



**Fully-Implicit Formulation of the Capacitance  
Resistance Model to Simulate Two-Phase  
Immiscible Flow**

BY

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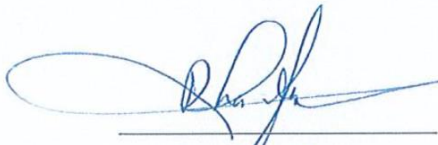
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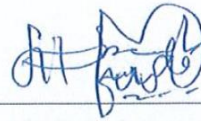
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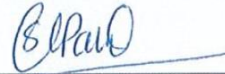
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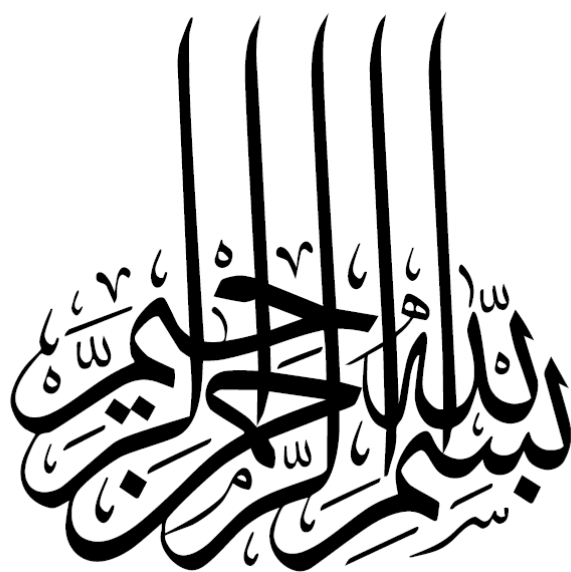
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I dedicate this effort to my mother; the reason of my success in every step of my life.

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## LIST OF ABBREVIATIONS

|                 |                                                                          |
|-----------------|--------------------------------------------------------------------------|
| <b>CRM</b>      | Capacitance Resistance Model                                             |
| <b>FDM</b>      | Finite-Difference Method                                                 |
| <b>SA-CRMP</b>  | Semi Analytical - Capacitance Resistance Model for<br>Producer           |
| <b>FD-CRMP</b>  | Finite Difference - Capacitance Resistance Model<br>for Producer         |
| <b>TP-CRMIP</b> | Two-Phase - Capacitance Resistance Model for<br>Injector – Producer pair |
| <b>PDE</b>      | Partial Differential Equations                                           |
| $C_o$           | Oil Compressibility                                                      |
| $C_w$           | Water Compressibility                                                    |
| $C_f$           | Rock Compressibility                                                     |
| $\phi$          | Porosity                                                                 |

## ABSTRACT

Full Name : Ahmed Sadeed  
Thesis Title : Fully Implicit Formulation of the capacitance resistance model to simulate two-phase immiscible flow  
Major Field : Petroleum Engineering  
Date of Degree : May 2023.

The Capacitance-Resistance Model (CRM) is a reduced physics, data-driven technique that is used to model waterflooding performance of a reservoir. It is based on the theory of signal processing which dictates that any changes applied at the injectors have a direct influence on the producers. The CRM model is computationally inexpensive as it does not solve the full physics models used in traditional reservoir simulations to determine well production rates. Instead, it relies on historical data of production and injection rates along with well bottom hole pressures to estimate these rates. The CRM model consists of a material balance equation coupled with the inflow equation. Traditionally, a semi-analytical solution is used to solve the differential equation representing the CRM model. In this study, however, finite-difference schemes are used to solve the ODE representing the CRM model.

In the first phase of this study, a producer-based representation of the CRM model (CRMP) is solved to estimate total production rates using finite-difference methods. In CRMP, separate control volumes are defined for each producer coupled with all the injectors in the reservoir. Since the resulting ODE for CRMP consists of time-dependent parameters, both the explicit and implicit finite-difference schemes are used to solve the problem. Stability

and consistency analyses are performed to evaluate the applicability of the resulting finite-difference solutions. The implicit finite-difference scheme was found to be unconditionally stable and consistent whereas the explicit finite-difference scheme was found to be conditionally stable. A constrained optimization technique such as Find MINimum of CONstrained nonlinear multivariable function (FMINCON) is implemented in order to estimate certain parameters in the CRMP model such that a good match is obtained between the observed data and the modeled solution. The CRMP model is applied on different reservoir models and results indicate that the CRMP model is able to accurately predict the production rates.

In the second phase of this study, capacitance resistance model for injector-producer pair (CRMIP) is used to assess the individual phase rates. The CRMIP model is an injector-producer pair representation of the CRM model where separate control volumes are defined for each injector-producer pair in the reservoir. A constrained optimization technique (FMINCON) is implemented to estimate the unknown parameters in the CRMIP model. In this study, a fully-implicit finite-difference scheme for a two-phase flow (TP-CRMIP) is developed to quantify the effect of injection rate on the production of individual oil and water phases. TP-CRMIP model is applied on different reservoir models and results indicate that the TP-CRMIP model is able to provide a reasonably good match with the individual phase rates.

Although the Capacitance-Resistance Model (CRM) has numerous advantages and has proven to be an effective tool, it has limitations when it comes to managing and planning field development, especially in terms of adding future wells. The CRM model does not provide a straightforward way of incorporating new wells into the model. The final phase

of this study addresses the issue. Two-phase capacitance resistance model for an injector-producer pair (TP-CRMIP) is used to initially history match the production rates to estimate model parameters. A newly developed cosine-based model is then used to generate the unknown model parameters for the injectors and/or producers using the known model parameters of existing wells. Once the model parameters of all injectors and producers are calculated, TP-CRMIP is used to calculate the production rates. Results indicated that the unknown model parameters generated using the cosine-based model when used along with the TP-CRMIP model provide reasonable estimates of the production rates.

## ملخص الرسالة

الاسم الكامل: أحمد سديد

عنوان الرسالة: صياغة ضمنية كاملة لنموذج مقاومة السعة لمحاكاة التدفق غير القابل للامتزاج على مرحلتين

التخصص: هندسة البترول

تاريخ الدرجة العلمية: مايو ٢٠٢٣

إن نموذج مقاومة السعة (CRM) هو تقنية حديثة لنمذجة غمر المياه وتحسين معدلات الحقن للحاقيات الحالية لزيادة معدلات الإنتاج. يعتمد على نظرية معالجة الإشارات حيث يكون لأي تغييرات مطبقة على الحاقيات تأثير مباشر على المنتجين. يوفر تقنية سريعة لنمذجة أداء تدفق المياه لخزان. السبب الرئيسي لعدم التكلفة الحسابية لنموذج CRM هو أن معدلات إنتاج الآبار المستقبلية لا يتم تحديدها من خلال حل نماذج الفيزياء الكاملة كما هو الحال مع مشاكل محاكاة المكامن التقليدية. بدلاً من ذلك ، يتم تحديد المعدلات بناءً على معدلات الإنتاج والحقن الملحوظة تاريخياً وضغوط قاع البئر. تتكون نماذج CRM من معادلة توازن المواد مقترنة بمعادلة التدفق الداخلي. تقليدياً ، يتم استخدام حل شبه تحليلي لحل المعادلة التفاضلية التي تمثل نموذج CRM. ومع ذلك ، في هذه الدراسة ، يتم استخدام مخططات الفروق المحدودة لحل ODE الذي يمثل نموذج CRM.

في المرحلة الأولى من هذه الدراسة ، تم حل التمثيل القائم على المنتج لنموذج CRM (CRMP) لتقدير معدلات الإنتاج الإجمالية باستخدام طريقة الفروق المحدودة. في CRMP ، يتم تحديد أحجام تحكم منفصلة لكل منتج إلى جانب جميع الحاقيات الموجودة في الخزان. نظراً لأن ODE الناتج لـ CRMP يتكون من معلمات تعتمد على الوقت ، يتم استخدام كل من مخططات الفروق المحدودة الصريحة والضمنية لحل المشكلة. يتم إجراء تحليلات الاستقرار والاتساق لتقييم إمكانية تطبيق حلول الفروق المحدودة الناتجة. تم العثور على نظام الفروق المحدودة الضمنية لتكون مستقرة ومتسقة بشكل غير مشروط بينما وجد أن مخطط الفرق المحدود الصريح مستقر مشروطاً. يتم تنفيذ تقنية تحسين مقيدة مثل البحث عن الحد الأدنى من وظيفة المتغيرات غير الخطية المقيدة (FMINCON) من أجل تقدير معلمات معينة في نموذج CRMP بحيث يتم الحصول على تطابق جيد بين البيانات المرصودة والحل النموذجي. يتم تطبيق نموذج CRMP على نماذج مكملة مختلفة وتشير النتائج إلى أن نموذج CRMP قادر على التنبؤ بدقة بمعدلات الإنتاج.

في المرحلة الثانية من هذه الدراسة ، يتم تقييم معدلات الطور الفردي بمساعدة نموذج مقاومة السعة لزوج الحاقن والمنتج (CRMIP). نموذج CRMIP عبارة عن تمثيل زوجي للحاقن والمنتج لنموذج CRM حيث يتم تحديد أحجام تحكم منفصلة لكل زوج من الحاقن والمنتج في الخزان. يتم تنفيذ تقنية التحسين المقيدة (FMINCON) لتقدير المعلمات غير المعروفة في نموذج CRMIP. في هذه الدراسة ، تم تطوير نموذج الفروق المحدود الضمني الكامل لنموذج التدفق ثنائي الطور (TP-CRMIP) لتقدير تأثير معدل الحقن على إنتاج مراحل الزيت والماء الفردية. يتم تطبيق نموذج TP-CRMIP على نماذج مكملة مختلفة وتشير النتائج إلى أن نموذج TP-CRMIP قادر على توفير تطابق جيد إلى حد معقول مع معدلات المرحلة الفردية.

بصرف النظر عن الفوائد العامة والتطبيقات الفعالة لنموذج CRM ، لا يزال من الصعب للغاية إدارة وتخطيط تطوير الحقل بشكل صحيح بسبب محدودية إضافة الآبار المستقبلية. في المرحلة النهائية ، تتناول هذه الدراسة مسألة إضافة آبار مستقبلية في نموذج CRM. يتم استخدام نموذج مقاومة السعة ثنائية الطور لزوج منتج الحاقن (TP-CRMIP) لمطابقة التاريخ المبدئي لمعدلات الإنتاج لتقدير معلمات النموذج. بعد ذلك ، يتم استخدام النموذج المعتمد على جيب التمام المطور حديثاً لتوليد معلمات النموذج غير المعروفة للحاقيات الجديدة و / أو المنتجين باستخدام معلمات النموذج المعروفة للآبار الموجودة. بمجرد حساب معلمات النموذج لجميع المحاقن والمنتجين ، يتم استخدام TP-CRMIP لحساب معدلات الإنتاج. ثم تتم مقارنة معدلات الإنتاج المحسوبة والمحاكاة لاختبار دقة النموذج القائم على جيب التمام. تم تقديم ثلاث حالات لتقييم الوظيفة المستندة إلى جيب التمام. يوفر النموذج المستند إلى جيب التمام تقديرات معقولة لمعدلات الإنتاج مقارنة بالمعدلات المحاكاة في جميع الحالات المدروسة. سيعمل نموذج TP-CRMIP الذي تم تطويره حديثاً على أساس جيب التمام كأداة سريعة وموثوقة لتحسين غمر المياه وتخطيط التطوير الميداني.

# **CHAPTER 1**

## **INTRODUCTION**

Reservoir simulation studies are essential for exploration and production companies for production planning, decision making and reservoir surveillance. Reservoir simulation honors the full physics of fluid flow in porous media and the accuracy is comparatively higher when compared to available other analytical and empirical models. Unfortunately, the lack of accurate data, missing information, model uncertainty and non-unique solutions makes the reservoir simulation model imperfect in many cases.

Traditional reservoir simulations typically involve creating complex reservoir geological models that consist of numerous grid blocks. These models are generated using geostatistical reservoir modeling, which can result in multiple realizations for a given property without a unique solution which can be tedious and time consuming.

To avoid delays in decision making and planning, other techniques are also employed for fast computation and estimation of future production rates. Such available models are analytical models, empirical models, proxy models and capacitance resistance models. Classic analytical models are mainly used for direct estimation of fluid in place and water encroachment from production data. They were initially developed using macroscopic material balance to understand the reservoir performance. For a homogenous control volume, the expansion of oil, water, gas, and rock at any time is equal to fluid production. This model is independent of temperature, and compositional gradients within the system and sometimes referred to as Tank Model.

Empirical models such as decline curve models are also available to provide direct and rapid estimation of ultimate reserves and are often used in production planning. They are data driven models and the model accuracy is highly dependent on data size and quality. Since many decline curve models are available for different reservoir conditions and fluid types, the selection is highly important for reasonable predicted output.

Capacitance Resistance Model (CRM) is based on the theory of electrical circuits consisting of resistors and capacitors. The input is applied via electric current which is either stored in a capacitor or transmitted to the output via a resistor. The same phenomenon is applied in waterflooding applications where the injected water rates are the source of input signals which are transmitted toward producers and reflected in the production rates as output signals. The injected water or input signals are either attenuated due to reservoir storage or transmitted to the producer similar to the capacitor and resistors in the electric circuit.

In this research work, the capacitance resistance model is used to study phase production rates. The model is solved using finite-difference methods instead of traditional semi analytical methods. The model solution is tested for consistency, convergence, and stability. The novel model is applied on different simulated cases and compared with the available models in the literature. In the latter part of this study, a new approach is presented to address the limitation of adding future wells in the CRM model. This approach is also tested on different cases to assess the accuracy.

The literature review of available CRM models is discussed in Chapter 2. It summarizes the continuous improvements and modifications to the original CRM model. It also

highlights the use of CRM in different reservoir conditions and the benefits of the model parameters in understanding the fluid dynamics.

Chapter 3 discusses the problem statement and the research objectives of this dissertation.

Chapter 4 assesses the value added and applicability of finite-difference methods (FDM) to CRM model. CRMP model was used in this chapter to evaluate the FDM solution. Convergence, consistency, and stability of the model solution were evaluated. Three different cases were studied to observe the model accuracy.

Chapter 5 proposes a two-phase finite-difference solution of the capacitance resistance model for an injector-producer pair (TP-CRMIP). This model provides a direct estimation of oil and water production rates. Three different simulated cases were studied to observe the model accuracy of TP-CRMIP.

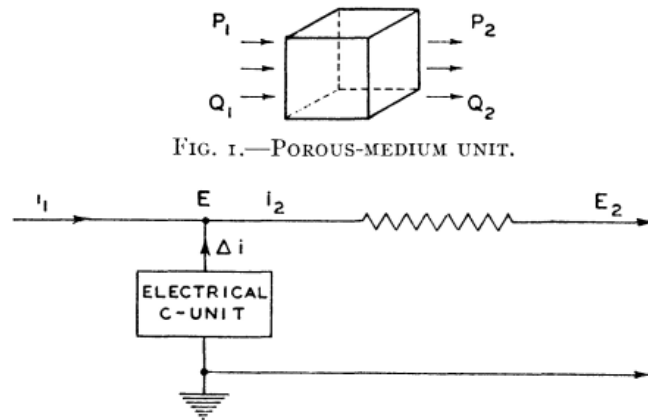
Chapter 6 proposes a solution of a challenging issue of CRM to effectively model new injectors and producers. This problem was resolved using a newly developed cosine-based model. This new model was tested on three different simulated cases to examine the accuracy.

Chapter 7 presents conclusions of this work

## CHAPTER 2

### LITERATURE REVIEW

The idea to relate reservoir fluid flow to the flow of current in electrical circuits was first introduced by (Bruce 1943). Instead of solving partial differential equations for fluid flow, he developed an apparatus of electrical units wired together to simulate fluid flow in porous media using the concept of current flow through conductive media as shown in Figure 1.



**Figure 1: A pictorial representation of analogy between porous media and electrical circuits**  
Wahl et al. (1962) used the concept of (Bruce 1943) and made an electrical system consisting of input and output devices, along with a resistance-capacitance network and a central control to model four reservoirs in Saudi Arabia. The resistance-capacitance network consisted of 4,900 resistance decades interlinked with 2,501 capacitance decades. A rectangular network of 2,501 mesh points is formed with components in a 41 x 61 mesh array.

Heffer (1997) studied injector-producer interaction using the distribution of in situ stresses on the flow directionalities of injectors toward the producer. They introduced a correlation coefficient called Spearman rank to model connectivity between injector-producer.

Panda and Chopra (1998) introduced an artificial neural network (ANN) to assess water flooding efficiency. A simulated case was presented to observe the effect of reservoir heterogeneity between an injector producer-pair using artificial neural networks applied on injection and production data. However, because of the intrinsic nature, ANN model cannot be used for sensitivity analysis of data outside the range of trained network.

The early research to optimize water flooding pattern and injection scheme for production enhancement was mainly focused on experimental investigation and statistical tools. Albertoni and Lake (2002, 2003) were first to mathematically model the injector-producer interaction using the analogy of resistor-capacitor networks. They used multivariate linear regression to account for the connectivity between injectors and producers. They reported that the total production rate for a particular well is a weighted linear combination of the individual injection rates. They used diffusivity filters to account for the attenuation and dissipation of injection signals due to reservoir heterogeneity.

Gentil (2005) was first to present the two-phase flow rates equation using an empirical fraction flow model to calculate individual oil production rate from the total production rate. He also investigated the physical meaning of injector-producer connectivity constant (regression weights) related to the reservoir transmissibility. He reported that the regression weights are the ratios of inverse distance average permeabilities of an injector-producer pair corresponding to each injector.

Yousef et al. (2005 & 2006) were the first to derive a physical model for an injector-producer connectivity using the continuity equation. They also incorporated the effect of change in bottom hole pressure of a producer in their model. The model was then solved using discretization in time to obtain the total production rate at the producer. They used two parameters such as weight constant to account for the attenuation of injection signals to the producer and the time constant to account for the compressibility and mobility of the fluid. The technique was successfully applied to the synthetic field models and two producing fields in Argentina and the North Sea. Since the defined parameters have direct analogy with the electrical circuits in terms of energy storage such as capacitors and energy transmittance such as resistors, this model was termed the Capacitance Resistance Model (CRM).

CRM is developed from signal processing techniques in which injection and production values are considered as input and output signals, respectively. With the standard CRM method, two parameters are estimated for each injector-producer pair during the initial history matching stage. These parameters are 1) inter-well connectivity that measures the connection between pairs and 2) time-constant indicating the time delay of the injection rate affecting the producer. These parameters provide some understanding of the underlying reservoir physics. For example, the inter-well connectivity represents the contribution of fluid flow from each injector towards each producer and the distribution of certain facies can be identified by CRM (Kaviani & Jensen, 2010). The authors observed that the inter-well connectivity is related to change in facies and some facies act like barriers to flow direction while some facies provide a continuous flow path which is reflected by high value of inter-well connectivity factor. On the other hand, the time

constant provides the response time from injectors toward the producer, and it is generally affected by the inter-well distance, fluid properties and the producer productivity index.

Sayarpour et al. (2009); Morteza Sayarpour, Kabir, and Lake (2008, 2009) further investigated the physics-based modelling of CRM and solved the governing mass balance differential equation semi-analytically (SA) to three different reservoir control volumes, such as single tank model (CRMT), a single producer model (SA-CRMP) and an injector-producer model (CRMIP), denoted as SA-CRMT, SA-CRMP and SA-CRMIP in this study. They also incorporated the change in bottom hole pressure of producers. The general solution of the differential mass balance equation was solved using superposition in time instead of numerical integration to find a semi-analytical solution for total production rates. This approach was validated using a numerical reservoir simulation model and applied successfully on a case involving water/CO<sub>2</sub> flooding. Once the total production rates are obtained, they used a fractional flow model to predict the oil production rate at each producer. Following the empirical model presented by Gentil 2005, more empirical fractional flow models were reported for immiscible and miscible flows to use along with CRM (Cao, Luo, and Lake 2014; Nguyen et al. 2011; Weber et al. 2009)

Weber et al. (2009) presented an approach to model a large data set consisting of several injectors and producers with fluctuations in injection and production rate data. The presented study was applicable with shut-in wells, intermittent wells and producers that become injectors. Different optimization techniques were reported to remove outliers in field data and reduce model parameters to improve and expand the capability of CRM to handle large field data sets.

Izgec, Kabir, and Etc (2009, 2010) presented a study to model injector-producer interaction before water breakthrough using the continuity equation and showed similarity of CRM to the pressure transient testing. The presented study can be used for immature fields where the saturation changes in the control volumes significantly affect the flow transmissibility, consequently, reflecting in the production rates. In 2010, Inzgec presented another study to model differential aquifer influx at individual producers using the combination of CRM model and unsteady state analytical aquifer model such as Van Everdingen and Hurst (VEH).

Kaviani and Jensen (2010) presented an approach similar to conventional CRM and called it as multi-well productivity Index (MPI). Instead of weight constant and time lag, they used influence matrices for each producer and injector-producer pair. This technique was successfully applied on two synthetic models.

Nguyen et al. (2011) presented an integrated approach to model the CRM. The CRM model was solved for total cumulative production using the total water injected in the control volume. The developed model was validated on synthetic cases and an oil field in Oman.

Zhao et al. (2015) presented an Interwell Numerical Simulation Model (INSIM) to study the injector-producer interaction. The interval between the injector and producer is discretized into small segments and each segment has two control parameters such as transmissibility and control volume. The model is solved using front tracking and the inter-well parameter at each segment is obtained using history matching. Once the saturation and inter-well parameters are known, phase production rates can be determined. For model prediction, they claimed that saturation and pressure profile generated from the INSIM model are sufficient.

Altaheini, Al-Towijri, and Ertekin (2016) addressed the associated challenges of the available CRM model and proposed a modified CRM model to accommodate the existing limitations. The limitations were frequent shut-in or addition of new wells, aquifer influx, variation in fluid compressibility and change in well productivity index due to workover operation. They resolved these limitations by introducing a pseudo injection rate and keeping the CRM parameters constant to address the aquifer support and shut-in periods. The change in well productivity index due to recompletion/workover was addressed by considering two separate producers having productivity index values before and after completion.

Eshraghi et al. (2016) studied the change in reservoir properties and production data fluctuation in the CRM model. They used a synthetic reservoir model with varying porosity and permeability. They observed a strong effect of changing porosity than permeability on CRM parameters. A linear trend was also observed between error to mean production ratio (EMPR) and reservoir porosity or permeability.

Jamali and Ettehad Tavakkot (2017) expanded the scope of CRM model to apply in large scale mature oil fields. They presented an approach to minimize the computing cost and increase convergence rate. They used global optimization for the CRM objective function incorporated into the gradient vector and Hessian matrix.

T. Zhang, IZGEC, and Sayarpour (2018) highlighted that the time constant and inter-well connectivity in CRM varies with time for heavy oil waterflooding. They considered the effect of viscous fingering in heavy oil and observed that CRM parameters varies with viscosity ratio of displacing and displaced fluid, storage and flow capacity contrast and the

flood maturity. They observed that the evenly distributed injected fluid among producers can produce constant CRM parameters after breakthrough.

K. Liu, Wu, and Ling (2019) and X. Liu et al. (2020) applied the CRM model on a field case of Gaither Draw Units (GDU) in Wyoming, USA, to estimate reservoir heterogeneity and inter-well connectivity. The field was injected with 1.7 MMSTB of water with no clear impact of water flooding on production performance. CRM technique was used to assess the reservoir heterogeneity and future well planning. They observed low inter well connectivity values and large time constant for some producers. This implied a weak connection between injectors and producers. They also verified the findings of CRM model using a full-scale reservoir simulation model on a commercial software.

Salehian et al. (2018); Salehian and Çınar (2019) and Temizel et al. (2018) presented another study to manage the frequent shut-in of wells in CRM model. They introduced a modified connectivity parameter to account for the frequent shut-in and derived a dynamic CRM model (D-CRM). The developed CRM model was in good agreement with the results generated from a synthetic reservoir model using commercial software.

Naudomsup and Lake (2019) extended the application of CRM model to get insight of the reservoir properties using CRM model coupled with tracer models, based on miscible displacement theory. Three tracer models were utilized in this study which are dispersion-only model, a Koval model, and a combination of both. The CRM model and tracer models were coupled together using two techniques such as serial fitting (CRM then tracers) and simultaneous fitting (CRM and tracers). The developed method was successfully applied on Lawrence field and found good agreement of inter-well connectivity with the observed peak tracer concentration.

Xuewu Wang, Wang, and Xia (2020) improved the available CRM model and introduced interface coefficients between producers to address the issue of well shut-in and pressure fluctuations. The model was validated by a synthetic reservoir model and the effect of changing well locations, frequent well switching from production well to injectors and vice versa was also reported. In another study, (Xiang Wang et al. 2020) addressed the limitation of low permeability in CRM model. They reported that the fluid flow in low permeability reservoirs requires a certain pressure gradient called a threshold pressure gradient and this pressure gradient was introduced in Darcy's equation to modify CRM model.

Kim (2019) further expanded the scope of CRM to overcome the limitation of CRM application in viscous oil and lack of bottom hole pressure. They observed a significant difference in inter-well connectivity observed from conventional CRM model and tracer test conducted in Lost Hills field, which is a low permeability, high porosity, and heavy oil formation. To overcome the limitation, they introduced a statistical approach in CRM model to analyze the CRM parameters using bootstrap method. A confidence interval is defined and if the initial solution is exceeding the confidence interval, then the parameter values are substituted with P50 values and injector-producer connectivity and connection-pair is updated.

Olenchikov and Posvyanskii (2019) studied the challenges of active aquifer and changes in operation mode on the available CRM model to predict the flow rates. They emphasized the use of productivity coefficient instead of well influence coefficient as tuning parameters for the model. The active aquifer was adjusted in the mass balance equation and a new modified CRM model was derived. The developed CRM provided flexibility to model crossflow, change in operating conditions and aquifer influence.

Mamghaderi, Aminshahidy, and Bazargan (2021) studied the model uncertainty of CRM to mimic the reservoir fluid flow behavior. They used a time correlated error coupled with the CRM model, including stochastic and non-stochastic parameters. The use of stochastic parameters generates multiple possible results, and the numbers of generated stochastic runs reflects the discrepancy in the matching problem. The intended model profile can then be obtained by averaging the possible results.

It is evident that the CRM is a powerful tool to determine connectivity between injector-producer pairs. However, the proper implementation and application of CRM in a complex subsurface system and varying surface operating conditions requires alteration and modification in the general solution of the CRM.

## **CHAPTER 3**

### **PROBLEM STATEMENT**

#### **3.1 Current Limitations**

As highlighted in Chapter 2, the current CRM models are based on the direct estimation of total production rate from the solution of the continuity equation eliminating the saturation changes of individual oil and water phases. Thus, total liquid production rates are calculated using CRM model provided that the model parameters are obtained beforehand by history matching a set of production and injection data from the reservoir control volume. The individual oil and water rates are then calculated indirectly by using available fraction flow models such as Koval method and Gentil method. Consequently, the current CRM models match the total production rates however, they lack the correct estimation of individual rates of each phase.

In all the available models, the limitation of adding future wells is not effectively addressed or resolved. Some of the available models adjust the new injectors and producers as a part of the history matching process while others provide solutions to introduce new injectors. However, there is no current model available in the literature that can efficiently model both new injectors and producers in the CRM model. Therefore, it is crucial to develop a more efficient CRM model to accommodate future wells that can estimate both the total production rates and the individual rates of each phase accurately.

### **3.2 Research Objectives**

In this study, a two-phase CRM model is developed to quantify the effect of injection rate on the production of individual oil and water phases. Oil and water rates are calculated using simultaneous solution of continuity equations for each phase at each time step. The newly developed CRM model reduces the deficiencies of the previous models and enhances the accuracy of estimating production rate of each phase.

The CRM provides a rapid technique to model the waterflooding performance of a reservoir. The primary reason for the computational inexpensiveness of the CRM model is that the future well production rates are not determined by solving the full physics models as is the case with traditional reservoir simulation problems. Rather, the rates are determined based on the historically observed production and injection rates and the well bottom hole pressures. The CRM models consist of a material balance equation coupled with the inflow equation. In this study, however, finite-difference schemes are used to solve the ODE representing the CRMP model.

To address the limitation of new wells, a new CRM model is proposed that can efficiently incorporate future wells into the model. In addition, the model can incorporate the injection and production data from the new wells into the existing model and adjust the inter-well connectivity and time constant accordingly. The proposed model is expected to significantly improve the accuracy of the CRM model and optimize production from reservoirs.

### **3.3 Scope of work**

This study aims to study the producer-based representation of the CRM model where separate control volumes are defined for each producer coupled with all the injectors in the reservoir. Since the resulting ordinary differential equation (ODE) for CRMP consists of time-dependent parameters, both the explicit and implicit finite-difference schemes are used to solve the problem.

Stability and consistency analyses are performed to test the applicability of the resulting finite-difference solutions of CRM model. A constrained optimization technique (FMINCON) will be used to estimate certain parameters in the CRMP model such that a good match is obtained between the observed data and the modeled solution. Computation improvement will also be evaluated with the available CRM models.

First, a single-phase CRM model is derived based on the solution of continuity equation for the combined rates and compared with the available semi-analytical solution of total production rates. In the second phase of this study, a two-phase CRM model is derived using the solution of continuity equation for each phase separately and direct estimation of saturation profile and production rate of each phase.

The study will be extended to determine the methodology to accommodate future wells in the developed two-phase CRM model.

# **CHAPTER 4**

## **A FINITE-DIFFERENCE APPROACH TO THE CAPACITANCE RESISTANCE MODEL**

### **4.1 Introduction**

Reservoir simulation studies are of vital importance for E&P companies for production planning, field development and reservoir management. Reservoir simulation studies honor the full physics of the fluid flow in porous media and therefore require extensive computational resources to accurately predict the reservoir production. Several models have been developed to efficiently predict the reservoir performance based on total liquid production rates instead of individual phase rates, considered as single-phase models in this study (B. Chen et al. 2017; B. Chen and Reynolds 2016; Fonseca et al. 2013; Isebor, Durlofsky, and Echeverría Ciaurri 2014; Oliveira and Reynolds 2014). One such model is the reduced order proxy model which attempts to reduce the computational complexity by simplifying the physics of the simulation model (cardoso and Durlofsky 2010; van Doren, Markovinović, and Jansen 2006; Gildin et al. 2013; He and Durlofsky 2014). However, multiple runs of the full physics reservoir simulation are required to develop a reduced order model. Proxy models are used to accelerate the history matching process as they offer computationally cheaper alternatives to full-scale reservoir simulation models (Mamghaderi, Aminshahidy, and Bazargan 2021). Appropriate use of orthogonal decomposition is a popular procedure to develop a proxy model (van Doren, Markovinović and Jansen, 2006; Cardoso and Durlofsky, 2010; Gildin et al., 2013; He and Durlofsky,

2014). These reduced-order models significantly improve computational performance by reducing the calculation time spent on forward simulation runs. However, the development of reduced orthogonal-decomposition models requires a full physics reservoir model which is simulated several times to capture snapshots of different property fields such as pressures and saturations (Zhao et al. 2015).

Data-driven proxy models provide a faster alternative to reduced order models as they require only the historical input (well injection rates) and output (well production and well bottom hole pressures) data to develop a proxy model. Two main categories of the data-driven statistical models are: 1) black-box models and 2) physics-based models.

The black-box proxy models are developed by a computer algorithm using the input and output response. The user has limited understanding of how the different functions and variables are combined to produce the output. Machine learning and artificial intelligence-based proxy models fall under this category and have recently been widely implemented in the industry to develop proxy models (X. Liu et al. 2020; Ng, Jahanbani Ghahfarokhi, and Nait Amar 2021). The disadvantage of this particular method is that the fluid flow system in the reservoir is not really taken into consideration and therefore the model may replicate the results during the history matching stage while producing very different responses during the prediction stage (Rowan and Clegg 1963; Wanderley de Holanda, Gildin, and Jensen 2018). These models may only work for limited cases and for a limited period of time thus compelling the user to update the model which further increases the computational costs (van Essen, van den Hof, and Jansen 2013; Heijn, Markovinic, and Jansen 2004).

In contrast to the black-box methods, the physics-based methods incorporate limited physics of fluid flow in the developed proxy models. One of the recent data driven physics-based model techniques for the estimate of total production rates is capacitance resistance model as discussed earlier in the literature review. It is based on the theory of electrical circuit consisting of a resistor and a capacitor (Bruce 1943) applied on waterflooding applications. The injected water rates are the source of input signals which are transmitted toward producers and reflect in the production rates as output signals.

## 4.2 Capacitance Resistance Model for Single Phase Fluid Flow

The differential equation for CRM is derived using material balance equations of individual phases in a control volume  $V_p$ . The continuity equations of each phase are added up to eliminate phase saturations. The average reservoir pressure in the resulting differential equation is then substituted by the drawdown equation. The derivation of capacitance resistance model (CRM) is provided in Appendix A.

The first mathematical model to account for the interaction of injector-producer pair is given by Alejandro and Lake, (2002) and Albertoni and Lake, (2003),

$$q_j(t) = \beta_{0_j} + \sum_{i=1}^N \beta_{ij} I_i(t) (j=1, 2, 3, \dots, N_p), \quad (4.1)$$

where  $q_j(t)$  is the total production rate at producer  $j$  at timestep  $t$ ,  $\beta_{0_j}$  and  $\beta_{ij}$  are the regression coefficients account for the unbalance and weighting factors between injector-producer pair, respectively.  $N_p$  represents the number of producers and  $I_i$  represents the injector rate at injector  $i$ .

Yousef et al., (2005), (2006), derived a physical model for an injector-producer connectivity using the continuity equation,

$$q(t) = q(t_0)e^{-\left(\frac{t-t_0}{\tau}\right)} + e^{-\left(\frac{t}{\tau}\right)} \int_{\xi=t_0}^{\xi=t} \left(\frac{1}{\tau}\right) e^{\xi} i(\xi) \partial \xi - e^{-\left(\frac{t}{\tau}\right)} \int_{\xi=t_0}^{\xi=t} J e^{\xi} \frac{dp_{wf}}{d\xi} \partial \xi. \quad (4.2)$$

where  $q(t)$  is the total production rate at time  $t$ ,  $q(t_0)$  is the total production rate at time  $t_0$ ,  $\tau$  is the time constant,  $i$  is the injection rate,  $J$  is the well productivity index and  $p_{wf}$  is the bottom hole pressure,  $\xi$  is the dummy variable for  $t$ .

Sayarpour, Kabir and Lake, (2008), (2009) and Sayarpour et al., (2009) extended the investigation of CRM and solved the physical model of Yousef et al., (2005), (2006), using a semi-analytical approach for three different cases to estimate total production rates.

$$q_t = q_{t-1} e^{-\left(\frac{\Delta t}{\tau}\right)} + \left(1 - e^{-\left(\frac{\Delta t}{\tau}\right)}\right) \left(f I_t - J \tau \frac{p_{wf}^t - p_{wf}^{t-1}}{\Delta t}\right) \quad (4.3)$$

where  $q_t$  and  $q_{t-1}$  is the total production rate at time  $t$  and  $(t-1)$ ,  $\tau$  is the time constant,  $f$  represents the inter-well connectivity,  $J$  is the well productivity index and  $p_{wf}$  is the bottom hole pressure.

Consider a control volume  $V_p$  covering the  $N_i$  number of injectors and  $N_j$  number of producers. Here,  $i$  and  $j$  are the index of each injector and producer, respectively. The CRMP model for the total production rate  $q_t$  at producer  $j$  is given by (Liang 2010):

$$\tau_j \frac{dq_t(t)}{dt} + q_t(t) = \sum_{i=1}^N f_{ij} I_i - \tau_j J_j \frac{dp_{wf}}{dt}, \quad (4.4)$$

where at any given time  $t$ ,  $q_t(t)$  is the total production rate,  $p_{wf}$  is the bottom hole flowing pressure at producer  $j$ ,  $I_i$  is the injection rate at injector  $i$ ,  $J_j$  is the productivity index of producer  $j$ ,  $\tau_j$  is the time constant for producer  $j$  and  $f_{ij}$  represents the fraction of injection rate of injector  $i$  flowing toward producer  $j$ .  $f_{ij}$  and  $\tau_j$  are the CRMP model parameters and are given by:

$$\tau_j = \frac{V_p c_t}{J_j}, \quad (4.5)$$

and

$$f_{i,j} = \frac{q_{i,j}}{I_i}, \quad (4.6)$$

where  $f_{ij}$  represents the inter-well connectivity between injector  $i$  and producer  $j$  and ranges between zero and one. A zero value indicates no transmissibility while a value of one indicates the injecting fluid from injector  $i$  contributes only to producer  $j$ .  $\tau$  represents the time delay from injector response toward the producer because of permeability in the producer drainage volume, the total compressibility of injecting fluid and reservoir formation,  $c_t$  and the productivity index  $J_j$  at producer  $j$ .  $f_{ij}$  and  $\tau_j$  are the unknown model parameters of the CRMP model and are obtained by minimizing the error between the observed total production rates and CRMP calculated total production rates. The objective function is defined as:

$$\min \sqrt{\frac{(\vec{q}_{t,obs} - \vec{q}_{t,cal})^T \cdot (\vec{q}_{t,obs} - \vec{q}_{t,cal})}{n_{dpts}}}, \quad (4.7)$$

such that

$$\sum_{j=1}^{N_j} f_{i,j} \leq 1, \text{ for all injectors } i, \quad (4.8)$$

and

$$f_{i,j}, \tau_j \geq 0 . \quad (4.9)$$

Here,  $n$ , represents the total number of data points used for the history matching,  $q_{t,obs}$  is the total observed production rate, and  $q_{t,cal}$  is the total CRMP calculated production rate.

### 4.3 Finite-difference approximation of CRMP model (FD-CRMP)

In this study, the differential equation obtained from the CRMP model was solved using the finite-difference method (FDM). FDM is a classical numerical technique for solving ordinary or partial differential equations which are difficult or impossible to solve analytically. FDM replaces the derivatives in a differential equation with difference quotients (Forward, Backward or Central). The dependent variables such as time and/or space are discretized into a finite number of steps and the resulting algebraic or finite-difference equations are solved at each step size.

The explicit finite-difference form of the CRMP model is given by:

$$\tau_j \left( \frac{q_j^{n+1} - q_j^n}{t^{n+1} - t^n} \right) + q_j^n = \sum_i^{N_i} f_{i,j} I_i^n - \tau_j J_j \left( \frac{p_{wf,j}^{n+1} - p_{wf,j}^n}{t^{n+1} - t^n} \right) \quad (4.10)$$

where  $q_j^{n+1}$  and  $q_j^n$  are the total production rates at producer  $j$  at time  $n+1$  and  $n$ , respectively.  $p_{wf,j}$  is the bottom hole flowing pressure at producer  $j$ ,  $I_i$  is the injection

rate at injector  $i$ ,  $J_j$  is the productivity index of producer  $j$ ,  $\tau_j$  is the time constant at producer  $j$  and  $f_{ij}$  represents the fraction of injection rate of injector  $i$  flowing toward producer  $j$ .

Upon rearranging,

$$q_j^{n+1} = q_j^n + \frac{1}{\tau_j} \left( \sum_{i=1}^{N_i} f_{i,j} I_i^n (t^{n+1} - t^n) - q_j^n (t^{n+1} - t^n) \right) - J_j (p_{wf,j}^{n+1} - p_{wf,j}^n) \quad (4.11)$$

The implicit finite-difference form of the CRMP model is given by:

$$\frac{q_j^{n+1} - q_j^n}{t^{n+1} - t^n} + \frac{1}{\tau_j} q_j^{n+1} = \frac{1}{\tau_j} \sum_{i=1}^{N_i} f_{i,j} I_i - J_j \frac{P_{wf,j}^{n+1} - P_{wf,j}^n}{t^{n+1} - t^n} \quad (4.12)$$

and upon rearranging,

$$q_j^{n+1} = \frac{\frac{1}{\tau_j} \sum_{i=1}^{N_i} f_{i,j} I_i^{n+1} + \frac{q_j^n}{t^{n+1} - t^n} - J_j \frac{P_{wf,j}^{n+1} - P_{wf,j}^n}{t^{n+1} - t^n}}{\frac{1}{t^{n+1} - t^n} + \frac{1}{\tau_j}} \quad (4.13)$$

Equations (4.11) and (4.13) show the explicit and implicit forms of the CRMP model, respectively. When differential equations are numerically solved, each derivative term is substituted by Taylor series. The terms from Taylor series expansion can be arranged to get the equivalent finite-difference scheme and the neglected higher order terms are called truncation error. The neglected terms, round off values, grid size or timestep can lead to consistency, stability and convergence problems in a finite-difference solution. These aspects will be discussed fully in the upcoming sections.

#### 4.3.1 Consistency

A finite-difference solution of a differential equation is consistent if the truncation error approaches zero by decreasing the grid size or reducing the time step. Thus, the finite-

difference solution would approach the exact solution of the differential equation. To evaluate the consistency of the FD-CRMP, the initial equation is given by,

$$\tau \frac{dq(t)}{dt} + q_t(t) = I(t) - \tau J \frac{dp_{wf}}{dt} \quad (4.14)$$

Separating derivative and non-derivative terms gives,

$$\frac{dq(t)}{dt} + J \frac{dp_{wf}}{dt} = \frac{1}{\tau} I(t) - \frac{1}{\tau} q_t(t) \quad (4.15)$$

Let  $f(t) = i(t) - q(t)$ , then,

$$\frac{dq(t)}{dt} + J \frac{dp_{wf}}{dt} = \frac{1}{\tau} f(t) \quad (4.16)$$

Discretizing Equation (4.16) in time yields the following:

$$\frac{q^{n+1} - q^n}{\Delta t} + J \frac{p_{wf}^{n+1} - p_{wf}^n}{\Delta t} = \frac{1}{\tau} f^{n+1,n}, \quad (4.17)$$

and using Taylor series expansion for  $\frac{dq}{dt}$  we arrive at:

$$q^{n+1} = q^n + \Delta t \frac{dq}{dt} + \frac{\Delta t^2}{2} \frac{d^2 q}{dt^2} + O(\Delta t^3), \quad (4.18)$$

and for  $\frac{dp_{wf}}{dt}$ ,

$$p_{wf}^{n+1} = p_{wf}^n + \Delta t \frac{dp_{wf}}{dt} + \frac{\Delta t^2}{2} \frac{d^2 p_{wf}}{dt^2} + O(\Delta t^3). \quad (4.19)$$

Substituting  $q^{n+1}$  and  $p_{wf}^{n+1}$  in Eq. 4.17 and rearranging,

$$\frac{dq}{dt} + \frac{\Delta t}{2} \frac{d^2 q}{dt^2} + O(\Delta t^2) + J \frac{dp_{wf}}{dt} + J \frac{\Delta t}{2} \frac{d^2 p_{wf}}{dt^2} + \frac{d^2 p_{wf}}{dt^2} + O(\Delta t^2) = \frac{1}{\tau} f^{n+1,n} \quad (4.20)$$

Substituting Eq. 4.16 and rearranging,

$$\frac{\Delta t}{2} \frac{d^2 q}{dt^2} + O(\Delta t^2) + J \frac{\Delta t}{2} \frac{d^2 p_{wf}}{dt^2} + O(\Delta t^2) = \frac{1}{\tau} f^{n+1,n} - \frac{1}{\tau} f(t) \quad (4.21)$$

At any timestep  $n$  or  $n+1$ , if  $\Delta t \rightarrow 0$  then,

$$\frac{1}{\tau} f^{n+1,n} - \frac{1}{\tau} f(t) \rightarrow 0 \quad (4.22)$$

hence, the formulation is consistent.

### 4.3.2 Stability

A finite-difference approximation is stable if the truncation or round off errors decrease from one time-step to another as the computation proceeds. A finite-difference scheme is unstable if the errors continue to increase uncontrollably with time. An amplification factor ( $G$ ) is defined as

$$G = \frac{\epsilon_j^{n+1}}{\epsilon_j^n} \quad (4.23)$$

where,  $\epsilon_j^n$  is the round-off error at the timestep  $n$ , and  $\epsilon_j^{n+1}$  is the round-off error at the timestep  $n+1$ . The amplification factor's absolute value needs to be less than or equal to one to stabilize the finite-difference scheme, i.e.,  $|G| \leq 1$ .

First, we consider the stability of the explicit scheme which is given by Eq. 4.10.

$$\tau_j \left( \frac{q_j^{n+1} - q_j^n}{t^{n+1} - t^n} \right) + q_j^n = \sum_i^{N_i} f_{i,j} J_i^n - \tau_j J_j \left( \frac{p_{wf,j}^{n+1} - p_{wf,j}^n}{t^{n+1} - t^n} \right) \quad (4.24)$$

To perform the stability analysis for the explicit scheme of CRMP, the RHS of the equations consisting of all the elements which are independent of the unknown variables are dropped from the equation.

Eq. 4.10 can therefore be written as

$$q_j^{n+1} = \left( 1 - \frac{\Delta t}{\tau_j} \right) q_j^n \quad (4.25)$$

Equation 4.23 can also be written as round-off error ( $\epsilon$ ) because the error has to satisfy the discretized equation (Anderson 1995).

$$\epsilon_j^{n+1} = \left(1 - \frac{\Delta t}{\tau_j}\right) \epsilon_j^n \quad (4.26)$$

Re-arranging Eq. 4.25 and introducing the amplification factor,

$$|G| = \left|1 - \frac{\Delta t}{\tau_j}\right| \quad (4.27)$$

Since,  $\Delta t \geq 0$ , and  $\tau_j \geq 0$ , therefore, for the explicit scheme of CRMP to be stable,  $\Delta t \leq \tau_j$

Next, we consider the stability of the implicit scheme of CRMP shown in Eq. 4.12

$$\frac{q_j^{n+1} - q_j^n}{t^{n+1} - t^n} + \frac{1}{\tau_j} q_j^{n+1} = \frac{1}{\tau_j} \sum_{i=1}^{N_i} f_{i,j} I_i - J_j \frac{P_{wf,j}^{n+1} - P_{wf,j}^n}{t^{n+1} - t^n} \quad (4.28)$$

After dropping the RHS which consists of terms which are independent of the unknown variable we get

$$q_j^{n+1} = \frac{q_j^n}{1 + \frac{\Delta t}{\tau_j}} \quad (4.29)$$

Equation 4.26 can also be written as round-off error ( $\epsilon$ ) as

$$\epsilon_j^{n+1} = \frac{\epsilon_j^n}{1 + \frac{\Delta t}{\tau_j}} \quad (4.30)$$

Re-arranging Eq. 4.28 and introducing the amplification factor we get,

$$|G| = \left| \frac{1}{1 + \frac{\Delta t}{\tau_j}} \right| \quad (4.31)$$

Since,  $\Delta t \geq 0$ , and  $\tau_j \geq 0$ , the implicit scheme of CRMP is unconditionally stable.

Therefore, the explicit finite-difference scheme of the CRM model is conditionally stable with a CFL condition  $\Delta t \leq \tau_j$ , while the implicit form is unconditionally stable.

### 4.3.3 Convergence

A finite-difference equation is convergent if the finite-difference solution of the differential equation approaches the true solution. Convergence is related to consistency and stability and according to Dahlquist's equivalence theorem for ordinary differential equations:

$$\text{consistency} + \text{stability} \Leftrightarrow \text{convergence} \quad (4.32)$$

Because of convergence assurance, the implicit finite scheme will be used henceforth.

## 4.4 Application of Finite-Difference Capacitance Resistance Model for Producer (FD-CRMP)

The CRMP model consists of the following two steps.

1. History matching of the FD-CRMP model with the observed data to estimate the unknown model parameters such as  $f_{ij}$  and  $\tau_j$ .
2. Predict the production rate using the history matched model parameters.

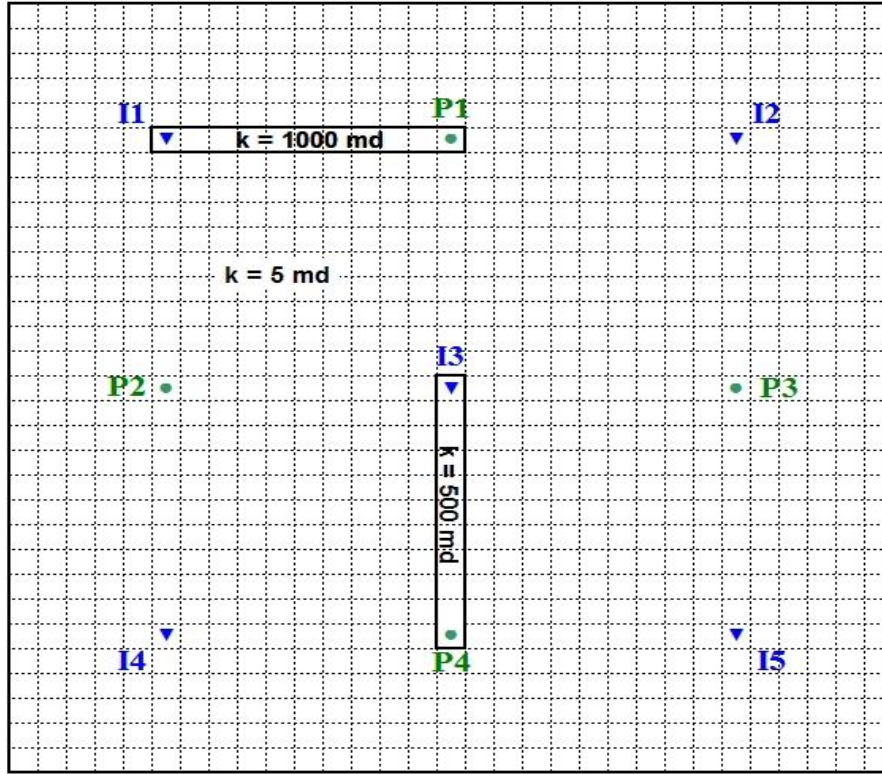
In this study, a local built-in optimization tool of MATLAB, `fmincon` is used to history match the production data and estimate the model parameters. `Fmincon` (Holanda, Gildin, and Jensen 2015; Pinto, Gildin, and Schiozer 2015) is a non-linear multivariable gradient-based method designed to work on problems where the objective and constraint functions are both continuous and have continuous first derivative.

## **4.5 Example Cases**

Three separate cases are presented to test the efficiency of the newly developed finite-difference scheme for the CRMP model. The first case uses field production data from the literature to test the accuracy of the model. The second and third cases use synthetic production data obtained by simulating a synthetic reservoir model in a commercial simulator. Efficiency is tested by comparing the results of the model with both the semi-analytical solution of the CRMP model, and the actual available data.

### **4.5.1 Case 1: Data available in the literature**

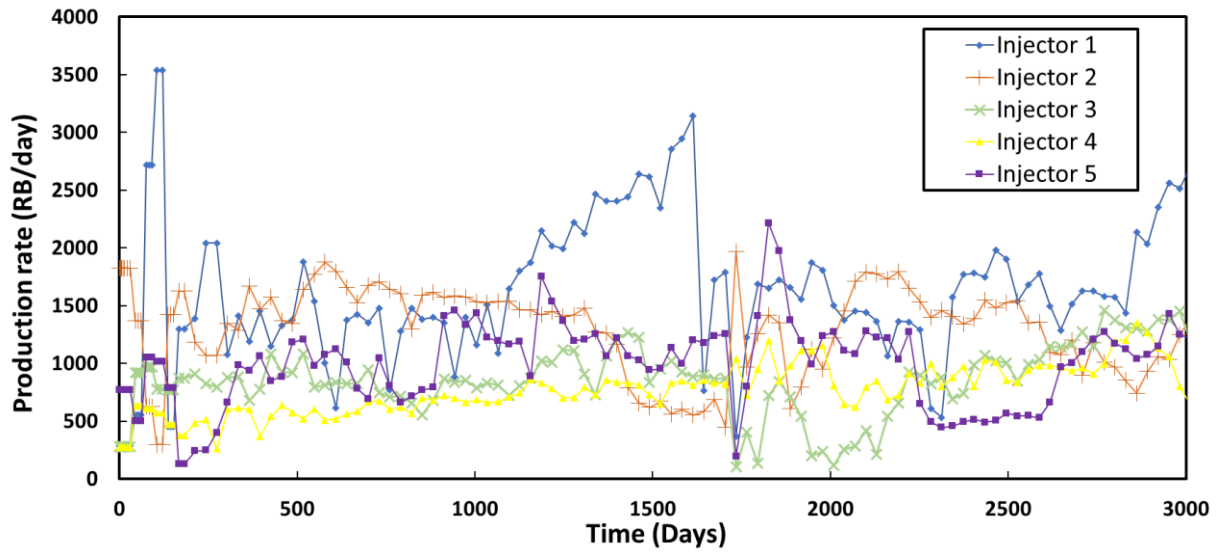
The first example uses the streak case presented by M. Sayarpour (2008). The reservoir consists of a total of five injectors and four producers (Figure 2). There is a high permeability streak between injector 1 - producer 1, and injector 3 - producer 4. The general reservoir properties and injection data are given in Table 1 and Figure 3, respectively. The bottom hole pressure is constant at 200 psi for each producer.



**Figure 2: Model representing the location of injectors/producers and the high permeability streaks for Case 1**

**Table 1: Model parameters and fluid properties to generate data for the Case 1**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 5                        |
| No. of Producers  | 4                        |
| $K_h$ (md)        | 5 md except two streaks  |
| $\phi$ (fraction) | 0.18                     |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $1 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 250                      |
| Grid size (ft)    | $80 \times 80 \times 12$ |
| Model dimension   | $31 \times 31 \times 5$  |



**Figure 3: Injection Rates data for Case 1**

The FD-CRMP and SA-CRMP models were applied to 50% of the observed production rates data of each producer and the matched model parameters were used for predicting production rates. The results were compared with the production rates calculated by the semi-analytical model (SA-CRMP) and the simulated data, presented in Figures 4-7. The matched parameters and mean square error between observed and estimated data are listed in Table 2.

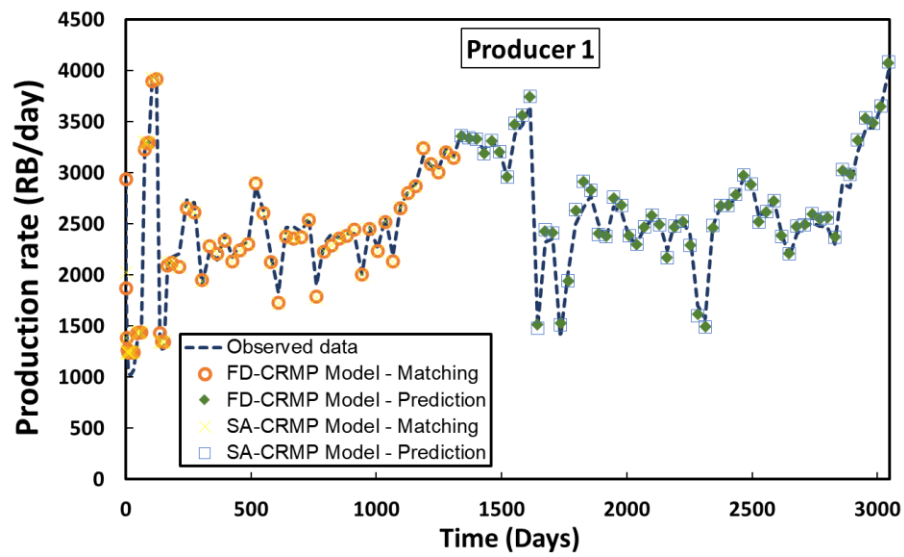


Figure 4: Producer 1 production rate comparison between SA-CRMP and FD-CRMP for Case 1

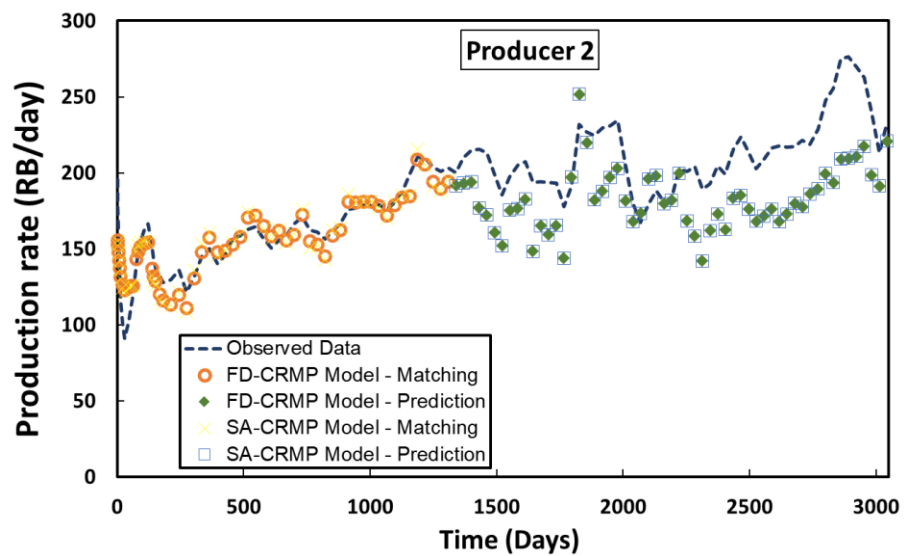
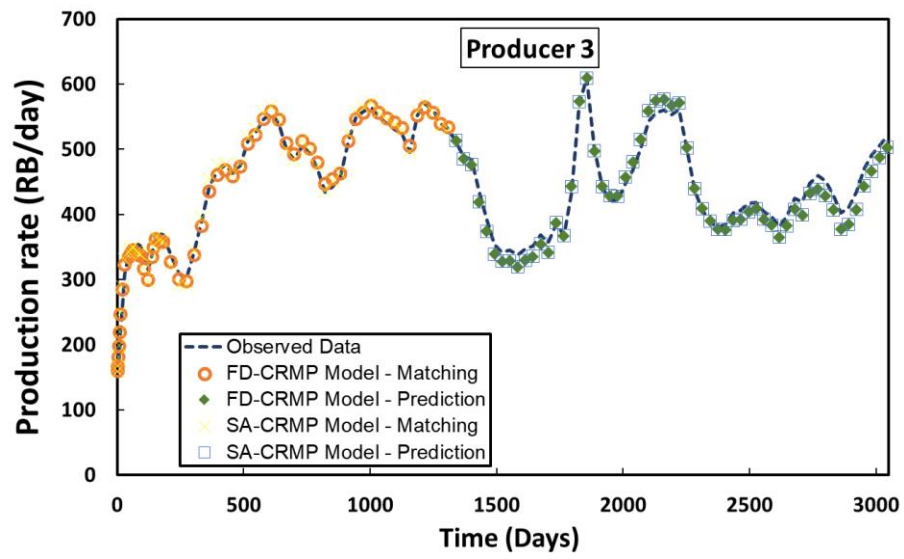
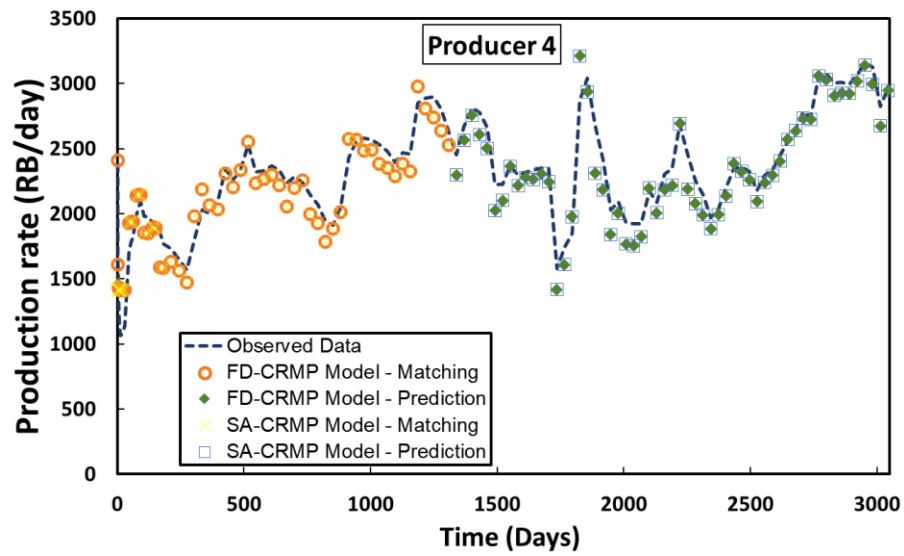


Figure 5: Producer 2 production rate comparison between SA-CRMP and FD-CRMP for Case 1



**Figure 6: Producer 3 production rate comparison between SA-CRMP and FD-CRMP for Case 1**



**Figure 7: Producer 4 production rate comparison between SA-CRMP and FD-CRMP for Case 1**

**Table 2: Values of CRMP parameters given by SA-CRMP (Sayarpour,2008) and FD-CRMP.**

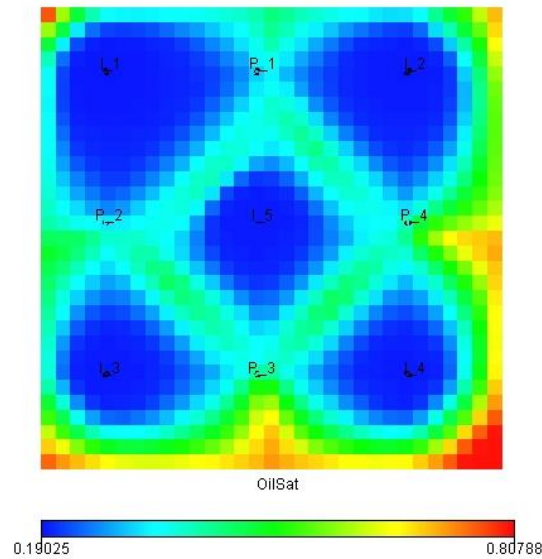
|                        | FD-CRMP model |       |       |        | SA-CRMP Model |       |       |        |
|------------------------|---------------|-------|-------|--------|---------------|-------|-------|--------|
|                        | P1            | P2    | P3    | P4     | P1            | P2    | P3    | P4     |
| $f_{1,j}$              | 0.96          | 0.02  | 0.00  | 0.02   | 0.96          | 0.02  | 0.00  | 0.03   |
| $f_{2,j}$              | 0.41          | 0.03  | 0.20  | 0.28   | 0.41          | 0.03  | 0.20  | 0.28   |
| $f_{3,j}$              | 0.09          | 0.00  | 0.00  | 0.91   | 0.09          | 0.00  | 0.00  | 0.91   |
| $f_{4,j}$              | 0.30          | 0.08  | 0.00  | 0.62   | 0.30          | 0.08  | 0.00  | 0.62   |
| $f_{5,j}$              | 0.15          | 0.04  | 0.19  | 0.63   | 0.16          | 0.04  | 0.19  | 0.62   |
| $\tau j, \text{ Days}$ | 0.56          | 11.50 | 44.55 | 0.23   | 0.97          | 16.17 | 49.33 | 0.60   |
| Error (Matching)       | 75.32         | 11.81 | 11.56 | 143.15 | 77.72         | 11.86 | 11.07 | 141.95 |
| Error (Prediction)     | 74.67         | 34.69 | 16.15 | 127.52 | 75.83         | 34.28 | 15.72 | 125.68 |

As can be seen from the results in case 1, the inter-well connectivity between injector-producer pair injector 1- producer 1 and Injector 3 – producer 4 is above 0.9 and the reservoir model shows a high permeability streak between these injector-producer pairs. This exhibits a strong relation of reservoir characteristic and fluid flow behavior with the inter-well connectivity  $f_{i,j}$ . The  $f_{i,j}$  values of other producer pairs corresponding to injector I1 and I3 are almost zero which shows that most of the injected water from I1 and I3 is translated toward P1 and P4 respectively and the contribution is minimal toward other producers. The  $f_{i,j}$  values of injector producer pair expect I1-P1 and I3-P4 are varying based on the distance from injector-producer pair and the injection rates variation. Furthermore, it can also be observed from Table 2 that both SA-CRMP and FD-CRMP model parameters and mean square errors are comparable and in good agreement with the

observed production rates. Since the results of both models are comparable, FD-CRMP can be used as an alternate approach for solving CRM.

#### 4.5.2 Case 2: Synthetic data using a commercial simulator.

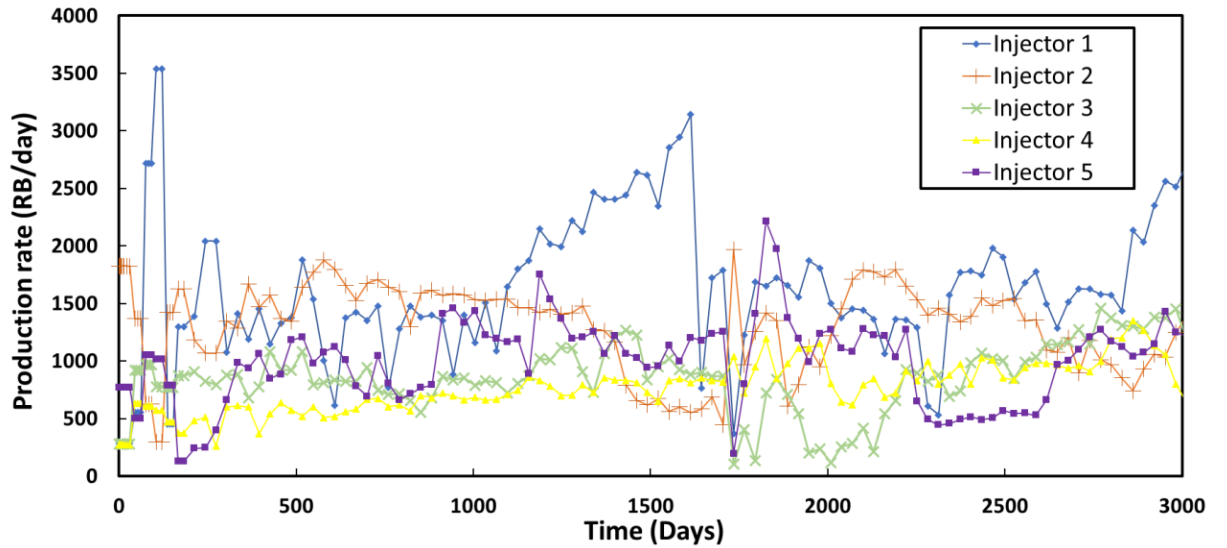
The second case uses data obtained by simulating a synthetic homogenous reservoir (Figure 8) using a commercial simulator. The general reservoir properties are presented in Table 3 and injection rates are shown in Figure 9.



**Figure 8: Saturation map for Case 2, showing distribution of injectors and producers at the end of the simulation run**

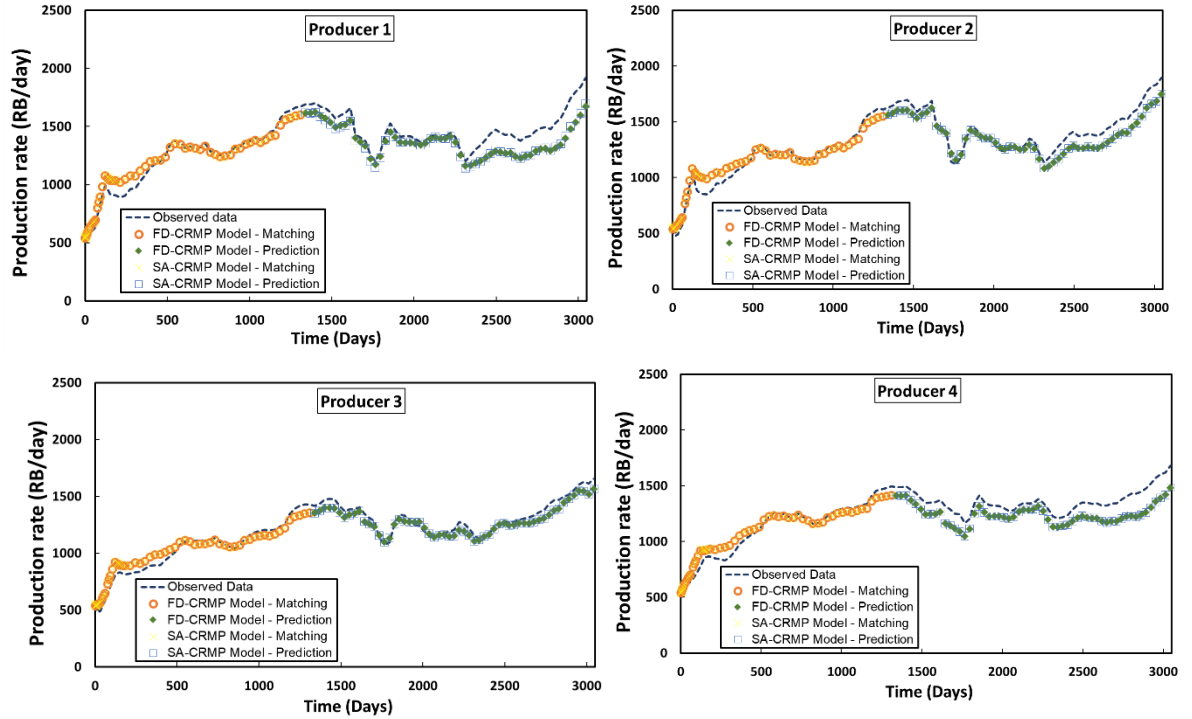
**Table 3: Model parameters and fluid properties to generate data for Case 2**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 5                        |
| No. of Producers  | 4                        |
| $K_h$ (md)        | 40                       |
| $K_v$ (md)        | 4                        |
| $\phi$ (fraction) | 0.18                     |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $1 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | $80 \times 80 \times 12$ |
| Model dimension   | $31 \times 31 \times 5$  |



**Figure 9: Injection Rates data for Case 2**

Again, the FD-CRMP and SA-CRMP models were applied to 50% of the observed data of each producer and the matched model parameters were used for predicting production rates. The results were compared with the production rates calculated by the semi-analytical model (SA-CRMP) and simulated data, shown in Figure 10. The matched parameters and mean square error between observed and estimated data are listed in Table 4.



**Figure 10: Comparison of total production rate of each producer with FD-CRMP and SA-CRMP for Case 2**

**Table 4: Values of CRMP parameters obtained by the FD-CRMP model and SA-CRMP**

|                       | FD-CRMP model |        |        |        | SA-CRMP Model |        |        |        |
|-----------------------|---------------|--------|--------|--------|---------------|--------|--------|--------|
|                       | P1            | P2     | P3     | P4     | P1            | P2     | P3     | P4     |
| $f_{1,j}$             | 0.32          | 0.31   | 0.18   | 0.19   | 0.31          | 0.31   | 0.18   | 0.19   |
| $f_{2,j}$             | 0.32          | 0.15   | 0.14   | 0.34   | 0.31          | 0.15   | 0.15   | 0.34   |
| $f_{3,j}$             | 0.05          | 0.19   | 0.22   | 0.09   | 0.05          | 0.18   | 0.20   | 0.07   |
| $f_{4,j}$             | 0.05          | 0.22   | 0.53   | 0.20   | 0.08          | 0.21   | 0.50   | 0.21   |
| $f_{5,j}$             | 0.33          | 0.28   | 0.15   | 0.24   | 0.32          | 0.28   | 0.16   | 0.24   |
| $\tau j$ , Days       | 100.00        | 100.00 | 100.00 | 100.00 | 100.00        | 100.00 | 100.00 | 100.00 |
| Error<br>(Matching)   | 65.87         | 67.88  | 61.24  | 66.05  | 68.96         | 70.31  | 63.98  | 68.92  |
| Error<br>(Prediction) | 136.55        | 87.09  | 42.16  | 134.94 | 134.01        | 90.26  | 50.41  | 135.24 |

It is observed from Figure 10 that the FD-CRMP model shows a good history match and prediction with SA-CRMP and the observed simulated production data which is evident from the root mean square errors as listed in Table 4. It is also highlighted that the values of inter-well connectivity factors,  $f$ , are higher between the nearest well pairs and the value is low if the distance between injectors are producers is large. In the case of FD-CRMP, for injector 1, the nearest producers are P1 and P2 and the values are 0.32 and 0.31, respectively which are comparatively higher than 0.18 and 0.19 for producers P3 and P4, respectively. Similarly for injector 2, the nearest producers are P1 and P4 and the values are 0.32 and 0.34, respectively. Other injectors also showed a similar pattern of flow which indicates that the major contribution of injected fluid is usually toward the nearest producer

in a homogenous reservoir provided that the bottom hole pressures for each producer is constant at each time step.

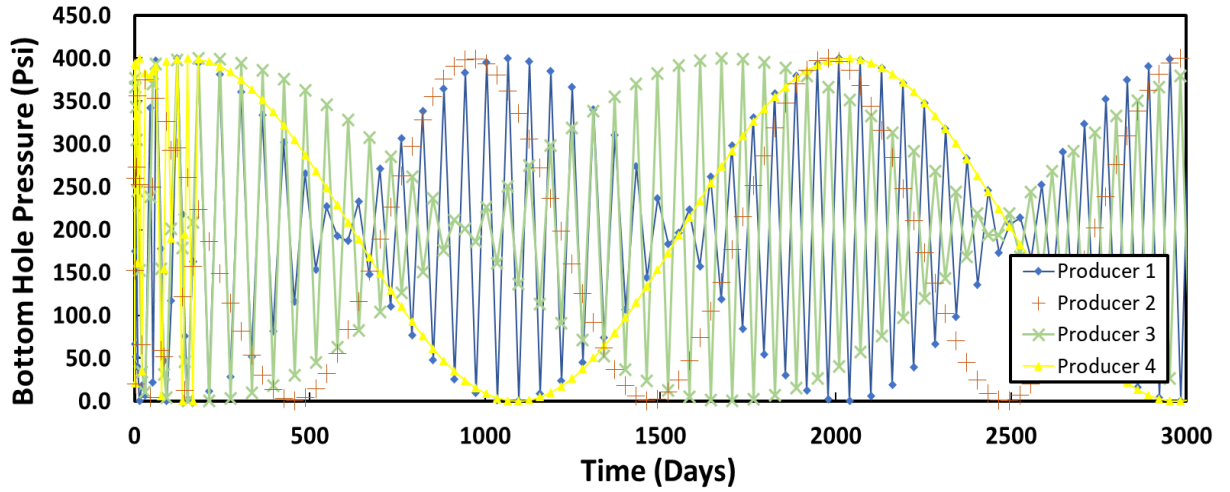
Since the model parameters and calculated mean square error of both models SA-CRMP and FD-CRMP are similar, there is a need to compute the run time of both models to evaluate both models. The computational cost of the FD-CRMP model is assessed by comparing the total simulation run time of the FD-CRMP model to that of the SA-CRM model and full-scale simulation as shown in Table 5. All simulations in this work were carried out on a system having an Intel(R) Core (TM) i7-10510U CPU @ 1.80GHz 2.30 GHz processor with 1TB Solid State Drive (SSD) and 16 GB RAM. The comparison of simulation run time presented in Table 5 indicates a substantial increase in the computational efficiency during forward simulation (prediction) as compared to SA-CRMP (semi-analytical method) and as expected CRM models provides faster solution as compared to the full-scale simulation model.

**Table 5: Comparison of computation cost of semi-analytical model (Sayarpour,2008) and FD-CRMP with full-scale simulation model**

| <b>Model</b>                           | <b>Runtime (Sec.)</b>                                                      |                | <b>Percentage decrease of computational time compared to Full Scale Simulation</b> |
|----------------------------------------|----------------------------------------------------------------------------|----------------|------------------------------------------------------------------------------------|
| <b>FD-CRMP Model</b>                   | <b>History Matching Period:</b> 0.95<br><b>Prediction Period:</b> 2.79E-03 | Total:<br>0.95 | 67.58%                                                                             |
| <b>SA-CRMP Model (Sayarpour, 2008)</b> | <b>History Matching Period:</b> 1.10<br><b>Prediction Period:</b> 4.36E-03 | Total:<br>1.11 | 62.12%                                                                             |
| <b>Full Scale Simulation</b>           | <b>Total: 2.93</b>                                                         |                |                                                                                    |

#### **4.5.3 Case 3: FD-CRMP applied to a synthetic reservoir model with varying BHP.**

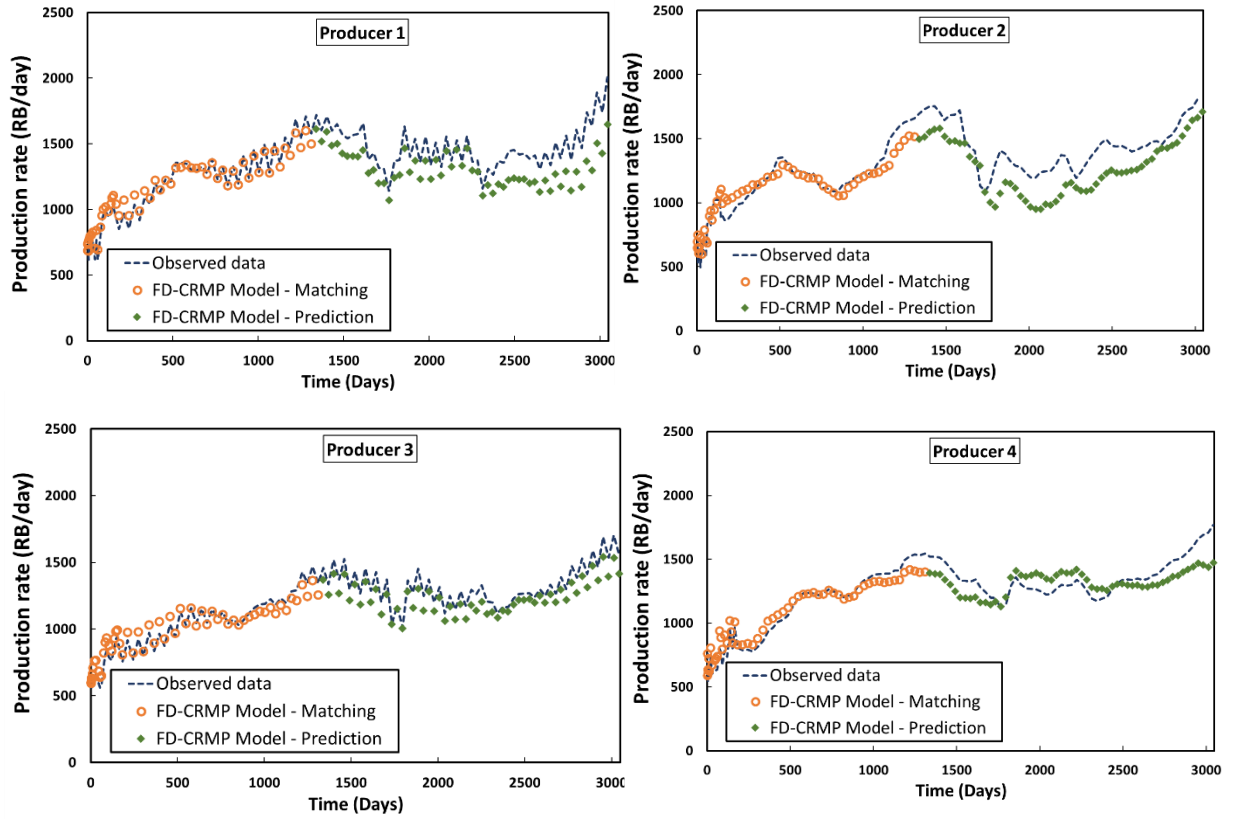
In this case, the synthetic model presented in case 2 was used as a base case. The bottom hole pressure at each producer was varied using a cosine function defined in (Awotunde 2014) with maximum BHP of 400 psi as shown in Figure 11. The model was run for approximately 3000 days with a timestep size of 30 days. The first 50% of the data was used for history matching and the remaining 50% of data was used for prediction. The FD-CRMP requires a prior knowledge of the productivity index if BHP at the producer is not constant. In this regard, there are three possible approaches to adjust the productivity index in the CRMP for prediction of flow rates which are discussed in the next sections.



**Figure 11: Bottom hole pressures for each producer used in Case 3**

#### 4.5.3.1 Method 1:

If the productivity index is known at discrete time intervals, a mean value of productivity index can be used in the FD-CRMP for history matching and prediction. The comparison of FD-CRMP results with the full-scale model is shown in Figure 12 and the matched parameters are shown in Table 6. The FD-CRMP shows a good history match and prediction with average mean square errors of 67.14 and 146.36, respectively.



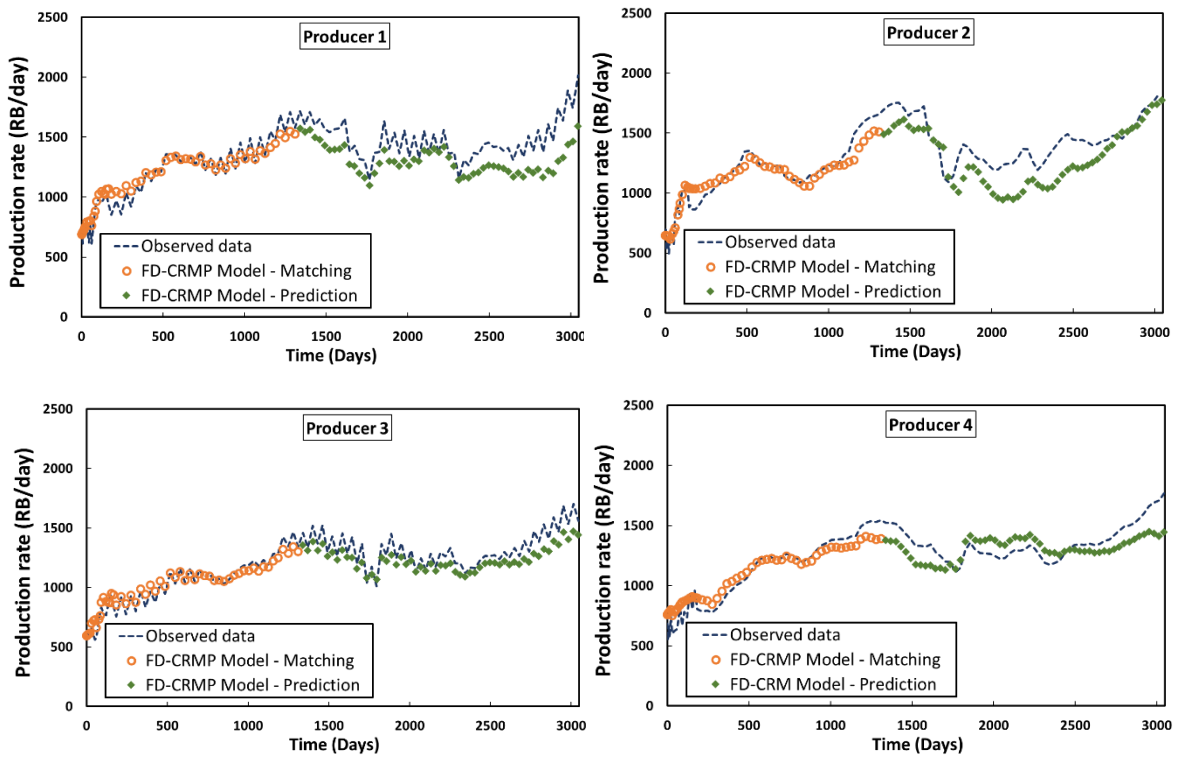
**Figure 12: Comparison of total production rate of each producer with FD-CRMP  
for method 1**

**Table 6: Values of CRMP parameters obtained by FD-CRMP model developed in this study.**

|                    | <b>P1</b> | <b>P2</b> | <b>P3</b> | <b>P4</b> |
|--------------------|-----------|-----------|-----------|-----------|
| $f_{1,j}$          | 0.29      | 0.19      | 0.15      | 0.10      |
| $f_{2,j}$          | 0.37      | 0.14      | 0.19      | 0.31      |
| $f_{3,j}$          | 0.01      | 0.63      | 0.22      | 0.00      |
| $f_{4,j}$          | 0.00      | 0.00      | 0.44      | 0.56      |
| $f_{5,j}$          | 0.34      | 0.22      | 0.16      | 0.28      |
| $\tau j$ , Days    | 100       | 100       | 100       | 100       |
| Error (Matching)   | 58.43     | 76.10     | 62.66     | 71.36     |
| Error (Prediction) | 187.88    | 183.94    | 94.66     | 118.94    |

#### 4.5.3.2 Method 2

Alternatively, the last known value of productivity index can be used in the FD-CRMP for history matching and prediction. The comparison of the FD-CRMP results with the full-scale model is shown in Figure 13 and the matched parameters are shown in Table 7. The FD-CRMP model shows a good history match and prediction with average mean square errors of 75.05 and 154.59, respectively.



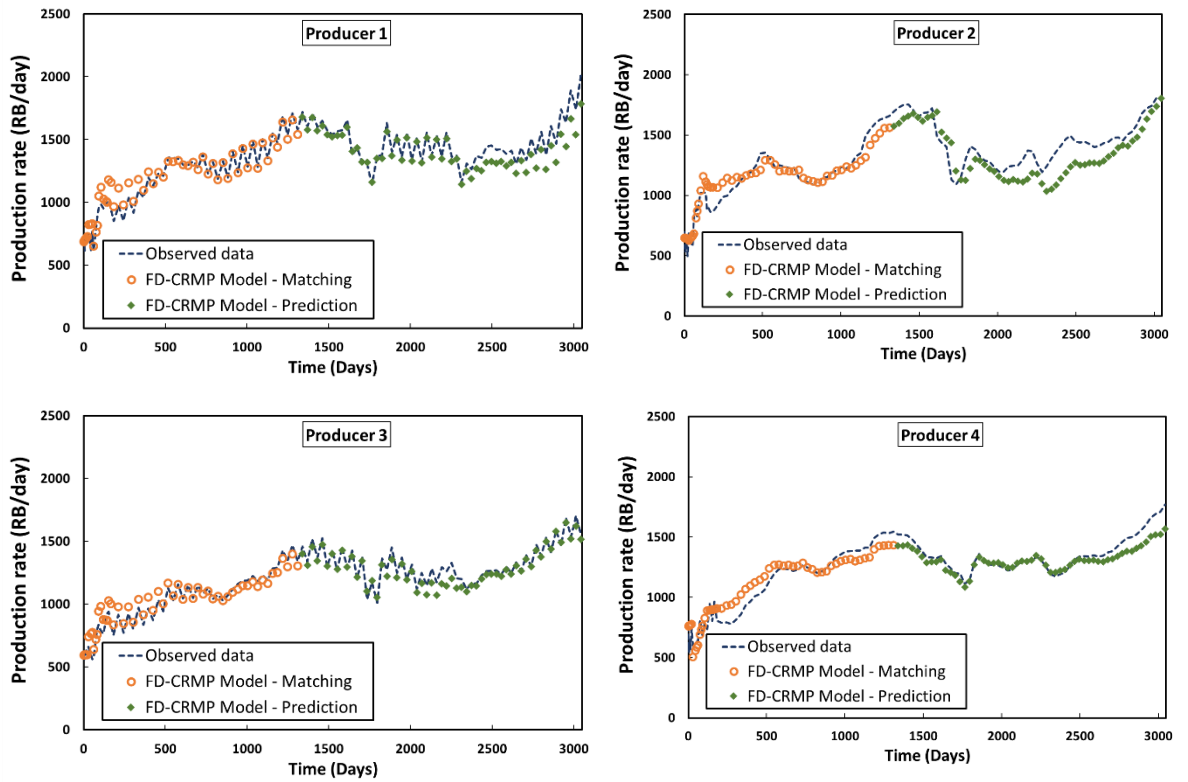
**Figure 13: Comparison of total production rate of each producer with FD-CRMP for Method 2**

**Table 7: Values of CRMP parameters obtained by FD-CRMP model developed in this study.**

|                    | <b>P1</b> | <b>P2</b> | <b>P3</b> | <b>P4</b> |
|--------------------|-----------|-----------|-----------|-----------|
| $f_{1,j}$          | 0.31      | 0.19      | 0.18      | 0.07      |
| $f_{2,j}$          | 0.40      | 0.07      | 0.21      | 0.32      |
| $f_{3,j}$          | 0.00      | 0.70      | 0.15      | 0.00      |
| $f_{4,j}$          | 0.00      | 0.00      | 0.40      | 0.60      |
| $f_{5,j}$          | 0.28      | 0.27      | 0.18      | 0.27      |
| $\tau j, Days$     | 100.00    | 100.00    | 100.00    | 100.00    |
| Error (Matching)   | 76.16     | 77.43     | 56.88     | 89.72     |
| Error (Prediction) | 200.66    | 178.01    | 108.57    | 131.12    |

#### 4.5.3.3 Method 3:

In this case, the productivity index is unknown, and it is determined during history matching along with the other matching parameters such as  $f_{ij}$  and  $\tau_j$ . The comparison of FD-CRMP results with the full-scale model is shown in Figure 14. The matched parameters are listed in Table 8. The FD-CRMP model shows a good history match and prediction with average mean square errors of 49.56 and 108.52, respectively. By comparing the root mean square error of all methods, it is observed that optimizing the productivity index along with other unknown FD-CRMP parameters provides better history matching and prediction performance of observed data.



**Figure 14: Comparison of total production rate of each producer with FD-CRMP for method 3**

**Table 8: Values of CRMP parameters obtained by FD-CRMP model developed in this study.**

|                    | <b>P1</b> | <b>P2</b> | <b>P3</b> | <b>P4</b> |
|--------------------|-----------|-----------|-----------|-----------|
| $f_{1,j}$          | 0.32      | 0.35      | 0.14      | 0.18      |
| $f_{2,j}$          | 0.29      | 0.11      | 0.11      | 0.27      |
| $f_{3,j}$          | 0.02      | 0.40      | 0.31      | 0.12      |
| $f_{4,j}$          | 0.18      | 0.00      | 0.49      | 0.33      |
| $f_{5,j}$          | 0.31      | 0.24      | 0.19      | 0.26      |
| $\tau j$ , Days    | 100.00    | 100.00    | 100.00    | 100.00    |
| Productivity Index | 0.50      | 0.11      | 0.42      | 0.84      |
| Error (Matching)   | 38.28     | 61.62     | 41.60     | 56.76     |
| Error (Prediction) | 137.27    | 118.16    | 51.14     | 127.52    |

## 4.6 Conclusion

In this chapter, a novel technique is presented to solve the capacitance resistance model using finite-difference method. Finite-difference explicit and implicit methods were examined for consistency, stability and convergence. It was shown that implicit method was unconditionally stable. The finite-difference approach was observed to be computationally faster than the available semi-analytical solution. FDM-CRM is a faster technique and inexpensive to use. It can be successfully applied to field case studies for history-matching and optimization of reservoir performance which exhibits its unique capability as a powerful engineering tool. The studied cases demonstrated that the FD-CRM is capable of successfully mimic the solution from synthetic numerical simulation model and a good quality match is observed. FD-CRM requires only injection and production data to effectively evaluate the inter-well connectivity, the reservoir

characteristic and future rates predictions. Three different methods to obtain the productivity index for prediction in cases where the bottom hole pressures are varying was presented. The three different methods are: 1) calculating a mean value of the known productivity index obtained from the historical data, 2) using the last known value of productivity index obtained from the historical data, and 3) setting the productivity index as an unknown and estimating it during the history matching process. Results indicated that optimizing the productivity index along with other unknown FD-CRMP parameters provides better history matching and prediction performance of observed data. The newly developed finite-difference CRMP model will serve as a fast and reliable tool for water flooding optimization.

## **CHAPTER 5**

# **A FULLY-IMPLICIT FINITE-DIFFERENCE CAPACITANCE RESISTANCE MODEL FOR MODELING TWO-PHASE FLOW**

### **5.1 Introduction**

Two of the most widely used physics-based models for rapid estimated of production rates are the Capacitive Resistance Models (CRM) (A. A. Yousef et al. 2006) and the Interwell Simulation Model (INSIM) (Guo, Reynolds, and Zhao 2018). A related interwell model for advection dispersion equations (ISADE) was recently developed by (Jamal et al. 2023). CRM has attracted a lot of attention as a proxy reservoir management tool (Mamghaderi, Aminshahidy, and Bazargan 2021; Salazar et al. 2012; M. Sayarpour 2008; M. Sayarpour et al. 2009; Morteza Sayarpour, Kabir, and Lake 2008, 2009; Weber et al. 2009; K. Zhang et al. 2022; T. Zhang, IZGEC, and Sayarpour 2018).

Different approaches have been implemented to solve the differential equation of CRM for the total production rates. Santos Oliveira, Horowitz and Rojas Tueros, 2021, extended the work of producer base representation of CRM (CRMP) and improved prediction of individual oil and water rates. They used a combination of Koval and Gentil fractional flow models, called Kogen model to estimate oil and water rate separately.

Several researchers assume a slightly-compressible reservoir system while ignoring changes in the fluid saturations (M. Sayarpour 2008; M. Sayarpour et al. 2009; Morteza Sayarpour, Kabir, and Lake 2008, 2009; Weber et al. 2009; T. Zhang, IZGEC, and Sayarpour 2018). Cao, Luo and Lake, 2014, highlighted that the existing CRM models are

based exclusively on single-phase fluid flow equations. These single-phase models provide good approximation for mature water flooding where change in saturation is minimal. However, they fail to capture the physics when the system has high dependence on fluid saturations such as in the case of immature water flood. Cao, Luo and Lake, 2014, presented first the two-phase CRM model to account for the individual oil and water rates. The fluid mobility and total compressibility was coupled with the time constant to account for the saturation changes. Thus, instead of using a constant time constant, the time constant varies with the average saturation of the control volume and the effect becomes significant at low water cut. The authors used an implicit-rate explicit-saturation solution approach. In this regard, the oil phase CRM equation was added to the water-phase CRM equation to obtain a single equation for the total phase which was solved for the total well rate.

As highlighted from the above literature, most of the current CRM models are based on the direct estimation of total production rate from the solution of continuity equation eliminating the effect of saturation on individual oil and water phase. Thus, total production rates are calculated by solving CRM model using only injection rate data, provided that the model parameters such as inter well connectivity and time constant are obtained beforehand by history matching a set of injection and production data. The individual oil and water rates are then calculated indirectly by using available fraction flow models such as Koval method and Gentil method. Consequently, the current CRM models match the total production rates however it lacks the correct estimation of individual rates of each phase.

In this study, a fully-implicit finite-difference solution to the two-phase CRMIP (TP-CRMIP) model is developed to quantify the effect of injection rate on oil and water rates

at the production wells. Oil and water rates are calculated using simultaneous solution of continuity equations for the two phases at each time-step. The approach presented improves the accuracy of the estimated oil and water production rates.

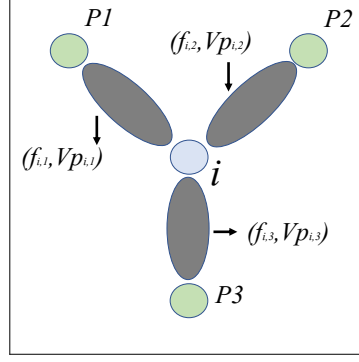
## 5.2 Mathematical Model for Two-Phase Flow

This section describes the governing fluid flow differential equations for two-phase fluid in a reservoir and the finite-difference formulations to solve those equations.

### 5.2.1 Two-Phase Capacitance Resistance Model for Injector-Producer pair (TP-CRMIP)

The differential equation for capacitance resistance model is derived using material balance of individual phase in separate control volume ( $V_{Pi,j}$ ) for each injector ( $i$ )-producer ( $j$ ) pair. The average reservoir pressure from the resulting differential equation is then substituted by the drawdown equation. The individual oil rate and water rates are then calculated by solving two equations for two unknowns which are water saturation and oil rate.

Consider a cluster of three producers  $P1$ ,  $P2$ , and  $P3$  and one injector  $i$  as shown in Figure 15. The contribution of injection rate of injector  $i$  toward production rate of each of the producers is denoted by  $f_{i,j}$ , where  $i$  represents the index of injectors and  $j$  represents the index of producer.



**Figure 15: Injector-Producer pair showing control volume between injector  $i$  and producers 1, 2 and 3.**

The water-phase equation is given by,

$$V_{p_{i,j}} \left( \frac{d\bar{S}_{w_{i,j}}}{dt} + \bar{S}_{w_{i,j}} c_{fw} \frac{dp_{wf_j}}{dt} + \frac{1}{J_{o_{i,j}}} \bar{S}_{w_{i,j}} c_{fw} \frac{dq_{o_{i,j}}(t)}{dt} \right) = f_{i,j} I_i(t) - q_{o_{i,j}}(t) \frac{J_{w_{i,j}}}{J_{o_{i,j}}} \quad (5.1)$$

and the oil phase equation is given by,

$$V_{p_{i,j}} \left( -\frac{d\bar{S}_{w_{i,j}}}{dt} + (1 - \bar{S}_{w_{i,j}}) c_{fo} \frac{dp_{wf_j}}{dt} + \frac{1}{J_{o_{i,j}}} (1 - \bar{S}_{w_{i,j}}) c_{fo} \frac{dq_{o_{i,j}}(t)}{dt} \right) = -q_{o_{i,j}}(t), \quad (5.2)$$

where  $c_{fw} = c_f + c_w$ ,  $c_{fo} = c_f + c_o$ ,  $c_f$  is the rock compressibility,  $c_w$  is the water compressibility and  $c_o$  is the oil compressibility. Also,  $V_{p_{i,j}}$  represents the control volume between injector  $i$  and producer  $j$ ,  $\bar{S}_{w_{i,j}}$  is the average water saturation in the control volume between injector  $i$  and producer  $j$ ,  $q_{o_{i,j}}(t)$  is the oil production rate at producer  $j$  contributed by injector  $i$ ,  $p_{wf_j}$  and  $J_{o_j}$  are the bottomhole pressure and the productivity index, respectively, of producer  $j$ . A detailed derivation of Eqs. 5.1 and 5.2 is presented in Appendix B. The productivity index  $J_o$  is defined as.

$$J_o = \frac{2\pi kh}{\ln\left(\frac{r_e}{r_w}\right)} \frac{k_{ro}}{\mu_o}, \quad (5.3)$$

where  $k$  is the absolute permeability of the formation,  $h$  is the formation thickness,  $r_e$  is the reservoir radius,  $r_w$  is the wellbore radius,  $k_{ro}$  is the oil relative permeability and  $\mu_o$  is the oil phase viscosity.

In Eq. 5.3, the term  $\frac{2\pi kh}{\ln\left(\frac{r_e}{r_w}\right)}$  can be considered as constant and denoted by  $J_G$ . Thus, we

may write,

$$J_G = \frac{2\pi kh}{\ln\left(\frac{r_e}{r_w}\right)} \quad (5.4)$$

In the TP-CRMIP,  $V_{p,i,j}$ ,  $f_{i,j}$  and  $J_G$  are considered as unknown model parameters. These parameters are estimated using constrained nonlinear regression.

### 5.2.2 Finite-difference solution of TP-CRMIP model

In this study, a fully-implicit finite-difference method is used to solve the differential equation obtained from the TP-CRMIP model. Finite-difference method (FDM) is a classical numerical technique of solving ordinary or partial differential equations which are difficult or impossible to solve analytically. FDM replaces the ordinary or the partial differential equation by a system of discrete linear equations.

The finite-difference approximation of water-phase equation is given by,

$$V_{p_{i,j}} \left( \frac{\bar{S}_{w_{i,j}}^{n+1} - \bar{S}_{w_{i,j}}^n}{\Delta t} + c_{fw} \bar{S}_{w_{i,j}}^{n+1} \frac{p_{wf_j}^{n+1} - p_{wf_j}^n}{\Delta t} + \frac{1}{J_{o_{i,j}}^{n+1}} c_{fw} \bar{S}_{w_{i,j}}^{n+1} \frac{q_{o_{i,j}}^{n+1} - q_{o_{i,j}}^n}{\Delta t} \right) = f_{i,j} I_i^{n+1} - q_{o_{i,j}}^{n+1} \frac{J_{w_{i,j}}^{n+1}}{J_{o_{i,j}}^{n+1}} \quad (5.5)$$

Similarly, for the oil phase,

$$V_{p_{i,j}} \left( -\frac{\bar{S}_{w_{i,j}}^{n+1} - \bar{S}_{w_{i,j}}^n}{\Delta t} + c_{fo} (1 - \bar{S}_{w_{i,j}}^{n+1}) \frac{p_{wf_j}^{n+1} - p_{wf_j}^n}{\Delta t} + \frac{1}{J_{o_{i,j}}^{n+1}} c_{fo} \bar{S}_{w_{i,j}}^{n+1} \frac{q_{o_{i,j}}^{n+1} - q_{o_{i,j}}^n}{\Delta t} \right) = -q_{o_{i,j}}^{n+1} \quad (5.6)$$

Since Eqs. 5.5 and 5.6 are nonlinear, the discretized equations have to be written as residual functions and the Newton-Raphson method is used for solving the resulting nonlinear simultaneous equations. A Jacobian matrix is formed from the derivatives of the residual functions with respect to water saturation and oil rate. A detailed derivation of residuals and derivatives are presented in Appendix C. The residual vector  $\vec{R}^{\gamma+1}$  is given by the following equation.

$$\vec{R}^{\gamma+1} = \begin{bmatrix} R_{w_{i,j}}^{\gamma+1} \\ R_{o_{i,j}}^{\gamma+1} \end{bmatrix} \quad (5.7)$$

where  $R_{w_{i,j}}^{\gamma+1}$  and  $R_{o_{i,j}}^{\gamma+1}$  are the residual functions of the water and oil phases, respectively. The

Jacobian matrix  $\mathfrak{J}^{\gamma+1}$  is given by,

$$\mathfrak{J}^{\gamma+1} = \begin{bmatrix} \frac{\partial R_{w_{i,j}}^{\gamma+1}}{\partial q_{o_{i,j}}^{\gamma+1}} & \frac{\partial R_{w_{i,j}}^{\gamma+1}}{\partial \bar{S}_{w_{i,j}}^{\gamma+1}} \\ \frac{\partial R_{o_{i,j}}^{\gamma+1}}{\partial q_{o_{i,j}}^{\gamma+1}} & \frac{\partial R_{o_{i,j}}^{\gamma+1}}{\partial \bar{S}_{w_{i,j}}^{\gamma+1}} \end{bmatrix} \quad (5.8)$$

Once the Jacobian matrix and the residuals are formed, the linear system,

$$\mathfrak{J}^{\gamma+1} \cdot \delta \vec{\alpha}^{\gamma+1} = -\vec{R}^{\gamma+1} \quad (5.9)$$

is solved to obtain  $\delta \vec{\alpha}$  and the vector  $\vec{\alpha}$  is updated by,

$$\vec{\alpha}^{\gamma+1} = \vec{\alpha}^{\gamma} + \delta \vec{\alpha}^{\gamma+1}. \quad (5.10)$$

In Eq. 5.10, the vector  $\vec{\alpha}^{\gamma+1}$  contains the unknown variables and is given by,

$$\vec{\alpha}^{\gamma+1} = \begin{bmatrix} q_{o_{i,j}}^{\gamma+1} \\ \bar{S}_{w_{i,j}}^{\gamma+1} \end{bmatrix}. \quad (5.11)$$

At each time-step, the unknown variables  $q_{o_{i,j}}^{n+1}$  and  $\bar{S}_{w_{i,j}}^{n+1}$  are obtained at the last iteration of the Newton-Raphson method. The oil production rate  $q_{o_j}^{n+1}$  at producer  $j$  at each time-step  $n+1$  can be calculated using,

$$q_{o_j}^{n+1} = \sum_{j=1}^{N_j} q_{o_{i,j}}^{n+1} \quad (5.12)$$

and the average water saturation at the producer  $j$  at each time-index  $n+1$  is calculated using,

$$\bar{S}_{w_j}^{n+1} = \sum_{j=1}^{N_j} (\bar{S}_{w_{i,j}}^{n+1} V_{p_{i,j}}) / \sum_{j=1}^{N_j} V_{p_{i,j}} \quad (5.13)$$

Once, oil production rate  $q_{o_j}^{n+1}$  is known, the water production rate is evaluated at each time-step using,

$$q_{w_j}^{n+1} = q_{o_j}^{n+1} \frac{J_w^{n+1}}{J_o^{n+1}} \quad (5.14)$$

The model parameters  $V_{p_{i,j}}$ ,  $f_{i,j}$  and  $J_{G_j}$  in Eqs. 5.5 and 5.6 are determined by history matching the observed and simulated fluid rates. The objective function minimized is defined as the sum of individual root mean square errors (RMSE) of observed and simulated phase rates (Eq. 5.15).

$$\min \left( \sum_{j=1}^{N_p} \sqrt{\frac{(\bar{q}_{o,obs_j} - \bar{q}_{o,cal_j})^T (\bar{q}_{o,obs_j} - \bar{q}_{o,cal_j})}{n_{dpts}}} + \sqrt{\frac{(\bar{q}_{w,obs_j} - \bar{q}_{w,cal_j})^T (\bar{q}_{w,obs_j} - \bar{q}_{w,cal_j})}{n_{dpts}}} \right) \quad (5.15)$$

where  $\bar{q}_{o,obs_j}$  is the vector of observed oil production rates at producer  $j$ ,  $\bar{q}_{o,cal_j}$  is the vector of calculated oil rates (from TP-CRMIP) at producer  $j$ ,  $\bar{q}_{w,obs_j}$  is the vector of observed water production rate at producer  $j$  and  $\bar{q}_{w,cal_j}$  is the vector of calculated water production rate (from TP-CRMIP) at producer  $j$ .

The objective function is constrained by,

$$f_{i,j} \geq 0 \quad (5.16)$$

and

$$\sum_{j=1}^{N_j} f_{i,j} \leq 1 \quad \text{for each injector } i \quad (5.17)$$

The TP-CRMIP model consists of the following two steps:

1. History matching to estimate the unknown model parameters  $V_{p,i,j}$ ,  $f_{i,j}$  and  $J_{G_j}$ .
2. Prediction of flow rates for each phase using the parameters estimated in the first step.

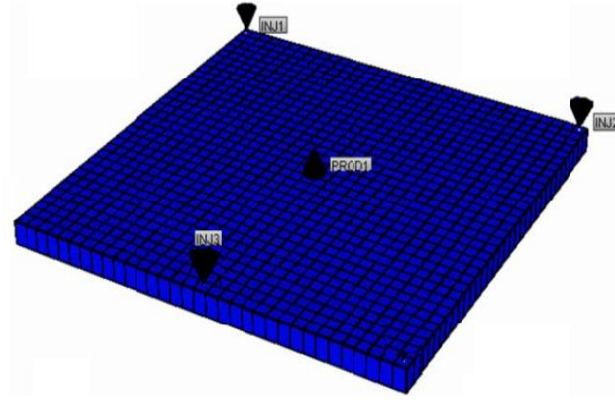
In this study, a built-in optimization tool in MATLAB, *fmincon*, is used to history match the production data and estimate the model parameters. *fmincon* is a nonlinear multivariable gradient-based optimization tool designed for problems related to continuous objective and constraint functions. The MATLAB function *fmincon* is selected because of constraints in our model and because this function *fmincon* is computationally more efficient than global optimization routines (Y. Chen, He, and Liu 2021)

### 5.3 Example Cases

Three separate cases are presented to test the efficiency and accuracy of the TP-CRMIP model. In each of the cases the model is initially simulated using a commercial simulator to generate the observed data. The first case is a synthetic homogenous reservoir consisting of three injectors and one producer placed at random locations in the reservoir. The accuracy of the TP-CRMIP model was tested by comparing the results with the observed data and the results obtained when using the two-phase model developed by Fei Cao (Cao, Luo, and Lake 2014). In all cases, 50% of the simulated data was used for history matching and the remaining 50% of the data was predicted using TP-CRMIP. In the second case, the TP-CRMIP model was assessed for its prediction accuracy. Four injectors and one producer were placed in a synthetic heterogeneous reservoir to generate the synthetic production data. The third case uses a more realistic reservoir model available in the literature.

#### **5.3.1 Case 1. TP-CRMIP model applied on a homogenous reservoir model with three injectors and one producer (comparison with the Fei Cao model)**

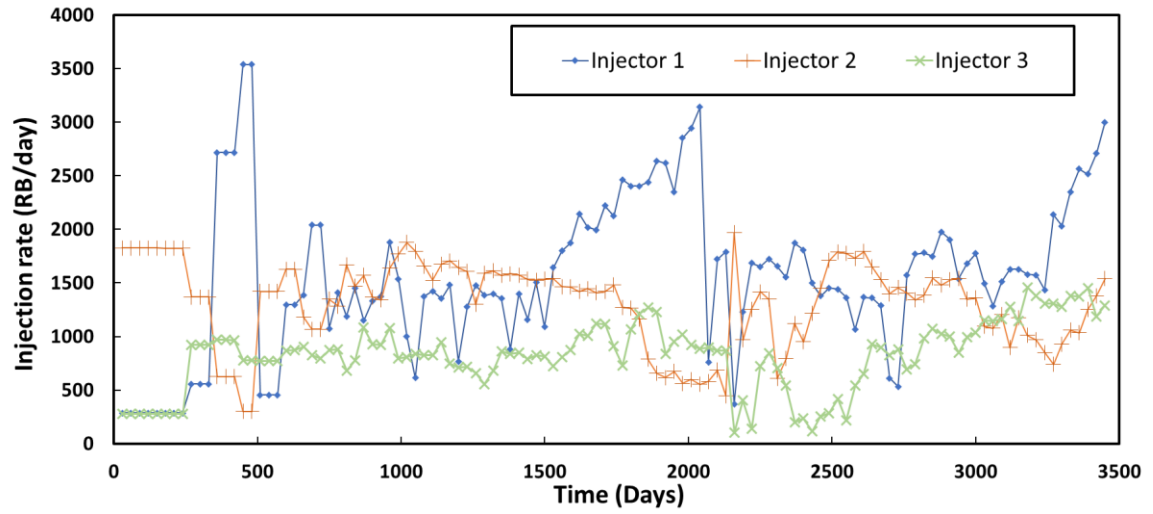
A synthetic model (Figure 16) is simulated on a commercial simulator to test the accuracy of TP-CRMIP. One producer is placed in the middle of the reservoir surrounded by three injectors. The reservoir dimensions, properties, and injection rates data are shown in Table 9 and Figure 17.



**Figure 16:Reservoir model showing the location of single producer at the middle and Injectors at the corners for the Case 1.**

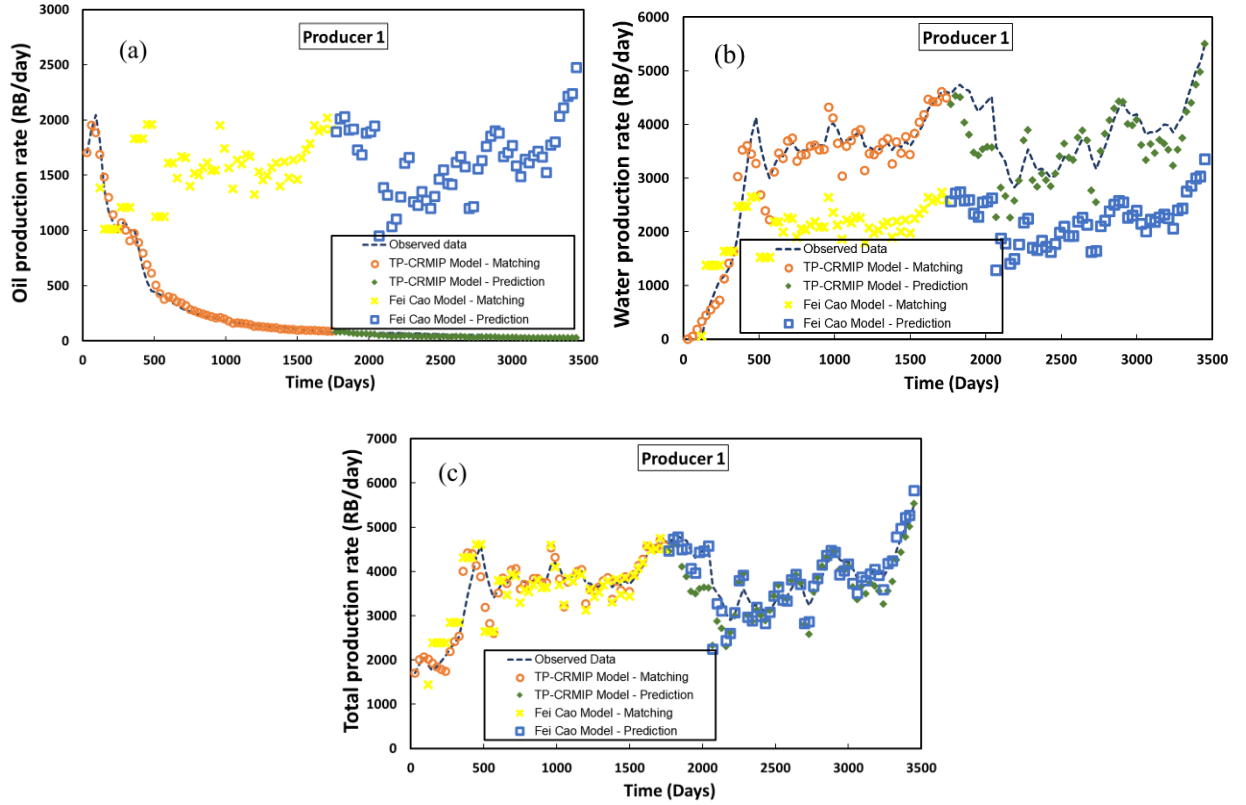
**Table 9: Model parameters and fluid properties to generate data for the Case 1.**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 3                        |
| No. of Producers  | 1                        |
| $K_h$ (md)        | 200                      |
| $K_v$ (md)        | 20                       |
| $\phi$ (fraction) | 0.2                      |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $3 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | $80 \times 80 \times 65$ |
| Model dimension   | $33 \times 33 \times 1$  |



**Figure 17: Injection rates data for the Case 1.**

The TP-CRMIP model was applied on the first 50% of the observed data to compute the model parameters and the remaining 50% of the data is predicted using the matched model. The results were then compared with the simulated data and the two-phase CRM model developed by Fei Cao (Cao, Luo, and Lake 2014) as shown in Figure 18.



**Figure 18: Comparison of oil, water, and total production rate with TP-CRMIP model and two-phase CRM model developed by Fei Cao.**

It is evident from Fig. 18c that both Fei Cao and TP-CRMIP model produces a good match for the total production rate. However, the Fei Cao model fails to produce satisfactory matches for the phase rates (Figures 18a & 18b). The TP-CRMIP model, however, produces a very good match for the individual phase rates (oil and water). This is further evident from the values of root mean square error (RMSE) obtained for each of the phase rates as listed in Table 10. The root mean square error is 43.53 for TP-CRMIP and 62.25 for the two-phase Fei Cao model during history march and 83.18 and 74.67 during the prediction using TP-CRMIP and Fei Cao model respectively. The accuracy of

the newly developed TP-CRMIP model is significantly higher in predicting the phase rates than the model available in the literature.

**Table 10: Mismatch values for producer 1 (Example 1).**

| Error Values |            | RMSE using TP-CRMIP |       |            | RMSE using Fei Cao |        |            |
|--------------|------------|---------------------|-------|------------|--------------------|--------|------------|
|              |            | Oil                 | Water | Total Rate | Oil                | Water  | Total Rate |
| Producer 1   | History    | 7.28                | 44.64 | 43.53      | 182.29             | 182.47 | 62.25      |
|              | Prediction | 7.68                | 75.49 | 83.18      | 278.46             | 276.95 | 74.67      |

Table 11 lists the values of the parameters estimated during the history matching process. The first and the second column list the parameters obtained when the TP-CRMIP model is used. Columns 3 and 4 list the parameters and their corresponding values when the two-phase model developed by Fei Cao is used.

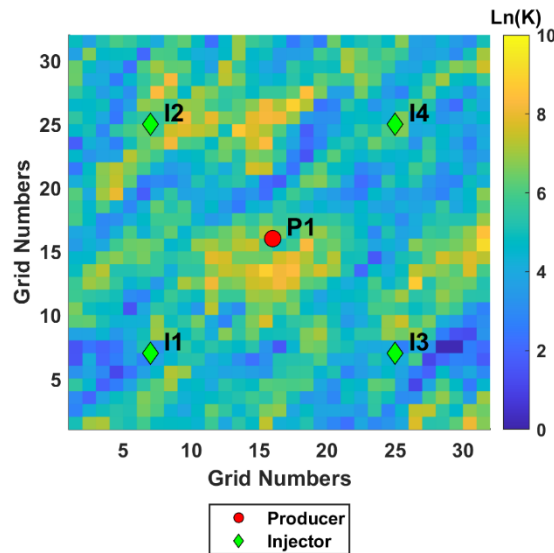
**Table 11: Estimated values of the unknown parameters (Example 1).**

| TP-CRMIP                         | Producer           | Fei Cao               | Producer           |
|----------------------------------|--------------------|-----------------------|--------------------|
| $f_{1,j}$ , injector 1           | 1.0                | $f_{1,j}$             | 1.0                |
| $f_{2,j}$ , injector 2           | 1.0                | $f_{2,j}$             | 1.0                |
| $f_{3,j}$ , injector 3           | 1.0                | $f_{3,j}$             | 1.0                |
| $V_{p_{1,j}}$ , bbl., injector 1 | $7.54 \times 10^5$ | $\tau'_j$             | 144.7              |
| $V_{p_{2,j}}$ , bbl., injector 2 | $7.40 \times 10^5$ | $V_P$ , bbl.          | $8.74 \times 10^5$ |
| $V_{p_{3,j}}$ , bbl., injector 3 | $3.68 \times 10^5$ | $\bar{S}_{o,initial}$ | 0.4                |
| $J_G$ , md-ft                    | 3.06               |                       |                    |

It is observed from Table 11 that the value of inter-well connectivity coefficient is one for each injector-producer pair, which is obvious considering there is no boundary dominated flow (i.e., there is no aquifer present).

### 5.3.2 Case 2: TP-CRMIP model applied on a heterogeneous reservoir model with four injectors and one producer.

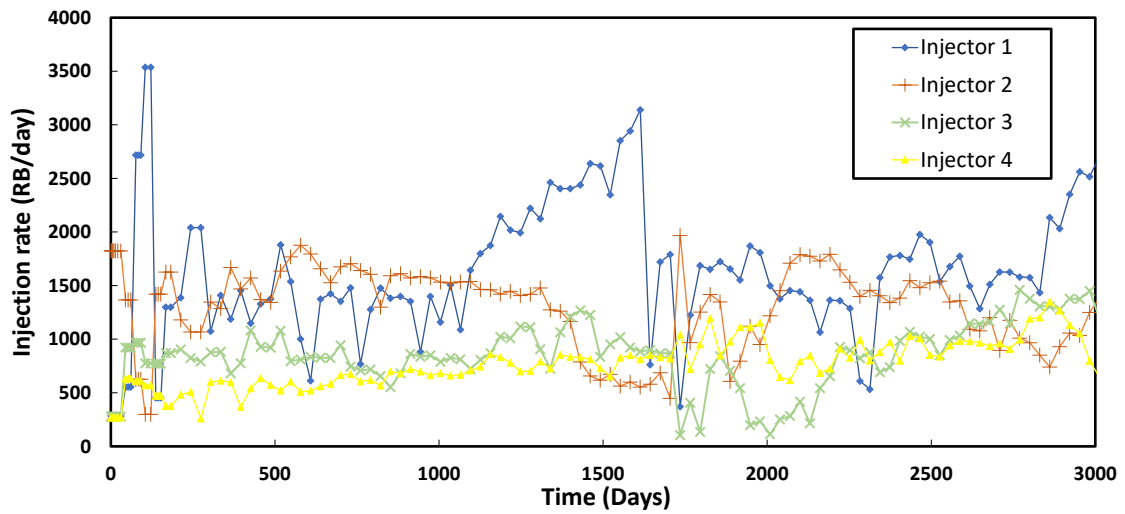
The second case considers a heterogeneous reservoir with non-uniform permeability distribution (Figure 19). A single producer is placed in the middle of a heterogeneous reservoir surrounded by four injectors. The bottom hole pressure at the producer is constant and the injection rates in all injectors are fluctuating. The general reservoir properties and injection rates data are given in Table 12 and Figure 20.



**Figure 19: Reservoir model showing the location of single producer at the middle surrounded by four injectors for Case 2**

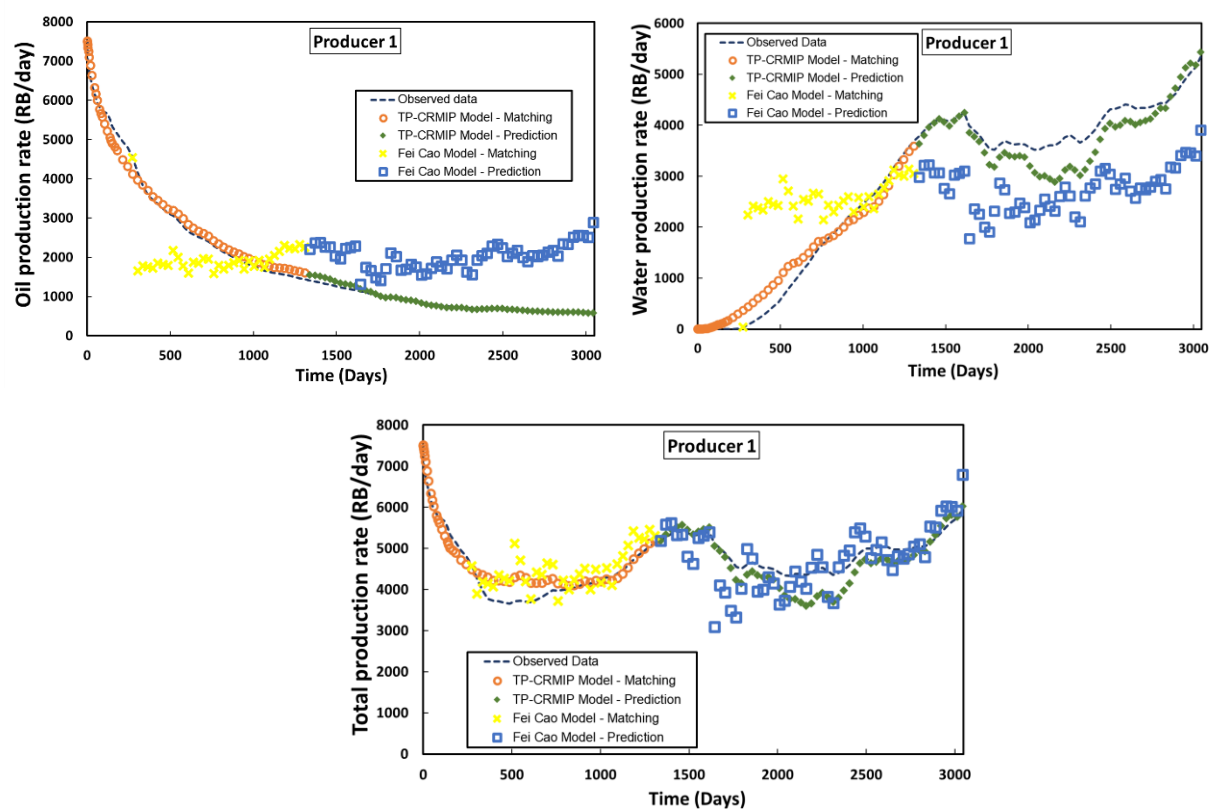
**Table 12: Model parameters and fluid properties to generate data for the Case 2**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 4                        |
| No. of Producers  | 1                        |
| $K_h$ (md)        | 150-250                  |
| $K_v$ (md)        | 15-25                    |
| $\phi$ (fraction) | 0.2                      |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $1 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | $80 \times 80 \times 65$ |
| Model dimension   | $33 \times 33 \times 1$  |



**Figure 20: Injection Rates data for the Case 2**

The TP-CRMIP model was applied on the first 50% of the observed data to compute the model parameters and the remaining 50% of the data is predicted using the matched model. The results were then compared with the simulated data and the two-phase CRM model developed by Fei Cao (Cao, Luo, and Lake 2014). The results are presented in Figure 21 and Table 13.



**Figure 21: Comparison of Oil, Water & total production rate with TP- CRMIP model and two-phase CRM model developed by Fei Cao for the Case 2**

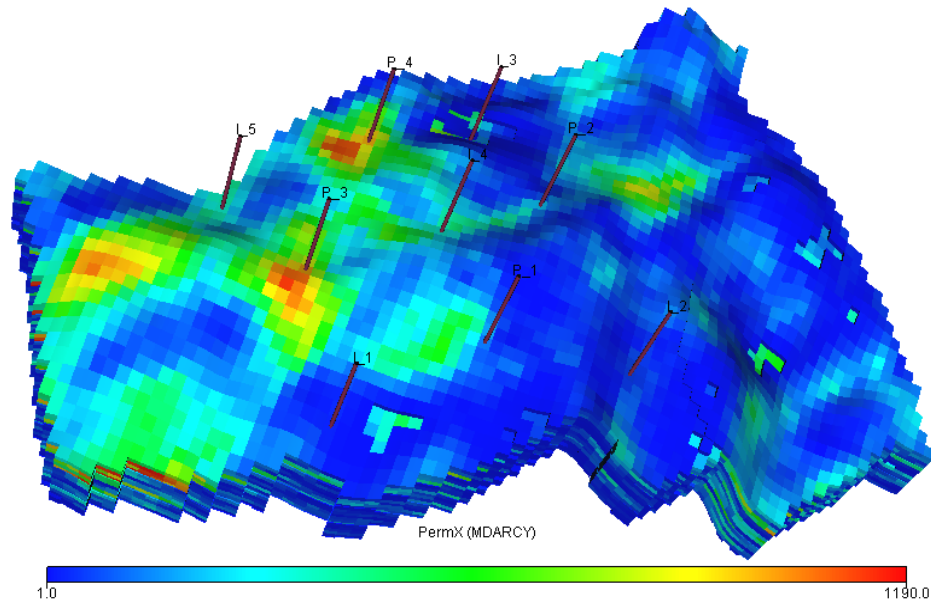
**Table 13: Values of CRMIP parameters obtained by two-phase CRMIP model derived in this study**

|                      | <b>P1</b>          |
|----------------------|--------------------|
| $f_{1,j}$            | 1.00               |
| $f_{2,j}$            | 1.00               |
| $f_{3,j}$            | 1.00               |
| $f_{4,j}$            | 1.00               |
| $V_{p_{1,j}}$ , bbl. | $6.00 \times 10^6$ |
| $V_{p_{2,j}}$ , bbl. | $1.99 \times 10^6$ |
| $V_{p_{3,j}}$ , bbl. | $4.00 \times 10^6$ |
| $V_{p_{4,j}}$ , bbl. | $3.99 \times 10^6$ |
| $J_G$ , md-ft        | 3.633              |
| Error (Matching)     | 38.69              |
| Error (Prediction)   | 60.78              |

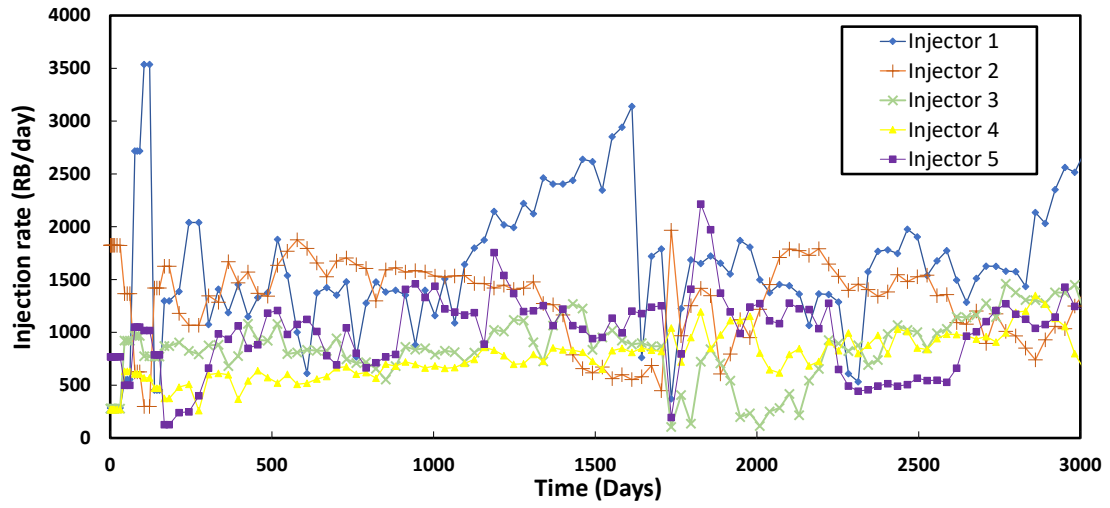
It can be observed from Table 13 that the newly developed TP-CRMIP provided good prediction of the phase rates. The value of inter-well connectivity factor is one for all injector-producer pair which exhibits full contribution of injectors towards the producer. Since, the reservoir is produced in a controlled environment with no aquifer support or boundary dominated flow, the injectors are expected to contribute 100% toward the producer. It is worth mentioning that the TP-CRMIP can be successfully applied to any injection producer pattern. The predicted flow rates are in good agreement with the observed rates.

### 5.3.3 Case 3: UNISIM Model

In this final case, TP-CRMIP model was applied on the UNISIM reservoir model (Figure 22) available in the literature. UNISIM-1D model was simulated with 5 injectors and 4 producers to generate the production data. The first 50% of the production data was used for history matching and the remaining data was used to compare the prediction obtained from the TP-CRMIP model. The general reservoir dimensions, properties and injection rates data are shown in Figure 23 and Table 14.



**Figure 22: Reservoir model showing the location of producers and injectors for the Case 3**

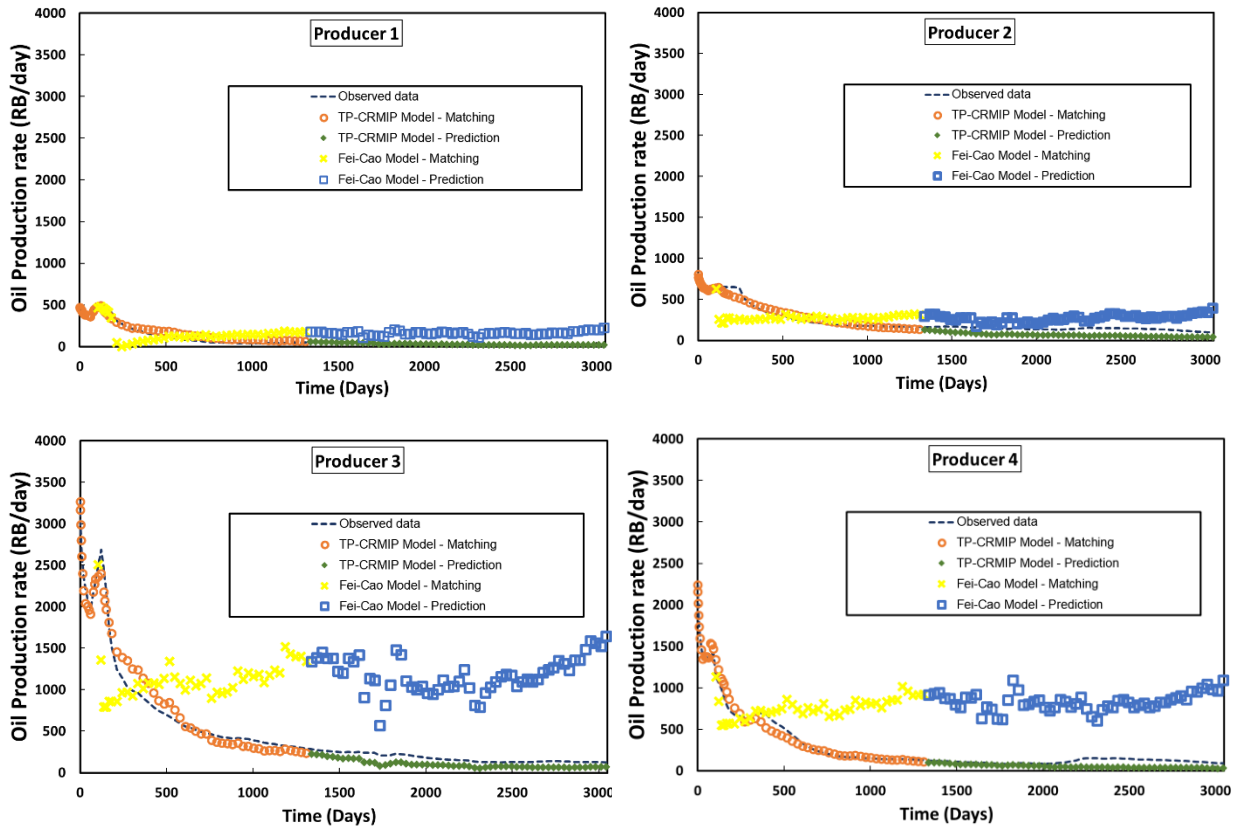


**Figure 23: Injection Rates data for the Case 3.**

**Table 14: Model parameters and fluid properties to generate data for the Case 3.**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 5                        |
| No. of Producers  | 4                        |
| $K_h$ (md)        | 1-1190                   |
| $K_v$ (md)        | 0.1-119                  |
| $\phi$ (fraction) | 0.2                      |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $1 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | variable                 |
| Model dimension   | $81 \times 58 \times 20$ |

The results are shown in Figures. 24-26. Results show a satisfactory match between the observed data and the production rates obtained from the TP-CRMIP model.



**Figure 24: Comparison of oil production rate between TP-CRMIP and Fei-Cao model.**

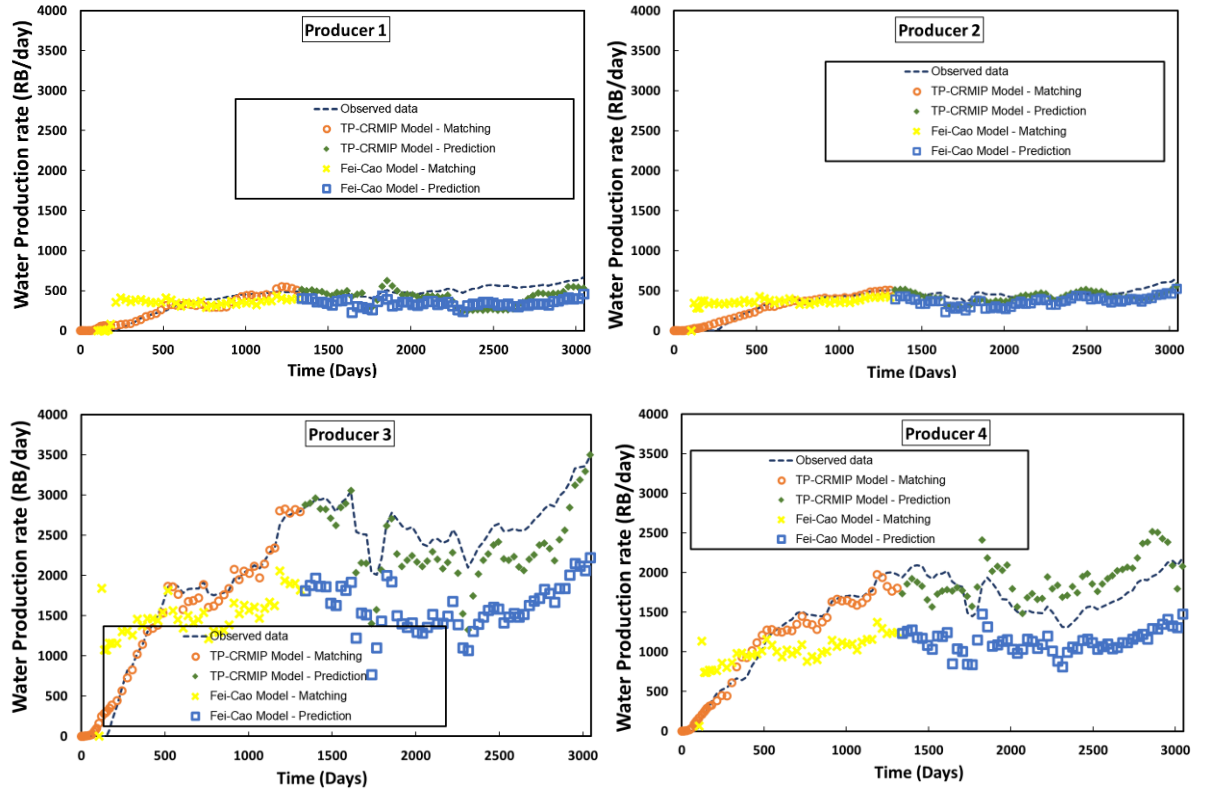


Figure 25: Comparison of water production rate between TP-CRMIP and Fei-Cao model.

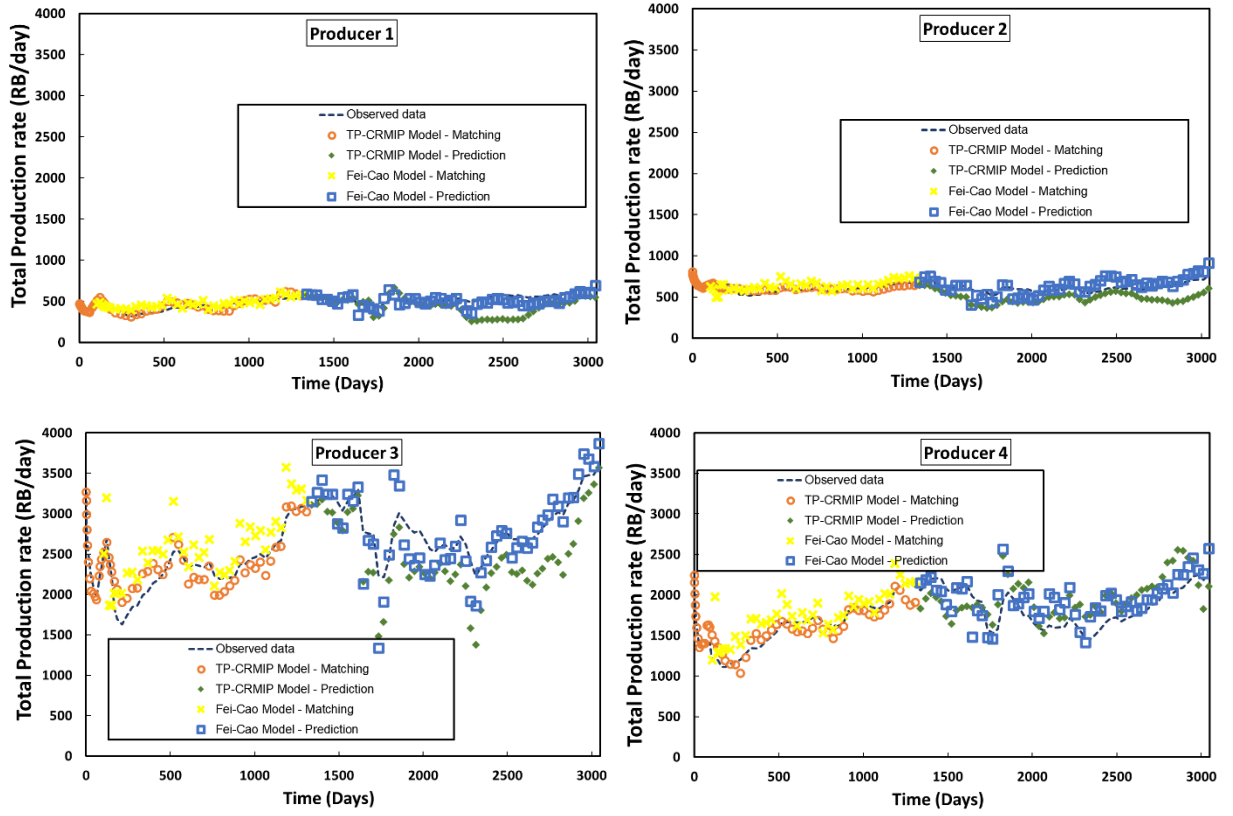


Figure 26: Comparison of total production rate between TP-CRMIP and Fei-Cao model.

**Table 15: Values of CRM parameters obtained by two-phase CRM model derived in Case 3.**

| TP-CRMIP             | P1 (j =1)          | P2 (j =2)          | P3 (j =3)          | P4 (j =4)          | Sum  |
|----------------------|--------------------|--------------------|--------------------|--------------------|------|
| $f_{1,j}$            | 0.11               | 0.18               | 0.49               | 0.22               | 1.00 |
| $f_{2,j}$            | 0.00               | 0.12               | 0.57               | 0.30               | 1.00 |
| $f_{3,j}$            | 0.06               | 0.08               | 0.59               | 0.27               | 1.00 |
| $f_{4,j}$            | 0.00               | 0.00               | 0.00               | 1.00               | 1.00 |
| $f_{5,j}$            | 0.20               | 0.04               | 0.52               | 0.24               | 1.00 |
| $V_{p_{1,j}}$ , bbl. | $1.91 \times 10^5$ | $5.60 \times 10^5$ | $1.94 \times 10^6$ | $1.07 \times 10^5$ |      |
| $V_{p_{2,j}}$ , bbl. | $1.00 \times 10^3$ | $7.15 \times 10^5$ | $9.31 \times 10^5$ | $2.42 \times 10^5$ |      |
| $V_{p_{3,j}}$ , bbl. | $6.49 \times 10^4$ | $1.00 \times 10^5$ | $4.43 \times 10^5$ | $2.64 \times 10^5$ |      |
| $V_{p_{4,j}}$ , bbl. | $1.01 \times 10^3$ | $1.04 \times 10^3$ | $1.00 \times 10^3$ | $1.06 \times 10^6$ |      |
| $V_{p_{5,j}}$ , bbl. | $4.42 \times 10^5$ | $1.00 \times 10^3$ | $8.54 \times 10^5$ | $3.31 \times 10^5$ |      |
| $J_G$ , md-ft        | 0.57               | 0.61               | 8.23               | 3.31               |      |

It is observed from Table 15 that the newly developed two-phase CRMIP provided good prediction of the phase rate with reasonable confidence interval. The value of interval connectivity factor is below one for most injector-producer pair which exhibits partial contribution of injectors towards the producer.

Table 16 shows the mismatch between production rates obtained for each of the phase rates. The root mean square error for TP-CRMIP is comparable and in some producers significantly lower than the two-phase Fei Cao model.

**Table 16: Mismatch values for each producer (Example 3).**

|            | Phase      | RMSE using TP-CRMIP | RMSE using Fei Cao |
|------------|------------|---------------------|--------------------|
| Producer 1 | Oil rate   | 2.61                | 13.44              |
|            | Water rate | 10.31               | 16.14              |
| Producer 2 | Oil rate   | 5.72                | 16.72              |
|            | Water rate | 4.99                | 14.12              |
| Producer 3 | Oil rate   | 10.52               | 92.49              |
|            | Water rate | 24.86               | 91.23              |
| Producer 4 | Oil rate   | 6.79                | 63.83              |
|            | Water rate | 21.56               | 57.48              |

## 5.4 Conclusions

The oil and gas industry relies on accurate estimation of oil and water rates to optimize the production of hydrocarbons. In this study, a fully-implicit finite-difference approach is presented to solve the capacitance resistance model and provide a direct and more accurate estimation of oil and water rates.

TP-CRMIP provides advantage over available conventional CRM models in that it solves the individual phase rates simultaneously instead of the conventional way of estimating the total production rate and then using empirical fractional flow models for individual phase rates.

Results from this study reveal that the indirect approach of computing individual phase production rates is highly dependent on the total liquid production rates data. Such an indirect approach provides a good estimate of the total production rate but gives poor estimates of individual phase rates because it ignores the changes in saturation. Thus, the indirect estimate fails to satisfy and mimic the trend for the individual oil and water rates. On the contrary, TP-CRMIP provides far better results of individual oil and water rates. It is worth mentioning that the TP-CRMIP also performed well during prediction of flow rates. The newly developed finite-difference CRMIP model will serve as a fast and reliable tool for water flooding optimization and prediction of production rates.

# **CHAPTER 6**

## **A COSINE-BASED CORRELATION TO ESTIMATE CONNECTIVITY FACTORS OF NEW WELLS IN CAPACITANCE RESISTANCE MODEL**

### **6.1 Introduction**

The planning and management of petroleum fields using reservoir simulation typically requires the estimation of various parameters such as permeability, porosity, water and oil cut to deploy the most effective production scheme. Reservoir simulation models offer physics-based solutions to obtain these parameters, but their computational cost and required data collection can make them unfavorable to use in many cases. To address this issue, CRM models have become a popular alternative. These models offer a low-cost and efficient technique to analyze fluid flow dynamics.

With capacitance resistance models being heuristic and requiring only a few simple data inputs, many limitations are expected. Some of these limitations are attributed to the model itself while others are different cases of data input. Examples of limitations due to the nature of the model include highly varying fluid compressibility. This restricts reservoir fluids to water and oil due to these fluids being incompressible and slightly compressible, respectively. Another limitation is altering well productivity index. This restricts the use of data whereby wells are recompleted and/or well skin values are altered. Data restrictions, however, are experienced in modeling a time interval whereby new production wells are

introduced, producers are frequently shut-in, and presence of external sinks/sources (A. Yousef et al. 2005; A. A. Yousef et al. 2006).

Kim, 2019 introduced a technique to overcome the limitation of CRM in predicting flow rates when new injectors are added to the reservoir. They proposed a linear relationship between inter-well connectivity and distance between injector-producer pairs. The coefficients of the linear equation were determined by curve fitting on available inter-well connectivities and distances. The proposed methodology is only applicable to homogeneous reservoirs, and they assumed that the inter-well connectivity of existing wells does not vary with the introduction of new injectors.

Weber D. (2009) presented two approaches to overcome the injector well placement limitation. In the first approach, they proposed to generate simulated response of new injector placed at different locations in a simple grid-based reservoir model. Then, CRM and fractional flow model parameters are calculated for each defined location to create a surface map of model parameters, which is then fitted by a polynomial equation. In the second approach, a logistic regression model is presented to fit the CRM model parameters and estimate the coefficients of the regression model based on distance between injectors and producers. Once regression model is trained, it can be used for the prediction of model parameters for new wells. Once the model parameters with the new injector have been computed using this regression model,  $f_{ij}$ 's are normalized to satisfy  $f_{ij} \leq 1$ , if necessary.

S. Chitsiripanich (2015) developed a technique to identify potential infill drilling sites for producers. The technique includes a qualitative examination of maps showing normalized inter-well connectivities, permeability, porosity, net-pay thickness, and oil saturation. Based on the analysis, they proposed to drill new producers in areas with small normalized

inter-well connectivity, large oil saturation, and porosity. This approach was successfully applied to a deltaic sandstone reservoir that had interfingered lacustrine shales in Southeast Asia. Although the technique does not incorporate a flow simulation model or optimization algorithm, it can still provide useful insights into infill locations based on static and dynamic information.

In this study, a modified version of CRMIP model is used, which is a two-phase capacitance resistance model for injector producer pair. CRMIP has been widely used in recent studies (Lins, Horowitz, and Tueros 2017; Morteza Sayarpour, Kabir, and Lake 2008, 2009; Tueros et al. 2018). In this study, CRMIP solution was modified to simultaneously solve the continuity equations for each phase to determine each phase rates. The modified version of CRMIP is termed as two-phase CRMIP (TP-CRMIP). TP-CRMIP has three unknown model parameters: inter-well connectivity  $f_{i,j}$ , that represents the contribution of injectors  $i$  toward producer  $j$ ; inter-well pore volume  $V_{p_{i,j}}$ , that represent the pore volume between injector producer pair; and geometric constant, that indicates the drainage volume around a producer affecting its rates.

## **6.2 Mathematical Models for adding new wells.**

This section describes the methodology to add new wells in an existing reservoir producing for a certain period of time.

### **6.2.1 Methodology**

Consider a field in production with a set of producers and injectors, there is likely some level of connectivity already established between them. By using the existing well data, it

may be possible to avoid traditional simulation methods when adding new injectors or producers in the existing network. In this study, inter-well connectivity from existing wells is limited to between zero and one, and negative values are not considered. This is because allowing for negative connectivity would suggest that an injector could decrease production in a given producer, which is physically impossible. While connectivity exceeding unity may occur in cases where there is external injection or aquifer support, such scenarios were not considered in this study.

TP-CRMIP is first applied to the injection and production rates of existing wells. This provides estimate for the inter-well connectivity  $f_{i,j}$ , inter-well pore volume  $Vp_{i,j}$  and geometric constants  $J_G$  of the existing wells in the reservoir. The existing and future well spatial location or coordinates in the reservoir must be known. It is assumed that all wells are drilled vertically to ignore  $z$ -coordinate. Inter-well connectivities for new wells are calculated using cosine model as

$$f_{i,j} = C_1 \left( C_2 + \cos t_{i,j}^{ploc} + \cos t_{i,j}^{iloc} + \cos t_j^{prate} \right), \quad (6.1)$$

where,

$$t_{i,j}^{ploc} = \alpha_0 + \alpha_1 \log(x_j) + \alpha_2 \log(y_j) + \alpha_3 \log(d_{i,j}), \quad (6.2)$$

and

$$t_{i,j}^{iloc} = \beta_0 + \beta_1 \log(x_i) + \beta_2 \log(y_i) + \beta_3 \log(d_{i,j}), \quad (6.3)$$

and,

$$t_j^{prate} = \gamma_0 + \gamma_1 \dot{q}_{avg,j'}^n + \gamma_2 \dot{q}_{avg,j'}^{n-1}, \quad (6.4)$$

whereas,

$$d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}, \quad (6.5)$$

and

$$\dot{q}_{avg,j'}^n = \log_{10} (q_{avg,j'}^n), \quad (6.6)$$

Similarly for the inter-well pore volume,

$$\dot{V}_{i,j} = C_1 (C_2 + \cos t_{i,j}^{ploc} + \cos t_{i,j}^{iloc} + \cos t_j^{prate}), \quad (6.7)$$

where  $\dot{V} = \frac{V_b}{V_{b,max}}$ ,

and for geometric constant, if required

$$\dot{J}_G = C_1 (C_2 + \cos t_{i,j}^{ploc} + \cos t_{i,j}^{iloc} + \cos t_j^{prate}), \quad (6.8)$$

where  $\dot{J}_G = \frac{J_G}{J_{Gb,max}}$ ,

Note that  $0 \leq \alpha \leq 2\pi, 0 \leq \beta \leq 2\pi$  and  $0 \leq \gamma \leq 2\pi$ .

$x_i$  and  $y_i$  are the distances in the  $x$  and  $y$  direction of the existing injectors and  $x_j$  and  $y_j$  are the distance in the  $x$  and  $y$  direction of the existing producers. The inter-well distance of all wells are the independent factors that influence the cosine model.

The regression coefficients obtained from the cosine model are utilized to calculate future injector/producer inter-well connectivity based on their distance from existing wells and their  $x$  and  $y$  directions. If the predicted connectivity for a specific injector exceeds one, the parameters are normalized to ensure that no single injector dominates. On the other hand, if the predicted connectivities are below one, normalization is not applied to avoid falsely increasing connectivities for poorly connected locations. TP-CRMIP requires initial known

values of production rates to calculate production rates for the next timestep. Thus, for the estimation of initial production rates of new producers, the equation 6.9 can be used.

$$q_j^n = q_{avg,j}^n \left\{ 3 + \cos(\alpha_0 + \alpha_1 \dot{x}_j) + \cos(\alpha_2 + \alpha_3 \dot{y}_j) + \cos \left[ \alpha_4 - \alpha_5 \left( \dot{q}_{j^1}^n / \dot{d}_{j,j^1} \right) + \alpha_6 \left( \dot{I}_{i^1}^n / \dot{d}_{j,i^1} \right) \right] \right\}, \quad (6.9)$$

where  $\dot{x} = \log_{10} x$ ,  $\dot{q} = \log_{10} q$ ,  $j^1$  is the nearest producer to Producer  $j$  and  $i^1$  is the nearest injector to Producer  $j$ .  $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$  and  $\alpha_6$  are the regression coefficients.

### **6.3 Application of New Cosine Based Function to estimate TP-CRMIP parameter values for new injectors and producers.**

This approach consists of the following steps:

1. History matching of existing wells to estimate the unknown model parameters such as  $Vp_{i,j}$ ,  $f_{i,j}$  and  $J_G$ .
2. Estimation of cosine function coefficients using the estimated model parameters.
3. Estimate inter-well connectivity and inter-well pore volume for new wells with the existing wells using cosine function.
4. Using the matched and estimated values of model parameters, predict flow rates for each phase for existing wells and new wells.

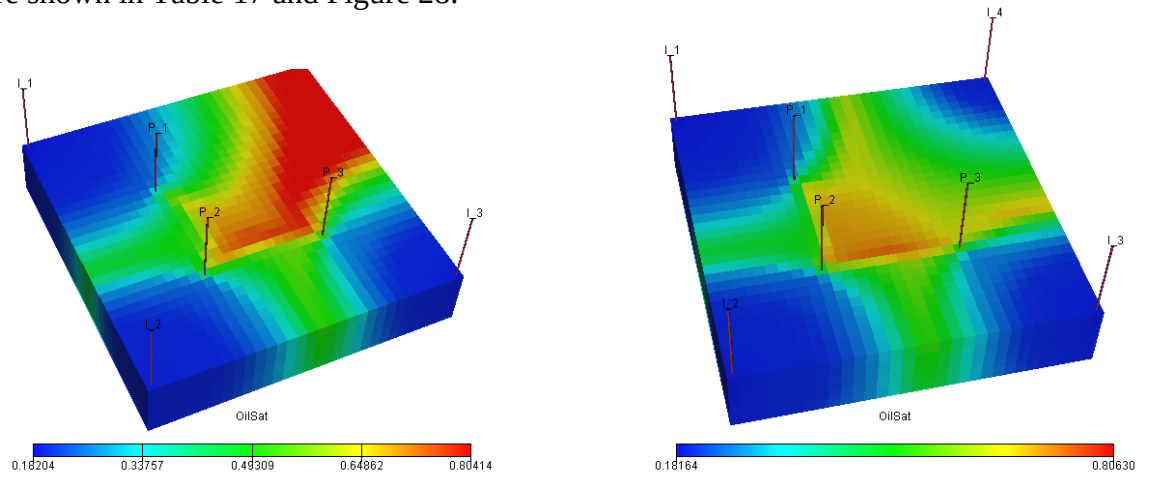
In this chapter, a MATLAB function, `fmincon`, is used to history match the production data and estimate the model parameters for TP-CRMIP. Once the model parameters are known, `fmincon` is used to fit the cosine model with known model parameters to extract regression coefficients of cosine models.

## 6.4 Example Cases

Three separate cases are presented to test the efficiency and accuracy of the new cosine-based approach to estimate model parameters of new wells for TP-CRMIP model. In each of the cases the reservoir is initially simulated using a commercial simulator to generate the observed data with a definite number of wells for a given duration. Then, injector and/or producers are added in the same reservoir and produce for more years. The first case is a synthetic homogenous reservoir consisting of three injectors and three producers placed at different locations. The reservoir is produced for four years with constant bottom hole pressure of all producers and variable injection rates. After four years, a new injector is introduced in the reservoir with variable injection rates. In the second case, the reservoir is produced the same as in the first case but after four years, a new producer is introduced in the reservoir. Finally in the third case, a more realistic heterogenous reservoir model, UNISIM is used. The reservoir initially produced for four years with two producers and three injectors. After that, two more injectors and producers were added to the reservoir. The new cosine-based function was applied to determine the model parameters of new wells using the prior history matched model parameters of existing wells. The accuracy of the new cosine-based approach was tested by comparing the results with the observed data and the results obtained when using the logistic model approach presented by Weber (2009).

#### 6.4.1 Case 1: Addition of a single injector in an existing reservoir producing with three injectors and three producers.

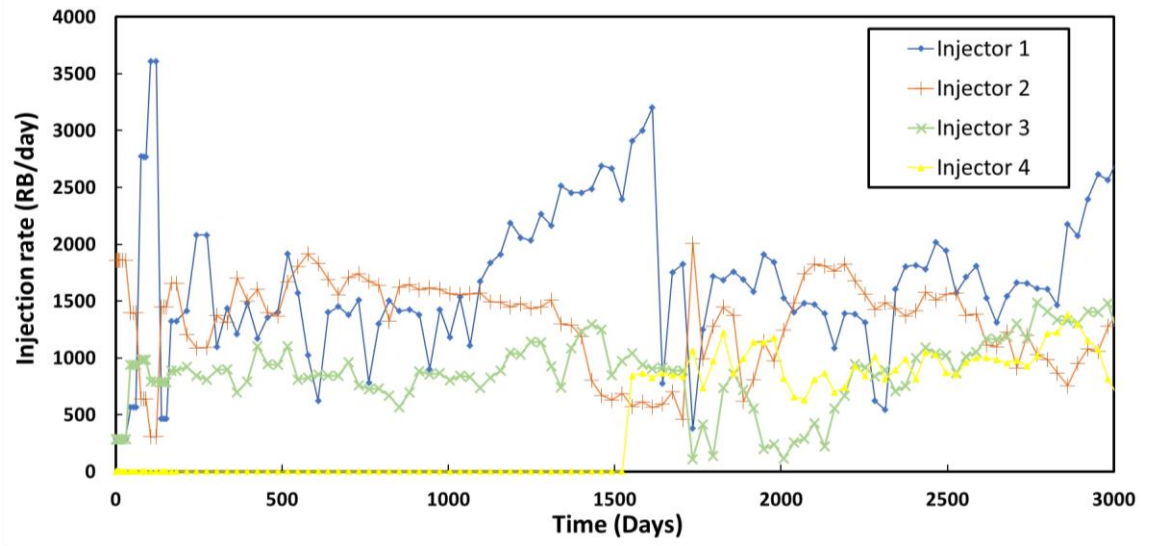
A synthetic model (Figure 27) is simulated on a commercial simulator to test the accuracy of new cosine-based function to predict future rates after addition of a single injector. Three producers and three injectors were placed in the reservoir initially and production was taken for four years. After four years of production, a single injector is placed, and the reservoir is produced for another four years. The reservoir dimensions, properties, and injection rates data are shown in Table 17 and Figure 28.



**Figure 27: Oil saturation distribution with three injectors and three producers after four years of production (left). Oil saturation distribution after addition of new injector for another four years (right) for the Case 1**

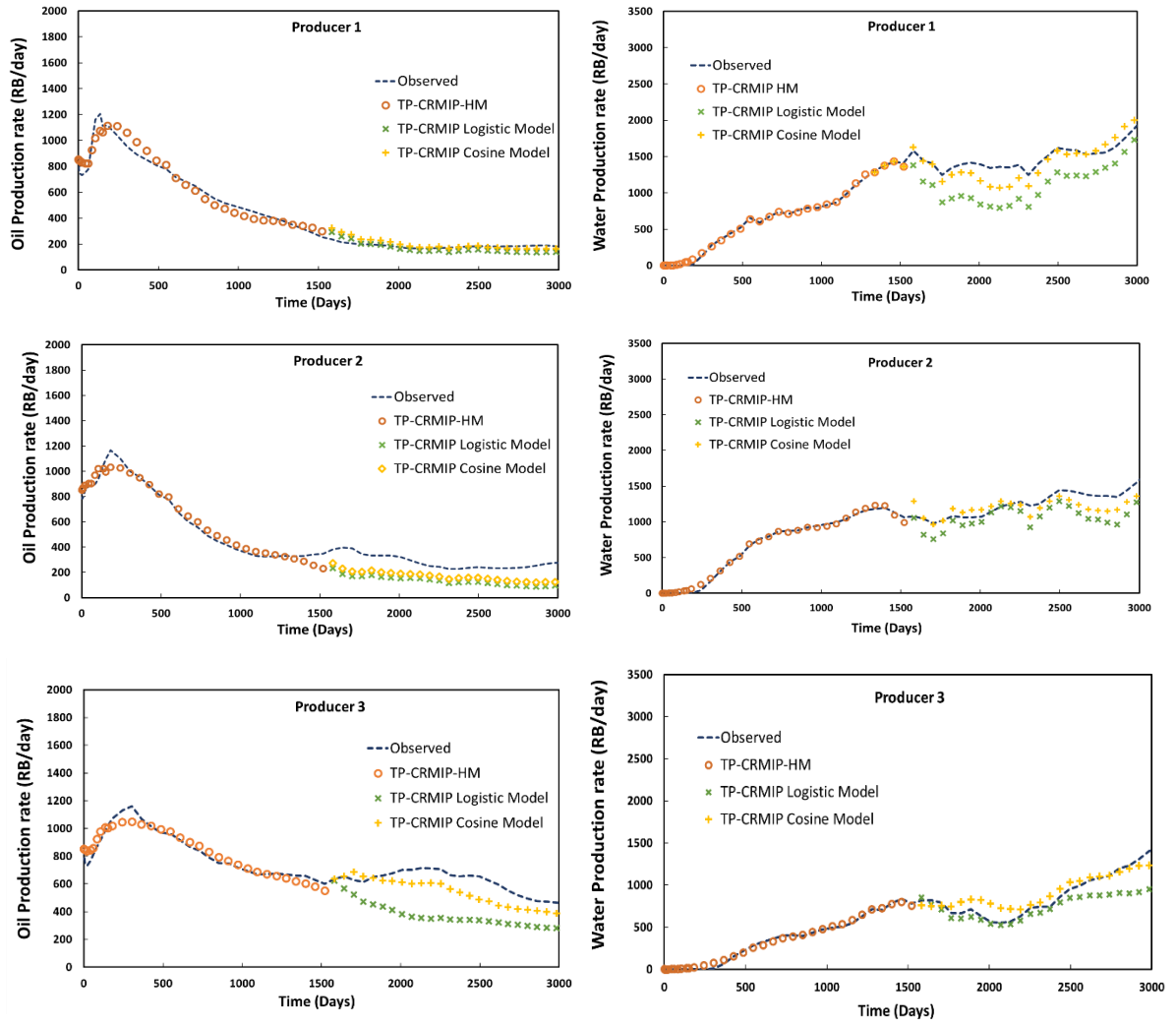
**Table 17: Model parameters and fluid properties to generate data for the Case 1**

| <b>Property</b>   | <b>Value</b>             |
|-------------------|--------------------------|
| No. of Injectors  | 4                        |
| No. of Producers  | 1                        |
| $K_h$ (md)        | 200                      |
| $K_v$ (md)        | 20                       |
| $\phi$ (fraction) | 0.2                      |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $3 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | $80 \times 80 \times 65$ |
| Model dimension   | $33 \times 33 \times 1$  |

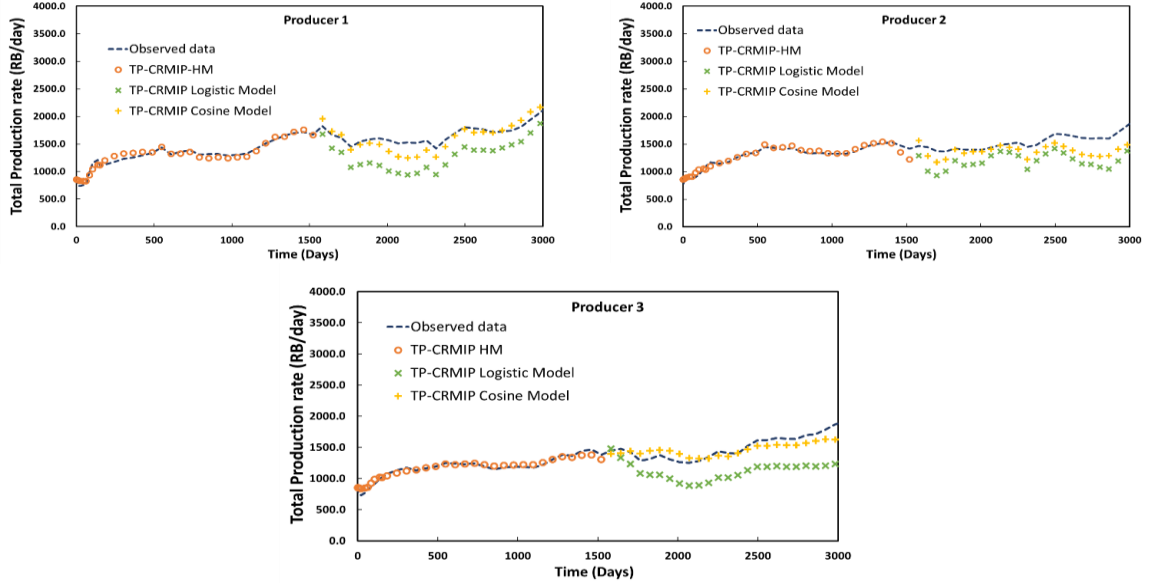


**Figure 28: Injection Rates data for the Case 1**

The TP-CRMIP model was applied on the observed data to obtain the model parameters of TP-CRMIP. The model parameters of TP-CRMIP and distance of injectors and producers are then used in cosine-based function to obtain coefficients of cosine-based function. Once the coefficients are obtained, the cosine-based function is then used to obtain inter-well connectivity, inter-well pore volume of new injector with existing producers. The results of the production rates using new cosine-based function were compared with the observed data from reservoir simulation and logistic model presented by Weber (2009) and shown in Figures 29-30.



**Figure 29: Comparison of simulated oil and water production rate with TP-CRMIP Cosine model and logistic model developed by Weber (2009) for the Case 1**



**Figure 30: Comparison of simulated total production rate with TP-CRMIP Cosine model and logistic model developed by Weber (2009) for the Case 1**

It is evident from Figures 29 and 30 that the new cosine-based model for prediction of TP-CRMIP model parameters produces a closest match for the total production rates than the logistic model presented by weber (2009). It is also worth mentioning that this cosine-based function also performed better in predicting the individual phase rates (oil and water) than the available models in the literature. This is further evident from the values of root mean square error (RMSE) of production rates obtained for each producer as shown in Table 18.

**Table 18: Mismatch values for each producer (Example 1)**

| <b>Producer</b> | <b>TP-CRMIP Cosine model</b> | <b>TP-CRMIP Logistic model</b> |
|-----------------|------------------------------|--------------------------------|
| P1              | 8.01                         | 33.61                          |
| P2              | 12.21                        | 21.02                          |
| P3              | 7.57                         | 15.91                          |

Table 19 lists the values of the parameters estimated when only three injectors and producers were initially active. While Table 20 lists the model parameters using cosine and logistic based models. The first and the second column lists the parameters obtained when the cosine-based model is used. Columns 3 and 4 list the parameters and their corresponding values when the logistic model developed by Weber (2009) is used.

**Table 19: Estimated values of the unknown parameters for three injectors and three producers (Example 1)**

|                      | <b>TP-CRMIP Cosine Based Model</b> |                    |                    |
|----------------------|------------------------------------|--------------------|--------------------|
|                      | <b>P1</b>                          | <b>P2</b>          | <b>P3</b>          |
| $f_{1,j}$            | 0.51                               | 0.28               | 0.21               |
| $f_{2,j}$            | 0.28                               | 0.53               | 0.19               |
| $f_{3,j}$            | 0.19                               | 0.00               | 0.81               |
| $V_{p_{1,j}}$ , bbl. | $1.84 \times 10^6$                 | $1.66 \times 10^6$ | $1.16 \times 10^6$ |
| $V_{p_{2,j}}$ , bbl. | $6.73 \times 10^5$                 | $1.94 \times 10^6$ | $2.82 \times 10^6$ |
| $V_{p_{3,j}}$ , bbl. | $1.49 \times 10^6$                 | $1.00 \times 10^2$ | $1.84 \times 10^6$ |
| J_G, md-ft           | 0.35                               | 0.52               | 0.37               |

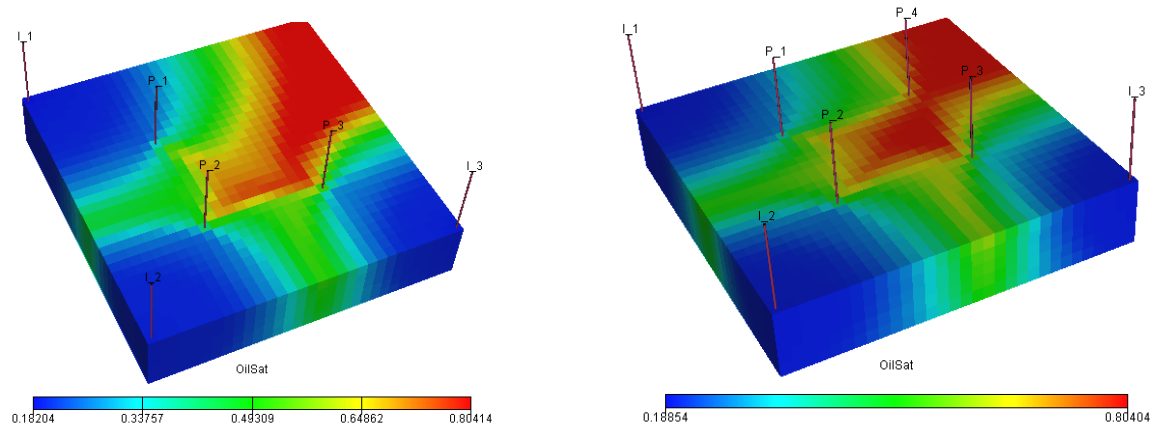
**Table 20: Estimated values of the unknown parameters after new injector is introduced (Example 1)**

|                      | TP-CRMIP Cosine Based Model |                    |                    | TP-CRMIP Logistic Model |                    |                    |
|----------------------|-----------------------------|--------------------|--------------------|-------------------------|--------------------|--------------------|
|                      | P1                          | P2                 | P3                 | P1                      | P2                 | P3                 |
| $f_{1,j}$            | 0.51                        | 0.28               | 0.21               | 0.51                    | 0.28               | 0.21               |
| $f_{2,j}$            | 0.28                        | 0.53               | 0.19               | 0.28                    | 0.53               | 0.19               |
| $f_{3,j}$            | 0.19                        | 0.00               | 0.81               | 0.19                    | 0.00               | 0.81               |
| $f_{4,j}$            | 0.22                        | 0.14               | 0.14               | 0.00                    | 0.95               | 0.28               |
| $V_{p_{1,j}}$ , bbl. | $1.84 \times 10^6$          | $1.66 \times 10^6$ | $1.16 \times 10^6$ | $1.86 \times 10^6$      | $1.58 \times 10^6$ | $1.19 \times 10^6$ |
| $V_{p_{2,j}}$ , bbl. | $6.73 \times 10^5$          | $1.94 \times 10^6$ | $2.82 \times 10^6$ | $6.53 \times 10^5$      | $1.71 \times 10^6$ | $2.93 \times 10^6$ |
| $V_{p_{3,j}}$ , bbl. | $1.49 \times 10^6$          | $1.00 \times 10^2$ | $1.84 \times 10^6$ | $1.57 \times 10^6$      | 0.00               | $1.90 \times 10^6$ |
| $V_{p_{4,j}}$ , bbl. | $1.80 \times 10^6$          | $1.91 \times 10^6$ | $1.82 \times 10^6$ | 0.00                    | 0.00               | $1.12 \times 10^5$ |
| J_G, md-ft           | 0.35                        | 0.52               | 0.37               | 0.35                    | 0.52               | 0.37               |

The sum of root mean square error is 27.79 for TP-CRMIP cosine-based model and 70.54 for the TP-CRMIP logistic based model (weber, 2009). The accuracy of the newly developed TP-CRMIP cosine-based model is twice than the model available in the literature.

#### 6.4.2 Case 2: Addition of a single producer in an existing reservoir producing with three injectors and three producers.

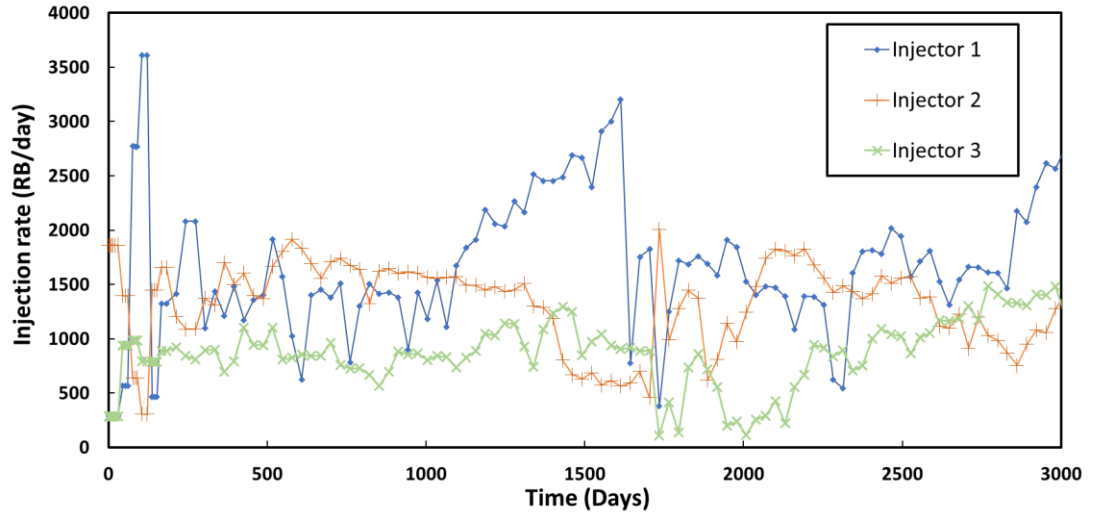
A synthetic model (Figure 31) is simulated on a commercial simulator to test the accuracy of new cosine-based function to predict future rates after addition of a single injector. Three producers and three injectors were placed in the reservoir initially and production was taken for four years. After four years of production, a single producer is placed, and reservoir is produced for another four years. The reservoir dimensions, properties, and injection rates data are shown in Table 21 and Figure 32.



**Figure 31: Oil saturation distribution with three injectors and three producers after four years of production (left). Oil saturation distribution after addition of new producer for another four years (right) for the Case 2**

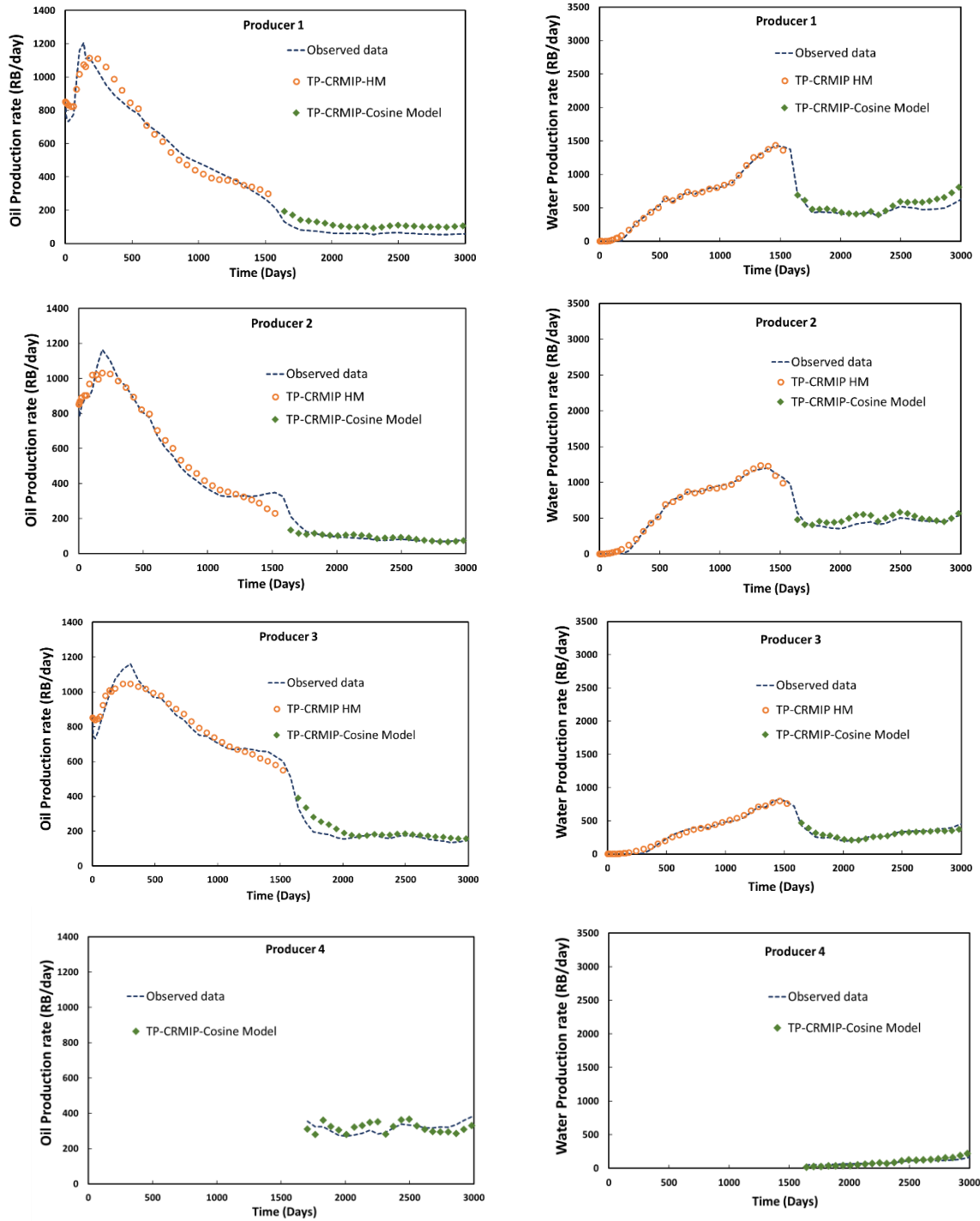
**Table 21: Model parameters and fluid properties to generate data for the Case 2**

| <b>Property</b>   | <b>Value</b>             |
|-------------------|--------------------------|
| No. of Injectors  | 3                        |
| No. of Producers  | 4                        |
| $\phi$ (fraction) | 0.2                      |
| $K_h$ (md)        | 200                      |
| $K_v$ (md)        | 20                       |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $3 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Model dimension   | $33 \times 33 \times 1$  |
| Grid size (ft)    | $80 \times 80 \times 65$ |

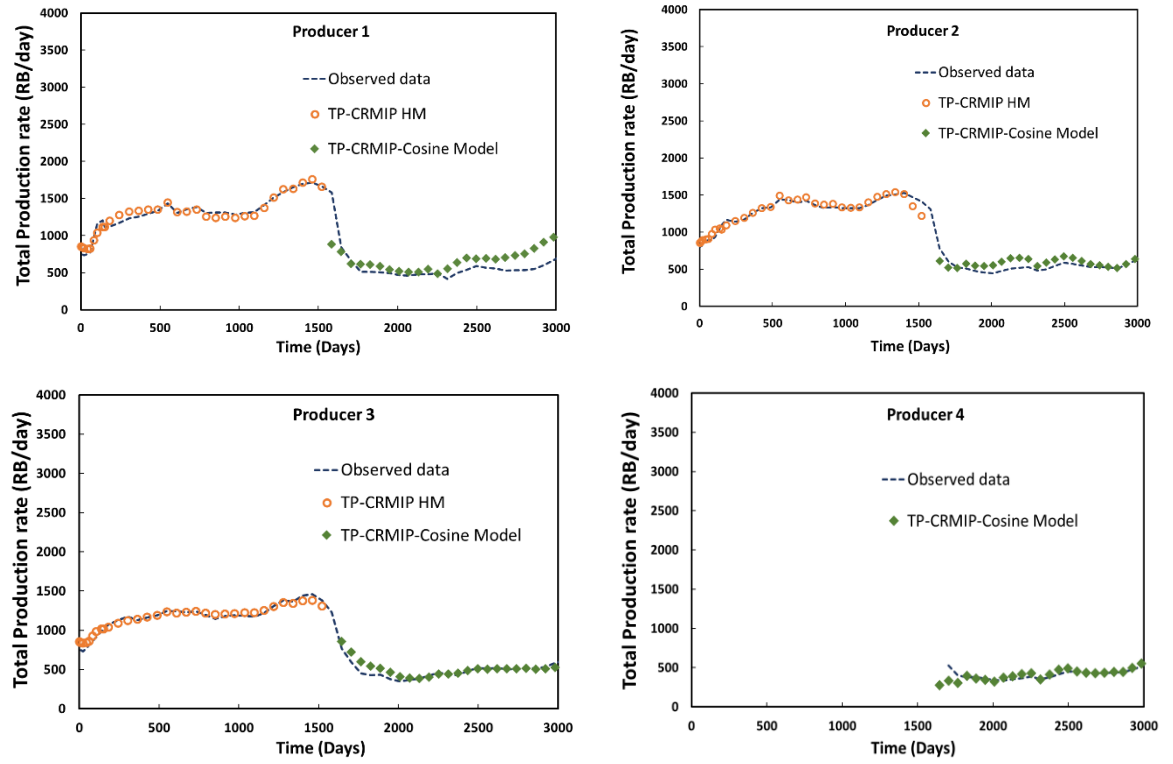


**Figure 32: Injection Rates data for the Case 2**

Similar to the first case, TP-CRMIP model was applied on the observed data to obtain the model parameters of TP-CRMIP. The model parameters of TP-CRMIP and distance of injectors and producers are then used in new cosine-based function to obtain regression coefficients of cosine-based function. Once the coefficients are obtained, the cosine-based function is then used to obtain inter-well connectivity, inter-well pore volume and geometric constant of new producer with existing injectors. The results of the production rates using new cosine-based function were compared with the observed data from reservoir simulation. The results are presented in Figures 33-34 and Table 22.



**Figure 33: Comparison of oil and water production rate with TP-CRMIP cosine-based model for the Case 2**



**Figure 34: Comparison of total production rate with TP-CRMIP cosine-based model for the Case 2**

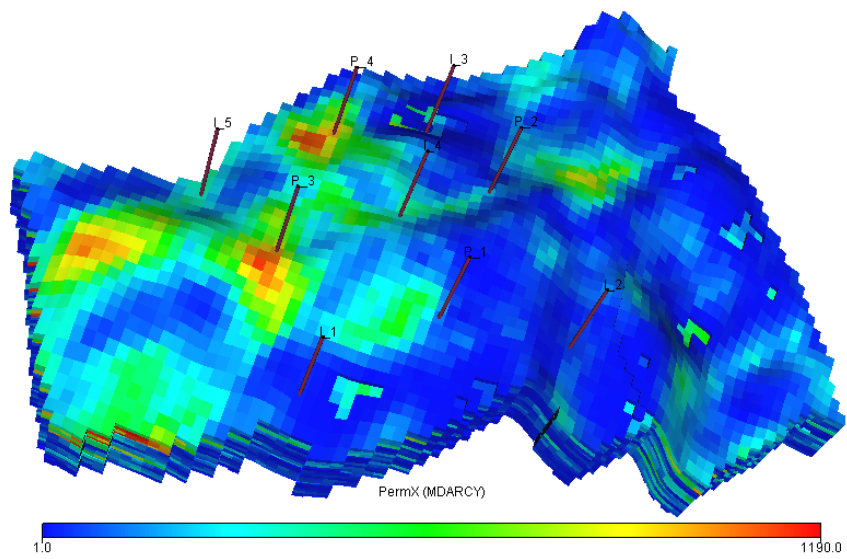
**Table 22: Values of CRMIP parameters obtained by two-phase CRMIP model derived in this study.**

|                      | <b>P1</b>          | <b>P2</b>          | <b>P3</b>          | <b>P4</b>          | <b>Sum</b> |
|----------------------|--------------------|--------------------|--------------------|--------------------|------------|
| $f_{1,j}$            | 0.45               | 0.26               | 0.11               | 0.18               | 1.00       |
| $f_{2,j}$            | 0.24               | 0.42               | 0.14               | 0.21               | 1.00       |
| $f_{3,j}$            | 0.28               | 0.00               | 0.69               | 0.03               | 1.00       |
| $V_{p_{1,j}}$ , bbl. | $1.84 \times 10^6$ | $1.84 \times 10^6$ | $1.16 \times 10^6$ | $1.84 \times 10^6$ |            |
| $V_{p_{2,j}}$ , bbl. | $6.73 \times 10^5$ | $1.94 \times 10^6$ | $2.83 \times 10^6$ | $2.12 \times 10^6$ |            |
| $V_{p_{3,j}}$ , bbl. | $1.49 \times 10^6$ | $1.00 \times 10^2$ | $1.95 \times 10^6$ | $2.21 \times 10^6$ |            |
| J_G, md-ft           | 1.81               | 2.39               | 1.57               | 0.81               |            |
| Error                | 14.7               | 11.8               | 5.9                | 6.8                |            |

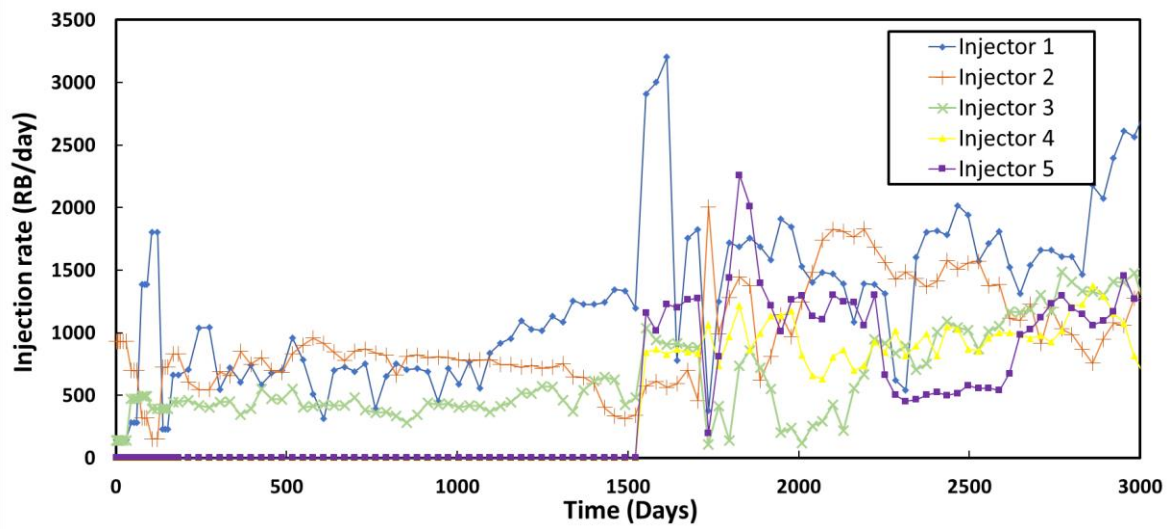
It can be observed from Figures 33-34 that the newly developed TP-CRMIP cosine based provided good prediction of the phase rates. The value of interval connectivity factor is one for all injector-producer pair which exhibits full contribution of injectors towards the producer. It is worth mentioning that the TP-CRMIP can be successfully applied to any injection producer pattern. The predicted flow rates are in good agreement with the observed rates.

#### **6.4.3 Case 3: UNISIM Model**

In this final case, TP-CRMIP model was applied on UNISIM reservoir model (Figure 35) available in the literature. UNISIM-1D model was simulated with three injectors and two producers to generate the production data for four years. Then, two new injectors and two producers were added to the reservoir and the reservoir is produced for another four years. The general reservoir dimensions, properties and injection rates data are shown in Figure 36 and Table 23.



**Figure 35: Reservoir model showing the location of producers and injectors for the Case 3**

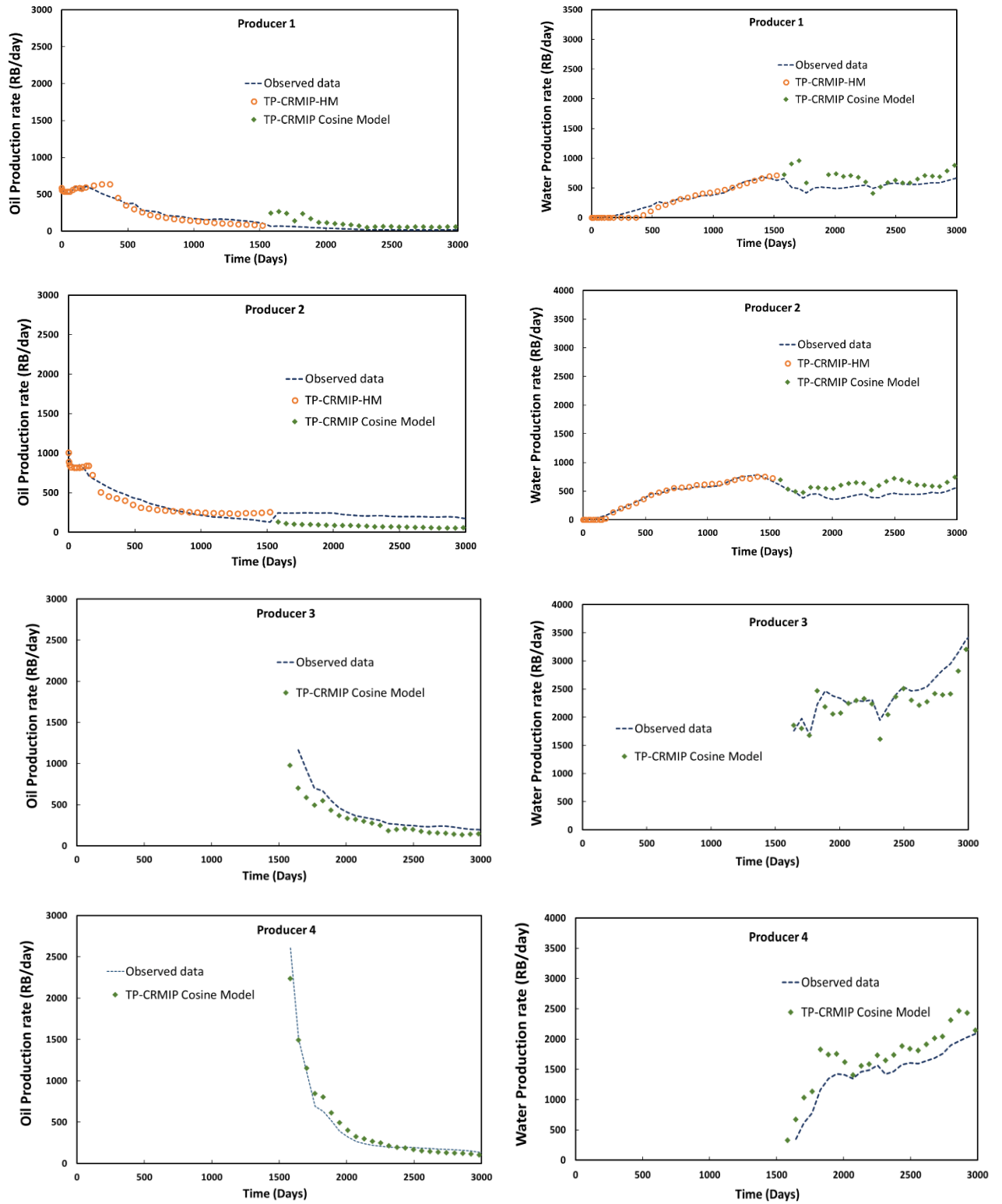


**Figure 36: Injection rates data for the Case 3**

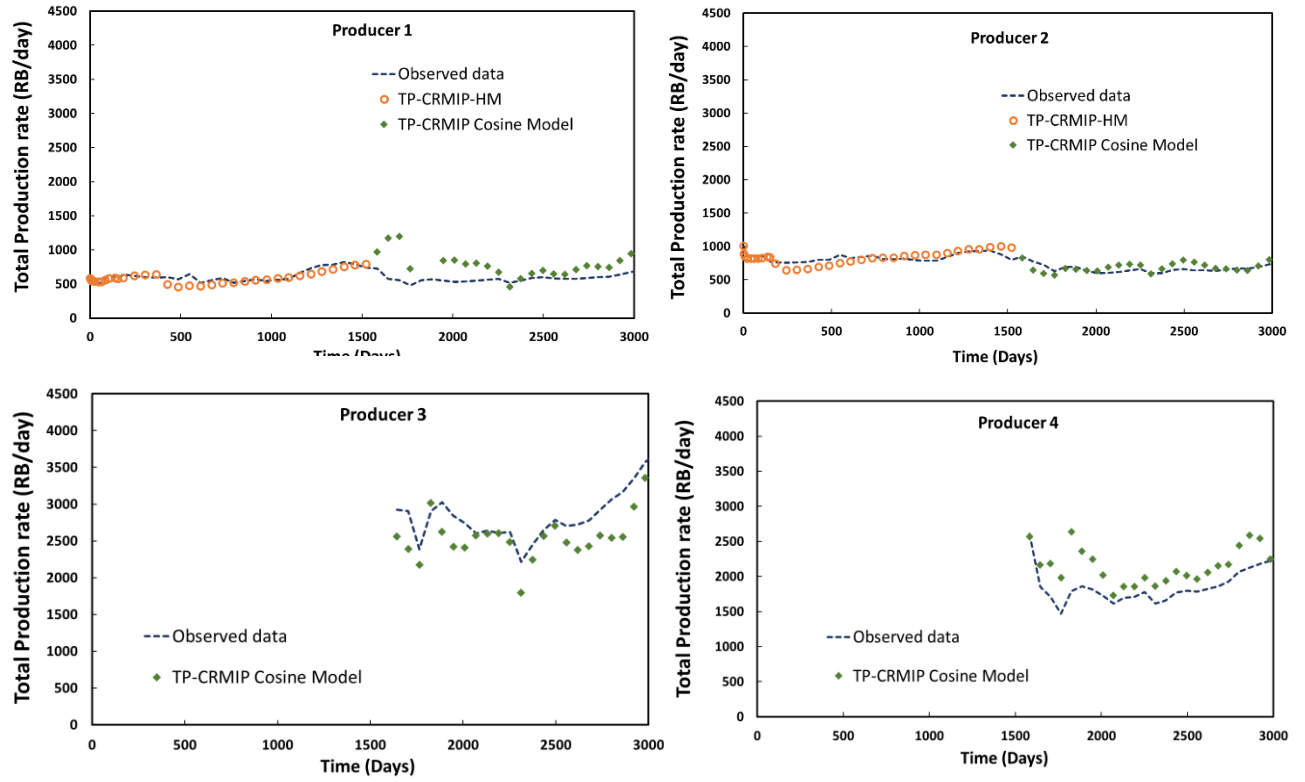
**Table 23: Model parameters and fluid properties to generate data for the Case 3**

| Property          | Value                    |
|-------------------|--------------------------|
| No. of Injectors  | 5                        |
| No. of Producers  | 4                        |
| $K_h$ (md)        | 1-1190                   |
| $K_v$ (md)        | 0.1-119                  |
| $\phi$ (fraction) | 0.2                      |
| $C_o$ (1/psi)     | $5 \times 10^{-6}$       |
| $C_w$ (1/psi)     | $1 \times 10^{-6}$       |
| $C_f$ (1/psi)     | $1 \times 10^{-6}$       |
| $P_{wf}$ (psi)    | 500                      |
| Grid size (ft)    | Variable                 |
| Model dimension   | $81 \times 58 \times 20$ |

The results are shown in Figures. 37-38. Results show a satisfactory match between the observed data and the production rates obtained from the TP-CRMIP cosine-based model.



**Figure 37: Comparison of oil and water production rate for each producer with TP-CRMIP cosine-based model for the Case 3**



**Figure 38: Comparison of total production rate for each producer with TP-CRMIP cosine- based model for the Case 3**

**Table 24: Values of CRM parameters obtained by TP-CRM model cosine model derived in Case 3**

|                      | <b>P1</b>          | <b>P2</b>          | <b>P3</b>          | <b>P4</b>          | <b>Sum</b> |
|----------------------|--------------------|--------------------|--------------------|--------------------|------------|
| $f_{1,j}$            | 0.11               | 0.18               | 0.52               | 0.19               | 1.00       |
| $f_{2,j}$            | 0.13               | 0.15               | 0.48               | 0.24               | 1.00       |
| $f_{3,j}$            | 0.10               | 0.43               | 0.32               | 0.15               | 1.00       |
| $f_{4,j}$            | 0.00               | 0.00               | 0.00               | 1.00               | 1.00       |
| $f_{5,j}$            | 0.21               | 0.05               | 0.49               | 0.25               | 1.00       |
| $V_{p_{1,j}}$ , bbl. | $1.18 \times 10^6$ | $8.76 \times 10^5$ | $1.94 \times 10^6$ | $1.07 \times 10^5$ |            |
| $V_{p_{2,j}}$ , bbl. | $6.91 \times 10^5$ | $7.44 \times 10^5$ | $9.31 \times 10^5$ | $2.42 \times 10^5$ |            |
| $V_{p_{3,j}}$ , bbl. | $2.49 \times 10^5$ | $1.47 \times 10^5$ | $4.43 \times 10^5$ | $2.64 \times 10^5$ |            |
| $V_{p_{4,j}}$ , bbl. | $1.00 \times 10^3$ | $1.00 \times 10^3$ | $1.00 \times 10^3$ | $1.06 \times 10^6$ |            |
| $V_{p_{5,j}}$ , bbl. | $4.42 \times 10^5$ | $1.00 \times 10^3$ | $8.54 \times 10^5$ | $3.30 \times 10^5$ |            |
| J_G, md-ft           | 5.23               | 0.80               | 5.55               | 2.42               |            |
| Error                | 19.22              | 8.12               | 21.31              | 22.66              |            |

It is observed from Table 24 that the newly developed TP-CRMIP cosine-based model provided good prediction of the phase rate. It is worth noting that the cosine-based model also provides good estimates of production rates when multiple injectors and producers were introduced in the reservoir.

## 6.5 Conclusion

The Capacitance Resistance Model (CRM) has become a widely used and reliable tool for observing fluid flow in water flooding. Its ability to optimize injection patterns has enabled reservoir engineers to effectively manage reservoirs and improve recovery rates. However, there is a significant limitation to the CRM in its inability to account for the addition of future wells. Thus, it is quite challenging to properly manage and plan the field development.

This study addresses the issue of adding future wells in the CRM model. Two-phase capacitance resistance model for injector producer pair (TP-CRMIP) is used to initially history match the production rates to estimate model parameters. Then, newly developed cosine-based model is used to generate the unknown model parameters of new injectors and/or producers using the known model parameters of existing wells. Once the model parameters of all injectors and producers are calculated, TP-CRMIP is used to calculate the production rates. The calculated and simulated production rates are then compared to test the accuracy of cosine-based model.

The cosine-based model has been tested in three different cases, and in all three, it provided reasonable estimates of production rates when compared to simulated rates. The accuracy of the cosine-based model demonstrates that it is a reliable tool for estimating unknown model parameters of TP-CRMIP for new wells.

The newly developed cosine-based TP-CRMIP model will serve as a fast and reliable tool for water flooding optimization and field development planning. This model represents a significant advancement in the field of reservoir engineering. Its ability to account for the addition of future wells offers a more comprehensive understanding of reservoir dynamics, enabling reservoir engineers to optimize production rates and improve recovery rates. With this model, reservoir engineers can simulate the effects of adding new wells in different locations, generating accurate production rate estimates for each scenario. By comparing the simulated production rates for each scenario, engineers can identify the most effective locations for new wells, optimizing the field development plan and improving overall recovery rates.

## CHAPTER 7

### CONCLUSIONS AND RECOMMENDATIONS

#### 7.1 Conclusions

The newly developed finite-difference CRMIP model serves as a fast and reliable tool for water flooding optimization and prediction of production rates. This is particularly important because accurate estimation of oil and water rates is crucial for optimizing production and increasing recovery from oil reservoirs. TP-CRMIP provides a significant advantage over available conventional CRM models, and its accuracy and speed make it an ideal tool for oil and gas companies. Thus, the development of TP-CRMIP represents a significant advancement in the modeling of two-phase flow in oil reservoirs. The new model provides a more direct and accurate estimation of oil and water rates. The success of TP-CRMIP highlights the importance of developing new and innovative approaches to modeling and optimizing oil and gas production.

The primary outcomes of this research work are as follows:

- In this work, a novel technique is presented to solve the capacitance resistance model using finite-difference method. Finite-difference explicit and implicit methods were examined for consistency, stability and convergence. It was shown that implicit method was unconditionally stable.
- The finite-difference approach was observed to be computationally faster than the available semi-analytical solution. FDM-CRM is a faster technique and inexpensive to use. It can be successfully applied to field case studies for history-

matching and optimization of reservoir performance which exhibits its unique capability as a powerful engineering tool.

- The studied cases demonstrated that the FD-CRM is capable of successfully matching the solution from synthetic numerical simulation model. FD-CRM requires only injection and production data to effectively evaluate the inter-well connectivity, the reservoir characteristic and future rates predictions.
- Three different methods are presented in this work to obtain the productivity index for prediction in cases where the bottom hole pressures vary. These methods are: 1) calculating a mean value of the known productivity index obtained from the historical data, 2) using the last known value of productivity index obtained from the historical data, and 3) setting the productivity index as an unknown and estimating it during the history matching process. Results indicated that optimizing the productivity index along with other unknown FD-CRMP parameters provides better history matching and prediction performance of observed data.
- In this study, a fully-implicit finite-difference approach is presented to solve the capacitance resistance model and provide a direct and more accurate estimation of oil and water rates.
- TP-CRMIP provides advantage over available conventional CRM models in that it solves the individual phase rates simultaneously instead of the conventional way of estimating the total production rate and then using empirical fractional flow models for individual phase rates.
- Results from this study reveal that the indirect approach of computing individual phase production rates is highly dependent on the total liquid production rates. Such

an indirect approach provides a good estimate of the total production rate but gives poor estimates of individual phase rates because it ignores the changes in saturation. Thus, the indirect estimate fails to satisfy and mimic the trend for the individual oil and water rates.

- On the contrary, TP-CRMIP provides far better results of individual oil and water rates. It is worth mentioning that the TP-CRMIP also performed well during prediction of flow rates. The newly developed finite-difference CRMIP model will serve as a fast and reliable tool for water flooding optimization and prediction of production rates.
- Capacitance resistance model is a fast and reliable tool to observe the fluid flow in water flooding. This method has been used widely to optimize injection pattern. However, because of the limitation of adding future wells, it is quite challenging to properly manage and plan the field development. This study addresses the issue of adding future wells in the CRM model.
- Two-phase capacitance resistance model for injector producer pair (TP-CRMIP) is used to initially history match the production rates to estimate model parameters. Then, newly developed cosine-based model is used to generate the unknown model parameters of new injectors and/or producers using the known model parameters of existing wells. Once the model parameters of all injectors and producers are calculated, TP-CRMIP is used to calculate the production rates. The calculated and simulated production rates are then compared to test the accuracy of cosine-based model.

- Three cases are presented to assess the cosine-based function. Cosine-based model provided reasonable estimates of production rates compared to simulated rates in all studied cases. The newly developed cosine-based TP-CRMIP model will serve as a fast and reliable tool for water flooding optimization and field development planning.

## **7.2 Recommendations**

Based on the insights obtained from this research work, we propose the implementation of the following items to further improve upon the methods that have already been discussed in this dissertation:

- The TP-CRMIP can be applied to horizontal and multilateral wells.
- The work can be extended to three phase flow for oil/water and gas.
- This work can also be extended to accommodate water influx model.
- The capillary pressure effect is ignored in the current study. It will be interesting to observe and quantify the effect of capillary pressure even though it may be negligible.

## APPENDIX A

### DERIVATION OF THE CONTINUITY EQUATIONS FOR FD-CRMP

#### MODEL

For a water-oil displacement process, the material balance equations for each phase in a control volume are given as follows:

The continuity equation for water phase is given by

$$V_p \left( \frac{d\bar{S}_w}{dt} + \bar{S}_w (c_f + c_w) \frac{d\bar{P}}{dt} \right) = I(t) - q_w(t), \quad (\text{A.1})$$

Similarly for oil phase

$$V_p \left( \frac{d\bar{S}_o}{dt} + \bar{S}_o (c_f + c_o) \frac{d\bar{P}}{dt} \right) = -q_o(t) \quad (\text{A.2})$$

Adding Eq A.1 and Eq A.2 gives,

$$V_p \left( \frac{d\bar{S}_w}{dt} + \bar{S}_w (c_f + c_w) \frac{d\bar{P}}{dt} + \frac{d\bar{S}_o}{dt} + \bar{S}_o (c_f + c_o) \frac{d\bar{P}}{dt} \right) = I(t) - q_w(t) - q_o(t) \quad (\text{A.3})$$

Since,

$$q_w(t) + q_o(t) = q_t(t) \quad (\text{A.4})$$

then,

$$V_p \left( \frac{d(\bar{S}_w + \bar{S}_o)}{dt} + \frac{d\bar{P}}{dt} (\bar{S}_w c_f + \bar{S}_o c_f + \bar{S}_o c_o + \bar{S}_w c_w) \right) = I(t) - q_t(t) \quad (\text{A.5})$$

Since the CRM model considers only two-phase then  $\bar{S}_w + \bar{S}_o = 1$  and the total compressibility is given by  $c_t = \bar{S}_o c_o + \bar{S}_w c_w + c_f$ . Thus,

$$V_p \left( c_t \frac{d\bar{P}}{dt} \right) = I(t) - q_t(t) \quad (\text{A.6})$$

Using the drawdown equation,

$$q_t(t) = J_t (\bar{P} - p_{wf}) \quad (\text{A.7})$$

Substitute  $\bar{P}$  from Eq A.7 into Eq A.6,

$$V_p c_t \frac{d \left( p_{wf} + \frac{q_t(t)}{J_t} \right)}{dt} = I(t) - q_t(t), \quad (\text{A.8})$$

Upon rearranging,

$$V_p c_t \frac{dp_{wf}}{dt} + \frac{V_p c_t}{J_t} \frac{dq_t(t)}{dt} = I(t) - q_t(t) \quad (\text{A.9})$$

Assume  $\frac{V_p c_t}{J_t}$  is constant and defined as a time constant  $\tau$ , then the final equation for

capacitance resistance model (CRM) becomes,

$$\tau \frac{dq_t(t)}{dt} + q_t(t) = I(t) - \tau J_t \frac{dp_{wf}}{dt} \quad (\text{A.10})$$

For a producer-based representation of capacitance resistance model (CRMP):

$$\tau_j \frac{dq_t(t)}{dt} + q_t(t) = \sum_{i=1}^N f_{i,j} I_i^{n+1}(t) - \tau_j J_j \frac{dp_{wf_j}}{dt} \quad (\text{A.11})$$

## APPENDIX B

### DERIVATION OF THE CONTINUITY EQUATIONS FOR TP-CRMIP MODEL

For oil-water displacement, the continuity equation for the water phase is given by

$$V_p \left( \frac{d\bar{S}_w}{dt} + \bar{S}_w (c_f + c_w) \frac{d\bar{p}}{dt} \right) = I(t) - q_w(t) \quad (\text{B.12})$$

If we replace  $c_f + c_w$  with  $c_{fw}$ , then Eq. B.1 can be written as

$$V_p \left( \frac{d\bar{S}_w}{dt} + c_{fw} \bar{S}_w \frac{d\bar{p}}{dt} \right) = I(t) - q_w(t) \quad (\text{B.13})$$

Using the well productivity equation given by

$$q_w(t) = J_w (\bar{p} - p_{wf}) \quad (\text{B.14})$$

Substituting  $\bar{p}$  from Eq. B.3 into Eq. B.2, we obtain

$$V_p \left( \frac{d\bar{S}_w}{dt} + c_{fw} \bar{S}_w \frac{d \left( p_{wf} + \frac{q_w(t)}{J_w} \right)}{dt} \right) = I(t) - q_w(t) \quad (\text{B.15})$$

Since,

$$\frac{q_w(t)}{J_w} = \frac{q_o(t)}{J_o} \quad (\text{B.16})$$

then eq. B.4 can be written as

$$V_p \left( \frac{d\bar{S}_w}{dt} + c_{fw} \bar{S}_w \frac{d \left( p_{wf} + q_o(t) / J_o \right)}{dt} \right) = I(t) - q_o(t) \frac{J_w}{J_o} \quad (\text{B.17})$$

Considering the productivity index  $J_o$  is not constant, Eq. B.6 becomes

$$V_p \left[ \left( 1 - \frac{c_{fw} \bar{S}_w q_o}{J_o^2} \frac{dJ_o}{d\bar{S}_w} \right) \frac{d\bar{S}_w}{dt} + c_{fw} \bar{S}_w \frac{dp_{wf}}{dt} + \frac{c_{fw} \bar{S}_w}{J_o} \frac{dq_o(t)}{dt} \right] = I(t) - q_o(t) \frac{J_w}{J_o} \quad (\text{B.18})$$

This is the final form of CRM equation for the water phase. In term of capacitance resistance model for injector-producer pair CRMIP, the equation can be written as

$$V_{p_{i,j}} \left[ \left( 1 - \frac{c_{fw} \bar{S}_{w_{i,j}} q_{o_{i,j}}}{J_{o_{i,j}}^2} \frac{dJ_{o_{i,j}}}{d\bar{S}_{w_{i,j}}} \right) \frac{d\bar{S}_{w_{i,j}}}{dt} + \bar{S}_{w_{i,j}} c_{fw} \frac{dp_{wf_j}}{dt} + \frac{c_{fw} \bar{S}_{w_{i,j}}}{J_{o_{i,j}}} \frac{dq_{o_{i,j}}(t)}{dt} \right] = f_{i,j} I_i(t) - q_{o_{i,j}}(t) \frac{J_{w_{i,j}}}{J_{o_{i,j}}} \quad (\text{B.19})$$

The continuity equation for the oil phase is given by

$$V_p \left( \frac{d\bar{S}_o}{dt} + \bar{S}_o (c_f + c_o) \frac{d\bar{p}}{dt} \right) = -q_o(t) \quad (\text{B.20})$$

Defining  $c_f + c_o = c_{fo}$ , Eq. B.9 can be expressed as

$$V_p \left( \frac{d\bar{S}_o}{dt} + c_{fo} \bar{S}_o \frac{d\bar{p}}{dt} \right) = -q_o(t) \quad (\text{B.21})$$

Using drawdown equation,

$$q_o(t) = J_o (\bar{p} - P_{wf}) \quad (\text{B.22})$$

Substituting  $\bar{p}$  from Eq. B.11 in Eq. B.10, we obtain

$$V_p \left( \frac{d\bar{S}_o}{dt} + c_{fo} \bar{S}_o \frac{d \left( p_{wf} + q_o(t) / J_o \right)}{dt} \right) = -q_o(t) \quad (\text{B.23})$$

Since we consider the productivity index  $J_o$  not constant, Eq. B.12 becomes,

$$V_p \left[ \frac{d\bar{S}_o}{dt} + c_{fo} \bar{S}_o \frac{dp_{wf}}{dt} + \frac{c_{fo} \bar{S}_o}{J_o} \frac{dq_o(t)}{dt} - \frac{c_{fo} \bar{S}_o q_o}{J_o^2} \frac{dJ_o}{d\bar{S}_w} \frac{d\bar{S}_w}{dt} \right] = -q_o(t) \quad (B.24)$$

Using  $\bar{S}_o = 1 - \bar{S}_w$  in B.13, we obtain,

$$V_p \left[ - \left( 1 + \frac{c_{fo} \bar{S}_o q_o}{J_o^2} \frac{dJ_o}{d\bar{S}_w} \right) \frac{d\bar{S}_w}{dt} + c_{fo} (1 - \bar{S}_w) \frac{dp_{wf}}{dt} + \frac{c_{fo} (1 - \bar{S}_w)}{J_o} \frac{dq_o(t)}{dt} \right] = -q_o(t) \quad (B.25)$$

Equation B.14 is the final form of CRM equation for the oil phase. In terms of capacitance resistance model for injector-producer pair CRMIP, the equation can be modified as

$$V_{p_{i,j}} \left\{ - \left[ 1 + \frac{c_{fo} (1 - \bar{S}_{w_{i,j}}) q_{o_{i,j}}}{J_{o_{i,j}}^2} \frac{dJ_{o_{i,j}}}{d\bar{S}_{w_{i,j}}} \right] \frac{d\bar{S}_{w_{i,j}}}{dt} + (1 - \bar{S}_{w_{i,j}}) c_{fo} \frac{dp_{wf_j}}{dt} + \frac{(1 - \bar{S}_{w_{i,j}}) c_{fo}}{J_{o_{i,j}}} \frac{dq_{o_{i,j}}(t)}{dt} \right\} = -q_{o_{i,j}}(t) \quad (B.26)$$

The term  $\frac{c_{fw} \bar{S}_{w_{i,j}} q_{o_{i,j}}}{J_{o_{i,j}}^2} \frac{dJ_{o_{i,j}}}{d\bar{S}_{w_{i,j}}}$  in Eq. B.8 and the term  $\frac{c_{fo} (1 - \bar{S}_{w_{i,j}}) q_{o_{i,j}}}{J_{o_{i,j}}^2} \frac{dJ_{o_{i,j}}}{d\bar{S}_{w_{i,j}}}$  in Eq. B-15

are much smaller than 1. Thus, both terms have been neglected in deriving the phase residuals presented in Appendix C.

## APPENDIX C

### RESIDUALS AND COMPONENTS OF THE JACOBIAN MATRIX

Equations B.8 and B.15 are discretized and written in terms of residuals for water and oil phases, respectively. In these two equations, the unknown variables are  $q_{o_{i,j}}$  and  $\bar{S}_{w_{i,j}}$ . Given that the values of these two primary variables are known at a time-level  $n$ , we seek their values at a future time-level  $n+1$ . However, due to the nonlinearity of these two equations, Newton-Raphson iterations are employed within each pair of consecutive time-levels. Thus, between time-level  $n$  and time-level  $n+1$ , we introduce a Newton-Raphson iteration index  $\gamma$  such that the residual of the water phase is given by

$$R_{w_{i,j}}^{\gamma+1} = \frac{V_{p_{i,j}}}{\Delta t} \left[ \frac{\bar{S}_{w_{i,j}}^{\gamma+1} - \bar{S}_{w_{i,j}}^n + \bar{S}_{w_{i,j}}^{\gamma+1} c_{fw} (p_{wf_j}^{n+1} - p_{wf_j}^n)}{J_{o_{i,j}}^{\gamma+1} \bar{S}_{w_{i,j}}^{\gamma+1} c_{fw} (q_{o_{i,j}}^{\gamma+1} - q_{o_{i,j}}^n)} \right] - f_{i,j} I_i^{n+1} + q_{o_{i,j}}^{\gamma+1} \frac{J_{w_{i,j}}^{\gamma+1}}{J_{o_{i,j}}^{\gamma+1}} \quad (C.1)$$

and the residual for the oil phase is given by

$$R_{o_{i,j}}^{\gamma+1} = \frac{V_{p_{i,j}}}{\Delta t} \left[ -(\bar{S}_{w_{i,j}}^{\gamma+1} - \bar{S}_{w_{i,j}}^n) + (1 - \bar{S}_{w_{i,j}}^{\gamma+1}) c_{fo} (p_{wf_j}^{n+1} - p_{wf_j}^n) \right] + q_{o_{i,j}}^{\gamma+1} \quad (C.2)$$

The derivative of water-phase residual equation  $R_{w_{i,j}}^{\gamma+1}$  with respect to the oil rate  $q_{o_{i,j}}^{\gamma+1}$  is

$$\frac{dR_{w_{i,j}}^{\gamma+1}}{dq_{o_{i,j}}^{\gamma+1}} = \frac{V_{p_{i,j}}}{\Delta t} \left( \frac{1}{J_{o_{i,j}}^{\gamma+1}} \bar{S}_{w_{i,j}}^{\gamma+1} c_{fw} \right) + \frac{J_{w_{i,j}}^{\gamma+1}}{J_{o_{i,j}}^{\gamma+1}} \quad (C.3)$$

and the derivative of water-phase residual equation with respect to average water saturation is given as

$$\begin{aligned} \frac{\partial R_{w_{i,j}}^{\gamma+1}}{\partial \bar{S}_{w_{i,j}}^{\gamma+1}} = & \frac{c_{fw} V_{p_{i,j}}}{\Delta t} \left[ \frac{1}{c_{fw}} + \left( p_{wf_j}^{n+1} - p_{wf_j}^n \right) + \frac{\left( q_{o_{i,j}}^{\gamma+1} - q_{o_{i,j}}^n \right)}{J_{o_j}^{\gamma+1}} \left( 1 - \frac{\bar{S}_{w_{i,j}}^{\gamma+1}}{J_{o_{i,j}}^{\gamma+1}} \frac{dJ_{o_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} \right) \right] \\ & + \frac{q_{o_{i,j}}^{\gamma+1}}{J_{o_{i,j}}^{\gamma+1}} \left( \frac{dJ_{w_j}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} - \frac{J_{w_{i,j}}^{\gamma+1}}{J_{o_{i,j}}^{\gamma+1}} \frac{dJ_{o_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} \right) \end{aligned} \quad (C.4)$$

The derivative of oil-phase residual equation with respect to oil rate  $q_{o_{i,j}}^{\gamma+1}$  is given by

$$\frac{dR_{o_{i,j}}^{\gamma+1}}{dq_{o_{i,j}}^{\gamma+1}} = \frac{V_{p_{i,j}}}{\Delta t} \left[ \left( 1 - \bar{S}_{w_{i,j}}^{\gamma+1} \right) \frac{c_{fo}}{J_{o_{i,j}}^{n+1}} \right] + 1 \quad (C.5)$$

and the derivative of oil-phase residual equation with respect to the average water saturation is

$$\frac{\partial R_{o_{i,j}}^{\gamma+1}}{\partial \bar{S}_{w_{i,j}}^{\gamma+1}} = - \frac{c_{fo} V_{p_{i,j}}}{\Delta t} \left\{ \frac{1}{c_{fo}} + \left( p_{wf_j}^{n+1} - p_{wf_j}^n \right) + \frac{\left( q_{o_{i,j}}^{\gamma+1} - q_{o_{i,j}}^n \right)}{J_{o_{i,j}}^{\gamma+1}} \left[ 1 + \left( 1 - \bar{S}_{w_{i,j}}^{\gamma+1} \right) \frac{1}{J_{o_{i,j}}^{\gamma+1}} \frac{dJ_{o_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} \right] \right\} \quad (C.6)$$

The derivative of oil productivity index with respect to the average water saturation is given by,

$$\frac{dJ_{o_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} = \frac{d \left[ J_{G_j} \frac{k_{ro_{i,j}}^{\gamma+1}}{\mu_o} \right]}{d\bar{S}_{w_{i,j}}^{\gamma+1}} \quad (C.7)$$

Since  $\mu_o$  and  $J_G$  are constant, Eq. C.7 becomes

$$\frac{dJ_{o_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} = \frac{J_{G_j}}{\mu_o} \frac{dk_{ro_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} \quad (C.8)$$

where

$$k_{ro,i,j}^{\gamma+1} = k_{ro}^0 \left( \frac{1 - \bar{S}_{w_{i,j}}^{\gamma+1} - S_{or}}{1 - S_{wr} - S_{or}} \right)^2. \quad (C.9)$$

Similarly, the derivative of water productivity index with respect to the average water saturation is given by,

$$\frac{dJ_{w_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}} = \frac{J_{G_j}}{\mu_w} \frac{dk_{rw_{i,j}}^{\gamma+1}}{d\bar{S}_{w_{i,j}}^{\gamma+1}}, \quad (C.10)$$

where

$$k_{rw_{i,j}}^{\gamma+1} = k_{rw}^0 \left( \frac{\bar{S}_{w_{i,j}}^{\gamma+1} - S_{wr}}{1 - S_{wr} - S_{or}} \right)^2 \quad (C.11)$$

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