

**OPTIMIZATION OF WELL PLACEMENT AND PATH IN
HYDROCARBON RESERVOIRS**

BY

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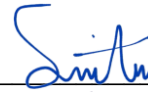
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2023

|This work is dedicated to my family and friends |

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LIST OF ABBREVIATIONS

CMA-ES	:	Covariance Matrix Adaptation Evolutionary Strategy
COP	:	Cumulative Oil Production
DE	:	Differential Evolution
GA	:	Genetic Algorithm
HGA	:	Hybrid Genetic Algorithm
LGR	:	Local Grid Refinement
LWD	:	Logging While Drilling
MWD	:	Measurement While Drilling
NPV	:	Net Present Value
PSO	:	Particle Swarm Optimization
PVT	:	Pressure Volume Temperature
RMSE	:	Root Mean Squared Error
SL	:	Straight-Line
USD	:	US dollars (\$)
VRR	:	Voidage Replacement Ratio

ABSTRACT

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[The optimization of well placement is an integral element of the development plan of a field. Its essence thrives with more complex problems and, in such cases, requires further advanced techniques and imposed constraints to obtain the optimal solution. The optimization algorithms, e.g., Genetic Algorithm (GA) and Differential Evolution (DE), have proven their success in well placement applications as can be observed by the numerous research papers available in the literature. These algorithms optimize an objective function (or sometimes called cost function), by finding its minimum or maximum, and it is usually represented by an economical factor, e.g., net present value (NPV), that illustrates the plan's profitability.

The objective of this study is to optimize the placement of wells of any shape, e.g., vertical, horizontal, s-shaped, irregular, etc., using DE optimization algorithm with NPV defined as the objective function. The study involves both well types (producers and injectors) and incorporates operational and economical constraints, such as minimum well spacing, maximum inclination and azimuth angle changes, placement within reservoir boundaries, and operating bottomhole pressure for both injectors and producers. Nonlinear inequality formulations have been developed to ensure the satisfaction of the minimum spacing and placement within reservoir boundaries constraints.

The novelty of this research is demonstrated in the integration of each well segment directional information (inclination, azimuth, and length) as decision parameters. These parameters are sufficient to represent the segment's starting and ending points and can be assembled to create the complete well path. Local Grid Refinement was employed to better represent the changes in the path on a smaller scale (within a grid block). In this study, three different reservoir examples were investigated: a reservoir with channels, a reservoir with distributed permeability field, and Brugge field, in which in each case, a comparison with an existing method of representing the well as a straight line was performed. Overall, the proposed method has shown better if not comparable results to the straight-line path method in most scenarios. Additionally, further analysis emphasized the importance of using an appropriate number of segments that suffices to create freedom in designing the well path while not causing extensive simulation time or convergence problems. |

ملخص الرسالة

الاسم الكامل: عمر قاسم الحربي

عنوان الرسالة: استثمار مواقع الآبار ومساراتها في مكامن الهيدروكربون

التخصص: هندسة البترول

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يعد اختيار مواقع الآبار عنصر أساسي في خطة تطوير الحقل. تزداد أهميته مع الأوضاع الأكثر تعقيداً وفي مثل هذه الحالات يتطلب تقنيات متقدمة وقيود مفروضة للحصول على الحل الأمثل. لقد أثبتت خوارزميات الاستثمار والتحسين مثل الخوارزمية الوراثية والخوارزمية التطورية نجاحها في تطبيقات اختيار مواقع الآبار المثلى كما يمكن ملاحظته من العديد من الأوراق البحثية المنشورة. تحسن هذه الخوارزميات دالة الهدف (أو ما تسمى أحياناً دالة التكلفة)، من خلال إيجاد القيمة الصغرى أو القصوى لها، وتمثل عادة بعامل اقتصادي مثل القيمة الحالية الصافية (NPV) التي تعكس ربحية الخطة. من البارز أيضاً أن عدة دراسات استخدمت أكثر من دالة هدف، والمعروف بالتحسين متعدد الأهداف، لتحسين التخطيط الأمثل لاختيار مواقع الآبار.

هدف هذه الدراسة هو استثمار مواقع الآبار عامة، على سبيل المثال الآبار المحفورة رأسياً، أفقياً، بشكل غير منتظم، وغيرها، باستخدام خوارزميات التحسين المطلقة بتعريف NPV كدالة هدف. تتضمن الدراسة كلا من أنواع الآبار (الإنتاجية والحقيقية) وتدمج القيود العملية والاقتصادية، مثل الحد الأدنى للمسافة بين الآبار، والحد الأقصى لتغير زاوية الميلان والسمت للبئر، ووضع الآبار داخل حدود المكن، وضغط قاع البئر لكلا من الآبار الإنتاجية والحقيقية. تم تطوير معادلات غير خطية للتأكد من تحقيق متطلبات الحد الأدنى للمسافة بين الآبار ووضع الآبار داخل حدود المكن. يتفرد هذا البحث عن طريق تقسيم البئر إلى أجزاء يتم تعريف كل منها بمعلومات تعكس موقع واتجاه الجزء. يتم استعمال ودمج هذه المعلومات في خوارزمية التحسين مما يساعد في تخطيط مواقع ومسارات الآبار. من البارز أيضاً في هذا البحث هو استعمال "Local Grid Refinement" لتمثيل نقاط الآبار والتغيرات في بيانات الموقع بدقة أعلى. تم توضيح ثلاثة أمثلة مختلفة في هذا البحث، حيث تم إجراء مقارنة مع طريقة أخرى وهي تمثيل الآبار كخط مستقيم، وتمت دراسة تأثير عدد أجزاء البئر المحددة باستثمار حالات مع اختيار عدد مختلف من الأجزاء.

CHAPTER 1

INTRODUCTION

One of the most important factors for the success of a field development is the placement of its wells. The placement configuration affects the amount of hydrocarbon to be produced daily and cumulatively, the time of water breakthrough occurrence and water cut percentages, and of course the economic value (e.g., net present value “NPV”). Furthermore, recent developments have allowed for more advanced drilling techniques, such as horizontal and deviated wells. Generally, these wells outperform their counterparts, vertical wells, due to the large exposure to the hydrocarbon resources they can attain. In addition, horizontal and deviated wells can be very effective in scenarios where vertical wells would be considered unfavorable, e.g., producing from thin reservoirs. Nowadays, these types of wells can be drilled, geo-steered (navigated) following planned trajectories, and monitored as well in real-time. However, with complex reservoir properties, such as formation heterogeneity and fault presence, well-trajectory planning requires advanced investigation for better ultimate field recovery.

Moreover, there are effective algorithms that allow solving problems with one or more optimization objectives. These algorithms have been employed extensively in the literature for various applications, e.g., history matching, production forecasting, and well placement. These techniques can be classified mainly into gradient-based optimization algorithms, e.g., gradient descent, and stochastic optimization algorithms, e.g., Genetic

Algorithm (GA) and Differential Evolution (DE). Gradient-based optimizers aim to find a local solution which could or could not be the global optimum while the stochastic optimizers attempt to obtain the global optimal solution. Furthermore, it is very important to comprehend the problem at hand and assign a relative objective function to acquire the best possible solution. For instance, in history matching problems, the objective function could be an error metric, e.g., Root Mean Squared Error “RMSE”, which the algorithm would attempt to minimize as much as possible by varying several parameters like permeability and skin. Likewise, well placement problems involve variation in selected parameters, such as well location, trajectory, length, number of laterals, etc., to minimize the negative value of NPV, hence obtaining as maximum NPV as possible.

Usually, for this example of problems, NPV or total hydrocarbon produced are commonly chosen as objective functions. However, optimization algorithms do not adhere to common or engineering sense, such that it could provide results that are invalid and illogical. For example, in a well placement problem, the optimization algorithm could place a well outside the reservoir boundaries, could place two wells in the exact location, could place two intersecting wells, etc. These failures are not comprehended by the program as its objective was only to minimize or maximize a certain cost function. Therefore, it is vital to set conditions, or constraints, to guide the algorithm to achieve an appropriate solution.

In this thesis report, chapter 2 reviews the literature work regarding well placement optimization problems and the algorithms employed to acquire the results.

Chapter 3 introduces the problem statement, problem formulation, associated theory, and the objective of this thesis research.

Chapter 4 reviews the optimization algorithm used, and additional theoretical background utilized in this research.

Chapter 5 presents the investigated case examples and discusses the results obtained from each case.

Chapter 6 sums up and concludes this thesis report.

|

CHAPTER 2

LITERATURE REVIEW

2.1 Well Placement Optimization

Pan (1995) studied one of the early applications of optimization algorithms in planning the optimum well placement. In his work, he was able develop a strategy (**Figure 1**) for an offshore oil field to maximize its net present value subjected to constraints of facility capacity. He studied 35x35 combination of cases with variation in the starting time of development (from 0 to 204 months, for every 6 months) and variation in the required time to produce oil at full capacity of the facility (from 5 to 175 months, for every 5 months). He solved this problem with two different approaches. The first method was to perform 1225 (35x35) simulation runs to obtain the exhaustive data set. The second approach was to perform 35 simulation runs and utilize them to generate the 1225 realizations employing multivariate interpolation techniques, namely Least Squares and Kriging (Pan, 1995).

Bittencourt & Horne (1997) employed a Genetic Algorithm (GA) to optimize the location, direction (vertical or horizontal), and orientation (if horizontal) of 33 wells in a real reservoir development problem. Two cases were studied: one case was conducted with pre-knowledge of wells locations proposed in the real project development plan, while the other case was investigated without prior information of wells locations. The two cases were solved using a hybrid GA to maximize the NPV of the field. Optimization of the former case, which involved the proposed locations, has shown premature convergence at early generations but proceeded to enhance at later modified generations. On the other hand, the

latter case showed a better regulated generation of new realizations. The profits of the two cases were \$1,203.83 million and \$1,189.92 million of the former and latter cases, respectively (Bittencourt & Horne, 1997).

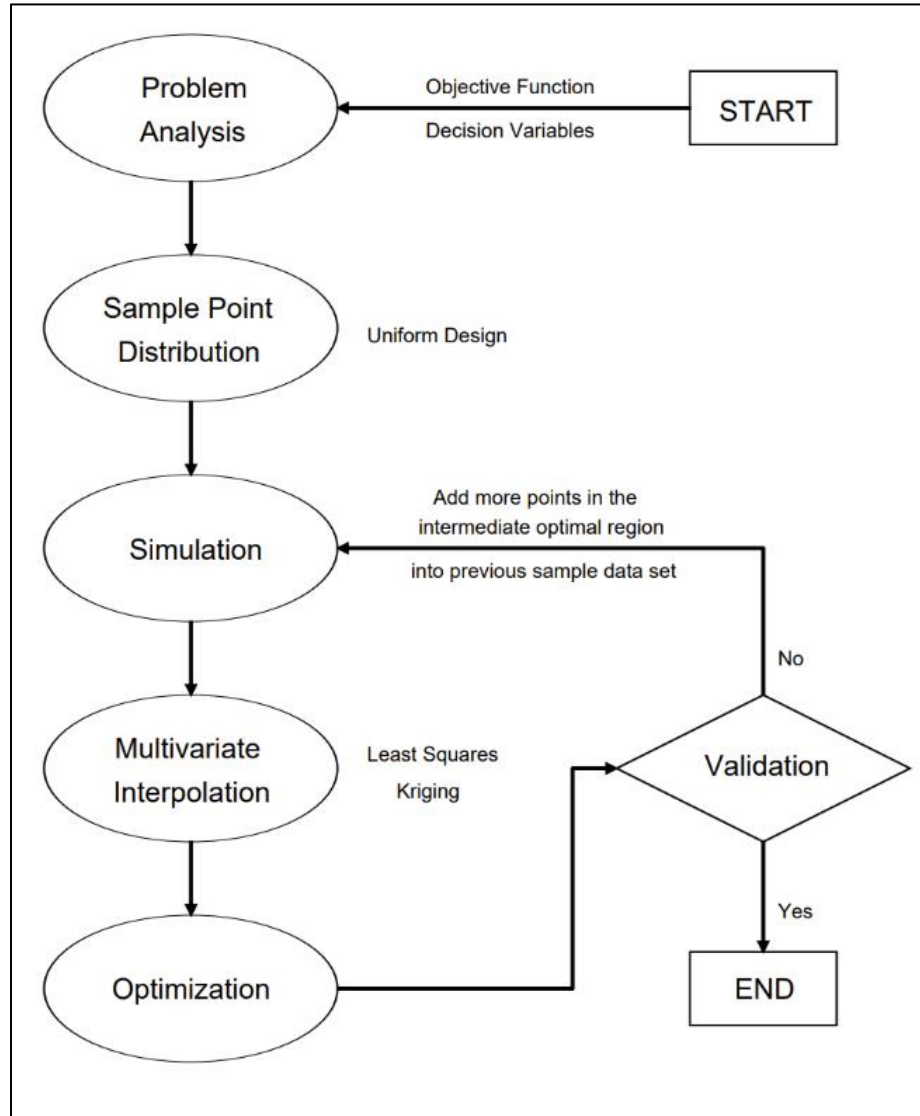


Figure 1: Optimization Strategy Developed by (Pan, 1995)

Guyaguler & Horne (2000) conducted a study to select the optimal wells coordinates that result in the maximum oil production over a two-year period. The optimization algorithm

applied was a hybrid GA (HGA). Their research involved six different runs, in which each has a different number of injectors and/or producers, as well as the freedom to which how many wells are optimizable. In addition, reservoir grid and petrophysical properties are varied, such as cases with 16 x 16 grid system or degree of heterogeneity of the reservoir. The summarized description of these runs is presented in **Table 1** (Guyaguler & Horne, 2000).

Table 1: Optimization runs summary (Guyaguler & Horne, 2000)

Run No.	No. Producers	No. Injectors	Reservoir Grid System
1	1	-	16 x 16
2	1	1	16 x 16
3	2	-	16 x 16
4	3 (fixed location) 1 (optimized by HGA)	-	32 x 32
5	3 (fixed location)	1 (optimized by HGA)	32 x 32
6	2 (fixed location)	2 (optimized by HGA)	32 x 32

Yeten et al. (2003) investigated the placement of nonconventional wells considering the wells' type, location, and trajectory. The research studied both single and multi-lateral wells and utilized GA alongside three "helper" algorithms to maximize the NPV. The trajectory of each well lateral is assumed as a straight line in a 3D system and is represented by a vector τ , which connects the coordinate points of the well lateral's toe (t) and heel (h). Basically, the algorithm approaches the solution by optimizing the heel and toe

coordinates (i, j, k) , the inclination angle (θ_{inc}) , and the well length in the x - y plane. The authors discussed three cases with different reservoir properties: a dual-drive reservoir with a gas cap and a bottom aquifer, an oil/water layered reservoir, and a single-phase fluvial channel reservoir. The authors were able to successfully optimize the location of the wells and their additional laterals, if any, providing minimum NPV while obeying the imposed constraints (Yeten et al., 2003).

Wang et al. (2007) proposed a solution to optimize the location of an additional well (an injector) to a synthetic reservoir with existing producers using gradient based algorithm. Gradient based algorithms are local optimization algorithms and are not commonly used in well placement problems as global optimization algorithms, e.g., GA. This is attributed to the vulnerability local optimization algorithms have to being trapped within a local optimal solution without approaching the ultimate optimum. Nevertheless, the authors investigate their adopted method first with an easy case of a homogenous reservoir with four producers at each corner. Expectedly, the solution approached by the method was locating the injector at the center. Moreover, the authors tested the method on another case of a heterogeneous reservoir with two producers placed arbitrarily. The optimization technique realistically selected a location of the injector which is reasonable with the heterogeneity of the reservoir and favorable flow from injector to producers (Wang et al., 2007).

Emerick et al. (2009) developed a program that optimizes well placement considering multiple nonlinear constraints, such as maximum well length, minimum spacing between wells, and inactive grid cells. Genetic Algorithm was applied as the optimization method, the cost function defined was NPV, and the decision parameters used were heel point, toe

point, type (injector or producer), and the status (active or inactive) of each well. In this study, several field applications were examined. For instance, Field 1 is an offshore field with one existing producer and an injector. The reservoir engineer plan was to add 13 more wells (8 producers and 5 water injectors) of a maximum 800 meters length. The study compared two strategies approached: one with initial wells locations as proposed by the engineer, and the other with the starting points of wells being random. The results showed that the first strategy gave higher NPV with less number of simulations as compared with the second strategy. Field 2, which is also an offshore field, had similar strategies as Field 1, meaning two optimizations were tested one with initial points suggested by the engineer and another with random initial locations. However, this latter plan outperformed the former. Thus, it is important to select wisely the initial locations of wells, otherwise, the chances of attaining the optimal solution will decline (Emerick et al., 2009).

Awotunde & Sibaweihi (2014) solved a well-placement optimization problem through a multi-objective approach. Previously mentioned work discussed mostly the use of single objective, which is commonly to maximize the NPV. However, the authors included an additional objective to the problem to be solved, which is to optimize VRR, or i.e., to minimize $|VRR - 1|$. Their study conducted three cases: a case where NPV was solely used in a single objective problem, a second case where VRR was the sole objective of the problem, and a third case where both NPV and VRR considered in a multi-objective problem. The three cases were approached using two different optimization algorithms: Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) and Differential Evolution (DE) (Awotunde & Sibaweihi, 2014).

Awotunde & Naranjo (2014) integrated well spacing as a constraint to optimize placement of wells. The minimum spacing constraint was determined from the minimum circular area imposed between any two wells in the field and is represented as follows:

$$r_{\min} = \sqrt{\frac{43560A}{\pi}} \quad (2.1)$$

The number of constraints increases as more wells are placed into the field. The authors formulated an equation to compute the number of constraints as in the following:

$$N_c = \frac{N_{\text{wells}}(N_{\text{wells}} - 1)}{2} \quad (2.2)$$

The cost function defined in the study is the net present value, and the optimization of the wells locations was achieved using a global optimization algorithm, Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES). In addition, a penalty approach was adopted in which whenever the selected locations violate the minimum well spacing constraint, a negative NPV is added. A couple of case studies were conducted, e.g., a reservoir with four facies of different permeability and porosity values. In this case example, 18 producers and 12 injectors were used to develop the reservoir. Several operational and economical constraints were defined: minimum operating bottomhole pressure of 2000 psi of all producers, maximum water cut of 97%, minimum oil production of 100 STB/day, constant water injection rate of 3000 STB/day, and maximum bottomhole pressure of 6500 psi of all injectors. This case was approached using CMA-ES and several other unconstrained optimization techniques. The results showed that CMA-ES was able to find optimal well location while satisfying all the constraints while the unconstrained methods violated the proposed non-linear constraints (Awotunde & Naranjo, 2014).

Naderi & Khamehchi (2017) compared three different optimization algorithms performance in solving four different cases. The optimization algorithms used were Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Bat Algorithm (BA). The objective was to optimize the location of six production wells by maximizing the objective function, NPV. Overall, the results showed that PSO needed less computational time and less function evaluations. However, its best NPV was not as good as the NPV retrieved using BA, which needed relatively higher computational time to reach its optimal NPV. Notably, GA featured the lowest average time per function evaluation among the three techniques. **Table 2** summarizes the results obtained from the three optimization algorithms (Naderi & Khamehchi, 2017).

Table 2: Optimization results summary (Naderi & Khamehchi, 2017)

Algorithm	No. Function Evaluations	Computational Time (seconds)	Average Time (seconds)	NPV (MM\$)
GA	800	4989	6.24	1.638
PSO	625	4624	7.40	1.854
BA	800	5040	6.30	1.993

Awotunde (2019) examined the performance of six dimension-reduction techniques: constant-control (Const) approach, piecewise-constant (PWC) approach, trigonometric approach, Bessel-function (Bess) approach, polynomial approach, and the data-decomposition approach. These methodologies were utilized in five different case examples to find the optimal well-location or flow rates, with the cost function defined as

the NPV of the field and was maximized using a global-optimization algorithm. The objective of the first two case studies was to optimize the wells rates, while fixing other parameter such as location or number of wells, in which the first case involved a synthetic reservoir with 15 wells while the second case involved 30 wells in the Brugge Field. The remaining three examples were aimed to find the optimal wells rates, wells locations, types (injector or producer), and number of wells (Awotunde, 2019).

Ding et al. (2020) utilized particle swarm optimization algorithm to efficiently optimize well placement. They combined two objective functions in a sequential manner, which they referred to as hybrid objective function. They first used productivity potential value (PPV), which is a static value estimated without the need to run simulation. They run the program using this objective for early iterations, and then switched to a dynamic-measure cost function, which is the cumulative oil production (COP). The incorporation of the static function resulted in computational time reduction while the integration of the dynamic objective function preserved the accuracy of the obtained solution. The process workflow is illustrated in **Figure 2** (Ding et al., 2020).

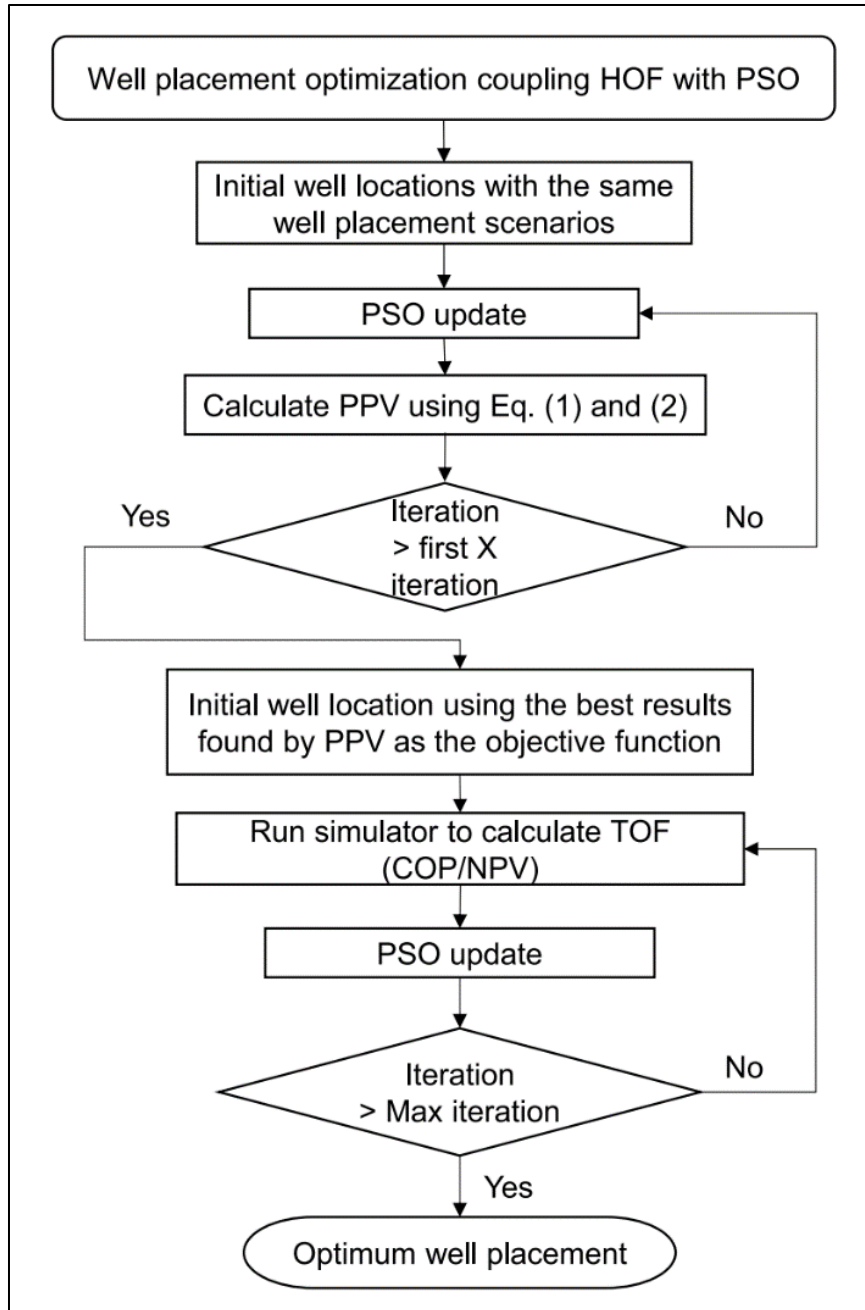


Figure 2: The optimization workflow of HOF with PSO (Ding et al., 2020)

2.2 Optimization Algorithms

2.2.1 Genetic Algorithm (GA)

Genetic Algorithm is a very effective algorithm. It was developed in 1970 by John Holland on the basis of survival traits of strong species in nature. Initially, this technique commences randomly. However, subsequent generations (iterations) are a result of mutation, cross-over, and recombination processes of its ancestors. Ultimately, the algorithm achieves an optimal solution. **Figure 3** depicts the general flowchart of genetic algorithms (Islam et al., 2020).

During the GA process, parent chromosomes are selected, based on fitness quality, and evolved through recombination process of mutation and cross-overs, in which child chromosomes are generated. The process repeats as child chromosomes are tested for fitness and then subsequent selection to produce the next generation. Iteratively, successive generations are evolving in terms of chromosome average fitness until the program breaks upon a stopping criterion yielding the best solution (McCall, 2005).

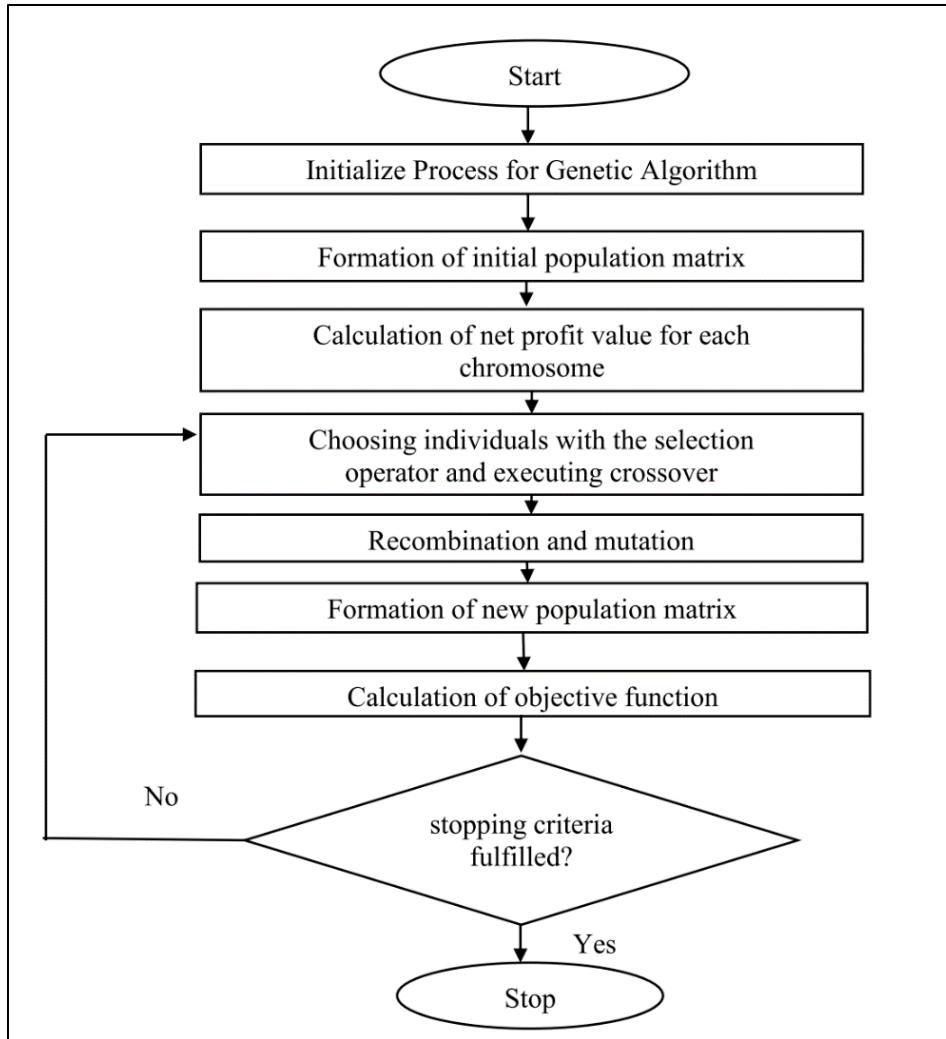


Figure 3: Typical genetic algorithm flowchart (Islam et al., 2020).

2.2.2 Particle Swarm Optimization (PSO)

Particle swarm optimization was developed by Kennedy and Eberhart in 1995. In this method, particles fly around in the search space, in which their position and velocity are variant. Initially, the particles velocity and position are randomly assigned. However, with time elapsing, particles adjust their position and velocity according to calculated fitness measure of probable solutions. Finally, the particles reach their final destination, which

exhibits the best fitness, and the algorithm declares that location as the optimal solution.

Figure 4 shows the general flow chart of a PSO algorithm (Islam et al., 2020).

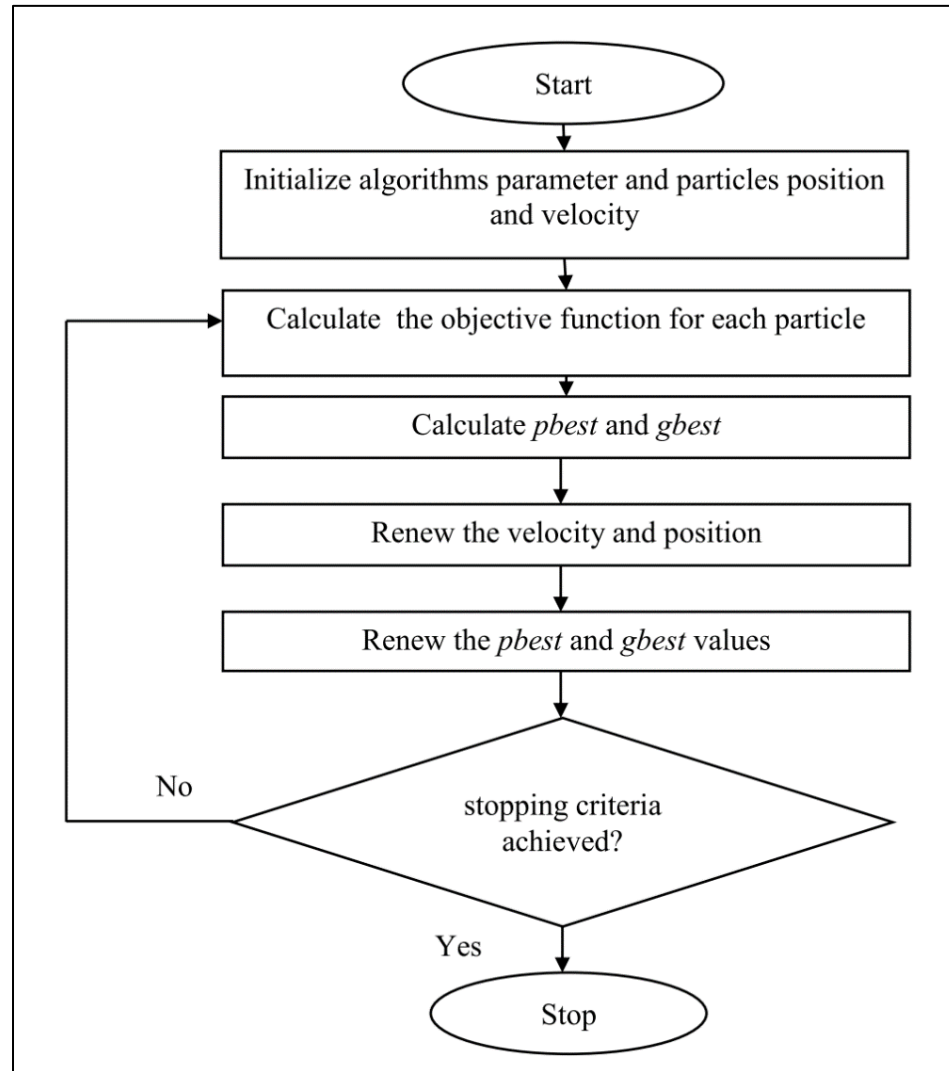


Figure 4: Particle swarm optimization algorithm flowchart (Islam et al., 2020).

2.2.3 Differential Evolution (DE)

Differential Evolution is a metaheuristic optimization algorithm that is similar in concept to GA. However, DE differs in that it relies more on mutation operations rather than crossover. **Figure 5** demonstrates the typical workflow chart of DE (Islam et al., 2020).

Another key difference between the two algorithms is that DE uses real-valued vectors rather than binary string representation (Tušar & Filipič, 2007).

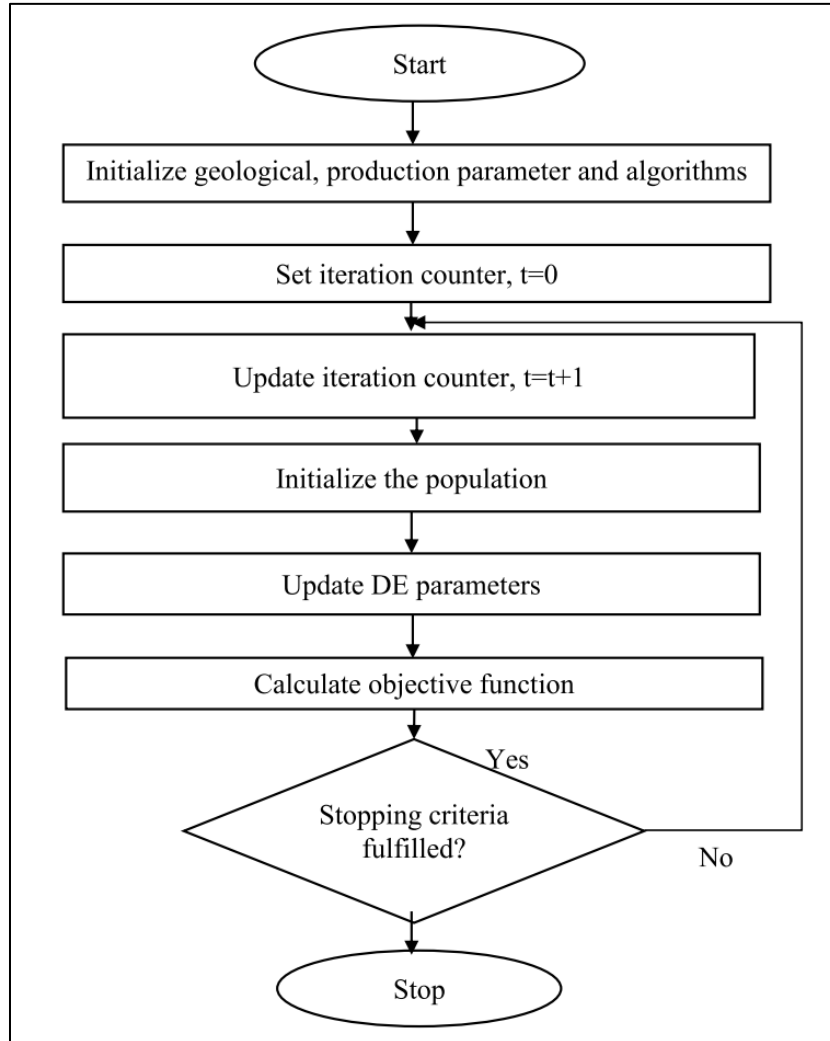


Figure 5: Typical differential evolution flowchart (Islam et al., 2020).

CHAPTER 3

PROBLEM STATEMENT & FORMULATION

3.1 Problem Statement

Modern technologies, such as geo-steering, have allowed drilling wells with more complicated trajectories as compared to the past. Thus, designing advanced wells, e.g., wells that are changing in inclination and azimuth angles as more path is drilled, is now feasible (**Figure 6**). Yet, literature work does not study the optimization of the placement of such wells where continuous path changes in azimuth and inclination angles matter. Alternatively, it considers the well path as perfectly straight, whether the path is vertical or with a constant angle of deviation. Therefore, the need for a study which integrates such angular changes is arising. **Figure 7** shows the directional components, most importantly the azimuth and inclination angles.

The proposed notion in this work is to divide each well to a number of segments, in which each segment is defined with directional information (location points and angle directions) that are adequate to create back the original well when re-assembled with the remaining segments. This segmentation creates freedom in designing the well path. Thus, this work investigates the placement of wells of any drilled path, i.e., not necessarily drilled in a specific pattern, e.g., s-shaped, vertical, undulated, etc. The research studies well placement incorporating the option of changes in the inclination and azimuth angles within practical and logical limitations. This work comprises both well types, producers and injectors, and

integrate critical constraints to provide reasonable solution endorsing petroleum engineering logic.

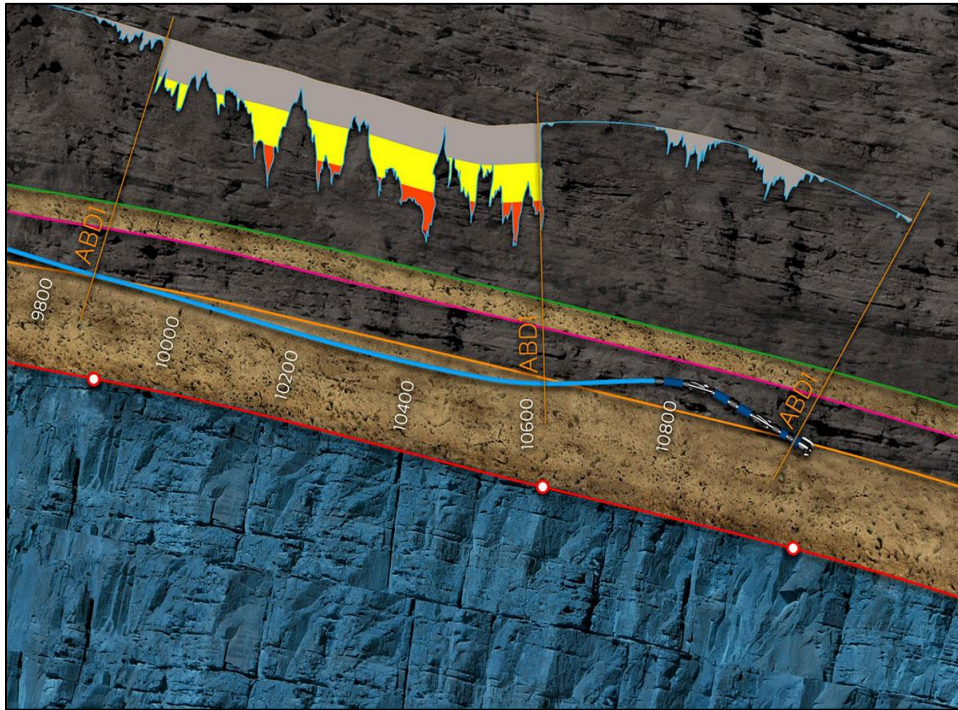


Figure 6: An example of a well path with angle changes (Sarraj, 2019)

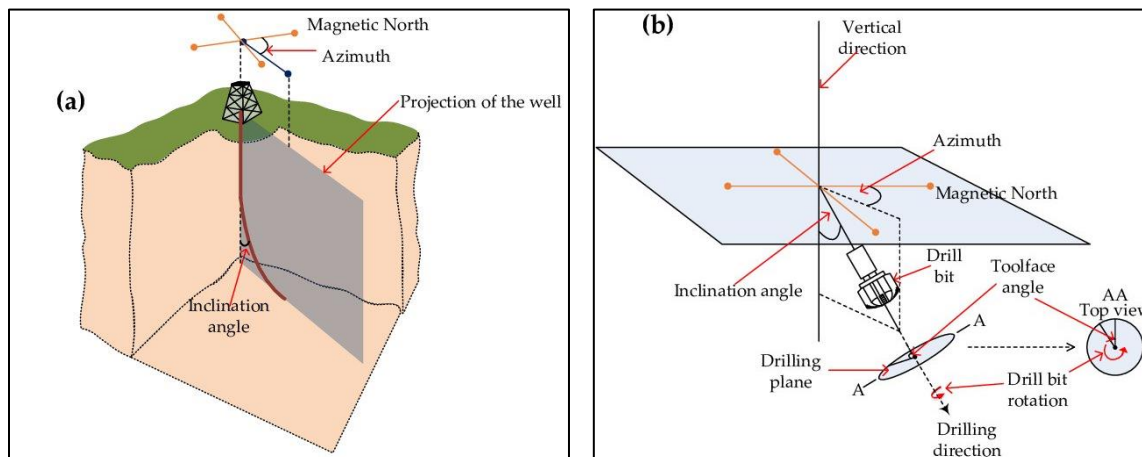


Figure 7: Directional Survey Components (Gooneratne, 2017)

3.2 Decision Parameters

The parameters to be optimized in the under-investigated problem are the wells' initial location coordinate points (i_1, j_1, k_1) and angle directions $(\theta_{azi}, \theta_{inc})$, the changes in azimuth and inclination angles of each segment compared to its preceding segments $(\overline{\Delta\theta}_{azi}, \overline{\Delta\theta}_{inc})$, and the total length of the well inside the reservoir targeted section (L). Additionally, there are other parameters which are pre-defined and not left to the optimizer for the purpose of simplicity and avoiding complicating the problem further, e.g., the number of segments and the number of producers and injectors. Further, several constraints are imposed on these optimizable parameters, namely, well starting points within reservoir boundaries, maximum changes in azimuth and inclination angles of a segment relative to its preceding segment, and a maximum inclination value of any segment.

The reason for choosing the changes of angles for each segment rather than the typical (i, j, k) is mainly for simplifying and reducing the problem dimension. If we assume that we want to optimize the placement of a well with five segments, then we can represent it using the typical (i, j, k) representation. Alternatively, we can represent it using the first segment heel point information $(i, j, k, \theta_{azi}, \theta_{inc})$ and only consider the changes in the azimuth and inclination angles $(\Delta\theta_{azi}, \Delta\theta_{inc})$ for the remaining points. **Table 3** compares the number of parameters required to define a well path with five segments.

Table 3: Required information to define a multi-segment well ((i, j, k) vs Proposed representations)

Method	(i, j, k) Representation			Proposed Representation				
<i>Parameter</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>i</i>	<i>j</i>	<i>k</i>	θ_{azi}	θ_{inc}
<i>Initial Location (Segment 1 heel)</i>	9	17	2	9	17	2	40°	75°
<i>Parameter</i>	<i>i</i>	<i>j</i>	<i>k</i>	-	-	-	$\Delta\theta_{azi}$	$\Delta\theta_{inc}$
<i>Segment 2 heel</i>	11.47	19.35	2.51	-	-	-	3.51°	-4.80°
<i>Segment 3 heel</i>	14.38	21.95	3.08	-	-	-	-1.65°	-2.57°
<i>Segment 4 heel</i>	17.68	25.00	3.57	-	-	-	0.84°	3.22°
<i>Segment 5 heel</i>	21.58	28.15	3.96	-	-	-	-3.78°	4.05°
<i>Segment 5 toe</i>	25.52	31.84	4.39	-	-	-	-	-

Therefore, the (i, j, k) representation needed 18 values to define such well path, i.e.:

$$(3 + 3 \times N_{segments}) \times N_{wells} \quad (3.1)$$

while the proposed representation required 13 values, i.e.:

$$(3 + 2 \times N_{segments}) \times N_{wells} \quad (3.2)$$

Figure 8 compares the number of decision parameters required for the algorithm to optimize for two conditions using the two representation methods. Condition (a) compares the number of decision parameters needed with respect to variation in the number of segments (consistent among all wells) given a fixed number of wells. On the other hand,

Condition (b) compares the required optimizable parameters given a fixed number of segments while varying the number of wells. It can be deduced from these two conditions that the proposed method requires smaller number of optimizable parameters in comparison with the (i, j, k) representation, and the difference between the two methods increases with higher number of segments or wells. Thus, using the proposed representation would ideally reduce the burden on the optimization algorithm since it will have a smaller number of decision parameters to optimize. Ultimately, this is anticipated to yield better optimization results. Further, the (i, j, k) representation is almost impossible to be directly used in the optimization algorithm and yield unfeasible results. This could be attributed to the angular constraints that will be violated when attempting to optimize the (i, j, k) points of each segment. For example, it may result in a solution of wells trajectories that go back in direction or wells that have impractically high values of azimuth or inclination angles created between two consecutive segments. Therefore, the proposed representation solves this problem by incorporating these constraints within its decision parameters as it uses the changes of azimuth and inclination angles between each two consecutive segments as decision parameters. These angular changes could be easily bounded with acceptable value limits during the optimization process to achieve practical well trajectories. In addition to the (i, j, k) representation, alternative prospect representations were investigated for feasibility. However, each has its own restrictions that prevent it from being adopted in the optimization algorithm. Further information on these representations and their restrictions is documented in **Appendix A**.

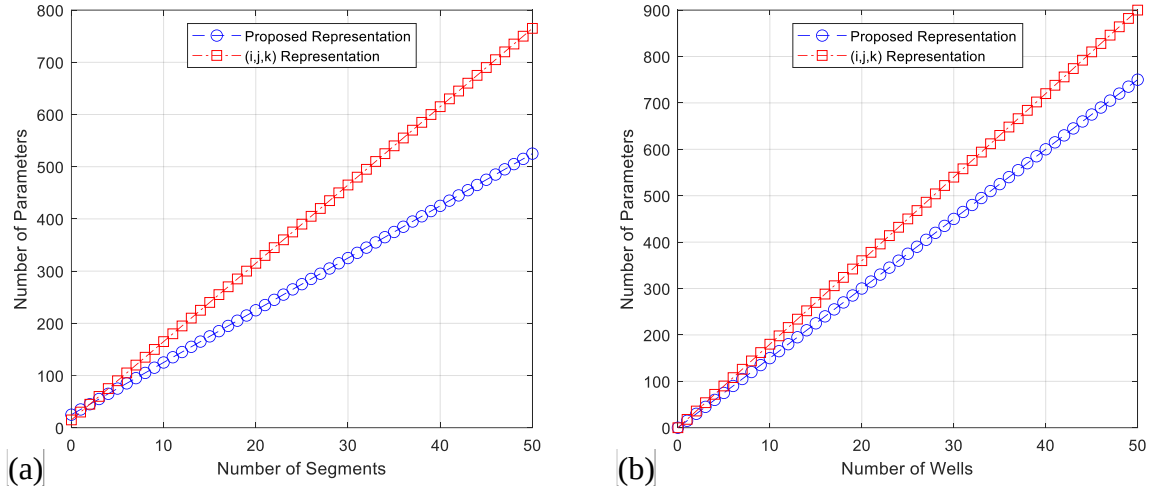


Figure 8: Comparing the required number of decision parameters using the (i, j, k) representation vs the proposed representation. (a) fixing the number of segments while varying the number of wells. (b) fixing the number of wells while varying the number of segments

However, since the simulator only accepts (i, j, k) representation, a conversion from our proposed representation to the (i, j, k) representation is a must. **Figure 9** shows one segment represented with (i, j, k) . Here, we can use the Pythagorean formula, the initial point (heel point) of the segment, and **Figure 7** to calculate the toe point (or the heel point of the next segment), given the azimuth angle, inclination angle, and the segment length.

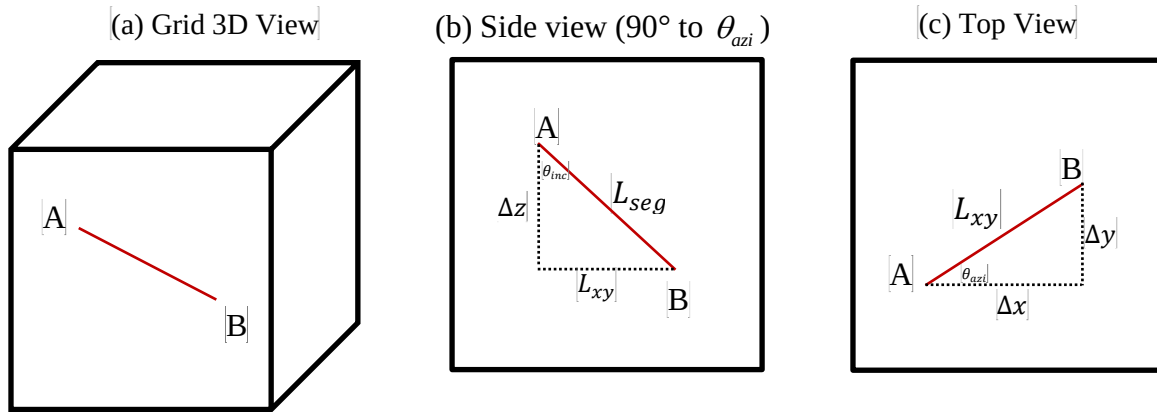


Figure 9: Segment directional components

Therefore, the following equations (**Equations 3.3 to 3.14**) are derived from **Figure 9** illustrations, where m stands for the point index, with $m=1$ meaning the heel point of Segment 1, $m=2$ is the heel point of Segment 2, ..., $m=N_{seg}$ is the heel point of the last segment, and lastly $m=N_{seg}+1$ is the toe point of the last segment. These equations utilize the selected decision parameters and provide (i, j, k) representation of the wells and its segments locations, meaning the (i, j, k) of each segment which is the acceptable form by the simulation software.

$$\theta_{azi}^{m+1} = \theta_{azi}^m + \Delta\theta_{azi} \quad (3.3)$$

$$\theta_{inc}^{m+1} = \theta_{inc}^m + \Delta\theta_{inc} \quad (3.4)$$

$$\Delta k = L_{seg} \cos(\theta_{inc}^{m+1}) \quad (3.5)$$

$$L_{xy} = L_{seg} \sin(\theta_{inc}^{m+1}) \quad (3.6)$$

$$\Delta i = L_{xy} \sin(\theta_{azi}^{m+1}) \quad (3.7)$$

$$\Delta j = L_{xy} \cos(\theta_{azi}^{m+1}) \quad (3.8)$$

$$i_{m+1} = i_m + \Delta i \quad (3.9)$$

$$j_{m+1} = j_m + \Delta j \quad (3.10)$$

$$k_{m+1} = k_m + \Delta k \quad (3.11)$$

To better convey the conversion process, the following brief example is explained. A well with five segments in a $50 \times 50 \times 10$ reservoir model with each grid dimension being 100 ft. can be defined using the following information:

- Well initial point location, $(i_1, j_1, k_1) = (9, 17, 2)$

- angle values, $(\theta_{azi}, \theta_{inc}) = (40^\circ, 75^\circ)$

- $\overline{\Delta\theta_{azi}} = \begin{bmatrix} 3.51^\circ \\ -1.65^\circ \\ 0.84^\circ \\ -3.78^\circ \end{bmatrix}$

- $\overline{\Delta\theta_{inc}} = \begin{bmatrix} -4.80^\circ \\ -2.57^\circ \\ 3.22^\circ \\ 4.05^\circ \end{bmatrix}$

- Well Length, $L = 750$ ft.

- Number of segments, $N_{seg} = 5$

- Segment Length, $L_{seg} = 150$ ft.

Using the given well information and utilizing **Equations 3.3 – 3.11**, the coordinate points of all segments can be calculated to yield **Table 4**.

However, the reservoir simulation software is limited to integers when defining segments' coordinate points (i, j, k) . Further, approximating the values to their nearest integer (which would be the center of the grid block) would not be practical and accurate in every scenario

since some reservoir models are defined with very large grid block sizes and the possibility of two or more segments to fall within the same grid block is possible in such cases. Hence, the well trajectory details could be mis-represented in such scenarios. Therefore, this paper attempts to solve this problem through the use of “Local Grid Refinement” or “LGR” which allows to create local grids within the original grids allowing better representation of the well trajectory and its segments’ locations.

Table 4: Calculated well’s (i, j, k) using the proposed representation

Description	(i, j, k)	$(\theta_{azi}, \theta_{inc})$
<i>Well initial point (Segment 1 heel point)</i>	(9 , 17 , 2)	(40° , 75°)
<i>Segment 2 heel point</i>	(11.47 , 19.35 , 2.51)	(43.51° , 70.20°)
<i>Segment 3 heel point</i>	(14.38 , 21.95 , 3.08)	(41.86° , 67.63°)
<i>Segment 4 heel point</i>	(17.68 , 25.00 , 3.57)	(42.70° , 70.85°)
<i>Segment 5 heel point</i>	(21.58 , 28.15 , 3.96)	(38.92° , 74.90°)
<i>Segment 5 toe point</i>	(25.52 , 31.84 , 4.39)	-

3.3 Local Grid Refinement (LGR)

Local Grid Refinement is the process of allocating local grids within the original defined grids (global grids). **Figure 10** demonstrates an example of a Cartesian local grid refinement within a Cartesian global grid system. The LGR process is especially beneficial for cases of reservoirs where the original grids are not sufficient to capture the changes in

the small-scale, e.g., changes of well path within a global grid. Thus, instead of defining smaller global grids in the reservoir model and increasing the complexity and computational time of the simulation, it is more efficient to create LGR where it is needed. However, the size, and hence number, of local grids should be judiciously selected. For instance, a very large size of grids may not suffice to capture small-scale changes. On the other hand, a very small size of local grids may yield undesirable convergence issues, e.g., the simulation would not be able to converge or would take an extremely long time to converge.

To better understand the usefulness of this approach, let us assume that we have the reservoir in **Figure 11**, and we want to place a well from Point A to Point B with the illustrated path in Part (a). Then, due to the limitations of the simulator, which only accepts well points in the center of the grids, the well path would look like in Part (b). On the other hand, **Figure 12** shows the corresponding simulator path should we have applied LGR. Though it has not exactly replicated the planned path, the simulator path assisted by LGR preserved more information of the planned path and the initial locations of points A and B than the case where LGR was not used. This method would be of more significance with more wells placed in the reservoir and more complicated path designs.

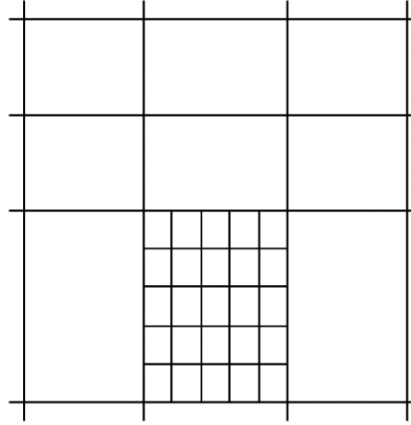
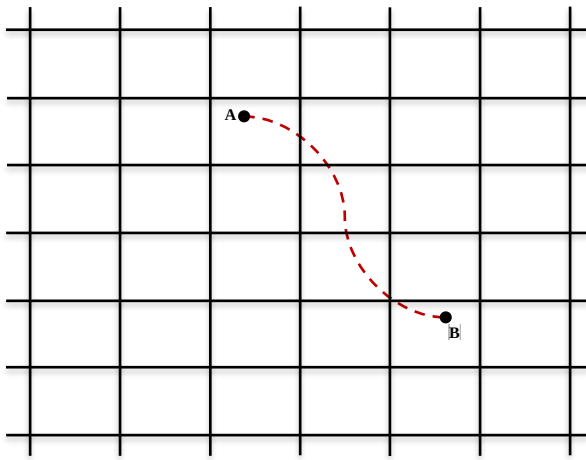
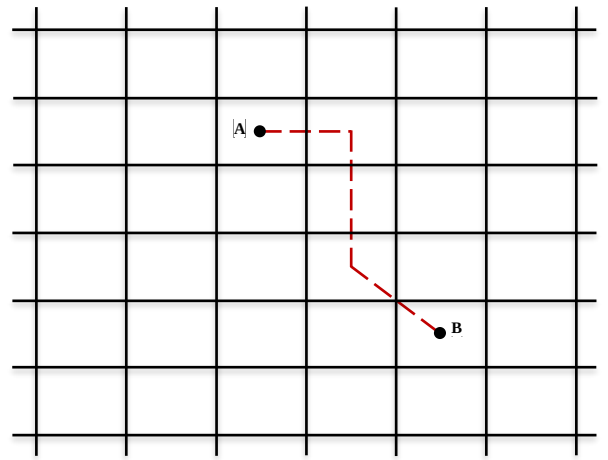


Figure 10: Cartesian grid refinement (Schlumberger 2021)

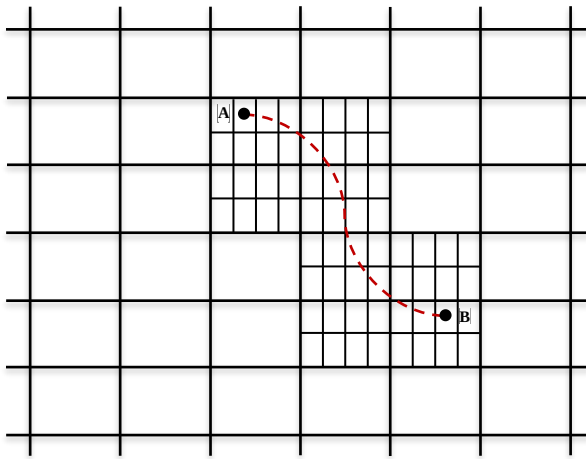


(a) Planned Path

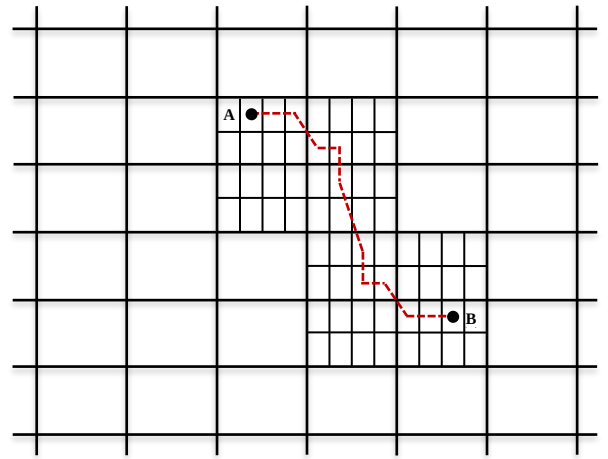


(b) Corresponding Simulator Path

Figure 11: Planned vs Simulator Path [No LGR used]



(a) Planned Path



(b) Corresponding Simulator Path

Figure 12: Planned vs Simulator Path [with LGR used]

CHAPTER 4

THE OPTIMIZATION THEORY

4.1 Differential Evolution (DE)

Differential evolution (DE) is an iterative approach that attempts to solve a problem through finding the optimum value. This metaheuristic algorithm is best known for its capability of solving high-dimensional problems while maintaining a low number of assumptions made. Nonetheless, although metaheuristic algorithms, including DE, try to solve for the optimal solution, the solution is not guaranteed to be a global optimum.

The DE process starts with the first generation of initial guess (population) in which contains the initial candidate solutions. The algorithm evaluates the candidate solutions for fitness and uses these to generate subsequent candidates by combining existing ones (mutation), while keeping the best candidates in hand (selection). New candidates' members are selected and combined randomly, as well as mixed with target vector (recombination). The new candidates are considered trial vectors and would only be accepted should they exhibit better fitness value than the original vector.

More clearly, let's assume that the objective is to optimize a function Φ with M number of real parameters. Then, DE randomly generate the initial population of N_p agents or parameter vectors symbolized as $\vec{x}_j, j \in (1, 2, \dots, N_p)$. The generation process of the agents' values is restricted between defined lower and upper limits $(\vec{x}_L, \vec{x}_U)$. The DE then proceeds with the following steps:

1. At iteration k , the target vector $\vec{x}_j^k$ is updated using three other agents $(\vec{x}_{r_1}^k, \vec{x}_{r_2}^k, \vec{x}_{r_3}^k)$. Note that j, r_1, r_2, r_3 must be distinct from each other.
2. A mutation factor F , which is a constant between 0 and 2, is multiplied to the difference between two agents and then added to the third agent to yield a donor vector $\vec{v}_j^{k+1}$. This step is called mutation.

$$\vec{v}_j^{k+1} = \vec{x}_{r_1}^k + F(\vec{x}_{r_2}^k - \vec{x}_{r_3}^k) \quad (4.1)$$

3. The target vector $\vec{x}_j^k$ and the donor vector $\vec{v}_j^{k+1}$ are used to produce a trial vector $\vec{y}_j^{k+1}$ by replacing members of $\vec{x}_j^k$ with their correspondents in $\vec{v}_j^{k+1}$ given a pre-determined cross-over probability CR ,

$$\vec{y}_{m,j}^{k+1} = \begin{cases} \vec{v}_{m,j}^{k+1} & \text{if } \dot{r} \leq CR \text{ or } m = m_{rand} \\ \vec{x}_{m,j}^k & \text{if } \dot{r} > CR \text{ or } m \neq m_{rand} \end{cases} \quad (4.2)$$

where $m \in (1, 2, \dots, M)$, $\dot{r}$ is a random number sampled from a uniform distribution between 0 and 1, and m_{rand} is a random integer from 1 to M . The purpose of m_{rand} is to assure that $\vec{x}_j^k$ and $\vec{v}_j^{k+1}$ are distinct from each other. This step is called recombination.

4. A comparison between the target vector $\vec{x}_j^k$ and the trial vector $\vec{y}_j^{k+1}$, and whichever yields lower function value is selected for the next generation.

$$\vec{x}_j^{k+1} = \begin{cases} \vec{y}_j^{k+1} & \text{if } \Phi(\vec{y}_j^{k+1}) \leq \Phi(\vec{x}_j^k) \\ \vec{x}_j^k & \text{otherwise} \end{cases} \quad (4.3)$$

for $\bar{x}_j, j \in (1, 2, \dots, N_p)$. This step is called selection.

5. Steps 2,3, and 4 (mutation, recombination, and selection) are repeated until satisfying a predefined stopping criterion.
6. Finally, the agent from the last generation which yields the lowest function value is chosen and reported as the best solution.

The size of the population, N_p , is commonly determined using the following equation:

$$N_p = 4 + \text{floor}(3 \times \ln D) \quad (4.4)$$

where D is the dimension of the problem.

4.2 The Objective Function (Net Present Value)

It is very important for any optimization problem to select an appropriate objective function (sometimes referred to as cost function). Generally, for well optimization problems, it is more common to use the net present value (NPV) of the field as a cost function. Alternatively, cumulative oil production (COP) is another common metric used as an objective function for such problems. However, unlike COP where it only accounts for total oil produced regardless of the expenses paid prior, during, and post-production, the NPV is more favorable in most scenarios since it best reflects real-life economics. Hence, this paper employs NPV as the main and sole objective function in the optimization workflow. However, since the optimization algorithm approaches minimize the selected objective function, the objective function in this paper is set to use the negative value of NPV, yielding ultimately maximum NPV.

The net present value (NPV) is the net discounted cashflow from an initial point of time up to present. In well placement problems, the initial point of time could be the initial time of production up to a specific point of time defined. The NPV can be a positive or a negative value, in which a positive value means a profitable project while a negative value reflects unprofitable project (expenses are higher than revenue). When calculating NPV for such problems, this term is affected by many factors, such as the number and location of wells, their types whether producers or injectors, the operations applied from spud date up to completion, e.g., drilling and logging operations, rig equipment, etc. Thus, NPV is an excellent metric to indicate the profitability of the field.

The net present value in this project is defined as (Khan & Awotunde 2017).

$$NPV = \sum_{n=1}^N \frac{CF_n}{(1+r)^n} - C_{cap} \quad (4.5)$$

where N is the total number of discounting periods, which in this work is every month, n is the period index, r is the discount rate, CF_n is the cash flow in period n , and C_{cap} is the capital expenses, which is expressed as:

$$C_{cap} = C_{facility} + N_{prod}C_{prod} + N_{inj}C_{inj} + C_{traj} \quad (4.6)$$

where $C_{facility}$ is the installation cost of the facilities, C_{prod} and C_{inj} are the drilling costs of a producer and an injector, respectively, and N_{prod} and N_{inj} are the number of producers and injectors, respectively. C_{traj} is an additional cost term incorporated to account for the expenses accompanied by drilling modern well trajectories, such those requiring advanced tools like MWD/LWD. C_{traj} is defined as follows:

$$C_{traj} = TFD \times C_{traj, ft} \quad (4.7)$$

where TFD is the total feet drilled of all wells within the reservoir section, and $C_{traj, ft}$ is the average cost of an average foot drilled.

The term CF_n is defined as:

$$CF_n = R_n - E_n \quad (4.8)$$

where R_n is the total revenue in period n , while E_n is the total operating expenditures in period n . R_n and E_n are defined as the following, respectively:

$$R_n = C_n^{o, prod} Q_n^{o, prod} + C_n^{g, prod} Q_n^{g, prod} \quad (4.9)$$

$$E_n = C_n^{w, prod} Q_n^{w, prod} + C_n^{w, inj} Q_n^{w, inj} + C_{op, n} Q_n^{o, prod} \quad (4.10)$$

where $C_n^{o, prod}$ is the price of produced oil per barrel, $Q_n^{o, prod}$ is the total barrels of oil produced in period n , $C_n^{g, prod}$ is the price of produced gas per 1000 ft³, and $Q_n^{g, prod}$ is the total cubic feet of gas produced in period n . In addition, $C_n^{w, prod}$ is the price of producing and treating one barrel of water, $Q_n^{w, prod}$ is the total barrels of water produced in period n , $C_n^{w, inj}$ is the price of treating and injecting one barrel of water, and $Q_n^{w, inj}$ is the total barrels of water injected in period n . $C_{op, n}$ stands for the operation cost of one barrel of oil. In this project, the definitions and assumptions made to evaluate the NPV are listed in **Table 5**.

Table 5: Economical definitions for NPV calculation

Parameter	Value	Unit
$C_{facility}$	50×10^6	USD
C_{prod}	2.5×10^6	USD
C_{inj}	2.5×10^6	USD
$C_n^{o,prod}$	70	USD/bbl
$C_n^{g,prod}$	3	USD/MSCF
$C_n^{w,prod}$	5	USD/bbl
$C_n^{w,inj}$	10	USD/bbl
$C_{op,n}$	8	USD/bbl
$C_{traj,ft}$	150	USD/ft
r	10	% (Annual rate)
r	0.7974	% (Monthly rate)

4.3 The Penalty Approach

As emphasized earlier, this research aims to optimize the placement of wells and their path adhering to a number of imposed constraints, such as minimum well spacing and placement within reservoir boundaries. It is a common practice in literature to convert such constrained optimization problem (COP) into an unconstrained optimization problem (UOP) through integration of these constraints in the objective function. This integration can be in the form of penalty terms or Lagrange multipliers (Awotunde & Naranjo, 2014). This work employs the penalty approach to fulfill such purpose.

Initially, it is very important to formulate the constraints expressions. For the minimum well spacing constraint, a spherical volume with $r_{\min}$ around each segment is assumed, in which if any other segment from a different well falls within this volume it would be considered a violation. The actual process starts by assuming the first well is not violating this constraint, and so we move to the second well. Starting from the first segment of the second well, if any segment from the first well falls within spherical volume of that first segment, then the amount of length falling inside this sphere is stored. The process repeats with the other segments from the same well, and then the segments from the third well will check the first and second wells for such violation. Ultimately, the summation of all lengths that violated the constraint is calculated (G_c). The following nonlinear constraints express the minimum spacing constraints.

$$\left\{ \begin{aligned}
& g_{(2,1|1,1)} = r_{\min}^2 - (x_{2,1} - x_{1,1})^2 - (y_{2,1} - y_{1,1})^2 - (z_{2,1} - z_{1,1})^2 \leq 0 \\
& g_{(2,1|1,2)} = r_{\min}^2 - (x_{2,1} - x_{1,2})^2 - (y_{2,1} - y_{1,2})^2 - (z_{2,1} - z_{1,2})^2 \leq 0 \\
& \vdots \\
& g_{(2,1|m_1)} = r_{\min}^2 - (x_{2,1} - x_{1,m_1})^2 - (y_{2,1} - y_{1,m_1})^2 - (z_{2,1} - z_{1,m_1})^2 \leq 0 \\
& g_{(2,2|1,1)} = r_{\min}^2 - (x_{2,2} - x_{1,1})^2 - (y_{2,2} - y_{1,1})^2 - (z_{2,2} - z_{1,1})^2 \leq 0 \\
& g_{(2,2|1,2)} = r_{\min}^2 - (x_{2,2} - x_{1,2})^2 - (y_{2,2} - y_{1,2})^2 - (z_{2,2} - z_{1,2})^2 \leq 0 \\
& \vdots \\
& g_{(2,2|m_1)} = r_{\min}^2 - (x_{2,2} - x_{1,m_1})^2 - (y_{2,2} - y_{1,m_1})^2 - (z_{2,2} - z_{1,m_1})^2 \leq 0 \\
& \vdots \\
& g_{(2,m_2|1,m_1)} = r_{\min}^2 - (x_{2,m_2} - x_{1,m_1})^2 - (y_{2,m_2} - y_{1,m_1})^2 - (z_{2,m_2} - z_{1,m_1})^2 \leq 0 \\
& \vdots \\
& g_{(N_{\text{well}}+1|1,1)} = r_{\min}^2 - (x_{N_{\text{well}}+1} - x_{1,1})^2 - (y_{N_{\text{well}}+1} - y_{1,1})^2 - (z_{N_{\text{well}}+1} - z_{1,1})^2 \leq 0 \\
& \vdots \\
& g_{(N_{\text{well}}+1|1,m_1)} = r_{\min}^2 - (x_{N_{\text{well}}+1} - x_{1,m_1})^2 - (y_{N_{\text{well}}+1} - y_{1,m_1})^2 - (z_{N_{\text{well}}+1} - z_{1,m_1})^2 \leq 0 \\
& g_{(N_{\text{well}}+1|2,1)} = r_{\min}^2 - (x_{N_{\text{well}}+1} - x_{2,1})^2 - (y_{N_{\text{well}}+1} - y_{2,1})^2 - (z_{N_{\text{well}}+1} - z_{2,1})^2 \leq 0 \\
& \vdots \\
& g_{(N_{\text{well}}+1|2,m_2)} = r_{\min}^2 - (x_{N_{\text{well}}+1} - x_{2,m_2})^2 - (y_{N_{\text{well}}+1} - y_{2,m_2})^2 - (z_{N_{\text{well}}+1} - z_{2,m_2})^2 \leq 0 \\
& \vdots \\
& g_{(N_{\text{well}}+1|N_{\text{well}}-1,m_{N_{\text{well}}-1})} = r_{\min}^2 - (x_{N_{\text{well}}+1} - x_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 - (y_{N_{\text{well}}+1} - y_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 - (z_{N_{\text{well}}+1} - z_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 \leq 0 \\
& g_{(N_{\text{well}}+2|1,1)} = r_{\min}^2 - (x_{N_{\text{well}}+2} - x_{1,1})^2 - (y_{N_{\text{well}}+2} - y_{1,1})^2 - (z_{N_{\text{well}}+2} - z_{1,1})^2 \leq 0 \\
& \vdots \\
& g_{(N_{\text{well}}+m_{N_{\text{well}}}|N_{\text{well}}-1,m_{N_{\text{well}}-1})} = r_{\min}^2 - (x_{N_{\text{well}}+m_{N_{\text{well}}}} - x_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 - (y_{N_{\text{well}}+m_{N_{\text{well}}}} - y_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 - (z_{N_{\text{well}}+m_{N_{\text{well}}}} - z_{N_{\text{well}}-1,m_{N_{\text{well}}-1}})^2 \leq 0
\end{aligned} \right. \quad (4.11)$$

where N_{well} means the total number of wells, m_1 means the number of segments of Well 1 (which could be used to represent the last segment of that well also), while $m_{N_{\text{well}}-1}$ means the number of segments of Well $N_{\text{well}} - 1$ (the well before last). Additionally, the last expression in Equation 24, $g_{(N_{\text{well}}+m_{N_{\text{well}}}|N_{\text{well}}-1,m_{N_{\text{well}}-1})}$, means that the distance between Segment $m_{N_{\text{well}}}$ of well N_{well} to Segment $m_{N_{\text{well}}-1}$ of well $N_{\text{well}} - 1$ should be larger than the minimum imposed distance which is $r_{\min}$.

The resultant number of constraints equations $N_{c,G}$ can be estimated as

$$N_{c,G} = \sum_{n=2}^{N_{well}} (N_{seg,well(n)} \times N_{seg,well(n-1)} \times \cdots \times N_{seg,well(1)}) \quad (4.12)$$

where $N_{seg,well(n)}$ means the number of segments of Well n .

The second enforced constraint in this paper is to place all wells within reservoir boundaries. The process checks each segment of each well for their coordinate points. If that point is outside the reservoir boundaries, then the part that falls outside the reservoir is cut from the well path plan and recorded, and the total lengths of all segments that violated these constraints are summed (H_c). The following formulas represent the nonlinear constraints related to this restriction.

$$\left[\begin{array}{l}
\left\{ \begin{array}{ll} h_{(1,1)} = 0 & \text{if inside reservoir} \\ h_{(1,1)} = L_{h_{(1,1)}} & \text{if outside reservoir} \end{array} \right. \\
\left\{ \begin{array}{ll} h_{(1,2)} = 0 & \text{if inside reservoir} \\ h_{(1,2)} = L_{h_{(1,2)}} & \text{if outside reservoir} \end{array} \right. \\
\vdots \\
\left\{ \begin{array}{ll} h_{(1,m_1)} = 0 & \text{if inside reservoir} \\ h_{(1,m_1)} = L_{h_{(1,m_1)}} & \text{if outside reservoir} \end{array} \right. \\
\left\{ \begin{array}{ll} h_{(2,1)} = 0 & \text{if inside reservoir} \\ h_{(2,1)} = L_{h_{(2,1)}} & \text{if outside reservoir} \end{array} \right. \\
\vdots \\
\left\{ \begin{array}{ll} h_{(2,m_2)} = 0 & \text{if inside reservoir} \\ h_{(2,m_2)} = L_{h_{(2,m_2)}} & \text{if outside reservoir} \end{array} \right. \\
\vdots \\
\left\{ \begin{array}{ll} h_{(N_{well},1)} = 0 & \text{if inside reservoir} \\ h_{(N_{well},1)} = L_{h_{(N_{well},1)}} & \text{if outside reservoir} \end{array} \right. \\
\vdots \\
\left\{ \begin{array}{ll} h_{(N_{well},m_{N_{well}})} = 0 & \text{if inside reservoir} \\ h_{(N_{well},m_{N_{well}})} = L_{h_{(N_{well},m_{N_{well}})}} & \text{if outside reservoir} \end{array} \right.
\end{array} \right. \quad (4.13)$$

where $h_{(2,1)}$ is the penalty value for Segment 1 of Well 2, and $L_{h_{(2,1)}}$ is the length of Segment 1 of Well 2 that falls outside the reservoir boundary. And so, $h_{N_{well},m_{N_{well}}}$ is the penalty value for Segment $m_{N_{well}}$ of Well N_{well} , and $L_{h_{(N_{well},m_{N_{well}})}}$ is the total length of Segment $m_{N_{well}}$ of Well N_{well} that falls outside the reservoir. The resultant number of constraints equations $N_{c,H}$ can be estimated as

$$N_{c,H} = \sum_{n=1}^{N_{well}} N_{seg,well(n)} \quad (4.14)$$

where $N_{seg,well(n)}$ means the number of segments of Well n .

Further, to incorporate these two constraints to the objective function, a normalization of their values is required. Typically, the cost function using only NPV can be expressed as:

$$\Phi_{NPV,k} = -NPV_k \quad (4.15)$$

However, since we are using the penalty approach and two constraints are imposed, then two additional terms are needed. In addition, normalization of each term of the three (including NPV) is required so that all terms are scaled and of relevant importance. Furthermore, a scaling factor is multiplied to the two constraint terms to stress their significance. This factor is designed to monotonically increase with more function evaluations to assure obedience of the solution to the imposed constraints. **Equation 4.16** expresses the objective function used in this paper.

$$\Phi_k = -\frac{NPV_k}{\alpha_{NPV,k=1}} + \lambda_k \left[\frac{1}{\beta_{G_c,k=1}} G_c + \frac{1}{\gamma_{H_c,k=1}} H_c \right] \quad (4.16)$$

where k stands for iteration number in the search algorithm, $\alpha_{NPV,k=1}$ is the median NPV of the first iteration, $\beta_{G_c,k=1}$ and $\gamma_{H_c,k=1}$ are the maximum G_c and H_c , respectively, of the first iteration. The median and maximum referred to here are the median and maximum calculated from the solutions obtained in the first generation. λ_k here refers to the scaling factor which increases as more generations are produced, and as this value increases, the terms that reflect the violation increase and so the search for a non-violating solution is emphasized. As a result, the objective function behavior is to focus on the primary objective (minimizing $-NPV$ with moderate emphasis on constraints) at early iterations,

and stricter behavior to comply with the constraints is exhibited at later iterations. Moreover, the scaling factor should be wisely selected, a large value may yield to early convergence and suboptimal solution, while a very small value may result in an extensive search and a long time to find a solution. In this work, this value is selected to vary linearly between 1 and 10, with value of one at the initial iteration and 10 at the last, which can be expressed as:

$$\lambda_k = 9 \times \left(\frac{k}{k_{\max}} \right) + 1 \quad (4.17)$$

where k refers to the iteration index while $k_{\max}$ means the maximum number of iterations chosen.

CHAPTER 5

RESULTS AND DISCUSSION

Three different well placement optimization problems were investigated and solved, in which in each the proposed approach was evaluated using different reservoir example. For each problem, DE was employed with Equation 27 as its objective function, in which within the DE algorithm, F is defined as 0.85, CR is set to 0.95, the population size is selected to be 10, and the number of generations is 200. The economical parameters used to evaluate the objective function are defined in **Table 5**. For all the cases, the algorithm solved for the optimized initial wells' locations and direction $(i_1, j_1, k_1, \theta_{azi}, \theta_{inc})$, the angular changes of each segment relative to its preceding $(\overrightarrow{\Delta\theta_{azi}}, \overrightarrow{\Delta\theta_{inc}})$, and the total well length inside the reservoir (L).

The main scenario in each case example compares five different realizations performed using the proposed approach in this paper, termed as “Generalized Path” or “GP”, with another five realizations run using Straight-Line Path approach “SLP”, i.e., creating straight-line vertical wells or deviated wells with a constant degree angle. Using the results obtained from the main scenario using the GP approach, an additional scenario that connects the initial well point to the last well point creating a straight-line path (SLP created from GP) was studied. The purpose of such scenario is to examine the effect of assuming a straight-line path instead of a generalized path. In addition, two sensitivity analyses of two parameters were inspected. The first scenario highlights the impact of the chosen number of segments to represent the well path with. The second scenario analyzes the

influence of selecting an appropriate minimum well spacing distance. Three different values were used for each sensitivity analysis, the optimization algorithm was run for each, and the results were compared.

5.1 Example 1 – Channel Reservoir

The investigated reservoir in this example is synthetic and consists of four facies in four different layers. The reservoir is divided into $75 \times 75 \times 4$ grids, in which each grid's dimension is $150\text{ ft} \times 150\text{ ft} \times 150\text{ ft}$. **Figure 13** illustrates the permeability distribution maps of the four layers in a log-scale. **Table 6** provides the petrophysical properties of the four facies, and **Figure 14** shows the rock and rock-fluid properties in the reservoir. The presence of channels in the reservoir creates a potential challenge, as these channels can cause the injected water to move rapidly to the producers through these channels, leaving behind a significant amount of oil (resulting in a poor sweep efficiency). The objective in this example is to determine the best positions for seven wells (four producers and three injectors) since the initial development of the field and for a production period of ten years. For this example, the lower and upper limits of the decision parameters to be optimized are listed in **Table 7**.

Several constraints were enforced on each producer and injector. For each producer, a liquid rate of 15,000 bpd was imposed as long as the bottomhole pressure of that producer does not decrease below a minimum value of 2,500 psi. Meaning, if the producer is not capable to maintain the required production rate without falling below this value, the constraint on this producer switches from rate-constrained to pressure-constrained, and the well continues production at that minimum bottomhole pressure regardless of the liquid

rate. On the other hand, the imposed constraint on each injector was to keep the bottomhole pressure at a value of 4,000 psi, regardless of the amount of water injected, to maintain the reservoir's pressure and energy to push its fluids to surface.

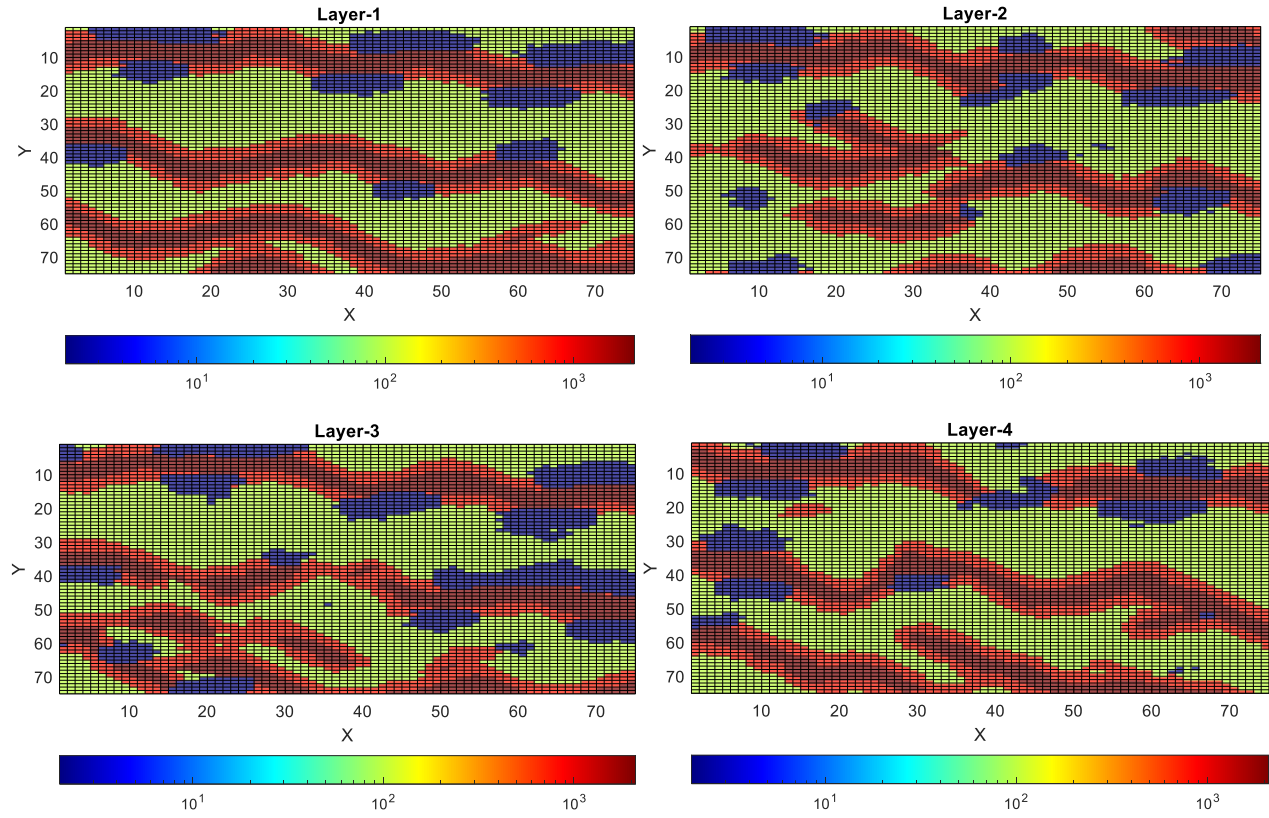


Figure 13: Channel reservoir permeability distribution maps

Table 6: Channel reservoir properties

Facies	Color	k (md)	ϕ
1	Green	100	0.12
2	Burgundy	2100	0.37
3	Red	750	0.23
4	Blue	2	0.05

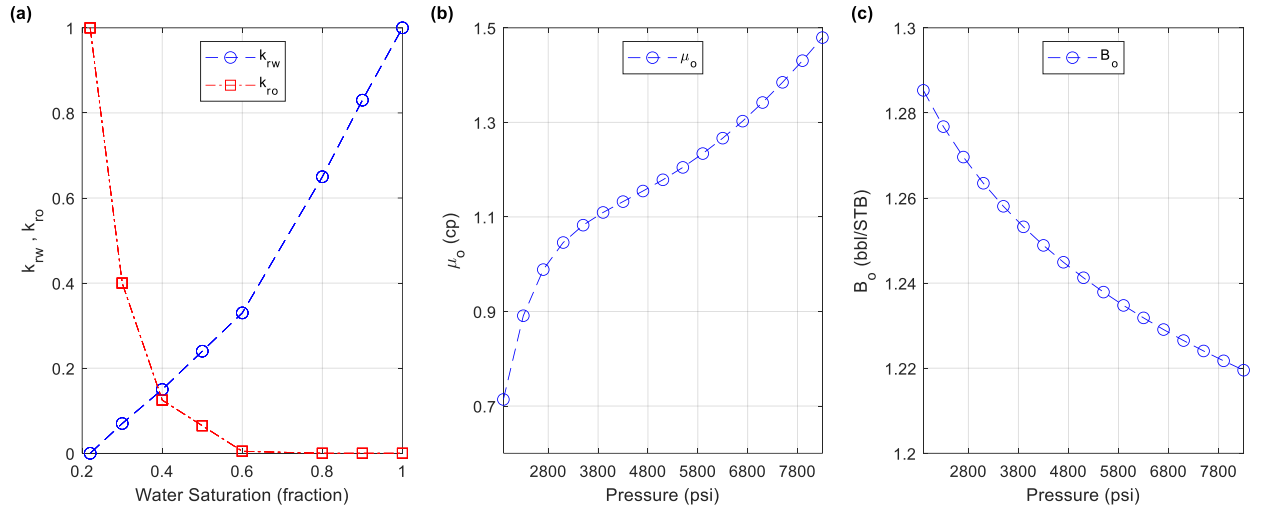


Figure 14: Fluid & Fluid-Rock Properties of Example 1. (a) oil/water relative permeability curves, (b) oil viscosity, and (c) oil formation volume factor

Table 7: Lower and upper limits of the decision parameters utilized.

Parameter	Lower limit	Upper limit
i	1	75
j	1	75
k	1	4
θ_{azi}	0°	360°
θ_{inc}	0°	95°
$\Delta\theta_{azi}$	-8°	8°
$\Delta\theta_{inc}$	-8°	8°

In this example, a spherical volume of radius equals to 1,500 feet around each segment was used as the minimum well spacing, in which if any segment of any other well falls within this volume will be accounted as a penalty to the objective function. When applying **Equations 4.12 and 4.14** to this problem of seven wells with 10 segments each, we needed to consider 1.11×10^7 nonlinear constraints (minimum spacing constraints) + 70 additional constraints (placement within reservoir boundaries constraints). Additionally, to further validate the proposed approach, a comparison with the approach of straight-line wells was investigated.

Five different realizations were performed of each path approach: this work's proposed approach Generalized Path or (GP) and the approach of Straight-Line Path or (SLP). The results from the five realizations were sorted from highest to lowest based on NPV, and the highest, median, and lowest scenarios were documented and plotted (as in **Figure 15**) to compare the results of the two approaches. It is worth noting that the NPV in **Figure 15** is not monotonically increasing as more functions were evaluated, and this is basically due to the objective function being a normalized term incorporating the NPV and the penalization terms. For this example, it can be noted that, generally, the GP approach outperformed the SLP approach. As a matter of fact, the median realization of the GP approach succeeded in providing better results than the best realization of the SLP approach. **Table 8** summarizes the results: NPV, RF, and penalty terms, obtained from the optimization algorithm employed for this example. Overall, the GP method succeeded in yielding better NPV with lower recovery factor percentages (%RF), reflecting the efficient placement design of wells locations and paths. Remarkably, all the cases managed to adhere to the minimum spacing constraint though the high number of nonlinear constraints

imposed. However, about half the scenarios violated considerably the other constraint imposed and placed some segments outside the reservoir boundaries. **Figures 16 – 17** illustrate the final placement location of the seven wells from the highest NPV case of each approach.

Table 8: Results summary of the two methods and their highest NPV, median NPV, and lowest NPV

Method	Realization	NPV (\$)	RF (%)	G_c (ft)	H_c (ft)
GP	Highest NPV	7.11×10^9	24.49	0	0
	Median NPV	6.97×10^9	25.51	0	0.2
	Lowest NPV	6.74×10^9	27.53	0	195.7
SLP	Highest NPV	6.89×10^9	28.03	0	0.01
	Median NPV	6.82×10^9	27.65	0	304.2
	Lowest NPV	5.71×10^9	27.57	0	185.9

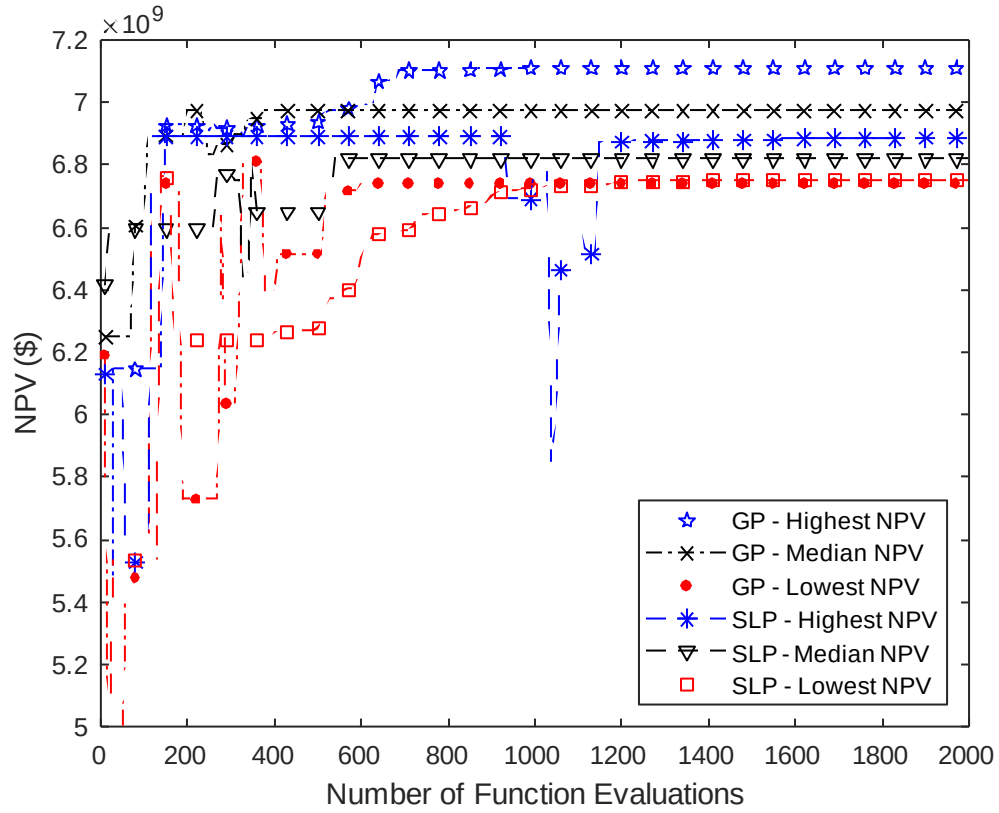
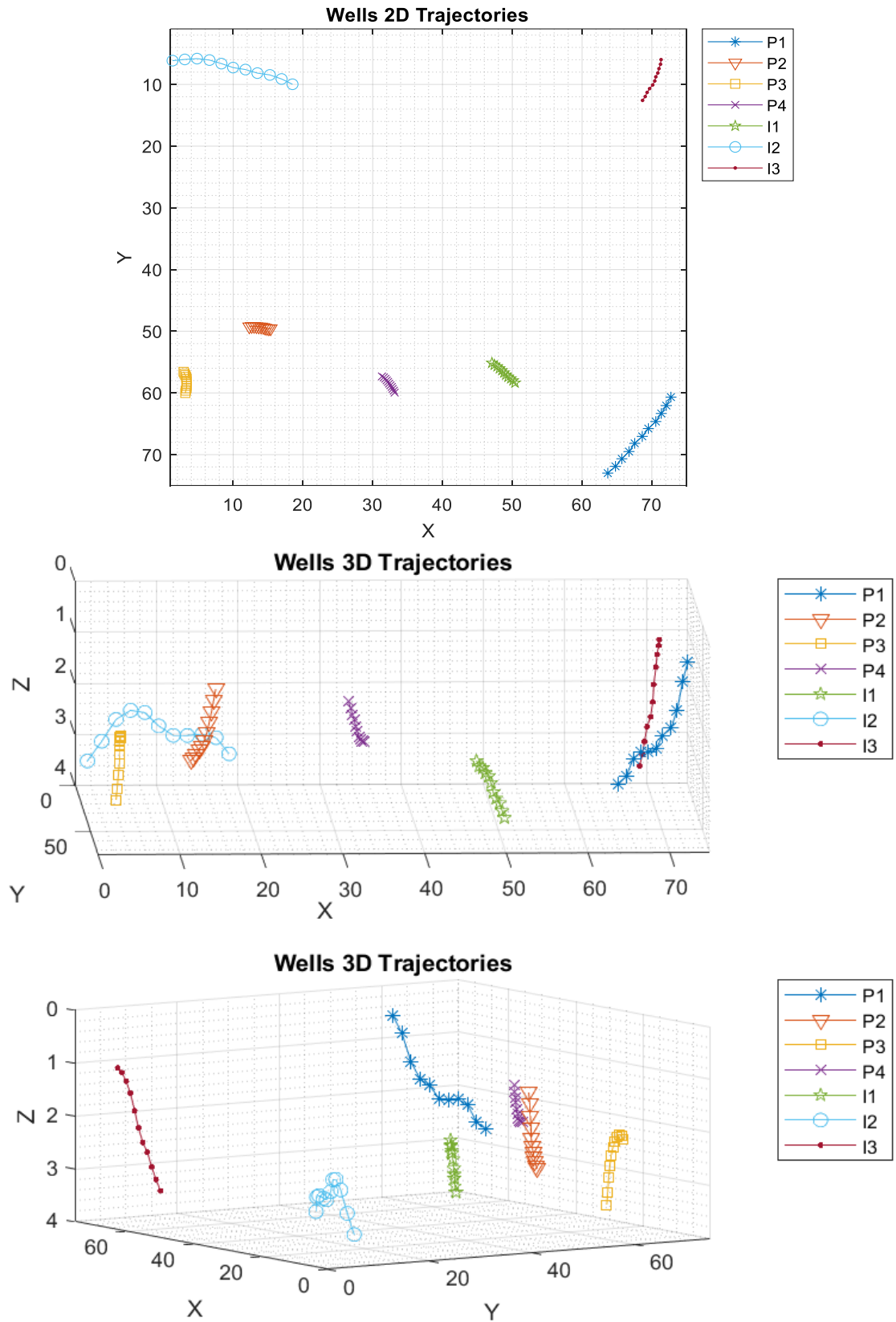


Figure 15: NPV vs Number of Function Evaluations for Channel Reservoir case



[Figure 16: Well Placement from top 2D view, and two 3D views (Generalized Path “GP”)]

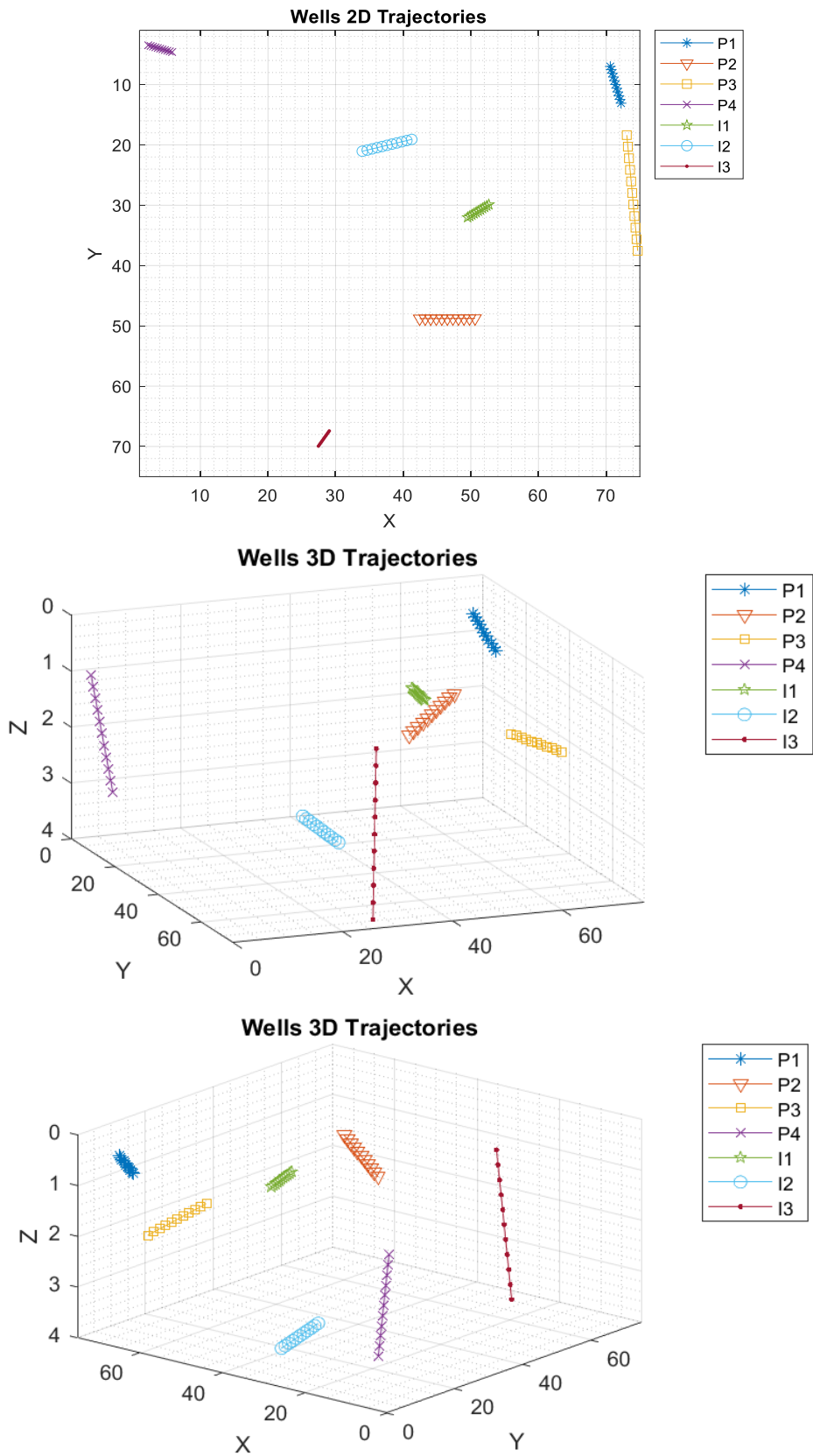


Figure 17: Well Placement from top 2D view, and two 3D views (Straight-Line Path “SLP”)

Moreover, the results obtained from the GP approach were used to create an alternative scenario with the same wells locations. However, a straight-line path was created connecting the initial and end points of each well, as illustrated in **Figure 18**. The objective of this comparison is to investigate if it is valid to assume a straight-line path from our already obtained solutions. **Table 9** presents the achieved results. It is evident that there is a significant difference between the NPV obtained from the two scenarios. This shows the significance of preserving the original well information, as assuming a straight-line could lead to inaccurate and misleading results.

Table 9: A comparison between the original and alternative scenarios based on the highest NPV, median NPV, and lowest NPV

Method	Realization	Original Scenario		Alternative Scenario (SLP created from GP)	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	7.11×10^9	24.49	6.13×10^9	11.86
	Median NPV	6.97×10^9	25.51	5.75×10^9	12.60
	Lowest NPV	6.74×10^9	27.53	5.57×10^9	13.83

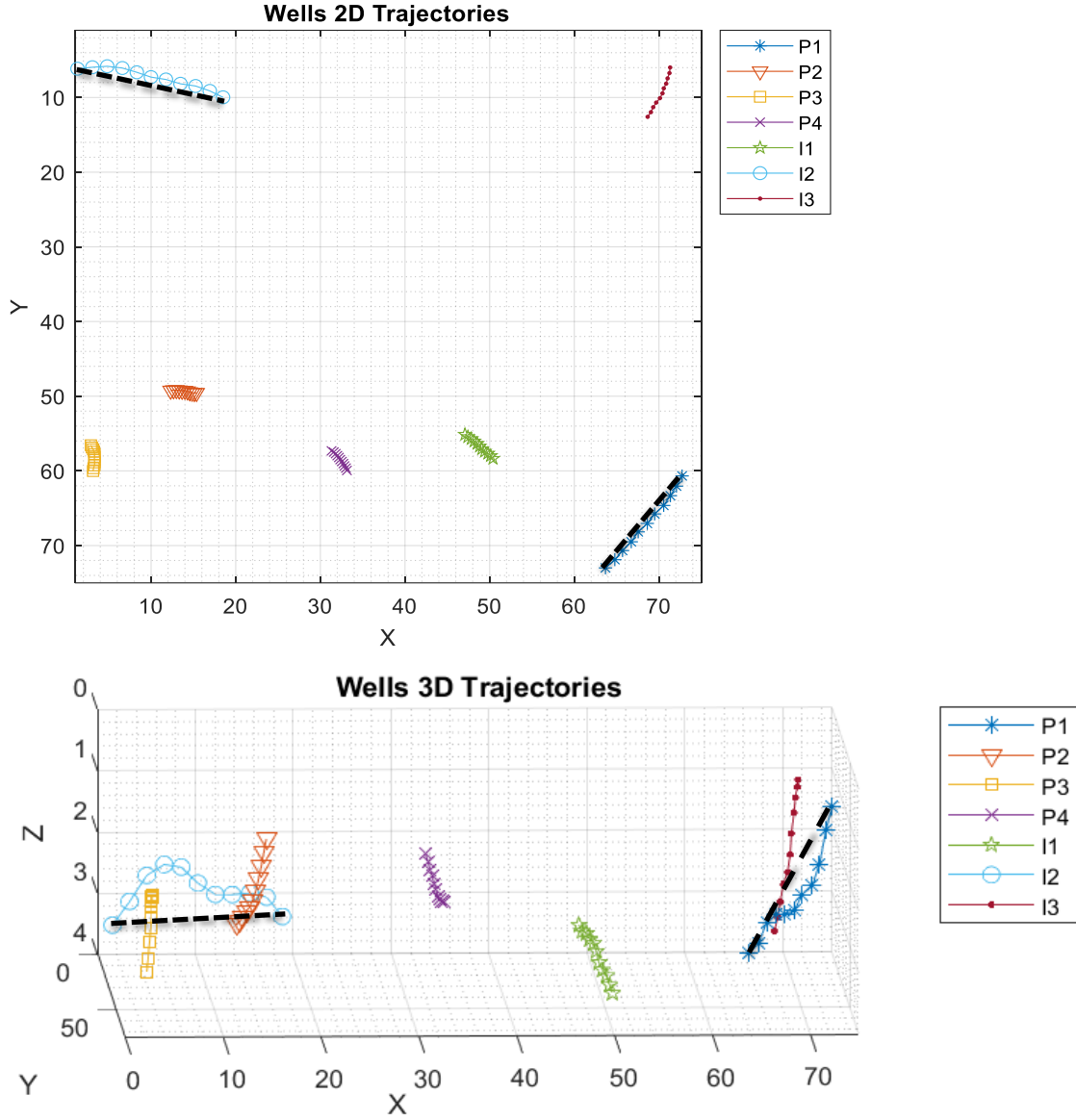


Figure 18: 2D and 3D views of the wells' configuration illustrating the straight-line path created connecting the initial point to the end point of the wells

To further investigate, five additional realizations were performed with a lower number of segments (5 segments) and another five with higher segments number (15 segments). This is to examine the effect of changing the number of decision parameters associated with the different number of segments in our problem. **Table 10** sums up the results obtained from the additional scenarios of different segments number and compares them with the original

scenario of 10 segments. It can be inferred from the table that the 10-segment scenario showed a slight enhancement in terms of NPV when looking at all realizations in general. However, it is observed that the highest NPV case of 5-segment scenario outperformed its counterpart using 10 segments in providing efficiently high NPV with lower %RF. In addition, selecting 15 segments resulted in less NPV overall, thus, reflecting the difficulty encountering the optimization algorithm dealing with more decision parameters.

Table 10: A comparison between three different number of segments scenarios based on their highest NPV, median NPV, and lowest NPV

Method	Realization	5 segments		10 segments		15 segments	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	7.10×10^9	22.55	7.11×10^9	24.49	6.96×10^9	26.08
	Median NPV	6.93×10^9	23.81	6.97×10^9	25.51	6.92×10^9	26.25
	Lowest NPV	6.44×10^9	29.00	6.74×10^9	27.53	5.95×10^9	25.94

A sensitivity analysis to understand the influence of minimum well spacing selection on the optimization results was performed. Two additional cases were investigated: 300 ft and 900 ft. **Table 11** provides the results the of the three cases: 300 ft, 900 ft, 1500 ft (original scenario). The results suggest that lower minimum well spacing distance has contributed in more consistent results, as observed from the smaller differences between highest and lowest NPV realizations among the three scenarios. When looking at SLP approach solely,

a general trend of increasing NPV is observed with less minimum well spacing distance selected. This could be attributed to lower values allowing the optimization algorithm more liberty in designing the wells locations and paths. However, it needs to be considered that the value selection should follow industry standards.

Table 11: A comparison between three different minimum well spacing distance scenarios according to their highest NPV, median NPV, and lowest NPV

Method	Realization	$r_{\min} = 300$ ft		$r_{\min} = 900$ ft		$r_{\min} = 1500$ ft	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	7.08×10^9	22.89	6.95×10^9	25.78	7.11×10^9	24.49
	Median NPV	7.07×10^9	23.27	6.92×10^9	26.89	6.97×10^9	25.51
	Lowest NPV	6.93×10^9	24.92	6.85×10^9	29.23	6.74×10^9	27.53
SLP	Highest NPV	7.07×10^9	23.13	7.03×10^9	23.34	6.89×10^9	28.03
	Median NPV	7.00×10^9	24.25	6.95×10^9	26.26	6.82×10^9	27.65
	Lowest NPV	6.67×10^9	28.57	6.74×10^9	28.89	5.71×10^9	27.57

Furthermore, a scenario that differs in the NPV calculation from the original example was performed. The notion in this case is to increase the expenses by 20% and reduce the revenues by 20%. This is expected to hinder both approaches, GP and SLP, with more negative effect expected on the GP approach, since its ability to design more complex well

paths will require higher expenses. The purpose here is to challenge our proposed method and examine if it still provides comparable results to the SLP method. **Equation 5.1** shows the modified cashflow relation accounting for the applied changes. **Table 12** provides the results obtained from this case and compares it to the original case. The output still show more promising results from the GP method. However, the difference between the two appraoched (GP and SLP) reduced for the modified scenario. This is expected given the modified scenario is in favor of the SLP method.

$$CF_n = 0.8R_n - 1.2E_n \quad (5.1)$$

Table 12: A comparison between the original and the modified scenarios based on the highest NPV, median NPV, and lowest NPV

Method	Realization	Original Scenario		Modified Scenario	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	7.11×10^9	24.49	4.55×10^9	22.62
	Median NPV	6.97×10^9	25.51	4.32×10^9	27.70
	Lowest NPV	6.74×10^9	27.53	4.21×10^9	26.66
SLP	Highest NPV	6.89×10^9	28.03	4.54×10^9	22.90
	Median NPV	6.82×10^9	27.65	4.25×10^9	23.89
	Lowest NPV	5.71×10^9	27.57	2.54×10^9	26.89

5.2 Example 2 – Reservoir with Distributed Permeability Field

In this example, a synthetic reservoir with a distributed permeability is studied. The reservoir is discretized into $64 \times 64 \times 3$ and each grid is of $150 \text{ ft} \times 150 \text{ ft} \times 150 \text{ ft}$ in the x , y , and z dimensions. The rock and rock-fluid properties are presented in **Figure 19**. The reservoir's three layers and log-permeability distribution maps are shown in **Figure 20**. The locations of seven wells, their trajectory, and their lengths are to be optimized for maximum NPV given a set of constraints. The wells are split up into four producers and three injectors, and the optimization runs from the beginning of the field production life and for a total production period of 10 years. Moreover, the lower and upper limits of the decision parameters used are shown in **Table 13**.

To ensure logic and engineering sense is followed through the optimization process, a number of constraints were imposed on the seven wells. For the producers, a liquid rate of 10,000 bpd is to be maintained given that the bottomhole pressure of the producer shall not fall below the given minimum, which was set at a value of 2,500 psi. To clarify, the producer will switch from aiming to target 10,000 bpd once it cannot maintain a bottomhole pressure of 2,500 psi. Oppositely, the injectors are set to inject water at a given pressure of 4,500 psi to assure pressure maintenance throughout the reservoir. Similar to the previous example, a minimum spacing constraint is imposed on the system. A radius of 1,500 feet surrounds each segment and if any other well's segment lies within this distance, the amount of length that violates this constraint is recorded and incorporated in the calculation of the objective function as a penalty. Once **Equations 4.12 and 4.14** were applied for the current example case, the resultant number of nonlinear constraints were 97,685, with

97,650 originating from the minimum spacing constraints and 35 from the reservoir boundaries constraints.

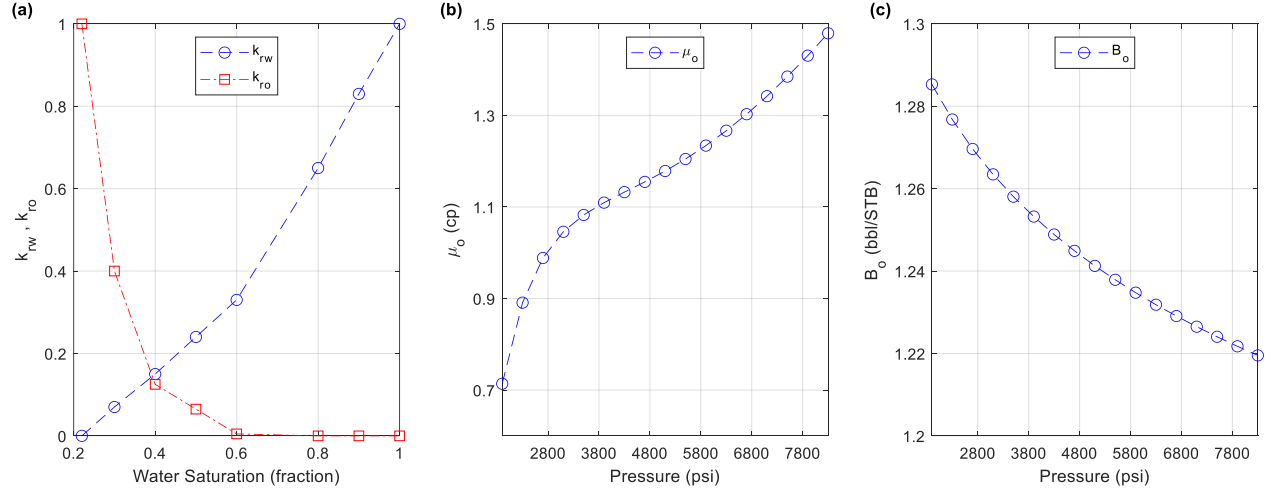


Figure 19: Fluid & Fluid-Rock Properties of Example 2. (a) oil/water relative permeability curves, (b) oil viscosity, and (c) oil formation volume factor

Table 13: Lower and upper limits of the decision parameters utilized.

Parameter	Lower limit	Upper limit
i	1	64
j	1	64
k	1	3
θ_{azi}	0°	360°
θ_{inc}	0°	95°
$\Delta\theta_{azi}$	-8°	8°
$\Delta\theta_{inc}$	-8°	8°

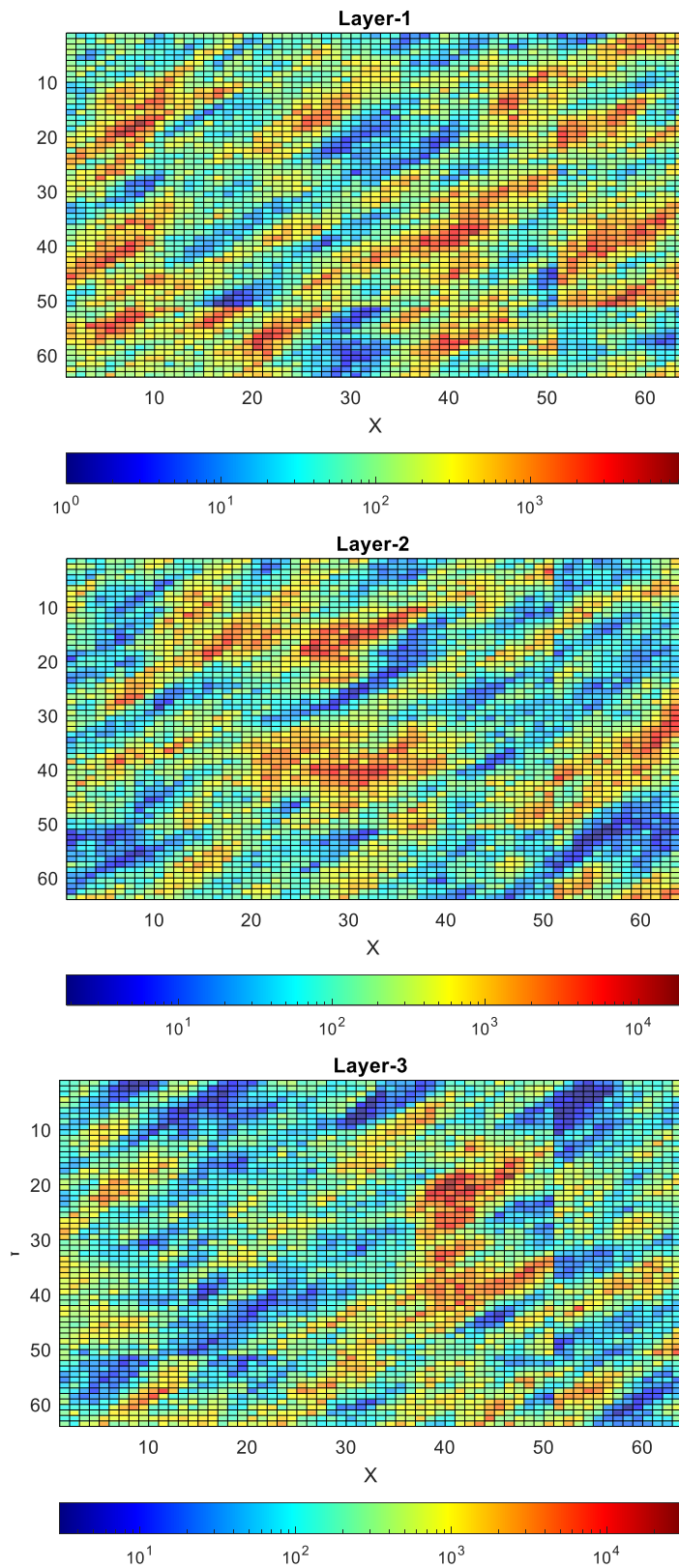


Figure 20: permeability distribution maps for case example 2

A comparison was performed with different scenarios including only straight-line wells to validate the proposed approach in this work. Five realizations are evaluated and ranked highest to lowest in terms of the resultant NPV. The best, median, and worst realizations were then plotted for the two approaches to compare, as can be shown in **Figure 21**. The figure reflects the net present value calculated at each function evaluation. Moreover, it can be inferred from these plots that the best realization obtained utilizing the GP method exceeds its counterpart achieved using SLP method by an approximate amount of 122 million dollars. The same applies for the median and lowest realizations as they show improved values when using the GP approach compared with SLP approach. Once again, a fluctuation is observed in the NPV plot resulting from the fact that the objective function is not $-NPV$, instead it is another normalized term accounting for NPV and penalty terms. A summary of the results obtained from the two methods and their three cases are shown in **Table 14**. Though that we observe small values of H_c , yet we can say that in this example, almost all the realization succeeded to avoid violating the imposed constraints as these small values are not of major significance, except for one case, the lowest scenario of the SLP method. This case has failed to honor this constraint and exhibited more than 300 ft of well section placed outside the boundaries of the reservoir, consequently, this has been reflected on the obtained relatively low NPV output. **Figures 23 – 24** show the optimal wells' locations from highest NPV case of each approach, respectively.

Table 14: Results summary of the two methods and their highest NPV, median NPV, and lowest NPV

Method	Realization	NPV (\$)	RF (%)	G_c (ft)	H_c (ft)
GP	Highest NPV	4.07×10^9	17.12	0	0.49
	Median NPV	3.92×10^9	16.81	0	0.14
	Lowest NPV	3.68×10^9	17.12	0	0.91
SLP	Highest NPV	3.94×10^9	17.13	0	0.45
	Median NPV	3.89×10^9	16.61	0	1.16
	Lowest NPV	3.54×10^9	15.50	0	309.9

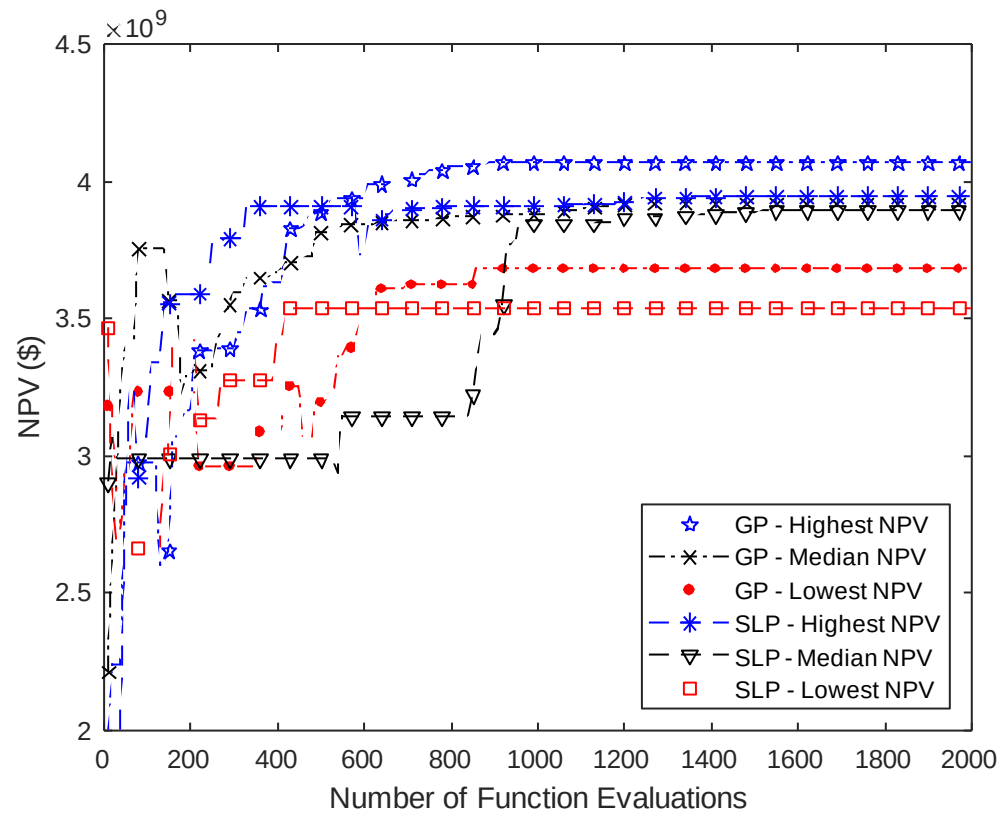


Figure 21: NPV vs Number of Function Evaluations for Distributed Perm. Reservoir case

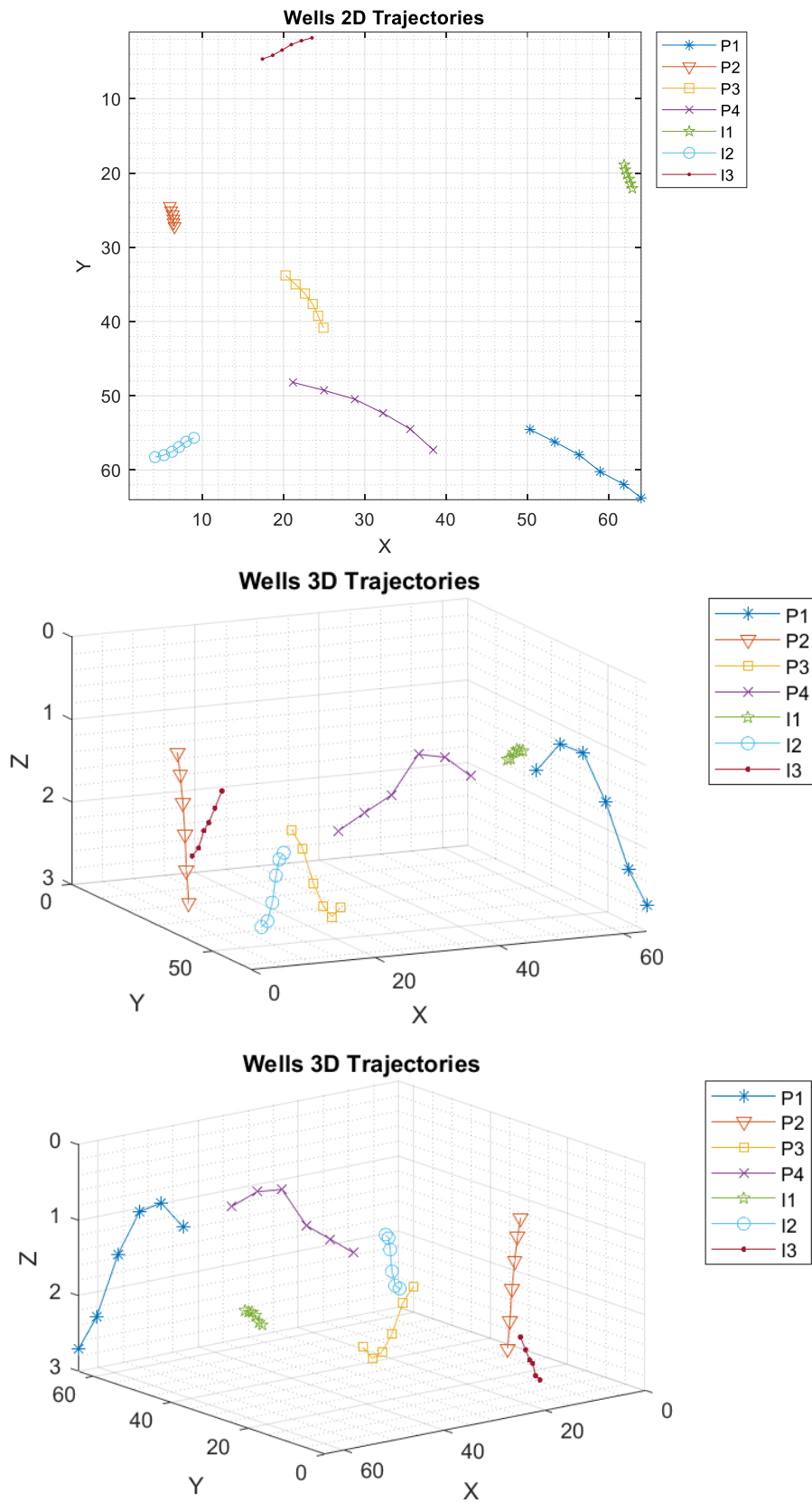


Figure 22: Well Placement from top 2D view, and two 3D views (Generalized Path “GP”)

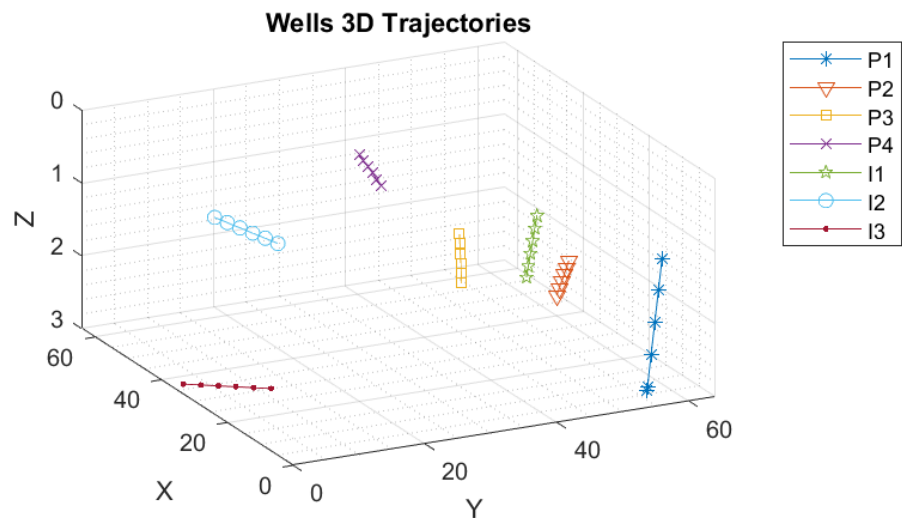
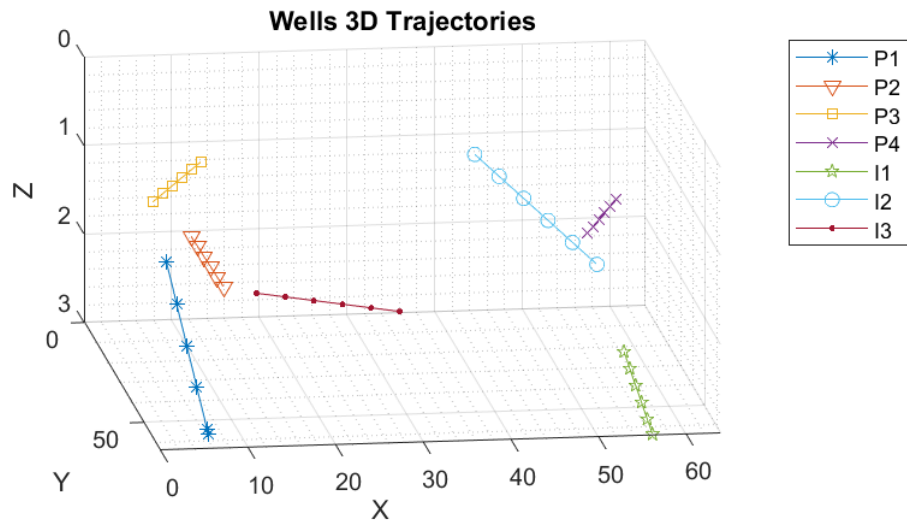
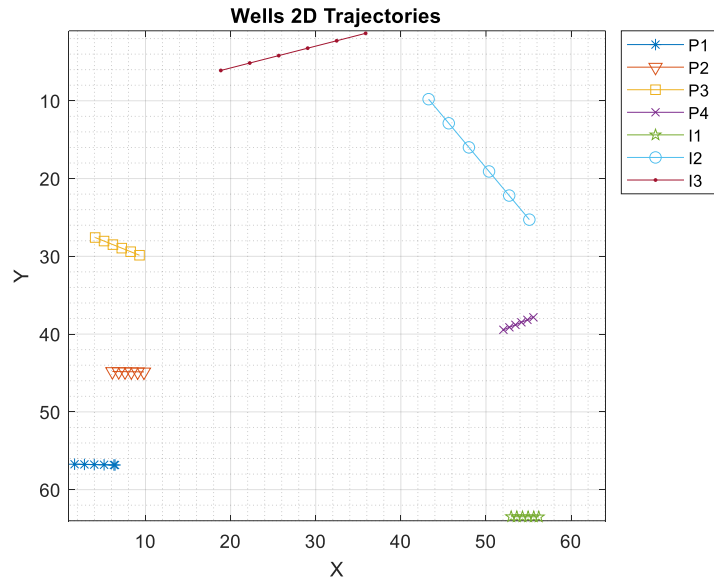


Figure 23: Well Placement from top 2D view, and two 3D views (Straight-Line Path “SLP”)

Similar to Case Example 1, SLP scenario was created from the obtained GP results. The straight line is connecting each well's initial point with its final point. A comparison was then performed with the original GP scenario as shown in **Table 15**. The results attained from this scenario shows better NPV for the original scenario as compared to the SLP-created-from-GP scenario. However, when comparing this change with the previous case example (Example 1 – Channel Reservoir), it is not as relevant. A possible explanation to this is that in this example, lower number of segments were used in the well design. Thus, resultant well paths in this example could be closer to SLP than Example 1, where 10 segments were used).

Table 15: A comparison between the original and alternative scenarios based on the highest NPV, median NPV, and lowest NPV

Method	Realization	Original Scenario		Alternative Scenario (SLP created from GP)	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	4.07×10^9	17.12	3.87×10^9	16.09
	Median NPV	3.92×10^9	16.81	3.86×10^9	16.73
	Lowest NPV	3.68×10^9	17.12	3.41×10^9	16.47

For the sensitivity analysis of different number of segments in this case example, only one lower case was performed. This is due to the original scenario being five segments, and the simulation model being prone to convergence issues and extreme simulation times when selecting number of segments of more than five. Thus, the analysis comprised the original case of five segments and a lower number which is three segments. **Table 16** compares the two values according to their NPV. The results obtained exhibited minor differences between the two cases. However, both cases outperformed the SLP approach in terms of NPV results.

Table 16: A comparison between two different number of segments scenarios based on the highest NPV, median NPV, and lowest NPV

Method	Realization	3 segments		5 segments	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	4.03×10^9	17.10	4.07×10^9	17.12
	Median NPV	3.96×10^9	17.14	3.92×10^9	16.81
	Lowest NPV	3.68×10^9	15.83	3.68×10^9	17.12

5.3 Example 3 – Brugge Field

Brugge is a synthetic field developed as a benchmark to evaluate closed-loop workflows of waterflooding optimization in combination with history matching. The field's design includes two elongated half domes in its east and west flanks. In addition, a major fault is present in its northern edge, and an internal fault of a slight deviation of about 20° to the northern fault (Peters et al. 2009). The reservoir in this field is discretized into $81 \times 58 \times 20$ with grid dimensions of $75m \times 75m \times 2.5m$. The properties of its rock and fluid are shown in **Figure 24**. The reservoir layers and their respective porosity maps are depicted in **Figure 25**. In addition, the main PVT data are presented in **Table 17**. The algorithm is programmed to find the optimum locations of seven wells (four producers and three injectors), the changes in azimuth and inclination of each segment, and the total length of each well. The optimization process starts at the initial time of field life and for a production period of 10 years. The employed lower and upper limits of the decision parameters are shown in **Table 18**.

Table 17: PVT data of the Brugge Field (Peters et al. 2010).

Fluid	Water	Oil
Density (lb./ft ³)	62.6	56
Compressibility (psi ⁻¹)	3e-6	9.26e-6
Viscosity (cp)	0.32	1.294

Various constraints are enforced on the system to avoid illogical and impractical outcomes. A liquid rate of 15,000 bpd is required for each producer to maintain unless the bottomhole pressure of that producer falls below 2,500 psi. Alternatively, the producer will maintain a

pressure above 2,500 psi regardless of liquid rate. The injectors are set to inject unfixed amount of water with the only condition being to satisfy a bottomhole pressure of 5,000 psi. These constraints are set to allow production of hydrocarbons while preserving reservoir's pressure at the same time. Furthermore, a minimum spacing constraint is enforced, in which a value of 1,500 feet surrounds each well segment in a spherical manner. In case any segment of another well enters this sphere, the amount of length that falls within this volume will be cut from the well path design and recorded, and so the objective function will be penalized by such amount. Since this case example is similar to the previous example where there are seven wells with five segments each, then the number of constraints is the same (97,685).

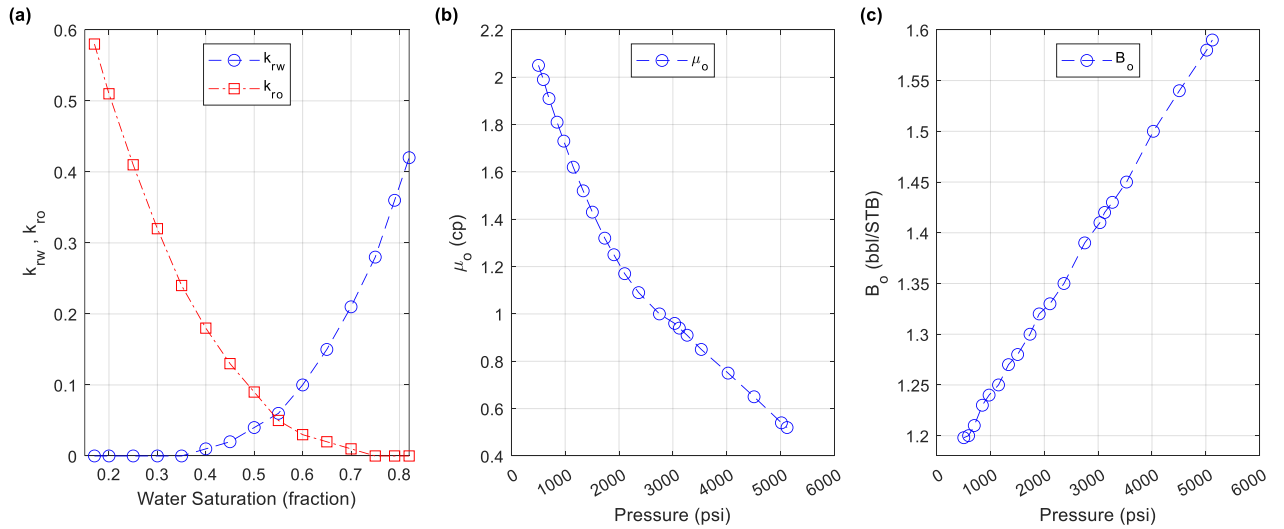
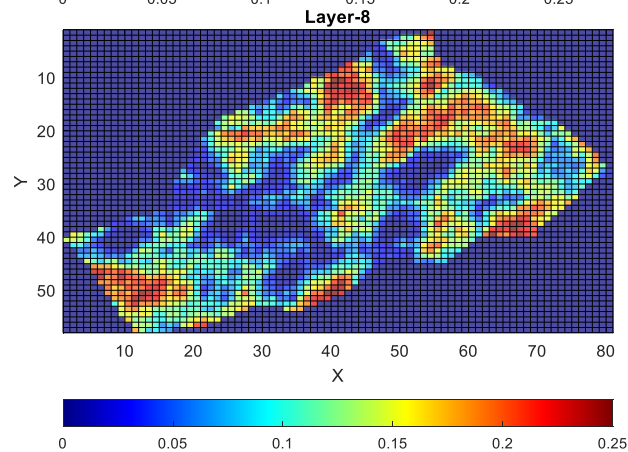
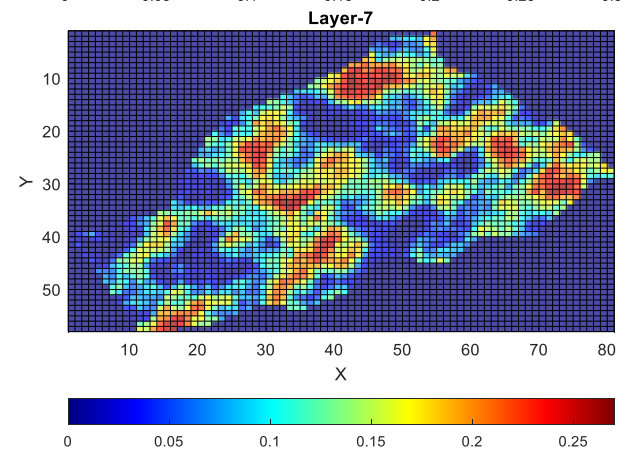
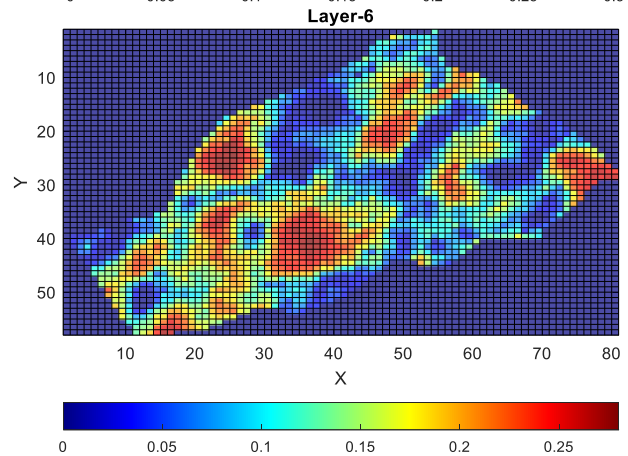
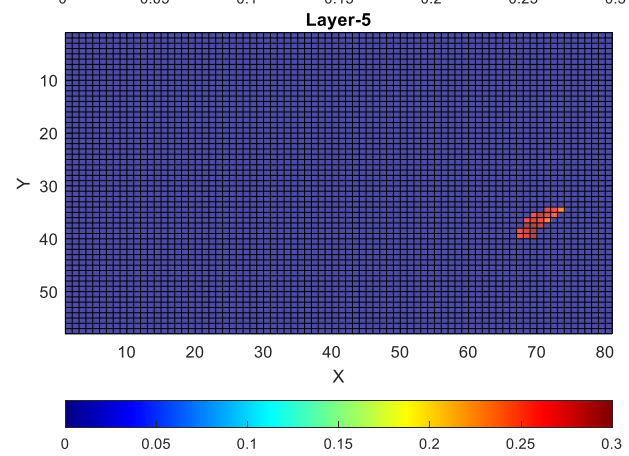
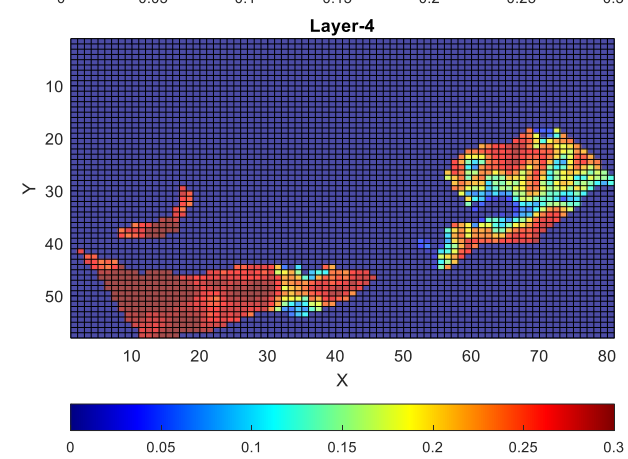
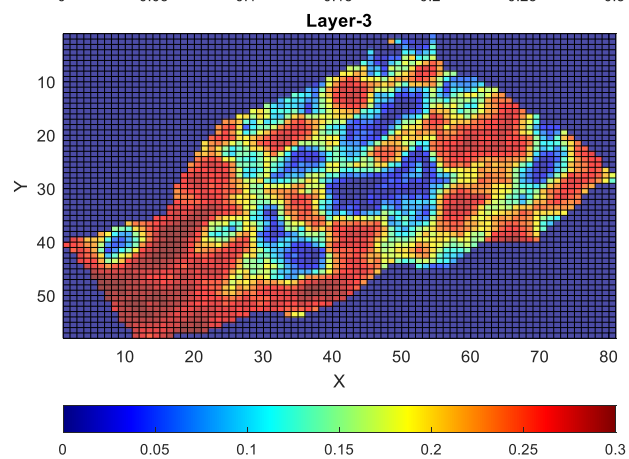
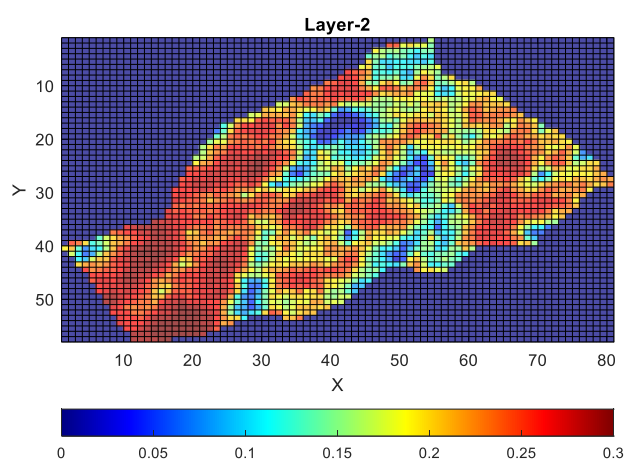
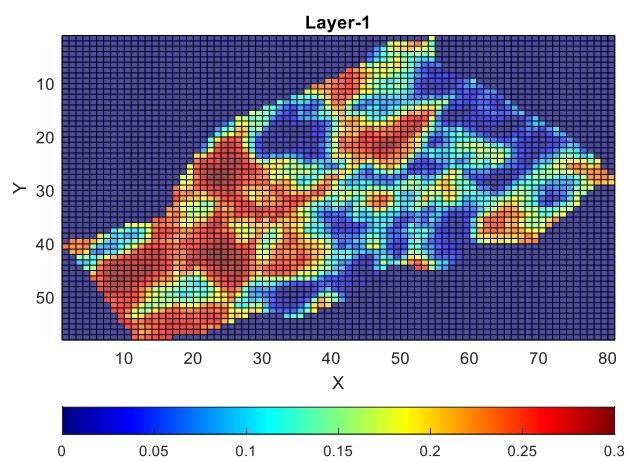
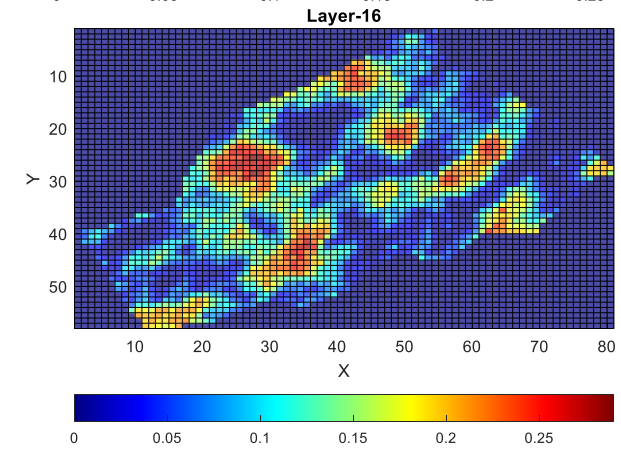
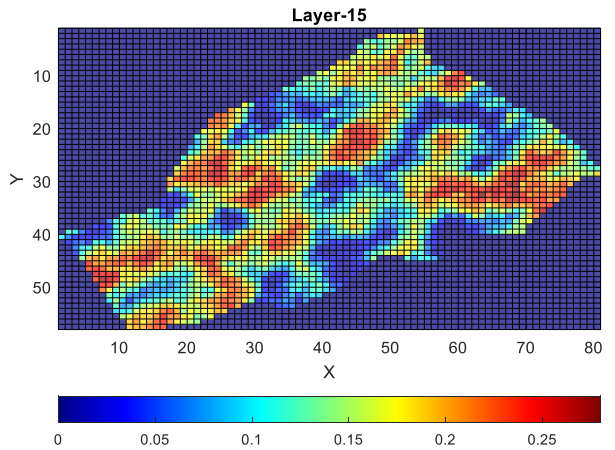
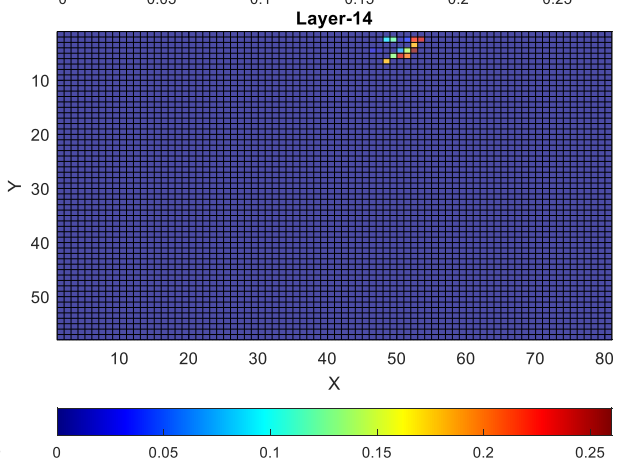
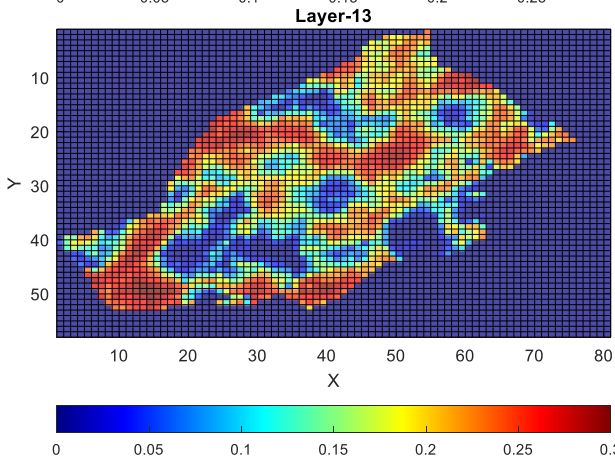
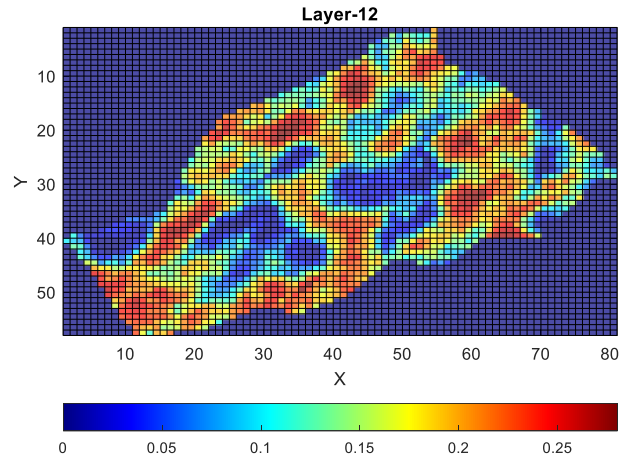
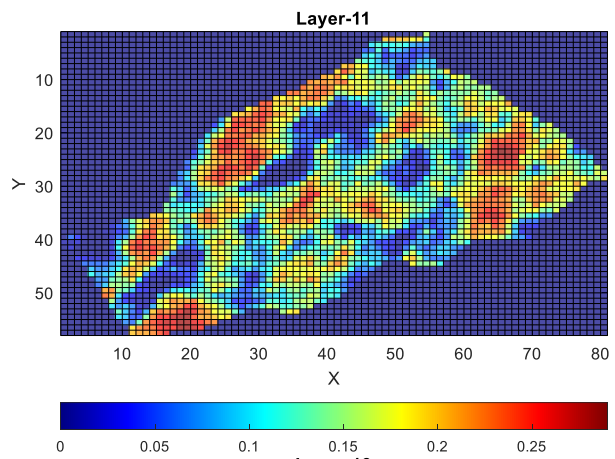
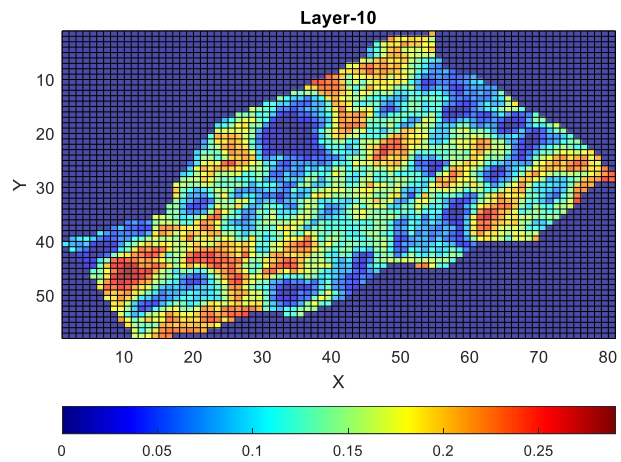
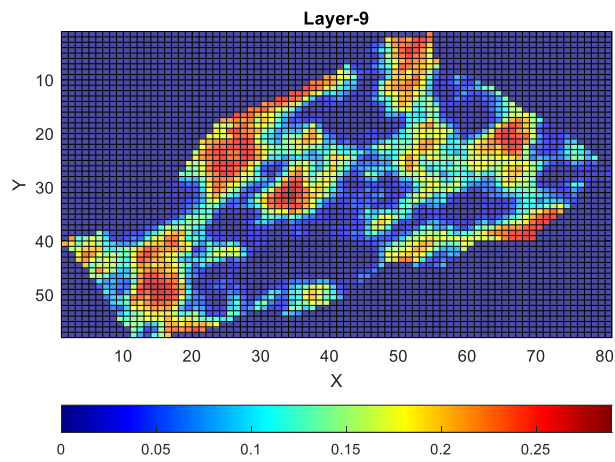
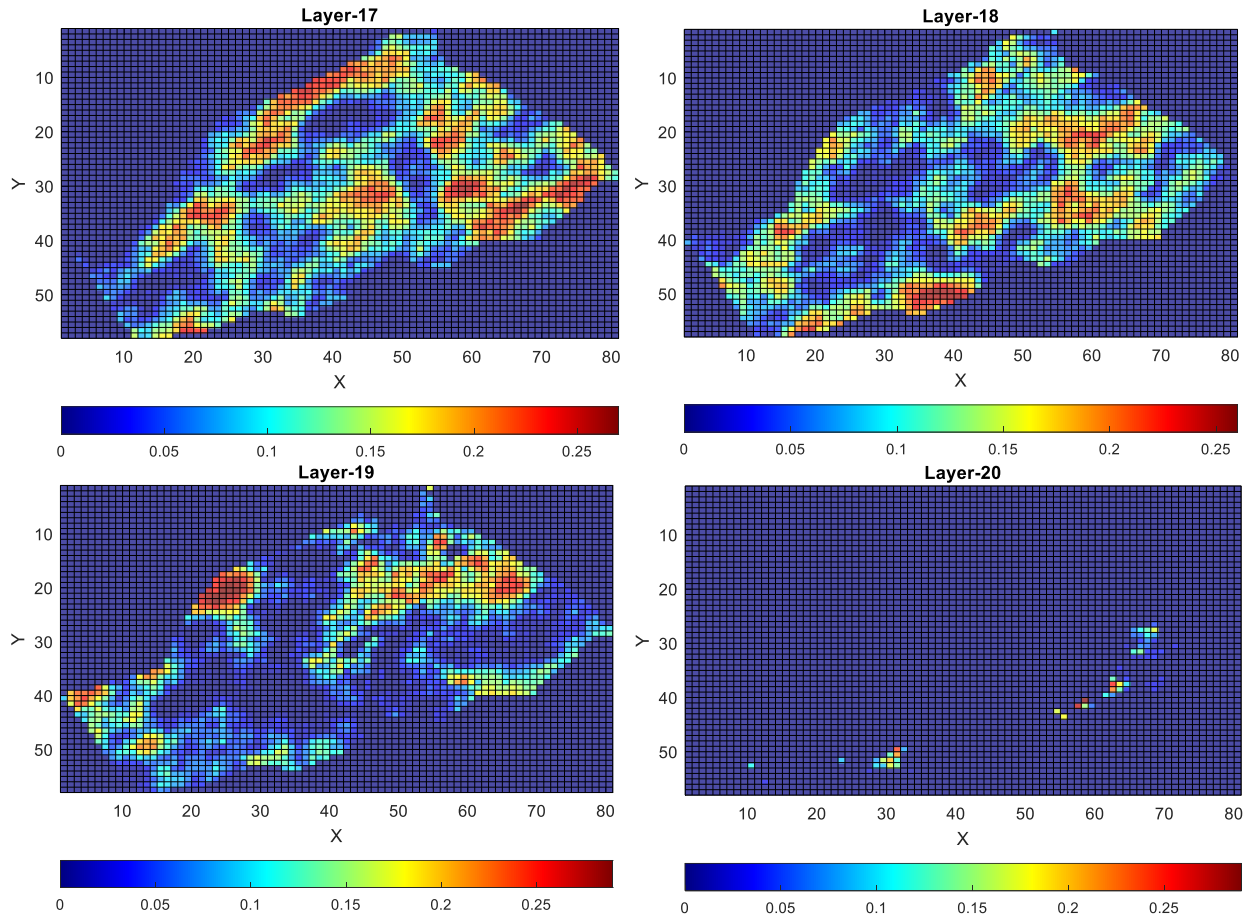


Figure 24: Fluid & Fluid-Rock Properties of Example 3. (a) oil/water relative permeability curves, (b) oil viscosity, and (c) oil formation volume factor







[Figure 25: Porosity distribution maps for case example 3]

Table 18: Lower and upper limits of the decision parameters utilized.

Parameter	Lower limit	Upper limit
i	1	81
j	1	58
k	1	20
θ_{azi}	0°	360°
θ_{inc}	0°	95°
$\Delta\theta_{azi}$	-8°	8°
$\Delta\theta_{inc}$	-8°	8°

In this case example, the proposed method (GP) is compared with SLP method. This is in order to assess the proposed method in such examples. Further, five different realizations have been performed and their results are sorted from highest to lowest, in which then the highest, the median, and the lowest are plotted in **Figure 26**. The figure shows the net present value behavior with more generations produced. Additionally, it illustrates relatively better high-NPV scenario performance using the GP method over its counterpart (SLP method), with an additional 166 million dollars. Further, the lowest NPV obtained using GP method is almost similar in value to the median of the SLP method. As a result, this adds credibility to the proposed method since it reflects that in most cases, the proposed method would output better results than the SLP method. **Table 19** provides a summary of the results of the three cases of both methods. It is worth mentioning that in this case study, the reservoir is very large, at least much larger than the two cases discussed previously in this paper. Hence, it is less prone than the previous case examples to violate such

constraints given the same number of wells and boundary limits of well length selected. Yet, one scenario has still failed to respect the minimum spacing and placement within boundaries constraints, which is the scenario with the lowest NPV obtained using the SLP approach. The final configuration of the wells and their respective paths for all the scenarios and for both methods are illustrated in **Figures 27 and 28**.

Table 19: Results summary of the two methods and their highest NPV, median NPV, and lowest NPV

Method	Realization	NPV (\$)	RF (%)	G_c (ft)	H_c (ft)
GP	Highest NPV	6.88×10^9	21.58	0	0.87
	Median NPV	6.01×10^9	20.38	0	1.42
	Lowest NPV	5.46×10^9	20.83	0	2.58
SLP	Highest NPV	6.71×10^9	21.05	0	0
	Median NPV	5.45×10^9	18.08	0	2.95
	Lowest NPV	4.90×10^9	21.91	457.2	83.5

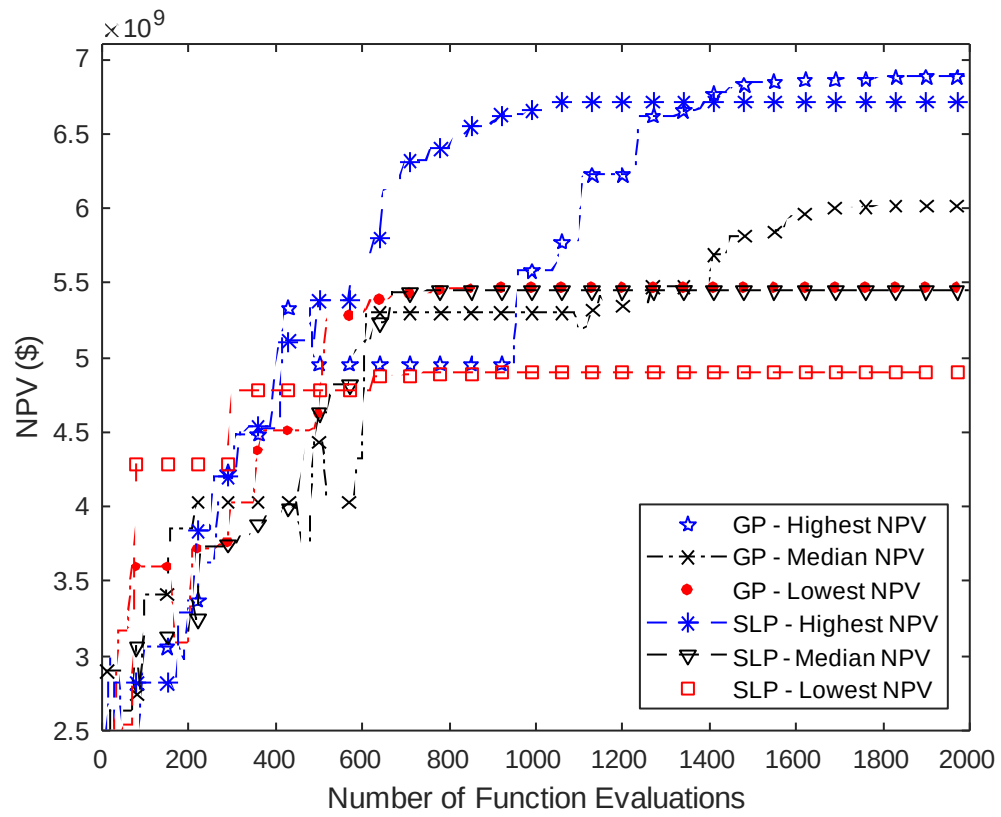


Figure 26: NPV vs Number of Function Evaluations for Brugge case

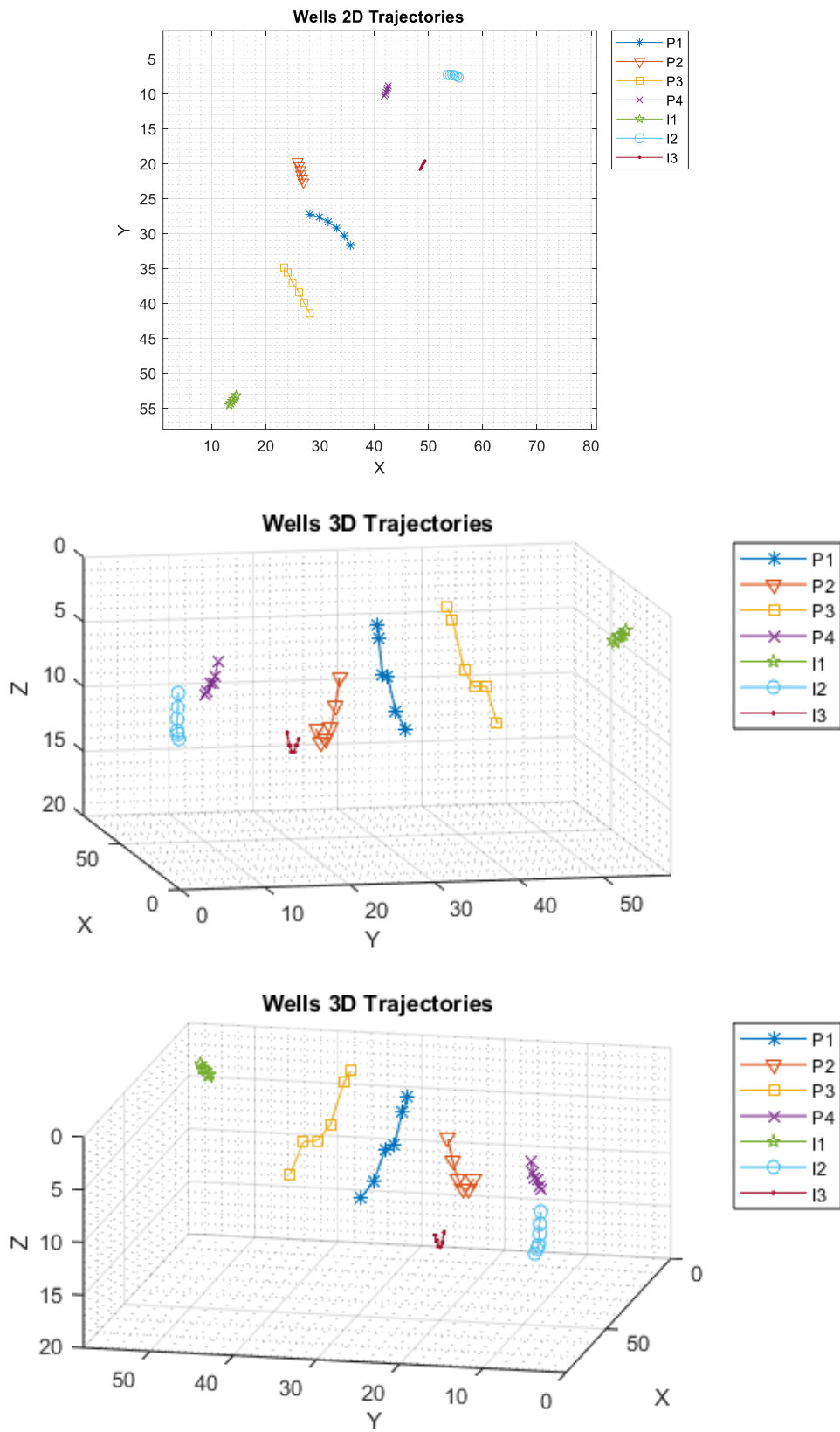


Figure 27: Well Placement from top 2D view, and two 3D views (Generalized Path “GP”)

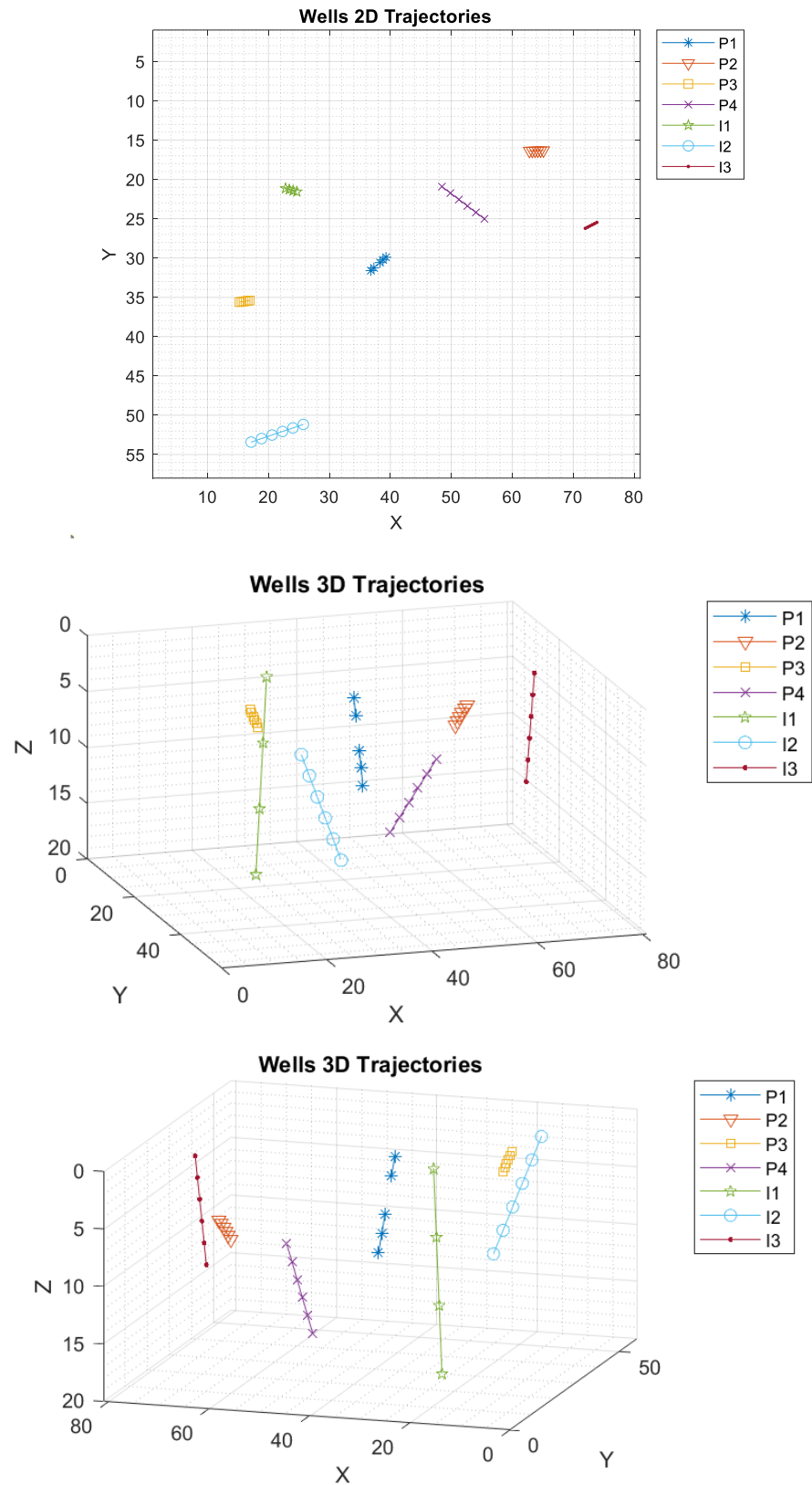


Figure 28: Well Placement from top 2D view, and two 3D views (Straight-Line Path “SLP”)

The results obtained using the GP method were utilized to generate an SLP solution, similar to the two previous reservoir examples. The initial and final points of each well were connected with a straight line. The results obtained from this scenario are presented in comparison with the original scenario in **Table 20**.

Table 20: A comparison between the original and alternative scenarios based on the highest NPV, median NPV, and lowest NPV

Method	Realization	Original Scenario		Alternative Scenario (SLP created from GP)	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	6.88×10^9	21.58	5.46×10^9	21.63
	Median NPV	6.01×10^9	20.38	4.92×10^9	18.92
	Lowest NPV	5.46×10^9	20.83	4.43×10^9	17.24

For further analysis, five additional realizations were performed for a case of wells with 3 segments and another of 7 segments. The reason for not selecting a number of segments higher than that is because the reservoir is very large in size and more segments will result in more extensive simulation time and most probably will encounter convergence issues. The results attained from the two additional scenarios as well as the original case of 5 segments are all documented in **Table 21**. The results showed when comparing the 5-segment scenario to 7-segment scenario, better NPV was observed for the 5-segment case. This could be attributed to the higher number of decision parameters the 7 segments scenario requires the algorithm to optimize. Overall, the scenario of 5 segments showed the most promising results emphasizing

the importance of selecting an appropriate segments number for the well design. An appropriate selection shall not be too low number with less path freedom nor too high resulting in more difficult optimization associated with the additional decision parameters.

Table 21: A comparison between three different number of segments scenarios based on their highest NPV, median NPV, and lowest NPV

Method	Realization	3 segments		5 segments		7 segments	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	5.51×10^9	24.50	6.88×10^9	26.22	6.15×10^9	25.91
	Median NPV	5.32×10^9	24.88	6.01×10^9	24.77	5.98×10^9	23.26
	Lowest NPV	4.33×10^9	24.26	5.46×10^9	25.31	5.21×10^9	24.55

To better comprehend the impact of well spacing, several cases of different minimum well spacings were analyzed. Additional to the original case (1500 ft), two more cases of 300 ft and 900 ft of well spacings were selected. **Table 19** exhibits the results of the three cases side to side for comparison. For both methods, GP and SLP, using 300 ft yielded the best NPV, with 1500 ft almost just as good for the GP approach. Remarkably, using 900 ft as a minimum spacing provided the highest median for both GP and SLP.

Table 22: A comparison between three different minimum well spacing distance scenarios according to their highest NPV, median NPV, and lowest NPV

Method	Realization	$r_{\min} = 300 \text{ ft}$		$r_{\min} = 900 \text{ ft}$		$r_{\min} = 1500 \text{ ft}$	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	6.94×10^9	26.68	6.43×10^9	26.60	6.88×10^9	21.58
	Median NPV	6.08×10^9	24.40	6.14×10^9	23.36	6.01×10^9	20.38
	Lowest NPV	5.39×10^9	26.51	4.94×10^9	26.15	5.46×10^9	20.83
SLP	Highest NPV	6.88×10^9	25.86	6.30×10^9	25.28	6.71×10^9	21.05
	Median NPV	5.77×10^9	25.58	6.02×10^9	25.11	5.45×10^9	18.08
	Lowest NPV	4.78×10^9	23.38	4.79×10^9	25.36	4.90×10^9	21.91

Similar to Example 1, a scenario that reduces the revenues and increases the expenses by 20% each (**Equation 5.1**) was performed and compared with the original case. **Table 20** compares the results achieved from this scenario with the original scenario. Unlike the original scenario, the modified scenario showed unfavourable output from GP method when compared to SLP results.

Table 23: A comparison between the original and the modified scenarios based the highest NPV, median NPV, and lowest NPV

Method	Realization	Original Scenario		Modified Scenario	
		NPV (\$)	RF (%)	NPV (\$)	RF (%)
GP	Highest NPV	6.88×10^9	21.58	4.80×10^9	25.02
	Median NPV	6.01×10^9	20.38	4.33×10^9	26.19
	Lowest NPV	5.46×10^9	20.83	4.10×10^9	24.49
SLP	Highest NPV	6.71×10^9	21.05	4.93×10^9	25.68
	Median NPV	5.45×10^9	18.08	4.45×10^9	24.43
	Lowest NPV	4.90×10^9	21.91	4.34×10^9	24.39

CONCLUSIONS & RECOMMENDATIONS

A new approach to optimize the placement of wells and their respective path was proposed. The novelty in this work was in the well representation, which in this project was to divide the well into multiple parts, or segments, to allow for more freedom in the path design. However, this does not mean that the path could not be of a straight-line. Instead, this approach added the possibility of infinite path designs, including vertical, horizontal, irregular, or any other design should the algorithm find that path to be optimum. Local grid refinement was utilized to accurately place the segments' coordinate points within the global grids and avoid any uncertainty in translating the planned path into the simulator.

A stochastic algorithm, namely DE, was employed with a single objective function comprising a negative NPV term and two additional penalty terms. The three terms were normalized with scaling factors, e.g., the median NPV of the first generation was utilized to scale down the NPV term. In addition, the penalization terms were moderated according to their significance with a scaling factor. This factor was selected to be monotonically increasing as more generations were produced, which has helped the algorithm to provide optimum results while avoiding violating the imposed constraints as best as possible.

The work has studied three case examples in which in each a general comparison with straight-line path wells were performed with a performance comparison of NPV behavior with more function evaluations. In addition, alternative comparison scenarios were also

analyzed for various purposes. The significance of the number of segments was investigated through a comparison between different realizations of low, medium, and high number of segments. Analysis of the results indicated the need for appropriate selection of number of segments. A high number of segments may lead to extensive simulation time or convergence issues, while a low number may not result in sufficient freedom of well path design.

The recommendations outlined from this work for future research are as follow:

- Applying upscaling techniques to grids where no wells are placed.
- Integrating more detailed calculations of the NPV for complex-path wells.
- Coupling the optimization with model reduction techniques.
- Coupling the optimization with techniques that help and guide the algorithm in the searching process.

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Appendix A – Alternative Well Representations

Prospect A:

The suggested decision parameters are: $(L_{well}, x, y, z, dx_1, dz_1, dx_2, dz_2, \dots)$. These parameters are sufficient to produce the remaining $(dy_1, dy_2, \dots)$ given that the segment length (L_{seg}) is known using $L_{seg} = \frac{L_{well}}{N_{seg}}$, in which the number of segments is set by the

user. The suggested lower limits in this representation would be:

$(L_{well,min}, i_{grid,min}, j_{grid,min}, z_{grid,min}, -L_{seg}, -L_{seg}, -L_{seg}, -L_{seg}, \dots)$; in which $L_{well,min}$ is the minimum well length selected by the user, $i_{grid,min}$, $j_{grid,min}$, and $z_{grid,min}$ are the minimum acceptable locations of (i, j, k) which basically would be (1,1,1), and L_{seg} is calculated using $L_{seg} = \frac{L_{well}}{N_{seg}}$.

The suggested upper limits in this representation would be:

$(L_{well,max}, i_{grid,max}, j_{grid,max}, z_{grid,max}, L_{seg}, L_{seg}, L_{seg}, L_{seg}, \dots)$; in which $L_{well,max}$ is the maximum well length selected by the user, $i_{grid,max}$, $j_{grid,max}$, and $z_{grid,max}$ are the maximum acceptable locations of (i, j, k) which basically mean the reservoir boundaries.

However, for different well lengths L_{well} , which could differ from one generation to

another since it is a decision parameter, (L_{seg}) will be different as it follows: $L_{seg} = \frac{L_{well}}{N_{seg}}$.

Thus, we will be unable to assign the lower and upper limits of L_{well} properly before running the algorithm.

Prospect B:

The suggested decision parameters are: $(L_{well}, x, y, z, dx_1, dz_1, dx_2, dz_2, \dots)$. These parameters are sufficient to produce the remaining $(dy_1, dy_2, \dots)$ given that the number of segments (N_{seg}) is known using $N_{seg} = \frac{L_{well}}{L_{seg}}$, in which the segment length is set by the

user. The suggested lower limits in this representation would be:

$(L_{well,min}, i_{grid,min}, j_{grid,min}, z_{grid,min}, -L_{seg}, -L_{seg}, -L_{seg}, -L_{seg}, \dots)$; in which $L_{well,min}$ is the minimum well length selected by the user, $i_{grid,min}$, $j_{grid,min}$, and $z_{grid,min}$ are the minimum acceptable locations of (i, j, k) which basically would be (1,1,1), and L_{seg} is calculated using $L_{seg} = \frac{L_{well}}{N_{seg}}$.

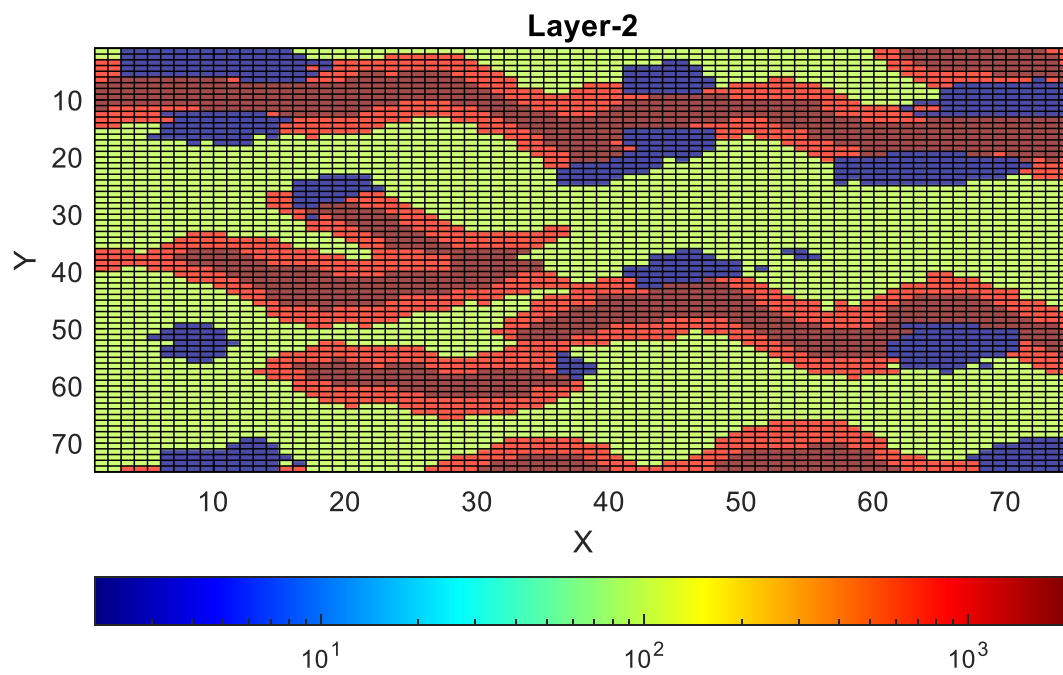
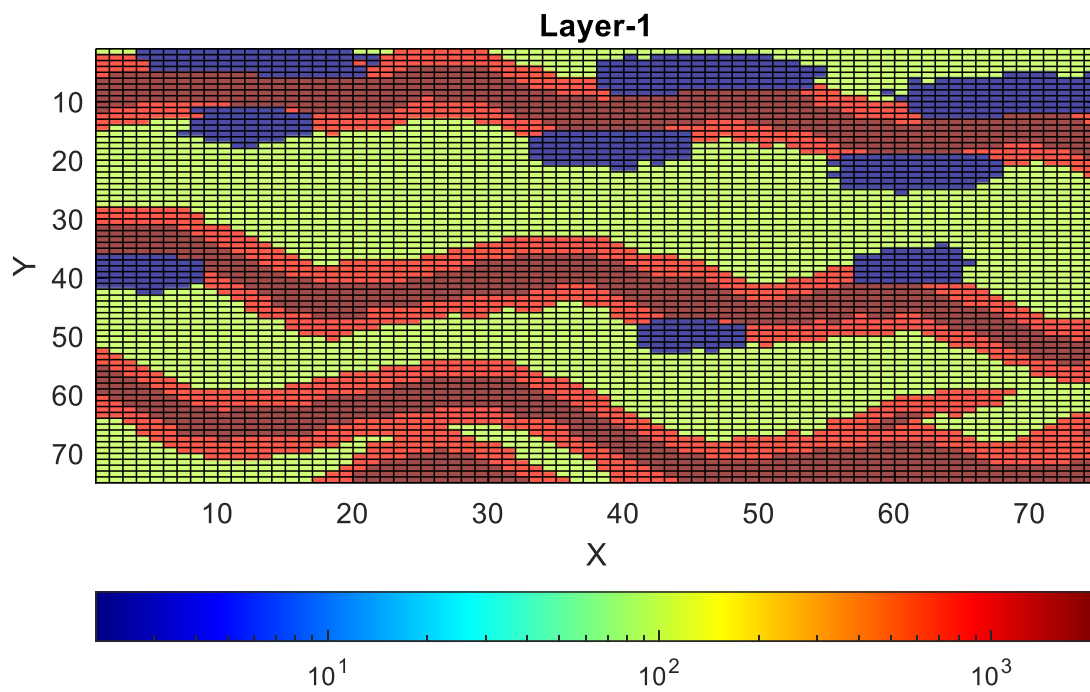
The suggested upper limits in this representation would be:

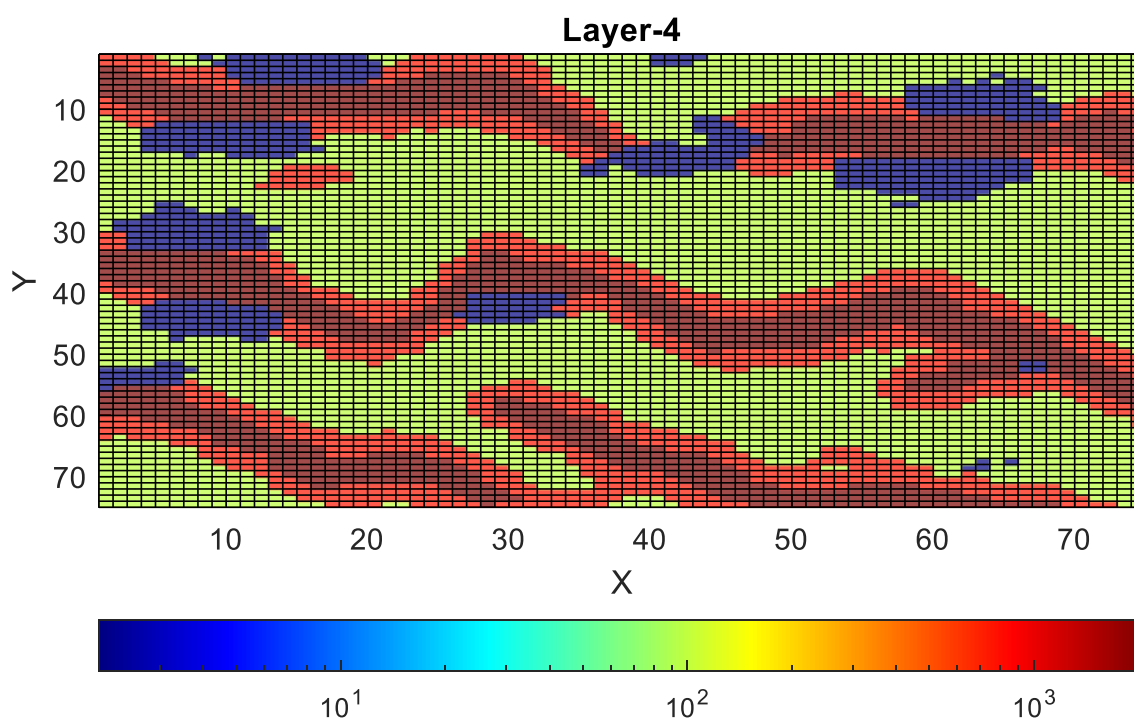
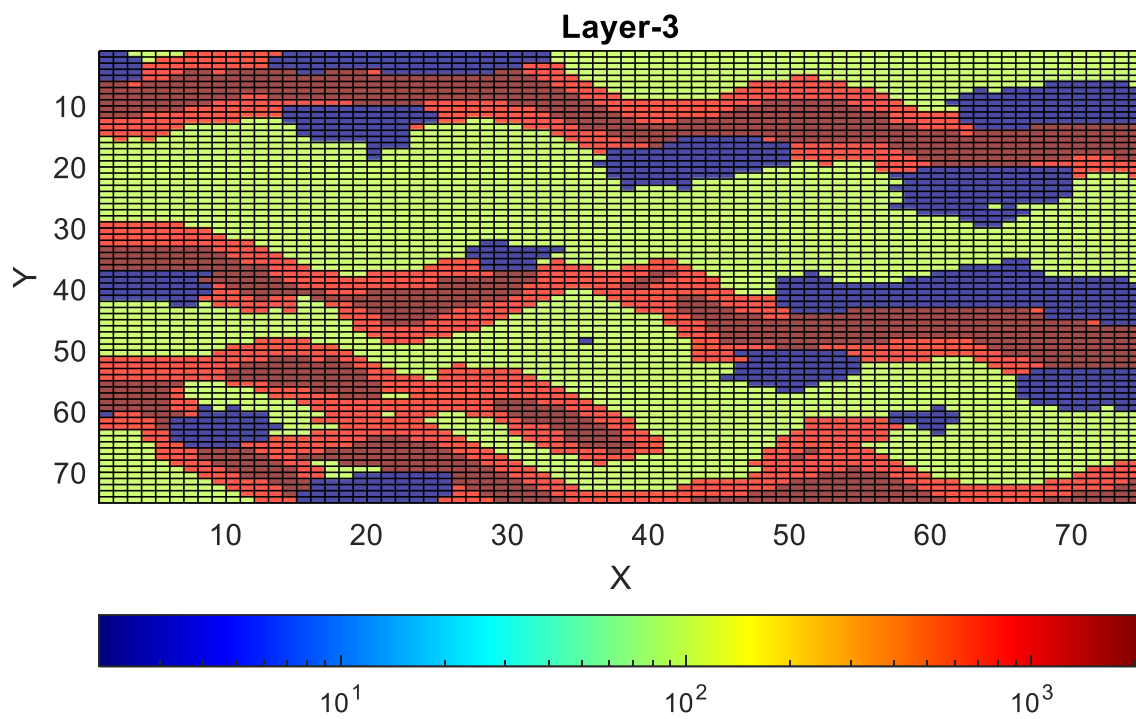
$(L_{well,max}, i_{grid,max}, j_{grid,max}, z_{grid,max}, L_{seg}, L_{seg}, L_{seg}, L_{seg}, \dots)$; in which $L_{well,max}$ is the maximum well length selected by the user, $i_{grid,max}$, $j_{grid,max}$, and $z_{grid,max}$ are the maximum acceptable locations of (i, j, k) which basically mean the reservoir boundaries.

However, for different well lengths L_{well} , which could differ from one generation to another since it is a decision parameter, N_{seg} will be different as it follows: $N_{seg} = \frac{L_{well}}{L_{seg}}$.

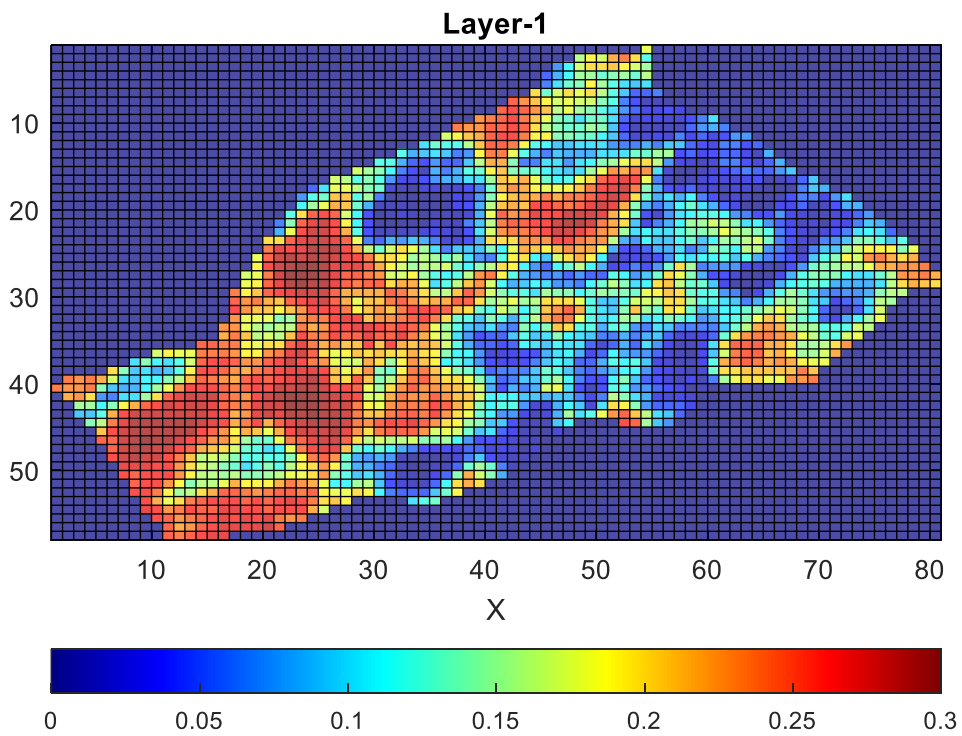
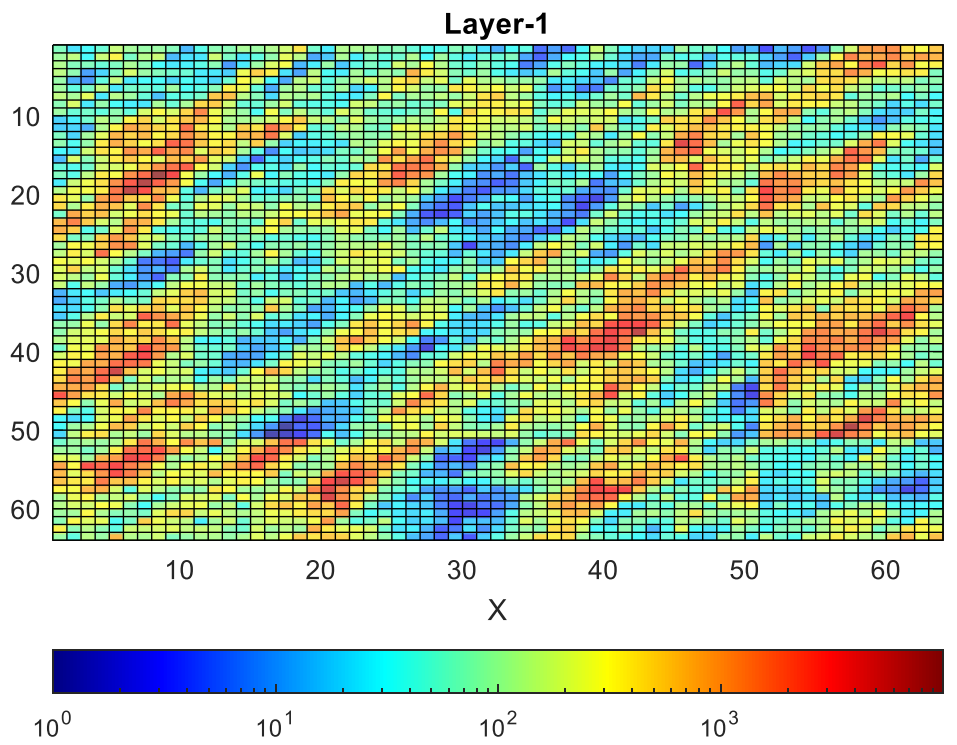
Thus, we will be unable to assign the lower and upper limits of L_{well} properly before running the algorithm.

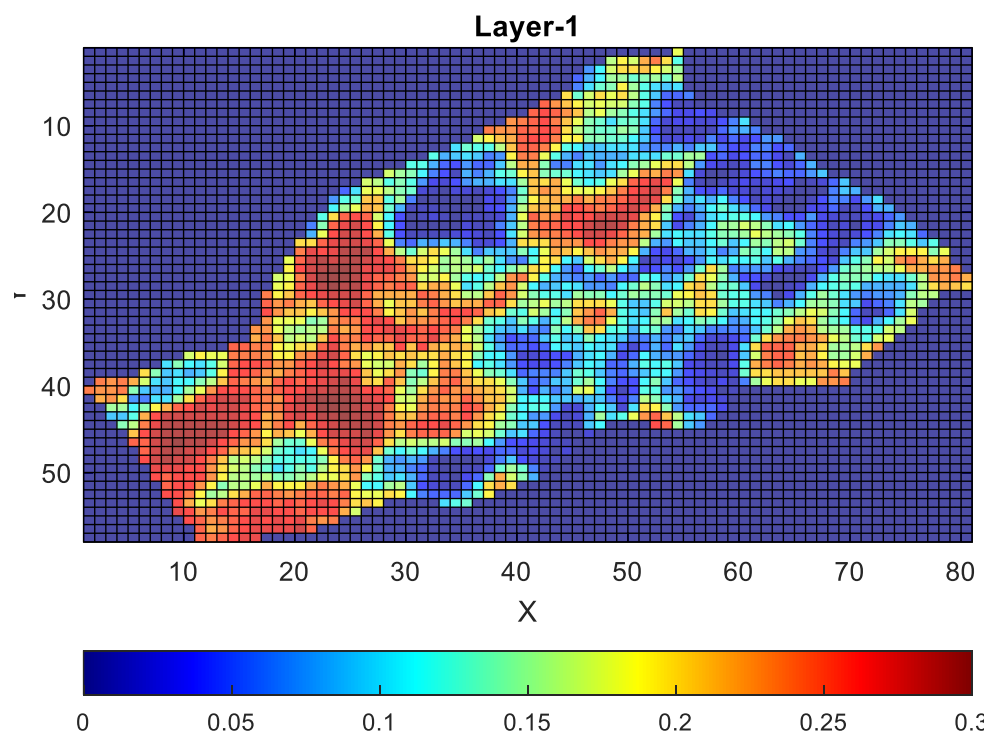
Appendix B – Example 1 Layers



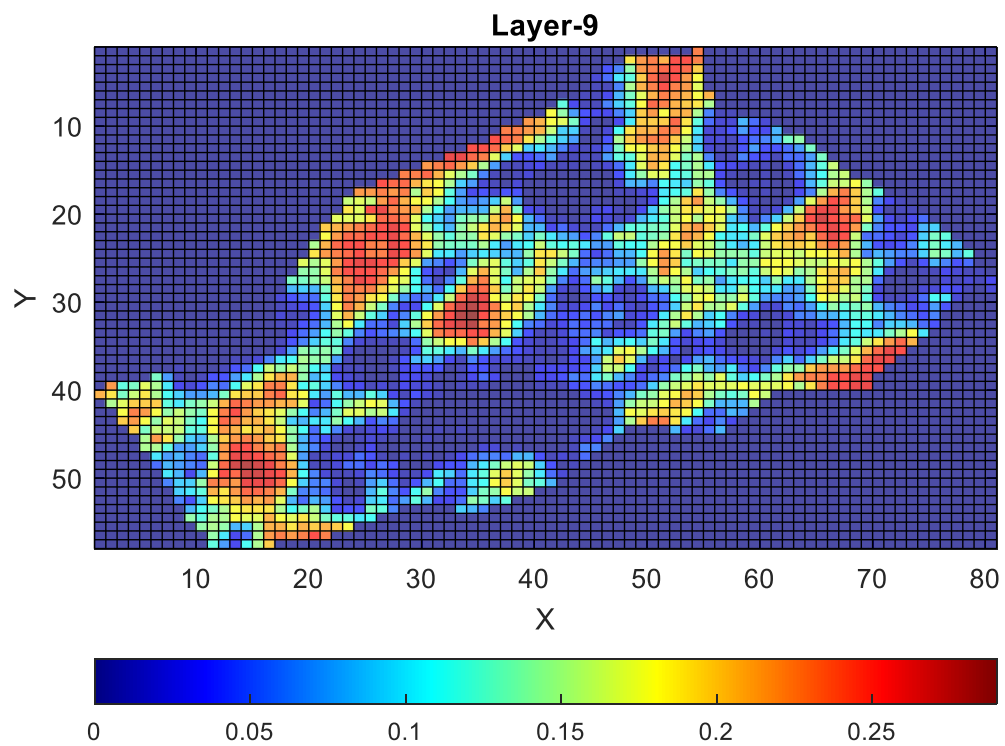
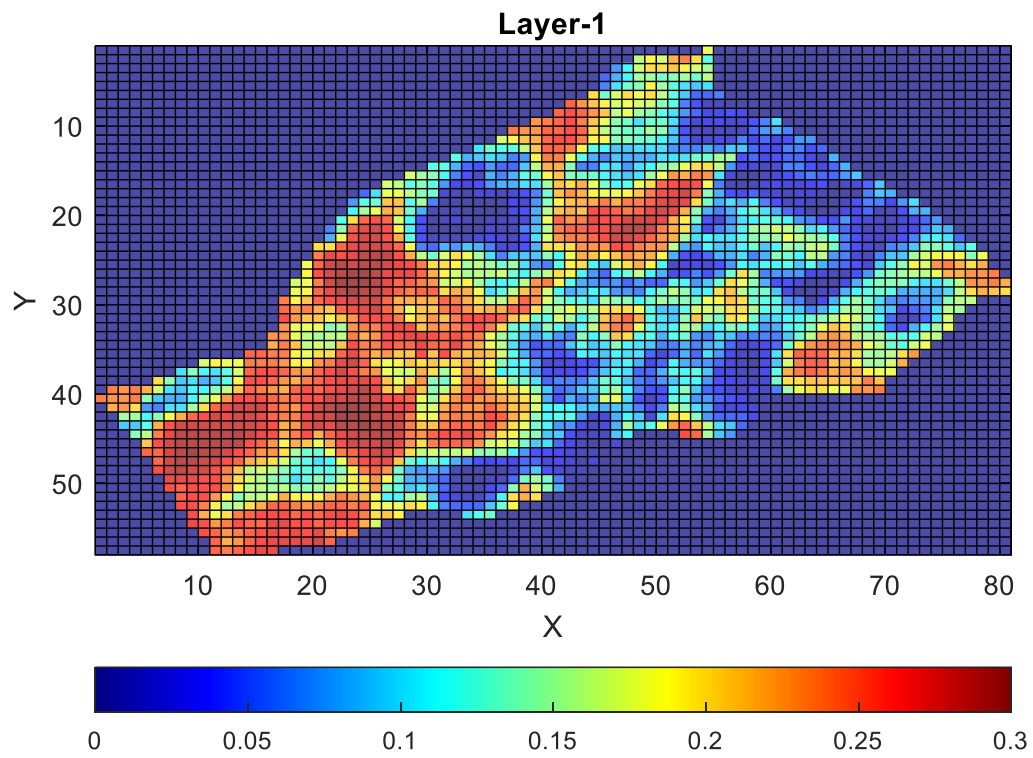


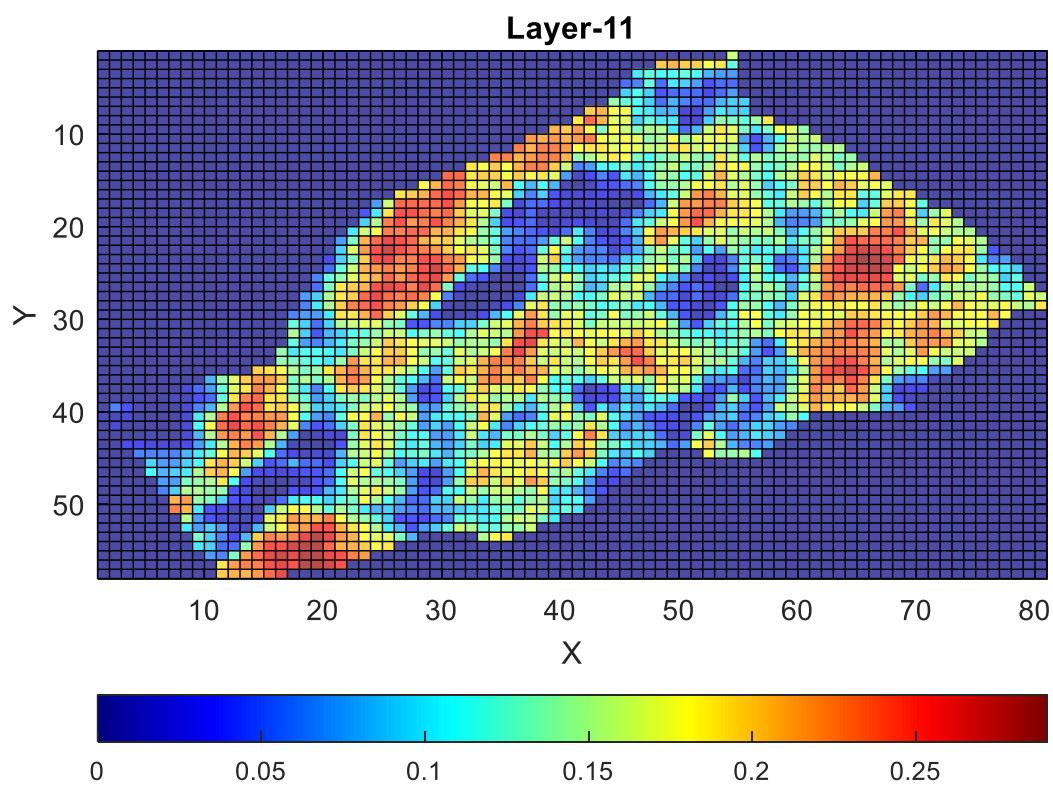
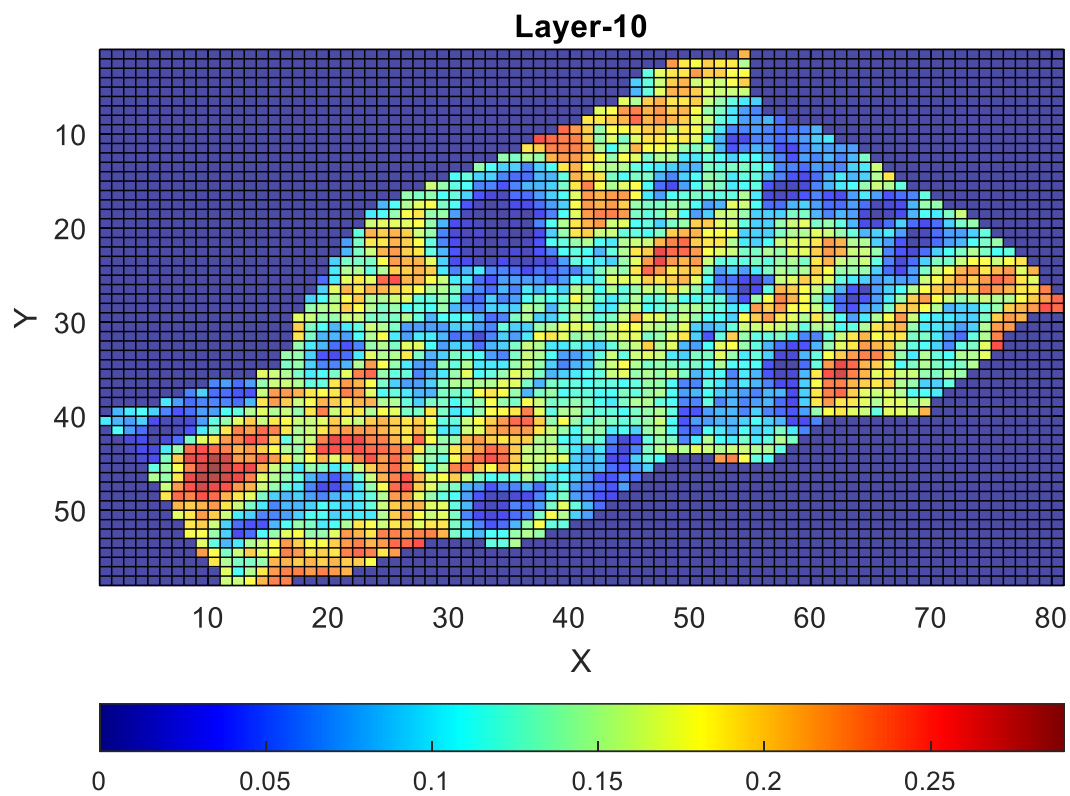
Appendix C – Example 2 Layers

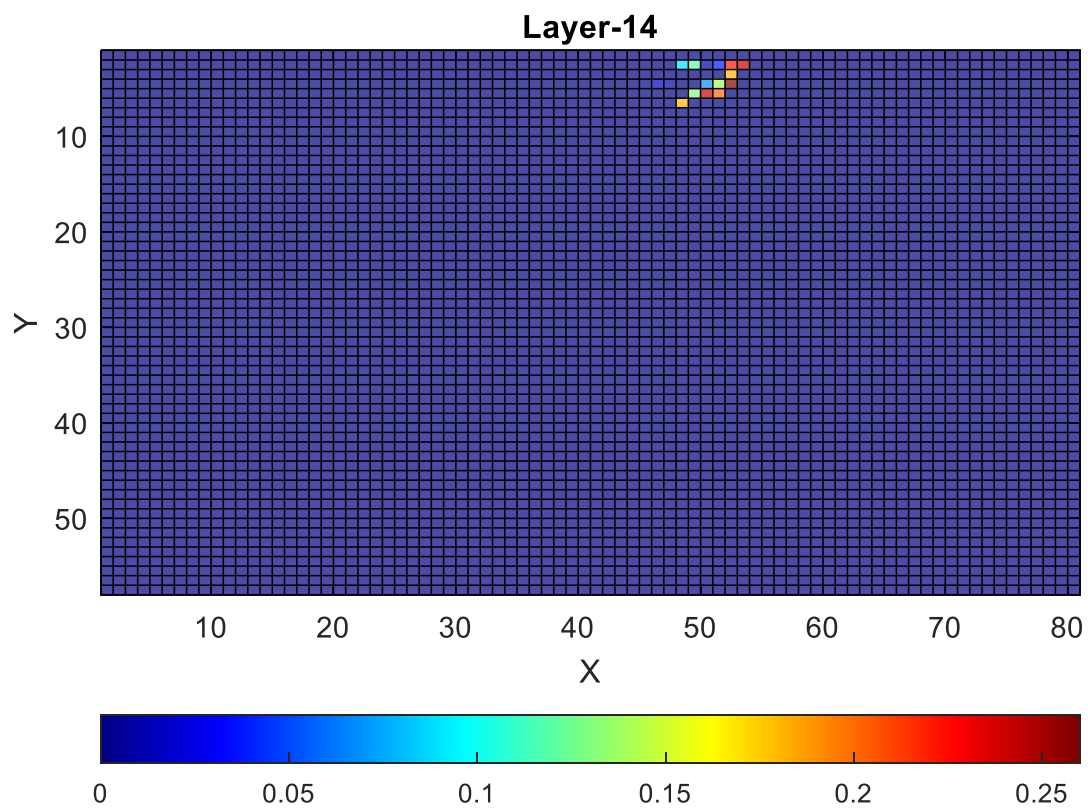
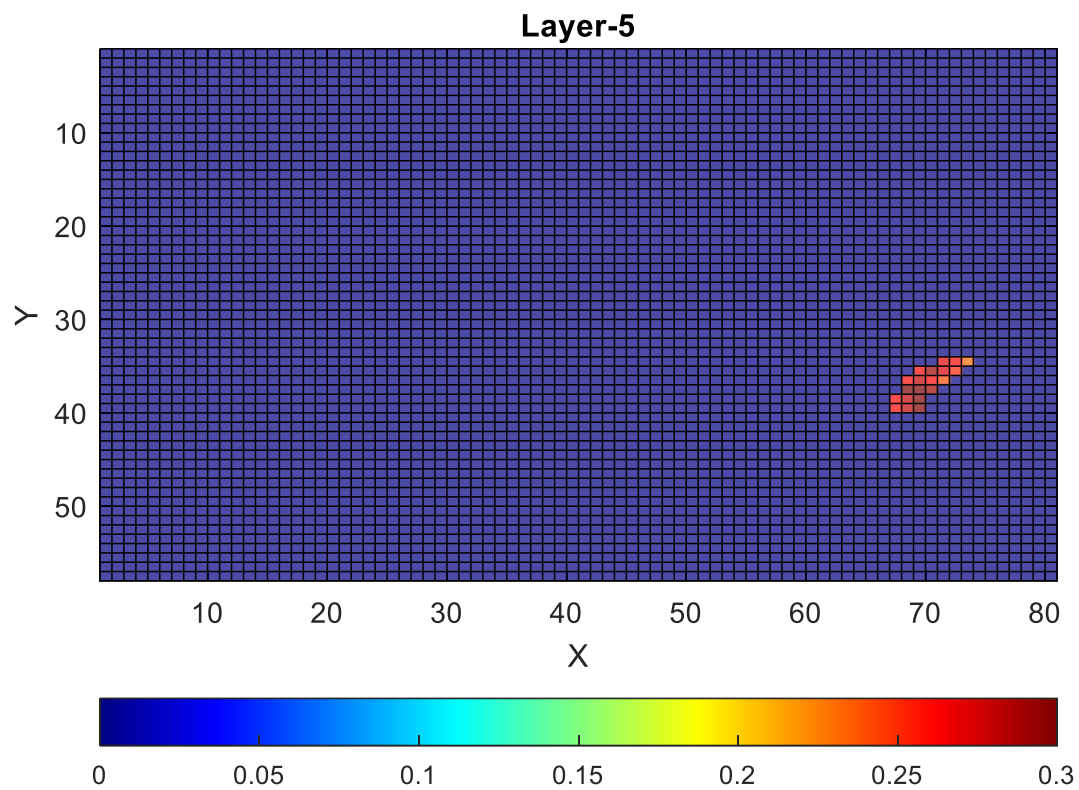


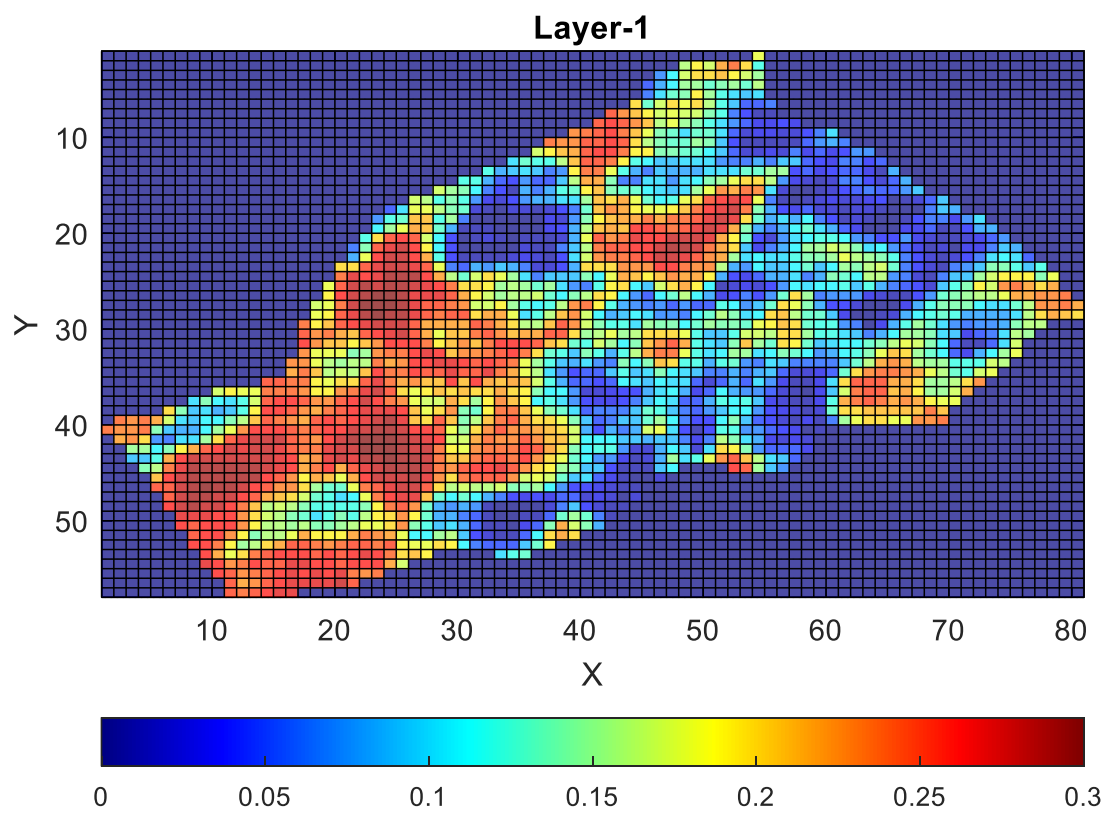
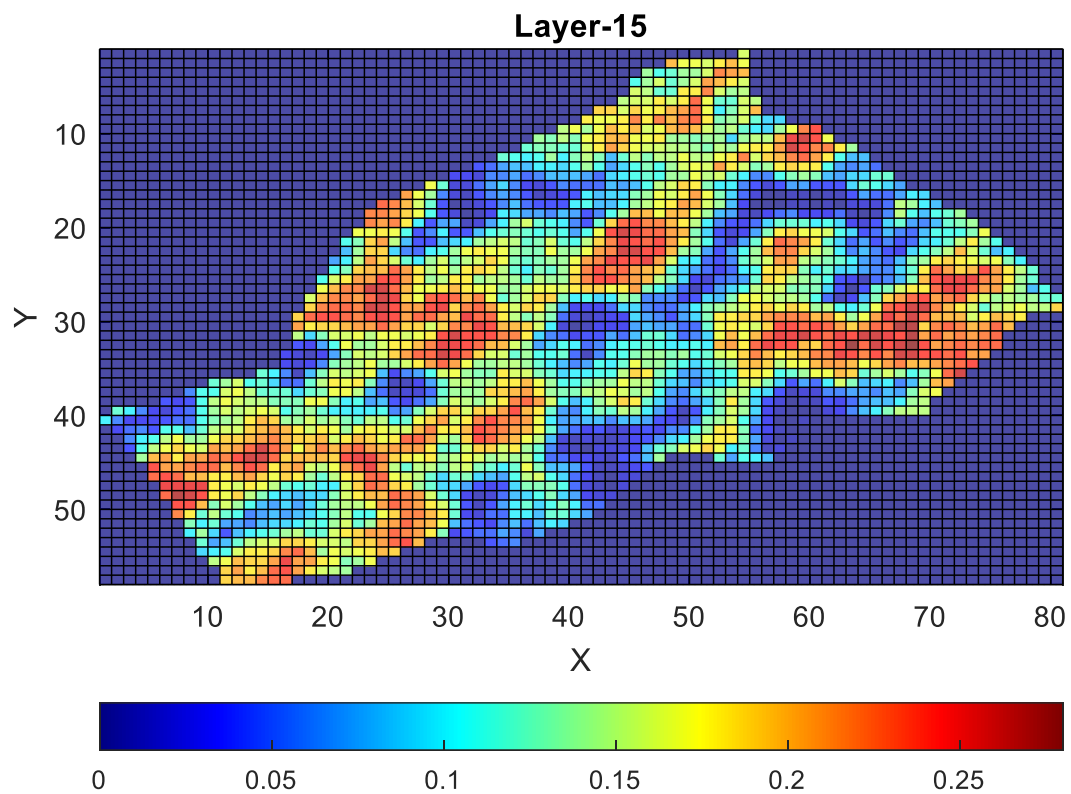


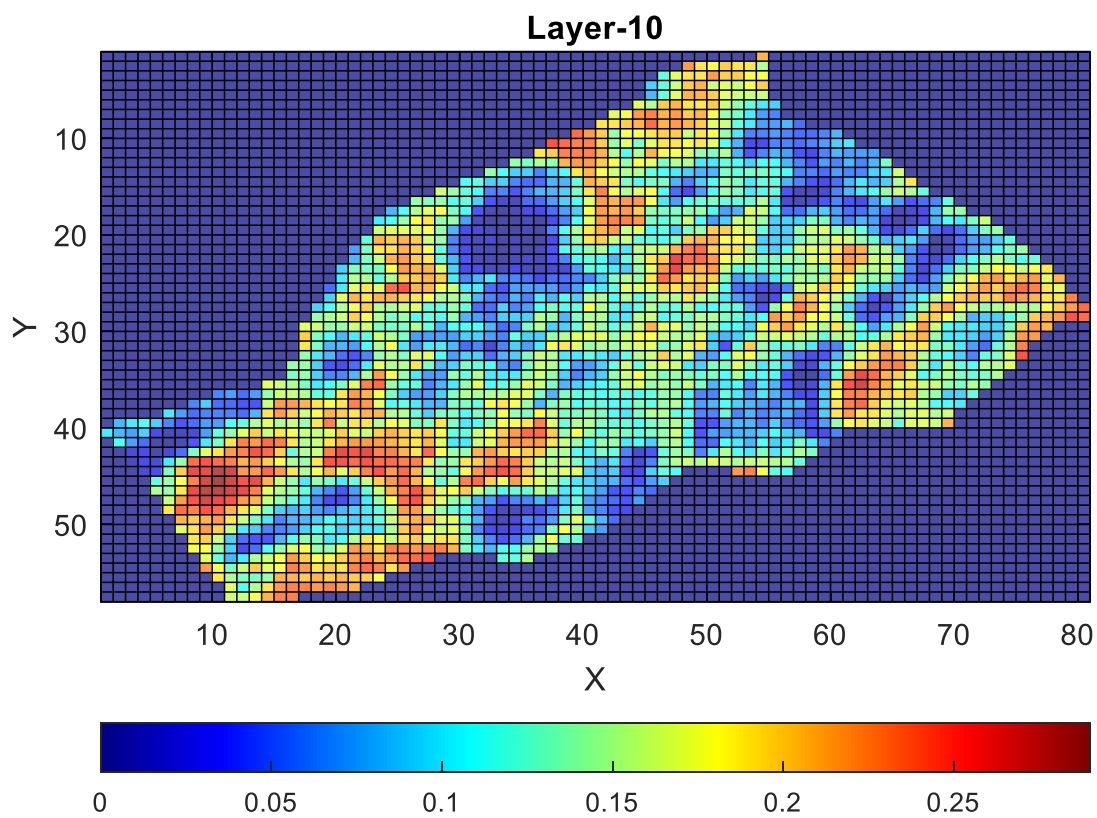
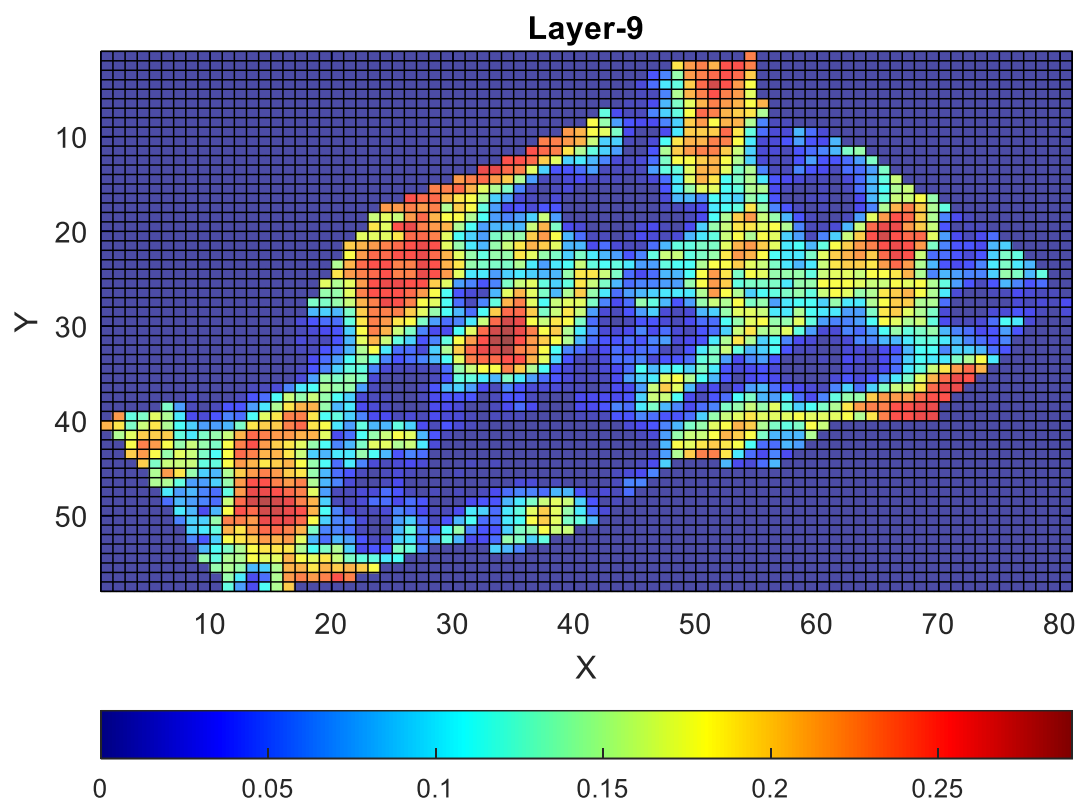
Appendix D – Example 3 Layers

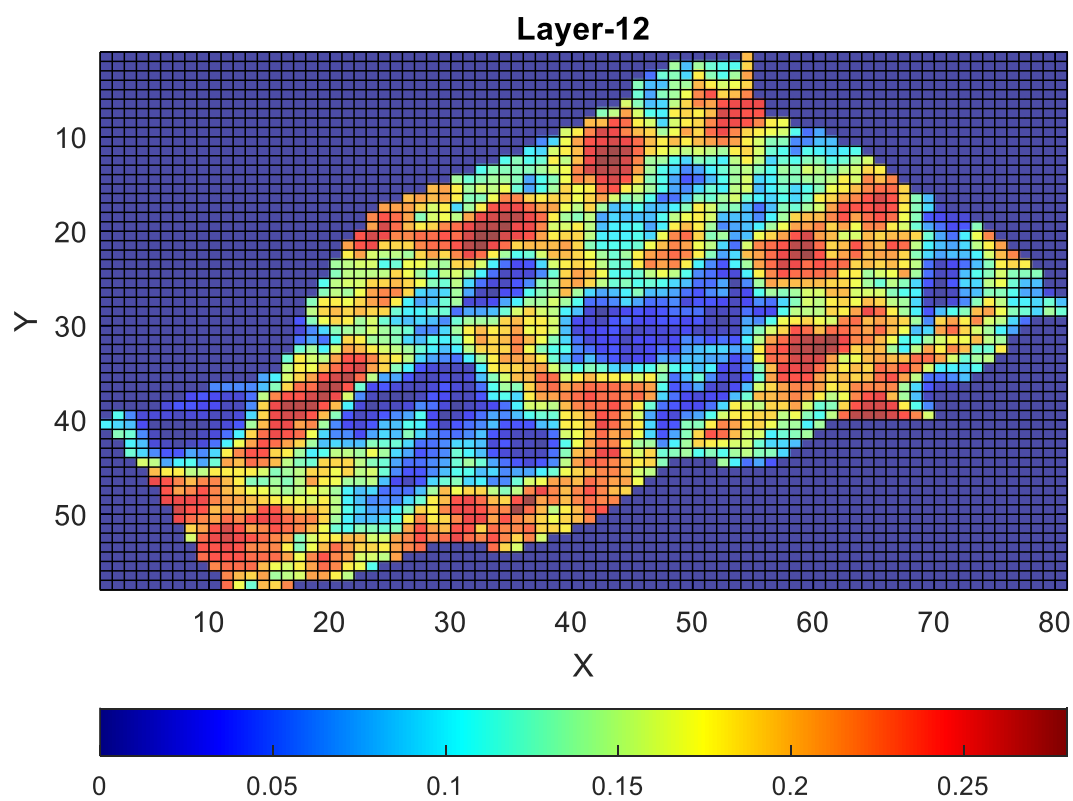
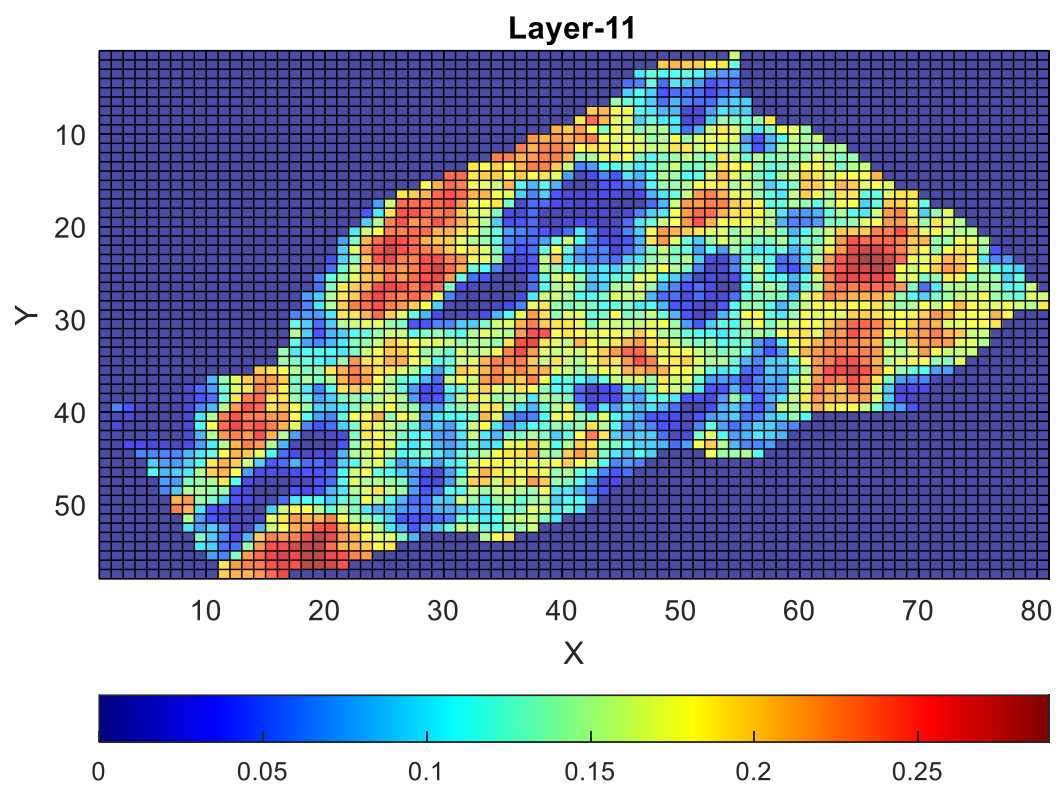


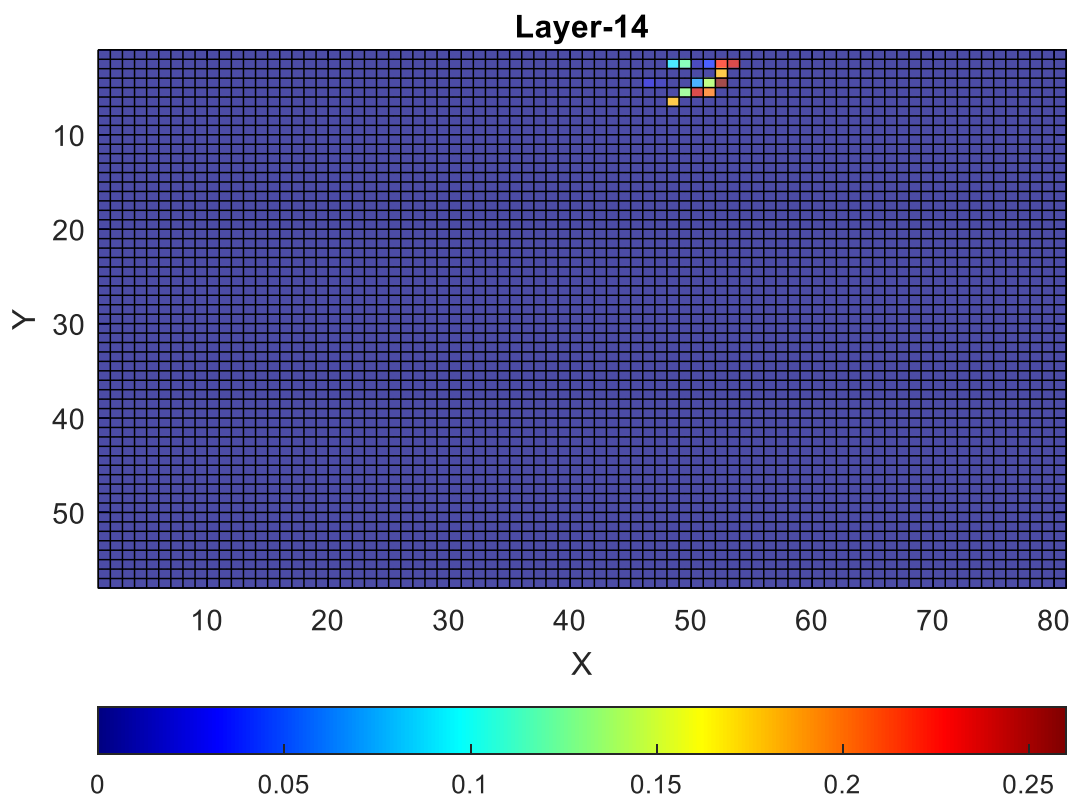
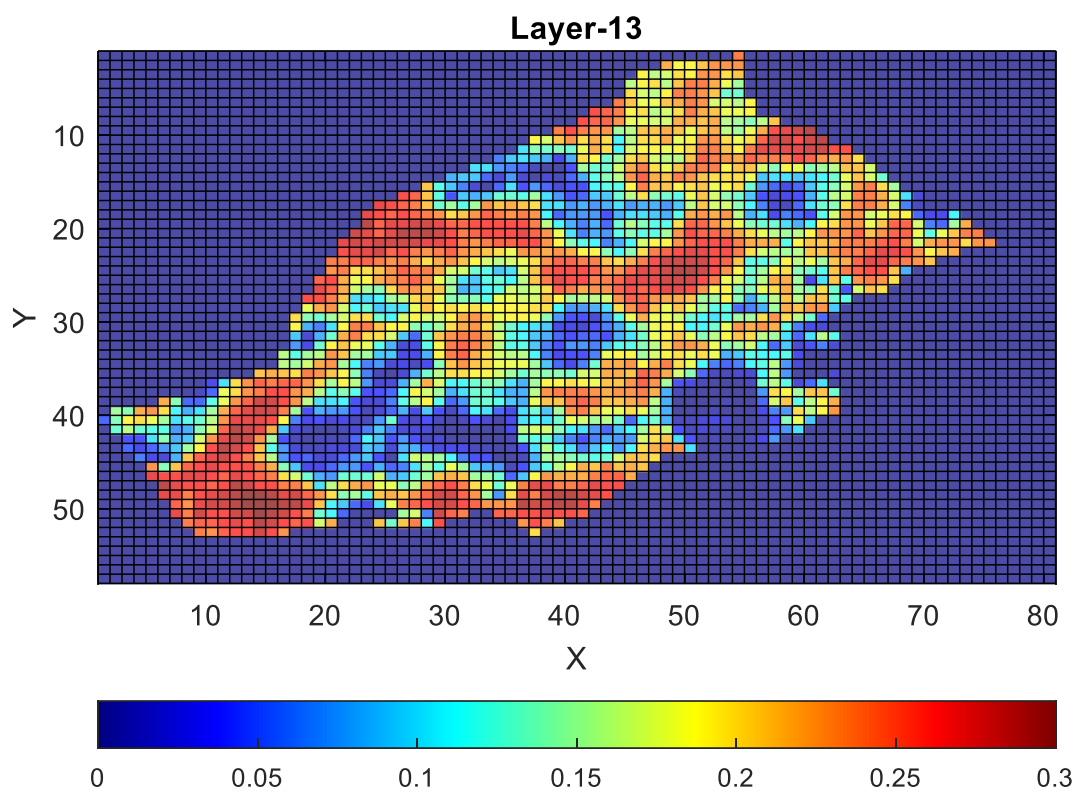




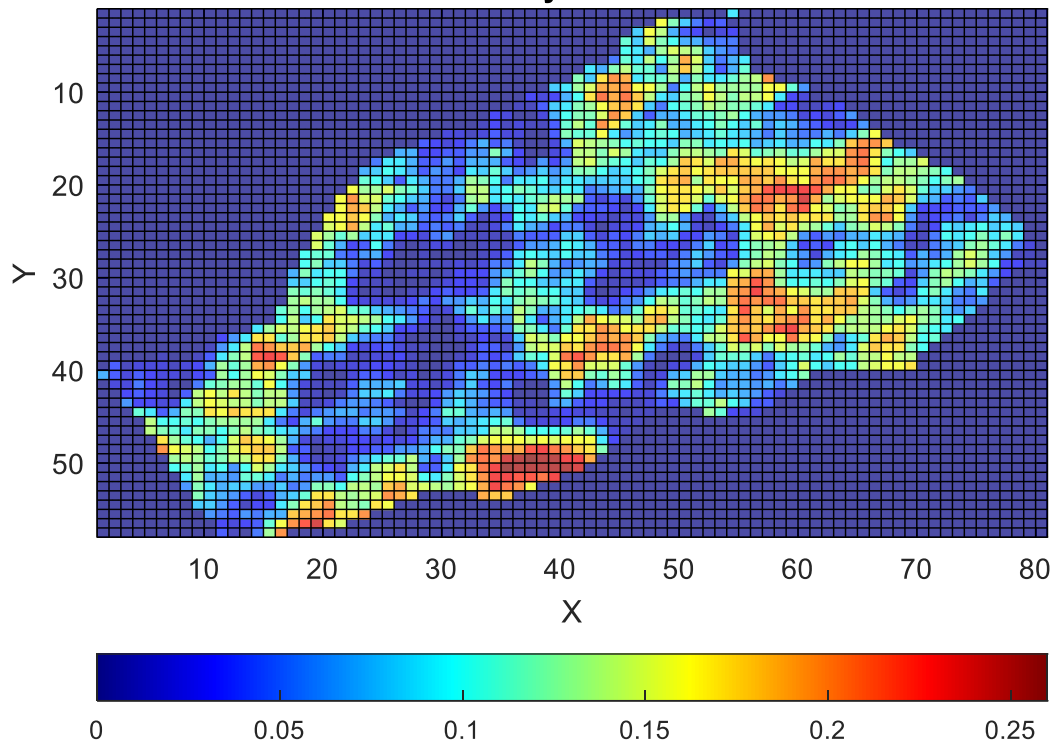




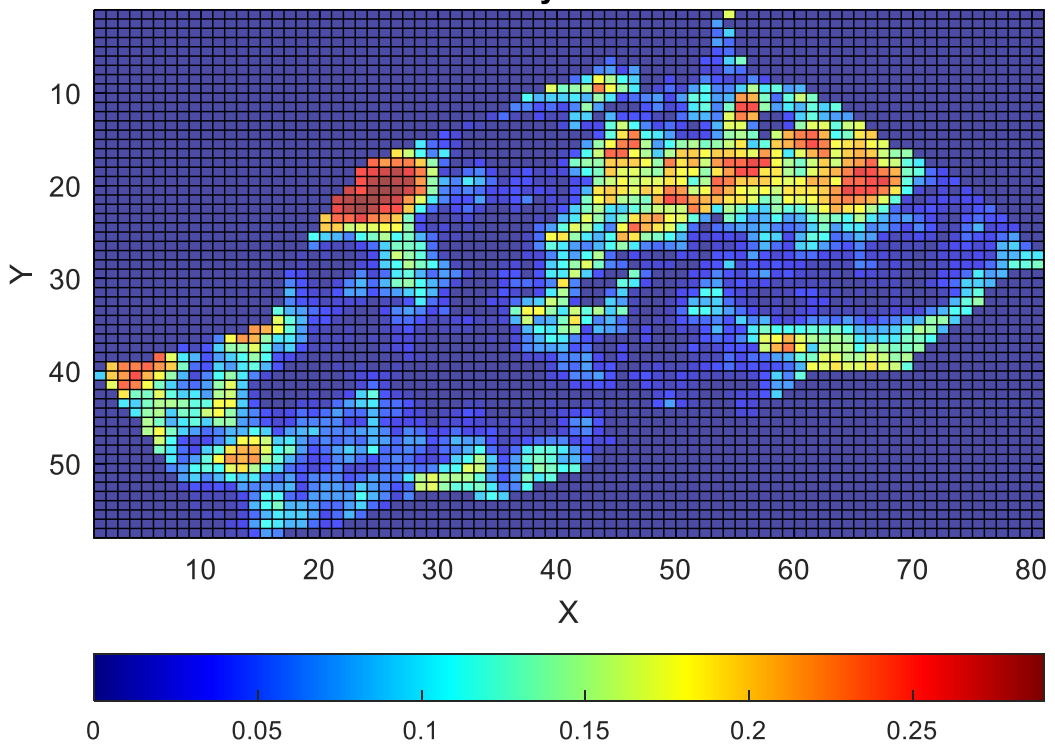


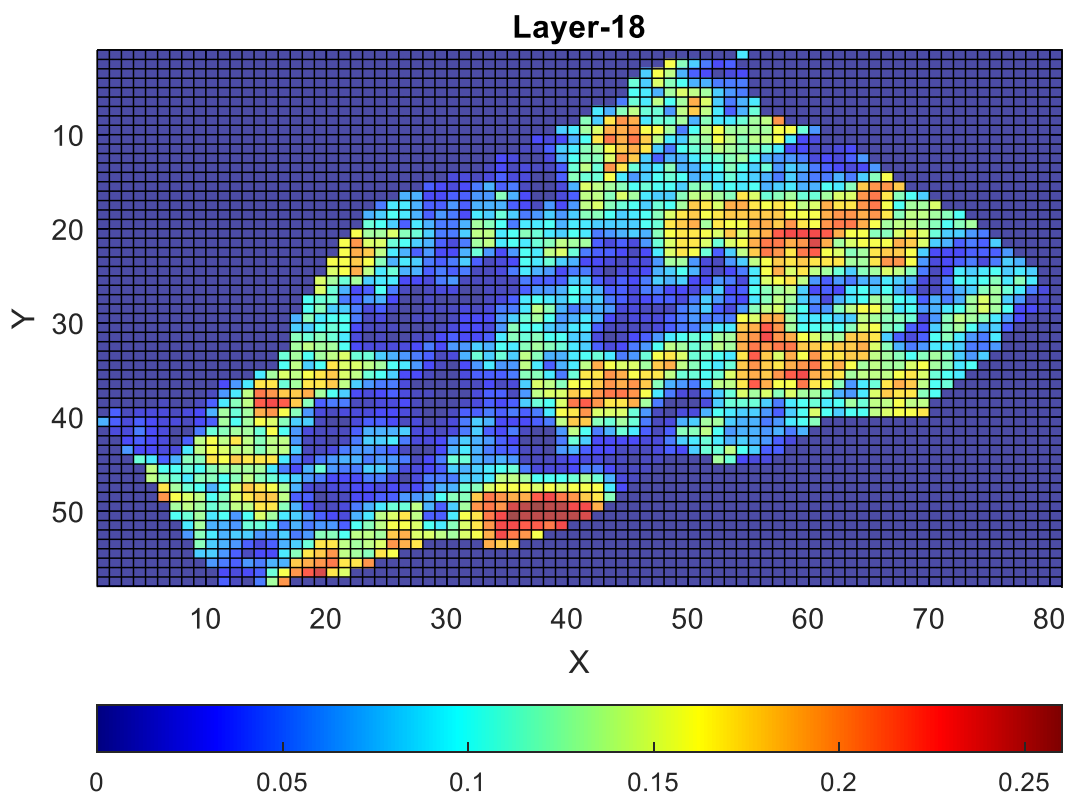
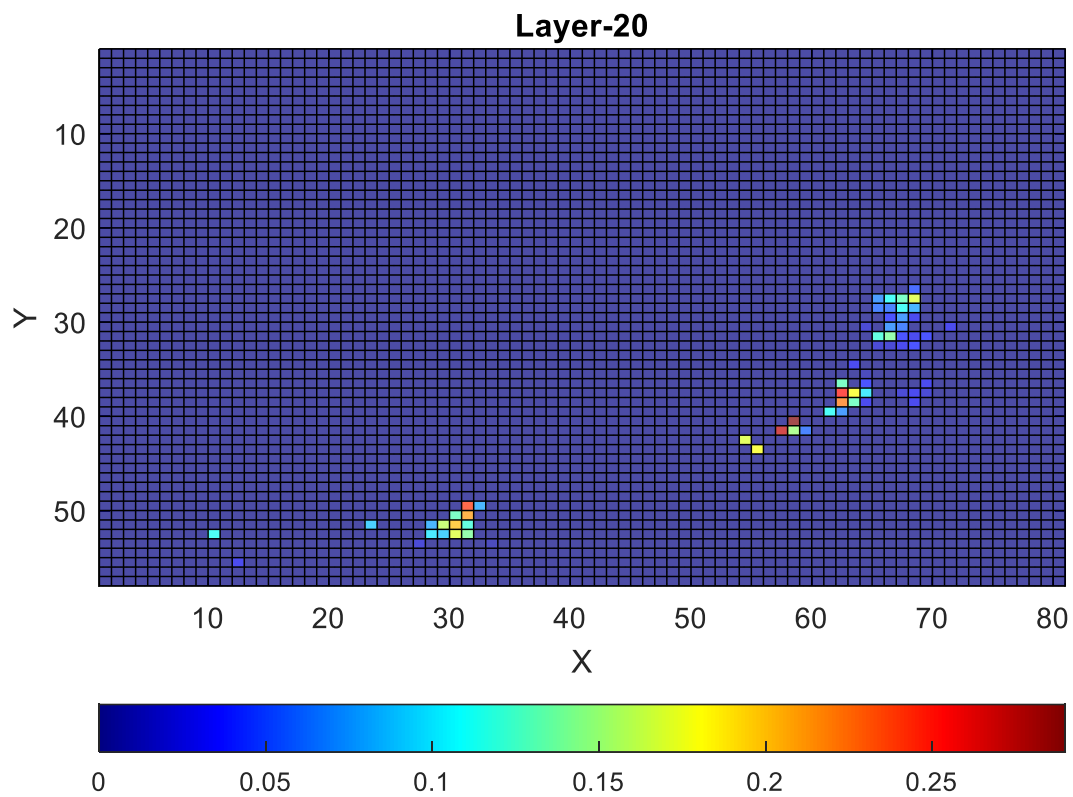


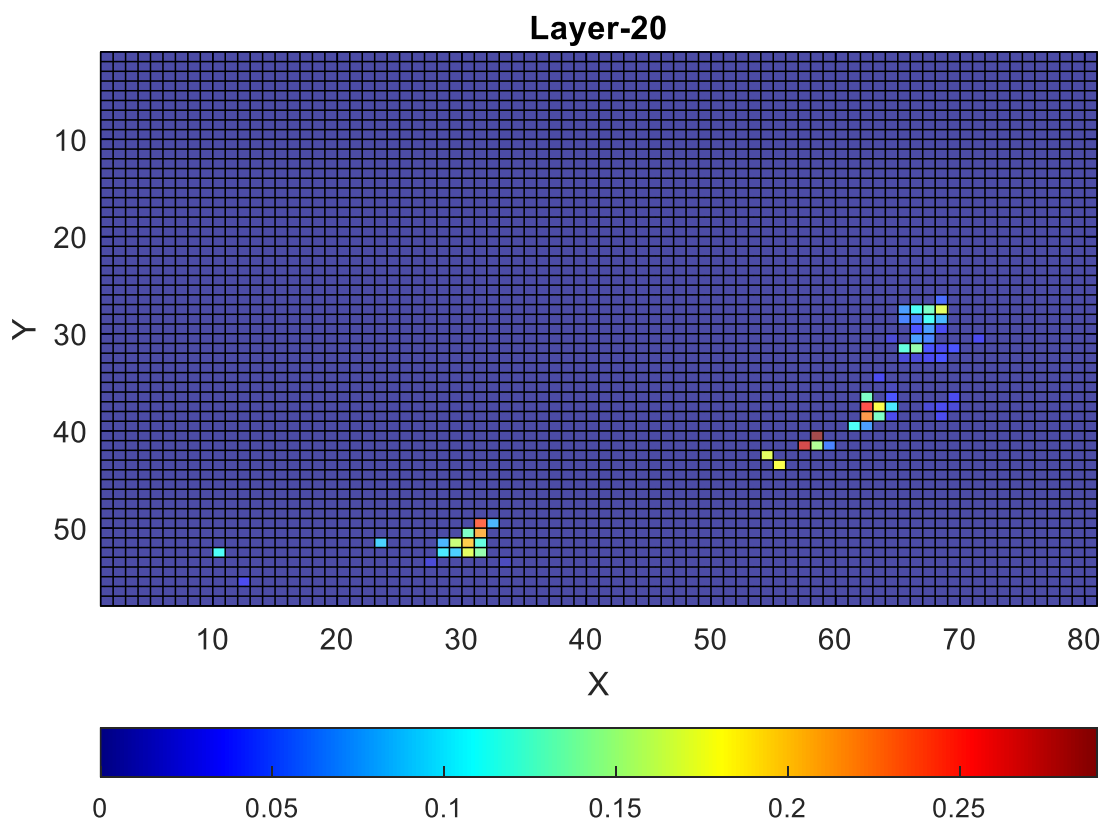
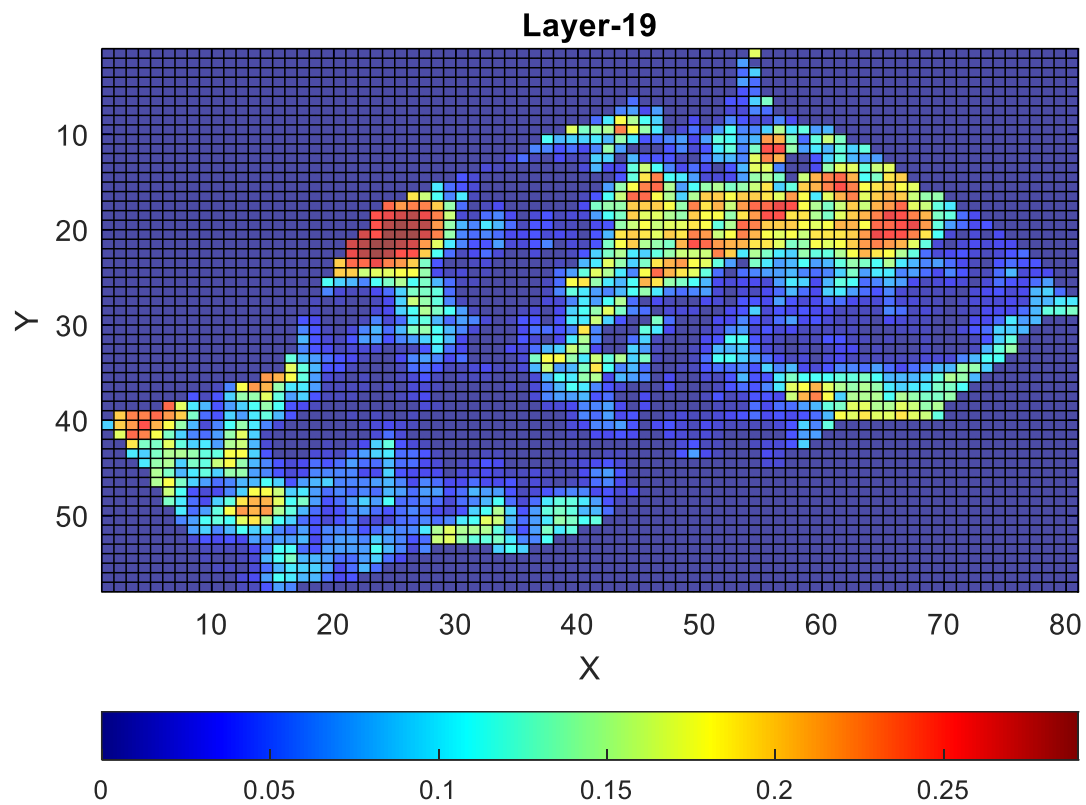
Layer-18



Layer-19







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Publications Alharbi, O. Q., & Alarifi, S. A. (2023). Productivity Index Prediction for Single-Lateral and Multilateral Oil Horizontal Wells Using Machine Learning Techniques. *ACS Omega*.