

**TEMPERATURE AND PRESSURE TRANSIENTS IN
LINEAR HOT WATER FLOODED HEAVY OIL
RESERVOIRS**

BY
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In

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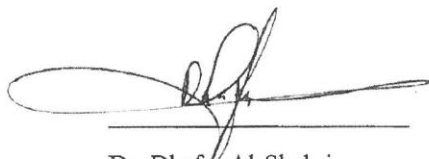
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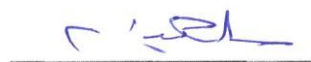
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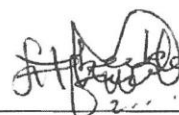


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To my beloved family

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LIST OF ABBREVIATIONS

T	Temperature
T_i	Initial Reservoir Temperature
u	Velocity
Pe	Peclet Number
κ	Coefficient of Thermal Conductivity
λ_{eff}	Effective Thermal Diffusivity
ρ	Density
c_p	Specific Heat Capacity
t	Time
z	Laplace Transform Variable
p	Pressure
p_i	Initial Pressure
k	Permeability
μ	Viscosity
ϕ	Porosity
c_t	Total Compressibility
M	Number of Grids
L	Total reservoir Length
f_A	Fractional Area
S_w	Water Saturation
D_i	Decline Coefficient
S_o	Oil Saturation

<i>BPD</i>	Barrel Per Day
<i>EOR</i>	Enhanced Oil Recovery

ABSTRACT

Full Name : Maksat Orayev
Thesis Title : Temperature and Pressure Transients in Linear Hot Water Flooded Heavy Oil Reservoirs
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As production of conventional oil declines gradually the oil industry has mainly focused on producing from heavy oil and bitumen reservoirs. These high viscosity hydrocarbon deposits account for a significant fraction of the world's oil reserves. The majority of thermal recovery techniques are utilized in heavy oil reservoirs to increase recovery by reducing oil viscosity. The most widely used thermal EOR methods to extract heavy and extra-heavy oil are steam/hot water flooding, electromagnetic radiation, electrical heating, and in-situ combustion. In most cases, isothermal conditions are assumed when solving fluid flow equations. During thermal EOR processes, it is important to consider heat flow equations as viscosity of heavy oil is strongly temperature dependent.

In this research, we model pressure and temperature transients in heavy oil reservoirs undergoing hot water injection by using mathematical tools and MATLAB. Here, we consider a moving boundary between injection and production wells. Therefore, we also consider it to be a composite reservoir with two regions. For each region, fluid properties such as density, viscosity, specific heat capacity, thermal conductivity, and compressibility are considered to be different. Heat flow equation is solved to find temperature at several time steps and then viscosity is updated in the fluid flow model to solve for pressure. Fluid flow and heat transfer equations are solved by converting them to

dimensionless form and applying Laplace transform. To convert the solution from Laplace space to real space, Gaver-Stehfest method is used to numerically inverse Laplace transform. Finally, inverse modelling is utilized to estimate the reservoir parameters and to match the pressure transient of non-isothermal reservoirs that undergo hot water injection and deduced that integrating both fluid flow and heat flow models can yield accurate results.

ملخص الرسالة

الاسم الكامل : فاجيف سليمانوف

عنوان الرسالة: درجة الحرارة والضغط العابرين في الخزانات النفطية الثقيلة المغمورة بالمياه الساخنة الخطية.

التخصص: هندسة البترول

تاريخ الدرجة العلمية: ديسمبر 2023

مع انخفاض إنتاج النفط التقليدي تدريجياً، ركزت صناعة النفط بشكل أساسي على الإنتاج من النفط الثقيل وخزانات البيتومين. تمثل هذه الرواسب الهيدروكربونية عالية اللزوجة جزءاً كبيراً من احتياطات النفط العالمية. يتم استخدام غالبية تقنيات الاسترداد الحراري في مكامن النفط الثقيل لزيادة الاستخلاص عن طريق تقليل لزوجة النفط. إن طرق الاستخلاص المعزز للنفط الحرارية EOR الأكثر استخداماً لاستخراج النفط الثقيل والتقليل للغاية هي حقن البخار/الماء الساخن، والإشعاع الكهرومغناطيسي، والتدفئة الكهربائية، والاحتراق في الموقع. في معظم الحالات، يتم افتراض الظروف الحرارية عند حل معادلات تدفق الموائع. أثناء عمليات الاستخلاص المعزز للنفط بالحرارة EOR، من المهم مراعاة معادلات التدفق الحراري حيث أن لزوجة النفط الثقيل تعتمد بشدة على درجة الحرارة. في هذا البحث، قمنا بنمذجة الضغط العابر ودرجة الحرارة العابرة في مكامن النفط الثقيل التي تخضع لحقن الماء الساخن باستخدام الأدوات الرياضية وبرنامج MATLAB. وهنا، نعتبر الحدود المتحركة بين آبار الحقن والإنتاج. ولذلك، فإننا نعتبره أيضاً كمكناً مركباً ذو منطقتين. بالنسبة لكل منطقة، نعتبر خصائص السوائل مثل الكثافة واللزوجة والسعة الحرارية المحددة والتوصيل الحراري والانضغاط مختلفة. تم حل معادلة التدفق الحراري لإيجاد درجة الحرارة في عدة خطوات زمنية ومن ثم يتم تحديث اللزوجة في نموذج تدفق السوائل لحل المعادلات للضغط. يتم حل معادلات تدفق الموائع وانتقال الحرارة عن طريق تحويلها إلى صيغة بلا أبعاد وتطبيق تحويل Laplace. لتحويل الحل من مجال Laplace إلى المجال الحقيقي ذو الوقت، يتم استخدام طريقة Gaver-Stehfest لعكس تحويل Laplace رقمياً. أخيراً، يتم استخدام النمذجة العكسية لتقدير بارامترات المكمن ومطابقة الضغط العابر للخزانات غير متساوية الحرارة التي تخضع لحقن الماء الساخن واستنتاج أن تكامل كل من نماذج تدفق السوائل وتدفق الحرارة يمكن أن يؤدي إلى نتائج دقيقة.

CHAPTER 1

INTRODUCTION

Oil and gas are the primary resources that drive the development of many countries across the globe. However, due to industrialization and development of global economy, there has been a surge in oil consumption, prompting oil-producing nations to improve their production processes for optimal output. With conventional oil reserves on the decrease, the importance of heavy oil reservoirs is expected to grow, playing a very important role in satisfying the world's energy needs. These reservoirs are likely to become a reliable source of energy in the near future. (Souraki et al., 2011). Canada, Venezuela, and the United States have the highest heavy oil in place globally. The key characteristic of these reservoirs is their extremely high viscosity, which can range between 10,000cp and 6,000,000cp (K.Alikhlalov et al., 2011).

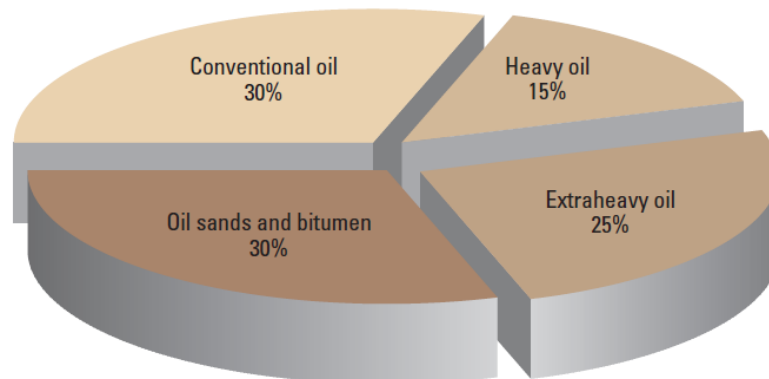


Figure 1.1 Total world oil reserves according to API classification (Alboudwarej et al., 2006).

As can be seen from the Figure 1.1, heavy oil reserves comprise approximately 70 % of total world oil reserves. Thermal EOR methods are in general applied to heavy oil

reservoirs that possess high viscosity. By introducing heat, viscosity and density of heavy oils decrease and mobility of reservoir fluids increases.

In this research thesis, we limit ourselves to performing pressure and temperature transients while injecting hot water into heavy oil reservoir. Because of the increase in temperature, there will be a change in viscosity, so while solving the pressure model, a changing viscosity model will be used. Due to temperature change in the heavy oil reservoir, at different time steps, viscosity values will be updated in the fluid flow model. Carrying out accurate pressure transient analysis will enable us to determine some critical reservoir parameters.

1.1 Problem statement

In most cases, fluid flow equation is solved independently of the heat flow equation. Since viscosity is highly temperature-dependent, in heavy oil reservoirs, it is very important to couple heat transport and fluid transport models to deal with the issue of viscosity change.

1.2 Research Objective

Our main goal is to perform transients in temperature and pressure in heavy oil reservoirs that are being injected with hot water. Therefore, only a producer-injector pair and a linear flow is considered between them. To observe pressure and temperature responses, sensitivity analysis will be conducted. Also, reservoir parameters such as permeability, pay thickness, will be estimated by using inverse modelling. This work aims to integrate both heat flow and fluid flow models and include conduction and advection heat transfer mechanisms.

1.3 Literature Review

1.3.1 Heavy Oil Reservoirs

The bulk of the world's oil reserves are composed of costly to obtain and refine heavy, viscous oils. Generally, the denser or heavier crude oil is, the less valuable it is economically. The focus of industry is changing to the exploitation of heavy oil in many parts of the world due to increasing oil demand and prices, as well as the decrease in production of the majority of conventional oil reservoirs. API values of 22.3° or less are considered as heavy oils.

Table 1.1 Total world oil reserves according to API classification (Lake et al., 2014).

Crude type	°API	Viscosity, cP	Remarks
Light and medium/Conventional oil	>22	<100	
Heavy oil	<22	>100	
Extra-Heavy crude	<10	>10,000	
Natural bitumen	<10	>1000,000	also called Tar or Oil sands

Because of their high viscosity and density, oils with an API rating of 10 or below are referred to as extraheavy, ultraheavy, or superheavy oils. According to Goodarzi et al. (2009) heavy oil is the class of oils having viscosities changing between 50 and 5000 cP. At initial reservoir temperature and pressure, it is difficult for the oil to flow because of high viscosity. According to Ancheyta and Speight (2007) heavy oil is an extremely high viscosity petroleum that has a sulfur level higher than ordinary oil. According to Meyer et al. (2007), heavy oil becomes dense due to the removal of light fractions through natural processes that occur post-evolution from natural source materials. The predominant constituents in heavy oil consist of asphaltic molecules and the incorporation of heteroatoms such as oxygen, sulfur, and nitrogen into the carbon network. Consequently,

irrespective of the oil's origin, it consistently includes high proportions of asphaltenes, heavy metals, sulfur, and nitrogen. Conventional crudes, including West Texas Intermediate or Brent, have densities between 38 and 40 API. Oil density is crucial for determining the value of the resource and for calculating the output and expenses of refining, while oil viscosity has a greater impact on producibility and recovery. Oil production is more challenging the more viscous the oil is. Density and viscosity are not directly correlated, despite the fact that the terms "viscous" and "heavy" are occasionally used to describe oils with a high density. As shown in Table 1.1, poor API gravity as well as high viscosity ($>100\text{cP}$) are characteristics of heavy oils. Heavy oils are often more viscous than conventional oils. Water has a viscosity of 1 centipoise (cP), whereas conventional oil can have a viscosity of up to 10 cP. Heavy and extraheavy oils can have viscosities between 20 cP to more than 1,000,000 cP. At ambient temperature, bitumen, the most viscous hydrocarbon, is a solid; yet, when heated, it swiftly melts. The question of why oil corporations are interested in investing resources to extract heavy oil emerges since it is less profitable, more challenging to produce, and more complex to refine than conventional oils. First of all, numerous heavy oil reservoirs may now be accessed profitably given the current state of the economy. Secondly, there are plenty of these resources. It is estimated that there are nine to thirteen trillion barrels of oil in the planet. Only around 30% of that total is made up of conventional oil; the rest is made up of bitumen, extraheavy oil, and heavy oil (Alboudwarej Joao Felix Shawn Taylor Edmonton et al., 2006).

Early research in numerical simulation of hot water injection includes mainly one-dimensional models with two-phase fluid flow in reservoir and with the assumption of no

heat loss to the surroundings. (Enamul et al., 2007) solved one dimensional energy balance equation to find temperature profile in reservoir. In their work, convection and conduction heat transfer mechanisms were employed. Next, they considered two cases where reservoir rock and fluid have (1) same temperature, (2) different temperatures. Temperature profile in reservoir undergoing hot water injection is also strongly dependent on both injected fluid and formation fluid velocity. The difference in temperature values between reservoir rock and fluid was found to be insignificant by using simultaneous iteration. Because fluid velocity in porous media was very small, convective term did not yield a significant change in temperature distribution in reservoir. However, it strongly depends on injected hot fluid velocity. Also, their findings show that temperature distribution inside the reservoir is greatly impacted by injection rate and time. By using production data of the well, (K. Wu et al., 2011) estimated the time it would take for injected fluid to breakthrough in the production well. Considering the introduced model had distinct features than numerical simulation, it would yield better results if model is used for early phases of hot water injection and simulation is used for later phases. A three-dimensional numerical simulation model introduced by (Mozaffari et al., 2013) can be used to study hot water or steam injection into heavy oil reservoirs. This model employs a finite difference method for simulation purpose. The developed model included relative permeabilities for all phases, reservoir fluid characteristics that change with temperature. However, the model did not account for change in rock characteristics due to temperature change. Other assumptions comprised oil that is non-volatile, and vapor gas that cannot be dissolved in oil phase. Additionally, heavy oil recovery can be 30 % when hot water is injected into reservoir.

(Ashrafi et al., 2011b) conducted laboratory experiments on Athabasca heavy oil containing cores to know how viscosity, density, and interfacial tension changes versus temperature. Then, they performed a numerical simulation on heterogeneous reservoir rock sample which is saturated with heavy oil undergoing steam injection. Porosity and permeability data that were obtained from experiments were applied in numerical simulation. They also carried out sensitivity analyses by changing temperature, rate, and quality of injected steam, positions of barriers inside core. From experimental and numerical studies, they found that, when the temperature and quality of steam were selected as 200 °C and 85 % respectively, optimum oil recovery was obtained. Also, residual oil saturation changed dramatically as barrier position is shifted inside the core. Moreover, porosity, permeability, injection rate and thickness also had a significant effect on oil recovery. In order to decrease the large amount of residual oil saturation left after CSS (Cyclic Steam Stimulation) process, hot water injection right after CSS is proposed by (Z. Wu & Liu, 2019). They used both experimental and numerical simulation to optimize the process. From experimental studies, they concluded that when oil recovery is 20 % after CSS process, a 120 °C hot water injection can be employed to further improve the heavy oil recovery. Another experimental work carried out by (Willman et al., 1961) showed how hot water injection mechanisms affected oil recovery. In their work, it has been shown that oil recovery is very high when steam or hot water is injected, whereas very low recovery was noted when waterflooding used on the core.

(Ghaffari & Ambastha, 1995) performed a numerical simulation study in a heavy oil reservoir to investigate pressure transient analysis for a horizontal well undergoing steam injection. The authors assumed that the reservoir is composite, containing two

regions: swept and unswept sections. By assigning no flow outer and constant rate inner boundary conditions, the three-dimensional diffusivity equation in cartesian coordinate system is solved. To simulate proposed the model, they used a block-centered finite difference approximation. Moreover, the pseudosteady-state method is utilized in order to calculate the swept volume after steam injection. Sensitivity study is carried out by estimating the impact of mobility ratio, shape of swept section, storativity ratio, and location of well in x,y, and z directions. To calculate the volume that was swept by steam injection, (Abdurrahman Satman et al., 1980) used pressure transient calculations and the considered the model to be a composite reservoir. The composite reservoir consisted of two regions: the inner (swept by steam) section and outer section. Both inner and outer sections had different properties such as permeability, compressibility, and porosity. They solved dimensionless diffusivity equations for both inner and outer sections. To find the solution for dimensionless pressure equation, Laplace space solution is used followed by Stehfest algorithm. They found out that impact of wellbore storage was very small. A Pseudosteady line that shows volume of the swept region comes after the semi-log line.

Heat efficiency of steam injection into reservoir can be calculated by considering only total amount of steam injected (Messner & Williams, 1982). By knowing the steam affected area in reservoir, heat losses and steam performance efficiency can be estimated. After carrying out pressure falloff test followed by injection of steam into reservoir, authors obtained very necessary parameters. Some of these parameters are kh (permeability thickness), porosity of swept area, and heat loss. Wellbore storage, transient and pseudosteady state, area swept by steam defined most parts of the falloff test curves on the graph.

1.3.2 Conventional Thermal Recovery Methods

1.3.2.1 Cyclic Steam Stimulation technique

The huff and puff technique, also known as cyclic steam stimulation, has three phases, which include steam injection phase, soaking phase, and oil production phase. With this method, the oil temperature can be increased by injecting steam into the well for an extended period of time. This lowers the oil's viscosity and improves its mobility. Once it is confirmed that there is sufficient steam in the well, it is shut in for a few weeks to ensure that the heat generated is evenly distributed, as illustrated in Figure 1.2. Because of the pressure created by the injected steam, the oil extraction can continue from the well (Mokheimer et al., 2019). Due to heat losses and production decrease over time, at some point, it becomes necessary to restart the steam injection cycle. The same prior processes are performed repeatedly, however after a certain number of cycles, the effectiveness of the process inevitably declines.

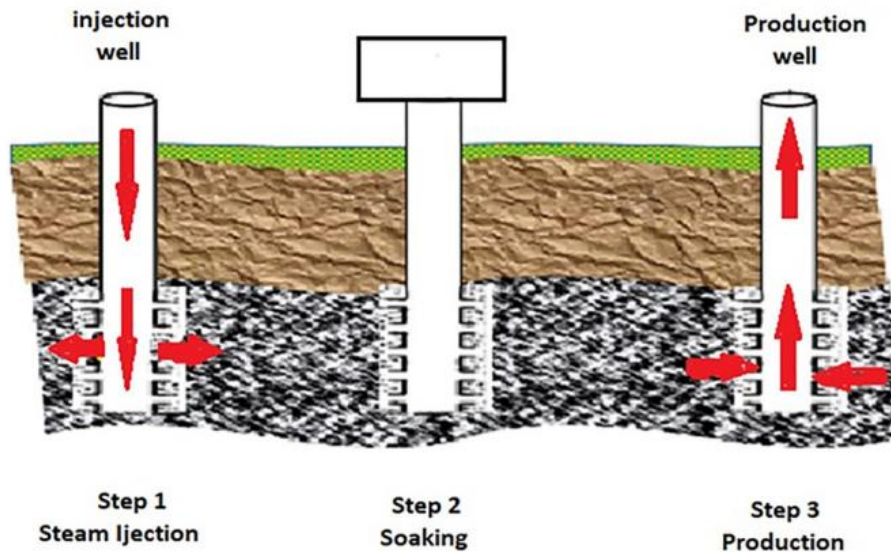


Figure 1.2 Schematic diagram showing Cyclic Steam Stimulation (Mokheimer et al.,2019).

1.3.2.2 Steam Assisted Gravity Drainage technique (SAGD)

By employing this method, a set of horizontal wells is drilled vertically aligned within the reservoir. Similar to other thermal processes, the injection of hot water serves to decrease the viscosity of the heavy oil. The heavy oil is removed from the sand due to its decreased viscosity and is gravity-driven into well below (a horizontal production well) as shown in Figure 1.3. The two parallel and horizontal wells are crucial to this technique, and directional drilling is the sole thing that makes this feasible. According to Hosseini et al., production of heavy oil in carbonate reservoirs is the optimum use for this technique. Although most of these reservoirs are naturally fractured, hardly much research has been done in this field. It is thought to be a very promising approach, but in order to reach its full potential, additional work and research are required.

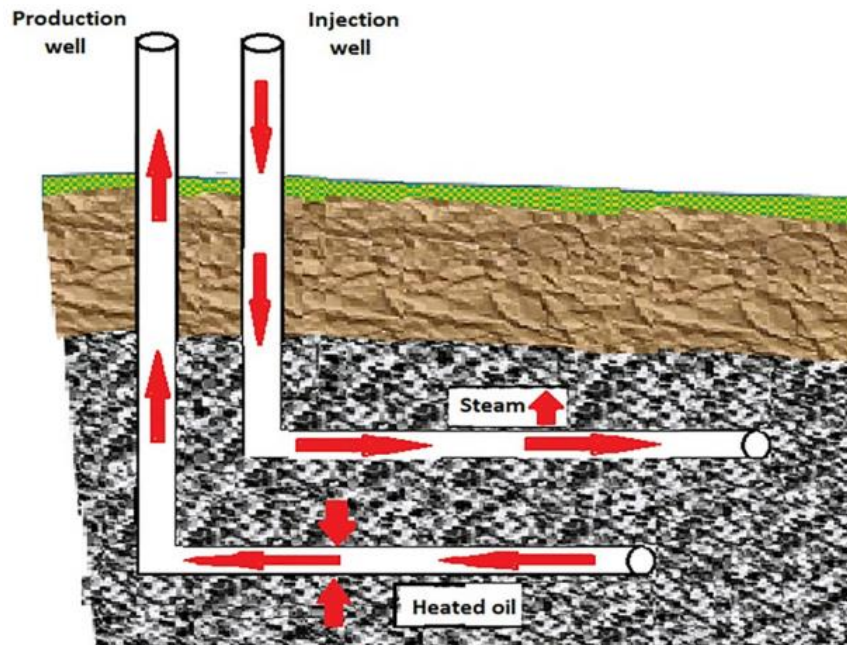
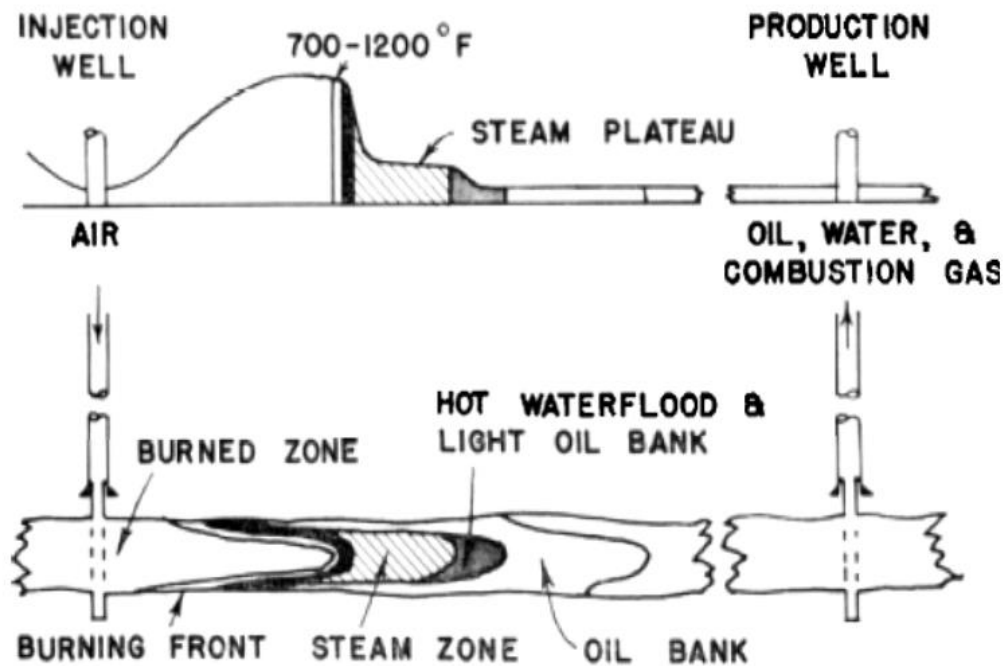


Figure 1.3 Schematic diagram showing steam-assisted gravity drainage (Mokheimer et al.,2019).

1.3.2.3 In-Situ Combustion technique

In the realm of thermal Enhanced Oil Recovery (EOR) methods, in situ combustion stands out as having the highest potential for development. However, akin to other thermal EOR approaches, in situ combustion demands substantial quantities of fossil fuels, typically in the form of gas. Given that every thermal EOR technique carries an environmental footprint due to associated CO₂ emissions, in situ combustion is regarded as the next generation in thermal EOR methods. Fuel, oxygen, and igniter are required, just like in other traditional heating techniques. However, as Figure 1.4 shows, in ISC, the ignition happens within the well. The process cannot be finished until an adequate volume of squeezed heated up enhanced oxygen is delivered into an oil reservoir at pressures equivalent to the reservoir's internal pressure. Up to 319 psi of initial reservoir pressure are



typical (Glatz et al., 2011).

Figure 1.4 Schematic diagram of in situ combustion (McNiel and Moss, 1958).

1.3.2.4 Electrical and Electromagnetic Heating methods

This Enhanced Oil Recovery method heats the heavy oil in the reservoir by means of electrical current. The heating process of the petroleum will create a vapor chamber in the formation, which will increase the mobility of the crude. The low-frequency Ohmic heating, radiofrequency/microwave heating, and the inductive heating are the three primary categories of electrical Enhanced Oil Recovery techniques. A potential difference is created across two electrodes, one of which serves as the anode and the other as the cathode, in the process of ohmic heating. This can be implemented on the field by employing two wells, one of which is used as the cathode and the other as the anode (Yuan et al., 2003).

The newly developed EOR method known as electromagnetic heating (EM) is receiving a lot of attention nowadays. This process uses electromagnetic waves to create electrical energy, which is then converted to thermal energy and used in EOR for the resistive and dielectric substances that are utilized in EOR, electromagnetic heating method thus transforms electromagnetic energy into thermal energy (Sahni et al., 2000).

1.3.2.5 Steam and Hot Water Injection methods

As depicted in Figure 1.5, the thermal Enhanced Oil Recovery (EOR) technique involving the injection of steam and heated water involves the substantial pumping of both water and steam into the well. This process aims to elevate the temperature of the oil, thereby lowering the viscosity of heavy oil.

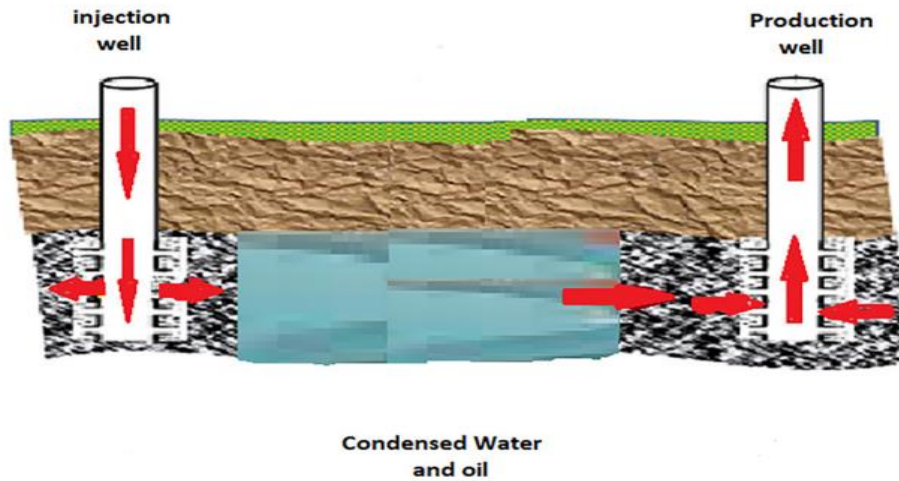


Figure 1.5 Schematic diagram of in situ combustion (Mokheimer et al.,2019).

The methods of heated water injection were simulated using a 3D sand-packed displacement model along with experimental testing, as detailed by Zhou, D., and Yang, 2017. Additionally, it is found that, primarily due to a low transport ratio between CO₂ and heavy oil, early gas breakthroughs restrict enhanced heavy oil recovery with immiscible CO₂ injecting. According to published studies, hot water flooding is less successful than steam injection because water has a lower thermal energy content than steam (Bera and Babadagli, 2015). Water, however, has a stronger dragging power than steam at high temperatures. Steam injecting is the process of continuously injecting steam into an oil-containing deposit. As a result, a slow-moving hot water zone with almost constant temperature is created, which greatly reduces the viscosity of the heavy oil and boosts its ability to move. Within the heated zone, this strongly mobilized oil is driven by a vaporizing gas, which lowers the initial oil saturation to as low as 10%. In a unique approach, (Mohebbifar et al., 2015) integrated the use of nanotechnology with biomaterials. The maximum efficiency of 78% was noted when biopolymer and SiO₂ nanoparticles were injected into one pore volume. The recovery factor of steam flooding normally ranges from 50% to 60% of OOIP. But because it is a pattern-driven process, the

size of the pattern and the geology will ultimately determine how well it performs (Ali, S., 1982). The AL-1 Block in the Shengli Oilfield in China is one example of an extra-heavy oil reserve that has been studied experimentally using heated steam flooding approach (Liu et al., 2017). Numerical research on steam flooding in heavy oil, which boosts the extraction of oil by 58 percent during a predetermined period of time, indicates that only 32 percent of oil initially in place may be obtained utilizing the hot water injection technique (Mozaffari et al., 2013). Hossain et al. (2018) introduced dimensionless scaling factors specifically tailored for thermal flooding in porous media. They also put forth methodologies for gauging the hydraulic and thermal diffusivity of a fluid within a porous medium.

Hadabi et al. (2016) carried out a study concentrating on the heterogeneous heavy oil field in southern Oman. The aim was to pinpoint the optimal thermal exchange parameters for steam flooding within the reservoir. For the study, a CMG-STARs simulator was built to simulate heated steam flooding procedure using the steam trap subcooled valve control. They came to the conclusion that, following steam flooding, valve control provides a quicker exchange of heat. Additionally, it yields greater productivity with less hot water needed.

The research project conducted by Pang, Z., Liu, H., and Zhu, L. (2015) incorporated the application of nitrogen thermal foam flooding to tackle issues related to steam overriding and steam fingering. The findings revealed that the utilization of nitrogen foaming could significantly enhance the displacement effectiveness of steam flooding, increasing it from 43 percent to 81 percent in a single sand-pack study.

CHAPTER 2

WORKFLOW

2.1 Assumptions

The following set of assumptions are considered in our research:

- Linear flow from injection to production well.
- Isotropic and homogeneous reservoir.
- Reservoir fluid is comprised of oil and water.
- Oil and water are at liquid phase all the time.
- Both oil and injected water are flowing.
- No heat loss to the adjacent formations.
- We will consider the reservoir to be a composite reservoir with two regions. Zone 1 is a mixture of heavy oil and injected hot water. And the heavy oil region is Zone 2 which is unaffected by the injected heat.

2.2 Initial Condition (IC) and Boundary Conditions (BCs)

2.2.1 Initial and Boundary conditions for the heat flow model

Since we consider a moving boundary between injector and producer, we also consider our model to be a composite reservoir. Therefore, in our heat model 1 IC and two BCs are assigned. The following IC, and BCs are employed to solve the heat flow model:

Initially, the reservoir is at a temperature T_i

$$T_j(x, t = 0) = T_i. \quad (2.1)$$

Left Boundary Condition (at the injector) is given as:

$$T(x=0, t) = T_{inj} \quad (2.2)$$

And Right Boundary Condition (at the producer) is given as:

$$T(x=L, t) = T_{prod} \quad (2.3)$$

2.2.2 Initial, Boundary and Interface conditions for the fluid flow model

Initially, the reservoir is at a pressure p_i :

$$p_j(x, t=0) = p_i \quad ; \quad j = 1, 2 \quad (2.4)$$

Left Boundary Condition (at the injector) is given as:

$$\text{LBC} \Rightarrow -\left(\frac{kA}{\mu B}\right)_1 \left(\frac{\partial p_1}{\partial x}\right)_{x=0} = q_{inj} \quad (2.5)$$

And Right Boundary Condition (at the producer) is given as:

$$\text{RBC} \Rightarrow \left(\frac{kA}{\mu B}\right)_3 \left(\frac{\partial p_3}{\partial x}\right)_{x=L} = q_{prod} \quad (2.6)$$

Additionally, two interface conditions (IFCs) at the interfaces are assigned to solve the fluid flow model:

$$\text{IFC 1a} \Rightarrow \frac{k_1}{\mu_1} \left(\frac{\partial p_1}{\partial x}\right)_{x=x_{int1}} = \frac{k_2}{\mu_2} \left(\frac{\partial p_2}{\partial x}\right)_{x=x_{int1}} \quad (2.7)$$

$$\text{IFC 1b} \Rightarrow p_1(x=x_{int1}, t) = p_2(x=x_{int1}, t) \quad (2.8)$$

2.3 Methodology

The methodology for our research is given below:

1. Heat Transport and Fluid Flow equations in 1D are considered.

2. Relevant Initial Conditions (ICs), Boundary Conditions (BCs), and Interface Conditions (IFCs) are assigned for both heat flow and fluid flow models.
3. Dimensionless variables are defined for both heat flow and fluid flow models.
4. Heat Transport and Fluid Flow equations, ICs, BCs, and IFCs are converted into dimensionless form.
5. Laplace transformation is applied to Heat Transport and Fluid Flow equations, ICs, BCs, and IFCs in the dimensionless form.
6. Solutions to the Heat Transport and Fluid Flow in Laplace domain are found.
7. BCs and IFCs are substituted in the solution.
8. Gavel-Stehfest algorithm is utilized to numerically invert the solutions to obtain their values in the real domain.
9. Fluid flow and heat transport problems are solved sequentially to update fluid viscosity at each time interval.
10. Sensitivity analysis is conducted to observe pressure and temperature responses.
11. By using inverse modeling, reservoir parameters are estimated.

2.4 Heat Model Derivations

2.4.1 Energy balance for the proposed model

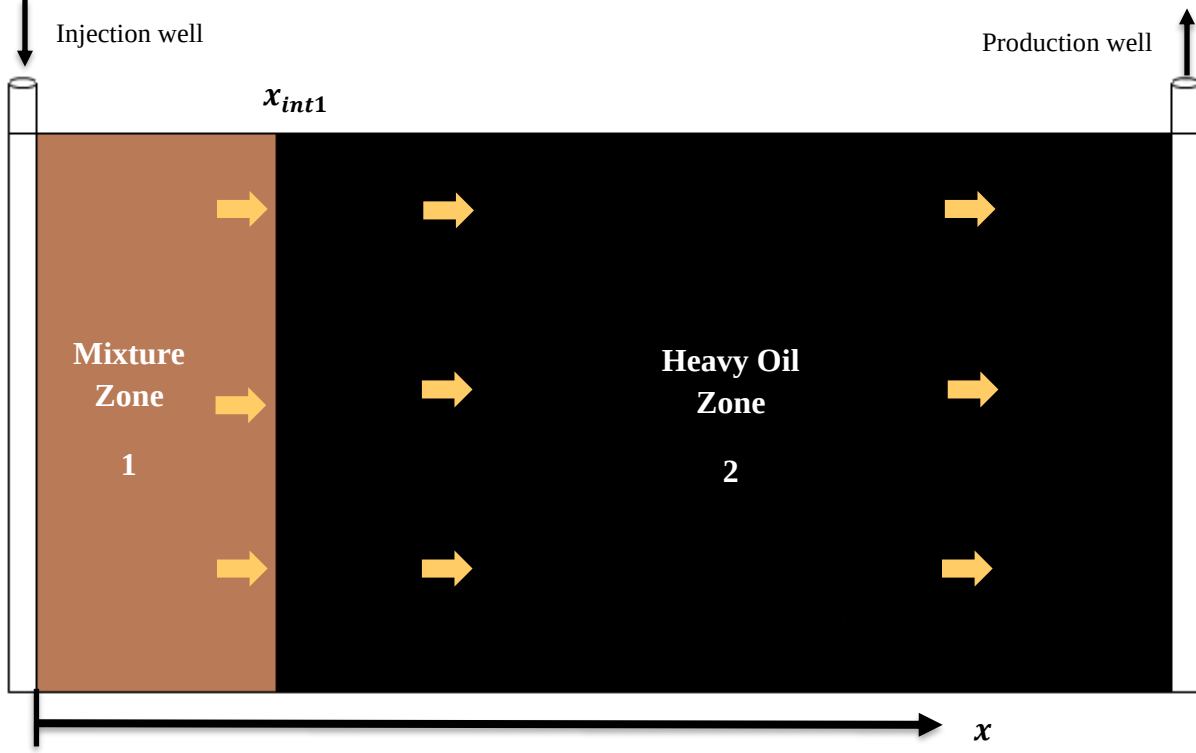


Figure 2.1 Schematic illustration of the proposed model containing two zones

Figure 2.1 given above represents a composite reservoir model with two distinct zones and linear flow from injector towards the producer. First, we consider energy balance equation for both solid and fluid phase which can be written as follows:

$$\begin{aligned} & \nabla \cdot \left[(1-\phi) \kappa_s \cdot \nabla T_s + \phi \kappa_f \cdot \nabla T_f \right] - \nabla \cdot (\rho_f c_{pf} \bar{u}_f T_f) + q_T \\ &= \frac{\partial}{\partial t} \left[(1-\phi) \rho_s c_{vs} T_s + \phi \rho_f c_{vf} T_f \right] \end{aligned} \quad (2.9)$$

After simplification and some assumptions, final form of linear heat transport equation in 1D cartesian coordinates (only linear flow) can be written as shown below:

$$\lambda_{eff_j} \frac{\partial^2 T_j}{\partial x^2} - \dot{u}_{f_j} \frac{\partial T_j}{\partial x} = \frac{\partial T_j}{\partial t} \quad \text{for } j = 1, 2 \quad (2.10)$$

Where $\lambda_{eff} = \frac{\kappa_{eff}}{(\rho c_v)_{eff}}$ and $\dot{u}_f = \frac{(\rho c_p)_f}{(\rho c_v)_{eff}} u_f$.

Detailed derivation of the Equation 2.10 is provided in the Appendix A.

2.4.2 Dimensionless Form of Energy Balance

Heat flow equation in 1D is transformed into dimensionless form by using following variables:

$$x_D = \frac{x}{L}, \quad (2.11)$$

$$t_D = \frac{\dot{u}_{1,ini} t}{L}, \quad (2.12)$$

and

$$T_D = \frac{T}{T_i} \quad (2.13)$$

Hence, the Heat Flow equation in dimensionless form can be written as follows:

$$\frac{1}{Pe_j} \frac{\partial^2 T_{Dj}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial T_{Dj}}{\partial x_D} = \frac{\partial T_{Dj}}{\partial t_D} ; \quad j = 1, 2 \quad (2.14)$$

Where $Pe_j = \frac{\dot{u}_{1,ini} L}{\lambda_{eff_j}}$. Detailed derivation of Equation 2.14 is provided in Appendix C.

The dimensionless form of initial and boundary conditions are as follows:

$$T_{D_j}(x_D, t_D = 0) = 1, \quad (2.15)$$

$$T_D(x_D = 0, t_D) = \frac{T_{inj}}{T_i} = T_{D_{inj}}, \quad (2.16)$$

and

$$T_D(x_D = 1, t_D) = \frac{T_{prod}}{T_i} = T_{D_{prod}}. \quad (2.17)$$

2.4.3 Analytical Solution

A full derivation of analytical solution is provided in the Appendix D. We can take Laplace transformation of dimensionless energy balance equation with respect to time, and we can obtain the following equation:

$$\frac{1}{Pe_j} \frac{\partial^2 \hat{T}_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial \hat{T}_{D_j}}{\partial x_D} - z \hat{T}_{D_j} = -1 \quad (2.18)$$

Now we can apply Laplace transformation to Dirichlet boundary conditions as follows:

$$\hat{T}_D(x_D = 0, z) = \frac{T_{D_{inj}}}{z}, \quad (2.19)$$

and

$$\hat{T}_D(x_D = 1, z) = \frac{T_{D_{prod}}}{z} \quad (2.20)$$

Equation 2.18 has a particular solution, and it is given by:

$$\hat{T}_{D_j}(x_D, z) = A_j e^{\frac{1}{2} x_D Pe_j \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} - \sqrt{\frac{4z}{Pe_j} + \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} \right)^2} \right)} + B_j e^{\frac{1}{2} x_D Pe_j \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} + \sqrt{\frac{4z}{Pe_j} + \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} \right)^2} \right)} + \frac{1}{z} \quad (2.21)$$

Where $j = 1, 2$.

Equation 2.21 is computationally expensive, unstable, and too complicated to solve.

2.4.4 Semi-Analytical solution

In Appendix E, the full derivation of semi-analytical solution is provided. First, let's consider dimensionless 1D heat flow equation in Laplace domain:

$$\frac{1}{Pe_j} \frac{\partial^2 \hat{T}_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial \hat{T}_{D_j}}{\partial x_D} - z \hat{T}_{D_j} = -1 \quad (2.22)$$

It can be discretized into grids as follows:

$$\frac{1}{Pe_{j,m}} \left[\frac{\hat{T}_{D_{j,m+1}} - 2\hat{T}_{D_{j,m}} + \hat{T}_{D_{j,m-1}}}{(\Delta x_D)^2} \right] - \dot{\omega}_{j,m} \left[\frac{\hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m-1}}}{\Delta x_D} \right] - z \hat{T}_{D_{j,m}} = -1 \quad (2.23)$$

After rearranging and simplifications, we get:

$$-\left(1 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D\right) \hat{T}_{D_{j,m-1}} + \left(2 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2\right) \hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m+1}} = Pe_{j,m} (\Delta x_D)^2 \quad (2.24)$$

Left Boundary condition (LBC) for Injection well in dimensionless Laplace domain is given by below equation:

$$\hat{T}_D(x_D = 0, z) = \frac{T_{D_{inj}}}{z} \quad (2.25)$$

After substituting Left boundary condition, discretized equation for First Grid can be written as:

$$\left[3 + 2\dot{\omega}_j Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2 \right] \hat{T}_{D_1} - \hat{T}_{D_2} = Pe_{j,m} (\Delta x_D)^2 + \frac{2}{z} (1 + \dot{\omega}_j Pe_{j,m} \Delta x_D) T_{D_{inj}} \quad (2.26)$$

Right Boundary condition (LBC) for Production well in dimensionless Laplace domain is given by:

$$\hat{T}_D(x_D = 1, z) = \frac{T_{D_{prod}}}{z} \quad (2.27)$$

Again, after substituting Reft boundary condition, discretized equation for Final Grid can be written as:

$$-\left(1 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D\right) \hat{T}_{D_{M-1}} + \left[3 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D + z Pe_{j,M} (\Delta x_D)^2 \right] \hat{T}_{D_M} = Pe_{j,M} (\Delta x_D)^2 + \frac{2T_{D_{prod}}}{z} \quad (2.28)$$

2.5 Derivation of Flow Model

Full derivation equations of the fluid flow model are provided in Appendix G. The diffusivity equation in 1D cartesian coordinates for linear flow in a composite reservoir with two regions can be written as follows:

$$\frac{\partial^2 p_1}{\partial x^2} = \left(\frac{\mu \phi c_t}{k} \right)_1 \frac{\partial p_1}{\partial t} \quad \text{for } 0 \leq x \leq x_{int1} \quad (2.29)$$

$$\frac{\partial^2 p_2}{\partial x^2} = \left(\frac{\mu \phi c_t}{k} \right)_2 \frac{\partial p_2}{\partial t} \quad \text{for } x_{\text{int1}} \leq x \leq L \quad (2.30)$$

Where L is total reservoir length and x_{int1} is distance to interface. Initially, reservoir is at constant pressure p_i :

$$p_j(x, t = 0) = p_i \quad ; \quad \text{for } j = 1, 2 \quad (2.31)$$

However, we have left boundary condition and right boundary condition of constant flux, and equal flux and equal pressure at the interface.

Left Boundary Condition (flux boundary condition at the injector) is given as:

$$\text{LBC} \Rightarrow - \left(\frac{kA}{\mu B} \right)_1 \left(\frac{\partial p_1}{\partial x} \right)_{x=0} = q_{\text{inj}} \quad (2.32)$$

And Right Boundary Condition (flux boundary condition at the producer) is given as:

$$\text{RBC} \Rightarrow \left(\frac{kA}{\mu B} \right)_3 \left(\frac{\partial p_3}{\partial x} \right)_{x=L} = q_{\text{prod}} \quad (2.33)$$

Also, equal flux and equal pressure interface conditions (IFCs) at the interface are shown below:

$$\text{IFC 1a} \Rightarrow \frac{k_1}{\mu_1} \left(\frac{\partial p_1}{\partial x} \right)_{x=x_{\text{int1}}} = \frac{k_2}{\mu_2} \left(\frac{\partial p_2}{\partial x} \right)_{x=x_{\text{int1}}} \quad (2.34)$$

$$\text{IFC 1b} \Rightarrow p_1(x = x_{\text{int1}}, t) = p_2(x = x_{\text{int1}}, t) \quad (2.35)$$

Dimensionless variables are used to convert boundary and interface conditions into dimensionless form:

$$x_D = \frac{x}{L} \quad (2.36)$$

$$t_D = \left(\frac{k}{\mu \phi c_t} \right)_{1ini} \frac{t}{L^2} = \frac{\eta_{1ini} t}{L^2} \quad (2.37)$$

$$p_D = \frac{(p_i - p)}{p_i} \quad (2.38)$$

Therefore, we can get the dimensionless form of the flow equations:

$$\frac{\partial^2 p_{D_1}}{\partial x_D^2} = b_1 \frac{\partial p_{D_1}}{\partial t_D} \quad \text{for } 0 \leq x \leq x_{int1} \quad (2.39)$$

$$\frac{\partial^2 p_{D_2}}{\partial x_D^2} = b_2 \frac{\partial p_{D_2}}{\partial t_D} \quad \text{for } x_{int1} \leq x \leq L \quad (2.40)$$

Dimensionless form of initial condition is provided below:

$$p_{D_j}(x_D, t_D = 0) = 0 \quad \text{for } j = 1, 2 \quad (2.41)$$

Dimensionless form of the left boundary condition:

$$\left(\frac{\partial p_{D_1}}{\partial x_D} \right)_{x_D=0} = q_{D,inj} \quad (2.42)$$

Dimensionless form of the right boundary condition:

$$\left(\frac{\partial p_{D_2}}{\partial x_D} \right)_{x_D=1} = q_{D,prod} \quad (2.43)$$

Dimensionless form of the interface conditions (equal flux and equal pressure at the interface):

$$\frac{k_1}{\mu_1} \left(\frac{\partial p_{D_1}}{\partial x_D} \right)_{x_D=x_{Dint1}} = \frac{k_2}{\mu_2} \left(\frac{\partial p_{D_2}}{\partial x_D} \right)_{x_D=x_{Dint1}} \quad (2.44)$$

$$p_{D_1}(x_D = x_{D_{\text{int}1}}, t_D) = p_{D_2}(x_D = x_{D_{\text{int}1}}, t_D) \quad (2.45)$$

Next, we apply Laplace transformation to dimensionless fluid flow equation, initial, boundary and interface conditions:

$$\frac{\partial^2 \hat{p}_{D_j}}{\partial x_D^2} - z b_j \hat{p}_{D_j} = 0 \quad \text{for } j = 1, 2 \quad (2.46)$$

$$\left(\frac{\partial \hat{p}_{D_1}}{\partial x_D} \right)_{x_D=0} = \frac{q_{D, \text{inj}}}{z} \quad (2.47)$$

$$\left(\frac{\partial \hat{p}_{D_2}}{\partial x_D} \right)_{x_D=1} = \frac{q_{D, \text{prod}}}{z} \quad (2.48)$$

$$\frac{k_1}{\mu_1} \left(\frac{\partial \hat{p}_{D_1}}{\partial x_D} \right)_{x_D=x_{D_{\text{int}1}}} = \frac{k_2}{\mu_2} \left(\frac{\partial \hat{p}_{D_2}}{\partial x_D} \right)_{x_D=x_{D_{\text{int}1}}} \quad (2.49)$$

$$\hat{p}_{D_1}(x_D = x_{D_{\text{int}1}}, z) = \hat{p}_{D_2}(x_D = x_{D_{\text{int}1}}, z) \quad (2.50)$$

General solution to Equation 2.46 can be written as:

$$\hat{p}_{D_1}(x_D, z) = A e^{-x_D \sqrt{z b_1}} + B e^{x_D \sqrt{z b_1}} \quad (2.51)$$

$$\hat{p}_{D_2}(x_D, z) = C e^{-x_D \sqrt{z b_2}} + D e^{x_D \sqrt{z b_2}} \quad (2.52)$$

2.6 Water saturation and fractional area of mixing/penetration

As mentioned earlier, in our model, we have two zones: mixture zone and heavy oil zone. Heavy oil zone contains only heavy oil, unaffected by the injected heat, however, mixture zone contains both heavy oil and injected hot water, hence, we will have water saturation in mixture zone. It is necessary to explain how mixture zone is formed and how

interface between two zones advances as injection continues. Equation 2.53 explains how fractional area of penetration changes with water saturation. As water injection continues, fractional area of penetration decreases, because water saturation increases, hence, hot water has less space to penetrate.

$$f_A = f_{A_{\max}} - \frac{(f_{A_{\max}} - f_{A_{\min}}) D_i S_w}{1 + D_i S_w} \quad (2.53)$$

Where f_A is fraction of the area that water can penetrate, $f_{A_{\max}}$ is maximum fractional area that water can penetrate, $f_{A_{\min}}$ is minimum fractional area that water can penetrate, S_w is water saturation, and D_i is decline coefficient.

First, we assume that there is a 40 feet-long mixture zone with 12 percent initial water saturation before starting the simulation.

$$A_w = f_A * A \quad (2.54)$$

Where A is the cross-sectional area of the reservoir, A_w is the area that water can penetrate.

Equations 2.55 to 2.57 show how mixture zone length is calculated. Equation 2.55 describes how we calculate the amount of water that is being injected into the reservoir for every timestep.

$$V_{wat} = 0.23394 * q_{inj} * \Delta t \quad (2.55)$$

$$L_p = \frac{V_{wat}}{A_w} \quad (2.56)$$

$$L_{mix_{new}} = L_{mix} + L_p \quad (2.57)$$

Equations 2.58 to 2.60 show water saturation in the mixture region is being calculated.

$$V_{p(old)} = \phi^* A^* L_{mix} \quad (2.58)$$

$$V_{p(new)} = V_{p(old)} + \phi^* A^* L_p \quad (2.59)$$

$$S_w = \frac{S_{w,ini} * V_{p(old)} + V_{wat}}{V_{p(new)}} \quad (2.60)$$

2.7 Input Data

The table provided below depicts the input data that we used in our model.

Table 2.1 Input data into our model

Parameter	Unit	Value	Parameter	Unit	Value
T_i	F	250	B_{hvo}	-	0.9948
L	ft	1000	B_{hotw}	-	0.9926
w	ft	60	M	-	100
h	ft	50	p_i	psi	2500
ϕ	-	0.3	k	mD	3000
q_{inj}	bbl/day	80	ρ_{hotw}	$\frac{lbm}{ft^3}$	62.428
q_{prod}	bbl/day	-70	$C_{p_{hotw}}$	$\frac{Btu}{lbm \cdot F}$	0.9993
c_t	$\frac{1}{psi}$	$7 * 10^{-6}$	κ_{hotw}	$\frac{Btu}{hr \cdot ft \cdot F}$	0.606
ρ_{hvo}	$\frac{lbm}{ft^3}$	63.1771	ρ_s	$\frac{lbm}{ft^3}$	167
$C_{p_{hvo}}$	$\frac{Btu}{lbm \cdot F}$	0.4419	C_{p_s}	$\frac{Btu}{lbm \cdot F}$	0.21
κ_{hvo}	$\frac{Btu}{hr \cdot ft \cdot F}$	$69.38 * 10^{-6}$	κ_s	$\frac{Btu}{hr \cdot ft \cdot F}$	1.65

CHAPTER 3

VALIDATION

For validation, a heat flow equation is converted to dimensionless form and then discretized. Full derivation of numerical model is provided in Appendix F. Discretized main equation is shown below:

$$-\left(1 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D\right) T_{D_{j,m-1}}^{n+1} + \left(2 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D + C_m\right) T_{D_{j,m}}^{n+1} - T_{D_{j,m+1}}^{n+1} = C_m T_{D_{j,m}}^n \quad (3.1)$$

By using Dirichlet left boundary condition at the injection well and after simplifications we get the following discretized equation for the first grid:

$$\left(3 + 2\dot{\omega}_1 Pe_1 \Delta x_D + C_1\right) T_{D_1}^{n+1} - T_{D_2}^{n+1} = C_1 T_{D_1}^n + 2\left(1 + \dot{\omega}_1 Pe_1 \Delta x_D\right) T_{D_{inj}} \quad (3.2)$$

By using Dirichlet right boundary condition at the production well and after simplifications we get the following discretized equation for the final grid:

$$-\left(1 + \dot{\omega}_M Pe_M \Delta x_D\right) T_{D_{M-1}}^{n+1} + \left(3 + \dot{\omega}_1 Pe_1 \Delta x_D + C_M\right) T_{D_M}^{n+1} = C_M T_{D_M}^n + 2T_{D_{prod}} \quad (3.3)$$

For validation purposes, we compare outputs of semi-analytical and fully numerical models. In the figures below, we plot temperature against distance at 1 hour, 500 hours and 3000 hours for both semi-analytical and fully numerical model. As we can see from the graph, outputs of semi-analytical and fully numerical models perfectly match each other.

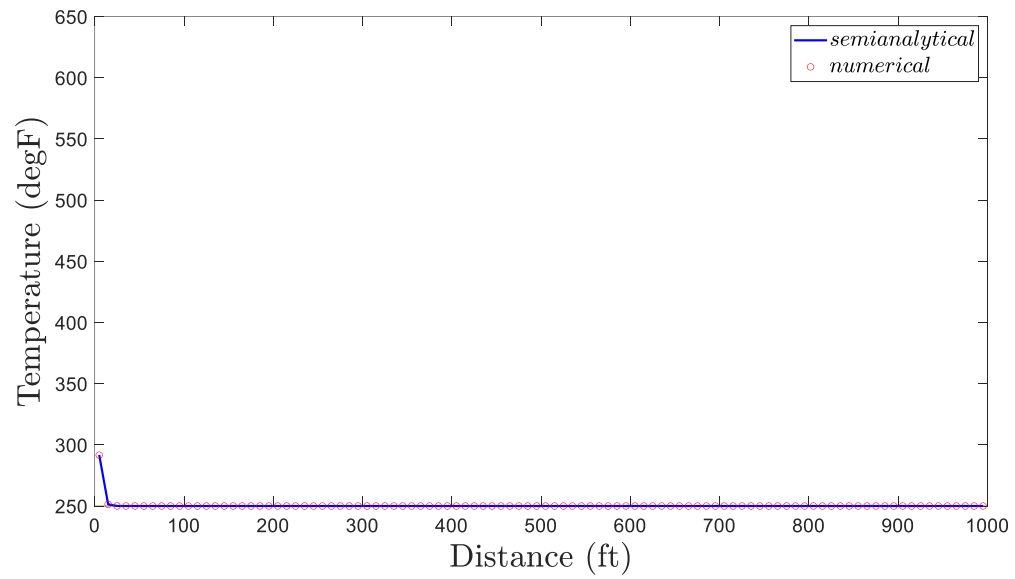


Figure 3.1 Temperature against distance at 1 hour

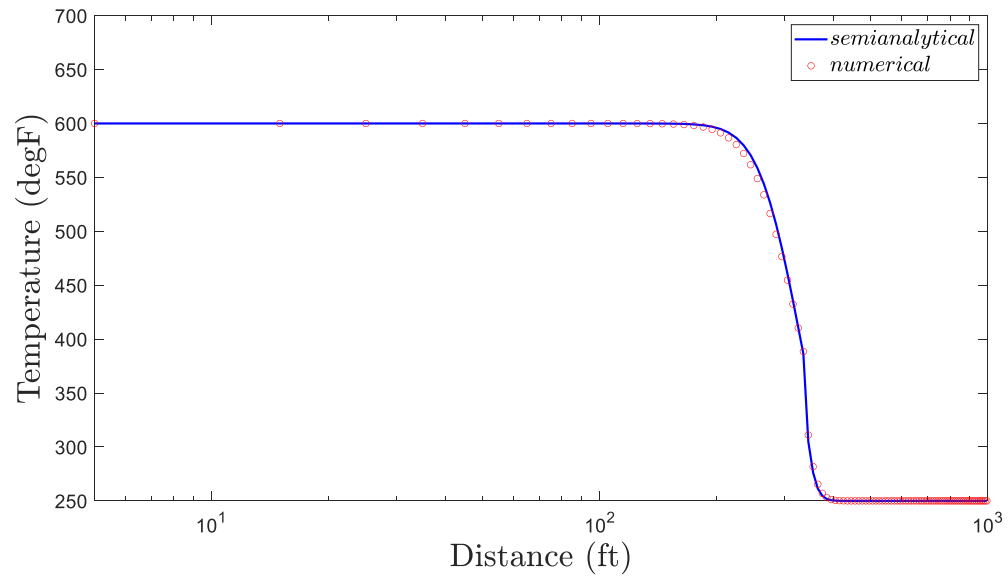


Figure 3.2 Temperature against distance at 500 hours

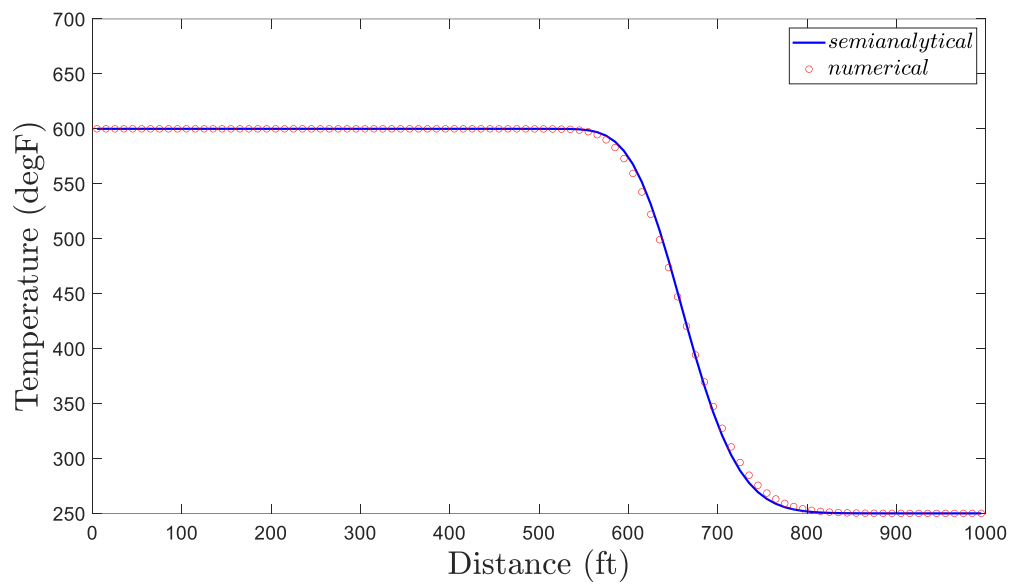


Figure 3.3 Temperature against distance at 3000 hours

CHAPTER 4

SENSITIVITY ANALYSIS

In this section, sensitivity analysis is carried out to observe pressure, temperature, and interface movement responses by changing certain set of parameters. Such parameters include injection rate, production rate, total reservoir length, pay thickness, initial reservoir temperature, maximum fractional area, minimum fractional area, and decline coefficient.

4.1 Temperature Model

4.1.1 Temperature vs time

In this section, temperature curve increase versus time is depicted in graphs provided below. In these graphs, temperature against time at several points from injection well is shown. Figures 4.1 through 4.3 display temperature profiles over time at various linear points from the injection well within the reservoir, specifically at points 5 feet, 15 feet, and 25 feet away. As we can easily see from the figures, the temperature differential diminishes rapidly as distance increases from the injection well. As observed and anticipated, the injection rate is directly linked to the temperature. Figure 4.1 depicts the temperature versus time for three different injection rates at 5 ft distance away from injection well. As evident from the figure, the endpoint temperature values at the end of 3000 hours are very close. When the injection rate is 120 BPD, temperature rises higher than that of injection rates of 100 BPD and 80 BPD. The temperature plotted against time in Figure 4.2 corresponds to a distance of 15 feet from the injection well. We note a very similar pattern to the previous scenario.

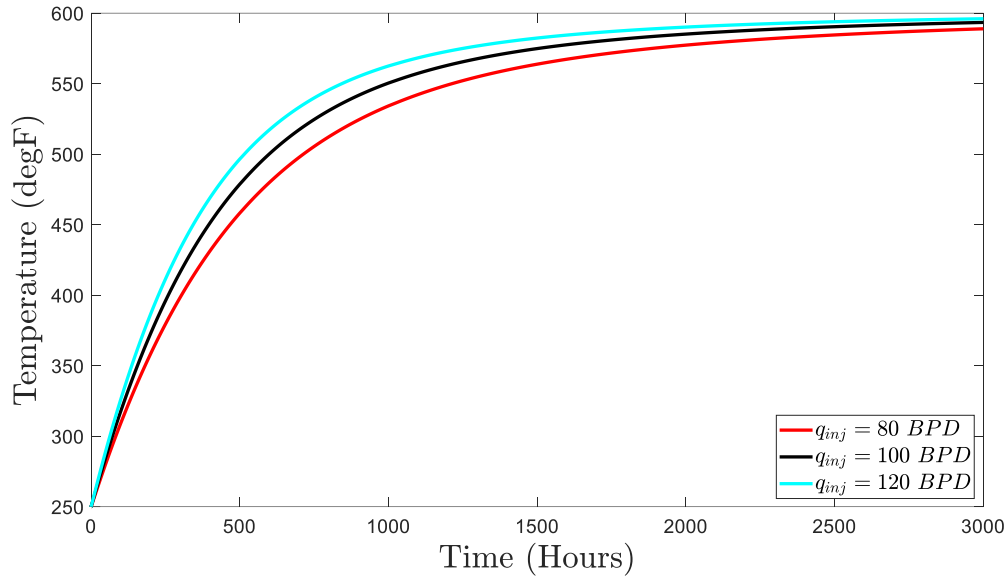


Figure 4.1 Temperature against Time at $x = 5 \text{ ft}$ for $q_{prod} = -70 \text{ BPD}$ with varying injection rates

However, in Figure 4.3, we observe a distinct temperature profile. As we can see, the endpoint temperature values at the end of 3000 hours are far away from each other compared to 5 feet and 15 feet cases.

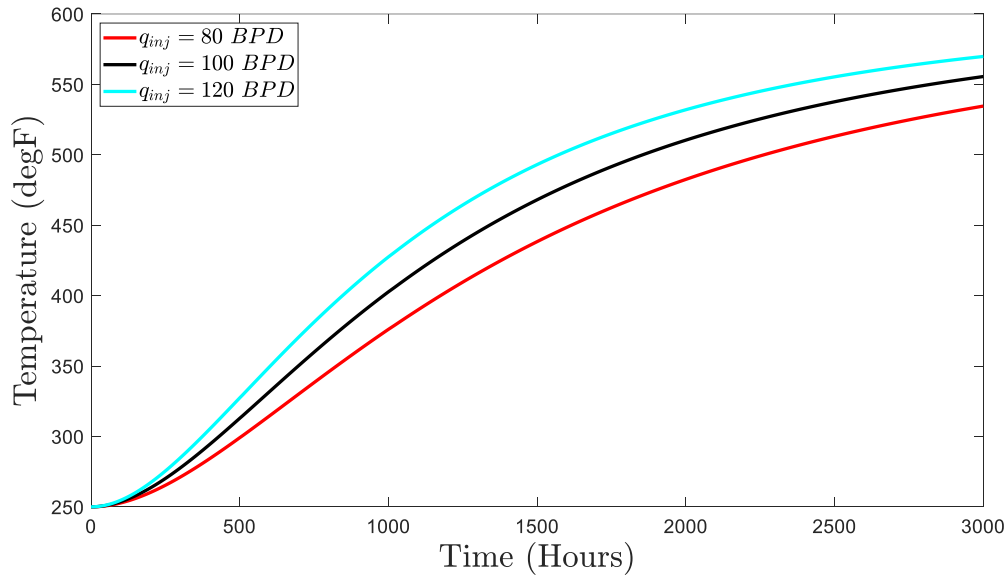


Figure 4.2 Temperature against Time at $x = 15 \text{ ft}$ for $q_{prod} = -70 \text{ BPD}$ with varying injection rates

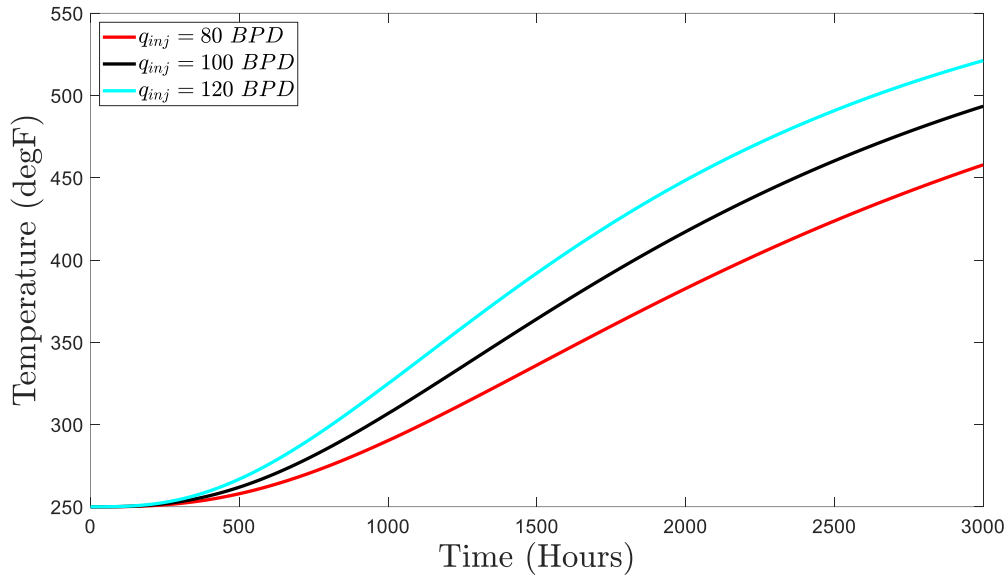


Figure 4.3 Temperature against Time at $x = 25$ ft for $q_{prod} = -70$ BPD with varying injection rates

4.1.2 Temperature vs distance

Figure 4.4 to 4.6 depicts temperature versus distance at 1 hour, 500 hours, and 3000 hours for only varying injection rates. As we can see from Figure 4.4, after 1 hour, there is almost no visible difference between temperature profiles for varying injection rates.

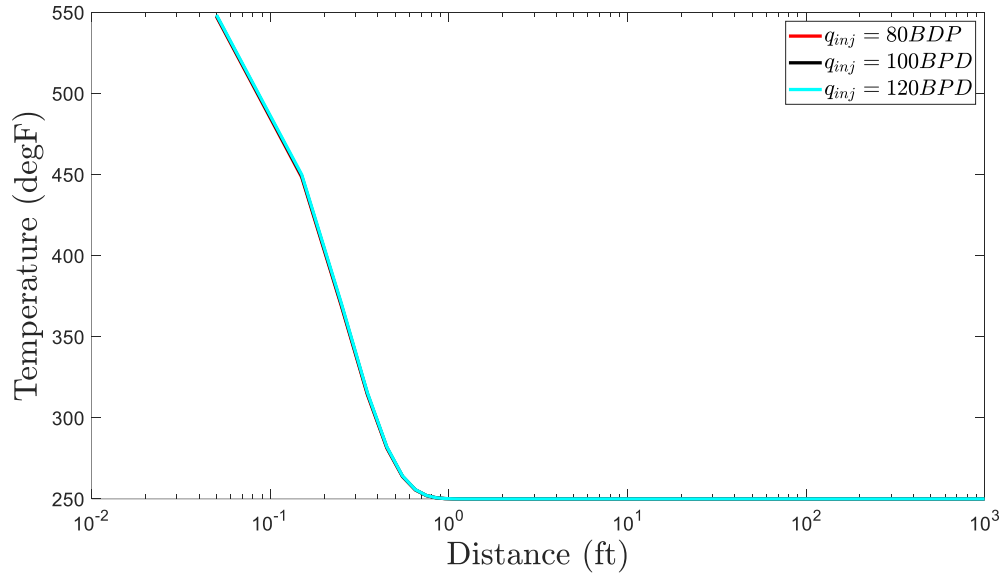


Figure 4.4 Temperature against Distance at 1 hour for $q_{prod} = -70$ BPD with varying injection rates

However, if we look at the Figure 4.5, we can observe a difference in temperatures for different injection rates. As expected, higher injection flowrates result in higher temperatures.

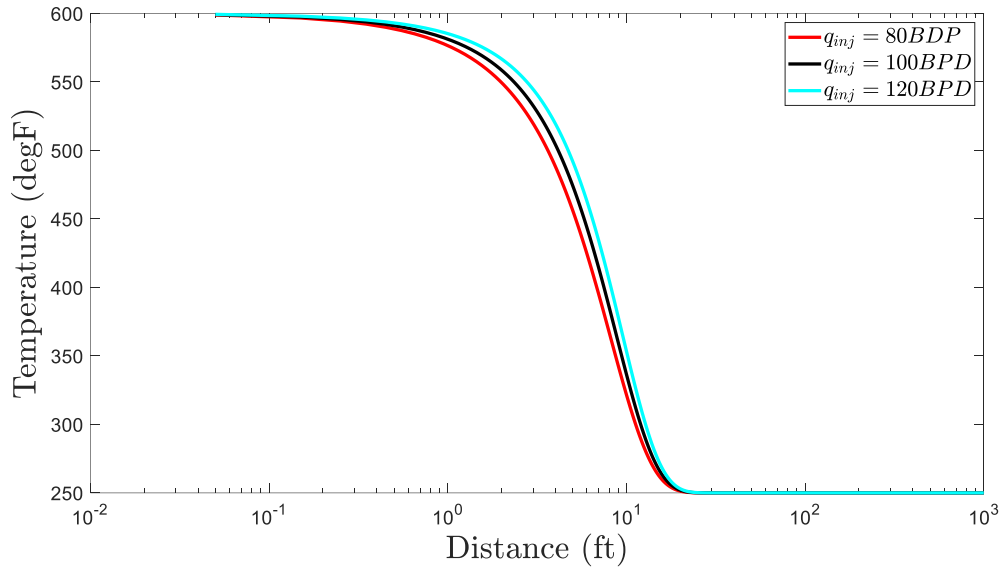


Figure 4.5 Temperature against Distance at 500 hours for $q_{prod} = -70$ BPD with varying injection rates

At 500 hours, as we can see from the graph, injected heat affects only as far as approximately 20 ft, and the rest of the reservoir is at initial temperature of 250 F.

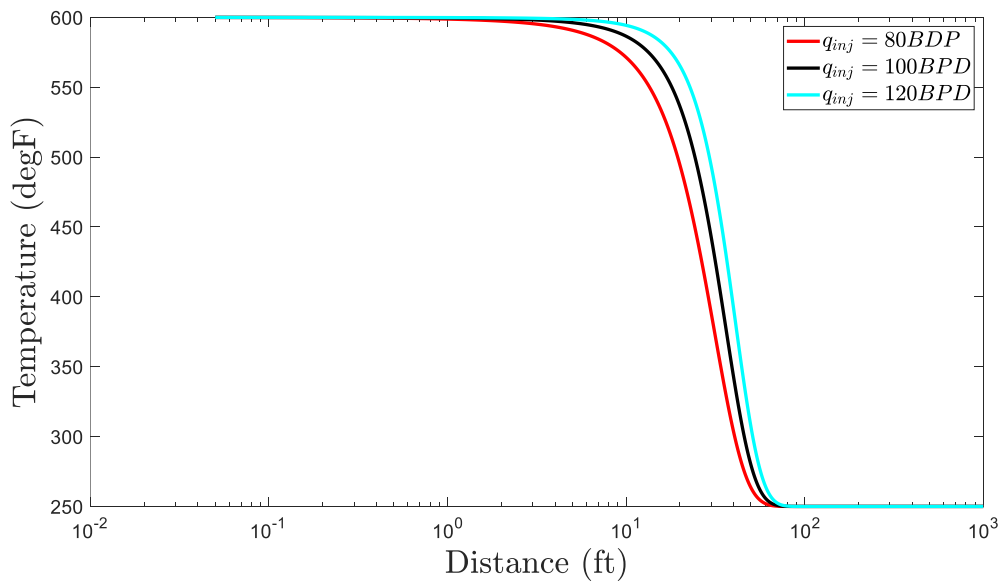


Figure 4.6 Temperature against Distance at 3000 hours for $q_{prod} = -70$ BPD with varying injection rates

Looking at Figure 4.6, after 300 hours, we can observe that injected heat affects as far as almost 100 ft.

4.2 Pressure Model

4.2.1 Effect of Flowrate

Figures 4.7 and 4.8 display how injection and production well pressures change with varying injection and production rates. Solid lines represent injection well pressures and the dotted lines depict production well pressures. In Figure 4.7, production rate is kept at constant value of -70 BPD, as we can see there is an increase in both injection and production well pressures with increasing injection rates.

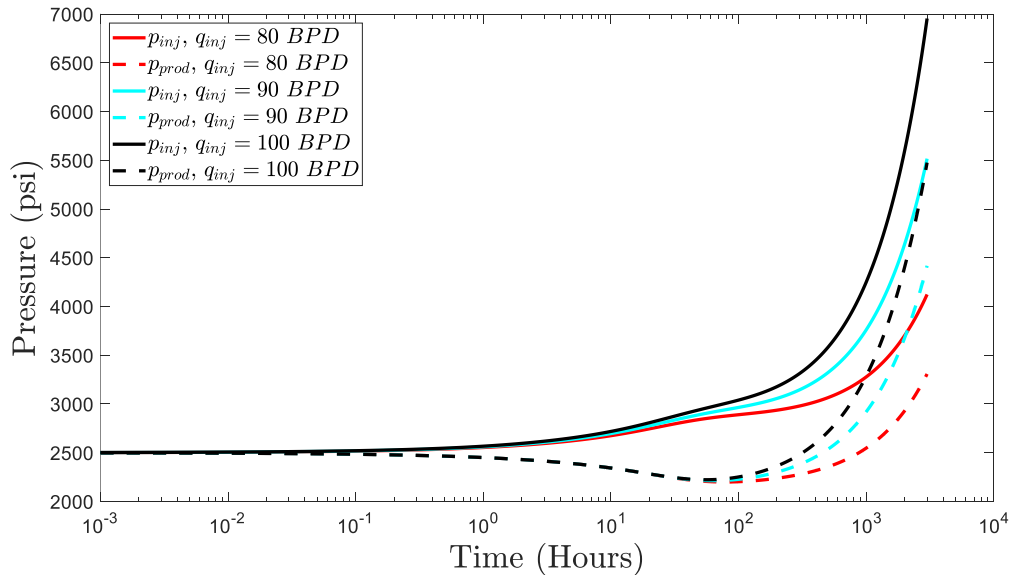


Figure 4.7 Pressure against Time for $q_{prod} = -70$ BPD with varying injection rates

However, in Figure 4.8, injection rate is kept at constant value of 80 BPD, and we change production rates to see how pressure profile is affected. As can be seen from the graph below, increasing production rates reduces both injection and production well pressures over time. Also, for production rates of -70 and -75 BPD, we can see an upward trend in

both injection and production pressures. In contrast, when production rate is -80 BPD, both injection and production well pressures show a slight downward trend.

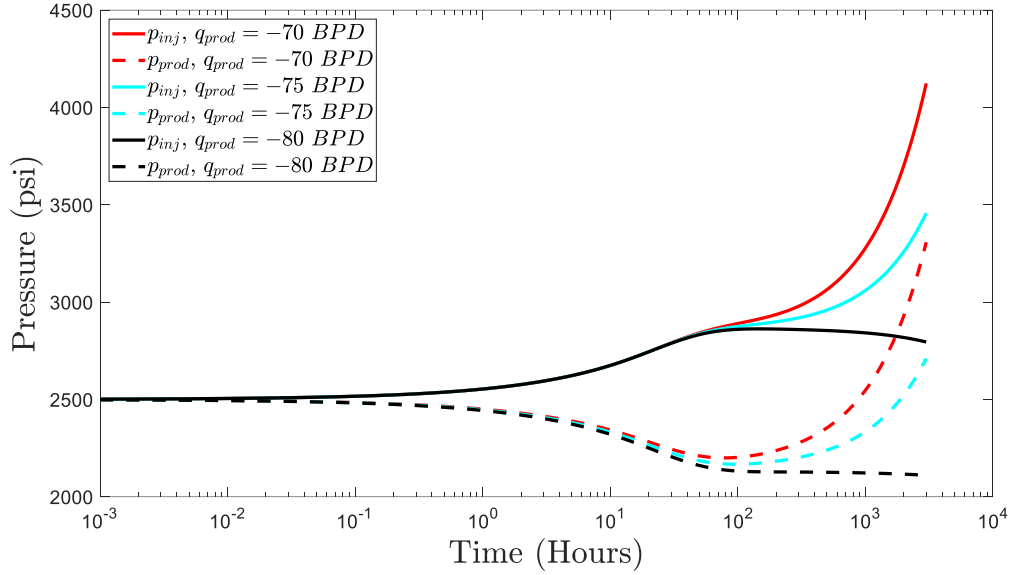


Figure 4.8 Pressure against Time for $q_{inj} = 80$ BPD with varying production rates

4.2.2 Effect of Total Reservoir Length

Below Figure 4.9 depicts the effect of different total reservoir lengths. As we can see, as

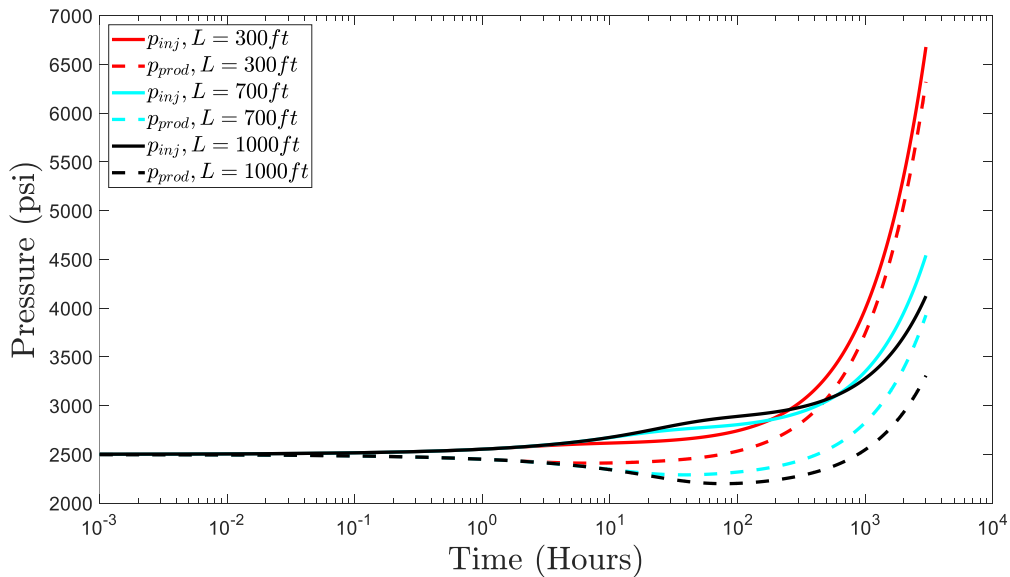


Figure 4.9 Pressure against Time for $q_{inj} = 80$ BPD & $q_{prod} = -70$ BPD with varying Lengths

reservoir length gets shorter, both injection and production well pressures become very high because production well feels the injection well earlier than that of longer reservoir lengths. For example, when total reservoir length is 300 ft, both injection and production pressures reach more than 6000 psi.

4.2.3 Effect of Pay Thickness

Figure 4.10 given below illustrates the effect of pay thickness on pressure profile at injection and production wells. As we can see from the graph, as the thickness becomes

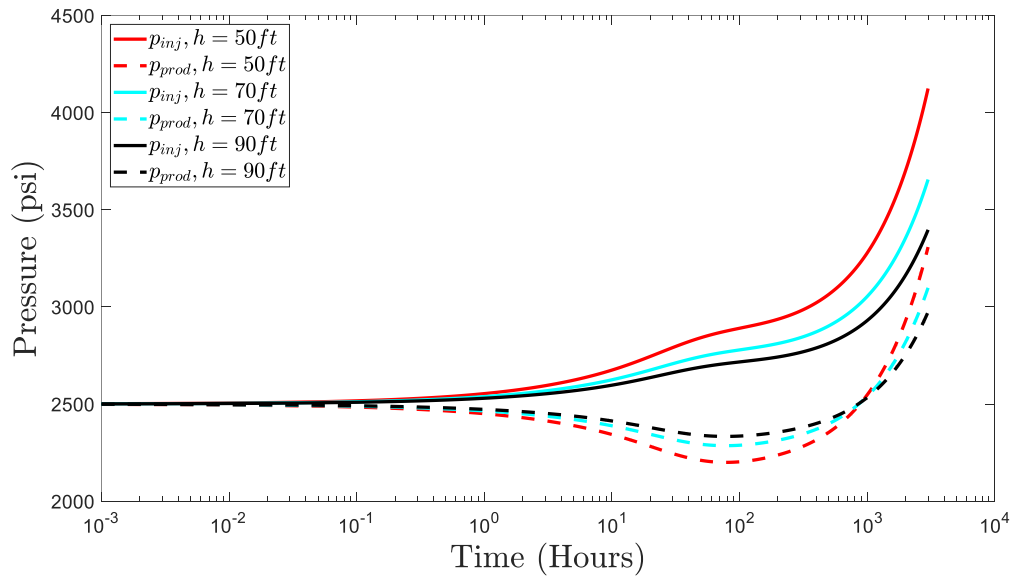


Figure 4.10 Pressure against Time for $q_{inj} = 80 \text{ BPD}$ & $q_{prod} = -70 \text{ BPD}$ with varying Pay Thickness

larger, injection and production well pressures reduce due to increasing cross sectional area. As cross-sectional area increases with increasing thickness, injected fluid penetrates more area and thus causes reduction in pressure profile.

4.2.4 Effect of Initial Temperature

Figure 4.11 shows how varying initial reservoir temperature affects injection and production well pressure profiles. A higher initial temperature lowers viscosity and in turn, it results in lower pressure profile at the wells due to increased mobility. However, lower initial reservoir temperature yields higher viscosity, which in turn causes reduced mobility of heavy oil and high pressure profile at injector and producer.

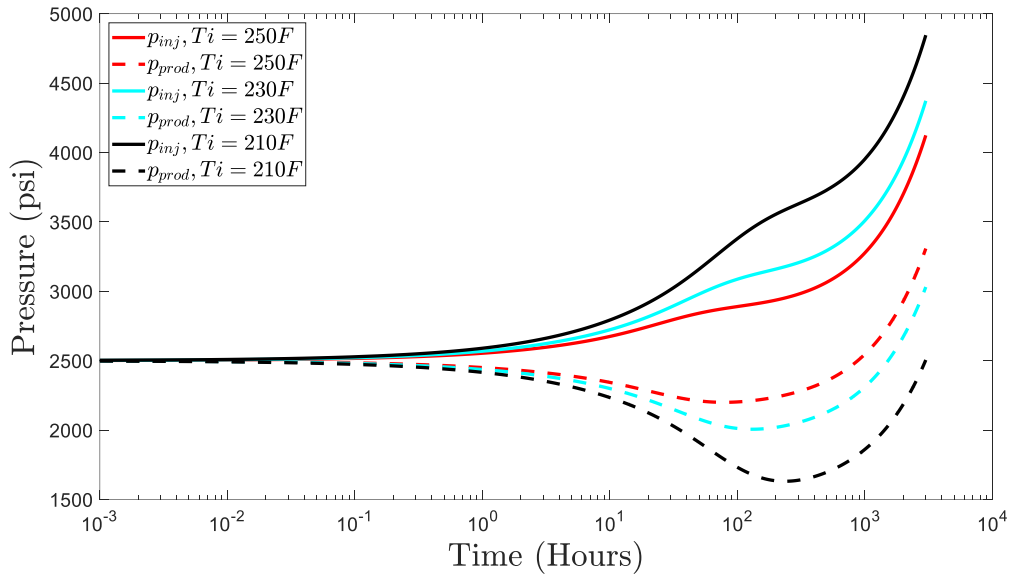


Figure 4.11 Pressure against Time for $q_{inj} = 80 \text{ BPD}$ & $q_{prod} = -70 \text{ BPD}$ with varying Initial Temperature

4.3 Sensitivity analysis on Interface Movement

4.3.1 Effect of Injection Flow Rate

Figure 4.12 shows how varying injection rates affect interface movement. Before starting our simulation, we already have 40 ft of mixture zone. Therefore, in the figures, it starts from 40 ft and goes up with time. As we can see from the graph below, higher injection rates cause interface to move further away from injection well.

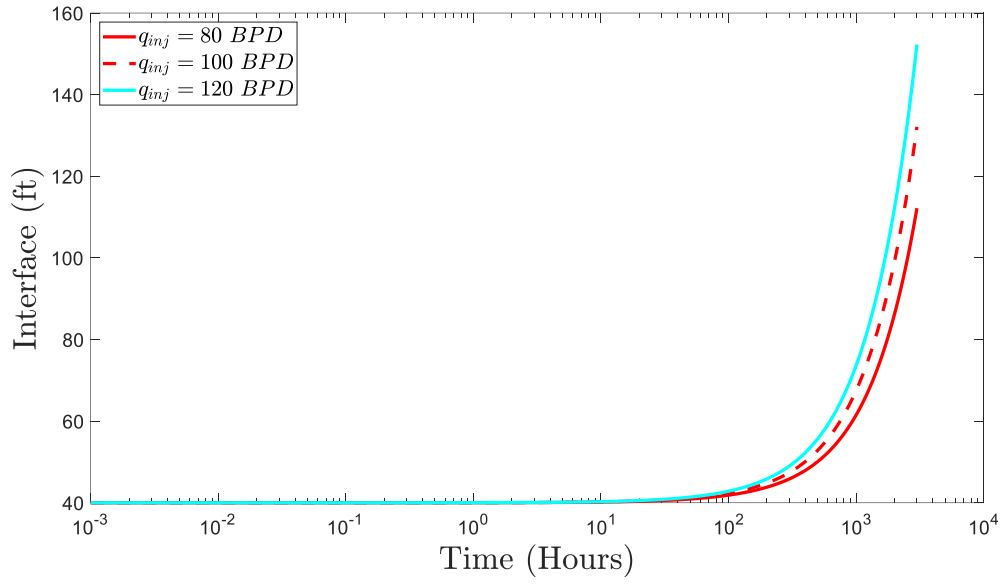


Figure 4.12 Interface movement against Time for $q_{prod} = -70$ BPD with varying Injection Rates

4.3.2 Effect of f_{Amin}

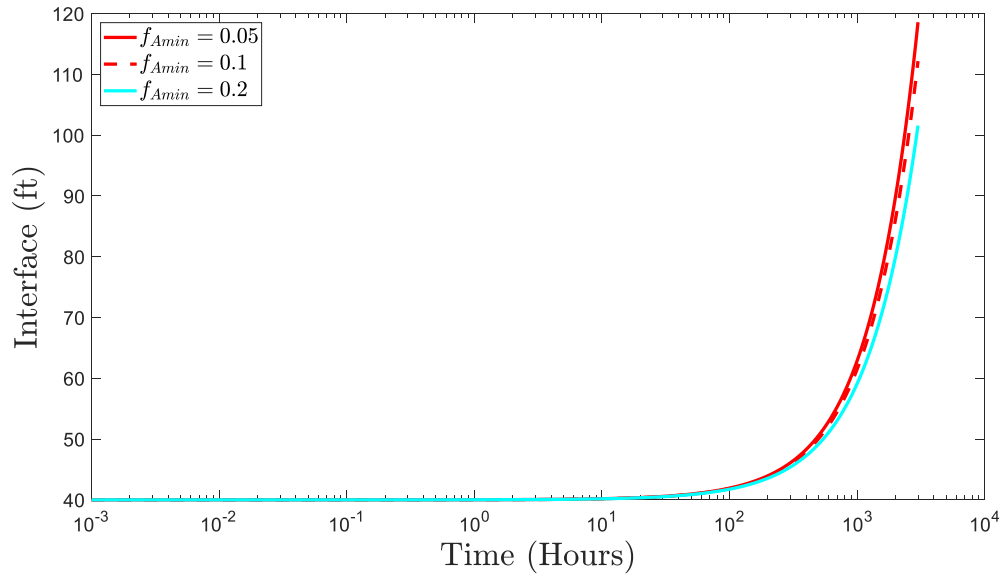


Figure 4.13 Interface movement against Time for $q_{inj} = 80$ BPD & $q_{prod} = -70$ BPD with varying f_{Amin}

In our model, f_{Amin} indicates the minimum area that injected hot water can penetrate. As we can observe from Figure 4.13, as minimum penetrable area decreases, interface moves

further away from the injection wellbore, because there is a smaller space for water to enter.

4.3.3 Effect of f_{Amax}

In our model, f_{Amax} indicates the maximum area that injected hot water can penetrate. If we look at Figure 4.14, we can see that as maximum penetrable area increases, interface is closer to injection wellbore after 3000 hours, because there is more space for water to enter.

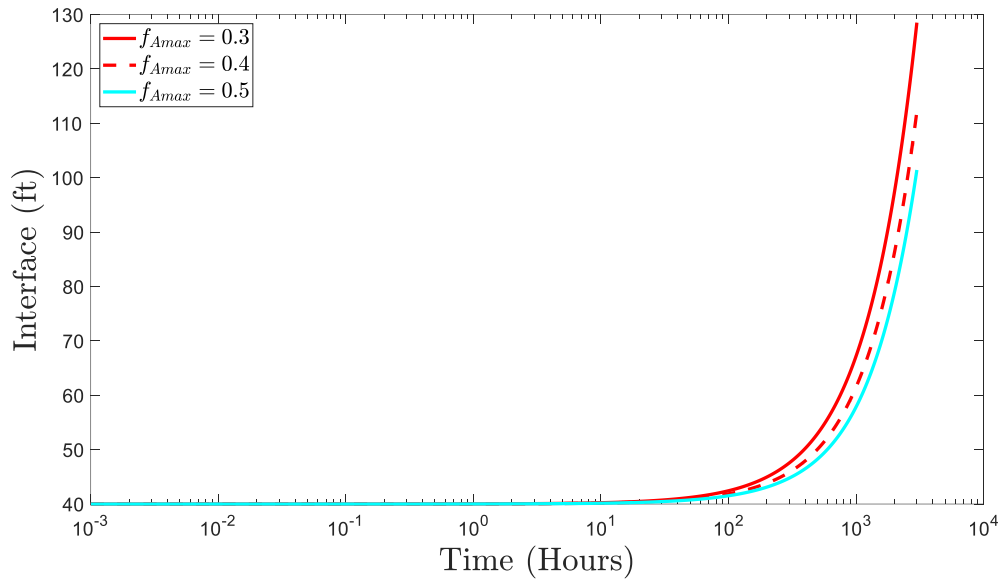


Figure 4.14 Interface movement against Time for $q_{inj} = 80 \text{ BPD}$ & $q_{prod} = -70 \text{ BPD}$ with varying f_{Amax}

4.3.4 Effect of D_i

In our model, D_i is utilized to model how water saturation and interface changes over time.

We can see from Figure 4.15, as D_i increases, interface moves further away from injection well.

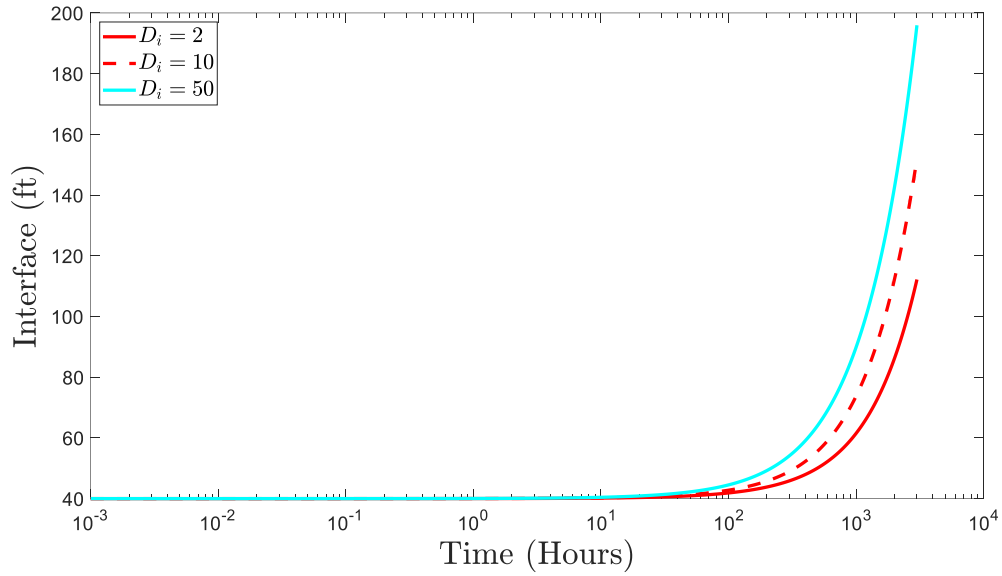


Figure 4.15 Interface movement against Time for $q_{inj} = 80$ BPD & $q_{prod} = -70$ BPD with varying D_i

CHAPTER 5

INVERSE MODELING RESULTS

5.1 Methods of Parameter Matching

The primary goal in pressure transient analysis is to identify a set of parameters (reservoir/well) that can match the measured pressure data with minimal error. The objective function can be expressed as:

$$\Phi(\vec{\gamma}) = \frac{1}{2\xi} \left\| \vec{d}_{cal} - \vec{d}_{meas} \right\|_2^2 \quad (5.1)$$

Where $\vec{\gamma}$ is the vector of unknown parameters, $\vec{d}_{cal}$ is the vector of calculated (modelled) pressure data, $\vec{d}_{meas}$ is the vector of measured data, and ξ is a scaling factor as the product of data points and the maximum member in the set.

Differential evolution (DE) is the inverse modeling technique employed in this thesis utilizing the objective function. An evolutionary technique called differential evolution uses an evolutionary process to iteratively improve a candidate solution in order to solve global optimization challenges. DE first initializes a population of candidate solutions, then creates new ones by merging the existing ones, which are then crossed over, and finally chooses the best candidate solution based on a condition. When a better candidate solution is discovered in a later generation, its value is updated. To move forward from generation to generation, every candidate goes through mutation, cross-over, and selection.

5.2 Initial Data and Parameters

Table 5.1 consists of the parameters of interest that we matched in inverse modeling. Also, lower limits and upper limits as well as actual values that we used in the matching are provided in the table below.

Table 5.1 Parameters Match Targets and Boundaries

<i>Parameter</i>	<i>Lower Limit</i>	<i>Upper Limit</i>	<i>Target</i>
$k (md)$	2000	5000	3000
$h(ft)$	45	58	50
f_{Amin}	0.01	0.3	
f_{Amax}	0.3	0.6	
D_i	1	20	
$x_{int}(ft)$	20	100	

Figure 5.1 illustrates the injection and production well pressure data used in the matching process. The same model was built by using CMG STARS software, and pressure data from CMG STARS software is considered as our measured data. The blue solid lines represent the data from the model that we created by using CMG STARS software, whereas the red circles depict the same data with added noise, treated as the measured data. The Gaussian white noise technique was utilized to add the noise to our measured data.

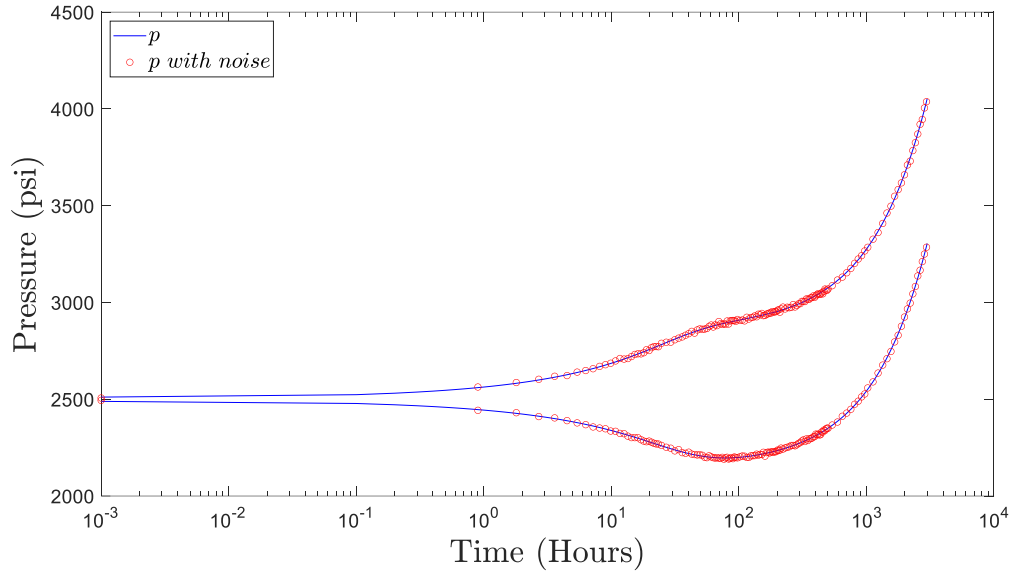


Figure 5.1 Injection and Production data used in Inverse Modeling

5.2.1 Case 1

Case 1 involves matching solely the measured injection well pressure values using composite reservoir flow model coupled with heat model. Table 5.2 represents results of this case for best, median, and worst realizations. In Figure 5.2, the best realization of calculated injection well pressure values versus measured pressure values are depicted. As we can easily see from the graph, it is a very good match because we consider heat added composite model with moving interface.

Table 5.2 Case 1 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
k (md)	2585.223	2584.909	2584.685
h (ft)	53.964	53.992	54
f_{Amin}	0.013046	0.0103363	0.293267
f_{Amax}	0.300134	0.3021086	0.557688
D_i	19.99974	1.007723	19.999105

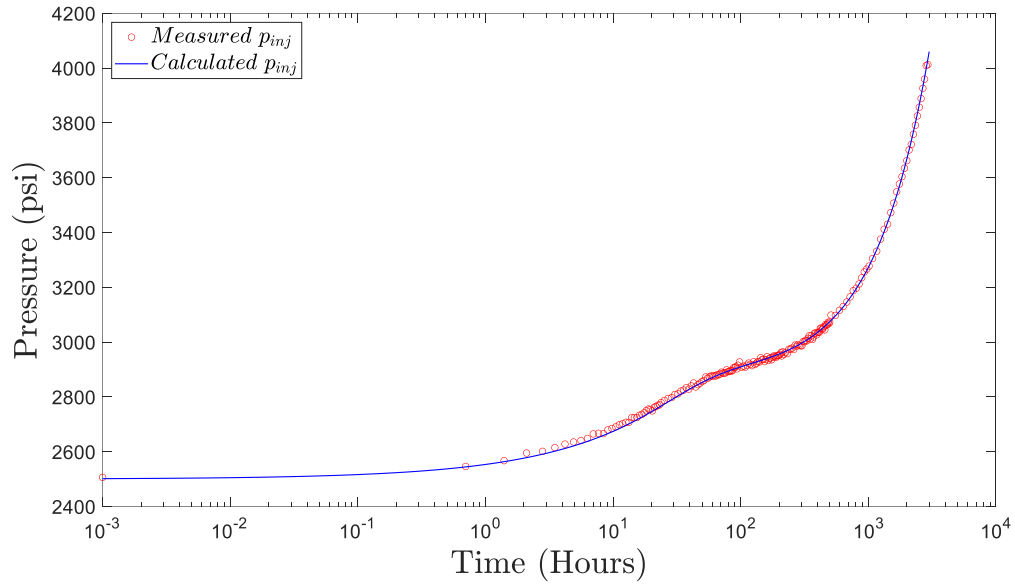


Figure 5.2 Case 1 Injection Well Pressure Match versus Measured Data

5.2.2 Case 2

Case 2 involves matching only the measured production well pressure values using composite reservoir flow model coupled with heat model. Table 5.3 represents results of this case for best, median, and worst realizations. In Figure 5.3, the best realization of calculated production well pressure values versus measured pressure values are depicted. As we can observe from the graph, it is an excellent match because we consider heat added composite model with moving interface.

Table 5.3 Case 2 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
$k \text{ (md)}$	2966.97	2964.45	2960.015
$h \text{ (ft)}$	49.934	49.949	50.057
f_{Amin}	0.240673	0.281419	0.032624
f_{Amax}	0.599704	0.566356	0.420916
D_i	2.04	9.986	3.244

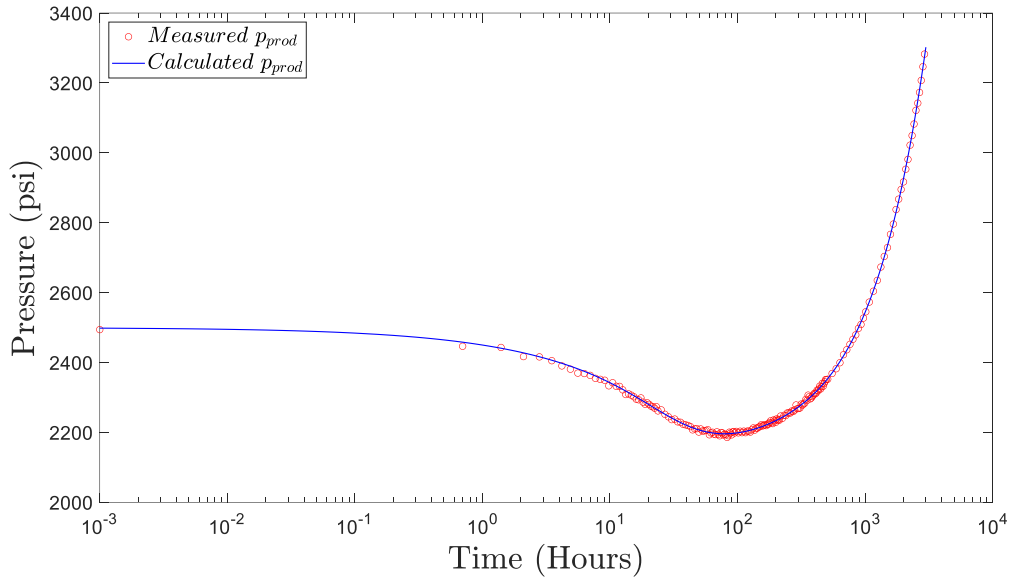


Figure 5.3 Case 2 Production Well Pressure Match versus Measured Data

5.2.3 Case 3

Case 3 involves matching both measured production and injection well pressure values combined using composite reservoir flow model coupled with heat model. Table 5.4 represents results of this case for best, median, and worst realizations. In Figure 5.4, the best realization of calculated both injection and production well pressure values versus measured pressure values are depicted. As we can observe from the graph, it is an excellent match because we consider heat added composite model with moving interface.

Table 5.4 Case 3 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
k (md)	2882.458	2880.973	2890.545
h (ft)	51.1282	51.143	51.129
f_{Amin}	0.010005	0.01003075	0.0380106
f_{Amax}	0.300003	0.30053877	0.5999954
D_i	19.948	19.838532	19.881

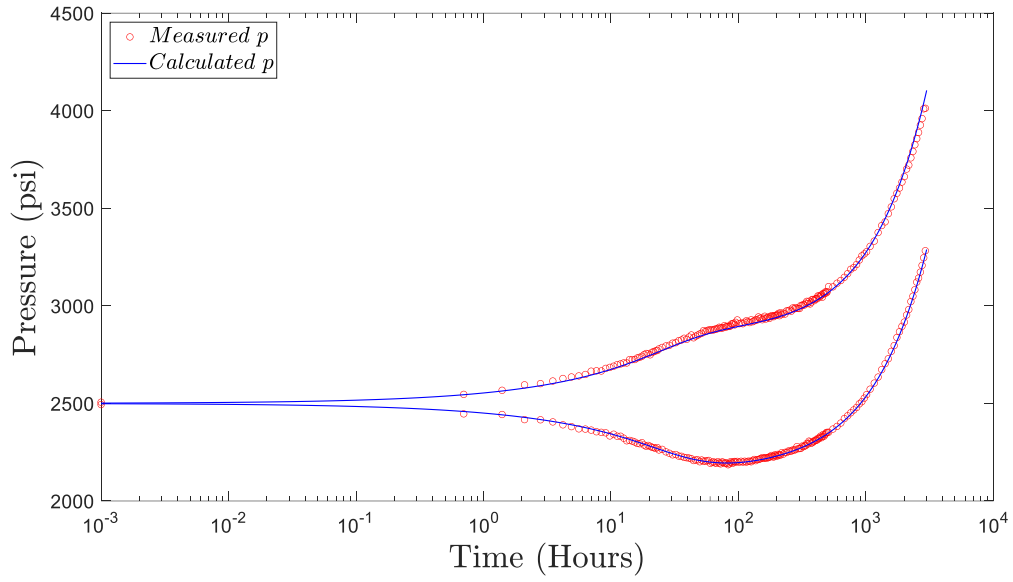


Figure 5.4 Case 3 Combined Injection and Production Well Pressure Match versus Measured Data

Figures 5.5 to 5.7 display errors for Case 3, indicating the best realization, median realization, and worst realization. Since we are matching two distinct variables, errors arise from each of them, and the inverse model aims to minimize the total error. A noticeable

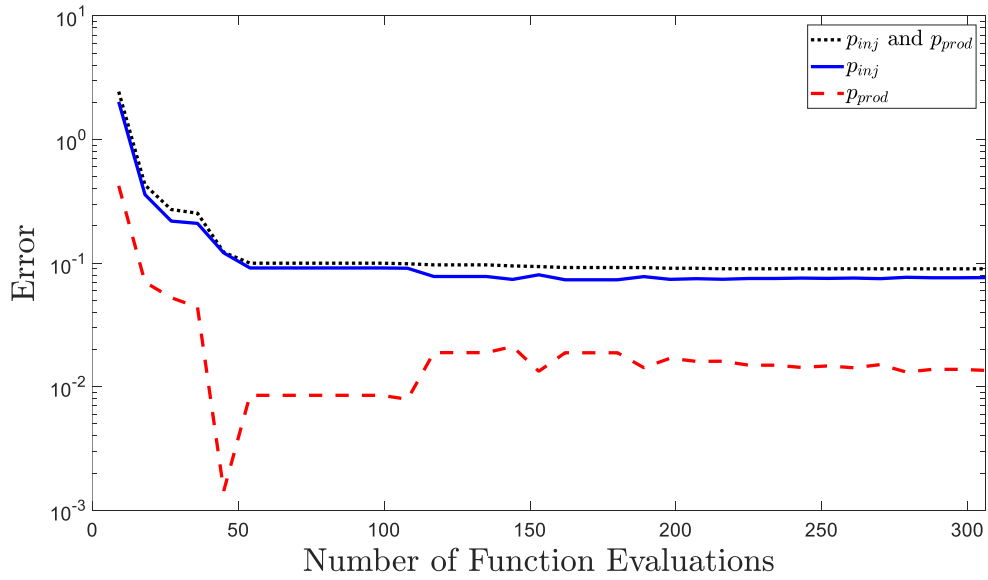


Figure 5.5 Best Realization Errors Comparison for Case 3

trend is the decreasing absolute error, with injection pressure values exhibiting higher errors compared to production pressure values.

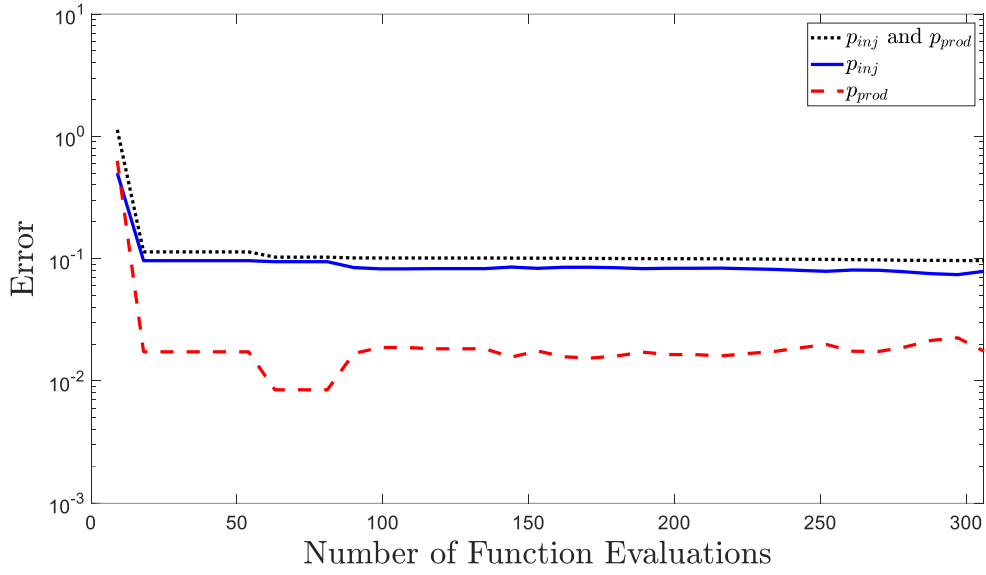


Figure 5.6 Median Realization Errors Comparison for Case 3

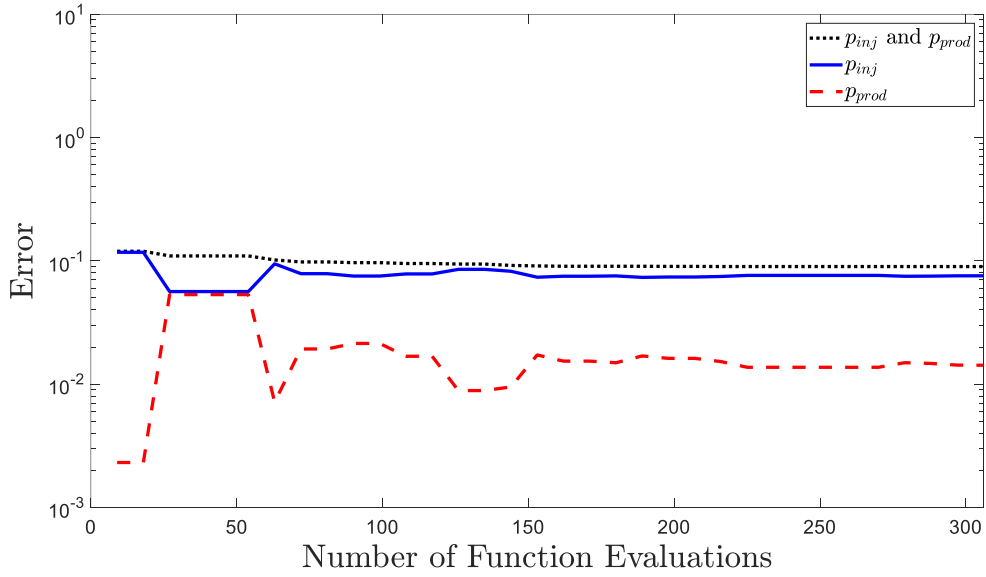


Figure 5.7 Worst Realization Errors Comparison for Case 3

5.2.4 Case 4

In the 4th, 5th, and 6th Cases we carry out inverse modeling by considering a fixed interface without adding heat model. Case 4 involves matching solely the measured injection well pressure values using composite reservoir flow and fixed interface, but

without adding heat model. Table 5.5 represents results of this case for best, median, and worst realizations. In Figure 5.8, the best realization of calculated injection well pressure values versus measured pressure values are depicted. As we can easily see from the graph, even though we consider a fixed boundary condition, and we did not couple heat model it is a very good match.

Table 5.5 Case 4 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
k (md)	2674.234	2675.687	2626.862
h (ft)	52.769	52.754	52.804
x_{int}	20.002	20.207	92.173

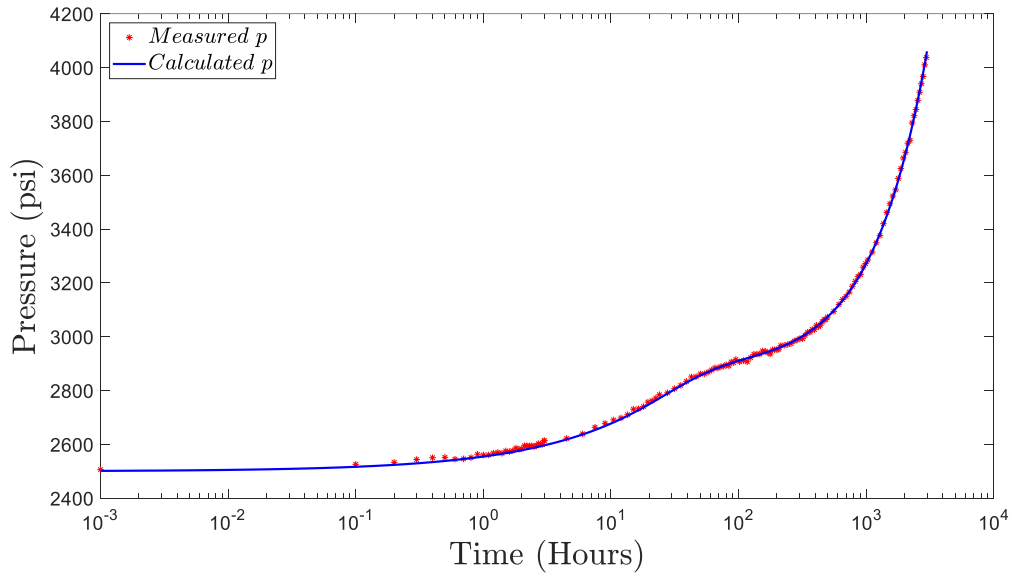


Figure 5.8 Case 4 Injection Well Pressure Match versus Measured Data

5.2.5 Case 5

Case 5 involves matching only the measured production well pressure values using composite reservoir flow and fixed interface, but without adding heat model. Table 5.6 represents results of this case for best, median, and worst realizations. In Figure 5.9, the

best realization of calculated production well pressure values versus measured pressure values are depicted. As we can easily see from the graph, even though we consider a fixed boundary condition, and we did not couple heat model it is a very good match.

Table 5.6 Case 5 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
$k\text{ (md)}$	2750.962	2751.463	2751.945
$h\text{ (ft)}$	53.972	53.952	53.953
x_{int}	99.874	99.958	99.083

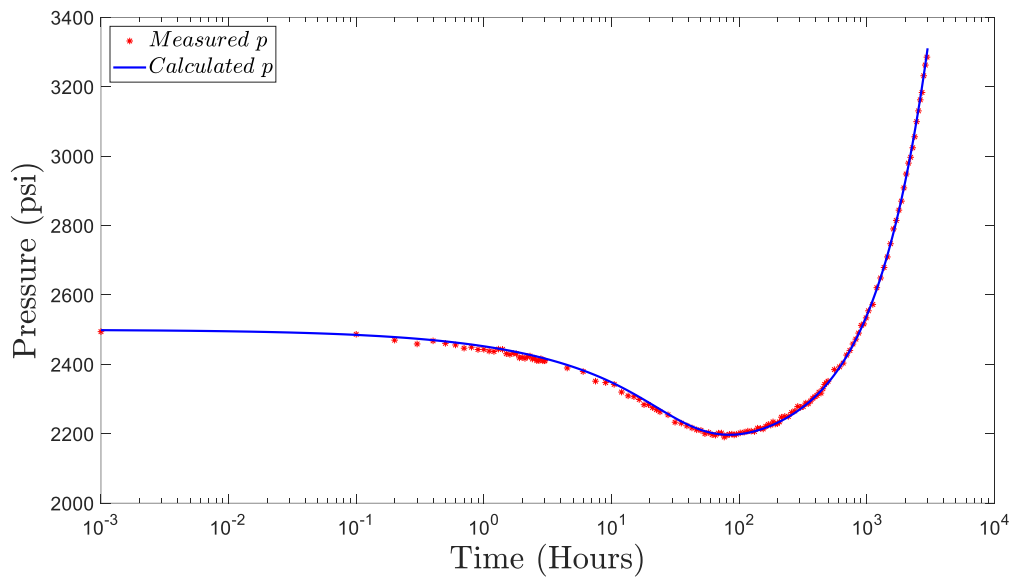


Figure 5.9 Case 5 Production Well Pressure Match versus Measured Data

5.2.6 Case 6

Case 6 involves matching both measured production and injection well pressure values combined using composite reservoir flow and fixed interface, but without adding heat model. Table 5.7 represents results of this case for best, median, and worst realizations. In Figure 5.10, the best realization of calculated production well pressure values versus

measured pressure values are depicted. As we can easily see from the graph, even though we consider a fixed boundary condition, and we did not couple heat model it is a very good match.

Table 5.7 Case 6 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
$k \text{ (md)}$	2733.272	2738.256	2750.641
$h \text{ (ft)}$	52.64	52.576	52.31
x_{int}	20.014	20.396	50.742

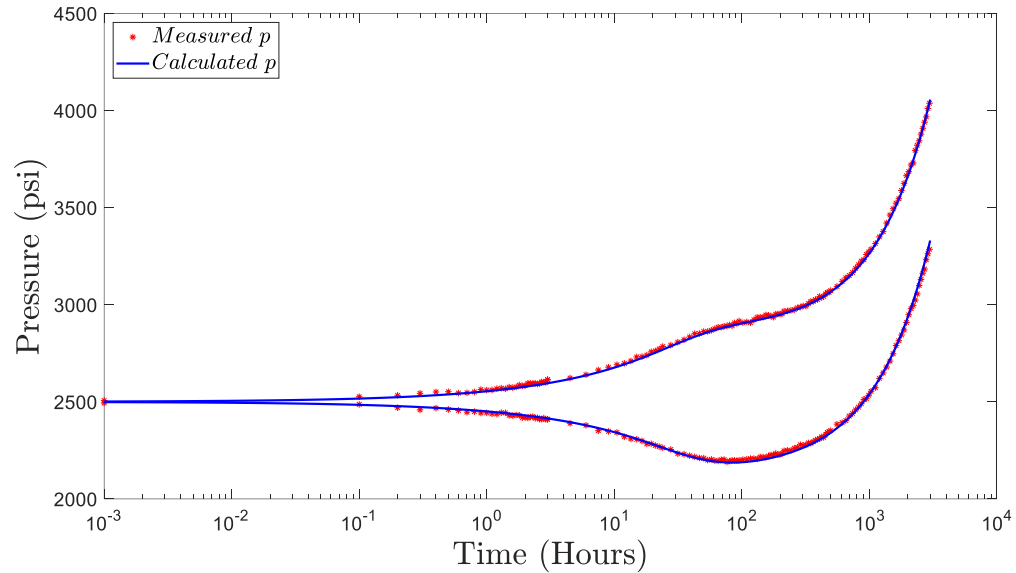


Figure 5.10 Case 6 Combined Injection and Production Well Pressure Match versus Measured Data

Figures 5.11 to 5.13 display errors for Case 6, indicating the best realization, median realization, and worst realization. Since we are matching two distinct variables, errors arise from each of them, and the inverse model aims to minimize the total error. A noticeable trend is the decreasing absolute error, with injection pressure values exhibiting higher errors compared to production pressure values.

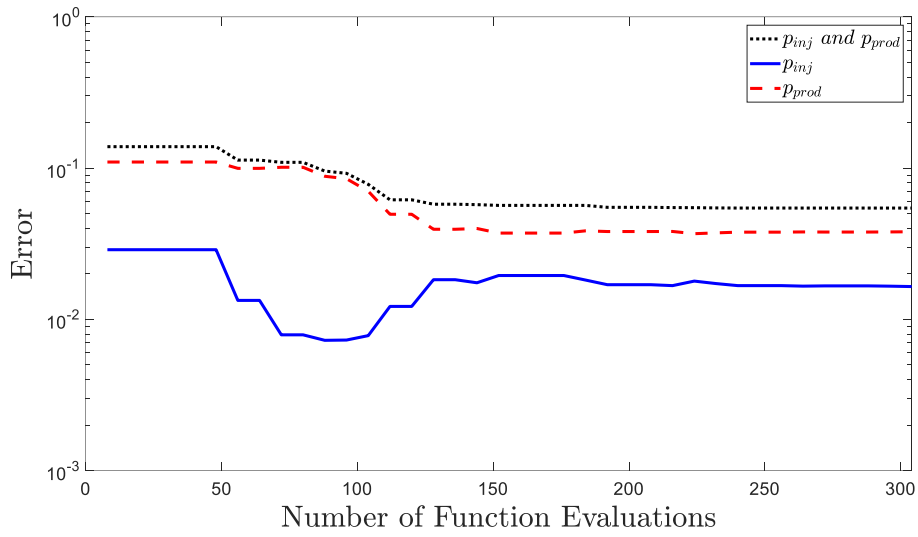


Figure 5.11 Best Realization Errors Comparison for Case 6

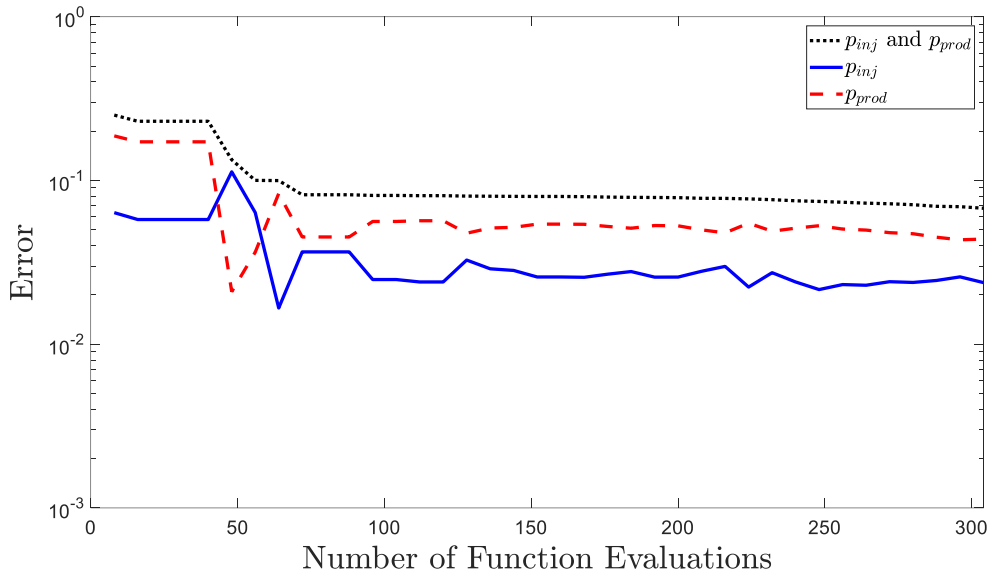


Figure 5.12 Median Realization Errors Comparison for Case 6

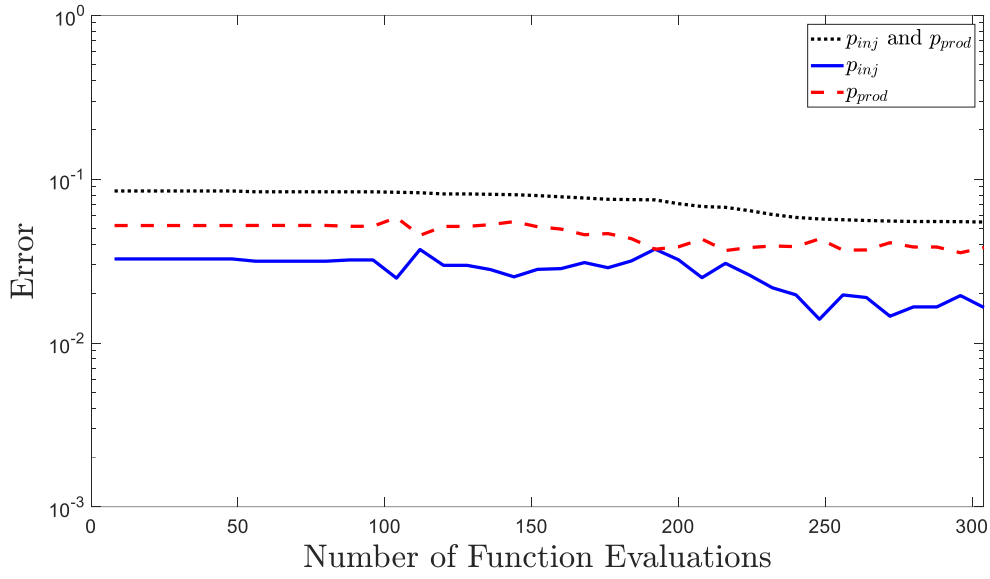


Figure 5.13 Case 3 Worst Realization Errors Comparison for Case 6

5.2.7 Cases comparison

Table 5.8 given below shows the best realization values for permeability and pay thickness for our model in which we consider a moving boundary adding heat flow and fixed interface model in which we ignore heat flow.

Table 5.8 Comparison between our model and fixed interface model

<i>Parameter</i>	<i>Case 3</i>		<i>Case 6</i>		<i>Target</i>
	<i>Best Realization</i>	<i>Error %</i>	<i>Best Realization</i>	<i>Error %</i>	
<i>k (md)</i>	2882.46	3.92	2733.27	8.89	3000
<i>h(ft)</i>	51.13	2.26	52.64	5.28	50
<i>kh</i>	147,380.18	1.75	143,879.33	4.08	150,000

As can be seen from the table, the error from the target for each parameter in our best realization case (Case 3) is much smaller than the model with fixed interface (Case 6) that ignores heat flow.

CHAPTER 6

CONCLUSION

Heavy oil is one of the most viable forms of energy that can be recovered from reservoirs worldwide, although it is challenging to extract. Around half of the world's reserves are comprised of bitumen, heavy oils, and oil sands. Enhanced oil recovery techniques are the primary means of achieving heavy oil recovery. Thermal methods are often considered because they increase recovery and lower viscosity. Thermal processes include in-situ burning, electrical heating, steam and hot water injection, and electromagnetic radiation.

In heavy oil reservoirs, in most cases, isothermal conditions are assumed when solving fluid flow equations. During thermal EOR processes, it is important to consider heat flow equations as viscosity of heavy oil is strongly temperature dependent.

In this work, we modeled pressure and temperature transients in heavy oil reservoirs undergoing hot water injection by using mathematical tools and MATLAB. Here, we consider a moving boundary between injector and producer. Hence, we also consider it to be a composite reservoir with two regions. Fluid characteristics like density, viscosity, specific heat capacity, thermal conductivity, and compressibility have been proposed to vary depending on the zone. The heat flow equation is solved to find temperature at several time steps and then viscosity is updated in the fluid flow model to solve for pressure. Fluid flow and heat transfer equations are solved by converting them to dimensionless form and applying Laplace transform. To convert the solution from Laplace space to real space, Gaver-Stehfest method is used to numerically inverse Laplace transform.

Finally, inverse modelling is utilized to estimate the reservoir parameters and to match the pressure transient of non-isothermal reservoirs that undergo hot water injection and conclude that integrating both fluid flow and heat flow models yields more accurate results.

APPENDICES

Appendix A

We begin with energy balance for fluxes:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{s/s} = \dot{E}_{acc}$$

Now we can use the following expressions:

$$\dot{E}_{in} - \dot{E}_{out} = - \int_A \vec{F}_T \cdot \vec{n} dA$$

$$\vec{F}_T = \vec{F}_{disp} + \vec{F}_{conv}$$

$$\vec{F}_{disp} = -\phi \kappa_f \cdot \nabla T_f$$

$$\vec{F}_{conv} = \rho_f h_f \vec{u}_f$$

$$h_f = c_{pf} T_f$$

$$\dot{E}_{s/s} = \int_V q_{Tf} dV + \int_V \gamma_{sf} (T_s - T_f) dV$$

$$\dot{E}_{acc} = \frac{\partial}{\partial t} \int_V (\phi \rho_f e_f) dV$$

$$e_f = c_v T$$

Combining all the terms given above, we get:

$$-\int_A \vec{F}_T \cdot \vec{n} dA + \int_V q_{Tf} dV + \int_V \gamma_{sf} (T_s - T_f) dV = \frac{\partial}{\partial t} \int_V (\phi \rho_f c_{vf} T_f) dV$$

Applying the Gauss Divergence Theorem, we have:

$$-\int_V \nabla \cdot \vec{F}_T dV + \int_V q_{Tf} dV + \int_V \gamma_{sf} (T_s - T_f) dV = \frac{\partial}{\partial t} \int_V (\phi \rho_f c_{vf} T_f) dV$$

For any arbitrary control volume V, the integrand must vanish identically. Therefore,

$$-\nabla \cdot (-\phi \kappa_f \cdot \nabla T_f + \rho_f c_{pf} \vec{u}_f T) + q_{Tf} + \gamma_{sf} (T_s - T_f) = \frac{\partial (\phi \rho_f c_{vf} T_f)}{\partial t}$$

$$\nabla \cdot \phi \kappa_f \cdot \nabla T_f - \nabla \cdot (\rho_f c_{pf} \vec{u}_f T) + q_{Tf} + \gamma_{sf} (T_s - T_f) = \frac{\partial (\phi \rho_f c_{vf} T_f)}{\partial t}$$

Now the same equation can be written for the solid phase:

$$\nabla \cdot [(1-\phi) \kappa_s \cdot \nabla T_s] + q_{Ts} + \gamma_{sf} (T_f - T_s) = \frac{\partial [(1-\phi) \rho_s c_{vs} T_s]}{\partial t}$$

Now equations for fluid and solid phases can be combined:

$$\nabla \cdot [(1-\phi) \kappa_s \cdot \nabla T_s + \phi \kappa_f \cdot \nabla T_f] - \nabla \cdot (\rho_f c_{pf} \vec{u}_f T) + q_T = \frac{\partial [(1-\phi) \rho_s c_{vs} T_s + \phi \rho_f c_{vf} T_f]}{\partial t}$$

Where $q_T = q_{Tf} + q_{Ts}$.

By assuming thermal equilibrium between rock and fluid phase ($T = T_f = T_s$), we

obtain:

$$\nabla \cdot \left\{ \left[(1-\phi) \kappa_s + \phi \kappa_f \right] \cdot \nabla T \right\} - \nabla \cdot \left(\rho_f c_{pf} \bar{u}_f T \right) + q_T = \frac{\partial}{\partial t} \left\{ \left[(1-\phi) \rho_s c_{vs} + \phi \rho_f c_{vf} \right] T \right\}$$

Now we can substitute effective parameters:

$$\kappa_{eff} = \phi \kappa_f + (1-\phi) \kappa_s$$

$$(\rho c_v)_{eff} = \phi \rho_f c_{vf} + (1-\phi) \rho_s c_{vs}$$

$$\nabla \cdot \left[\kappa_{eff} \cdot \nabla T \right] - \nabla \cdot \left(\rho_f c_{pf} \bar{u}_f T \right) + q_T = \frac{\partial}{\partial t} \left[(\rho c_v)_{eff} T \right]$$

For cartesian coordinate system we have:

$$\bar{\kappa}_{eff} = \begin{pmatrix} \kappa_{eff_{xx}} & \kappa_{eff_{xy}} & \kappa_{eff_{xz}} \\ \kappa_{eff_{yx}} & \kappa_{eff_{yy}} & \kappa_{eff_{yz}} \\ \kappa_{eff_{zx}} & \kappa_{eff_{zy}} & \kappa_{eff_{zz}} \end{pmatrix}; \quad \nabla T = \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix}$$

$$\bar{\kappa}_{eff} \cdot \nabla T = \begin{pmatrix} \kappa_{eff_{xx}} & \kappa_{eff_{xy}} & \kappa_{eff_{xz}} \\ \kappa_{eff_{yx}} & \kappa_{eff_{yy}} & \kappa_{eff_{yz}} \\ \kappa_{eff_{zx}} & \kappa_{eff_{zy}} & \kappa_{eff_{zz}} \end{pmatrix} \cdot \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} = \begin{bmatrix} \kappa_{eff_{xx}} \frac{\partial T}{\partial x} + \kappa_{eff_{xy}} \frac{\partial T}{\partial y} + \kappa_{eff_{xz}} \frac{\partial T}{\partial z} \\ \kappa_{eff_{yx}} \frac{\partial T}{\partial x} + \kappa_{eff_{yy}} \frac{\partial T}{\partial y} + \kappa_{eff_{yz}} \frac{\partial T}{\partial z} \\ \kappa_{eff_{zx}} \frac{\partial T}{\partial x} + \kappa_{eff_{zy}} \frac{\partial T}{\partial y} + \kappa_{eff_{zz}} \frac{\partial T}{\partial z} \end{bmatrix}$$

Putting together all the terms we get:

$$\begin{aligned}\nabla \cdot (\kappa_{eff} \cdot \nabla T) &= \frac{\partial}{\partial x} \left(\kappa_{eff_{xx}} \frac{\partial T}{\partial x} + \kappa_{eff_{xy}} \frac{\partial T}{\partial y} + \kappa_{eff_{xz}} \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial y} \left(\kappa_{eff_{yx}} \frac{\partial T}{\partial x} + \kappa_{eff_{yy}} \frac{\partial T}{\partial y} + \kappa_{eff_{yz}} \frac{\partial T}{\partial z} \right) + \\ &\quad + \frac{\partial}{\partial z} \left(\kappa_{eff_{zx}} \frac{\partial T}{\partial x} + \kappa_{eff_{zy}} \frac{\partial T}{\partial y} + \kappa_{eff_{zz}} \frac{\partial T}{\partial z} \right)\end{aligned}$$

The convection term is written as follows:

$$\rho_f c_{pf} \vec{u}_f \cdot \nabla T = (\rho c_p)_f u_{f,x} \frac{\partial T}{\partial x} + (\rho c_p)_f u_{f,y} \frac{\partial T}{\partial y} + (\rho c_p)_f u_{f,z} \frac{\partial T}{\partial z}$$

We are dealing with linear case. So, only x – direction (1D) is considered:

$$\frac{\partial}{\partial x} \left(\kappa_{eff_{xx}} \frac{\partial T}{\partial x} \right) - (\rho c_p)_f u_{f,x} \frac{\partial T}{\partial x} + q_T = \frac{\partial}{\partial t} [(\rho c_v)_{eff} T]$$

We can assume following:

1. Incompressible flow.
2. c_p and c_v are constant.
3. κ_{eff} and ϕ are not function of temperature (T)

Hence, the equation can be written as follows:

$$\kappa_{eff} \frac{\partial^2 T}{\partial x^2} - (\rho c_p)_f u_f \frac{\partial T}{\partial x} + q_T = (\rho c_v)_{eff} \frac{\partial T}{\partial t}$$

Let's divide both sides by $(\rho c_v)_{eff}$:

$$\frac{\kappa_{eff}}{(\rho c_v)_{eff}} \frac{\partial^2 T}{\partial x^2} - \frac{(\rho c_p)_f u_f}{(\rho c_v)_{eff}} \frac{\partial T}{\partial x} + \frac{q_T}{(\rho c_v)_{eff}} = \frac{\partial T}{\partial t}$$

$$\lambda_{eff} \frac{\partial^2 T}{\partial x^2} - \dot{u}_f \frac{\partial T}{\partial x} + \dot{q}_T = \frac{\partial T}{\partial t}$$

Where $\kappa_{eff} = \phi \kappa_f + (1 - \phi) \kappa_s$,

$$\lambda_{eff} = \frac{\kappa_{eff}}{(\rho c_v)_{eff}},$$

$$\dot{u}_f = \frac{(\rho c_p)_f}{(\rho c_v)_{eff}} u_f$$

$$\dot{q}_T = \frac{q_T}{(\rho c_v)_{eff}}$$

Let's assume there is no distributed sink/source, we get:

$$\lambda_{eff} \frac{\partial^2 T}{\partial x^2} - \dot{u}_f \frac{\partial T}{\partial x} = \frac{\partial T}{\partial t}$$

Therefore, the final form of linear heat transport equation in 1D is written as follows for two regions:

$$\lambda_{eff_j} \frac{\partial^2 T_j}{\partial x^2} - \dot{u}_j \frac{\partial T_j}{\partial x} = \frac{\partial T_j}{\partial t} ; \quad j = 1, 2$$

Appendix B

Let's start form the mass balance over a regular representative elementary control volume:

$$\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{s/s} = \dot{m}_{acc}$$

Where $\dot{m}_{in} = \sum (\rho \vec{u} A)_{in}$ and $\dot{m}_{out} = \sum (\rho \vec{u} A)_{out}$

The Flux Term is given by:

$$\dot{m}_{in} - \dot{m}_{out} = - \int_A (\rho \vec{u}) \cdot \vec{n} dA$$

The Sink and Source Term:

$$\dot{m}_{s/s} = \int_V \frac{\rho q_{well}}{V} dV$$

For the Accumulation Term:

$$\dot{m}_{acc} = \int_V \frac{\partial(\phi \rho)}{\partial t} dV$$

From the Gauss Divergence Theorem, we have the following equation:

$$\int_A \vec{a} \cdot \vec{n} dA = \int_V \nabla \cdot \vec{a} dV$$

Substituting all the terms in the main equation gives:

$$- \int_V \nabla \cdot (\rho \vec{u}) dV + \int_V \frac{\rho q_{well}}{V} dV - \int_V \frac{\partial(\phi \rho)}{\partial t} dV = 0$$

$$\Rightarrow \int_V \left[-\nabla \cdot (\rho \vec{u}) + \frac{\rho q_{well}}{V} - \frac{\partial(\phi \rho)}{\partial t} \right] dV = 0$$

For any arbitrary control volume V , the integrand must vanish identically. Therefore,

$$-\nabla \cdot (\rho \vec{u}) + \frac{\rho q_{well}}{V} = \frac{\partial(\phi \rho)}{\partial t}$$

For Cartesian coordinates we have:

$$\nabla \cdot \vec{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

Using the definition of the divergence operator in the Cartesian coordinates, we obtain

$$-\frac{\partial(\rho u_x)}{\partial x} - \frac{\partial(\rho u_y)}{\partial y} - \frac{\partial(\rho u_z)}{\partial z} + \frac{\rho q_{well}}{V} = \frac{\partial(\phi \rho)}{\partial t}$$

Also, the velocity is defined in 3D as:

$$\vec{u} = -\frac{\bar{k}}{\mu} \cdot \nabla p$$

$$\bar{k} = \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{pmatrix} \quad \nabla p = \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix}$$

Darcy velocity is given by equations shown below:

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = -\frac{\bar{k}}{\mu} \cdot \nabla p = -\frac{1}{\mu} \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{pmatrix} \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix}$$

$$u_x = -\frac{1}{\mu} \left[k_{xx} \frac{\partial p}{\partial x} + k_{xy} \frac{\partial p}{\partial y} + k_{xz} \frac{\partial p}{\partial z} \right]$$

$$u_y = -\frac{1}{\mu} \left[k_{yx} \frac{\partial p}{\partial x} + k_{yy} \frac{\partial p}{\partial y} + k_{yz} \frac{\partial p}{\partial z} \right]$$

$$u_z = -\frac{1}{\mu} \left[k_{zx} \frac{\partial p}{\partial x} + k_{zy} \frac{\partial p}{\partial y} + k_{zz} \frac{\partial p}{\partial z} \right]$$

We may write the continuity equation in terms of pressure as:

$$\nabla \cdot \left[\left(\frac{\rho}{\mu} \right) \bar{k} \cdot \nabla p \right] + \frac{\rho q_{well}}{V} = \frac{\partial(\phi \rho)}{\partial t}$$

Let $\bar{k}$ be a diagonal tensor (in case the coordinate system is aligned the principal directions if the permeability tensor):

$$\bar{k} = \begin{pmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{pmatrix}$$

$$\frac{\partial}{\partial x} \left(\frac{k_x \rho}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k_y \rho}{\mu} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{k_z \rho}{\mu} \frac{\partial p}{\partial z} \right) + \frac{\rho q_{well}}{V} = \frac{\partial(\phi \rho)}{\partial t}$$

Now let $\bar{k}$ be isotropic so that $\bar{k} = kI$ or $k = k_x = k_y = k_z$. Then, we may write the permeability tensor as $\bar{k} = kI$

$$\frac{\partial}{\partial x} \left(\frac{k\rho}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k\rho}{\mu} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{k\rho}{\mu} \frac{\partial p}{\partial z} \right) + \frac{\rho q_{\text{well}}}{V} = \frac{\partial(\phi\rho)}{\partial t}$$

Consider 1D (only in the x -direction) and no sink or source term, then we obtain:

$$\frac{\partial}{\partial x} \left(\frac{k\rho}{\mu} \frac{\partial p}{\partial x} \right) = \frac{\partial(\phi\rho)}{\partial t}$$

Appendix C

Converting 1D heat flow equation into dimensionless form. Dimensionless variables are provided below:

$$x_D = \frac{x}{L} \quad ; \quad t_D = \frac{\dot{u}_{1,ini} t}{L} \quad ; \quad T_D = \frac{T}{T_i}$$

$$\partial x = L \partial x_D$$

$$\partial t = \frac{L}{\dot{u}_{1,ini}} \partial t_D$$

$$\frac{\partial T}{\partial t} = T_i \frac{\partial T_D}{\partial t} = \frac{T_i \dot{u}_{1,ini}}{L} \frac{\partial T_D}{\partial t_D}$$

$$\frac{\partial T}{\partial x} = T_i \frac{\partial T_D}{\partial x} = \frac{T_i}{L} \frac{\partial T_D}{\partial x_D}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{T_i}{L} \frac{\partial T_D}{\partial x_D} \right) = \frac{T_i}{L^2} \frac{\partial^2 T_D}{\partial x_D^2}$$

After substituting dimensionless variables and simplifying we obtain:

$$\frac{1}{Pe_j} \frac{\partial^2 T_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial T_{D_j}}{\partial x_D} = \frac{\partial T_{D_j}}{\partial t_D} ; \quad j = 1, 2$$

Where $Pe_j = \frac{\dot{u}_{1,ini} L}{\lambda_{eff_j}}$.

Appendix D

Taking Laplace transformation of the dimensionless heat flow equation is shown below:

$$\mathcal{L} \left\{ \frac{1}{Pe_j} \frac{\partial^2 T_{D_j}}{\partial x_D^2} \right\} - \mathcal{L} \left\{ \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial T_{D_j}}{\partial x_D} \right\} = \mathcal{L} \left\{ \frac{\partial T_{D_j}}{\partial t_D} \right\}$$

$$\frac{1}{Pe_j} \frac{\partial^2 \hat{T}_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial \hat{T}_{D_j}}{\partial x_D} = z \hat{T}_{D_j} - T_{D_j}(x_D, t_D = 0)$$

Note that $T_{D_j}(x_D, t_D = 0) = 1$

$$\frac{1}{Pe_j} \frac{\partial^2 \hat{T}_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial \hat{T}_{D_j}}{\partial x_D} - z \hat{T}_{D_j} = -1$$

The general solution to the equation given above is given by:

$$\hat{T}_{D_j}(x_D, z) = A_j e^{\frac{1}{2}x_D Pe_j \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} - \sqrt{\frac{4z}{Pe_j} + \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} \right)^2} \right)} + B_j e^{\frac{1}{2}x_D Pe_j \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} + \sqrt{\frac{4z}{Pe_j} + \left(\frac{\dot{u}_j}{\dot{u}_{1,ini}} \right)^2} \right)} + \frac{1}{z};$$

Where $j = 1, 2$.

Appendix E

ODE Discretization for Heat Flow model

First, we start with the dimensionless heat flow equation in Laplace domain:

$$\frac{1}{Pe_j} \frac{\partial^2 \hat{T}_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial \hat{T}_{D_j}}{\partial x_D} - z \hat{T}_{D_j} = -1$$

The equation given above can be discretized as follows:

$$\frac{1}{Pe_{j,m}} \left[\frac{\hat{T}_{D_{j,m+1}} - 2\hat{T}_{D_{j,m}} + \hat{T}_{D_{j,m-1}}}{(\Delta x_D)^2} \right] - \dot{\omega}_{j,m} \left[\frac{\hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m-1}}}{\Delta x_D} \right] - z \hat{T}_{D_{j,m}} = -1$$

Multiplying both sides by $Pe_{j,m} (\Delta x_D)^2$ and expanding:

$$-\hat{T}_{D_{j,m+1}} + 2\hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m-1}} + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D (\hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m-1}}) + z Pe_{j,m} (\Delta x_D)^2 \hat{T}_{D_{j,m}} = Pe_{j,m} (\Delta x_D)^2$$

Rearranging for the general equation:

$$-(1 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D) \hat{T}_{D_{j,m-1}} + (2 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2) \hat{T}_{D_{j,m}} - \hat{T}_{D_{j,m+1}} = Pe_{j,m} (\Delta x_D)^2$$

Dirichlet Boundary Condition for Injection well (LBC)

$$T(x=0, t) = T_{inj}$$

$$T_i T_D(x_D = 0, t_D) = T_{inj}$$

$$T_D(x_D = 0, t_D) = \frac{T_{inj}}{T_i} = T_{D_{inj}}$$

$$\mathcal{L}\{T_D\} = \mathcal{L}\{T_{D_{inj}}\}$$

$$\hat{T}_D(x_D = 0, z) = \frac{T_{D_{inj}}}{z}$$

Discretized Equation for First Grid

$$\frac{1}{Pe_{j,m}} \frac{1}{\Delta x_D} \left[\frac{\hat{T}_{D_2} - \hat{T}_{D_1}}{\Delta x_D} - \frac{\hat{T}_{D_1} - \hat{T}_{D_{inj}}}{\frac{\Delta x_D}{2}} \right] - \dot{\omega}_j \left[\frac{\hat{T}_{D_1} - \hat{T}_{D_{inj}}}{\frac{\Delta x_D}{2}} \right] - z \hat{T}_{D_1} = -1$$

$$\frac{1}{Pe_{j,m}} \frac{1}{(\Delta x_D)^2} \left[\hat{T}_{D_2} - 3\hat{T}_{D_1} + 2\hat{T}_{D_{inj}} \right] - \frac{2\dot{\omega}_j}{\Delta x_D} \left[\hat{T}_{D_1} - \hat{T}_{D_{inj}} \right] - z \hat{T}_{D_1} = -1$$

Multiplying both sides by $Pe_{j,m}(\Delta x_D)^2$ and expanding:

$$\hat{T}_{D_2} - 3\hat{T}_{D_1} + 2\hat{T}_{D_{inj}} - 2\dot{\omega}_j Pe_{j,m} \Delta x_D (\hat{T}_{D_1} - \hat{T}_{D_{inj}}) - z Pe_{j,m} (\Delta x_D)^2 \hat{T}_{D_1} = -Pe_{j,m} (\Delta x_D)^2$$

Rearranging gives us:

$$-2(1 + \dot{\omega}_j Pe_{j,m} \Delta x_D) \hat{T}_{D_{inj}} + \left[3 + 2\dot{\omega}_j Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2 \right] \hat{T}_{D_1} - \hat{T}_{D_2} = Pe_{j,m} (\Delta x_D)^2$$

Substituting LBC gives us:

$$-2\left(1+\dot{\omega}_j Pe_{j,m} \Delta x_D\right) \frac{T_{D_{inj}}}{z} + \left[3+2\dot{\omega}_j Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2\right] \hat{T}_{D_1} - \hat{T}_{D_2} = Pe_{j,m} (\Delta x_D)^2$$

Therefore:

$$\left[3+2\dot{\omega}_j Pe_{j,m} \Delta x_D + z Pe_{j,m} (\Delta x_D)^2\right] \hat{T}_{D_1} - \hat{T}_{D_2} = Pe_{j,m} (\Delta x_D)^2 + \frac{2}{z} \left(1+\dot{\omega}_j Pe_{j,m} \Delta x_D\right) T_{D_{inj}}$$

Diritchlet Boundary Condition for Production well (RBC)

$$T(x=L, t) = T_{prod}$$

$$T_i T_D(x_D=1, t_D) = T_{prod}$$

$$T_D(x_D=1, t_D) = \frac{T_{prod}}{T_i} = T_{D_{prod}}$$

$$\mathcal{L}\{T_D\} = \mathcal{L}\{T_{D_{prod}}\}$$

$$\hat{T}_D(x_D=1, z) = \frac{T_{D_{prod}}}{z}$$

Discretized Equation for Grid M

$$\frac{1}{Pe_{j,M}} \frac{1}{\Delta x_D} \left[\frac{\hat{T}_{D_{prod}} - \hat{T}_{D_M}}{\frac{\Delta x_D}{2}} - \frac{\hat{T}_{D_M} - \hat{T}_{D_{M-1}}}{\Delta x_D} \right] - \dot{\omega}_{j,M} \left[\frac{\hat{T}_{D_M} - \hat{T}_{D_{M-1}}}{\Delta x_D} \right] - z \hat{T}_{D_M} = -1$$

$$\frac{1}{Pe_{j,M} (\Delta x_D)^2} \left[2(\hat{T}_{D_{prod}} - \hat{T}_{D_M}) - (\hat{T}_{D_M} - \hat{T}_{D_{M-1}}) \right] - \frac{\dot{\omega}_{j,M}}{\Delta x_D} [\hat{T}_{D_M} - \hat{T}_{D_{M-1}}] - z \hat{T}_{D_M} = -1$$

Multiplying both sides by $Pe_{j,M} (\Delta x_D)^2$ and expanding:

$$2\hat{T}_{D_{prod}} - 3\hat{T}_{D_M} + \hat{T}_{D_{M-1}} - \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D (\hat{T}_{D_M} - \hat{T}_{D_{M-1}}) - z Pe_{j,M} (\Delta x_D)^2 \hat{T}_{D_M} = -Pe_{j,M} (\Delta x_D)^2$$

Rearranging gives us:

$$-(1 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D) \hat{T}_{D_{M-1}} + [3 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D + z Pe_{j,M} (\Delta x_D)^2] \hat{T}_{D_M} - 2\hat{T}_{D_{prod}} = Pe_{j,M} (\Delta x_D)^2$$

Substituting RBC gives us:

$$-(1 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D) \hat{T}_{D_{M-1}} + [3 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D + z Pe_{j,M} (\Delta x_D)^2] \hat{T}_{D_M} - 2 \frac{T_{D_{prod}}}{Z} = Pe_{j,M} (\Delta x_D)^2$$

After further simplification, we get the following equation in discretized form:

$$-(1 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D) \hat{T}_{D_{M-1}} + [3 + \dot{\omega}_{j,M} Pe_{j,M} \Delta x_D + z Pe_{j,M} (\Delta x_D)^2] \hat{T}_{D_M} = Pe_{j,M} (\Delta x_D)^2 + \frac{2T_{D_{prod}}}{Z}$$

Appendix F

Fully numerical model

First, we start with the dimensionless heat flow equation:

$$\frac{1}{Pe_j} \frac{\partial^2 T_{D_j}}{\partial x_D^2} - \frac{\dot{u}_j}{\dot{u}_{1,ini}} \frac{\partial T_{D_j}}{\partial x_D} = \frac{\partial T_{D_j}}{\partial t_D}$$

The equation given above can be discretized as follows:

$$\frac{1}{Pe_{j,m}} \frac{1}{\Delta x_D} \left[\frac{T_{D_{j,m+1}}^{n+1} - T_{D_{j,m}}^{n+1}}{\Delta x_D} - \frac{T_{D_{j,m}}^{n+1} - T_{D_{j,m-1}}^{n+1}}{\Delta x_D} \right] - \dot{\omega}_{j,m} \left[\frac{T_{D_{j,m}}^{n+1} - T_{D_{j,m-1}}^{n+1}}{\Delta x_D} \right] = \frac{T_{D_{j,m}}^{n+1} - T_{D_{j,m-1}}^n}{\Delta t_D}$$

Multiplying both sides by $Pe_{j,m} (\Delta x_D)^2$ and expanding:

$$T_{D_{j,m+1}}^{n+1} - 2T_{D_{j,m}}^{n+1} + T_{D_{j,m-1}}^{n+1} - \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D (T_{D_{j,m}}^{n+1} - T_{D_{j,m-1}}^{n+1}) = C_{j,m} (T_{D_{j,m}}^{n+1} - T_{D_{j,m}}^n)$$

Where $C_{j,m} = \frac{Pe_{j,m} (\Delta x_D)^2}{\Delta t_D}$.

Rearranging for the general equation we get:

$$-(1 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D) T_{D_{j,m-1}}^{n+1} + (2 + \dot{\omega}_{j,m} Pe_{j,m} \Delta x_D + C_m) T_{D_{j,m}}^{n+1} - T_{D_{j,m+1}}^{n+1} = C_m T_{D_{j,m}}^n$$

Diritchlet Boundary Condition for Injection well (LBC)

$$T(x=0, t) = T_{inj}$$

$$T_i T_D(x_D=0, t_D) = T_{inj}$$

$$T_D(x_D=0, t_D) = \frac{T_{inj}}{T_i} = T_{D_{inj}}$$

Discretized Equation for First Grid

$$\frac{1}{Pe_1} \frac{1}{\Delta x_D} \left[\frac{T_{D_2}^{n+1} - T_{D_1}^{n+1}}{\Delta x_D} - \frac{T_{D_1}^{n+1} - T_{D_{inj}}^{n+1}}{\frac{\Delta x_D}{2}} \right] - \dot{\omega}_1 \left[\frac{T_{D_1}^{n+1} - T_{D_{inj}}^{n+1}}{\frac{\Delta x_D}{2}} \right] = \frac{T_{D_1}^{n+1} - T_{D_1}^n}{\Delta t_D}$$

After multiplying both sides by $Pe_{j,m}(\Delta x_D)^2$, rearranging and substituting LBC gives us:

$$(3 + 2\dot{\omega}_1 Pe_1 \Delta x_D + C_1) T_{D_1}^{n+1} - T_{D_2}^{n+1} = C_1 T_{D_1}^n + 2(1 + \dot{\omega}_1 Pe_1 \Delta x_D) T_{D_{inj}}$$

Dirichlet Boundary Condition for Production well (RBC)

$$T(x = L, t) = T_{prod}$$

$$T_i T_D(x_D = 1, t_D) = T_{prod}$$

$$T_D(x_D = 1, t_D) = \frac{T_{prod}}{T_i} = T_{D_{prod}}$$

Discretized Equation for Grid M

$$\frac{1}{Pe_M} \frac{1}{\Delta x_D} \left[\frac{T_{D_{prod}}^{n+1} - T_{D_M}^{n+1}}{\frac{\Delta x_D}{2}} - \frac{T_{D_M}^{n+1} - T_{D_{M-1}}^{n+1}}{\Delta x_D} \right] - \dot{\omega}_M \left[\frac{T_{D_M}^{n+1} - T_{D_{M-1}}^{n+1}}{\Delta x_D} \right] = \frac{T_{D_M}^{n+1} - T_{D_M}^n}{\Delta t_D}$$

After multiplying both sides by $Pe_{j,m}(\Delta x_D)^2$, rearranging and substituting LBC gives us:

$$-(1 + \dot{\omega}_M Pe_M \Delta x_D) T_{D_{M-1}}^{n+1} + (3 + \dot{\omega}_1 Pe_1 \Delta x_D + C_M) T_{D_M}^{n+1} = C_M T_{D_M}^n + 2T_{D_{prod}}$$

Appendix G

Fluid flow equation in 1D cartesian coordinates after simplifications is provided below:

$$\frac{\partial^2 p_j}{\partial x^2} = \frac{1}{\eta_j} \frac{\partial p_j}{\partial t} \quad ; \quad \text{for} \quad j = 1, 2$$

Where $\eta_j = \frac{k}{\mu_j \phi c_{t_j}}$.

Initial, boundary and interface conditions are given as follows:

Initially, reservoir is at constant pressure p_i :

$$p_j(x, t = 0) = p_i \quad ; \quad j = 1, 2$$

Left Boundary Condition (flux boundary condition at the injector) is given as:

$$\text{LBC} \Rightarrow -\left(\frac{kA}{\mu B}\right)_1 \left(\frac{\partial p_1}{\partial x}\right)_{x=0} = q_{inj}$$

Also, equal flux and equal pressure interface conditions (IFCs) at the interface are shown below:

$$\text{IFC 1a} \Rightarrow \frac{k_1}{\mu_1} \left(\frac{\partial p_1}{\partial x}\right)_{x=x_{int1}} = \frac{k_2}{\mu_2} \left(\frac{\partial p_2}{\partial x}\right)_{x=x_{int1}}$$

$$\text{IFC 1b} \Rightarrow p_1(x = x_{int1}, t) = p_2(x = x_{int1}, t)$$

And Right Boundary Condition (flux boundary condition at the producer) is given as:

$$\text{RBC} \Rightarrow \left(\frac{kA}{\mu B}\right)_2 \left(\frac{\partial p_2}{\partial x}\right)_{x=L} = q_{prod}$$

Dimensionless variables are defined as follows:

$$x_D = \frac{x}{L} \quad t_D = \left(\frac{k}{\mu \phi c_t}\right)_{1ini} \frac{t}{L^2} = \frac{\eta_{1ini} t}{L^2} \quad p_D = \frac{(p_i - p)}{p_i}$$

We can easily convert fluid flow equation, initial, boundary and interface conditions into dimensionless form by using dimensionless variables. Dimensionless form of diffusivity equation in 1D is given below:

$$\frac{\partial^2 p_{D_j}}{\partial x_D^2} = b_j \frac{\partial p_{D_j}}{\partial t_D} ; \quad j = 1, 2$$

Dimensionless form of initial condition is given below:

$$p_{D_j}(x_D, t_D = 0) = 0 ; \quad j = 1, 2$$

Dimensionless form of left boundary condition is given below:

$$\left(\frac{\partial p_{D_1}}{\partial x_D} \right)_{x_D=0} = q_{D,inj}$$

Dimensionless form of equal flux interface condition is given below:

$$\frac{k_1}{\mu_1} \left(\frac{\partial p_{D_1}}{\partial x_D} \right)_{x_D=x_{Dint1}} = \frac{k_2}{\mu_2} \left(\frac{\partial p_{D_2}}{\partial x_D} \right)_{x_D=x_{Dint1}}$$

Dimensionless form of equal pressure interface condition is given below:

$$p_{D_1}(x_D = x_{Dint1}, t_D) = p_{D_2}(x_D = x_{Dint1}, t_D)$$

Dimensionless form of right boundary condition is given below:

$$\left(\frac{\partial p_{D_2}}{\partial x_D} \right)_{x_D=1} = q_{D,prod}$$

Where $q_{D,inj} = \frac{q_{inj}L}{p_i} \left(\frac{\mu B}{kA} \right)_1$, and $q_{D,prod} = -\frac{q_{prod}L}{p_i} \left(\frac{\mu B}{kA} \right)_2$

Next, we apply Laplace transformation to dimensionless fluid flow equation, initial, boundary and interface conditions. Laplace transformation of diffusivity equation in 1D is given below:

$$\frac{\partial^2 \hat{p}_{D_j}}{\partial x_D^2} - z b_j \hat{p}_{D_j} = 0 \quad \text{for } j = 1, 2$$

Dimensionless form of initial condition in Laplace space is given below:

$$\hat{p}_{D_j}(x_D, t_D = 0) = 0 \quad ; \quad j = 1, 2$$

Dimensionless form of left boundary condition in Laplace space is given below:

$$\left(\frac{\partial \hat{p}_{D_1}}{\partial x_D} \right)_{x_D=0} = \frac{q_{D,inj}}{z}$$

Dimensionless form of equal flux boundary condition in Laplace space is given below:

$$\frac{k_1}{\mu_1} \left(\frac{\partial \hat{p}_{D_1}}{\partial x_D} \right)_{x_D=x_{Dint1}} = \frac{k_2}{\mu_2} \left(\frac{\partial \hat{p}_{D_2}}{\partial x_D} \right)_{x_D=x_{Dint1}}$$

Dimensionless form of equal pressure boundary condition in Laplace space is given below:

$$\hat{p}_{D_1}(x_D = x_{Dint1}, z) = \hat{p}_{D_2}(x_D = x_{Dint1}, z)$$

Dimensionless form of right boundary condition in Laplace space is given below:

$$\left(\frac{\partial \hat{p}_{D_2}}{\partial x_D} \right)_{x_D=1} = \frac{q_{D,prod}}{z}$$

General solution to Dimensionless diffusivity equation in Laplace space for each zone can be written as:

$$\hat{p}_{D_1}(x_D, z) = Ae^{-x_D \sqrt{zb_1}} + Be^{x_D \sqrt{zb_1}}$$

$$\hat{p}_{D_2}(x_D, z) = Ce^{-x_D \sqrt{zb_2}} + De^{x_D \sqrt{zb_2}}$$

Obtaining the derivatives for both solutions:

$$\frac{\partial \hat{p}_{D_1}}{\partial x_D} = \sqrt{zb_1} (Be^{x_D \sqrt{zb_1}} - Ae^{-x_D \sqrt{zb_1}})$$

$$\frac{\partial \hat{p}_{D_2}}{\partial x_D} = \sqrt{zb_2} (De^{x_D \sqrt{zb_2}} - Ce^{-x_D \sqrt{zb_2}})$$

Now we can substitute the solutions given above into the boundary and interface conditions to get following set of equations:

$$-A + B = \frac{q_{D,inj}}{z\sqrt{zb_1}}$$

$$-\frac{k_1 \sqrt{b_1}}{\mu_1} Ae^{-x_{Dint1} \sqrt{zb_1}} + \frac{k_1 \sqrt{b_1}}{\mu_1} Be^{x_{Dint1} \sqrt{zb_1}} + \frac{k_2 \sqrt{b_2}}{\mu_2} Ce^{-x_{Dint1} \sqrt{zb_2}} - \frac{k_2 \sqrt{b_2}}{\mu_2} De^{x_{Dint1} \sqrt{zb_2}} = 0$$

$$Ae^{-x_{Dint1} \sqrt{zb_1}} + Be^{x_{Dint1} \sqrt{zb_1}} - Ce^{-x_{Dint1} \sqrt{zb_2}} - De^{x_{Dint1} \sqrt{zb_2}} = 0$$

$$-Ce^{-\sqrt{zb_2}} + De^{\sqrt{zb_2}} = \frac{q_{D,prod}}{z\sqrt{z}}$$

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