

**ON PROCESS MONITORING USING CONDITIONAL
AGGREGATE MEAN CONTROL CHARTS**

BY

ALAA ALSAEED

A Thesis Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

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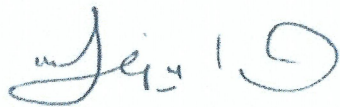
DEANSHIP OF GRADUATE STUDIES

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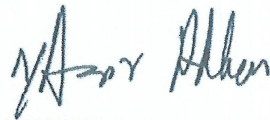
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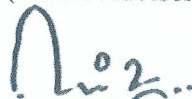
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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

To my parents,
my family, and
Hanan & Amjad, my cousins.

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THESIS ABSTRACT

FULL NAME OF STUDENT: Alaa Abdulaziz Suleman AlSaeed

TITLE OF STUDY:

On Process Monitoring Using Conditional Aggregate Mean Control Charts

MAJOR FIELD: Applied Statistics

DATE OF DEGREE: May 2023

This thesis focuses on the field of Statistical Process Control (SPC), which aims to study the stability of process parameters to ensure the quality of the process. The statistical tool "Control Chart" is used to accomplish this purpose of studying stability by classifying processes under in-control or out-of-control based on the analysis of the process variation. The control chart monitors the process to detect any shift that may occur in the process parameters from the target parameters.

The thesis aims to adapt the control chart structure proposed in the literature, known as Conditional Aggregate Mean (CAM), to the conditions of real-world applications. It involves adapting the CAM structure to monitor the stability of process dispersion employing the strategy of using a transformed variable to represent dispersion parameter behavior. The purpose of using this strategy is to monitor a variable that approximately follows a normal distribution, simplifying the process of determining statistical values and, consequently, facilitating the construction of a control chart.

Additionally, the thesis incorporates the CAM control chart into a Non-Parametric (NP) framework to monitor process location using the nonparametric sign-test statistic. NP approaches allow the release of assumptions made during chart construction, particularly the assumption of normality, providing the flexibility to accommodate data with non-normal characteristics. They offer robustness in the face of outliers and skewed distributions. The adoption of NP approaches in control charts extends the applicability of control charts to a wider range of processes, making them more suitable for the variability encountered in practical applications.

ملخص الرسالة

الاسم الكامل: الاء بنت عبد العزيز بن سليمان السعيد

عنوان الرسالة: مراقبة العمليات باستخدام مخططات تحكم "المتوسط التراكمي المشروط"

التخصص: إحصاء تطبيقي

تاريخ الدرجة العلمية: مايو ٢٠٢٣

تركز هذه الرسالة على مجال المراقبة الإحصائية للعمليات والذي يُعنى لدراسة استقرار معلمات عملية ما لضمان جودة سير العملية بلا معوقات أو أخطاء. حيث يتم استخدام الأداة الإحصائية "مخطط التحكم" لدراسة استقرار العملية وذلك بتصنيفها تحت السيطرة / خارج السيطرة بناءً على تحليل تباين قراءات تلك العملية. يعمل "مخطط التحكم" على مراقبة سير العملية وإعطاء مؤشر في حال ملاحظة حدوث تحول في قيم معلمات العملية عن القيم المتوقعة لعملية تحت السيطرة .

الهدف من هذه الرسالة هو دراسة مخطط التحكم المقترح في الأدبيات المسمى بـ "المتوسط التراكمي المشروط" باعتبار الحالات العملية للعمليات في صورة تلبي حاجة التطبيقات الواقعية. قامت الرسالة بمواءمة المخطط المذكور من كونه أداة لمراقبة موضع العمليات إلى أداة لمراقبة مدى استقرار تباين العمليات، وذلك بالاستعانة باستراتيجية مراقبة "المتغير المتحول". حيث يمثل هذا المتغير المتحول سلوك معلمة التشتت المراد دراسة استقرارها. يهدف استخدام هذه الاستراتيجية الى مراقبة متغير ذو توزيع طبيعي لتسهيل ايجاد القيم الإحصائية؛ مُسهله بذلك بناء مخطط التحكم.

درست الرسالة ايضًا مخطط "المتوسط التراكمي المشروط" لمراقبة الموضع باعتبار المنهج اللا معلمي لمخططات التحكم. حيث يمتاز المنهج اللا معلمي بمتانة نتائجه التحليلية في حال الإخلال بالافتراضات التي يتم فرضها خلال مرحلة بناء المخططات، على سبيل المثال: افتراض أن بيانات العمليات موزعة بشكل طبيعي. تقوم مخططات التحكم اللامعلمية على إحصائيات الاختبارات اللا معلمية باستخدامها كإحصائيات تخطيط لمخططات التحكم. في هذا الصدد، استخدمت الرسالة إحصائية اختبار الإشارة اللا معلمي في بناء مخطط التحكم المقترح.

CHAPTER 1

Introduction

1.1 Statistical Process Control

Statistical Process Control (SPC) was developed by Walter Shewhart at Bell Laboratories in the early 1920s. It is a data-driven field concerned with monitoring a process's output and verifying its conformity with certain specifications to ensure process stability and improve its quality. SPC involves the use of statistical methods that rely on the variation analysis of process parameters. SPC techniques are typically used in manufacturing applications, such as automobile manufacturing, chemical production, and the packaging industry. However, they can also be used in non-manufacturing applications, which are known as services, such as healthcare, public transportation, and education.

Quality of design and quality of conformance are two fundamental aspects that impact product quality, whether the product is a manufactured good or a service. The quality of design is the intentional planning and engineering of a product to satisfy customer requirements while reducing risks, guaranteeing fitness for use, and placing a priority on safety. It entails the proactive steps taken during the design stage, which may involve either creating new products or improving existing ones. The second aspect, the quality of conformance, focuses on the consistent adherence of a product to established standards, resulting in reliable performance, effective functionality, ease of maintenance, and the reduction of undesirable variations. SPC is concerned with improving process quality by addressing this second aspect of quality.

1.1.1 Process variation

The term quality in this context refers to the modern meaning of quality, which embraces the idea that some level of variation is natural, inherent, and acceptable in processes and products. This modern definition of quality differs from the traditional conception, which focuses on achieving uniformity and consistency in products.

The modern definition of quality has evolved and expanded significantly upon Shewhart's initial concept, which categorizes variation in processes into two types: natural and unnatural. These are often referred to as "common cause variation" and "special cause variation", respectively. Below is an explanation of these two types of variations:

- The common cause variation is inherent in any process and will always be present. It is caused by the combined effect of numerous small and often uncontrollable factors that influence the process, such as the nature of the processed material, the poor maintenance of machines, inappropriate procedures, the environmental conditions, etc. A process

that features only common causes variation is considered statistically in-control (IC), or by another term, is stable.

- In contrast, the special cause variation (also known as assignable cause variation) is which stem from external sources of the process and can produce unexpected and undesirable changes in process outputs. It is embedded in operator mistakes, machine malfunction, calibration issues, etc. The process that exhibits the two types of variation, common cause and special cause variations, is considered statistically out-of-control (OOC), unstable.

The presence of special cause variation affects the underlying probability distribution of the quality characteristic by introducing non-random, identifiable factors that influence the data. These factors can lead to shifts or changes in the distribution's parameters. Considering these consequences of special cause variation, the distinguishing between common cause and special cause variations becomes crucial for providing insights into process behavior, determining whether the process is stable or not, reducing waste in process materials, and preventing defects. Unambiguously, distinguishing between the types of variation contributes to enhancing process quality.

SPC has many established tools for studying process variation, including histograms, Pareto charts, cause-and-effect diagrams, scatter plots, and control charts. The primary tool and the subject of this thesis is the control chart. This thesis provides a comprehensive introduction to the control chart, reviews related literature and enhancement techniques for the control chart as well as suggests two chart structures.

1.2 Control chart

The most widely used tool in SPC is the control chart. Control charts monitor the behaviour of process parameters, such as location, dispersion, shape parameters (e.g., skewness, kurtosis), etc., for the purpose of detecting any unnatural variation that is considered as a shift and may impede the process from being statistically IC. Control chart (Figure 1.1) is defined as a graph that depicts the behaviour of a quality characteristic, say X , over time to confirm its stability. Two components are needed to construct the control chart: a plotting statistic and control limits. The plotting statistic (also known by the chart statistic) is shown as a series of points in the chart, each point represents the test statistic of a specific sample after applying a particular structure to it. Where the test statistic is selected in a way that it represents the behavior of the concerned parameter of the quality characteristic distribution. The control limits: Lower Control Limit (LCL) and Upper Control Limit (UCL) are the lines at which the process gives an OOC signal if a value of the plotting statistic falls on or beyond them; Central Line (CL) represents the expected (target) value of the plotting

statistic. For the IC process, observations of the plotting statistic are distributed around the central line. The central line is called also as the target line or the reference line.

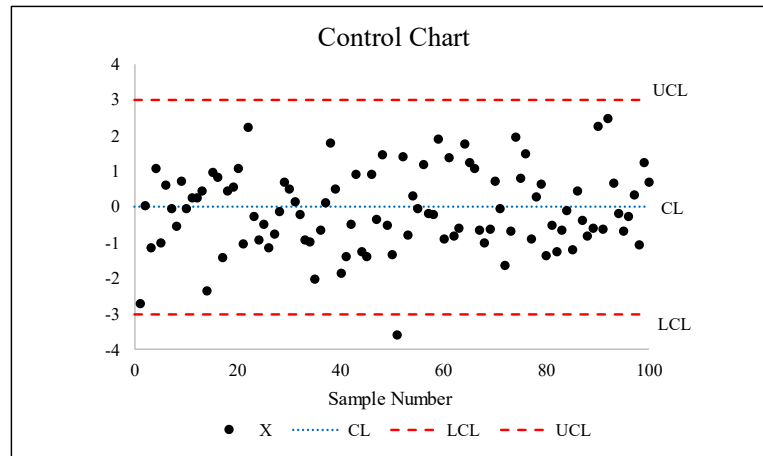


Figure 1. 1 Control chart.

1.2.1 Formulating hypotheses to assess process stability

To assess the stability of a quality characteristic's behaviour, it is imperative to systematically monitor the parameters that define its distribution. This involves calculating a representative statistic, known as a test statistic, for each sample. The key parameters that are typically monitored include the location, dispersion, and shape parameters. They can be described as follows.

1. **Location Parameter:** Also referred to as the central parameter, this parameter denotes the centre or location of a probability distribution. It signifies the value around which the observations tend to cluster. The location parameter is commonly studied using test statistics like the sample mean or sample median.
2. **Dispersion Parameter:** Often termed the scale parameter, the dispersion parameter characterizes the spread or variability of a probability distribution. It quantifies how data points are scattered or dispersed around the location parameter. A smaller value of the dispersion parameter indicates that observations are closely clustered around the process location, signifying greater stability. Test statistics like sample variance or sample range are typically employed to study the process dispersion.
3. **Shape Parameter:** The shape parameter describes the form or shape of a probability distribution. It influences specific attributes of the distribution, such as symmetry, skewness, or the presence of multiple peaks. Test statistics such as the coefficient of skewness, kurtosis, or moments can be employed to study the shape parameters.

Monitoring these parameters enables a comprehensive assessment of the behaviour of a quality characteristic over time and helps ensure process stability and, ultimately, products quality.

Understanding these key process parameters lays the foundation for selecting appropriate control charts, which can be categorized based on whether they monitor these parameters separately or jointly, as well as whether they are designed for univariate (individual) or multivariate analysis. When monitoring separately, the focus is given to one parameter at a time, such as the location, dispersion, or shape parameter, using specific control chart designed for that parameter. On the other hand, when monitoring jointly, the consideration is given to multiple parameters simultaneously. These joint charts analyze the relationships between parameters and can signal when any combination of parameters deviates from the expected behaviour. They are particularly valuable when changes in one parameter may affect others, providing a more comprehensive view of process stability.

The second dimension of control chart types comprises univariate (individual) and multivariate charts. Univariate charts are tailored to focus on a single quality characteristic (a study variable) at a time with one process parameter (e.g., location, dispersion) per chart. They assume independence between quality characteristics being monitored. On the other hand, multivariate control charts are designed to monitor and control multiple quality characteristics (multiple study variables) simultaneously. Multivariate charts consider potential dependencies or correlations between the quality characteristics (the study variables).

The standard approach to assess process stability and quality is to express it in the form of a hypothesis. The Null Hypothesis (H_0) states that the process is IC, meaning any observed variations are within the expected range due to common causes. The Alternative Hypothesis (H_1), on the other hand, suggests that the process is OOC, indicating significant variations beyond what is expected, primarily due to special causes.

Let shift amount be denoted by δ for shift in location, γ for shift in dispersion, the target location by μ_0 , and the target dispersion by σ_0 , then mathematically, based on the type of concern, the hypotheses for testing the stability of a univariate (individual) quality characteristic ignoring the effect of the sample size n can be expressed as follows.

The test hypothesis for monitoring the process location:

$$H_0: \mu = \mu_0$$

$$H_1: \mu = \mu_1 = \mu_0 + \delta$$

The test hypothesis for monitoring the process dispersion:

$$H_0: \sigma = \sigma_0$$

$$H_1: \sigma = \sigma_1 = \gamma\sigma_0$$

The test hypothesis for monitoring the process location and dispersion simultaneously:

$$H_0: \delta = 0 \text{ \& } \gamma = 1$$

$$H_1: \delta \neq 0 \text{ or } \gamma > 1$$

for $X \sim f(\mu_0 + \delta\sigma_0, \gamma\sigma_0)$, where $f(\cdot)$ is the distribution of the quality characteristic X .

1.2.2 Performance evaluation

The effectiveness of a control chart is typically determined by evaluating its ability to detect shifts, i.e., the OOC signals, promptly using a small number of samples and maintaining this efficiency across various monitoring scenarios. This evaluation involves using measures that can be categorized as either individual or overall performance measures. The individual measure assesses the efficiency of a control chart for a single value of shift magnitude, and the most widely used measures of this kind are based on Run Length (RL). RL is the number of samples before an OOC signal is produced. It has the characteristic of the random variables, and its properties are used as performance measures, such as Average of Run Length (ARL), Standard Deviation of Run Length ($SDRL$), Median of Run Length ($MDRL$), k^{th} Percentile of Run Length (P_kRL), Average Time to Signal (ATS), and RL-curve. Alternatively, the performance measure that assesses the overall effectiveness of a control chart for a range of shift magnitudes is defined as an overall measure, such as Extra Quadratic Loss (EQL). EQL is described as the average of weighted ARL 's that are corresponding to a range of δ 's, where the weights are determined by the square of the shifts. Notationally,

$$EQL = E_{\delta}(\delta^2 ARL(\delta)) = \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta) f_{\delta}(\delta) d\delta$$

where $E_{\delta}(\cdot)$ and $f_{\delta}(\delta)$ are respectively the expected and probability functions of the shift.

The purpose of weighting by δ^2 is to ensure the precise representation of the influence of each value of ARL to the loss, and this is important since ARL values decrease as δ increases. When the distribution of δ is uniform (i.e., equidistant shifts) in a domain of $(\delta_{min}, \delta_{max})$, then its density function is given by $f_{\delta}(\delta) = \frac{1}{\delta_{max} - \delta_{min}}$, and hence EQL is computed as follows.

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta) d\delta$$

For the equidistant shifts, the integration in EQL can be approximated using Riemann Sum, see (Appendix 1-A) for details, the resulting formula becomes as follows.

$$EQL \approx \frac{1}{m+1} \sum_{i=0}^m \delta_i^2 ARL_{\delta_i} \quad (\delta = 0 \text{ included})$$

$$EQL \approx \frac{1}{m} \sum_{i=1}^m \delta_i^2 ARL_{\delta_i} \quad (\delta = 0 \text{ excluded})$$

where, m is the number of shifts considered over the whole process shifts' domain $\delta_{min} < \delta < \delta_{max}$.

While there is no single term explicitly represents "loss" in the EQL formula, EQL itself serves as a measure of process instability. Therefore, when EQL is higher, there is a greater potential for quality losses due to process variability. According to this concept of EQL and in the context of comparing the performance of two charts, the chart with a smaller EQL is considered superior.

ARL is the expected value of the run length and comes in two forms: in-control ARL (ARL_0) and out-of-control ARL (ARL_1). ARL_0 indicates the expected number of samples required for an OOC signal to appear when the process is actually in control, i.e., the signal is a false alarm. A large ARL_0 is desirable in this scenario. On the other hand, ARL_1 indicates the expected number of samples required for an OOC signal to appear when the process is already out of control. In this case, it is desirable for ARL_1 to be as small as possible. The same forms of in-control and out-of-control notations are applicable to the other metrics, e.g., $SDRL_0$, $SDRL_1$, and $MDRL_0$, $MDRL_1$.

The term change-point, denoted by τ , refers to the sample number at which a shift occurred. Controlling a process of unity change-point, i.e., $\tau = 1$, is considered under zero-state monitoring. Whereas controlling a process with shift occurring at a sample other than the first sample, i.e., $\tau > 1$, is considered under steady-state monitoring. It was observed in the literature that, using the measure of ARL for the steady-state, gives a deceptive performance due to the impact of the false alarms before the change-point. Alternatively, the measure of Conditional Expected Delay (CED) is used to evaluate the chart performance in the case of steady-state, which is simply an average of run length that has been counted after the change-point τ , conditioned by no false alarm has been raised before the change-point. Mathematically, $CED(\tau) = E[RL - \tau + 1 \mid RL \geq \tau]$.

To gain a thorough understanding of control chart theory and its practical application, there is a compelling need to explore and analyze it through simulation. Several simulation techniques are available for this purpose. The most essential method is Monte Carlo Simulation. This approach allows for delving into control chart performance under diverse scenarios, providing valuable insights for optimizing their design, parameter settings, and overall effectiveness in maintaining process quality and stability.

Using Monte Carlo Simulation to study the performance of the control chart allows using a measure of Probability to Signal (PTS). It is the probability to alarm the process as OOC and this alarm could be false or correct alarm. When the signal is raised for IC process, it is a false alarm, but if the signal raised for OOC process, it is a correct alarm. In hypothesis

terms, considering the independence between observations, the *PTS* can be defined as follows.

- *PTS* under H_0 is the probability of type-I errors, α . "False Alarm Probability (*FAP*)"
- *PTS* under H_1 is the power, $1 - \beta$, where β is the probability of type-II errors.

While *FAP* is a user-specified parameter that represents the probability of having a false alarm when the process is in control, False Alarm Rate (*FAR*), on the other hand, represents the rate at which false alarms occur, typically expressed as the number of false alarms per a number of observations. In the simulation context, *FAR* is calculated by counting how many false alarms occurred relative to the total number of simulations. The *FAR* is a way to assess how well the control chart, under a given set of conditions, conforms with the intended *FAP*. *FAP* can be considered as a theoretical value, while *FAR* is a performance metric calculated from real or simulated data to evaluate the performance of charts.

In Figure 1.2, the *PTS* is the shaded area outside the control limits, where the process raises an OOC signal.

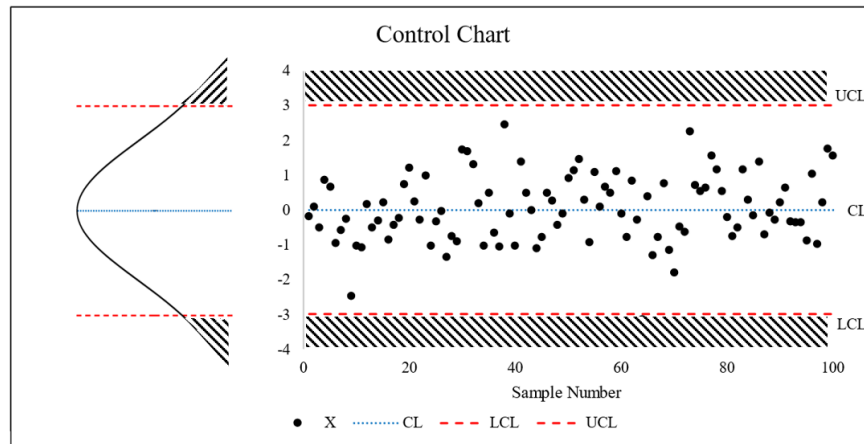


Figure 1. 2 Illustrating PTS area.

1.2.3 Implementing a control chart

As mentioned earlier, control charts require two primary components for implementation: the plotting statistic and the control limits. To construct the control limits, an additional component known as the acceptance amount is needed where it represents the accepted amount to have false alarms and regulates the power of the chart. For some charts the acceptance amount is the probability of type-I errors whereas for others it is known by sensitivity parameter as it will be illustrated later. The following is an explanation of each of these components.

- The plotting statistic represents the observations of the quality characteristic (study variable) after a specific designed structure is applied to their test statistics (variable of interest), as it will be illustrated in the next section.
- The sensitivity parameter determines an acceptable amount of deviation in the observations from the target to be proceed as IC. A smaller value of the sensitivity parameter results in a more powerful control chart, and the opposite holds true. Typically, it is pre-defined at a specific value based on a desired ARL_0 .
- The control limits are determined using a limit coefficient, denoted as L , which is derived by compromising between the desirable ARL_0 and the value of the sensitivity parameter. "L-sigma limits" is a method of constructing limits and it is preferable if the plotting statistic has a symmetric distribution. It formulates the lower and upper control limits by:

$$E(P) \pm L SD(P)$$

where $E(\cdot)$ and $SD(\cdot)$ are, respectively, the expected value and the standard deviation of a variable, P is the plotting statistic, and L is the limits coefficient.

It is important to note that a larger L results in wider limits, increasing the probability of type-II errors, while a smaller L leads to narrower limits, increasing the probability of type-I errors.

The central line is always given by $CL = E(P)$.

If the plotting statistic has an asymmetric distribution, the method of the "probability limits" is more preferable for constructing the control limits. It computes the limits based on a given level of type-I error (α), by finding LCL & UCL that satisfy the following equations.

$$\int_{-\infty}^{LCL} f_P(P) dP = \frac{\alpha}{2} \quad \& \quad \int_{UCL}^{\infty} f_P(P) dP = \frac{\alpha}{2}$$

where $f_P(P)$ is the probability function of the plotting statistic P .

Simply, the LCL & UCL using the probability limits method are the quantiles of the $f_P(P)$ that associated with $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$, respectively.

Control charts possess several characteristics and structures designed to address real-world applications across various scenarios. These variations in application requirements have led to the development of different types of control charts. Understanding the different types of control charts is crucial for selecting the most appropriate one based on the specific needs

of a particular application. The next section provides a comprehensive exploration of these control chart types, offering their structures and distinct features.

1.3 Types of control chart

The control charts can be classified into two major types based on the size of shift to be detected, memory and memoryless charts, where the memory chart concerns detecting small/moderate disturbances in a process by designing the plotting statistic in a way that accumulates the shifts of the samples through allocating weights to the current and past information of samples in the plotting statistic. On the contrary, the memoryless chart is used to detect shifts of large size by using only the current sample information.

1.3.1 Shewhart's control chart

Shewhart's control chart (Montgomery, 2020) forms the foundation of memoryless charts. Its plotting statistic is essentially the test statistic derived from the samples, which serves to represent the behaviour of the parameter of concern. For example, let process observations X be identically and independently distributed normally with known parameters, i.e., $X \sim N(\mu_0, \sigma_0)$ where μ_0 and σ_0 are the target IC mean and standard deviation, respectively. If the variable of interest (the test statistic) for the process is denoted by $\hat{\theta}$, then the Shewhart- $\hat{\theta}$ control chart is defined by the following characteristics.

The plotting statistic: $\hat{\theta}_i$, i is the sample number.

The control limits, based on L -sigma limits: $E(\hat{\theta}) \pm L SD(\hat{\theta})$

The limit coefficient L in the memoryless charts can be found theoretically since the RL in the memoryless charts follows the geometric distribution based on the following details.

- RL is the number of samples to reach 1st OOC signal.
- Samples are independent.
- Probability to get first OOC signal is fixed at α , the probability of type-I errors.

Therefore, in the memoryless charts, RL follows the geometric distribution.

i.e.,

$$RL \sim \text{Geometric}(\alpha)$$

Hence, the average run length ARL is given by $ARL = \frac{1}{\alpha}$.

This relation, $ARL = \frac{1}{\alpha}$, allows for determining the value of α by a given desirable ARL_0 . Thus, the limit coefficient L can be computed by the $\left(\frac{\alpha}{2}\right)^{th}$ quantile of the distribution of the plotting statistic for the memoryless control charts.

The idea of Shewhart's structure (the memoryless structure) is to directly monitor the current sample information without considering the past information. Each point in the memoryless chart represents a single sample, this grants the chart to specifically detect shifts of large size.

On the other hand, the control charts of Cumulative Sum CUSUM (Page, 1954) and Exponential Weighted Moving Average EWMA (Roberts, 1959) are the popular memory charts for monitoring the behaviour of the process parameters to detect small/moderate shifts. Each of them has a different structure. Let the process observations X be identically and independently distributed normally, i.e., $X \sim N(\mu_0, \sigma_0)$, and the test statistic $\hat{\theta}$ is computed for each sample to study the stability of process parameter θ , then their structures for monitoring process location, i.e., $\theta = \mu$, are as follows.

1.3.2 Cumulative sum CUSUM

The cumulative sum CUSUM (Figure 1.3), as the name says, its plotting statistic is the cumulative sum of deviations on the test statistic $\hat{\theta}$ from the target parameter θ_0 over the samples. Mathematically, the CUSUM statistic is $C_i = (\hat{\theta}_i - \theta_0) + C_{i-1}$, where i is the number of sample. Since this simple formula will cumulate both kinds of shifts, positive/negative shifts, then the different directions of shift, positive/negative, that may occur will pull each other causing a delay in detection. To avoid such this situation, each direction of shifts should be monitored individually by the following formulas,

$$C_i^+ = \text{Max}[0, (\hat{\theta}_i - \theta_0) - K + C_{i-1}^+]$$

$$C_i^- = \text{Min}[0, (\hat{\theta}_i - \theta_0) + K + C_{i-1}^-]$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

Where $K = k\sigma_{\hat{\theta}}$ and is known by the reference value (or the allowance value). k is the sensitivity parameter of the chart, and to minimize the average run length of shifted process ARL_1 , k is recommended to be small. It is usually defined by:

$$k = \frac{\delta}{2} \sigma$$

δ is the amount of shift to be detected in σ units. For the location parameter, $\theta = \mu$, and hence, the shift size can be expressed in sigma unit based on the formula $\mu_1 = \mu_0 + \delta\sigma$. Therefore, the sensitivity parameter can be given in the form $k = \frac{1}{2} \frac{|\mu_1 - \mu_0|}{\sigma} \sigma = \frac{|\mu_1 - \mu_0|}{2}$, for more details see (Bissell, 1969). If the influence of the sample size n has to be taken into consideration, σ equals $\sigma_{\hat{\theta}}$. Instead, σ_0 represents σ when the sample size n is not of interest.

Once k has been selected, the chart statistics are plotted against the control limit $\pm H = \pm h\sigma$, where h is the limits coefficient and is chosen such that, it gives the desired ARL_0 . These limits are expressed based on the method of L -sigma limits by $0 \pm h\sigma$, where h is equivalent to L . And as the stability refers to have small deviations on the test statistic of a sample i from the target parameter (i.e., $(\hat{\theta}_i - \theta_0) \rightarrow 0$), the CUSUM statistics are expected to be around zero for an IC process, and hence, the central line is at zero (i.e., $CL = 0$).

It is to be noted that the upper CUSUM statistic (C_i^+) monitors the positive shift of the process observations and the lower one (C_i^-) monitors the negative shift, and they will be equal to zero if there is no deviation has been observed from the target parameter.

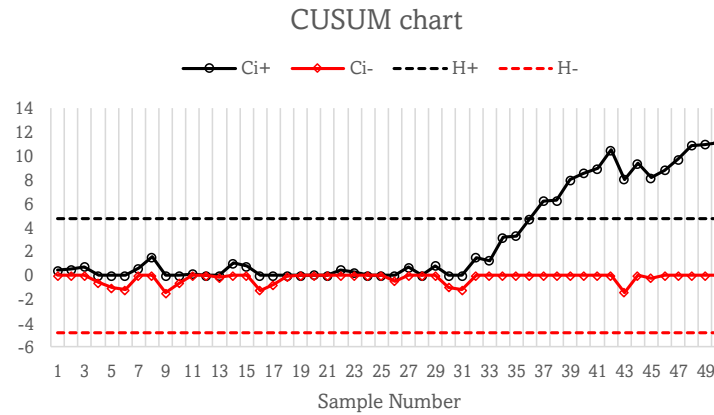


Figure 1. 3 The CUSUM control chart.

1.3.3 Exponential weighted moving average EWMA

The alternative memory structure is the EWMA control chart (Figure 1.4) which has the advantage that it is simpler to set up and use than the CUSUM control chart.

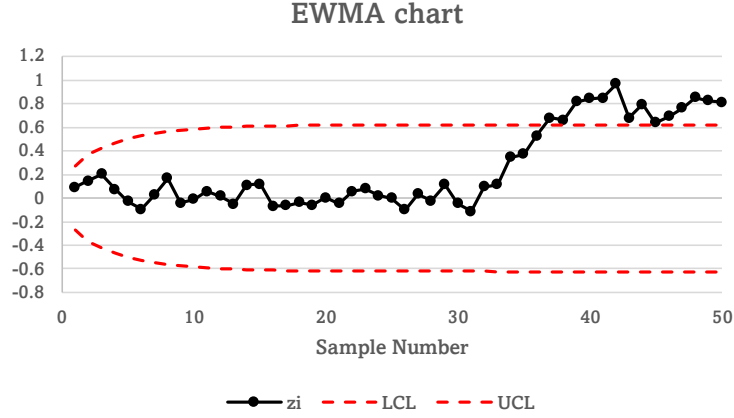


Figure 1. 4 The EWMA control chart.

The EWMA structure, as its name suggests, involves continuously averaging data points by allocating a specific weight to the most recent observation, and the remaining weights are given to the past data, where the weights for earlier data decrease exponentially. Incorporating the past information by giving a small amount of weight to the recent data makes the chart respond to small to moderate shifts efficiently. Conversely, placing a large amount of weight on recent data points makes the EWMA more responsive to large shifts and less sensitive to small to moderate disturbances.

The EWMA plotting statistic is defined by the formula:

$$Z_i = \lambda \hat{\theta}_i + (1 - \lambda)Z_{i-1}$$

for $i = 1, 2, 3, \dots$

with starting value $Z_0 = \mu_0$.

Where λ is known as a weight parameter that is given to the information of the current sample $\hat{\theta}_i$, and it is known also as a smoothing parameter, its range lies between the interval $0 < \lambda \leq 1$. The remaining weights $(1 - \lambda)$ is given to the past samples Z_{i-1} .

Following the steps in (Appendix 1-B), the EWMA plotting statistic can be written as:

$$Z_i = \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j \hat{\theta}_{i-j} + (1 - \lambda)^i Z_0$$

This form of the EWMA statistic facilitates deriving the mean and standard deviation of the EWMA statistic and allows concluding the following.

- The weights $\lambda(1 - \lambda)^j$ decrease geometrically over time (i), so that, the EWMA chart is sometimes called as a Geometric Moving Average (GMA).
- The sum of the weights is equal to 1. See (Appendix 1-C).

The mean and standard deviation of the EWMA statistic are needed to construct the chart limits based on the method of L -sigma limits, i.e., $E(Z_i) \pm L SD(Z_i)$. The following parts are the derivations of the mean and variance of the EWMA statistic.

Mean of the EWMA plotting statistic:

Since $Z_i = \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j \hat{\theta}_{i-j} + (1 - \lambda)^i Z_0$, then

$$E(Z_i) = E \left[\lambda \sum_{j=0}^{i-1} (1 - \lambda)^j \hat{\theta}_{i-j} + (1 - \lambda)^i Z_0 \right] = \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j E(\hat{\theta}_{i-j}) + (1 - \lambda)^i Z_0$$

Assuming $Z_0 = \mu_0$,

$$E(Z_i) = \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j \mu_0 + (1 - \lambda)^i \mu_0 = \mu_0 \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j + (1 - \lambda)^i \mu_0$$

Replacing $\lambda \sum_{j=0}^{i-1} (1 - \lambda)^j$ by $1 - (1 - \lambda)^i$ (as the sum is of finite geometric series with $r = (1 - \lambda)$ and $n = i - 1 + 1 = i$), then

$$E(Z_i) = \mu_0 [1 - (1 - \lambda)^i] + (1 - \lambda)^i \mu_0 = \mu_0 [1 - (1 - \lambda)^i + (1 - \lambda)^i] = \mu_0$$

It is concluded that $E(Z_i) = \mu_0$.

Variance of the plotting statistic:

$$\begin{aligned} Var(Z_i) &= Var \left[\lambda \sum_{j=0}^{i-1} (1 - \lambda)^j \hat{\theta}_{i-j} + (1 - \lambda)^i Z_0 \right] = \lambda^2 Var \left[\sum_{j=0}^{i-1} (1 - \lambda)^j \hat{\theta}_{i-j} \right] \\ &= \lambda^2 \sum_{j=0}^{i-1} Var[(1 - \lambda)^j \hat{\theta}_{i-j}] = \lambda^2 \sum_{j=0}^{i-1} (1 - \lambda)^{2j} Var[\hat{\theta}_{i-j}] = \lambda^2 \sum_{j=0}^{i-1} (1 - \lambda)^{2j} \sigma_{\hat{\theta}}^2 \end{aligned}$$

$$\begin{aligned}
&= \sigma_{\theta}^2 \lambda^2 \sum_{j=0}^{i-1} [(1-\lambda)^2]^j = \sigma_{\theta}^2 \lambda^2 \left[\frac{1 - (1-\lambda)^{2i}}{1 - (1-\lambda)^2} \right] = \sigma_{\theta}^2 \lambda^2 \left[\frac{1 - (1-\lambda)^{2i}}{1 - (1-2\lambda+\lambda^2)} \right] \\
&= \sigma_{\theta}^2 \lambda^2 \left[\frac{1 - (1-\lambda)^{2i}}{2\lambda - \lambda^2} \right] = \sigma_{\theta}^2 \lambda^2 \left[\frac{1 - (1-\lambda)^{2i}}{\lambda(2-\lambda)} \right] = \frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}] \sigma_{\theta}^2
\end{aligned}$$

Therefore,

$$Var(Z_i) = \frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}] \sigma_{\theta}^2$$

Since asymptotically $[1 - (1-\lambda)^{2i}]$ goes to 1, see (Appendix 1-D), then

$$Var(Z_i) = \frac{\lambda}{2-\lambda} \sigma_{\theta}^2$$

At $\lambda = 1$, the maximum possible value of λ , the Shewhart setup is generated,

$$Var(Z_i) = \sigma_{\theta}^2$$

EWMA control limits:

Based on the mean value and the variance of the EWMA statistic, the limits of the EWMA control chart are as follows.

The central line:

$$CL = E(Z_i) = \mu_0$$

The upper and lower control limits using the L -sigma, i.e., $E(Z_i) \pm L SD(Z_i)$, are:

$$\mu_0 \pm L \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}] \sigma_{\theta}^2}$$

These limits are called "Time Varying Limits" and sometimes expressed asymptotically by employing the fact that $[1 - (1-\lambda)^{2i}] \rightarrow 1$, which yields the following control limits:

$$\mu_0 \pm L \sqrt{\frac{\lambda}{2-\lambda} \sigma_{\theta}^2}$$

In case the weight is given entirely to the current sample (i.e., $\lambda = 1$) the Shewhart setup limits are generated:

$$\mu_0 \pm L \sigma_{\theta}^2$$

The CUSUM and EWMA control charts can be established also to analyze process dispersion. The details of monitoring process dispersion will be provided in Chapter 3.

In addition to the essential control charts, i.e., Shewhart, CUSUM, and EWMA, researchers in the field of SPC have introduced a diverse array of modern control charts. These contemporary charts are designed to expand the techniques for effective quality control. The following subsections introduce only two of these modern structures for their relevance to the thesis topic.

1.3.4 Progressive mean PM

The modern Progressive Mean (PM) control chart (N. Abbas et al., 2013) has arisen as a competitor to the existing memory control charts, CUSUM and EWMA, for its superior capacity to detect shifts by small numbers of samples as well as it is simpler to implement. To monitor a process location, the PM statistic is defined as a cumulative average of sample statistic, say $\bar{X}$, over time by allocating equal weights to each sample, mathematically:

$$PM_i = \frac{\sum_{j=1}^i \bar{X}_j}{i}, i = 1, 2, \dots$$

It exhibits a good performance detecting process shifts by small number of samples, but the authors mentioned an important note that the PM chart is not efficient in case of the steady-state monitoring – it shows a poor performance in detecting process shifts when the occurrence of shift is delayed from the start of monitoring. In this regard, (Knoth et al., 2021) criticized the PM performance by illustrating the *CED* curve of the chart at different change-points for different shift sizes against the EWMA chart performance (Figure 1.5). It is apparent that the PM control chart's *CED* curve never stabilizes at a particular level.

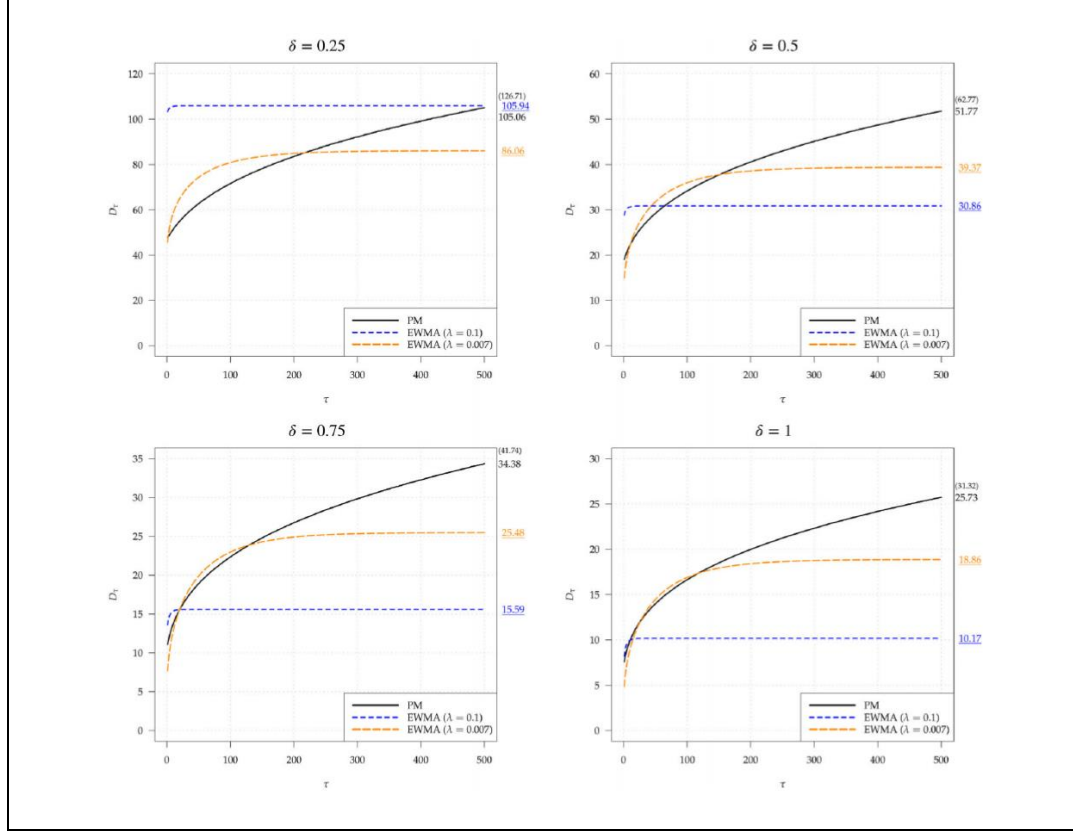


Figure 1.5 CED curves of PM control chart against EWMA control chart, provided by Knoth et al. (2021).

The reason of giving a poor performance in the steady-state case can be explained by having a closer look at the PM's plotting statistic:

$$PM_i = \frac{\sum_{j=1}^i \bar{X}_j}{i} = \frac{\bar{X}_1}{i} + \frac{\bar{X}_2}{i} + \frac{\bar{X}_3}{i} + \dots + \frac{\bar{X}_i}{i}$$

The PM's plotting statistic assigns an identical amount of weight ($1/i$) to every sample of data from the beginning of monitoring until the i^{th} sample. That whenever a shift occurs at the i^{th} sample, the PM's plotting statistic always includes all the information from the IC observations before the shift with a weight of $(1 - 1/i)$. This forces the plotting statistic to be within the control limits, delaying the chart's declaration of the shifted process as OOC. Evidently, as the shift delays from the start of the monitoring, the PM's plotting statistic carries heavier weights of IC observation, making the PM chart significantly less effective. Denoting change-point, τ , as the sample number at which the shift occurred. It is noticeable that as τ gets larger the PM performance becomes inferior, i.e., the CED curve of the PM becomes larger and never gets stabilized. Floating PM control chart (N. Abbas et al., 2014b) follows the same structure and drawback of the PM control chart, but it is used for monitoring process dispersion, and its details will be shown in Chapter 3.

The prevention state of declaring OOC signal due to heavy weights of IC observations that are carried in the chart statistic is a known phenomenon in SPC called by the inertia effect. To counteract this inertia effect, the Fast Initial Response (FIR) feature, sometimes also

referred to as the head-start feature, is utilized. It involves assigning non-zero values to the starting values of a chart's structure. This initial non-zero setting enables the control chart to respond more quickly to shifts or deviations from the expected process behavior, thus reducing the possibility of delayed detection caused by the inertia effect. The next modern chart is considered as an adjustment of the PM chart, and in somehow, it utilizes the concept of the head-start feature, the clarification will be provided later.

1.3.5 Conditional aggregate mean CAM

(N. Abbas, 2023) suggested addressing the aforementioned drawback of the PM chart by appending a condition to start aggregating the samples information only if a deviation in the samples is noticed. For that, a counting variable is needed to start counting the number of samples when noticing a deviation and to reset the memory of the chart statistic in case no deviation has been noticed; hence, reset the control limits. This counter variable is also used as a weighting parameter for the chart. It works based on an indicator variable that indicates if a deviation of the samples' means from the target location is observed, and as an appropriate choice, the CUSUM statistics are used as indicator variables.

$$\begin{aligned} C_i^+ &= \text{Max} \left[0, \bar{X}_i - \left(\mu_0 + k \frac{\sigma_0}{\sqrt{n}} \right) + C_{i-1}^+ \right] \\ C_i^- &= \text{Max} \left[0, \left(\mu_0 - k \frac{\sigma_0}{\sqrt{n}} \right) - \bar{X}_i + C_{i-1}^- \right] \end{aligned} \quad (1.1)$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

Where the lower CUSUM statistic monitors the negative deviation and the upper one monitors the positive deviation. The counter variables can be defined as follows.

$$\begin{aligned} I_i^+ &= \begin{cases} 1 & , \text{if } C_{i-1}^+ = 0 \\ I_{i-1}^+ + 1 & , \text{otherwise} \end{cases} \\ I_i^- &= \begin{cases} 1 & , \text{if } C_{i-1}^- = 0 \\ I_{i-1}^- + 1 & , \text{otherwise} \end{cases} \end{aligned} \quad (1.2)$$

for $i = 1, 2, 3, \dots$

where C_{i-1}^+ and C_{i-1}^- are defined in equations (1.1).

Hence, the structure of the Conditional Aggregate Mean (CAM) control chart is given by the following plotting statistics.

$$CAM_i^+ = \frac{1}{I_i^+} \bar{X}_i + \left(1 - \frac{1}{I_i^+}\right) CAM_{i-1}^+$$

$$CAM_i^- = \frac{1}{I_i^-} \bar{X}_i + \left(1 - \frac{1}{I_i^-}\right) CAM_{i-1}^-$$

where I_i^+ and I_i^- are defined in equations (1.2).

They are plotted against the upper and lower control limits of the L -sigma limits form, i.e., $E(CAM_i) \pm L SD(CAM_i)$ where L is the limits coefficient, $E(CAM_i) = \mu_0$ and $SD(CAM_i) = \frac{\sigma_X}{\sqrt{nI_i}}$.

Therefore,

$$UCL_i = \mu_0 + L \frac{\sigma_X}{\sqrt{nI_i^+}}$$

$$LCL_i = \mu_0 - L \frac{\sigma_X}{\sqrt{nI_i^-}}$$

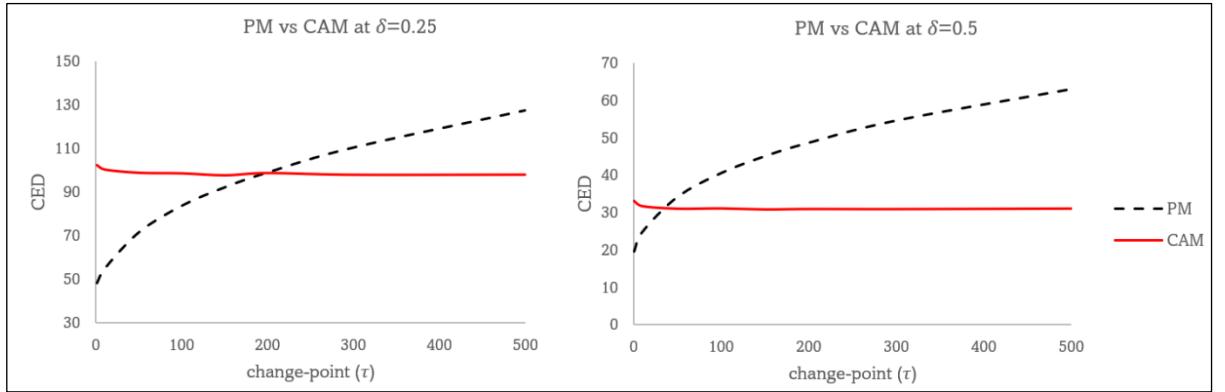


Figure 1. 6 PM vs CAM performance.

Comparing the CED curves of the PM chart to the CAM chart for the steady-state (Figure 1.6) shows that the proposed idea of the CAM chart is effective in fixing the PM's drawback as it is noticeable how the CED 's of the CAM stabilizes as the change-point τ increases.

The approach of imposing a condition on the PM structure is an implicit utilization of the head-start feature, where the condition constantly determines which sample serves as the head-start. This determination is achieved through the indicator variable, which designates samples with non-zero values as the initial point for aggregating information in the chart statistic.

1.4 Thesis outline

This thesis concentrates the study in the CAM control chart for monitoring process dispersion and implements CAM structure under non-parametric setup for monitoring process location. The outline of the thesis is as follows.

Chapter 2 - Literature review of control charts

In this chapter, a review on the literature of control chart is conducted, presenting the recent developments and modifications suggested for the control chart to enhance its effectiveness in assessing the stability and quality control of processes.

Chapter 3 - CAM control chart for monitoring process dispersion

This chapter expands the application of the CAM structure to encompass the monitoring of process dispersion, addressing the practical necessity in various applications. This chapter explores the methods for monitoring dispersion parameters and employs the transformation method to monitor the non-normal test statistic that represents the dispersion parameter of interest. The transformation is applied in this context to convert the non-normal test statistic to an approximately normal variable, streamlining the development of the proposed chart. Within this chapter, the variance test statistic is employed as a representative variable for the dispersion parameter, producing the CAM- S^2 control chart. The proposed chart is then compared with existing essential charts, offering valuable insights into its potential advantages and benefits. To illustrate its effectiveness, this chapter includes a case study involving the monitoring of windmill rotor speeds.

Chapter 4 - Non-parametric CAM for monitoring process location

This chapter tackles situations where the underlying distribution of data is unknown or violates the normality assumption. This chapter adapts the CAM structure, originally designed with the assumption of normality, for nonparametric monitoring of the location parameter. It employs nonparametric analysis approaches, especially the sign test, to construct a nonparametric control chart. The sign test statistic serves as a plotting statistic for the chart, enabling robust process monitoring. Additionally, this chapter includes a

comparative analysis of the proposed chart against existing nonparametric EWMA control chart.

Chapter 5 - Summary and conclusions

In this chapter, the main findings and conclusions of this thesis are consolidated, offering a cohesive overview of the research's significance and implications. This chapter guides the reader through the study's key points.

CHAPTER 2

Literature reviews and enhancements history

Over the years, control charts have evolved through various developments and enhancements, leading to improve their performance and ability to meet clients' demands for highly accurate control charts. These advancements aim to save materials and time while elevating process standards. Furthermore, there is a growing recognition of the need to release some of the theoretical assumptions, that were initially established when designing control charts, in order to meet the practical demands of real-world processes. This chapter delves into the relevant literature, exploring key suggestions proposed to enhance the effectiveness of control charts.

2.1 Combining charts

The integration of various control chart methodologies is a strategic step to elevate charts performance. This involves the concept of combining a memory control chart with a memoryless one, resulting in a control chart effectively monitoring both small to moderate and large shift sizes. Shewhart's control chart serves as the foundational structure for memoryless charts, and in this context, it has been combined with various memory charts as follows.

(Lucas, 1982) proposed the combined Shewhart-CUSUM control chart, where the standardized CUSUM statistic and standardized current observation are used simultaneously as the statistic of the proposed chart and plotted against the limits of the standardized CUSUM to detect the small/moderate shifts, and the limits of Shewhart to detect large shifts, respectively. It offers advantages in detecting significant shifts in process means with minimal impact on the detection of smaller shifts and ARL_0 . However, the susceptibility of the combined chart to trigger an OOC signal with a single large observation sacrifices the robustness of the original CUSUM chart against outliers. Consequently, the combined scheme is most effective in laboratory settings where all extreme observations must be investigated. In contrast, in scenarios where resistance to outliers is crucial, and investigating all suspected outliers is not feasible, choosing CUSUM scheme may be more suitable.

For the EWMA control chart, the same producer has been suggested by (Lucas and Saccucci 1990), where Shewhart limits are added to the EWMA scheme so that an OOC signal is raised when the EWMA statistic exceeds the EWMA control limits or when the current observation goes beyond the Shewhart limits. This improvement proves equally effective for the EWMA control chart as it does for the CUSUM control chart. However, the combined Shewhart EWMA, like the combined Shewhart CUSUM, should not be

employed when the process occasionally produces outliers, as the Shewhart limits causes the combined chart to be less sensitive to outliers.

(N. Abbas & Riaz, 2022) suggested using the idea of combining Shewhart with the PM chart to address the drawback of the PM structure for the case of steady-state monitoring. In the proposed combined chart, the Shewhart statistic targets the delayed shifts whereas the PM targets the shifts that occur right from the start of the monitoring. The combined chart performs significantly better in steady-state than the classic PM chart, but with a slight sacrifice in zero-state for detecting smaller shifts compared to the classic PM chart.

2.2 Run rules

Instead of using usual control limits, the run rules strategy aims to identify shifts using two lines that called Action Line (AL) and Warning Line (WL). It makes use of the concept of a single-sample inspection where the action to declare a process as OOC is taken based on a single point. Run rules strategy is based on counting samples observations that locate between the warning and action lines, AL and WL, and declaring the process as OOC if k consecutive points out of n points lie between these lines, where k and n are pre-selected. The idea of run rules works in resistance of having a pattern in the process observation. (Page, 1955) presented the Shewhart with four types of the run rules in details. Run rules with Shewhart improves the chart performance as a memory property becomes appended to the Shewhart chart. For more about Shewhart control chart with run rules, see (Wheeler, 1983) and (Roberts, 1958). The run rules can be used also for memory charts, CUSUM and EWMA, to boost their performance in detection. (Riaz et al., 2011) and (N. Abbas et al., 2011) applied two run rules schemes to the CUSUM and EWMA control chart, respectively. The first scheme is, declaring the process as OOC if two consecutive points lie between the lines AL and WL. The second scheme is, declaring the process as OOC if two out of three consecutive points are plotted between the lines AL and WL. The CUSUM and EWMA with run rules give a reliable performance with the domination to EWMA of second run rules scheme.

2.3 Using adaptive limits

The CUSUM control chart is effective at detecting a specific size of shift, but it experiences a decrease in its ability to identify intervals of shifts; which is the case of real-world scenarios where shifts tend to be unpredictable. An adaptive CUSUM (ACUSUM) control chart, initially proposed by (Sparks, 2000) and extended by (Jiang et al., 2008), enhances the CUSUM's effectiveness in detecting a range of shift sizes. This enhancement is achieved by estimating the behavior of the sample statistic using the EWMA statistic to estimate the amount of shift to be detected. Consequently, it adapts the CUSUM sensitivity parameter accordingly using the formula $k = \delta/2$, where k represents the sensitivity

parameter. For multivariate process mean monitoring, (Dai et al., 2011) introduced the use of adaptive property with CUSUM. (Capizzi & Masarotto, 2003) proposed the adaptive EWMA (AEWMA) control chart, incorporating (Huber, 2011) and (Hampel et al., 2011) as weighting functions to predict the amount of shift.

2.4 Fast initial response feature

In memory control charts, when a shift occurs in a sample, the previous IC information that is stored in the memory of the chart statistic negatively affects the performance of the chart by delaying the shift detection. Hence, the strategy of using Fast Initial Response (FIR) is motivated by (Lucas & Crosier, 1982). It aims to counter the delay in detection caused by previous IC samples by assigning nonzero values for the starting values of a chart statistic. This ensures a rapid and more accurate response to shifts in the process, reducing the chances of overlooking significant deviations from the expected behavior. (Lucas & Crosier, 1982) examined the use of FIR property with the CUSUM control chart with different starting values: $C_0^+ = C_0^- = 0, \frac{h}{4}, \frac{h}{2}, \frac{3h}{4}$, where h is the limit coefficient. The nonzero starting values are known as head-start values, the authors recommended using a head-start of $h/2$ based on an inspection on ARL properties. They concluded that the FIR feature guarantees charts a faster detection of an OOC process, but for the IC process, the chart performance may be slightly reduced. Following the same details of the proposed idea of FIR in CUSUM, (Lucas & Saccucci, 1990) introduced FIR with EWMA and found that the FIR feature is effective for EWMA with λ less than or equal to 0.25.

2.5 Auxiliary information based control charts

The idea of incorporating the relationship of the variable of interest with another correlated variable in control charting was first inspired by (Mandel, 1969). Control charts that utilize an additional variable corresponding to the variable of interest are known as Auxiliary Information Based (AIB) control charts. (Riaz, 2008a) and (Riaz, 2008b) presented AIB-Shewhart control charts for monitoring process parameters, location and dispersion, respectively. (Sanusi et al., 2018) and (N. Abbas et al., 2014a) designed AIB-CUSUM and AIB-EWMA control charts, respectively, demonstrating the superior performance of these AIB charts compared to classical charts. The fact that must be highlighted here, is that, the aforementioned AIB control charts are constructed under the assumption that auxiliary variable is consistently stable. Some processes follow this assumption. However, in many processes the auxiliary variable might shift, which has a significant impact on the effectiveness of AIB control charts, particularly when the variables have a strong correlation. In this regard, (N. Abbas et al., 2022) proposed an innovative structure that compromises the efficiency and robustness of the AIB-chart against the shifts in the

auxiliary variable. The proposed modified AIB chart represents a generalized form of Shewhart and AIB charts and proven its reliability.

2.6 Phase I analysis and Phase II monitoring

In real-world applications, it is common to encounter situations where a process needs to be monitored, but there is a lack of information about its underlying process parameters. In this situation, statistical analysis employs inference techniques to retrospectively analyze a ‘reference data’ and utilize them to deduce estimates of the target process parameters, where the reference data is defined as the initial samples of an ongoing process after confirming that they are in control. This procedure is referred to as Phase I (retrospective) analysis, and in this context, control charting comprises two main steps: estimating unknown parameters and then initiating the process stability monitoring. On the other hand, treating the process parameters in the control charts as known is considered in Phase II (prospective) monitoring, where the underlying process parameters are chosen based on: intended outcomes, past studies, or the estimated values derived from Phase I analysis.

Extending the discussion about considering charting as hypotheses testing in Chapter 1, (Woodall, 2000) suggested that control charting resembles repeated hypothesis testing in the case of monitoring under Phase II, when the distribution and in-control parameters of the process are assumed to be known. However, for Phase I monitoring, where the parameters or distribution are unknown, control charting is more about exploratory data analysis and not directly equivalent to traditional hypothesis testing.

(Montgomery, 2020) pointed out that the focus in Phase I is on assessing the probability of detecting real problems rather than quantifying the expected time between false alarms. So, during Phase I, the emphasis is on understanding the process, setting up control limits, and ensuring that genuine issues are not missed. Hence, *ARL*, which measures the time between false alarms, may not align with these objectives, for that, the probability of detecting assignable causes is preferable to be a performance measure of Phase I analysis.

(Ghosh et al., 1981) explore how the Shewhart- $\bar{X}$ chart behaves when estimating the process variance from sample data. It reveals that the chart’s properties and the distribution of run lengths differ from cases where the variance is known, highlighting the impact of variance estimation on control chart performance.

(Jensen et al., 2006) review existing literature that explicitly examines the impact of parameter estimation on control chart properties. They offer some recommendations and suggest potential areas for future research in this field.

(Human et al., 2010) explored Shewhart structure for monitoring S^2 , S , and R charts when the mean and variance of a normally distributed process are unknown and estimated. They mentioned that the current applications often ignore the statistical dependence of signaling events and simultaneous comparisons, posing issues. The study proposes a methodology to

address these issues by calculating proper control limits to restrict the false alarm probability (FAP) to a specified nominal value (FAP_0).

The paper of (Schoonhoven et al., 2011) focuses on designing and analyzing the Shewhart control chart for monitoring standard deviation with estimated limits. It covers a wide range of statistical methods for estimating the in-control standard deviation during Phase I and then designing the control chart for real-time process monitoring in Phase II. The study assesses the performance of these design schemes in scenarios where Phase I data are both uncontaminated and normally distributed, as well as when they are contaminated. The paper introduces a robust estimation method based on the mean absolute deviation from the median, supplemented with a simple screening method. Notably, this approach proves to be efficient under normality and outperforms traditional estimators and several other robust methods in the presence of data contamination.

The (Zwetsloot et al., 2014) study investigates how the choice of Phase I estimation method influences the performance of the EWMA control chart in Phase II monitoring.

2.7 Non-parametric control charts

Parametric control charts are designed based on specific data distribution assumptions, such as normality. Non-parametric control charts are employed when either the assumptions of parametric control charts cannot be satisfied or when there is uncertainty about these assumptions. In case data distribution violates the assumed distribution, the use of parametric control charts can lead to inaccurate process monitoring by increasing likelihood of false alarms or missed deviations.

Non-parametric control charts utilize statistics of non-parametric tests to be as their chart statistics. These non-parametric tests are statistical techniques that do not rely on assumptions about the underlying data distribution. Instead, they assess data based on their order or ranks. Examples of non-parametric tests include sign-test, Mann-Whitney U-test, Kruskal-Wallis test, and Wilcoxon signed-rank test.

(Amin & Searcy, 1991) introduced the non-parametric EWMA control chart, utilizing Wilcoxon signed-rank statistics and ranking within groups. Their simulation results revealed that the efficiency of this chart varies with the data distribution. It may be less efficient than parametric methods for normal distributions but notably more efficient for heavy-tailed distributions. Moreover, their procedure demonstrates minimal sensitivity to the choice of the weighting parameter. (Hackl & Ledolter, 1991) explores the performance of a rank-based EWMA control chart. The results suggest that the rank-based EWMA exhibits only a slight increase in the average run length compared to the traditional EWMA when the data conforms to a normal distribution. However, this minor reduction in efficiency is negligible when considering the substantial improvements offered by the rank-based EWMA for datasets with heavy-tailed distributions. Despite its advantages, the rank-

based EWMA chart does have certain limitations. While the traditional EWMA can detect shifts with a single extreme observation, making it sensitive to outliers, the rank-based EWMA is less sensitive to outliers but may require multiple observations to signal a shift, even if that shift is substantial. (Amin et al., 1995) implemented non-parametric Shewhart charts using sign-test statistic for monitoring location and dispersion parameters, and the idea of improving Shewhart by introducing the warning limits is employed, as well as suggested non-parametric CUSUM chart using sign-test statistic for monitoring location. They also discussed the features of sign control charts and the impact of non-normality on the $\bar{X}$ chart. For more of non-parametric control charts, see (Chakraborti et al., 1999), (S. Bakir, 2001) , and (Chakraborti & Graham, 2019).

CHAPTER 3

CAM- S^2 Control Chart

This chapter aims to study the Conditional Aggregate Mean (CAM) chart for monitoring process dispersion using sample variance. The resulting structure is denoted as the CAM- S^2 control chart. It is developed by following the same structure of the CAM control chart (introduced in Chapter 1), which consistently demonstrates its effectiveness in detecting shifts in both potential scenarios of shift occurrence, zero and steady states.

Process dispersion, in general, can be controlled by the classical method which directly applies the chart structure on a dispersion statistic, (R , S^2 , S , ... etc), see CUSUM- R by (Page, 1954), CUSUM- S by (Tuprah & Ncube, 1987), EWMA- R by (Ng & Case, 1989), and EWMA- S^2 by (Chang & Gan, 1994). Alternatively, the process dispersion can also be monitored by using transformation methods, which is the idea that has been motivated by (Chang & Gan, 1995). The strategy of monitoring the process dispersion using transformation methods aims to deal with a test statistic that approximately follows normal distribution, which will facilitate constructing the chart and its control limits. These transformations are defined in such a way that a shift in the dispersion of the study variable causes a change in the mean of the transformed variable. There are many charts that have been implemented by the transformation methods to monitor the process dispersion using the variance statistic such as CUSUM- $\ln S^2$ based on logarithmic transformation (Chang & Gan, 1995), EWMA- S^2 based on three-parameter logarithmic transformation (Castagliola, 2005), CUSUM- S^2 based on three-parameter logarithmic transformation (Castagliola et al., 2009), EWMA- S^2 based on four-parameter logarithmic (Johnson SB) transformation (Castagliola et al., 2010), and Floating PM based on three and four-parameter logarithmic transformation (N. Abbas et al., 2014b).

The proposed CAM- S^2 control chart will be constructed using two transformation methods for monitoring process dispersion using the samples variances, which are, three-parameter logarithmic transformation and four-parameter logarithmic (Johnson SB) transformation.

3.1 The proposed control chart

Assuming that the quality characteristic of interest is normally distributed with mean μ and standard deviation σ , identically and independently distributed samples of size n at each time point $i = 1, 2, \dots$ are selected. Notationally,

$$X_{i,j} \sim N(\mu_0, \sigma^2)$$

where i is the sample number, $j = 1, 2, \dots, n$ is the observation number within a sample,

μ_0 is the mean of an IC study variable and assumed to be known,

$\sigma^2 = \gamma^2 \sigma_0^2$ is the variance, σ_0^2 is the target IC variance of the process and is assumed to be known,

and finally, γ is the amount of shift in process standard deviation.

Here, $\gamma = 1$ refers to an IC situation for the process dispersion, $\gamma > 1$ indicates that the process variance has increased from the target level σ_0^2 , this case of shift is typically referred to as a positive shift in dispersion. Where $0 < \gamma < 1$ indicates that the process variance has decreased from the target level σ_0^2 , this case of shift is typically referred to as a negative shift in dispersion, it often refers to enhanced stability in the process dispersion.

To assess the stability of process parameters, the behaviour of the samples' statistics needs to be studied. The stability of process dispersion can be assessed by using the variance of the process samples, s_i^2 . In other words, denoting sample size by $n \geq 2$, the test statistic is:

$$s_i^2 = \sum_{j=1}^n \frac{(X_{i,j} - \bar{X}_i)^2}{n-1} \sim \frac{\sigma_0^2}{n-1} \chi_{(n-1)}^2 \quad , \quad i = 1, 2, \dots$$

Utilizing the concept of transformation that is instead of monitoring a statistic that follows a non-normal distribution, the normalized version of that statistic can be monitored. Consequently, when a shift occurs in the process variance, the mean of the normalized variable will be shifted accordingly.

Common transformation methods are used to convert a chi-squared distributed variable into approximately normally distributed, such as $\ln$ -transformation, three-parameter logarithmic transformation, four-parameter logarithmic (Johnson SB) transformation, and Chi-squared transformation.

Aiming to transform the statistic of the sample variance, s_i^2 , that follows the chi-squared distribution, the following part shows the resulting transformed statistic by using two different transformation methods, three-parameter logarithmic transformation and four-parameter logarithmic (Johnson SB) transformation.

3.1.1 Transforming the test statistic

The transformed statistic obtained by applying the three-parameter logarithmic transformation method becomes approximately distributed as normal using the formula:

$$T_i = a_T + b_T \ln (s_i^2 + c_T) \sim N(\mu_T, \sigma_T)$$

with $a_T = A_T(n) - 2B_T(n)\ln(\sigma_0)$, $b_T = B_T(n)$, and $c_T = C_T(n)\sigma_0^2$,

where $A_T(n)$, $B_T(n)$, and $C_T(n)$ are functions that depend only on the sample size n , and can be found in Table 3.1 along with the mean and variance of T_i , $\mu_T(n)$ and $\sigma_T(n)$.

Table 3. 1 Values of three-parameter logarithmic transformation variables depending on the sample size, n .

n	$\mu_T(n)$	$\sigma_T(n)$	$A_T(n)$	$B_T(n)$	$C_T(n)$
3	0.02472	0.9165	-0.6627	1.8136	0.6777
4	0.01266	0.9502	-0.7882	2.1089	0.6261
5	0.00748	0.9670	-0.8969	2.3647	0.5979
6	0.00485	0.9765	-0.9940	2.5941	0.5801
7	0.00335	0.9825	-1.0827	2.8042	0.5678
8	0.00243	0.9864	-1.1647	2.9992	0.5588
9	0.00182	0.9892	-1.2413	3.1820	0.5519
10	0.00141	0.9912	-1.3135	3.3548	0.5465
11	0.00112	0.9927	-1.3820	3.5189	0.5421
12	0.00090	0.9938	-1.4473	3.6757	0.5384
13	0.00074	0.9947	-1.5097	3.8260	0.5354
14	0.00062	0.9955	-1.5697	3.9705	0.5327
15	0.00052	0.9960	-1.6275	4.1100	0.5305

The transformed statistic obtained by applying the four-parameter logarithmic transformation method becomes approximately distributed as normal using the formula:

$$U_i = a_U + b_U \ln \left(\frac{s_i^2 - c_U}{c_U + d_U - s_i^2} \right) \sim N(\mu_U, \sigma_U)$$

with $a_U = A_U(n)$, $b_U = B_U(n)$, $c_U = C_U(n)\sigma_0^2$, and $d_U = D_U(n)\sigma_0^2$

where $A_U(n)$, $B_U(n)$, $C_U(n)$, and $D_U(n)$ are functions that depend only on the sample size n , and can be found in Table 3.2 along with the mean and variance of U_i , $\mu_U(n)$ and $\sigma_U(n)$.

Table 3. 2 Values of four-parameter logarithmic transformation variables depending on the sample size, n .

n	$\mu_U(n)$	$\sigma_U(n)$	$A_U(n)$	$B_U(n)$	$C_U(n)$	$D_U(n)$
2	0.0766	0.8566	3.0517	0.9382	-0.2936	22.214
3	0.0184	0.9475	3.1936	1.1952	-0.2588	15.077
4	0.0078	0.9739	3.3657	1.3983	-0.2438	12.591
5	0.0039	0.9852	3.5402	1.5727	-0.2352	11.312
6	0.0022	0.9908	3.7111	1.7281	-0.2295	10.530
7	0.0014	0.9940	3.8768	1.8698	-0.2254	10.000
8	0.0009	0.9958	4.0369	2.0010	-0.2224	9.618
9	0.0006	0.9970	4.1918	2.1238	-0.2200	9.328
10	0.0004	0.9978	4.3417	2.2396	-0.2181	9.100
11	0.0003	0.9983	4.4869	2.3495	-0.2166	8.917
12	0.0002	0.9987	4.6279	2.4544	-0.2152	8.766
13	0.0002	0.9989	4.7648	2.5549	-0.2141	8.640
14	0.0001	0.9991	4.8981	2.6515	-0.2132	8.532
15	0.0001	0.9993	5.0279	2.7446	-0.2123	8.440

If $c_U + d_U - s_i^2 \leq 0$, then the sample observation can be taken directly as an OOC signal as it gives a very large value of s_i^2 compared to the value of σ_0^2 . See (Castagliola et al., 2010) for more detail.

3.1.2 The CUSUM structure

Recall the CUSUM control chart that was developed by (Page, 1954), where it is used to detect small/moderate shifts in the process mean and works by accumulating deviations of sample statistic, sample mean $\bar{X}_i$, from the target mean μ_0 using the following plotting statistics,

$$C_i^+ = \text{Max}[0, \bar{X}_i - (\mu_0 + K) + C_{i-1}^+]$$

$$C_i^- = \text{Max}[0, (\mu_0 - K) - \bar{X}_i + C_{i-1}^-]$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

where $K = k\sigma_{\bar{X}} = k \frac{\sigma_0}{\sqrt{n}}$ is usually known by the reference value (or the allowance value), and k is the sensitivity parameter and usually defined to be one half of the δ in σ units, i.e., $k = \frac{\delta}{2} \sigma$; δ is the amount of shift to be detected in σ units. If the effect of the sample size needs to be considered, then σ is represented as $\left(\frac{\sigma_0}{\sqrt{n}}\right)$. Otherwise, when the sample size is not a concern, σ_0 represents σ . The CUSUM chart statistics of $\bar{X}$ are plotted against a single control limit $H = h\sigma_{\bar{X}}$, where h is the limit coefficient of the chart and obtained based on the desired value of ARL_0 .

It is to be noted that the lower CUSUM statistic (C_i^-) monitors the negative shift and the upper one (C_i^+) monitors the positive shift, and they will be equal to zero if there is no deviation from the target mean is observed. The CUSUM statistics will be needed as indicator variables for the proposed charts to indicate if there is a deviation in the samples, T_i and U_i , from the target, i.e., μ_T and μ_U .

The CUSUM statistics as an indicator variable of the CAM- S^2 using three-parameter logarithmic transformation are given by:

$$\begin{cases} C_i^+ = \text{Max}[0, T_i - (\mu_T + k\sigma_T) + C_{i-1}^+] \\ C_i^- = \text{Max}[0, (\mu_T - k\sigma_T) - T_i + C_{i-1}^-] \end{cases} \quad (3.1)$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

The CUSUM statistics as an indicator variable of the CAM- S^2 using four-parameter logarithmic transformation are given by:

$$\begin{cases} C_i^+ = \text{Max}[0, U_i - (\mu_U + k\sigma_U) + C_{i-1}^+] \\ C_i^- = \text{Max}[0, (\mu_U - k\sigma_U) - U_i + C_{i-1}^-] \end{cases} \quad (3.2)$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

3.1.3 The proposed structure

Defining a counter variable, I_i , that depends on the CUSUM statistics for each i^{th} sample by:

$$I_i^+ = \begin{cases} 1 & , \text{if } C_{i-1}^+ = 0 \\ I_{i-1}^+ + 1 & , \text{otherwise} \end{cases} \quad (3.3)$$

$$I_i^- = \begin{cases} 1 & , \text{if } C_{i-1}^- = 0 \\ I_{i-1}^- + 1 & , \text{otherwise} \end{cases} \quad (3.4)$$

for $i = 1, 2, 3, \dots$

where C_i^+ and C_i^- are computed from equation (3.1) for the CAM- S^2 using three-parameter logarithmic transformation method, or from equation (3.2) for the CAM- S^2 using four-parameter logarithmic transformation method.

The idea behind the counter variable for the proposed chart is, to start counting the number of samples when noticing a deviation in a sample statistic from the target parameter, and to reset the memory of the CAM- S^2 plotting statistic when no deviation has been observed, hence reset the corresponding control limits. It is also used for the weighting scheme of the chart.

Now, the proposed chart can be implemented based on the normalized statistics, T_i and U_i , using the counter variable by allocating equal weights of $1/I_i$ to the last sample for which the counter was equal to 1 and the subsequent samples until the i^{th} sample as the following details.

Based on the three-parameter logarithmic transformation, the CAM- S^2 plotting statistics take the form of

$$CT_i^+ = \frac{1}{I_i^+} T_i + \left(1 - \frac{1}{I_i^+}\right) CT_{i-1}^+ \quad , \quad i = 1, 2, \dots$$

$$CT_i^- = \frac{1}{I_i^-} T_i + \left(1 - \frac{1}{I_i^-}\right) CT_{i-1}^- \quad , \quad i = 1, 2, \dots$$

with starting values $CT_0^+ = CT_0^- = \mu_T$, and I_i^+ and I_i^- are from equations (3.3) and (3.4), respectively.

Since $E(CT_i^+) = E(CT_i^-) = \mu_T$, $Var(CT_i^+) = \frac{\sigma_T^2}{I_i^+}$, and $Var(CT_i^-) = \frac{\sigma_T^2}{I_i^-}$, then CT_i^+ is plotted against the upper control limit $\mu_T + L \frac{\sigma_T}{\sqrt{I_i^+}}$, and CT_i^- against the lower control limit $\mu_T - L \frac{\sigma_T}{\sqrt{I_i^-}}$. Where L is the limits coefficient and determined based on the desired ARL_0 .

Based on the four-parameter logarithmic transformation, the CAM- S^2 plotting statistics are given by

$$CU_i^+ = \frac{1}{I_i^+} U_i + \left(1 - \frac{1}{I_i^+}\right) CU_{i-1}^+ \quad , \quad i = 1, 2, \dots$$

$$CU_i^- = \frac{1}{I_i^-} U_i + \left(1 - \frac{1}{I_i^-}\right) CU_{i-1}^- \quad , \quad i = 1, 2, \dots$$

with starting values $CU_0^+ = CU_0^- = \mu_U$, and I_i^+ and I_i^- are from equations (3.3) and (3.4), respectively.

Since $E(CU_i^+) = E(CU_i^-) = \mu_U$, $Var(CU_i^+) = \frac{\sigma_U^2}{I_i^+}$, and $Var(CU_i^-) = \frac{\sigma_U^2}{I_i^-}$, then CU_i^+ is plotted against the upper control limit $\mu_U + L \frac{\sigma_U}{\sqrt{I_i^+}}$, and CU_i^- against the lower control limit $\mu_U - L \frac{\sigma_U}{\sqrt{I_i^-}}$. Where L is the limits coefficient and determined based on the desired ARL_0 .

3.2 Performance results

This section presents the performance of the proposed chart for the cases of zero and steady state using ARL measure, and the CED curve, (Kenett & Pollak, 2012). Where the change-point, denoted by τ , has been studied at a group of sample numbers, $\tau = \{1, 3, 5, 10, 25, 25, 50, 50, 100, 150, 200\}$. The EQL measure, which quantifies the overall effectiveness of a control chart against each value of the sensitivity parameter k , has been also used for the performance evaluation. The EQL is described as an average ARL that is weighted using the square of the shift over the entire domain of process shifts, more details on its formula can be found in Chapter 1.

Table 3.3 illustrates the values of the CAM- S^2 chart limit coefficient, L , corresponding to the allowance value, K , by setting $ARL_0 = 370$ and using Monte Carlo simulation with 10^5 replications.

Table 3.3 L values corresponding to K and n for CAM- S^2 control chart.

	K	0.25	0.5	0.75	1
3-Parameter transformation	$n = 3$	3.283	3.352	3.353	3.330
	$n = 5$	3.212	3.265	3.249	3.206
	$n = 10$	3.214	3.269	3.252	3.210
4-Parameter transformation	$n = 3$	3.228	3.289	3.279	3.243
	$n = 5$	3.216	3.272	3.257	3.216
	$n = 10$	3.241	3.305	3.298	3.267

3.2.1 CAM- S^2 chart performance for zero-state

For the zero-state shift monitoring, the performance of CAM- S^2 chart has been evaluated by ARL measure applying the Monte Carlo simulations with 10^5 replications. Table 3.4.a, Table 3.4.b and Table 3.4.c for $n = 3, 5, 10$ respectively show the resulting ARL using three-parameter logarithmic transformation and fixing $ARL_0 = 370$ with their EQL values in the last column for $\lambda > 1$. Table 3.5.a, Table 3.5.b and Table 3.5.c for $n = 3, 5, 10$ respectively show the resulting ARL using four-parameter logarithmic transformation and fixing $ARL_0 = 370$ with their EQL values in the last column for $\lambda > 1$.

Table 3. 4. a *ARL* and *EQL* values using three-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 3$.

<i>K</i>	γ	<i>ARL</i>	<i>SDRL</i>	<i>P</i> ₀₁ <i>RL</i>	<i>P</i> ₀₅ <i>RL</i>	<i>P</i> ₁₀ <i>RL</i>	<i>P</i> ₂₅ <i>RL</i>	<i>P</i> ₅₀ <i>RL</i>	<i>P</i> ₇₅ <i>RL</i>	<i>P</i> ₉₀ <i>RL</i>	<i>P</i> ₉₅ <i>RL</i>	<i>P</i> ₉₉ <i>RL</i>	<i>EQL</i>
0.25	0.25	6.42	0.61	5	6	6	6	6	7	7	7	8	39.18
	0.50	12.19	3.17	7	8	8	10	12	14	16	18	21	
	0.75	41.58	19.28	11	16	20	27	38	52	67	77	100	
	0.90	241.06	207.88	16	37	54	96	179	322	510	655	988.01	
	1.05	195.21	190.58	4	13	24	60	137	269	444	573	880	
	1.10	103.41	96.45	3	8	14	35	76	142	228	295	447	
	1.25	29.75	24.61	1	3	5	11	23	41	63	78	112	
	1.50	10.36	8.19	1	2	2	4	8	14	21	27	38	
	2.00	4.00	2.94	1	1	1	2	3	5	8	10	14	
	3.00	1.97	1.23	1	1	1	1	2	2	4	4	6	
0.5	0.25	6.65	0.65	6	6	6	6	7	7	7	8	8	53.53
	0.50	12.67	3.26	7	8	9	10	12	15	17	19	22	
	0.75	59.29	42.95	12	17	21	29	46	76	116	145	212	
	0.90	564.99	548.19	18	43	73	173	399	779	1279	1659	2528	
	1.05	185.07	181.49	4	13	23	56	130	255	421	544	841	
	1.10	100.22	96.11	3	8	14	32	71	138	226	293	445	
	1.25	28.60	24.83	2	3	5	11	22	38	61	77	117	
	1.50	10.09	7.74	1	2	2	4	8	14	20	25	36	
	2.00	4.03	2.86	1	1	1	2	3	5	8	10	14	
	3.00	1.99	1.23	1	1	1	1	2	3	4	4	6	
0.75	0.25	6.66	0.65	6	6	6	6	7	7	7	8	8	82.70
	0.50	13.34	4.57	7	8	9	10	12	15	19	22	30	
	0.75	162.11	151.25	12	18	26	54	116	220	358	464	708	
	0.90	1129.57	1119.73	19	65	124	331	787	1557	2599	3375	5167	
	1.05	185.27	182.82	4	12	22	55	129	256	425	552	845	
	1.10	102.29	99.34	3	8	13	32	72	141	232	301	460	
	1.25	29.25	26.57	2	3	5	10	21	40	64	82	125	
	1.50	9.91	7.90	1	2	2	4	8	13	20	26	37	
	2.00	3.94	2.76	1	1	1	2	3	5	8	9	13	
	3.00	1.98	1.21	1	1	1	1	2	3	4	4	6	
1	0.25	6.60	0.67	6	6	6	6	7	7	7	8	8	132.94
	0.50	22.12	15.48	7	8	9	11	17	28	42	53	78	
	0.75	707.29	694.76	14	44	82	211	496	977	1617	2097	3209	
	0.90	1749.89	1745.59	21	94	188	504.8	1214	2422	4026	5225	8022	
	1.05	188.70	186.26	4	12	22	56	131	261	432	560	860	
	1.10	106.25	104.32	3	7	13	32	74	147	243	314	481.01	
	1.25	30.37	28.22	1	3	5	10	22	41	67	86	132	
	1.50	10.02	8.42	1	2	2	4	8	13	21	27	40	
	2.00	3.87	2.75	1	1	1	2	3	5	7	9	13	
	3.00	1.95	1.18	1	1	1	1	2	2	3	4	6	

Table 3. 4. b ARL and EQL values using three-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 5$.

K	γ	ARL	SDRL	$P_{01}RL$	$P_{05}RL$	$P_{10}RL$	$P_{25}RL$	$P_{50}RL$	$P_{75}RL$	$P_{90}RL$	$P_{95}RL$	$P_{99}RL$	EQL
0.25	0.25	3.14	0.35	3	3	3	3	3	3	4	4	4	23.79
	0.50	5.93	1.57	3	4	4	5	6	7	8	9	10	
	0.75	20.05	9.18	5	8	10	13	19	25	32	37	47	
	0.90	102.60	77.03	10	20	28	49	83	134	203	254	371	
	1.05	160.37	151.65	4	13	23	53	115	220	358	463	708	
	1.10	69.74	59.89	3	7	12	27	54	95	148	188	281	
	1.25	17.44	12.95	1	3	4	8	14	24	35	43	60	
	1.50	6.10	4.23	1	1	2	3	5	8	12	14	20	
	2.00	2.49	1.54	1	1	1	1	2	3	5	5	8	
	3.00	1.36	0.63	1	1	1	1	1	2	2	3	3	
0.5	0.25	3.21	0.41	3	3	3	3	3	3	4	4	4	27.35
	0.50	6.09	1.58	3	4	4	5	6	7	8	9	11	
	0.75	21.31	11.10	6	8	10	14	19	26	35	43	59	
	0.90	178.93	166.86	11	21	30	60	128	243	397	512	782	
	1.05	161.35	156.15	4	12	21	50	113	223	365	472	722	
	1.10	72.24	66.68	3	7	12	25	52	99	159	205	312	
	1.25	16.99	12.99	1	3	4	8	14	23	34	42	62	
	1.50	6.06	4.08	1	1	2	3	5	8	12	14	19	
	2.00	2.51	1.51	1	1	1	1	2	3	5	5	7	
	3.00	1.38	0.64	1	1	1	1	1	2	2	3	4	
0.75	0.25	3.18	0.39	3	3	3	3	3	3	4	4	4	33.77
	0.50	6.03	1.59	3	4	4	5	6	7	8	9	11	
	0.75	29.35	22.14	5	8	10	14	23	38	58	74	109	
	0.90	312.27	307.34	10	23	39	95	217	428	710	923	1430	
	1.05	162.50	158.95	4	12	20	49	114	224	370	481	735	
	1.10	76.69	73.38	3	7	11	24	54	105	172	223	339	
	1.25	17.52	14.69	1	3	4	7	13	23	37	47	70	
	1.50	5.91	4.07	1	1	2	3	5	8	11	14	20	
	2.00	2.47	1.48	1	1	1	1	2	3	4	5	7	
	3.00	1.36	0.62	1	1	1	1	1	2	2	3	3	
1	0.25	3.13	0.34	3	3	3	3	3	3	4	4	4	43.78
	0.50	6.03	1.85	3	4	4	5	6	7	8	9	12	
	0.75	53.01	47.64	5	8	10	19	38	72	115	148	225	
	0.90	510.98	509.14	10	31	59	150	353	706	1173	1522	2354	
	1.05	165.09	162.94	4	11	20	49	115	229	377	487	748	
	1.10	80.32	77.98	3	6	11	25	56	111	181	236	364	
	1.25	18.52	16.45	1	3	4	7	14	25	40	51	78	
	1.50	5.83	4.26	1	1	2	3	5	8	11	14	21	
	2.00	2.41	1.44	1	1	1	1	2	3	4	5	7	
	3.00	1.35	0.61	1	1	1	1	1	2	2	3	3	

Table 3. 4. c *ARL* and *EQL* values using three-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 10$.

<i>K</i>	γ	<i>ARL</i>	<i>SDRL</i>	<i>P</i> ₀₁ <i>RL</i>	<i>P</i> ₀₅ <i>RL</i>	<i>P</i> ₁₀ <i>RL</i>	<i>P</i> ₂₅ <i>RL</i>	<i>P</i> ₅₀ <i>RL</i>	<i>P</i> ₇₅ <i>RL</i>	<i>P</i> ₉₀ <i>RL</i>	<i>P</i> ₉₅ <i>RL</i>	<i>P</i> ₉₉ <i>RL</i>	<i>EQL</i>
0.25	0.25	1.93	0.26	1	1	2	2	2	2	2	2	2	14.21
	0.50	2.82	0.74	2	2	2	2	3	3	4	4	5	
	0.75	9.33	4.09	3	4	5	6	9	12	15	17	21	
	0.90	45.92	28.92	5	10	15	25	40	61	84	100	139	
	1.05	112.25	99.90	4	11	19	42	84	152	242	312	470	
	1.10	40.21	30.25	2	6	9	18	33	55	80	98	142	
	1.25	9.36	6.19	1	2	3	5	8	13	18	21	29	
	1.50	3.37	1.99	1	1	1	2	3	4	6	7	10	
	2.00	1.52	0.73	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	1	2	
0.5	0.25	1.98	0.13	1	2	2	2	2	2	2	2	2	15.47
	0.50	2.89	0.76	2	2	2	2	3	3	4	4	5	
	0.75	9.52	4.12	3	4	5	6	9	12	15	17	22	
	0.90	59.93	50.16	6	11	15	25	45	79	125	160	240	
	1.05	123.91	118.44	4	11	18	40	88	169	278	359	551	
	1.10	43.11	37.27	3	6	9	17	32	57	91	117	177	
	1.25	9.30	6.05	1	2	3	5	8	12	17	21	29	
	1.50	3.40	1.98	1	1	1	2	3	4	6	7	10	
	2.00	1.53	0.73	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	2	2	
0.75	0.25	1.97	0.17	1	2	2	2	2	2	2	2	2	17.44
	0.50	2.87	0.75	2	2	2	2	3	3	4	4	5	
	0.75	9.72	4.88	3	4	5	6	9	12	16	19	27	
	0.90	90.56	84.82	5	10	15	30	65	123	201	260	396	
	1.05	133.78	129.95	4	10	17	41	94	184	303	395	602	
	1.10	48.65	44.89	2	6	9	17	35	66	107	138	210	
	1.25	9.30	6.62	1	2	3	5	8	12	18	22	32	
	1.50	3.33	1.93	1	1	1	2	3	4	6	7	9	
	2.00	1.53	0.73	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	2	2	
1	0.25	1.92	0.27	1	1	2	2	2	2	2	2	2	19.92
	0.50	2.81	0.74	2	2	2	2	3	3	4	4	5	
	0.75	11.33	7.55	3	4	5	6	9	14	21	26	38	
	0.90	132.08	127.42	5	11	18	41	93	181	298	385	590	
	1.05	141.90	139.07	4	10	17	43	100	196	323	418	643	
	1.10	54.44	51.80	2	5	8	18	39	74	122	158	241	
	1.25	9.70	7.59	1	2	3	4	8	13	19	25	37	
	1.50	3.27	1.93	1	1	1	2	3	4	6	7	9	
	2.00	1.51	0.71	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	1	2	

Table 3. 5. a *ARL* and *EQL* values using four-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 3$.

<i>K</i>	γ	<i>ARL</i>	<i>SDRL</i>	<i>P</i> ₀₁ <i>RL</i>	<i>P</i> ₀₅ <i>RL</i>	<i>P</i> ₁₀ <i>RL</i>	<i>P</i> ₂₅ <i>RL</i>	<i>P</i> ₅₀ <i>RL</i>	<i>P</i> ₇₅ <i>RL</i>	<i>P</i> ₉₀ <i>RL</i>	<i>P</i> ₉₅ <i>RL</i>	<i>P</i> ₉₉ <i>RL</i>	<i>EQL</i>
0.25	0.25	5.17	0.71	4	4	4	5	5	6	6	6	7	38.38
	0.50	10.86	3.37	5	6	7	8	10	13	15	17	20	
	0.75	39.12	19.78	9	13	17	25	36	50	65	76	99	
	0.90	214.53	187.25	13	30	45	84	158	287	459	589	880	
	1.05	201.32	194.89	4	14	25	63	142	278	458	592	898	
	1.10	106.88	99.25	3	9	15	36	78	146	236	302	461	
	1.25	30.10	24.68	1	3	6	12	24	42	63	79	112	
	1.50	10.56	8.26	1	2	2	4	8	14	22	27	38	
	2.00	4.02	2.94	1	1	1	2	3	5	8	10	14	
	3.00	1.97	1.23	1	1	1	1	2	2	4	4	6	
0.5	0.25	5.35	0.71	4	4	5	5	5	6	6	7	7	46.93
	0.50	11.21	3.46	5	6	7	9	11	13	16	18	21	
	0.75	53.33	39.89	9	14	18	26	41	69	105	133	198	
	0.90	404.97	392.47	14	32	54	125	285.5	560	914	1184	1819	
	1.05	195.41	191.88	4	14	24	59	137	268	443	575	896	
	1.10	105.65	101.02	3	9	15	34	75	145	238	309	469	
	1.25	29.42	25.39	1	4	6	12	22	39	62	80	119	
	1.50	10.23	7.85	1	2	2	5	8	14	21	25	37	
	2.00	4.02	2.87	1	1	1	2	3	5	8	10	14	
	3.00	1.98	1.22	1	1	1	1	2	2	4	4	6	
0.75	0.25	5.32	0.71	4	4	5	5	5	6	6	7	7	61.37
	0.50	11.65	4.45	5	6	7	9	11	14	17	20	27	
	0.75	110.34	102.05	9	14	19	38	79	150	243	315	481	
	0.90	680.65	669.33	14	41	77	203	476	943	1555	2025	3081	
	1.05	193.50	191.06	4	13	23	57	135	268	441	576	885	
	1.10	107.11	104.11	3	8	14	33	75	148	242	317	479	
	1.25	30.10	27.39	1	3	5	11	22	41	66	85	128	
	1.50	10.10	8.07	1	2	2	4	8	13	21	26	39	
	2.00	3.94	2.80	1	1	1	2	3	5	8	9	13	
	3.00	1.97	1.20	1	1	1	1	2	2	4	4	6	
1	0.25	5.23	0.73	4	4	4	5	5	6	6	6	7	83.72
	0.50	16.15	10.61	5	6	7	9	13	20	30	37	54	
	0.75	271.14	265.18	9	20	34	83	189	374	616	799	1237	
	0.90	1030.01	1023.14	15	57	112	300	718	1431	2355	3052	4697	
	1.05	194.39	193.24	4	12	23	57	135	267	444	580	899	
	1.10	109.97	107.52	3	8	13	33	77	152	250	323	497	
	1.25	31.64	29.69	1	3	5	11	22	43	70	91	138	
	1.50	10.17	8.61	1	2	2	4	8	14	21	27	41	
	2.00	3.87	2.80	1	1	1	2	3	5	7	9	14	
	3.00	1.94	1.19	1	1	1	1	2	2	3	4	6	

Table 3. 5. b ARL and EQL values using four-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 5$.

K	γ	ARL	SDRL	$P_{01}RL$	$P_{05}RL$	$P_{10}RL$	$P_{25}RL$	$P_{50}RL$	$P_{75}RL$	$P_{90}RL$	$P_{95}RL$	$P_{99}RL$	EQL
0.25	0.25	2.67	0.49	2	2	2	2	3	3	3	3	4	24.30
	0.50	5.43	1.68	3	3	3	4	5	6	8	8	10	
	0.75	19.52	9.51	4	7	9	12	18	25	32	37	47	
	0.90	100.15	76.87	8	17	26	47	81	131	199	252	371	
	1.05	169.88	160.71	4	13	24	56	122	232	379	490	742	
	1.10	73.43	62.87	3	8	13	28	57	100	155	197	295	
	1.25	18.10	13.36	1	3	4	8	15	25	36	44	62	
	1.50	6.21	4.31	1	1	2	3	5	8	12	15	20	
	2.00	2.50	1.56	1	1	1	1	2	3	5	6	8	
	3.00	1.36	0.63	1	1	1	1	1	2	2	3	3	
0.5	0.25	2.78	0.46	2	2	2	3	3	3	3	3	4	27.34
	0.50	5.57	1.69	3	3	4	4	5	7	8	9	10	
	0.75	20.70	11.48	5	7	9	13	18	26	35	43	60	
	0.90	159.23	149.78	8	18	26	53	113	216	353	458	696	
	1.05	174.52	169.80	5	13	23	54	122	241	396	514	781	
	1.10	78.36	73.10	3	8	13	27	56	107	173	225	343	
	1.25	17.77	13.56	1	3	4	8	14	24	35	44	65	
	1.50	6.18	4.17	1	1	2	3	5	8	12	14	20	
	2.00	2.50	1.55	1	1	1	1	2	3	5	5	8	
	3.00	1.37	0.64	1	1	1	1	1	2	2	3	4	
0.75	0.25	2.75	0.47	2	2	2	2	3	3	3	3	4	31.77
	0.50	5.52	1.69	3	3	4	4	5	7	8	9	10	
	0.75	27.27	20.86	4	7	9	13	21	35	54	69	102	
	0.90	242.45	236.15	8	18	31	74	171	335	548	711	1090	
	1.05	179.73	176.51	4	12	22	54	126	248	411	534	817	
	1.10	84.12	81.14	3	7	12	26	59	116	190	246	378	
	1.25	18.59	15.68	1	3	4	8	14	25	39	50	75	
	1.50	6.05	4.20	1	1	2	3	5	8	11	14	20	
	2.00	2.48	1.51	1	1	1	1	2	3	4	5	7	
	3.00	1.36	0.63	1	1	1	1	1	2	2	3	3	
1	0.25	2.68	0.49	2	2	2	2	3	3	3	3	4	37.27
	0.50	5.51	1.90	3	3	3	4	5	6	8	9	12	
	0.75	43.67	39.10	4	7	9	16	32	59	94	122	185	
	0.90	343.10	339.39	8	22	40	101	240	472	786	1019	1564	
	1.05	181.87	179.80	4	12	21	54	126	251	415	539	836	
	1.10	89.54	87.51	3	7	12	27	63	123	203	266	404	
	1.25	19.71	17.51	1	3	4	7	14	26	43	55	83	
	1.50	6.03	4.46	1	1	2	3	5	8	12	15	22	
	2.00	2.43	1.49	1	1	1	1	2	3	4	5	7	
	3.00	1.35	0.62	1	1	1	1	1	2	2	3	3	

Table 3. 5. c *ARL* and *EQL* values using four-parameter logarithmic transformation – zero state ($\tau = 1$), for $n = 10$.

<i>K</i>	γ	<i>ARL</i>	<i>SDRL</i>	<i>P</i> ₀₁ <i>RL</i>	<i>P</i> ₀₅ <i>RL</i>	<i>P</i> ₁₀ <i>RL</i>	<i>P</i> ₂₅ <i>RL</i>	<i>P</i> ₅₀ <i>RL</i>	<i>P</i> ₇₅ <i>RL</i>	<i>P</i> ₉₀ <i>RL</i>	<i>P</i> ₉₅ <i>RL</i>	<i>P</i> ₉₉ <i>RL</i>	<i>EQL</i>
0.25	0.25	1.23	0.42	1	1	1	1	1	1	2	2	2	14.45
	0.50	2.63	0.78	1	2	2	2	3	3	4	4	5	
	0.75	9.20	4.24	2	3	4	6	9	12	15	17	22	
	0.90	46.27	29.46	5	10	14	25	41	62	85	102	140	
	1.05	116.61	103.18	4	12	20	43	87	158	252	323	480	
	1.10	41.39	30.74	2	6	9	19	35	57	82	100	143	
	1.25	9.59	6.31	1	2	3	5	8	13	18	22	30	
	1.50	3.37	2.02	1	1	1	2	3	4	6	7	10	
	2.00	1.50	0.73	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.21	1	1	1	1	1	1	1	1	2	
0.5	0.25	1.31	0.46	1	1	1	1	1	2	2	2	2	15.93
	0.50	2.70	0.79	1	2	2	2	3	3	4	4	5	
	0.75	9.44	4.30	2	4	4	6	9	12	15	17	22	
	0.90	59.92	50.24	5	10	15	25	45	80	125	159	240	
	1.05	132.34	126.71	4	12	20	43	94	180	296	385	590	
	1.10	45.40	39.11	2	6	10	18	34	61	97	123	185	
	1.25	9.57	6.22	1	2	3	5	8	13	18	21	29	
	1.50	3.44	2.03	1	1	1	2	3	4	6	7	10	
	2.00	1.53	0.74	1	1	1	1	1	2	3	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	1	2	
0.75	0.25	1.30	0.46	1	1	1	1	1	2	2	2	2	18.03
	0.50	2.70	0.79	1	2	2	2	3	3	4	4	5	
	0.75	9.69	5.02	2	4	4	6	9	12	16	19	27	
	0.90	87.23	81.94	5	10	14	29	62	119	195	252	384	
	1.05	146.64	143.08	4	11	19	45	102	202	332.1	434	658	
	1.10	52.94	48.96	2	6	9	18	38	72	117	151	229	
	1.25	9.67	6.88	1	2	3	5	8	13	19	23	34	
	1.50	3.39	1.99	1	1	1	2	3	4	6	7	10	
	2.00	1.52	0.73	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.22	1	1	1	1	1	1	1	1	2	
1	0.25	1.27	0.44	1	1	1	1	1	2	2	2	2	20.49
	0.50	2.65	0.78	1	2	2	2	3	3	4	4	5	
	0.75	11.14	7.56	2	3	4	6	9	14	21	26	38	
	0.90	122.21	118.52	4	10	16	38	86	169	277	358	546	
	1.05	158.83	156.87	4	10	19	47	110	221	363	473	724	
	1.10	60.55	57.88	2	6	9	19	43	83	136	177	270	
	1.25	10.23	8.02	1	2	3	5	8	13	21	26	39	
	1.50	3.32	2.00	1	1	1	2	3	4	6	7	10	
	2.00	1.51	0.71	1	1	1	1	1	2	2	3	4	
	3.00	1.05	0.21	1	1	1	1	1	1	1	1	2	

According to the tables, it is clearly noted that enlarging the sample size has a positive effect on the performance of the charts, this is very clear by comparing their overall measure *EQL*'s. Further, it can be inferred, especially for larger n , that the smaller values of the allowance parameter K grants better performance of the charts. The two transformation methods, three-parameter logarithmic transformation and four-parameter logarithmic transformation give almost similar performance results when detecting shifts of size $\gamma > 1.25$. The CAM- S^2 (3-Patameter) chart outperforms the CAM- S^2 (4-Patameter) chart when the shift lies in the interval $0.9 < \gamma \leq 1.25$, whereas the CAM- S^2 (4-Patameter) chart exhibits a higher performance in detecting shifts of size $\gamma \leq 0.9$.

3.2.2 CAM- S^2 chart performance for steady-state

For the steady-state shift monitoring, the performance of CAM- S^2 chart has been evaluated by the CED metric, which is obtained by using Monte Carlo simulations with 10^5 replications. The allowance values of K at 0.25 and 0.5 have been selected to be studied as they give the smallest values of EQL .

The conditional expected delay is defined by:

$$CED(\tau) = E(d \mid RL \geq \tau)$$

where τ is the change-point position,

RL is the number of samples to detect 1st OOC signal, conditioned by that the signal is given after the change-point, and

$d = RL - \tau + 1$ is the delay in detecting the first OOC signal.

For fixed in control average run length $ARL_0 = 370$ when $n = 3, 5, 10$ and for each $\gamma = \{0.25, 0.5, 0.75, 0.9, 1.05, 1.1, 1.25, 1.5, 2, 3\}$, Figure 3.1 and Figure 3.2 show the resulting CED curves using three-parameter logarithmic transformation and four-parameter logarithmic transformation, respectively.

It is observed that the CAM- S^2 control chart stabilizes as the change-point increases. This means that the proposed control chart keeps its high performance under the steady-state shift monitoring. Hence, the CAM- S^2 is considered as an efficient control chart for its ability to detect shifts for both possible cases of monitoring, zero and steady state, with small number of samples. To prove the merit of the proposed structure, the next section provides a comparison of its performance against existing control charts.

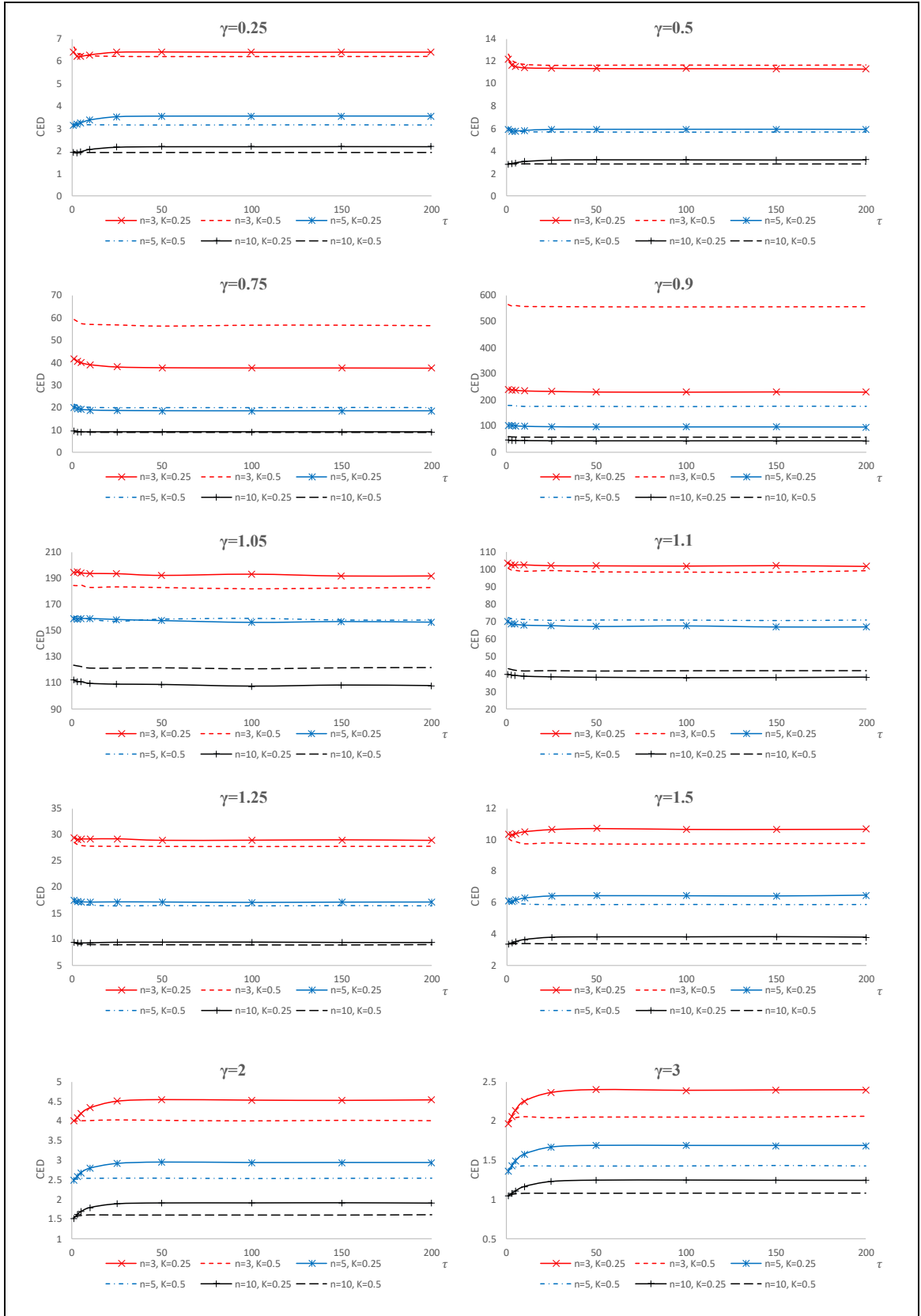


Figure 3.1 CED curves of CAM-S² using three-parameter transformation method.

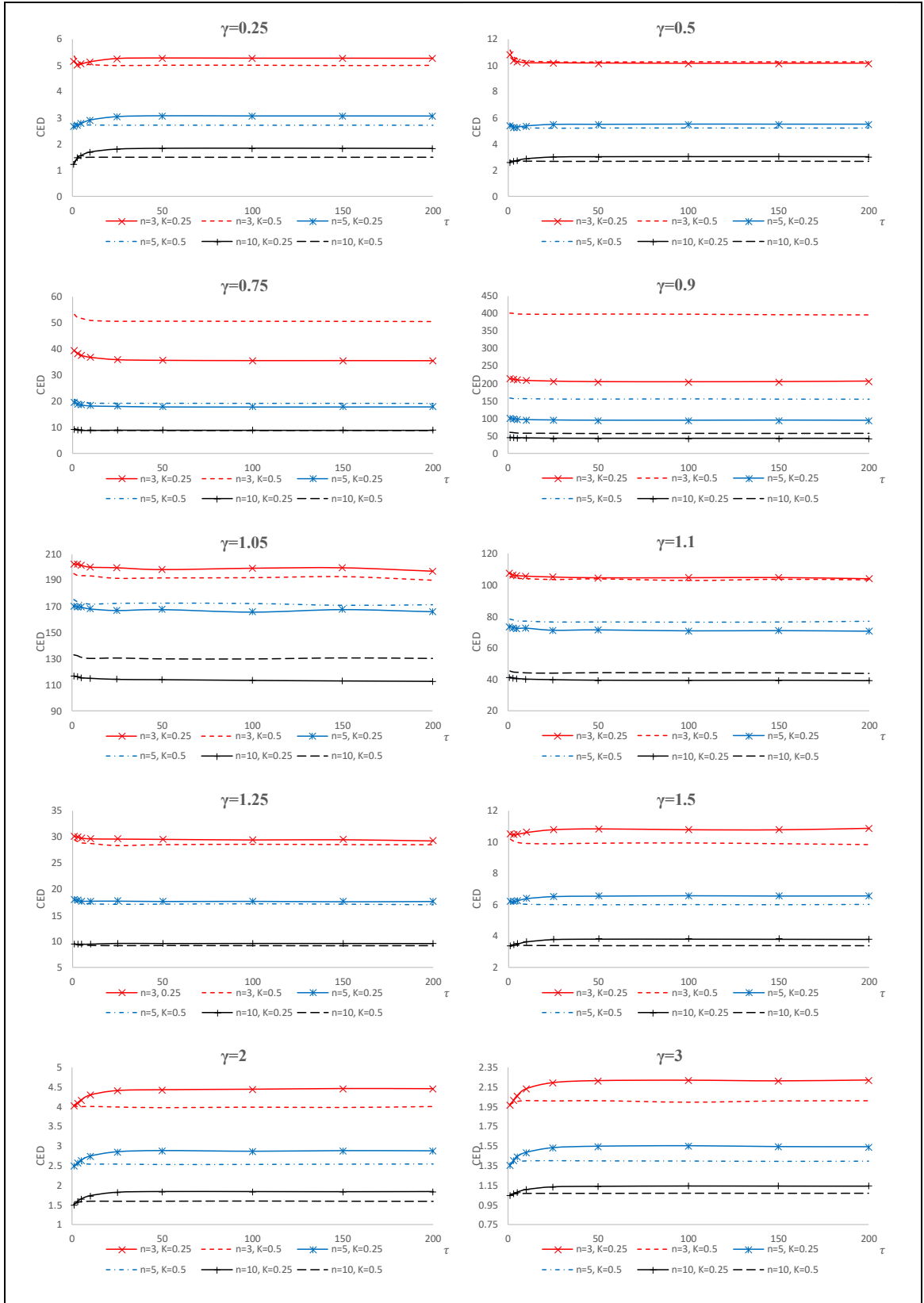


Figure 3. 2 CED curves of CAM-S² using four-parameter transformation method.

3.3 Comparison

The performance of the existing Shewhart- S^2 (Montgomery, 2020), EWMA- S^2 (Castagliola, 2005), CUSUM- S^2 (Castagliola et al., 2009), and Floating PM (N. Abbas et al., 2014b) charts are presented in this section against the proposed CAM- S^2 chart for $n = 3, 5, 10$ at different values of γ considering the two states of shift monitoring: zero and steady states. The evaluation is conducted based on ARL tables, CED curves, and EQL values. The comparison of the charts is studied based on the charts' performance using their optimal parameters.

3.3.1 Comparison of charts' performance for zero-state

The following tables (Table 3.6, Table 3.7, and Table 3.8) with $n = 3, 5, 10$ respectively show the ARL values of the charts: Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , Floating PM (3-Parameter), and CAM- S^2 (3-Parameter and 4-Parameter) at different values of shift size, γ .

Table 3. 6 ARL values for $n = 3$, zero state ($\tau = 1$).

γ	Shewhart S^2	EWMA S^2	CUSUM S^2	Floating PM	CAM S^2	CAM S^2 (Johnson SB)
0.25	46.77	11.12	7.23	3.37	6.42	5.17
0.50	184.97	16.33	11.26	5.60	12.19	10.86
0.75	415.36	39.00	32.37	14.90	41.58	39.12
0.90	513.61	156.26	177.38	52.24	241.06	214.53
1.05	267.52	154.77	194.57	108.24	195.21	201.32
1.10	186.06	64.99	99.20	50.41	103.41	106.88
1.25	64.92	15.38	28.98	15.68	29.75	30.10
1.50	18.66	5.93	12.17	6.55	10.36	10.56
2.00	5.19	2.89	6.00	3.00	4.00	4.02
3.00	2.08	1.71	3.43	1.70	1.97	1.97

Table 3. 7 ARL values for $n = 5$, zero state ($\tau = 1$).

γ	Shewhart S^2	EWMA S^2	CUSUM S^2	Floating PM	CAM S^2	CAM S^2 (Johnson SB)
0.25	4.81	7.02	3.50	2.04	3.14	2.67
0.50	51.20	10.02	5.62	3.39	5.93	5.43
0.75	239.78	22.24	20.70	8.89	20.05	19.52
0.90	444.71	81.11	177.73	32.20	102.60	100.15
1.05	252.87	121.51	164.52	74.37	160.37	169.88
1.10	159.61	45.19	73.70	32.16	69.74	73.43
1.25	43.45	11.21	17.11	9.67	17.44	18.10
1.50	10.47	4.85	6.40	4.09	6.10	6.21
2.00	2.87	2.54	3.09	1.99	2.49	2.50
3.00	1.35	1.54	1.80	1.25	1.36	1.36

Table 3. 8 *ARL* values for $n = 10$, zero state ($\tau = 1$).

γ	Shewhart S^2	EWMA S^2	CUSUM S^2	Floating PM	CAM S^2	CAM S^2 (Johnson SB)
0.25	1.02	4.01	3.06	1.06	1.93	1.23
0.50	6.19	5.90	4.68	2.03	2.82	2.63
0.75	81.68	12.42	10.31	4.95	9.33	9.20
0.90	311.69	40.52	38.61	18.06	45.92	46.27
1.05	228.01	78.23	100.65	45.04	112.25	116.61
1.10	119.29	27.02	36.98	18.42	40.21	41.39
1.25	22.67	7.73	11.13	5.54	9.36	9.59
1.50	4.76	3.73	5.40	2.45	3.37	3.37
2.00	1.52	2.13	2.97	1.33	1.52	1.50
3.00	1.04	1.29	1.98	1.03	1.05	1.05

From these *ARL* tables, it can be observed that the Floating PM control chart has smallest values of ARL_1 that are rapidly decreasing as γ gets larger, which means that the Floating PM chart has the highest performance in detecting the shifts occurring at the beginning of a process. The EWMA- S^2 control chart also has a reliable ability in detecting shifts that is superior to the CUSUM- S^2 and CAM- S^2 when the process dispersion shifted positively, i.e., $\gamma > 1$.

The CUSUM- S^2 and the proposed CAM- S^2 charts have almost similar results at $1.05 \leq \gamma \leq 1.25$, whereas for γ larger than 1.25, the proposed CAM- S^2 chart is superior. For $\gamma < 1$, the charts performance is incomparable because the bias existence for negative shifts. Relying on the *EQL* (Figure 3.3), it is fair to say that the CAM- S^2 exceed the CUSUM- S^2 . So, according to a more thorough analysis using other metrics, CAM- S^2 is more effective than CUSUM- S^2 in detecting a range of shifts. Therefore, it is recommended to use CAM- S^2 over CUSUM- S^2 for monitoring shifts of unknown size. Although the CUSUM- S^2 and the proposed CAM- S^2 charts experience a poor performance comparing with the *ARL* of the other charts, the right decision is to judge the charts after testing their detection abilities in the steady-state shift monitoring.

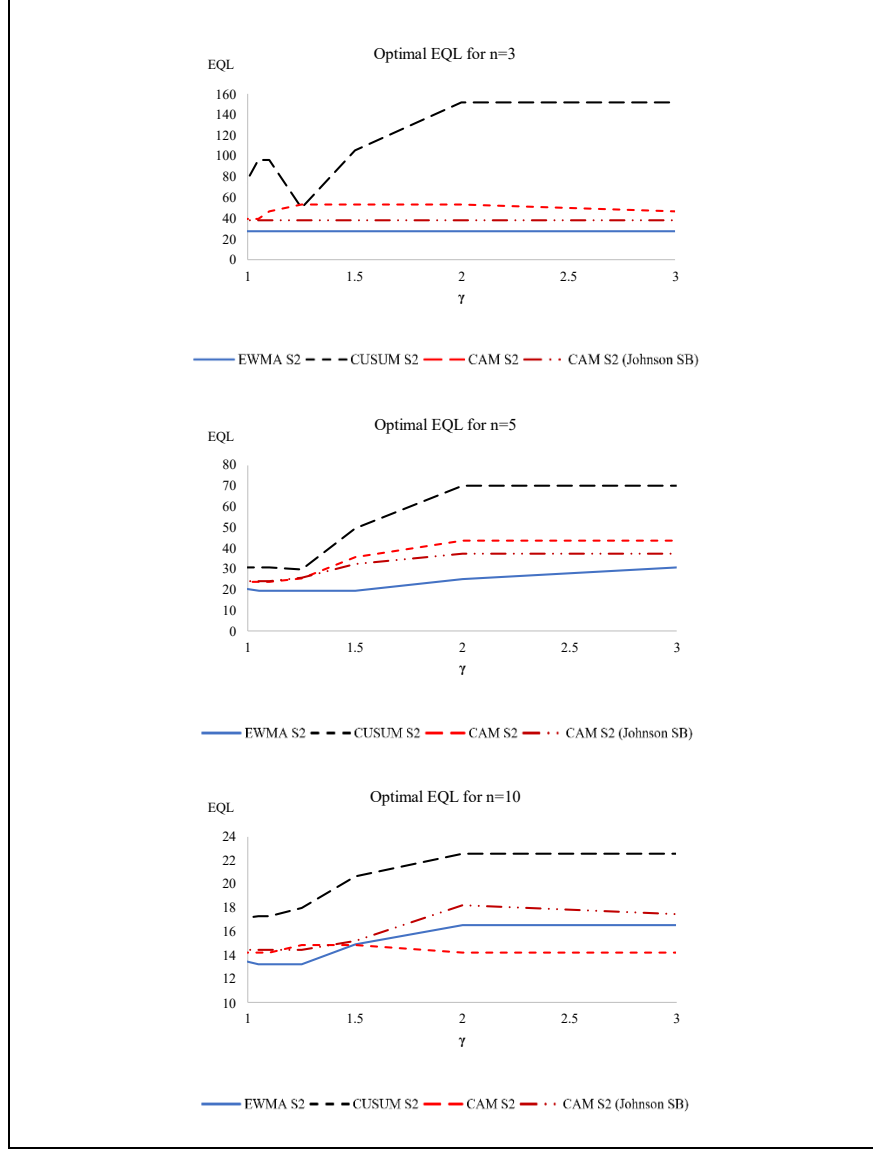


Figure 3.3 *EQL* curves.

3.3.2 Comparison of charts' performance for steady-state

The following figures (Figure 3.4, Figure 3.5, and Figure 3.6) for $n = 3, 5, 10$, respectively display the *CED* curves at $\gamma = \{0.25, 0.5, 0.75, 0.9, 1.05, 1.1, 1.25, 1.5, 2, 3\}$ for the charts: Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , Floating PM (3-Parameter), and CAM- S^2 (3-Parameter and 4-Parameter). The chart with invisible curve means that it has a large value of *CED*. The results can be organized as follows.

- CAM- S^2 versus Shewhart- S^2

As Shewhart- S^2 is designed to detect shifts of large size (i.e., $\gamma > 1.25$), the comparison here has been conducted based on figures of $\gamma = 2$ and $\gamma = 3$. It is

observed that the CAM- S^2 performs slightly better than the Shewhart- S^2 for small sample sizes, $n = 3, 5$, in detecting shifts of size $\gamma = 2$ where they have almost similar results for $\gamma = 3$. The superior performance of CAM- S^2 in detecting shifts of size $\gamma = 2$ with smaller sample sizes (i.e., $n = 3, 5$) highlights its effectiveness in situations where data collection may be constrained or costly, and where the ability to detect moderate shifts remains crucial. In contrast, the Shewhart- S^2 control chart demonstrates its strength for larger sample sizes (i.e., $n = 10$), outperforming CAM- S^2 . This suggests that for processes with abundant data, Shewhart- S^2 may be the preferred choice. These findings emphasize the versatility of CAM- S^2 , making it a valuable chart for applications where smaller sample sizes and shifts of medium size are encountered, while recognizing Shewhart- S^2 as the preferred chart for larger sample sizes and large shifts.

- CAM- S^2 versus EWMA- S^2

The charts tend to have similar results for large negative shifts ($\gamma = 0.25, 0.5$) as the sample size n increases. This similarity suggests that when dealing with negative shifts, the choice between CAM- S^2 and EWMA- S^2 may be less critical, and either chart can perform adequately. For $\gamma = 1.05, 1.1$, the EWMA- S^2 outperforms CAM- S^2 especially when the shift is occurred at the beginning of the monitoring. However, it is obvious that EWMA- S^2 loses its performance in detecting positive shifts for $\tau > 20$, especially with small sample sizes, which may give misleading results. For moderate and large positive shifts $\gamma \geq 1.25$, the performance of CAM- S^2 is quite robust, and it provides more accurate results even for small sample sizes n . Therefore, CAM- S^2 can be a better alternative to EWMA- S^2 when dealing with both sizes of shifts, moderate and large.

- CAM- S^2 versus CUSUM- S^2

In terms of the CED measure, it is evident that these two charts exhibit similar performance for $\gamma \geq 1.25$ as well as for $\gamma \leq 0.5$. In these cases, both CAM- S^2 and CUSUM- S^2 demonstrate comparable efficiency in detecting shifts of these magnitudes. However, the scenario changes when considering shifts within the range of $0.75 \leq \gamma \leq 1.1$, where within this range, the CUSUM- S^2 exceeds the CAM- S^2 . In fact, the CUSUM- S^2 can be considered as the dominant chart in case of steady-state for a given size of shift as it has the minimum values of CED in most cases.

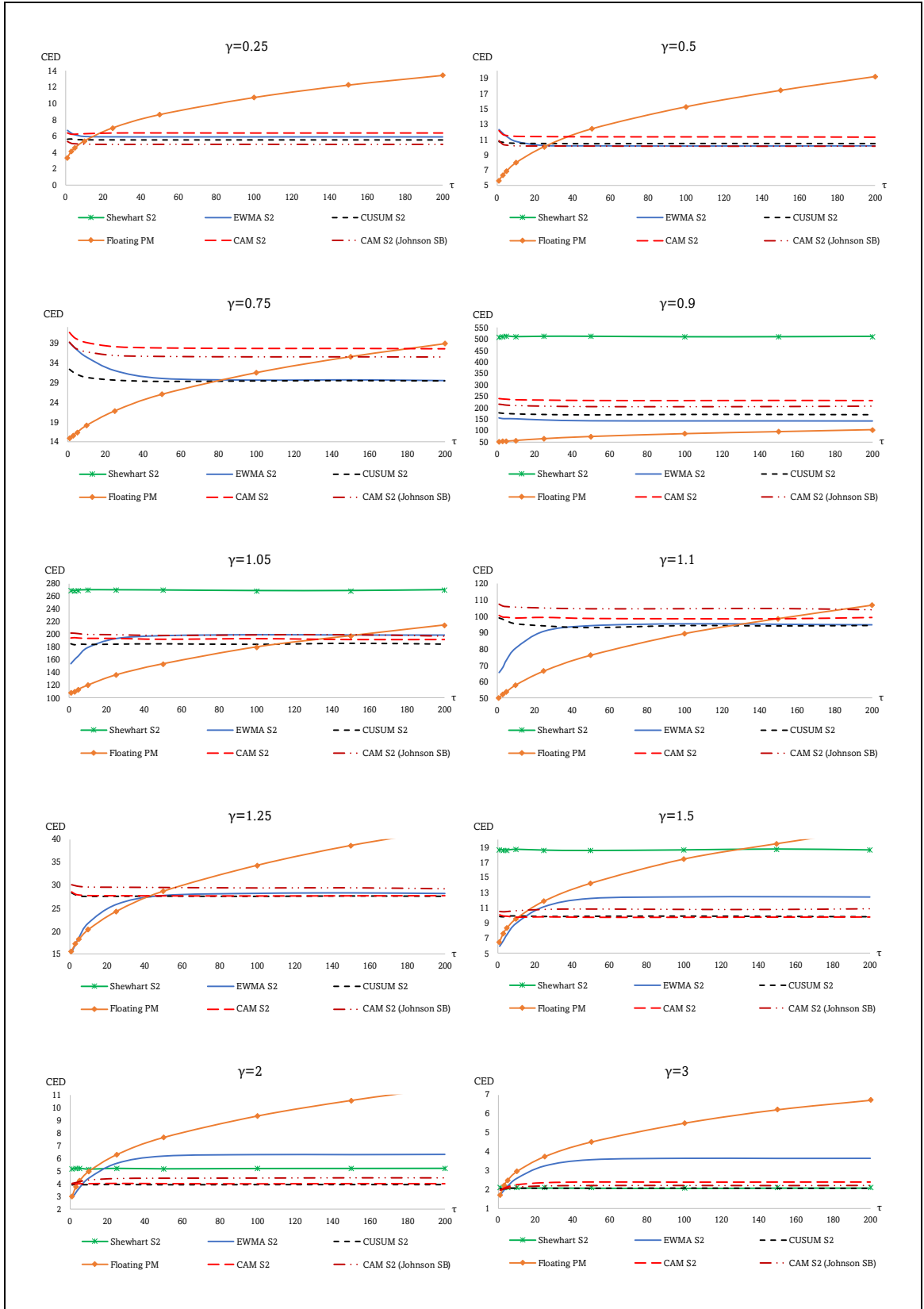


Figure 3. 4 CED curves for $n = 3$.



Figure 3.5 CED curves for $n = 5$.

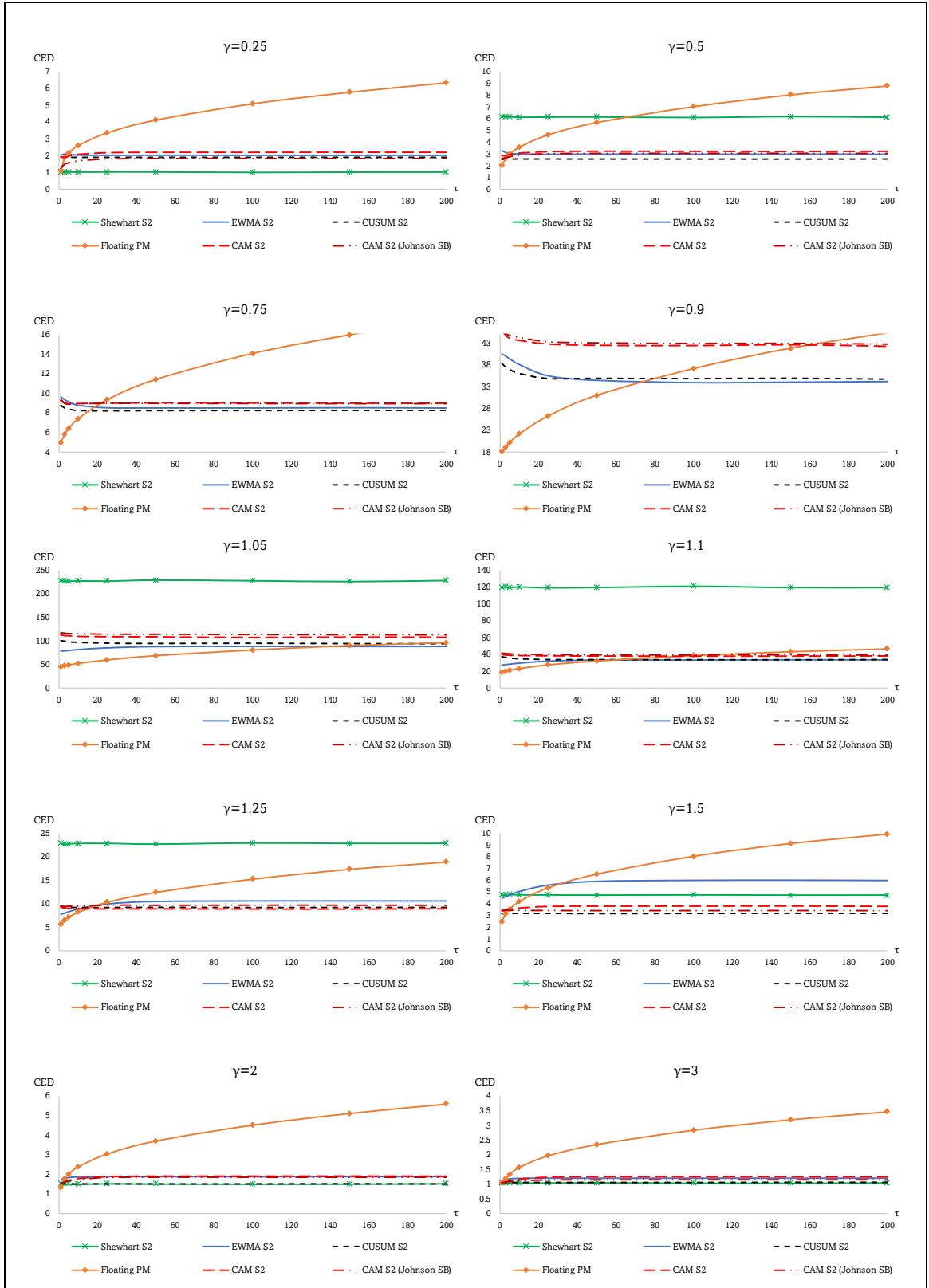


Figure 3.6 CED curves for $n = 10$.

3.4 Illustrative example

In this section an illustrative example is constructed by generating 100 samples of size $\{3,5,10\}$, first 50 samples are drawn from the standard normal i.e., $X \sim N(0,1)$ as in-control part of the process, then, the last 50 samples are drawn from the normal distribution with a shift on σ_0 of size 1.5, i.e., $X \sim N(0,1.5)$. Figure 3.7, Figure 3.8, and Figure 3.9 show the charts performance for each sample size $n = 3,5,10$, respectively. The results were as follows.

For $n = 3$, Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , and CAM- S^2 detected the shift at the sample number 78, 72, 63, 50, respectively.

For $n = 5$, Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , and CAM- S^2 detected the shift at the sample number 57, 56, 54, 53, respectively.

For $n = 10$, the charts gave similar results where Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , and CAM- S^2 detected the shift at the sample number 56.

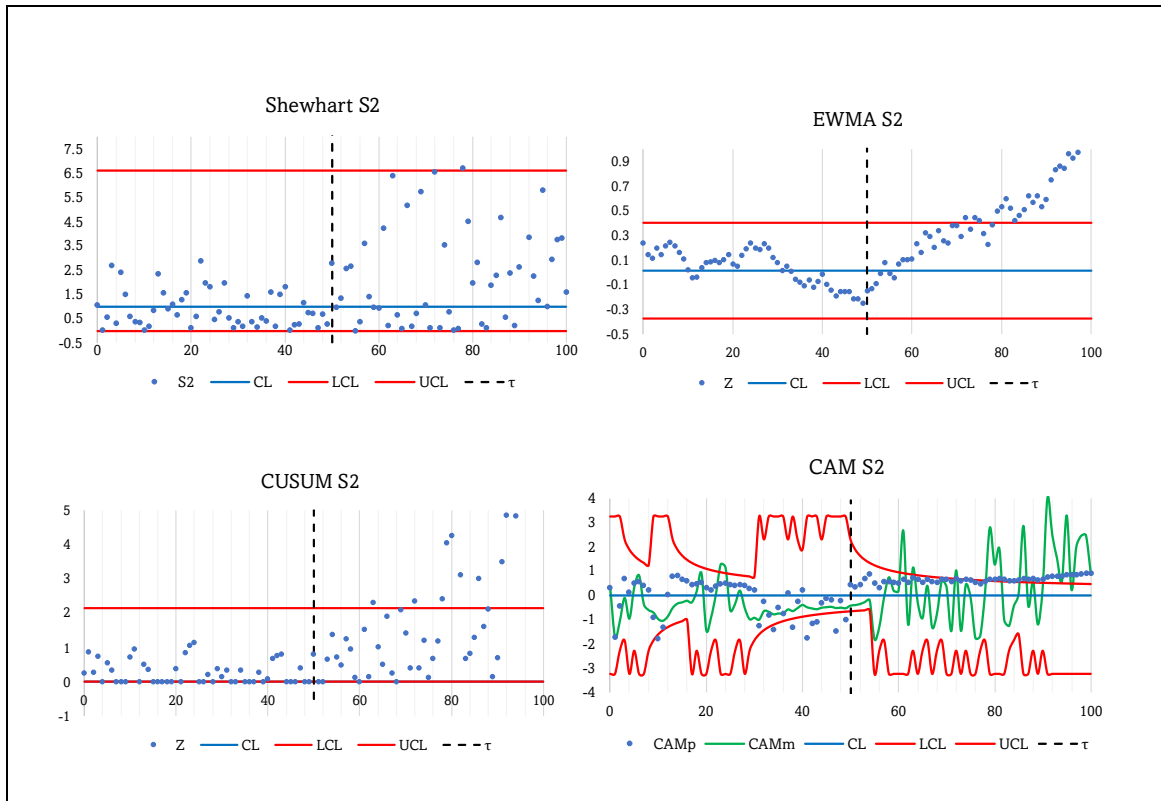


Figure 3. 7 Example for $n = 3$.

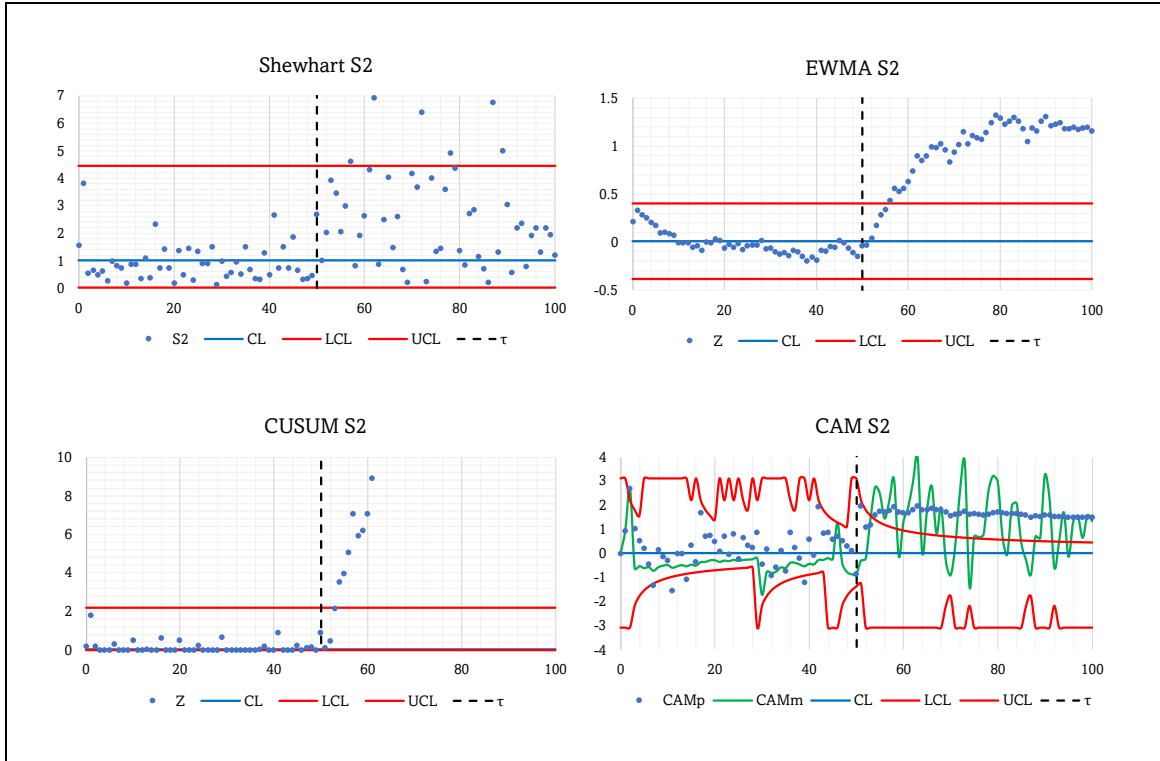


Figure 3. 8 Example for $n = 5$.

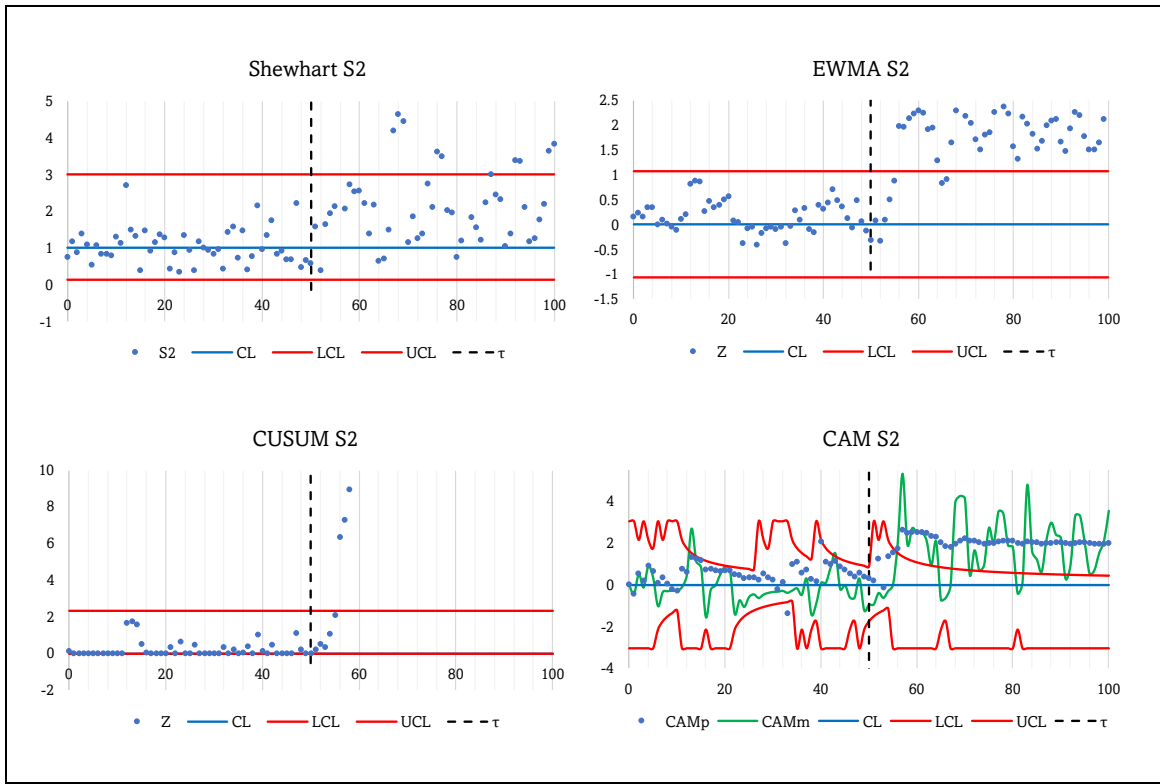


Figure 3. 9 Example for $n = 10$.

3.5 Application; windmill rotor speed

This section examines the variation of a real-world dataset of a windmill rotor speed under a primary braking system (Pankaj, 2021) using the control charts of Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , and CAM- S^2 (3-Parameter). The primary braking system in a wind turbine is a mechanical braking system that uses friction to slow down or stop the rotor. The braking force is produced by brake pads or calipers that are connected to the wind turbine's rotor or hub. The mechanical braking system is designed to be fail-safe, meaning that it will automatically engage in case of power failure or other emergencies.

The mechanical braking system is typically activated by a control system that monitors various factors such as wind speed, rotor speed, and temperature to ensure the stability of the windmill's operation. For instance, if the wind speed exceeds the maximum operating speed, or if there is a mechanical failure, then, the control system will activate the mechanical braking system to slow down or stop the rotor.

Wind turbines with control systems typically use sensors to measure wind speed and direction, as well as other parameters that may affect the turbines operation such as temperature and humidity. The information gathered through the sensors is used by the control system for altering the wind turbine blades' pitch angle by a controller. The pitch angle is the angle produced by the wind turbine blades and the plane of rotation. By altering the pitch angle, the control system can regulate the amount of lift and drag produced by the blades, which can affect the rotor speed. In general, as the pitch angle increases, lift and drag will also increase, which could boost rotor speed. Conversely, a decreased pitch angle will result in decreased lift and drag, that will slow the rotor speed.

Windmill rotor speed is defined as the rotating speed of a wind turbine's blades. It is a crucial factor that influences the turbine's safety and power generation effectiveness. The turbine may not produce enough power to be viable if the rotor speed is too low. In contrast, if the rotor speed is too high, it could cause damage to the electrical grid and the turbine. The optimal rotor speed for a wind turbine depends on the wind speed, blade design, and turbine capacity. Control systems in windmills are used to optimize the productivity of wind turbines and maintain a consistent rotor speed. They adjust the rotor speed to coincide with the wind speed and direction, ensuring that the wind turbine performs at its most effective level. The control system may also limit the rotor speed in high wind conditions to prevent damage to the turbine.

The targeted average windmill rotor speed for wind turbines with control systems depends on various factors, including the specific wind turbine model, environmental conditions, and the settings of the control system. Typically, the control system will aim to maintain a consistent rotor speed within a certain range, depending on the wind turbine model and operating conditions. For the given dataset, the data is the normalized speed variation in the primary optimal range of 50-55 m/s at a time interval of 10 minutes. For the study purpose, it has been taken 100 samples of size 5 from the data and made an increasing in the sample dispersion by a shift of size 1.25 from the sample of number 55 (i.e., $\tau = 55$). The

performance of the charts (Figure 3.10) was obtained using their optimal sensitivity parameter and the result was as follows. The Shewhart- S^2 detected the shift at the sample of number 73, but the delay, here, is expected as this chart has been designed for large shifts. For the EWMA- S^2 , it had a delay amount of 68 samples. Whereas the CUSUM- S^2 was able to detect the shift at the 59th sample, which is referred as a good performance that makes CUSUM- S^2 superior to EWMA- S^2 . The CAM- S^2 also was able to detect the shift at the 59th sample, and that indicates the reliability of the proposed CAM- S^2 control chart. The example conclusion consists with the CED results (Figure 3.5; $\gamma = 1.25$).

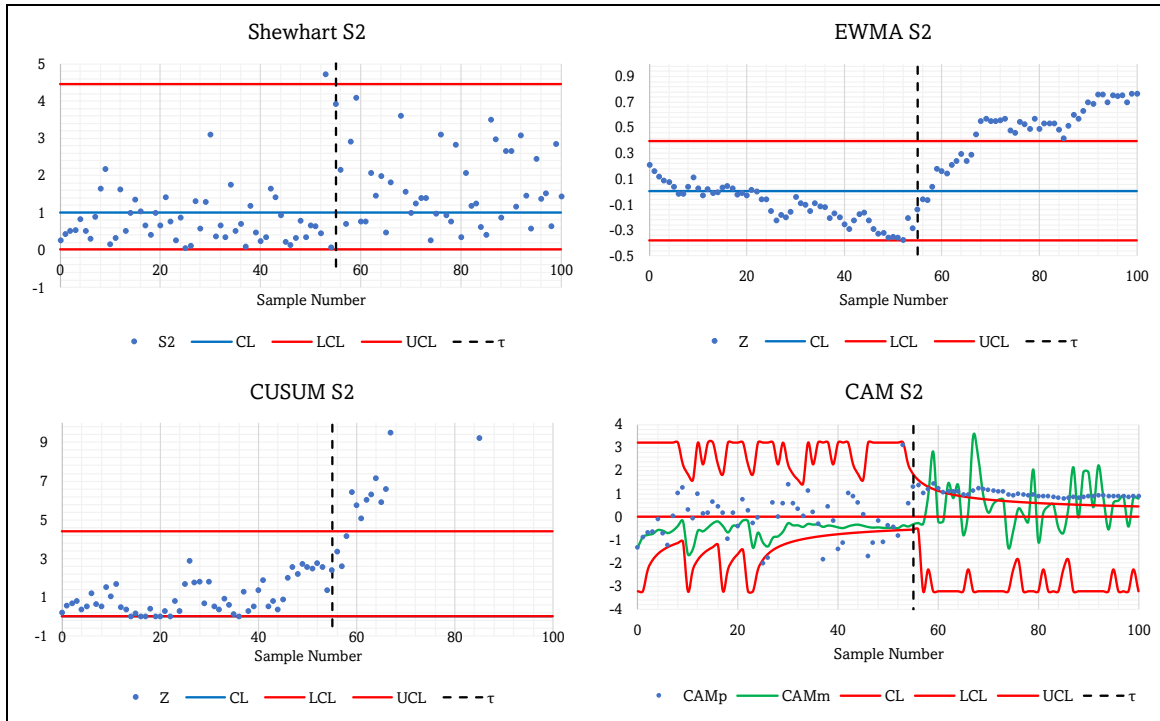


Figure 3. 10 Control charts performance to detect an increased shift of size 0.25 in the dispersion at $\tau = 300$.

3.6 Summary and conclusions

This chapter presented two charts of CAM- S^2 to detect shifts in a process dispersion. Their structure utilizes the sample variance to ensure the stability of process dispersion. For monitoring the behaviour of the sample variance, that follows the chi-squared distribution, the strategy of transforming the non-normal sample statistic to approximately normal distributed variable is employed in the proposed structure. This strategy of the transformation is typically used to facilitate constructing charts' limits. Two transformation methods are used for the proposed charts, that are, three-parameter logarithmic transformation method and four-parameter logarithmic transformation (Johnson SB) method, producing two transformed variables that serve as sample statistics, where the

charts apply their structure on these sample statistics. The proposed charts have been constructed in accordance with the original CAM structure, where it implements an equally weighting scheme on the samples statistic in case a deviation is observed. Otherwise, when no deviation is observed, all the weight is given to the current sample. The stage of searching for potential shifts (noticing deviations) is conducted based on variables known as indicator variables where they indicate if there is a deviation in the samples from target. The CUSUM statistics are used in this context to be the indicator variables and computed for each sample to signal any positive deviation by its upper statistic or any negative deviation by its lower one. An additional variable is a part of the CAM structure, which is known by the counter variable. The purpose of that counter variable is to reset the plotting statistic memory in case of no deviation is observed, otherwise, the chart statistic accumulates samples' information using equally weighting scheme. The proposed charts have been evaluated for the cases of zero and steady state, and they have been compared with Shewhart- S^2 , EWMA- S^2 , CUSUM- S^2 , and Floating PM control charts. The conclusion is that the proposed scheme demonstrates reliable performance, as evidenced by the measures of ARL , EQL , and CED , especially, for steady-state monitoring case and samples of small size.

CHAPTER 4

NP-CAM Control Chart

In many real-life situations, when monitoring a process using control charts, the process observations follow an unknown distribution, which may violate the normality assumption that has been assumed when the Conditional Aggregate Mean (CAM) control chart has been constructed. For these situations, the need for non-parametric CAM control chart arises. This chapter applies the non-parametric sign-test to the CAM statistic for monitoring the location parameter of a process with an unknown distribution.

In SPC, most of the suggested structures of the control charts are constructed under the uncontaminated and normality assumptions of process observations, which in practice may be violated or even be, the underlying distribution of the observations, unknown. Consequently, the chart performance will be negatively affected and hence loses its ability to detect shifts by a small number of samples. In this regard, the study of non-parametric control charts is needed.

Non-parametric tests provide reliable results even when the assumptions are violated, which makes the non-parametric analysis to be more robust. (Gibbons & Chakraborti, 2003) mentioned that it is sensible to consider robustness as a performance criterion for parametric tests and power as a performance criterion for nonparametric, however, they claimed that nonparametric tests often exhibit comparable levels of power, particularly with small sample sizes. Consequently, in cases where assumptions are uncertain, nonparametric tests may be favored.

Non-parametric (NP) control chart is considered as a non-parametric analysis that is used to study the stability of a process variation when the process observations have a population of an asymmetric, heavy-tailed, or even unknown distribution. These types of distributions cannot be always studied using parametric control charts, that depend on the properties of sample mean statistic, for many reasons: the non-existence possibility of some moments (mean or variance) for the heavy-tailed distributions (Markovich, 2007), the outliers and extreme values can cause some sample statistics (mean or variance) to become biased, poor quality estimates, and consequently, losing chart efficiency.

There are two ideas that may be contested to avoid using the non-parametric analysis, which are, employing the central limit theorem and transforming the distribution of observations into normal distribution. Discussing these two ideas, the central limit theorem is not always possible to be used since the assumption of finite variance may become violated, particularly with the heavy-tailed distributions, and whenever it is applicable, it may not work effectively if the sample size is not sufficiently large. (Chakraborti et al., 1999) stated that the central limit theorem may be considered by practitioners as a reliable means to rescue the performance of control charts. This perspective may hold true for some control charts based on the averages of specific statistics

from well-behaved processes. However, it cannot be generalized to all control charts. Furthermore, it is not always possible to invoke the central limit theorem, particularly when using control charts for individual data.

Whereas for the transformation idea, it can often make the distribution appropriate for the parametric analyses, but the non-parametric analysis is more than dealing with the normality violation, it also has further properties that are included in outliers' resistance, no need for estimating variance to construct the control limits, more efficiency when the sample size is small.

The essential control chart structures of Shewhart's charts (Montgomery, 2020), the CUSUM (Page, 1954), and the EWMA (Roberts, 1959) have been constructed under the parametric assumptions of normality and no outliers. Later, many charts have been proposed to study the performance of these essential charts releasing the assumption of normality by using different non-parametric tests. To implement such these NP charts, the statistic of the non-parametric tests is used as a test statistic of the chart. That means for each sample, its test statistic is represented by a non-parametric test statistic. This chapter presents a non-parametric control chart based on the non-parametric sign-test.

Based on the sign-test, many charts in the literature are constructed to study the performance of the essential charts releasing the assumption of normality. (Farnum & Stanton, 1986) presented Shewhart using counts to monitor process location. (Amin et al., 1995) developed basic Shewhart and CUSUM sign charts. (Amin & Widmaier, 2000) added some features to the Shewhart sign chart to enhance its performance in monitoring the process location. (Amin & Sattler, 2001) used the property of curtailed sampling on Shewhart and CUSUM sign charts for monitoring the process location. (Z. Abbas et al., 2021) studied the PM control chart for individual samples using sign-test to monitor process location.

There are other charts that depend on a test other than the sign-test. (S. T. Bakir & Reynolds, 1979) used the within group signed-ranks to develop a non-parametric CUSUM for monitoring process location, (Amin & Searcy, 1991) used the within-group signed-ranks to develop EWMA. (S. T. Bakir, 2004) developed a Shewhart signed-ranks control chart of grouped observations to monitor a process location. (Chakraborti & Eryilmaz, 2007) developed run rules Shewhart signed-ranks control chart under the known case. For more of non-parametric control charts, see (Chakraborti et al., 1999), (S. Bakir, 2001) , and (Chakraborti & Graham, 2019).

4.1 The proposed control chart

Let the sample size is denoted by n , and the process observations, denoted by X , follow a continuous unknown distribution, F , such that

$$X_{ij} \sim F(\theta)$$

where i is the sample number, j is the observation number, and $\underline{\theta}$ is a vector of the distribution parameters.

4.1.1 The stability concern – Test statistic

The sample median, which is considered as a robust statistic, will be used as a variable of the stability concern to control the process location. Assuming the target median of X is denoted by M_0 , then, the test statistic for each i^{th} sample using the sign-test is computed by:

$$SN_i = \sum_{j=1}^n \text{sign}(X_{ij} - M_0)$$

where j is the observation number at the i^{th} sample, and

$$\text{sign}(X_{ij} - M_0) = \begin{cases} 1 & , \text{if } X_{ij} > M_0 \\ -1 & , \text{if } X_{ij} < M_0 \end{cases}$$

The case of $X_{ij} = M_0$ is discarded as the distribution of the process observations is assumed to be continuous, which leads to have $P(X_{ij} = M_0) = 0$.

To identify the distribution of the test statistic SN_i , a random variable has been suggested to be defined for each sample i , (Z. Abbas et al., 2021), such that

$$B_i = \frac{SN_i + n}{2} \sim \text{Binomial}(n, p)$$

This variable gives the number of positive signs in a sample, and it is characterized as a binomial random variable for the following reasons: the sample size n is fixed, each observation is independent and either greater than the target median M_0 or less than it, and the probability p to exceed the target median M_0 is predefined at a desired rate, i.e., p is the same for each observation.

Therefore, the expected value and the variance of the variable B_i are given by:

$$E(B_i) = np, \text{ and } \text{Var}(B_i) = np(1 - p)$$

This implies the following results for the sign-test statistic:

$$SN_i = 2B_i - n$$

with $E(SN_i) = n(2p - 1)$ and $Var(SN_i) = 4np(1 - p)$.

The process is considered statistically IC if the SN_i are symmetrically distributed around its expected value. In other words, the probability of having observations below the target median is equal to the probability of having observations above it, i.e., $P(X > M_0) = 0.5$. Hence, the parameter of success probability for the SN distribution is $p = 0.5$ and this leads to have the following.

$$E(SN_i) = 0 \text{ and } Var(SN_i) = n \quad (4.1)$$

4.1.2 The proposed NP-CAM structure

To construct the non-parametric CAM control chart, as the parametric CAM chart, there are two pre-requisite variables:

- The indicator variable; Non-parametric CUSUM statistics:

$$\begin{aligned} C_i^+ &= \text{Max}[0, (SN_i - E(SN_i)) - k * SD(SN_i) + C_{i-1}^+] \\ C_i^- &= \text{Min}[0, (SN_i - E(SN_i)) + k * SD(SN_i) + C_{i-1}^-] \end{aligned}$$

for $i = 1, 2, 3, \dots$

with $C_0^+ = C_0^- = 0$. Where k is the sensitivity parameter of the chart.

Using equations (4.1), we get

$$\begin{aligned} C_i^+ &= \max[0, SN_i - k\sqrt{n} + C_{i-1}^+] \\ C_i^- &= \min[0, SN_i + k\sqrt{n} + C_{i-1}^-] \end{aligned} \quad (4.2)$$

for $i = 1, 2, 3, \dots$

with starting values $C_0^+ = C_0^- = 0$.

- The counter variables:

$$\begin{aligned} D_i^+ &= \begin{cases} 1 & \text{if } C_{i-1}^+ = 0 \\ 1 + D_{i-1}^+ & \text{otherwise.} \end{cases} \\ D_i^- &= \begin{cases} 1 & \text{if } C_{i-1}^- = 0 \\ 1 + D_{i-1}^- & \text{otherwise.} \end{cases} \end{aligned} \quad (4.3)$$

for $i = 1, 2, 3, \dots$

where C_i^+ and C_i^- are the CUSUM statistics from equations (4.2).

Now, the plotting statistics of the non-parametric CAM control chart using the sign-test statistic can be computed by these formulas:

$$NP.CAM_i^+ = \frac{1}{D_i^+} SN_i + \left(1 - \frac{1}{D_i^+}\right) NP.CAM_{i-1}^+$$

$$NP.CAM_i^- = \frac{1}{D_i^-} SN_i + \left(1 - \frac{1}{D_i^-}\right) NP.CAM_{i-1}^-$$

for $i = 1, 2, 3, \dots$

where D_i^+ and D_i^- are the counter variables from equations (4.3).

The initial values are $NP.CAM_0^+ = NP.CAM_0^- = E(SN) = 0$.

To construct the control limits, based on the method of L -sigma limits, the mean and variance of the plotting statistics are needed and can be found as follows.

$$E(NP.CAM^+) = E(NP.CAM^-) = E(SN) = 0$$

$$Var(NP.CAM^+) = \frac{\sigma_{SN}^2}{D_i^+} = \frac{n}{D_i^+} \quad \text{and} \quad Var(NP.CAM^-) = \frac{\sigma_{SN}^2}{D_i^-} = \frac{n}{D_i^-}$$

Therefore, the non-parametric CAM plotting statistics are plotted against the control limits that are given by:

The upper control limit: $UCL_i = L \sqrt{\frac{n}{D_i^+}}$, the central line: $CL = 0$, and the lower control limit: $LCL_i = -L \sqrt{\frac{n}{D_i^-}}$.

Where L is the limits coefficient and chosen based on the desired ARL_0 , the in-control ARL .

4.2 Performance results

This section presents the performance of the proposed NP-CAM chart for zero and steady-states. It has been implemented for unknown distribution, by drawing SN_i randomly from the binomial distribution (i.e., $SN \sim Bin(1, 0.5)$); and evaluated by the measure of the average run length with fixing $ARL_0 = 510$ and implemented also for different kinds of distributions: $Normal(0, 1)$, $Laplace(0, 1)$, $Logistic(0, \sqrt{3}/\pi)$, $t(4)$, $t(8)$, $Gamma(0.5, 1)$, $Gamma(1, 1)$, and $Gamma(3, 1)$.

Figure 4.1 shows the skewness of the distributions that has been studied for the NP-CAM chart. The Normal, Laplace, Logistic, $t(4)$, and $t(8)$ distributions are all categorized as symmetric distributions, whereas the Gamma distributions are right-skewed. The symmetric non-normal distributions differ from the normal distribution by featuring probability functions with heavy tails. These distributions are ranked based on the heaviness of their tails, with the Laplace distribution possessing the heaviest tail, followed by $t(4)$, $t(8)$, and then the Logistic distribution. In terms of skewness, the gamma distribution with the smallest location parameter exhibits higher skewness.

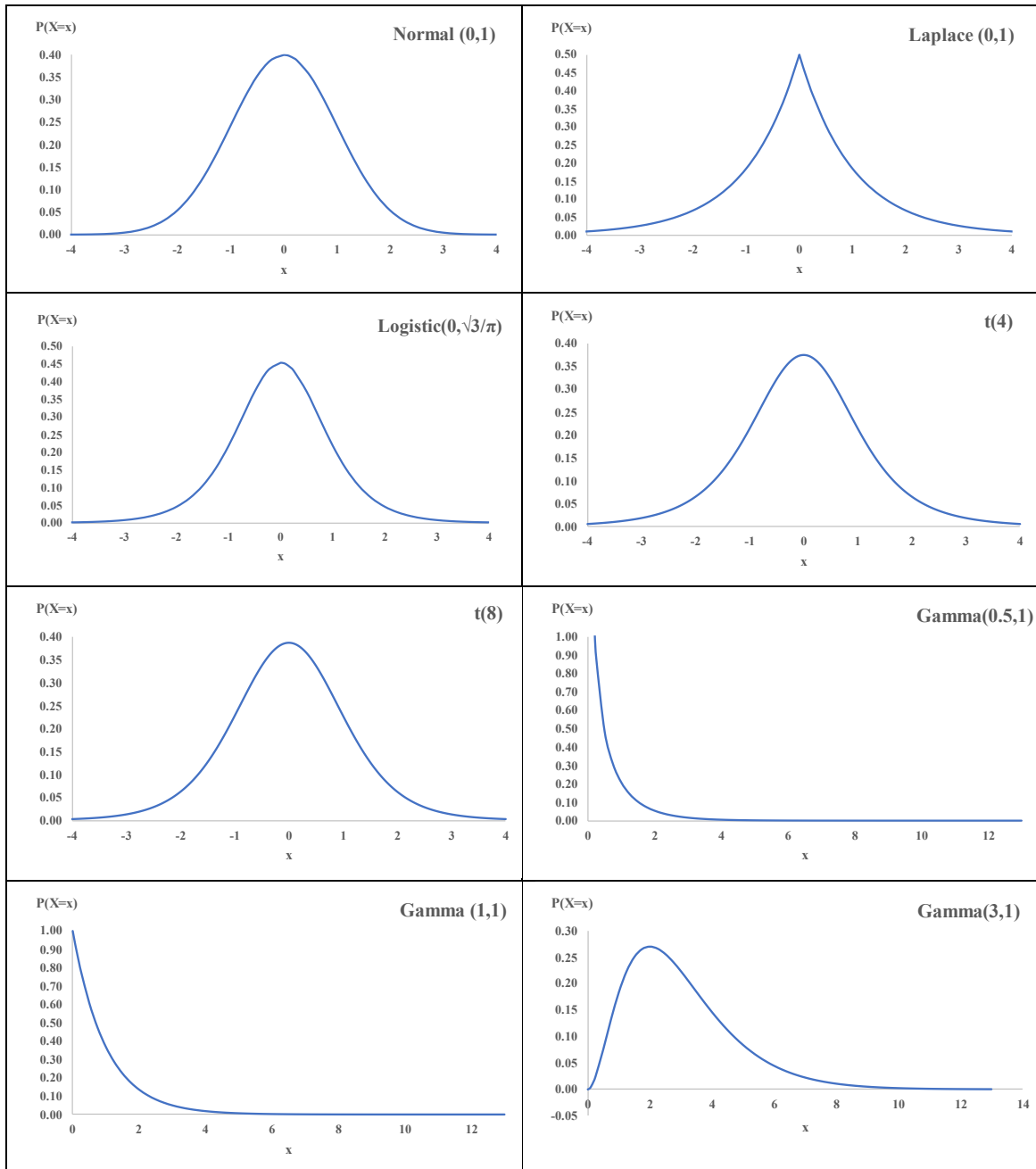


Figure 4. 1 Skewness of the distributions.

4.2.1 The NP-CAM chart performance for zero-state

▪ Free-distribution results

Table 4.1 presents the results obtained by implementing the NP-CAM control chart with a fixed $ARL_0 = 510$. These results represent the chart's performance under IC conditions. The table includes metrics of $SDRL$ and P_kRL (k^{th} Percentile of Run Length).

Remarkably, the results for $SDRL$, P_5RL , $P_{25}RL$, $P_{50}RL$, $P_{75}RL$, and $P_{95}RL$ show minimal variation across different sensitivity parameter values denoted as k . This consistency in NP-CAM performance across various sensitivity values k underscores its robustness and highlights the effectiveness of non-parametric techniques in maintaining control chart performance.

Table 4. 1 Free-distribution results of NP-CAM control chart.

n=1							
k	ARL	$SDRL$	P_5RL	$P_{25}RL$	$P_{50}RL$	$P_{75}RL$	$P_{95}RL$
0.2	510.28	492.73	40.00	159.00	359.00	703.00	1494.05
0.3	510.36	494.17	40.00	158.00	358.00	702.00	1498.00
0.4	510.18	497.87	38.00	154.00	357.00	705.25	1503.00
0.5	510.74	500.08	37.00	154.00	357.00	704.00	1504.00
0.8	510.86	504.70	33.00	152.00	355.00	705.00	1520.00
0.9	510.92	503.37	33.00	154.00	356.00	707.00	1513.00

▪ Distributional results

This part delves into the performance analysis of the proposed control chart across various data distributions. The goal is to assess how the CAM- S^2 control chart responds to different data natures and to understand its performance characteristics under diverse distribution scenarios.

Table 4. 2 *ARL* and *EQL* values of NP-CAM for $Normal(0,1)$, zero-state $\tau = 1$.

<i>Normal(0,1)</i>						
<i>K</i>	0.2	0.3	0.4	0.5	0.8	0.9
<i>L</i>	3.087	3.1298	3.12	3	2.835	2.83
δ	<i>ARL</i>					
-3	10.0456	10.0463	10.0417	10.0379	9.0597	9.0573
-2.75	10.1026	10.1032	10.0995	10.0849	9.1397	9.1333
-2.5	10.2212	10.2223	10.2145	10.1757	9.2909	9.2854
-2.25	10.4405	10.4347	10.4289	10.3617	9.5739	9.5802
-2	10.8313	10.8461	10.8103	10.7041	10.1051	10.1361
-1.75	11.5245	11.5253	11.4819	11.3113	11.0945	11.1082
-1.5	12.7405	12.7337	12.6657	12.4203	12.9393	12.9354
-1.25	14.8626	14.8433	14.7214	14.5407	16.3450	16.3682
-1	18.8334	18.8113	18.6598	18.8642	23.4978	23.5158
-0.75	27.0744	27.1068	27.3614	29.0672	40.1895	40.2384
-0.5	48.3427	49.9720	53.2786	59.9998	86.2203	86.0834
-0.25	136.9506	150.6051	165.5782	185.1944	239.5783	239.0004
0.25	137.8541	151.6859	165.1668	186.0443	239.1907	240.3568
0.5	48.1989	49.6624	53.1094	59.6454	85.8865	86.4384
0.75	27.0692	27.1091	27.2752	29.1723	40.1726	40.1613
1	18.8024	18.7975	18.6196	18.8528	23.5091	23.5086
1.25	14.8546	14.8350	14.7212	14.5301	16.4107	16.3860
1.5	12.7388	12.7346	12.6651	12.4255	12.9060	12.9324
1.75	11.5313	11.5290	11.4875	11.3076	11.0892	11.0914
2	10.8311	10.8337	10.8104	10.6900	10.1212	10.1199
2.25	10.4389	10.4464	10.4276	10.3569	9.5763	9.5845
2.5	10.2216	10.2187	10.2163	10.1773	9.2832	9.2898
2.75	10.1051	10.1039	10.1020	10.0849	9.1346	9.1389
3	10.0456	10.0486	10.0456	10.0380	9.0573	9.0622
<i>EQL</i>	35.6546	35.7943	35.9120	36.0973	36.2941	36.3145

Table 4. 3 *ARL* and *EQL* values of NP-CAM for *Laplace*(0,1), zero-state $\tau = 1$.

<i>Laplace</i> (0, 1)						
<i>K</i>	0.2	0.3	0.4	0.5	0.8	0.9
<i>L</i>	3.087	3.1302	3.1241	3.0502	2.84	2.82905
δ	<i>ARL</i>					
-3	10.2559	10.2541	10.2426	10.2116	9.3334	9.3269
-2.75	10.3648	10.3639	10.3535	10.2976	9.4704	9.4875
-2.5	10.5234	10.5305	10.5026	10.4347	9.6964	9.6909
-2.25	10.7530	10.7611	10.7391	10.6277	9.9918	9.9965
-2	11.1029	11.0878	11.0754	10.9288	10.4644	10.4866
-1.75	11.6149	11.6088	11.5729	11.3905	11.2210	11.2410
-1.5	12.4088	12.3981	12.3604	12.1103	12.4318	12.4099
-1.25	13.6835	13.6920	13.5948	13.3411	14.3997	14.4439
-1	15.8987	15.8454	15.7179	15.6320	18.1961	18.2344
-0.75	20.2246	20.1482	20.0479	20.5087	26.1711	26.0882
-0.5	30.6759	30.6561	31.2880	33.9926	47.6685	47.9853
-0.25	72.4009	77.4122	84.2689	97.1383	135.5561	136.3558
0.25	72.7504	76.9864	84.9195	96.4249	135.2983	135.4000
0.5	30.7887	30.8115	31.3907	34.0337	47.7589	47.6909
0.75	20.2364	20.1653	20.0592	20.4513	26.2009	26.1830
1	15.8684	15.8732	15.7457	15.6080	18.1948	18.2060
1.25	13.6575	13.6619	13.5882	13.3608	14.4431	14.4409
1.5	12.3788	12.3942	12.3378	12.1276	12.4105	12.4337
1.75	11.6193	11.6144	11.5656	11.3760	11.2559	11.2230
2	11.1094	11.1089	11.0701	10.9360	10.4966	10.4950
2.25	10.7586	10.7531	10.7325	10.6235	10.0109	10.0214
2.5	10.5311	10.5238	10.5082	10.4282	9.6831	9.6973
2.75	10.3667	10.3689	10.3601	10.2970	9.4843	9.4693
3	10.2566	10.2547	10.2510	10.2046	9.3404	9.3231
<i>EQL</i>	34.6491	34.6773	34.6637	34.5195	34.1043	34.1165

Table 4. 4 *ARL* and *EQL* values of NP-CAM for $Logistic(0, \sqrt{3}/\pi)$, zero-state $\tau = 1$.

<i>Logistic(0, $\sqrt{3}/\pi$)</i>						
<i>K</i>	0.2	0.3	0.4	0.5	0.8	0.9
<i>L</i>	3.087	3.1298	3.121	3	2.835	2.83
δ	<i>ARL</i>					
-3	10.1539	10.1466	10.1486	10.1244	9.1901	9.2054
-2.75	10.2466	10.2450	10.2336	10.2001	9.3112	9.3123
-2.5	10.3825	10.3821	10.3664	10.3049	9.5026	9.5003
-2.25	10.5936	10.6045	10.5837	10.4927	9.7903	9.7864
-2	10.9630	10.9545	10.9278	10.8062	10.2926	10.2813
-1.75	11.5307	11.5191	11.4843	11.3218	11.1102	11.1047
-1.5	12.5010	12.4734	12.4298	12.1968	12.5323	12.5310
-1.25	14.1236	14.1502	14.0370	13.8387	15.1802	15.2497
-1	17.2523	17.2449	17.0914	17.1414	20.6432	20.6646
-0.75	23.8703	23.7810	23.8490	24.8658	33.4682	33.2310
-0.5	40.8262	41.6000	43.6319	48.4496	70.2018	69.9467
-0.25	114.4737	125.7401	137.7948	157.0169	208.3231	206.8559
0.25	114.4524	125.7801	137.6682	156.6524	208.0409	207.6619
0.5	41.0002	41.5941	43.5413	48.5924	70.0012	70.5107
0.75	23.8120	23.8015	23.8546	24.7565	33.3822	33.5048
1	17.2442	17.2344	17.1098	17.0890	20.6772	20.5452
1.25	14.1290	14.1613	14.0345	13.8293	15.1709	15.2284
1.5	12.4875	12.4838	12.4255	12.2309	12.5759	12.5555
1.75	11.5375	11.5426	11.4944	11.3132	11.1084	11.1047
2	10.9505	10.9535	10.9269	10.7880	10.2775	10.2980
2.25	10.5988	10.6010	10.5848	10.4902	9.7853	9.7874
2.5	10.3757	10.3777	10.3700	10.3135	9.4967	9.5081
2.75	10.2425	10.2402	10.2364	10.1898	9.3142	9.3209
3	10.1522	10.1517	10.1429	10.1220	9.1959	9.1970
EQL	35.2190	35.3181	35.3797	35.4208	35.3621	35.3643

Table 4. 5 *ARL* and *EQL* values of NP-CAM for $t(4)$, zero-state $\tau = 1$.

	$t(4)$					
K	0.2	0.3	0.4	0.5	0.8	0.9
L	3.09	3.1304	3.1	3.03	2.83	2.88
δ	<i>ARL</i>					
-3	10.2351	10.2311	10.2245	10.1942	9.3055	9.3048
-2.75	10.3138	10.3129	10.3076	10.2550	9.4178	9.4129
-2.5	10.4357	10.4425	10.4235	10.3546	9.5668	9.5741
-2.25	10.6133	10.6085	10.5916	10.4993	9.7937	9.7951
-2	10.8690	10.8631	10.8458	10.7252	10.1611	10.1504
-1.75	11.2780	11.2980	11.2530	11.1004	10.7412	10.7478
-1.5	11.9764	11.9733	11.9357	11.7244	11.7549	11.7925
-1.25	13.1755	13.1702	13.1201	12.8769	13.6621	13.6299
-1	15.4421	15.4269	15.3046	15.1724	17.3509	17.4467
-0.75	20.3448	20.2715	20.1901	20.6087	26.4064	26.5297
-0.5	33.4893	33.5193	34.3771	37.8174	53.4096	53.6520
-0.25	91.0100	98.7669	108.4255	123.9477	169.7010	169.4647
0.25	90.9800	98.3188	108.2900	124.1775	170.2756	169.1186
0.5	33.3994	33.4850	34.4362	37.6884	53.4179	53.4958
0.75	20.3782	20.3436	20.1213	20.6050	26.4183	26.3296
1	15.4553	15.4248	15.3000	15.1881	17.3623	17.4094
1.25	13.1927	13.1631	13.0903	12.8869	13.6223	13.6601
1.5	11.9879	11.9727	11.9326	11.7437	11.7601	11.7868
1.75	11.2955	11.2956	11.2619	11.1015	10.7471	10.7464
2	10.8727	10.8798	10.8572	10.7278	10.1591	10.1547
2.25	10.6075	10.5981	10.5827	10.5051	9.7974	9.8078
2.5	10.4341	10.4279	10.4255	10.3523	9.5667	9.5688
2.75	10.3168	10.3097	10.3064	10.2603	9.4134	9.4051
3	10.2303	10.2292	10.2319	10.1877	9.3062	9.3092
<i>EQL</i>	34.3661	34.4143	34.4446	34.3819	33.7735	33.7840

Table 4. 6 *ARL* and *EQL* values of NP-CAM for $t(8)$, zero-state $\tau = 1$.

	$t(8)$					
K	0.2	0.3	0.4	0.5	0.8	0.9
L	3.09	3.1304	3.12	3.03	2.83	2.88
δ	<i>ARL</i>					
-3	10.1529	10.1510	10.1423	10.1182	9.1989	9.1969
-2.75	10.2255	10.2330	10.2248	10.1873	9.3013	9.3137
-2.5	10.3612	10.3656	10.3573	10.2990	9.4817	9.4729
-2.25	10.5744	10.5776	10.5536	10.4794	9.7517	9.7349
-2	10.9152	10.9220	10.8887	10.7654	10.2265	10.2385
-1.75	11.4913	11.4877	11.4341	11.2664	11.0379	11.0385
-1.5	12.4589	12.4622	12.3802	12.1506	12.4704	12.4792
-1.25	14.1294	14.1109	14.0433	13.7994	15.1874	15.1969
-1	17.3040	17.2993	17.1349	17.1442	20.6659	20.6624
-0.75	24.0151	24.0375	23.9938	25.1587	33.7844	33.6914
-0.5	41.6899	42.1643	44.1308	49.7008	71.7820	71.2469
-0.25	116.9516	128.2222	141.1772	160.6616	211.7383	211.2891
0.25	116.6106	128.9946	142.5207	161.1547	210.8583	211.2189
0.5	41.5345	42.2415	44.5932	49.8439	71.2548	71.4061
0.75	24.0206	24.0165	24.0941	25.2001	33.7216	33.8124
1	17.3312	17.2810	17.1640	17.1924	20.7296	20.7729
1.25	14.1061	14.1410	14.0017	13.7988	15.1849	15.1888
1.5	12.4542	12.4304	12.3944	12.1481	12.4820	12.4930
1.75	11.4985	11.4719	11.4402	11.2692	11.0642	11.0495
2	10.9128	10.9111	10.8842	10.7642	10.2425	10.2334
2.25	10.5764	10.5760	10.5612	10.4770	9.7637	9.7527
2.5	10.3643	10.3588	10.3557	10.2955	9.4693	9.4746
2.75	10.2294	10.2345	10.2234	10.1897	9.3031	9.3017
3	10.1460	10.1495	10.1446	10.1235	9.1948	9.1959
<i>EQL</i>	35.2039	35.3075	35.3797	35.4435	35.3590	35.3542

Tables 4.2 through 4.6 reveal a consistent pattern that the NP-CAM control chart demonstrates robust and stable performance with the heaviest-tailed distributions, rendering it highly effective for monitoring extreme values or smaller shifts. In contrast, the traditional parametric control charts, when applied to data of heavy-tailed distributions, tend to become overly sensitive to extreme values while becoming less responsive to smaller shifts in the process mean. These issues are effectively managed by the NP-CAM control chart, as evident in the tables. The chart exhibits its most reliable performance with the heaviest-tailed distribution (such as Laplace, followed by $t(4)$, $t(8)$, and then Logistic) by effectively balancing sensitivity to shifts and robustness to extreme values. This balance contributes to maintaining process stability and reducing false alarms, making the NP-CAM chart well-suited for monitoring heavy-tailed data.

Table 4. 7 *ARL* and *EQL* values of NP-CAM for $\text{Gamma}(0.5,1)$, zero-state $\tau = 1$.

<i>Gamma</i>(0.5,1)						
<i>K</i>	0.2	0.3	0.4	0.5	0.8	0.9
<i>L</i>	3.087	3.1304	3.121	3	2.84	2.82905
δ	<i>ARL</i>					
-3	11.1269	11.1303	11.0978	10.9514	10.5368	10.5433
-2.75	11.4117	11.3940	11.3614	11.2112	10.9323	10.9311
-2.5	11.7791	11.7699	11.7179	11.5244	11.4579	11.4408
-2.25	12.2359	12.2510	12.1994	11.9725	12.1743	12.1910
-2	12.8935	12.8653	12.8267	12.5958	13.1899	13.1715
-1.75	13.7721	13.7556	13.6490	13.4364	14.5742	14.5921
-1.5	15.0432	15.0298	14.9102	14.7609	16.6959	16.6991
-1.25	16.9123	16.9050	16.7649	16.7717	20.0580	20.0647
-1	20.0531	20.0561	19.8689	20.3085	25.8792	25.8559
-0.75	25.8228	25.8374	25.9360	27.4732	37.3743	37.3640
-0.5	38.9591	39.3587	41.1492	45.8245	65.6624	66.1724
-0.25	88.7140	95.0527	104.9349	119.8695	164.4536	164.9787
0.25	31.3451	31.4787	32.0743	34.7476	49.2255	49.2753
0.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
0.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.25	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.25	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
3	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
<i>EQL</i>	35.0899	35.1072	35.0801	34.9997	34.8978	34.9040

Table 4. 8 *ARL* and *EQL* values of NP-CAM for $\text{Gamma}(1,1)$, zero-state $\tau = 1$.

<i>Gamma</i>(1, 1)						
<i>K</i>	0.2	0.3	0.4	0.5	0.8	0.9
<i>L</i>	3.09	3.1304	3.1	3.03	2.83	2.88
δ	<i>ARL</i>					
-3	10.9038	10.9136	10.8821	10.7682	10.2360	10.2281
-2.75	11.2104	11.2023	11.1642	11.0209	10.6153	10.6289
-2.5	11.5822	11.5740	11.5334	11.3545	11.1666	11.1476
-2.25	12.0952	12.0729	12.0274	11.7961	11.9013	11.9119
-2	12.7720	12.7657	12.7021	12.4773	12.9937	13.0081
-1.75	13.7870	13.7576	13.6398	13.4579	14.5718	14.6095
-1.5	15.2225	15.2344	15.1115	14.9336	16.9883	17.0028
-1.25	17.4713	17.5010	17.3336	17.3824	21.1182	21.1661
-1	21.3425	21.3327	21.2668	21.8491	28.4845	28.5055
-0.75	28.7140	28.7687	29.1740	31.3003	43.7547	43.6058
-0.5	46.2347	47.3289	50.3642	56.6547	81.9298	82.1072
-0.25	115.8662	127.1402	139.4239	158.8848	210.3542	209.4845
0.25	78.1309	83.1089	91.8834	105.5156	147.2378	146.5079
0.5	20.4279	20.4273	20.3008	20.8122	26.7187	26.6543
0.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.25	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
1.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.25	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
3	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
<i>EQL</i>	35.5333	35.5997	35.6329	35.6711	35.8458	35.8499

Table 4. 9 *ARL* and *EQL* values of NP-CAM for $\text{Gamma}(3,1)$, zero-state $\tau = 1$.

$\text{Gamma}(3,1)$						
K	0.2	0.3	0.4	0.5	0.8	0.9
L	3.087	3.1302	3.1241	3.0502	2.84	2.82905
δ	<i>ARL</i>					
-3	10.5537	10.5486	10.5383	10.4546	9.7313	9.7288
-2.75	10.7786	10.7728	10.7571	10.6485	10.0398	10.0441
-2.5	11.1108	11.0996	11.0779	10.9341	10.4865	10.4749
-2.25	11.5724	11.5473	11.5334	11.3393	11.1294	11.1654
-2	12.2082	12.2295	12.1622	11.9540	12.1369	12.1365
-1.75	13.2064	13.1909	13.1176	12.9024	13.7130	13.6494
-1.5	14.7258	14.7135	14.6097	14.4246	16.1266	16.1288
-1.25	17.0950	17.1660	16.9965	16.9867	20.4400	20.4585
-1	21.4081	21.4415	21.2712	21.9337	28.5718	28.4977
-0.75	29.8843	29.8796	30.4109	32.8057	45.8812	45.8215
-0.5	50.7246	52.1768	55.8099	63.2751	90.9134	90.7384
-0.25	134.3694	147.7816	162.8336	182.9854	235.3532	234.7665
0.25	113.9486	124.9321	138.0996	157.1559	207.1001	206.9335
0.5	36.2569	36.4801	38.0595	41.9844	60.0170	60.1513
0.75	18.9611	18.9606	18.8473	19.0765	23.8811	23.8970
1	12.8839	12.8806	12.8096	12.5701	13.1866	13.1552
1.25	10.5495	10.5422	10.5295	10.4537	9.7141	9.7160
1.5	10.0026	10.0027	10.0021	10.0017	9.0028	9.0035
1.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.25	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.5	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
2.75	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
3	10.0000	10.0000	10.0000	10.0000	9.0000	9.0000
<i>EQL</i>	35.6796	35.7871	35.8937	36.0312	36.2479	36.2380

Based on the chart results, it is evident that positive shifts are promptly detected by the chart when monitoring data from a gamma distribution, which is a positively skewed distribution. This ability for rapid detection can be attributed to the distinct characteristics of positively skewed distributions, where the right tail extends further than the left tail. This makes the distribution to have relatively few exceptionally high values on the right side, with the majority of data points clustered on the left side. Consequently, any data point surpassing the mean (center) of the distribution naturally falls on the right side of the mean, allowing the quick detection of positive shifts. Conversely, detecting negative shifts becomes slower as most data points are concentrated on the higher side of the distribution. It takes time for data points to drop significantly below the mean, resulting in delayed detection. This delay in detection becomes significant as the distribution is less skewed, these results can be interpreted using Table 4.7, Table 4.8, and Table 4.9.

4.2.2 The NP-CAM chart performance for steady-state

▪ Distributional results

Figures 4.2 and 4.3 display the *CED* curves, a reliable metric for steady-state monitoring, of NP-CAM control charts for positive and negative shift detection, respectively. These

charts are applied to various distributions, including symmetric ones such as $Normal(0,1)$, $Laplace(0,1)$, $Logistic(0, \sqrt{3}/\pi)$, $t(4)$, and $t(8)$, as well as skewed distributions like $Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$. The CED curves are generated using the sensitivity parameter k of optimal performance corresponding to each shift size δ .

The analysis of the steady-state performance is consistent with the findings in the ARL tables and also indicates that all distributions exhibit a noticeable decrease in the CED curve until $\tau = 50$. Beyond this point, the curves stabilize as the change-point τ is delayed. The chart gives a high detection performance for the heavy-tailed distributions compared with the normal distribution for small values of shift, whereas for large shifts, the performance of the NP-CAM for all distributions is almost similar.

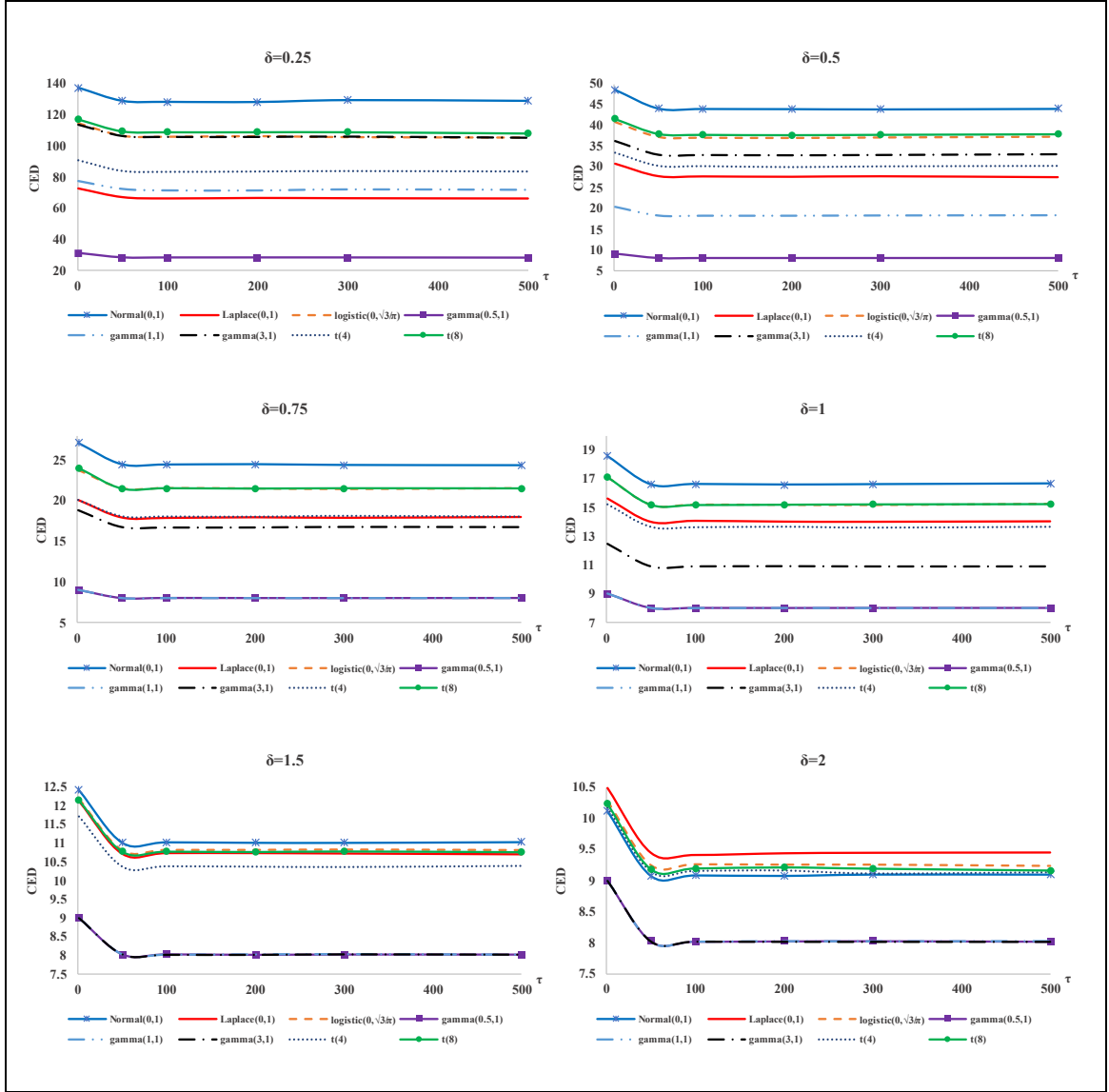


Figure 4. 2 CED curves of NP-CAM chart for different distributions to monitor positive shifts.

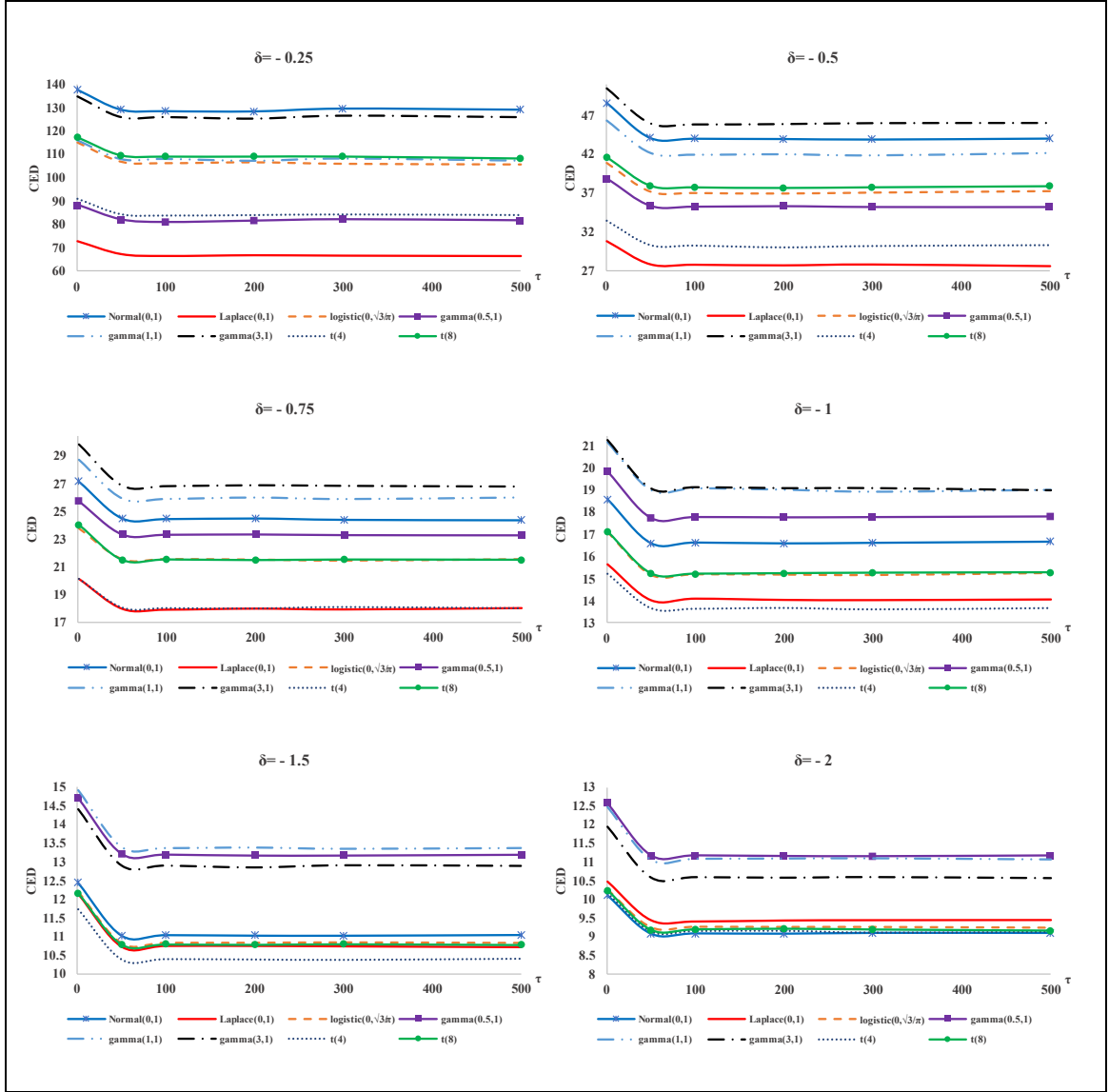


Figure 4.3 CED curves of NP-CAM chart for different distributions to monitor negative shifts.

4.3 Comparison

The proposed NP-CAM is examined in this section against the non-parametric exponential weighted moving average (NP-EWMA) for zero and steady states. The process distributions of $Normal(0,1)$, $Laplace(0,1)$, $Logistic(0, \sqrt{3}/\pi)$, $t(4)$, $t(8)$, $Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$ are studied for the comparison.

4.3.1 Comparison of charts' performance for zero-state

The metrics of ARL and EQL are used for performance analysis, considering the case of zero-state monitoring. Tables 4.10 to 4.14 show the charts' performance for the symmetric distributions: $Normal(0,1)$, $Laplace(0,1)$, $Logistic(0, \sqrt{3}/\pi)$, $t(4)$, and $t(8)$. Tables 4.15, 4.16, and 4.17 display the charts' performance for the gamma distributions: $Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$, respectively.

For the symmetric distributions, for small to moderate shift values, i.e., $|\delta| \leq 1.5$, the NP-EWMA detects shifts faster than the NP-CAM, whereas for large shifts, $|\delta| > 1.5$, NP-EWMA and NP-CAM exhibit almost similar abilities to detect shifts. For $|\delta| \geq 1.5$, the NP-CAM becomes slightly better. This conclusion is based on the highest sensitivity parameters (i.e., the minimum values of k) for each chart. However, when comparing their EQL values corresponding to the highest sensitivity value, it indicates that the NP-CAM is more effective in detecting shifts of unknown size.

Likewise, for gamma distributions (the right-skewed distributions), the results of ARL and EQL analysis are consistent with the results for symmetric distributions.

Table 4. 10 ARL and EQL values of the charts for $Normal(0,1)$, zero-state $\tau = 1$.

<i>Normal(0,1)</i>						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.0366	9.0389	9.0474	10.0456	10.0463	10.0417
-2.75	11.0782	9.0857	9.1029	10.1026	10.1032	10.0995
-2.5	11.1675	9.1872	9.2186	10.2212	10.2223	10.2145
-2.25	11.3422	9.3771	9.4347	10.4405	10.4347	10.4289
-2	11.6507	9.7338	9.8287	10.8313	10.8461	10.8103
-1.75	12.2002	10.3622	10.5718	11.5245	11.5253	11.4819
-1.5	13.1905	11.4672	11.9098	12.7405	12.7337	12.6657
-1.25	14.8815	13.5556	14.4612	14.8626	14.8433	14.7214
-1	18.0653	17.6458	19.6672	18.8334	18.8113	18.6598
-0.75	24.7284	27.0262	32.1632	27.0744	27.1068	27.3614
-0.5	42.4378	54.3615	67.6884	48.3427	49.9720	53.2786
-0.25	120.1590	168.8781	202.6429	136.9506	150.6051	165.5782
0.25	120.1270	168.9485	203.1273	137.8541	151.6859	165.1668
0.5	42.4233	54.2921	67.6789	48.1989	49.6624	53.1094
0.75	24.7289	26.9489	32.0373	27.0692	27.1091	27.2752
1	18.0756	17.6278	19.7140	18.8024	18.7975	18.6196
1.25	14.9134	13.5469	14.4561	14.8546	14.8350	14.7212
1.5	13.1808	11.4792	11.8837	12.7388	12.7346	12.6651
1.75	12.2069	10.3565	10.5692	11.5313	11.5290	11.4875
2	11.6582	9.7324	9.8385	10.8311	10.8337	10.8104
2.25	11.3415	9.3732	9.4349	10.4389	10.4464	10.4276
2.5	11.1661	9.1858	9.2204	10.2216	10.2187	10.2163
2.75	11.0804	9.0929	9.1042	10.1051	10.1039	10.1020
3	11.0353	9.0406	9.0469	10.0456	10.0486	10.0456
<i>EQL</i>	37.6206	32.8479	34.1415	35.6546	35.7943	35.9120

Table 4. 11 *ARL* and *EQL* values of the charts for *Laplace*(0,1), zero-state $\tau = 1$.

<i>Laplace</i> (0, 1)						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.2024	9.2177	9.2518	10.2559	10.2541	10.2426
-2.75	11.2843	9.3156	9.3572	10.3648	10.3639	10.3535
-2.5	11.4127	9.4490	9.5138	10.5234	10.5305	10.5026
-2.25	11.5908	9.6662	9.7758	10.753	10.7611	10.7391
-2	11.8578	9.9720	10.1089	11.1029	11.0878	11.0754
-1.75	12.2851	10.4578	10.6723	11.6149	11.6088	11.5729
-1.5	12.9188	11.1822	11.5489	12.4088	12.3981	12.3604
-1.25	13.9415	12.4087	13.0400	13.6835	13.692	13.5948
-1	15.6963	14.5771	15.7879	15.8987	15.8454	15.7179
-0.75	19.1725	19.1075	21.6483	20.2246	20.1482	20.0479
-0.5	27.7083	31.2733	37.8357	30.6759	30.6561	31.2880
-0.25	63.3690	86.8029	108.4937	72.4009	77.4122	84.2689
0.25	63.3762	87.0493	109.1690	72.7504	76.9864	84.9195
0.5	27.6324	31.1903	37.5991	30.7887	30.8115	31.3907
0.75	19.1938	19.1196	21.6440	20.2364	20.1653	20.0592
1	15.7202	14.6073	15.8325	15.8684	15.8732	15.7457
1.25	13.9229	12.4158	13.0495	13.6575	13.6619	13.5882
1.5	12.9113	11.1648	11.5697	12.3788	12.3942	12.3378
1.75	12.2679	10.4333	10.6618	11.6193	11.6144	11.5656
2	11.8675	9.9646	10.1151	11.1094	11.1089	11.0701
2.25	11.5912	9.6636	9.7653	10.7586	10.7531	10.7325
2.5	11.3996	9.4576	9.5182	10.5311	10.5238	10.5082
2.75	11.2810	9.3121	9.3608	10.3667	10.3689	10.3601
3	11.1972	9.2211	9.2453	10.2566	10.2547	10.2510
<i>EQL</i>	36.7437	31.4694	32.3716	34.6491	34.6773	34.6637

Table 4. 12 *ARL* and *EQL* values of the charts for $Logistic(0, \sqrt{3}/\pi)$, zero-state $\tau = 1$.

<i>Logistic</i> (0, $\sqrt{3}/\pi$)						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.1188	9.1298	9.1458	10.1539	10.1466	10.1486
-2.75	11.1879	9.2094	9.2401	10.2466	10.2450	10.2336
-2.5	11.2947	9.3296	9.3748	10.3825	10.3821	10.3664
-2.25	11.4643	9.5179	9.5964	10.5936	10.6045	10.5837
-2	11.7494	9.8290	9.9702	10.9630	10.9545	10.9278
-1.75	12.2067	10.3527	10.5814	11.5307	11.5191	11.4843
-1.5	12.9615	11.2641	11.6546	12.5010	12.4734	12.4298
-1.25	14.3270	12.8610	13.6088	14.1236	14.1502	14.0370
-1	16.8137	15.9684	17.5763	17.2523	17.2449	17.0914
-0.75	22.0901	23.1007	27.0187	23.8703	23.7810	23.8490
-0.5	36.2064	44.3497	54.9203	40.8262	41.6000	43.6319
-0.25	100.1845	140.8540	171.7462	114.4737	125.7401	137.7948
0.25	100.0314	141.2652	171.9303	114.4524	125.7801	137.6682
0.5	36.2158	44.1960	54.7736	41.0002	41.5941	43.5413
0.75	22.1077	23.0701	27.1133	23.8120	23.8015	23.8546
1	16.8393	15.9919	17.6929	17.2442	17.2344	17.1098
1.25	14.3195	12.8591	13.6210	14.1290	14.1613	14.0345
1.5	12.9716	11.2602	11.6366	12.4875	12.4838	12.4255
1.75	12.1987	10.3661	10.5917	11.5375	11.5426	11.4944
2	11.7476	9.8306	9.9698	10.9505	10.9535	10.9269
2.25	11.4706	9.5240	9.6006	10.5988	10.6010	10.5848
2.5	11.2992	9.3310	9.3765	10.3757	10.3777	10.3700
2.75	11.1866	9.2062	9.2344	10.2425	10.2402	10.2364
3	11.1188	9.1321	9.1491	10.1522	10.1517	10.1429
<i>EQL</i>	37.2427	32.2479	33.3890	35.2190	35.3181	35.3797

Table 4. 13 *ARL* and *EQL* values of the charts for $t(4)$, zero-state $\tau = 1$.

		$t(4)$					
The chart:		NP-EWMA			NP-CAM		
K/λ		0.05	0.14	0.25	0.2	0.3	0.4
δ		<i>ARL</i>					
-3		11.1807	9.2049	9.2378	10.2351	10.2311	10.2245
-2.75		11.2406	9.2683	9.3164	10.3138	10.3129	10.3076
-2.5		11.3376	9.3710	9.4237	10.4357	10.4425	10.4235
-2.25		11.4728	9.5296	9.6084	10.6133	10.6085	10.5916
-2		11.6879	9.7671	9.8747	10.8690	10.8631	10.8458
-1.75		12.0236	10.1385	10.3224	11.2780	11.2980	11.2530
-1.5		12.5654	10.7720	11.0669	11.9764	11.9733	11.9357
-1.25		13.5421	11.9116	12.4469	13.1755	13.1702	13.1201
-1		15.3573	14.1542	15.2186	15.4421	15.4269	15.3046
-0.75		19.2850	19.2961	21.7678	20.3448	20.2715	20.1901
-0.5		29.9046	34.6109	42.2391	33.4893	33.5193	34.3771
-0.25		79.2611	110.9023	137.4011	91.0100	98.7669	108.4255
0.25		79.4008	110.9309	137.3263	90.9800	98.3188	108.2900
0.5		29.8509	34.5966	42.1057	33.3994	33.4850	34.4362
0.75		19.2866	19.2595	21.8293	20.3782	20.3436	20.1213
1		15.3884	14.1081	15.2538	15.4553	15.4248	15.3000
1.25		13.5262	11.9064	12.4257	13.1927	13.1631	13.0903
1.5		12.5761	10.7764	11.0513	11.9879	11.9727	11.9326
1.75		12.0108	10.1397	10.3242	11.2955	11.2956	11.2619
2		11.6709	9.7761	9.8771	10.8727	10.8798	10.8572
2.25		11.4734	9.5316	9.6012	10.6075	10.5981	10.5827
2.5		11.3373	9.3761	9.4289	10.4341	10.4279	10.4255
2.75		11.2464	9.2732	9.3066	10.3168	10.3097	10.3064
3		11.1821	9.2033	9.2295	10.2303	10.2292	10.2319
<i>EQL</i>		36.5305	31.2811	32.1551	34.3661	34.4143	34.4446

Table 4. 14 *ARL* and *EQL* values of the charts for $t(8)$, zero-state $\tau = 1$.

$t(8)$						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.1164	9.1310	9.1470	10.1529	10.1510	10.1423
-2.75	11.1779	9.1996	9.2325	10.2255	10.2330	10.2248
-2.5	11.2784	9.3167	9.3592	10.3612	10.3656	10.3573
-2.25	11.4487	9.4956	9.5709	10.5744	10.5776	10.5536
-2	11.7225	9.8059	9.9273	10.9152	10.9220	10.8887
-1.75	12.1691	10.3201	10.5296	11.4913	11.4877	11.4341
-1.5	12.9441	11.2080	11.5958	12.4589	12.4622	12.3802
-1.25	14.2970	12.8385	13.5575	14.1294	14.1109	14.0433
-1	16.8591	16.0671	17.6620	17.3040	17.2993	17.1349
-0.75	22.2303	23.3623	27.1780	24.0151	24.0375	23.9938
-0.5	36.7890	45.1257	56.4054	41.6899	42.1643	44.1308
-0.25	101.6359	144.2182	175.4809	116.9516	128.2222	141.1772
0.25	101.6955	144.2920	176.3848	116.6106	128.9946	142.5207
0.5	36.7536	45.4454	56.1703	41.5345	42.2415	44.5932
0.75	22.2488	23.4206	27.2717	24.0206	24.0165	24.0941
1	16.8694	16.0985	17.6675	17.3312	17.2810	17.1640
1.25	14.3004	12.8293	13.5946	14.1061	14.1410	14.0017
1.5	12.9369	11.1996	11.6028	12.4542	12.4304	12.3944
1.75	12.1649	10.3122	10.5257	11.4985	11.4719	11.4402
2	11.7190	9.7996	9.9348	10.9128	10.9111	10.8842
2.25	11.4459	9.5054	9.5768	10.5764	10.5760	10.5612
2.5	11.2813	9.3153	9.3567	10.3643	10.3588	10.3557
2.75	11.1778	9.1984	9.2334	10.2294	10.2345	10.2234
3	11.1138	9.1333	9.1450	10.1460	10.1495	10.1446
<i>EQL</i>	37.2251	32.2612	33.3964	35.2039	35.3075	35.3797

Table 4. 15 *ARL* and *EQL* values of the charts for $Gamma(0.5,1)$, zero-state $\tau = 1$.

<i>Gamma(0.5, 1)</i>						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.8843	9.9844	10.1255	11.1269	11.1303	11.0978
-2.75	12.1033	10.2336	10.4491	11.4117	11.3940	11.3614
-2.5	12.4042	10.5830	10.8507	11.7791	11.7699	11.7179
-2.25	12.7991	11.0298	11.3818	12.2359	12.2510	12.1994
-2	13.2968	11.6380	12.1240	12.8935	12.8653	12.8267
-1.75	14.0125	12.4899	13.1180	13.7721	13.7556	13.6490
-1.5	15.0492	13.6965	14.7141	15.0432	15.0298	14.9102
-1.25	16.5569	15.6889	17.1198	16.9123	16.9050	16.7649
-1	19.0589	18.9851	21.3508	20.0531	20.0561	19.8689
-0.75	23.7237	25.4243	29.9643	25.8228	25.8374	25.9360
-0.5	34.6035	41.7380	51.3745	38.9591	39.3587	41.1492
-0.25	77.4823	107.7908	133.5856	88.7140	95.0527	104.9349
0.25	28.2690	32.2403	38.6979	31.3451	31.4787	32.0743
0.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
0.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.25	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.25	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
3	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
<i>EQL</i>	37.1175	31.9103	32.9297	35.0899	35.1072	35.0801

Table 4. 16 *ARL* and *EQL* values of the charts for $Gamma(1,1)$, zero-state $\tau = 1$.

<i>Gamma(1, 1)</i>						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	<i>ARL</i>					
-3	11.7179	9.8089	9.9388	10.9038	10.9136	10.8821
-2.75	11.9400	10.0497	10.2257	11.2104	11.2023	11.1642
-2.5	12.2197	10.3924	10.6262	11.5822	11.5740	11.5334
-2.25	12.6423	10.8742	11.1910	12.0952	12.0729	12.0274
-2	13.2031	11.5197	11.9672	12.7720	12.7657	12.7021
-1.75	14.0112	12.4675	13.1152	13.7870	13.7576	13.6398
-1.5	15.2216	13.9198	14.9295	15.2225	15.2344	15.1115
-1.25	17.0222	16.2901	17.8840	17.4713	17.5010	17.3336
-1	20.0868	20.3724	23.3431	21.3425	21.3327	21.2668
-0.75	26.0994	29.0114	34.4699	28.7140	28.7687	29.1740
-0.5	40.6907	51.5961	63.9281	46.2347	47.3289	50.3642
-0.25	101.6337	142.7006	174.3274	115.8662	127.1402	139.4239
0.25	68.2823	95.0939	117.4276	78.1309	83.1089	91.8834
0.5	19.4257	19.4992	21.9527	20.4279	20.4273	20.3008
0.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.25	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
1.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.25	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
3	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
<i>EQL</i>	37.4847	32.5103	33.6900	35.5333	35.5997	35.6329

Table 4. 17 ARL and EQL values of the charts for $Gamma(3,1)$, zero-state $\tau = 1$.

$Gamma(3,1)$						
The chart:	NP-EWMA			NP-CAM		
K/λ	0.05	0.14	0.25	0.2	0.3	0.4
δ	ARL					
-3	11.4276	9.4776	9.5441	10.5537	10.5486	10.5383
-2.75	11.6037	9.6864	9.7819	10.7786	10.7728	10.7571
-2.5	11.8608	9.9715	10.1077	11.1108	11.0996	11.0779
-2.25	12.2402	10.3788	10.6167	11.5724	11.5473	11.5334
-2	12.7739	11.0162	11.3236	12.2082	12.2295	12.1622
-1.75	13.5531	11.9473	12.4640	13.2064	13.1909	13.1176
-1.5	14.7781	13.4433	14.3111	14.7258	14.7135	14.6097
-1.25	16.7411	15.9132	17.4931	17.0950	17.1660	16.9965
-1	20.1646	20.5176	23.3279	21.4081	21.4415	21.2712
-0.75	26.9842	30.4022	36.0523	29.8843	29.8796	30.4109
-0.5	44.2631	57.2660	71.3662	50.7246	52.1768	55.8099
-0.25	117.0267	164.9495	199.1787	134.3694	147.7816	162.8336
0.25	99.6337	140.5805	173.1934	113.9486	124.9321	138.0996
0.5	32.3818	38.5549	47.1784	36.2569	36.4801	38.0595
0.75	18.2036	17.8642	19.9878	18.9611	18.9606	18.8473
1	13.3171	11.6170	12.0947	12.8839	12.8806	12.8096
1.25	11.4228	9.4758	9.5432	10.5495	10.5422	10.5295
1.5	11.0019	9.0021	9.0026	10.0026	10.0027	10.0021
1.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.25	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.5	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
2.75	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
3	11.0000	9.0000	9.0000	10.0000	10.0000	10.0000
EQL	37.6280	32.8089	34.0599	35.6796	35.7871	35.8937

4.3.2 Comparison of charts' performance for steady-state

In this section, the performance of NP-CAM and NP-EWMA across various shift sizes δ is analyzed using the *CED* curve, the reliable metric for steady-state monitoring. The *CED* curves for the following distributions are depicted in Figures 4.4 to 4.11, respectively: $Normal(0,1)$, $Laplace(0,1)$, $Logistic(0, \sqrt{3}/\pi)$, $t(4)$, $t(8)$, $Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$. The results are obtained by considering different change-points, including $\tau = 1, 50, 100, 200, 300, 500$. The charts performance is based on their optimal sensitivity parameter k .

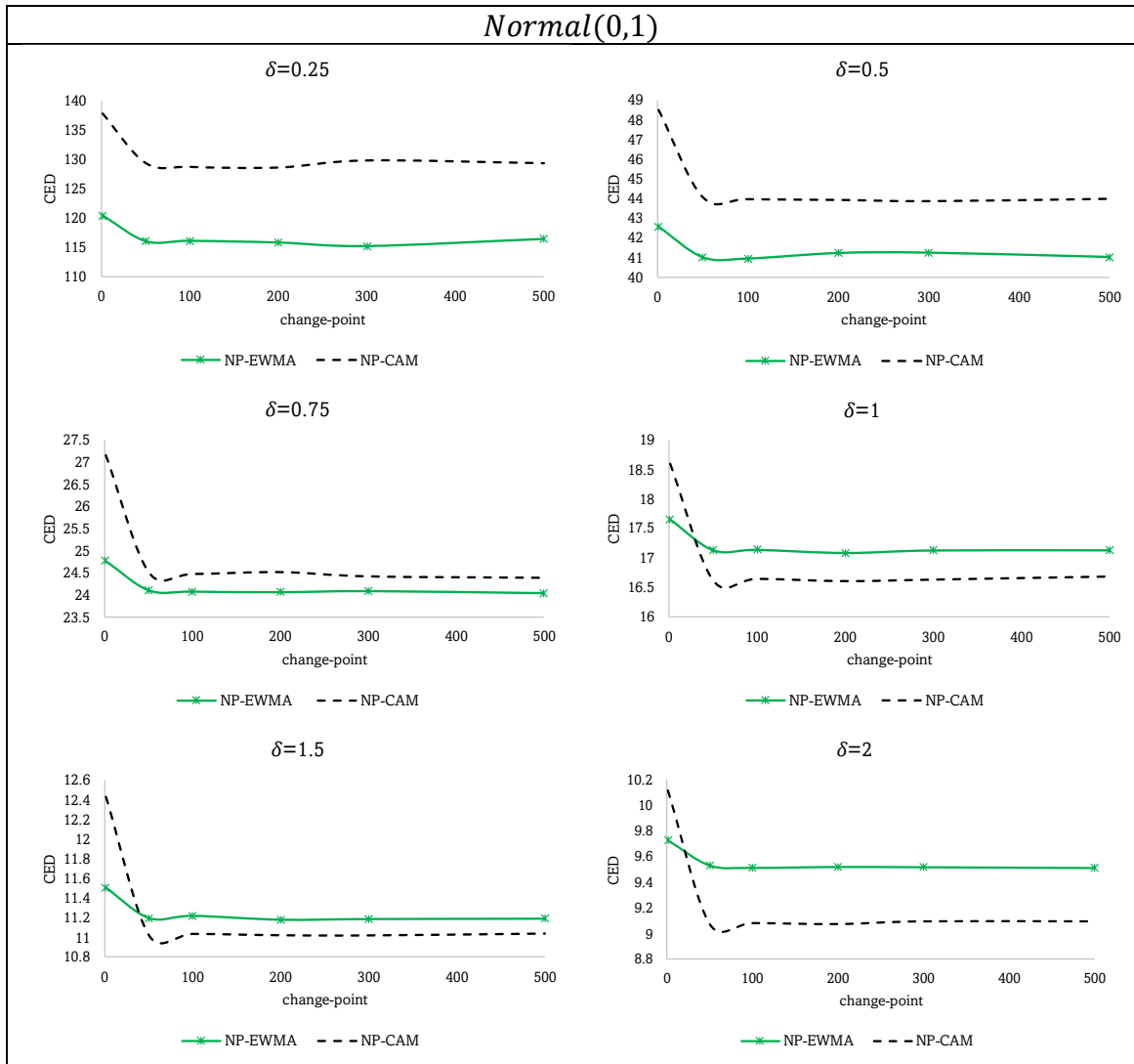


Figure 4. 4 CED curves of the non-parametric charts for $Normal(0,1)$.

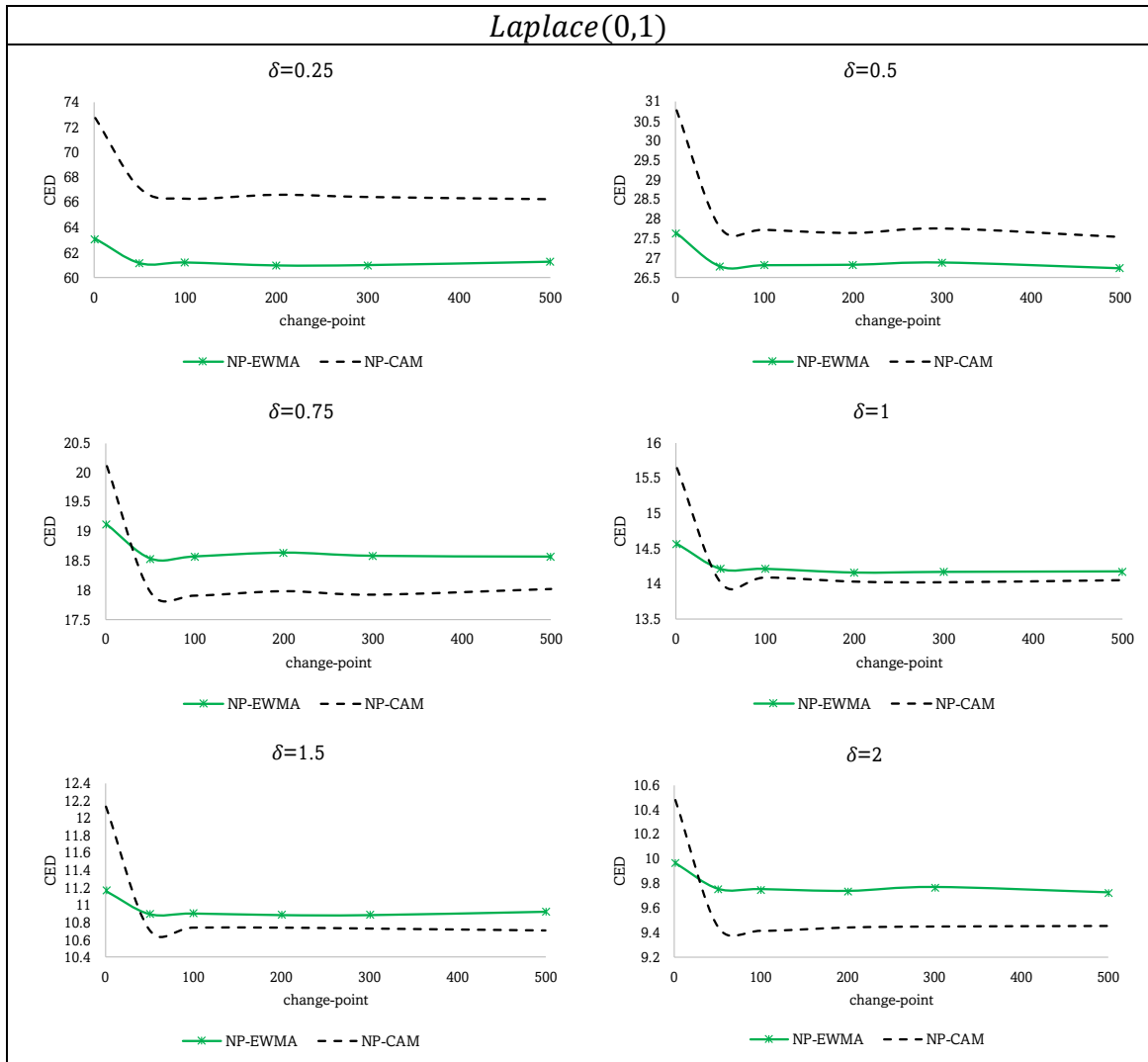


Figure 4. 5 CED curves of the non-parametric charts for $Laplace(0,1)$.

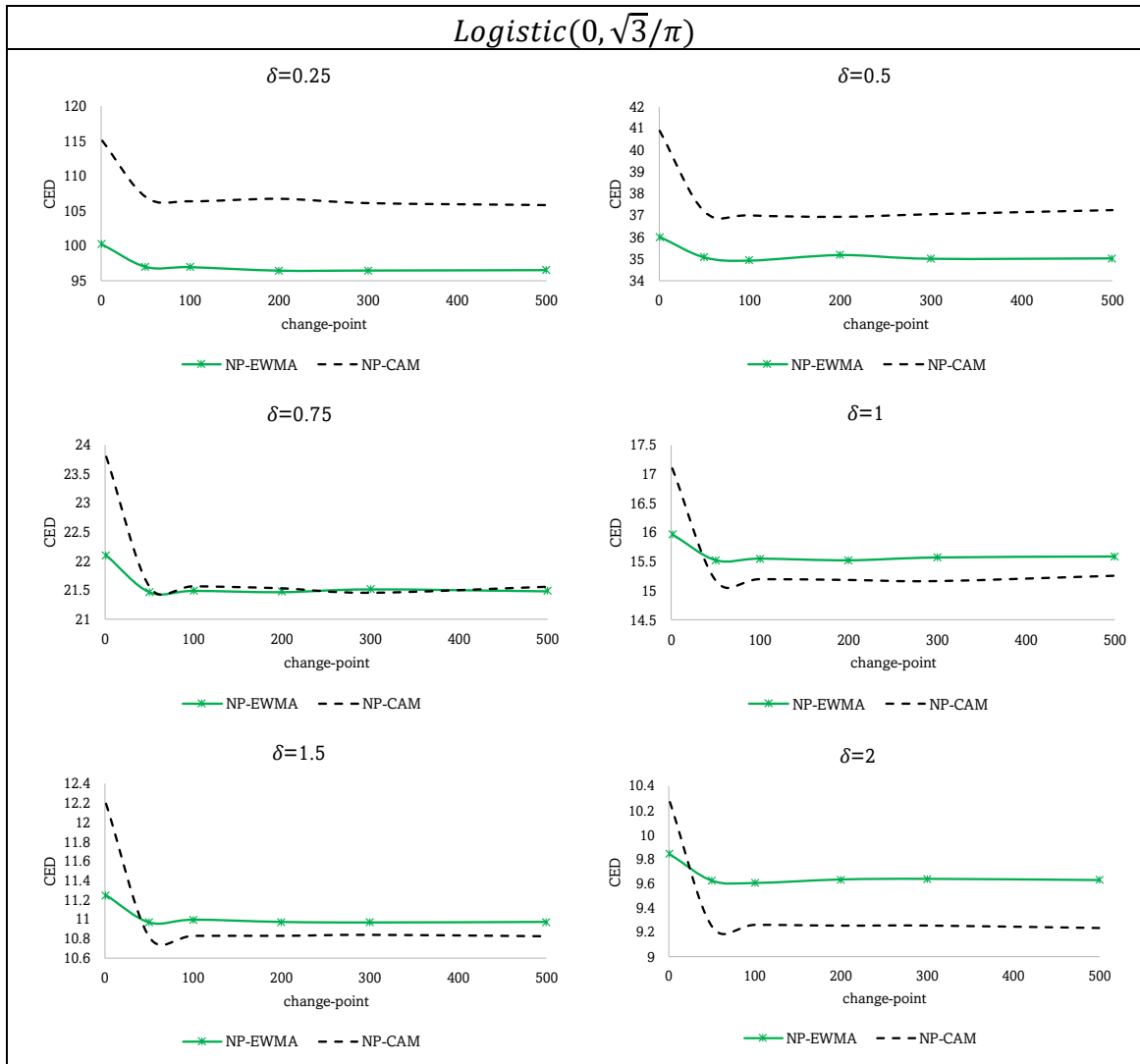


Figure 4. 6 CED curves of the non-parametric charts for $Logistic(0, \sqrt{3}/\pi)$.

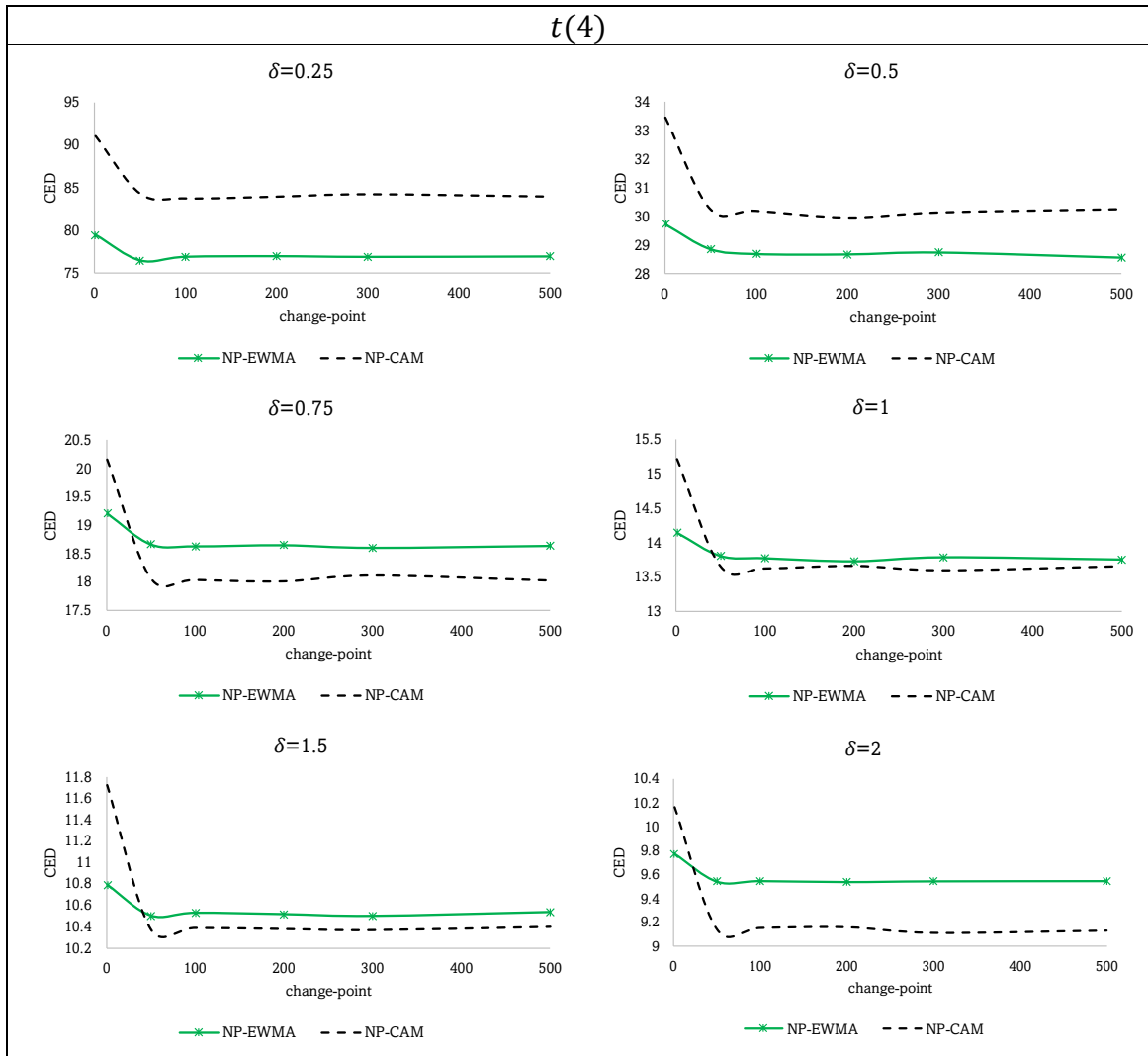


Figure 4. 7 CED curves of the non-parametric charts for $t(4)$.

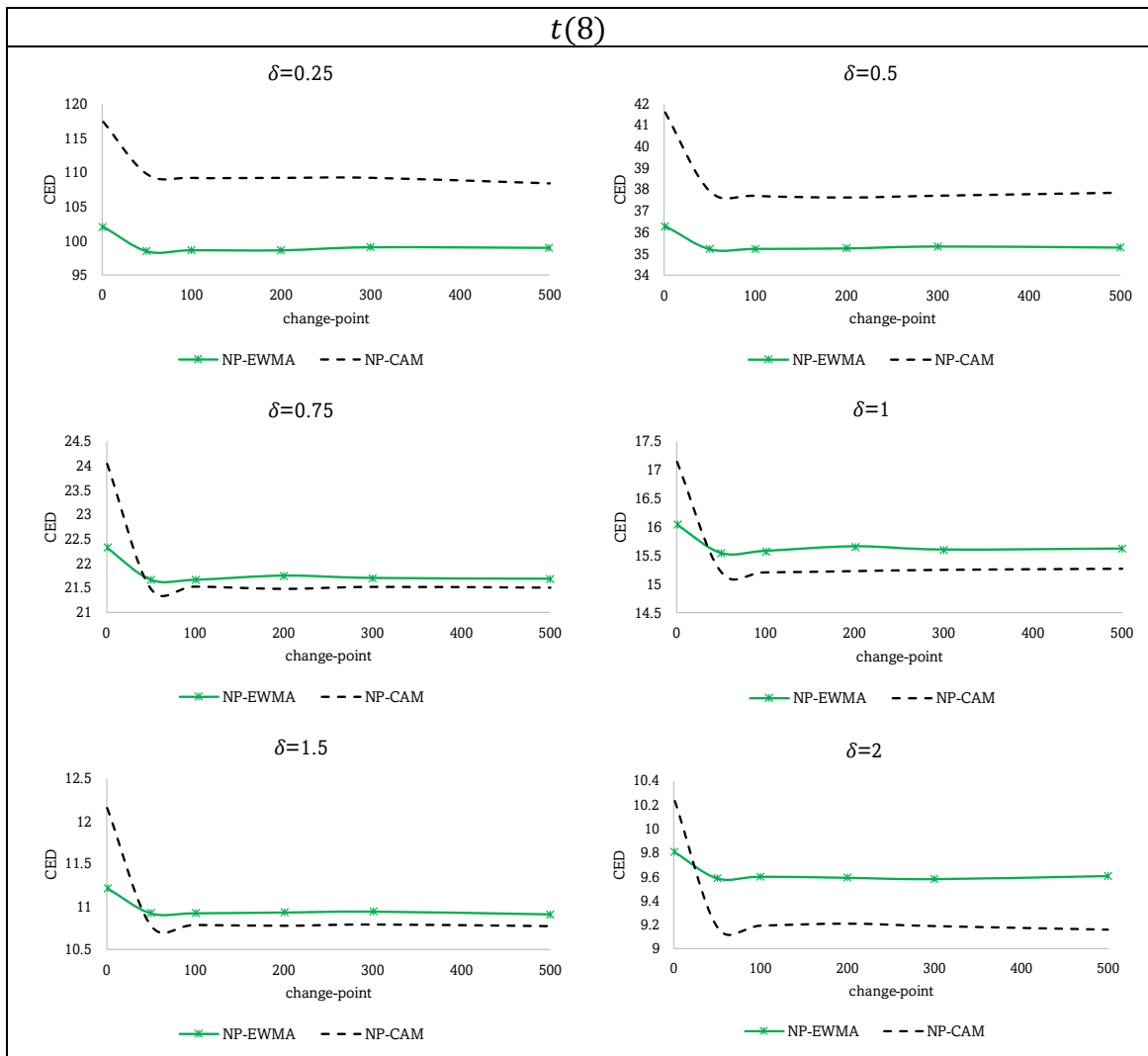


Figure 4. 8 CED curves of the non-parametric charts for $t(8)$.

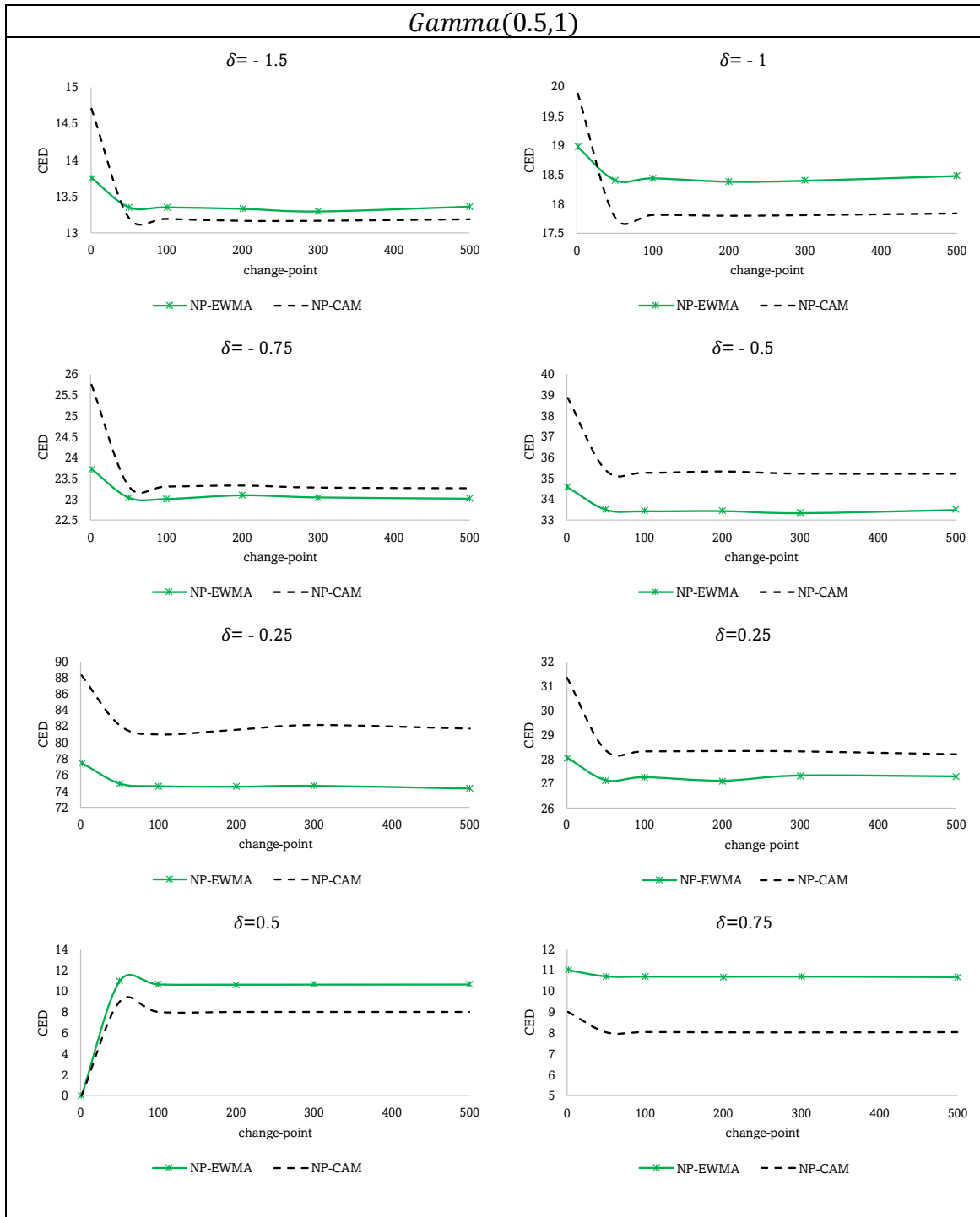


Figure 4. 9 CED curves of the non-parametric charts for $Gamma(0.5,1)$.

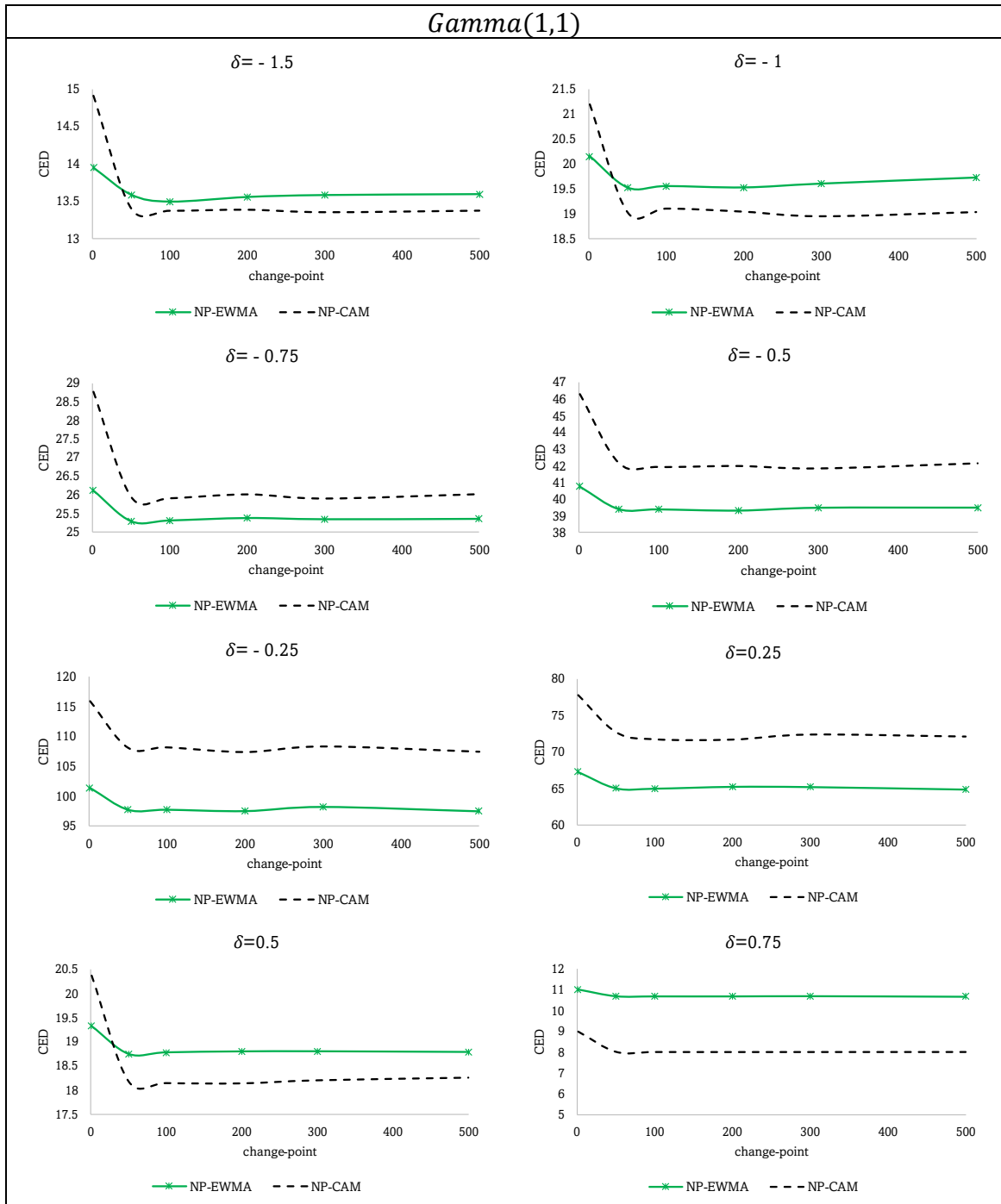


Figure 4. 10 CED curves of the non-parametric charts for $Gamma(1,1)$.

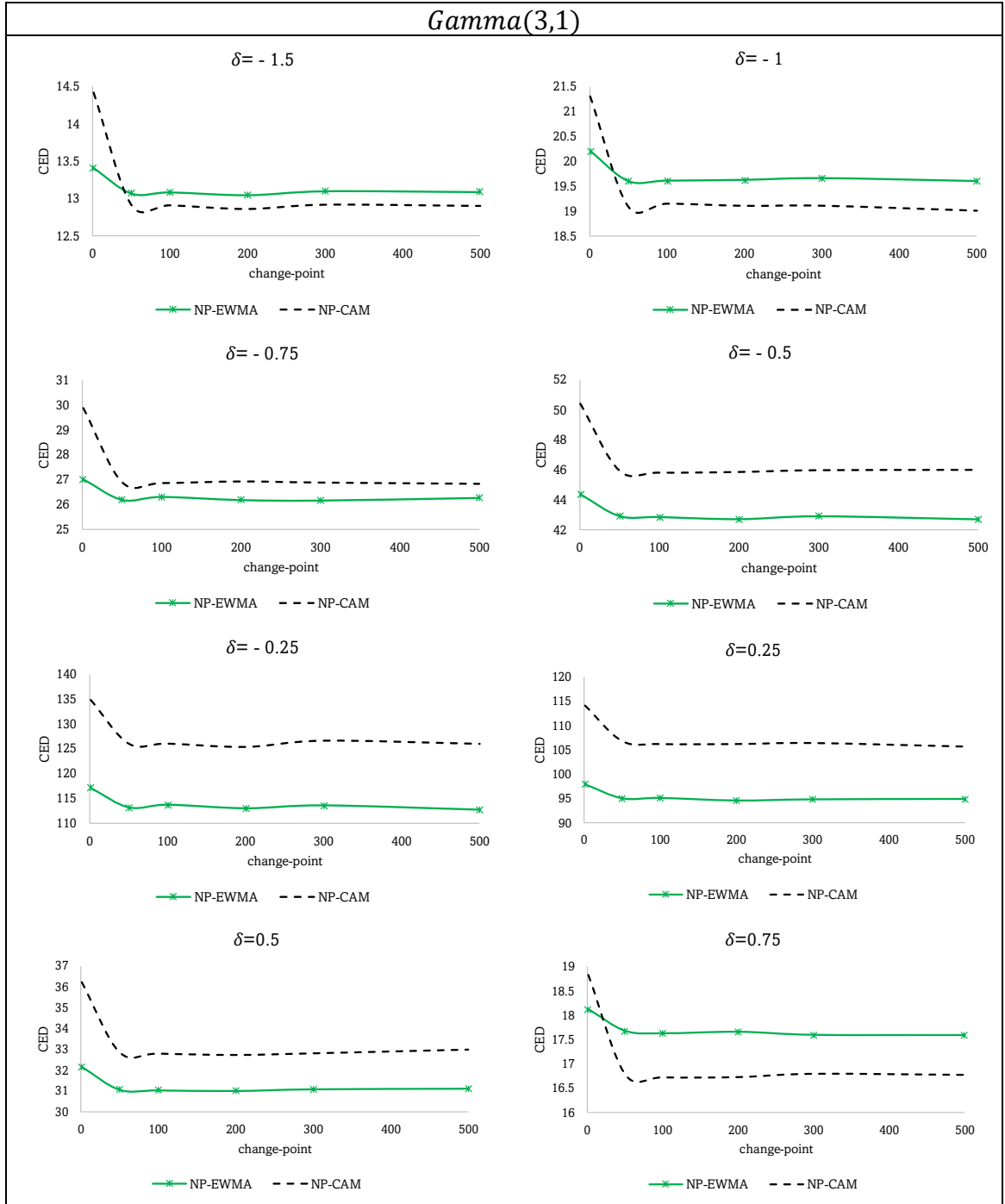


Figure 4. 11 CED curves of the non-parametric charts for $Gamma(3,1)$.

Figure 4.4 illustrates the chart performance for the normal distribution, $N(0,1)$. In the case of small shifts ($\delta \leq 0.25$), NP-EWMA exhibits slightly faster detection times compared to NP-CAM, suggesting that NP-EWMA is more effective in detecting minor changes in a normal distribution. However, as the shift size increases ($\delta = 0.5, 0.75, 1, 1.5, 2$), the

advantage of NP-EWMA diminishes, and for larger shifts, both NP-EWMA and NP-CAM perform similarly, with no significant difference in terms of detection times.

Figures 4.5, 4.6, 4.7, and 4.8, respectively, depict the chart performance for the heavy-tailed distributions: $Laplace(0,1)$, $Logistic(0,\sqrt{3}/\pi)$, $t(4)$, and $t(8)$. NP-EWMA is consistently better at detecting small shifts ($\delta \leq 0.5$) compared to NP-CAM. This means NP-EWMA outperforms NP-CAM in quickly detecting minor changes in heavy-tailed distributions. However, for moderate to large shifts ($\delta = 0.75, 1, 1.5, 2$), the NP-CAM exhibit higher detection in heavy-tailed distributions. In other words, when it comes to moderate and large shifts, the NP-CAM has superior performance compared to NP-EWMA and this superiority becomes significant with heavier-tailed distributions.

Figures 4.9, 4.10, and 4.11 illustrate that, for all three types of Gamma distributions ($Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$), the results indicate that NP-CAM performs better for positive moderate shifts as well as for large shifts (both positive and negative), i.e., the NP-CAM is effective for shifts of size $\delta \geq 0.5$ and $\delta < -0.75$; while NP-EWMA is more effective for small shifts (both positive and negative) and negative moderate shifts, i.e., $-0.75 \leq \delta \leq 0.25$. This finding suggests that the choice between NP-CAM and NP-EWMA should be guided by the nature of the process shift, particularly its size and direction.

4.4 Illustrative example

This section resembles a real-life application by drawing 30 random samples from each distribution of $Normal(0,1)$, $Laplace(0,1)$, $Logistic(0,\sqrt{3}/\pi)$, $t(4)$, $t(8)$, $Gamma(0.5,1)$, $Gamma(1,1)$, and $Gamma(3,1)$. Assuming these samples represent a process to be monitored, an increased shift of size 0.5 is added to the process location from sample number 15, i.e., the change-point (τ) = 15. The proposed NP-CAM chart and NP-EWMA are applied to the data to detect the shift.

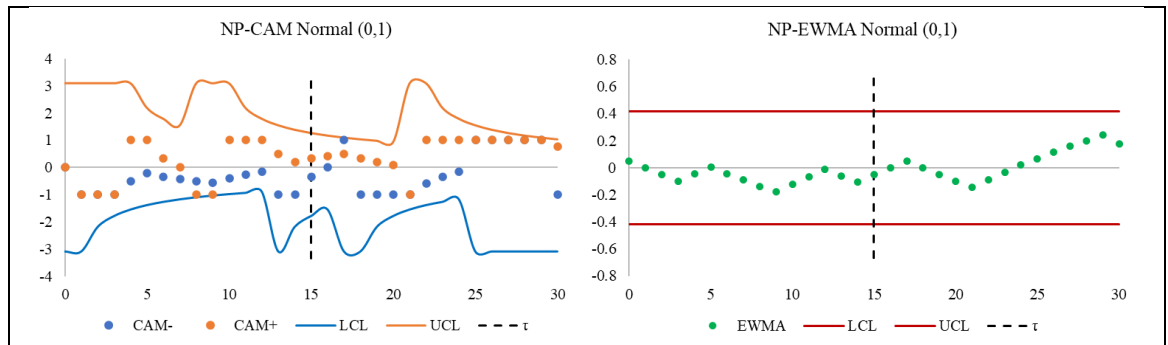


Figure 4. 12 Example of NP-CAM and NP-EWMA control charts to monitor process of the normal distribution.

For the case of symmetric distribution (Figure 4.12 & 4.13), starting with the normal distribution, it is notable that the charts exhibit similar results. However, as the distribution's tails become heavier, the superiority goes to the NP-CAM, referring the efficiency of the proposed chart.

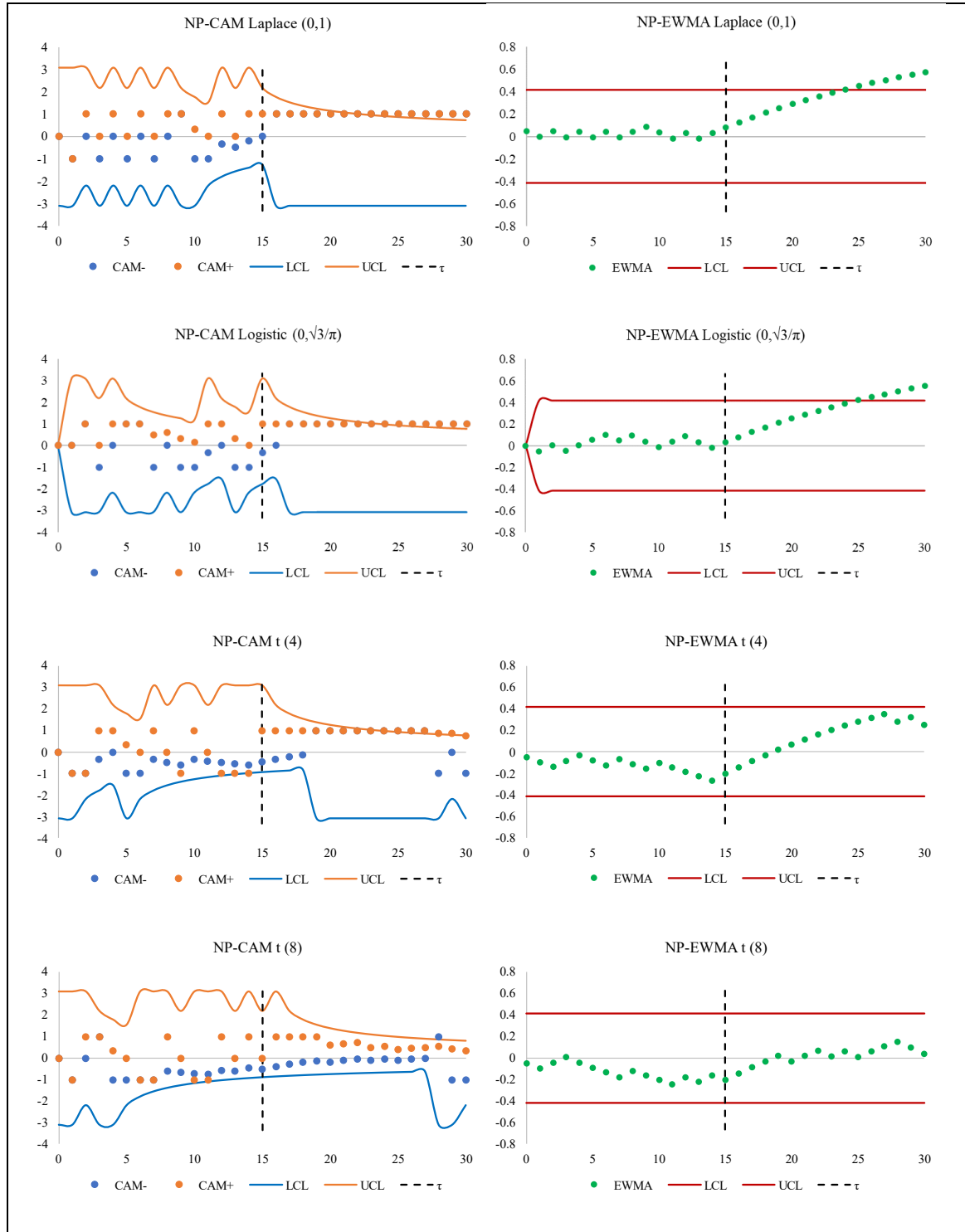


Figure 4. 13 Example of NP-CAM and NP-EWMA control charts to monitor process of symmetric distribution.

The performance of the Gamma distributions (Figure 4.14) is expected to be in such manner as they are right skewed, where an increase in the location by a shift size of 0.5 is highly changed to go beyond the expected area of gamma variable.

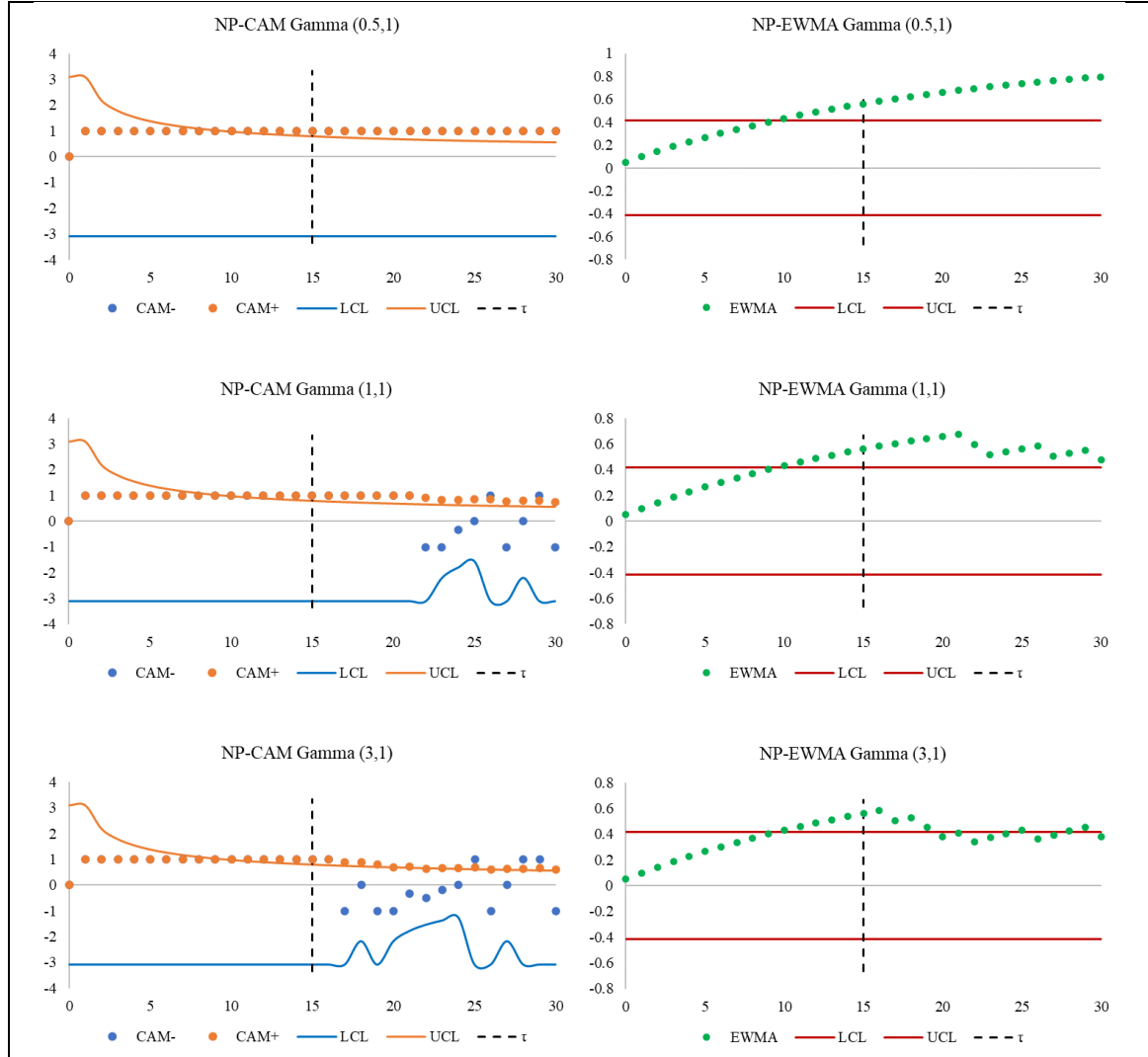


Figure 4. 14 Example of NP-CAM and NP-EWMA control charts to monitor process of skewed distribution.

4.5 Summary and conclusions

This chapter addresses the practical challenges of monitoring processes with non-normal or unknown distributions by introducing the Non-Parametric Conditional Aggregate Mean (NP-CAM) control chart. Non-parametric control charts, in general, leverage non-parametric test statistics as test statistics for the monitoring. This approach enhances the robustness of charts against distributional assumption violations and improves their resistance to outliers. The proposed NP-CAM control chart applies the CAM structure to

the test statistic of the non-parametric sign-test. Throughout the chapter, various distribution types are considered for chart evaluation, including symmetric, heavy-tailed symmetric, and asymmetric distributions. The key finding is that NP-CAM consistently performs well across different scenarios, establishing itself as a reliable choice for quality control. It offers a robust and consistent approach for detecting process deviations, regardless of the underlying distribution type. The comparative analysis conducted in this chapter reveals that NP-CAM can emerge as the preferred chart for detecting moderate to large shifts in processes, while NP-EWMA is preferable for processes with smaller shifts. These results provide valuable guidance for practitioners in selecting the most suitable control chart based on the expected shift size in their specific processes. Overall, NP-CAM demonstrates its effectiveness as a versatile tool for process monitoring and quality control in the presence of non-normal or unknown distributions.

CHAPTER 5

Summary and conclusions

This chapter concludes the thesis by summarizing the essential concepts on which it stood and what it has contributed to the literature on Statistical Process Control (SPC). Additionally, suggestions for further study are provided.

The objective of this thesis is to investigate the Conditional Aggregate Mean (CAM), an innovative control chart structure, to confirm its applicability in addressing the real-world requirements. The CAM chart emerges as a robust alternative to address the limitations encountered with the Progressive Mean (PM) chart, particularly in steady-state scenarios. This research delves into the profound significance of integrating the CAM chart into statistical process control, offering valuable insights into its applications and contributions in the realm of process monitoring.

Progressive Mean (PM) and Conditional Aggregate Mean (CAM) Control Charts

The PM control chart (N. Abbas et al., 2013) was introduced as an inventive approach for process monitoring, exhibiting remarkable performance when shifts in processes occur at or near the beginning of the monitoring period, often referred to as zero-state monitoring. In fact, it surpasses traditional control charts like the Shewhart, Cumulative Sum (CUSUM), and Exponentially Weighted Moving Average (EWMA) charts under such circumstances. However, it has been observed that its performance diminishes when the shift occurrence is delayed from the beginning of the monitoring, a scenario known as steady-state monitoring (Knoth et al., 2021). To address these limitations, the CAM control chart (N. Abbas, 2023) was developed, offering enhanced performance and robustness across various shift scenarios. The CAM chart is initially proposed for monitoring process location under the assumptions of the process normality and known parameters.

CAM for Monitoring Process Dispersion

Process stability entails monitoring both location and dispersion parameters to ensure a consistent and predictable process performance. Changes in dispersion, even when the location parameter remains stable, can significantly impact product quality and process reliability. While a consistent process mean suggests that the averages of process outputs are centered around the target value, variations in process output dispersion can render the process unstable. This instability increases the likelihood of defects and non-conforming products, resulting in higher costs, customer complaints, and other issues. Consequently, the importance of studying control charts for monitoring process dispersion becomes evident.

Chapter 3 in this thesis extends the analysis of the CAM structure, with a particular focus on the monitoring of process dispersion, constructing CAM- S^2 control chart. The CAM- S^2 chart leverages transformation methods to convert the non-normal test statistics that represent dispersion parameter into approximately normal variables to facilitate chart implementation. Applying the CAM structure to these transformed variables produces the CAM- S^2 . The analysis results, considering the zero and steady states of monitoring, demonstrate the effectiveness of the CAM- S^2 chart for monitoring process dispersion. The chart is constructed with the assumption that the process follows a normal distribution with known parameters.

CAM for Monitoring Process Location Under Non-Parametric (NP) Setup

Parametric control charts are typically constructed with the assumption of a normal distribution for process data. However, when this assumption is violated, the performance of these charts can be seriously compromised, leading to inaccurate results. In such cases, parametric charts may incorrectly interpret natural variations as signals of process shifts, resulting in more false alarms that disrupt efficient monitoring. Conversely, parametric control charts may fail to detect actual process shifts in non-normal data, resulting in a loss of control over the process.

Overall, the improper use of parametric charts in non-normal data scenarios can lead to a misinterpretation of the process behavior with significant consequences. To avoid these issues and ensure accurate and reliable process monitoring, non-parametric (NP) control charts should be considered. They are especially useful for the process with unknown distribution, small sample sizes, or highly skewed observations.

In Chapter 4 of this thesis, the CAM Chart for monitoring location parameter was adapted to accommodate non-normally distributed observations, addressing practical needs in process monitoring. Non-parametric methods, particularly the sign-test, were employed to achieve this objective. Realising the assumption of normality in process observations broadens the applicability of the CAM chart across a wider range of scenarios. The findings emphasize the versatility and robustness of the NP-CAM chart, further expanding the capabilities of the CAM structure.

Concluding Insights and Future Directions in Control Chart Research

In conclusion, this thesis represents a comprehensive exploration of novel control chart structures and their applications. It highlights the critical importance of recognizing and addressing the limitations of existing methods while striving for continuous improvement in process monitoring techniques.

The findings presented here refine existing methods and enable practitioners and researchers to make informed decisions regarding the selection of the most suitable control

chart for their specific applications, whether it involves monitoring process location or dispersion. Exploring the techniques of statistical process control in greater depth guarantees the development of more accurate, adaptable, and effective quality control methods in diverse industrial settings.

To push the boundaries of quality control and as part of future research recommendations, enhancing the CAM structure by incorporating the modification ideas discussed in Chapter 2 holds the potential for improving its performance and potentially surpassing its competitors. Additionally, in Chapter 4, the chart was introduced using the sign-test statistic; however, exploring the application of other non-parametric tests to assess chart performance under non-normal conditions could yield valuable insights.

Appendixes

Appendix (1-A), using Riemann Sum to approximate the value of EQL .

Since δ is distributed uniformly, i.e., $\delta \sim \text{Uniform}(\delta_{min}, \delta_{max})$, then its probability density function PDF is given by $f(\delta) = \frac{1}{\delta_{max} - \delta_{min}}$ for $\delta_{min} \leq \delta \leq \delta_{max}$.

Considering m partitions as it is the number of shifts, the width of the rectangles in Riemann Concept of Approximation is:

$$\Delta \delta = \frac{\delta_{max} - \delta_{min}}{m}$$

Since we have

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta) d\delta$$

Then, the approximated EQL using Reimann Sum is given by

$$EQL \approx \frac{1}{\delta_{max} - \delta_{min}} \sum_{i=1}^m \delta_i^2 ARL(\delta_i) \Delta \delta$$

Substituting,

$$EQL \approx \frac{1}{\delta_{max} - \delta_{min}} \sum_{i=1}^m \delta_i^2 ARL(\delta_i) \frac{\delta_{max} - \delta_{min}}{m}$$

Simplifying,

$$EQL \approx \frac{1}{m} \sum_{i=1}^m \delta_i^2 ARL(\delta_i)$$

Appendix (1-B), deriving the second form of the EWMA plotting statistic.

$$\begin{aligned} Z_i &= \lambda X_i + (1 - \lambda)Z_{i-1} \\ &= \lambda X_i + (1 - \lambda)[\lambda X_{i-1} + (1 - \lambda)Z_{i-2}] \\ &= \lambda X_i + \lambda(1 - \lambda)X_{i-1} + (1 - \lambda)^2 Z_{i-2} \\ &= \lambda X_i + \lambda(1 - \lambda)X_{i-1} + (1 - \lambda)^2 [\lambda X_{i-2} + (1 - \lambda)Z_{i-3}] \\ &= \lambda X_i + \lambda(1 - \lambda)X_{i-1} + \lambda(1 - \lambda)^2 X_{i-2} + (1 - \lambda)^3 Z_{i-3} \end{aligned}$$

$$\begin{aligned}
&= \lambda(1-\lambda)^0 X_{i-0} + \lambda(1-\lambda)^1 X_{i-1} + \lambda(1-\lambda)^2 X_{i-2} + \cdots + \\
&\lambda(1-\lambda)^{i-1} X_{i-(i-1)} + (1-\lambda)^i Z_{i-i} \\
&= \lambda[(1-\lambda)^0 X_{i-0} + (1-\lambda)^1 X_{i-1} + (1-\lambda)^2 X_{i-2} + \cdots + \\
&(1-\lambda)^{i-1} X_1] + (1-\lambda)^i Z_0
\end{aligned}$$

$$Z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j X_{i-j} + (1-\lambda)^i Z_0$$

Appendix (1-C), proof the unity of the weights.

To find the sum of the weights, we have $\sum_{j=0}^{i-1} (1-\lambda)^j$ is a geometric series with $r = 1 - \lambda$, and $n = i$, Therefore, the sum of the weights can be computed as follows

$$\begin{aligned}
&\lambda \sum_{j=0}^{i-1} (1-\lambda)^j + (1-\lambda)^i = \lambda \left[\frac{1 - (1-\lambda)^i}{1 - (1-\lambda)} \right] + (1-\lambda)^i \\
&= \lambda \left[\frac{1 - (1-\lambda)^i}{\lambda} \right] + (1-\lambda)^i = 1 - (1-\lambda)^i + (1-\lambda)^i = 1
\end{aligned}$$

Appendix (1-D), proof of $\lim_{i \rightarrow \infty} [1 - (1-\lambda)^{2i}] = 1$.

For $\lambda = 1$, the proof is directly completed by substitution.

For $0 < \lambda < 1$:

$$\begin{aligned}
\lim_{i \rightarrow \infty} [1 - (1-\lambda)^{2i}] &= 1 - \lim_{i \rightarrow \infty} (1-\lambda)^{2i} = 1 - \lim_{i \rightarrow \infty} e^{\ln(1-\lambda)^{2i}} = 1 - \lim_{i \rightarrow \infty} e^{2i \ln(1-\lambda)} \\
&= 1 - e^{\lim_{i \rightarrow \infty} [2i \ln(1-\lambda)]} = 1 - e^{2 \ln(1-\lambda) \lim_{i \rightarrow \infty} [i]} = 1 - e^{2 \ln(1-\lambda) \infty}
\end{aligned}$$

Since $0 < \lambda < 1$, this implies $0 < 1 - \lambda < 1$, hence $\ln(1-\lambda) < 0$, then

$$= 1 - e^{-2 \infty} = 1 - 0 = 1$$

Nomenclature

SPC	Statistical Process Control
CUSUM	Cumulative Sum
EWMA	Exponential Weighted Moving Average
GMA	Geometric Moving Average
PM	Progressive Mean
CAM	Conditional Aggregate Mean
IC	In-Control
OOC	Out-Of-Control
LCL	Lower Control Limit
UCL	Upper Control Limit
CL	Central Line
RL	Run Length
ARL	Average of Run Length
ARL_0	In-Control ARL
ARL_1	Out-Of-Control ARL
SDRL	Standard Deviation of Run Length
MDRL	Median of Run Length
P_kRL	k^{th} Percentile of Run Length
ATS	Average Time to Signal
EQL	Extra Quadratic Loss
CED	Conditional Expected Delay
PTS	Probability to Signal
FAP	False Alarm Probability
FAR	False Alarm Rate
NP	Non-Parametric
FIR	Fast Initial Response
AIB	Auxiliary Information-Based
WL	Warning Line
AL	Action Line
θ	A Process Parameter
$\underline{\theta}$	A Vector of Process Parameters
$\hat{\theta}$	An Estimator of θ
θ_0	The Target Parameter
$\sigma_{\hat{\theta}}$	The Standard Deviation of $\hat{\theta}$
μ	Location Parameter
μ_0	Target Location
μ_1	Shifted Location
σ	Standard Deviation
σ_0	Target Standard Deviation
σ_1	Shifted Standard Deviation
δ	Shift Amount in Location Parameter
γ	Shift Amount in Dispersion Parameter
X	A random variable

$\bar{X}$	Sample Mean
S	Sample Standard Deviation
S^2	Sample Variance
R	Sample Range
P	The Plotting Statistic
λ	Weight Parameter of EWMA
K	The Reference Value (Or the Allowance Value)
k	Sensitivity Parameter
L	Limits Coefficient
h	Limits Coefficient of CUSUM
H_0	The Null Hypothesis
H_1	The Alternative Hypothesis
n	Sample Size
α	Probability of Type-I Errors
β	Probability of Type-II Errors
τ	Change-Point
$E(\cdot)$	Expected Value of a Random Variable
$SD(\cdot)$	Standard Deviation of a Random Variable

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