

**FRACTIONAL-ORDER CONTROL OF QUADROTOR BASED ON
FRACTIONAL ORDER MODEL**

BY

KHALED SAEED BIN GAUFAN

A Thesis Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEM AND CONTROL ENGINEERING

DECEMBER 2021

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN- 31261, SAUDI ARABIA
DEANSHIP OF GRADUATE STUDIES

This thesis, written by **KHALED SAEED BIN GAUFAN** under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS AND CONTROL ENGINEERING**.

3 (1) 22
بجاء

Dr. Mujahed AIDhaifallah
Department Chairman

سليمان

Prof. Suliman S. Al-Homidan
Dean of Graduate Studies



Jan 3/2022

Prof. Abdulwahed Saif
(Advisor)

04/01/2022

Prof. Sami Elferik
(Member)

3 (1) 22
بجاء

Dr. Mujahed AIDhaifallah
(Member)

Date



© Khaled Bin Gaufan

2021

[This work is dedicated to future me]

ACKNOWLEDGMENTS

All praise and thanks to Almighty Allah for providing me with the wisdom, courage, health, and patience required to complete this work. All my gratitude goes to King Fahd University of Petroleum and Minerals (KFUPM) and the control and instrumentation engineering department for supporting me as an undergraduate and graduate student for the past seven and half years. I want to thank my supervisor, Prof. Abdulwahed Saif, whose knowledge and experience were crucial in developing the research objectives and methods. Your informative remarks encouraged me to improve my thoughts and raise the quality of my work. He also has provided me with tremendous encouragement, advice, insightful comments, and mentoring. I remain grateful to Prof. Sami El-Ferik and Dr. Mujahed AlDhaifallah, members of my committee, for their incredibly useful assistance to this project and who were always available when I needed them and contributed to my academic and professional achievements. I would like to thank the Interdisciplinary Center for Smart Mobility and Logistics for supporting me on this thesis work.

Finally, I owe a debt of appreciation to my family: my father for establishing in me a love of creativity and hardworking, my mother for teaching me how to be a good person, and my sisters for always being there for me. Without the help of my friends, I would not have been able to finish this dissertation. My uncles, cousins deserve special mention for their unflinching care and constant support. I give everything, including this, to everyone who supports me. |

Table of Contents

ACKNOWLEDGMENTS	v
LIST OF TABLES.....	x
LIST OF FIGURES.....	xi
LIST OF ABBREVIATIONS.....	xxiii
ABSTRACT	xxiv
ملخص الرسالة	xxv
1 CHAPTER 1 INTRODUCTION	1
1.1 Introduction and Motivation	1
1.2 Thesis Objectives and Contribution	2
1.3 Methodologies	4
1.4 Thesis Organization	4
2 CHAPTER 2 LITERATURE REVIEW	6
2.1 Unmanned Aerial Vehicles (UAVs)	6
2.2 UAV Control Designs.....	7
3 CHAPTER 3 QUADROTOR DYNAMIC MODEL	10
3.1 Introduction	10
3.2 Preliminaries	10
3.2.1 Gamma Function	10
3.2.2 Caputo Fractional Derivative	10

3.2.3	Properties of Fractional order Derivative (for $n = 1$)	11
3.3	Integer Quadrotor Dynamic Model	12
3.4	Fractional Order Transformation Scheme	16
3.5	Validation of Fractional Order NLQM	19
3.5.1	Pseudo Random Binary Signal (PRBS)	20
3.5.2	Step Input Signal (SIS)	23
3.5.3	Simulation Results	27
3.6	Conclusion	35
4	CHAPTER 4 PID and FOPID Controls Designs	36
4.1	Introduction	36
4.2	Closed Loop Simulation	39
4.2.1	Altitude Controller	39
4.2.2	Attitude and Heading Controllers	39
4.2.3	x and y Positions Controllers	40
4.3	Controller Parameters Tuning Employing GA	42
4.4	PID Control Design	45
4.4.1	Introduction	45
4.4.2	PID Control Design of Integer System	45
4.4.3	PID Control Design of Fractional System	51
4.4.4	PID Control Simulation	56
4.4.5	PID Results and Discussion	81

4.5	FOPID Control Design	82
4.5.1	Introduction	82
4.5.2	FOPID Control Design of Integer System	83
4.5.3	FOPID Control Design of Fractional System	84
4.5.4	FOPID Control Simulation.....	85
4.5.5	FOPID Control Results and Discussion	108
5	CHAPTER 5 SMC AND FOSMC DESIGNS.....	109
5.1	Sliding Mode Control (SMC) Design	109
5.1.1	Introduction	109
5.1.2	SMC Design of Integer System.....	111
5.1.3	Sliding Mode Control (SMC) Design of Fractional System	121
5.1.4	SMC Simulation.....	131
5.1.5	SMC Results and Discussion	145
5.2	Fractional Order Sliding Mode Control (FOSMC) Design	146
5.2.1	Introduction	146
5.2.2	FOSMC Design of Integer System	147
5.2.3	Fractional Ordre Sliding Mode Control (FOSMC) Design of Fractional System	155
5.2.4	FOSMC Simulation	164
5.2.5	FOSMC Results and Discussion	179
6	Conclusion and Future work	181
6.1	Future work.....	182

7	References.....	183
8	Vitae.....	190

LIST OF TABLES

Table 3-1: System Parameters	27
Table 4-1: Case Studies with Reference Trajectories (RT)	38
Table 4-2: Optimal Values of the Objective Functions as PID Controllers Applied.....	57
Table 4-3: PID Controller Parameters with System States	57
Table 4-4: Integer and Fractional Systems Performance as PID Controllers Applied	58
Table 4-5: Optimal Values of the Objective Functions as FOPID Controllers Applied ..	86
Table 4-6: FOPID Controller Parameters with System States.....	86
Table 4-7: Integer and Fractional Systems Performance as FOPID Controllers Applied	87
Table 5-1: Case Studies with Reference Trajectories (RT)	110
Table 5-2: Integer and Fractional Systems Performance when SMCs are Applied	131
Table 5-3: Integer and Fractional Systems Performance when FOSMCs are Applied ..	164

LIST OF FIGURES

Figure 3-1: Quadrotor Dynamic.....	12
Figure 3-2: The Quadrotor Movement Mechanism	13
Figure 3-3: PRBS Control Inputs $U1$ and $U1f$	21
Figure 3-4: PRBS Control Inputs $U2$ and $U2f$	21
Figure 3-5: PRBS Control Inputs $U3$ and $U3f$	22
Figure 3-6: PRBS Control Inputs $U4$ and $U4f$	22
Figure 3-7: Virtual Control Inputs Ux and Uxf	23
Figure 3-8: Virtual Control Inputs Uy and Uyf	23
Figure 3-9: Step Control Inputs $U1$ and $U1f$	24
Figure 3-10: Step Control Inputs $U2$ and $U2f$	24
Figure 3-11: Step Control Inputs $U3$ and $U3f$	25
Figure 3-12: Step Control Inputs $U4$ and $U4f$	25
Figure 3-13: Virtual Control Inputs Ux and Uxf	26
Figure 3-14: Virtual Control Inputs Uy and Uyf	26
Figure 3-15: x and x_f Positions Responses with PRBS Inputs and the error between them	28
Figure 3-16: y and y_f Positions Responses with PRBS Inputs and the error between them	29

Figure 3-17: z and zf Altitudes Responses with PRBS Inputs and the error between them	29
Figure 3-18: ϕ and ϕf Angles Responses with PRBS Inputs and the error between them	30
Figure 3-19: θ and θf Angles Responses with PRBS Inputs and the error between them	30
Figure 3-20: ψ and ψf Angles Responses with PRBS Inputs and the error between them	31
Figure 3-21: x and xf Positions Responses with Step Inputs and the error between them	31
Figure 3-22: y and yf Positions Responses with Step Inputs and the error between them	32
Figure 3-23: z and zf Positions Responses with Step Inputs and the error between them	32
Figure 3-24: ϕ and ϕf Angles Responses with Step Inputs and the error between them	33
Figure 3-25: θ and θf Angles Responses with Step Inputs and the error between them.	33
Figure 3-26: ψ and ψf Angles Responses with PRBS Inputs and the error between them	34
Figure 4-1: Rotational and Transational subsystems	37
Figure 4-2:Altitude Controller Block Diagram	39
Figure 4-3:Attitude and Heading Controllers Block Diagram	40
Figure 4-4: All System Controllers Block Diagram	41

Figure 4-5: Linear Analysis Tool in Simulink.....	43
Figure 4-6: PID Block Diagram.....	45
Figure 4-7: x and x_f Position States with Desired Reference (1 m) and the Error Between Them.....	59
Figure 4-8: y and y_f Position States with Desired Reference (1 m) and the Error Between Them.....	59
Figure 4-9: z and z_f Altitude States with Desired Reference (2 m) and the Error Between Them.....	60
Figure 4-10: φ and φ_f Angle States with (0°) and the Error Between Them.....	60
Figure 4-11: θ and θ_f Angle States with (0°) and the Error Between Them.....	61
Figure 4-12: ψ and ψ_f Angle States with Desired Reference (5°) and the Error Between Them.....	61
Figure 4-13: The fractional Roll Angle State (φ_f) with $\varphi_{fd} = 0^\circ$	62
Figure 4-14: The fractional Pitch Angle State (θ_f) with $\theta_{fd} = 0^\circ$	62
Figure 4-15: PID Altitude Control Inputs ($U1$ and $U1_f$) of Integer and Fractional Systems and the Error Between Them.....	63
Figure 4-16: PID Attitude Control Input ($U2$ and $U2_f$) of Roll Angle of Integer and Fractional Systems and the Error Between Them	63
Figure 4-17: PID Attitude Control Input ($U3$ and $U3_f$) of Pitch Angle of Integer and Fractional Systems and the Error Between Them	64
Figure 4-18: PID Attitude Control Input ($U4$ and $U4_f$) of Yaw Angle of Integer and Fractional Systems and the Error Between Them	64

Figure 4-19: PID x Position Control Input (U_x and U_{xf}) of Integer and Fractional Systems and the Error Between Them	65
Figure 4-20: PID y Position Control Input (U_y and U_{yf}) of Integer and Fractional Systems.....	65
Figure 4-21: 3D Plot of Circular Trajectory with PID Controllers.....	67
Figure 4-22: Roll Angle Responses (φ , φ_f) with PID Controllers and the Error Between Them.....	67
Figure 4-23: Pitch Angle Responses (θ , θ_f) with PID Controllers and the Error Between Them.....	68
Figure 4-24: Yaw Angle Responses (ψ and ψ_f) (with 1°) with PID Controller and the Error Between Them	68
Figure 4-25: PID Control Inputs (U_2 and U_{2f}) and the Error Between Them.....	69
Figure 4-26: PID Control Inputs (U_3 and U_{3f}) and the Error Between Them.....	69
Figure 4-27: PID Control Inputs (U_4 and U_{4f}) and the Error Between Them	70
Figure 4-28: x Position Responses (x and x_f) with PID Controller and the Error Between Them.....	70
Figure 4-29: y Position Responses (y and y_f) with PID Controller and the Error Between Them.....	71
Figure 4-30: z Altitude Responses (z and z_f) with PID Controller and the Error Between Them.....	71
Figure 4-31: PID Control Inputs (U_1 and U_{1f}) and the Error Between Them.....	72
Figure 4-32: PID Virtual Control Inputs (U_x and U_{xf}) and the Error Between Them ..	72

Figure 4-33: PID Virtual Control Inputs (U_y and U_{yf}) and the Error Between Them ..	73
Figure 4-34: 3D Plot of Helical Trajectory with PID Controllers	74
Figure 4-35: Roll Angle Responses (φ , φ_f) with PID Controllers and the Error Between Them.....	74
Figure 4-36: Pitch Angle Responses (θ , θ_f) with PID Controllers and the Error Between Them.....	75
Figure 4-37: Yaw Responses (ψ , ψ_f) (with 1°) with PID Controllers and the Error Between Them.....	75
Figure 4-38: x Position Responses (x and x_f) with PID Controller and the Error Between Them.....	76
Figure 4-39: y Position Responses (y and y_f) with PID Controller and the Error Between Them.....	76
Figure 4-40: z Position Responses (z and z_f) with PID Controller and the Error Between Them.....	77
Figure 4-41: PID Control Inputs (U_1 and U_{1f}) and the Error Between Them.....	77
Figure 4-42: PID Control Inputs (U_2 and U_{2f}) and the Error Between Them.....	78
Figure 4-43: PID Control Inputs (U_3 and U_{3f}) and the Error Between Them.....	78
Figure 4-44: PID Control Inputs (U_4 and U_{4f}) and the Error Between Them.....	79
Figure 4-45: PID Virtual Control Inputs (U_x and U_{xf}) and the Error Between Them ..	79
Figure 4-46: PID Virtual Control Inputs (U_y and U_{yf}) and the Error Between Them ..	80
Figure 4-47: FOPID Block Diagram.....	82
Figure 4-48: Expansion of FOPID from Point to Plane.....	83

Figure 4-49: x and x_f Position States with Desired Reference (1 m) with FOPID Controller and the Error Between Them.....	88
Figure 4-50: y and y_f Position States with Desired Reference (1 m) with FOPID Controller and the Error Between Them.....	88
Figure 4-51: z and z_f Position States with Desired Reference (2 m) with FOPID Controller and the Error Between Them.....	89
Figure 4-52: φ and φ_f Angle States with (0°) with FOPID Controller and the Error Between Them.....	89
Figure 4-53: θ and θ_f Angle States with (0°) with FOPID Controller and the Error Between Them.....	90
Figure 4-54: and ψ_f Angle States with Desired Reference (5°) with FOPID Controller and the Error Between Them	90
Figure 4-55: The fractional Roll Angle State (φ_f) with $\varphi_{fd} = 0^\circ$ with FOPID Controller	91
Figure 4-56: The fractional Pitch Angle State (θ_f) with $\theta_{fd} = 0^\circ$ with FOPID Controller	91
Figure 4-57: FOPID Altitude Control Inputs ($U1$, $U1_f$) of Integer and Fractional Systems and the Error Between Them.....	92
Figure 4-58: FOPID Attitude Control Inputs ($U2$, $U2_f$) of Integer and Fractional Systems and the Error Between Them.....	92

Figure 4-59: FOPID Attitude Control Inputs ($U3, U3f$) of Integer and Fractional Systems and the Error Between Them.....	93
Figure 4-60: FOPID Heading Control Inputs ($U4, U4f$) of Integer and Fractional Systems and the Error Between Them.....	93
Figure 4-61: FOPID x Position Control Input (Ux, Uxf) of Integer and Fractional Systems and the Error Between Them.....	94
Figure 4-62: FOPID y Position Control Input (Uy, Uyf) of Integer and Fractional Systems and the Error Between Them.....	94
Figure 4-63: 3D Plot of Circular Trajectory with FOPID Controllers.....	95
Figure 4-64: Roll Angle Responses ($\varphi, \varphi f$) with FOPID Controllers and the Error Between Them.....	96
Figure 4-65: Yaw Angle Responses (ψ and ψf) (with 1°) with FOPID Controller and the Error Between Them	96
Figure 4-66: FOPID Control Inputs ($U2$ and $U2f$) and the Error Between Them	97
Figure 4-67: FOPID Control Inputs ($U3$ and $U3f$) and the Error Between Them	97
Figure 4-68: FOPID Control Inputs ($U4$ and $U4f$) and the Error Between Them	98
Figure 4-69: x Position Responses (x and xf) with FOPID Controller and the Error Between Them.....	98
Figure 4-70: y Position Responses (y and yf) with FOPID Controller and the Error Between Them.....	99
Figure 4-71: z Altitude Responses (z and zf) with FOPID Controller and the Error Between Them.....	99

Figure 4-72: FOPID Control Inputs ($U1$ and $U1f$) and the Error Between Them	100
Figure 4-73: FOPID Virtual Control Inputs (Ux and Uxf) and the Error Between Them	100
Figure 4-74: FOPID Virtual Control Inputs (Uy and Uyf) and the Error Between Them	101
Figure 4-75: 3D Plot of Helical Trajectory with FOPID Controllers	102
Figure 4-76: Roll Angle Responses ($\varphi , \varphi f$) with FOPID Controllers and the Error Between Them.....	102
Figure 4-77: Yaw Responses ($\psi , \psi f$) (with 1°) with FOPID Controllers and the Error Between Them.....	103
Figure 4-78: x Position Responses (x and xf) with FOPID Controller and the Error Between Them.....	103
Figure 4-79: y Position Responses (y and yf) with FOPID Controller and the Error Between Them.....	104
Figure 4-80: z Position Responses (z and zf) with FOPID Controller and the Error Between Them.....	104
Figure 4-81: FOPID Control Inputs ($U1$ and $U1f$) and the Error Between Them	105
Figure 4-82: FOPID Control Inputs ($U2$ and $U2f$) and the Error Between Them	105
Figure 4-83: FOPID Control Inputs ($U3$ and $U3f$) and the Error Between Them	106
Figure 4-84: FOPID Control Inputs ($U4$ and $U4f$) and the Error Between Them	106
Figure 4-85: FOPID Virtual Control Inputs (Ux and Uxf) and the Error Between Them	107

Figure 4-86: FOPID Virtual Control Inputs (Uy and Uyf) and the Error Between Them	107
Figure 5-1: x and x_f Position States with Desired Reference (1 m) under SMCs and the Error Between Them	132
Figure 5-2: y and y_f Position States with Desired Reference (1 m) under SMCs and the Error Between Them	132
Figure 5-3: z and z_f Altitude States with Desired Reference (2 m) under SMCs and the Error Between Them	133
Figure 5-4: φ and φ_f Angle States with (2°) under SMCs and the Error Between Them	133
Figure 5-5: θ and θ_f Angle States with (2°) under SMCs and the Error Between Them	134
Figure 5-6: ψ and ψ_f Angle States with (5°) under SMCs and the Error Between Them	134
Figure 5-7: SMC Inputs ($U1$ and $U1f$) in both Integer and Fractional Systems and the Error Between Them	135
Figure 5-8: SMC Inputs ($U2$ and $U2f$) in both Integer and Fractional Systems and the Error Between Them	135
Figure 5-9: SMC Inputs ($U3$ and $U3f$) in both Integer and Fractional Systems and the Error Between Them	136

Figure 5-10: SMC Inputs (U_4 and U_{4f}) in both Integer and Fractional Systems and the Error Between Them	136
Figure 5-11: SMC Inputs (U_x and U_{xf}) in both Integer and Fractional Systems and the Error Between Them	137
Figure 5-12: SMC Inputs (U_y and U_{yf}) in both Integer and Fractional Systems and the Error Between Them	137
Figure 5-13: Circular Trajectory Response under SMC - Case 2	139
Figure 5-14: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under SMC - Case 2	139
Figure 5-15: Angle SMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2	140
Figure 5-16: x Position Response in Integer and Fractional Systems under SMC - Case 2	140
Figure 5-17: Position SMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2	141
Figure 5-18: Helical Trajectory Response under SMC – Case 3	142
Figure 5-19: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under SMC – Case 3	143
Figure 5-20: Angle SMC Inputs in Integer and Fractional Systems and the Error Between Them – Case 3	143
Figure 5-21: x Position Responses in Integer and Fractional Systems under SMC and the Error Between Them – Case 3	144

Figure 5-22: Position SMC Inputs in Integer and Fractional Systems – Case 3	144
Figure 5-23: x and x_f Position States with Desired Reference (1 m) under FOSMCs and the Error Between Them	165
Figure 5-24: y and y_f Position States with Desired Reference (1 m) under FOSMCs and the Error Between Them	166
Figure 5-25: z and z_f Altitude States with Desired Reference (2 m) under FOSMCs and the Error Between Them	166
Figure 5-26: φ and φ_f Angle States with (2°) under FOSMCs and the Error Between Them	167
Figure 5-27: θ and θ_f Angle States with (2°) under FOSMCs and the Error Between Them	167
Figure 5-28: ψ and ψ_f Angle States with (5°) under FOSMCs and the Error Between Them	168
Figure 5-29: FOSMC Inputs ($U1$ and $U1_f$) in both Integer and Fractional Systems and the Error Between Them	168
Figure 5-30: FOSMC Inputs ($U2$ and $U2_f$) in both Integer and Fractional Systems and the Error Between Them	169
Figure 5-31: FOSMC Inputs ($U3$ and $U3_f$) in both Integer and Fractional Systems and the Error Between Them	169
Figure 5-32: FOSMC Inputs ($U4$ and $U4_f$) in both Integer and Fractional Systems and the Error Between Them	170

Figure 5-33: FOSMC Inputs (U_x and U_{xf}) in both Integer and Fractional Systems and the Error Between Them	170
Figure 5-34: FOSMC Inputs (U_y and U_{yf}) in both Integer and Fractional Systems and the Error Between Them	171
Figure 5-35: Circular Trajectory Response under FOSMC - Case 2.....	172
Figure 5-36: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under FOSMC - Case 2	173
Figure 5-37: Angle FOSMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2.....	173
Figure 5-38: x Position Responses in Integer and Fractional Systems under FOSMC - Case 2	174
Figure 5-39: Positions (z and z_f) with different fractional orders γ	174
Figure 5-40: Position FOSMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2.....	175
Figure 5-41: Helical Trajectory Response under FOSMC – Case 3.....	176
Figure 5-42: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under FOSMC – Case 3.....	176
Figure 5-43: Angle FOSMC Inputs in Integer and Fractional Systems – Case 3.....	177
Figure 5-44: x Position Responses in Integer and Fractional Systems under FOSMC – Case 3	177
Figure 5-45: Positions (x and x_f) plots with different fractional γ	177
Figure 5-46: Position FOSMC Inputs in Integer and Fractional Systems and the Error Between Them – Case 3	178

LIST OF ABBREVIATIONS

IONL	Integer Order Non-linear
FONL	Fractional Order Non-linear
IONLQM	Integer Order Non-linear Quadrotor Model
FONLQM	Fractional Order Non-linear Quadrotor Model
INLMM	Integer Non-linear Mathematical Model
FNLMM	Fractional Non-linear Mathematical Model
PRBS	Pseudo Random Binary Signal
SIS	Step Input Signal
x and x_f	Integer and Fractional States of position x
y and y_f	Integer and Fractional States of position y
z and z_f	Integer and Fractional States of position y
φ and φ_f	Integer and Fractional States of Roll angle
θ and θ_f	Integer and Fractional States of Pitch angle
ψ and ψ_f	Integer and Fractional States of Yaw angle
PID Controller	Proportional, Integral and Derivative Controller
FOPID Controller	Fractional Order Proportional, Integral and Derivative Controller
PIDTT	PID Tuner Tool
AC	Altitude Controller

ABSTRACT

Full Name: [Khaled Saeed Salmen Bin Gaufan]

Thesis Title: Fractional-Order Control of Quadrotor Based on Fractional Order Model

Major Field: [Systems and Control Engineering]

Date of Degree: [2021]

The dynamic fractional order equations are used to present several real systems. In addition, the exact model in many real systems has been shown to be more precise with fractional differential equations. This research has been dealt with a Nonlinear Quadrotor Model (NLQM) that have become the major topic for control engineering researchers. Firstly, transforming NLQM to fractional model and compare its states with the integer system. Then, proposing both integer and fractional controllers to the fractional order model to show their performance and which perform better. Also, the same work will be done on the linearized forms of the two systems. Simulation results are verified using MATLAB.

ملخص الرسالة

الاسم:

خالد سعيد سالمين بن جوفان

عنوان الرسالة:

التحكم الكسري في الطائرة بدون طيار رباعية المرواح بناءً على نموذج كسري
للطائرة بدون طيار رباعية المرواح
هندسة نظم التحكم

التخصص:

تاريخ الدرجة العلمية: ديسمبر 2021

تستخدم معادلات الترتيب الكسري الديناميكي لتقديم عدة أنظمة حقيقية. بالإضافة إلى ذلك ، فقد ثبت أن النموذج الدقيق في العديد من الأنظمة الحقيقية يكون أكثر دقة مع المعادلات التفاضلية الكسرية. في هذه الأطروحة البحثية سيتم استعراض نموذج جديد و واقعي لطائرة بدون طيار رباعية المرواح و سيتم مقارنته مع النموذج القديم و السائد. و ستتم المفاضلة بين النموذجين من خلال عدة متحكمات صحيحة و كسرية و اختبارها معا من أجل إظهار أدائها وأيها يعمل بشكل أفضل. تم التحقق من نتائج المحاكاة باستخدام (الماتلاب).

CHAPTER 1 INTRODUCTION

1.1 Introduction and Motivation

Unmanned Vehicle Systems (UVS) are important for different areas nowadays because they can be controlled and operated remotely without human interference. UVS is a research key because of the increase in demand of remote sensing and control in wide range of applications such as scientific surveys, traffic surveillance, transportation aids, and inspection in addition to operation in harsh environments [1]. UVS have various configurations, characteristics, shapes and sizes which will be reflected on system dynamics [2]. The development in miniaturization of UVS offers high potential effort for small size and low cost of UVS compared to manned applications especially in certain applications. Rapid growing of UVS comes with promising future because of its size, cost, construction simplicity and maneuverability [3]–[5]

Developing UVS in the absence of the operator is costly in the controller complexity for tracking and vision. The controller is demanded to overcome many drawbacks, starting with stabilizing the system, driving the system to the desired trajectory in the shortest possible time, adapt against any variations of system dynamics and finally be robust against any disturbances. All these requirements ended up making the control design as an important issue and an interested subject to be investigated

In order to design a controller for UVS, accurate models are needed to reflect system dynamics either by precise modeling or real time identification. UVS have a framework of

rigid body dynamics and can be described by a set of differential equations using Euler-Lagrange.

In recent years, the exact model in many real systems has been shown to be more precise with fractional differential equations like viscoelastic systems [6], [7], electromagnetic theory [8], [9], economics [10], mechatronics [11]. In certain cases, dynamic fractional order equations are used to present several real systems. Over the last two decades, fractional system models were widely explored [12]–[18]. Thus, the conduct of too many existing systems, like viscoelastic systems, has been shown to be more reliable by fractional orders differential equations. In addition, Fractional systems were used widely in various scientific fields as successful approaches for modelling physical processes in the real world. In [19], non-integer systems for the explanation of long memory and inherited properties of complex phenomena have been investigated over the past years in various fields, including energy fuel [20], imaging science[21], biomedicine [22] and accident investigation[23]. A major example of fractional order systems is vibration systems that have many implementations in the industry and they modelled by fractional order equations. According to [14] Vibration systems with fractional order differential equations can be more accurately modeling and controlling. By modelling a system of vibration, we should formulate a fractional order input control for fractional order equations. Up to date, numerous studies were carried out to explore the vibration system fractional order control.

1.2 Thesis Objectives and Contribution

This study makes the following contributions: deriving an accurate and comprehensive mathematical model of the quadrotor UAV by applying the fractional order scheme

transformation on the transitional equations of the quadrotor model, developing different control strategies, and applying them to the resulting mathematical model in MATLAB. The thesis will conclude with an assessment of the controller design schemes in terms of dynamic performance, improving tracking accuracy and response speed, and system stability.

The primary objective of this research is to test the improvements obtained in transforming the dynamics of a given nonlinear system (Quadrotor) into a fractional order one by implementing an integer and fractional control techniques.

The specific objectives of the proposed research project are as follows:

1. Transforming the nonlinear Quadratic model into a fractional order one.
2. Compare between the States of the Quadrotor Non-linear model (integer/fractional nonlinear ones).
3. Design two linear control algorithms for both integer and fractional systems.
4. Design two nonlinear control algorithms for integer and fractional systems and compare them.
5. Evaluate the performance of the nonlinear controllers by benchmarking the results to the results of linear controllers.
6. Extensive Simulations will be carried to show the effectiveness of the proposed scheme.

1.3 Methodologies

Developing thesis objective as mentioned in the previous section will go through several steps as following

1. Transforming the nonlinear Quadratic model into a fractional order one.
2. Compare between the States of the Quadrotor Non-linear model (integer/fractional nonlinear ones).
3. Design two linear control algorithms for both integer and fractional systems.
4. Design two nonlinear control algorithms for integer and fractional systems and compare them.
5. Evaluate the performance of the nonlinear controllers by benchmarking the results to the results of linear controllers.
6. Extensive Simulations will be carried to show the effectiveness of the proposed scheme.

1.4 Thesis Organization

The thesis is organized as the following

Chapter 1 presents an introduction to the primary work, motivation, thesis purpose, methodology, and lastly thesis organization.

Chapter 2 provides a survey of the literature on UAVs and the most extensively employed control systems in the area of quadrotors, especially the FOPID control and FOSMC for nonlinear systems.

Chapter 3 presents the quadrotor dynamic model based on the Newton-Euler concept in full details including the rotor dynamics and aerodynamic effects acting on the quadrotor body. Also, the fractional order scheme transformation on the quadrotor model is presented in this chapter.

Chapter 4 presents the developed PID and FOPID approaches to control and stabilize the position and angle states of the quadrotor in both integer and fractional systems. Computer simulations are used to validate the controllers, and the obtained results are shown there.

Chapter 5 covers the proposed SMC and FOSMC algorithms for controlling and stabilizing the quadrotor's position and angle states in both integer and fractional systems. The controllers are validated using computer simulations, and the results are shown.

Chapter 6 shows the conclusion and future work for this thesis.

CHAPTER 2 LITERATURE REVIEW

2.1 Unmanned Aerial Vehicles (UAVs)

The concept of unmanned aerial vehicles (UAVs) changes from one reference to another. For our objectives, unmanned aerial vehicles (UAVs) are miniature aircrafts that operate without the assistance of a pilot. They can be driven remotely by a person or autonomously; autonomous vehicles are controlled by an onboard computer that can be preprogrammed to do a single job or a broad variety of duties as described in the specification [2]. In other publications, UAVs may refer to either motorized or automatic shutoff, tethered or free aerial aircraft [24]. Unmanned aerial vehicles (UAVs) are gaining interest to their employment in the armed services, business, and public sectors, and are especially desirable in places where a human operator is not required. Miniature UAVs are more appealing since they perform better due to their shorter take-off range and longer flying endurance.

UAVs were formerly utilized primarily in military purposes, but they are now being used in civil applications as well. [25]. The advancement of unmanned aerial vehicles (UAVs), which began around World War II, has progressed significantly for both armed and non-military activities. UAVs are increasingly being used in areas where having a human operator is either inconvenient and dangerous, or is not necessary at all. UAV operations and missions today range from agricultural inspection [26], meteorological data collecting [27], and catastrophe monitoring [28], with sizes ranging from tiny birds to helicopters. Recent trends and studies indicate that the number of UAVs for diverse purposes will rise

by a few thousand in the near future [29], [30], with the ultimate shape and size of the UAV becoming task dependent. Fixed wing and rotorcraft are the most common types of UAVs available in the market. Fixed-wing UAVs use a stiff, aerodynamically streamlined wing structure to create lift as they move. Rotorcraft UAVs, on the other hand, produce lift by the use of propellers. This distinction enables rotorcraft UAVs to remain stationary and hover at a single location, but fixed wing UAVs must continually move. Flapping wing UAVs, often known as ornithopters, are becoming increasingly popular among academics [1]. The design of such UAVs is inspired by nature, mimicking birds and insects. Flapping wing UAVs have various potential advantages over fixed wing devices, including improved maneuverability, low speed, vertical take-off and landing [31].

2.2 UAV Control Designs

The quadrotor is an unmanned aerial vehicle (UAV) that moves vertically to take off and land. Numerous research and studies have been conducted in recent years to model and regulate Quadrotors. Quadrotor UAVs are widely employed in a variety of applications because to their numerous advantages like as speed, smoothness, small size, and environmental friendliness [32], [33].

[34] introduced the use of PID and LQ control schemes on an indoor micro quadrotor, and it was discovered that these two types of controllers performed comparably and were capable of stabilizing the quadrotor's attitude around its hover position when subjected to minor disturbances [34], [35]. [36] employed a traditional PID controller to regulate the position and orientation of a quadrotor, and it was able to stable in a low-speed wind condition. a self-tuning PID controller based on adaptive pole placement to govern the

quadrotor's attitude and heading [37]. The suggested controller worked effectively with online parameter tuning, according to simulation [37]. H_∞ is a linear robust controller; robust controllers take parametric uncertainties and highly nonlinear dynamics into account. It is said to be utilized for comprehensive helicopter control [38].

Sliding mode control (SMC) consists of an algorithm that is fundamentally resilient in parameter, nonlinear, external and insecure adjustments. It is introduced where the robustness criterion in drive systems is highly necessary and in the face of strong uncertainty [39]. Backstepping is an iterative control approach that works on both linear and non-linear problems. [38]. In a more recent paper, [40] proposed the use of backstepping and sliding-mode nonlinear control methods to control the quadrotor which gave better performance in the presence of disturbances. [41] proposed developing controllers that can stabilize the quadrotor in an outdoors environment, they compared the performance of an integral sliding-mode controller vs. a reinforcement learning controller and they reached a conclusion that both controllers were able to stabilize the quadrotor outdoors with an improved performance over classical control technique.

In the field of rotor aircraft, different methods of typical control like PID and backstepping have been implemented [42],. The attitude control systems were separated and defined as a first-order plus the lag time system and then the VTOL UAV fractional PI and [PI] flight controller design method [43]. The authors in [44] developed an autoregressive exogenous input (ARX) model to regulate VTOL UAV's pitch loop and converted them to FOPTD (first order plus time delay). In [45] proposed a sliding fractional order mode control method with an effective attitude controller which has a greater degree of freedom to

achieve desired performance. Using Black–Nichols approach to control the quadrotor`s position and attitude by implementing new approach of fraction order $PI^{\gamma}D^{\mu}$ [42]. In order to minimize the chattering and maximize the robustness of the dynamical response of UAV quad-rotor model, a fractional-order backstepping sliding mode control technique is implemented [46].

As represented in [47]–[49], sliding-mode controllers were used with fractional-order derivatives to theoretically mitigate chatter impacts. In [50], it was stated that a fractional order disruption observer could approximate the disorder, and a new fractional order sliding mode controller on the observer basis was recommended to minimize the chattering effect and monitoring errors. The authors in [51] proposed an adaptive SMC, including a slide surface with a fractional order integral part and a switching form with a less discord., means has less chattering. As shown in [52], to stabilize an uncertain non-linear non-integer system, a sliding integral surface was formulated.. In [51], [52], two suggested approaches implemented integral fractional order sliding surface and various reaching laws to decrease the reach time and increase system performance. In [53], to control the speed of a permanent magnet motor in terms of reducing the chattering of the desired value and to maximize robustness, a sliding surface using the fractional order integral was implemented. For the control of an elastic robot link-manipulator to boost performance, a sliding surface was presented in [54] with two fractional order derivatives terms.

CHAPTER 3 QUADROTOR DYNAMIC MODEL

3.1 Introduction

This chapter shows the quadrotor's integer and fractional mathematical non-linear models. The integer one is obtained utilizing theoretical method. Then this model is transformed into fractional one. Both models are tested using different control techniques

3.2 Preliminaries

This section provides some preliminary details about the fractional calculus that is to be found in our research.

3.2.1 Gamma Function

This function extends the definition of factor numbers and plays a special part in fractional mathematics. This function has a general definition as indicated by $\Gamma(x)$ as below [55].

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt \quad (3.1)$$

3.2.2 Caputo Fractional Derivative

Utilizing Gamma function, the common operator in fractional-order dynamic equations is Caputo fractional derivative operator and it's defined as follows

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t - \tau)^{\alpha-n+1}} d\tau, \quad 0 < n \in \mathbb{Z} \quad (3.2)$$

Where $n - 1 < \alpha < n$.

3.2.3 Properties of Fractional order Derivative (for $n = 1$)

1. Commutative Property

For any function, the fractional order derivative (D^α) of the fractional order integral ($D^{-\alpha}$) equals the function itself

$$D^\alpha(D^{-\alpha}f(t)) = f(t) \quad (3.3)$$

On the other hand, the fractional order integral ($D^{-\alpha}$) of the fractional order derivative (D^α) for any function is

$$D^{-\alpha}(D^\alpha f(t)) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{k!} (t-a)^k \quad (3.4)$$

2. Linearity Property

$$D^\alpha(f_1 + f_2) = D^\alpha f_1 + D^\alpha f_2 \quad (3.5)$$

$$D^\alpha(cf) = cD^\alpha f \quad (3.6)$$

3. Zero Rule Property

$$D^0(f(t)) = f(t) \quad (3.7)$$

3.3 Integer Quadrotor Dynamic Model

The quadrotor consists of four spinning propellers powered by four-dc engines that control vehicle motion. The quadrotor's position depends on the Euler angles, which are the angle of Roll (ϕ), the angle of Pitch (θ) and the angle of Yaw (ψ). Figure 3-1 describes the dynamics of the quadrotor Equations (3.8) and (3.9). The system consists of the R^b and R^m inertial frames as shown. The torques of Roll, Pitch and Yaw and the overall thrust force appear in the quadrotor.

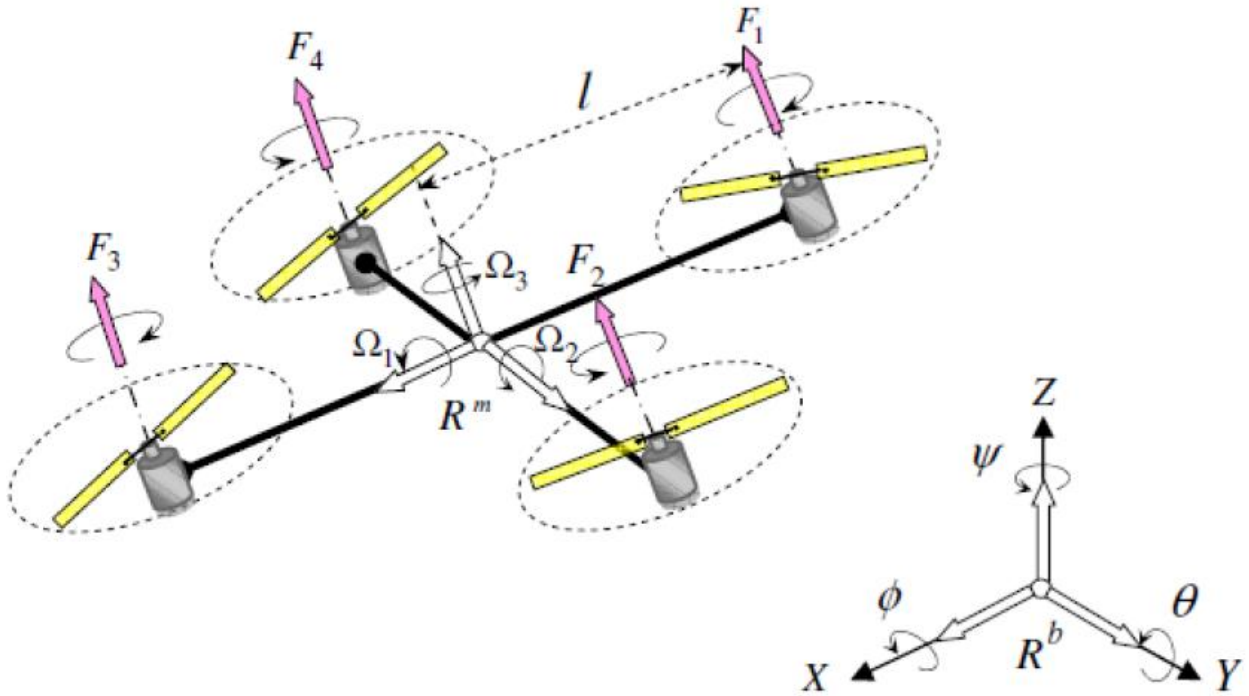


Figure 3-1: Quadrotor Dynamic

Rotors (1, 3) and rotors (2, 4) rotate in various ways to negate the moments produced by each other, as discussed previously. The rolling velocity can be accomplished in the x-axis direction of the vehicle by increased the angular momentum of the second rotor and decreasing its angular velocity by maintaining the entire rotation steady. Likewise, the

pitch velocity along the y - axis is accomplished by increasing the third rotor angular velocity and reducing the first rotor angular velocity. And the motion of the yaw along the z-axis will increase the rotor velocity (1, 3) and reduce the rotor's velocity (2, 4). Figure 3-2 illustrates this mechanism.

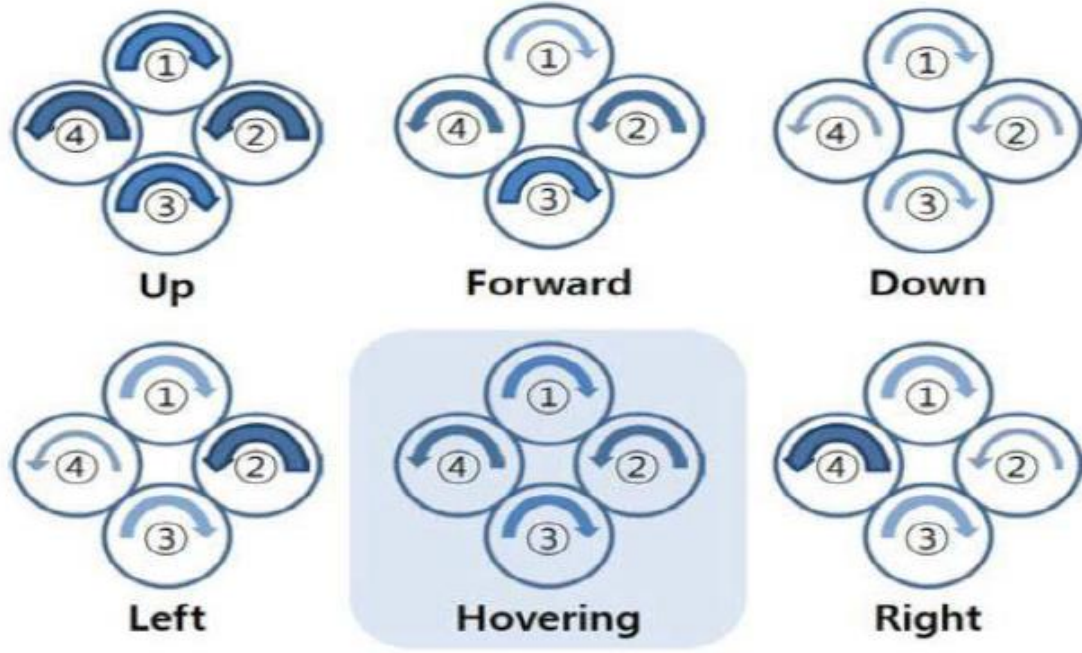


Figure 3-2: The Quadrotor Movement Mechanism

$$R = \begin{pmatrix} c\psi c\theta & s\phi s\theta c\psi - s\psi c\phi & c\phi s\theta c\psi + s\psi s\phi \\ s\psi c\theta & s\phi s\theta s\psi + c\psi c\theta & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix}$$

Depending on the angles of Euler, we can obtain the transformation matrix that transforms the systems from the frame of inertia or ground to the body frame as follows:

where $s(x)$ and $c(x)$ indicate $\sin(x)$ and $\cos(x)$ respectively.

Now, taking into account the Euler rotation angles and the Newton-Euler or Lagrange modeling method, the motion equations are obtained from the translational and rotational parameters which are $[\varphi, \theta, \psi, x, y, z]$. Hence, the nonlinear equations set derived are:

$$\begin{aligned}\ddot{\varphi} &= a_1 \dot{\psi} \dot{\theta} + b_1 \Omega_r \dot{\theta} + c_1 \dot{\varphi}^2 + d_1 U_2 \\ \ddot{\theta} &= a_2 \dot{\psi} \dot{\varphi} + b_2 \Omega_r \dot{\varphi} + c_2 \dot{\theta}^2 + d_2 U_3\end{aligned}\quad (3.8)$$

$$\begin{aligned}\ddot{\psi} &= a_3 \dot{\varphi} \dot{\theta} + b_3 \dot{\psi}^2 + c_3 U_4 \\ \ddot{x} &= a_4 \dot{x} + b_4 (c_\varphi s_\theta c_\psi + s_\varphi s_\psi) U_1 \\ \ddot{y} &= a_5 \dot{y} + b_5 (c_\varphi s_\theta s_\psi - s_\varphi c_\psi) U_1 \\ \ddot{z} &= a_6 \dot{z} - g + b_6 (c_\varphi c_\theta) U_1\end{aligned}\quad (3.9)$$

where,

$$a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}; \quad b_1 = -\frac{J_r}{I_{xx}}; \quad c_1 = -\frac{K_{fx}}{I_{xx}}; \quad d_1 = \frac{l}{I_{xx}};$$

$$a_2 = \frac{I_{zz} - I_{xx}}{I_{yy}}; \quad b_2 = \frac{J_r}{I_{yy}}; \quad c_2 = -\frac{K_{fy}}{I_{yy}}; \quad d_2 = \frac{l}{I_{yy}};$$

$$a_3 = \frac{I_{xx} - I_{yy}}{I_{zz}}; \quad b_3 = -\frac{K_{fz}}{I_{zz}}; \quad c_3 = \frac{d}{I_{zz}};$$

$$a_4 = -\frac{K_{tx}}{m}; \quad b_4 = \frac{1}{m}$$

$$a_5 = -\frac{K_{ty}}{m}; \quad b_5 = \frac{1}{m}$$

$$a_6 = -\frac{K_{tz}}{m}; \quad b_6 = \frac{1}{m}$$

where l indicates the length of each arm and m is the quadrotor mass. I_x , is the inertia moments around x – axis , I_y is the inertia moments around y – axis and I_z is the moment of inertia around z - axis. U_x and U_y are the virtual inputs whereas the control inputs are U_1, U_2, U_3 and U_4 and they are considered by the rotor's angular velocities relation as following:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ -b & 0 & b & 0 \\ d & -d & d & -d \end{pmatrix} \begin{pmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{pmatrix} \quad (3.10)$$

$$U_x = c_\varphi s_\theta c_\psi + s_\varphi s_\psi$$

$$U_y = c_\varphi s_\theta s_\psi - s_\varphi c_\psi$$

where b is the thrust and d is the parameter of the drag. ω^i is the angular velocity of the i^{th} rotor. The interpretation of these equations in details is shown in [56],[57].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + d_1 U_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= a_2 x_2 x_6 + b_2 \Omega_r x_2 + c_2 x_4^2 + d_2 U_3 \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= a_3 x_2 x_4 + b_3 x_6^2 + c_3 U_4 \\ \dot{x}_7 &= x_8 \end{aligned} \quad (3.11)$$

$$\dot{x}_8 = a_4 x_8 + b_4 U_x U_1$$

$$\dot{x}_9 = x_{10}$$

$$\dot{x}_{10} = a_5 x_{10} + b_5 U_y U_1$$

$$\dot{x}_{11} = x_{12}$$

$$\dot{x}_{12} = a_6 x_{12} - g + b_6 (c_\varphi c_\theta) U_1$$

Therefore, the integer state space representation will be as follows:

3.4 Fractional Order Transformation Scheme

Given the following dynamic system:

$$M\ddot{Y}(t) + C\dot{Y}(t) + KY(t) = f \quad (3.12)$$

Where $Y = [y_1, y_2, \dots, y_n]^T \in R^n$ is the displacement vector and $M \in R^{n \times n}$ is the mass matrix and $C \in R^n$ is the damping matrix and the stiffness matrix is $K \in R^n$ and finally, $f \in R^r$ represents the input vector.

As shown in [14], this system can be transformed into a linear pseudo state space fractional order system as follows with $\alpha=0.5$ (fractional order):

Firstly, taking the state variables as:

$$\begin{aligned} y_1(t) &= y(t) \\ y_2(t) &= D^{0.5}y(t) \\ y_3(t) &= \dot{y}(t) \\ y_4(t) &= D^{1.5}y(t) \end{aligned} \quad (3.13)$$

Hence, equation (1) can be rewritten as follows using equation (2):

$$\begin{bmatrix} D^{0.5}y_1(t) \\ D^{0.5}y_2(t) \\ D^{0.5}y_3(t) \\ D^{0.5}y_4(t) \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M^{-1}K & 0 & -M^{-1}C & 0 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \end{bmatrix} f \quad (3.14)$$

Then, the linear fractional order system form is:

$$D^{0.5}y(t) = AY(t) + Bf$$

Where,

$$A = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M^{-1}K & 0 & -M^{-1}C & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \end{bmatrix}$$

The same procedure has been done on the integer quadrotor model using the following changes of variables to obtain the fractional state space form of the translational equations Eq 1.3-3 of the quadrotor model and will keep the rotational equations Eq 1.3-2 as integer equations:

$$x_1 = x$$

$$x_2 = D^{0.5}x$$

$$x_3 = \dot{x}$$

(3.15)

$$x_5 = y$$

$$x_6 = D^{0.5}y$$

$$x_7 = \dot{y}$$

$$x_9 = z$$

$$x_{10} = D^{0.5}z$$

$$x_{11} = \dot{z}$$

Hence, based on above equation, the translational equations will be as follows:

$$D^{0.5}x_1 = x_2$$

$$D^{0.5}x_2 = x_3$$

$$D^{0.5}x_3 = x_4$$

$$D^{0.5}x_4 = a_4x_3 + b_4U_xU_1$$

$$D^{0.5}x_5 = x_6$$

(3.16)

$$D^{0.5}x_6 = x_7$$

$$D^{0.5}x_7 = x_8$$

$$D^{0.5}x_8 = x_9$$

$$D^{0.5}x_9 = x_{10}$$

$$D^{0.5}x_{10} = x_{11}$$

$$D^{0.5}x_{11} = x_{12}$$

Finally, the whole fractional system of the quadrotor will be as follows:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a_1x_4x_6 + b_1\Omega_rx_4 + c_1x_2^2 + d_1U_2$$

$$\dot{x}_3 = x_4$$

$$\begin{aligned}
\dot{x}_4 &= a_2 x_2 x_6 + b_2 \Omega_r x_2 + c_2 x_4^2 + d_2 U_3 \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= a_3 x_2 x_4 + b_3 x_6^2 + c_3 U_4 \\
D^{0.5} x_{f_7} &= x_{f_8} \\
D^{0.5} x_{f_8} &= x_{f_9} \\
D^{0.5} x_{f_9} &= x_{f_{10}} \\
D^{0.5} x_{f_{10}} &= a_4 x_{f_9} + b_4 U_x U_1 \\
D^{0.5} x_{f_{11}} &= x_{f_{12}} \\
D^{0.5} x_{f_{12}} &= x_{f_{13}} \\
D^{0.5} x_{f_{13}} &= x_{f_{14}} \\
D^{0.5} x_{f_{14}} &= a_5 x_{f_{13}} + b_5 U_y U_1 \\
D^{0.5} x_{f_{15}} &= x_{f_{16}} \\
D^{0.5} x_{f_{16}} &= x_{f_{17}} \\
D^{0.5} x_{f_{17}} &= x_{f_{18}} \\
D^{0.5} x_{f_{18}} &= a_6 x_{f_{17}} - g + b_6 (c_\varphi c_\theta) U_1
\end{aligned} \tag{3.17}$$

3.5 Validation of Fractional Order NLQM

Many research scholars prove that the models of real systems can be represented accurately by fractional order equations. This section shows a comparison between the integer order NLQM and the fractional one. To validate the FONLQM, we use a Pseudo Random Binary

Signal (PRBS) and apply as input to both IONLQM and FONLQM. Then the two systems are simulated in parallel and the results are compared.

3.5.1 Pseudo Random Binary Signal (PRBS)

As defined in [58], the PRBS is a signal produced by employing a pulse rectangle modulated signal with a random length. This signal can be approximated as discrete white noise, with a mean of zero and a variance close to one. Furthermore, PRBS is high in frequency and does not deviate the system from its operational point. It is usually applied in identification and validation jobs.

The pseudorandom signal of the PRBS is a long series characterized by a random change in the pulse width, but it is periodic across a wide span of time. The length of the sequence determines the duration. Its amplitude can be very low, but it must be above the residual noise. If the signal to noise ratio is too low, it is necessary to increase the duration of the simulation to get a good estimate of the parameters. A typical value of the amplitude of the PRBS is 0.5% to 5% of the value of the operating point on which it is applied as in [58]

$$AMP_{PRBS} = W_{Eq} \pm 0.005 * W_{Eq}; \text{ where } W_{Eq} = \sqrt{\frac{mg}{4b}}$$

The PRBS can be generated in MATLAB Simulink using the predefined function block called Band Limited White Noise. The signals are generated for both systems as show in (Figure 3-3, Figure 3-4, Figure 3-5, Figure 3-6, Figure 3-7 and Figure 3-8)

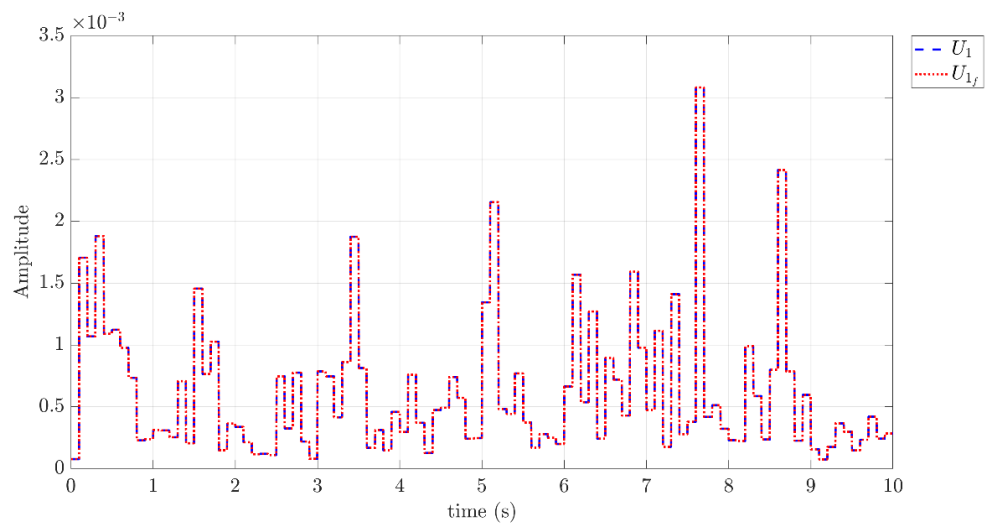


Figure 3-3: PRBS Control Inputs U_1 and U_{1f}

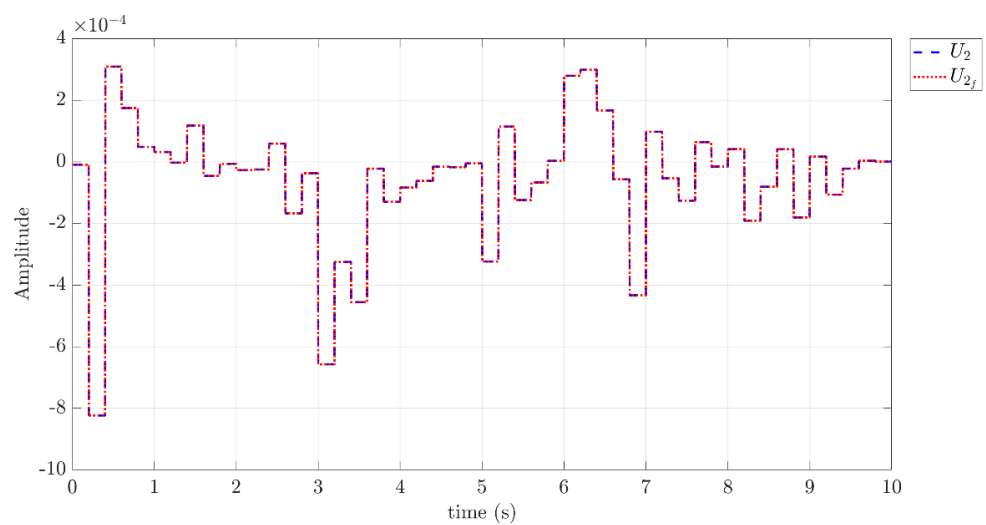


Figure 3-4: PRBS Control Inputs U_2 and U_{2f}

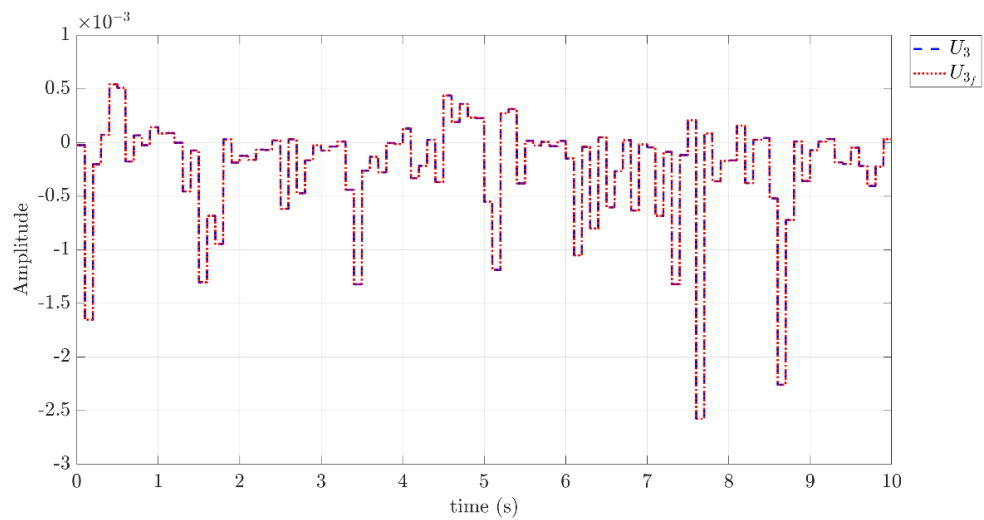


Figure 3-5: PRBS Control Inputs U_3 and U_{3f}

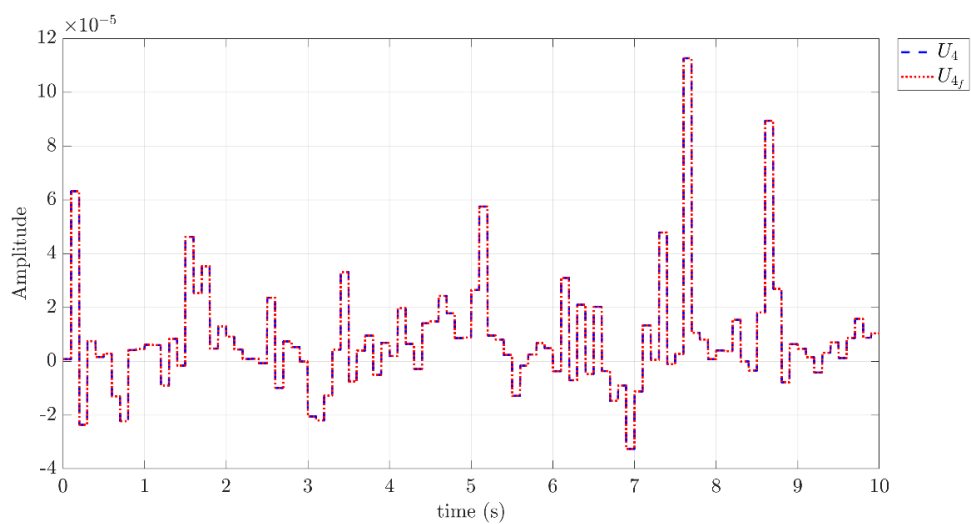


Figure 3-6: PRBS Control Inputs U_4 and U_{4f}

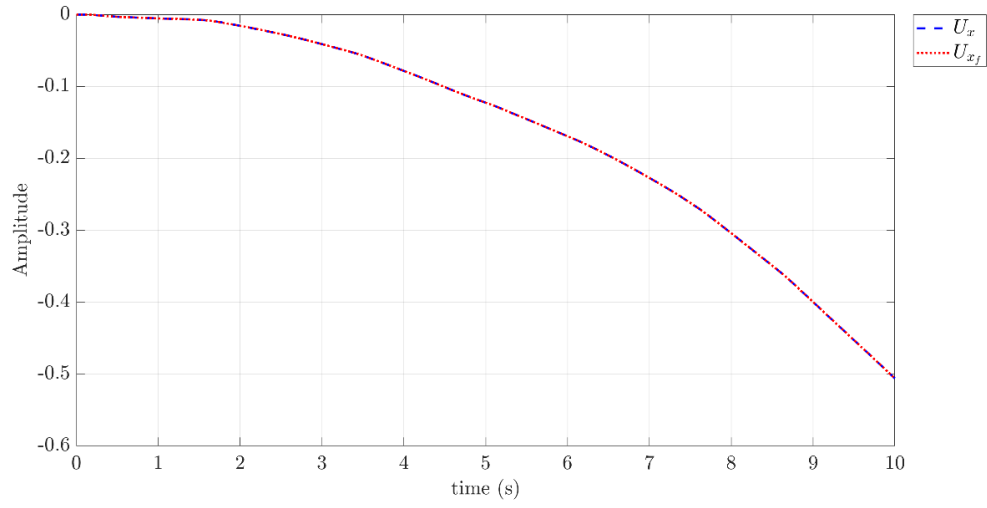


Figure 3-7: Virtual Control Inputs U_x and U_{x_f}

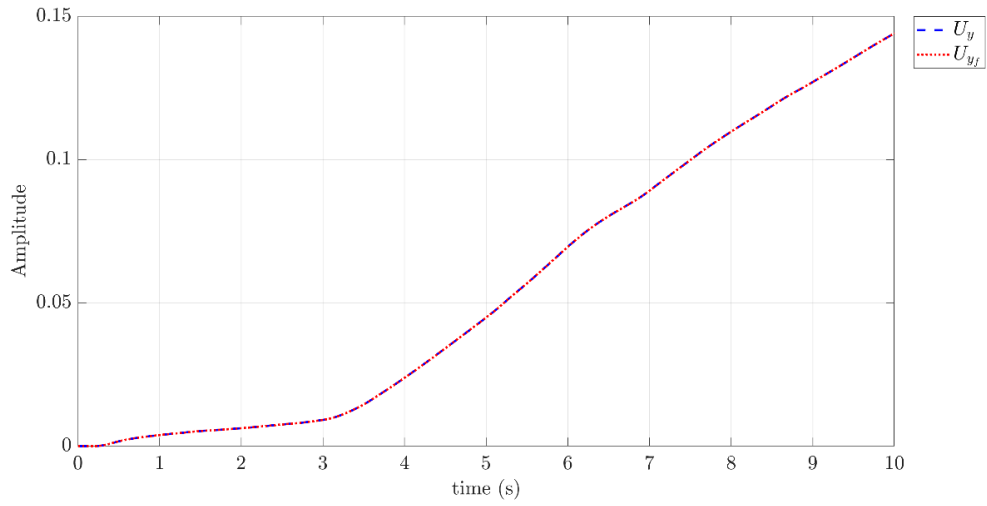


Figure 3-8: Virtual Control Inputs U_y and U_{y_f}

3.5.2 Step Input Signal (SIS)

The SIS can be generated in MATLAB Simulink using the predefined function block called Step with step time = 1, and final values (5, 10, 15, 20). (Figure 3-9, Figure 3-10, Figure 3-11, Figure 3-12, Figure 3-13 and Figure 3-14) show the signals of both systems.

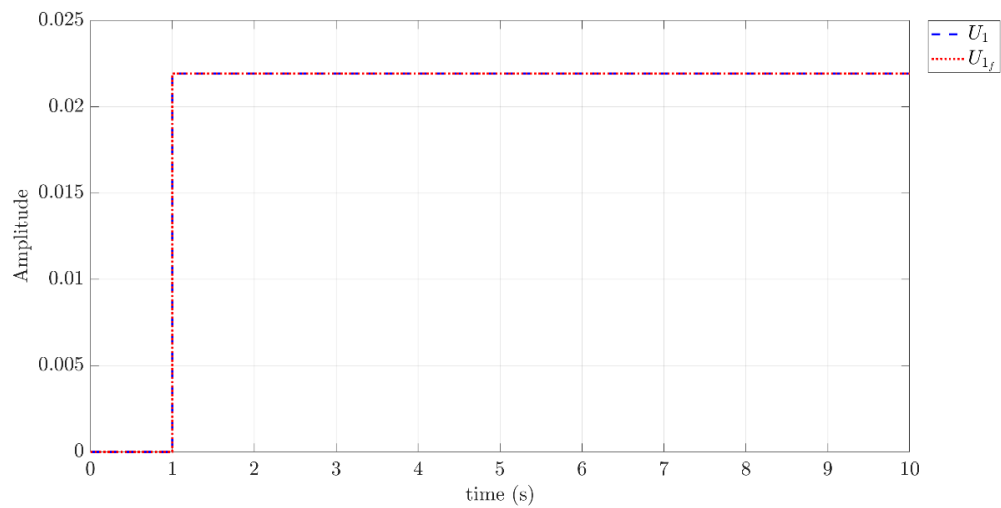


Figure 3-9: Step Control Inputs U_1 and U_{1f}

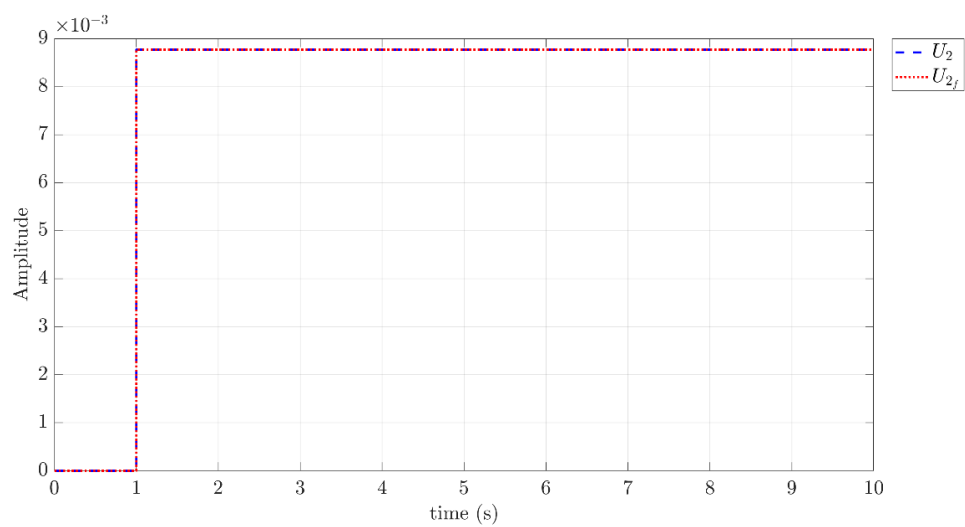


Figure 3-10: Step Control Inputs U_2 and U_{2f}

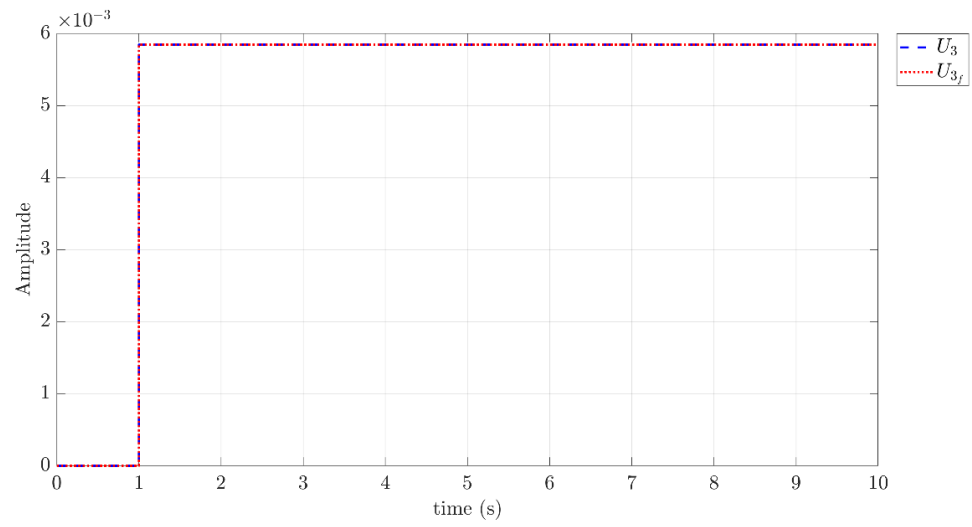


Figure 3-11: Step Control Inputs U_3 and U_{3f}

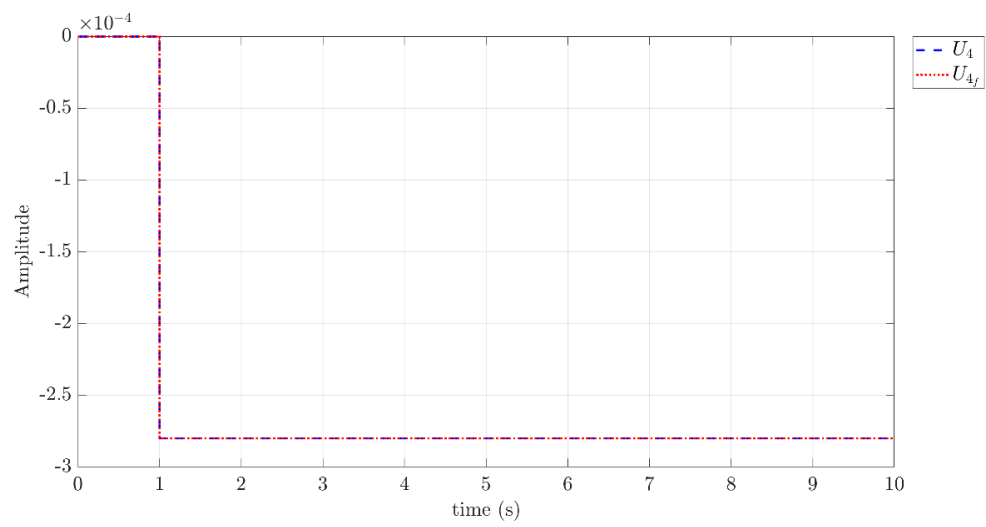


Figure 3-12: Step Control Inputs U_4 and U_{4f}

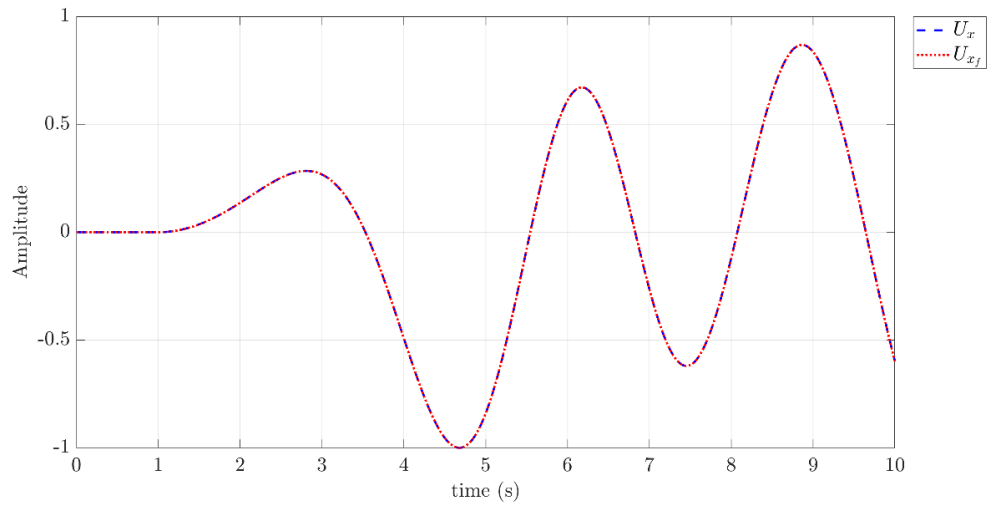


Figure 3-13: Virtual Control Inputs U_x and U_{x_f}

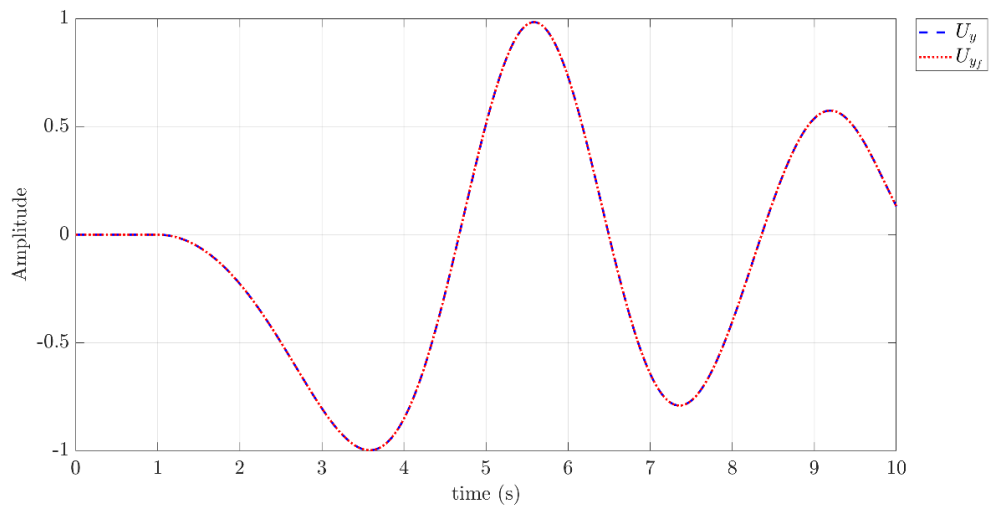


Figure 3-14: Virtual Control Inputs U_y and U_{y_f}

3.5.3 Simulation Results

As given in [59], Table 3-1 shows the quadrotor parameters that they are used in the simulation.

Table 3-1: System Parameters

Parameter Symbol	Description	Value	Units
g	The acceleration due to gravity	9.81	m/s^2
m	Quadrotor Mass	0.486	Kg
L	Quadrotor Arm Length	0.225	m
I_{xx}	The Moments of Inertia around x	4.856 E-03	Kg. m^2
I_{yy}	The Moments of Inertia around y	4.856 E-03	Kg. m^2
I_{zz}	The Moments of Inertia around z	8.801 E-03	Kg. m^2
K_{fx}	The Rotational Drag Coefficient in x	5.5670 E-04	N/rad/s
K_{fy}	The Rotational Drag Coefficient in y	5.5670 E-04	N/rad/s
K_{fz}	The Rotational Drag Coefficient in z	6.3540 E-04	N/rad/s
K_{tx}	The Linear Drag Coefficient in x	5.5670 E-04	N/m/s
K_{ty}	The Linear Drag Coefficient in y	5.5670 E-04	N/m/s
K_{tz}	The Linear Drag Coefficient in z	6.3540 E-04	N/m/s
J_r	The Inertia Moment of Rotor	2.8385 E-05	$N.m/rad/s^2$

b	The Lift Coefficient	2.923 E-05	N.m/rad/s
d	The Drag Coefficient	1.12 E-06	N.m/rad/s

The simulation of the IONLQ and the FONLQ models with PRBS and step inputs are carried out in MATLAB using all the parameters described. The behavior of all the states with the given PRBS inputs are obtained as shown in (Figure 3-15, Figure 3-16, Figure 3-17, Figure 3-18, Figure 3-19 and Figure 3-20), also the same figures show the errors between the integer and fractional states.

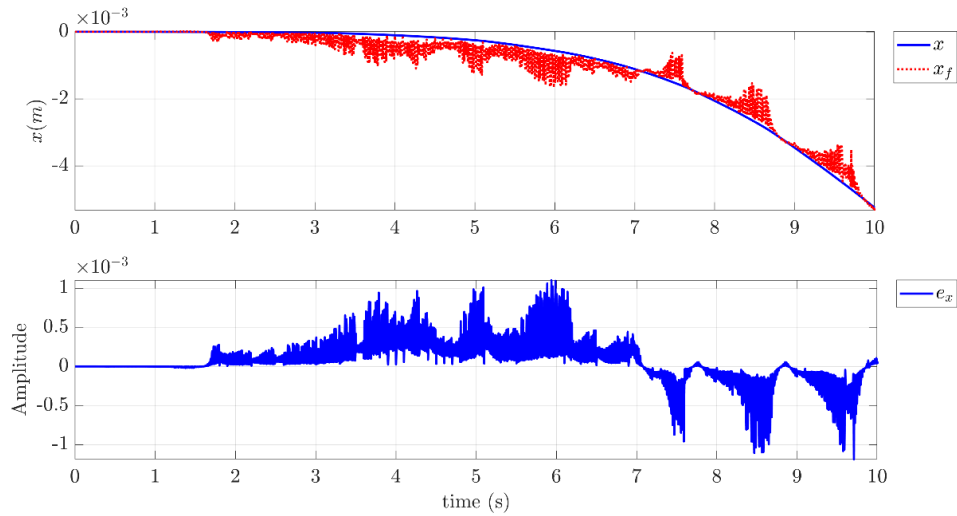


Figure 3-15: x and x_f Positions Responses with PRBS Inputs and the error between them

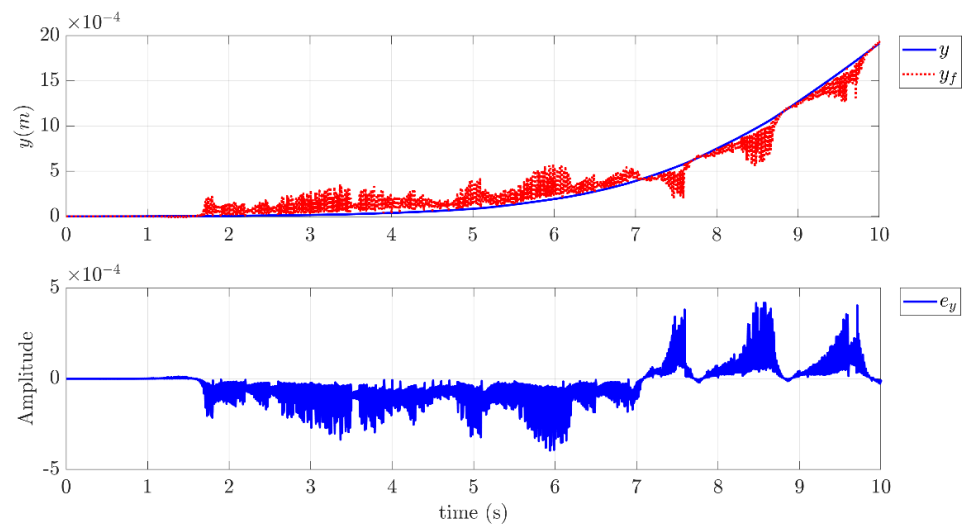


Figure 3-16: y and y_f Positions Responses with PRBS Inputs and the error between them

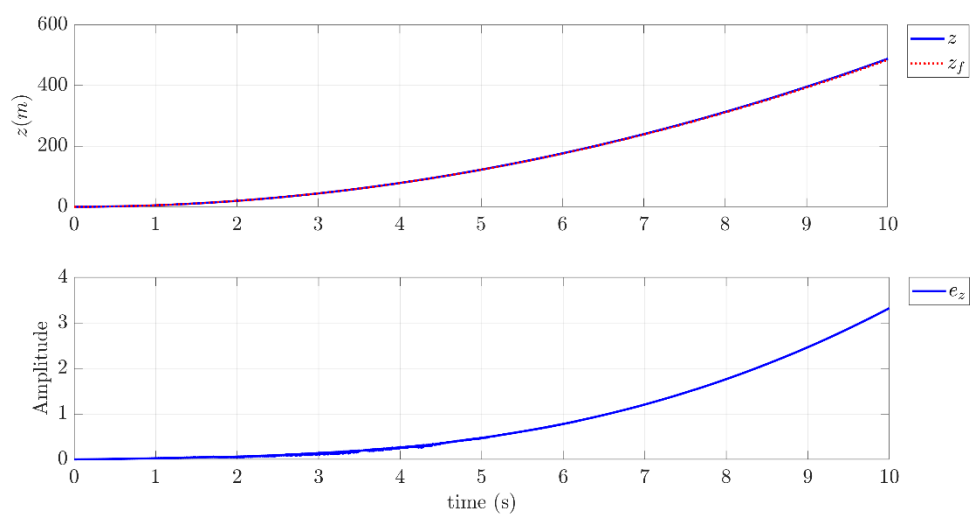


Figure 3-17: z and z_f Altitudes Responses with PRBS Inputs and the error between them

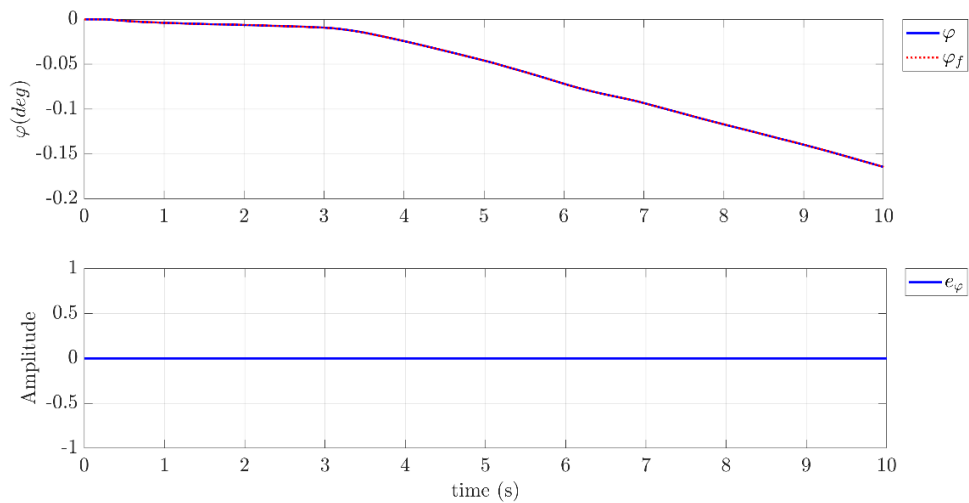


Figure 3-18: ϕ and ϕ_f Angles Responses with PRBS Inputs and the error between them

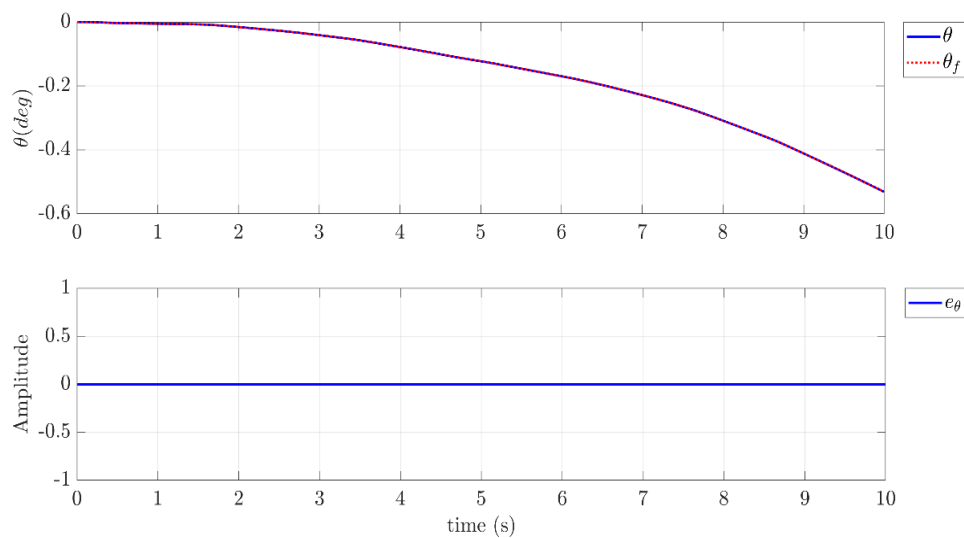


Figure 3-19: θ and θ_f Angles Responses with PRBS Inputs and the error between them

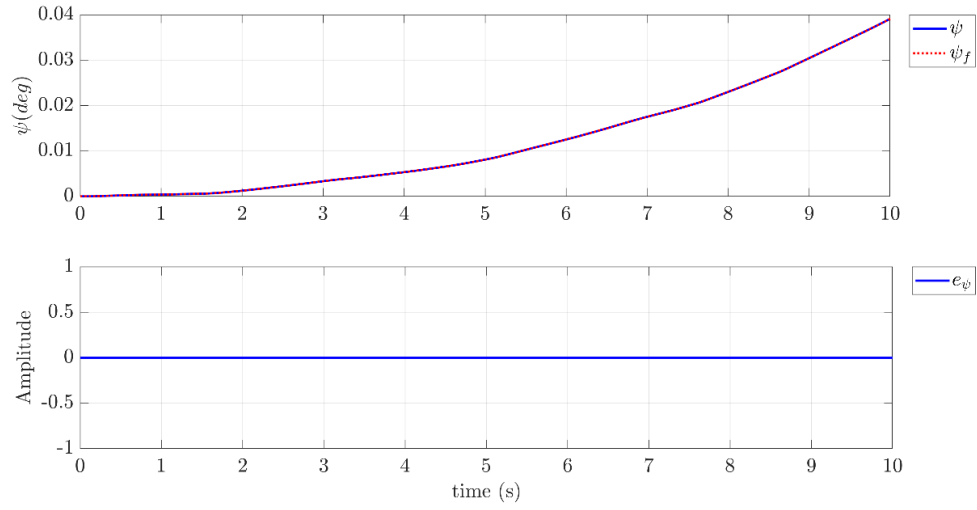


Figure 3-20: ψ and ψ_f Angles Responses with PRBS Inputs and the error between them

(Figure 3-21, Figure 3-22, Figure 3-23, Figure 3-24, Figure 3-25 and Figure 3-26) show the behavior of all the states of IONLQM and the FONLQM with SIS inputs, also the same figures show the errors between the integer and fractional states.

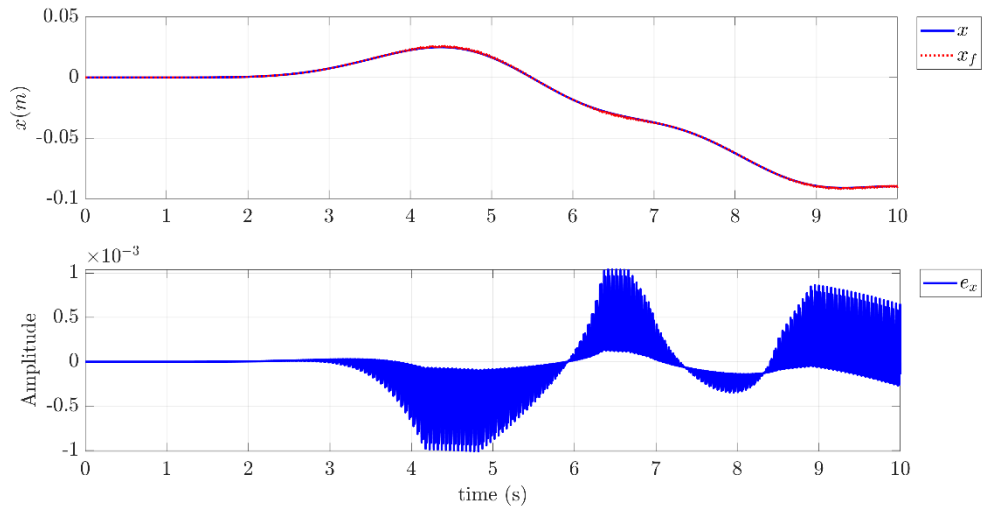


Figure 3-21: x and x_f Positions Responses with Step Inputs and the error between them

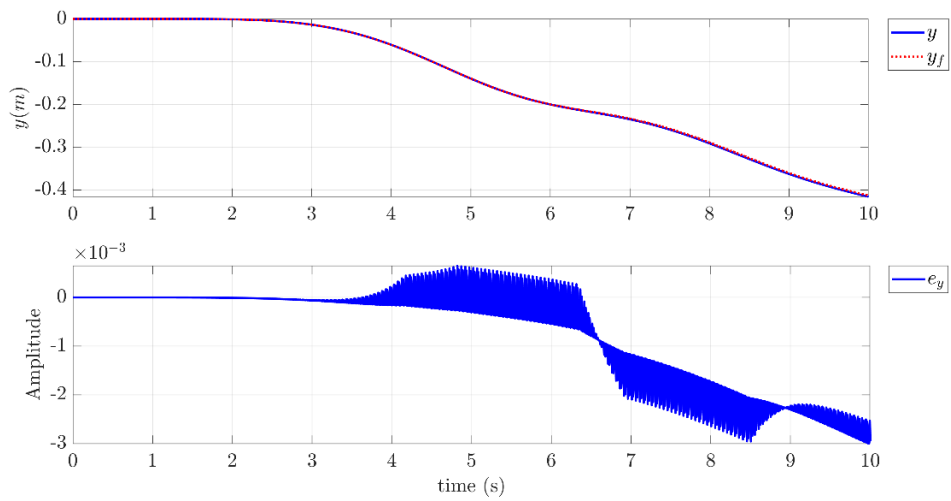


Figure 3-22: y and y_f Positions Responses with Step Inputs and the error between them

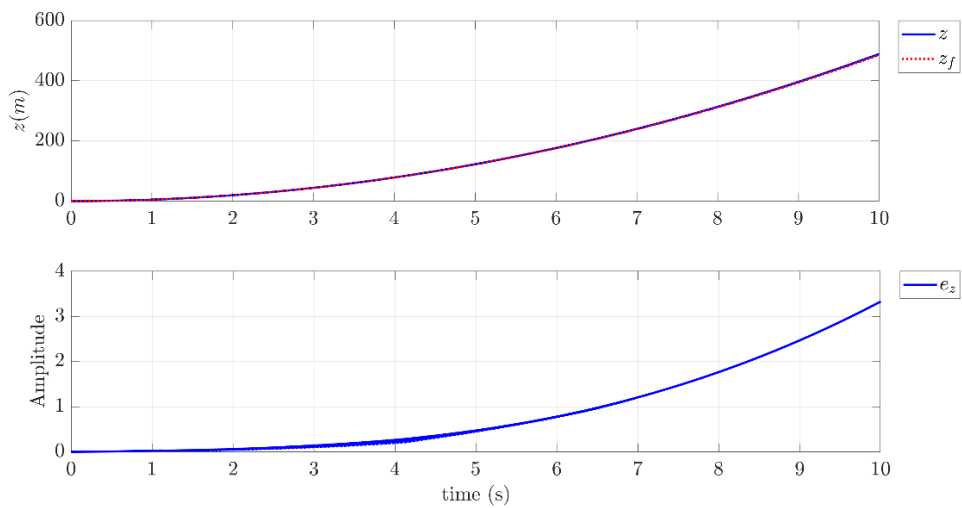


Figure 3-23: z and z_f Positions Responses with Step Inputs and the error between them

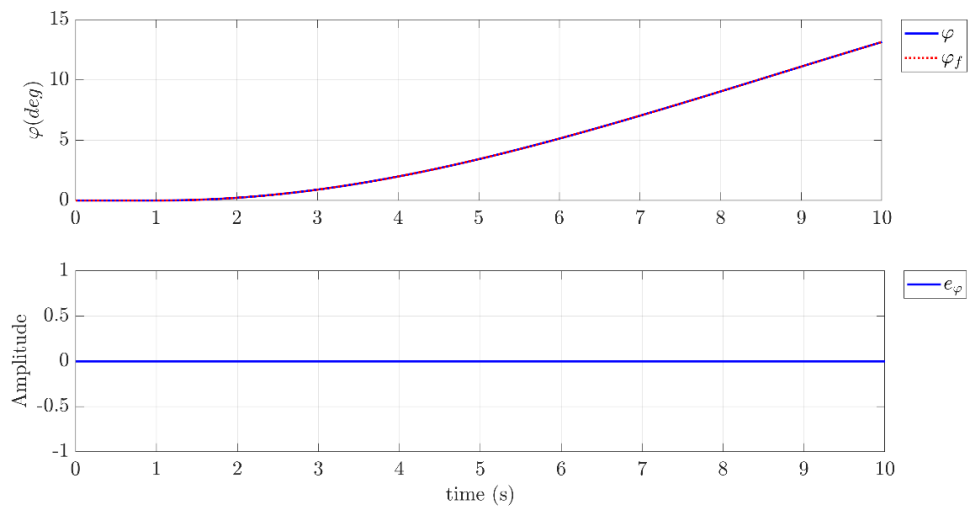


Figure 3-24: ϕ and ϕ_f Angles Responses with Step Inputs and the error between them

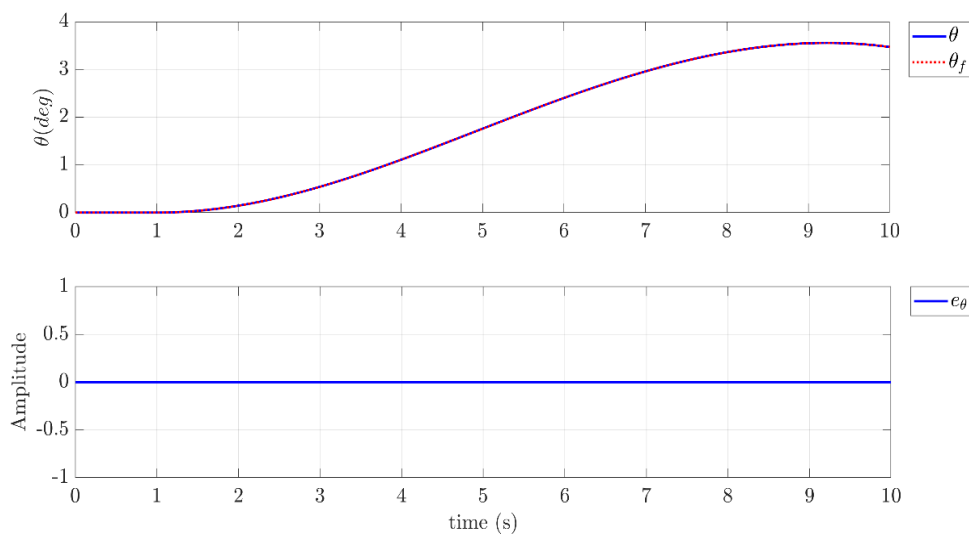


Figure 3-25: θ and θ_f Angles Responses with Step Inputs and the error between them

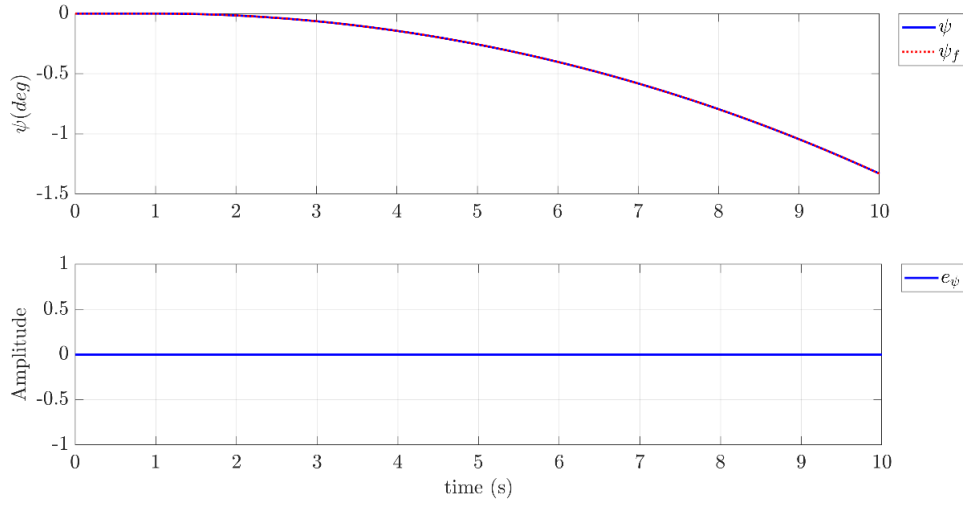


Figure 3-26: ψ and ψ_f Angles Responses with PRBS Inputs and the error between them

Therefore, it can be observed that the responses of position states in both integer and fractional systems are consistent, while the angles states responses in both systems are exactly the same. Furthermore, the fractional order model gives a good approximation of the dynamics of the non-linear quadrotor model.

3.6 Conclusion

In this chapter, the dynamics and the operation of the quadrotor was explained. Then the integer non-linear mathematical model (INLMM) was formulated and the state equations were obtained. This model showed the non-linear, multivariable and under actuated behavior of the quadrotor which makes it relatively difficult to control.

Then, the INLMM transformed into a fractional non-linear mathematical model (FNLMM) using fractional calculus. After that, the simulations were carried out to compare the INLMM and the FNLMM by giving PRBS inputs and SIS. The results confirmed that the FNLMM provides a good approximation of the system. Also, we provided the errors that compare the position states of both integer and fractional models to indicate that the systems will not be that same.

CHAPTER 4 PID and FOPID Controls Designs

4.1 Introduction

A quadrotor is a type of rotary-winged aircraft. The quadrotor's control difficulties stem mostly from its complicated dynamics, non-linearity, multivariable structure, and, most importantly, its underactuated states. As a result, the problem of controlling a quadrotor has received much interest in the previous few years. Many techniques have been applied to successfully control the quadrotor and can be found in the literature which have been discussed earlier in chapter 2.

As observed from Equations (3.8) and (3.9), the quadrotor system is a 6 DOF because it has six outputs $(\phi, \theta, \psi, x, y, z)$ but it has four inputs that can be manipulated (U_1, U_2, U_3, U_4) ; hence it considers as a type of underactuated systems because the number of controlled variables (outputs) is more than the number of manipulated variables (inputs). Also, we can observe in same Equations that the rotational dynamics do not depend on the translational states; hence we can control the quadrotor attitude and heading (angular states ϕ, θ, ψ) separately by implementing three controllers (U_2, U_3, U_4) for three outputs (ϕ, θ, ψ) and form a rotational subsystem as an inner loop. While, the outer loop will govern the quadrotor system's position by delivering the altitude and position (translational states) with the controlled angles. (see Figure 4-1)

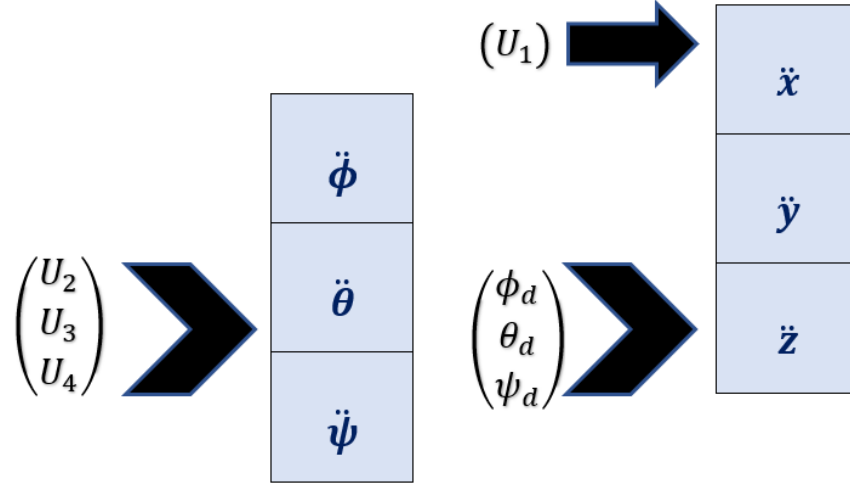


Figure 4-1: Rotational and Transational subsystems

In this chapter, we first study about the theories of two majors control techniques, PID control and FOPID control. Then, we define these controllers for our systems and also the control laws are developed. The simulations are performed to control the IONL and FONL models of the quadrotor using the developed controllers and the tracking of a desired trajectory is done. The performance of the proposed control designs will be assessed by implementing these controllers on MATLAB/Simulink.

Also, in this section, we propose three case studies that have been investigated to highlight the issues that the quadcopter control systems may face in achieving the required tracking. The first case study was the step input, the circular path was the second case study and the third case study was the helical path. The states with reference trajectories of these case studies are presented in Table 4-1.

Table 4-1: Case Studies with Reference Trajectories (RT)

Cases with RT States	Case (1)	Case (2)	Case (3)
	Reference Trajectory	Reference Trajectory	Reference Trajectory
x	$u(t)$	$\cos(t)$	$\cos(t)$
y	$u(t)$	$\sin(t)$	$\sin(t)$
z	$2u(t)$	$u(t)$	$0.2t$
ψ	$5u(t)$	$u(t)$	$u(t)$

4.2 Closed Loop Simulation

PID, FOPID Controllers are used to control the transitional and rotational states of the quadrotor in both integer and fractional systems.

4.2.1 Altitude Controller

Figure 4-2 shows the block diagram of the altitude controller (AC). The input of AC is the error signal e_z where $e_z = z - z_d$ and it produces the output U_1

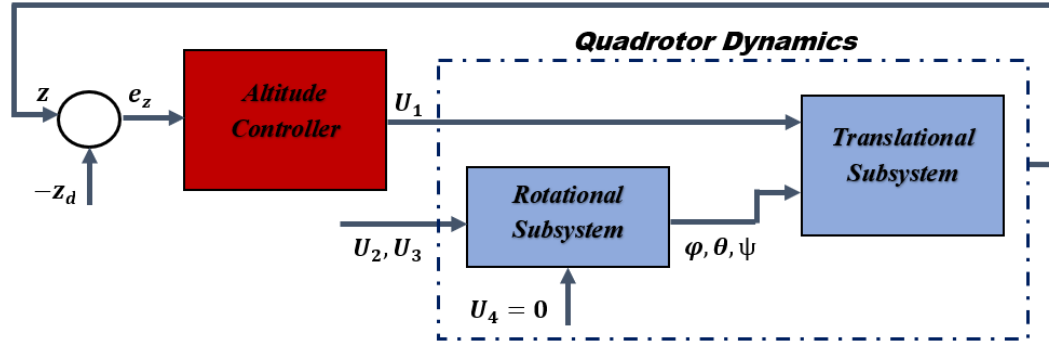


Figure 4-2:Altitude Controller Block Diagram

4.2.2 Attitude and Heading Controllers

The block diagram of attitude and heading controller is shown in Figure 4-3. Attitude controller has e_φ and e_θ as inputs and U_2 and U_3 as outputs where $e_\varphi = \varphi - \varphi_d$ and $e_\theta = \theta - \theta_d$. The input of the heading controller is e_ψ where $e_\psi = \psi - \psi_d$ and it produces the output U_4

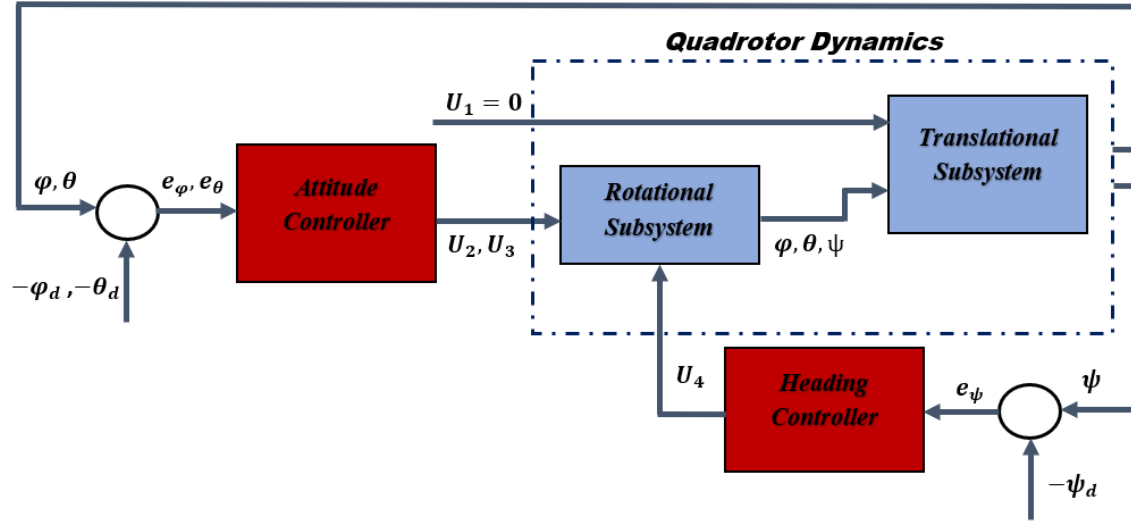


Figure 4-3: Attitude and Heading Controllers Block Diagram

4.2.3 x and y Positions Controllers

We observe from the translational equations of quadrotor model that x and y positions are not decoupled and they cannot be controlled directly by the system's controls U_1 through U_4 . Therefore, we assume having two virtual control inputs U_x and U_y that control x and y positions.

The position controller is introduced to the closed loop simulation for the quadrotor altitude and attitude controllers, as illustrated in Figure 4-4.

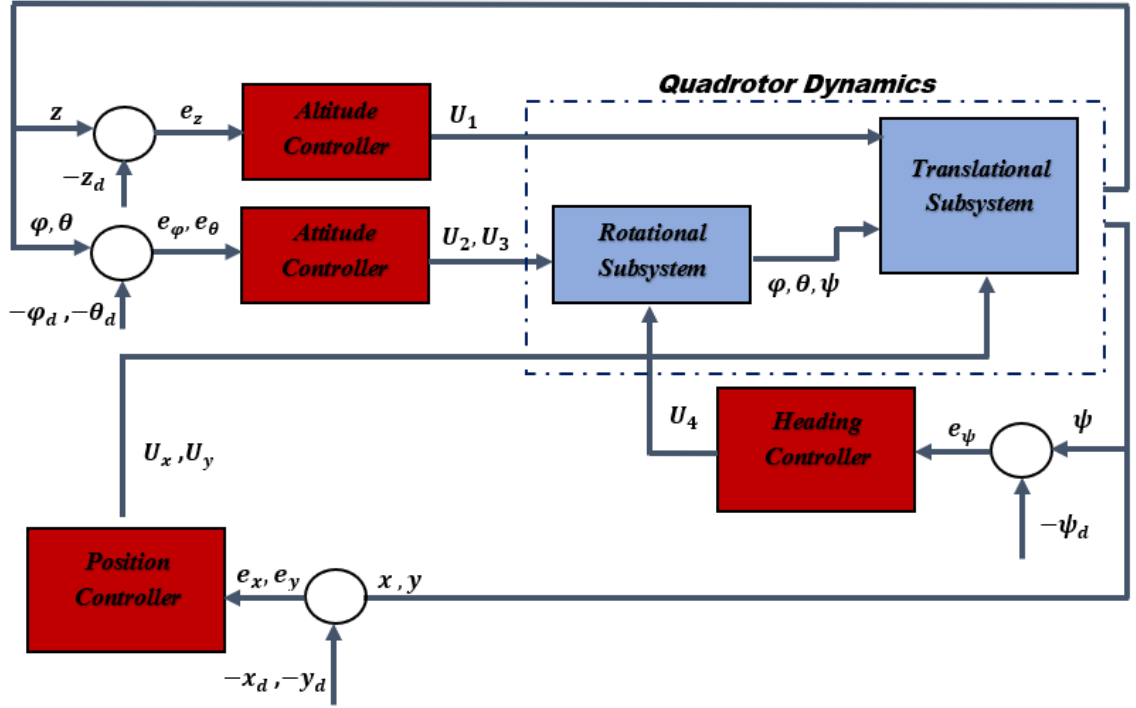


Figure 4-4: All System Controllers Block Diagram

The controller blocks in the preceding block diagrams can include any type of control method, whether linear or nonlinear. All controller inputs are errors connected to some of the quadrotor states and provide an output that is either one or more control inputs U_1 through U_4 or ϕ_d and θ_d with U_x and U_y if it is the position controller.

4.3 Controller Parameters Tuning Employing GA

In this section, we propose a technique to tune the parameters of PID and FOPID controllers using the optimization approach, GA. The stated method involves computationally calculating the proportional gain (K_p), integral gain (K_i), and the derivative gain (K_d) for PID and FOPID controlled systems; integral constant (λ) and derivative constant (μ) for FOPID controller whose process modeled as a linear system using Linear Analysis Tool in Simulink around the initial conditions as shown in Figure 4-5. A MATLAB software is utilized to find the optimal value of the parameters of PID and FOPID controllers that can meet the majority of the system requirements, such as reducing rising time, maintaining a high system response and keeping robustness. The objective function of GA was chosen so that the Integral of Time Absolute Error (ITAE) performance index is minimized. After that we use the obtained optimal parameters of PID and FOPID controllers to find the objective functions of integral absolute error (IAE), integral square error (ISE) and the integral time square error (ITSE). The formulas of IAE, ITAE, ISE, ITSE are shown in Equations (4.1), (4.2), (4.3) and (4.4).

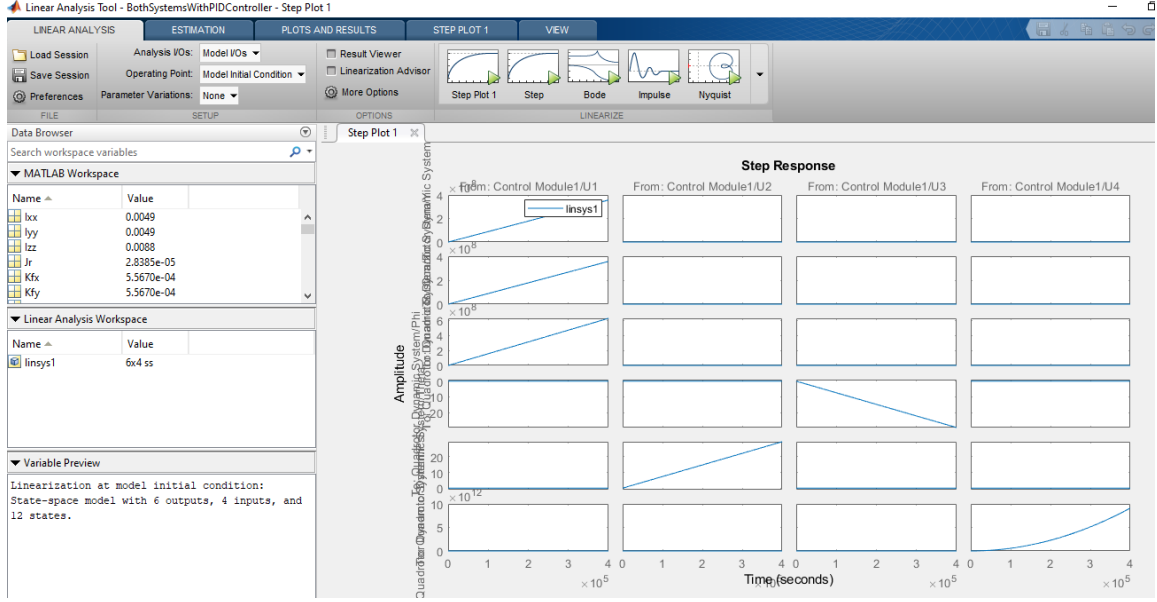


Figure 4-5: Linear Analysis Tool in Simulink

$$IAE = \int |e(t)|dt \quad (4.1)$$

$$ITAE = \int t|e(t)|dt \quad (4.2)$$

$$ISE = \int e^2(t)dt \quad (4.3)$$

$$ITSE = \int te^2(t)dt \quad (4.4)$$

As defined in [58], the GA method is an iterative optimization technique that works as follows:

- Firstly, it generates a random “population” comprising numerous individuals which will be a vector for the controller parameters in our scenario.
- The individuals fitness of the population is assessed by use of an objective function which minimize the ITAE performance index to such values as control gains.
- New population is then derived from the existing population via genetic operations such as evolution, mutation and crossover and also their fitness is assessed.
- The "elite" of the two populations are then nominated to form a third population.

- The term "elite" refers to individuals who have the best fitness or have the lowest value of the objective function (minimizing the ITAE performance index in our case). The algorithm is applied until it achieves a population with all (or most) of its members being elite, at which point it returns the individual (the value for the control gains) with the lowest possible fitness (produces the least possible ITAE performance index for the system).

Throughout this work, we employed the MATLAB optimization toolbox, which provides a built-in function for GA optimization.

4.4 PID Control Design

4.4.1 Introduction

In this section, we designed PID controllers that control the transitional and rotational states of the quadrotor in integer system. Figure 4-6 shows the PID controller's block diagram.

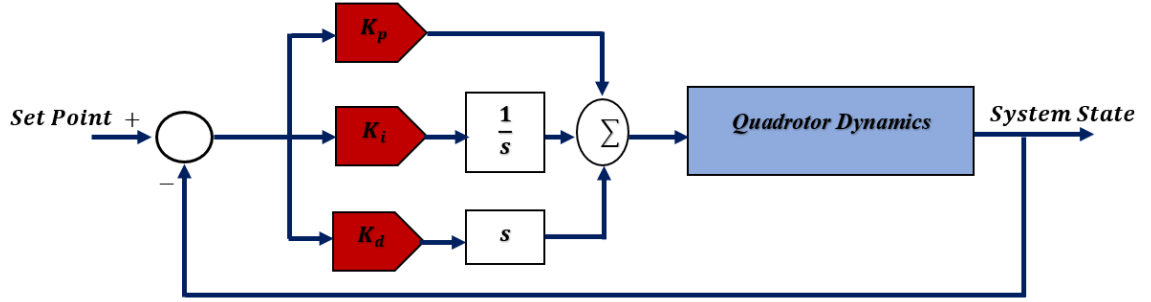


Figure 4-6: PID Block Diagram

4.4.2 PID Control Design of Integer System

4.4.2.1 Altitude PID of Integer System

To govern the quadrotor's altitude, we created a PID controller. It produces the control input U_1 , which is also responsible of the quadrotor's altitude, as shown in Equation (4.5).

Now, we can derive the control law as follows:

$$U_1 = \frac{m}{c\varphi c\theta} \left[K_p e_z + K_D \dot{e}_z + K_i \int e_z dt + g \right] \quad (4.5)$$

Where

z Actual Altitude

z_d Desired Altitude

$e_z = (z - z_d)$ Altitude Error

K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain
$c\varphi$	$\cos \varphi$
$c\theta$	$\cos \theta$
g	Gravity Value
m	Quadrotor Mass

4.4.2.2 Attitude and Heading Controllers in Integer System

4.4.2.2.1 Roll Angle Controller of Integer system

A PID controller is designed to control the quadrotor's Roll angle φ in integer system which generates the control input U_2 as shown in Equation (4.6). The control law can be derived as follows:

$$U_2 = I_{xx} \left[K_p e_\varphi + K_D \dot{e}_\varphi + K_i \int e_\varphi dt \right] \quad (4.6)$$

Where

φ	Actual Roll Angle
φ_d	Desired Roll Angle
e_φ	$= (\varphi - \varphi_d)$ Roll Angle Error
K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain

I_{xx} The Moments of Inertia around x

4.4.2.2.2 Pitch Angle Controller of Integer system

A PID controller is designed to control the quadrotor's Pitch angle θ in integer system and it generates the control input U_3 as represented in Equation (4.7). The derived control law is as follows:

$$U_3 = I_{yy} \left[K_p e_\theta + K_D \dot{e}_\theta + K_i \int e_\theta dt \right] \quad (4.7)$$

Where

θ Actual Pitch Angle

θ_d Desired Pitch Angle

$e_\theta = (\theta - \theta_d)$ Pitch Angle Error

K_p Proportional Gain

K_D Derivative Gain

K_i Integral Gain

I_{yy} The Moments of Inertia around y

4.4.2.2.3 Yaw Angle Controller of Integer system

Another PID controller is designed to control the quadrotor's Yaw angle ψ in integer system and it produces the control input U_4 as indicated in Equation (4.8). The derived control law is as follows:

$$U_4 = I_{zz} \left[K_p e_\psi + K_D \dot{e}_\psi + K_i \int e_\psi dt \right] \quad (4.8)$$

Where

ψ Actual Yaw Angle

ψ_d Desired Yaw Angle

$e_\psi = (\psi - \psi_d)$ Yaw Angle Error

K_p Proportional Gain

K_D Derivative Gain

K_i Integral Gain

I_{zz} The Moments of Inertia around z

4.4.2.3 Positions Controller of Integer system

Because the system is underactuated, the quadrotor position (x, y) cannot be driven directly. It can be controlled by manipulating the Roll and Pitch angles (φ, θ) . As a result, the position is controlled by specifying desired Roll and Pitch angles (φ_d, θ_d) and applying them to their respective controllers. The steps of finding desired Roll and Pitch angles (φ_d, θ_d) can be summarized as follows:

Finding desired Roll Angle(φ_d)

First, as in Equation (3.10) the virtual control inputs are defined as follow,

$$U_x = c_\varphi s_\theta c_\psi + s_\varphi s_\psi \quad (4.9)$$

$$U_y = c_\varphi s_\theta s_\psi - s_\varphi c_\psi \quad (4.10)$$

Now, solve for s_θ in Equation (4.9),

$$s_{\theta} = \frac{1}{c_{\varphi} c_{\psi}} [U_x - s_{\varphi} s_{\psi}] \quad (4.11)$$

Substituting Equation (4.11) in Equation (4.10)

$$U_y = \frac{s_{\psi}}{c_{\psi}} [U_x - s_{\varphi} s_{\psi}] - s_{\varphi} c_{\psi} \quad (4.12)$$

Solve for s_{φ} in Equation (4.12) to find $\boldsymbol{\varphi}_d$,

$$\frac{s_{\varphi} s_{\psi}^2}{c_{\psi}} + s_{\varphi} c_{\psi} = \frac{s_{\psi}}{c_{\psi}} U_x - U_y$$

$$s_{\varphi} \left[\frac{s_{\psi}^2}{c_{\psi}} + c_{\psi} \right] = \frac{s_{\psi}}{c_{\psi}} U_x - U_y$$

$$s_{\varphi} \left[\frac{s_{\psi}^2 + c_{\psi}^2}{c_{\psi}} \right] = \frac{s_{\psi}}{c_{\psi}} U_x - U_y$$

$$\text{Since } (s_{\psi}^2 + c_{\psi}^2 = 1) \rightarrow s_{\varphi} = s_{\psi} U_x - c_{\psi} U_y$$

$$\boldsymbol{\varphi}_d = \mathbf{sin}^{-1}[s_{\psi_d} U_x - c_{\psi_d} U_y] \quad (4.13)$$

Finding desired Pitch angle (θ_d)

Now, solve for s_{φ} in Equation (4.9),

$$s_{\varphi} = \frac{1}{s_{\psi}} [U_x - c_{\varphi} s_{\theta} c_{\psi}] \quad (4.14)$$

Substituting Equation (4.14) in Equation (4.10)

$$U_y = c_{\varphi} s_{\theta} s_{\psi} - \frac{c_{\psi}}{s_{\psi}} [U_x - c_{\varphi} s_{\theta} c_{\psi}] \quad (4.15)$$

Solve for s_{θ} in Equation (4.15) to find $\boldsymbol{\theta}_d$,

$$s_{\theta} [c_{\varphi} s_{\psi} + \frac{c_{\varphi} c_{\psi}^2}{s_{\psi}}] = U_y + \frac{c_{\psi}}{s_{\psi}} U_x$$

$$s_{\theta} [c_{\varphi} (\frac{s_{\psi}^2 + c_{\psi}^2}{s_{\psi}})] = U_y + \frac{c_{\psi}}{s_{\psi}} U_x$$

Since $(s_\psi^2 + c_\psi^2 = 1) \rightarrow s_\theta(\frac{c_\phi}{s_\psi}) = U_y + \frac{c_\psi}{s_\psi} U_x$

$$s_\theta = \frac{1}{c_\phi} [c_\psi U_x + s_\psi U_y]$$

$$\theta_d = \sin^{-1}[\frac{1}{c_{\phi_d}} (c_{\psi_d} U_x + s_{\psi_d} U_y)] \quad (4.16)$$

Therefore, after obtaining stable PID controllers for the quadrotor's altitude and orientation.

Now, we can design PID controllers that control x and y positions.

4.4.2.3.1 x Position Controller in Integer System

A PID controller is designed to govern the x position in integer system. It produces the virtual control input U_x . Therefore, the derived control law will be as follows in Equation (4.17):

$$U_x = \frac{m}{U_1} \left[K_p e_x + K_D \dot{e}_x + K_i \int e_x dt \right] \quad (4.17)$$

Where

x	Actual x Position
x_d	Desired x Position
e_x	$= (x - x_d)$ x Position Error
K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain
U_1	Altitude Controller
m	Quadrotor Mass

4.4.2.3.2 y Position Controller in Integer System

Another PID controller is designed to govern the y position in integer system. It produces the virtual control input U_y . Hence, the derived control law will be as follows in Equation (4.18):

$$U_y = \frac{m}{U_1} \left[K_p e_y + K_D \dot{e}_y + K_i \int e_y dt \right] \quad (4.18)$$

Where

y	Actual y Position
y_d	Desired y Position
e_y	$= (y - y_d)$ y Position Error
K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain
U_1	Altitude Controller
m	Quadrotor Mass

4.4.3 PID Control Design of Fractional System

4.4.3.1 Introduction

In this section, we designed PID controllers based on the fractional dynamic equation (3.17) that control the altitude, attitude, heading, and positions in fractional system. The PID controller's block diagram is shown previously in Figure 4-6.

4.4.3.2 Altitude PID of Fractional System

We designed a PID controller to control the quadrotor's altitude in fractional system. we can derive the control law as follows:

$$U_{1f} = \frac{m}{c\varphi c\theta} \left(K_p e_{z_f} + K_D \dot{e}_{z_f} + K_i \int e_{z_f} dt + g \right) \quad (4.19)$$

Where

z_f Fractional Actual Altitude

z_d Desired Altitude

$e_{z_f} = (z_f - z_d)$ Fractional Altitude Error

K_p Proportional Gain

K_D Derivative Gain

K_i Integral Gain

$c\varphi$ $\cos \varphi$

$c\theta$ $\cos \theta$

g Gravity Value

m Quadrotor Mass

4.4.3.3 Attitude and Heading Controllers in Fractional System

4.4.3.3.1 Roll Angle Controller of Fractional system

A PID controller is designed to govern the quadrotor's Roll angle φ in fractional system which generates the control input U_{2f} . The control law can be derived as follows:

$$U_{2f} = I_{xx} \left(K_p e_{\varphi_f} + K_D \dot{e}_{\varphi_f} + K_i \int e_{\varphi_f} dt \right) \quad (4.20)$$

Where

- φ_f Fractional Actual Roll Angle
- φ_d Desired Roll Angle
- $e_{\varphi_f} = (\varphi_f - \varphi_d)$ Fractional Roll Angle Error
- K_p Proportional Gain
- K_D Derivative Gain
- K_i Integral Gain
- I_{xx} The Moments of Inertia around x

4.4.3.3.2 Pitch Angle Controller of Fractional system

A PID controller is designed to govern the quadrotor's Pitch angle θ in fractional system and it generates the control input U_{3f} . The control law can be derived as follows:

$$U_{3f} = I_{yy} \left(K_p e_{\theta_f} + K_D \dot{e}_{\theta_f} + K_i \int e_{\theta_f} dt \right) \quad (4.21)$$

Where

- θ_f Fractional Actual Pitch Angle
- θ_d Desired Pitch Angle
- $e_{\theta_f} = (\theta_f - \theta_d)$ Fractional Pitch Angle Error
- K_p Proportional Gain
- K_D Derivative Gain
- K_i Integral Gain

I_{yy} The Moments of Inertia around y

4.4.3.3 Yaw Angle Controller of Fractional system

Another PID controller is designed to control the quadrotor's Yaw angle ψ in fractional system and it produces the control input U_{4f} . The control law can be derived as follows:

$$U_{4f} = I_{zz} \left(K_p e_{\psi_f} + K_D \dot{e}_{\psi_f} + K_i \int e_{\psi_f} dt \right) \quad (4.22)$$

Where

ψ_f Fractional Actual Yaw Angle

ψ_d Desired Yaw Angle

$e_{\psi_f} = (\psi_f - \psi_d)$ Fractional Yaw Angle Error

K_p Proportional Gain

K_D Derivative Gain

K_i Integral Gain

I_{zz} The Moments of Inertia around z

4.4.3.4 Positions Controller of Fractional system

4.4.3.4.1 x Position Controller in Fractional System

A PID controller is designed to govern the x position in fractional system. It produces the virtual control input U_{x_f} . Therefore, the derived control law will be as follows:

$$U_{x_f} = \frac{m}{U_1} \left(K_p e_{x_f} + K_D \dot{e}_{x_f} + K_i \int e_{x_f} dt \right) \quad (4.23)$$

Where

x_f	Fractional Actual x Position
x_d	Desired x Position
e_{x_f}	$= (x_f - x_d)$ Fractional x Position Error
K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain
U_1	Altitude Controller
m	Quadrotor Mass

4.4.3.4.2 y Position Controller in Fractional System

Another PID controller is designed to govern the y position in fractional system. It produces the virtual control input U_{y_f} . Hence, the derived control law will be as follows:

$$U_{y_f} = \frac{m}{U_1} \left(K_p e_{y_f} + K_D \dot{e}_{y_f} + K_i \int e_{y_f} dt \right) \quad (4.24)$$

Where

y_f	Fractional Actual y Position
y_d	Desired y Position
e_{y_f}	$= (y_f - y_d)$ Fractional y Position Error
K_p	Proportional Gain
K_D	Derivative Gain
K_i	Integral Gain
U_1	Altitude Controller

4.4.4 PID Control Simulation

We used the GA to obtain the gains of altitude PID controller (K_p, K_i and K_D) with a desired value z_d of 2 m. The control parameters produced by the GA were $K_p = 0.0127$, $K_i = 0.000524$ and $K_D = 10$. The objective function utilized to assess the GA was minimizing the ITAE performance criteria. The process being tested was the obtained transfer function ($\frac{z(s)}{U_1(s)}$) in Equation (4.25) which have been found by using Linear Analysis tool (LAT) as shown in Figure 4-5 and this tool depends on selecting the system inputs (U_1, U_2, U_3 and U_4) and outputs ($\varphi, \theta, \psi, x, y$ and z) as shown Simulink block diagram (see Figure 4-4)

$$\frac{z(s)}{U_1(s)} = \frac{2.058}{s^2 + 0.001307s} \quad (4.25)$$

Correspondingly, we obtained the PID controller gains of attitude, heading and positions controllers using GA. Equations (4.26), (4.27) and (4.28) indicate the obtained transfer functions ($\frac{x(s)}{U_1(s)}, \frac{y(s)}{U_1(s)}, \frac{\varphi(s)}{U_2(s)}, \frac{\theta(s)}{U_3(s)}$ and $\frac{\psi(s)}{U_4(s)}$) that were tested with GA.

$$\frac{x(s)}{U_1(s)} = \frac{y(s)}{U_1(s)} = \frac{1.815e-8}{s^2 + 0.001145s} \quad (4.26)$$

$$\frac{\varphi(s)}{U_2(s)} = \frac{\theta(s)}{U_3(s)} = \frac{51.48}{s^2 + 1.318e06} \quad (4.27)$$

$$\frac{\psi(s)}{U_4(s)} = \frac{0.0001273}{s^2} \quad (4.28)$$

Also, Table 4-2 shows the optimal values of the objective functions that have been used to assess the GA approach of tuning the PID controller parameters and getting the optimal ones.

Table 4-2: Optimal Values of the Objective Functions as PID Controllers Applied

Optimal Objective Functions System States	IAE	ITAE	ISE	ITSE
Altitude (z)	0.05376	0.002353	0.02964	0.000582
Position (x and y)	1.010	0.5050	1.010	0.5050
Attitude (φ and θ)	1.0085	0.5042	1.0071	0.5034
Heading (ψ)	1.0077	0.5043	1.0055	0.5018

Table 4-3 represents the summary of obtained altitude, attitude, heading and positions PID controllers using GA. As a result of quadrotor symmetry, the Roll and Pitch controllers have the same gains of PID controller as shown in Table 4-3. Similarly, x and y position controllers are the same. Therefore, they will have the same parameters of PID controller as represented in Table 4-3.

Table 4-3: PID Controller Parameters with System States

System States	K_p	K_i	K_d
Altitude (z)	0.0127	0.000524	10
Position (x and y)	9.998	10	10
Attitude (φ and θ)	24.999	25	25
Heading (ψ)	25	24.998	25

Case Study 1: Step Inputs Trajectories (with PID)

We study both integer and fractional systems of quadrotor by applying PID control approach and setting the references or the inputs (x, y, z and ψ) as step inputs as shown in Table 4-1.

Table 4-4 shows the summary of the systems (integer/fractional ones) performance in terms of overshoot OS, and rising time RT when PID controllers applied and the step inputs stated as desired references.

Table 4-4: Integer and Fractional Systems Performance as PID Controllers Applied

System States	Desired Values	Integer System		Fractional System	
		OS%	RT (sec)	OS %	RT (sec)
Altitude (z)	2 m	0.486	0.194	0.505	0.189
Position (x and y)	1 m	9.341	0.158	9.341	0.154
Heading (ψ)	5°	2.234	0.167	2.234	0.167

(Figure 4-7, Figure 4-8 and Figure 4-9) show the responses of x and y positions and altitude states of both integer and fractional systems with the desired values (1 m, 1 m and 2 m) respectively. While, the responses of angle states of both integer and fractional systems are shown in (Figure 4-10, Figure 4-11 and Figure 4-12). Also, the fractional angle states (φ_f and θ_f) with ($\varphi_{df} = \theta_{df} = 0^\circ$) are shown in (Figure 4-13 and Figure 4-14).

The PID control inputs of both integer and fractional systems that controlled the quadrotor altitude, attitude, heading and positions are shown in (Figure 4-15, Figure 4-16, Figure 4-17 and Figure 4-18) and the PID virtual control inputs (U_x and U_{x_f} , U_y and U_{y_f}) of both integer and fractional systems are shown in (Figure 4-19 and Figure 4-20).

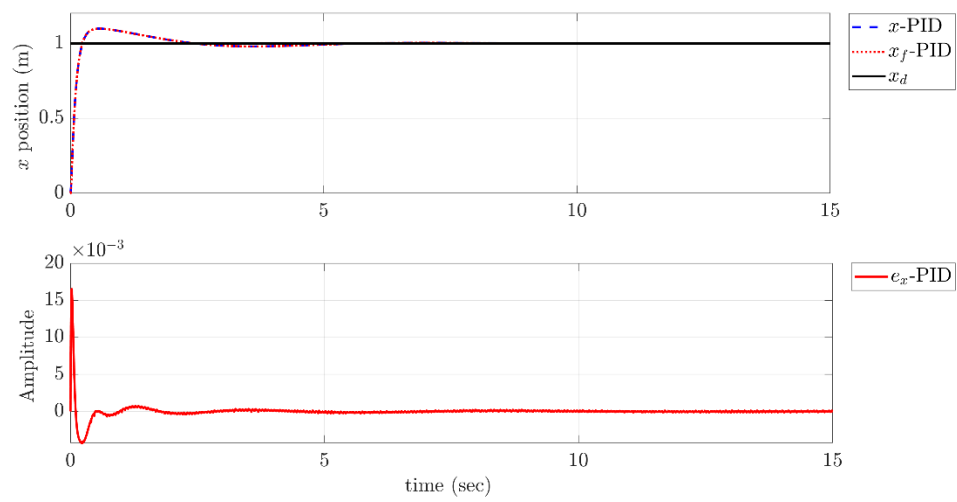


Figure 4-7: x and x_f Position States with Desired Reference (1 m) and the Error Between Them

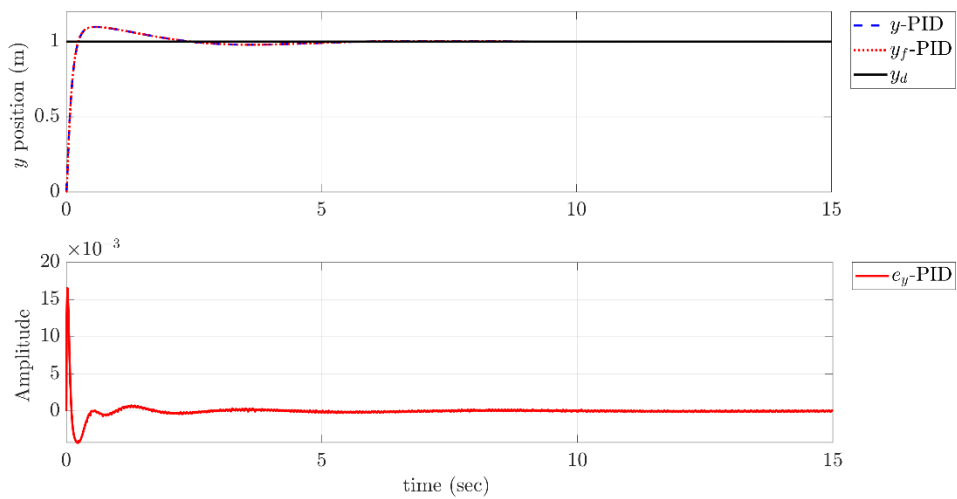


Figure 4-8: y and y_f Position States with Desired Reference (1 m) and the Error Between Them

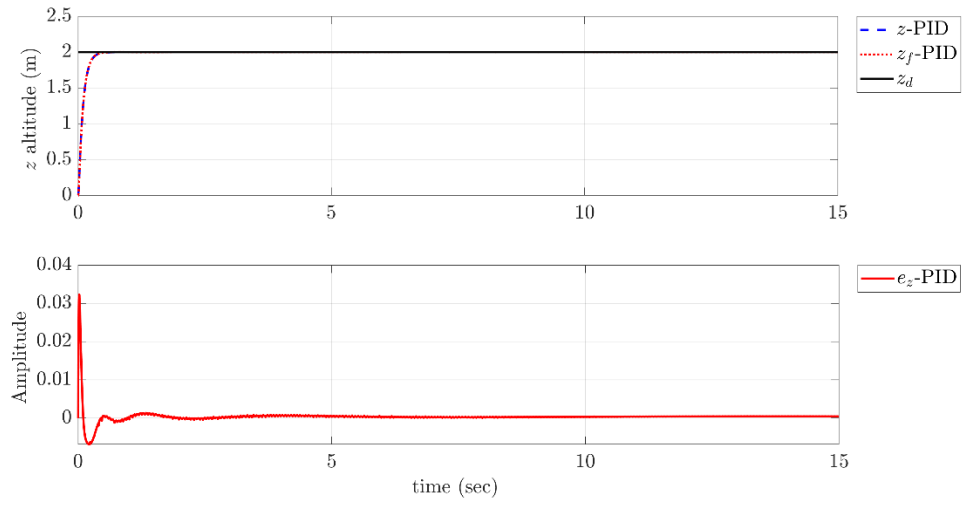


Figure 4-9: z and z_f Altitude States with Desired Reference (2 m) and the Error Between Them

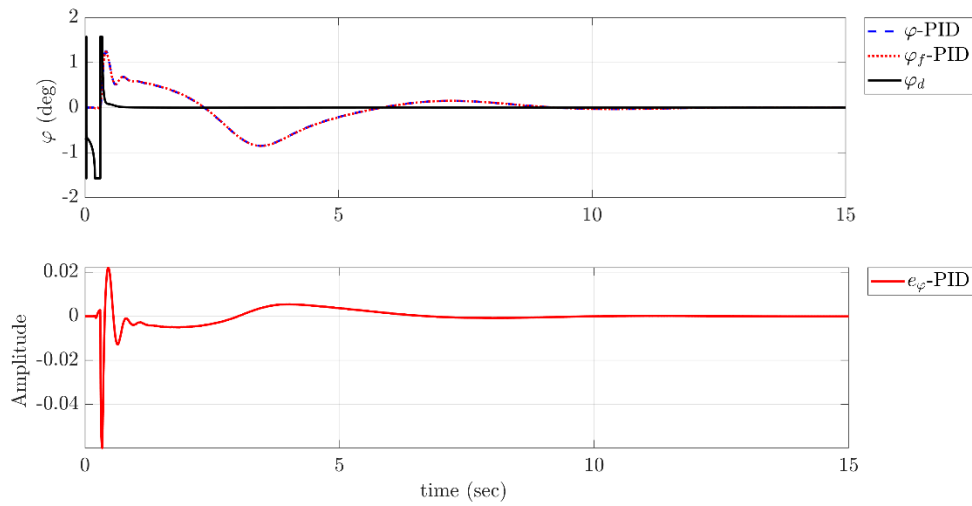


Figure 4-10: φ and φ_f Angle States with (0°) and the Error Between Them

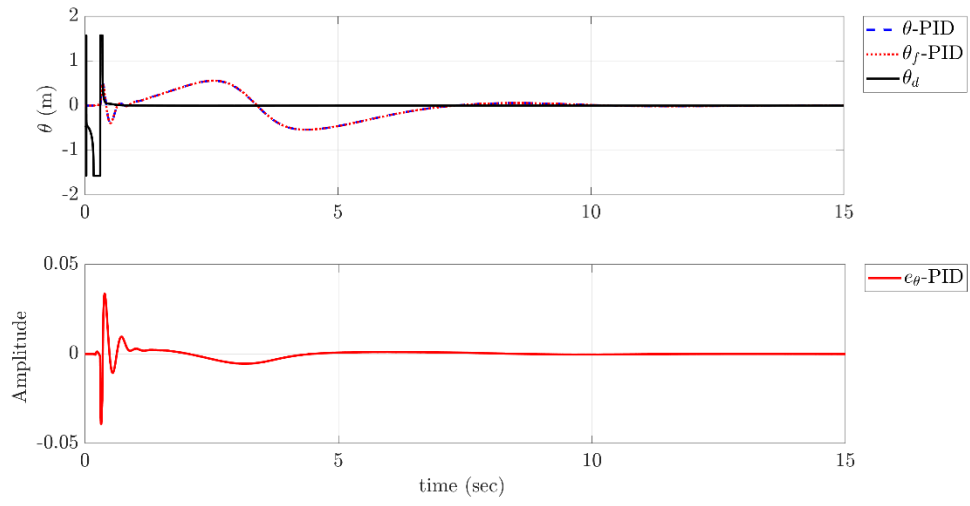


Figure 4-11: θ and θ_f Angle States with (0°) and the Error Between Them

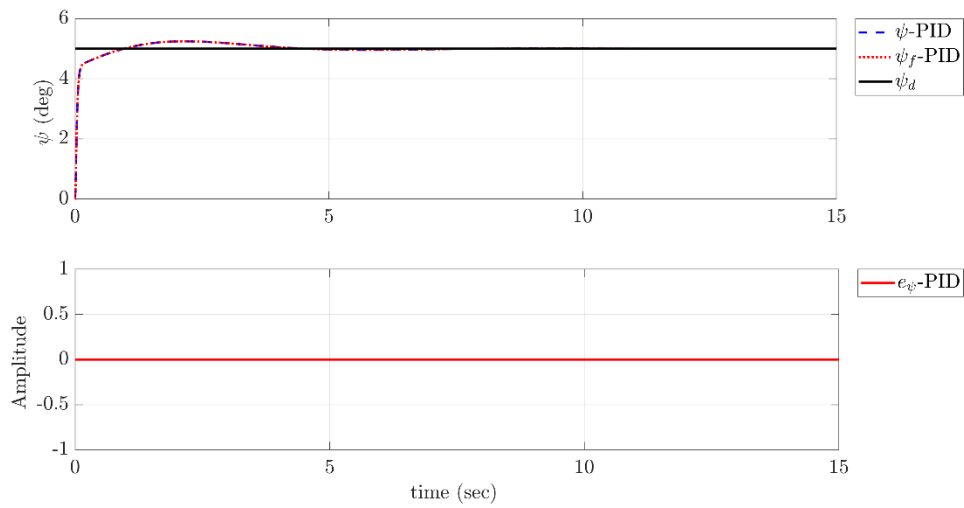


Figure 4-12: ψ and ψ_f Angle States with Desired Reference (5°) and the Error Between Them

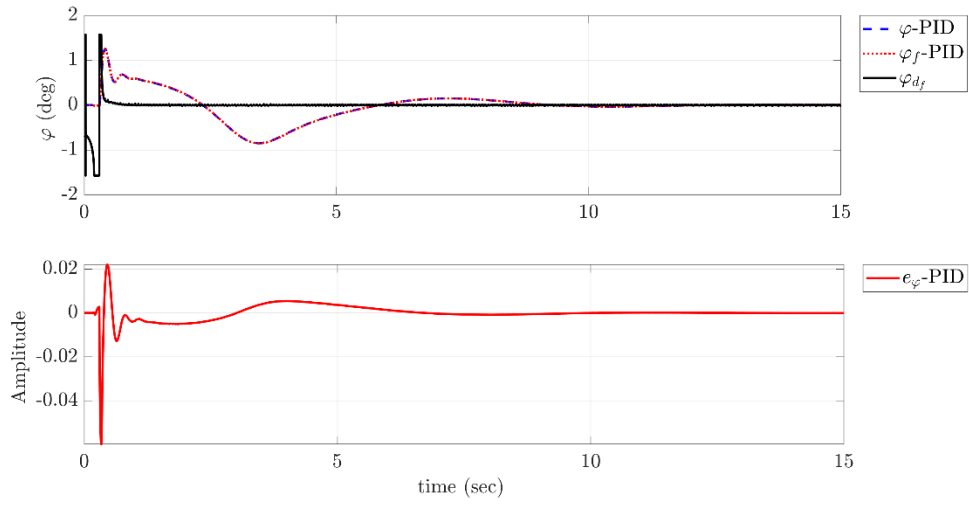


Figure 4-13: The fractional Roll Angle State (φ_f) with $\varphi_{fd} = 0^\circ$

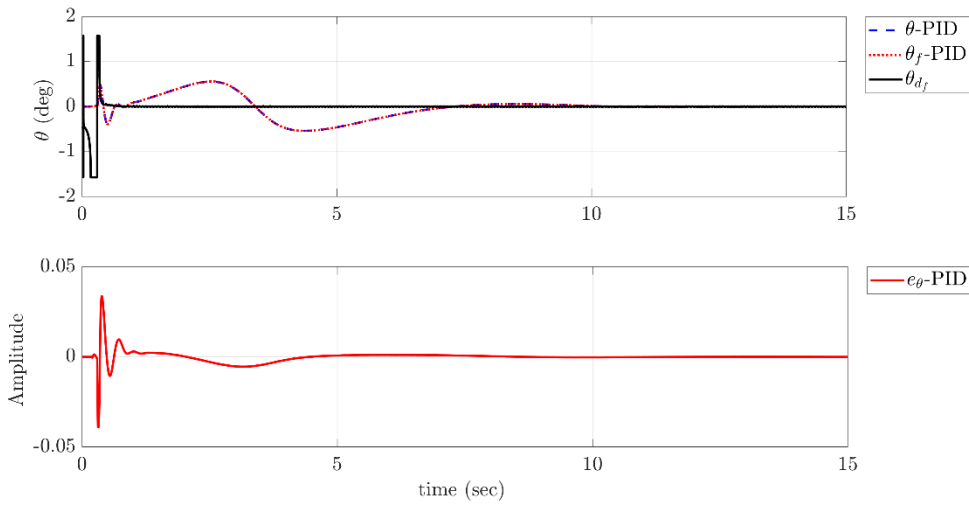


Figure 4-14: The fractional Pitch Angle State (θ_f) with $\theta_{fd} = 0^\circ$

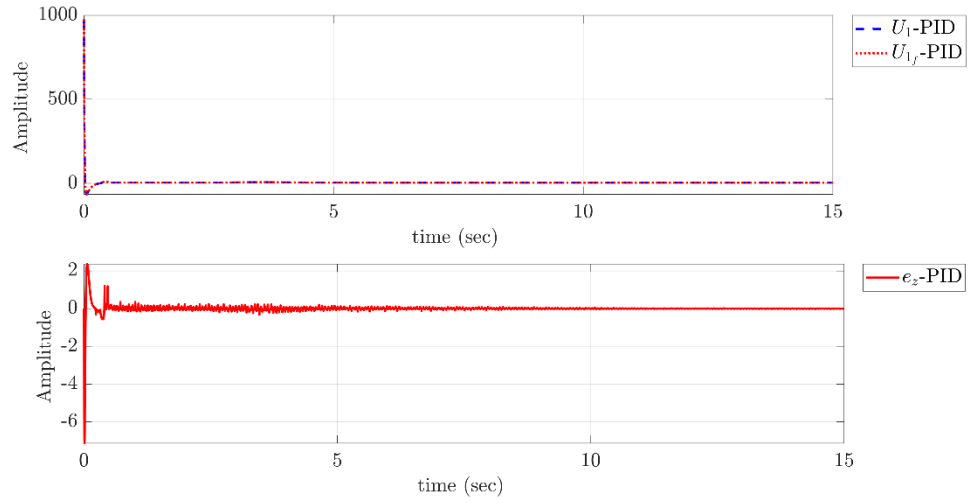


Figure 4-15: PID Altitude Control Inputs (U_1 and U_{1f}) of Integer and Fractional Systems and the Error Between Them

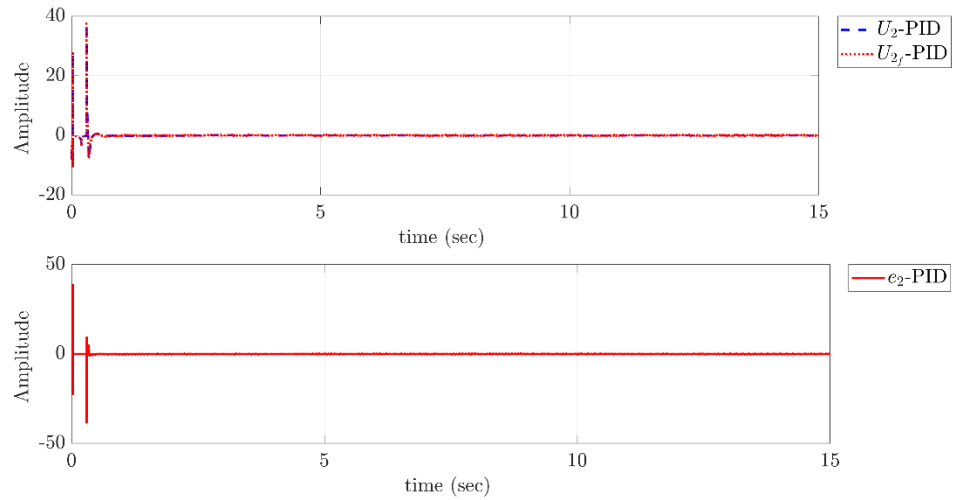


Figure 4-16: PID Attitude Control Input (U_2 and U_{2f}) of Roll Angle of Integer and Fractional Systems and the Error Between Them

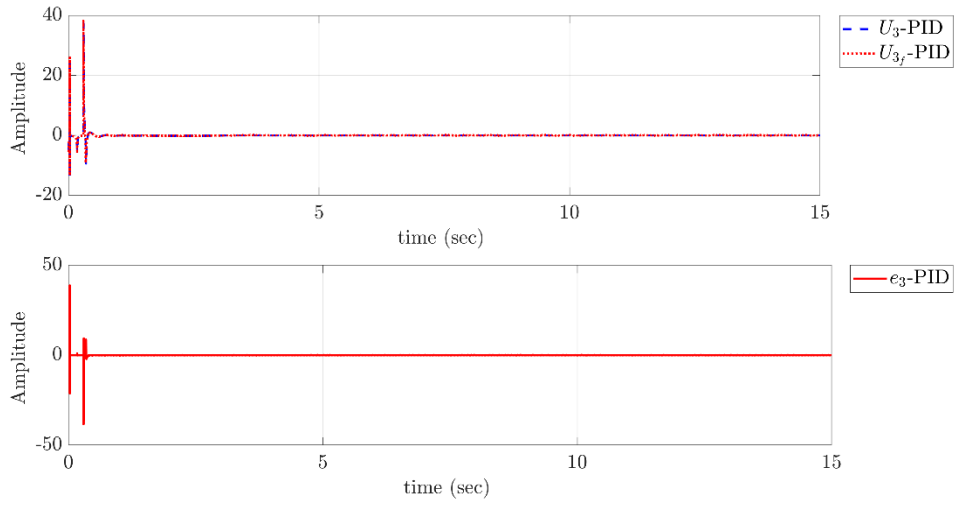


Figure 4-17: PID Attitude Control Input (U_3 and U_{3f}) of Pitch Angle of Integer and Fractional Systems and the Error Between Them

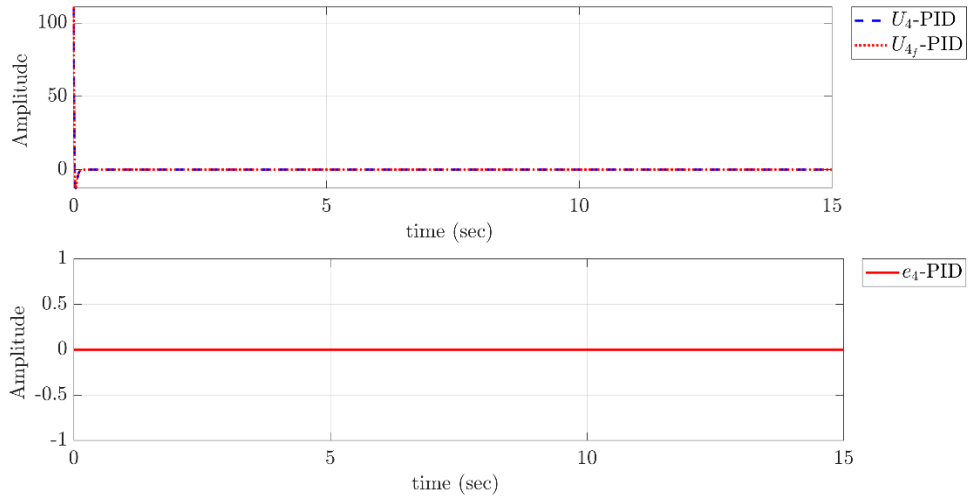


Figure 4-18: PID Attitude Control Input (U_4 and U_{4f}) of Yaw Angle of Integer and Fractional Systems and the Error Between Them

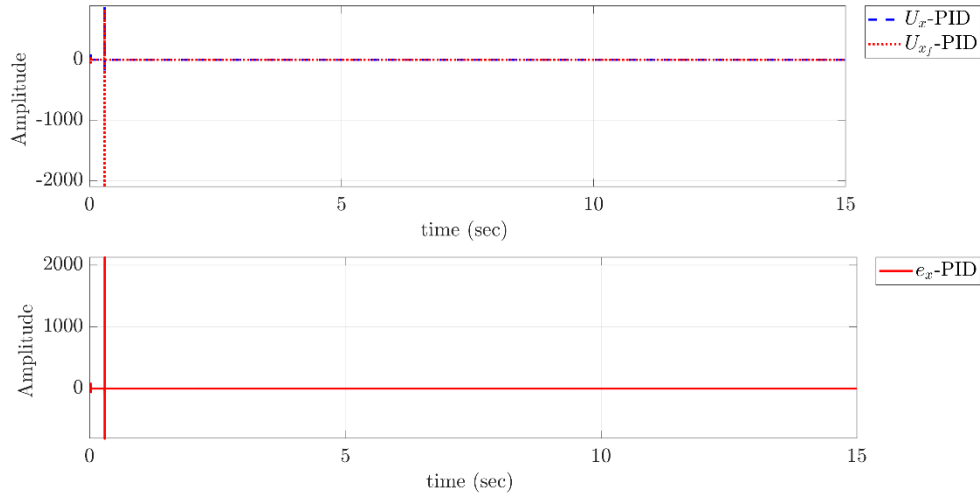


Figure 4-19: PID x Position Control Input (U_x and U_{x_f}) of Integer and Fractional Systems and the Error Between Them

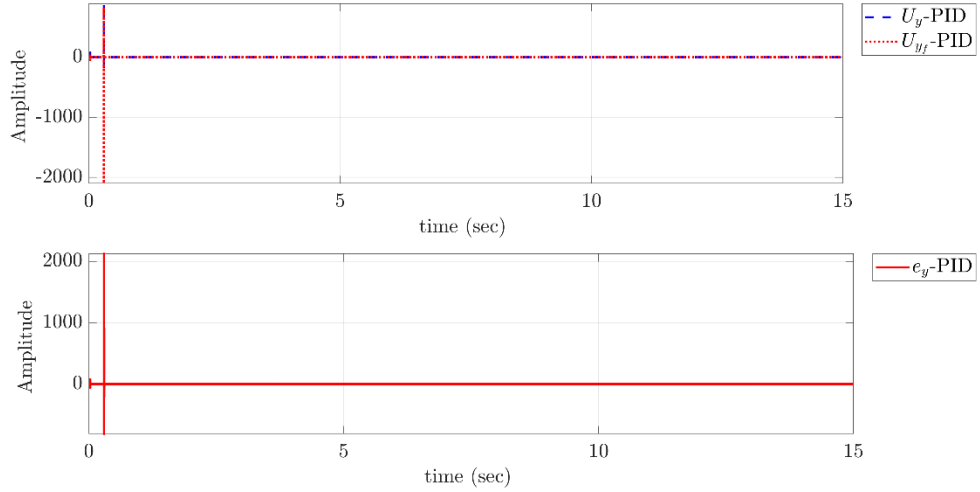


Figure 4-20: PID y Position Control Input (U_y and U_{y_f}) of Integer and Fractional Systems.

Observation:

It can be observed that all the fractional responses (x_f, y_f, z_f) have oscillations comparing with integer responses (x, y, z) because of applying the fractional scheme transformation on transitional subsystem which becomes a subsystem with memory effect as described in

[60]–[62]. However, the fractional transformation doesn't implement on rotational subsystem, the fractional responses of Roll and Pitch angles (φ_f, θ_f) have oscillations because they are affected by the virtual control inputs (U_x, U_y) which are used to obtain the desired Roll and Pitch angle (φ_d, θ_d) that control the quadrotor's positions. Also, we observed that $U_{1f}, U_{2f}, U_{3f}, U_{xf}$ and U_{yf} have oscillations due to fractional transformation, whereas U_{4f} doesn't has oscillations because it doesn't affect by the fractional transformation.

On the other hand, as observed in Table 4-4, the transient states of fractional responses (x_f, y_f, z_f) are improved in terms of rising time comparing with integer responses (x, y, z) . While the overshoots of (x_f, y_f) and (x, y) are the same and the overshoot of (z_f) comparing with (z) doesn't improved.

Case Study 2: Circular Shape Trajectory (With PID)

We move the quadrotor of both the integer and the factional systems along a circular path which was presented in Table 4-1 as second case study (Case 2) and it is shown in Figure 4-21 as 3D plot. The responses of integer and fractional angles of quadrotor are shown in Figure 4-22 and Figure 4-24. The Yaw angle (ψ) is set to be as a step function; however, the Roll and Pitch angles (φ, θ) follow their desired angles, that are provided by the outer loop which controls the quadrotor's position, because changing those angles changes the positions x and y. Also, the PID control inputs $(U_2$ and U_{2f}, U_3 and U_{3f}, U_4 and $U_{4f})$ that control the quadrotor angles of both integer and fractional systems are shown in separate figures (Figure 4-25, Figure 4-26 and Figure 4-27).

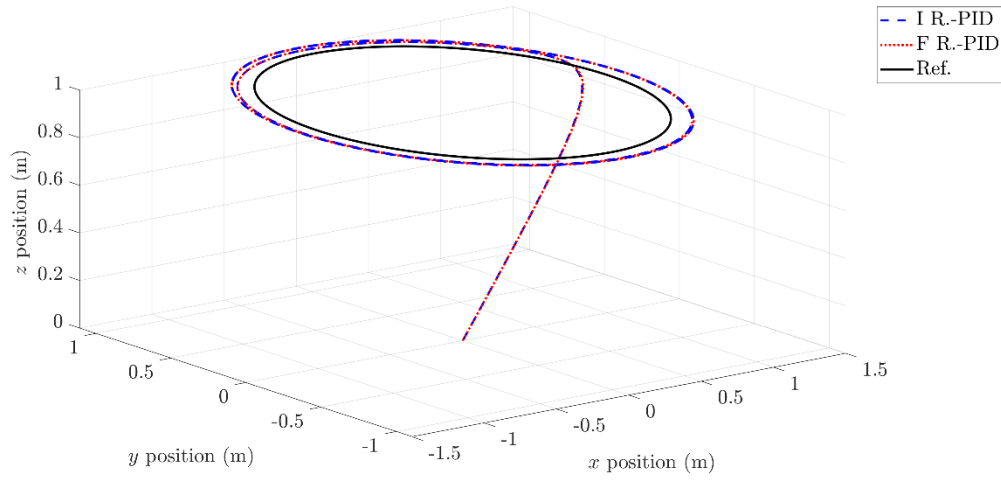


Figure 4-21: 3D Plot of Circular Trajectory with PID Controllers

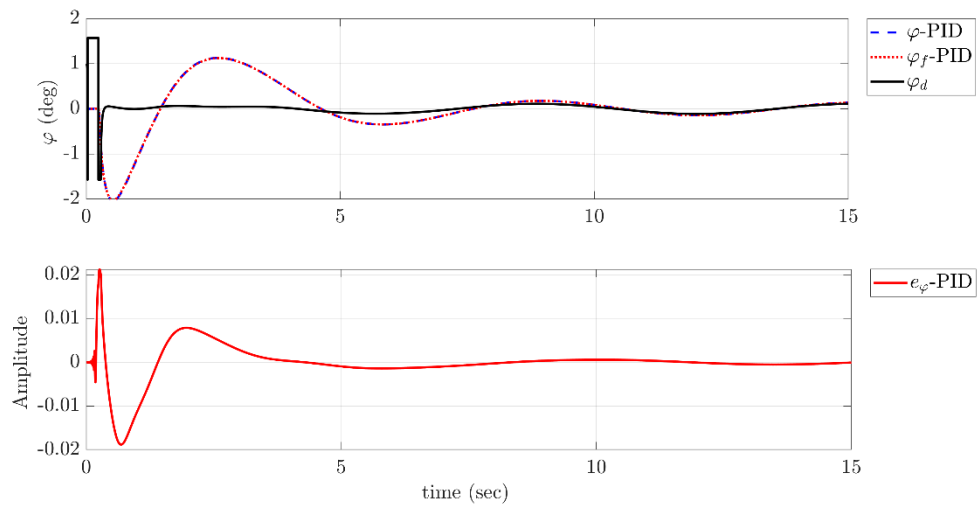


Figure 4-22: Roll Angle Responses (φ , φ_f) with PID Controllers and the Error Between Them

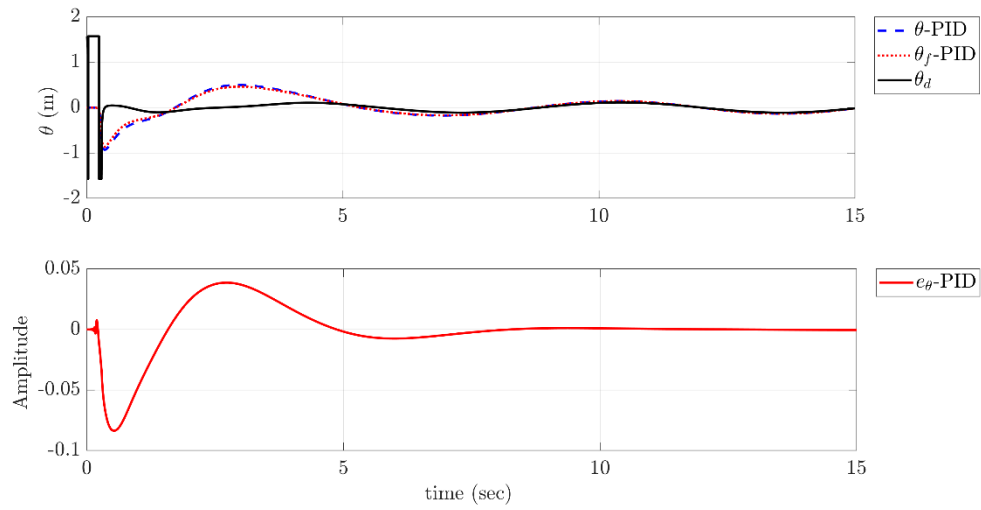


Figure 4-23: Pitch Angle Responses (θ , θ_f) with PID Controllers and the Error Between Them

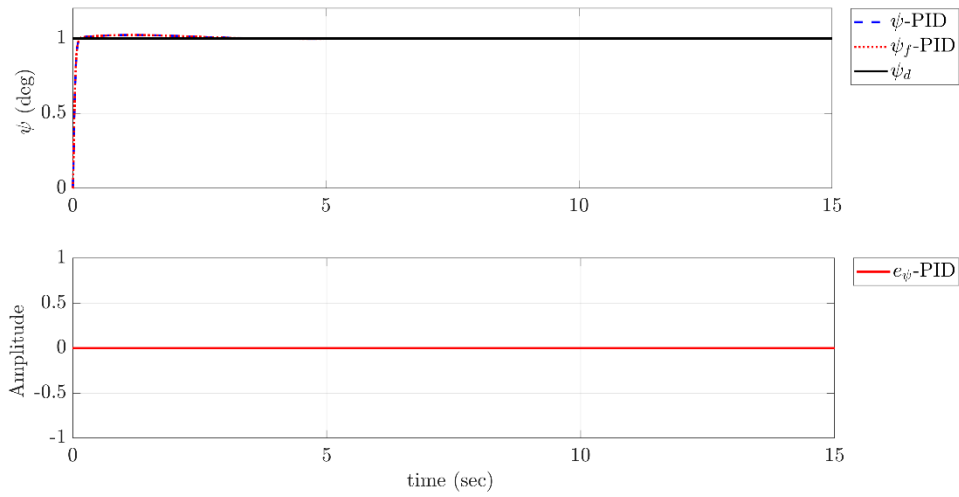


Figure 4-24: Yaw Angle Responses (ψ and ψ_f) (with 1°) with PID Controller and the Error Between Them

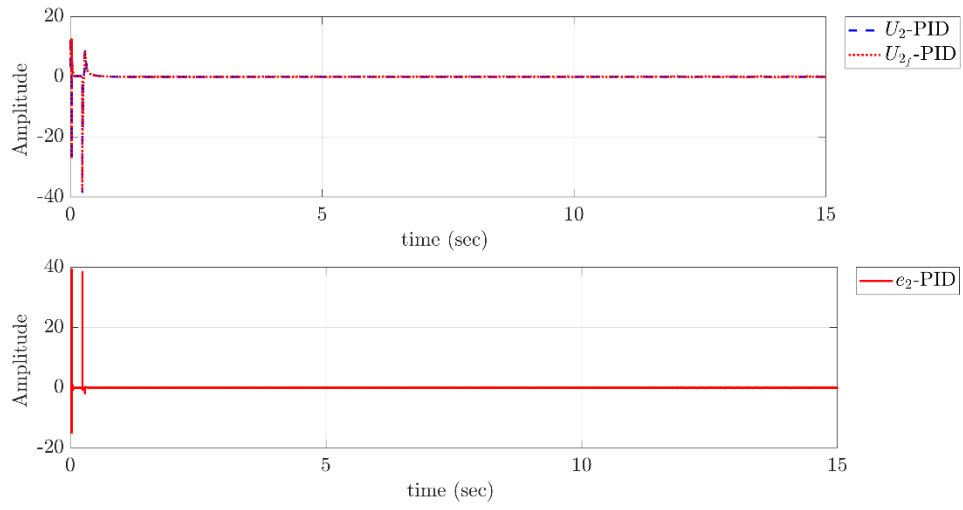


Figure 4-25: PID Control Inputs (U_2 and U_{2f}) and the Error Between Them

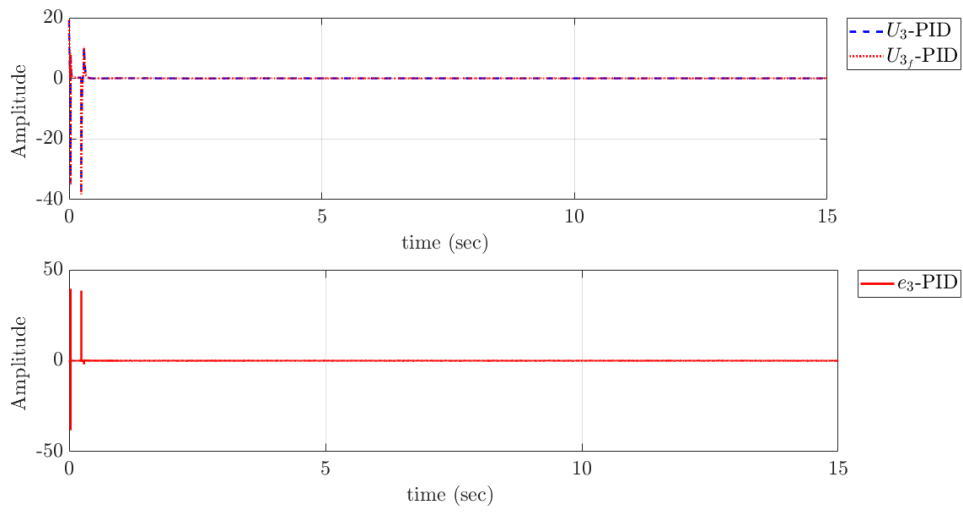


Figure 4-26: PID Control Inputs (U_3 and U_{3f}) and the Error Between Them

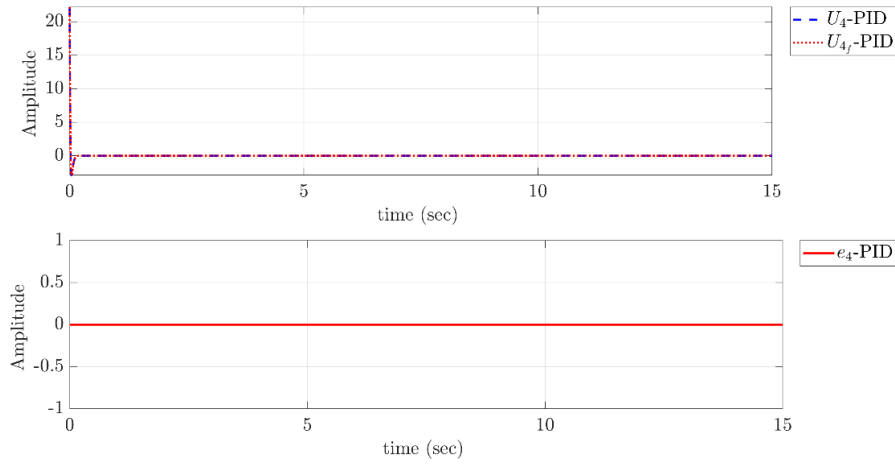


Figure 4-27: PID Control Inputs (U_4 and U_{4f}) and the Error Between Them

On the other hand, the responses of integer and fractional positions and altitudes of quadrotor are illustrated in separate figures (Figure 4-28, Figure 4-29 and Figure 4-30). Furthermore, the PID control and virtual inputs that control the quadrotor altitude and positions of both integer and fractional systems are shown in separate figures (Figure 4-31, Figure 4-32 and Figure 4-33).

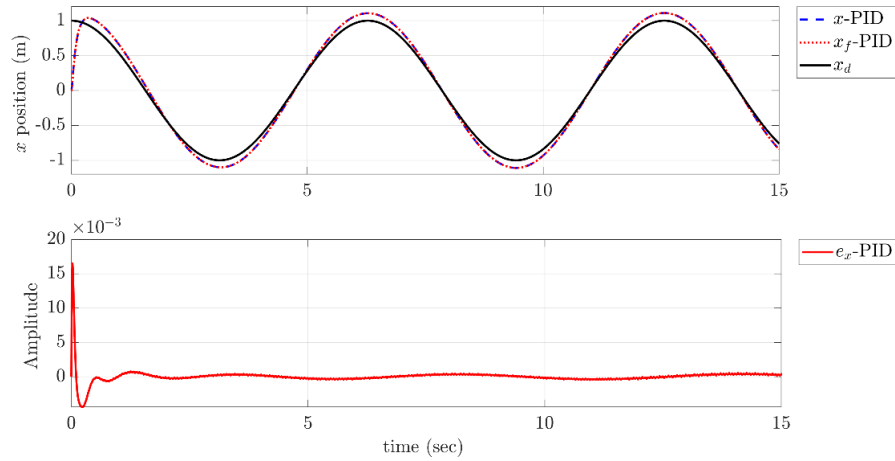


Figure 4-28: x Position Responses (x and x_f) with PID Controller and the Error Between Them

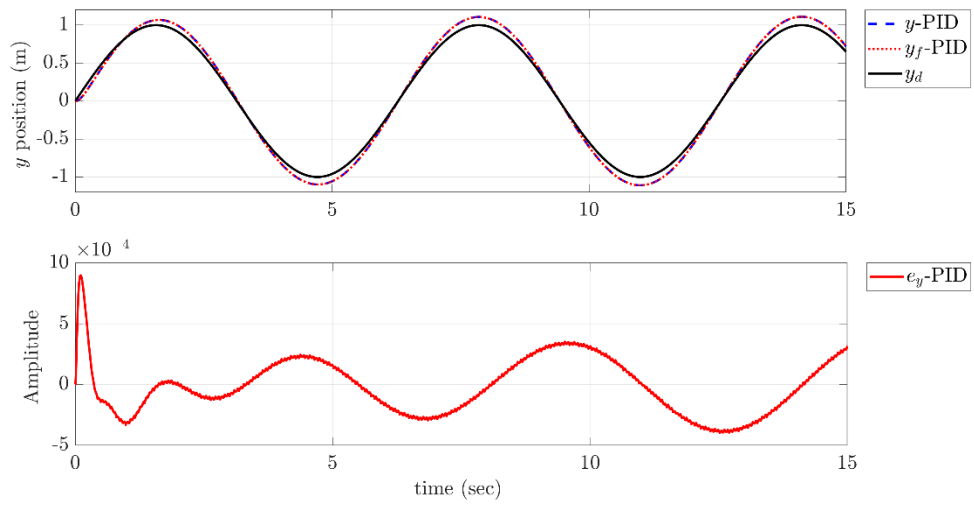


Figure 4-29: y Position Responses (y and y_f) with PID Controller and the Error Between Them

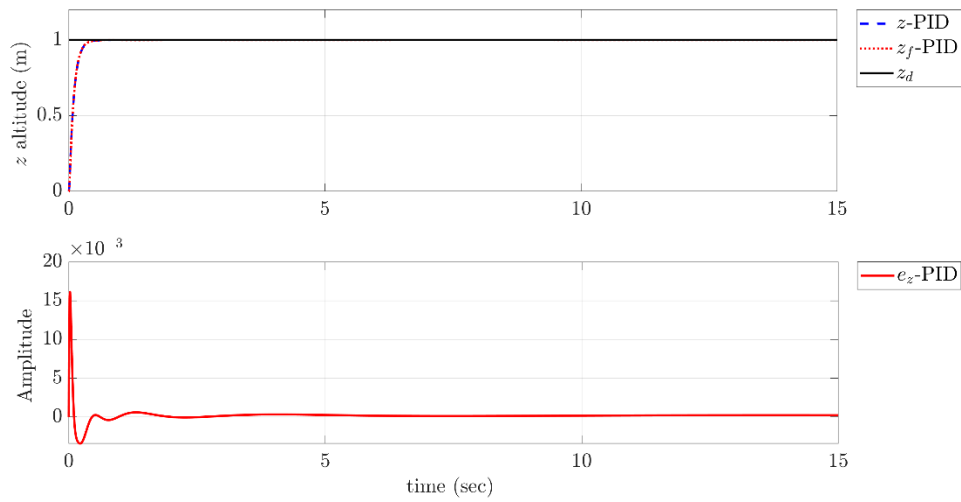


Figure 4-30: z Altitude Responses (z and z_f) with PID Controller and the Error Between Them

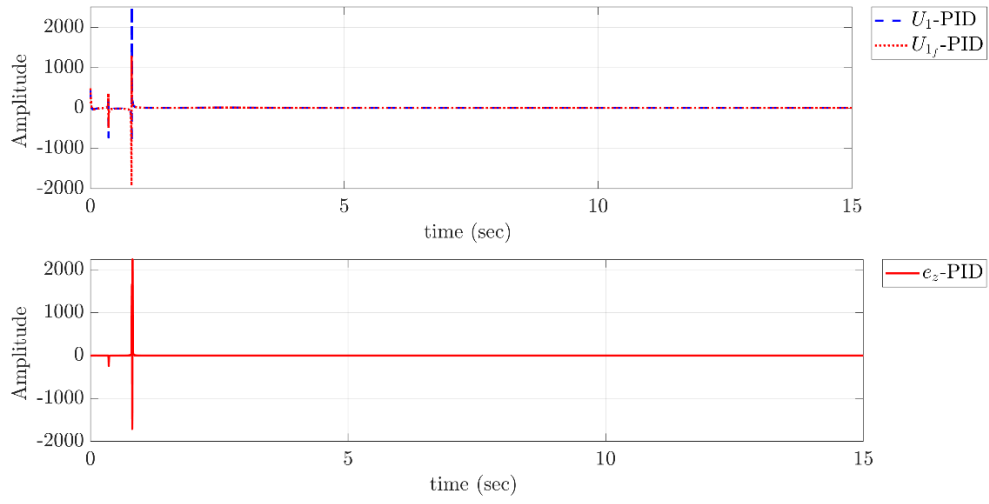


Figure 4-31: PID Control Inputs (U_1 and U_{1f}) and the Error Between Them

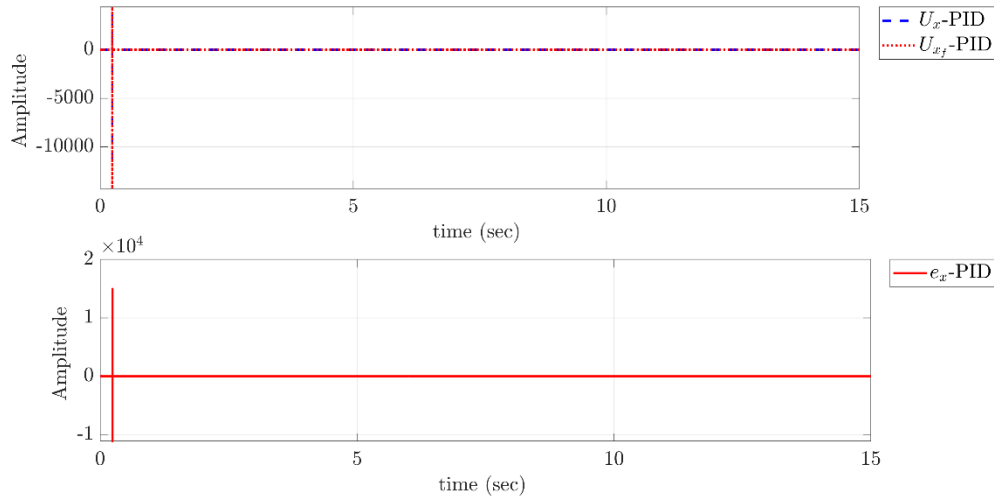


Figure 4-32: PID Virtual Control Inputs (U_x and U_{xf}) and the Error Between Them

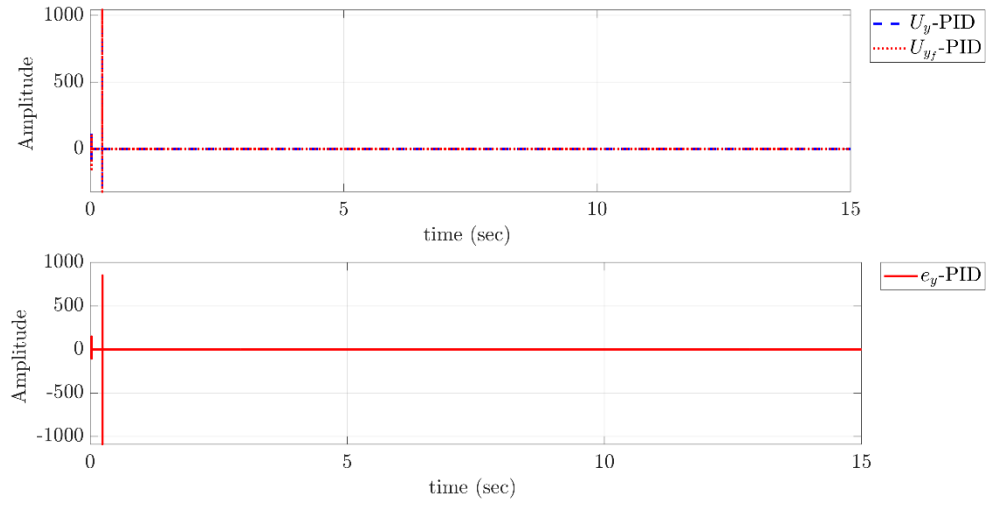


Figure 4-33: PID Virtual Control Inputs (U_y and U_{yf}) and the Error Between Them

Helical Shape Trajectory (With PID)

We move the quadrotor of both the integer and the fractional systems along a helical path (Case 3) and it is as shown as 3D plot in Figure 4-34. The responses of integer and fractional angles, positions and altitude states of quadrotor are shown respectively in separate figures (Figure 4-35, Figure 4-37, Figure 4-38, Figure 4-39 and Figure 4-40).

Also, the PID control and virtual inputs are shown respectively in separate figures (Figure 4-41, Figure 4-42, Figure 4-43, Figure 4-44, Figure 4-45 and Figure 4-46).

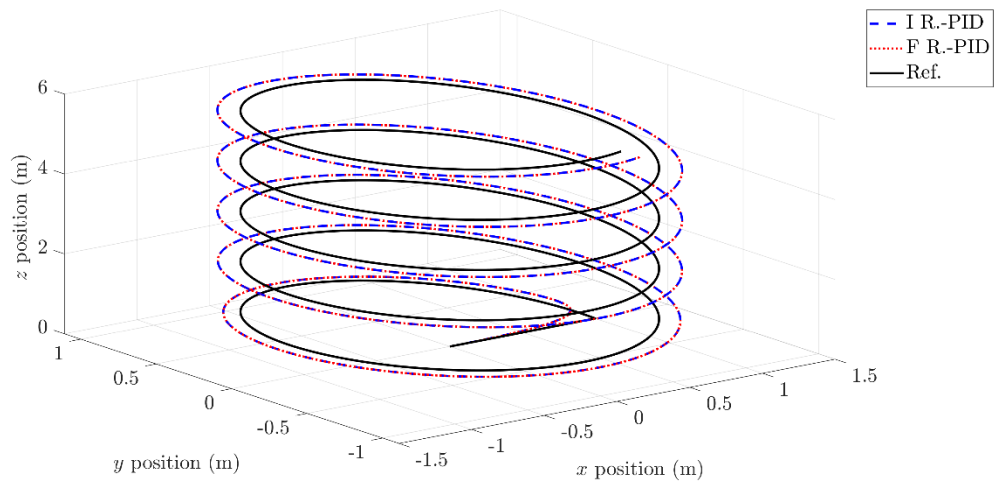


Figure 4-34: 3D Plot of Helical Trajectory with PID Controllers

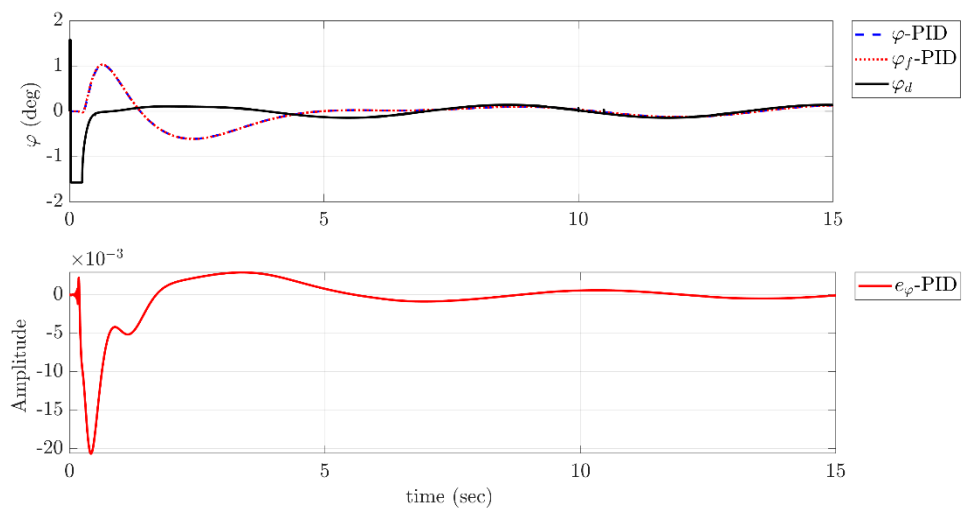


Figure 4-35: Roll Angle Responses (φ, φ_f) with PID Controllers and the Error Between Them

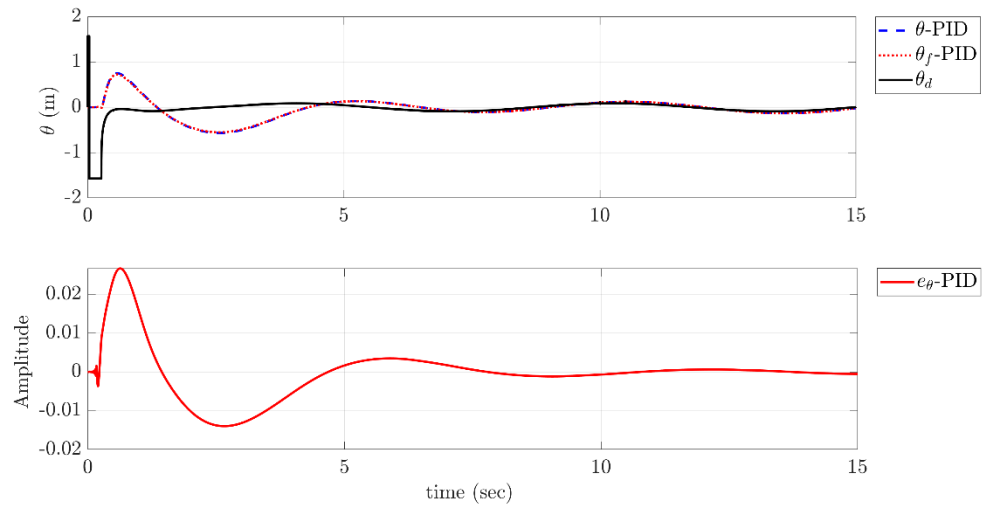


Figure 4-36: Pitch Angle Responses (θ , θ_f) with PID Controllers and the Error Between Them

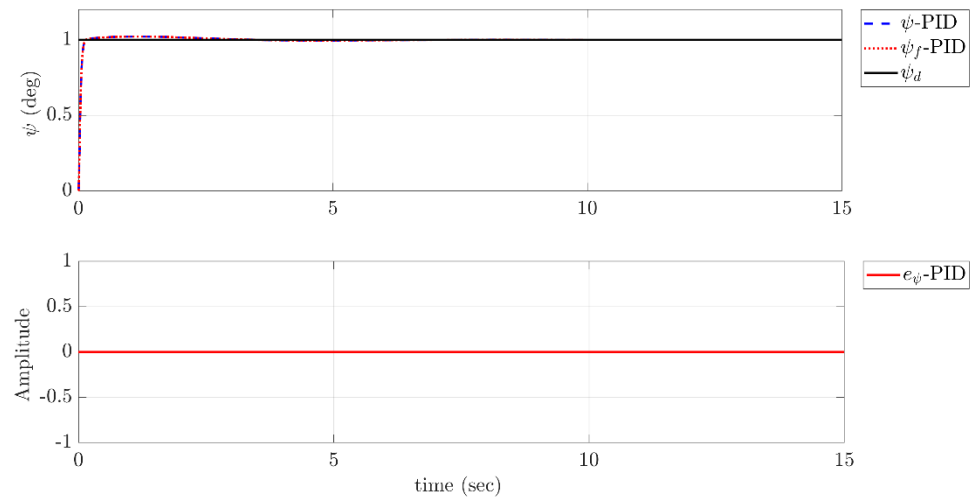


Figure 4-37: Yaw Responses (ψ , ψ_f) (with 1°) with PID Controllers and the Error Between Them

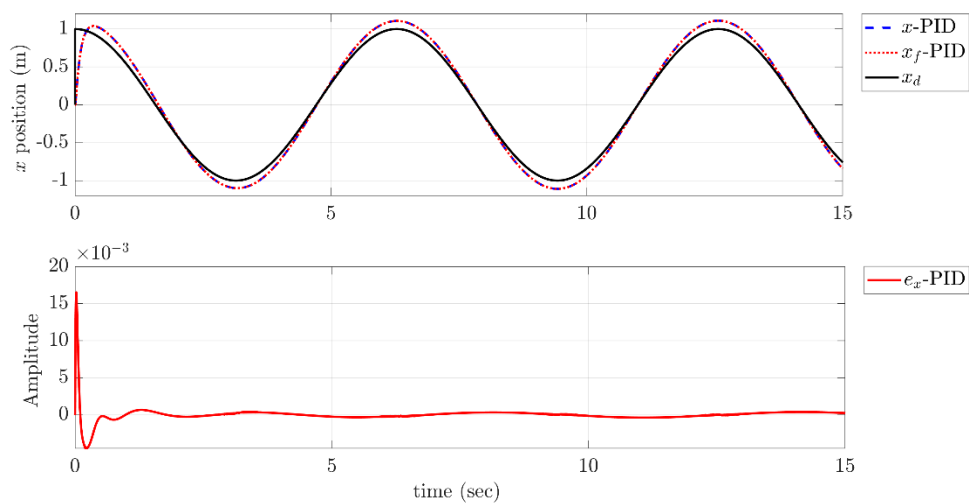


Figure 4-38: x Position Responses (x and x_f) with PID Controller and the Error Between Them

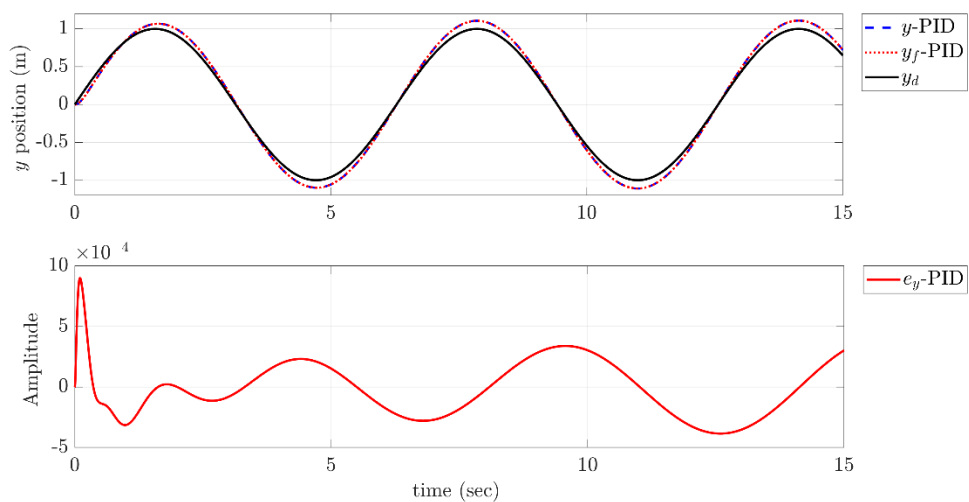


Figure 4-39: y Position Responses (y and y_f) with PID Controller and the Error Between Them

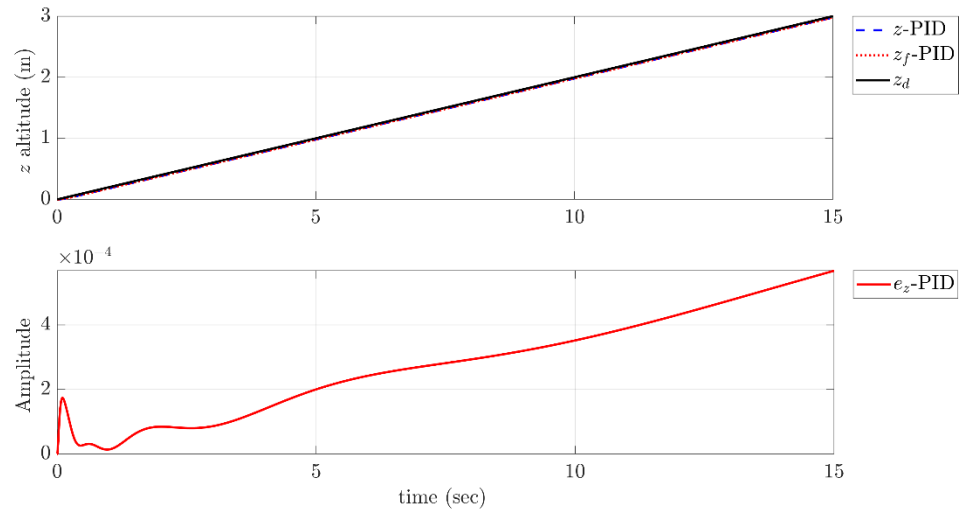


Figure 4-40: z Position Responses (z and z_f) with PID Controller and the Error Between Them

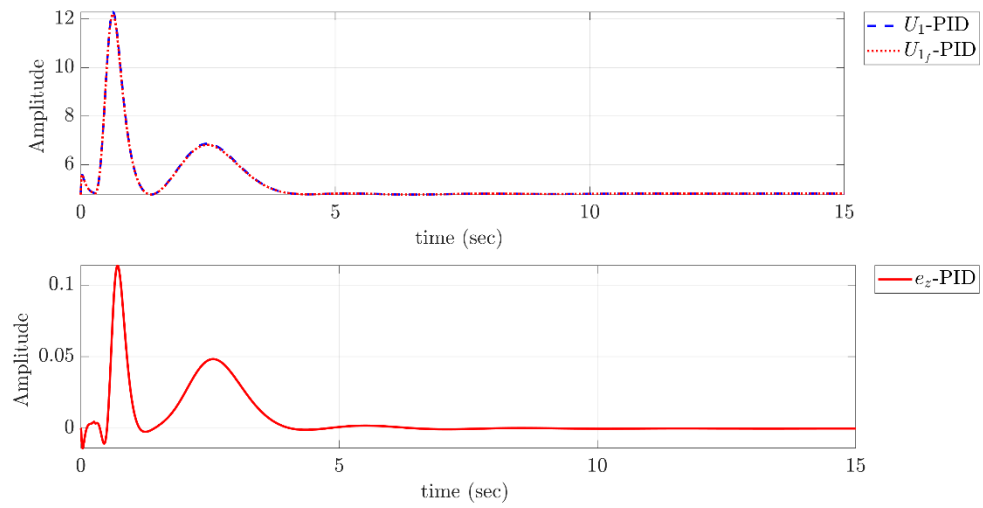


Figure 4-41: PID Control Inputs (U_1 and U_{1f}) and the Error Between Them

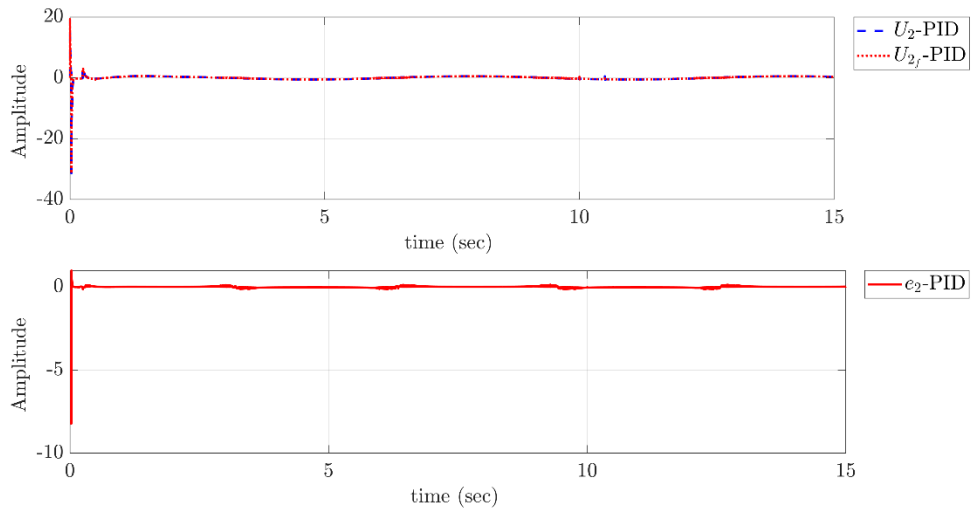


Figure 4-42: PID Control Inputs (U_2 and U_{2f}) and the Error Between Them

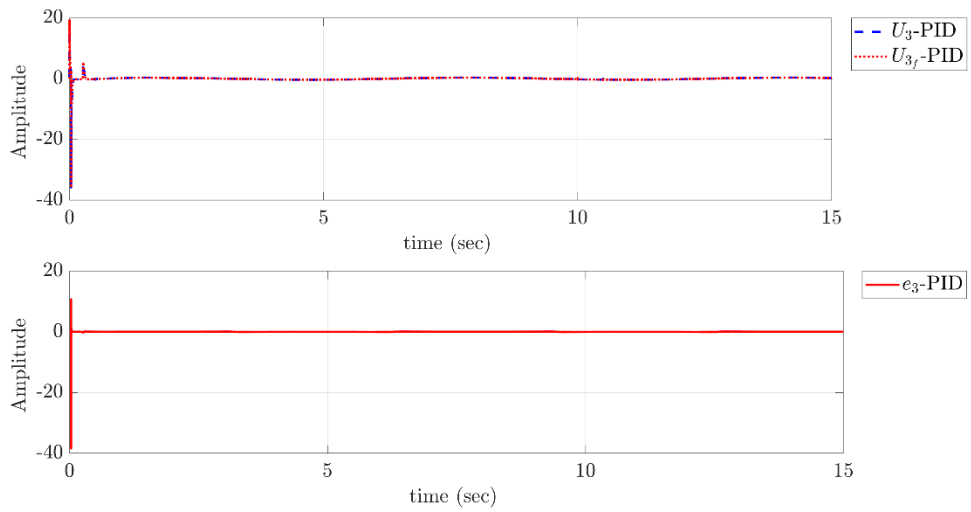


Figure 4-43: PID Control Inputs (U_3 and U_{3f}) and the Error Between Them

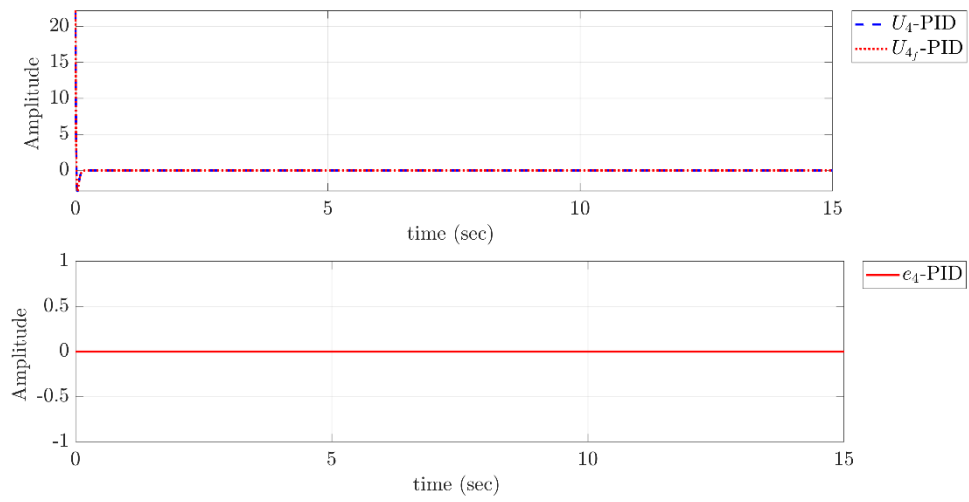


Figure 4-44: PID Control Inputs (U_4 and U_{4f}) and the Error Between Them

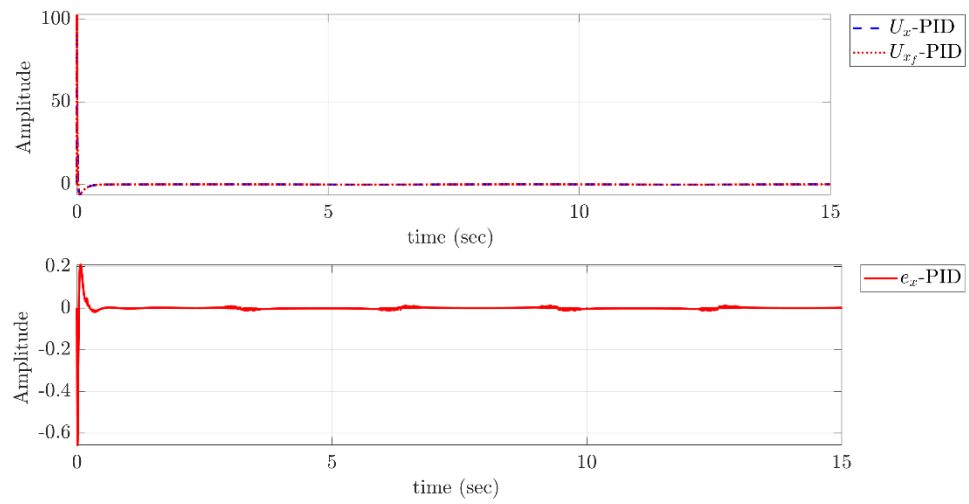


Figure 4-45: PID Virtual Control Inputs (U_x and U_{xf}) and the Error Between Them

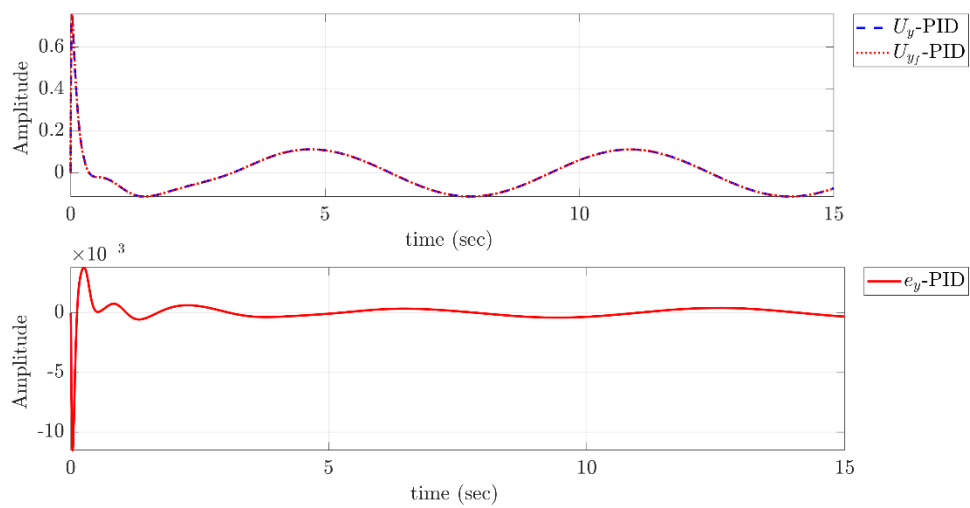


Figure 4-46: PID Virtual Control Inputs (U_y and U_{yf}) and the Error Between Them

4.4.5 PID Results and Discussion

The proposed PID controllers have been tested and simulated using MATLAB/SIMULINK software. FOMCON Toolbox has been used for the fractional calculus.

The main objective of implementing PID controller is to stabilize the UAV of both systems (integer and fractional ones) as well as follows a desired reference with better performance.

We observed that the stated PID controllers stabilized both the integer and the fractional systems of UAV and followed the desired references as shown in above figures.

Because of transforming the translational equations to fractional ones and keeping the rotational equations as integer ones, the simulated and obtained results of applying step inputs as desired references show that the fractional position states (x_f, y_f and z_f) are improved in terms of (Rising Time) comparing with integer position states (x, y and z), while the overshoots of (x and x_f) and (y and y_f) are the same and the overshoot of (z and z_f) is not improved as shown in Table 4-4.

Also, the obtained results of driving the quadrotor along circular and helical trajectories reveal that the quadrotor of both integer and fractional systems doesn't really follow the path well and also, they have poor transient and steady states.

Furthermore, we employed the GA technique to tune and obtain the optimal parameters of the proposed PID controllers, and the objective functions used to evaluate the GA were minimizing IAE, ITAE, ISE, and ITSE, as shown in Table 4-2. When compared to the others, the ITSE was the best.

4.5 FOPID Control Design

4.5.1 Introduction

In this section, we will implement a Fractional Order PID (FOPID) controller for both integer and fractional systems. Figure 4-47 shows the block diagram of the FOPID.

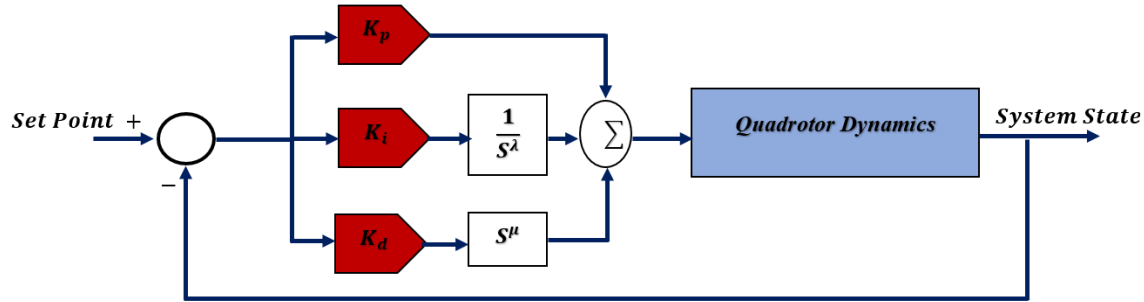


Figure 4-47: FOPID Block Diagram

PID Controllers are improved in a fractional order technique of form as $PI^\lambda D^\mu$ controllers.

The most typical structure of the fractional order PID (FOPID) can be stated as:

$$P(s) = K_p + K_I \frac{1}{s^\lambda} + K_D s^\mu \quad (4.29)$$

Where

K_p Proportional Gain

K_I Integral Gain

K_D Derivative Gain

λ Non-integer Order of Integral Term

μ Non-integer Order of Derivative Term

FOPID controller will be an integer order PID controller when λ and $\mu = 1$.

Furthermore, it can be observed that the FOPID controller adjusted by tuning five

parameters (K_p, K_I, K_D, λ and μ) comparing with integer order PID which regulates by three parameters (K_p, K_I, K_D). This develops FOPID controller from point to plane - as shown in Figure 4-48 - and adding more accuracy and flexibility to the controller design.

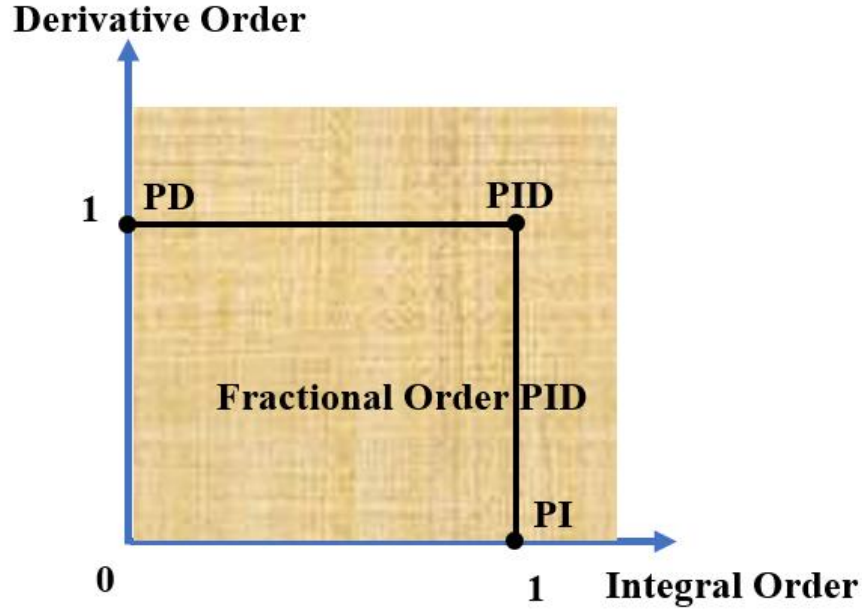


Figure 4-48: Expansion of FOPID from Point to Plane

4.5.2 FOPID Control Design of Integer System

The same procedure that we have done with integer order PID in integer system as shown in Equations (4.5), (4.6), (4.7) and (4.8), we will design four FOPID control inputs ($U_{1FOPID}, U_{2FOPID}, U_{3FOPID}$ and U_{4FOPID}) that govern the altitude, orientation of the quadrotor with two more parameters (λ and μ) as shown below in Equations (4.30), (4.31), (4.32) and (4.33). Also virtual FOPID control inputs (U_{xFOPID}, U_{yFOPID}) that control the quadrotor position are shown in Equations (4.34) and (4.35).

$$U_{1FOPID} = \frac{m}{c\varphi c\theta} \left[\left(K_p e_z + K_i (I^\lambda(e_z)) + K_d (D^\mu(e_z)) \right) + g \right] \quad (4.30)$$

$$U_{2FOPID} = I_{xx} \left[\left(K_p e_\varphi + K_i (I^\lambda(e_\varphi)) + K_d (D^\mu(e_\varphi)) \right) \right] \quad (4.31)$$

$$U_{3FOPID} = I_{yy} \left[\left(K_p e_\theta + K_i (I^\lambda(e_\theta)) + K_d (D^\mu(e_\theta)) \right) \right] \quad (4.32)$$

$$U_{4FOPID} = I_{zz} \left[\left(K_p e_\psi + K_i (I^\lambda(e_\psi)) + K_d (D^\mu(e_\psi)) \right) \right] \quad (4.33)$$

$$U_{x_{FOPID}} = \frac{m}{U_{1FOPID}} \left[\left(K_p e_x + K_i (I^\lambda(e_x)) + K_d (D^\mu(e_x)) \right) \right] \quad (4.34)$$

$$U_{y_{FOPID}} = \frac{m}{U_{1FOPID}} \left[\left(K_p e_y + K_i (I^\lambda(e_y)) + K_d (D^\mu(e_y)) \right) \right] \quad (4.35)$$

where,

I^λ Fractional Integral with order λ

D^μ Fractional derivative with order μ

4.5.3 FOPID Control Design of Fractional System

Using the same technique as stated in Equations (4.19), (4.20), (4.21) and (4.22) for integer order PID in fractional systems, we constructed four FOPID control inputs that regulate the quadrotor's altitude, attitude, and heading with two additional parameters (λ and μ) as shown below in Equations (4.36), (4.37), (4.38) and (4.39). Also, Equations (4.40) and (4.41) indicate the virtual FOPID control inputs that govern the quadrotor position.

$$U_{1_f FOPID} = \frac{m}{c\varphi c\theta} \left[\left(K_p e_{z_f} + K_i (I^\lambda(e_{z_f})) + K_d (D^\mu(e_{z_f})) \right) + g \right] \quad (4.36)$$

$$U_{2_f FOPID} = I_{xx} \left[\left(K_p e_{\varphi_f} + K_i (I^\lambda(e_{\varphi_f})) + K_d (D^\mu(e_{\varphi_f})) \right) \right] \quad (4.37)$$

$$U_{3f_{FOPID}} = I_{yy} \left[\left(K_p e_{\theta_f} + K_i \left(I^\lambda(e_{\theta_f}) \right) + K_d(D^\mu(e_{\theta_f})) \right) \right] \quad (4.38)$$

$$U_{4f_{FOPID}} = I_{zz} \left[\left(K_p e_{\psi_f} + K_i \left(I^\lambda(e_{\psi_f}) \right) + K_d(D^\mu(e_{\psi_f})) \right) \right] \quad (4.39)$$

$$U_{x_f_{FOPID}} = \frac{m}{U_{1f_{FOPID}}} \left[\left(K_p e_{x_f} + K_i \left(I^\lambda(e_{x_f}) \right) + K_d(D^\mu(e_{x_f})) \right) \right] \quad (4.40)$$

$$U_{y_f_{FOPID}} = \frac{m}{U_{1f_{FOPID}}} \left[\left(K_p e_{y_f} + K_i \left(I^\lambda(e_{y_f}) \right) + K_d(D^\mu(e_{y_f})) \right) \right] \quad (4.41)$$

4.5.4 FOPID Control Simulation

We used the GA approach to obtain the optimal parameters (K_p, K_i, λ, K_d and μ) of FOPID controllers. The objective function was minimizing the ITAE performance criteria. The processes being tested were the transfer functions in Equations (4.25), (4.26), (4.27) and (4.28).

Table 4-5 shows the optimal values of objective functions that have been used to assess the GA approach of tuning the FOPID controller parameters and obtaining the optimal ones.

Table 4-5: Optimal Values of the Objective Functions as FOPID Controllers Applied

Optimal Objective Functions System States	IAE	ITAE	ISE	ITSE
Altitude (z)	0.06083	0.005175	0.03344	0.000761
Position (x and y)	1.010	0.5050	1.010	0.5050
Attitude (φ and θ)	1.0072	0.5037	1.0044	0.5024
Heading (ψ)	1.0076	0.5033	1.0052	0.5016

Case Study 1: Step Inputs Trajectories (with FOPID)

We study both integer and fractional systems of quadrotor by applying FOPID control technique and setting the references or the inputs (x, y, z and ψ).

Table 4-6 represents the summary of obtained altitude, attitude, heading and positions PID controllers using GA.

Table 4-7 shows the summary of the systems (integer/fractional ones) performance in terms of overshoot OS, and rising time RT with FOPID controllers and step inputs as desired references.

Table 4-6: FOPID Controller Parameters with System States

System States	K_p	K_i	λ	K_d	μ
Altitude (z)	3.27e-05	0.00294	0.9800	10	0.9800
Position (x and y)	9.9999	9.9865	0.5075	9.9998	0.9787
Attitude (φ and θ)	24.9976	25	0.5001	24.9989	0.5006
Heading (ψ)	24.9992	24.9945	0.5009	24.9999	0.9800

Table 4-7: Integer and Fractional Systems Performance as FOPID Controllers Applied

System States	Desired Values	Integer System		Fractional System	
		OS%	RT (sec)	OS %	RT (sec)
Altitude (z)	2 m	0.505	0.213	0.505	0.209
Position (x and y)	1 m	11.80	0.166	11.80	0.162
Heading (ψ)	5°	0.639	0.151	0.639	0.151

(Figure 4-49, Figure 4-50 and Figure 4-51) show the responses of x and y positions and altitude states of both integer and fractional systems with the desired values (1 m, 1 m and 2 m) respectively with FOPID controllers. Whereas, the responses of angle states of both integer and fractional systems with FOPID controllers are shown in (Figure 4-52, Figure 4-53 and Figure 4-54). Also, the fractional angle states (φ_f and θ_f) with (0°) with FOPID controllers are shown in (Figure 4-55 and Figure 4-56).

The FOPID control inputs of both integer and fractional systems that controlled the altitude, attitude and heading of the quadrotor are shown in (Figure 4-57, Figure 4-58, Figure 4-59 and Figure 4-60). Also, the FOPID virtual control inputs of both integer and fractional systems are shown in (Figure 4-61 and Figure 4-62)

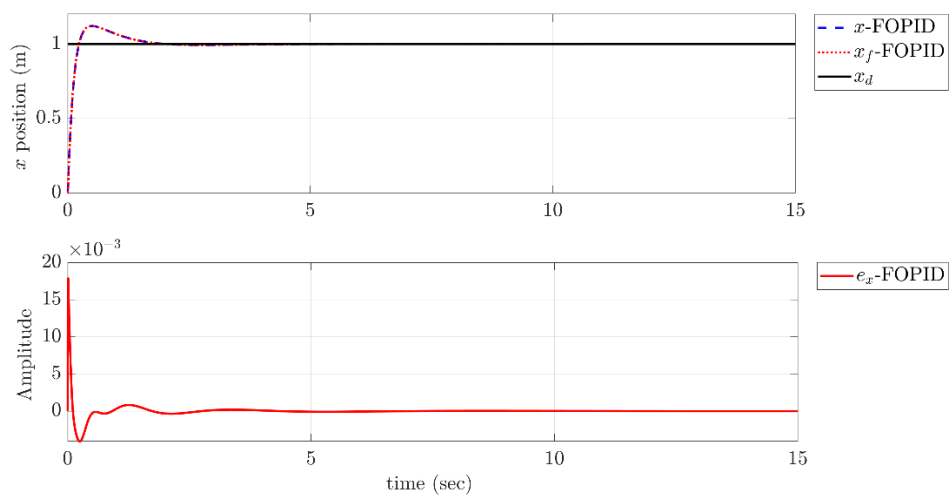


Figure 4-49: x and x_f Position States with Desired Reference (1 m) with FOPID Controller and the Error Between Them

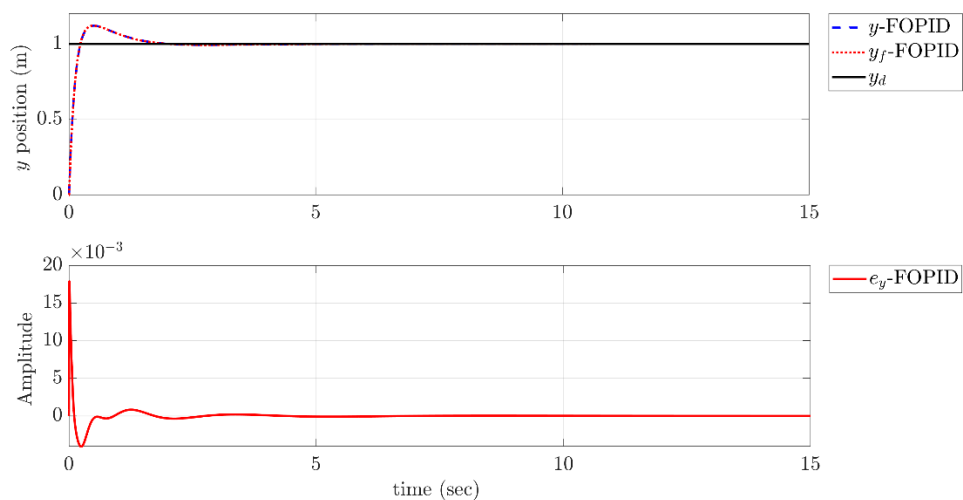


Figure 4-50: y and y_f Position States with Desired Reference (1 m) with FOPID Controller and the Error Between Them

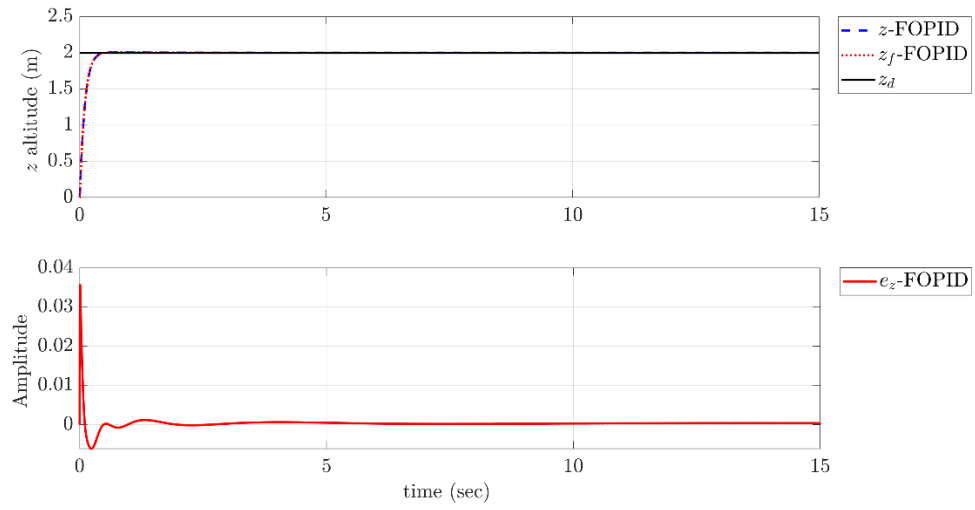


Figure 4-51: z and z_f Position States with Desired Reference (2 m) with FOPID Controller and the Error Between Them

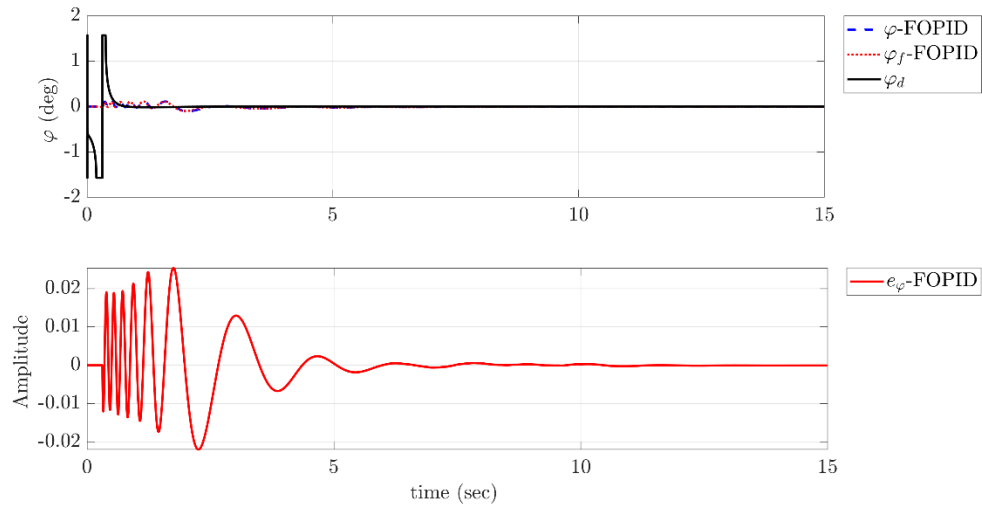


Figure 4-52: ϕ and ϕ_f Angle States with (0°) with FOPID Controller and the Error Between Them

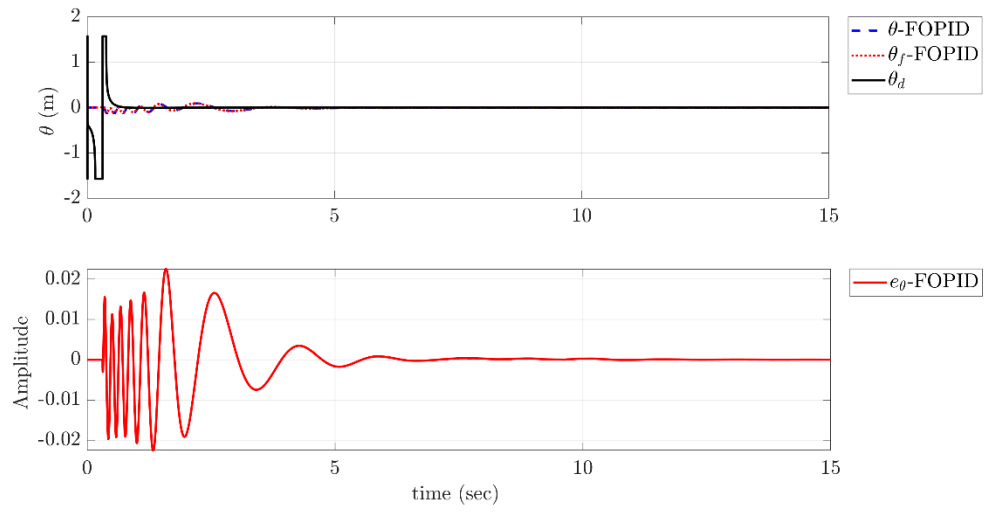


Figure 4-53: θ and θ_f Angle States with (0°) with FOPID Controller and the Error Between Them

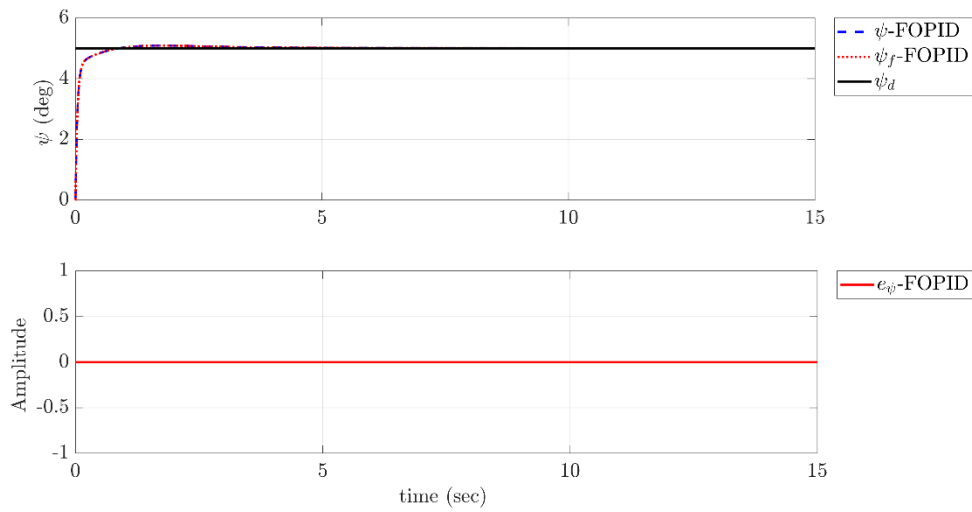


Figure 4-54: ψ and ψ_f Angle States with Desired Reference (5°) with FOPID Controller and the Error Between Them

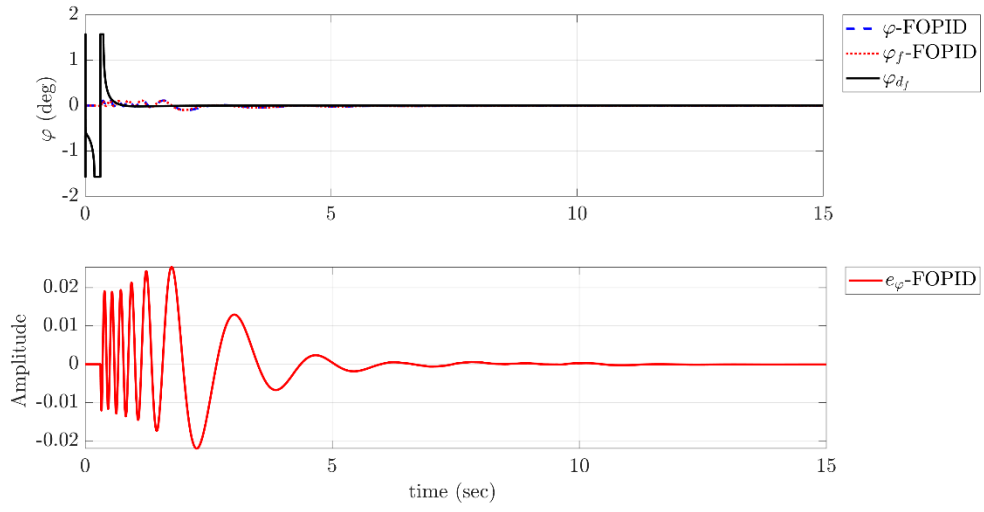


Figure 4-55: The fractional Roll Angle State (φ_f) with $\varphi_{f_d} = 0^\circ$ with FOPID Controller

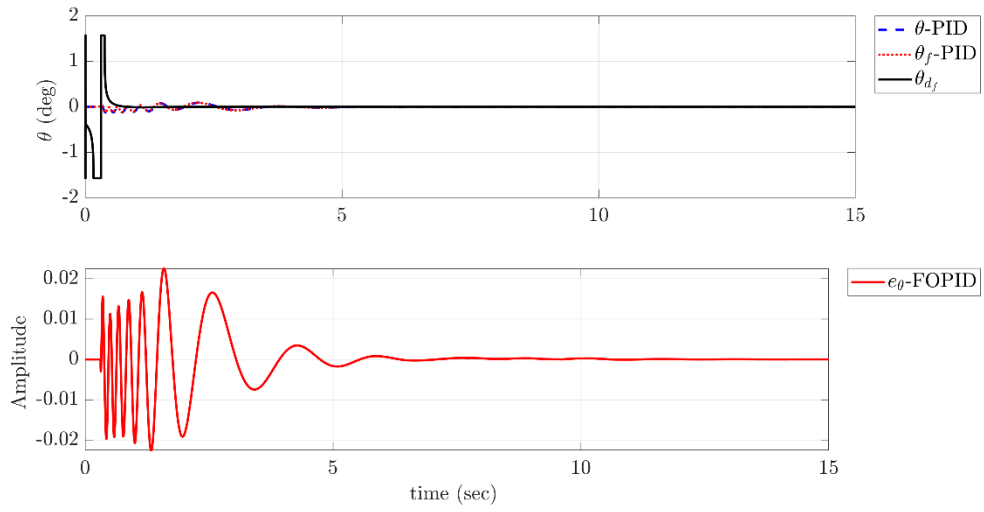


Figure 4-56: The fractional Pitch Angle State (θ_f) with $\theta_{f_d} = 0^\circ$ with FOPID Controller

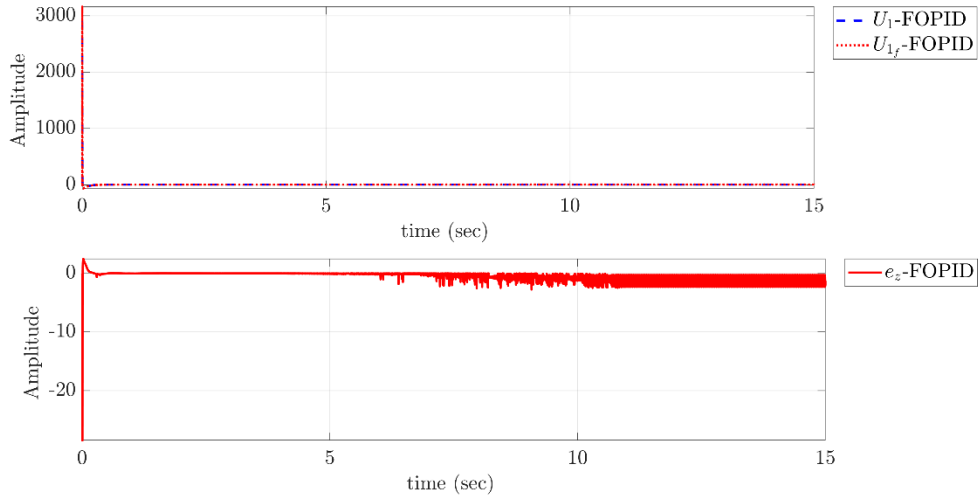


Figure 4-57: FOPID Altitude Control Inputs (U_1, U_{1f}) of Integer and Fractional Systems and the Error Between Them

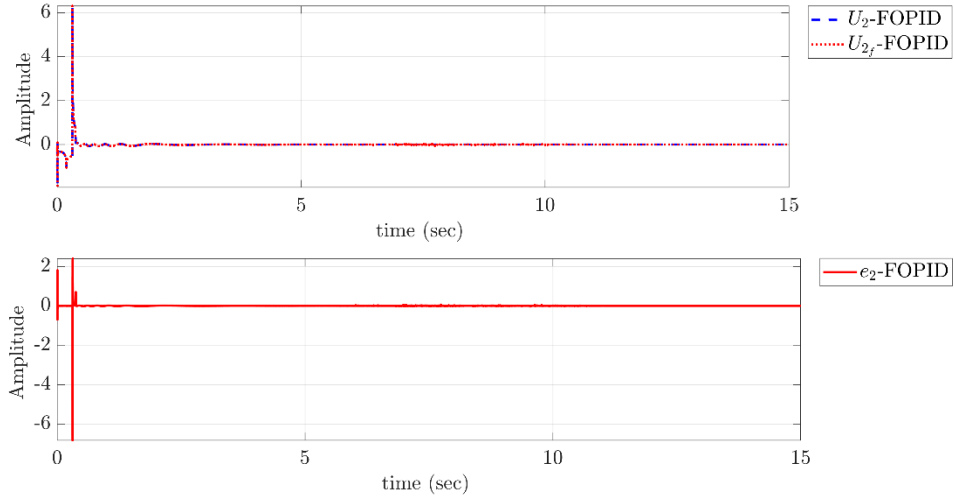


Figure 4-58: FOPID Attitude Control Inputs (U_2, U_{2f}) of Integer and Fractional Systems and the Error Between Them

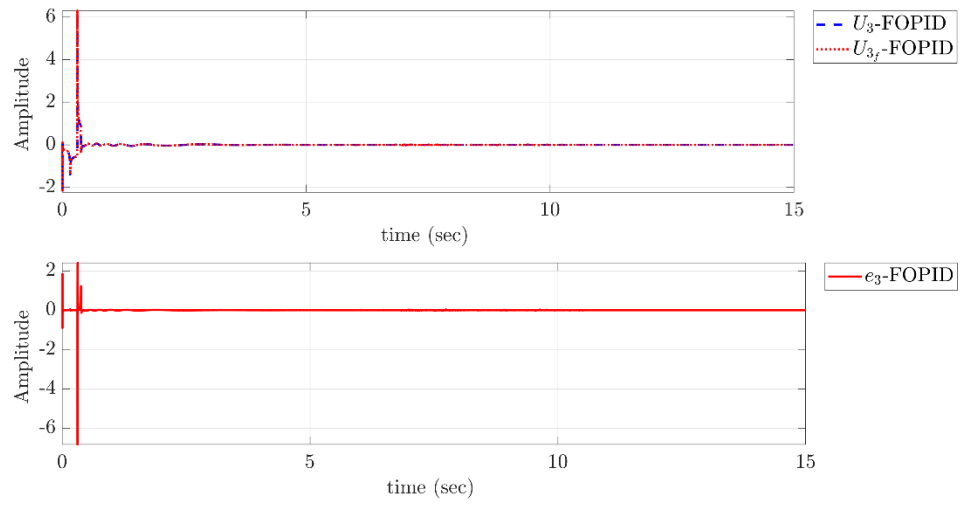


Figure 4-59: FOPID Attitude Control Inputs (U_3 , U_{3f}) of Integer and Fractional Systems and the Error Between Them

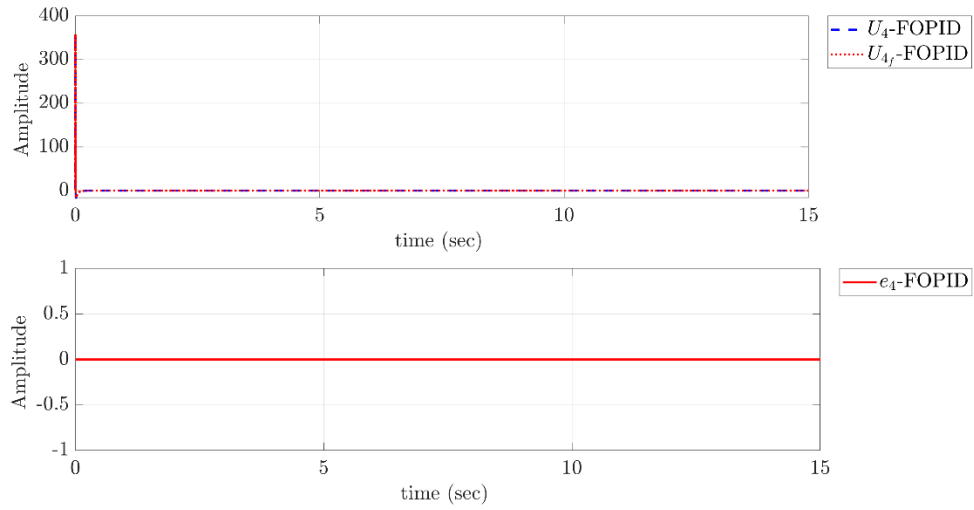


Figure 4-60: FOPID Heading Control Inputs (U_4 , U_{4f}) of Integer and Fractional Systems and the Error Between Them

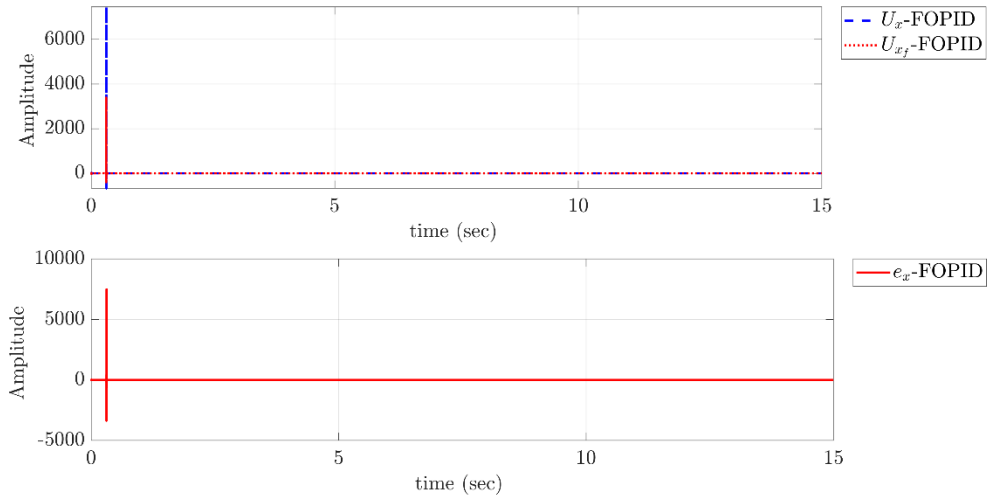


Figure 4-61: FOPID x Position Control Input (U_x , U_{x_f}) of Integer and Fractional Systems and the Error Between Them

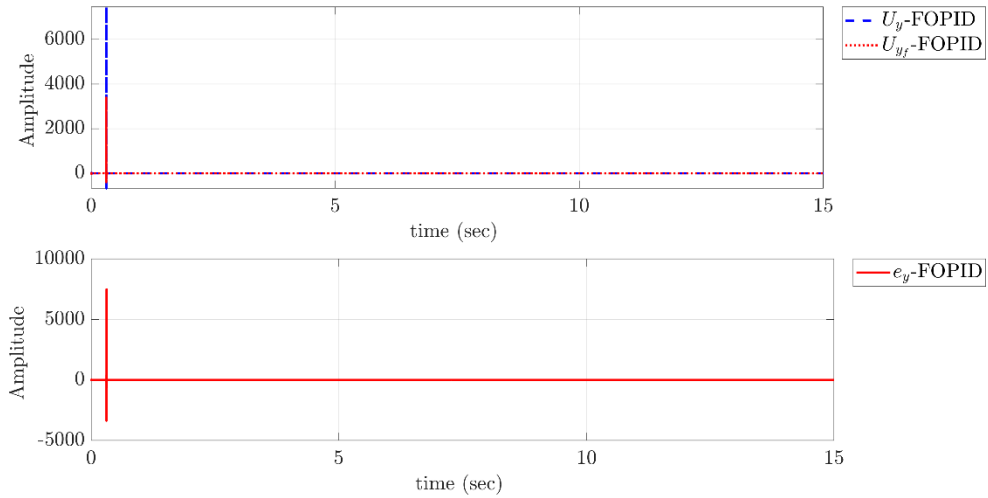


Figure 4-62: FOPID y Position Control Input (U_y , U_{y_f}) of Integer and Fractional Systems and the Error Between Them

Observation:

It can be observed that the oscillations that appear in the fractional responses (z_f , x_f , y_f) with PID controllers have been eliminated when the FOPID controllers are implemented

however, the transient states of fractional responses (x_f, y_f, z_f) are not improved in terms of rising time comparing with integer responses (x, y, z) as shown in Table 4-7.

Also, FOPID controllers remove the oscillations that appear in the fractional responses of Roll and Pitch angles (φ_f, θ_f). Moreover, the FOPID control inputs in fractional system have small oscillations comparing with FOPID control inputs in integer system.

Case Study 2: Circular Shape Trajectory (With FOPID)

We build FOPID controllers that regulate the altitude, positions, attitude, and heading of a quad-rotor in both integer and fractional systems while moving it in a circular route case (2) and it is seen in Figure 4-63 as 3D plot. The responses of integer and fractional angles of quadrotor are seen in (Figure 4-64 and Figure 4-65). The FOPID control inputs that control the quadrotor angles in both integer and fractional systems are shown in separate figures (Figure 4-66, Figure 4-67 and Figure 4-68).

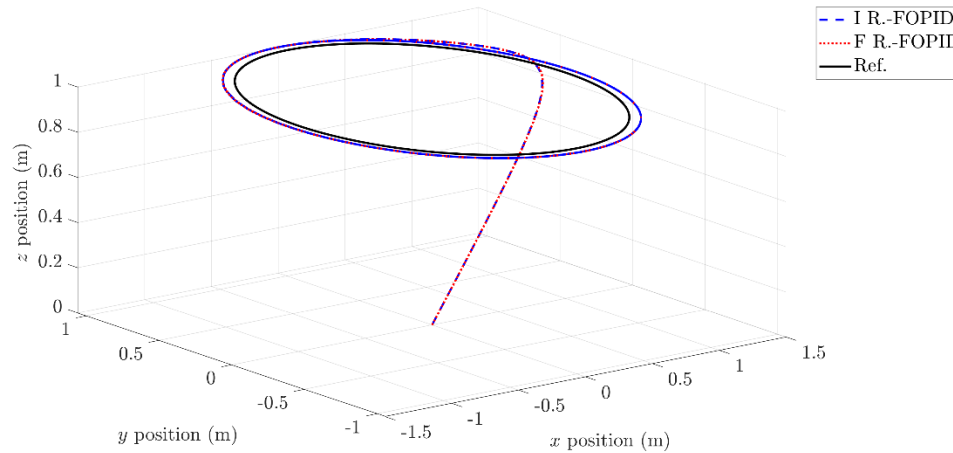


Figure 4-63: 3D Plot of Circular Trajectory with FOPID Controllers

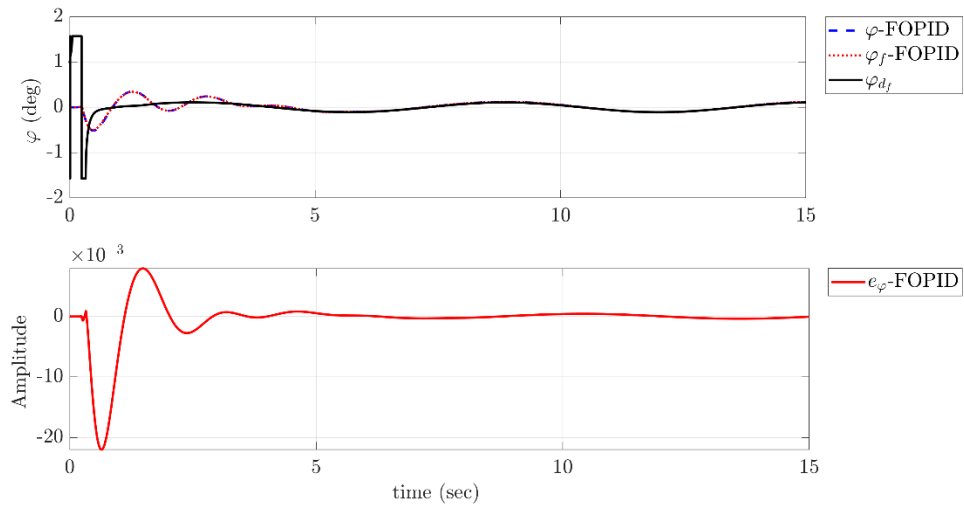


Figure 4-64: Roll Angle Responses (φ, φ_f) with FOPID Controllers and the Error Between Them

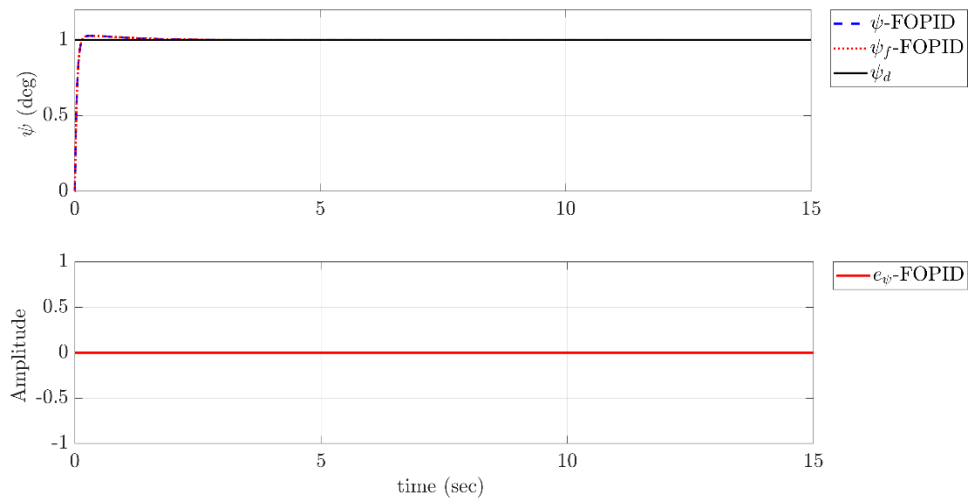


Figure 4-65: Yaw Angle Responses (ψ and ψ_f) (with 1°) with FOPID Controller and the Error Between Them

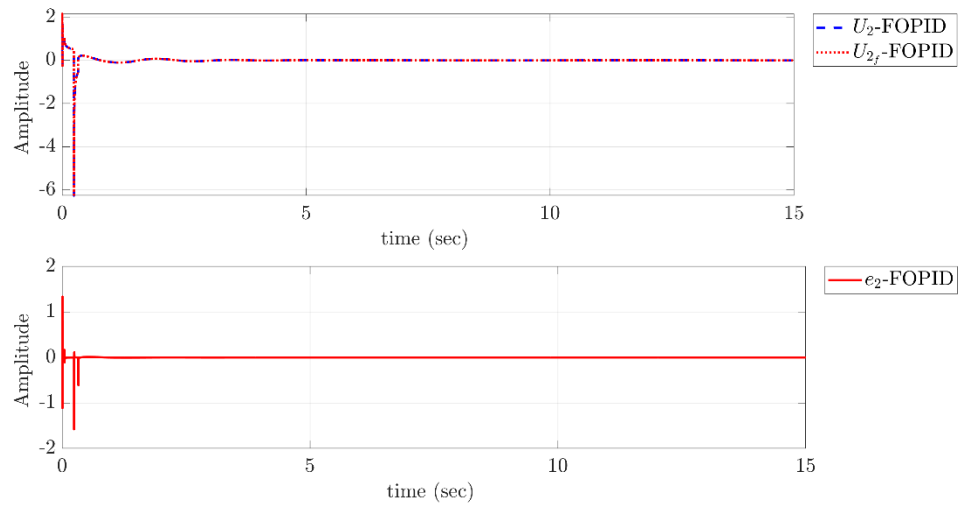


Figure 4-66: FOPID Control Inputs (U_2 and U_{2f}) and the Error Between Them

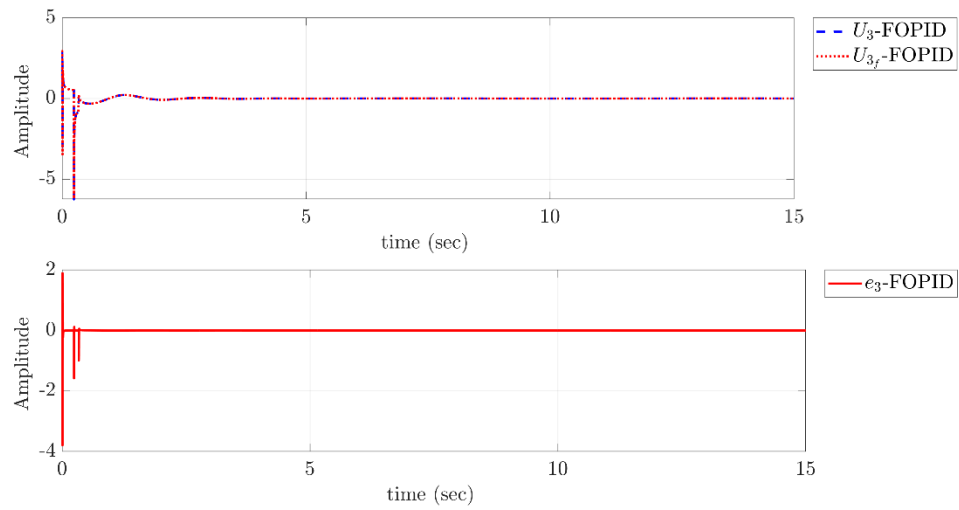


Figure 4-67: FOPID Control Inputs (U_3 and U_{3f}) and the Error Between Them

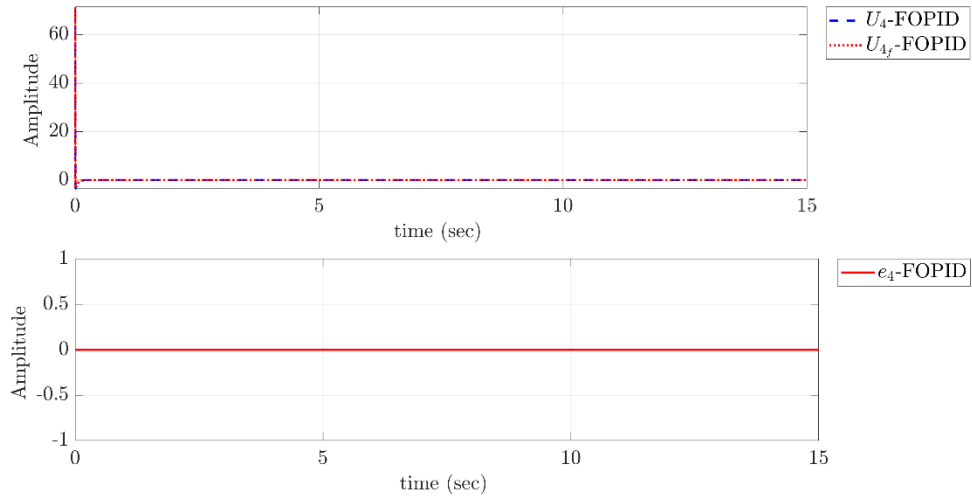


Figure 4-68: FOPID Control Inputs (U_4 and U_{4f}) and the Error Between Them

On the other hand, the responses of integer and fractional positions and altitudes of quadrotor are illustrated in separate figures (Figure 4-69, Figure 4-70 and Figure 4-71). Furthermore, the FOPID control and virtual inputs of both integer and fractional systems are shown in separate figures (Figure 4-72, Figure 4-73 and Figure 4-74).

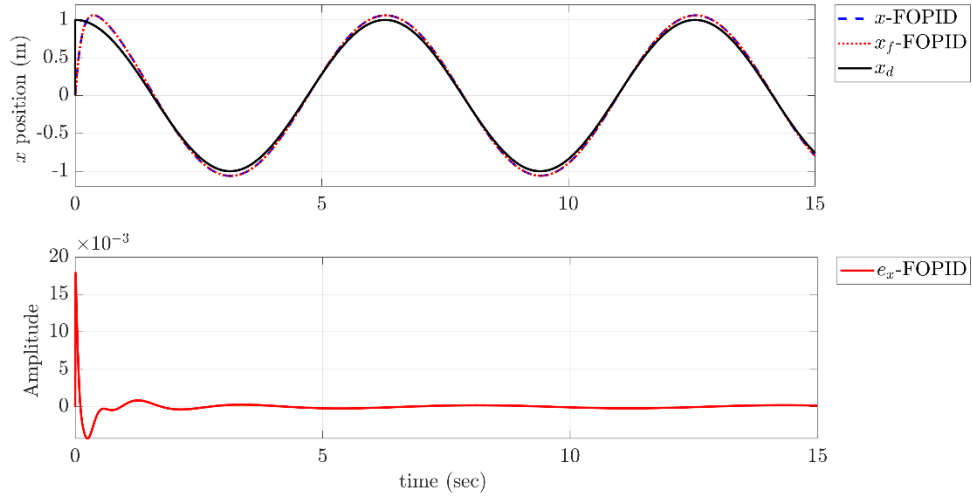


Figure 4-69: x Position Responses (x and x_f) with FOPID Controller and the Error Between Them

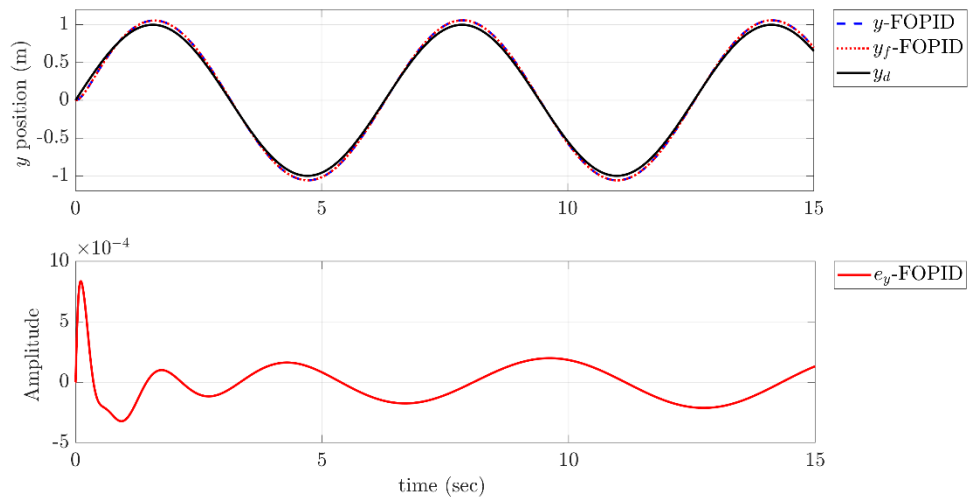


Figure 4-70: y Position Responses (y and y_f) with FOPID Controller and the Error Between Them

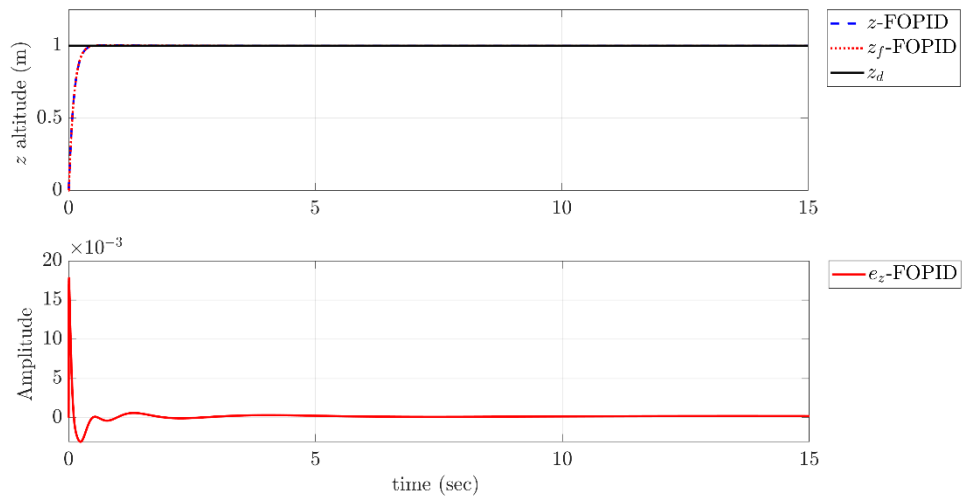


Figure 4-71: z Altitude Responses (z and z_f) with FOPID Controller and the Error Between Them

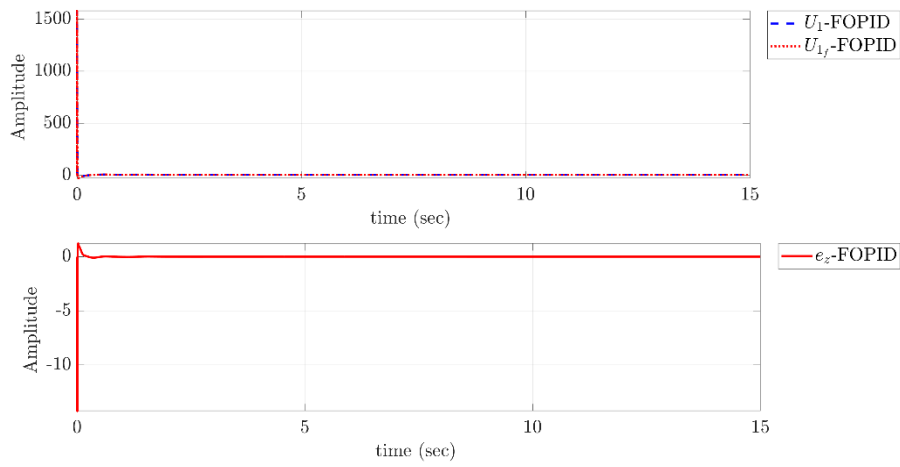


Figure 4-72: FOPID Control Inputs (U_1 and U_{1f}) and the Error Between Them

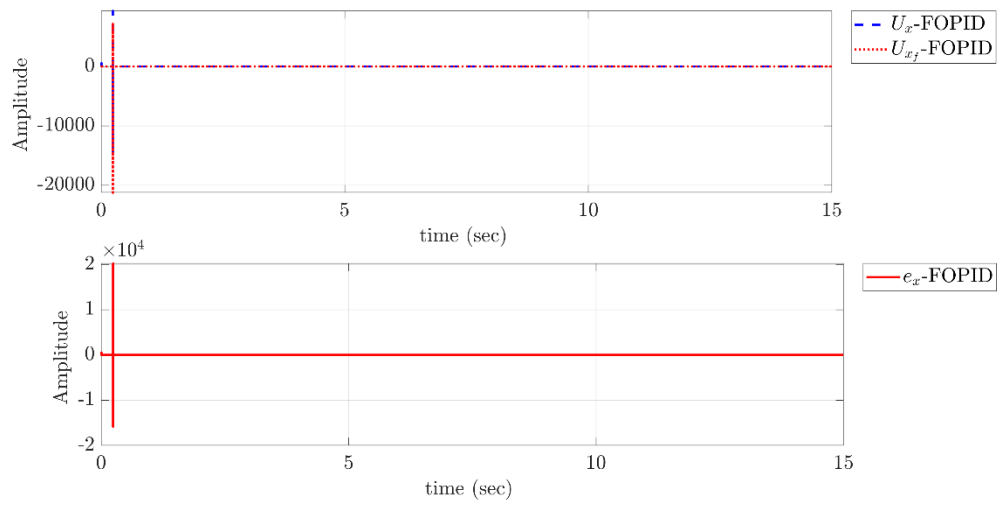


Figure 4-73: FOPID Virtual Control Inputs (U_x and U_{xf}) and the Error Between Them

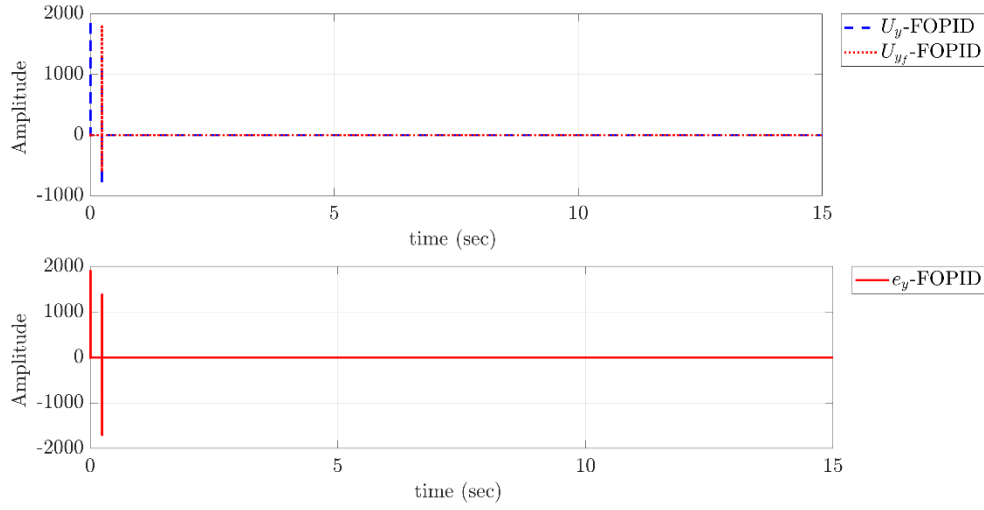


Figure 4-74: FOPID Virtual Control Inputs (U_y and U_{y_f}) and the Error Between Them

Case Study 3: Helical Shape Trajectory (With FOPID)

The quadrotor of both the integer and fractional systems is moved along a helical trajectory, (Case 3) and it is shown in Figure 4-75 as 3D plot. The responses of the quadrotor's integer and fractional angle, position, and altitude states are presented in separate figures (Figure 4-76, Figure 4-77, Figure 4-78, Figure 4-79 and Figure 4-80). Also, the FOPID control and virtual inputs are shown in separate figures (Figure 4-81, Figure 4-82, Figure 4-83, Figure 4-84, Figure 4-85 and Figure 4-86).

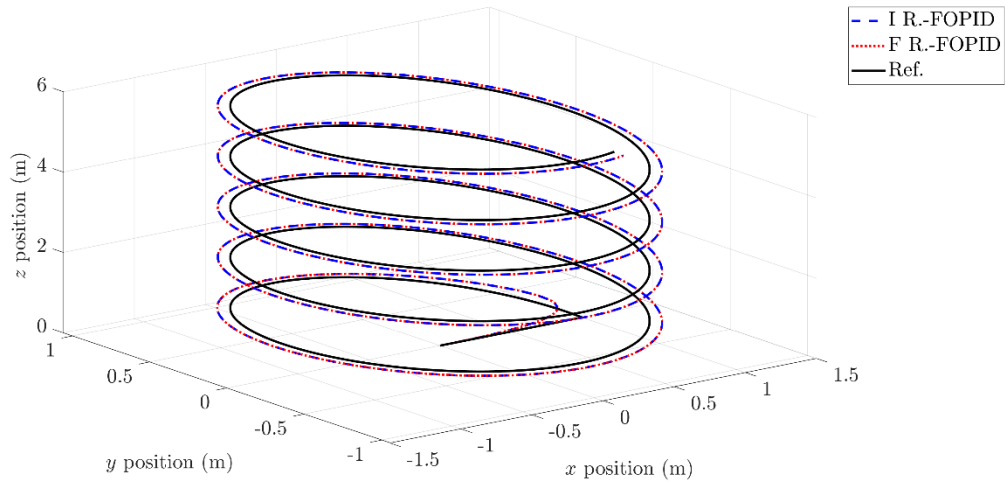


Figure 4-75: 3D Plot of Helical Trajectory with FOPID Controllers

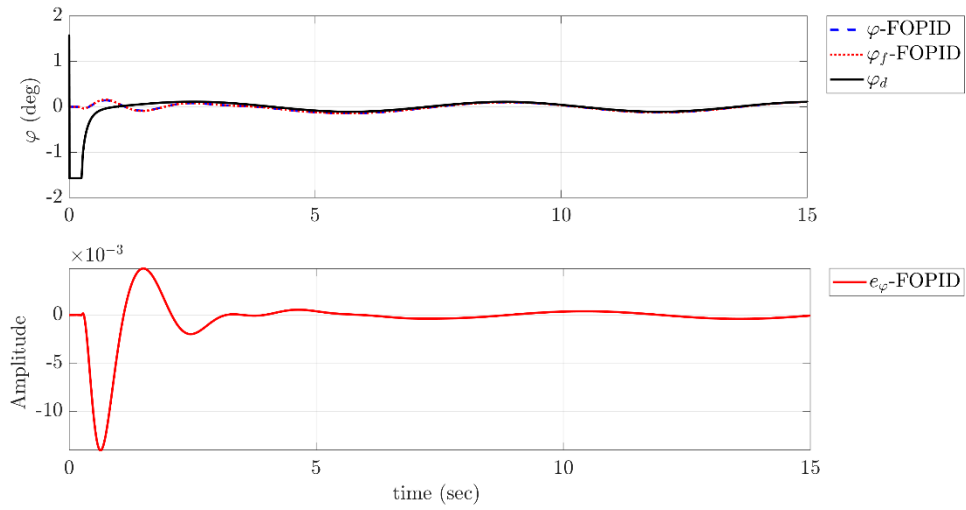


Figure 4-76: Roll Angle Responses (φ , φ_f) with FOPID Controllers and the Error Between Them

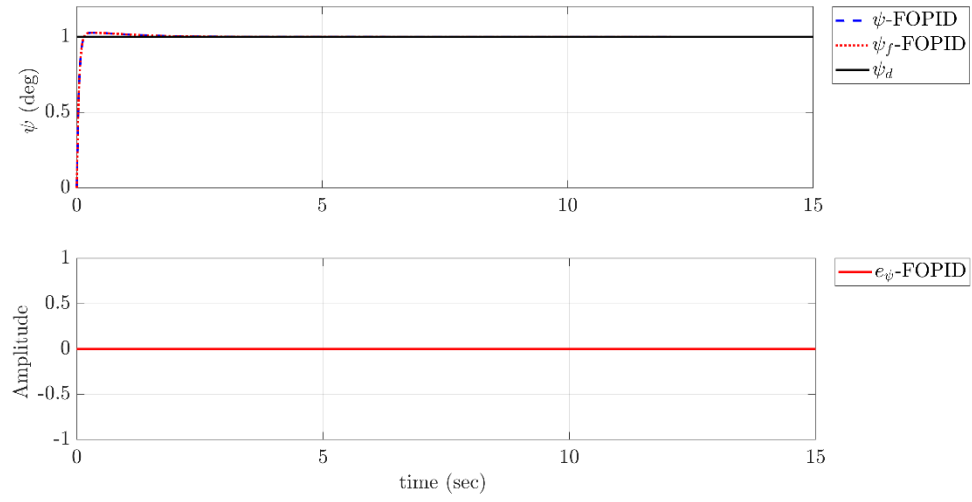


Figure 4-77: Yaw Responses (ψ, ψ_f) (with 1°) with FOPID Controllers and the Error Between Them

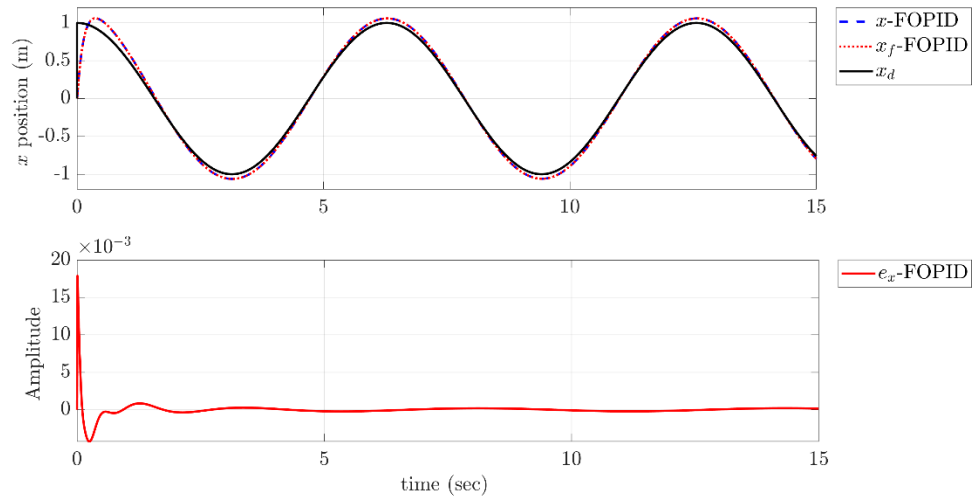


Figure 4-78: x Position Responses (x and x_f) with FOPID Controller and the Error Between Them

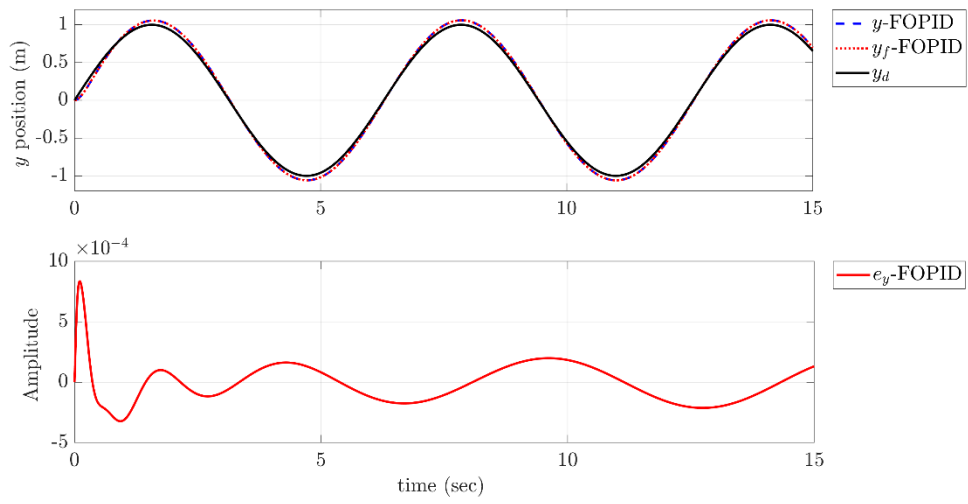


Figure 4-79: y Position Responses (y and y_f) with FOPID Controller and the Error Between Them

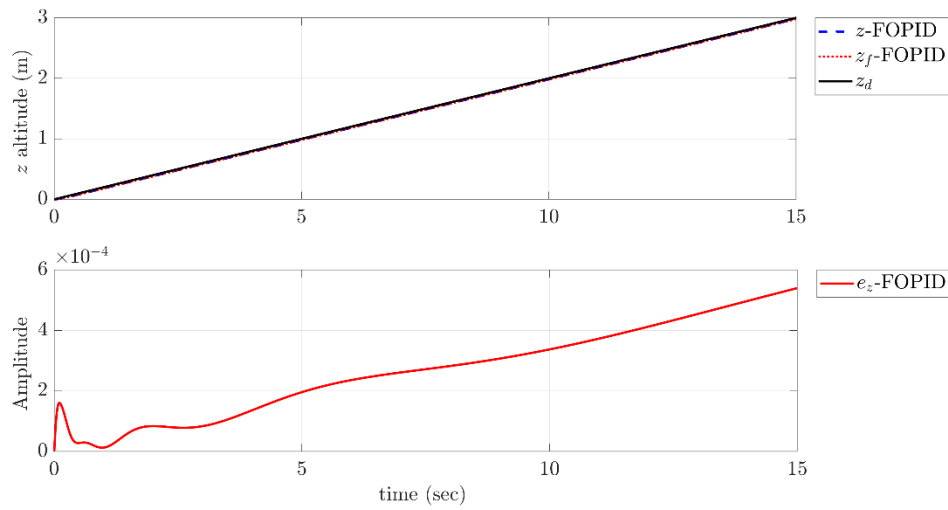


Figure 4-80: z Position Responses (z and z_f) with FOPID Controller and the Error Between Them

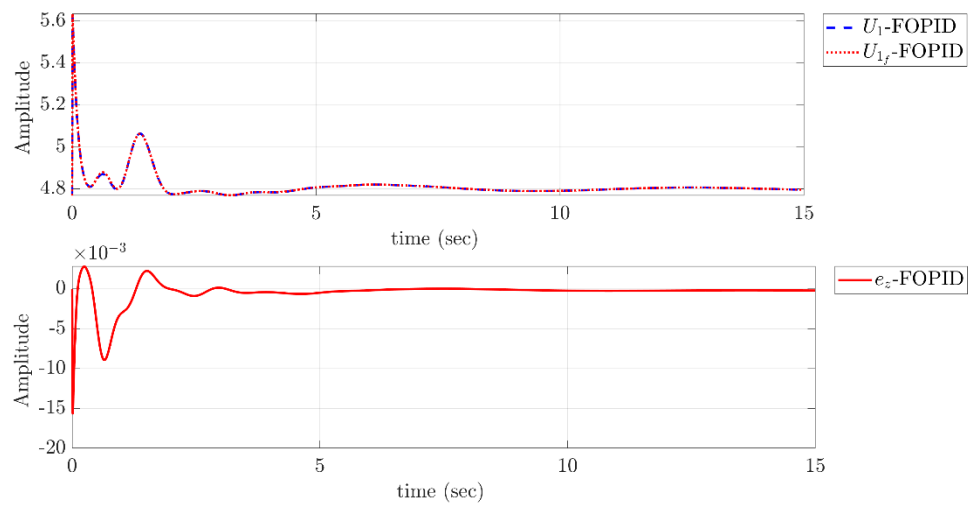


Figure 4-81: FOPID Control Inputs (U_1 and U_{1f}) and the Error Between Them

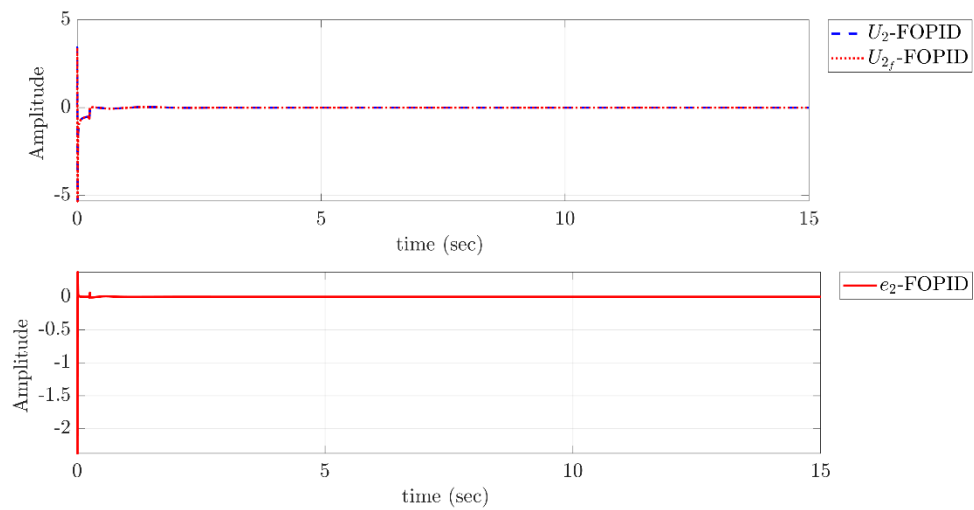


Figure 4-82: FOPID Control Inputs (U_2 and U_{2f}) and the Error Between Them

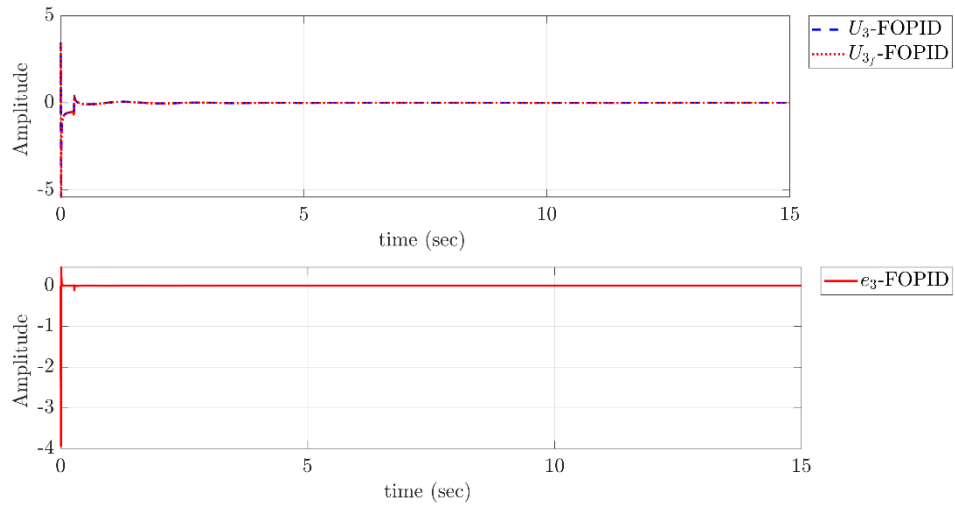


Figure 4-83: FOPID Control Inputs (U_3 and U_{3f}) and the Error Between Them

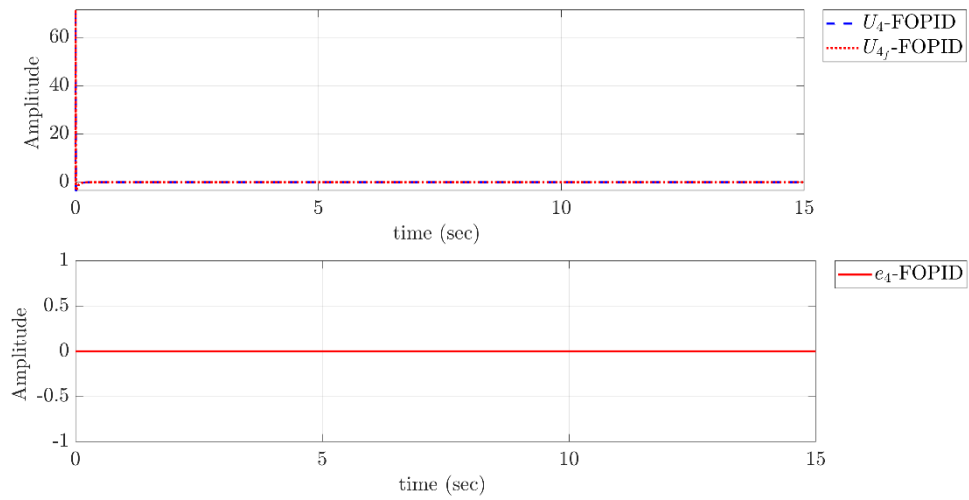


Figure 4-84: FOPID Control Inputs (U_4 and U_{4f}) and the Error Between Them

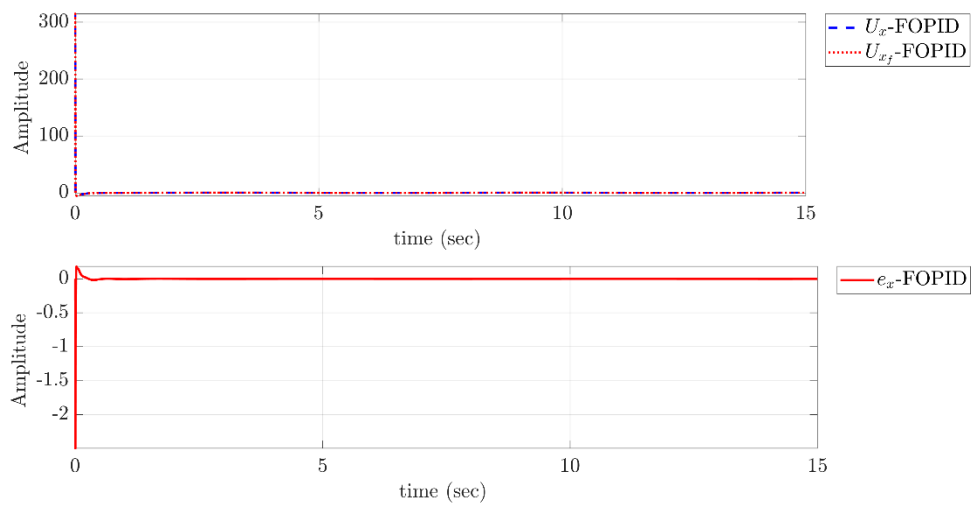


Figure 4-85: FOPID Virtual Control Inputs (U_x and U_{x_f}) and the Error Between Them

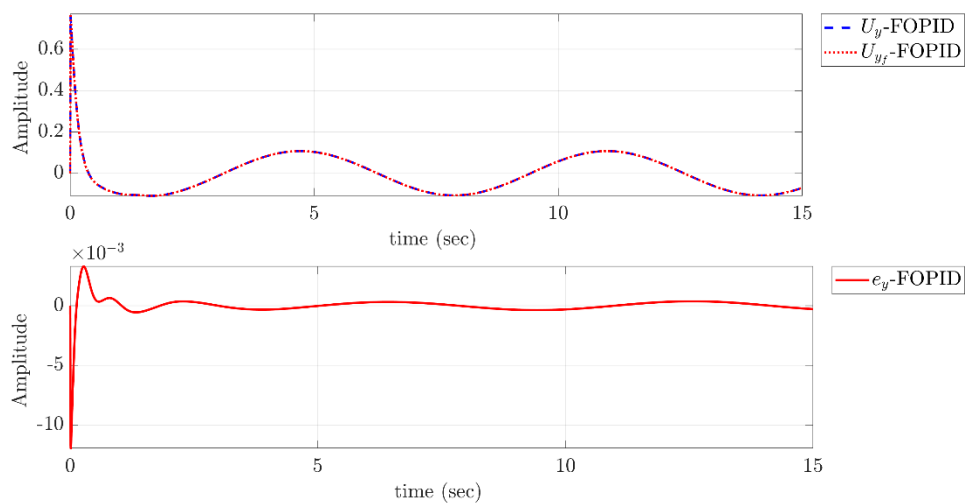


Figure 4-86: FOPID Virtual Control Inputs (U_y and U_{y_f}) and the Error Between Them

4.5.5 FOPID Control Results and Discussion

MATLAB/SIMULINK was used to test and simulate the proposed FOPID controllers. The fractional calculus was accomplished with the FOMCON Toolbox.

The primary goal of the FOPID controller design is to stabilize the UAV as well as follows a desired reference with greater performance than a PID controller.

We observed that the stated FOPID controllers stabilize both the integer and the fractional systems of UAV.

Also, the major advantage of implementing FOPID controllers are eliminating the oscillations that appear when the PID controllers are applied with the quadrotor's systems (integer and fractional ones) as shown in above figures. However, the transient states of integer and fractional responses with FOPID controllers aren't enhanced comparing with PID controllers when we apply the step inputs as desired references as shown in Table 4-7.

Moreover, the obtained results of moving the quadrotor along circular and helical trajectories show that the quadrotors of both integer and fractional systems with FOPID controllers follow the trajectories better than PID controllers and have considerably superior transient and steady states.

In addition, we utilized the GA approach to tune and get the optimal parameters of the proposed FOPID controllers, with the goal functions minimizing IAE, ITAE, ISE, and ITSE, as shown in Table 4-5. The ITSE was the best when compared to the others.

CHAPTER 5 SMC AND FOSMC DESIGNS

5.1 Sliding Mode Control (SMC) Design

In this section, we proposed a nonlinear SMC to control the quadrotor states because the quadrotor system is a type of nonlinear systems.

5.1.1 Introduction

Sliding mode control (SMC) consists of an algorithm that is fundamentally resilient in parameter, nonlinear, external and insecure adjustments. It is introduced where the robustness criterion in drive systems is highly necessary and in the face of strong uncertainty [39]. It employs a fast switching control rule to drive state courses to track and maintain a predefined, customer surface in the states space and to keep them there [63]. As in [58], the control law of SMC (U_{CL}) has two rules of control as shown in Equation (5.1); an equivalent control rule (u_{EQ}) and a corrective control rule (u_C). To achieve the corrective control function, any deviations in the state trajectories from the sliding surface will be compensated while, the equivalent control ensures that the state routes remain on the sliding surface by maintained the surface rate to zero.

$$U_{CL} = u_C + u_{EQ} \quad (5.1)$$

Where

U_{CL} Sliding Mode Control Law

u_C Corrective Control

We introduce three case studies that have also been examined in order to highlight the issues that quadcopter control systems may meet when attempting to achieve the required tracking. The first case study was the step input, the second case study was the circular path, and the third case study was the helical path. Table 5-1 shows the states of these case studies together with their reference trajectories.

Table 5-1: Case Studies with Reference Trajectories (RT)

Cases with RT States	Case (1)	Case (2)	Case (3)
	Reference Trajectory	Reference Trajectory	Reference Trajectory
φ	$2u(t)$	$2u(t)$	$2u(t)$
θ	$2u(t)$	$2u(t)$	$2u(t)$
ψ	$5u(t)$	$u(t)$	$u(t)$
x	$u(t)$	$\cos(t)$	$\cos(t)$
y	$u(t)$	$\sin(t)$	$\sin(t)$
z	$2u(t)$	$u(t)$	$0.2t$

5.1.2 SMC Design of Integer System

In this section we proposed fractional sliding mode controllers that control the integer quadrotor states in Equation (3.11). The SMC of fractional order nonlinear systems can be built using two methods, as shown in [60], [64], [65]. The first relates to a class of fractional nonlinear systems, while the second applies to nonlinear systems of both fractional and integer order. The second strategy, which was outlined in [65], will be used in this research.

Given the following system as a general form of integer order nonlinear systems,

$$\dot{x} = w(x) + v(x)u(t) \quad (5.2)$$

Where

$w(x)$ and $v(x)$ Functions of States as Vectors

$u(t)$ Control Input

Now, defining the sliding surface as,

$$s = q(x - x_d) \quad (5.3)$$

Where

q Positive Constant Vector ($q^T \in \mathbb{R}^n$) and (n the system order)

x Vector States

x_d Desired Vector States to be Tracked

Let,

$$\dot{s} = -qsgn(s) \quad ; q > 0 \quad (5.4)$$

We select $\dot{s} = -qsgn(s)$ to guarantee that the sliding condition ($s \dot{s} < 0$) for $s \neq 0$. And Selecting Lyapunov function as $V = \frac{1}{2}s^2$ and its rate is $\dot{V} = s \dot{s}$ which ensures the stability and therefore s is asymptotically stable. If we select $\dot{s}$ as,

$$\dot{s} = -qsgn(s) - ks \quad ; q \text{ and } k > 0 \quad (5.5)$$

The sliding condition ($s \dot{s}$) will be also less than zero ($s \dot{s} < 0$) and increasing the term (k) will increase $\dot{s}$ and this will result in quicker approach towards $s = 0$.

Therefore, in order to design control law of the given system Equation (5.2), taking 1st derivative on both sides of the defined sliding surface in Equation (5.3) as follows,

$$\dot{s} = q(\dot{x} - \dot{x}_d) \quad (5.6)$$

Comparing Equation (5.6) and Equation (5.5) indicates

$$-qsgn(s) - ks = q(\dot{x} - \dot{x}_d) \quad (5.7)$$

Substituting ($\dot{x}$) from Equation (5.2) implies,

$$\begin{aligned} -qsgn(s) - ks &= q [w(x) + v(x)u(t)] - qx_d \\ u(t) &= \frac{1}{qv(x)} [-qsgn(s) - ks - qw(x) + qx_d] \end{aligned} \quad (5.8)$$

Based on above steps, we can design the sliding mode controllers that control altitude, attitude and heading and x , y positions of the integer quadrotor dynamic model as represented in the following sections. Firstly, we define the sliding surfaces in Equation (5.9) and the Lyapunov functions that ensure the stability of the proposed SMCs in Equation (5.10).

$$\begin{aligned} S_i &= \{ k_{i-1}(x_{i-1} - x_{(i-1)d}) + k_i(x_i - x_{id}) \quad / i \\ &\in \{2, 4, 6, 8, 10, 12\} \end{aligned} \quad (5.9)$$

With

k Positive Constant Vector ($k^T \in \mathbb{R}^n$) and (n the system order)

x Vector States

x_d Desired Vector States to be Tracked

$$V_i = \left\{ \frac{1}{2} S_i^2 \quad / i \in \{2, 4, 6, 8, 10, 12\} \right. \quad (5.10)$$

Let,

$$\dot{S} = -q \operatorname{sgn}(S) \quad ; q > 0 \quad (5.11)$$

We select $\dot{S} = -q \operatorname{sgn}(S)$ to guarantee that the sliding condition ($S\dot{S} < 0$) for $s \neq 0$. Also, we choose $V = \frac{1}{2} S^2$ and its rate $\dot{V} = S\dot{S}$ which ensures the stability and therefore S is asymptotically stable.

As discussed in [60], selecting the sliding rate as,

$$\dot{S} = -q \operatorname{sgn}(S) - kS \quad ; q \text{ and } k > 0 \quad (5.12)$$

The sliding condition ($S\dot{S}$) will be also less than zero ($S\dot{S} < 0$) and increasing the term (k) will increase $\dot{S}$ and this will result in quicker approach towards $S = 0$.

5.1.2.1 Altitude SMC Design in Integer System

In this part, the altitude sliding mode controller (ASMC) is developed to track a reference trajectory z_d .

From Equation (3.11), the altitude dynamic (z) is

$$\dot{x}_{11} = x_{12} \quad (5.13)$$

$$\dot{x}_{12} = a_6 x_{12} - g + b_6 (c_\varphi c_\theta) U_1 \quad (5.14)$$

Therefore, and based on Equation (5.9), the altitude sliding surface is defined as,

$$S_z = S_{12} = k_{11}(x_{11} - x_{11d}) + k_{12}(x_{12} - x_{12d}) \quad (5.15)$$

Taking the derivative of the sliding surface (S_z) would results:

$$\dot{S}_z = -q_z \text{sgn}(S_z) - k_z S_z \quad (5.16)$$

Where

- ($q_z \text{sgn}(S_z) + k_z S_z$) represents the discontinuous part that satisfies the reachability condition.

q_z and $k_z > 0$

$$\dot{S}_z = k_{11}(\dot{x}_{11} - \dot{x}_{11d}) + k_{12}(\dot{x}_{12} - \dot{x}_{12d}) = -q_z \text{sgn}(S_z) - k_z S_z \quad (5.17)$$

Substituting the altitude dynamic equations ($\dot{x}_{11}$ and $\dot{x}_{12}$) implies,

$$\begin{aligned} k_{11}(x_{12} - \dot{x}_{11d}) + k_{12}(a_6 x_{12} - g + b_6 c_\varphi c_\theta U_1 - \dot{x}_{12d}) \\ = -q_z \text{sgn}(S_z) - k_z S_z \end{aligned} \quad (5.18)$$

Therefore, the sliding law control of altitude will be calculated as follows:

$$\begin{aligned} U_1 = \frac{1}{k_{12} b_6 c_\varphi c_\theta} [-q_z \text{sign}(S_z) - k_z S_z - k_{12}(a_6 x_{12} - g - \dot{x}_{12d}) \\ - k_{11}(x_{12} - \dot{x}_{11d})] \end{aligned} \quad (5.19)$$

Stability Analysis

The primary goal of any control system design is to stabilize the original system. To ensure the stability of the proposed controller, a Lyapunov function is applied as a positive definite function, and its rate must be negative definite or semi-negative definite.

Therefore, and based on Equation (5.10), the altitude Lyapunov function is defined as,

$$V_z = V_{12} = \frac{1}{2} S_z^2 \quad (5.20)$$

Taking its time derivative as,

$$\dot{V}_z = \dot{V}_{12} = S_z \dot{S}_z \quad (5.21)$$

$$\dot{V}_z = \dot{V}_{12} = S_z [k_{11}(x_{12} - \dot{x}_{11d}) + k_{12}(a_6 x_{12} - g + b_6 c_\varphi c_\theta U_1 - \dot{x}_{12d})] \quad (5.22)$$

Now, substitute (U_1) from Equation ((5.19) and simplify the result,

$$\begin{aligned} \dot{V}_z = \dot{V}_{12} &= S_z [k_{11}(x_{12} - \dot{x}_{11d}) + [-q_z \text{sign}(S_z) - k_z S_z - k_{11}(x_{12} - \dot{x}_{11d})]] \\ \dot{V}_z = \dot{V}_{12} &= S_z [-q_z \text{sign}(S_z) - k_z S_z] \\ \dot{V}_z = \dot{V}_{12} &= -q_z |S_z| - k_z S_z^2 \leq 0 \end{aligned} \quad (5.23)$$

Which is negative $\forall t$, since $q_z, k_z > 0$

5.1.2.2 Attitude SMC Design in Integer System

The quadrotor's orientation is determined by the rotation angles. To govern the roll, pitch, and yaw angles, three different controllers can be designed. Based on Equation ((5.9), the

sliding surface is constructed in the same way as the altitude control design, and the control law is stated as follows:

5.1.2.2.1 Roll Angle SMC in Integer System

From Equation (3.11), the roll angle dynamics (φ) are

$$\dot{x}_1 = x_2 \quad (5.24)$$

$$\dot{x}_2 = a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + d_1 U_2 \quad (5.25)$$

From Equation (5.9), the sliding surface can be defined as

$$S_\varphi = S_2 = k_1(x_1 - x_{1d}) + k_2(x_2 - x_{2d}) \quad (5.26)$$

Taking the derivative of the sliding surface (S_φ) would results:

$$\dot{S}_\varphi = \dot{S}_2 = -q_\varphi \text{sgn}(S_\varphi) - k_\varphi S_\varphi \quad (5.27)$$

→

$$\dot{S}_\varphi = \dot{S}_2 = k_1(\dot{x}_1 - \dot{x}_{1d}) + k_2(\dot{x}_2 - \dot{x}_{2d}) = -q_\varphi \text{sgn}(S_\varphi) - k_\varphi S_\varphi \quad (5.28)$$

Substituting the roll angle dynamic equations ($\dot{x}_1$ and $\dot{x}_2$) implies,

$$\begin{aligned} k_1(x_2 - \dot{x}_{1d}) + k_2(a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + d_1 U_2 - \dot{x}_{2d}) \\ = -q_\varphi \text{sgn}(S_\varphi) - k_\varphi S_\varphi \end{aligned} \quad (5.29)$$

Now, solving for U_2

$$\begin{aligned} U_2 = \frac{1}{k_2 d_1} [-q_\varphi \text{sgn}(S_\varphi) - k_\varphi S_\varphi - k_2(a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 - \dot{x}_{2d}) \\ - k_1(x_2 - \dot{x}_{1d})] \end{aligned} \quad (5.30)$$

Stability Analysis

From Equation (5.10), the roll angle Lyapunov function is defined as,

$$V_\varphi = V_2 = \frac{1}{2} S_\varphi^2 \quad (5.31)$$

Taking its time derivative as,

$$\dot{V}_\varphi = \dot{V}_2 = S_\varphi \dot{S}_\varphi \quad (5.32)$$

$$\dot{V}_\varphi = \dot{V}_2 = S_\varphi \left[k_1(x_2 - \dot{x}_{1d}) + k_2(a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + d_1 U_2 - \dot{x}_{2d}) \right] \quad (5.33)$$

Now, substitute (U_2) from Equation ((5.30) and simplify the result,

$$\begin{aligned} \dot{V}_\varphi = \dot{V}_2 &= S_\varphi \left[k_1(x_2 - \dot{x}_{1d}) + \left[-q_\varphi \operatorname{sgn}(S_\varphi) - k_\varphi S_\varphi - k_1(x_2 - \dot{x}_{1d}) \right] \right] \\ \dot{V}_\varphi = \dot{V}_2 &= S_\varphi \left[-q_\varphi \operatorname{sign}(S_\varphi) - k_\varphi S_\varphi \right] \\ \dot{V}_\varphi = \dot{V}_2 &= -q_\varphi |S_\varphi| - k_\varphi S_\varphi^2 \leq 0 \end{aligned} \quad (5.34)$$

Which is negative $\forall t$, since $q_\varphi, k_\varphi > 0$

5.1.2.2.2 Pitch Angle SMC in Integer System

Similarly, from Equation (3.11), the pitch angle dynamics are

$$\dot{x}_3 = x_4 \quad (5.35)$$

$$\dot{x}_4 = a_2 x_2 x_6 + b_2 \Omega_r x_2 + c_2 x_4^2 + d_2 U_3 \quad (5.36)$$

Using the same procedures as roll angle and altitude SMC designs, the designed controllers (U_3) for pitch angle can be derived as follows,

$$U_3 = \frac{1}{k_4 d_2} [-q_\theta \text{sign}(S_\theta) - k_\theta S_\theta - k_4(a_2 x_2 x_6 + b_2 \Omega_r x_2 + c_2 x_4^2 - \dot{x}_{4d}) - k_3(x_4 - \dot{x}_{3d})] \quad (5.37)$$

Stability Analysis

Based on Equation (5.10), the pitch angle Lyapunov function is designed as,

$$V_\theta = V_4 = \frac{1}{2} S_\theta^2 \quad (5.38)$$

After simplification, the time derivative is

$$\dot{V}_\theta = \dot{V}_4 = -q_\theta |S_\theta| - k_\theta S_\theta^2 \leq 0 \quad (5.39)$$

5.1.2.2.3 Yaw Angle SMC in Integer System

From Equation (3.11), the yaw angle dynamics are

$$\dot{x}_5 = x_6 \quad (5.40)$$

$$\dot{x}_6 = a_3 x_2 x_4 + b_3 x_6^2 + c_3 U_4 \quad (5.41)$$

Using the same procedures as roll angle and altitude SMC designs, the designed controller (U_4) for yaw angle can be derived as follows,

$$U_4 = \frac{1}{k_6 c_3} [-q_\psi \text{sign}(S_\psi) - k_\psi S_\psi - k_6(a_3 x_2 x_4 + b_3 x_6^2 - \dot{x}_{6d}) - k_5(x_6 - \dot{x}_{5d})] \quad (5.42)$$

Stability Analysis

Based on Equation (5.10), the yaw angle Lyapunov function is designed as,

$$V_\psi = V_6 = \frac{1}{2} S_\psi^2 \quad (5.43)$$

After simplification, the time derivative is

$$\dot{V}_\psi = \dot{V}_6 = -q_\psi |S_\psi| - k_\psi S_\psi^2 \leq 0 \quad (5.44)$$

5.1.2.3 Position SMC Design in Integer System

In this section we designed the virtual control inputs (U_x and U_y) which controlled the quadrotor x and y positions. Based on Equation (5.9), the sliding surfaces of both (x, y) are defined as:

$$S_x = S_8 = k_7(x_7 - x_{7d}) + k_8(x_8 - x_{8d}) \quad (5.45)$$

$$S_y = S_{10} = k_9(x_9 - x_{9d}) + k_{10}(x_{10} - x_{10d}) \quad (5.46)$$

After substituting and simplification, the control signals (U_x, U_y) are

$$U_x = \frac{1}{k_8 b_4 U_1} [-q_x \text{sign}(S_x) - k_x S_x - k_8(a_4 x_8 - \dot{x}_{8d}) - k_7(x_8 - \dot{x}_{7d})] \quad (5.47)$$

$$U_y = \frac{1}{k_{10} b_5 U_1} [-q_y \text{sign}(S_y) - k_y S_y - k_{10}(a_5 x_{10} - \dot{x}_{10d}) - k_9(x_{10} - \dot{x}_{9d})] \quad (5.48)$$

Stability Analysis

Based on Equation (5.10), the position Lyapunov functions are designed as,

$$V_x = V_8 = \frac{1}{2} S_x^2 \quad (5.49)$$

$$V_y = V_{10} = \frac{1}{2} S_y^2 \quad (5.50)$$

After simplification, the time derivative is

$$\dot{V}_x = \dot{V}_8 = -q_x |S_x| - k_x S_x^2 \leq 0 \quad (5.51)$$

$$\dot{V}_y = \dot{V}_{10} = -q_y |S_y| - k_y S_y^2 \leq 0 \quad (5.52)$$

Which are negative $\forall t$, since $q_x, q_y, k_x, k_y > 0$

5.1.3 Sliding Mode Control (SMC) Design of Fractional System

In this section we will use the second method of designing SMC that can be applied to fractional order and integer order nonlinear systems, it was discussed in [60], [64], [65].

The proposed integer sliding mode controllers was applied to the fractional states (transitional equations) of Equation (3.17)

Given the following system as a general form of fractional order nonlinear systems,

$$\mathfrak{D}^{\beta}x = w(x) + v(x)u(t) \quad (5.53)$$

β Fractional Order (where $0 < \beta < 1$)

$w(x)$ and $v(x)$ Functions of States as Vectors

$u(t)$ Control Input

Now, defining the sliding surface as,

$$s = q(x - x_d) \quad (5.54)$$

q Positive Constant Vector ($q^T \in \mathbb{R}^n$) and (n the system order)

x Vector States

x_d Desired Vector States to be Tracked

Let,

$$\mathfrak{D}^{\beta}s = -qsgn(s) \quad ; q > 0 \quad (5.55)$$

For integer order nonlinear systems, $\beta = 1$, $\dot{s} = -qsgn(s)$ which guarantees that the sliding condition ($s\dot{s} < 0$) for $s \neq 0$. Selecting Lyapunov function as $V = \frac{1}{2}s^2$ and its rate is $\dot{V} = s\dot{s}$ which ensures the stability and therefore s is asymptotically stable.

For fractional order nonlinear systems,

$$\mathfrak{D}^\beta s = -qsgn(s) \quad ; q > 0 \quad (5.56)$$

Taking β integral of both sides $\rightarrow s = -q\mathfrak{D}^{-\beta}sgn(s)$

Taking 1st derivative $\rightarrow \dot{s} = -q\mathfrak{D}^{1-\beta}sgn(s)$

For $0 < \beta < 1$, $\mathfrak{D}^{1-\beta}sgn(s)$ will stay positive, and this ensures that $s\dot{s} < 0$. If we select $\mathfrak{D}^\beta s$ as,

$$\mathfrak{D}^\beta s = -qsgn(s) - ks \quad ; q \text{ and } k > 0 \quad (5.57)$$

The sliding condition ($s\dot{s}$) will be also less than zero ($s\dot{s} < 0$) and increasing the term (k) will increase $\dot{s}$ and this will result in quicker approach towards $s = 0$.

Therefore, in order to design control law of the given system Equation ((5.53), taking β^{th} derivative on both sides of the defined sliding surface as follows,

$$\begin{aligned} s &= q(x - x_d) \\ \mathfrak{D}^\beta s &= q\mathfrak{D}^\beta x - q\mathfrak{D}^\beta x_d \end{aligned} \quad (5.58)$$

Comparing Equation (5.57) and Equation (5.58) indicates

$$-qsgn(s) - ks = q\mathfrak{D}^\beta x - q\mathfrak{D}^\beta x_d \quad (5.59)$$

Substituting ($\mathfrak{D}^\beta x$) from Equation (5.53) implies,

$$\begin{aligned} -qsgn(s) - ks &= q[w(x) + v(x)u(t)] - q\mathfrak{D}^\beta x_d \\ u(t) &= \frac{1}{qv(x)} [-qsgn(s) - ks - qw(x) + q\mathfrak{D}^\beta x_d] \end{aligned} \quad (5.60)$$

Based on above steps, we can design the sliding mode controllers that control altitude, and x, y positions of the fractional quadrotor dynamic model as represented in the following

sections. Firstly, we define the sliding surfaces in Equation (5.61) and the Lyapunov functions that ensure the stability of the proposed SMCs in Equation (5.62).

$$\begin{aligned} S_{f_i} = & \left\{ q_{i-3} \left(x_{f_{i-3}} - x_{f_{(i-3)d}} \right) + q_{i-2} \left(x_{f_{i-2}} - x_{f_{(i-2)d}} \right) \right. \\ & + q_{i-1} \left(x_{f_{i-1}} - x_{f_{(i-1)d}} \right) + q_i \left(x_{f_i} - x_{f_{id}} \right) \\ & \left. \backslash i \in \{ \mathbf{10, 14, 18} \} \right\} \end{aligned} \quad (5.61)$$

With

q Positive Constant Vector ($q^T \in \mathbb{R}^n$) and (n the system order)

x_f Vector States

x_{f_d} Desired Vector States to be Tracked

$$V_{f_i} = \left\{ \frac{1}{2} S_{f_i}^2 \quad / i \in \{ \mathbf{10, 14, 18} \} \right\} \quad (5.62)$$

Let,

$$\mathcal{D}^\beta S = -\gamma \text{sgn}(S) \quad ; \gamma > 0 \quad (5.63)$$

Taking β integral of both sides $\rightarrow S = -\gamma \mathcal{D}^{-\beta} \text{sgn}(S)$

Taking 1st derivative $\rightarrow \dot{S} = -\gamma \mathcal{D}^{1-\beta} \text{sgn}(S)$

For $0 < \beta < 1$, $\mathcal{D}^{1-\beta} \text{sgn}(S)$ will stay positive, and this ensures that $S\dot{S} < 0$. If we select

$\mathcal{D}^\beta S$ as,

$$\mathcal{D}^\beta S = -\gamma \text{sgn}(S) - \sigma S \quad ; \gamma \text{ and } \sigma > 0 \quad (5.64)$$

The sliding condition ($S\dot{S}$) will be also less than zero ($S\dot{S} < 0$) and increasing the term (σ)

will increase $\dot{S}$ and this will result in quicker approach towards $S = 0$. **In our systems and**

designs we assume that $\alpha = 0.5$

5.1.3.1 Altitude SMC Design in Fractional System

The altitude sliding mode controller (ASMC) of the fractional system is designed to track a fractional order reference trajectory z_{fd} .

From Equation (3.17), The fractional dynamics of the quadrotor altitude are

$$\begin{aligned} D^{0.5}x_{f_{15}} &= x_{f_{16}} \\ D^{0.5}x_{f_{16}} &= x_{f_{17}} \\ D^{0.5}x_{f_{17}} &= x_{f_{18}} \end{aligned} \tag{5.65}$$

$$D^{0.5}x_{f_{18}} = a_6x_{f_{17}} - g + b_6(c_\varphi c_\theta)U_1$$

Based on Equations (5.61), the fractional altitude sliding surface can be defined as,

$$\begin{aligned} S_{f_z} = S_{18} &= q_{15}(x_{f_{15}} - x_{f_{15d}}) + q_{16}(x_{f_{16}} - x_{f_{16d}}) \\ &+ q_{17}(x_{f_{17}} - x_{f_{17d}}) + q_{18}(x_{f_{18}} - x_{f_{18d}}) \end{aligned} \tag{5.66}$$

Taking half derivative (for $\alpha = 0.5$) of both sides,

$$\begin{aligned} D^{0.5}S_{f_z} = D^{0.5}S_{18} &= q_{15}(D^{0.5}x_{f_{15}} - D^{0.5}x_{f_{15d}}) \\ &+ q_{16}(D^{0.5}x_{f_{16}} - D^{0.5}x_{f_{16d}}) + q_{17}(D^{0.5}x_{f_{17}} - D^{0.5}x_{f_{17d}}) \\ &+ q_{18}(D^{0.5}x_{f_{18}} - D^{0.5}x_{f_{18d}}) \end{aligned} \tag{5.67}$$

Now, let

$$D^{0.5}S_{f_z} = -\gamma_z \text{sgn}(S_{f_z}) - \sigma_z S_{f_z} \quad ; \gamma_z \text{ and } \sigma_z > 0 \tag{5.68}$$

Comparing Equation (5.68) and Equation (5.67) implies,

$$\begin{aligned}
& -\gamma_z \text{sgn}(S_{f_z}) - \sigma_z S_{f_z} = q_{15} (D^{0.5} x_{f_{15}} - D^{0.5} x_{f_{15d}}) \\
& + q_{16} (D^{0.5} x_{f_{16}} - D^{0.5} x_{f_{16d}}) + q_{17} (D^{0.5} x_{f_{17}} - D^{0.5} x_{f_{17d}}) \\
& + q_{18} (D^{0.5} x_{f_{18}} - D^{0.5} x_{f_{18d}})
\end{aligned} \tag{5.69}$$

Substituting ($D^{0.5} x_{f_{15}}, D^{0.5} x_{f_{16}}, D^{0.5} x_{f_{17}}, D^{0.5} x_{f_{18}}$) from the fractional dynamic altitude of the quadrotor Equation (5.65)

$$\begin{aligned}
& -\gamma_z \text{sgn}(S_{f_z}) - \sigma_z S_{f_z} \\
& = q_{15} (x_{f_{16}} - D^{0.5} x_{f_{15d}}) + q_{16} (x_{f_{17}} - D^{0.5} x_{f_{16d}}) \\
& + q_{17} (x_{f_{18}} - D^{0.5} x_{f_{17d}}) + q_{18} (a_6 x_{f_{17}} - g + b_6 c_\varphi c_\theta U_1 - D^{0.5} x_{f_{18d}})
\end{aligned} \tag{5.70}$$

Now, solving for U_1 ,

$$\begin{aligned}
U_1 = \frac{1}{q_{18} b_6 c_\varphi c_\theta} & [-\gamma_z \text{sgn}(S_{f_z}) - \sigma_z S_{f_z} - q_{15} (x_{f_{16}} - D^{0.5} x_{f_{15d}}) \\
& - q_{16} (x_{f_{17}} - D^{0.5} x_{f_{16d}}) - q_{17} (x_{f_{18}} - D^{0.5} x_{f_{17d}}) \\
& - q_{18} (a_6 x_{f_{17}} - g - D^{0.5} x_{f_{18d}})]
\end{aligned} \tag{5.71}$$

Stability Analysis

Based on Equation (5.62), the Lyapunov function can be defined as follows,

$$V_{f_z} = V_{f_{18}} = \frac{1}{2} S_{f_z}^2 \tag{5.72}$$

Taking the first-time derivative of the Lyapunov function as

$$\dot{V}_{f_z} = \dot{V}_{f_{18}} = S_{f_z} \dot{S}_{f_z} \tag{5.73}$$

Now, we define $\mathfrak{D}^{0.5} S_{f_z}$ as,

$$\mathfrak{D}^{0.5}S_{f_z} = -\gamma_z \text{sgn}(S_{f_z}) - \sigma_z S_{f_z} \quad ; \gamma_z \text{ and } \sigma_z > 0 \quad (5.74)$$

Taking *half* integral of both sides $\rightarrow S_{f_z} = -\gamma_z \mathfrak{D}^{-0.5} \text{sgn}(S_{f_z}) - \sigma_z \mathfrak{D}^{-0.5} S_{f_z}$

Taking 1st derivative $\rightarrow \dot{S}_{f_z} = -\gamma_z \mathfrak{D}^{0.5} \text{sgn}(S_{f_z}) - \sigma_z \mathfrak{D}^{0.5} S_{f_z}$

Therefore,

$$\begin{aligned} \dot{V}_{f_z} &= \dot{V}_{f_{18}} = S_{f_z} [-\gamma_z \mathfrak{D}^{0.5} \text{sgn}(S_{f_z}) - \sigma_z \mathfrak{D}^{0.5} S_{f_z}] \\ \dot{V}_{f_z} &= \dot{V}_{f_{18}} = -\gamma_z \mathfrak{D}^{0.5} |S_{f_z}| - \sigma_z \mathfrak{D}^{0.5} S_{f_z}^2 \leq 0 \end{aligned} \quad (5.75)$$

$\mathfrak{D}^{0.5} |S_{f_z}|$ and $\mathfrak{D}^{0.5} S_{f_z}^2$ always positive as proved and discussed in [60], [65], and this ensures that $S_{f_z} \dot{S}_{f_z} < 0$

5.1.3.2 Position SMC Design in Fractional System

We developed the position sliding mode controllers (U_x, U_y) which are designed to track fractional order reference trajectories x_{f_d}, y_{f_d} respectively.

5.1.3.2.1 SMC of Position x in Fractional System

From Equation (3.17), The fractional dynamics of the quadrotor x position are

$$\begin{aligned} D^{0.5}x_{f_7} &= x_{f_8} \\ D^{0.5}x_{f_8} &= x_{f_9} \\ D^{0.5}x_{f_9} &= x_{f_{10}} \\ D^{0.5}x_{f_{10}} &= a_4 x_{f_9} + b_4 U_x U_1 \end{aligned} \quad (5.76)$$

Based on Equations (5.61), the fractional sliding surface of x position can be defined as,

$$\begin{aligned}
S_{f_x} = S_{10} = & q_7 (x_{f_7} - x_{f_{7d}}) + q_8 (x_{f_8} - x_{f_{8d}}) + q_9 (x_{f_9} - x_{f_{9d}}) \\
& + q_{10} (x_{f_{10}} - x_{f_{10d}})
\end{aligned} \tag{5.77}$$

Taking half derivative (for $\alpha = 0.5$) of both sides,

$$\begin{aligned}
D^{0.5}S_{f_x} = D^{0.5}S_{10} = & q_7 (D^{0.5}x_{f_7} - D^{0.5}x_{f_{7d}}) \\
& + q_8 (D^{0.5}x_{f_8} - D^{0.5}x_{f_{8d}}) + q_9 (D^{0.5}x_{f_9} - D^{0.5}x_{f_{9d}}) \\
& + q_{10} (D^{0.5}x_{f_{10}} - D^{0.5}x_{f_{10d}})
\end{aligned} \tag{5.78}$$

Now, let

$$D^{0.5}S_{f_x} = -\gamma_x \text{sgn}(S_{f_x}) - \sigma_x S_{f_x} \quad ; \gamma_x \text{ and } \sigma_x > 0 \tag{5.79}$$

Comparing Equation (5.79) and Equation (5.78) implies,

$$\begin{aligned}
-\gamma_x \text{sgn}(S_{f_x}) - \sigma_x S_{f_x} = & q_7 (D^{0.5}x_{f_7} - D^{0.5}x_{f_{7d}}) \\
& + q_8 (D^{0.5}x_{f_8} - D^{0.5}x_{f_{8d}}) + q_9 (D^{0.5}x_{f_9} - D^{0.5}x_{f_{9d}}) \\
& + q_{10} (D^{0.5}x_{f_{10}} - D^{0.5}x_{f_{10d}})
\end{aligned} \tag{5.80}$$

Substituting ($D^{0.5}x_{f_7}$, $D^{0.5}x_{f_8}$, $D^{0.5}x_{f_9}$, $D^{0.5}x_{f_{10}}$) the fractional dynamics of the quadrotor x position Equation (5.76)

$$\begin{aligned}
-\gamma_x \text{sgn}(S_{f_x}) - \sigma_x S_{f_x} = & q_7 (x_{f_8} - D^{0.5}x_{f_{7d}}) \\
& + q_8 (x_{f_9} - D^{0.5}x_{f_{8d}}) + q_9 (x_{f_{10}} - D^{0.5}x_{f_{9d}}) \\
& + q_{10} (a_4 x_{f_9} + b_4 U_x U_1 - D^{0.5}x_{f_{10d}})
\end{aligned} \tag{5.81}$$

Now, solving for U_x ,

$$\begin{aligned}
U_x = \frac{1}{q_{10} b_4 U_1} [& -\gamma_x \text{sgn}(S_{f_x}) - \sigma_x S_{f_x} - q_7 (x_{f_8} - D^{0.5} x_{f_{7d}}) \\
& - q_8 (x_{f_9} - D^{0.5} x_{f_{8d}}) - q_9 (x_{f_{10}} - D^{0.5} x_{f_{9d}}) \\
& - q_{10} (a_4 x_{f_9} - D^{0.5} x_{f_{10d}})]
\end{aligned} \tag{5.82}$$

Stability Analysis

Based on Equation (5.62), the Lyapunov function can be defined as follows,

$$V_{f_x} = V_{f_{10}} = \frac{1}{2} S_{f_x}^2 \tag{5.83}$$

Taking the first-time derivative of the Lyapunov function as

$$\dot{V}_{f_x} = \dot{V}_{f_{10}} = S_{f_x} \dot{S}_{f_x} \tag{5.84}$$

Now, we define $\mathfrak{D}^{0.5} S_{f_x}$ as,

$$\mathfrak{D}^{0.5} S_{f_x} = -\gamma_x \text{sgn}(S_{f_x}) - \sigma_x S_{f_x} \quad ; \gamma_x \text{ and } \sigma_x > 0 \tag{5.85}$$

Taking *half* integral of both sides $\rightarrow S_{f_x} = -\gamma_x \mathfrak{D}^{-0.5} \text{sgn}(S_{f_x}) - \sigma_x \mathfrak{D}^{-0.5} S_{f_x}$

Taking 1st derivative $\rightarrow \dot{S}_{f_x} = -\gamma_x \mathfrak{D}^{0.5} \text{sgn}(S_{f_x}) - \sigma_x \mathfrak{D}^{0.5} S_{f_x}$

Therefore,

$$\begin{aligned}
\dot{V}_{f_x} = \dot{V}_{f_{10}} &= S_{f_x} [-\gamma_x \mathfrak{D}^{0.5} \text{sgn}(S_{f_x}) - \sigma_x \mathfrak{D}^{0.5} S_{f_x}] \\
\dot{V}_{f_x} = \dot{V}_{f_{10}} &= -\gamma_x \mathfrak{D}^{0.5} |S_{f_x}| - \sigma_x \mathfrak{D}^{0.5} S_{f_x}^2 \leq 0
\end{aligned} \tag{5.86}$$

$\mathfrak{D}^{0.5} |S_{f_x}|$ and $\mathfrak{D}^{0.5} S_{f_x}^2$ always positive and this ensures that $S_{f_x} \dot{S}_{f_x} < 0$

5.1.3.2.2 SMC of Position y in Fractional System

From Equation (3.17), The fractional dynamics of the quadrotor y position are

$$\begin{aligned}
 D^{0.5}x_{f_{11}} &= x_{f_{12}} \\
 D^{0.5}x_{f_{12}} &= x_{f_{13}} \\
 D^{0.5}x_{f_{13}} &= x_{f_{14}} \\
 D^{0.5}x_{f_{14}} &= a_5x_{f_{13}} + b_5U_yU_1
 \end{aligned} \tag{5.87}$$

Following the same steps as altitude and x position SMC designs, the designed controller (U_y) for the quadrotor y position in fractional system can be derived as follows,

$$\begin{aligned}
 U_y = \frac{1}{q_{14}b_5U_1} &[-\gamma_y \text{sgn}(S_{f_y}) - \sigma_y S_{f_y} - q_{11}(x_{f_{12}} - D^{0.5}x_{f_{11d}}) \\
 &- q_{12}(x_{f_{13}} - D^{0.5}x_{f_{12d}}) - q_{13}(x_{f_{14}} - D^{0.5}x_{f_{13d}}) \\
 &- q_{14}(a_5x_{f_{13}} - D^{0.5}x_{f_{14d}})]
 \end{aligned} \tag{5.88}$$

Stability Analysis

Based on Equation (5.62), the Lyapunov function is designed as,

$$V_{f_y} = V_{f_{10}} = \frac{1}{2} S_{f_y}^2 \tag{5.89}$$

After simplification, the time derivative is

$$\dot{V}_{f_y} = \dot{V}_{f_{14}} = -\gamma_y \mathfrak{D}^{0.5} |S_{f_y}| - \sigma_y D^{0.5} S_{f_y}^2 \leq 0 \tag{5.90}$$

5.1.3.3 Attitude SMC Design in Fractional System

Because the rotating subsystem was not subjected to the fractional scheme transformation, the sliding mode controllers for the fractional system are the same as for the integer system, as shown in Equations (5.30), (5.37) and (5.42).

5.1.4 SMC Simulation

In this section, we present the simulation results of the proposed SMCs, which are applied for integer and fractional order systems, and we investigate them using three different case studies.

Case Study 1: Step Inputs Trajectories (with SMC)

We study both integer and fractional systems of quadrotor by applying SMC approach and setting the references or the inputs (φ, θ and ψ) and (x, y, z) as step inputs as shown in

Table 5-1.

Table 5-2 shows the summary of the systems (integer/fractional ones) performance in terms of overshoot OS, and rising time RT when SMCs applied and the step inputs stated as desired references.

Table 5-2: Integer and Fractional Systems Performance when SMCs are Applied

System States	Desired Values	Integer System		Fractional System	
		OS%	RT (sec)	OS %	RT (sec)
Altitude (z)	2 m	0.505	0.217	0.881	0.008
Position (x and y)	1 m	0.505	0.216	0.892	0.008
Attitude (φ and θ)	2°	0.505	0.864	0.505	0.864
Heading (ψ)	5°	0.505	1.303	0.505	1.303

Figure 5-1, Figure 5-2 and Figure 5-3 show the responses of integer and fractional systems' x and y positions, as well as altitude states, with the desired values (1 m, 1 m and 2 m) respectively when the SMCs are applied. While, the responses of angle states in both integer and fractional systems are shown in (Figure 5-4, Figure 5-5 and Figure 5-6) with the desired values ($2^\circ, 2^\circ$ and 5°) respectively.

Figure 5-7, Figure 5-8, Figure 5-9 and Figure 5-10 depict the sliding mode control inputs in both integer and fractional systems that controlled the quadrotor's altitude, attitude and heading, while the virtual sliding mode control inputs in both integer and fractional systems are shown in (Figure 5-11 and Figure 5-12)

Figure 5-1: x and x_f Position States with Desired Reference (1 m) under SMCs and the Error Between Them

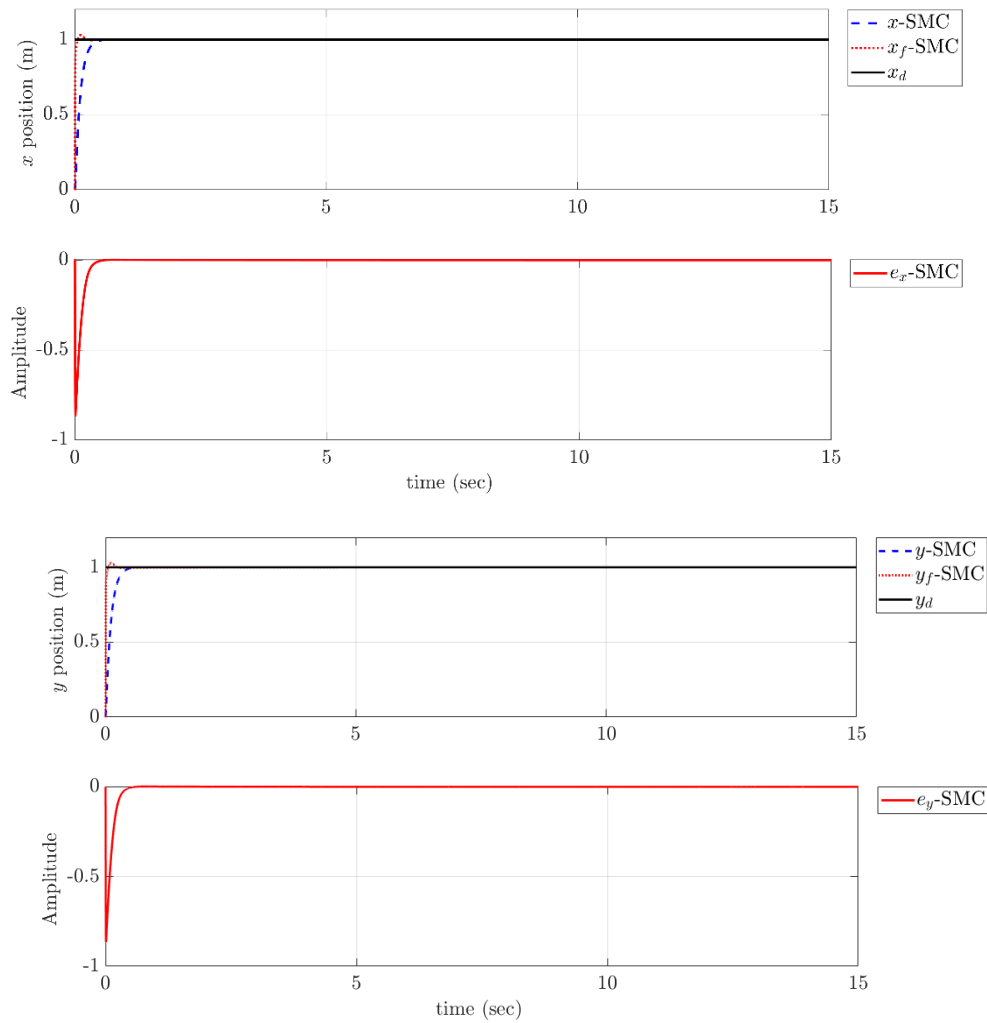


Figure 5-2: y and y_f Position States with Desired Reference (1 m) under SMCs and the Error Between Them

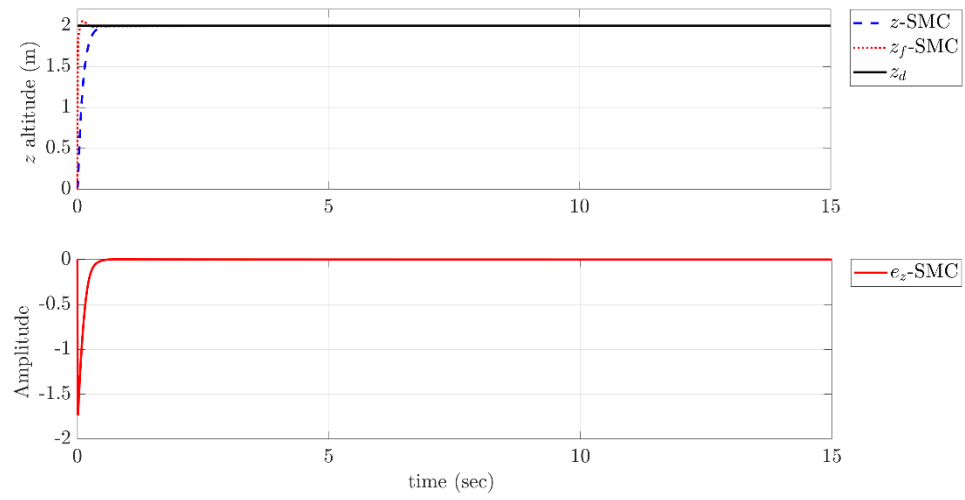


Figure 5-3: z and z_f Altitude States with Desired Reference (2 m) under SMCs and the Error Between Them

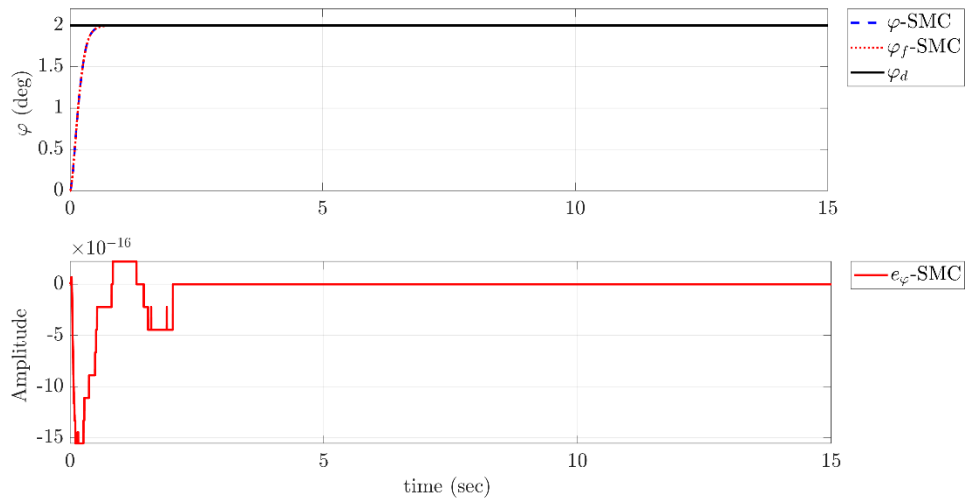


Figure 5-4: φ and φ_f Angle States with (2°) under SMCs and the Error Between Them

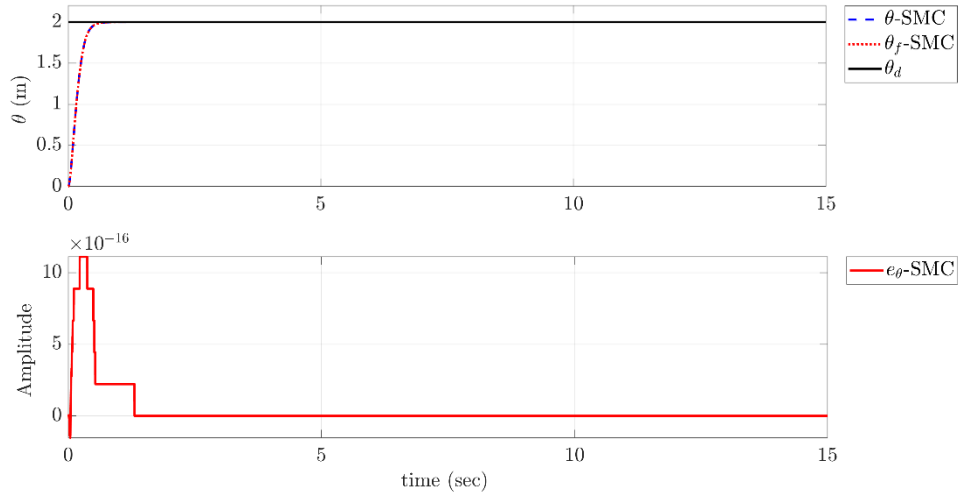


Figure 5-5: θ and θ_f Angle States with (2°) under SMCs and the Error Between Them

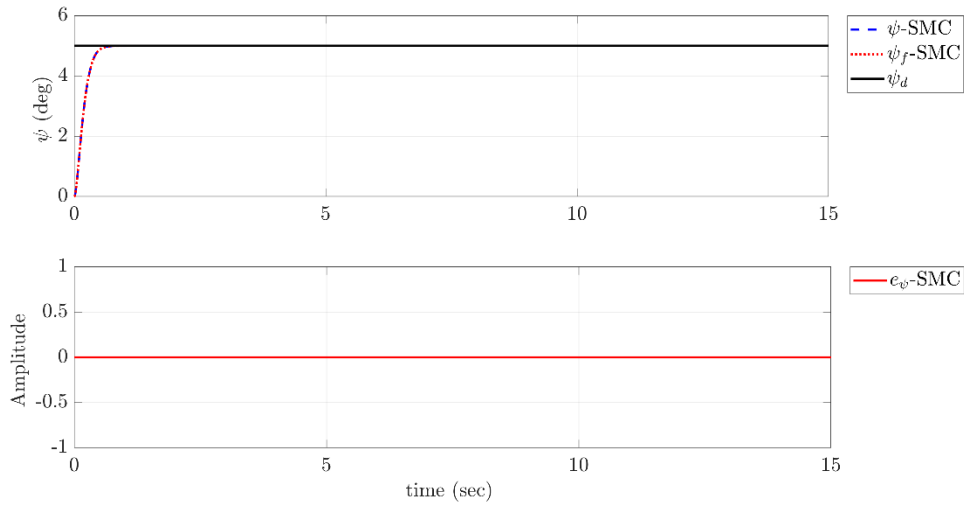


Figure 5-6: ψ and ψ_f Angle States with (5°) under SMCs and the Error Between Them

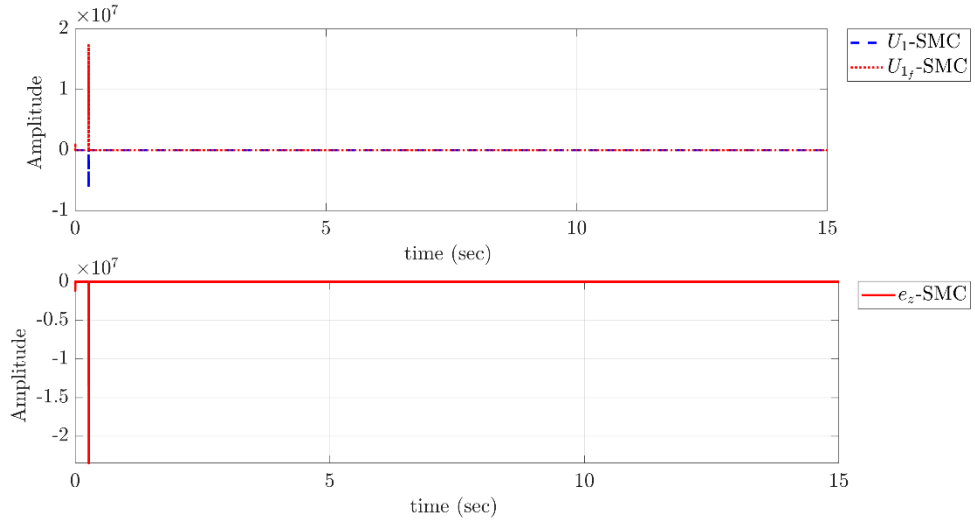


Figure 5-7: SMC Inputs (U_1 and U_{1f}) in both Integer and Fractional Systems and the Error Between Them

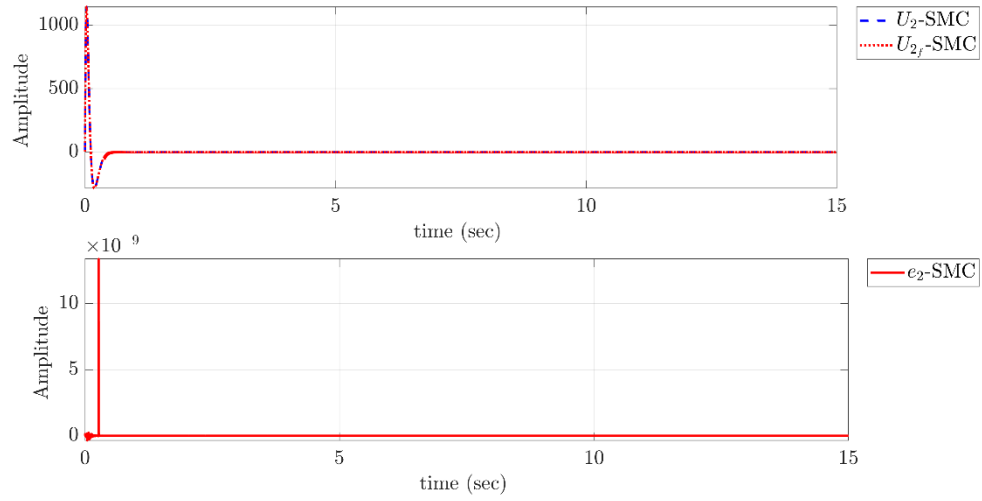


Figure 5-8: SMC Inputs (U_2 and U_{2f}) in both Integer and Fractional Systems and the Error Between Them

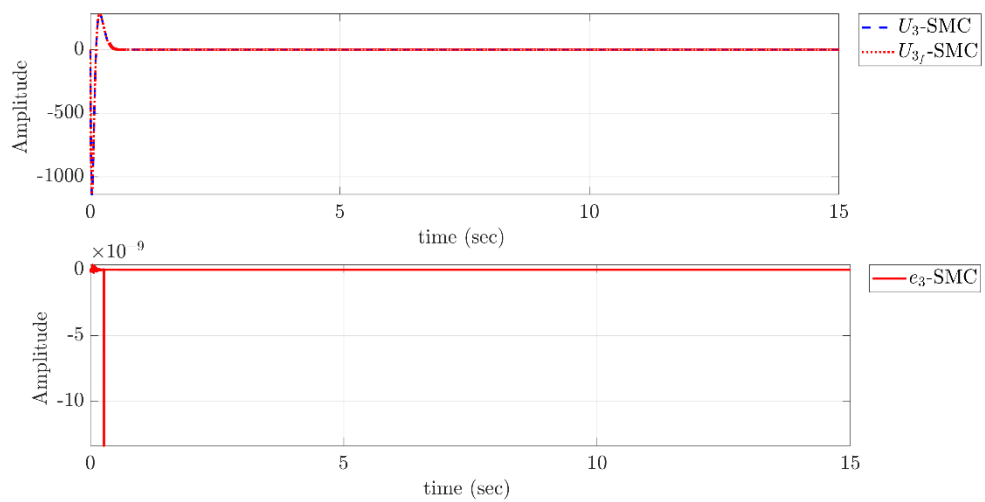


Figure 5-9: SMC Inputs (U_3 and U_{3f}) in both Integer and Fractional Systems and the Error Between Them

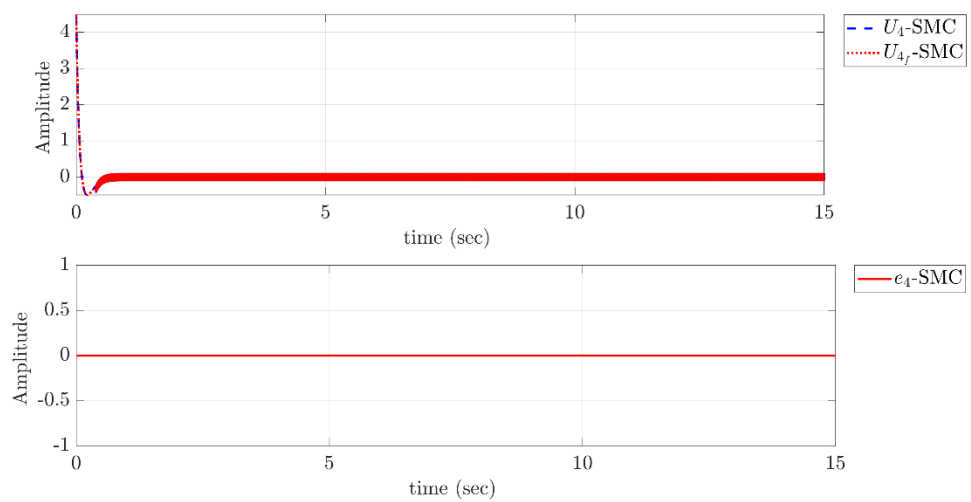


Figure 5-10: SMC Inputs (U_4 and U_{4f}) in both Integer and Fractional Systems and the Error Between Them

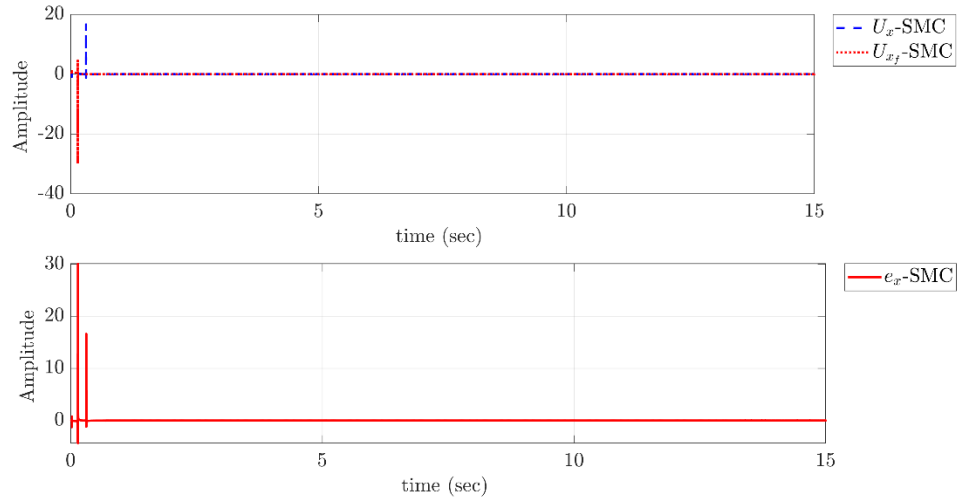


Figure 5-11: SMC Inputs (U_x and U_{x_f}) in both Integer and Fractional Systems and the Error Between Them

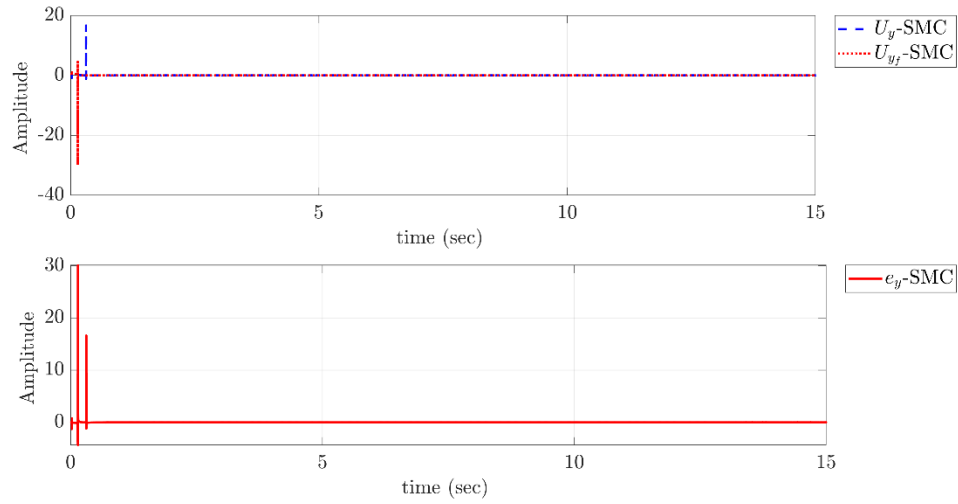


Figure 5-12: SMC Inputs (U_y and U_{y_f}) in both Integer and Fractional Systems and the Error Between Them

Observation:

It can be observed that all the fractional responses (x_f, y_f, z_f) are improved in terms of rising time (RT) comparing with integer responses (x, y, z) as shown in figures (Figure 5-1, Figure 5-2 and Figure 5-3) and Table 5-2. While, the responses of angle states in both integer and fractional systems are the same because we do not apply the fractional scheme

transformation on the rotational subsystem of the quadrotor model. Also, the SMC inputs in fractional system has less chattering comparing with the SMC inputs in integer system, while the SMC inputs that control the quadrotor angles are the same in both systems and they have high chattering.

Case Study 2: Circular Shape Trajectory (With SMC)

We move the quadrotor of both the integer and the fractional systems along a circular path which is listed in Table 5-1 as second case study (Case 2) and it is shown in Figure 5-13 as 3D plot. The roll, pitch and yaw angles (φ, θ, ψ) are set to be as step functions $(2^\circ, 2^\circ, 5^\circ)$ and their responses in both integer and fractional systems are shown in Figure 5-14. Also, the SMC inputs that control the quadrotor angles in both integer and fractional systems are shown in Figure 5-15.

On the other hand, the responses of quadrotor positions in both integer and fractional systems are shown in Figure 5-16. Furthermore, the SMC inputs that control the quadrotor positions in both systems are also shown in Figure 5-17.

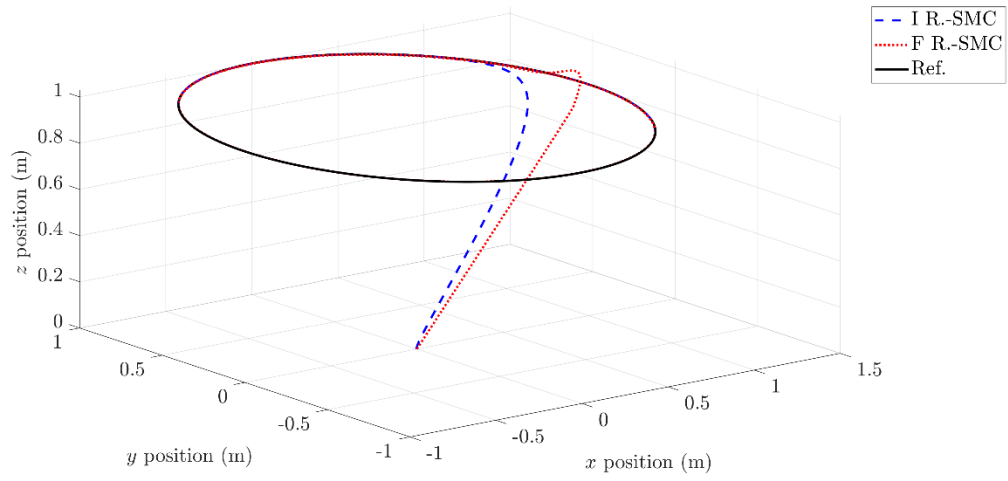


Figure 5-13: Circular Trajectory Response under SMC - Case 2

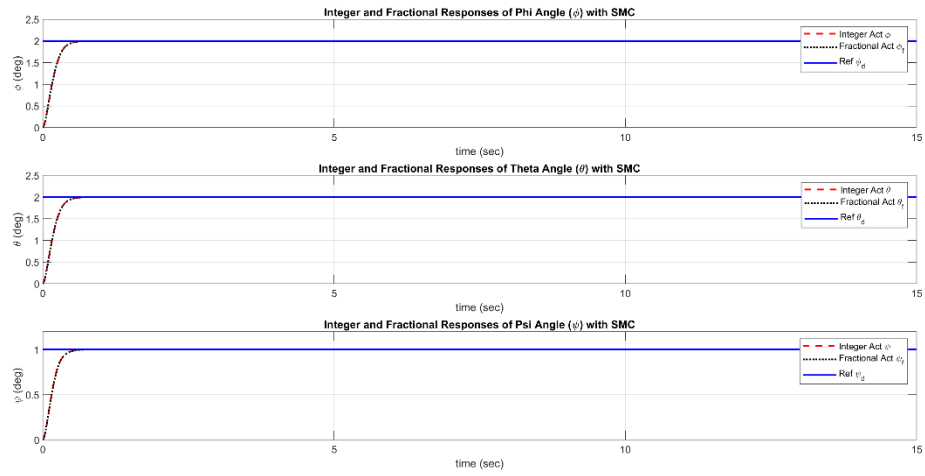
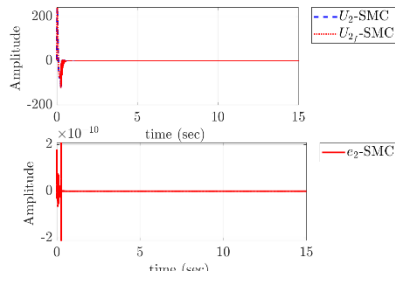
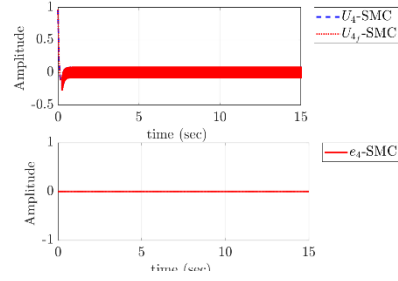


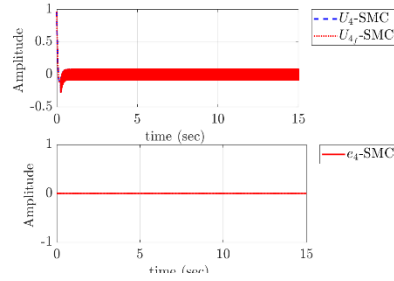
Figure 5-14: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under SMC - Case 2



(a) Control Inputs U_2 and U_{2f}



(b) Control Inputs U_3 and U_{3f}



(c) Control Inputs U_4 and U_{4f}

Figure 5-15: Angle SMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2

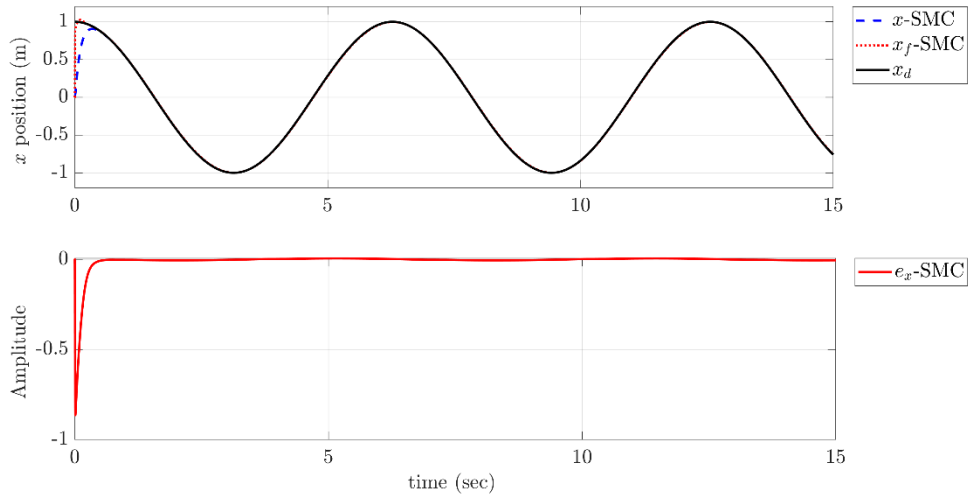
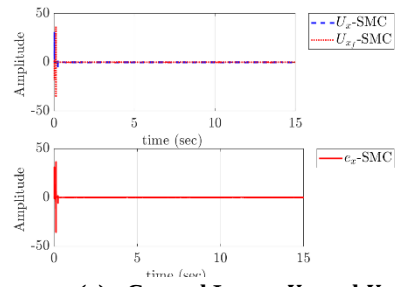
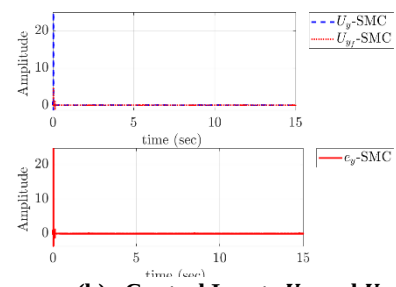


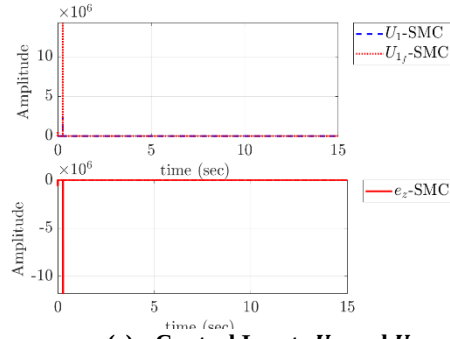
Figure 5-16 x Position Response in Integer and Fractional Systems under SMC - Case 2



(a) Control Inputs U_x and U_{x_f}



(b) Control Inputs U_y and U_{y_f}



(c) Control Inputs U_1 and U_{1_f}

Figure 5-17: Position SMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2

Case Study 3: Helical Shape Trajectory (With SMC)

We drive the quadrotors of both the integer and factional systems along a helical route, which is stated as third case study (Case 3) in Table 5-1 and shown as a 3D plot in Figure 5-18. The roll, pitch and yaw angles (φ, θ, ψ) are defined as step functions ($2^\circ, 2^\circ, 5^\circ$) and their responses in both integer and fractional systems are shown in Figure 5-19. Also, the SMC inputs that control the angles of the quadrotor in both integer and fractional systems are shown in Figure 5-20.

On the other hand, the responses of quadrotor positions in both integer and fractional systems are shown in Figure 5-21. Furthermore, Figure 5-22 depicts the SMC inputs that regulate the quadrotor positions in both systems.

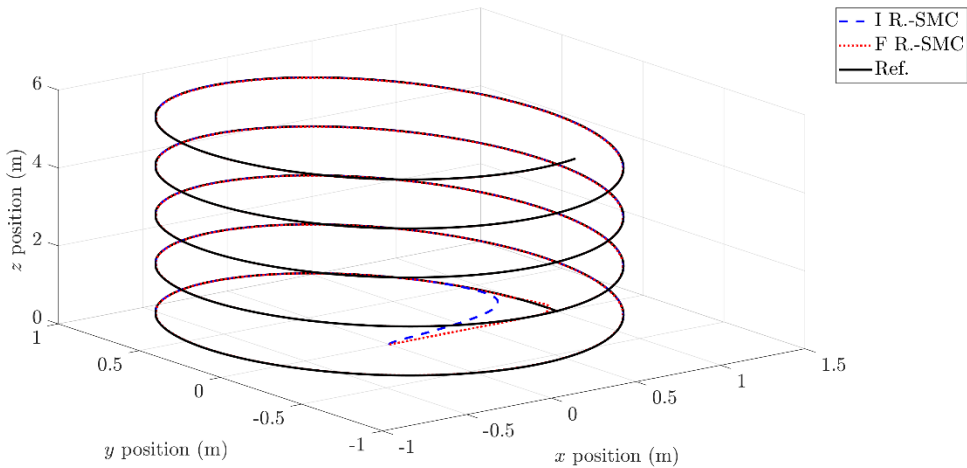


Figure 5-18: Helical Trajectory Response under SMC – Case 3

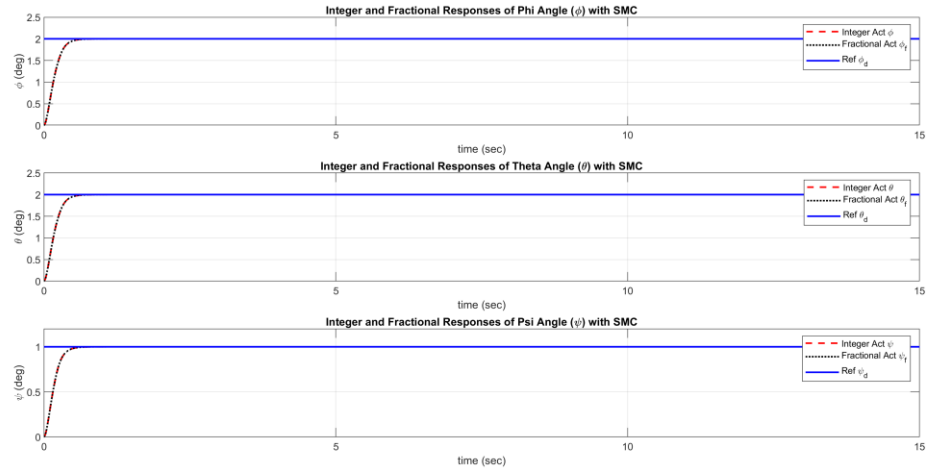


Figure 5-19: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under SMC – Case 3

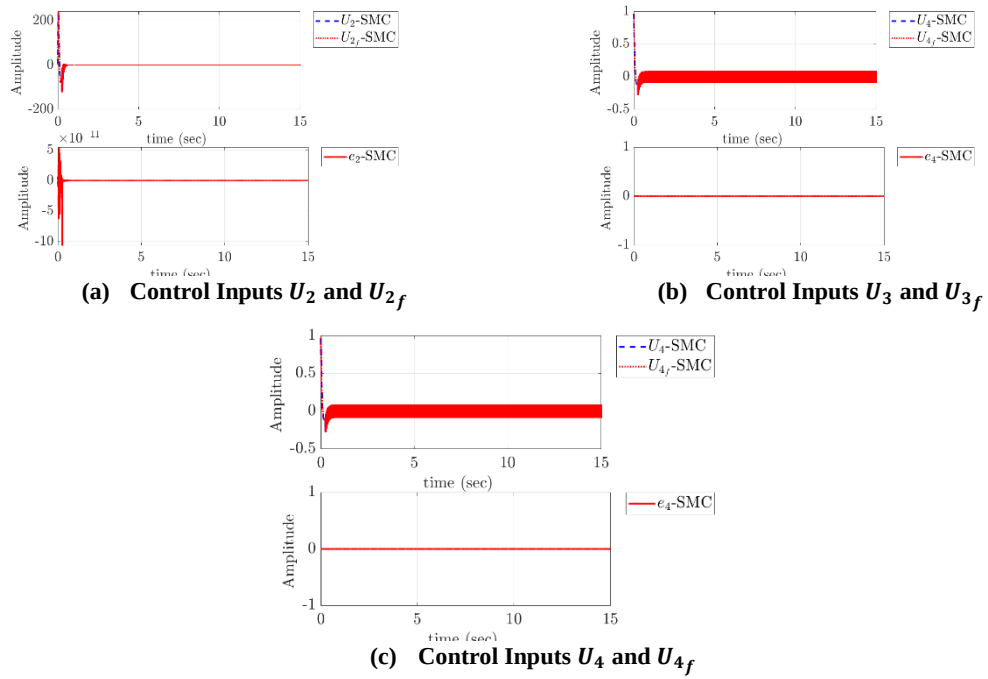


Figure 5-20: Angle SMC Inputs in Integer and Fractional Systems and the Error Between Them – Case 3

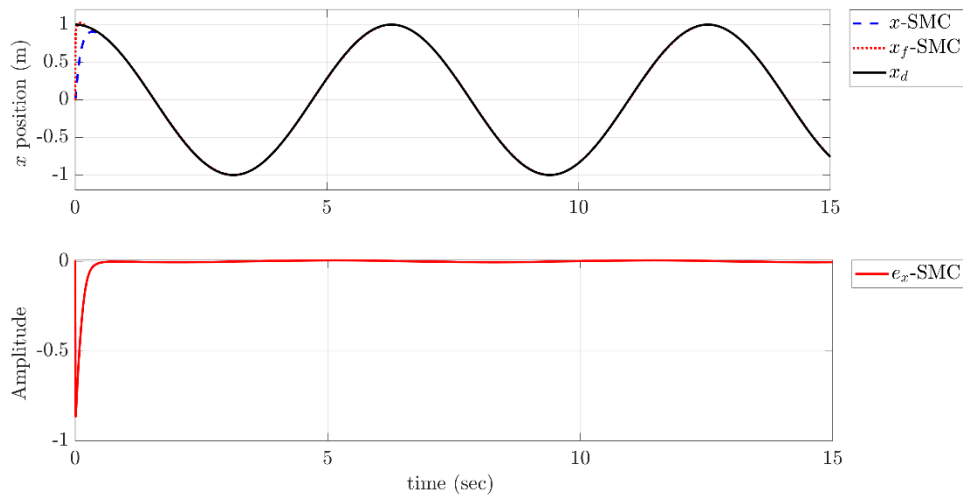


Figure 5-21: x Position Responses in Integer and Fractional Systems under SMC and the Error Between Them – Case 3

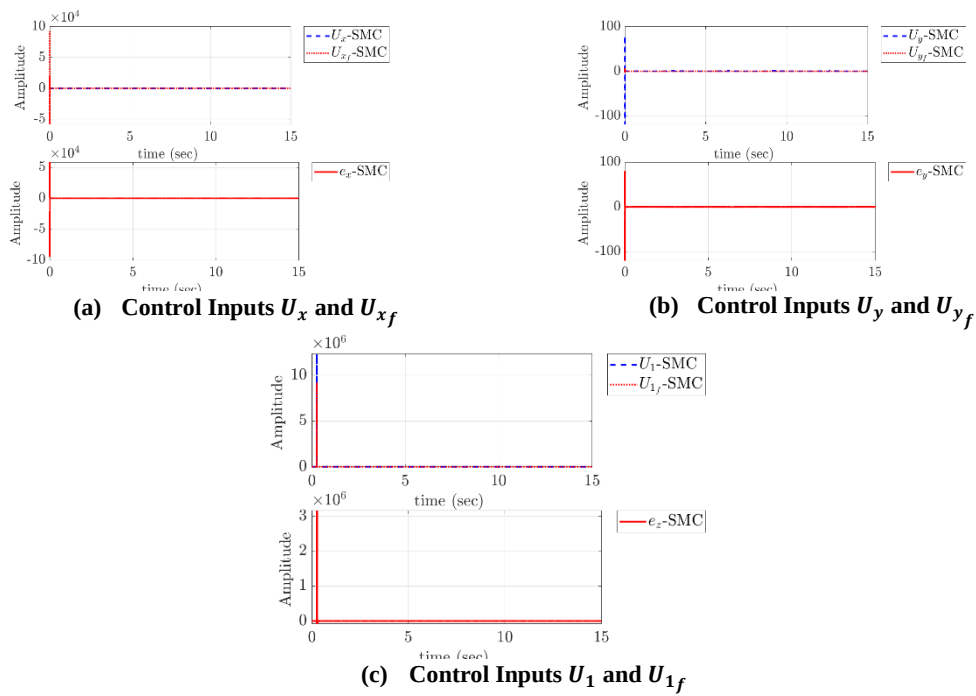


Figure 5-22: Position SMC Inputs in Integer and Fractional Systems – Case 3

5.1.5 SMC Results and Discussion

The primary goal of designing SMC is to improve the performance in both integer and fractional systems by stabilizing them and following a desired reference.

We observed that the proposed SMCs stabilize both the integer and the fractional systems and followed the desired trajectories as shown above in the stated case studies.

The simulated results of the all the case studies show that the responses of the position states in fractional system are enhanced in term of tracking accuracy and response speed. Also, The SMC introduces high chattering due to the discontinuity in the controller but, the designed SMCs that control the positions of quadrotor in fractional system have less chattering comparing with those in integer system as shown in above figures.

5.2 Fractional Order Sliding Mode Control (FOSMC) Design

In this section, we proposed a nonlinear FOSMC to control the quadrotor states because the quadrotor system is a type of nonlinear systems.

5.2.1 Introduction

Sliding mode control (SMC) is a form of variable structure control system that provides discontinuous feedback by switching once the system reaches the sliding surface and then forcing the system to stay there. The sliding variable is a function of the system states, and when the controlled states enter the sliding surface, it represents the behavior of the controlled system, resulting in a lower dynamics order than the original system. As a result, SMC is classified as robust control since the lowered order has the benefit of being insensitive to parameter uncertainty and disturbances.

Recently, the fractional order was added to the SMC to enhance the control system, and it is known as the FOSMC.

5.2.2 FOSMC Design of Integer System

As shown in the next sections, we propose fractional order sliding mode controllers to control the integer quadrotor dynamic model's altitude, attitude, heading, and x, y positions. To begin, we define the sliding surfaces in Equation (5.91) and the Lyapunov functions in Equation (5.92) that ensure the stability of the proposed FOSMCs.

$$\begin{aligned} S_{i_f} = \{ & k_{i-1}(x_{i-1} - x_{(i-1)d}) + k_i D^\gamma(x_i - x_{id}) \quad / i \\ & \in \{2, 4, 6, 8, 10, 12\} \end{aligned} \quad (5.91)$$

With

- k Positive Constant Vector ($k^T \in \mathbb{R}^n$) and (n the system order)
- x Vector States
- x_d Desired Vector States to be Tracked
- D^γ Fractional derivative (γ the fractional order)

$$V_{i_f} = \left\{ \frac{1}{2} S_{i_f}^2 \quad / i \in \{2, 4, 6, 8, 10, 12\} \right. \quad (5.92)$$

5.2.2.1 Altitude FOSMC Design in Integer System

In this part, the altitude fractional order sliding mode controller (AFOSMC) in integer system is developed to track a reference trajectory z_d and based on Equation (5.91), it can be defined as:

$$S_{z_f} = S_{12_f} = k_{11}(x_{11} - x_{11d}) + k_{12} D^\gamma(x_{12} - x_{12d}) \quad (5.93)$$

Taking the derivative of the sliding surface (S_z) would results:

$$\dot{S}_{z_f} = -q_{z_f} \text{sgn}(S_{z_f}) - k_{z_f} S_{z_f} \quad (5.94)$$

Where

- $(q_{z_f} \text{sgn}(S_{z_f}) + k_{z_f} S_{z_f})$ represents the discontinuous part that satisfies the reachability condition.

q_{z_f} and $k_{z_f} > 0$

$$\begin{aligned} \dot{S}_{z_f} &= k_{11}(\dot{x}_{11} - \dot{x}_{11d}) + k_{12} D^\gamma(\dot{x}_{12} - \dot{x}_{12d}) \\ &= -q_{z_f} \text{sgn}(S_{z_f}) - k_{z_f} S_{z_f} \end{aligned} \quad (5.95)$$

Substituting the altitude dynamic equations ($\dot{x}_{11}$ and $\dot{x}_{12}$) implies,

$$\begin{aligned} k_{11}(x_{12} - \dot{x}_{11d}) + k_{12} D^\gamma(a_6 x_{12} - g + b_6 c_\varphi c_\theta U_1 - \dot{x}_{12d}) \\ = -q_{z_f} \text{sgn}(S_{z_f}) - k_{z_f} S_{z_f} \end{aligned} \quad (5.96)$$

Using the linear property of the fractional order operator,

$$\begin{aligned} k_{11}(x_{12} - \dot{x}_{11d}) + k_{12} D^\gamma(a_6 x_{12} - g) + k_{12} b_6 D^\gamma(c_\varphi c_\theta U_1) - k_{12} D^\gamma \dot{x}_{12d} \\ = -q_{z_f} \text{sgn}(S_{z_f}) - k_{z_f} S_{z_f} \end{aligned} \quad (5.97)$$

Now, solving for the controller U_1

$$\begin{aligned} k_{12} b_6 D^\gamma(U_1 c_\varphi c_\theta) \\ = -q_{z_f} \text{sgn}(S_{z_f}) - k_{z_f} S_{z_f} - k_{12} D^\gamma(a_6 x_{12} - g) + k_{12} D^\gamma \dot{x}_{12d} \\ - k_{11}(x_{12} - \dot{x}_{11d}) \end{aligned} \quad (5.98)$$

Using the commutative property of the fractional order operator, we multiply both sides by $D^{-\gamma}$ to cancel the existing D^γ

$$k_{12}b_6\mathbf{D}^{-\gamma}D^\gamma(U_1c_\varphi c_\theta) = -q_{z_f}\mathbf{D}^{-\gamma}sgn(S_{z_f}) - k_{z_f}\mathbf{D}^{-\gamma}S_{z_f} \quad (5.99)$$

$$\begin{aligned} & -k_{12}\mathbf{D}^{-\gamma}D^\gamma(a_6x_{12} - g) + k_{12}\mathbf{D}^{-\gamma}D^\gamma\dot{x}_{12d} - k_{11}\mathbf{D}^{-\gamma}(x_{12} - \dot{x}_{11d}) \\ k_{12}b_6(U_1c_\varphi c_\theta) & = -q_{z_f}\mathbf{D}^{-\gamma}sgn(S_{z_f}) - k_{z_f}\mathbf{D}^{-\gamma}S_{z_f} - k_{12}(a_6x_{12} - g) \\ & + k_{12}\dot{x}_{12d} - k_{11}\mathbf{D}^{-\gamma}(x_{12} - \dot{x}_{11d}) \end{aligned} \quad (5.100)$$

Therefore, the fractional sliding mode control law of altitude in integer system will be calculated as follows:

$$\begin{aligned} U_1 & = \frac{1}{k_{12}b_6c_\varphi c_\theta} [-q_{z_f}\mathbf{D}^{-\gamma}sgn(S_{z_f}) - k_{z_f}\mathbf{D}^{-\gamma}S_{z_f} \\ & - k_{12}(a_6x_{12} - g - \dot{x}_{12d}) - k_{11}\mathbf{D}^{-\gamma}(x_{12} - \dot{x}_{11d})] \end{aligned} \quad (5.101)$$

Stability Analysis

A Lyapunov function is built as a positive definite function, with its time derivative being negative definite or semi-negative definite, in order to guarantee the stability of the designed controller.

Based on Equation (5.92), the Lyapunov function can be defined as,

$$V_{z_f} = V_{12_f} = \frac{1}{2}S_{z_f}^2 \quad (5.102)$$

Taking its time derivative as,

$$\dot{V}_{z_f} = \dot{V}_{12_f} = S_{z_f} \dot{S}_{z_f} \quad (5.103)$$

$$\begin{aligned} \dot{V}_{z_f} = \dot{V}_{12_f} & = S_{z_f} [k_{11}(x_{12} - \dot{x}_{11d}) + k_{12}D^\gamma(a_6x_{12} - g + b_6c_\varphi c_\theta U_1 \\ & - \dot{x}_{12d})] \end{aligned} \quad (5.104)$$

Now, substitute (U_1) from Equation (5.101) and simplify the result,

$$\dot{V}_{12f} = S_{zf} [k_{11}(x_{12} - \dot{x}_{11d}) + D^\gamma(a_6 x_{12} - g - q_{zf} D^{-\gamma} \text{sgn}(S_{zf})) \quad (5.105)$$

$$- k_{zf} D^{-\gamma} S_{zf} - (a_6 x_{12} - g - \dot{x}_{12d}) - k_{11} D^{-\gamma} (x_{12} - \dot{x}_{11d}) - \dot{x}_{12d}]$$

$$\dot{V}_{12f} = S_{zf} D^\gamma [-q_{zf} D^{-\gamma} \text{sgn}(S_{zf}) - k_{zf} D^{-\gamma} S_{zf}] \quad (5.106)$$

$$\dot{V}_{12f} = S_{zf} [-q_{zf} \text{sgn}(S_{zf}) - k_{zf} S_{zf}] \quad (5.107)$$

$$\dot{V}_{12f} = -q_{zf} |S_{zf}| - k_{zf} S_{zf}^2 \leq 0 \quad (5.108)$$

5.2.2.2 Attitude FOSMC Design in Integer System

This part shows the fractional order sliding mode controllers that govern the roll, pitch and yaw angles. Using Equation (5.91), the sliding surfaces can be defined in the same way as the altitude FOSMC design, and the control laws are stated as follow:

5.2.2.2.1 Roll Angle FOSMC in Integer System

The sliding surface can be defined as

$$S_{\varphi_f} = S_{2f} = k_1(x_1 - x_{1d}) + k_2 D^\gamma(x_2 - x_{2d}) \quad (5.109)$$

Taking the derivative of the sliding surface (S_{φ_f}) would results:

$$\dot{S}_{\varphi_f} = \dot{S}_{2f} = -q_{\varphi_f} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} S_{\varphi_f} \quad (5.110)$$

→

$$\dot{S}_{\varphi_f} = k_1(\dot{x}_1 - \dot{x}_{1d}) + k_2 D^\gamma(\dot{x}_2 - \dot{x}_{2d}) \quad (5.111)$$

$$= -q_{\varphi_f} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} S_{\varphi_f}$$

Substituting the roll angle dynamic equations ($\dot{x}_1$ and $\dot{x}_2$) implies,

$$\begin{aligned}
& k_1(x_2 - \dot{x}_{1d}) + k_2 D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + d_1 U_2 - \dot{x}_{2d}) \\
& = -q_{\varphi_f} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} S_{\varphi_f}
\end{aligned} \tag{5.112}$$

Using the linear property of the fractional order operator,

$$\begin{aligned}
& k_1(x_2 - \dot{x}_{1d}) + k_2 D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) + (d_1 k_2 D^\gamma U_2) \\
& - (k_2 D^\gamma \dot{x}_{2d}) = -q_{\varphi_f} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} S_{\varphi_f}
\end{aligned} \tag{5.113}$$

Now, solving for the controller U_2

$$\begin{aligned}
d_1 k_2 D^\gamma U_2 &= -q_{\varphi_f} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} S_{\varphi_f} - k_2 D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) \\
&+ k_2 D^\gamma \dot{x}_{2d} - k_1(x_2 - \dot{x}_{1d})
\end{aligned} \tag{5.114}$$

Using the commutative property of the fractional order operator, we multiply both sides by $D^{-\gamma}$ to cancel the existing D^γ

$$\begin{aligned}
d_1 k_2 \mathbf{D}^{-\gamma} D^\gamma U_2 &= -q_{\varphi_f} \mathbf{D}^{-\gamma} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} \mathbf{D}^{-\gamma} S_{\varphi_f} \\
&- k_2 \mathbf{D}^{-\gamma} D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) + k_2 \mathbf{D}^{-\gamma} D^\gamma \dot{x}_{2d} - k_1 \mathbf{D}^{-\gamma} (x_2 - \dot{x}_{1d})
\end{aligned} \tag{5.115}$$

$$\begin{aligned}
d_1 k_2 U_2 &= -q_{\varphi_f} \mathbf{D}^{-\gamma} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} \mathbf{D}^{-\gamma} S_{\varphi_f} \\
&- k_2 (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) + k_2 \dot{x}_{2d} - k_1 \mathbf{D}^{-\gamma} (x_2 - \dot{x}_{1d})
\end{aligned} \tag{5.116}$$

Therefore, the sliding mode control law of roll angle will be calculated as follows:

$$\begin{aligned}
U_2 &= \frac{1}{d_1 k_2} [-q_{\varphi_f} \mathbf{D}^{-\gamma} \text{sgn}(S_{\varphi_f}) - k_{\varphi_f} \mathbf{D}^{-\gamma} S_{\varphi_f} \\
&- k_2 (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) + k_2 \dot{x}_{2d} - k_1 \mathbf{D}^{-\gamma} (x_2 - \dot{x}_{1d})]
\end{aligned} \tag{5.117}$$

Stability Analysis

Based on Equation (5.92), the Lyapunov function can be defined as,

$$V_{\phi_f} = V_{2f} = \frac{1}{2} S_{\phi_f}^2 \quad (5.118)$$

Taking its time derivative as,

$$\dot{V}_{\phi_f} = \dot{V}_{2f} = S_{\phi_f} \dot{S}_{\phi_f} \quad (5.119)$$

$$\begin{aligned} \dot{V}_{\phi_f} = \dot{V}_{2f} = S_{\phi_f} [k_1(x_2 - \dot{x}_{1d}) + k_2 D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 + \\ d_1 U_2 - \dot{x}_{2d})] \end{aligned} \quad (5.120)$$

Now, substitute (U_2) from Equation (5.117) and simplify the result,

$$\begin{aligned} \dot{V}_{12f} = S_{z_f} [k_1(x_2 - \dot{x}_{1d}) + D^\gamma (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2 \\ - q_{\phi_f} D^{-\gamma} \text{sgn}(S_{\phi_f}) - k_{\phi_f} D^{-\gamma} S_{\phi_f} \\ - (a_1 x_4 x_6 + b_1 \Omega_r x_4 + c_1 x_2^2) + \dot{x}_{2d} - k_1 D^{-\gamma} (x_2 - \dot{x}_{1d}) \\ - \dot{x}_{2d})] \end{aligned} \quad (5.121)$$

$$\dot{V}_{2f} = S_{\phi_f} D^\gamma [-q_{\phi_f} D^{-\gamma} \text{sgn}(S_{\phi_f}) - k_{\phi_f} D^{-\gamma} S_{\phi_f}] \quad (5.122)$$

$$\dot{V}_{2f} = S_{\phi_f} [-q_{\phi_f} \text{sgn}(S_{\phi_f}) - k_{\phi_f} S_{\phi_f}] \quad (5.123)$$

$$\dot{V}_{2f} = -q_{\phi_f} |S_{\phi_f}| - k_{\phi_f} S_{\phi_f}^2 \leq 0 \quad (5.124)$$

5.2.2.2.2 Pitch Angle FOSMC in Integer System

Using the same procedures as roll angle and altitude FOSMC designs, the designed fractional order sliding model controller (U_3) for pitch angle can be derived as follows,

$$\begin{aligned} U_3 = \frac{1}{d_2 k_4} [-q_{\theta_f} D^{-\gamma} \text{sgn}(S_{\theta_f}) - k_{\theta_f} D^{-\gamma} S_{\theta_f} \\ - k_4 (a_2 x_2 x_6 + b_2 \Omega_r x_2 + c_2 x_4^2 - \dot{x}_{4d}) - k_3 D^{-\gamma} (x_4 - \dot{x}_{3d})] \end{aligned} \quad (5.125)$$

Stability Analysis

Based on Equation (5.92) the Lyapunov function is designed as,

$$V_{\theta_f} = V_{4_f} = \frac{1}{2} S_{\theta_f}^2 \quad (5.126)$$

After simplification, the time derivative is

$$\dot{V}_{\theta_f} = \dot{V}_{4_f} = -q_{\theta_f} |S_{\theta_f}| - k_{\theta_f} S_{\theta_f}^2 \leq 0 \quad (5.127)$$

5.2.2.2.3 Yaw Angle FOSMC in Integer System

The designed fractional order sliding model controller (U_4) for yaw angle can be derived using the same steps as the altitude, roll and pitch angles FOSMC designs:

$$U_4 = \frac{1}{c_3 k_6} [-q_{\psi_f} D^{-\gamma} \text{sgn}(S_{\psi_f}) - k_{\psi_f} D^{-\gamma} S_{\psi_f} - k_6 (a_3 x_2 x_4 + b_3 x_6^2 - \dot{x}_{6d}) - k_5 D^{-\gamma} (x_6 - \dot{x}_{5d})] \quad (5.128)$$

Stability Analysis

Based on Equation (5.92) the Lyapunov function is designed as,

$$V_{\psi_f} = V_{6_f} = \frac{1}{2} S_{\psi_f}^2 \quad (5.129)$$

After simplification, the time derivative is

$$\dot{V}_{\psi_f} = \dot{V}_{6_f} = -q_{\psi_f} |S_{\psi_f}| - k_{\psi_f} S_{\psi_f}^2 \leq 0 \quad (5.130)$$

5.2.2.3 Position FOSMC Design in Integer System

In this section, we developed the virtual control inputs that controlled the x and y positions of the quadrotor. According to Equation (5.91), the sliding surfaces of both (x, y) are defined as follows:

$$S_{x_f} = S_{8_f} = k_7(x_7 - x_{7d}) + k_8 D^\gamma (x_8 - x_{8d}) \quad (5.131)$$

$$S_{y_f} = S_{10_f} = k_9(x_9 - x_{9d}) + k_{10} D^\gamma (x_{10} - x_{10d}) \quad (5.132)$$

After substituting and simplification, the control signals (U_x, U_y) are

$$U_x = \frac{1}{k_8 b_4 U_1} [-q_{x_f} D^{-\gamma} \text{sign}(S_{x_f}) - k_{x_f} D^{-\gamma} S_{x_f} - k_8 (a_4 x_8 - \dot{x}_{8d}) - k_7 D^{-\gamma} (x_8 - \dot{x}_{7d})] \quad (5.133)$$

$$U_y = \frac{1}{k_{10} b_5 U_1} [-q_{y_f} D^{-\gamma} \text{sign}(S_{y_f}) - k_{y_f} D^{-\gamma} S_{y_f} - k_{10} (a_5 x_{10} - \dot{x}_{10d}) - D^{-\gamma} k_9 (x_{10} - \dot{x}_{9d})] \quad (5.134)$$

Stability Analysis

Based on Equation (5.92), the Lyapunov functions are designed as,

$$V_{x_f} = V_{8_f} = \frac{1}{2} S_{x_f}^2 \quad (5.135)$$

$$V_{y_f} = V_{10_f} = \frac{1}{2} S_{y_f}^2 \quad (5.136)$$

After simplification, the time derivative is

$$\dot{V}_{x_f} = \dot{V}_{8_f} = -q_{x_f} |S_{x_f}| - k_{x_f} S_{x_f}^2 \leq 0 \quad (5.137)$$

$$\dot{V}_{y_f} = \dot{V}_{10_f} = -q_{y_f} |S_{y_f}| - k_{y_f} S_{y_f}^2 \leq 0 \quad (5.138)$$

Which are negative $\forall t$, since $q_{x_f}, q_{y_f}, k_{x_f}, k_{y_f} > 0$

5.2.3 Fractional Ordre Sliding Mode Control (FOSMC) Design of Fractional System

In this section, we proposed three fractional order sliding mode controllers (U_1, U_x and U_y) that control altitude, and x, y positions of the fractional quadrotor dynamic model. While, the fractional order sliding mode controllers (U_2, U_3 and U_4) that controlled the quadrotor's attitude in the fractional system remain unchanged from those in the integer system as shown in Equations [(5.117), (5.125) and (5.128)].

To begin, we define the fractional order sliding surfaces in Equation (5.139) and the Lyapunov functions that ensure the stability of the proposed FOSMCs in Equation (5.140).

$$\begin{aligned} S_{FOSMC_i} = & \left\{ q_{i-3} \left(x_{f_{i-3}} - x_{f_{(i-3)d}} \right) + q_{i-2} \left(x_{f_{i-2}} - x_{f_{(i-2)d}} \right) \right. \\ & \left. + q_{i-1} \left(x_{f_{i-1}} - x_{f_{(i-1)d}} \right) + q_i D^\gamma \left(x_{f_i} - x_{f_{id}} \right) \right\} \\ & \backslash i \in \{ \mathbf{10, 14, 18} \} \end{aligned} \quad (5.139)$$

With

q Positive Constant Vector ($q^T \in \mathbb{R}^n$) and (n the system order)

x_f Vector States

x_{f_d} Desired Vector States to be Tracked

D^γ Fractional derivative (γ the fractional order)

$$V_{FOSMC_i} = \left\{ \frac{1}{2} S_{FOSMC_i}^2 \right\} / i \in \{ \mathbf{10, 14, 18} \} \quad (5.140)$$

5.2.3.1 Altitude FOSMC Design in Fractional System

The altitude fractional order sliding mode controller (AFOSMC) in fractional system is designed to track a fractional order reference trajectory z_{fd} .

Based on Equations (5.139), the fractional order altitude sliding surface can be defined as,

$$\begin{aligned} S_{FOSMC18} = & q_{15} (x_{f15} - x_{f15d}) + q_{16} (x_{f16} - x_{f16d}) \\ & + q_{17} (x_{f17} - x_{f17d}) + q_{18} \mathbf{D}^\gamma (x_{f18} - x_{f18d}) \end{aligned} \quad (5.141)$$

Taking half derivative (for $\beta = 0.5$) of both sides,

$$\begin{aligned} D^{0.5} S_{FOSMC18} = & q_{15} (D^{0.5} x_{f15} - D^{0.5} x_{f15d}) \\ & + q_{16} (D^{0.5} x_{f16} - D^{0.5} x_{f16d}) + q_{17} (D^{0.5} x_{f17} - D^{0.5} x_{f17d}) \\ & + q_{18} \mathbf{D}^\gamma (D^{0.5} x_{f18} - D^{0.5} x_{f18d}) \end{aligned} \quad (5.142)$$

Now, let

$$\begin{aligned} D^{0.5} S_{FOSMC18} = & -\rho_z \operatorname{sgn}(S_{FOSMC18}) - \kappa_z S_{FOSMC18} \quad ; \rho_z \text{ and } \kappa_z \\ & > 0 \end{aligned} \quad (5.143)$$

Comparing Equation (5.143) and Equation (5.142) implies,

$$\begin{aligned} -\rho_z \operatorname{sgn}(S_{FOSMC18}) - \kappa_z S_{FOSMC18} = & q_{15} (D^{0.5} x_{f15} - D^{0.5} x_{f15d}) \\ & + q_{16} (D^{0.5} x_{f16} - D^{0.5} x_{f16d}) + q_{17} (D^{0.5} x_{f17} - D^{0.5} x_{f17d}) \\ & + q_{18} \mathbf{D}^\gamma (D^{0.5} x_{f18} - D^{0.5} x_{f18d}) \end{aligned} \quad (5.144)$$

Substituting $(D^{0.5}x_{f_{15}}, D^{0.5}x_{f_{16}}, D^{0.5}x_{f_{17}}, D^{0.5}x_{f_{18}})$ from the fractional dynamic altitude of the quadrotor Equation (5.65)

$$\begin{aligned}
& -\rho_z \text{sgn}(S_{FOSMC_{18}}) - \kappa_z S_{FOSMC_{18}} = q_{15} (x_{f_{16}} - D^{0.5}x_{f_{15d}}) \\
& + q_{16} (x_{f_{17}} - D^{0.5}x_{f_{16d}}) + q_{17} (x_{f_{18}} - D^{0.5}x_{f_{17d}}) \\
& + q_{18} \mathbf{D}^\gamma (a_6 x_{f_{17}} - g + b_6 c_\varphi c_\theta U_1 - D^{0.5}x_{f_{18d}})
\end{aligned} \tag{5.145}$$

Using the linear property of the fractional order operator,

$$\begin{aligned}
& -\rho_z \text{sgn}(S_{FOSMC_{18}}) - \kappa_z S_{FOSMC_{18}} = q_{15} (x_{f_{16}} - D^{0.5}x_{f_{15d}}) \\
& + q_{16} (x_{f_{17}} - D^{0.5}x_{f_{16d}}) + q_{17} (x_{f_{18}} - D^{0.5}x_{f_{17d}}) \\
& + q_{18} \mathbf{D}^\gamma (a_6 x_{f_{17}} - g - D^{0.5}x_{f_{18d}}) + b_6 q_{18} \mathbf{D}^\gamma c_\varphi c_\theta U_1
\end{aligned} \tag{5.146}$$

Now, solving for the controller U_1

$$\begin{aligned}
& b_6 q_{18} \mathbf{D}^\gamma c_\varphi c_\theta U_1 = -\rho_z \text{sgn}(S_{FOSMC_{18}}) - \kappa_z S_{FOSMC_{18}} \\
& - q_{15} (x_{f_{16}} - D^{0.5}x_{f_{15d}}) - q_{16} (x_{f_{17}} - D^{0.5}x_{f_{16d}}) \\
& - q_{17} (x_{f_{18}} - D^{0.5}x_{f_{17d}}) - q_{18} \mathbf{D}^\gamma (a_6 x_{f_{17}} - g - D^{0.5}x_{f_{18d}})
\end{aligned} \tag{5.147}$$

Using the commutative property of the fractional order operator, we multiply both sides by $D^{-\gamma}$ to cancel the existing D^γ

$$\begin{aligned}
b_6 q_{18} \mathbf{D}^{-\gamma} \mathbf{D}^{\gamma} c_{\varphi} c_{\theta} U_1 &= -\rho_z \mathbf{D}^{-\gamma} \text{sgn}(S_{FOSMC18}) - \kappa_z \mathbf{D}^{-\gamma} S_{FOSMC18} \\
&- q_{15} \mathbf{D}^{-\gamma} (x_{f16} - D^{0.5} x_{f15d}) - q_{16} \mathbf{D}^{-\gamma} (x_{f17} - D^{0.5} x_{f16d}) \\
&- q_{17} \mathbf{D}^{-\gamma} (x_{f18} - D^{0.5} x_{f17d}) - q_{18} \mathbf{D}^{-\gamma} \mathbf{D}^{\gamma} (a_6 x_{f17} - g \\
&- D^{0.5} x_{f18d})
\end{aligned} \tag{5.148}$$

$$\begin{aligned}
b_6 q_{18} c_{\varphi} c_{\theta} U_1 &= -\rho_z \mathbf{D}^{-\gamma} \text{sgn}(S_{FOSMC18}) - \kappa_z \mathbf{D}^{-\gamma} S_{FOSMC18} \\
&- q_{15} \mathbf{D}^{-\gamma} (x_{f16} - D^{0.5} x_{f15d}) - q_{16} \mathbf{D}^{-\gamma} (x_{f17} - D^{0.5} x_{f16d}) \\
&- q_{17} \mathbf{D}^{-\gamma} (x_{f18} - D^{0.5} x_{f17d}) - q_{18} (a_6 x_{f17} - g \\
&- D^{0.5} x_{f18d})
\end{aligned} \tag{5.149}$$

Hence, the fractional order sliding mode control law of altitude in fractional system will be calculated as follows:

$$\begin{aligned}
U_1 &= \frac{1}{b_6 q_{18} c_{\varphi} c_{\theta}} [-\rho_z \mathbf{D}^{-\gamma} \text{sgn}(S_{FOSMC18}) - \kappa_z \mathbf{D}^{-\gamma} S_{FOSMC18} \\
&- q_{15} \mathbf{D}^{-\gamma} (x_{f16} - D^{0.5} x_{f15d}) - q_{16} \mathbf{D}^{-\gamma} (x_{f17} - D^{0.5} x_{f16d}) \\
&- q_{17} \mathbf{D}^{-\gamma} (x_{f18} - D^{0.5} x_{f17d}) - q_{18} (a_6 x_{f17} - g \\
&- D^{0.5} x_{f18d})]
\end{aligned} \tag{5.150}$$

Stability Analysis

Based on Equation (5.140), the Lyapunov function can be defined as follows,

$$V_{FOSMC18} = \frac{1}{2} S_{FOSMC18}^2 \tag{5.151}$$

Taking the first-time derivative of the Lyapunov function as

$$\dot{V}_{FOSMC_{18}} = S_{FOSMC_{18}} \dot{S}_{FOSMC_{18}} \quad (5.152)$$

Now, we define $\mathfrak{D}^{0.5} S_{FOSMC_{18}}$ as,

$$\begin{aligned} D^{0.5} S_{FOSMC_{18}} &= -\rho_z \operatorname{sgn}(S_{FOSMC_{18}}) - \kappa_z S_{FOSMC_{18}} \quad ; \rho_z \text{ and } \kappa_z \\ &> 0 \end{aligned} \quad (5.153)$$

Taking *half* integral of both sides $\rightarrow S_{FOSMC_{18}} = -\rho_z D^{-0.5} \operatorname{sgn}(S_{FOSMC_{18}}) - \kappa_z D^{-0.5} S_{FOSMC_{18}}$

Now, taking 1st derivative of both sides;

$$\rightarrow \dot{S}_{FOSMC_{18}} = -\rho_z D^{0.5} \operatorname{sgn}(S_{FOSMC_{18}}) - \kappa_z D^{0.5} S_{FOSMC_{18}}$$

Now, substituting ($\dot{S}_{FOSMC_{18}}$) in Equation (5.152),

$$\begin{aligned} \dot{V}_{FOSMC_{18}} &= S_{FOSMC_{18}} [-\rho_z D^{0.5} \operatorname{sgn}(S_{FOSMC_{18}}) - \kappa_z D^{0.5} S_{FOSMC_{18}}] \\ \dot{V}_{FOSMC_{18}} &= -\rho_z D^{0.5} |S_{FOSMC_{18}}| - \kappa_z S_{FOSMC_{18}}^2 \end{aligned} \quad (5.154)$$

$D^{0.5} |S_{FOSMC_{18}}|$ and $S_{FOSMC_{18}}^2$ always positive and as shown in [60], [65], and this ensures that

$$S_{FOSMC_{18}} \dot{S}_{FOSMC_{18}} < 0$$

5.2.3.2 Position FOSMC Design in Factional System

5.2.3.2.1 x Position FOSMC in Factional System

Based on Equations (5.139), the fractional order sliding surface of x position can be defined as,

$$\begin{aligned}
S_{FOSMC10} &= q_7 (x_{f_7} - x_{f_{7d}}) + q_8 (x_{f_8} - x_{f_{8d}}) + q_9 (x_{f_9} - x_{f_{9d}}) \\
&\quad + q_{10} \mathbf{D}^\gamma (x_{f_{10}} - x_{f_{10d}})
\end{aligned} \tag{5.155}$$

Taking half derivative (for $\beta = 0.5$) of both sides,

$$\begin{aligned}
D^{0.5} S_{FOSMCx} &= q_7 (D^{0.5} x_{f_7} - D^{0.5} x_{f_{7d}}) \\
&\quad + q_8 (D^{0.5} x_{f_8} - D^{0.5} x_{f_{8d}}) + q_9 (D^{0.5} x_{f_9} - D^{0.5} x_{f_{9d}}) \\
&\quad + q_{10} \mathbf{D}^\gamma (D^{0.5} x_{f_{10}} - D^{0.5} x_{f_{10d}})
\end{aligned} \tag{5.156}$$

Now, let

$$\begin{aligned}
D^{0.5} S_{FOSMC10} &= -\rho_x \text{sgn}(S_{FOSMC10}) - \kappa_x S_{FOSMC10} \quad ; \rho_x \text{ and } \kappa_x \\
&> 0
\end{aligned} \tag{5.157}$$

Comparing Equation (5.157) and Equation (5.156) implies,

$$\begin{aligned}
-\rho_x \text{sgn}(S_{FOSMC10}) - \kappa_x S_{FOSMC10} &= q_7 (D^{0.5} x_{f_7} - D^{0.5} x_{f_{7d}}) \\
&\quad + q_8 (D^{0.5} x_{f_8} - D^{0.5} x_{f_{8d}}) + q_9 (D^{0.5} x_{f_9} - D^{0.5} x_{f_{9d}}) \\
&\quad + q_{10} \mathbf{D}^\gamma (D^{0.5} x_{f_{10}} - D^{0.5} x_{f_{10d}})
\end{aligned} \tag{5.158}$$

Substituting ($D^{0.5} x_{f_7}$, $D^{0.5} x_{f_8}$, $D^{0.5} x_{f_9}$, $D^{0.5} x_{f_{10}}$) the fractional dynamics of the quadrotor x position, Equation (5.76);

$$\begin{aligned}
-\rho_x \text{sgn}(S_{FOSMC10}) - \kappa_x S_{FOSMC10} &= q_7 (x_{f_8} - D^{0.5} x_{f_{7d}}) \\
&\quad + q_8 (x_{f_9} - D^{0.5} x_{f_{8d}}) + q_9 (x_{f_{10}} - D^{0.5} x_{f_{9d}}) \\
&\quad + q_{10} \mathbf{D}^\gamma (a_4 x_{f_9} + b_4 U_x U_1 - D^{0.5} x_{f_{10d}})
\end{aligned} \tag{5.159}$$

Using the linear property of the fractional order operator,

$$\begin{aligned}
& -\rho_x \text{sgn}(S_{FOSMC_{10}}) - \kappa_x S_{FOSMC_{10}} = q_7 (x_{f_8} - D^{0.5} x_{f_{7d}}) \\
& + q_8 (x_{f_9} - D^{0.5} x_{f_{8d}}) + q_9 (x_{f_{10}} - D^{0.5} x_{f_{9d}}) \\
& + q_{10} \mathbf{D}^\gamma (a_4 x_{f_9} - D^{0.5} x_{f_{10d}}) + q_{10} b_4 \mathbf{D}^\gamma (U_x U_1)
\end{aligned} \tag{5.160}$$

Now, solving for U_x ,

$$\begin{aligned}
q_{10} b_4 \mathbf{D}^\gamma (U_1 U_x) &= [-\rho_x \text{sgn}(S_{FOSMC_{10}}) - \kappa_x S_{FOSMC_{10}} \\
& - q_7 (x_{f_8} - D^{0.5} x_{f_{7d}}) - q_8 (x_{f_9} - D^{0.5} x_{f_{8d}}) \\
& - q_9 (x_{f_{10}} - D^{0.5} x_{f_{9d}}) \\
& - q_{10} \mathbf{D}^\gamma (a_4 x_{f_9} - D^{0.5} x_{f_{10d}})]
\end{aligned} \tag{5.161}$$

Using the commutative property of the fractional order operator, we multiply both sides by $D^{-\gamma}$ to cancel the existing D^γ

$$\begin{aligned}
q_{10} \mathbf{D}^{-\gamma} \mathbf{D}^\gamma b_4 U_1 U_x &= [-\rho_x \text{sgn} \mathbf{D}^{-\gamma} (S_{FOSMC_{10}}) - \kappa_x \mathbf{D}^{-\gamma} S_{FOSMC_{10}} \\
& - q_7 \mathbf{D}^{-\gamma} (x_{f_8} - D^{0.5} x_{f_{7d}}) - q_8 \mathbf{D}^{-\gamma} (x_{f_9} - D^{0.5} x_{f_{8d}}) \\
& - q_9 \mathbf{D}^{-\gamma} (x_{f_{10}} - D^{0.5} x_{f_{9d}}) - q_{10} \mathbf{D}^{-\gamma} \mathbf{D}^\gamma (a_4 x_{f_9} - D^{0.5} x_{f_{10d}})]
\end{aligned} \tag{5.162}$$

$$\begin{aligned}
q_{10} b_4 U_1 U_x &= [-\rho_x \text{sgn} \mathbf{D}^{-\gamma} (S_{FOSMC_{10}}) - \kappa_x \mathbf{D}^{-\gamma} S_{FOSMC_{10}} \\
& - q_7 \mathbf{D}^{-\gamma} (x_{f_8} - D^{0.5} x_{f_{7d}}) - q_8 \mathbf{D}^{-\gamma} (x_{f_9} - D^{0.5} x_{f_{8d}}) \\
& - q_9 \mathbf{D}^{-\gamma} (x_{f_{10}} - D^{0.5} x_{f_{9d}}) - q_{10} (a_4 x_{f_9} - D^{0.5} x_{f_{10d}})]
\end{aligned} \tag{5.163}$$

Hence, the fractional order sliding mode control law of x position in fractional system will be calculated as follows:

$$\begin{aligned}
U_x = \frac{1}{q_{10}b_4U_1} & [-\rho_x \text{sgn} D^{-\gamma}(S_{FOSMC_{10}}) - \kappa_x D^{-\gamma} S_{FOSMC_{10}} \\
& - q_7 D^{-\gamma} (x_{f_8} - D^{0.5} x_{f_{7d}}) - q_8 D^{-\gamma} (x_{f_9} - D^{0.5} x_{f_{8d}}) \\
& - q_9 D^{-\gamma} (x_{f_{10}} - D^{0.5} x_{f_{9d}}) - q_{10} (a_4 x_{f_9} - D^{0.5} x_{f_{10d}})]
\end{aligned} \tag{5.164}$$

Stability Analysis

Based on Equation (5.140), the Lyapunov function can be defined as follows,

$$V_{FOSMC_{10}} = \frac{1}{2} S_{FOSMC_{10}}^2 \tag{5.165}$$

Taking the first-time derivative of the Lyapunov function as

$$\dot{V}_{FOSMC_{10}} = S_{FOSMC_{10}} \dot{S}_{FOSMC_{10}} \tag{5.166}$$

Now, we define $\mathcal{D}^{0.5} S_{FOSMC_{10}}$ as,

$$\begin{aligned}
D^{0.5} S_{FOSMC_{10}} &= -\rho_x \text{sgn}(S_{FOSMC_{10}}) - \kappa_x S_{FOSMC_{10}} \quad ; \rho_x \text{ and } \kappa_x \\
&> 0
\end{aligned} \tag{5.167}$$

Taking *half* integral of both sides $\rightarrow S_{FOSMC_{10}} = -\rho_x D^{-0.5} \text{sgn}(S_{FOSMC_{10}}) - \kappa_x D^{-0.5} S_{FOSMC_{10}}$

Now, taking 1st derivative of both sides;

$$\rightarrow \dot{S}_{FOSMC_{10}} = -\rho_x D^{0.5} \text{sgn}(S_{FOSMC_{10}}) - \kappa_x D^{0.5} S_{FOSMC_{10}}$$

Now, substituting ($\dot{S}_{FOSMC_{10}}$) in Equation (5.166),

$$\begin{aligned}\dot{V}_{FOSMC_{10}} &= S_{FOSMC_{10}}[-\rho_x D^{0.5} \text{sgn}(S_{FOSMC_{10}}) - \kappa_x D^{0.5} S_{FOSMC_{10}}] \\ \dot{V}_{FOSMC_{10}} &= -\rho_x D^{0.5} |S_{FOSMC_{10}}| - \kappa_x S_{FOSMC_{10}}^2\end{aligned}\quad (5.168)$$

$D^{0.5}|S_{FOSMC_{10}}|$ and $S_{FOSMC_{10}}^2$ always positive and as shown in [60], [65], and this ensures that

$$S_{FOSMC_{10}} \dot{S}_{FOSMC_{10}} < 0$$

5.2.3.2.2 y Position FOSMC in Factional System

Following the same steps as altitude and x position FOSMC designs, the designed controller (U_y) for the quadrotor y position in fractional system can be derived as follows,

$$\begin{aligned}U_y &= \frac{1}{q_{14} b_5 U_1} [-\rho_x \mathbf{D}^{-\gamma} \text{sgn}(S_{FOSMC_{14}}) - \kappa_x \mathbf{D}^{-\gamma} S_{FOSMC_{14}} \\ &\quad - q_{11} \mathbf{D}^{-\gamma} (x_{f_{12}} - D^{0.5} x_{f_{11d}}) \\ &\quad - q_{12} \mathbf{D}^{-\gamma} (x_{f_{13}} - D^{0.5} x_{f_{12d}}) \\ &\quad - q_{13} \mathbf{D}^{-\gamma} (x_{f_{14}} - D^{0.5} x_{f_{13d}}) \\ &\quad - q_{14} \mathbf{D}^{-\gamma} (a_5 x_{f_{13}} - D^{0.5} x_{f_{14d}})]\end{aligned}\quad (5.169)$$

Stability Analysis

Based on Equation (5.140), the Lyapunov function can be defined as follows,

$$V_{FOSMC_{14}} = \frac{1}{2} S_{FOSMC_{14}}^2 \quad (5.170)$$

After simplification, the time derivative is

$$\dot{V}_{FOSMC_{14}} = -\rho_y D^{0.5} |S_{FOSMC_{14}}| - \kappa_y S_{FOSMC_{14}}^2 \quad (5.171)$$

5.2.4 FOSMC Simulation

In this part, we show the simulation results of the proposed FOSMCs that are applied for integer and fractional order systems and we study it for three different case studies. **The used fractional order of the proposed FOSMC is ($\beta = 0.3$).**

Case Study 1: Step Inputs Trajectories (with FOSMC)

We study both integer and fractional systems of quadrotor by applying FOSMC technique and setting the references or the inputs (φ, θ and ψ) and (x, y, z) as step inputs as shown in Table 5-3.

Table 5-3 shows the summary of the systems (integer/fractional ones) performance in terms of overshoot OS, and rising time RT when FOSMCs are applied and the step inputs stated as desired references.

Table 5-3: Integer and Fractional Systems Performance when FOSMCs are Applied

System States	Desired Values	Integer System		Fractional System	
		OS%	RT (sec)	OS %	RT (sec)
Altitude (z)	2 m	0.505	0.210	0.871	0.0066
Position (x and y)	1 m	0.505	0.210	0.974	0.0067
Attitude (φ and θ)	2°	2.58	0.282	0.505	0.290
Heading (ψ)	5°	2.58	0.298	0.505	0.310

(Figure 5-23, Figure 5-24 and Figure 5-25) show the responses of x and y positions, as well as altitude states, in both integer and fractional with the desired values (1 m, 1 m and 2 m) respectively when the FOSMCs are applied. While, the responses of angle states in both

integer and fractional systems are shown in (Figure 5-26, Figure 5-27 and Figure 5-28) with the desired values (2° , 2° and 5°) respectively.

(Figure 5-29, Figure 5-30, Figure 5-31 and Figure 5-32) depict the fractional sliding mode control inputs of both integer and fractional systems that controlled the quadrotor's altitude, attitude and heading, while the fractional sliding mode control inputs (U_x and U_{x_f} , U_y and U_{y_f}) in both integer and fractional systems are shown in (Figure 5-33 and Figure 5-34)

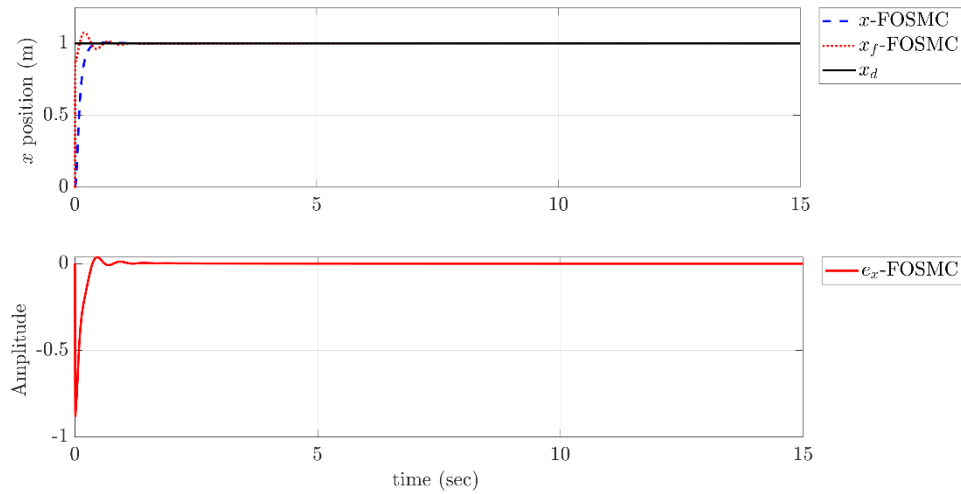


Figure 5-23: x and x_f Position States with Desired Reference (1 m) under FOSMCs and the Error Between Them

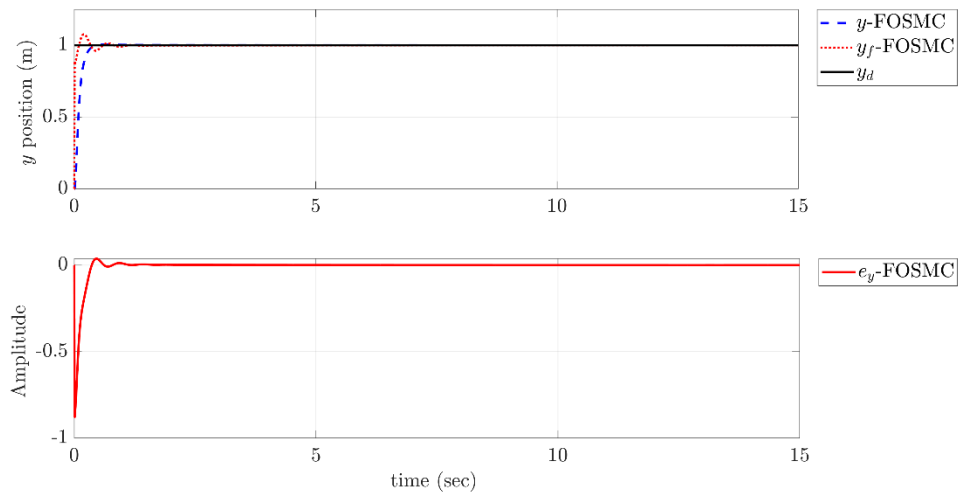


Figure 5-24: y and y_f Position States with Desired Reference (1 m) under FOSMCs and the Error Between Them

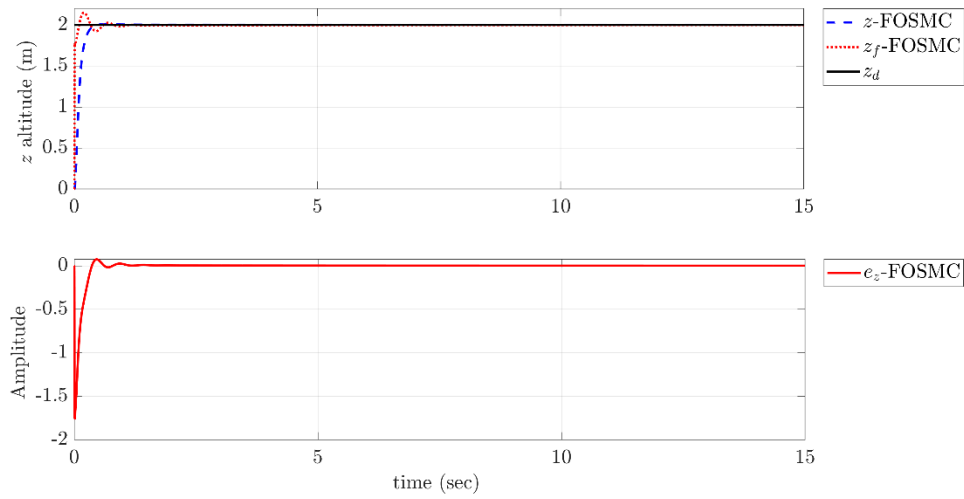


Figure 5-25: z and z_f Altitude States with Desired Reference (2 m) under FOSMCs and the Error Between Them

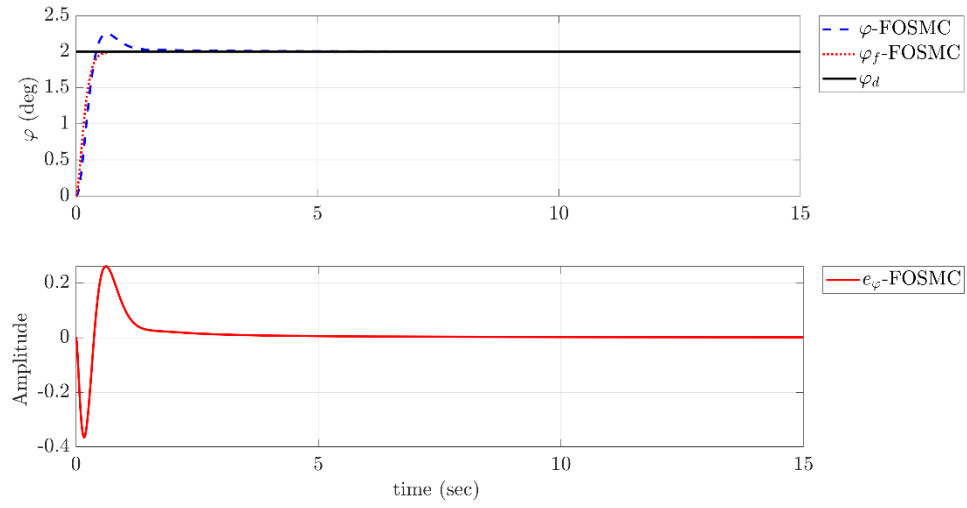


Figure 5-26: φ and φ_f Angle States with (2°) under FOSMCs and the Error Between Them

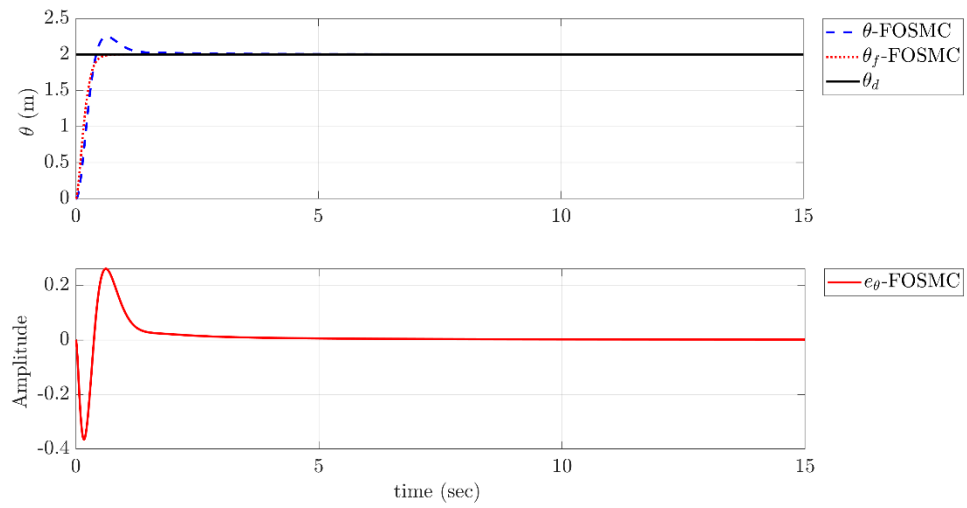


Figure 5-27: θ and θ_f Angle States with (2°) under FOSMCs and the Error Between Them

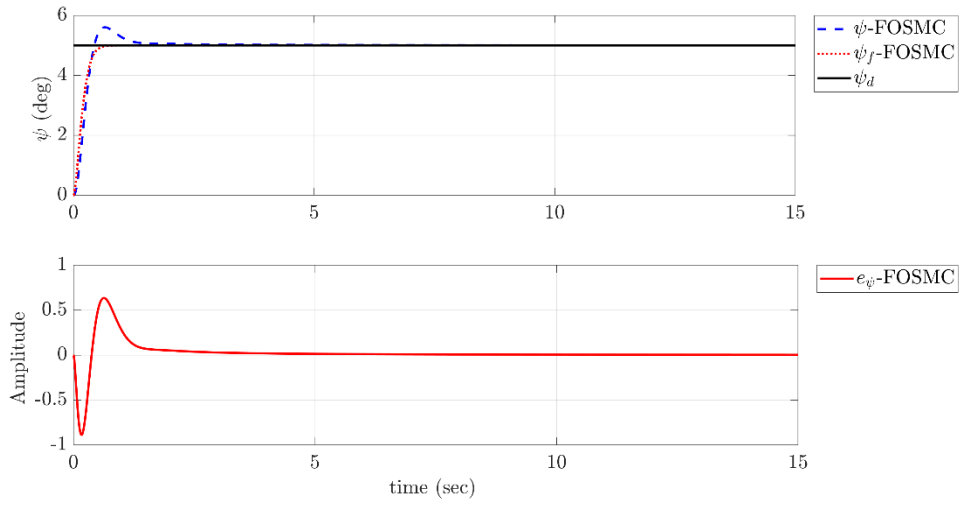


Figure 5-28: ψ and ψ_f Angle States with (5°) under FOSMCs and the Error Between Them

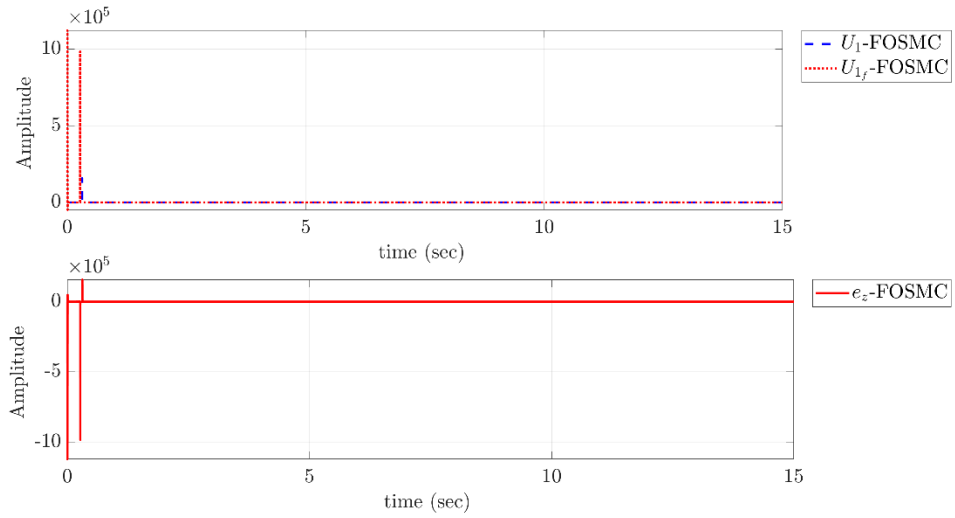


Figure 5-29: FOSMC Inputs (U_1 and U_{1_f}) in both Integer and Fractional Systems and the Error Between Them

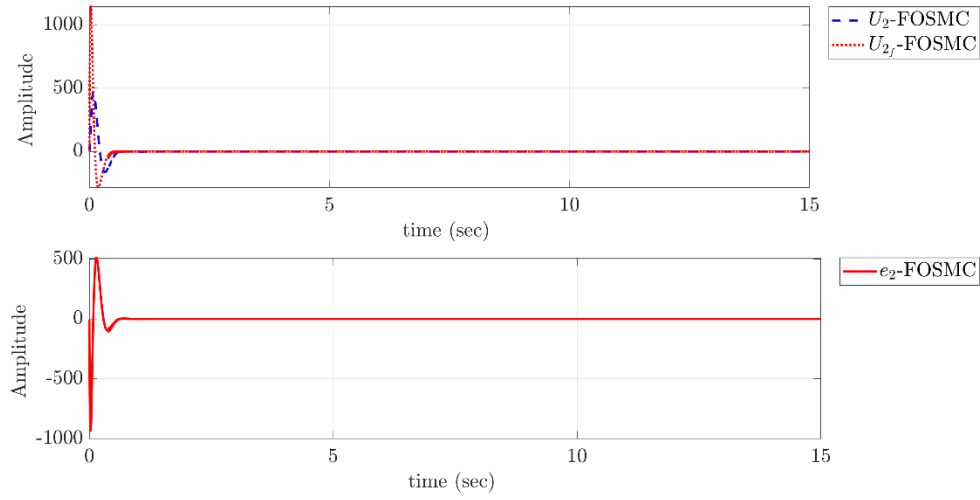


Figure 5-30: FOSMC Inputs (U_2 and U_{2f}) in both Integer and Fractional Systems and the Error Between Them

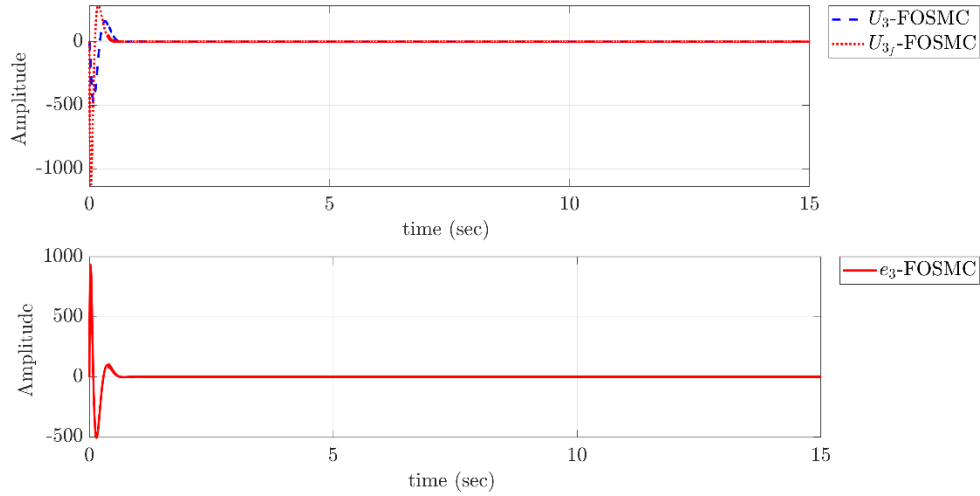


Figure 5-31: FOSMC Inputs (U_3 and U_{3f}) in both Integer and Fractional Systems and the Error Between Them

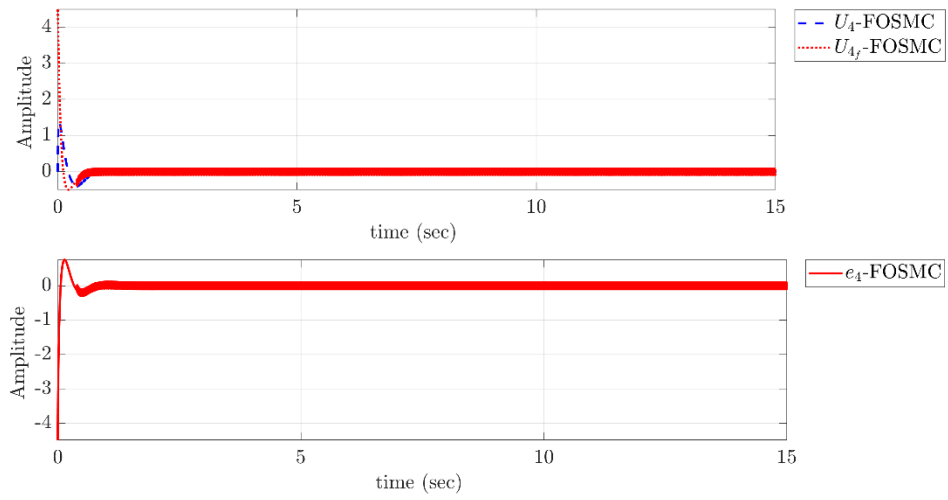


Figure 5-32: FOSMC Inputs (U_4 and U_{4f}) in both Integer and Fractional Systems and the Error Between Them

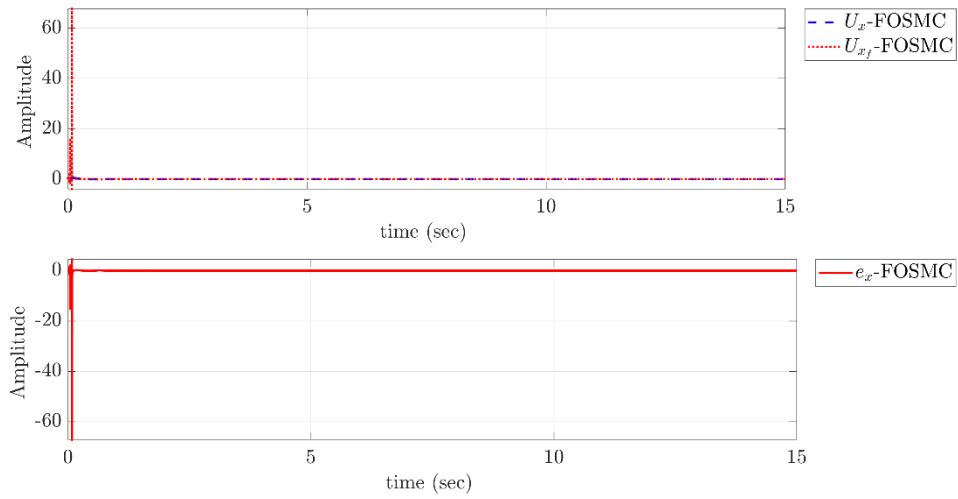


Figure 5-33: FOSMC Inputs (U_x and U_{xf}) in both Integer and Fractional Systems and the Error Between Them

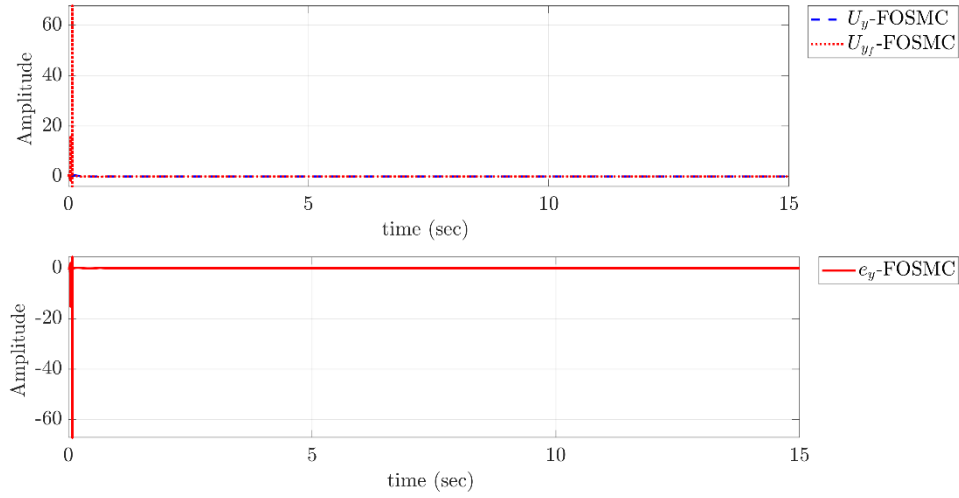


Figure 5-34: FOSMC Inputs (U_y and U_{y_f}) in both Integer and Fractional Systems and the Error Between Them

Observation:

It can be observed that all the fractional responses (x_f, y_f, z_f) are improved in terms of tracking accuracy and rising time (RT) comparing with integer responses (x, y, z) as shown in (Figure 5-23, Figure 5-24, and Figure 5-25) and Table 5-3. Also, the FOSMC inputs in integer system have huge chattering comparing with the FOSMC inputs in fractional system.

Case Study 2: Circular Shape Trajectory (With FOSMC)

We move the quadrotor of both the integer and the fractional systems along a circular path which is listed in Table 5-1 as second case study (Case 2) and it is shown in Figure 5-35 as 3D plot. The roll, pitch and yaw angles (φ, θ, ψ) are set to be as step functions ($2^\circ, 2^\circ, 5^\circ$) and their responses in both integer and fractional systems are shown in Figure

5-36. Also, the FOSMC inputs that control the quadrotor angles in both integer and fractional systems are shown in Figure 5-37.

On the other hand, the responses of quadrotor positions in both integer and fractional systems are shown in Figure 5-38. Also, Figure 5-39 shows the positions (z and z_f) plots with different fractional orders. Furthermore, the FOSMC inputs that control the quadrotor positions in both systems are also shown in Figure 5-40.

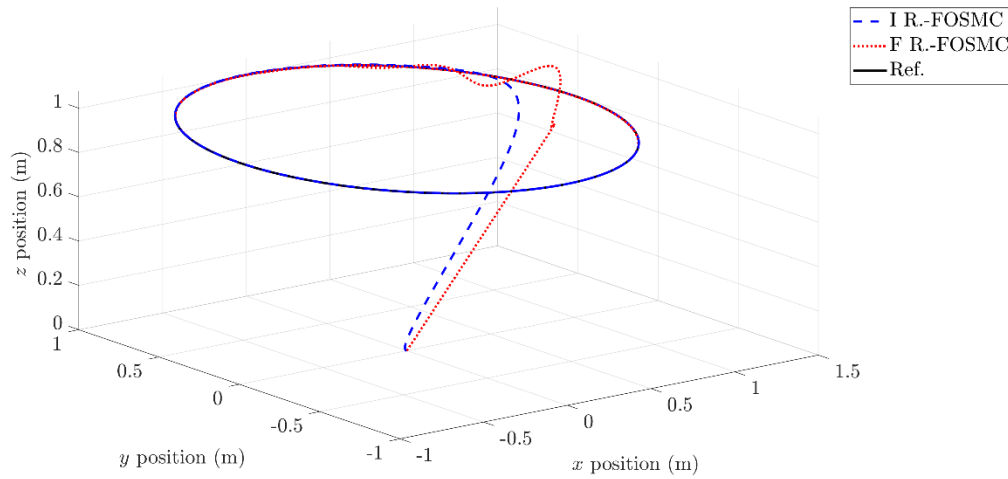


Figure 5-35: Circular Trajectory Response under FOSMC - Case 2

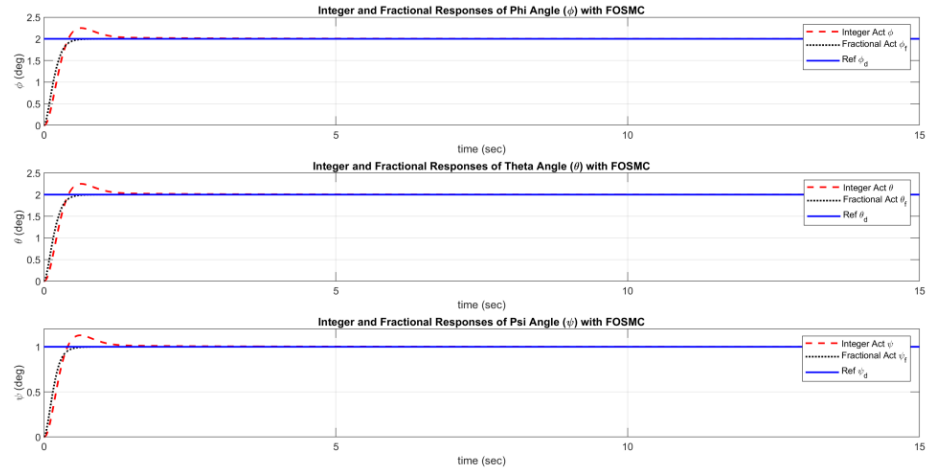


Figure 5-36: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under FOSMC - Case 2

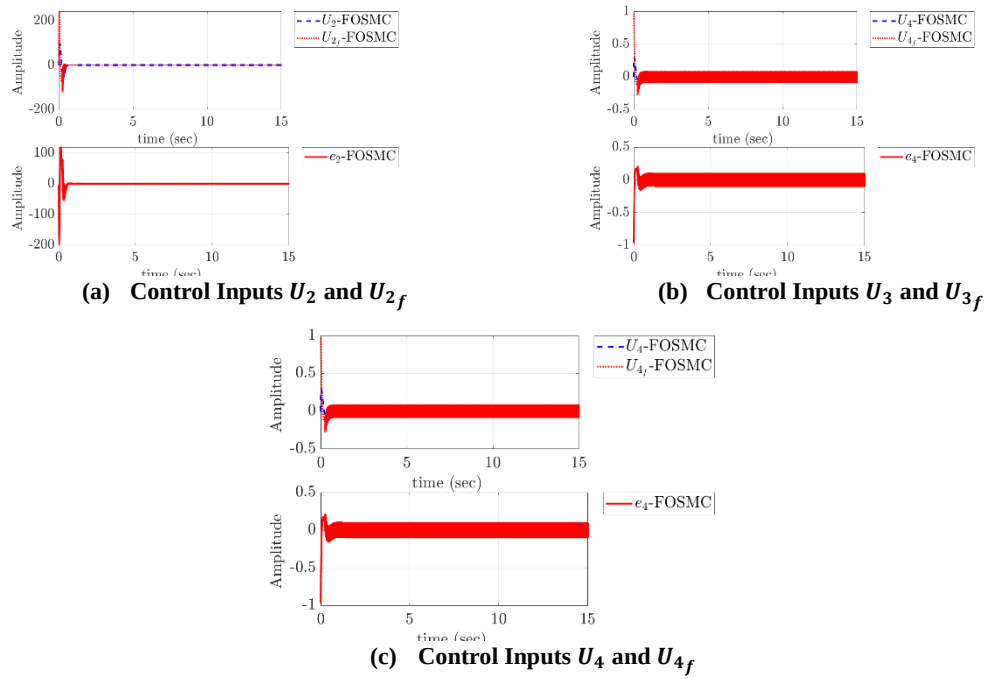


Figure 5-37: Angle FOSMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2

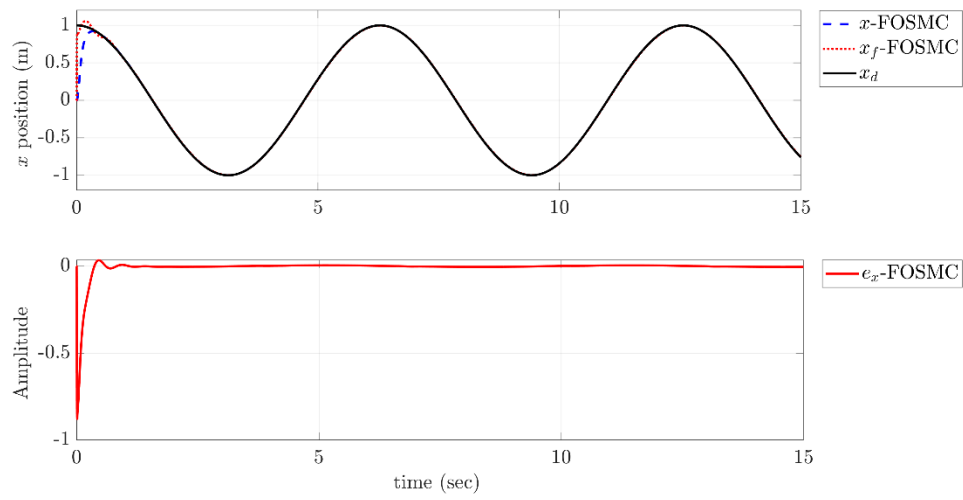


Figure 5-38: x Position Responses in Integer and Fractional Systems under FOSMC - Case 2

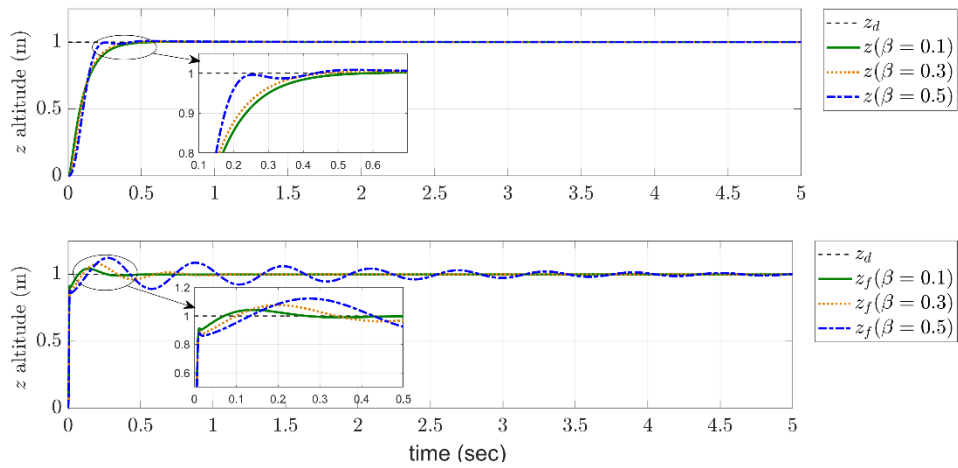


Figure 5-39: Positions (z and z_f) with different fractional orders γ

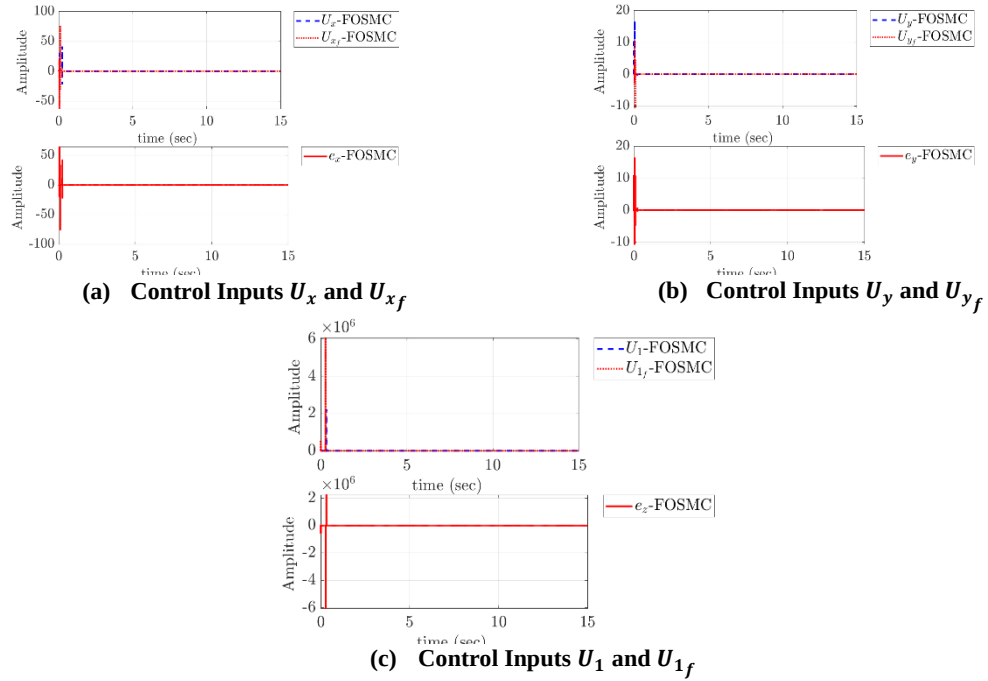


Figure 5-40: Position FOSMC Inputs in Integer and Fractional Systems and the Error Between Them - Case 2

Case Study 3: Helical Shape Trajectory (With FOSMC)

We drive the quadrotors of both the integer and fractional systems along a helical course, which is stated as third case study (Case 3) in Table 5-1 and shown as a 3D plot in Figure 5-41. The roll, pitch and yaw angles (φ, θ, ψ) are defined as step functions ($2^\circ, 2^\circ, 5^\circ$) and their responses in both integer and fractional systems are shown in Figure 5-42. Also, the FOSMC inputs that control the angles of the quadrotor in both integer and fractional systems are shown in Figure 5-43.

On the other hand, the responses of quadrotor positions in both integer and fractional systems are shown in Figure 5-44. Also, Figure 5-45 shows the positions (x and x_f) plots with different fractional orders. Furthermore, Figure 5-46 depict the FOSMC inputs that regulate the quadrotor positions in both systems.

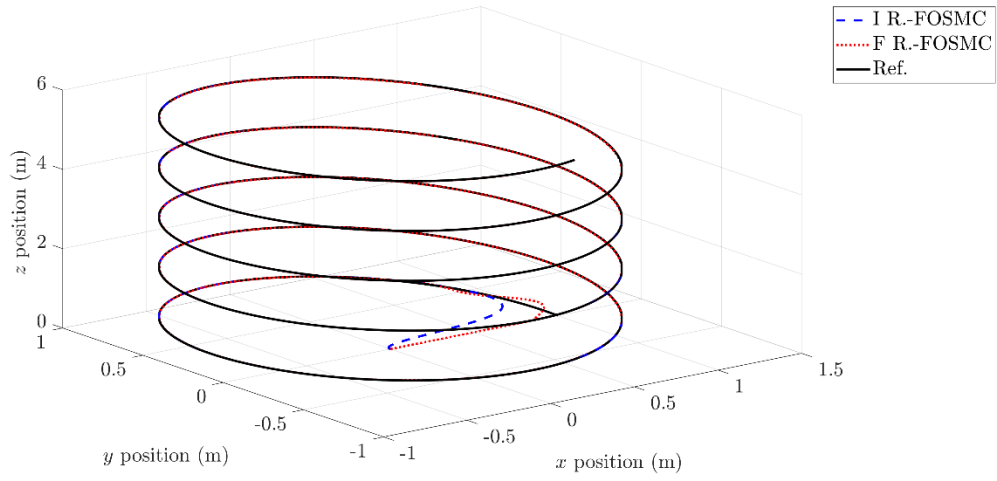


Figure 5-41: Helical Trajectory Response under FOSMC – Case 3

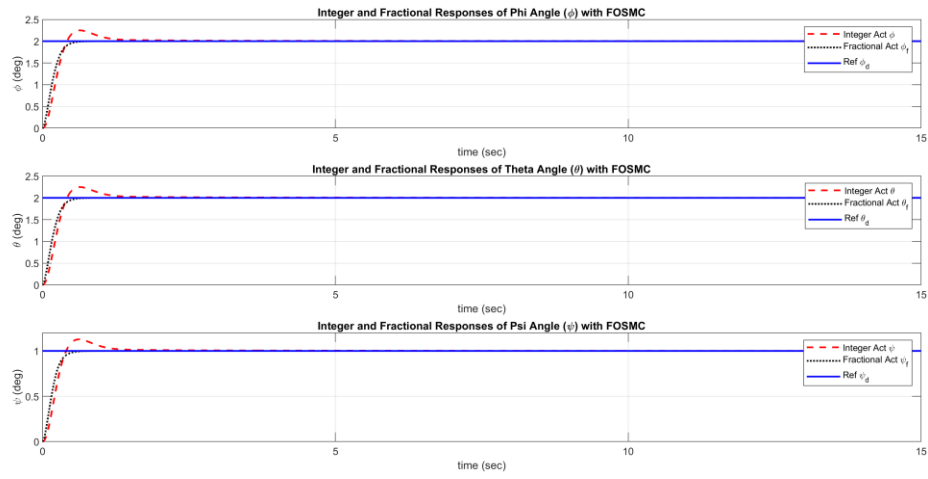
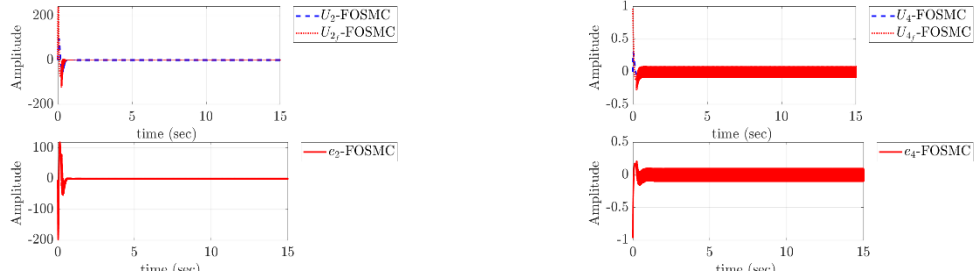
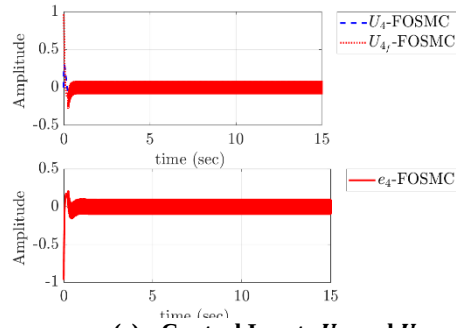


Figure 5-42: Roll, Pitch and Yaw Angles Responses in Integer and Fractional Systems under FOSMC – Case 3



(a) Control Inputs U_2 and U_{2f}



(b) Control Inputs U_3 and U_{3f}

(c) Control Inputs U_4 and U_{4f}

Figure 5-43: Angle FOSMC Inputs in Integer and Fractional Systems – Case 3

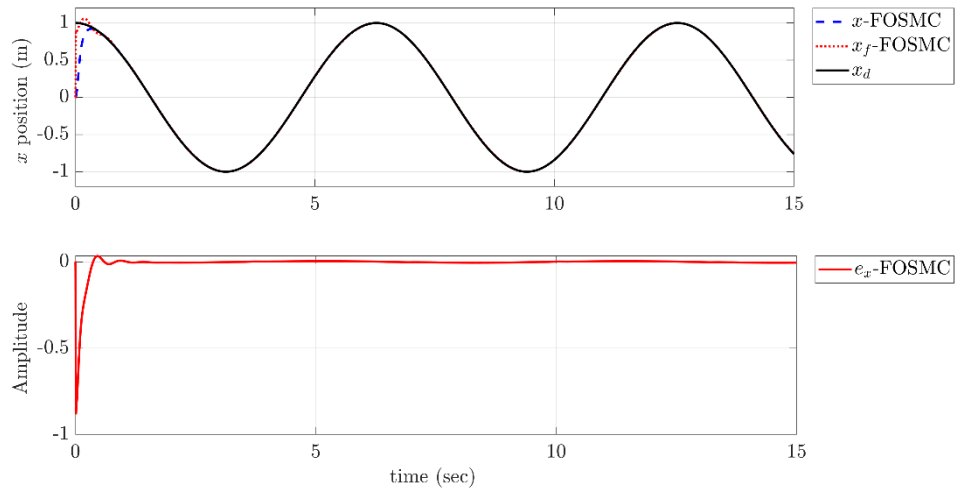


Figure 5-44: x Position Responses in Integer and Fractional Systems under FOSMC – Case 3

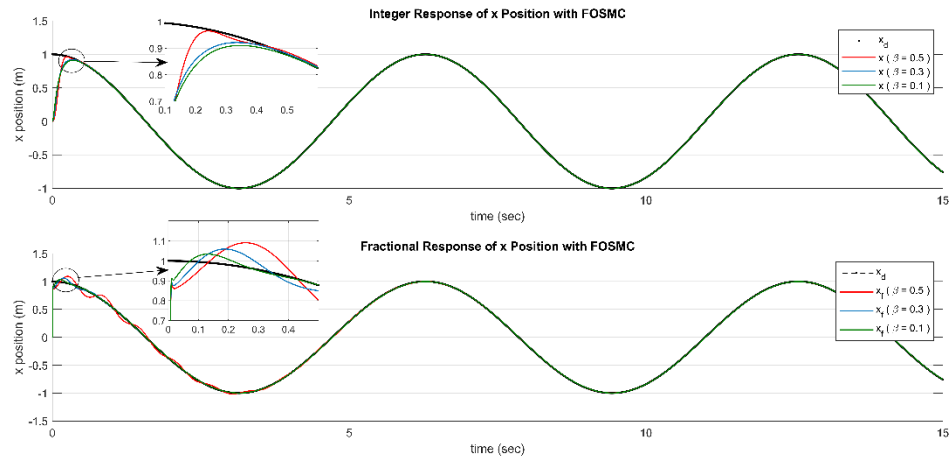
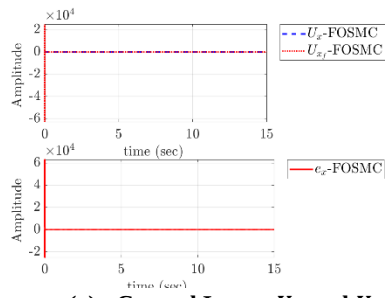
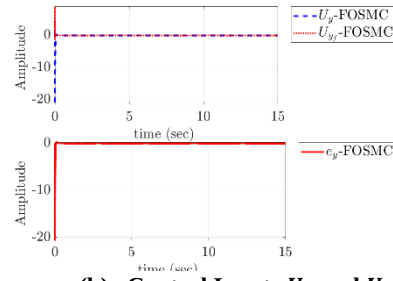


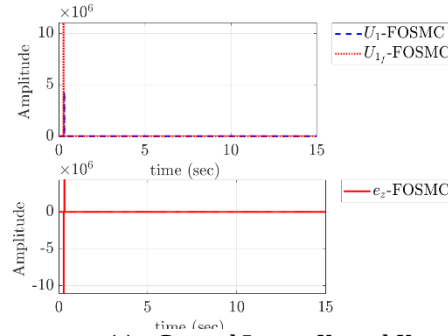
Figure 5-45: Positions (x and x_f) plots with different fractional γ



(a) Control Inputs U_x and U_{x_f}



(b) Control Inputs U_y and U_{y_f}



(c) Control Inputs U_1 and U_{1_f}

Figure 5-46: Position FOSMC Inputs in Integer and Fractional Systems and the Error Between Them – Case 3

5.2.5 FOSMC Results and Discussion

The major purpose of FOSMC is to boost performance in both integer and fractional systems by stabilizing them and following a desired reference, and as shown in the simulation section, the FOSMCs stabilize both systems and follow the desired references in all the stated case studies.

The simulated results of the all the case studies show that the responses of the position states in fractional system are enhanced in term of tracking accuracy, response speed and chattering comparing with those in integer systems.

One of the drawbacks of sliding mode control is the chattering issue, and the high amplitude and frequency of the chattering may cause serious damage to mechanical components of the system, such as the quadrotor's motors. Therefore, fractional order sliding mode control has a significant effect on lowering controller chattering. The altitude and the x position FOSM controllers in both integer and fractional systems were tested by varying the fractional order (β) with three different values and comparing the results. As can be seen in the above figures (Figure 5-39 and Figure 5-45), the fractional order has significantly reduced the chattering of the controller. However, this increase in the fractional order affects the transient performance of the outputs which are the altitude and x position of the quadrotor. Furthermore, the results reveal that decreasing the value of fractional order in FOSMC performs well in the fractional system but poorly in the integer system, however increasing that value of fractional order yields the opposite results.

The fractional order parameters (β) were chosen to produce high output performance as well as better control input behavior. They were not chosen to reduce the chattering at the expense of system performance, and vice versa. Hence, there is a trade-off between both behaviors.

Conclusion and Future work

The purpose of this study was to use fractional calculus to derive an accurate mathematical model for the quadrotor Unmanned Aerial Vehicle (UAV), as well as to design linear and nonlinear control techniques to stabilize the transitional and rotational states of quadrotor UAV and to evaluate the results of these control systems using simulation software (MATLAB).

The quadrotor dynamic model was constructed in precise details, including aerodynamic effects and rotor dynamics, which we observed absent in various literature reviews. Four control approaches were then designed and synthesized; a linear Proportional-Integral-Derivative (PID) controller, a Fractional Order PID controller, a nonlinear Sliding Mode Controller (SMC) and a nonlinear Fractional Order SMC. On the basis of the developed mathematical model of the quadrotor, a comprehensive simulation was therefore implemented in MATLAB/Simulink. The simulation environment was utilized to assess and compare the dynamic performances of the stated controllers under various input scenarios.

We employed the Genetic Algorithm (GA) to tune the parameters of the PID and FOPID controllers where the objective function was minimizing the ITAE. The four proposed controllers perform perfectly to stabilize both the integer and fractional systems of the quadrotor and the fractional order states of the quadrotor's altitude, x and y positions are improved in terms response speed. When the quadrotor was ordered to fly in a different trajectories, the SMC and FOSMC in fractional systems performed significantly better than

the typical PID and FOPID controllers. The FOSMC comparing with others in fractional system produced the best results in terms of tracking accuracy, response speed, and chattering reduction.

6.1 Future work

1. Transform the whole integer quadrotor model into a fractional one.
2. Test the obtained fractional model in a load drop application.
3. Propose a linear optimal controller such as LQR and compare it with the proposed controllers in this thesis.
4. Develop adaptive control algorithms to count for the system uncertainties would enhance the performance of the quadrotor when operating in a real environment.

References

- [1] G. Goh, S. Agarwala, G. Goh, V. Dikshit, ... S. S.-A. S. and, and undefined 2017, "Additive manufacturing in unmanned aerial vehicles (UAVs): Challenges and potential," *Elsevier*, Accessed: Nov. 20, 2021. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S127096381630503X>.
- [2] M. Habib, W. Abdelaal, and M. Saad, "Dynamic modeling and control of a quadrotor using linear and nonlinear approaches," 2014, Accessed: Nov. 20, 2021. [Online]. Available: <http://dar.aucegypt.edu/handle/10526/3965>.
- [3] Y. Liu and R. Bucknall, "A survey of formation control and motion planning of multiple unmanned vehicles," *Robotica*, vol. 36, no. 7, pp. 1019–1047, 2018, doi: 10.1017/S0263574718000218.
- [4] D. Chabot, "Trends in drone research and applications as the journal of unmanned vehicle systems turns five," *J. Unmanned Veh. Syst.*, vol. 6, no. 1, pp. Vi–xv, 2018, doi: 10.1139/juvs-2018-0005.
- [5] "Improved robust adaptive control of high-order nonlinear systems with guaranteed performance - ProQuest." <https://www.proquest.com/docview/2013321917?pq-origsite=gscholar&fromopenview=true> (accessed Nov. 20, 2021).
- [6] A. Al Jarbough, "Rheological Behaviour Modelling of Viscoelastic Materials by using Fractional Model," *Ali Al Jarbough / Energy Procedia*, vol. 19, pp. 143–157, 2012, doi: 10.1016/j.egypro.2012.05.194.
- [7] J. T. Katsikadelis and N. G. Babouskos, "Post-buckling analysis of viscoelastic plates with fractional derivative models," *Eng. Anal. Bound. Elem.*, vol. 34, no. 12, pp. 1038–1048, Dec. 2010, doi: 10.1016/j.enganabound.2010.07.003.
- [8] D. Baleanu, A. K. Golmankhaneh, and A. K. Golmankhaneh, "On electromagnetic field in fractional space," *Nonlinear Anal. Real World Appl.*, vol. 11, no. 1, pp. 288–292, Feb. 2010, doi: 10.1016/j.nonrwa.2008.10.058.
- [9] S. Das and S. Das, "Application of Generalized Fractional Calculus in Electrical Circuit Analysis and Electromagnetics," in *Functional Fractional Calculus*, Springer Berlin Heidelberg, 2011, pp. 387–436.
- [10] Z. Wang, X. Huang, and H. Shen, "Control of an uncertain fractional order economic system via adaptive sliding mode," *Neurocomputing*, vol. 83, pp. 83–88, Apr. 2012, doi: 10.1016/j.neucom.2011.11.018.
- [11] R. Martin, J. J. Quintana, A. Ramos, and I. De La Nuez, "Modeling electrochemical double layer capacitor, from classical to fractional impedance," in *Proceedings of the Mediterranean Electrotechnical Conference - MELECON*, 2008, pp. 61–66, doi:

10.1109/MELCON.2008.4618411.

- [12] H. S. Ahn, Y. Q. Chen, and I. Podlubny, "Robust stability test of a class of linear time-invariant interval fractional-order system using Lyapunov inequality," *Appl. Math. Comput.*, vol. 187, no. 1 SPEC. ISS., pp. 27–34, Apr. 2007, doi: 10.1016/j.amc.2006.08.099.
- [13] A. T. Azar and F. E. Serrano, "Robust control for asynchronous switched nonlinear systems with time varying delays," in *Advances in Intelligent Systems and Computing*, 2017, vol. 533, pp. 891–899, doi: 10.1007/978-3-319-48308-5_85.
- [14] S. Balochian and N. Rajaei, "Fractional-Order Optimal Control of Fractional-Order Linear Vibration Systems with Time Delay," *Int. J. Syst. Dyn. Appl.*, vol. 7, no. 3, pp. 72–93, 2018, doi: 10.4018/ijdsda.2018070104.
- [15] Y. Q. Chen, H. S. Ahn, and I. Podlubny, "Robust stability check of fractional order linear time invariant systems with interval uncertainties," *Signal Processing*, vol. 86, no. 10, pp. 2611–2618, Oct. 2006, doi: 10.1016/j.sigpro.2006.02.011.
- [16] T. T. Hartley and C. F. Lorenzo, "Dynamics and control of initialized fractional-order systems," *Nonlinear Dyn.*, vol. 29, no. 1–4, pp. 201–233, Jul. 2002, doi: 10.1023/A:1016534921583.
- [17] T.-C. Lin and C.-H. Kuo, "Adaptive Hybrid Intelligent Tracking Control for Uncertain Fractional Order Chaotic Systems," *Int. J. Syst. Dyn. Appl.*, vol. 1, no. 1, pp. 1–16, 2012, doi: 10.4018/ijdsda.2012010101.
- [18] S. H. Nabavi and S. Balochian, "The Stability of a Class of Fractional Order Switching System with Time-Delay Actuator," *Int. J. Syst. Dyn. Appl.*, vol. 7, no. 1, pp. 85–96, 2018, doi: 10.4018/ijdsda.2018010105.
- [19] E. S. Alaviyan Shahri, A. Alfi, and J. A. Tenreiro Machado, "Stability analysis of a class of nonlinear fractional-order systems under control input saturation," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 7, pp. 2887–2905, 2018, doi: 10.1002/rnc.4055.
- [20] T. J. Freeborn, B. Maundy, and A. S. Elwakil, "Fractional-order models of supercapacitors, batteries and fuel cells: A survey," *Materials for Renewable and Sustainable Energy*, vol. 4, no. 3. SpringerOpen, pp. 1–7, Sep. 01, 2015, doi: 10.1007/s40243-015-0052-y.
- [21] Q. Yang, D. Chen, T. Zhao, and Y. Chen, "Fractional Calculus In Image Processing: A Review," *Fract. Calc. Appl. Anal.*, vol. 19, no. 5, pp. 1222–1249, Aug. 2016, Accessed: Dec. 06, 2020. [Online]. Available: <http://arxiv.org/abs/1608.03240>.
- [22] T. J. Freeborn, "A survey of fractional-order circuit models for biology and biomedicine," *IEEE J. Emerg. Sel. Top. Circuits Syst.*, vol. 3, no. 3, pp. 416–424, 2013, doi: 10.1109/JETCAS.2013.2265797.

- [23] N. Eluru, V. Chakour, M. Chamberlain, and L. F. Miranda-Moreno, "Modeling vehicle operating speed on urban roads in Montreal: A panel mixed ordered probit fractional split model," *Accid. Anal. Prev.*, vol. 59, pp. 125–134, 2013, doi: 10.1016/j.aap.2013.05.016.
- [24] M. Bailey, "Unmanned aerial vehicle path planning and image processing for orthoimagery and digital surface model generation," 2012, Accessed: Nov. 20, 2021. [Online]. Available: <https://core.ac.uk/download/pdf/216051590.pdf>.
- [25] A. Azzam, X. W.-2010 2nd I. A. C. on, and undefined 2010, "Quad rotor arial robot dynamic modeling and configuration stabilization," *ieeexplore.ieee.org*, Accessed: Nov. 20, 2021. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/5456804/>.
- [26] C. Zhang and J. M. Kovacs, "The application of small unmanned aerial systems for precision agriculture: A review," *Precis. Agric.*, vol. 13, no. 6, pp. 693–712, Dec. 2012, doi: 10.1007/S11119-012-9274-5/FIGURES/3.
- [27] T. Spiess, J. Bange, M. Buschmann, and P. Vörsmann, "First application of the meteorological Mini-UAV 'M2AV,'" *Meteorol. Zeitschrift*, vol. 16, no. 2, pp. 159–169, May 2007, doi: 10.1127/0941-2948/2007/0195.
- [28] S. Adams, C. F.-9th international workshop on remote, and undefined 2011, "A survey of unmanned aerial vehicle (UAV) usage for imagery collection in disaster research and management," *researchgate.net*, Accessed: Nov. 20, 2021. [Online]. Available: https://www.researchgate.net/profile/Carol-Friedland-2/publication/266465037_A_Survey_of_Unmanned_Aerial_Vehicle_UAV_Usage_for_Imagery_Collection_in_Disaster_Research_and_Management/links/54d8d9ab0cf24647581c9aa0/A-Survey-of-Unmanned-Aerial-Vehicle-UAV-Usage-for-Imagery-Collection-in-Disaster-Research-and-Management.pdf.
- [29] J. T. K. Ping, A. E. Ling, T. J. Quan, and C. Y. Dat, "Generic unmanned aerial vehicle (UAV) for civilian application-A feasibility assessment and market survey on civilian application for aerial imaging," *2012 IEEE Conf. Sustain. Util. Dev. Eng. Technol. STUDENT 2012 - Conf. Bookl.*, pp. 289–294, 2012, doi: 10.1109/STUDENT.2012.6408421.
- [30] "U.M. Research, U.S. Military Unmanned Aerial Vehicles... - Google Scholar." https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=U.M.+Research%2C+U.S.+Military+Unmanned+Aerial+Vehicles+%28UAV%29+Market+Forecast+2010-2015%2C+Available+at+UAV+Market+Research%3A+http%3A%2F%2Fwww.uavmarketresearch.com%2F2011.&btnG= (accessed Nov. 20, 2021).
- [31] J. W. Gerdes, S. K. Gupta, and S. A. Wilkerson, "A review of bird-inspired flapping wing miniature air vehicle designs," *J. Mech. Robot.*, vol. 4, no. 2, Mar. 2012, doi: 10.1115/1.4005525.

- [32] T. Han, M. Chi, Z. H. Guan, B. Hu, J. W. Xiao, and Y. Huang, "Distributed Three-Dimensional Formation Containment Control of Multiple Unmanned Aerial Vehicle Systems," *Asian J. Control*, vol. 19, no. 3, pp. 1103–1113, May 2017, doi: 10.1002/asjc.1445.
- [33] B. Zhu, K. Guo, L. X.-I. T. on A. Control, and undefined 2018, "A new distributed model predictive control for unconstrained double-integrator multiagent systems," *ieeexplore.ieee.org*, Accessed: Dec. 08, 2020. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/8325302/>.
- [34] S. Bouabdallah, A. Noth, and R. Siegwart, "PID vs LQ control techniques applied to an indoor micro Quadrotor," *2004 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, vol. 3, pp. 2451–2456, 2004, doi: 10.1109/IROS.2004.1389776.
- [35] P. Edward Ian Pounds, "Design, Construction and Control of a Large Quadrotor Micro Air Vehicle," 2007.
- [36] J. Li and Y. Li, "Dynamic analysis and PID control for a quadrotor," *2011 IEEE Int. Conf. Mechatronics Autom. ICMA 2011*, pp. 573–578, 2011, doi: 10.1109/ICMA.2011.5985724.
- [37] J. Yang, Z. Cai, Q. Lin, Y. W.-2013 C. Automation, and undefined 2013, "Self-tuning PID control design for quadrotor UAV based on adaptive pole placement control," *ieeexplore.ieee.org*, Accessed: Nov. 20, 2021. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/6775734/>.
- [38] F. Kendoul, "Survey of advances in guidance, navigation, and control of unmanned rotorcraft systems," *J. F. Robot.*, vol. 29, no. 2, pp. 315–378, Mar. 2012, doi: 10.1002/ROB.20414.
- [39] "Sliding Mode Control - an overview | ScienceDirect Topics." <https://www.sciencedirect.com/topics/engineering/sliding-mode-control> (accessed Dec. 07, 2020).
- [40] S. Bouabdallah, R. S.-P. of the 2005 IEEE, and undefined 2005, "Backstepping and sliding-mode techniques applied to an indoor micro quadrotor," *ieeexplore.ieee.org*, Accessed: Nov. 20, 2021. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/1570447/>.
- [41] S. Waslander, G. Hoffmann, ... J. J.-... on I. R., and undefined 2005, "Multi-agent quadrotor testbed control design: Integral sliding mode vs. reinforcement learning," *ieeexplore.ieee.org*, Accessed: Nov. 20, 2021. [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/1545025/>.
- [42] R. Ayad, W. Nouibat, M. Zareb, and Y. Bestaoui Sebanne, "Full Control of Quadrotor Aerial Robot Using Fractional-Order FOPID," *Iran. J. Sci. Technol. - Trans. Electr. Eng.*, vol. 43, no. April 2019, pp. 349–360, 2019, doi: 10.1007/s40998-018-0155-4.

- [43] Y. Luo, L. Di, J. Han, and H. Chao, "VTOL UAV altitude flight control using fractional order controllers," 2010, Accessed: Dec. 08, 2020. [Online]. Available: https://digitalcommons.usu.edu/aggieair_pres/15/.
- [44] J. Han *et al.*, "Pitch Loop Control of a VTOL UAV Using Fractional Order Controller," *J Intell Robot Syst*, vol. 73, no. 1–4, pp. 187–195, 2014, doi: 10.1007/s10846-013-9912-9.
- [45] Y. Guo, Z. Deng, L. Zu, and Y. Lv, "Trajectory tracking control of a quad-rotor using fractional-order sliding mode," in *Chinese Control Conference, CCC*, Sep. 2017, pp. 6414–6419, doi: 10.23919/ChiCC.2017.8028375.
- [46] X. Shi, Y. Cheng, C. Yin, S. Dadras, and X. Huang, "Design of Fractional-Order Backstepping Sliding Mode Control for Quadrotor UAV," *Asian J. Control*, vol. 21, no. 1, pp. 156–171, Jan. 2019, doi: 10.1002/asjc.1946.
- [47] C. Yin, S. Zhong, W. C. N. S. and N. Simulation, and undefined 2012, "Design of sliding mode controller for a class of fractional-order chaotic systems," *Elsevier*, Accessed: Dec. 06, 2020. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1007570411002188>.
- [48] N. Djeghali, S. Djennoune, M. Bettayeb, M. G.-I. transactions, and undefined 2016, "Observation and sliding mode observer for nonlinear fractional-order system with unknown input," *Elsevier*, Accessed: Dec. 06, 2020. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0019057816000562>.
- [49] M. Yousefi and T. Binazadeh, "Delay-independent sliding mode control of time-delay linear fractional order systems," *Trans. Inst. Meas. Control*, vol. 40, no. 4, pp. 1212–1222, Feb. 2018, doi: 10.1177/0142331216678059.
- [50] J. Wang, C. Shao, and Y. Q. Chen, "Fractional order sliding mode control via disturbance observer for a class of fractional order systems with mismatched disturbance," *Mechatronics*, vol. 53, pp. 8–19, Aug. 2018, doi: 10.1016/j.mechatronics.2018.05.006.
- [51] C. Yin, Y. Cheng, Y. Q. Chen, B. Stark, and S. Zhong, "Adaptive fractional-order switching-type control method design for 3D fractional-order nonlinear systems," *Nonlinear Dyn.*, vol. 82, no. 1–2, pp. 39–52, Oct. 2015, doi: 10.1007/s11071-015-2136-8.
- [52] C. Yin, X. Huang, Y. Chen, S. Dadras, S. ming Zhong, and Y. Cheng, "Fractional-order exponential switching technique to enhance sliding mode control," *Appl. Math. Model.*, vol. 44, pp. 705–726, Apr. 2017, doi: 10.1016/j.apm.2017.02.034.
- [53] W. ZHANG, Z. MIAO, X. YU, T. H.-E. M. & Control, and undefined 2018, "Fractional Order Integral Sliding Mode Control for Permanent Magnet Synchronous Motor Based on Improved Sliding Mode Observer," *en.cnki.com.cn*, Accessed: Dec. 07, 2020. [Online]. Available:

http://en.cnki.com.cn/Article_en/CJFDTotal-ZXXD201807002.htm.

- [54] H. Delavari, P. Lanusse, and J. Sabatier, “Fractional Order Controller Design for A Flexible Link Manipulator Robot,” *Asian J. Control*, vol. 15, no. 3, pp. 783–795, May 2013, doi: 10.1002/asjc.677.
- [55] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, “Preface,” *North-Holland Mathematics Studies*, vol. 204, no. C. Elsevier, 2006, doi: 10.1016/S0304-0208(06)80001-0.
- [56] A. Sánchez, I. Fantoni, R. Lozano, and J. De León Morales, “OBSERVER-BASED CONTROL OF A PVTOL AIRCRAFT,” *IFAC Proc. Vol.*, vol. 38, no. 1, pp. 121–126, 2005, doi: 10.3182/20050703-6-CZ-1902.01981.
- [57] H. Bouadi, M. Bouchoucha, and M. Tadjine, “Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor,” pp. 22–27, 2007.
- [58] H. talla M. ElKholy, “Dynamic Modeling and Control of a Quadrotor Using Linear and Nonlinear Approaches,” 2014.
- [59] S. Wen, M. Z. Q. Chen, Z. Zeng, X. Yu, and T. Huang, “Fuzzy Control for Uncertain Vehicle Active Suspension Systems via Dynamic Sliding-Mode Approach,” *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 47, no. 1, pp. 24–32, Jan. 2017, doi: 10.1109/TSMC.2016.2564930.
- [60] M. W. Khan, M. Abid, A. Q. Khan, G. Mustafa, M. Ali, and A. Khan, “Sliding mode control for a fractional-order non-linear glucose-insulin system,” *IET Syst. Biol.*, vol. 14, no. 5, pp. 223–229, 2020, doi: 10.1049/iet-syb.2020.0030.
- [61] M. Farman, A. Akgül, M. U. Saleem, S. Imtiaz, and A. Ahmad, “Dynamical behaviour of fractional-order finance system,” *Pramana - J. Phys.*, vol. 94, no. 1, pp. 1–10, 2020, doi: 10.1007/s12043-020-02030-8.
- [62] L. C. de Barros, M. M. Lopes, F. S. Pedro, E. Esmei, J. P. C. dos Santos, and D. E. Sánchez, “The memory effect on fractional calculus: an application in the spread of COVID-19,” *Comput. Appl. Math.*, vol. 40, no. 3, pp. 1–21, 2021, doi: 10.1007/s40314-021-01456-z.
- [63] J. Liu and X. Wang, “Advanced Sliding Mode Control Sliding Mode Control Based on a Linear Matrix Inequality for Inverted Pendulum,” *Adv. Sliding Mode Control Mech. Syst.*, pp. 81–96, 2011.
- [64] B. Wang, J. Ding, F. Wu, and D. Zhu, “Robust finite-time control of fractional-order nonlinear systems via frequency distributed model,” *Nonlinear Dyn.*, vol. 85, no. 4, pp. 2133–2142, 2016, doi: 10.1007/s11071-016-2819-9.
- [65] M. Ö. Efe, “Fractional order sliding mode control with reaching law approach,” *Turkish J. Electr. Eng. Comput. Sci.*, vol. 18, no. 5, pp. 731–747, 2010, doi: 10.3906/elk-0906-3.

Vitae

Name	:Khaled Saeed Bin Gaufan
Nationality	:Yemeni
Date of Birth	:9/19/1992
Email	:jofanee1992@gmail.com
Contact Details	:00967-777490323, 00966-533270309
Address	:Aljawadeh village, Al-Oaten, Hadramout, Yemen
Academic Background	:B.Sc. Control and Instrumentation System Engineering, (2013-2018), KFUPM, Dhahran, Saudi Arabia. M.S Control and Systems Engineering, (2019 - 2021), KFUPM, Dhahran, Saudi Arabia.