

**INVESTIGATION OF WAVE PROPAGATION IN ADVANCED
FUNCTIONALLY GRADED CERAMIC-METAL PLATES
WITH POROSITIES RESTING ON VISCO ELASTIC
FOUNDATION**

BY

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MASTER OF SCIENCE

In

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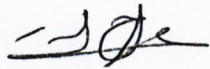
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KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

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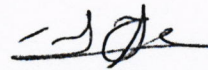


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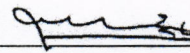
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2021

DEDICATED TO
MY BELOVED PARENTS, BROTHERS, SISTERS AND ALL FAMILY MEMBERS

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LIST OF ABBREVIATIONS

C	:	Phase velocity of a wave
C_g	:	Group velocity of a wave
C_{ij}	:	Elastic stiffness coefficients
C_t	:	Damping coefficient
$C(z)$	:	Moisture concentration
C_0	:	Reference moisture concentration
C_b	:	Moisture concentration at the plate's bottom surface
E	:	Young's modulus
$f(z)$	:	Shear shape function
$g(z)$	:	Derivative of the shape function
h	:	Plate thickness
h_0	:	Coordinate of the bottom surface
h_1	:	Coordinate of the core's bottom surface
h_2	:	Coordinate of the core's upper surface
h_3	:	Coordinate of the upper surface
H_1	:	Thickness of the lower layer

H_2	:	Thickness of the core layer
H_3	:	Thickness of the upper layer
I_n	:	Mass inertia
k_1	:	Plate's curvature in the x-direction
k_2	:	Plate's curvature in the y-direction
K_w	:	Winkler's parameter
K_p	:	Pasternak's parameter
$[K]$	:	Stiffness Matrix
l	:	Core-to-thickness ratio
M	:	Resultant moment
$[M]$	:	Mass Matrix
N	:	Resultant normal force
N_0	:	Pure axial force due to hygro-thermal changes
PM	:	Porosity model
$Pr(z)$	:	Effective property of the composite FGM
Pr_c	:	Property of the homogeneous ceramic
Pr_m	:	Property of the homogeneous metal

r	:	Temperature and moisture variation exponent
S	:	Resultant shear force
$T(z)$	:	Temperature change
T_0	:	Reference Temperature
T_b	:	Temperature at the plate's bottom surface
u	:	Displacement in the x-direction
u_0	:	Unknown for the mid-plane displacement in the x-direction
U	:	A constant for the solution of u_0
v	:	Displacement in the y-direction
v_0	:	Unknown for the mid-plane displacement in the y-direction
V	:	A constant for the solution of v_0
V_c	:	Ceramic volume fraction
V_m	:	Metal volume fraction
w	:	Displacement in the z-direction
w_0	:	Unknown for the mid-plane displacement in the z-direction
W	:	A constant for the solution of w_0
X	:	A constant for the solution of θ

α	:	Temperature expansion coefficient
β	:	Moisture expansion coefficient
γ	:	Shear strain
δ	:	The operation of first variation
$\delta^{(n)}$	:	Layer's porosity coefficient
δK	:	First variation of the kinetic energy
δU	:	First variation of the strain energy
δV	:	First variation external work
ΔC	:	Moisture concentration change between the plate's surfaces
ΔT	:	Temperature difference between the plate's surfaces
ε	:	Normal strain
θ	:	Unknown for the mid-plane displacement due to shear
κ	:	Wave number
λ	:	Porosity volume fraction
$\{\Lambda\}$	:	Constants of the solution form
ν	:	Poisson's ratio
ξ	:	Porosity distribution factor

ρ	:	Mass density
ς	:	Odd and even porosity distribution coefficient
σ	:	Normal Stress
τ	:	Shear stress
ϕ_x	:	Rotation in the xz-plane
ϕ_y	:	Rotation in the yz-plane
ω	:	Circular frequency
∇^2	:	Laplace operator

|

ABSTRACT

Full Name : Saeed Ibrahim Osman Tahir

Thesis Title : INVESTIGATION OF WAVE PROPAGATION IN ADVANCED
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Due to their superior performance and the increasing demand for their characteristics, functionally graded materials' (FGMs) applications are increasing in various modern industries like aeronautics, civil infrastructures, structural health monitoring, damage detection and nanostructures. Propagating waves in FG plates are considered common in structural applications. This thesis is meant to focus on investigating wave propagation in infinite advanced functionally graded ceramic-metal plates with porosity and resting on the viscoelastic foundation. A simple hyperbolic higher-order shear deformation theory (hyperbolic HSDT) with undetermined integral variables is implemented to formulate and solve the waves' dispersion relations analytically. The theory applicability is verified by numerical comparison with previous works. The effects of gradation parameter, porosity parameter and porosity type on wave propagation in FG plates are examined and illustrated. Propagation phase velocities are studied for plates without foundation and plates resting on Winkler, Winkler-Pasternak, visco-Winkler and visco-Pasternak foundations. Additionally, the effect of hygro-thermal environment on the phase velocity is investigated. |

ملخص الرسالة

الاسم الكامل: سعيد ابراهيم عثمان طاهر

عنوان الرسالة: دراسة انتشار الموجات في الصفائح المتقدمة المتدرجة دالياً من معدن وسيراميك وذات مسامات ومستندة على قاعدة فيسكو مرنة

التخصص: الهندسة المدنية

تاريخ الدرجة العلمية: أبريل 2021

نظراً لأدائها الفائق وازدياد الحاجة لخصائصها، أصبحت تطبيقات المواد المتدرجة دالياً تتزايد في القطاعات الحديثة مثل الطيران والبنية التحتية المدنية ومراقبة صحة المنشآت وكشف الأضرار والهياكل النانوية. انتشار الموجات في الصفائح المتدرجة دالياً (FG) يعتبر شائعاً في التطبيقات الإنشائية. هذه الرسالة تدرس انتشار الموجات عبر صفائح متقدمة متدرجة دالياً ولانهائية مكونة من معدن وسيراميك مع الأخذ بعين الاعتبار تأثيرات المسامات والقواعد الفيسكو مرنة. تم استخدام نظرية قص تكاملية مبسطة ذات درجة عليا ودالة زائدية لصياغة وحل علاقات انتشار الموجات تحليلياً. تم التحقق من صحة تطبيق النظرية بالمقارنة مع أوراق علمية منشورة. تمت دراسة تأثيرات معاملات التدرج، والمسامية، ونوع المسامية، وقاعدة وينكلر، وقاعدة وينكلر-باسترنالك، وقاعدة فيسكو وينكلر، وقاعدة فيسكو باسترنالك على انتشار الموجات في الصفائح المتدرجة دالياً.

CHAPTER 1

INTRODUCTION

Materials have always played an important role in manmade structures. People used a variety of structural materials in order to sustain the different loads and environmental conditions that exist in all engineering applications. Among the material types, composites are being used in many fields because they can combine the beneficial properties of two, or more, different materials. The main objective of using composite materials was to produce lightweight materials that have desirable properties like strength, stiffness, toughness and heat resistance. For example, steel and ceramic are usually combined together to produce a composite material that has steel's strength and toughness as well as ceramic's hardness and resistance against heat and moisture effects.

Composite materials are growing continuously in terms of types, applications and analytical studies. The main two traditional classes of composites are reinforced materials and laminated materials. Reinforcement can be in the forms of large particles, fibers, or both like in reinforced concrete. On the other hand, laminated materials are composed of two or more materials joined together such as sandwich plates. In laminated plates, there are locations of discontinuity of the material properties, which lead to a disturbance in stress distribution. To overcome this disadvantage, functionally graded material (FGM) is developed.

FGM is a new class of composite materials that are manufactured to provide high mechanical performance. These materials are usually made with ceramic and metal. The volume fraction, which is occupied by each of the two components, changes gradually according to a certain mathematical function. This gradual variation in the material composition is reflected in smooth changes of the mechanical properties through one or more directions. Gradation functions are selected to provide adequate performance depending on the loading conditions that is be applied to the element.

The large variety of loading conditions has attracted researchers' efforts to study the mechanics and behaviors of functionally graded (FG) beams and plates extensively. Many theories were developed to analyze this category of materials. A significant number of research publications focused lately on testing and developing plate theories and validating their applicability for FG plates under static loads, moving loads, vibration, moisture, the thermal effect and other conditions.

This thesis focuses on studying the wave propagation in advanced functionally graded (AFG) ceramic-metal plates. Porosity resulting from the manufacturing process is considered in the mechanical properties functions. In addition to a plate without foundation, plates resting on Winkler, Winkler-Pasternak, visco-Winkler and visco-Pasternak foundations are investigated. The solution is based on an integral hyperbolic higher-order shear deformation theory (hyperbolic HSDT) that expresses the rotation angles by indeterminate integral functions. As a four-variable plate theory, the used hyperbolic HSDT is efficient for practical applications.

1.1 The Concept of FGM

Engineers are always fascinated by using materials of high strength-weight ratio and high stiffness-weight ratio to overcome the difficulties related to structures' high weight. Furthermore, in some modern industries, such as aerospace structures, aeronautics, and nanostructures, high performance and low weight are mandatory. This led to the development of composite materials.

A composite material is an integration of two or more different materials that yields of a structural element that has the advantageous characteristics of its components. A laminated plate, i.e. a composite of discrete joined layers, is designed to have adequate properties in each level of its thickness depending on the internal load at that level. However, the locations near material discontinuity in laminated plates becomes under stress concentrations that lead to cracking and delamination. These issues are solved by inventing the new generation of composites known by functionally graded materials.

A functionally graded plate has a continuous variation of its composing materials ratios through its thickness. The smooth gradation alleviates the concentration of stress at locations of properties change. Like other composite materials, FGM is manufactured to achieve superior properties, like high strength, high stiffness and greater resistance to fatigue and environmental conditions at lower weight. Figure 1 compares the thermal properties and material structures of FGM and laminated plates [1]. FGM particles ratios of ceramic and metal change gradually which results in smooth and relieved stress distribution.

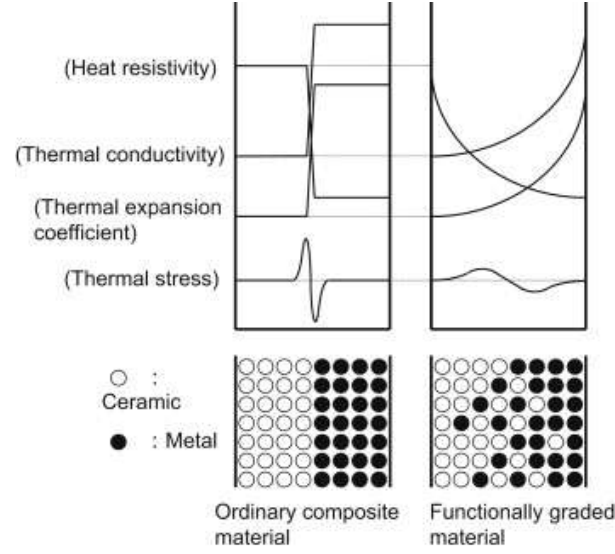


Figure 1 Thermal properties and material structures of FGM and laminated plates [1]

1.2 Objectives of the Study

Functionally graded plates are used in many applications to resist extreme loading conditions. Plates under high frequency mechanical vibrations are analyzed not only based on the first mode associated with the minimum natural frequency, but also higher modes are studied to achieve economically efficient design. When the plates are subjected to high-frequency vibrations, which may exceed 100s of kHz, wave propagation analysis is necessary.

This thesis considers an infinite ceramic-metal FG plate resting on visco elastic foundation. Figure 2 illustrates the gradation of the plate, its thickness, and the foundation parameters. k_w , k_s and c_t represent Winkler's parameter, Pasternak's parameter and damping coefficient, respectively. The effect of each of these parameters on the wave propagation problem is investigated.

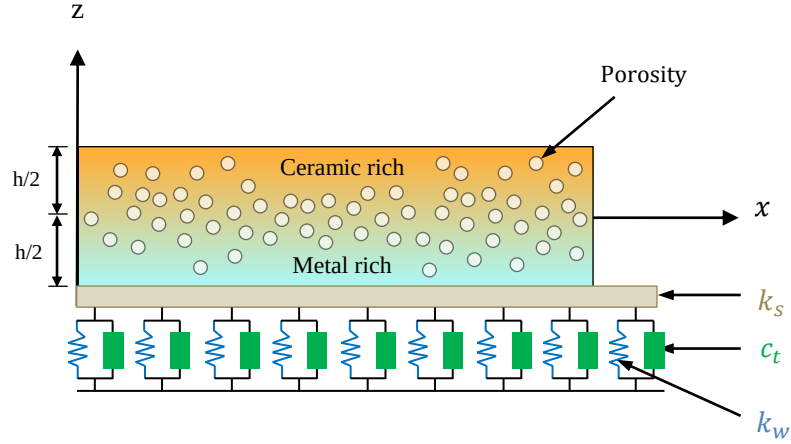


Figure 2 Plate configuration

The objective of this proposed thesis is to derive the governing differential equations (GDE) of the wave propagation problem, solve the GDE analytically, and to provide numerical examples for the solved problems. The solution mainly represents the frequencies and phase velocities of different modes of the wave propagation as functions of wave number. The specific objectives of this study are:

- Proposing a simple and efficient shear deformation plate theory.
- Using this theory to study wave propagation of FG plate with porosity.
- Studying the effect of viscoelastic foundation on wave propagation of FG plate.

Studying the effects of different parameters on wave propagation of FG plate such as porosity volume fraction and distribution function along thickness direction, Winkler's parameter, Pasternak's parameter, damping coefficient, material gradation power index and plate thickness.

The wave propagation study mainly concerns about frequency and phase velocity of of high wave numbers. Wave propagation behavior in the described plate is to be studied analytically based on a Hyperbolic HSDT and using Hamilton's principle. The Hyperbolic HSDT is given in Eq. 12. In which the shape function $f(z)$ is a hyperbolic function of z given in Eq. 13.

The imperfection that appears in the form of porosities is considered based on three different porosity models that are

- $\xi = \frac{\lambda}{2}$
- $\xi = 1 - e^{-\frac{\lambda}{2}}$
- $\xi = \ln\left(1 + \frac{\lambda}{2}\right)$

In which λ is the porosity volume fraction and ξ represent the distribution of the porosity volume along thickness.

CHAPTER 2

LITERATURE REVIEW

2.1 Effective Properties of FG Plates

FGM plates are manufactured in several forms. The most common types used are single-phase ceramic-metal FG plate and sandwich metal-ceramic-metal FG plate. Figures 3 and 4 illustrate these two types. This thesis aims to study a single phase FG plate that has pure ceramic top surface.

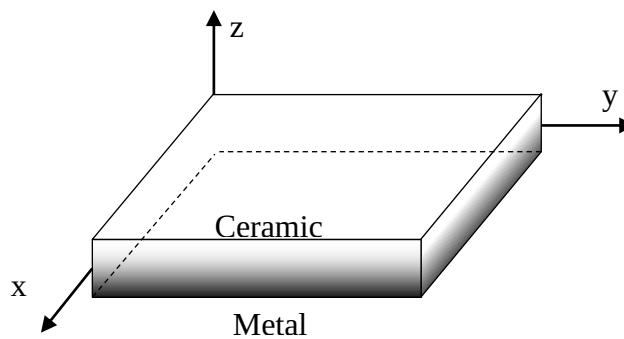


Figure 3 Ceramic-metal FG plate model

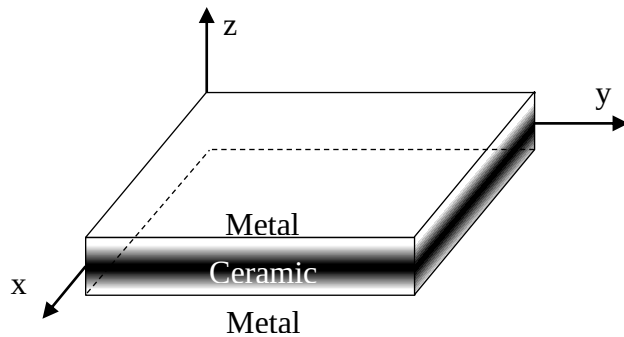


Figure 4 Ceramic-metal FG Sandwich plate model

The gradation from metal to ceramic is described by the function of volume fraction. The volume fractions $V_c(z)$ and $V_m(z)$ are equal to the percentages of each of ceramic and metal contents, respectively, by volume. They depend only on z because the gradation is applied only in z -direction. At any location of the plate, $V_c(z) + V_m(z) = 1$.

The most commonly implemented method to estimate the overall properties of the FG plate in terms of its constituents' properties is the rule of mixture model. Based on the rule of mixture, all material properties are assumed to vary smoothly by a function of the volume fraction and the constituents' properties.

A property P of the composite material, that can be modulus of elasticity, Poisson's ratio, density, thermal expansion coefficient, moisture expansion coefficient, or any other property, is found by [2]

$$P(z) = P_c V_c + P_m V_m \quad (1)$$

The property function $P(z)$ is implemented in the constitutive relations as a homogenous property for the FGM plate. The variation of Poisson's ratio has a negligible influence on the FG plate behavior [3]. Hence, for simplicity, it is considered to be constant through the all directions of the plate. Power-law distribution [4], [5] is used to express the volume fractions

$$V_c = \left(\frac{z}{h} + \frac{1}{2} \right)^N \quad (2)$$

$$V_m = 1 - V_c \quad (3)$$

where h is the plate thickness and the exponent N is the material power index. The domain of z is $(-h/2, h/2)$. At $z = -h/2$ the material is pure metal and $V_m = 1$, while at $z = h/2$ the material is pure ceramic and $V_c = 1$. The variation of volume fraction along the thickness direction is plotted in figure 5 for different material power indexes.

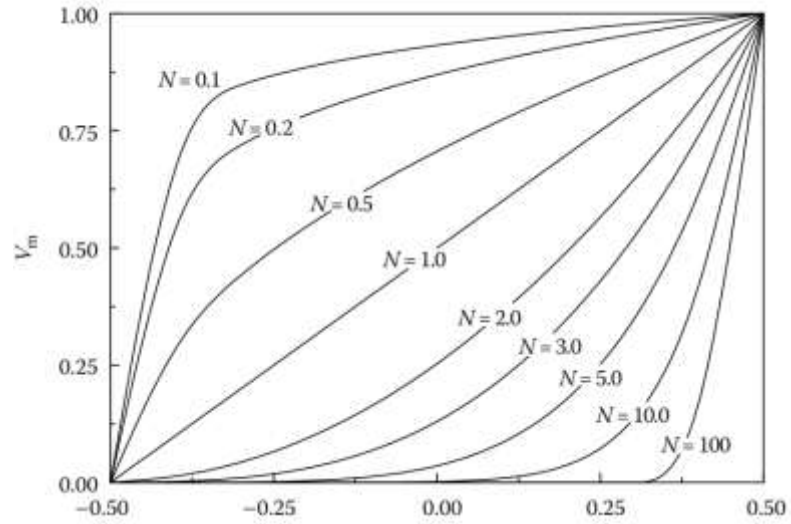


Figure 5 Variation of volume fraction along thickness direction for different material power indexes [6]

The manufacturing process produces imperfect ceramic-metal plates that have inevitable porosities spreading through the thickness which causes a reduction in the effective properties of the functionally graded material. This reduction is calculated based on the assumed porosity volume fraction, denoted by λ . Value of λ equals zero for a plate without porosity. The factor of porosity distribution, denoted by ξ , is a function of λ that describes how the voids are distributed through the direction of gradation.

Three types of porosity distribution factor are to be considered in this study. These types are defined by the following functions [7]–[9] . The three functions are plotted in figure 6.

$$\text{Type 1: } \xi = 1 - e^{-\frac{\lambda}{2}} \quad (4)$$

$$\text{Type 2: } \xi = \ln\left(1 + \frac{\lambda}{2}\right) \quad (5)$$

$$\text{Type 3: } \xi = \frac{\lambda}{2} \quad (6)$$

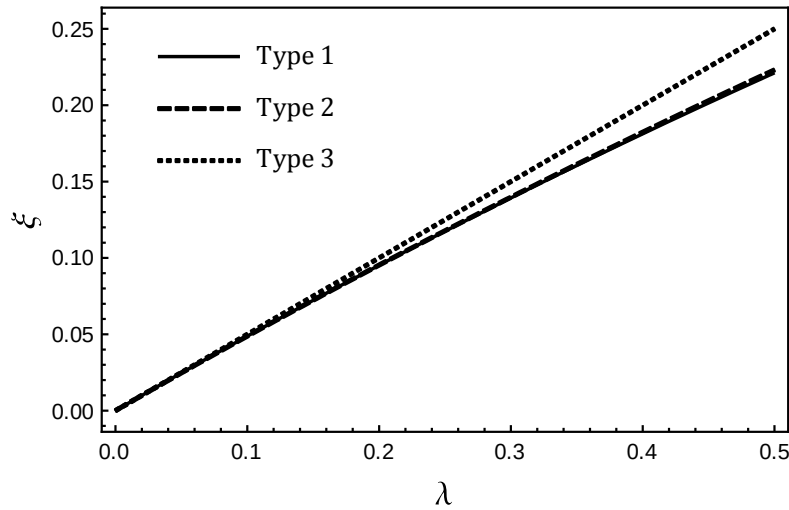


Figure 6 Types of porosity distribution factor versus porosity volume fraction

2.2 Elastic Plate Theories

Plate theories are developed to describe the behavior of plates subjected to transverse actions. They are based on an educated guess, in which the form of the solution is assumed, according to certain behavioral assumptions, and then the governing equations are derived to correlate the variables of the assumed solution. There are two approaches

adopted in developing plate theories. The first approach, which is commonly used, assumes the form of displacement field equations with certain unknown variables. Then strains and stresses are found in terms of these variables via constitutive relations. From stresses and strain, all internal forces are determined as well. Then energy principles come into the picture to be applied for derivation of governing equations. The second approach assumes the form of stress field equations with certain unknown variables. Then Hooke's law is applied to find the displacement field and the same principles are applied to derive governing equations. Governing equations are then solved by series expansion of the unknown variables. Navier's solution is to be used to solve the governing equations.

The following subsections are to overview some famous plate theories. The equations are typed in accordance with the coordinate system and dimensions illustrated in Figure 7.

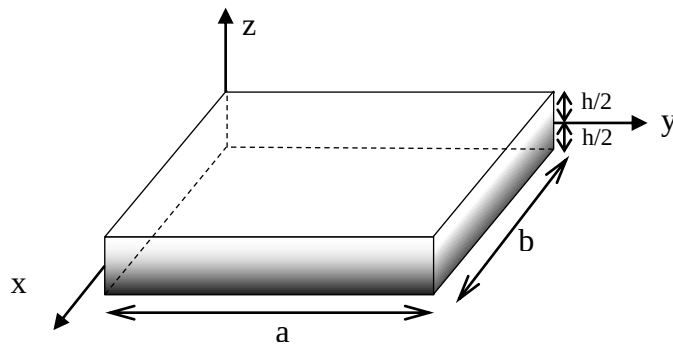


Figure 7 Coordinate system and dimensions of the plate

2.2.1 Classical Plate Theory (CPT)

Classical plate theory is the simplest theory of plates. Its displacement field is formulated according to the assumptions of Kirchhoff plate which state the following

- Mid-surface deformation is neglected because it is assumed to be very small, which makes its strain even smaller and negligible.
- Mid-surface level is assumed to remain the mid-surface after deformation.
- Planes perpendicular to the mid-surface are assumed to remain perpendicular after deformation. This is equivalent to saying that the transverse shear strain is negligible which also leads to neglecting the transverse normal strain.
- The plate is under transverse loading only, which hardly cause any normal stresses in the transverse direction. Hence, σ_z is neglected.

Omitting the negligible dimensions, CPT results in the following displacement field that has only three variables.

$$\begin{aligned}u(x, y, z, t) &= u_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial x} \\v(x, y, z, t) &= v_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial y} \\w(x, y, z, t) &= w_0(x, y, t)\end{aligned}\tag{7}$$

where $u_0(x, y, t)$ and $v_0(x, y, t)$ represent the deformations at the mid-surface along x and y directions, respectively. $w_0(x, y, t)$ represents the transversal deformations and depends on the z coordinate. CPT is sufficient for thin plates, i.e. plates satisfying $h/a < 0.05$.

2.2.2 First-Order Shear Deformation Theory (FSDT)

In the FSDT, the third assumption of Kirchhoff plate is canceled so that transverse shear strain is considered. The shear strain angles will lead to additional terms in u and v . The displacement field becomes of five variables as given in Eq. 8 [10].

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) + z\phi_x(x, y, t) \\ v(x, y, z, t) &= v_0(x, y, t) + z\phi_y(x, y, t) \\ w(x, y, z, t) &= w_0(x, y, t) \end{aligned} \quad (8)$$

ϕ_x and ϕ_y represent the angles of deflection in xz -plane and yz -plane, respectively.

They are given by

$$\phi_x = -\frac{\partial w_0}{\partial x} \quad (9)$$

$$\phi_y = -\frac{\partial w_0}{\partial y} \quad (10)$$

2.2.3 Third-Order Shear Deformation Theory (TSDT)

In addition to removing the normality assumption, the transversal planes curvature after deformation is considered. Displacement field is assumed to be of the third degree in terms of z .

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) + z\phi_x(x, y, t) - cz^3 \left(\phi_x + \frac{\partial w}{\partial x} \right) \\ v(x, y, z, t) &= v_0(x, y, t) + z\phi_y(x, y, t) - cz^3 \left(\phi_y + \frac{\partial w}{\partial y} \right) \\ w(x, y, z, t) &= w_0(x, y, t) \end{aligned} \quad (11)$$

TSDT provides more reliable solutions when compared with simpler theories. Figure 8 summarizes and illustrates the differences between CPT, FSDT and TSDT.

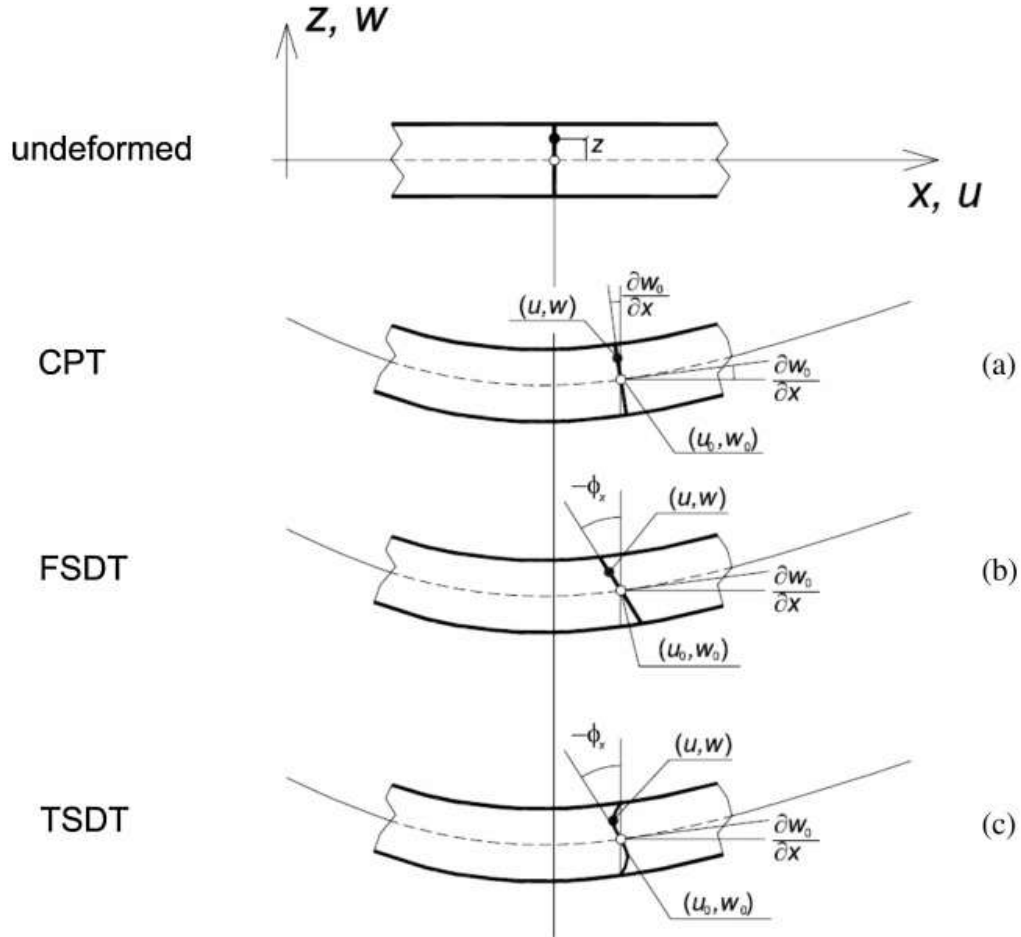


Figure 8 Transverse deformation based on CPT, FSDT and TSDT [4]

2.2.4 Hyperbolic Higher-Order Shear Deformation Theory (Hyperbolic HSDT)

Many other higher order shear deformation theories are developed to catch plates' deformation behavior accurately. In this proposed thesis, hyperbolic HSDT with undetermined integral variables is applied. The displacement field is given in Eq. 12 [11].

$$\begin{aligned}
u(x, y, z, t) &= u_0(x, y, t) - z \frac{\partial w_0}{\partial x} + k_1 f(z) \int \theta(x, y, t) dx \\
v(x, y, z, t) &= v_0(x, y, t) - z \frac{\partial w_0}{\partial y} + k_2 f(z) \int \theta(x, y, t) dy \\
w(x, y, z, t) &= w_0(x, y, t)
\end{aligned} \tag{12}$$

In this theory only four unknown variables exist, that are u_0, v_0, w_0 and θ . They represent the plate deflection at the middle plane. $f(z)$ is called shape function. It describes the variation of transverse shear strain along the thickness direction. In this hyperbolic HSDT, the shape function is

$$f(z) = z - \frac{2z \sinh\left(\frac{z^2}{h^2}\right)}{2 \sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)} \tag{13}$$

The derivative of $f(z)$ is denoted by $g(z)$, which is evaluated to be

$$g(z) = 1 - \frac{2 \sinh\left(\frac{z^2}{h^2}\right)}{2 \sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)} - \frac{4z^2 \cosh\left(\frac{z^2}{h^2}\right)}{h^2 \left(2 \sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)\right)} \tag{14}$$

Variation of $f(z)$ and $g(z)$ along the thickness direction is shown in figure 9. It can be noticed that zero shear strain at the two faces of the plate is forced by $g(z)$.

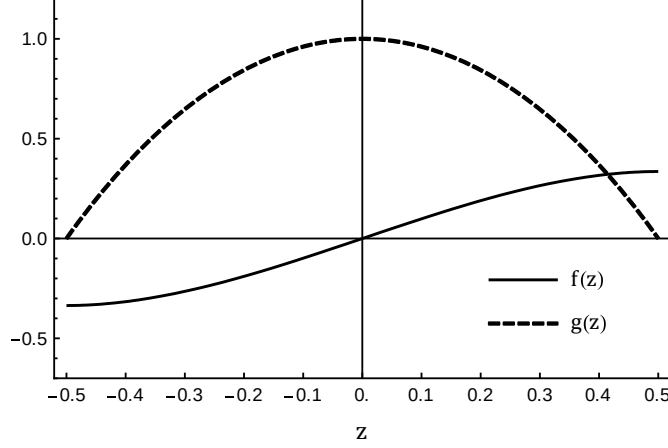


Figure 9 The hyperbolic shape function and its derivative variation along z-axis

The angles of deflection in xz -plane and yz -plane are given by

$$\phi_x = \int \theta(x, y, t) dx \quad (15)$$

$$\phi_y = \int \theta(x, y, t) dy \quad (16)$$

2.3 Concepts in Calculus of Variation

In this section, some of the variational calculus concepts that are used in GDEs formulation are explained briefly.

2.3.1 The First Variation

The first variation is a differential operator that acts on functionals. Extreme values of a function are found by equating its first derivative to zero. However, a functional, i.e. function that has a domain of admissible functions and a range of real numbers, has extreme values where its first variation is zero.

This operator acts only on the strain and displacement expressions, but not on stresses, resultant forces coming from internal stresses and foundation reaction, even though they both can be expressed as functions of displacement variables. So the stresses must be converted to resultant forces, instead of expressing them in terms of displacement variables, before applying the first variation operator.

2.3.2 The Fundamental Lemma of Calculus of Variations

In a simple form, the lemma says that for an arbitrary function $\eta(x)$ the following implication is true for any integrable function $G(x)$

$$\int_a^b G(x) \cdot \eta(x) dx = 0 \implies G(x) = 0 \text{ for } x \in (a, b) \quad (17)$$

The fundamental lemma of calculus of variations is used to obtain the governing motion equations from the integral form of Hamilton principle.

2.3.3 Resolving Variations of Derived Functions

Only $(\delta u_0, \delta v_0, \delta w_0 \text{ and } \delta \theta)$ are arbitrary functions of (x, y, t) , so the variation of $(u_0, v_0, w_0 \text{ and } \theta)$ derivatives shall be converted to expressions containing only variations of the non-derived four functions. This is done by the following technique.

Considering the arbitrary function $\delta \xi(x, y, t)$, a function $\psi(x, y, t)$ and the $[a, b]$ as functions' domain. Note that $\xi(x, y, t)$ is either u_0, v_0, w_0, θ , while $\psi(x, y, t)$ may include a

combination of u_0, v_0, w_0, θ and any other function. The integration by parts formula implies:

$$\begin{aligned} \int_a^b \psi(x, y, t) \delta \frac{\partial \xi(x, y, t)}{\partial i} di \\ = \left[\int_a^b \psi(x, y, t) \delta \xi(x, y, t) \right]_b^a - \int_a^b \frac{\partial \psi(x, y, t)}{\partial i} \delta \xi(x, y, t) di \end{aligned} \quad (18)$$

Where $i = x, y, \text{ or } t$

By definition, the first variation is zero at the boundaries. Hence, the formula becomes

$$\int_a^b \psi(x, y, t) \delta \frac{\partial \xi(x, y, t)}{\partial i} di = - \int_a^b \frac{\partial \psi(x, y, t)}{\partial i} \delta \xi(x, y, t) di \quad (19)$$

Based on the boundary conditions, this literally means that each first derivative can be moved from the arbitrary function to its coefficient with additional minus sign. The process is repeated for higher order and mixed derivatives.

2.4 Methodology and Formulation

2.4.1 Kinematics Formulation

Kinematics is formulated based on the hyperbolic HSDT described previously. The theory involves four unknowns that are u_0, v_0, w_0 and θ . Strains are found in terms of these unknowns using linear strain-displacement relationships. Stresses are therefore

found in terms of the four unknowns using the stress-strain constitutive relation shown below.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (20)$$

It is essential for the solution process to express all energy and work related variables in terms of the plate theory unknowns only. Hence, these steps are helpful to find work and energy terms. The stresses and any other function of z shall be resolved before applying the δ operator.

The implemented shear deformation theory includes undetermined integrals of θ . These integrals are handled by relating them to the derivatives of θ as follow

$$\int \theta \, dx = -\frac{1}{\kappa_1^2} \frac{\partial \theta}{\partial x} \quad (21)$$

$$\int \theta \, dy = -\frac{1}{\kappa_2^2} \frac{\partial \theta}{\partial y} \quad (22)$$

These relations are true because the Navier type solution that is applied expresses θ by

$$\theta = X \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \quad (23)$$

2.4.2 Governing Differential Equations

The governing differential equations are derived by applying Hamilton's principle, which states that

$$\int_0^t (\delta U + \delta V - \delta K) dt = 0 \quad (24)$$

δ is a differential operator called the first variation and U , V and K are the strain energy, external work and kinetic energy, respectively. Once these variations are plugged into the equation in terms of u_0, v_0, w_0, θ and their variation, the fundamental lemma of calculus of variations is applied.

Fundamental lemma of calculus of variations states that the sum of coefficients of an arbitrary function within the integral of a zero-functional is equal to zero. Hence, the coefficients of $\delta u_0, \delta v_0, \delta w_0$ and $\delta \theta$ are equated to zero to get four GDEs, one equation from each.

2.4.3 Dispersion Relations

The dispersion relations relate the frequency of wave propagation to the plate theory unknowns. u_0, v_0, w_0 and θ are unknown functions of x, y and t . These functions' forms are assumed based on a Navier type solution that is shown below

$$\begin{Bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \\ \theta(x, y, t) \end{Bmatrix} = \begin{Bmatrix} U \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ V \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ W \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ X \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \end{Bmatrix} \quad (25)$$

U, V, W and X are constants. κ_1 and κ_2 are the wave numbers in the directions of x-axis and y-axis, respectively. ω is the wave frequency and $i = \sqrt{-1}$.

This solution is plugged into the derived GDEs and resulting equations are rewritten into a matrix form.

$$([K] - \omega^2[M]) \{\Lambda\} = 0 \quad (26)$$

K is called the stiffness matrix and it contains the terms related to strain energy and external work. M is called the mass matrix and it contains the terms related to kinetic energy. Λ is a vector that consists of the four unknown constants U, V, W and X .

The dispersion relations are solved numerically by finding eigenvalues that represent the frequency squared. Each eigenvalue represents the square of a single mode frequency. By varying the wave numbers, ω is plotted as a function of κ . The phase velocity value is found numerically for each data point and mode i by

$$C_i = \frac{W_i(\kappa)}{\kappa}, \quad (i = 1, 2, 3, 4) \quad (27)$$

2.4.4 Results Validation

Results are validated by comparing with previous published works from literature. This is a common practice between researchers in this field and is followed to make sure that the calculations were carried out correctly, to give our results reliability and to appreciate and

honor the great works done by pioneer researchers. Additionally, the results are reviewed by peers who are expert in the field.

2.4.5 Parametric Studies

The dispersion relations are solved for different data groups to perform parametric studies. The parametric studies conclude the effect of each of material gradation, dimension, porosity, and foundation parameters on the wave propagation frequency and phase velocity.

2.5 Studies from Literature

2.5.1 FGM Plate Theories

As a solution for delamination and stress concentration in laminated composites, a new generation of composites, known by functionally graded material, was introduced by Japanese material scientists in 1984 [12]. The classical plate theory (CPT) is applicable under certain conditions such as Kirchhoff's plate assumptions and width/thickness greater than 20 [4]. This makes the CPT inaccurate for FGM plates, which are mostly thick plates. Reddy [13] analyzed rectangular FG plate using his third-order shear deformation theory (TSDT). He compared with results obtained by the first-order shear deformation theory (FSDT) and showed that the higher order theory is more accurate. Unlike Reddy's TSDT, FSDT required shear correction factors because it assumes

constant shear stress distribution along the thickness direction and violate the zero-shear-strain boundary condition.

Other higher-order shear deformation theories (HSDT), which describe a larger range of admissible shear stress distributions, were implemented for FG plates. Ait et al. [14] studied the vibration in FG plate using a HSDT that has four variables. Phung-Van et al. [15] conducted static and dynamic studies on composite FG plates based on HSDT and showed that the theory can be applied for any degree of continuity.

More advanced HSDTs were developed to provide more accurate analysis with fewer variables. Abdelhakim et al., Elmossouess et al. and Menasria et al. [16]–[18] applied a new HSDT that represents the shear deformation by undetermined integrations of the rotation angle to analyze FG plates under various loads and environmental conditions. The theory has four variables. The undetermined integral form of the shear displacement part was resolved by a novel method. Belkhodja et al. [19] extended the theory to become an exponential-trigonometric quasi-3D HSDT and verified its applicability for bending, buckling and free vibration of FG plates. A new hyperbolic HSDT was developed by Mahi et al. [20] and applied for FGM and laminated sandwich plates.

2.5.2 Vibration and Dynamic Analysis of FG Plates

Researchers also put great efforts on implementing the new theories to understand and to measure the behavior of FGM plates under vibration. Ait Atmane et al. [21] provided a HSDT based solution for the free vibration problem of FGM plate resting on Winkler–Pasternak foundation. The solution was generalized for power law, exponential law, and

other gradation laws. Benachour et al. [22] applied the refined four-variable HSDT for FGM plate subjected to free vibration. They verified that their simplified theory can give more accurate results. Bennoun et al. [23] proposed a new five-variable shear and normal deformation theory for vibration in sandwich FG plates. They found that the results accuracy could be enhanced by considering the normal deformation as well without going for higher number of variables.

2.5.3 Wave Propagation in FG Plates

Clearer perception of the dynamic behavior of FGM plates at high frequencies is achieved by investigating their wave propagation characteristics. Han et al. [24] developed a numerical method to study wave propagation in point loaded FGM cylindrical shell. The behavior of elastic waves dispersion in FG plate was studied by Chen et al. [25] using two approaches. The first approach was based on approximating the FGM by an adequate number of laminated homogeneous layers, and the other solution was obtained considering continuity of the material gradation. Considering continuity provided accurate results, but not for very low frequencies. Boukhari et al. [26] investigated the wave propagation in FGM beams. Their piece of work also included a detailed study of the effect of temperature on propagation of waves.

Fourn et al. [27] investigated the problem wave propagation of infinite FGM plate using HSDT with undetermined integrals and verified the high accuracy of the theory that involves less computational cost. Sun and Luo [28] analyzed wave propagation and dynamic response of fully clamped FGM plate subjected to impulse load using a theory

that considers rotary inertia in addition to shear deformation. They also investigated the propagation of FGM plates under a point impact load and thermal environment [29], [30].

Sintering process of FGM production may insert porous volumes inside the plate, which reduces its mechanical properties. Mourad et al. [31] developed an efficient method to consider the porosity in FGM plates in wave propagation analysis. The increase of porosity volume fraction can slow down propagating waves. FGM plates with porosity was investigated for wave propagation by Bennai et al. [7] using HSDT with a hyperbolic shape function. Three different models of porosity distribution along thickness direction were suggested by Bennai et al., Gupta et al., Wattanasakulpong and Ungbhakorn [7]–[9].

CHAPTER 3

WAVE PROPAGATION OF A CERAMIC-METAL FGM

PLATE WITH POROSITIES RESTING ON A

VISCOELASTIC FOUNDATION

This chapter studies the dispersion relations of wave propagation in infinite advanced functionally graded (FG) ceramic-metal plates. A simple integral hyperbolic higher-order shear deformation theory (HSDT), with undetermined integral terms and only four unknowns, is used to formulate a solution for the waves' dispersion relations. The effective functionally graded materials' (FGM) properties follow the power-law with three different types of uneven porosity distributions. The effect of foundation viscosity is investigated by considering the damping coefficient in addition to Winkler's and Pasternak's parameters. The wave propagation's governing equations are derived based on the present integral hyperbolic HSDT using Hamilton's principle. The eigenvalue problem describing the porous FG plate dispersion relations resting on a viscoelastic foundation is analytically determined. The theory accuracy is validated by numerically comparing the results with previously published works. Finally, the influences of gradation power, porosity parameters, and the viscoelastic foundation parameters on wave propagation in an FG plate are examined and discussed.

3.1 Introduction

Functionally graded material (FGM) is a category of composite materials in which the properties vary continuously along a certain direction. The material gradation function's continuity results in overcoming the stress concentrations and delamination disadvantages in laminates. FGM has applications in many different industries such as thermo-generators, industrial construction, civil infrastructure, defense, nuclear reactors, aeronautics, mining, nanostructures, and biosensors [19], [32]–[34]. In many of these applications, the plates can be subjected to high frequencies. Consequently, an accurate prediction of the wave propagation behavior in imperfect FGM structures under various loading conditions is important for design considerations.

Plate theories started with the classical plate theory (CPT) proposed by Kirchhoff [35]. CPT does not consider the shear deformation, so it estimates thick and moderately thick plates' behaviors inaccurately. Other shear deformation theories were later developed and implemented for analyzing FGM plates. The deflection response, bending behavior, and vibration characteristics of the FGM plate were studied using the third-order shear deformation theory (TSDT) [13], [36], [37]. Recently developed higher-order shear deformation theories (HSDT) describing a wider range of admissible transverse shear deformations were used to analyze the behavior of functionally graded (FG) plates. Various novel HSDT were applied to study FGM beams and plates [15], [31], [38], [39].

Novel HSDTs with only four variables were developed for analyzing composite plates under various conditions [17], [40]. New three-variable HSDT have also been introduced recently for studying FGM plates [41], [42]. The limitation of 2D plate theories is that

they do not consider the thickness stretching effect as it can be negligible. However, there are many articles investigated this effect using quasi 3D plate theories [43]–[46].

The theoretical mechanical properties of the FGM plates are reduced by the voids inserted during the sintering process. Many studies in the literature have described the variation of FGM plate properties through the thickness [3], [4], [10], [47]–[50]. The manufacturing imperfection of FGM plates was also investigated in terms of distributed porosities [7]–[9], [51], [52] and initial geometric imperfection [53]–[56].

A general solution for the free vibration of FGM plates based on the power law, exponential law, and other material variation laws while resting on a Winkler–Pasternak foundation was provided by [21]. Fenjan et al. [57] used a differential quadrature method with a scale factor describing the size effect to investigate the free vibration of FG nanobeams considering the material's porosity and the environmental temperature. The free vibration of porous FG beams was studied for two gradation functions using the beam's classical theory and first-order shear deformation theory [58]. Selmi [59] presented an exact solution for exponential FG beams' nonlinear buckling behavior with different axial loads and vibration amplitudes using the Galerkin procedure. The buckling and vibration of sandwich FG micro beams with porosity and geometric curvature were analyzed [60]. Modified couple stress theory was used to study the effect of the thermal environment and the magnetic field. Madenci [61] presented an alternative procedure to analyze thin and thick FG beams using the mixed-finite element method.

Gupta & Talha [62] also considered the initial geometric imperfection while studying a porous FG plate's static characteristics using Pasternak's elastic foundation reaction.

Singh & Harsha [63] applied Galerkin Vlasov's method to investigate the vibration and buckling responses of a porous sandwich S-FGM plate resting on a Pasternak foundation. They concluded that the high vibration modes are sensitive to the porosity distribution type. The effect of the visco-Pasternak foundation on the bending, vibration, and dynamic FG plate's responses has been investigated by many researchers [64]–[66]. Hadji [67] developed a new simple model for shear deformable FG plate static analysis considering porosities while avoiding shear correction factors. Vinyas [68] used a finite element method to explore the FG magneto-electro-elastic circular and annular plates' vibrational behavior, considering the effect of the material porosities. Different FGM porosity distribution shapes were investigated in the literature [69]–[71].

Wave propagation in FGM structures has also attracted considerable research attention. Han et al. [24] analyzed the wave propagation of point-loaded functionally graded cylindrical shells by a numerical approach. Elastic waves' dispersion in FGM plates were investigated by approximating a sufficient number of homogeneous layers and, alternatively, by considering the continuous material change [25]. Boukhari et al. [26] presented a detailed investigation of the temperature effect on the FG beams' wave propagation.

FGM plates have also been investigated for wave propagation in many other works in the open literature [7], [72]–[77]. Fourn et al. [27] presented the four modes of infinite FG plates' dispersion relations based on an integral HSDT involving fewer computations and verified the theories' accuracy. Sun & Luo [28] worked on dynamic and wave propagation analysis to study a clamped FG plate under an impulse load, in addition to FG plates' subjected to a point-load impact and a thermal environment [29], [30]. They

introduced an HSDT that considers rotary inertia in addition to shear deformation. Mourad et al. [31] considered the FGM plates' porosities using an efficient approach for wave propagation analysis. Liang & Wang [78] also considered the FGM's porosities in the wave propagation analysis of a sandwich FG plate using a quasi-3D HSDT.

This article deals with the wave propagation characteristics of infinite FG ceramic-metal plates with porosity resting on a viscoelastic foundation. A simple four-variable integral hyperbolic shear deformation plate theory is proposed for the solution. This theory implies two undetermined integral terms with a hyperbolic shape function in the displacement field to describe the shear rotations in both the xz -plane and the yz -plane with a single unknown, while satisfying the boundary conditions in both faces of the plate without the need for shear correction coefficients. The material constituents' volume fractions are assumed to be changing from metal to ceramic by following a power law along the thickness direction only. The governing differential equations are derived based on the proposed plate theory and the three-parameter viscoelastic foundation model. Finally, the influences of the gradation power, the porosity parameters, and the viscoelastic foundation parameters on the wave propagation in a functionally graded plate are examined and discussed.

3.2 Effective Properties of the Porous FGM

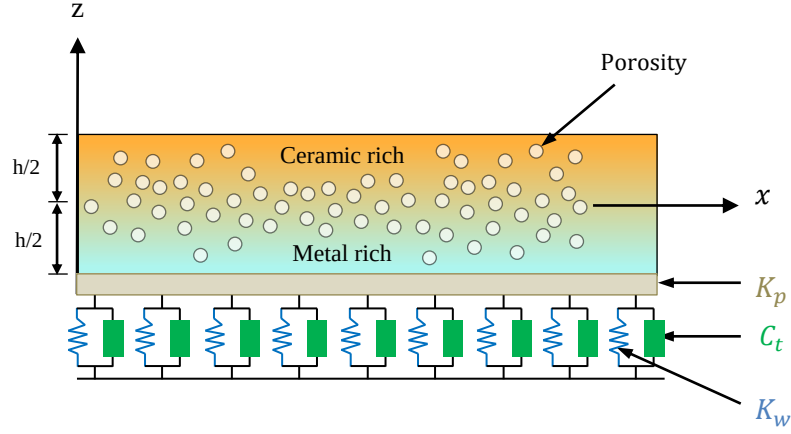


Figure 10 Material gradation of the porous FG plate resting on a viscoelastic foundation

The porous ceramic-metal FGM plate considered here is shown in Figure10. The mechanical properties are functions of the ceramic and metal properties, in addition to the constituents' and porosities' volume fractions, all of which are expressed by [48]

$$Pr(z) = Pr_m V_m + Pr_c V_c - (Pr_c + Pr_m) \xi \varsigma \quad (28)$$

Pr is a property of the FG material such as Young's modulus (E), Poisson's ratio (ν), and mass density (ρ). Where Pr_m represents this property in pure metal and Pr_c represents it in pure ceramic. ξ is a factor of porosity distribution, which is a function of the porosity volume fraction (λ). Three types of porosity distribution factors are shown in Table 1 and studied in this chapter. ς implies unevenly distributed porosities over the thickness, and it is given by:

$$\varsigma = 2 \left(\frac{1}{2} - \frac{|z|}{h} \right) \quad (29)$$

V_c and V_m are the volume fractions of ceramic and metal, respectively, given by [4], [5]

$$V_c = (1 - V_m) = \left(\frac{1}{2} + \frac{z}{h}\right)^P \quad (30)$$

P is the gradient index with a zero value for a homogeneous ceramic plate, infinity for the homogeneous metallic plate, or a positive value for different FG plates. Poisson's ratio is considered to be constant as its variation effect on the plate's behavior is negligible [3]. However, the mass density and Young's modulus are expressed by

$$\rho(z) = \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h}\right)^P - \xi(\rho_c + \rho_m) \left(1 - \frac{2|z|}{h}\right) \quad (31)$$

$$E(z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h}\right)^P - \xi(E_c + E_m) \left(1 - \frac{2|z|}{h}\right) \quad (32)$$

Table 1 Types of porosity distribution factor ξ [7]–[9]

Type-1	$\xi = 1 - e^{-\lambda/2}$
Type-2	$\xi = \text{Log}(1 + \lambda/2)$
Type-3	$\xi = \lambda/2$

3.3 Mathematical Formulation

3.3.1 Kinematics

In the present article, the displacement field is expressed as

$$u = u_0(x, y, t) - z \frac{\partial w}{\partial x} + k_1 f(z) \int \theta(x, y, t) dx \quad (33)$$

$$v = v_0(x, y, t) - z \frac{\partial w}{\partial y} + k_2 f(z) \int \theta(x, y, t) dy$$

$$w = w_0(x, y, t)$$

The four unknown variables u_0 , v_0 , w_0 and θ represent the plate displacements at the middle plane (at $z = 0$). The shear shape function $f(z)$ represents the variation of the shear stress and strain along the thickness direction. For the proposed solution

$$f(z) = z - \frac{2z \sinh\left(\frac{z^2}{h^2}\right)}{2 \sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)} \quad \text{and} \quad g(z) = \frac{df(z)}{dz} \quad (34)$$

Applying linear strain–displacement relations, the strain components are evaluated in terms of u_0 , v_0 , w_0 and θ as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} - z \begin{Bmatrix} \varepsilon_x^b \\ \varepsilon_y^b \\ \gamma_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \gamma_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} \quad (35)$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^b \\ \varepsilon_y^b \\ \gamma_{xy}^b \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad (36)$$

$$\begin{Bmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \gamma_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \int \frac{\theta}{\partial y} dx + k_2 \int \frac{\theta}{\partial x} dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \int \theta dx \\ k_2 \int \theta dy \end{Bmatrix}$$

Since a Navier type solution will be applied, the following relations can simplify the undetermined integrals.

$$\begin{aligned} \int \theta \, dx &= A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y}, \\ \frac{\partial}{\partial y} \int \theta \, dx &= A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x} \int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y} \end{aligned} \quad (37)$$

Hence, the strain terms associated with shear deformation become

$$\begin{pmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \gamma_{xy}^s \end{pmatrix} = \begin{pmatrix} k_1 \theta \\ k_2 \theta \\ A' k_1 \frac{\partial^2 \theta}{\partial x \partial y} + B' k_2 \frac{\partial^2 \theta}{\partial x \partial y} \end{pmatrix}, \quad \begin{pmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{pmatrix} = \begin{pmatrix} A' k_1 \frac{\partial \theta}{\partial x} \\ B' k_2 \frac{\partial \theta}{\partial y} \end{pmatrix} \quad (38)$$

A', B', k_1 and k_2 are expressed in terms of the wavenumbers of wave propagation as

$$A' = -\frac{1}{\kappa_1^2}, \quad B' = -\frac{1}{\kappa_2^2}, \quad k_1 = \kappa_1^2, \quad k_2 = \kappa_2^2 \quad (39)$$

Stress components are expressed using the constitutive relations

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} \quad (40)$$

The elastic stiffness coefficients for the FGM plate are given by

$$C_{11} = C_{22} = \frac{E(z)}{1 - \nu^2}, C_{12} = \frac{\nu E(z)}{1 - \nu^2} \quad (41)$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2\nu + 2}$$

3.3.2 The Viscoelastic Foundation

The present work deals with an FGM plate resting on a viscoelastic foundation. The viscoelastic foundation model considers three different foundation actions: the spring-like action, shear interaction, and time-dependent action of the unglued viscous foundation medium. These three actions are illustrated in Figure 10 and considered in the formulation by applying the following three-parameter reaction [65] on the FG plate

$$R_f = \left(K_w - K_p \nabla^2 + C_t \frac{\partial}{\partial t} \right) w \quad (42)$$

Where K_w is Winkler's parameter, K_p is Pasternak's parameter, C_t is the damping coefficient, w is the displacement in the z -direction and ∇^2 is a Laplace operator.

3.3.3 Equations of Motion

The equations of motion are derived via Hamilton's principle, which states the following:

$$\int_0^t (\delta U + \delta V - \delta K) dt = 0 \quad (43)$$

U, V , and K represent the internal energy, the external work, and the kinetic energy, respectively. The first variation operator is denoted by δ .

The first variation of the internal energy in the FGM plate is given by

$$\begin{aligned}
\delta U &= \int_A \int_{-h/2}^{h/2} (\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}) dz dA \\
&= \int_A (N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta \varepsilon_x^b + M_y^b \delta \varepsilon_y^b + M_{xy}^b \delta \varepsilon_{xy}^b \\
&\quad + M_x^s \delta \varepsilon_x^s + M_y^s \delta \varepsilon_y^s + M_{xy}^s \delta \varepsilon_{xy}^s + S_{xz}^s \delta \gamma_{xz}^0 + S_{yz}^s \delta \gamma_{yz}^s) dA
\end{aligned} \tag{44}$$

Where A is the surface area and N, M and N are the stresses resultants. These resultant forces and moments are defined by

$$\begin{bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} \cdot (\sigma_x, \sigma_y, \tau_{xy}) dz \tag{45}$$

And

$$\begin{Bmatrix} S_{xz} \\ S_{yz} \end{Bmatrix} = \int_{-h/2}^{h/2} g(z) \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{46}$$

By employing the stress-strain relations, the normal stresses resultants can be expressed by

$$\begin{Bmatrix} N_x \\ N_y \\ M_x^b \\ M_y^b \\ M_x^s \\ M_y^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{11}^s & B_{12}^s \\ A_{12} & A_{22} & B_{12} & B_{22} & B_{12}^s & B_{22}^s \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{11}^s & D_{12}^s \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{12}^s & D_{22}^s \\ B_{11}^s & B_{12}^s & D_{11}^s & D_{12}^s & H_{11}^s & H_{12}^s \\ B_{12}^s & B_{22}^s & D_{12}^s & D_{22}^s & H_{12}^s & H_{22}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_x^b \\ \varepsilon_y^b \\ \varepsilon_x^s \\ \varepsilon_y^s \end{Bmatrix} \quad (47)$$

While shear stresses resultants are expressed by

$$\begin{Bmatrix} N_{xy} \\ M_{xy}^b \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{66} & B_{66} & B_{66}^s \\ B_{66} & D_{66} & D_{66}^s \\ B_{66}^s & D_{66}^s & H_{66}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xy}^0 \\ \gamma_{xy}^b \\ \gamma_{xy}^s \end{Bmatrix} \quad (48)$$

$$\begin{Bmatrix} S_{xz} \\ S_{yz} \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix}$$

Where

$$\begin{bmatrix} A_{11} & B_{11} & B_{11}^s & D_{11} & D_{11}^s & H_{11}^s \\ A_{22} & B_{22} & B_{22}^s & D_{22} & D_{22}^s & H_{22}^s \\ A_{12} & B_{12} & B_{12}^s & D_{12} & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & B_{66}^s & D_{66} & D_{66}^s & H_{66}^s \end{bmatrix} = \int_{-h/2}^{h/2} (1, z, f, z^2, zf, f^2) \begin{Bmatrix} C_{11} \\ C_{22} \\ C_{12} \\ C_{66} \end{Bmatrix} dz \quad (49)$$

And

$$\begin{Bmatrix} A_{44}^s \\ A_{55}^s \end{Bmatrix} = \int_{-h/2}^{h/2} g(z)^2 \begin{Bmatrix} C_{44} \\ C_{55} \end{Bmatrix} dz \quad (50)$$

The external work on the FGM plate is exerted by the foundation reaction only. Hence, the first variation of the external work is given by

$$\delta V = R_f \delta w = \int_A (K_w w_0 - K_p \nabla^2 w_0 + C_t \dot{w}_0) \delta w_0 dA \quad (51)$$

The first variation of the kinetic energy is given for the wave propagation problem by

$$\begin{aligned}
\delta U &= \int_A \int_{-h/2}^{h/2} \rho(z) (\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}) dz dA \\
&= \int_A \left(I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + I_2 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \right) \delta u_0 \\
&\quad + \left(I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + I_2 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \right) \delta v_0 \\
&\quad + \left(-I_1 \frac{\partial \ddot{u}_0}{\partial x} + I_3 \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_1 \frac{\partial \ddot{v}_0}{\partial y} + I_3 \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right. \\
&\quad \left. - I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} + I_0 \ddot{w}_0 \right) \delta w_0 \\
&\quad + \left(I_2 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} - I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \right. \\
&\quad \left. + I_2 k_2 B' \frac{\partial \ddot{v}_0}{\partial y} - I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} + I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \delta \theta dA
\end{aligned} \tag{52}$$

Where the mass inertias introduced in the equation are defined as

$$(I_0, I_1, I_2, I_3, I_4, I_5) = \int_{-h/2}^{h/2} (1, z, f, z^2, zf, f^2) \rho(z) dz \tag{53}$$

Substituting δU , δV and δK in Hamilton's principle, Eq. (15) is expanded to

$$\begin{aligned}
& \int_0^t \int_A \left(-\frac{\partial N_x}{\partial x} - \frac{\partial N_{xy}}{\partial y} - I_0 \ddot{u}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial x} - I_2 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \right) \delta u_0 \\
& + \left(-\frac{\partial N_y}{\partial y} - \frac{\partial N_{xy}}{\partial x} - I_0 \ddot{v}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial y} - I_2 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \right) \delta v_0 \\
& + \left(-\frac{\partial^2 M_x^b}{\partial x^2} - 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} - \frac{\partial^2 M_y^b}{\partial y^2} + I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial x^2} \right. \\
& + I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_1 \frac{\partial \ddot{v}_0}{\partial y} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial y^2} + I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} - I_0 \ddot{w}_0 \\
& \left. + K_w w_0 - K_p \nabla^2 w_0 + C_t \dot{w}_0 \right) \delta w_0 \\
& + \left(A' k_1 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + B' k_2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 M_x^s + k_2 M_y^s - A' k_1 \frac{\partial S_{xz}}{\partial x} \right. \\
& - B' k_2 \frac{\partial S_{yz}}{\partial y} - I_2 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \\
& \left. - I_2 k_2 B' \frac{\partial \ddot{v}_0}{\partial y} + I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} - I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \delta \theta dA dt \\
& = 0
\end{aligned} \tag{54}$$

The equations of motion are obtained by setting each coefficient of the unknown variations to zero.

$$\begin{aligned}
\delta u_0 : \quad & \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -I_0 \ddot{u}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial x} - I_2 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \\
\delta v_0 : \quad & \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = -I_0 \ddot{v}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial y} - I_2 k_2 B' \frac{\partial \ddot{\theta}}{\partial y}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\delta w_0 : \quad & \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} \\
& = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_1 \frac{\partial \ddot{v}_0}{\partial y} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial y^2} \\
& \quad + I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} - I_0 \ddot{w}_0 + K_w w_0 - K_p \nabla^2 w_0 + C_t \dot{w}_0 \\
\delta \theta : \quad & -A' k_1 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - B' k_2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - k_1 M_x^s - k_2 M_y^s + A' k_1 \frac{\partial S_{xz}}{\partial x} \\
& \quad + B' k_2 \frac{\partial S_{yz}}{\partial y} \\
& = -I_2 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \\
& \quad - I_2 k_2 B' \frac{\partial \ddot{v}_0}{\partial y} + I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} - I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}
\end{aligned}$$

3.3.4 Dispersion Relations

The solution representing the wave propagation in the FG plate is assumed to have the following form

$$\begin{Bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \\ \theta(x, y, t) \end{Bmatrix} = \begin{Bmatrix} U \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ V \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ W \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \\ X \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \end{Bmatrix} \quad (56)$$

U, V, W and X are the constant amplitude coefficients of the propagating wave, κ_1, κ_2 and ω are the wavenumbers in the x and y directions and the wave frequency, and i is the imaginary unit.

This solution is substituted into the equations of motion. The resulting equations have five unknowns that are ω, U, V, W and X . For each term, ω either appears as squared or does not appear outside the exponential function. The four equations of motion are simplified in the following matrix form

$$([K] - \omega^2[M]) \{\Lambda\} = 0 \quad (57)$$

Where the stiffness matrix $[K]$ and the mass matrix $[M]$ are expressed as

$$[K] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}, \quad [M] = \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}, \quad (58)$$

$$\{\Lambda\} = \begin{Bmatrix} U \\ V \\ W \\ X \end{Bmatrix}$$

Where

$$k_{11} = \kappa_1^2 A_{11} + \kappa_2^2 A_{66}$$

$$k_{12} = \kappa_1 \kappa_2 (A_{12} + A_{66})$$

$$k_{13} = -i \kappa_1 (\kappa_1^2 B_{11} + \kappa_2^2 (B_{12} + 2B_{66}))$$

$$k_{14} = -i \kappa_1 (k_1 B_{11}^s + k_2 B_{12}^s - (A'k_1 + B'k_2) \kappa_2^2 B_{66}^s)$$

$$k_{22} = \kappa_2^2 A_{22} + \kappa_1^2 A_{66}$$

$$k_{23} = -i \kappa_2 (\kappa_2^2 B_{22} + \kappa_1^2 (B_{12} + 2B_{66}))$$

$$k_{24} = -i \kappa_2 (k_2 B_{22}^s + k_1 B_{12}^s - (A'k_1 + B'k_2) \kappa_1^2 B_{66}^s)$$

$$k_{33} = -\kappa_1^4 D_{11} - \kappa_2^4 D_{22} - 2\kappa_1^2 \kappa_2^2 (D_{12} + 2D_{66}) + K_w - K_p(\kappa_1^2 + \kappa_2^2) + i C_t \omega_f$$

$$k_{34} = -k_1 \kappa_1^2 D_{11}^s - k_2 \kappa_1^2 D_{12}^s - k_1 \kappa_2^2 D_{12}^s - k_2 \kappa_2^2 D_{22}^s + 2A' k_1 \kappa_1^2 \kappa_2^2 D_{66}^s \\ + 2B' k_2 \kappa_1^2 \kappa_2^2 D_{66}^s$$

$$k_{44} = -k_1^2 H_{11}^s - 2k_1 k_2 H_{12}^s - k_2^2 H_{22}^s - (A'^2 k_1^2 + 2A'B' k_1 k_2 + B'^2 k_2^2) \kappa_1^2 \kappa_2^2 H_{66}^s \\ + A' k_1^2 A_{44}^s + B' k_2^2 A_{55}^s$$

$$m_{11} = m_{22} = I_0$$

$$m_{13} = -i\kappa_1 I_1$$

$$m_{14} = iA' k_1 \kappa_1 I_2$$

$$m_{23} = -i\kappa_2 I_1$$

$$m_{24} = -iB' k_2 \kappa_2 I_3$$

$$m_{33} = -I_0 - i(\kappa_1^2 + \kappa_2^2) I_3$$

$$m_{34} = (A' k_1 \kappa_1^2 + B' k_2 \kappa_2^2) I_4$$

$$m_{44} = -((A' k_1 \kappa_1)^2 + (B' k_2 \kappa_2)^2) I_5$$

The dispersion relations are given for the wave propagation in the FGM plates by the following eigenvalue problem

$$|[K] - \omega^2[M]| = 0 \quad (59)$$

The four eigenvalues are expressed as functions of the wavenumber by

$$\omega_i = W_i(\kappa), \quad (i = 1,2,3,4) \quad (60)$$

ω_1 and ω_4 correspond to the flexural wave modes M_1 and M_4 , respectively, while ω_2 and ω_3 correspond to the extensional wave modes M_2 and M_3 , respectively.

The phase velocity corresponding to each wave mode is given by

$$C_i = \frac{W_i(\kappa)}{\kappa}, \quad (i = 1,2,3,4) \quad (61)$$

3.4 Numerical Results and Discussion

The ceramic-metal FGM plate shown in Figure 10 is considered, with the constituents' volume fractions varied based on the power law. A zero-power index defines a homogeneous ceramic plate, and the higher power index represents a higher metallic concentration. The Si3N4 and SUS304 properties reported by Belkorissat et al. [79] are listed in Table 2 and considered in the numerical calculations for ceramic and metal, respectively. The circular frequencies calculated based on the present theory are compared with the available literature values [80], [81] to verify the present theory's accuracy in studying wave propagation. Table 3 compares the results for the circular frequencies (ω) between the present study and other researchers' results.

The influences of gradation power, porosity parameters, and the three parameters of the viscoelastic foundation model on wave propagation are examined and discussed. Calculations are made with the proper units that are shown in the figures and tables.

Table 2 The properties of the materials used [79]

Materials	E (GPa)	ρ (kg/m ³)	ν
Si3N4	348.43	2370	0.3
SUS304	201.04	8166	0.3

Table 3 Comparison of present results for the circular frequencies (ω) with the results of [80], [81] ($\nu = 0.3, E = 210$ GPa, $\rho = 7480$ Kg/m³, $h = 0.01$ m, $\kappa_1 = \kappa_2$)

Wavenumber κ (m ⁻¹)	2	5	8	11	14	17	20	23
Hooman Aminipour [80]	128.46	802.28	2050.90	3869.44	6250.88	9186.18	12664.47	16673.20
Mohammad Rahim Nami [81]	128.27	801.71	2052.38	3880.28	6285.41	9267.77	12827.37	16964.19
Present	128.26	800.97	2047.55	3863.09	6240.55	9170.85	12643.1	16644.5

The dispersion relations between the principal circular frequency and the wavenumber are shown in Figure 11 for the three types of porosity distribution factor. It is noticed that the relation between frequency and wavenumber in the three types is identical. It is clear that the principal frequency increases as the power index is reduced, and it reaches the maximum values in the homogeneous ceramic plate.

Figure 12 shows the dispersion relations between the phase velocity and the porous FGM plate's principal circular frequency. The phase velocity is high at low natural frequencies because the associated wavenumber is low, and it slows down as the frequency increases. The phase velocity is higher in a stiffer ceramic-rich FGM plate that has a lower power index.

The effect of porosity on the wave propagation of the power-law model FGM plate is changed by changing the material power index. Figure 13 shows the phase velocity curves versus the porosity volume fraction for the three types of porosity distribution and

different power indices. The phase velocity is increased with the increase of porosities in a stiffer, ceramic-rich, FGM plate ($P < 0.7$), and the opposite influence is noticed with higher metallic content ($P > 0.7$). FGM plates with P values around 0.7 behave like stiff plates as the porosity content is low. However, the plate stiffness is reduced if more porosity voids are induced, and the plate's response becomes more like a metallic-rich FGM plate. This mixed behavior is shown in Figure 13 (b). The absolute value of the average slope of the phase velocity versus the λ curve increases as P moves further from 0.7. It is also noticed that the porosity distribution factor of type-3 reports a more sensitive phase velocity change of λ , regardless of the power index value. At the same time the other two types have similar behavior.

The effect of different foundation models on the relation between the phase velocity and the wavenumber is plotted in Figure 14. The frequency and the phase velocity increase with the increase of either the Winkler's or the Pasternak's parameters or with the decrease of the damping coefficient. Hence, the highest phase velocity is observed in the plate resting on the Winkler-Pasternak foundation. The damping coefficient effect on the phase velocity dispersion relations with the wavenumber and the circular frequency are shown for the porous FGM plate in Figures 15 and 16. It can be seen that the damping action of the foundation slows down the propagating waves. The influence of the damping coefficient is felt more at lower wavenumbers.

The influences of each of the Winkler's parameter, the Pasternak's parameter, and the damping coefficient on the variation of the phase velocity versus the porosity volume fraction λ are examined separately, as demonstrated in Figures 17-19. The greater damping coefficient leads to shifting the curve of the velocity versus λ downward with a

negligible change in the slope. On the other hand, as the value of K_w or K_p increases, the curve is shifted upward and flattened more. If either the value of K_w exceeds $2.05 \times 10^{10} \text{ kN/m}^3$ or K_p exceeds $1.02 \times 10^6 \text{ kN/m}$, the phase velocity will no longer be influenced by the porosity.

Tables 4 and 5 present the circular frequencies and the phase velocities for the FGM plate resting on a three-parameter viscoelastic foundation with different thicknesses, power indices, porosity volume fractions, damping coefficients, and wavenumbers. In these two tables $\kappa_1 = \kappa_2 = \kappa$, $K_w = 10^9 \text{ kN/m}^3$, $K_p = 10^5 \text{ kN/m}$. It can be noticed that the influence of the damping coefficient decreases until it disappears with the increase of either the wavenumber or the FGM plate thickness.

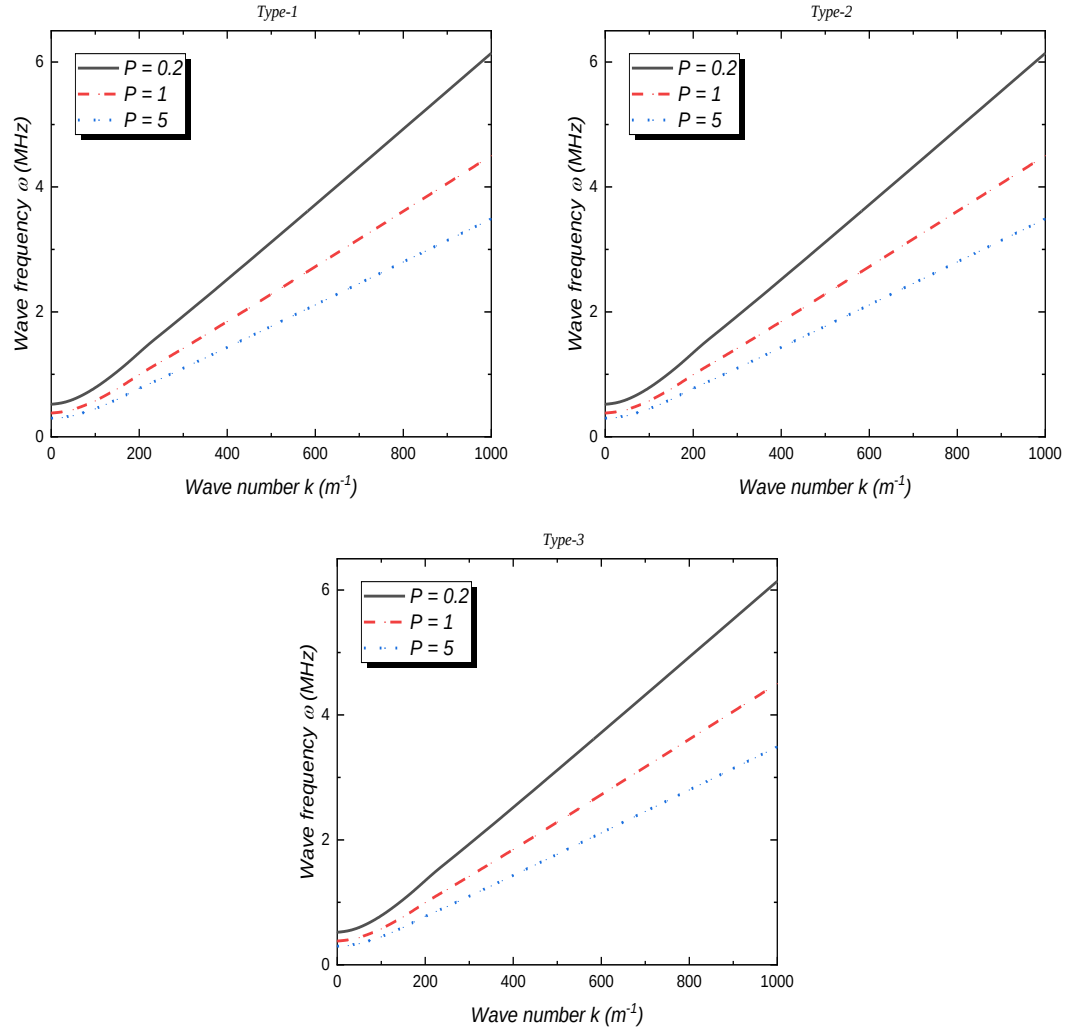


Figure 11 Porous FG plate's dispersion relation between the wave frequency and the wavenumber for different material power indices with $h = 0.05\text{ m}$, $\lambda = 0.1$, $\kappa_2 = 100\text{ m}^{-1}$, $K_w = 0$, $K_p = 0$, $C_t = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3)

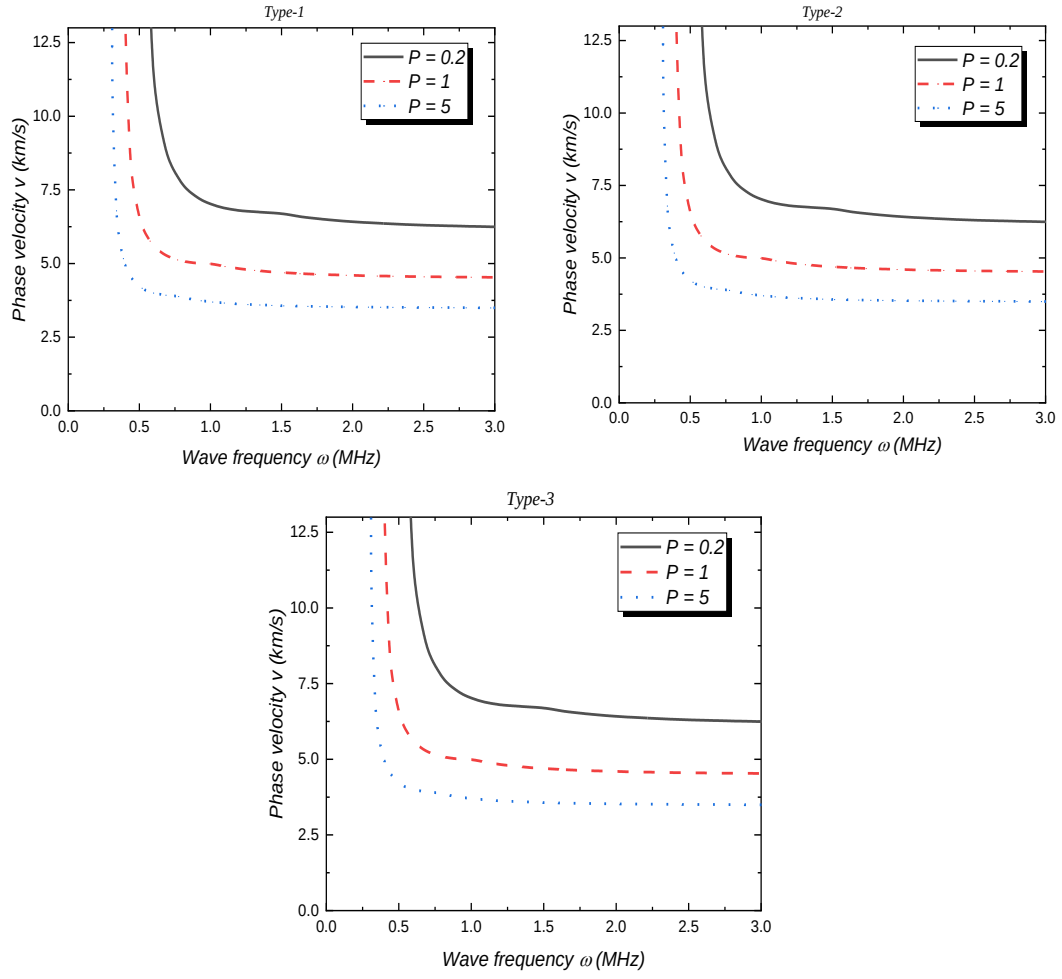


Figure 12 Porous FG plate's dispersion relation between the phase velocity and the wave frequency for different material power indices with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $\kappa_2 = 100 \text{ m}^{-1}$, $K_w = 0$, $K_p = 0$, $C_t = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3)

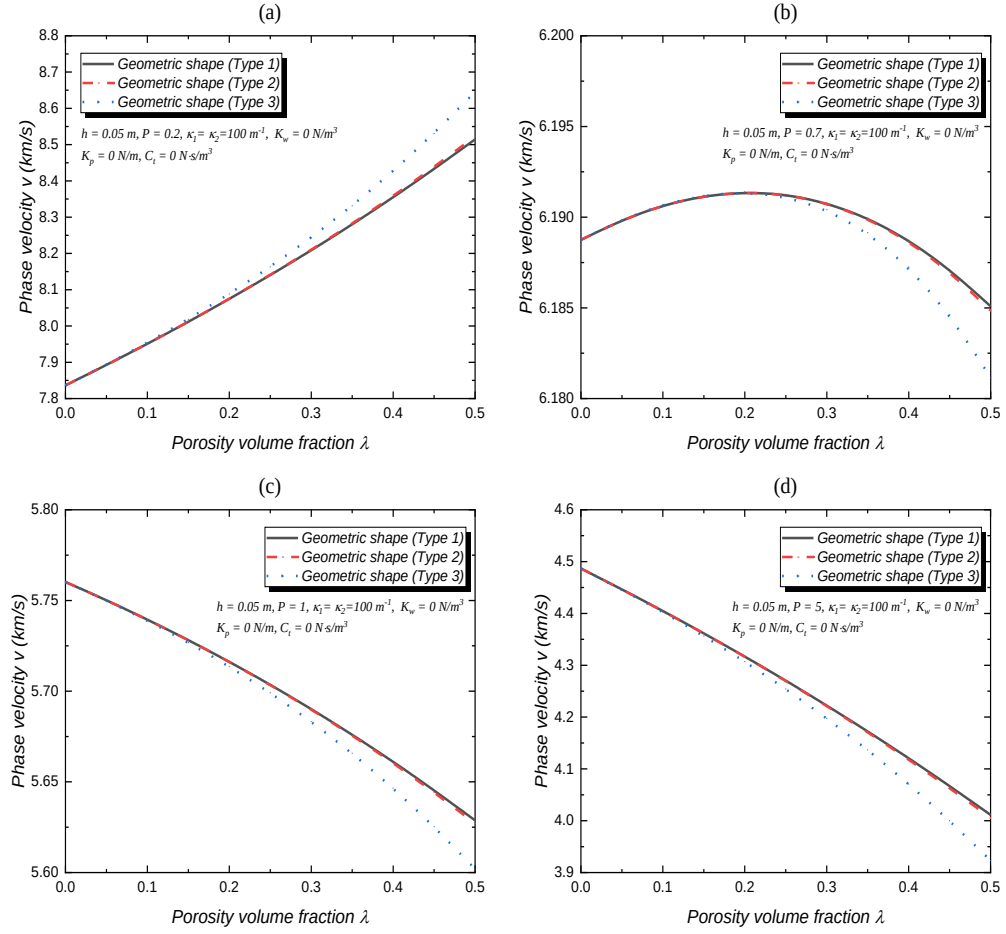


Figure 13 Porous FG plate's phase velocity versus porosity volume fraction λ : (material power index (a) $P = 0.2$, (b) $P = 0.7$), (c) $P = 1$ and (d) $P = 5$).

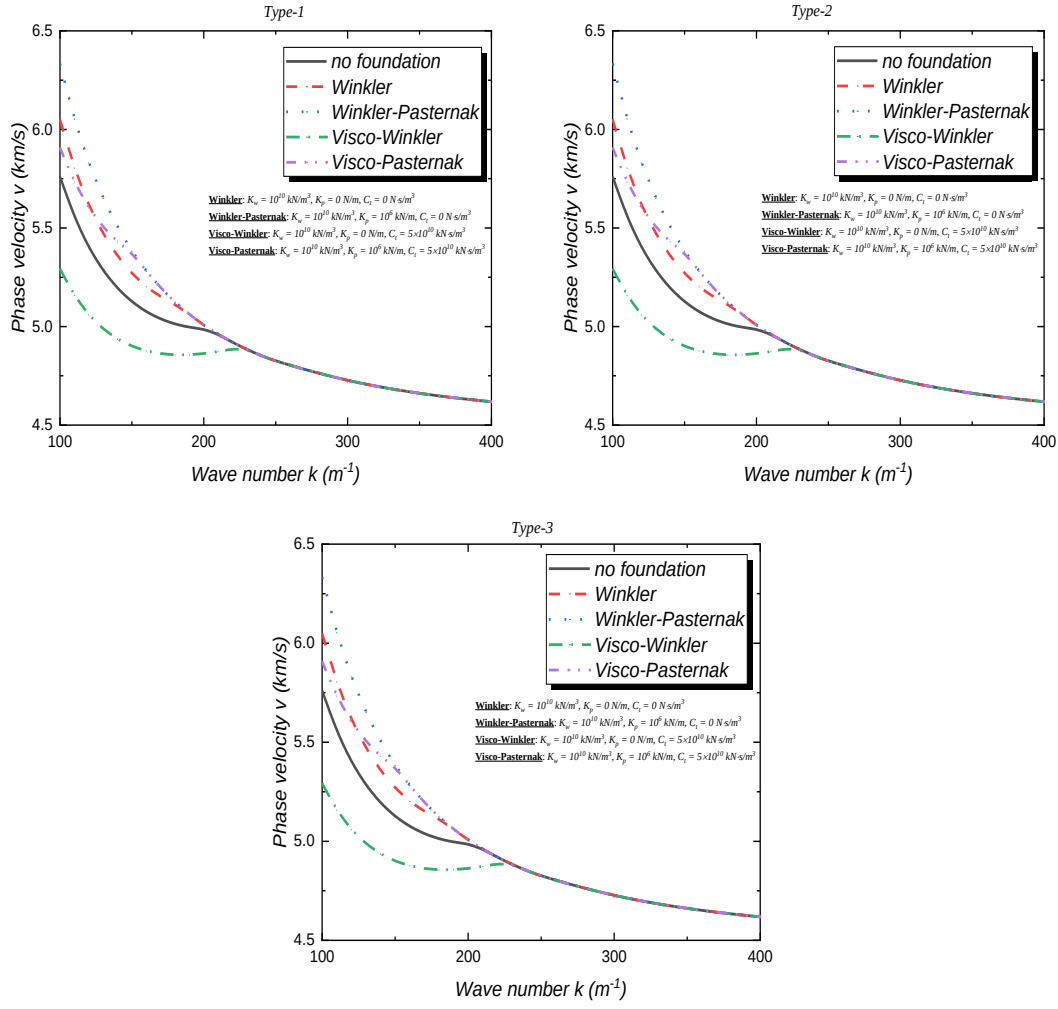


Figure 14 Porous FG plate's dispersion relation between the phase velocity and the wavenumber for different foundations, with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $P = 1$, $\kappa_2 = 100 \text{ m}^{-1}$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3)

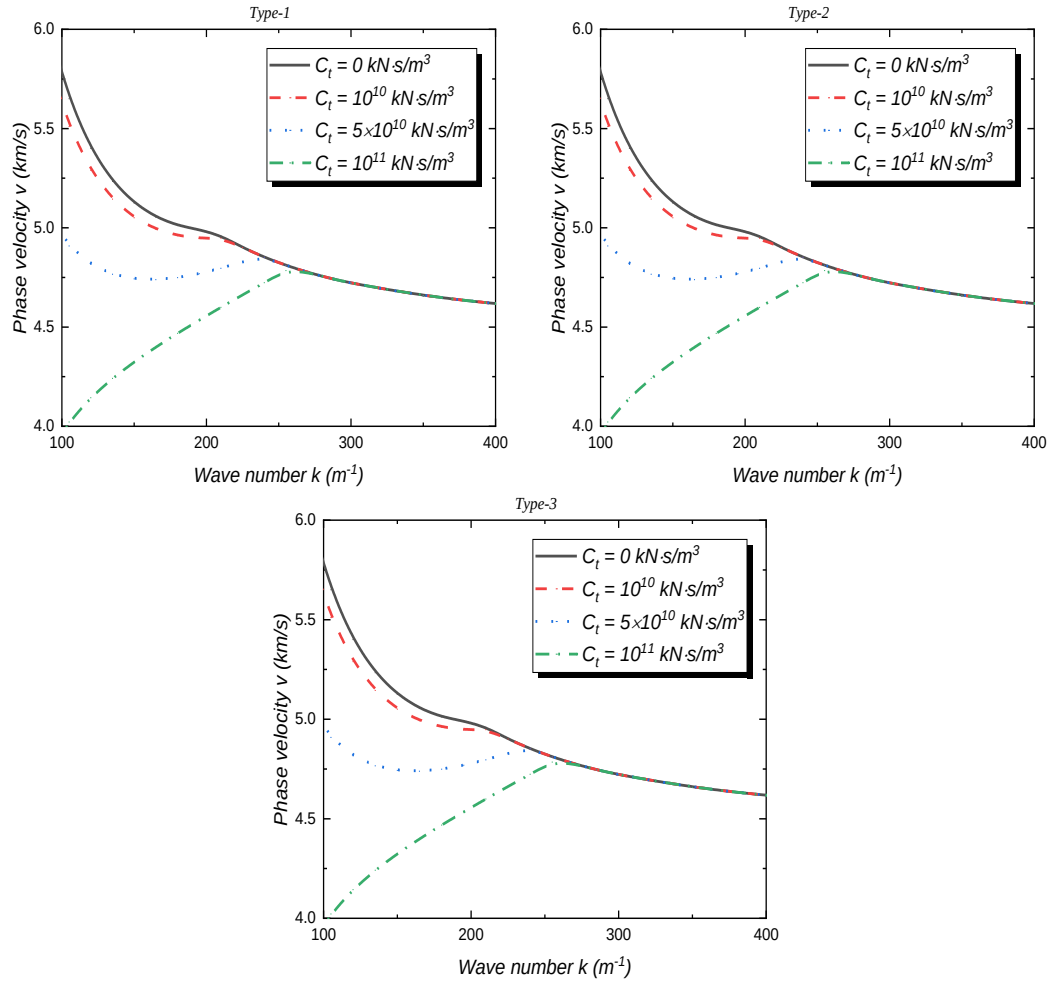


Figure 15 The effect of the foundation's damping coefficient on the porous FG plate's dispersion relation between the phase velocity and the wavenumber with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $P = 1$, $\kappa_2 = 100 \text{ m}^{-1}$, $K_w = 0$, $K_p = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3)

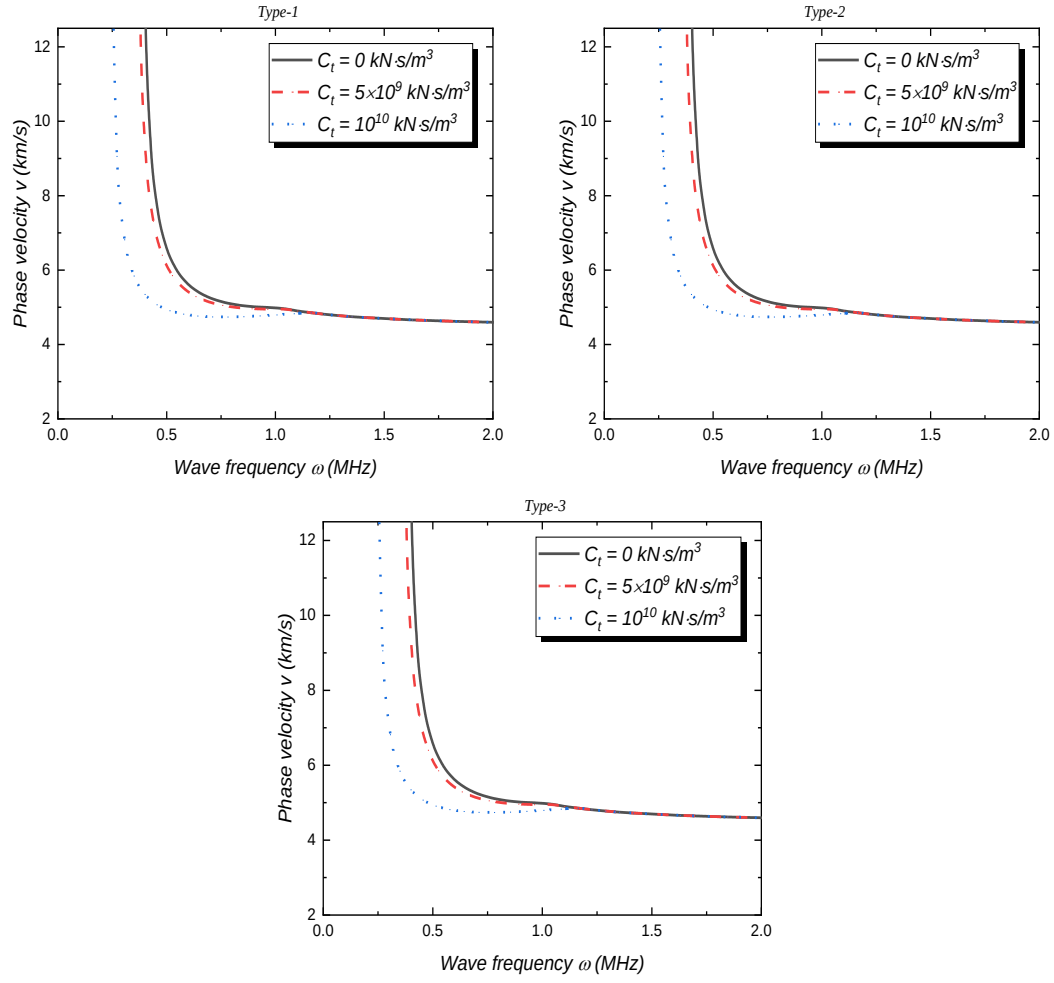


Figure 16 The effect of the foundation's damping coefficient on the porous FG plate's dispersion relation between the phase velocity and the wave frequency with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $P = 1$, $K_w = 0$, $K_p = 0$, $\kappa_2 = 100 \text{ m}^{-1}$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3).

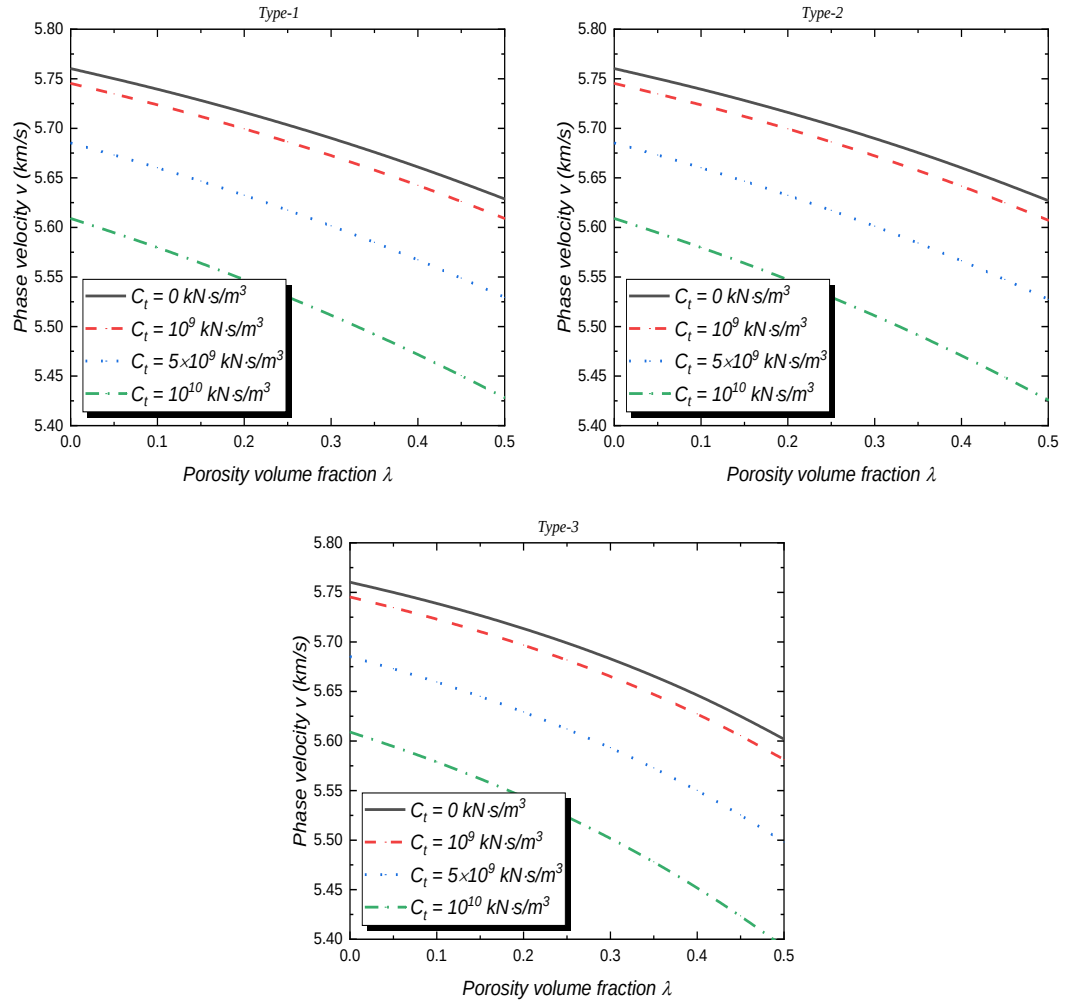


Figure 17 The effect of the foundation's damping coefficient on the porous FG plate's dispersion relation between the phase velocity and the porosity volume fraction λ with $h = 0.05 \text{ m}$, $\kappa_1 = \kappa_2 = 100 \text{ m}^{-1}$, $P = 1$, $K_w = 0$, $K_p = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3).

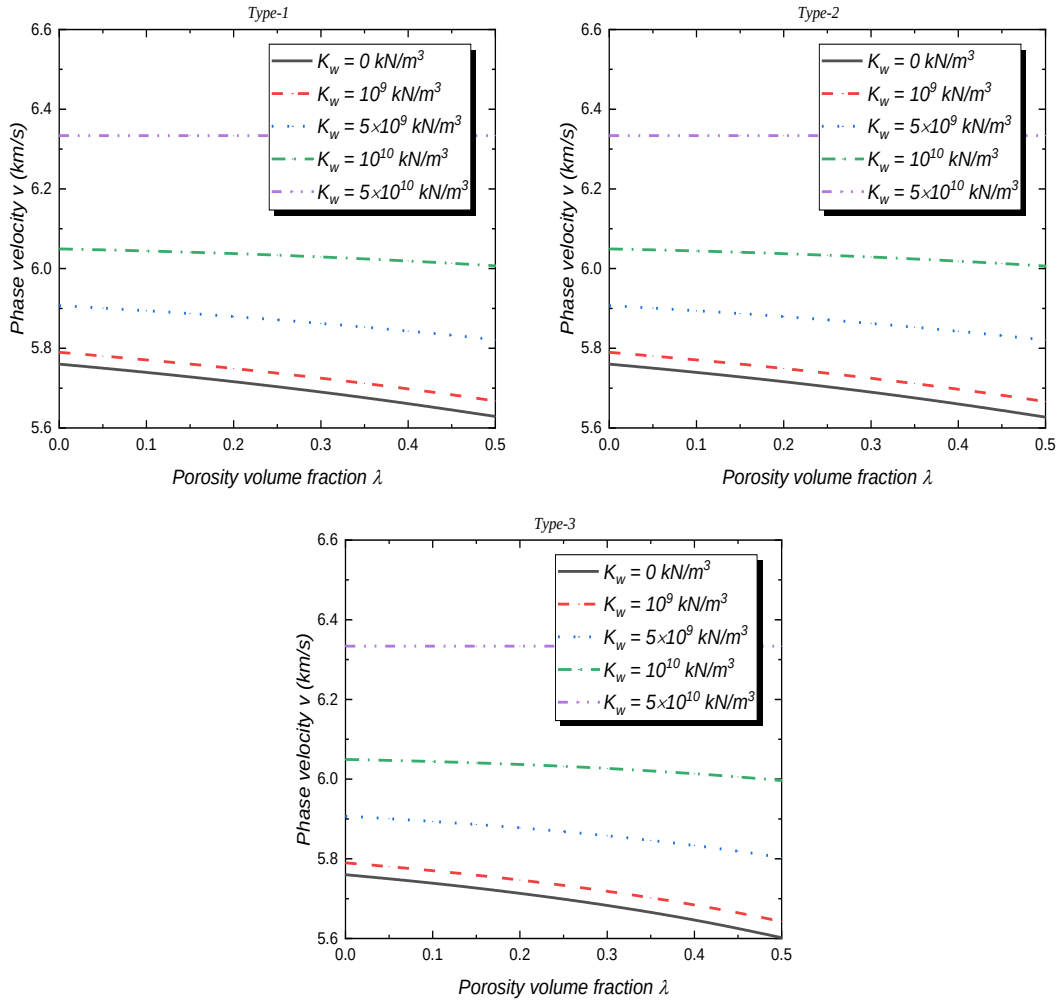


Figure 18 The effect of Winkler's parameter on the porous FG plate's dispersion relation between the phase velocity and the porosity volume fraction λ with $h = 0.05 \text{ m}$, $P = 1$, $\kappa_1 = \kappa_2 = 100 \text{ m}^{-1}$, $C_t = 0$, $K_p = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3).

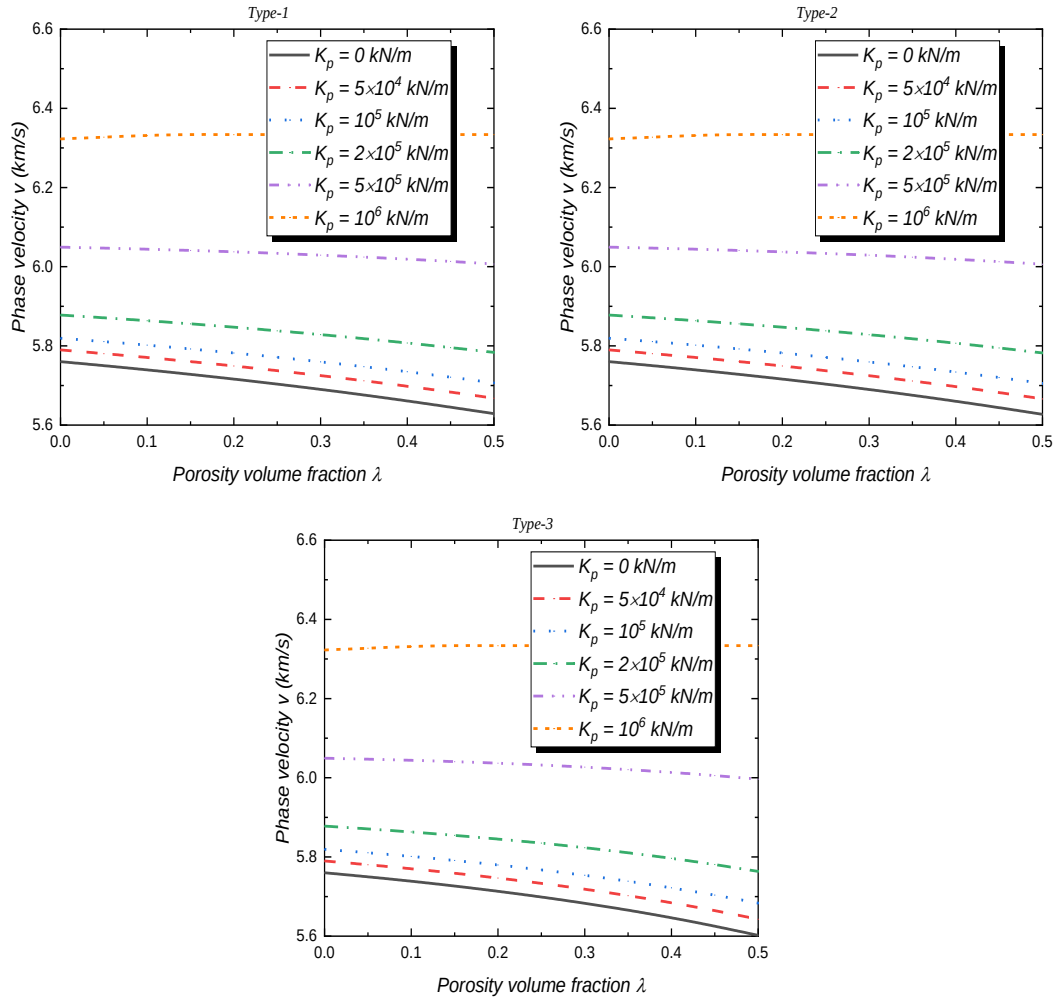


Figure 19 The effect of Pasternak's parameter on the porous FG plate's dispersion relation between the phase velocity and the porosity volume fraction λ with $h = 0.05$ m, $P = 1$, $\kappa_1 = \kappa_2 = 100$ m⁻¹, $K_w = 0$, $C_t = 0$: (distribution of porosity (a) type-1, (b) type-2 and (c) type-3)

Table 4 Naturel frequencies (MHz) of type-1 porous FGM plate for different thicknesses (m), material power indices, foundations' damping coefficients ($10^8 \text{kN}\cdot\text{s}/\text{m}^3$), wavenumbers (m^{-1}) and porosity volume fractions

h	P	C_t	$\lambda = 0$			$\lambda = 0.1$			$\lambda = 0.2$		
			κ			κ			κ		
			100	200	300	100	200	300	100	200	300
0.02	0.2	0	0.6650	1.5463	2.4788	0.6781	1.5712	2.5173	0.6922	1.5981	2.5588
		10	0.6599	1.5441	2.4775	0.6727	1.5689	2.5159	0.6865	1.5956	2.5573
		50	0.6392	1.5353	2.4721	0.6507	1.5595	2.5102	0.6631	1.5857	2.5513
		100	0.6122	1.5243	2.4654	0.6220	1.5478	2.5031	0.6326	1.5732	2.5437
	1	0	0.4899	1.1394	1.8324	0.4909	1.1374	1.8289	0.4918	1.1350	1.8247
		10	0.4856	1.1375	1.8313	0.4864	1.1354	1.8277	0.4871	1.1329	1.8235
		50	0.4678	1.1299	1.8267	0.4677	1.1274	1.8228	0.4674	1.1245	1.8183
		100	0.4445	1.1204	1.8209	0.4432	1.1174	1.8167	0.4415	1.1139	1.8119
	5	0	0.3902	0.8946	1.4377	0.3864	0.8809	1.4152	0.3822	0.8661	1.3912
		10	0.3860	0.8928	1.4365	0.3820	0.8790	1.4141	0.3776	0.8641	1.3900
		50	0.3689	0.8854	1.4320	0.3640	0.8712	1.4093	0.3586	0.8558	1.3849
		100	0.3463	0.8761	1.4264	0.3400	0.8613	1.4033	0.3332	0.8454	1.3787
0.05	0.2	0	0.7939	1.7283	2.5925	0.8060	1.7614	2.6421	0.8190	1.7977	2.6965
		10	0.7922	1.7283	2.5925	0.8042	1.7614	2.6421	0.8171	1.7977	2.6965
		50	0.7854	1.7283	2.5925	0.7969	1.7614	2.6421	0.8094	1.7977	2.6965
		100	0.7767	1.7283	2.5925	0.7877	1.7614	2.6421	0.7996	1.7977	2.6965
	1	0	0.5849	1.2668	1.9001	0.5833	1.2668	1.9001	0.5815	1.2668	1.9001
		10	0.5834	1.2668	1.9001	0.5817	1.2668	1.9001	0.5799	1.2668	1.9001
		50	0.5775	1.2668	1.9001	0.5755	1.2668	1.9001	0.5733	1.2668	1.9001
		100	0.5700	1.2668	1.9001	0.5676	1.2668	1.9001	0.5649	1.2668	1.9001
	5	0	0.4573	0.9819	1.4728	0.4497	0.9698	1.4546	0.4414	0.9572	1.4358
		10	0.4559	0.9819	1.4728	0.4481	0.9698	1.4546	0.4398	0.9572	1.4358
		50	0.4501	0.9819	1.4728	0.4420	0.9698	1.4546	0.4333	0.9572	1.4358
		100	0.4428	0.9819	1.4728	0.4343	0.9698	1.4546	0.4251	0.9572	1.4358
0.1	0.2	0	0.8642	1.7283	2.5925	0.8807	1.7614	2.6421	0.8988	1.7977	2.6965
		10	0.8642	1.7283	2.5925	0.8807	1.7614	2.6421	0.8988	1.7977	2.6965
		50	0.8642	1.7283	2.5925	0.8807	1.7614	2.6421	0.8988	1.7977	2.6965
		100	0.8642	1.7283	2.5925	0.8807	1.7614	2.6421	0.8988	1.7977	2.6965
	1	0	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001
		10	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001
		50	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001
		100	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001	0.6334	1.2668	1.9001
	5	0	0.4909	0.9819	1.4728	0.4849	0.9698	1.4546	0.4786	0.9572	1.4358
		10	0.4909	0.9819	1.4728	0.4849	0.9698	1.4546	0.4786	0.9572	1.4358
		50	0.4909	0.9819	1.4728	0.4849	0.9698	1.4546	0.4786	0.9572	1.4358
		100	0.4909	0.9819	1.4728	0.4849	0.9698	1.4546	0.4786	0.9572	1.4358

Table 5 Phase velocities (km/s) of type-1 porous FGM plate for different thicknesses (m), material power indices, foundations' damping coefficients ($10^8 \text{ kN}\cdot\text{s}/\text{m}^3$), wavenumbers (m^{-1}) and porosity volume fractions

h	P	C_t	$\lambda = 0$			$\lambda = 0.1$			$\lambda = 0.2$		
			κ			κ			κ		
			100	200	300	100	200	300	100	200	300
0.02	0.2	0	6.6497	7.7316	8.2628	6.7807	7.8561	8.3910	6.9215	7.9906	8.5294
		10	6.5990	7.7206	8.2583	6.7269	7.8445	8.3862	6.8645	7.9782	8.5244
		50	6.3917	7.6766	8.2405	6.5070	7.7977	8.3673	6.6310	7.9285	8.5043
		100	6.1219	7.6213	8.2181	6.2203	7.7389	8.3436	6.3259	7.8659	8.4792
	1	0	4.8994	5.6969	6.1081	4.9095	5.6868	6.0962	4.9182	5.6748	6.0825
		10	4.8561	5.6875	6.1043	4.8640	5.6769	6.0922	4.8705	5.6644	6.0782
		50	4.6784	5.6497	6.0889	4.6773	5.6371	6.0760	4.6742	5.6224	6.0611
		100	4.4452	5.6020	6.0696	4.4316	5.5868	6.0556	4.4152	5.5693	6.0396
	5	0	3.9019	4.4732	4.7922	3.8639	4.4046	4.7175	3.8217	4.3306	4.6374
		10	3.8603	4.4640	4.7885	3.8202	4.3949	4.7135	3.7757	4.3204	4.6332
		50	3.6891	4.4271	4.7735	3.6399	4.3559	4.6977	3.5856	4.2791	4.6165
		100	3.4626	4.3805	4.7546	3.4004	4.3066	4.6778	3.3319	4.2270	4.5955
0.05	0.2	0	7.9390	8.6417	8.6417	8.0597	8.8069	8.8069	8.1902	8.9884	8.9884
		10	7.9221	8.6417	8.6417	8.0417	8.8069	8.8069	8.1710	8.9884	8.9884
		50	7.8538	8.6417	8.6417	7.9691	8.8069	8.8069	8.0938	8.9884	8.9884
		100	7.7675	8.6417	8.6417	7.8773	8.8069	8.8069	7.9961	8.9884	8.9884
	1	0	5.8487	6.3338	6.3338	5.8329	6.3338	6.3338	5.8149	6.3338	6.3338
		10	5.8341	6.3338	6.3338	5.8175	6.3338	6.3338	5.7986	6.3338	6.3338
		50	5.7751	6.3338	6.3338	5.7552	6.3338	6.3338	5.7328	6.3338	6.3338
		100	5.7003	6.3338	6.3338	5.6763	6.3338	6.3338	5.6494	6.3338	6.3338
	5	0	4.5734	4.9095	4.9095	4.4965	4.8488	4.8488	4.4140	4.7859	4.7859
		10	4.5590	4.9095	4.9095	4.4814	4.8488	4.8488	4.3980	4.7859	4.7859
		50	4.5013	4.9095	4.9095	4.4203	4.8488	4.8488	4.3333	4.7859	4.7859
		100	4.4280	4.9095	4.9095	4.3427	4.8488	4.8488	4.2511	4.7859	4.7859
0.1	0.2	0	8.6417	8.6417	8.6417	8.8069	8.8069	8.8069	8.9884	8.9884	8.9884
		10	8.6417	8.6417	8.6417	8.8069	8.8069	8.8069	8.9884	8.9884	8.9884
		50	8.6417	8.6417	8.6417	8.8069	8.8069	8.8069	8.9884	8.9884	8.9884
		100	8.6417	8.6417	8.6417	8.8069	8.8069	8.8069	8.9884	8.9884	8.9884
	1	0	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338
		10	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338
		50	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338
		100	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338	6.3338
	5	0	4.9095	4.9095	4.9095	4.8488	4.8488	4.8488	4.7859	4.7859	4.7859
		10	4.9095	4.9095	4.9095	4.8488	4.8488	4.8488	4.7859	4.7859	4.7859
		50	4.9095	4.9095	4.9095	4.8488	4.8488	4.8488	4.7859	4.7859	4.7859
		100	4.9095	4.9095	4.9095	4.8488	4.8488	4.8488	4.7859	4.7859	4.7859

3.5 Conclusions

In this chapter, the dispersion relations of wave propagation in power-law-based functionally graded plate with porosities and resting on a three-parameter viscoelastic foundation are investigated using a simple integral four-variable hyperbolic shear deformation theory. The exponential Navier type solution is used to formulate the symbolic equations. The principal frequencies of wave propagation are calculated by solving the eigenvalue problem. This is formulated based on the present theory by considering the foundation's time-dependent effect and the FG plate's porosity. The dispersion relations between the wave propagation components are presented. The influences of gradation power, porosity parameters, and the viscoelastic foundation parameters on the various dispersion relations are examined. The dimensional results for the frequencies and phase velocities are tabulated.

The obtained results indicate that the porosities and the foundation's viscosity significantly affect the FG plates' wave propagation. The porosity content effect depends on the material exponent as the porosities slow down the phase velocity of the high-material-exponent plates. The type-3 porosity distribution factor reports a more sensitive phase velocity change, regardless of the power index value, while the other two types have similar behavior. The foundation's damping coefficient and the wavenumber do not influence the wave propagation phase velocity of thicker plates. Hence, it is recommended to consider the effect of the surrounding medium's viscosity in the wave propagation analysis of thin FGM plates.

CHAPTER 4

WAVE PROPAGATION ANALYSIS OF CERAMIC-METAL FUNCTIONALLY GRADED SANDWICH PLATES WITH DIFFERENT POROSITY DISTRIBUTIONS IN A HYGRO-THERMAL ENVIRONMENT

Wave propagation analysis of porous functionally graded (FG) sandwich plate in a hygro-thermal environment is presented in this chapter. The sandwich plates' composing materials change through three layers that are either homogeneous ceramic, homogeneous metal, or power-law-based functionally graded ceramic-metal. Six different porosity models are considered in the analysis to express the porosities' distribution factor and uniformity. The study is conducted using a simple four-unknown integral higher-order shear deformation theory (HSDT). The effect of moisture and temperature on wave propagation in porous FG sandwich plates is investigated by considering their role on the materials' expansion. The governing equations are derived for the wave propagation problem based on the presented theory via Hamilton's principle. A generalized solution for wave propagation is applied to formulate the

stiffness and mass matrix that describes the dispersion relations. The numerical results are obtained by solving an eigenvalue problem. The effects of core-to-thickness ratio, FGM power index, porosity volume fraction, temperature, and moisture change are illustrated and discussed. The presented results can be utilized as a benchmark for further studies on wave propagation in FGM plates.

4.1 Introduction

Composite materials are developed to provide high-level performance and reliability. The conventional laminates' main disadvantage is the weak borders between layers in which the stress concentration causes delamination and failure. Functionally graded (FG) plates have their properties changing continuously through the thickness without any interfaces between layers. Due to their high-level performance, functionally graded materials (FGM) are involved in many applications from different fields. For instance, the FGM is used in the power generation systems' emitters to reduce the systems' thermal stresses and the emitters' tendency to crack [82]. In many applications, these advanced materials are subjected to environments of high temperature and high moisture concentration. Several plate theories that vary in terms of complexity and reliability are being developed to study the functionally graded plates.

Many studies are conducted on the analysis of FG plates, beams, shells, and nanostructures. Various higher-order shear deformation theories (HSDT) were developed to provide more helpful analysis methods with less computational cost. Demirhan and Taskin [83] analyzed the free vibration and bending behaviors of FG plates with porosities and simple supports on two opposite edges using a four-variable HSDT. Yuda

and Xiaoguang [84] presented a nonlinear solution for the parametric vibration and stability of a simply supported FG rectangular plate. They applied the Galerkin method to derive the nonlinear vibration equation and studied the steady-state principal resonance. Ansari et al. [85] investigated the free vibration and flexural behaviors of CNT-reinforced FG plate using finite element method of nine-node 2-D elements and third-order Taylor's series expansion. The vibration and flexural characteristics of an FG plate resting on a two-parameter foundation were studied using the Lagrangian quadrilateral nine-node finite elements with eight degrees of freedom per node formulated based on HSDT that includes the normal transverse deformation [86]. A dynamic stiffness method was developed to analyze the free vibration of sigmoid FG Levy-type plates and show the effect of the FGM properties on natural frequencies [87].

Singh and Harsha [88] studied the nonlinear bending behavior of FG plates using the classical plate theory with Von-Karman strains analytically. Karami, Janghorban, and Rabczuk [89] investigated the static response of simply-supported hexagonal beryllium crystals FG nanoplates using a five-variable HSDT with the thickness stretching effect and nonlocal strain gradient. Tati [90] modified the FSDT to develop a higher-order shear deformation element to study FG plates under uniform and sinusoidal loadings, assuming a quadratic distribution of transverse shear strain. Karami et al. [91] presented a mixed form 3-D solution for the static analysis of FG Levy-type plate using the mixed-field multiterm (MM) extended Kantorovich method. They utilized the modified power series method to produce the results.

Wave propagation in FGM structures is studied in many researches for different loading conditions using various higher-order plate theories. The wave propagation in FG doubly-

curved shells was studied using an exponential-trigonometric HSDT considering thin and thick shells in addition to anisotropic constituent materials [92]. Gao, Qin, and Chu [75] examined the wave propagation in graphene platelets reinforced functionally graded foam plates. The properties are based on the Halpin–Tsai model, using several shear deformation plate theories. Karami, Shahsavari, and Janghorban [93] analyzed the wave propagation in FG nanoplates analytically using a refined four-variable plate theory. They presented the size effect based on a two-parameter nonlocal strain gradient theory in addition to the magnetic field's influence on the plate. Zhang, Yu, and Zhang [94] found the dispersion curves of guided Lamb and SH waves propagation in hexagonal quasi-crystal FG plates considering the effects of phonon-phason coupling. The wave propagation in FG cylindrical shell with an internal fixed cylinder filled with a compressible fluid and subjected to impulsive load was studied using the 3-D theory of elasticity [95].

Shahsavari, Talebitooti, and Kornokar [96] investigated the propagation of acoustic waves through cylindrical shells made of FGM with porosity and showed that increasing the porosities helps improve the sound insulation characteristics of the structure. The porosity effect on the propagation of guided waves was also studied for FG nanoplates using the first-order shear deformation theory [97]. She et al. [98] used Reddy's higher-order shear deformation beam theory and the nonlocal strain gradient theory to examine the wave propagation characteristics for porous FG nanobeams.

Researchers also investigated the response of functionally graded materials in moisture and thermal environments. She, Ren, and Yuan [99] analyzed the wave propagation in nanotubes of two layers using the nonlocal strain gradient theory and showed the

influences of porosities and hygro-thermal changes. The influence of temperature change on wave propagation was also investigated for FG beams with simple supports resting on an elastic foundation [100]. Jouneghani et al. [101] studied the hygro-thermoelastic response of FGM nanobeams with internal porosities. In addition, Dastjerdi et al. [102] investigated the bending behavior of moderately thick porous FGM plates in the hygro-thermal environment using a quasi 3D plate theory. The studies considering the effects of the hygro-thermal environment on FGM plates are few, especially for wave propagation problems.

Based on the literature review, no research work is conducted on the wave propagation in porous FG sandwich plates and subjected to a hygro-thermal environment. Therefore, this work investigates the wave propagation in infinite porous FG sandwich plates in a hygro-thermal environment. The analysis is conducted using a simple integral hyperbolic HSDT with only four unknowns. The transverse shear strains are described by undetermined integrals and a hyperbolic shape function that satisfy the boundary conditions without any correction factors. The four governing differential equations are obtained by Hamilton's principle using the presented plate theory. Six different porosity distribution models are described and analyzed in this chapter. A power-law-based functionally graded plate made of alumina and aluminum is used to present the numerical results of frequencies and phase velocities. Two sandwich configurations are considered: a sandwich plate with a homogeneous ceramic core layer and a sandwich plate with an FG core layer. Influences of the porosity, temperature, moisture, material gradation, and sandwich layers thickness on the wave propagation in the sandwich FG plates are examined.

4.2 Effective Properties of the Porous FGM

Figure 20 shows the configurations of the FG sandwich plates with porosities. The effective material properties depend on z -coordinate and are given by

$$Pr(z) = Pr_m + (Pr_c - Pr_m)V_c^{(n)} - \xi(Pr_c + Pr_m)(1 - \varsigma) \delta^{(n)} \quad (62)$$

where $Pr(z)$ is the effective property of the composite FGM as a function of z , Pr_m and Pr_c are the properties of pure ceramic and pure metal, respectively, $V_c^{(n)}$ is the ceramic volume fraction in layer n , ξ is the porosity distribution factor, $\delta^{(n)}$ determines whether the layer n is porous or not, and ς determines whether the porosity is evenly or unevenly distributed.

$$\delta^{(n)} \equiv n \bmod 2 \quad (63)$$

where $\bmod$ represents the modulo operator, and $\equiv$ is the congruence relation.

Six porosity distribution models (PM) are shown in Table 6 and studied in this chapter. These porosities are considered to be distributed in the functionally graded layers only, as illustrated in Figure 20. The ceramic volume fractions of the three layers are given here for different types of sandwich plates.

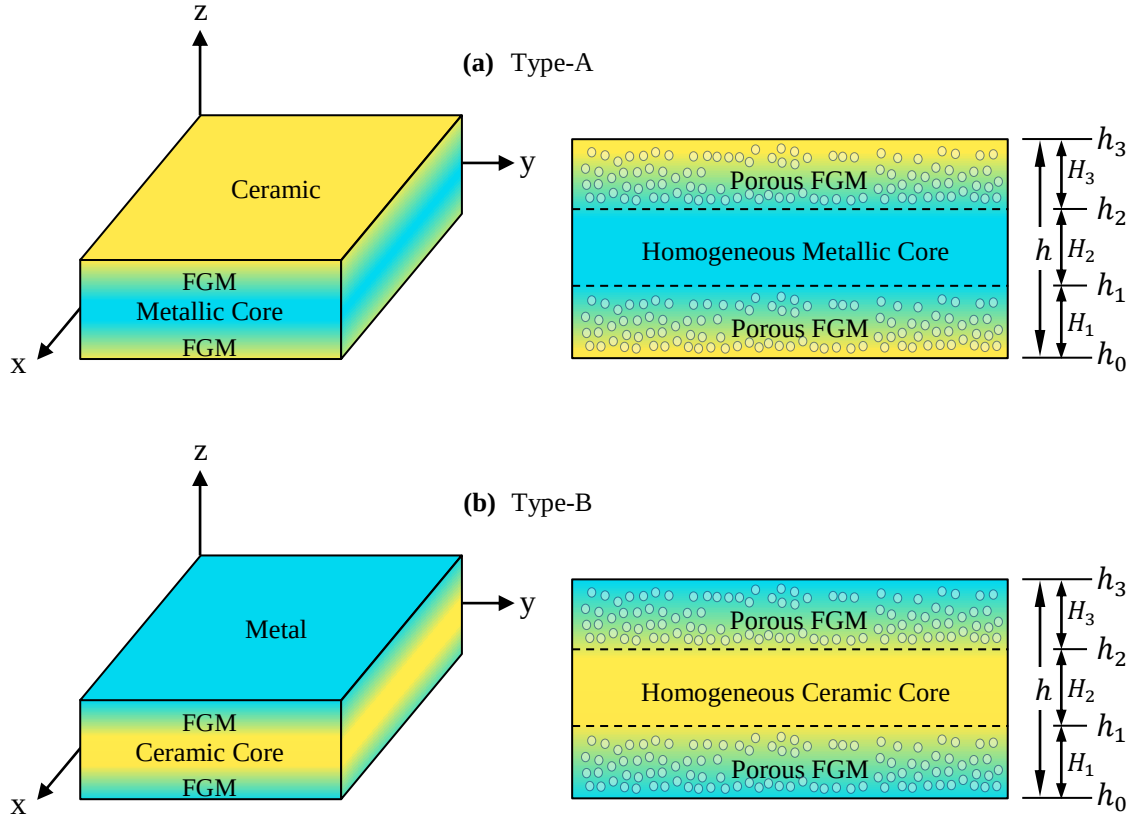


Figure 20 FGM sandwich plates configurations: (a) Type-A, (b) Type-B

Table 6 Porosity models (PM) of the functionally graded material

Distribution factor ξ	Distribution over z-direction	
	Even ($\zeta = 0$)	Uneven ($\zeta = 2 z /h$)
$1 - e^{-\lambda/2}$	PM1	PM4
$\ln(1 + \lambda/2)$	PM2	PM5
$\lambda/2$	PM3	PM6

Type-A: the plate is composed of a ceramic core sandwiched by two functionally graded layers.

$$V_c^{(1)} = \left(\frac{z - h_0}{h_1 - h_0} \right)^P$$

$$V_c^{(2)} = 1$$

(64)

$$V_c^{(3)} = \left(\frac{z - h_3}{h_2 - h_3} \right)^P$$

Type-B: the plate is composed of a ceramic core sandwiched by two functionally graded layers.

$$V_c^{(1)} = \left(\frac{z - h_0}{h_1 - h_0} \right)^P$$

$$V_c^{(2)} = 1$$

(65)

$$V_c^{(3)} = \left(\frac{z - h_3}{h_2 - h_3} \right)^P$$

where h_0, h_1, h_2 , and h_3 are the z -coordinates of layers boundaries as illustrated in Figure 21, and P is the power-law exponent of the material gradation function.

4.3 Mathematical Formulation

4.3.1 Kinematics

An integral higher-order shear deformation theory with four unknowns, i.e. the mid-plane displacements u_0, v_0, w_0 and θ , is implemented to express the displacement field by

$$\begin{aligned}
u &= u_0(x, y, t) - z \frac{\partial w}{\partial x} + k_1 f(z) \int \theta(x, y, t) dx \\
v &= v_0(x, y, t) - z \frac{\partial w}{\partial y} + k_2 f(z) \int \theta(x, y, t) dy
\end{aligned} \tag{66}$$

$$w = w_0(x, y, t)$$

where: u, v , and w represent the displacements of the plate toward the x, y , and z directions, respectively, at any location in the plate.

The transverse shear stresses and strains distributions through the thickness direction are described by the shape function $f(z)$, which is given by

$$f(z) = z - \frac{4 e^{1/4} z \sinh\left(\frac{z^2}{h^2}\right)}{-1 + 3\sqrt{e}} \tag{67}$$

The normal strain on z -direction is neglected. The other strain components are given by

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} - z \begin{Bmatrix} \varepsilon_x^b \\ \varepsilon_y^b \\ \gamma_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \gamma_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} \tag{68}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^b \\ \varepsilon_y^b \\ \gamma_{xy}^b \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{69}$$

$$\begin{Bmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \gamma_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \int \frac{\theta}{\partial y} dx + k_2 \int \frac{\theta}{\partial x} dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \int \theta dx \\ k_2 \int \theta dy \end{Bmatrix}$$

And

$$\begin{aligned} \int \theta dx &= A' \frac{\partial \theta}{\partial x}, \quad \int \theta dy = B' \frac{\partial \theta}{\partial y}, \\ \frac{\partial}{\partial y} \int \theta dx &= A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x} \int \theta dy = B' \frac{\partial^2 \theta}{\partial x \partial y} \end{aligned} \tag{70}$$

These relations are true because of the applied solution type. Therefore, the shear strains parts related to transverse shear deformation are simplified to

$$\begin{Bmatrix} \gamma_{xy}^s \\ \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} A' k_1 \frac{\partial^2 \theta}{\partial x \partial y} + B' k_2 \frac{\partial^2 \theta}{\partial x \partial y} \\ A' k_1 \frac{\partial \theta}{\partial x} \\ B' k_2 \frac{\partial \theta}{\partial y} \end{Bmatrix} \tag{71}$$

where, A' , B' , k_1 and k_2 are related to the wavenumbers of wave propagation κ_1 and κ_2 by

$$A' = -\frac{1}{\kappa_1^2}, \quad B' = -\frac{1}{\kappa_2^2}, \quad k_1 = \kappa_1^2, \quad k_2 = \kappa_2^2 \tag{72}$$

The five stresses components are related to the strains by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{Q_{55}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - T(z) - C(z) \\ \varepsilon_y - T(z) - C(z) \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (73)$$

The elastic stiffness coefficients Q_{ij} are given for the FGM plate by

$$Q_{11} = Q_{22} = \frac{E(z)}{1 - \nu^2}, Q_{12} = \frac{\nu E(z)}{1 - \nu^2} \quad (74)$$

$$Q_{44} = C_{Q_{55}} = Q_{66} = \frac{E(z)}{2\nu + 2}$$

4.3.2 Equations of Motion

The governing equations are obtained by implementing Hamilton's principle.

$$\int_0^t (\delta U + \delta V - \delta K) dt = 0 \quad (75)$$

U, V , and K are the strain energy, the work, and the kinetic energy, respectively, and δ is the first variation operator. $\delta U, \delta V$, and δK are expressed in terms of the four unknowns of the theory and their first variations as follows.

$$\begin{aligned}
\delta U = \int_S & (N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + N_0 \nabla^2 w \delta w_0 + M_x^b \delta \varepsilon_x^b + M_y^b \delta \varepsilon_y^b \\
& + M_{xy}^b \delta \varepsilon_{xy}^b + M_x^s \delta \varepsilon_x^s + M_y^s \delta \varepsilon_y^s + M_{xy}^s \delta \varepsilon_{xy}^s + S_{xz} \delta \gamma_{xz}^0 \\
& + S_{yz} \delta \gamma_{yz}^s) dS
\end{aligned} \tag{76}$$

where S upper surface of the FG sandwich plate, and N_i, M_i^j and S_i are the stresses resultant forces and moments that are evaluated by

$$\begin{bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{bmatrix} = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} \cdot (\sigma_x, \sigma_y, \tau_{xy}) dz \tag{77}$$

And

$$\begin{Bmatrix} S_{xz} \\ S_{yz} \end{Bmatrix} = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} g(z) \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{78}$$

Using the constitutive relations to put the stresses in terms of strains, the resultants are given by

$$\begin{Bmatrix} N_x \\ N_y \\ M_x^b \\ M_y^b \\ M_x^s \\ M_y^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{11}^s & B_{12}^s \\ A_{12} & A_{22} & B_{12} & B_{22} & B_{12}^s & B_{22}^s \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{11}^s & D_{12}^s \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{12}^s & D_{22}^s \\ B_{11}^s & B_{12}^s & D_{11}^s & D_{12}^s & H_{11}^s & H_{12}^s \\ B_{12}^s & B_{22}^s & D_{12}^s & D_{22}^s & H_{12}^s & H_{22}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_x^b \\ \varepsilon_y^b \\ \varepsilon_x^s \\ \varepsilon_y^s \end{Bmatrix} \tag{79}$$

$$\begin{Bmatrix} N_{xy} \\ M_{xy}^b \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{66} & B_{66} & B_{66}^s \\ B_{66} & D_{66} & D_{66}^s \\ B_{66}^s & D_{66}^s & H_{66}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xy}^0 \\ \gamma_{xy}^b \\ \gamma_{xy}^s \end{Bmatrix}$$

$$\begin{Bmatrix} S_{xz} \\ S_{yz} \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix}$$

Where

$$\begin{bmatrix} A_{11} & B_{11} & B_{11}^s & D_{11} & D_{11}^s & H_{11}^s \\ A_{22} & B_{22} & B_{22}^s & D_{22} & D_{22}^s & H_{22}^s \\ A_{12} & B_{12} & B_{12}^s & D_{12} & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & B_{66}^s & D_{66} & D_{66}^s & H_{66}^s \end{bmatrix} \quad (80)$$

$$= \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} (1, z, f, z^2, zf, f^2) \begin{Bmatrix} C_{11} \\ C_{22} \\ C_{12} \\ C_{66} \end{Bmatrix} dz$$

And

$$\begin{Bmatrix} A_{44}^s \\ A_{55}^s \end{Bmatrix} = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} g(z)^2 \begin{Bmatrix} C_{44} \\ C_{55} \end{Bmatrix} dz \quad (81)$$

The pure axial force resulted from the change in temperature and moisture is given for both directions by

$$N_0 = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} (c_{11}^{(n)} + c_{12}^{(n)}) [\alpha^{(n)}(z) T(z) + \beta^{(n)}(z) C(z)] dz \quad (82)$$

where $\alpha^{(n)}(z)$ and $\beta^{(n)}(z)$ are the effective thermal and moisture expansion coefficients of the FGM sandwich plate at layer n and distance z from the mid-plane. $T(z)$ and $C(z)$ are the change in temperature and moisture in the plate, respectively, which is assumed to follow the following power law

$$T(z) = \Delta T \left(\frac{2z + h}{2h} \right)^r + T_b - T_0 \quad (83)$$

$$C(z) = \Delta C \left(\frac{2z + h}{2h} \right)^r + C_b - C_0$$

where T_b and C_b are the changes in temperature and moisture of the bottom surface, while ΔT and ΔC are the differences in temperature and moisture changes between the top and bottom surfaces. The exponent r is a non-negative value that determines the curvature of the temperature and moisture changes over the thickness.

$$T_b - T_0 = 25 \text{ }^\circ\text{C} \quad (84)$$

where T_0 and C_0 represent the reference temperature and the reference moisture concentration.

The external work is zero since the FGM plate is not subjected to any external forces.

δK is given by

$$\begin{aligned}
\delta K = \int_A & \left(I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + I_2 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \right) \delta u_0 \\
& + \left(I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + I_2 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \right) \delta v_0 \\
& + \left(-I_1 \frac{\partial \ddot{u}_0}{\partial x} + I_3 \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_1 \frac{\partial \ddot{v}_0}{\partial y} + I_3 \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right. \\
& \left. - I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} + I_0 \ddot{w}_0 \right) \delta w_0 \\
& + \left(I_2 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} - I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \right. \\
& \left. + I_2 k_2 B' \frac{\partial \ddot{v}_0}{\partial y} - I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} + I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \delta \theta \, dA
\end{aligned} \tag{85}$$

Where the mass inertias introduced in the equation are defined as

$$(I_0, I_1, I_2, I_3, I_4, I_5) = \sum_{n=1}^3 \int_{h_{n-1}}^{h_n} (1, z, f, z^2, zf, f^2) \rho(z) \, dz \tag{86}$$

The equations of motion are obtained by substituting δU and δK in Hamilton's principle and applying the fundamental lemma of the calculus of variations.

$$\begin{aligned}
\delta u_0 : \quad & \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -I_0 \ddot{u}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial x} - I_2 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \\
\delta v_0 : \quad & \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = -I_0 \ddot{v}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial y} - I_2 k_2 B' \frac{\partial \ddot{\theta}}{\partial y}
\end{aligned} \tag{87}$$

$$\begin{aligned}
\delta w_0 : \quad & \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_0 \nabla^2 w_0 \\
& = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_1 \frac{\partial \ddot{v}_0}{\partial y} - I_3 \frac{\partial^2 \ddot{w}_0}{\partial y^2} \\
& \quad + I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} - I_0 \ddot{w}_0 \\
\delta \theta : \quad & -A' k_1 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - B' k_2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - k_1 M_x^s - k_2 M_y^s + A' k_1 \frac{\partial S_{xz}}{\partial x} \\
& \quad + B' k_2 \frac{\partial S_{yz}}{\partial y} \\
& = -I_2 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \\
& \quad - I_2 k_2 B' \frac{\partial \ddot{v}_0}{\partial y} + I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} - I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}
\end{aligned}$$

4.3.3 Dispersion Relations

The solution representing the wave propagation in the infinite functionally graded plate is assumed to have the following form

$$\begin{Bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \\ \theta(x, y, t) \end{Bmatrix} = \begin{Bmatrix} U \\ V \\ W \\ \Theta \end{Bmatrix} \exp[i(\kappa_1 x + \kappa_2 y - \omega t)] \quad (88)$$

U, V, W and Θ are the amplitudes of the waves, ω is the circular frequency, and i is the imaginary number. Using this solution, the four governing equations are rewritten in the following matrix form

$$([K] - \omega^2[M]) \{\Lambda\} = 0 \quad (89)$$

Where the stiffness matrix [K] and the mass matrix [M] are expressed as

$$[K] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}, [M] = \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}, \quad (90)$$

$$\{\Lambda\} = \begin{Bmatrix} U \\ V \\ W \\ X \end{Bmatrix}$$

Where

$$k_{11} = A_{11}\kappa_1^2 + A_{66}\kappa_2^2$$

$$k_{12} = \kappa_1\kappa_2(A_{12} + A_{66})$$

$$k_{13} = -i\kappa_1(B_{11}\kappa_1^2 + \kappa_2^2(B_{12} + 2B_{66}))$$

$$k_{14} = -i\kappa_1(k_1B_{11}^s + k_2B_{12}^s - (A'k_1 + B'k_2)B_{66}^s\kappa_2^2)$$

$$k_{22} = A_{22}\kappa_2^2 + A_{66}\kappa_1^2$$

$$k_{23} = -i\kappa_2(B_{22}\kappa_2^2 + \kappa_1^2(B_{12} + 2B_{66}))$$

$$k_{24} = -i\kappa_2(k_2B_{22}^s + k_1B_{12}^s - (A'k_1 + B'k_2)B_{66}^s\kappa_1^2)$$

$$k_{33} = -D_{11}\kappa_1^4 - D_{22}\kappa_2^4 - 2(D_{12} + 2D_{66})\kappa_1^2\kappa_2^2 + N_0\nabla^2 w_0$$

$$k_{34} = -(k_1D_{11}^s + k_2D_{12}^s)\kappa_1^2 - (k_1D_{12}^s + k_2D_{22}^s)\kappa_2^2 + 2(A'k_1D_{66}^s + B'k_2D_{66}^s)\kappa_1^2\kappa_2^2$$

$$k_{44} = -k_1^2 H_{11}^s - 2k_1 k_2 H_{12}^s - k_2^2 H_{22}^s - (A'^2 k_1^2 + 2A'B'k_1 k_2 + B'^2 k_2^2) H_{66}^s \kappa_1^2 \kappa_2^2 \\ + A'k_1^2 A_{55}^s + B'k_2^2 A_{44}^s$$

and

$$m_{11} = m_{22} = I_0$$

$$m_{13} = -iI_1 \kappa_1$$

$$m_{14} = iA'k_1 I_2 \kappa_1$$

$$m_{23} = -iI_1 \kappa_2$$

$$m_{24} = iB'k_2 I_3 \kappa_2$$

$$m_{33} = -I_0 - I_3(\kappa_1^2 + \kappa_2^2)$$

$$m_{34} = I_4(A'k_1 \kappa_1^2 + B'k_2 \kappa_2^2)$$

$$m_{44} = -I_5((A'k_1 \kappa_1)^2 + (B'k_2 \kappa_2)^2)$$

The dispersion relations are obtained by finding the eigenvalues.

$$|[K] - \omega^2[M]| = 0 \quad (91)$$

Four frequencies are found as functions of the wavenumbers

$$\omega_i = W_i(\kappa), \quad (i = 1, 2, 3, 4) \quad (92)$$

ω_1 and ω_4 represent the frequencies of flexural modes M_1 and M_4 . ω_2 and ω_3 represent the frequencies of extensional modes M_2 and M_3 . The phase velocity is simply the frequency divided by the corresponding wavenumber.

$$C_i = \frac{W_i(\kappa)}{\kappa}, \quad (i = 1, 2, 3, 4) \quad (93)$$

4.4 Numerical Results and Discussion

The analytical solution of wave propagation in porous FGM sandwich plate is presented considering the hygro-thermal environment effect. Next, the numerical results are given for the three types of FGM sandwich plates composed of ceramic (alumina) and metal (aluminum) whose properties are retrieved from [103] and listed in Table 7. The frequencies are obtained by solving the eigenvalue problem and used to find the corresponding phase velocities.

Table 7 The properties of the ceramic and metal [103]

Materials	E (GPa)	ρ (kg/m ³)	α (1/°C)	β (wt% H_2O) ⁻¹	ν
Alumina	380	3800	7×10^{-6}	0.001	0.3
Aluminum	70	2707	23×10^{-6}	0.44	0.3

Many examples are solved to investigate the effects of core-to-thickness ratio, FG material power index, porosity volume fraction, temperature change, and moisture change on various dispersion relations.

Figure 21 shows the variations in the principal wave frequencies for the wavenumbers of porous FGM Sandwich plates in a hygro-thermal environment for different core-to-thickness ratios. The increase of the metallic core thickness generates more flexible plates with lower principal frequency, while increasing the ceramic core generates stiffer plates with higher principal frequency. The effect of changing the core-to-thickness ratio on the principal wave frequency of a porous FGM sandwich plate is more significant at high wavenumbers.

The relations between the phase velocities and the material power indices are shown in Figure 22 for different core thicknesses. In all cases, the phase velocity is reduced by increasing the value of P . The figure also shows that the effect of the core-to-thickness ratio on phase velocities in Type-A can be reversed when $P > 4$.

In Figure 23, the relations between phase velocity and porosity volume fraction are plotted. In Type-A, the phase velocity increases slightly by increasing the evenly distributed porosities while decreasing unevenly distributed porosities. This is because more porosity volume is considered in the ceramic rich side of the FGM layers. In Type-B, the plate becomes more flexible, and the phase velocity decreases with increasing the porosity volume fraction in all studied models.

Figures 24 and 25 show the influence of temperature and moisture concentration changes on the relation between the phase velocity and porous FGM Sandwich plates the core-to-thickness ratio. It can be seen that the phase velocity is reduced due to the hygro-thermal effects. The change in moisture concentrations has a significant effect that increases when the metallic content is higher. However, the influence of temperature changes is

negligible for $\Delta T \leq 150^\circ\text{C}$. The influence of the hygro-thermal environments is noticed when the wavenumber is less than 400 m^{-1} .

Figure 26 shows that the change in temperature and moisture concentration reduces the porous FGM sandwich plate's stiffness, and hence, the phase velocity is reduced.

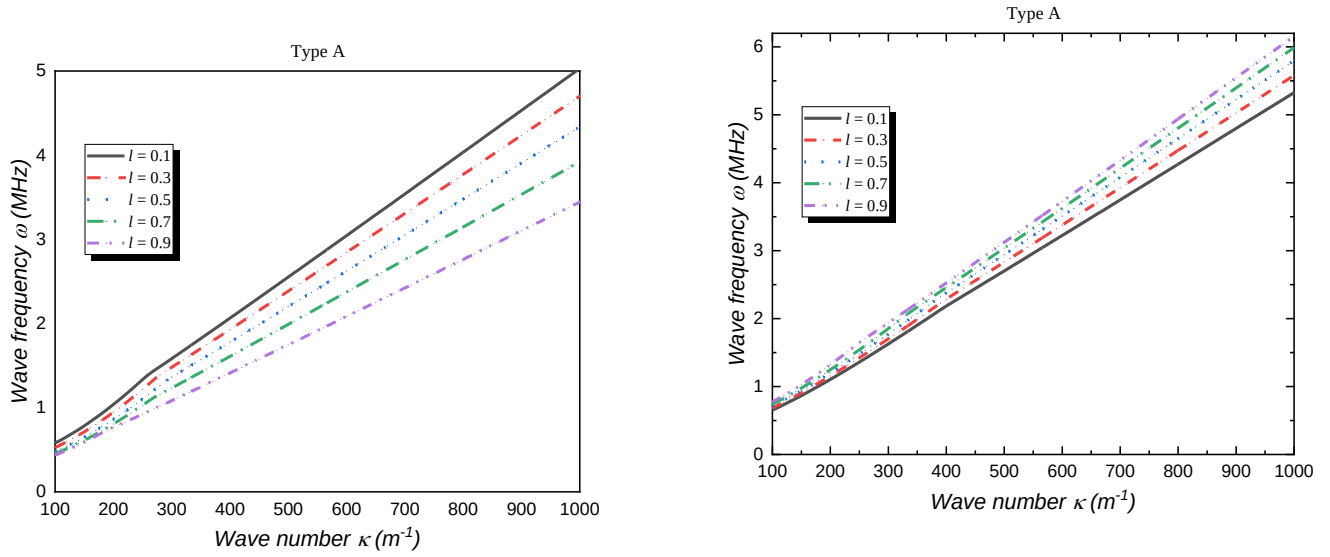


Figure 21 The influence of core-to-thickness ratio on the wave frequency and the wavenumber dispersion relation in porous FG plate in the hygro-thermal environment with $h = 0.05 \text{ m}$, $\lambda = 0.1 \text{ PM1}$, $P = 1$, $\kappa_2 = 100 \text{ m}^{-1}$, $H_1 = H_3$, $\Delta C = 10\%$, $\Delta T = 50^\circ\text{C}$: (a) type-A, (b) type-B.

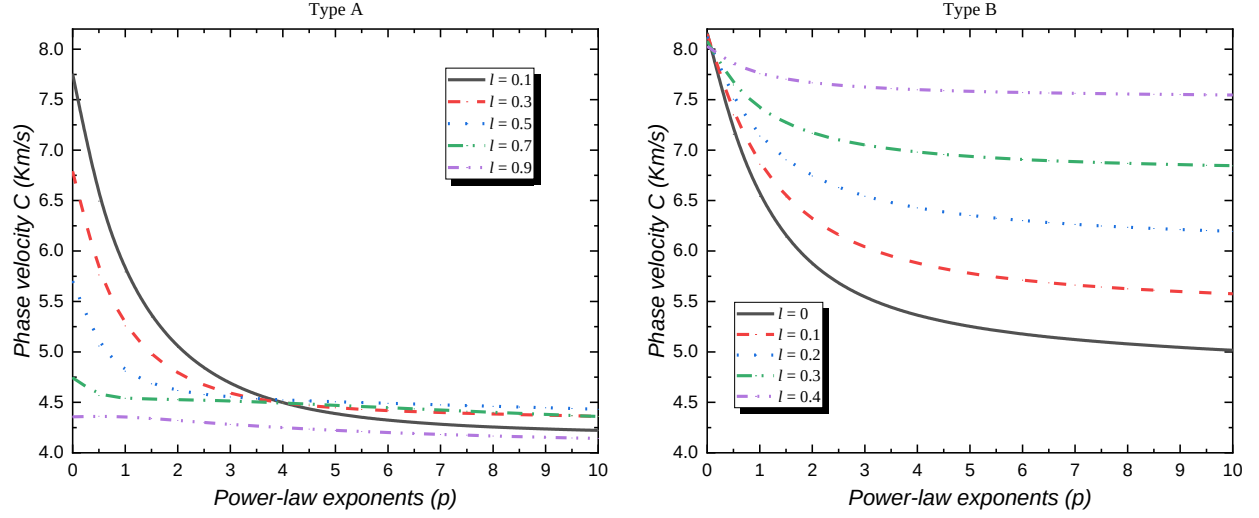


Figure 22 The influence of core-to-thickness ratio on the phase velocity and the power-law index relation for porous FG plate in the hygro-thermal environment with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $PM1$, $\kappa_1 = \kappa_2 = 100 \text{ m}^{-1}$, $H_1 = H_3$, $\Delta C = 10\%$, $\Delta T = 50^\circ\text{C}$: (a) type-A, (b) type-B.

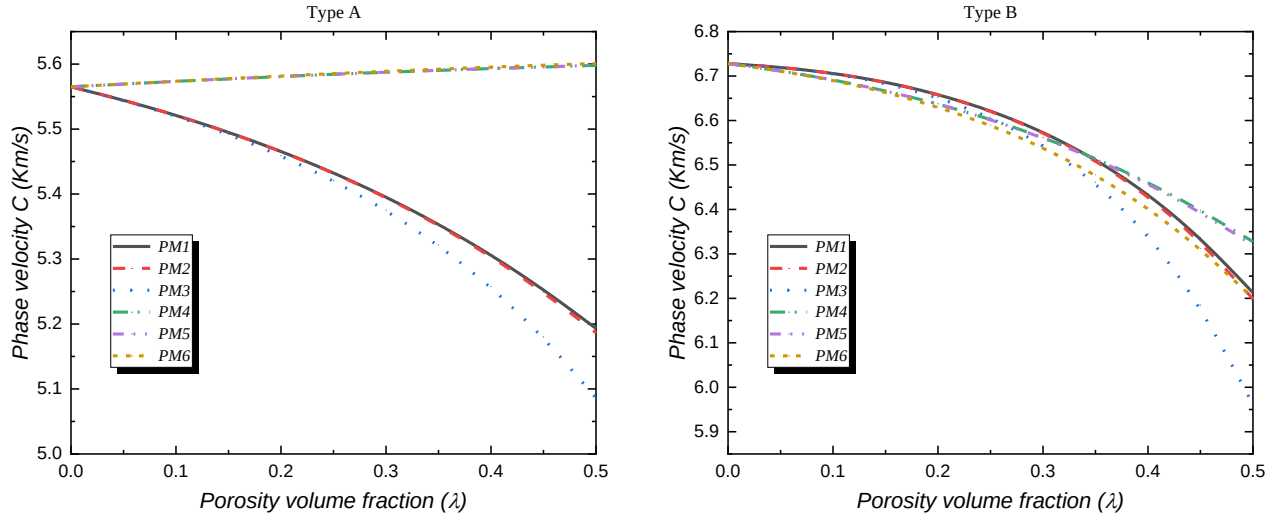


Figure 23 The relation between the phase velocity and the porosity volume fraction of FG plate in the hygro-thermal environment using various porosity models with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $PM1$, $\kappa_1 = \kappa_2 = 100 \text{ m}^{-1}$, $l = 0.2$, $\Delta C = 10\%$, $\Delta T = 50^\circ\text{C}$: (a) type-A, (b) type-B.

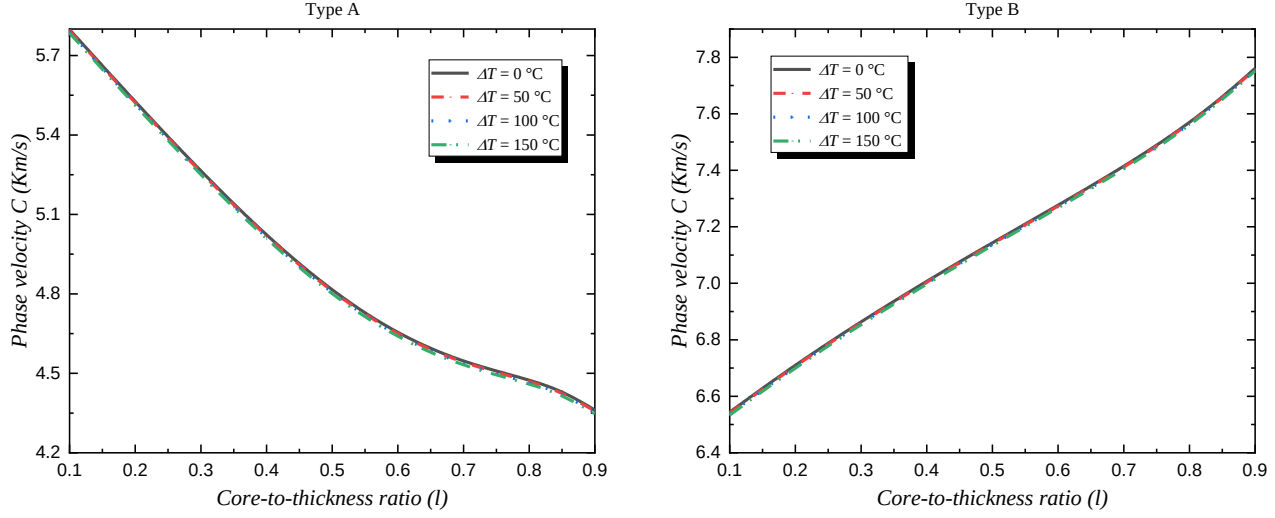


Figure 24 The influence of temperature change on the relation between the phase velocity and the core thickness of porous FG plate in the hygro-thermal environment with $h = 0.05\text{ m}$, $\lambda = 0.1$, $PM1$, $\kappa_1 = \kappa_2 = 100\text{ m}^{-1}$, $P = 1$, $\Delta C = 10\%$: (a) type-A, (b) type-B.

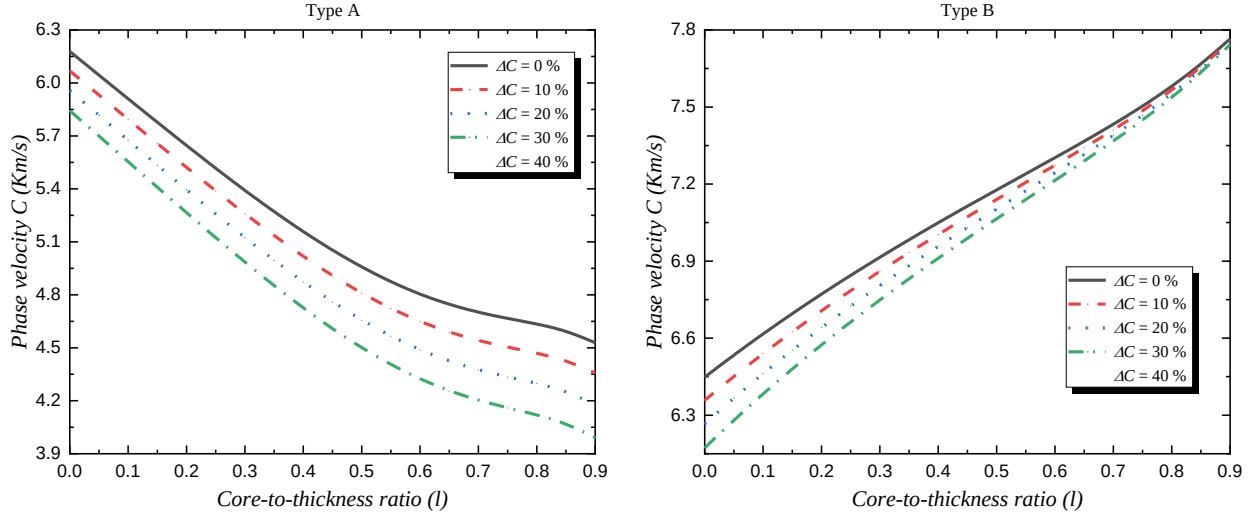


Figure 25 The influence of moisture change on the relation between the phase velocity and the core thickness of porous FG plate in the hygro-thermal environment with $h = 0.05\text{ m}$, $\lambda = 0.1$, $PM1$, $\kappa_1 = \kappa_2 = 100\text{ m}^{-1}$, $P = 1$, $\Delta T = 50^\circ\text{C}$: (a) type-A, (b) type-B.

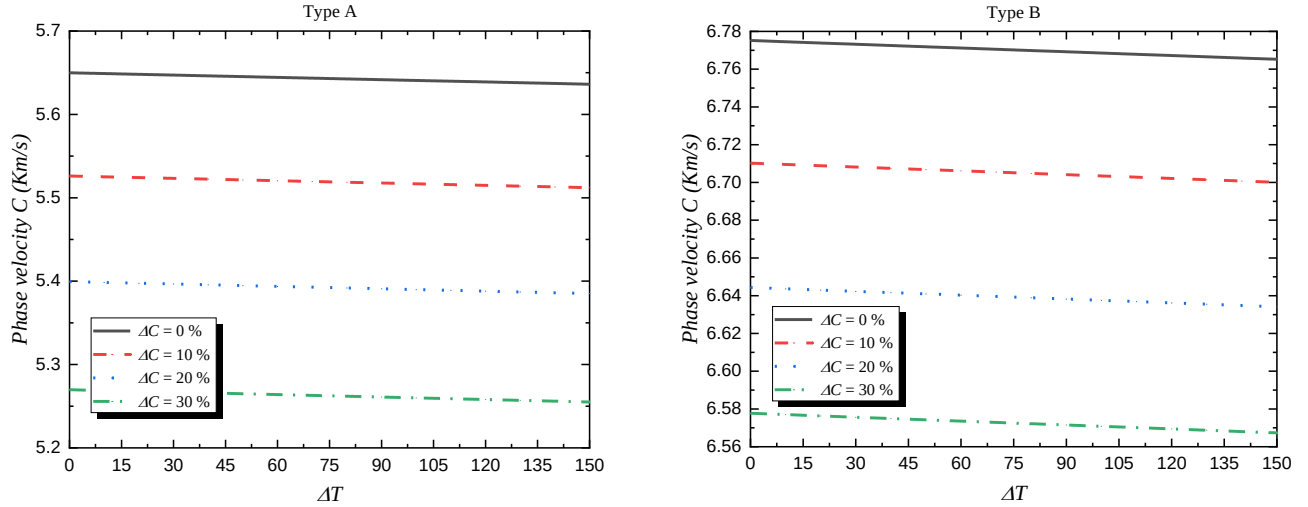


Figure 26 The Influences of the change in temperature ΔT and moisture concentration ΔC on the phase velocity of porous FGM sandwich plate with $h = 0.05 \text{ m}$, $\lambda = 0.1$, $PM1$, $\kappa_1 = \kappa_2 = 100 \text{ m}^{-1}$, $P = 1$: (a) type-A, (b) type-B

4.5 Conclusions

In conclusion, the analysis of wave propagation was conducted for porous FGM sandwich plate in the hygro-thermal environment using an integral four-variable HSDT. The study included three types of sandwich configurations and six porosity models. The temperature and moisture cause material expansions that induce additional stresses in the FG plate. The governing equations of the problem are presented and solved by formulating an eigenvalue problem using a Navier-type solution. The chapter examined and discussed the effects of core-to-thickness ratio, FGM power index, porosity volume fraction, temperature change, and moisture change. The hygro-thermal stresses have significant influences on the dispersion relations of porous FGM sandwich plates. Different sandwich types behave differently when subjected to such stresses. The effect of these stresses on the phase velocities mostly appears when the wavenumber is less than 400 m^{-1} . The change in moisture concentrations has a significant effect that

increases when the metallic content is higher. However, the influence of temperature changes is small.

CHAPTER 5

CONCLUSIONS

In conclusion, the dispersion relations of wave propagation were formulated analytically based on the presented simple hyperbolic higher-order shear deformation theory (hyperbolic HSDT) with undetermined integral variables. The theory applicability was verified by numerical comparison with previous works. The effects of gradation parameter, porosity parameter and porosity type on wave propagation in FG plates were examined and their effects on the dispersion relations were illustrated. Propagation phase velocities were plotted for plates without foundation and plates resting on Winkler, Winkler-Pasternak, visco-Winkler and visco-Pasternak foundations. Six different porosity models were investigated in the study. Additionally, the effect of hygro-thermal environment on the phase velocity is investigated.

5.1 Summary of the Approach

The solution started with the plate theory, which assumes the displacement field. The HSDT used included a hyperbolic shape function and undetermined integrals to express shear deformations.

Then linear strain is found based on the theory. In this step the undetermined integrals are converted to differential expressions. Stresses are then found using the constitutive relations and used to express the resultant forces.

After expressing the strains and stresses in terms of the plate theory unknowns, Hamilton's principle is applied to derive the equations of motion. Hamilton's principle is solved using the fundamental lemma of calculus of variation to derive the governing differential equations (GED).

A Navier type solution for wave propagation is implemented and the GED is expressed in a matrix form. Then these matrices are solved for numerical examples to find the various dispersion relations.

This solution approach can be divided into two parts:

1. Symbolic formulation
2. Numerical computations

The solution approach is summarized in Figures 27 and 28.

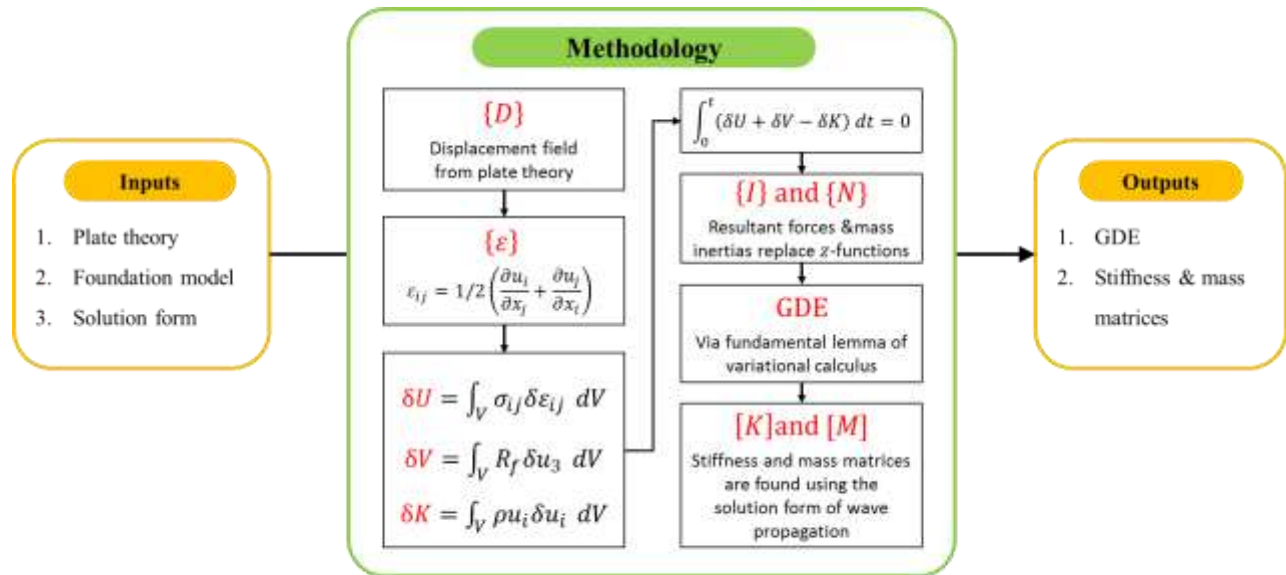


Figure 27 Summary of the solution approach: symbolic formulation

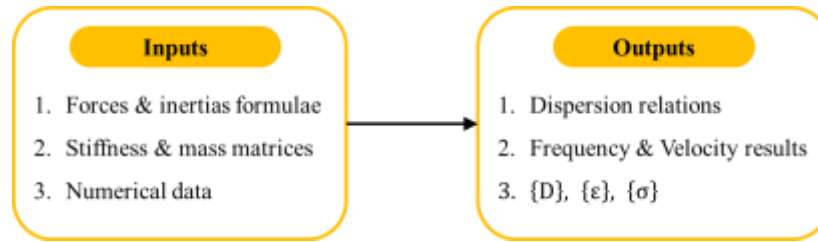


Figure 28 Summary of the solution approach: numerical computations

5.2 Summary of the Results

The following are the some notes on the effects of the studied parameters on wave propagation on ceramic-metal FGM plates and ceramic-metal FGM sandwich plates with various porosity distributions, foundations, and hygro-thermal environments.

- The influences of gradation on the various dispersion relations are examined to find that the relations are highly influenced by the material power index
- The influences of the viscoelastic foundation parameters are illustrated. Their influences appear in certain situations depending on the intensity of the foundation parameters.
- The porosity parameters have small effect on the dispersion relations but this effect changes significantly based on the material power index.
- The type-3 porosity distribution factor reports a more sensitive phase velocity change
- Type-1 and type-2 porosity distributions have a similar behavior.
- The foundation's damping coefficient and the wavenumber do not influence the wave propagation phase velocity of thicker plates.
- The hygro-thermal stresses have significant influences on the dispersion relations of porous FGM sandwich plates.

- Different sandwich types behave differently when subjected to such stresses.
- The effect of these stresses on the phase velocities mostly appears when the wavenumber is less than 400 m^{-1} .
- The influence of temperature changes is small.
- The change in moisture concentrations has a significant effect that increases when the metallic content is higher.

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