

**PRESSURE TRANSIENT ANALYSIS ON HEAVY OIL
RESERVOIRS UTILIZING MICROWAVE HEATING**

BY

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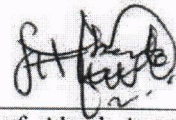
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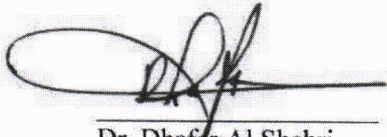
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*This work is dedicated to my family.
For their endless love and support.*

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TABLE OF CONTENT

ACKNOWLEDGMENTS	v
TABLE OF CONTENT	vi
LIST OF TABLES	ix
LIST OF FIGURES	x
ABSTRACT	xiii
ملخص الرسالة	xiv
CHAPTER 1 INTRODUCTION	1
1.1 PROBLEM STATEMENT.....	4
1.2 OBJECTIVE.....	4
CHAPTER 2 LITERATURE REVIEW	5
2.1 Heavy Oil Reservoirs	5
2.2 Conventional Thermal Recovery Methods	9
2.3 Electrical Heating Methods	13
2.4 Heat Transfer Models	23
2.5 Existing Heat Transfer Solution Methodologies	28
2.6 Fluid Flow Models.....	30
2.7 Combined Heat and Flow Model.....	33
2.8 Pressure Transient Analysis for non-Isothermal Reservoirs	33
CHAPTER 3 METHODOLOGY	41
3.1 Assumptions	41
3.2 Boundary Conditions & Initial Conditions.....	41
3.3 Workflow.....	42

3.4	Heat Model Derivations	43
3.5	Flow Model Derivation	49
3.6	Input Data	52
CHAPTER 4 Validation.....		53
4.1	No Flow.....	53
4.2	Constant Flow	55
4.3	Discussion.....	57
CHAPTER 5 Sensitivity Analysis		58
5.1	Temperature Model	58
5.2	Pressure Model	67
CHAPTER 6 Results.....		76
6.1	Methods of Parameter Matching.....	76
6.2	Initial Data & Parameters	77
6.3	Case 1.....	79
6.4	Case 2.....	80
6.5	Case 3.....	81
6.6	Cases Comparison.....	84
CONCLUSION.....		87
Appendix A.....		88
Appendix B.....		89
Appendix C.....		90
Appendix D		91
Appendix E.....		92
Appendix F.....		94
Appendix G		95

<i>Appendix H</i>	97
<i>Appendix I</i>	102
<i>NOMENCLATURE</i>	105
<i>REFERENCES</i>	107
<i>VITAE</i>	115

LIST OF TABLES

Table 1, Heavy Oils & Bitumen Sands Global Distribution	3
Table 2, Heavy Oil Viscosities and Gravities Examples.....	7
Table 3, Screening Criteria for EOR Methods	8
Table 4, Screening Criteria for Thermal EOR Methods	8
Table 5, Field Applications of EM Heating (Li, D 2019)	22
Table 6, Experiments of EM Heating (Li, D 2019).....	22
Table 7, Skin Factor Analysis for Pressure Transient Data in Figure 16	35
Table 8, Parameters Match Targets and Boundaries	77
Table 9, Case 1 Parameters Matches.....	79
Table 10, Case 2 Parameters Matches.....	80
Table 11, Case 3 Parameters Matches.....	81

LIST OF FIGURES

Figure 1, Energy Consumption Worldwide forecasted up to 2050.....	1
Figure 2, Forecast and History of Conventional Oil	2
Figure 3, Distribution of Heavy Oil Resources Worldwide	6
Figure 4, Viscosity and Temperature Relationship for Oils from Several Fields..	7
Figure 5, Cyclic Steam Injection Process.....	9
Figure 6, Steam Drive Process Scheme and Zones	10
Figure 7, Schematic Illustration of Steam-Assisted Gravity Drainage	11
Figure 8, Schematic Illustration of Dry Forward In-situ Combustion	12
Figure 9, Induction Heating in a Vertical Well	15
Figure 10, Schematic of Resistive Heating	16
Figure 11, Molecules Behaviour in an Alternating Electric Field.	18
Figure 12, Super Heat and Boiling Schematic in a Microwave Oven	18
Figure 13, Schematic of Microwave Heating Using a Single Production Well.....	19
Figure 14, Schematic of Microwave Heating Using a Nearby Drilled Hole	20
Figure 15, PTA for Cold Injection into a Hot Reservoir (Variable Skin)	34
Figure 16, PTA for Injection of Cold Water into a Hot Reservoir (Variable r_{inj})	35
Figure 17, PTA for Injection of Cold Water into a Hot Reservoir.....	35
Figure 18, Stimulated Pressure Data for SRT Injection Test	36
Figure 19, Comparison of Derivatives for Various Heat Models	37
Figure 20, Comparison of Sand-face Flowrate Various Heat Models	37
Figure 21, Comparison of Wellbore Storage Coefficient for Various Heat Models.....	38
Figure 22, Wellbore Temperature Profile with and without Joule-Thomson Expansion ..	39

Figure 23, Evolution of Temperature Profile in The Reservoir for 6,200 BPD.....	39
Figure 24, Effect of Productivity Index Caused by Pressure and Temperature Change....	40
Figure 25, Effect of Viscosity the Pressure Derivative and Pressure Measurements	40
Figure 26, Sketch of the System.....	43
Figure 27, Radial Comparision of Temperature No-flow Results for 720 hours.....	54
Figure 28, Time Comparison of No-flow Results at the Wellbore.	55
Figure 29, Radial Comparision of Temperature Constant-flow Results for 720 hours.....	56
Figure 30, Time Comparison of Constant-flow Results at the Wellbore.....	56
Figure 31, Temperature Profile vs Time at r_w	58
Figure 32, Temperature Profile vs Time at $r = 1'$	59
Figure 33, Temperature Profile vs Time at $r = 5'$	60
Figure 34, Temperature Profile vs Time at $r = 10'$	60
Figure 35, Average temperature of the Heated Region vs Time.....	61
Figure 36, Averaging Criterion vs Heated Region Radius.....	61
Figure 37, Temperature Profile vs Distance at 1 Hour.....	62
Figure 38, Temperature Profile vs Distance at 10 hours.....	63
Figure 39, Temperature Profile vs Distance at 100 Hours	64
Figure 40, Temperature Profile vs Distance at 240 Hours	64
Figure 41, Temperature Profile vs Distance at 480 Hours	65
Figure 42, Temperature Profile vs Distance at 720 Hours	66
Figure 43, Temperature Profile vs Distance at 720 Hours between 3 and 20 Feet.	66
Figure 44, Dimensionless Field Plot for 1 KW	67
Figure 45, Dimensionless Field Plot for 5 KW	68
Figure 46, Dimensionless Field Plot for 10 KW	68

Figure 47, Dimensionless Field Plot for -5 BPD.....	69
Figure 48, Dimensionless Field Plot for -20 BPD.....	69
Figure 49, Dimensionless Field Plot for -50 BPD.....	70
Figure 50, Dimensionless Field Plot with varying r_e	71
Figure 51, Dimensionless Field Plot with Varying h	72
Figure 52, Dimensionless Field Plot with Varying s	72
Figure 53, Dimensionless Field Plot with varying T_i	73
Figure 54, Dimensionless Field Plot with Constant/Varying r_{em}	74
Figure 55, Dimensionless Field Plot with Specified Mobility Ratios and r_{em}	74
Figure 56, Dimensionless Field Plot with Different T_{avg} Criterion.....	75
Figure 57, Pressure Data used in Inverse Modelling.....	78
Figure 58, Temperature Data used in Inverse Modelling.....	78
Figure 59, Case 1 Pressure Match against Measured Data	79
Figure 60, Case 2 Pressure Match against Measured Data	80
Figure 61, Case 3 Pressure Match against Measured Data	82
Figure 62, Case 3 Temperature Match against Measured Data	82
Figure 63, Case 3 Errors Comparison for Best Realization	83
Figure 64, Case 3 Errors Comparison for Median Realization	83
Figure 65, Case 3 Errors Comparison for Worst Realization.....	84
Figure 66, Comparison of all Cases Pressure Match against Measured Data.....	85
Figure 67, Error in Best Realizations of All Cases	85
Figure 68, Error in Median Realizations of All Cases	86
Figure 69, Error in Worst Realizations of All Cases.....	86

ABSTRACT

Name : Ali Redha Alnasser

**Title : Pressure Transient Analysis on Heavy Oil Reservoirs Utilizing
Microwave Heating**

Major Field : Petroleum Engineering

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Heavy oil is a part of the most feasible energy sources extracted from petroleum in the world despite its challenging nature to produce. Thermal processes are often considered as enhanced oil recovery methods as they decrease viscosity and increase recovery. Thermal methods include steam injection, electrical heating, electromagnetic radiation, and in-situ combustion. In these reservoirs, the fluid flow equation is often solved independently of the heat flow equation despite non-isothermal conditions; thus, pressure transient analysis is not commonly performed. This work investigated and evaluated the benefits of microwave heaters (electromagnetic radiations) to enhance productivity in a composite reservoir using mathematical tools and MATLAB to perform pressure transient analysis. A mathematical approach was utilised by converting the energy balance partial differential equation to an ordinary differential equation in Laplace space. The equation was then solved numerically to find the temperature to determine viscosity at multiple time steps and finally substituting viscosity in the pressure model to solve for pressures and other parameters of interest.

This work utilised inverse modelling to match the pressure response of non-isothermal reservoirs that utilise microwave heaters and concluded that both temperature and pressure modelling must be used to get accurate results and a proper representation of the reservoir.

ملخص الرسالة

الاسم: علي رضا الناصر

عنوان الرسالة: فحص الضغط الانتقالي على مكامن الزيت الثقيل التي تستخدم تسخين الميكروويف

التخصص: هندسة البترول

تاريخ التخرج: ديسمبر ٢٠٢٢م

الزيت الثقيل جزء من أكثر مصادر الطاقة جدوى المستخرجة من البترول في العالم على الرغم من طبيعة إنتاجه الصعبة. تم العثور على كمية كبيرة من الاحتياطيات العالمية في شكل رمال نفطية وقار وزيتوت ثقيلة. يتم استخدام طرق محسنة لاستخراج النفط لاستخراج كمية كبيرة من الزيت الثقيل. غالبًا ما يتم استخدام العمليات الحرارية لأنها تقلل اللزوجة وتزيد من الاسترداد. تشمل الطرق الحرارية الحقن البخار والتسخين الكهربائي والإشعاع الكهرومغناطيسي والاحتراق الموقعي.

في هذه المكامن، غالبًا ما يتم حل معادلة تدفق المائع بشكل مستقل عن معادلة تدفق الحرارة على الرغم من تغير الحرارة؛ وبالتالي، لا يتم إجراء تحليل الضغط الانتقالي بالشكل الصحيح. من الضروري دمج كل من نماذج التدفق ونماذج الحرارة لمعالجة اللزوجة المتغيرة.

تقوم هذه الرسالة بفحص وتقييم فوائد سخانات الميكروويف (الإشعاعات الكهرومغناطيسية) لتعزيز الإنتاجية في مكن مركب باستخدام الأدوات الرياضية وبرنامج MATLAB لإجراء تحليل الضغط الانتقالي. تم استخدام نهج رياضي عن طريق تحويل المعادلة التفاضلية الجزئية لتوازن الطاقة إلى معادلة تفاضلية عادية في فضاء لابلاس. ثم تم حل المعادلة رقميًا لإيجاد درجة الحرارة لتحديد اللزوجة في خطوات زمنية متعددة واستبدال اللزوجة أخيرًا في نموذج الضغط لحل الضغوط والمعلومات الأخرى ذات الأهمية.

استخدم هذا العمل النمذجة العكسية لمطابقة استجابة الضغط للمكامن متغيرة الحرارة التي تستخدم سخانات الميكروويف ولخصت إلى أنه يجب استخدام نمذجة درجة الحرارة والضغط معًا للحصول على نتائج دقيقة ومثثلة للمكن.

CHAPTER 1

INTRODUCTION

The never-ending growth in the world energy demand due to continuous population growth and steady economic development in the past decades has caused a decrease in the availability of petroleum resources despite improvements and increases in efficiency in production and refining. The past consumption of petroleum resources is mainly composed of conventional easily produced oils with a high market value. Nevertheless, the increasing energy demand will grow until 2050, as indicated by the U.S Energy Information Administration in Figure 1.

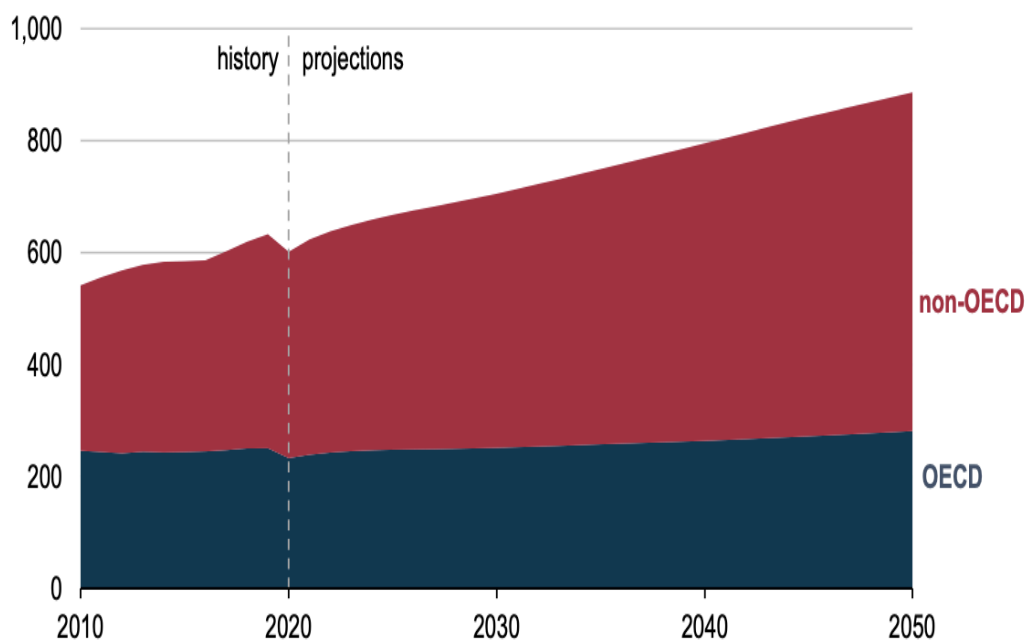


Figure 1, Energy Consumption Worldwide forecasted up to 2050.

Oil fields productions history has shown that the discovery of conventional oil reserves constituting light and medium oil peaked around 1970 (Campbell & Laherrère, 1998). After that, conventional oil followed a constant decline, and with the continuation of this trend, it will represent a much smaller portion of available petroleum resources in the future, as can be seen in Figure 2.

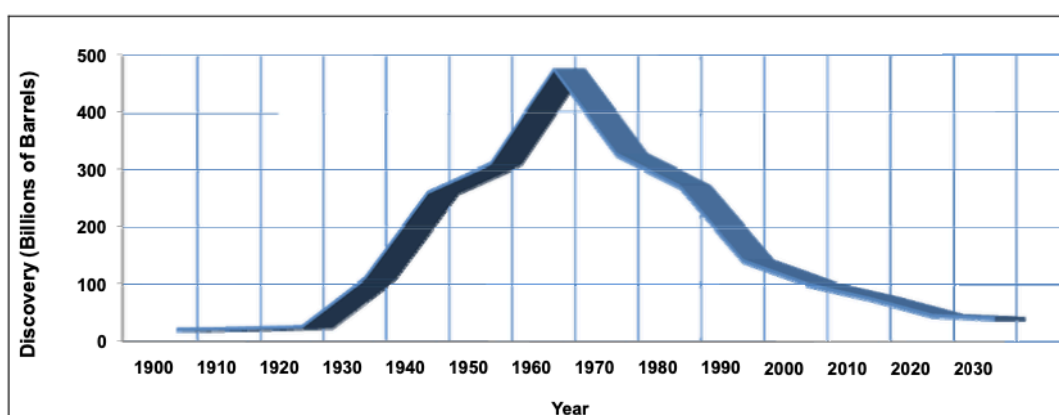


Figure 2, *Forecast and History of Conventional Oil*

The worldwide oil industry must transform more potential resources into viable energy options to fill the energy demand gap. Hence, new technologies must be developed to produce unconventional resources such as heavy or extra-heavy oil economically (Suslick et al., 2003).

Studies estimate that heavy and extra-heavy oils account for more than 6 trillion barrels, equivalent to 70% of all energy derived from petroleum sources worldwide (Oilfield Review Summer, 2006).

The past consumption of petroleum resources is mainly composed of conventional oil reserves whose market value is high and whose production is relatively simple. Conventional oil production has reached a plateau since 2004, and the recovery is from non-conventional oil, as seen in Figure 2 (Tverberg, 2008; Santos, 2013). North America and South America have a significant amount of heavy

oil and bitumen, specifically from Venezuela and Canada, as shown in Table 1 (Adapted from Meyer, 2003).

Table 1, Heavy Oils & Bitumen Sands Global Distribution

Region	Heavy Oil			Bitumen		
	Oil in Place BBO	Recovery Factor	Reserve BBO	Oil in Place BBO	Recovery Factor	Reserve BBO
North America	185.8	0.19	35.3	1659.1	0.32	530.9
South America	2043.8	0.13	265.7	1.1	0.09	0.1
Africa	40.0	0.18	7.2	430.0	0.10	43.0
Europe	32.7	0.15	4.9	1.4	0.14	0.2
Middle East	651.7	0.12	78.2	0.0	0.10	0.0
Asia	211.4	0.14	29.6	267.5	0.16	42.8
Russia	103.1	0.13	13.4	259.2	0.13	33.7
Total	3268.5	-	434.3	2618.3	-	650.7

Over 50% of global reserves can be found in oil sands, bitumen, and heavy oils. The largest and most significant heavy oil reserves can be found in Canada and Venezuela (Meyer et al., 2007).

Even though viscosity is a function of both temperature and pressure, these heavy oil reservoirs usually utilize thermal methods that decrease viscosity to enhance the overall recovery.

This proposal investigates these heavy oil reservoirs with high viscosity and low API gravity. Out of all thermal recovery methods, the investigation will be on microwave heating (electromagnetic radiation). This work will pay attention to conduction and advection as heat transport mechanisms.

Due to microwave heating, there will be a change in viscosity, so when solving for fluid flow, a variable viscosity model must be utilized. The process can be repeated for multiple time steps and obtain multiple viscosity values. These various values of viscosities will

yield different pressure values and enable us to determine some critical reservoir parameters by utilizing pressure transient analysis.

Integrating the heat transport and variable-viscosity fluid flow models will allow us to estimate well-test parameters for these challenging heavy oil reservoirs under non-isothermal conditions.

1.1 PROBLEM STATEMENT

The fluid flow equation is solved independently of the heat flow equation; this leads to pressure transient analysis, a vital tool that enables us to estimate important reservoir parameters, to be not commonly performed in reservoirs that utilize microwave heaters. In these reservoirs, it is essential to integrate flow and heat models to tackle the problem of changing viscosity.

1.2 OBJECTIVE

This work aims to evaluate the effect of a microwave heater placed at the sand face of a well over the pressure and temperature to estimate various well-test parameters including skin factor, permeability, wellbore storage, and the radius of investigation. Heat balance and flow balance will be used collectively. This thesis plans to integrate both processes and include all radiation, conduction, and advection heat transfer elements.

CHAPTER 2

LITERATURE REVIEW

2.1 Heavy Oil Reservoirs

Almost 50% of oil reserves globally take the form of heavy oil, oil sand bitumen, or extra heavy oil. Therefore, bitumen and heavy oil are some of the most significant resources that can be utilized for future energy demand. The most significant amount of heavy oil and oil sands can be found in Canada, Alberta, and Eastern Venezuela (Meyer et al., 2007). Figure 3 shows the world's distribution of heavy oil (IEA, 2006 as cited in Acar, 2007)

The high viscosity, in addition to the low API gravity of these oils, leads to inadequate oil recovery, reaching 10% by primary production (Butler & Yee, 2002). The main driver for fluid flow is viscosity, a measure of fluids resistant to flow where higher viscosity implies more resistance to flow. Viscosity is dependent on pressure and temperature but more significantly on the latter.

Crude oil with high specific gravity and high viscosity in situ is defined as heavy oil. There is no agreement in the industry for the exact value, but oils with viscosity from 10 cp up to 10,000 are classified as heavy oil. Another way to mention these oils is by stating the API gravity (less than 22.3 according to the world energy council). Moreover, oils with an API gravity of less than ten are labelled “extra-heavy”. In addition to high viscosity and low specific gravity, these heavy oils usually have high asphaltene content, low hydrogen-carbon ratio, high heavy-metal contents, and even Sulphur (Acar, 2007)

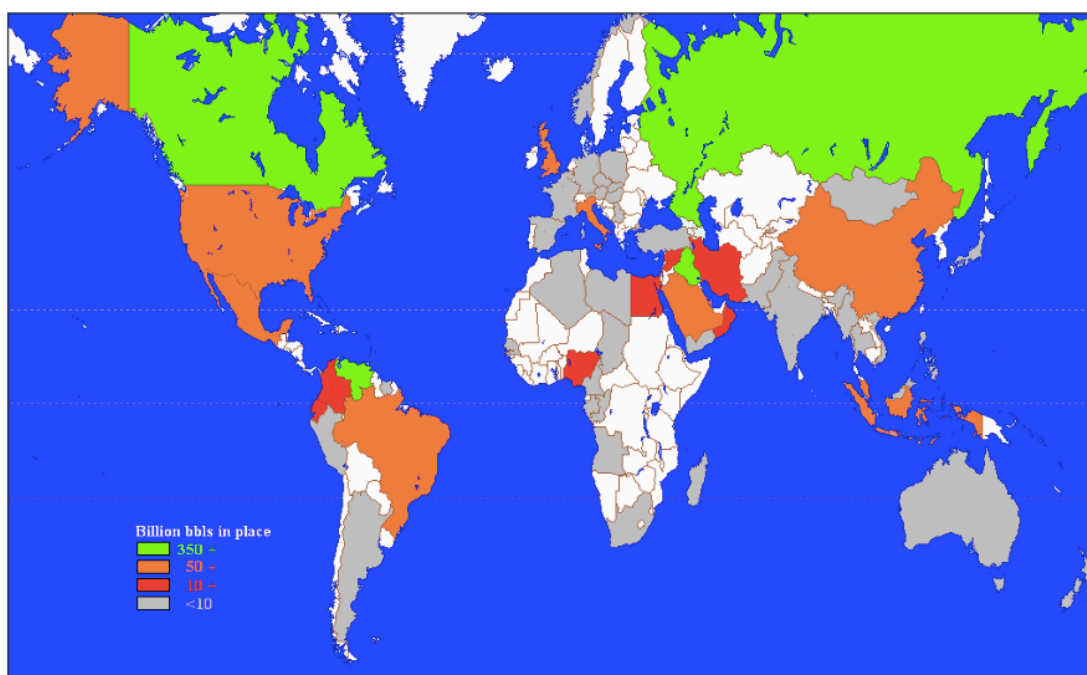


Figure 3, Distribution of Heavy Oil Resources Worldwide

Viscosity is greatly affected by temperature, especially for oils compared to gasses; a temperature change may cause a tenfold viscosity reduction. The actual effect of temperature on viscosity depends on the type of oil because of different compositions and even volatility (Speight, 1991; Santos, 2013).

It is worth noting that even though low API gravity describes heavy oil or even extra-heavy oil, similar API gravity oils can have much different viscosity. Table 2 shows that for Bachaquero oil and Cold-Lake oil, even though the API gravity difference between them is only 1 degree, the viscosity difference is hundred-fold higher (Briggs, 1988).

Farouq (2003) studied the relationship between viscosity and temperature for oil from several fields, as shown in Figure 4 (Farouq Ali 2003). Even a 20 C change in temperature can decrease viscosity from 30,000 cp to 1,000 cp (Athabasca oil).

Table 2, Heavy Oil Viscosities and Gravities Examples

Field	Location	Gravity (°API)	Reservoir Viscosity, cp
Bachaquero	Venezuela	13	150
Emlichheim	Germany	24.5	175
Lost Hills	California	14	400
Cold Lake	Canada	10 to 12	10,000 to 100,000

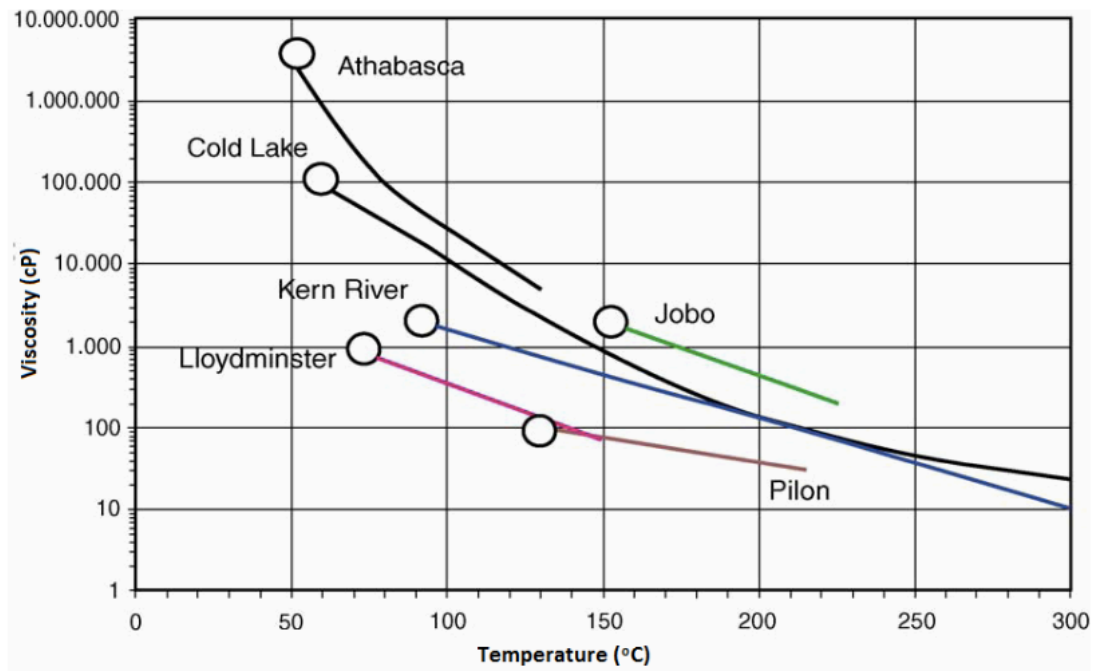


Figure 4, Viscosity and Temperature Relationship for Oils from Several Fields..

As viscosity is mainly a function of temperature, heavy oil recovery methods are thermal, thus increasing temperature, decreasing viscosity, and increasing flow capacity. Conventional thermal recovery methods include hot water injection, steam-assisted gravity drainage, continuous steam flooding, cyclic steam injection, and fire flooding, also known as in-situ combustion (Li, 2019; Acar, 2007). These methods pose an environmental risk due to the extensive need for steam generation, which comes from the combustion of natural gas and thus may not consistently achieve economic criteria. Furthermore, they may only sometimes apply to some reservoirs, such as those with low injectivity, low pressure, high heterogeneity, high clay content, or thin sections (Bera & Babadagli, 2015).

Table 3, Screening Criteria for EOR Methods

Screening Parameters	Steam injection	<i>In-situ</i> combustion	CO ₂ Flood	Surfactant-polymer	Polymer	Alkaline Waterflood	Hydrocarbon Miscible
1. Oil viscosity, cP	n.c.	n.c.	< 12	< 20	< 200	< 200	< 5
2. Oil gravity, API	10-25	10-45	> 30	< 25	> 18	15-35	> 30
3. Depth, ft	200-5000	> 500	> 2300	< 8500	< 8500	n.c.	n.c.
4. Reservoir temperature, °F	n.c.	n.c.	n.c.	< 250	< 200	< 200	n.c.
5. Initial reservoir pressure, psig	n.c.	n.c.	> 1200	n.c.	n.c.	n.c.	n.c.
6. Net pay, ft	> 20	> 10	n.c.	n.c.	n.c.	n.c.	n.c.
7. Permeability, md	n.c.	n.c.	n.c.	> 20	> 20	> 50	n.c.
8. Residual oil saturation, %	50	50	25	> 25	> 50	> 25	> 25
9. Transmissibility, md ft/cp	> 100	> 20	n.c.	n.c.	n.c.	n.c.	n.c.
10. Porosity, %	> 10	> 10	n.c.	n.c.	n.c.	n.c.	n.c.
11. Salinity (TDS), ppm	n.c.	n.c.	n.c.	< 50,000	n.c.	< 2500	n.c.
12. Hardness (Ca and Mg)	n.c.	n.c.	n.c.	< 1000	n.c.	-	n.c.
13. Operating pressure, psi	< 2500	n.c.	> 1100	n.c.	n.c.	n.c.	> 1300
14. Target oil, bbl/acre-ft	> 500	> 400	n.c.	n.c.	n.c.	n.c.	n.c.
15. Lithology	n.c.	n.c.	n.c.	Sandstone	n.c.	Sandstone	n.c.
16. Well spacing	< 10	< 20	n.c.	n.c.	n.c.	n.c.	n.c.

Table 4, Screening Criteria for Thermal EOR Methods

Method	API Gravity	Depth, m	Porosity	Thickness, m	Viscosity, cp	Permeability, md
Steam flood	8-13.5	<1500	>20%	>6	200-3000	250-1000
In-situ Combustion	10-16	<3500	>20%	>3	1500-5000	>50
Electrical Heating	>7	NC	NC	Thin	>50	NC

Thus, there has been an interest in an alternative to conventional thermal methods to address those challenges that have impacted the feasibility of recovery processes that utilize steam. The alternative is electrical methods (Vermeulen & McGee, 2000). The primary objective of electrical heating is merely to increase reservoir temperature, thus reducing oil viscosity and increasing oil mobility as a transformation of electrical energy can heat reservoirs into heat energy (Ritchey, 1956). A general Enhanced Oil Recovery (EOR) screening criteria is seen in Table 3, while a general thermal EOR screening criteria can be seen in Table 4 (Donaldson, 1985; Taber 1997). Electrical methods are indeed applicable in most cases, unlike conventional thermal methods, with depth, porosity, and permeability not crucial.

2.2 Conventional Thermal Recovery Methods

2.2.1 Cyclic Steam Injection

Cyclic steam injection involves an alternating pattern of steam injection and well production, often referred to as Huff & Puff. Steam is usually injected for 2-6 weeks into a producing well at very high rates, and then the well is put into production for a couple of months. The process is repeated until an economic limit is hit either due to a decrease in pressure/production or a significant increase in water cut; the process is seen in Figure 5 below (Acar, 2007; Pizzaro, 1990).

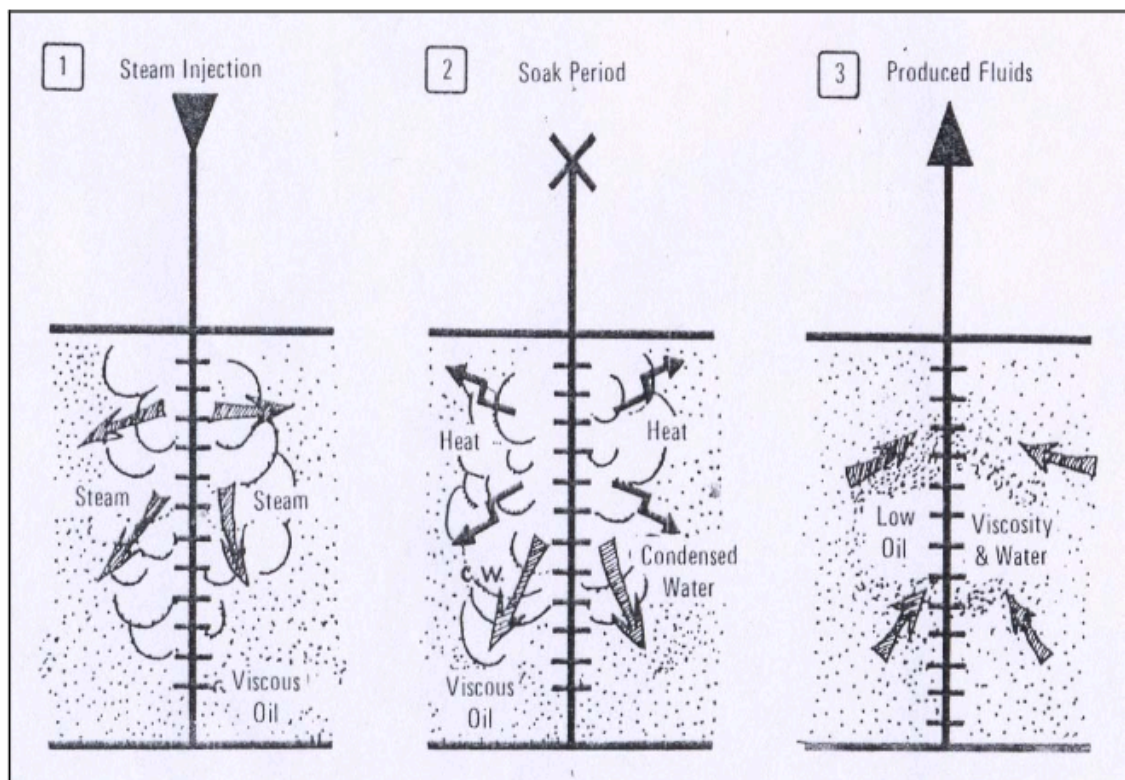


Figure 5, Cyclic Steam Injection Process

2.2.2 Steam flooding

Steam-flooding or steam drive is when an injection well continuously injects steam. The steam pushes the fluids toward production wells while increasing/maintaining reservoir pressure and heating the formation in parallel. During steam flooding, multiple zones are formed within the reservoir. First, the steam zone, then a zone with high steam saturation with oil saturation decreased to less than 15%, and then a water transition zone for hot injected water and colder water approaching reservoir temperature, and finally, an oil zone that was pushed by the steam. Steam separates due to gravity and overrides oil, which might cause early breakthroughs; hence, it is recommended that perforations are placed at the bottom of the producing zones. Steam-flooding schematic and regions formed are observed in Figure 6 below (Acar, 2007; Pizzaro, 1990).

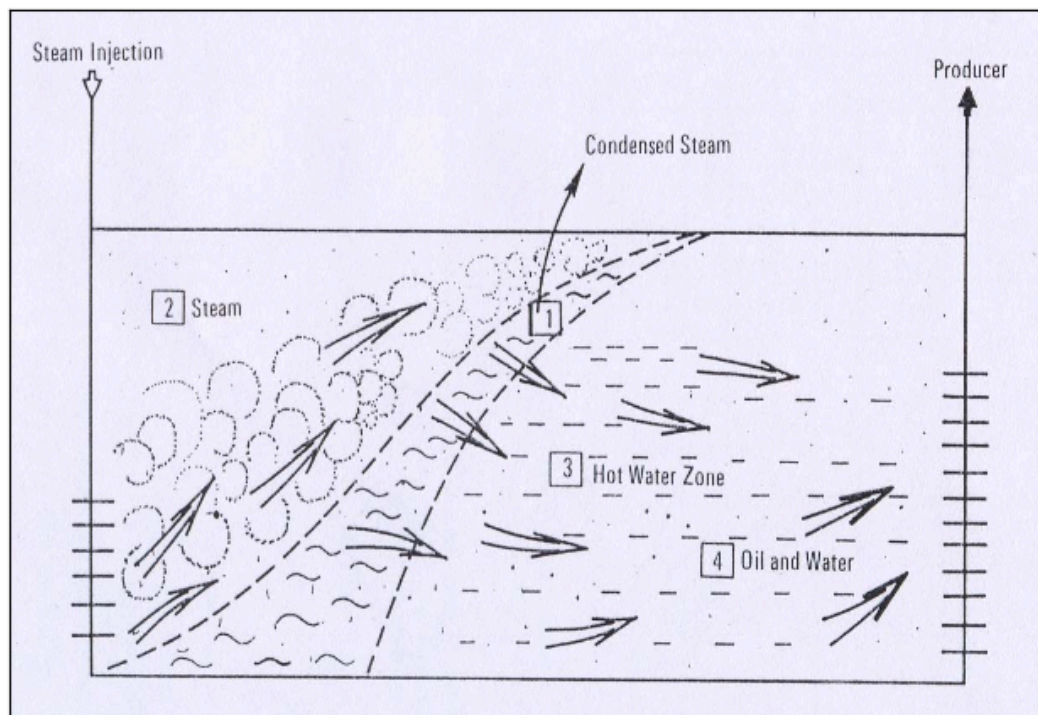


Figure 6, Steam Drive Process Scheme and Zones

2.2.3 Steam-Assisted-Gravity-Drainage (SAGD)

Dr Butler first recommended SAGD in the 1980s (Butler et al. 1985, 1987, 1994). The process involves two parallel horizontal wells at the reservoir's upper zone and bottom. The top well performs steam injection while the latter well acts as a producer, the vertical distance between them is merely 5-7m. The recovery involves a pre-heating phase and a SAGD phase. The Pre-heating creates a thermal connection between the two wells and can be a cyclic-steam cycle, electrical heating, fracturing, or even solvent injection (Al Bahlani, 2008; Dong X, 2014; Xi C, 2017). After that, steam is continuously injected from the upper well, creating a steam chamber that undergoes several stages, namely vertical expansion, horizontal expansion, and exhaustion (Butler, 1991; Dong, 2019). The steam travels upwards due to density difference until it reaches the boundary and accumulates to expand horizontally. Once the steam reaches the reservoir boundary vertically and horizontally, the steam depletes (exhaustion). During this operation, heated oil, and condensed water flow to the production well (Butler, 1997; Speight, 2009). Figure 7 shows the schematic of SAGD. (Guo et al. 2016)

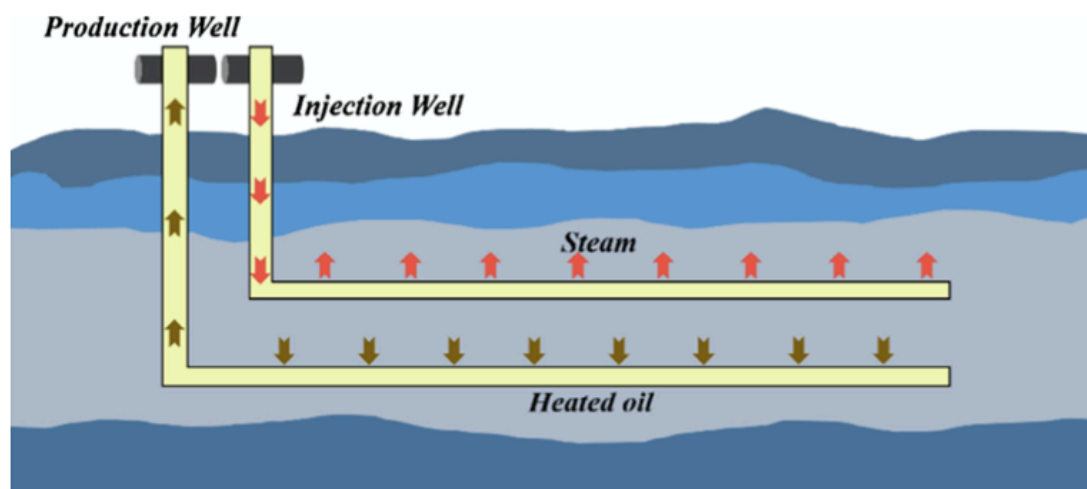


Figure 7, Schematic Illustration of Steam-Assisted Gravity Drainage

2.2.4 In-Situ Combustion

heavier hydrocarbons. The fire front moves and pushes the hot mixtures of gas, steam, and hot water. The fire and the hot mixture reduce the oil viscosity and increase its mobility while pushing it toward production wells. In-situ combustion is more effective than steam flood operations for moderately thick reservoirs containing viscous oils. In-situ combustion is divided into dry, wet, and reverse (Pizzaro, 1990; Acar, 2007). Figure 8 shows a typical air-injection (dry combustion) schematic (Guo et al, 2016).

Dry Forward Combustion

This is a straightforward method involving injecting solely air.

Wet Combustion

In this process, water is injected and passes through the combustion front along with air. The combustion zone's water may be vapour, liquid, or both. Optimally, the water is injected simultaneously with air but intermittently with air when there is too high of flow resistance in the near wellbore region of the injector.

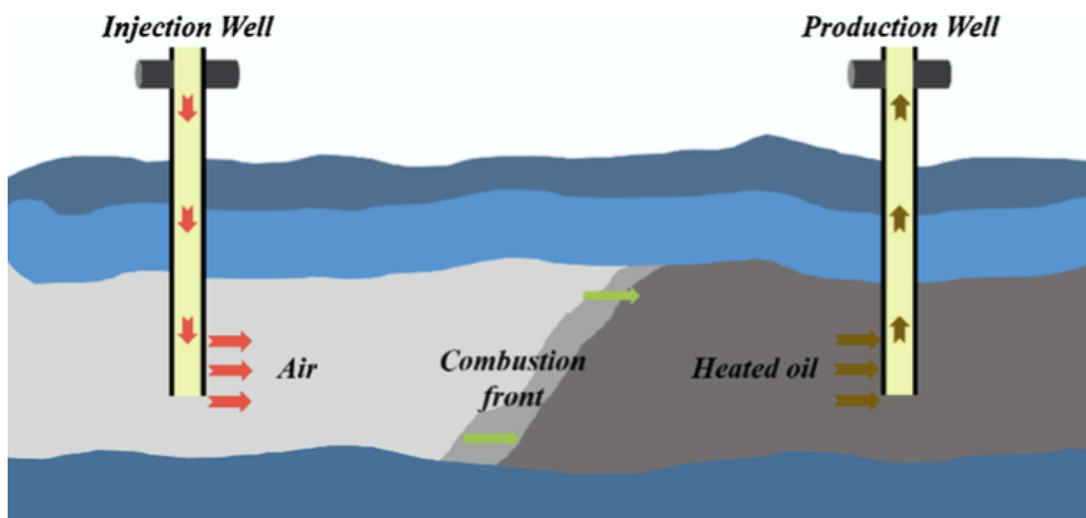


Figure 8, Schematic Illustration of Dry Forward In-situ Combustion

Reverse Combustion

During reverse combustion, the air is injected through injection wells that eventually become production wells. In the beginning, forward combustion takes place. After the combustion zone moves away from the ignition well, injection stops and is started in adjacent wells. The adjacent wells push oil towards wells that were initially ignition wells. If the oil in the adjacent wells ignites spontaneously, the air injection for the reverse combustion process is stopped, and the process turns into forward combustion.

2.3 Electrical Heating Methods

Electrical heating methods are divided into three categories based on the heat source. First, downhole resistive heaters generate heat as per Joule's effect leading to a temperature increase near the wellbore (Rodriguez et al., 2008), this method mainly depends on conduction, and it takes a significant amount of time; this is also known as inductive heating. Secondly, ohmic heating or resistive heating uses low-frequency electric current; in this method, electrical current flows in the formation via conductive formation water in the interconnected pore-spaces and heats the water, thus heating the formation and oil (Hasanvand & Golpravar, 2015). Finally, heating the formation by high-frequency radiation such as radiofrequency and microwaves, dipoles are formed by molecules that align with the electric field. This molecular movement produces heat in the reservoir through friction (Bera & Babadagli, 2015).

There are a few advantages of electrical heating compared with conventional thermal recovery methods. Based on experiments in Venezuela, Russia, Canada, Netherlands, Brazil, and the USA, most field tests show the feasibility of electric heating to recover oil

sand and heavy oil and its potential as an alternative to steam injection. The advantages of using electricity include but are not limited to eliminating operation costs and equipment associated with the injection. Eliminating injection of water or steam correspond to a reduction in water-cut, reduction of swelling or reactions with water-sensitive clays, increased sweep efficiency, avoiding viscous fingering, and elimination of pressure limitation (Li, D 2019).

However, there are limitations to electrical heating. The invasion depth or depth of heating is not significantly high, so the application of the technology requires tight well spacing, especially in electromagnetic heating, which leads to high drilling and completion costs limiting the application in big reservoirs. Furthermore, if the heating mechanism is water-dependent, limitations may occur due to vaporizations. (Li, D 2019)

2.3.1 Inductive Heating

An alternating electric current flow through conductors, which induces a magnetic field surrounding it; the change of magnetic fields creates secondary currents, which then, in turn, heat the medium of circulation. The system heats the production casing using several inductors, mainly used for vertical wells, and the schematics are shown in Figure 9 below (Cambon, 1978; Kumar, 2000; Acar, 2007).

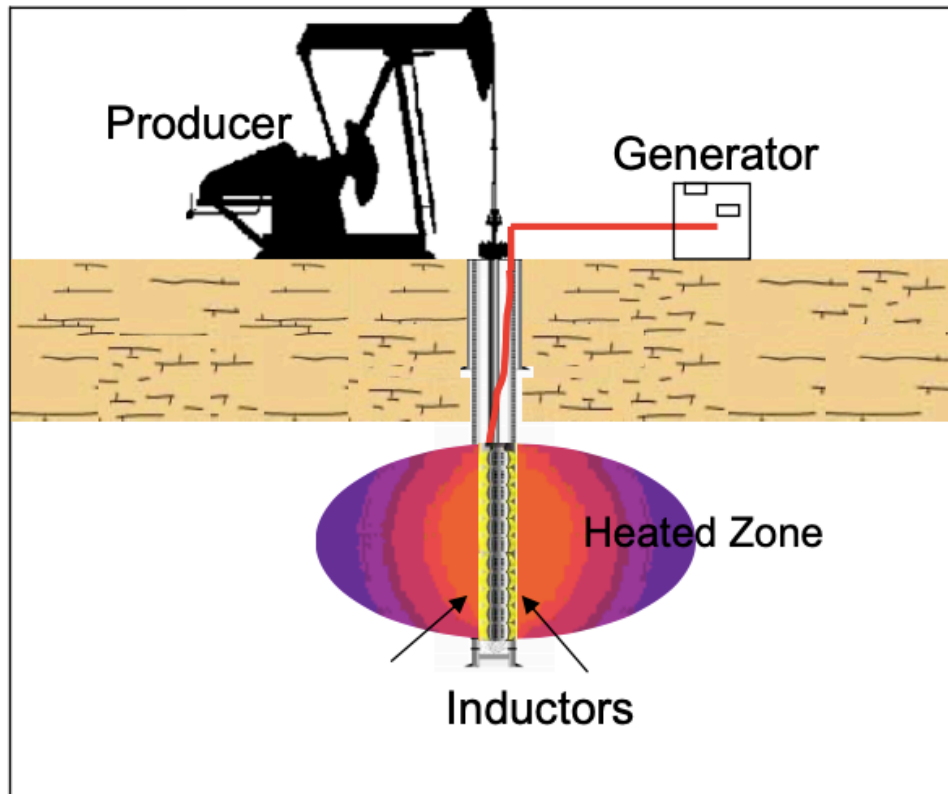


Figure 9, Induction Heating in a Vertical Well

2.3.2 Low-Frequency Electric Heating

Low-frequency electric heating, also known as low-frequency resistive heating and even ohmic heating, occurs when a low-frequency current flows through the reservoir, and electrical energy is converted into heat. Heat generation near the wellbore reduces oil viscosity, thus increasing its mobility and reducing the pressure drop in that region. This is used in the industry by placing the equipment in two neighbouring production wells, which serve as an anode and a cathode, a potential difference is applied, and an electric path through formation water (if salty) is formed, which can be seen in Figure 10 below (Chhetri, 2008). This mechanism indicates a limitation as the formation temperature must not exceed

the boiling point of water at reservoir conditions, or there will be a loss of conductivity (Cambon, 1978; Kumar, 2000; Acar, 2007).

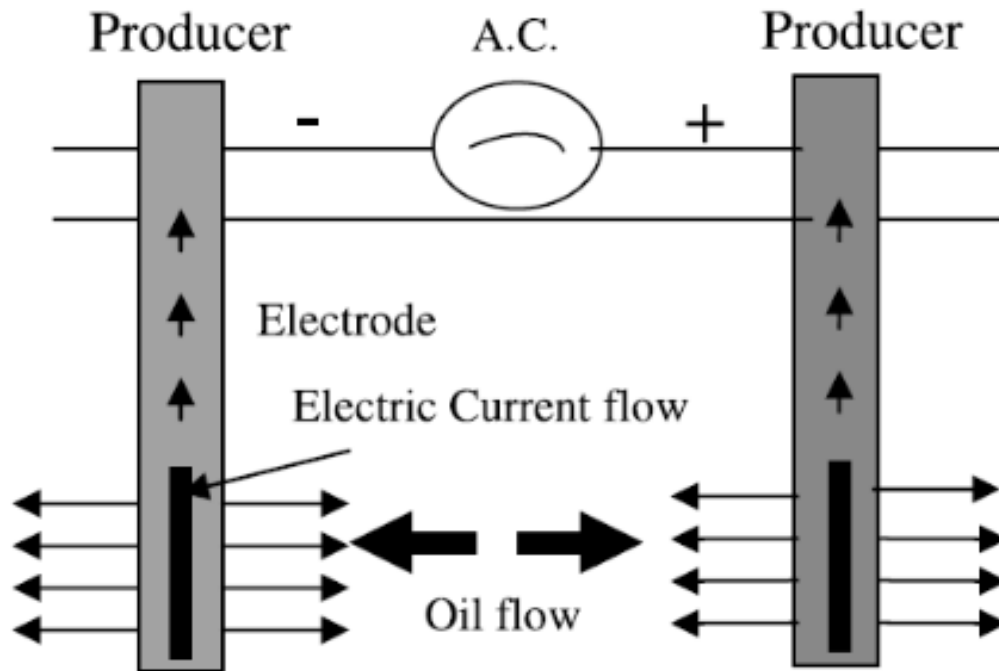


Figure 10, Schematic of Resistive Heating

2.3.3 Electromagnetic Heating

A disturbance in the environment that propagates through space is named a wave. If this disturbance occurs within magnetic space, it yields a magnetic wave. As such, for electromagnetic waves, the magnetic space and electric space are occupied with waves that are perpendicular to each other. Thus, generating two fields, an electric field, and a magnetic field, perpendicular to each other and perpendicular to wave propagation, creates an electromagnetic waves field that oscillates in time and space to keep electricity and magneticity constant (Taheri-Shakib, 2021; Kong, 1975)

The microwave region of the electromagnetic spectrum falls between a frequency of 300 MHz and 300,000 MHz. Microwave is often a loosely applied term for waves above 1000

MHz because of the “micro” or small wavelengths of waves at these frequencies (Acar, 2007. Jackson, 2002. Milan, 1978. Fanchi, 1990)

Microwaves heat many different materials, including but not limited to foods and ceramics. In the field, microwaves have applications in heating oil reservoirs for enhanced oil recovery, especially for heavy oils (Acar, 2007). Electromagnetic heating is also used for shale oil recovery (Taheri-Shakib, 2021)

There are three modes of interaction between microwaves and materials, transmission, reflection, and absorption, where maximum heating occurs. Electromagnetic wave interactions depend on the material; the interactions are strong with some and weaker with others. For example, crude oil does not absorb microwaves, but some of its constituents do. Some microwave receptors that are often mixed with crude are activated carbon, methanol, and iron oxides (Acar, 2007. Jackson, 2002. Milan, 1978. Fanchi, 1990. Ovallas, 2002).

Microwave heating also depends on the water saturation of the formation; the water molecules heat up due to a strong electromagnetic field and then transfer the heat to the formation, which is then absorbed by the oil, causing a viscosity decrease and less flow resistance. Of course, microwave heating is affected by the source design and the electromagnetic properties of the material (which depend on the frequency, temperature, and electrical properties (Acar, 2007; Jackson, 2002; Milan, 1978; Fanchi, 1990; Ovallas, 2002).

In reservoir conditions, dielectric heating overwhelms, and molecules dipoles align with the electric field; this causes rotational movement of the dipole and bipolarity in the molecules with a velocity proportional to frequency, resulting in significant heating, as can be seen in Figure 11 (Taheri-Shakib, 2021).

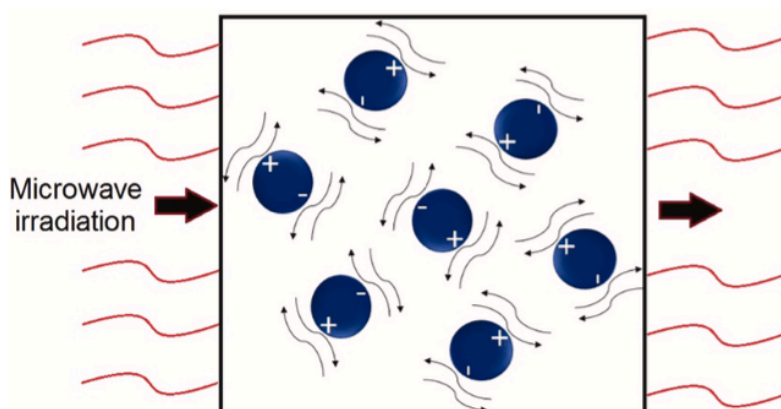


Figure 11, Molecules Behaviour in an Alternating Electric Field.

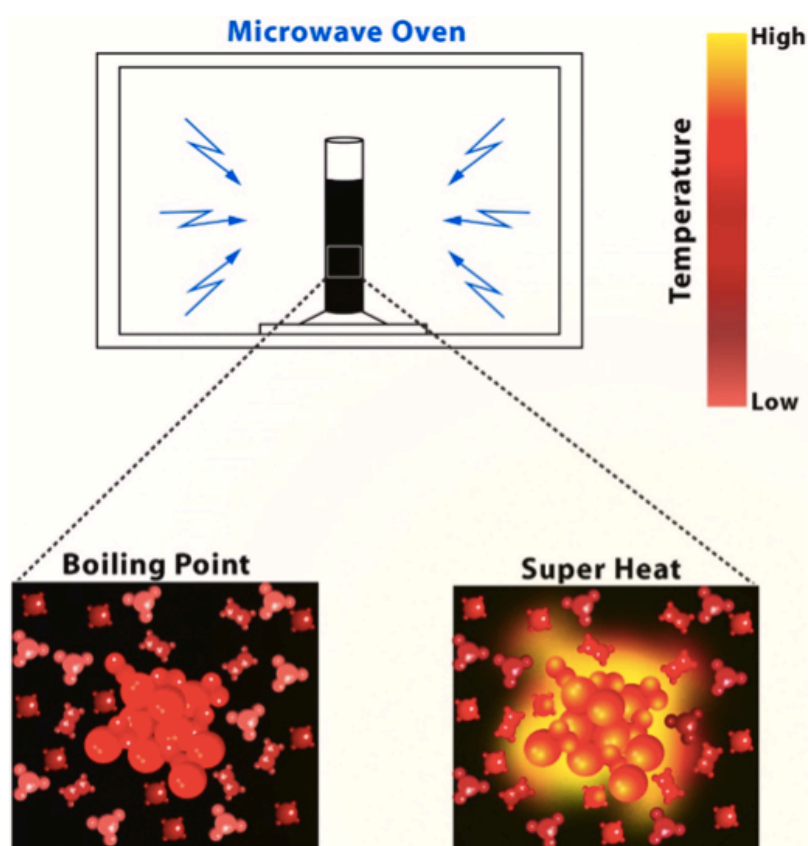


Figure 12, Super Heat and Boiling Schematic in a Microwave Oven

Hydrocarbon with sands can absorb microwaves and attain high temperatures of up to 400 degrees Celsius due to superheated zones because of high polarity and receptibility, which can be seen in Figure 12 (Taheri-Shakib 2018, 2021). These superheated regions create

micro explosions in the smallest connected pore spaces and open them wider, leading to an effective porosity increase (i.e., higher permeability) (Acar, 2007. Jackson, 2002. Milan, 1978. Fanchi, 1990. Owallas, 2002).

Microwave heating may be utilized in the field by placing a microwave antenna at the production well or in a nearby hole close to the production well. The fluids are heated continuously and are then pushed to the production well. The schematic of both methods can be seen in Figure 13 and Figure 14 (Acar, 2007; Bientinesi et al., 2013).

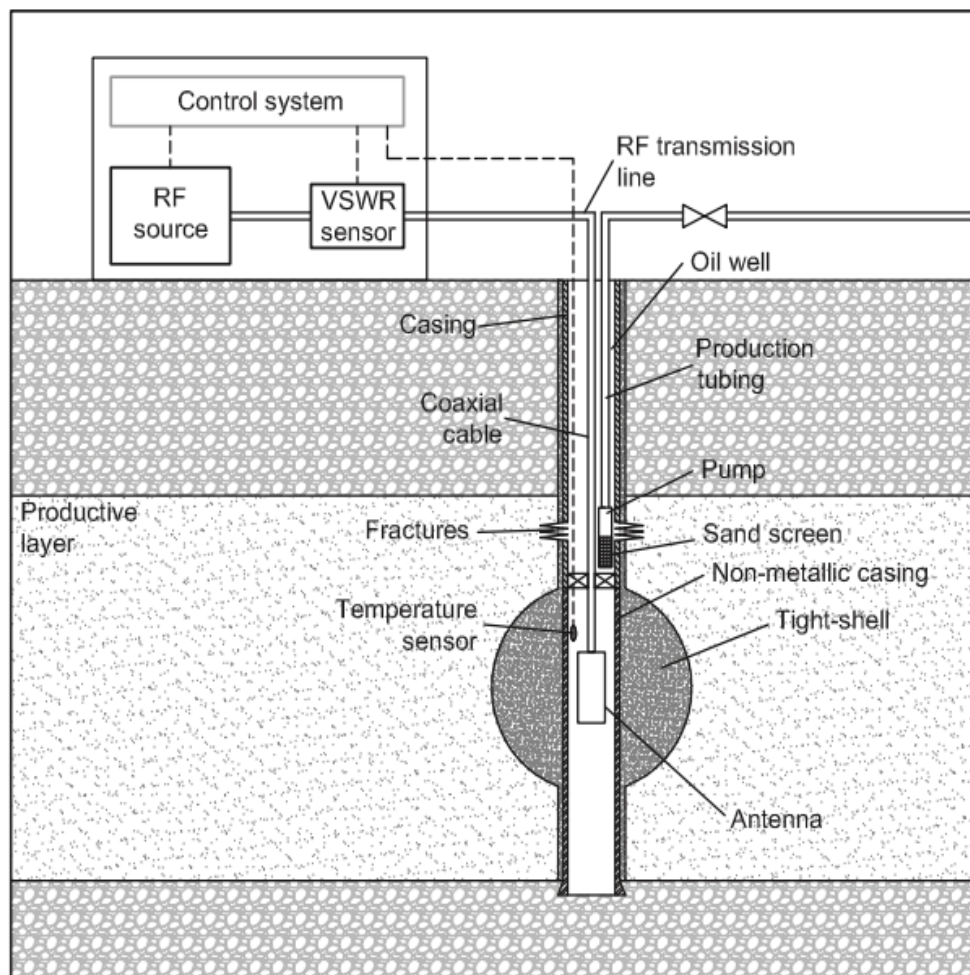


Figure 13, Schematic of Microwave Heating Using a Single Production Well

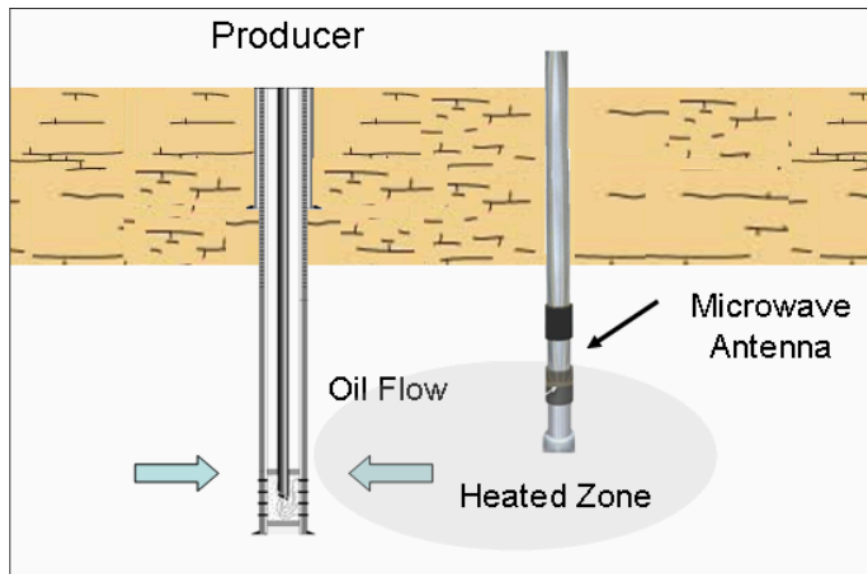


Figure 14, Schematic of Microwave Heating Using a Nearby Drilled Hole

The penetration depth of microwaves is usually tiny; the microwave energy rapidly heats fluids and causes further fluids to be drawn to the producing well. Penetration depth is a function of microwave frequency. Milan AC (1978) found that the maximum value of electromagnetic wave propagation is 15m with a frequency of 2450 MHz (Acar, 2007. Jackson, 2002. Milan, 1978. Fanchi, 1990. Owallas, 2002).

2.3.3.1 Case Studies utilizing Microwaves

Case 1

Cambon et al. presented up to 86% yield in lighter products, like conventional oil recovery methods, after studying Canadian tar sands production utilizing microwave heating at a frequency of 2450 MHz.

Case 2

In a modelling study, Kumar et al. used microwave heating in an oil reservoir with very poor permeability and 33 cp viscosity. Microwave heating was conducted with a 9m microwave antenna and a frequency of 915 MHz around 9m from the production well. The

heating increased the temperature by 150 °C near the antenna after one year. Within five years of heating, the temperature increased by 18m away from the source by 90 °C. It has been estimated that when placing two 60-kW sources 500m and 550m away from the production well, an 80% improvement in cumulative oil recovery of 10 years can be achieved.

Case 3

A numerical simulation of microwave heating at a frequency of 915 MHz was done by Warren et al. for an aquifer drive in a Saskatchewan reservoir. A reported increase in cumulative oil production by 9% was shown compared to cold production (without heating) while decreasing water viscous fingering.

Case 4

Numerical simulations of dielectric heating in wells of both vertically and horizontally oriented wells were performed by Kumar et al. The authors found that the steam flooding process alongside radiofrequency and microwave heating results in a better temperature distribution than steam injection alone. The model showed that a 60 kW microwave antenna 9m away from a vertical production well resulted in a higher recovery of cumulative oil by 80% when compared to cold production.

Case 5

Three conceptual reservoirs which contain crude oils of varying viscosity (medium to extra heavy) were studied by Ovallas et al. Simulation showed a significant increase in oil production due to microwave heating because of viscosity reduction. The simulations showed that microwave heating could produce almost ten times more energy in the reservoir than it uses as electricity, thus encouraging the use of microwave heat technology in EOR applications of unconventional oils.

2.3.3.2 Summary of Literature Field Studies on Electrical Heating Methods

Table 5, Field Applications of EM Heating (Li, D 2019)

Field name	Heating method	Results	Reservoir properties	References
Ishimbayskoye oil field in Bashkortostan and Yul'timirovskoye bitumen field, Russia	High frequency heating of electromagnetic radiation at setting temperature of 110 °C.	Oil rate is increased by 2.5 times and water cut is reduce to 1/3.	-	Sayakhov, 1970
Avintaquin Canyon and Asphalt Ridge, Utah, USA	13.56 MHz at power from 5 to 20 kW was applied at Avintaquin Canyon, 13.56 MHz at 200 kW was applied at Asphalt Ridge	Avintaquin Canyon was heated up to 637 K and Asphalt Ridge was heated up to 473 K.	-	Bridges et al., 1979
Asphalt Ridge tar sand near Vernal, Utah, USA	RF power carried from 40 to 75 kW and the frequency was changed from 2.2875 to 13.56 MHz.	About 25 m ³ of the tar sand column was heated. Average deposit temperature was about 200 °C. Bitumen recovery was expected in the range from 50 to 80%.	-	Sresty et al., 1986
Rio Panan oil field, Brazil	Low frequency at 30 kW is applied between neighboring wells, 328 feet apart.	Oil rate increases from 1.2 to 10 bbl/day after 70 days of heating.	$h = 27 \text{ ft}; \phi = 0.27; k = 4 \text{ darcies}; \mu_o = 2452 \text{ cp at } 100^\circ\text{F}; \mu_o = 100 \text{ cp at } 220^\circ\text{F } P_i = 284 \text{ psi}$	Pizarro and Trevisan, 1990
Lashburn and Northminster, Lloydminster heavy oil area, Canada	Heating with 30 kW power. Single well heating, current flows into formation from electrically isolated casing and up through the overburden to surface ground wells to complete the circuit.	Quick response heating but failed due to electricity delivery system and sand production. Oil rate is 1.27 and 3.75 times of primary production under heating of Northminster and Lashburn, respectively.	-	Davison 1992; Davison 1995
North Midway field, California, USA	13.56 MHz at 25 kW is applied between neighboring wells, 328 feet apart.	Oil rate increases from 1.2 to 10 bbl/day after 70 days of heating.	$h = 27 \text{ ft}; \phi = 0.27; k = 4 \text{ darcies}; \mu_o = 2452 \text{ cp at } 100^\circ\text{F}; \mu_o = 100 \text{ cp at } 220^\circ\text{F } P_i = 284 \text{ psi}$	Kasevich et al., 1994
Wildmere field, Alberta, Canada.	EM heating	Oil rate was increased to 3.18 tonnes/day under EM heating from 0.95 tonnes/day of primary production.	-	Mukhametshina and Martynova, 2013; Bera and Babadagli, 2015
Dover site, Alberta, Canada.	EM heating with solvent (ESEIEH)	For an oil production rate of 10,000 bbl/day, ESEIEH was estimated to cost \$50/bbl while under similar economic assumptions SAGD would cost \$70/bbl	-	Wise and Patterson, 2016; Sadeghi et al., 2017a

h is reservoir thickness, ϕ is porosity, k is permeability, μ_o is oil viscosity.

2.3.3.3 Summary of Literature Laboratory Experiments on Microwave Heating

Table 6, Experiments of EM Heating (Li, D 2019)

Experimental model	RF/MW frequency and source power	Results	Oil type	Rock matrix	Reference
20×20×10cm Plexiglass filled with sand	5, 10, 20 MHz	29, 32 and 37% of OOIP produced for 5, 10, 20 MHz, respectively. The recovery is 45% under combination of EM heating and N ₂ injection.	Oil viscosity is 1000 cp at room temperature.	Reservoir sands	Chakma and Jha, 1992; Jha and Chakma, 2010;
55 gallon drum with diatomaceous earth and soil	1kW with 50.55 MHz and 200W with 144 MHz	Diatomaceous earth could be efficiently heated to the temperature of 150 °C.	Texaco oil in Bakersfield, California.	Diatomite reservoir	Kasevich et al., 1994
Crushed rock packed with crude and water in core holder	Fixed frequency of 2450 MHz with 10 levels of power. The maximum power is 700 W.	Carbonate formation was heated up with irradiation without the presence of water. Asphaltenes cause permanent changes in crude oil rheology under irradiation.	United Arab Emirates(UAE) crude with viscosity of 5.27cp under room temperature. The initial asphaltene concentration is 3% by weight.	Carbonate rock from Jabel Hafet region of Al-Ain, UAE.	Gunal and Islam, 2000
Samples were placed in the oven in 125 ml Erlenmeyer flasks.	6700+-507.5 Mhz	Upgrading using microwave energy produced oil with pipeline specification viscosity and density.	Crude oil from Polver Lake reservoir in the Lloydminster of Saskatchewan. The viscosity is 11300 cp at room temperature.	-	Jackson, 2002
300 ml cylindrical stainless-steel reactor in which EM energy penetrates from the top.	2.45 GHz	Determination of microwave heating on heavy oil reservoir and validation with mathematical model.	Lake Maracaibo heavy oil and Orinoco Basin extra-heavy crude	Silica	Ovalles et al., 2002

Table 6-Continued, Experiments of EM Heating (Li, D 2019)

Experimental model	RF/MW frequency and source power	Results	Oil type	Rock matrix	Reference
Graphite core holder (Ø5.2×8.5cm) packed with crushed limestone premixed with crude oil and water.	Microwave oven with 1450 MHz and 1400 W.	High water saturations lead to higher oil production and high salinity promotes microwave heating.	Oil samples from conceptual reservoirs (Bati Raman 9.5°API, Garzan 12°API, Camurlu 18°API) in southeast Turkey.	Limestone	Hascakir et al., 2009
Polyvinyl chloride pipes of 0.5 m in length and 22.5 mm in inner diameter was filled with silica sand.	81.36 MHz with 6 kW used to heat up sample.	Microwave heating is more effective on small size sand particles.		Dry silica sand	Kovaleva et al., 2010a
A cubical stainless case with open top and edge length of 7 cm was filled with 16 °API heavy oil in crushed sandstone.	3.0 GHz with 1000W microwave oven.	Sample temperature was raised up to 100°C. Effect of initial water saturation, wettability, porosity and permeability on dielectric heating was discussed.	16 °API heavy oil	Sandstone	Jha et al., 2011
Core holder (Ø2×12inch)	1100W microwave oven.	10.34 to 20.79% of oil was produced.	Heavy oil with viscosity of 540.52 cp at room temperature.	Berna sandstone, glass beads, sand-pack	Alomair et al., 2012
Sandbox of 1.25×1.25×0.75m.	2.45 GHz with variable power from 1 to 2 kW.	Heating temperature is up to 200 °C.	Oil sands	Oil sands	Bientinesi et al., 2013
Buchner filter tunnel	2.45 GHz microwave with variable power from 100 to 1000 W.	Nano-nickel catalysts increase the efficiency of microwave heating. Up to 90% oil recovery depending on operation conditions.	Heavy oil from northern Alberta, Canada. The viscosity is 120cp at 60 °C.	Glass beads	Greff and Babadagli, 2013

2.4 Heat Transfer Models

2.4.1 Energy Balance

Appendix A contains the derivation of energy balance which may be written as:

$$\begin{aligned} \nabla \cdot [(1 - \phi)\tilde{\kappa}_s \cdot \nabla T_s + \phi\tilde{\kappa}_f \cdot \nabla T_f] + \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{Ts} \\ = \frac{\partial}{\partial t} \left((1 - \phi)\rho_s C_{vs} T_s + \phi\rho_f C_{vf} T_f \right). \end{aligned} \quad (1)$$

Where $\tilde{\kappa}_s$ is the thermal conductivity tensor, T_s and T_f are the solid and fluid temperatures, C_{ps} and C_{pf} are the isobaric heat capacities of solids and fluids, q_{Ts} is the heat rate at the source, and C_{vs} is the isometric heat capacity of the solid matrix. A similar expression is derived by Bientinesi et al. (2013):

$$\rho_T C_T \frac{\partial T}{\partial t} - \nabla \cdot (\kappa_{eff} \nabla T) = q_{em}. \quad (2)$$

The equation above does not account for advection, we can account by adding the term:

$$\rho_T C_T \frac{\partial T}{\partial t} - \nabla \cdot (\kappa_{eff} \nabla T) + \nabla \cdot [-\nabla \cdot [\rho_f c_{pf} T \vec{u}_f]] = q_{em}. \quad (3)$$

Bientinesi also defined electromagnetic radiation and electromagnetic flux as:

$$q_{em} = -\nabla \cdot \vec{F}_{em}, \quad (4)$$

and

$$F(r) = \frac{P(r)}{r^2}. \quad (5)$$

Where $F(r)$ is the electromagnetic flux. If we consider only radial coordinates, fluids, and rock temperature to be equal, we rewrite the heat balance as:

$$q_{em} + \rho_o q_o C_{po} \left(\frac{\partial T}{\partial r} \right) + 2\pi h \kappa \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = 2\pi r h \rho_T C_{pT} \left(\frac{\partial T}{\partial t} \right). \quad (6)$$

2.4.2 Accounting for Electromagnetic Radiation

Multiple methods often approximate radiated power per unit volume. Microwaves are an electric current that varies with time in a linear antenna that generates a varying circulating magnetic field; the magnetic field then produces a circulating electric field as per Faraday's law. Then, the electric field also produces a magnetic field described by Ampere's law. The electric field and magnetic fields are cross-linked perpendicular to each other and propagate away from the source; most work in analytical modelling of microwave heating attempts to predict a temperature profile outside the wellbore.

2.4.2.1 Abernethy (1976)

Abernethy (1976) showed that the radiation in a linear homogeneous conductivity medium the radiation propagation in the +x direction follows:

$$\frac{d\psi(x)}{dx} = -\alpha\psi(x). \quad (7)$$

Where $\psi(x)$ $\left(\frac{Watts}{m^2}\right)$ is the power density, and α $\left(\frac{1}{m}\right)$ is the power absorption coefficient.

Abernethy assumed a cylindrical element with a microwave at the centre to estimate absorbed power, which led to the development of a power adsorption equation to describe power density in radial coordinates (Abernethy, 1976. Fanchi, 1990. Soliman, 1997):

$$\frac{d\psi(r)}{dr} = -\left(\alpha + \frac{1}{r}\right)\psi(r). \quad (8)$$

The power absorption coefficient depends on the properties of the absorbing materials present in the medium and can be presented as:

$$\alpha = 0.02 \alpha_e, \quad (9)$$

Such that:

$$\alpha_e = \sqrt{\left(\frac{\omega^2 k_{em} \varepsilon}{2} \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon^2}}\right)} - 1. \quad (10)$$

Where α_e is the electrical field absorption coefficient $\left(\frac{1}{m}\right)$, ω is the angular frequency, k_{em} is the electromagnetic permeability $\left(\frac{Henries}{m}\right)$, σ is conductivity $\left(\frac{mho}{m}\right)$, ε is permittivity $\left(\frac{Farad}{m}\right)$.

A detailed derivation for radiated power as a function of radius as per equation 11 is included in Appendix B, the relationship is written as:

$$P(r) = P_0 e^{-\alpha(r-r_0)}. \quad (11)$$

The energy balance may be established with unit volume energy change to calculate the temperature profiles. Abernethy's model neglects heat loss and utilizes the sum of convection and radiation to create a simplified model. Abernethy obtains the following simplified energy balance model (q_o is assumed positive for production):

$$[\alpha P_0 e^{-\alpha(r-r_0)}] + \rho_o q_o C_{po} \left(\frac{\partial T}{\partial r} \right) + 2\pi h \kappa_T \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = 2\pi r h \rho_T C_{pT} \left(\frac{\partial T}{\partial t} \right). \quad (12)$$

Abernethy neglected conduction heat transfer to obtain following heat balance equation:

$$\left(\frac{\partial T}{\partial t} \right) = \frac{1}{2\pi r h \rho_T C_{pT}} \left\{ \alpha [P_0 e^{-\alpha(r-r_0)}] + \rho_o q_o C_{po} \left(\frac{\partial T}{\partial r} \right) \right\}. \quad (13)$$

Abernethy then obtained the steady-state solution presented by:

$$T(r) = T_0 + \frac{P_0 e^{-\alpha(r-r_0)}}{\rho_o q_o C_{po}}, \quad (14)$$

And the transient state constant flow solution as:

$$T(r, t) = T_0 + \frac{P_0 e^{\alpha r_o}}{\rho_o q_o C_{po}} \left(e^{-\alpha r} - e^{-\alpha \sqrt{r^2 + 2At}} \right), \quad (15)$$

Where:

$$A = \frac{\rho_o q_o C_{po}}{2\pi h \rho_T C_{pT}}. \quad (16)$$

Transient state zero-flow solution was obtained as:

$$T(r, t) = T_0 + \frac{P_0 e^{-\alpha(r-r_0)} t}{2\pi r h \rho_T C_{pT}}. \quad (17)$$

Which is identical to the steady state solution shown in equation 14.

2.4.2.2 Fanchi (1990)

Many studies have been carried out using simplified Maxwell's equations assuming an electromagnetically linear and isotropic porous medium. The simplified Maxwells' equations are:

$$\nabla \times E = - \frac{\partial B}{\partial t}, \quad (18)$$

$$\nabla \times H = - \frac{\partial D}{\partial t}, \quad (19)$$

$$\nabla \cdot D = 0, \quad (20)$$

and

$$\nabla \cdot B = 0. \quad (21)$$

Where E and H are the electric and magnetic field intensities, D and B are the electric and magnetic flux densities, respectively.

Electromagnetic dissipated power per unit of volume as based on Maxwell's equation is described as (Fanchi, 1990):

$$P_{avg} = \omega \varepsilon_0 \varepsilon'' E_o^2. \quad (22)$$

Where ω is the angular frequency, ε'' is the loss factor, and E_o^2 is the root mean square of electric flux density,

For the radial direction on a cylinder and based on Poynting's vector for electromagnetic radiation, Fanchi found that:

$$P(r) = \frac{2\beta_o h E_o^2}{\omega k_{em} \delta} e^{-2\alpha_o r}. \quad (23)$$

Where β_o is the phase shift constant, h is the thickness of the irradiated zone, k_{em} is the electromagnetic permeability, δ is defined as $\sqrt{\alpha_o^2 + \beta_o^2}$, α_o is the electric field absorption coefficient (analogous to α_e in Abernethy's solution)

Solving for r_w , then solving for E_o will give:

$$E_o^2 = \frac{P_o \omega k_{em} \delta}{2\beta_o h} e^{2\alpha_o r_w}. \quad (24)$$

Substituting equation 24 back in equation 23 will yield:

$$P_{avg} = P_o e^{-\alpha(r-r_w)}, \quad (25)$$

where:

$$\alpha = 2\alpha_o \left(\frac{1}{m} \right), \text{ or } \alpha = 0.02\alpha_o \left(\frac{1}{cm} \right). \quad (26)$$

Equation 26 is identical to Abernethy's solution in equation 11.

2.4.2.3 Taheri-Shakib (2021)

Taheri-Shakib used another model to estimate the average dissipated power in 2021:

$$P_{avg} = \frac{\gamma + \omega \epsilon \tan \delta}{2} |\vec{E}|^2, \quad (27)$$

such that:

$$\tan \delta = \frac{\epsilon}{\epsilon_{eff}}. \quad (28)$$

Where γ is the effective conductivity, ω is the angular frequency, ϵ is the relative permittivity, $\tan \delta$ is the tangent loss angle, ϵ_{eff} is the dielectric loss parameters exhibit the heat conversion efficiency of absorbed energy and $\vec{E}$ is the magnitude of the electric field. The temperature change corresponding to electromagnetic radiation alone is described as (Taheri-Shakib, 2021):

$$\frac{\Delta T}{\Delta t} = \frac{2\pi f \epsilon_0 \epsilon_{eff} |E|^2}{\rho_m C_{pm}}. \quad (29)$$

Where subscript m stands for the medium and ϵ_0 is the permittivity of free space.

2.5 Existing Heat Transfer Solution Methodologies

Most literature work is based on Maxwell's equations or Abernethy's power. Thermal models are solved using a commercial thermal simulator or self-made numerical or analytical models. There has been no analytical solution to the problem except when one of the heat transfer mechanisms is neglected. Commonly, conduction heat transfer is neglected, but in some studies, conduction is considered, no-flow cases are evaluated and,

thus, neglecting conduction. The boundary condition used at the inner boundary is commonly that heat flux on the axis of symmetry corresponding to the temperature derivative with respect to distance at zero equals zero.

In 1976, Abernethy's published his solution to the electromagnetic irradiation problem in a heat reservoir, thus showing the simplified relationship between electromagnetic power and radial distance. Abernethy neglected conduction and used the steady solution as a basis for the transient solution with constant flow. When Abernethy neglected conduction, the heat transfer problem became of the first order in space, and thus only one boundary condition was needed to solve it. Abernethy used a constant temperature at the outer boundary. Fanchi, in 1990 evaluated Abernethy's solution and rederived from electromagnetic irradiation but did not proceed to solve it in any different method. Carrizales et al. (2008) provided solutions for steady-state linear flow (co-current and counter-current) and steady state radial flow. They utilized constant heat flux at the wellbore to obtain an expression for radiated energy and not to solve the PDE and then used a symmetry of flow at the inner boundary such as:

$$\left(r \frac{\partial T}{\partial r} \right)_{r=r_w} = 0. \quad (30)$$

Various authors use the same inner boundary condition indicated by Equation 30 to solve the heat transfer problem (Davletbaev et al., 2010; Bogdanov et al., 2011; Carrizales et al., 2012; Peraset al., 2012; Davletbaev et al., 2016; Sadeghi et al., 2017; Hu et al., 2020)

Bientinesi et al. (2013) conducted experiments to evaluate the effect of electromagnetic heating shell tank of quartz with oil and water and then used a simple numerical model that includes conduction and radiation with no flow. They utilized the boundary condition:

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_w} = 0.5 \frac{K}{mm} \quad (31)$$

The value of 0.5 Kelvin per mm constant boundary condition at the well was measured experimentally and used in their numerical model. It remarkably matches experimental data when the temperature is below 100 degrees Celsius (the boiling temperature of water). Their results show that the inner boundary condition of specified heat flux should be used.

2.6 Fluid Flow Models

2.6.1 Conservation of Mass (Continuity Equation)

The mass balance is often modelled with the continuity equation given by:

$$-\nabla \cdot (\rho_f \vec{u}) + \frac{\rho_f q_{sc}}{V} = \frac{\partial(\rho_f \phi)}{\partial t}. \quad (32)$$

Where ∇ is the del operator, ρ_f is the fluid density, $\vec{u}$ is the fluid velocity, ϕ is the system's porosity, q is the flow rate, V is the system bulk volume, and t is the time variable. The derivation of the continuity equation can be seen in Appendix C.

2.6.2 Darcy's Law

The simplest transport model is given by Darcy law (Darcy, 1856):

$$\vec{u} = -\frac{1}{\mu} \bar{\bar{K}} \cdot \nabla p = -\frac{1}{\mu} \bar{\bar{K}} \cdot \nabla p. \quad (33)$$

Where $\bar{\bar{K}}$ is the permeability tensor of the medium, μ is the viscosity of the flowing fluid, and p is the fluid pressure.

2.6.3 Radial Diffusivity Equation

Appendix D includes the radial diffusivity equation derivation. For radial dominated flow and assuming no sink/source term, the diffusivity equation may be written as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \rho_f \frac{k_r}{\mu} \frac{\partial p}{\partial r} \right) = \frac{\partial(\rho\phi)}{\partial t}. \quad (34)$$

Where r is the radius, k_r is the radial permeability.

2.6.4 Constant Viscosity Incompressible fluids

The following equations of states indicated are used in the radial diffusivity equation to obtain the constant viscosity incompressible fluids radial diffusivity equation.

$$\rho_f = \frac{\rho_{sc}}{B_f}, \quad (35)$$

$$c_f = -\frac{1}{B_f} \frac{dB_f}{dp}, \quad (36)$$

and

$$c_r = \frac{1}{\phi} \frac{d\phi}{dp}. \quad (37)$$

B_f is the formation volume factor, c_f is fluid isothermal compressibility, and c_r is the rock isothermal compressibility. Thus, the radial diffusivity equation may be written as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r k_r \frac{\partial p}{\partial r} \right) = \mu \phi c_t \frac{\partial p}{\partial t}. \quad (38)$$

Where c_t is the system total compressibility

2.6.5 Variable Viscosity Compressible Fluids

As shown before, the radial diffusivity equation may be written as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{k}{\mu} \rho_f r \frac{\partial p}{\partial r} \right) = \frac{\partial (\phi \rho_f)}{\partial t}. \quad (39)$$

The variable viscosity compressible fluids equation derivation uses the following relationships in the radial diffusivity equation:

$$c_r = \frac{1}{\phi} \left(\frac{\partial \phi}{\partial p} \right)_T, \quad (40)$$

$$\beta_r = \frac{1}{\phi} \left(\frac{\partial \phi}{\partial T} \right)_p, \quad (41)$$

$$c_f = \frac{1}{\rho_f} \left(\frac{\partial \rho_f}{\partial p} \right)_T, \quad (42)$$

$$\beta_f = -\frac{1}{\rho_f} \left(\frac{\partial \rho_f}{\partial T} \right)_p, \quad (43)$$

and

$$v_f = \frac{\mu}{\rho_f}. \quad (44)$$

Where β_r and β_f are the rock and fluid isobaric compressibility, and v_f is the kinematic viscosity of the fluid.

Splitting the RHS of equation 3 with the chain rule, then utilizing the multi-variable chain rule, using equations 8 to 12, and assuming constant permeability with respect to radius the following derivatives can be obtained:

$$\frac{\partial p}{\partial t} = \frac{k}{\phi \mu (c_f + c_r)} \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right) - \frac{k}{\phi \mu (c_f + c_r)} \frac{1}{v_f} \frac{\partial v_f}{\partial T} \frac{\partial p}{\partial r} \frac{\partial T}{\partial r} + \frac{\beta_f - \beta_r}{c_f + c_r} \frac{\partial T}{\partial t}, \quad (45)$$

and

$$\frac{\partial T}{\partial t} = \frac{k}{\phi\mu(\beta_f - \beta_r)} \left(\frac{\partial^2 p}{\partial r^2} \right) + \frac{k}{\phi\mu(\beta_f - \beta_r)} \left(\frac{1}{r} - \frac{1}{v_f} \frac{\partial v_f}{\partial T} \frac{\partial T}{\partial r} \right) \frac{\partial p}{\partial r} - \frac{c_f + c_r}{\beta_f - \beta_r} \frac{\partial p}{\partial t}. \quad (46)$$

2.7 Combined Heat and Flow Model

Oil production can be estimated by considering the temperature change effect on oil viscosity and obtaining a 1D expression for the density of heat sources (Abernethy, 1976; Davletbaev et al., 2011):

$$q_o = 2\alpha_d J_b \frac{r_d}{r} e^{(-2\alpha_d(r-r_d))}. \quad (47)$$

Where α_d the damping factor of the microwave, J_b is the radiation intensity at a well bottom, and r_d is the radius of a microwave radiator.

Lamber's equation was used in other approaches to calculating energy absorption where reflection and interference were neglected. However, its application was limited in semi-infinite cases (Ayappa, 1970).

2.8 Pressure Transient Analysis for non-Isothermal Reservoirs

In the literature, non-isothermal pressure transient analysis is focused on three situations. The first is a moving-front dominated problem with the problem being dealt with as a composite reservoir (Benson, 1982; Benson & Bodvarsson, 1982). The second focuses on high-pressure non-Isothermal gas reservoirs (App, 2010; Bahonar, 2011a; Bahonar, 2011b), and the final one investigates high-pressured oil-gas-water reservoirs and puts a strong emphasis on Joule-Thomson's Expansion.

2.8.1 Injection Wells

Benson and Bodvarsson's (1982) work investigate a moving front after a single-rate injection of cold water into a hotter reservoir. Their work investigates different radii cold spots and outlines the differences in calculated skin for non-isothermal behaviour and apparent skin for isothermal behaviour. The main driver for variation is the viscosity changes due to temperature change which in turn causes flow resistance to change and hence pressure depletion near-wellbore to change. They have developed a specific procedure to analytically calculate skin for those wells by first finding the cold spot (injection front) radius. The hot spot begins at the time corresponding to the injection front radius. Then, they measured the slope, obtained the "thermal" skin factor due to the cold spot and pressure drop due to the hot spot, and then calculated the corrected skin factor. Figures 15-18 and Table 7 below show a summary of their results (Benson and Bodvarsson, 1982).

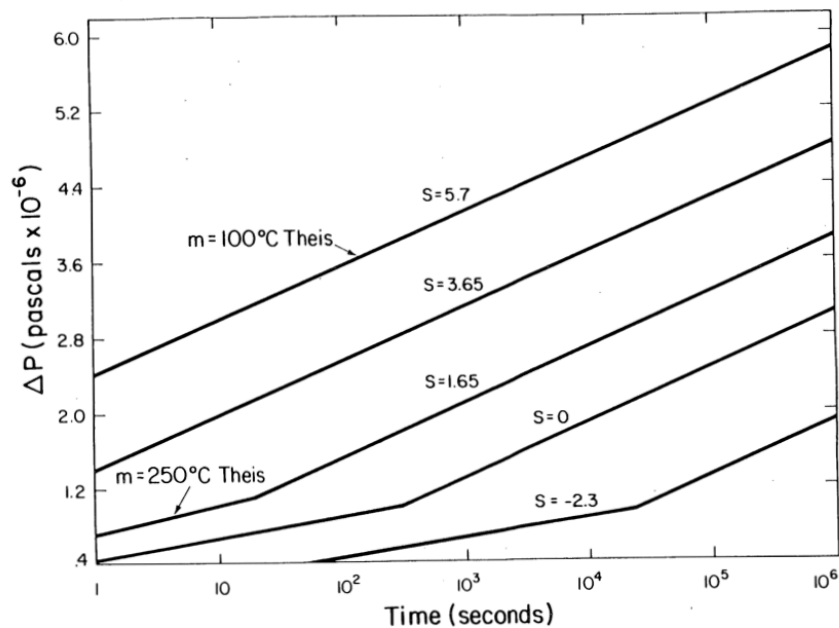


Figure 15, PTA for Cold Injection into a Hot Reservoir (Variable Skin)

Table 7, Skin Factor Analysis for Pressure Transient Data in Figure 16

Input Skin	Calculated Skin (Non-Isothermal)	Apparent Skin
5.7	5.6	3.0
3.6	3.6	0.9
1.6	1.6	-1.1
0.0	0.1	-2.7
-2.3	-2.3	-4.9

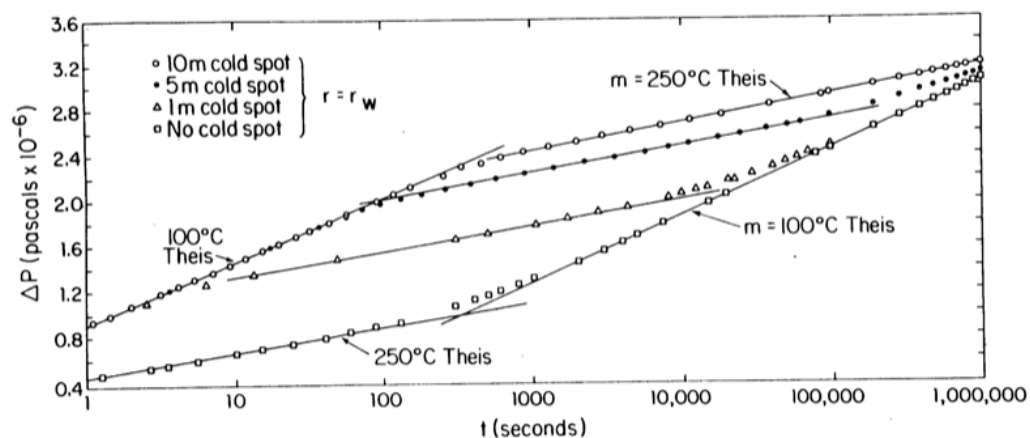
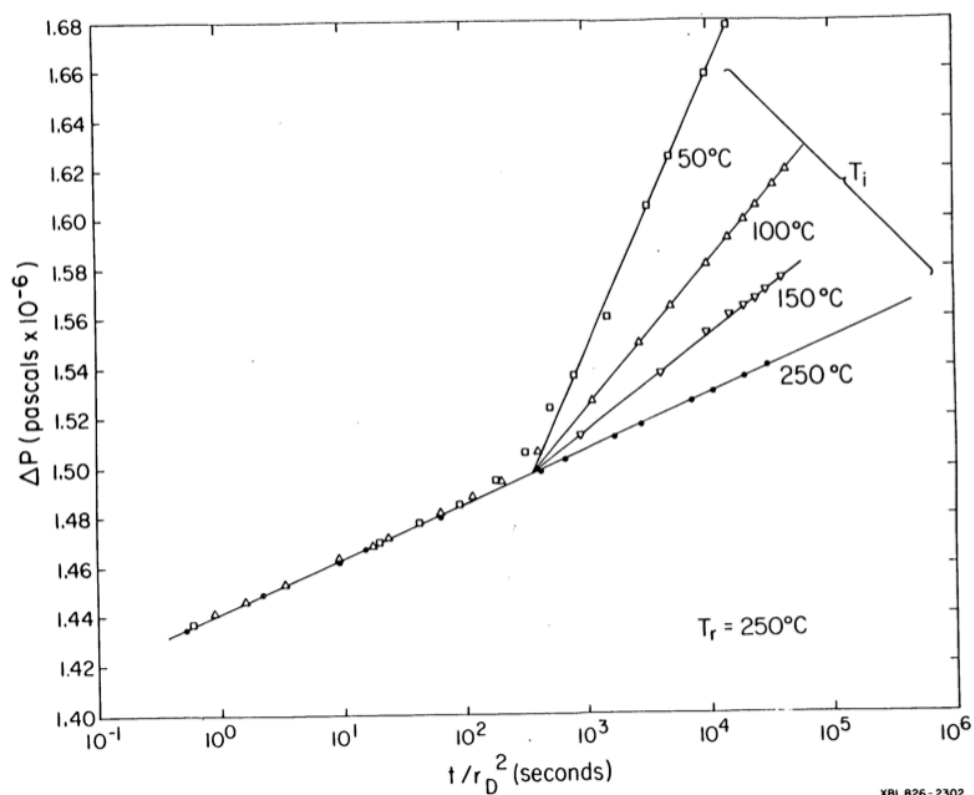
Figure 16, PTA for Injection of Cold Water into a Hot Reservoir (Variable r_{inj})

Figure 17, PTA for Injection of Cold Water into a Hot Reservoir

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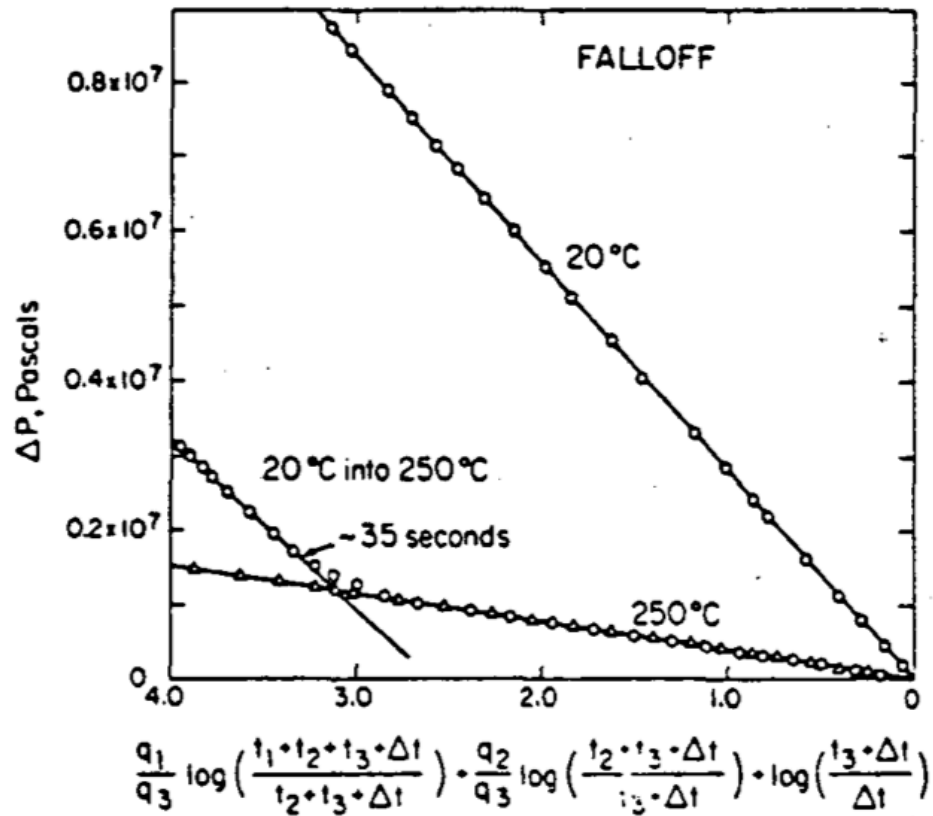


Figure 18, Stimulated Pressure Data for SRT Injection Test

2.8.2 High-Pressure Gas Reservoirs

Bahonar et al. (2011a, 2011b) developed a numerically fully implicit model utilized for non-isothermal wellbore/reservoir simulation that considers transient mass, momentum, and energy balance in the reservoir and wellbore, including the cement and tubular heat loss. Their results show the importance of modelling non-Isothermal behaviour, as shown in Figure 19-21 below for drawdown tests. Isothermal and non-isothermal refer to their numerical simulator, while well-testing-software is counted as the baseline analytical model.

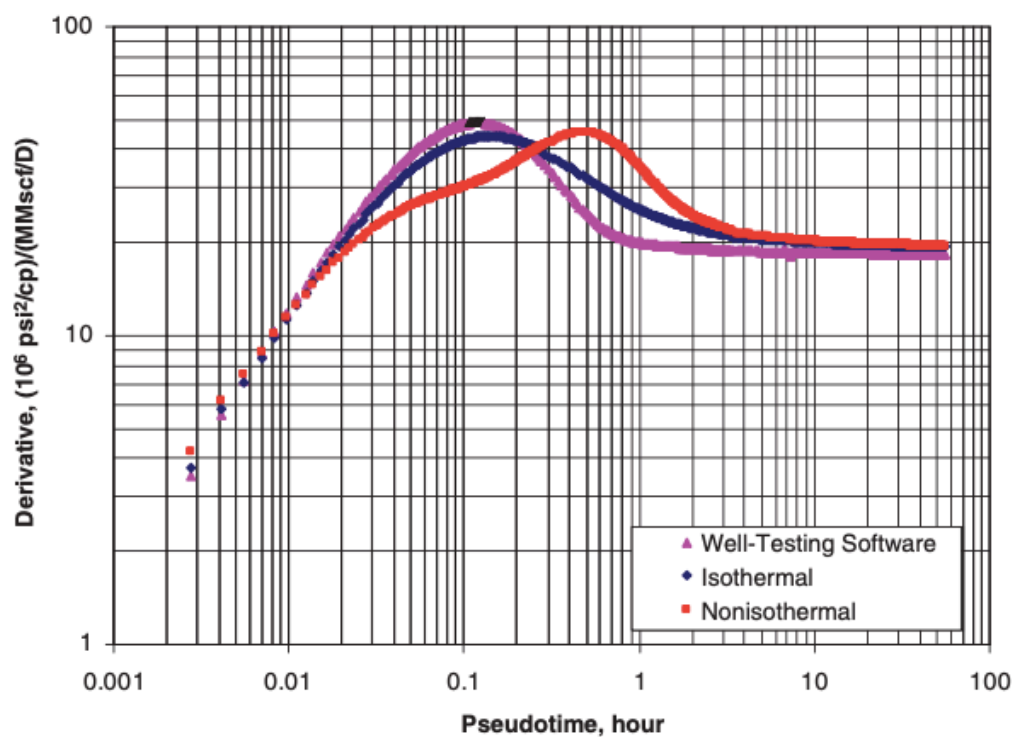


Figure 19, Comparison of Derivatives for Various Heat Models

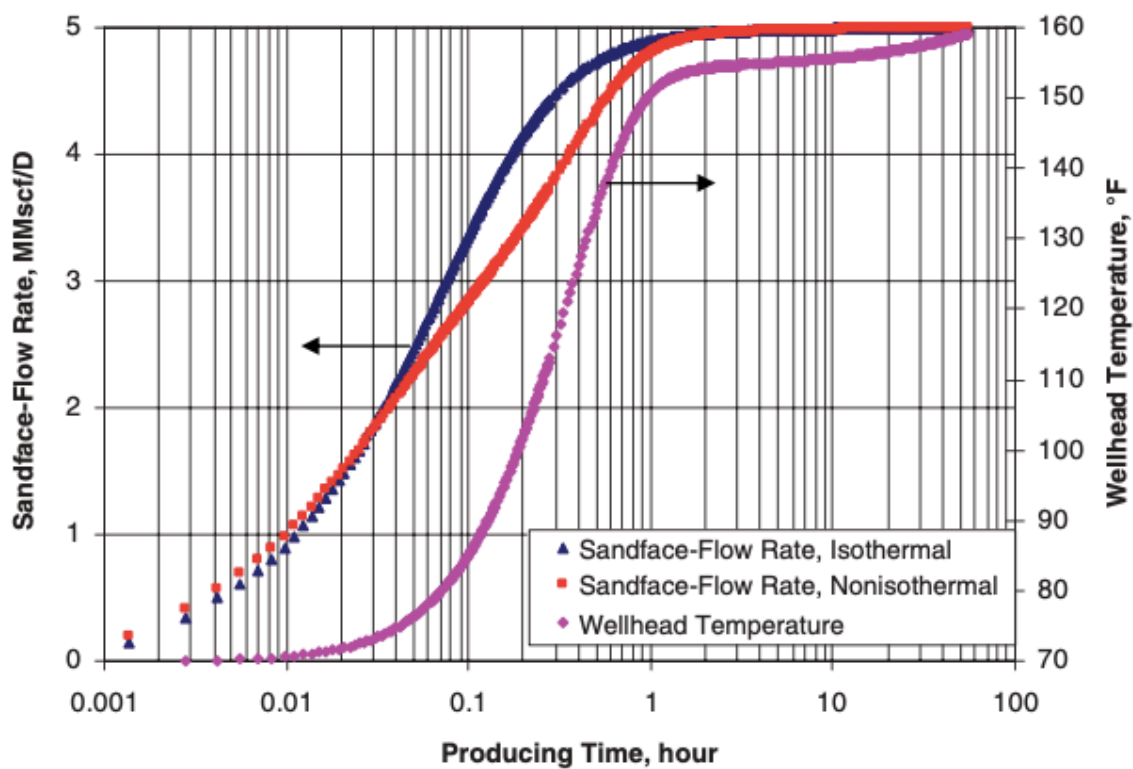


Figure 20, Comparison of Sand-face Flowrate Various Heat Models

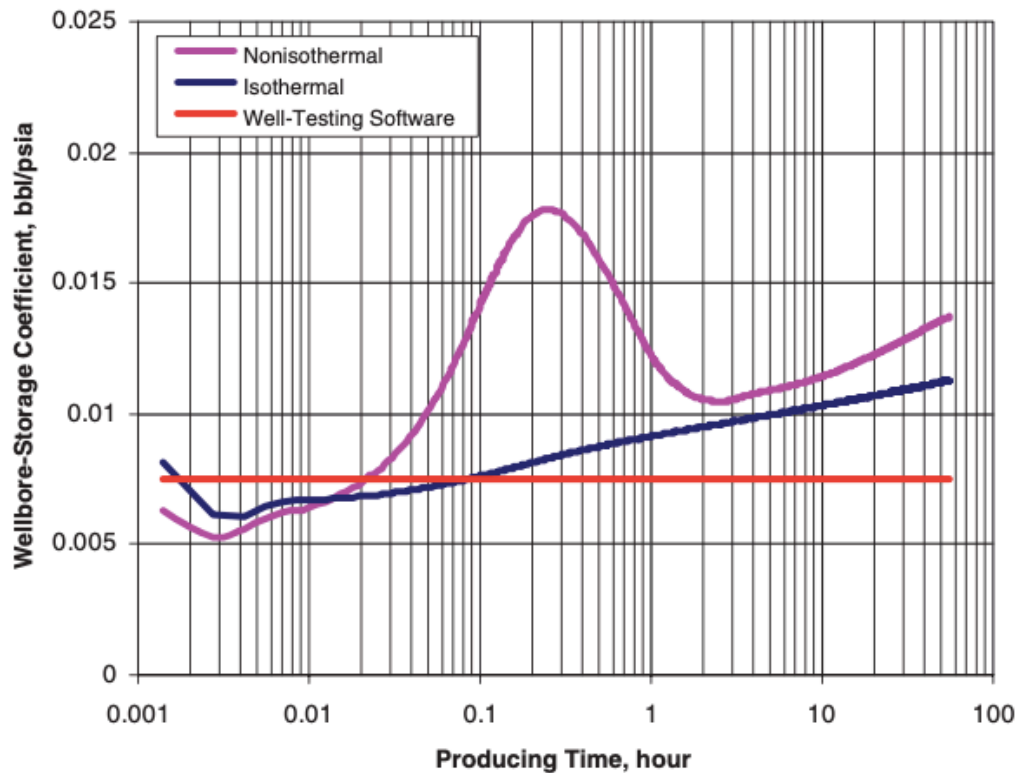


Figure 21, Comparison of Wellbore Storage Coefficient for Various Heat Models

2.8.3 High-Pressure Non-Isothermal Oil Reservoirs

App (2010) investigated the effect of non-isothermal behaviour on the Joule-Thomson Expansion on oil-gas-water reservoirs. Joule-Thomson expansion is a phenomenon in which pressure changes causes a temperature change; this is often described as “frictional heating”. This friction reduces pressure due to the temperature increase from Joule-Thomson fluid expansion. App used a numerical model and created synthetic well-tests to highlight the impact of non-is thermal behaviour on oil reservoirs.

App’s study concluded that the most significant increase in temperature due to Joule-Thomson expansion was in the near-wellbore region. Hence, erroneous results in initial reservoir temperature may be observed if it was based on flowing bottom hole temperature.

The productivity index is observed to be much higher due to high drawdown and increase in temperature. Rate change causes a heating/cooling effect because of the expansion or compression of reservoir fluids. Finally, that well-test analysis overestimates near wellbore skin and underestimates skin if changing viscosity is ignored due to pressure and temperature effects. A summary of the results is available in Figures 22-25 below.

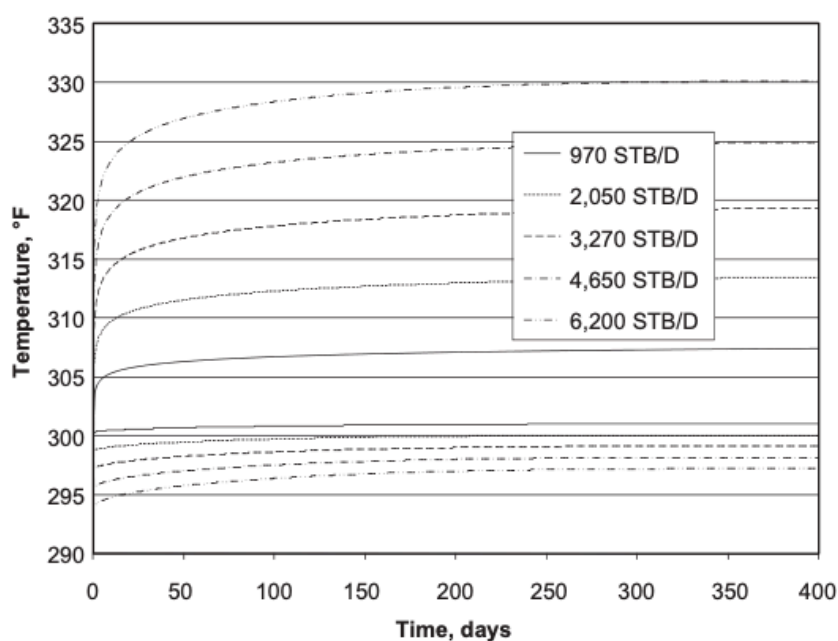


Figure 22, Wellbore Temperature Profile with and without Joule-Thomson Expansion

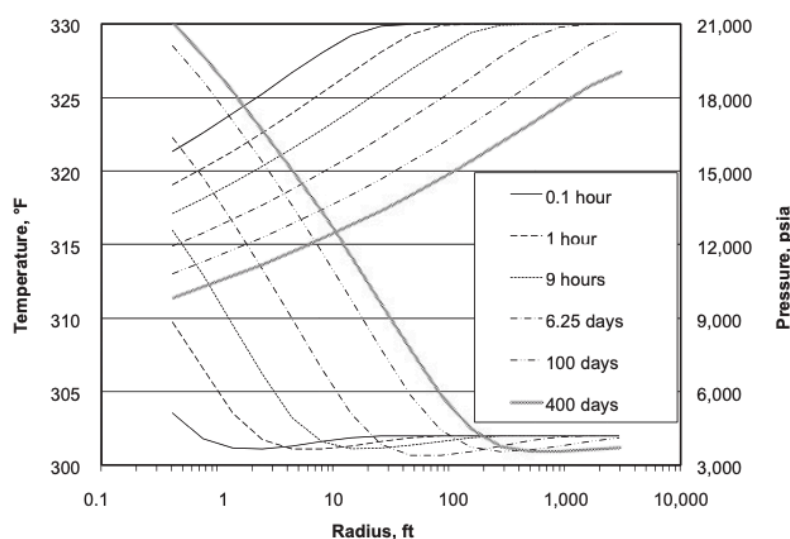


Figure 23, Evolution of Temperature Profile in The Reservoir for 6,200 BPD

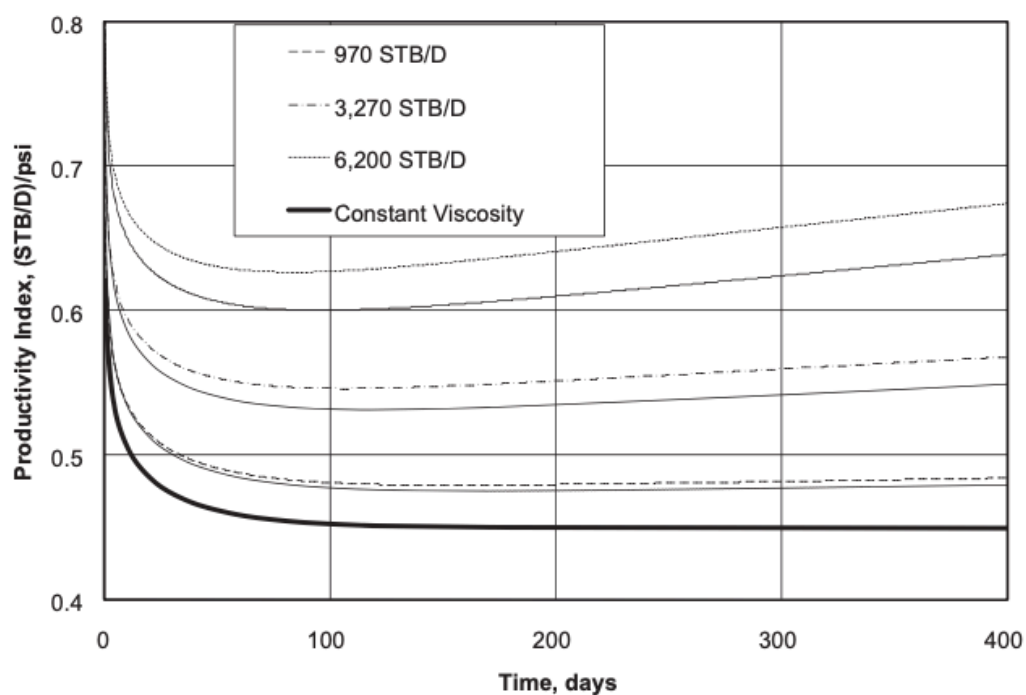


Figure 24, Effect of Productivity Index Caused by Pressure and Temperature Change

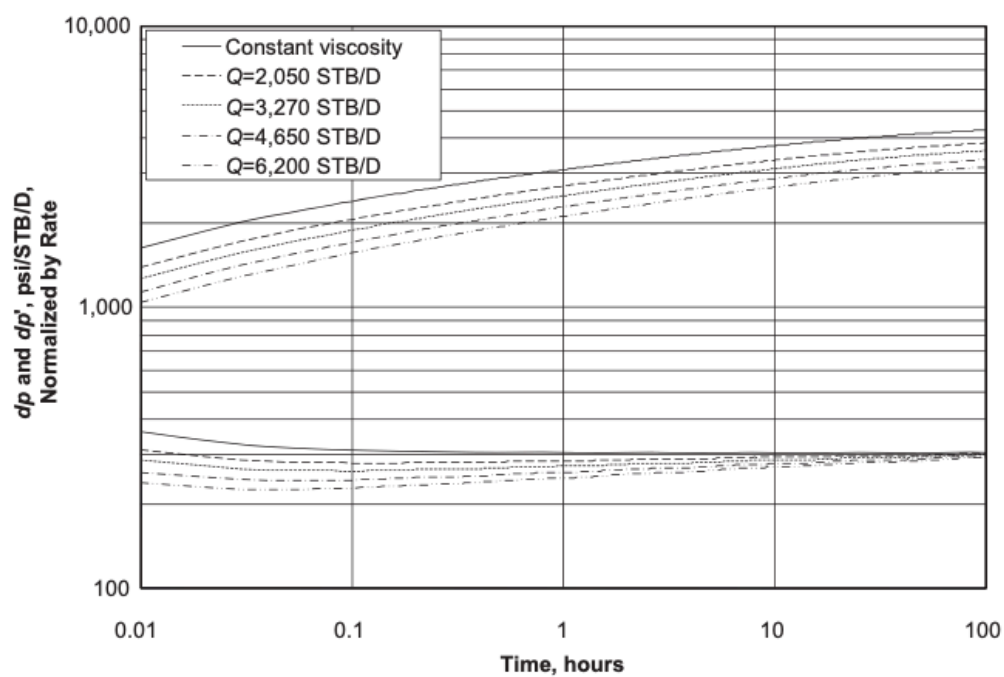


Figure 25, Effect of Viscosity the Pressure Derivative and Pressure Measurements

CHAPTER 3

METHODOLOGY

3.1 Assumptions

- Purely radial flow.
- Isotropic reservoir.
- Reservoir fluid consists of oil and water only.
- The reservoir is at irreducible water saturation.
- No phase change for water.
- Single phase flow (oil only).
- No heat loss to adjacent formations.
- This problem will be treated as a composite reservoir problem. Zone 1 is close to the wellbore, which is affected by Microwave heating, and Zone 2 is further away, which is not affected.

3.2 Boundary Conditions & Initial Conditions

The following boundary and initial conditions are used (Davletbaev et al., 2011; Bientinesi et al., 2013):

- 1) The reservoir is initially at a known temperature T_i
$$\Rightarrow T(r, t = 0) = T_i. \quad (48)$$

$$\Rightarrow \text{The reservoir is initially at a known pressure } p_i$$

$$\Rightarrow p(r, t = 0) = p_i. \quad (49)$$

Composite Reservoir

$$\Rightarrow T(r, t) = T_{avg}(t) \text{ for } r_w \leq r \leq r_{em}, \quad (50)$$

$$\Rightarrow p(r, t) = p_1(t) \text{ for } r_w \leq r \leq r_{em}, \quad (51)$$

$\Rightarrow$ and

$$\Rightarrow p(r, t) = p_2(t) \text{ for } r_{em} \leq r \leq r_e. \quad (52)$$

2) Constant heat loss at outer boundary

$$\Rightarrow \left\{ \kappa \frac{\partial T}{\partial r} \right\}_{r=r_e} = q_h T_{ext}. \quad (53)$$

3) At $r = r_w$, the heat flux is specified (IBC)

$$\Rightarrow \left\{ \kappa_T \frac{\partial T}{\partial r} \right\}_{r=r_w} - \frac{q}{2\pi r_w h} \rho c_p \Delta T = -\frac{P_0}{2\pi r_w h}. \quad (54)$$

4) The reservoir pressure is constant at the boundary

$$\Rightarrow p(r_e, t) = p_i, \quad (55)$$

$\Rightarrow$ or

$$\Rightarrow \left\{ \frac{\partial p}{\partial t} \right\}_{r=r_e} = 0. \quad (56)$$

3.3 Workflow

- 1) Energy balance is derived for the system, and critical energy balance transfer processes are considered (conduction, advection, and radiation) and then converted to Laplace domain.
- 2) Energy balance is solved in the Laplace domain in MATLAB to obtain T at different time steps.
- 3) T at different time steps is used to obtain viscosity.
- 4) Viscosity is substituted in the radial composite reservoir flow equation.
- 5) A flow rate/pressure profile is obtained.

- 6) The flow/pressure profile is used in PTA to determine reservoir properties such as
- a. Permeability
 - b. Skin factor
 - c. Wellbore storage
 - d. Radius of investigation
 - e. Pay Thickness

3.4 Heat Model Derivations

3.4.1 Obtaining Energy Balance for System

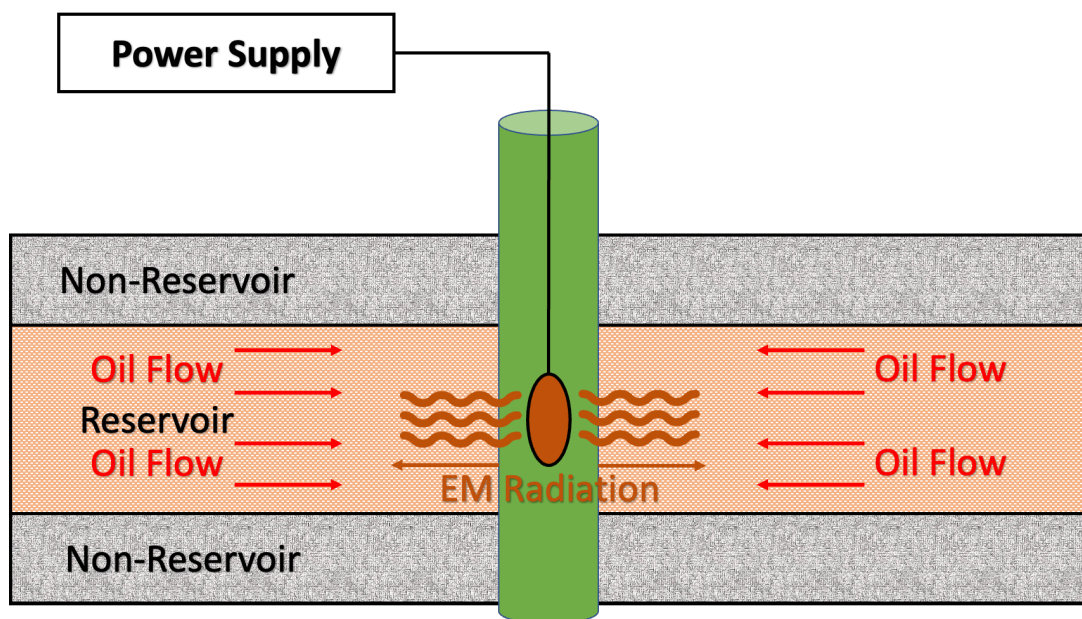


Figure 26, Sketch of the System

Figure 26 shows the overall system sketch with electromagnetic radiation into the reservoir and oil flow towards the wellbore (counter-current). We start with the energy balance

equation shown before in equation 1. An important note is that our convention is that flow towards the wellbore is negative, yielding a negative sign for the advection term. Thus, the energy balance equation can be written as follows:

$$\begin{aligned} \nabla \cdot [(1 - \phi)\tilde{\kappa}_s \cdot \nabla T_s + \phi\tilde{\kappa}_f \cdot \nabla T_f] - \nabla \cdot [\rho_f C_f T_f \vec{u}_f] + q_{em} \\ = \frac{\partial}{\partial t} \left((1 - \phi)\rho_s C_s T_s + \phi\rho_f C_f T_f \right), \end{aligned} \quad (57)$$

We can write equation 57 in one-dimensional form in cylindrical coordinates system, the detailed derivation of the process is found in Appendix E. Thus, equation 57 may be rewritten as:

$$\lambda \frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda}{r} - \frac{\dot{q}}{2\pi r h} \right) \frac{\partial T}{\partial r} + \dot{\alpha} \frac{e^{-\alpha(r-r_w)}}{r} = \frac{\partial T}{\partial t}, \quad (58)$$

where:

$$\lambda = \frac{\tilde{\kappa}_{eff}}{(\rho C_v)_{eff}}, \quad (59)$$

$$\dot{q} = 2\pi r h \dot{u}_f, \quad (60)$$

$$\dot{u}_f = \frac{\rho_f C_{pf} u_f}{(\rho C_v)_{eff}}, \quad (61)$$

and:

$$\dot{\alpha} = \frac{\alpha P_0}{2\pi h (\rho C_v)_{eff}}. \quad (62)$$

The boundary conditions and initial conditions used for the problem described above are shown in equations 48, 50, and 54.

3.4.2 Energy Balance Dimensionless Form

The energy balance equation is then converted to dimensionless form, the detailed derivations are available in Appendix F. We utilize the following dimensionless variables:

$$r_D = \frac{r}{r_w}, \quad (63)$$

$$q_D = \frac{\dot{q}}{2\pi h \lambda}, \quad (64)$$

$$t_D = \frac{\lambda t}{r_w^2}, \quad (65)$$

and:

$$T_D = \frac{T}{T_i}. \quad (66)$$

Then, the dimensionless energy balance equation is written as:

$$\frac{\partial^2 T_D}{\partial r_D^2} + \left(\frac{1}{r_D} [1 - q_D] \right) \frac{\partial T_D}{\partial r_D} + \frac{\dot{\alpha} r_w}{\lambda} \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D} = \frac{\partial T_D}{\partial t_D}. \quad (67)$$

To simplify the equation, using the following identity (Al-Niami & Rushton, 1978):

$$q_D = (2\varepsilon + 1), \quad (68)$$

We can rewrite the equation as:

$$\frac{\partial^2 T_D}{\partial r_D^2} + \left(\frac{-2\varepsilon}{r_D} \right) \frac{\partial T_D}{\partial r_D} + \frac{\dot{\alpha} r_w}{\lambda} \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D} = \frac{\partial T_D}{\partial t_D}. \quad (69)$$

The dimensionless forms of the initial and boundary conditions shown in equations 48, 50, and 54 are:

$$T_D(r_D, t_D = 0) = 1, \quad (70)$$

$$\left\{ \frac{\partial T_D}{\partial r_D} \right\}_{r_D=1} - q_D(T_D - 1) = -q_{hD_{int}}, \quad (71)$$

and:

$$\left(\frac{\partial T_D(r_D, t_D)}{\partial r_D} \right)_{r_D=r_{eD}} = q_{hT_{Dext}}, \quad (72)$$

where:

$$q_{hD_{int/ext}} = \frac{q_{hT_{int/ext}} r_w}{T_i K_{eff}}. \quad (73)$$

3.4.3 Analytical Solution

More details are shown in Appendix H regarding the derivation of the analytical solution, by taking the Laplace transform of equation 69 with respect to time we obtain:

$$\frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} - \frac{2\varepsilon}{r_D} \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} - z \hat{T}_D(r_D, z) + \dot{\beta} \frac{e^{-\alpha(r_w(r_D-1))}}{z r_D} + 1 = 0, \quad (74)$$

where z is the Laplace transform variable, and:

$$\dot{\beta} = \frac{\dot{\alpha} r_w}{\lambda T_i}. \quad (75)$$

We then convert initial condition and boundary conditions outlined in equations 70 to 72 to Laplace domain as:

$$\hat{T}_D(r_D, z(t_D = 0)) = \frac{1}{z}, \quad (76)$$

$$\left\{ \frac{\partial \hat{T}_D}{\partial r_D} \right\}_{r_D=1} - 2\eta \left(\hat{T}_D - \frac{1}{z} \right) = -\frac{q_{hD_{int}}}{z}, \quad (77)$$

and:

$$\left(\frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} \right)_{r_D=r_{eD}} = \frac{q_{hD_{ext}}}{z}. \quad (78)$$

Equation 74 has the general solution (Al-Niami & Rushton, 1978):

$$\hat{T}_D(r_D, z) = r_D^\eta [A I_\eta(\sqrt{z} r_D) + B K_\eta(\sqrt{z} r_D)], \quad (79)$$

where $\eta = \varepsilon + 0.5$, I & K are the modified Bessel functions of first kind and second kind respectively. The particular solution of equation 74 is given by:

$$\begin{aligned} \hat{T}_D(r_D, z) = & r_D^\eta [AI_\eta(\sqrt{z}r_D) + BK_\eta(\sqrt{z}r_D)] \\ & + r_D^\eta \left\{ \left[I_\eta(\sqrt{z}r_D) \int_1^{r_D} \frac{\zeta^{-\eta} \dot{\beta} e^{-\alpha r_w(\zeta-1)} \pi K_\eta(\sqrt{z}\zeta)}{2z} d\zeta \right] \right. \\ & \left. + \left[K_\eta(\sqrt{z}r_D) \int_1^{r_D} -\frac{\zeta^{-\eta} \dot{\beta} e^{-\alpha r_w(\zeta-1)} \pi I_\eta(\sqrt{z}\zeta)}{2z} d\zeta \right] \right\} + \frac{1}{z}. \end{aligned} \quad (80)$$

Equation 80 is computationally taxing, unstable, and too complicated to solve.

3.4.4 Semi-Analytical Solution

The full derivation of the numerical (semi-analytical) solution is available in Appendix H.

The semi-analytical discretization is done using the following terms for grids m to M:

$$\varepsilon + 0.5 = \eta, \quad (81)$$

$$x_D = \ln(r_D), \quad (82)$$

$$r_D = e^{x_D}, \quad (83)$$

and:

$$\tau_m = \ln\left(\frac{r_{Dm}}{r_{Dm-1}}\right). \quad (84)$$

The discretised form of the initial condition and boundary conditions shown in equations 76 to 78 are:

$$(\hat{T}_D)_{t_D=0} = \frac{1}{z}, \quad (85)$$

$$\omega \hat{T}_{wD} - \hat{T}_{D1} = \zeta, \quad (86)$$

and:

$$\tau_{m+\frac{1}{2}}(\hat{T}_{Dm+1} - \hat{T}_{Dm}) = \frac{qh_{D_{ext}}r_{eD}}{z}, \quad (87)$$

where:

$$\omega = [2\eta(r_{D1} - r_{wD}) + 1], \quad (88)$$

and:

$$\zeta = \frac{(2\eta + qh_{D_{int}})(r_{D1} - r_{wD})}{z}. \quad (89)$$

General Equation

By using equations 81 to 84 in equation in equation 74 we can obtain the general heat balance equation for all grids as:

$$\begin{aligned} -\tau_m \hat{T}_{Dm-1} + \left[\tau_{m+1} + \tau_m - (1 + 2\varepsilon) + \frac{z\Delta r_{Dm}^2}{2} \right] \hat{T}_{Dm} + [-\tau_{m+1} + (1 + 2\varepsilon)] \hat{T}_{Dm+1} \\ = \frac{\Delta r_{Dm}^2}{2} + \Delta r_{Dm} \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z}. \end{aligned} \quad (90)$$

All values are evaluated at the grid's centres. For example, if we were to evaluate grid 1 then the model evaluates at the centre of grid 1 and grid 2. Therefore, we need to adjust our model to consider inner boundary conditions at the wellbore and the outer boundary condition. Thus, we have discretised equations specifically for the first and final grid that considers the boundary conditions (equation 86 and 87 in equation 90) as:

$$\begin{aligned} \left[\left(1 - \frac{1}{\omega}\right) \tau_{wf} + \tau_{1\frac{1}{2}} - \eta + \frac{z}{2} \Delta r_{D1}^2 \right] \hat{T}_{D1} - \left[\tau_{1\frac{1}{2}} - \eta \right] \hat{T}_{D2} \\ = \left[\frac{\Delta r_{D1}^2}{2} + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} \Delta r_{D1} \right] + \frac{\tau_{wf} \zeta}{\omega}, \end{aligned} \quad (91)$$

and:

$$\begin{aligned}
& -\tau_{m-\frac{1}{2}}\hat{T}_{D_{m-1}} + \left[\tau_{m-\frac{1}{2}} + \frac{Z}{2}\Delta r_{D_m}^2 \right] \hat{T}_{D_m} \\
& = \left[\frac{\Delta r_{D_m}^2}{2} + \frac{\dot{\beta} e^{-\alpha r_w(r_{D_m}-1)}}{Z} \Delta r_{D_m} \right] + \frac{q_{hD_{ext}} r_{eD}}{Z} \left[1 - \frac{\eta}{\tau_{m+\frac{1}{2}}} \right]. \quad (92)
\end{aligned}$$

Semi-Analytical Solution (no Conduction)

No conduction cases are modelled by assuming $\kappa_{eff} \approx 0$.

3.5 Flow Model Derivation

The following equations describe a radial composite reservoir:

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \frac{\partial p_1}{\partial r} = \left(\frac{\phi c_t \mu}{k} \right)_1 \frac{\partial p_1}{\partial t} \quad \text{for } r_w \leq r \leq r_{em}, \quad (93)$$

And:

$$\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial p_2}{\partial r} = \left(\frac{\phi c_t \mu}{k} \right)_2 \frac{\partial p_2}{\partial t} \quad \text{for } r_{em} \leq r \leq r_e. \quad (94)$$

Where r_{em} = *Interface radius*. We use an initial condition of constant pressure, inner boundary condition of constant flux, an outer boundary condition of closed reservoir, equal flux, and pressure conditions at the interface (equations 55, 56, 95-97)

$$\left(\frac{dp_1}{dr} \right)_{r_w} = -1, \quad (95)$$

$$\left(\frac{dp_1}{dr} \right)_{r_{em}} = M \left(\frac{dp_2}{dr} \right)_{r_{em}}, \quad (96)$$

and:

$$p_1 = p_2 \text{ when } r = r_{em}. \quad (97)$$

We utilize following dimensionless variables to obtain the dimensionless form in addition to the previously defined dimensionless radius (equation 63):

$$r_{eD} = \frac{r_e}{r_w}, \quad (98)$$

$$R_D = \frac{r_{em}}{r_w}, \quad (99)$$

$$t_D = 0.000264 \frac{t}{r_w^2} \left(\frac{k}{\phi c_t \mu} \right)_1, \quad (100)$$

$$p_{D1} = \frac{0.00708 k_1 h(p_i - p_1)}{q \mu_1}, \quad (101)$$

and:

$$p_{D2} = \frac{0.00708 k_2 h(p_i - p_2)}{q \mu_2}. \quad (102)$$

The following variables are used for simplification purposes:

$$F = \frac{\left(\frac{k}{\phi \mu c_t} \right)_1}{\left(\frac{k}{\phi \mu c_t} \right)_2}, \quad (103)$$

and:

$$M = \frac{\left(\frac{k}{\mu} \right)_1}{\left(\frac{k}{\mu} \right)_2}. \quad (104)$$

Thus, we obtain the dimensionless form of the flow equations:

$$\frac{\partial^2 p_{D1}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{D1}}{\partial r_D} = \frac{\partial p_{D1}}{\partial t_D} \quad \text{for } 1 \leq r_D \leq R_D, \quad (105)$$

and:

$$\frac{\partial^2 p_{D2}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{D2}}{\partial r_D} = F \frac{\partial p_{D2}}{\partial t_D} \quad \text{for } R_D \leq r_D \leq r_{eD}, \quad (106)$$

Initial conditions:

$$p_{D1}(r_D, t_D = 0) = p_{D2}(r_D, t_D = 0) = 0. \quad (107)$$

Inner boundary condition:

$$\left(\frac{dp_{D1}}{dr_D}\right)_{r_D=1} = -1. \quad (108)$$

Outer boundary condition:

$$\left(\frac{dp_{D2}}{dr_D}\right)_{r_D=r_{eD}} = 0. \quad (109)$$

Interface conditions, equal flux, and equal pressure:

$$\left(\frac{dp_{D1}}{dr_D}\right)_{r_D=R_D} = M \left(\frac{dp_{D2}}{dr_D}\right)_{r_D=R_D}, \quad (110)$$

and:

$$p_{D1} = p_{D2} \text{ when } r_D = R_D. \quad (111)$$

The solution for equations 105 and 106 by using equations 107 to 111 is and LaPlace transform is:

$$\hat{p}_{D1} = C_1 I_0(r_D \sqrt{z}) + C_2 K_0(r_D \sqrt{z}) \quad (111)$$

$$\hat{p}_{D2} = C_3 I_0(r_D \sqrt{Fz}) + C_4 K_0(r_D \sqrt{Fz}) \quad (112)$$

The full derivation of the solution and the identifies for the constants are available in Appendix I. When accounting for wellbore storage and skin, we use:

$$\hat{p}_D = \frac{[z\hat{p}_D + s]}{z\{1 + C_D z[z\hat{p}_D + s]\}} \quad (113)$$

3.6 Input Data

Parameter	Unit	Value	Parameter	Unit	Value
T_i	K	323.15	r_w	ft	0.33
r_e	ft	1000	h	ft	50
C_s	$\frac{bbl}{psi}$	0.0004	B	-	1.05
c_t	$\frac{1}{psi}$	$7 * 10^{-6}$	ϕ	-	0.3
q_{sc}	$\frac{bbl}{day}$	-20	q_{hext}	W	-10^{-40}
P_0	W	5000	α	$\frac{1}{m}$	0.22
ρ_o	$\frac{kg}{m^3}$	1012	κ_o	$\frac{W}{m.K}$	0.12
C_{po}	$\frac{J}{kg.K}$	1850	ρ_s	$\frac{kg}{m^3}$	2640
κ_s	$\frac{W}{m.K}$	2.75	C_{ps}	$\frac{J}{kg.K}$	712
k	mD	2000	s	-	2.0
M	-	1000	N	-	270

CHAPTER 4

Validation

For validation, three different temperature curves are generated from the numerical model. Abernethy's analytical solution neglects conduction and uses only an outer boundary condition of constant temperature as the radius approaches infinity (infinite-acting reservoir). Hence, the internal boundary condition of constant heat flux is omitted. Furthermore, other sources in the literature also omit internal heat source as an inner boundary condition and use no temperature change in the inner boundary (Sources). Therefore, the three temperature curves generated are for the cases of 1) $q_{h_{int}} \neq 0, \kappa \neq 0$, which is our original model, 2) $q_{h_{int}} = 0, \kappa \neq 0$ to compare our results and the analytical solution with the conduction effect, 3) $q_{h_{int}} \neq 0, \kappa \approx 0$, and 4) $q_{h_{int}} = 0, \kappa \approx 0$. The results in the 3rd and 4th curves are generated to compare our solution without conduction to Abernethy's solution when inner-boundary heat flux is considered and omitted respectively.

4.1 No Flow

Figure 27 shows the various comparison curves for temperature profile with distance when the flow rate is set to 0, while Figure 28 compares wellbore temperatures against time. Abernethy's results perfectly overlap the numerical model with conductivity set to a minimal value almost equal to zero whether the inner heat flux is set to 0 or set to a constant value, as expected. The main difference between this numerical model and Abernethy's

solution is the neglect of conduction, the differential equation solved is one order in space and thus only one boundary condition was required (outer boundary condition). In the near wellbore region, Abernethy's results exceed the numerical model results with conduction. However, after $r \approx 0.6 \text{ ft}$, the numerical model results are higher than Abernethy's because thermal diffusivity is set to zero when neglecting the conduction term. All the heat stays within the near wellbore region and diffuses into the reservoir. Similarly, when setting heat-flux at the inner boundary to zero and considering conduction, the temperature barely increases from T_i as most heat diffuses into the reservoir.

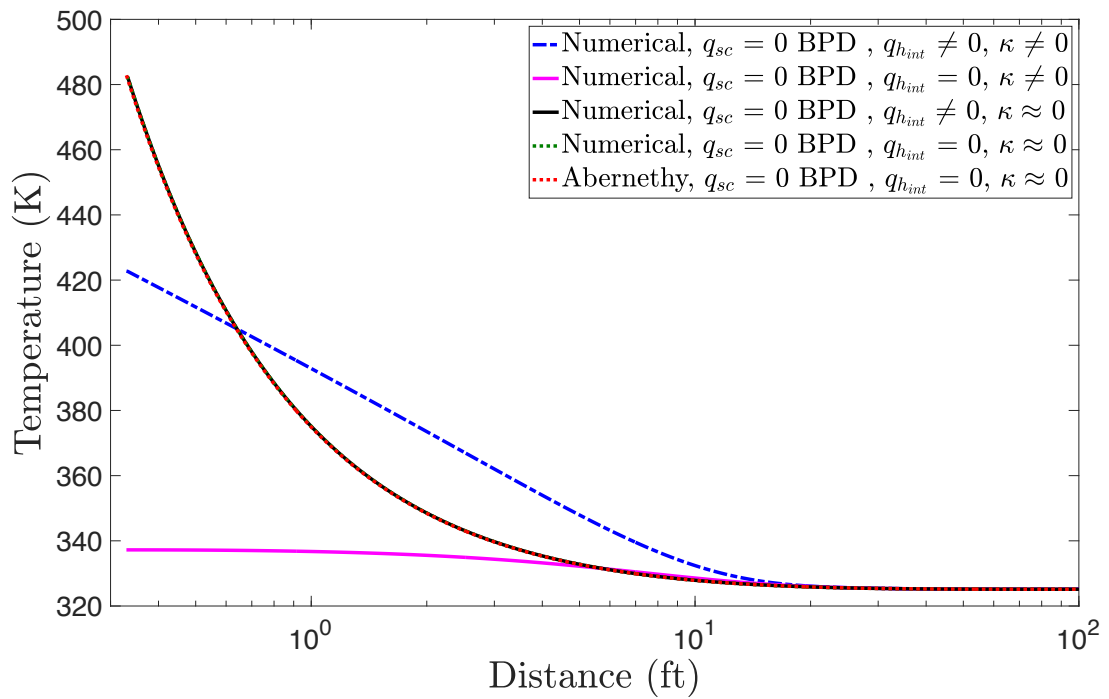


Figure 27, Radial Comparison of Temperature No-flow Results for 720 hours.

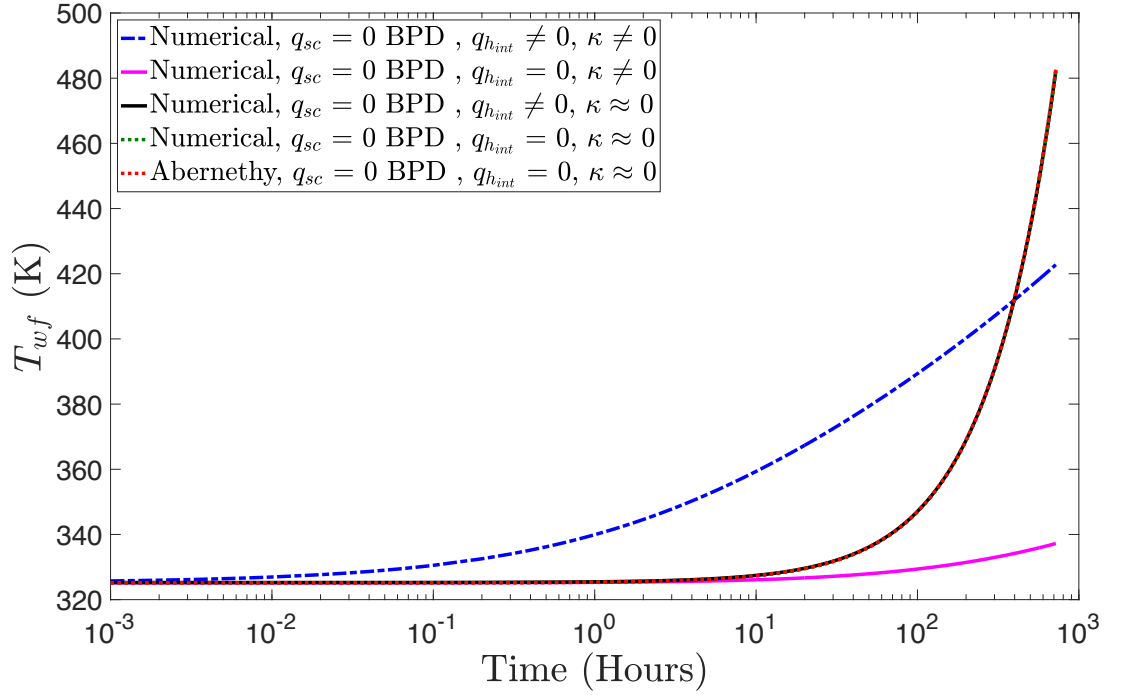


Figure 28, Time Comparison of No-flow Results at the Wellbore.

4.2 Constant Flow

Figure 29 shows the various comparison curves for the temperature profile with distance when the flow rate is set to -20 BPD, while Figure 30 shows wellbore temperature against time. Like the zero-flow rate case, Abernethy's results perfectly overlap the numerical model when conduction is approximately zero. The numerical model results with a constant heat flux inner boundary and conduction are higher than all other curves. This occurs because as oil flows counter-flow towards the wellbore and opposite to the electromagnetic radiations, it brings the diffused heat from further away to the regions closer to the wellbore. Similarly, when setting heat-flux at the inner boundary to zero and considering conduction, the temperature barely increases from T_i as most of the heat is diffusing away into the reservoir with the minimum difference between these results and Abernethy's ($\sim 10\text{K}$).

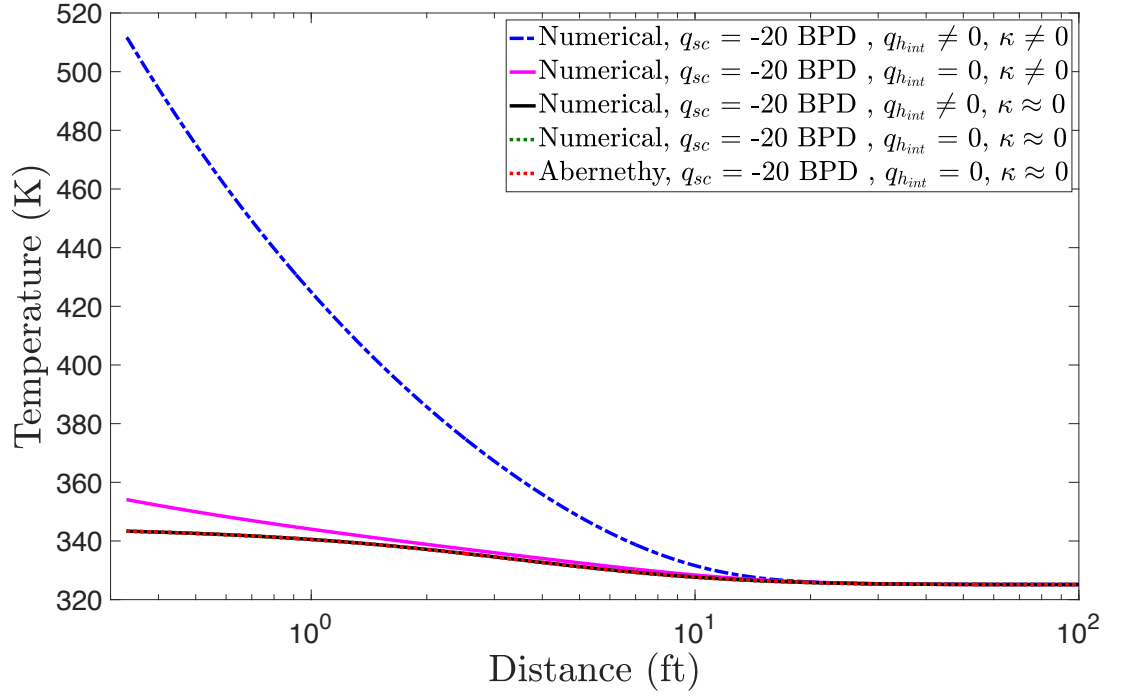


Figure 29, Radial Comparison of Temperature Constant-flow Results for 720 hours.

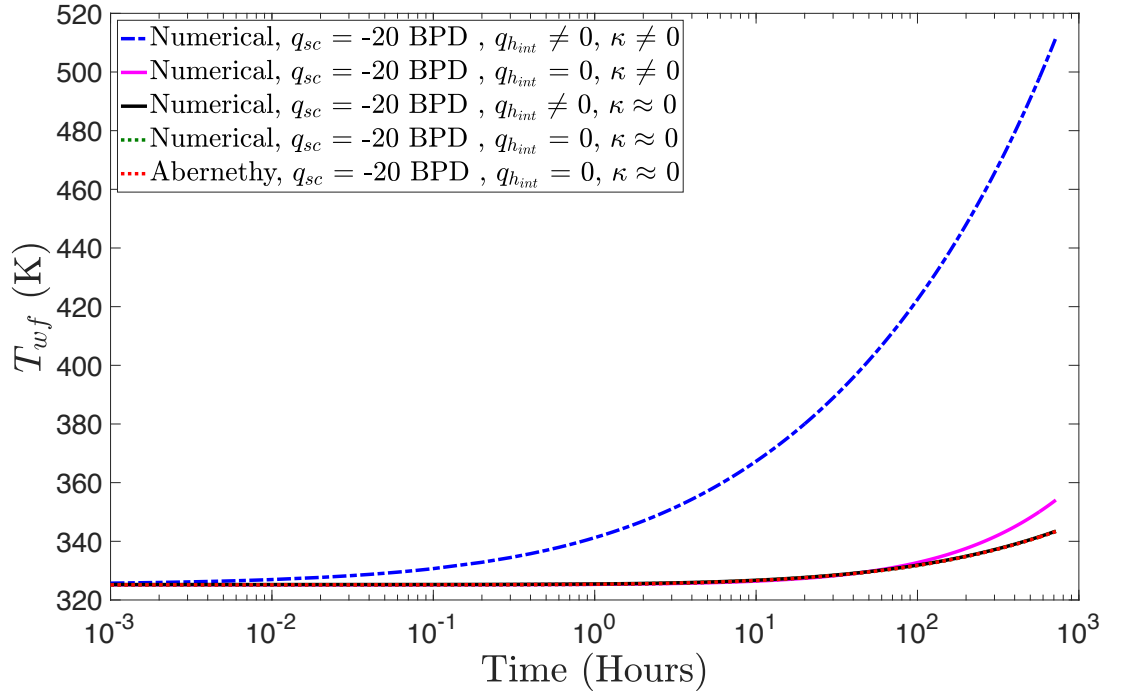


Figure 30, Time Comparison of Constant-flow Results at the Wellbore.

4.3 Discussion

The findings in this section indicate that this numerical model perfectly matches Abernethy's Analytical solution when conduction is neglected, no matter what inner boundary condition was used. This numerical model has an advantage over other models used in literature and indicates that conduction heat transfer should not be neglected due to its immense effect on the near wellbore region, whether a constant heat flux was used at the inner boundary or not. Furthermore, previous discussion experimental work (Bientinesi et al., 2013) shows that the constant heat flux at the wellbore must be considered. This numerical model is the first to obtain the transient state solution and does not neglect any heat transfer mechanisms.

CHAPTER 5

Sensitivity Analysis

5.1 Temperature Model

5.1.1 Temperature versus Time

The temperature curve increase with time is illustrated by a logarithmic growth function. Figure 31 to Figure 34 show the temperature profiles with time at multiple points radially from the wellbore centre in the reservoir such as the wellbore, 1', 5', and 10' away. The temperature difference decreases rapidly further away from the wellbore. As we can see and as expected, power and production rate are directly proportional to the temperature.

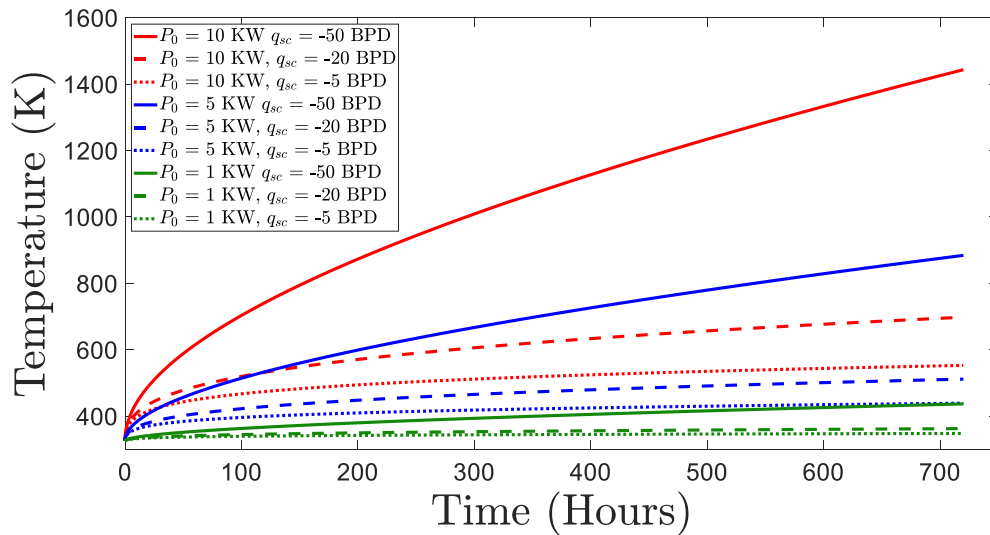


Figure 31, Temperature Profile vs Time at r_w

Figure 31 shows the temperature profile against time at the wellbore, in the case of 5 KW and -50BPD, it is observed that after around 120 hours the temperature curve exceeds the 10 KW and -5 BPD curve. In other words, we required 120 hours and ten times the flow rate to match the heating induced by twice the power

Figure 32 is the temperature against time for a radial distance of 1' from the center of the wellbore. We observe a similar behavior to the wellbore figure except that for 5KW and -20 BPD case the temperature matches the 10KW and -5BPD case later at 370 hours. As the region heated is further away from the wellbore, it takes longer for it to be heated.

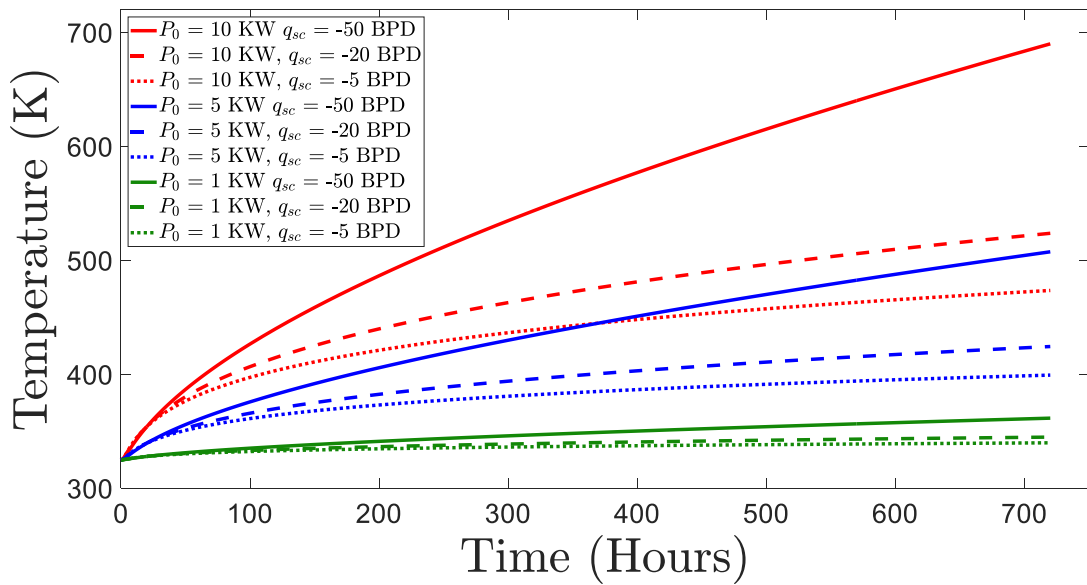


Figure 32, Temperature Profile vs Time at $r = 1'$

However, in Figure 33 we notice that the temperature profile at 5' is inverted, meaning that a higher production rate for the same wattage has less temperature. This is because for higher flow rate cases the heat is being conducted from regions further away in the reservoir to the wellbore and being produced. Thus, we notice a higher temperature difference in near regions compared to regions further away. This also suggests there is a certain distance at

which the temperature profiles intersect with each other which will be shown in the next section.

The temperature profile at 10' in Figure 34 follows the identical trend to the temperature profile at 5' in Figure 33 except that the temperature range is lower which is expected as the region is much farther away.

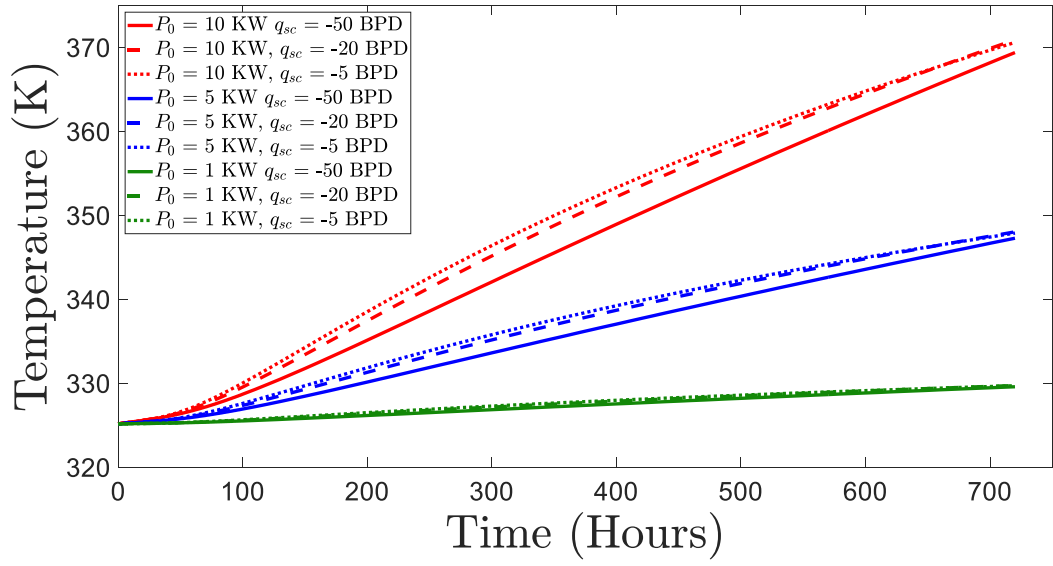


Figure 33, Temperature Profile vs Time at $r = 5'$

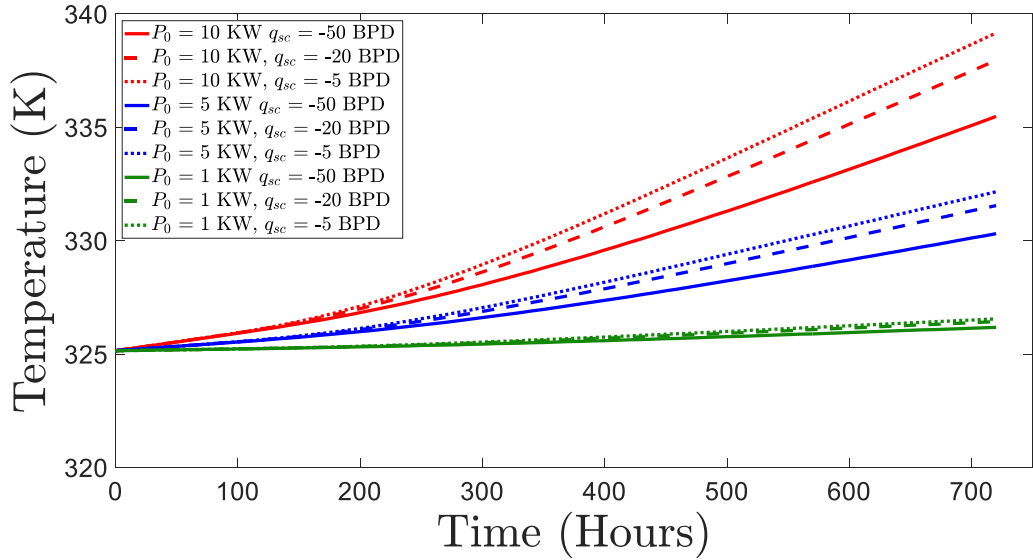


Figure 34, Temperature Profile vs Time at $r = 10'$

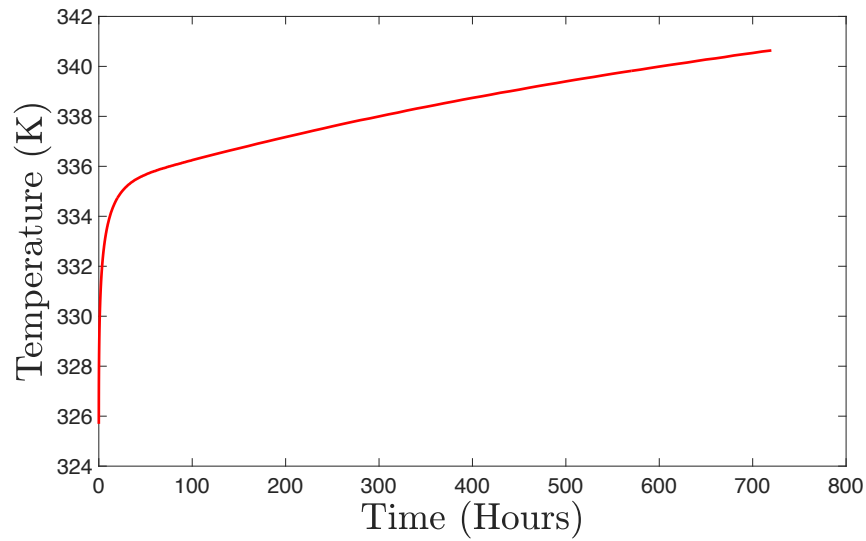


Figure 35, Average temperature of the Heated Region vs Time

The average radial-thickness weighted temperature is shown above in Figure 35. The average temperature sharply increases in early time and then follows a logarithmic growth trend. The average temperature is subject to the averaging criterion used. In the case above, T_{avg} is computed for the region with $\Delta T \geq 0.5$ K. The higher the averaging criterion the higher the temperature but the lower the computed heated region would appear to be.

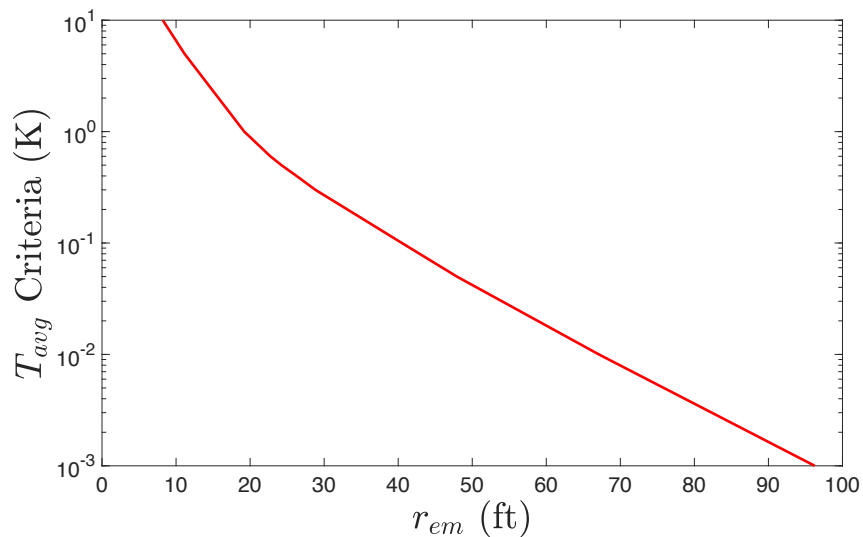


Figure 36, Averaging Criterion vs Heated Region Radius

Figure 36 shows the relationship between the temperature averaging criteria and the radius of the electromagnetic heating zone. As expected, a lower averaging criterion would cause a higher thickness of the heated zone extending from r_w to r_{em} . Thus, in turn, affecting the results of the model.

5.1.2 Temperature versus Distance

Temperature profiles against distance are generated for multiple periods below in Figure 37 to Figure 43 for 1, 10, 100, 240, 480, and 720 hours. The profiles follow an exponential decay trend with a higher temperature closer to the wellbore as expected. All the curves have two intersection points extremely close to each other at the same power and varying flow rates, for all radii less than r_x the temperature profiles for -50 BPD are higher than -20 BPD higher than -5 BPD. For all distances greater than r_x the temperature profiles are higher for -5 BPD than -20BPD than -50 BPD. In other words, after r_x the temperature profiles for the same power are flipped.

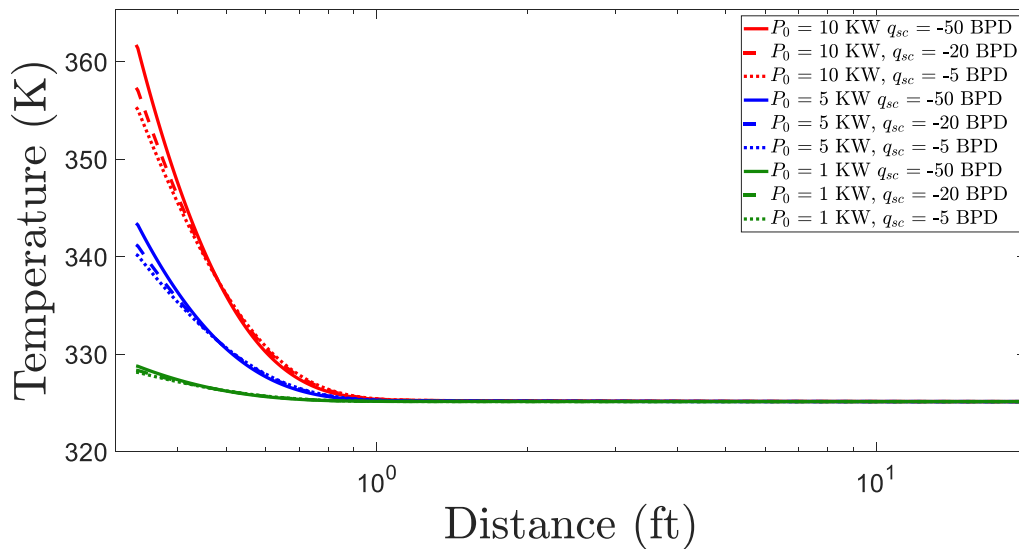


Figure 37, Temperature Profile vs Distance at 1 Hour

Figure 37 shows the temperature profile against distance for $t = 1$ hour. As expected, higher power and higher flow rate induces higher temperature difference. The intersection points r_x here are between 0.4 and 0.5 ft for all cases.

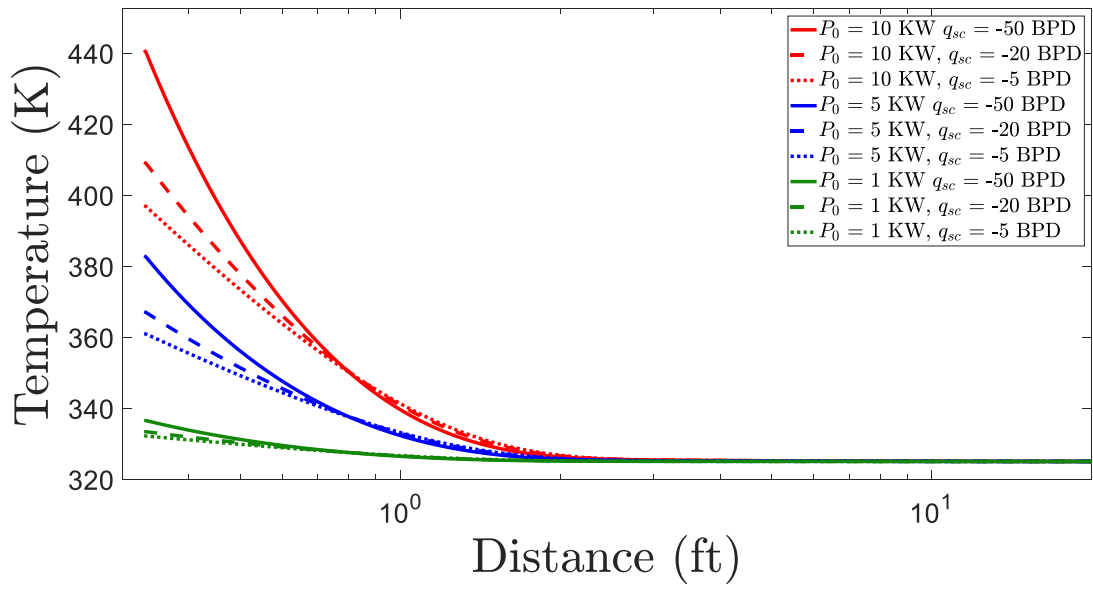


Figure 38, Temperature Profile vs Distance at 10 hours

Figure 38 shows the temperature profile against distance for $t = 10$ hour. The trend is like figure xx but with higher temperatures. The intersection points r_x here are between 0.7 and 0.8 ft for all cases.

Figure 39 shows the temperature profile against distance for $t = 100$ hour. The trend here is different than the previous two figures. The 5KW,-20BPD case temperature is higher than 10KW,-5BPD case for the region between r_w and $r \cong 0.56 ft$. Furthermore, the intersection points r_x here are between 1 and 2 ft for all cases.

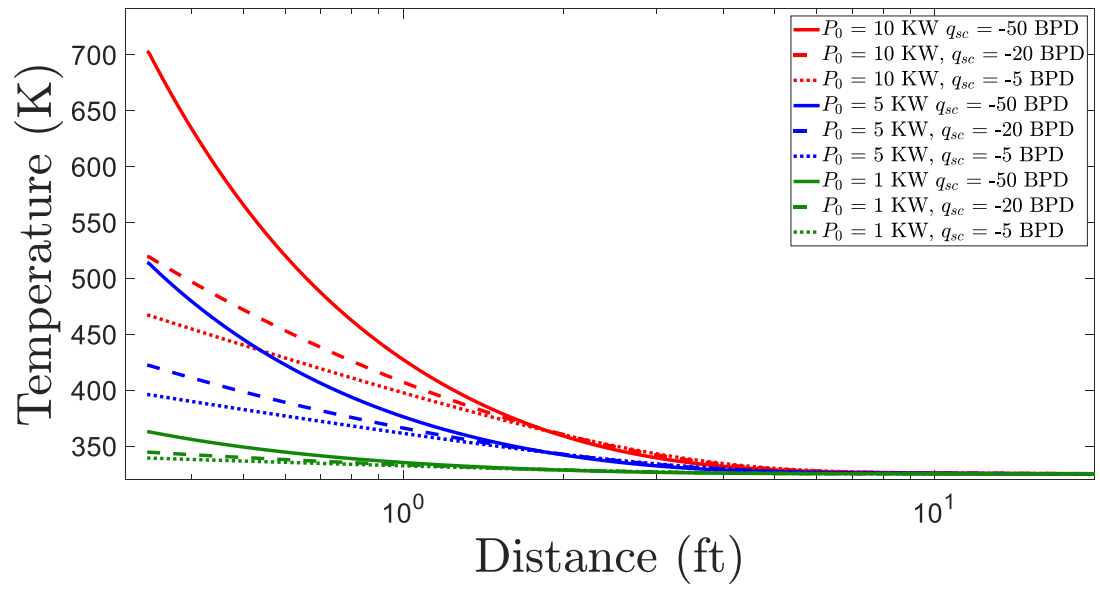


Figure 39, Temperature Profile vs Distance at 100 Hours

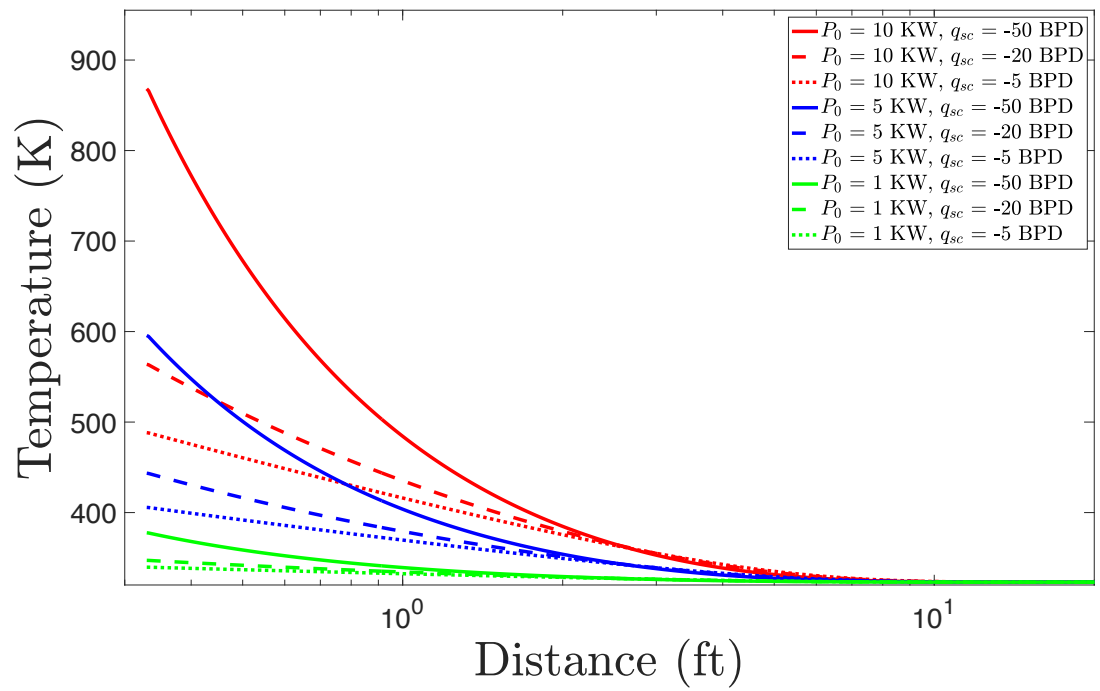


Figure 40, Temperature Profile vs Distance at 240 Hours

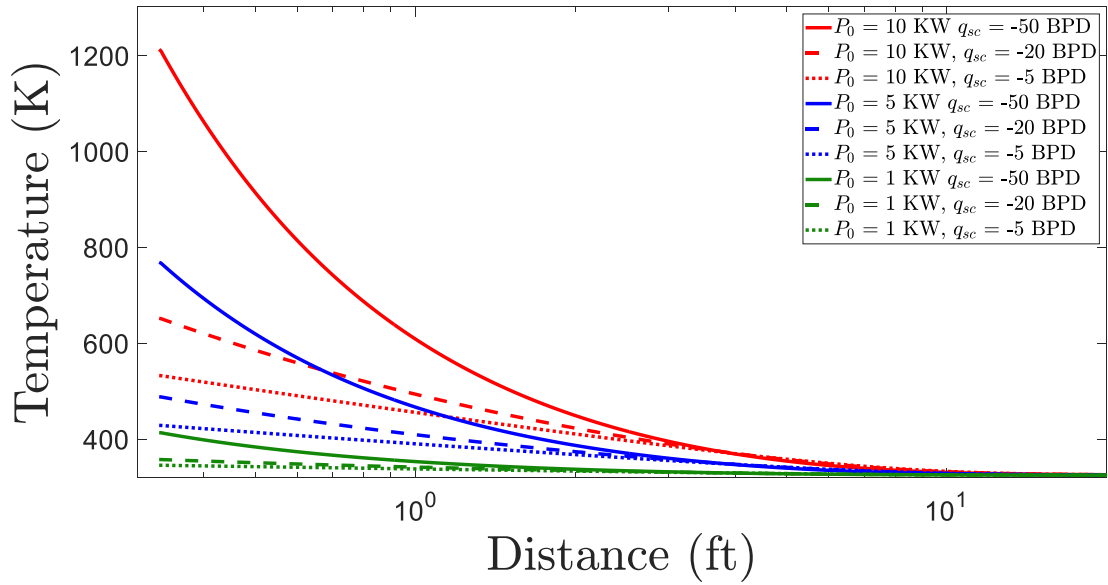


Figure 41, Temperature Profile vs Distance at 480 Hours

Figure 40 shows the temperature profile against distance for $t = 240$ hour. The trend here is like the previous two figures. The 5KW, -20BPD case temperature is higher than 10KW, -5BPD & 10KW,-20BPD. The heating induced by conduction-convection is significant for the region between r_w and $r \cong 0.8$ ft. Furthermore, the intersection points r_x here lay further between 2 and 3 ft for all cases.

Figure 41 shows the temperature profile against distance for $t = 480$ hour. The trend here is like the previous two figures. The heating at the wellbore of 1KW, -50 BPD case almost matches the heating induced by 5KW, -5BPD case. Furthermore, the intersection points r_x here lay further between 3 and 4 ft for all cases.

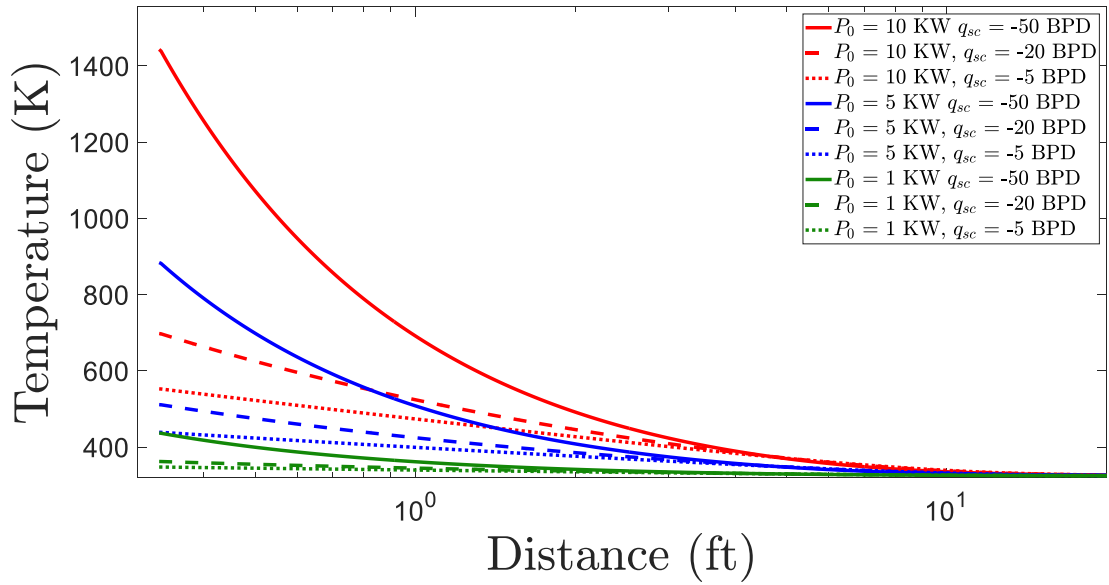


Figure 42, Temperature Profile vs Distance at 720 Hours

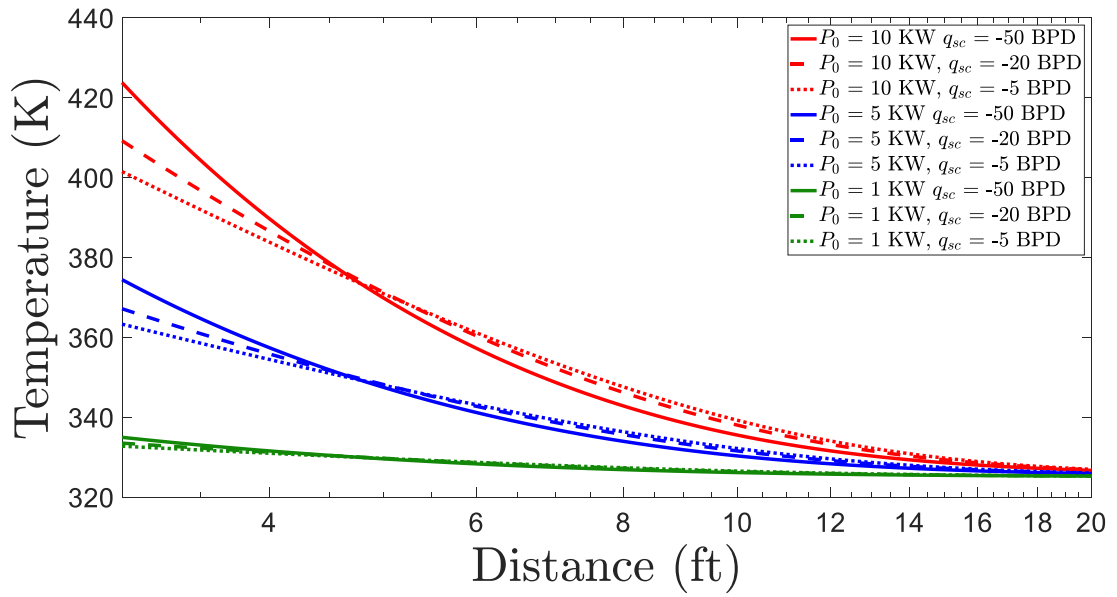


Figure 43, Temperature Profile vs Distance at 720 Hours between 3 and 20 Feet.

Figure 42 shows the temperature profile against distance for $t = 720$ hour. The trend here is like the previous two figures. The heating at the wellbore of 1KW, -50 BPD case finally matches the heating induced by 5KW, -5BPD case. Furthermore, the intersection points r_x here lay further between 4 and 5 ft for all cases. The zoomed in plot is shown in Figure 43.

5.2 Pressure Model

5.2.1 Effect of Power

Figure 44 to Figure 46 below contain the dimensionless field plots for varying flow rates at a constant power. The dotted lines are the derivative curves while the solid lines are dimensionless pressure curves. As both power and flow-rate increases, the faster that we observe a boundary effect to higher mobility and diffusivity induced by higher temperature in the heated region. Furthermore, higher flow rate induces a higher draw-down which in turn induces higher drawdown.

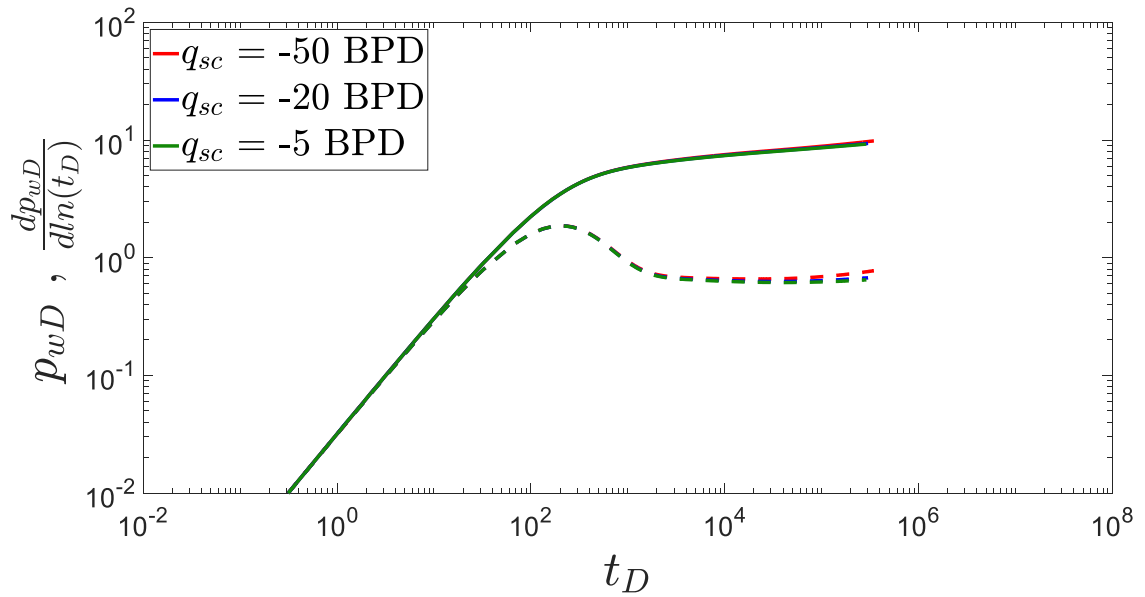


Figure 44, Dimensionless Field Plot for 1 KW

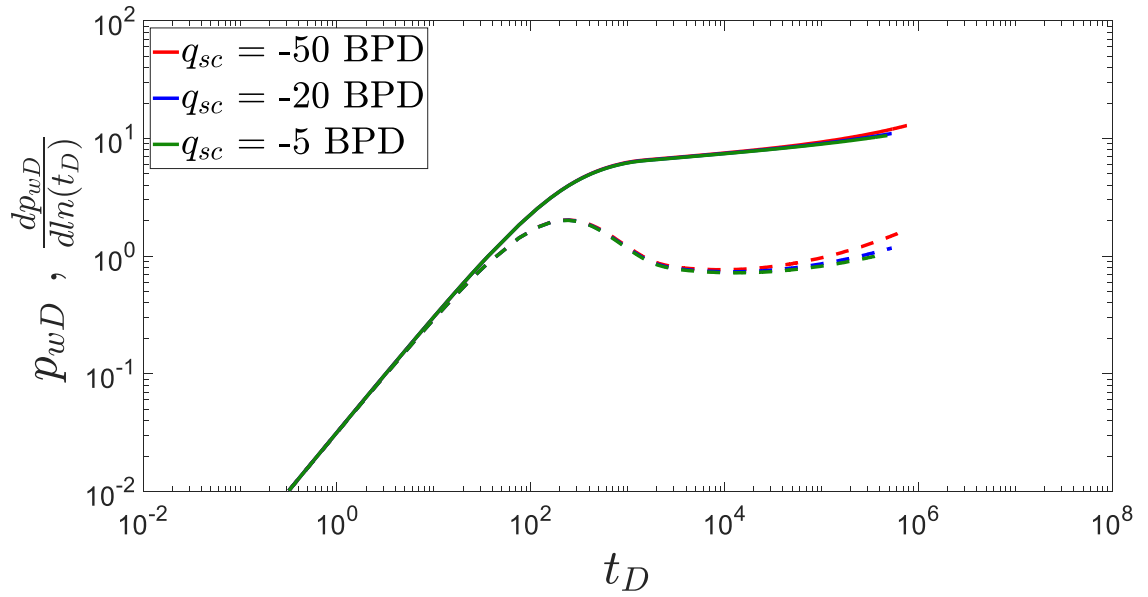


Figure 45, Dimensionless Field Plot for 5 KW

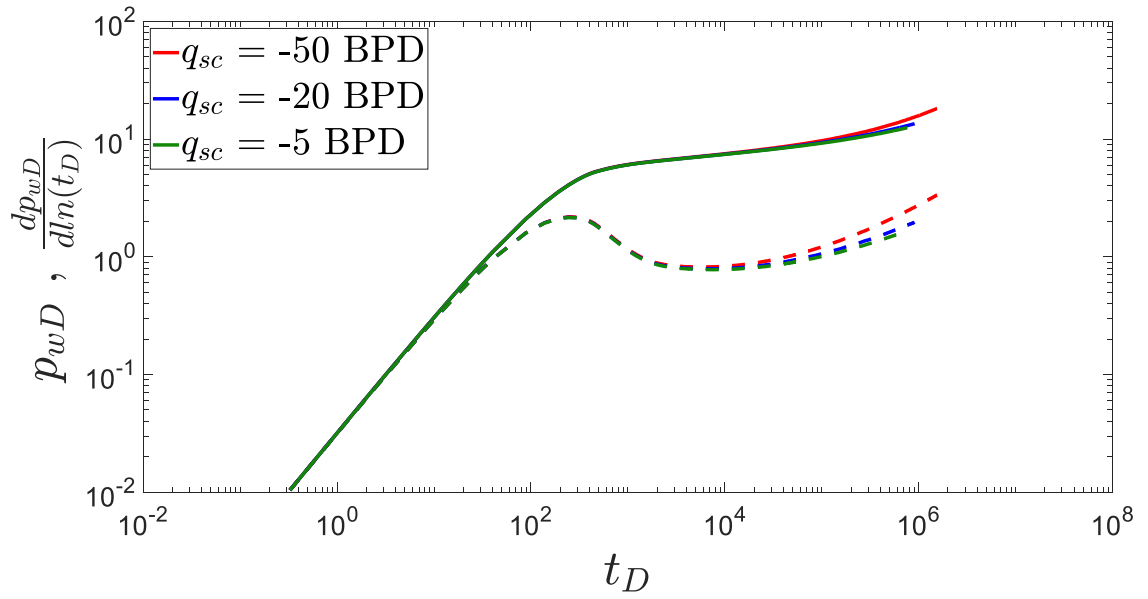


Figure 46, Dimensionless Field Plot for 10 KW

5.2.2 Effect of Flowrate

Figure 47 to Figure 49 below contain the dimensionless field plots for varying power at a constant flowrate. The dotted lines are the derivative curves while the solid lines are

dimensionless pressure curves. As both power and flow-rate increases, the faster that we observe a boundary effect to higher mobility and diffusivity induced by higher temperature in the heated region. Furthermore, higher flow rate induces a higher draw-down which in turn induces higher drawdown.

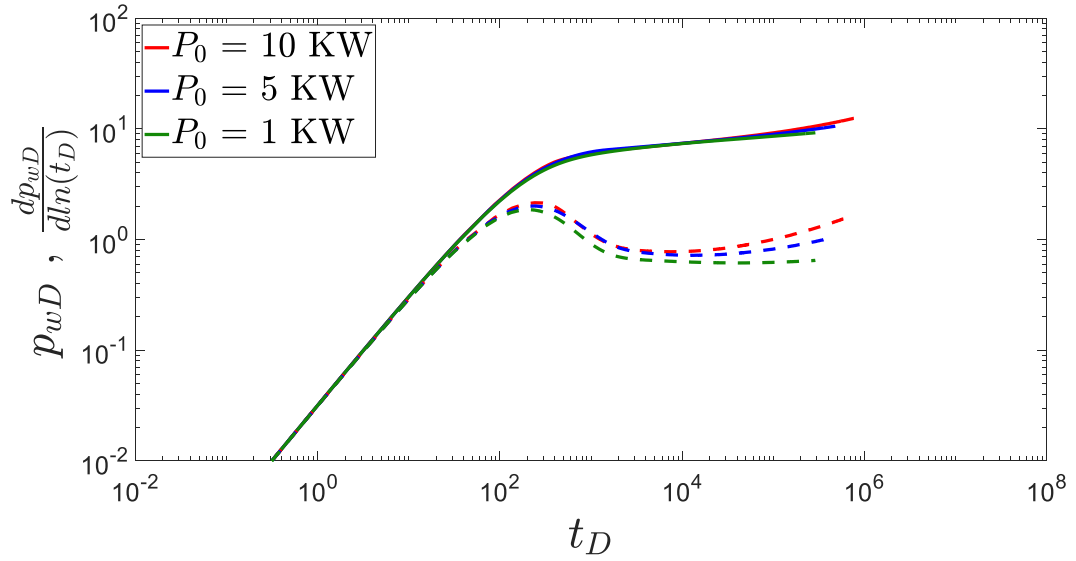


Figure 47, Dimensionless Field Plot for -5 BPD

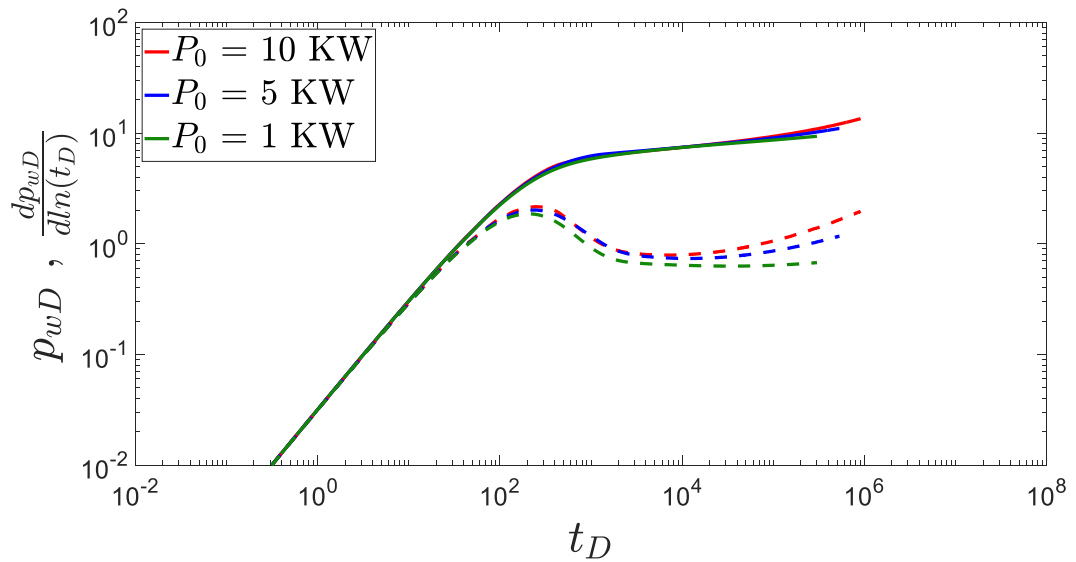


Figure 48, Dimensionless Field Plot for -20 BPD

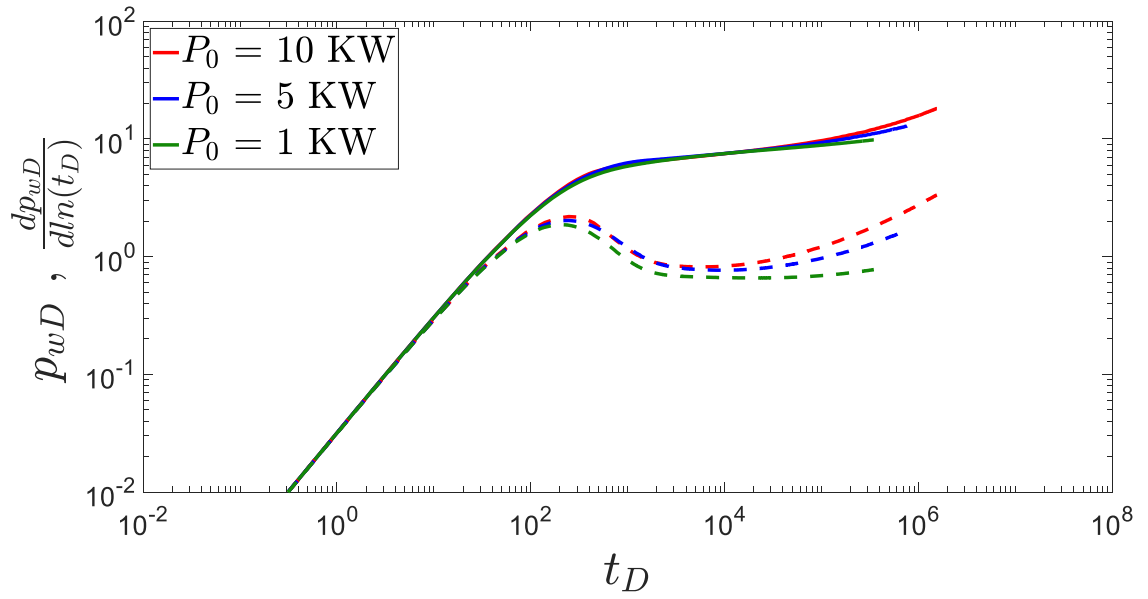


Figure 49, Dimensionless Field Plot for -50 BPD

5.2.3 Effect of Effective Radius

Below Figure 50 shows the effect of the external boundary radius on the dimensionless field plot for the case of 5 KW and -20 BPD. As the reservoir becomes smaller, the heating caused by electromagnetic waves is distributed over a smaller region causing a higher temperature to increase and thus increasing mobility further and inducing more drawdown. Furthermore, the smaller the reservoir the earlier we can observe the boundary response such in the case of $r_e = 75'$.

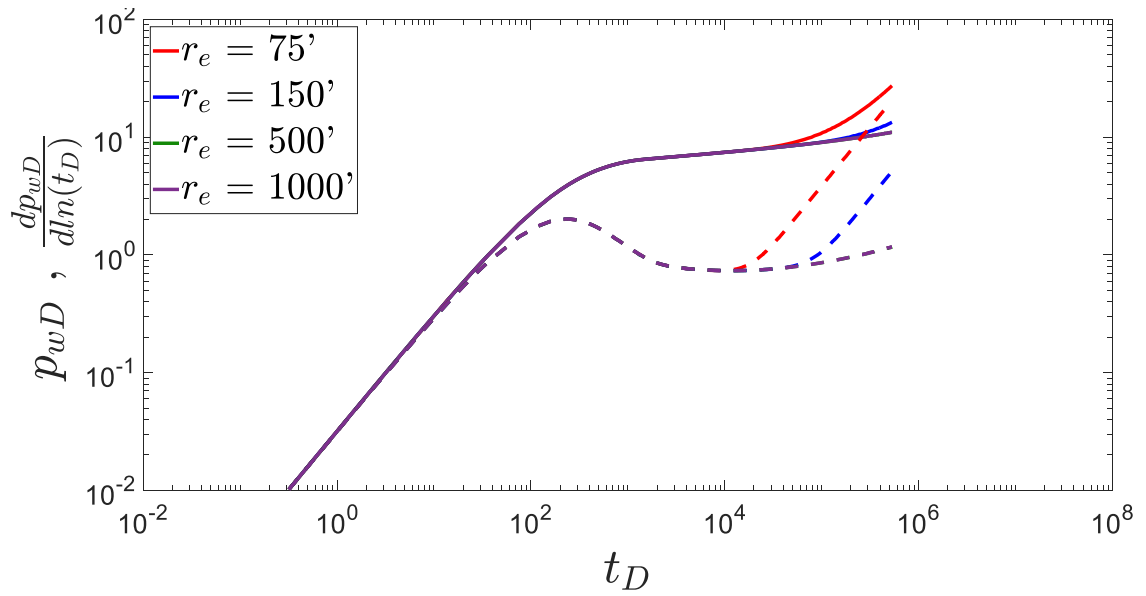


Figure 50, Dimensionless Field Plot with varying r_e

5.2.4 Effect of Pay Thickness

Below Figure 51 shows the effect of the reservoir thickness on the dimensionless field plot for the case of 5 KW and -20 BPD. As the thickness becomes smaller, the heating caused by electromagnetic waves is distributed over a smaller region causing a higher temperature to increase and thus increasing mobility further and inducing more drawdown. Furthermore, the smaller the reservoir the earlier we can observe the boundary response. It is important to note that dimensionless time ranges are different as temperature is different for each case thus μ differs which is used in dimensionless time derivation.

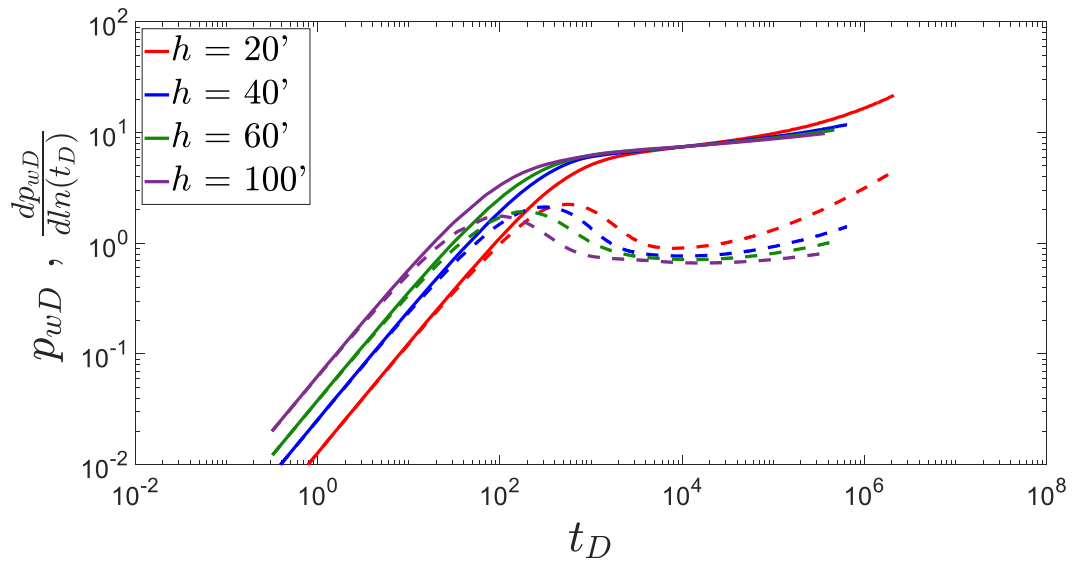


Figure 51, Dimensionless Field Plot with Varying h

5.2.5 Effect of Skin

Below Figure 52 shows the effect of skin on the dimensionless field plot for the case of 5 KW and -20 BPD. Skin does not affect our temperature model. Lower skin allows the effect of wellbore storage to be more evident.

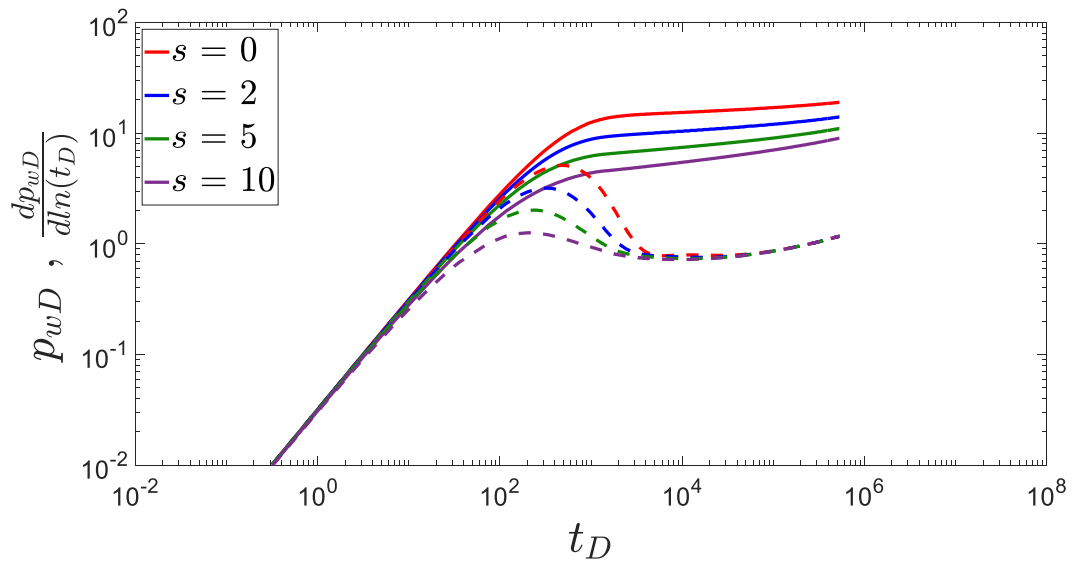


Figure 52, Dimensionless Field Plot with Varying s

5.2.6 Effect of Initial Temperature

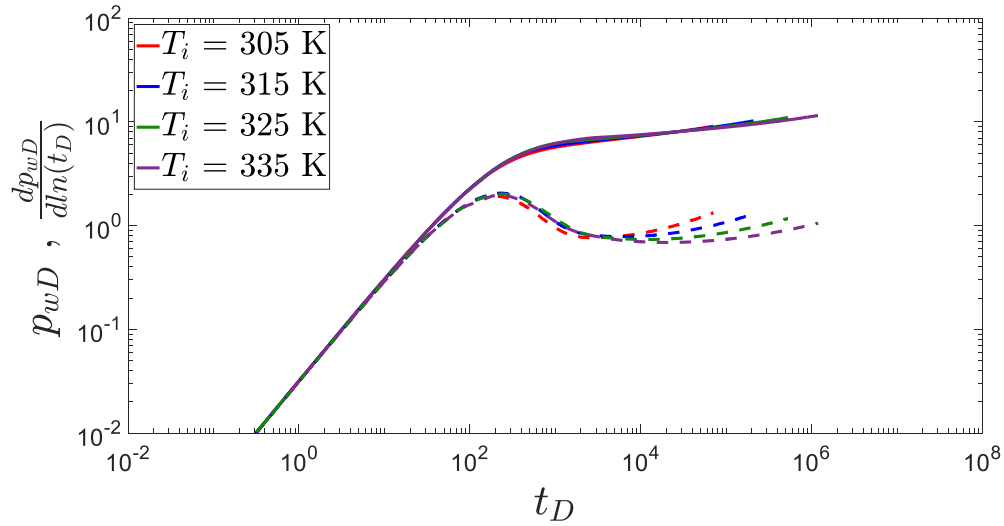


Figure 53, Dimensionless Field Plot with varying T_i

Above Figure 53 shows the effect of the initial temperature on the dimensionless field plot for the case of 5 KW and -20 BPD. Higher initial temperature induces a higher drawdown to a higher initial viscosity.

5.2.7 Effect of Specified Boundary

Below Figure 54 shows the effect of specifying the boundary for the electromagnetic heated region on the dimensionless field plot for the case of 5 KW and -20 BPD. It is important to note that the boundary changes with time as the reservoir is heated for longer. The final radius of the heated region is approximately 40 ft.

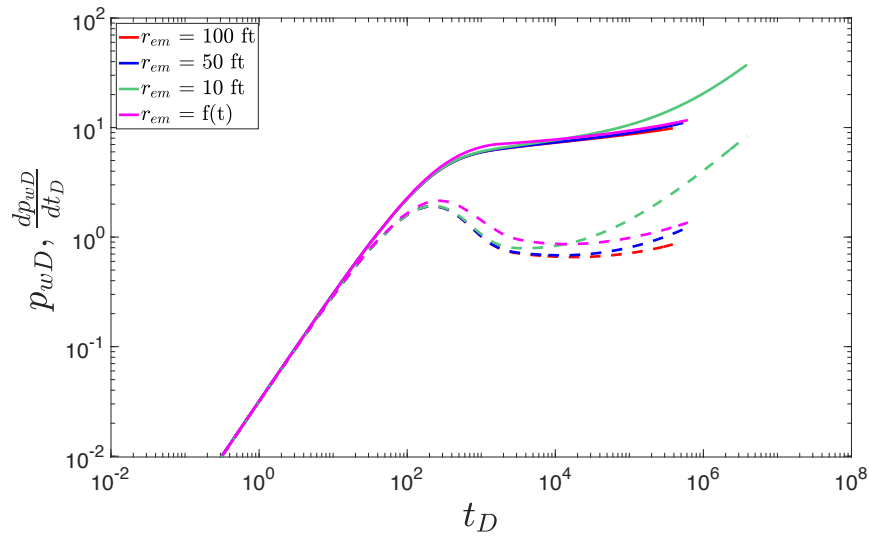


Figure 54, Dimensionless Field Plot with Constant/Varying r_{em}

5.2.8 Effect of Specified Mobility Ratio

Below Figure 55 shows the effect of specifying the boundary and the mobility ratio and thus eliminating the heat model. The boundary picked here is 40' and the figure below shows the radial composite behavior.

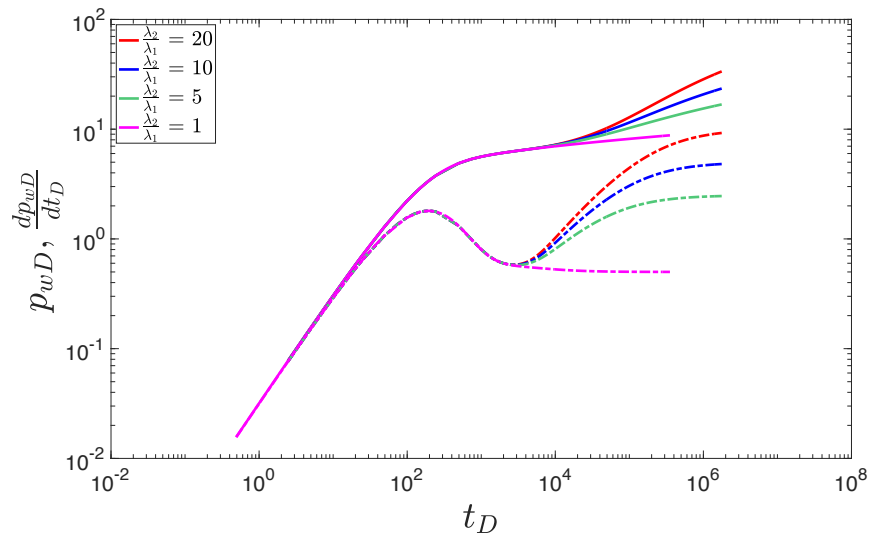


Figure 55, Dimensionless Field Plot with Specified Mobility Ratios and r_{em}

5.2.9 Effect of Averaging Criterion

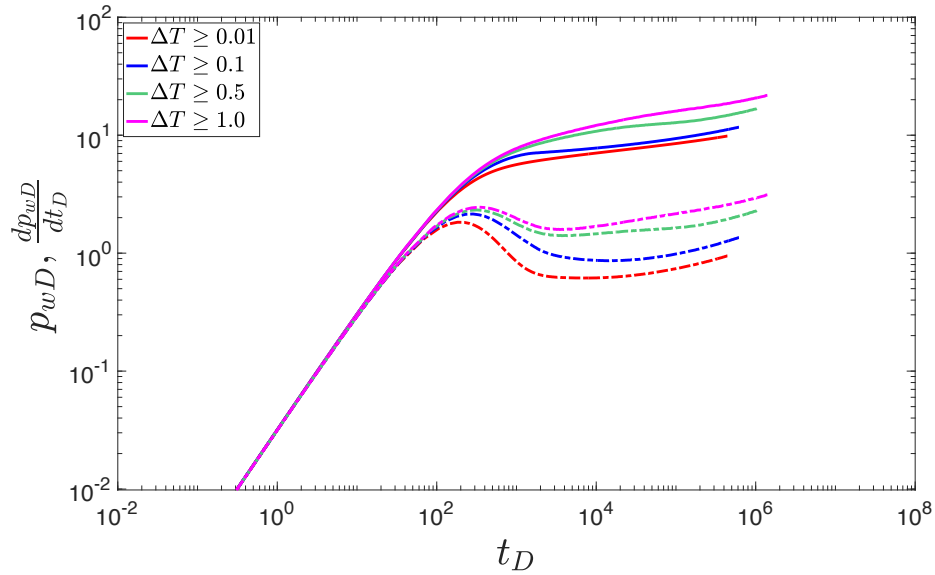


Figure 56, Dimensionless Field Plot with Different T_{avg} Criterion

Above Figure 56 shows the effect of different T_{avg} criterion on dimensionless field plots for 5 KW and -20 BPD. When a higher T_{avg} criterion is picked, the electromagnetic heated region computed becomes smaller, but the average temperature of that region is higher with higher mobility.

CHAPTER 6

Results

6.1 Methods of Parameter Matching

Pressure transient analysis main objective is to find a set of reservoir/well parameters that matches a set of measured data with the least possible error. The objective function may be written as:

$$\Phi(\vec{\gamma}) = \frac{1}{2\xi} \left\| \vec{d}_{cal} - \vec{d}_{meas} \right\|_2^2, \quad (114)$$

Where $\vec{\gamma}$, $\vec{d}_{cal}$, $\vec{d}_{meas}$ are vectors containing unknown parameters, calculated data from the model, and measured data respectively. ξ is a scaling factor set as the product of data points and the mean of the members in the set (Awotunde, 2015).

The method of inverse modelling used in this thesis using the objective function is called differential evolution (DE). This algorithm type is evolutionary which means that it searches for an optimal solution repetitively and continuously update the stored value of best match, DE uses several candidate solutions called agents. In this algorithm, these agents are initially generated using a uniform distribution. The values for different agents of the population are evaluated to determine which are the best agents. The best agent, along with its corresponding output value, is stored externally as the best solution for the current generation. This value is updated when a better agent is found in subsequent generations (Awotunde, 2015).

6.2 Initial Data & Parameters

Table 8 contains the parameters of interest that were matched in inverse modelling and the lower limits, upper limits, and actual values used in the matching and modelling respectively.

Table 8, Parameters Match Targets and Boundaries

<i>Parameter</i>	<i>Lower Limit</i>	<i>Upper Limit</i>	<i>Target</i>
$k \text{ (md)}$	1000	3000	2000
s	0.00	3.00	2.00
$C_s \text{ (psi}^{-1}\text{)}$	0.000100	0.000600	0.000400
$h \text{ (ft)}$	30	100	50
$r_e \text{ (ft)}$	500	2000	1000

Figure 57 & Figure 58 show the pressure and temperature data used in matching. The solid lines are the model output for our selected case of 5 KW & -20 BPD while the red circles are the calculated data with added noise which we treated as the measured data.

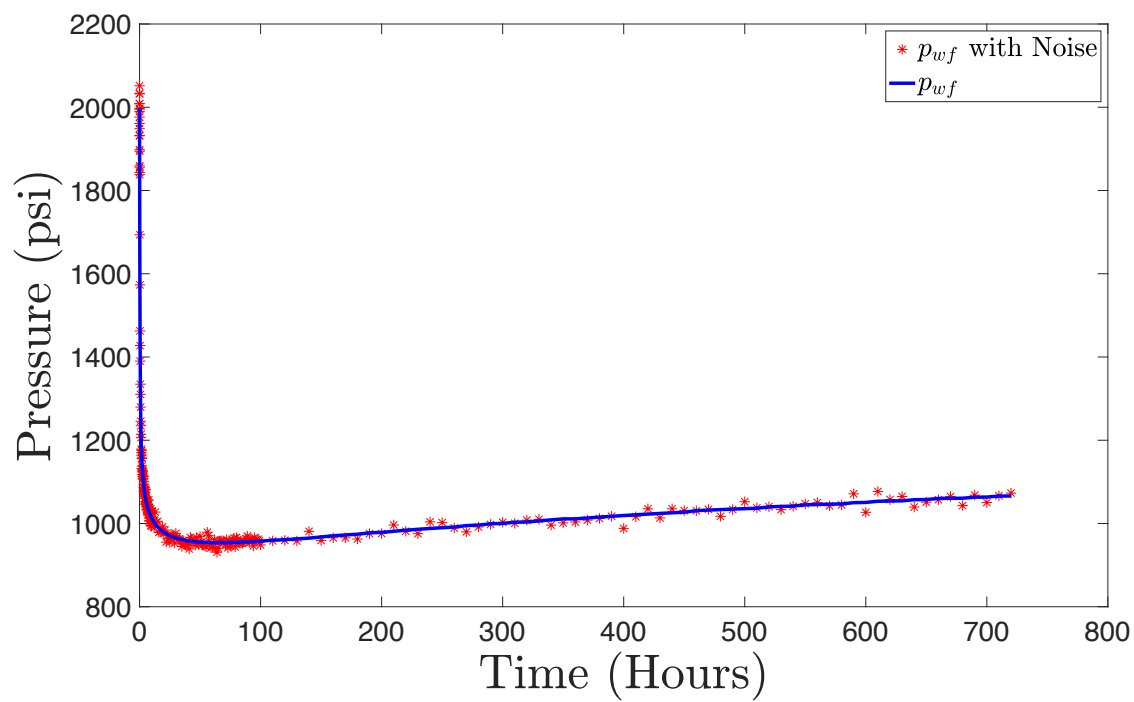


Figure 57, Pressure Data used in Inverse Modelling

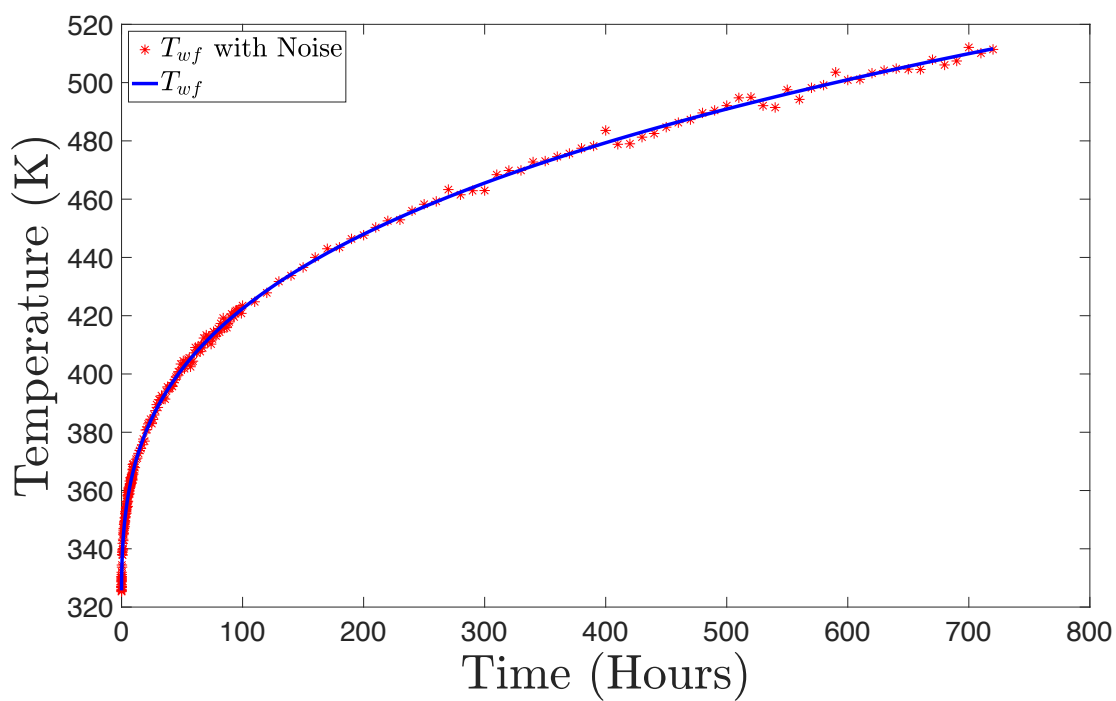


Figure 58, Temperature Data used in Inverse Modelling

6.3 Case 1

Table 9, Case 1 Parameters Matches

Parameter	Best Realization	Median Realization	Worst Realization
k (md)	2971	2945	1923
s	3.00	2.98	3.00
C_s (psi ⁻¹)	0.000318	0.000306	0.000292
h (ft)	63	63	95
r_e (ft)	500	517	689

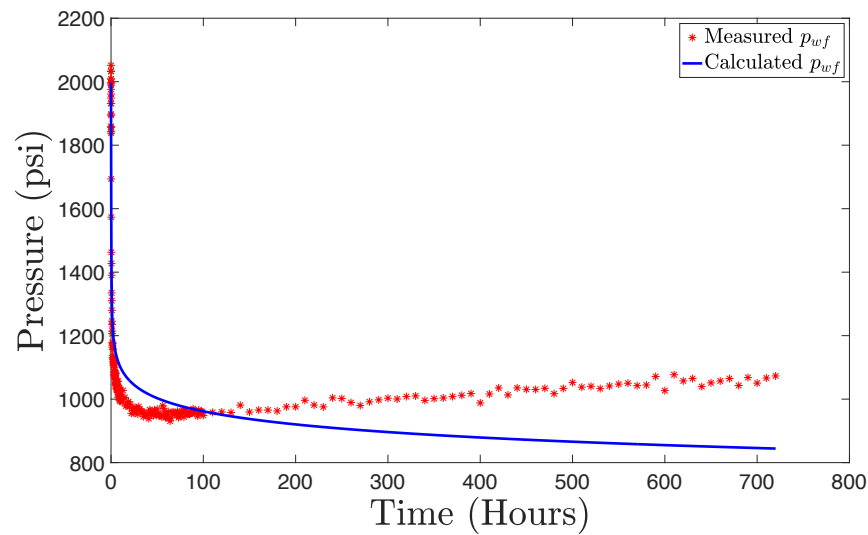


Figure 59, Case 1 Pressure Match against Measured Data

Case 1 matches the measured pressure values using a homogeneous reservoir flow model. Table 9 shows the results of Case 1 match for the best realization, median realization, and worst realization. Figure 59 shows the best realization calculated pressure values against the measured pressure values. We can see that it is a poor match, and accounting for the composite behaviour is necessary. It is important to note that for all cases and matches

discussed in the later section, the reservoir effective radius r_e is not matched accurately because the boundary response is not observed in the pressure results.

6.4 Case 2

Table 10, Case 2 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
k (md)	1982	1961	1429
s	2.26	1.91	3.00
C_s (psi ⁻¹)	0.000424	0.000393	0.000276
h (ft)	52	51	83
r_e (ft)	1832	683	500

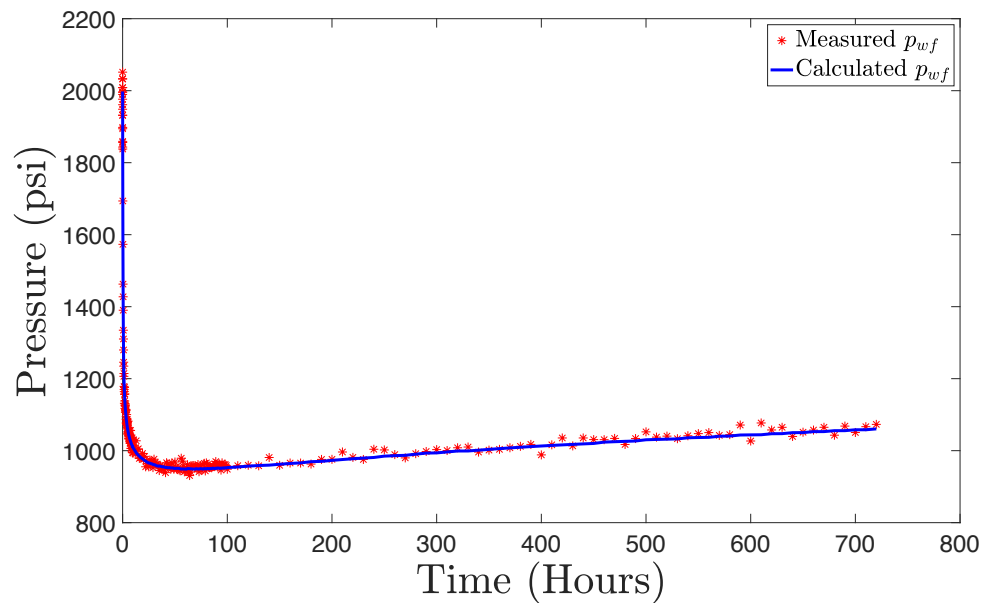


Figure 60, Case 2 Pressure Match against Measured Data

Case 2 corresponds to matching only the measured pressure values using a composite reservoir flow model & heat model. Table 10 shows the results of Case 2 match for the best

realization, median realization, and worst realization. Figure 60 shows the best realization of calculated pressure values against the measured pressure values. We can see that it is an excellent match because of accounting for the composite behavior and heat.

6.5 Case 3

Case 3 corresponds to matching both the measured pressure values and temperature values using a composite reservoir flow model & heat model. Table 11 shows the results of Case 2 match for the best realization, median realization, and worst realization, and Figure 61 shows the best realization calculated pressure values against the measured pressure values. In contrast, Figure 62 shows the calculated temperature values against the measured temperature values. We can see that it is an excellent match because of accounting for the composite behavior and heat.

Table 11, Case 3 Parameters Matches

<i>Parameter</i>	<i>Best Realization</i>	<i>Median Realization</i>	<i>Worst Realization</i>
k (md)	2016	2112	2252
s	2.12	2.42	3.00
C_s (psi ⁻¹)	0.000421	0.000418	0.000373
h (ft)	50	50	51
r_e (ft)	631	1359	660

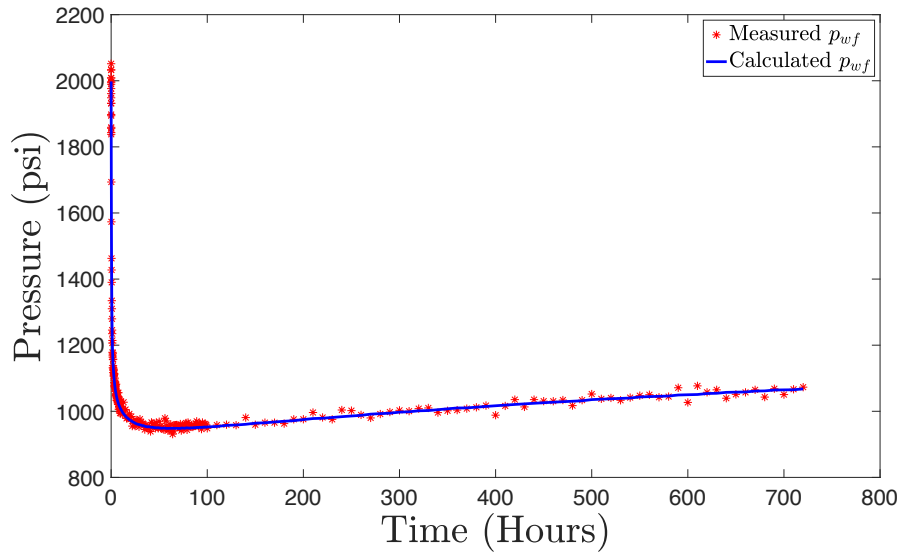


Figure 61, Case 3 Pressure Match against Measured Data

Figure 63 to Figure 65 shows Case 3 errors for best realization, median realization, and worst realization. As we are matching two different variables, errors come from each of them, and the inverse model sets to minimize the total error, not the specific error. We can see a decreasing trend of absolute error with higher errors in pressure values as compared to temperature values.

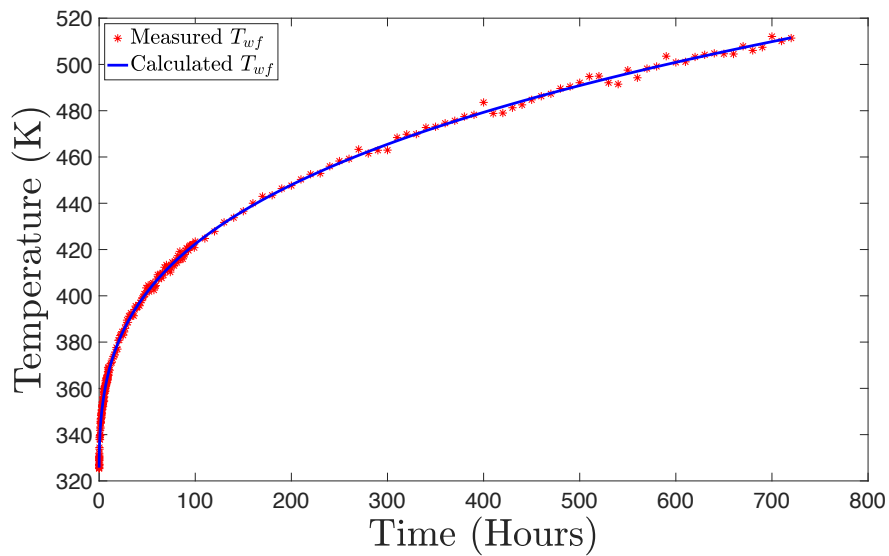


Figure 62, Case 3 Temperature Match against Measured Data

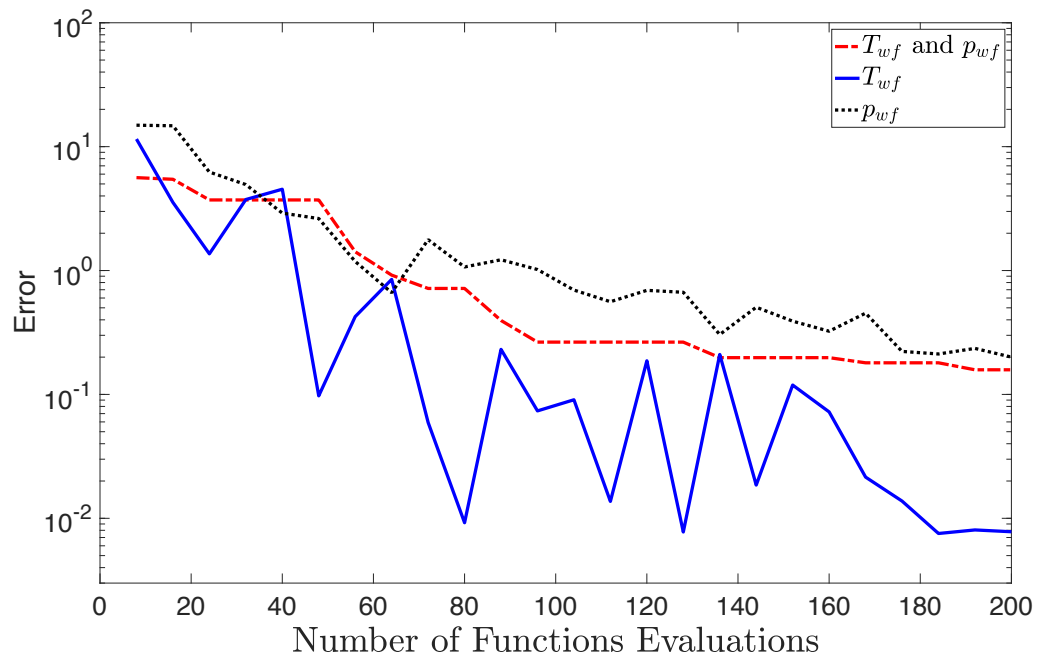


Figure 63, Case 3 Errors Comparison for Best Realization

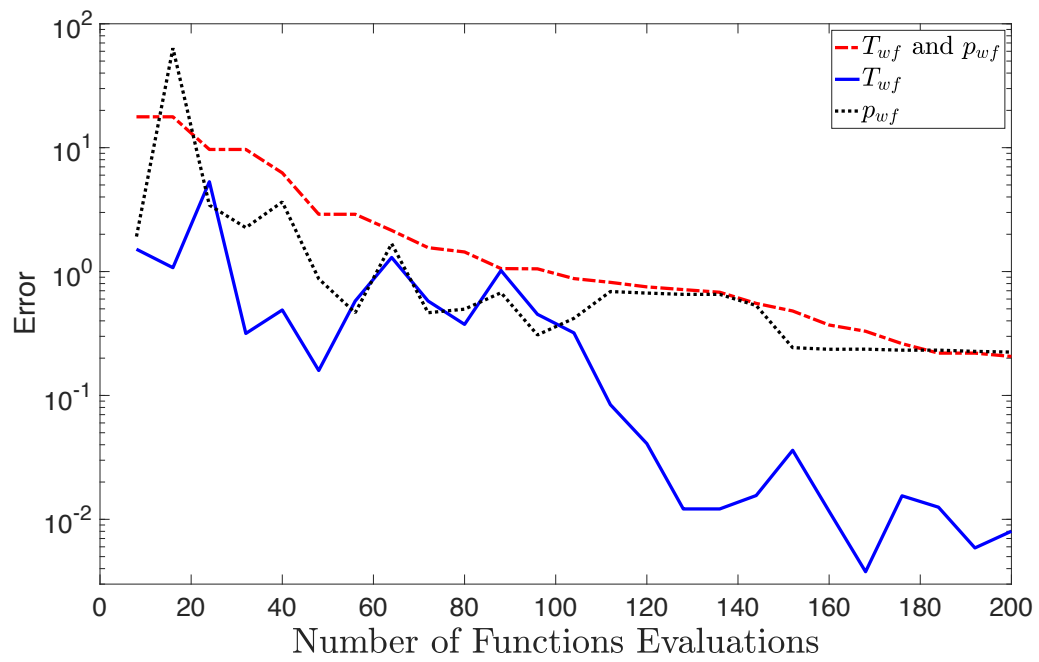


Figure 64, Case 3 Errors Comparison for Median Realization

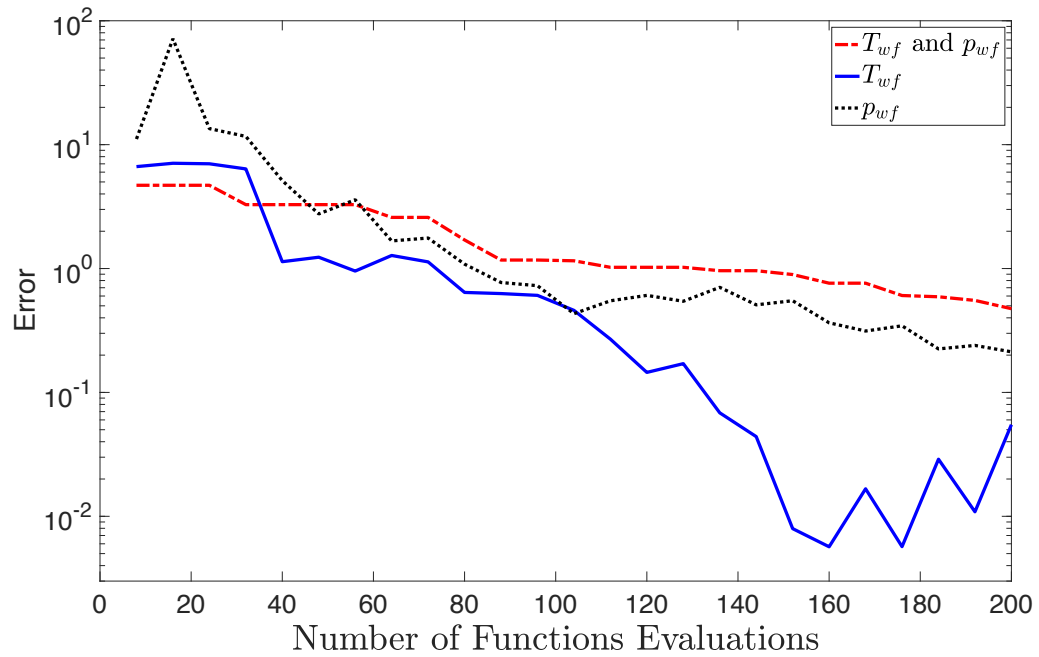


Figure 65, Case 3 Errors Comparison for Worst Realization

6.6 Cases Comparison

Figure 66 compares the results of the best realization in terms of pressure for all cases. We can see that the best match is reached by matching firstly only the pressure with the flow and heat model, secondly the pressure and temperature using both the flow and heat model and lastly the pressure assuming a homogeneous model.

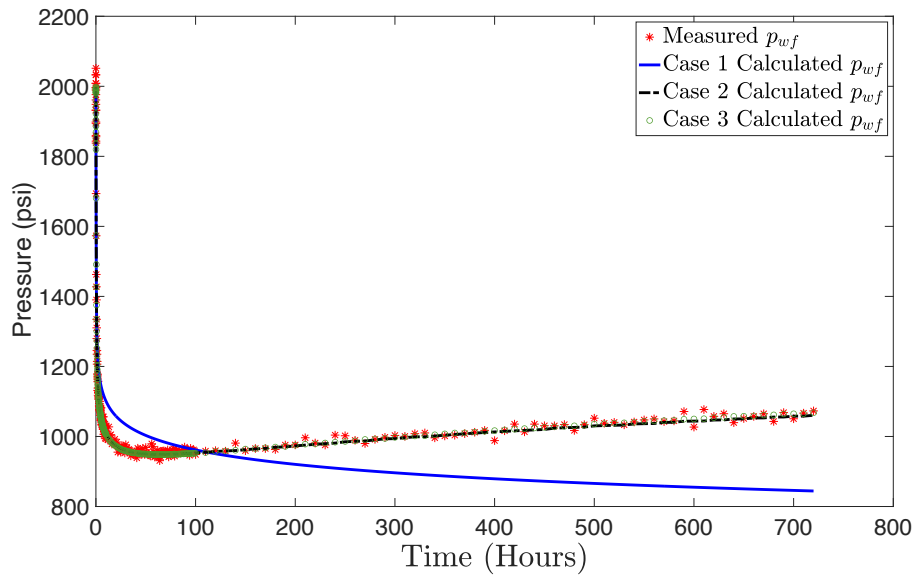


Figure 66, Comparison of all Cases Pressure Match against Measured Data

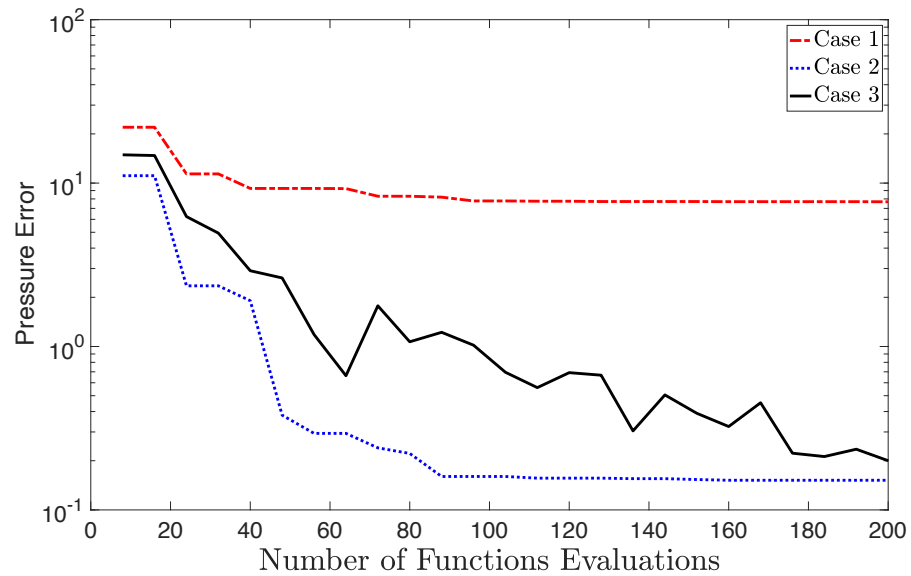


Figure 67, Error in Best Realizations of All Cases

Figure 67 to Figure 69 compare the errors of all cases for best realization, median realization, and worst realization. Case 2 has the lowest errors in all figures except the worst realization. In the worst realization figure, case 3 has the lowest pressure errors. These

results show that it is more accurate to match only pressure values, and it is necessary to use both a composite flow model and a heat model to match the results well.

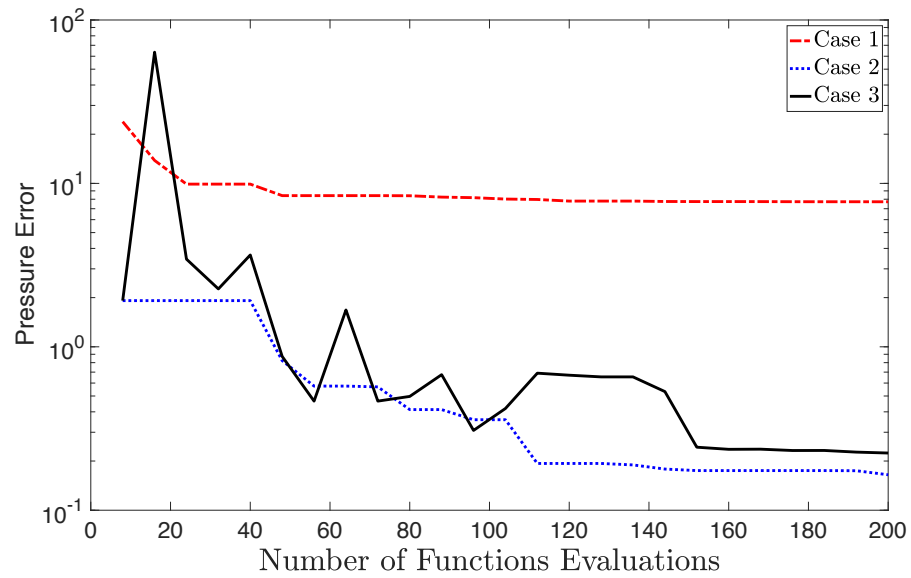


Figure 68, Error in Median Realizations of All Cases

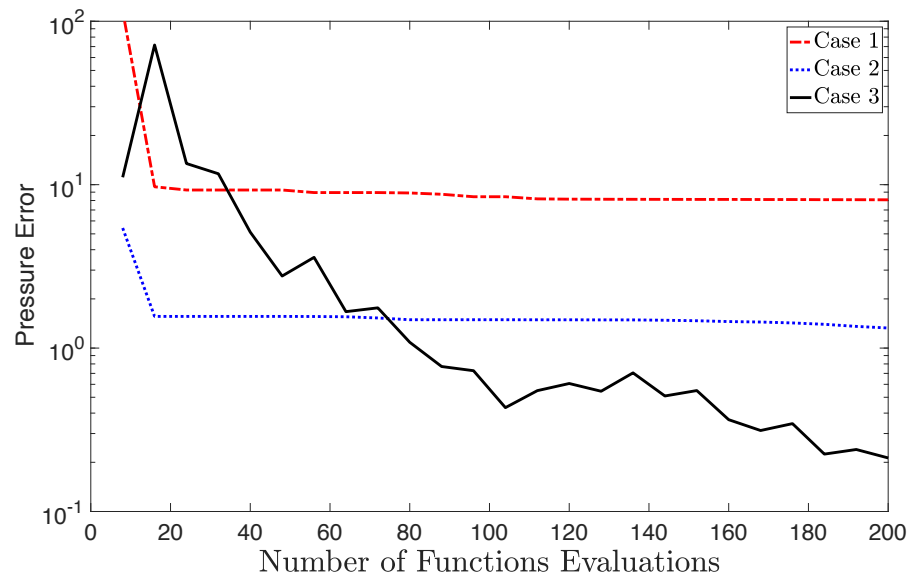


Figure 69, Error in Worst Realizations of All Cases

CONCLUSION

Heavy oil makes up a significant portion of oil reserves in the world. Enhancing recovery methods mainly affecting temperature are utilised to efficiently extract heavy oil, such as steam injection, electrical heating, in-situ combustion, and electromagnetic radiation.

In these reservoirs, pressure transient analysis is not commonly performed as the fluid flow equation is solved independently of the heat flow equation. Thus, the integration of flow and heat models is vital for changing viscosity.

This work investigated the effect of placing a microwave heater at the wellbore to enhance productivity using mathematical tools and MATLAB to perform inverse pressure modelling of pressure data and estimate reservoir parameters.

Unlike previous literature, this work does not assume a steady state nor neglect any heat transfer mechanisms. It has been shown that conduction significantly affects the temperature of the near-wellbore region and should not be neglected. Furthermore, accurate pressure response matching requires utilising both the pressure and heat models.

Appendix A

We start with energy balance of fluxes:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{\frac{s}{s}} = \dot{E}_{acc}, \quad (A.1)$$

Using the following identities:

$$\dot{E}_{in} - \dot{E}_{out} = - \int_A \vec{F}_T \cdot \vec{n} dA, \quad (A.2)$$

$$\vec{F}_T = \vec{F}_{disp} + \vec{F}_{conv} + \vec{F}_{em}, \quad (A.3)$$

$$\vec{F}_{disp} = \text{Heat Flux Conducted} = -\phi \tilde{\kappa}_f \cdot \nabla T_f, \quad (A.4)$$

$$\vec{F}_{conv} = \text{Heat Flux Convected} = \rho_f h_f \vec{u}_f, \quad (A.5)$$

$$\vec{F}_{em} = \text{Heat Flux Radiated} = \frac{1}{2} Re [\vec{E}_s \times \vec{H}_s^*], \quad (A.6)$$

$$h_f = C_{pf} T_f, \quad (A.7)$$

$$C_{pf} = \left(\frac{dh_f}{dT} \right)_p, \quad (A.8)$$

$$\dot{E}_{s/s} = \int_V q_{Tf} dV + \int_V \gamma_{sf} (T_s - T_f) dV, \quad (A.9)$$

and:

$$\dot{E}_{acc} = \frac{d}{dt} \int_V (\phi \rho_f e_f) dV. \quad (A.10)$$

Thus, we obtain:

$$- \int_A \vec{F}_T \cdot \vec{n} dA + \int_V q_{Tf} dV + \int_V \gamma_{sf} (T_s - T_f) dV = \int_V \frac{\partial (\phi \rho_f e_f)}{\partial t} dV. \quad (A.11)$$

Using Gauss Divergence Theorem, rearranging, and considering that the integrand vanishes identically.

$$\nabla \cdot [\phi \tilde{\kappa}_f \cdot \nabla T_f] - \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{em} + \gamma_{sf} (T_s - T_f) = \frac{\partial (\phi \rho_f e_f)}{\partial t}. \quad (A.12)$$

Noting that specific internal energy:

$$e = C_v T. \quad (A.13)$$

Repeating a similar process for solids, we obtain:

$$\nabla \cdot [\phi \tilde{\kappa}_f \cdot \nabla T_f] - \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{em} + \gamma_{sf} (T_s - T_f) = \frac{\partial (\phi \rho_f C_{vf} T_f)}{\partial t}, \quad (A.14)$$

and:

$$\nabla \cdot [(1 - \phi) \tilde{\kappa}_s \cdot \nabla T_s] + \gamma_{sf} (T_f - T_s) + q_{em} = \frac{\partial}{\partial t} ((1 - \phi) \rho_s C_{vs} T_s). \quad (A.15)$$

Combining fluid and solid we obtain:

$$\begin{aligned} & \nabla \cdot [(1 - \phi) \tilde{\kappa}_s \cdot \nabla T_s + \phi \tilde{\kappa}_f \cdot \nabla T_f] - \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{em} \\ & = \frac{\partial}{\partial t} ((1 - \phi) \rho_s C_{vs} T_s + \phi \rho_f C_{vf} T_f). \end{aligned} \quad (A.16)$$

Appendix B

$P(r)$ is defined as the total radiated power in a cylinder of height h :

$$P(r) = 2\pi r h \psi(r) \quad (B.1)$$

Rearranging (B.1) for power density:

$$\frac{P(r)}{2\pi r h} = \psi(r) \quad (B.2)$$

Differentiating (B.2) with respect to radius:

$$\frac{dP(r)}{dr} = 2\pi h \psi(r) + 2\pi r h \frac{d\psi(r)}{dr} \quad (B.3)$$

Substituting (B.2) into (B.3) the equations:

$$\frac{dP(r)}{dr} = 2\pi h \psi(r) - 2\pi r h \left(\alpha + \frac{1}{r} \right) \psi(r) \quad (B.4)$$

$$\frac{dP(r)}{dr} = -\alpha P(r) \quad (B.5)$$

By integrating (B.5) we obtain:

$$e^{-\alpha(r-r_w)} = \frac{P(r)}{P_0} \quad (B.6)$$

Therefore, the radiated power can be obtained as a function of radius as:

$$P(r) = P_0 e^{-\alpha(r-r_w)} \quad (B.7)$$

Appendix C

We start from mass balance:

$$m_{in} - m_{out} + m_{sink} + m_{source} = m_{accumulation} \quad (C.1)$$

Where m stands for mass rate.

Noting that,

$$m_{in} - m_{out} = - \int_A (\rho_f \vec{u}) \cdot \vec{n} dA \quad (C.2)$$

Gauss Divergence Theorem States:

$$\int_A \vec{u} \cdot \vec{n} dA = \int_V \nabla \cdot \vec{u} dV \quad (C.3)$$

Also, the sink and source terms:

$$m_{sink} + m_{source} = m_{s/s} = \int_V \frac{\rho_f q}{v} dV \quad (C.4)$$

For the accumulation term:

$$m_{accumulation} = \frac{d}{dt} \int_V \phi \rho_f dV = \int_V \frac{\partial(\rho_f \phi)}{\partial t} dV \quad (C.5)$$

Therefore:

$$- \int_V \nabla \cdot (\rho_f \vec{u}) dV + \int_V \frac{\rho_f q}{v} dV = \int_V \frac{\partial(\rho_f \phi)}{\partial t} dV \quad (C.6)$$

Appendix D

For cylindrical coordinates:

$$\nabla p = \begin{bmatrix} \frac{\partial p}{\partial r} \\ \frac{1}{r} \frac{\partial p}{\partial \theta} \\ \frac{\partial p}{\partial z} \end{bmatrix} \quad (D.1)$$

$$\bar{\bar{K}} = \begin{bmatrix} k_{rr} & k_{r\theta} & k_{rz} \\ k_{\theta r} & k_{\theta\theta} & k_{\theta z} \\ k_{zr} & k_{z\theta} & k_{zz} \end{bmatrix} \quad (D.2)$$

$$\nabla \cdot \vec{u} = \frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \quad (D.3)$$

Combining equations and assuming no crossflow between directions we obtain

$$\left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \rho_f \frac{k_r}{\mu} \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\rho_f \frac{k_\theta}{\mu r} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(\rho_f \frac{k_z}{\mu} \frac{\partial p}{\partial z} \right) \right) + \frac{\rho_f q}{v} = \frac{\partial (\rho_f \phi)}{\partial t} \quad (D.4)$$

Appendix E

Assuming thermal equilibrium:

$$T_s = T_f = T$$

Thus,

$$\begin{aligned} \nabla \cdot \{ [(1 - \phi)\tilde{\kappa}_s + \phi\tilde{\kappa}_f] \cdot \nabla T \} + \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{em} \\ = \frac{\partial}{\partial t} \{ [(1 - \phi)\rho_s C_{vs} + \phi\rho_f C_{vf}] T \} \end{aligned} \quad (E.1)$$

Substituting for effective parameters:

$$\{ [1 - \phi]\tilde{\kappa}_s + \phi\tilde{\kappa}_f \} = \tilde{\kappa}_{eff} \quad (E.2)$$

$$(1 - \phi)\rho_s C_{vs} + \phi\rho_f C_{vf} = (\rho C_v)_{eff} \quad (E.3)$$

To obtain:

$$\nabla \cdot \{ \tilde{\kappa}_{eff} \cdot \nabla T \} + \nabla \cdot [\rho_f C_{pf} T \vec{u}_f] + q_{Ts} + q_{em} = \frac{\partial}{\partial t} \{ [(\rho C_v)_{eff}] T \} \quad (E.4)$$

Assuming no heat source $q_{Ts} = 0$ and assuming isotropic reservoir:

$$\kappa_{eff} \nabla^2 T + \rho_f C_{pf} \vec{u}_f \cdot \nabla T + q_{em} = (\rho C_v)_{eff} \frac{\partial T}{\partial t} \quad (E.5)$$

Dividing both sides by $(\rho C_v)_{eff}$

$$\frac{\kappa_{eff} \nabla^2 T}{(\rho C_v)_{eff}} + \frac{\rho_f C_{pf} \vec{u}_f \cdot \nabla T}{(\rho C_v)_{eff}} + \frac{q_{em}}{(\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.9)$$

Simplifying using:

$$\frac{\kappa_{eff}}{(\rho C_v)_{eff}} = \lambda \quad (E.10)$$

$$\frac{\rho_f C_{pf} \vec{u}_f}{(\rho C_v)_{eff}} = \vec{u}_f \quad (E.11)$$

We obtain:

$$\lambda \nabla^2 T + \vec{u}_f \cdot \nabla T + \frac{q_{em}}{(\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.12)$$

The total electromagnetic power as a function of radius is obtained with:

$$P(r) = P_0 e^{-\alpha(r-r_w)} \quad (E.13)$$

A cylindrical element in the radial direction, $(r, r + dr)$ will gain as per:

$$q_{em}(r) = \alpha P_0 e^{-\alpha(r-r_w)} = \alpha P(r) \quad (E.14)$$

Therefore, the gain in purely the radial direction is obtained by dividing with the cylindrical surface area:

$$\frac{q_{em}(r)}{2\pi r h} = \frac{\alpha P_0 e^{-\alpha(r-r_w)}}{2\pi r h} = \frac{\alpha P(r)}{2\pi r h} \quad (E.15)$$

Then solving equation 76 for radial coordinates:

$$\lambda \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \vec{u}_f \cdot \frac{\partial T}{\partial r} + \frac{\alpha P(r)}{2\pi r h (\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.16)$$

Separating the first term:

$$\lambda \frac{\partial^2 T}{\partial r^2} + \frac{\lambda}{r} \frac{\partial T}{\partial r} + \dot{u}_f \frac{\partial T}{\partial r} + \frac{\alpha P(r)}{2\pi r h (\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.17)$$

Grouping the second and third term:

$$\lambda \frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda}{r} + \dot{u}_f \right) \frac{\partial T}{\partial r} + \frac{\alpha P(r)}{2\pi r h (\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.18)$$

Or

$$\lambda \frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda}{r} + \dot{u}_f \right) \frac{\partial T}{\partial r} + \frac{\alpha P_0 e^{-\alpha(r-r_w)}}{2\pi r h (\rho C_v)_{eff}} = \frac{\partial T}{\partial t} \quad (E.19)$$

Let:

$$\frac{\alpha P_0}{2\pi h (\rho C_v)_{eff}} = \dot{\alpha} \quad (E.20)$$

$$\dot{u}_f = \frac{\dot{q}}{2\pi r h} \quad (E.21)$$

Noting that, our sign convection is negative for production and thus:

$$\lambda \frac{\partial^2 T}{\partial r^2} + \left(\frac{\lambda}{r} - \frac{\dot{q}}{2\pi r h} \right) \frac{\partial T}{\partial r} + \dot{\alpha} \frac{e^{-\alpha(r-r_0)}}{r} = \frac{\partial T}{\partial t} \quad (E.22)$$

Appendix F

$$\begin{aligned} \lambda \frac{\partial^2 (T_i(T_D + 1))}{\partial (r_D r_w)^2} + \left(\frac{\lambda}{r_D r_w} - \frac{2\pi h \lambda q_D}{2\pi h r_D r_w} \right) \frac{\partial (T_i(T_D + 1))}{\partial r_D r_w} + \dot{\alpha} \frac{e^{-\alpha(r_D r_w - r_w)}}{r_D r_w} \\ = \frac{\partial (T_i(T_D + 1))}{\partial \frac{t_D r_w^2}{\lambda}} \end{aligned} \quad (F.1)$$

Multiplying both sides by $\frac{r_w^2}{T_i}$:

$$\begin{aligned} \lambda \frac{\partial^2 (T_D + 1)}{\partial (r_D r_w)^2} r_w^2 + \left(\frac{\lambda}{r_D r_w} - \frac{2\pi h \lambda q_D}{2\pi h r_D r_w} \right) \frac{\partial (T_D + 1)}{\partial r_D r_w} r_w^2 + \dot{\alpha} \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D r_w} r_w^2 \\ = \frac{\partial (T_D + 1)}{\partial \frac{t_D r_w^2}{\lambda}} r_w^2 \end{aligned} \quad (F.2)$$

$$\lambda \frac{\partial^2 (T_D)}{\partial r_D^2} + \left(\frac{\lambda}{r_D} - \frac{\lambda q_D}{r_D} \right) \frac{\partial (T_D)}{\partial r_D} + \dot{\alpha} r_w \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D} = \lambda \frac{\partial (T_D)}{\partial t_D} \quad (F.3)$$

Divide both sides by λ :

$$\frac{\partial^2 T_D}{\partial r_D^2} + \left(\frac{1}{r_D} - \frac{q_D}{r_D} \right) \frac{\partial T_D}{\partial r_D} + \frac{\dot{\alpha} r_w}{\lambda} \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D} = \frac{\partial T_D}{\partial t_D} \quad (F.4)$$

Finally,

$$\frac{\partial^2 T_D}{\partial r_D^2} + \left(\frac{1}{r_D} [1 - q_D] \right) \frac{\partial T_D}{\partial r_D} + \frac{\dot{\alpha} r_w}{\lambda} \frac{e^{-\alpha(r_D r_w - r_w)}}{T_i r_D} = \frac{\partial T_D}{\partial t_D} \quad (F.5)$$

Appendix G

$$\frac{\partial^2 T_D}{\partial r_D^2} + \left(\frac{-2\varepsilon}{r_D}\right) \frac{\partial T_D}{\partial r_D} + \frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D} = \frac{\partial T_D}{\partial t_D} \quad (G.1)$$

Term by term breakdown:

$$\mathcal{L}\left\{\frac{\partial^2 T}{\partial r_D^2}\right\} = \frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} \quad (G.2)$$

$$\mathcal{L}\left\{\left(\frac{-2\varepsilon}{r_D}\right) \frac{\partial T_D}{\partial r_D}\right\} = \left(\frac{-2\varepsilon}{r_D}\right) \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} \quad (G.3)$$

$$\mathcal{L}\left\{\frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D}\right\} = \frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D} \frac{1}{z} \quad (G.4)$$

$$\mathcal{L}\left\{\frac{\partial T_D}{\partial t_D}\right\} = z \hat{T}_D(r_D, z) - T_D(r_D, t_D = 0) \quad (G.5)$$

Therefore,

$$\begin{aligned} \frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} + \left(\frac{-2\varepsilon}{r_D}\right) \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} + \frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D} \frac{1}{z} \\ = z \hat{T}_D(r_D, z) - T_D(r_D, t_D = 0) \end{aligned} \quad (G.6)$$

Note that $T_D(r_D, t_D = 0) = 1$

$$\frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} + \left(\frac{-2\varepsilon}{r_D}\right) \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} + \frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D} \frac{1}{z} = z \hat{T}_D(r_D, z) - 1 \quad (G.7)$$

Rearranging:

$$\frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} - \frac{2\varepsilon}{r_D} \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} - z \hat{T}_D(r_D, z) + \frac{\dot{\alpha} r_w e^{-\alpha(r_D r_w - r_w)}}{\lambda T_i r_D} \frac{1}{z} + 1 = 0 \quad (G.8)$$

Let,

$$\frac{\dot{\alpha} r_w}{\lambda T_i} = \beta \quad (G.9)$$

Thus:

$$\frac{\partial^2 \hat{T}_D(r_D, z)}{\partial r_D^2} - \frac{2\varepsilon}{r_D} \frac{\partial \hat{T}_D(r_D, z)}{\partial r_D} - z \hat{T}_D(r_D, z) + \beta \frac{e^{-\alpha(r_w(r_D - 1))}}{z r_D} + 1 = 0 \quad (G.10)$$

Equation (G.10) has the general solution:

$$\hat{T}_D(r_D, z) = r_D^\eta [A I_\eta(\sqrt{z} r_D) + B K_\eta(\sqrt{z} r_D)], \quad (G.11)$$

where $\eta = \varepsilon + 0.5$, I & K are the modified Bessel functions of first kind and second kind respectively. The particular solution of equation (G.10) is given by:

$$\begin{aligned}
\hat{T}_D(r_D, z) = & r_D^\eta [A I_\eta(\sqrt{z} r_D) + B K_\eta(\sqrt{z} r_D)] \\
& + r_D^\eta \left\{ \left[I_\eta(\sqrt{z} r_D) \int_1^{r_D} \frac{\zeta^{-\eta} \dot{\beta} e^{-\alpha r_w(\zeta-1)} \pi K_\eta(\sqrt{z} \zeta)}{2z} d\zeta \right] \right. \\
& \left. + \left[-K_\eta(\sqrt{z} r_D) \int_1^{r_D} \frac{\zeta^{-\eta} \dot{\beta} e^{-\alpha r_w(\zeta-1)} \pi I_\eta(\sqrt{z} \zeta)}{2z} d\zeta \right] \right\} + \frac{1}{z} \quad (G.12)
\end{aligned}$$

Appendix H

Note the following:

$$\varepsilon + 0.5 = \eta \quad (H.1)$$

Equivalently:

$$\varepsilon = \eta - 0.5 \quad (H.2)$$

Equivalently:

$$2\varepsilon = 2\eta - 1 \quad (H.3)$$

Equivalently:

$$1 + 2\varepsilon = 2\eta \quad (H.4)$$

And:

$$\tau_m = \ln \left(\frac{r_{D_m}}{r_{D_{m-1}}} \right) \quad (H.5)$$

General Equation Discretization

First, we start with:

$$\frac{\partial^2 \hat{T}_D}{\partial r_D^2} - \frac{2\varepsilon}{r_D} \frac{\partial \hat{T}_D}{\partial r_D} - z \hat{T}_D + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z r_D} + 1 = 0 \quad (H.6)$$

Equivalently:

$$\frac{\partial^2 \hat{T}_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \hat{T}_D}{\partial r_D} - \frac{1}{r_D} \frac{\partial \hat{T}_D}{\partial r_D} - \frac{2\varepsilon}{r_D} \frac{\partial \hat{T}_D}{\partial r_D} - z \hat{T}_D + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z r_D} + 1 = 0 \quad (H.7)$$

Therefore,

$$\frac{1}{r_D} \frac{d}{dr_D} \left(r_D \frac{d\hat{T}_D}{dr_D} \right) - \frac{(1+2\varepsilon)}{r_D} \frac{d\hat{T}_D}{dr_D} - z \hat{T}_D + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z r_D} + 1 = 0 \quad (H.8)$$

Let

$$x_D = \ln(r_D) \rightarrow r_D = e^{x_D} \quad (H.9)$$

$$dx_D = \frac{1}{r_D} dr_D \rightarrow dr_D = r_D dx_D \quad (H.10)$$

So that

$$\frac{dx_D}{dr_D} = \frac{1}{r_D} \quad (H.11)$$

$$\frac{d\hat{T}_D}{dr_D} = \frac{d\hat{T}_D}{dx_D} \frac{dx_D}{dr_D} = \frac{1}{r_D} \frac{d\hat{T}_D}{dx_D} \quad (H.12)$$

Therefore,

$$\frac{1}{r_D} \frac{d}{dr_D} \left(r_D \frac{d\hat{T}_D}{dr_D} \right) - \frac{(1+2\varepsilon)}{r_D} \frac{d\hat{T}_D}{dr_D} - z \hat{T}_D + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z r_D} + 1 = 0 \quad (H.13)$$

Becomes:

$$\frac{1}{r_D} \frac{d}{dr_D} \left(r_D \frac{1}{r_D} \frac{d\hat{T}_D}{dx_D} \right) - \frac{(1+2\varepsilon)}{r_D} \frac{1}{r_D} \frac{d\hat{T}_D}{dx_D} - z \hat{T}_D + \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z r_D} + 1 = 0 \quad (H.14)$$

Then,

$$\frac{1}{r_D} \frac{d}{dr_D} \left(\frac{d\hat{T}_D}{dx_D} \right) - \frac{(1+2\varepsilon)}{r_D^2} \frac{d\hat{T}_D}{dx_D} - z\hat{T}_D + \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{zr_D} + 1 = 0 \quad (H.15)$$

And

$$\frac{1}{r_D} \frac{d}{r_D dx_D} \left(\frac{d\hat{T}_D}{dx_D} \right) - \frac{(1+2\varepsilon)}{r_D^2} \frac{d\hat{T}_D}{dx_D} - z\hat{T}_D + \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{zr_D} + 1 = 0 \quad (H.16)$$

Then

$$\frac{1}{r_D^2} \frac{d}{dx_D} \left(\frac{d\hat{T}_D}{dx_D} \right) - \frac{(1+2\varepsilon)}{r_D^2} \frac{d\hat{T}_D}{dx_D} - z\hat{T}_D + \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{zr_D} + 1 = 0 \quad (H.17)$$

And

$$\frac{1}{e^{2x_D}} \frac{d}{dx_D} \left(\frac{d\hat{T}_D}{dx_D} \right) - \frac{(1+2\varepsilon)}{e^{2x_D}} \frac{d\hat{T}_D}{dx_D} - z\hat{T}_D + \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{ze^{x_D}} + 1 = 0 \quad (H.18)$$

Or in discrete form:

$$\begin{aligned} \frac{1}{e^{2x_D}} \frac{1}{\Delta x_{D_m}} \left(\frac{\hat{T}_{D_{m+1}} - \hat{T}_{D_m}}{x_{D_{m+1}} - x_{D_m}} - \frac{\hat{T}_{D_m} - \hat{T}_{D_{m-1}}}{x_{D_m} - x_{D_{m-1}}} \right) - \frac{(1+2\varepsilon)}{e^{2x_D} \Delta x_{D_m}} [\hat{T}_{D_{m+1}} - \hat{T}_{D_m}] - z\hat{T}_D \\ = - \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{ze^{x_D}} - 1 \end{aligned} \quad (H.19)$$

Recall:

$$x_D = \ln(r_D) \quad (H.20)$$

$$r_D = e^{x_D} \quad (H.21)$$

We obtain:

$$\begin{aligned} \frac{1}{e^{2x_D}} \frac{1}{\Delta x_{D_m}} \left(\frac{\hat{T}_{D_{m+1}} - \hat{T}_{D_m}}{\ln\left(\frac{r_{D_{m+1}}}{r_{D_m}}\right)} - \frac{\hat{T}_{D_m} - \hat{T}_{D_{m-1}}}{\ln\left(\frac{r_{D_m}}{r_{D_{m-1}}}\right)} \right) - \frac{(1+2\varepsilon)}{e^{2x_D} \Delta x_{D_m}} [\hat{T}_{D_{m+1}} - \hat{T}_{D_m}] - z\hat{T}_{D_m} \\ = - \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{ze^{x_D}} - 1 \end{aligned} \quad (H.22)$$

Multiply both sides by $e^{2x_D} \Delta x_{D_m}$ to obtain:

$$\begin{aligned} \frac{\hat{T}_{D_{m+1}} - \hat{T}_{D_m}}{\ln\left(\frac{r_{D_{m+1}}}{r_{D_m}}\right)} - \frac{\hat{T}_{D_m} - \hat{T}_{D_{m-1}}}{\ln\left(\frac{r_{D_m}}{r_{D_{m-1}}}\right)} - (1+2\varepsilon)[\hat{T}_{D_{m+1}} - \hat{T}_{D_m}] - ze^{2x_D} \Delta x_{D_m} \hat{T}_{D_m} \\ = -e^{x_D} \Delta x_{D_m} \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{z} - e^{2x_D} \Delta x_{D_m} \end{aligned} \quad (H.23)$$

Note that:

$$\begin{aligned} e^{2x_D} \Delta x_D &= \int_{x_{D_{m-\frac{1}{2}}}}^{x_{D_{m+\frac{1}{2}}}} e^{2x_D} dx_D = \frac{1}{2} [e^{2x_D}]_{x_{D_{m-\frac{1}{2}}}}^{x_{D_{m+\frac{1}{2}}}} = \frac{1}{2} [e^{2x_{D_{m+\frac{1}{2}}}} - e^{2x_{D_{m-\frac{1}{2}}}}] \\ &= \frac{1}{2} \left[e^{2\ln\left(r_{D_{m+\frac{1}{2}}}\right)} - e^{2\ln\left(r_{D_{m-\frac{1}{2}}}\right)} \right] = \frac{1}{2} [r_{D_{m+\frac{1}{2}}}^2 - r_{D_{m-\frac{1}{2}}}^2] \end{aligned} \quad (H.24)$$

Similarly,

$$\begin{aligned}
e^{x_D} \Delta x_D &= \int_{x_{Dm-\frac{1}{2}}}^{x_{Dm+\frac{1}{2}}} e^{x_D} dx_D = [e^{x_D}]_{x_{Dm-\frac{1}{2}}}^{x_{Dm+\frac{1}{2}}} = \left[e^{x_{Dm+\frac{1}{2}}} - e^{x_{Dm-\frac{1}{2}}} \right] \\
&= \left[e^{\ln(r_{Dm+\frac{1}{2}})} - e^{\ln(r_{Dm-\frac{1}{2}})} \right] = \left[r_{Dm+\frac{1}{2}} - r_{Dm-\frac{1}{2}} \right]
\end{aligned} \tag{H.25}$$

Therefore,

$$\begin{aligned}
&\frac{\hat{T}_{Dm+1} - \hat{T}_{Dm}}{\ln\left(\frac{r_{Dm+1}}{r_{Dm}}\right)} - \frac{\hat{T}_{Dm} - \hat{T}_{Dm-1}}{\ln\left(\frac{r_{Dm}}{r_{Dm-1}}\right)} - (1 + 2\varepsilon)[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - ze^{2x_D} \Delta x_{Dm} \hat{T}_{Dm} \\
&= -e^{x_D} \Delta x_{Dm} \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} - e^{2x_D} \Delta x_{Dm}
\end{aligned} \tag{H.26}$$

Becomes:

$$\begin{aligned}
&\frac{\hat{T}_{Dm+1} - \hat{T}_{Dm}}{\ln\left(\frac{r_{Dm+1}}{r_{Dm}}\right)} - \frac{\hat{T}_{Dm} - \hat{T}_{Dm-1}}{\ln\left(\frac{r_{Dm}}{r_{Dm-1}}\right)} - (1 + 2\varepsilon)[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - z\hat{T}_{Dm} \frac{1}{2} \left[r_{Dm+\frac{1}{2}}^2 - r_{Dm-\frac{1}{2}}^2 \right] \\
&= - \left[r_{Dm+\frac{1}{2}} - r_{Dm-\frac{1}{2}} \right] \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} - \frac{1}{2} \left[r_{Dm+\frac{1}{2}}^2 - r_{Dm-\frac{1}{2}}^2 \right]
\end{aligned} \tag{H.27}$$

Let

$$\frac{1}{\ln\left(\frac{r_{Dm+1}}{r_{Dm}}\right)} = \tau_{m+1} \tag{H.28}$$

$$\frac{1}{\ln\left(\frac{r_{Dm}}{r_{Dm-1}}\right)} = \tau_m \tag{H.29}$$

Therefore,

$$\begin{aligned}
&\tau_{m+1}[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - \tau_m[\hat{T}_{Dm} - \hat{T}_{Dm-1}] - (1 + 2\varepsilon)[\hat{T}_{Dm+1} - \hat{T}_{Dm}] \\
&\quad - z\hat{T}_{Dm} \frac{1}{2} \left[r_{Dm+\frac{1}{2}}^2 - r_{Dm-\frac{1}{2}}^2 \right] \\
&= - \left[r_{Dm+\frac{1}{2}} - r_{Dm-\frac{1}{2}} \right] \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} - \frac{1}{2} \left[r_{Dm+\frac{1}{2}}^2 - r_{Dm-\frac{1}{2}}^2 \right]
\end{aligned} \tag{H.30}$$

And

$$\begin{aligned}
&\tau_{m+1}[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - \tau_m[\hat{T}_{Dm} - \hat{T}_{Dm-1}] - (1 + 2\varepsilon)[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - z\hat{T}_{Dm} \frac{1}{2} [\Delta r_{Dm}^2] \\
&= -[\Delta r_{Dm}] \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} - \frac{1}{2} [\Delta r_{Dm}^2]
\end{aligned} \tag{H.31}$$

Then

$$\begin{aligned}
&\tau_{m+1}[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - \tau_m[\hat{T}_{Dm} - \hat{T}_{Dm-1}] - (1 + 2\varepsilon)[\hat{T}_{Dm+1} - \hat{T}_{Dm}] - \frac{z\hat{T}_{Dm} \Delta r_{Dm}^2}{2} \\
&= -\Delta r_{Dm} \frac{\dot{\beta} e^{-\alpha r_w(r_D-1)}}{z} - \frac{\Delta r_{Dm}^2}{2}
\end{aligned} \tag{H.32}$$

Multiply both sides by -1

$$\begin{aligned} -\tau_{m+1}[\hat{T}_{D_{m+1}} - \hat{T}_{D_m}] + \tau_m[\hat{T}_{D_m} - \hat{T}_{D_{m-1}}] + (1 + 2\varepsilon)[\hat{T}_{D_{m+1}} - \hat{T}_{D_m}] + \frac{z\hat{T}_{D_m}\Delta r_{D_m}^2}{2} \\ = \Delta r_{D_m} \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{z} + \frac{\Delta r_{D_m}^2}{2} \end{aligned} \quad (H.33)$$

Expanding:

$$\begin{aligned} [-\tau_{m+1}\hat{T}_{D_{m+1}} + \tau_{m+1}\hat{T}_{D_m}] + [\tau_m\hat{T}_{D_m} - \tau_m\hat{T}_{D_{m-1}}] + [(1 + 2\varepsilon)\hat{T}_{D_{m+1}} - (1 + 2\varepsilon)\hat{T}_{D_m}] \\ + \frac{z\hat{T}_{D_m}\Delta r_{D_m}^2}{2} \\ = \Delta r_{D_m} \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{z} + \frac{\Delta r_{D_m}^2}{2} \end{aligned} \quad (H.34)$$

Rearranging for the general equation:

$$\begin{aligned} -\tau_m\hat{T}_{D_{m-1}} + \left[\tau_{m+1} + \tau_m - (1 + 2\varepsilon) + \frac{z\Delta r_{D_m}^2}{2} \right] \hat{T}_{D_m} + [-\tau_{m+1} + (1 + 2\varepsilon)]\hat{T}_{D_{m+1}} \\ = \frac{\Delta r_{D_m}^2}{2} + \Delta r_{D_m} \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{z} \end{aligned} \quad (H.35)$$

Inner Boundary Condition

$$\left\{ \frac{\partial \hat{T}_D}{\partial r_D} \right\}_{r_D=1} - (2\eta)\hat{T}_D + \frac{2\eta}{z} = -\frac{qh_{D_{int}}}{z} \quad (H.36)$$

Like the general equation:

$$\begin{aligned} \hat{T}_{D1} - \hat{T}_{wD} = 2\eta(r_{D1} - r_{wD})\hat{T}_{wD} - \frac{2\eta(r_{D1} - r_{wD})}{z} \\ - \frac{qh_{D_{int}}(r_{D1} - r_{wD})}{z} \end{aligned} \quad (H.37)$$

$$[2\eta(r_{D1} - r_{wD}) + 1]\hat{T}_{wD} - \hat{T}_{D1} = \frac{(2\eta + qh_{D_{int}})(r_{D1} - r_{wD})}{z} \quad (H.38)$$

Let:

$$\omega = [2\eta(r_{D1} - r_{wD}) + 1] \quad (H.39)$$

$$\zeta = \frac{(2\eta + qh_{D_{int}})(r_{D1} - r_{wD})}{z} \quad (H.40)$$

So that the IBC becomes:

$$\omega\hat{T}_{wD} - \hat{T}_{D1} = \zeta \quad (H.41)$$

Outer Boundary Condition

$$\left\{ \kappa \frac{\partial T}{\partial r} \right\}_{r=r_e} = q_{h_{T_{ext}}} \quad (H.42)$$

Or:

$$\left(\frac{\partial \hat{T}_D}{\partial r_D} \right)_{reD} = \frac{qh_{D_{ext}}}{z} \quad (H.43)$$

Or:

$$\left(r_D \frac{\partial \hat{T}_D}{\partial r_D}\right)_{reD} = \frac{qh_{D_{ext}}r_{eD}}{Z} \quad (H.44)$$

Therefore:

$$\tau_{m+\frac{1}{2}}(\hat{T}_{D_{m+1}} - \hat{T}_{D_m}) = \frac{qh_{D_{ext}}r_{eD}}{Z} \quad (H.45)$$

Discretised Equation for First and Final Grid

Initializing from grid 1 and not the wellbore using equation 84 and 85:

$$\begin{aligned} \left[\left(1 - \frac{1}{\omega}\right)\tau_{wf} + \tau_{\frac{1}{2}} - \eta + \frac{Z}{2}\Delta r_{D_1}^2\right]\hat{T}_{D_1} - \left[\tau_{\frac{1}{2}} - \eta\right]\hat{T}_{D_2} \\ = \left[\frac{\Delta r_{D_1}^2}{2} + \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{Z}\Delta r_{D_1}\right] + \frac{\tau_{wf}\zeta}{\omega} \end{aligned} \quad (H.46)$$

Equation for final grid block:

$$\begin{aligned} -\tau_m\hat{T}_{D_{m-1}} + \left[\tau_m + \frac{Z\Delta r_{D_m}^2}{2}\right]\hat{T}_{D_m} \\ = \frac{\Delta r_{D_m}^2}{2} + \Delta r_{D_m} \frac{\dot{\beta}e^{-\alpha r_w(r_D-1)}}{Z} \\ + [\tau_{m+1} - \eta] \frac{qh_{D_{ext}}}{Z}(r_{D_{m+1}} - r_{D_m}) \end{aligned} \quad (H.47)$$

Therefore:

$$\begin{aligned} -\tau_{m-\frac{1}{2}}\hat{T}_{D_{m-1}} + \left[\tau_{m-\frac{1}{2}} + \frac{Z}{2}\Delta r_{D_m}^2\right]\hat{T}_{D_m} \\ = \left[\frac{\Delta r_{D_m}^2}{2} + \frac{\dot{\beta}e^{-\alpha r_w(r_{D_m}-1)}}{Z}\Delta r_{D_m}\right] + \frac{qh_{D_{ext}}r_{eD}}{Z} \left[1 - \frac{\eta}{\tau_{m+\frac{1}{2}}}\right] \end{aligned} \quad (H.48)$$

Appendix I

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \frac{\partial p_1}{\partial r} = \left(\frac{\phi c_t \mu}{k} \right)_1 \frac{\partial p_1}{\partial t} \quad \text{for } r_w \leq r \leq R \quad (I.1)$$

$$\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial p_2}{\partial r} = \left(\frac{\phi c_t \mu}{k} \right)_2 \frac{\partial p_2}{\partial t} \quad \text{for } R \leq r \leq r_e \quad (I.2)$$

$$\left(\frac{dp_1}{dr} \right)_{r_w} = -1, \quad (I.2)$$

$$\left(\frac{dp_1}{dr} \right)_{r_{em}} = M \left(\frac{dp_2}{dr} \right)_{r_{em}}, \quad (I.3)$$

and:

$$p_1 = p_2 \text{ when } r = r_{em}. \quad (I.4)$$

The following variables are used to derive the dimensionless form:

$$r_D = \frac{r}{r_w}, \quad (I.5)$$

$$r_{eD} = \frac{r_e}{r_w} \quad (I.6)$$

$$R_D = \frac{R}{r_w} \rightarrow R = \text{Interface radius} \quad (I.7)$$

$$t_D = 0.000264 \frac{t}{r_w^2} \left(\frac{k}{\phi c_t \mu} \right)_1 \quad (I.8)$$

$$p_{D1} = \frac{0.00708 k_1 h (p_i - p_1)}{q \mu_1} \quad (I.9)$$

$$p_{D2} = \frac{0.00708 k_2 h (p_i - p_2)}{q \mu_2} \quad (I.10)$$

$$F = \frac{\left(\frac{k}{\phi \mu c_t} \right)_1}{\left(\frac{k}{\phi \mu c_t} \right)_2} \quad (I.11)$$

$$M = \frac{\left(\frac{k}{\mu} \right)_1}{\left(\frac{k}{\mu} \right)_2} \quad (I.12)$$

Thus, by using (I.3) to (I.12) in (I.1 – I.2), we obtain:

$$\frac{\partial^2 p_{D1}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{D1}}{\partial r_D} = \frac{\partial p_{D1}}{\partial t_D} \quad \text{for } 1 \leq r_D \leq R_D \quad (I.13)$$

And

$$\frac{\partial^2 p_{D2}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{D2}}{\partial r_D} = F \frac{\partial p_{D2}}{\partial t_D} \quad \text{for } R \leq r_D \leq r_{eD} \quad (I.14)$$

And initial conditions:

$$p_{D1}(r_D, t_D = 0) = p_{D2}(r_D, t_D = 0) = 0 \quad (I.15)$$

Inner boundary condition:

$$\left(\frac{d\hat{p}_{D1}}{dr_D} \right)_{r_D=1} = \frac{-1}{z} \quad (I.16)$$

OBC, closed outer boundary:

$$\left(\frac{d\hat{p}_{D2}}{dr_D} \right)_{r_D=r_{eD}} = 0 \quad (I.17)$$

Interface condition, equal flux:

$$\left(\frac{d\hat{p}_{D2}}{dr_D} \right)_{r_D=R_D} = M \left(\frac{d\hat{p}_{D1}}{dr_D} \right)_{r_D=R_D} \quad (I.18)$$

Interface condition, equal pressure:

$$p_{D1} = p_{D2} \text{ when } r_D = R_D \quad (I.19)$$

The solution for I.13

$$\hat{p}_{D1} = C_1 I_0(r_D \sqrt{z}) + C_2 K_0(r_D \sqrt{z}) \quad (I.20)$$

Obtaining the derivative:

$$\frac{d\hat{p}_{D1}}{dr_D} = C_1 \sqrt{z} I_1(r_D \sqrt{z}) - C_2 \sqrt{z} K_1(r_D \sqrt{z}) \quad (I.21)$$

The solution for I.14

$$\frac{\partial^2 p_{D2}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{D2}}{\partial r_D} = F \frac{\partial p_{D2}}{\partial t_D} \quad \text{for } R \leq r_D \leq r_{eD} \quad (I.22)$$

The solution is

$$\hat{p}_{D2} = C_3 I_0(r_D \sqrt{Fz}) + C_4 K_0(r_D \sqrt{Fz}) \quad (I.23)$$

Obtaining the derivative:

$$\frac{d\hat{p}_{D2}}{dr_D} = C_3 \sqrt{Fz} I_1(r_D \sqrt{Fz}) - C_4 \sqrt{Fz} K_1(r_D \sqrt{Fz}) \quad (I.24)$$

Using inner boundary condition:

$$C_1 \sqrt{z} I_1(\sqrt{z}) - C_2 \sqrt{z} K_1(\sqrt{z}) = -\frac{1}{z} \quad (I.25)$$

Or:

$$C_1 = \frac{z^{\frac{3}{2}} C_2 K_1(\sqrt{z}) - 1}{z^{\frac{3}{2}} I_1(\sqrt{z})} \quad (I.26a)$$

Or:

$$C_2 = \left(\frac{C_1 z^{\frac{3}{2}} I_1(\sqrt{z}) + 1}{z^{3/2} K_1(\sqrt{z})} \right) \quad (I.26b)$$

Using OBC, closed outer boundary (I.17) in (I.24)

$$C_3 = \frac{C_4 K_1(r_{eD} \sqrt{FZ})}{I_1(r_{eD} \sqrt{FZ})} \quad (I.27)$$

Using interface condition, equal flux:

$$\left(\frac{d\hat{p}_{D2}}{dr_D} \right)_{r_D=R_D} = M \left(\frac{d\hat{p}_{D1}}{dr_D} \right)_{r_D=R_D} \quad (I.28)$$

$$C_3 \sqrt{FZ} I_1(r_D \sqrt{FZ}) - C_4 \sqrt{FZ} K_1(r_D \sqrt{FZ}) = MC_1 \sqrt{Z} I_1(r_D \sqrt{Z}) - MC_2 \sqrt{Z} K_1(r_D \sqrt{Z}) \quad (I.29)$$

$$[C_4] = \frac{[I_1(r_{eD} \sqrt{FZ})] (MC_1 \sqrt{Z} I_1(R_D \sqrt{Z}) - MC_2 \sqrt{Z} K_1(R_D \sqrt{Z}))}{K_1(r_{eD} \sqrt{FZ}) \sqrt{FZ} I_1(R_D \sqrt{FZ}) - \sqrt{FZ} K_1(R_D \sqrt{FZ}) I_1(r_{eD} \sqrt{FZ})} \quad (I.30)$$

Substituting equations I.20 and I.23 into Interface condition, equal pressure (I.19)

$$C_1 I_0(R_D \sqrt{Z}) + C_2 K_0(R_D \sqrt{Z}) = C_3 I_0(R_D \sqrt{FZ}) + C_4 K_0(R_D \sqrt{FZ}) \quad (I.31)$$

By substituting equations for constants (I.26b, I.27, I.30) into (I.31), expanding, and simplifying:

$$\frac{z^{\frac{3}{2}} C_2 K_1(\sqrt{Z}) I_0(R_D \sqrt{Z}) + C_2 K_0(R_D \sqrt{Z}) z^{\frac{3}{2}} I_1(\sqrt{Z})}{z^{\frac{3}{2}} I_1(\sqrt{Z})} = C_2 XY - \left[\frac{M \sqrt{Z} I_1(R_D \sqrt{Z}) X - I_0(R_D \sqrt{Z})}{z^{\frac{3}{2}} I_1(\sqrt{Z})} \right] \quad (I.32)$$

Where:

$$X = \frac{[K_1(r_{eD} \sqrt{FZ}) I_0(R_D \sqrt{FZ}) + I_1(r_{eD} \sqrt{FZ}) K_0(R_D \sqrt{FZ})]}{\sqrt{FZ} [K_1(r_{eD} \sqrt{FZ}) I_1(R_D \sqrt{FZ}) - I_1(r_{eD} \sqrt{FZ}) K_1(R_D \sqrt{FZ})]} \quad (I.33)$$

$$Y = M \sqrt{Z} \frac{K_1(\sqrt{Z}) I_1(R_D \sqrt{Z}) - I_1(\sqrt{Z}) K_1(R_D \sqrt{Z})}{I_1(\sqrt{Z})} \quad (I.34)$$

Thus:

$$[C_2] \left[-\frac{K_1(\sqrt{Z}) I_0(R_D \sqrt{Z}) + K_0(R_D \sqrt{Z}) I_1(\sqrt{Z})}{I_1(\sqrt{Z})} + XY \right] = \left[\frac{M \sqrt{Z} I_1(R_D \sqrt{Z}) X - I_0(R_D \sqrt{Z})}{z^{\frac{3}{2}} I_1(\sqrt{Z})} \right] \quad (I.35)$$

Rearranging and simplifying:

$$C_2 = \left[\frac{M \sqrt{Z} I_1(R_D \sqrt{Z}) X - I_0(R_D \sqrt{Z})}{z^{\frac{3}{2}} I_1(\sqrt{Z}) (XY - V)} \right] \quad (I.36)$$

Where:

$$V = \frac{K_1(\sqrt{Z}) I_0(R_D \sqrt{Z}) + K_0(R_D \sqrt{Z}) I_1(\sqrt{Z})}{I_1(\sqrt{Z})} \quad (I.37)$$

NOMENCLATURE

∇ : Del operator

p : *Pressure*

p_i : *Initial reservoir pressure*

q_{sc} : *Flow rate at standard conditions*

V : *System bulk volume*

P : *Power*

t : *Time*

T : *Temperature*

T_i : *Initial reservoir temperature*

ε : *Permittivity*

ε_0 : *Permittivity of free space*

σ : *Conductivity*

γ : *Effective conductivity*

δ : *Loss angle*

$\vec{E}$: *Electric field*

$\dot{E}$: *Energy Flowrate*

ϕ : *Porosity*

S : *Saturation*

ω : *Angular frequency*

κ : *Conductivity*

k : *Permeability*

k_{em} : *Electromagnetic permeability*

K : *Permeability Tensor*

α_e : *Electrical field absorption coefficient*

u : *Velocity*

ν : *Kinematic viscosity*

e : *Specific internal energy*

ρ : *density*

ψ : *Power density*

$\vec{F}$: *Heat Flux*

h_f : *Fluid Enthalpy*

C_p : *Heat Capacity at constant pressure*

C_V : *Heat Capacity at constant temperature*

C_{pT} : *Total Heat Capacity at constant pressure*

c_r : *Rock Isothermal Compressibility*

β_r : *Rock Isobaric Compressibility*

c_f : *Fluid Isothermal Compressibility*

β_f : *Fluid Isobaric Compressibility*

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Academic Background :

- **University:** University of New South Wales
- **Degree:** Bachelor of Petroleum Engineering (Honours)
- **Graduation Date:** December 2018
- **GPA:** Distinction, Honours Class 2 Division 1

Experiences :

- **Company** : Saudi Aramco
- **Position** : Petroleum Engineer
- **Duration** : December 2018 – Present
- **Job Roles & Duties** :
 - Gas Facilities Engineer (March 2022 – Present)
 - Planning of both materials and budget for gas wells requirements across various fields. This includes but is not limited to material forecasting and project management tasks.
 - Coordinating with various entities within and outside Saudi Aramco to ensure appropriate completion of all projects milestones as planned.
 - Evaluating and coordination various projects overlapping the same field from different Saudi Aramco entities such that they do not come

into conflict with one another while simultaneously meet all safety constraints.

- Planning, and coordinating various tie-in of gas wells which require extensive technical analysis and optimization to minimize cost.
- Subsurface Technology Engineer (March 2021 – February 2022)
 - Continuously monitoring a fit-for-purpose subsurface technology portfolio for Saudi Aramco uses to be utilized in both oil and gas reservoirs for various vendors maximizing quality, vendor diversification, and utility. Such technologies include but are not limited to inflow control devices, stimulation downhole equipment such as ports, packers, balls, and complete systems with various pressure ratings and functionalities.
 - Approaching various vendors to plan and bring new technologies to Saudi Aramco. In charge of reviewing each technology extensively and following up with the vendor for additional testing, simulation, and clarifications.
 - Coordinating and following up on various joint projects with various research institutes relating to downhole equipment and stimulation fluids/additives.
 - Designing acid stimulation programs including the pump stages, pump volumes, and pump rates.
- Gas Reservoir Engineer (December 2018 – February 2021)
 - Planning future wells based on multi-well performance analysis, geology, geophysics, well testing results, and simulation models all in conjunction with both surface facilities and safety constraints

- Production allocation across fields based on demand while ensuring uniform drawdown, optimal production strategy, and avoiding any potential production issues such as hydrates and sands production.
- Point of contact and decision-maker on various gas operations including drilling vertical and horizontal wells, workovers, and various rigless stimulation operations in both sandstone and carbonate reservoirs.
- Optimizing individual well completion strategy and identifying best completion intervals with state-of-the-art technologies including stimulation requirement based on analysis of petrophysical logs, offsets, simulation models, and pressure transient responses.
- Planning future reservoir surveillance requirements in addition to continuous monitoring and comparison of various well/reservoir characteristics including but not limited to fluid mobilities, isobaric maps, material balance, total produced volumes, and gas-in-place.

Interests :

- Reservoir Engineering
- Well Test Analysis
- Artificial Intelligence
- Optimization
- Computational Methods