
<i>P</i>-value	<2.20E-16				
Coefficients	(Intercept)	Urbanization	GDP	Female Labor	Female Education
Estimate	3.276	0.4022	-1.55E-12	-0.1484	-0.3848
Std. Error	0.5754	0.03263	8.353E-14	0.01087	0.02957
Pr(> t)	0.000000475	<2.00E-16	<2.00E-16	<2.00E-16	<2.00E-16

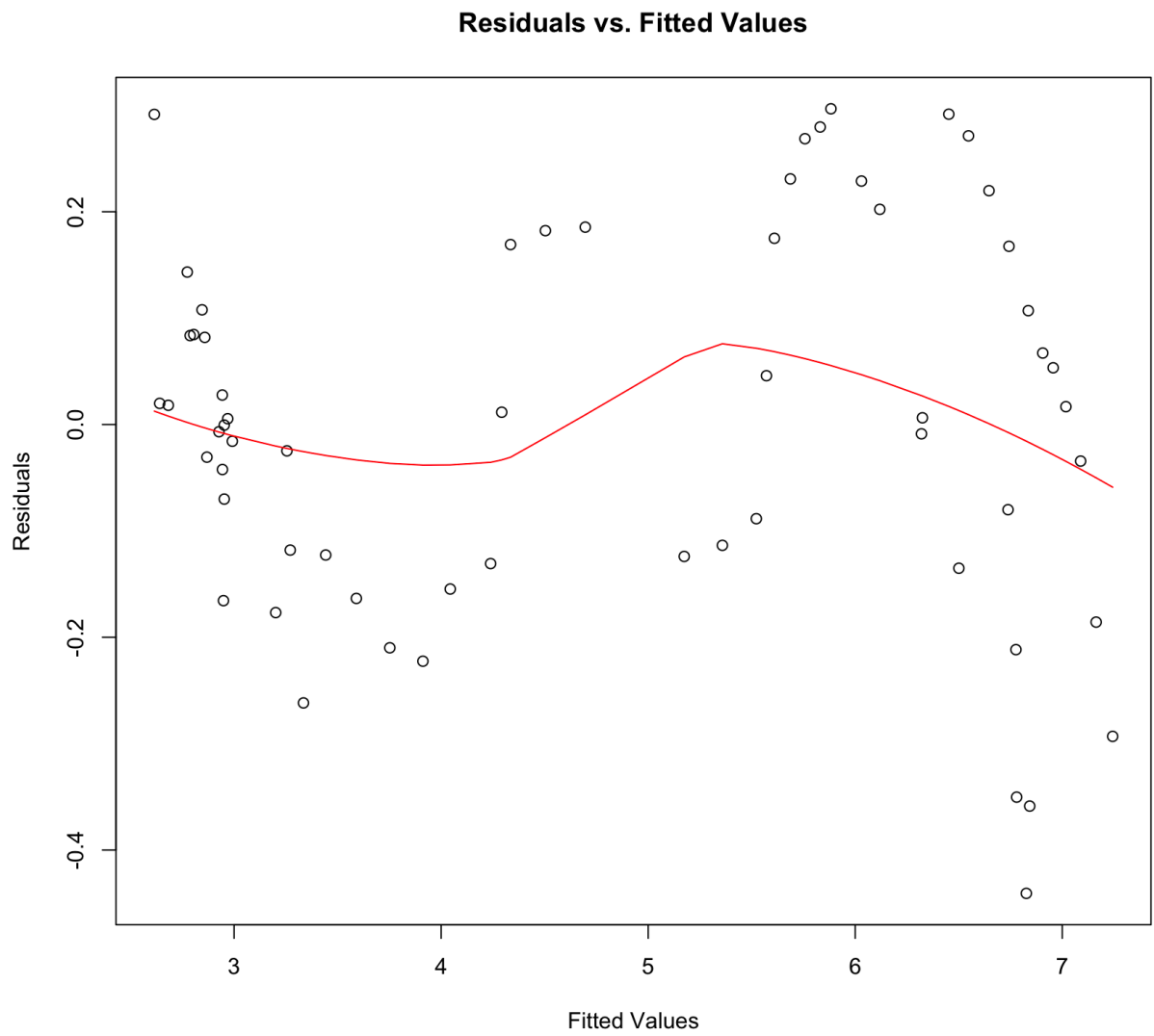


Figure 5-5 Model 1 Residuals vs. Fitted Values Plot

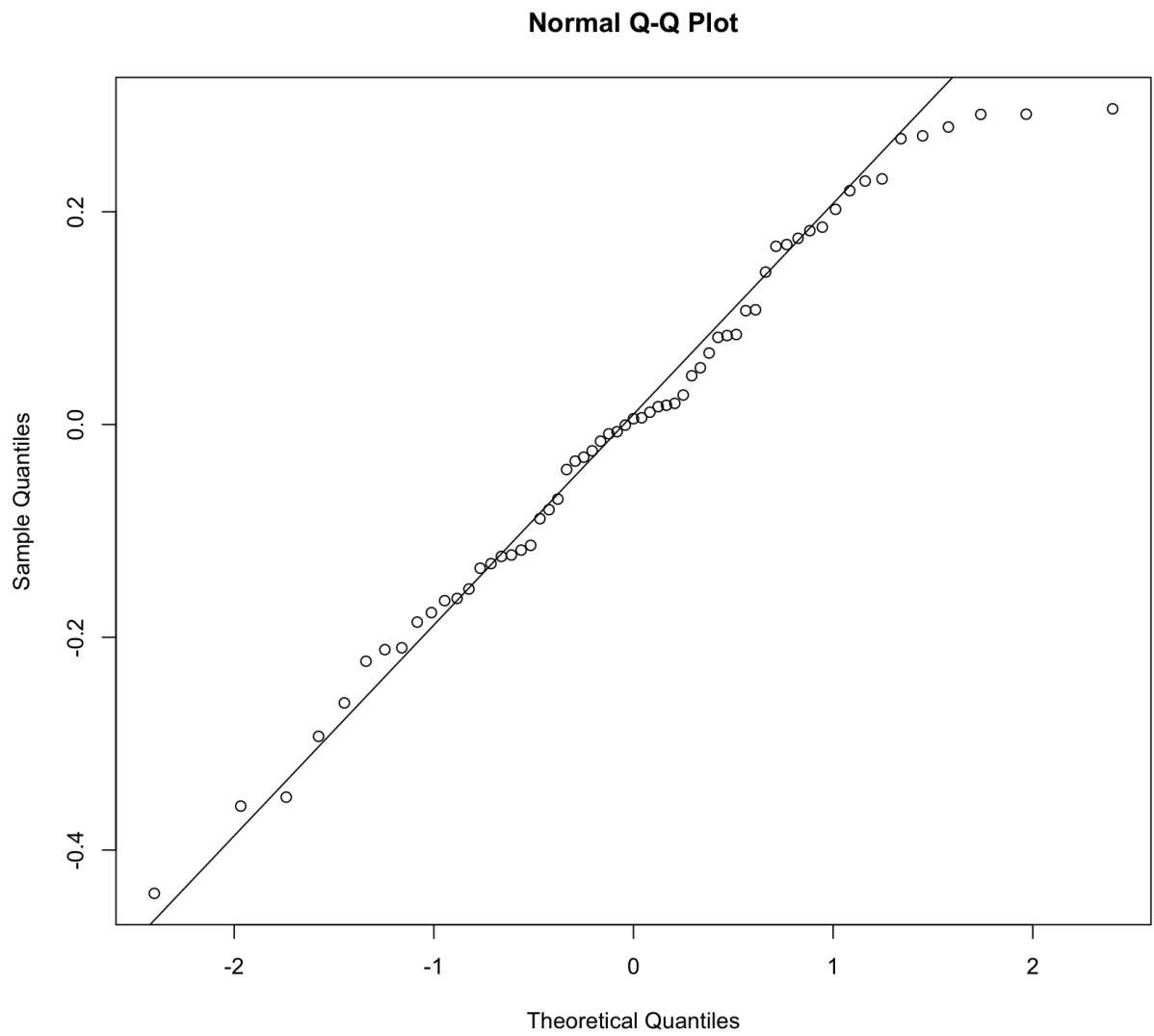


Figure 5-6 Model 1 QQ Plot

TABLE 5-2 Model 1 Test Results of Regression Assumptions

Assumption	Test	<i>P</i>-value
Linearity	RESET test	<2.2e-16
Normality of residuals	Shapiro-Wilk normality test	3.31E-01
Homoscedasticity	studentized Breusch-Pagan test	2.19E-03
Independence of errors	Durbin Watson Test	0

Now, another model will be fit but without interpolating the missing data to check if the results would be different.

The model summary (Table 5-3) suggests that Urbanization, Female Labor, and Female Education significantly influence the Total Fertility Rate in the given context, while GDP appears to have a negligible impact. The model exhibits a high level of explanatory power, as indicated by the high *R*-squared values and the significant F-statistic.

Next, the assumptions of regression model are checked:

- **Linearity:** In Figure 5-7, the scatter of points around the horizontal axis is not completely random. There is curvature in the residuals which might indicate the violation of the linearity assumption. This is confirmed by the small *p*-value (< 0.05) of RESET test (Table 5-3) provides strong evidence against the null hypothesis. Therefore, the result suggests that there are omitted nonlinearities in your regression model, indicating a lack of fit.
- **Normality of Residuals:** The Q-Q plot in Figure 5-8 does not show serious departure from normality. This is confirmed by a *p*-value of 0.1088 (greater than the typical significance level of 0.05) of the Shapiro-Wilk Normality Test (Table 5-4). There is not enough evidence to conclude that the residuals significantly deviate from a normal distribution. Therefore, based on this test, it can be assumed that the residuals are approximately normally distributed.
- **Homoscedasticity:** from the residuals vs. fitted values plot (Figure 5-7), it can be observed that the residuals follow a pattern which might indicate heteroscedasticity. The result of Breusch-Pagan test (Table 5-4) confirms that. With a small *p*-value (0.002191), there is evidence to reject the null hypothesis of homoscedasticity. This

suggests the presence of heteroscedasticity in the residuals of model 2. In practical terms, it means that the variance of the residuals is not constant across all levels of the independent variables.

- **Independence of Errors:** The p -value of 0 of the Durbin Watson test (Table 5-4) suggests strong evidence against the null hypothesis. Therefore, there is evidence of first-order autocorrelation in the residuals of the regression model.
- Lastly, high VIF values (Table 5-4) for Urbanization and GDP could be a concern due to multicollinearity. It might be worth investigating the correlation between Urbanization and GDP and considering whether one of them can be excluded from the model to address multicollinearity issues. The VIF for FemaleLabor and FemaleEducation is less concerning.

Accordingly, the next model will be fit without the GDP since it is insignificant predictor and might be a cause of the high VIF for urbanization.

TABLE 5-3 Model 2 Summary

Model	TFR ~ Urbanization + GDP + FemaleLabor + FemaleEducation				
Multiple <i>R</i>-squared	0.9846				
Adjusted <i>R</i>-squared	0.9822				
F-static	414.8				
<i>P</i>-value	<2.20E-16				
Coefficients	(Intercept)	Urbanization	GDP	Female Labor	Female Education
Estimate	2.71E+01	-2.47E-01	9.30E-13	-1.11E-01	-1.91E-01
Std. Error	3.72E+00	1.05E-01	3.93E-13	3.32E-02	4.51E-02
Pr(t)	9.74E-08	0.026086	0.025542	0.002518	0.000247

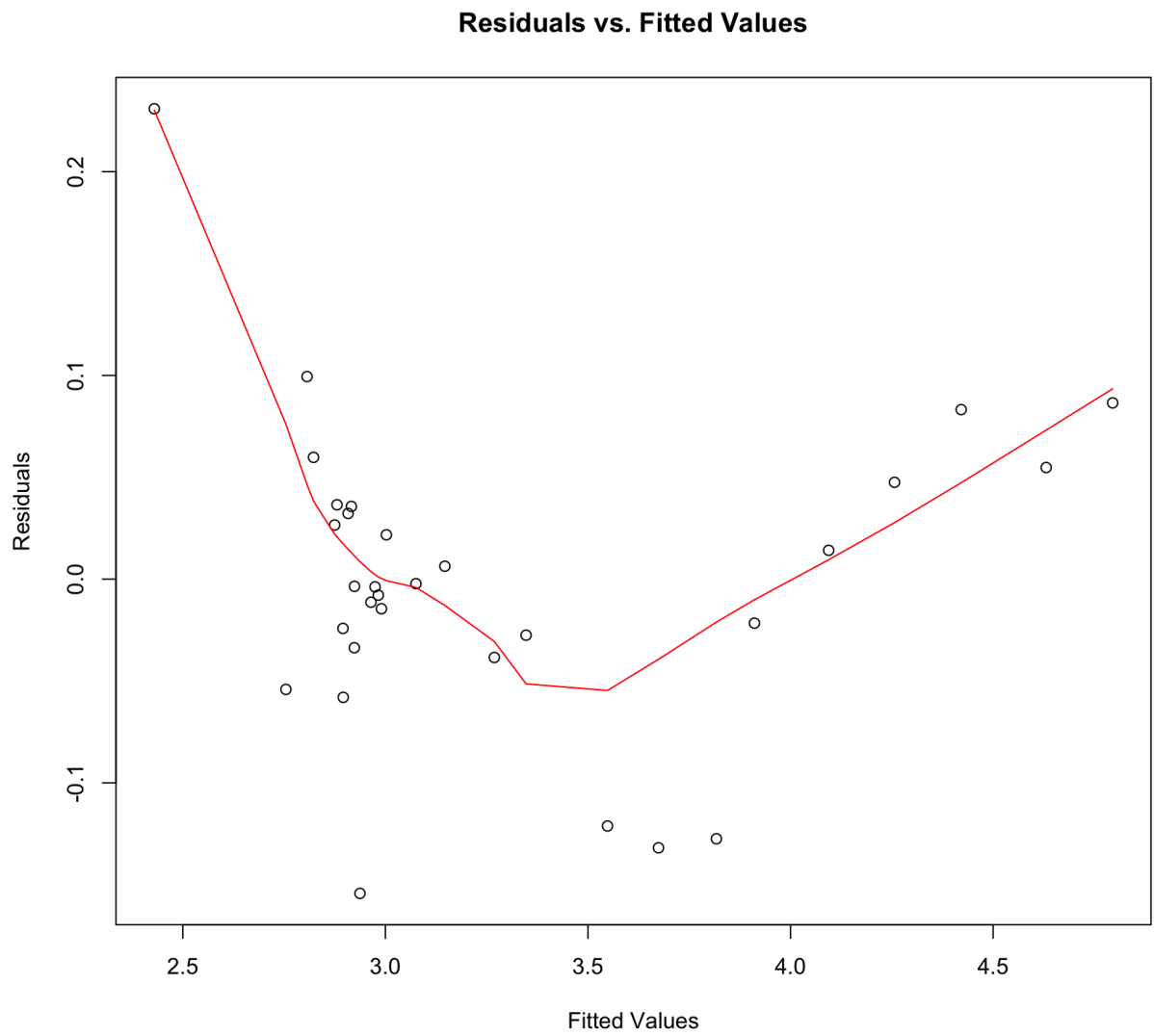


Figure 5-7 Model 2 Residuals vs. Fitted Values

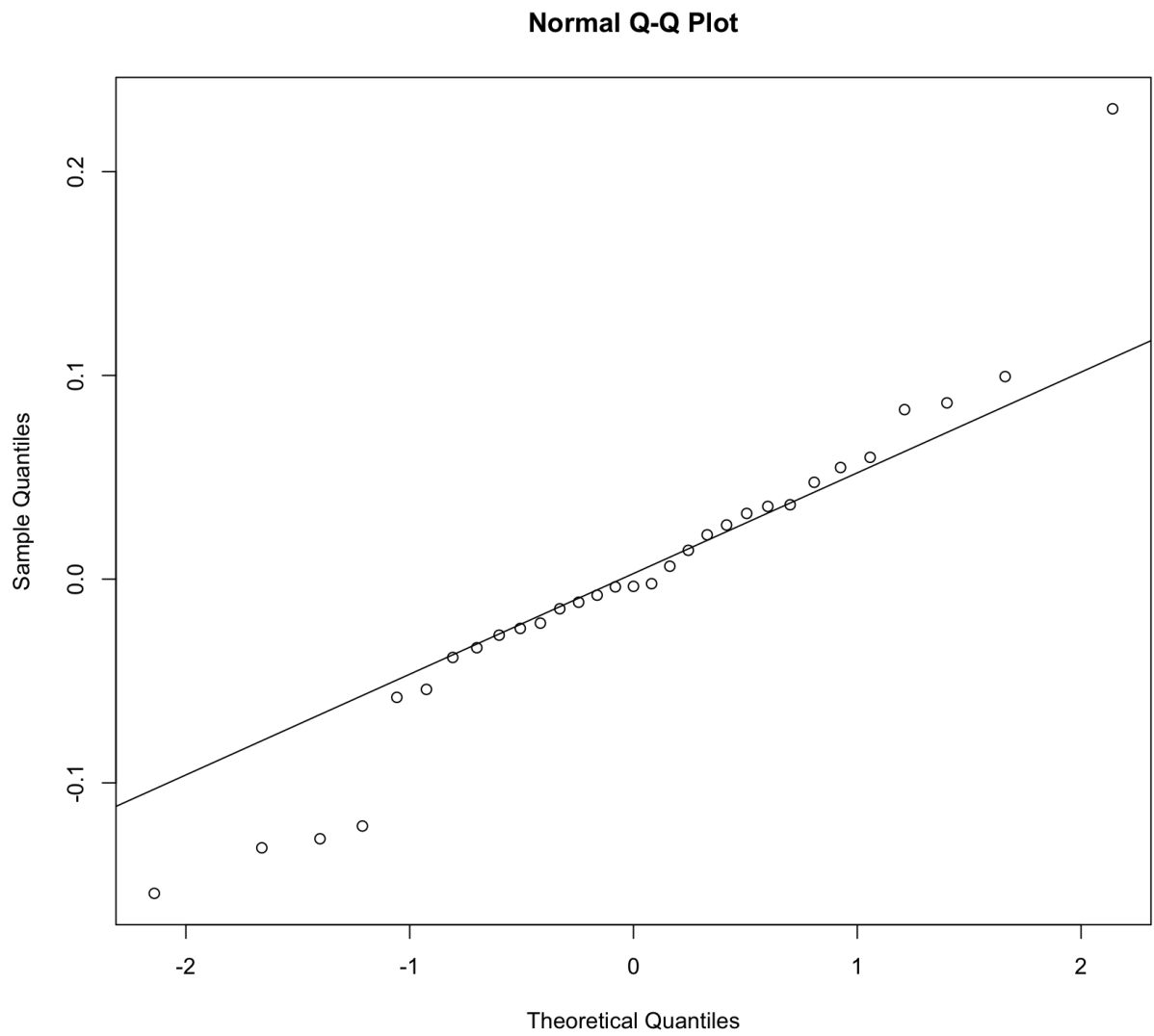


Figure 5-8 Model 2 Q-Q Plot

TABLE 5-4 Model 2 Test Results of Regression Assumptions

Assumption	Test	<i>P</i>-value
Linearity	RESET test	3.88E-09
Normality of residuals	Shapiro-Wilk normality test	0.31088
Homoscedasticity	studentized Breusch-Pagan test	0.002191
Independence of errors	Durbin Watson Test	0
No multicollinearity	VIF	VIF value
	Urbanization	548.90
	GDP	387.56
	Female Labor	4.801233
	Female Education	31.038863

The summary of the model in Table 5-5 shows that Female Labor and Female Education have statistically significant effects on *TFR*, with negative coefficients, implying that as Female Labor and Female Education increase, *TFR* tends to decrease. The model with Urbanization included does not perform significantly better than a model without it, as indicated by its non-significant coefficient.

The model, in general, has a high *R*-squared value, indicating a good fit to the data.

Next, the assumptions of regression model are checked:

- **Linearity:** In Figure 5-9, the scatter of points around the horizontal axis is not completely random. There is curvature in the residuals which might indicate the violation of the linearity assumption. This is confirmed by the small *p*-value (< 0.05) of RESET test (Table 5-6) provides strong evidence against the null hypothesis. Therefore, the result suggests that there are omitted nonlinearities in your regression model, indicating a lack of fit.
- **Normality of Residuals:** The Q-Q plot in Figure 5-10 does not show serious departure from normality. This is confirmed by a *p*-value of 0.5388 (greater than the typical significance level of 0.05) of the Shapiro-Wilk Normality Test (Table 5-6). There is not enough evidence to conclude that the residuals significantly deviate from a normal distribution. Therefore, based on this test, it can be assumed that the residuals are approximately normally distributed.
- **Homoscedasticity:** from the residuals vs. fitted values plot (Figure 5-9), it can be observed that the residuals do not follow a pattern which might indicate homoscedasticity. The result of Breusch-Pagan test (Table 5-6) confirms that. With a *p*-value (0.06715), there is not enough evidence to reject the null hypothesis of

homoscedasticity.. In practical terms, it means that the variance of the residuals is constant across all levels of the independent variables.

- **Independence of Errors:** The p -value of 0 of the Durbin Watson test (Table 5-6) suggests strong evidence against the null hypothesis. Therefore, there is evidence of first-order autocorrelation in the residuals of the regression model.
- Lastly, the VIF values (Table 5-6) for Urbanization, Female Labor, and Female Education are all below 5, suggesting that there is no severe multicollinearity among these predictors. These values indicate a moderate level of correlation, which is generally acceptable. The VIF results suggest that the predictors in the model are not highly correlated, and multicollinearity is not a major concern.

In summary, although the normality, homoscedasticity, and multicollinearity assumptions seem to be met, the model shows indications of issues related to linearity and autocorrelation. To improve the model, interactions between the variables can be considered.

TABLE 5-5 Model 3 Summary

Model	TFR ~ Urbanization + FemaleLabor + FemaleEducation			
Multiple <i>R</i>-squared	0.9812			
Adjusted <i>R</i>-squared	0.9792			
F-static	470.8			
<i>P</i>-value	<2.20E-16			
Coefficients	(Intercept)	Urbanization	Female Labor	Female Education
Estimate	1.83E+01	-2.77E-05	-8.62E-02	-2.89E-01
Std. Error	4.23E-01	1.03E-02	3.41E-02	2.01E-02
Pr(t)	<2.00E-16	0.9979	0.0176	3.42E-14

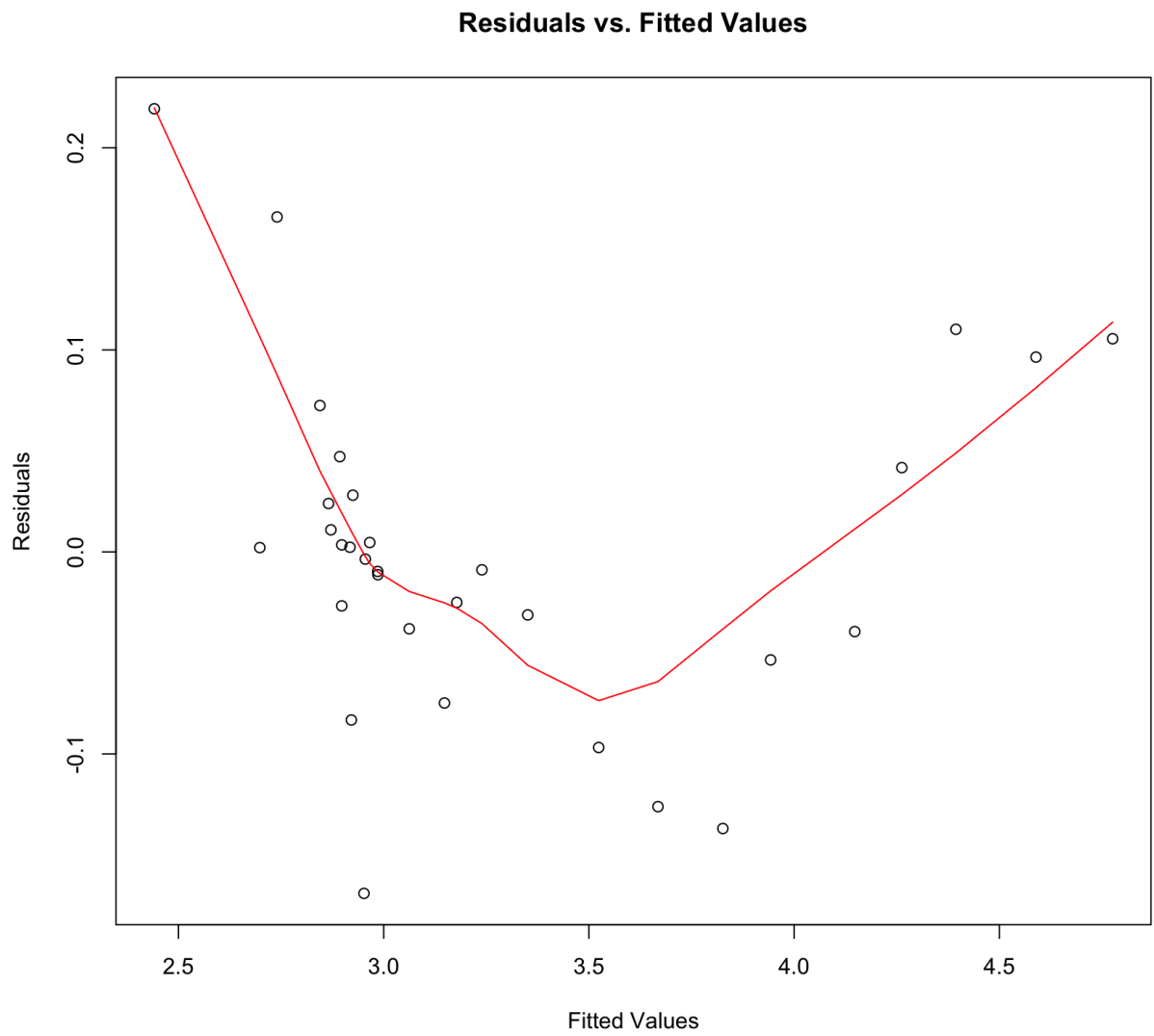


Figure 5-9 Model Residuals vs. Fitted Values

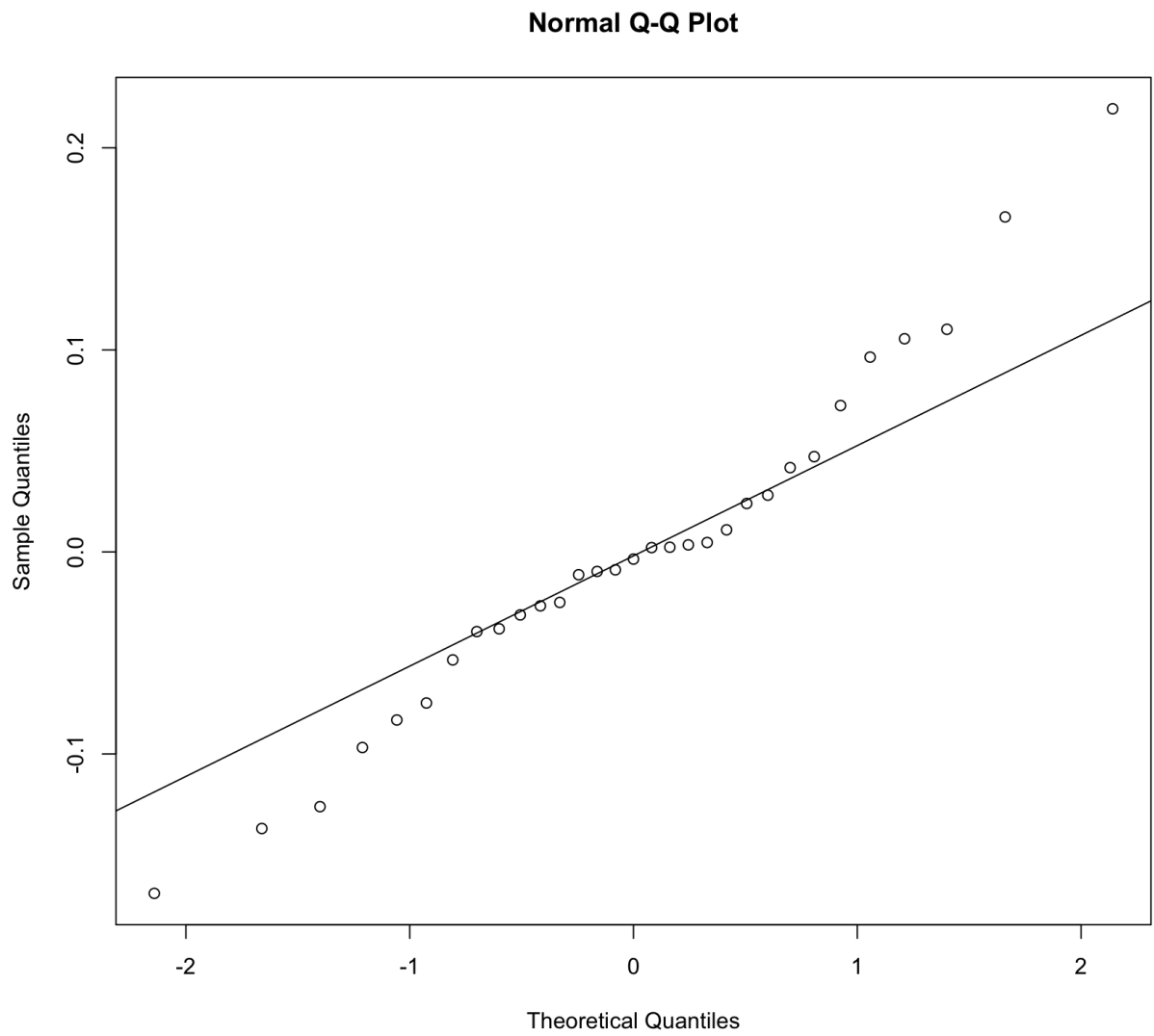


Figure 5-10 Model 3 Q-Q Plot

TABLE 5-6 Model 3 Test Results of Regression Assumptions

Assumption	Test	<i>P</i>-value
Linearity	RESET test	8.59E-10
Normality of residuals	Shapiro-Wilk normality test	0.53880
Homoscedasticity	studentized Breusch-Pagan test	0.067150
Independence of errors	Durbin Watson Test	0
No multicollinearity	VIF	VIF value
	Urbanization	4.534859
	Female Labor	4.322893
	Female Education	5.239348

The summary of the model in Table 5-7 shows that Urbanization and Female Labor, individually, do not appear to be significant predictors of *TFR* in this model while Female Education has a significant negative effect on *TFR*. There are significant interactions between Urbanization and Female Education, suggesting that the relationship between *TFR* and Urbanization depends on the level of Female Education. The model seems to fit the data very well based on the high *R*-squared values 99.5%

Next, the assumptions of regression model are checked:

- **Linearity:** In Figure 5-11, the scatter of points around the horizontal axis is random. Also, The *p*-value of RESET (Table 5-8) 0.099 is greater than the typical significance level of 0.05. Therefore, there is no enough evidence to reject the null hypothesis. This suggests that there is no strong evidence of a specification error or departure from linearity in the model based on the RESET test. In practical terms, it implies that the linear regression model adequately captures the linear relationship between the predictors and the response variable.
- **Normality of Residuals:** The Q-Q plot in Figure 5-12 does not show serious departure from normality. This is confirmed by a *p*-value of 0.2448 (greater than the typical significance level of 0.05) of the Shapiro-Wilk Normality Test (5-8). There is not enough evidence to conclude that the residuals significantly deviate from a normal distribution. Therefore, based on this test, it can be assumed that the residuals are approximately normally distributed.
- **Homoscedasticity:** From the residuals vs. fitted values plot (Figure 5-11), it can be observed that the residuals do not follow a pattern which might indicate homoscedasticity. The result of Breusch-Pagan test (5-8) confirms that. With a *p*-

value (0.05193), there is not enough evidence to reject the null hypothesis of homoscedasticity. In practical terms, it means that the variance of the residuals is constant across all levels of the independent variables.

- **Independence of Errors:** The p -value of 0 of the Durbin Watson test (5-8) suggests strong evidence against the null hypothesis. Therefore, there is evidence of first-order autocorrelation in the residuals of the regression model.
- Lastly, the VIF values (Table 5-8) for Urbanization, Female Labor, and Female Education are all 1, suggesting that there is no multicollinearity among these predictors. The VIF results suggest that the predictors in the model are not correlated, and multicollinearity is not a concern.

TABLE 5-7 Model 4 Summary

Model	TFR ~ Urbanization + FemaleLabor + FemaleEducation+Urbanization:FemaleLabor+Urbanization:FemaleEducation+ FemaleLabor:FemaleEducation						
Multiple R-squared	0.996						
Adjusted R-squared	0.9949						
F-static	985.7						
P-value	<2.20E-16						
Coefficients	(Intercept)	Urbanization	Female Labor	Female Education	Urbanization: FemaleLabor	Urbanization: FemaleEducation	FemaleLabor :FemaleEducation
Estimate	69.713742	-0.892902	0.228703	-1.536196	-0.005804	0.020851	0.002869
Std. Error	12.567698	0.385428	1.197572	0.20941	0.011384	0.00583	0.024679
Pr(> t)	1.05E-05	0.02938	0.85015	1.41E-07	0.6148	0.00152	0.90842

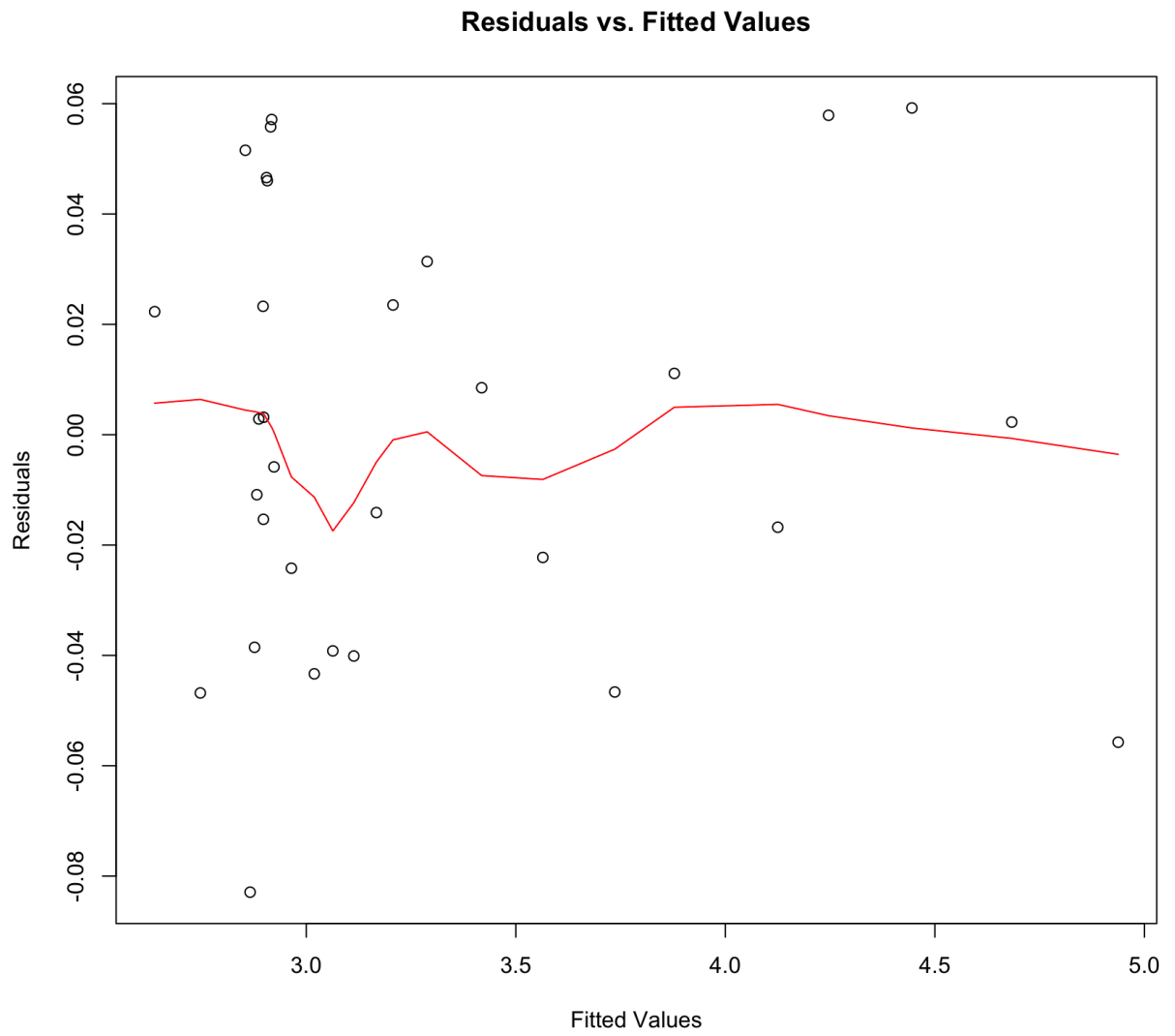


Figure 5-11 Model 4 Residuals vs. Fitted Values

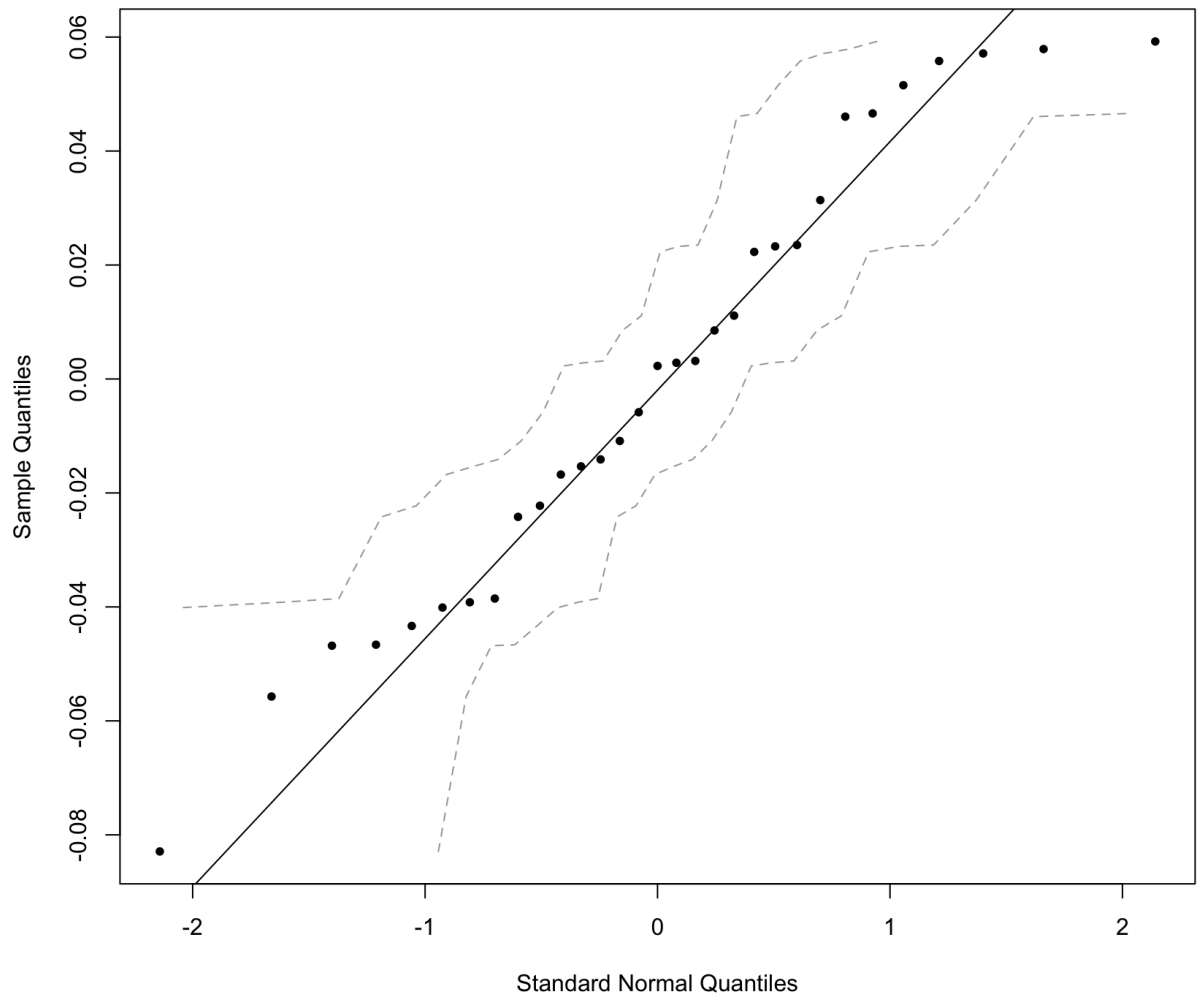


Figure 5-12 Model 4 Q-Q Plot

TABLE 5-8 Model 4 Test Results of Regression Assumptions

Assumption	Test	<i>P-value</i>
Linearity	RESET test	0.099
Normality of residuals	Shapiro-Wilk normality test	0.24480
Homoscedasticity	studentized Breusch-Pagan test	0.051930
Independence of errors	Durbin Watson Test	0
No correlations	VIF	VIF value
	Urbanization	1
	Female Labor	1
	Female Education	1

In summary, although adding the interactions in the model improved the *R*-squared and overcame the issue of linearity, the issue of autocorrelation is still there. Considering other approaches like transfer functions can be considered at this stage.

5.2.3 Transfer Functions

Transfer function models, particularly in the context of time series analysis, can potentially address issues of autocorrelation. Transfer function models are designed to capture the dynamic relationships between variables, allowing for the inclusion of lagged values and exogenous input variables.

5-9 is the result of an automatic model selection process for an ARIMA (AutoRegressive Integrated Moving Average) time series model using the `auto.arima` function in R. The function evaluates different combinations of ARIMA orders and selects the model with the lowest Akaike Information Criterion (AIC). In this case, the best model selected by the `auto.arima` function is **ARIMA(0,2,0)**. The numbers in the parentheses represent the order of the autoregressive (*p*), differencing (*d*), and moving average (*q*) components of the ARIMA model, respectively.

The AIC values associated with each model are negative, and the model with the lowest (most negative) AIC is considered the best fit. In this case, ARIMA(0,2,0) has the lowest AIC among the models considered.

So, the best model chosen is ARIMA(0,2,0), which indicates a second-order differencing (*d*=2) without any autoregressive or moving average components. This implies that the time series data might have a trend that needs differencing twice to make it stationary.

TABLE 5-9 Competing Models

Model	AIC
ARIMA(2,2,2)	-249.8694
ARIMA(0,2,0)	-255.7156
ARIMA(1,2,0)	-253.8297
ARIMA(0,2,1)	-253.7899
ARIMA(1,2,1)	-251.9388

Below is the summary of model 1 for the ARIMA(0,2,0) model fit to a time series and an interpretation of the key components.

ARIMA Model Information

ARIMA(0,2,0): This indicates an AutoRegressive Integrated Moving Average model with zero autoregressive (AR) terms, two differences (indicating second-order differencing), and zero moving average (MA) terms.

Model Fit Information

- $\sigma^2 = 0.0007429$: The estimated variance of the residuals.
- Log likelihood = 128.89: The log likelihood of the model given the data. Higher values are better.
- AIC (Akaike Information Criterion) = -255.79: A measure of the model's goodness of fit. Lower values are better.
- AICc = -255.72: AIC corrected for small sample sizes.
- BIC (Bayesian Information Criterion) = -253.71: Similar to AIC but penalizes model complexity more. Lower values are better.

Training Set Error Measures

- ME (Mean Error) = -0.001206543: The average of the forecast errors.
- RMSE (Root Mean Squared Error) = 0.02680568: A measure of the average magnitude of the forecast errors.
- MAE (Mean Absolute Error) = 0.02028274: The average of the absolute forecast errors.
- MPE (Mean Percentage Error) = 0.01770858: The average percentage difference between the forecast and the actual values.

- MAPE (Mean Absolute Percentage Error) = 0.4816512: A percentage version of MAE, providing a relative measure of accuracy.
- MASE (Mean Absolute Scaled Error) = 0.2588318: Scaled relative to the mean absolute error of a naïve forecast.
- ACF1 (Autocorrelation of Forecast Errors at Lag 1) = -0.0776905: A measure of the autocorrelation in the forecast errors.

In summary, the model seems to have a good fit to the training data based on the given error measures and information criteria.

Now, the model will be assessed after adding the predictors: female education, female labor, urbanization and GDP.

Below is the summary for model 2 that includes a regression component with exogenous variables along with ARIMA(0,2,0) errors and interpretation of the key components.

Regression Coefficients:

- exog_urbanization: -0.0057
- exog_GDP: 0 (coefficient is zero)
- exog_FemaleLabor: 0.0111
- exog_FemaleEducation: -0.0056

Standard errors (s.e.) are also provided for each coefficient.

- exog_urbanization: 0.0106
- exog_GDP: 0
- exog_FemaleLabor: 0.0116
- exog_FemaleEducation: 0.0144

ARIMA Model Information:

ARIMA(0,2,0): This indicates an AutoRegressive Integrated Moving Average model with zero autoregressive (AR) terms, two differences (indicating second-order differencing), and zero moving average (MA) terms.

Model Fit Information:

- $\sigma^2 = 0.0007822$:
- Log likelihood = 129.45
- AIC = -248.9
- AICc = -247.77
- BIC (Bayesian Information Criterion) = -238.51

Training Set Error Measures:

- ME (Mean Error) = -0.001137512
- RMSE (Root Mean Squared Error) = 0.02655678
- MAE (Mean Absolute Error) = 0.01933895
- MPE (Mean Percentage Error) = 0.02425205
- MAPE (Mean Absolute Percentage Error) = 0.4586414
- MASE (Mean Absolute Scaled Error) = 0.2467879
- ACF1 (Autocorrelation of Forecast Errors at Lag 1) = -0.06131313

In summary, the model includes a regression component with coefficients for exogenous variables, and the ARIMA errors follow a (0,2,0) structure. The model seems to have a good fit to the training data based on the given error measures and information criteria.

Since the GDP is not adding any effect, the model will be assessed without the GDP. The summary of model 3 indicates the following:

Regression Coefficients:

- `exog_urbanization`: -0.0013
- `exog_FemaleLabor`: 0.0112
- `exog_FemaleEducation`: -0.0049

Standard errors (s.e.) are also provided for each coefficient.

- `exog_urbanization`: 0.0821
- `exog_FemaleLabor`: 0.0117
- `exog_FemaleEducation`: 0.0144

ARIMA Model Information:

ARIMA(0,2,0): This indicates an AutoRegressive Integrated Moving Average model with zero autoregressive (AR) terms, two differences (indicating second-order differencing), and zero moving average (MA) terms.

Model Fit Information:

- $\sigma^2 = 0.0007703$
- $\log \text{likelihood} = 129.37$
- $\text{AIC (Akaike Information Criterion)} = -250.74$
- $\text{AICc} = -249.99$:
- $\text{BIC (Bayesian Information Criterion)} = -242.42$

Training Set Error Measures:

- $\text{ME (Mean Error)} = -0.001151452$
- $\text{RMSE (Root Mean Squared Error)} = 0.02659263$
- $\text{MAE (Mean Absolute Error)} = 0.01943455$
- $\text{MPE (Mean Percentage Error)} = 0.02246131$
- $\text{MAPE (Mean Absolute Percentage Error)} = 0.4593915$

- MASE (Mean Absolute Scaled Error) = 0.2480079
- ACF1 (Autocorrelation of Forecast Errors at Lag 1) = -0.06614485

In summary, the model includes a regression component with coefficients for exogenous variables, and the ARIMA errors follow a (0,2,0) structure. The model appears to have a better than the model with the GDP included based on the AIC value.

Model Forecasting

To evaluate the accuracy of a Transfer Function model in predicting average *TFR* in MENA region, the dataset is divided into training (1960-2015) and testing (2016-2020) sets, encompassing 90% and 10% of the available data, respectively.

Transfer Function models will be applied to the entire set of predictors collectively and individually. The objective is to identify the model with the best fit for *TFR* prediction.

- a) **Model Evaluation:** The accuracy of each model will be assessed by comparing the AIC and errors. The model demonstrating the highest accuracy during the evaluation phase will be selected.
- b) **Validation:** The selected model will be used to predict *TFR* for the validation period (2016-2020).
- c) **Comparison and Analysis:** Forecasted *TFR* values will be compared to actual data for the validation period, and the accuracy of the overall forecasting will be analyzed.
- d) **Forecasting:** The selected model will be used to forecast the average MENA *TFR*

Table 5-10 summarizes the results of the Transfer Function with ARIMA (0,2,0) models fitted on the *TFR* data from 1960-2015:

- Model 1: This model serves as the baseline without any regressors. It provides an AIC value and error metrics that can be used for comparison.
- Model 2: This model includes all regressors. It generally improves AIC and error metrics compared to the baseline, indicating that including these predictors contributes to a better fit.
- Model 3: This model excludes GDP as a predictor. It shows slightly improved AIC and some error metrics compared to model 2, suggesting that GDP might not be a significant contributor.
- Model 4: This model considers only Urbanization. While AIC is similar to model 2, it performs slightly worse in some error metrics.
- Model 5: This model considers only Female Education. It performs similarly to model 4 but with different error patterns.
- Model 6: This model considers only Female Labor. It has a similar AIC to model 1 and performs well in some error metrics.

Based on the AIC and error measures, model 1 and model 6 are considered the best fit with a slight difference between them. Thus, both models will be used to forecast the average *TFR* from 2016-2020. Then the model with the smallest difference from the actual average *TFR* will be used to forecast the average *TFR* from 2021-2026.

TABLE 5-10 Transfer Function with ARIMA (0,2,0) Models from 1960-2015

Model	Regressors	Coefficients	AIC	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Model 1: no regressors	None	NA	-241.09	-0.00101	0.02506	0.01857	0.02894	0.39778	0.23151	-0.01335
Model 2: all regressors	Urbanization	-0.029	-239.81	-0.002	0.024	0.016	0.019	0.354	0.206	0.078
	GDP	0								
	Female education	0.0009								
	Female labor	0.0184								
Model 3: excluding GDP	Urbanization	-0.015	-237.9	-0.00112	0.02442	0.01779	0.02821	0.38400	0.22184	0.00659
	Female education	0.0063								
	Female labor	0.018								
Model 4: Urbanization	Urbanization	0.0008	-239.09	-0.00100	0.02506	0.01856	0.02903	0.39763	0.23143	-0.01298
Model 5: Female education	Female education	0.0095	-239.47	-0.00084	0.02497	0.01850	0.03300	0.39669	0.23073	-0.01385
Model 6: Female Labor	Female labor	0.0184	-241.7	-0.00114	0.02446	0.01760	0.02715	0.37802	0.21953	0.01444

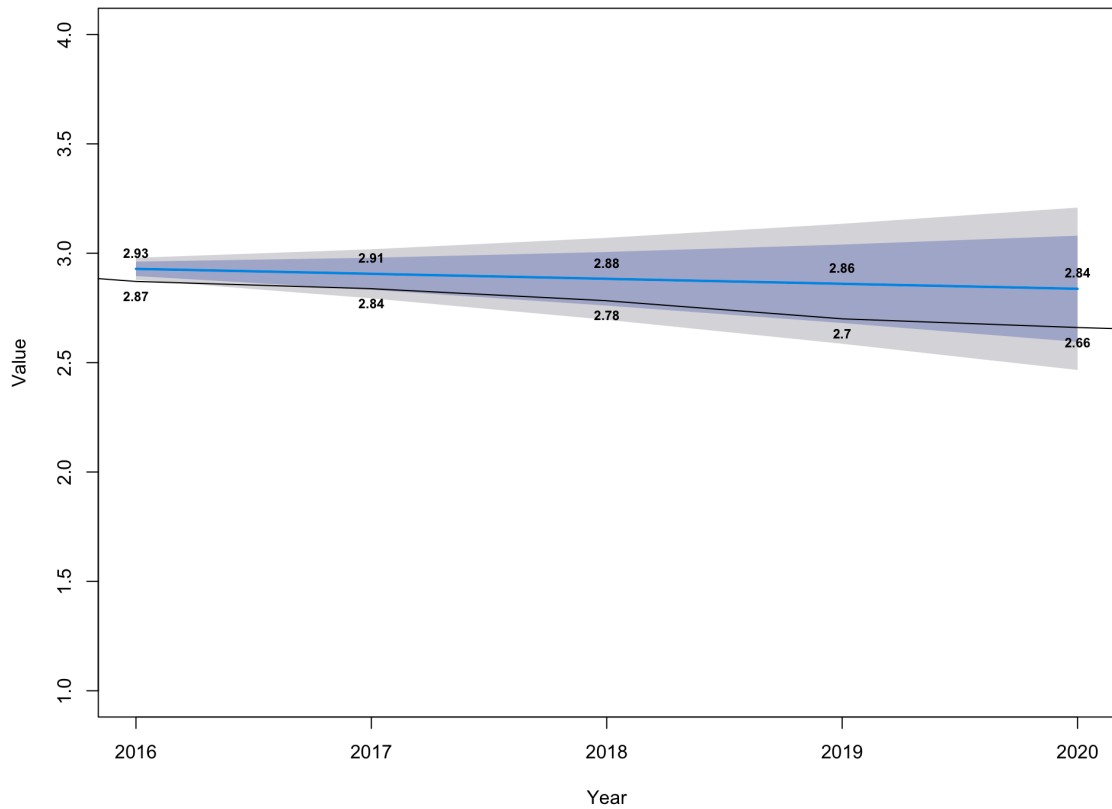
Table 5-11 and Figure 5-13 show the forecasted values using model 1 (no regressors). The blue range represents the 80% prediction interval, which means there is an 80% probability that the true value will fall within this interval. The grey range represents the 95% prediction interval, providing a wider range with a 95% probability. It can be observed that the model consistently overestimates the average *TFR* across the years, as indicated by the negative differences.

On the other hand, Table 5-12 and Figure 5-13 show the forecasted values using model 6 (female labor only). It can be observed that the model consistently underestimates the average *TFR* across the years, as indicated by the positive differences.

Based on the provided metrics, model 1 shows smaller differences and percentage errors in the forecasted values compared to the actual values. Thus, although model 6 has slightly a lower AIC, when considering both the AIC and the forecast accuracy, model 1 seems to be a better fit for forecasting the MENA average *TFR*. Hence, model 1 will be used to forecast average *TFR* from 2021-2026.

**TABLE 5-11 Difference Between Actual Average TFR and Forecasted Average
TFR Using Model 1**

Year	Actual Average TFR	Forecasted Average TFR	Difference	Difference %	Lo 80	Hi 80	Lo 90	Hi 90
2016	2.87	2.93	-0.06	-2.0%	2.90	2.96	2.88	2.98
2017	2.84	2.91	-0.07	-2.4%	2.83	2.98	2.79	3.02
2018	2.78	2.88	-0.10	-3.6%	2.76	3.01	2.70	3.07
2019	2.70	2.86	-0.16	-5.9%	2.68	3.04	2.59	3.13
2020	2.66	2.84	-0.18	-6.7%	2.60	3.08	2.47	3.21

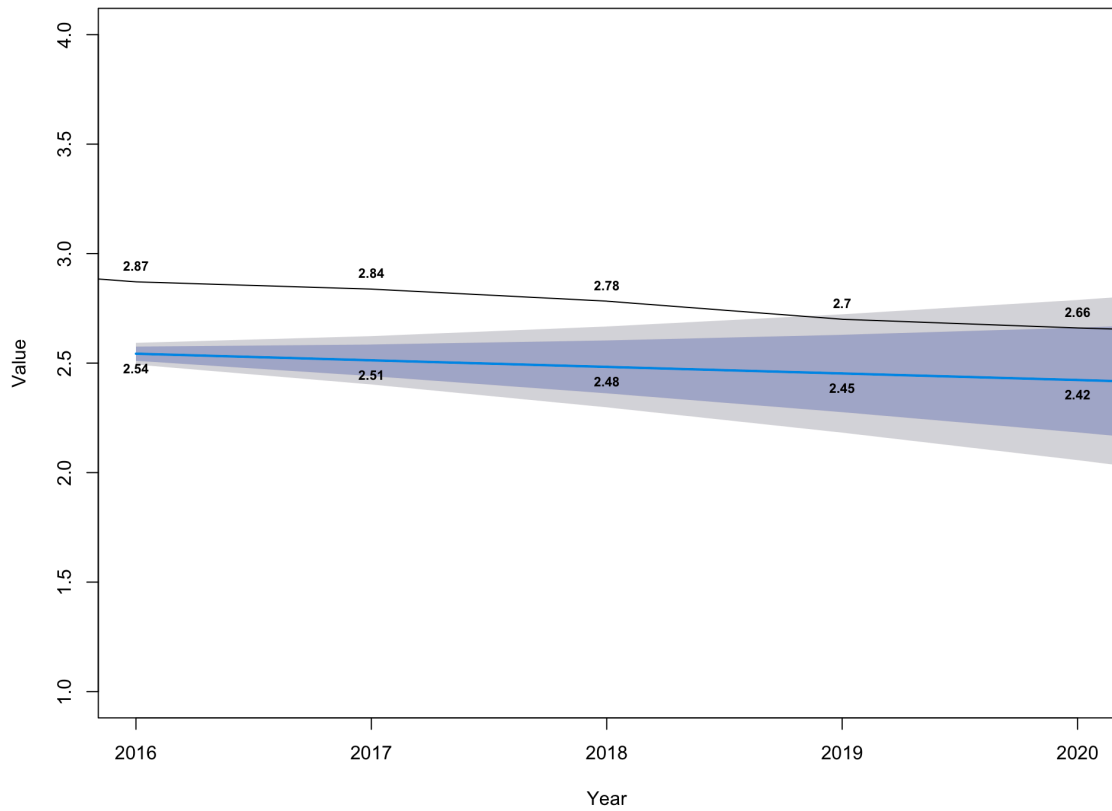


— Forecasted Average *TFR* 80% Prediction Interval
— Actual *TFR* 95% Prediction Interval

Figure 5-13 Actual vs. Forecasted Average TFR Using Model 1

TABLE 5-12 Difference Between Actual Average TFR and Forecasted Average TFR**Using Model 6**

Year	Actual Average TFR	Forecasted Average TFR	Difference	Difference %	Lo 80	Hi 80	Lo 90	Hi 90
2016	2.87	2.54	0.33	11.4%	2.51	2.58	2.49	2.59
2017	2.84	2.51	0.33	11.5%	2.44	2.58	2.40	2.62
2018	2.78	2.48	0.30	10.8%	2.36	2.60	2.30	2.67
2019	2.70	2.45	0.25	9.2%	2.28	2.63	2.18	2.72
2020	2.66	2.42	0.24	8.9%	2.18	2.66	2.06	2.79



— Forecasted Average *TFR* ■ 80% Prediction Interval
— Actual *TFR* ■ 95% Prediction Interval

Figure 5-14 Actual vs. Forecasted Average TFR Using Model 6

Figure 5-14 represents the estimated value of the average *TFR* from 2021-2026 along with prediction intervals using model 1. The blue range represents the 80% prediction interval, and the grey range represents the 95% prediction interval.

For example, taking the year 2021:

- Point Forecast: 2.5826
- Lo 80: 2.5470, Hi 80: 2.6182
- Lo 95: 2.5282, Hi 95: 2.6370

These values suggest that, based on the model, the expected average *TFR* in 2021 is estimated to be around 2.5826, and there is an 80% probability that the true value will be between 2.5470 and 2.6182, and a 95% probability that it will be between 2.5282 and 2.6370. The same interpretation applies to subsequent years in the forecast.

Looking at the provided forecast, if the point forecasts for future years are consistently decreasing, it suggests a declining trend in the predicted *TFR*.

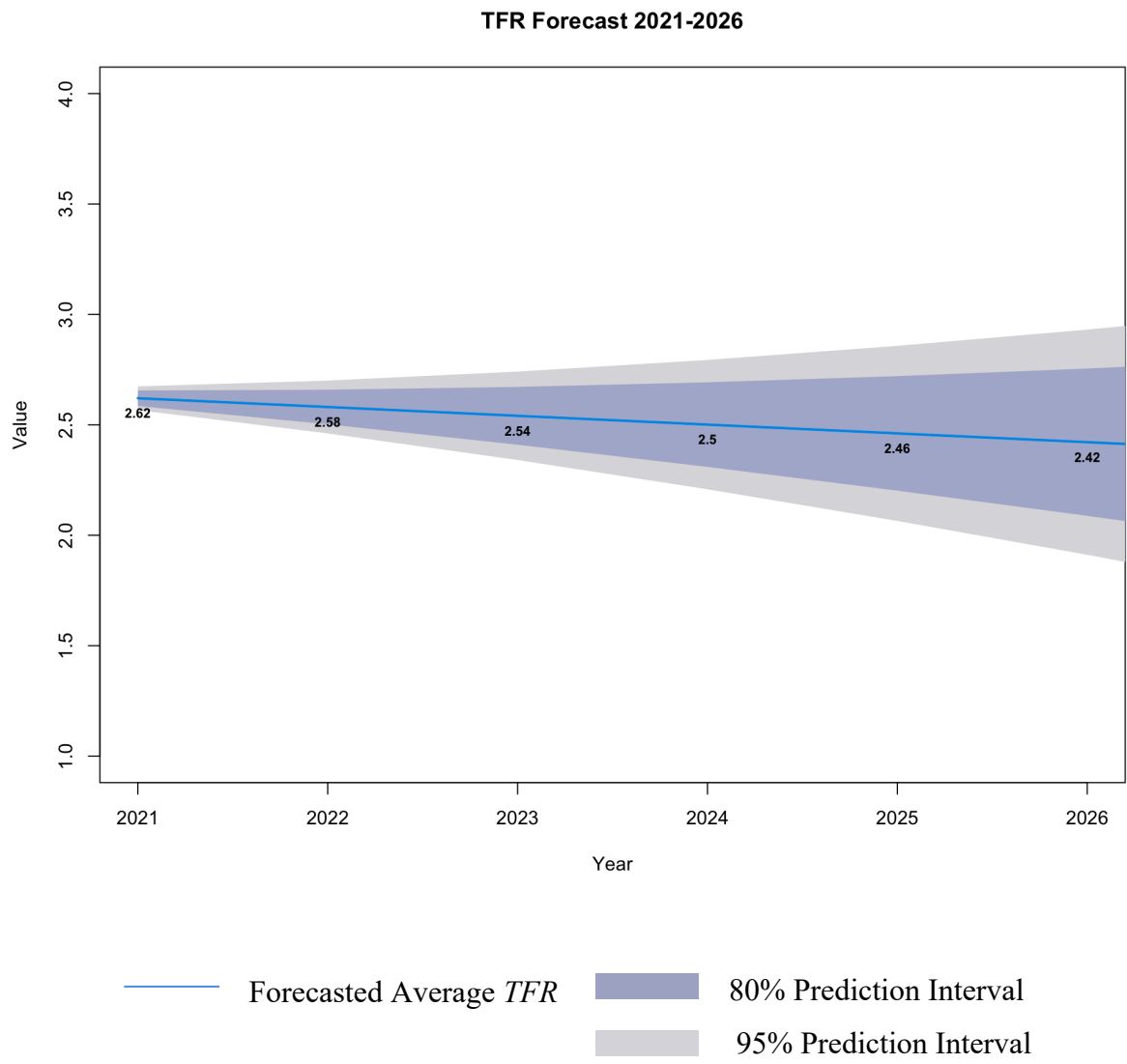


Figure 5-15 Forecasted Average TFR from 2021-2026 Using Model 1

CHAPTER 6

RESULTS AND CONCLUSION

This chapter summarizes the conclusions of the three research questions in addition to the limitations and recommendations for future research.

6.1 Distribution of Fertility Rates

The first research question aimed to provide an insightful exploration into the distribution of fertility rates in the Middle East and North Africa (MENA) region. The diversity in fertility rates revealed not only the heterogeneity within MENA but also the dynamic nature of reproductive behaviors. In summary, this analysis of fertility rate data in the Middle East and North Africa (MENA) region from 1960 to 2021 has yielded valuable insights into the changing demographic landscape. Fertility rates have exhibited a consistent and notable decline over the decades, indicative of evolving population dynamics. The data initially displayed characteristics of a left-skewed and leptokurtic distribution, but over time, it transitioned towards a more symmetrical and less heavy-tailed pattern.

However, it's important to acknowledge the limitations of this study. First and foremost, the analysis relies on publicly available demographic databases and World Bank reports. As such, the accuracy and completeness of the data are contingent on the sources themselves. Variabilities or inconsistencies in data collection methodologies among different regions within the MENA area may introduce uncertainties. Furthermore, this study does not delve into the underlying causes of the observed changes in fertility rates.

While the data shows a consistent decline, further research is needed to explore the complex socio-cultural, economic, and policy factors that drive these demographic shifts.

Despite these limitations, these findings have far-reaching implications for policy planning, resource allocation, healthcare services, and family planning programs in the MENA region. Policymakers can use this information to make informed decisions regarding healthcare, education, social services, and labor markets. Demographers and researchers can leverage these insights to better understand demographic trends and anticipate future population shifts.

The analysis reaffirms the suitability of the Generalized Extreme Value (GEV) distribution in modeling *TFR* as extreme values, aligning with the presence of outlier events within the fertility rate data. The suitability of the GEV for any years above other distributions tend to make us view fertility rates as natural extreme value or the maximum rates of fertility. This is crucial for risk assessment and management in demography and other various fields, such as finance, insurance, and economics, where understanding extreme events is of paramount importance.

As the MENA region continues to undergo demographic changes, this study provides a foundational understanding of fertility rate distribution that can inform future policies and initiatives, contributing to the region's socio-economic development and well-being. These findings and their implications serve as a steppingstone for further in-depth research into the underlying factors driving these changes and their broader societal impacts.

6.2 Statistical Modeling of Fertility Rates

In the rapidly evolving landscape of the Middle East and North Africa (MENA) region, characterized by swift demographic transitions, the application of statistical modeling emerges as an indispensable tool. This chapter has undertaken a journey to explore and analyze fertility patterns in MENA through the lens of statistical modeling, aiming to provide a nuanced understanding of the region's fertility trends over time.

The integration of regression and time series modeling approaches has provided a comprehensive understanding of fertility dynamics in the MENA region. The declining trend in fertility rates, as indicated by both models, aligns with global patterns of demographic transition.

However, the identified limitations and model violations underscore the complexity of fertility dynamics. Policymakers and researchers should approach findings with caution, recognizing the need for ongoing refinement and exploration of alternative models. The combination of regression and time series modeling offers a more nuanced understanding of the multifaceted factors influencing fertility in MENA.

As the MENA region continues its demographic transition, future research endeavors could delve deeper into specific sub-regional variations, consider additional variables, and explore the evolving impact of global trends on reproductive behaviors. The collaboration between statistical modelers, demographers, and policymakers remains crucial for developing targeted interventions that address demographic challenges and promote sustainable development in the MENA region.

In conclusion, this chapter serves as a testament to the power of statistical modeling in unraveling the intricate tapestry of fertility rates in MENA. While the journey has provided

valuable insights, it is a call to action for continued exploration, refinement, and collaboration to navigate the evolving demographic landscape of the MENA region.

6.3 Determinants of Fertility in MENA

For the third research question, an in-depth exploration of the determinants influencing fertility in MENA was conducted. Urbanization emerged as a critical factor, reflecting the complex interplay between urban development and demographic outcomes. Female empowerment, as manifested through education and labor force participation, also proved pivotal.

The final model incorporates exogenous variables (urbanization, female labor, female education) and ARIMA(0,2,0) model. GDP is excluded based on its lack of significance. The model provides forecasts for future fertility rates with associated prediction intervals, offering insights into expected trends. The forecasted values suggest a declining trend in predicted average *TFR*, providing insights into potential future fertility rate patterns in the MENA region.

In summary, this study contributes to understanding the complex dynamics of fertility rates in the MENA region, providing a foundation for future research and policy considerations. The nuanced understanding gained from this analysis contributes to the broader discourse on reproductive behaviors, offering policymakers and researchers a comprehensive view of the forces shaping fertility in the region. As we navigate the evolving landscape of societal norms and economic structures, these determinants serve as guideposts for formulating targeted interventions to address demographic challenges and promote sustainable development.

6.4 Limitations:

- **Data Limitations:** The study relies on available data from the World Bank, and data completeness and accuracy may be subject to limitations inherent in international databases. Lack of granularity in certain variables, such as detailed educational attainment or specific urbanization factors, may restrict a more nuanced analysis.
- **Regional Variability:** The MENA region is diverse in terms of cultural, economic, and political contexts. Generalizing findings to the entire region may overlook sub-regional variations and unique country-level dynamics.
- **Omitted Variables:** The models may not include all relevant determinants of fertility rates. Factors such as cultural shifts, government policies, and social norms, which are challenging to quantify, may contribute significantly.
- **Policy Dynamics:** The study does not explicitly consider the impact of policy changes or interventions on fertility rates. A more explicit exploration of policy variables and their effects could enhance the practical applicability of the research.

6.5 Recommendations for Future Research:

- **Qualitative Studies:** Conduct qualitative studies to complement quantitative findings, capturing the cultural, social, and contextual nuances that may influence fertility decisions.
- **Sub-Regional Analyses:** Break down the analysis into sub-regional or country-level studies to account for variations in fertility determinants within the MENA region.

- **Policy Impact Assessment:** Investigate the direct impact of specific policies on fertility rates, considering variations in policy implementation across countries.
- **Forecast Validation:** Validate model forecasts against real-time data as they become available, refining the model over time and ensuring its relevance for policy and decision-making.

In addressing these limitations and pursuing these recommendations, future research can contribute to a more comprehensive and nuanced understanding of fertility determinants in the MENA region.

6.6 Overall Conclusion

In conclusion, our journey through the distribution, statistical modeling, and determinants of fertility rates in MENA has unveiled a multifaceted landscape. The spatial analysis revealed geographical variations, prompting a deeper examination of localized factors influencing reproductive behaviors. Our statistical models, despite encountering challenges, provided valuable insights into the relationships between urbanization, female empowerment, and fertility rates. The determinants explored, including urbanization and female empowerment through education and labor force participation, emerged as crucial forces shaping demographic patterns in MENA.

As we synthesize these conclusions, it becomes evident that a holistic understanding of fertility dynamics in MENA requires an interdisciplinary approach. The combination of spatial analysis, statistical modeling, and exploration of determinants offers a comprehensive perspective that is crucial for policymakers, researchers, and practitioners. The findings contribute not only to the academic discourse on demography but also provide

actionable insights for formulating evidence-based policies aimed at addressing the unique challenges and opportunities within the MENA region.

This study, while comprehensive, is a steppingstone in the ongoing journey to unravel the complexities of fertility dynamics. Future research endeavors could delve deeper into specific sub-regional variations, consider additional variables, and explore the evolving impact of global trends on reproductive behaviors in MENA. The quest for knowledge continues, guided by the commitment to advancing our understanding of the intricate interplay between socioeconomic, cultural, and policy factors shaping fertility rates. Further investigations should embrace a multidisciplinary approach, incorporating insights from sociology, demography, economics, and anthropology to capture the rich tapestry of influences on family planning decisions.

As we move forward, researchers should actively engage with policymakers, translating empirical findings into practical recommendations that can inform targeted interventions and strategies. Ultimately, this ongoing scientific exploration stands as a testament to the collective dedication to unraveling the mysteries surrounding fertility dynamics in the MENA region and contributing to the global discourse on population dynamics and sustainable development.

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