

**ANALYSIS AND CONTROL OF DYNAMIC RESPONSE OF BEAMS WITH NON-
CLASSICAL BOUNDARY CONDITIONS SUBJECTED TO A RECIPROCATING LOAD
USING FIXED AND MOVING LINEAR SPRING-MASS ABSORBER**

BY

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DHAHRAN, SAUDI ARABIA

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In

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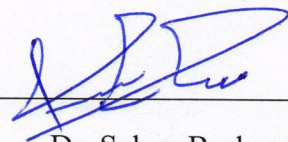
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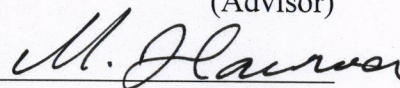
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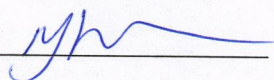
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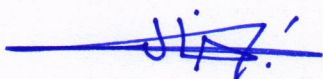
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*In the name of Allah, the Most
Gracious and the
Most Merciful*

Dedicated

to

My Beloved Parents, Grandmother, Siblings,

and Friends

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First and foremost, all praises and my deepest gratitude to ALLAH ALMIGHTY for his endless blessings, bounties, and mercy and for bestowing wisdom, health and patience upon me to finish this research. Many peace and blessings be upon Prophet Muhammad (PBUH), his family and his companions.

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THESIS ABSTRACT (ENGLISH)

NAME: OMER NADEEM

TITLE: ANALYSIS AND CONTROL OF DYNAMIC RESPONSE OF BEAMS WITH NON-CLASSICAL BOUNDARY CONDITIONS DUE TO SINUSOIDAL RECIPROCATING LOAD USING FIXED AND MOVING LINEAR SPRING-MASS ABSORBER

MAJOR FIELD: MECHANICAL ENGINEERING

DATE OF DEGREE: (DEC 2021)

The present work is focused on analysis and control of dynamic responses of the beam having non-classical boundary conditions, which are enhanced in such a way that they are simulating eigenvalues of classically established beams. The elastic beam in this study is excited by a moving load reciprocating continuously (back and forth) along full length or partial length of beam at multiple cycles. Linear spring-mass absorber are employed by fixing them at various locations or by setting absorber in motion at optimum speed or by traversing absorber to optimum travel length coupled with optimum speed (maintaining synchronized or unsynchronized motion with mentioned load). In terms of analysis, the beams with flexible support are analysed using the Euler–Bernoulli theory and for solving methodology, discretization methods are used to solve initial-boundary value partial differential equations by discretizing with respect to all but one of the independent variables, resulting in a system of ODEs, which is then solved numerically. The dynamic defections obtained, are then analysed to obtain set of critical speeds that is causing

resonance or cancellation of the free response and then analysing how different absorber configurations affect the dynamic amplitudes and critical speeds. The performance of the absorber is evaluated through the variation of the maximum deflections during vibration. The dynamic deflections are utilized to obtain an optimum set of speed and travel length parameters for moving linear spring-mass absorber. The prospective uses for current load circumstances (sinusoidal and reciprocating) offer us a notion of how it could be used to reduce vibration, as a fixed or moving vibration absorber, or for amplification, in energy recycling or harvesting processes. The results obtained are reviewed to have insight into dynamic deflection variation, critical speeds estimation and absorber optimum configurations for multiple cycles of reciprocating load and travel lengths in non-classical beams.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

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THESIS ABSTRACT (ARABIC)

ملخص الرسالة

الاسم: عمر نديم

عنوان الرسالة: تحليل والتحكم في الاستجابة الديناميكية للأعمدة غير التقليدية الخاضعة لحمل متبادل باستخدام مثبت كتلة

زبركية خطية ثابتة ومتحركة

التخصص: الهندسة الميكانيكية

تاريخ التخرج: 1442 هـ - (ديسمبر 2021 م)

يركز العمل الحالي على التحليل والتحكم في الاستجابات الديناميكية للحزمة ذات الشروط الحدودية غير الكلاسيكية ، والتي يتم تحسينها بطريقة تحاكي القيم الذاتية للحزم التقليدية. يتم تحفيز الحزمة المرنة في هذه الدراسة من خلال حمل متحرك يتكرر باستمرار (ذهابًا وإيابًا) بطول كامل أو جزئي للحزمة في دورات متعددة. يتم استخدام المثبط الخطي للكتلة الزبركية مع نفس خصائص المواد ولكن مع تكوينات مختلفة عن طريق تثبيتها في مواقع مختلفة أو عن طريق ضبط المثبط في الحركة على السرعة المثلى أو عن طريق عبور المثبط إلى الطول الأمثل مقترنًا بالسرعة المثلى (الحفاظ على حركة متزامنة أو غير متزامنة مع الحمل المذكور).

أما من حيث التحليل ، فيتم تحليل حزم الدعم المرنة باستخدام نظرية أويلر-برنولي ولحل المنهجية ، يتم استخدام طرق التقدير لحل المعادلات التفاضلية الجزئية لقيمة الحدود الأولية عن طريق التقدير فيما يتعلق بجميع المتغيرات المستقلة باستثناء واحد ، مما يؤدي إلى نظام ODEs ، والذي يتم حله بعد ذلك عدديًا .

يتم بعد ذلك تحليل الانشقاقات الديناميكية التي تم الحصول عليها للحصول على مجموعة من السرعات الحرجة التي تسبب صدى أو إلغاء للاستجابة الحرة ثم تحليل كيفية قيام تكوينات المثبط المختلفة بتحويل السعات الديناميكية والسرعات الحرجة. يتم تقدير أداء المثبط من خلال المؤشر الرئيسي لانحراف أقصى انحرافات أثناء الاهتزاز ، يتم استخدام هذه المؤشرات المذكورة لتطوير مجموعة مثلى من معلمات السرعة وطول الحركة لتحريك المثبط الخطي النابض.

تقدم لنا الاستخدامات المستقبلية لظروف الحمل الحالية (الجيبية والتبادلية) فكرة عن كيفية استخدامها لتقليل الاهتزاز ، كمثبط اهتزاز ثابت أو متحرك ، أو للتضخيم ، في إعادة تدوير الطاقة أو عمليات الحصاد. تتم مراجعة النتائج التي تم الحصول عليها للحصول على نظرة ثاقبة للتباين الديناميكي للخلل وتقدير السرعات الحرجة والتكوينات المثلى للمثبط لدورات متعددة من الحمل المتبادل و الأطوال في الحزم غير الكلاسيكية.

شهادة ماجستير علوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

NOMENCLATURE

A	Cross section area of the beam
b	Width of beam
I	Moment of inertia
m	Mass of beam
h	Height of the beam
ρ	Density of beam
E	Elastic modulus of beam
w_o	Static deflection of beam
k_d	Spring stiffness of absorber
m_d	Mass of absorber
c_d	Damping coefficient
δ	Dirac delta function
$v = V_f$	Dimensional Speed of moving load
V_d	Dimensional speed of moving absorber
V_c	Critical Velocity
m_a	Ratio of absorber speed to load speed
$F(x, t)$	Force vector in spatial and temporal field
$G(x, t)$	Absorber force vector in spatial and temporal field
$Z(t)$	Absorber velocity vector in time domain as a function of p

$H(t)$	Absorber velocity vector in time domain as a function of η
$k_r=k_\theta$	Torsional spring stiffness
$k_l=k_m$	Linear spring stiffness
ω_n	Eigen value for mode shape
q	No of cycles (1,2,3 and 4) with $q=0.5$ for half cycle
P	Force of moving load
r	Dimensionless speed ratio for load
ρ	Dimensionless length parameter ratio for load travelling
q	No of reciprocating cycles ($q = n$, where $n=0.5, 1, 2, 3, 4\cdots$)
γ	Non-Dimensional Geometrical and Material Properties of beam
c_1	Dimensionless damping coefficient of beam
c_2	Dimensionless damping coefficient of absorber
k_1	Dimensionless spring stiffness of absorber
c_3	Dimensionless ratio of damping to mass of absorber
k_2	Dimensionless natural frequency of absorber
u	Displacement vector
M	Moment
w	Transverse beam displacement at given spatial position (x) and time (t) - Dimensional
$\hat{w}$	Transverse beam displacement at given spatial position (x) and time (t) - Non-dimensional
$\hat{x}$	Dimensional spatial position of load

x	Non-dimensional spatial position of load
$\hat{y}$	Dimensional absorber displacement in spatial domain
y	Non- dimensional absorber displacement in spatial domain
$\hat{t}$	Dimensional time domain
t	Non-dimensional time domain
d_t	Absorber travel length
η	Dimensionless ratio of absorber travel length
i	Multiple factor for travelling length
j	Multiple factor for dimensionless critical speed

Chapter 1 : INTRODUCTION

The dynamics of beams and their responses to different types of forces excitation have always been an intriguing factor on research. Especially the dynamic deflection, their control and along with their utilization into effective manner remains a source of interest for the engineers, researchers, and scientists. With the passage of time as the structures becoming stiffer and light weight and the moving loads becoming more fast, periodic, and heavy, the analysis of these responses and their controlling is becoming more and more crucial in extensive research. Euler Bernoulli beams have always been adopted for the analysis for the reason of their simplicity and simplified solving. Many case studies have been discussed, has fast trains introduced which has continuous load along with speed surpassing the excitation response resistance, so it became the need of time to have analysis on the moving loads (either single or multiple) at high speed along with different damping and controlling conditions.

In any sector of civil, mechanical, and aeronautical engineering, beams as elements play a very crucial role. Study of amplitudes of vibrations in beams with classical boundary conditions are developed and researched in detail in several contributions. Though there are also quite contributions that dealt with the vibrations of beam systems with flexible boundary conditions, but most of them are done with constant or harmonic forward travelling load only (single or multiple). Researchers have been studying the behaviour of a flexible support beam with a moving load or a mass-spring system for decades. Because of its widespread use in a variety of engineering applications, it has become a prominent topic in the literature. – e.g., high-speed railways, wind displaced transmission wires, bridges (suspension type), overhead cranes and tethered satellite

systems. To describe the system's characteristics and the influence of moving load or force on the dynamic response of the beam, several models have been proposed [1, 2].

1.1 Background

The study of vibration, its effective control and transmission has been conducted through ages especially in the beams. As beams are the main components in almost all mechanical and civil structures with examples ranging from the rotor beam element of helicopters to the suspended bridge structures, it signifies the importance of research on beam response analysis and finding new methods for controlling and suppressing these responses.

The beam considered in the analysis is the Euler Bernoulli beam due to fact, that its analysis is simple as compared to other beams especially Timoshenko beam because of assumptions that simplify our analysis to accommodate more complex loading mechanism. The Euler-Bernoulli beam theory is the most elementary in regard to other beams theory. In Euler Bernoulli Beam theory, shear deformations are usually neglected, as a result constraints are lifted to have more convenient analysis, and the plane sections remain plane and perpendicular to the longitudinal axis. In the matrix's equations of Euler Bernoulli beam, we have only 2 degrees of freedom at the beam section level, and 4 degrees of freedom at nodal element. The constitutive law is applicable for 4 x 4 matrix of Euler Bernoulli beam in solving for the above degrees of freedoms [3].

Elastic (non-classical) supports in beams can provide us with flexibility of studying various configuration of beams by changing linear and torsional springs stiffness attached as the supports for the beam. These supports are called non-classical boundary conditions because they provide us with general equations for the analysis of beam, different than those studied with classical

boundary conditions. As a common starting point, simple beam traversed by a constant travelling force are studied by Kryloff to obtain the closed form solutions [4]. These solutions obtained has become most famous for its utilization in simply supported bridges and being further investigated by many authors [5–14]. Not only being researched by authors, but these solutions had also been adopted on theoretical basis by several regulations with Eurocode1 being one of them [15].

1.2 Euler Bernoulli beam

The Euler Bernoulli beam as shown in Figure 1.1, is the most convenient for the analysis of moving loads under many variable conditions and a complete review regarding it can be found in Fryba [1]. In comparison to the other beams e.g., Timoshenko beam, we neglect the shear deformation, and the planar section is considered to stay plane and perpendicular to the longitudinal axis, so it provides more simplified results because things get complex when shearing is added in the existing equations and its equations are also of the lower order. In this work, entire focus will be on Euler-Bernoulli beam theory because it does not include the shear displacement factor, simplifying the equation of motion [8].

The torsional effect will also be neglected in the Euler Bernoulli beam, the only rotation that can be considered will be due to bending and general boundary conditions applied on the beam will be main factor in providing variable response analysis for the periodic reciprocation load. Because vibration analysis is based on equations of motion, a thorough understanding of these equations is required. Many equations of motions have been developed considering the beam theories and methods.

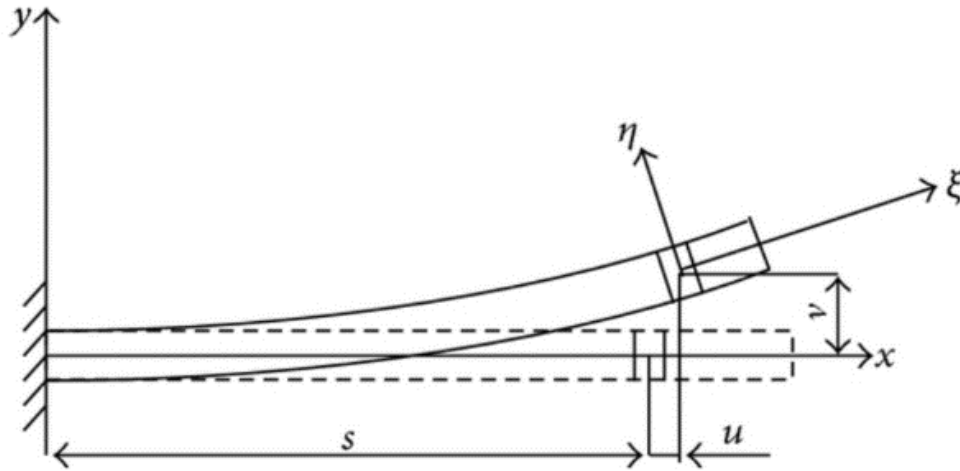


Figure 1.1 Euler Bernoulli Beam Section

In general theories while studying Euler-Bernoulli beam, following assumptions are made:

- A. Material is considered elastic, isotropic, and homogenous.
- B. The beam is having symmetry and is prismatic about the principal axis
- C. Deflection in traverse direction are small.

With these assumptions, material and geometrical properties of beam are assumed to be constants and removed outside equations integrals. It provides us further simplifications for the upcoming derivations.

1.3 Critical velocity and cancellation speeds:

Critical Velocity is one of the most crucial factors, in the beam as it plays a significant role for dynamic analysis and attaining the optimisation. The current study is done to analyse the dynamic deflection when the single sinusoidal load is set to reciprocate along the beam. The studies that examine moving loads series are the most relevant [6, 14, 16, 17]. There are several critical speeds, as will be shown later, that are having lower values when observed with single moving load.

Furthermore, it is only one load and because the load departs from the beam after a completing travel length in fixed period, so the greatest deflection cannot go beyond 1.7 times the static deflection, as a single moving load does not create unbounded response at resonance. Whereas in case of moving loads succession or reciprocating single load, resonance is induced at numerous speeds and the vibration deflection becomes higher in value than the static one and single forward travel load only. Many scholars have lately investigated the free vibration response of a simply supported beam under only one moving load – i.e., the dynamic deflection obtained as a response following the departure of load from beam. In other studies, numerous speeds have also been observed where the amplitude attains its peak value and other speeds where the amplitude cancels out and becomes equal to the static response [18–20].

1.4 Classical and Non-Classical Boundary Conditions

The beams having ends that are elastically constrained for rotation and translation, or with ends that are using masses (concentrated) or rotational inertia, being subjected to dynamic analysis is a non-classic structural problem that is becoming increasingly significant in mechanical and aeronautical engineering. Several researchers attempt to solve differential equation of fourth order, by analytically solving coefficients and assuming beam to be sufficiently slender to be termed as Euler-Bernoulli beam and then solving numerically for relatively basic boundary conditions. But it has been observed that this exact technique has difficulties in finding roots of the characteristic equation and establishing the system's normal modes.

As stated before, the most common boundary conditions in the vibration analysis of beams are classical boundary conditions and most common configurations for these boundary conditions are free, clamped, and supported ends with corresponding equations shown in Table. 1-1.

On contrary, non-classical boundary conditions are considered less common, but in recent research it is proven fact that their importance is similar in impact as classical conditions because of the flexibility they provide in analysis. The impact of change in stiffness of springs being used as beam boundaries on natural frequencies can also be analysed widening our area of research. These non-classical boundary conditions consist of masses, springs, rotational inertia or even absorbers. For these non-classical boundary conditions, corresponding equations are presented in Table. 1-2, where coefficient values for left end of beam are $a = -1$ and $b = 1$, and for right end of the beam are $a = 1$ and $b = -1$. $k_m=k_l$ and $k_r=k_\theta$ are linear and torsional spring stiffnesses respectively, similarly $c_d=c_m$ and $c_r=c_\theta$ are linear and torsional spring damping coefficient, m_c is concentrated mass value and I_c is rotational inertia value for concentrated mass [21].

Table 1-1: Classical supports boundary conditions [21]

Boundary Condition	Equations
Free end	$EI \frac{\partial^2 v(x, t)}{\partial x^2} = 0$ and $EI \frac{\partial^3 v(x, t)}{\partial x^3} = 0$
Supported end	$v(x, t) = 0$ and $EI \frac{\partial^2 v(x, t)}{\partial x^2} = 0$
Clamped end	$v(x, t) = 0$ and $EI \frac{\partial v(x, t)}{\partial x} = 0$

Table 1-2: Non-classical supports boundary conditions [21]

Boundary Condition	Equations
Linear Spring	$EI \frac{\partial^2 v(x, t)}{\partial x^2} = 0$ and $EI \frac{\partial^3 v(x, t)}{\partial x^3} = ak_m v(x, t)$
Torsional Spring	$EI \frac{\partial^3 v(x, t)}{\partial x^3} = 0$ and $EI \frac{\partial^2 v(x, t)}{\partial x^2} = bk_r \frac{\partial v(x, t)}{\partial x}$
Concentrated Mass	$EI \frac{\partial^2 v(x, t)}{\partial x^2} = 0$ and $EI \frac{\partial^3 v(x, t)}{\partial x^3} = am_c \frac{\partial^2 v(x, t)}{\partial t^2}$
Rotational Inertia	$EI \frac{\partial^3 v(x, t)}{\partial x^3} = 0$ and $EI \frac{\partial^2 v(x, t)}{\partial x^2} = bI_c \frac{\partial^3 v(x, t)}{\partial x \partial t^2}$
Damper	$EI \frac{\partial^2 v(x, t)}{\partial x^2} = 0$ and $EI \frac{\partial^3 v(x, t)}{\partial x^3} = ac_m \frac{\partial v(x, t)}{\partial t}$
Torsional Damper	$EI \frac{\partial^3 v(x, t)}{\partial x^3} = 0$ and $EI \frac{\partial^2 v(x, t)}{\partial x^2} = bc_r \frac{\partial^2 v(x, t)}{\partial x \partial t}$

1.5 Fixed and Moving -Active and Passive Spring Mass Absorber

A spring mass absorber is absorber consisting of simple combination of spring and mass with specific parameters, is relatively smaller in magnitude (size) than larger resonant or vibrating systems with sole purpose of reducing dynamic deflection. Even though size of absorber is smaller, but it consists of sufficient weight to adequately shift the resonant frequency, dynamic deflection as well as critical speed and has such a design that will not fatigue the spring of absorber. The dynamic absorber can be fixed at different locations (in middle of beam being most optimum) along the length of the beam, and it reduces or eliminates the vibration of a system that is harmonically excited. As design consideration of absorber is important, it is to be noted that to

have zero amplitude of primary system, vibration absorber to be designed with working frequency close to the excitation frequency.

Considering the common mechanisms of damping to reduce vibration, passive vibration damping devices typically use a mechanical device or a fluid, however viscoelastic materials can also be used. In either instance, the vibration's kinetic energy is transformed to heat as part of conservation of energy.

Several types of damping systems contain inherent resonances; therefore, they work best at frequencies higher than 4 Hz [22]. If requirement is to have damping in multiple directions simultaneously, passive vibration absorbers become least effective in that case which means they are optimum for vibrations induced in X, Y, or Z direction. Figure 1-2 shows simple example of passive vibration damping system, with a rigid mass and a spring as simple harmonic oscillator and an added absorber as well Active vibration damper with feedback control mechanism. The purpose of added absorber is to convert mechanical energy into heat, which can be removed from system.

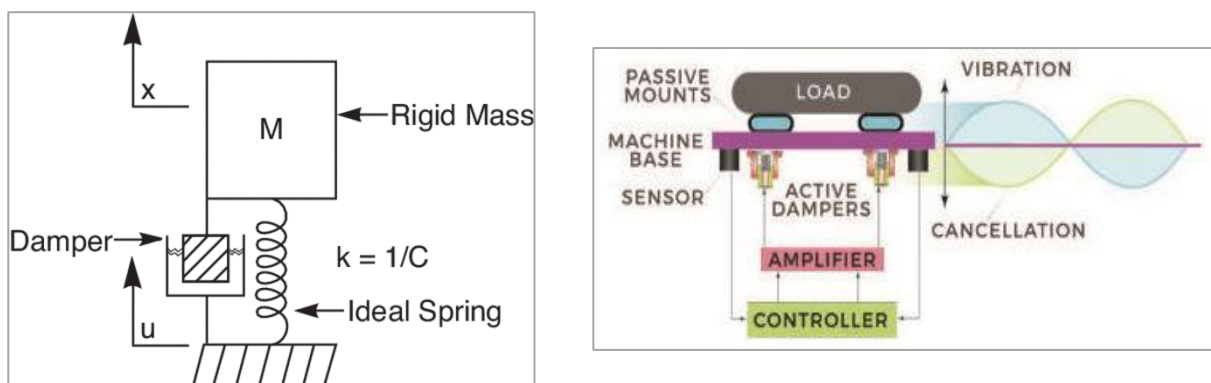


Figure 1-2 Passive and Active Vibration Damping Systems [22]

Active vibration damping systems use additional power in the form of electronically controlled sensors and actuators in a closed-loop system to counter vibrations and cease oscillations. Active vibration damping systems, unlike passive systems, can be modified to mitigate resonances, allowing them to operate at lower frequencies due to structural resonances. Also, vibrations and interference induced by the equipment itself, such as cabling noise and acoustic noise can also be reduced using active system.

By employing feed-forward control, controlling proactive damping response can be much better than reactive damping response. Whereas traditional closed-loop setups offer same mechanism but utilizes opposite forces to counteract vibrations entering the system and coupled with feed-forward enables the system to produce and send cancelling signals in advance of the expected vibrations.

1.6 Thesis Motivations and Objectives

Beam models have been studied extensively using different absorber configurations as linear systems. Majority of the studies listed assumed traveling load as a single load travelling at a constant speed along the beam's span. Even though several studies sought to investigate effects of variety of elements, which includes a base with elastic properties or spring mass absorber, it was also determined that treating the moving load and its velocity as constants was sufficient [5, 18, 19, 23–26]. Fryba et al. studied that critical speed is achieved when the driving frequency overlaps with the beam's initial natural frequency for harmonic moving load [1]. Other scientists investigated it and enhanced system complexity by incorporating various degrees of freedom and by varying velocities to simulate moving vehicles, and all these investigations were conducted with forward traveling loads and mostly for beams with classical boundary conditions [5, 23, 24, 27–30]. Some authors recently utilized reciprocating load to analyse the effect of dynamic responses, but load considered in most of these studies are constant travelling loads [31]. Other researchers studied inertia effects by studying about moving mass or spring-mass systems [4, 32].

The current research looks at the dynamic response of a single sinusoidal load reciprocating back and forth along a beam for prospective uses such as vibration reduction (as a dynamic vibration absorber) and amplification (for energy extraction). Focus will be on Euler Bernoulli beam analysis having flexible boundary conditions utilizing linear spring and torsional spring as variable flexible supports which can be varied to simulate any type of beam with main interest in simply supported and cantilever beam. In these non-classical beams of variable flexible supports, linear spring mass absorber which is passive type having mass of 5% - 10% of total beam mass will be utilized. The term “reciprocating and sinusoidal load” considered in ongoing study refers to the

load that increases its speed at start, reaches to maximum and then decreases to zero when about to reach end point and maintaining this oscillatory motion while reciprocating to and from along the length (span) of the beam at multiple cycles. In these studies, attached linear spring mass absorber will either be fixed or performing same sinusoidal motion depending on type of configuration required in analysis. The objectives of this thesis are,

1. Analysing flexible supports behaviour with different linear and torsional spring stiffnesses and increasing the stiffness value to attain eigen values and mode shapes like Simply supported beam and Cantilever beam.
2. Studying the effect of a constant load reciprocating sinusoidally with multiple cycles at different velocities. Dynamic analysis for resonance and cancellation of amplitudes for beams with non-classical end supports.
3. Analysis of dynamic response cancellation and amplification for non-classical beam with utilization of fixed absorber at least and most effective locations and with utilization of synchronized and unsynchronized moving absorber at optimum or least effective absorber speeds and travel lengths.

1.7 Organization of the work

Current thesis is categorized into seven chapters as follows:

- **Chapter 1: Introduction:** This section gives introduction about the topic analysed in the thesis. It will give information about Euler Bernoulli beam models with travelling load types, followed by some information about absorber configurations utilized and then the research objectives of the thesis.
- **Chapter 2: Literature Review:** In this chapter, literature review is done on various type of moving loads and forces, beam models adopted, solving techniques, absorber configurations, different types of boundary conditions and critical velocities resulting in resonances and cancellation effects in dynamic amplitudes.
- **Chapter 3: Mathematical Modelling:** In this chapter mathematical models are developed for Euler Bernoulli beam having dynamic amplitudes due to sinusoidal load reciprocating at multiple cycles. Mathematical models for different beam configurations with no absorber, fixed absorber and moving absorber are developed in this section.
- **Chapter 4: Validation of Results:** In this chapter numerical solutions are carried out for already established cases of dynamic amplitude analysis and comparison is done with verified previous studies to authenticate the developed mathematical models.
- **Chapter 5: Numerical analysis of dynamic deflection for non-classical simply supported beam:** In this chapter numerical solutions are carried out for dynamic amplitude analysis of non-classical simply supported beam without absorber and with stationary and moving absorber having different travelling behaviours and generated results are analysed.

- **Chapter 6: Numerical analysis of dynamic deflection for non-classical cantilever beam:** In this chapter like Chapter 5, numerical solutions are carried out for dynamic amplitude but for non-classical cantilever (fixed-free) beam without absorber or with stationary and moving absorber having different travelling behaviours and generated results from these configurations are analysed.
- **Chapter 7: Conclusions and Recommendations:** In final chapter, conclusion and summarization will be done for the work that has been carried out in the thesis against the objectives set for the study. Also, some recommendations are highlighted based on the obtained results.

Chapter 2 : LITERATURE REVIEW

Oscillations in bridges caused by traveling loads have been documented since the mid-nineteenth century, when first trains were built; these applications are then considered most important instances of traveling loads. For complete analysis of structures traversed by moving loads, one should read Fryba [1], which has various applications with clear demonstrated examples. As previously stated, Fryba's monograph is part of the first complete studies in this area, and solutions solved analytically for dynamic responses due to various load types, such as load with constant velocity, dynamic load with harmonic motion, and moving masses, were reviewed in great details in this monograph [1].

2.1 Moving Loads

When moving load influence is considered, problem of forced motion of a beam traversed by a dynamic load incorporates some complexities. Also, it is observed that results seem more realistic when moving masses are considered instead of moving loads; however, in Ref. [23] it was shown that when the ratio of the moving mass to beam mass is quite small, the beams dynamic responses under moving loads or moving masses shows quite similarity. Since load traversing structure is changing in spatial and temporal domains, the resulted deformation and stress values are significantly larger in comparison to those examined with stationary loads. So, for simplification, the most common type of analyses is with beam as Euler-Bernoulli model and moving load that is constant in its motion [18, 19, 25, 26]. Olsson et al. provided detail fundamental characteristics of the moving load problem utilizing analytical and finite element solutions [5]. Yau et al. studied response of simple beam in vertical direction due to passage of single load considering light damping [28].

Law et al. studied the travelling force in both temporal and frequency-based domains. In other work [29] . Lu et al. [30] analysed that the frequency characteristics for train load (also known as driving frequencies) showed significant influence on the railway bridge dynamic response and this conclusion was made when railway bridge was subjected to single excitation load. Several scholars have examined and reported on resonances and cancellation events in beams under changing loads, with some credible work [1, 5, 28–30, 33, 34].

Even though one of the first modern studies for finding an analytical solution and deriving an expression of critical velocity for moving loads that are traversing with constant motion in Timoshenko beam is by Timoshenko et al. [3]. Still the analysis of simple Euler-Bernoulli beams crossed by harmonic loads is of interest from both points of view (theoretical and practical), with the aim of assessing the involved parameters influence on the dynamics, and from a technical point of view, to provide efficient tools to estimate easily the maximum dynamic response and the dynamic amplification factor. Moving harmonic loads instead of constant load can represent, for instance, a component of the load transmitted to rails structure by moving trains, the load applied by vehicles resulting from the rough surface of pavements and vehicle mechanical systems, or the load exerted on a footbridge by a walking pedestrian [35–37].

Euler-Bernoulli beam is the most convenient for the analysis of moving loads under many variable conditions and many credible authors considered the Euler-Bernoulli beam models in their studies of moving loads, critical velocity analysis and absorber optimization [2, 5, 27–30].

The Euler–Bernoulli theory is used to analyse simply supported and cantilever beam, the Galerkin–Bubnov method utilized for reduction of equations from PDE to an ODE set, and multimode expansion for displacement field is utilized in numerical analysis of the problem. The maximum amplitude and the percentage of dissipated energy by the attached absorber are used to evaluate

dynamic absorber performance in vibration reduction. For designing an optimization strategy, same indicators are employed as prime factors of analysis [23]. Detailed dynamic analysis for the cantilever beam is also undertaken for modal analysis and observing natural frequencies [38].

2.2 Non-classical boundary conditions

Flexible (non-classical) boundary conditions, in comparison with general boundary conditions, are efficient for various configuration studies of excitation forces, and at the appropriate stiffness of these boundary conditions they can provide wide range of statistical study to understand the frequency range, but no numerical analysis has been done in this regard for the reciprocating moving loads having sinusoidal load along-with stationary and moving absorber. Yang et al. conducted an analytical study and found that beams show resonant (real) speeds at lower value than simply supported beams when they are elastically supported, but their cancellation (real) speeds are comparable, but comparison can be done for cancellation (real) speeds with beams having simple supports [28]. Comprehensive study with closed form solutions was done for the general boundary conditions for the harmonically excited vibrations due to sinusoidal moving load, traversed single force and mass with constant motion without using any absorber and some analysis was done for these beams include fixed-fixed, simply supported, fixed-free and pinned-fixed configurations. In addition to this, force motion of three types is considered which are: uniform motion, accelerated and decelerated, beams response is determined with equations in closed forms and comparison analysis done with established results. The effects of the moving force frequency and speed on beams dynamic behaviour are researched in detail [39, 40].

Another important factor is that flexible boundary conditions have shown effect on the resonant frequency and critical speed and are considered special case of simply supported, clamped or

cantilever beams boundary conditions. Beams dynamic analysis with boundaries restricted elastically against translation and rotation, or with concentrated masses or rotational inertia linked to ends, is a classic problem of structural studies that is becoming increasingly significant in mechanical and aeronautical engineering. Several writers have attempted to solve differential equation of fourth order with changing coefficients analytically by assuming sufficiently slender beam to be termed a Euler-Bernoulli beam [41].

Most of the work with flexible or non-classical boundary conditions is done on the free vibration of the Euler Bernoulli beam and for studying various mode shapes with different stiffnesses, material and rotational inertias of the linear, concentrated masses, torsional springs etc. attached at the ends [42]. The study of elastic structure stability-initiated interest in non-classical mechanical boundary conditions, and a handy reference for many mechanical loadings examined till 1963 are in Bolotin's book [43].

Many authors used different solving techniques to obtain accurate results for free vibration response of the non-classical boundary conditions. For various nonclassical boundary conditions, many numerical results were also provided [44]. Cantilever beam frequencies of free vibrations with different cross-sectional areas and constraining springs as boundary were also examined in some studies [45]. Free vibration responses have been done for different beam configurations and cross-sectional areas utilizing various boundary conditions by many authors [21, 46–49].

Zhiwie et al. to examine the vibration properties of flexural wave in the compound plate, which is periodic in nature, had used the flexible boundary conditions. In his work, he adopted boundary conditions general model by using the specific combination of linear and torsional springs. And

these combination in accordance with stiffness were used to analyse all the non-dissipation boundary condition which also included the classical boundary conditions [50].

2.3 Spring Mass Vibration Absorbers

Vibrations in structures can be regulated by using absorbers of active or passive type mounted on beams, that are particularly useful assuming the structure cannot be modified. Passive devices provide several advantages, including low maintenance and the lack of a need for external power. Beams provide a fundamental model for the structural elements' comparison analysis with attached absorber for many engineering applications [41]. Moving mass-spring absorber systems are also used in more accurate models by considering the inertia and bounce effect of moving object [19, 24, 26]. To solve the system's equations of motion, various analytical and numerical methodologies were applied. – such as finite element methodology [6], penalty method formulation [51] and Lagrange multiplier analysis [52]. Ouyang provided a brief overview of efforts on shifting loads and their applications [7]. More recently, Svedholm et al. studied about beam response with non-classical boundary conditions and non-proportional damping [53].

For beams subjected to moving loads, Wu suggested optimum location of linear dynamic absorber in centre of the beam span [54]. Following Den Hartog's [55] technique, the simplified model was also utilized to derive values for ideal stiffness of absorber and damping ratio. It should be noted that the absorber equation in the Wu model was changed to account for the spring mass [24].

Kwon et al. discovered that for bridge traversed by high-speed train, placement of tuned mass absorber (TMD) in middle, resulted in maximum displacement reduction in vertical direction by 21% and the free vibration dampened more quickly [56].

Ref. [25] describes a new technique for decreasing resonance of simply supported beams under shifting loads. The load carrying main beam, was connected to an auxiliary beam positioned beneath the main one using viscous absorbers in this treatment. The results suggest that with this type of device, main beam resonant response can be considerably reduced [57]. Xia et al. shown that bridge damping influences the cancellation effect, and that as damping increases, cancellation efficiency decreases [58]. Greco and Santini investigated beam subjected to moving loads and having ends constrained by two rotational viscous absorbers and found that the absorber's efficacy is highly impacted by the speed of moving load [59]. Similarly, influence of linear and nonlinear absorber Euler Bernoulli beam dynamic responses, when subjected to moving loads was studied by Samani and Pellicano [26]. When compared to linear absorber, nonlinear absorber does not give a considerable improvement in vibration response [19]. Later, the research is expanded to look at the response of moving loads series [24, 26]. Some of the authors used moving absorber also to analyse the effect of it, on beam dynamic response and how it is related with the speed of the moving load. Similar work is done by Soares et al. for reducing vibration of the beam by utilizing a moving absorber [60]. Recently, dynamic response of a non-classically damped beam traversed by moving mass-spring-absorber system with generic boundary conditions was investigated [61].

2.4 Critical velocity

Critical Velocity is one of the most important factors in the beam as it plays a major role in cancellation and resonance phenomena for deflection in the beam. Structural vibrations can become extremely severe when moving load velocity exceeds a specific value known as critical velocity, which corresponds to the minimum phase velocity of the bending wave propagating within the beam-support system for an infinite beam[33]. In another study, it is observed that

bending wave propagation is the lowest between the modal resonant velocities for finite beams [34]. Diego et al. investigated the elastic beam critical velocities and calculated the harmonic-variable magnitude over time. This harmonic variable magnitude is characterized in terms of the oscillation's frequency, amplitude mean value. Using these variables, influence on critical velocities were explored using parametric analysis [62].

Vladimir et al. utilized the maximally allowed critical velocity to determine stability conditions in critical systems [63]. The approaching of moving load to the critical velocity results in the increased deterioration of the supporting structure which can result in negative effect on surrounding structures and not only that, if the critical velocity is exceeded, significant problems can occur, with train derailment and fatigue failure as a result [64]. Recently, Bashmal et al. determined Euler–Bernoulli beam critical and cancellation speeds when subjected to moving load having continuous motion [31]. Since critical velocity is important factor in the moving load beam analysis, its analysis was taken into consideration using the flexible boundary conditions with the reciprocating moving load.

From the literature review there is a lot of research work done for classical beams, especially for fixed absorber and one way may moving load (either single or multiple or harmonic) on structures. Studies on maximum deflection in beam with non-classical boundary conditions can also be found, but with different interests and mostly those are limited to identification of natural frequencies in free vibration or with fixed absorber. None of these studies dealt with the effect of continuously reciprocating absorber and reciprocating moving load to study the effect of maximum deflection and time response in beams. It is hoped that the results of this proposed work will assist in understanding the reciprocating load and reciprocating absorber effect on the maximum deflection of beams and their correlation with the critical velocities.

In this work, the Euler-Bernoulli linear beam theory and non-classical boundary conditions are used to study the vibrations control of non-classical simply supported beams and cantilever beam having linear and torsional springs of critically high stiffness as a support, thus simulating the effect of simply supporting beam subjected to reciprocating loads with multiple periodic cycles and controlled by a fixed, moving, or reciprocating absorber. In this study, absorber is characterized as a linear spring-mass-absorber system moving with a defined velocity along linear elastic continuous beam. The lateral displacements of the beam are modelled using a three-mode modal expansion, and the Galerkin method is used to create equations of motions sets discretised in nature, which are then solved using the Runge-Kutta method. The preliminary findings demonstrate the importance of the absorber's position and velocity in beam vibration management and can be used by engineers to optimize the position, travel speed, and travel length of vibration absorber.

Chapter 3 : MATHEMATICAL MODELING

The equations of motion govern the behaviour of the beam with flexible boundary conditions under moving loads and a moving absorber derived here following the Euler Bernoulli's beam equation model.

3.1 Euler-Bernoulli Beam on Non-Classical Supports

Consider the Euler Bernoulli beam with dimensional length “ L ” and specific mass “ m ” as shown in Figure 3.1 and applying Glarekin and Runge-Kutta method to examine the maximum deflection at different percentages of critical velocities. The mathematical modelling of the non-classical beam will be done as follows,

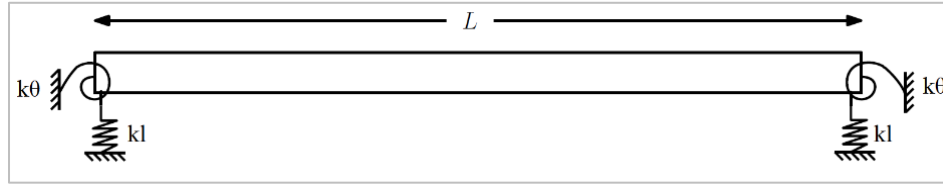


Figure 3.1 : Euler-Bernoulli beam with linear spring and torsional spring as flexible supports

Dimensional boundary conditions of Euler Bernoulli beam with linear springs having stiffness k_l and torsional spring with stiffness k_θ is given in dimensional form as follows [21],

$$EI \frac{\partial^3 \hat{w}(0, \hat{t})}{\partial \hat{x}^3} = k_l \hat{w}(0, \hat{t}) \quad EI \frac{\partial^3 \hat{w}(L, \hat{t})}{\partial \hat{x}^3} = -k_l \hat{w}(L, \hat{t}) \quad (3.1)$$

$$EI \frac{\partial^2 \hat{w}(0, \hat{t})}{\partial \hat{x}^2} = k_\theta \frac{\partial \hat{w}(0, \hat{t})}{\partial \hat{x}} \quad EI \frac{\partial^2 \hat{w}(L, \hat{t})}{\partial \hat{x}^2} = -k_\theta \frac{\partial \hat{w}(L, \hat{t})}{\partial \hat{x}} \quad (3.2)$$

Applying non-dimensional parameters given in Eq. (3.3),

$$x = \frac{\hat{x}}{L} \quad w = \frac{\hat{w}}{w_o} \quad (3.3)$$

Now substituting Eq. (3.3) in Eq. (3.1) and Eq. (3.2) results in,

$$\frac{EIw_o}{L^3} \frac{\partial^3 w(0, t)}{\partial x^3} = k_l w(0, t) \quad \frac{EIw_o}{L^3} \frac{\partial^3 w(L, t)}{\partial x^3} = -k_l w(L, t) \quad (3.4)$$

$$\frac{EIw_o}{L^3} \frac{\partial^2 w(0, t)}{\partial x^2} = k_\theta \frac{\partial w(0, t)}{\partial x} \quad \frac{EIw_o}{L^3} \frac{\partial^2 w(0, t)}{\partial x^2} = -k_\theta \frac{\partial w(0, t)}{\partial x} \quad (3.5)$$

Re-arranging the above boundary conditions from Eq. (3.4) and Eq. (3.5) to obtain,

$$\frac{\partial^3 w(0, t)}{\partial x^3} = \frac{k_m L^3}{EIw_o} w(0, t) \quad \frac{\partial^2 w(0, t)}{\partial x^2} = \frac{k_\theta L^3}{EIw_o} \frac{\partial w(0, t)}{\partial x} \quad (3.6)$$

$$\frac{\partial^3 w(1, t)}{\partial x^3} = -\frac{k_m L^3}{EIw_o} w(1, t) \quad \frac{\partial^2 w(1, t)}{\partial x^2} = -\frac{k_\theta L^3}{EIw_o} \frac{\partial w(1, t)}{\partial x} \quad (3.7)$$

3.2 Mathematical modelling of beam with moving load and without attached spring mass absorber

In this section, there is no linear spring-mass absorber is attached on the beam and there is a force $F(x, t)$ acting on the beam as shown in Figure 3.2,

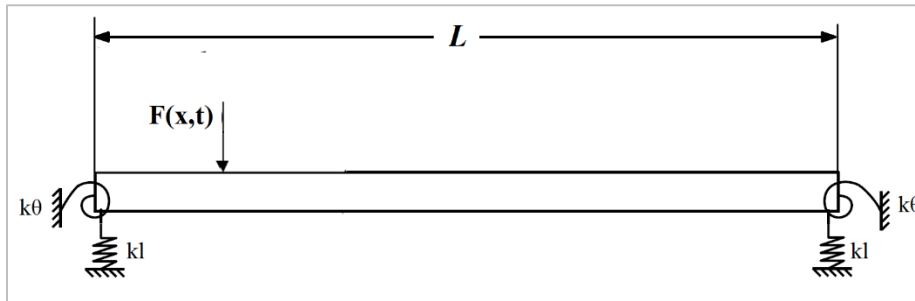


Figure 3.2: Beam with linear spring and torsional spring as flexible supports with force $F(x, t)$ and no absorber attached

For the beam (Dimensional form) without absorber, mathematical model can be written as [11],

$$EI \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + c_b \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} = \hat{F}(\hat{x}, \hat{t}) \quad (3.8)$$

Dividing Eq. (3.8) by $\rho b h$ yields,

$$\frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} = \frac{\hat{F}(\hat{x}, \hat{t})}{\rho b h} \quad (3.9)$$

3.2.1 Continuous Load (Forward Travel only)

In this section, force $F(x, t)$ is moving the beam with velocity V_f . To reduce complexity in equations we will utilize $v = V_f$ in all equations, so initial modelling will be done for this moving force on beam without absorber attached as shown in Figure 3.3.

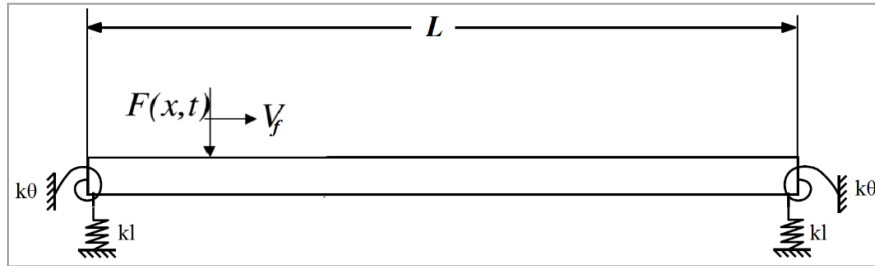


Figure 3.3: Beam with force $F(x, t)$ traveling with velocity V_f

Considering $F(x, t)$ for forward travel load as below,

$$\hat{F}(\hat{x}, \hat{t}) = P \delta(\hat{x} - v\hat{t}) \quad (3.10)$$

Substituting Eq. (3.10) in Eq. (3.9) results in,

$$\frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} = P \frac{\delta(\hat{x} - v \hat{t})}{\rho b h} \quad (3.11)$$

Applying non-dimensional parameters of time,

$$t = \frac{\hat{t}}{L} v \quad (3.12)$$

Substituting non-dimensional values from Eq. (3.3) and Eq. (3.12) in Eq. (3.11) yields,

$$\frac{EI w_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h} \frac{w_o v}{L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} = P \frac{\delta(x - t)}{L \rho b h} \quad (3.13)$$

Dividing Equation (3.13) by $\frac{w_o v^2}{L^2}$ and rearranging,

$$\begin{aligned} \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b L}{\rho b h v} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\ = P \frac{\delta(x - t)}{\rho b h} \frac{L}{w_o v^2} \end{aligned} \quad (3.14)$$

Simplifying using below parameters,

$$\gamma = \sqrt{\frac{EI}{\rho b h L^2 v^2}} \quad c1 = \frac{c_b L}{\rho b h v} \quad (3.15)$$

Substituting parameters from Equation (3.15) in Equation (3.14),

$$\frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} = P \frac{\delta(x - t)}{\rho b h} \frac{L}{w_o v^2} \quad (3.16)$$

For a simply supported beam, static deflection is given as [1, 2],

$$w_o = \frac{PL^3}{48EI} \quad (3.17)$$

For a cantilever beam, static deflection is given as [1, 2],

$$w_o = \frac{PL^3}{3EI} \quad (3.18)$$

The final mathematical model for the beam equations for simply supported and cantilever beam are given in Eq. (3.19) and Eq. (3.20) respectively,

$$\frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} = \frac{48EI}{\rho b h L^2 v^2} \delta(x - t) \quad (3.19)$$

$$\frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} = \frac{3EI}{\rho b h L^2 v^2} \delta(x - t) \quad (3.20)$$

3.2.2 Reciprocating moving load (Sinusoidal)

In this section, force $F(x, t)$ is reciprocating along the beam with velocity $v = V_f$ as shown in Figure 3.4 and having sinusoidal motion which means load will accelerate at start, attain maximum velocity and then deaccelerate while approaching the end point of its travel length, so initial modelling will be done for this moving force on a beam without absorber and then in next section with linear spring-mass absorber attached.

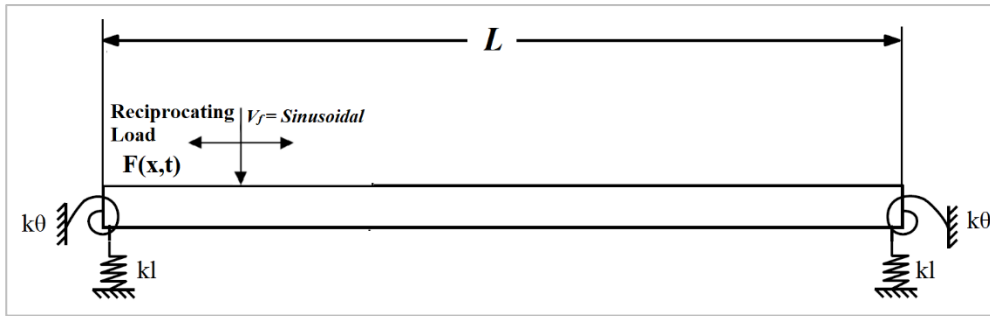


Figure 3.4: Beam with force $F(x, t)$ traveling with velocity $V_f = \text{Sinusoidal}$ and Reciprocating at multiple cycles (q) along length of beam ($0 \leq x \leq L$)

Following below mentioned equation of $F(x, t)$ will be utilized for reciprocating and sinusoidal load,

$$\hat{F}(\hat{x}, \hat{t}) = P \delta\left(\hat{x} - \frac{\rho L}{2} \left(1 - \cos\left(\frac{\pi v}{\rho L} \hat{t}\right)\right)\right) \quad (3.21)$$

In Eq. (3.21). ρ is the travel length of the load on the beam, Substituting Eq. (3.21) in Eq. (3.8) results in,

$$\begin{aligned} \frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\ = \frac{P \delta\left(\hat{x} - \frac{\rho L}{2} \left(1 - \cos\left(\frac{\pi v}{\rho L} \hat{t}\right)\right)\right)}{\rho b h} \end{aligned} \quad (3.22)$$

Substituting non-dimensional parameters from Eq. (3.3) and Eq. (3.12) in Eq. (3.22) and rearranging,

$$\begin{aligned} \frac{EI w_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b w_o v}{\rho b h L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} \\ = \frac{P}{\rho b h L} \delta\left(x - \frac{\rho}{2} \left(1 - \cos\left(\frac{\pi}{\rho} t\right)\right)\right) \end{aligned} \quad (3.23)$$

Dividing Eq. (3.23) by $\frac{w_o v^2}{L^2}$ and rearranging yields,

$$\begin{aligned} \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b L}{\rho b h v} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\ = P \frac{\delta\left(x - \frac{\rho}{2} \left(1 - \cos\left(\frac{\pi}{\rho} t\right)\right)\right)}{\rho b h} \frac{L}{w_o v^2} \end{aligned} \quad (3.24)$$

Again, substituting parameters from Eq. (3.15), Eq. (3.17) and Eq. (3.18) in Eq. (3.24) and rearranging, following equations are obtained for non-classical simply supported and cantilever beam models respectively,

$$\begin{aligned} \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} \\ = \frac{48EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \end{aligned} \quad (3.25)$$

$$\begin{aligned} \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} \\ = \frac{3EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \end{aligned} \quad (3.26)$$

3.3 A beam with fixed spring mass absorber

In this section, linear spring-mass absorber is incorporated in the beam model as shown in Figure 3.5, and there is a same force $F(x, t)$ acting on this beam as previous model, but now with the addition of $G(x, t)$ in this model.

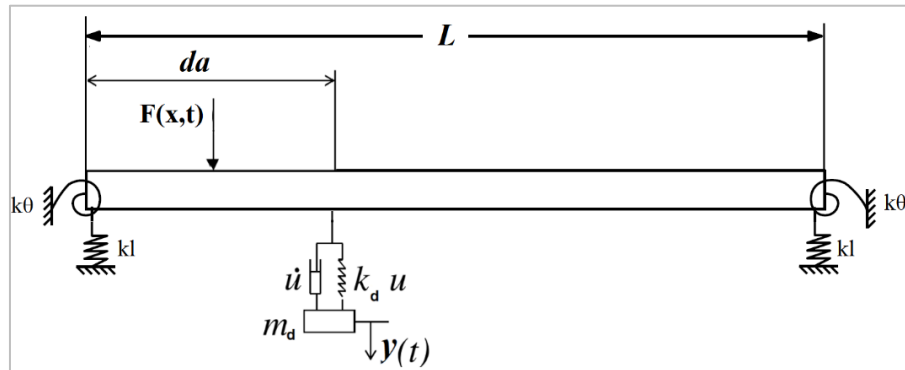


Figure 3.5: Beam with linear spring and torsional spring as flexible supports with force $F(x, t)$ and spring mass absorber attached

For beam (Dimensional Form) mathematical model with added absorber will be written as follows [31],

$$EI \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + c_b \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}} + \rho b h \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} + \left(k_d \hat{u}(t) + c_d \frac{\partial \hat{u}(t)}{\partial \hat{t}} \right) \hat{G}(\hat{x}, \hat{t}) = \hat{F}(\hat{x}, \hat{t}) \quad (3.27)$$

For spring-mass absorber (Dimensional Form),

$$m_d \frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - c_d \frac{\partial \hat{u}(da, \hat{t})}{\partial \hat{t}} - k_d \hat{u}(da, \hat{t}) = 0 \quad (3.28)$$

Dividing Equation (3.28) by $\rho b h$,

$$\frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} + \left(\frac{k_d}{\rho b h} \hat{u}(\hat{t}) + \frac{c_d}{\rho b h} \frac{d\hat{u}(\hat{t})}{d\hat{t}} \right) \hat{G}(\hat{x}, \hat{t}) = \frac{\hat{F}(\hat{x}, \hat{t})}{\rho b h} \quad (3.29)$$

Dividing Equation (3.29) by m_d ,

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \frac{d\hat{u}(\hat{t})}{d\hat{t}} - \frac{k_d}{m_d} \hat{u}(\hat{t}) = 0 \quad (3.30)$$

Now the value of $u(x, t)$ and its derivative is given by,

$$\hat{u}(\hat{t}) = \hat{w}(da, \hat{t}) - \hat{y}(\hat{t}) \quad (3.31)$$

$$\frac{d\hat{u}(\hat{t})}{d\hat{t}} = \frac{\partial \hat{w}(da, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \quad (3.32)$$

Substituting these values from Eq. (3.31) and Eq. (3.32) in Eq. (3.29) and Eq. (3.30), the following equations are obtained,

$$\begin{aligned}
& \frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\
& + \left(\frac{k_d}{\rho b h} (\hat{w}(da, \hat{t}) - \hat{y}(\hat{t})) \right. \\
& \left. + \frac{c_d}{\rho b h} \left(\frac{\partial \hat{w}(da, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) \right) \hat{G}(x, t) = \frac{\hat{F}(\hat{x}, \hat{t})}{\rho b h}
\end{aligned} \tag{3.33}$$

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \left(\frac{\partial \hat{w}(da, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) - \frac{k_d}{m_d} (\hat{w}(da, \hat{t}) - \hat{y}(\hat{t})) = 0 \tag{3.34}$$

3.3.1 Continuous Load (Forward Travel only)

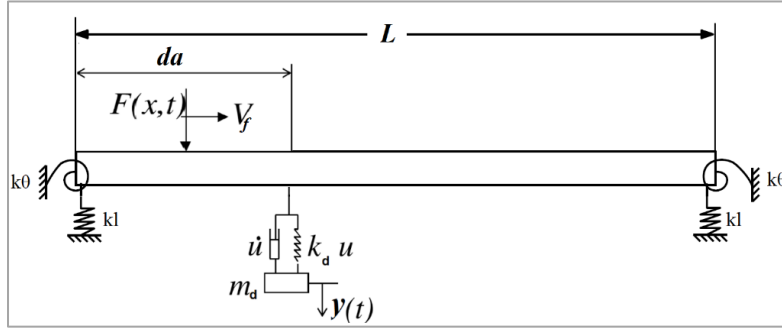


Figure 3.6: Beam with force $F(x, t)$ traveling with velocity V_f = continuous and forward travel only with attached spring mass absorber

Consider the system with forward travel load and fixed absorber as shown in Figure 3.6. Utilization of $F(x, t)$ values from Eq. (3.10) and incorporating $G(x, t)$ as given below,

$$\hat{G}(\hat{x}, \hat{t}) = \delta(\hat{x} - da) \tag{3.35}$$

Eq. (3.33) and Eq. (3.34) can be written as,

$$\begin{aligned}
& \frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\
& + \left(\frac{k_d}{\rho b h} (\hat{w}(d, \hat{t}) - \hat{y}(\hat{t})) + \frac{c_d}{\rho b h} \left(\frac{\partial \hat{w}(d, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) \right) \delta(\hat{x} \\
& - da) = P \frac{\delta(\hat{x} - v\hat{t})}{\rho b h}
\end{aligned} \quad (3.36)$$

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \left(\frac{\partial \hat{w}(da, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) - \frac{k_d}{m_d} (\hat{w}(da, \hat{t}) - \hat{y}(\hat{t})) = 0 \quad (3.37)$$

Applying additional non-dimensional parameters,

$$y = \frac{\hat{y}}{w_o} \quad d = \frac{\hat{d}}{L} \quad (3.38)$$

Substituting remaining non-dimensional from Eq. (3.38) with first set of parameters from Eq. (3.3)

and Eq. (3.12) in Eq. (3.36) and Eq. (3.37), following set of equations are obtained,

$$\begin{aligned}
& \frac{EI w_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h} \frac{w_o v}{L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} (w_o w(d, t) - w_o y(t)) \right. \\
& \left. + \frac{c_d}{\rho b h} \left(\frac{w_o v}{L} \frac{\partial w(d, t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \right) \frac{\delta}{L} (x - d) \\
& = P \frac{\delta(x - t)}{L \rho b h}
\end{aligned} \tag{3.39}$$

$$\begin{aligned}
& \frac{w_o v^2}{L^2} \frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \left(\frac{w_o v}{L} \frac{\partial w(d, t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \\
& - \frac{k_d}{m_d} (w_o w(d, t) - w_o y(t)) = 0
\end{aligned} \tag{3.40}$$

Dividing Eq. (3.39) and Eq. (3.40) by $\frac{w_o v^2}{L^2}$ and re-arranging,

$$\begin{aligned}
& \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h v} \frac{L}{\partial t} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} \frac{L}{v^2} (w(d, t) - y(t)) \right. \\
& \left. + \frac{c_d}{\rho b h v} \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \delta(x - d) \\
& = P \frac{\delta(x - t)}{\rho b h} \frac{L}{w_o v^2}
\end{aligned} \tag{3.41}$$

$$\frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \frac{L}{v} \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) - \frac{k_d}{m_d} \frac{L^2}{v^2} (w(d, t) - y(t)) = 0 \tag{3.42}$$

Utilizing assumed parameters from Eq. (3.15) and others described in Eq. (3.43),

$$c_2 = \frac{c_d}{\rho b h v} \quad k_1 = \frac{k_d L}{\rho b h v^2} \quad c_3 = \frac{c_d L}{m_d v} \quad k_2 = \frac{k_d L^2}{m_d v^2} \quad (3.43)$$

ubstituting parameters from Eq. (3.15), Eq. (3.17), Eq. (3.18), Eq. (3.43) in Eq. (3.41) and Eq. (3.42), non-dimensional equation models for non-classical simply supported and cantilever beam and spring mass absorber are obtained as below,

$$\begin{aligned} & \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} \\ & + \left(k_1 (w(d, t) - y(t)) + c_2 \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \delta(x - d) \\ & = \frac{48EI}{\rho b h L^2 v^2} \delta(x - t) \end{aligned} \quad (3.44)$$

$$\begin{aligned} & \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c_1 \frac{\partial w(x, t)}{\partial t} \\ & + \left(k_1 (w(d, t) - y(t)) + c_2 \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \delta(x - d) \\ & = \frac{3EI}{\rho b h L^2 v^2} \delta(x - t) \end{aligned} \quad (3.45)$$

$$\frac{d^2 y(t)}{dt^2} - c_3 \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) - k_2 (w(d, t) - y(t)) = 0 \quad (3.46)$$

3.3.2 Reciprocating moving load (Sinusoidal)

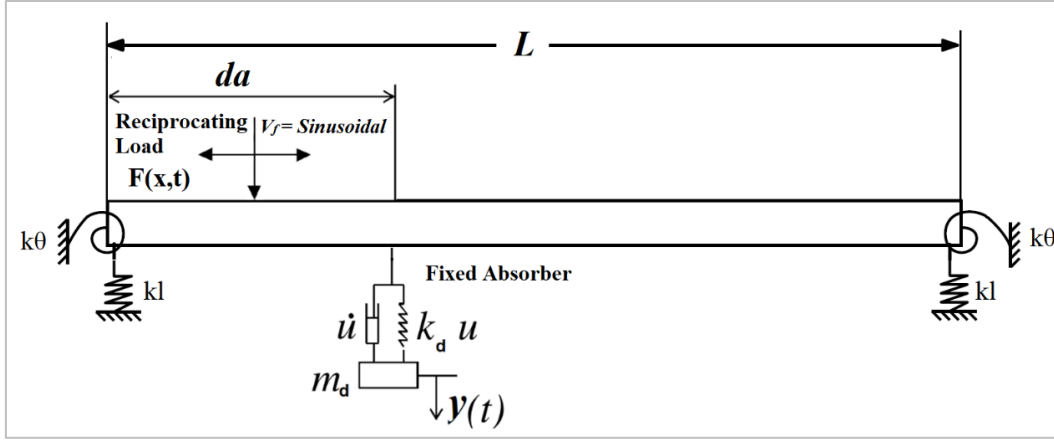


Figure 3.7: Beam with force $F(x, t)$ traveling with velocity $V_f = \text{Sinusoidal}$ and Reciprocating at multiple cycles (q) along length of beam ($0 \leq x \leq L$)

In this sub-section, the linear spring mass absorber (stationary) is fixed while the load is travelling with sinusoidal behaviour and completing multiple travelling cycles (q) along the beam as shown in Figure 3.7.

Utilizing $F(x, t)$ value as below,

$$\hat{F}(\hat{x}, \hat{t}) = P \delta\left(\hat{x} - \frac{\rho L}{2} \left(1 - \cos\left(\frac{\pi v}{\rho L} \hat{t}\right)\right)\right) \quad (3.47)$$

Substituting $F(x, t)$ values from Eq. (3.47) and $G(x, t)$ value from Eq. (3.35) in Eq. (3.33) and Eq. (3.34), following equations are obtained,

$$\begin{aligned}
& \frac{EI}{\rho b h} \frac{\partial^4 \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\
& + \left(\frac{k_d}{\rho b h} (\widehat{w}(da, \hat{t}) - \hat{y}(\hat{t})) \right) \\
& + \frac{c_d}{\rho b h} \left(\frac{\partial \widehat{w}(da, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) \delta(\hat{x} - da) \\
& = \frac{P \delta(\hat{x} - \frac{\rho L}{2} \left(1 - \cos\left(\frac{\pi v}{\rho L} \hat{t}\right) \right))}{\rho b h}
\end{aligned} \tag{3.48}$$

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \left(\frac{\partial \widehat{w}(d, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) - \frac{k_d}{m_d} (\widehat{w}(da, \hat{t}) - \hat{y}(\hat{t})) = 0 \tag{3.49}$$

Substitution of non-dimensional parameters from Eq. (3.12) and Eq. (3.38) in Eq. (3.48) and Eq. (3.49) results in,

$$\begin{aligned}
& \frac{EI w_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h} \frac{w_o v}{L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} (w_o w(da, t) - w_o y(t)) \right) \\
& + \frac{c_d}{\rho b h} \left(\frac{w_o v}{L} \frac{\partial w(da, t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \frac{\delta}{L} (x \\
& - da) = \frac{P}{\rho b h L} \delta \left(x - \frac{\rho}{2} \left(1 - \cos\left(\frac{\pi}{\rho} t\right) \right) \right)
\end{aligned} \tag{3.50}$$

$$\begin{aligned}
& \frac{w_o v^2}{L^2} \frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \left(\frac{w_o v}{L} \frac{\partial w(da, t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \\
& - \frac{k_d}{m_d} (w_o w(da, t) - w_o y(t)) = 0
\end{aligned} \tag{3.51}$$

Dividing Eq. (3.50) and Eq. (3.51) by $\frac{w_o v^2}{L^2}$ and re-arranging,

$$\begin{aligned}
& \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b L}{\rho b h v} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h v^2} (w(da, t) - y(t)) + \frac{c_d}{\rho b h v} \left(\frac{\partial w(da, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \\
& * \delta(x - da) = P \frac{\delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)}{\rho b h} \frac{L}{w_o v^2}
\end{aligned} \tag{3.52}$$

$$\frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \frac{L}{v} \left(\frac{\partial w(da, t)}{\partial t} - \frac{dy(t)}{dt} \right) - \frac{k_d}{m_d} \frac{L^2}{v^2} (w(da, t) - y(t)) = 0 \tag{3.53}$$

For making equation less complex, substituting simplified parameters from Eq. (3.15) and Eq. (3.43) and w_o values from Eq. (3.17) and Eq. (3.18) in Eq. (3.52) and Eq. (3.53) and re-arranging, non-dimensional models for stated configuration of non-classical simply supported beam and cantilever beam respectively are obtained as final equations below, whereas for linear spring mass absorber it is same as in Eq. (3.45)

The final mathematical model for the equations is as follows,

$$\begin{aligned}
& \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\
& + \left(k1(w(da, t) - y(t)) + c2 \left(\frac{\partial w(da, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \delta(x \\
& - da) = \frac{48EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)
\end{aligned} \tag{3.54}$$

$$\begin{aligned}
& \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\
& + \left(k1(w(da, t) - y(t)) + c2 \left(\frac{\partial w(da, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \delta(x \\
& - da) = \frac{3EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)
\end{aligned} \tag{3.55}$$

3.4 Beam with moving spring mass absorber

3.4.1 Reciprocating load with reciprocating absorber – Synchronized travel

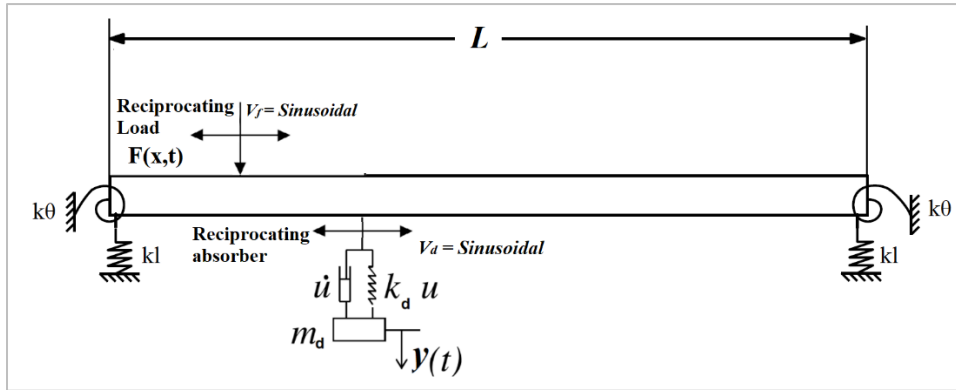


Figure 3.8: Beam with force $F(x, t)$ traveling with velocity $V_f = \text{Sinusoidal}$ and Reciprocating at multiple cycles ($q = cn$) along length of beam ($0 \leq x \leq L$)

In this section, linear absorber is also performing sinusoidal motion which is synchronized with load travelling at same or different speeds as shown in Figure 3.8. Here, $\hat{G}(\hat{x}, \hat{t})$ and $Z(\hat{t})$ is given in Eq. (3.56) as follows,

$$\begin{aligned}\hat{G}(\hat{x}, \hat{t}) &= \delta\left(\hat{x} - \frac{\rho L}{2}\left(1 - \cos\left(\frac{\pi v}{\rho L}\hat{t}\right)\right)\right) \\ \hat{G}(\hat{x}, \hat{t}) &= \delta(\hat{x} - Z(\hat{t}))\end{aligned}\tag{3.56}$$

Where,

$$Z(\hat{t}) = \frac{\rho L}{2}\left(1 - \cos\left(\frac{\pi v}{\rho L}\hat{t}\right)\right)$$

Adding Eq. (3.47) and Eq. (3.56) in Eq. (3.33) and Eq. (3.34),

$$\begin{aligned}&\frac{EI}{\rho b h} \frac{\partial^4 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \hat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \hat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\&+ \left(\frac{k_d}{\rho b h} (\hat{w}(\hat{x}, \hat{t}) - \hat{y}(\hat{t})) \right. \\&+ \left. \frac{c_d}{\rho b h} \left(\frac{\partial \hat{w}(Z(\hat{t}), \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) \right) \delta(\hat{x} - Z(\hat{t})) \\&= \frac{P \delta(\hat{x} - Z(\hat{t}))}{\rho b h}\end{aligned}\tag{3.57}$$

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \left(\frac{\partial \hat{w}(Z(\hat{t}), \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) - \frac{k_d}{m_d} (\hat{w}(Z(\hat{t}), \hat{t}) - \hat{y}(\hat{t})) = 0\tag{3.58}$$

Substituting non-dimensional parameters from Eq. (3.3), Eq. (3.12) and Eq. (3.38) in Eq. (3.57) and Eq. (3.58), following equations are obtained,

$$\begin{aligned}
& \frac{EIw_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h} \frac{w_o v}{L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} (w_o w(Z(t), t) - w_o y(t)) \right) \\
& + \frac{c_d}{\rho b h} \left(\frac{w_o v}{L} \frac{\partial w(Z(t), t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) * \frac{\delta}{L} \delta(x - Z(t)) \\
& = \frac{P}{\rho b h L} \delta(x - Z(t))
\end{aligned} \tag{3.59}$$

Where $Z(t)$ is non-dimensional parameter of $Z(\hat{t})$ given as,

$$Z(t) = \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right)$$

$$\begin{aligned}
& \frac{w_o v^2}{L^2} \frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \left(\frac{w_o v}{L} \frac{\partial w(Z(t), t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \\
& - \frac{k_d}{m_d} (w_o w(Z(t), t) - w_o y(t)) = 0
\end{aligned} \tag{3.60}$$

Dividing Eq. (3.59) and Eq. (3.60) by $\frac{w_o v^2}{L^2}$, and re-arranging yields,

$$\begin{aligned}
& \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b L}{\rho b h v} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} \frac{L}{v^2} (w(Z(t), t) - y(t)) \right. \\
& + \left. \frac{c_d}{\rho b h v} \left(\frac{\partial w(Z(t), t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) * \frac{\delta}{L} (x - Z(t)) \\
& = P \frac{\delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)}{\rho b h} \frac{L}{w_o v^2}
\end{aligned} \tag{3.61}$$

$$\frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \frac{L}{v} \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) - \frac{k_d}{m_d} \frac{L^2}{v^2} (w(Z(t), t) - y(t)) = 0 \tag{3.62}$$

Substituting simplified parameters from Eq. (3.15) and Eq. (3.43) and w_o values from Eq. (3.17) and Eq. (3.18) in Eq. (3.61) and Eq. (3.62) and re-arranging the terms, Eq. (3.63), Eq. (3.64) and Eq. (3.65) are obtained which are non-dimensional model for non-classical simply supported beam and cantilever beam and moving spring mass absorber respectively,

The final mathematical non-dimensional model for the utilized beam with mentioned absorber configuration is as follows,

$$\begin{aligned}
& \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\
& + \left(k1(w(Z(t), t) - y(t)) + c2 \left(\frac{\partial w(Z(t), t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \\
& * \frac{\delta}{L}(x - Z(t)) = \frac{48EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)
\end{aligned} \tag{3.63}$$

$$\begin{aligned}
& \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\
& + \left(k1(w(d, t) - y(t)) + c2 \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) * \frac{\delta}{L}(x \\
& - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right)) \\
& = \frac{3EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)
\end{aligned} \tag{3.64}$$

Non-dimensional mathematical model for moving beam is given in Eq. (3.65),

$$\frac{d^2 y(t)}{dt^2} - c3 \left(\frac{\partial w(d, t)}{\partial t} - \frac{dy(t)}{dt} \right) - k2(w(d, t) - y(t)) = 0 \tag{3.65}$$

3.4.2 Reciprocating load with reciprocating absorber – Unsynchronized Travelling

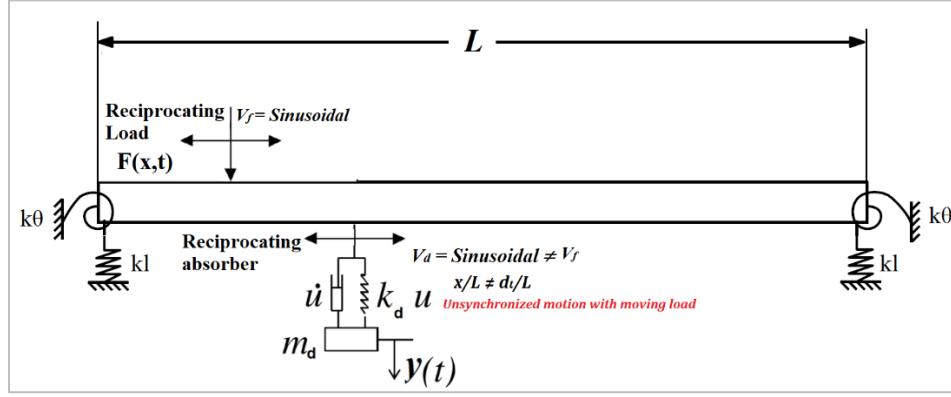


Figure 3.9: Beam with force $F(x, t)$ traveling with velocity $V_f = \text{Sinusoidal}$ and Reciprocating at multiple cycles ($q = cn$) along length of beam ($0 \leq x \leq L$)

In this subsection, linear absorber is also performing sinusoidal motion but in contrary to previous section, it is unsynchronized having different travel length in comparison to load travelling at same or different speeds as shown in Figure 3.9. Here, $\hat{G}(\hat{x}, \hat{t})$ and $H(\hat{t})$ is given as follows,

$$\hat{G}(\hat{x}, \hat{t}) = \delta\left(\hat{x} - \frac{\eta L}{2} \left(1 - \cos\left(\frac{\pi m_a}{\eta L} \hat{t}\right)\right)\right) \quad (3.66)$$

$$\hat{G}(\hat{x}, \hat{t}) = \delta(\hat{x} - H(\hat{t}))$$

Where,

$$H(\hat{t}) = \frac{\eta L}{2} \left(1 - \cos\left(\frac{\pi m_a}{\eta L} \hat{t}\right)\right) \quad (3.67)$$

In the modelling η is the travel length of the absorber on the beam, so it can be stated as travel length covered by absorber which is different than the travel length of the reciprocating load. The ratio of absorber speed to load speed m_a is defined as,

$$m_a = \frac{V_d}{V_f} \quad (3.68)$$

Substituting Eq. (3.47) and Eq. (3.66) in Eq. (3.33) and Eq. (3.34) and re-arranging,

$$\begin{aligned}
& \frac{EI}{\rho b h} \frac{\partial^4 \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{x}^4} + \frac{c_b}{\rho b h} \frac{\partial \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{x}} + \frac{\partial^2 \widehat{w}(\hat{x}, \hat{t})}{\partial \hat{t}^2} \\
& + \left(\frac{k_d}{\rho b h} (\widehat{w}(d, \hat{t}) - \hat{y}(\hat{t})) \right) \\
& + \frac{c_d}{\rho b h} \left(\frac{\partial \widehat{w}(d, \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) \delta(\hat{x} - H(\hat{t})) \\
& = \frac{P \delta(\hat{x} - \frac{\rho L}{2} \left(1 - \cos\left(\frac{\pi v}{\rho L} \hat{t}\right) \right))}{\rho b h}
\end{aligned} \tag{3.69}$$

$$\frac{d^2 \hat{y}(\hat{t})}{d\hat{t}^2} - \frac{c_d}{m_d} \left(\frac{\partial \widehat{w}(H(\hat{t}), \hat{t})}{\partial \hat{t}} - \frac{d\hat{y}(\hat{t})}{d\hat{t}} \right) - \frac{k_d}{m_d} (\widehat{w}(H(\hat{t}), \hat{t}) - \hat{y}(\hat{t})) = 0 \tag{3.70}$$

Substituting non-dimensional parameters from Eq. (3.12) and Eq. (3.15) in Eq. (3.69) and Eq.

(3.70) and utilizing $t = \frac{\hat{t} v_d}{L}$, which after further re-arranging are given as follows,

$$\begin{aligned}
& \frac{EI w_o}{\rho b h L^4} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b}{\rho b h} \frac{w_o v}{L} \frac{\partial w(x, t)}{\partial t} + \frac{w_o v^2}{L^2} \frac{\partial^2 w(x, t)}{\partial t^2} \\
& + \left(\frac{k_d}{\rho b h} (w_o w(d, t) - w_o y(t)) \right) \\
& + \frac{c_d}{\rho b h} \left(\frac{w_o v}{L} \frac{\partial w(d, t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \frac{\delta}{L} \delta \left(x \right. \\
& \left. - \frac{\eta}{2} \left(1 - \cos\left(\frac{\pi}{\eta v f} t\right) \right) \right) = \frac{P}{\rho b h L} \delta \left(x - \frac{\rho}{2} \left(1 - \cos\left(\frac{\pi}{\rho} t\right) \right) \right)
\end{aligned} \tag{3.71}$$

Where $H(t)$ is non-dimensional parameter of $H(\hat{t})$ given as,

$$H(t) = \frac{\eta}{2} \left(1 - \cos \left(\frac{\pi}{\eta v f} t \right) \right)$$

$$\begin{aligned} \frac{w_o v^2}{L^2} \frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \left(\frac{w_o v}{L} \frac{\partial w(H(t), t)}{\partial t} - \frac{w_o v}{L} \frac{dy(t)}{dt} \right) \\ - \frac{k_d}{m_d} (w_o w(H(t), t) - w_o y(t)) = 0 \end{aligned} \quad (3.72)$$

Dividing Eq. (3.71) and Eq. (3.72) by $\frac{w_o v^2}{L^2}$ and re-arranging,

$$\begin{aligned} \frac{EI}{\rho b h v^2 L^2} \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{c_b L}{\rho b h v} \frac{\partial w(x, t)}{\partial t} + \frac{\partial^2 w(x, t)}{\partial t^2} \\ + \left(\frac{k_d L}{\rho b h v^2} (w(H(t), t) - y(t)) \right. \\ \left. + \frac{c_d}{\rho b h v} \left(\frac{\partial w(H(t), t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \left(\frac{\delta}{L} (x \right. \\ \left. - \frac{\eta}{2} \left(1 - \cos \left(\frac{\pi}{\eta v} t \right) \right) \right) \\ = P \frac{\delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right)}{\rho b h} \frac{L}{w_o v^2} \end{aligned} \quad (3.73)$$

$$\frac{d^2 y(t)}{dt^2} - \frac{c_d}{m_d} \frac{L}{v} \left(\frac{\partial w(H(t), t)}{\partial t} - \frac{dy(t)}{dt} \right) - \frac{k_d L^2}{m_d v^2} (w(H(t), t) - y(t)) = 0 \quad (3.74)$$

Substituting simplified parameters from Eq. (3.15) and Eq. (3.43) and w_o values from Eq. (3.17) and Eq. (3.18) in Eq. (3.73) and Eq. (3.74) and re-arranging, mathematical models are obtained for non-classical simply supported beam and cantilever beam and moving spring mass absorber respectively,

The final mathematical model of the equations is,

$$\begin{aligned} \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\ + \left(k1(w(x, t) - y(t)) + c2 \left(\frac{\partial w(x, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \left(\frac{\delta}{L} \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \right) \\ - \frac{\eta}{2} \left(1 - \cos \left(\frac{\pi}{\eta v f} t \right) \right) = \frac{48EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \end{aligned} \quad (3.75)$$

$$\begin{aligned} \frac{\partial^2 w(x, t)}{\partial t^2} + \gamma^2 \frac{\partial^4 w(x, t)}{\partial x^4} + c1 \frac{\partial w(x, t)}{\partial t} \\ + \left(k1(w(x, t) - y(t)) + c2 \left(\frac{\partial w(x, t)}{\partial t} - \frac{dy(t)}{dt} \right) \right) \left(\frac{\delta}{L} \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \right) \\ - \frac{\eta}{2} \left(1 - \cos \left(\frac{\pi}{\eta v f} t \right) \right) = \frac{3EI}{\rho b h L^2 v^2} \delta \left(x - \frac{\rho}{2} \left(1 - \cos \left(\frac{\pi}{\rho} t \right) \right) \right) \end{aligned} \quad (3.76)$$

For absorber (Non-Dimensional Form) following is the equation as shown below,

$$\frac{d^2 y(t)}{dt^2} - c3 \left(\frac{\partial w(H(t), t)}{\partial t} - \frac{dy(t)}{dt} \right) - k2(w(H(t), t) - y(t)) = 0 \quad (3.77)$$

Chapter 4 : Validation of Mathematical Modelling

In this section, the current study findings are compared to those found in the literature [1, 2, 19, 21, 54, 60] to show accuracy of developed model. Beam material qualities and dimensions are chosen to be same as those used in the published studies [19], and verifications are done based on the reported data. To generalize plotted results, dimensionless parameters of results are used. Because there is no data for a sinusoidal load reciprocating across the beam, comparison can only be made for only one load traveling only once from one end of the beam to the other as a continuous load.

In discussing this section, first cross comparison will be done for beam with non-classical boundary conditions using same parameters of beam and supports spring stiffnesses as in previous literature [21]. Comparison analysis will be done for mode shapes and eigen values with results being tabulated for clear comparison.

Then in next validation, classical model of simply supported will be compared with non-classical model of simply supported beam and both results will be compared with literature [2] to ensure that eigen values agree.

Later dynamic responses of forward travel moving load introduced in modelling, whose speed is constant will be studied for beam without fixed absorber or with fixed absorber at different damping conditions and detailed comparison will be performed. Then finally, for moving absorber and load travelling together reaching to end of beam will be compared with reference studies [61].

4.1.1 Validation with beam natural frequencies

In this section using flexible boundary conditions, modal shapes will be plotted. Eigen values and modal shapes will be compared with authorized results [21]. The beams parameters utilized for the boundary conditions are mentioned in Table 4-1,

Table 4-1: Beam flexural rigidity and spring stiffness [21]

Parameters	Values
Flexural Rigidity of Beam (EI)	8000 N.m ²
Spring Stiffness (k_s)	1000 N/m

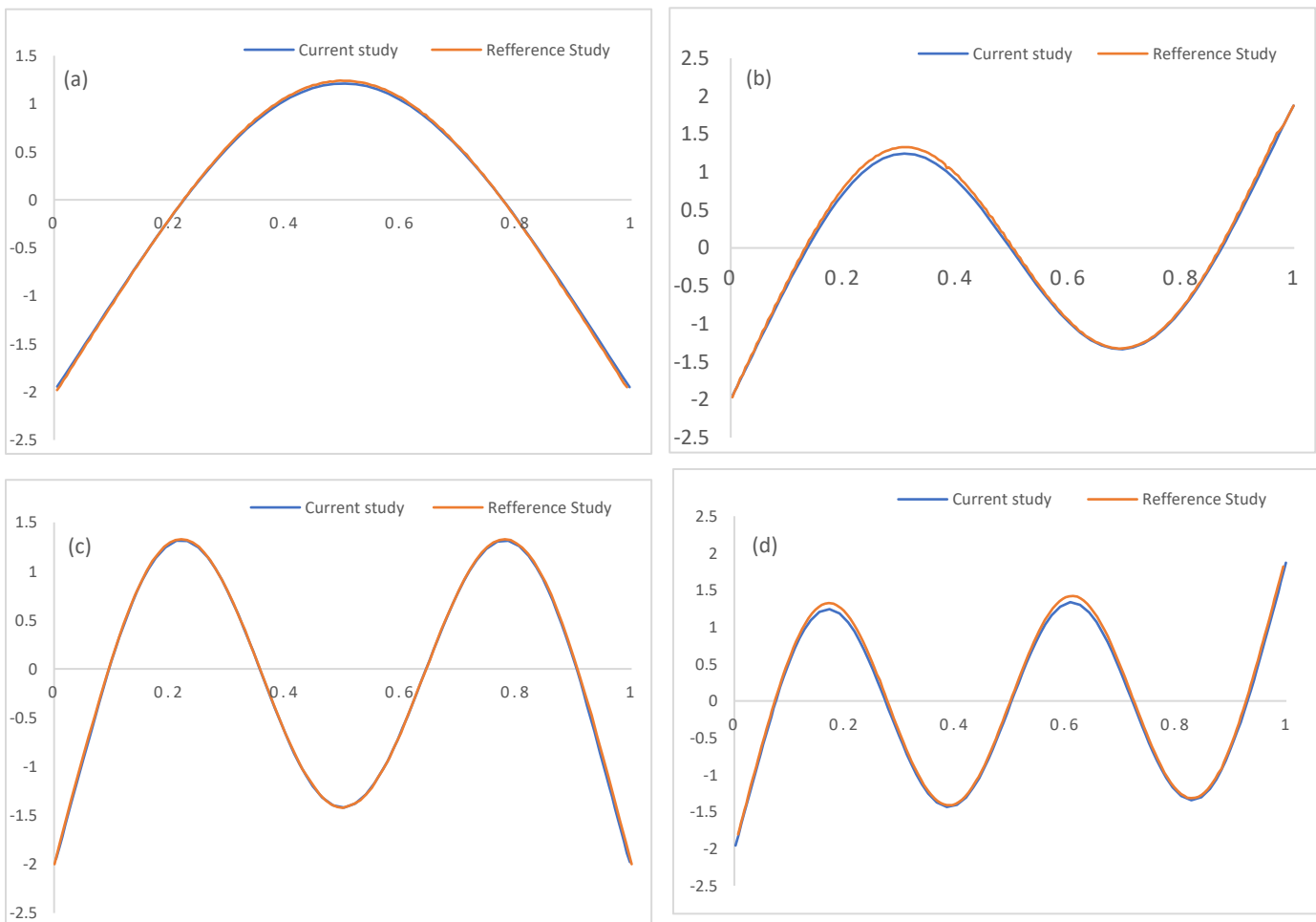


Figure 4.1: Mode shapes of beams with non-classical boundary condition in comparison with Ref. [21] (a) First Modal expansion (b) Second Modal Expansion (c) Third Modal Expansion (d) Fourth Modal Expansion in comparison to Reference study [21]

The comparison of Eigen values is shown in below Table 4-2, and it is observed that they are in excellent agreement with each other.

Table 4-2: Current study and reference Eigen values for four modes with flexible supports [21]

Mode (n)	ω_n value	Ref. Eigen Values [21]	Current study Eigen Values ($\sqrt{\omega_n}$)
1 st Mode	22.3956	4.7324	4.7324
2 nd Mode	61.6809	7.8537	7.8537
3 rd Mode	120.9081	10.9958	10.9958
4 th Mode	199.8621	14.1373	14.1372

The modal shapes and eigen values plotted in Figure 4.1 are in perfect agreement with the mode shapes plotted by the author [21] for beam having boundary conditions of linear spring (non-classical). In next section, we will increase the springs stiffnesses value to sufficiently large quantity, that it starts simulating Simply supported or Cantilever beam depending on the values of stiffness for both torsional and linear spring connected.

To further validate our analysis, at both ends of beam we will equate stiffness of torsional spring (k_θ) to zero and will increase the stiffness of linear spring (k_l) to sufficiently large values as shown in Figure 4.1 until mode shapes and eigen values of beams are similar to simply supported beam. Further analysis will be carried out in the next chapter that can be regarded as validation. As shown in Figure 5.1 from next chapter, to generate modal behaviour of cantilever beam, we will equate stiffness of one side of linear and torsional spring to zero and for the other side stiffness of both linear and torsion spring will be increased to sufficient large value. Modal shapes of both beams will be plotted and will be compared with classical modal shapes of simply supported and cantilever beam and comparison analysis of eigen values will be performed. Mode shapes for both Cantilever beam and simply supported with classical boundary conditions are shown in Figure 4.2.

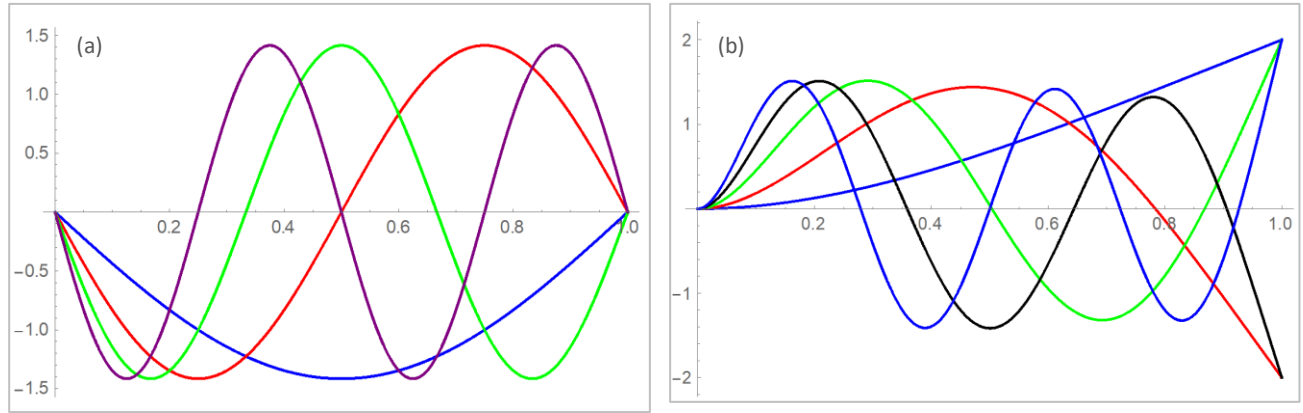


Figure 4.2: Mode shapes of beam with non-classical boundary conditions (a) Non-classical simply supported beam (b) Non-classical cantilever beam

Table 4-3 and 4-4, illustrate the eigen value comparison analysis of both beams in terms of Classical and Non-Classical boundary conditions with the Reference Values [2].

Table 4-3: Comparison of eigen values of classical and non-classical simply supported beam with reference value [2]

Mode (n)	ω_n value Non-classical beam	ω_n values Classical beam	Ref. Eigen Values [2]	Non-classical Beam Eigen Values ($\frac{\omega_n}{n\pi}$)	Classical Beam Eigen Values ($\frac{\omega_n}{n\pi}$)
1 st Mode	9.8696	9.8696	$\pi = 3.1416$	3.1416	3.1416
2 nd Mode	39.4784	39.4784	$2\pi = 6.2832$	6.2832	6.2832
3 rd Mode	88.8264	88.8264	$3\pi = 9.4248$	9.4248	9.4248
4 th Mode	157.914	157.914	$4\pi = 12.5664$	12.5664	12.5664

Table 4-4: Comparison of eigen values of classical and non-classical cantilever beam with reference value [2]

Mode (n)	ω_n value Non-classical beam	ω_n values Classical beam	Ref. Eigen Values [2]	Non-classical Beam Eigen Values ($\sqrt{\omega_n}$)	Non-classical Beam Eigen Values ($\sqrt{\omega_n}$)
1 st Mode	3.5160	3.5160	1.87510	1.87510	1.87510
2 nd Mode	22.0345	22.0345	4.69409	4.69409	4.69409
3 rd Mode	61.6972	61.6972	7.85475	7.85475	7.85475
4 th Mode	120.902	120.902	10.99554	10.99554	10.99554

The eigen values calculated in the current study for non-classical boundary conditions are identical to the eigen values calculated for those of classical boundary conditions which are equal to the

eigen values obtained from reference [2]. The modal analysis performed in this section perfectly validate our case of utilization of flexible boundary conditions.

4.1.2 Validation with and without fixed absorber and moving load..

4.1.2.1 Analytical Validation

In this case of validation, beam coupled with linear dynamic absorber at fixed position and traversed by a moving load travelling at 61 % of critical velocity is now explored and compared with Ref. [19] to have confidence in accuracy of the existing model.

To check validity, the stiffness of linear springs used as a boundary condition is increased to such a large value that it generates mode shapes of simply supported beam with fixed absorber. We will utilize code of both non-classical and classical SS model and will study three cases with specified parameters to perform the analysis. The beam parameters taken for this validation are,

Table 4-5: Reference parameters of Simply Supported Beam [19]

E	ρ	L	h	b
206.8 GPa	7820Kg/m ³	4 m	0.3 m	0.3 m

(a) Validation with first set of parameters of absorber for classical and non-classical beam model with reference study [19].

The parameters utilized for linear spring mass absorber and its location along length of beam are shown in Table 4-6.

Table 4-6: Parameters of Linear Mass Spring Dynamic Absorber [19]

m absorber	K absorber	Viscous damping	Dynamic absorber location
1.4076 kg	877.8 N/m	12.98 Ns/m	2 m (x/L=0.5)

Table 4-7: Reference results of deflection and time [19]

Beam and dynamic absorber condition	Maximum deflection (mm)	Position of maximum deflection (m)	Time of maximum deflection (s)
Undamped beam with linear dynamic absorber, optimal values	1.6016	2.11	0.1402

Since we are dealing with dimensionless parameters, we will convert dimensional results of reference [19] into non-dimensional values using Eq. (3.3) and Eq. (3.20) and dimensionless values are shown in Table 4-8.

Table 4-8: Reference parameters with results of deflection and time

Observed Value	Dimensional value from Reference [19]	Dimensionless value [19]
Maximum deflection	1.6016 mm	1.709 mm
Time of maximum deflection	0.1402 s	0.753 s

Classical and non-classical simply supported beam maximum dynamic deflection values (dimensionless) are shown in Table 4-9,

Table 4-9: Comparison of Classical and non-classical beam measurements with Reference values [19]

Parameter	Current Values (Non-classical boundary conditions)	Current values (Classical boundary conditions)	Reference values [19]
Maximum deflection	0.752 s	0.752 s	0.753 s
Time of maximum deflection	1.662 mm	1.662 mm	1.709 mm

By analysing the results, we notice slight difference in the numerical value of deflection whereas time is almost similar. The slight deviation is due to difference in mathematical modelling of beam and absorber which will be further checked in next two cases by changing parameters of beams as per Ref. [19].

(b) Validation with another set of parameters of absorber with reference study [19]

The parameters utilized for linear spring mass absorber and its location along length of beam [19] are shown in Table 4-10.

Table 4-10: Parameters of linear spring-mass absorber [19]

m absorber	K absorber	Viscous damping	Dynamic absorber location
1.4076 kg	900 N/m	10.5 Ns/m	2.2 m ($x/L = 0.55$)

Table 4-11: Reference results of deflection and time [19]

Beam and dynamic absorber condition	Maximum deflection (mm)	Position of maximum deflection (m)	Time of maximum deflection (s)
Damped beam with linear dynamic absorber	1.5306	2.11	0.1402

Since we are dealing with dimensionless parameters, we will convert dimensional results of reference [19] into non-dimensional values using Eq. (3.3) and Eq. (3.20) and dimensionless values are shown in Table 4-12.

Table 4-12: Reference parameters with results of deflection and time [19]

Observed Value	Dimensional value from Reference	Dimensionless value
Maximum deflection	1.5306 mm	1.633 mm
Time of maximum deflection	0.1401 s	0.753 s

Classical and non-classical simply supported beam maximum dynamic deflection values (dimensionless) are shown in Table 4-13.

Table 4-13: Comparison of classical and non-classical beam measurements with Ref. values [19].

Parameter	Current Values (Non-classical boundary conditions)	Current values (Classical boundary conditions)	Reference values [19]
Maximum deflection	0.750 s	0.750	0.753
Time of maximum deflection	1.65843 mm	1.65843	1.709

In observing the results, we again notice slight deviation in the numerical value of deflection, but it is improving then previous case and getting closer to current values whereas time is almost similar. Since in this section we reduced damping value as per Ref. [19], we see slight improvement in results. As per this analogy, the difference in mathematical modelling for beam and absorber has role in this deviation but still our results accuracy is above 98%. In the final case of this section, we would utilize the minimal absorber used by author [19].

(c) Validation with minimal set of parameters of absorber with reference study [19]

The new minimal parameters utilized for linear spring mass absorber and its location along length of beam [19] are shown in Table 4-14.

Table 4-14: Parameters of Linear Mass Spring Dynamic Absorber [19]

m absorber	K absorber	Viscous damping	Dynamic absorber location
1.4076 kg	1795 N/m	0.1 Ns/m	2.12 m ($x/L = 0.53$)

Utilizing the reference values [19] for maximum deflection, its position and time are shown in Table 4-15.

Table 4-15: Reference parameters with results of deflection and time [19]

Beam and dynamic absorber condition	Maximum deflection (mm)	Position of maximum deflection (m)	Time of maximum deflection (s)
Undamped beam with linear dynamic absorber	1.5260	2.12	0.1403

Converting dimensional results of reference [19] into non-dimensional values using Eq. (3.3) and Eq. (3.20) and dimensionless values are shown in Table 4-16.

Table 4-16: Reference parameters with results of deflection and time

Observed Value	Dimensional value from Reference [19]	Dimensionless value [19]
Maximum deflection	1.526 mm	1.628
Time of maximum deflection	0.140 s	0.754

Classical and non-classical simply supported beam maximum dynamic deflection values (dimensionless) are shown in Table 4-17,

Table 4-17: Comparison of Classical and non-classical beam measurements with Reference values

Parameter	Current Values (Non-classical)	Current values (Classical)	Reference values [19]
Maximum deflection	0.753	0.753	0.754
Time of maximum deflection	1.631	1.631	1.628

Results are improved significantly when damping parameters are reduced but again as stated before it is due to mathematical modelling difference and results of maximum dynamic deflection agree with each other with precision of 98% and above. Since numerical analysis at maximum deflection value is completed, now in next section we will perform graphical comparison with published studies [19, 54] to ensure our results are completely accurate for utilization.

4.1.2.2 Graphical Validation

In this section, the maximum dynamic deflection is observed in terms of change of dimensionless speed (r). The validation using graphical analysis is done to have more insight into slight deviation being observed in previous section and we are suspecting different approach to mathematical modelling is the reason here. To ensure the accuracy of result, the plotted results will be compared with another author [54] to perform verification. Since authors did not study oscillating motion of

the load rather continuous load has been studied so we will change our load travel mode to continuous mode and will perform analysis to validate our results. The beam parameters utilized will be from Table 4-5.

(a) Without fixed absorber

First, we will perform our analysis without absorber and will analyse the results with previous authors work [19, 54]. The parameters adopted for beam will be same as utilized in Section 1.1.

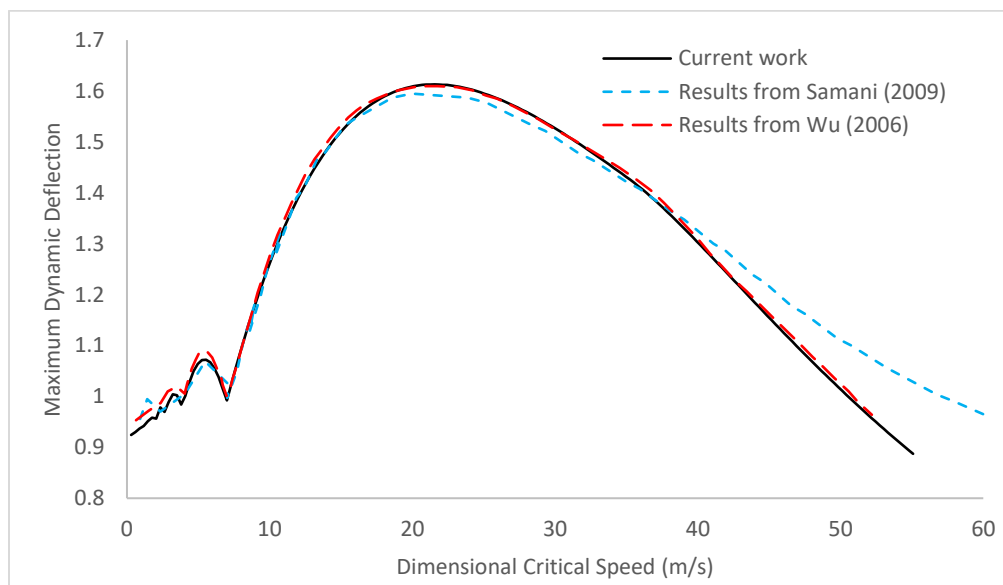


Figure 4.3: Dynamic deflection of non-classical simply supported beam at mid-span as a function of dimensionless speed (r) for $q=0.5$; — : Current work, - - - Previous results [19], - - - Previous results [54]

Results shown in Figure 4.3 without absorber are in excellent agreement with results from literature [19, 54]. Just at higher critical speed, deviation in results observed in comparison to Samani [19] from Wu results [54] because of difference in mathematical modelling used by Samani [19], whereas our results are in perfect agreement with Wu paper [54].

(b) With fixed spring mass absorber

In this section, the linear mass dynamic absorber is introduced in the system and results are plotted for maximum dynamic deflection. Parameters adopted for Linear Spring Mass Absorber are same as in Table 4-6.

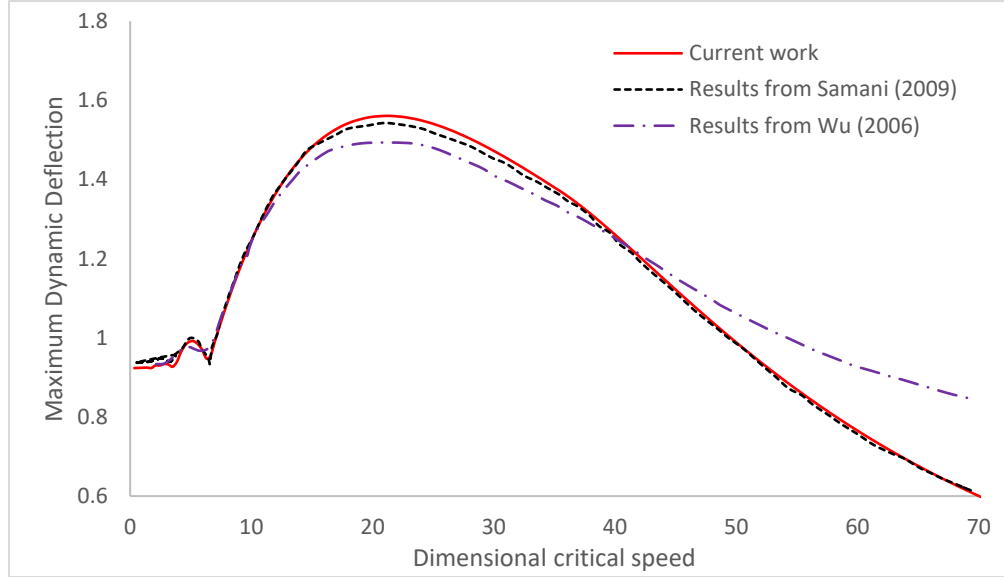


Figure 4.4: Dynamic deflection of non-classical simply supported beam at mid-span as a function of dimensionless speed (r) for $q=0.5$; — : Current work, ----- Previous results [19], - . - . Previous results [54]

As seen from Figure 4.4, the deviation of results are occurring at higher critical speeds when compared with Samani [19] but in excellent agreement with Wu [54]. As we expected, current work is valid and difference in results are due to mathematical modelling difference, increasing our confidence in results.

In continuation to this section validation, now performing dynamic amplitude analysis by varying spring mass absorber location along the entire length of the beam ($0 \leq x/L \leq 1$). The results

have been compared with Ref. [19] and are found in agreement with the results. The slight deviation in results is expected due to the reason stated before. Plotted results are shown in Figure 4.5.

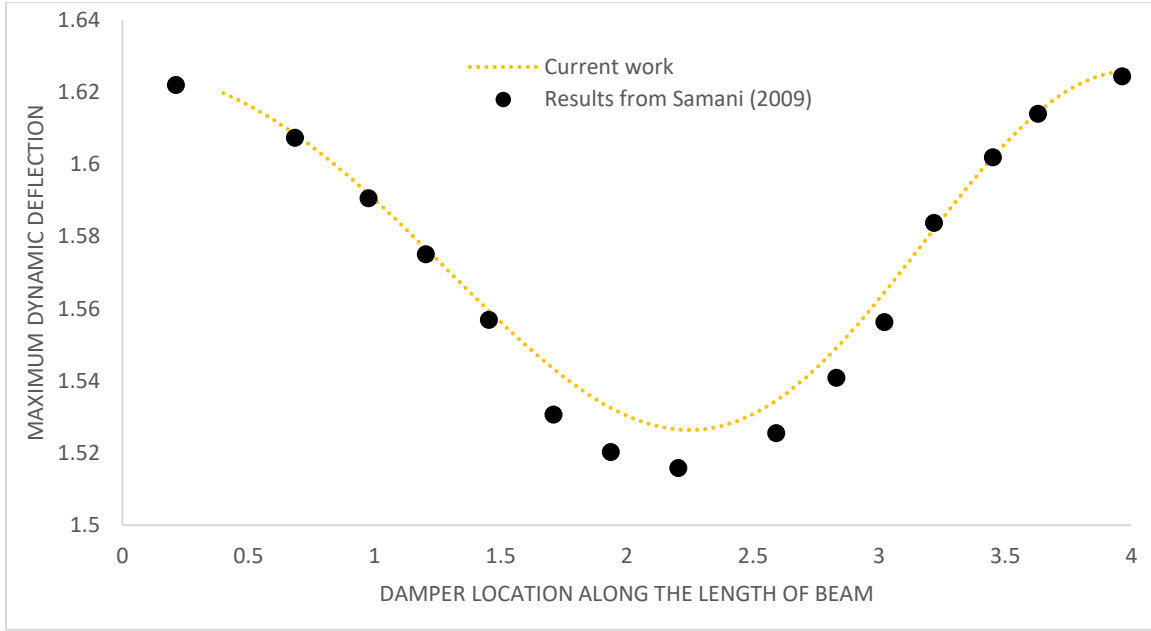


Figure 4.5: Dynamic deflection of non-classical simply supported beam at mid-span as a function of absorber location along the beam; : Current work, ● Previous results [19]

4.1.3 Validation with moving spring-mass absorber

In this section, we will be focusing on moving absorber to study its effect on dynamic amplitude of the vibrating beam. So, we will consider continuous load traversing the length of beam with absorber covering the same length of beam but at different speed. To validate our study, results plotted will be compared with results of Soares [60]. We will utilize parameters of beam for lengths of $L = 4$ m and different speeds of absorber ($m = V_d/V_f = 0.1, 1.0$ and 2.0). The parameters adopted for beams will be from Table 4-5 and parameters utilized for spring mass absorber are from Table 4-6 except that beam is in synchronized motion with the load,

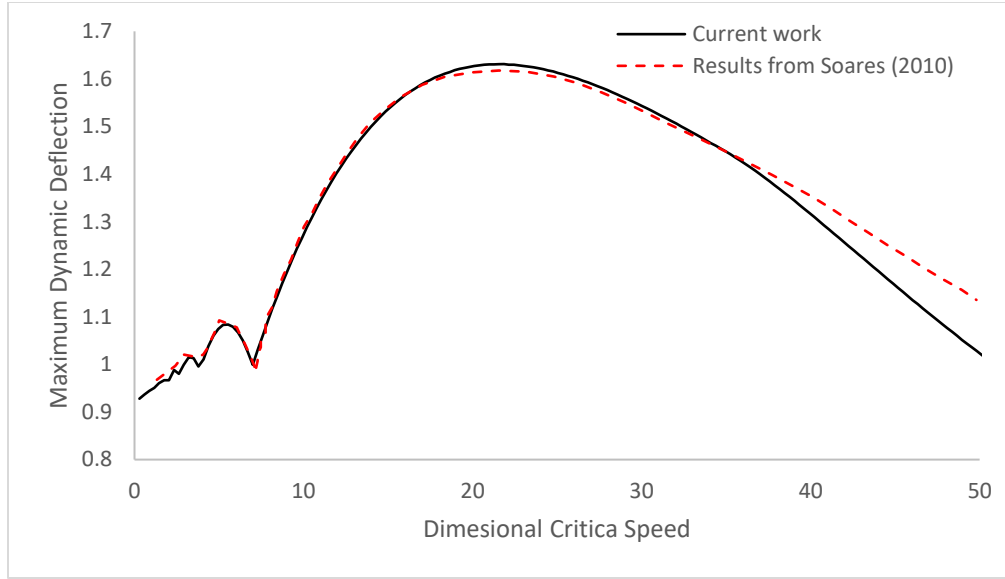


Figure 4.6: Dynamic deflection of non-classical simply supported beam at mid-span as a function of load speed (V_f) with moving absorber speed ($V_d = 0.1V_f$); — : Current work, ----- Previous results [60]

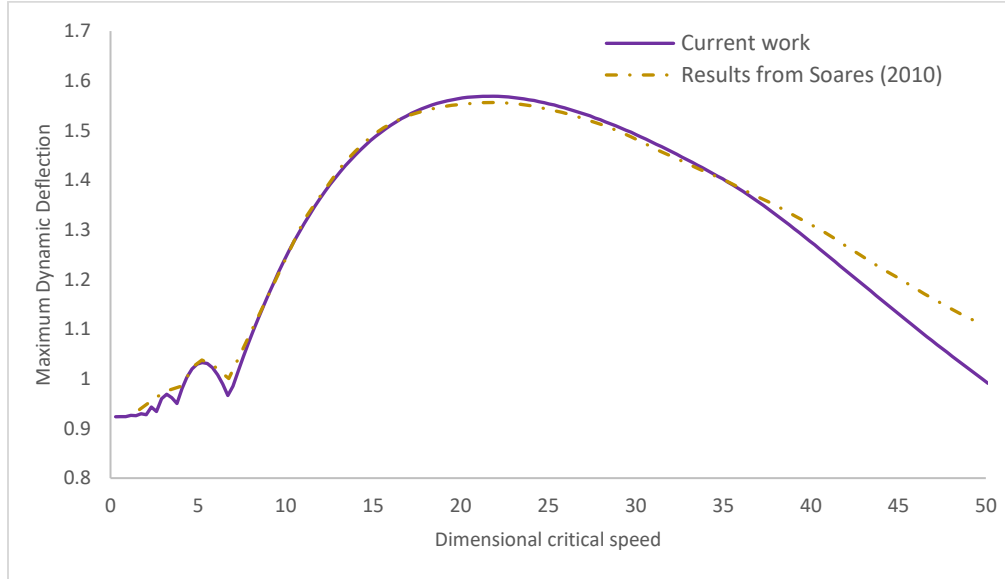


Figure 4.7: Dynamic deflection of non-classical simply supported beam at mid-span as a function of load speed (V_f) with moving absorber speed ($V_d = 1.0 V_f$); — : Current work, ----- Previous results [60]

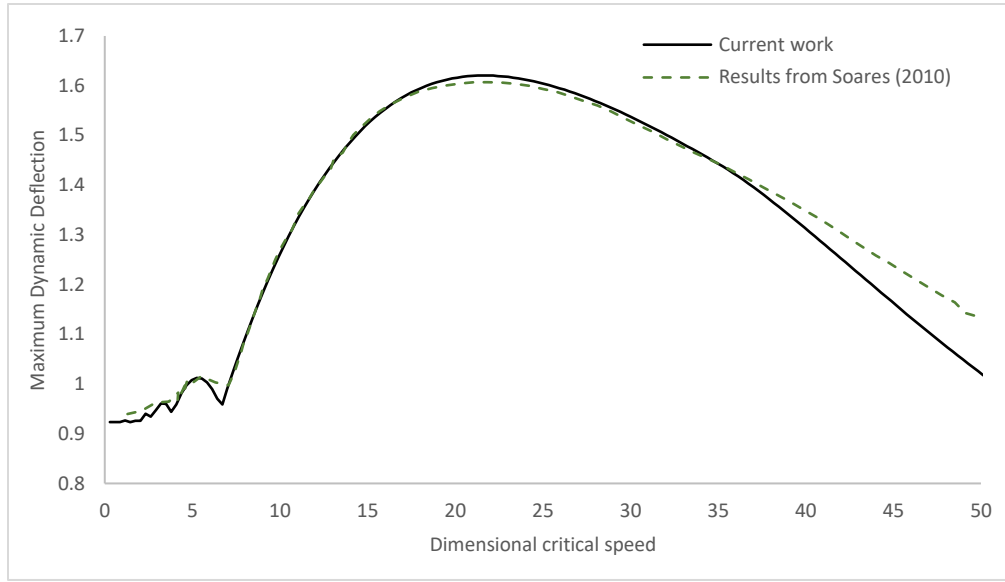


Figure 4.8: Dynamic deflection of non-classical simply supported beam at mid-span as a function of load speed (V_f) with moving absorber speed ($V_d = 2.0 V_f$); — : Current work, - - - - Previous results [60]

As per analysis from Figure 4.6, 4.7, 4.8, they are in best comparison with the results from literature [60], the only slight difference at higher speeds is due to modelling difference and they are of no concern to us as the critical speed at which max. dynamic deflection is occurring is important in our study and higher critical speeds are unrealistic in approach but for sake of analysis, we will analyse them in coming results sections. This section validates our current results and provide us confidence in utilizing them for analysing dynamic amplitudes of selected beams traversed by reciprocating sinusoidal load and with different damping configurations.

Chapter 5 : RESULTS ANALYSIS-NON-CLASSICAL SIMPLY SUPPORTED BEAM

5.1 Results

The derived equations from Section 3.0, are used to study non-classical beams responses with different conditions of oscillating load reciprocating at multiple cycles. The Euler Bernoulli beam parameters properties as follows:

Table 5-1: Properties of Beam (Aluminium)

E	ρ	L	b	h
206.9 GPa	7820 Kg/m ³	6 m	0.03 m	0.03 m

Following are the properties for the linear spring-mass absorber under investigation:

Table 5-2: Properties of Linear Spring Mass Absorber [19]

m absorber	k absorber	c absorber
2.1114 kg	286.11 N/m	9.076 N-s/m

5.1.1 Free vibration response analysis

As a starting case of modelling, let's consider an Euler Bernoulli beam with linear spring as supports of beam and we will vary stiffness of spring to see behaviour of natural frequencies. In several practical circumstances, the study of vibrating beams with elastically constrained ends against rotation and translation is of tremendous importance as discussed in literature review.

In study by other authors, the spring stiffness variation is not enhanced to sufficiently large value to replicate eigen values or natural frequencies of simply supported or cantilever beam. But we performed this analysis, and while enhancing stiffness values for this effect we observed that after certain value of spring stiffness, the natural frequency (ω) values become constant, and that is the value for spring stiffness where beam starts simulating either simply supported beam, cantilever or free-free depending on stiffness values of torsional springs and linear springs.

Figure 5.1 shows the variation of natural frequencies with increase in stiffness of linear springs and torsional springs attached at the ends. In below case, the torsional spring stiffness has been made zero and linear spring stiffness is made to increase from zero to certain value after which natural frequency value of beam becomes constant.

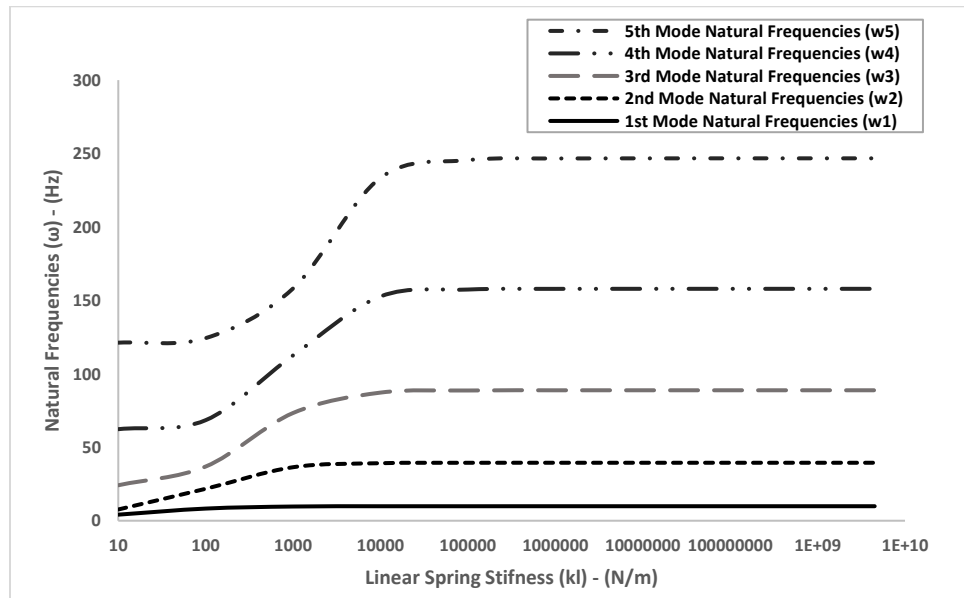


Figure 5.1: Natural Frequencies Variation with Increase in Spring Stiffness-Five Mode Shapes (Non-classical Simply Supported Beam: Torsional Spring Stiffness =0 and Linear Spring Stiffness= varying)

It can be seen from the Figure 5.1, that for each mode shape, the value of stiffness (kl) at which the natural frequency (ω) value becomes constant is different, lower mode shapes are attaining constant values of natural frequencies around 10,000 N/m spring stiffness whereas higher modes at around 100,000 N/m and above, but at the end they all converge to constant value for sufficient high spring stiffness.

5.1.2 Forced vibration response analysis

a) Without linear spring-mass absorber

5.1.2.1 Full travel oscillating load ($p=1$)

The load travelling on the beam has a sinusoidal motion in which load accelerates at start reaches maximum speed and then decelerates until reaches zero at the end of the beam. The load will not reach at end only, but it will also reciprocate to and from with multiple cycles.

First, we will observe dynamic response for load travelling to full distance ($p=1$) without absorber and will analyse the maximum deflection that will occur during this travelling motion for half cycle ($q=0.5$). Figure 5.2 shows response obtained for oscillating load travelling to end of beam and stopping at the end point.

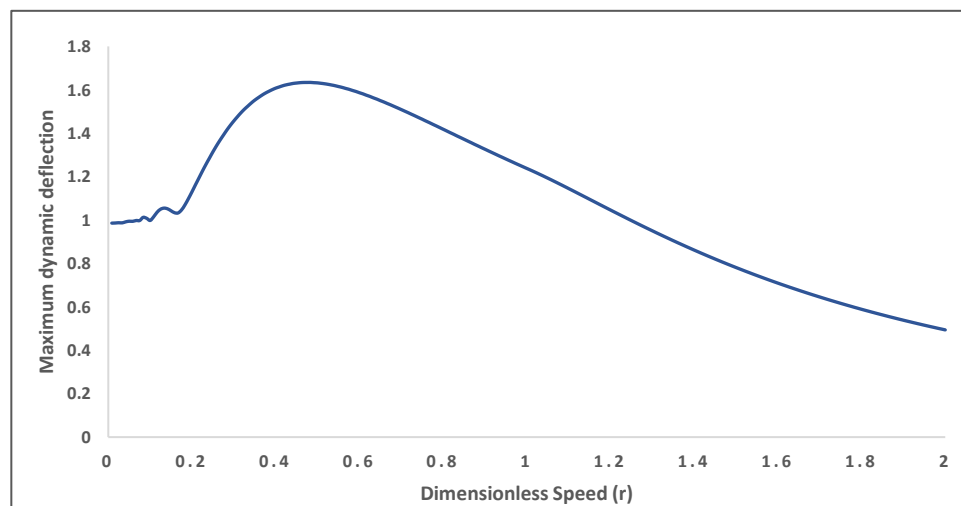


Figure 5.2: Dynamic deflection of non-classical simply supported beam at mid-span as a function of dimensionless speed (r) for half cycle (forward travel only)- $q=0.5$

As evident from Figure 5.2, the maximum dynamic deflection is occurring around $r = \frac{vf}{vc} = 0.475$ and then the dynamic deflection starts decreasing moderately with the increase in value of r .

Now we will increase our analysis further by considering multiple travelling cycles in which load will move to end ($p = \frac{x}{L} = 1$) and will come back to origin ($p = \frac{x}{L} = 0$) completing one cycle as $q = 1$. So, two complete travels to end of beam and coming back to origin will be $q = 2$ cycle and similar trend goes for $q = 3$ and $q = 4$ cycles. The reciprocation of load in multiple cycles amplify the dynamic deflection of beam which has been shown in Figure 5.3.

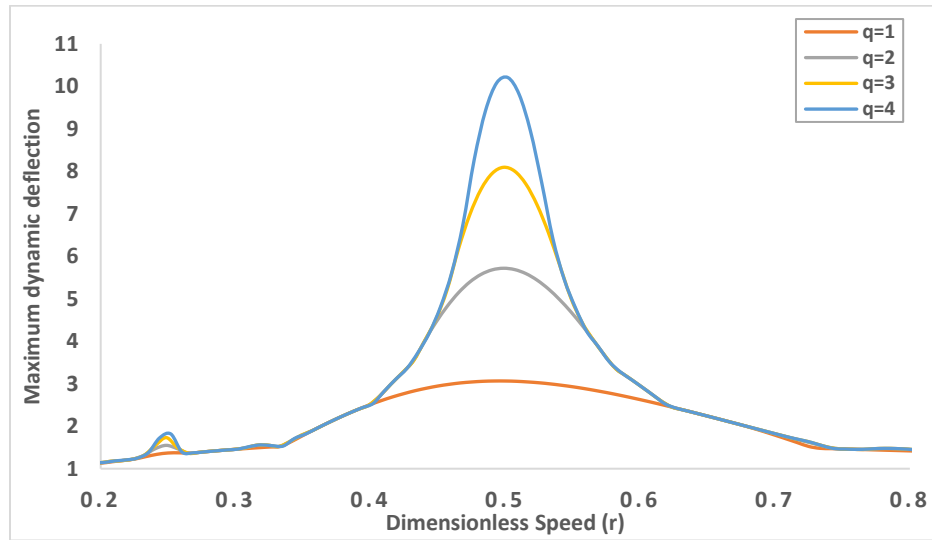


Figure 5.3: Variation of dynamic deflection of the mid-span of a non-classical simply supported beam as a function of traveling speeds for multiple cycles: $q = n$ (where $n = 1, 2, 3, 4$)

The trend for maximum dynamic deflection with increase in no. of complete travelling cycles (q) which is equal to $q = 1, 2, 3, 4$ is shown in Figure 5.4 (a). The maximum dynamic deflection is occurring around a particular range of dimensionless speed ratio r which can be observed in Figure 5.4 (b)

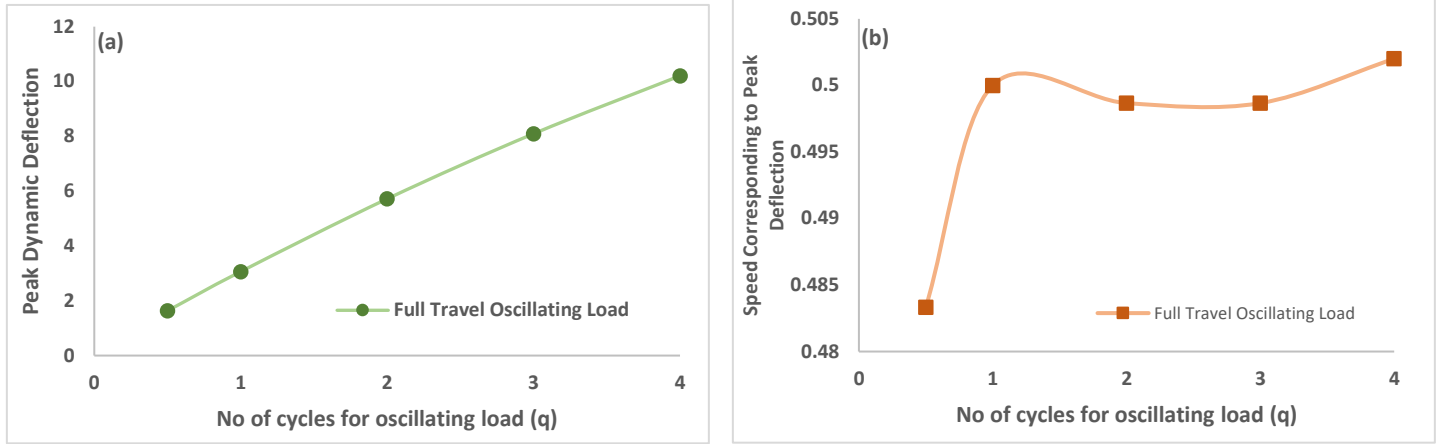


Figure 5.4: Variation observed as a function of multiple cycles q for (a) Peak Dynamic Deflection- (b) Speed Corresponding to Peak Dynamic Deflection

We observed the fact that when load is travelling to forward distance only and then stopping at end, the maximum deflection peak is more flattened as cancellation phenomena due to returning load is not affecting the vibrating beam. But when we increased the cycles ($q=n>0.5$), the graph attained shaper peak for dynamic deflection and immediately decreases down to lower dynamic deflection as the damping increases due to cancellation effect by returning load.

Since the peaks deflection is function of travelling length parameter $p=0 \leq x/L \leq 1$, so we will analyse the effect of partial travelling of load on peak dynamic deflection of beams in next section.

5.1.2.2 Partial travel oscillating load ($p \neq 1$)

In this section, the oscillating load travelling on the beam will cover length of beam partially. The load starts at $p=\frac{x}{L}=0$ and travels a distance $\frac{x}{L} < 1$ along the beam, then travels back to the origin at $p=\frac{x}{L}=0$ covering one whole cycle as $q = 1$.

To understand the response behavior as the dimensionless length parameter p is varied, the dynamic responses after one cycle are plotted versus dimensionless critical speed ratios r for different values of dimensionless length parameter p .

As stated earlier, load will not be continuous rather it will be sinusoidal load in which load accelerates at start reaches maximum speed and then decelerates until reaches zero at the end of the beam.

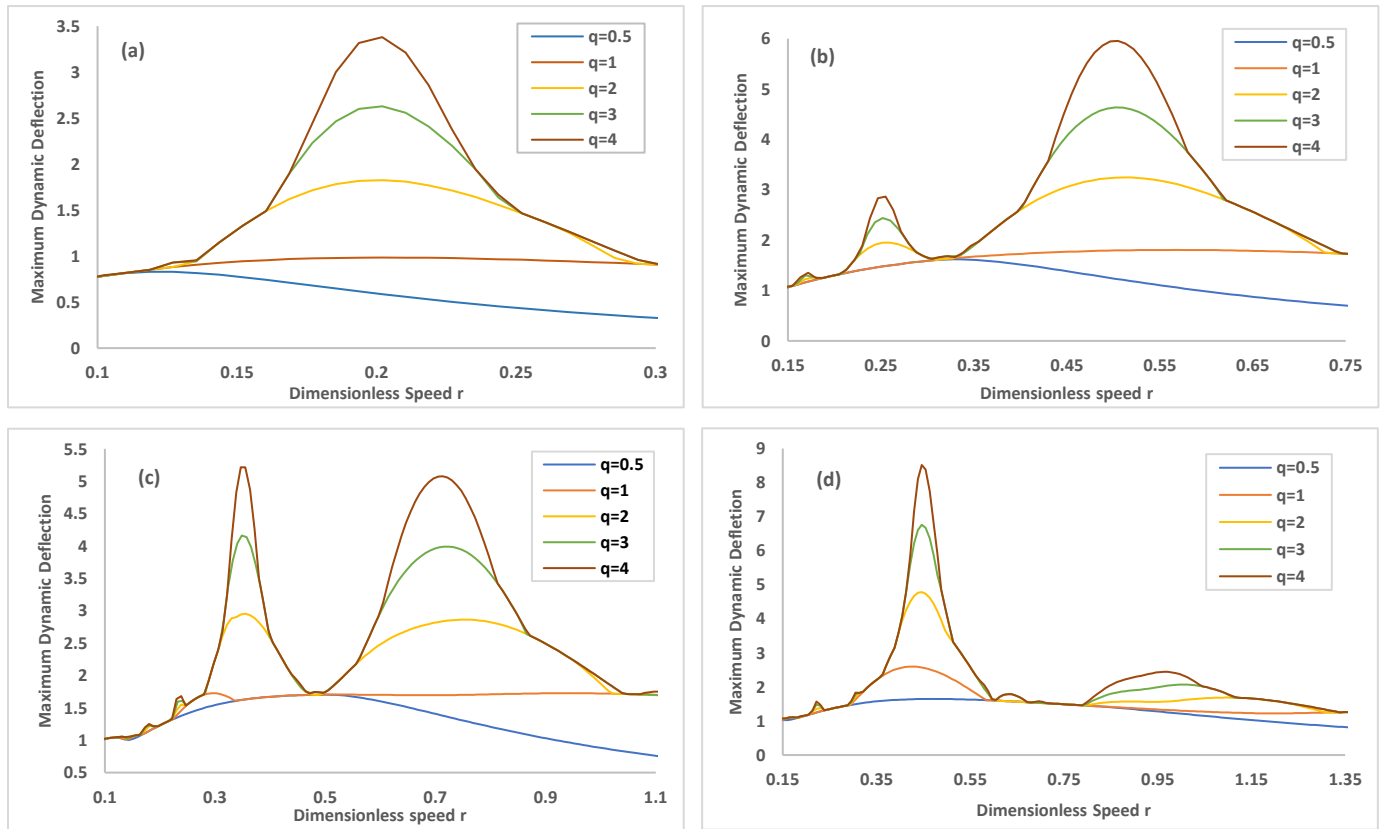


Figure 5.5: Maximum Dynamic Deflection vs Dimensionless Speed ratio r : at $q = 0.5, 1, 2, 3, 4$ for (a) $p = 0.2$ (b) $p = 0.5$ (c) $p = 0.7$ (d) $p = 0.9$

As observed from Figure 5.5, more than one peak is observed for dynamic deflection when load is travelling to partial distance with multiple cycles q . To have more insight into variation of peak

dynamic deflection with number of cycles, the peak values, and their corresponding speed ratios (r) are plotted against multiple number of cycles (q) as shown in Figure 5.6.

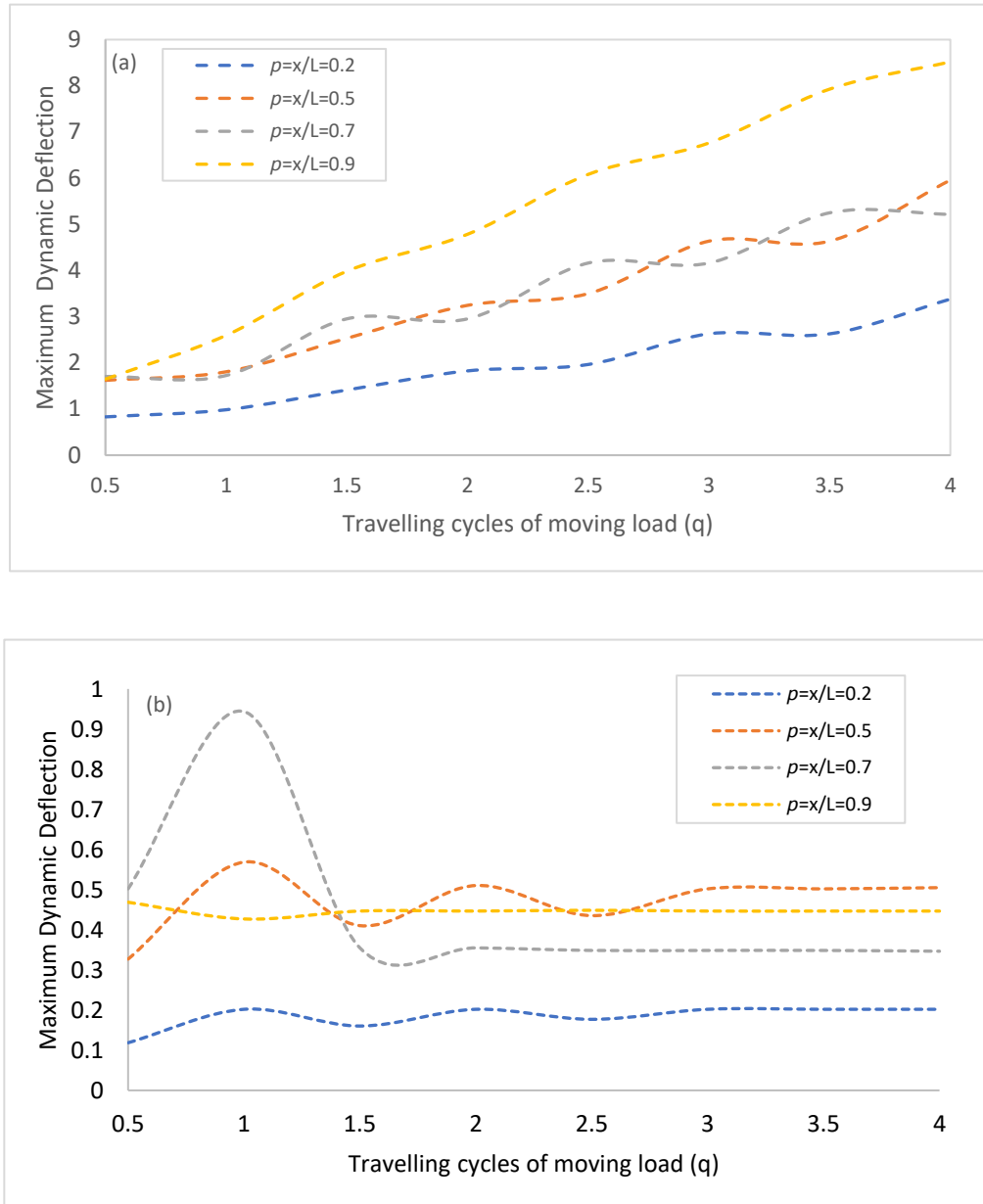


Figure 5.6: Variations of observed as a function of (a) Peak Dynamic Deflection (b) Dimensionless speed corresponding to Peak dynamic deflection for multiple cycles: $q = n$ (where $n = 0.5$ to 4)

We observed that for travelling load up to $p = 0.5$ and 0.7 , the peak dynamic deflection is in synchronous with each other but are of opposite phases as seen in Figure 5.6(a). The speed corresponding to maximum dynamic deflection have different variations in reference to multiple cycles as observed in Figure 5.6(b)

When data analysis is done for partial travel to $p = 0.2$, the average dimensionless $r = \frac{vf}{vc}$ at which maximum deflection occurs for all cycles (converging at higher cycles) is about $r = 0.202$ of the dimensionless speed and only one peak is observed which is at this speed ratio. For the travelling distance of $p = 0.5$ the velocity (common) for higher cycles at which maximum deflection is occurring is taken which is $r = 0.502$ (Approximate around 0.5) and we are observing peak at this speed and another peak at half multiple of this speed ($j = 0.5$). Similar strategy goes for travelling distance p of 0.7 and 0.9 , giving us velocity ratio (r) of 0.35 and 0.45 respectively. In our analysis for travelling distance of $p = 0.7$, we observe one peak at original speed ratio 0.35 and other peak at multiple factor ($j = 2$) and finally for travelling distance of $p = 0.9$, higher peak is observed at original speed ratio and small peak observed at half multiple of this speed ($j = 0.5$).

5.1.2.3 Effect of critical dimensionless speed $r = 0.202, 0.502, 0.35$ and 0.45 on rest of load travelling length (p).

If we analyze the trend of the numerical values of dimensionless ratio speed (r), we see that numerical value analogy is such that for travel distance less than midpoint of beam $p < 0.5$, the numerical value of speed ratio for peak deflection is same as travel distance but when load travels to distance more than midpoint of beam $p > 0.5$, the numerical value of ratio for peak deflection is half of travel distance on beam. If we state facts as per this analogy, then there

should be two peaks observed at mid span of beam, one when load traversed the original travel length p and other at multiple ($i = \text{Constant multiple factor}$) of p .

To analyze this observation further we will use $r=0.202, 0.502, 0.35$ and 0.45 and will check its effect on the rest of travel distance of beam. Figure 5.7 (a, b, c, d) provides us insight into this observation.

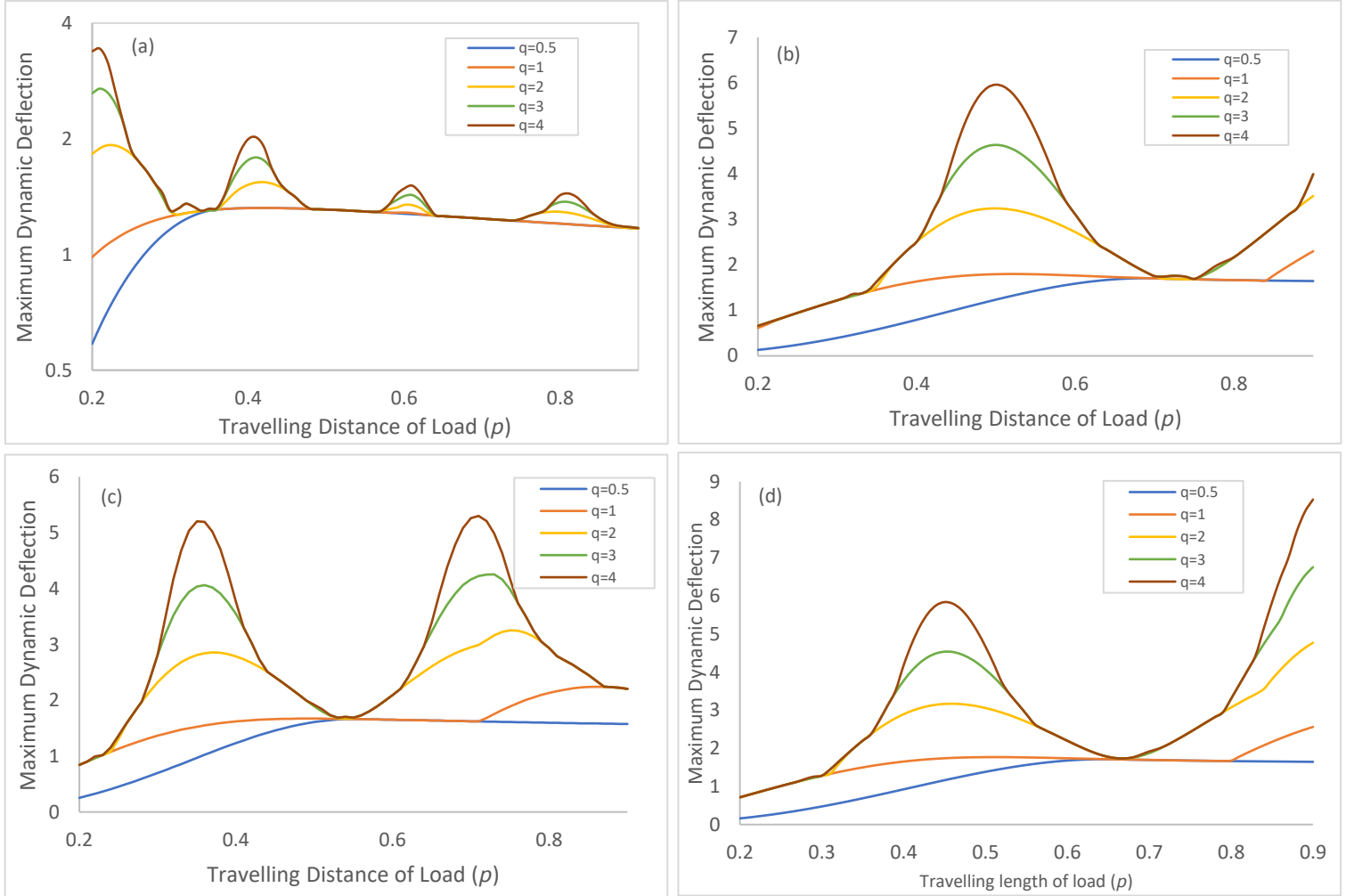


Figure 5.7: Variation of maximum dynamic deflection as a function of multiple cycles (q) for dimensionless speed for remaining travel length (p) on beam (a) $r = 0.202$ (b) $r = 0.502$ (c) 0.35 (d) 0.45

In Figure 5.7 (a) analysis, maximum deflection peaks are observed at $p = 0.2$ of travel distance as expected but also smaller peaks observed at even multiples of travelling length of 0.2 ($i = 2, 4, 6$) along the remaining length parameter of beam whereas cancellation observed at odd multiples of travelling length of 0.2 ($i = 1, 3, 5$)

In Figure 5.7 (b), we are using 0.502 (Dimensionless speed providing maximum deflection when load is reciprocating to midpoint) and will check its effect on the rest of travel distance of beam.

As expected, the peak deflection for all cycles is occurring at $p = \frac{x}{L} = 0.5$ and we see another trend of increase of dynamic deflection when load reached to $p = \frac{x}{L} = 0.9$ which would increase more if load reached to end of beam. As stated before, we should expect two peaks of deflection for critical speed corresponding to partial travel $p = 0.5$, one at original point (for $\frac{x}{L} = 0.5$) and other at two times of original point (for $\frac{x}{L} > 0.9$). Now in last two cases, this analogy will be discussed when we use speed ratio corresponding to $\frac{x}{L} = 0.7$ and $\frac{x}{L} = 0.9$.

In Figure 5.7 (c), we are using 0.35 (Dimensionless speed r providing maximum deflection when load is reciprocating to range $0.5 \leq \frac{x}{L} \leq 1.0$ and the exact travel length at which maximum deflection is occurring is 0.7) and we will check its effect on the rest of travel distance $0.2 \leq \frac{x}{L} \leq 0.9$ of beam. On analyzing, we are seeing two peaks of deflection for critical speed corresponding to partial travel, one at original point (for $\frac{x}{L} = 0.7$) and other at half of original point ($i = 0.5$, for $\frac{x}{L} = 0.35$).

Referring to last Figure 5.7(d), we should expect similar trend of two peaks as observed with speed ratio 0.35, we are using 0.45 (Dimensionless speed providing maximum deflection when load is reciprocating to range $0.5 \leq \frac{x}{L} \leq 1.0$ and the exact travel length at which maximum

deflection is occurring is 0.9) and we will check its effect on the rest of travel distance $0.2 \leq \frac{x}{L} \leq 0.9$ of beam. As predicted, two peaks observed one at original travel and other at multiple of it ($i=0.5$).

This proves our point that partial travel of oscillating load completing reciprocating cycle along the beam provides peaks of deflection as per analogy mentioned above, which is evident for all cases of partial travel.

b) With fixed linear spring-mass absorber

The linear absorbers acting on the beam defined in the previous section (Wu model [54]) are investigated in this part, and different locations ($d=\frac{da}{L}$) approaches are utilized to analyze the optimal location for linear absorber. Absorber location is critical factor when the load has oscillating (sinusoidal) movement and is completing travel by reciprocating with multiple cycles (q) along the length of the beam. Indicator considered for evaluation of its performance based on absorber location, is the damping effect on maximum vibration amplitude as a function of no of cycles q . Since it is the most common variation assumed in understanding dynamic amplitudes for forward travel loads, hence it is critical for our analysis, as it will give us response control insight along the entire length of beam for multiple cycles.

To perform analysis, same system of beam is considered as in previous case and the absorber is fixed on the beam at definite multiple locations of $d = 0.2, 0.5, 0.7$ and 0.9 to see dynamic responses. In the subsection of this case, oscillation load will be covering the entire span of beam with multiple cycles $q = n$ (Min:0.5 to Max:4).

5.1.2.4 Full travel oscillating load ($p=1$)

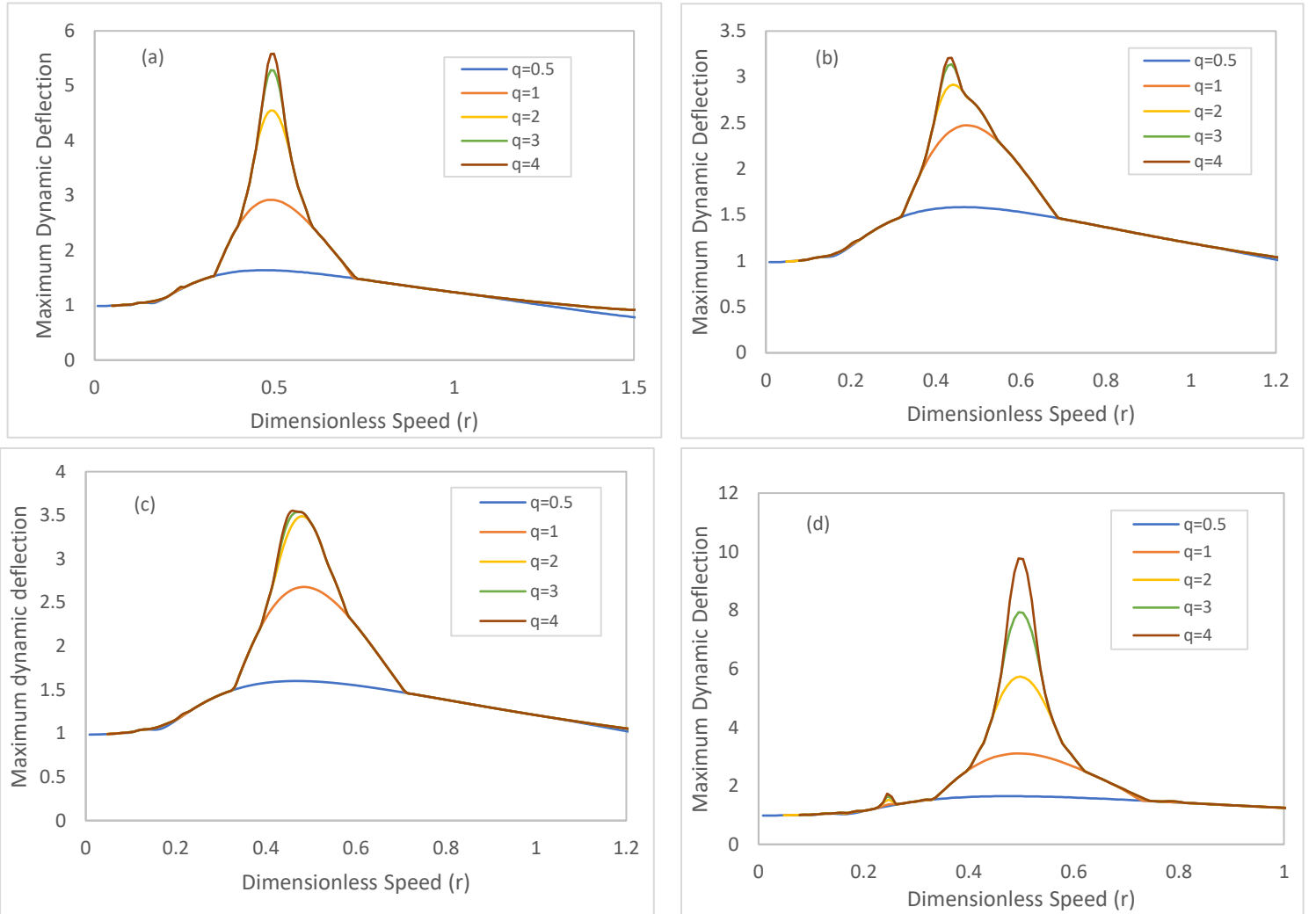


Figure 5.8: Variation of maximum dynamic amplitude of non-classical simply supported beam at mid-span as a function of Dimensionless Speed at $d = \frac{da}{L} =$ (a) 0.2 (b) 0.5 (c) 0.7 and (d) 0.9

In Figure 5.8 analysis, multiple deflection peaks are diminished with the usage of absorber at different locations. Singular peaks are observed which are dampened as compared to dynamic deflections when no absorber is introduced in the system. Discrete locations taken for absorbers are $d = \frac{da}{L} = 0.2, 0.5, 0.7$ and 0.9 . Extracting peak deflections values from these damped values

and comparing them with no absorber deflections in Figure 5.15 will provide us more insight into dynamic analysis with absorber when load is reciprocating for multiple higher cycles.

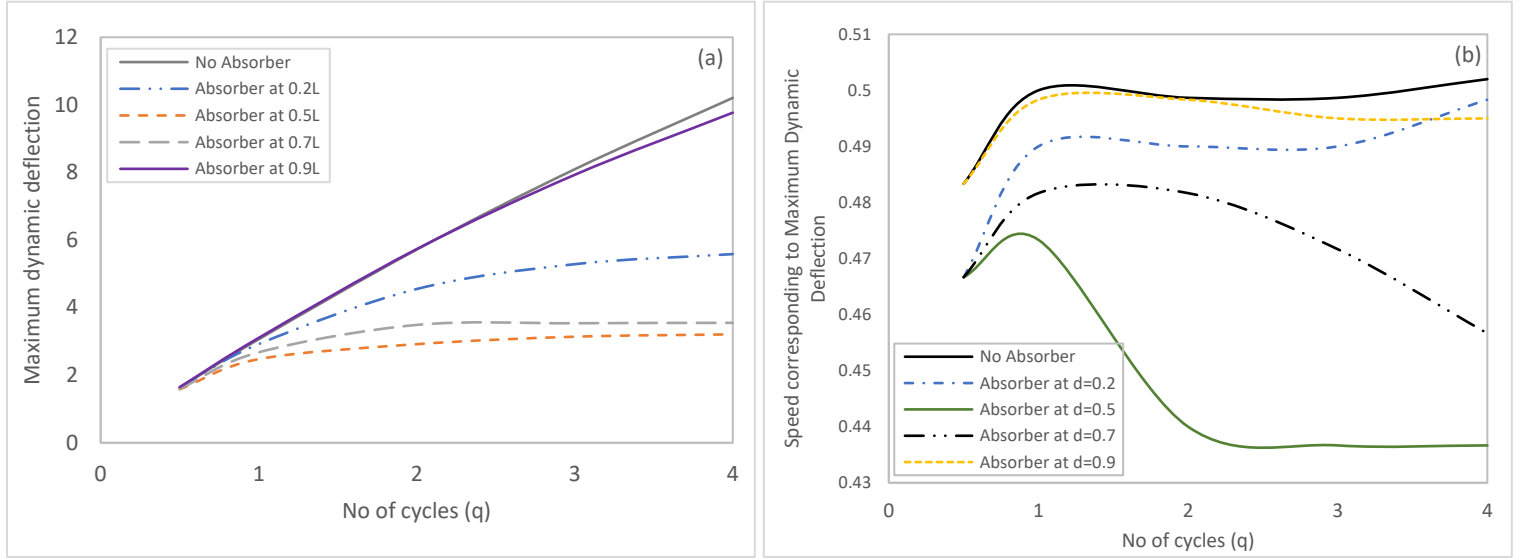


Figure 5.9: Variation for (a) Peak Values of Dynamic Deflection (b) Speed corresponding to peak dynamic deflection (vs) No of cycles for absorber positions at $d=\frac{da}{L}=0.2, 0.5, 0.7, 0.9$

Figure 5.9, gives insight into change of maximum displacement with and without absorber at the mid-span of the beam as a function of dimensionless speed r . The inclusion of the absorber at $d=\frac{da}{L}=0.2$, reduced the maximum dynamic deflection of the beam up to 45% for maximum cycle of $q=4$. The absorber will now be moved to further locations and effect on maximum dynamic amplitude will be observed. It also shows that by inclusion of the absorber at $d=\frac{da}{L}=0.5$, maximum dynamic deflection of the beam reduced by 69% for maximum cycle of $q=4$. For forward only travelling load, the effect of absorber location looks negligible being smaller change in damping, and but here damping is increasing with the increase of reciprocating cycle for the oscillation load and absorber location is providing more reduction in dynamic amplitudes in comparative analysis,

but fact shall not be ignored that dimensionless speed r is different in case of forward travel load. As stated before, studying of location change is assumed very basic variation by other authors but it is playing very critical role in load reciprocating with multiple cycles. Similarly with inclusion of absorber at $d = \frac{da}{L} = 0.7$, damping achieved up to 65% for maximum cycle of $q=4$. Now in next case absorber will be fixed at $d = \frac{da}{L} = 0.9$ to evaluate final damping effect at higher cycles of reciprocating load. Whereas the change of maximum displacement at inclusion of the absorber at $d = \frac{da}{L} = 0.9$, reduces the maximum displacement of the beam up to only 4% for maximum cycle of $q=4$. So, for oscillating load, the maximum damping effect is obtained when absorber is at mid-location of the beam.

Figure 5.9 also shows the maximum reduction in displacement is for the highest cycle of $q=4$ and the best absorber position at which this damping effect is more significant than other absorber positions is at $d = \frac{da}{L} = 0.5$ and the significant shift of dimensionless speed r for different absorber positions is seen to be more significant when absorber is at $d = \frac{da}{L} = 0.5$, whereas this shift of dimensionless r is least for $d = \frac{da}{L} = 0.9$. As shown in the plots above, the absorber position has a significant impact on the beam response. The drift of dimensional speed value and variation of the maximum displacement value with the linear absorber location sheds light on its importance in reciprocating load with multiple cycles ($q = n$) but in this section, we used discrete values of absorber location which missed the effects of absorber location for other points along the length of beam ($0 \leq p = \frac{x}{L} \leq 1$).

To confirm the analysis for absorber allocation at rest of beam length, we selected the dimensionless speed of 0.4965 which is the average ratio value at which maximum deflection is

occurring accommodating the dimensionless speeds r corresponding to maximum deflection occurring for all the cycles of reciprocating load. This speed is producing higher peaks of amplitude when no absorber is considered in the beam system. Now, we will utilize this speed and will get the effect of dynamic amplitude on all the positions of absorber along the length of the beam. Figure 5.16 is plotted to analyze the effect of absorber located at other points along the length of the beam ($0 < d = \frac{da}{L} \leq 0.9$)

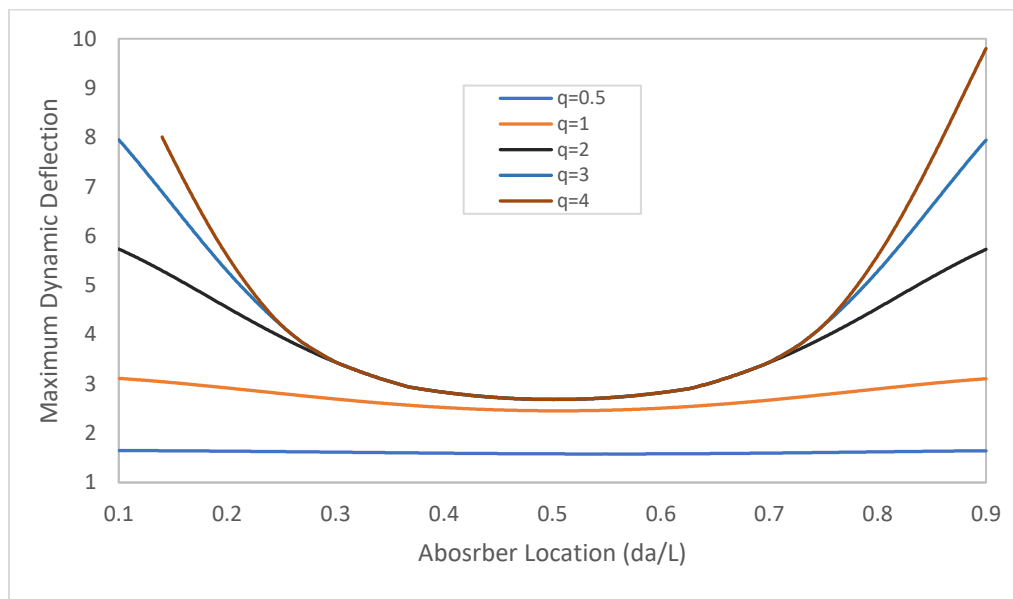


Figure 5.10: Maximum dynamic deflection at mid span of the beam as function of absorber location along the length of the beam ($0 < d = \frac{da}{L} \leq 0.9$).

As per analysis along the entire length of beam, it is noted the maximum damping is occurring when absorber is fixed between 0.5 and $0.55 L$ as shown in Figure 5.10,. Another thing noticed in analysis is that for forward travel load only ($q = 0.5$), absorber does not have a significant effect and dynamic deflection is reduced only from 1.64 to 1.58mm where in case of reciprocating cycle of $q = 4$, dynamic deflection reduced from 10.2 to 3.2 mm showing significance impact when load

is reciprocating. As per Figure 5.10, the best absorber location for $q = 0.5$ to 1 is $0.5 < d = \frac{da}{L} \geq 0.55$ whereas for $q > 1$ it is $d = \frac{da}{L} = 0.5$, which proves the fact that best absorber location is around the middle of the beam.

5.1.2.5 Partial travel oscillating load ($p \neq 1$)

In the previous section, we studied the absorber location impact on the full travel of oscillating load. In this section, we will study the effect of absorber location on partial travel of oscillating load. As studied in section of partial travel without absorber, we noted that ratio of critical velocities for partial travels of moving load reciprocating to $p = x/L = 0.2, 0.5, 0.7$ and 0.9 are $r = 0.202, 0.502, 0.35$ and 0.45 respectively. We will observe effect of location of absorber on maximum displacement of beam for these partial travels considering these ratios of speeds. These speeds are producing higher peaks of amplitude when no absorber is considered in the beam system. Now, we will utilize these speeds and will get the effect of dynamic amplitude on all the positions of absorber along the length of the beam. Figure 5.11 is plotted to analyse the effect of absorber located at other points along the length of the beam ($0 \leq p = \frac{x}{L} \leq 1$).

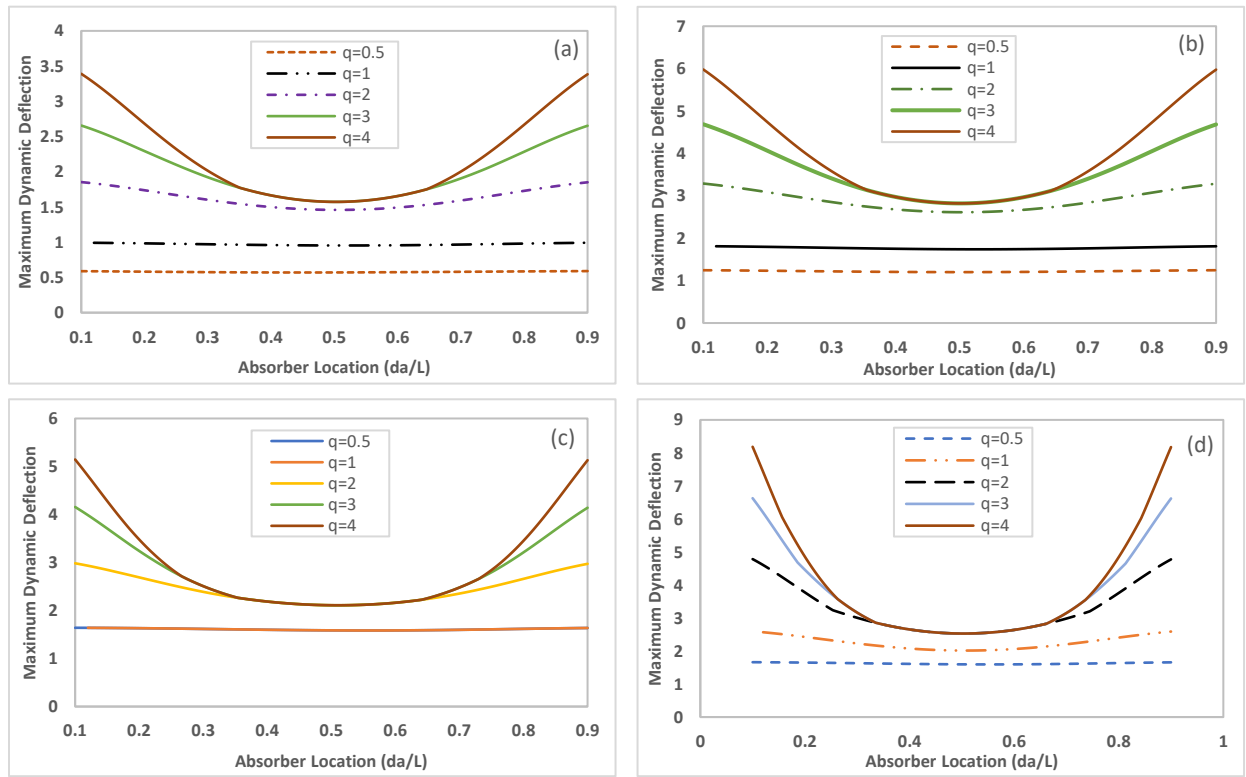


Figure 5.11: Maximum Dynamic Deflection at mid span of the beam as function of absorber location along the length of beam ($0.1 \leq da/L \leq 0.9$) when load subjected to partial travel of $d=x/L=$ (a) 0.2, (b) 0.5, (c) 0.7, (d) 0.9.

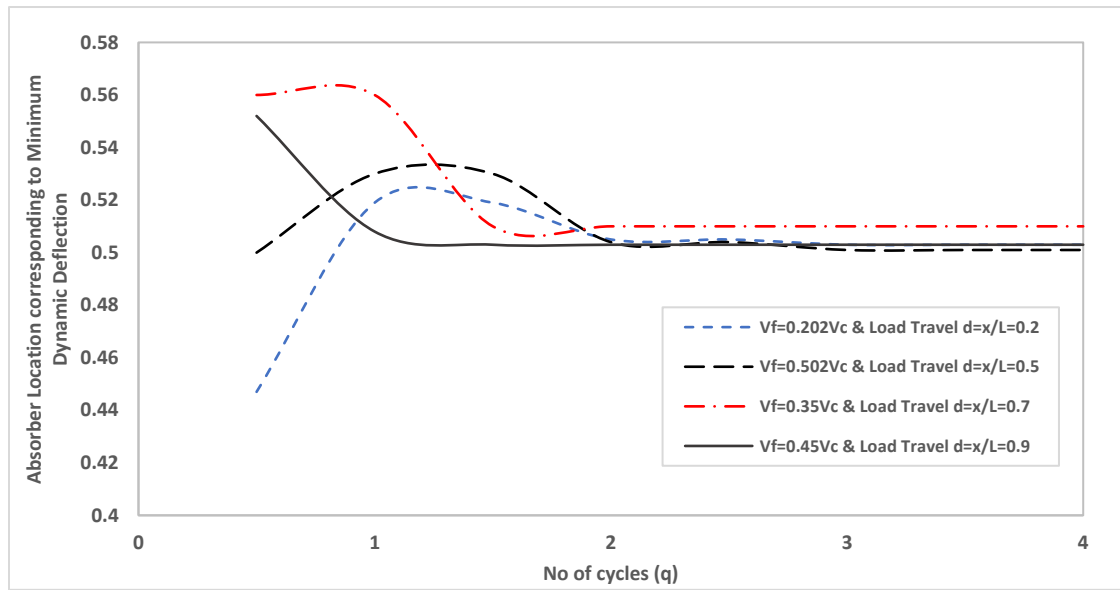


Figure 5.12: Best absorber location along the length of beam ($0.1 \leq da/L \leq 0.9$) when load perform partial travel of $x/L=$ (a) 0.2, (b) 0.5, (c) 0.7, (d) 0.9 with critical velocities corresponding to partial travel vs No. of cycles (q).

We analyzed for the partial travel of load, that absorber location is again critical when we have load travelling for multiple cycles even if it is for short travel distance along the length of the beam ($0.1 \leq p \leq 0.9$). As seen from Figure 5.12, for partial travel of load with corresponding critical speed ratios, maximum damping effect of the displacement of the beam is between 0.45 and 0.55 which is evident further from Figure 5.12. Best absorber locations for partial travel of loads are between $0.44 \leq d_a/L \leq 0.56$ with trend lines for best absorber location are converging between $0.44 \leq d_a/L \leq 0.56$ when reciprocating cycle for oscillating load is increased from $q=2$ to $q=4$.

In the previous sections, we studied about the reciprocating load effect on the displacement of beam as a function of reciprocating cycles of load without absorber and with absorber at different locations along the length of the beam ($0.1 \leq x/L \leq 0.9$). In the next section we will study the effect of moving absorber on the displacement of the beam when the absorber is moving with the load.

c) With moving linear spring-mass absorber

In the present section, spring mass absorbers acting on the beam defined in the previous section (Soares model [60]) are analyzed, and different travelling speeds $m = V_d/V_f$ ($V_d = d_t/t$) approaches are applied to find the optimal speed of moving absorber. The load utilized is same as in previous cases which will be sinusoidal load and reciprocating at multiple cycles (q). The cases that will be considered based on moving absorber speed and travel length (absorber configurations) to analyze its impact on dynamic deflection of the beam. With variations considered, load and absorber with sinusoidal motion are reciprocating at multiple cycles (q),

- i. To full length of beam ($p = \eta = 1$) with same speed $m=1$ (Absorber motion synchronized with load motion).
- ii. To full length of beam ($p = \eta = 1$) with different speed $m \neq 1$.

- iii. To same partial travel length ($\rho=\eta\neq 1$) reciprocating at multiple cycles with same speed $m=1$.
- iv. To same partial travel length ($\rho=\eta\neq 1$) reciprocating at multiple cycles with different speed $m\neq 1$.
- v. To different partial travel length ($\rho\neq\eta\neq 1$) reciprocating at multiple cycles with optimum speed.

5.1.2.6 Load and absorber reciprocating to same full length ($\rho=\eta=1$) and with same speed ($m=1$).

Consider now the system of Figure 3.8 and 3.9, with a moving load and a moving absorber with velocity $V_f(t) = V_d(t) = \text{Sinusoidal}$. The absorber mass considered in this study is 5% of the mass of the beam and agrees with absorber values utilized [19]. Absorber velocity and travel length is synchronized with load critical velocity $V_f(t)$ and travel length $\rho=1$. Figure 5.13 shows the maximum displacement at the mid-span of the beam for the considered model as a function of the dimensionless speed of load.

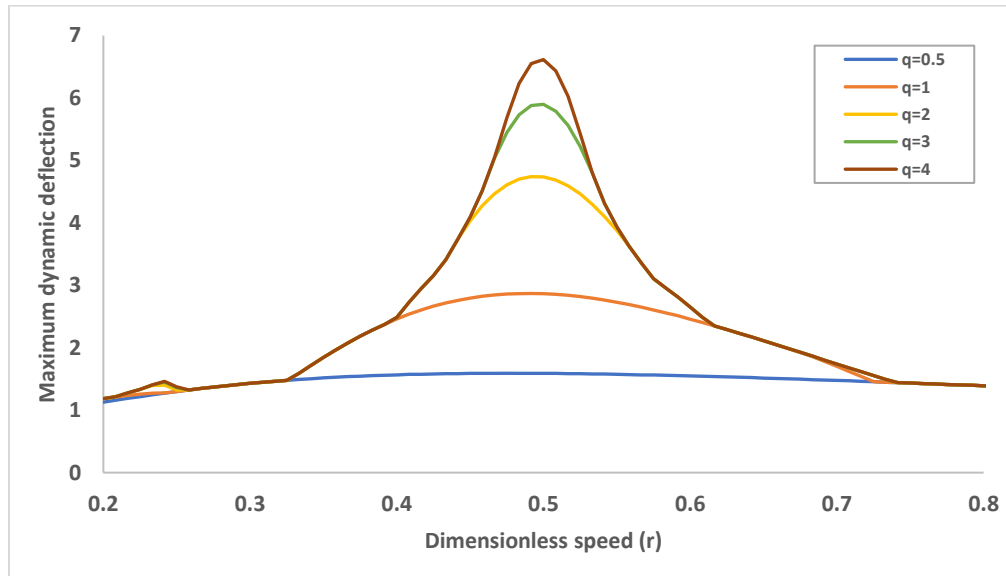


Figure 5.13: Variation of dynamic deflection amplitude of non-classical simply supported beam at mid-span as a function of Dimensionless speed (r)

For mid-span defection of simply supported beam, maximum dynamic deflection trends for full travel length are almost similar as studied in previous section, when no absorber is fixed to the beam. As stated before, absorber utilized is 5% of the mass of the beam and is providing considerable damping effect at the mid-span. In the section of analysis for absorber with fixed beam, we have optimum absorber location at $d = d_a/L = 0.5$ to 0.55 and less effective absorber location equivalent to $d = 0.9$. Dynamic deflection for these locations will be compared with peak dynamic deflection obtained for synchronized motions and no absorber configuration.

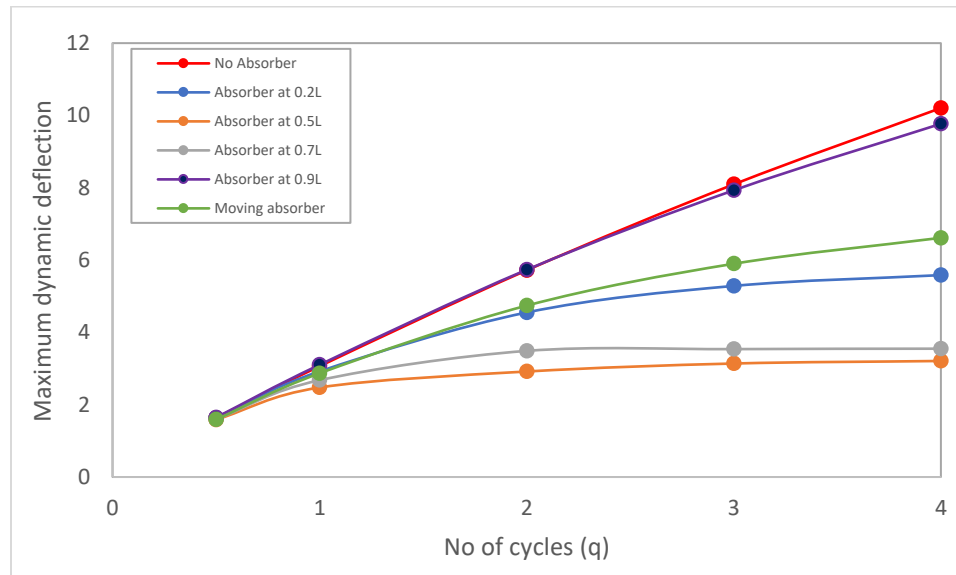


Figure 5.14: Variation of peak dynamic deflection of non-classical simply supported beam at mid-span as a function of no of cycles q for no absorber, absorber at $d = 0.2, 0.5, 0.7, 0.9$ and synchronized moving absorber

The results obtained in above Figure 5.14, shows dynamic deflection in red curve for varying critical velocity when travelling load movement is completely synced with motion of dynamic absorber. The displacement amplitude is reduced by 32% which is better as compared to absorber fixed at $d=d_a/L = 0.9$. But still fixing stationary absorber at $d=d_a/L \leq 0.5$ and 0.7 is providing us better damping effect of maximum amplitude at mid-span of the beam.

5.1.2.7 Load and absorber reciprocating to same full length ($\rho=\eta=1$) and with different speed ($m \neq 1$)

In this section, we will study about full travel reciprocation of load to entire length of beam. But absorber will be reciprocating at different speed ratios ($m = V_d/V_f$). In analysis, we will study which of absorber speed factor (m) is providing the maximum damping effect to displacement of beam when load is completing travel along entire length of the beam ($0 < \rho \leq 1$). We will vary the absorber speed factor ($0 < m \leq 2$) to obtain the corresponding dynamic responses. Figure 5.15 is obtained for these dynamic responses as a function of speed factor m .

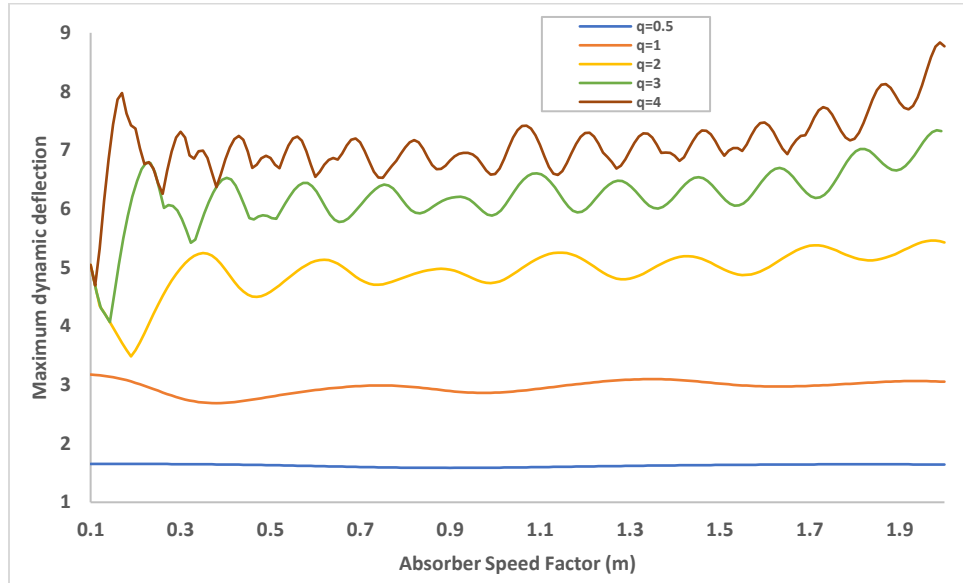


Figure 5.15: Variation of dynamic deflection amplitude of non-classical simply supported beam at mid-span as a function of absorber speed factor (m)

Figure 5.15 displays the variation of the maximum displacement at the beam mid-span for varied absorber speed factor m and a dimensionless speed r that induced the greatest displacement. When data is extracted for optimum absorber speed at which we are getting minimum dynamic deflection we obtain Figure 5.16.

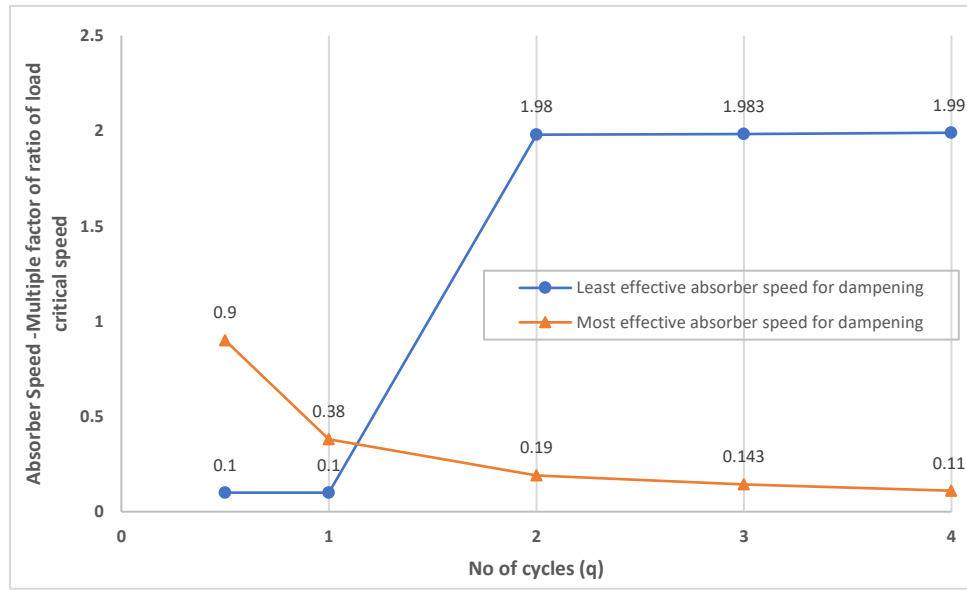


Figure 5.16: Variation of absorber speed factor (m) as a function of no of oscillating cycles (q)

As per Figure 5.16, the dynamic deflection of beam is minimum when absorber is travelling at speed less than the speed of load ($m < 1$). As number of reciprocating cycles (q) increases, the optimum absorber speed becomes lesser than load speed to provide efficient energy dissipation. To make the comparison evident, we will plot the Figure 5.17 to compare the dynamic amplitude damping for least effective and optimum speed of absorber at various cycles with peak deflection curves obtained in Figure 5.4 (a).

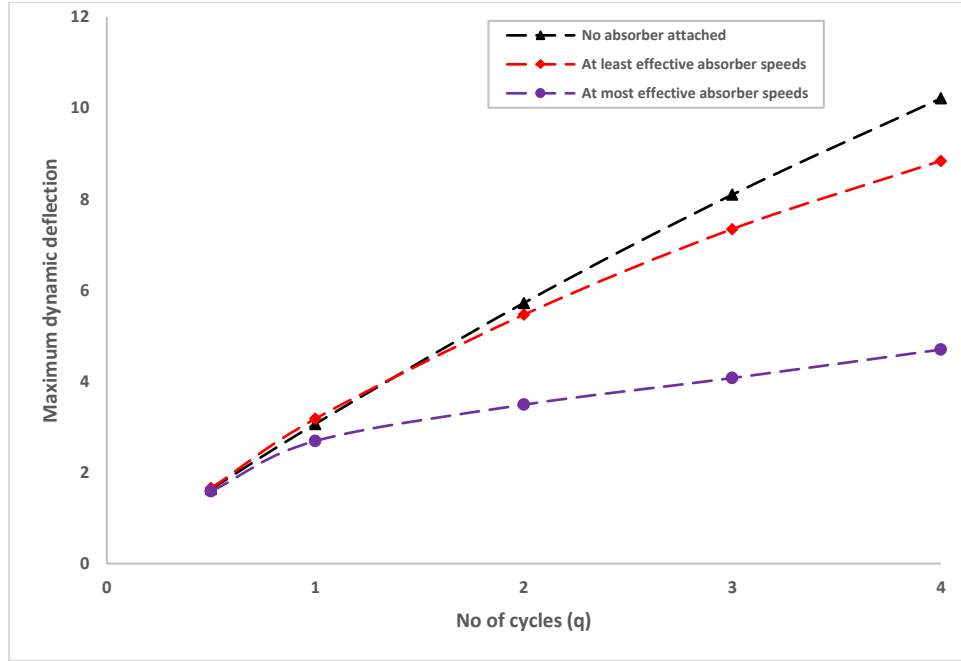


Figure 5.17: Variation of maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q)

Figure 5.17 shows that optimum absorber speeds are providing damping of deflection amplitudes at $q=0.5, 1, 2, 3$ and 4 cycles due to energy dissipation of about 2.9%, 12.2%, 39%, 49.7% and 54% respectively. At least effective absorber speeds amplification is observed at $q=0.5$ and $q=1$, whereas amplification at $q=0.5$ is very minimal that can be ignored. Slightly high amplification at $q=1$ of 4% is due to energy buildup, $q=1$ is the first cycle when load starts its return motion during beam first deflection of forward travel of load, and that too accompanied with synchronized absorber motion, so this results in adding amplification to overall response which becomes normalized as cycles increases. In case of least optimum absorber speed, we have very moderate damping at $q=2, 3$ and 4 . The amplitude damping produced at $q=2, 3$ and 4 cycles are 4.5%, 9.3% and 13.4% respectively due to minimal dissipation of energy.

5.1.2.8 Load and absorber reciprocating to same partial travel length ($p=\eta \neq 1$) and with same or different speed.

Dynamic responses due to load travelling to fixed partial distances of $p=x/L = 0.2, 0.5, 0.7, 0.9$ are already studied in previous sections with and without fixed absorber. In this section we will analyze load and absorber both travelling to same partial distances but at certain range of speed ratio and will analyze which absorber speed factor (m) is providing more impact on controlling dynamic deflection. Already calculated dimensionless speed ratios (r) for partial travel at which maximum deflection is occurring, will be utilized to see impact of absorber speed on the beam deflection.

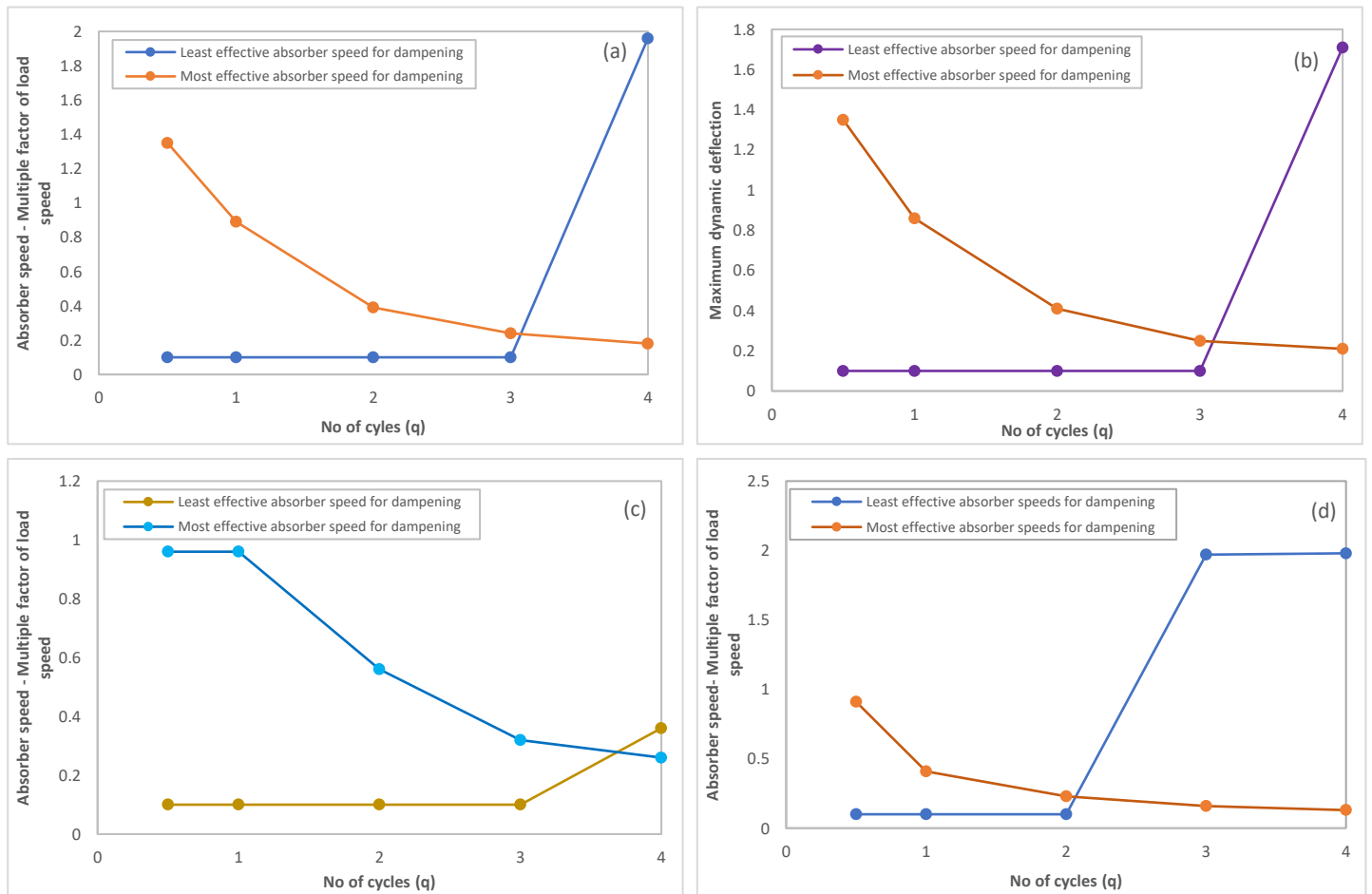


Figure 5.18: Variation of (a) absorber speed factor m -- (b) maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for p (a) $x/L = 0.2$, (b) $x/L = 0.5$, (c) $x/L = 0.7$ and (d) $x/L = 0.9$

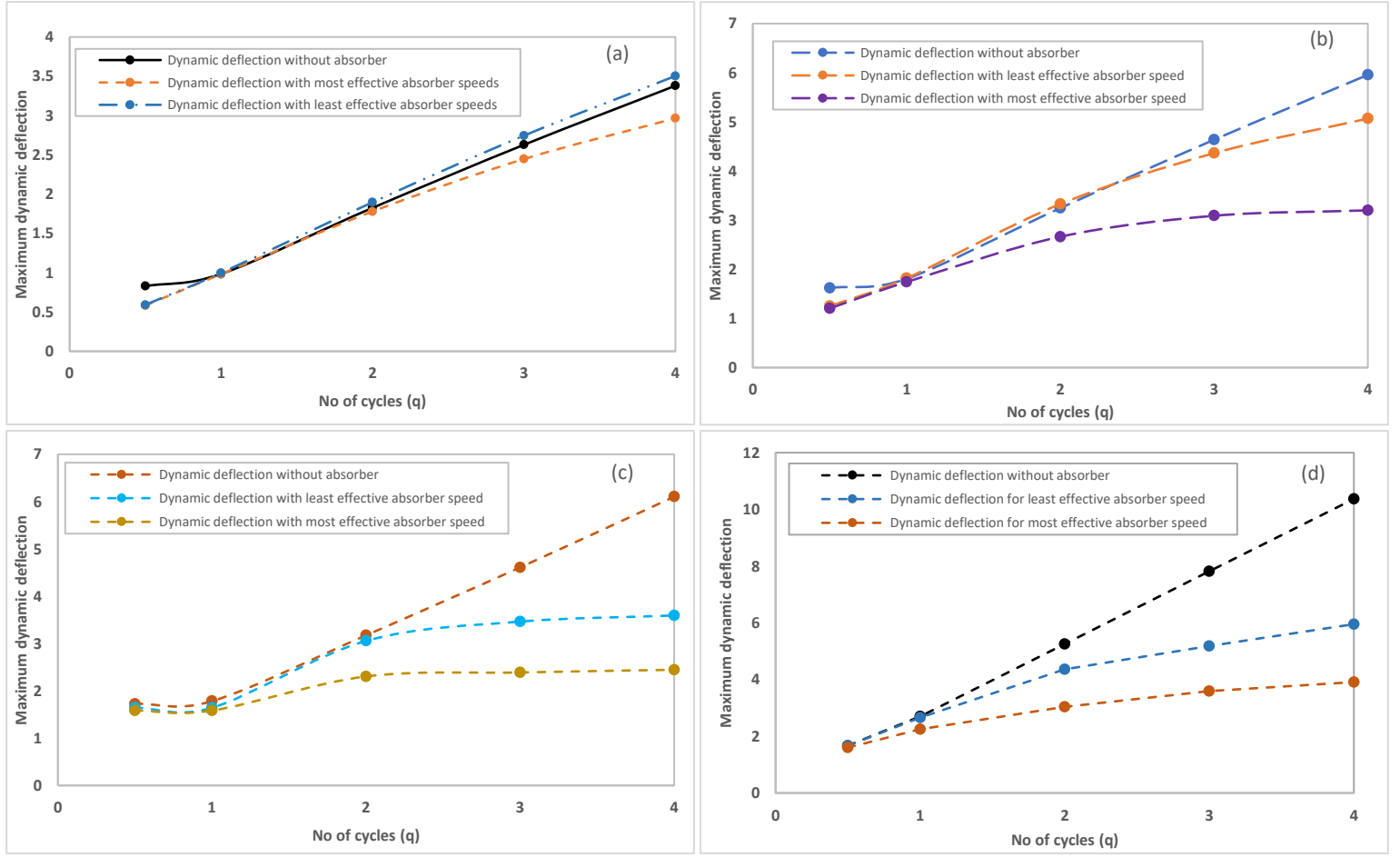


Figure 5.19: Variation of (a) absorber speed factor m -- (b) maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for p (a) $x/L = 0.2$, (b) $x/L = 0.5$, (c) $x/L = 0.7$ and (d) $x/L = 0.9$

Figure 5.18 shows absorber speed factor (m) trend for partial travel lengths $x/L = 0.2, 0.5, 0.7, 0.9$, it is observed that most effective absorber speeds have consistent trend, for all cycles. The curve shows that as reciprocating cycle increases, speed factor (m) decreases in value for satisfactory damping results. In contrast, least effective speed factors (m) are constant (0.1) for lower cycles up to $q=3$ (not for partial travel $x/L = 0.9$) and then for $q=4$ cycle jumps to speed factor value above $m > 1.5$ for partial travels of $x/L = 0.2, 0.5$ and 0.9 . Only in case for $x/L = 0.7$ both least and optimum speed factors (m) are below value of 1.

Table 5-3: Least effective and optimum absorber speed factors for partial travels ($p = 0.2, 0.5, 0.7$ and 0.9) and no of cycles ($q = 0.5, 1, 2, 3, 4$)

No of cycles (q)	Least effective absorber speed factor (m)				Most effective absorber speed factor (m)			
	x/L = 0.2	x/L = 0.5	x/L = 0.7	x/L = 0.9	x/L = 0.2	x/L = 0.5	x/L = 0.7	x/L = 0.9
1	0.1	0.1	0.1	0.1	1.35	1.35	0.96	0.91
2	0.1	0.1	0.1	0.1	0.89	0.86	0.96	0.41
4	0.1	0.1	0.1	0.1	0.39	0.25	0.56	0.23
6	0.1	0.1	0.1	1.97	0.24	0.25	0.32	0.16
8	1.96	1.71	0.36	1.98	0.18	0.21	0.26	0.13

From dynamic deflection results for $q=0.5, 1, 2, 3$ and 4 cycles plotted in Figure 5.19, we will first analyze $q=0.5$ which is forward travel only. For partial travel of $x/L = 0.2$ and 0.5 , the damping produced by both least and optimum absorber speed factors (m) are similar and providing very effective damping at $q=0.5$. When forward travel load deflection analyzed for partial travel of $p = x/L = 0.7$ and 0.9 , damping effects are very minimal due to usage of critical velocity ratio, which is best suited for higher cycles than the lower cycles.

For deflection at cycles $1 \leq q \leq 2$ we observe slight amplification for partial travel of $x/L = 0.2$ and 0.5 which can be due to reason of reciprocating cycle being starting at this point of $q=1$, so load return after $q=0.5$ along with absorber return, increases momentum due to short travel length along beam resulting in slight amplification in these ranges and then when deflection gets normalized for remaining cycles, and absorber starts providing energy dissipation instead of buildup.

For partial travels to $x/L = 0.7$ and 0.9 , the curve deflection obtained, have consistent reducing trend along the maximum dynamic deflection curve. At $q=4$ reciprocating cycle, the damping provided in comparison to undamped dynamic deflection is 60% for synced optimum absorber speed of $m=0.26$ at partial travel of $x/L = 0.7$ and 62% for synced optimum absorber speed of

$m=0.13$ at partial travel of $x/L = 0.9$. Noticeable point is that for $p = x/L \geq 0.5$, both least effective and optimum absorber speeds are providing staggering damping.

The least and most effective absorber speeds impact on dynamic deflection studied in previous section are for synchronized load which means absorber is travelling to same partial travel length as the travel length of load ($\eta=1$) but with different speeds from which least and most optimum speeds are opted out.

5.1.2.9 Load reciprocating to partial lengths with optimum speed of absorber and different travel lengths.

In this section, optimum speeds will now be used and will be coupled with unsynchronized absorber having varying travel length $\eta = dt/L$ to study about the most effective absorber travel length for partial travels of load to $p = x/L = 0.2, 0.5, 0.7$ and 0.9 .

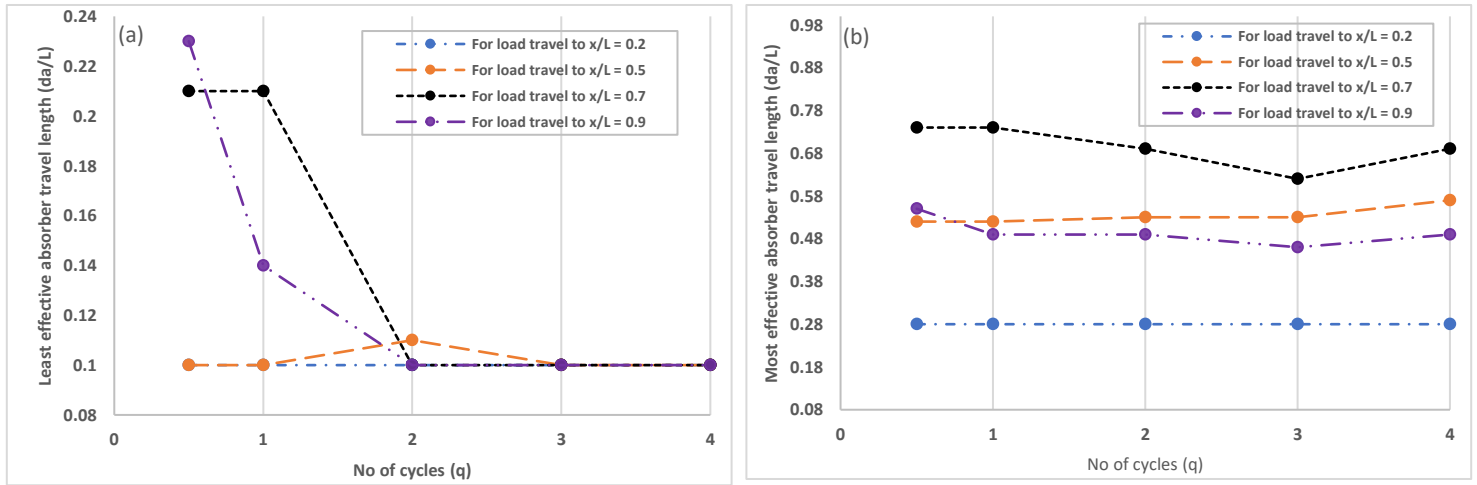


Figure 5.20: Variation of (a) Least effective η - (b) Most effective η as a function of no of oscillating cycles (q) for $p = x/L = 0.2, 0.5, 0.7, 0.9$

It is evident from Figure 5.20, the combination of optimum absorber speed with unsynchronized travel is least effective when absorber travel length is $\eta \leq 0.24$. For optimum η , our effective range

is $0.28 \leq \eta \leq 0.75$ depending on partial travel lengths. To have more clear insight into absorber travel length effectiveness, values are tabulated in Table 5-4.

Table 5-4: Least effective and optimum absorber travel lengths (η) for partial travels ($p=0.2, 0.5, 0.7$ and 0.9) and no of cycles ($q = 0.5, 1, 2, 3, 4$) at corresponding optimum absorber speed ratios.

No of cycles (n)	Least effective travel length η				Most effective travel length η			
	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$
1	0.1	0.1	0.21	0.23	0.28	0.52	0.74	0.55
2	0.1	0.1	0.21	0.14	0.28	0.52	0.74	0.49
4	0.1	0.11	0.1	0.1	0.28	0.53	0.69	0.49
6	0.1	0.1	0.1	0.1	0.28	0.53	0.62	0.46
8	0.1	0.1	0.1	0.1	0.28	0.57	0.69	0.49

We have calculated the best absorber speeds and corresponding to these optimum absorber speed factor m , we obtained best travel length η . With combination of both these optimum values incorporated in the non-classical simply supported beam analysis, we will plot dynamic deflection produced at mid-span of the beam and results obtained will be compared with maximum dynamic deflection obtained when no absorber is incorporated in the system. Figures 5.21, 5.22, 5.23, 5.24 are plotted for dynamic deflections for partial travel lengths obtained at 5 different analysis conditions which are as follows and they will be stated in same order while explaining trends of curves,

- Least effective absorber speed with same travel length
- Least effective travel length with optimum absorber speed
- No absorber attached
- Most effective absorber speed with same travel length
- Most effective travel length with optimum absorber speed

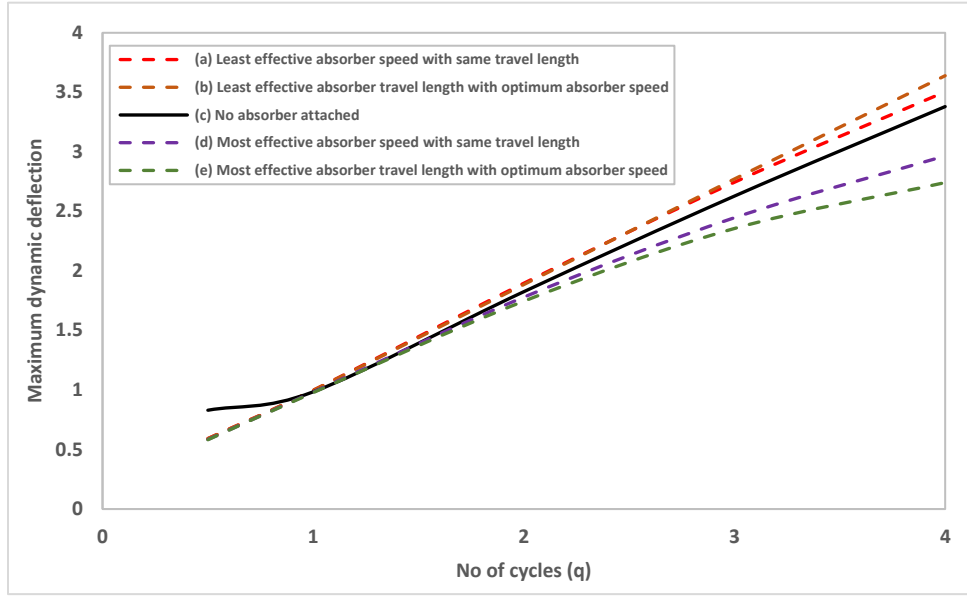


Figure 5.21: Variation of maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for $p=x/L = 0.2$

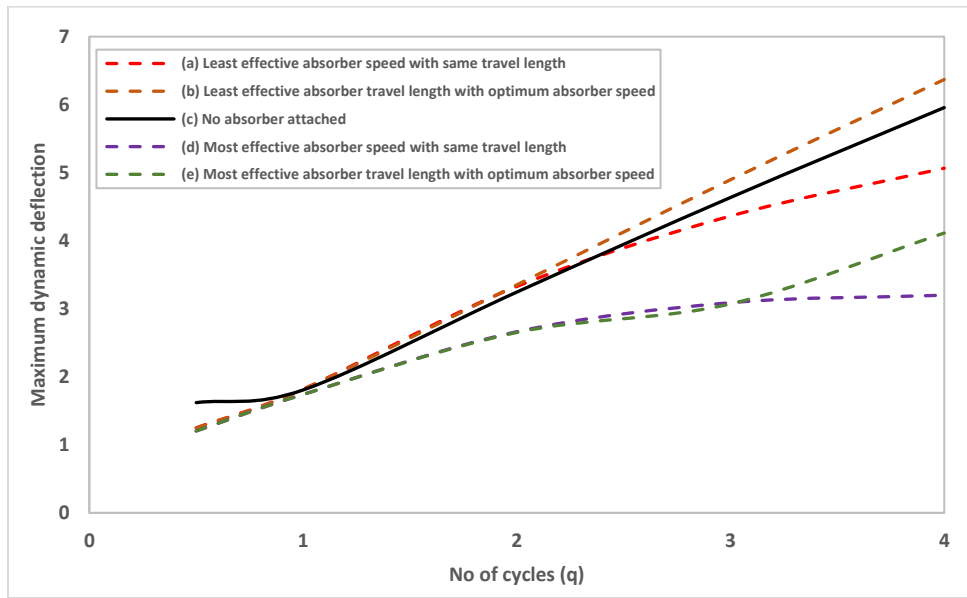


Figure 5.22: Variation of maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for $p=x/L = 0.5$

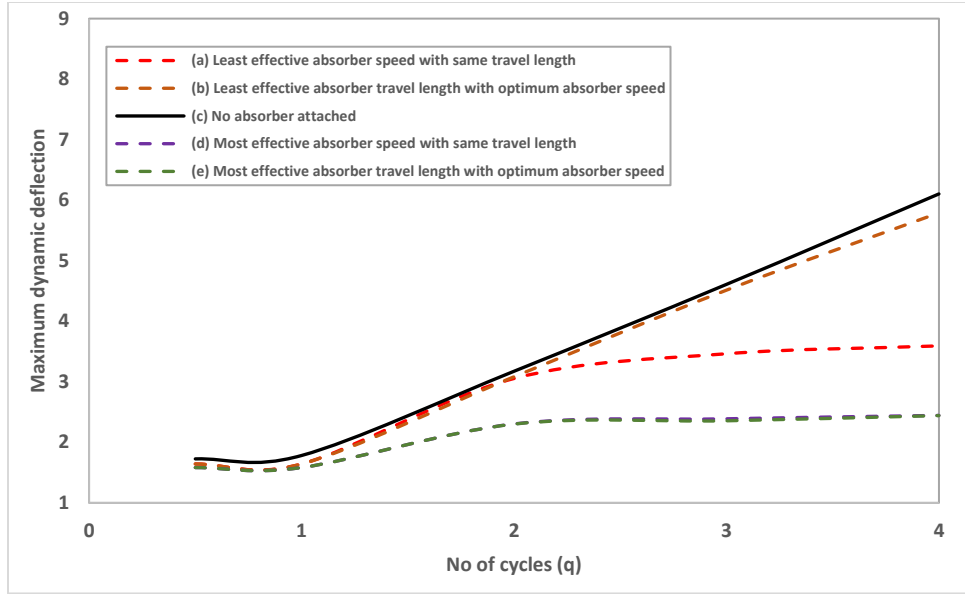


Figure 5.23: Variation of maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for $p=x/L = 0.7$

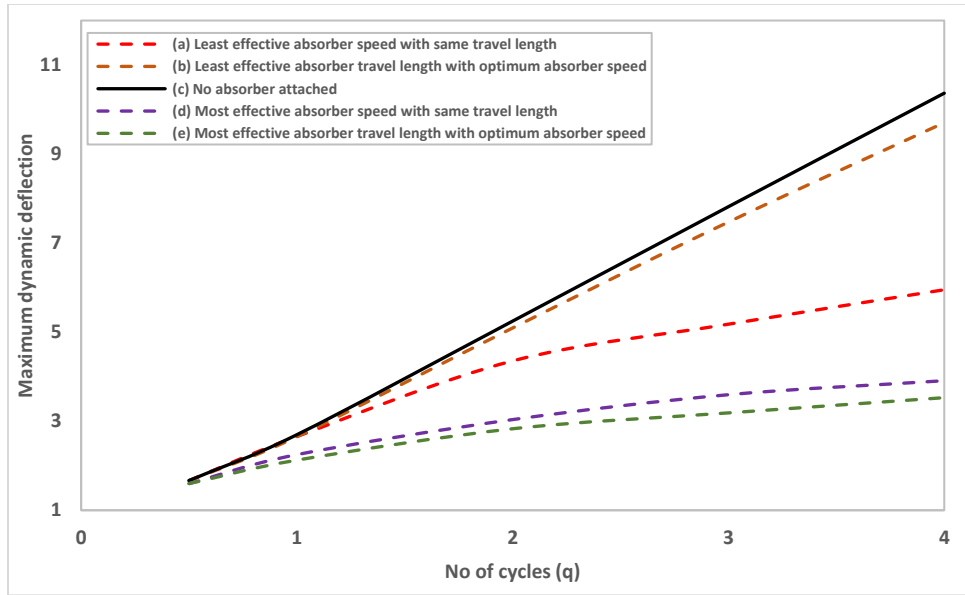


Figure 5.24: Variation of maximum dynamic deflection at mid-span of the beam as a function of no of oscillating cycles (q) for $p=x/L = 0.9$

As observed from Figures 5.21, for partial travel of $x/L = 0.2$, we see that analysis configuration of (a), (b) are resulting in amplification due to energy buildup. Possible reason for this amplification in dynamic deflection for $q \leq 1$ is because travel length is too short for both load and absorber, and while beam is going through first vibration the returning reciprocating load coupled with speeding dynamic absorber results in amplification of deflection in beam. For forward travel load all configurations are resulting in damping due to reason that there is no amplification due to returning load.

For partial travel of $x/L = 0.5$ in Figures 5.22, configuration (b) is resulting in amplification of dynamic amplitude at all the cycles of reciprocating load, whereas for configuration (a), amplification is observed for $1 \leq q \leq 2$ and then damping effect is produced at higher q cycles. As observed maximum damping at $q=4$ ($x/L = 0.5$) for most effective m and η are 37% and 31% for configuration (d) and (e) respectively.

Finally for partial travel of $x/L = 0.7$ and 0.9 in Figures 5.23 and Figures 5.24, no amplification observed at any of the absorber configuration utilized. Damping is observed in all the least effective and optimum moving absorber dynamic configurations. In terms of optimum reduction percentage, (d) and (e) are providing reduction in dynamic deflection with higher damping at $q \geq 2$ cycles with reduction percentage at $q=4$ cycle of about 60% ($x/L = 0.7$) and 70% ($x/L = 0.9$).

Chapter 6 : RESULTS ANALYSIS-NON-CLASSICAL

CANTILEVER BEAM

6.1 Results

The derived equations in Section 3.0, are used to study non-classical cantilever beams responses with different movements of oscillating load reciprocating at multiple cycles (q).

The Euler Bernoulli beam parameters properties and corresponding calculated parameters are same as utilized in Table 5-1 and 5-3:

The following are the properties for the linear spring mass absorber under investigation:

Table 6-1: Properties of Linear Spring Mass Absorber

m absorber	k absorber	c absorber
2.1114 kg	286.11 N/m	9.076 N-s/m

6.1.1 Free vibration response analysis

In this section we will focus on analysis of cantilever beam with non-classical end supports. During spring stiffness variation, we observed that after certain value of spring stiffness of linear spring (k_l) and torsional spring (k_θ), the natural frequency values become constant which is similar to values of classical cantilever beam [$\beta_1 = 1.875104$, $\beta_2 = 4.694091$, $\beta_3 = 7.854757$, $\beta_4 = 10.995541$].

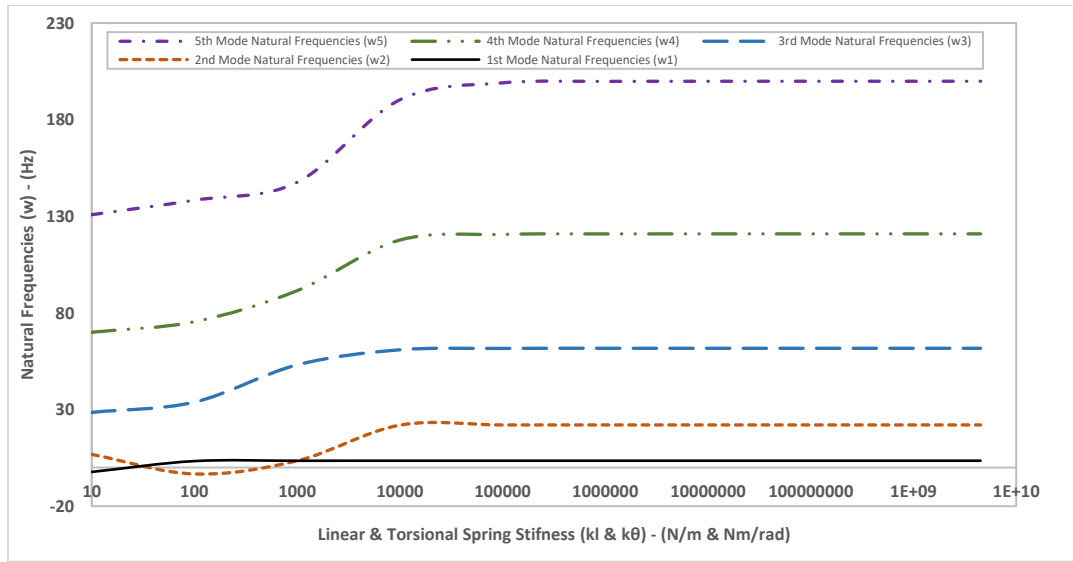


Figure 6.1: Natural frequencies variation with increase in spring stiffness of linear and torsional spring-five mode shapes (Non-classical Cantilever Beam: Linear and Torsional Spring Stiffness =varying to max. value at one end and Linear and Torsional Spring Stiffness =0 at other end)

It is seen from the above Figure 6.1 that for each mode shape, the value of stiffness at which the natural frequency value becomes constant is different, lower mode shapes are attaining constant values of natural frequencies earlier than the higher mode shapes and but at the end they all converge to constant value for sufficient high spring stiffnesses.

It can be seen from the Figure 6.1, that for each mode shape, the value of stiffness (kl) at which the natural frequency (ω) value becomes constant is different, first mode shapes is attaining constant values of natural frequencies around 500 N/m, whereas 2nd and 3rd mode shapes are attaining at around 10,000 N/m spring stiffness whereas higher modes at around 100,000 N/m and above, but at the end they all converge to constant value for sufficient high spring stiffness.

6.1.2 Forced vibration response analysis

a) Without linear spring-mass absorber

6.1.2.1 Full travel oscillating load ($p=1$)

The load adopted has a sinusoidal motion in which load accelerates at start reaches maximum speed till mid of travel length and then decelerates until reaches zero at the end of the beam. The loads studied by other authors are based on constant moving load ($V_f = \text{Constant}$). In our study, load will not reach at end only, but it will also reciprocate back and forth with multiple cycles ($q=n$).

First, we will see for load travelling to full distance without absorber and will analyse the maximum deflection that will occur during this travelling motion. Figure 6.2 shows response at the tip of cantilever beam obtained for oscillating load travelling to end of beam ($p=x/L=1$) and stopping at the end point.

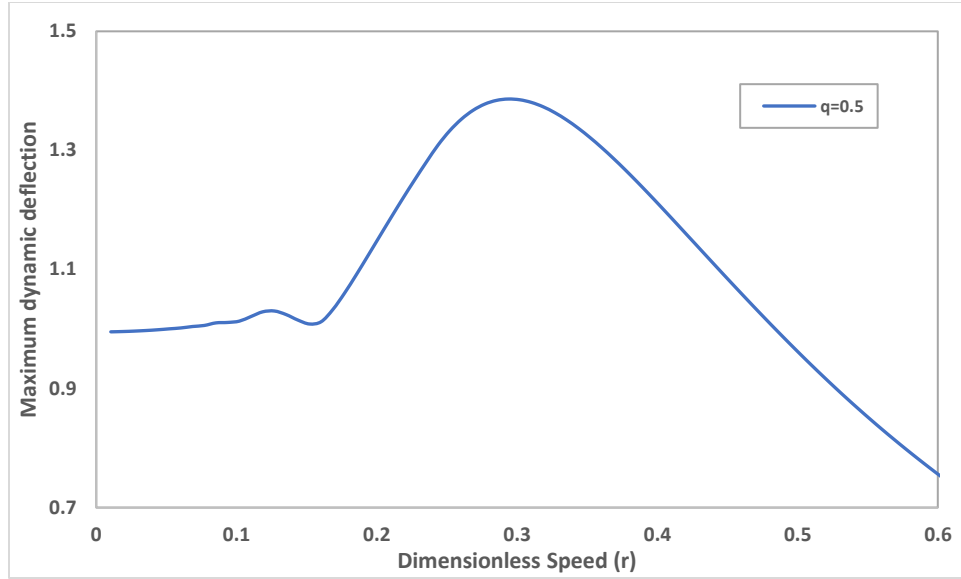


Figure 6.2: Variation of maximum dynamic deflection of the free end of a non-classical cantilever beam as a function of dimensionless speeds (r) for Single Cycle ($q = 0.5$)

As evident from the Figure 6.2, the maximum dynamic deflection for only forward travel of load is occurring around 29.3 % of Critical velocity and then the dynamic deflection starts decreasing moderately with the increase in ratio of critical velocity.

Now we will increase our analysis further, by considering multiple travelling cycles in which load will move to end and will come back to origin completing one cycle as $q=1$. So, two complete travels to end of beam and coming back to origin will be $q=2$ cycle and similar trend goes for $q=3$ and $q=4$ cycles. The reciprocation of load in multiple cycles amplify the dynamic deflection of beam which has been shown in Figure 6.3.

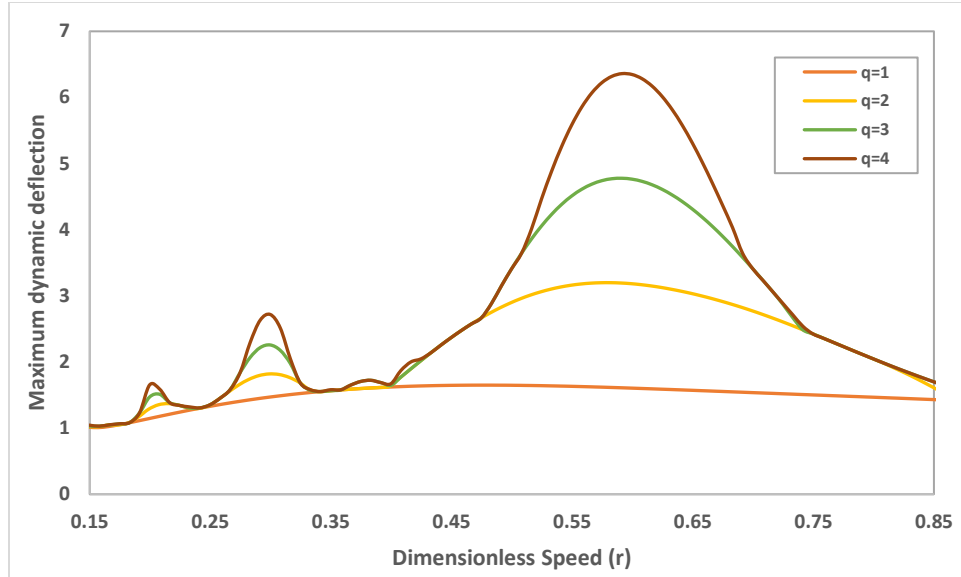


Figure 6.3: Variation of maximum dynamic deflection of the free end of a non-classical cantilever beam as a function of traveling speeds for multiple cycles ($q = 0.5, 1, 2, 3, 4$)

In Figure 6.3, the maximum dynamic deflection for multiple cycles $q=1$ and $q=2$ are 47% and 51.3% respectively whereas for $q=3$ and $q=4$ reciprocation of load is same and occurring around 59 % of velocity ratio and then the dynamic deflection starts decreasing at high slope with the increase in dimensionless speed ratio. It can be noticed that there are major speed ratios ($r = V_t/V_c$) where the amplitude increases significantly, while at other local peaks, the amplitude ratio increases at lower rates.

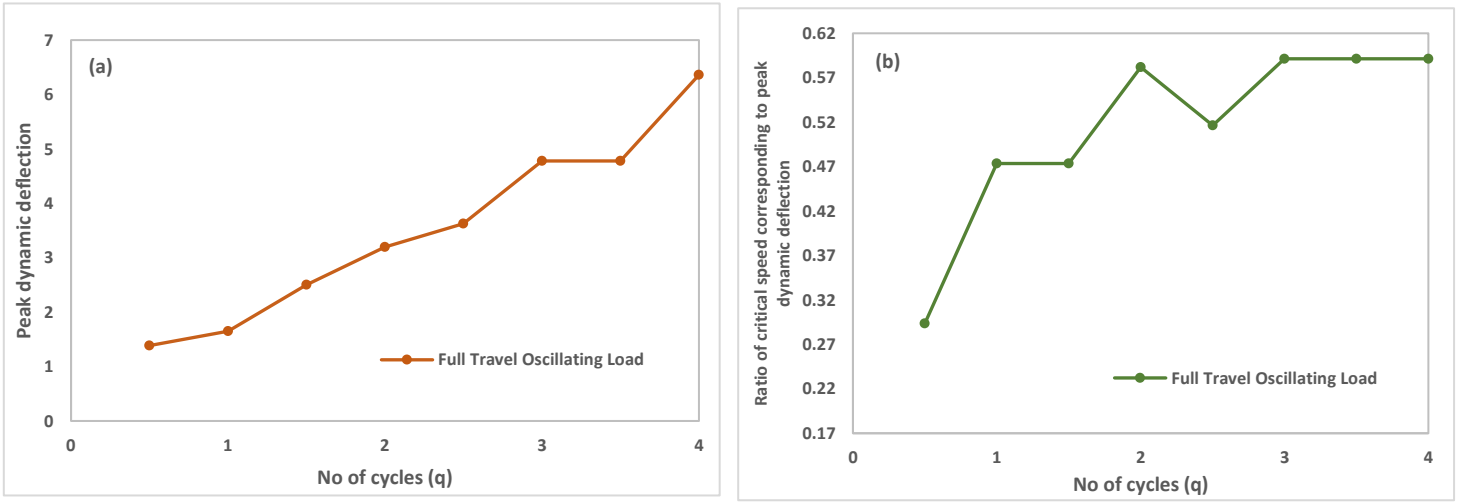


Figure 6.4: Variation observed as a function of multiple cycles: $q = 0.5 - 4$ for (a) Peak Dynamic Deflection (b) Dimensionless Speed r Corresponding to Peak Dynamic Deflection

The trend for maximum dynamic deflection with increase in no of travelling cycles is shown in Figure 6.4(a). The maximum dynamic deflection is occurring around a particular range of dimensionless speed ($0.27 < r < 0.60$) with variation based on no. of cycles (q) and can be observed in Figure 6.4(b)

6.1.2.2 Partial travel oscillating load $p=x/L<1$

In this section, the oscillating load travelling on the beam will cover the length of beam partially. The load starts at $x=0$ and travels a distance $p=x/L<1$ along the beam, then travels back to the support at $x=0$. The period for one whole cycle is $q=1$. To understand the response behaviour as the length parameter p is varied, the absorber free responses after one cycle are plotted versus dimensionless speed r for different values of p . As stated earlier, load will not be continuous rather it will be sinusoidal load in which load accelerates at start reaches maximum speed and then

decelerates until reaches zero at the end of the beam. And definite partial travel lengths of $x/L = 0.2, 0.5, 0.7$ and 0.9 are selected for the analysis.

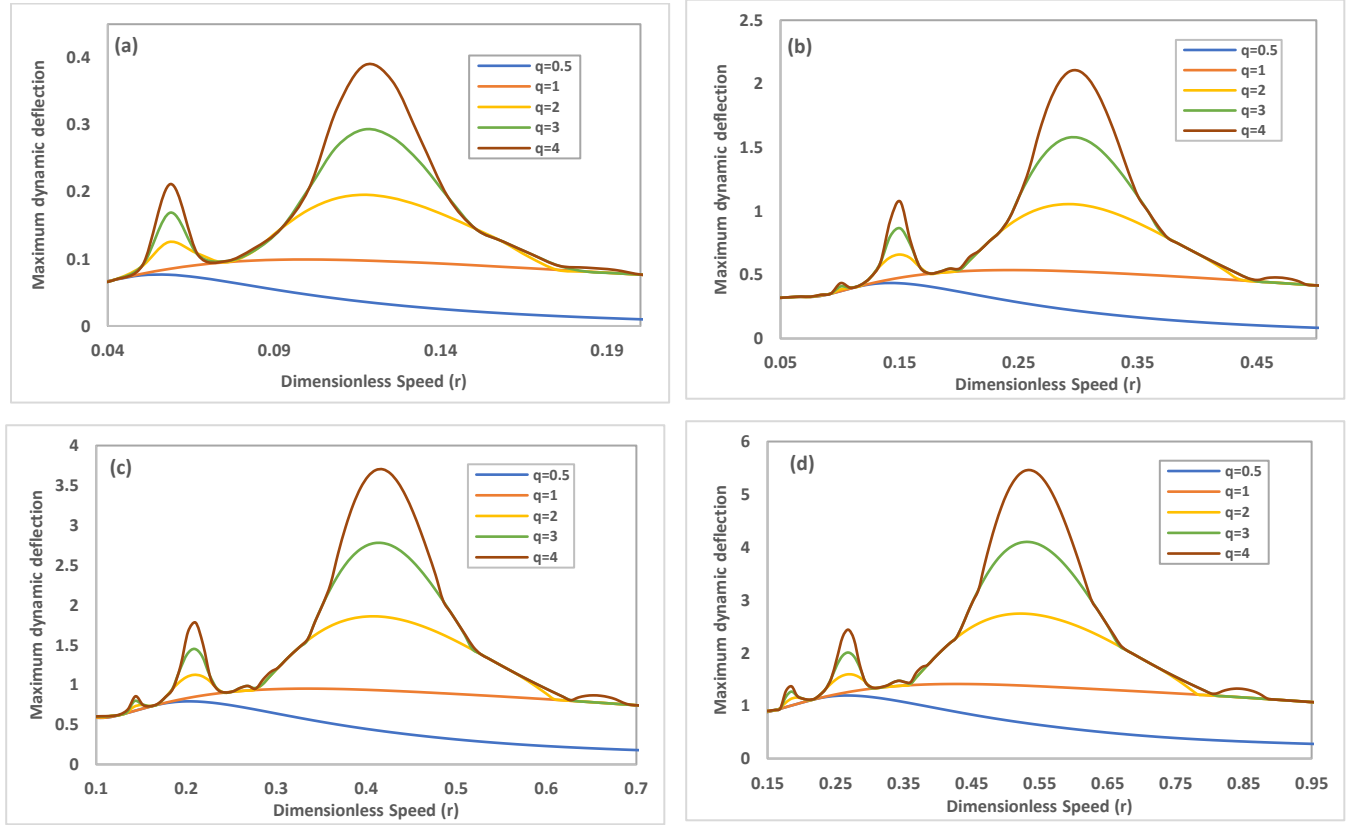


Figure 6.5: Maximum Dynamic Deflection vs Dimensionless Speed r : at $q=0.5, 1, 2, 3, 4$ cycle for (a) $p = 0.2$ (b) $p = 0.5$ (c) $p = 0.7$ (d) $p = 0.9$

As observed from Figure 6.5, dynamic amplitude has peak values at certain range of speed ratio, when load is travelling to partial distance with multiple cycles. To have more insight into variation of peak dynamic deflection at tip of cantilever beam with number of cycles ($q = n$), the peak values and their corresponding speed are plotted against multiple number of cycles.

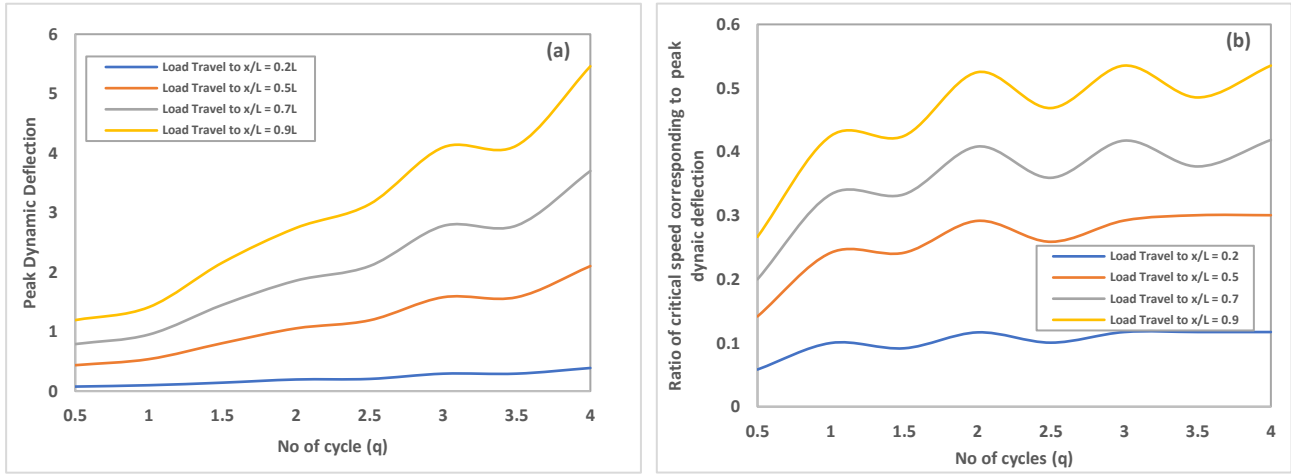


Figure 6.6: Variations observed as a function of multiple cycles: $q=0.5, 1, 2, 3, 4$ cycles for (a) Peak Dynamic Deflection- (b) Dimensionless Speed Corresponding to Peak Dynamic Deflection r

It is observed that the speed corresponding to maximum dynamic deflection and dimensionless speed have been increasing for all cycles with increasing of partial travelling distance. For load partial travel to $p = 0.2$, maximum deflection occurs for lower cycles is around loci of $r = 0.05$ and for higher cycles is about $r = 0.11$. For the travelling distance of $\frac{x}{L} = 0.5$ the velocity (common) for higher cycles at which maximum deflection is occurring is taken to be 0.29 which is common dimensionless speed ratio for reciprocating cycles of $q=2$ and higher. Dimensionless speed providing maximum deflection when load is reciprocating to the travel length of $\frac{x}{L} = 0.7$ is 0.40. Similarly, for partial travel point of $\frac{x}{L} \geq 0.9$ we will use dimensionless speed $r = 0.5$. The values of r calculated, are those at which maximum deflections are occurring for higher cycles and these values will be utilized in next sections to check how these speeds impact dynamic amplitudes when partial travel length of load varied.

i. **Effect of critical dimensionless speed r for $p= 0.2, 0.5, 0.7$ and 0.9 on rest of load travelling length**

If we analyze the trend of the numerical values of dimensionless ratio of critical speeds, we see that numerical value analogy is such that maximum dynamic deflection at tip of beam occurs depending on where partial travelling load stops its motion and return to origin and at which dimensionless speed ratio r . So, to analyze how definite ratio of dimensionless speeds obtained in previous section provide resonance and cancellations on the tip of the beam for different partial travel lengths, these travel lengths and their corresponding critical speed ratios are utilized for rest of beam travel length.

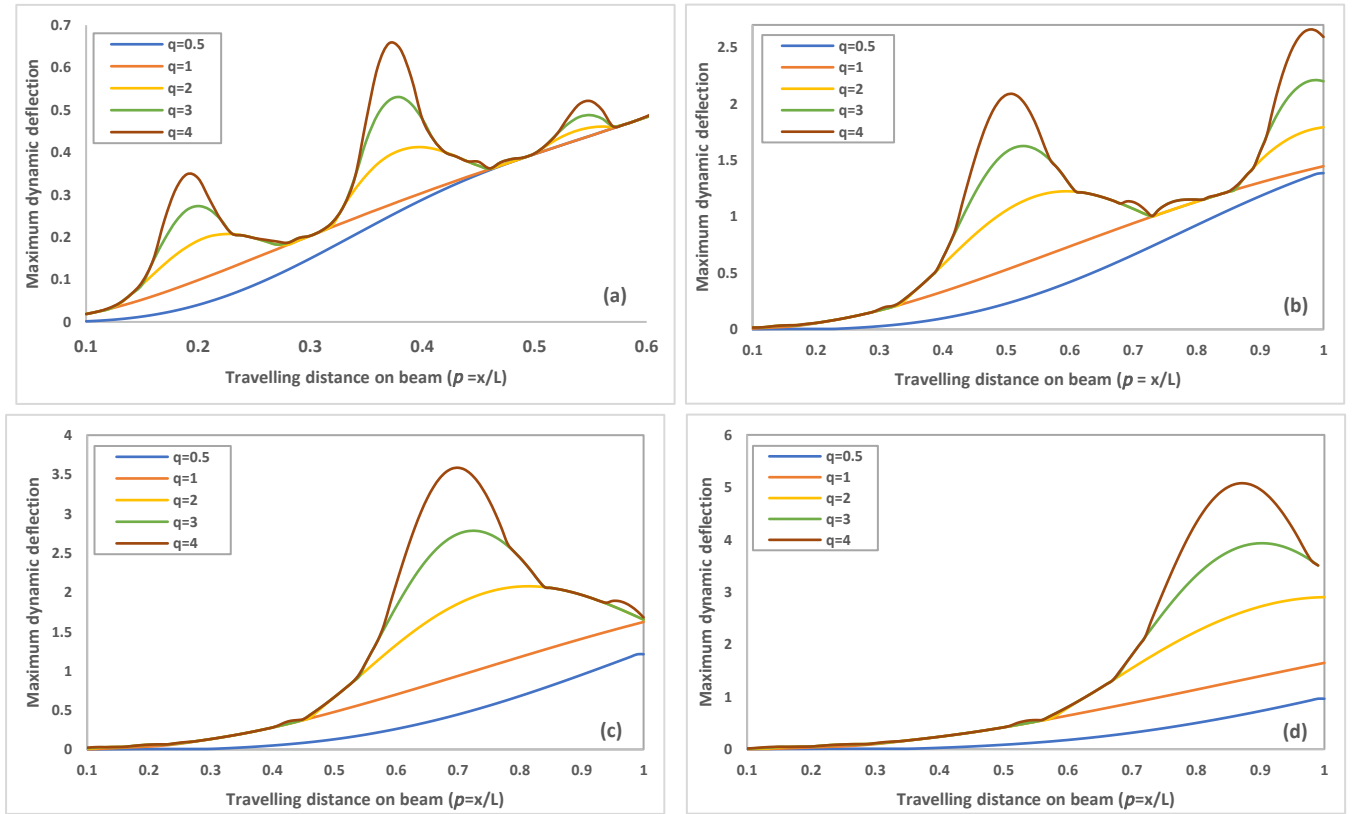


Figure 6.7: Variation of maximum dynamic deflection as a function of multiple cycles for Dimensionless speed of (a) $r = 0.11$ (b) $r = 0.29$ (c) $r = 0.4$ (d) $r = 0.5$ for travel length of beam ($0.1 \leq x/L \leq 1.0$)

In first case below, speed ratio of 11% is utilized which is resonance speed at higher cycles, when load travels to partial length $\frac{x}{L} = 0.2$, and its impact is being checked on dynamic deflection for the rest of travel distance of beam.

In Figure 6.7 analysis, deflection peaks with sharp slopes are observed around $\frac{x}{L} = 0.2$ and $\frac{x}{L} = 0.7$ because of selection of ratio of critical speed. So, resonance peaks occurring at free end of beam when load with 11% of critical speed either complete partial travel to $p = 0.2$ or around $p = 0.4$ of total length of beam. The analogy observed here is that for symmetrical beam larger and smaller peaks are occurring at tip of beam for $q=2$ cycle and higher cycles when beam is travelling to multiple of partial travel length of $x/L = 0.2$.

In this case, for the travelling distance of $\frac{x}{L} = 0.5$ the velocity (common) for higher cycles at which maximum deflection is occurring is taken to be 29% which is common dimensionless speed ratio for reciprocating cycles of $q=2$ cycle and higher and its impact is checked on dynamic deflection for the rest of travel distance of beam. Figure 6.8 provides insight into this observation.

As expected, the peak deflection for all cycles occurs when oscillating load reaches to $\frac{x}{L} = 0.5$ and we see another trend of increase of dynamic deflection at the free end when load is reaching to $\frac{x}{L} = 1.0$. Multiple peaks of deflection are analysed for dimensionless speed $r = 0.29$, one at original point (for $\frac{x}{L} \leq 0.5$) and other at two times of original point (for $\frac{x}{L} \geq 0.5$).

For this case, we are using 0.40 (Dimensionless speed providing maximum deflection when load is reciprocating to the travel length of $\frac{x}{L} = 0.7$) and we will check its effect on the rest of travel distance $0.1 \leq \frac{x}{L} \leq 1.0$ of beam. Figure 6.9 utilized to analyze this aspect of peak deflection in beam.

In Figure 6.9, peak dynamic deflection trend is observed for $q=2$ to 4 cycle and trend of peak rising is around $\frac{x}{L} = 0.7$ and no other multiple peaks can be seen. In this case we should expect similar trend of only single peak at free end of beam as observed in Figure 6.8 but for partial travel point of $\frac{x}{L} \geq 0.9$ because of utilization of dimensionless speed $r = 0.5$ (50% of Load critical speed) and we will check its effect on the rest of travel distance $0.1 \leq \frac{x}{L} \leq 1.0$ of beam. Figure 6.10 plotted to analyze this aspect of peak deflection in beam.

This proves our point that partial travel of sinusoidal load completing reciprocating cycle along the beam provides peaks of deflection as per analogy mentioned above, which is evident for all cases of partial travel.

b) With fixed linear spring-mass absorber

The linear absorbers acting on the non-classical cantilever beam with the beam model specified in the previous part are investigated in this section, and different locations $d=da/L$ approaches are applied to find the optimal location for linear dynamic absorber. Absorber location is critical factor when the load has oscillating movement, and it is also completing travel by reciprocating with multiple cycles along the length of the beam. Indicator considered for evaluation of the performance based on absorber location in non-classical cantilever beam is the damping effect on maximum vibration amplitude (occurring at tip of beam) as a function of no of cycles.

6.1.2.3 Full travel oscillating load ($p=1$) with absorber attached to beam at different locations ($d = da/L$)

In this section, we will use load speed of 59% of critical velocity we obtained for higher cycles ($q = n > 2$) when load is traversing full distance from $0 \leq x/L \leq 1$ without any absorber attached. Utilizing this speed, we will put absorber at different locations to see the dynamic response for at

multiple cycles. From dynamic responses, least and most effective absorber locations will be opted out to utilize for comparison with no absorber beam dynamic response.

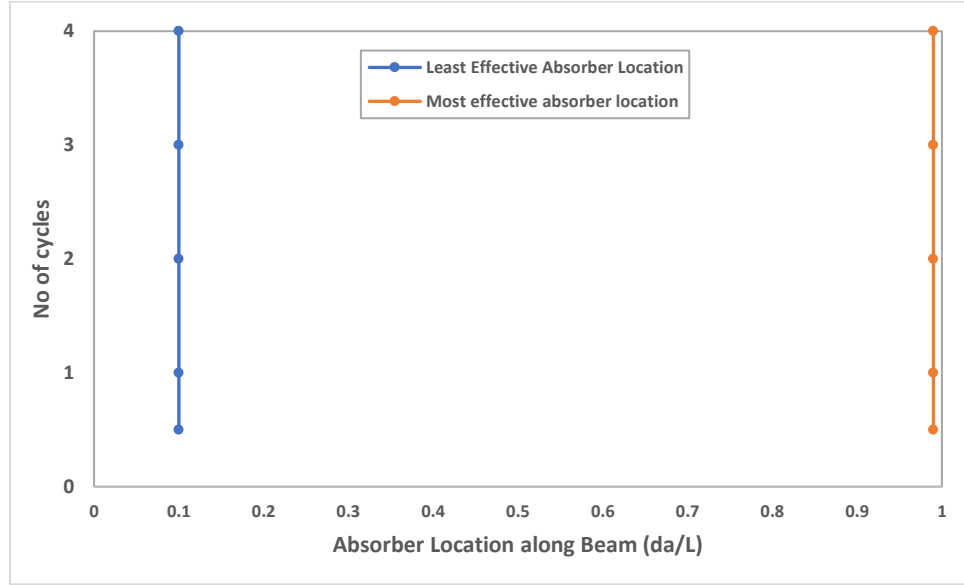


Figure 6.8: Absorber most and least effective locations based on no of cycles of full travel oscillating load reciprocating with multiple cycles of ($q = 0.5$ to 4)

It can be seen from Figure 6.8, that when load travels with speed of 59% of critical speed, the absorber location at which we should expect more damping for full travel ($p=1$) is $d=da/L = 0.99$ (at approximately tip or free end of beam). And the location at which it is providing least effective damping for all cycles is $d=da/L = 0.1$.

To have more detailed review we will attach absorber at least effective location and will obtain dynamic responses. Then we will obtain our results for absorber at best location and comparison will be done for both dynamic responses with no absorber attached and least and most effective attached absorber locations.

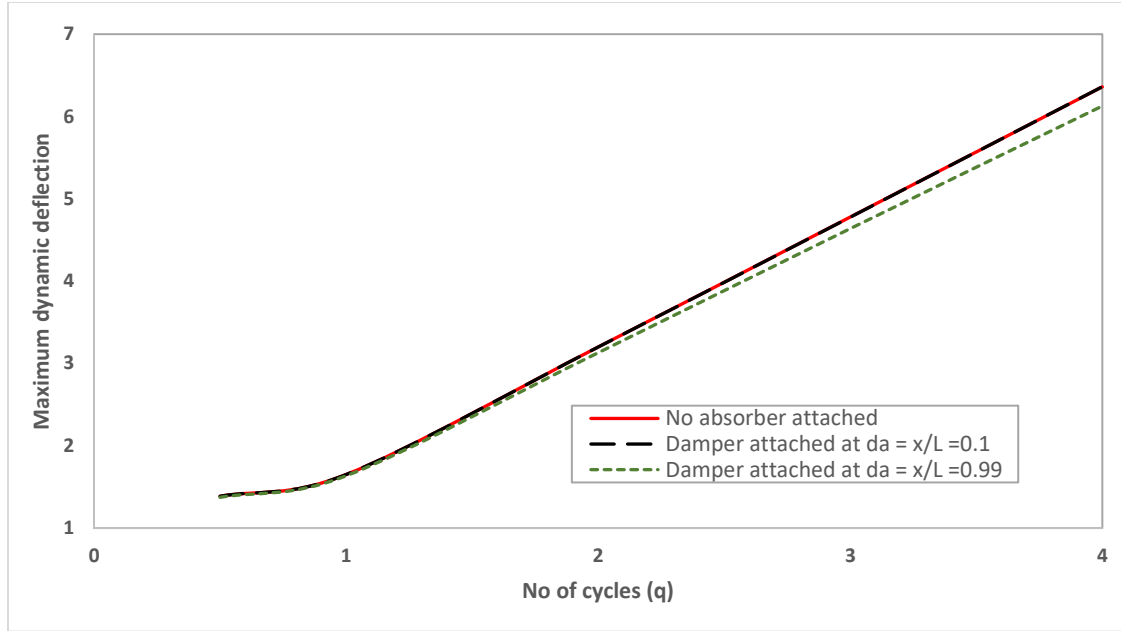


Figure 6.9: Variation of maximum dynamic amplitude of the free end of a cantilever beam as a function of no of oscillating cycles (q)

As evident from Figure 6.9, the damping effect provided by the least effective absorber location is so negligible that it is not clearly distinguishable in the results plotted. But in the numerical results it is seen that deflection instead of being reduced, increased very slightly when absorber attached at least effective location making it totally unfit for damping purpose. But when absorber fixed at the tip of the cantilever beam, the damping can be observed throughout the multiple cycles. At $q=4$ cycle, the dynamic deflection is reduced by 3.7% in comparison with $q=4$ cycle of beam with no absorber attached.

6.1.2.4 Partial travel oscillating load with load travelling to fixed lengths of $p = 0.2, 0.5, 0.7$ and 0.9 .

In this section of our study, we would take definite travel lengths which will be partial of total length of beam ($p = x/L \leq 1$). The partial travel lengths utilized will be $p = x/L = 0.2, 0.5, 0.7$ and

0.9 to analyse dynamic responses when absorber attached at various locations. In previous case of absorber-less study for same partial travel lengths, dimensionless speed of 11%, 29%, 40% and 50% are obtained for these partial lengths respectively (obtained as best fit for higher cycles). Least and most effective absorber locations studies will be performed for these travel lengths with corresponding dimensionless speed r .

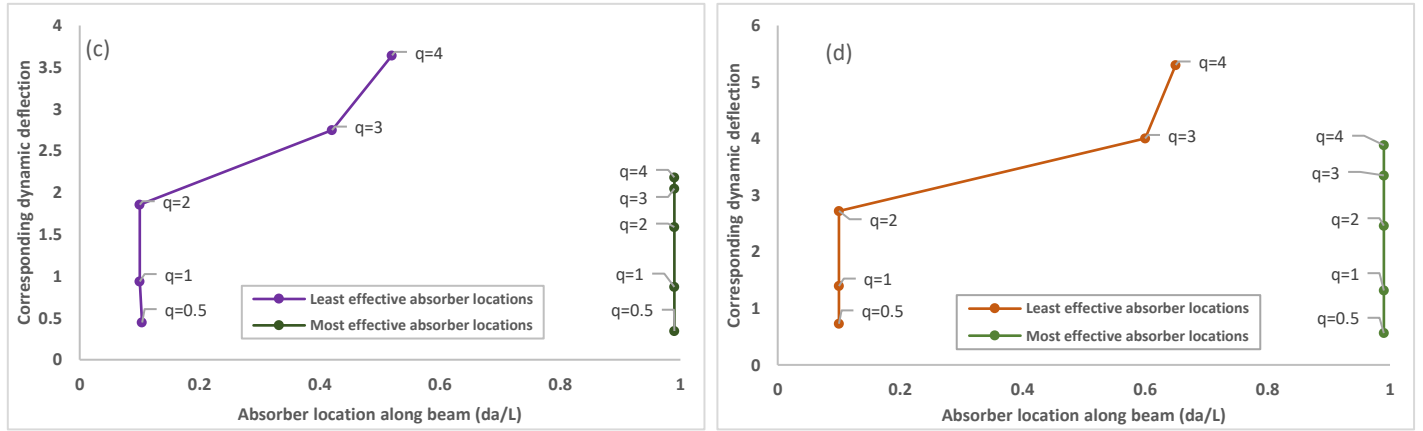


Figure 6.10: Variation of corresponding dynamic amplitude of the free end of a cantilever beam as a function of absorber location along the beam at $q=0.5, 1, 2, 3, 4$ cycle for (a) $p=0.2$ (b) $p=0.5$ (c) $p=0.7$ (d) $p=0.9$

Figure 6.13 shows that when load travels with the corresponding dimensionless speed ratio r for definite partial length p , least effective absorber location is varying depending on reciprocating cycles q and travel lengths p , but most effective absorber location is same for all cycles and travel lengths. It is analyzed that no matter the partial travel length traversed by the load is, the most effective absorber location is at the free end of the cantilever beam $d=da/L = 0.99$.

So, most effective absorber location of these definite partial travel lengths will be utilized and comparison analysis with no-absorber cantilever beam will be performed to see impact on the overall maximum dynamic deflection of the beam.

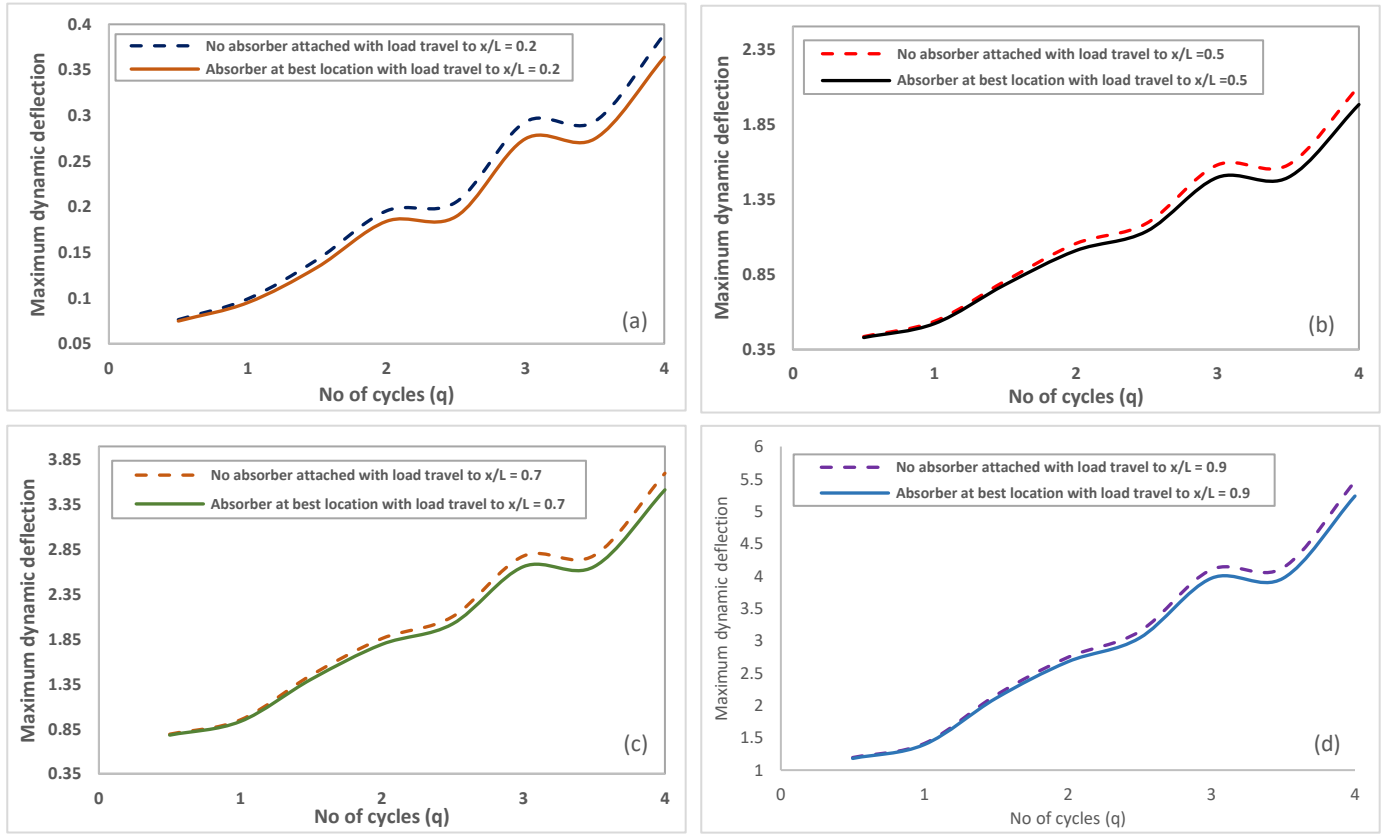


Figure 6.11: Variation of maximum dynamic amplitude of the free end of a non-classical cantilever beam as a function of no. of cycles q for $p =$ (a) 0.2 (b) 0.5 (c) 0.7 (d) 0.9

In Figure 6.11, it is seen that at optimum absorber location it is providing measurable reduction in responses for minimum to maximum partial travel length, The percentage of reduction observed for $p = x/L = 0.2, 0.5, 0.7$ and 0.9 are 6%, 5.8%, 5% and 4% respectively showing that the damping is reducing slightly as the partial travel length is increased. When travel is increased to full travel load, the damping is reduced by 3.7% of the total deflection of the beam in comparison to when there is no absorber attached to beam.

c) With moving linear spring-mass absorber

In the present section, spring mass absorbers moving on the beam are analyzed, and different travelling speeds $V_d = dt/t$ approaches are applied to find the optimal moving absorber speed for spring-mass absorber. The load utilized will be same as previous cases which will be sinusoidal

load reciprocating at multiple cycles ($q = n$). The cases that will be considered with moving absorber to analyze its impact on dynamic deflection of the beam are as follows,

- i. Load and absorber with sinusoidal motion reciprocating at multiple cycles to full length of beam ($p = \eta = 1$) with same speed $m = 1$ (Absorber motion synchronized with load motion).
- ii. Load and absorber with sinusoidal motion reciprocating at multiple cycles to full length of beam ($p = \eta = 1$) with different speed $m \neq 1$.
- iii. Load and absorber with sinusoidal motion traversing same partial travel length ($p = \eta \neq 1$) reciprocating at multiple cycles with same speed $m = 1$.
- iv. Load and absorber with sinusoidal motion traversing same partial travel length ($p = \eta \neq 1$) reciprocating at multiple cycles with different speed $m \neq 1$.
- v. Load and absorber with sinusoidal motion traversing different partial travel length ($p \neq \eta \neq 1$) reciprocating at multiple cycles with optimum speed.

6.1.2.5 Load and absorber reciprocating periodically to same full length ($p = \eta = 1$) and same speed ratio ($m = 1$)

Consider now the system of Figure 3.8 and 3.9, with a moving load and a moving absorber with velocity $V_f(t) = V_d(t) = \text{Sinusoidal}$. The absorber mass considered in this study is 10% of the mass of the beam and agrees with absorber values utilized [19]. Load dimensionless speed r is synchronized with the absorber dimensionless speed ($m = V_d/V_f$) and travel length η is also synchronized which is first case of analysis in this section. Figure 6.12 shows the maximum displacement at the free end of the beam for the considered model as a function of the dimensionless speed r of load motion.

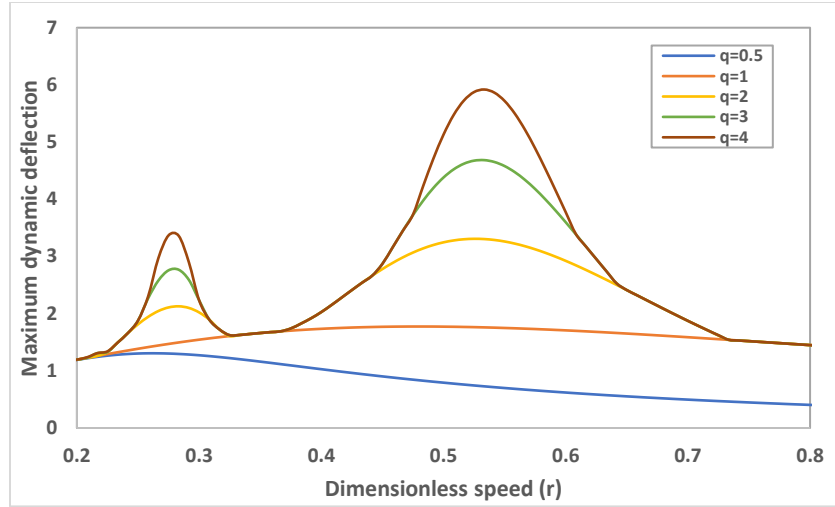


Figure 6.12: Max. Dynamic Deflection vs Dimensionless Speed r : at $q = 0.5, 1, 2, 3, 4$ cycle for $p = 1.0$ with speeds of load and absorber synchronized ($m = 1$)

The maximum dynamic deflection trends for full travel length are almost similar as studied in previous sections but with slight decrease in value due to introduction of synchronized moving load. Peak dynamic deflection obtained here for considered synchronized motions will be compared with peak dynamic deflection obtained when beam has no absorber or has stationary absorber at most effective fixed location.

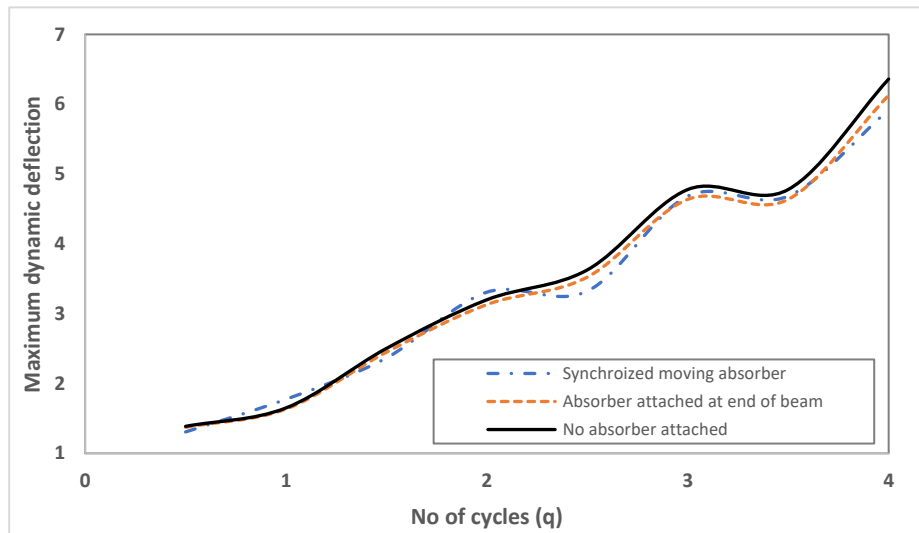


Figure 6.13: Max. Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 1.0$ at different no damping condition or different damping configuration

The results obtained in Figure 6.13 are comparison results for non-classical cantilever beam (same parameters) with synced moving absorber, optimum location fixed absorber ($d_a/L = 0.99$) and no absorber. Dynamic deflection trend with increase of no of cycles, is quite consistent for fixed absorber beam and is less than absorber-less beam deflection throughout the curve.

Whereas for beam with synced motion absorber, the deflection is providing amplification instead of damping effect at some certain cycles (i.e $q = 1$ and 2) and providing damping better than fixed absorber at higher cycle of load reciprocation (i.e $q = 4$). Since in this study higher cycle is main priority, we see that for $q=4$ cycle the displacement amplitude is reduced by 6.9%, which is double of damping 3.7% provided when absorber is fixed at $d=d_a/L = 0.99$. But still fixing stationary absorber is providing us consistent damping effect of maximum amplitude at free end of the beam.

6.1.2.6 Load and absorber reciprocating to same full length ($p = \eta = 1$) and different speed ratio ($m \neq 1$)

In this section, we will study about full travel of load to entire length of beam $p = 1.0$ and reciprocating back and forth completing cycles up to $q=4$. But absorber will be oscillating at different speed ($m = \frac{v_d}{v_f} \neq 1$) then travelling speed of the load but to same length $p = \eta = 1$. In analysis we will study which absorber speed factor (m) is providing maximum damping effect to displacement of beam when load is traversing along entire length of the beam ($p = \frac{x}{L} = 1$). Figure 6.14 is obtained for this velocity variation to obtain the optimum speed ratio m for the damping effect.

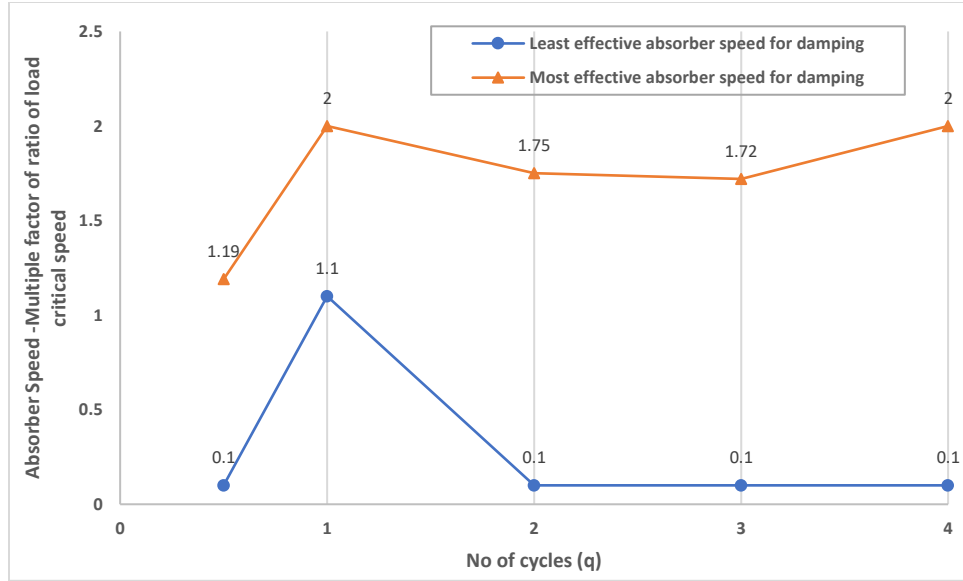


Figure 6.14: Max. Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 1$, $\eta = 1$ and $m = 0.1$ to 2 ($\neq 1$)

Figure 6.20 displays the least and most effective speed ratio (m) of absorber as a function of no of cycles (q). Since speed higher than $m = 1$, is unrealistic approach for absorber having mass which is 10% of beam of mass but for sake of analysis, it will be considered to see impact on dynamic deflection of the beam. The least and most effective speeds values are shown in Table 6-2.

Table 6-2: Values of least effective and optimum speed factors for load full travel ($p = 1$) at different no of cycles ($q = 0.5, 1, 2, 3, 4$)

No of cycles (q)	Optimum speed ratio (m)	Least effective speed ratio (m)
1	1.19	0.1
2	2	1.1
4	1.75	0.1
6	1.72	0.1
8	2	0.1

It can be seen from absorber speed ratio (m) table that optimum absorber speeds are above $m = 1.15$ and for least effective speed ratio $m = 0.1$ is same for all cycles except for $q=1$ which can be due to reason that while load is observing its first peak due to start first reciprocating, the returning load acts as new load entering from free end accompanied with high speed absorber results in energy buildup at this point resulting in amplification, so high speed of $m=1.1$ is least effective for $q=1$ cycle and then response amplitude becomes consistent as reciprocation continues bringing speed ratio m equal to 0.1 in category of least effective absorber speeds,

Optimum and least effective speed ratios of absorber will be utilized, and dynamic deflection will be obtained to have comparison analysis when no absorber is attached to beam. As again stated before, the optimum speed ratios obtained are unrealistic higher values of absorber but will be used to generate results to provide insight into amplitudes damping. To make the comparison evident, we will plot results to compare the dynamic amplitude damping for least effective and optimum absorber speed ratio m for $q=0.5, 1, 2, 3, 4$ cycles.

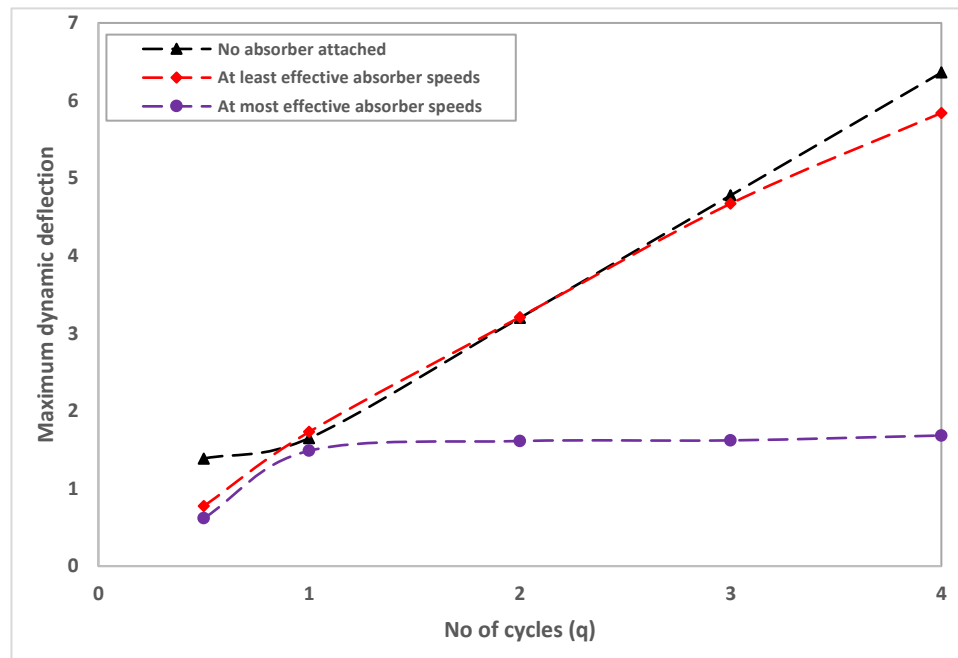


Figure 6.15: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 1$ and $\eta = 1$ at different configurations (m = Optimum Speed and Non-optimum speed)

Figure 6.15 shows that least effective absorber speeds are providing realistic damping of deflection amplitudes at $q=0.5$, 3 and 4 due to energy dissipation with damping of about 12%, 2.3% and 8% respectively. Whereas amplifications produced at $q=1$ cycle are due to energy buildup which is about 5% and at $q=2$ cycle of about 0.31%, which is lower as compared to $q=1$ because $q=1$ cycle is the first cycle when load and absorber starts return motion during beam first deflection after forward travel of load is completed. In case of optimum absorber speed, we have exceptionally large damping, which is unrealistic, but it makes sense as speed of spring mass absorber is unrealistic too. The amplitude damping produced at $q=2$, 3 and 4 cycles are 50%, 66% and 73% respectively due to remarkably high dissipation of energy.

6.1.2.7 Load and absorber reciprocating periodically to same partial travel length ($p = \eta \neq 1$) and same or different speed ratio

In this section, the load is travelling to fixed partial distances of $p = x/L = 0.2, 0.5, 0.7, 0.9$ as already studied in previous sections with and without fixed absorber. In this section we will analyze load and absorber both travelling to partial distances but with synchronized ($m = 1$) and unsynchronized speeds ($m \neq 1$) and will analyze which speed of absorber is providing more impact on controlling dynamic deflection for definite partial travel lengths. In previous sections, we already established the speed ratios r for partial travel at which maximum deflection is occurring, we will utilize that to see impact of absorber speed ratio m on the beam deflection. So, absorber speed ratio m will be function of maximum dynamic deflection for partial travel lengths.

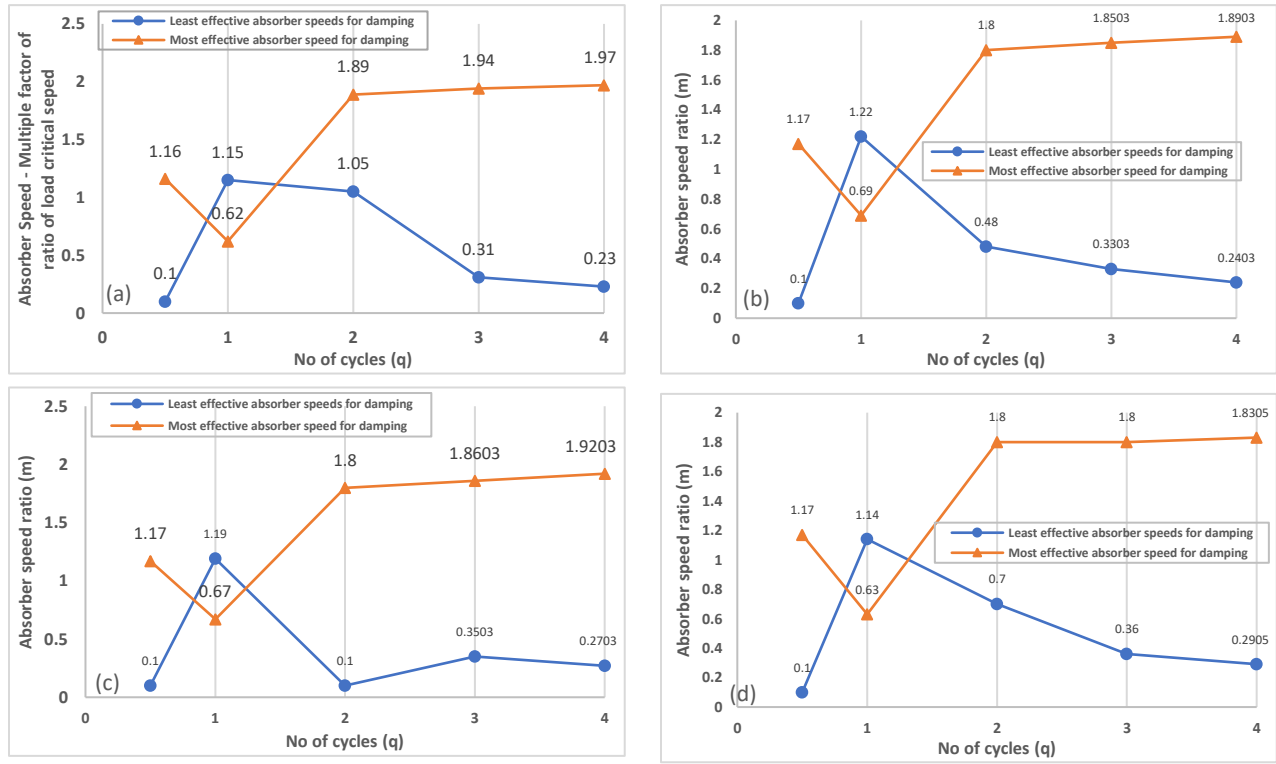


Figure 6.16: Absorber speed ratio m vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle at $\eta=1$ for p = (a) 0.2 (b) 0.5 (c) 0.7 (d) 0.9

Figure 6.16 shows absorber speed ratio m trend for partial travel lengths $x/L = 0.2, 0.5, 0.7, 0.9$, and it is observed that most effective absorber speeds are higher than $m = 1$, for all cycles except $q=1$ due to same fact as discussed before, that high speed of absorber at start phase of reciprocating cycle will amplify the deflection so absorber higher speed will be least effective for $q=1$ cycle rather slow speed will be more effective in controlling dynamic amplitude. The least and most effective speeds values are shown in Table 6-3.

Table 6-3: Values of least effective and optimum speed factors for partial travels ($p=0.2, 0.5, 0.7$ and 0.9) and no. of cycles ($q = 0.5, 1, 2, 3, 4$)

No of cycles (n)	Least effective absorber speed ratio				Most effective absorber speed ratio			
	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$
1	0.1	0.1	0.1	0.1	1.16	1.17	1.17	1.17
2	1.15	1.22	1.19	1.14	0.62	0.69	0.67	0.63
4	1.05	0.48	0.1	0.7	1.89	1.80	1.80	1.80
6	0.31	0.33	0.35	0.36	1.94	1.85	1.86	1.80
8	0.23	0.24	0.27	0.29	1.97	1.89	1.92	1.83

Dynamic deflection results will be plotted for $q=0.5, 1, 2, 3$ and 4 cycles when absorber is moving with least or optimum absorber speed ratio m and comparison analysis will be performed with amplitudes when no absorber is attached on the beam. The moving absorber travel length is same as the partial travel length of load $p = \eta$ but with different speed ratios of minimum and maximum impact. The dynamic deflection results are plotted in Figure 6.23 (a, b, c, d).

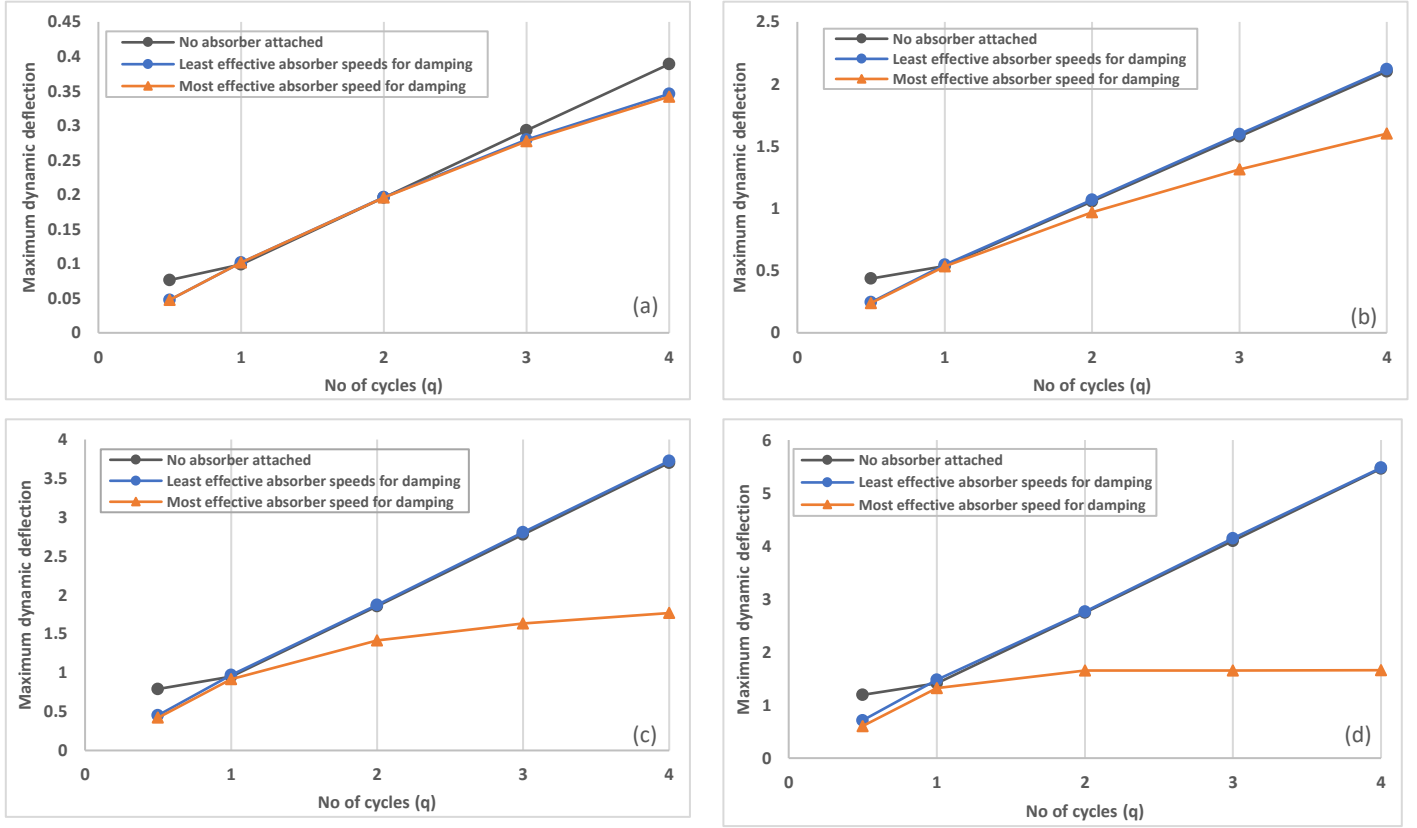


Figure 6.17: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 with $\eta = 1$ at different absorber speed ratios m cycle for $p =$ (a) 0.2 (b) 0.5 (c) 0.7 (d) 0.9

For partial travel length of $p = x/L = 0.2$, it is evident from Figure 6.17 (a) that least and optimum absorber speed ratio m are providing almost similar damping for all selected cycles. The reduction of 11% provided with optimum absorber speeds and load reciprocating to partial travel length of $p < 1$.

As load travel length increased for load to $p = x/L = 0.5$, it is observed from Figure 6.17 (b), the least effective absorber speed is not providing us ample damping and the trend line for no-absorber beam dynamic deflection is almost overlapping with least effective absorber speed trend line except for $q=0.5$ cycle (forward travel load) where beam is having amplitude reduction of 9%.

Whereas for optimum absorber speed ratio, at $q=4$ cycle damping of beam deflection is 11% due to energy dissipation with moving absorber having optimum absorber speed ratio m .

From Figure 6.17 (c) and 6.17 (d), for load partial travel lengths of $p = x/L = 0.7$ and 0.9 , least effective absorber speed ratio has slightly increased the dynamic deflection instead of reducing as travel length of absorber ($\eta \neq 1$) increased too, except for $q=0.5$ in which it is behaving like most effective absorber speed ratio m .

The least and most effective absorber speeds impact on dynamic deflection studied in this section are for synchronized load which means load is travelling to same partial travel length as the travel length of load $p=\eta$, but with same or different speeds from which least and most optimum speeds are opted out. Optimum speeds will now be used and will be coupled with unsynchronized absorber travel length having varying travel length $p \neq \eta < 1$ to analyze for the most effective absorber travel length ratio for partial travel of load to $p = x/L = 0.2, 0.5, 0.7$ and 0.9 .

6.1.2.8 Partial travel with load and absorber moving to optimum and non-optimum travel length ($\eta \neq 1$) and optimum speeds

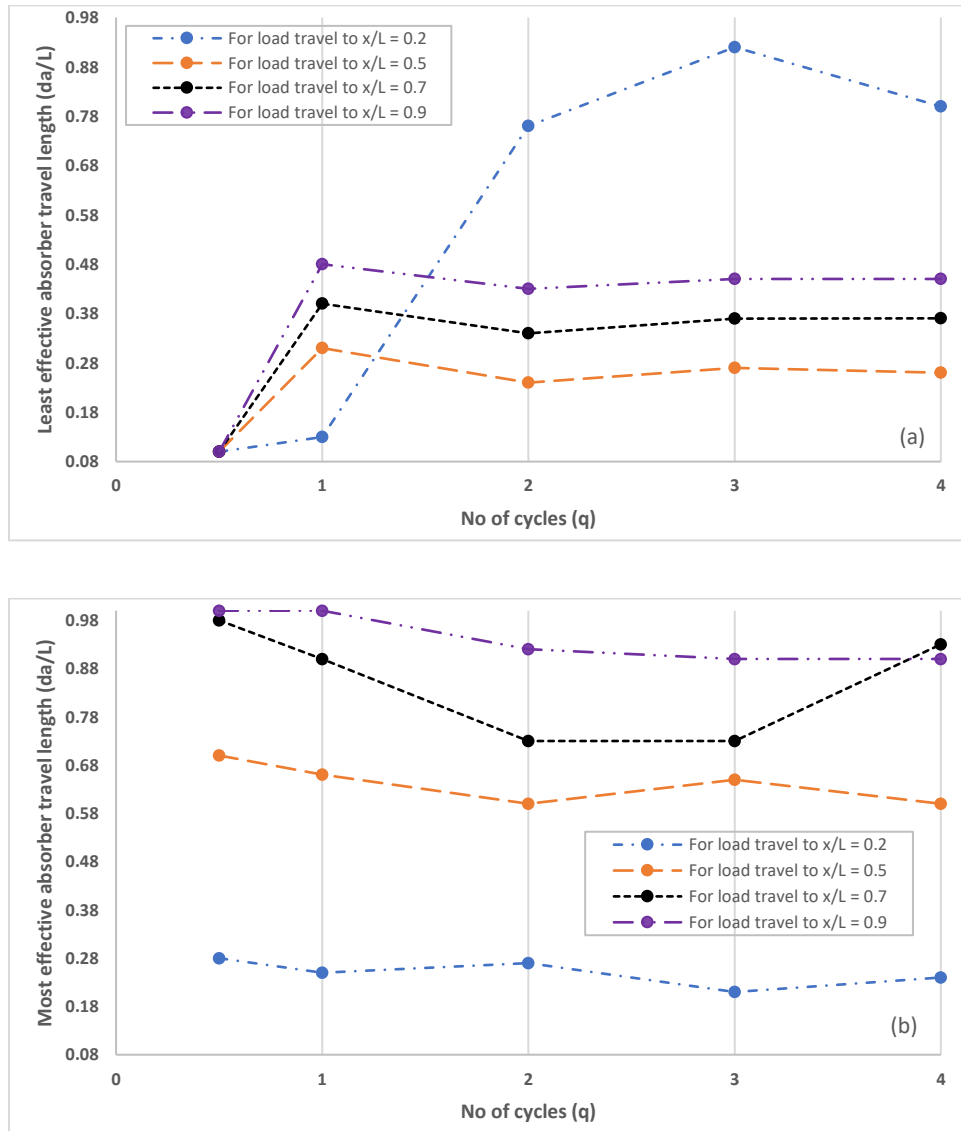


Figure 6.18: Absorber travel length ratio η vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $\eta \neq 1$ for $p = 0.2, 0.5, 0.7, 0.9$ (a) Least effective absorber travel length (b) Most effective absorber travel length

On analyzing Figure 6.18, the travel length curve for load partial travel to $x/L = 0.2$ is different than other partial travel lengths. It is seen that most effective absorber travel length is around $0.20 \leq \eta \leq 0.30$ for all cycles considered whereas least effective for $q=0.5$ and $q=1$ cycle is $\eta \leq 0.2$ and for higher cycles is $\eta \geq 0.55$. It indicates when load travels to 20% of length of beam, absorber travel length to be around 20 – 30% of beam length for efficient damping. The least effective damping for higher cycles is when load travels $\eta \geq 0.6$. For other load partial travel lengths of $x/L = 0.5, 0.7$ and 0.9 , the trend is quite explanatory, with least damping takes place when absorber travels to $\eta \leq 0.5$ which is mid-point of symmetric beam and in comparison, high damping takes place when load travels to $0.6 \leq \eta \leq 0.7$ for load partial travel of $x/L = 0.5$, in analogy $0.6 \leq \eta \leq 0.9$ for load partial travel of $x/L = 0.7$ and $0.9 \leq \eta \leq 1.0$ for load partial travel of $x/L = 0.9$. The optimum and non-optimum travel lengths for selected optimum absorber speeds are tabulated in Table 6-4.

Table 6-4: Values for least effective and optimum absorber travel lengths (η) for partial travels ($p=0.2, 0.5, 0.7$ and 0.9) and no of cycles ($q = 0.5, 1, 2, 3, 4$) at corresponding optimum absorber speed ratios.

No of cycles (n)	Least effective travel length				Most effective travel length			
	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$	$x/L = 0.2$	$x/L = 0.5$	$x/L = 0.7$	$x/L = 0.9$
1	0.1	0.1	0.1	0.1	0.28	0.7	0.98	1.0
2	0.13	0.31	0.4	0.48	0.25	0.66	0.90	1.0
4	0.76	0.24	0.34	0.43	0.27	0.60	0.73	0.92
6	0.92	0.27	0.37	0.45	0.21	0.65	0.73	0.90
8	0.98	0.26	0.37	0.45	0.24	0.60	0.93	0.90

We have calculated the best absorber speeds and corresponding to these optimum absorber speeds, we obtained best travel length of absorber η . With combination of both these optimum values incorporated in the non-classical cantilever beam analysis, we will plot dynamic deflection and results obtained which will be compared with maximum dynamic deflection obtained when no absorber is incorporated in the system. Figures 6.19 – 6.22 are plotted for dynamic deflection

obtained at 5 different analysis conditions which are as follows and they will be stated in same order while explaining trends of curves,

- Least effective absorber speed with same travel length
- Least effective travel length with optimum absorber speed
- No absorber attached
- Most effective absorber speed with same travel length
- Most effective travel length with optimum absorber speed

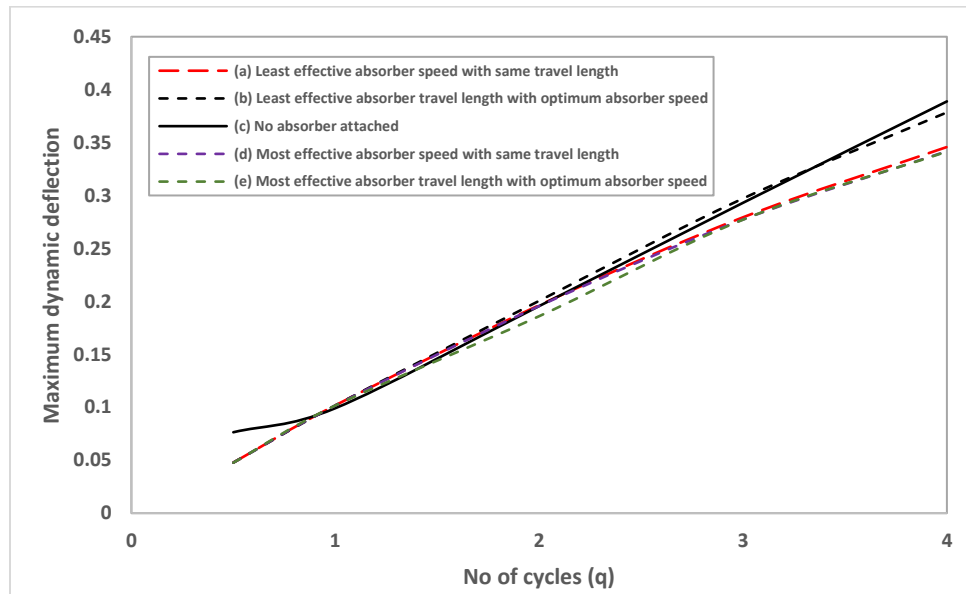


Figure 6.19: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 0.2$ at different configurations

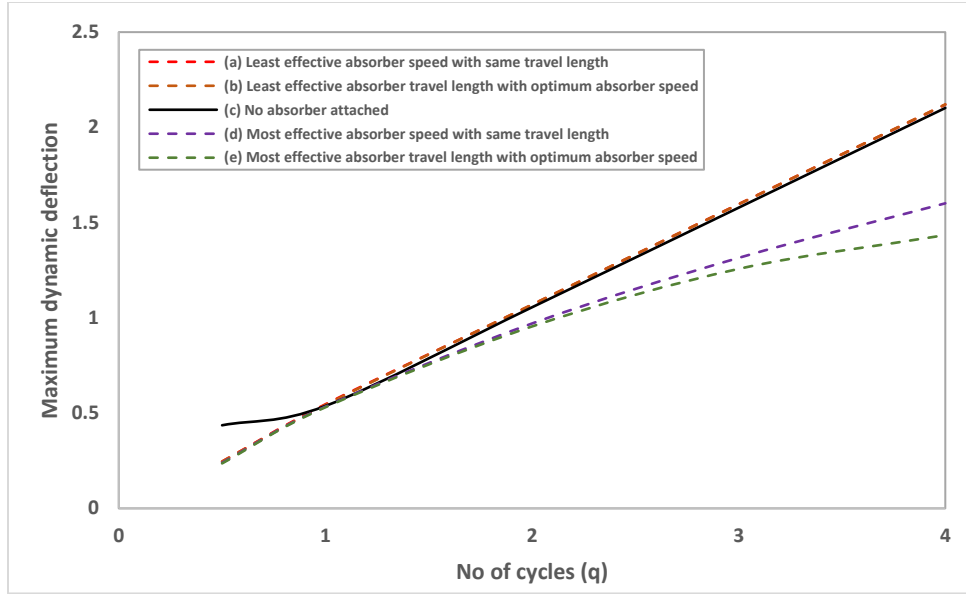


Figure 6.20: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 0.5$ at different configurations

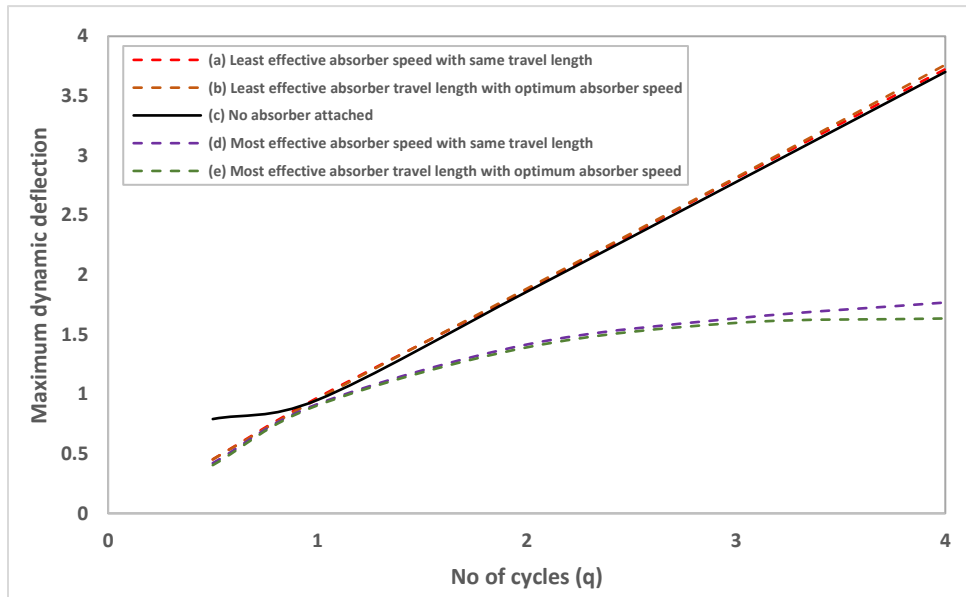


Figure 6.21: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 0.7$ at different configurations

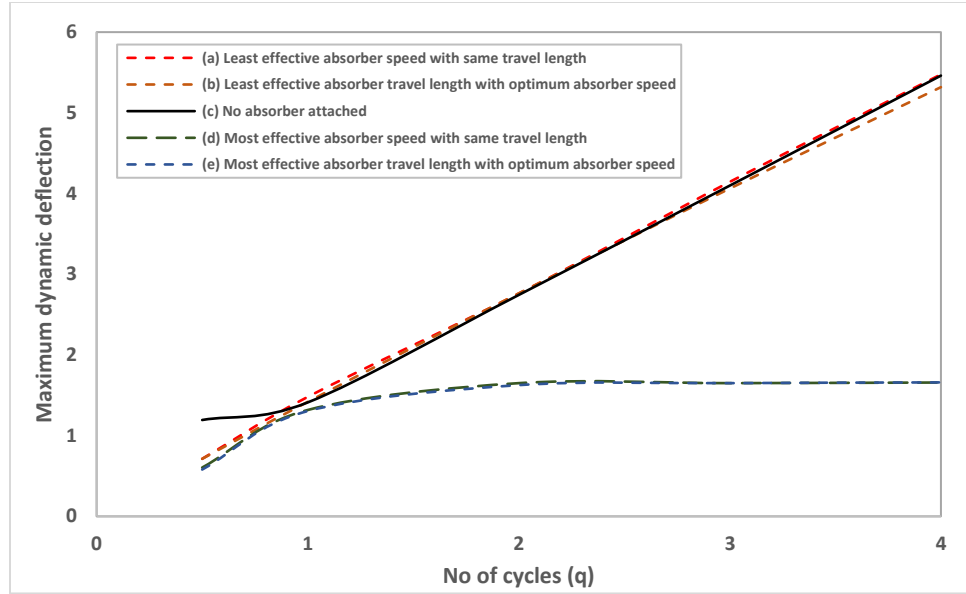


Figure 6.22: Maximum Dynamic Deflection vs No of reciprocating cycles q : at 0.5, 1, 2, 3, 4 cycle for $p = 0.9$ at different configurations

As observed from Figure 6.19, for partial travel of $p = 0.2$ we see that analysis configuration of (d) and (e) are providing noticeable reduction in dynamic deflection for $q=0.5, 2, 3, 4$ cycles whereas for $q=1$ cycle it is resulting in amplification of amplitude. Maximum damping being provided by configuration (e) for $q=4$ cycle is around 12% in comparison to maximum dynamic deflection when no absorber is attached to beam. For partial travel of $p = 0.5$ as in Figure 6.20, (a) and (b) configurations are resulting in slight amplification of beam deflection except for $q=0.5$. Whereas (d) and (e) are resulting in damping for all cycles except for $q=1$ cycle at which curve is converging with absorber-less beam deflection curve. The damping provided by (d) and (e) configurations are 24% and 32% in comparison to configuration (c) but for cycles $q \leq 2$ curves for these two configurations are almost overlapping. Finally for partial travel of $p = 0.7$ and 0.9 in Figure 6.21 and Figure 6.22, (d) and (e) are providing reduction in dynamic deflection with little damping at $q=0.5$ and higher damping at $q > 1$ cycles with reduction percentage at $q=4$ cycle of about 56% ($p = 0.7$) and 70% ($p = 0.9$).

Chapter 7 : CONCLUSION

7.1 Conclusion

In this thesis, dynamic response of an elastic beam having flexible boundary conditions being subjected to a sinusoidal load which is reciprocating at multiple cycles to full length and partial length is studied. Closed-form expressions are derived. To anticipate the amplification and cancellation speeds for various multiple cycles (q) and different dimensionless length ratio (p). Following are the summarized conclusions:

It is observed that when the moving force reciprocates across entire span or partial travel length of the beam, there are many critical speeds that exist depending on travel length and they can be seen increasing in set of multiples for both non-classical simply supported beam and cantilever beam. Another thing observed is that the critical speed defined in previous studies (classical cases) of a single direction moving load is acting as cancellation speeds depending on travel length in current study of reciprocating sinusoidal load.

In terms of resonance, critical speed value for full travel is half of dimensionless length traversed in terms of value and it stays same for higher cycles but varies slightly when moving to lowers cycle until forward travel load only cycle, and they are classical critical speed fraction values when observed at higher cycle. The common analysis is that dynamic deflection and critical speeds increase as the dimensionless length ratio (p) increases for both non-classical simply supported and cantilever beam. These observations are noted without any absorber attached in the system. With absorber attached, shift was observed in amplification inducing critical speeds with damping improving at higher cycles of load reciprocating cycle and noted that even if load is sinusoidal and reciprocating, still the best location of spring mass absorber for the simply supported beam is in

the middle of the beam. Whereas for the cantilever beam, most optimum location is at tip of the beam.

Similarly, while using moving absorber for damping of dynamic deflection in beam, the absorber velocity can be tuned to produce the largest displacement reduction, and as shown in the results, the maximum displacement reduction achieved is for all combinations of both absorber and load velocities when dimensionless ratio $m = V_d/V_f < 1$ for non-classical simply supported beam and $m = V_d/V_f > 1$ for cantilever beam. These first findings demonstrate the significant impact of a moving absorber on beam vibrations, for multiple cycles moving load and the maximum beam displacement occurs close to the mid-span being depending on both the absorber velocity and the load velocity in non-classical simply supported beam. These optimum travel speeds, when opted to get optimum travel length of absorber, it is observed that for simply supported beam, optimum absorber travel length is close to load travel length except above $x/L > 0.9$ where absorber optimum travel length is around middle of the beam. In cantilever beam for $x/L = 0.2$, the optimum absorber travel length is similar as simply supported beam whereas for $x/L > 0.4$ the absorber optimum travel length is $\eta \geq 0.6$. In case of least effective travel lengths, we observed amplifications in dynamic deflection that can also be utilized for certain applications.

This research could be useful in vibration suppression or energy harvesting. The dimensionless speed ratio, deflection magnitude and dimensionless length ratio of the applied load can all be adjusted to manage the dynamic deflection and considerable dynamic deflection may be achieved, when the length ratio can be kept low to reduce displacement in vertical directions at the load application range. The magnification and damping control of vibrations control of flexible simply supported beams and cantilever beam traversed by moving loads and controlled by a fixed or moving absorber with synchronized or unsynchronized travel motion with absorber plays critical

role in efficient energy consumption and harvesting or recycling applications in eco-friendly structural machineries, bridges or railway tracks.

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