

On The Mechanical Behavior of Functionally Graded Carbon Nanotube-Reinforced Composite Beam

BY

Abdulmajeed Mohammed Al-Subaie

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This thesis, written by **ABDULMAJEED MOHAMMED AL-SUBAIE** under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN CIVIL ENGINEERING**.



Prof. Shamsad Ahmad
Department Chairman



Prof. Suliman Saleh Al-Homidan
Dean of Graduate Studies



Dr. Mohammed Al-Osta
(Advisor)



Chair Prof. Abdelouahed Tounsi
(Member)



Dr. Salah Al-Dulaijan
(Member)

Date



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2022

This work is dedicated to my beloved family.

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LIST OF ABBREVIATIONS

FGM	:	Functionally Graded Material
CNTRC	:	Higher-Order Shear Deformation Theory
TSD	:	Third-Order Shear Deformation
CBT	:	Classical Beam Theory

ABSTRACT

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Functionally graded materials (FGMs) are a particular category of composites in a way that, based on applications' demands, different materials are arranged and graded. The gradation varies along the material depending upon the either is continuous or detached. (FGMs) applications are rising in diverse recent industries such as aviation, structural health control, urban infrastructures, harm disclosure, and nanostructures due to their powerful performance and the growing desire for their specifications. The aim of the thesis is to study the mechanical behavior of functionally graded carbon nanotube-reinforced composite (CNTRC) beams. The beam is supported on a viscoelastic foundation. The problem is solved using the Hamilton principle. In this study, several patterns of reinforcement of single-walled carbon nanotubes are aligned and disturbed in the polymeric matrix. The (CNTRC) beam material properties are evaluated by applying the mixture rule. The theory of third shear deformation is developed to deal with the bending, buckling, and vibration effect. The effective functionally graded materials' (FGM) properties are studied by considering three different uneven distribution types of porosity. The damping coefficient and Winkler's and Pasternak's parameters are considered to investigate the viscosity effect on the foundation. The accuracy of the current theory is inspected with multiple comparison works. After that, the theory's applicability is validated by numerical comparison with previously published studies.

ملخص الرسالة

الاسم الكامل: عبدالمجيد بن محمد بن عبدالله السبيعي

عنوان الرسالة: السلوك الميكانيكي لشعاع الكربون المسامي المصنف وظيفيًا والمدعوم بأنابيب نانوية

التخصص: هندسة مدنية وبيئية

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المواد المصنفة وظيفيًا (FGMs) هي فئة معينة من المركبات بطريقة يتم فيها ترتيب المواد المختلفة وتصنيفها بناءً على متطلبات التطبيقات. يختلف التدرج على طول المادة اعتمادًا على إما المستمر أو المنفصل. تتزايد تطبيقات (المواد المصنفة وظيفيًا في الصناعات الحديثة المتنوعة مثل الطيران، والرقابة الصحية الهيكلية، والبنى التحتية الحضرية، والكشف عن الأضرار، والبنى النانوية بسبب أدائها القوي والرغبة المتزايدة في مواصفاتها. الهدف من الأطروحة هو دراسة السلوك الميكانيكي لشعاع مركب مقوى بأنابيب كربونية متدرجة وظيفيًا (CNTRC). سيتم دعم الحزمة على أساس لزج مطاطي. سيتم حل المشكلة باستخدام مبدأ هاملتون. في هذه الدراسة سيتم محاذاة العديد من أنماط تقوية الأنابيب النانوية الكربونية أحادية الجدار وتعطيلها في المصفوفة البوليمرية. سيتم تقييم خصائص مادة الحزمة (CNTRC) من خلال تطبيق قاعدة الخليط. تم تطوير نظرية تشوه القص الثالث للتعامل مع تأثير الانحناء والالتواء والاهتزاز. تمت دراسة خصائص المواد المتدرجة وظيفيًا (FGM) من خلال النظر في ثلاثة أنواع مختلفة من التوزيع غير المتكافئ للمسامية. يعتبر معامل التخميد ومعلمات Winkler و Pasternak للتحقق من تأثير اللزوجة على الأساس. يتم فحص دقة النظرية الحالية من خلال أعمال مقارنة متعددة. بعد ذلك تم التحقق من قابلية تطبيق النظرية من خلال المقارنة العددية مع الدراسات المنشورة سابقًا.

CHAPTER 1

INTRODUCTION

1.1 Background

Composite materials are growing continuously in terms of types, applications, and analytical studies. The two main traditional classes of composites are reinforced materials and laminated materials. Reinforcement can be in the form of large particles, fibers, or both, like in reinforced concrete. On the other hand, laminated materials are composed of two or more materials joined together, such as sandwich plates. This gradual variation in the material composition is reflected in smooth changes in the mechanical properties in one or more directions. Gradation functions are selected to provide adequate performance depending on the loading conditions applied to the element [1]. Multiple applications of composite materials with aligned stages, such as bio-composites, carbon fiber-reinforced polymer composites (CFRP), and aligned crystals in liquid. The aligned structure mostly has large length/diameter ratios. As the measures of these stages shrink to sub-atomic dimensions, new properties become clear. However, control and adjusting become troublesome with existing innovation. Carbon nanotubes certainly stand out enough to be noticed in light of their size and their anticipated construction delicate properties. Albeit enormous quantities of unsorted nanotubes are accessible, tests that examine properties or show applications are missing a direct result of the issues of dealing with individual nanotubes and of actually consolidating individual nanotubes into gadgets [2]. Carbon nanotubes (CNT) have become another type of fiber support in polymer matrix composites because of the prevalent mechanical, electrical, and thermal properties of the CNT, which have turned into another type of fiber support in polymer framework composites. Compared with the old-style carbon fiber-supported CNTRC can further develop, resulting in a

rise in stiffness and strength. Numerous hypotheses were created to study and analyze this type of material.

1.2 Thesis Objectives

This proposed thesis aims to derive the governing differential equations (GDE) of (the CNTRC) beam, solve the GDE analytically, and provide numerical examples for the solved problems. The specific objectives of this study are:

1. Proposing a simple and efficient shear deformation beam theory.
2. The mechanical behavior of (CNTRC) structures is examined.
3. The effects of some geometric and material parameters are investigated.
4. The free vibration responses, buckling, and bending of (CNTRC) beam controlled the results and are investigated and discussed.

1.3 Significance of the Research

- 2 Composite materials are growing continuously in terms of types, applications, and analytical studies. Functionally graded material (FGM) has drawn a high interest over the last two decades from researchers. This is due to the advanced FGM mechanical behavior, such as weight-to-strength ratio and their ability to resist extreme loading conditions. A special type of this material is the CNTRC which has enhanced properties. The researchers focused on three main effects: bending, vibration, and buckling.

1.4 Thesis organization

The body of the thesis contains six chapters. It starts with an introduction about the topic in general and the statement of the problem while defining the study's objectives. The second chapter presents the literature about the topic, explaining important concepts needed for understanding the approach to solving the problem. Chapter three contains un

under submitting journal article about the bending and buckling behavior, addressing the solution to the problem, while chapter four presents another contains un under submitting journal article regarding the vibration behavior. The thesis concludes in chapter five by summarizing the important points of the study and suggesting future work on the area of CNTRC beams. |

CHAPTER 2

LITERATURE REVIEW

2.1 Classical Beam Theory (CBT)

Classical beam theory is the simplest theory of beams. Its displacement field is formulated according to the assumptions of the Kirchhoff beam, which state the following

- I. Mid-surface deformation is neglected because it is assumed to be very small, which makes its strain even smaller and negligible.
- II. The mid-surface level is assumed still be the mid-surface after deformation.
- III. Planes perpendicular to the mid-surface are supposed to be perpendicular next to the deformation still. That is equal to saying that the transverse shear strain is insignificant, which prompts ignoring the normal transverse strain.
- IV. The beam is under transverse loading only, which hardly causes any normal stresses in the transverse direction. Hence, σ_z is neglected.

Omitting the negligible dimensions, CBT results in the number of displacement field that has only three variables.

2.2 The concept of FGM

FGM alludes to a heterogeneous composite material with an angle compositional variety of the constituents from one surface of the material to the next, which brings about consistently shifting material properties. As of now, progressed strategies to bring compositional gradients into different material systems are being created by materials researchers [3]. A functionally graded plate has a nonstop variety of its creating materials proportions over its thickness. The smooth gradation reduces the concentration of pressure at areas of properties alteration. Like other composites, FGM is fabricated to accomplish unrivaled properties, similar to high stiffness, strength, and

more protection from fatigue and natural condition, at a lower weight. Fig. 1 analyzes the thermal properties and material constructions of FGM and covered plates [4].

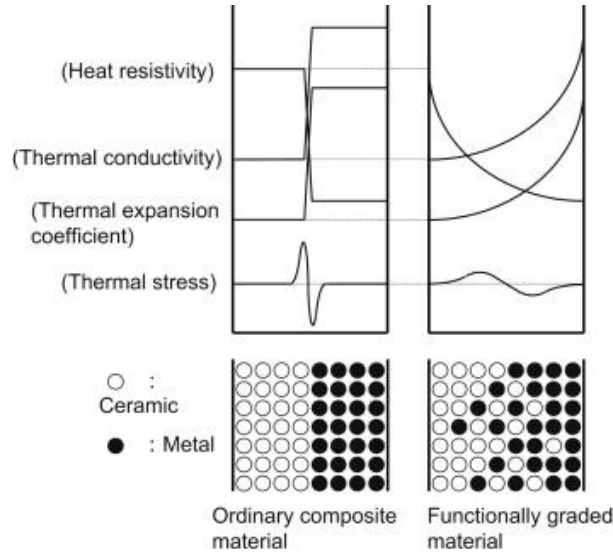


Figure 1 Thermal properties and material structures of FGM and laminated plates [19]

2.3 Bending and buckling of CNTRC

As a result, carbon nanotubes are a powerful reinforcement element in polymer composites, significantly improving their electro-thermomechanical properties as [5] stated. It has been proposed to develop functionally graded CNT-based composite materials using FG reinforcement patterns. Carbon nanotubes come in two forms: single and double-walled. Using the FGM concept, multiple materials are gradually merged in a specified direction, eliminating stress concentration between layers which can cause delamination and cracking [6]. As the use of FGMs and carbon nanotubes reinforced (CNTR) composites have grown over the past decade, they have received much more attention for analysis. Furthermore, the Unified Formulation carried out by [7] facilitates the development of refined beam, plate, and shell theories.

A wide range of studies have explored the behavior of FG beams for bending, buckling, and dynamic behaviors considering various material distributions and manufacturing defects. The viscoelastic foundation model was adopted, and porosity was introduced to

account for the manufacturing deficiencies. The study performed many parameters investigations to show their effect on the nanoplate vibrational response. A new exponential function and a new numerical tool were proposed for calculating beam volume fractions. Carbon nanotube-reinforced composite (CNTRC) beams were studied for their bending, buckling, and vibration characteristics [8]. In addition to the shear layer and Winkler spring, the beams rest on the Pasternak elastic foundation. Polymeric matrixes with multiple patterns of reinforcement align and distribute the single-walled carbon nanotubes (SWCNTs). The dynamic buckling of a nanocomposite FG plate was investigated by [9]. The plate is reinforced with carbon nanotubes which were distributed in various configurations. The Kelvin-Voigt model was used to model the viscoelastic foundation giving more realistic behavior to the plate. A differential transformation method (DTM) was applied by [10] for the investigation of porous FGM beams with different elastic supports. As a result of applying a higher-order shear deformation theory, graphene sheets were studied for the first time for longitudinal wave propagation [11]. In their study, Sugano et.al [12] evaluated the propagation of waves on porous FG curved beams subjected to thermal factors. For the study of porosity, mass density and Young's modulus were used in a reciprocal relationship. Carbon nanotubes (CNTs) pose attractive properties such as their low density (1.8 g/cm^3) with outstanding strength (150 GPa) and modulus of (1 TPa) as studied by [13]. They are used as reinforcing agents for composite materials. Functionally Graded Materials (FGMs), a new form of composites, provide a solution to the stress concentration between the plays by means of varying the constituting materials through the thickness of the structure, which was investigated by [14].

2.4 Vibration of CNTRC

Structures' vibrational response is of high interest to researchers since it plays a critical role in the design aspects of beams, plates, and shells. Research has advanced to consider structures reinforced with carbon nanotubes. An article by Ding et.al [15] provided an exact mathematical solution for the free vibration of a carbon nanotube functionally graded (FG-CNTs) beam addressing five types of CNTs distributions. The solution adopted a theory of first-order shear deformation and mixed boundary conditions. A modified form Fourier

series was used in the model formulation, and the Rayleigh-Ritz method was applied to solve the problem, which was in good agreement with the result obtained by [16]. The researches done on the static and dynamic responses of composite beams were reviewed by [17], where the various displacement fields used in the literature were described and compared. Using Winkler Pasternak's elastic foundation, Zhao et.al [18] studied CNT composite beam dynamics (free vibrational response). Single-wall carbon nanotubes (SWCNTs) reinforcements of infinite length were distributed throughout a polymer matrix to reinforce the affected beam. Edge lines were considered to support the beam. In order to summarize the refined beam theory, displacement along the transverse section of a beam was divided into shear and bending components, and Mori-Tanaka's method was used to estimate the material properties of composite beams. An FG nanocomposite double-beam reinforced with single-walled CNTs was examined for its free vibration properties by [17]. A double beam on a linear and shear layer is coupled by an interlayer spring and supported in thermal environments by a linear and shear layer. In order to deduce the motion equations incorporating the thermal effects, the authors applied the first-order shear deformation theory and the Euler-Lagrange variational principle. Other important studies of the vibration of various structures include ([9], [19], [20], and [21]). The vibration response is modeled and investigated via various deformation theories. A study on the beams' dynamic and static behaviors was presented by [22] using three displacement models. With consideration of the "broken line" theory, a classical sandwich beam model was formulated. As a generalization of the first model, the second model only considers the nonlinear shear effect in the core. Nonlinear shear effects in the core and at the faces are taken into account in a third model. With the FG-CNTRC face sheet as the face sheet of a rotating sandwich beam, Lai et.al [23] presented a numerical solution to the problem of free vibration.

A close relationship has been found between boundary relaxation and geometric imperfections in FG-CNTRC beams that can influence vibration behavior. Results showed that geometric imperfection's amplitude and modes determine how much influence boundary relaxation has on frequency, but it is not affected by its location. It is apparent from the literature cited above that no studies have been done on how viscoelastic foundation affects the vibration response of FG-CNT beams. Consequently, this study uses

a simple HSDT with four unknowns to evaluate the vibrational response of the FG beam reinforced with CNTs and standing on a viscoelastic foundation. Different distributions of CNTs along with different parameters are studied, including length-to-thickness ratios, foundation perimeter, Winkler parameters, damping effects, etc. We hope the presented results will enhance the design aspect of FG-CNT beams and will serve as rigid references for future research.

CHAPTER 3

Journal Article 1

Influences of Porosity Distributions on Bending and Buckling Behavior of Functionally Graded Carbon Nanotube-Reinforced Composite Beam

3.1 Abstract

The bending and buckling effect for carbon nanotube-reinforced composite (CNTRC) beams can be evaluated by developing the theory of third shear deformation (TSDT). In this work, beams rest on a viscoelastic foundation in which the single-walled carbon nanotubes (SWCNTs) are distributed and aligned in a matrix polymer. Four patterns of reinforcement are used for the CNTRC beams. The rule of mixtures is assessed for CNTRC beams' material properties. The effective functionally graded materials (FGM) properties are studied by considering three different uneven distribution types of porosity. The damping coefficient and Winkler's and Pasternak's parameters are considered to investigate the viscosity effect on the foundation. The accuracy of the current theory is inspected with multiple comparison works. Moreover, beam different parameters' effects on the CNTRC beam bending and buckling over viscoelastic foundation are discussed. The results demonstrated that the weakest type of CNTRC beams to resist the buckling and flexure loads is O-Beam, whereas X-Beam is the strongest. Moreover, it is indicated that the presence of porosity in the beams decreases the stiffness and increases deflection. In comparison, the deflection was reduced in the presence of a viscoelastic foundation.

Keywords: Bending and buckling; carbon nanotube-reinforced composite; FGM; Viscoelastic foundation

3.2 Introduction

Several properties of CNTs, such as resilience, stiffness, and high strength, make them

ideal materials for mechanical, electrical, and thermal applications, as stated by [24]. As a result, carbon nanotubes are a powerful reinforcement element in polymer composites, significantly improving their electro-thermomechanical properties, as represented by [5]. It has been proposed to develop functionally graded CNT-based composite materials using FG reinforcement patterns. Carbon nanotubes come in two forms: single and double-walled. A seamless rolling process produces CNT made of 1 (nm) in diameter and centimeters in length called SWCNTs, which was discussed by [25]. Multiwalled carbon nanotubes (MWCNT) have (2-100) (nm) diameters and lengths of tens of microns that are formed concentrically and separated by 0.35 (nm) as carried out by [26]. Material systems with functionally graded properties solve many practical problems. Using the FGM concept, multiple materials are gradually merged in a specified direction, eliminating stress concentration between layers which can cause delamination and cracking, as illustrated by Abdelbaki and C [6]. FGMs and CNTR composites have grown over the past decade, so they have received much more attention for analysis [27]. The classical beam theory (CBT) is known to be able to foresee the behavior of sandwich beams [28]. Other theories give more accurate predications such as the high order shear deformation theories (HSDT), the first order shear theory (FSDT), which are studied by ([16], [29], [30] and [31]), or the 3D elasticity theory which is studied by [32]. Furthermore, the Unified Formulation carried out by [7] facilitates the evolution of refined shell, plate, and beam theories. This formulation addressed various structural problems, but we only cite a few examples ([33], [34], and [35]).

3.2.1 Studies on various FGM structures

Research has therefore been conducted on beams, plates, and shells with FGM composites or CNTR composite materials for buckling, dynamic, and static analysis. HSDTs have been developed by Zerrouki et.al [14], who used a displacement field having constant transverse displacement to investigate the FGM beams dynamic and static behaviors. Through the application of rotational spring theory (RST) and Bernoulli-Euler beam theory (PEBT), Yang et.al [36] investigated FG beams with open edge cracks of elastic buckling and free vibration. The numerical results were obtained using mixed boundary conditions such as clamped-clamped, hinged-hinged, and clamped-free

boundaries. Different models of beam theories have been used by [37] to study the FG beams' dynamic behaviors. Under uniform and sinusoidal loading, the FGM beams static response was investigated by Chikh and A [38] using various HSDTs. The method of the Navier solution has been used by Khelifa et.al [39] to analyze the CNTRC beams buckling with a stretching effect. CNTRC beams with simple support, including a Winkler spring and shear layer, are considered on a Pasternak elastic foundation. The refined SDT with stretching effect was used to present new solutions to buckling loads. The layer-wise theory is also powerful in the analysis of FG structures.

3.2.2 Static and dynamic studies of beams

Several studies have explored the behavior of FG beams for bending, buckling, and dynamic behaviors considering various material distributions and manufacturing defects. FG beams, free vibration, and static analysis were conducted by [40] using a new HSDT model that takes into account porosities that may occur during fabrication. Nonlocal strain theory was utilized by Khazaei et.al [27] to search the vibration behavior of FG plates reinforced with CNTs. The viscoelastic foundation model was adopted, and porosity was introduced to account for the manufacturing deficiencies. The study investigated many parameters to show their effect on the nanoplate vibrational response. The logarithmic-uneven porosity pattern was discussed, along with two types of porosity distributions that are even and uneven. As part of the [41] study, graphene platelets reinforced porous nanocomposite curved beams were analyzed for through-thickness stretching caused by elastic stability and static bending. An isotropic FG sandwich beam analysis under buckling and free vibration is presented by Nguyen et.al [42] using a new HSDT. The transverse shear stresses are distributed hyperbolically in this theory, and the boundary condition is traction-free. Variations in the nonlinear CNTs of FG nanobeams were studied by Zerrouki et.al [14]. A new exponential function and a new numerical tool were proposed for calculating beam volume fractions. CNTRC beams were studied for their bending, buckling, and vibration characteristics [8]. In addition to the shear layer and Winkler spring, the beams rest on the Pasternak elastic foundation. Polymeric matrixes with multiple reinforcement patterns align and distribute the SWCNTs. The dynamic buckling

of a nanocomposite FG plate was investigated by Mudhaffar et.al [21]. The plate is reinforced with carbon nanotubes which were distributed in various configurations. The Kelvin-Voigt model was utilized to model the viscoelastic foundation giving more realistic behavior to the plate. A differential transformation method (DTM) was applied by [8] for the investigation of porous FGM beams with different elastic supports. As a result of applying HSDT, graphene sheets were studied for the first time for longitudinal wave propagation by Eyvazian et.al [10]. The propagation of waves on porous FG curved beams subjected to thermal factors has been evaluated by Xu et.al [11]. For the study of porosity, mass density and Young's modulus were used in a reciprocal relationship.

3.2.3 Numerical studies on FGM beams

The numerical investigation of FG beams using finite-element methods has also been studied by Kallannavar et.al [32], who examined the doubly-curved skew laminated sandwich composites (DSLSCs) vibration response with different temperatures and porosities. 3D printed polylactic acid (PLA) and graphite-epoxy face sheet core properties dependent on temperature were considered to perform the numerical analysis. FSDT-based combined thermoelastic finite element (FE) model was evolved to study the modal behavior of DSLSC shells when their temperature rises uniformly. Utilizing the generalized finite element method (GFEM), Pham et.al [43] studied the behavior of hygro-thermal vibration for porous curved beams made of bidirectional functionally graded porous (BDFGP) materials. A rotating FGM cantilever beam was studied using the finite element method by Almitani et.al [44]. Continuous and discontinuous contact problems between the FG layer pressed with a uniformly distributed load and homogeneous half-plane were investigated by Yaylaci et.al [45] using an analytical formulation and FEM. Using the Timoshenko beam theory (TBT), the equation for axially loading the beam has been derived. Free vibration analysis of CNTRC beams was predicted by Vo-Duy et.al [46] with the help of FSDT. CNTRC layers are perfectly bonded to create the laminated beam. CNTs with single walls are assumed to be distributed functionally graded (FG) or uniformly (UD) within each layer. It is clear from the above synthesis that no recent research has been conducted on the buckling and bending behavior of porous sandwich FG beams that rest on viscoelastic foundations. Thus, this article presents a buckling and

bending analysis of an FG sandwich beam on viscoelastic substrates addressing various porosity distributions caused by manufacturing. Using a trigonometric function, beam materials varied linearly across the thickness, ensuring zero surface shear stress. This article shows the parametric influence of length-to-thickness ratios, volume fractions, porosity distributions, damping coefficients, and foundation parameters using numerical examples.

3.3 Porous CNTRC beams characteristics

3.3.1 Geometrical configuration and material properties of CNTRC beams

The present study is all about a CNTRC porous beam made of SWCNTs and an isotropic polymer matrix mixture. Fig. 2 represents a CNTRC beam having a damping coefficient (C_t), length (L), thickness (h), displacement (x), Pasternak's parameter (K_p) and Winkler's parameter (K_w).

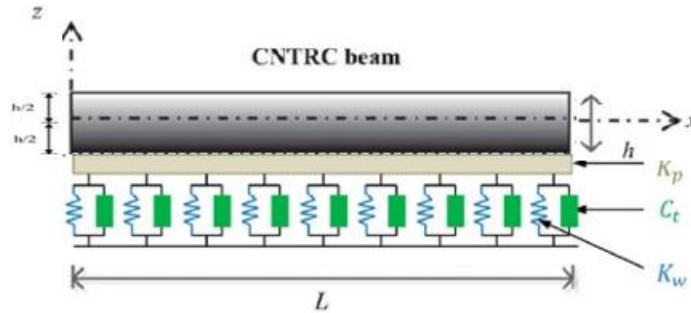


Figure 2 CNTRC beam on viscoelastic foundation geometry

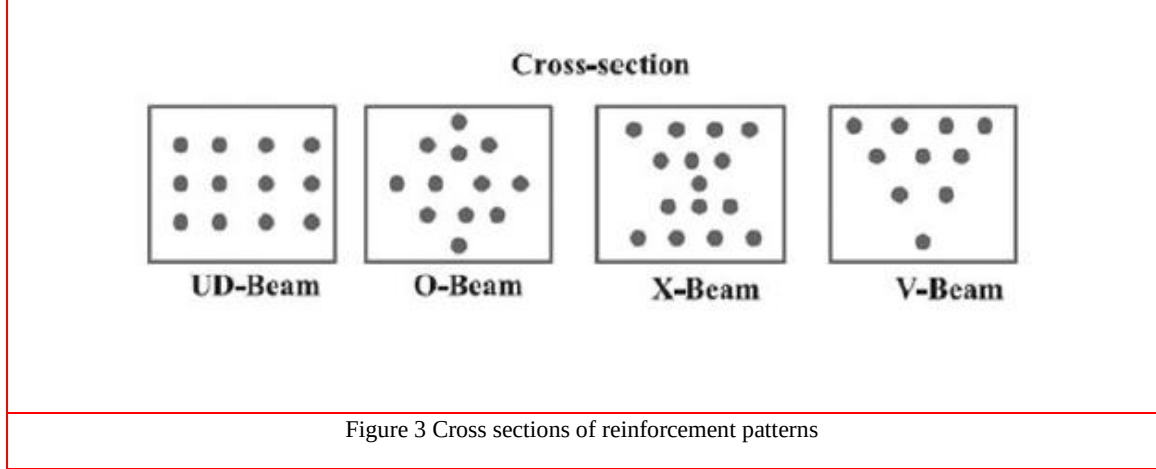


Fig. 3 indicates four reinforcement patterns supported by the Pasternak viscoelastic foundation over the cross sections, which are considered in this study. The effective shear modulus, young's modulus, and material properties of CNTRC beams can be computed by applying the rule of the mixture as follows

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \quad (1)$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \quad (2)$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \quad (3)$$

where E_{11}^{cnt} , E_{22}^{cnt} and G_{12}^{cnt} are the Young's and shear modulus of the SWCNT, E_p and G_p are the polymer matrix's corresponding material properties, V_{cnt} and V_p are the polymer matrix and CNT volume fractions using the relation of ($V_{cnt} + V_p = 1$). η_i ($i = 1, 2, 3$) which are the CNT efficiency parameters, are considered to introduce the size-dependent material properties of SWCNT. ν and ρ are the CNTRC beam Poisson's ratio and mass density, respectively, and they could be expressed by employing the same rule as

$$\nu = V_{cnt} \nu^{cnt} + V_p \nu^p, \quad \rho = V_{cnt} \rho^{cnt} + V_p \rho^p \quad (4)$$

where v^{cnt} and ρ^{cnt} are the CNT Poisson's ratio and density, respectively while v^p and ρ^p are the polymer matrix Poisson's ratio and density, respectively. In a way to represent the material distributions for different patterns of CNTRC shown in Fig. 3 within beams cross sections, the following formulas are expressed

UD-Beam

$$V_{cnt} = V_{cnt}^* \quad (5)$$

O-Beam

$$V_{cnt} = 2 \left(1 - 2 \frac{|z|}{h} \right) V_{cnt}^* \quad (6)$$

X-Beam

$$V_{cnt} = 4 \frac{|z|}{h} V_{cnt}^* \quad (7)$$

V-Beam

$$V_{cnt} = \left(1 + \frac{2z}{h} \right) V_{cnt}^* \quad (8)$$

where (V_{cnt}^*) is the CNT volume fraction and it is expressed as

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + (\rho^{cnt}/\rho^m)(1 - W_{cnt})} \quad (9)$$

where W_{cnt} is the CNT mass fraction. Moreover, it could be seen from Eqs. (5)-(8) that O, X, and V-Beams are some types of FGM that have a constant variation of their material constituents within their thicknesses, while the UD Beam has a uniform distribution of CNT. Table 1 shows the considered η_i associated with V_{cnt}^* .

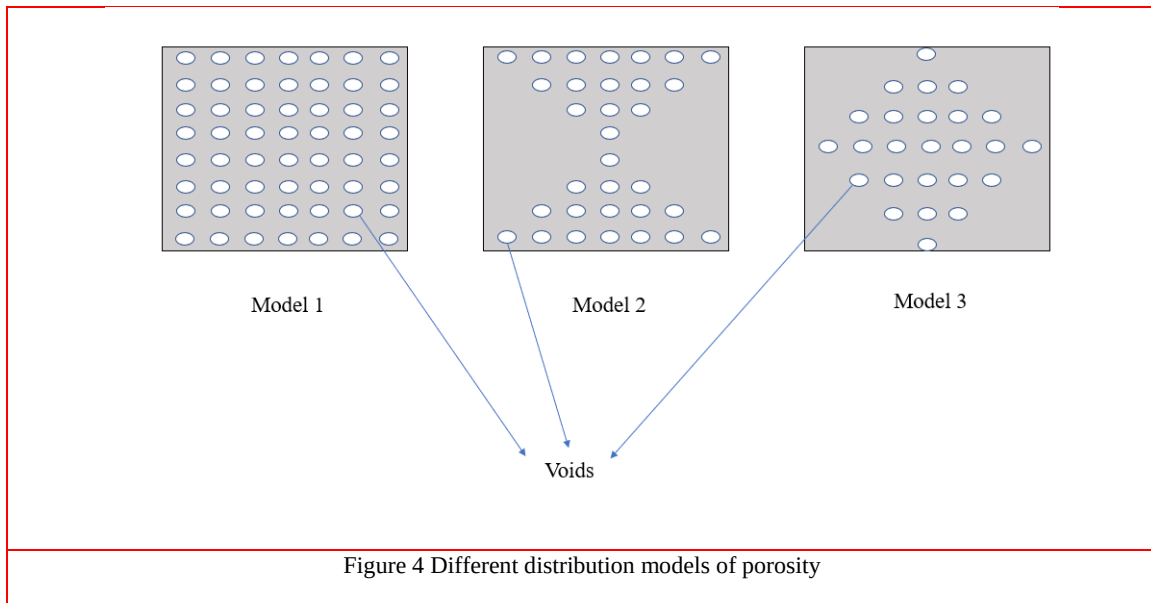
Table 1 CNT efficiency parameters (η_i) related to the considered volume fraction (V_{cnt}^*) [47].

Case	V_{cnt}^*	η_1	$\eta_2 = \eta_3$
1	0.12	1.2833	1.0566
2	0.17	1.3414	1.7101
3	0.28	1.3238	1.7380

3.3.2 Porosity effect on CNTRC beam property

Porosity issues are highly significant in the composite's mechanical behavior. The mechanical behavior of composites varies dramatically in the case of porosity presence. Hence, three different porosity distribution models are used in this study, as shown in Fig.

4



In the first model, the voids are uniformly spread along the height. For the second model, the voids stack in the lower and upper surfaces. For the third model, the voids stack in the middle. Effective material properties P , such as young's modulus and Poisson's ratio for those models, can be computed as follows;

Model 1

$$P(a) = P(1 - a) \quad (10)$$

Model 2

$$P(a) = P\left(1 - a \frac{2|z|}{h}\right) \quad (11)$$

Model 3

$$P(a) = P\left(1 - \frac{a}{2}\left(1 - \frac{2|z|}{h}\right)\right) \quad (12)$$

In Eqs. (10)-(12) and (12), a ($a \leq 1$) refers to the porosity volume fraction.

3.4 Formulations and theories

3.4.1 Kinematics and constitutive equations

The displacement field of the present paper can be obtained as

$$u = u_0(x, t) - z \frac{\partial w}{\partial x} + k_1 f(z) \int \theta(x, t) dx \quad (13)$$

$$w = w_0(x, t) \quad (14)$$

Where u_0 , w_0 and θ are the beam displacement along the beam mid-plane at ($z = 0$). In this paper, the shape function $f(z)$, which indicates the variation of the shear stress and strain within the direction of thickness, is chosen based on a trigonometric function as

$$f(z) = \frac{h}{\pi} \sin\left(\frac{\pi}{h} z\right) \quad \text{and} \quad g(z) = \frac{df(z)}{dz} \quad (15)$$

The strain components are evaluated in terms of u_0 , w_0 by using the relations of linear strain displacement and expressed as

$$\varepsilon_x = \varepsilon_x^0 - z\varepsilon_x^b + f(z)\varepsilon_x^s \quad (16)$$

$$\gamma_{xz} = g(z)\gamma_{xz}^s \quad (17)$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad \varepsilon_x^b = -\frac{\partial^2 w_0}{\partial x^2}, \quad \varepsilon_x^s = k_1\theta, \quad \gamma_{xz}^s = k_1 \int \theta dx \quad (18)$$

The following undetermined integrals simplification is derived after applying Navier type solution

$$\int \theta dx = A' \frac{\partial \theta}{\partial x} \quad (19)$$

In terms of the Navier type A' and k_1 are expressed as

$$A' = -\frac{1}{\kappa_1^2}, \quad k_1 = \kappa_1^2 \quad (20)$$

Assuming that CNTRC beam material is under Hooke's law, stresses are expressed as

$$\sigma_x = Q_{11}(z)\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z)\gamma_{xz} \quad (21)$$

$$Q_{11}(z) = \frac{E_{11}(z)}{1-\nu^2} \quad \text{and} \quad Q_{55}(z) = G_{12}(z) \quad (22)$$

3.4.2 The viscoelastic foundation

This work studies the CNTRC beam resting on a viscoelastic foundation which takes into account three foundation actions: the spring-like, shear interaction, and time-dependent of

the non-adhesive viscous foundation medium. Hence, the foundation effect can be obtained as

$$R_f = \left(K_w - K_p \nabla^2 + C_t \frac{\partial}{\partial t} \right) w_0 \quad (23)$$

where K_w is Winkler's parameter, K_p is Pasternak's parameter, ∇^2 is a Laplace operator. C_t is the damping coefficient and can be obtained from ($C_t = c_t A_{110} 10^3 / L^4$) where c_t is the damping constant. K_w and K_p can be determined from $K_w = \beta_w A_{110} / L^2$ and $K_p = \beta_p A_{110}$ in which β_w and β_p are the corresponding spring constant factors. Moreover, A_{110} is defined as the stiffness extension or A_{11} values of a homogeneous beam that is made of pure matrix material and is expressed as

$$A_{110} = \int_{-h/2}^{h/2} \frac{E_p}{1 - \nu^2} dz \quad (24)$$

3.4.3 Equations of motion

The equation of motion can be derived utilizing Hamilton's principle for static loading (bending and buckling) as follows

$$\int_{t_1}^{t_2} (\delta U + \delta V) dt = 0 \quad (25)$$

where δU and δV are the virtual variation of the strain and potential energy, respectively. The beam strain energy variation is stated as

$$\begin{aligned}
\delta U &= \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dz dx \\
&= \int_0^L \left(N \frac{d\delta u_0}{dx} - M_b \frac{d^2 \delta w_0}{dx^2} + M_s k_1 \delta \theta + Q A' k_1 \frac{d\delta \theta}{dx} \right) dx
\end{aligned} \tag{26}$$

where N , M_b , M_s and Q are stress resultants, and they are defined as

$$(N, M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f) \sigma_x dz \quad \text{and} \quad Q = \int_{-h/2}^{h/2} g \tau_{xz} dz \tag{27}$$

The potential energy variation made by transverse load q , the axial compressive force N , and the density of reaction force of the foundation R_f can be written as

$$\delta V = \int_0^L \left[(R_f - q) \delta w_0 + N \frac{dw_0}{dx} \frac{d\delta w_0}{dx} \right] dx \tag{28}$$

By substituting δU and δV expressions in Eqs. (26) and (28) into Hamilton principal ingredients Eq. (25), then resolving the variations of the unknowns' derivatives (using by parts integration formula) and using fundamental calculus lemma of variations for each unknown, the equations of motion can be computed by setting each unknown variations coefficient to zero.

$$\delta u_0: \frac{\partial N}{\partial x} = 0 \tag{29}$$

$$\delta w_0: \frac{\partial^2 M_b}{\partial x^2} = K_w w_0 - K_p \nabla^2 w_0 + C_t \dot{w}_0 - q + N \nabla^2 w_0 \tag{30}$$

$$\delta \theta: -k_1 M_s + A' k_1 \frac{\partial Q}{\partial x} = 0 \tag{31}$$

By substituting Eqs. (13) and (14) into Eqs. (16)-(18) and the subsequent results into Eq. (27), the stress resultants constitutive equations are obtained as

$$N = A_{11} \frac{du_0}{dx} - B_{11} \frac{d^2w_0}{dx^2} + D_{11} k_1 \theta \quad (32)$$

$$M_b = B_{11} \frac{du_0}{dx} - E_{11} \frac{d^2w_0}{dx^2} + F_{11} k_1 \theta \quad (33)$$

$$M_s = D_{11} \frac{du_0}{dx} - F_{11} \frac{d^2w_0}{dx^2} + G_{11} k_1 \theta \quad (34)$$

$$Q = A_{55}^s A' k_1 \frac{\partial \theta}{\partial x} \quad (35)$$

where A_{11} , etc., are the beam stiffness which can be expressed as

$$(A_{11}, B_{11}, D_{11}, E_{11}, F_{11}, G_{11}) = \int_{-h/2}^{h/2} Q_{11}(1, z, f(z), z^2, zf(z), f^2(z)) dz \quad (36)$$

where

$$A_{55}^s = \int_{-h/2}^{h/2} Q_{55}[g(z)]^2 dz \quad (37)$$

Eqs. (32)-(35) are expressed in terms of displacements (u_0, w_0, θ) by using Eqs. (29)-(31) and Eqs. (32)-(35) as

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_0}{\partial x^3} + D_{11} k_1 \frac{\partial \theta}{\partial x} = 0 \quad (38)$$

$$B_{11} \frac{\partial^3 u_0}{\partial x^3} - E_{11} \frac{\partial^4 w_0}{\partial x^4} + F_{11} k_1 \frac{\partial^2 \theta}{\partial x^2} - K_w w_0 + K_p \frac{\partial^2 w_0}{\partial x^2} - C_t \dot{w}_0 + q - N \frac{\partial^2 w_0}{\partial x^2} = 0 \quad (39)$$

$$-D_{11} k_1 \frac{du_0}{dx} + F_{11} k_1 \frac{d^2 w_b}{dx^2} - G_{11} k_1^2 \theta + A'^2 k_1^2 A_{55}^s \frac{\partial^2 \theta}{\partial x^2} = 0 \quad (40)$$

3.4.4 Analytical solution

Navier type solution is employed to obtain analytical solutions of a simply supported CNTRC beam. The solution is assumed to be as

$$\begin{Bmatrix} u_0(x, t) \\ w_0(x, t) \\ \theta(x, t) \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ W_m \sin(\lambda x) e^{i\omega t} \\ \theta_m \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \quad (41)$$

where U_m , W_m , and θ_m are the arbitrary determining parameters, ω is the eigenfrequency associated with m eigenmode, and $(\lambda = k_1 = m\pi / L)$. q , which is the transverse load, is expanded in the Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \quad (42)$$

Where Q_m is the load amplitude obtained from

$$Q_m = \frac{2}{L} \int_0^L q(x) \sin(\lambda x) dx \quad (43)$$

The coefficients Q_m is given below for the uniform distributed load case as

$$Q_m = \frac{4q_0}{m\pi}, (m = 1, 3, 5 \dots) \quad (44)$$

Substituting u_0 , w_0 , θ and q from Eqs. (40) and (41) into the equations of motion Eqs. (38)-(40), the analytical solutions can be derived by the following equations

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ 0 \end{Bmatrix} \quad (45)$$

and

$$\begin{aligned} S_{11} &= \lambda^2 A_{11}, & S_{12} &= -B_{11} \lambda^3, & S_{13} &= -k_1 D_{11} \lambda \\ S_{22} &= E_{11} \lambda^4 + K_w + K_p \lambda^2 + 0.5 C_t - N \lambda^2 \\ S_{23} &= k_1 F_{11} \lambda^2, & S_{33} &= k_1^2 G_{11} + k_1^2 A'^2 A_{55}^s \lambda^2 \end{aligned} \quad (46)$$

3.5 Results and discussion

This part presents and discusses the bending and buckling behavior of CNTRC beams and numerical results. The effective material characteristics at the ambient temperature employed in this study are poly methyl methacrylate (PMMA). The armchair (10, 10) SWCNTs are selected for the reinforcement material. Table 2 illustrates the material parameters used in the numerical analysis.

Table 2 (PMMA) and reinforcement material properties [47].

(PMMA) properties	Reinforcement properties
$\nu^p = 0.3$	$\nu^{cnt} = 0.19$
$\rho^p = 1190 \text{ kg/m}^3$	$\rho_{cn} = 1400 \text{ kg/m}^3$

$E^p = 2.5 \text{ GPa}$	$E_{11}^{cnt} = 600 \text{ GPa}$
-	$E_{22}^{cnt} = 10 \text{ GPa}$
-	$G_{12}^{cot} = 17.2 \text{ GPa}$

Following nondimensionalizations are adopted

bending analysis

$$\bar{w} = 100 \frac{E_p h^3}{q_0 L^4} w$$

$$\bar{\sigma}_x = \frac{h}{q_0 L} \sigma_x \left(\frac{L}{2}, \frac{h}{2} \right)$$

$$\bar{\tau}_{xz} = \frac{h}{q_0 L} \tau_{xz}(0,0)$$

For buckling

$$\bar{N} = \frac{N}{A_{110}}$$

Tables 3 and 4 show the bending analysis of UD beams and buckling with different types of beams with/without elastic foundations. It can be observed that this work agrees with the bending and buckling results of [47], who utilized the theory of third shear deformation in the case of missing porosity and damping constants. Table 3, which discusses bending analysis, represents that the displacement decreases for both elastic and no elastic foundation in the case of having a damping constant and without porosity. Then, in the case of having porosity and no damping constant, the displacement increases. The last case contains a damping constant and porosity and displacement values between the two cases. Moreover, an increase in the amount of V_{cnt}^* would result in increasing the stiffness of the CNTRC beams.

Table 4 discusses buckling analysis, which shows that the buckling critical load increases for both elastic and no elastic foundation in the case of having a damping constant and no porosity. Then, in the case of having porosity and a damping constant, the critical buckling load decreases. In the last case, which contains a damping constant and porosity, buckling critical load values are between the two cases. As represented, O-beam is the

weakest to resist deflection and buckling, while X-Beam is the strongest.

Table 3 Displacement of UD-Beam with and without elastic foundation under uniform loads, different damping constants, and porosity coefficients.

V_{cnt}^*	L/h	Theory	c_t	a (Model 1)	w	
					$k_w=0, k_p=0$	$k_w=0.1, k_p=0.02$
0.12	10	Ref*	0	0	0.703	0.593
		Present	0	0	0.702	0.59259
			0.1	0	0.535	0.46826
			0	0.2	0.777	0.64829
			0.1	0.2	0.581	0.50606
	20	Ref*	0	0	0.460	0.311
		Present	0	0	0.460	0.3108
			0.1	0	0.383	0.27325
			0	0.2	0.489	0.32733
			0.1	0.2	0.405	0.28713
0.17	10	Ref*	0	0	0.448	0.401
		Present	0	0	0.448	0.401
			0.1	0	0.374	0.340
			0	0.2	0.492	0.437
			0.1	0.2	0.406	0.368
	20	Ref*	0	0	0.307	0.232
		Present	0	0	0.307	0.232
			0.1	0	0.270	0.211
			0	0.2	0.324	0.244
			0.1	0.2	0.285	0.221
0.28	10	Ref*	0	0	0.324	0.299
		Present	0	0	0.324	0.299
			0.1	0	0.283	0.264
			0	0.2	0.359	0.329
			0.1	0.2	0.311	0.288
	20	Ref*	0	0	0.203	0.167
		Present	0	0	0.203	0.167
			0.1	0	0.186	0.156
			0	0.2	0.214	0.176
			0.1	0.2	0.196	0.164

*Taken from [47]

Table 4 Comparison of buckling loads for CNTRC beam with and without elastic foundation, different damping constants, and porosity coefficients. ($L/h=15$, $V_{cnt}^*=0.12$).

Theory	c_t	a (Model 1)	$k_w=0, k_p=0$			$k_w=0.1, k_p=0.02$		
			UD	O	X	UD	O	X
Ref [47]	0	0	0.0985	0.0575	0.1291	0.1286	0.0876	0.1592
Present	0	0	0.0985	0.0575	0.1292	0.1287	0.0876	0.1593
	0.1	0	0.1211	0.0800	0.1517	0.1512	0.1102	0.1818
	0	0.2	0.0914	0.0538	0.1187	0.1215	0.0839	0.1489
	0.1	0.2	0.11389	0.0763	0.1412	0.1440	0.1064	0.1714

3.5.1 Porosity impact on bending and buckling

The porosity effect has been introduced with different models of voids distribution in Lamina and showed that porosity has a negative impact where it increases bending and buckling. As expected, the beam stiffness gets reduced in porosity presence. Fig. 5(a) shows that model 1 has the highest deflection compared to other models since it has the largest voids, leading to loss of stiffness and increased bending. Fig. 5(b) shows the opposite relation can be observed for the critical buckling load for the model in which it decreases with increasing porosity coefficient since the beam stiffness is reduced. It can be seen that as much as the model has voids as less, the beam resists buckling load. It is also observed in Fig. 6(a) that the strongest beam with the smallest deflection is the X-Beam, followed by the (UD, V, and O-Beam). However, because of its symmetry, the bending coupling characteristic in V-Beam causes bending moments and more compressive loading to the beam, leading to more deflections. Furthermore, the X-Beam is the strongest beam that supports the largest buckling load with increasing porosity, as shown in Fig. 6(b). It is observed in Fig. 7(a) that the shear parameters of the Winkler and the Pasternak get higher. The transverse displacement gets decreases. This decreasing trend is related to the elastic medium stiffness. Fig. 7(b) shows that increasing the constant spring factors leads to linearly increased buckling loads.

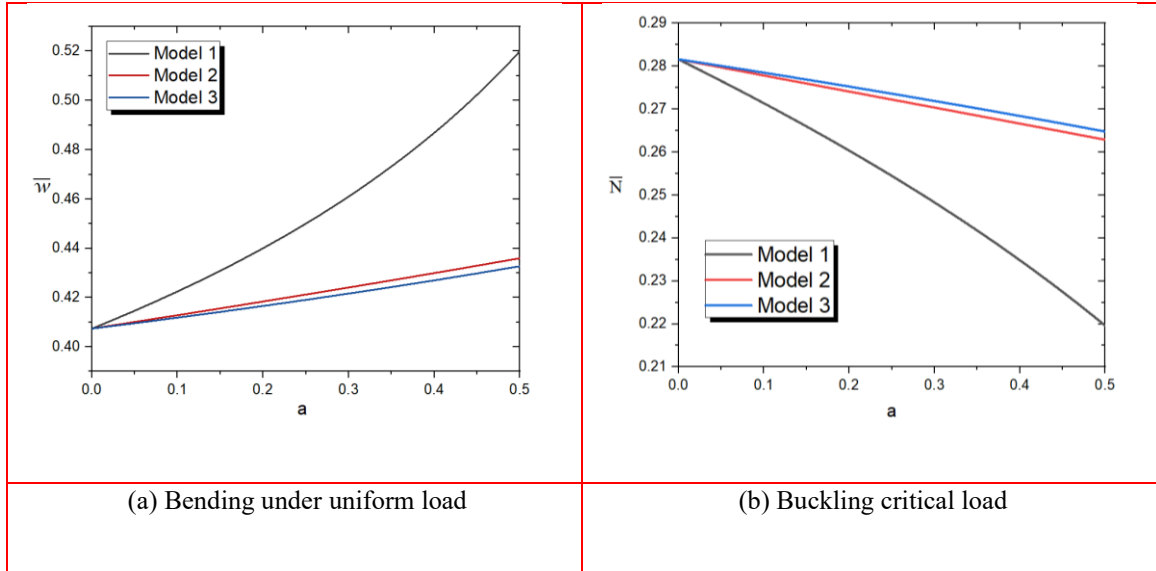
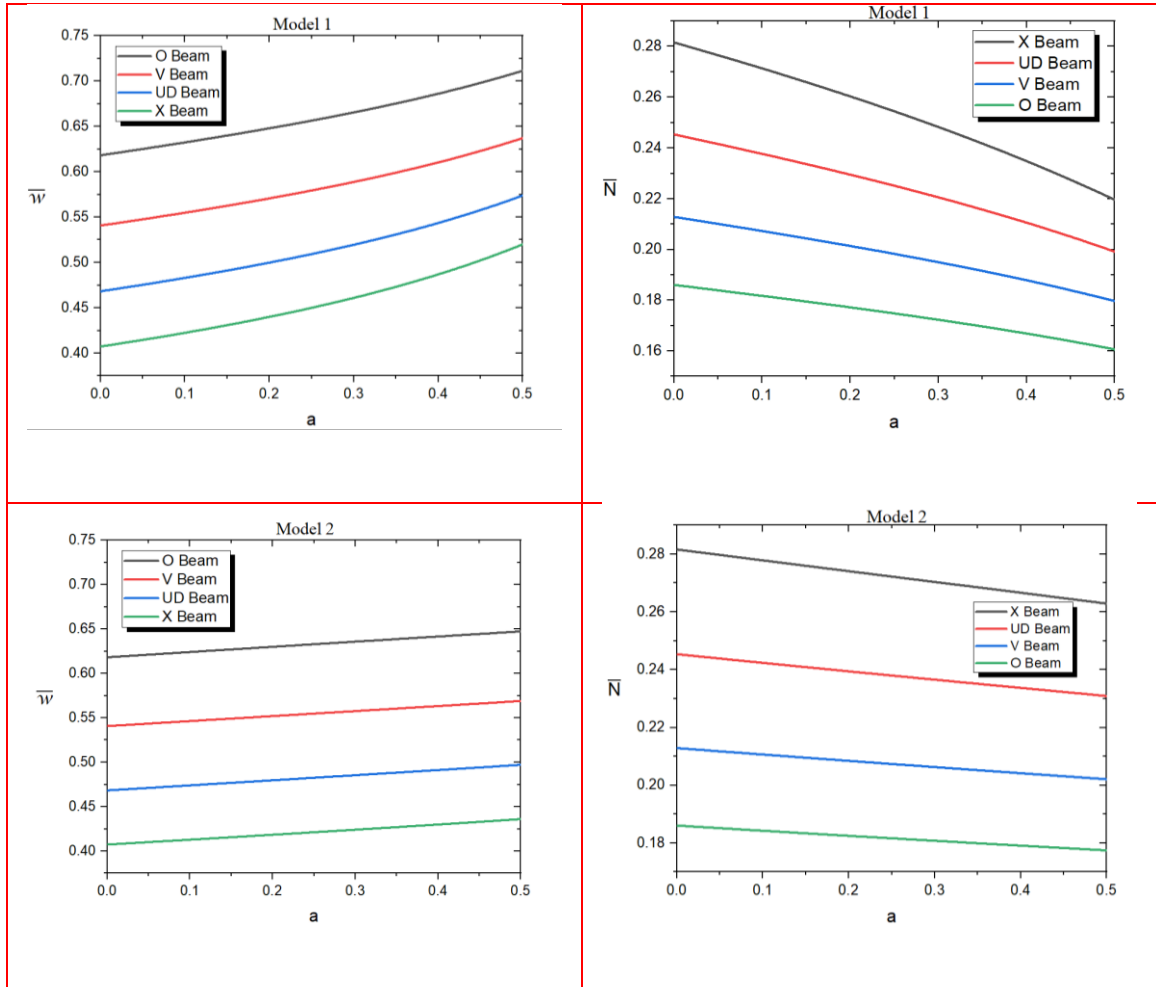
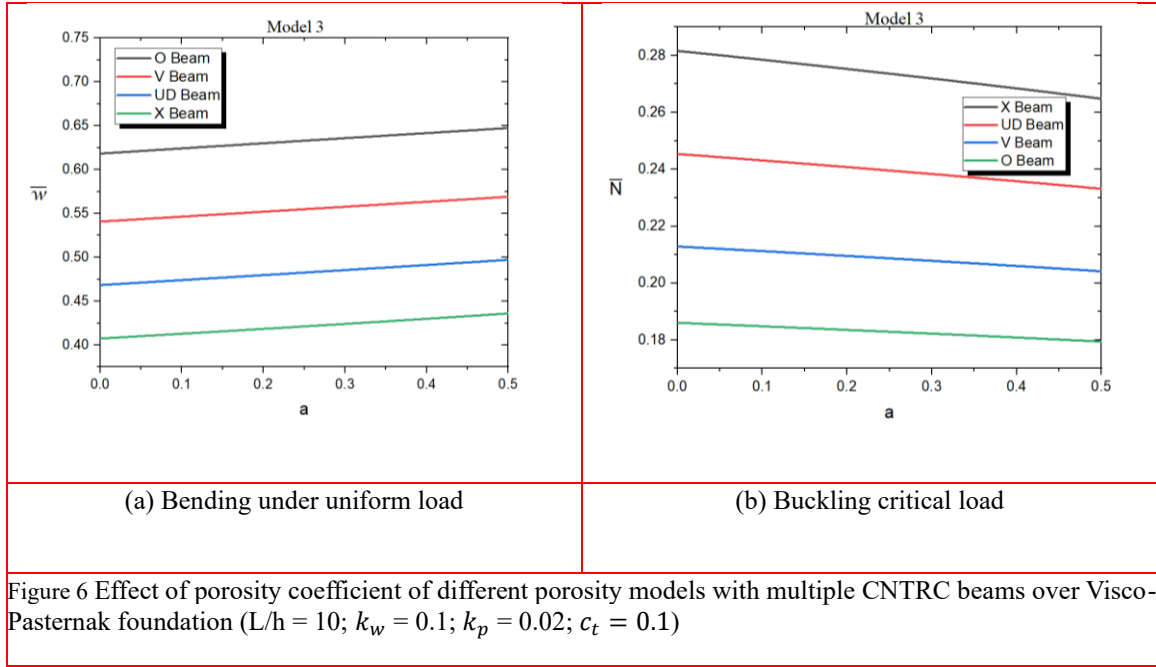


Figure 5 Effect of porosity coefficient for CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $c_t = 0.1$)





The effect of porosity coefficient for different porosity models of CNTRC X-beam over different types of foundations is considering the following parameters

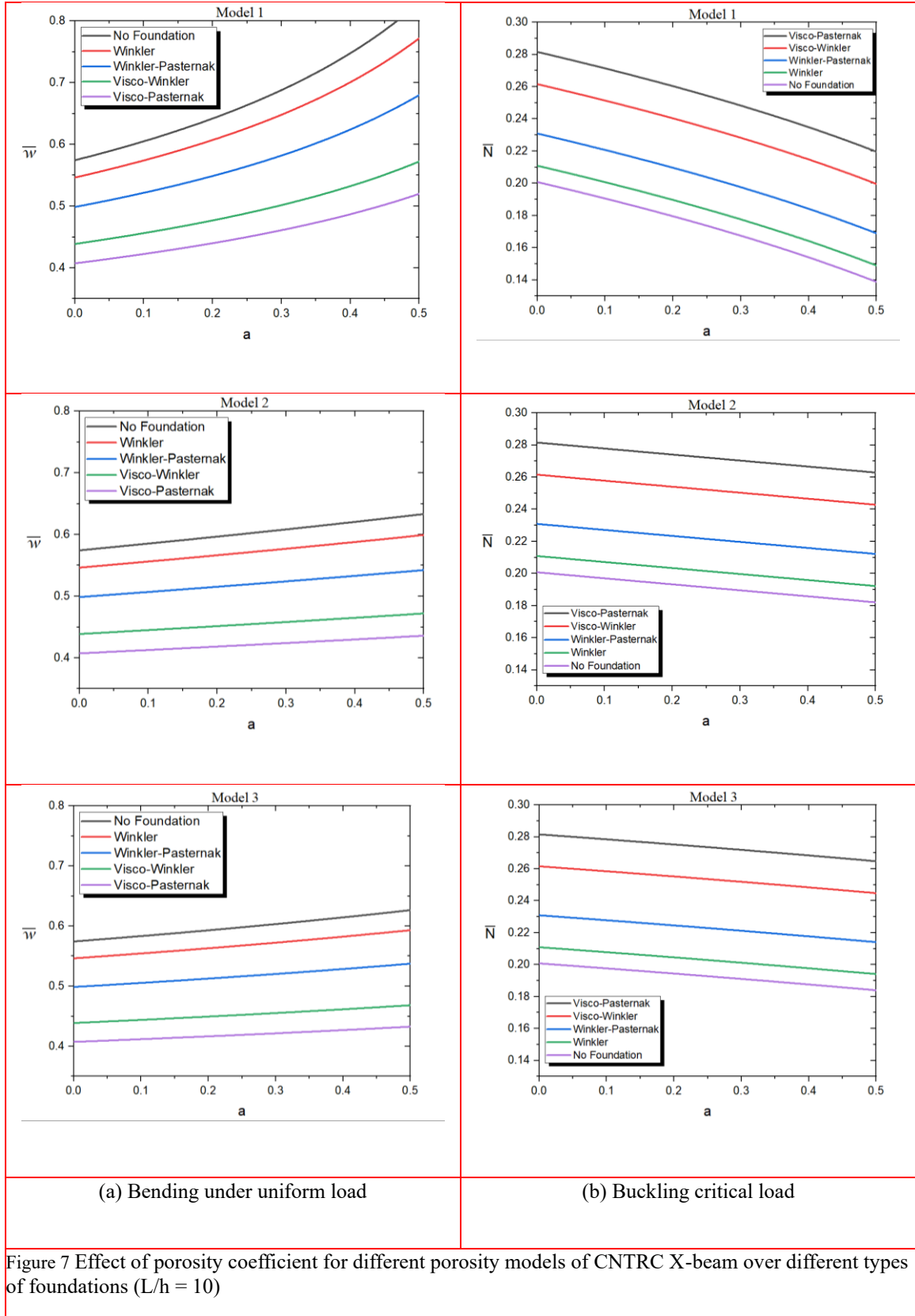
No-Foundation ($L/h = 10$; $k_w = 0$; $k_p = 0$; $c_t = 0$).

Winkler ($L/h = 10$; $k_w = 0.1$; $k_p = 0$; $c_t = 0$).

Winkler- Pasternak ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $c_t = 0$).

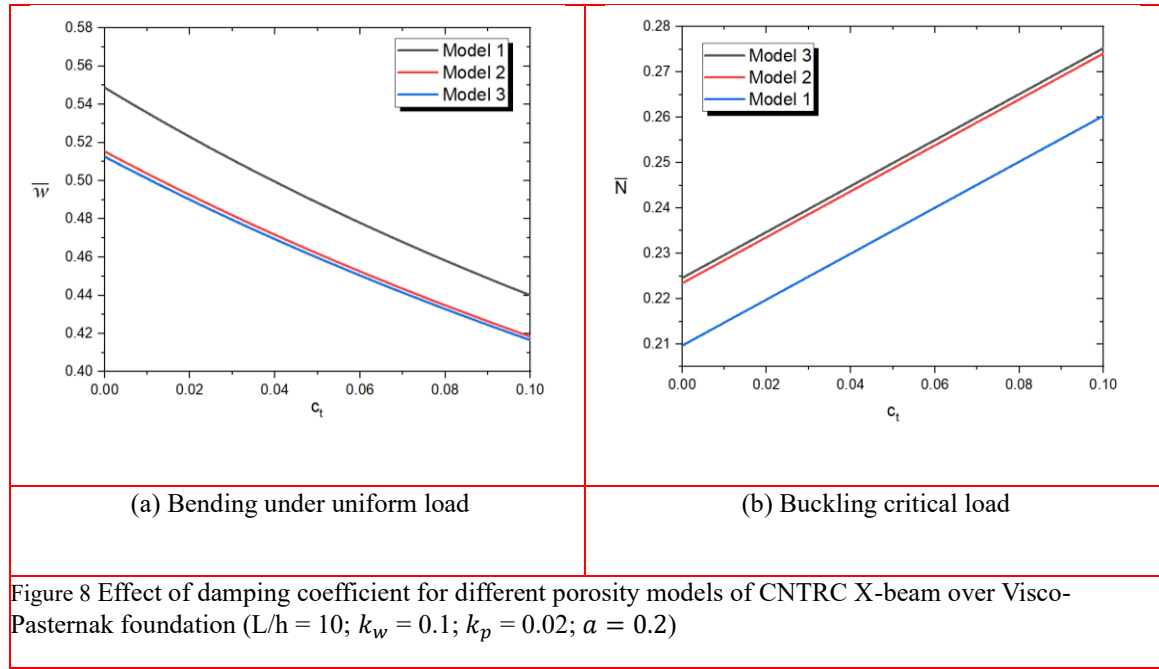
Visco-Winkler ($L/h = 10$; $k_w = 0.1$; $k_p = 0$; $c_t = 0.1$).

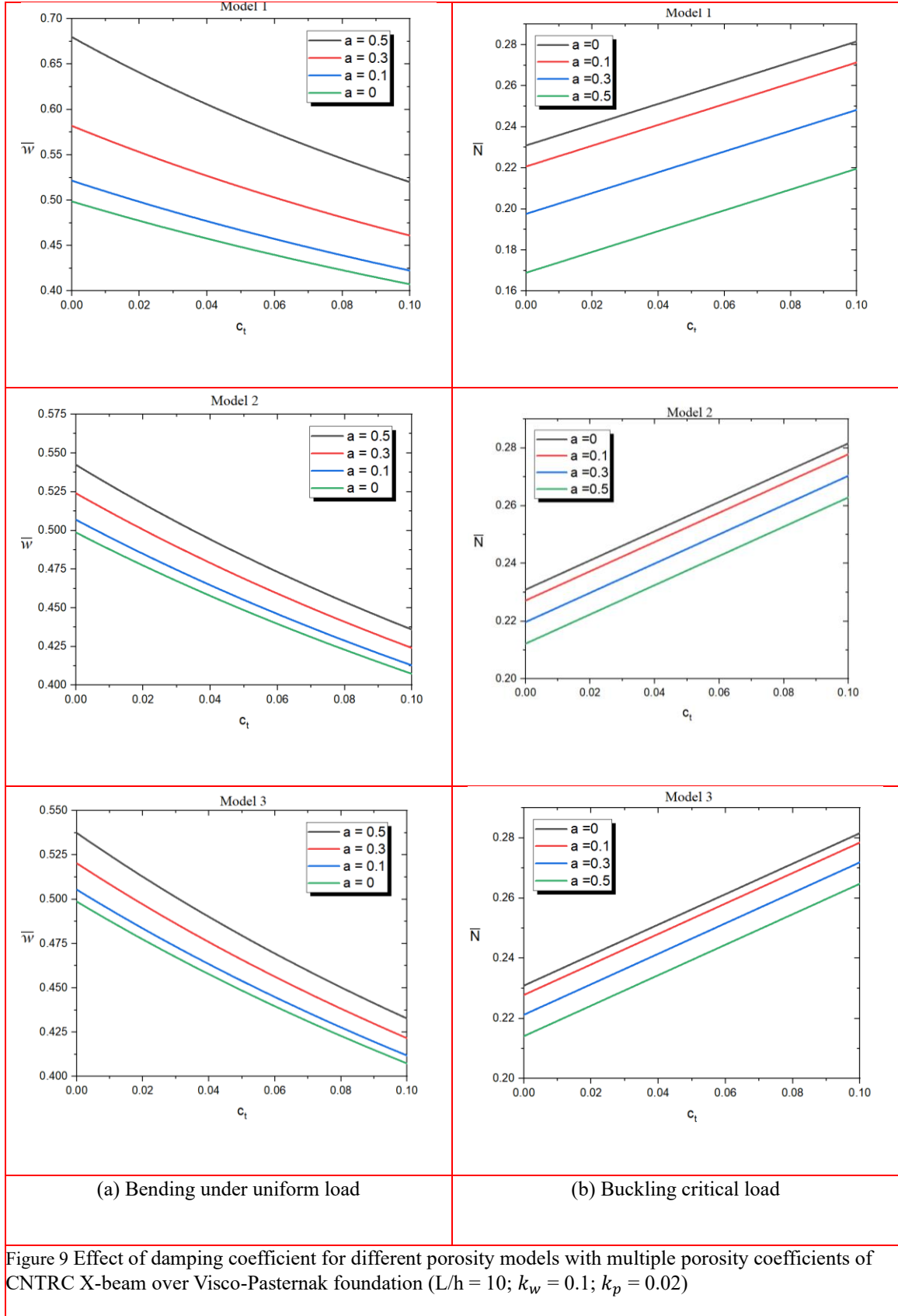
Visco-Pasternak ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $c_t = 0.1$).



3.5.2 Damping coefficient impact on bending and buckling

The damping coefficient has been introduced, and the analysis in Figs. (8)-(10) showed that increasing the damping constant enhance the bending and buckling effect due to increasing spring constant factors, which affects transverse buckling. In bending, adding the damping constant enhance the foundation's elastic medium, which raises stiffness and supports more flexural loads. Porosity presence, as shown in Figs. (8)-(9) affected the bending load but increasing the damping constant reduced their influence. It is also observed in Fig. 10 that the O-Beam is the weakest to resist deflections and buckling load, whereas the X-Beam is the strongest beam that supports the largest buckling load and deflections with increasing porosity.





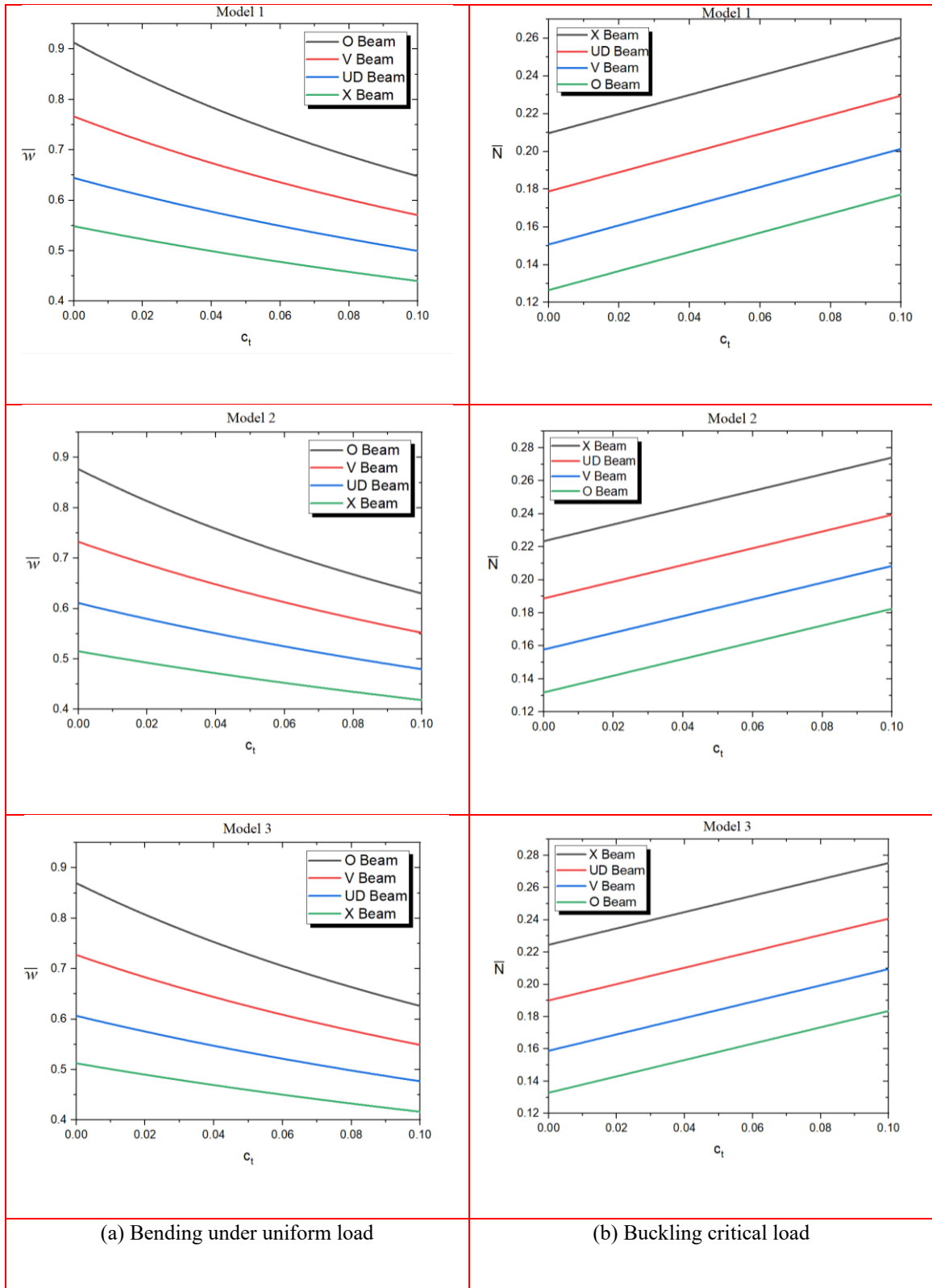
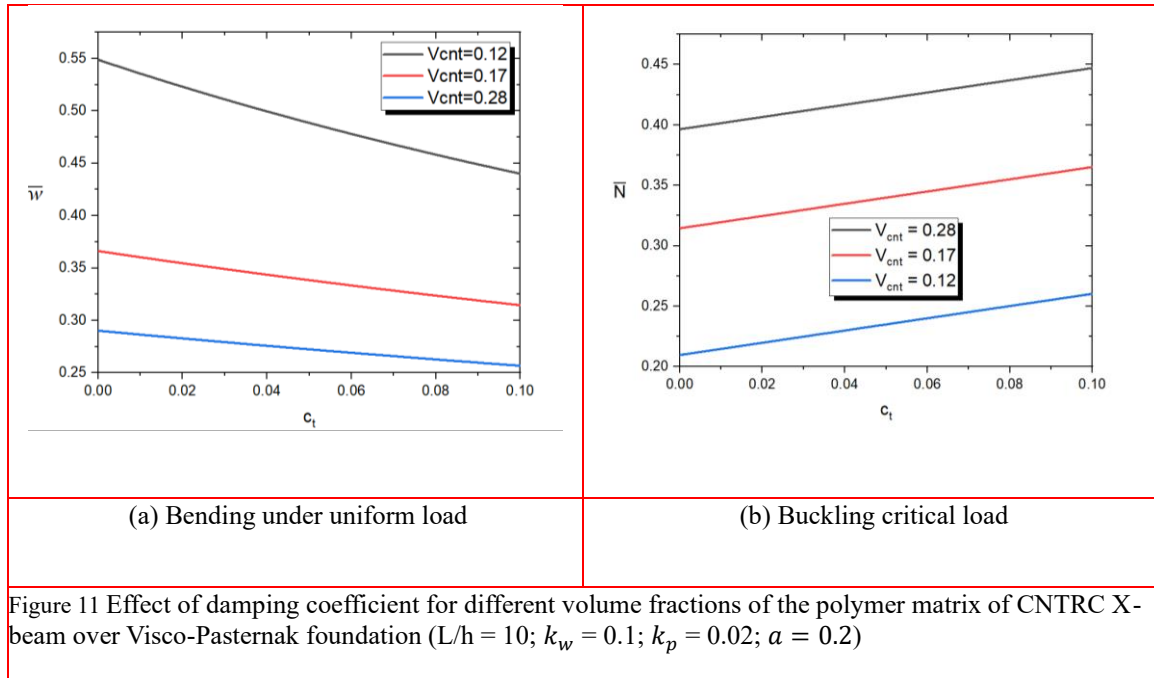
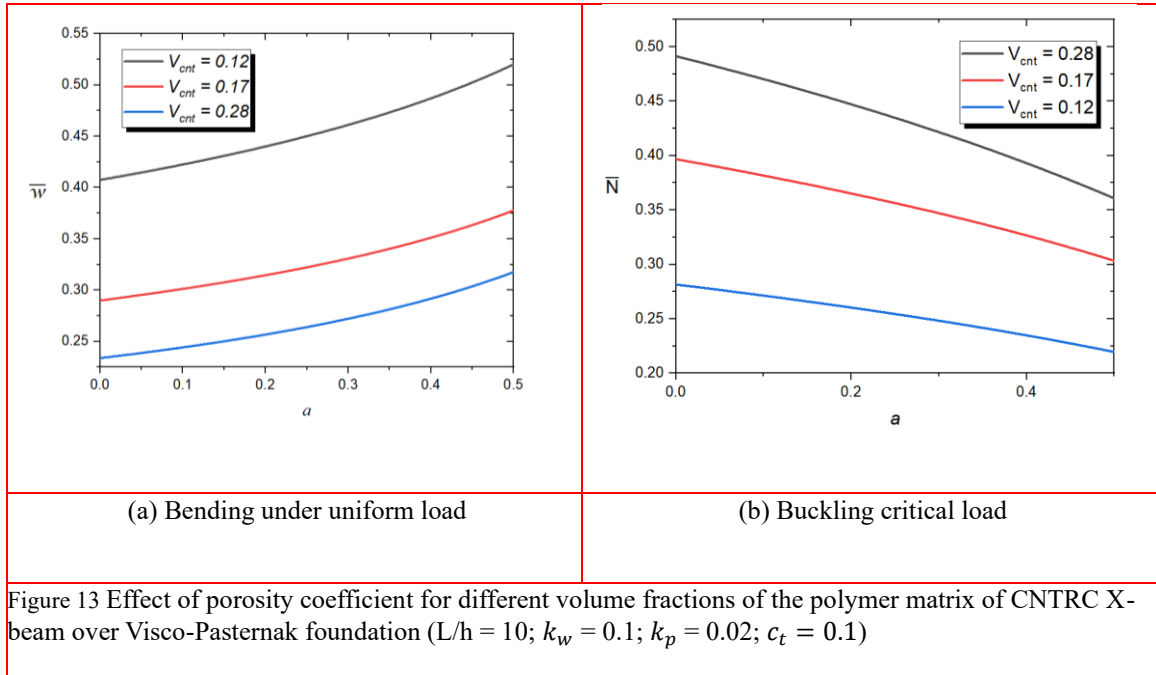
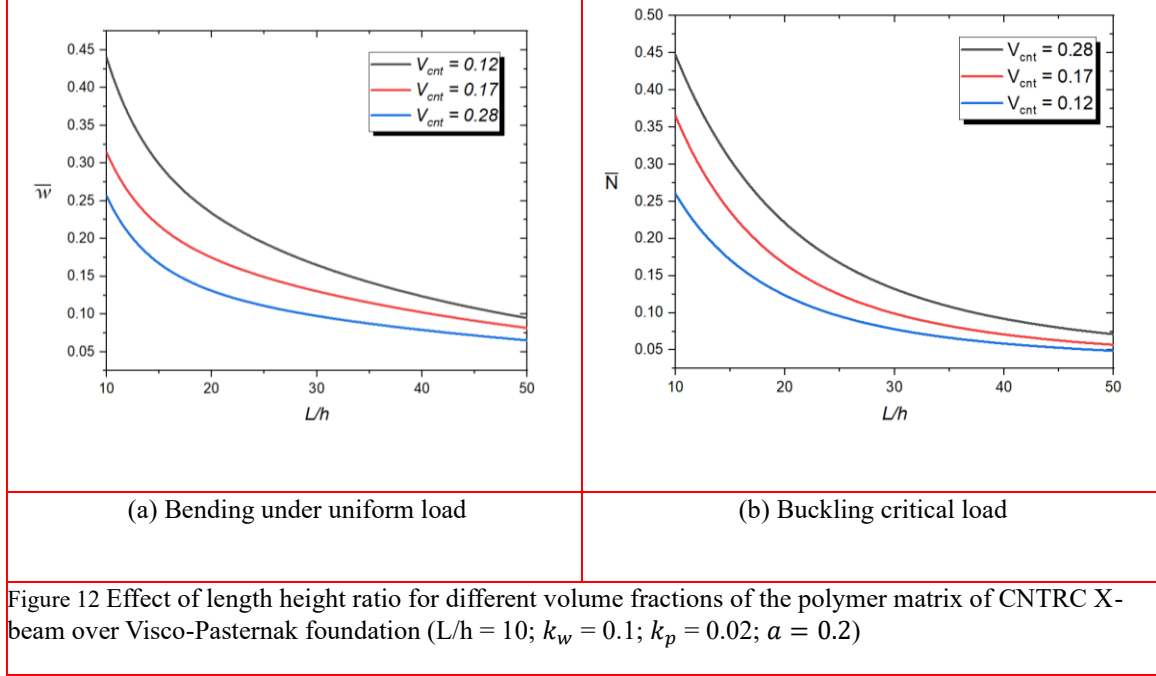


Figure 10 Effect of damping coefficient for different porosity models with multiple CNTRC beams over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $a = 0.2$)

3.5.3 Volume fractions of CNT impact on bending and buckling

Volume fractions of CNTs influence the deflection, and buckling load of the strongest beam (X-beam), as shown in Figs. (11)-(13). The reduction in the deflections and the buckling loads caused by increasing the volume fractions of CNTs leads to increased beam stiffness. Fig 11 shows that the damping parameter enhances the bending and buckling effect. Fig. 12 represents the dramatic reduction of both critical buckling loads and deflections, which occur in the range of $L / h = 10$ to 30. It is also observed in Fig. 13 that porosity affects deflection and buckling load but increasing the volume fraction reduces its effect.





3.6 Conclusion

This work investigates the bending and buckling of CNTRC beams resting on a viscoelastic foundation using the third shear deformation theory. The porosity effect has

been introduced with different models of voids distribution in Lamina. Hamilton's principle has been used to derive equations of motion. A damping constant is added in the formulation to satisfy viscosity needs. By making the comparison with some available results, the accuracy of the present work is numerically checked, and many figures with multiple parameters change have been discussed. Numerical results showed that the O-Beam is the weakest among different types of CNTRC beams, while X-Beam is the strongest to support the buckling and flexure loads. Moreover, the presence of the viscoelastic foundation is found to enhance the bending and buckling effect. The damping coefficient enhanced the deflection in bending and increased the critical buckling load. Porosity is affected negatively by decreasing beam stiffness and increased bending and buckling effects.

CHAPTER 4

Journal Article 2

Porosity-Dependent Vibration Investigation of Functionally Graded Carbon Nanotube-Reinforced Composite Beam

4.1 Abstract

This work utilizes simplified higher-order shear deformation beam theory (HSDBT) to investigate the vibration response for functionally graded carbon nanotube-reinforced composite (CNTRC) beam. Single-walled carbon nanotubes (SWCNTs) are distributed and aligned in a matrix of polymer throughout the beam, resting on a viscoelastic foundation. Four un-similar patterns of reinforcement distribution functions are investigated for the CNTRC beam. Porosity is another consideration taken into account due to its significant effect on functionally graded materials (FGMs) properties. Three types of uneven porosity distributions are studied in this study. The damping coefficient and Winkler's and Pasternak's parameters are considered in investigating the viscosity effect on the foundation. Moreover, the impact of different parameters on the vibration of the CNTRC beam supported by a viscoelastic foundation is discussed. A comparison to other works is made to validate numerical results in addition to analytical discussions.

Keywords: Free vibration; functionally graded materials; CNT-reinforced beam; porosity; viscoelastic foundation

4.2 Introduction

Carbon nanotubes (CNTs) pose attractive properties such as their low density (1.8 g/cm^3) with outstanding strength (150 GPa) and modulus of elasticity (1 TPa) [14]. They are used as reinforcing agents for composite materials. FGMs, a new form of composites, provide a solution to the stress concentration between the plays by means of

varying the constituting materials through the thickness of the structure, as represented by [48] and [13]. Reinforcing the FGM structures with CNTs is desirable to produce materials capable of sustaining extreme mechanical and thermal loadings that could be used in manufacturing nuclear reactors, airplanes' bodies, spacecraft, lightweight civil structures, etc., as represented by [49] and [50].

4.2.1 Vibrational studies on FGM structures

Structures' vibrational response is of high interest to researchers since it has a critical role in the design of beams, plates, and shells. Research has advanced to consider structures reinforced with carbon nanotubes. An article by Yang et.al [36] provided an exact free vibration mathematical solution for FG CNTs beam by addressing five types of CNTs distributions. The solution adopted a theory of mixed boundary conditions and first-order shear deformation (FSDT). A modified form Fourier series was used in the model formulation, and the Rayleigh-Ritz rule was used to resolve the problem, which was in high agreement with the result taken by Wu et.al [51]. The researches on composite beams' dynamic and static responses were reviewed by Sayyad et.al [16]. To reinforce the affected beam, SWCNTs reinforcements of infinite length were distributed throughout a polymer matrix. Edge lines were considered to support the beam. In order to summarize the refined beam theory, displacement along the transverse section of a beam was divided into shear and bending components, and Mori-Tanaka's formula was applied to estimate the properties of composite beams material. An FG nanocomposite double-beam reinforced with SWCNTs was examined by Zhao et.al [18] for its free vibration properties. A double beam on a linear and shear layer is coupled by an interlayer spring and supported in thermal environments by a linear and shear layer. FSDT and Euler-Lagrange variational methods were applied to derive equations of motion, including thermal effects. Other important studies of the vibration of various structures include ([9], [15], [17], [19], [20], [21], and [52]).

4.2.2 Theories used in the vibration analysis

The vibration response is modeled and investigated via various deformation theories. A study on the beams' dynamic and static behaviors was presented by [22] using three displacement models. With consideration of the "broken line" theory, a classical sandwich beam model was formulated. As a generalization of the first model, the second model only considers the nonlinear shear effect in the core. Nonlinear shear effects in the core and at the faces are considered in a third model by the FG CNTRC face sheet, as the face sheet of a rotating sandwich beam was presented by Lai et.al [23] for the problem of a numerical solution of free vibration. By using artificial springs, the rotating sandwich beam's boundary conditions were replicated classically and non-classically. Temperature-dependent materials were considered for both cores and face sheets. Hamilton's principle and FSDT were used to derive the beam's vibration equation, and the problem was numerically resolved by the differential quadrature method (DQM). With the aid of finite element (FE) models generated in the ANSYS environment utilizing calculated properties of the material, Kumar et.al [53] investigated FG-CNT beams' harmonic response. Three types of FG beams were evaluated for the vibration response: beams of O type (CNT - FG Δ), X type (CNT -FGX), and Δ type. It was found that the Δ type has a maximum amplitude peak, and the X type showed a low peak with the highest natural frequency, thus explaining small hardening behavior. Various theories used in the review of literature include [42] and [46].

4.2.3 Importance of the viscoelastic foundation

The vibration damping of beams is investigated in literature either by viscoelastic substrate or by including constituting damping materials. With the help of local/nonlocal integral viscoelastic models, thermal vibration damping of porous FG microbeams with varying porosity distributions was investigated by Song et.al [54]. To predict the vibration for FG-CNTRCDS using higher-order shear deformation theory (HSDT), Wang et.al [55] developed a dynamic analytical model. A Rayleigh-Ritz principle was applied to obtain an analytical solution satisfying clamped boundary conditions. FSDT was used by Peng et.al [56] to study the impact of boundary relaxation on FG CNTRC imperfect beams' free vibration characteristics. The boundary spring method was used in addition to arbitrary

boundary conditions to analyze imperfect FG CNTRC beams. The boundary relaxation degree was examined using parameters of relaxation that are simulated with varying spring stiffness. Governing equations have been derived by utilizing the Rayleigh-Ritz formula for beams with geometric disadvantages and relaxed conditions. Then the equations were solved for the frequency of the beams. A close relationship has been found between boundary relaxation and geometric imperfections in FG-CNTRC beams that can influence vibration behavior. Results showed that geometric imperfection's amplitude and modes determine how much influence boundary relaxation has on frequency, but it is not affected by its location. Various types of beams are studied in the context of vibration. Having piezoelectric layers, the FG CNTRC micro-beams have been studied by [57] for their nonlinear free vibration characteristics in thermal environments. The study examined four different patterns of CNT reinforcement distributions, including uniform and nonuniform distributions. Using von Karman's assumption with the beam theory of Euler-Bernoulli, the motion equations were derived utilizing Hamilton's method and the theory of nonlocal strain gradient. The FG CNTRC microbeams' approximate nonlinear frequencies were determined for simple boundary conditions and clamped-clamped utilizing two analytical rules: analogous linearization and the rule of energy balance. In thermal environments, FG beams pre-twisted were studied by Shen et.al [58]. Modeled using HOSDT, the beam is reinforced with CNTs and has temperature-dependent properties. It is apparent from the literature cited above that no studies have been done on how viscoelastic foundation affects the vibration response of FG-CNT beams. Consequently, this study uses a simple HSDBT with three unknowns to examine the vibrational behavior of the FG beam reinforced with CNTs and supported by a viscoelastic foundation. Different distributions of CNTs along with different parameters are studied, including length-to-thickness ratios, foundation perimeter, Winkler parameters, and damping effects. The presented results are beneficial to enhance the design aspect of FG CNT beams and help as rigid references for future research.

4.3 Properties of (CNTRC) beams

4.3.1 Geometrical pattern and material properties of CNTRC beams

This study considers a porous CNTRC beam that is manufactured of SWCNTs with an isotropic polymer mixture. Fig. 14 shows the CNTRC beam that has a length (L) and thickness (h) and is supported by a viscoelastic foundation. The foundation has many parameters, such as a damping coefficient (C_t), Winkler's parameter (K_w), and Pasternak's parameter (K_p)

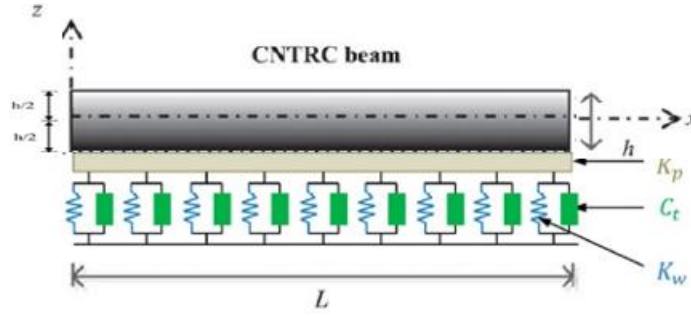


Figure 14 Geometry of CNTRC beam supported by viscoelastic foundation

This paper considers four cross-sections of patterns of reinforcement resting on the Pasternak viscoelastic foundations, which are represented in Fig. 15.

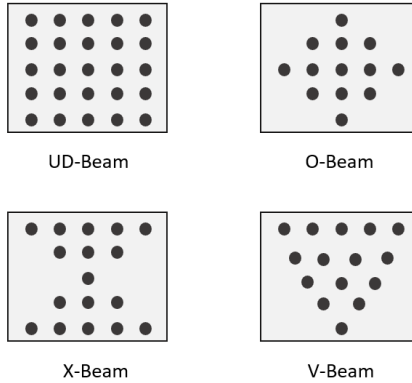


Figure 15 Cross-sections of reinforcement configurations

CNTRC beams' material properties are derived utilizing the mixture rules as follows

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \quad (1)$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \quad (2)$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \quad (3)$$

where G_{12}^{cnt} , E_{11}^{cnt} and E_{22}^{cnt} are the SWCNT shear and Young's modulus, respectively. G_p and E_p are the shear and Young's modulus of the polymer matrices. V_p and V_{cnt} are volume fractions of polymer matrix and CNT, which are linked to each other using the following relation

$$(V_{cnt} + V_p = 1). \quad (4)$$

The efficiency parameters of CNT refer to the SWCNT size-dependent material properties η_i ($i = 1, 2, 3$). However, CNTRC mass density ρ and beams Poisson's ratio ν are also expressed by the rule of the mixture as

$$\nu = V_{cnt} \nu^{cnt} + V_p \nu^p, \quad (5)$$

$$\rho = V_{cnt} \rho^{cnt} + V_p \rho^p \quad (6)$$

where ρ^{cnt} and ν^{cnt} are the CNT density and Poisson's ratio. Similarly, ρ^p and ν^p are the polymer matrix density and Poisson's ratio.

Material distributions for cross sections configurations of reinforcement of CNTRC beams which are represented in Fig. 15, can be represented by the following mathematical functions

UD-Beam

$$V_{cnt} = V_{cnt}^* \quad (7)$$

O-Beam

$$V_{cnt} = 2 \left(1 - 2 \frac{|z|}{h} \right) V_{cnt}^* \quad (8)$$

X-Beam

$$V_{cnt} = 4 \frac{|z|}{h} V_{cnt}^* \quad (9)$$

V-Beam

$$V_{cnt} = \left(1 + \frac{2z}{h} \right) V_{cnt}^* \quad (10)$$

in which V_{cnt} represents the volume fraction of CNT. As represented in Eq. (7)-(10), (V, O, and X-Beam) are some types of FGM in which their material constituents constantly vary across the thicknesses. Unlike other beams, the UD-Beam distributes CNT constituents uniformly. V_{cnt} can be obtained by

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + (\rho^{cnt}/\rho^m)(1 - W_{cnt})} \quad (11)$$

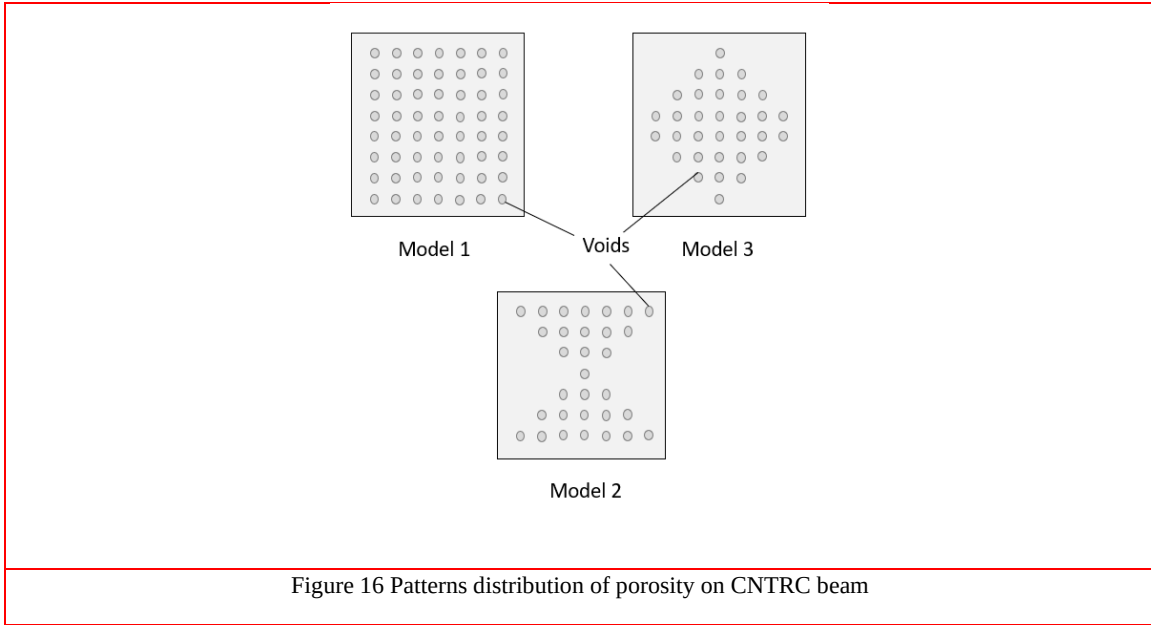
in which W_{cnt} denotes the mass fraction of CNT. However, the CNT η_i used with the considered V_{cnt}^* in this work are shown in Table 5.

Table 5 The CNT η_i used with the considered V_{cnt}^*

Case	V_{cnt}^*	η_1	$\eta_2 = \eta_3$
1	0.12	1.2833	1.0566
2	0.17	1.3414	1.7101
3	0.28	1.3238	1.7380

4.3.2 Porosity patterns on CNTRC beam

Porosities in matters have essential considerations in the mechanical response of composites. In the case of porosity presence, the composites' mechanical behavior varies largely. In this work, three models of porosity distribution are used to deal with the impact of porosity, as represented in Fig. 16



Model 1 represents the distribution of the voids uniformly along the height. Model 2 denotes the distribution of the voids in the laminas' lower and upper surfaces and graded in between. Model 3 indicates the distribution of the voids in the middle of the laminas. Therefore, material property (P) based on the three models can be determined as follows:

Model 1

$$P(a) = P(1 - a) \quad (12)$$

Model 2

$$P(a) = P \left(1 - a \frac{2|z|}{h} \right) \quad (13)$$

Model 3

$$P(a) = P\left(1 - \frac{a}{2}\left(1 - \frac{2|z|}{h}\right)\right) \quad (14)$$

In Eqs. (12)-(14), a ($a \leq 1$) indicates the volume fraction of porosity.

4.4 Formulations

4.4.1 Constitutive and kinematic equations

This work's displacement field is expressed as

$$u = u_0(x, t) - z \frac{\partial w}{\partial x} + k_1 f(z) \int \theta(x, t) dx \quad (15)$$

$$w = w_0(x, t) \quad (16)$$

in which u_0 , w_0 and θ are the displacement through the beam thickness at ($z = 0$).

The shear strain and stress variation along thickness's direction, which is the shape function $f(z)$, is selected using a trigonometric function as follows:

$$f(z) = \frac{h}{\pi} \sin\left(\frac{\pi}{h} z\right) \quad (17)$$

$$g(z) = \frac{df(z)}{dz} \quad (18)$$

The components of strain are obtained in terms of u_0 , w_0 after applying linear strain–displacement relations and given as follows:

$$\varepsilon_x = \varepsilon_x^0 - z \varepsilon_x^b + f(z) \varepsilon_x^s \quad (19)$$

$$\gamma_{xz} = g(z)\gamma_{xz}^s \quad (20)$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^b \\ \varepsilon_x^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial^2 w_0}{\partial x^2} \\ k_1 \theta \\ k_1 \int \theta dx \end{Bmatrix} \quad (21)$$

By applying a Navier-type solution, the undetermined integrals could be simplified to the following relation

$$\int \theta dx = A' \frac{\partial \theta}{\partial x} \quad (22)$$

A' and k_1 can be obtained in terms of the Navier as follows:

$$A' = -\frac{1}{\kappa_1^2} \quad , \quad k_1 = \kappa_1^2 \quad (23)$$

Since the material follows Hooke's law, CNTRC beam stresses can be found as follows:

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \\ Q_{11}(z) \\ Q_{55}(z) \end{Bmatrix} = \begin{Bmatrix} Q_{11}(z)\varepsilon_x \\ Q_{55}(z)\gamma_{xz} \\ \frac{E_{11}(z)}{1-\nu^2} \\ G_{12}(z) \end{Bmatrix} \quad (24)$$

4.4.2 The viscoelastic foundation

This work aims to investigate the response of the CNTRC beam supported by viscoelastic foundations that implements three actions for the foundation: the shear interaction, spring-like, and time-dependent of the nonadhesive viscous medium of the foundation. Consequently, the foundation effect can be obtained as

$$R_f = \left(K_w - K_p \nabla^2 + C_t \frac{\partial}{\partial t} \right) w_0 \quad (25)$$

where K_w and K_p are Winkler- Pasternak's parameter, respectively. ∇^2 denotes a Laplace operator. C_t indicates the damping coefficient that can be determined from

$$(C_t = c_t A_{110} 10^3 / L^4) \quad (26)$$

where c_t implies the damping constant. K_w and K_p can be found from

$$(K_w = \beta_w A_{110} / L^2) \quad (27)$$

$$(K_p = \beta_p A_{110}) \quad (28)$$

where β_w and β_p are the constant factors corresponding to spring and shear actions, respectively. Moreover, A_{110} refers to the stiffness extension or A_{11} values of a homogeneous beam that is manufactured by pure matrix material and is expressed as

$$A_{110} = \int_{-h/2}^{h/2} \frac{E_p}{1 - \nu^2} dz \quad (29)$$

4.4.3 Equations of motion

The motion equations are derived from Hamilton's principle as follows

$$\int_{t_1}^{t_2} (\delta U + \delta V + \delta K) dt = 0 \quad (30)$$

where δU , δV and δK are the strain, potential, and kinetic energy virtual variation, respectively. The beam strain energy variation can be computed as

$$\begin{aligned} \delta U &= \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dz dx \\ &= \int_0^L \left(N \frac{d\delta u_0}{dx} - M_b \frac{d^2 \delta w_0}{dx^2} + M_s k_1 \delta \theta + Q A' k_1 \frac{d\delta \theta}{dx} \right) dx \end{aligned} \quad (31)$$

Where N , M_b , M_s and Q are the stress resultants which are stated as

$$\begin{Bmatrix} N \\ M_b \\ M_s \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \begin{Bmatrix} 1 \\ z \\ f \end{Bmatrix} dz \quad \text{and} \quad Q = \int_{-h/2}^{h/2} g \tau_{xz} dz \quad (32)$$

The axially compressive force N , transverse load q , and the density of reaction force of the foundation R_f which cause the potential energy variation, can be represented as

$$\delta V = \int_0^L \left[(R_f - q) \delta w_0 + N \frac{dw_0}{dx} \frac{d\delta w_0}{dx} \right] dx \quad (33)$$

The kinetic energy variation is expressed as

$$\delta K = \int_0^L \int_{-h/2}^{h/2} \rho(z) (\ddot{u} \delta u + \ddot{w} \delta w) dz dy dx \quad (34)$$

By substituting variation expressions in Eqs. (31), (33), and (34) into Hamilton principal Eq. (30), then applying (by parts integration formula) to resolve the variations of the unknowns' derivatives, the motion equations can be calculated by utilizing fundamental calculus lemma of variations for each unknown and equating each unknown variations coefficient to zero.

$$\delta u_0: \frac{\partial N}{\partial x} - I_1 \frac{\partial^2 u_0}{\partial t^2} + I_2 \frac{\partial^3 w_0}{\partial t^2 \partial x} - I_3 \frac{\partial^3 \theta}{\partial t^2 \partial x} = 0 \quad (35)$$

$$\begin{aligned} \delta w_0: \frac{\partial^2 M_b}{\partial x^2} + K_w w_0 - K_p \nabla^2 w_0 + C_t \dot{w}_0 - q + N \nabla^2 w_0 - I_1 \frac{\partial^2 w_0}{\partial t^2} - I_2 \frac{\partial^3 u_0}{\partial t^2 \partial x} \\ + I_4 \frac{\partial^4 w_0}{\partial t^2 \partial x^2} + I_5 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} = 0 \end{aligned} \quad (36)$$

$$\delta \theta: -k_1 M_s + A' k_1 \frac{\partial Q}{\partial x} + I_3 \frac{\partial^2 u_0}{\partial t^2 \partial x} - I_5 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} - I_6 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} = 0 \quad (37)$$

The constitutive equations for the stress resultants are obtained by substituting Eqs. (15) and (16) into Eqs. (19)-(21) and the subsequent results into Eq. (32) as follows

$$\begin{Bmatrix} N \\ M_b \\ M_s \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & D_{11} \\ B_{11} & E_{11} & F_{11} \\ D_{11} & F_{11} & G_{11} \end{bmatrix} \begin{Bmatrix} \frac{du_0}{dx} \\ -\frac{d^2 w_0}{dx^2} \\ k_1 \theta \end{Bmatrix} \quad (38)$$

$$Q = A_{55}^s A' k_1 \frac{\partial \theta}{\partial x} \quad (39)$$

Where A_{11} , B_{11} , etc., are the beam stiffness that are expressed as

$$\begin{Bmatrix} A_{11} \\ B_{11} \\ D_{11} \\ E_{11} \\ F_{11} \\ G_{11} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11} \begin{Bmatrix} 1 \\ z \\ f(z) \\ z^2 \\ zf(z) \\ f^2(z) \end{Bmatrix} dz \quad (40)$$

where

$$A_{55}^s = \int_{-h/2}^{h/2} Q_{55}[g(z)]^2 dz \quad (41)$$

$$\begin{Bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{Bmatrix} = \int_{-h/2}^{h/2} \rho(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ z^2 \\ zf(z) \\ f(z)^2 \end{Bmatrix} dz \quad (42)$$

Eqs. (38)-(39) can be expressed in displacements terms (u_0, w_0, θ) by using Eqs. (35)-(37) and Eqs. (38)-(39) as

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_0}{\partial x^3} + D_{11} k_1 \frac{\partial \theta}{\partial x} - I_1 \frac{\partial^2 u_0}{\partial t^2} + I_2 \frac{\partial^3 w_0}{\partial t^2 \partial x} - I_3 \frac{\partial^3 \theta}{\partial t^2 \partial x} = 0 \quad (43)$$

$$\begin{aligned} B_{11} \frac{\partial^3 u_0}{\partial x^3} - E_{11} \frac{\partial^4 w_0}{\partial x^4} + F_{11} k_1 \frac{\partial^2 \theta}{\partial x^2} - K_w w_0 + K_p \frac{\partial^2 w_0}{\partial x^2} - C_t \dot{w}_0 + q - N \frac{\partial^2 w_0}{\partial x^2} \\ - I_1 \frac{\partial^2 w_0}{\partial t^2} - I_2 \frac{\partial^3 u_0}{\partial t^2 \partial x} + I_4 \frac{\partial^4 w_0}{\partial t^2 \partial x^2} + I_5 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} = 0 \end{aligned} \quad (44)$$

$$\begin{aligned} -D_{11} k_1 \frac{du_0}{dx} + F_{11} k_1 \frac{d^2 w_b}{dx^2} - G_{11} k_1^2 \theta + A'^2 k_1^2 A_{55}^s \frac{\partial^2 \theta}{\partial x^2} \\ + I_3 \frac{\partial^2 u_0}{\partial t^2 \partial x} - I_5 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} - I_6 \frac{\partial^4 \theta}{\partial t^2 \partial x^2} = 0 \end{aligned} \quad (45)$$

4.4.4 Analytical solution

The analytical solutions for a simply supported CNTRC beam are obtained by employing the Navier solution method, and it is assumed to be as follows

$$\begin{Bmatrix} u_0(x, t) \\ w_0(x, t) \\ \theta(x, t) \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ W_m \sin(\lambda x) e^{i\omega t} \\ \theta_m \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \quad (46)$$

where U_m , W_m , and θ_m are arbitrary parameters to be determined, ω is the eigenfrequency related to m eigenmode, and $(\lambda = k_1 = m\pi / L)$. Fourier series is used to expand transverse load q as follows

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \quad (47)$$

where the load amplitude Q_m is calculated from

$$Q_m = \frac{2}{L} \int_0^L q(x) \sin(\lambda x) dx \quad (48)$$

The coefficients Q_m are introduced for typical loads. In the case of a uniform distributed load, Q_m equals to the following

$$Q_m = \frac{4q_0}{m\pi}, \quad (m = 1, 3, 5 \dots) \quad (49)$$

The analytical solutions can be obtained by substituting the expansions of $(u_0, w_0, \theta$ and $q)$ from Eqs. (46)-(47) into the equations of motion Eqs. (43)-(45) as the following

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \right) \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ 0 \end{Bmatrix} \quad (50)$$

where

$$\begin{Bmatrix} S_{11} \\ S_{12} \\ S_{13} \\ S_{22} \\ S_{23} \\ S_{33} \end{Bmatrix} = \begin{Bmatrix} \lambda^2 A_{11} \\ -B_{11} \lambda^3 \\ -k_1 D_{11} \lambda \\ E_{11} \lambda^4 + K_w + K_p \lambda^2 + 0.5 C_t + N \lambda^2 \\ k_1 F_{11} \lambda^2 \\ k_1^2 G_{11} + k_1^2 A'^2 A_{55}^s \lambda^2 \end{Bmatrix} \quad (51)$$

$$\begin{Bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{22} \\ m_{23} \\ m_{33} \end{Bmatrix} = \begin{Bmatrix} I_1 \\ -I_2 \lambda \\ A' I_3 k_1 \lambda \\ I_1 + I_4 \lambda^2 \\ -A' I_5 k_1 \lambda^2 \\ A'^2 I_6 k_1^2 \lambda^2 \end{Bmatrix} \quad (52)$$

4.5 Results and discussion

Vibration behavior and numerical results of CNTRC beams are presented and discussed in this section. Polymethyl methacrylate (PMMA) is the effective material used at ambient temperature in this work. The armchair (10, 10) SWCNTs are chosen for reinforcement material. The following table shows the parameters of the material

Table 6 (PMMA) and reinforcement material properties [47]

(PMMA) properties	Reinforcement properties
$\nu^p = 0.3$	$\nu^{cnt} = 0.19$
$\rho^p = 1190 \text{ kg/m}^3$	$\rho_{cn} = 1400 \text{ kg/m}^3$
$E^p = 2.5 \text{ GPa}$	$E_{11}^{cnt} = 600 \text{ GPa}$
	$E_{22}^{cnt} = 10 \text{ GPa}$

	$G_{12}^{cot} = 17.2 \text{ GPa}$
--	-----------------------------------

For vibration analysis

$$I_{00} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho^p dz$$

$$\bar{\omega} = \omega L \sqrt{\frac{I_{00}}{A_{110}}}$$

Table 7 Comparison of fundamental frequencies for CNTRC beam with and without elastic foundation (L / h = 15, $\nu^{cnt} = 0.12$)

Theory	c_t	a (Model 1)	$k_w=0, k_p=0$				$k_w=0.1, k_p=0.02$			
			UD	O	X	V	UD	O	X	V
Ref ^(a)	0	0	0.9749	0.7446	1.1163	0.8442	1.1140	0.9192	1.2397	1.0015
Present	0	0	0.9750	0.7446	1.1164	0.8444	1.1141	0.9192	1.2399	1.0017
	0.1	0	1.0807	0.8784	1.2099	0.9643	1.2077	1.0306	1.3246	1.1047
	0	0.2	1.0321	0.7917	1.1767	0.8968	1.1901	0.9888	1.3176	1.0748
	0.1	0.2	1.1522	0.9429	1.2834	1.0327	1.2957	1.1136	1.4137	1.1906

^(a)Taken from [47]

This work is validated by the vibration results of [52] in the case of missing porosity and damping constant utilizing the derived HSDBT. Table 7 shows vibration analysis for different types of beams with and without elastic foundations, damping constant, and porosity. As represented, having a damping constant and no porosity increases the fundamental frequency for both elastic and no-elastic foundations. Then, in the case of having porosity and no damping constant, the fundamental frequency decreases. In the last case, which contains a damping constant and porosity, the fundamental frequency values are between the two cases. Moreover, increasing ν^{cnt} amount stiffens the CNTRC beams and increases the fundamental frequency, which can be observed in Figs. 22-23, and 24. However, O-beam has the lowest natural frequency while X-beam has the highest.

4.5.1 Porosity impact on fundamental frequency

The effect of porosity on fundamental frequency has been investigated with three voids distribution models in lamina for different parameters. The porosity enhanced the beam's fundamental frequency as it increased with increasing porosity coefficient. Fig. 17 illustrates that model 1 has the highest fundamental frequency compared to other models due to its large number of voids, which leads to a rise in beam vibration capacity. It is also observed in Fig. 18 that the beam with the highest fundamental frequency is the X-Beam and followed by the (UD, V, and O-Beam). It is observed in Fig. 19 shows that Visco-Pasternak resists high fundamental frequency compared to any other foundation. It can be observed that as the foundation nears the Visco-Pasternak condition as the frequency increases with increasing porosity coefficient. While model 1 shows a nonlinear change, other models are almost linear.

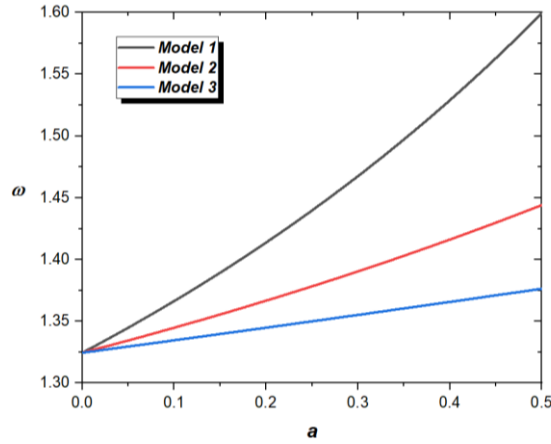


Figure 17 Porosity coefficient effect on the fundamental frequencies for CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w=0.1$, $k_p=0.02$; $c_t = 0.1$).

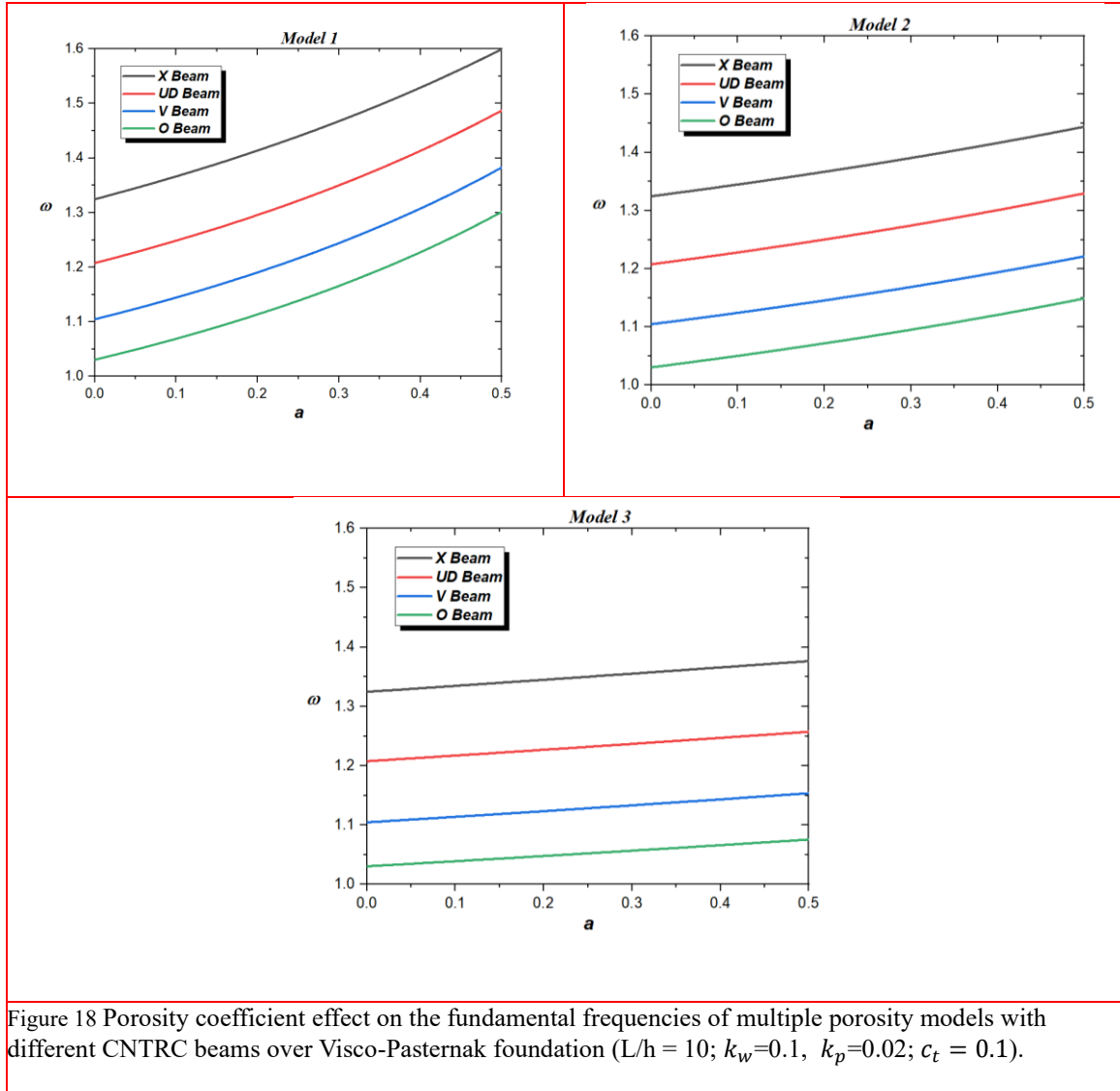


Figure 18 Porosity coefficient effect on the fundamental frequencies of multiple porosity models with different CNTRC beams over Visco-Pasternak foundation ($L/h = 10$; $k_w=0.1$, $k_p=0.02$; $c_t = 0.1$).

The following table represents the considered parameters of the porosity coefficient effect for multiple porosity models of CNTRC X-beam over different types of foundations

Table 8 Considered parameters of the porosity coefficient effect for multiple porosity models of CNTRC X-beam

Type of foundation	L/h	k_w	k_p	c_t
No-Foundation	10	0	0	0
Winkler	10	0.1	0	0

Winkler- Pasternak	10	0.1	0.02	0
Visco-Winkler	10	0.1	0	0.1
Visco- Pasternak	10	0.1	0.02	0.1

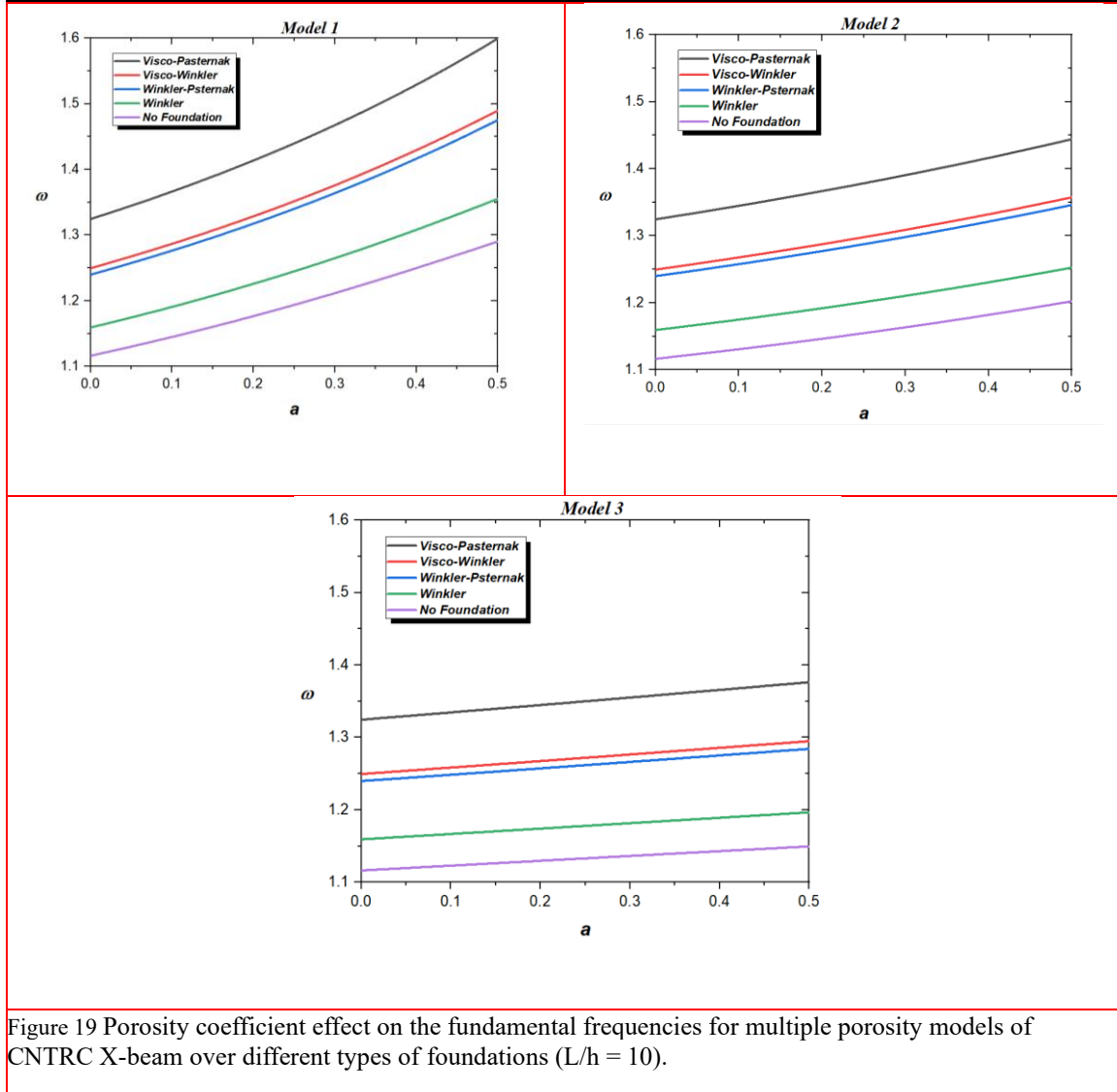


Figure 19 Porosity coefficient effect on the fundamental frequencies for multiple porosity models of CNTRC X-beam over different types of foundations ($L/h = 10$).

4.5.2 Damping coefficient impact on fundamental frequency

The damping coefficient has been investigated, and the analysis in Figs. (20)-(22) showed that increasing the damping constant enhances the vibration effect due to

increasing spring constant factors. Adding the damping constant enhances the foundation's elastic medium, which supports more dynamic loads by increasing beam stiffness. Figs. (20)-(21) represents that increasing porosity, with increasing the damping constant, increases the beam's fundamental frequency. It is also observed in Fig. 22 that the X-Beam has the largest fundamental frequency, while O-Beam has the lowest.

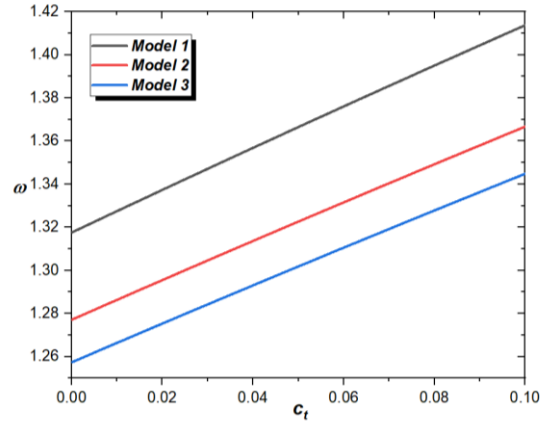
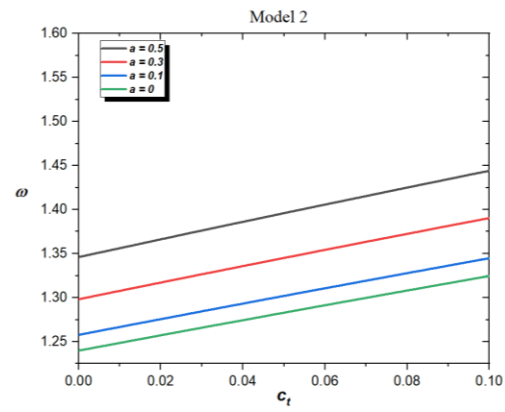
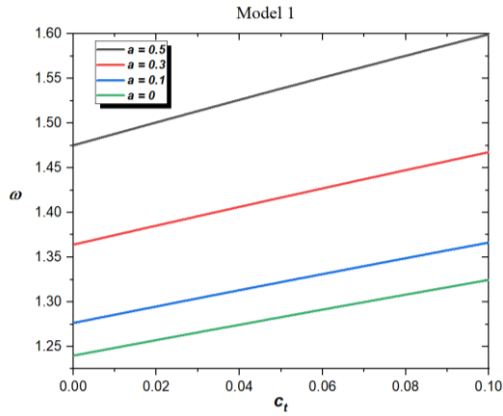


Figure 20 Effect of damping coefficient on the fundamental frequencies for multiple porosity models of CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $a = 0.2$).



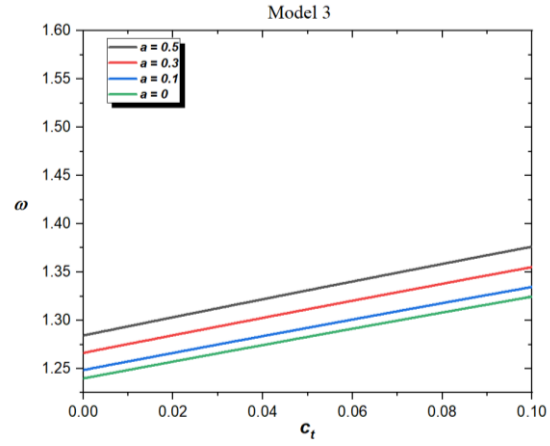
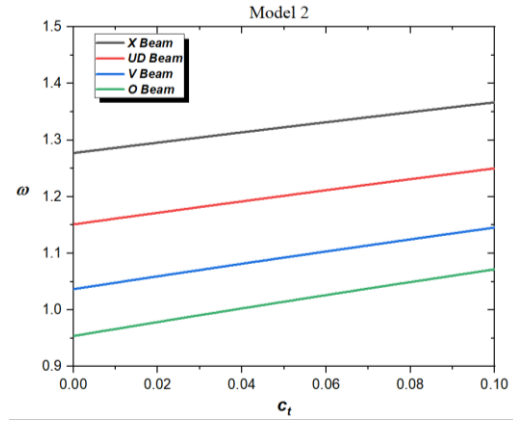
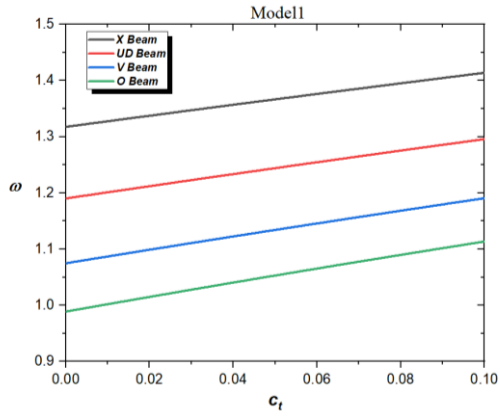


Figure 21 Damping coefficient effect on the fundamental frequencies for multiple porosity models with different porosity coefficients of CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$).



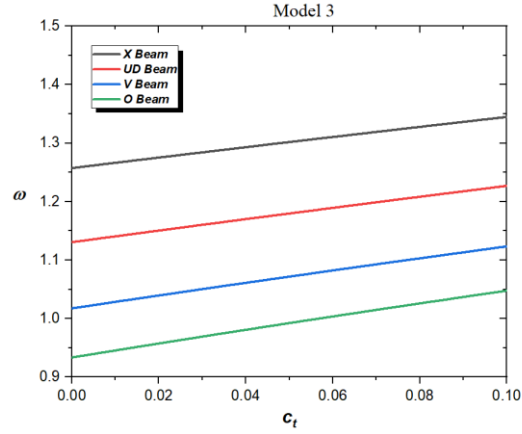


Figure 22 Damping coefficient effect on the fundamental frequencies for multiple porosity models with different CNTRC beams over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $\alpha = 0.2$).

4.5.3 Volume fractions of CNT impact on fundamental frequency

Fig. 23 shows that increasing volume fractions of the polymer matrix increases the fundamental frequency. Adding constant damping leads to enhancing the beam stiffness, increasing the beam's ability to resist more dynamic loads. Fig. 24 shows that increasing the polymer matrix volume fractions increases the fundamental frequency with an increasing length-height ratio. Fig. 25 shows that increasing the polymer matrix volume fractions increases the fundamental frequency with increasing porosity coefficient.

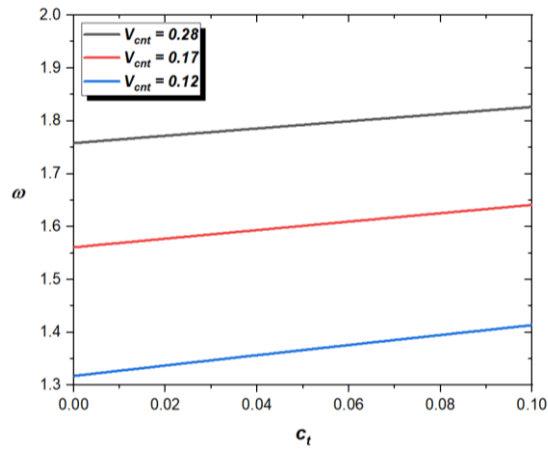


Figure 23 Damping coefficient effect on the fundamental frequencies for multiple volume fractions of the polymer matrix of CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $a = 0.2$).

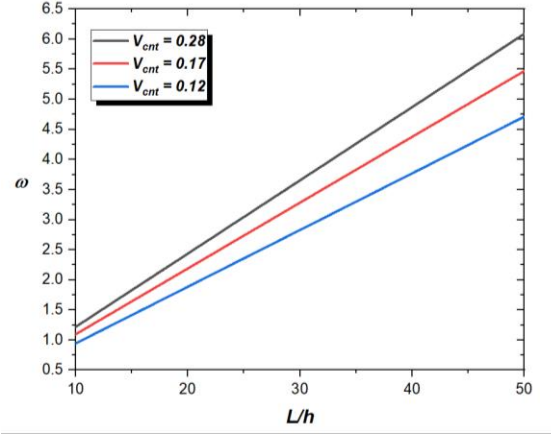


Figure 24 Thickness effect on the fundamental frequencies for multiple volume fractions of the polymer matrix of CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $a = 0.2$).

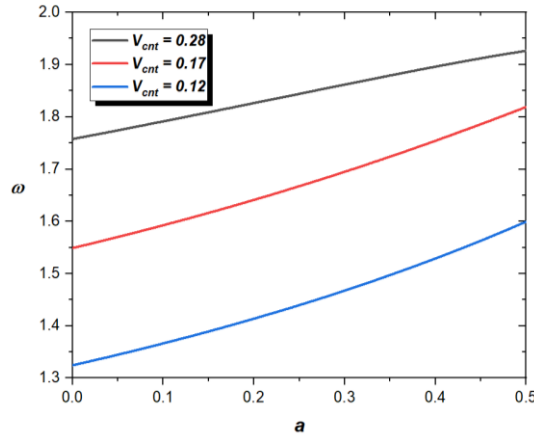


Figure 25 Porosity coefficient effect on the fundamental frequencies for multiple volume fractions of the polymer matrix of CNTRC X-beam over Visco-Pasternak foundation ($L/h = 10$; $k_w = 0.1$; $k_p = 0.02$; $c_t = 0.1$).

4.6 Conclusion

This work studied the vibration behavior of CNTRC beams resting on a viscoelastic foundation utilizing a simplified HSDBT. The porosity effect has been investigated

for multiple models of voids distribution in the lamina. Equations of motion have been derived using Hamilton's principle. A damping constant is added in the formulation to count for viscosity requirements. The accuracy of the present work is numerically validated by making a comparison with available results from the literature. Different parameters' effect has been discussed with many representing figures. Numerical results show that the X-Beam has the highest frequency among different types of CNTRC beams, while O-Beam has the lowest fundamental frequency. Moreover, using the damping coefficient enhanced the vibration effect due to increasing spring constant factors. Furthermore, porosity enhanced the beam's fundamental frequency as it increased with increasing porosity coefficient.

Chapter 5

Conclusions and Recommendations

5.1 Conclusions

This work investigates the bending, buckling, and vibration of CNTRC beams resting on a viscoelastic foundation utilizing a simplified HSDBT. The porosity effect has been introduced with different models of voids distribution in Lamina. Hamilton's principle has been used to derive equations of motion. A damping constant is added in the formulation to satisfy viscosity needs. By making the comparison with some available results, the accuracy of the present work is numerically checked, and many figures with multiple parameters change have been discussed. Numerical results showed that the X-Beam has the highest frequency among different types of CNTRC beams and is the strongest to support the buckling and flexural loads, while O-Beam is the weakest and has the lowest fundamental frequency. Moreover, the presence of the viscoelastic foundation is found to enhance the bending, buckling, and vibration effect. The damping coefficient enhanced the deflection in bending, increased the critical buckling load, and enhanced the vibration effect due to increasing spring constant factors. Porosity has been affected negatively by decreasing beam stiffness and increased bending and buckling effects while enhancing the beam's fundamental frequency as it increased with increasing porosity coefficient.

5.2 Recommendations

This work investigated the mechanical behavior of CNTRC beams under various porosity conditions and different types of CNT distribution. Future work could address the behavior in the dynamic aspects. Future work could also study the following:

- 1- Studying the wave propagation of CNTRC beams with porosity and viscoelastic foundation or other types of material.
- 2- Extending the work to the behavior of CNTRC beams over viscoelastic foundation using FEM.

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Vitae

Name	:Abdulmajeed Mohammed Al-Subaie
Nationality	:Saudi
Date of Birth	:4/7/1997
Email	:Abdulmajeed1009@gmail.com
Address	:Ishbilia, Riyadh, Saudi Arabia. Zip code: 13226
Academic Background	BS Degree (2020): Third honors (GPA 3.16/4.00) (Major Civil & Environmental Engineering), KFUPM University, Dhahran, Saudi Arabia. MS Degree (2022): Second honors (GPA 3.28/4.00) (Major: Civil Engineering – Structures), KFUPM, Dhahran, Saudi Arabia
Submitted Papers	Abdulmajeed Al-Subaie, Mohammed A. Al-Osta, Ibrahim Al-Faqeeh, Abdelouahed Tounsi, Abdelbaki Chikh, Ismail M. Mudhaffar and Salah U. Al-Dulaijan, 2022. Influences of Porosity Distributions on Bending and Buckling Behavior of Functionally Graded Carbon Nanotube-Reinforced Composite Beam, <i>Concrete and Computer</i> Abdulmajeed Al-Subaie, Mohammed A. Al-Osta, Ibrahim A Al-Faqeeh, Abdelouahed Tounsi, Abdelbaki Chikh, Ismail M. Mudhaffar and Salah U. Al-Dulaijan, 2022. Porosity-Dependent Vibration Investigation of Functionally Graded Carbon Nanotube-Reinforced Composite Beam, <i>Concrete and Computer</i>
Software Skills	Mathematica, STAAD Pro, AutoCAD and MS Office