

**STOCHASTIC AND ROBUST
OPTIMIZATION MODELS FOR A
DOWNSTREAM PETROLUEM SUPPLY
CHAIN**

BY

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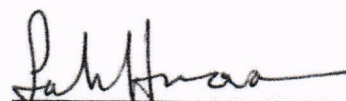


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*I humbly and wholeheartedly dedicate this thesis to
my beloved **parents and sisters**, who have been my source of inspiration and
for their endless love and support throughout my entire life,
my **friends**, who shared their words of advice and encouragement to finish
this study and were the source of joy during this fruitful journey.*

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ABSTRACT

Full Name Mohamed Elfadel Eltagani Osman

Thesis Title STOCHASTIC AND ROBUST OPTIMIZATION MODELS FOR A
DOWNSTREAM PETROLUEM SUPPLY CHAIN

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The refined products distribution network in Saudi Arabia is highly complex due to the vast area the country occupies. In addition, demand uncertainty adds to the complexity of long-term planning. The thesis aims at developing an optimization model for strategic planning of the refined products supply chain in Saudi Arabia considering demand uncertainty.

The objective of the model is to minimize both the operational and fixed costs in the long term. The model covers a period of 5 years (2022 – 2026), and considers two products, Diesel and Gasoline. In addition, three modes of transportation are considered, trucks, pipelines, and ships. A mathematical programming approach is employed to formulate the deterministic model and GAMS language is used to code the model and obtain a near optimal investment plan. In addition, the solution time is reduced via implementing a penalty function approach.

To cope with the demand uncertainty, we consider two models, a two-stage stochastic optimization model and a robust optimization model. In the two-stage stochastic model, the uncertainty is captured by means of scenario planning approach, where the demand growth rate values are drawn from the distribution of the historical data. In the robust model, the uncertainty is captured via optimizing the worst-case scenario for the demand realization from the historical data. In terms of complexity, the two-stage stochastic model is the most complex model because of the different scenarios considered. In addition, the stochastic model results in the highest optimal cost due to the high variation in demand growth rates between scenarios, followed by the robust model and finally, the deterministic model.

ملخص الرسالة

الاسم بالكامل: محمد الفاضل التجاني عثمان
عنوان الرسالة: نماذج رياضية إحصائية لتحسين سلسلة إمداد المنتجات النفطية المكررة
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شبكة توزيع المنتجات النفطية في المملكة العربية السعودية معقدة جدا نظراً لمساحة المملكة الشاسعة. بالإضافة لذلك، تصنيف عشوائية الطلب على هذه المنتجات إلى صعوبة التخطيط الطويل المدى. تهدف هذه الرسالة إلى تطوير نموذج رياضي للتخطيط الاستراتيجي لسلسلة إمداد المنتجات النفطية أخذاً بالاعتبار عشوائية الطلب على هذه المنتجات.

تكمّن الغاية من النموذج في تقليل التكاليف الثابتة والتشغيلية لسلسلة الإمداد على المدى الطويل. يغطي النموذج فترة خمس سنوات (2022 - 2026)، ويأخذ بالاعتبار منتجين البنزين والديزل. بالإضافة لذلك، يتيح النموذج استخدام ثلاث وسائل نقل، الشاحنات، خطوط الأنابيب، والسفن. تم اتباع أسلوب البرمجة الرياضية لصياغة النموذج الحتمي واستخدام لغة برمجة قامز (GAMS) لبرمجة النموذج والحصول على خطة توزيع قريبة للمثلى. بالإضافة لذلك، تم تقليل زمن الحل باستخدام أسلوب دالة الجزاء.

للتعامل مع عشوائية الطلب، تم تطوير نموذجين مختلفين، نموذج عشوائي ذو مرحلتين، ونموذج متين إحصائياً. في النموذج العشوائي ذو مرحلتين، يتم التعامل مع عشوائية الطلب باستخدام أسلوب السيناريوهات، حيث يتم استخراج قيم معدلات نمو الطلب من التوزيع الإحصائي للبيانات التاريخية. في النموذج المتين إحصائياً، يتم التعامل مع عشوائية الطلب بحل النموذج لأسوء سيناريو محتمل من البيانات التاريخية. من ناحية تعقيد الحل، فقد كان النموذج العشوائي ذو المرحلتين الأكثر تعقيداً نظراً لعدد السيناريوهات المعتبرة في النموذج. كما نتج النموذج العشوائي ذو المرحلتين

عن أعلى تكلفة إجمالية بسبب الطلب المرتفع في بعض السيناريوهات، يتبعه النموذج المتين إحصائياً، ثم النموذج
الاحتمالي.

Chapter 1

INTRODUCTION

Petroleum industries play an important role in the world economy as they supply the necessary products to sustain the world energy supply. As a result, the petroleum supply chain PSC was significantly studied and explored in the literature. The PSC consists of three segments, upstream, midstream, and downstream. The upstream PSC is responsible for exploring and exporting oil and gas from the wells. The midstream PSC activities include refining the crude oil and gas to their final refined products. The downstream PSC transports these refined products from the refineries to the demand markets.

The refined petroleum products distribution network in Saudi Arabia is a highly integrated and complex system that transports these products from supply centers (refineries and import terminals) to demand centers using a multi-mode transportation system. The modes of transportation are pipelines, trucks, and ships. The distribution network consists of various supply centers that supply refined/processed products to the distribution centers (Bulk Plants). The Distribution Centers (DC) in turn supply Demand Markets (cities) with the required amounts of the refined products according to their demand. A distribution center can also supply another distribution center, which in turn supplies a demand center. Figure 1 provides an abstract view of the distribution system network.

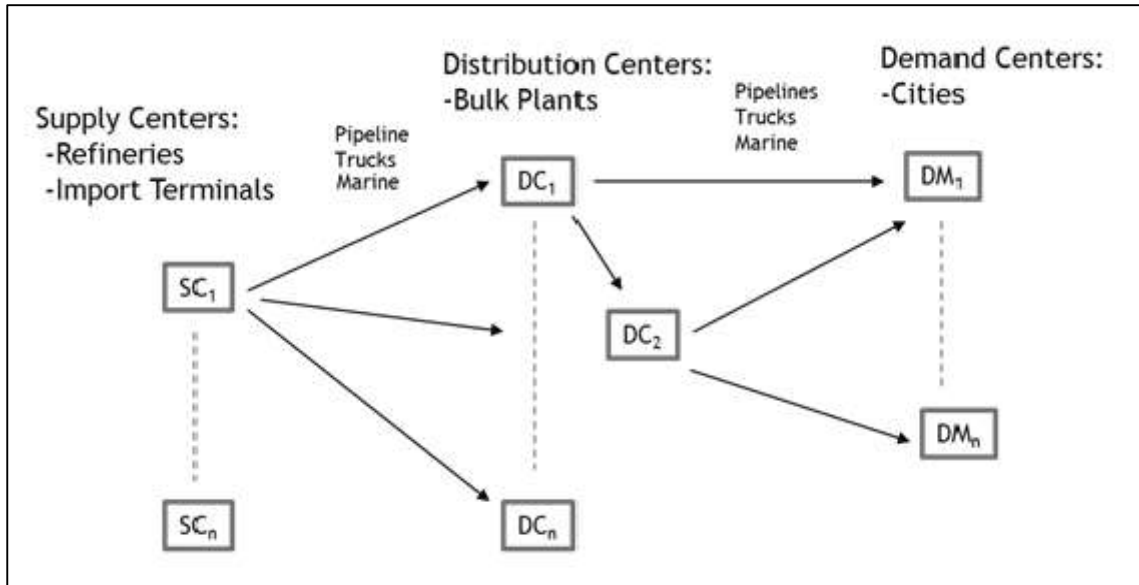


Figure 1: Superstructure of the Distribution System Network

The distribution network requires strategic, tactical, and operational planning. Strategic planning addresses issues such as building new facilities and increasing the capacity of the existing facilities, in addition to planning the annual production and distribution quantities of all products. The planning horizon for strategic planning is in terms of years. Tactical planning refers to the mid-term decisions such as the manufacturing processes required to produce the products, and the inventory levels required to meet the demand. The planning horizon is in terms of months or quarters. Operational planning refers to the short-term decisions required to satisfy the business needs. These include shop floor scheduling decisions. The planning horizon is in the order of days or weeks.

The petroleum supply chain is subject to many uncertainties including demand uncertainty, prices fluctuations, and the variability in production and refining quantities (Oliveira et al., 2014a). These uncertainties represent potential risks for the entire petroleum supply chain and add to the complexity of planning for the long term (Lima et al., 2018). To reduce the

effects of these uncertainties, many stochastic mathematical programming approaches have been utilized such as stochastic Programming and robust Optimization. Stochastic programming assigns a probability distribution for the uncertain parameters. On the other hand, robust optimization assigns uncertainty sets for the uncertain parameters without assuming a certain distribution. The distribution of the demand for petroleum products is dependent on other factors such as the price, the population size and the laws and regulations and thus may not be accurately estimated from historical data. In this work, we apply robust optimization and stochastic optimization to incorporate the uncertainty of the demand and obtain an optimal or near-optimal solution.

The aim of this work is to develop a model to address the strategic planning of the refined products distribution network in Saudi Arabia considering demand uncertainty. The model will be used to determine the optimal investment plan for the configuration of the refined products distribution system. The factors that will be taken into consideration are the capacity of the existing distribution centers, quantities transported from supply centers to distribution centers and then to demand centers, and modes of transportation used. The method employed will use a mathematical programming approach in which the transport of material in the distribution system is described using relevant constraints. A two-stage stochastic optimization model is then formulated by considering a probable range for the demand forecasts. In addition, a robust optimization model is developed for the worst-case realization of the demand.

The model will minimize the total fixed and operational costs. The model will also assist in defining the optimal investment plan for the configuration and operation of the refined products distribution system over the planning horizon 2022-2026. The thesis objectives

will be accomplished by conducting several tasks that include problem formulation, model development, developing a penalty function approach to reduce the problem complexity and solution time, developing a two-stage stochastic optimization model and a robust model.

The rest of this Thesis is ordered as follows. Chapter 2 contains a comprehensive literature review that is divided into two types of relevant literature. First, deterministic models developed for the distribution network design and production planning for petroleum and petrochemical products are reviewed. Subsequently, stochastic programming and robust optimization literature for petroleum supply chains is reviewed. In addition, the chapter discusses the contribution of the thesis. Chapter 3 defines the problem under study and lists the assumptions used for the models. Chapter 4 will introduce the deterministic model with the formulation. In addition, Chapter 4 includes the application of the model on a case study in Saudi Arabia. Furthermore, the penalty function approach will be discussed in Chapter 4 alongside its application on the before mentioned case study. Chapter 5 will discuss the Two-Stage Stochastic Model including the definition, formulation, scenarios generation, and application on the case study. Chapter 6 will discuss the Robust Model including the definition, formulation, scenarios generation, and application on the case study. Finally, chapter 7 concludes the thesis and provides some recommendations for future work.

Chapter 2

LITERATURE REVIEW

2.1 Petroleum Transportation Networks and Production Planning Literature

Duffuaa, 1991 has developed an integer programming model to optimize the chemical production at the Saudi Arabian Fertilizer Company (SAFCO) for two of their major products, ammonia, and urea. For each product, the amount produced was determined by the reactor setting and the operating parameters.

Al-Saleh et al., 1991 have used linear programming to assess the impact of oil production on the petrochemical industry in Saudi Arabia. The solution was optimized for each level considering the crude type needed and gas to oil ratio.

Duffuaa et al., 1992 have developed a linear programming model that integrates oil fields, GOSPs, gas plants, fractionation plants, and petrochemical plants. The model schedules the oil production to maximize the produced associated gas required as input to sustain the existing petrochemical industry.

Al-Amer et al., 1998 and Alfares and Al-Amer, 2002 have formulated a mixed integer programming model for planning the future investments of the Saudi petrochemical industry. The models select the optimum set of products and technologies and production

levels. The objective is to maximize the total profit, subject to raw material availability and budget.

Neiro & Pinto, 2004 have considered a Petroleum Supply Chain (PSC) problem that includes crude oil suppliers, refineries, terminals, intermediate and final products, and distribution through pipelines. The decisions considered include selecting crude oil types, determining production levels, and developing transportation plans for a multi-period planning horizon. Three mathematical models (for refineries, storage tanks, and pipelines) were proposed and integrated into a mixed integer nonlinear model to maximize the revenue. A case study of a PSC in Brazil composed of 4 refineries, 5 terminal facilities, 20 different crude oils, and 32 refined products has been presented.

Kim et al., 2008 have proposed an integrated model that combines oil supply network design with operations planning at refineries and distribution centers. Considering a single-period and a single mode of transportation, the model determines transportation flows in the network, production rates of products, and flow rates at refineries, as well as the decisions on establishing distribution centers. A real case of 3 refineries, 46 distribution centers, and 124 markets is used to illustrate the advantages of the model.

Al-Qahtani & Elkamel, 2008 have studied the refinery stage of PCS. A mathematical model was proposed to determine the optimal integration within a network of refineries located at different sites. Each refinery is composed of different production units that can be operated on a range of operating modes. The model found the optimal feedstock at each refinery, and the amounts transferred from one refinery to another. The model also specifies

the expansion at each production unit required to improve the production flexibility across the network.

Alfares, 2009 has presented a mixed-integer linear programming model for the optimum production and procurement planning of multi-grade petrochemicals, considering several production plants and several raw material suppliers. For each plant, the model determines the quantity and production sequence of all grades, and the amounts of raw materials purchased from all suppliers. Mrad et al., 2016 have extended the work of Alfares, 2009, by developing a multiple-period model for the optimum production, procurement, and inventory planning of multi-grade, multi-supplier, and multi-plant chemical production.

Guyonnet et al., 2009 have considered the oil procurement, processing, and distribution problems as one optimization problem and constructed an integrated model for this comprehensive problem. They then investigated the benefits of such an integrated model versus tackling the three problems in three sequential models.

Fernandes et al., 2013 have studied a strategic design problem of a PSC composed of a set of petroleum companies that produce or import different oil products through a set of refineries. They proposed a mixed-integer linear model that was solved by CPLEX to maximize the total SC profit. The model was applied to a real case study of a Portuguese PSC to find the optimal network design considering a 12-year planning horizon, two refineries, eleven storage depots, eight oil products, and three transportation modes.

Kazemi & Szmerekovsky, 2015 have studied a downstream PSC consisting of three stages; refineries, bulk plants, and demand markets. A multi-echelon, multi-product, and

single-period model was used to optimize the locations and capacities of bulk plants, transportation modes, and the flow of material through the PSC to minimize the total cost. The model was tested in a case study from the U.S. petroleum industry involving 62 refineries, 57 distribution centers, and 180 demand nodes.

Moradi Nasab & Amin-Naseri, 2016 have studied an integrated five-stage PSC that includes oil fields, oil storage, refineries, distribution centers, and customer zones. The model aims to maximize the total profit of the multi-period, multi-product, and multi-mode SC. The model was applied to an Iranian PSC case that includes three oil fields, three storage tanks, three refineries, three distribution centers, three products, and three customer zones.

Ghaithan et al., 2017 have developed an integrated multi-objective, multi-period oil and gas supply chain (OGSC) model for the tactical planning of the downstream segment. Attia et al. (2017) have developed a multi-objective mathematical programming model for the tactical planning of the upstream hydrocarbon supply chains (HCSCs). Both models share two objectives: minimizing cost and maximizing revenue. The downstream model also aims to maximize the service level, while the upstream model aims to maximize the sustainability by minimizing the depletion rate. Realistic case studies from Saudi Arabia were used to demonstrate the practicability of the two models.

Ghaithan et al., 2021 considered the interconnection between the oil and gas supply chains under price and demand uncertainty. They developed a multi-objective stochastic model for downstream oil and gas supply chain. They used a two-stage approach to formulate the model and Augmented- ϵ method to solve the multi-objective problem.

2.2 Stochastic and Robust Optimization Literature for Petroleum Supply Chains

2.2.1 Stochastic Programming Literature for Petroleum Supply Chains

Oliveira et al., 2013 developed a Lagrangian Relaxation approach for oil supply chain network design under demand uncertainty considering risk. They developed a multi-product, multi-period Stochastic Optimization model to determine the optimal investment plan. The proposed Lagrangian Relaxation approach presents a new method for updating the Lagrangian multipliers.

Oliveira et al., 2014b have studied a stochastic PSC with uncertain demand and two stages: the refinery/source stage and the distribution base stage. A multi-period, multi-product, and single-mode of transportation SC model was formulated for two decision levels: strategic and tactical. A stochastic Benders' decomposition method is used to determine the strategic decisions by solving a master problem and then tactical decisions by solving sub-problems. The approach was applied on a realistic case study that involves 4 products, 13 distribution bases, 1 refinery, and 2 external supply sources, considering a 6-year planning horizon.

Yousefi-Babadi et al., 2017 have designed a reliable multi-objective queuing model PSC under uncertainty environments, namely, the risks of disruptions and unknown parameters. The objectives are minimizing the average tardiness, total cost, and transportation cost. The solution was obtained in three stages, determining the parameters values using Jackson network, converting the multi-objective queuing model into single

objective using linear programming LP-metric approach, solving the single objective model using Lagrangian relaxation.

Lima et al., 2018 have considered a multi-stage stochastic programming approach for the optimization of tactical planning of oil supply chain. They utilized the Autoregressive Integrated Moving Average- ARIMA methodology and tree analysis to deal with the uncertainty in oil demand and price. The model was applied to the oil supply chain in Portugal.

Wang et al., 2019 have studied the optimization of the downstream oil supply chain with the possibility of planning a new pipeline route. A two-stage stochastic Mixed Integer Linear Programming- MILP model was developed to consider the impact of demand uncertainty. The model was applied to a case study in China.

Zang et al., 2020 developed a stochastic two-stage MILP model for the combination of midstream and downstream petroleum supply chain. They considered price and demand as the uncertain parameters, and they proposed a novel method to generate the scenarios and they determine the optimum number to consider for scenarios.

Pudasaini, 2021 developed a stochastic multi-objective model for the downstream petroleum supply chain for strategic and tactical planning. The considered objectives are minimizing transportation cost and minimizing product loss. The paper considers demand uncertainty and proposes a two-stage scenario analysis approach to obtain the optimal solution.

Lima et al., 2021 developed an MILP model to tackle the strategic and tactical planning of downstream petroleum supply chain under uncertainty. The model objective is to minimize the total costs by obtaining the optimum network design and distribution assignment. The model developed used fuzzy parameters with chance constrained programming to tackle the uncertainties.

2.2.2 Robust Optimization Literature for Petroleum Supply Chains

Ribas et al., 2010 developed a strategic planning model for petroleum supply chain considering three uncertainties, supply, demand, and price. They propose three solution approaches, stochastic programming, Min-Max robust model to minimize the maximum regret from the pre-scenario optimal solution, Min-Max model to maximize the worst-case solution. The model was applied on a case study in Brazil and the time horizon was 10 years.

Leiras et al., 2010 extended the work of Al-Qahtani and Elkamel (2008) by developing a robust optimization model for the strategic planning of the multi-refinery networks. The uncertainty sources are raw material costs, products prices, and demand. They applied Bertsimas and Sim robust model to control the robustness of the solution and solve it efficiently. The model was applied on a case study to illustrate its benefits.

Lima C et al., 2018 developed a robust optimization model for the tactical planning of downstream petroleum supply chain. They applied ARIMA technique and Monte Carlo simulation to obtain the random parameters. They tested their model on a real petroleum supply chain in Portugal to confirm its applicability.

Sangaiah et al., 2020 developed a robust optimization MILP model for the strategic planning of liquefied natural gas sales to minimize vendor's costs. The uncertainty is considered for the manufacturers' production. The robust model they applied is Bertsimas and Sim robust model which allows for controlling the robustness of the model. They apply Cuckoo optimization algorithm to obtain a near-optimal solution efficiently.

Lima et al., 2019 developed an adjustable robust optimization model for the tactical and operational planning of downstream petroleum supply chain. They considered demand uncertainty and compare their approach to non-adjustable robust optimization and stochastic programming for the Portuguese petroleum supply chain. Their results indicate a small difference in solution quality. However, the adjustable robust optimization approach showed great advantage in terms of solution time.

Attia, 2021 developed a multi-objective robust optimization model for the tactical planning of petroleum supply chain. The uncertainty sources are international oil price and demand. The model considers the three robustness criterion proposed by Kouvelis and Yu, 1997, absolute robustness criteria, robust deviation criteria, and relative robustness criteria. The considered objectives are minimizing cost, maximizing revenue, and minimizing depletion rate. The model is applied on a case study in Saudi Arabia.

2.3 Contribution of the Thesis

The main contribution of the thesis is that it develops two-stage stochastic and robust optimization models for the downstream petroleum supply chain with uncertain demand for multiple years, transportation modes, and products with the option of expanding

existing distribution centers, refineries, and transportation links. As can be seen in Table 1, Robust Optimization was not applied in the literature for the downstream petroleum supply chain. The application of Robust Optimization considers a more realistic case where the demand for petroleum products is uncertain. In addition, unlike stochastic programming, no assumption is made regarding the distribution of the demand, which aids in obtaining a more robust solution. As can be seen in later sections, the model considered in this work is very complex and unique due to the high number of binary variables needed to represent realistic assumptions such as the fixed and variable cost components, upper and lower bounds for transportation amounts, and capacity expansion lower bounds and upper bounds for capacity expansion frequencies. This complexity is handled by applying a penalty approach to reduce the solution time and optimality gap.

Table 1: Literature on the Downstream Petroleum Supply Chain

Author(year)	Level			Uncertainty		Approach	SC Decisions					Objective		
	S	T	O	D	ST		LA	CE	I	P	TM	Profit	Cost	Other
Kim et al. (2008)	✓	✓	✓	✓		MINLP+ MILP	✓				✓	✓		
Alqahatani and Elkamel (2008)	✓			✓		MILP		✓					✓	
Guyonne et al. (2009)		✓	✓	✓		NLP			✓	✓			✓	
Fernandes et al. (2013)	✓	✓		✓		MILP	✓				✓	✓		
Oliveira et al.(2013)	✓	✓			✓	LR	✓	✓	✓				✓	
Oliveira et al.(2014)	✓	✓			✓	Two-stage MILP	✓	✓	✓				✓	
Kazemi and Szmerekovsky (2015)	✓	✓		✓		MILP	✓	✓			✓		✓	
Nasab and Amin-Naseri (2016)	✓	✓		✓		MILP	✓	✓	✓		✓	✓		
Gaithan et al. (2017)		✓		✓		Multi objective MILP			✓	✓		✓	✓	✓
Yousefi-Babadi et al. (2017)		✓	✓		✓	Multi objective MINLP	✓			✓			✓	✓
Lima et al. (2018)		✓			✓	Multi-stage MILP	✓		✓	✓	✓	✓		
Wang et al. (2019)	✓	✓			✓	Two-stage MILP	✓		✓		✓		✓	
This Work	✓				✓	RO + Two-stage MILP	✓	✓			✓		✓	

S Strategic T Tactical O Operational

D Deterministic ST Stochastic

LR Lagrangian Relaxation RO Robust Optimization

LA Location Allocation CE Capacity Expansion I Inventory P Production TM Transportation Modes

Chapter 3

PROBLEM DEFINITION AND ASSUMPTIONS

3.1 Problem Definition

The proposed model considers the distribution of two products, namely, Gasoline and Diesel, using three modes of transportation, trucks, ships, and pipelines. As illustrated in Figure 1, the process begins from the supply centers, the products are shipped to the distribution centers. From the distribution centers, products can be shipped to the demand markets (final destinations) or to other distribution centers. The products can be shipped between two nodes only if there exists a transportation link between them and the amounts shipped are within the capacity of the transportation link. The capacities of transportation links can be established and expanded for a limited number of times during the planning period. Supply centers and distribution centers have limited capacities that they can ship per period, and they can be expanded for a limited number of times. The objective of the model is to determine:

- 1- The amounts shipped from each supply center to the distribution centers, between the distribution centers, and from the distribution centers and the demand markets.
- 2- The capacities expansion amounts for the transportation links, supply centers, and distribution centers.

Such that the total, fixed and variable, cost is minimized.

3.2 Assumptions

1. The flow can only be between:
 - a. Supply centers and distribution centers.
 - b. Distribution centers
 - c. Distribution centers and demand markets
2. Multiple shipping modes are allowed between two points.
3. Distribution centers' locations are fixed and known during the planning period, i.e., it is not allowed to establish a new distribution center.
4. Supply centers and demand markets are fixed and known during the planning period.
5. The current capacities of the distribution centers are known.
6. The current capacities of the transportation links are known.
7. The minimum additional capacities and the maximum capacities for the distribution centers and the transportation links are known.
8. The number of extensions for the distribution centers and the transportation links are limited.
9. The demand for each product at any period cannot exceed the **current** supply center's capacity for that product.

10. Shortages and backorders are not allowed.
11. There is a minimum amount of each product that can be transported by each transportation mode.
12. The variable and fixed costs of transporting products by each transportation mode are fixed and known during the planning period.
13. The variable and fixed costs of expanding distribution centers and transportation links are known during the planning horizon.
14. The variable costs of transporting products are linear w.r.t the amount transported.
15. The variable costs of expanding distribution centers and transportation links are linear w.r.t the amounts of products added.
16. Distances between supply centers, distribution centers and demand markets are known.
17. The expansion of distribution centers and transportation links requires two years, and the cost is paid after the expansion project.
18. The expansions of distribution centers and transportation links are permanent.
19. The costs are returned to the present value (worth).

Chapter 4

DETERMINISTIC MODEL

4.1 Notations

4.1.1 Indices

Supply center (refinery, or import terminal), $i : 1, \dots, I$

j : Distribution center (bulk plant), $j : 1, \dots, J$

k : Demand center (city), $k : 1, \dots, K$

t : Time period (year), $t : 1, \dots, T$

p : Product (diesel, gasoline), $p : 1, \dots, P$

m : Transportation mode (pipeline, truck, ship), $m : 1, \dots, M$

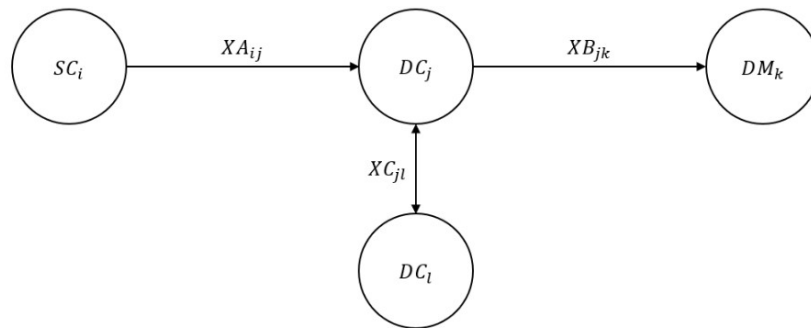


Figure 2: Illustration of the Indices Used in the Model

4.1.2 Parameters

R_i : Capacity of SC_i

S_j : Current capacity of DC_j

D_{kpt} : Demand of DM_k for product p during year t

WA_{ijm} : Current transportation capacity between SC_i and DC_j by mode m

WB_{jkm} : Current transportation capacity between DC_j and DM_k by mode m

WC_{jlm} : Current transportation capacity between DC_j and DC_l by mode m

ESL_j : Min additional capacity of new/existing DC_j

ESU_j : Max additional capacity of existing DC_j

ERL_i : Min additional capacity of new/existing SC_i

ERU_i : Max additional capacity of existing SC_i

$EWAL_{ijm}$: Min new/additional transportation capacity between SC_i & DC_j by mode m

$EWBL_{jkm}$: Min new/additional transportation capacity between DC_j & DM_k by mode m

$EWCL_{jlm}$: Min new/additional transportation capacity between DC_j & DC_l by mode m

EU_j : Max additional capacity of existing DC_j .

$EWAU_{ijm}$: Max new/additional transportation capacity between SC_i & DC_j by mode m

$EWBU_{jkm}$: Max new/additional transportation capacity between DC_j & DM_k by mode m

$EWC_{U_{jlm}}$: Max new/additional transportation capacity between DC_j & DC_l by mode m

WAL_{ijm} : Min. amount of product p that can be transported per year by mode m between SC_i and DC_j ,

WBL_{jkm} : Min. amount of product p that can be transported per year by mode m between DC_j and DM_k ,

WCL_{jlm} : Min. amount of product p that can be transported per year by mode m between DC_j and DC_l ,

CA_{ijm} : Unit transportation cost by mode m between SC_i and DC_j

CB_{jkm} : Unit transportation cost by mode m between DC_j and DM_k

CC_{jlm} : Unit transportation cost by mode m between DC_j and DC_l

CS_j : Unit cost of added capacity in DC_j

CR_i : Unit cost of added capacity in SC_i

CWA_{ijm} : Unit cost of new/added transport capacity by mode m between SC_i and DC_j

CWB_{jkm} : Unit cost of new/added transport capacity by mode m between DC_j and DM_k

CWC_{jlm} : Unit cost of new/added transport capacity by mode m between DC_j and DC_l

$CFES_{jt}$: Capacity expansion fixed cost in DC_j at period t

$CFER_{jt}$: Capacity expansion fixed cost in SC_i at period t

$CFEWA_{ijmt}$: Fixed cost of capacity expansion in the link connecting SC_i with DC_j for transportation mode m at period t .

$CFEWB_{jkmt}$: Fixed cost of capacity expansion in the link connecting DC_j with DM_k for transportation mode m at period t .

$CFEWC_{jlmt}$: Fixed cost of capacity expansion in the link connecting DC_j with DC_l for transportation mode m at period t .

$CFWA_{ijmpt}$: Transportation fixed cost of product p between SC_i and DC_j through transportation mode m at period t

$CFWB_{jkmpt}$: Transportation fixed cost of product p between DC_j and DM_k through transportation mode m at period t

$CFWC_{jlmpt}$: Transportation fixed cost of product p between DC_j and DC_l through transportation mode m at period t

NW : Maximum number of expansions of a transportation link during the plan

NS : Maximum number of expansions of a DC

NR : Maximum number of expansions of a SC

r : The annual effective interest rate.

4.1.3 Decision Variables

XA_{ijmpt} : Amount of product p transported by mode m between SC_i and DC_j in year t ,

$$XA_{ijmpt} \geq 0$$

XB_{jkmp} : Amount of product p transported by mode m between DC_j and DM_k in year t ,

$$XB_{jkmp} \geq 0$$

XC_{jlmp} : Amount of product p transported by mode m between DC_j and DC_l in year t ,

$$XC_{jlmp} \geq 0$$

YA_{ijmp} : Equals 1 if $XA_{ijmp} > 0$; 0 otherwise,

YB_{jkmp} : Equals 1 if $XB_{jkmp} > 0$; 0 otherwise,

YC_{jlmp} : Equals 1 if $XC_{jlmp} > 0$; 0 otherwise,

YS_{jt} : Equals 1, if capacity of DC_j is expanded in year t ; and 0 otherwise.

YR_{it} : Equals 1, if capacity of SC_i is expanded in year t ; and 0 otherwise.

YEA_{ijmt} : Equals 1, if mode m transport between SC_i & DC_j is established/expanded in year t , and 0 otherwise.

YEB_{jkmt} : Equals 1, if mode m transport between DC_j & DM_k is established/expanded in year t , and 0 otherwise.

YEC_{jlmt} : Equals 1, if mode m transport between DC_j & DC_l is established/expanded in year t , and 0 otherwise.

XES_{jt} : Added capacity in DC_j at year t ,

XER_{it} : Added capacity in SC_i at year t ,

$XEWA_{ijmt}$: New/added transportation capacity by mode m between SC_i and DC_j in year t .

$XEWB_{jkmt}$: New/added transportation capacity by mode m between DC_j and DM_k in year t . $XEWC_{jlm t}$: New/added transportation capacity by mode m between DC_j and DC_l in year t .

4.2 Mathematical Model

4.2.1 Objective Function

Minimize the Present Value of the Total Cost: Operating cost (of transporting products) + Capital cost (of establishing and expanding DCs and transportation links) + fixed cost (of DCs & transportation links).

Minimize Z =

$$\sum_{t=1}^T \left[\sum_{p=1}^P \sum_{m=1}^M \sum_{j=1}^J \left(\sum_{i=1}^I CA_{ijm} XA_{ijmpt} + \sum_{k=1}^K CB_{jkm} XB_{jkmpt} + \sum_{l=1(l \neq j)}^J CC_{jlm} XC_{jlmpt} \right) \right] * (1 + r)^{-t} \quad (1. a)$$

$$+ \left(\sum_{t=3}^T \left[\sum_{j=1}^J CS_j XES_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CR_i XER_{it} \right] \right) * (1 + r)^{-t+2} \quad (1. b)$$

$$+ \sum_{t=3}^T \left[\sum_{m=1}^M \sum_{j=1}^J \left(\sum_{i=1}^I CWA_{ijm} XEW A_{ijmt} + \sum_{k=1}^K CWB_{jkm} XEW B_{jkmt} + \sum_{l=1(l \neq j)}^J CWC_{jlm} XEW C_{jlmt} \right) \right] * (1+r)^{-t+2} \quad (1.c)$$

$$+ \sum_{t=3}^T \left[\sum_{i=1}^I \sum_{j=1}^J \sum_{m=1}^M CFEWA_{ijmt} YEW A_{ijmt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M CFEWB_{jkmt} YEW B_{jkmt} + \sum_{j=1}^J \sum_{l \neq j}^J \sum_{m=1}^M CFEWC_{jlm} YEW C_{jlmt} \right] * (1+r)^{-t+2} \quad (1.d)$$

$$+ \sum_{t=3}^T \left[\sum_{j=1}^J CFES_{jt} YS_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CFER_{it} YR_{it} \right] * (1+r)^{-t+2} \quad (1.e)$$

$$+ \sum_{t=1}^T \left[\sum_{i=1}^I \sum_{j=1}^J \sum_{m=1}^M \sum_{p=1}^P CFWA_{ijmpt} YA_{ijmpt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M \sum_{p=1}^P CFWB_{jkmpt} YB_{jkmpt} + \sum_{j=1}^J \sum_{l \neq j}^J \sum_{m=1}^M \sum_{p=1}^P CFWC_{jlmpt} YC_{jlmpt} \right] * (1+r)^{-t} \quad (1.f)$$

Equation (1.a) represents the cost of transporting the products from supply centers to distribution centers, between distribution centers and from distribution centers to demand markets. Equation (1.b) is the total variable cost of expanding distribution centers and supply centers. Equation (1.c) is the total variable cost of expanding the transportation links from supply centers to distribution centers, between distribution centers and from distribution centers to demand markets. Equation (1.d) is the total fixed cost of expanding

distribution centers and supply centers. Equation (1.e) represents the total fixed cost of expanding transportation links. Equation (1.f) represents the total fixed cost of transporting the products between the nodes.

Note that the term $(1 + r)^{-t}$ is used to return the costs to the present value and the term $(1 + r)^{-t+2}$ is for expansion projects that require two years to complete and are paid after the project end as mentioned in assumption 17.

4.2.2 Constraints

4.2.2.1 Supply Centers (Refineries) Capacities

Sum of all outputs from SC_i cannot exceed its capacity + expanded capacity

$$\sum_{j=1}^J \sum_{p=1}^P \sum_{m=1}^M XA_{ijmpt} \leq R_i + \sum_{\tau=3}^t XER_{i\tau},, \quad (2)$$

$$i = 1, \dots, I, t = 1, \dots, T$$

Equation (3) ensures that total capacity expansions for a SC cannot exceed the maximum allowed quantity ERU_i .

$$\sum_{t=3}^T XER_{it} \leq ERU_i, \quad (3)$$

$$i = 1, \dots, I,$$

Equation (4) is a logical constraint to ensure $YR_{it} = 1$ if $XER_{it} > 0$.

$$XER_{it} \leq ERU_i YR_{it} \quad (4)$$

$$i = 1, \dots, I, \quad t = 3, \dots, T$$

SC_i expansion is above the minimum allowed capacity.

$$XER_{it} \geq ERL_i YR_{it}, \quad (5)$$

$$i = 1, \dots, I, \quad t = 3, \dots, T$$

Number of expansions of SC_i cannot exceed NR .

$$\sum_{t=3}^T YR_{it} \leq NR, \quad (6)$$

$$i = 1, \dots, I$$

4.2.2.2 Distribution Centers (Bulk Plants) Capacities

Sum of all inputs of product p into DC_j cannot exceed its current + expanded capacity

$$\sum_{i=1}^I \sum_{p=1}^P \sum_{m=1}^M XA_{ijmpt} + \sum_{l=1, l \neq j}^J \sum_{p=1}^P \sum_{m=1}^M XC_{l j m p t} \leq S_j + \sum_{\tau=3}^t XES_{j\tau}, \quad (7)$$

$$j = 1, \dots, J, \quad t = 1, \dots, T$$

Equation (8) ensures that total capacity expansions for a DC can not exceed the maximum allowed quantity ESU_j .

$$\sum_{t=3}^T XES_{jt} \leq ESU_j, \quad (8)$$

$$j = 1, \dots, J$$

Equation (9) is a logical constraint to ensure $YS_{jt} = 1$ if $XES_{jt} > 0$.

$$XES_{jt} \leq EU_j YS_{jt} \quad (9)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

DC_j expansion is above the minimum allowed capacities.

$$XES_{jt} \geq EL_j YS_{jt}, \quad (10)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

Number of expansions of DC_j cannot exceed NS .

$$\sum_{t=3}^T YS_{jt} \leq NS, \quad (11)$$

$$j = 1, \dots, J,$$

4.2.2.3 Distribution Center Material Balance

Sum of product p inputs ($A + C$) into DC_j is equal to sum of its outputs ($B + C$)

$$\sum_{m=1}^M \left(\sum_{i=1}^I XA_{ijmpt} + \sum_{l \neq j} XC_{ljmpt} \right) = \sum_{m=1}^M \left[\sum_{k=1}^K XB_{jkmpt} + \sum_{l \neq j} XC_{jlmpt} \right], \quad (12)$$

$$j = 1, \dots, J, p = 1, \dots, P, t = 1, \dots, T$$

4.2.2.4 Demand Markets (Cities)

Sum of inputs of product p into DM_k must satisfy its demand

$$\sum_{j=1}^J \sum_{m=1}^M XB_{jkmpt} \geq D_{kpt}, \quad (13)$$

$$k = 1, \dots, K, p = 1, \dots, P, t = 1, \dots, T$$

4.2.2.5 Transportation Capacity from SC to DC

Amounts of all products transported by mode m between SC_i and DC_j cannot exceed link i - j mode m capacity

$$\sum_{p=1}^P XA_{ijmpt} \leq WA_{ijm} + \sum_{\tau=3}^t XEWA_{ijm\tau}, \quad (14)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 1, \dots, T$$

Expansion of link i - j cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWA_{ijmt} \leq EWAU_{ijm}, \quad (15)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (16) is a logical constraint to ensure $YEWA_{ijmt} = 1$ if $XEWA_{ijmt} > 0$.

$$XEWA_{ijmt} \leq EWAU_{ijm} YEWA_{ijmt}, \quad (16)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link i - j is above the minimum allowed capacity

$$XEWA_{ijmt} \geq EWAL_{ijm} YEWA_{ijmt}, \quad (17)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link i - j cannot exceed NW

$$\sum_{t=3}^T YEW A_{ijmt} \leq NW, \quad (18)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (19) is a logical constraint to ensure $YA_{ijmpt} = 1$ if $XA_{ijmpt} > 0$.

$$XA_{ijmpt} \leq (WA_{ijm} + EWAU_{ijm})YA_{ijmpt}, \quad (19)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Amounts transported between i and j is above the minimum allowed quantities.

$$XA_{ijmpt} \geq WAL_{ijmp}YA_{ijmpt}, \quad (20)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

4.2.2.6 Transportation Capacity from DC to DM

Amounts of all products transported by mode m between DC_j and DM_k cannot exceed link j - k mode m capacity

$$\sum_{p=1}^P XB_{jkmpt} \leq WB_{jkm} + \sum_{\tau=3}^t XEWB_{jkm\tau}, \quad (21)$$

$$j = 1, \dots, J, \quad k = 1, \dots, K, \quad m = 1, \dots, M, \quad t = 1, \dots, T$$

Expansion of link j - k cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWB_{jkm t} \leq EWB U_{jkm}, \quad (22)$$

$$j = 1, \dots, J, \quad k = 1, \dots, K, \quad m = 1, \dots, M$$

Equation (23) is a logical constraint to ensure $YEWB_{jkm t}$ is = 1 if $XEWB_{jkm t} > 0$.

$$XEWB_{jkm t} \leq EWB U_{jkm} YEWB_{jkm t}, \quad (23)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link j - k is above the minimum capacity expansion.

$$XEWB_{jkm t} \geq EWB L_{jkm} YEWB_{jkm t}, \quad (24)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link j - k cannot exceed NW

$$\sum_{t=3}^T YEWB_{jkm t} \leq NW, \quad (25)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M$$

Equation (26) is a logical constraint to ensure $YB_{jkm p t}$ is = 1 if $XB_{jkm p t} > 0$.

$$XB_{jkm p t} \leq (WB_{jkm} + EWB U_{jkm}) YB_{jkm p t}, \quad (26)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Transportation in link $j-k$ is above the minimum quantity.

$$XB_{jkmpt} \geq WBL_{jkmp}YB_{jkmpt}, \quad (27)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

4.2.2.7 Transportation Capacity from DC to DC

Amounts of all products transported by mode m between DC_j and DC_l cannot exceed link $j-l$ mode m capacity

$$\sum_{p=1}^P XC_{jlmpt} \leq WC_{jlm} + \sum_{\tau=3}^t XEW C_{jlm\tau}, \quad (28)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, t = 1, \dots, T$$

Expansion of link $j-l$ cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEW C_{jlm\tau} \leq EWC U_{jlm}, \quad (29)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M,$$

Equation (26) is a logical constraint to ensure $YEW C_{jlm\tau}$ is = 1 if $XEW C_{jlm\tau} > 0$:

$$XEW C_{jlm\tau} \leq EWC Y_{jlm} YEW C_{jlm\tau}, \quad (30)$$

$$j = 1, \dots, J, l (l \neq j) = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link $j-l$ is above the minimum capacity expansion.

$$XEWC_{jlm t} \geq EWCL_{jlm} YEW C_{jlm t}, \quad (31)$$

$$j = 1, \dots, J, l (l \neq j) = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link j - l cannot exceed NW

$$\sum_{t=3}^T YEW C_{jlm t} \leq NW, \quad (32)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M,$$

Equation (29) is a logical constraint to ensure YC_{jlmpt} is = 1 if $XC_{jlmpt} > 0$

$$XC_{jlmpt} \leq (WC_{ijm} + EWC U_{jlm}) YC_{jlmpt}, \quad (33)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Transportation in link j - l is above the minimum quantity.

$$XC_{jlmpt} \geq WCL_{jlm p} YC_{jlmpt}, \quad (34)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

4.3 Data

4.3.1 Supply Centers

Table 2 (Saudi Aramco, 2017) shows the capacities of the refineries in Saudi Arabia. For the purpose of modeling, and because Yanbu, SAMREF and YASREF are all located in Yanbu, they are considered as one refinery with a capacity of 381.4 million barrels per

year. Similarly, SASREF and SATORP in Jubail with a total capacity of 255.5 ,million barrels per year. The refineries' capacities are displayed in Table 3.

Table 2:List of Refineries

	Refinery	Total Capacity(Million Barrels/Year)
1	Ras Tanura	200.75
2	SAMREF- Yanbu	146
3	YASREF-Yanbu	146
4	Yanbu	89.4
5	Riyadh	45.26
6	Jeddah	32.12
7	Rabigh	146
8	SASREF-Jubail	109.5
9	SATROP-Jubail	146

Table 3:List of Refineries Used in the Model

	Refinery	Total Capacity (Million Barrels/Year)
1	Ras Tanura	200.75
2	Yanbu	381.43
3	Riyadh	45.26
4	Jeddah	32.12
5	Rabigh	146
6	Jubail	255.5

4.3.2 Distribution Centers

Table 4 (Saudi Aramco,2016) shows the locations of the 17 bulk plants in Saudi Arabia. We could not obtain the capacities of the bulk plants; thus, they are all assumed to be 50 million barrels per year. Figure 3 shows the locations of the distribution centers and the refineries.

Table 4:List of Bulk Plants

		Bulk Plant
Central	1	Jawf
	2	Qassim
	3	Riyadh
Western	4	Tabuk
	5	Al-Sulayyil
	6	Rabigh
	7	Jazan
	8	Najran
	9	Abha
	10	Yanbu
	11	Jeddah
	12	Duba
Eastern	13	Turaif
	14	Alhafuf
	15	Dhahran
	16	Qatif
	17	Saffaniya



Figure 3: Locations of Bulk Plants and Refineries

4.3.3 Demand Markets

The demand markets considered in this paper are the cities with a population of 100,000 or more, cities within 50 Km are combined. The demand by region, illustrated in Table 5 (Aramco, 2017). In addition, the populations of the demand markets, were used to find the demand at each of the 20 demand markets. Table 6 (SAMA,2021) displays the total petroleum refined products consumption from 2011-2020 and the yearly growth rates. To estimate the demand during the planning horizon, i.e. 2022-2026, the average growth rate from 2011-2020 was used. Table 7 shows the cities considered and their demand by product in 2022.

Table 5:Demand Per Region

Region	Gasoline(MB/Year)	Diesel (MB/Year)
Central	75.92	67.525
Wester	102.2	89.06
Eastern	40.515	51.1

Table 6:Refined Products Consumption (2011-2020)

Year	Consumption (MB)	Growth rate
2011	1321.4	4.96
2012	1408.6	6.60
2013	1423.8	1.08
2014	1516.8	6.53
2015	1586.5	4.59
2016	1590.3	0.24
2017	1609.9	1.23
2018	1562.3	-2.96
2019	1566.1	0.25
2020	1527.8	-2.45

Table 7:List of demand Markets with Demand Values

		City	Population	Gasoline	Diesel
Central	1	Riyadh		54.51	48.48
	2	Kharj		5.51	4.90
	3	Qassim		5.07	4.51
	4	Hail		3.46	3.08
	5	Sakakah		1.66	1.48
	6	Qurayyat		1.33	1.19
Western	7	Jeddah		31.78	27.69
	8	Medina (Sultanah)		24.90	21.70
	9	Mecca		14.67	12.78
	10	Taif		5.88	5.13
	11	Khamis Mushait		6.63	5.78
	12	Tabuk		5.05	4.40
	13	Najran		2.87	2.50
	14	Jazan		2.31	2.01
	15	Yanbu		2.22	1.93
Eastern	16	Dammam		19.41	24.49
	17	Alhafuf		8.83	11.13
	18	Hafr Albaten		4.11	5.18
	19	Jubail		3.59	4.52
	20	Arar		2.25	2.83

4.3.4 Transportation Links

The values of the transportation links capacities could not be found. Thus, we assumed that there is a transportation link capacity of 50 million barrels per year by trucks between all the refineries and bulk plants, and between all the bulk plants, and between the bulk plants and the cities. In addition, there is a transportation link capacity of 50 million barrels per year by pipelines if a pipeline route exists, and 0 otherwise. Finally, there is a transportation link capacity of 50 million barrels per year by marine between the cities in the coasts, and 0 otherwise. Tables A1 to A3 in Appendix A display the values of the capacity parameters.

4.3.5 Transportation Costs

The variable costs of transporting 1 million barrels are assumed to be SR 10,000/KM by truck, SR 5,000/KM by pipelines, and SR 5,000/KM by ships. Tables A4 to A6 display the variable transportation costs parameters. The fixed transportation cost is assumed to be SR 2,000,000 for all the links.

4.3.6 Expansion Costs

The variable cost of adding a capacity of 1 million barrels to a bulk plant is assumed to be SR 5,000,000. The fixed cost of expanding a bulk plant is assumed to be SR 20,000,000. The variable cost of adding a capacity of 1 million barrels to a transportation link is assumed to be SR 2,000,000. The fixed cost of expanding a transportation link is assumed to be SR 5,000,000.

4.3.7 Transportation and Expansion Limits

The minimum quantity that can be transported between two links is assumed to be 250,000 barrels. The minimum and maximum amounts for expanding a transportation link are assumed to be 500,000 and 100,000,000 barrels respectively. The minimum and maximum amount for distribution centers and refineries capacity expansions are assumed to be 10,000,000 and 100,000,000 barrels respectively. The maximum numbers of distribution centers capacity expansions and transportation links expansions during the planning horizon are both assumed 1.

4.3.8 Effective Interest Rate

The effective interest rate used in this study is assumed to be 10%, this we believe is a sufficient value to cover for both interest and inflation rates.

4.4 Results and Analysis

The model was coded in GAMS 24.7.4 and was solved using the CPLEX solver on a personal computer with 16 GB RAM and Intel Core i7 10th generation. The code used to run the model is attached in Appendix E. After experimenting with the model, we note that a near optimal solution, i.e., optimality gap of 5% can be obtained in much less time, thus, we set a stopping criterion at 5% optimality gap. Table 8 shows the model summary. A near optimal solution is found to be = SR 2.113 billion. The relative gap from the best possible solution is 4.87%. The solution time = 16,500 seconds (4.5 hours).

Table 8: Full Model Summary

	Number of Constraints	Number of Variables	Number of Binary Variables
Full Model	79,449	65,252	28,569

The total amount distributed in the planning horizon is 3.719 billion barrels. Figure 4 displays the breakdown of the consumed amounts per product each year. Figure 5 displays the amounts distributed between refineries and distribution centers per transportation mode each year. It can be seen that trucks are the most used mode followed by sea and finally pipelines. Which can be anticipated given that pipelines are available for few routes. Table 9 shows a breakdown of distributed amounts by each refinery per mode each year, it is worth noticing that refineries Riyadh and Jeddah consumed nearly their total capacities.

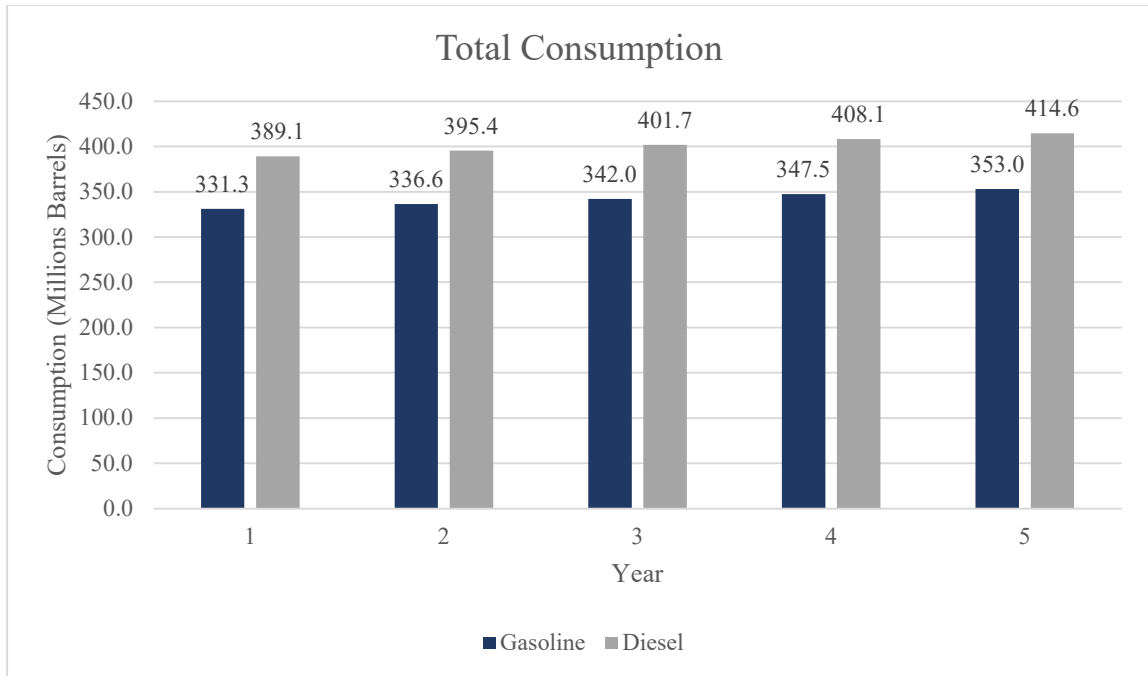


Figure 4: Products Consumption Per Year

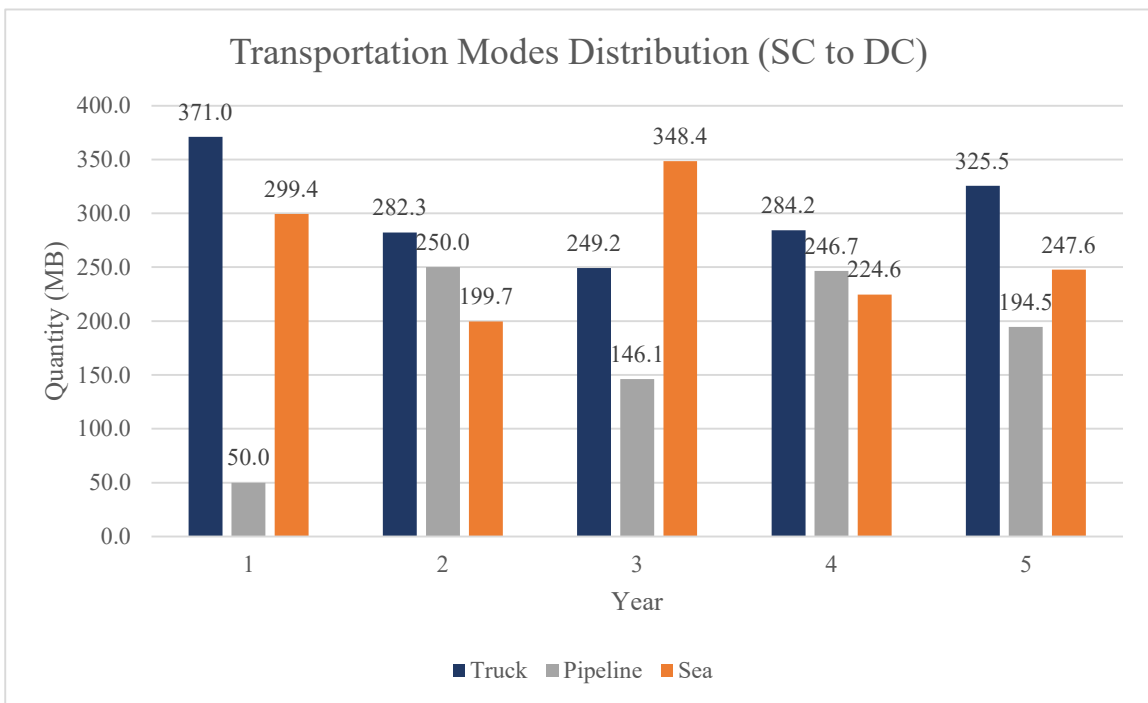


Figure 5: Distributed Amounts by Transportation Mode

Table 9: Breakdown of Distributed Amounts Between SC and DC

		Year				
Refinery	Mode	1	2	3	4	5
Ras Tanura	Truck	29.0	34.5	0.0	16.0	50.0
	Pipeline	50.0	150.0	0.0	50.0	99.2
	Sea	50.0	0.0	50.0	0.0	0.0
Jubail (2)	Truck	21.0	15.5	54.7	38.6	3.8
	Pipeline	0.0	50.0	50.0	100.0	50.0
	Sea	100.0	0.0	100.0	50.0	50.0
Rabigh	Truck	63.9	30.7	18.7	52.4	94.3
	Pipeline	0.0	0.0	50.0	0.0	0.0
	Sea	82.1	104.5	76.8	93.0	50.0
Yanbu (3)	Truck	180.2	124.2	99.2	99.9	100.0
	Pipeline	0.0	50.0	46.1	96.7	45.3
	Sea	67.4	95.2	121.6	81.7	147.6
Riyadh	Truck	45.3	45.3	45.3	45.3	45.3
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Jeddah	Truck	31.6	32.1	31.3	32.1	32.1
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0

Figure 6 shows the amounts distributed between distribution centers and demand markets by transportation modes each year, Table 10 displays the breakdown of these amounts for distribution centers. It can be clearly seen that trucks are the most dominant mode used to distribute the products unlike for the distribution centers and refineries, where sea transportation was close to trucks. This could be because 5 of the refineries have access to the sea (all but Riyadh).

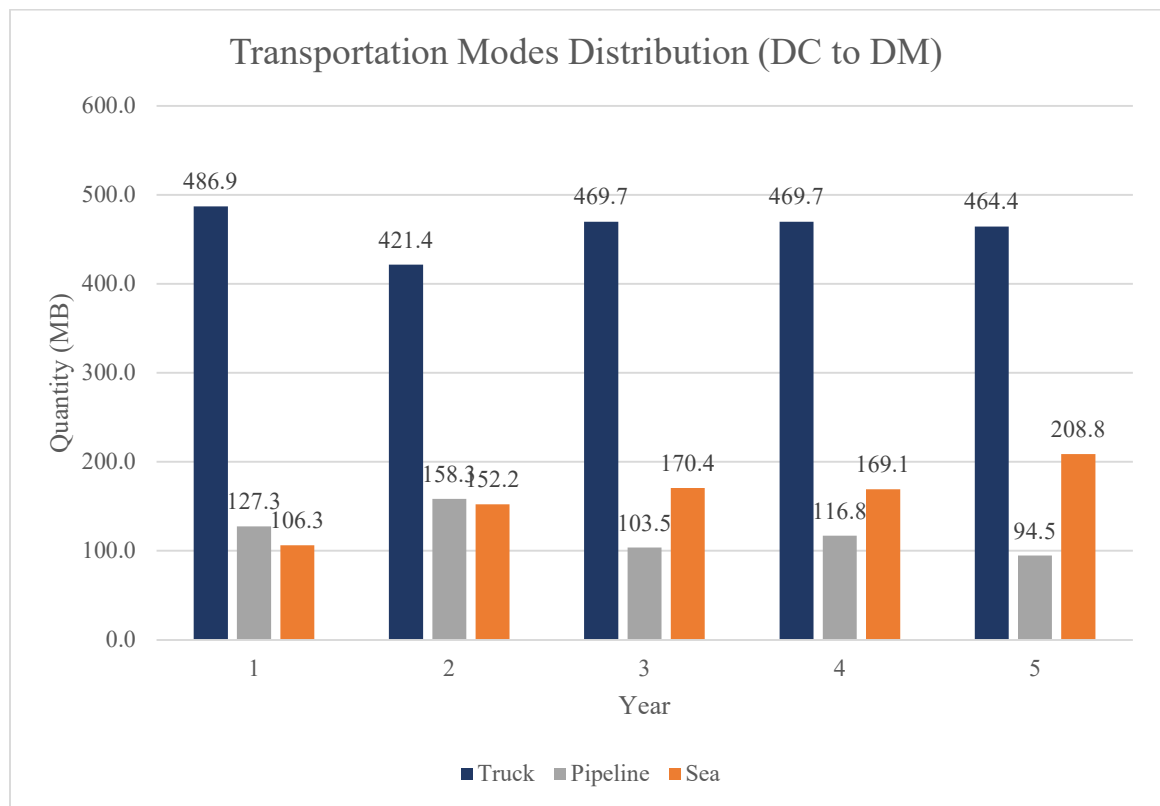


Figure 6: Transportation Modes Distribution (DC to DM)

Table 10: Breakdown of Distributed Amounts Between DC and DM

		Year				
Distribution Center	Mode	1	2	3	4	5
Jawf	Truck	32.7	26.1	46.1	46.8	45.3
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Tabuk	Truck	48.8	49.6	49.2	49.9	50.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Turaif	Truck	0.0	0.0	0.0	0.0	0.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Salil	Truck	45.3	49.5	50.0	49.9	49.1
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Rabigh	Truck	46.5	9.3	24.2	37.1	19.8
	Pipeline	3.5	40.7	25.8	8.4	30.2
	Sea	0.0	0.0	0.0	4.4	0.0
Jazan	Truck	9.9	8.1	4.6	15.5	0.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	39.5	41.6	43.8	34.5	47.6
Najran	Truck	0.0	9.4	0.0	10.4	27.5
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Abha	Truck	45.5	49.1	50.0	48.7	48.9
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
	Truck	47.6	48.3	40.1	49.9	45.5

Yanbu	Pipeline	2.4	1.7	9.9	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	4.5
Riyadh	Truck	50.0	50.0	50.0	50.0	50.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Jeddah	Truck	50.0	11.4	21.1	0.0	28.2
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	38.6	28.9	50.0	21.8
Duba	Truck	11.8	12.0	0.0	11.5	0.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	38.2	38.0	50.0	38.5	50.0
Qassim	Truck	48.7	48.5	50.0	50.0	50.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Alhafuf	Truck	50.0	50.0	50.0	50.0	50.0
	Pipeline	0.0	0.0	0.0	0.0	0.0
	Sea	0.0	0.0	0.0	0.0	0.0
Dhahran	Truck	0.0	0.0	0.0	0.0	0.0
	Pipeline	21.4	50.0	2.3	14.9	14.3
	Sea	28.6	0.0	47.7	35.1	35.7
Qatif	Truck	0.0	0.0	34.6	0.0	0.0
	Pipeline	50.0	16.0	15.4	50.0	0.0
	Sea	0.0	34.0	0.0	0.0	49.2
Saffaniya	Truck	0.0	0.0	0.0	0.0	0.0
	Pipeline	50.0	50.0	50.0	43.5	50.0
	Sea	0.0	0.0	0.0	6.5	0.0

The full results of the model can be found in tables B1 to B3 in Appendix B. The model suggests establishing a pipeline route between Yanbu refinery and Jawf distribution center at the third year (earliest time allowed by the model) with the capacity of 46.8 million barrels. The results indicate that no expansion is required for refineries and distribution centers. The rest of the results show that the model follows a logical approach in which each node distributes to the nodes that are in close proximity to it. The model did not require distribution between distribution centers. Finally, we note the impact of the effective interest rate as the optimal value after solving the model without considering the time value of money is 2.797 billion SR. The full results of the model without the effective interest rate are displayed in Table B7 to B12. The results shows that whilst the objective function value is different from the model with the effective interest rate, the values of the decision variables are very close. For instance, the model also suggests establishing a pipeline route between Yanbu refinery and Jawf distribution center but with a capacity of 49.7 million barrels instead of 46.8 million barrels.

The main advantage of the model is its simplicity from a formulation point of view which makes it more explainable to stakeholders and decision makers. However, the fixed expected growth rate reduces its practicality for strategic planning.

4.5 Penalty Function Approach

As shown in 4.4, the model takes long time to obtain a near optimal solution. The complexity will even increase when applying the two-stage stochastic model. Thus, we apply solution techniques that reduce the model's complexity utilizing the problem

structure or by applying heuristic methods. Of these methods, the penalty function has shown good results in terms of performance and applicability.

The penalty function approach works by relaxing the complicating constraints and adding them to the objective function and multiplying them by penalty parameters. The parameters' values are then increased until these constraints are satisfied. A general mathematical programming problem can be formulated as:

$$\begin{aligned} \min f(x) \\ \text{s.t. } a_i(x) \leq 0, \forall i \in I \end{aligned} \quad (35)$$

Where f and a_i are real-valued smooth functions, and I is a finite set.

Assuming a subset $K \in I$ such that $a_i(x) \leq 0, \forall i \in K$ are the complicating constraints,

$$\begin{aligned} \min f(x) + \sum_{i \in K} \mu_i g_i(a_i(x)) \\ \text{s.t. } a_i(x) \leq 0, \forall i \in I, i \notin K \end{aligned} \quad (36)$$

Where μ_i is the penalty function multiplier, and g_i represents a real-valued penalty function.

There are many types and implementations of the penalty function such as the max function, the square function, and the absolute value function. After experimenting with the model, the max function was found to be the most appealing. The max function can be implemented using this formulation:

$$\begin{aligned}
& \min f(x) + \sum_{i \in K} \mu_i \max(a_i(x), 0) \\
& s. t. \quad a_i(x) \leq 0, \quad \forall i \in I, i \notin K
\end{aligned} \tag{37}$$

After experimenting with the model under study, constraints (19) and (23) were found to be the complicating constraints, and thus, are penalized and added to the objective function.

The model resulted in an optimal solution with a total cost = SR 2.112 billion and an optimality gap of 4.9% and a solution time of 7200 seconds (2 hours). This shows the performance of the penalty function approach as it reduces the solution time to 50%. The full results of the model with penalty function are displayed in Tables B4 to B6 in Appendix B. The results are very similar to the original model and also suggest establishing a pipeline route between Yanbu and Jawf with capacity of 46.09 million barrels. The small variation in the results is because optimality (0% optimality gap) was not reached for both models.

Chapter 5

TWO-STAGE STOCHASTIC MODEL

Stochastic optimization is a well-studied approach in the literature for handling uncertainty. In stochastic optimization, uncertain parameters are assumed to follow a certain distribution and from that the expected objective function is calculated. Stochastic optimization was shown to have good performance when the data follows a certain distribution Azadeh et al., 2017. There are two main approaches for handling uncertain parameters in stochastic optimization: handling the continuous uncertain parameters and discretizing the uncertain parameters values into different scenarios. It was shown that scenario-based stochastic optimization generally yields more tractable models, Lima et al., 2016. A general formulation for a stochastic model can be as:

$$\begin{aligned} \min E[c(\zeta)^T x] \\ \text{s.t. } a_i(\zeta)x \leq b_i(\zeta), \quad \forall i \end{aligned} \tag{38}$$

Where $c \in \mathbb{R}^n$ represents the cost parameters vector, $a_i \in \mathbb{R}^n$ is the coefficients vector, $b_i \in \mathbb{R}$ is the resource value, and ζ represents the distribution/ scenario from which the data was collected.

For the model on-hand, we consider demand as the uncertain parameter. The demand for refined products is random and is dependent on many factors such as the price, policy

changes, and technological advances. Thus, considering all possible realizations of demand is very essential from the strategic perspective.

In this study, we develop a scenario-based two-stage stochastic model to capture the demand uncertainty of the refined products. In the two-stage programming framework, some decisions, first stage decisions, are made before the realization of uncertainty and the second stage decisions are made after the uncertainty is revealed. A general formulation of the scenario-based two-stage stochastic model is as follows:

$$\begin{aligned}
 & \min c_1^T x_1 + E[c_2(s)^T x_2] \\
 & s. t. \quad a_{i1} x_1 \leq b_{i1} \quad \forall i \\
 & \quad \quad a_{j2}(s) x_2 \leq b_{j2}(s) \quad \forall j
 \end{aligned} \tag{39}$$

Where 1,2 are suffixes for the first and second stage decisions respectively, $s \in S$ represent the set of scenarios for the model.

For the model under study, first stage decisions represent the investment decisions of expanding the supply and distribution centers, and expanding and establishing the transportation links. The second stage decisions are the amounts of products distributed between the nodes.

5.1 Two-Stage Stochastic Model Formulation

5.1.1 Objective Function

In the two-stage stochastic model, the objective function represents the present value of the total expected cost across all scenarios $s \in S$ with scenarios probabilities of $pr(s)$:

Minimize $Z =$

$$\left(\sum_{t=3}^T \left[\sum_{j=1}^J CS_j XES_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CR_i XER_{it} \right] \right) * (1 + r)^{-t+2} \quad (40.a)$$

$$+ \sum_{t=3}^T \left[\sum_{m=1}^M \sum_{j=1}^J \left(\sum_{i=1}^I CW A_{ijm} XEW A_{ijmt} + \sum_{k=1}^K CW B_{jkm} XEW B_{jkmt} \right. \right. \\ \left. \left. + \sum_{l=1(l \neq j)}^J CW C_{jlm} XEW C_{jlmt} \right) \right] * (1 + r)^{-t+2} \quad (40.b)$$

$$+ \sum_{t=3}^T \left[\sum_{i=1}^I \sum_{j=1}^J \sum_{m=1}^M CFEW A_{ijmt} YEW A_{ijmt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M CFEW B_{jkm} YEW B_{jkmt} \right. \\ \left. + \sum_{j=1}^J \sum_{l \neq j}^J \sum_{m=1}^M CFEW C_{jlm} YEW C_{jlmt} \right] * (1 + r)^{-t+2} \quad (40.c)$$

$$+ \sum_{t=3}^T \left[\sum_{j=1}^J CFES_{jt} YS_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CFER_{it} YR_{it} \right] * (1 + r)^{-t+2} \quad (40.d)$$

$$\begin{aligned}
& + \sum_{s=1}^S pr(s) \left[\sum_{t=1}^T \left[\sum_{p=1}^P \sum_{m=1}^M \sum_{j=1}^J \left(\sum_{l=1}^I CA_{ijm} XA_{ijmpt}(s) \right. \right. \right. \\
& \quad \left. \left. + \sum_{k=1}^K CB_{jkm} XB_{jkmpt}(s) + \sum_{l=1(l \neq j)}^J CC_{jlm} XC_{jlmpt}(s) \right) \right] * (1+r)^{-t} \right]
\end{aligned}
\tag{40.e}$$

$$\begin{aligned}
& + \sum_{s=1}^S pr(s) \left[\sum_{t=1}^T \left[\sum_{l=1}^I \sum_{j=1}^J \sum_{m=1}^M \sum_{p=1}^P CFWA_{ijmpt} YA_{ijmpt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M \sum_{p=1}^P CFWB_{jkmpt} YB_{jkmpt} \right. \right. \\
& \quad \left. \left. + \sum_{j=1}^I \sum_{l \neq j}^J \sum_{m=1}^M \sum_{p=1}^P CFWC_{jlmpt} YC_{jlmpt} \right] * (1+r)^{-t} \right]
\end{aligned}
\tag{40.f}$$

Where equations (40.a), (40.b), (40.c), and (40.d) correspond to the first stage decisions which are the investment and expansion decisions. Equations (40.e), and (40.f) represent the second stage decisions of distributing and transporting the products.

Note that the term $(1+r)^{-t}$ is used to return the costs to the present value and the term $(1+r)^{-t+2}$ is for expansion projects that require two years to complete and are paid after the project end as mentioned in assumption 17.

5.1.2 Constraints

5.1.2.1 Supply Centers (Refineries) Capacities

Sum of all outputs in scenario s from SC_i cannot exceed its capacity + expanded capacity

$$\sum_{j=1}^J \sum_{p=1}^P \sum_{m=1}^M XA_{ijmpt}(s) \leq R_i + \sum_{\tau=3}^t XER_{i\tau},, \quad (41)$$

$$i = 1, \dots, I, t = 1, \dots, T, s = 1, \dots, S$$

Equation (42) ensures that total capacity expansions for a SC can not exceed the maximum allowed quantity ERU_i .

$$\sum_{t=3}^T XER_{it} \leq ERU_i, \quad (42)$$

$$i = 1, \dots, I,$$

Equation (43) is a logical constraint to ensure $YR_{it} = 1$ if $XER_{it} > 0$.

$$XER_{it} \leq ERU_i YR_{it} \quad (43)$$

$$i = 1, \dots, I, t = 3, \dots, T$$

SC_i expansion is above the minimum allowed capacity.

$$XER_{it} \geq ERL_i YR_{it}, \quad (44)$$

$$i = 1, \dots, I, t = 3, \dots, T$$

Number of expansions of SC_i cannot exceed NR .

$$\sum_{t=3}^T YR_{it} \leq NR, \quad (45)$$

$$i = 1, \dots, I,$$

5.1.2.2 Distribution Centers (Bulk Plants) Capacities

Sum of all inputs in scenario s of product p into DC_j cannot exceed its current + expanded capacity

$$\sum_{i=1}^I \sum_{p=1}^P \sum_{m=1}^M XA_{ijmpt}(s) + \sum_{l=1, l \neq j}^J \sum_{p=1}^P \sum_{m=1}^M XC_{ljmpt}(s) \leq S_j + \sum_{\tau=3}^t XES_{j\tau}, \quad (46)$$

$$j = 1, \dots, J, t = 1, \dots, T, s = 1, \dots, S$$

Equation (47) ensures that total capacity expansions for a DC can not exceed the maximum allowed quantity ESU_j .

$$\sum_{t=3}^T XES_{jt} \leq ESU_j, \quad (47)$$

$$j = 1, \dots, J,$$

Equation (48) is a logical constraint to ensure $YS_{jt} = 1$ if $XES_{jt} > 0$.

$$XES_{jt} \leq EU_j YS_{jt} \quad (48)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

DC_j expansion is above the minimum allowed capacities.

$$XES_{jt} \geq EL_j YS_{jt}, \quad (49)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

Number of expansions of DC_j cannot exceed NS .

$$\sum_{t=3}^T YS_{jt} \leq NS, \quad (50)$$

$$j = 1, \dots, J,$$

5.1.2.3 Distribution Center Material Balance

Sum of product p inputs ($A + C$) under scenario s into DC_j is equal to sum of its outputs ($B + C$)

$$\sum_{m=1}^M \left(\sum_{i=1}^I XA_{ijmpt}(s) + \sum_{l \neq j} XC_{ljmpt}(s) \right) = \sum_{m=1}^M \left[\sum_{k=1}^K XB_{jkmpt}(s) + \sum_{l \neq j} XC_{jlmpt}(s) \right], \quad (51)$$

$$j = 1, \dots, J, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

5.1.2.4 Demand Markets (Cities)

Sum of inputs of product p under scenario s into DM_k must satisfy its demand

$$\sum_{j=1}^J \sum_{m=1}^M XB_{jkmpt}(s) \geq D_{kpt}, \quad (52)$$

$$k = 1, \dots, K, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

5.1.2.5 Transportation Capacity from SC to DC

Amounts of all products transported by mode m between SC_i and DC_j under scenario s cannot exceed link i - j mode m capacity

$$\sum_{p=1}^P XA_{ijmpt}(s) \leq WA_{ijm} + \sum_{\tau=3}^t XEW A_{ijm\tau}, \quad (53)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 1, \dots, T, s = 1, \dots, S$$

Expansion of link i - j cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWA_{ijmt} \leq EWAU_{ijm}, \quad (54)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (55) is a logical constraint to ensure $YEWA_{ijmt} = 1$ if $XEWA_{ijmt} > 0$.

$$XEWA_{ijmt} \leq EWAU_{ijm} YEWA_{ijmt}, \quad (55)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link i - j is above the minimum allowed capacity

$$XEWA_{ijmt} \geq EWAL_{ijm} YEWA_{ijmt}, \quad (56)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link i - j cannot exceed NW

$$\sum_{t=3}^T YEWA_{ijmt} \leq NW, \quad (57)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (58) is a logical constraint to ensure $YA_{ijmpt}(s) = 1$ if $XA_{ijmpt}(s) > 0$.

$$XA_{ijmpt}(s) \leq (WA_{ijm} + EWAU_{ijm}) YA_{ijmpt}(s), \quad (58)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

Amounts transported between i and j under scenario s is above the minimum allowed quantities.

$$XA_{ijmpt}(s) \geq WAL_{ijmp}YA_{ijmpt}(s), \quad (59)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, \quad t = 1, \dots, T, \quad s = 1, \dots, S$$

5.1.2.6 Transportation Capacity from DC to DM

Amounts of all products transported by mode m between DC_j and DM_k in scenario s cannot exceed link j - k mode m capacity

$$\sum_{p=1}^P XB_{jkmpt}(s) \leq WB_{jkm} + \sum_{\tau=3}^t XEWB_{jkm\tau}, \quad (60)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, \quad t = 1, \dots, T, s = 1, \dots, S$$

Expansion of link j - k cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWB_{jkm\tau} \leq EWB_{jkm}, \quad (61)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M$$

Equation (62) is a logical constraint to ensure $YEWB_{jkm\tau}$ is = 1 if $XEWB_{jkm\tau} > 0$.

$$XEWB_{jkm\tau} \leq EWB_{jkm}YEWB_{jkm\tau}, \quad (62)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 4, \dots, T$$

Expansion of link $j-k$ is above the minimum capacity expansion.

$$XEWB_{jkmt}(s) \geq EWBL_{jkm}YEWB_{jkmt}, \quad (63)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 4, \dots, T, s = 1, \dots, S$$

Number of expansions of link $j-k$ cannot exceed NW

$$\sum_{t=3}^T YEWB_{jkmt} \leq NW, \quad (64)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M$$

Equation (65) is a logical constraint to ensure $YB_{jkmp}(s)$ is = 1 if $XB_{jkmp}(s) > 0$.

$$XB_{jkmp}(s) \leq (WB_{jkm} + EWBU_{jkm})YB_{jkmp}(s), \quad (65)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

Transportation in link $j-k$ in scenario s is above the minimum quantity.

$$XB_{jkmp}(s) \geq WBL_{jkmp}YB_{jkmp}(s), \quad (66)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

Transportation Capacity from DC to DC

Amounts of all products transported by mode m between DC_j and DC_l in scenario s cannot exceed link $j-l$ mode m capacity

$$\sum_{p=1}^P XC_{jlmpt}(s) \leq WC_{jlm} + \sum_{\tau=3}^t XEW C_{jlm\tau}, \quad (67)$$

$$j = 1, \dots, J, \quad l = 1, \dots, J \ (l \neq j), \quad m = 1, \dots, M, \quad t = 1, \dots, T, \quad s = 1, \dots, S$$

Expansion of link j - l cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEW C_{jlm t} \leq EWC U_{jlm}, \quad (68)$$

$$j = 1, \dots, J, \quad l = 1, \dots, J \ (l \neq j), \quad m = 1, \dots, M,$$

Equation (69) is a logical constraint to ensure $YEW C_{jlm t}$ is = 1 if $XEW C_{jlm t} > 0$:

$$XEW C_{jlm t} \leq EWC Y_{jlm} YEW C_{jlm t}, \quad (69)$$

$$j = 1, \dots, J, \quad l \ (l \neq j) = 1, \dots, J, \quad m = 1, \dots, M, \quad t = 3, \dots, T$$

Expansion of link j - l is above the minimum capacity expansion.

$$XEW C_{jlm t} \geq EWC L_{jlm} YEW C_{jlm t}, \quad (70)$$

$$j = 1, \dots, J, \quad l \ (l \neq j) = 1, \dots, J, \quad m = 1, \dots, M, \quad t = 3, \dots, T$$

Number of expansions of link j - l cannot exceed NW

$$\sum_{t=3}^T YEW C_{jlm t} \leq NW, \quad (71)$$

$$j = 1, \dots, J, \quad l = 1, \dots, J \ (l \neq j), \quad m = 1, \dots, M,$$

Equation (72) is a logical constraint to ensure $YC_{jlmpt}(s)$ is = 1 if $XC_{jlmpt}(s) > 0$

$$XC_{jlmpt}(s) \leq (WC_{ijm} + EWC_{U_{jlm}})YC_{jlmpt}(s), \quad (72)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

Transportation in link j - l under scenario s is above the minimum quantity.

$$XC_{jlmpt}(s) \geq WCL_{jlmpt}YC_{jlmpt}(s), \quad (73)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T, s = 1, \dots, S$$

5.2 Scenarios Generation

There are several approaches to generate the scenarios for stochastic optimization, Al-Othman et al., 2008 considered varying the uncertain parameters by $\pm 20\%$ and calculating the Expected Value of Imperfect Information EVPI to find the difference between the optimum costs between the stochastic model and the deterministic models with $\pm 20\%$ variation in the uncertain parameters. Leiras et al., 2013 generated a scenario tree with 9 scenarios based on expert's judgment and historical data. Zhao et al., 2017 followed a simple scenarios generation approach with three levels of realizations for each uncertain parameter with their respective probabilities.

However, these approaches result in a limited number of scenarios per parameter, which can lead to an inaccurate solution. As the number of scenarios increases, the obtained solution approaches the continuous stochastic model solution. However, as the number of scenarios increases, the model complexity increases, and an optimal solution may not be

found. Zang et al., 2020 proposed dividing the whole probability space for a parameter into N parts and then solving for different values of N until the optimal value approaches a certain limit.

Following the approach of . Zang et al., 2020, we discretize the probability space of the demand growth rate and calculate two measures for N, the optimal solution, and the optimality gap after 8 hours of running the model. The data in Table 11 (SAMA, 2021), shows the demand values and the yearly demand growth rates of refined products in Saudi Arabia from 1991 to 2020. The data was taken back to 1991 to allow for estimating a distribution to the data with higher certainty.

Table 11: Demands and Growth Rates of Refined Products in Saudi Arabia from1991-2020

Year	Demand	Growth Rate
1991	485,592	0.98
1992	524,365	7.98
1993	557,409	6.30
1994	587,735	5.44
1995	588,300	0.10
1996	625,478	6.32
1997	636,794	1.81
1998	673,849	5.82
1999	685,287	1.70
2000	711,704	3.85
2001	737,639	3.64
2002	772,441	4.72
2003	807,792	4.58

2004	863,862	6.94
2005	923,928	6.95
2006	956,315	3.51
2007	1,018,684	6.52
2008	1,110,703	9.03
2009	1,151,245	3.65
2010	1,258,929	9.35
2011	1,321,411	4.96
2012	1,408,590	6.60
2013	1,423,796	1.08
2014	1,516,804	6.53
2015	1,586,498	4.59
2016	1,590,300	0.24
2017	1,609,939	1.23
2018	1,562,274	-2.96
2019	1,566,118	0.25
2020	1,527,753	-2.45

As can be seen from Figure 7, the histogram of the yearly demand growth rate resembles a normal distribution histogram. In addition, from the normal probability plot in Figure 8, we can conclude that the data follows a normal distribution ($\mu = 3.98$ and $\sigma = 3.18$) with high value of $R^2 = 0.97$.

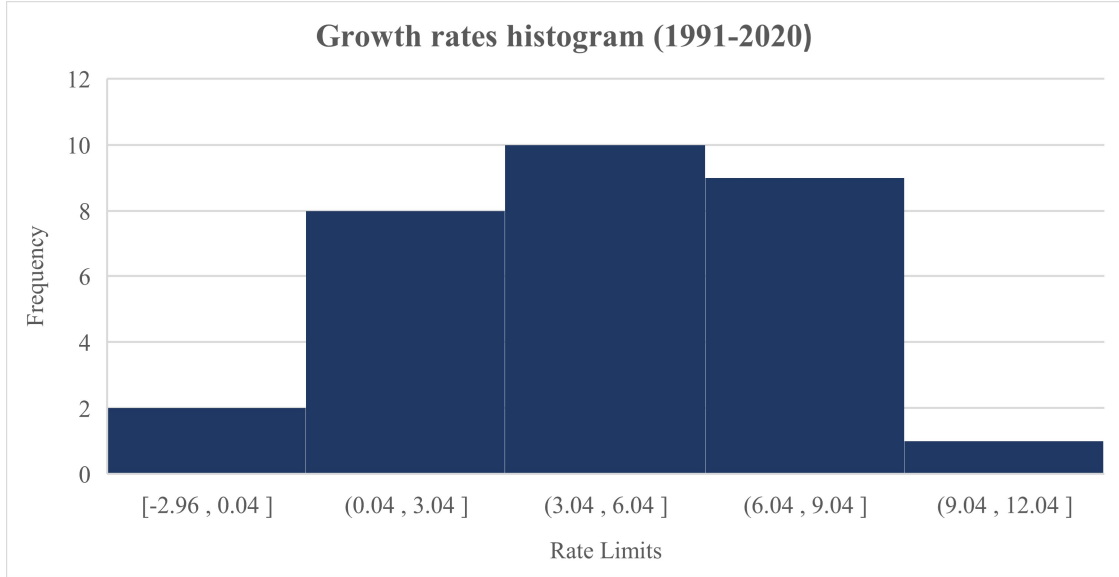


Figure 7: Growth Rates Histogram (1991-2020)

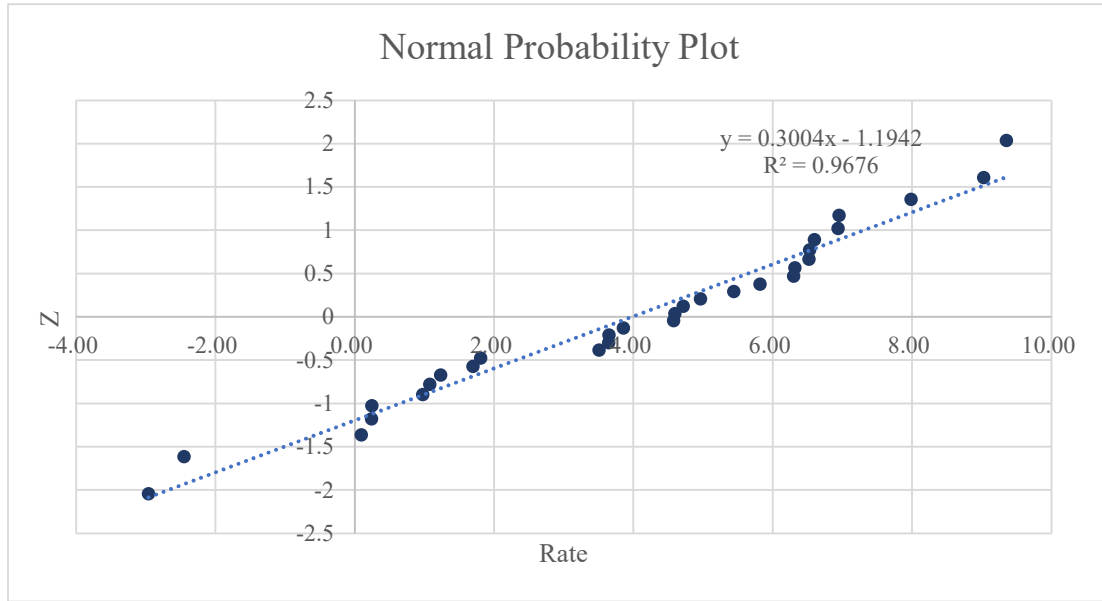


Figure 8: Normal Probability Plot of demand growth rate

To discretize the probability space of the demand growth rate, we make use of the symmetry of the Normal distribution. Consider for example the case for $N = 5$, we can use $\delta = \sigma$ to discretize the probability space to 5 segments as shown in Figure 9.

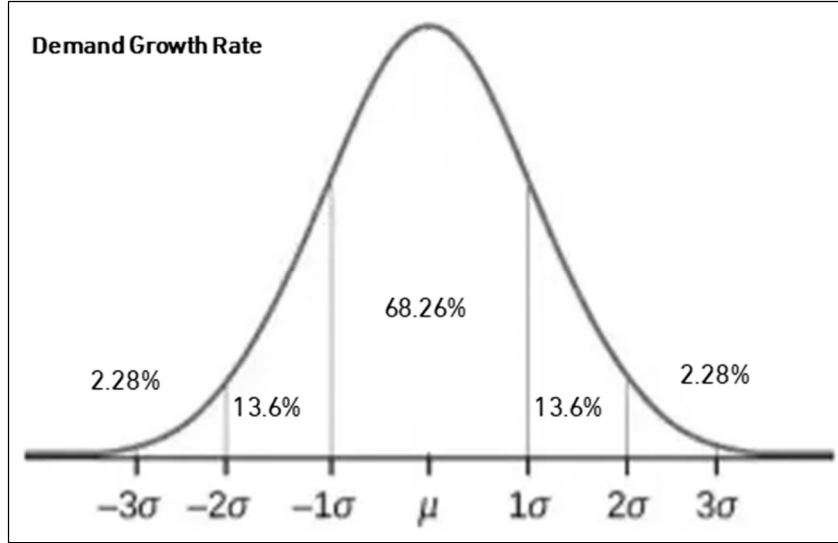


Figure 9: Demand growth rate discretization for $N=5$

As we increase the value of N , we approach the optimal solution for the continuous stochastic problem. However, as N increases, the computational complexity increases, and the model may require long time to execute. Thus, a proper value of N should be obtained to balance solution quality and computational complexity. To determine the value of N , the model was run with values of N ranging from 1 to 13 for 8 hours or until the optimality is below 5%. We select odd values for N to always reflect the mean demand growth rate at one scenario. Table 12 shows the model statistics for the executed models in addition to the results of the optimal solution and optimality gap.

Table 12: Models Statistics and Results for Different Values of N

N	1	3	5	7	9	11	13
Constraints	96,959	222,369	347,779	473,189	598,599	724,009	849,419
Variables	65,252	151,952	238,652	325,352	412,052	498,752	585,452
Binary Variables	28,569	72,429	116,289	160,149	204,009	247,869	291,729
Objective Function (Billions SR)	2.35	2.65	2.98	3.17	3.27	3.41	3.55
Optimality Gap	4.80%	4.95 %	6.30%	5.70%	4.76 %	6.40%	19.80%

Figure 10 shows the optimal solutions and optimality gaps of the model runs with different values for N , it can be seen that the optimal solution trend starts to stabilize at $N = 9$. However, at $N = 13$, there is a leap in the optimality gap. Thus, we consider $N = 11$ as the proper value for N to balance both the optimal solution and computational complexity.

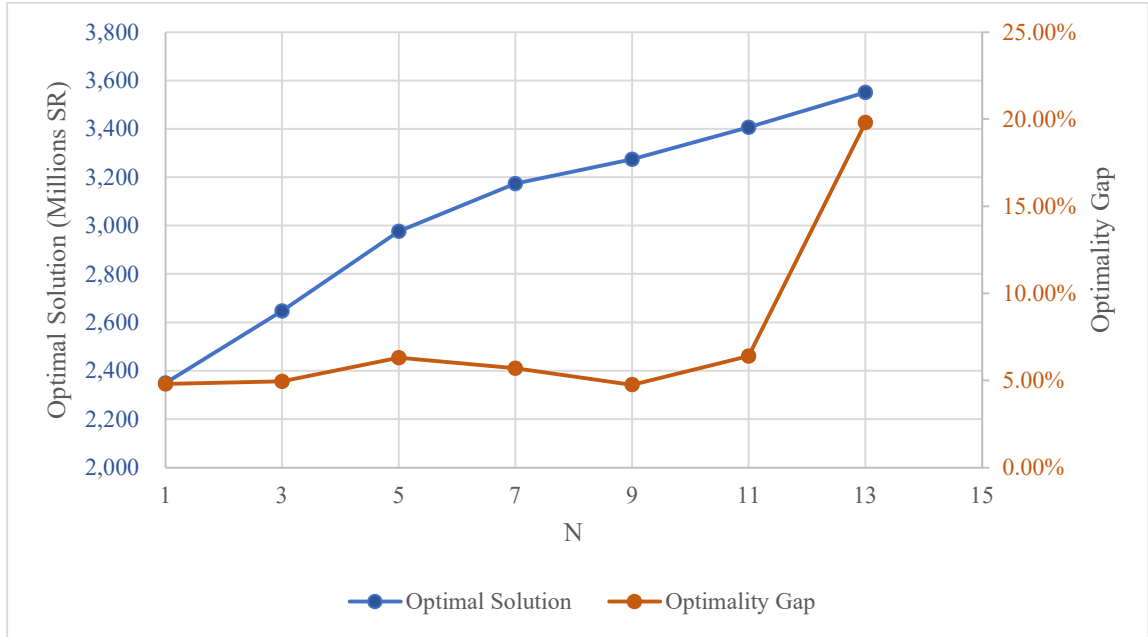


Figure 10: Optimal Solutions and Optimality Gaps for Different Values of N

5.3 Results and Analysis

The model was coded in GAMS 24.7.4 and was solved using the CPLEX solver with implementing the penalty function approach discussed in 4.5. The code used to run the model is attached in Appendix F. A near optimal solution is found to be = SR 3.373 billion. The relative gap from the best possible solution is 5%. The solution time = 42000 seconds (12 hours). The model statistics and results are displayed in Table 13. The optimal solution of the two-stage stochastic model is 59% higher than it for the deterministic model because the model is solving for high demand scenarios.

Table 13: Two-stage Stochastic Model Summary

	Number of Constraints	Number of Variables	Number of Binary Variables
Full Model	724,009	498,752	247,869

The total consumed amounts of each product under the different scenarios are displayed in Figure 11, Table 14 shows the breakdown of these amounts by year. A smooth increasing trend can be depicted due to the higher growth rates. Figure 12 shows the amounts distributed by each transportation mode under the different scenarios across the planning horizon. Unlike the deterministic model, the amounts distributed from refineries to distribution centers through sea, with total distribution across scenarios = 16.2 billion barrels, are higher than trucks, with total distribution across scenarios = 15.4 billion with variations through different scenarios. This is because of the high volume considered in this model for high demand scenarios, which leads to choosing the sea mode to reduce the

distribution costs. It's also worth noting that there is no clear link between the scenario and the used modes for transportation.

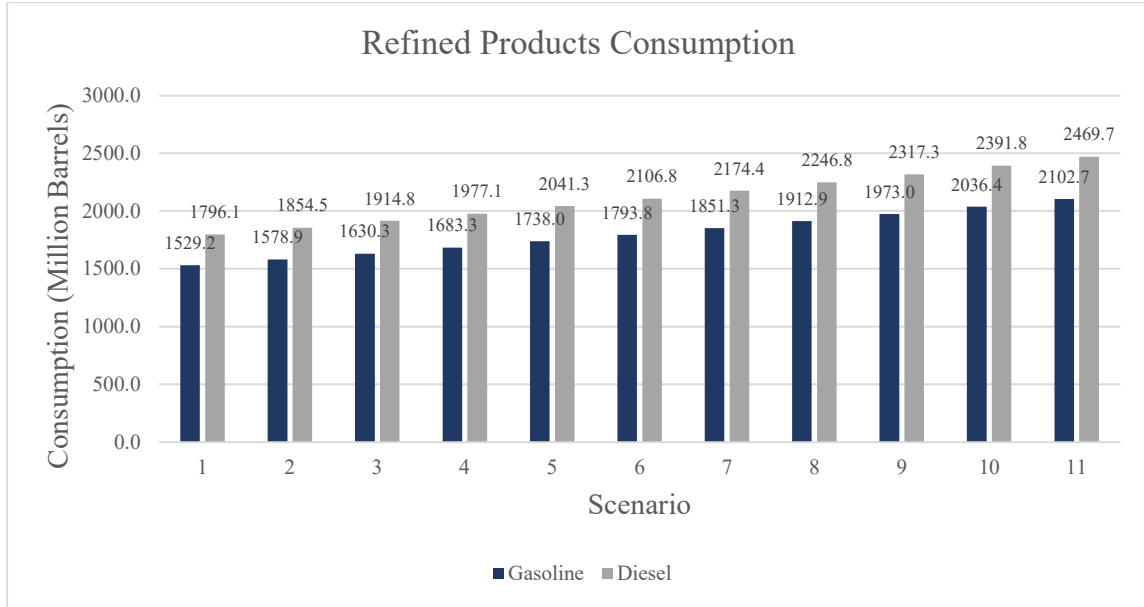


Figure 11: Products Consumption Per Scenario

Table 14: Breakdown of Products Consumption Per Year

Product	Year/Scenario	1	2	3	4	5	6	7	8	9	10	11
Gasoline	1	331.3	331.3	331.3	331.3	331.3	331.3	331.3	331.3	331.3	331.3	331.3
	2	318.1	323.4	328.7	334.0	339.3	344.5	349.7	355.2	360.3	365.6	370.9
	3	305.3	315.6	326.0	336.6	347.4	358.2	369.2	380.7	391.8	403.4	415.2
	4	293.1	308.0	323.4	339.3	355.7	372.5	389.7	408.2	426.1	445.1	464.8
	5	281.4	300.6	320.8	342.0	364.3	387.3	411.4	437.5	463.4	491.1	520.4
Diesel	1	389.1	389.1	389.1	389.1	389.1	389.1	389.1	389.1	389.1	389.1	389.1
	2	373.6	379.8	386.0	392.2	398.5	404.6	410.8	417.2	423.2	429.4	435.6
	3	358.6	370.7	382.9	395.4	408.0	420.7	433.6	447.2	460.2	473.8	487.7
	4	344.3	361.8	379.9	398.5	417.8	437.5	457.7	479.4	500.5	522.8	546.0
	5	330.5	353.1	376.8	401.7	427.9	454.9	483.2	513.9	544.3	576.8	611.2

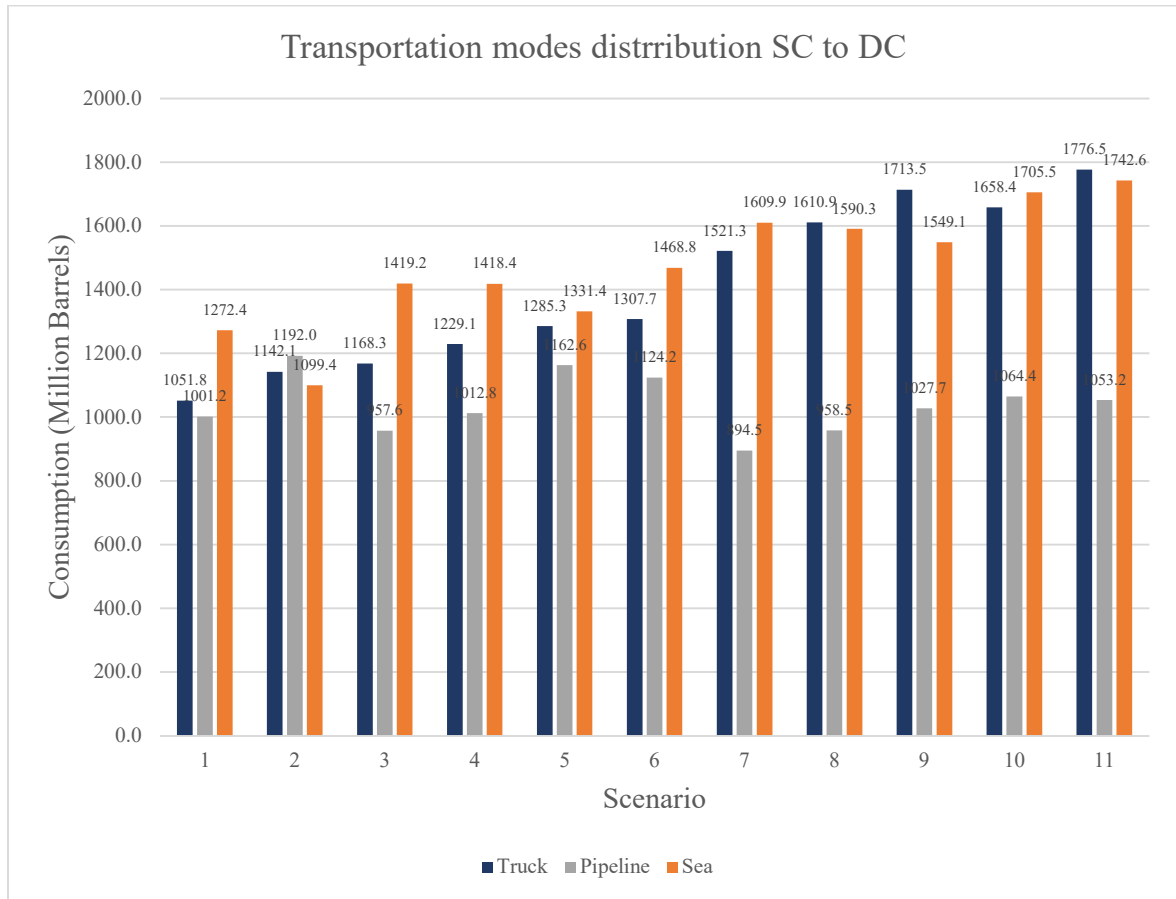


Figure 12: Transportation Modes Per Scenario (SC to DC)

Samples of the full model results are displayed in Tables C1 to C10 in Appendix C. As shown in Table C2, the optimal solution of the two-stage stochastic model suggests transporting products between some distribution centers. Figure 13 displays amounts transported between distribution centers under each scenario. It can be seen that most of this distribution occurred through sea (256.7 million barrels out of 324.6 million barrels) which could be due to the low fixed and variable costs. In addition, we can generally see that as we move to higher scenarios, these amounts decrease because the high demand

values means that distribution centers capacities will be used to satisfy the near demand markets only.

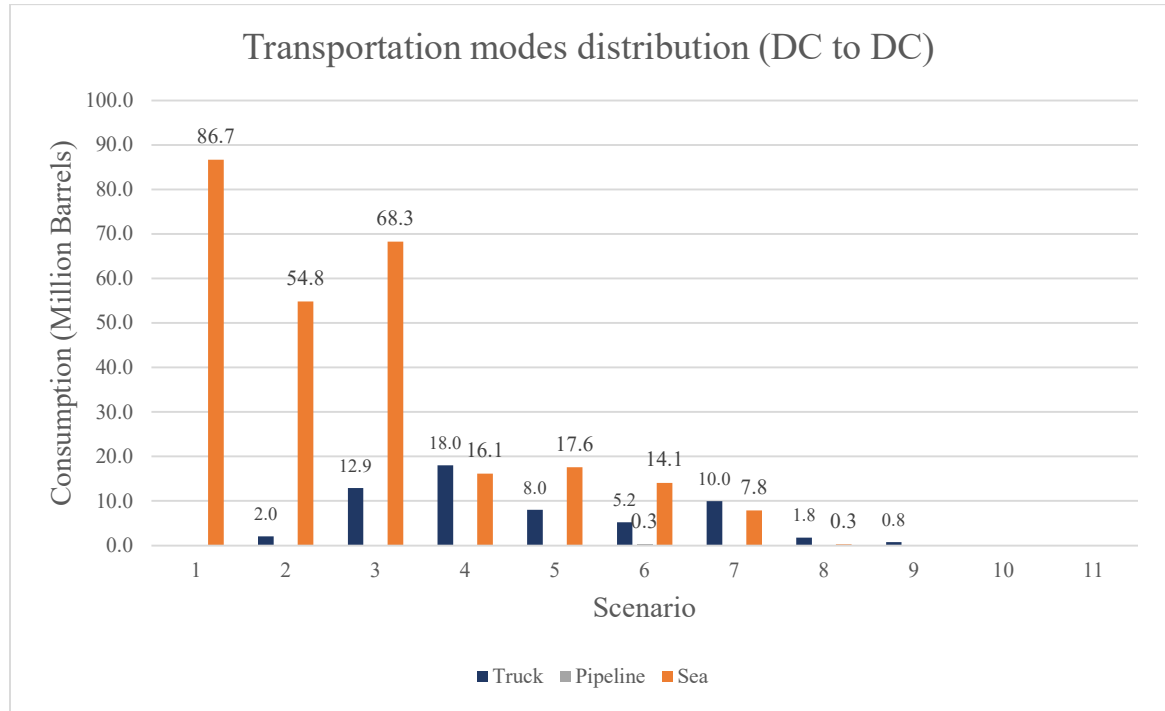


Figure 13: Transportation Modes Distribution (DC to DC)

Figure 14 displays the amounts transported between distribution centers and demand markets by transportation mode under the different scenarios. We can see that, similar to the deterministic model, trucks are the most dominant mode used. In addition, in this model, we note that pipelines were used more than trucks to distribute the products to the cities. This is because most of demand markets do not have access to sea.

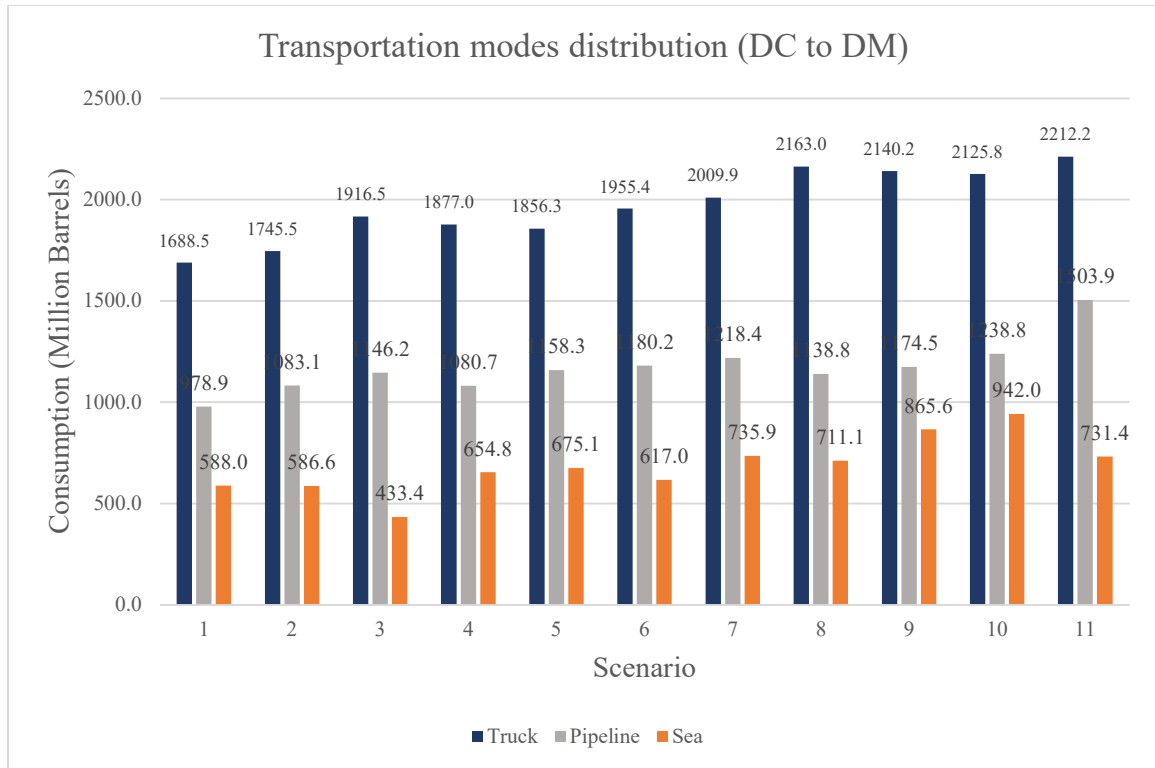


Figure 14: Transportation Modes Distribution (DC to DM)

The model suggests expanding two refineries at the fifth year, namely Riyadh (20.5 million barrels) and Jeddah (50 million barrels). The model selects the fifth year because the total demand at the fifth year for high scenarios exceeds the total capacity of all the refineries and the fifth year leads to the lowest net present value. The model also suggests expanding three distribution centers, namely Rabigh (94.7 million barrels), Qatif (100 million barrels) at the third year, and Yanbu (86.9 million barrels) at the fourth year. The three distribution centers have sea transportation and were used to transport the products between distribution centers. The model does not suggest establishing or expanding any of the transportation links.

The main advantage of the model is that it considers wide range for the expected demand growth rate covering data of 30 years. This is especially useful in the two-stage model as it adjusts by expanding the nodes and links on the first stage for all the possible scenarios. On the other hand, the high number of considered scenarios adds to the complexity of the problem and increases the solution time. In addition, some considered scenarios might be anomalies which cause high variations and increase the optimal total cost drastically.

Chapter 6

ROBUST OPTIMIZATION

Robust optimization is another approach for handling uncertainty via the use of uncertainty sets. The optimal solution in robust optimization is feasible to all possible realizations of the uncertain parameter within the uncertainty set. This approach is more applicable in many real-world applications since the distribution of parameters' values is not assumed. In addition, robust optimization often leads to more tractable and computationally efficient models than stochastic optimization.

The general formulation of the robust optimization is as follows:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & a_i(u_i)x \leq b_i, \quad \forall u_i \in U, \forall i \end{aligned} \tag{74}$$

Where $u_i \in \mathbb{R}^k$ represent the uncertainty set of the coefficients. The goal of formulation (74) is to find the optimal solution that satisfies all possible realizations of the uncertainty set U .

Formulation (74) can have infinitely many constraints depending on the problem formulation and the type of uncertainty set. In general, Robust optimization problems are intractable, however, for specific formulations and uncertainty sets, an efficient solution can be obtained.

6.1 Uncertainty Sets

In this study, we consider the uncertainty set to be polyhedral. When U is polyhedral and the associated decision variables within U are linear, the resulting sup-problem is still linear. Consider formulation (74) with polyhedral uncertainty set:

$$\begin{aligned} \min c^T x \\ s. t. \quad \max_{V_i a_i \leq v_i} a_i x \leq b_i, \quad \forall i \end{aligned} \quad (75)$$

Where $V_i \in \mathbb{R}^{n \times n}$ and $v_i \in \mathbb{R}^n$, are the coefficients of the polyhedral set.

Solving the problem requires solving the sub-problem:

$$\begin{aligned} \max a_i^T x \\ s. t. \quad V_i a_i \leq v_i \quad \forall i \end{aligned} \quad (76)$$

Considering the dual of sub-problem (76):

$$\begin{aligned} \min p_i^T v_i \\ s. t. \quad p_i V_i = x, \quad \forall i \\ p_i \geq 0, \quad \forall i \end{aligned} \quad (77)$$

And thus, the robust optimization problem becomes:

$$\begin{aligned} \min c^T x \\ s. t. \quad p_i v_i \leq b_i, \quad \forall i \end{aligned}$$

$$p_i V_i = x, \quad \forall i$$

$$p_i \geq 0, \quad \forall i \tag{78}$$

Which is linear and can be solved in a polynomial time.

Note that when the uncertain parameter is b_i , i.e., on the right side of the constraint, a new variable $x_{n+1} = 1$, can be introduced to bring b_i to the left side and equate the sum to 0.

For the problem under study, we assume an upper limit for the demand value D_{kpt} that is obtained from the historical growth rate data in Table 11 (SAMA,2021). That is

$$D_{kpt}^{upper\ limit} = D_{kp} * (1 + maximum\ growth\ rate)^{t-1},$$

$$D_{kpt}^{upper\ limit} = D_{kp1} * (1 + 0.094)^{t-1}$$

And thus, we have the coefficients of the polyhedral uncertainty set as $V_{kpt} = I \in$

$$\mathbb{R}^{k.p.t \times k.p.t} \text{ and } v_{kpt} = D_{kpt}^{upper\ limit}.$$

6.2 Robust Model Formulation

6.2.1 Objective Function

The objective function of the robust model is the same as the objective function of the deterministic model.

$$Minimize\ Z =$$

$$\sum_{t=1}^T \left[\sum_{p=1}^P \sum_{m=1}^M \sum_{j=1}^J \left(\sum_{i=1}^I CA_{ijm} XA_{ijmpt} + \sum_{k=1}^K CB_{jkm} XB_{jkmpt} + \sum_{l=1(l \neq j)}^J CC_{jlm} XC_{jlmpt} \right) \right]$$

$$* (1 + r)^{-t}$$

(79. a)

$$+ \left(\sum_{t=3}^T \left[\sum_{j=1}^J CS_j XES_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CR_i XER_{it} \right] \right) * (1 + r)^{-t+2}$$

(79. b)

$$+ \sum_{t=3}^T \left[\sum_{m=1}^M \sum_{j=1}^J \left(\sum_{i=1}^I CWA_{ijm} XEWA_{ijmt} + \sum_{k=1}^K CWB_{jkm} XEWB_{jkm} \right. \right. \\ \left. \left. + \sum_{l=1(l \neq j)}^J CWC_{jlm} XEWC_{jlm} \right) \right] * (1 + r)^{-t+2}$$

(79. c)

$$+ \sum_{t=3}^T \left[\sum_{i=1}^I \sum_{j=1}^J \sum_{m=1}^M CFEWA_{ijmt} YEWA_{ijmt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M CFEWB_{jkm} YEWB_{jkm} \right. \\ \left. + \sum_{j=1}^J \sum_{l \neq j}^J \sum_{m=1}^M CFEWC_{jlm} YEWC_{jlm} \right] * (1 + r)^{-t+2} \quad (79. d)$$

$$+ \sum_{t=3}^T \left[\sum_{j=1}^J CFES_{jt} YS_{jt} \right] + \sum_{t=3}^T \left[\sum_{i=1}^I CFER_{it} YR_{it} \right] * (1 + r)^{-t+2} \quad (79. e)$$

$$\begin{aligned}
& + \sum_{t=1}^T \left[\sum_{i=1}^I \sum_{j=1}^J \sum_{m=1}^M \sum_{p=1}^P CFW A_{ijmpt} Y A_{ijmpt} + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M \sum_{p=1}^P CFW B_{jkmpt} Y B_{jkmpt} \right. \\
& \quad \left. + \sum_{j=1}^I \sum_{l \neq j}^J \sum_{m=1}^M \sum_{p=1}^P CFW C_{jlmpt} Y C_{jlmpt} \right] * (1 + r)^{-t}
\end{aligned}
\tag{79.f}$$

6.2.2 Constraints

6.2.2.1 Supply Centers (Refineries) Capacities

Sum of all outputs from SC_i cannot exceed its capacity + expanded capacity.

$$\sum_{j=1}^J \sum_{p=1}^P \sum_{m=1}^M X A_{ijmpt} \leq R_i + \sum_{\tau=3}^t XER_{i\tau},,
\tag{80}$$

$$i = 1, \dots, I, t = 1, \dots, T$$

Equation (81) ensures that total capacity expansions for a SC can not exceed the maximum allowed quantity ERU_i .

$$\sum_{t=3}^T XER_{it} \leq ERU_i,
\tag{81}$$

$$i = 1, \dots, I.$$

Equation (82) is a logical constraint to ensure $YR_{it} = 1$ if $XER_{it} > 0$.

$$XER_{it} \leq ERU_i YR_{it}
\tag{82}$$

$$i = 1, \dots, I, t = 3, \dots, T.$$

SC_i expansion is above the minimum allowed capacity.

$$XER_{it} \geq ERL_i YR_{it}, \quad (83)$$

$$i = 1, \dots, I, \quad t = 3, \dots, T.$$

Number of expansions of SC_i cannot exceed NR .

$$\sum_{t=3}^T YR_{it} \leq NR, \quad (84)$$

$$i = 1, \dots, I.$$

6.2.2.2 Distribution Centers (Bulk Plants) Capacities

Sum of all inputs of product p into DC_j cannot exceed its current + expanded capacity.

$$\sum_{i=1}^I \sum_{p=1}^P \sum_{m=1}^M XA_{ijmpt} + \sum_{l=1, l \neq j}^J \sum_{p=1}^P \sum_{m=1}^M XC_{ljmpt} \leq S_j + \sum_{\tau=3}^t XES_{j\tau}, \quad (85)$$

$$j = 1, \dots, J, \quad t = 1, \dots, T$$

Equation (86) ensures that total capacity expansions for a DC can not exceed the maximum allowed quantity ESU_j .

$$\sum_{t=3}^T XES_{jt} \leq ESU_j, \quad (86)$$

$$j = 1, \dots, J.$$

Equation (87) is a logical constraint to ensure $YS_{jt} = 1$ if $XES_{jt} > 0$.

$$XES_{jt} \leq EU_j YS_{jt} \quad (87)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

DC_j expansion is above the minimum allowed capacities.

$$XES_{jt} \geq EL_j YS_{jt}, \quad (88)$$

$$j = 1, \dots, J, t = 3, \dots, T$$

Number of expansions of DC_j cannot exceed NS.

$$\sum_{t=3}^T YS_{jt} \leq NS, \quad (89)$$

$$j = 1, \dots, J,$$

6.2.2.3 Distribution Center Material Balance

Sum of product p inputs ($A + C$) into DC_j is equal to sum of its outputs ($B + C$).

$$\sum_{m=1}^M \left(\sum_{i=1}^I XA_{ijmpt} + \sum_{l \neq j} XC_{ljmpt} \right) = \sum_{m=1}^M \left[\sum_{k=1}^K XB_{jkmpt} + \sum_{l \neq j} XC_{jlmpt} \right], \quad (90)$$

$$j = 1, \dots, J, \quad p = 1, \dots, P, \quad t = 1, \dots, T$$

6.2.2.4 Demand Markets (Cities)

We introduce a new variable $x_v = 1$ for the right-side terms in equation (91) and split the polyhedral uncertainty set coefficients into two parts $[p_{jkmpt}^1, V_{jkmpt}^1, v_{jkmpt}^1]$ and $[p_{kpt}^2, V_{kpt}^2, v_{kpt}^2]$ for the right and left sides respectively.

$$\sum_{j=1}^J \sum_{m=1}^M p_{jkmpt}^1 * v_{jkmpt}^1 - p_{kpt}^2 * v_{kpt}^2 \geq 0, \quad (91)$$

$$k = 1, \dots, K, p = 1, \dots, P, t = 1, \dots, T$$

$$p_{jkmpt}^1 * V_{jkmpt}^1 = XB_{jkmpt} \quad (92)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

$$p_{kpt}^2 * V_{kpt}^2 = x_v \quad (93)$$

$$k = 1, \dots, K, p = 1, \dots, P, t = 1, \dots, T$$

6.2.2.5 Transportation Capacity from SC to DC

Amounts of all products transported by mode m between SC_i and DC_j cannot exceed link i - j mode m capacity.

$$\sum_{p=1}^P XA_{ijmpt} \leq WA_{ijm} + \sum_{\tau=3}^t XEWA_{ijm\tau}, \quad (94)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 1, \dots, T$$

Expansion of link i - j cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWA_{ijmt} \leq EWAU_{ijm}, \quad (95)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (96) is a logical constraint to ensure $YEWA_{ijmt} = 1$ if $XEWA_{ijmt} > 0$.

$$XEW A_{ijmt} \leq EWAU_{ijm} YEW A_{ijmt}, \quad (96)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link i - j is above the minimum allowed capacity

$$XEW A_{ijmt} \geq EWAL_{ijm} YEW A_{ijmt}, \quad (97)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link i - j cannot exceed NW

$$\sum_{t=3}^T YEW A_{ijmt} \leq NW, \quad (98)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M,$$

Equation (99) is a logical constraint to ensure $YA_{ijmpt} = 1$ if $XA_{ijmpt} > 0$.

$$XA_{ijmpt} \leq (WA_{ijm} + EWAU_{ijm}) YA_{ijmpt}, \quad (99)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Amounts transported between i and j is above the minimum allowed quantities.

$$XA_{ijmpt} \geq WAL_{ijmp} YA_{ijmpt}, \quad (100)$$

$$i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

6.2.2.6 Transportation Capacity from DC to DM

Amounts of all products transported by mode m between DC_j and DM_k cannot exceed link j - k mode m capacity

$$\sum_{p=1}^P XB_{jkmpt} \leq WB_{jkm} + \sum_{\tau=3}^t XEWB_{jkm\tau}, \quad (101)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 1, \dots, T$$

Expansion of link j - k cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEWB_{jkm\tau} \leq EWBU_{jkm}, \quad (102)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M$$

Equation (103) is a logical constraint to ensure $YEWB_{jkm\tau}$ is = 1 if $XEWB_{jkm\tau} > 0$.

$$XEWB_{jkm\tau} \leq EWBU_{jkm} YEWB_{jkm\tau}, \quad (103)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 4, \dots, T$$

Expansion of link j - k is above the minimum capacity expansion.

$$XEWB_{jkm\tau} \geq EWBL_{jkm} YEWB_{jkm\tau}, \quad (104)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, t = 4, \dots, T$$

Number of expansions of link j - k cannot exceed NW

$$\sum_{t=3}^T YEWB_{jkm t} \leq NW, \quad (105)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M$$

Equation (106) is a logical constraint to ensure YB_{jkmpt} is = 1 if $XB_{jkmpt} > 0$.

$$XB_{jkmpt} \leq (WB_{jkm} + EWBU_{jkm})YB_{jkmpt}, \quad (106)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Transportation in link j - k is above the minimum quantity.

$$XB_{jkmpt} \geq WBL_{jkm p}YB_{jkmpt}, \quad (107)$$

$$j = 1, \dots, J, k = 1, \dots, K, m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

6.2.2.7 Transportation Capacity from DC to DC

Amounts of all products transported by mode m between DC_j and DC_l cannot exceed link j - l mode m capacity

$$\sum_{p=1}^P XC_{jlmpt} \leq WC_{jlm} + \sum_{\tau=3}^t XEW C_{jlm\tau}, \quad (108)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, t = 1, \dots, T$$

Expansion of link j - l cannot exceed the maximum capacity expansion throughout the planning horizon.

$$\sum_{t=3}^T XEW C_{jlm t} \leq EWC U_{jlm}, \quad (109)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M,$$

Equation (110) is a logical constraint to ensure $YEW C_{jlm t}$ is = 1 if $XEW C_{jlm t} > 0$:

$$XEW C_{jlm t} \leq EWC Y_{jlm} YEW C_{jlm t}, \quad (110)$$

$$j = 1, \dots, J, l (l \neq j) = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Expansion of link $j-l$ is above the minimum capacity expansion.

$$XEW C_{jlm t} \geq EWC L_{jlm} YEW C_{jlm t}, \quad (111)$$

$$j = 1, \dots, J, l (l \neq j) = 1, \dots, J, m = 1, \dots, M, t = 3, \dots, T$$

Number of expansions of link $j-l$ cannot exceed NW

$$\sum_{t=3}^T YEW C_{jlm t} \leq NW, \quad (112)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M,$$

Equation (113) is a logical constraint to ensure $YC_{jlm p t}$ is = 1 if $XC_{jlm p t} > 0$

$$XC_{jlm p t} \leq (WC_{ijm} + EWC U_{jlm}) YC_{jlm p t}, \quad (113)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

Transportation in link $j-l$ is above the minimum quantity.

$$XC_{jlmpt} \geq WCL_{jlmpt}YC_{jlmpt}, \quad (114)$$

$$j = 1, \dots, J, l = 1, \dots, J (l \neq j), m = 1, \dots, M, p = 1, \dots, P, t = 1, \dots, T$$

6.3 Results and Analysis

The model was coded in GAMS 24.7.4 and was solved using the CPLEX solver with implementing the penalty function approach discussed in 4.5. The code used to run the model is attached in Appendix G. The model resulted in an optimal total cost of SR 3.096 billion and the solution time was 8,000 seconds (2.2 hours) with 4.9% relative gap from the optimal solution using the penalty function approach. Table 14 displays the robust model statistics.

	Number of Constraints	Number of Variables	Number of Binary Variables
Full Model	107,359	75,852	28,569

Table 15: Robust Model Summary

The optimal solution of the robust model is 46% higher than the solution for the deterministic model because of the high demand the model must satisfy. In addition, it's 10% lower than the optimal solution of the two-stage stochastic model. This is because the two-stage stochastic model accommodates for high variations in the growth rates based on the number of scenarios as opposed to the robust model which considers only one scenario.

The total amount consumed under the robust model is 4.311 billion barrels during the planning horizon. Figure 15 shows the consumption of each product per year. Figure 16 shows a breakdown for the transportation modes used for transporting the products from the refineries to the distribution centers. Table 15 displays the amounts refineries supplied to distribution centers by transportation mode at each year. We note a similar pattern to the deterministic model where trucks and sea were the most dominant modes used for transportation and their values are close.

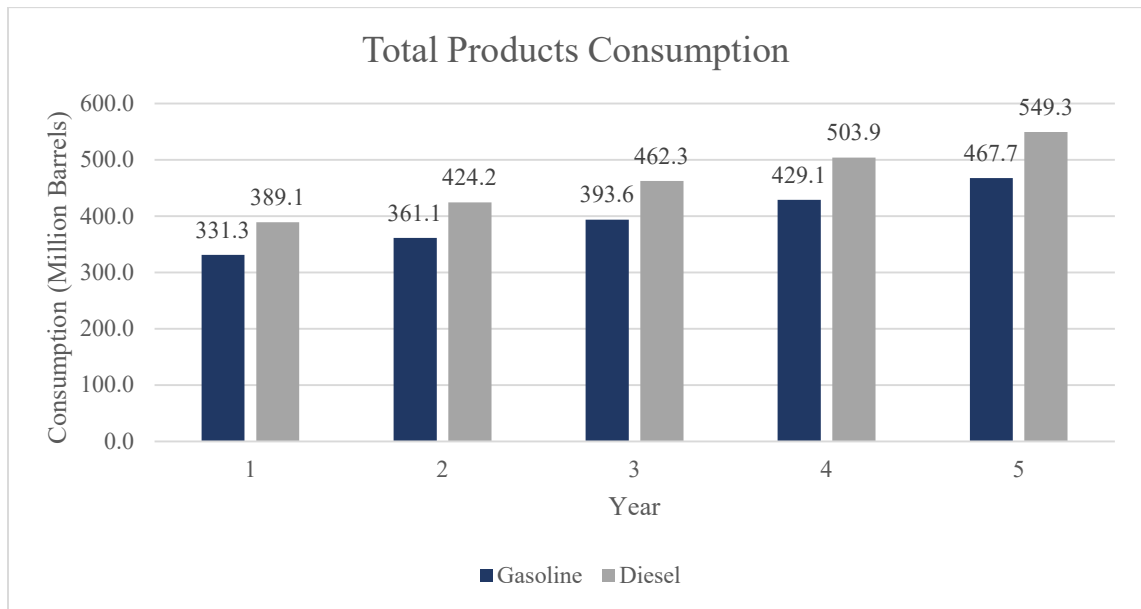


Figure 15: Total Products Consumption Per Year

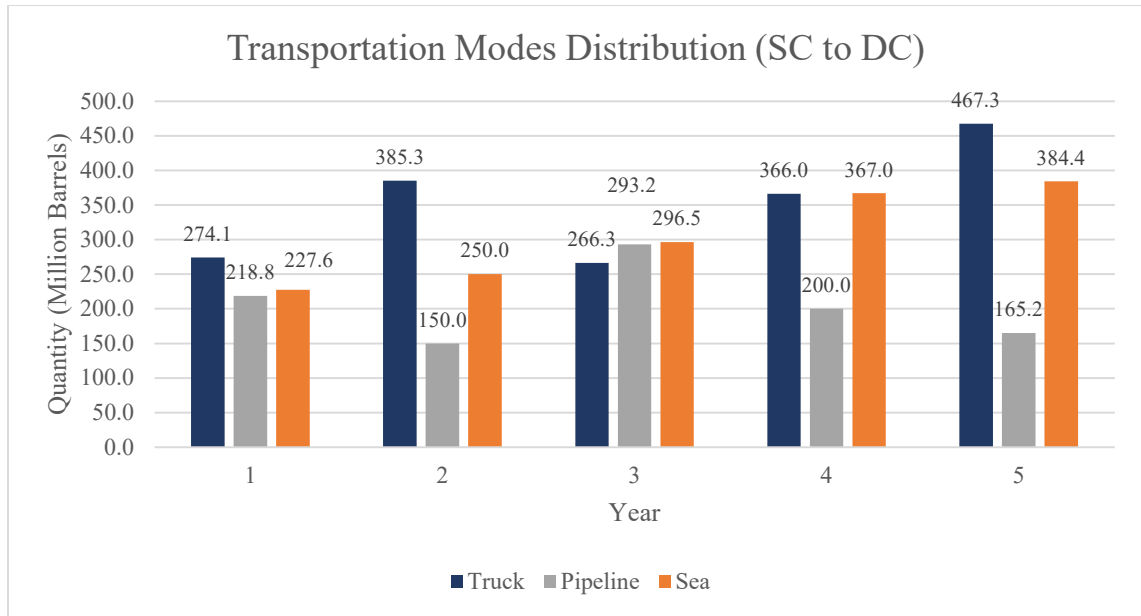


Figure 16: Transportation Modes Distribution (SC To DC)

Table 16: Refineries Distribution by Transportation Mode

Refinery	Mode	Year				
		1	2	3	4	5
Ras Tanura	Truck	50.0	0.0	24.9	0.0	31.0
Ras Tanura	Pipeline	100.0	100.0	100.0	66.4	100.0
Ras Tanura	Sea	50.0	50.0	50.0	100.0	48.4
Jubail(2)	Truck	4.4	53.6	25.1	54.7	84.1
Jubail(2)	Pipeline	50.0	50.0	147.1	33.6	0.0
Jubail(2)	Sea	0.0	0.0	0.0	97.1	148.7
Rabigh	Truck	18.4	58.0	39.5	40.7	74.9
Rabigh	Pipeline	0.0	0.0	0.0	50.0	0.0
Rabigh	Sea	127.6	82.1	100.0	50.0	71.1
Yanbu (3)	Truck	124.4	196.3	123.4	198.5	200.0
Yanbu (3)	Pipeline	68.8	0.0	46.1	50.0	65.2
Yanbu (3)	Sea	50.0	117.9	123.9	119.8	116.2

Riyadh	Truck	45.3	45.3	43.9	45.3	45.3
Riyadh	Pipeline	0.0	0.0	0.0	0.0	0.0
Riyadh	Sea	0.0	0.0	0.0	0.0	0.0
Jeddah	Truck	31.6	32.1	9.6	26.9	32.1
Jeddah	Pipeline	0.0	0.0	0.0	0.0	0.0
Jeddah	Sea	0.0	0.0	22.6	0.0	0.0

Figure 17 shows the amounts distributed from distribution centers to demand markets by transportation for each year. Table 16 provides a breakdown for these amounts by refinery. We also note the same pattern as in the deterministic and stochastic models were trucks are by far the most dominant mode of distribution.

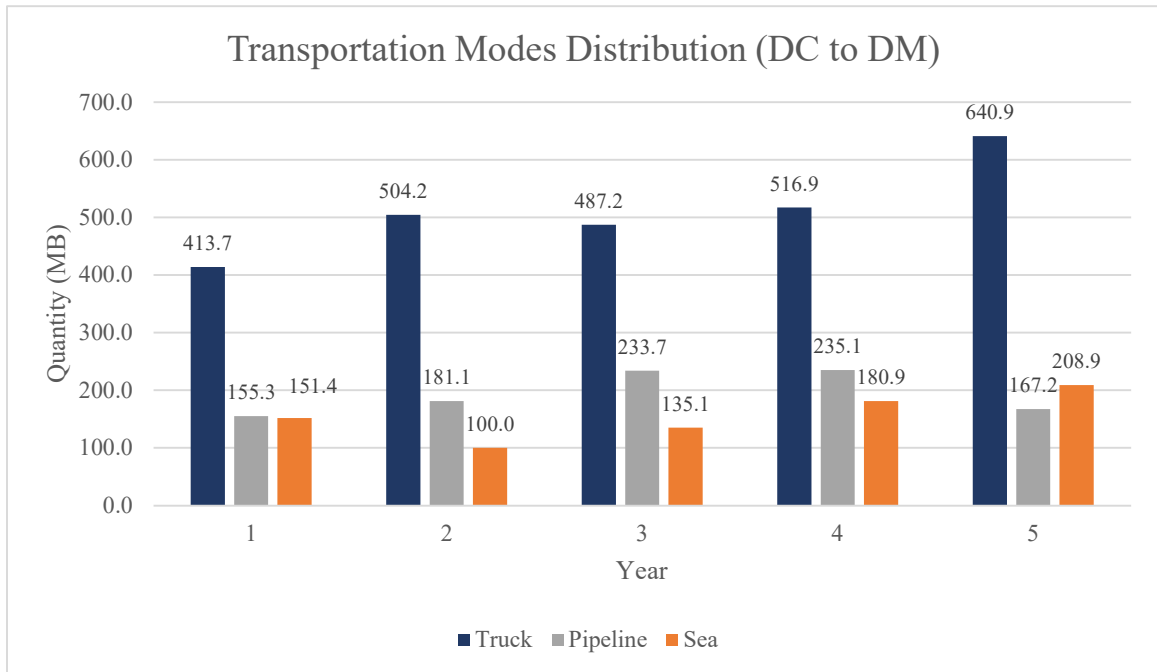


Figure 17: Transportation Modes Distribution (DC to DM) for the Robust Model

Table 17: Distribution Centers Amounts

		Year				
Distribution Center	Mode	1	2	3	4	5
Jawf	Truck	25.7	46.3	0.0	48.6	50.0
Jawf	Pipeline	0.0	0.0	0.0	0.0	0.0
Jawf	Sea	0.0	0.0	0.0	0.0	0.0
Tabuk	Truck	48.8	50.0	49.7	49.9	50.0
Tabuk	Pipeline	0.0	0.0	0.0	0.0	0.0
Tabuk	Sea	0.0	0.0	0.0	0.0	0.0
Turaif	Truck	0.0	0.0	0.0	0.0	50.0
Turaif	Pipeline	0.0	0.0	0.0	0.0	0.0
Turaif	Sea	0.0	0.0	0.0	0.0	0.0
Salil	Truck	49.7	49.8	43.9	50.0	50.0
Salil	Pipeline	0.0	0.0	0.0	0.0	0.0
Salil	Sea	0.0	0.0	0.0	0.0	0.0
Rabigh	Truck	11.2	26.6	50.0	50.0	50.0
Rabigh	Pipeline	38.8	23.4	0.0	0.0	0.0
Rabigh	Sea	0.0	0.0	0.0	0.0	0.0
Jazan	Truck	9.9	0.0	26.8	5.7	6.2
Jazan	Pipeline	0.0	0.0	0.0	0.0	0.0
Jazan	Sea	38.9	50.0	19.6	44.3	43.8
Najran	Truck	0.0	40.1	0.0	17.5	50.0
Najran	Pipeline	0.0	0.0	0.0	0.0	0.0
Najran	Sea	0.0	0.0	0.0	0.0	0.0
Abha	Truck	50.0	50.0	49.0	50.0	50.0

Abha	Pipeline	0.0	0.0	0.0	0.0	0.0
Abha	Sea	0.0	0.0	0.0	0.0	0.0
Yanbu	Truck	47.6	42.3	51.2	59.3	54.7
Yanbu	Pipeline	0.0	7.7	28.9	38.0	18.5
Yanbu	Sea	0.0	0.0	39.7	22.5	46.7
Riyadh	Truck	50.0	50.0	50.0	50.0	50.0
Riyadh	Pipeline	0.0	0.0	0.0	0.0	0.0
Riyadh	Sea	0.0	0.0	0.0	0.0	0.0
Jeddah	Truck	9.2	50.0	24.3	35.8	28.8
Jeddah	Pipeline	0.0	0.0	0.0	0.0	0.0
Jeddah	Sea	40.8	0.0	25.7	14.2	21.2
Duba	Truck	11.8	0.0	0.0	0.0	0.0
Duba	Pipeline	0.0	0.0	0.0	0.0	0.0
Duba	Sea	38.2	50.0	50.0	50.0	50.0
Qassim	Truck	49.9	50.0	50.0	50.0	50.0
Qassim	Pipeline	0.0	0.0	0.0	0.0	0.0
Qassim	Sea	0.0	0.0	0.0	0.0	0.0
Alhafuf	Truck	50.0	49.1	50.0	50.0	50.0
Alhafuf	Pipeline	0.0	0.0	0.0	0.0	0.0
Alhafuf	Sea	0.0	0.0	0.0	0.0	0.0
Dhahran	Truck	0.0	0.0	0.0	0.0	0.0
Dhahran	Pipeline	16.5	50.0	50.0	50.0	50.0
Dhahran	Sea	33.5	0.0	0.0	0.0	0.0
Qatif	Truck	0.0	0.0	39.8	0.0	51.1
Qatif	Pipeline	50.0	50.0	107.3	97.1	48.7

Qatif	Sea	0.0	0.0	0.0	50.0	47.3
Saffaniya	Truck	0.0	0.0	2.6	0.0	0.0
Saffaniya	Pipeline	50.0	50.0	47.4	50.0	50.0
Saffaniya	Sea	0.0	0.0	0.0	0.0	0.0

The full model results are displayed in Tables D1 to D3 in Appendix D. As shown in table D3, the model suggests expanding two distribution centers, namely Qatif (69.8 million barrels) , and Yanbu (97.12). Note that these two distribution centers are both at sea and were both expanded for the two-stage stochastic model. The model does not suggest expanding a refinery, which means that the total capacity is sufficient for the planning horizon. The model also does not suggest expanding or establishing a transportation route.

The main advantage of the robust model is that it provides a statistically robust solution without significantly increasing the solution complexity. However, the solution is only near optimal for high demand growth rate values and feasible but highly expensive for low demand growth rate values.

Chapter 7

CONCLUSION

In this work, we developed optimization models for the downstream petroleum supply chain that considers two products and three modes of transportation to minimize the total fixed and operational costs of transporting these products over a period of five years. The model was applied to the petroleum supply chain in Saudi Arabia. The downstream supply chain includes 9 supply centers, 17 distribution centers, and the 20 cities that have population above 100,000.

As the model complexity is high, there was a need for a solution technique to reduce the solution time and obtain an optimal solution. Thus, we devised a penalty function approach that penalizes the complicating constraints and adds them to the objective function to reduce the complexity of the model. The penalty function approach resulted in at least 50% reduction in solution time.

To capture the effect of demand uncertainty, two approaches were utilized, two-stage stochastic programming model, and robust model. In the two-stage stochastic model, the uncertainty is captured by means of scenario planning, where the demand growth rate values are drawn from the distribution of the historical data. The model aims at minimizing the expected total cost. In the robust model, the uncertainty is captured via optimizing the worst-case scenario for the demand realization from the historical data.

Table 17 shows a comparison between the results of the three models . The number of variables and constraints in the stochastic model is much higher than both the robust and deterministic models due to the 11 scenarios generated. As a result, the solution time of the stochastic model was also much higher than it for the robust and deterministic model. The stochastic model solution has the highest cost because of the high variations of the demand growth rates, followed by the robust model which considers a scenario with high demand growth rate, and then, far from them, the deterministic model that considers a scenario with an expected demand growth rate.

Table 18: Models Comparison

Model	Deterministic	Stochastic	Robust
Constraints	79,449	724,009	107,359
Variables	65,252	498,752	75,852
Binary Variables	28,569	247,869	28,569
Optimal Solution (Billions SR)	2.113	3.373	3.096
Optimality Gap	4.80%	5%	4.9%

The stochastic model considers a wide range of scenarios and thus, can be very effective for highly variable conditions. On the other hand, the robust model considers only the worst-case scenario and thus, is more appropriate for worst case analysis and modeling. Although the deterministic model does not deal with uncertainty, it can be very effective for modelling standard and normal conditions.

The results of the three models follow a logical approach in which refineries and distribution centers distribute to the nodes with proximity to them. In addition, in all the three models, the most used transportation modes between refineries and distribution centers are trucks and sea, while the most dominant modes between distribution centers and demand markets are trucks. The deterministic model suggests establishing a new pipeline route, the stochastic and robust models suggest expanding some of the distribution centers and refineries.

7.1 Future Work

This work can be extended by considering the refined products exports and maximizing the total profit instead of minimizing the total cost. This direction can include studying the relation between the demand and the price, and then mathematically representing it. This is specifically interesting since both parameters are uncertain in real life.

The results of the developed models can be significantly improved and be more practical by acquiring real data for the capacities of distribution centers and transportation links, and the fixed and variable costs. For instance, the assumption of equal capacities for the distribution centers is oversimplifying, as in reality, it's expected that distribution centers that are near from large cities should have higher capacities.

Another direction for extending this work could be by considering the integration of the upstream and downstream petroleum supply chains starting from the extraction of the crude oil up to the distribution to the final markets. The resulting model can be very complicated

considering the number of stages and the number of variables and constraints required to model the problem.

Finally, multi-objective optimization can be considered to allow for studying other factors such as the environmental and social impacts of different distribution strategies. There is an increasing interest in the petroleum literature in studying the impact of these products on global warming and greenhouse gases emissions since the petroleum products are responsible for a significant fraction of these emissions.

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Appendix A

MODEL PARAMETERS

WA_{ijm} :

Table A1: Transportation Links Capacities Between Refineries and Distribution Centers (MB)

i	m/j	Jawf	Tabuk	Turaif	Salil	Rabigh	Jazan	Najran	Abha	Yanbu	Riyadh	Jeddah	Duba	Qassim	Hafuf	Dhahran	Qatif	Saffaniya
Ras Tanura	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	50	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Jubail(2)	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	50	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Rabigh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Yanbu (3)	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Riyadh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Jeddah	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50

WC_{jlm} :

Table A2: Transportation Links Capacities Between Distribution Centers (MB)

j	m/l	Jawf	Tabuk	Turaif	Salil	Rabigh	Jazan	Najran	Abha	Yanbu	Riyadh	Jeddah	Duba	Qassim	Hafuf	Dhahran	Qatif	Saffaniya
Jawf	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tabuk	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turaif	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Salil	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rabigh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	50	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	50	50	50	50
Jazan	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	50	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Najran	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Abha	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Yanbu	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	50	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	50	50	50	50
Riyadh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Jeddah	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	50	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Duba	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	50	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Qasim	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hasa	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Dhahran	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Qatif	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50
Saffaniya	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	50	0	0	0	50	0	0	0	0	0	50	50	50
	Sea	0	0	0	0	50	50	0	0	50	0	50	50	0	0	50	50	50

WB_{jkm} :

Table A3: Transportation Links Capacities Between Distribution Centers and Demand Markets (MB)

j	m/k	Sakakah	Tabuk	Hail	Qurayyat	Medina	Jizan	Najran	Abha	Yanbu	Riyadh	Jeddah	Mecca	Qassim	Hafuf	Taif	Dammam	Kharj	Hafr	Albaten	Jubail	Arar
Jawf	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tabuk	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turaif	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Salil	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rabigh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	50	50	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	50	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0
Jazan	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0
Najran	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Abha	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Yanbu	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	50	50	0	0	0	0	0	0	50	0	0	50	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0
Riyadh	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Jeddah	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0
Duba	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	50	50	0	0
Qasim	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hasa	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sea	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Dhahran	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	50	0	0	50	50	0	0	0	0	0	0	50	0	0	50	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0
Qatif	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	50	0	0	50	50	0	0	0	0	0	0	50	0	0	50	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0
Saffaniya	Truck	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	Pipeline	0	0	0	0	0	50	0	0	50	50	0	0	0	0	0	0	50	0	0	50	0
	Sea	0	0	0	0	0	50	0	0	50	0	50	0	0	0	0	0	50	0	0	50	0

CA_{ijm} :

Table A4: Variable Transportation Costs Between Distribution Centers and Refineries (SR '000)

i	m/j	Jawf	Tabuk	Turaif	Salil	Rabigh	Jazan	Najran	Abha	Yanbu	Riyadh	Jeddah	Duba	Qassim	Hafuf	Dhahran	Qatif	Saffaniya
Ras Tanura	Truck	1400	1700	1200	1000	1400	1600	1300	1400	1500	450	1400	1800	800	200	100	50	200
	Pipeline	700	850	600	500	700	800	650	700	750	225	700	900	400	100	50	25	100
	Sea	Inf	Inf	Inf	Inf	700	800	Inf	Inf	750	Inf	700	900	Inf	Inf	50	25	100
Jubail(2)	Truck	1400	1600	1200	900	1400	1600	1300	1400	1500	450	1400	1800	800	200	100	80	200
	Pipeline	700	800	600	450	7000	800	650	700	750	225	700	900	400	100	50	40	100
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	750	Inf	700	900	Inf	Inf	50	40	100
Rabigh	Truck	1200	800	1400	1000	0	850	1000	750	200	1000	150	650	750	1400	1400	1400	1400
	Pipeline	600	400	700	500	0	425	500	375	100	500	75	325	375	700	700	700	700
	Sea	Inf	Inf	Inf	Inf	0	425	Inf	Inf	100	Inf	75	325	Inf	Inf	700	700	700
Yanbu (3)	Truck	1000	650	1300	1100	200	1000	1200	900	0	1000	350	500	700	1300	1450	1450	1400
	Pipeline	500	325	650	550	100	500	600	450	0	500	175	250	350	650	725	725	700
	Sea	Inf	Inf	Inf	Inf	100	500	Inf	Inf	0	Inf	175	250	Inf	Inf	725	725	700
Riyadh	Truck	1400	1300	1300	500	1000	1100	900	950	1000	0	950	1300	450	300	350	350	500
	Pipeline	700	650	650	250	500	550	450	475	500	0	475	650	225	150	175	175	250
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Jeddah	Truck	1400	1000	1500	850	150	700	900	600	300	950	0	800	900	1300	1300	1300	1500
	Pipeline	700	500	750	425	75	350	450	300	150	475	0	400	450	650	650	650	750
	Sea	Inf	Inf	Inf	Inf	75	350	Inf	Inf	150	Inf	0	400	Inf	Inf	650	650	750

CC_{jlm} :

Table A5: Variable Transportation Costs Between Distribution Centers (SR ‘000)

j	m/l	Jawf	Tabuk	Turaif	Salil	Rabigh	Jazan	Najran	Abha	Yanbu	Riyadh	Jeddah	Duba	Qassim	Hafuf	Dhahran	Qatif	Saffaniya
Jawf	Truck	0	350	400	1500	1000	2000	2000	1700	900	900	1200	550	650	1300	1400	1400	1100
	Pipeline	0	175	200	750	700	1000	1000	850	450	450	600	275	325	650	700	700	550
	Sea	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Tabuk	Truck	350	0	700	1500	800	1600	1900	1600	650	1200	1000	200	950	1600	1600	1600	1500
	Pipeline	175	0	350	750	400	800	950	800	325	600	500	100	475	800	800	800	750
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Turaif	Truck	400	700	0	1800	1400	2200	2200	2000	1200	1200	1500	900	1000	1400	1400	1400	1000
	Pipeline	200	350	0	900	700	1100	1100	1000	600	600	750	450	500	700	700	700	500
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Salil	Truck	1500	1500	1800	0	1000	700	400	500	1100	550	850	1500	900	850	950	950	1000
	Pipeline	750	750	900	0	500	350	200	250	550	275	425	750	450	425	475	475	500
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Rabigh	Truck	1000	800	1400	1000	0	900	1000	750	200	1000	150	650	750	1400	1400	1400	1400
	Pipeline	500	400	700	500	0	450	500	375	100	500	75	325	375	700	700	700	700
	Sea	Inf	Inf	Inf	Inf	0	450	Inf	Inf	100	Inf	75	325	Inf	Inf	700	700	700
Jazan	Truck	2000	1600	2200	700	900	0	300	200	1000	1100	700	1800	1350	1500	1500	1500	1650
	Pipeline	1000	800	1100	350	450	0	150	100	500	550	350	900	675	750	750	750	825
	Sea	Inf	Inf	Inf	Inf	450	0	Inf	Inf	500	Inf	350	900	Inf	Inf	750	750	825
Najran	Truck	2000	1900	2200	400	1000	300	0	300	1250	900	900	1700	1300	1300	1300	1300	1500
	Pipeline	1000	950	1100	200	500	150	0	150	625	450	450	850	650	650	650	650	750
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Abha	Truck	1700	1600	2000	500	750	200	300	0	1000	1000	650	1450	1100	1250	1350	1350	1450
	Pipeline	850	800	1000	250	375	100	150	0	500	500	325	725	550	625	675	675	725
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Yanbu	Truck	900	650	1200	1100	200	1000	1250	1000	0	1000	300	500	650	1350	1450	1450	1350
	Pipeline	450	325	600	550	100	500	625	500	0	500	150	250	325	675	725	725	675
	Sea	Inf	750	Inf	Inf	100	500	Inf	Inf	0	Inf	150	150	Inf	Inf	725	725	675
Riyadh	Truck	900	1200	1200	550	1000	1100	900	1000	1000	0	1000	1300	400	300	400	400	500
	Pipeline	450	600	600	275	500	550	450	500	500	0	500	650	200	150	200	200	250
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Jeddah	Truck	1200	1000	1500	850	150	700	900	650	300	1000	0	800	850	1250	1350	1350	1500
	Pipeline	600	500	750	425	75	350	450	325	150	500	0	400	425	625	675	675	750
	Sea	Inf	Inf	Inf	Inf	Inf	350	Inf	Inf	150	Inf	0	400	Inf	Inf	675	675	750
Duba	Truck	550	200	900	1500	650	1800	1700	1450	500	1300	800	0	1000	1650	1550	1550	1450
	Pipeline	275	100	450	750	325	900	850	725	250	650	400	0	500	825	775	775	725
	Sea	Inf	Inf	Inf	Inf	Inf	900	Inf	Inf	250	Inf	400	0	Inf	Inf	775	775	725
Qasim	Truck	650	950	1000	900	750	1350	1300	1100	650	400	850	1000	0	700	800	800	700
	Pipeline	325	475	500	450	375	675	650	550	325	200	425	500	0	350	400	400	350
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Hasa	Truck	1300	1600	1400	850	1400	1500	1300	1250	1350	300	1250	1650	700	0	150	150	400
	Pipeline	650	800	700	425	700	750	650	625	675	150	625	825	350	0	75	75	200
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Dhahran	Truck	1400	1600	1400	950	1400	1500	1300	1350	1450	400	1350	1550	800	150	0	50	250
	Pipeline	700	800	700	475	700	750	650	675	725	200	675	775	400	75	0	25	125
	Sea	Inf	Inf	Inf	Inf	Inf	750	Inf	Inf	725	Inf	675	775	Inf	Inf	0	25	125
Qatif	Truck	1400	1600	1400	950	1400	1500	1300	1350	1450	400	1350	1550	800	150	50	0	250
	Pipeline	700	800	700	475	700	750	650	675	725	200	675	775	400	75	25	0	125
	Sea	Inf	Inf	Inf	Inf	Inf	750	Inf	Inf	725	Inf	675	775	Inf	Inf	25	0	125
Saffaniya	Truck	1100	1500	1000	1000	1400	1650	1500	1450	1350	500	1500	1450	700	400	250	250	0
	Pipeline	550	750	500	500	700	825	750	725	675	250	750	725	350	200	125	125	0
	Sea	Inf	Inf	700	Inf	Inf	825	Inf	Inf	675	Inf	750	725	Inf	Inf	125	125	0

CB_{jkm} :

Table A6: Variable Transportation Costs Between Distribution Centers and Demand Markets (SR '000)

j	m/k	Sakakah	Tabuk	Hail	Qurayyat	Medina	Jazan	Najran	Abha	Yanbu	Riyadh	Jeddah	Mecca	Qassim	Hafuf	Taif	Dammam	Kharj	Hafr Albaten	Jubail	Arar
Jawf	Truck	0	350	400	800	1000	2000	2000	1700	900	900	1200	1250	650	1300	1200	1400	1350	800	1250	250
	Pipeline	0	175	200	400	700	1000	1000	850	450	450	600	625	325	650	600	700	675	400	625	125
	Sea	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Tabuk	Truck	350	0	650	550	650	1600	1900	1600	650	1200	1000	1000	950	1600	1100	1600	1400	1200	1600	600
	Pipeline	175	0	325	275	325	800	950	800	325	600	500	500	475	800	550	800	700	600	800	300
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Turaif	Truck	400	700	800	150	1200	2200	2200	2000	1200	1200	1500	1600	1000	1400	1600	1400	1400	800	1200	250
	Pipeline	200	350	400	75	600	1100	1100	1000	600	600	750	800	500	700	800	700	700	400	600	125
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Salil	Truck	1500	1500	1200	1800	900	700	400	500	1100	550	850	200	900	850	700	950	500	1000	1000	1600
	Pipeline	750	750	600	900	450	350	200	250	550	275	425	100	450	425	350	475	250	500	500	800
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Rabigh	Truck	1000	800	700	1200	300	900	1000	750	200	1000	150	200	750	1400	300	1400	1000	1100	1500	1250
	Pipeline	500	400	350	600	150	450	500	375	100	500	75	100	375	700	150	700	500	550	750	625
	Sea	Inf	Inf	Inf	Inf	Inf	450	Inf	Inf	100	Inf	75	Inf	Inf	Inf	Inf	700	Inf	Inf	750	Inf
Jazan	Truck	2000	1600	1500	2100	1100	0	300	200	1000	1100	700	900	1350	1500	700	1500	1200	1600	1600	2100
	Pipeline	1000	800	750	1050	550	0	150	100	500	550	350	450	675	750	350	750	600	800	800	1050
	Sea	Inf	Inf	Inf	Inf	Inf	0	Inf	Inf	500	Inf	350	Inf	Inf	Inf	750	Inf	Inf	Inf	800	Inf
Najran	Truck	2000	1900	1550	2200	1250	300	0	300	1250	900	900	700	1300	1300	750	1300	900	1450	1400	2000
	Pipeline	1000	950	775	1100	625	150	0	150	625	450	450	350	650	650	375	650	450	725	700	1000
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Abha	Truck	1700	1600	1350	2000	1000	200	300	0	1000	1000	650	350	1100	1250	600	1350	1000	1400	1400	2000
	Pipeline	850	800	675	1000	500	100	150	0	500	500	325	175	550	625	300	675	500	700	700	1000
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Yanbu	Truck	900	650	600	1150	200	1000	1250	1000	0	1000	300	850	650	1350	450	1450	1150	1000	1500	1200
	Pipeline	450	325	300	575	100	500	625	500	0	500	150	425	325	675	225	575	500	750	600	600
	Sea	Inf	Inf	Inf	Inf	Inf	500	Inf	Inf	0	Inf	150	Inf	Inf	Inf	Inf	725	Inf	Inf	750	Inf
Riyadh	Truck	900	1200	650	1450	800	1100	900	1000	1000	0	1000	850	400	300	800	400	100	500	450	1050
	Pipeline	450	600	325	725	400	550	450	500	500	0	500	425	200	150	400	200	50	250	225	525
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Jeddah	Truck	1200	1000	833	1500	400	700	900	650	300	1000	0	100	850	1250	150	1350	1000	1300	1400	1400
	Pipeline	600	500	416.5	750	200	350	450	325	150	500	0	50	425	625	75	675	500	650	700	700
	Sea	Inf	Inf	Inf	Inf	Inf	350	Inf	Inf	150	Inf	0	Inf	Inf	Inf	Inf	675	Inf	Inf	700	Inf
Duba	Truck	550	200	900	750	650	1800	1700	1450	500	1300	800	850	1000	1650	900	1550	1450	1300	1650	800
	Pipeline	275	100	450	375	325	900	850	725	250	650	400	425	500	825	450	775	725	650	825	400
	Sea	Inf	Inf	Inf	Inf	Inf	900	Inf	Inf	250	Inf	400	Inf	Inf	Inf	Inf	775	Inf	Inf	825	Inf
Qasim	Truck	650	950	250	900	450	1350	1300	1100	650	400	850	800	0	700	700	800	500	400	850	800
	Pipeline	325	475	125	450	225	675	650	550	325	200	425	400	0	350	350	400	250	200	425	400
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Hasa	Truck	1300	1600	900	1500	1150	1500	1300	1250	1350	300	1250	1200	700	0	1100	150	350	600	250	1100
	Pipeline	650	800	450	750	575	750	650	625	675	150	625	600	350	0	550	75	175	300	125	550
	Sea	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Dhahran	Truck	1400	1600	1000	1450	1250	1500	1300	1350	1450	400	1350	1250	800	150	1200	50	450	500	100	1000
	Pipeline	700	800	500	725	625	750	650	675	725	200	675	625	400	75	600	25	225	250	50	500
	Sea	Inf	Inf	Inf	Inf	Inf	750	Inf	Inf	725	Inf	675	Inf	Inf	Inf	Inf	25	Inf	Inf	500	Inf
Qatif	Truck	1400	1600	1000	1450	1250	1500	1300	1350	1450	400	1350	1250	800	150	1200	0	450	500	100	1000
	Pipeline	700	800	500	725	625	750	650	675	725	200	675	625	400	75	600	0	225	250	50	500
	Sea	Inf	Inf	Inf	Inf	Inf	750	Inf	Inf	725	Inf	675	Inf	Inf	Inf	Inf	0	Inf	Inf	500	Inf
Saffaniya	Truck	1100	1500	800	1250	1150	1650	1500	1450	1350	500	1500	1400	700	400	1300	250	600	350	200	850
	Pipeline	550	750	400	625	575	825	750	725	675	250	750	700	350	200	650	125	300	175	100	425
	Sea	Inf	Inf	Inf	Inf	Inf	825	Inf	Inf	675	Inf	750	725	Inf	Inf	Inf	125	Inf	Inf	100	Inf

Appendix B

DETERMINISTIC MODEL RESULTS

Notes:

The optimal values of binary variables = 1, if their continuous counterparts > 0 , and 0 otherwise.

Decision variables not displayed in the tables below (other than binary variable) have optimal values of 0.

XA_{ijmpt} :

Table B1: Distributed Amounts Between Refineries and Distribution Centers in the Deterministic Model (MB)

I	J	m	p	t=1	t=2	t=3	t=4	t=5
Ras Tanura	Riyadh	Pipeline	Gasoline	50				50
Ras Tanura	Riyadh	Pipeline	Diesel		50			
Ras Tanura	Hafuf	Truck	Gasoline	29.00242			15.96546	21.89226
Ras Tanura	Hafuf	Truck	Diesel		34.53343			28.10774
Ras Tanura	Dhahran	Pipeline	Diesel		50			
Ras Tanura	Qatif	Pipeline	Gasoline		34.01361			
Ras Tanura	Qatif	Pipeline	Diesel		15.98639		50	49.20427
Ras Tanura	Qatif	Sea	Gasoline			50		
Ras Tanura	Qatif	Sea	Diesel	50				
Jubail(2)	Salil	Truck	Gasoline			4.74		
Jubail(2)	Salil	Truck	Diesel				4.600352	3.842418

Jubail(2)	Riyadh	Pipeline	Gasoline			50		
Jubail(2)	Riyadh	Pipeline	Diesel				50	
Jubail(2)	Hafuf	Truck	Gasoline	15.46657		15.71404		
Jubail(2)	Hafuf	Truck	Diesel	20.99758		34.28596	34.03454	
Jubail(2)	Dhahran	Pipeline	Gasoline				50	50
Jubail(2)	Dhahran	Sea	Gasoline	50				
Jubail(2)	Dhahran	Sea	Diesel			50		
Jubail(2)	Saffaniya	Pipeline	Gasoline		50			
Jubail(2)	Saffaniya	Sea	Gasoline				50	
Jubail(2)	Saffaniya	Sea	Diesel	50		50		50
Rabigh	Rabigh	Truck	Gasoline	50			25.35344	50
Rabigh	Rabigh	Pipeline	Diesel			50		
Rabigh	Rabigh	Sea	Gasoline		50			
Rabigh	Rabigh	Sea	Diesel				24.64656	
Rabigh	Jazan	Sea	Gasoline	32.05649		26.83193		
Rabigh	Jazan	Sea	Diesel		4.485008		18.30743	
Rabigh	Najran	Truck	Gasoline				4.686064	18.06542
Rabigh	Najran	Truck	Diesel				5.742165	9.484424
Rabigh	Abha	Truck	Gasoline	13.94351	30.65124		6.045375	16.77272
Rabigh	Abha	Truck	Diesel			18.73207	10.56351	
Rabigh	Jeddah	Sea	Gasoline			38.39704		16.25747
Rabigh	Jeddah	Sea	Diesel	50	50	11.60296	50	33.74253
Yanbu (3)	Jawf	Truck	Gasoline	19.21072	12.32373			
Yanbu (3)	Jawf	Truck	Diesel	13.52723	13.74367			
Yanbu (3)	Jawf	Pipeline	Gasoline			19.83038	20.14766	20.47003
Yanbu (3)	Jawf	Pipeline	Diesel			26.24533	26.66525	24.82138
Yanbu (3)	Tabuk	Truck	Gasoline	48.84239	49.62387	10.34275	39.74913	32.24644
Yanbu (3)	Tabuk	Truck	Diesel			38.83021	10.11424	17.75356

Yanbu (3)	Jazan	Sea	Gasoline	7.465216	45.22746		31.69257	
Yanbu (3)	Jazan	Sea	Diesel	9.88951		21.60673		47.63505
Yanbu (3)	Yanbu	Truck	Diesel	50				
Yanbu (3)	Yanbu	Pipeline	Diesel		50		49.89257	
Yanbu (3)	Yanbu	Sea	Gasoline			50		
Yanbu (3)	Yanbu	Sea	Diesel					50
Yanbu (3)	Duba	Sea	Gasoline		12.28907		50	50
Yanbu (3)	Duba	Sea	Diesel	50	37.71093	50		
Yanbu (3)	Qassim	Truck	Gasoline	30.79276	15.49914	33.71099	8.572658	8.709821
Yanbu (3)	Qassim	Truck	Diesel	17.87063	33.0416	16.28901	41.42734	41.29018
Riyadh	Salil	Truck	Gasoline		7.836911	11.1653	45.26	
Riyadh	Salil	Truck	Diesel	45.26	37.42309	34.0947		45.26
Jeddah	Salil	Truck	Gasoline		4.282075			
Jeddah	Najran	Truck	Gasoline		9.400852			
Jeddah	Abha	Truck	Gasoline			31.26793		18.61775
Jeddah	Abha	Truck	Diesel	31.59008	18.43707		32.12	13.50225

XB_{jkmpt} :

Table B2: Distributed Amounts Between Distribution Centers and Demand Markets in the Deterministic Model (MB)

J	k	m	P	t=1	t=2	t=3	t=4	t=5
Jawf	Sakakah	Truck	Gasoline	2.68	2.72	2.77	2.81	2.86
Jawf	Sakakah	Truck	Diesel	2.66	2.70	2.74	2.79	2.83
Jawf	Hail	Truck	Gasoline	5.58	5.67	5.76	5.85	5.94
Jawf	Hail	Truck	Diesel	5.53	5.62	5.71	5.80	5.89
Jawf	Qurayyat	Truck	Diesel			2.20	2.23	
Jawf	Hafr Albatan	Truck	Gasoline	7.08		7.31	7.43	7.55
Jawf	Hafr Albatan	Truck	Diesel			10.08	10.24	10.41
Jawf	Arar	Truck	Gasoline	3.87	3.93	4.00	4.06	4.13
Jawf	Arar	Truck	Diesel	5.34	5.43	5.51	5.60	5.69
Tabuk	Tabuk	Truck	Gasoline	7.87	8.00	8.12	8.25	8.39
Tabuk	Tabuk	Truck	Diesel			9.95	10.11	10.28
Tabuk	Qurayyat	Truck	Gasoline	2.15	2.18	2.22		2.29
Tabuk	Qurayyat	Truck	Diesel					2.27
Tabuk	Medina	Truck	Gasoline	38.82	39.44		31.50	21.57
Tabuk	Medina	Truck	Diesel			28.88		5.21
Salil	Riyadh	Truck	Gasoline				33.73	
Salil	Riyadh	Truck	Diesel	36.45				19.24
Salil	Mecca	Truck	Gasoline		3.09	6.74	2.21	
Salil	Mecca	Truck	Diesel		28.48	25.00		29.86
Salil	Kharj	Truck	Gasoline		9.03	9.17	9.32	
Salil	Kharj	Truck	Diesel	8.81	8.95	9.09	4.60	
Rabigh	Medina	Truck	Gasoline					19.80

Rabigh	Medina	Truck	Diesel			20.23		
Rabigh	Yanbu	Pipeline	Gasoline	3.46				
Rabigh	Yanbu	Sea	Diesel				4.45	
Rabigh	Riyadh	Pipeline	Gasoline		40.68			30.20
Rabigh	Riyadh	Pipeline	Diesel			25.84	8.41	
Rabigh	Jeddah	Truck	Gasoline	14.50				
Rabigh	Mecca	Truck	Gasoline	22.87			15.73	
Rabigh	Mecca	Truck	Diesel			3.93		
Rabigh	Taif	Truck	Gasoline	9.17	9.32		9.62	
Rabigh	Taif	Truck	Diesel				11.79	
Jazan	Jazan	Truck	Gasoline		3.66			
Jazan	Jazan	Truck	Diesel		4.49		4.63	
Jazan	Jazan	Sea	Gasoline				3.78	
Jazan	Jazan	Sea	Diesel	4.41		4.56		4.70
Jazan	Najran	Truck	Gasoline	4.47		4.61		
Jazan	Najran	Truck	Diesel	5.48				
Jazan	Abha	Truck	Gasoline				10.85	
Jazan	Jeddah	Sea	Gasoline	35.05	41.57	22.22	17.07	
Jazan	Jeddah	Sea	Diesel			17.05	13.68	42.93
Najran	Jazan	Truck	Gasoline					3.84
Najran	Najran	Truck	Gasoline		4.54		4.69	4.76
Najran	Najran	Truck	Diesel				5.74	5.83
Najran	Riyadh	Truck	Gasoline		4.86			
Najran	Kharj	Truck	Gasoline					9.47
Najran	Kharj	Truck	Diesel					3.65

Abha	Jazan	Truck	Gasoline	3.60		3.72		
Abha	Najran	Truck	Diesel		5.56	5.65		
Abha	Abha	Truck	Gasoline	10.34	10.51	10.67		11.02
Abha	Abha	Truck	Diesel	12.67	12.87	13.08	13.29	13.50
Abha	Mecca	Truck	Gasoline		20.14	16.87	6.05	24.37
Abha	Mecca	Truck	Diesel	18.92			29.39	
Yanbu	Medina	Truck	Gasoline			40.08		
Yanbu	Medina	Truck	Diesel	47.57	48.33		49.89	45.48
Yanbu	Yanbu	Pipeline	Gasoline			3.57		
Yanbu	Yanbu	Pipeline	Diesel		1.67			
Yanbu	Yanbu	Sea	Diesel					4.52
Yanbu	Riyadh	Pipeline	Gasoline			6.35		
Yanbu	Riyadh	Pipeline	Diesel	2.43				
Riyadh	Riyadh	Truck	Gasoline	50.00		50.00		50.00
Riyadh	Riyadh	Truck	Diesel		50.00		50.00	
Jeddah	Jeddah	Truck	Gasoline					6.48
Jeddah	Jeddah	Truck	Diesel	29.65				
Jeddah	Jeddah	Sea	Gasoline			28.93		
Jeddah	Jeddah	Sea	Diesel		38.58		50.00	21.77
Jeddah	Mecca	Truck	Diesel	9.11				
Jeddah	Taif	Truck	Gasoline			9.47		9.77
Jeddah	Taif	Truck	Diesel	11.24	11.42	11.60		11.98
Duba	Tabuk	Truck	Diesel	9.64	9.80			
Duba	Qurayyat	Truck	Gasoline				2.25	
Duba	Qurayyat	Truck	Diesel	2.13	2.16			

Duba	Medina	Truck	Gasoline				9.22	
Duba	Yanbu	Sea	Gasoline		3.51		3.63	3.69
Duba	Yanbu	Sea	Diesel	4.24	2.64	4.38		
Duba	Jeddah	Sea	Gasoline		8.77		34.90	46.31
Duba	Jeddah	Sea	Diesel	31.07	23.11	45.62		
Duba	Dammam	Sea	Diesel	2.92				
Qassim	Riyadh	Truck	Gasoline	22.62		25.27		
Qassim	Riyadh	Truck	Diesel		14.89	7.92	32.93	32.66
Qassim	Qassim	Truck	Gasoline	8.17	8.30	8.44	8.57	8.71
Qassim	Qassim	Truck	Diesel	8.10	8.23	8.36	8.50	8.63
Qassim	Hafr Albaten	Truck	Gasoline		7.19			
Qassim	Hafr Albaten	Truck	Diesel	9.77	9.92			
Hafuf	Riyadh	Truck	Gasoline					5.67
Hafuf	Riyadh	Truck	Diesel		13.20	12.61		
Hafuf	Hafuf	Truck	Gasoline	15.22	15.47	15.71	15.97	16.22
Hafuf	Hafuf	Truck	Diesel	21.00	21.33	21.67	22.02	22.37
Hafuf	Dammam	Truck	Gasoline	4.90				
Hafuf	Dammam	Truck	Diesel				7.38	
Hafuf	Kharj	Truck	Gasoline	8.88				
Hafuf	Kharj	Truck	Diesel				4.64	5.73
Dhahran	Riyadh	Pipeline	Gasoline	15.23			14.89	7.74
Dhahran	Riyadh	Pipeline	Diesel		10.40			
Dhahran	Dammam	Pipeline	Diesel		30.93			
Dhahran	Dammam	Sea	Gasoline	28.58			35.11	35.67
Dhahran	Dammam	Sea	Diesel			47.67		

Dhahran	Jubail	Pipeline	Gasoline	6.19		6.59
Dhahran	Jubail	Pipeline	Diesel		8.67	2.33
Qatif	Riyadh	Pipeline	Gasoline			9.06
Qatif	Dammam	Truck	Gasoline			34.56
Qatif	Dammam	Pipeline	Diesel	43.26	15.99	41.05
Qatif	Dammam	Sea	Gasoline		34.01	
Qatif	Dammam	Sea	Diesel			49.20
Qatif	Jubail	Pipeline	Gasoline			6.38
Qatif	Jubail	Pipeline	Diesel	6.74		8.95
Saffaniya	Riyadh	Pipeline	Gasoline		43.72	43.51
Saffaniya	Riyadh	Pipeline	Diesel	48.21		43.53
Saffaniya	Jubail	Pipeline	Gasoline		6.28	
Saffaniya	Jubail	Pipeline	Diesel	1.79	6.47	9.09
Saffaniya	Jubail	Sea	Gasoline			6.49

$XEWA_{ijmt}$:

Table B3: Transportation Link Expansion Values From the Deterministic Model

i	j	M	t=3
Yanbu (3)	Jawf	Pipeline	46.81292

XA_{ijmpt} (Penalty Function Approach):

Table B4: Distributed Amounts Between Refineries and Distribution Centers in the Penalty Function Model (MB)

I	J	m	p	t=1	t=2	t=3	t=4	t=5
Ras Tanura	Turaif	Truck	Diesel					2.270518
Ras Tanura	Riyadh	Pipeline	Gasoline			50		8.994359
Ras Tanura	Riyadh	Pipeline	Diesel				50	
Ras Tanura	Hafuf	Truck	Gasoline		15.46657			27.62597
Ras Tanura	Hafuf	Truck	Diesel		34.53343		34.03454	
Ras Tanura	Dhahran	Pipeline	Gasoline				41.59759	
Ras Tanura	Dhahran	Pipeline	Diesel			8.806597	7.376898	49.20427
Ras Tanura	Dhahran	Sea	Gasoline			40.94251		
Ras Tanura	Qatif	Pipeline	Gasoline					30.85808
Ras Tanura	Qatif	Pipeline	Diesel			50	50	19.14192
Ras Tanura	Qatif	Sea	Diesel	50	50			
Ras Tanura	Saffaniya	Pipeline	Gasoline			8.238695		
Ras Tanura	Saffaniya	Pipeline	Diesel			41.7613		
Jubail(2)	Turaif	Truck	Diesel				2.234762	
Jubail(2)	Salil	Truck	Gasoline		3.082856	11.33455		3.454037
Jubail(2)	Salil	Truck	Diesel				4.74	
Jubail(2)	Riyadh	Pipeline	Diesel	50	50			41.00564
Jubail(2)	Hafuf	Truck	Gasoline	18.62601		15.71404	15.96546	
Jubail(2)	Hafuf	Truck	Diesel	31.37399		34.28596		22.37403
Jubail(2)	Dhahran	Pipeline	Gasoline		40.29774			
Jubail(2)	Dhahran	Pipeline	Diesel		9.702257			
Jubail(2)	Dhahran	Sea	Gasoline	50				

Jubail(2)	Saffaniya	Pipeline	Gasoline	50	50			
Jubail(2)	Saffaniya	Pipeline	Diesel					9.090663
Jubail(2)	Saffaniya	Sea	Gasoline					40.90934
Jubail(2)	Saffaniya	Sea	Diesel	50				
Rabigh	Rabigh	Truck	Gasoline	48.57708				
Rabigh	Rabigh	Truck	Diesel					27.27561
Rabigh	Rabigh	Pipeline	Gasoline					22.72439
Rabigh	Rabigh	Pipeline	Diesel			47.60784		
Rabigh	Rabigh	Sea	Gasoline	35.7205				47.84883
Rabigh	Rabigh	Sea	Diesel					2.151169
Rabigh	Jazan	Sea	Gasoline			8.436791	13.81678	
Rabigh	Jazan	Sea	Diesel	41.92937	10.04774	41.56321		
Rabigh	Najran	Truck	Gasoline		4.539634		13.89377	
Rabigh	Najran	Truck	Diesel					5.83404
Rabigh	Abha	Truck	Gasoline				16.00276	35.39047
Rabigh	Abha	Truck	Diesel	18.14673	12.87434	13.08033		
Rabigh	Jeddah	Sea	Gasoline			7.076782		16.25747
Rabigh	Jeddah	Sea	Diesel	50	50	28.23505	50	33.74253
Yanbu (3)	Jawf	Truck	Gasoline	12.12965	17.1486			
Yanbu (3)	Jawf	Truck	Diesel	13.52723	13.74367			
Yanbu (3)	Jawf	Pipeline	Gasoline			22.04909	20.14766	20.47003
Yanbu (3)	Jawf	Pipeline	Diesel			24.04576	24.43049	24.82138
Yanbu (3)	Tabuk	Truck	Gasoline	38.20744	49.62387	30.64582	38.12059	39.03309
Yanbu (3)	Tabuk	Truck	Diesel	11.77472		18.02855	10.11424	10.96691
Yanbu (3)	Rabigh	Pipeline	Diesel	14.2795				

Yanbu (3)	Rabigh	Sea	Gasoline					2.392161
Yanbu (3)	Jazan	Sea	Gasoline	8.070627	39.95226			
Yanbu (3)	Jazan	Sea	Diesel				35.83821	50
Yanbu (3)	Abha	Truck	Gasoline		5.00566			
Yanbu (3)	Yanbu	Pipeline	Gasoline				50	
Yanbu (3)	Yanbu	Pipeline	Diesel	50				50
Yanbu (3)	Yanbu	Sea	Diesel		50		49.89257	
Yanbu (3)	Jeddah	Sea	Diesel				14.68817	
Yanbu (3)	Duba	Sea	Gasoline	49.54949	12.28907	50	45.555	50
Yanbu (3)	Duba	Sea	Diesel		37.71093		4.445002	
Yanbu (3)	Qassim	Truck	Gasoline	41.89651	10.67427	8.437656	41.50129	8.709821
Yanbu (3)	Qassim	Truck	Diesel	8.103487	39.32573	41.56234	8.498711	41.29018
Riyadh	Salil	Truck	Gasoline	45.26	7.836911		9.316678	6.011708
Riyadh	Salil	Truck	Diesel		37.42309	38.02186	35.94332	39.24829
Riyadh	Najran	Truck	Gasoline			4.612268		
Jeddah	Najran	Truck	Gasoline					17.46872
Jeddah	Abha	Truck	Gasoline	31.85327	32.12	32.12	18.83039	
Jeddah	Abha	Truck	Diesel				13.28961	13.50225

XB_{jkmp} (Penalty Function Approach):

Table B5: Distributed Amounts Between Distribution Centers and Demand Markets from the Penalty Function Model (MB)

j	k	m	p	t=1	t=2	t=3	t=4	t=5
Jawf	Sakakah	Truck	Gasoline	2.68	2.72	2.77	2.81	2.86
Jawf	Sakakah	Truck	Diesel	2.66	2.70	2.74	2.79	2.83
Jawf	Hail	Truck	Gasoline	5.58	5.67	5.76	5.85	5.94
Jawf	Hail	Truck	Diesel	5.53	5.62	5.71	5.80	5.89
Jawf	Qurayyat	Truck	Gasoline			2.22		
Jawf	Hafr Albatan	Truck	Gasoline		4.82	7.31	7.43	7.55
Jawf	Hafr Albatan	Truck	Diesel			10.08	10.24	10.41
Jawf	Arar	Truck	Gasoline	3.87	3.93	4.00	4.06	4.13
Jawf	Arar	Truck	Diesel	5.34	5.43	5.51	5.60	5.69
Tabuk	Tabuk	Truck	Gasoline	7.87	8.00	8.12	8.25	8.39
Tabuk	Tabuk	Truck	Diesel	9.64		9.95	10.11	10.28
Tabuk	Qurayyat	Truck	Gasoline	2.15	2.18		2.25	2.29
Tabuk	Qurayyat	Truck	Diesel	2.13		2.20		
Tabuk	Medina	Truck	Gasoline	28.19	39.44	22.52	27.61	28.36
Tabuk	Medina	Truck	Diesel			5.87		0.69
Turaif	Qurayyat	Truck	Diesel				2.23	2.27
Salil	Riyadh	Truck	Gasoline	35.02				
Salil	Riyadh	Truck	Diesel				2.05	
Salil	Jeddah	Truck	Gasoline		1.89			
Salil	Mecca	Truck	Gasoline	1.36		2.16		
Salil	Mecca	Truck	Diesel		28.48	28.93	29.39	29.86

Salil	Kharj	Truck	Gasoline	8.88	9.03	9.17	9.32	9.47
Salil	Kharj	Truck	Diesel		8.95	9.09	9.24	9.38
Rabigh	Medina	Truck	Gasoline	10.63			13.10	13.01
Rabigh	Medina	Truck	Diesel			43.23		
Rabigh	Yanbu	Pipeline	Diesel			4.38		2.15
Rabigh	Yanbu	Sea	Gasoline	3.46				
Rabigh	Yanbu	Sea	Diesel	1.81				
Rabigh	Riyadh	Pipeline	Gasoline	12.45	39.26			34.84
Rabigh	Riyadh	Pipeline	Diesel				27.28	
Rabigh	Jeddah	Truck	Diesel	1.23				
Rabigh	Taif	Truck	Gasoline	9.17	9.32	2.39	9.62	
Rabigh	Taif	Truck	Diesel	11.24				
Jazan	Jazan	Truck	Gasoline			3.72		
Jazan	Jazan	Truck	Diesel		4.49			
Jazan	Jazan	Sea	Gasoline	3.60	0.28		3.78	
Jazan	Jazan	Sea	Diesel	4.41		4.56	4.63	4.70
Jazan	Najran	Truck	Gasoline	4.47				
Jazan	Najran	Truck	Diesel		5.56	5.65	5.74	
Jazan	Yanbu	Sea	Diesel					2.36
Jazan	Jeddah	Sea	Gasoline		39.67	4.72	10.04	
Jazan	Jeddah	Sea	Diesel	37.51		31.35	25.47	42.93
Najran	Jazan	Truck	Gasoline					3.84
Najran	Najran	Truck	Gasoline		4.54	4.61	4.69	4.76
Najran	Najran	Truck	Diesel					5.83
Najran	Riyadh	Truck	Gasoline				9.21	8.87

Abha	Jazan	Truck	Gasoline	3.38				
Abha	Najran	Truck	Diesel	5.48				
Abha	Abha	Truck	Gasoline	10.34	10.51	10.67	10.85	11.02
Abha	Abha	Truck	Diesel	12.67	12.87	13.08	13.29	13.50
Abha	Mecca	Truck	Gasoline	21.51	23.24	21.45	23.99	24.37
Yanbu	Medina	Truck	Gasoline			17.55		
Yanbu	Medina	Truck	Diesel	47.57	48.33		49.89	50.00
Yanbu	Yanbu	Truck	Diesel	2.43				
Yanbu	Yanbu	Sea	Diesel		1.67			
Yanbu	Riyadh	Pipeline	Gasoline			32.45		
Riyadh	Riyadh	Truck	Gasoline			50.00		8.99
Riyadh	Riyadh	Truck	Diesel	50.00	50.00		50.00	41.01
Jeddah	Jeddah	Truck	Gasoline					6.48
Jeddah	Jeddah	Truck	Diesel	21.97		31.32	38.21	
Jeddah	Jeddah	Sea	Diesel		38.58			21.77
Jeddah	Mecca	Truck	Diesel	28.03				
Jeddah	Taif	Truck	Gasoline			7.08		9.77
Jeddah	Taif	Truck	Diesel		11.42	11.60	11.79	11.98
Duba	Tabuk	Truck	Diesel		9.80			
Duba	Qurayyat	Truck	Diesel		2.16			
Duba	Yanbu	Sea	Gasoline		3.51	3.57	3.63	3.69
Duba	Yanbu	Sea	Diesel		2.64		4.45	
Duba	Jeddah	Sea	Gasoline	49.55	8.77	46.43	41.93	46.31
Duba	Jeddah	Sea	Diesel		23.11			
Qassim	Riyadh	Truck	Gasoline	26.64			32.93	

Qassim	Riyadh	Truck	Diesel		21.17	33.20		32.66
Qassim	Qassim	Truck	Gasoline	8.17	8.30	8.44	8.57	8.71
Qassim	Qassim	Truck	Diesel	8.10	8.23	8.36	8.50	8.63
Qassim	Hafr Albatan	Truck	Gasoline	7.08	2.37			
Qassim	Hafr Albatan	Truck	Diesel		9.92			
Hafuf	Riyadh	Truck	Gasoline	3.40				
Hafuf	Riyadh	Truck	Diesel		13.20	12.61	12.01	
Hafuf	Hafuf	Truck	Gasoline	15.22	15.47	15.71	15.97	16.22
Hafuf	Hafuf	Truck	Diesel	21.00	21.33	21.67	22.02	22.37
Hafuf	Dammam	Truck	Gasoline					11.41
Hafuf	Kharj	Truck	Diesel	8.81				
Hafuf	Jubail	Truck	Diesel	1.57				
Dhahran	Riyadh	Pipeline	Gasoline	10.34				
Dhahran	Riyadh	Pipeline	Diesel		4.12			
Dhahran	Dammam	Pipeline	Gasoline	33.48			35.11	
Dhahran	Dammam	Pipeline	Diesel					49.20
Dhahran	Dammam	Sea	Gasoline		34.01	34.56		
Dhahran	Dammam	Sea	Diesel				7.38	
Dhahran	Jubail	Pipeline	Gasoline	6.19	6.28	6.38	6.49	
Dhahran	Jubail	Pipeline	Diesel		5.58	8.81		
Qatif	Riyadh	Pipeline	Diesel			2.33		19.14
Qatif	Dammam	Truck	Diesel		46.92			
Qatif	Dammam	Pipeline	Diesel	46.18		47.67	41.05	
Qatif	Dammam	Sea	Gasoline					24.27
Qatif	Jubail	Pipeline	Gasoline					6.59

Qatif	Jubail	Pipeline	Diesel	3.82	3.08	8.95	
Saffaniya	Riyadh	Pipeline	Gasoline		50.00	8.24	50.00 40.91
Saffaniya	Riyadh	Pipeline	Diesel	37.09		41.76	
Saffaniya	Hafr Albatn	Truck	Diesel	9.77			
Saffaniya	Jubail	Pipeline	Diesel	3.14			9.09

$XEWA_{ijmt}$ (Penalty Function Approach):

Table B6: Transportation Link Expansion Values from the Deterministic Model

i	j	m	t=3
Yanbu (3)	Jawf	Pipeline	46.09

Appendix C

TWO-STAGE STOCHASTIC MODEL RESULTS

XA_{ijmpts} (Sample):

Table C1: Distributed Amounts Between Refineries and Distribution Centers in the Stochastic Model (MB)

i	j	m	p	t	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
Ras	Turai	Truc	Gasol	2										2.371	2.406
Ras	Turai	Truc	Gasol	3							2.395				
Ras	Turai	Truc	Gasol	4											3.015
Ras	Riyad	Pipeli	Gasol	2	21.54	46.47								50	
Ras	Riyad	Pipeli	Gasol	3					40.76						
Ras	Riyad	Pipeli	Gasol	4				4.74	9.538						
Ras	Riyad	Pipeli	Diesel	3	19.74									50	
Ras	Riyad	Pipeli	Diesel	4						9.900					3.010
Ras	Riyad	Pipeli	Diesel	5						50			50		
Ras	Hafuf	Truc	Gasol	2		29.50	17.73	24.29		27.91	27.58		26.69	16.79	19.59
Ras	Hafuf	Truc	Gasol	3	7.528		0.25	19.56	25.58	16.45	25.12	25.36		18.53	23.68
Ras	Hafuf	Truc	Gasol	4		1.891		15.34	0.25			23.63		21.79	21.35
Jubail(2	Turai	Truc	Diesel	2									2.317		4.135
Jubail(2	Turai	Truc	Diesel	5											29.41
Jubail(2	Salil	Truc	Gasol	1	4.74	4.74								4.74	4.74
Jubail(2	Salil	Truc	Gasol	2				12.68		9.236			4.74		4.74

Jubail(2	Salil	Truc	Gasol	3										4.74
Jubail(2	Riyad	Pipeli	Gasol	4	6.627								50	46.98
Jubail(2	Riyad	Pipeli	Gasol	5									50	
Jubail(2	Riyad	Pipeli	Diesel	1						50				
Jubail(2	Riyad	Pipeli	Diesel	2		3.520			44.43	50				14.51
Jubail(2	Hafuf	Truc	Diesel	3	19.35	20.84	35.01	30.28	24.41			24.38	31.21	26.31
Jubail(2	Hafuf	Truc	Diesel	4	15.53	19.52	33.80	33.21	49.5					28.64
Jubail(2	Hafuf	Truc	Diesel	5	19.64	18.69	39.49	34.60		24.79		30.63	28.70	27.43
Jubail(2	Dhah	Truc	Gasol	5									26.65	
Jubail(2	Dhah	Truc	Diesel	1								10.33		
Jubail(2	Dhah	Pipeli	Gasol	1		50							41.46	
Rabigh	Tabu	Truc	Gasol	2				7.933	0.25					
Rabigh	Tabu	Truc	Gasol	3								5.138		
Rabigh	Tabu	Truc	Diesel	1						11.52				
Rabigh	Tabu	Truc	Diesel	3						10.42				
Rabigh	Turai	Truc	Gasol	2					2.200					
Rabigh	Rabig	Truc	Diesel	4				50				50		32.90
Rabigh	Rabig	Truc	Diesel	5				45.09	48.81					34.02
Rabigh	Rabig	Pipeli	Gasol	1		26.68	45.77					50		46.09
Rabigh	Rabig	Pipeli	Gasol	2	50					49.75				
Rabigh	Jazan	Sea	Gasol	5		5.004	7.503	1.849			15.49			
Rabigh	Jazan	Sea	Diesel	1	27.75	23.58	50	36.05	18.14	50	28.12	22.31	26.37	24.14
Rabigh	Jazan	Sea	Diesel	2	31.74	36.25								
Rabigh	Jazan	Sea	Diesel	3				6.044					1.277	
Rabigh	Jazan	Sea	Diesel	4		20.97			19.48					

Rabigh	Abha	Truc	Gasol	2	14.25	6.836	15.10	2.559	10.66	35.60	17.88				
Rabigh	Abha	Truc	Gasol	3				1.245			20.67				
Rabigh	Abha	Truc	Gasol	4							12.84				
Rabigh	Abha	Truc	Diesel	1				31.58	31.58	14.05	3.439	17.88			
Rabigh	Abha	Truc	Diesel	2		2.355	45.49	39.16	22.47	16.51	38.50	17.88			
Rabigh	Jedda	Truc	Diesel	3			17.88								
Rabigh	Jedda	Truc	Diesel	5				4.428	4.249						
Rabigh	Jedda	Sea	Gasol	1	38.75	38.75				36.04	38.75	38.75			
Rabigh	Jedda	Sea	Gasol	2	9.694		0.25		31.29	16.60	46.69	23.28	41.03		
Yanbu	Jawf	Truc	Gasol	3						2.986					
Yanbu	Jawf	Truc	Gasol	4								17.01			
Yanbu	Jawf	Truc	Gasol	5						3.539	28.47	39.48			
Yanbu	Tabu	Truc	Gasol	3					21.40	19.04	23.68	21.28	30.62		
Yanbu	Tabu	Truc	Gasol	4			8.060			11.78	21.31	23.43	11.04		
Yanbu	Tabu	Truc	Gasol	5						20.32	16.77	33.53	19.21	34.85	
Yanbu	Tabu	Truc	Diesel	3			1.816	17.11		24.79	22.47	23.38	27.92	19.37	
Yanbu	Tabu	Truc	Diesel	4							14.90	21.47	26.56	38.95	
Yanbu	Tabu	Truc	Diesel	5							28.20	16.46	30.78	15.14	
Yanbu	Duba	Sea	Diesel	2		4.136	50	50	48.55	49.75	50		4.676	31.69	
Yanbu	Duba	Sea	Diesel	3	22.30	27.34	14.20	12.84		12.66	11.22	47.08	36.21	5.311	
Yanbu	Duba	Sea	Diesel	4	1.417	11.43	12.00	20.25	24.52	26.19	45.44	24.82	50	31.54	29.71
Yanbu	Duba	Sea	Diesel	5			3.643	13.41	30.24	24.97	50	50	25.85	50	
Yanbu	Qassi	Truc	Gasol	1	15.25	32.12	41.89	40.57		40.25		31.62		15.25	32.12
Yanbu	Qassi	Truc	Gasol	2	32.84	14.88	42.21	46.55	31.70	8.249	41.19	50	8.889	9.019	9.150
Yanbu	Qassi	Truc	Gasol	3							23.92	23.10	22.32	25.36	20.67

Yanbu	Qassi	Truc	Gasol	4						9.716			27.06	20.30	10.27	
Yanbu	Qassi	Truc	Gasol	5						33.07		7.274				
Riyadh	Salil	Truc	Gasol	3						9.121			34.84	22.56	11.13	
Riyadh	Salil	Truc	Gasol	4								39.12	10.45	33.42	37.64	
Riyadh	Salil	Truc	Gasol	5								12.42		13.49	22.24	
Riyadh	Salil	Truc	Diesel	1	39.26	36.25	45.26	4.566	37.08	22.88		45.26	4.587	15.81	31.62	
Riyadh	Qassi	Truc	Diesel	4	7.169						27.31					
Riyadh	Qassi	Truc	Diesel	5	7.847					17.26	15.68	15.76		3.646		
Riyadh	Hafuf	Truc	Gasol	4	0.25											
Jeddah	Jazan	Sea	Diesel	2	21.75					4.590						
Jeddah	Jazan	Sea	Diesel	3	22.20					24.39						
Jeddah	Jazan	Sea	Diesel	4	15.83	18.26							29.46	10.60		
Jeddah	Jazan	Sea	Diesel	5	19.16	20.47	21.84	18.73	16.10	15.40	24.80	3.718	31.92	6.543	6.933	
Jeddah	Najra	Truc	Gasol	1	0.368											

$XB_{jk m p t s}$ (Sample):

Table C2: Distributed Amounts Between Refineries and Distribution Centers in the Stochastic Model (MB)

j	k	m	p	t	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
Jawf	Sakakah	Truck	Gasoli	1	2.68	2.68		2.68	2.68		2.68	2.68	2.68	2.68	2.68
Jawf	Hail	Truck	Gasoli	1				5.58	5.58		5.58	5.58	5.58		0.37
Jawf	Riyadh	Truck	Gasoli	5											26.5
Jawf	Hafr	Truck	Diesel	5										1.47	
Jawf	Arar	Truck	Gasoli	5										5.74	
Tabuk	Sakakah	Truck	Gasoli	2	2.57		2.66								
Tabuk	Tabuk	Truck	Gasoli	4				8.06			9.26		10.1	10.5	11.0
Tabuk	Hail	Truck	Gasoli	2	5.35	5.44	5.53								
Tabuk	Qurayyat	Truck	Gasoli	4							2.53		2.76	2.89	
Tabuk	Medina	Truck	Gasoli	5								1.27	5.56	7.55	22.4
Tabuk	Arar	Truck	Diesel	1									5.34		
Turaif	Qurayyat	Truck	Gasoli	4											3.02
Turaif	Hafr	Truck	Diesel	5										13.0	15.3
Turaif	Arar	Truck	Diesel	5											8.39
Salil	Najran	Truck	Diesel	1					0.25						
Salil	Riyadh	Truck	Gasoli	2					38.0		19.2	39.7	25.0		
Salil	Taif	Truck	Diesel	2						2.14					
Salil	Kharj	Truck	Gasoli	4								10.9	10.4	11.9	12.4
Salil	Hafr	Truck	Gasoli	2			3.36								
Rabigh	Sakakah	Truck	Gasoli	4							3.15				
Rabigh	Medina	Truck	Diesel	3		26.4	1.61	3.84	6.04			50.0	16.3	23.8	9.62
Rabigh	Jazan	Sea	Diesel	4					4.74						

Rabigh	Yanbu	Pipelin	Diesel	3											4.87		
Rabigh	Riyadh	Pipelin	Gasoli	1	32.2	10.4	0.25	29.4	9.11						21.4	4.21	30.2
Rabigh	Jeddah	Truck	Diesel	4			1.20			35.7	5.03	3.10					
Rabigh	Mecca	Truck	Diesel	2											28.7		
Rabigh	Taif	Truck	Gasoli	3	8.45		9.03	9.32		9.92	10.2	10.5					
Jazan	Jazan	Truck	Gasoli	2					3.69		3.80	3.86					4.03
Jazan	Najran	Truck	Diesel	1	3.61	5.48	5.48	5.48	5.22	5.48	5.48	5.48			3.61	5.48	
Jazan	Abha	Truck	Gasoli	2				10.4		10.7							
Najran	Kharj	Truck	Gasoli	1											1.74		
Abha	Jazan	Truck	Gasoli	5											3.87		
Abha	Najran	Truck	Gasoli	3												5.44	
Yanbu	Hail	Truck	Diesel	4	4.89	5.14	5.40	5.66									
Yanbu	Medina	Truck	Gasoli	3		27.8	2.31		6.16			44.6	12.2	10.4			
Yanbu	Jeddah	Truck	Diesel	5								7.29					
Yanbu	Taif	Truck	Diesel	1								11.2					
Yanbu	Arar	Truck	Diesel	5		0.38											
Riyadh	Riyadh	Truck	Gasoli	2	21.5	46.4	50.0			1.07			50.0	50.0	35.4		
Riyadh	Kharj	Truck	Gasoli	4	7.86	8.26	8.67	9.10	9.54	9.99							
Jeddah	Medina	Truck	Diesel	5						5.61							
Jeddah	Jazan	Sea	Diesel	4				4.52									
Duba	Tabuk	Truck	Gasoli	5	6.68	7.14	7.62	8.13	8.65	9.20							
Duba	Medina	Truck	Diesel	3											2.18		
Duba	Yanbu	Sea	Gasoli	1	3.46	3.46	3.46	3.46	3.46				3.46	3.46	3.46		
Qassim	Hail	Truck	Diesel	5						6.46	6.86	7.30	1.97				
Qassim	Riyadh	Truck	Gasoli	2	18.2		34.1	34.6	16.0		32.5	41.2					

Qassim	Qassim	Truck	Gasoli	3	7.53	7.79	8.04	8.31	8.57	8.84	9.11	9.39	9.67	9.95	10.2
Hafuf	Riyadh	Truck	Diesel	5			19.4					2.91			
Hafuf	Hafuf	Truck	Gasoli	5					16.7	17.8	18.6	19.3	21.2	22.5	23.9
Hafuf	Dammam	Truck	Diesel	2					1.54				0.22		6.90
Hafuf	Kharj	Truck	Gasoli	2				8.95							2.55
Dhahra	Riyadh	Pipelin	Gasoli	3	1.64	2.86	50.0	50.0	50.0	43.3	43.1			34.9	
Dhahra	Hafuf	Truck	Gasoli	1			14.9								
Qatif	Riyadh	Pipelin	Gasoli	2								5.47			
Qatif	Hafuf	Truck	Gasoli	4	13.4	12.2	14.6		16.3		17.9				
Qatif	Dammam	Truck	Gasoli	1	33.4			33.4			33.4				3.82
Qatif	Dammam	Pipelin	Gasoli	4		31.1	32.6		35.9	37.6			43.0		46.9
Qatif	Dammam	Sea	Gasoli	4	29.6						39.3			44.9	
Qatif	Kharj	Truck	Gasoli	1			2.76								
Qatif	Jubail	Pipelin	Diesel	2	5.67		8.46								
Saffaniy	Riyadh	Truck	Gasoli	5							3.76				

XC_{jlmpts} :

Table C3: Distributed Amounts Between Distribution Centers (MB)

j	l	m	p	t	S1	S2	S3	S4	S5	S6	S7	S8	S9
Tabuk	Jazan	Truck	Gasoline	2					0.25				
Tabuk	Duba	Truck	Gasoline	2								0.25	
Salil	Jawf	Truck	Gasoline	1									0.25
Salil	Tabuk	Truck	Gasoline	1					0.25				
Salil	Tabuk	Truck	Gasoline	2								0.25	
Salil	Abha		Gasoline	2							0.25		
Salil	Abha	Truck	Diesel	1				0.25					
Salil	Qassim	Truck	Diesel	2			0.25				0.25		
Salil	Hafuf	Truck	Gasoline	3									0.25
Rabigh	Jazan	Sea	Gasoline	4				0.25					
Rabigh	Jeddah	Sea	Gasoline	4	18.06		9.42						
Rabigh	Jeddah	Sea	Gasoline	5		8.32		2.55					
Rabigh	Jeddah	Sea	Diesel	4		21.49	11.67	3.96	3.62				
Rabigh	Jeddah	Sea	Diesel	5	34.95	2.34	6.89	4.56					
Jazan	Tabuk	Truck	Gasoline	5							0.25		
Jazan	Abha	Truck	Gasoline	3						2.21			
Jazan	Abha	Truck	Gasoline	4			10.05						
Jazan	Abha	Truck	Diesel	5							1.83		
Jazan	Jeddah	Sea	Gasoline	3						6.23			
Jazan	Jeddah	Sea	Gasoline	4							7.84	0.25	
Jazan	Duba	Truck	Diesel	4							0.25		
Jazan	Duba	Sea	Gasoline	5					0.25				

Najran	Salil	Truck	Gasoline	2				10.37	
Abha	Jawf	Truck	Diesel	1				0.25	
Abha	Najran	Truck	Diesel	2				0.25	
Abha	Hafuf	Truck	Gasoline	2			0.42		
Yanbu	Jeddah	Sea	Gasoline	5			8.88		
Yanbu	Jeddah	Sea	Diesel	4			7.38		
Yanbu	Jeddah	Sea	Diesel	5			12.79		
Yanbu	Duba	Sea	Gasoline	5	11.90	11.52			
Yanbu	Duba	Sea	Diesel	4	11.35				
Yanbu	Duba	Sea	Diesel	5	10.45	11.16	10.33	4.80	
Riyadh	Hafuf	Truck	Gasoline	3				0.25	
Jeddah	Jazan	Pipeline	Gasoline	5				0.25	
Jeddah	Qassim	Truck	Gasoline	2				0.25	
Duba	Tabuk	Truck	Gasoline	3	2.05			3.38	
Duba	Tabuk	Truck	Gasoline	5			2.77		
Duba	Tabuk	Truck	Diesel	1				0.25	
Duba	Tabuk	Truck	Diesel	2				0.25	
Duba	Tabuk	Truck	Diesel	3				0.25	
Duba	Tabuk	Truck	Diesel	4				5.73	
Duba	Jazan	Truck	Diesel	4				0.25	
Duba	Yanbu	Sea	Gasoline	3		0.92		6.16	
Duba	Yanbu	Sea	Diesel	4				6.24	
Duba	Jeddah	Sea	Gasoline	3				6.00	
Duba	Jeddah	Sea	Diesel	5				1.54	1.61
Duba	Qassim	Truck	Gasoline	4					0.25

Duba	Qassim	Truck	Diesel	5		0.25	
Duba	Hafuf	Truck	Gasoline	3		0.25	
Duba	Hafuf	Truck	Diesel	5		0.25	
Qassim	Salil	Truck	Gasoline	1		0.25	
Qassim	Jazan	Truck	Diesel	1			0.25
Qassim	Riyadh	Truck	Diesel	4		2.27	
Qassim	Duba	Truck	Diesel	2		0.25	
Qassim	Hafuf	Truck	Gasoline	4		0.25	
Hafuf	Jawf	Truck	Gasoline	1		0.25	
Hafuf	Tabuk	Truck	Gasoline	3		0.25	0.25
Hafuf	Tabuk	Truck	Diesel	3			0.25
Hafuf	Tabuk	Truck	Diesel	4			0.25
Hafuf	Jazan	Truck	Diesel	5		0.25	
Hafuf	Najran	Truck	Gasoline	2			0.25
Hafuf	Abha	Truck	Gasoline	2		0.25	
Hafuf	Abha	Truck	Diesel	2			0.25
Hafuf	Riyadh	Truck	Diesel	4	1.70		
Hafuf	Riyadh	Truck	Diesel	5		4.12	
Hafuf	Jeddah	Truck	Diesel	2	0.25		
Hafuf	Qassim	Truck	Gasoline	4		0.25	
Hafuf	Qassim	Truck	Diesel	4		0.25	
Hafuf	Qatif	Truck	Gasoline	2			0.25
Dhahran	Hafuf	Truck	Gasoline	1	0.25		
Qatif	Qassim	Truck	Gasoline	1		0.25	
Qatif	Hafuf	Truck	Gasoline	1		0.25	

Qatif	Hafuf	Truck	Gasoline	5	0.25
Saffaniya	Hafuf	Truck	Gasoline	4	0.25
Saffaniya	Hafuf	Truck	Diesel	4	0.25

XER_{it} :

Table C4: Refineries Expansion Values from the Stochastic Models (MB)

i	t=5
Riyadh	20.50
Jeddah	50.05

XES_{jt} :

j	t=3	t=4
Rabigh	94.72	
Yanbu		86.89
Qatif	100	

Table C5: Distribution centers expansion values from the stochastic model (MB)

Appendix D

ROBUST MODEL RESULTS

XA_{ijmpts} :

Table D1: Distributed Amounts Between Refineries and Distribution Centers in the Robust Model (MB)

i	j	m	p	t=1	t=2	t=3	t=4	t=5
Ras Tanura	Turaif	Truck	Gasoline					18.50
Ras Tanura	Salil	Truck	Diesel					4.74
Ras Tanura	Najran	Truck	Diesel					7.73
Ras Tanura	Riyadh	Pipeline	Gasoline	46.96			16.36	25.94
Ras Tanura	Riyadh	Pipeline	Diesel	3.04	50	50		24.06
Ras Tanura	Hafuf	Truck	Gasoline	29				
Ras Tanura	Hafuf	Truck	Diesel	21		24.95		
Ras Tanura	Dhahran	Sea	Gasoline	50				
Ras Tanura	Dhahran	Sea	Diesel				50.00	
Ras Tanura	Qatif	Pipeline	Gasoline				50.00	
Ras Tanura	Qatif	Pipeline	Diesel	50	50	50.00		50.00
Ras Tanura	Qatif	Sea	Gasoline			50.00		48.39
Ras Tanura	Qatif	Sea	Diesel				50.00	
Ras Tanura	Saffaniya	Sea	Gasoline		50			
Jubail(2)	Turaif	Truck	Diesel					31.50
Jubail(2)	Salil	Truck	Diesel	4.41	4.49		4.74	

Jubail(2)	Najran	Truck	Gasoline					2.57
Jubail(2)	Riyadh	Pipeline	Diesel				33.64	
Jubail(2)	Hafuf	Truck	Gasoline	16.59	25.05	19.71		20.36
Jubail(2)	Hafuf	Truck	Diesel	32.52		30.29		29.64
Jubail(2)	Dhahran	Pipeline	Gasoline	43.23				
Jubail(2)	Dhahran	Pipeline	Diesel	6.77	50.00			
Jubail(2)	Dhahran	Sea	Diesel					50.00
Jubail(2)	Qatif	Pipeline	Gasoline		47.12			
Jubail(2)	Qatif	Sea	Gasoline				47.12	
Jubail(2)	Qatif	Sea	Diesel					48.74
Jubail(2)	Saffaniya	Pipeline	Gasoline		50.00			
Jubail(2)	Saffaniya	Pipeline	Diesel	50.00				
Jubail(2)	Saffaniya	Sea	Gasoline					50.00
Jubail(2)	Saffaniya	Sea	Diesel				50.00	
Rabigh	Rabigh	Truck	Diesel					17.32
Rabigh	Rabigh	Pipeline	Gasoline				50.00	
Rabigh	Rabigh	Sea	Diesel	28.79	50	50.00		
Rabigh	Jazan	Sea	Gasoline					21.09
Rabigh	Jazan	Sea	Diesel	48.80				
Rabigh	Najran	Truck	Gasoline	14.55				39.70
Rabigh	Najran	Truck	Diesel	25.54			7.09	
Rabigh	Abha	Truck	Gasoline	18.41	4.07	39.46	33.59	17.88
Rabigh	Abha	Truck	Diesel	13.81				
Rabigh	Jeddah	Sea	Gasoline	50.00		10.90	23.59	34.13
Rabigh	Jeddah	Sea	Diesel	32.07	39.10	26.41		15.87

Yanbu (3)	Jawf	Truck	Gasoline	12.13	20.94		15.71	11.66
Yanbu (3)	Jawf	Truck	Diesel	13.53	25.39		32.93	38.34
Yanbu (3)	Tabuk	Truck	Gasoline	48.84	10.92	19.69	12.98	50.00
Yanbu (3)	Tabuk	Truck	Diesel		39.08	30.01	36.88	
Yanbu (3)	Rabigh	Pipeline	Gasoline					32.68
Yanbu (3)	Rabigh	Pipeline	Diesel	21.21				
Yanbu (3)	Jazan	Sea	Gasoline		50			
Yanbu (3)	Jazan	Sea	Diesel			23.89	50.00	28.91
Yanbu (3)	Yanbu	Truck	Gasoline		50			
Yanbu (3)	Yanbu	Truck	Diesel			23.72	50.00	50.00
Yanbu (3)	Yanbu	Pipeline	Gasoline			46.13	34.80	32.55
Yanbu (3)	Yanbu	Pipeline	Diesel	47.57			15.20	
Yanbu (3)	Yanbu	Sea	Gasoline				19.85	
Yanbu (3)	Yanbu	Sea	Diesel			50.00		37.30
Yanbu (3)	Jeddah	Sea	Gasoline		17.93			
Yanbu (3)	Duba	Sea	Gasoline	12.18		50.00	50.00	
Yanbu (3)	Duba	Sea	Diesel	37.82	50.00			50.00
Yanbu (3)	Qassim	Truck	Gasoline	32.03	41.17	22.16	39.51	50.00
Yanbu (3)	Qassim	Truck	Diesel	17.87	8.83	27.80	10.49	
Riyadh	Salil	Truck	Gasoline	31.76	9.60	10.55	5.39	
Riyadh	Salil	Truck	Diesel	13.50	35.66	33.30	39.87	45.26
Jeddah	Jazan	Sea	Gasoline			22.57		
Jeddah	Najran	Truck	Gasoline				10.45	
Jeddah	Abha	Truck	Gasoline		32.12			12.24
Jeddah	Abha	Truck	Diesel	31.59		9.55	16.41	19.88

XB_{jkmpts} :

Table D2: Distributed Amounts Between Distribution Centers and Demand Markets in the Robust Model (MB)

j	k	m	p	t=1	t=2	t=3	t=4	t=5
Jawf	Sakakah	Truck	Gasoline	2.68	2.92		3.47	3.78
Jawf	Sakakah	Truck	Diesel	2.66	2.90		3.44	3.75
Jawf	Tabuk	Truck	Diesel					13.61
Jawf	Hail	Truck	Gasoline	5.58	6.08		7.22	7.87
Jawf	Hail	Truck	Diesel	5.53	6.03		7.16	7.80
Jawf	Qurayyat	Truck	Diesel				2.76	
Jawf	Riyadh	Truck	Diesel					8.91
Jawf	Hafr Albatan	Truck	Gasoline		7.72			
Jawf	Hafr Albatan	Truck	Diesel		10.65		12.65	
Jawf	Arar	Truck	Gasoline	3.87	4.22		5.01	
Jawf	Arar	Truck	Diesel	5.34	5.82		6.92	4.27
Tabuk	Sakakah	Truck	Gasoline			3.18		
Tabuk	Sakakah	Truck	Diesel			3.16		
Tabuk	Tabuk	Truck	Gasoline	7.87	8.58	9.35	10.19	11.11
Tabuk	Tabuk	Truck	Diesel		10.51	11.46	12.49	
Tabuk	Qurayyat	Truck	Gasoline	2.15	2.34	2.55	2.78	
Tabuk	Qurayyat	Truck	Diesel		2.32	2.53		
Tabuk	Medina	Truck	Gasoline	38.82				38.89
Tabuk	Medina	Truck	Diesel		25.26	6.52	24.39	
Tabuk	Taif	Truck	Diesel		0.99			
Tabuk	Arar	Truck	Gasoline			4.60		
Tabuk	Arar	Truck	Diesel			6.35		

Turaif	Qurayyat	Truck	Gasoline					3.03
Turaif	Qurayyat	Truck	Diesel					3.01
Turaif	Qassim	Truck	Diesel					11.44
Turaif	Hafr Albaten	Truck	Gasoline					10.00
Turaif	Hafr Albaten	Truck	Diesel					13.79
Turaif	Arar	Truck	Gasoline					5.47
Turaif	Arar	Truck	Diesel					3.27
Salil	Riyadh	Truck	Gasoline		9.59			
Salil	Mecca	Truck	Gasoline	22.87	0.01			
Salil	Mecca	Truck	Diesel	9.11	30.55	33.30	36.30	37.57
Salil	Kharj	Truck	Gasoline	8.88		10.55	5.39	
Salil	Kharj	Truck	Diesel	8.81	9.60		8.31	12.43
Rabigh	Medina	Truck	Gasoline				38.12	15.91
Rabigh	Medina	Truck	Diesel		26.60	50.00		17.32
Rabigh	Riyadh	Pipeline	Diesel	38.76	23.40			
Rabigh	Mecca	Truck	Gasoline					16.76
Rabigh	Taif	Truck	Gasoline				11.88	
Rabigh	Taif	Truck	Diesel	11.24				
Jazan	Jazan	Truck	Gasoline			4.28		
Jazan	Jazan	Truck	Diesel	4.41		5.24	5.72	6.23
Jazan	Jazan	Sea	Gasoline		3.93			
Jazan	Najran	Truck	Gasoline			5.31		
Jazan	Najran	Truck	Diesel	5.48		6.51		
Jazan	Abha	Truck	Diesel			5.50		
Jazan	Jeddah	Sea	Gasoline		46.07	12.98		21.09

Jazan	Jeddah	Sea	Diesel	38.91	6.64	44.28	22.68
Najran	Jazan	Truck	Gasoline			4.67	5.09
Najran	Jazan	Truck	Diesel	4.81			
Najran	Najran	Truck	Gasoline	4.87		5.79	6.31
Najran	Najran	Truck	Diesel	5.97		7.09	7.73
Najran	Riyadh	Truck	Gasoline				18.34
Najran	Riyadh	Truck	Diesel	14.76			
Najran	Kharj	Truck	Gasoline	9.68			12.54
Abha	Jazan	Truck	Gasoline	3.60			
Abha	Najran	Truck	Gasoline	4.47			
Abha	Abha	Truck	Gasoline	10.34	11.27	12.29	13.39
Abha	Abha	Truck	Diesel	12.67	13.81	9.55	16.41
Abha	Mecca	Truck	Gasoline		24.92	27.17	20.20
Abha	Mecca	Truck	Diesel	18.92			1.99
Yanbu	Medina	Truck	Gasoline		42.32	46.13	12.16
Yanbu	Medina	Truck	Diesel	47.57			37.22
Yanbu	Yanbu	Truck	Gasoline				4.48
Yanbu	Yanbu	Truck	Diesel			5.04	5.49
Yanbu	Yanbu	Pipeline	Gasoline	3.77			
Yanbu	Riyadh	Pipeline	Gasoline	3.91			38.01
Yanbu	Riyadh	Pipeline	Diesel			28.94	18.46
Yanbu	Jeddah	Sea	Gasoline				27.67
Yanbu	Jeddah	Sea	Diesel			39.75	22.49
Riyadh	Riyadh	Truck	Gasoline	46.96			16.36
Riyadh	Riyadh	Truck	Diesel	3.04	50.00	39.54	33.64

Riyadh	Kharj	Truck	Diesel				10.46	
Jeddah	Jeddah	Truck	Gasoline			7.94		
Jeddah	Jeddah	Truck	Diesel			20.80		11.85
Jeddah	Jeddah	Sea	Gasoline	40.83				14.17 21.18
Jeddah	Jeddah	Sea	Diesel			25.75		
Jeddah	Mecca	Truck	Gasoline					9.42
Jeddah	Taif	Truck	Gasoline	9.17	10.00	10.90		12.95
Jeddah	Taif	Truck	Diesel		11.27	13.35	14.56	15.87
Duba	Tabuk	Truck	Diesel	9.64				
Duba	Qurayyat	Truck	Diesel	2.13				
Duba	Yanbu	Sea	Gasoline	3.46		4.11		
Duba	Yanbu	Sea	Diesel	4.24	4.62			5.98
Duba	Jeddah	Sea	Gasoline	8.72		45.89	50.00	
Duba	Jeddah	Sea	Diesel	21.81	45.38			44.02
Qassim	Hail	Truck	Gasoline			6.63		
Qassim	Hail	Truck	Diesel			6.57		
Qassim	Riyadh	Truck	Gasoline	16.78	32.26		13.64	38.46
Qassim	Qassim	Truck	Gasoline	8.17	8.91	9.71	10.59	11.54
Qassim	Qassim	Truck	Diesel	8.10	8.83	9.63	10.49	
Qassim	Kharj	Truck	Gasoline				6.11	
Qassim	Hafr Albatan	Truck	Gasoline	7.08		5.82	9.17	
Qassim	Hafr Albatan	Truck	Diesel	9.77		11.60		
Hafuf	Riyadh	Truck	Gasoline	13.78		6.97		
Hafuf	Hafuf	Truck	Gasoline	15.22	16.59	18.09	19.71	20.36
Hafuf	Hafuf	Truck	Diesel	21.00	22.89	24.95	27.19	29.64

Hafuf	Dammam	Truck	Diesel	9.63			
Hafuf	Kharj	Truck	Diesel			3.09	
Dhahran	Riyadh	Pipeline	Gasoline	10.34			
Dhahran	Riyadh	Pipeline	Diesel	6.77	35.00	29.15	50.00
Dhahran	Dammam	Pipeline	Gasoline	36.49			
Dhahran	Dammam	Pipeline	Diesel		4.86	9.80	
Dhahran	Dammam	Sea	Gasoline	33.48			
Dhahran	Jubail	Pipeline	Gasoline	6.19	6.74		
Dhahran	Jubail	Pipeline	Diesel		10.14	11.05	
Qatif	Riyadh	Pipeline	Gasoline		50.00	45.76	
Qatif	Riyadh	Pipeline	Diesel				21.51
Qatif	Hafuf	Truck	Gasoline				1.13
Qatif	Dammam	Truck	Gasoline		39.78		
Qatif	Dammam	Truck	Diesel				50.00
Qatif	Dammam	Pipeline	Gasoline			43.35	
Qatif	Dammam	Pipeline	Diesel	46.18	40.70	50.00	15.18
Qatif	Dammam	Sea	Gasoline				47.26
Qatif	Dammam	Sea	Diesel			50.00	
Qatif	Jubail	Pipeline	Gasoline		7.35	8.01	
Qatif	Jubail	Pipeline	Diesel	3.82	9.30		12.04
Saffaniya	Riyadh	Pipeline	Gasoline	50.00	47.41		41.27
Saffaniya	Riyadh	Pipeline	Diesel	45.29		50.00	
Saffaniya	Hafr Albatan	Truck	Gasoline		2.59		
Saffaniya	Jubail	Pipeline	Gasoline				8.73
Saffaniya	Jubail	Pipeline	Diesel	4.71			

XES_{jt} :

Table D3: Distribution Centers Expansion Values from the Robust Model (MB)

j	t=3
Yanbu	69.84
Qatif	97.12

Appendix E

DETERMINISTIC MODEL CODE

sets

i supply centers / 1*6/

j distribution centers / 1*17/

k demand markets / 1*20/

t time period / 1*5/

p product / 1*2/

m transportation mode / 1*3/ ;

alias (j,l)

alias (t,tp);

parameters

R(i) : current capacity of SC(i)

/1 200.75 , 2 255.5, 3 146, 4 381.43, 5 45.26, 6 32.12 /

S(j) : current capacity of DC(j)

$D(k,p,t)$: demand of DM(k) for product p during year t

$WA(i,m,j)$: current transportation capacity between SC(i) and DC(j) by mode m

$WB(j,m,k)$: current transportation capacity between DC(j) and DM(k) by mode m

$WC(j,m,l)$: current transportation capacity between DC(j) and DC(l) by mode

$ESL(j)$: min new-additional capacity of new-existing DC(j)

$ERL(i)$: min new-additional capacity of existing R(i)

$EWAL(i,j,m)$: min new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBL(j,k,m)$: min new-additional transportation capacity between DC(j) & DM(k) by mode m

$EWCL(j,l,m)$: min new-additional transportation capacity between DC(j) & DC(l) by mode m

$ESU(j)$: max new-additional capacity of new-existing DC(j)

$ERU(i)$: max new-additional capacity of existing R(i)

$EWAU(i,j,m)$: max new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBU(j,k,m)$: max new-additional transportation capacity between DC(j) & DM(k) by mode m

$EWCU(j,l,m)$: max new-additional transportation capacity between $DC(j)$ & $DC(l)$ by mode m

$WAL(i,j,m,p)$: min. amount of product p that can be transported per year by mode m between $SC(i)$ and $DC(j)$

$WBL(j,k,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DM(k)$

$WCL(j,l,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DC(l)$

$CA(i,m,j)$: cost per Million Barrel of transportation by mode m between $SC(i)$ and $DC(j)$

$CB(j,m,k)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DM(k)$

$CC(j,m,l)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DC(l)$

$CS(j)$: cost per Million Barrel of new-added capacity in $DC(j)$

$CR(i)$: cost per Million Barrel of new-added capacity in $R(i)$

$CWA(i,j,m)$: cost per Million Barrel of new-added transport capacity by mode m between $SC(i)$ and $DC(j)$

$CWB(j,k,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DM(k)$

$CWC(j,l,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DC(l)$

$CFES(j,t)$: fixed cost of capacity expansion in $DC(j)$ at period t

$CFER(i,t)$: fixed cost of capacity expansion in $R(i)$ at period t

$CFEWA(i,j,m,t)$: fixed cost of capacity expansion in the link connecting $SC(i)$ with $DC(j)$ for transportation mode m at period t .

$CFEWB(j,k,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DM(k)$ for transportation mode m at period t .

$CFEWC(j,l,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DC(l)$ for transportation mode m at period t .

$CFWA(i,j,m,p,t)$: fixed cost of transporting product p between $SC(i)$ and $DC(j)$ through transportation mode m at period t

$CFWB(j,k,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DM(k)$ through transportation mode m at period t

$CFWC(j,l,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DC(l)$ through transportation mode m at period t

NW : maximum number of expansions of a transportation link during the plan

/ 1 /

NS : maximum number of expansions of a DC

/ 1 /

NR : maximum number of expansions of a R

/ 1 /

DR(p) : Demand growth rate

/1 0.016, 2 0.016 /

DY1(k,p) : Demand of the first year

;

\$onecho > inputs8.txt

dset=k rng=a2:a21 rdim=1

dset=p rng=b1:c1 rdim=1

par=DY1 rng=sheet4!a1 rdim=1 cdim=1

\$offecho

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load DY1

\$GDXIN

\$onecho > inputs8.txt

dset=i rng=a2:a19 rdim=1

dset=m rng=b2:b19 rdim=1

dset=j rng=c1:s1 cdim=1

par=CA rng=sheet1!a1 rdim=2 cdim=1

par= WA rng=sheet5!a1 rdim=2 cdim=1

\$offecho

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load CA,WA

\$GDXIN

\$onecho > inputs8.txt

dset=j rng=a2:a52 rdim=1

dset=m rng=b2:b52 rdim=1

dset=l rng=c1:s1 cdim=1

dset=k rng=c1:v1 cdim=1

par=CB rng=sheet3!a1 rdim=2 cdim=1

par= WB rng=sheet7!a1 rdim=2 cdim=1

par=CC rng=sheet2!a1 rdim=2 cdim=1

par= WC rng=sheet6!a1 rdim=2 cdim=1

\$offecho

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load CB,WB,CC,WC

\$GDXIN

$S(j) = 50$;

$ESL(j) = 10$;

$ERL(i) = 10$;

$EWAL(i,j,m) = 0.5$;

$EWBL(j,k,m) = 0.5$;

$EWCL(j,l,m) = 0.5$;

$ESU(j) = 100$;

$ERU(i) = 100$;

$EWAU(i,j,m) = 100$;

$EWBU(j,k,m) = 100$;

$EWCU(j,l,m) = 100$;

$WAL(i,j,m,p) = 0.25$;

$WBL(j,k,m,p) = 0.25$;

$WCL(j,l,m,p) = 0.25$;

$*CA(i,j,m) = \text{uniform}(200000,500000)$;

$*CB(j,k,m) = \text{uniform}(200000,500000)$;

*CC(j,l,m) = uniform(200000,500000) ;

CS(j) = 5000 ;

CR(i) = 5000 ;

CWA(i,j,m) = 1000 ;

CWB(j,k,m) = 1000 ;

CWC(j,l,m) = 1000 ;

CFES(j,t) = 20000 ;

CFER(i,t) = 20000 ;

CFEWA(i,j,m,t) = 5000 ;

CFEWB(j,k,m,t) = 5000 ;

CFEWC(j,l,m,t) = 5000 ;

CFWA(i,j,m,p,t) = 2000 ;

CFWB(j,k,m,p,t) = 2000 ;

CFWC(j,l,m,p,t) = 2000 ;

D(k,p,t) = DY1(k,p) * (1+DR(p)) ** (ord(t)-1) ;

Scalar ir ;

ir = 0.1 ;

positive variables

$XA(i,j,m,p,t)$: amount of product p transported by mode m between $SC(i)$ and $DC(j)$ in year t

$XB(j,k,m,p,t)$: amount of product p transported by mode m between $DC(j)$ and $DM(k)$ in year t

$XC(j,l,m,p,t)$: amount of product p transported by mode m between $DC(j)$ and $DC(l)$ in year t

$XES(j,t)$: new-added capacity in $DC(j)$ at year t

$XER(i,t)$: new-added capacity in $R(i)$ at year t

$XEWA(i,j,m,t)$: new-added transport capacity by mode m between $SC(i)$ and $DC(j)$ in year

$XEWB(j,k,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DM(k)$ in year t

$XEWC(j,l,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DC(l)$ in year t

binary variables

$YA(i,j,m,p,t)$: equals 1 if $XA_{ijmpt} > 0$ and 0 otherwise

$YB(j,k,m,p,t)$: equals 1 if $XB_{jkmpt} > 0$ and 0 otherwise

$YC(j,l,m,p,t)$: equals 1 if $XC_{jlmpt} > 0$ and 0 otherwise

$YS(j,t)$: equals 1 if capacity of DC(j) is established-expanded in year t and 0 otherwise

$YR(i,t)$: equals 1 if capacity of R(i) is expanded in year t and 0 otherwise

$YEWA(i,j,m,t)$: equals 1 if mode m transport between SC(i) & DC(j) is established-expanded in year t and 0 otherwise

$YEWB(j,k,m,t)$: equals 1 if mode m transport between DC(j) & DM(k) is established-expanded in year t and 0 otherwise

$YEW(j,l,m,t)$: equals 1 if mode m transport between DC(j) & DC(l) is established-expanded in year t and 0 otherwise ;

$YS.fx(j,'1') = 0$;

$YS.fx(j,'2') = 0$;

$*YS.fx(j,'3') = 0$;

$YR.fx(i,'1') = 0$;

$YR.fx(i,'2') = 0$;

$*YR.fx(i,'3') = 0$;

$YEWA.fx(i,j,m,'1') = 0$;

$YEWA.fx(i,j,m,'2') = 0$;

$*YEWA.fx(i,j,m,'3') = 0$;

YEWB.fx(j,k,m,'1')= 0 ;

YEWB.fx(j,k,m,'2')= 0 ;

*YEWB.fx(j,k,m,'3')= 0 ;

YEWB.fx(j,l,m,'1')= 0 ;

YEWB.fx(j,l,m,'2')= 0 ;

*YEWB.fx(j,l,m,'3')= 0 ;

equations con1, con2, con3, con4,con5, con6, con7, con8, con33,con34,con35, con36 ;

****Supply Center Capacity ****

con1(i,t).. sum((j,m,p), XA(i,j,m,p,t)) =l= R(i) + sum((tp)\$ (ord(tp) <= ord(t)), XER(i,tp));

* Sum of all outputs from SC(i) cannot exceed its capacity

****Distribution Center Capacity ****

con2(j,t).. sum((i,m,p), XA(i,j,m,p,t)) + sum((l,m,p)\$ (ord(l) <> ord(j)), XC(j,l,m,p,t)) =l= S(j) + sum((tp)\$ (ord(tp) <= ord(t)), XES(j,tp));

*Sum of all inputs of product p into DC(j) cannot exceed its new/expanded capacity

con3(j).. sum((t), XES(j,t))=l= ESU(j);

con4(j,t).. XES(j,t) =l= ESU(j) * YS(j,t) ;

con5(j,t).. XES(j,t) =g= ESL(j) * YS(j,t) ;

* Expansions of SC(j) are between the maximum and minimum expansion capacity

$$\text{con33}(i) \dots \sum((t), \text{XER}(i,t)) = \text{ERU}(i);$$

$$\text{con34}(i,t) \dots \text{XER}(i,t) = \text{ERU}(i) * \text{YR}(i,t) ;$$

$$\text{con35}(i,t) \dots \text{XER}(i,t) = \text{ERL}(i) * \text{YR}(i,t) ;$$

* Expansions of R(i) are between the maximum and minimum expansion capacity

$$\text{con6}(j) \dots \sum((t), \text{YS}(j,t)) = \text{NS} ;$$

* Number of expansions of DC(j) cannot exceed NS

$$\text{con36}(i) \dots \sum((t), \text{YR}(i,t)) = \text{NR} ;$$

* Number of expansions of DC(j) cannot exceed NS

**** Distribution Center Material Balance ****

$$\text{con7}(j,p,t) \dots \sum((m), (\sum(i, \text{XA}(i,j,m,p,t)) + \sum(l) (\text{ord}(l) \triangleleft \text{ord}(j)), \text{XC}(l,j,m,p,t)))) \\ = e =$$

$$\sum((m), (\sum(k, \text{XB}(j,k,m,p,t)) + \sum(l) (\text{ord}(l) \triangleright \text{ord}(j)), \text{XC}(j,l,m,p,t)))) ;$$

* Sum of product p inputs (A + C) into DC(j) is equal to sum of its outputs (B + C)

**** Demand Center Material Balance ****

$$\text{con8}(k,p,t) \dots \sum((j,m), \text{XB}(j,k,m,p,t)) = \text{D}(k,p,t) ;$$

* Sum of inputs of product p into D(m,k) cannot be less than its demand

**** Transportation Capacity from SC to DC****

equations con9, con10, con11, con12, con13, con14, con15 ;

con9(i,j,m,t).. $\sum(p, XA(i,j,m,p,t)) = WA(i,m,j) + \sum(tp, XEWA(i,j,m,tp))$;
 $XEWA(i,j,m,tp)$;

* Amounts of all products transported by mode m between SC(i) and DC(j) cannot exceed link i-j new/expanded mode m capacity

con10(i,j,m).. $\sum(t, XEWA(i,j,m,t)) = EWAU(i,j,m)$;

con11(i,j,m,t).. $XEWA(i,j,m,t) = EWAU(i,j,m) * YEWA(i,j,m,t)$;

con12(i,j,m,t).. $XEWA(i,j,m,t) = EWAL(i,j,m) * YEWA(i,j,m,t)$;

* Expansions of link i-j are between the maximum and minimum expansion capacity

con13(i,j,m).. $\sum(t, YEWA(i,j,m,t)) = NW$;

* Number of expansions of link i-j cannot exceed NW

con14(i,j,m,p,t).. $XA(i,j,m,p,t) = ((WA(i,m,j) + EWAU(i,j,m)) * YA(i,j,m,p,t))$;

con15(i,j,m,p,t).. $XA(i,j,m,p,t) = WAL(i,j,m,p) * YA(i,j,m,p,t)$;

* Transportation in link i-j is between the min quantity and the max capacity

**** Transportation Capacity from DC to DM ****

equations con16, con17, con18, con19, con20, con21, con22 ;

con16(j,k,m,t).. $\sum(p, XB(j,k,m,p,t)) = WB(j,m,k) + \sum(tp) (\text{ord}(tp) \leq \text{ord}(t)), XEWB(j,k,m,tp));$

* Amounts of all products transported by mode m between DC(i) and DM(k) cannot exceed link j-k new/expanded mode m capacity

con17(j,k,m).. $\sum(t, XEWB(j,k,m,t)) = EWB(j,k,m);$

con18(j,k,m,t).. $XEWB(j,k,m,t) = EWB(j,k,m) * YEWB(j,k,m,t);$

con19(j,k,m,t).. $XEWB(j,k,m,t) = EWB(j,k,m) * YEWB(j,k,m,t);$

* Expansions of link j-k are between the maximum and minimum expansion capacity

con20(j,k,m).. $\sum(t, YEWB(j,k,m,t)) = NW ;$

* Number of expansions of link i-j cannot exceed NW

con21(j,k,m,p,t).. $XB(j,k,m,p,t) = ((WB(j,m,k) + EWB(j,k,m)) * YB(j,k,m,p,t)) ;$

con22(j,k,m,p,t).. $XB(j,k,m,p,t) = WBL(j,k,m,p) * YB(j,k,m,p,t) ;$

* Transportation in link j-k is between the min quantity and the max capacity

** Transportation Capacity from DC to DC **

equations con23, con24, con25, con26, con27, con28, con29, con30, con31, con32;

con23(j,l,m,t) $(\text{ord}(l) < \text{ord}(j))$.. $\sum(p, XC(j,l,m,p,t)) = WC(j,m,l) + \sum(tp) (\text{ord}(tp) \leq \text{ord}(t)), XEWC(j,l,m,tp));$

* Amounts of all products transported by mode m between DC(j) and DC(l) cannot exceed link j-l new/expanded mode m capacity

$$\text{con24}(j,l,m) \$(\text{ord}(l) \triangleleft \text{ord}(j)).. \sum(t, \text{XEWC}(j,l,m,t)) \leq \text{EWCU}(j,l,m);$$

$$\text{con25}(j,l,m,t) \$(\text{ord}(l) \triangleleft \text{ord}(j)).. \text{XEWC}(j,l,m,t) \leq \text{EWCU}(j,l,m) * \text{YEWC}(j,l,m,t);$$

$$\text{con26}(j,l,m,t) \$(\text{ord}(l) \triangleleft \text{ord}(j)).. \text{XEWC}(j,l,m,t) \leq \text{EWCL}(j,l,m) * \text{YEWC}(j,l,m,t);$$

* Expansions of link j-k are between the maximum and minimum expansion capacity

$$\text{con27}(j,l,m) \$(\text{ord}(l) > \text{ord}(j)).. \sum(t, \text{YEWC}(j,l,m,t)) \leq \text{NW} ;$$

* Number of expansions of link i-j cannot exceed NW

$$\text{con31}(j,l,m,t).. \text{YEWC}(j,l,m,t) \leq \text{YEWC}(l,j,m,t) ;$$

* Expansions of links of DCs are in two directions

$$\text{con32}(j,l,m,t).. \text{XEWC}(j,l,m,t) \leq \text{XEWC}(l,j,m,t) ;$$

* Expansions of links of DCs in two directions are equal

$$\text{con28}(j,l,m,p,t) \$(\text{ord}(l) \triangleleft \text{ord}(j)).. -1 * \text{XC}(j,l,m,p,t) \leq -1 * ((\text{WC}(j,m,l) + \text{EWCU}(j,l,m))$$

$$* \text{YC}(j,l,m,p,t)) ;$$

$$\text{con29}(j,l,m,p,t) \$(\text{ord}(l) \triangleleft \text{ord}(j)).. \text{XC}(j,l,m,p,t) \leq \text{WCL}(j,l,m,p) * \text{YC}(j,l,m,p,t) ;$$

* Transportation in link j-k is between the min quantity and the max capacity

$$\text{con30}(j,l,m,p,t).. \text{YC}(j,l,m,p,t) + \text{YC}(l,j,m,p,t) \leq 1 ;$$

* Transportation between DC's are in one direction for every period

free variables P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, Z;

equations E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, OBJ;

* Variable Transportation Cost

E1.. P1 =e= sum((t,m,p), sum((j), (sum(i, CA(i,m,j) * XA(i,j,m,p,t)) + sum(k, CB(j,m,k) * XB(j,k,m,p,t)) + sum((l)\$ (ord(l) <> ord(j)), CC(j,m,l) * XC(j,l,m,p,t))) * (1+ir)**(-ord(t)))) ;

* DC Variable Expansion Cost

E2.. P2 =e= sum ((t,j), (CS(j) * XES(j,t))*((1+ir)**(-ord(t)+2))));

* Variable Transportation Expansions Cost

E3.. P3 =e= sum((t,j,m), (sum(i, CWA(i,j,m) * XEWA(i,j,m,t)) + sum(k, CWB(j,k,m) * XEWB(j,k,m,t)) + sum((l)\$ (ord(l) > ord(j)), CWC(j,l,m) * 0.5*(XEWC(j,l,m,t)+ XEWC(l,j,m,t))))*(1+ir)**(-ord(t)+2)) ;

* Fixed Cost of Capacity Expansion of a DC

E4.. P4 =e= sum((t,j), CFES(j,t) * YS(j,t)*(1+ir)**(-ord(t)+2)) ;

* Fixed cost of Capacity expansion of a Link between SC and DC

E5.. P5 =e= sum((t,i,j,m), CFEWA(i,j,m,t) * YEWA(i,j,m,t)*(1+ir)**(-ord(t)+2)) ;

* Fixed cost of Capacity expansion of a Link between DC and DM

$$E6.. P6 = e = \text{sum}((t,j,k,m), \text{CFEWB}(j,k,m,t) * \text{YEWB}(j,k,m,t) * (1+ir)^{**(-ord(t)+2)});$$

* Fixed cost of Capacity expansion of a Link between DC and DC

$$E7.. P7 = e = \text{sum}((t,j,l,m) \$ (ord(l) > ord(j)), \text{CFEWC}(j,l,m,t) * 0.5 * (\text{YEWB}(j,l,m,t) + \text{YEWB}(j,j,m,t)) * (1+ir)^{**(-ord(t)+2)});$$

* Fixed cost of Transportation between SC and DC

$$E8.. P8 = e = \text{sum}((t,i,j,m,p), \text{CFWA}(i,j,m,p,t) * \text{YA}(i,j,m,p,t) * (1+ir)^{**(-ord(t))});$$

* Fixed cost of Transportation between DC and DM

$$E9.. P9 = e = \text{sum}((t,j,k,m,p), \text{CFWB}(j,k,m,p,t) * \text{YB}(j,k,m,p,t) * (1+ir)^{**(-ord(t))});$$

* Fixed cost of Transportation between DC and DC

$$E10.. P10 = e = \text{sum}((t,j,l,m,p) \$ (ord(l) < ord(j)), \text{CFWC}(j,l,m,p,t) * \text{YC}(j,l,m,p,t) * (1+ir)^{**(-ord(t))});$$

$$E11.. P11 = e = \text{sum}((t,i), (\text{CR}(i) * \text{XER}(i,t)) * (1+ir)^{**(-ord(t)+2)});$$

$$*E12.. P12 = e = \text{sum}((t,j,k,m,p), \text{XB}(j,k,m,p,t) * \text{Pr}(p) * (1+ir)^{**(-ord(t))});$$

** Objective Function **

$$\text{OBJ.. } Z = e = P1 + P2 + P3 + P4 + P5 + P6 + P7 + P8 + P9 + P10 + P11 ;$$

**** Model ****

Option Reslim=30000;

option optcr = 0;

option solvelink = 0 ;

option mip = cplex;

model part /ALL / ;

solve part using MIP minimizing Z ;

display Z.l;

execute_unload "TVM_Results.gdx" XA.L XB.L XC.L XES.L XEWA.L XEWB.L
XEWCL YAL YBL YCL YS.L YEWA.L YEWB.L YEWCL XER.L YR.L

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XA.L'

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XB.L rng=Sheet2!'

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XC.L rng=Sheet3! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XES.L rng=Sheet4! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XEWA.L rng=Sheet5! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XEWB.L rng=Sheet6! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XEWCL rng=Sheet7! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YA.L rng=Sheet8! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YB.L rng=Sheet9! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YC.L rng=Sheet10! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YS.L rng=Sheet11! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YEWA.L rng=Sheet12!'

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YEWB.L rng=Sheet13!'

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YEWC.L rng=Sheet14!'

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=XER.L rng=Sheet15! '

execute 'gdxxrw.exe TVM_Results.gdx o=TVM_Results.xls var=YR.L rng=Sheet16! '

Appendix F

TWO-STAGE STOCHASTIC MODEL CODE

sets

i supply centers / 1*6/

j distribution centers / 1*17/

k demand markets / 1*20/

t time period / 1*5/

p product / 1*2/

m transportation mode / 1*3/

s Scenarios /1*11/ ;

alias (j,l)

alias (t,tp);

parameters

R(i) : current capacity of SC(i)

/1 200.75 , 2 255.5, 3 146, 4 381.43, 5 45.26, 6 32.12 /

$SD(j)$: current capacity of DC(j)

$D(k,p,t,s)$: demand of DM(k) for product p during year t

$WA(i,m,j)$: current transportation capacity between SC(i) and DC(j) by mode m

$WB(j,m,k)$: current transportation capacity between DC(j) and DM(k) by mode m

$WC(j,m,l)$: current transportation capacity between DC(j) and DC(l) by mode m

$ESL(j)$: min new-additional capacity of new-existing DC(j)

$ERL(i)$: min new-additional capacity of existing R(i)

$EWAL(i,j,m)$: min new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBL(j,k,m)$: min new-additional transportation capacity between DC(j) & DM(k) by mode m

$EWCL(j,l,m)$: min new-additional transportation capacity between DC(j) & DC(l) by mode m

$ESU(j)$: max new-additional capacity of new-existing DC(j)

$ERU(i)$: max new-additional capacity of existing R(i)

$EWAU(i,j,m)$: max new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBU(j,k,m)$: max new-additional transportation capacity between $DC(j)$ & $DM(k)$ by mode m

$EWCU(j,l,m)$: max new-additional transportation capacity between $DC(j)$ & $DC(l)$ by mode m

$WAL(i,j,m,p)$: min. amount of product p that can be transported per year by mode m between $SC(i)$ and $DC(j)$

$WBL(j,k,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DM(k)$

$WCL(j,l,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DC(l)$

$CA(i,m,j)$: cost per Million Barrel of transportation by mode m between $SC(i)$ and $DC(j)$

$CB(j,m,k)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DM(k)$

$CC(j,m,l)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DC(l)$

$CS(j)$: cost per Million Barrel of new-added capacity in $DC(j)$

$CR(i)$: cost per Million Barrel of new-added capacity in $R(i)$

$CWA(i,j,m)$: cost per Million Barrel of new-added transport capacity by mode m between $SC(i)$ and $DC(j)$

$CWB(j,k,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DM(k)$

$CWC(j,l,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DC(l)$

$CFES(j,t)$: fixed cost of capacity expansion in $DC(j)$ at period t

$CFER(i,t)$: fixed cost of capacity expansion in $R(i)$ at period t

$CFEWA(i,j,m,t)$: fixed cost of capacity expansion in the link connecting $SC(i)$ with $DC(j)$ for transportation mode m at period t .

$CFEWB(j,k,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DM(k)$ for transportation mode m at period t .

$CFEWC(j,l,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DC(l)$ for transportation mode m at period t

$CFWA(i,j,m,p,t)$: fixed cost of transporting product p between $SC(i)$ and $DC(j)$ through transportation mode m at period t

$CFWB(j,k,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DM(k)$ through transportation mode m at period t

$CFWC(j,l,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DC(l)$ through transportation mode m at period t

NW : maximum number of expansions of a transportation link during the plan

/ 1 /

NS : maximum number of expansions of a DC

/ 1 /

NR : maximum number of expansions of a R

/ 1 /

DR(s) : Demand growth rate

/ 1 -4, 2 -2.4, 3 -0.8, 4 0.8, 5 2.4, 6 3.98, 7 5.56,

8 7.2, 9 8.75, 10 10.34, 11 11.95 /

pr(s) : Probability of scenario s

/ 1 0.006, 2 0.016, 3 0.044, 4 0.091, 5 0.15, 6 0.37, 7 0.15,

8 0.09, 9 0.044, 10 0.016, 11 0.006/

DY1(k,p) : Demand of the first year

;

\$onecho > inputs8.txt

dset=k rng=a2:a21 rdim=1

dset=p rng=b1:c1 rdim=1

par=DY1 rng=sheet4!a1 rdim=1 cdim=1

\$offecho

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load DY1

\$GDXIN

\$onecho > inputs8.txt

dset=i rng=a2:a19 rdim=1

dset=m rng=b2:b19 rdim=1

dset=j rng=c1:s1 cdim=1

par=CA rng=sheet1!a1 rdim=2 cdim=1

par= WA rng=sheet5!a1 rdim=2 cdim=1

\$offecho

```
$call GDXXRW.exe inputs8.xlsx @inputs8.txt
```

```
$GDXIN inputs8.gdx
```

```
*****
```

```
$load CA,WA
```

```
$GDXIN
```

```
*****
```

```
$onecho > inputs8.txt
```

```
dset=j rng=a2:a52 rdim=1
```

```
dset=m rng=b2:b52 rdim=1
```

```
dset=l rng=c1:s1 cdim=1
```

```
dset=k rng=c1:v1 cdim=1
```

```
par=CB rng=sheet3!a1 rdim=2 cdim=1
```

```
par= WB rng=sheet7!a1 rdim=2 cdim=1
```

```
par=CC rng=sheet2!a1 rdim=2 cdim=1
```

```
par= WC rng=sheet6!a1 rdim=2 cdim=1
```

```
*****
```

```
$offecho
```

```
$call GDXXRW.exe inputs8.xlsx @inputs8.txt
```

```
$GDXIN inputs8.gdx
```

```
*****
```

```
$load CB,WB,CC,WC
```

```
$GDXIN
```

```
SD(j) = 50 ;
```

```
ESL(j) = 10 ;
```

```
ERL(i) = 10 ;
```

```
EWAL(i,j,m) = 0.5 ;
```

```
EWBL(j,k,m) = 0.5 ;
```

```
EWCL(j,l,m) = 0.5 ;
```

```
ESU(j) = 100 ;
```

```
ERU(i) = 100 ;
```

```
EWAU(i,j,m) = 100 ;
```

```
EWBU(j,k,m) = 100 ;
```

```
EWCU(j,l,m) = 100 ;
```

$WAL(i,j,m,p) = 0.25$;
 $WBL(j,k,m,p) = 0.25$;
 $WCL(j,l,m,p) = 0.25$;
 $*CA(i,j,m) = \text{uniform}(200000,500000)$;
 $*CB(j,k,m) = \text{uniform}(200000,500000)$;
 $*CC(j,l,m) = \text{uniform}(200000,500000)$;
 $CS(j) = 5000$;
 $CR(i) = 5000$;
 $CWA(i,j,m) = 1000$;
 $CWB(j,k,m) = 1000$;
 $CWC(j,l,m) = 1000$;
 $CFES(j,t) = 20000$;
 $CFER(i,t) = 20000$;
 $CFEWA(i,j,m,t) = 5000$;
 $CFEWB(j,k,m,t) = 5000$;
 $CFEWC(j,l,m,t) = 5000$;
 $CFWA(i,j,m,p,t) = 2000$;

$$CFWB(j,k,m,p,t) = 2000 \quad ;$$

$$CFWC(j,l,m,p,t) = 2000 \quad ;$$

$$D(k,p,t,s) = DY1(k,p) * (1+(DR(s)/100))^{** (ord(t)-1)} \quad ;$$

Scalar ir ;

$$ir = 0.1 \quad ;$$

positive variables

$XA(i,j,m,p,t,s)$: amount of product p transported by mode m between $SC(i)$ and $DC(j)$ in year t for scenario s

$XB(j,k,m,p,t,s)$: amount of product p transported by mode m between $DC(j)$ and $DM(k)$ in year t for scenario s

$XC(j,l,m,p,t,s)$: amount of product p transported by mode m between $DC(j)$ and $DC(l)$ in year t for scenario s

$XES(j,t)$: new-added capacity in $DC(j)$ at year t

$XER(i,t)$: new-added capacity in $R(i)$ at year t

$XEWA(i,j,m,t)$: new-added transport capacity by mode m between $SC(i)$ and $DC(j)$ in year t

$XEWB(j,k,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DM(k)$ in year t

$XEWC(j,l,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DC(l)$ in year t

binary variables

$YA(i,j,m,p,t,s)$: equals 1 if $XA_{ijmpt} > 0$ and 0 otherwise

$YB(j,k,m,p,t,s)$: equals 1 if $XB_{jkmpt} > 0$ and 0 otherwise

$YC(j,l,m,p,t,s)$: equals 1 if $XC_{jlmpt} > 0$ and 0 otherwise

$YS(j,t)$: equals 1 if capacity of $DC(j)$ is established-expanded in year t and 0 otherwise

$YR(i,t)$: equals 1 if capacity of $R(i)$ is expanded in year t and 0 otherwise

$YEWA(i,j,m,t)$: equals 1 if mode m transport between $SC(i)$ & $DC(j)$ is established-expanded in year t and 0 otherwise

$YEWB(j,k,m,t)$: equals 1 if mode m transport between $DC(j)$ & $DM(k)$ is established-expanded in year t and 0 otherwise

$YEW(j,l,m,t)$: equals 1 if mode m transport between $DC(j)$ & $DC(l)$ is established-expanded in year t and 0 otherwise ;

$YS.fx(j,'1') = 0$;

$YS.fx(j,'2') = 0$;

$*YS.fx(j,'3') = 0$;

$YR.fx(i,'1') = 0$;

YR.fx(i,'2') = 0 ;

*YR.fx(i,'3') = 0 ;

YEWA.fx(i,j,m,'1')= 0 ;

YEWA.fx(i,j,m,'2')= 0 ;

*YEWA.fx(i,j,m,'3')= 0 ;

YEWB.fx(j,k,m,'1')= 0 ;

YEWB.fx(j,k,m,'2')= 0 ;

*YEWB.fx(j,k,m,'3')= 0 ;

YEWB.fx(j,l,m,'1')= 0 ;

YEWB.fx(j,l,m,'2')= 0 ;

*YEWB.fx(j,l,m,'3')= 0 ;

equations con1, con2, con3, con4,con5, con6, con7, con8, con33,con34,con35, con36 ;

****Supply Center Capacity ****

con1(i,t,s).. sum((j,m,p), XA(i,j,m,p,t,s)) =l= R(i) + sum((tp)\$ (ord(tp) <= ord(t)), XER(i,tp));

* Sum of all outputs from SC(i) cannot exceed its capacity

****Distribution Center Capacity ****

$$\text{con2}(j,t,s) \dots \text{sum}((i,m,p), \text{XA}(i,j,m,p,t,s)) + \text{sum}((l,m,p) \$ (\text{ord}(l) \triangleleft \text{ord}(j)), \text{XC}(j,l,m,p,t,s)) \\ = \text{SD}(j) + \text{sum}((tp) \$ (\text{ord}(tp) \leq \text{ord}(t)), \text{XES}(j,tp));$$

*Sum of all inputs of product p into DC(j) cannot exceed its new/expanded capacity

$$\text{con3}(j) \dots \text{sum}((t), \text{XES}(j,t)) = \text{ESU}(j);$$

$$\text{con4}(j,t) \dots \text{XES}(j,t) = \text{ESU}(j) * \text{YS}(j,t) ;$$

$$\text{con5}(j,t) \dots \text{XES}(j,t) = \text{ESL}(j) * \text{YS}(j,t) ;$$

* Expansions of SC(j) are between the maximum and minimum expansion capacity

$$\text{con33}(i) \dots \text{sum}((t), \text{XER}(i,t)) = \text{ERU}(i);$$

$$\text{con34}(i,t) \dots \text{XER}(i,t) = \text{ERU}(i) * \text{YR}(i,t) ;$$

$$\text{con35}(i,t) \dots \text{XER}(i,t) = \text{ERL}(i) * \text{YR}(i,t) ;$$

* Expansions of R(i) are between the maximum and minimum expansion capacity

$$\text{con6}(j) \dots \text{sum}((t), \text{YS}(j,t)) = \text{NS} ;$$

* Number of expansions of DC(j) cannot exceed NS

$$\text{con36}(i) \dots \text{sum}((t), \text{YR}(i,t)) = \text{NR} ;$$

* Number of expansions of DC(j) cannot exceed NS

** Distribution Center Material Balance **

con7(j,p,t,s).. $\sum((m), (\sum(i, XA(i,j,m,p,t,s)) + \sum((l)$(ord(l) \leq ord(j)), XC(l,j,m,p,t,s)))) = e =$

$\sum((m), (\sum(k, XB(j,k,m,p,t,s)) + \sum((l)$(ord(l) \leq ord(j)), XC(j,l,m,p,t,s)))) ;$

* Sum of product p inputs (A + C) into DC(j) is equal to sum of its outputs (B + C)

** Demand Center Material Balance **

con8(k,p,t,s).. $\sum((j,m), XB(j,k,m,p,t,s)) = g = D(k,p,t,s) ;$

* Sum of inputs of product p into D(m,k) cannot be less than its demand

** Transportation Capacity from SC to DC**

equations con9, con10, con11, con12, con13, con14, con15 ;

con9(i,j,m,t,s).. $\sum(p, XA(i,j,m,p,t,s)) = l = WA(i,m,j) + \sum((tp)$(ord(tp) \leq ord(t)), XEWA(i,j,m,tp));$

* Amounts of all products transported by mode m between SC(i) and DC(j) cannot exceed link i-j new/expanded mode m capacity

con10(i,j,m).. $\sum(t, XEWA(i,j,m,t)) = l = EWAU(i,j,m);$

con11(i,j,m,t).. $XEWA(i,j,m,t) = l = EWAU(i,j,m) * YEWA(i,j,m,t);$

con12(i,j,m,t).. $XEWA(i,j,m,t) = g = EWAL(i,j,m) * YEWA(i,j,m,t);$

* Expansions of link i-j are between the maximum and minimum expansion capacity

con13(i,j,m).. sum(t, YEWA(i,j,m,t)) =l= NW ;

* Number of expansions of link i-j cannot exceed NW

con14(i,j,m,p,t,s).. XA(i,j,m,p,t,s) =l= ((WA(i,m,j) + EWAU(i,j,m)) * YA(i,j,m,p,t,s)) ;

con15(i,j,m,p,t,s).. XA(i,j,m,p,t,s) =g= WAL(i,j,m,p) * YA(i,j,m,p,t,s) ;

* Transportation in link i-j is between the min quantity and the max capacity

** Transportation Capacity from DC to DM **

equations con16, con17, con18,con19, con20,con21, con22 ;

con16(j,k,m,t,s).. sum(p, XB(j,k,m,p,t,s)) =l= WB(j,m,k) + sum((tp)\$ (ord(tp) <= ord(t)), XEWB(j,k,m,tp));

* Amounts of all products transported by mode m between DC(i) and DM(k) cannot exceed link j-k new/expanded mode m capacity

con17(j,k,m).. sum(t,XEWB(j,k,m,t)) =l= EWBU(j,k,m);

con18(j,k,m,t).. XEWB(j,k,m,t) =l= EWBU(j,k,m) * YEWB(j,k,m,t);

con19(j,k,m,t).. XEWB(j,k,m,t) =g= EWBL(j,k,m) * YEWB(j,k,m,t);

* Expansions of link j-k are between the maximum and minimum expansion capacity

con20(j,k,m).. sum(t, YEWB(j,k,m,t)) =l= NW ;

* Number of expansions of link i-j cannot exceed NW

con21(j,k,m,p,t,s).. $XB(j,k,m,p,t,s) = ((WB(j,m,k) + EWBU(j,k,m)) * YB(j,k,m,p,t,s))$;

con22(j,k,m,p,t,s).. $XB(j,k,m,p,t,s) = WBL(j,k,m,p) * YB(j,k,m,p,t,s)$;

* Transportation in link j-k is between the min quantity and the max capacity

** Transportation Capacity from DC to DC **

equations con23, con24, con25, con26, con27, con28, con29, con30, con31, con32;

con23(j,l,m,t,s)\$(ord(l) < ord(j)).. $\sum(p, XC(j,l,m,p,t,s)) = WC(j,m,l) + \sum((tp)$(ord(tp) <= ord(t)), XEWC(j,l,m,tp));$

* Amounts of all products transported by mode m between DC(j) and DC(l) cannot exceed link j-l new/expanded mode m capacity

con24(j,l,m)\$(ord(l) < ord(j)).. $\sum(t, XEWC(j,l,m,t)) = EWCU(j,l,m);$

con25(j,l,m,t)\$(ord(l) < ord(j)).. $XEWC(j,l,m,t) = EWCU(j,l,m) * YEWC(j,l,m,t);$

con26(j,l,m,t)\$(ord(l) < ord(j)).. $XEWC(j,l,m,t) = EWCL(j,l,m) * YEWC(j,l,m,t);$

* Expansions of link j-k are between the maximum and minimum expansion capacity

con27(j,l,m)\$(ord(l) > ord(j)).. $\sum(t, YEWC(j,l,m,t)) = NW$;

* Number of expansions of link i-j cannot exceed NW

con31(j,l,m,t).. $YEWC(j,l,m,t) = YEWC(l,j,m,t)$;

* Expansions of links of DCs are in two directions

$$\text{con32}(j,l,m,t).. \text{XEWC}(j,l,m,t) = \text{XEWC}(l,j,m,t) \quad ;$$

* Expansions of links of DCs in two directions are equal

$$\text{con28}(j,l,m,p,t,s)$(\text{ord}(l) \leq \text{ord}(j)).. -1*\text{XC}(j,l,m,p,t,s) = -1*((\text{WC}(j,m,l) + \text{EWCU}(j,l,m)) * \text{YC}(j,l,m,p,t,s)) \quad ;$$

$$\text{con29}(j,l,m,p,t,s)$(\text{ord}(l) < \text{ord}(j)).. \text{XC}(j,l,m,p,t,s) = \text{WCL}(j,l,m,p) * \text{YC}(j,l,m,p,t,s) \quad ;$$

* Transportation in link j-k is between the min quantity and the max capacity

$$\text{con30}(j,l,m,p,t,s).. \text{YC}(j,l,m,p,t,s) + \text{YC}(l,j,m,p,t,s) = 1 \quad ;$$

* Transportation between DC's are in one direction for every period

free variables P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, Z;

equations E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, OBJ;

* Variable Transportation Cost

$$\text{E1.. } P1 = \sum(s, pr(s) * (\sum((t,m,p), \sum((j), (\sum(i, CA(i,m,j) * XA(i,j,m,p,t,s)) + \sum(k, CB(j,m,k) * XB(j,k,m,p,t,s)) + \sum(l)$(\text{ord}(l) < \text{ord}(j)), CC(j,m,l) * XC(j,l,m,p,t,s)))) * (1+ir)^{(-ord(t))}))) \quad ;$$

* DC Variable Expansion Cost

$$\text{E2.. } P2 = \sum((t,j), (CS(j) * XES(j,t)) * ((1+ir)^{(-ord(t)+2)}));$$

* Variable Transportation Expansions Cost

$$E3.. P3 = e = \sum((t,j,m), (\sum(i, CWA(i,j,m) * XEWA(i,j,m,t)) + \sum(k, CWB(j,k,m) * XEWB(j,k,m,t)) + \sum(l) \$ (ord(l) > ord(j)), CWC(j,l,m) * 0.5 * (XEWC(j,l,m,t) + XEWC(l,j,m,t)))) * (1+ir)^{-(ord(t)+2)}) ;$$

* Fixed Cost of Capacity Expansion of a DC

$$E4.. P4 = e = \sum((t,j), CFES(j,t) * YS(j,t) * (1+ir)^{-(ord(t)+2)}) ;$$

* Fixed cost of Capacity expansion of a Link between SC and DC

$$E5.. P5 = e = \sum((t,i,j,m), CFEWA(i,j,m,t) * YEWA(i,j,m,t) * (1+ir)^{-(ord(t)+2)}) ;$$

* Fixed cost of Capacity expansion of a Link between DC and DM

$$E6.. P6 = e = \sum((t,j,k,m), CFEBW(j,k,m,t) * YEWB(j,k,m,t) * (1+ir)^{-(ord(t)+2)}) ;$$

* Fixed cost of Capacity expansion of a Link between DC and DC

$$E7.. P7 = e = \sum((t,j,l,m) \$ (ord(l) > ord(j)), CFELC(j,l,m,t) * 0.5 * (YELC(j,l,m,t) + YELC(j,j,m,t)) * (1+ir)^{-(ord(t)+2)}) ;$$

* Fixed cost of Transportation between SC and DC

$$E8.. P8 = e = \sum(s, pr(s) * (\sum((t,i,j,m,p), CFWA(i,j,m,p,t) * YA(i,j,m,p,t,s) * (1+ir)^{-(ord(t))}))) ;$$

* Fixed cost of Transportation between DC and DM

$$E9.. P9 = e = \sum(s, pr(s) * (\sum((t,j,k,m,p), CFWB(j,k,m,p,t) * YB(j,k,m,p,t,s) * (1+ir)^{-(ord(t))}))) ;$$

* Fixed cost of Transportation between DC and DC

E10.. P10 =e= sum(s,pr(s)*(sum((t,j,l,m,p)\$ (ord(l) <> ord(j)), CFWC(j,l,m,p,t) *
YC(j,l,m,p,t,s)*(1+ir)**(-ord(t))))) ;

E11.. P11 =e= sum ((t,i), (CR(i) * XER(i,t))*((1+ir)**(-ord(t)+2))));

*E12.. P12 =e= sum((t,j,k,m,p), XB(j,k,m,p,t,s) * Pr(p)*(1+ir)**(-ord(t))) ;

** Objective Function **

OBJ.. Z =e= P1 + P2 + P3 + P4 + P5 + P6 + P7 + P8 + P9 + P10 + P11 ;

** Model **

Option Reslim=50000;

option optcr = 0;

option solvelink = 0 ;

option mip = cplex;

model part /ALL / ;

File soft / '%emp.info%' /;

put soft;

\$onput

adjustequ

con14 maxz 10000

con21 maxz 10000

\$offput

putclose;

solve part using EMP minimizing Z ;

parameters

slack1(i,j,m,p,t,s)

slack2(j,k,m,p,t,s)

slack3(j,l,m,p,t,s)

total_slack1

total_slack2 ;

slack1(i,j,m,p,t,s) = (WA(i,m,j) + EWAU(i,j,m)) * YA.l(i,j,m,p,t,s) - XA.l(i,j,m,p,t,s) ;

slack2(j,k,m,p,t,s) = ((WB(j,m,k) + EWBU(j,k,m)) * YB.l(j,k,m,p,t,s)) - XB.l(j,k,m,p,t,s);

slack3 (j,l,m,p,t,s) \$(ord(l) <> ord(j)) = ((WC(j,m,l) + EWCU(j,l,m)) * YC.l(j,l,m,p,t,s)) - XC.l(j,l,m,p,t,s) ;

```
total_slack1 = sum((i,j,m,p,t,s), min(0, (WA(i,m,j) + EWAU(i,j,m)) * YA.l(i,j,m,p,t,s) -
XA.l(i,j,m,p,t,s))) ;
```

```
total_slack2 = sum((j,k,m,p,t,s), min(0, ((WB(j,m,k) + EWBU(j,k,m)) * YB.l(j,k,m,p,t,s))
- XB.l(j,k,m,p,t,s)));
```

```
display  Z.l, slack1, slack2, slack3, total_slack1, total_slack2, XA.l, XB.l, XC.l,
YA.L,YB.L, YC.l;
```

```
execute_unload "Stochastic_Results.gdx" XA.L XB.L XC.L XES.L XEWA.L XEWB.L
XEW.C L YA.L YB.L YC.L YS.L YEWA.L YEWB.L YEW.C L XER.L YR.L
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XA.L'
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XB.L
rng=Sheet2!'
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XC.L
rng=Sheet3! '
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XES.L
rng=Sheet4! '
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XEWA.L
rng=Sheet5! '
```

```
execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XEWB.L
rng=Sheet6! '
```

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XEWC.L
rng=Sheet7! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YA.L
rng=Sheet8! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YB.L
rng=Sheet9! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YC.L
rng=Sheet10! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YS.L
rng=Sheet11! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YEWA.L
rng=Sheet12! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YEWB.L
rng=Sheet13! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YEWC.L
rng=Sheet14! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=XER.L
rng=Sheet15! '

execute 'gdxxrw.exe Stochastic_Results.gdx o=Stochastic_Results.xls var=YR.L
rng=Sheet16! '

Appendix G

ROBUST MODEL CODE

sets

i supply centers / 1*6/

j distribution centers / 1*17/

k demand markets / 1*20/

t time period / 1*5/

p product / 1*2/

m transportation mode / 1*3/ ;

alias (j,l)

alias (t,tp);

parameters

R(i) : current capacity of SC(i)

/1 200.75 , 2 255.5, 3 146, 4 381.43, 5 45.26, 6 32.12 /

$S(j)$: current capacity of DC(j)

$D(k,p,t)$: demand of DM(k) for product p during year t

$WA(i,m,j)$: current transportation capacity between SC(i) and DC(j) by mode m

$WB(j,m,k)$: current transportation capacity between DC(j) and DM(k) by mode m

$WC(j,m,l)$: current transportation capacity between DC(j) and DC(l) by mode m

$ESL(j)$: min new-additional capacity of new-existing DC(j)

$ERL(i)$: min new-additional capacity of existing R(i)

$EWAL(i,j,m)$: min new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBL(j,k,m)$: min new-additional transportation capacity between DC(j) & DM(k) by mode m

$EWCL(j,l,m)$: min new-additional transportation capacity between DC(j) & DC(l) by mode m

$ESU(j)$: max new-additional capacity of new-existing DC(j)

$ERU(i)$: max new-additional capacity of existing R(i)

$EWAU(i,j,m)$: max new-additional transportation capacity between SC(i) & DC(j) by mode m

$EWBU(j,k,m)$: max new-additional transportation capacity between $DC(j)$ & $DM(k)$ by mode m

$EWCU(j,l,m)$: max new-additional transportation capacity between $DC(j)$ & $DC(l)$ by mode m

$WAL(i,j,m,p)$: min. amount of product p that can be transported per year by mode m between $SC(i)$ and $DC(j)$

$WBL(j,k,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DM(k)$

$WCL(j,l,m,p)$: min. amount of product p that can be transported per year by mode m between $DC(j)$ and $DC(l)$

$CA(i,m,j)$: cost per Million Barrel of transportation by mode m between $SC(i)$ and $DC(j)$

$CB(j,m,k)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DM(k)$

$CC(j,m,l)$: cost per Million Barrel of transportation by mode m between $DC(j)$ and $DC(l)$

$CS(j)$: cost per Million Barrel of new-added capacity in $DC(j)$

$CR(i)$: cost per Million Barrel of new-added capacity in $R(i)$

$CWA(i,j,m)$: cost per Million Barrel of new-added transport capacity by mode m between $SC(i)$ and $DC(j)$

$CWB(j,k,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DM(k)$

$CWC(j,l,m)$: cost per Million Barrel of new-added transport capacity by mode m between $DC(j)$ and $DC(l)$

$CFES(j,t)$: fixed cost of capacity expansion in $DC(j)$ at period t

$CFER(i,t)$: fixed cost of capacity expansion in $R(i)$ at period t

$CFEWA(i,j,m,t)$: fixed cost of capacity expansion in the link connecting $SC(i)$ with $DC(j)$ for transportation mode m at period t .

$CFEWB(j,k,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DM(k)$ for transportation mode m at period t .

$CFEWC(j,l,m,t)$: fixed cost of capacity expansion in the link connecting $DC(j)$ with $DC(l)$ for transportation mode m at period t .

$CFWA(i,j,m,p,t)$: fixed cost of transporting product p between $SC(i)$ and $DC(j)$ through transportation mode m at period t

$CFWB(j,k,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DM(k)$ through transportation mode m at period t

$CFWC(j,l,m,p,t)$: fixed cost of transporting product p between $DC(j)$ and $DC(l)$ through transportation mode m at period t

NW : maximum number of expansions of a transportation link during the plan

/ 1 /

NS : maximum number of expansions of a DC

/ 1 /

NR : maximum number of expansions of a R

/ 1 /

DR(p) : Demand growth rate

/1 0.090, 2 0.090 /

DY1(k,p) : Demand of the first year

sd(j,k,m,p,t) : small d

v(k,p,t) : v

cd(j,k,m,p,t) : capital d

vc(k,p,t) : vc

;

\$onecho > inputs8.txt

dset=k rng=a2:a21 rdim=1

dset=p rng=b1:c1 rdim=1

par=DY1 rng=sheet4!a1 rdim=1 cdim=1

\$offecho

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load DY1

\$GDXIN

\$onecho > inputs8.txt

dset=i rng=a2:a19 rdim=1

dset=m rng=b2:b19 rdim=1

dset=j rng=c1:s1 cdim=1

par=CA rng=sheet1!a1 rdim=2 cdim=1

par= WA rng=sheet5!a1 rdim=2 cdim=1

\$offecho

```
$call GDXXRW.exe inputs8.xlsx @inputs8.txt
```

```
$GDXIN inputs8.gdx
```

```
*****
```

```
$load CA,WA
```

```
$GDXIN
```

```
*****
```

```
$onecho > inputs8.txt
```

```
dset=j rng=a2:a52 rdim=1
```

```
dset=m rng=b2:b52 rdim=1
```

```
dset=l rng=c1:s1 cdim=1
```

```
dset=k rng=c1:v1 cdim=1
```

```
par=CB rng=sheet3!a1 rdim=2 cdim=1
```

```
par= WB rng=sheet7!a1 rdim=2 cdim=1
```

```
par=CC rng=sheet2!a1 rdim=2 cdim=1
```

```
par= WC rng=sheet6!a1 rdim=2 cdim=1
```

```
*****
```

```
$offecho
```

\$call GDXXRW.exe inputs8.xlsx @inputs8.txt

\$GDXIN inputs8.gdx

\$load CB,WB,CC,WC

\$GDXIN

S(j) = 50 ;

ESL(j) = 10 ;

ERL(i) = 10 ;

EWAL(i,j,m) = 0.5 ;

EWBL(j,k,m) = 0.5 ;

EWCL(j,l,m) = 0.5 ;

ESU(j) = 100 ;

ERU(i) = 100 ;

EWAU(i,j,m) = 100 ;

EWBU(j,k,m) = 100 ;

EWCU(j,l,m) = 100 ;

WAL(i,j,m,p) = 0.25 ;

$$WBL(j,k,m,p) = 0.25 \quad ;$$

$$WCL(j,l,m,p) = 0.25 \quad ;$$

$$*CA(i,j,m) = \text{uniform}(200000,500000) \quad ;$$

$$*CB(j,k,m) = \text{uniform}(200000,500000) \quad ;$$

$$*CC(j,l,m) = \text{uniform}(200000,500000) \quad ;$$

$$CS(j) = 5000 \quad ;$$

$$CR(i) = 5000 \quad ;$$

$$CWA(i,j,m) = 1000 \quad ;$$

$$CWB(j,k,m) = 1000 \quad ;$$

$$CWC(j,l,m) = 1000 \quad ;$$

$$CFES(j,t) = 20000 \quad ;$$

$$CFER(i,t) = 20000 \quad ;$$

$$CFEWA(i,j,m,t) = 5000 \quad ;$$

$$CFEWB(j,k,m,t) = 5000 \quad ;$$

$$CFEWC(j,l,m,t) = 5000 \quad ;$$

$$CFWA(i,j,m,p,t) = 2000 \quad ;$$

$$CFWB(j,k,m,p,t) = 2000 \quad ;$$

$$CFWC(j,l,m,p,t) = 2000 \quad ;$$

$$D(k,p,t) = DY1(k,p) * (1+DR(p))^{** (ord(t)-1)} \quad ;$$

$$sd(j,k,m,p,t) = 1 \quad ;$$

$$cd(j,k,m,p,t) = 1 \quad ;$$

$$v(k,p,t) = D(k,p,t) \quad ;$$

$$vc(k,p,t) = 1 \quad ;$$

Scalar ir ;

$$ir = 0.1 \quad ;$$

positive variables

$XA(i,j,m,p,t)$: amount of product p transported by mode m between $SC(i)$ and $DC(j)$ in year t

$XB(j,k,m,p,t)$: amount of product p transported by mode m between $DC(j)$ and $DM(k)$ in year t

$XC(j,l,m,p,t)$: amount of product p transported by mode m between $DC(j)$ and $DC(l)$ in year t

$XES(j,t)$: new-added capacity in $DC(j)$ at year t

$XER(i,t)$: new-added capacity in $R(i)$ at year t

$XEWA(i,j,m,t)$: new-added transport capacity by mode m between $SC(i)$ and $DC(j)$ in year t

$XEWB(j,k,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DM(k)$ in year t

$XEWC(j,l,m,t)$: new-added transport capacity by mode m between $DC(j)$ and $DC(l)$ in year t

$xv(k,p,t)$: new dual variable#1

$pc(j,k,m,p,t)$: new dual variable#2

$p_v(k,p,t)$: new dual variable#3

binary variables

$YA(i,j,m,p,t)$: equals 1 if $XA_{ijmpt} > 0$ and 0 otherwise

$YB(j,k,m,p,t)$: equals 1 if $XB_{jkmpt} > 0$ and 0 otherwise

$YC(j,l,m,p,t)$: equals 1 if $XC_{jlmpt} > 0$ and 0 otherwise

$YS(j,t)$: equals 1 if capacity of $DC(j)$ is established-expanded in year t and 0 otherwise

$YR(i,t)$: equals 1 if capacity of $R(i)$ is expanded in year t and 0 otherwise

$YEWA(i,j,m,t)$: equals 1 if mode m transport between $SC(i)$ & $DC(j)$ is established-expanded in year t and 0 otherwise

YEWB(j,k,m,t) : equals 1 if mode m transport between DC(j) & DM(k) is established-
expanded in year t and 0 otherwise

YEWB(j,l,m,t) : equals 1 if mode m transport between DC(j) & DC(l) is established-
expanded in year t and 0 otherwise ;

YS.fx(j,'1') = 0 ;

YS.fx(j,'2') = 0 ;

*YS.fx(j,'3') = 0 ;

YR.fx(i,'1') = 0 ;

YR.fx(i,'2') = 0 ;

*YR.fx(i,'3') = 0 ;

YEWA.fx(i,j,m,'1')= 0 ;

YEWA.fx(i,j,m,'2')= 0 ;

*YEWA.fx(i,j,m,'3')= 0 ;

YEWB.fx(j,k,m,'1')= 0 ;

YEWB.fx(j,k,m,'2')= 0 ;

*YEWB.fx(j,k,m,'3')= 0 ;

$$YEW.C.fx(j,l,m,'1')=0 ;$$

$$YEW.C.fx(j,l,m,'2')=0 ;$$

$$*YEW.C.fx(j,l,m,'3')=0 ;$$

$$xv.fx(k,p,t) = 1 ;$$

equations con1, con2, con3, con4,con5, con6, con7, con8, con33,con34,con35, con36,
con37, con38 ;

****Supply Center Capacity ****

$$con1(i,t).. \sum((j,m,p), XA(i,j,m,p,t)) = R(i) + \sum((tp)$(ord(tp) \leq ord(t)), XER(i,tp));$$

* Sum of all outputs from SC(i) cannot exceed its capacity

****Distribution Center Capacity ****

$$con2(j,t).. \sum((i,m,p), XA(i,j,m,p,t)) + \sum((l,m,p)$(ord(l) < ord(j)), XC(j,l,m,p,t)) = S(j) + \sum((tp)$(ord(tp) \leq ord(t)), XES(j,tp));$$

*Sum of all inputs of product p into DC(j) cannot exceed its new/expanded capacity

$$con3(j).. \sum((t), XES(j,t)) = ESU(j);$$

$$con4(j,t).. XES(j,t) = ESU(j) * YS(j,t) ;$$

$$con5(j,t).. XES(j,t) = ESL(j) * YS(j,t) ;$$

* Expansions of SC(j) are between the maximum and minimum expansion capacity

$$\text{con33}(i) \dots \sum((t), \text{XER}(i,t)) = \text{ERU}(i);$$

$$\text{con34}(i,t) \dots \text{XER}(i,t) = \text{ERU}(i) * \text{YR}(i,t) ;$$

$$\text{con35}(i,t) \dots \text{XER}(i,t) = \text{ERL}(i) * \text{YR}(i,t) ;$$

* Expansions of R(i) are between the maximum and minimum expansion capacity

$$\text{con6}(j) \dots \sum((t), \text{YS}(j,t)) = \text{NS} ;$$

* Number of expansions of DC(j) cannot exceed NS

$$\text{con36}(i) \dots \sum((t), \text{YR}(i,t)) = \text{NR} ;$$

* Number of expansions of DC(j) cannot exceed NS

** Distribution Center Material Balance **\

$$\text{con7}(j,p,t) \dots \sum((m), (\sum(i, \text{XA}(i,j,m,p,t)) + \sum((l) \$(\text{ord}(l) < \text{ord}(j)), \text{XC}(l,j,m,p,t)))) \\ = e =$$

$$\sum((m), (\sum(k, \text{XB}(j,k,m,p,t)) + \sum((l) \$(\text{ord}(l) > \text{ord}(j)), \text{XC}(j,l,m,p,t)))) ;$$

* Sum of product p inputs (A + C) into DC(j) is equal to sum of its outputs (B + C)

** Demand Center Material Balance **

$$\text{con8}(k,p,t) \dots \sum((j,m), \text{pc}(j,k,m,p,t) * \text{sd}(j,k,m,p,t)) - \text{pv}(k,p,t) * \text{v}(k,p,t) = g = 0 ;$$

* Sum of inputs of product p into D(m,k) cannot be less than its demand

$$\text{con37}(j,k,m,p,t) \dots \text{pc}(j,k,m,p,t) * \text{cd}(j,k,m,p,t) = \text{XB}(j,k,m,p,t) ;$$

$$\text{con38}(k,p,t) \dots \text{pv}(k,p,t) * \text{vc}(k,p,t) = \text{xv}(k,p,t) ;$$

**** Transportation Capacity from SC to DC****

equations con9, con10, con11, con12, con13, con14, con15 ;

$$\text{con9}(i,j,m,t) \dots \sum(p, \text{XA}(i,j,m,p,t)) = \text{WA}(i,m,j) + \sum((tp) \$ (\text{ord}(tp) \leq \text{ord}(t)), \text{XEWA}(i,j,m,tp));$$

* Amounts of all products transported by mode m between SC(i) and DC(j) cannot exceed link i-j new/expanded mode m capacity

$$\text{con10}(i,j,m) \dots \sum(t, \text{XEWA}(i,j,m,t)) = \text{EWAU}(i,j,m);$$

$$\text{con11}(i,j,m,t) \dots \text{XEWA}(i,j,m,t) = \text{EWAU}(i,j,m) * \text{YEWA}(i,j,m,t);$$

$$\text{con12}(i,j,m,t) \dots \text{XEWA}(i,j,m,t) = \text{EWAL}(i,j,m) * \text{YEWA}(i,j,m,t);$$

* Expansions of link i-j are between the maximum and minimum expansion capacity

$$\text{con13}(i,j,m) \dots \sum(t, \text{YEWA}(i,j,m,t)) = \text{NW} ;$$

* Number of expansions of link i-j cannot exceed NW

$$\text{con14}(i,j,m,p,t) \dots \text{XA}(i,j,m,p,t) = ((\text{WA}(i,m,j) + \text{EWAU}(i,j,m)) * \text{YA}(i,j,m,p,t)) ;$$

$$\text{con15}(i,j,m,p,t) \dots \text{XA}(i,j,m,p,t) = \text{WAL}(i,j,m,p) * \text{YA}(i,j,m,p,t) ;$$

* Transportation in link i-j is between the min quantity and the max capacity

**** Transportation Capacity from DC to DM ****

equations con16, con17, con18,con19, con20,con21, con22 ;

con16(j,k,m,t).. $\sum(p, XB(j,k,m,p,t)) = WB(j,m,k) + \sum(tp)(ord(tp) \leq ord(t)),$
XEWB(j,k,m,tp));

* Amounts of all products transported by mode m between DC(i) and DM(k) cannot exceed
link j-k new/expanded mode m capacity

con17(j,k,m).. $\sum(t, XEWB(j,k,m,t)) = EWBU(j,k,m);$

con18(j,k,m,t).. $XEWB(j,k,m,t) = EWBU(j,k,m) * YEWB(j,k,m,t);$

con19(j,k,m,t).. $XEWB(j,k,m,t) = EWB(j,k,m) * YEWB(j,k,m,t);$

* Expansions of link j-k are between the maximum and minimum expansion capacity

con20(j,k,m).. $\sum(t, YEWB(j,k,m,t)) = NW ;$

* Number of expansions of link i-j cannot exceed NW

con21(j,k,m,p,t).. $XB(j,k,m,p,t) = ((WB(j,m,k) + EWBU(j,k,m)) * YB(j,k,m,p,t)) ;$

con22(j,k,m,p,t).. $XB(j,k,m,p,t) = WBL(j,k,m,p) * YB(j,k,m,p,t) ;$

* Transportation in link j-k is between the min quantity and the max capacity

**** Transportation Capacity from DC to DC ****

equations con23, con24, con25,con26, con27,con28, con29, con30, con31, con32;

$$\text{con23}(j,l,m,t) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } \sum(p, \text{XC}(j,l,m,p,t)) = \text{WC}(j,m,l) + \sum(tp) \text{ } (\text{ord}(tp) \leq \text{ord}(t)), \text{XEWC}(j,l,m,tp));$$

* Amounts of all products transported by mode m between DC(j) and DC(l) cannot exceed link j-l new/expanded mode m capacity

$$\text{con24}(j,l,m) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } \sum(t, \text{XEWC}(j,l,m,t)) = \text{EWCU}(j,l,m);$$

$$\text{con25}(j,l,m,t) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } \text{XEWC}(j,l,m,t) = \text{EWCU}(j,l,m) * \text{YEW C}(j,l,m,t);$$

$$\text{con26}(j,l,m,t) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } \text{XEWC}(j,l,m,t) = \text{EWCL}(j,l,m) * \text{YEW C}(j,l,m,t);$$

* Expansions of link j-k are between the maximum and minimum expansion capacity

$$\text{con27}(j,l,m) \text{ } (\text{ord}(l) > \text{ord}(j)) \text{.. } \sum(t, \text{YEW C}(j,l,m,t)) = \text{NW} ;$$

* Number of expansions of link i-j cannot exceed NW*

$$\text{con31}(j,l,m,t) \text{.. } \text{YEW C}(j,l,m,t) = \text{YEW C}(l,j,m,t) ;$$

* Expansions of links of DCs are in two directions

$$\text{con32}(j,l,m,t) \text{.. } \text{XEWC}(j,l,m,t) = \text{XEWC}(l,j,m,t) ;$$

* Expansions of links of DCs in two directions are equal

$$\text{con28}(j,l,m,p,t) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } -1 * \text{XC}(j,l,m,p,t) = -1 * ((\text{WC}(j,m,l) + \text{EWCU}(j,l,m))$$

* $\text{YC}(j,l,m,p,t) ;$

$$\text{con29}(j,l,m,p,t) \text{ } (\text{ord}(l) < \text{ord}(j)) \text{.. } \text{XC}(j,l,m,p,t) = \text{WCL}(j,l,m,p) * \text{YC}(j,l,m,p,t) ;$$

* Transportation in link j-k is between the min quantity and the max capacity

$$\text{con30}(j,l,m,p,t) \cdot YC(j,l,m,p,t) + YC(l,j,m,p,t) = 1 ;$$

* Transportation between DC's are in one direction for every period

free variables P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, Z;

equations E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, OBJ;

* Variable Transportation Cost

$$\begin{aligned} E1.. P1 = & \sum((t,m,p), \sum((j), (\sum(i, CA(i,m,j) * XA(i,j,m,p,t)) + \sum(k, CB(j,m,k) * \\ & XB(j,k,m,p,t)) + \sum((l) \$ (ord(l) < ord(j)), CC(j,m,l) * XC(j,l,m,p,t))) * (1+ir)^{(-ord(t))})) \\ & ; \end{aligned}$$

* DC Variable Expansion Cost

$$E2.. P2 = \sum((t,j), (CS(j) * XES(j,t)) * ((1+ir)^{(-ord(t)+2)}));$$

* Variable Transportation Expansions Cost

$$\begin{aligned} E3.. P3 = & \sum((t,j,m), (\sum(i, CWA(i,j,m) * XEWA(i,j,m,t)) + \sum(k, CWB(j,k,m) * \\ & XEWB(j,k,m,t)) + \sum((l) \$ (ord(l) > ord(j)), CWC(j,l,m) * 0.5 * (XEWC(j,l,m,t) + \\ & XEWC(l,j,m,t))) * (1+ir)^{(-ord(t)+2)}) ; \end{aligned}$$

* Fixed Cost of Capacity Expansion of a DC

$$E4.. P4 = \sum((t,j), CFES(j,t) * YS(j,t) * (1+ir)^{(-ord(t)+2)}) ;$$

* Fixed cost of Capacity expansion of a Link between SC and DC

$$E5.. P5 = e = \sum((t,i,j,m), CFEWA(i,j,m,t) * YEWA(i,j,m,t)*(1+ir)**(-ord(t)+2)) ;$$

* Fixed cost of Capacity expansion of a Link between DC and DM

$$E6.. P6 = e = \sum((t,j,k,m), CFEWB(j,k,m,t) * YEWB(j,k,m,t)*(1+ir)**(-ord(t)+2)) ;$$

* Fixed cost of Capacity expansion of a Link between DC and DC

$$E7.. P7 = e = \sum((t,j,l,m)\$(ord(l) > ord(j)), CFEWC(j,l,m,t) * 0.5*(YEW C(j,l,m,t)+ YEW C(j,j,m,t))*(1+ir)**(-ord(t)+2)) ;$$

* Fixed cost of Transportation between SC and DC

$$E8.. P8 = e = \sum((t,i,j,m,p), CFWA(i,j,m,p,t) * YA(i,j,m,p,t)*(1+ir)**(-ord(t))) ;$$

* Fixed cost of Transportation between DC and DM

$$E9.. P9 = e = \sum((t,j,k,m,p), CFWB(j,k,m,p,t) * YB(j,k,m,p,t)*(1+ir)**(-ord(t))) ;$$

* Fixed cost of Transportation between DC and DC

$$E10.. P10 = e = \sum((t,j,l,m,p)\$(ord(l) < ord(j)), CFWC(j,l,m,p,t) * YC(j,l,m,p,t)*(1+ir)**(-ord(t))) ;$$

$$E11.. P11 = e = \sum((t,i), (CR(i) * XER(i,t))*((1+ir)**(-ord(t)+2)));$$

** Objective Function **

$$OBJ.. Z = e = (P1 + P2 + P3 + P4 + P5 + P6 + P7 + P8 + P9 + P10 + P11) ;$$

**** Model ****

Option Reslim=30000;

option optcr = 0;

option solvelink = 0 ;

option mip = cplex;

model part /ALL / ;

File soft / '%emp.info%' /;

put soft;

\$onput

adjustequ

con14 maxz 20000

con21 maxz 20000

\$offput

putclose;

solve part using EMP MINIMIZING Z ;

parameters

slack1(i,j,m,p,t)

slack2(j,k,m,p,t)

slack3(j,l,m,p,t)

total_slack1

total_slack2 ;

slack1(i,j,m,p,t) = (WA(i,m,j) + EWAU(i,j,m)) * YA.l(i,j,m,p,t) - XA.l(i,j,m,p,t) ;

slack2(j,k,m,p,t) = ((WB(j,m,k) + EWBU(j,k,m)) * YB.l(j,k,m,p,t)) - XB.l(j,k,m,p,t);

slack3 (j,l,m,p,t) \$(ord(l) <> ord(j)) = ((WC(j,m,l) + EWCU(j,l,m)) * YC.l(j,l,m,p,t)) - XC.l(j,l,m,p,t) ;

total_slack1 = sum((i,j,m,p,t), min(0, (WA(i,m,j) + EWAU(i,j,m)) * YA.l(i,j,m,p,t) - XA.l(i,j,m,p,t))) ;

total_slack2 = sum((j,k,m,p,t), min(0, ((WB(j,m,k) + EWBU(j,k,m)) * YB.l(j,k,m,p,t)) - XB.l(j,k,m,p,t)));

display Z.l, slack1, slack2, slack3, total_slack1, total_slack2, XA.l, XB.l, XC.l, YA.L,YB.L, YC.l, D;

execute_unload "Robust_Results.gdx" XA.L XB.L XC.L XES.L XEWA.L XEWB.L XEWC.L YA.L YB.L YC.L YS.L YEWA.L YEWB.L YEWC.L XER.L YR.L

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XA.L'

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XB.L rng=Sheet2!'

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XC.L rng=Sheet3! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XES.L rng=Sheet4!

‘

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XEWA.L
rng=Sheet5! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XEWB.L
rng=Sheet6! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XEWC.L
rng=Sheet7! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YA.L rng=Sheet8! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YB.L rng=Sheet9! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YC.L rng=Sheet10!

,

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YS.L rng=Sheet11! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YEWA.L
rng=Sheet12! '

execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YEWB.L
rng=Sheet13! '

```
execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YEWC.L  
rng=Sheet14! '
```

```
execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=XER.L rng=Sheet15!  
,
```

```
execute 'gdxxrw.exe Robust_Results.gdx o=Robust_Results.xls var=YR.L rng=Sheet16!
```

Vitae

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Biography Mohamed Osman is a graduate student at King Fahd University of Petroleum and Minerals (KFUPM), under the Industrial and Systems Engineering Department. He received his B.S. from the same college. His areas of interest include Supply Chain Modelling, Optimization, and Statistical and Machine Learning.