An optimization of a multi-period single-item capacitated lot-sizing problem with remanufacturing and outsourcing

BY

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A Thesis Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

INDUSTRIAL AND SYSTEMS ENGINEERING

December 2021

AN OPTIMIZATION OF A MULTI-PERIOD SINGLE-ITEM CAPACITATED LOT-SIZING PROBLEM WITH REMANUFACTURING AND OUTSOURCING

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December 2021

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DHAHRAN - 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by **OMAR HAMDY ELDALGAMOUNY** under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN INDUSTRIAL AND SYSTEMS ENGINEERING**

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Dedication

This research is dedicated to Almighty Allah, my beloved creator and master, who has been the greatest source of strength in my whole life and in my period of achieving this study. Also, my work is dedicated to my most beloved person; our Prophet Mohammad peace be upon him, who has always been our guidance to Allah and our teacher of all good manners. The first revealed word from Qura'an to him was "Iqra" which literally means "read" emphasizing on vitality of the continuous learning process.

I would like also to dedicate my research to my beloved parents and my whole family. No word can express their limitless support and precious prayers which elevated me to be on the right path for my entire life. May Almighty Allah bless and protect them all.

I would like also to thank Dr. Mohammad Abdel-Aal; The first one to teach me the basics of academical researching skills which allowed me to be what I am today as a researcher. I would like also to thank whoever taught me any single valuable information which, in turn, helped me later to build my knowledge and to achieve this degree.

I wish that this work contributes to the current body of knowledge and to promote the growth and prosperity within the humanity.

Acknowledgement

I would like to very much thank Dr. Mohammad Abdel-Aal (My M.SC Advisor, SE Department, KFUPM) for his cooperation, consultations, directive instructions and sharing of his valuable knowledge and suggestions during the thesis preparation, and implementation. No word can express my deepest gratitude. His continual support and endless guidance enlightened my road to accomplish my study. Most importantly, I never felt that he is my advisor only, it goes beyond that to treat me as if I was his brother.

I would like also to thank Dr. Essam Kaoud for his valuable knowledge and cooperation during the implementation of this work. His instructive comments and support are invaluable.

I also extend my sincere appreciation to my committee members; Dr. Ahmed Alhanbali (Associate professor, SE Department, KFUPM), and Dr. Ahmed Attia (Assistant professor, SE Department, KFUPM) for their support and contribution to the success of this study.

Lastly, I would like to thank King Fahd University of Petroleum and Minerals (KFUPM) for providing me with all the resources I needed for conducting the entire study and for providing such a great master program.

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ABSTRACT

Full Name: [OMAR HAMDY ELDALGAMOUNY]

Thesis Title: [AN OPTIMIZATION OF A MULTI-PERIOD SINGLE-ITEM CAPACITATED LOT-SIZING PROBLEM WITH REMANUFACTURING AND OUTSOURCING]

Major Field: [INDUSTRIAL AND SYSTEMS ENGINEERING]

Date of Degree: [DECEMBER 2021]

Due to the characteristics of the different supply chains nowadays, reverse logistics has attracted a lot of researchers' attention lately. In this context, many industries seek for acquiring a sustainable production system that is environmentally friendly. Therefore, these industries have to deal with the customer returned items which have several types. This study investigates and optimizes Mixed Integer Linear Programming (MILP) models for a single-item multi-period capacitated lot-sizing problem with remanufacturing and outsourcing. The outsourcing is one of the cost reduction options used to satisfy a part of the customer demands when the company encounters a limited capacity, and it is performed by hiring an outside party to perform some of the services related to the manufacturing of new products. Four different MILP models were developed in this study considering different criteria and extensions, and a comparison between their outputs was also accomplished. This problem is capacitated in terms of production and in terms of storage. The production capacity was overcome by allowing for some overtime and outsourcing in some periods. Moreover, the customer deterministic demands are satisfied by either producing new items or remanufacturing returned items. These different scopes were considered in the models' formulations, and the combination of all those criteria was merged in one single model. The objective is to obtain an optimal plan on a limited period of time. In this study, separate

set-up costs with overtime were considered along with the environmental aspect, which make this problem strongly NP-hard problem. A state-of-art metaheuristic approach was developed, and different multi-sizes of instances were examined. An efficient search methodology of neighborhood structures was implemented which is called Variable Neighborhood Search (VNS) and it was used to build up distance-based neighborhood structures for this lot sizing problem. A powerful mathematical and optimization software program called GAMS with built-in CPLEX solver were used to solve these optimization models. The results obtained from the metaheuristic approach were compared with each original analytical model formulations. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps and in terms of computational time deductions.

ARABIC ABSTRACT

ملخص الرسالة

الاسم الكامل: عمر حمدي الدلجموني.

عنوان الرسالة: مَنْهَجِيَّةٌ مُثْلَى لِتَعْيِينِ الْكَمِّيَّاتِ الْمُنَاسِبَةِ لِمَسْأَلَةٍ مَحْدُودَةٍ مُتَعَدِّدَةِ الْفَتَرَاتِ أُحَادِيَّةِ الْمُنْتَجِ بِاسْتِخْدَامِ إِعَادَةِ التَّصْنِيعِ وَبَعْضِ الْإِمْدَادَاتِ الْخَارِجِيَّةِ.

التخصص: الهندسة الصناعية والنظم.

تاريخ الدرجة العلمية: ديسمبر 2021.

نظرًا لخصائص سلاسل الإمداد المختلفة في الوقت الحاضر، فقد جذبت الخدمات اللوجستية العكسية الكثير من انتباه الباحثين مؤخرًا. وفي هذا السياق، تسعى العديد من الصناعات للحصول على نظام إنتاج مستدام وصديق للبيئة. لذلك، فإنه يتعين على هذه الصناعات التعامل مع المنتجات التي يتم إرجاعها من قبل العملاء والتي لها أنواع عدة. تقوم هذه الدراسة ببحث وتحسين نماذج البرمجة الخطية الصحيحة المختلطة لحل مشكلة تعيين الكميات المناسبة لمسألة محدودة متعددة الفترات أحادية المنتج باستخدام إعادة التصنيع وبعض الإمدادات الخارجية. يعد الاستعانة بمصادر إمداد خارجية أحد خيارات خفض التكلفة المستخدمة لتلبية جزء من طلبات العملاء عندما تواجه الشركة قدرة إنتاجية محدودة، ويتم إجراؤها عن طريق تعيين طرف خارجي لأداء بعض الخدمات المتعلقة بتصنيع المنتجات الجديدة. تم تطوير أربعة نماذج مختلفة من البرمجة الخطية الصحيحة المختلطة في هذه الدراسة مع مراعاة معايير وأبعاد مختلفة، كما تم إجراء مقارنة بين نتائجها المختلفة. تتفاقم هذه المشكلة من حيث محدودية الإنتاج ومحدودية التخزين. تم التغلب على الطاقة الإنتاجية المحدودة من خلال السماح ببعض أوقات العمل الإضافي وبعض الإمدادات الخارجية في بعض الفترات. علاوة على ذلك، يتم تلبية طلبات العملاء المحددة إما عن طريق إنتاج عناصر جديدة أو إعادة تصنيع المنتجات التي تم إرجاعها. هذه النطاقات المختلفة تم أخذها بعين الاعتبار في صيغة نماذج رياضية، وتم دمج كل تلك المعايير المختلفة في نموذج واحد. الهدف من هذه النمذجة هو الحصول على خطة مثالية في فترة زمنية محدودة. في هذه الدراسة، تم الأخذ بعين الاعتبار تكاليف تجهيزية منفصلة مع السماح بالعمل الإضافي إضافة إلى

مراعاة الجانب البيئي مما يجعل هذه المسألة تتزايد صعوبتها بشكل فائق. وقد تم تطوير نهج اجتهادي مختصر وعلى أحدث طراز، وتم فحصه عن طريق اختبار أمثلة مختلفة متعددة الأحجام. كما تم تطوير منهجية بحث فعالة جديدة لهيكلة الحلول القريبة والتي تسمى طريقة البحث المتغيرة لإيجاد الحلول القريبة، وقد تم استخدامها لإيجاد الحلول القريبة على اعتبار المسافة الفاصلة بينهم وذلك لحل مسألة تعيين الكميات المناسبة هذه. تم استخدام برنامج تحسين رياضي قوي يسمى (جامز) وبداخله تم اختيار (سيبلكس) في حل نماذج التحسين هذه. وقد تمت مقارنة النتائج التي تم الحصول عليها من الطريقة المختصرة المقترحة مع كل نتائج النماذج الرياضية الأصلية تلك. وقد تفوقت الطريقة الاجتهادية المختصرة المقترحة على كل تلك النماذج الرياضية الأصلية من حيث قربها من الحل الأمثل ووصولها إليه في فترة زمنية أقل بكثير.

CHAPTER ONE

INTRODUCTION

1.1 Overview of the Study

A lot of researchers' attention has been drawn lately by reverse logistics. This is due to the characteristics of different supply chains nowadays and the shortage of the natural resources. In this context, acquiring a sustainable production system that is environmentally friendly is the goal that many industries seek for. Therefore, these industries have to deal with the customer returned items which have several types. The process of returning items could be related to customer online unsatisfaction purchase, recyclable components, products that are defective and therefore have to be repairable and re-utilizable materials as well. This recovery process usually contains disassembly, reassembly, cleaning, testing, and replacement/repair of the recollected items. Figure1. shows a simple overview of the reverse logistics process and Figure2. shows a simple example of the remanufacturing life cycle.

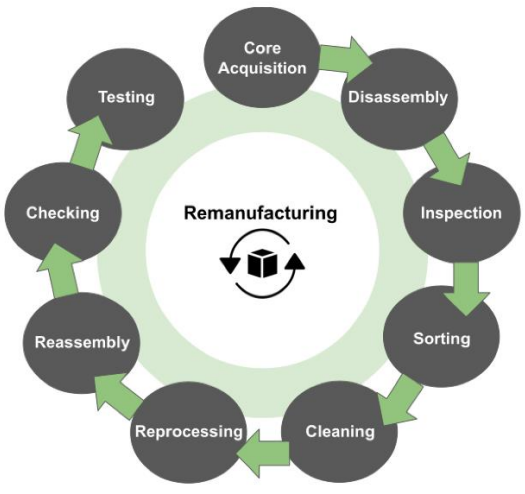

Figure 1. A simple overview of the reverse logistics process

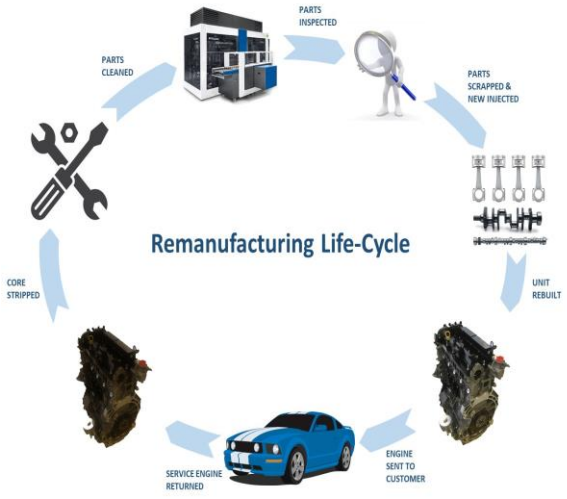

Figure 2. A simple example of the remanufacturing life cycle

MILP models for a single-item multi-period capacitated lot-sizing problem with remanufacturing and outsourcing were investigated and optimized in this proposed study. The outsourcing is one of the cost reduction options used to satisfy a part of the customer demands when the company encounters a limited capacity, and it is performed by hiring an outside party to perform some of the services related to the manufacturing of new products. In this study, a development of four different MILP models was accomplished considering different criteria and extensions, and a comparison between their outputs was also performed. This problem is capacitated in terms of production and in terms of storage. By allowing for some overtime and outsourcing in some periods, the production capacity was overcome. Moreover, the customer deterministic demands are satisfied by two different options: the production of new products or the remanufacturing of these returned items. A consideration of these different scopes was conducted through models' formulations, and all those criteria were combined in one single model. Obtaining an optimal plan over a limited planning period is the aim of this study. In addition, separate set-up costs with over time were considered in this work along with the environmental aspect and outsourcing, which make this problem strongly NP-hard problem. A state-of-art metaheuristic approach was developed, and different multi-sizes of instances were examined. Shaking technique is an efficient search methodology of neighborhood structures that was implemented and used to build up distance-based neighborhood structures for this lot sizing problem. This search methodology falls under the category of VNS algorithm. A powerful mathematical and optimization software program called GAMS was used to solve these optimization models. A comparison between the results obtained from the metaheuristic approach and each of the original analytical models was accomplished. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps and in terms of

time deductions. Figure3. gives an overview of the economic lot sizing problem with remanufacturing and deterministic demand, where x^m , x^r are the manufacturing and remanufacturing quantities respectively, y^m , y^r are the binary decision variables of manufacturing and remanufacturing respectively, I^s, I^r are the inventories of manufactured units and returns respectively, and *d* is the customer demand.

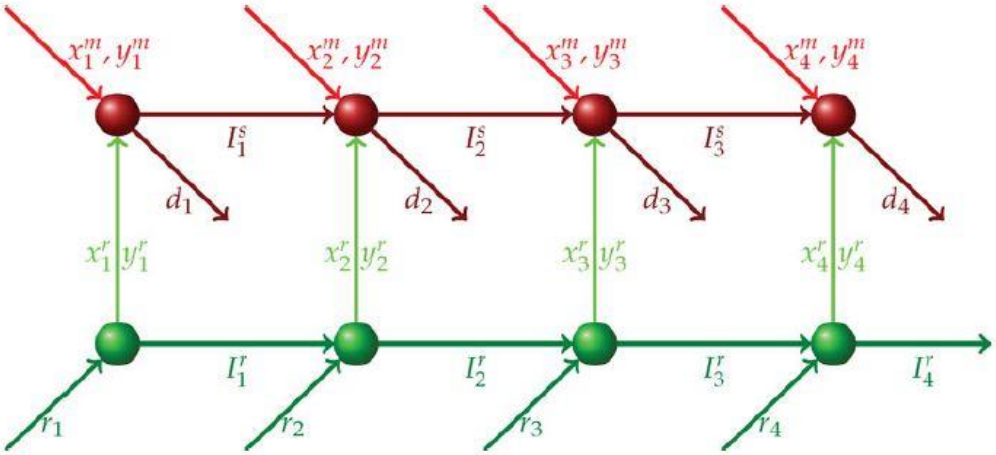

Figure 3. An overview of the economic lot sizing problem with remanufacturing
Helmrich et al., (2014)

1.2 Problem Statement

This study investigates and optimizes MILP models for a single-item multi-period capacitated lot-sizing problem involving remanufacturing and outsourcing. The outsourcing is one of the cost reduction options used to satisfy a part of the customer demands when the company encounters a limited capacity, and it is performed by hiring an outside party to perform some of the services related to the manufacturing of new products. This problem is capacitated in terms of production and in terms of storage. The production capacity was overcome by allowing for some overtime and outsourcing in some periods. However, customer deterministic demands are fulfilled

by the production of the brand-new products or the remanufacturing of current returned units acquired from customers. In this study, a development of four different MILP models was accomplished considering different criteria and extensions, and a comparison between their outputs was also performed. The major goal is to obtain an optimal plan of such a problem of the lot-sizing involving remanufacturing along with outsourcing on a limited period of time. In this proposed study, separate set-up costs with over time were considered along with the integrating the environmental aspect into the problem making it a green supply chain problem. One of the assumptions here in this model is that the remanufacturing items are as good as new. In this context, a good point to consider is the tradeoff between remanufacturing items to be as good as new, which may affect the environmental aspect, and selling those items as batches in a secondary market with lower price. Another point mentioned earlier is that the demand is deterministic, so no correlation between variables would be considered in the models. To add to this, all the previous problems published in this field were NP-hard problems which, in turn, restrict the model solution capability and therefore require no exact solution method to solve the problem. Considering the contribution of outsourcing and making a restriction on the storage capacity make our model strongly NP-hard and this would complicate the problem more. In addition, this restriction on the storage capacity was not performed before which makes this problem more powerful. A state-of-art metaheuristic approach was developed to overcome this NP-hard feature, and different huge multi-sizes of instances were examined to test the power of this proposed metaheuristic. An efficient search methodology of neighborhood structures was implemented which is called shaking method and it was used to build up distance-based neighborhood structures for this lot sizing problem. This search methodology falls under the category of VNS algorithm. A powerful mathematical and optimization software program called GAMS was used to solve these

optimization models with CPLEX solver. The results obtained from the metaheuristic approach were compared with each original analytical models' formulation. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps in the large-scale problems and in terms of time deductions in the small-scale problems. This can be illustrated in a sense that the proposed metaheuristics sometimes reaches to better solutions with almost 74% gap compared to the original analytical model in the large-scale problems, with a deduction of time by more than 92% in the small-scale problems compared to the time the original mathematical models take to find a solutions within 1500 sec time range. To sum up, the consideration of outsourcing, green supply chain, separation of set-up costs, overtime and the involvement of the storage & production capacities make the models strongly NP-hard and make this problem very challenging unique problem.

1.3 Research Primary Objectives

Primarily, the major objectives of this research are demonstrated as follows:

- Formulating the lot-sizing problem as MILP models with single item and multi-period case.
- Considering the remanufacturing of returns and outsourcing.
- Building up four different mathematical models.
- Considering the cases of separate set-up costs, storage capacity, over time, and environmental impact.
- Solving different instances of these models using reliable data set.
- Developing a metaheuristics solution approach and comparing it with the original analytical models.

1.4 Significance of the Study

This study investigates and optimizes MILP models for a single-item multi-period capacitated lot-sizing problem involving remanufacturing and outsourcing. As will be shown later in the literature review section, the papers related to the topic of this study have many gaps. There is not any previous work in the literature considering these scopes examined in this study. To add to this, this problem is capacitated in terms of production and in terms of storage, and this makes it a unique real case problem. To overcome the production capacity, the study allows for some overtime and outsourcing in some periods. One more unique contribution is that, in this study, a development of four different MILP real case models was accomplished considering different criteria and extensions, and a comparison between their outputs was also performed, which also was not considered before in the literature. In this proposed study, separate set-up costs with over time were considered along with the integrating the environmental aspect into the problem making it a green supply chain problem. Moreover, all the previous problems published in this field were NP-hard problems which, in turn, restrict the model solution capability and therefore require no exact solution method to solve the problem. Considering the contribution of outsourcing and making a restriction on the storage capacity make our model strongly NP-hard and this would complicate the problem more. In addition, this restriction on the storage capacity was not performed before which makes our problem more powerful. A state-of-art metaheuristic approach was developed to overcome this NP-hard feature, and different multi-sizes of instances were examined to test the power of this proposed metaheuristic. An efficient search methodology of neighborhood structures was implemented which is called shaking method and it was used to build up distance-based neighborhood structures for this lot sizing problem. A powerful mathematical and optimization software program called GAMS was used to solve these optimization models

with CPLEX solver. The use of GAMS software is also unique to the literature solving such a problem. Previous studies utilize other softwares, such as Python, C++, Java, and MATLAB. The results obtained from this metaheuristic approach were compared with each original analytical model formulations. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps in the large-scale problems and in terms of time deductions in the small-scale problems. This can be illustrated in a sense that the proposed metaheuristics sometimes reaches to better solutions with almost 74% gap compared to the original analytical model in the large-scale problems, with a deduction of time by more than 92% in the small-scale problems compared to the time the original mathematical models take to find a solutions within 1500 sec time range. To sum up, the consideration of outsourcing, green supply chain, separation of set-up costs, overtime and the involvement of the storage & production capacities make the models strongly NP-hard and make this problem very challenging unique problem. This study will fill a significant gap in this field of research.

1.5 Scope and Future work

The scope of this proposed study is to is to find the best optimal plan that investigates and optimizes MILP models for a single-item multi-period capacitated lot-sizing problem involving remanufacturing and outsourcing. Another objective is to satisfy customer demands by the production of the brand-new products, or the remanufacturing of current returned units acquired from customers. To add to this, dealing with storage capacity, separate set-up costs and overtime are also objectives of this study. Moreover, considering the environmental impact is another aspect in this proposed study. The future work could be accomplished by considering stochastic models which will elevate the difficulty level of this problem. Another future work could be conducted by considering multi-objectives, and multi-items also. Moreover, considering uncapacitated case is one of the possible extensions to this study. Finally, applying hybrid heuristics and/or matheuristics that are designed especially for this problem would enhance the solution outputs and time and will be a unique valuable contribution to the field of this research study.

CHAPTER TWO

LITERATURE REVIEW (LR)

During the latest years, the problem of the lot-sizing with remanufacturing has been investigated by several researchers. A lot of those researchers have conducted their study based on the very basic work that was performed by Wagner and Whitin, (1958). This model discusses the economic lot size dynamic problem. Researchers have relied on that model and they have proposed many extensions based on that model.

2.1 Lot-Sizing with Remanufacturing

2.1.1 Deterministic Demand & MILP Model

The models proposed earlier by Richter and Sombrutzki, (2000); Richter and Weber, (2001); Teunter et al., (2006) discuss the lot sizing case with a single item that includes manufacturing only and excludes outsourcing. The model proposed by Wang et al., (2011) is a possible extension of these earlier proposed models. It investigates a dynamic single-item, multi-period, lot-sizing problem considering remanufacturing along with outsourcing. This model is based on deterministic demand and finite planning horizon. The new items (re)manufacturing, and outsourcing are the sources of satisfying customer demand without backlogging. Minimizing the total cost resulted from determining the optimal plan for corresponding (re)manufacturing, along with outsourcing is the objective of that study. The total cost is decomposed into the returns' holding costs and (re)manufactured items, setup separate costs and the costs of outsourcing. Also, to obtain the optimal solution, the dynamic programming approach was conducted, and it has a complexity of order O (T_2), where T is the number of periods in the planning horizon. In addition,

the paper proposed by Fazle Baki et al., (2014) also addresses the problem of lot-sizing considering remanufacturing of returned products. The model deals with deterministic demand and limited planning horizon of the returns. Determining the optimal plan which minimizes the overall cost consisting of setup along with holding costs is the aim of this study. The demand is then satisfied by either remanufacturing returned units or manufacturing new products. The paper formulates a MILP model after proving, generally, that an example of introduced problem is NP-Hard. Also, a dynamic programming-based heuristic was developed, and general numerical examples were performed. Moreover, Cunha and Melo, (2016) investigated the economic problem of lot-sizing including remanufacturing. This type of problem deals with a single item and deterministic demand on a limited period. The customer demand is also satisfied by two different options: the production of new products or the remanufacturing of these returned items. A multicommodity (MIP) model and a reference on Wagner and Whitin, (1958) formulation were developed aiming to minimize the overall cost. A dynamic heuristic was conducted, and larger numerical instances were examined. Interestingly, in a remanufacturing system, the problem of the dynamic capacitated time lot sizing considering the returns of customer products is addressed in the paper proposed by Roshani et al., (2016). The deterministic demand of the products of single level type is satisfied by remanufacturing the customers return. For the sake of minimizing the costs of remanufacturing over a limited time period, a mathematical formulation is developed, and it is an NP-hard problem. Thus, the latter paper proposed a simulated annealing approach and obtained an efficient solution by generating a set of experimental examples. Considering the study investigated by Zouadi et al., (2016), the problem of lot sizing with remanufacturing under a constraint on the carbon emission was addressed. For every operation, the system considers unrelated set-up costs whether it is a production or remanufacturing operation. The latter paper also considers a

deterministic return's quantity over a limited planning horizon. The reverse of Wagner and Whitin, (1958) problem was also extended along with a Hybrid Algorithm. Also, addressing the economic problem of lot-sizing considering remanufacturing is also investigated by Piñeyro and Viera, (2009). The remanufacturing of the old returned items is the way of satisfying the demand. Relying on the analysis of such a problem, a set of inventory policies was suggested and evaluated. The useful returns and useful remanufacturing are the new concepts introduced in this paper as well. Maximizing the utilization of returned products along with minimizing the overall costs are the aim of this study. The latter paper proposes a MILP formulation and showed that it is NP-hard problem. Thus, basic Tabu Search procedure and several inventory policies were developed. Interestingly, Helmrich et al., (2014) introduces the idea of having a process with a choice of having separate/joint setup costs. In their study, the satisfaction of customer deterministic demand by producing modern units or remanufacturing old units. Knowing that both choices are NP-hard, several different (MIP) formulations were developed. Also, more tight formulations, like the formulations of both shortest path and semi shortest path are proposed since classical lot-sizing formulations provide weak lower bounds. To add to this, in the paper proposed by Mohammadi and Tap, (2012), a MILP formulation for the multi-item uncapacitated deterministic lot-sizing problem was addressed. Finding the best order time/quantity which minimizes the overall cost is the aim of this paper. The total costs involve purchasing, ordering and holding cost accomplished on a limited period of time. Moreover, Ahn et al., (2011) proposed the lot sizes problem with remanufacturing and deterministic demand on a limited period of time. The goal is minimizing the overall costs consisting of setup/holding costs. A (MIP) model was suggested in this paper. The paper suggests two heuristics based on dynamic programming after proving that such a problem is NP-hard. They are called the aggregation heuristic and the decomposition heuristic.

Interestingly, in the paper proposed by Cunha et al., (2019), also a MILP matheuristics was developed for the problem of capacitated multi-item lot-sizing considering remanufacturing. The latter problem is only capacitated in terms of production, not in terms of storage, and it is recognized as NP-hard type problem. The demand is deterministic and is fulfilled by producing capacitated number of units or by remanufacturing them. The main goal is to determine an optimal plan over a discrete time horizon. An exact method called column generation followed by MILP along with relax-and-fix method are the constructive approaches that were proposed in this paper. A local search called fix-and-optimize is also tackled to improve the obtained solution quality. In the paper proposed by Sifaleras et al., (2015), the variable neighborhood search method for the ELSP with remanufacturing is addressed, which has been proved to be NP-hard. The paper investigates two VNS methodologies, some new neighborhoods and an efficient way for exploring them. Different instances are examined, and the robustness is also demonstrated. On the other hand, a VNS approach is addressed for an uncapacitated multi-level lot sizing problem in the research work proposed by Xiao et al., (2011). The paper defines a neighborhood structure and two different moving policies along with a shaking rule for better search approach.

2.1.2 Stochastic Demand & MILP Model

On the other hand, the study proposed by Kilic, (2013) extends the classical model by considering a (re)manufacturing systems dealing with both stochastic dynamic demands and returns with a constraint on the customer service level. The satisfaction of the demand is accomplished by producing brand new products and remanufacturing returned demand units. Building on a combination between static and dynamic stochastic strategies, the latter paper addresses a MILP based heuristic specified for this given problem. Also, in the article introduced by Jing et al., (2014), they address the problem of dynamic capacitated lot-sizing considering

remanufacturing. The latter problem is only capacitated in terms of production, not in terms of storage, and it is recognized as NP-hard type problem. The demand for remanufactured products could also be satisfied by new ones. A fuzzy MILP model was tackled, and it considers the demands uncertainties, and the capacity constraints. Genetic algorithm is the solution approach designed for that problem. Moreover, the research work proposed by Abdel-Aal, (2019) considers also a capacitated problem with uncertain demand and multi-item multi-period case. The latter problem is only capacitated in terms of production, not in terms of storage, and it is recognized as NP-hard type problem. The latter work also involves a consideration of setup and over times along with backordering. The model addresses a robust MILP optimization with a budget-uncertainty. After considering different cost structures and parameters, the study applies a robust optimization approach to tackle the uncertainty of the demand resulted from unknown probability distribution. This latter paper also examined some numerical experiments to verify the validity and power of this study.

2.1.3 Stochastic Demand & MINLP Model

An inventory system including activities such as production, remanufacturing along with refurbishing is examined on a limited period of time in the paper presented by Konstantaras, (2010). The demand is dynamic and stochastic, and the proposed paper model utilized a MINLP formulation. After products return form customers, products are distinguished as either “re-manufacturable” or “re-furbishable” items. The procedure of determining the order and the quantities that were remanufactured and the level of inventory of returned units is investigated in order to minimize the overall costs. The disassembly operations are considered in the contribution performed by Schulz and Ferretti, (2011). In addition, the work is extended to tackle the behavior

of the system in a random environment. A MINLP approach was tackled and three different heuristic approaches were examined as well.

2.2 Lot-Sizing with Different Approaches

The paper proposed by Zikopoulos, (2012) investigates a remanufacturing system considering a multi-period along with the uncertainty regarding the quality of returned units. The major goal of this research is to investigate the degree of advisability of considering the quality information to be utilized in the decision-making processes and make a comparison with the other alternative policies. On the other hand, the paper proposed by van den Heuvel and Wagelmans, (2008) shows the equivalency between three different models compared to a classical model. These three models are the model of lot-sizing considering a remanufacturing option, the model of lot-sizing considering production time windows, and a model of lot-sizing considering overall capacities. It has been shown in this paper that these three models are identical to the classical lot-sizing model with inventory limits. A single unit system of recovery is tackled in the paper proposed by Konstantaras, (2010). In this type of systems, utilized products are recovered later after being collected from customers. Customer deterministic constant demand is to be fulfilled by producing new units or remanufacturing these old units, with backlogging of excess demand. A specific analytical method is developed, and the optimal lot sizes as well as optimal set-up numbers are found. To add to this, the article introduced by Andrew-Munot and Ibrahim, (2013) investigated two remanufacturing decision-making ways. The lot-sizing problems considering inventory in a variable (re)manufacturing system is the first decision-making way addressed in this paper. Addressing a production/inventory problem in a remanufacturing system with non-similar remanufacturing policies is the second decision making tool. The remanufactured systems are a remanufacture-to-stock system and a remanufacture-to-order system. “The production and

inventory planning simulation models in a remanufacturing system are expected to become significant decision-making tools in remanufacturing operations" Andrew-Munot and Ibrahim, (2013). On the other hand, the analysis of the repair and waste EOQ disposal system was discussed in the paper proposed by Richter, (1997). The model considers different setup values for production/repair in some interval of time. The cost analysis is extended and the non-similar trend of the lowest cost, best setup values, lot sizes and aggregation periods are investigated as well. The paper proposed by Piñeyro and Viera, (2012) tries to find the best solution of the (ELSR) problem. They showed that the overall remanufacturing number of units corresponding to a remanufacturing plan of optimal cost can be obtained apart making an assumption of known periods where a carry out of remanufacturing is performed. The acquired results can be utilized to obtain a solvable algorithm that is effective for acquiring a solution to the ELSR problem. Moreover, the paper proposed by Suzanne et al., (2020) considers the challenges and opportunities encountered when aiming to achieve a circular economy in production planning. Considering discrete time plans, this latter paper addresses a valuable comprehensive review of the scopes related to the reverse logistics which is correlated with the circular economy concept. Figure 4. gives an overview of the circular economy systems proposed by Suzanne et al., (2020).

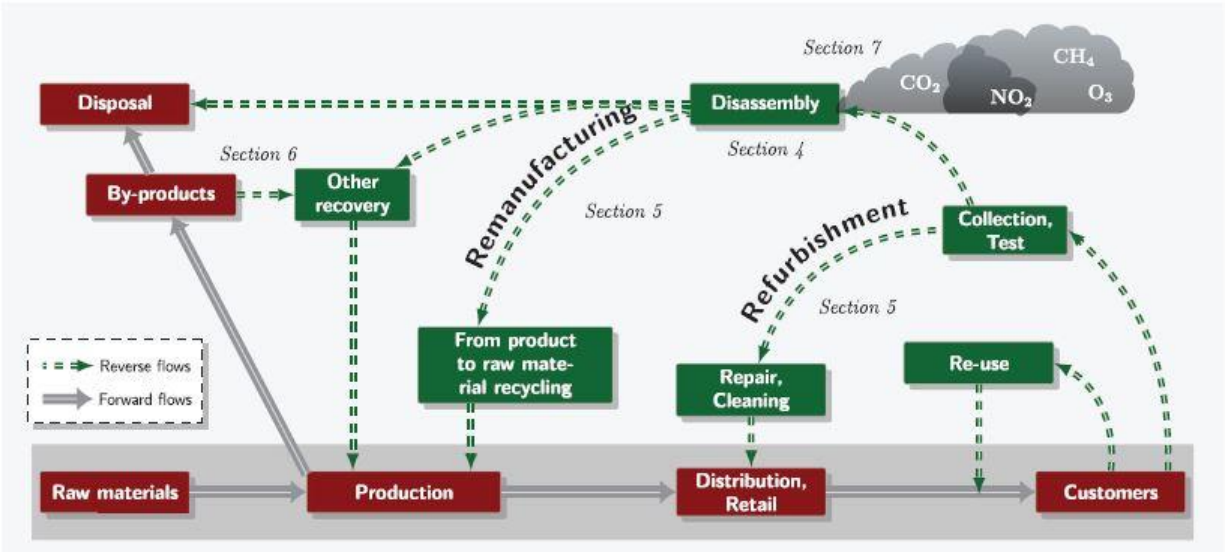

Figure 4. An overview of the circular economy systems
Suzanne et al., (2020)

The paper proposed by Meyr and Mann, (2013) investigates the general lot-sizing and scheduling problem with a specific consideration of production lines that are parallel. The aim of this latter study is to minimize the overall costs. The study utilizes a decomposition approach to tackle the multi-line case and decompose into a group of single-line problems. The paper considers different approaches and those approaches are compared with existing solutions through different sizes of instances. Another interesting scope that could be classified also under this section of the literature review and would add a valuable contribution in this topic of research is the work accomplished by Zouadi et al., (2016). This latter paper introduces the environmental aspect to the economic lot sizing problem with remanufacturing. Different set-up costs are considered for both (re)manufacturing under a single production line. A mathematical model was considered in this latter paper along with two hybrid approaches (HM1 & HM2) which give approximate solutions, and a comparison of the results was performed as well. The following Table1. provides a valuable comprehensive Literature Review (LR) summary.

Paper Title\Covered Topics	# Items		# Period		Capacitated Problem	Demand		Finite Planning Horizon	Separate Set-Up	Model		Environment	Method
	Single	Multi	Single	Multi		Deterministic	Stochastic			MILP	MINLP		
01- Wagner and Whitin, (1958)	√			√		√		√		√			Dynamic version of ELSP
02- Richter, (1997)	√							√					Tracing the interaction between economic and ecological factors
03- Richter and Sombrutzki, (2000); Richter and Weber, (2001); Teunter et al., (2006)	√			√		√		√		√			Improvement of Wagner and Whitin, (1958) with Zero-Inventory-Property
04- van den Heuvel and Wagelmans, (2008)													Comparison between 4 models
05- Piñeyro and Viera, (2009)						√				√			Decomposition analysis & a basic Tabu Search procedure
06- Konstantaras, (2010)	√							√					An exact analytical approach is developed
07- (Konstantaras, 2010)							√	√			√		Two different policies related to items inspection, sorting & numb of batches
08- Schulz and Ferretti, (2011)							√				√		Three different heuristic approaches are examined
09- Wang et al., (2011)	√			√		√		√		√			Dynamic Programming
10- Ahn et al., (2011)						√		√		√			Two Dynamic Programming-Based Heuristics
11- Xiao et al., (2011)		√		√		√		√	√	√			VNS Metaheuristics

12- Zikopoulos, (2012)		√				√							Comparison with the other alternative policies
13- Mohammadi and Tap, (2012)		√				√		√		√			MIP-Based Heuristics
14- Piñeyro and Viera, (2012)						√		√					
15- Meyr and Mann, (2013)		√		√		√		√		√			A Decomposition approach
16- Kilic, (2013)				√			√			√			Static-Dynamic uncertainty heuristic strategy
17- Andrew-Munot and Ibrahim, (2013)													1-Four alternative policies were examined. 2-Multi-Criteria decision making tools to improve inventory management
18- Fazle Baki et al., (2014)						√		√		√			Dynamic Programming-Based Heuristic
19- Jing et al., (2014)					√		√			√			Dynamic & genetic algorithms
20- Helmrich et al., (2014)						√			√	√			Tighter formulations are proposed and heuristics
21- Sifaleras et al., (2015)	√			√		√		√	√	√			VNS Metaheuristics
22- Cunha and Melo, (2016)		√				√		√		√			Dynamic Heuristic
23- Roshani et al., (2016)	√			√	√ (P)	√		√		√			Simulated Annealing algorithm
24- Roshani et al., (2016)	√		√		√ (P)	√		√		√			Simulated Annealing

25- Zouadi et al., (2016)	√					√		√	√	√		√	1- An extension of the reverse Wagner and Whitin, (1958) 2- Dynamic manufacturing planning was proposed along with a Hybrid algorithm
26- Abdel-Aal, (2019)	√			√	√ (P)		√	√	√	√			1- Robust optimization approach for uncertain demand. 2- Robust counterpart model based on duality theory
27- Cunha et al., (2019)		√		√	√ (P)	√		√		√			Matheuristics
28- Suzanne et al., (2020)													Comprehensive review
Thesis Proposal	√			√	√ (P [T]) & (S)	√		√	√	√		√	A unique VNS Metaheuristic with 4 Models

Table 1. LR summary. P=Production T=Time S=Storage

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter provides the essential steps and methodologies needed to fulfill the objectives presented previously in chapter one of this study. It explains the models, pseudocodes, and approaches utilized to achieve the results obtained in this study. Moreover, the methods and mathematical models developed in this research work is going to be explained in detail. In a summary, this chapter acts as a roadmap guiding the researcher to accomplish the proposed study effectively and efficiently.

3.2 Models' Assumptions

The major assumptions of the proposed models are demonstrated as follows:

- Shortage is not allowed in any period and, therefore, no shortage cost is considered in the models' formulations.
- No backlogging is considered in the models.
- There is no upper limit on overtime of production and remanufacturing.
- Demand is deterministic with pre-known quantity of returns.
- No fixed cost of outsourcing and no outsourcing inventory are considered in the models' formulations.
- The outsourcing cost is assumed to be the highest between all costs and its value could be an estimated fraction of the demand based on the applied case study.

- The quantities of production and remanufacturing will be covered by both regular time and overtime, and this assures that gas emission during overtime is not missing.
- A uniform distribution is used only for generating data inputs since the problem size is significantly large, and this does not affect the deterministic feature of the models.
- The formulations and the data structure are unitless and they depend on the structure of the implemented case study.

3.3 Mathematical Models

3.3.1 Indices Set

Following is the set representing the indices of parameters and decision variables:

Set t = {1, 2, 3, ……, T} = set of periods spanning the planning horizon.

For each period $t \in T$, the models have a dynamic deterministic demand, where periods could be in any unit of time, such that hours, days or years depending on the structure of the implemented case study.

3.3.2 Parameters

Following are descriptions of parameters used in the models of this study:

f_t^p Fixed cost (set-up cost) of production in period $\boldsymbol{t}$.

v_t^p Variable cost of production in period $\boldsymbol{t}$.

f_t^r Fixed cost (set-up cost) of remanufacturing in period $\boldsymbol{t}$.

v_t^r Variable cost of remanufacturing in period $\boldsymbol{t}$.

v_t^{os} Variable cost of outsourcing in period $\boldsymbol{t}$.

oc_t^p Overtime cost of production in period $\boldsymbol{t}$.

oc_t^r Overtime cost of remanufacturing in period $\boldsymbol{t}$.

h_t^p Holding cost of produced units in period $\boldsymbol{t}$.

h_t^r Holding cost of returned units in period $\boldsymbol{t}$.

st_t^p Set-up times of production in period $\boldsymbol{t}$.

st_t^r Set-up times of remanufacturing in period $\boldsymbol{t}$.

pt_t^p Unitary Production times in period $\boldsymbol{t}$.

pt_t^r Unitary Remanufacturing times in period $\boldsymbol{t}$.

lt_t^p Limits the possible time for producing new units in period $\boldsymbol{t}$.

lt_t^r Limits the possible time for remanufacturing new units in period $\boldsymbol{t}$.

d_t Deterministic demand in period $\boldsymbol{t}$.

r_t Returned units in period $\boldsymbol{t}$.

e^p Unit emission per produced units.

e^r Unit emission per remanufactured units.

se^p Set-up emission of production.

se^r Set-up emission of remanufacturing.

$\overline{I_t^p}$ Max inventory level of production in period $\boldsymbol{t}$.

$\overline{I_t^r}$ Max inventory level of remanufacturing in period $\boldsymbol{t}$.

$\overline{OS_t}$ Max allowable amount of outsourcing in period $\boldsymbol{t}$.

$E_{\max(t)}$ Max unitary environmental impact allowed in period $\boldsymbol{t}$.

The overall demand and quantity of returns are computed as:

$$d_{kt} = \sum_{l=k}^{t} d_l \ , \ r_{kt} = \sum_{l=k}^{t} r_l \ , \ l = k, \dots, t \ , \ 1 \le k \le t \le T$$

Therefore, **upper bounds** of produced amounts and remanufactured amounts in period *t* can be comprehended as:

$M_t^p = \sum_{i=t}^{T} d_t \ and \ M_t^r = \sum_{i=t}^{T} r_t$, *where* ***i*** *is just a counter*. These upper bounds are measured based on the accumulated demand and returns from this period *t* until the end of the planning horizon *T*.

3.3.3 Decision Variables

Following are descriptions of decision variables used to optimize the models of this study:

y_t^p = $\begin{cases} 1, \text{if a production of a unit occurs in period } \boldsymbol{t}. \\ 0, \text{otherwise} \end{cases}$

y_t^r = $\begin{cases} 1, \text{if a remanufacturing of a unit occurs in period } \boldsymbol{t}. \\ 0, \text{otherwise} \end{cases}$

x_t^p The amount of production in period $\boldsymbol{t}$.

x_t^r The amount of remanufacturing in period $\boldsymbol{t}$.

x_t^{os} The amount of outsourcing in period $\boldsymbol{t}$.

I_t^p The inventory level of production at the end of period $\boldsymbol{t}$.

I_t^r The inventory level of remanufacturing at the end of period $\boldsymbol{t}$.

o_t^p The overtime amount of production in period $\boldsymbol{t}$.

o_t^r The overtime amount of remanufacturing in period $\boldsymbol{t}$.

3.3.4 First Model Formulation (F1)

In this mathematical formulation, the model examines the **overtime case** with a **limited storage capacity** restriction as well. This model also considers a **separation of the set-up costs** which, in turn makes this model NP-hard and this would complicate the problem more. In this case, as discussed before, the upper bounds of the produced amounts and remanufactured amounts in period t are computed as $M_t^p = \sum_{i=t}^{T} d_t \ and \ M_t^r = \sum_{i=t}^{T} r_t$. Following is the formulation of this model **F1**:

O.F

1) Min

$$\sum_t(h_t^p\ I_t^p + v_t^p\ x_t^p + f_t^p\ y_t^p) + \sum_t(h_t^r\ I_t^r + v_t^r\ x_t^r + f_t^r\ y_t^r) + \sum_t(oc_t^p\ o_t^p) + \sum_t(oc_t^r\ o_t^r)$$

S.t

2) $I_{t-1}^p + x_t^p + x_t^r - I_t^p = d_t$ $\forall t$

3) $I_{t-1}^r + r_t - x_t^r - I_t^r = 0$ $\forall t$

4) $x_t^p \leq M_t^p\ y_t^p$ $\forall t$

5) $x_t^r \leq M_t^r\ y_t^r$ $\forall t$

6) $pt_t^p\ x_t^p + st_t^p\ y_t^p \leq\ lt_t^p + o_t^p$ $\forall t$

7) $pt_t^r\ x_t^r + st_t^r\ y_t^r \leq\ lt_t^r + o_t^r$ $\forall t$

8) $I_t^p \leq \overline{I_t^p}$ $\forall t$

9) $I_t^r \leq \overline{I_t^r}$ $\forall t$

10) $I_0^p = 0$

11) $I_0^r\ = 0$

12) $x_t^p, x_t^r, I_t^p, I_t^r, o_t^p, o_t^r \geq 0$ $\forall t$

13) $y_t^p, y_t^r = \{0,1\}$ $\forall t$

3.3.5 Second Model Formulation (F2)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime** case. A **separation of the set-up** costs is also considered. This would complicate the model more and would make it NP-hard problem. Following is the formulation of this model **F2**:

O.F

1) Min

$$\sum_t(h_t^p\ I_t^p + v_t^p\ x_t^p + f_t^p\ y_t^p) + \sum_t(h_t^r\ I_t^r + v_t^r\ x_t^r + f_t^r\ y_t^r) + \sum_t(v_t^{os}\ x_t^{os}) + \sum_t(oc_t^p\ o_t^p) + \sum_t(oc_t^r\ o_t^r)$$

S.t

2) $I_{t-1}^p + x_t^p + x_t^r + x_t^{os} - I_t^p = d_t \qquad \forall t$

3) $I_{t-1}^r + r_t - x_t^r - I_t^r = 0 \qquad \forall t$

4) $x_t^p \le M_t^p\ y_t^p \qquad \forall t$

5) $x_t^r \le M_t^r\ y_t^r \qquad \forall t$

6) $x_t^{os} \le \overline{OS_t} \qquad \forall t$

7) $pt_t^p\ x_t^p + st_t^p\ y_t^p \le\ lt_t^p + o_t^p \qquad \forall t$

8) $pt_t^r\ x_t^r + st_t^r\ y_t^r \le\ lt_t^r + o_t^r \qquad \forall t$

9) $I_0^p = 0$

10) $I_0^r\ = 0$

11) $x_t^p, x_t^r, x_t^{os}, I_t^p, I_t^r, o_t^p, o_t^r \ge 0 \qquad \forall t$

12) $y_t^p, y_t^r = \{0,1\} \qquad \forall t$

3.3.6 Third Model Formulation (F3)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime case**. A restriction on the **storage capacity** and a **separation of the set-up** costs are also considered. This would complicate the model more and would make it strongly NP-hard as well. Following is the formulation of this model **F3**:

O.F

1) Min

$$\sum_t(h_t^p\ I_t^p + v_t^p\ x_t^p + f_t^p\ y_t^p) + \sum_t(h_t^r\ I_t^r + v_t^r\ x_t^r + f_t^r\ y_t^r) + \sum_t(v_t^{os}\ x_t^{os}) + \sum_t(oc_t^p\ o_t^p) + \sum_t(oc_t^r\ o_t^r)$$

S.t

2) $I_{t-1}^p + x_t^p + x_t^r + x_t^{os} - I_t^p = d_t$ $\forall t$

3) $I_{t-1}^r + r_t - x_t^r - I_t^r = 0$ $\forall t$

4) $x_t^p \le M_t^p\ y_t^p$ $\forall t$

5) $x_t^r \le M_t^r\ y_t^r$ $\forall t$

6) $x_t^{os} \le \overline{OS_t}$ $\forall t$

7) $pt_t^p\ x_t^p + st_t^p\ y_t^p \le\ lt_t^p + o_t^p$ $\forall t$

8) $pt_t^r\ x_t^r + st_t^r\ y_t^r \le\ lt_t^r + o_t^r$ $\forall t$

9) $I_t^p \le \overline{I_t^p}$ $\forall t$

10) $I_t^r \le \overline{I_t^r}$ $\forall t$

11) $I_0^p = 0$

12) $I_0^r = 0$

13) $x_t^p, x_t^r, x_t^{os}, I_t^p, I_t^r, o_t^p, o_t^r \ge 0$ $\forall t$

14) $y_t^p, y_t^r = \{0,1\}$ $\forall t$

3.3.7 Fourth Model Formulation (F4)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime case**. A restriction on the **storage capacity** and **a separation of the set-up** costs are also considered. To add to this, carbon emission is added in this formulation to consider the **environmental case** as well. These extensions combined are significant to be considered together, and this would complicate the model more and would make it strongly NP-hard as well. Following is the formulation of this model **F4**:

O.F

1) Min

$$\sum_t(h_t^p\, I_t^p + v_t^p\, x_t^p + f_t^p\, y_t^p) + \sum_t(h_t^r\, I_t^r + v_t^r\, x_t^r + f_t^r\, y_t^r) + \sum_t(v_t^{os}\, x_t^{os}) + \sum_t(oc_t^p\, o_t^p) + \sum_t(oc_t^r\, o_t^r)$$

S.t

2) $I_{t-1}^p + x_t^p + x_t^r + x_t^{os} - I_t^p = d_t$ $\forall t$

3) $I_{t-1}^r + r_t - x_t^r - I_t^r = 0$ $\forall t$

4) $x_t^p \le M_t^p\, y_t^p$ $\forall t$

5) $x_t^r \le M_t^r\, y_t^r$ $\forall t$

6) $x_t^{os} \le \overline{OS_t}$ $\forall t$

7) $pt_t^p\, x_t^p + st_t^p\, y_t^p \le lt_t^p + o_t^p$ $\forall t$

8) $pt_t^r\, x_t^r + st_t^r\, y_t^r \le lt_t^r + o_t^r$ $\forall t$

9) $I_t^p \le \overline{I_t^p}$ $\forall t$

10) $I_t^r \le \overline{I_t^r}$ $\forall t$

11) $I_0^p = 0$

12) $I_0^r = 0$

13) $\sum_t(e^p\, x_t^p + se^p\, y_t^p + e^r\, x_t^r + se^r\, y_t^r) \le \sum_t(d_t\, E_{max(t)})$

14) $x_t^p, x_t^r, x_t^{os}, I_t^p, I_t^r, o_t^p, o_t^r \ge 0$ $\forall t$

15) $y_t^p, y_t^r = \{0,1\}$ $\forall t$

3.3.8 Models Explanation

The explanation of the models is going to be performed on the last fourth model (F4), since it combines the ideas and extensions of all previous models. Therefore, it can be considered as the general comprehensive model and that is why the explanation will be based on that specific model. The function representing the objective of this model is simply to minimize the overall costs consisting of production, remanufacturing, setup, holding, outsourcing and overtime costs. The 1st constraint along with the 2nd constraint represent the balance of the final products and the returned items. The 3rd constraint along with the 4th constraint represent the capacity constraints for both (re)manufacturing. In another word, in any period *t*, the production and return quantities cannot exceed the remaining deterministic demand which is to be fulfilled from period *t* to *T*. Thus, if there is a setup in a period *t*, its related cost will automatically be added to the objective function. The 5th constraint is developed to make an upper bound for the outsourcing quantity. The 6h constraint along with the 7th constraint represent the time capacity constraints for both (re)manufacturing. To illustrate this group of constraints, the allowed production time along with the used production overtime in a specific period *t* should be enough for accommodating the total production time and the setup time needed in that period *t*. Similarly, the allowed remanufacturing time along with the used remanufacturing overtime in a specific period *t* should be enough for accommodating the total remanufacturing time and the setup time needed in that period *t*. Inventory storage capacities of both (re)manufacturing are represented in the 8th and 9th constraints. The inventories' initializations are set in both 10th and 11th constraints. The 12th constraint is the carbon emission constraint which assures that the emissions resulted from both (re)manufacturing cannot exceed a specific assigned upper limit. The last two constraint which are the 13th and 14th constraints represent the nonnegativity and binary constraints respectively.

3.4 Metaheuristic Solution Methodology

3.4.1 Variable Neighborhood Search (VNS) Approach

A metaheuristic search methodology called Variable Neighborhood Search (VNS) is an excellent algorithm designed for dealing with many kinds of optimization problems. As mention in the literature review section, this methodology is investigated by many researchers, such as the works addressed by Xiao et al., (2011) and Sifaleras et al., (2015). This search strategy has a main major advantage compared to other local search techniques. This advantage is that, within a local search strategy, predefined neighborhoods are changed systematically. Thus, it can perform a highly efficient searching in nearby neighborhoods where better solutions are easy to find. Its way of searching is that it starts exploring about the predefined neighborhoods from nearest to farthest and, if no better solutions are found, it jumps out to explore farther neighborhoods. For those pre reasons and justifications, VNS algorithm distinguishes itself from other local search methods, such as Genetic algorithm, Tabu search algorithm and Simulated Annealing algorithm. The basic VNS scheme has two main phases. Neighborhood structure definition, sequence exploration selection, and incumbent solution initiation could be considered as the 1st phase of this basic VNS. A neighborhood search loop is considered as the second phase where better solutions are iteratively examined (nearest is first) and then assigned as a current incumbent solution. In this proposed thesis study, an advanced unique VNS algorithm is tackled, and a proper local search methodology is developed. Section 3.4.3 illustrates the pseudocode associated with the thesis study in hand.

3.4.2 The Neighborhood Local Search Structure

To implement the VNS metaheuristic algorithm, a Local Search (LS) approach has to be conducted. This proposed LS approach defines a neighborhood structure that is based on the distance between the neighborhood solutions $N_j(x)$ and the incumbent solution (x). Based on the differentiation between two solutions, a distance metric is developed. It could be stated as the number of different points between any two binary solutions, such as $\{x, x'\}\ \forall x \in X$, where X is the set of all possible solutions of the problem. This can be represented as follows:

$$\rho\,(x, x') = |x \setminus x'| = |x' \setminus x| \quad \forall x, x' \in X$$

where $|x \setminus x'|$ is the number of different points between the two solutions x and x'. To illustrate this, consider the following example of the two solutions x and x' in a problem with three items and three periods as follows:

$$x = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & \mathbf{0} \\ 1 & \mathbf{1} & 1 \end{vmatrix} \qquad x' = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & \mathbf{1} \\ 1 & \mathbf{0} & 1 \end{vmatrix}$$

Therefore, $\rho\,(x, x') = |x \setminus x'| = 2$. In this proposed search methodology, a new neighborhood solution is extracted from the current incumbent solution by a shaking mechanism which is performed by switching or reversing a single value or more of the incumbent solution randomly. By this technique, a new solution is defined and based on the changes of the number of values, new neighborhoods are created.

$$x\,(incumbent) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & \mathbf{1} \\ 1 & 1 & 1 \end{vmatrix} \quad x' \in N_1(x) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & \mathbf{1} & 1 \end{vmatrix} \quad x'' \in N_2(x) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

Another helping approach utilized in this study is accomplished by using the Zero-Inventory-Level (ZIL) property which is, simply, satisfies the following equation:

$$I_{t-1}^{p} \cdot y_t^p = 0$$

This property might not guarantee that a feasible solution is achieved, since it will force y_t^p to be zero even if I_{t-1}^{p} is very small. However, if an infeasible solution is generated due to the use of this property, it will be ignored automatically. Another point is that implementing this property in the solution code gives better time than shaking $y_t^p and\ y_t^r$ only. However, comparing the time difference between the possible combinations shows that the computational time gap is not significant compared to the solution gap. For this reason, this property was only utilized in the initialization phase and not in the implementation phase.

3.4.3 Pseudocode Development

Following is the pseudocode developed in this research work:

- Heuristic Initialization Method
    - **Procedure** heuristic-initialize
        - Set $y_t^p, y_t^r \leftarrow 1$
        - Set $y_t^p, y_t^r \leftarrow 0$ if $x_t^p, x_t^r \leftarrow 0$, otherwise set $y_t^p, y_t^r \leftarrow 1$
        - Solve the original analytical model as LP
        - Set $x_t^p, x_t^r, x_t^{os}, I_t^p, I_t^r, o_t^p, o_t^r \leftarrow outcome$
    - **End procedure**

- Local Search Algorithm

  - **Procedure** local search
    - **Repeat**
      - $Improvement \leftarrow 0$
      - **For** $J \leftarrow 1$, J_{max} ***do***
        - **For** $I \leftarrow 1$, I_{max} ***do***
          - Find the best neighborhood x' of x where $[x' \in N_1(x)]$
          - **If** x' is better than x **then**
            - Set $x \leftarrow x'$
            - Set $Improvement \leftarrow 1$
          - **End if**
        - **End for**
      - **End for**
    - **Until** improvement = = 0
  - **End procedure**

- VNS Algorithm

  - **Procedure** VNS
    - Apply the heuristic-initialize method
    - **While** $t < T$ **do**
      - Apply the local-search Algorithm
      - Shake k consecutive $N_j(J \in 1, \ldots, J_{max})$ one after the other.
      - Generate a random N_j order by $\rho\ (x, x')$ structure.
    - **End while**
  - **End procedure**

3.4.4 Environmental Policy

This research study integrates an environmental policy represented in the formulations as a carbon emission constraint. This constraint is derived from the global carbon emission regulation where the unitary carbon emission is capacitated by an upper limit of a max unitary environmental impact E_{max}. This holds over the whole planning horizons. The paper proposed by Absi et al., (2013) investigates different environmental policies and regulations.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This chapter discusses the results obtained from the developed previous four models and then a comparison will be performed between the CPLEX solver analytical outputs and the proposed metaheuristic methodology. Both solver and metaheuristic were implemented in GAMS optimization software. Some of the utilized data structure was taken from different prework performed in the literature and is mentioned in the reference section. The rest of the data are assumed based on the structure of the problem in hand. This data structure proves its reliability and efficiently when compared to other studies. Moreover, the upcoming section will show that the results obtained from the proposed metaheuristics methodology outperforms the original solver outputs in terms of time in the small-scale problems and in terms of solutions in the large-scale problems. The problems sizes are huge as will be shown in this chapter. The size depends on the number of periods, integer variables, continuous variables and constraints.

4.2 Data Structure

The data used in achieving this study is classified into models' parameters, scalars, and metaheuristic parameters. Most of the parameters utilized in this study are uniformly distributed and some of them are fixed. The values of these parameters and scalars are shown in Table2, Table3 and Table4 respectively. The data structure is unitless and it depends on the structure of the implemented case study.

Table 2. Models' parameters for the small-scale problems

Parameter	Value
f_t^p	uniform (8000, 10000)
v_t^p	1
f_t^r	uniform (7000, 8000)
v_t^r	1
v_t^{os}	600
oc_t^p	uniform (40, 45)
oc_t^r	uniform (40, 45)
h_t^p	uniform (1, 5)
h_t^r	uniform (1, 5)
st_t^p	uniform (150, 180)
st_t^r	uniform (150, 180)
pt_t^p	uniform (4, 6)
pt_t^r	uniform (4, 6)
lt_t^p	uniform (1200, 1800)
lt_t^r	uniform (1200, 1800)
d_t	uniform (150, 200)
$\overline{OS_t}$	uniform (100, 200)
r_t	uniform (50, 80)
$E_{\max(t)}$	uniform (700, 800)
$\overline{I_t^p}$	uniform (500, 600)
$\overline{I_t^r}$	uniform (500, 600)

Table 3. Models' parameters for the large-scale problems

Parameter	Value
f_t^p	uniform (49000, 50000)
v_t^p	1
f_t^r	uniform (1000, 2000)
v_t^r	1
v_t^{os}	600
oc_t^p	uniform (50, 55)
oc_t^r	uniform (45, 50)
h_t^p	1
h_t^r	1
st_t^p	uniform (150, 180)
st_t^r	uniform (150, 180)
pt_t^p	uniform (1, 1)
pt_t^r	uniform (1, 1)
lt_t^p	uniform (5000, 7000)
lt_t^r	uniform (5000, 7000)
d_t	uniform (1, 200)
$\overline{OS_t}$	uniform (600, 1000)
r_t	uniform (90, 100)
$E_{\max(t)}$	uniform (1000, 2000)
$\overline{I_t^p}$	uniform (5000, 10000)
$\overline{I_t^r}$	uniform (4000, 8000)

Table 4. Models' Scalars

Scalar	Value
e^p	10
e^r	8
se^p	6
se^r	4

4.3 Pre-Runs Combinations Selection

As discussed earlier in section 3.3.2, it has been shown that this metaheuristics methodology uses different approaches, which are the shaking approach and the ZIL property. The consideration of these approaches gives a plenty of combinations. In this section, different educationally selected combinations are examined in order to come up with the optimal combination that yields to the best possible outcomes. The considered combinations are accomplished by shaking y_t^p only, shaking y_t^r only, shaking both y_t^p, y_t^r, shaking y_t^p considering the ZIL property of I_t^p, and shaking y_t^r considering the ZIL property of I_t^r. The final combination considers all of those approaches which are shaking both y_t^p, y_t^r and considering the ZIL property of both I_t^p, I_t^r. Since the 4th model is the most comprehensive model between all models, those trials are performed on this 4th model (F4). Thirty iterations per each combination with 100 periods are performed in this section and then the results of those different combinations will be compared with the original solver output. The various combinations of these pre runs are illustrated in the coming two tables; Table5 and Table6.

Table 5. Shaking of yp only, yr only and both yp,yr

Serial	Yp only		Yr only		Yp & Yr	
	Cost	Time	Cost	Time	Cost	Time
1	200670.98	428.73	179838.5	449.51	169760.75	223.08
2	200515.9	338.62	183103.82	358.5	177102.92	277.54
3	218224.15	383.89	177823.49	343.98	171038.79	404.93
4	204359.16	427.39	185692.12	542.66	163896.63	505.52
5	214019.1	351.24	176566.11	594.61	169177.42	315.21
6	209773.93	233.78	172250.59	333.72	173251.51	261.64
7	207213.36	500.31	183492.37	422.95	176694.61	300.59
8	202305.24	541.87	193723.35	300.91	171319.25	355.24
9	203008.76	568.37	188686.86	271.98	164855.46	385.89
10	218282.35	349.12	178775.19	431.41	169779.11	312.95
11	202018.72	457.01	180860.28	248.73	167190.65	372.03
12	202054.37	360.32	188461.9	312.35	184019.97	343.94
13	214323.95	402.87	182232.31	265.13	166524.69	394.06
14	205262.96	674.42	184008.09	304.98	166419.82	303.24
15	209435.52	287.42	171890.85	498.3	167995.43	362.12
16	202909.97	446.75	189505.38	281.62	177816.38	287.91
17	210316.45	519.21	188037.03	371.93	172184.68	447.56
18	200958.02	574.36	178421.03	244.67	166092.62	402.07
19	207105.47	429.17	183020.42	258.43	167495.92	308.33
20	205004.37	518.81	185236.7	306.27	161803.51	367.92
21	206810.58	389.43	176725.11	417.94	170587.79	347.67
22	200549.87	613.98	185363.55	292.55	172954.89	381.07
23	209844.24	319.18	182953.29	676.1	173692.81	216.82
24	208775.96	487.47	181369.84	371.08	171475.21	255.08
25	212067.61	242.99	184933.79	280.86	163818.33	616.41
26	200550.68	417.87	187622.94	270.35	172050.65	336.05
27	205984.97	518.07	183597.29	497.26	168570.78	295.44
28	206474.99	389.38	184131.39	334.41	170693.14	237.65
29	206629.83	726.37	177073.81	593.45	158060.89	448.94
30	204256.37	651.89	187771.09	434.59	167466.35	518.8
Min Value	200515.9	233.78	171890.85	244.67	158060.89	216.82
Avg Value	206656.9277	451.6763333	182772.283	377.041	169793.032	352.8566667

Table 6. Shaking of yp with Ip ZIL, shaking of yr with Ir ZIL
and the combination of all

Serial	Yp & Ip		Yr & Ir		Yp, Yr & Ip, Ir	
	Cost	Time	Cost	Time	Cost	Time
1	210816.17	173.4	185150.53	310.59	178919.19	499.39
2	205873.18	493.5	195796.72	394.92	185162.35	268.67
3	218805.12	485.89	186276.02	438.9	183274.5	150.9
4	206183.31	377.58	189917.77	186.97	193709.06	196.85
5	210584.47	225.36	194428.89	516.26	181200.52	273.76
6	200490.99	398.84	191867.01	242.4	195188.05	199.41
7	207633.02	566.56	204143.06	364.77	177624.79	356.55
8	212175.59	867.94	193481.41	399.95	174723.89	310.68
9	210619.58	369.4	188363.76	367.83	184077.79	450.31
10	208728.75	478.97	182270.76	269.18	182066.55	350.91
11	206014.08	564.18	188138.39	288.75	189200.95	288.21
12	212113.62	228.2	192262.77	503.23	183274.82	377.51
13	212743.06	497.82	188421.61	467.32	187191.4	440.8
14	212470.44	261.32	191519.97	858.18	177956.93	331.75
15	207929.97	235.61	194308.46	250.32	180333.64	328.38
16	218671.11	245.65	215812.19	267.89	173822.91	384.48
17	207151.66	384.99	197756.71	281.84	186866.56	296.25
18	206149.6	330.67	194294.32	289.32	183652.16	321.18
19	210846.4	242.7	194109.39	321.66	183799.48	434.31
20	202527.56	466.85	181898.87	721.22	172694.59	342.66
21	215194.57	333.58	181339.93	660.4	180200.75	485.24
22	211131.34	449	205706.99	332.82	182332.58	393.25
23	205617.66	385.62	186007.07	556.83	189814.07	378.65
24	208589.38	388.86	191819.06	604.06	188068.39	362.57
25	211669.39	319.36	186635.88	362.53	176079.19	403.38
26	205278.3	476.28	204614.7	267.09	174375.72	318.7
27	207411.94	475.2	189565.08	535	173039.14	336.81
28	203310.26	613.37	185591.23	212.02	186548.67	331.25
29	204542.36	289.44	195410.83	878.79	184559.68	217.96
30	207679.26	336.11	188180.22	414.35	185097.02	550.59
Min Value	200490.99	173.4	181339.93	186.97	172694.59	150.9
Avg Value	208965.0713	398.7416667	192169.6533	418.84633	182495.178	346.0453333

The near-optimal value obtained from the solver is 152106.24 and this is reached with elapsed time 3600 sec. Thus, as can be comprehended from the previous runs, shaking both y_t^p, y_t^r gives better solutions. On the other hand, shaking both y_t^p, y_t^r and considering the ZIL property of both I_t^p, I_t^r give better times. However, shaking both y_t^p, y_t^r gives also reasonably good times. The min cost value of the latter approach is 158060.89 which is obtained in 216.82 sec. The average cost value of the latter approach is 169793.032 which is obtained in an average time of 352.857 sec. Comparing those values to the solver output proves that the proposed metaheuristic methodology with shaking both y_t^p, y_t^r outperforms the solver output in a sense that it gives a qualitative solution with 11.6% gap from the solver, but with approximately 90.2% average deduction in time. From this previous analysis, shaking both y_t^p, y_t^r is to be considered in all coming runs in this study.

4.4 Models' Experiments

These experiments are to be initiated for each model of the four mathematical models and then a brief comparison of their results will be accomplished. Different instances sizes are examined. For each model, five different problems with five different periods are examined. These problems are grouped into two different categories which are the small-scale problems (50, 100 and 1000 periods) and the large-scale problems (5000 and 10000 periods). The average and minimum percentage gaps of both cost and time between the solver and proposed metaheuristic are computed to show the power and efficiency of the proposed metaheuristic. The different experiments of those four models are shown in the following tables; Table7, Table8, Table9 and Table10 respectively.

4.4.1 First Model Experiments (F1)

In this mathematical formulation, the model examines the **overtime case** with a **limited storage capacity** restriction as well. This model also considers a **separation of the set-up costs** which, in turn makes this model NP-hard and this would complicate the problem more. Five different problems with five different periods are examined and the results of the different experiments performed on this model are shown in Table7.

Table 7. First model (F1) experiments

Period\Problem			1	2	3	4	5
50	Solver	Cost	104123.4531	99844.43835	102985.1823	101019.7924	98421.43367
		Time	303	314	274	267	278
	Heuristic	Min Cost	110753.22	108780.23	110725.5	105698.04	107167.77
		Avg Cost	115492.3	112841.38	115825.198	111472.95	111828.978
		Min Time	31.35	53.15	64.62	33.17	39.37
		Avg Time	54.572	70.792	75.676	55.454	66.278
	% Gap (Avg)	Cost	-10.91862267	-13.0171914	-12.46782831	-10.34763318	-13.62258588
		Time	81.98943894	77.45477707	72.3810219	79.23071161	76.15899281
	% Gap (Min)	Cost	-6.367217712	-8.949713968	-7.515952818	-4.631020763	-8.886617477
		Time	89.65346535	83.07324841	76.41605839	87.57677903	85.8381295
100	Solver	Cost	206970.5292	203243.1964	206458.8991	204180.3334	208067.9089
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	225149.67	223836.46	226308.6	238037.61	230216.76
		Avg Cost	231542.84	230861.098	236117.18	246659.03	235857.542
		Min Time	115.36	121.23	131.29	142.86	116.92
		Avg Time	155.862	159.03	181.836	163.866	142.138
	% Gap (Avg)	Cost	-11.87237181	-13.58859833	-14.36522282	-20.80449955	-13.35603998
		Time	89.6092	89.398	87.8776	89.0756	90.52413333
	% Gap (Min)	Cost	-8.78344411	-10.13232618	-9.614359552	-16.58204587	-10.64501066
		Time	92.30933333	91.918	91.24733333	90.476	92.20533333
1000	Solver	Cost	755927.8882	713706.1463	768680.6479	785446.8306	812697.953
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	851555.61	823501.61	863156.38	877833.64	914884.97
		Avg Cost	861962.414	828557.01	871287.258	888818.582	929056.016
		Min Time	422.37	425.96	381.24	390.11	417.88
		Avg Time	448.062	459.732	430.252	418.77	466.19

	% Gap (Avg)	Cost	-14.02706891	-16.09217804	-13.34840552	-13.16088465	-14.31750413
		Time	70.1292	69.3512	71.31653333	72.082	68.92066667
	% Gap (Min)	Cost	-12.65037622	-15.3838473	-12.29063491	-11.76232506	-12.57380022
		Time	71.842	71.60266667	74.584	73.99266667	72.14133333
5000	Solver	Cost	6061600.137	6285569.863	6175035.025	6081185.7	6157241.874
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	2229919.77	2002341.324	2572750.629	2214996.678	2779054.723
		Avg Cost	2484012.863	2553828.726	2798033.18	2816689.929	2940865.792
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	59.02050932	59.36997309	54.6879788	53.68189581	52.23728656
		Time	0	0	0	0	0
	% Gap (Min)	Cost	63.2123578	68.14383791	58.33625851	63.57623682	54.8652663
		Time	0	0	0	0	0
10000	Solver	Cost	12332055.16	12522791.75	12624280.44	12033007.31	13837830.34
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	5317878.054	5167623.537	5518271.115	5511570.322	5582483.318
		Avg Cost	5598507.048	5427010.932	5958614.057	5874081.175	6263682.755
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	54.60199475	56.66293075	52.80036685	51.18359839	54.73508056
		Time	0	0	0	0	0
	% Gap (Min)	Cost	56.8776008	58.73425318	56.28843054	54.19623558	59.65781354
		Time	0	0	0	0	0

As can be seen from the previous runs, considering 100 periods for example as a small-scale problem, the near-optimal value obtained from the solver for the first problem is 231542.84, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 2484012.863 which is obtained in an average time of 155.862 sec. Moreover, the minimum cost value of the proposed metaheuristic approach is 225149.67 which is obtained in a minimum time of 115.36 sec. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a near qualitative average solution with almost -11.8% gap from the solver,

but with approximately 89.6% average deduction in time. Considering another example of a large-scale problem of 10000 periods which is 10 times bigger than the previous example, the solution value obtained from the solver is 12332055.16, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 5598507.048 which is obtained in an average time of 1500 sec, while the minimum cost value of the proposed metaheuristic approach is 5317878.054. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a much better average qualitative solution with 54.6% gap from the solver, while fixing the time. This is due to the capacity and feature limitations of the computer used to run these experiments. Also, when the minimum values of the performed runs are considered, the improvements are much better. The rest of the runs for this model are summarized in the previous table7.

4.4.2 Second Model Experiments (F2)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime** case. A **separation of the set-up** costs is also considered. This would complicate the model more and would make it NP-hard problem. Five different problems with five different periods are examined and the results of the different experiments performed on this model are shown in Table8.

Table 8. Second model (F2) experiments

Period\Problem			1	2	3	4	5
50	Solver	Cost	315718.2498	308343.6955	319651.3657	313088.2071	313147.1517
		Time	267	301	325	259	452
	Heuristic	Min Cost	342306.91	340995.81	352300.79	339281.68	343713.42
		Avg Cost	351879.406	407856.242	362079.14	349041.23	358315.558
		Min Time	36.59	122.99	33.2	33.29	63.91
		Avg Time	62.358	147.136	83.654	63.956	82.048
	% Gap (Avg)	Cost	-11.45361607	-32.2732548	-13.27314033	-11.48335264	-14.42401954
		Time	76.64494382	51.11760797	74.26030769	75.30656371	81.84778761
	% Gap (Min)	Cost	-8.42164183	-10.58951909	-10.21407315	-8.366164007	-9.760991975
		Time	86.29588015	59.13953488	89.78461538	87.14671815	85.86061947
100	Solver	Cost	593363.5975	607969.6151	611435.2388	625403.1359	613918.8066
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	679448.62	689354.44	706237.94	708769.39	696635.56
		Avg Cost	694577.622	696544.828	720565.4	720363.134	704518.954
		Min Time	124.74	126.02	109.46	113.28	105.62
		Avg Time	145.65	146.998	203.568	144.5	122.464
	% Gap (Avg)	Cost	-17.0576734	-14.56901969	-17.84819623	-15.18380587	-14.75767583
		Time	90.29	90.20013333	86.4288	90.36666667	91.83573333
	% Gap (Min)	Cost	-14.50797165	-13.38633098	-15.50494561	-13.33000257	-13.47356564
		Time	91.684	91.59866667	92.70266667	92.448	92.95866667
1000	Solver	Cost	1266105.965	1384454.39	1369851.811	1342580.621	1231061.397
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	1583802.26	1649588.6	1614042.25	1576018.17	1607934.51
		Avg Cost	1661681.246	1725279.268	1665463.83	1631595.14	1650836.8
		Min Time	389.67	391.68	403.4	390.18	412.77
		Avg Time	411.728	451.082	444.068	427.97	428.51
	% Gap (Avg)	Cost	-31.24345766	-24.61799247	-21.5798539	-21.52679057	-34.09865702
		Time	72.55146667	69.92786667	70.39546667	71.46866667	71.43266667
	% Gap (Min)	Cost	-25.09239384	-19.15080854	-17.82604787	-17.38722762	-30.61367324
		Time	74.022	73.888	73.10666667	73.988	72.482
5000	Solver	Cost	26760108.19	34104381.76	34798970.95	30328289.13	20918133.28
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	8805073.982	8873475.48	8807580.526	9456675.635	9616685.007
		Avg Cost	9182692.441	9185352.696	9182393.198	9766466.313	10083768.98
		Min Time	1500	1500	1500	1500	1500

		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	65.68514456	73.06694266	73.61303238	67.79750328	51.79412598
		Time	0	0	0	0	0
	% Gap (Min)	Cost	67.09626912	73.98142109	74.69011214	68.81896108	54.02704018
		Time	0	0	0	0	0
10000	Solver	Cost	64458488.42	69252914.62	60012517.21	60578232.66	62228655.5
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	25821872.63	22141342.07	22918807.91	22139584.97	22090625.9
		Avg Cost	28354945.63	24060364.12	24207065.09	24072136.63	23466182.42
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	56.01053279	65.25725415	59.66330657	60.26272875	62.29039141
		Time	0	0	0	0	0
	% Gap (Min)	Cost	59.94030691	68.0282885	61.80995402	63.4529038	64.50087869
		Time	0	0	0	0	0

As can be seen from the previous runs, considering 100 periods for example as a small-scale problem, the near-optimal value obtained from the solver for the first problem is 593363.5975, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 694577.622 which is obtained in an average time of 145.65 sec. Moreover, the minimum cost value of the proposed metaheuristic approach is 679448.62 which is obtained in a minimum time of 124.74 sec. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a near qualitative average solution with almost -17% gap from the solver, but with approximately 90.2% average deduction in time. Considering another example of a large-scale problem of 10000 periods which is 10 times bigger than the previous example, the solution value obtained from the solver is 64458488.42, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 28354945.63 which is obtained in an average time of 1500 sec, while the minimum cost

value of the proposed metaheuristic approach is 25821872.63. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a much better average qualitative solution with 54.6% gap from the solver, while fixing the time. This is due to the capacity and feature limitations of the computer used to run these experiments. Also, when the minimum values of the performed runs are considered, the improvements are much better. The rest of the runs for this model are summarized in the previous table8.

4.4.3 Third Model Experiments (F3)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime case**. A restriction on the **storage capacity** and a **separation of the set-up** costs are also considered. Five different problems with five different periods are examined and the results of the different experiments performed on this model are shown in Table9.

Table 9. Third model (F3) experiments

Period\Problem			1	2	3	4	5
50	Solver	Cost	315718.2498	308343.6955	319651.3657	313088.2071	313147.1517
		Time	1003	614	574	567	478
	Heuristic	Min Cost	350514.48	343216.48	347170.65	345586.35	350561.34
		Avg Cost	355355.974	346072.618	358593.094	354153.478	357400.956
		Min Time	102.92	112.58	65.44	66.56	101.76
		Avg Time	151.584	173.894	144.572	129.52	136.558
	% Gap (Avg)	Cost	-12.55477763	-12.23599608	-12.18256279	-13.11619855	-14.13195174
		Time	84.88693918	71.67850163	74.81324042	77.15696649	71.43138075
	% Gap (Min)	Cost	-11.02129199	-11.30971219	-8.609155875	-10.37986809	-11.94779775
		Time	89.09745763	81.0789916	88.20900901	87.12572534	78.30277186
100	Solver	Cost	594886.3146	606696.5406	611435.2388	625996.7741	616952.0874
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	682918.53	681717.02	692785.83	705161.67	701934.64
		Avg Cost	700304.026	696000.474	712644.954	733497.584	714771.572
		Min Time	144.9	115.9	115.09	146.06	130.1

		Avg Time	168.542	154	161.08	159.034	161.368
	% Gap (Avg)	Cost	-17.72064826	-14.71970375	-16.55281031	-17.17274184	-15.85528059
		Time	88.76386667	89.73333333	89.26133333	89.39773333	89.24213333
	% Gap (Min)	Cost	-14.79815777	-12.36540418	-13.30485815	-12.64621468	-13.77457899
		Time	90.34	92.27333333	92.32733333	90.26266667	91.32666667
1000	Solver	Cost	1394705.194	1423716.489	1335543.347	1383746.007	1321341.906
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	1638620.57	1735402.79	1686042.6	1566454.95	1630360.87
		Avg Cost	1702277.656	1773845	1719734.444	1694783.572	1712731.628
		Min Time	354.65	378.04	397.44	325.16	397.11
		Avg Time	464.142	450.64	431.02	468.34	459.238
	% Gap (Avg)	Cost	-22.05286562	-24.59257257	-28.7666513	-22.47793765	-29.62062431
		Time	69.0572	69.95733333	71.26533333	68.77733333	69.38413333
	% Gap (Min)	Cost	-17.48866909	-21.89244159	-26.24394441	-13.2039364	-23.38675269
		Time	76.35666667	74.79733333	73.504	78.32266667	73.526
5000	Solver	Cost	34547154.91	31756669.76	36685308.86	32356346.79	31063575.05
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	9703768.089	9284781.301	9752477.781	9532865.91	9778516.251
		Avg Cost	9857104.509	9582229.649	10076333	10070778.65	10374613.71
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	71.46768081	69.82608781	72.53305665	68.87541504	66.60199707
		Time	0	0	0	0	0
	% Gap (Min)	Cost	71.91152755	70.76273623	73.41584933	70.53787941	68.52095667
		Time	0	0	0	0	0
10000	Solver	Cost	67579723.04	72904122.42	63261016.71	63594876.88	65669429.15
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	26038545.85	27144449.81	25271143.71	24031696.43	25273777.31
		Avg Cost	27629159.6	27928227.45	26318672.58	25152896	26045771.79
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	59.11619883	61.69184057	58.39669681	60.4482354	60.33805665
		Time	0	0	0	0	0
	% Gap (Min)	Cost	61.46988375	62.76692057	60.05258052	62.21126982	61.5136333
		Time	0	0	0	0	0

As can be seen from the previous runs, considering 100 periods for example as a small-scale problem, the near-optimal value obtained from the solver for the first problem is 594886.314613, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 700304.026 which is obtained in an average time of 168.542 sec. Moreover, the minimum cost value of the proposed metaheuristic approach is 682918.53 which is obtained in a minimum time of 144.9 sec. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a near qualitative average solution with almost -17.7% gap from the solver, but with approximately 88.7% average deduction in time. Considering another example of a large-scale problem of 10000 periods which is 10 times bigger than the previous example, the solution value obtained from the solver is 67579723.04, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 27629159.6 which is obtained in an average time of 1500 sec, while the minimum cost value of the proposed metaheuristic approach is 26038545.85. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a much better average qualitative solution with 59.1% gap from the solver, while fixing the time. This is due to the capacity and feature limitations of the computer used to run these experiments. Also, when the minimum values of the performed runs are considered, the improvements are much better. The rest of the runs for this model are summarized in the previous table9.

4.4.4 Fourth Model Experiments (F4)

In this mathematical formulation, the model examines the **outsourcing case** along with the **overtime case**. A restriction on the **storage capacity** and **a separation of the set-up** costs are also considered. To add to this, carbon emission is added in this formulation to consider the **environmental case** as well. These extensions combined are significant to be considered together, and this would complicate the model more and would make it strongly NP-hard as well. For this comprehensive model, five different problems with five different periods are examined and the results of the different experiments performed on this model are shown in Table10.

Table 10. Forth model (F4) experiments

Period\Problem			1	2	3	4	5
50	Solver	Cost	315718.2498	308343.6955	319651.3657	313088.2071	313147.1517
		Time	944	595	555	517	467
	Heuristic	Min Cost	346333.52	340505.24	352653.16	345540.84	337855.23
		Avg Cost	355861.852	343387.226	360153.968	349900.936	348815.456
		Min Time	91.58	98.69	80.12	136.35	106.98
		Avg Time	110.566	142.832	108.454	175.054	148.512
	% Gap (Avg)	Cost	-12.71500847	-11.36508741	-12.67086791	-11.75794171	-11.39026945
		Time	88.2875	75.99462185	80.45873874	66.14042553	68.1987152
	% Gap (Min)	Cost	-9.697022649	-10.43042065	-10.32430888	-10.36533225	-7.890245276
		Time	90.29872881	83.41344538	85.56396396	73.62669246	77.18976546
100	Solver	Cost	594893.73	606696.5406	611435.2388	625996.7741	617236.4091
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	677039.71	692955.69	677482.75	720788.32	701934.64
		Avg Cost	693983.91	700282.264	698617.602	727260.718	714771.572
		Min Time	109.16	113.38	104.89	140.97	130.1
		Avg Time	153.424	154.426	149.66	167.838	161.368
	% Gap (Avg)	Cost	-16.65678676	-15.42545855	-14.25864224	-16.17643223	-15.80191342
		Time	89.77173333	89.70493333	90.02266667	88.8108	89.24213333
	% Gap (Min)	Cost	-13.80851333	-14.21784099	-10.80204526	-15.14249751	-13.72217026
		Time	92.72266667	92.44133333	93.00733333	90.602	91.32666667
1000	Solver	Cost	1325983.132	1426891.045	1343397.703	1349779.832	1328403.615
		Time	1500	1500	1500	1500	1500

	Heuristic	Min Cost	1619911.53	1665082.86	1599256.63	1573162.04	1627886.15
		Avg Cost	1676117.894	1709549.696	1675102.082	1624397.644	1684377.572
		Min Time	418.57	424.43	394.31	383.73	410.41
		Avg Time	475.06	515.66	430.808	457.98	456.41
	% Gap (Avg)	Cost	-26.40567242	-19.80940675	-24.69145051	-20.34537826	-26.79712351
		Time	68.32933333	65.62266667	71.27946667	69.468	69.57266667
	% Gap (Min)	Cost	-22.16682784	-16.69306257	-19.04565762	-16.54952928	-22.54454385
		Time	72.09533333	71.70466667	73.71266667	74.418	72.63933333
5000	Solver	Cost	34547100	35361842.1	36685251.53	32356327.72	30562580.49
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	9971490.497	9576607.03	9482227.348	9734848.843	9793014.231
		Avg Cost	10997690.22	9881203.025	10118244.11	9895688.229	10263128.51
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	68.16609725	72.05687702	72.41876862	69.41652861	66.41929985
		Time	0	0	0	0	0
	% Gap (Min)	Cost	71.1365339	72.91824616	74.15248103	69.9136165	67.95750203
		Time	0	0	0	0	0
10000	Solver	Cost	77579723.37	72904122.42	73261016.71	73594876.88	72729229.15
		Time	1500	1500	1500	1500	1500
	Heuristic	Min Cost	28407525.59	28224957.31	27191081.61	25831005.28	24567649.57
		Avg Cost	29273111.48	29192792.79	28808339.38	26829424.79	25356820.05
		Min Time	1500	1500	1500	1500	1500
		Avg Time	1500	1500	1500	1500	1500
	% Gap (Avg)	Cost	62.2670587	59.95728112	60.67712315	63.54443961	65.13531032
		Time	0	0	0	0	0
	% Gap (Min)	Cost	63.38279597	61.28482674	62.88465158	64.90108229	66.22039054
		Time	0	0	0	0	0

In section 4.3, the 4th model was discussed with different combinations and different problems with 100 periods. Thus, considering 100 periods for example as a small-scale problem, the near-optimal value obtained from the solver for the first problem is 594893.73, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 693983.91 which is obtained in an average time of 153.424 sec. Moreover, the minimum cost value of the proposed metaheuristic approach is 677039.71

which is obtained in a minimum time of 109.16 sec. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a near qualitative average solution with almost -16.6% gap from the solver, but with approximately 89.7% average deduction in time. Considering another example of a large-scale problem of 10000 periods which is 10 times bigger than the previous example, the solution value obtained from the solver is 77579723.37, which is reached with elapsed time 1500 sec. However, the average cost value of the proposed metaheuristic approach is 29273111.48 which is obtained in an average time of 1500 sec, while the minimum cost value of the proposed metaheuristic approach is 28407525.59. Comparing those values to the solver output proves that the proposed metaheuristic methodology outperforms the solver output in a sense that it gives a much better average qualitative solution with 62.3% gap from the solver, while fixing the time. This is due to the capacity and feature limitations of the computer used to run these experiments. Also, If the minimum values of the performed runs are considered, the improvements would be much better. The rest of the runs for this model are summarized in the previous table10. The different characteristics between the models and the randomness feature of the proposed metaheuristic give an interpretation for the solutions differences and % gaps between the models. Thus, even though the 4^{th} model is the most comprehensive model between all models, it is not necessarily that the results of this model are better in terms of solutions gaps since the model characteristics and its structure are significantly different from the other models.

4.4.5 Models Comparison & Summary

As can be comprehended from the previous runs, shaking both y_t^p, y_t^r gives better solutions than the other alternative combinations. The four different models have proven their robustness and efficiency. The detailed comparisons between costs and times of the different proposed models were illustrated in the previous related sections. The results obtained from the metaheuristic approach were compared with each original analytical models' formulation. For each model, five different problems and runs with five different periods were examined. The proposed metaheuristic outperformed the solver in terms of time in the small-scale problems (50, 100 and 1000 periods) and in terms of solution in the large-scale problems (5000 and 10000 periods). The average and minimum percentage gaps of both cost and time between the solver and proposed metaheuristic are computed to show the power and efficiency of the proposed metaheuristic. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps (74% gap) in the large-scale problems while fixing the computational time and in terms of time deductions (92%) in the small-scale problems. The different characteristics between the models and the randomness feature of the proposed metaheuristic give an interpretation for the solutions differences and % gaps between the models. Thus, even though the 4th model is the most comprehensive model between all models, it is not necessarily that the results of this model are better in terms of solutions gaps since the model characteristics and its structure are significantly different from the other models.

CHAPTER FIVE

CONCLUSION

A lot of researchers' attention has been drawn lately by reverse logistics due to the characteristics of different supply chains nowadays. Thus, acquiring a sustainable production system that is environmentally friendly is the goal that many industries seek for. Different MILP models for a single-item multi-period capacitated lot-sizing problem with remanufacturing and outsourcing were investigated and optimized in this study. Four different MILP models were developed in this study considering different criteria and extensions, and a comparison between their outputs was also accomplished. This problem is capacitated in terms of production and in terms of storage. However, customer deterministic demands are fulfilled by the production of the brand-new products or the remanufacturing of current returned units acquired from customers. Overtime and outsourcing in some periods were allowed in order to overcome the production capacity problem. The objective is to obtain an optimal plan on a limited period of time. In this study, separate set-up costs with overtime were considered along with the environmental aspect, which make this problem strongly NP-hard problem. A state-of-art metaheuristic approach was implemented, and different multi-sizes of instances were examined. An efficient search methodology of neighborhood structures was implemented which is called Variable Neighborhood Search (VNS) to build up distance-based neighborhood structures for this lot sizing problem. A powerful mathematical and optimization software program called GAMS was used to solve these optimization models. The results obtained from the metaheuristic approach were compared with each original analytical model formulations. The proposed metaheuristic method outperformed all those original mathematical models in terms of solutions gaps in the large-scale problems and in terms of time deductions in the small-scale problems. This can be

illustrated in a sense that the proposed metaheuristics sometimes reaches to better solutions with almost (74%) gap compared to the original analytical model in the large-scale problems, with a deduction of time by more than (92%) in the small-scale problems compared to the time the original mathematical models take to find a solutions within 1500 sec time range. The future work could be accomplished by considering a stochastic demand that will elevate the difficulty level of this problem. Another future wore could be conducted by considering multi-objectives, and multi-items also. Moreover, considering uncapacitated case is one of the possible valuable extensions to this study. Finally, applying hybrid heuristics and/or matheuristics that are designed especially for this problem would enhance the solution outputs and time and will be a unique valuable contribution to the field of this research study.

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Due to his excellent records of bachelor's degree, Omar has been awarded fully funded scholarship by Saudi government which covers his study of master's degree at King Fahad University of Petroleum & Minerals (KFUPM), the top engineering school in Middle East and North Africa. Being passionate about systems optimization, he selected to study Industrial & Systems Engineering (ISE) as his major study field. Omar has developed a robust knowledge in industrial engineering concepts including operation research, supply chains and production systems. Beside the manual optimization techniques, Omar works on various count of computer-aided programs such as GAMS, AMPL, C++ and MATLAB. His research and project interests primarily include industrial systems optimization, supply chain systems modeling, production planning, inventory control, scheduling, decision making, operation research and mathematical programming. While his time studying at the college, Omar ensured to enhance his personal skills through participating in various social events and clubs while maintaining a second honor degree level. He worked as a QA/QC Engineer at Heba Fire Fighting Equip. Mfg. Co. Ltd. He also worked as a part-time Academic Assistant for some major courses. He served as a Charter Member for one of KFUPM TOASTMASTER Clubs. He also became a member of Saudi Council of Engineers (SCE) and Industrial Engineering and Operations Management Society (IEOM).