

**CONDITION-BASED MAINTENANCE (CBM) OPTIMIZATION
FOR MULTI-COMPONENT SYSTEM CONSIDERING
MULTIPLE DEPENDENCIES**

BY

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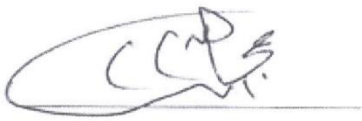
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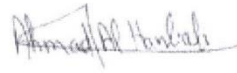
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Dedication

This thesis is dedicated to all my family members especially my father Dr. Hafiz Afzal , my mother, and my elder brother Dr Muhammad Akram , without their support, advice and encouragement it would not have been possible.

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ABSTRACT

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Thesis Title : Condition-Based Maintenance (CBM) Optimization for Multi-Component System Considering Multiple Dependencies

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A large amount of contribution towards the total production cost in the industry is associated with maintenance costs of systems. For single-unit systems, condition-based maintenance (CBM) strategy can be successfully implemented by introducing a deterioration threshold, upon reaching which, maintenance activity is performed on a system. However, for complex systems involving multiple (units) components, this strategy may not be straightforward, as several dependencies may exist between these components, and failing to consider them may not give an optimal solution. Despite the growing interest in CBM strategies for complex systems in both theoretical and practical domain, it is clear from the literature that no recent effort has been made by researcher to examine CBM policies for complex systems subject to various types of dependencies. In this research, it is aimed to provide an extensive literature review on CBM strategies for complex systems which incorporate all types of dependencies. The research also aims to investigate the current research gaps and to provide a strong link between theory and practice by empirically validating the different dependencies in different practical applications. The objective of this study is to investigate the effect of joint dependencies such as economic and structural dependencies with other key

dependencies between multiple components of system. In this regard, a detailed modelling approach based on discrete event simulations and analytical modelling will be used to evaluate the performance of CBM strategy and optimize them. Furthermore, the proposed model is extended by incorporating spare-part policy along with maintenance policy. The proposed model is optimized using OptQuest in Arena simulation and optimal preventive thresholds and time for performing the scheduled maintenance activities is determined in order to optimize the system by maximizing availability of systems and minimizing the total maintenance costs of the systems.

ملخص الرسالة

الاسم الكامل: عبدالرحمن حافظ محمد افضل

عنوان الرسالة: تحسين الصيانة حسب الحالة لنظام متعدد المكونات مع مراعاة التبعيات المتعددة

التخصص: هندسة النظم

تاريخ الدرجة العلمية: ابريل 2021 م

يرتبط قدر كبير من المساهمة في تكلفة الإنتاج الإجمالية في الصناعة بتكاليف صيانة الأنظمة. بالنسبة للأنظمة أحادية الوحدة ، يمكن تنفيذ استراتيجية الصيانة المستندة إلى الحالة (CBM) بنجاح من خلال إدخال حد تدهور ، عند الوصول إلى أي نشاط صيانة يتم تنفيذه على النظام. ومع ذلك ، بالنسبة للأنظمة المعقدة التي تشتمل على مكونات (وحدات) متعددة ، قد لا تكون هذه الإستراتيجية مباشرة ، حيث قد توجد العديد من التبعيات بين هذه المكونات ، وقد لا يؤدي عدم أخذها في الاعتبار إلى الحل الأمثل. على الرغم من الاهتمام المتزايد باستراتيجيات CBM للأنظمة المعقدة في كل من المجالين النظري والعملي ، فمن الواضح من الأدبيات أنه لم يبذل أي جهد مؤخرًا من قبل الباحث لفحص سياسات CBM للأنظمة المعقدة الخاضعة لأنواع مختلفة من التبعيات. يهدف هذا البحث إلى تقديم مراجعة شاملة للأدبيات حول استراتيجيات CBM للأنظمة المعقدة التي تتضمن جميع أنواع التبعيات. يهدف البحث أيضًا إلى استكشاف فجوات البحث الحالية وتوفير رابط قوي بين النظرية والتطبيق من خلال التحقق التجريبي من التبعيات المختلفة في التطبيقات العملية المختلفة. الهدف من هذه الدراسة هو التحقيق في تأثير التبعيات المشتركة مثل التبعيات الاقتصادية والهيكلية مع التبعيات الرئيسية الأخرى بين مكونات متعددة للنظام. في هذا الصدد ، سيتم استخدام نهج نمذجة مفصلة يعتمد على محاكاة الأحداث المنفصلة والنمذجة التحليلية لتقييم أداء استراتيجية تدابير بناء الثقة وتحسينها. علاوة على ذلك ، تم تمديد النموذج المقترح من خلال دمج سياسة قطع الغيار إلى جانب سياسة الصيانة. تم تحسين النموذج المقترح باستخدام OptQuest in Arena المحاكاة ويتم تحديد الحدود الوقائية المثلى والوقت لأداء أنشطة الصيانة المجدولة من أجل تحسين النظام من خلال زيادة توافر الأنظمة وتقليل إجمالي تكاليف الصيانة للأنظمة

Chapter 1: Introduction

1.1 Introduction

A large part of the total production cost in an industry can be associated with the maintenance costs of systems. In the power industry, around 30 percent of the total production cost consists of maintenance cost (Graber,2004). Whereas, in manufacturing industries, around 60 percent of the total production cost consists of maintenance cost and up to 30 percent of these costs are due to unnecessary maintenance or poor maintenance policies.

To come up with the best maintenance strategy for the systems, many studies have been published in the past. In general, maintenance activities can be divided into two main categories: corrective and preventive (Wang and Pham, 2006). Corrective Maintenance (CM) is the maintenance that is done when a complete system failure occurs. CM means performing actions as a result of failure, to restore a component to a specific condition. Preventive maintenance (PM) is the maintenance that is done when the system is still operating. PM means performing actions in order to restore a component required condition through systematic inspection, detection, and prevention of incipient failures. Pure corrective maintenance (CM) strategy results in high failure costs, whereas preventive maintenance strategy results in high (preventive) maintenance costs. Condition-Based Maintenance (CBM) strategy not only aims to balance and minimize these costs simultaneously but also increases availability, productivity, and safety of the

systems by minimizing the number of failures (Camci, 2009).

For single-unit systems, the CBM strategy can be successfully implemented by introducing a deterioration threshold, upon reaching which, maintenance activity is performed on the system. However, for complex systems involving multiple components, this strategy may not be straightforward, as several dependencies may exist between these components, and failing to consider them may not give close-to-optimal solution. Nowadays, more extant of research on CBM focuses on system with complex structures and various dependencies existing between the components. It is evident from the literature in the past, many authors have reviewed maintenance strategies for multi-component systems. A list of these studies can be seen in Table 1. Earlier, researchers such as Thomas (1986), Dekker et al. (1997), and Nicolai and Dekker (2008) introduced three types of dependencies, namely, structural, stochastic, and economic. Structural dependence refers to the scenario where the repair or replacement of a component requires some other components to be dismantled or replaced as well. On the other hand, stochastic dependence represents the situation where the deterioration process of one component is (partly statistically) dependent on other components. Finally, economic dependence refers to the situation where the combined and joint maintenance, including replacement and inspection actions, of several components, leads to a mostly lower cost than maintaining each component separately. Later on, Keizer et al. (2017) argued that the classification of the aforementioned three dependencies is no longer sufficient and that further types of dependencies need to be introduced. For that reason, these latter authors proposed an additional type of dependency called resource dependence. This dependency

represents the situation where the same maintenance resources such as workers, spare parts, and toolings are shared by all components, and the component maintenance is initiated only if a maintenance resource is available.

Despite the growing interest in CBM strategies for complex systems in both theoretical and practical domain, it is clear from the literature that no recent effort has been made by researcher to examine CBM policies for complex systems subject to various types of dependencies. Therefore, in this thesis, it is aimed to provide an extensive literature review on CBM policies for multi-component systems which incorporate the aforementioned dependencies. It also aims to investigate the current research gaps and to provide a strong link between theory and practice by empirically validating the different dependencies in different practical applications.

1.2 Problem Statement

This section presents the general problem statement of the research addressed in this thesis.

1.2.1 Maintenance policy for multiple-component system with various dependencies

Consider a CBM model for a multiple-component system whose deterioration rate follows Gamma process. The proposed CBM model is for a two-component system where the deterioration rate of each component depends on both its own deterioration rate and on the other component. Various dependencies such as stochastic, structural, and economic dependencies are considered among the components. Inspections are

required to reveal the system's state; however, measurement errors may occur due to imperfect inspections thus leading toward incorrect maintenance decisions. When the system requires maintenance, necessary actions are carried out. These actions may restore the system to as good as new or restore it to an imperfect state in-between its current state and the as-good-as-new state. Therefore, the proposed model takes into account both imperfect inspections and imperfect maintenance actions. Actions are either one of these: replace none, replace either component 1 or 2, or replace both. The maintenance actions are decided according to a CBM plan in which preventive and opportunistic maintenance rule is applied. The objective of the model is to obtain the best maintenance policy which minimizes the total operating and maintenance cost of the system.

1.2.2 Joint spare-part and Maintenance policy for multiple-component system with various dependencies

In this section, CBM and spare-part policy are incorporated jointly. The proposed joint spare-part and CBM model is for a two-component system where the deterioration rate of each component depends on both its own deterioration rate and of the other component. All the dependencies which are stochastic, structural, economic, and resource dependencies are considered among the components. Inspection occurs at regular time periods with inter-inspection intervals ΔT (decision parameter) to reveal the deterioration level of the component as well as spare-part inventory which follows (0,1) provisioning policy. To optimize the model, decision variables which are maintenance threshold, spare-part ordering threshold, and inspection times are

optimized such that the total operating and maintenance cost of the system is minimized.

1.3 Motivation for Research

The motivation behind pursuing the research in this topic is as follow :

- A large amount of contribution towards the total production cost in an industry are associated with maintenance costs of systems. Condition-based maintenance is one of the maintenance strategies that aims to balance and minimize the failure and preventive maintenance costs, and to increase availability, productivity, and safety of the systems by minimizing the number of failures.
- Most of the existing CBM policies do not consider complex systems such as multiple component systems or dependencies between them. However, systems nowadays are very complex and composed of multiple components. These components are not independent and may have many dependencies existing between them which can be mainly classified in four classes (discussed later)
- Despite the growing interest for CBM strategies for complex systems in both theoretical and practical aspect , very few papers in the literature exist on CBM strategies for multi-component systems subject to various types of dependencies.

- Most of the existing CBM studies that consider more than one type of dependencies usually center around two very common dependencies (positive economic dependency & structural dependency) and very little attention has been given to the combined effect of other types of dependencies.
- Therefore, an important area of research interest would be to consider joint effects of other major dependencies along with either economic or structural dependencies. Furthermore, the applications of these dependencies can empirically be validated in different applied settings including windfarm or gearbox systems.

1.4 Research Objectives

This research aims to achieve the following objectives :

- To investigate the current research gaps and to provide a strong link between theory and practice by empirically validating the different dependencies in different practical applications.
- To investigate the effect of joint dependencies such as economic and structural dependencies with other key dependencies (stochastic, resource) and validate the model in practical applications.
- To develop a condition-based maintenance policy that incorporates multi-component systems and considers various dependencies among them.

- To provide a detailed modelling approach based on an analytical model and validate it by modelling it on a discrete event simulation system to obtain best performance of condition-based maintenance strategy, and to identify optimal thresholds for maintenance to optimize the system by maximizing availability and minimizing total maintenance costs of the systems.

1.5 Contributions of this research

The main contributions of this research are as follow :

- It provides an extensive literature review on CBM policies for multi-component systems which incorporate various types of dependencies.
- A condition-based maintenance policy for the multi-component system is considered with various dependencies existing between them. The objective of the model is to obtain optimal maintenance and operating cost of the system by optimizing thresholds for preventive maintenance. The proposed model will also determine the optimal time for performing the scheduled maintenance activities on the system.
- A simulation model is developed for the proposed model to measure the accuracy of approximations made in the analytical model.
- To reflect more realistic situation, Inspection and repair are considered to be imperfect and their impact on the model is investigated.
- The simulation model is further extended by incorporating spare-part policy into

the existing model and considering all the dependencies between the components. The objective is to obtain an optimal joint spare-part & maintenance strategy.

- To validate the results obtained from Simulation model and to speed up the computation and facilitate the optimization of CBM, an analytical approach based on Markov chains is developed and modeled on Matlab

1.6 Thesis Organization

The remainder of the thesis is structured as follow. A brief literature review on maintenance policies with various dependencies is provided in section 2. Section 3 presents the proposed maintenance policy for the multi-component system with further extensions and incorporation of spare-part. A Numerical example is presented in section 4 along with sensitivity analysis to get various insights. An analytical model approach for the same maintenance system is proposed in section 5. The thesis is then concluded in section 6 along with implications and potential research directions outlined.

Chapter 2: Literature Review

This chapter provides brief literature on existing CBM policies considering various types of dependencies. To discuss the findings of this research with the relevant body of knowledge, this chapter first covers basic definitions in the field of maintenance and a brief introduction of condition-based maintenance.

2.1 Types of Maintenance

Maintenance can be performed through several actions, and it is classified into several types (Shin and Jun, 2015). As illustrated in figure 2, Maintenance is classified into two main policies, corrective and preventive maintenance.

This broad classification of maintenance is adopted in many literatures such as Duffuaa et al. (2001), in which CBM belongs to preventive maintenance policies. Whereas, several studies, such as Shin and Jun (2015), classify it into three types : preventive maintenance, corrective maintenance, and Condition-based maintenance.

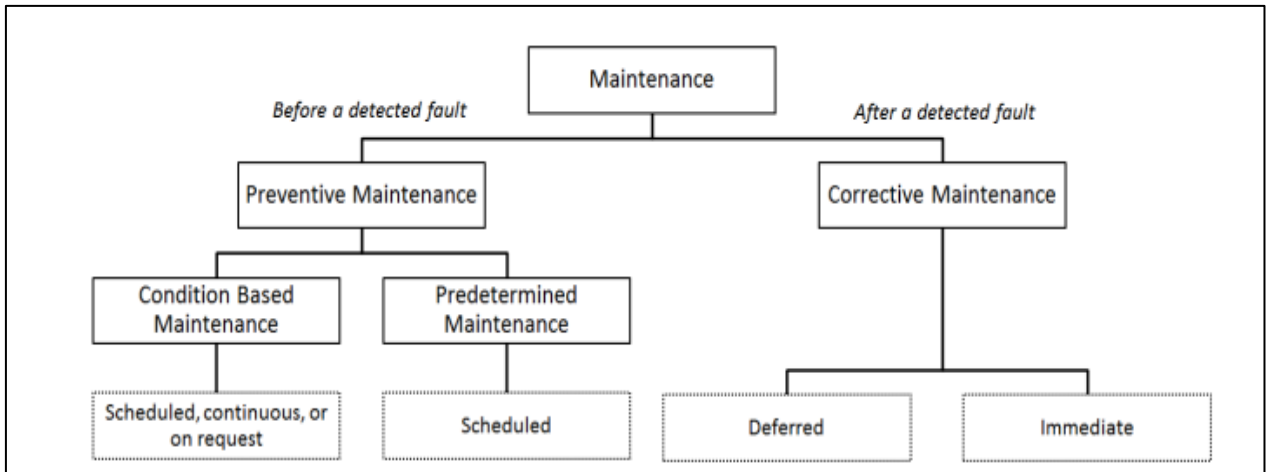


Figure 1 Classification of Maintenance

Corrective Maintenance (CM) is the maintenance that is done when complete failure of the system occurs. This maintenance means all actions performed as a result of failure, to restore an item to a specified condition. Preventive maintenance (PM) is the maintenance that occurs while the system is still operating. It means all actions performed in an attempt to retain an item in a specified condition by providing systematic inspection, detection, and prevention of incipient failures. CM policy, known as run-to-failure, is one of the maintenance strategies that results in high failure costs, whereas preventive maintenance strategy results in high maintenance costs. Condition-based maintenance (CBM) is a maintenance strategy that monitors the actual condition of a system and makes respective maintenance decisions. CBM dictates that maintenance should only be performed when a certain degradation threshold that indicates system deterioration is reached. This strategy not only aims to balance and minimize failure and maintenance costs of the system simultaneously but also increases availability, productivity, and safety of the systems by minimizing the number of failures (Camci, 2009).

2.2 Literature Review

The survey of existing literature on condition-based maintenance policy considering various types of dependencies has two main objectives. The first objective is to show the existing and recent developments in this area of research. The second objective is to relate the proposed model to existing works in order to provide relevant contributions, and to highlight the gaps in the current research. The focus will be on recent CBM models, especially those models that take into account dependencies between the components.

As mentioned earlier, Keizer et al. (2017) classified four types of dependencies that can exist between components which are structural, stochastic, resource, and economic dependency. This research will adopt the same classification when providing the review of recent literature.

Structural dependency applies when maintenance action on one of the components requires dismantling or maintenance action on other components as well (Dekker et al,1997). According to recent literature, it can be further classified into three types: maintenance, usage, and performance. Maintenance structural dependence exists when performing a maintenance action on one component either requires or prohibits maintenance on other components. For example, airplane are required to have same thickness for all the tires, and therefore even if one tire requires maintenance, all tires are replaced simultaneously. Similarly, usage type exists when a failure or maintenance action on a component requires other components to be halted. Performance type

exists because the system's performance can be determined only by the performance of components and the way they are configured within the system. For example, components in a system can be connected in series and parallel configurations.

Stochastic dependency implies that the deterioration process of a component not only depends on its own state but also on the states of the other components. This type of dependency can occur in different ways and can be categorized into three types accordingly: load-sharing, common-deterioration mode and failure-induced damage. Failure-induced damage occurs when failure of one component causes major and one-time damage to other components leading toward sudden failure or an immediate increase in their deterioration level. The second type occurs when more than one component share the whole load of the system and as one of them fails, systems keep working but with an increased load on other working components to keep up the same output level, and hence they deteriorate faster. Apart from this, component deterioration can also increase the deterioration rate of other components, which is referred to as degradation-based load sharing. Similarly, the third type, common-deterioration mode, exists when several components fail or deteriorate at the same time, e.g., due to similar working conditions. An example of this could be a windfarm where all windmills face similar deterioration because of a heavy storm.

Economic dependency implies that joint maintenance on multiple components results in a different cost than the sum of individual maintenance costs of these components. It can be further classified into two types: positive and negative dependency. Positive economic dependency occurs when performing joint maintenance results in lesser cost

than maintaining them separately whereas negative dependency occurs when joint maintenance results in more cost than maintaining them separately. Moreover, resource dependency is applied when multiple components share the same set of maintenance resources such as tools, spares, and manpower. Based on these four dependencies, the findings of recent literature are summarized below. Table 1 depicts all four types of dependencies and other relevant information.

Camci (2009) considers structural & resource dependency along with economic dependency. In structural dependency, the author restricted joint maintenance of certain components by implementing a penalty cost on them, whereas author also implemented resource dependency by restricting the set of possible maintenance activities and considering resources to be consumed during each maintenance activity. Similarly, Koochaki et al. (2013) considered resource dependency besides economic & structural dependency for a series system and studied multiple performance criteria, which are system availability & maintenance cost

Table 1: Contributions done in CBM considering various types of dependencies

		Type of Dependency								
		Stochastic-Dep			Economic-Dep	Structural-Dep			Resource-Dep	
#	Contributions	Failure-Induced damage	Load-Sharing	Common Deterioration	Positive	Maintenance	Usage	Performance	Workers	Spares
1	Camci (2009)				✓	✓				✓
2	Tan et al. (2011)							✓	✓	
3	Zhang et al. (2011)				✓			✓		
4	Koochaki et al.(2011)				✓			✓		
5	Van Horenbeek and Pintelon (2013)	✓			✓			✓		
6	Koochaki et al.(2013)				✓			✓	✓	
7	Hong et al (2014)			✓				✓		
8	Liu et al.(2014)							✓	✓	
9	Mercier and Pham (2014)			✓	✓			✓		
10	Nguyen et al. (2014)				✓		✓	✓		
11	Zhang et al. (2014)									
12	Do et al.(2015		✓		✓			✓		
13	Feng et al. (2015)			✓				✓		
14	Huynh et al. (2015)				✓			✓		
15	Nguyen et al. (2015)				✓		✓	✓		
16	Shafiee and Finkelstein (2015)				✓			✓		
17	Zhang et al.(2015)		✓		✓					
18	Li et al. (2016)			✓	✓			✓		
19	Olde Keizer et al (2016)		✓		✓			✓		
20	Do et al (2018)		✓		✓			✓		
21	Zhang et al (2020)	✓						✓		
22	Dinh et al (2020)				✓	✓		✓		
23	Shen et al (2020)	✓			✓					
24	Rabghi (2020)		✓		✓			✓	✓	✓
25	Olde Keizer et al (2017)		✓		✓			✓		
26	Zhou et al (2019)				✓					
27	Dinh et al (2020)					✓		✓		
28	Wang et al (2020)		✓		✓			✓		
29	Hu et al (2020)		✓		✓			✓		
30	Karatas & Kutanoglu (2020)									✓

In resource dependency, Koochaki et al. considered several cases such as single-maintenance worker resource, without worker constraints, and multiple workers with certain response time. Similarly, Mercier and Pham (2014) considered stochastic dependency (common deterioration mode) along with economic and structural dependencies for a two-component system connected either in series or parallel and aimed to optimize both inspection interval & preventive maintenance threshold. They considered a bivariate degradation process where the degree of stochastic dependency is explained by the correlation coefficient of the process. Similarly, Do et al (2015) also considered stochastic dependency (load sharing) along with both economic & structural dependency for a two-component series system and aimed to optimize opportunistic & preventive replacement threshold along with inspection interval. Li et al. (2016) studied stochastic dependency (common deterioration mode) besides economic and structural dependencies for a parallel system and aimed toward optimizing preventive maintenance threshold along with probability thresholds for grouping replacements while also optimizing inspection interval for each component separately. Furthermore, Nguyen et al.(2015) considered structural & economic dependencies for a complex system where in structural dependence, he considered both performance and usage types. They optimized inspection intervals along with system failure risk thresholds to decide on maintenance initiation.

Olde Keizer et al (2017) considered a parallel system subject to stochastic dependency (load sharing) & economic dependency and optimized replacement decisions to obtain minimum long-run total cost. Do et al. (2018) also proposed a condition-based maintenance policy for a two-component series system considering stochastic and

economic dependencies. they proposed opportunistic & preventive maintenance rules to obtain minimum total cost and applied their model on a gear-box application. Alrabghi (2020) presented a model for a two-component system considering all four kinds of dependencies and used discrete simulation to optimize the maintenance policy by minimizing the total long-run cost. Zhang et al.(2020) modelled maintenance policy for K-out-of-N deteriorating systems considering stochastic (failure-induced damage) and structural dependency and used simulation to obtain optimal maintenance costs. Similarly, Dinh et al(2020) developed a cost optimization model for maintenance model considering economic and structural dependency and implemented it on an application of milling machine. Shen et al. (2020) studied a system with two subsystems critical and protective auxiliary subsystem considering stochastic dependency along with economic dependency and obtained optimal inspection & preventive replacement threshold for the model. They implemented their model on a boring machine tool application.

2.3 Research Gap and Proposed Work

From the extensive literature review, several notable research gaps are identified. It is evident from the literature (see Table 1) that few dependencies are yet to be applied or considered in CBM models, e.g., a type of structural dependency where maintenance action on one component requires maintenance actions on other components simultaneously has not yet investigated under any CBM policy (See Table 1). It is also noted that, despite its practical significance, the applications of resource dependency are scarce and have been not considered very much in the literature and hence this provides an opportunity for further research. Few scenarios where resource

dependency can be further investigated by considering spare parts availability for each component in the system which in normal case are considered to be available all the time. Furthermore, second type of economic dependency, negative dependence is also very less incorporated in literature. Also, most of the literatures considers replacement / repair time to be negligible in their models, including them also provides future research opportunity.

Another significant research gap is the fact that existing CBM studies consider more than one type of dependencies usually center around between positive economic dependency & structural dependency and very less attention has been given to the combined effect of other types of dependencies like stochastic, resource etc. Therefore an important area of research interest would be to consider joint effects of other dependencies along with either economic or structural dependencies. Furthermore, the applications of these dependencies can empirically be validated in different applied settings including windfarm or gearbox.

An interesting application for the future research using joint dependencies in CBM as proposed earlier could be windpower sector. Windpower has become an important source for generating renewable energy and produces a large amount of electricity in many countries (Tian et al., 2011). The generation of power by wind turbine is generally affected by turbine failures and weather conditions. 25% of total cost of wind power accounts for operating and maintenance cost whereas around 15% of total cost of energy is lost due to maintenance costs plus revenue loss due to downtime of wind turbine and hence provides great potential for cost reduction by enhancing reliability of

wind power (Rockmann, et al., 2017). From the extant literature, it is found that existing maintenance policies presently adopted by wind turbines are mainly corrective maintenance (CM), preventive maintenance (PM) and condition-based maintenance (GdeNP et al. 2018).

CBM plays an important role in windpower sector now a days as new turbines usually consist of various sensors which can be used to collect data easily. It determines maintenance actions based on information on deterioration state which is obtained by visual inspections, condition monitoring technology using sensors or using online monitoring system that can continuously monitor components of wind turbine. There are primarily three types of degradation state information which is used to conduct condition based maintenance activities for a turbine components: monitored degradation state, fault diagnosis and prediction of remaining useful life. Some of the CBM methods have been proposed in past such as Besnard and Bertling (2010) considered a mathematical model to optimize CBM policy for wind turbine blades by classifying deterioration state according to the severity of damage while modeling process with continuous markov chain. These conventional CBM approaches were mainly focused on single wind turbine component, means, only individual turbine components were focused instead of the whole system. Whereas, In general, windfarm consists of many wind turbines and each of which contains blades, gear box, generator etc. It is a multiple component system where maintenance activities and operation are interdependent. Not all the dependencies exist have been considered in windpower sector and only limited literatures have considered dependencies between the turbines while considering condition monitoring information. For example, Shafiee et al. (2015) considered a

condition-based maintenance policy for a multi-bladed wind turbine system where they considered the dependencies between components, but did not consider the economic and stochastic dependency between the components.

From the above, it is therefore proposed that to investigate the joint effects of dependencies such as economic or structural dependencies with other key dependencies and apply it on a numerical example.

2.4 Methodology

The objective of this thesis is to investigate the effect of joint dependencies such as economic or structural dependencies with other key dependencies and validate it in different applied settings such as windfarm or gear box etc. In this regard, a detailed modelling approach based on analytical modelling and validated by discrete-event simulation will be used to obtain condition-based maintenance strategy and to identify optimal preventive & corrective thresholds in order to optimize the system by maximizing the availability of wind turbines and minimizing total maintenance costs of the systems. The proposed model will also determine the optimal time for performing the scheduled maintenance activities.

For simulation, this study will be adopting Arena simulation software. This software provides an integrated framework for developing simulation models in a wide variety of applications. It also allows the user to do analysis of the current system, which improves decision-making and helps to make decisions on capital investment with minimal cost. It also helps in identifying limitations and constraints of the system and allows to

consider various 'what if' scenarios and provides system-specific quantities to measure performance.

Chapter 3: Model Development

The objective of this section is to develop a discrete-event simulation model for a two-component system that follows CBM policy. Various dependencies between these components are considered. The model is further extended by integrating spare-part policy into the existing maintenance model.

3.1 Discrete Event Simulation Model

Simulation refers to a vast collection of applications & systems that are used to depict the behavior of real-life systems by the use of appropriate simulation software. The proposed two-component system is modelled using discrete-event simulation which is nowadays a very important and effective method to model maintenance systems that are very complex and difficult to study analytically (Alrabghi and Tiwari, 2016; Atashgar and Abdollahzadeh, 2017) and Optquest is used to optimize the proposed system

3.2 Proposed Maintenance Policy

3.2.1 System Description

A system is considered with two components that have structural dependency between them implying that when one or both components fail, the whole system fails. Every component, i , undergoes incessant deterioration that adds up over time, which is assigned a random variable X_t^i . Once the deterioration level of component i reaches L^i , failure limit, the component fails. There also exists a stochastic dependency between

the components which implies, deterioration rate of a component not only depends on its own deteriorate rate but also on the deterioration of the other component. Similarly, Economic dependency is also considered between the components, that is, combining maintenance activities on both components results in a lower cost than performing them separately. It is assumed that a failed component is immediately detected i.e., self-announcing mechanism. If due to failure, a component stops functioning, the degradation level of the other component stays unaffected for the time it does not function i.e., until it is maintained. Since incessant examining is not possible, components inspected at every τ unit of time. The model is applied on a case study and various scenarios are studied to obtain the best maintenance policy which minimizes the system's long-run annual cost.

3.2.2 Deterioration modelling

The degradation level of component i at time $t + 1$ can be written as expressed by Do et al. (2019) :

$$X_{t+1}^i = X_t^i + f(X_t^j) + \Delta X^i \quad \text{for } j = 1, 2 \text{ and } j \neq i \quad (1)$$

where,

$f(X_t^j)$ – signifies how the degradation speed of component i is affected by component j and is the function of the degradation level of component j at time t . We suppose that $f(X_t^j)$ can be written as:

$$f(X_t^j) = \mu^j \times (X_t^j)^{\sigma^j} \quad (2)$$

where σ^j & μ^j are positive real numbers that express the effect of component j on the degradation rate of component i . In the scenario when component j does not affect the degradation of component i , implies that $\mu^j = 0$. The two components undergo steady degradation independently when $\mu^1 = \mu^2 = 0$. And finally, the effect of component j on the degradation of component i is independent of the degradation level of component j i.e. $f(X_t^j) = \mu^j$.

ΔX^i – is the random amount of degradation per time unit of the i th component when the component is independent of the system. Gamma distribution is used a lot in the literature to model fatigue, corrosion, and crack growth degradations. Therefore, we also suppose that ΔX^i follows a Gamma distribution with probability density function:

$$f_{\alpha^i, \beta^i}(x) = \frac{1}{\Gamma(\alpha^i)} (\beta^i)^{\alpha^i} x^{\alpha^i-1} e^{-\beta^i x} I_{\{x \geq 0\}},$$

where, $I_{\{x \geq 0\}}$ is an indicator function $I_{\{x \geq 0\}} = 1$ if $x \geq 0$, $I_{\{x \geq 0\}} = 0$ otherwise.

When the degradation of a component reaches the defined failure threshold, the component is assumed to be correctively maintained, and while the maintenance is carried out for the failed component the degradation of the other component stops. The proposed maintenance plan introduces two types of preventive maintenance plans and takes advantage of the positive economic dependency. The decision parameters to be optimized are also identified and formulated using the proposed maintenance plan.

3.2.3 Maintenance Strategy

The maintenance strategy of the system is outlined as follow:

- If component i fails before an inspection time (*between* T_{k-1} & T_k) it will be replaced when the next inspection occurs (T_k). This is called corrective maintenance action.
- When an inspection occurs at time T_k and component i is operating then the component will be inspected, and maintenance decision is made depending on the inspection results.

Preventive maintenance at each component level has two types of rules: 1) individual preventive maintenance. 2) opportunistic preventive maintenance.

- Individual preventive maintenance – We assign a decision parameter m_p^i , which is the preventive threshold limit. If at inspection time T_k the degradation level of component i is equal to or larger than m_p^i ($x_{T_k}^i \geq m_p^i$) then the component is instantly preventively replaced.
- Opportunistic preventive maintenance - This maintenance proposes that for every component i , an opportunistic threshold, m_o^i (decision parameter) is assigned. If at inspection time T_k a component j ($j \neq i$) is found to need correctively or preventively maintenance, component i will be preventively maintained simultaneously if its degradation is equal to or greater than the opportunistic threshold ($x_{T_k}^i \geq m_o^i$).

- To make an optimal selection of the maintenance tasks, the degradation of the components is monitored periodically in a joint way for both components every ΔT unit of times.

3.2.4 Cost Model Under Maintenance policy

The cost of inspection for each component i can be decomposed as follows:

$$C_I^i = c_I^0 + c_I^i$$

where,

- c_I^0 – denotes the setup cost of inspection which can be shared.
- c_I^i – denotes the inspection cost of a specific component i .

When corrective or preventive maintenance occurs, the cost can be expressed as follows:

For preventive maintenance cost of a component i :

$$C_P^i = c_P^0 + c_P^i$$

where,

- c_P^0 – denotes the setup cost of performing preventive maintenance which can be shared when preventive maintenance is performed on the other component at the same time.
- c_P^i – denotes the cost of a specific component i preventive maintenance.

For corrective maintenance cost of a component i , the cost is decomposed as follows:

$$C_c^i = c_c^0 + c_c^i$$

where,

- c_c^0 – denotes the setup cost of performing corrective maintenance which can be shared when multiple corrective actions are performed at the same time ($c_c^0 \geq c_p^0$).
- c_c^i – denotes the cost of a specific component i corrective maintenance ($c_c^i \geq c_p^i$).

Additional to the above-mentioned costs, a downtime cost of the component, c_d , is incurred per unit time of downtime. This cost occurs when a failed component is not replaced immediately. c_d is the downtime cost rate added per time unit.

3.2.5 Maintenance optimization plan

The decision parameters of the proposed model are: $\Delta T, m_o^1, m_o^2, m_p^1, m_p^2$, which have to be optimized to obtain an optimal maintenance plan. A cost model can be expressed to find the optimal values of the decision parameters. The long-run expected total cost per unit time is defined as:

$$C^\infty(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2) = \lim_{t \rightarrow \infty} \frac{C^t(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2)}{t - t_{down}} \quad (3)$$

where $C^t(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2)$ is the cumulative total cost and t_{down} is the cumulative downtime between $(0, t]$. From the renewal theory stated by Ross [1996], (3) can be expressed as:

$$C^\infty(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2) = \frac{\mathbb{E}[C^{T_{re}}(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2)]}{\mathbb{E}[T_{re} - T_{down}]} \quad (4)$$

where $\mathbb{E}$ is the mathematical expectation, T_{re} is the length of the system's first maintenance cycle, i.e., it can be defined as the time at which all the components were replaced. The total downtime in first maintenance cycle is known as T_{down} . We can assume that $T_{re} = \Delta T \cdot m$, where m is a positive integer, leading us to:

$$C^{T_{re}}(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2) = \frac{\sum_{k=1}^m (C_{ins}^k + C_{main}^k) + T_{down} \cdot C_d}{\Delta T \cdot m - T_{down}} \quad (5)$$

Here, c_d denotes the downtime cost for each time unit of the two-component system.

$C_{ins}^k = \sum_{i=1}^u c_I^i + I_{\{u \geq 1\}} c_I^0$, where u is the number of components inspected at time T_k and C_{ins}^k denotes the total inspection cost.

$C_{main}^k = c_p^1 + c_p^2 + c_p^0$ if both components are preventively maintained.

$C_{main}^k = c_p^i + c_p^0$ if only component i is preventively maintained.

$C_{main}^k = c_p^i + c_c^j + c_c^0$ if component i is preventively maintained and j is correctively maintained.

$C_{main}^k = c_c^i + c_c^0$ if only component i is correctively maintained.

C_{main}^k is the total maintenance cost at the time T_k .

The decision variables ($\Delta T, m_o^1, m_o^2, m_p^1, m_p^2$) are optimized using OptQuest in Rockwell Arena simulation in order to obtain the minimum cost rate from eq (4).

3.3 Incorporating imperfect inspection and maintenance in the system

In this section, the proposed model is extended by incorporating imperfect inspections and maintenance to represent a more realistic situation. Inspections are necessary to reveal the system's state; however, in reality, measurement errors may occur due to imperfect inspections thus leading toward incorrect maintenance decisions. Similarly, when the system requires maintenance, necessary maintenance actions are carried out. These actions may restore the system to as good as new or restore it to an imperfect state in-between its current state and as good as a new state (Do et al.,2015) . The causes of imperfect maintenance action can be due to e.g., human factors (lack of skills, lack of attention), lack of spare parts, lack of repair time or due to the use of low-cost maintenance policy , low quality spare parts , maintenance resources etc. Most of the existing literature on CBM policies have considered imperfect inspections apart from imperfect maintenance actions for single-component systems, whereas in practice, systems are complex and may have multiple components. The proposed discrete-event simulation-based model for a two-component system is extended by considering both imperfect inspections and imperfect maintenance actions along with various dependencies between the components. The model is applied on a numerical example and decision variables were optimized to obtain the best maintenance policy which

minimizes the system's long-run annual cost.

3.3.1 Impact of Imperfect Maintenance on the existing model

In this section, it is considered that imperfect preventive maintenance actions lead the system to an as good as new state or restore it to an imperfect state in-between its current state and as good as new state. If at time T_i , k th imperfect preventive action is performed on a component, then the maintenance gain which is defined as quantity by which deterioration level is reduced due to imperfect maintenance is described by a continuous random variable Z^k . the value of Z^k is bounded between 0 and X_{T_i} which is the deterioration level of component i at time t . It is assumed that Z^k follows uniform distribution and after every imperfect maintenance action, the deterioration level of the component becomes

$$X^k = X_{T_i} - Z^k.$$

In real systems, the intervention gain Z^k are measurable (e.g. crack length of the system after imperfect maintenance).

3.3.2 Impact of Imperfect Maintenance on Deterioration speed

The impact of imperfect action on the deterioration of each component is modelled by assuming that each imperfect maintenance action affects the deterioration speed of the component. In real systems, imperfect action may affect the deterioration speed of the systems, e.g., welding a component may reduce its crack length but impacts other properties of the material, removing parts improperly from the system may damage other parts and increase their deterioration. Hence, after imperfect maintenance, the

deterioration level of the system may reduce but it would impact the deterioration speed and may increase it.

This impact of imperfect action can be modelled by positive continuous random-variable ϵ^k which is described by exponential distribution with the following probability density function :

$$h(x) = \gamma e^{-\gamma x} I_{[x \geq 0]}$$

γ represents a non-negative real number and the mean value of ϵ^k is given as $E[\epsilon^k] = 1/\gamma$.

If the maintenance action performed on a component is perfect (e.g. perfect preventive maintenance or corrective), the mean deterioration speed of components per unit time resets to

$$v_k = v_0 = \frac{\alpha_0}{\beta}$$

However, if the maintenance action is imperfect, then the mean deterioration speed of the component becomes

$$v_k = v_{k-1} + \epsilon^k$$

3.3.3 Impact on maintenance action cost

In the case of imperfect maintenance action, in general, lower maintenance costs is incurred as compared to the perfect maintenance action. The reason behind lower imperfect cost can be due to limitations and use of lower-quality resources such as low-

skilled maintenance workers, low quality spare parts etc. or due to partial maintenance action instead of replacement. In practice, high-quality maintenance actions require more level of resources allocated to it which usually results in higher maintenance action costs (Lie et al. ,1986 ;Liu Y et al ,2010).This imperfect action cost can be independent of intervention gain Z^k and bounded by perfect preventive action cost.

When imperfect preventive maintenance occurs, the cost can be expressed as follows:

$$C_{imp}^i = c_p^0 + c_{MP}^i$$

where $C_{imp}^i \leq C_p^i$

The impact of imperfect preventive cost higher than perfect PM cost is also studied ($C_{imp}^i \geq C_p^i$) and it is found that increasing the imperfect cost has almost no impact on the maintenance policy.

3.3.4 Impact of imperfect inspection on the existing model

Inspections are very necessary to reveal the exact state of the system; however, in reality, measurement errors may occur due to poor quality of inspections thus leading toward incorrect maintenance decisions. By comparing actual and inspected degradation levels, these maintenance decisions can be classified into four types (Hao et al. 2020) as shown in table (2). “Negative” implies that inspected degradation level is less than the preventive threshold whereas “Positive” indicates that the inspected degradation is greater than the preventive threshold. A “True Negative” implies that the inspected degradation is observed to be less than the preventive threshold whereas, in reality, it is less than the preventive threshold. Similarly, “False Negative” implies that

the inspected degradation is observed to be less than the preventive threshold whereas in reality, the system has crossed the preventive threshold.

table 2: Possible outcome for any imperfect inspection

Inspection Result	Actual Degradation level	
Inspected degradation level	Positive	Negative
Positive	TP	FP
Negative	FN	TN

The imperfect inspection is incorporated in the model by introducing an error term that represents the measurement error at time t , $\mathcal{E}(t)$. So imperfectly inspected degradation level is given as :

$$Z(t) = X(t) + \mathcal{E}(t)$$

where, t is the inspection time, $X(t)$ represents the actual deterioration level of the component, and $Z(t)$ is the inspected deterioration level, $\mathcal{E}(t)$ is the measurement error at time t that follows the normal distribution $N(0, \sigma_E^2)$. The cost of the imperfect inspection is lower than the perfect inspection cost and is given as

$$C_{ImpI}^i = c_I^0 + c_{Imp}^i$$

where $(C_{ImpI}^i \leq C_I^i)$.

3.4 Simulation Model Development

Rockwell Arena software is used to implement the maintenance approach described in the previous section. The simulation model is composed of several steps that are defined in Figure 2. The decision variables of the system are optimized by using OptQuest which

is a very powerful optimization tool embedded in Arena software. This optimization engine combines Tabu search strategies along with scatter search to obtain optimal solutions globally (Alrabghi,2020). Like any other software, OptQuest provides users with the option to define decision variables, constraints, and objective functions. To study the sensitivity to input parameters different scenarios were considered and their impact on the model was studied.



Figure 2:Steps involved in simulation model (Adapted from Alrabghi,2020)

Following scenarios were considered in the system:

- 1) Perfect Inspections and Repairs
- 2) Imperfect inspections and Perfect Repairs
- 3) Perfect inspections and Imperfect Repairs
- 4) Imperfect Inspections and Repairs

3.4.1 Simulation Model Assumptions

This section enlists assumptions on which simulation model for each scenario is based.

A. Perfect Inspection and Maintenance

Simulation model for the perfect inspection and maintenance system is based on following main assumptions:

- Deterioration rate of each component follows Gamma distribution.
- If one component stops working due to failure, whole system will stop working and deterioration level of other component stays unchanged until the maintenance action is performed.
- Inspection is done on both components simultaneously at discrete time periods with inter-inspection intervals ΔT (decision parameter).
- At every inspection period, state of each component is observed and based on degradation level for each component i ($X_{T_k}^i = x_{T_k}^i$), maintenance decision is made.

- Inspections are accurate and perfect, i.e. they reveal the exact deterioration level of the component.
- Inspection time and maintenance action time is considered to be negligible.
- To perform maintenance actions, all the respective tools and materials are assumed to be available at every inspection time.
- maintenance action is perfect i.e it sets the state of component to as good as new.

B. Imperfect Inspection and Perfect Maintenance

The assumptions of the simulation model for the scenario when imperfect inspection and perfect maintenance repair is considered are based on the following:

- Deterioration rate of each component follows Gamma distribution.
- If one component stops working due to failure, whole system will stop working and deterioration level of other component stays unchanged until the maintenance action is performed.
- Inspection is done on both components simultaneously at discrete time periods with inter-inspection intervals ΔT (decision parameter).
- At every inspection period, state of each component is observed and based on inspected degradation level for each component i ($X_{T_k}^i = x_{T_k}^i$), maintenance decision is made.

- Inspections are imperfect i.e., it doesn't reveal the exact deterioration level of component, instead it reveals deterioration level with certain measurement error that follows normal distribution.
- Inspection and maintenance times are considered to be negligible.
- To perform maintenance actions, all the respective tools and materials are assumed to be available at every inspection time.
- Maintenance action is perfect, i.e. it sets the state of component to as good as new.

C. Perfect Inspection and Imperfect Maintenance

The assumptions of the scenario where perfect inspection and imperfect maintenance is considered are as follow:

- Deterioration rate of each component follows Gamma distribution.
- If one component stops working due to failure, whole system will stop working and deterioration level of other component stays unchanged until the maintenance action is performed.
- Inspection is done on both components simultaneously at discrete time periods with inter-inspection intervals ΔT (decision parameter).
- At every inspection period, state of each component is observed and based on inspected degradation level for each component i ($X_{T_k}^i = x_{T_k}^i$), maintenance decision is made.

- Inspections are perfect i.e., it reveals the exact deterioration level of component.
- Inspection and maintenance times are considered to be negligible.
- To perform maintenance actions, all the respective tools and materials are assumed to be available at every inspection time.
- Imperfect maintenance actions lead the system to a as good as new state or restore it to an imperfect state in-between its current state and as good as new state.
- Imperfect maintenance action have a negative impact on system which results in increased deterioration rate of component i.e $v_k = v_{k-1} + \epsilon^k$
- Corrective maintenance actions are considered to perfect i.e system becomes as good as new and deterioration speed resets to zero i.e $v_k = v_0 = \frac{\alpha_0}{\beta}$

D. Perfect Inspection and Imperfect Maintenance

The assumptions for the scenario when both inspection and maintenance action are considered to be imperfect are as follow:

- Deterioration rate of each component follows Gamma distribution.
- If one component stops working due to failure, whole system will stop working and deterioration level of other component stays unchanged until the maintenance action is performed.

- Inspection is done on both components simultaneously at discrete time periods with inter-inspection intervals ΔT (decision parameter).
- At every inspection period, state of each component is observed and based on inspected degradation level for each component i ($X_{T_k}^i = x_{T_k}^i$), maintenance decision is made.
- Inspections are imperfect i.e., the inspected deterioration level involve measurement error that follows normal distribution
- Inspection and maintenance times are considered to be negligible.
- It is assumed that to perform maintenance actions, all the respective tools and materials are available at every inspection time.
- Imperfect maintenance actions lead the system to a as good as new state or restore it to an imperfect state in-between its current state and as good as new state.
- Imperfect maintenance action have a negative impact on system which results in increased deterioration rate of component i.e $v_k = v_{k-1} + \epsilon^k$
- Corrective maintenance actions are considered to perfect i.e system becomes as good as new and deterioration speed resets to zero i.e $v_k = v_0 = \frac{\alpha_0}{\beta}$

3.4.2 Model Parameters

The proposed maintenance model and its scenarios are run for 10000 atu (arbitrary time unit) and 10 replications were done for each scenario to account for variation in the model. Figure 3 shows settings of Run setup of the model. The Simulation program is run on a system with AMD Ryzen 5 (3550) CPU @ 2.1 GHz and 16 GB of RAM. No warm-up period is considered in the model since each component is assumed to be as good as new and has zero deterioration level at the beginning of each simulation, so warm-up period is not required. The deterioration of system components follows Gamma distribution. Structural and stochastic dependencies are considered between the components, which means, deterioration of component is not only dependent on itself, but also on the deterioration rate of other component. In Arena, deterioration rate of both components is modelled as shown in Figure 4.

Inspections are done at every τ discrete time periods and depending on the quality of inspections and deterioration rates of both components, maintenance decisions are made, and system costs are updated accordingly. Inspection process of the system is represented in figure 4.

Run Setup

Run Speed

Run Control

Reports

Project Parameters

Replication Parameters

Array Sizes

Arena Visual Designer

Establish replication-related options for the current model. Settings include the number of simulation replications to be run, the length of the replication, the start date and time of the simulation, warm-up time length, time units, and the type of initialization to be performed between replications.

Replication Parameters

Number of Replications:

10

Start Date and Time:

☐ Sunday
,
April
11, 2021
12:56:39 AM

Warm-up Period:

0

Minutes

Replication Length:

10000

Minutes

Hours Per Day:

24

Terminating Condition:

TNow >= 10000 && EntitiesWIP(Inspections) == 0

Base Time Units:

Minutes

Parallel Replications

☐ Run Replications in Parallel

Number of Parallel Processes:

1

Parallel Replication Input Data Files:

Data File

Add

Edit

Remove

☐ Disable Auto Generated Folder Deletion

Initialize Between Replications

☒ Statistics

☒ System

OK

Cancel

Apply

Help

Figure 3:Run Setup Parameters for model

41

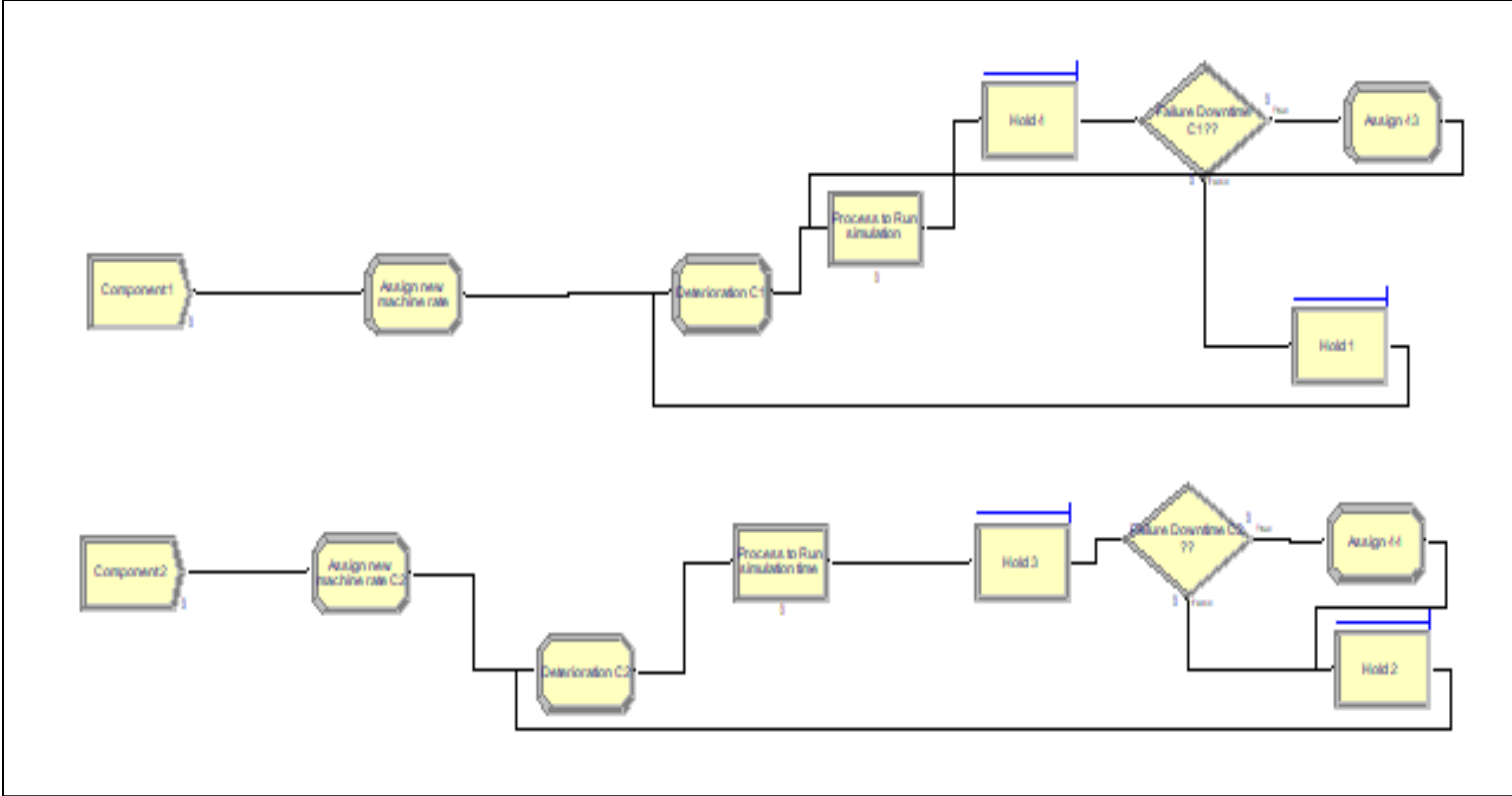


Figure 4 :Deterioration process of both component

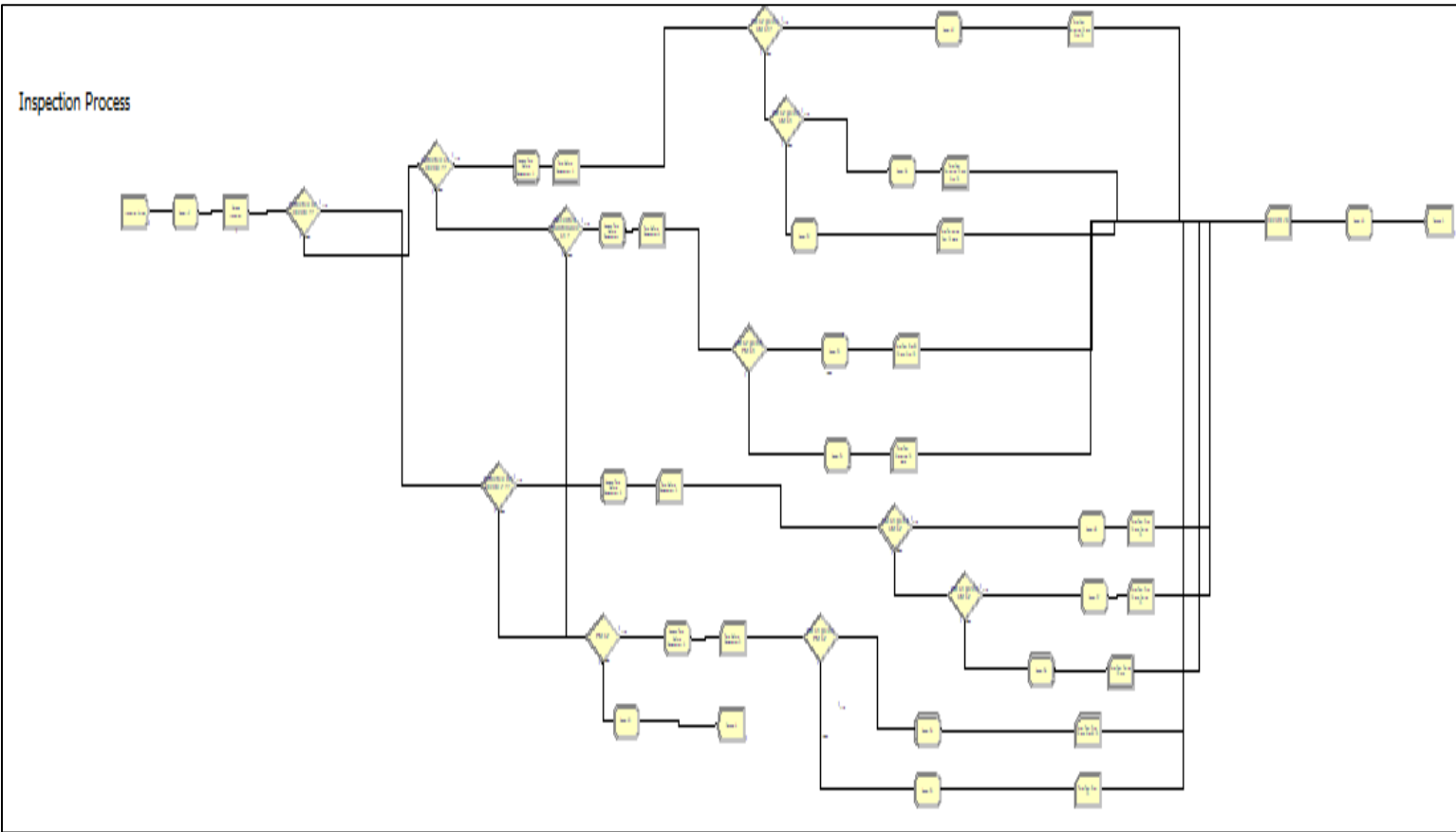


Figure 5:Inspection process of the system

A. Verification and validation of the model

The model's animation combined with dynamic statistics depicted a general view of the system's behavior. In order to verify the model's accuracy and validate it, following steps were taken.

- Simulation model is thoroughly examined to see whether the model imitates the real system.
- Expert consultation is done to identify gaps of the model and necessary suggestions were adapted.
- The process of validation of the simulation model is conducted by reproducing the results of numerical examples existing in published literature for similar models.
- The simulation model is applied on a numerical problem and meaningful results were generated.
- An analytical approach is developed using Markov chains and modelled on Matlab to validate the results obtained from simulation.

3.5 Joint Spare-part Inventory & Maintenance Policy

3.5.1 System Description & Assumptions

In this section, we extend the previously presented maintenance model by incorporating spare-part policy into existing maintenance policy. A system is considered with two components which have structural dependency between them implying that when one or both components fail, the whole system fails. Every component, i , undergoes

incessant deterioration that adds up over time, which is assigned a random variable X_t^i . Once the deterioration level of component i reaches L^i , failure limit, the component fails. There also exists a stochastic dependency between the components which implies, deterioration rate of a component not only depends on its own deteriorate rate but also on the deterioration of other component. Similarly, Economic dependency is also considered between the components, that is, combining maintenance activities on both components results in lower cost than performing them separately. Moreover, Resource dependency is also incorporated in the model implying that maintenance action on a component is performed only if its spare part is available. Inspection occurs at regular time periods with inter-inspection intervals ΔT (decision parameter) to reveal deterioration level of component as well as spare-part inventory which follows (0,1) provisioning policy. The inspection for maintenance and spare-part can be performed at the same time for both simultaneously or at different times depending on the requirement. The state of each component is observed at every inspection period and the degradation level of component i ($i = 1,2$) at inspection times $T_k = k\Delta T$ ($k = 1,2,3, \dots$) can be expressed as $X_{T_k}^i = x_{T_k}^i$. Similarly, spare-part inventory is also monitored at every inspection period which reveals the inventory level u_i (0,1) for each component. At each inspection period, maintenance decision is made depending on the current deterioration level and spare-part availability. For each component i , if its spare part is available, i.e., $u_i=1$, a maintenance action is carried out according to maintenance policy explained in previous sections. Once spare parts shortage occurs upon maintenance requests, maintenance activities are carried out as follow :

- If maintenance is required only on one component i and the requirement is CM, the system will stop working and downtime cost C_d will incur per time unit until its spare part become available and maintenance action is carried out.
- If the required maintenance action is PM and no spare-part is available, the system will continue to operate, and maintenance action is postponed until next inspection period.
- If maintenance is required on both components and it is PM, a spare part will be ordered on urgent basis and maintenance action will be delayed until next inspection period. If at least one component requires CM, system will shut down at a cost rate C_d per time unit and until spare part becomes available and maintenance is performed.
- If one component requires PM/CM and other has reached opportunistic threshold, but no spare part for it is currently available, only PM/CM action will be carried out on thus missing the opportunity of OM for other component.

3.5.2 Spare-part provisioning policy

At each inspection period for spare part inventory, following spare-part provisioning policy for is applied for each component :

- If $u_i=0$ and degradation level of component i (X_i) < Spare-part threshold (m_s), no spare part ordering take place

- If $u_i=0$ and degradation level of component i (X_i) $>$ Spare-part threshold (m_s), spare part is ordered on a normal basis with ordering cost OC_{normal} and lead time UNIF (2,5). Once spare part is delivered, it is held in the inventory at a cost of C_h per unit time.
- If $u_i=0$ and degradation level of component i (X_i) $>$ Preventive Maintenance threshold (m_p), the component either requires PM or CM. Therefore, an urgent spare part order is placed with ordering cost OC_{urgent} and lead time UNIF(1,3). In case of CM, since all delivered spare parts will be consumed immediately, hence. no holding cost will incur.

3.5.3 Cost Model & Maintenance optimization plan

The decision parameters for proposed joint maintenance & spare-part policy are : $\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2$ which have to be optimized in order to obtain optimal maintenance plan. A cost model can be expressed to find the optimal values of the decision parameters. We define the long run expected total cost per unit time as:

$$C^\infty(\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2) = \lim_{t \rightarrow \infty} \frac{C^t(\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2)}{t - t_{down}} \quad (3)$$

where $C^t(\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2)$ is the cumulative total cost and t_{down} is the cumulative downtime between $(0, t]$. From the renewal theory stated by Ross [1996], (3) can be expressed as:

$$C^\infty(\Delta T, \Delta Ts, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2) = \frac{\mathbb{E}[C^{Tre}(\Delta T, \Delta Ts, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2)]}{\mathbb{E}[T_c - T_{down}]} \quad (4)$$

where $\mathbb{E}$ is the mathematical expectation and the length of the system first life cycle is known as T_c , it can also be defined as the time at which all the components were replaced. The total downtime in first life cycle is known as T_{down} . We can assume that $T_c = \Delta T \cdot m$, where m is a positive integer, which leads us to:

$$C^{Tc}(\Delta T, \Delta Ts, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2) = \frac{\sum_{k=1}^m (C_{ins}^k + C_{hold}^k + C_{main}^k + C_{spare}^k + C_{spare\ insp}^k) + T_{down} \cdot C_d}{\Delta T \cdot m - T_{down}} \quad (5)$$

where C_d denotes the downtime cost for each time unit.

$C_{ins}^k = \sum_{i=1}^u c_i^i + I_{\{u \geq 1\}} c_I^0$, where u is the number of components inspected at time T_k and C_{ins}^k denotes the total inspection cost. Similarly, $C_{spare\ insp}^k = \sum_{i=1}^u c_s^i$, where u is the spare-part inspected for component i at time T_k and $C_{spare\ insp}^k$ denotes the total spare-part inspection cost. Total maintenance cost of components is as follow:

$$C_{main}^k = c_p^1 + c_p^2 + c_p^0 \text{ if both components are preventively maintained}$$

$$C_{main}^k = c_p^i + c_p^0 \text{ if only component } i \text{ is preventively maintained.}$$

$$C_{main}^k = c_p^i + c_c^j + c_c^0 \text{ if component } i \text{ is preventively maintained and } j \text{ is correctively maintained.}$$

$$C_{main}^k = c_c^i + c_c^0 \text{ if only component } i \text{ is correctively maintained.}$$

C_{main}^k is the total maintenance cost at time T_k .

To find the expected spare-part ordering cost, two different ordering costs are considered, normal ordering cost OC_{normal}^i & urgent ordering cost OC_{urgent}^i for each component $i(i=1,2)$. Therefore, total spare-part ordering cost is computed as :

$$C_{spare}^k = \sum_{i=1}^2 P_{normal}^i * OC_{normal}^i + \sum_{i=1}^2 P_{urgent}^i * OC_{urgent}^i$$

Where P_{normal}^i represent number of parts that are ordered on normal basis for component i and OC_{normal}^i represents the normal ordering cost for component i . Similarly, P_{urgent}^i represents the number of parts that are ordered urgently and OC_{urgent}^i is the urgent cost of ordering spare-part for component i . Inventory holding cost occurs for the parts which are ordered on a regular basis with lead time following uniform distribution and are stored at inventory before they are consumed. However, inventory cost does not occur for the parts which are ordered on an urgent since they will be consumed as soon as they arrive. Therefore, total inventory holding cost is given as :

$$C_{hold}^k = \sum_{i=1}^2 HI^i * HC^i$$

Where HI^i is number of spare-parts in inventory for component i and HC^i is holding cost for component i .

3.5.4 Simulation model for Joint spare-part and maintenance policy

In this section, Simulation model was developed for joint spare-part and maintenance policy. With the help of OptQuest, optimal values for the decision variables

$(\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2)$ were identified and minimum cost of the system was obtained.

A. Simulation Model Assumptions

The simulation model for joint spare-part and maintenance strategy is based on following assumptions :

- Each component deteriorates with gamma distribution.
- If one component stops working due to failure, whole system will stop working and deterioration level of other component stays unchanged until the maintenance action is performed.
- Inspection is done on both components simultaneously at discrete time periods with inter-inspection intervals ΔT (decision parameter).
- Spare parts are monitored on regular time periods and depending on the spare-part threshold and available inventory, a spare-part provisioning decision is made.
- If spare-part shortage occurs during maintenance inspection, then it is checked if an order is already in process, if not, an order is made on urgent basis with higher cost than normal ordering cost.
- Spare-part lead time follows uniform distribution. However, lead time for orders made on urgent basis are shorter than normal orders.
- Inspection and maintenance times are considered to be negligible.

- It is assumed that to perform maintenance actions, all the respective tools and materials are available at every inspection time.
- If maintenance is required only on one component and it is CM, the system will stop working and downtime cost C_d will incur per time unit until its spare part become available and maintenance action is carried out.
- If the required maintenance action is PM and no spare-part is available, the system will check if the order is already made, if not, an order will be based on normal and system will continue to operate, thus postponing the maintenance action to next inspection period.
- Spare-part provisioning is based on (0,1) policy for each component.
- At beginning of simulation, there is one spare-part available for each component.
- Holding cost occurs for each spare-part inventory per time unit.

B. Model Parameters

The proposed joint spare-part and maintenance model is run for 10000 atu (arbitrary time unit) and 10 replications were done to account for variation in the model. No warm-up period is considered in the model since each component is assumed to be as good as new at the beginning of simulation and spare parts for each component are available. All four kinds of dependencies (structural, stochastic, economic and resource) are considered in the model. Figure 6 and 7 illustrates the process of spare-part ordering on normal and urgent basis respectively. Inspections for both spare parts and maintenance are done at discrete time periods. Maintenance decisions are made according to

availability of spare-parts and deterioration level for each component and system cost is updated accordingly . Inspection process of the joint system is shown in figure 8. Similarly, spare-part inspection process for both components is shown in figure 9.

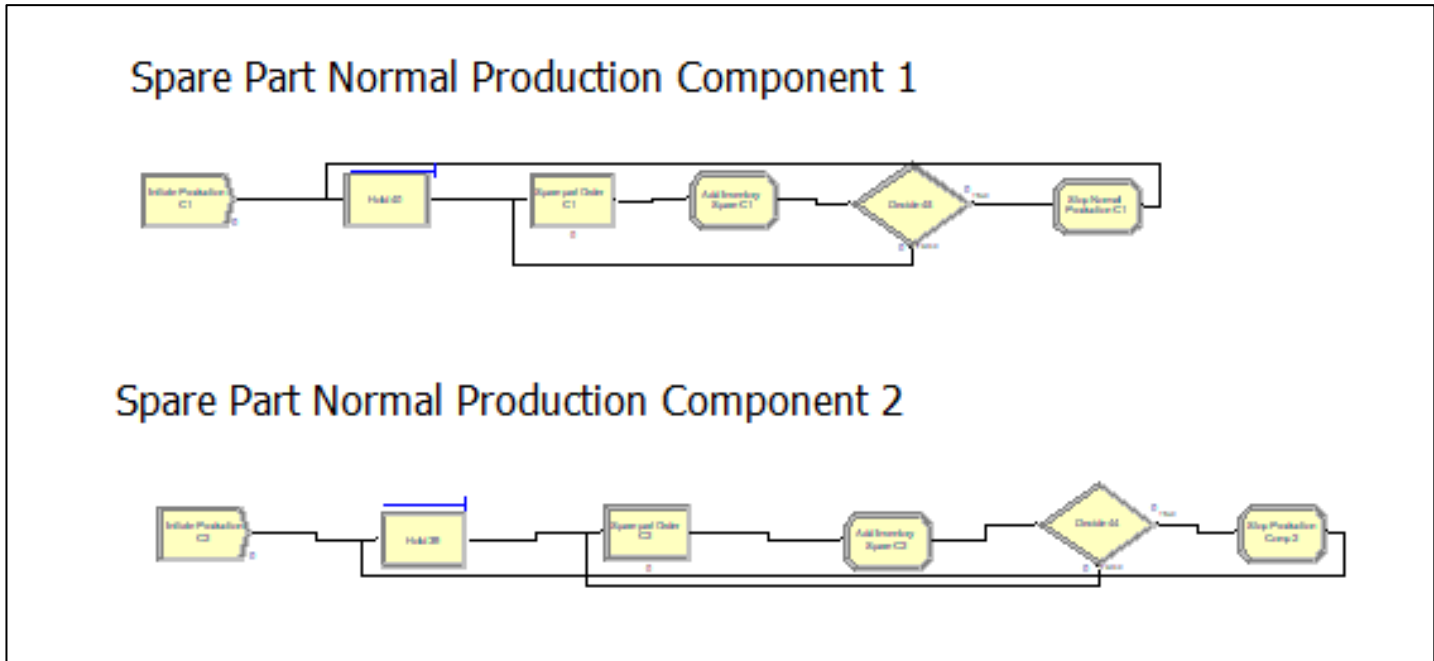


Figure 6 : Spare-part normal ordering process for both components

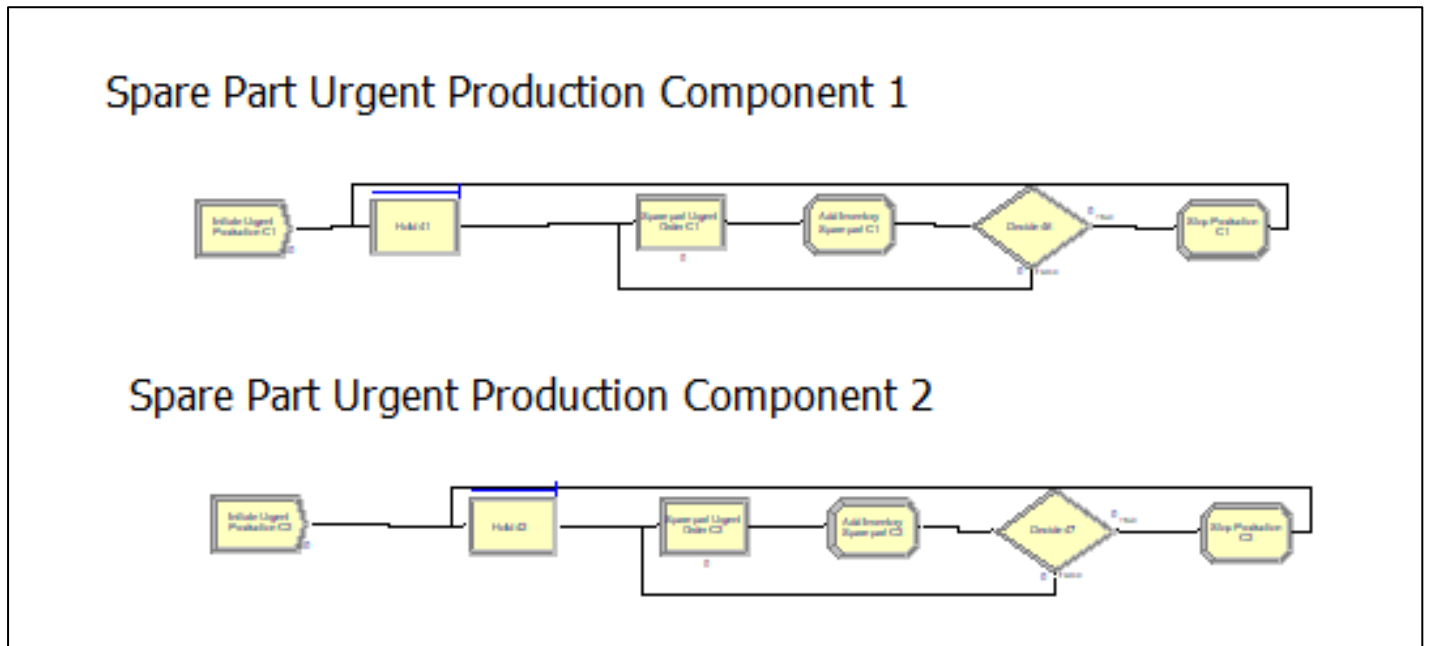


Figure 7: Spare-part urgent ordering process for both components

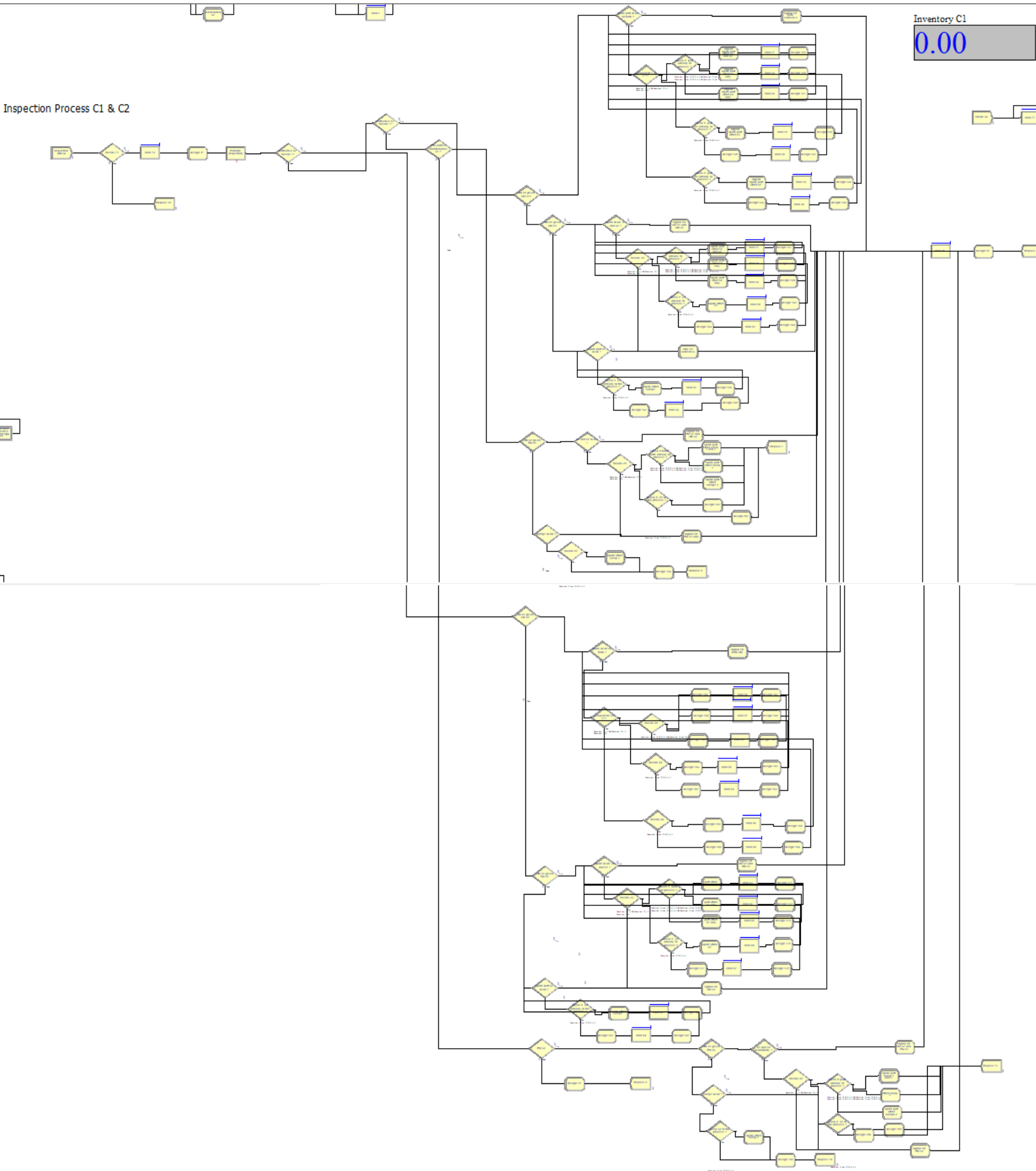


Figure 8 :Inspection process of the joint system

Spare Part Inspection Component 1



Spare Part Inspection Component 2



Figure 9:Spare-part Inspection process for both components

Chapter 4: Numerical Example and Sensitivity Analysis

This chapter provides an application of proposed CBM policies by applying it on real-life numerical problems obtained from existing literature. The objective is to demonstrate the applicability and practical value of the proposed model. For the validation purpose of simulation model, numerical example adapted from (Do et al., 2015) paper is reproduced and similar results were obtained.

4.1 Numerical Example for Model Validation

In this section, Maintenance policy is applied on a numerical example which is adapted from (Do et al., 2015) paper. They consider a system that consists of two components. The deterioration rate of every component is not only dependent to its own state but also on the state of the other component. The data containing deterioration rates and maintenance costs is given in table below.

Table 3: Data of two-component system

Component	α	β	σ	μ	Inspection Cost	PM cost	CM cost	Corrective Threshold
1	2	1	0.5	0.1	4	50	60	30
2	1	1	0.5	0.1	4	60	70	30
PM setup cost	10	CM setup cost	10	Inspection setup cost	2	Downtime Cost	70	

The authors considered two cases, in first case, both stochastic and economic dependencies were considered. In the second case, stochastic dependency was not considered among the components. The same values of inter-inspection time,

preventive and opportunistic maintenance are used in this example as adapted by (Do et al,2015) which are given in table 4.

Table 4: Values of Decision variables used by reference paper

Component	Preventive threshold	Opportunistic threshold	Corrective threshold	Inter-inspection time
1	7	6	30	10
2	18	16	30	

Table 5: Total system costs (Arena-simulation vs Actual)

	Cases	Our Model	Do et al's Paper
Total Cost of the system	Both dependencies considered	10.822	10.82
	Considering only economic dependency	12.22	12.55

Total long-run cost of the system was calculated on Arena simulation using the proposed model and the results were compared with was (Do et al., 2015) for above mentioned cases. From table 5, it can be observed that the proposed simulation model produced almost exactly the same results as in the referenced paper. The total cost varied by a % difference of 2.7 % in the 2nd case where only economic dependency is considered. Such difference may cause due to the use of different simulation software.

4.2 Numerical Example on Maintenance Policy

The proposed simulation model along with different scenarios is illustrated by applying it on a case study presented by Do et al (2015) . The data values of numerical example are slightly modified to account for extra parameters considered in the proposed maintenance policy. The problem considers a system that consists of two components

connected in series, so whenever one of the components fails, whole systems fail. The deterioration rate of every component is not only dependent to its own state but also on the state of the other component. The data containing deterioration rates and maintenance costs is given in table 6 below. The decision variables of the problem were optimized, and minimum long-run maintenance cost of the system for each scenario was obtained. Moreover, to study the impact of input parameters on the model and its scenarios, sensitivity analysis is also performed by altering input parameters.

A very large number of simulation realizations is done in order to obtain long-run cost rates of the model. To obtain optimal cost of the system, the model was run on OptQuest. The optimal values for decision variables $(\Delta T, m_o^1, m_o^2, m_p^1, m_p^2)$ were obtained along with long-run system cost for each scenario which is given in table 7.

The optimal results obtained from OptQuest are meaningful and can be explained. For the scenario 1 when both inspections and maintenance action are perfect, long-run system cost is = 12.967 atu. For the scenario 2 when inspection is imperfect and maintenance actions are perfect, long-run cost is = 12.602 atu. The long-run system cost is lower in this case because imperfect inspection cost is very less as compared to perfect scenario and the chances of making wrong maintenance error is small since preventive and corrective maintenance thresholds are far from each other.

Similarly, for scenario 3, when inspections are perfect but preventive maintenance action is imperfect, long-run system cost is around 18.175 atu. The long-run cost of the system has increased as preventive maintenance replacements are not perfect and they do not restore the state of component to as good as new but in between the current

and as good as new state. Also, these imperfect actions causes the deterioration speed of the component to increase which means more system replacements. In this case, the optimal preventive threshold is almost equal to corrective maintenance, it is because since imperfect preventive actions have impact in terms of failures which leads toward higher costs, it is better to replace the system correctively instead of replacing it preventively and causing high replacement costs along with increased deterioration rate of the components. The last scenario which represents both inspections and maintenance actions to be imperfect, has long-run system cost =18.143 atu. The long-run cost of the system has reduced as compared to 3rd scenario since inspection cost is now lower than the perfect case. Similarly, all the maintenance thresholds are equal to almost corrective thresholds for component 1, it is because when imperfect action is performed, it increases the deterioration rate of the system thus increasing the chances of making incorrect maintenance decisions which may result in more frequent imperfect replacements. Therefore, it is better to replace the system whenever corrective failure occurs since it leads toward lower cost and brings system to as good as new condition.

Table 6: Model Parameters

Model Dataset		
Parameters	Description	Value
D1	Deterioration rate for component 1	Gamma(2,1)
D2	Deterioration rate for component 2	Gamma(1,1)
C_l^0	Inspection Setup cost for each component i (can be shared)	2
c_l^1	Perfect inspection cost for each component i	6
c_{impl}^i	Imperfect Inspection cost for each component i	2
c_p^0	Preventive maintenance setup cost (can be shared)	10
c_p^1	Perfect Preventive Maintenance cost for Comp 1	60
c_p^2	Perfect Preventive Maintenance cost for Comp 2	80
c_{imp}^1	Imperfect Preventive Maintenance Cost Comp 1	40
c_{imp}^2	Imperfect Preventive Maintenance Cost Comp 2	50
c_c^0	Corrective Maintenance setup cost (can be shared)	10
c_c^1	Corrective Maintenance cost Comp 1	120
c_c^2	Corrective Maintenance cost Comp 2	180
ΔT	Time between inspections for each component	8
m_p^1	Preventive Maintenance threshold Comp 1	8
m_p^2	Preventive Maintenance threshold Comp 2	15
Cth	Corrective Maintenance threshold Comp 1 & 2	30
mo^1	Opportunistic threshold Comp 1	8
mo^2	Opportunistic threshold Comp 2	10
z^k	Quantity by which deterioration level is reduced due to an imperfect maintenance	Uniform(0,1)
ϵ_k	impact of imperfect action	Expo (0.2)
$\epsilon(t)$	Imperfect inspection measurement error standard deviation	0.67
T	Simulation Run time	10000

Table 7: Optimal Solution for All Scenarios of the model

Opt-Quest Optimal Solutions for Each Scenario							
Scenario 1 : Perfect Inspection and Repair							
Simulation #	Long-run System Cost	Feasibility	OMC1	OMC2	PMC1	PMC2	ΔT
95	12.96712	Feasible	4.011893	17.023435	21.03329	24.99119	3
Scenario 2 : Imperfect Inspection and Perfect Repair							
Simulation #	Long-run System Cost	Feasibility	OMC1	OMC2	PMC1	PMC2	ΔT
68	12.60212	Feasible	2.756455	19.302107	21.80743	25.53126	3
Scenario 3 : Perfect Inspection and Imperfect Repair							
Simulation #	Long-run System Cost	Feasibility	OMC1	OMC2	PMC1	PMC2	ΔT
99	18.175	Feasible	20.359604	27.50132	30	29.42922	1
Scenario 4 : Imperfect Inspection and Imperfect Repair							
Simulation #	Long-run System Cost	Feasibility	OMC1	OMC2	PMC1	PMC2	ΔT
100	18.14368	Feasible	30	21.078864	30	30	1

4.3 Sensitivity Analysis

In this section, sensitivity analysis is performed on a numerical example illustrated above to study the impact of input parameters on the simulation model. The value of the input parameters were changed, one at a time, while keeping all other parameters unchanged and their impact on long-run cost of all the described scenarios were observed. This is done for the parameters that directly affect the model. Specifically, these relevant parameters are: Corrective maintenance cost for component 1 C_c^1 , Corrective maintenance cost for component 2 C_c^2 , Perfect preventive maintenance cost for component 1 C_p^1 , Perfect preventive maintenance cost for component 2 C_p^2 , Imperfect

preventive maintenance cost for component 1 C_{imp}^1 , Imperfect preventive maintenance cost for component 2 C_{imp}^2 , Perfect inspection cost for both components C_I^1 and C_I^2 , Deterioration rate for component 1 D_1 and Deterioration rate for component 2 D_2 . The value of each parameter was individually changed up and down by two distinct steps between (25-50)% each. Table 5 shows the results obtained from sensitivity analysis of model.

Sensitivity analysis reveals the following key insights of input parameters :

- As depicted in figure 10 , the corrective cost for component 1 has little impact on optimal long-run cost of the system in the scenarios when maintenance repair is perfect. However, in scenarios where it is imperfect, change in corrective cost has major impact on the long-run optimal cost of the system. It is due to the fact that in the latter two scenarios, imperfect repair is avoided by the system since it has high impact on deterioration as well as on cost. So instead of preventively replacing the components, it is preferred to let system run until failure occurs. This causes in more corrective replacements than preventive, thus increasing the corrective replacement cost of the system.
- Similarly, As shown in figure 11, the corrective cost for component 2 has less impact on the optimal long-run system cost when maintenance action is considered to be perfect. However, when maintenance action is imperfect, corrective cost has major impact on the long-run cost since maintenance action is only performed when system reaches failure.

Table 8: Sensitivity Analysis results for all Scenarios

Parameter	Original value	New Values	S#1 : Both Perfect						S#2 : Imperfect Insp and Perfect Repair						S#3 : Perfect Insp and Imperfect Repair						S#4 : Both Imperfect					
			OM C1	OM C2	PM C1	PM C2	ΔT	Long-run	OM C1	OM C2	PM C1	PM C2	ΔT	Long-run	OM C1	OM C2	PM C1	PM C2	ΔT	Long-run	OM C1	OM C2	PM C1	PM C2	ΔT	Long-run Cost
$C_1^{1,2}$	8	3	2.6563	17.1996	21.1306	25.1864	3	12.5308							26.3489	27.1015	30	30	1	18.0612						
		6	2.8625	16.9615	21.1623	25.2351	3	12.84688							26.3489	27.1015	30	30	1	18.1336						
		12	2.6162	19.1541	22.0697	25.737	3	13.03604							26.3489	27.1015	30	30	1	18.2784						
		6	2.6426	19.2874	22.3401	25.8425	3	13.22424							19.0749	18.8909	30	29.9931	1	18.27916						
C_c^1	130	70	1	15.0842	30	27.5639	1	10.86356	15.141	14.024	29.966	28.713	1.0202	11.10772	30	27.5448	30	30	1	13.77964	27.267	19.01	30	29.97	1	14.39688
		100	2.5707	19.9768	22.1426	24.9171	3.05	12.645	2.8	19.3	21.9	25.531	3	12.344	30	27.5448	30	30	1	16.02364	30	21.85	30	30	1	16.11352
		160	3	17	21	24	3	13.13044	3.471	17.03	19.446	26.379	2.4362	12.869	19.9143	24.1291	30	30	1	20.08476	13.279	22.555	30	30	1	19.86328
		190	4	17	20	25	3	13.40104	8.994	12.969	12.014	15.031	8.9947	14.43512	13.7445	27.7995	30	29.106	1	22.24432	15.397	23.603	28.665	30	1	21.38392
C_c^2	190	90	2.2291	22.3999	23.1167	28.4623	2.44	12.35516	23.112	26.097	25.76	30	1.0254	11.24416	22.0508	29.9362	30	30	1	13.86368	28.746	22.856	30	30	1	15.178
		140	1.3233	22.8841	27.0096	30	1	12.92536	4.211	18.716	26.116	24.646	1	12.32552	24.3261	22.3905	30	29.852	1	16.29232	18.49	28.56	30	30	1	16.44528
		240	3	17	21	24	3	13.06304	1.207	19.527	24.753	26.206	1	12.18744	1.94098	25.0317	29.998	29.0864	1.0024	20.13904	30	19.748	30	30	1	19.50228
		290	2.008	16.0561	23.9973	23.8699	2.98	13.23228	2	16	22.241	24	3	12.81308	2.02155	25.0712	29.988	28.9021	1.0034	21.58916	1	23.268	30	29.533	1	21.39112
C_{im}^1	50	20													13.9736	27.6602	27.092	29.9846	1.0109	16.57836	8.6831	29.403	26.283	30	1.0197	15.89144
		30													1	25.9732	30	30	1	17.72312	18	28	27	30	1	16.98696
		70													29.833	18.8602	30	29.988	1	18.2398	27.63	19.97	30	29.973	1	18.7348
		90													30	17.7431	30	29.9803	1	18.3132	30	17.257	30	30	1	19.43924
C_{im}^2	60	20													3.3244	20.9887	29.979	28.2797	1.0163	16.58564	24.154	25.153	30	26.25	1	15.5834
		40													1.78521	24.9861	29.985	29.0769	1.0017	17.84568	18.647	24.188	30	30	1	17.20212
		90													1	25.9825	30	30	1	18.75016	16.59	26.913	30	30	1	19.19756
		110													15.7934	29.9705	29.968	30	1	18.39744	1	27.71	30	30	1	20.2082
C_p^1	70	20	18.454	26.3166	21.7079	26.6923	2.32	8.9848	12.279	18.925	18.233	20.784	3.4017	8.11816												
		40	1.9186	20.9296	23.012	28.5209	2.08	10.62728	9.1464	13.087	12.015	14.855	9.0365	11.01916												
		100	1	16.8127	24.1938	27.0121	1.44	15.0784	11.305	16.663	29.862	27.141	1.579	15.52364												
		130	11.757	15.0269	30	27.2	1	15.3724	15.432	17.543	29.995	27.087	1	15.47196												
C_p^2	90	30	1.9781	13.3476	22.7974	21.813	2.71	9.46092	1.9599	11.103	22.921	20.899	2.628	9.0324												
		60	3.403	14.9438	21.4236	24.2624	2.69	11.37456	3.3038	16.851	21.793	23.769	2.5168	10.94316												
		120	4.1518	18.4625	27.2421	28.3926	1.02	13.83516	5.7498	22.925	25.187	26.554	2.0505	13.99156												
		150	3.2563	21.7007	23.7138	27.7464	2.74	15.77972	1	25.941	25.056	28.526	1	15.02724												
D_1	(2,1)	(1,0.5)	15.1	6.2	29.6	24.4	1.8	7.72804	17.605	16.658	27.793	24.945	1.0147	7.30784	30	27.1547	30	30	1	12.18652	30	22.108	30	30	1	12.77744
		(1.5,0.75)	12.07	16.9048	27.1209	27.024	1.01	8.93932	13.392	14.591	29.546	25.855	1	8.9986	22.1205	26.4513	30	30	1	15.10576	30	23.65	30	30	1	15.18764
		(2.5,1.25)	3.3388	18.5064	18.6745	23.5465	3.64	17.79536	2.9692	18.995	23.293	22.881	1.991	16.3728	1	26.0717	30	30	1	21.42732	1	26.328	30	30	1	20.9178
		(3,1.5)	3	19	22	22	2	22.13204	1	23.551	24.739	28.401	1	20.22776	15.8776	29.3549	27.632	29.9272	1.0269	24.07448	14.135	29.921	27.713	30	1.0094	22.9876
D_2	(1,1)	(0.5,0.5)	1.0499	22.8702	26.1973	30	1	9.46636	3.0642	22.079	23.832	27.919	1.8205	9.25092	4.85431	29.4648	30	30	1	13.96696	12.525	28.514	30	30	1	14.81536
		(0.75,0.75)	1.9887	20.4157	26.6978	26.7026	1	10.52808	1	22.99	25.013	28.978	1	10.36516	21.4282	24.6626	30	29.9741	1	15.86212	23.428	22.557	30	29.972	1	16.4116
		(2,2)	16.517	5.87491	29.587	23.6624	1	25.30592	18.857	5.55	29.54	23.189	1	24.5762	9.48411	16.0466	29.814	26.9045	1.7421	26.90828	11.993	14.402	28.496	25.954	1.3824	25.35728
		(3,3)	19.769	5.38634	29.5012	22.932	1	49.95116	19.918	5.2992	29.581	19.762	1	47.95368	18.7649	5.87429	30	23.0358	1.0102	29.80936	18	19	30	25	1	26.86452

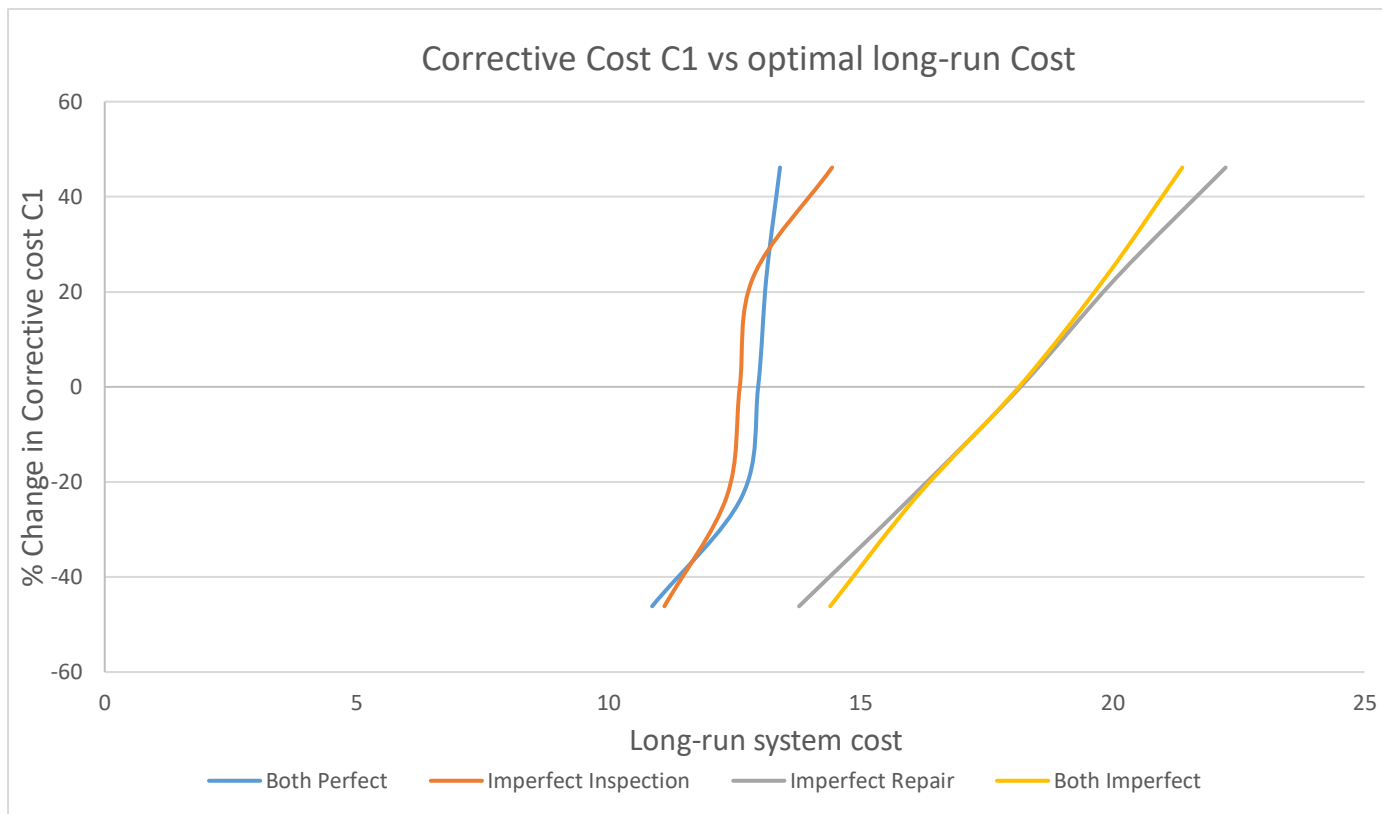


Figure 10: Sensitivity Analysis of long-run cost w.r.t Corrective cost for comp 1

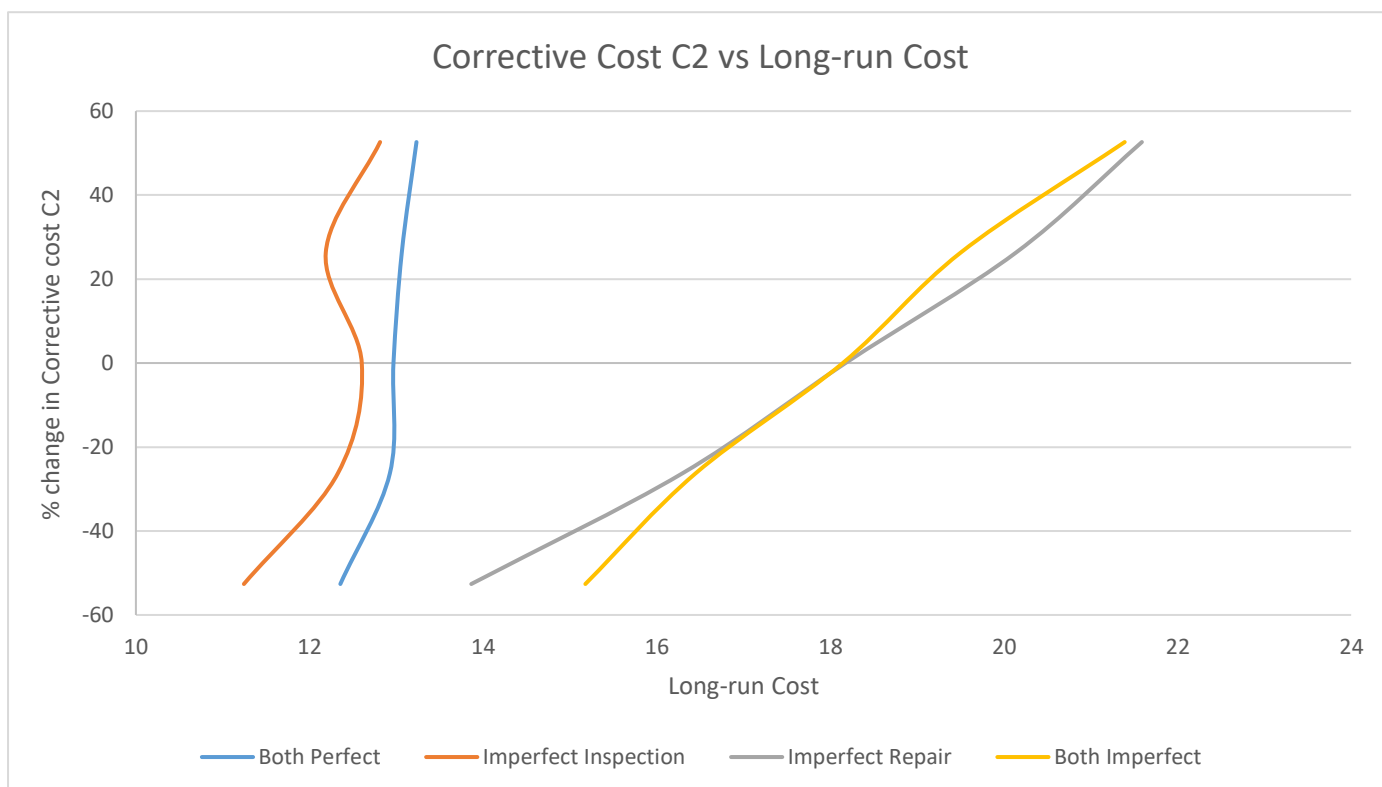


Figure 11 : Sensitivity Analysis of long-run cost w.r.t Corrective cost for component 2

- Figure 12 represents the % change in optimal long-run cost of system due to change in imperfect repair cost for component 1. It can be seen that increasing imperfect preventive cost has very small impact on the long-run cost. This is because when imperfect repair scenario is considered, it is cheaper for the model to let component run until system failure and perform only corrective replacement on it.
- Figure 13 depicts the % changes in the long-run system cost as change in imperfect preventive cost for component 2 occurs. The parameter has no impact on the long-run cost when only imperfect repair scenario is considered. The reason behind it is the same as provided above. However, in other scenario, the cost parameter has minor impact on the long-run cost. This is because imperfect inspections result in incorrect maintenance decisions due to which unnecessary preventive maintenance actions are performed, thus increasing the long-run cost of the system.
- Figure 14 & 15 represents the sensitivity analysis for the perfect preventive costs for component 1 and 2 respectively. It can be seen that % change in cost parameters increases the optimal long-run cost of the system. It is due to the fact that since the maintenance action is perfect, so to avoid system failure, it is replaced preventively.

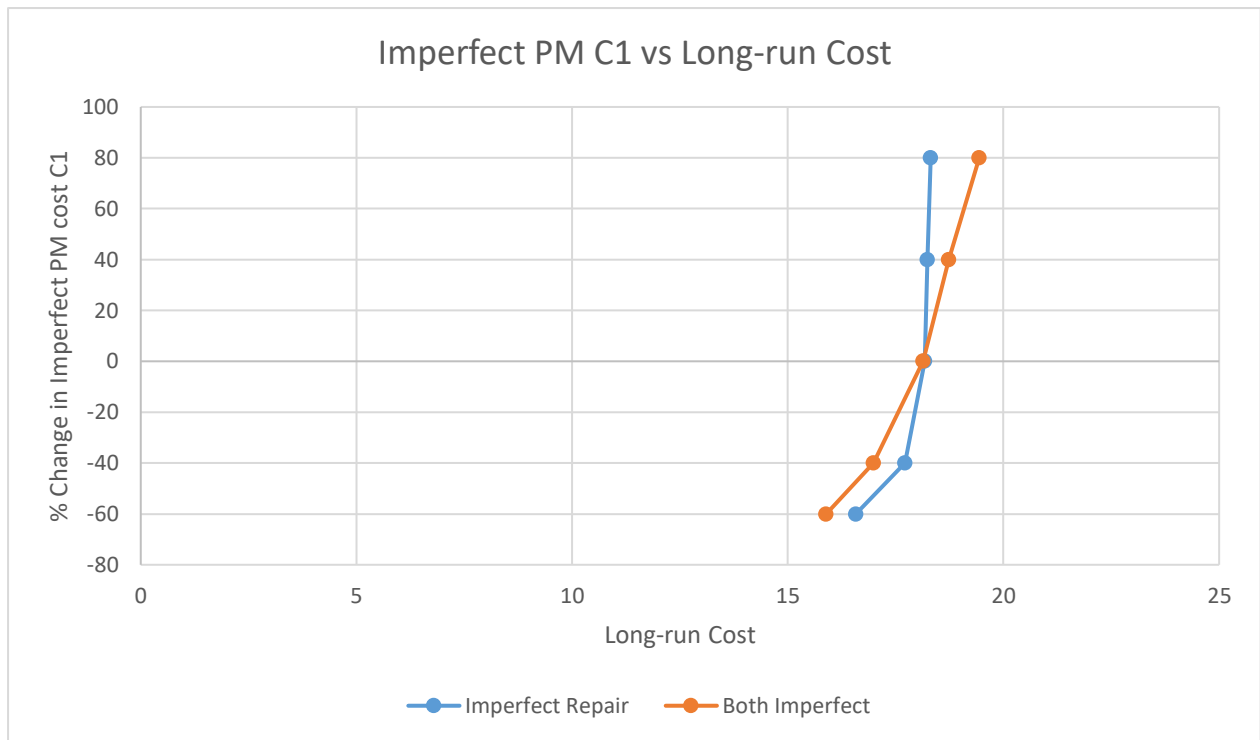


Figure 12: Sensitivity Analysis of long-run cost w.r.t Corrective cost for C 2

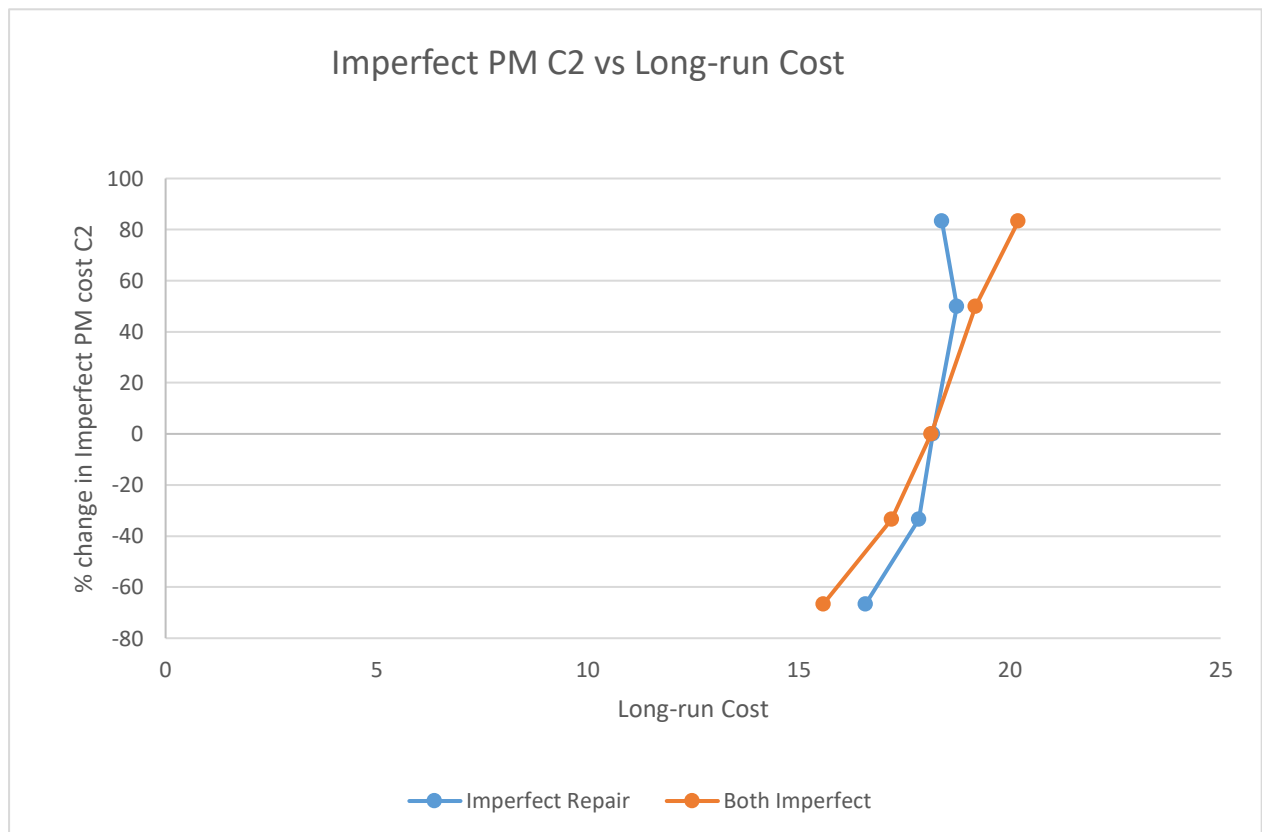


Figure 13: Change in long-run cost due to % change in Imperfect PM cost C2

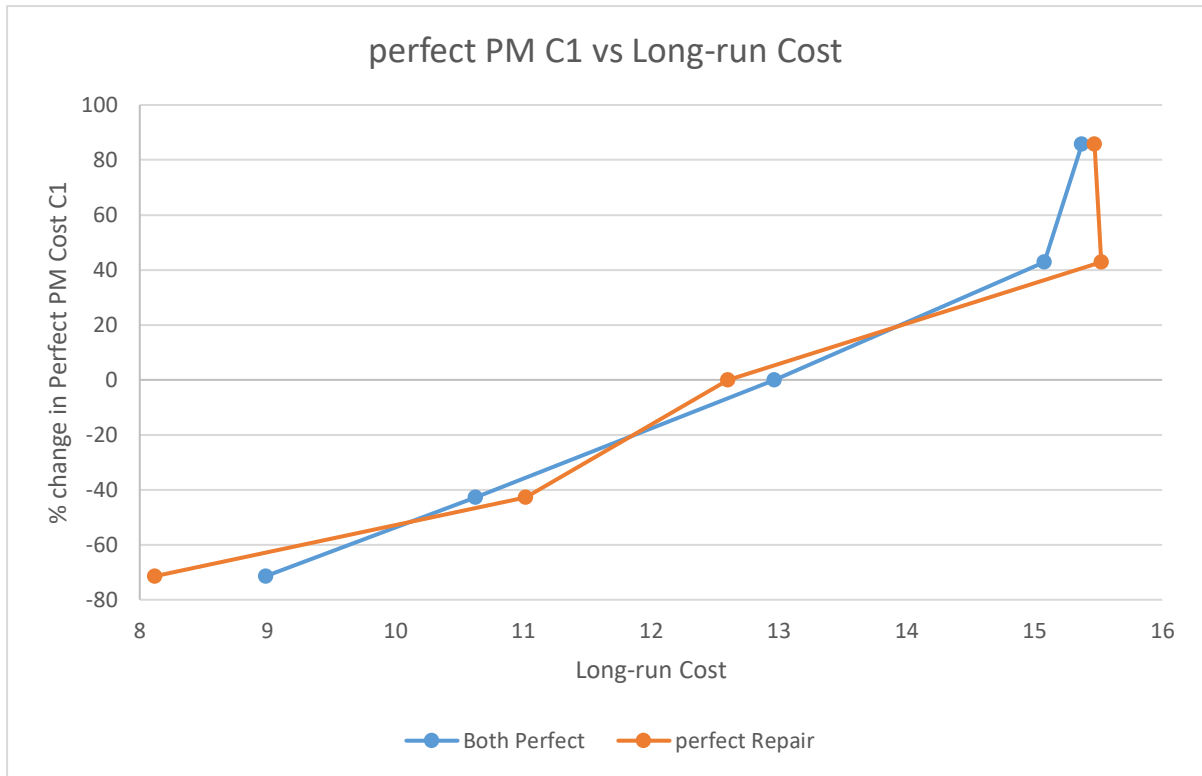


Figure 14:Sensitivity analysis of perfect preventive cost of C1

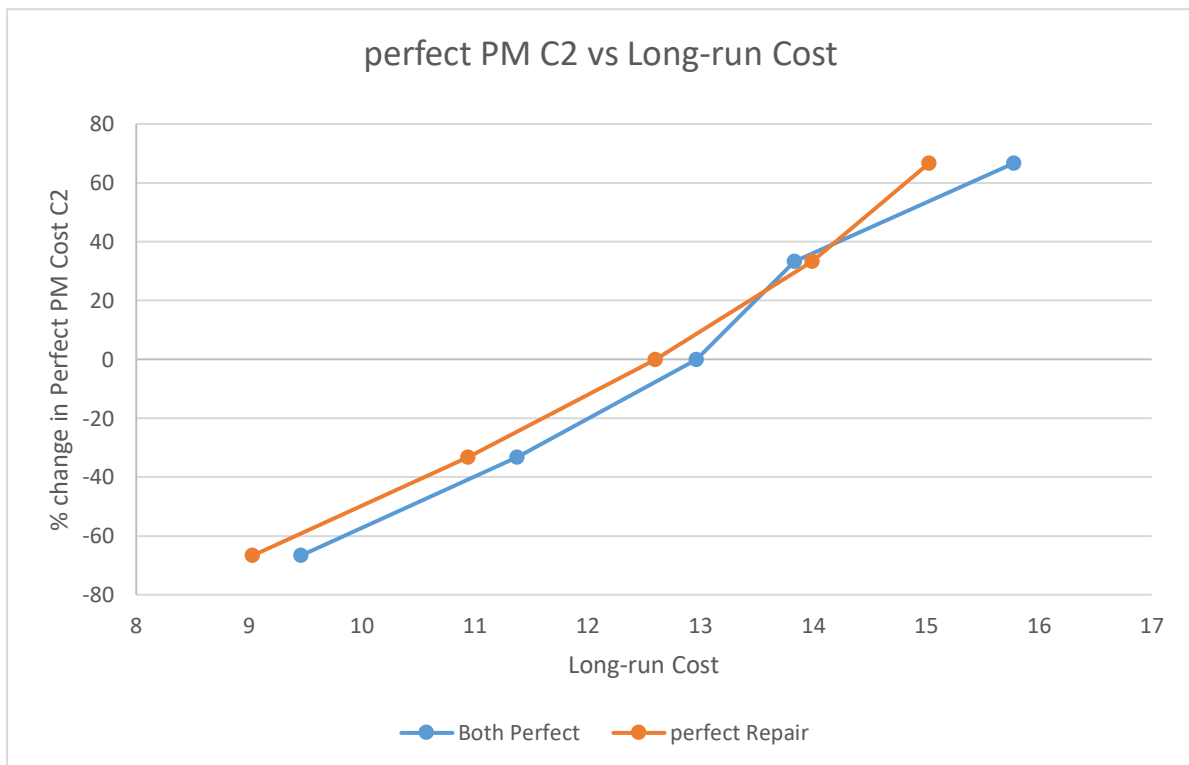


Figure 15 :Sensitivity analysis of perfect preventive cost of C2

- To study the impact of shape and scale parameter on the deterioration rate of the component, values of α and β were changed in such a way that the average deterioration rate stayed the same and their impact on the optimal long-run cost for all the scenarios were studied. As illustrated in figure 16 and 17, changes in these parameters for both components results in increased optimal long-run cost of the system. However, for component 2, the % change impact of these parameters was noted to be very high for the scenarios where perfect maintenance action is considered. This is because increase in α and β means high deterioration rate of the component, which results in more frequent replacements of the system. Since maintenance actions are perfect, so more preventive maintenance has done causing spike in the optimal long-run system cost.

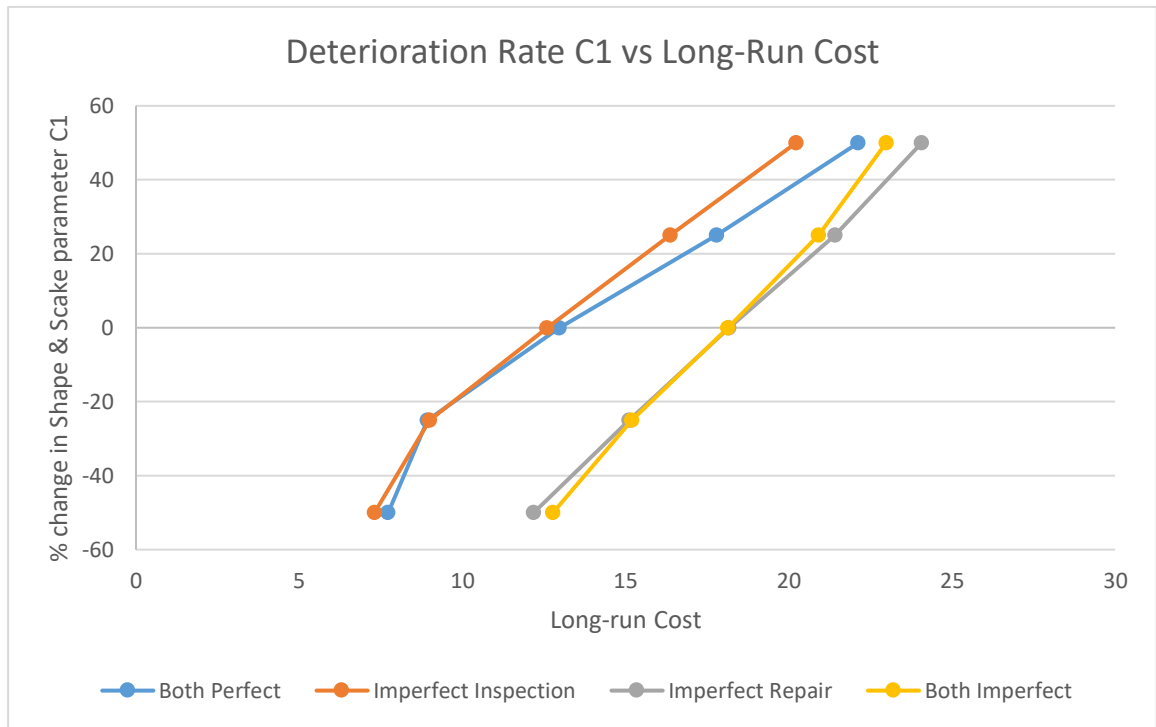


Figure 16: Sensitivity analysis of long-run cost w.r.t (α, β) Component 1

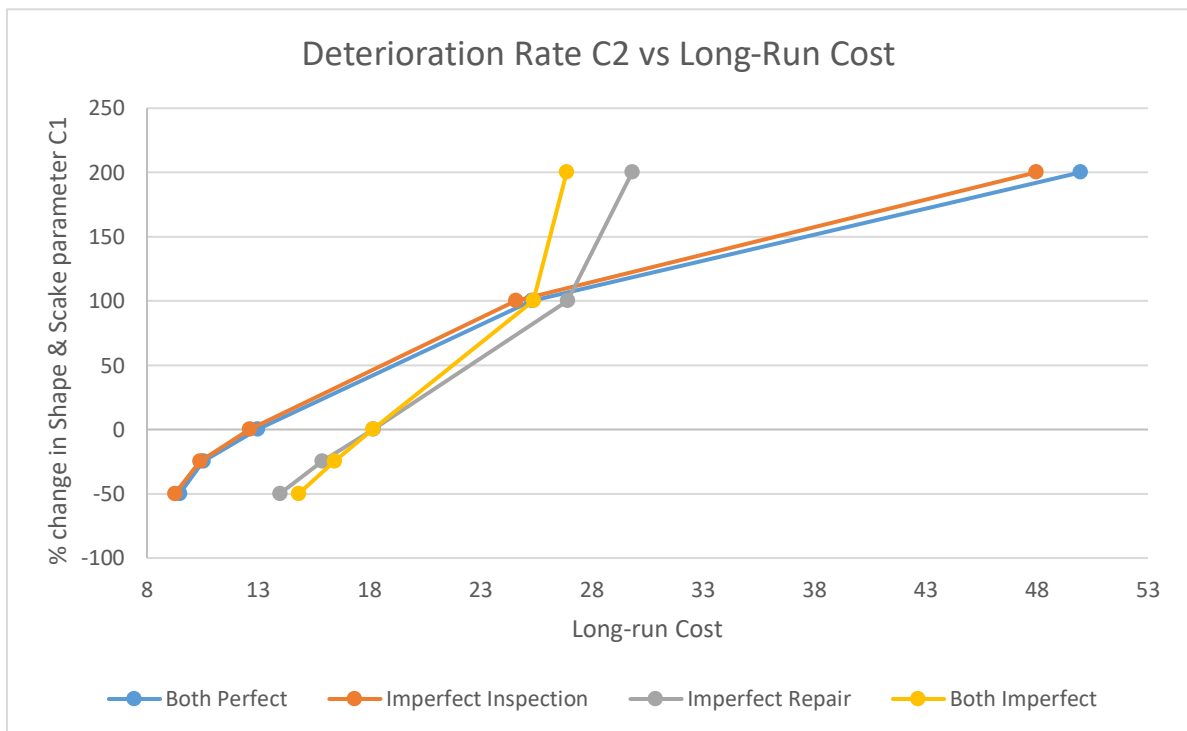


Figure 17: Sensitivity analysis of long-run cost w.r.t (α, β) Component 2

4.4 Numerical Example on Joint Spare-part and Maintenance

policy

The proposed joint spare-part and maintenance model is illustrated by applying it on a numerical problem. Considers a system that consists of two components connected in series, so whenever one of the components fails, whole system fails. The spare-part inventory for each component follows (0,1) provisioning policy. The data containing deterioration rates for components and maintenance costs is given in table 9 below. It is assumed that at the beginning, one spare-part is available for each component. The decision variables of the problem were optimized, and best maintenance policy with minimum long-run maintenance cost of the system is obtained. To obtain optimal cost of the system, the model was run on OptQuest. The optimal values for decision variables $(\Delta T, \Delta T_s, m_o^1, m_o^2, m_p^1, m_p^2, m_s^1, m_s^2)$ is obtained along with long-run system cost is given in table 10.

Table 9 :Parameters for Joint policy

Component	α	β	σ	μ	Insp Cost	PM cost	CM cost	OM th	PM th	CT h	NO Cost	UC cost	HC cost	LN	LU
1	2	1	0.5	0.1	6	50	120	6	8	30	8	20	2	Unif(2,5)	Unif(1,3)
2	1	1	0.5	0.1	6	60	180	10	15	30	5	18	2	Unif(2,5)	Unif(1,3)
PM set cost	10	CM set cost	10	Inspset cost	2	Down Cost	70								

Table 10 : Optimal Cost for Joint maintenance and spare-part policy

Opt-Quest Optimal Solution										
Joint Spare-Part & Maintenance Policy										
S#	Long-run Cost	Feasibility	OMC1	OMC2	PMC1	PMC2	Sth 1	Sth 2	ΔT	ΔTs
889	16.348	Feasible	13.056	12.342	25.030	25.180	6.363	9.538	5	1

As illustrated in table 10, optimal long-run cost of system is obtained at 889 simulation run which is around 16.348 atu .In the optimal case, spare-part inspection is done after every 1-time unit and maintenance inspection is performed after every 5-time unit. The spare-part of component 1 is ordered whenever its deterioration rate reaches 6.36 value whereas for component 2, spare-parts are ordered when its deterioration rate crosses 9.54 value. The preventive threshold for both components is almost same which is equal to 25.

Chapter 5: Analytical Model Development

5.1 Introduction

The objective of this chapter is to develop an analytical model for the maintenance strategy proposed in the previous chapter to validate the results of the model obtained from simulation, and to provide an alternative modelling framework to speed up the computation in the optimization.

5.2 System Description

A two-component system is considered which have structural and economic dependency between them. Structural dependency implies that when one or both components fail, the whole system fails. Every component, i , undergoes a continuous deterioration that adds up over time, which is assigned a random variable X_t^i . Once the deterioration level of component i reaches L^i , failure limit, the component fails. Economic dependency is also considered between the components, that is, combining maintenance activities on both components results in a lower cost than performing them separately. Inspections are performed at discrete times periodically.

5.3 Maintenance Strategy

The maintenance strategy of the system is outlined as follow:

- If a component i fails before an inspection time (*between* T_{k-1} & T_k) it will be replaced when the next inspection occurs (T_k).
- When an inspection occurs at time T_k and component i is operating then the component will be inspected, and maintenance decision is made depending on the inspection results.

Preventive maintenance at each component level has two types of rules: 1) individual preventive maintenance. 2) opportunistic preventive maintenance.

- Individual preventive maintenance – We assign a decision parameter m_p^i , which is the preventive threshold limit. If at inspection time T_k the degradation level of component i is equal to or larger than m_p^i ($x_{T_k}^i \geq m_p^i$) then the component is instantly replaced.
- Opportunistic preventive maintenance - for every component i , an opportunistic threshold, m_o^i (decision parameter) is assigned. If at inspection time T_k a component j ($j \neq i$) is selected to be either correctively or preventively maintained, component i will be preventively maintained simultaneously if the degradation level of component i is equal to or greater than the opportunistic threshold ($x_{T_k}^i \geq m_o^i$).

To make an optimal selection of the maintenance tasks, the present condition of components is monitored periodically every τ .

5.4 Developing Analytical Model

The proposed two-component system is modelled using two approaches. First, it is developed using a two-dimensional Markov chain to obtain the exact analytical solution, then a heuristic method is presented in order to obtain an approximated solution for the system.

5.4.1 Two-Dimensional Markov-Chain Approach

A. Assumptions for Two-Dimensional Markov-Chain System

The proposed two-dimensional MC system is subject to the following assumptions :

- Inspection is performed on the system periodically after every τ period.
- Each inspection reveals the system deterioration state perfectly.
- Based on the current state (i,j) of the Markov chain, one of the following decisions are made at each inspection :

- If $0 \leq i \leq y_{o,1-1}$ and $0 \leq j \leq y_{o,2}$, No maintenance action is taken on both components.
- If $0 \leq i \leq y_{o,1}$ and $y_{p,2} \leq j \leq y_{p,2-1}$, Preventive maintenance action is performed on component 2.
- If $0 \leq i \leq y_{o,1}$ and $j \geq y_{c,2}$, Corrective maintenance action is performed on component 2.
- If $y_{o,1} \leq i \leq y_{p,1-1}$ and $y_{p,2} \leq j \leq y_{p,2-1}$ or $j \geq y_{c,2}$, Both

components are jointly maintained (opportunistic maintenance for component 1).

- If $y_{p,1} \leq i \leq y_{p,1-1}$ and $0 \leq j \leq y_{o,2-1}$, Preventive maintenance action is performed only on component 1.
- if $i \geq y_{c,1}$ and $0 \leq j \leq y_{o,2-1}$, Corrective maintenance action is performed on component 1.
- If $y_{p,1} \leq i \leq y_{p,1-1}$ or $i \geq y_{c,1}$ and $y_{o,2} \leq j \leq y_{p,2-1}$, Both components are jointly replaced (opportunistic maintenance for component 2).

- Any Maintenance action (preventive or corrective) takes τ time units to replace the component.
- Downtime cost of the system is considered to be negligible.

B. Developing Two-Dimensional Markov-chain system

In this section, a two-dimensional discrete-time Markov-chain is used to model the two-component system to obtain the closed-form solution. The Markov-chain corresponding to CBM model is illustrated in figure 18. The 1st dimension of the Markov chain represents component 1 degradation at an inspection whereas the 2nd dimension represents component 2 degradation at an inspection. The degradation level of each component i at time $t + 1$ can be given as:

$$X_{t+1}^i = X_1^i + \Delta X^i \quad \text{for } j = 1, 2 \text{ and } j \neq 1 \quad (1)$$

where, ΔX^i – is the random amount of degradation per inspection time of the i th

component. We suppose that ΔX^i follows a Gamma probability density function:

$$f_{\alpha^i, \beta^i}(x) = \frac{1}{\Gamma(\alpha^i \tau)} (\beta^i)^{\alpha^i \tau} x^{\alpha^i \tau - 1} e^{-\beta^i x} I_{\{x \geq 0\}},$$

where, $I_{\{x \geq 0\}}$ is an indicator function $I_{\{x \geq 0\}} = 1$ if $x \geq 0$, $I_{\{x \geq 0\}} = 0$ otherwise.

The continuous non-decreasing deterioration process is discretized by introducing δ , an amount of degradation typically small. The result of discretization is a discrete-time discrete-state non-decreasing deterioration process with increments in deterioration $\{\delta(t): t \in N\}$ where N is a state space. Let ΔX denote the amount of deterioration during an inspection time τ , then ΔX follows Gamma distribution with parameters $(\alpha\tau, \beta)$.

The next step is to define the transition probabilities of the DTMC. The probability of a one-step transition from state i to state $i + 1$ in the DTMC is given as $P(\delta i \leq \Delta Y \leq \delta(i + 1))$, whereas for the last state of the state n_c , since it does not have any upper bound, so it is given as $P(\Delta Y \geq \delta(n_c))$. Once the state probabilities are calculated for both components, a two-dimensional transition matrix is generated. The steady-state probabilities are calculated using the equations $\pi = \pi P$ and $\sum_{i=1}^N \pi_i = 1$. The proposed system is modelled in MATLAB, as shown in figure 19, and converged steady-state probabilities were obtained for the components jointly. Since it is difficult to solve the system with more than two components using this approach, as it will result in a Markov chain with a large state space and transition probability matrices which makes calculations very tedious, heuristic is proposed in the next section to obtain an approximate solution for the model.

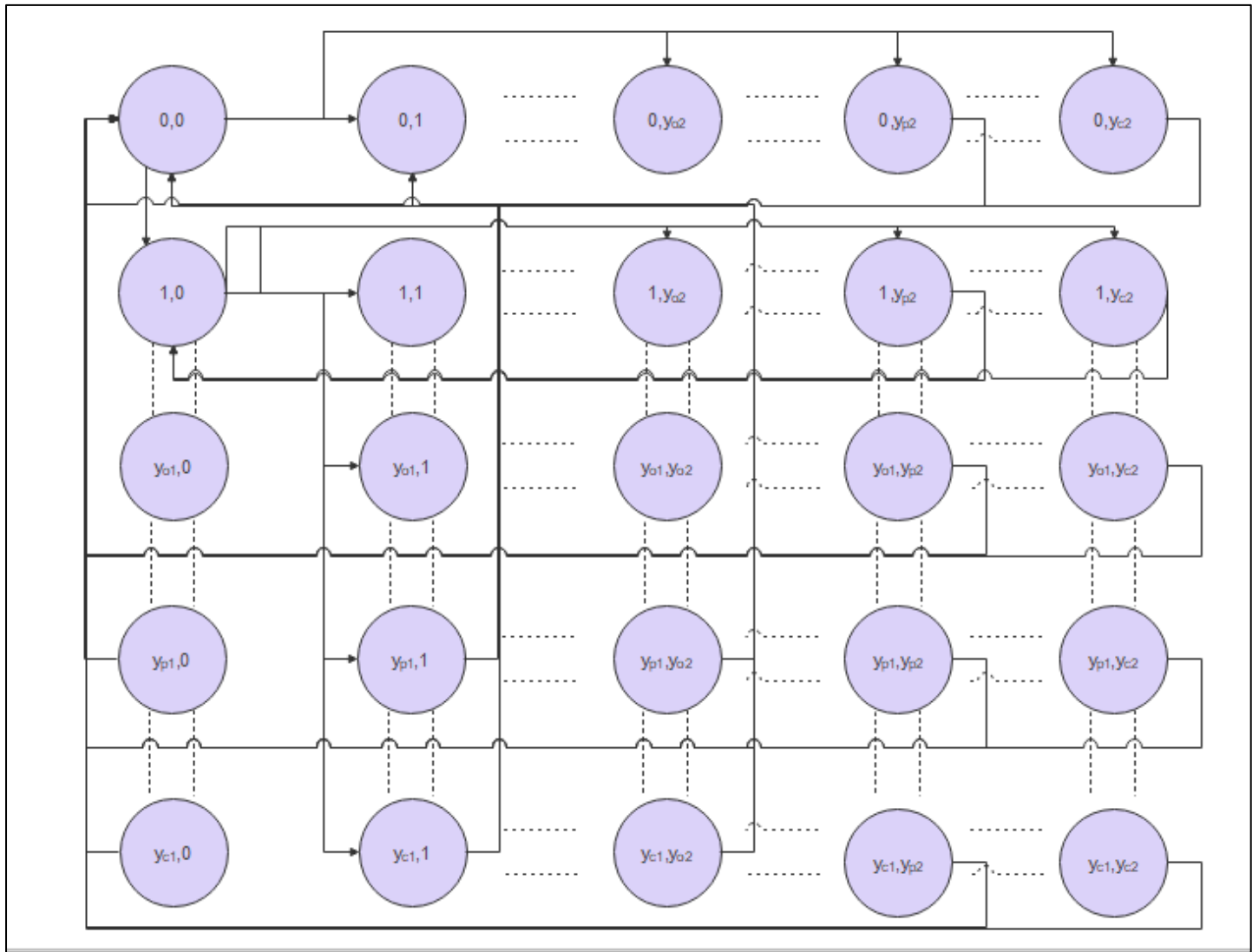


Figure 18 : Two-Dimensional Matrix for CBM system

```

DTMC2dv1.m
3 - N =2;% number of items
4 - tau = 5;
5 - alpha = [2,1];
6 - beta = [1,1];
7 - sigma =1;
8 - Main = 2*tau;
9 - yp = [5,4];
10 - yo = [4,3];
11 - yc = [6,5];
12 - maxnc = round(max(yc)/sigma);
13 - MM = zeros(N,maxnc+1,maxnc+1);
14 - % constructing the transp proba matrices for the items
15 - nc = zeros(1,N);
16 - np = zeros(1,N);
17 - no = zeros(1,N);
18 - for i=1:N...
25 - stdpp = zeros(N,max(nc(i)+1),1);
26 - % Computing steady state proba with using pi = pi*P and pi*ones=1 which
27 - % gives pi=(1,0,...,0)*inv([I-P]*), matrix [I-P]* is I-P with 1st column
28 - % equal to 1
29 - for i=1:N...
41 - % steady states probability for item 1 and 2 given in lines
42 - %Joint DTMC chains for 2 components
43 - t1 = zeros(nc(1)+1,nc(1)+1);
44 - t1(:, :) = MM(1,1:nc(1)+1,1:nc(1)+1);
45 - t2 = zeros(nc(2)+1,nc(2)+1);
46 - t2(:, :) = MM(2,1:nc(2)+1,1:nc(2)+1);
47 - t2d = kron( t1 , t2 );
48 - row_e1 = zeros(1, (nc(1)+1)*(nc(2)+1) );
49 - row_e1(1) = 1;
50 - for k=no(1):np(1) ...
56 - for k=np(1):nc(1) ...
62 - %sum(transpose(t2d))
63 - size_2d = (nc(1)+1)*(nc(2)+1);
64 - t2d = eye(size_2d)- t2d; % I - P
65 - t2d(:,1) = ones(size_2d,1); %replacing first column I-P by vector of ones
66 - % steady state is e0*inv(I-P)
67 - stdpp2d = transpose(t2d)\transpose(row_e1);
68 - % steady state matrix 2dimension
69 - A=reshape(stdpp2d,nc(2)+1,[]);
70 - D=transpose(A);
71 - D
72 - for i=1:yc(1)+1...
77 - %Cost Values
78 - CC1=120
79 - CC2=180
80 - PCC1=60
81 - PCC2=80
82 - ICC1=6
83 - ICC2=6
84 - Csetup=10
85 - Psetup=10
86 - InspSetup=2
87 - %Cost Factors
88 - pc1=sum(D(nc(1)+1,1:nc(2)+1))
89 - pc2=sum(D(1:nc(1)+1,nc(2)+1))
90 - pml=sum(sum(D(np(1)+1:nc(1),1:nc(2)+1)))
91 - pm2=sum(sum(D(1:nc(1)+1,np(2)+1:nc(2))))
92 - pol=sum(sum(D(no(1)+1:np(1),1:nc(2)+1)))
93 - pclandpc2 = D(nc(1)+1,nc(2)+1)
94 - onlypc1 = sum(D(nc(1)+1,1:no(2)))
95 - onlypc2 = sum(D(1:no(1),nc(2)+1))
96 - onlypml=sum(sum(D(np(1)+1:nc(1),1:no(2))))
97 - onlypmc2 =sum(sum(D(1:no(1),np(2)+1:nc(2))))
98 - pclandpo2=sum(D(nc(1)+1,no(2)+1:np(2)))
99 - pclandpm2=sum(D(nc(1)+1,np(2)+1:nc(2)))
100 - pmlandpo2=sum(sum(D(np(1)+1:nc(1),no(2)+1:np(2))))
101 - pmlandpm2=sum(sum(D(np(1)+1:nc(1),np(2)+1:nc(2))))
102 - pc2andpol=sum(D(no(1)+1:np(1),nc(2)+1))
103 - pc2andpml=sum(D(np(1)+1:nc(1),nc(2)+1))
104 - pm2andpol=sum(sum(D(no(1)+1:np(1),np(2)+1:nc(2))))
105 - %Cost Calculation
106 - CM1andCMC2=(CC1+CC2+Csetup)*pclandpc2
107 - CMClandOMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*pclandpo2
108 - CMClandPMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*pclandpm2
109 - OnlyCM1=(CC1+Csetup+ICC2+InspSetup)*onlypc1
110 - PMClandOMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pmlandpo2
111 - PMClandPMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pmlandpm2

```

Figure 19 : MATLAB Code for the Proposed Two-Dimension markov chain system

5.4.2 Heuristic Markov-Chain approach

A. Assumptions of Proposed Heuristic Model

The proposed system is subject to the following assumptions :

- Inspection is performed on the system periodically after every discrete τ time.
- Each inspection reveals the perfect deterioration state of the system.
- Preventive maintenance action takes τ time units to maintain the component.
- Corrective maintenance action takes M_c time units to replace the component.
- Both preventive and corrective replacement makes the system as good as new
- The deterioration rate of a component is independent of the other component (no stochastic dependence).
- The downtime cost of the system is considered to be negligible.
- Failure of one component results in failure of the whole system
- When failure occurs on one component, the deterioration rate of the other component stays the same until the component is replaced and the system starts to run.
- Based on the current state i of the Markov chain for each component, one of the following decisions are made at each inspection time:
- If $0 \leq i \leq y_o$ No maintenance action is taken.

- If $y_o \leq i \leq y_{p-1}$ and maintenance action is carried on another component (PM or CM), Preventive maintenance action is performed on the component.
- if $y_p \leq i \leq y_{c-1}$, the component is preventively replaced.
- If $i \geq y_c$, system failure occurs, and component is replaced correctively.
- If $y_{p,1} \leq i \leq y_{p,1-1}$ and $0 \leq j \leq y_{o,2-1}$, preventive maintenance action is performed only on component 1.

B. Developing Heuristic Markov-chain model

In this section, a heuristic approach is developed to model the two-component system in order to obtain the approximate solution. Each component is represented by a Markov chain which has its own probability of transition from i state to other state $i + j$ as illustrated in figure 20. Structural Dependency is considered along with economic dependency between the components. The deterioration rate is discretized by introducing δ , a small amount of degradation. Let ΔX be the amount of deterioration during an inspection time τ , then ΔX follows Gamma distribution with parameters $(\alpha\tau, \beta)$. Both Markov chains are independent in terms of deterioration rate of the component, however, there exists a correlation between them due to economic dependency. In order to consider this correlation between the two Markov chains, opportunistic states of each component (states between opportunistic and preventive thresholds) can either move forward or to state 0 with certain probabilities. E.g., when component 1 is at state i in its opportunistic zone, the chain can either move to the next state with probability $(1 - MP_2) * P_i$ or it can move to state 0 (indicates maintenance action is performed)

with probability MP_2 , where P_i is the probability to make a $i\delta$ jump to a next state and MP_2 is the probability of performing maintenance action on component 2 (Preventive or Corrective). Similarly, component 2 at state i in its opportunistic zone can either move to the next state with probability $(1 - MP_1) * P_i$ or it can move to state 0 with probability MP_1 , where MP_1 is the probability of performing maintenance action on component 1 (Preventive or Corrective) .

Similarly, to consider structural dependency between two independent Markov chains, all the transition probabilities of states of component 1 Markov chain that are between 0 and preventive threshold are updated with the probability of maintenance on component 2. In this way, for state i of component 1 ($0 \leq i \leq y_{p1}$) the Markov chain can jump forward with probability $(1 - PC_2) * P_i$ or it can stay at state i with probability $(PC_2 * P_i)$, where P_i is the probability of moving to the next state from state i and PC_2 is the probability of corrective maintenance action on component 2. Similarly, state i of component 2 ($0 \leq i \leq y_{p2}$) can jump forward with probability $(1 - PC_1) * P_i$ or it can stay at state i with probability $(PC_1) * P_i$, where PC_1 represents the probability of corrective maintenance on component 1. The transition state diagram for component 1 and 2 Markov chains is shown in figure 21 and figure 22 respectively.

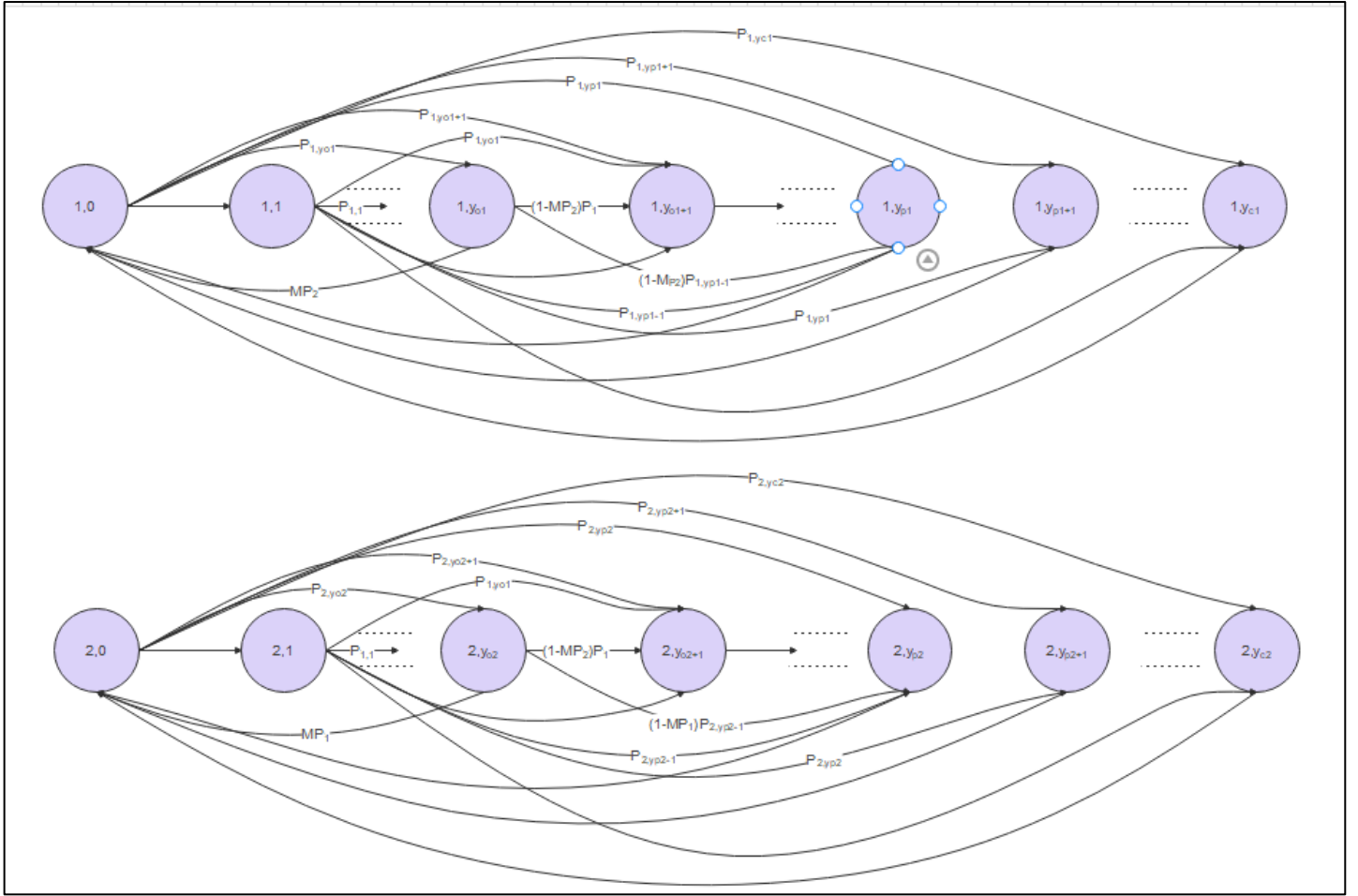


Figure 20 : Transition states of both Components

To obtain the steady state probabilities of the model incorporating both structural and economic dependencies, the following iterative solution algorithm was developed:

Solution Algorithm

Step 1: Set $MP_1 = MP_2 = 0$

where MP_1 : the probability of maintenance action (PM or CM) on component 2 and

MP_2 : the probability of maintenance on component 2

Step 2: Find the initial steady-state probability of Markov chains for both components

(π_1 and π_2)

Step 3 : Set $MP_1 = \sum_{i=y_{p1}}^{y_{c1}} \pi_i^1$ and $MP_2 = \sum_{i=y_{p2}}^{y_{c2}} \pi_i^2$

where $y_{p1}, \dots, y_{c1} - 1$ are the preventive states of components 1, and y_{c1} is the failure (last) state of component 1. Similarly, $y_{p2}, \dots, y_{c2} - 1$ are the preventive states of component 2, and y_{c2} is the failure (last) state of component 2.

Step 4: Compute the new steady-state probabilities for both components (π_1 and π_2).

Step 5: Check for steady-state probabilities convergence? if yes stop, otherwise go to step 3.

The proposed algorithm is implemented in MATLAB software (illustrated in figure 23) and the respective steady-state probabilities for both components were obtained.

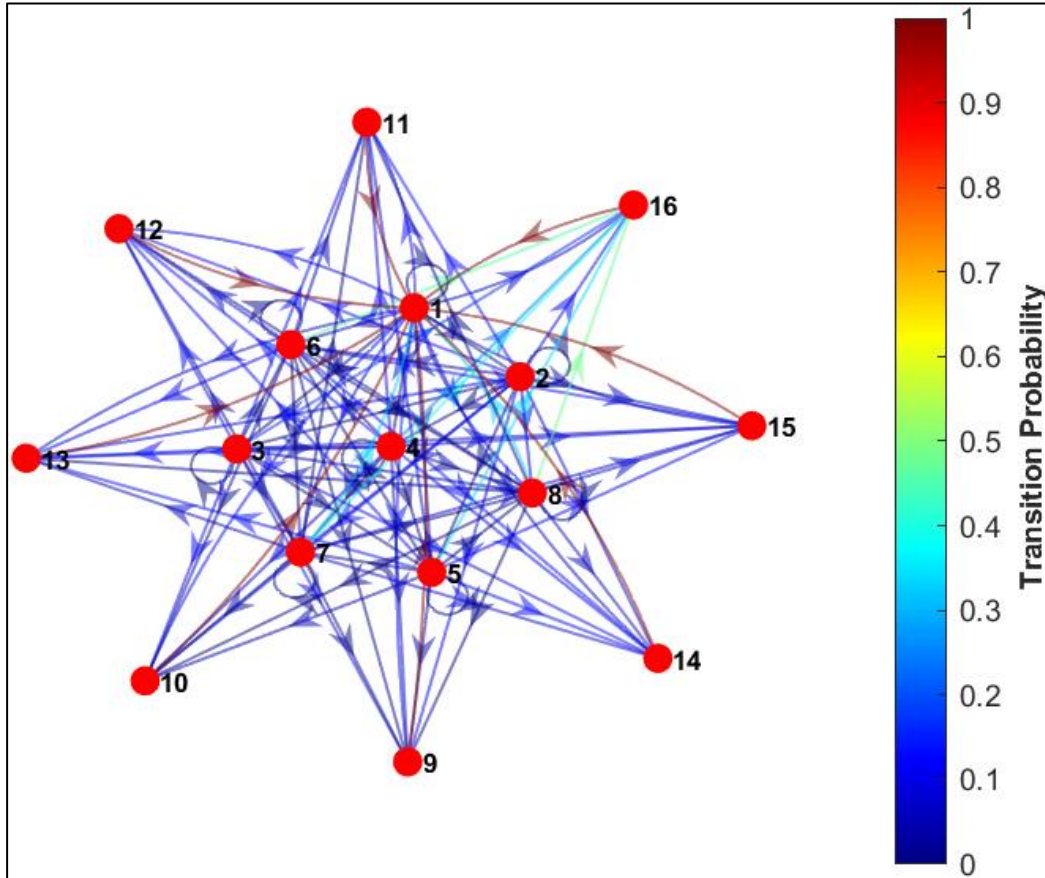


Figure 21: Transition state diagram of Component 1

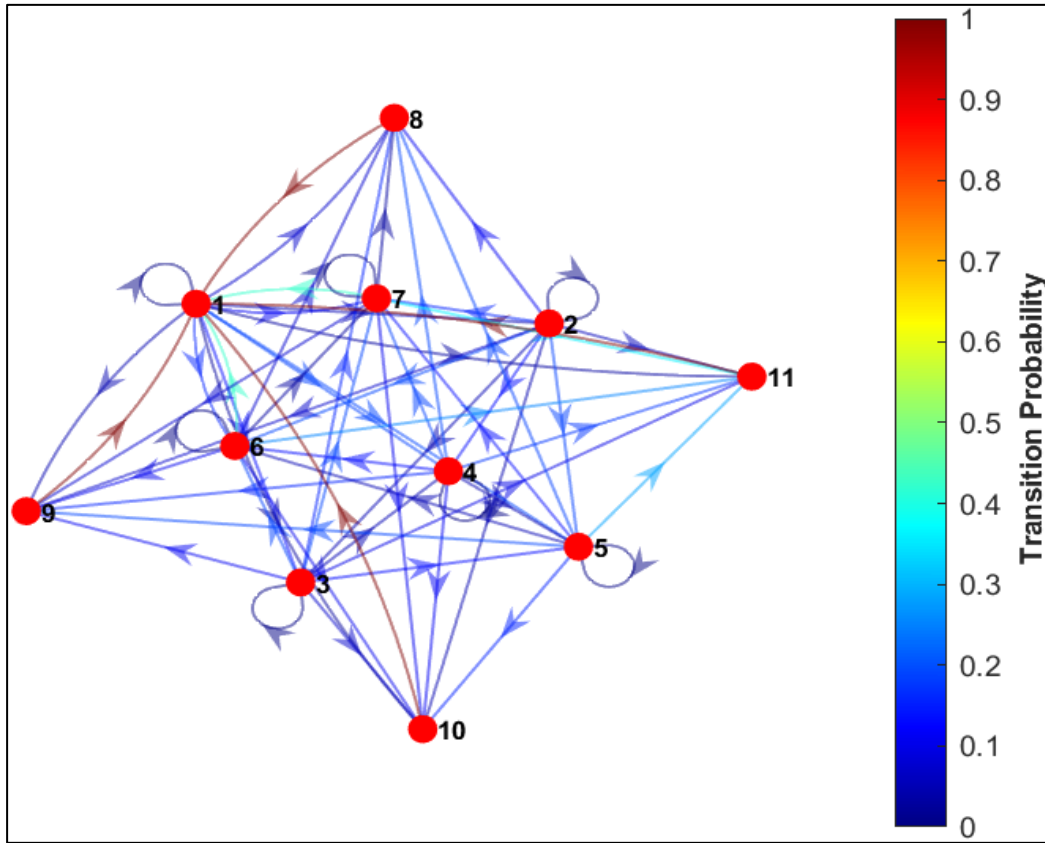


Figure 22: Transition state diagram of Component 2

The obtained steady-state probabilities for the two-component system considers inspection and maintenance repair time to be same which is equal to τ . Therefore, to incorporate the actual maintenance repair time for corrective replacement in the model, the obtained steady-state probabilities were scaled according to the given maintenance time. Assume P_i to be the obtained steady-state probability state i for any component, then the scaled steady-state probability of state i according to given corrective replacement time (M_c) is calculated as :

$$P'_i = \frac{\tau P_i}{\tau(P_0 + P_1 + \dots + P_{c-1}) + M_c(P_c)} \quad \text{when } 0 \leq i \leq y_{c-1}$$

$$P'_i = \frac{M_c P_i}{\tau(P_0 + P_1 + \dots + P_{c-1}) + M_c(P_c)} \quad \text{when } i \geq y_c$$

```

2 - N = 2; % number of items
3 - tau = 5;
4 - alpha = [2,1] ;
5 - beta = [1,1];
6 - sigma = 1;
7 - yp = [8,7];
8 - yo = [6,5];
9 - yc = [15,10];
10 - maxnc = round(max(yc)/sigma);
11 - MM = zeros(N,maxnc+1,maxnc+1);
12 - % constructing the transp proba matrices for the items
13 - nc = zeros(1,N);
14 - np = zeros(1,N);
15 - no = zeros(1,N);
16 - for i=1:N
17 -     nc(i)=round(yc(i)/sigma);
18 -     np(i)=round(yp(i)/sigma);
19 -     no(i)=round(yo(i)/sigma);
20 -     pp = starting_prob(tau,yc(i),sigma,alpha(i),beta(i)); % = ones(1,nc(i))/( nc(i) ); %
21 -     MM(i,1:nc(i)+1,1:nc(i)+1) = Trans_prob(np(i),nc(i),pp);
22 - end
23 -
24 - stdpp = zeros(N,max(nc(i)+1),1);
25 - % Computing steady state proba with using pi = pi*P and pi*ones=1 which
26 - % gives pi=(1,0,...,0)*inv([I-P]*), matrix [I-P]* is I-P with 1st column
27 - % equal to 1
28 - for i=1:N
29 -     ee = ones(nc(i)+1,1);
30 -     e0 = zeros(nc(i)+1,1);
31 -     e0(1) = 1;
32 -     t = zeros(nc(i)+1,nc(i)+1);
33 -     t(:,1) = MM(i,1:nc(i)+1,1:nc(i)+1); % t is transp proba matrix for item i
34 -     t = eye(nc(i)+1)- t; % I - P
35 -     t(:,1) = ee; %replacing first column I-P by vector of ones
36 -     % steady state is e0*inv(I-P)
37 -     stdpp(i,1:nc(i)+1) = transpose(t)\e0;
38 - end
39 -
40 - % steady states probability for item 1 and 2 given in lines
41 - stdpp
42 -
43 -
44 - %diffe is variable used for to compute difference between stdpp new and old
45 - diffe = 1; %initial difference
46 - error = .00001;
47 -
48 - % total probability of pm and cm for all items
49 - while (diffe>= error)
50 -     stdpp_old = stdpp;
51 -     for i=1:N
52 -
53 -         t = zeros(nc(i)+1,nc(i)+1);
54 -         t(:,1) = MM(i,1:nc(i)+1,1:nc(i)+1); % t is transp proba matrix for item i
55 -
56 -         % probability that other than i item is on pm or cm maintenance
57 -         tot_pm_cm = 0;
58 -         for j = 1:N
59 -             tot_pm_cm = tot_pm_cm + sum(stdpp(j,np(j)+1:nc(j)+1));
60 -         end
61 -         tot_pm_cm_i = tot_pm_cm - sum(stdpp(i,np(i)+1:nc(i)+1));
62 -
63 -         % updating the transition probability matrix of item i with prob of cm and pm
64 -         % the other i items
65 -
66 -         for j=no(i)+1:np(i)
67 -             t(j,:) = (1-tot_pm_cm_i) * t(j,:);
68 -             t(j,1) = tot_pm_cm_i;
69 -         end
70 -         t
71 -         mc = dtmc(t)
72 -         figure;
73 -         %numSteps=20
74 -         graphplot(mc,'ColorEdges',true);
75 -         t = eye(nc(i)+1)- t; % I - P
76 -         t(:,1) = ones(nc(i)+1,1); %replacing first column I-P by vector of ones

```

Figure 23:Modelling of Maintenance system on Matlab Software

5.5 Cost Model Under Proposed Models

The cost of inspection for each component i can be decomposed as follows:

$$C_I^i = c_I^0 + c_I^i$$

where,

- c_I^0 – denotes the setup cost of inspection which can be shared.
- c_I^i – denotes the inspection cost of a specific component i .

When corrective or preventive maintenance occurs, the cost can be expressed as follows:

For preventive maintenance cost of a component i :

$$C_P^i = c_P^0 + c_P^i$$

where,

- c_P^0 – denotes the setup cost of performing preventive maintenance which can be shared when multiple preventive maintenance actions are performed at the same time.
- c_P^i – denotes the cost of a specific component i .

For corrective maintenance cost of a component i :

$$C_C^i = c_C^0 + c_C^i$$

where,

- c_c^0 – denotes the setup cost of performing corrective maintenance which can be shared when multiple preventive maintenance actions are performed at the same time ($c_c^0 \geq c_p^0$).
- c_c^i – denotes the cost of a specific component i ($c_c^i \geq c_p^i$).

5.6 Numerical Example

The purpose of this section is to validate the results generated from the above-developed models by comparing them with simulation model. The numerical example considers a system that has two components which are structurally and economically dependent. Each component deteriorates following gamma process. The data containing deterioration rates and maintenance costs is given in table 11 below. The long-run cost of the system obtained from the analytical model and proposed heuristics model is compared with the cost obtained from the simulation model and presented in table 9.

Table 11 : Datasets for Analytical Model

Component	α	β	Insp Cost	PM cost	CM cost	OMth	PMth	CTh	PM setup cost	Inspection setup cost	CM setup cost
1	2	1	6	50	120	6	8	15	10	2	10
2	1	1	6	60	180	10	15	30	10	2	10

Table 12 :Result Comparison between Methods

Approach	Long-run System Cost(per time unit)	% difference
Two-Dimensional Markov Chain	16.88996	0.081839847
Heuristic Approximation Method	18.5877	0.010451503
Simulation Model	18.39544	

As presented in table 12, the long-run system costs obtained from the analytical and heuristic approximation method are found very close to the cost obtained from the simulation model. The minor difference in two-Dimensional model result is because of the structural dependency which is not considered in this approach. Therefore, the results obtained from Rockwell Arena Simulation are validated by the above-proposed methods with a small percentage difference.

Chapter 6: Conclusion, Implication and Future Directions

6.1 Conclusion

This work studied multi-component system problem considering condition-based maintenance policy and various dependencies between the components. It also provided an extensive literature review on CBM policies for multi-component systems which incorporate various types of dependencies. In this work, a condition-based maintenance policy for multi-component system is considered with stochastic, structural, and economic dependencies present among them. The objective of the model was to obtain optimal maintenance and operating cost of the system by optimizing the thresholds of the preventive maintenance. It also determined the optimal time for performing the scheduled maintenance activities on the system. To reflect more realistic situation, imperfect inspections and maintenance actions were incorporated in the model and their impact on the model was also investigated. Through numerical example, it was shown that imperfect maintenance action has major impact on the model and the scenarios where it is not considered yields low system costs. Therefore, to avoid this, it is recommended to select high quality resources for maintenance actions such as repairmen, spare parts for the components and avoid the use of low-cost maintenance resources. Furthermore, sensitivity analysis was performed on key parameters to investigate their effect on decision-variables and long-run system cost. It is found that corrective maintenance cost has high impact on the system when imperfect

maintenance action scenarios are considered. Similarly, imperfect preventive maintenance cost has very little impact on the system cost since in imperfect maintenance action case, run to failure policy is chosen by the system. Similarly, perfect maintenance cost has large impact when perfect maintenance action scenario is considered. The proposed maintenance model was further extended by incorporating spare-part policy into it. In the joint policy, all the dependencies (stochastic, structural, economic and resource) were considered between the components. The objective was to obtain an optimal joint spare-part & maintenance strategy. Furthermore, to validate the results obtained from Simulation model and to speed up the computation time and facilitate the optimization of CBM, an exact analytical approach and heuristics based on Markov chains were developed and modeled on Matlab .

6.2 Research Implications and Future Research Directions

The contribution of this thesis has both practical and academic implications. Practically, the models presented in this study can help decision makers in determining the optimal maintenance and inventory actions for multi-component systems having various dependencies among them. E.g., minimizing the long-run system maintenance and inventory cost. Besides, this thesis also focuses on imperfect inspection and maintenance action. Thus, this finding of thesis can be applied in many practical problems. For example, to detect the crack size in railway tracks or bridges, engineers have many options for inspection which may give accurate or inaccurate reading depending on the quality of inspection, examples of such are ultrasonic testing, visual inspection, and magnetic particle inspection.

Finally, the model proposed in the thesis can also be used to take optimal decisions of inspection and replacement intervals in an inspection-repair replacement policy, which provides practical guidance for engineers.

The work done in this research can be further extended in following ways:

- Few dependencies that are yet to be applied or considered in CBM models can be considered. e.g., a type of structural dependency where maintenance action on one component requires maintenance actions on other components simultaneously.
- Few directions of resource dependency which are not yet explored can also be investigated such as considering shared set of tools or maintenance action on certain component requires certain combination of tools etc.
- The proposed Joint-spare part and maintenance policy can be investigated by extending provisioning policy to (s,S) policy. Similarly, non-periodic inspections can also be considered in the model.
- Furthermore, crucial aspects such as random lead times and backordering can be investigated in Joint spare-part and maintenance policy.
- A data-driven maintenance and spare-part policy can be investigated by incorporating real-time condition monitoring data.

- Another interesting direction for imperfect inspection and maintenance action would be to consider different inspection and maintenance repair options each having different cost and quality characteristic.
- This thesis considers deterioration rate of the components to follow Gamma process. However, the proposed inventory and maintenance policies can also be applied on other types of systems having different deterioration behaviors such as deteriorations modeled by Wiener, random coefficient, and Markov processes.

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- Minitab
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- GAMS
- Rockwell Arena Simulation
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Appendix A1

Arena Simulation

This appendix describes some of the very important modules of the Arena simulation software . These modules were used to build the Maintenance and Joint spare-part and Maintenance policies on Rockwell Arena software . Arena software is used to mimic a real-life system to analyze the effect of changes on it that might require large expenses and complex redesigning of system. This software is used to study the processes associated with logistics, supply chain, maintenance processes and manufacturing etc. It gives a platform to analyze a system in its actual configuration as well as under numerous possible alternatives to come up with the best system configuration and improve it. There are three basic panels in Arena that contain the variety of modules used in defining wide range of processes which are basic process, advanced process and advance transfer process .Some of the very important modules of the Arena software are described below :

1 Basic Process Modules

- **Create Module:** This module create entities according to a schedule or inter-arrival time. The entities arrives from this module to initiate a process through the system. Example such entities are customers, materials , inspection arrivals etc depending on the type of system that is being simulated.
- **Dispose Module:** This module is used to cut the process flow. Entities arrived in the system leaves the process through this module .
- **Process Module:** This module is used for performing activities on an

entities by seizing and delaying the resources under a specific completion time

- **Decision Module:** This module provides decision options during a process flow according to a set condition or by chance.
- **Assign module:** This module is used for assigning new values to variables, entity attributes, entity types, entity pictures, or other system variables
- **Record Module :** This module is used to collect statistics in the simulation model. Various types of observational statistics can be recorded including time between exits through the module, entity statistics (time, costing, etc.)

2 Advanced Process Module

- **Hold Module:** This module is used to hold entities in a queue until a set condition is met or a signal is received from another module.
- **Signal Module:** This module sends a signal value to the hold module to release the entities hold in there .

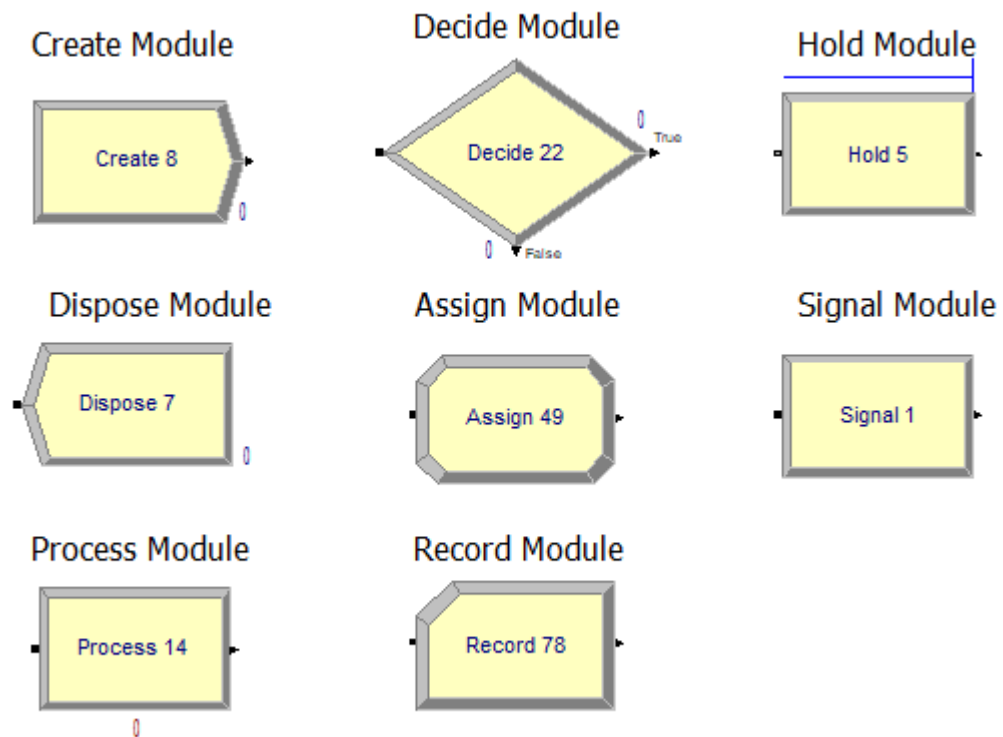


Figure 1 Symbolic Representation of Modules used in maintenance policies

Appendix A2

Simulation Model of Maintenance Policy

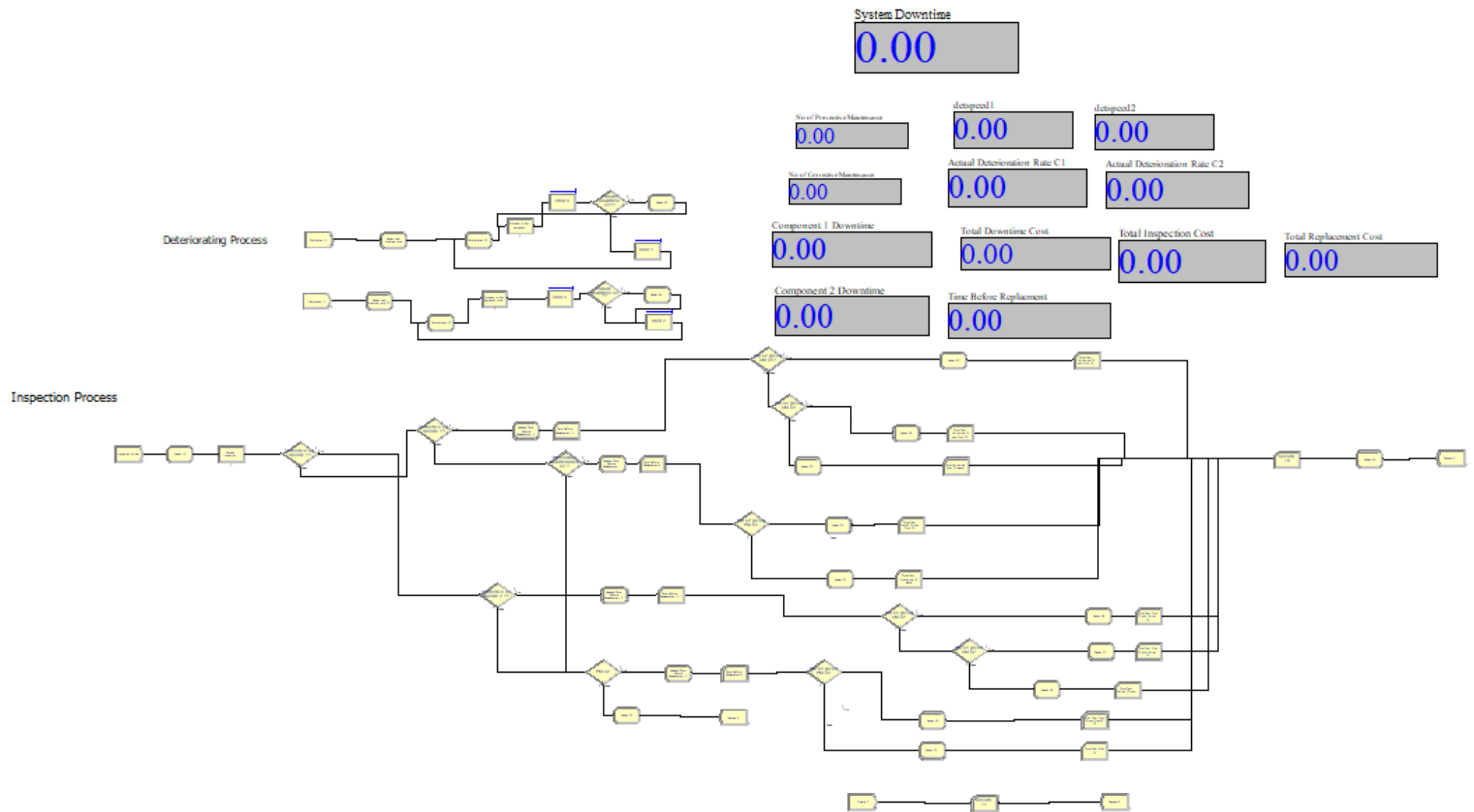


Figure 2 : Simulation Model of Maintenance policy

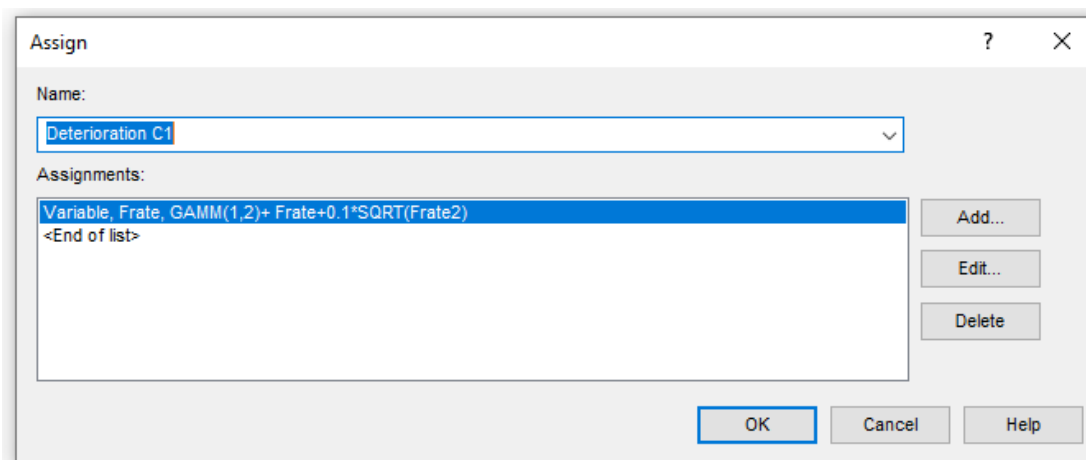


Figure 3 : Assigning Deterioration rate to Component 1

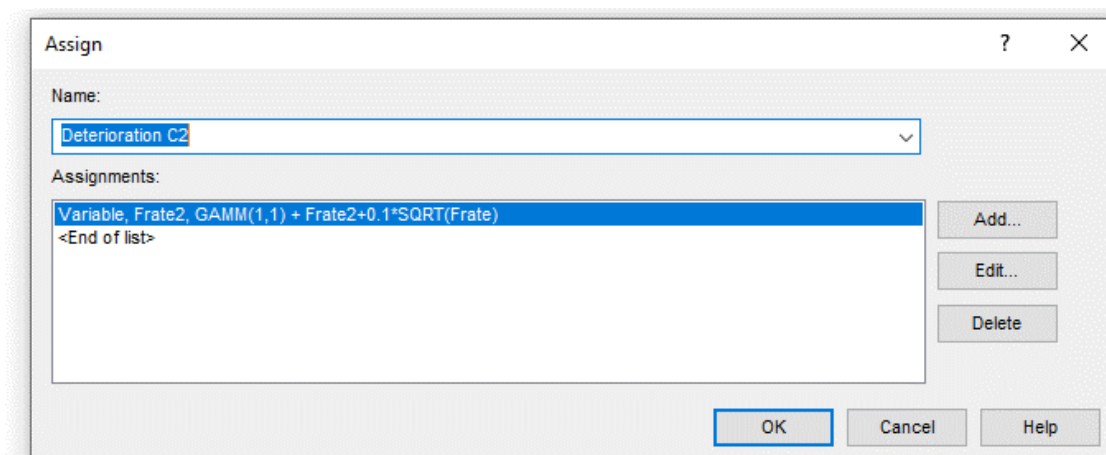


Figure 4 : Assigning Deterioration rate to Component 2

Create ? X

Name: Entity Type:

Time Between Arrivals

Type: Value: Units:

Entities per Arrival: Max Arrivals: First Creation:

Figure 4 : Arrival of Inspection entity at after every τ unit of time

Assign ? X

Name:

Assignments:

- Variable, Frate, 0
- Variable, Frate2, 0
- Variable, IP, 0
- Variable, CC1 and CC2, Corrective C1 cost+Corrective C2 cost+Corrective setup cost
- Variable, IC CC1 and CC2, 0
- <End of list>

Figure 4 Assigning Cost attributes to each maintenance activity

User Specified

Tally

Expression	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Downtime Comp 1	14.9000	3.17	8.0000	22.0000	8.0000	22.0000
Downtime Comp 2	0.1000	0.23	0.00	1.0000	0.00	1.0000
No of CM component 1	37.1000	4.64	31.0000	50.0000	31.0000	50.0000
No of CM component 2	0.6000	0.50	0.00	2.0000	0.00	2.0000
No of PM Component 1	1195.10	6.20	1182.00	1205.00	1182.00	1205.00
No of PM component 2	826.40	5.80	815.00	839.00	815.00	839.00
System Availability	0.9985	0.00	0.9977	0.9992	0.9977	0.9992
Time Before Replacement	8.1085	0.02	8.0776	8.1634	8.0000	16.0000
Total Down. Cost of System	0.1050	0.02	0.05600000	0.1610	0.05600000	0.1610
Total Insp. Cost of System	1.7803	0.01	1.7652	1.7908	1.7652	1.7908
Total Replac. Cost of System	15.4700	0.07	15.2920	15.6440	15.2920	15.6440
Total System Cost	17.3553	0.09	17.1520	17.5644	17.1520	17.5644

Figure 5 : Cost calculation obtained from Simulation model

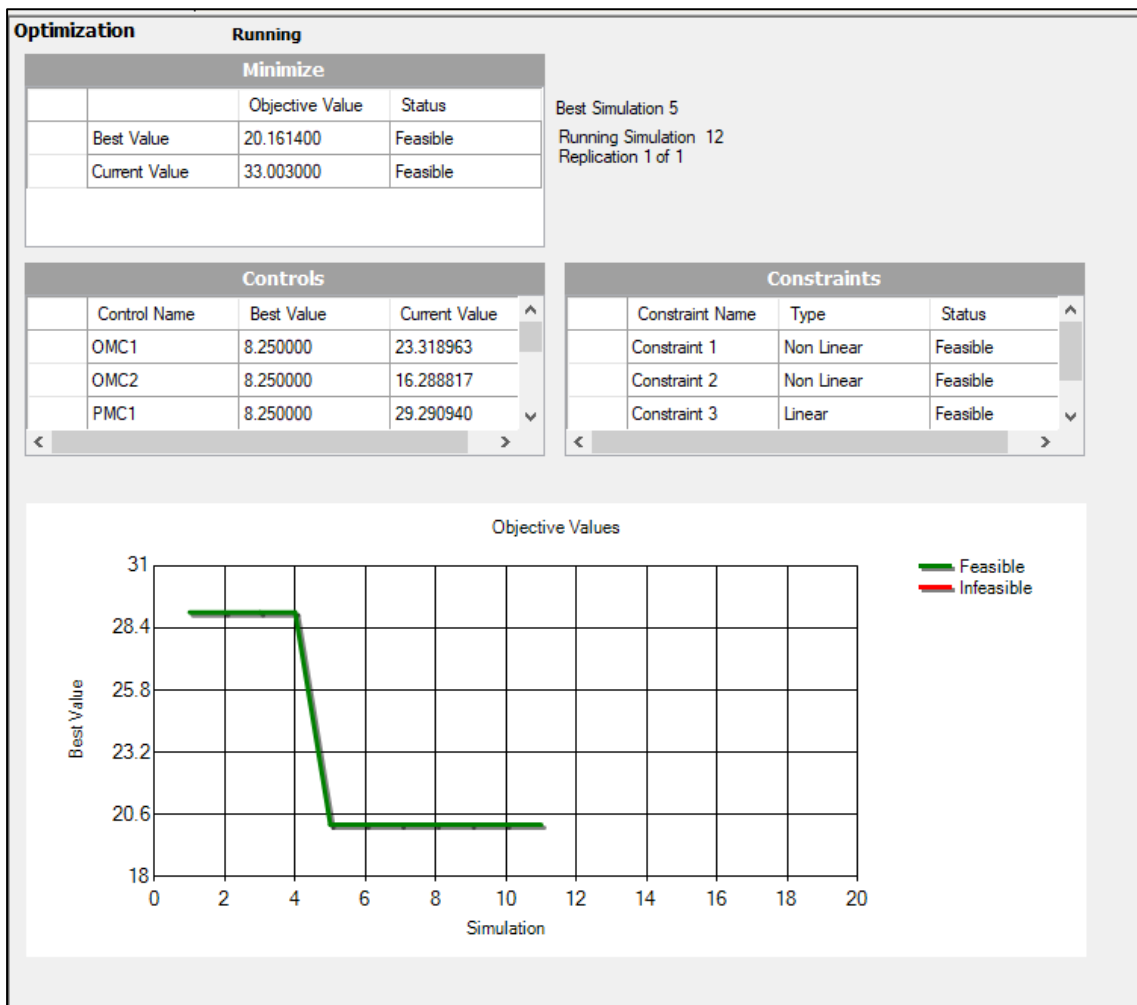


Figure 5 : Simulation Model Optimization On OptQuest

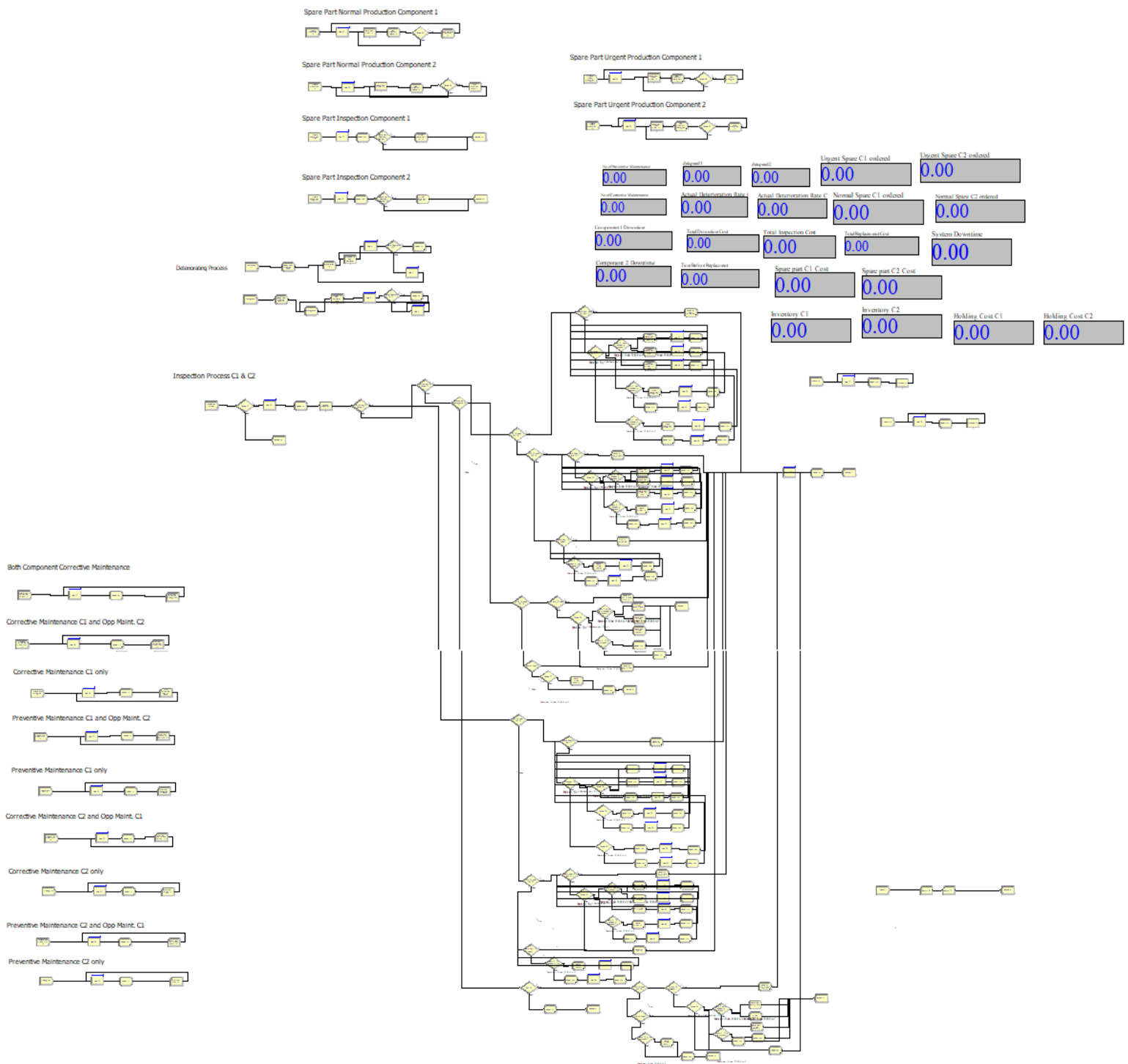


Figure 6 : Joint spare-part & Maintenance policy

Appendix A3

Analytical Model of Maintenance Policy

```

Editor - C:\Users\96656\Desktop\Thesis\Model\Matlab Code - Copy - CopyDTMCCost1.m
DTMCCost1.m
2 - clear all
3 - N = 2; % number of items
4 - tau = 5;
5 - alpha = [2,1];
6 - beta = [1,1];
7 - sigma = 0.1;
8 - Main = 1*tau;
9 - yp = [8,7];
10 - yo = [6,5];
11 - yc = [15,10];
12 - maxnc = round(max(yc)/sigma);
13 - MM = zeros(N,maxnc+1,maxnc+1);
14 - % constructing the trans proba matrices for the items
15 - nc = zeros(1,N);
16 - np = zeros(1,N);
17 - no = zeros(1,N);
18 - for i=1:N
19 - nc(i)=round(yc(i)/sigma);
20 - np(i)=round(yp(i)/sigma);
21 - no(i)=round(yo(i)/sigma);
22 - pp = starting_prob(tau,yc(i),sigma,alpha(i),beta(i)); % = ones(1,nc(i))/(nc(i)); %
23 - MM(i,1:nc(i)+1,1:nc(i)+1) = Trans_prob(np(i),nc(i),pp);
24 - end
25 - stdpp = zeros(N,max(nc(i)+1),1);
26 - % Computing steady state proba with using pi = pi*P and pi*ones=1 which
27 - % gives pi=[1,0,...,0]*inv([I-P]*), matrix [I-P]* is I-P with 1st column
28 - % equal to 1
29 - for i=1:N
30 - ee = ones(nc(i)+1,1);
31 - e0 = zeros(nc(i)+1,1);
32 - e0(1) = 1;
33 - t = zeros(nc(i)+1,nc(i)+1);
34 - t(:, :) = MM(i,1:nc(i)+1,1:nc(i)+1); % t is trans proba matrix for item i
35 - t = eye(nc(i)+1)- t; % I - P
36 - t(:,1) = ee; %replacing first column I-P by vector of ones
37 - % steady state is e0*inv(I-P)
38 -
39 - stdpp(i,1:nc(i)+1) = transpose(t)\e0;
40 - end
41 - % steady states probability for item 1 and 2 given in lines
42 - % diffe is variable used for compute difference between stdpp new and old
43 - diffe = 1; %initial difference
44 - error = .00001;
45 - % total probability of pm and cm for all items
46 - while (diffe>= error)
47 - stdpp_old = stdpp;
48 - for i=1:N
49 - t = zeros(nc(i)+1,nc(i)+1);
50 - t(:, :) = MM(i,1:nc(i)+1,1:nc(i)+1); % t is trans proba matrix for item i
51 - % probability that other than i item is on pm or cm maintenance
52 - tot_pm_cm = 0;
53 - pc2=0;
54 - for j = 1:N
55 - pc2=pc2+(stdpp(j,nc(j)+1));
56 - tot_pm_cm = tot_pm_cm + sum(stdpp(j,np(j)+1:nc(j)+1));
57 - end
58 - tot_pm_cm_i = tot_pm_cm - sum(stdpp(i,np(i)+1:nc(i)+1));
59 - % updating the transition probability matrix of item i with prob of cm and pm
60 - % the other i items
61 - for j=no(i)+1:np(i)
62 - t(j,:) = (1-tot_pm_cm_i) * t(j,:);
63 - t(j,1) = tot_pm_cm_i;
64 - end
65 - %Structural Dependency
66 - pc2=pc2-stdpp(i,nc(i)+1);
67 - t(1:np(i), :) = (1-pc2)*t(1:np(i),:);
68 - dd=pc2*ones(1,nc(i)+1);
69 - dd(np(i)+1:nc(i)+1)=zeros(1,nc(i)-np(i)+1)
70 - t=t+diag(dd,0)
71 - t = eye(nc(i)+1)- t; % I - P
72 - t(:,1) = ones(nc(i)+1,1); %replacing first column I-P by vector of ones
73 - % steady state is e0*inv(I-P)
74 - e0 = zeros(nc(i)+1,1);
75 - e0(1) = 1;
76 - stdpp(i,1:nc(i)+1) = transpose(t)\e0;
77 - end
78 - diffe = max(abs(stdpp-stdpp_old));
79 - end
80 - %Scaling of stdpp
81 - stdpp1=zeros(N,max(nc(i)+1));
82 - for i=1:N
83 - for j = 1:nc(i)
84 - stdpp1(i,j)=(tau*stdpp(i,j))/(tau*(sum(stdpp(i,1:nc(i))))+(Main*stdpp(i,nc(i)+1)));
85 - end
86 - stdpp1(i,nc(i)+1)=(Main*stdpp(i,nc(i)+1))/(tau*(sum(stdpp(i,1:nc(i))))+(Main*stdpp(i,nc(i)+1)));
87 - end
88 - pc1=stdpp1(1,nc(1)+1)
89 - pm1=sum(stdpp1(1,np(1)+1:nc(1)))
90 - pc2=stdpp1(2,nc(2)+1)
91 - pm2=sum(stdpp1(2,np(2)+1:nc(2)))
92 - po1=sum(stdpp1(1,no(1)+1:np(1))) * (pc2+pm2)
93 - po2=sum(stdpp1(2,no(2)+1:np(2))) * (pc1+pm1)
94 - %Cost Values
95 - CC1=120
96 - CC2=180
97 - PCC1=60
98 - PCC2=80
99 - ICC1=6
100 - ICC2=6
101 - Csetup=10
102 - Psetup=10
103 - InspSetup=2
104 - %Cost Calculation
105 - pc1andpc2=pc1*pc2
106 - onllypc1=pc1*sum(stdpp1(2,1:nc(2)))
107 - pc1andpo2=pc1*po2
108 - pc1andpm2=pc1*pm2
109 - pm1andpo2=pm1*po2
110 - pm1andpm2=pm1*pm2
111 - onllypm1=pm1*sum(stdpp1(2,1:nc(2)))
112 - pc2andpo1=pc2*po1
113 - pc2andpm1=pc2*pm1
114 - onllypc2=pc2*sum(stdpp1(1,1:nc(1)))
115 - pm2andpo1=pm2*po1
116 - onllypm2=pm2*sum(stdpp1(1,1:nc(1)))
117 - %Cost Calculation
118 - CM1andCMC2=(CC1+CC2+Csetup)*pc1andpc2
119 - CM1andOMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*pc1andpo2
120 - CM1andPMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*pc1andpm2
121 - OnlyCM1=(CC1+Csetup+ICC2+InspSetup)*onllypc1
122 - PM1andOMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pm1andpo2
123 - PM1andPMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pm1andpm2
124 - OnlyPM1=(PCC1+Psetup+ICC1+ICC2+InspSetup+InspSetup)*onllypm1
125 - CM2andOMC1=(CC2+Csetup+PCC1+ICC1+InspSetup)*pc2andpo1
126 - CM2andPMC1=(CC2+Csetup+PCC1+ICC1+InspSetup)*pc2andpm1
127 - OnlyCM2=(CC2+Csetup+ICC1+InspSetup)*onllypc2
128 - PM2andOMC1=(PCC2+Psetup+PCC1+InspSetup+ICC1+ICC2)*pm2andpo1
129 - OnlyPM2=(PCC2+Psetup+InspSetup+InspSetup+ICC1+ICC2)*onllypm2
130 - TotalCost=(CM1andCMC2 + CM1andOMC2 +CM1andPMC2+ CM2andOMC1 +CMC2and

```

Figure 6 : Matlab Code for Approximation method

```

Editor - C:\Users\96656\Desktop\Thesis\Matlab Code - Copy - Copy\DTMC2dv1.m
DTMC2dv1.m
1  clear all
2  N=2;% number of items
3  tau = 5;
4  alpha = [2,1];
5  beta = [1,1];
6  sigma=0.1;
7  yp = [8,7];
8  yo = [6,5];
9  yc = [15,10];
10 maxnc = round(max(yc)/sigma);
11 MM = zeros(N,maxnc+1,maxnc+1);
12 % constructing the transp proba matrices for the items
13 nc = zeros(1,N);
14 np = zeros(1,N);
15 no = zeros(1,N);
16 for i=1:N
17     nc(i)=round(yc(i)/sigma);
18     np(i)=round(yp(i)/sigma);
19     no(i)=round(yo(i)/sigma);
20     pp = starting_prob(tau,yc(i),sigma,alpha(i),beta(i)); % = ones(1,nc(i))/(nc
21     MM(i,1:nc(i)+1,1:nc(i)+1) = Trans_prob(np(i),nc(i),pp);
22 end
23 stdpp = zeros(N,max(nc(i)+1),1);
24 % Computing steady state proba with using pi = pi*P and pi*ones=1 which
25 % gives pi=(1,0,...,0)*inv([I-P]*), matrix [I-P]* is I-P with 1st column
26 % equal to 1
27 for i=1:N
28     ee = ones(nc(i)+1,1);
29     e0 = zeros(nc(i)+1,1);
30     e0(1) = 1;
31     t = zeros(nc(i)+1,nc(i)+1);
32     t(:, :) = MM(i,1:nc(i)+1,1:nc(i)+1); % t is transp proba matrix for item 1
33     t = eye(nc(i)+1) - t; % I - P
34     t(:,1) = ee; %replacing first column I-P by vector of ones
35     % steady state is e0*inv(I-P)
36     stdpp(i,1:nc(i)+1) = transpose(t)\e0;
37 end
38 % steady states probability for item 1 and 2 given in lines
39 %Joint DTMC chains for 2 components
40 t1 = zeros(nc(1)+1,nc(1)+1);
41 t1(:, :) = MM(1,1:nc(1)+1,1:nc(1)+1);
42 t2 = zeros(nc(2)+1,nc(2)+1);
43 t2(:, :) = MM(2,1:nc(2)+1,1:nc(2)+1);
44 t2d = kron( t1 , t2 );
45 row_e1 = zeros(1, (nc(1)+1)*(nc(2)+1) );
46 row_e1(1) = 1;
47 for k=no(1):np(1)
48     for j=np(2)+1:nc(2)+1
49         ll = k*(nc(2)+1);
50         t2d( ll+j, : ) = row_e1;
51     end
52 end
53 for k=np(1):nc(1)
54     for j=no(2)+1:np(2)
55         ll = k*(nc(2)+1);
56         t2d( ll+j, : ) = row_e1;
57     end
58 end
59 %sum(transpose(t2d))
60 size_2d = (nc(1)+1)*(nc(2)+1);
61 t2d = eye(size_2d) - t2d; % I - P
62 t2d(:,1) = ones(size_2d,1); %replacing first column I-P by vector of ones
63 % steady state is e0*inv(I-P)
64 stdpp2d = transpose(t2d)\transpose(row_e1);
65 % steady state matrix 2dimension
66 A=reshape(stdpp2d,nc(2)+1,[]);
67 D=transpose(A);
68 for i=1:yc(1)+1
69     for j=1:yc(2)+1
70         end
71     end
72 %Cost Values
73 CC1=120;
74 CC2=180;
75 PCC1=60;
76 PCC2=80;
77 ICC1=6;
78 ICC2=6;
79 Csetup=10;
80 Psetup=10;
81 InspSetup=2;
82 %Cost Factors
83 pc1=sum(D(nc(1)+1,1:nc(2)+1));
84 pc2=sum(D(1:nc(1)+1,nc(2)+1));
85 pm1=sum(sum(D(np(1)+1:nc(1),1:nc(2)+1)));
86 pm2=sum(sum(D(1:nc(1)+1,np(2)+1:nc(2)+1)));
87 pol1=sum(sum(D(no(1)+1,np(1),1:nc(2)+1)));
88 polandpc2 = D(nc(1)+1,nc(2)+1);
89 onlypc1 = sum(D(nc(1)+1,1:nc(2)+1));
90 onlypc2 = sum(D(1:nc(1),nc(2)+1));
91 onlypmc1=sum(sum(D(np(1)+1:nc(1),1:nc(2)+1)));
92 onlypmc2 = sum(sum(D(1:nc(1),np(2)+1:nc(2)+1)));
93 polandpo2=sum(D(nc(1)+1,np(2)+1:nc(2)+1));
94 polandpm2=sum(D(nc(1)+1,np(2)+1:nc(2)+1));
95 pmlandpo2=sum(sum(D(np(1)+1:nc(1),no(2)+1:np(2)+1)));
96 pmlandpm2=sum(sum(D(np(1)+1:nc(1),np(2)+1:nc(2)+1)));
97 pc2andpo1=sum(D(no(1)+1:np(1),nc(2)+1));
98 pc2andpml1=sum(D(np(1)+1:nc(1),nc(2)+1));
99 pm2andpo1=sum(sum(D(no(1)+1:np(1),np(2)+1:nc(2)+1)));
100 %Cost Calculation
101 CMlandCMC2=(CC1+CC2+Csetup)*polandpc2;
102 CMClanOMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*polandpo2;
103 CMClanPMC2=(CC1+PCC2+Csetup+ICC2+InspSetup)*polandpm2;
104 OnlyCM1=(CC1+Csetup+ICC2+InspSetup)*onlypc1;
105 PMClanOMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pmlandpo2;
106 PMClanPMC2=(PCC1+PCC2+Psetup+InspSetup+ICC1+ICC2)*pmlandpm2;
107 OnlyPM1=(PCC1+Psetup+ICC1+ICC2+InspSetup+InspSetup)*onlypmc1;
108 CMC2andOMC1=(CC2+Csetup+PCC1+ICC1+InspSetup)*pc2andpo1;
109 CMC2andPMC1=(CC2+Csetup+PCC1+ICC1+InspSetup)*pc2andpml1;
110 OnlyCM2=(CC2+Csetup+ICC1+InspSetup)*onlypc2;
111 PMC2andOMC1=(PCC2+Psetup+PCC1+InspSetup+ICC1+ICC2)*pm2andpo1;
112 OnlyPMC2=(PCC2+Psetup+InspSetup+InspSetup+ICC1+ICC2)*onlypmc2;
113 TotalCost= CMlandCMC2 + CMClanOMC2 + CMClanPMC2+ CMC2andOMC1 +CMC2andPMC1+ OnlyCM1 + OnlyCM2 +

```

Figure 6 : Matlab Code for Two-Dimensional Markov Chain method