COOPERATIVE AND COORDINATED CONTROL OF MULTIVEHICLE SYSTEMS

BY

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To My Parents

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DISSERTATION ABSTRACT

NAME:	Bilal Jafar Hamed Karaki
TITLE OF STUDY:	Cooperative and Coordinated Control of Multivehicle Systems
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Cooperative control of multiagent systems is an active and increasingly studied topic of research, with many engineering applications arising in multi-robot systems, rescue missions, and robotic manipulation. This thesis addresses the cooperative leader-following and leaderless control of multi-vehicle systems. In the first part of the thesis, given a geometric-pattern, the formation problem is investigated for discrete-time higher order multiagent systems under denial-of-service (DoS) attacks. The considered attack scenario compromises agents rather than edges of the communication topology. This leads to a severe behavior of the multiagent systems and most of the existing techniques studying arbitrary switching dynamics fail to investigate this situation. Besides, we propose a distributed scaled-consensus protocol that utilizes a probabilistic quantization algorithm to solve the practical mean-square scaled-consensus problem. The

second part of the thesis is focused on the leader-following and leaderless consensus problems of discrete-time multiagent systems. Employing an observer, the distributed control law of each agent is designed using local information to guarantee consensus, and the corresponding sufficient conditions are obtained by exploiting graph and Lyapunov theory approach. A modified distributed event-triggered consensus protocol is also designed to reduce transmissions over the network. Detailed analysis of the leaderless and the leader-following consensus is presented for both observer-based and full information protocols. Then the consensus problem for nonlinear multiagent systems with time delay is studied based on an asynchronous event-triggered mechanism. We select an agent as a leader that is constrained by its neighbors and controlled by the proposed protocol as other agents. It has been theoretically demonstrated that the consensus problem of multiagent systems with nonlinear perturbation and time delays can be achieved if the control gains and the triggering parameters are appropriately selected. Finally, output synchronization of heterogeneous time-delay discrete-time multiagent systems is investigated via output couplings. Sufficient conditions are obtained to guarantee dissipativity for agents modeled by descriptor dynamics. Then a distributed consensus protocol based on the outputs of the neighboring agents is proposed to show that the individual dissipativity leads to output synchronization of the overall multiagent system. The proposed synchronization scheme exhibits strong robustness facing norm-bounded uncertainties. The proposed cooperative protocols are validated through simulation examples and the results show considerable improvements.

ملخص الرسالة

الاسم: بلال جعفر حامد كركي

عنوان الدراسة: أنظمة التحكم التعاونية للمركبات المتعددة

التخصص: هندسة نظم التحكم

تاريخ الدرجة العلمية: أيار ٢٠٢١

ازداد الاهتمام البحثي لأنظمة التحكم التعاونية لارتباطها في العديد من التطبيقات الهندسية مثل التحكم في مجموعة من الروبوتات أو مهمات الإنقاذ الجماعي. هذه الرسالة تدرس أنظمة التحكم التعاونية للمركبات المتعددة بوجود وعدم وجود قائد للمركبات. في البداية، نقدم طريقة تحكم تُجبر المركبات على تغيير اماكنها لعمل شكل هندسي تم تعيينه مسبقا بالرغم من وجود حجب لخدمة الاتصال بين المركبات. تم اقتراح نظام تحكم متوزع بحيث يتم وصول المركبات الى أماكن وسرعات بنسب مختلفة عندما تكون القياسات غير مثالية. في الجزء الثاني من الرسالة، كان تركيز البحث على أنظمة التحكم التعاونية بوجود قائد للمركبات عن طريق استخراج معلومات عن المكان والسرعة دون قياسها. تم اشتقاق شروط كافية لضمان وصول المركبات الى حالة الاتفاق على المكان والسرعة باستخدام قواعد ليابانوف. تم تصميم برتوكول جديد بالإضافة الى مُنظم ارسال واستقبال القياسات لتقليل كميتها عبر شبكة الاتصالات. بالإضافة الى ذلك، تحليلات مفصلة قُدمت حول هذا البروتوكول لإثبات صحتها بحالتي المركبات الجماعية بوجود وعدم وجود قائد. بعدها تم تعميم الحالات السابقة لحل المشكلة عندما يكون النظام غير خطي. قمنا بدراسة تأثير وجود التأخير الزمني في نظام المركبة. جميع أنظمة التحكم التعاونية تمت محاكاتها عن طريق امثلة عملية لبيان فعاليتها وقدرتها على تحقيق الاتفاق بين المركبات.

CHAPTER 1

INTRODUCTION

This thesis mainly investigates the cooperation problem of multiagent systems, which has gained considerable attention in the last decade in the control community. Rapid advances in technologies, communication, and sensing have enabled various devices to be provided in a cooperative manner. In the field of cooperative systems, it is possible to use networking architectures in which agents communicate and cooperate to achieve a predefined common goal. The cooperative scenario in multivehicle systems promises much in terms of joint goal team, performance, and reliability. However, it requires expanding the conventional control theory, computation facilities, and communication policies. In this thesis, we are interested in cooperative multivehicle systems since they become very important and show up in new engineering applications being investigated these days. Moreover, they provide an interesting structure where control theory, communication, and computation facilities are integrated into unified settings.

1.1 Background and Motivation

Cooperative control appears to be of great importance when a single complex agent/vehicle is equivalently replaced by multiple yet simpler agents. In this scenario, two different control methodologies are commonly applied for controlling multiple agents: a centralized and a distributed control architecture. The centralized approach is established using a sufficiently powerful central control system such that it is responsible to control a whole group of agents. The limited bandwidth of communication, agent failure, and packet losses lead to drawbacks of this approach such as the lack of robustness. On the other hand, the distributed control architecture does not require a central controller at the cost of becoming far more complicated in structure and analysis point of view. Both techniques are of great importance depending on the circumstances of the real applications, the distributed approach is more advantageous to be used for multiagent systems due to the close relationship among agents in the group where information exchange plays a primary role. Solving the cooperation problem based on distributed techniques has several advantages such as less operational costs, strong robustness, adaptability, and flexibility.

In a cooperative control framework, consensus refers to the group behavior that all of the agents reach an agreement regarding a global goal of interest that is intimately related to the trajectories behavior of all agents. It is achieved through a consensus protocol that can be identified by an interaction law and the information exchange algorithm between the agents over a communication network. A formal investigation of reaching consensus which describes how groups of experts in manage-

ment reach agreement on a joint team objective in 1960s (see [2], [3], [4] and references therein). The idea behind consensus serves as a fundamental principle for the design of distributed multiagent cooperation and coordination algorithms. Consequently, the consensus problem has become an active research direction in the investigation of distributed multiagent coordination. To bridge the gap between the study of consensus algorithms and many physical properties inherited in practical systems, it is necessary and meaningful to study consensus by considering many practical factors, such as actuation, control, communication, computation, and vehicle dynamics, which characterize some important features of practical systems. This is the main motivation to study consensus.

One of the most important aspects that should be considered in the design of cooperative protocol is the communication limitation and the controller actuation schemes over the communication network. A practical design of multiagent systems usually equips each agent with a processor that is responsible for actuating the controller laws and collecting data from neighboring agents according to a specific protocol. It is of great interest to let the cooperative schemes to provide sampling procedure in order to decrease the number of transmissions between agents. The event-driven fashion is preferable for the application in hand due to the limited resources of embedded processors and band limited communication networks. However, reducing the number of information updates may degrade the performance of the overall system, this means that the design procedure should preserve desired properties such as stability and convergence. Although multiagent systems have many benefits, they pose one of the most challenging problems that is the vulnerability to cyber attacks. As the com-

munication links are introduced, the multiagent system becomes vulnerable to cyber attacks. Another important challenge in cooperative control of multi-vehicle systems is to cooperatively control the vehicles' positions such as the robots form a desired geometric pattern [5], [6]. There have been several control methodologies for multi-vehicle cooperation in the existing literature [7], [8], [9], [10], [11]. When one of the vehicles is designated as a leader and other robots are follower, the approach is called leader-following [12], [13], [14]. The follower vehicles have to position themselves in a cooperative manner such that their locations maintain the desired relative distance with respect to the leader vehicle. An example of cooperative control objective in multi-agent systems is the consensus, i.e. all the states of agents are supposed to converge to a global agreement on a state value. Formation control is also an important scheme where the objective is to make agents move in specified geometric shape [15], [16]. Coverage scheme occurs when a group of agents produces a maximal spread of their network without disconnection in the coverage goal. In addition, cooperative agents could be distributed as a group of sensors to collect information and estimate a phenomenon of interest. Flocking cooperation is used to force agents to behave like natural animals such as flocking birds [17]. Rendezvous is another well-known form of cooperation problems in space [18], [19]. This problem is equivalent to reaching an agreement in position by a number of vehicles within an interaction graph. The reader might see [20] and the references therein helpful for a more detailed discussion about the rendezvous problem. This kind of problems becomes challenging under variations in the graph topology. On the other hand, flocking is considered to be more challenging than rendezvous in space due to its requirements in both interactions between

agents and vehicle-to-obstacle collision avoidance.

1.2 Objectives of the Thesis

The long-term goal of this research aims at providing an effective cooperative and coordinated control of multivehicles systems. The research findings will contribute to the advancements of control theory to guarantee the stability of cooperated vehicles. This research also intends to investigate control over networks to utilize event triggered mechanisms for vehicles. Particularly, the research has the following core objectives:

1. To design a cooperative protocol to achieve consensus for multivehicle systems, which will guarantee cooperation in multivehicle systems under packet losses and/or adversarial attacks.

2. To analyze and synthesize an event triggering mechanism for cooperative multivehicle systems over communication networks, in face of uncertainties.

3. To investigate the problem of distributed leader-following consensus for general multivehicle systems and to provide cooperative observer-based methodology in order to force multivehicles reach consensus.

1.3 Steps and Control Methodology

The research approach of this thesis is intimately related to the rigorous mathematical analysis to guarantee cooperation and coordination among agents via distributed protocols. This approach is applied to the triggering mechanisms, the cooperation

process, and the impact of cyber attacks as well. Discrete-time analysis is investigated to show how the event-based structure should be designed so its performance matches the continuous-time counterpart.

To save the energy consumption involved in a consensus protocol, the event-triggering methodology should be investigated for multiagent systems. Based on an asynchronous event-triggered mechanism where local information of a communication graph is involved, a distributed consensus protocol will be used to reduce the network communication congestion for achieving state consensus for multiagent nonlinear systems. To avoid complications involving data fusion, the consensus protocol should allow every agent to broadcast its state when its own triggering condition instead of using continuous updates and without continuous monitoring of the triggering condition. We select an agent as a leader that is constrained by its neighbors and controlled by the proposed protocol as other agents.

By means of an observer, the distributed protocol of each agent is designed using local information to guarantee consensus. A modified distributed event-triggered consensus protocol is designed to reduce communication congestion. Detailed analysis of the leaderless and the leader- following consensus is presented for both observer-based and full information protocols.

For a secure consensus problem where multiple attacks can be seen as one channel attack, in this thesis, the adversaries could attack multiple agents independently/distributively. The considered attack scenario compromises agents rather than edges of the communication topology. A novel distributed consensus protocol is addressed to reduce the effects of such a DoS attack. The proposed protocol uses static

output-feedback.

A distributed consensus protocol based on the outputs of the neighboring agents is adapted to show that the individual dissipativity leads to output synchronization of the overall multiagent system. Moreover, the counterparts of the result of the cooperation based on dissipativity for the multiagent system with event triggering mechanism is also investigated.

Through the thesis, the cooperation problems are transformed into stability problems. The Lyapunov theory is used to guarantee cooperation among agents and Lassale's principle is not needed. Lyapunov stability theory hinges on the deployment of norms that physically represents the total energy of dynamical systems. By applying an appropriate transformation, the consensus/formation problem is transformed into a stability problem and necessary and/or sufficient conditions are given in terms of bilinear matrix inequalities and the consensus gain can be obtained based on linear matrix inequities (LMI). The feasibility of the LMI conditions can be checked easily using MATLAB-CVX-toolboxes. The norm $\|.\|$ denotes the Euclidean norm of a vector to measure the disagreement among agents. It should be noted that convergence speed could be changed when another norm is employed.

1.4 Thesis Outline and Statement of Contributions

This section gives a summary of this thesis. It describes each chapter's content and results. We also present journal papers in which material used in this thesis has been

or is going to be published. This dissertation provides a substantial number of results to the cooperated multivehicles in face of uncertainty/nonlinearity, packet losses in a communication network, and limited communication bandwidth. In addition, effective event-triggering that generates sporadic executions is investigated since it is an effective way of improving the efficiency of communication in real applications. The capability and features of the proposed cooperation strategies could be used for various vehicles with uncertainties, nonlinear dynamics, and time delays. Besides, generating sporadic executions of distributed protocols represents an effective way of improving the efficiency of communication in many real applications.

Chapter 2: DISTRIBUTED MULTIAGENT SYSTEMS: Chapter 2 provides relevant background for the topics considered in the thesis.

Chapter 3: DISTRIBUTED SECURE FORMATION: Output Feedback Approach In Chapter 3, the formation problem of multiagent systems is investigated for multiagent systems under denial-of-service (DoS) attacks. The considered attack scenario compromises agents rather than edges of the communication topology. A novel distributed consensus protocol is addressed to reduce the effects of such a DoS attack. The proposed protocol uses static output-feedback. By applying an appropriate transformation, the consensus/formation problem is transformed into a stability problem and necessary and sufficient conditions are given in terms of bilinear matrix inequalities. Then, a sufficient condition is derived to design the formation gain based on linear matrix inequity (LMI).

Chapter 4: QUANTIZED SCALED-CONSENSUS: In this Chapter, the practical mean-square scaled-consensus problem of multiagent systems is studied.

Higher-order general linear dynamics is investigated, where all agents are subject to faults or denial of service attacks (DoS). The considered network comprises agents taking quantized states under undirected communication channels. First, we propose a distributed scaled-consensus protocol that utilizes a probabilistic quantization algorithm. Second, scaled-disagreement analysis is provided via mean-square stability methodology. Finally, sufficient conditions in terms of linear matrix inequalities are presented in order to obtain scaled-consensus gain for both leaderless and leader-follower cases. The presented theoretical results are examined through two simulation examples.

Chapter 5: DISTRIBUTED EVENT-TRIGGERED CONSENSUS PROTOCOLS: This Chapter focuses on the leader-following and leaderless consensus problems of discrete-time multiagent systems. A distributed observer-based consensus protocol is proposed to investigate the consensus problem for multiagent systems of general discrete-time linear dynamics. By means of the observer, the distributed control law of each agent is designed using local information to guarantee consensus, and the corresponding sufficient conditions are obtained by exploiting graph and Lyapunov theory approach. A modified distributed event-triggered consensus protocol is designed to reduce communication congestion. Detailed analysis of the leaderless and the leader-following consensus is presented for both observer-based and full information protocols. Finally, some simulation examples are provided to demonstrate the effectiveness and capabilities of the established theories.

Chapter 6: LEADER-FOLLOWING CONSENSUS WITH TIME-VARYING DELAY: This Chapter studies the consensus problem for discrete time

nonlinear multiagent systems with time delay. Based on an asynchronous event-triggered mechanism where local information of a communication graph is involved, a distributed consensus protocol is proposed to reduce the network communication congestion for achieving state consensus for multiagent nonlinear systems. To avoid complications involving data fusion, the consensus protocol should allow every agent to broadcast its state when its own triggering condition instead of using continuous updates and without continuous monitoring of the triggering condition. To save the energy consumption involved in a consensus protocol, the proposed event-triggering guaranteed cost is investigated in this Chapter for multiagent systems. We select an agent as a leader that is constrained by its neighbors and controlled by the proposed protocol as other agents. It has been theoretically demonstrated that the consensus problem of multiagent systems with nonlinear perturbation and time delays can be achieved if the control gains and the triggering parameters are appropriately selected. Sufficient conditions are given to guarantee that the agents achieve consensus. Moreover, the gain matrix and triggering parameters of the proposed protocol are cast into matrix inequalities conditions. A simulation example is given to illustrate the effectiveness of the theoretical results.

Chapter 7: OUTPUT-SYNCHRONIZATION: Chapter 7 investigates output synchronization of heterogeneous time-delay discrete-time multiagent systems via output couplings. Firstly, sufficient conditions are provided to guarantee dissipativity for an individual agent modeled by the descriptor dynamics. A fully distributed consensus protocol based on the outputs of the neighboring agents is proposed to show that the individual dissipativity leads to output synchronization of the overall multia-

gent system. The proposed synchronization scheme exhibits strong robustness facing norm-bounded uncertainties. Moreover, the counterparts of the result of the cooperation based on dissipativity for the multiagent system with event triggering mechanism is also investigated. The proposed triggering mechanism and the consensus protocol solve the synchronization for all admissible uncertainties and time varying delay. Finally, a numerical heterogeneous model is carried out to illustrate the effectiveness of the proposed strategies.

Chapter 8: : CONCLUSIONS: In Chapter 8 we conclude the thesis and discuss future challenges, suggested problems, and new directions.

Publications: This thesis is primarily based on the publications:

- Bilal J. Karaki, and M. Mahmoud. "Quantized Scaled-Consensus of Linear Multiagent Systems on Faulty Networks", (International Journal of Systems Science, doi: 10.1080/00207721.2020.1869343), 15-Jan-2021.

- M. Mahmoud, Bilal J. Karaki. "Output-Synchronization of Discrete-Time Multiagent Systems: A Cooperative Event-Triggered Dissipative Approach", IEEE Transactions on Network Science and Engineering, doi: 10.1109/TNSE.2020.3029078, 14-Oct-2020.

- Bilal J. Karaki, and M. Mahmoud. "Scaled Consensus Design for General Linear-Multiagent Systems Under the effect of Attacks and Communication-Delays", (IEEE Transactions on Control of Network Systems, under review).

- Bilal J. Karaki, and M. Mahmoud. "Distributed Event-Triggered Consensus

Protocols for Discrete-Time Multiagent Systems", (IEEE Transactions on Systems, Man, and Cybernetics: Systems, under review).

- Bilal J. Karaki, and M. Mahmoud. "Leader-following Consensus for a Class of Nonlinear Multivehicle systems with Time-varying Delay", (IEEE Transactions on Cybernetics, under review).

CHAPTER 2

DISTRIBUTED MULTIAGENT SYSTEMS

In the past decades, cooperative control of multiagent systems has attracted much research attention due to its widely potential applications in unmanned aerial vehicles, power systems, sensor networks, mobile robots, etc., see [21]–[22]. In this research, all agents work cooperatively such that the group achieves desired collective behaviors by employing a cooperative control policy. One of the most typical aspects in cooperative control problems is the consensus problem, which is generally classified into leaderless consensus [23]-[24] and leader-following consensus [25]-[26]. The leaderless consensus approach requires all agents to reach a common value or variable. However, the leader-following scheme needs the followers to track the leader's state, where the leader can be considered as a reference generator and is not affected by the followers [27]-[28]. According to the existing research about multiagent systems, cooperation performance is primarily influenced by the behavior of the agents and the communication topology

[29].

2.1 Introduction to Cooperative Multiagent Systems

In recent years, cooperation and coordination of a group of vehicles have received increasing attention from various scientific communities. A central challenge for cooperative multiagent systems is to design a distributed protocol based on local measurements such that consensus is achieved. We call the multiagent system achieves consensus agreement if a prescribed common collective behavior is reached and maintained by the networked system. Consensus has been used in a wide variety of applications including system control theory, statistical physics, biology, applied mathematics, and computer science [30]. Various results have been obtained on this problem [31], [32], [33]. For instance, three formation algorithms have been investigated for a group of autonomous vehicles in [31]. Li and Guan [32] used center manifold to achieve a consensus of nonlinear multiagent systems. In addition, the average consensus was solved for multiagent systems with switching topologies based on Lyapunov analysis techniques [33]. In the past decades, various consensus protocols have been proposed to achieve the agreement, such as linear first-order consensus protocol [33], linear second-order MASs consensus protocol [34], consensus protocol for high-order linear dynamics [35], nonlinear protocols [36], etc.

The consensus problem of multiagent systems is one of the challenging problems in cooperative control methodologies and has a wide variety of scientific applications,

such as formation problem [37], [38], [39], wireless sensor networks [40], intelligent traffic management [41], rendezvous of spacecrafts [42]. For the sake of consensus, many researchers have addressed several effective control strategies, such as robust control [43], observer-based methods [44], pining methodology [45], adaptive control [46]. Thus, multiagent systems have been researched extensively by various scientific communities in automatic control, computer science, and other fields [47]. Hence, profound results on the properties of multi-agent systems have been established [48], [49]. Generally speaking, the consensus objective is guaranteed by finding a dynamic interaction protocol that drives agents to reach an agreement on a common target. It is a fundamental demand for investigating cooperative control methodologies of multi-agent control systems [50]. Based on Lyapunov methods and Lasalle's invariance principle, together with the algebraic graph and matrix theories, consensus methodologies have been reported in the literature, see, e.g. [51]. For instance, it is demonstrated that the consensus of position and velocity second-order challenge is investigated for multiagent systems in [52].

Achieving consensus is an essential requirement for the cooperation of multiagent system [53]. For instance, leaderless-consensus is investigated for discrete-time first-order multiagent systems under stochastic communication noise [54]. In addition, several significant findings on consensus problems have been addressed such as synchronization [55], [56], flocking [57], [58], and swarming [59], [60]. On the other hand, the leader-following consensus of linear and nonlinear multiagent systems is addressed in [61], [62], where the states are updated in accordance with neighboring information exchange in order to track the behavior of the leader.

Synchronization and cooperation of multi-agent systems have attracted considerable attention due to their extensive engineering applications such as control theory, microgrids, computer science, etc. [63], [64], [65], [66], [67], [68]. In recent decades, the cooperation and coordination of multi-agent systems have been substantially addressed [69], [70], which is a basic requirement for studying multiagent systems [71]. Cooperation among agents without a centralized controller, while each agent can only access local/neighboring information is one of the main challenges in multiagent systems. This problem is one of the most fundamental issues of cooperation control, whose main duty is to implement a distributed protocol via local and neighboring information so that the states or the outputs of agents can reach a prescribed agreement [72]. Analytical and numerical methodologies to solve the synchronization problem have been reported in the literature, which is evident from several monographs ([73], [74]), and numerous articles in different settings, such as consensus ([75]) and the references therein.

2.1.1 Scaled-Consensus

Recently, various new consensus problems have emerged such as scaled consensus problem [76]. Scaled-consensus is considered as one of the important coordination problems in multiagent systems. Scaled-consensus is considered a generalization of the usual consensus. The scaled-consensus was first defined by S. Roy [76], wherein agents' states reach dictated ratios in the asymptote. In other words, the multiagent system is said to reach scaled-consensus if the ratios between the agents' trajectories converge to predefined known constants. With this observation, usual consensus [77],

[78], [79], and group consensus [80], [81], [82], etc. become special cases of the scaled-consensus by adjusting an appropriate agents' scales. Shang [83] derived sufficient and necessary conditions required to achieve finite-time scaled-consensus for first-order dynamics. Hou et al. [84] investigated a scaled cluster consensus for first-order discrete-time multiagent systems. There are several interesting problems on cluster consensus desired to be more investigated. For instance, most existing results imposed strict conditions to achieve cluster consensus (see [85], [86], [87], [88], [89], [90], [91], [92]). More precisely, the coupling gains should be negative or positive and zero-sum for inter-cluster desynchronization. However, the coupling weights are usually stored distributively over the graph topology. In addition, the zero-sum coupling constraint is usually difficult to meet. Inspired by the work of Roy [76], a scaled cluster consensus is investigated where no conditions are imposed on the network topology in [93].

2.1.2 Output Feedback Protocols

The state-feedback consensus protocols are mainly based on a restrictive assumption that all the agent's states are accessible. However, generally speaking, the state variables of each agent can not be directly measured in practical applications, especially for agents with higher-order dynamics. Thus, full state feedback algorithms in such cases should be replaced by output feedback-based techniques. However, the static output feedback consensus protocols of multi-agent systems is more difficult than that of state feedback and observer based protocols. The output feedback control scheme has been applied to the consensus problem in [94], [95], [96]. However, cooperation via static output feedback is generally restrictive (see [97], [98] for more details). An

alternative is to use observer-based output feedback protocols. Some results on this topic can be found in [99], [100], [101], [102], and the references therein.

2.1.3 Dissipativity Based Approach

Consensus protocols for linear and nonlinear multiagent systems are investigated with the help of the Lyapunov method, algebraic graph theory, and matrix theory in [74]. Moreover, various consensus problem of second-order multiagent systems is addressed where the consensus of velocity and position can be achieved [103]. Dissipativity as well as its special case passivity has been exploited to analyze the stability of multiagent systems. The finite-time synchronization problem is investigated based on the passivity approach by exploiting adaptive state feedback with linear and nonlinear couplings [104]. Sufficient delay-dependent conditions are derived to realize the cooperation of a class of nonlinear multiagent systems such that the trajectories converge to the convex hull with mixed $\mathcal{H}_\infty$ and passivity index [105]. Under switching topologies, consensus protocol with the help of the dissipativity is proposed to ensure that the agents can reach an agreement [106]. In [107], distributed suboptimal $\mathcal{H}_\infty$ consensus problem was established based on the dissipativity of uncertain systems.

2.2 Practical Issues in Multiagents

The integration of logical control, communication, and physical processes/agents have introduced the control community to cyber-physical systems. With recent progress in control theory and its applications, cyber-physical systems have become one of the

major subjects in the field. Its powerful capabilities provide significant advantages and have been widely used in industrial, aerospace, chemical processes, and critical applications

2.2.1 Cyber Attacks

Networked multivehicles systems can possess a changing topology or time-varying dynamics due to failures or creations of links and/or nodes in various scenarios. There has been an enormous surge of interest among theorists from various disciplines of science and engineering in challenges appear in multiagent and multivehicles cooperation problem over networked systems. Although multi agent systems have many benefits, they pose one of the most challenging problems that is the vulnerability to cyber-attacks. As the communication links are introduced, the Multi-agent system becomes vulnerable to cyber-attacks.

Multiagent systems are vulnerable to cyber-threats due to the deep integration of physical agents and networking. Consequently, as highlighted in [108], [109], secure control has become an essential aspect of research in networked systems. Therefore, it is vital to investigate the coordination and cooperation problem of multiagent systems under cyberattacks, and many preliminary results have been investigated in this regard [110], [111], [112], [113]. By interrupting significant data exchanged between agents, the adversary aims to affect the consensus among agents and steer the agents to an unacceptable operating point. This type of cyberattack is called denial of service attacks (DoS) which is the most common and simplest type of attacks. The attacks arrive in a random and intermittent manner due to the fluctuation in adversary's

statistical factors. DoS attacks are often modeled by the Markov process or Bernoulli process [114], [115]. The secure consensus algorithms presented in [116], [117], [118] were devoted to a class of attacks such that all channels are seen as one channel from the adversary's point of view. Moreover, multiagent systems are more vulnerable to multiple attacks with different and independent characteristics since the agents always communicate with each other individually [119]. Moreover, investigating multiple DoS attacks case is more realistic and practical. Multiple attack scenario, where the adversary can compromise partial or all agents at any time, is the major motivation of this study.

Besides, reliable distributed consensus protocols have been emerging as an important research area since failures and attacks on communication channels are unavoidable in multiagent systems. It is worth noting that the existence of DoS attacks in multiagent systems leads to adverse effects on consensus performance and may cause disagreement among agents. The DoS attack in cooperative systems is a challenging and critical problem because the adversaries can compromises each channel/node independently [120]. In such a scenario, when the attacker successfully launches a DoS attack, the destination agent will be prevented from data exchange [121]. Therefore, a secure control design is an efficient mechanism since it preserves the system from possible attacks. For example, the work in [121], investigated distributed secure consensus protocol for linear multiagent systems under DoS attacks. It is worth noting that, most of the investigated secure consensus protocols under DoS attacks have been devoted to a class of adversaries that compromises all channels as one channel [122], [123], [124]. However, addressing the consensus problem where the adversary

can compromise partial agents/channels or all agents/channels is more practical. A secure protocol that achieves asymptotic consensus against multiple adversaries has been developed in [125]. More importantly, a predictive secure control for multiagent systems under DoS attacks has been investigated over quantized networks [126].

2.2.2 Quantization

Most of the aforementioned studies primarily focus on ideal communication patterns where quantization is not considered. However, the precision of the controller and communication bandwidth are constrained and limited. Hence, practical measurements produce error which could affect the performance of the consensus algorithms. Since quantization error is inevitable, the consensus over quantized communication channels has become an active research topic (see[127], [128], [129], [130] and references therein). In [131] sampled-data based consensus for MASs under logarithmic quantizers is discussed.

2.2.3 Time-Delay

It is worth pointing out that time-delay among agents is inevitable in real applications of multiagent systems. Although it has growing attention in recent years, the problem is rarely investigated for consensus protocols. Most of the existing research focuses on multiagents without considering the time-delay. Consequently, time-delay has become an essential aspect of research in the coordination and cooperation problem of multiagent. Introducing time delay could affect the consensus performance and steer

the agents to an unacceptable operating point. Various theoretical approaches have been investigated to solve consensus problem for time-delay multiagent systems [132], [133], [134].

2.2.4 Heterogeneous agents

On the other hand, most of the attention had been paid to cooperation among homogeneous agents rather than heterogeneous systems. Whereas synchronization methodologies for homogeneous multiagent systems have been thoroughly investigated, synchronization of non-identical agents still remains a challenging problem. Interesting results have been addressed for the multiagent synchronization problems for heterogeneous dynamics, including the descriptor system [135], [136].

2.3 Discrete vs Continuous

The aforementioned results are all continuous, and the control law design requires state information continuously which is too ideal to achieve consensus for multiagent systems. By comparison, for the sampled-data control methods, which are based on difference equations, the trajectory information is just broadcasted at discrete instants among the agents and the control law remains unchanged within two consecutive sampling times. It is expected that in real applications, due to the limited bandwidth and energy constraints of wireless communication, continuous protocols are obsolete and can not be implemented [137],[138],[139]. Implementing a periodical sampling scheme or the time-triggered policy could waste resources such as network bandwidth due to

the numerous redundant sampling data. This leads to data congestion particularly when the sampling intervals are very short [140]. It is for this reason that efficient control schemes should be implemented to reduce data transmission and control updates. Therefore, the event-triggered policy is an effective method to mitigate the unnecessary waste of communication and computation resources, and the sampled-data is released to the shared communication network when a defined condition is violated [141].

It is worth mentioning that continuous protocols require agents to exchange information continuously over the communication networks. However, this is an ideal case and should be avoided since the information exchange among the agents (especially the agents/vehicles which are physically distributed) takes place over digital and wireless networks. This implies that the data is sampled and discrete-time systems are more appropriate to analyze the cooperation in multiagent systems. Nevertheless, continuous analysis can be used to solve this issue based on event-triggering schemes. For discrete-time multiagent systems, the consensus and formation conditions are considerably harder to derive than its continuous counterpart [142], [143], [144].

Most of the synchronization passivity-based methods have considered continuous-time multiagent systems; however, implementation of cooperative discrete-time multiagent systems is needed. Discrete-time output consensus for heterogeneous multiagent systems is addressed using the internal model principle [145]. The internal model is exploited to convert the synchronization problem into an output stabilization problem.

2.4 Event-Triggered vs Periodic Cooperative Protocols

One of the most important aspects that should be considered in the implementation of cooperative algorithms is the communication limitation and the controller actuation schemes over the communication network. A practical design of multiagent systems usually equips each agent with a processor that is responsible for actuating the controller laws and collecting data from neighboring agents according to a specific protocol. It is of great interest to let the cooperative schemes to provide protocols that reduce the number of transmissions between agents. The event-driven fashion is preferable for the applications in hand due to the limited resources of embedded processors and band-limited networks. However, reducing the number of measurement updates may degrade the performance of the overall system, this means that the design procedure should preserve desired properties such as stability and convergence. A time-triggered sampling mechanism wastes computation and communication resources of the network.

Two types of distributed protocols to solve the synchronization problem for discrete-time dynamical system is proposed using observer-based methodology [146]. The aforementioned results focus on continuously updated protocols such that every agent needs to continuously employ its state and neighbors' states. On the other hand, to avoid this problem, researchers have investigated the triggering algorithms to reduce communication congestion over networks. Since continuous communication between agents usually leads to the waste of energy and resources, event-triggered mechanisms

are usually used to reduce unnecessary transmissions in practical applications [147], [148], [149]. Event triggering mechanisms have been tailored to fit synchronization over networks [150], [151], and so far, a number of results have been widely developed to address this issue [152], [153], [154], [155]. Some theoretical approaches have been investigated to address event-triggering consensus control for time delay multiagent systems [132], [133], [134]. Moreover, a decentralized event-triggered protocol based on dissipativity approach is proposed for systems with multiple sensors to sample agents outputs [156].

A distributed event-triggered control was proposed for multiagent systems with combinational measurements in [157]. In this case, the control law of each agent is executed only at its own triggering time instants and it uses the combinational measurements based on the information received from its neighbors. The corresponding results on event-triggered consensus based on combinational measurements can be found in the works [158]–[159]. Many results have been investigated to enlarge the inter-event time intervals to decrease the transmission frequency [160]-[161]. It is worth mentioning that the integral-type of event-triggered mechanisms has been proved to be more efficient to generate samples with larger inter-event intervals [162],[163]. The expansion of triggering-intervals in the integral-type occurs due to the averaging mechanism of the error over the period between last update and the current time rather than checking the error value at discrete instants. The integral type triggering is addressed for first-order multiagent systems in the work reported in [164]. Event-triggered consensus with integral type condition was considered for linear multiagent systems by using the combination measurements in [165], [166].

In [167], the leader-following consensus is considered for multi-agent systems and novel results have been addressed by using event-triggered protocols. Mahmoud *et al.* developed a discrete-time protocol for leader-following scheme and designed a dynamic output feedback consensus protocol [168]. In [169], sufficient conditions were established to achieve synchronization with a desired performance for discrete-time multi-agent systems with higher-order linear dynamics. It should be noted that many of consensus techniques [170]-[171] require fulls state information and assumes that the variables are accessible. Nevertheless, in real applications, the state variables are not available for measurement due to practical limitations. In this scenario, it is essential to introduce an observer-based control in order to reach consensus [172]-[173]. For instance, a discrete-time observer-based protocol was proposed for leader-following consensus in [172]. Moreover, Chen *et. al.* [174] addressed the consensus problem for multi-agent systems with state observers. The tracking cooperative methodology was developed for a multiagent system by exploiting an observer-based controller under external disturbances [173]. Therefore, to implement practical control protocols for multiagent systems, it is compulsory to investigate the network limitations and consider the computation resources. This motivates us to seek the possible approach of event-triggered discrete-time protocols for multiagent systems. We have tried to introduce a well-organized solution to the leaderless and leader-following problem of the discrete-time multiagent system under fixed communication topology.

For achieving the consensus of multiagent system, continuous information exchange of agents through neighbor-based interactions may lead to data congestion in the network. When capacity limitation of the communication network and en-

ergy resources are limited, an effective mechanism called the event-triggered strategy should be adopted to reduce unnecessary sampling and data transmission among agents [175], [176], [177]. Unlike the classical time-triggered control mechanism, sampled data is transmitted only by the occurrence of a predefined condition instead of periodic broadcasting among agents. Motivated by these features, a considerable number of researchers have extended event-triggered control mechanism to the consensus of multiagent systems earlier by the group guided by Dimarogonas [178],[179], and so far, a substantial amount of research has been reported on this issue [180], [181]. For instance, output feedback consensus methodology is addressed to control disturbed multiagent systems via event-triggered mechanism in [182]. Furthermore, as proposed in the work of Yang et al [183], a distributed event-triggered strategy for nonlinear multiagent systems is addressed that effectively reduces the information exchange and actuation updates. Most of the above works focus on multiagents with certain dynamics and without considering the time delay. However, several event-triggered methodologies on uncertain multiagent systems with time-delay have been investigated [184]. In [185] and [186], leader-following consensus via event triggering scheme is investigated by considering the leader that is not controlled by the consensus protocol and can not receive information from the followers. On the other hand, in various practical applications, the leader should be updated and controlled by the state of other agents. For example, the leader should receive information from the control center in order to solve the formation of aircraft problems. Hence, one might select an agent as a "Similar Leader". Unlike the results in [185], [186], "Similar Leader" is selected, constrained, and controlled by the same consensus protocol

of other agents. Based on consensus protocol designed, agents reach consensus with "Similar Leader".

It is worth mentioning that most of the aforementioned literature on event-triggered consensus control of multiagent focus on achieving consensus when agents are governed by first/second-order dynamics. However, in some cases, for example, the agents have limited energy resources to carry out specific tasks such as movement and communication. Taking into account the energy consumption, the event-based consensus guaranteed cost is addressed, with a performance index involving energy consumption. Energy-consumption is a very appealing topic in multiagent systems that ought to be addressed simultaneously with the consensus problem [187]. In [188], [189], a cost performance index is introduced to obtain a tradeoff between consensus regulating performance and energy consumption. A powerful methodology for guaranteed performance-based control can accommodate an energy index with an upper bound. In [190], a guaranteed performance consensus protocol has been addressed for multiagent systems with Lipschitz nonlinear dynamics. Few findings have been stated on the guaranteed performance of multiagent systems under event-triggered strategies. State feedback and event-triggered guaranteed cost based control are addressed for uncertain switched linear systems in [191] and [192], respectively. However, it worth mentioning that no time delay is considered in the dynamics of the agents investigated in [191] and [192]. On the other hand, time-delay among agents is inevitable in real applications. A guaranteed performance is studied for a class of higher-order linear multiagent control systems with sampled-data information [193]. Suboptimal output consensus for a high dimensional multiagent system with constant delay is addressed

Research directions	Continuous Dynamics	Discrete Dynamics
First-Order	[197], [198], [199], [200], [201], [202],	[203], [204]
Second-Order	[198],[170], [30]	[205], [206]
High-Order	[95], [35]	
State-Feedback	[94], [99]	
Output-Feedback	[95], [5], [99]	
Observer-Based	[114], [146], [44], [166], [174]	[100], [172]
Event-Triggered	[156], [197], [94]	[207], [208]
Time-Delay	[209], [197], [94]	[210]
Nonlinear	[128], [211], [55], [30]	[212], [213]
Uncertainty	[43]	[210]
Homogeneous		[142], [143]
Heterogeneous	[197],[170]	
Leader-following	[170], [55], [5],[30]	[146], [172]
Leaderless	[166]	[142]
Quantized	[128]	[205], [206]
DoS Attacks	[116], [214], [215],[122], [126]	
Scaled-Consensus	[198], [216], [83], [76], [84], [93]	

Table 2.1: Literature survey

in [194], and time-varying delay [195]. Guaranteed cost consensus is also addressed in [196] for second-order multiagents with directed graphs where communication delays are introduced. However, the aforementioned research on guaranteed cost consensus is based on a time-triggered mechanism rather than a distributed event-triggered protocol. It is found that guaranteed cost control has gained more attention for time delay multiagent systems with time-triggered strategy. Although it has growing attention in recent years, the problem is rarely investigated for event-triggered consensus protocols of nonlinear and uncertain multiagent systems.

2.5 Notations

In the sequel, $\mathbb{N}$ and $\mathbb{R}$ are the sets of integers and real numbers, respectively. $\mathbb{R}^n$ denotes n-dimensional Euclidean space. $\mathbb{R}^n$ denote the n-dimensional vector space equipped with the Euclidean norm. We use $\mathcal{M}^{-1}$ and $\mathcal{M}^T$ to denote the inverse and the transpose of any matrix $\mathcal{M}$, respectively. The notation $\mathbf{1}$ is used to denote the column vector with all components equal to one. $\otimes$ is the Kronecker product. We use I to represent the $n \times n$ identity matrix. Matrices are assumed to be of compatible dimensions for matrix operations, if their dimensions are not explicitly stated. In symmetric matrices, we use the symbol $\bullet$ to represent a block matrix that is induced by symmetry. Let $\mathbf{1}$ denote the vector of ones. $\mathbb{E}[\cdot]$ is used to represent the mathematical expectation. We consider the convention that intersection over empty index $\bigcap_{(i,j)\in\emptyset} = [t_1, t_2)$ as the union of all subintervals of $[t_1, t_2)$. Given two sets A and B, $A \backslash B$ is the relative complement of B in A. For the set $\mathcal{D}(a, b)$ of disjoint subintervals $[a_i, b_i)$, we use the notation $|\mathcal{D}(t, s)|$ to represent the sum of subintervals lengths over $[a, b]$.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ denote an undirected graph of N order, where $\mathcal{V} = \{v_i, i = 1, \ldots, N\}$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, be the set of vertices, edges, and the adjacency matrix of the graph $\mathcal{G}$, respectively. The Laplacian matrix $\mathcal{L} = [\ell_{ij}]_{N\times N}$ with respect to the graph $\mathcal{G}$ is defined as $\mathcal{L}_{ii} = \sum_{j=1}^{N} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$ for $i \neq j$. It is assumed that each node represents an agent and the information exchange of neighbors of the agent v_i is denoted by $\mathcal{N}_i = \{j : (v_i, v_j) \in \mathcal{E}\}$. If there is an information exchange between agent i and agent j, that is, $(v_i, v_j) \in \mathcal{E}$, then $a_{ij} = a_{ji} = 1$

otherwise $a_{ij} = a_{ji} = 0$. $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ defines the degree of an agent v_i. A sequence of different consecutive nodes starting with v_i and ending with v_j represents a path in the graph $\mathcal{G}$ from v_i to v_j. If there exists a path between any two vertices, then the graph $\mathcal{G}$ is considered to be connected. When the leader-follower case is considered, we denote a diagonal matrix $\mathcal{B} = \text{diag}\{b_1, b_2, \cdots, b_N\}$, where $b_i = 1$ if the leader belongs to the neighboring set of the agent i and $b_i = 0$ otherwise.

CHAPTER 3

DISTRIBUTED SECURE FORMATION

In this Chapter, the formation problem of multiagent systems is investigated for discrete-time multiagent systems under denial-of-service (DoS) attacks. Different from the existing works on secure consensus where multiple attacks can be seen as one channel attack, the adversaries could attack multiple agents independently/distributively. The considered attack scenario compromises agents rather than edges of the communication topology. This leads to a severe behavior of the multiagent systems and most of the existing techniques studying arbitrary switching dynamics fails to investigate this situation. In addition, the communication topology becomes disconnected whenever an agent is attacked because the agent becomes isolated. A novel distributed consensus protocol is addressed to reduce the effects of such a DoS attack. The proposed protocol uses static output-feedback. By applying appropriate transformation, the consensus/formation problem is transformed into a stability problem and neces-

sary and sufficient conditions are given in terms of bilinear matrix inequalities. Then, a sufficient condition is derived to design the formation gain based on linear matrix inequity (LMI). Finally, simulation example is presented to demonstrate the effectiveness of the proposed theoretical results.

3.1 Introduction

The state-feedback consensus protocols are mainly based on a restrictive assumption that all the agent's states are accessible. However, generally speaking, the state variables of each agent can not be directly measured in practical applications, especially for agents with higher-order dynamics. Thus, full state feedback algorithms in such cases should be replaced by output feedback-based techniques. However, the static output feedback consensus protocols of multi-agent systems is more difficult than that of state feedback and observer based protocols. Output feedback control scheme has been applied to the consensus problem in [94], [95], [96].

Multiagent systems are vulnerable to cyber-threats due to the deep integration of the physical agents and networking. Consequently, s highlighted in [108], [109], secure control has become an essential aspect of research in networked systems. Therefore, it is vital to investigate the coordination and cooperation problem of multiagent systems under cyber attacks, and many preliminary results have been investigated in this regard [110], [111], [112], [113]. By interrupting significant data exchanged between agents, the adversary aims to affect the consensus among agents and steer the agents to an unacceptable operating point. This type of cyber attacks is called denial of service

attacks (DoS) which is the most common and simplest type of attacks. The attacks arrive in a random and intermittent manner due to the fluctuation in adversary's statistical factors. DoS attacks are often modeled by Markov process or Bernoulli process [114], [115]. Moreover, investigating multiple DoS attacks case is more realistic and practical. Multiple attack scenario, where the adversary can compromise partial or all agents at any time, is the major motivation of this study. Motivated by the existing works, this Chapter concentrates on static output feedback formation problem for high-order discrete-time linear multiagent systems under multiple DoS attacks. The considered attack leads to losses of nodes/agents and result in switchings and disconnecting communication topology. The main contributions of the Chapter can be concluded as follows:

1. In our proposed consensus algorithm, it is not required to introduce extra sensors and devices to continuously measure the state of agents. Different from the existing work [214], [217], [119], the proposed protocol is designed based only on the relative output information thereby it is more favorable in practical applications.

2. We investigate multiple DoS attacks with independent characteristics for different transmission channels. Therefore, the DoS attack model in [218] can be viewed as a special case of the considered model in this Chapter. In addition, our investigation solves the formation problem under attacks that always disconnect the communication topology, where most of the stabilizing techniques of arbitrary switching dynamics fail.

3. Sufficient and necessary condition in terms of a bilinear matrix inequality and a sufficient condition in terms of LMI are derived to solve the formation problem.

3.2 Problem Statement and Preliminaries

3.2.1 System Description

Consider a group of N agents which update their states based on information exchange. The communication topology $\mathcal{G}$ is used to represent the information exchange among agents. The dynamics of each agent is governed by the following linear discrete-time system:

$$x_i(k+1) = Ax_i(k) + Bu_i(k)$$

$$y_i(k) = Cx_i(k) \tag{3.1}$$

where $x_i(k) \in \mathbb{R}^n$, $y_i(k) \in \mathbb{R}^p$, and $u_i(k) \in \mathbb{R}^n$ are the state, output and input of agent i for $i = 1,\ 2, \cdots, N$, respectively. A, B, and C are constant real matrices with appropriate dimensions.

Assumption 1 *The pair (A, B) is assumed to be controllable and the output matrix C is a full row rank matrix.*

Remark 1 *Note that the controllability is a mild assumption. No conditions have been imposed on the eigenvalues of the state matrix A for the usual consensus case. However, we need at least an eigenvalue equals to one in order to achieve the state-formation problem as will be seen in the sequel.*

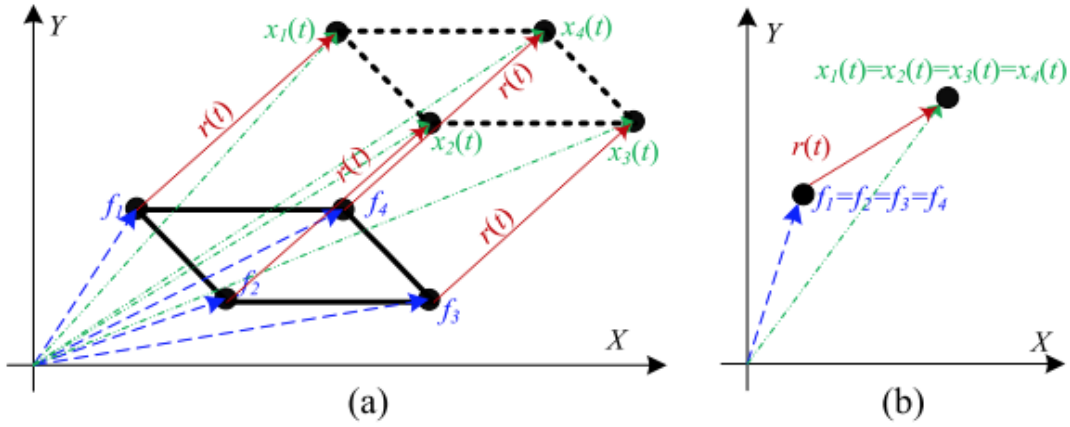

Figure 3.1: (a) Parallelogram formation and (b) point formation (consensus) [1].

The multiagent system (3.1) achieves formation when the agents collaborate such that their states reach prescribed constraints and form a geometric pattern/shape. We use relative vector constraint to describe the geometric patterns, see Figure 3.1. The vector variable h_i represents the desired state formation of agents. More specifically, the definition of point-formation and state formation are stated as follows.

Definition 3.1 *The multiagent system (3.1) achieves the usual consensus (point formation) if the following condition is satisfied for all agents $i \in \mathcal{G}$:*

$$\lim_{k\to\infty} \|x_i(k) - x_j(k)\| \to 0$$

The multiagent system (3.1) reaches the state formation if the following limit:

$$\lim_{k\to\infty} \|(x_i(k) - h_i) - (x_j(k) - h_j)\| \to 0$$

is satisfied for predefined formation constant vectors $h_i \in \mathbb{R}^N$ for all agents $i \in \mathcal{G}$.

Assumption 2 *For the state formation problem, the state matrix A has at least simple eigenvalue and equals to one. The formation vectors h_i are assumed to belong to the eigenspace of the state matrix A associated with the eigenvalue $\lambda(A) = 1$, i.e.*

$Ah_i = h_i \ \forall i \in \mathcal{G}$.

The following assumption on the graph topology $\mathcal{G}$ is standard in investigating cooperation and coordination of multiagent systems, see [119], [219].

Assumption 3 *The graph $\mathcal{G}$ is undirected and connected.*

Remark 2 *It is worth pointing out that point consensus can be formed via state formation definition with zero formation vectors $h_i = 0$. This implies that $Ah_i = h_i$ without an assumption on the eigenvalues of the state matrix A since h_i is trivial. Moreover, the communication graph $\mathcal{G}$ is assumed to be connected in normal operation only. Nevertheless, the graph losses this property under attack scenarios.*

3.2.2 Distributed Protocols

We are interested in designing distributed formation-protocol based on local measurements of the neighbors of each agent. To address the cooperation problem we consider the following formation protocol:

$$u_i(k) = K \sum_{j \in N_i} a_{ij} \left[y_i(k) - h_i^y - (y_j(k) - h_j^y) \right] \tag{3.2}$$

for all $i \in \mathcal{V}$, where $h_i^y = Ch_i$ and K is the formation gain matrix to be designed. It is worth noting that the formation protocol (3.2) does not explicitly depend on the vector constraint h_i. Consider the disagreement vector defined as follows

$$\xi_i(k) = x_i(k) - h_i - \frac{1}{N} \sum_{j=1}^{N} (x_j(k) - h_j) \tag{3.3}$$

which is used to convert the formation problem into stability analysis. Denote

$$u(k) = \begin{bmatrix} u_1^T(k) & u_2^T(k) & \dots & u_N^T(k) \end{bmatrix}^T \ x(k) = \begin{bmatrix} x_1^T(k) & x_2^T(k) & \dots & x_N^T(k) \end{bmatrix}^T,$$

$\xi(k) = \begin{bmatrix} \xi_1^T(k) & \xi_2^T(k) & \dots & \xi_N^T(k) \end{bmatrix}^T$, $\boldsymbol{h} = \begin{bmatrix} h_1^T & h_2^T & \dots & h_N^T \end{bmatrix}^T$. Then the compact form of the disagreement (3.3) can be expressed as follows

$$\xi(k) = (\mathcal{M} \otimes I_n)(x(k) - \boldsymbol{h}) \tag{3.4}$$

where $\mathcal{M} = I_n - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Hence, we can write the overall input $u(k)$ as follows

$$u(k) = (\mathcal{L} \otimes KC)\xi(k)$$

and the collective dynamics of the multiagent systems (3.1) is obtained:

$$x(k+1) = (I_N \otimes A)x(k) + (\mathcal{L} \otimes BKC)\xi(k) \tag{3.5}$$

Based on the transformation (3.4), Equation (3.5), and the fact that $\mathcal{M}\mathcal{L} = \mathcal{L}\mathcal{M} = \mathcal{L}$, one has

$$\begin{aligned} \xi(k+1) &= (\mathcal{M} \otimes I_n)(x(k+1) - \boldsymbol{h}) \\ &\overset{(a)}{=} (\mathcal{M} \otimes I_n)\big[(I_N \otimes A)x(k) + (\mathcal{L} \otimes BKC)\xi(k) - (I_N \otimes A)\boldsymbol{h}\big] \\ &= (\mathcal{M} \otimes A)x(k) + (\mathcal{L} \otimes BKC)\xi(k) - (\mathcal{M} \otimes A)\boldsymbol{h} \\ &= \big[(I_N \otimes A) + (\mathcal{L} \otimes BKC)\big]\xi(k) \end{aligned} \tag{3.6}$$

where (a) hods for that $Ah_i = h_i$. When the usual consensus (point formation) is considered, i.e. $h_i = 0$, then (a) holds without considering Assumption 2.

3.2.3 Denial of Service Attack

Denial of service attack is one of the most common attacks in cyber-physical systems due to its simplicity from the adversary's point of view. The adversary renders some or all the agents/channels inaccessible. In this Chapter, we assume that the agents are attacked and the attack is modeled as the lack of control signals and measurements at the same time, and each agent is compromised independently. This means that

all the edges of an attacked agent i are disconnected in both directions whenever an adversary attacks agent i. Different from existing models of DoS, this type of attack makes the communication graph disconnected if any agent is compromised by the adversary. Motivated by the notion of the average dwell time of switched systems [220], some limitations should be imposed on DoS duration, the following assumption is introduced.

Assumption 4 *[220] There exist positive constants μ_{ij} and $T_{ij} > 1$ satisfying*

$$\left|\mathcal{D}_{(i,j)}(k_1, k_2)\right| \leq \mu_{ij} + \frac{(k_2 - k_1)}{T_{ij}}$$

for all $k_1, k_2 \in \mathbb{N}$ with $k_2 \geq k_1$, where $\mathcal{D}_{(i,j)}(k_1, k_2)$ is the union of DoS intervals of each edge $(i, j) \in \mathcal{E}$ over the interval $[k_1, k_2]$. The average number of samples T_{ij} reflects the reciprocal of DoS attack strength.

Compared to the classical approach of DoS duration which neglects the individual characteristics of each edge [110], different attack modes are considered for different channels. As discussed in [110], it is reasonable for adversaries to stop attack action and switch to a sleep mode to supply its energy for next jamming. To define an index for the attack modes, the following set of attacked channels is given

$$\Gamma(k) = \left\{(i, j) \in \mathcal{E} | k \in \mathcal{D}_{(i,j)}(0, \infty)\right\} \tag{3.7}$$

When an agent i is subject ot an attack, then $(j, i) \in \Gamma(k)$ for all $j \in \mathcal{N}_i$. Define $\overline{\mathcal{D}}_{(i,j)}(k_1, k_2) = [k_1, k_2] \setminus \mathcal{D}_{(i,j)}(k_1, k_2)$ to denote the time intervals of channels which are not subject to the attack and indexed using the set $\mathcal{E}\setminus\Gamma$. The indexed set $\Gamma \subseteq \mathcal{E}$ represents disconnected channels by the attack and the index set $\mathcal{E}\setminus\Gamma$ represents the healthy edges $\left(k_1 < k_2 \text{ and } \overline{\mathcal{D}}_{(i,j)}(k_1, k_2) = [t_1, t_2] \setminus \mathcal{D}_{(i,j)}(k_1, k_2)\right)$. Consider the

partition $\Xi_\Gamma(k_1,k_2) = \left(\cap_{(i,j)\in\Gamma}\mathcal{D}_{(i,j)}(k_1,k_2)\right) \cap \left(\cap_{(i,j)\notin\Gamma}\overline{\mathcal{D}}_{(i,j)}(k_1,k_2)\right)$. Based on this particular structure of partition, we can represents the whole interval as the union of all partitions as follows:

$$\cup_{\Gamma\subseteq\varepsilon}\Xi_\Gamma(k_1,k_2) = [k_1,k_2] \tag{3.8}$$

In addition, the attack interval of each (i,j) channel can be expressed as follows:

$$\mathcal{D}_{(i,j)}(k_1,k_2) = \cup_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma}\Xi_\Gamma(k_1,k_2) \tag{3.9}$$

3.2.4 Distributed Protocol Under DoS Attacks

When the multiagent system (3.1), the distributed formation protocol (3.2) can be modeled as

$$u_i(k) = K\sum_{j\in\mathcal{N}_i,(j,i)\notin\Gamma(k)} a_{ij}\left[y_i(k) - h_i^y - (y_j(k) - h_j^y)\right]$$

Similarly, we can write the control law in the following compact form:

$$u(k) = ((\mathcal{L} - \mathcal{L}_\Gamma)\otimes KC)\xi(k) \tag{3.10}$$

where $\mathcal{L}$ denotes the Laplacian matrix of the communication graph $\mathcal{G}$ in normal operation without attacks. $\mathcal{L}_\Gamma = [l_{ij}^\Gamma]$ is the Laplacian matrix that represents the attacked channels, and its definition is given by

$$\begin{cases} l_{ij}^\Gamma = -1 & i\neq j \text{ and } (i,j)\in\Gamma(k) \\ l_{ii}^\Gamma = -\sum_i^N l_{ij}^\Gamma & i = j \end{cases} \tag{3.11}$$

It should be emphasized that $\mathcal{L}_\Gamma$ dose not associated with the graph subject to DoS attack. In other words, $\mathcal{L}_\Gamma$ is the Laplacian matrix of the complement of the graph under DoS attack. Similar to the derivation of Equation (3.6), substituting the augmented control law (3.10) in the multiagent system (3.1) gives the following overall

disagreement dynamics:

$$\xi(k+1) = \big[(I_N \otimes A) + ((\mathcal{L} - \mathcal{L}_\Gamma) \otimes BKC)\big]\xi(k) \tag{3.12}$$

Note that $\mathbf{1}$ is a common eigenvector of $\mathcal{L}$ and $\mathcal{L}_\Gamma$ associated with the simple zero eigenvalue. It is well known that the second eigenvalue of $\mathcal{L}$ is strictly positive for connected graphs. Hence, without loss of generality, the eigenvalues set of the Laplacian matrix can be represented in the following order $0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_N$. Consider the orthonormal matrix $\Psi = \left[\frac{\mathbf{1}}{\sqrt{N}},\ \nu_2,\ \nu_3, \cdots, \nu_N\right]$, where ν_i represents the orthonormal eigenvector set of $\mathcal{L}$, i.e. $\mathcal{L}\nu_i = \lambda_i\nu_i$ and $\|\nu_i\|^2 = 1$. Based on this construction, we have $\Psi^T\Psi = \Psi\Psi^T = I_n$,

$$\Psi^T\mathcal{L}\Psi = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda \end{bmatrix} \quad \text{and} \quad \Psi^T\mathcal{L}_\Gamma\Psi = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{R}_\Gamma \end{bmatrix}$$

where $\Lambda = \operatorname{diag}\{\lambda_2,\ ,\lambda_3, \cdots,\ \lambda_N\}$, and $\mathcal{R}_\Gamma = [\nu_2,\ \nu_3, \cdots, \nu_N]^T \mathcal{L}_\Gamma [\nu_2,\ \nu_3, \cdots, \nu_N]$. It is worth noting that the matrix $\mathcal{R}_\Gamma$ is not diagonal in general and its structure depends on the attack mode. Now, consider the similarity transformation:

$$\eta(k) = (I \otimes \Psi^T)\xi(k)$$

where $\eta(k) = \begin{bmatrix} \eta_1^T(k) & \eta_2^T(k) & \dots & \eta_N^T(k) \end{bmatrix}^T$. Since $\mathbf{1}^T$ is a left eigenvector of $\mathcal{M}$ corresponding to the zero eigenvalue, then $\eta_1(k) = (\frac{\mathbf{1}^T}{\sqrt{N}} \otimes I)\xi(k) = (\frac{\mathbf{1}^T}{\sqrt{N}} \otimes I)(\mathcal{M} \otimes I)(x(k) - \boldsymbol{h}) = 0$. Moreover, the rest of the dynamics is governed by the difference equation:

$$\eta^1(k+1) = \big[(I_{N-1} \otimes A) + ((\Lambda - \mathcal{R}_\Gamma) \otimes BKC)\big]\eta^1(k) \tag{3.13}$$

where $\eta^1(k) = \begin{bmatrix} \eta_2^T(k) & \dots & \eta_N^T(k) \end{bmatrix}^T$. Due to the fact that Ψ is unitary and

$\eta_1(k) = 0$ for all $k \geq 0$, one has

$$\eta^1(k) = \eta(k) = \|(\Psi^T \otimes I)\xi(k)\| = \|\xi(k)\|$$

Therefore, asymptotic stability of the dynamics (3.13) implies that $x(k) - \boldsymbol{h}$ belongs to the null space of $(\mathcal{M} \otimes I)$ which is spanned by the vectors $(\mathbf{1} \otimes q_i)$ for $i = 1, 2, \cdots, n$, where $q_i \in \mathbb{R}^n$ is the i-th column vector of the identity matrix I_n. Consequently, $x_i(k) - h_i(k) = \alpha_i$ for some constant α_i, $\forall i \in \mathcal{V}$, i.e. the formation problem is solved.

3.3 Output Feedback Approach

In this section, analysis and synthesis of static output feedback consensus for the multiagent system (3.1) will be derived. For the sake of simplicity, let

$$\boldsymbol{A} = \mathbb{T}A\mathbb{T}^{-1}, \quad \boldsymbol{B} = \mathbb{T}B, \quad \text{and} \quad \boldsymbol{C} = C\mathbb{T}^{-1}$$

where $\mathbb{T}$ is a nonsingular matrix such that $C\mathbb{T}^{-1} = \begin{bmatrix} I_p & \mathbf{0} \end{bmatrix}$. Using the similarity transformation $\tilde{\eta}(k) = (I_{N-1} \otimes \mathbb{T})\eta^1(k)$, then the following dynamics is obtained

$$\tilde{\eta}(k+1) = \big[(I_{N-1} \otimes \boldsymbol{A}) + ((\Lambda - \mathcal{R}_\Gamma) \otimes \boldsymbol{B}K\boldsymbol{C})\big]\tilde{\eta}(k) \tag{3.14}$$

According to Assumption 1, there always exists a transformation that satisfies $\begin{bmatrix} I_p & \mathbf{0} \end{bmatrix} = C\mathbb{T}^{-1}$. Moreover, this transformation can be obtained by

$$\mathbb{T} = \left(\begin{bmatrix} C \\ null(C^T) \end{bmatrix} \right)^{-1}. \tag{3.15}$$

3.3.1 Formation without DoS

Firstly, we consider the multiagent system without DoS attacks, i.e. $\mathcal{R} = 0$. The following result presents necessary and sufficient condition that guarantees the formation

algorithm.

Theorem 3.1 *Under Assumption 2, the formation problem is solved if and only if the symmetric matrices $\mathcal{S}_1 > 0$ and $\mathcal{S}_2 > 0$, and real matrices $\mathcal{H}$ and $\mathcal{Y}$ satisfy the following inequality:*

$$\begin{bmatrix} -(I_{N-1} \otimes \mathcal{S}) & \Upsilon_1 \\ \bullet & -(I_{N-1} \otimes \mathcal{S}) \end{bmatrix} < 0 \tag{3.16}$$

where $\mathcal{S} = \begin{bmatrix} \mathcal{S}_1 & 0 \\ 0 & \mathcal{S}_2 \end{bmatrix}$, $\mathcal{H} = \begin{bmatrix} I & 0 \\ H & I \end{bmatrix}$ *and* $\Upsilon_1 = (I_{N-1} \otimes \mathcal{S}\mathcal{H}^T \boldsymbol{A}^T \mathcal{H}^{-T}) + (\Lambda \otimes \boldsymbol{C}^T \mathcal{Y}^T \boldsymbol{B}^T \mathcal{H}^T)$ *Moreover, the formation gain is obtained by* $K = \mathcal{Y} S_1^{-1}$.

Proof. Consider the following Lyapunov candidate function:

$$V(k) = \tilde{\eta}^T(k)(I_{N-1} \otimes P)\tilde{\eta}^T(k)$$

where P is a positive definite matrix. Since the protocol under consideration is not subject to attack, we have $\mathcal{R} = 0$. The first difference of the Lyapunov function $\Delta V(k)$ along the dynamics (3.14) is negative if and only if the following inequality is satisfied:

$$\begin{bmatrix} -(I_{N-1} \otimes P) & \Upsilon_2 \\ \bullet & -(I_{N-1} \otimes P) \end{bmatrix} < 0 \tag{3.17}$$

where $\Upsilon_2 = (I_{N-1} \otimes \boldsymbol{A}^T P) + (\Lambda \otimes \boldsymbol{C}^T K^T \boldsymbol{B}^T P)$. Consider the partitioned matrices $X = \begin{bmatrix} X_1 & X_2 \\ \bullet & X_3 \end{bmatrix}$ and $\mathcal{V} = \begin{bmatrix} X_1 & X_2 \\ 0 & -X_2 X_1^{-1} X_2 + X_3 \end{bmatrix}$, where $X = P^{-1}$. Multiplying both sides of inequality (3.17) by the matrix $\begin{bmatrix} I_{N-1} \otimes \mathcal{V} & 0 \\ 0 & I_{N-1} \otimes \mathcal{V} \end{bmatrix}$ and its transpose,

respectively, we get the following equivalent inequality:

$$\begin{bmatrix} -(I_{N-1} \otimes \mathcal{V}P\mathcal{V}^T) & \Upsilon_3 \\ \bullet & -(I_{N-1} \otimes \mathcal{H}X\mathcal{H}^T) \end{bmatrix} < 0 \tag{3.18}$$

where $\Upsilon_3 = (I_{N-1} \otimes \mathcal{V}\boldsymbol{A}^T P\mathcal{V}^T) + (\Lambda \otimes \mathcal{V}\boldsymbol{C}^T K^T \boldsymbol{B}^T P\mathcal{V}^T)$. Define the following matrices $H = -X_2^T X_1^{-1}$, $\mathcal{S}_1 = X_1$, and $\mathcal{S}_2 = -X_2 X_1^{-1} X_2 + X_3$. Then we can write $\mathcal{H}$ and $\mathcal{V}$ as follows: $\mathcal{H} = \begin{bmatrix} I & 0 \\ -X_2^T X_1^{-1} & I \end{bmatrix}$ and $\mathcal{V} = \mathcal{H}X$. Moreover, one has $\mathcal{S} = \mathcal{V}P\mathcal{V}^T = \mathcal{H}^T X\mathcal{H}$, $P\mathcal{V}^T = \mathcal{H}^T$, $\mathcal{V}\boldsymbol{A}^T P\mathcal{V}^T = \mathcal{H}X\boldsymbol{A}^T X^{-1} X\mathcal{H}^T = \mathcal{H}X\mathcal{H}^T\mathcal{H}^{-T}\boldsymbol{A}^T\mathcal{H}^T = \mathcal{S}\mathcal{H}^T\boldsymbol{A}^T\mathcal{H}^{-T}$

$$\mathcal{V}\boldsymbol{C}^T K^T = \begin{bmatrix} X_1 \\ 0 \end{bmatrix} K^T = \begin{bmatrix} I_p \\ 0 \end{bmatrix} X_1 K^T = \boldsymbol{C}^T \mathcal{Y}^T$$

and

$$\mathcal{S} = \begin{bmatrix} X_1 & 0 \\ 0 & -X_2 X_1^{-1} X_2 + X_3 \end{bmatrix}$$

Substituting all these parameters in inequality 3.18 we get its equivalent form (3.16). We observe that $\mathcal{S} > 0$ if and only if $X > 0$ by Schur-complement, and $\Upsilon_1 = \Upsilon_3$, which completes the proof. ∎

Corollary 1 *Under Assumption (2), the formation problem is solved if the there exist positive definite matrices $\mathcal{S}_1 > 0$ and $\mathcal{S}_2 > 0$ satisfying the following inequality:*

$$\begin{bmatrix} -(I_{N-1} \otimes \mathcal{S}) & (I_{N-1} \otimes \mathcal{S}A^T) + (\Lambda \otimes \boldsymbol{C}^T \mathcal{Y}^T \boldsymbol{B}^T) \\ \bullet & -(I_{N-1} \otimes \mathcal{S}) \end{bmatrix} < 0$$

Proof. Similar to the proof of Theorem 3.1 with $X_2 = 0$, then we have $\mathcal{H} = I$, which completes the proof. ∎

Remark 3 *Note that the condition of Theorem 3.1 is a bilinear matrix inequality.*

However, fixing the variable H leads to an LMI inequality, which constitutes a sufficient condition only. It should be pointed out that Corollary 1 provides more conservative results compared with the condition in Theorem 3.1.

3.3.2 Formation Under DoS

We are now in a position to derive a sufficient condition on that guarantee state formation under DoS attacks. The main result is stated in the following theorem.

Theorem 3.2 *If there exist positive definite matrices $\mathcal{S}_1 > 0$, $\mathcal{S}_2 > 0$ and $\mathcal{Y}$ and scalars $\vartheta_\Gamma > 0$, φ_1^{ij} and φ_2^{ij} such that the following inequalities hold*

$$\begin{bmatrix} -\vartheta_\Gamma(I_{N-1} \otimes \mathcal{S}) & \Omega \\ \bullet & -I_{N-1} \otimes \mathcal{S} \end{bmatrix} < 0 \tag{3.19}$$

$$\varphi_1^{ij} - \varphi_2^{ij} \geq 0 \tag{3.20}$$

$$\log(\vartheta_\Gamma) \leq \left(\sum_{(i,j)\in\Gamma} \varphi_1^{ij} + \sum_{(i,j)\in\mathcal{E}\setminus\Gamma} \varphi_2^{ij} \right) \tag{3.21}$$

$$\nu \triangleq \sum_{(i,j)\in\mathcal{E}} \left(\left(-1 + \frac{1}{T_{ij}} \right) \varphi_2^{ij} - \frac{1}{T_{ij}} \varphi_1^{ij} \right) > 0 \tag{3.22}$$

where $\mathcal{S} = \begin{bmatrix} \mathcal{S}_1 & 0 \\ 0 & \mathcal{S}_2 \end{bmatrix}$, $\mathcal{H} = \begin{bmatrix} I & 0 \\ H & I \end{bmatrix}$, $\Omega = (I_{N-1} \otimes \mathcal{S}\mathcal{H}^T A^T \mathcal{H}^{-T}) + ((\Lambda + \mathcal{R}_\Gamma) \otimes \boldsymbol{C}^T \mathcal{Y}^T \boldsymbol{B}^T \mathcal{H}^T)$ *and* $\Gamma \subseteq \mathcal{E}$, *then under the DoS attack satisfying Assumption 4, the multiagent system (3.1) achieves state formation. Moreover, formation protocol gain matrix is given by* $K = \mathcal{Y}S_1^{-1}$.

Proof. Consider the following Lyapunov candidate function:

$$V(k) = \tilde{\eta}^T(k)(I_{N-1} \otimes P)\tilde{\eta}^T(k)$$

where P is a positive definite matrix. Similar to the proof of Theorem 3.1, the following inequality:

$$V(k+1) - \vartheta_\Gamma V(k) < 0 \tag{3.23}$$

is guaranteed if LMI (3.19) is satisfied. Note that ϑ_Γ should be strictly positive because $V(k)$ is positive-definite. However, ϑ_Γ could be greater than one, which leads to an unstable behavior due to a specific attack mode Γ. Let s_m denote the transition instants, i.e. at least one DoS is launched or removed at s_m. It follows from (3.23) that

$$\begin{aligned} V(k) \leq & (\vartheta_{\Gamma(s_m)})^{(k-s_m)} V(s_m) \\ \leq & (\vartheta_{\Gamma(s_m)})^{(k-s_m)} \prod_{i=0}^{m-1} (\vartheta_{\Gamma(s_i)})^{(s_{i+1}-s_i)} V(s_0) \end{aligned} \tag{3.24}$$

for all $k \in [s_m, s_{m+1})$, where $s_0 = 0$ is the initial time. Based on the properties of the natural logarithmic function, (3.24) is equivalent to

$$\log(V(k)) \leq D_m(k) + \log(V(s_0)) \tag{3.25}$$

where

$$D_m(k) = \log(\vartheta_{\Gamma(s_m)})(k - s_m) + \sum_{i=0}^{m-1} \log(\vartheta_{\Gamma(s_i)})(s_{i+1} - s_i) \tag{3.26}$$

Using the index set (3.7) and the definition of $\Xi_\Gamma(0,k)$, $D_m(k)$ can be written as follows:

$$D_m(k) = \sum_{\Gamma \subseteq \mathcal{E}} \log(\vartheta_{\Gamma(s_m)}) \left|\Xi_\Gamma(0,k)\right|$$

It follows from (3.21) that

$$D_m(k) \leq \sum_{\Gamma \subseteq \mathcal{E}} \left(\sum_{(i,j)\in\Gamma} \varphi_1^{ij} + \sum_{(i,j)\in\mathcal{E}\setminus\Gamma} \varphi_2^{ij} \right) \Xi_\Gamma(0,k)$$

$$= \sum_{(i,j)\in\mathcal{E}} \left(\varphi_1^{ij} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma} |\Xi_\Gamma(0,k)| + \varphi_2^{ij} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\notin\Gamma} |\Xi_\Gamma(0,k)| \right) \tag{3.27}$$

Based on Equations (3.8) and (3.9), one has

$$\sum_{\Gamma\subseteq\mathcal{E},(i,j)\notin\Gamma} |\Xi_\Gamma(0,k)| = \left|[0,k]/\mathcal{D}_{(i,j)}(0,k)\right| = k - \left|\mathcal{D}_{(i,j)}(0,k)\right| \tag{3.28}$$

$$\text{and} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma} |\Xi_\Gamma(0,k)| = \left|\mathcal{D}_{(i,j)}(0,k)\right| \tag{3.29}$$

Substituting Equations (3.28) and (3.29) in (3.27), and using condition (3.20) that is $\varphi_1^{ij} - \varphi_2^{ij} \geq 0$ with Assumption 4, one can obtain a new upper bound:

$$D_m(k) \leq \sum_{(i,j)\in\mathcal{E}} \left(\left(\varphi_1^{ij} - \varphi_2^{ij}\right) \left|\mathcal{D}_{(i,j)}(0,k)\right| + \varphi_2^{ij} k \right)$$

$$\leq \sum_{(i,j)\in\mathcal{E}} \left[\left(\varphi_1^{ij} - \varphi_2^{ij}\right) \mu_{ij} + \left(\varphi_2^{ij} \left(1 - \frac{1}{T_{ij}}\right) + \varphi_1^{ij} \frac{1}{T_{ij}} \right) k \right]$$

$$= \zeta - \nu k \tag{3.30}$$

where $\zeta = \sum_{(i,j)\in\mathcal{E}} \left(\left(\varphi_1^{ij} - \varphi_2^{ij}\right) \mu_{ij} \right)$. Then inequality (3.25) can be written in the following form:

$$\log(V(k)) \leq \zeta - \nu k + \log(V(s_0))$$

or equivalently:

$$V(k) \leq (e^{-\nu})^k e^{\zeta} V(s_0) \tag{3.31}$$

Since $\nu > 0$ by (3.22), we have $V(k) \to 0$ as $k \to \infty$ which yields $\lim_{k\to\infty} \|\tilde{\eta}(k)\| = 0$ and state formation is achieved. This completes the proof. ∎

Remark 4 *Note that the matrix inequality (3.19) is nonlinear and hence not easily tractable by the available convex optimization techniques. Different from Theorem* 3.1*, fixing the matrix H does not lead to an LMI condition. Nevertheless, we know that*

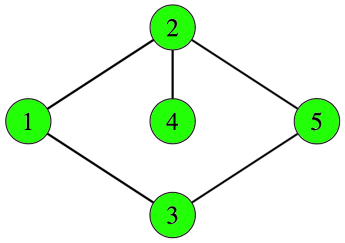

Figure 3.2: Communication topology of the agents without attacks, $\Gamma = \{\emptyset\}$.

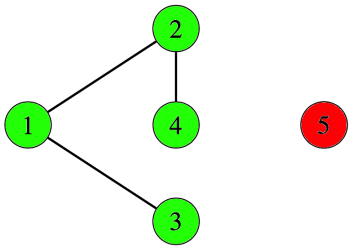

Figure 3.3: Communication topology of the agents when agent 5 is subject to attack, i.e. $\Gamma = \{(2,5),(5,2),(3,5),(5,3\ )\}$.

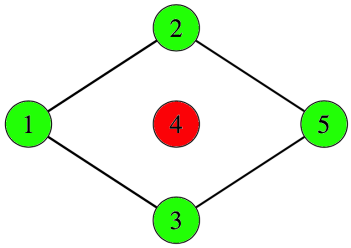

Figure 3.4: Communication topology when agent 4 is attacked, $\Gamma = \{(4,2),(2,4)\}$.

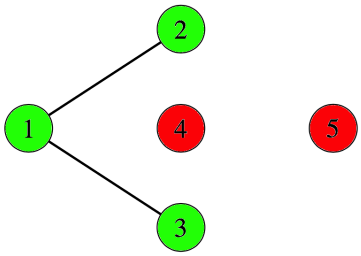

Figure 3.5: Communication topology of the agents when agents 5 and 4 are subject to DoS, $\Gamma = \{(2,5),(5,2),(3,5),(5,3),(2,4),(4,2)\}$.

the index Γ *comprises the case without attack scenario. Hence, we can use Theorem* 3.1 *or Corollary* 1 *to obtain the output feedback formation gain* K *and other matrix variables such as* $\mathcal{S}_1$ *and* $\mathcal{S}_2$*. Then the matrix inequality (3.19) becomes a set of LMI's in terms of the variables* ϑ_Γ*.*

Remark 5 *It is worth pointing out that the decay rates* θ_Γ *corresponding to the attack modes* Γ *and the DoS intervals of edges are unmatched. This constitutes a major difficulty for investigating the multiagent systems with multiple agents under attacks. Moreover, the techniques for switched dynamical systems with stable and unstable modes which has been addressed in [215] is not appropriate for such a structure. The presented conditions in Theorem 3.2 provide a class of decay rates* φ_1^{ij} *and* φ_2^{ij} *which can be considered as a stable and unstable rates of the edge* $(i,j) \in \mathcal{E}$ *in order to compensate the unstable behavior. In addition,* ϑ_Γ *could be greater than one when an attack mode is considered, and arbitrary switching methodologies can not be utilized here because the graph is disconnected by the attacks.*

3.4 Simulation Results

3.4.1 Formation for Quadrotor Vehicles

A quadrotor vehicle presented in [221], [222], can be modeled via two-decoupled dynamics along x and y directions as follows:

$$\ddot{p}_i^{\mathrm{x}} = g\theta_i^y$$

$$\dot{p}_i^y = -g\theta_i^x$$

$$\ddot{\theta}_i^x = k^m L u_i^x / I_x$$

$$\ddot{\theta}_i^y = k^m L u_i^y / I_y$$

where p_i^{x} and p_i^{x} denote the horizontal position along x and y directions of the i-th agent, respectively. g represents the gravitational acceleration. θ_i^x and θ_i^y are the pitch and roll angles, respectively. k^m is a force coefficient, L_i is the center distance, and I_x and I_y represent the moment of inertia in x and y directions, respectively.

Let $q_i^x(t) = \dot{p}_i^x(t)$, $q_i^y(t) = \dot{p}_i^y(t)$, $\omega_i^x(t) = \dot{\theta}_i^x(t)$, and $\omega_i^y(t) = \dot{\theta}_i^y(t)$ denote the linear and angular speeds for the i-th vehicle, respectively. Consider the parameter values in Table 3.4.1. With a sampling time 0.1s, the discrete-time model of the vehicle is obtained in the form of (3.1), where

$$x_i(k) = \begin{bmatrix} p_i^x & q_i^x & \theta_i^y & \omega_i^y & p_i^y & q_i^y & \theta_i^x & \omega_i^x \end{bmatrix}^T$$

$$A = \begin{bmatrix} 1 & 0.1 & 0.049 & 0.0016 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.98 & 0.049 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.1 & -0.049 & -0.0016 \\ 0 & 0 & 0 & 0 & 0 & 1 & -0.98 & -0.049 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.004 & -0.13 & 0.4 & 8 \\ 0.004 & 0.13 & 0.4 & 8 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.022 & 0 & -0.3 & 0 & 0 & 0.266 & 0 & -0.53 \\ 0 & -0.053 & 0 & -0.106 & 0.033 & 0 & -0.452 & 0 \\ 0 & 0 & 0 & 0 & 0.011 & 0 & -0.151 & 0 \\ -0.044 & 0 & -0.603 & 0 & 0 & 0.0266 & 0 & -0.053 \end{bmatrix}$$

Table 3.1: The parameters of quadrotor model.

Gravity Acceleration	g	9.8m/s^2
Force coefficient	k^m	12N.
Inertia of x -axis	I_x	0.03kg.m^2
Inertia of y -axis	I_y	0.03kg.m^2
Center Distance	L	0.2m

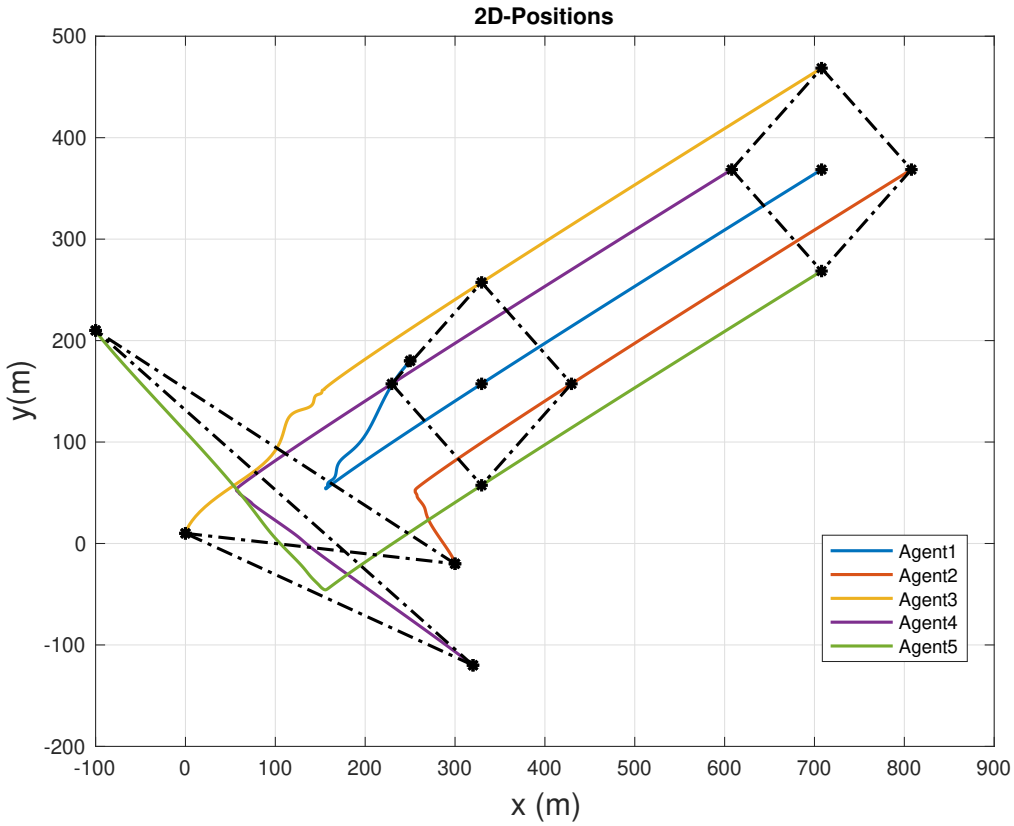

Figure 3.6: Two dimensional positions of the vehicles without failure/attacks (state formation is achieved).

The graph topology of 5 agents with no attacks is shown in Figure 3.2, and its corresponding Laplacian matrix can be easily obtained. The transformation $\mathbb{T}$ is obtained based on Equation (3.15). We apply this transformation on the matrix variables A, B and C to get the new state-space representation $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$. For this state representation, the LMI condition in Corollary 1 is feasible with the following

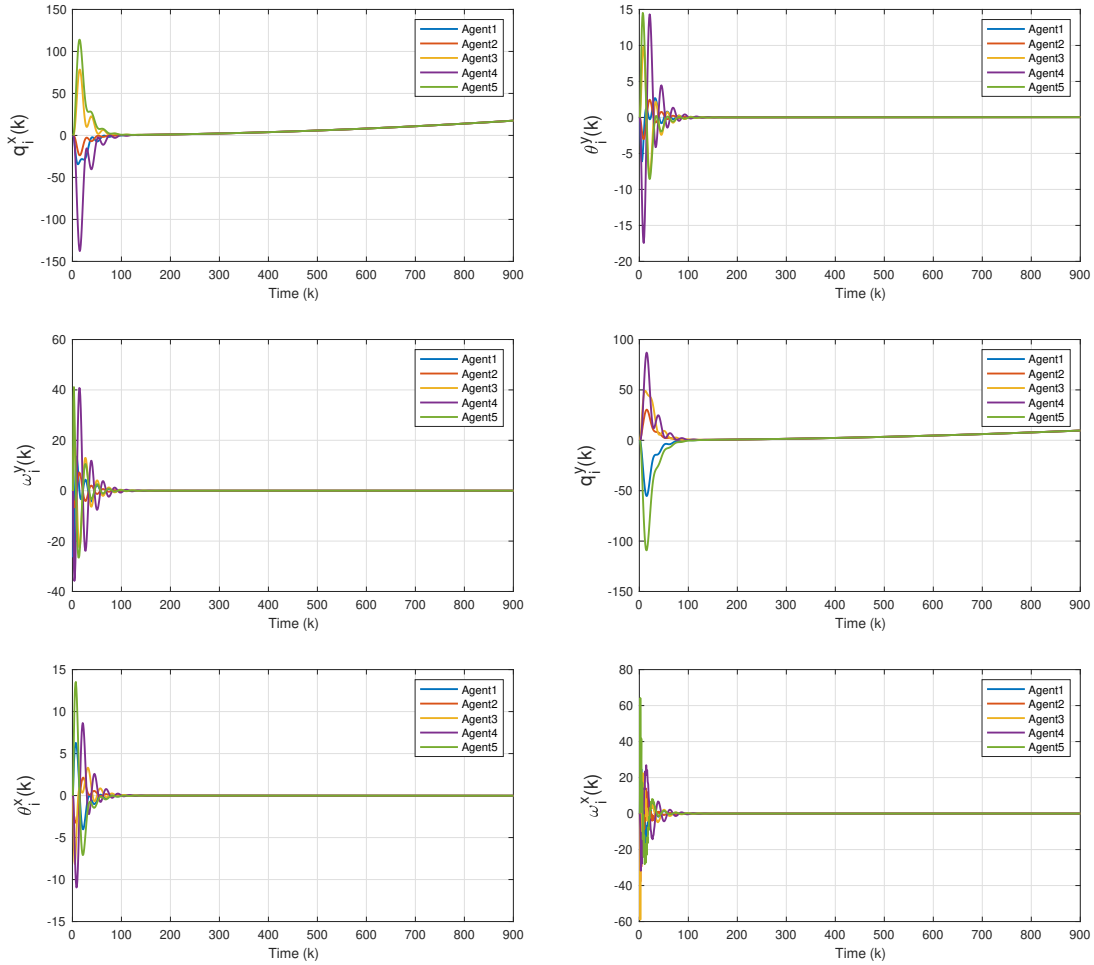

Figure 3.7: Linear speed, pitch angle, roll angle, and angular rates of the vehicles without DoS attack.

parameters:

$$\mathcal{S}_1 = \begin{bmatrix} 23.5481 & -7.5136 & -1.6065 & 3.1095 \\ -7.5136 & 30.8036 & -4.7956 & -1.3520 \\ -1.6065 & -4.7956 & 2.9318 & -5.5913 \\ 3.1095 & -1.3520 & -5.5913 & 24.9832 \end{bmatrix}$$

$$\mathcal{Y} = \begin{bmatrix} 0.7086 & -5.5154 & 3.0570 & -6.5788 \\ -1.1484 & 22.4372 & -8.2246 & 14.2036 \end{bmatrix}$$

and the state formation gain is obtained:

$$K = \mathcal{Y}\mathcal{S}_1^{-1} = \begin{bmatrix} 0.1053 & 0 & 1 & -0.0526 \\ -0.0263 & 0.5 & -1.5 & 0.2632 \end{bmatrix}$$

First, we will consider the formation gain for the multiagent system without attacks. Then we will consider the same gain according to Theorem 3.2 to test it and decide whether it is suitable to be implemented under DoS attack scenarios or not. Let the formation vectors be given as follows $h_1 = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^T$, $h_2 = [100,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^T$, $h_3 = [0,\ 0,\ 0,\ 0,\ 100,\ 0,\ 0,\ 0]^T$, $h_4 = [-100,\ 0,\ 0,\ 0,\ 00,\ 0,\ 0,\ 0]^T$, and $h_5 = [0,\ 0,\ 0,\ 0,\ -100,\ 0,\ 0,\ 0]^T$ which forms a diamond shape with an agent at its center. It can be seen that these formation vectors satisfy the constraint in Assumption 2. The initial conditions are

$$x_1 = \begin{bmatrix} 250 & 0 & 0 & 0 & 180 & 0 & -0.01 & 0 \end{bmatrix}^T$$

$$x_2 = \begin{bmatrix} 300 & 0 & 0 & 0.102 & -20 & 0 & 0 & -0.101 \end{bmatrix}^T$$

$$x_3 = \begin{bmatrix} 0 & 0 & 0.01 & 0 & 10 & 0 & 0 & 0 \end{bmatrix}^T$$

$$x_4 = \begin{bmatrix} 320 & 0 & 0 & -0.1 & -120 & 0 & 0 & 0.1 \end{bmatrix}^T$$

$$x_5 = \begin{bmatrix} -100 & 0 & 0 & -0 & 210 & 0 & 0 & 0 \end{bmatrix}^T$$

The proposed feedback protocol (3.2) is applied to the considered multivehicle system. The position of the UAV's are presented in a 2-dimensional view to show that the formation consensus is achieved, see Figure 3.6. Figure 3.7 shows the response of the remaining states $q_i^x(k)$, $q_i^y(k)$, θ_i^x, ω_i^x, θ_i^y, and ω_i^y without DoS attacks. Figure 3.6

shows that the vehicles can maintain the formation shape. The position error reaches zero, and pitch, roll angles and speeds of the vehicles achieve consistent behavior within 10 seconds.

Attack scenario: Consider the graph in Figure 3.2 which represents the communication topology under normal operation without DoS attack . Suppose that the adversary launches DoS attacks on agent 4 and agent 5 in independent manner. The attack strength is assumed to have the constraints $\mu_{42} = 3$, $\mu_{52} = \mu_{53} = 2.2$, $\Delta_{42} = 1.8$, and $\Delta_{52} = \Delta_{53} = 2$. Figure 3.3 and Figure 3.4 show the graph when agents 5 and 4 are under DoS, respectively, and the Laplacian of the attacked edges are given as

$$\mathcal{L}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \mathcal{L}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 2 \end{bmatrix}$$

$\mathcal{L}_4$ corresponds to the graph Laplacian of the removed edges due to simultanuous attacks on agents 4 and 5, see Figure 3.5.

$$\mathcal{L}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 2 \end{bmatrix}$$

It is worth noting that these Laplacian matrices can be seen as the complement of the graphs under attacks. Note that $\mathcal{R}_1 = 0$ corresponds to communication graph in Figure 3.2, and $\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are obtained based on $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_4$ with the use of the

unitary transformation Ψ given in Section II:

$$\mathcal{R}_2 = \begin{bmatrix} 0.4535 & 0 & -0.575 & 0.6089 \\ 0 & 0 & 0 & 0 \\ -0.575 & 0 & 0.7289 & -0.772 \\ 0.6089 & 0 & -0.772 & 0.8176 \end{bmatrix}$$

$$\mathcal{R}_3 = \begin{bmatrix} 0.1882 & 0.1502 & 0.2875 & -0.3045 \\ 0.1502 & 1 & -0.4605 & -1.3287 \\ 0.2875 & -0.4605 & 0.98 & 0.386 \\ -0.3045 & -1.3287 & 0.386 & 1.8318 \end{bmatrix}$$

$$\mathcal{R}_4 = \begin{bmatrix} 0.6417 & 0.1502 & -0.2875 & 0.3045 \\ 0.1502 & 1 & -0.4605 & -1.3287 \\ -0.2875 & -0.4605 & 1.7089 & -0.386 \\ 0.3045 & -1.3287 & -0.3860 & 2.6494 \end{bmatrix}$$

We employ these matrices and the formation gain obtained from Corollary 1 to check the LMI feasibility in Theorem 3.2. Conditions of Theorem 3.2 are satisfied with decay rates: $\alpha_1 = 0.92$, $\alpha_2 = 1.02$, $\alpha_3 = 1.02$, and $\alpha_4 = 1.03$. The position of the vehicles are shown in Figure 3.8. The vehicles maintain the formation shape under DoS attacks shown in Figure 3.9.

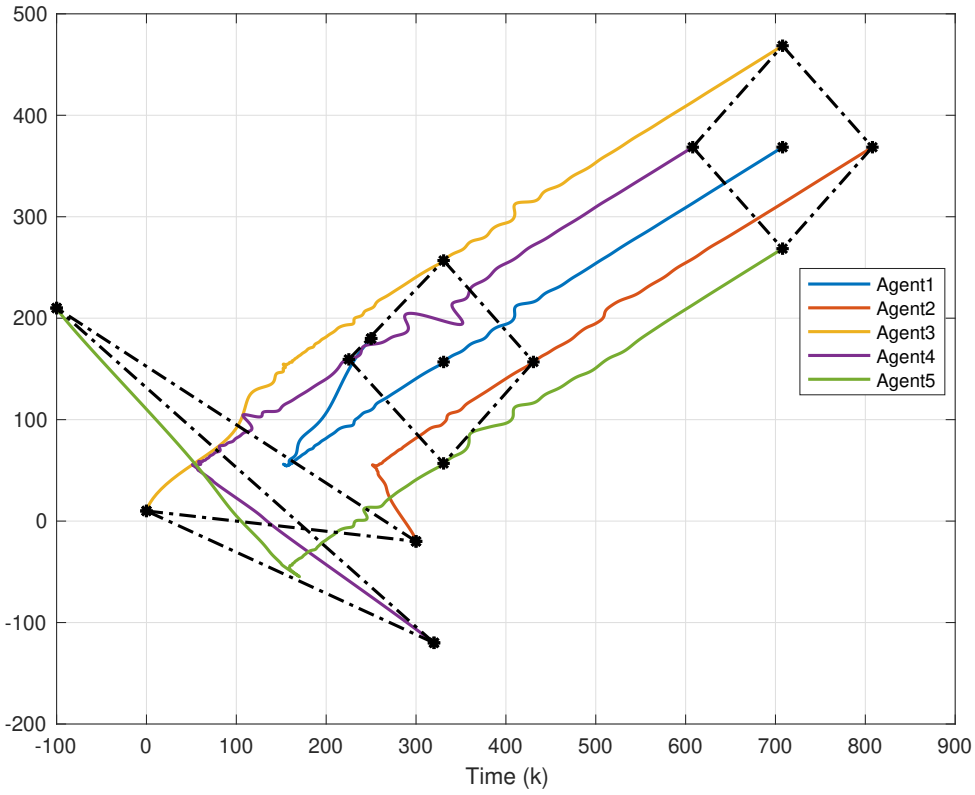

Figure 3.8: Two dimensional positions of the vehicles under DoS attacks (state formation is achieved).

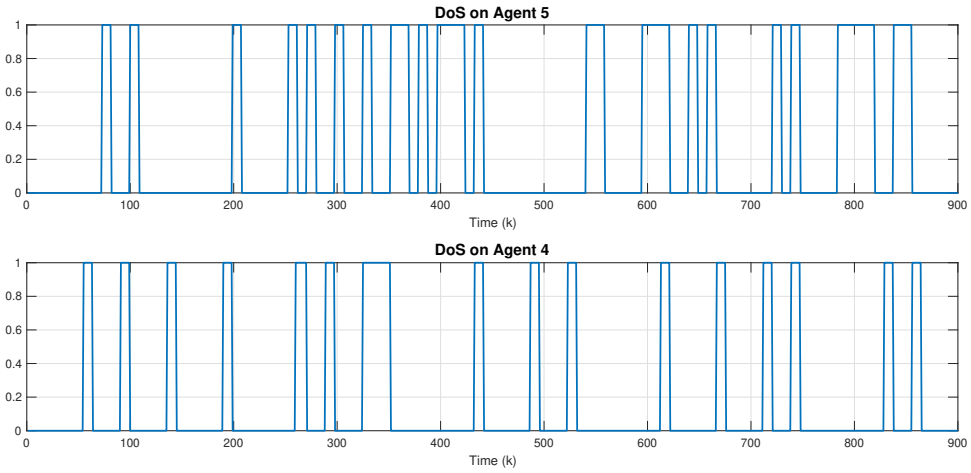

Figure 3.9: DoS signals (1 means that agent i is under attack).

CHAPTER 4

QUANTIZED SCALED-CONSENSUS

In this Chapter, the practical mean-square scaled-consensus problem of multiagent systems is studied. Higher-order general linear dynamics is investigated, where all agents are subject to faults or denial of service attacks (DoS). The considered network comprises agents taking quantized states under undirected communication channels. Different from the existing works, no strict conditions have been imposed on the eigenvalues of the agents' state matrix. First, we propose a distributed scaled-consensus protocol that utilizes a probabilistic quantization algorithm. Second, scaled-disagreement analysis is provided via mean-square stability methodology. Finally, sufficient conditions in terms of linear matrix inequalities are presented in order to obtain scaled-consensus gain for both leaderless and leader-follower cases. The presented theoretical results are examined through two simulation examples.

4.1 Introduction

Achieving consensus is an essential requirement for the cooperation of multiagent system [53]. It is to be pointed out that a great effort has been devoted by researchers in the investigation on consensus problems. Most of the aforementioned studies primarily focus on ideal communication patterns where quantization is not considered. However, the precision of the controller and communication bandwidth are constrained and limited. Hence, practical measurements produce errors that could affect the performance of the consensus algorithms. Since quantization error is inevitable, the consensus over quantized communication channels has become an active research topic (see[127], [128], [129], [130] and references therein). In [131] sampled-data based consensus for MASs under logarithmic quantizers is discussed.

Besides, reliable distributed consensus protocols have been emerging as an important research area since failures and attacks on communication channels are unavoidable in multiagent systems. It is worth noting that, most of the investigated secure consensus protocols under DoS attacks have been devoted to a class of adversaries that compromises all channels as one channel [122], [123], [124]. However, addressing the consensus problem where the adversary can compromise partial agents/channels or all agents/channels is more practical. A secure protocol that achieves asymptotic consensus against multiple adversaries has been developed in [125]. More importantly, a predictive secure control for multiagent systems under DoS attacks has been investigated over quantized networks [126]. It is worth pointing out that most of the existing results of consensus under DoS attacks and quantization effects are only developed for

usual consensus. On the other hand, scaled-consensus is considered as a generalization of the usual consensus.

Practically speaking, the cooperative protocols over communication networks are constrained where the loss of precision is introduced by the quantization process. Non-ideal measurements introduce error which deteriorates the performance of consensus convergence. Hence, quantization error should be considered for cooperative protocols. In addition, failures and attacks on communication channels are inevitable in multiagent systems. To the best of our knowledge, up to now, the scaled consensus with quantization and faulty networks has not been fully investigated, which is a major motivation of this Chapter. Motivated by the above discussion, our attention is devoted to the investigation of leaderless and leader-following scaled-consensus in the presence of DoS attacks and quantization error. The main contributions of this chapter are concluded as follows:

1. Compared with the existing results on scaled-consensus [198], this Chapter extends the results to higher-order dynamics. In [216], random packet dropouts have been investigated for the first time for scaled-consensus with ideal measurements. However, our proposed protocol generalizes the results with quantized measurements for higher-order rather than second-order dynamics. Besides, since the measurement exchange takes place in discrete instances of time in realistic communication networks, discrete-time-dynamics is considered.

2. A new distributed scaled-protocol is analyzed to schedule attacked agents. Different from most existing scaled-protocols that consider links failures, in the

presented case, the agent could be totally isolated from the communication topology by adversaries. Most of the existing results for consensus in the presence of link-failures/cyber-attacks have been investigated via dynamic protocols [223], [224]. However, the proposed static cooperative protocol is relatively simple to design as we do not need to implement complicated formulas.

3. Sufficient conditions in terms of linear matrix inequalities are used to obtain secure control gains that allow the proposed protocol to achieve leaderless and leader-follower practical mean-square scaled-consensus.

The rest of the Chapter is organized as follows. Section 4.2 is devoted to the problem formulation and preliminaries results. Section 4.3 presents sufficient conditions to achieve the mean-square practical scaled-consensus which are used to obtain the consensus gains as well. Corresponding theoretical results have been validated through numerical examples in section 4.4.

4.2 Problem Formulation and Preliminaries Results

The following lemmas are useful to prove theoretical results in this chapter:

Lemma 4.1 *[225] Consider a Lyapunov functional $V(x(k))$. If there exist positive scalars $\nu \geq 0, \theta_1 > 0, \theta_2 > 0,$ and $0 < \lambda_0 < 1$ such that*

$$\theta_1 \|x(k)\|^2 \leq V(x(k)) \leq \theta_2 \|x(k)\|^2$$

and $\mathrm{E}\left[V\left(x(k+1)\right) \mid x(k)\right] - V\left(x(k)\right) \leq \nu - \lambda_0 V\left(x(k)\right)$ *then the following inequality holds:*

$$\mathrm{E}\left[\|x(k)\|^2\right] \leq \frac{\theta_2}{\theta_1}\|x(0)\|^2(1-\lambda_0)^k + \frac{\nu}{\theta_1\lambda_0}$$

Lemma 4.2 *For any symmetric real matrix,* $\mathcal{Q}$ *of the form:*

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \bullet & \mathcal{Q}_3 \end{bmatrix} < 0$$

if and only if $\mathcal{Q}_3 < 0$ *and* $\mathcal{Q}_1 - \mathcal{Q}_2\mathcal{Q}_3^{-1}\mathcal{Q}_2^T < 0$.

4.2.1 System Description

Suppose that the multiagent system under consideration consists of N identical with general linear dynamics. Each agent is represented by the following state space form:

$$x_i(k+1) = Ax_i(k) + Bu_i(k),\ i = 1, \ldots, N \tag{4.1}$$

where $x_i(k) = \begin{bmatrix} x_{i1}(k) & x_{i2}(k) & \ldots & x_{in}(k) \end{bmatrix}^T \in \mathbb{R}^n$, $u_i(k) \in \mathbb{R}^m$ are the state, and input, respectively. A and B are constant matrices with appropriate dimensions. The collective dynamics of the multiagent system can be represented as

$$x(k+1) = (I_N \otimes A)\,x(k) + (I_N \otimes B)\,u(k) \tag{4.2}$$

where $u(k) = \begin{bmatrix} u_1^T(k) & u_2^T(k) & \ldots & u_N^T(k) \end{bmatrix}^T$ and $x(k) = \begin{bmatrix} x_1^T(k) & x_2^T(k) & \ldots & x_N^T(k) \end{bmatrix}^T$. In the leader-follower case, the leader agent has the following dynamics:

$$x_0(k+1) = A_0 x_0(k) \tag{4.3}$$

where $x_0(k) \in \mathbb{R}^n$ represents the state of the leader. It is worth noting that the multiagent system given in (4.1) and (4.3) is the generalization of the single and double integrator dynamics. The following assumptions are important in order to get

feasible solutions to the proposed conditions on scaled-consensus.

Assumption 5 *The pair (A, B) is assumed to be controllable.*

Assumption 6 *The graph $\mathcal{G}$ is assumed to be connected.*

Note that Assumption 5 is very mild in the investigation of cooperative problem of linear multiagent systems. In this chapter, compared to the existing results on consensus problem, no assumptions are imposed on the eigenvalues of the state matrix A.

4.2.2 Quantizer

In this note, we consider the effect of quantization of agent measurements on the scaled-consensus problem. We consider $x_{ij} \in \left[-\bar{\Delta}, \bar{\Delta}\right]$ is a scalar measurement required to be quantized. The transmitted data by each agent is assumed to have packets with m bits. Therefore, we have 2^m uniformly spaced quantization levels and each level is given as $\Delta_p = \frac{2\bar{\Delta}}{2^m}$. Similar to [205], the probabilistic quantizer $Q(.)$ adopted in this chapter is defined as follows

$$\begin{cases} \Pr[Q(x_{ij}) = k\Delta_p] = 1 - \frac{x_{ij} - k\Delta_p}{\Delta_p}, \\ \qquad\qquad\qquad\qquad k\Delta_p \leq x_{ij} < (k+1)\Delta_p \\ \Pr[Q(x_{ij}) = (k+1)\Delta_p] = \frac{x_{ij} - k\Delta_p}{\Delta_p}, \end{cases}$$

Denote the quantization error as $\Delta_{ij}(k) = Q(x_{ij}(k)) - x_{ij}(k)$. The expected value and the variance of $\Delta_{ij}(k)$ for such a quantizer are given in [205], [226], [206]:

$$E[Q(x_{ij})] = x_{ij}, \text{ and } E\left[(\Delta_{ij}(k))^2\right] \leqslant \frac{\Delta_p^2}{4}$$

We will use the following representation of the stacked quantization error of agent i: $\Delta_i(k) = Q(x_i(k)) - x_i(k)$, where $\Delta_i(k) = \begin{bmatrix} \Delta_{i1}(k) & \Delta_{i2}(k) & \dots & \Delta_{in}(k) \end{bmatrix}^T$.

4.2.3 Scaled-Consensus Protocol: Subject to Attacks

Consider a practical scenario where the data is quantized before transmission with the following distributed scaled-consensus protocol:

$$\begin{aligned} u_i(k) &= K \sum_{j \in N_i} \beta_{ij} a_{ij} \left(\alpha_{ij} Q(x_j(k)) - Q(x_i(k)) \right) \\ &= K \sum_{j \in N_i} \beta_{ij} a_{ij} \left(\alpha_{ij} x_j(k) - x_i(k) \right) + K \sum_{j \in N_i} \beta_{ij} a_{ij} \left(\alpha_{ij} \Delta_j(k) - \Delta_i(k) \right) \end{aligned} \tag{4.4}$$

where $\alpha_{ij} = \frac{\alpha_i}{\alpha_j}$ represents scale ratios, $\beta_{ij} = \beta_i(k)\beta_j(k)$, $\beta_i(k) \in \{0, 1\}$ is a Bernoulli distributed sequence, and K is the scaled-consensus gain matrix to be designed later. When a failure takes place between agent i and agent j then $\beta_{ij} = 0$ which implies that $\beta_i = 0$ or $\beta_j = 0$. For instance, let $\beta_i = 0$ then $\beta_{ij} = 0$ for all $j \in \mathcal{V}$ which indicates that the neighboring agents $\mathcal{N}_i$ are prevented to communicate with agent i in both directions. In other words, at least one agent is totally isolated if $\beta_{ij} = 0$ for some $i, j \in \mathcal{V}$. The sequence $\beta_i(k)$ is assumed to be mutually independent of $\beta_j(k)$, for $i \neq j$ with the following properties:

$$\text{Prob}\left\{\beta_i(k) = 1\right\} = \mathbb{E}\left[\beta_i(k)\right] := \bar{\beta}_i$$

$$\text{Prob}\left\{\beta_i(k) = 0\right\} = 1 - \mathbb{E}\left[\beta_i(k)\right] := 1 - \bar{\beta}_i$$

Since the considered Bernoulli sequences are mutually independent, then according to the definition of $\beta_{ij}(k)$, we have the following property:

$$\mathbb{E}\left[\beta_{ij}(k)\right] = \begin{cases} \bar{\beta}_i & \text{if } i = j \\ \bar{\beta}_i \bar{\beta}_j & \text{if } i \neq j \end{cases} \tag{4.5}$$

where $\bar{\beta}_i$ represents the isolation probability of agent i. It is worth pointing out that

due the occurrence of DoS attacks, the topology of the multiagent (5.1) is no longer fixed.

Assumption 7 *The probability distributions of the quantizers are independent of the Bernoulli distribution of the adversaries.*

Remark 6 *The channel fading considered in ([227]) prevents agents to receive data but allows them to send their measurements. However, in this Chapter,* $\beta_i(k) = 0$ *means that the agent* i *is totally isolated, i.e. it is prevented from sending and receiving information at the same time. This type of attack compromises agents rather than channels. Besides, the proposed attack scenario forms a disconnected topology whenever it happens and constitutes a severe type of DoS attack. It is worth mentioning that,* β_{ij} *can be modeled for each edge to model link failures of agent* i *independently similar to the work of ([228]). However, attacking channels might not lead to a disconnected graph. For instance, when the adversary compromises agent's channels with different probabilities, it can be considered as a mild attack because the possibility to isolate the agent is less probable.*

Based on (4.4), the stacked input vector is obtained:

$$u(k) = -\left(\Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes K\right)(x(k) + \Delta(k)) \tag{4.6}$$

where $\Delta(k) = \begin{bmatrix} \Delta_1^T(k) & \Delta_2^T(k) & \dots & \Delta_N^T(k) \end{bmatrix}^T$, $\Lambda = \text{diag}\{\alpha_1, \cdots, \alpha_N\}$, $\mathcal{L}$ represents the Laplacian of the multiagent system in normal operation without attacks, and $\mathcal{L}_\Theta = [l_{ij}^\Theta]$ is the Laplacian of the attacked graph, i.e. $l_{ij}^\Theta = -(1-\beta_{ij})a_{ij}$, and $l_{ii}^\Theta = -\sum_i^N l_{ij}^\Theta$. When an attack compromises agent i, $\beta_i = 0$, therefore $\beta_{ij} = 0$ for any $j = 1, 2, \cdots N$. This means that when the element $l_{ij}^\Theta = -1$, all channels connected to the agent i

are removed from the Laplacian matrix $\mathcal{L}$. Substituting the control law (4.6) in the overall dynamics (4.2), gives

$$x(k+1) = \left[(I_N \otimes A) - \Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK\right] x(k)$$
$$- (\Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK)\Delta(k) \tag{4.7}$$

4.2.4 Scaled-Consensus

Inspired by the definition of the mean-square robust consensus in [206]; [229], [227] and the definition of scaled consensus in [76], we state the following definition of quantized scaled-consensus.

Definition 4.1 *A group of N agents (multiagent system) is said to achieve practical mean-square scaled-consensus with the reciprocals* $\left(\alpha_1^{-1}, \ldots, \alpha_N^{-1}\right)$, *if there exists a monotonic increasing function* $c(\cdot)$ *with* $\lim_{\Delta_p \to 0} c(\Delta_p) = 0$ *such that*

$$\lim_{t\to\infty} \mathbb{E}\left[\left\|\frac{x_i(k)}{\alpha_i} - \frac{x_j(k)}{\alpha_j}\right\|^2\right] \leq c(\Delta_p), \qquad \forall\, i, j \in \mathcal{V},$$

where the scalars $\alpha_1, \ldots, \alpha_N$ *are assumed to be non-zero. When* $\Delta_p = 0$, *i.e. the quantization is not considered, the multiagent system is said to achieve mean-square scaled-consensus. In addition, mean-square practical Leader-follower practical scaled-consensus is said to be achieved if*

$$\lim_{t\to\infty} \mathbb{E}\left[\left\|\frac{x_i(k)}{\alpha_i} - \frac{x_0(k)}{\alpha_0}\right\|^2\right] \leq c(\Delta_p), \quad \forall\, i = 1,\ 2,\ \cdots, N$$

Achieving the scaled consensus implies that the states of the agents converge to dictated ratios in the asymptote. In other words, the multiagent system is said to reach scaled-consensus if the ratios between agents' trajectories converge to predefined known constants. For instance, the ration $x_i(k)/x_j(k)$ converges to $1/2$ when

the scales are selected as follows $\alpha_i = 1$ and $\alpha_j = 2$. The objective of this Chapter is to investigate practical mean-square leader-follower and leaderless scaled-consensus problem via distributed protocols.

4.2.5 Leaderless Disagreement Analysis

Consider the following scaled-disagreement vector:

$$\delta_i(k) = \frac{1}{\alpha_i} x_i(k) - \frac{1}{N} \sum_{j=1}^{N} \frac{1}{\alpha_j} x_j(k) \tag{4.8}$$

The augmented scaled-disagreement vector: $\delta(k) = \begin{bmatrix} \delta_1^T(k) & \delta_2^T(k) & \cdots & \delta_N^T(k) \end{bmatrix}^T$ can be written in the following compact form:

$$\delta(k) = (\mathcal{M} \otimes I_n)\, x(k) \tag{4.9}$$

where $\mathcal{M} = (I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T)\Lambda^{-1}$ and $\mathbf{1} = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}^T$. Choose a vector $\boldsymbol{r} = \begin{bmatrix} \alpha_1 & \cdots & \alpha_N \end{bmatrix}^T / \sqrt{\mu}$, where $\mu = \sum_{i=1}^{N} \alpha_i^2$. It is worth noting that $\boldsymbol{r}^T\boldsymbol{r} = 1$, and is an orthonormal right eigenvector of the matrix $\mathcal{M}$ associated with its simple zero-eigenvalue:

$$\mathcal{M}\boldsymbol{r} = (I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T)\Lambda^{-1}\boldsymbol{r} = (I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T)\frac{\mathbf{1}}{\sqrt{\mu}} = 0$$

It is easy to show that $rank(\mathcal{M}) = N - 1$. The null space of $\mathcal{M}$ is spanned by the vector $\boldsymbol{r}$. Since Λ is nonsingular, based on Kronecker properties, we characterize the null space of $(\mathcal{M} \otimes I_n)$ by the following representation:

$$x(k) = \sum_{i=1}^{N} \varphi_i(\boldsymbol{r} \otimes \boldsymbol{e}_i) = \left(\boldsymbol{r} \otimes \sum_{i=1}^{N} \varphi_i \boldsymbol{e}_i\right) = (\boldsymbol{r} \otimes \boldsymbol{\varphi}) \tag{4.10}$$

for an arbitrary vector $\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_N \end{bmatrix}^T$, where $\varphi_i \in \mathbb{R}$, and $\boldsymbol{e}_i$ is the i-th column of the identity matrix. From Equations (4.8) and (4.10), $\lim_{k\to\infty} \mathbb{E}[\|\delta(k)\|^2] = 0$ implies that $\lim_{k\to\infty} \mathbb{E}[\delta(k)] = \mathbf{0}$ and $\mathbb{E}[x(k)]$ belongs to the null space of $(\mathcal{M} \otimes I_n)$

that is $\mathbb{E}[x(k)] = (\boldsymbol{r} \otimes \boldsymbol{\varphi})$. This further gives that $\mathbb{E}[x_i(k)] = \alpha_i \boldsymbol{\varphi}$ for all $i = 1, 2, ..., N$, i.e. $\mathbb{E}[x_i(k)]/\alpha_i = \mathbb{E}[x_j(k)]/\alpha_j$. In other words, if the dynamics (4.29) is mean-square asymptotically stable, then $\lim_{k\to\infty} \mathbb{E}\left[\frac{x_i(k)}{\alpha_i} - \frac{x_j(k)}{\alpha_j}\right] = 0$. However, this statement is not enough to guarantee the mean-square scaled-consensus stability in terms of definition 4.1. Moreover, such an inadequate argument becomes worse for practical scaled-consensus and asymptotic behavior is more difficult to be satisfied under quantized networks. The following instrumental result will be used to guarantee practical consensus whenever the disagreement dynamics is mean-square stable.

Lemma 4.3 *Consider the disagreement vector $\delta(k)$ given in (4.9). If there exist positive scalars $\nu \geq 0, \theta_1 > 0, \theta_2 > 0$, and $0 < \lambda_0 < 1$ satisfying the following inequality*

$$\mathrm{E}\left[\|\delta(k)\|^2\right] \leq \frac{\theta_2}{\theta_1} \|\delta(0)\|^2 (1-\lambda_0)^k + \frac{\nu}{\theta_1 \lambda_0}$$

Then the multiagent system (4.1) achieves mean-square practical scaled-consensus. Moreover, if $\nu = 0$ then mean-square scaled-consensus is achieved.

Proof. From (4.10) we can write the agents' trajectories as: $x_i(k) = \alpha_i \boldsymbol{\varphi} + \alpha_i \zeta_i$, where ζ_i represents a consensus deviation of the agent i due to the quantization process and initial disagreement between agents. Hence, $x(k)$ can be written in a compact form: $x(k) = (\boldsymbol{r} \otimes \boldsymbol{\varphi}) + (\Lambda \otimes I)\zeta(k)$, where $\zeta(k) = \begin{bmatrix} \zeta_1^T(k) & \zeta_2^T(k) & \dots & \zeta_N^T(k) \end{bmatrix}^T$. As mentioned previously, $(\boldsymbol{r} \otimes \boldsymbol{\varphi})$ spans the null space of $(\mathcal{M} \otimes I_n)$, therefore

$$\begin{aligned} \delta(k) &= (\mathcal{M} \otimes I_n)\, x(k) \\ &= (\mathcal{M} \otimes I_n)\left((\boldsymbol{r} \otimes \boldsymbol{\varphi}) + (\Lambda \otimes I)\zeta(k)\right) \\ &= (\mathcal{M}\Lambda \otimes I_n)\, \zeta(k) \end{aligned} \tag{4.11}$$

Without loss of generality we assume that $(\Lambda \otimes I)\zeta(k)$ does not contain any element of the null space, i.e. it can be represented by the complement of the null space only. This holds because the $(\mathcal{M}\Lambda \otimes I_n)$ is a projection to the complement of its null space. Consider the eigenvalues of $\mathcal{M}^T\mathcal{M}$ defined as follows: $\mathcal{M}^T\mathcal{M}T_i = \sigma_i T_i$ with the eigenvector $T_1 = \boldsymbol{r}$ associated with the zero eigenvalue $\sigma_1 = 0$. Since the rank of $rank(\mathcal{M}^T\mathcal{M}) = N-1$ then $0 = \sigma_1 < \sigma_2 \leq \sigma_3 \leq \cdots \leq \sigma_N$. Setting the transformation:

$$T = [T_1 \; T_2 \; \cdots \; T_N] \tag{4.12}$$

which is a unitary matrix such that

$$\Sigma = T^T\mathcal{M}^T\mathcal{M}T = \text{diag}\{0, \sigma_2, \sigma_3, \cdots, \sigma_N\} \tag{4.13}$$

The set $\{T_1, T_2, ..., T_N\}$ is an orthonormal basis for R^N, as mentioned before $\{T_1 = \boldsymbol{r}\}$ spans the null space, and the range $(\Lambda \otimes I)\zeta(k)$ is spanned only by the vectors $(T_2 \otimes \boldsymbol{\varphi}), (T_3 \otimes \boldsymbol{\varphi}), ..., (T_N, \otimes\boldsymbol{\varphi}))$ which is the complement of the null space defined using equation (4.10), where $\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_N \end{bmatrix}^T$. Then, the following holds

$$(\Lambda \otimes I)\zeta(k) = \left(\left(\sum_{i=2}^{N} \vartheta_i T_i\right) \otimes \left(\sum_{i=1}^{N} \varphi_i \boldsymbol{e}_i\right)\right) = ((T\boldsymbol{\theta}) \otimes \boldsymbol{\varphi}) \tag{4.14}$$

for some constants $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_N \end{bmatrix}^T$, with $\theta_1 = 0$ that excludes elements

that are not in the range of $(\Lambda \otimes I)\zeta(k)$. Then, based on Equation (4.11), one has

$$\begin{aligned}
\|\delta(k)\|^2 &= \zeta^T(k)\left(\Lambda\mathcal{M}^T\mathcal{M}\Lambda \otimes I_n\right)\zeta(k) \\
&\overset{(a)}{=} \left(T\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right)^T\left(\mathcal{M}^T\mathcal{M} \otimes I_n\right)\left(T\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right) \\
&= \left(\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right)^T\left(T^T\mathcal{M}^T\mathcal{M}T \otimes I_n\right)\left(\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right) \\
&\overset{(b)}{=} \left(\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right)^T\left(\Sigma \otimes I_n\right)\left(\theta \otimes \boldsymbol{\varphi}\right) \\
&\overset{(c)}{\geq} \sigma_2\left(\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right)^T\left(I_N \otimes I_n\right)\left(\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right) \\
&\overset{(d)}{=} \sigma_2\left(T\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right)^T\left(T\boldsymbol{\theta} \otimes \boldsymbol{\varphi}\right) \\
&\overset{(e)}{=} \sigma_2\zeta^T(k)\left(\Lambda^2 \otimes I_n\right)\zeta(k) \\
&= \sigma_2 \min\left\{\alpha_i^2\right\}\|\zeta(k)\| \qquad (4.15)
\end{aligned}$$

(a) holds based on Equation (4.14), (b) is obtained by the decomposition in (4.13), (c) holds for that $\theta_1 = 0$ which eliminates the effect of the zero eigenvalue, (d) holds for that T is a unitary matrix, (e) holds based on Equation (4.14) similar to step (a), and last line holds from the definition of Λ. From the fact that $\|\zeta(k)\|^2 = \sum_{i=1}^{N}\|\zeta_i(k)\|^2$, Equation (4.15) can be rewritten as

$$\sum_{i=1}^{N}\|\zeta_i(k)\|^2 \leq \frac{1}{\sigma_2 \min\left\{\alpha_i^2\right\}}\|\delta(k)\|^2 \qquad (4.16)$$

Applying Cauchy Swartz inequality, gives

$$\left\|\frac{\zeta_i(k)}{\alpha_i} - \frac{\zeta_j(k)}{\alpha_j}\right\|^2 \leq 2\left\|\frac{\zeta_i(k)}{\alpha_i}\right\|^2 + 2\left\|\frac{\zeta_j(k)}{\alpha_j}\right\|^2 \qquad (4.17)$$

and straight forward calculation leads to

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left\|\frac{\zeta_i(k)}{\alpha_i}\right\|^2 = N\sum_{m=1}^{N}\left\|\frac{\zeta_m(k)}{\alpha_m}\right\|^2 \qquad (4.18)$$

Using Equations (4.17), (4.18), and (4.16), we obtain the following bound

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\|\frac{\zeta_i(k)}{\alpha_i}-\frac{\zeta_j(k)}{\alpha_j}\right\|^2 &\leq 4N\sum_{m=1}^{N}\left\|\frac{\zeta_m(k)}{\alpha_m}\right\|^2\\ &\leq \frac{4N}{\min\{\alpha_i^2\}}\sum_{m=1}^{N}\|\zeta_m(k)\|^2\\ &\leq \frac{4N}{\sigma_2\min\{\alpha_i^4\}}\|\delta(k)\|^2 \qquad (4.19)\end{aligned}$$

Taking the expected value and applying Lemma 4.1 completes the proof. ∎

Up to now, the mean square practical scaled-consensus has been converted to mean square stability problem. This step is inevitable for leaderless case. For the leader-follower case mean square stability can be directly utilized as will be seen in the next section. It will be shown how to obtain control parameters that solve the scaled-consensus problem.

4.3 Scaled-Consensus Subject to DoS Attacks

4.3.1 Leader-Follower Scaled-Consensus

Consider the following leader-follower distributed protocol:

$$\begin{aligned} u_i(k) &= K\sum_{j\in N_i}\beta_{ij}a_{ij}\left(\alpha_{ij}Q(x_j(k))-Q(x_i(k))\right) \\ &+ K\beta_{0i}b_i\left(Q(x_i(k))-\alpha_{i0}Q(x_0(k))\right) \\ &= K\sum_{j\in N_i}\beta_{ij}a_{ij}\Big[\alpha_{ij}x_j(k)-\alpha_{i0}x_0(k)-x_i(k)+\alpha_{i0}x_0(k)\Big] \\ &+ K\beta_{0i}b_i\left(x_i(k)-\alpha_{i0}x_0(k)\right) \\ &+ K\sum_{j\in N_i}\beta_{ij}a_{ij}\Big[\alpha_{ij}\Delta_j(k)-\alpha_{i0}\Delta_0(k)-\Delta_i(k)+\alpha_{i0}\Delta_0(k)\Big] \\ &+ K\beta_{0i}b_i\left(\alpha_{i0}\Delta_0(k)-\Delta_i(k)\right) \end{aligned} \tag{4.20}$$

where $b_i = 1$ if the agent i can acquire information from the leader and $b_i = 0$ otherwise, β_{0i} is a Bernoulli sequence which prevent agent i to communicate with the leader when it is subject to an attack, and the leader scale-ratio is given as $\alpha_{i0} = \frac{\alpha_i}{\alpha_0}$. Consider the disagreement vector: $\xi_i(k) = x_i(k) - \alpha_{i0}x_0(k)$, and the quantization difference: $\Gamma_i(k) = \alpha_{i0}\Delta_0(k) - \Delta_i(k)$. It is straight forward to conclude that $\mathbb{E}[\Gamma_i(k)] = 0$. Similar to leaderless case, we assume that the quantizer distribution of the leader is independent of the followers quantizers, i.e. $\mathbb{E}[\Delta_i^T(k)\Delta_0(k)] = 0$ for all $i \in \mathcal{V}$, which leads to an independence in the quantization difference: $\mathbb{E}[\Gamma_i^T(k)\Gamma_0(k)] = 0$, with the following variance:

$$\mathbb{E}[\|\Gamma_i(k)\|^2] = \mathbb{E}[\|\Delta_0(k)\|^2] + \mathbb{E}[\|\Delta_i(k)\|^2]$$

Note that $\alpha_{i0} = \alpha_{ij}\alpha_{j0}$, then according to the definition of leader-disagreement

$\xi_i(k)$ and the quantization difference $\Gamma_i(k)$, we come up with the following form of the protocol (4.20):

$$u_i(k) = K \sum_{j \in N_i} \beta_{ij} a_{ij} \left(\alpha_{ij} \xi_j(k) - \xi_i(k) \right) + K \beta_{0i} b_i \left(\xi_i(k) \right)$$

$$+ K \sum_{j \in N_i} \beta_{ij} a_{ij} (\alpha_{ij} \Gamma_j(k) - \Gamma_i(k)) + K \beta_{0i} b_i \Gamma_i(k) \tag{4.21}$$

Denote $\xi(k) = \begin{bmatrix} \xi_1^T(k) & \xi_2^T(k) & \dots & \xi_N^T(k) \end{bmatrix}^T$, and $\Gamma(k) = \begin{bmatrix} \Gamma_1^T(k) & \Gamma_2^T(k) & \dots & \Gamma_N^T(k) \end{bmatrix}^T$. Then the collective dynamics of the leader-follower disagreement is obtained:

$$\xi(k+1) = \left[(I_N \otimes A) - \Lambda \mathcal{H}_\Theta \Lambda^{-1} \otimes BK \right] \xi(k) - (\Lambda(\mathcal{H}_\Theta) \Lambda^{-1} \otimes BK) \Gamma(k) \tag{4.22}$$

where $\mathcal{H}_\Theta = [\mathcal{H}_{ij}]$ is defined as follows:

$$\mathcal{H}_{ij} = \begin{cases} -a_{ij} \beta_{ij} & i \neq j \\ -\sum_{j=1}^{N} \mathcal{H}_{ij} - \beta_{0i} b_i & i = j \end{cases}$$

Mean-stability of the collective dynamics (4.22) implies that the leader-follower scaled-consensus is achieved. The following Theorem gives a sufficient condition for leader-follower consensus case.

Theorem 4.1 *Leader-follower mean-square practical scaled-consensus in (5.1) with the leader (4.3) is achieved if there exist matrices $\mathcal{Z}$ and $\mathcal{X} > 0$ and scalars $\gamma_1 > 0$ and $0 < \gamma_0 < 1$ satisfying the linear matrix inequality:*

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} \\ \bullet & \Omega_{22} & \Omega_{23} \\ \bullet & \bullet & \Omega_{33} \end{bmatrix} < 0 \tag{4.23}$$

$$\textit{where } \Omega_{11} = (\gamma_0 - 1)(I_N \otimes \mathcal{X}),\ \Omega_{22} = -\gamma_1(I_N \otimes \mathcal{X})$$

$$\Omega_{13} = \left(I_N \otimes \mathcal{X}A^T\right) - \Lambda^{-1}\bar{\mathcal{H}}_\Theta \Lambda \otimes \mathcal{Z}^T B^T$$

$$\Omega_{23} = -\Lambda^{-1}\bar{\mathcal{H}}_\Theta \Lambda \otimes \mathcal{Z}^T B^T,\ \Omega_{33} = -(I_N \otimes \mathcal{X})$$

Moreover, the matrix gain of the protocol (4.20) is given by $K = \mathcal{Z}\mathcal{X}^{-1}$.

Proof. Consider the following Lyapunov function:

$$V(k) = \xi^T(k)(I \otimes P)\xi(k)$$

where P is a positive definite matrix. It follows from Equation (4.22) that

$$V(k+1) - V(k) + \gamma_0 V(k) - \gamma_1 \Gamma^T(k)\Gamma(k) = \kappa^T(k)\mathcal{W}\kappa(k) \tag{4.24}$$

where $F = \left[\begin{array}{c|c} (I_N \otimes A) - \Lambda\mathcal{H}_\Theta\Lambda^{-1} \otimes BK & -\Lambda\mathcal{H}_\Theta\Lambda^{-1} \otimes BK \end{array}\right]$, $\kappa(k) = \begin{bmatrix} \xi^T(k) & \Gamma^T(k) \end{bmatrix}^T$, and

$$\mathcal{W} = \begin{bmatrix} (\gamma_0 - 1)(I_N \otimes P) & 0 \\ 0 & -\gamma_1(I_N \otimes P) \end{bmatrix} + F^T(I_N \otimes P)F \tag{4.25}$$

Denote $\bar{\mathcal{W}} \triangleq \mathbb{E}[\mathcal{W}]$. Since the quantization difference $\Gamma(k)$ is a stochastic process that is independent of the attack-processes, then we have

$$\begin{aligned} \mathbb{E}\Big[\kappa^T(k)\mathcal{W}\kappa(k)\Big|\xi(k)\Big] &= \mathbb{E}\Big[\mathbb{E}\big[\kappa^T(k)\mathcal{W}\kappa(k)\big|\kappa(k)\big]\Big|\xi(k)\Big] \\ &= \mathbb{E}\Big[\kappa^T(k)\mathbb{E}\big[\mathcal{W}\big|\kappa(k)\big]\kappa(k)\Big|\xi(k)\Big] \\ &= \mathbb{E}\Big[\kappa^T(k)\mathbb{E}\big[\mathcal{W}\big]\kappa(k)\Big|\xi(k)\Big] \\ &= \mathbb{E}\Big[\kappa^T(k)\bar{\mathcal{W}}\kappa(k)\Big|\xi(k)\Big] \end{aligned} \tag{4.26}$$

Therefore, with the help of Equation (4.26), if $\bar{\mathcal{W}} \leq 0$ then the following hold by

taking the conditional expectation for (4.24):

$$\begin{aligned}\mathbb{E}\big[V(k+1)|\xi(k)\big]-V(k) &\leq -\gamma_0 V(k)+\gamma_1\mathbb{E}\left[\|\Gamma(k)\|^2\right]\\ &\overset{(a)}{=} -\gamma_0 V(k)+\gamma_1(N+n)\mathbb{E}[\|\Delta_0(k)\|^2]+\gamma_1\sum_{i=1}^{N}\sum_{j=1}^{n}\mathbb{E}[\|\Delta_{ij}\|^2]\\ &\overset{(b)}{=} -\gamma_0 V(k)+\frac{\gamma_1}{2}(N+n)\Delta_p^2 \end{aligned} \tag{4.27}$$

Based on the definition of quantization difference $\Gamma(k)$, (a) holds for that $\left\|\begin{bmatrix} a_1 & \cdots & a_N \end{bmatrix}^T\right\|^2 = \sum_{i=1}^{N}\|a_i\|^2$, and (b) holds for that the variance of the quantization process is given as Δ_p. We observe that the mean-square practical leader-follower scaled-consensus is achieved if $\bar{\mathcal{W}} < 0$. Based on its representation in (4.25) and using Schur complement, we have $\bar{\mathcal{W}} < 0$ if and only if

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & 0 & \tilde{\Omega}_{13} \\ \bullet & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} \\ \bullet & \bullet & \tilde{\Omega}_{33} \end{bmatrix} < 0 \tag{4.28}$$

where

$$\tilde{\Omega}_{11} = (\gamma_0 - 1)(I_N \otimes P),\ \ \tilde{\Omega}_{22} = -\gamma_1(I_N \otimes P)$$

$$\tilde{\Omega}_{13} = \left(I_N \otimes A^T\right) - \Lambda^{-1}\bar{\mathcal{H}}_\Theta \Lambda \otimes K^T B^T$$

$$\tilde{\Omega}_{23} = -\Lambda^{-1}\bar{\mathcal{H}}_\Theta \Lambda \otimes K^T B^T,\ \tilde{\Omega}_{33} = -(I_N \otimes P^{-1})$$

Substituting $K = \mathcal{Z}P$ and multiplying both sides of $\tilde{\Omega}$ by the congruent transformation diag $[I_N \otimes \mathcal{X},\ I_N \otimes \mathcal{X},\ I_N]$, where $\mathcal{X} = P^{-1}$ gives the equivalent LMI (4.23). This completes the proof. ∎

Remark 7 *The results of Theorem 4.1 can be used to achieve leaderless consensus by considering a virtual leader implemented in a specific agent. However, the initial condition of such virtual leader will restrict the final values by one agent which prevent*

negotiation/agreement between agents. This problem will be solved in the sequel.

Remark 8 *Equation (4.26) has been derived by using conditional expectation technique. Nevertheless, it can be proved using the fact that if X and Y are independent random variables then $\mathbb{E}[f(X)g(Y)] = \mathbb{E}[f(X)]\mathbb{E}[g(Y)]$.*

4.3.2 Leaderless Scaled-Consensus

It is easy to show the following properties:

$$\mathcal{M}\Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} = (\mathcal{L} - \mathcal{L}_\Theta)\mathcal{M} = (\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1}$$

Based on these properties and Equations (4.7) and (4.9), we have the following derivation of the overall leaderless disagreement dynamics:

$$\begin{aligned}
\delta(k+1) &= (\mathcal{M} \otimes I_n)\, x(k+1) \\
&= \left[(\mathcal{M} \otimes A) - \mathcal{M}\Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK\right] x(k) \\
&\quad - (\Lambda^{-1}\mathcal{M}\Lambda(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK)\Delta(k) \\
&= \left[(I_N \otimes A) - (\mathcal{L} - \mathcal{L}_\Theta) \otimes BK\right](\mathcal{M} \otimes I_n)\, x(k) \\
&\quad - ((\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK)\Delta(k) \\
&= \left[(I_N \otimes A) - (\mathcal{L} - \mathcal{L}_\Theta) \otimes BK\right]\delta(k) - ((\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes BK)\Delta(k)
\end{aligned} \tag{4.29}$$

According to Lemma 4.3, scaled-consensus is transformed into mean square-stability of dynamics (4.29). Consider the unitary matrix $\Psi = \begin{bmatrix} \mathbf{1}/\sqrt{N} & v_2 & \cdots & v_N \end{bmatrix}$, where v_i, $i = 2, 3, \cdots, N$ is the set of orthonormal eigenvectors associated with nonzero eigenvalues of the Laplacian $\mathcal{L}$, i.e.$\mathcal{L}v_i = \lambda_i v_i$, then we get

$$\Psi^T\Psi = \Psi\Psi^T = I_N \tag{4.30}$$

that is a unitary matrix. This matrix gives a diagonalizing transformation of the the Laplacian matrix:

$$\Psi^T \mathcal{L} \Psi = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{D} \end{bmatrix} \tag{4.31}$$

where $\mathcal{D} = \text{diag}\{\lambda_2, \lambda_3, ..., \lambda_N\}$. Since the vector $\mathbf{1}/N$ is also an eigenvector of $\mathcal{L}_\Theta$ associated with the zero eigenvalue, we have the following structure:

$$\Psi^T \mathcal{L}_\Theta \Psi = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{D}_\Theta \end{bmatrix} \tag{4.32}$$

where $\mathcal{D}_\Theta = S^T \mathcal{L}_\Theta S$, and $S = \begin{bmatrix} v_2 & \cdots & v_N \end{bmatrix}$. It is worth noting that $\mathcal{D}_\Theta$ is not diagonal in general because v_i, $i = 2, \cdots, N$ might not be eigenvectors of $\mathcal{D}_\Theta$. Based on Equations (4.30), (4.31), and (4.32), one has

$$\Psi^T (\mathcal{L} - \mathcal{L}_\Theta)^2 \Psi = \Psi^T (\mathcal{L} - \mathcal{L}_\Theta) \Psi \Psi^T (\mathcal{L} - \mathcal{L}_\Theta) \Psi = \begin{bmatrix} 0 & 0 \\ 0 & (\mathcal{D} - \mathcal{D}_\Theta)^2 \end{bmatrix} \tag{4.33}$$

Now, we are in a position to state the results for leaderless case.

Theorem 4.2 *Suppose that the scaled-consensus gain K and a scalar $0 < \gamma_0 < 1$ are given. The multiagent system (5.1) achieves mean-square practical scaled-consensus if there exist a positive definite matrix P, and scalar $\gamma_1 > 0$ satisfying the following linear matrix inequality*

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \bullet & \Xi_{22} \end{bmatrix} < 0 \tag{4.34}$$

where

$$\Xi_{11} = (I_{N-1} \otimes (A^T PA - P + \gamma_0 P) - (\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes (K^T B^T PA + A^T PBK)$$

$$+ (\mathcal{D} - \bar{\mathcal{D}}_\Theta)^2 \otimes K^T B^T PBK$$

$$\Xi_{12} = -(\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes A^T PBK + (\mathcal{D} - \bar{\mathcal{D}}_\Theta)^2 \otimes K^T B^T PBK$$

$$\Xi_{22} = (\mathcal{D} - \bar{\mathcal{D}}_\Theta)^2 \otimes K^T B^T PBK - \gamma_1 (I_{N-1} \otimes I_n)$$

and $\bar{\mathcal{D}}_\Theta = \mathbb{E}[\mathcal{D}_\Theta]$.

Proof. Consider the following Lyapunov function:

$$V(k) = \delta^T(k)(I \otimes P)\delta(k)$$

where P is positive definite matrix. It follows form (4.29) that

$$\Delta V(k) = \delta^T(k+1)(I \otimes P)\delta(k+1) - \delta^T(k)(I \otimes P)\delta(k) = \eta^T(k)\Upsilon\eta(k) \tag{4.35}$$

where $\eta(k) = \begin{bmatrix} \delta^T(k) & \Delta^T(k) \end{bmatrix}^T$

$$\Upsilon_{11} = (I \otimes (A^T PA - P) - (\mathcal{L} - \mathcal{L}_\Theta) \otimes K^T B^T PA + (\mathcal{L} - \mathcal{L}_\Theta)^2 \otimes K^T B^T PBK$$

$$\Upsilon_{12} = -(\mathcal{L} - \mathcal{L}_\Theta)\Lambda^{-1} \otimes A^T PBK + (\mathcal{L} - \mathcal{L}_\Theta)^2 \Lambda^{-1} \otimes K^T B^T PBK$$

$$\Upsilon_{22} = \Lambda^{-1}(\mathcal{L} - \mathcal{L}_\Theta)^2 \Lambda^{-1} \otimes K^T B^T PBK - (\gamma_1 I_N \otimes I_n)$$

Setting $\bar{\eta}(k) = \begin{bmatrix} (\Psi^T \otimes I) & 0 \\ 0 & (\Lambda\Psi^T \otimes I) \end{bmatrix} \eta(k)$, one has

$$\bar{\eta}_1(k) = \left(\frac{\mathbf{1}^T}{\sqrt{N}} \otimes I_n\right)\delta(k) = \left(\frac{\mathbf{1}^T}{\sqrt{N}} \otimes I_n\right)\left(\Lambda^{-1}\mathcal{M} \otimes I_n\right) x(k) = 0 \tag{4.36}$$

Then it follows from (4.31), (4.32) and (4.33) that

$$\Delta V(k) = \tilde{\eta}^T(k) U \tilde{\eta}(k) \tag{4.37}$$

where $U = \begin{bmatrix} U_{11} & U_{12} \\ \bullet & U_{22} \end{bmatrix}$, $\tilde{\eta}(k) = \begin{bmatrix} \bar{\eta}_2, & \cdots, & \bar{\eta}_N, & \bar{\eta}_{N+2}, & \cdots, & \bar{\eta}_{2N} \end{bmatrix}^T$,

$$U_{11} = (I_{N-1} \otimes (A^T P A - P) - (\mathcal{D} - \mathcal{D}_\Theta) \otimes K^T B^T P A$$

$$- (\mathcal{D} - \mathcal{D}_\Theta) \otimes A^T P B K + (\mathcal{D} - \mathcal{D}_\Theta)^2 \otimes K^T B^T P B K$$

$$U_{12} = -(\mathcal{D} - \mathcal{D}_\Theta) \otimes A^T P B K + (\mathcal{D} - \mathcal{D}_\Theta)^2 \otimes K^T B^T P B K$$

$$U_{22} = (\mathcal{D} - \mathcal{D}_\Theta)^2 \otimes K^T B^T P B K$$

Let $\mathcal{Y} = \begin{bmatrix} 0 & 0 \\ 0 & (\gamma_1 I_{N-1} \otimes I_n) \end{bmatrix}$. The following inequality holds for that Ψ is a unitary matrix with simple manipulations according to Kronecker properties:

$$\begin{aligned} \mathbb{E}[\tilde{\eta}^T(k) \mathcal{Y} \tilde{\eta}(k)] &\leq \gamma_1 \mathbb{E}[\Delta^T(k)(\Psi \Lambda^2 \Psi^T \otimes I_n)\Delta(k)] \\ &\leq \gamma_1 \mathbb{E}[\|\Psi \Lambda \otimes I_n\| \|\Delta(k)\|^2] \\ &\leq \gamma_1 \max\{\alpha_i\} \mathbb{E}[\|\Delta(k)\|^2] \end{aligned} \tag{4.38}$$

Based on (4.37) and (4.38), one has

$$\begin{aligned} \mathbb{E}\left[V(k+1)|\delta(k)\right] - V(k) \leq \mathbb{E}\left[\tilde{\eta}^T(k)(U - \mathcal{Y})\tilde{\eta}(k) + \gamma_0 V(k) \middle| \delta(k)\right] \\ + \gamma_1 \mathbb{E}[\|\Psi \Lambda \otimes I_n\| \|\Delta(k)\|^2] - \gamma_0 V(k) \end{aligned} \tag{4.39}$$

It is easy to verify that

$$\begin{aligned} \gamma_0 V(k) &= \gamma_0 \delta^T(k)(I_N \otimes P)\delta(k) \\ &= \eta^T(k) \begin{bmatrix} \gamma_0(I_N \otimes P) & 0 \\ 0 & 0 \end{bmatrix} \eta^T(k) = \tilde{\eta}(k) \begin{bmatrix} \gamma_0(I_{N-1} \otimes P) & 0 \\ 0 & 0 \end{bmatrix} \tilde{\eta}^T(k) \end{aligned} \tag{4.40}$$

and $\mathbb{E}[\|\Delta(k)\|^2] = \sum_{i=1}^{N} \sum_{j=1}^{n} \mathbb{E}[\|\Delta_{ij}\|^2] \leq (N+n)\frac{\Delta_p^2}{4}$. Noting that $\mathbb{E}\left[U - \mathcal{Y}\right] = \Xi$ which is a negative definite matrix, similar to the procedure in (4.26), we have

$$\mathbb{E}\left[V(k+1)|\delta(k)\right] - V(k) \leq -\gamma_0 V(k) + \bar{\nu} \tag{4.41}$$

where $\nu = \gamma_1 \max\{\alpha_i\}(N+n)\frac{\Delta_p^2}{4}$. Applying Lemma 4.3 completes the proof.

∎

Based on Lemma 4.1 and the definition of the proposed Lypaunov function, we can deduce the following bound on the scaled-disagreement:

$$\mathbb{E}\left[\|\delta(k)\|^2\right] \leq \frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}\|\delta(0)\|^2(1-\gamma_0)^k + \frac{\nu}{\underline{\lambda}(P)\gamma_0} \tag{4.42}$$

Corollary 2 *If there exist matrices $\mathcal{Z}$ and $\mathcal{X} > 0$ and scalars $\gamma_1 > 0$ and $0 < \gamma_0 < 1$ satisfying the linear matrix inequality:*

$$\mathcal{T} = \begin{bmatrix} \mathcal{T}_{11} & 0 & \mathcal{T}_{13} \\ \bullet & \mathcal{T}_{22} & \mathcal{T}_{23} \\ \bullet & \bullet & \mathcal{T}_{33} \end{bmatrix} < 0 \tag{4.43}$$

where $\mathcal{T}_{11} = (\gamma_0 - 1)(I_{N-1} \otimes \mathcal{X}), \quad \mathcal{T}_{22} = -\gamma_1(I_{N-1} \otimes \mathcal{X})$

$\mathcal{T}_{13} = (I_{N-1} \otimes \mathcal{X}A^T) - (\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes \mathcal{Z}^T B^T$

$\mathcal{T}_{23} = -(\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes \mathcal{Z}^T B^T, \quad \mathcal{T}_{33} = -(I_{N-1} \otimes \mathcal{X})$*, then the multiagent system (5.1) achieves mean-square practical scaled-consensus. Moreover, the consesus gain is given by* $K = \mathcal{Z}\mathcal{X}^{-1}$.

Proof. Inequality (4.34) can be decomposed as follows:

$$\Xi = \Theta + J^T(I_{N-1} \otimes P)J \tag{4.44}$$

where $\Theta = \begin{bmatrix} (\gamma_0 - 1)(I_{N-1} \otimes P) & 0 \\ 0 & -\gamma_1(I_{N-1} \otimes P) \end{bmatrix}$ and $J^T = \begin{bmatrix} (I_{N-1} \otimes A^T) - (\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes K^T B \\ -(\mathcal{D} - \bar{\mathcal{D}}_\Theta) \otimes K^T B \end{bmatrix}$. Applying Schur complement to Equation (4.44), multiplying both sides by the congruent transformation diag $[I_{N-1} \otimes \mathcal{X},\ I_{N-1} \otimes \mathcal{X},\ I_{N-1}]$ and substituting $KP = \mathcal{Z}$ gives LMI (4.43). ∎

4.4 Simulation Results

To demonstrate the effectiveness and feasibility of the proposed theoretical results, an illustrative example is provided. For both leaderless and leader-follower classes, we consider a group of ten agents whose communication topology is shown in Figure 4.1. The agents' dynamics are given as follows:

$$x_i(k+1) = \begin{bmatrix} 1 & 0.0952 & 0.2094 \\ 0 & 0.9048 & 0.2784 \\ 0 & 0 & 0.9512 \end{bmatrix} x_i(k) + \begin{bmatrix} 0.0108 \\ 0.0238 \\ 0.0975 \end{bmatrix} u_i(k)$$

First, for the leaderless case, we select the scaling values $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = 3$, and $\alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 1$. Now, assuming the expected values of the DoS attacks as follows: $\bar{\beta}_1 = 0.5, \bar{\beta}_2 = 0.7, \bar{\beta}_3 = 0.5, \bar{\beta}_4 = 0.6, \bar{\beta}_5 = 0.45, \bar{\beta}_6 = 0.75, \bar{\beta}_7 = 0.6, \bar{\beta}_8 = 0.8, \bar{\beta}_9 = 0.2$, and $\bar{\beta}_{10} = 0.7$. With the scalars $\gamma_0 = 0.01651$ and $\gamma_1 = 1.511$, by using the LMI (4.43) and the MATLAB software (with CVX package), we obtain the following scaled-consensus gain:

$$K = [-0.2644, \ -0.2678, \ -1.9018]$$

The initial conditions are given as:

$$x_1(0) = [0, \ 1, \ 2]^T, \qquad x_2(0) = [1, \ 2, \ 4]^T,$$

$$x_3(0) = [-1, \ 4, \ 1]^T, \qquad x_4(0) = [2, \ 5, \ 0]^T,$$

$$x_5(0) = [3, \ -5, \ 0]^T, \qquad x_6(0) = [1, \ -4, \ -1]^T,$$

$$x_7(0) = [4, \ 1, \ -3]^T, \qquad x_8(0) = [0, \ 5, \ 6]^T,$$

$$x_9(0) = [-3, \ 3, \ 3]^T, \qquad x_{10}(0) = [-2, \ 1, \ 2]^T$$

Under the effect of DoS attacks and quantization with $\Delta_p = 0.1$, the simulation results of the average state response using the protocol (4.4) are shown in Figure 4.2. The

average of state response is simulated over 10^4 times. Figure 4.3 displays the simulation results when ideal measurement is considered. We observe that scaled-consensus is achieved in both cases with small deviation due to the quantization effects.

For the leader-follower case, consider the same probabilities of DoS attacks and scaling parameters $\alpha_0 = -1$, $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = 3$, and $\alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 1$. The dynamics of the leader (4.3) is represented using the state-matrix A of the followers. The leader is connected to the agents $i = 1, 3$ only. The DoS probability against the leader agent is given $\bar{\beta}_{01} = 0.75$ and $\bar{\beta}_{02} = 0.6$. By using the LMI (4.23) and the MATLAB software (with CVX package), we have the following leader-follower scaled-consensus gain:

$$K = [-0.3797,\ -0.3997,\ -3.0401]$$

With the same initial conditions of the leaderless case, and the protocol (4.20), the states response of one sample path is displayed in Figure 4.4. Since the leader scale α_0 has negative value and the scales of the agents have positive scale ratios, we observe that opposite behavior has been achieved. Moreover, the agents that have the same scales reach the same common values, and each group reaches dedicated ratios with respect to other groups. We note that an oscillation appears due to the DoS attack. According to the input matrix $B = \begin{bmatrix} 0.0108 & 0.0238 & 0.0975 \end{bmatrix}$, one can see that the last element $B_3 \approxeq 9B_1$. An intense oscillation occurs in the behavior of x_3 due to the large value of the component $B_3 = 0.0975$ compared with B_2 and B_1. In addition, as can be seen in Figure 4.2, the oscillation becomes more intense because of the sever impact of the quantization error. It can be seen that the disagreement states are practical mean-square stable, see Figure 4.5. Despite of the link failures, the

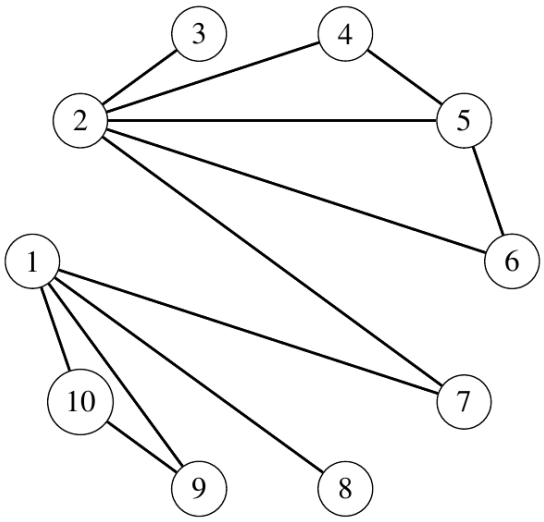

Figure 4.1: Communication topology.

disagreement converges to zero when the quantization is not considered, see Figure 4.3. On the other hand, practical mean-square convergence has been satisfied even though the DoS and quantized measurements are considered.

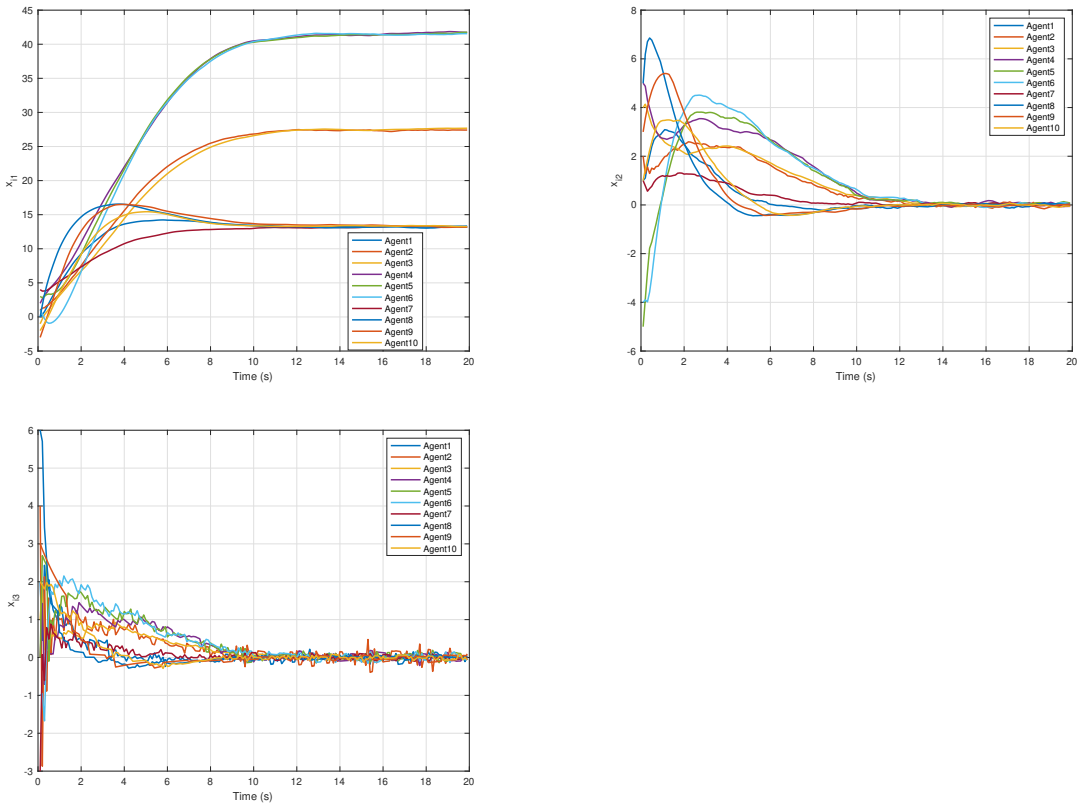

Figure 4.2: Average of the states' response over 10^4 simulations of leaderless MAS (5.1) subject to DoS attacks and quantization, with scales $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = 3$, and $\alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 1$.

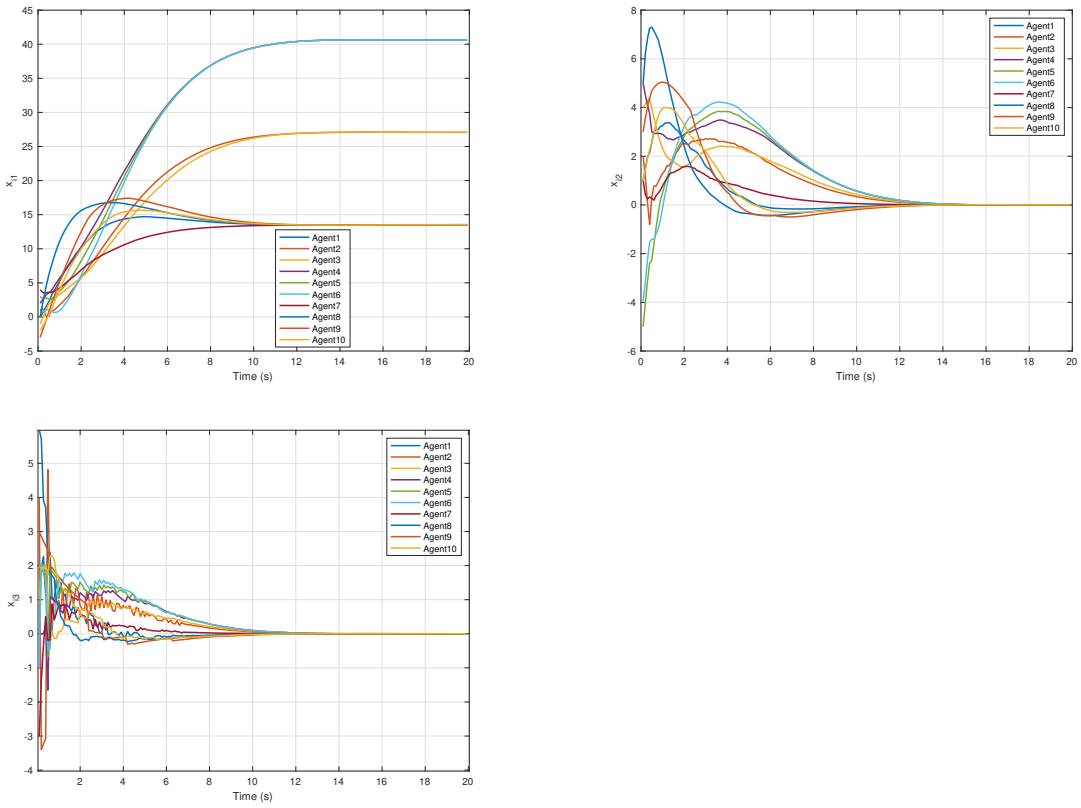

Figure 4.3: Average of the states' response over 10^4 simulations, of leaderless MAS subject to DoS attacks with ideal measurements, with scales $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = 3$, and $\alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 1$.

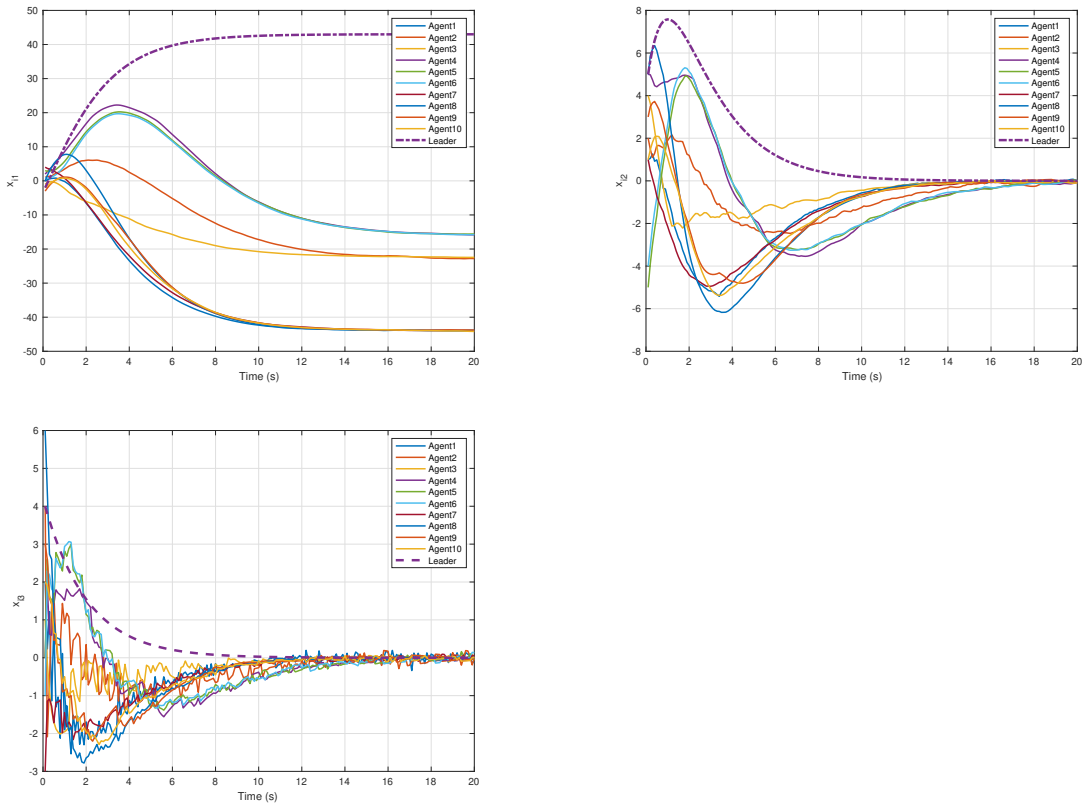

Figure 4.4: States' response of leader-follower case, with scales $\alpha_0 = -1$, $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, $\alpha_4 = \alpha_5 = \alpha_6 = 3$, and $\alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 1$.

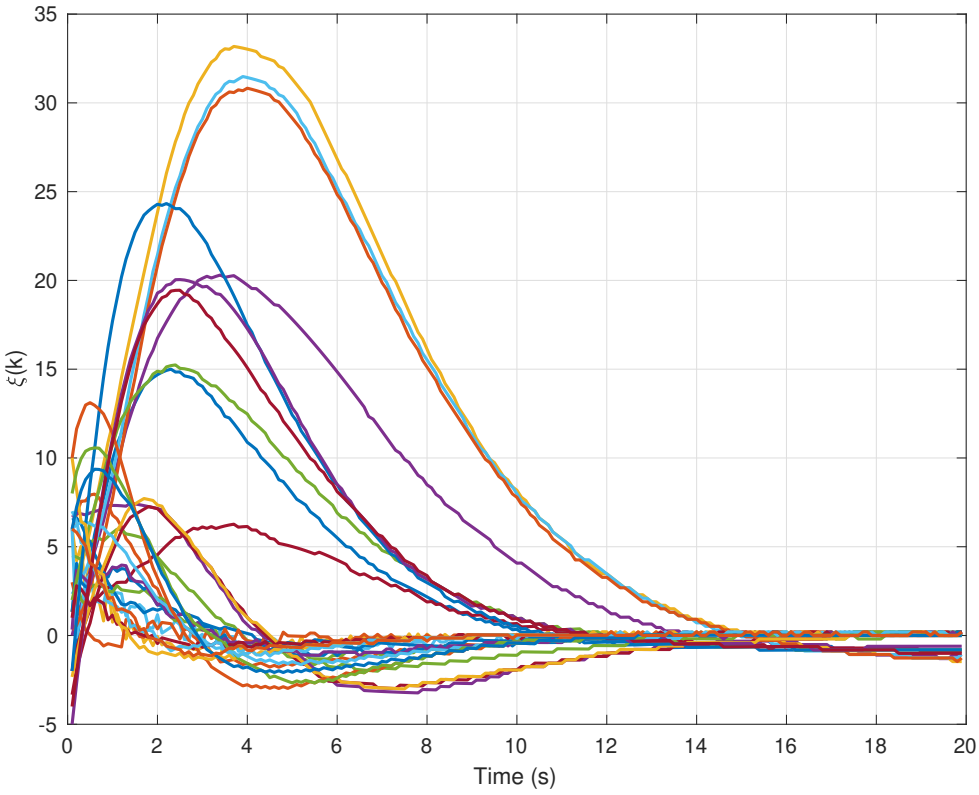

Figure 4.5: Tracking-error $\xi_i(k)$ evolution of of the leader.

CHAPTER 5

DISTRIBUTED EVENT-TRIGGERED CONSENSUS PROTOCOLS

This Chapter focuses on leader-following and leaderless consensus problems of discrete-time multiagent systems. A distributed observer-based consensus protocol is proposed to investigate the consensus problem for multiagent systems of general discrete-time linear dynamics. By means of the observer, the distributed control law of each agent is designed using local information to guarantee consensus, and the corre- sponding sufficient conditions are obtained by exploiting graph and control theory approach. A modified distributed event-triggered consensus protocol is designed to reduce communication congestion. Detailed analysis of the leaderless and the leader-following consensus is presented for both observer- based and full-information protocols. Finally, two simulation examples are provided to demonstrate the effectiveness and capabilities

of the established theories.

5.1 Introduction

The leaderless consensus approach requires all agents reach to a common value or variable. However, the leader-following scheme needs the followers to track the leader's state, where the leader can be considered as a reference generator and is not affected by the followers [27]-[28]. For the sake of consensus, many researches have been addressed several effective control strategies, such as robust control [43], observer based methods [44], pining methodology [45], adaptive control [46]. The aforementioned results are all continuous, and the control law design requires state information continuously which is too ideal to achieve consensus for multiagent systems. Implementing periodical sampling scheme or the time-triggered policy could waste resources such as networks bandwidth due to the numerous redundant sampling data. This leads to data congestion particularly when the sampling intervals are very short [140]. It is for this reason that efficient control schemes should be implemented to reduce data transmission and control update. Therefore, the event-triggered policy is an effective method to mitigate the unnecessary waste of communication and computation resources, and the sampled-data is released to the shared communication network when a defined condition is violated [141]. Many results have been investigated to enlarge the inter-event time intervals to decrease the transmission frequency [160]-[161].

To implement a practical control protocols for multiagent systems, it is compulsory to investigate the network limitations, and consider the computation resources. This

motivates us to seek the possible approach of event triggered discrete-time protocols for multiagent systems. We have tried to introduce a well organized solution to the leaderless and leader-following problem of discrete-time multiagent system under fixed communication topology. The main contributions and primary distinctions of this Chapter with other results can be summarized as follows:

1. Leaderless consensus is investigated with a modified version of distributed event-triggering protocols via combination measurement without the need of continuous communication. Moreover, the results are extended to observer-based leaderless consensus for the case where all agents' states are not directly accessible.

2. Leader-following consensus criterion is given to achieve the cooperative control using distributed observer-based for general linear multiagent systems. Unlike triggering policy based on model-based mechanisms, the control law is executed discretely such that it reduces the computational resources and controller's energy.

3. In [166] a modified integral-type was considered for continuous-time multiagent systems, the case we consider is a modified discrete-time version for multiagent system. Implementing such integral-type triggering policy requires short internal sampling to compute the integral which increases energy consumption of the controllers. However, the proposed discrete counterpart does not produce error due to sampling, excludes the Zeno phenomenon inherently, and is more easier to implement.

5.2 Problem Formulation

Consider a group of N agents with the following general discrete-time linear state space representation:

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + Bu_i(k) \\ y_i(k) &= Cx_i(k), \quad i = 1, \ldots, N \end{aligned} \tag{5.1}$$

where $x_i(k) \in \mathbb{R}^n$ represents the state of agent i, $u_i(k) \in \mathbb{R}^m$ represents its control law, and $y_i(k) \in \mathbb{R}^p$ denotes its output. In the sequel, the leader-following consensus will be investigated with the following leader dynamics:

$$\begin{aligned} x_0(k+1) &= Ax_0(k) \\ y_0(k) &= Cx_0(k), \end{aligned} \tag{5.2}$$

where $x_0(k)$ and $y_0(k)$ denote the state and output variable of the leader, respectively. Note that the presented dynamics of the follower-agents and the leader are a more general of the first and second order integrators. The following definition is used to distinguish between the leader-following and leaderless consensus.

Definition 5.1 *The discrete-time multiagent dynamics of (5.1) achieves leaderless consensus if*

$$\lim_{k\to\infty} \|x_i(k) - x_j(k)\| = 0 \quad \forall i, j \in \mathcal{G}$$

and the leader-following consensus is said to be achieved if

$$\lim_{k\to\infty} \|x_i(k) - x_0(k)\| = 0 \quad \forall i \in \mathcal{G}$$

Assumption 8 *Both leaderless and leader-following consensus will be addressed in this Chapter. For analysis convenience, the interaction topology is assumed to be undirected for both the leaderless and the leader-following cases. In addition, in the*

leader-following case, the graph $\mathcal{G}$ is assumed to contain a spanning tree with the leader agent as a root.

First, the following full-information protocol is considered for achieving the leaderless consensus:

$$u_i(k) = -Kz_i\left(k^i_{l^i}\right), \quad k \in [k^i_{l^i}, k^i_{l^i+1}) \tag{5.3}$$

where $z_i\left(k^i_{l^i}\right) = \sum_{j\in N_i} a_{ij}\left(x_i\left(k^i_{l^i}\right) - x_j\left(k^j_{l^j}\right)\right)$, K is the consensus gain to be designed, and $k^i_{l^i}$ is an integer that represents the triggering instant of the agent i.

Denote the triggering-error $e_i(k) = x_i\left(k^i_{l^i}\right) - x_i(k)$, the following represents the modified triggering rule: $k^i_{l^i+1} = \inf\left\{T \in \mathbb{N} | T > k^i_{l^i} : f_i(z_i, e_i) \leq 0\right\}$ with the following triggering function:

$$f_i(z_i, e_i) \;=\; \sum_{r=k^i_{l^i}}^{T}\left(\kappa_i\left\|z_i\left(k^i_{l^i}\right)\right\|^2 + \beta\alpha^r\right) - \sum_{r=k^i_{l^i}}^{T}\|e_i(r)\|^2 \tag{5.4}$$

where β is a positive scalar, $0 < \alpha < 1$ and κ_i is a positive scalar to be designed.Substituting the consensus protocol (5.3) into the multiagent dynamics (5.1) yields that following overall dynamics:

$$x(k+1) = (I_N \otimes A)\, x(k) - (I_N \otimes BK)\, z\left(k_l\right) \tag{5.5}$$

where $x(k) = \left[x_1^T(k), \ldots, x_N^T(k)\right]^T$, $z\left(k_l\right) = \left[z_1^T\left(k^1_{l^1}\right), \ldots, z_N^T\left(k^N_{l^N}\right)\right]^T$. Denote the disagreement vector $\delta_i(k) = x_i(k) - \bar{x}(k)$, and $\bar{x}(k) = \frac{1}{N}\sum_{i=1}^{N} x_i(k)$ where $\bar{x}(k)$ is the average of agents' states. Then, the compact form of disagreement error vector can be represented as

$$\delta(k) = (\mathcal{M} \otimes I_n)\, x(k), \tag{5.6}$$

where $\delta(k) = \left[\delta_1^T(k), \ldots, \delta_N^T(k)\right]^T$, and $\mathcal{M} = I_N - \frac{1}{N}\mathbf{1}\ \mathbf{1}^T$. It is straightforward to see that $\mathcal{L}\mathcal{M} = \mathcal{M}\mathcal{L} = \mathcal{L}$. Based on (5.5) and (5.6), the disagreement dynamics is

expressed as:

$$\delta(k+1) = (I_N \otimes A)\,\delta(k) - (\mathcal{M} \otimes BK) z\,(k_l) \tag{5.7}$$

From the relation between the augmented state and the disagreement vector given in (5.6), $\delta(t) = 0$ implies that $x(t)$ lies in the null space of $(\mathcal{M} \otimes I)$. Based on the definition of $\mathcal{M}$, the null space is spanned by the vector $(\mathbf{1} \otimes I)$. Therefore, the stability of the disagreement dynamics (5.7) implies that the leaderless consensus of the multiagent system (5.1) is achieved. The stability analysis of such dynamics will be investigated in the sequel under the modified triggering policy. We are now ready to introduce the observer based leaderless consensus method for the multiagent system (5.1). Consider the following observer based protocol:

$$\begin{aligned} \hat{x}_i(k+1) &= A\hat{x}_i(k) + Bu_i(k) + F\left(y_i(k) - \hat{y}_i(k)\right) \\ u_i(k) &= -\hat{K}\hat{z}_i\left(k^i_{l^i}\right) = -\hat{K}\sum_{j\in N_i} a_{ij}\left(\hat{x}_i\left(k^i_{l^i}\right) - \hat{x}_j\left(k^j_{l^j}\right)\right) \end{aligned} \tag{5.8}$$

based on the triggering condition:

$$f_i(z_i, \varpi_i) = \sum_{r=k^i_{l^i}}^{T}\left(\kappa_i \left\|\hat{z}_i\left(k^i_{l^i}\right)\right\|^2 + \beta\alpha^r\right) - \sum_{r=k^i_{l^i}}^{T}\left\|\varpi_i(r)\right\|^2 \tag{5.9}$$

where $\varpi_i(k) = \hat{x}_i(k^l_{l^i}) - \hat{x}_i(k)$ is the triggering error for agent i, and $\hat{x}_i(k), \hat{y}_i(k)$ represent the observer state and its output for each agent i, respectively. The matrices L and $\hat{K}$ are the observer and consensus gains to be designed later. Similar to leaderless problem, we address the full-information event-triggering protocol for the leader-following case with the following control law:

$$u_i(k) = -\tilde{K}\tilde{z}_i\left(k^i_{l^i}\right) \quad k \in [k^i_{l^i}, k^i_{l^i+1}) \tag{5.10}$$

where the combination error $\tilde{z}_i\left(k^i_{l^i}\right)$ is given as follows

$$\tilde{z}_i\left(k^i_{l^i}\right) = \left[\sum_{j\in N_i} a_{ij}\left(x_i\left(k^i_{l^i}\right) - x_j\left(k^j_{l^j}\right)\right) + d_i\left(x_i\left(k^i_{l^i}\right) - x_0(k)\right)\right] \tag{5.11}$$

based the following triggering function:

$$f_i(z_i, e_i) = \sum_{r=k_{l^i}^i}^{T} \left(\kappa_i \left\| \tilde{z}_i \left(k_{li}^i \right) \right\|^2 + \beta \alpha^r \right) - \sum_{r=k_{li}^i}^{T} \left\| e_i(r) \right\|^2 \tag{5.12}$$

Substituting the leader-following protocol (5.10) into the multiagent dynamics (5.1) yields that following overall dynamics:

$$x(k+1) = (I_N \otimes A)\, x(k) - (I_N \otimes BK)\, \tilde{z}\,(k_l) \tag{5.13}$$

where $\tilde{z}(k_l) = \left[\tilde{z}_1^T(k_{l^1}^1), \ldots, \tilde{z}_N^T(k_{l^N}^N) \right]^T$. Denote $\varepsilon_i(k) = x_i(k) - x_0(k)$ as the tracking error for agent i and the augmented error vector $\varepsilon(k) = \left[\varepsilon_1^T(k), \ldots, \varepsilon_N^T(k) \right]^T$. Using the dynamics of the leader (5.2) and the overall dynamics of the followers (5.13), then the overall dynamics of the following-error is obtained:

$$\varepsilon(k+1) = (I_N \otimes A)\, \varepsilon(k) - \left(I_N \otimes B\tilde{K} \right) \tilde{z}\,(k_l) \tag{5.14}$$

The stability of (5.14) implies that the leader-following consensus of the multiagent system (5.1) is achieved with the leader represented in (5.2).

5.3 Consensus Results

Here we investigate the leaderless consensus problem of multiagent system (5.1). To analyze the leaderless-consensus, we need to define $z_i(k) = \sum_{i \in N} a_{ij} \left(x_i(k) - x_j(k) \right)$, and $\tilde{e}_i(k) = z_i \left(k_{li}^i \right) - z_i(k)$, then the following transformations can be obtained:

$$z_i(k) = \sum_{j \in N_i} a_{ij} \left(x_i(k) - \bar{x}(k) - x_j(k) + \bar{x}(k) \right) = \sum_{j \in N_i} a_{ij} \left(\delta_i(k) - \delta_j(k) \right) \tag{5.15}$$

and

$$\begin{aligned}\tilde{e}_i(k) &= \sum_{j\in N_i} a_{ij}\left(x_i\left(k_{li}^i\right) - x_j\left(k_{lj}^j\right)\right) - \sum_{j\in N_i} a_{ij}\Big(x_i(k) - x_j(k)\Big) \\ &= \sum_{j\in N_i} a_{ij}\left(x_i\left(k_{li}^i\right) - x_i(k)\right) - \sum_{j\in N_i} a_{ij}\Big(x_j\left(k_{lj}^j\right) - x_j(k)\Big) \\ &= \sum_{j\in N_i} a_{ij}\left(e_i(k) - e_j(k)\right) \qquad (5.16)\end{aligned}$$

Therefore, they can be written in the following compact forms:

$$\begin{aligned}z(k) &= (\mathcal{L}\otimes I_n)\,\delta(k) = (\mathcal{L}\otimes I_n)\,x(k) \\ e(k) &= \left[e_1^T(k),\ldots,e_N^T(k)\right]^T \qquad (5.17) \\ \tilde{e}(k) &= z(k_l) - z(k) = (\mathcal{L}\otimes I_n)\,e(k) \\ \tilde{e}(k) &= \left[\tilde{e}_1^T(k),\ldots,\tilde{e}_N^T(k)\right]^T \qquad (5.18)\end{aligned}$$

Based on 5.16, with algebraic manipulations, it is easy to show that the following inequality holds for any matrix V:

$$\begin{aligned}\|V\tilde{e}_i(k)\| &= \|V\sum_{j\in N_i} a_{ij}(e_i(k) - e_j(k))\| \\ &\le \|\sum_{j\in N_i} a_{ij}Ve_i(k)\| + \|\sum_{j\in N_i} a_{ij}Ve_j(k)\| \\ &\le \ell_{ii}\|Ve_i(k)\| + \sum_{j\in N_i} a_{ij}\|Ve_j(k)\| \qquad (5.19)\end{aligned}$$

Now, we are ready to state the first result of leaderless consensus based on the protocol (5.3).

Theorem 5.1 *Consider the multiagent system (5.1) with an undirected and connected topology G. If there exist symmetric matrices $P > 0$, $Q > 0$ and $S > 0$ with appropriate dimensions such that the following linear matrix inequality holds*

$$A^TPA - P - \lambda_2 S + Q \le 0 \qquad (5.20)$$

and the consensus gain K satisfies:

$$J_1 = \left(\lambda_2\lambda_{min}(Q) - 2\|(\mathcal{L}\otimes H)\|^2\right) - 2\bar{\Delta}\bar{\kappa}\|\mathcal{L}\|^2\left(\bar{\psi}\|(I\otimes V)\|^2 + 2\|(\mathcal{L}\otimes H)\|^2\right) > 0 \tag{5.21}$$

where $V^TV = \frac{A^TPBK+K^TB^TPA}{2} + 2S, H^TH = \lambda_{max}(\mathcal{L})K^TB^TPBK - A^TPBK + 2S, \frac{1}{\Delta} \triangleq \left(1 - \max\left\{2\kappa_i N l_{ii}^2 + 2\kappa_i N^2 \max_{i\in N_j}\left\{a_{ji}^2\right\}\right\}\right) > 0$ $\lambda_2 = \lambda_2(\mathcal{L}),$ $\psi_i = N\left[\ell_{ii}^2 + \sum_{j\in N_i} a_{ji}^2\right]$, *and* $\bar{\psi} = \max_i \psi_i,$ $\bar{\kappa} = \max_i\{\kappa_i\}$ *then the multiagent system (5.1) achieves leaderless consensus based on the full-information protocol (5.3) with the modified event-triggering function (5.4).*

Proof: Consider the following Lyapunov function: $V(k) = \delta^T(k)(\mathcal{L}\otimes P)\delta(k)$ where P is a positive definite matrix. The difference of the Lyapunov function $\Delta V(t)$ along the dynamics of (5.5) is given by

$$\begin{aligned}\Delta V = V(k+1) - V(k) &= \delta^T(k)\left[(I\otimes A)^T(\mathcal{L}\otimes P)(I\otimes A) - \mathcal{L}\otimes P\right]\delta(k)\\ &\quad - 2\delta^T(k)\left[(I\otimes A)^T(\mathcal{L}\otimes P)(\mathcal{M}\otimes BK)\right]z(k_l)\\ &\quad + z^T(k_l)\left[(\mathcal{M}\otimes BK)^T(\mathcal{L}\otimes P)(\mathcal{M}\otimes BK)\right]z(k_l)\\ &\quad - \delta^T(k)\left[(\mathcal{L}^2\otimes S)\right]\delta(k) + \delta^T(k)\left[(\mathcal{L}^2\otimes S)\right]\delta(k)\end{aligned} \tag{5.22}$$

for any matrix S with appropriate dimension. Based on (5.17), one has

$$\begin{aligned}\delta^T(k)\left[(\mathcal{L}^2\otimes S)\right]\delta(k) &= \delta^T(k)\left[(\mathcal{L}\otimes I)(I\otimes S)(\mathcal{L}\otimes I)\right]\delta(k)\\ &= z^T(k)\left[(I\otimes S)\right]z(k)\end{aligned} \tag{5.23}$$

Recalling that $\mathcal{L}\mathcal{M} = \mathcal{M}\mathcal{L} = \mathcal{L}$, and according to (5.23), we can represent (5.22) as

$$\begin{aligned}\Delta V &= \delta^T(k)\left[(\mathcal{L}\otimes(A^TPA - P) - (\mathcal{L}^2\otimes S))\right]\delta(k)\\ &- 2\delta^T(k)(\mathcal{L}\otimes A^TPBK)z(k_l) + z^T(k)(I\otimes S)z(k)\\ &+ z^T(k_l)(\mathcal{L}\otimes K^TB^TPBK)z(k_l)\end{aligned} \tag{5.24}$$

Let $\mathcal{U} = -2\delta^T(k)(\mathcal{L} \otimes A^T PBK)z(k_l)$. Based on the definition of the error $\tilde{e}_i(k) = z_i\left(k_{li}^i\right) - z_i(k)$, Young's inequality, (5.17) and (5.18), the upper bound of $\mathcal{U}$ is obtained:

$$\begin{aligned}
\mathcal{U} &= -2\delta^T(k)(\mathcal{L} \otimes I)(I \otimes A^T PBK)z(k_l) \\
&= -2z^T(k)(I \otimes A^T PBK)z(k_l) \\
&= -2(z(k_l) - \tilde{e}^T(k))(I \otimes A^T PBK)z(k_l) \\
&\leq -z^T(k_l)(I \otimes A^T PBK)z(k_l) + \tilde{e}^T(k))(I \otimes A^T PBK)\tilde{e}(k) \qquad (5.25)
\end{aligned}$$

Using the properties of Kronecker products, since $K^T B^T PBK$ is positive definite then the following bound holds:

$$z^T(k_l)\left[(\mathcal{L} \otimes K^T B^T PBK)\right] z(k_l) \leq z^T(k_l)\left[(I \otimes \lambda_{max}(\mathcal{L})K^T B^T PBK)\right] z(k_l) \quad (5.26)$$

Replacing the bounds in (5.25) and (5.26) with their corresponding parts in (5.24),

gives

$$\begin{aligned}
\Delta V &\leq \delta^T(k)\left[(\mathcal{L}\otimes(A^TPA-P)-(\mathcal{L}\mathcal{L}\otimes S))\right]\delta(k)\\
&+ \tilde{e}^T(k)(I\otimes A^TPBK)\tilde{e}(k)+z^T(k)(I\otimes S)z(k)\\
&+ z^T(k_l)\left[I\otimes\left(\lambda_{max}(\mathcal{L})K^TB^TPBK-A^TPBK\right)\right]z(k_l)\\
&= \delta^T(k)\left[\mathcal{L}\otimes(A^TPA-P)-(\mathcal{L}\mathcal{L}\otimes S)\right]\delta(k)\\
&+ \sum_{i=1}^{N}\tilde{e}_i^T(k))\left[A^TPBK\right]\tilde{e}_i(k))\\
&+ \sum_{i=1}^{N}z_i^T(k_l)\left[\lambda_{max}(\mathcal{L})K^TB^TPBK-A^TPBK\right]z_i(k_l)\\
&+ \sum_{i=1}^{N}(z_i(k_{l^i}^i)-\tilde{e}_i(k))S(z_i(k_{l^i}^i)-\tilde{e}_i(k))\\
&\leq \delta^T(k)\left[(\mathcal{L}\otimes(A^TPA-P)-(\mathcal{L}\mathcal{L}\otimes S))\right]\delta(k)\\
&+ \sum_{i=1}^{N}\tilde{e}_i^T(k))\left[A^TPBK+2S\right]\tilde{e}_i(k))\\
&+ \sum_{i=1}^{N}z_i^T(k_l)\left[\lambda_{max}(\mathcal{L})K^TB^TPBK-A^TPBK+2S\right]z_i(k_l) \qquad (5.27)
\end{aligned}$$

From the definitions of V and ψ_i, using (5.19), and the fact $\left(\sum_{i=1}^{N}x_i\right)^2\leq N\sum_{i=1}^{N}x_i^2$, one has

$$\begin{aligned}
\sum_{i=1}^{N}\tilde{e}_i^T(k)\left[A^TPBK+2S\right]\tilde{e}_i(k) &= \sum_{i=1}^{N}\|V\tilde{e}_i(k)\|^2\\
&\leq \sum_{i=1}^{N}\left[\ell_{ii}\|Ve_i(k)\|+\sum_{j\in N_i}a_{ij}\|Ve_j(k)\|\right]^2\\
&\leq \sum_{i=1}^{N}N\left[\ell_{ii}^2\|Ve_i(k)\|^2+\sum_{j\in N_i}a_{ij}^2\|Ve_j(k)\|^2\right]\\
&\leq \sum_{i=1}^{N}\psi_i\|Ve_i(k)\|^2 \qquad (5.28)
\end{aligned}$$

Since $\mathcal{L}$ has a simple zero eigenvalue and symmetric, there exists an orthogonal matrix $\mathbf{U}$ such that $\mathbf{U}^{-1}\mathcal{L}\mathbf{U}=\mathbf{U}^T\mathcal{L}\mathbf{U}=J=\mathbf{diag}\{0,\lambda_2(\mathcal{L}),\ldots,\lambda_N(\mathcal{L})\}$. Let

$\tilde{\delta}(k) = \left(\mathbf{U}^T \otimes I_n\right)\delta(k)$, where $\tilde{\delta}(k) = \left[\tilde{\delta}_1^T(k), \ldots, \tilde{\delta}_N^T(k)\right]^T$. Then, using the algebraic fact $(x+y)^2 \leq 2x^2 + 2y^2$, LMI (5.20), and substituting (5.28) into (5.27) results in

$$\begin{aligned}
\Delta V &\leq \sum_{i=1}^{N}\Big[\tilde{\delta}_i^T(k)\left[\lambda_i(\mathcal{L})(A^TPA - P) - \lambda_i^2(\mathcal{L})S\right]\tilde{\delta}_i(k) \\
&+ \psi_i e_i^T(k)V^TVe_i(k)\Big] + z^T(k_l)\left[I \otimes \left(H^TH\right)\right]z(k_l) \\
&\leq -\lambda_2\lambda_{min}(Q)\|\tilde{\delta}(k)\| + \bar{\psi}\|(I\otimes V)e(k)\|^2 + \|(I\otimes H)z(k_l)\|^2 \\
&= -\lambda_2\lambda_{min}(Q)\|\delta(k)\| + \bar{\psi}\|(I\otimes V)e(k)\|^2 + \|(I\otimes H)[z(k) - \tilde{e}(k)]\|^2 \\
&\leq -\lambda_2\lambda_{min}(Q)\|\delta(k)\| + \bar{\psi}\|(I\otimes V)e(k)\|^2 \\
&+ 2\|(I\otimes H)z(k)\|^2 + 2\|(I\otimes H)\tilde{e}(k)]\|^2 \qquad (5.29)
\end{aligned}$$

Employing the presentations in (5.17) and (5.18), we can further manipulate the upper bound of ΔV as:

$$\begin{aligned}
\Delta V &\leq -\lambda_2\lambda_{min}(Q)\|\delta(k)\| + \bar{\psi}\|(I\otimes V)e(k)\|^2 \\
&+ 2\|(\mathcal{L}\otimes H)\delta(k)\|^2 + 2\|(\mathcal{L}\otimes H)e(k)]\|^2 \\
&\leq -\left(\lambda_2\lambda_{min}(Q) - 2\|(\mathcal{L}\otimes H)\|^2\right)\|\delta(k)\| \\
&+ \left(\bar{\psi}\|(I\otimes V)\|^2 + 2\|(\mathcal{L}\otimes H)\|^2\right)\|e(k)\|^2 \qquad (5.30)
\end{aligned}$$

In order to give an upper bound to the last term $\|e(k)\|$, we need to find the bound of its components $\|e_i(k)\|$. Based on the modified event-triggered condition (5.4) and

using (5.16), it can be derived that

$$\begin{aligned}
\sum_{r=k_{l^i}^i}^{T} \|e_i(r)\|^2 &\leq \sum_{r=k_{li}^i}^{T} \left(\kappa_i \left\|z_i\left(k_{l^i}^i\right)\right\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{li}^i}^{T} \left(\kappa_i \|\tilde{e}_i(r) + z_i(r)\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{li}^i}^{T} \left(2\kappa_i \|\tilde{e}_i(r)\|^2 + 2\kappa_i \|z_i(r)\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{li}^i}^{T} \left[2\kappa_i N \left(l_{ii}^2 \left\|e_i^T(r)\right\|^2 + \sum_{j\in N_i} a_{ij}^2 \left\|e_j^T(r)\right\|^2\right) + 2\kappa_i \|z_i(r)\|^2 + \beta\alpha^r\right] \\
&\leq \sum_{r=k_{li}^i}^{T} \left[2\kappa_i N l_{ii}^2 \left\|e_i^T(r)\right\|^2 + 2\kappa_i N^2 \max_{i\in N_j}\left\{a_{ji}^2\right\} \left\|e_i^T(r)\right\|^2\right. \\
&+ \left. 2\kappa_i \|z_i(r)\|^2 + \beta\alpha^r\right] \qquad (5.31)
\end{aligned}$$

Using the fact $\|e(r)\|^2 = \sum_{i=1}^{N} \|e_i(r)\|^2$, and (5.31), we have

$$\begin{aligned}
\sum_{r=k_{l^i}^i}^{T} \|e(r)\|^2 &= \sum_{r=k_{li}^i}^{T} \sum_{i=1}^{N} \|e_i(r)\|^2 \\
&\leq \sum_{r=k_{li}^i}^{T} \sum_{i=1}^{N} \left[2\kappa_i N l_{ii}^2 \left\|e_i^T(r)\right\|^2 + 2\kappa_i N^2 \max_{i\in N_j}\left\{a_{ji}^2\right\} \left\|e_i^T(r)\right\|^2\right. \\
&\left. +2\kappa_i \|z_i(r)\|^2 + \beta\alpha^r\right] \\
&\leq \sum_{r=k_{li}^i}^{T} \left[\left(1 - \frac{1}{\bar{\Delta}}\right) \|e(r)\|^2 + 2\bar{\kappa}\|z(r)\|^2 + N\beta\alpha^r\right] \qquad (5.32)
\end{aligned}$$

It follows form (5.17) and (5.32) that

$$\begin{aligned}
\sum_{r=k_0}^{T} \|e(r)\|^2 &\leq \bar{\Delta} \sum_{r=k_0}^{T} \left[2\bar{\kappa}\|z(r)\|^2 + N\beta\alpha^r\right] \\
&\leq \bar{\Delta} \sum_{r=k_0}^{T} \left[2\bar{\kappa}\|\mathcal{L}\|^2\|\delta(r)\|^2 + N\beta\alpha^r\right] \qquad (5.33)
\end{aligned}$$

Summing (5.30) from k_0 to T and employing (5.33), one gets

$$\begin{aligned} V(k) - V(k_0) &\leq -\Big(\lambda_2\lambda_{min}(Q) - 2\|(\mathcal{L}\otimes H)\|^2\Big)\sum_{r=k_0}^{k}\|\delta(r)\|^2 \\ &\quad + \Big(\bar{\psi}\|(I\otimes V)\|^2 + 2\|(\mathcal{L}\otimes H)\|^2\Big)\sum_{r=k_0}^{k}\|e(r)\|^2 \\ &\leq -J_1\sum_{r=k_0}^{k}\|\delta(r)\|^2 + J_2(\alpha^{k_0}-\alpha^{k+1}) \end{aligned} \tag{5.34}$$

Since $V(k)$ and J_1 are positive, rearranging (5.34) gives an upper bound for the sum of disagreement vector $\|\delta(k)\|$ as follows:

$$\sum_{r=k_0}^{k}\|\delta(r)\|^2 \leq \frac{V(k_0) + J_2(\alpha^{k_0}-\alpha^{k+1})}{J_1} \tag{5.35}$$

where $J_2 = \frac{N\bar{\Delta}\beta}{1-\alpha}\big(\bar{\psi}\|(I\otimes V)\|^2 + 2\|(\mathcal{L}\otimes H)\|^2\big)$. Note that (5.35) implies that $\sum_{r=k_0}^{k}\|\delta(r)\|^2$ converges absolutely. Absolute convergence implies that $\|\delta(r)\| \to 0$ for complete normed spaces which proves that the consensus is achieved.

Remark 9 *The second condition (5.21) in Theorem 5.1 is a bilinear matrix inequality. However, one can easily obtain the variables P, Q, and S by solving the LMI (5.20). The next step should include searching a state feedback gain K that satisfies (5.21), which becomes an LMI after solving the first LMI (5.20). This process can be repeated iteratively by decreasing the norm of the matrices V and H which in turn maximize the value of J_1. In addition, appropriate tuning of the triggering parameters κ_i is very essential to force J_1 to be positive. Moreover, if the eigenvalues of the state matrix A lie in the unit disk, one can chose S to be negative-definite in order to reduce the effect of the matrices V and H on the triggering parameters very efficiently.*

5.3.1 Leaderless observer

Here, we extend the results of state feedback leaderless-consensus to observer-based Leaderless-consensus. Denote

$$\begin{aligned}
\hat{x}(k) &= \begin{bmatrix} \hat{x}_1(k) \\ \vdots \\ \hat{x}_{\mathrm{N}}(k) \end{bmatrix}, \hat{z}\left(k_l\right) = \begin{bmatrix} \hat{z}_1\left(k_{l^1}^1\right) \\ \vdots \\ \hat{z}_{\mathrm{N}}\left(k_{l^N}^N\right) \end{bmatrix}, \hat{\delta}\left(k\right) = \begin{bmatrix} \hat{\delta}_1\left(k\right) \\ \vdots \\ \hat{\delta}_{\mathrm{N}}\left(k\right) \end{bmatrix}, \\
\zeta(k) &= \left[\zeta_1^T(k), \ldots, \zeta_{\mathrm{N}}^T(k)\right]^T, \ \zeta_i(k) = x_i(k) - \hat{x}_i(k) \\
\hat{\delta}_i(k) &= \hat{x}_i(k) - \hat{\bar{x}}_i(k), \ \ \hat{\bar{x}}_i(k) = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_i(k),
\end{aligned} \tag{5.36}$$

where $\hat{x}_i(k)$ represents the estimate of the state of i-th agent, $\zeta_i(k)$ represents the estimation error of agent i, $\hat{\bar{x}}(k)$ is the average of all estimates, and $\hat{\delta}(k)$ is the disagreement state. Based on the multiagent dynamics (5.1), and the observer-based dynamics (5.8), we obtain the following overall leaderless dynamics:

$$x(k+1) = \left(I_N \otimes A\right) x(k) - \left(I_N \otimes B\hat{K}\right) \hat{z}\left(k_l\right) \tag{5.37}$$

$$\hat{x}(k+1) = \left(I_N \otimes A\right) \hat{x}(k) - \left(I_N \otimes B\hat{K}\right) \hat{z}\left(k_l\right) + \left(I_N \otimes LC\right) \zeta(k) \tag{5.38}$$

$$\zeta(k+1) = \left[I_N \otimes (A - LC)\right] \zeta(k) \tag{5.39}$$

and the disagreement vector is obtained:

$$\hat{\delta}(k+1) = \left(I_N \otimes A\right) \hat{\delta}(k) - (\mathcal{M} \otimes B\hat{K})\hat{z}\left(k_l\right) + (\mathcal{M} \otimes LC)\zeta(k) \tag{5.40}$$

Based on (5.39), if we select an observer gain L such that the matrix $A - LC$ is Schur stable, then the estimation error will vanish and the disagreement vector is obtained as follows:

$$\hat{\delta}(k+1) = \left(I_N \otimes A\right) \hat{\delta}(k) - \left(\mathcal{M} \otimes B\hat{K}\right) \hat{z}\left(k_l\right) \tag{5.41}$$

Theorem 5.2 *Consider the multiagent system (5.1) with undirected topology G. If*

there exist symmetric matrices $P > 0$, $Q > 0$ and $S > 0$ with appropriate dimensions satisfying the conditions in Theorem 5.1 and $A - LC$ is Schur stable, then the multiagent system (5.1) achieves leaderless consensus under the observer-based controller with the modified event-triggering function (5.9)

Proof: The proof follows parallel line of Theorem 5.1.

5.3.2 Leader following Consensus

This part is devoted to the leader-following consensus of linear multiagent system (5.1) with the follower given in (5.2) based on the event-triggered function (5.12). Denote $\mathcal{H} = \mathcal{L} + B$, where B is the leader adjacency matrix and $\mathcal{L}$ is the Laplacian matrix of the graph G. Based on the definition of $\tilde{z}_i(k)$ in (5.11), then $\tilde{z}(k) = (\mathcal{H} \otimes I_n)\, \varepsilon(k)$. In addition, define $\bar{e}_i(k) = \tilde{z}_i\left(k_{l^i}^i\right) - \tilde{z}_i(k)$, and $\bar{e}(k) = \tilde{z}\left(k_l\right) - \tilde{z}(k)$, with the leader-following consensus protocol (5.11), one has

$$\begin{aligned} \bar{e}_i(k) &= \sum_{j \in N_l} a_{ij} \left(x_i\left(k_{l^i}^i\right) - x_j\left(k_{l^j}^j\right)\right) + d_i \left(x_i\left(k_{l^i}^i\right) - x_0(k)\right) \\ &- \sum_{j \in N_i} a_{ij} \left(x_i(k) - x_j(k)\right) - d_i \left(x_i(k) - x_0(k)\right) \\ &= \sum_{j \in N_i} a_{ij} \left(e_i(k) - e_j(k)\right) + d_i e_i(k) \end{aligned} \tag{5.42}$$

Hence the compact form can be represented as

$$\bar{e}(k) = \tilde{z}\left(k_l\right) - \tilde{z}(k) = (\mathcal{H} \otimes I_n)\, e(k) \tag{5.43}$$

$$\epsilon^T(k)(\mathcal{H}\mathcal{H} \otimes S)\epsilon(k) = \tilde{z}^T(k)(I \otimes S)\tilde{z}(k) \tag{5.44}$$

Algebraic manipulations for any matrix V and invoking (5.42), it is easy to establish the following inequality:

$$\begin{aligned}\|V\bar{e}_i(k)\| &= \|V\sum_{j\in N_i} a_{ij}(e_i(k)-e_j(k)) + d_iVe_i(k)\| \\ &\leq \|\sum_{j\in N_i} a_{ij}Ve_i(k)\| + \|\sum_{j\in N_i} a_{ij}Ve_j(k)\| + d_i\|Ve_i(k)\| \\ &\leq l_{ii}\|Ve_i(k)\| + a_{ij}\sum_{j\in N_i}\|Ve_j(k)\| + d_i\|Ve_i(k)\| \end{aligned} \tag{5.45}$$

which leads to the following result:

Theorem 5.3 *Consider the multiagent system (5.1) and the leader dynamics (5.2). The distributed event-triggered protocol (5.10) achieves leader-following consensus if there exist symmetric matrices $P>0$, $Q>0$ and $S>0$ with appropriate dimensions such that the following linear matrix inequality holds*

$$A^TPA - P - \lambda_2 S + Q \leq 0 \tag{5.46}$$

and the leader-following gain $\tilde{K}$ satisfies

$$J_1 = \left(\lambda_1\lambda_{min}(Q) - 2\|(\mathcal{H}\otimes H)\|^2\right) - 2\bar{\Gamma}\bar{\kappa}\|H\|^2\left(\bar{\psi}\|(I\otimes V)\|^2 + 2\|(\mathcal{H}\otimes H)\|^2\right) > 0 \tag{5.47}$$

where $\bar{\Gamma} = \frac{1}{(1-\bar{\nu})} > 0$, $\bar{\psi} = \max_i\{\psi_i\}, \bar{\kappa} = \max_i\{\kappa_i\}$, $\lambda_1 = \lambda_{min}(\mathcal{H}), \psi_i = (N+1)\left[\ell_{ii}^2 + d_i^2 + \sum_{j\in N_i} a_{ji}^2\right], \bar{\nu}_a = 2\kappa_i N(N+1)(l_{ii}^2 + d_i^2), \bar{\nu}_c = 2\kappa_i N(N+1)\max_{i\in N_j}\left\{a_{ji}^2\right\}, \bar{\nu} = \max\{\bar{\nu}_a, \bar{\nu}_c\}, H^TH = \lambda_{max}(\mathcal{H})\tilde{K}^TB^TPB\tilde{K} - A^TPB\tilde{K} + 2S, and V^TV = \frac{A^TPB\tilde{K}+\tilde{K}^TB^TPA}{2} + 2S$

proof: Consider the following Lyapunov function: $V(k) = \varepsilon^T(k)(\mathcal{H}\otimes P)\varepsilon(k)$. Using (5.43), and (5.44), the difference of the Lyapunov function $\Delta V(k)$ along the

dynamics of (5.14) can be obtained as follows

$$\begin{aligned}
\Delta V &= V(k+1) - V(k) \\
&= \epsilon^T(k) \left[(I \otimes A)^T (\mathcal{H} \otimes P)(I \otimes A) - \mathcal{H} \otimes P \right] \epsilon(k) \\
&- 2\epsilon^T(k) \left[(I \otimes A)^T (\mathcal{H} \otimes P)(I \otimes B\tilde{K}) \right] \tilde{z}(k_l) \\
&+ \tilde{z}^T(k_l) \left[(I \otimes B\tilde{K})^T (\mathcal{H} \otimes P)(I \otimes B\tilde{K}) \right] \tilde{z}(k_l) \\
&- \epsilon^T(k) \left[(\mathcal{H}\mathcal{H} \otimes S) \right] \epsilon(k) + \epsilon^T(k) \left[(\mathcal{H}\mathcal{H} \otimes S) \right] \epsilon(k) \\
&= \epsilon^T(k) \left[(\mathcal{H} \otimes (A^T P A - P) - (\mathcal{H}\mathcal{H} \otimes S)) \right] \epsilon(k) \\
&+ \mathcal{U}(k) + \mathcal{V}(k) + \tilde{z}^T(k) \left[(I \otimes S) \right] \tilde{z}(k) \qquad (5.48)
\end{aligned}$$

where $\mathcal{U}(k) = -2\epsilon^T(k) \left[(\mathcal{H} \otimes A^T P B\tilde{K}) \right] \tilde{z}(k_l)$ and $\mathcal{V}(k) = \tilde{z}^T(k_l) \left[(\mathcal{H} \otimes \tilde{K}^T B^T P B\tilde{K}) \right] \tilde{z}(k_l)$. Like the process in *Theorem 5.1*, incorporating the definition of $\bar{e}(k)$, (5.43), and Young's inequality lead to

$$\begin{aligned}
\mathcal{U}(k) &= -2\epsilon^T(k) \left[(\mathcal{H} \otimes I)(I \otimes A^T P B\tilde{K}) \right] \tilde{z}(k_l) \\
&= -2\tilde{z}^T(k) \left[(I \otimes A^T P B\tilde{K}) \right] \tilde{z}(k_l) \\
&= -2(\tilde{z}(k_l) - \bar{e}^T(k)) \left[(I \otimes A^T P B\tilde{K}) \right] \tilde{z}(k_l) \\
&\leq -\tilde{z}^T(k_l) \left[(I \otimes A^T P B\tilde{K}) \right] \tilde{z}(k_l) + \bar{e}(k))^T \left[(I \otimes A^T P B\tilde{K}) \right] \bar{e}(k)) \qquad (5.49)
\end{aligned}$$

and

$$\mathcal{V}(k) \leq \tilde{z}^T(k_l) \left[(I \otimes \lambda_{max}(\mathcal{H}) \tilde{K}^T B^T P B\tilde{K}) \right] \tilde{z}(k_l) \qquad (5.50)$$

Substituting (5.49) and (5.50) in (5.48) and using (5.44), we have

$$\begin{aligned}
\Delta V &\leq \epsilon^T(k) \left[(\mathcal{H} \otimes (A^T P A - P) - (\mathcal{H}\mathcal{H} \otimes S)) \right] \epsilon(k) \\
&+ \bar{e}^T(k)) \left[(I \otimes A^T P B\tilde{K}) \right] \bar{e}(k)) + \tilde{z}^T(k) \left[(I \otimes S) \right] \tilde{z}(k) \\
&+ \tilde{z}^T(k_l) \left[I \otimes \left(\lambda_{max}(\mathcal{H}) \tilde{K}^T B^T P B\tilde{K} - A^T P B\tilde{K} \right) \right] \tilde{z}(k_l)
\end{aligned}$$

Decomposing the Kronecker products into summation representations gives

$$\begin{aligned}\Delta V \leq & \epsilon^T(k)\left[(\mathcal{H}\otimes(A^TPA-P)-(\mathcal{H}\mathcal{H}\otimes S))\right]\epsilon(k)\\ &+\sum_{i=1}^{N}\bar{e}_i^T(k))\left[A^TPBK\right]\bar{e}_i(k))\\ &+\sum_{i=1}^{N}\tilde{z}_i^T(k_l)\left[\lambda_{max}(\mathcal{H})\tilde{K}^TB^TPB\tilde{K}-A^TPB\tilde{K}\right]\tilde{z}_i(k_l)\\ &+\sum_{i=1}^{N}(\tilde{z}_i(k_{li}^i)-\bar{e}_i(k))^TS(\tilde{z}_i(k_{li}^i)-\bar{e}_i(k)) \qquad (5.51)\end{aligned}$$

Applying Young's inequality to last term in (5.51), that is $(a-b)^TS(a-b)\leq 2a^Ta+2b^Tb$, we arrive at:

$$\begin{aligned}\Delta V \leq & \epsilon^T(k)\left[(\mathcal{H}\otimes(A^TPA-P)-(\mathcal{H}\mathcal{H}\otimes S))\right]\epsilon(k)\\ &+\sum_{i=1}^{N}\left[\bar{e}_i^T(k))\left(A^TPB\tilde{K}+2S\right)\bar{e}_i(k))+\tilde{z}_i^T(k_l)H^TH\tilde{z}_i(k_l)\right] \qquad (5.52)\end{aligned}$$

From the definition of the matrix V with $V^TV=\frac{A^TPB\tilde{K}+\tilde{K}^TB^TPA}{2}+2S$, then using (5.45), and the fact $\left(\sum_{i=1}^{N+1}x_i\right)^2\leq(N+1)\sum_{i=1}^{N+1}x_i^2$, one has

$$\begin{aligned}\sum_{i=1}^{N}\bar{e}_i^T(k)\left[A^TPB\tilde{K}+2S\right]\bar{e}_i(k) &= \sum_{i=1}^{N}\|V\bar{e}_i(k)\|^2\\ &\leq \sum_{i=1}^{N}\left[l_{ii}\|Ve_i(k)\|+\sum_{j\in N_i}a_{ij}\|Ve_j(k)\|+d_i\|Ve_i(k)\|\right]^2\\ &\leq \sum_{i=1}^{N}(N+1)\left[l_{ii}^2\|Ve_i(k)\|^2+\sum_{j\in N_i}a_{ij}^2\|Ve_j(k)\|^2\right.\\ &\quad\left.+d_i^2\|Ve_i(k)\|^2\right]\leq\sum_{i=1}^{N}\psi_i\|Ve_i(k)\|^2 \qquad (5.53)\end{aligned}$$

Since $\mathcal{H}$ is symmetric positive-definite, then there exists an orthogonal matrix U such that $U^{-1}\mathcal{H}U=U^T\mathcal{H}U=J=\text{diag}\{\lambda_1(\mathcal{H}),\lambda_2(\mathcal{H}),\ldots,\lambda_N(\mathcal{H})\}$. Let $\tilde{\epsilon}(k)=\left(U^T\otimes I_n\right)\epsilon(k)$, where $\tilde{\epsilon}(k)=\left[\tilde{\epsilon}_1^T(k),\ldots,\tilde{\epsilon}_N^T(k)\right]^T$. Substituting (5.53) in (5.52) gives

that

$$\Delta V \leq \sum_{i=1}^{N} \tilde{\epsilon}_i^T(k) \left[\lambda_i(\mathcal{H})(A^T P A - P) - \lambda_i^2(\mathcal{H}) S\right] \tilde{\epsilon}_i(k) + \sum_{i=1}^{N} \psi_i e_i^T(k) V^T V e_i(k) + \tilde{z}^T(k_l) \left(I \otimes H^T H\right) \tilde{z}(k_l) \tag{5.54}$$

Then, using inequality (5.46), and (5.43) we have

$$\begin{aligned}
\Delta V &\leq -\lambda_1 \lambda_{min}(Q) \|\tilde{\epsilon}(k)\| + \bar{\psi} \|(I \otimes V) e(k)\|^2 + \|(I \otimes H) \tilde{z}(k_l)\|^2 \\
&= -\lambda_1 \lambda_{min}(Q) \|\epsilon(k)\| + \bar{\psi} \|(I \otimes V) e(k)\|^2 + \|(I \otimes H)[\tilde{z}(k) - \bar{e}(k)]\|^2 \\
&\leq -\lambda_1 \lambda_{min}(Q) \|\epsilon(k)\| + \bar{\psi} \|(I \otimes V) e(k)\|^2 + 2\|(I \otimes H) \tilde{z}(k)\|^2 + 2\|(I \otimes H) \bar{e}(k)]\|^2 \\
&\leq -\lambda_1 \lambda_{min}(Q) \|\epsilon(k)\| + \bar{\psi} \|(I \otimes V) e(k)\|^2 + 2\|(\mathcal{H} \otimes H) \epsilon(k)\|^2 + 2\|(I \otimes H) \bar{e}(k)]\|^2 \\
&\leq -\lambda_1 \lambda_{min}(Q) \|\epsilon(k)\| + \bar{\psi} \|(I \otimes V) e(k)\|^2 + 2\|(\mathcal{H} \otimes H) \epsilon(k)\|^2 + 2\|(\mathcal{H} \otimes H) e(k)]\|^2 \\
&\leq -(\lambda_1 \lambda_{min}(Q) - 2\|(\mathcal{H} \otimes H)\|^2) \|\epsilon(k)\| \\
&+ (\bar{\psi} \|(I \otimes V)\|^2 + 2\|(\mathcal{H} \otimes H)\|^2) \|e(k)\|^2
\end{aligned} \tag{5.55}$$

Based on the event-triggering function of the leader-following case (5.12), and (5.42),

we have

$$\begin{aligned}
\sum_{r=k_{l^i}^i}^{T} \|e_i(r)\|^2 &\leq \sum_{r=k_{l^i}^i}^{T} \left(\kappa_i \left\|\tilde{z}_i\left(k_{l^i}^i\right)\right\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{l^i}^i}^{T} \left(\kappa_i \|\bar{e}_i(r) + \tilde{z}_i(r)\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{l^i}^i}^{T} \left(2\kappa_i \|\bar{e}_i(r)\|^2 + 2\kappa_i \|\tilde{z}_i(r)\|^2 + \beta\alpha^r\right) \\
&\leq \sum_{r=k_{l^i}^i}^{T} \Bigg[2\kappa_i(N+1)\Bigg((l_{ii}^2 + d_i^2)\left\|e_i^T(r)\right\|^2 + \sum_{j\in N_i} a_{ij}^2 \left\|e_j^T(r)\right\|^2\Bigg) \\
&\quad + 2\kappa_i \|\tilde{z}_i(r)\|^2 + \beta\alpha^r\Bigg] \\
&\leq \sum_{r=k_{l^i}^i}^{T} \Bigg[2\kappa_i(N+1)(l_{ii}^2 + d_i^2)\left\|e_i^T(r)\right\|^2 \\
&\quad + 2\kappa_i N(N+1)\max_{i\in N_j}\left\{a_{ji}^2\right\}\left\|e_i^T(r)\right\|^2 + 2\kappa_i \|\tilde{z}_i(r)\|^2 + \beta\alpha^r\Bigg] \qquad (5.56)
\end{aligned}$$

Using the fact $\|e(r)\|^2 = \sum_{i=1}^N \|e_i(r)\|^2$, and (5.56), we have

$$\begin{aligned}
\sum_{r=k_{l^i}^i}^{T} \|e(r)\|^2 &= \sum_{r=k_{l^i}^i}^{T}\sum_{i=1}^{N} \|e_i(r)\|^2 \\
&\leq \sum_{r=k_{l^i}^i}^{T}\sum_{i=1}^{N} \Bigg[2\kappa_i(N+1)(l_{ii}^2 + d_i^2)\left\|e_i^T(r)\right\|^2 \\
&\quad + 2\kappa_i N(N+1)\max_{i\in N_j}\left\{a_{ji}^2\right\}\left\|e_i^T(r)\right\|^2 + 2\kappa_i \|\tilde{z}_i(r)\|^2 + \beta\alpha^r\Bigg] \\
&= \sum_{r=k_{l^i}^i}^{T} \left[\left(1 - \frac{1}{\bar{\Gamma}}\right)\|e(r)\|^2 + 2\bar{\kappa}\|\tilde{z}(r)\|^2 + N\beta\alpha^r\right]
\end{aligned}$$

Since $\bar{\Gamma} > 0$, we have the equivalent bound:

$$\begin{aligned}
\sum_{r=k_{l^i}^i}^{T} \|e(r)\|^2 &\leq \bar{\Gamma}\sum_{r=k_{l^i}^i}^{T} \left[2\bar{\kappa}\|\tilde{z}(r)\|^2 + N\beta\alpha^r\right] \\
&\leq \bar{\Gamma}\sum_{r=k_{l^i}^i}^{T} \left[2\bar{\kappa}\|\mathcal{H}\|^2\|\epsilon(r)\|^2 + N\beta\alpha^r\right] \qquad (5.57)
\end{aligned}$$

Summing up (5.55) from k_0 to T and employing (5.57), one has

$$\begin{aligned} V(k) - V(k_0) &\leq -\left(\lambda_1 \lambda_{min}(Q) - 2\|(\mathcal{H} \otimes H)\|^2\right) \sum_{r=k_0}^{k} \|\epsilon(r)\|^2 \\ &\quad + \left(\bar{\psi}\|(I \otimes V)\|^2 + 2\|(\mathcal{H} \otimes H)\|^2\right) \sum_{r=k_0}^{k} \|e(r)\|^2 \\ &\leq -J_1 \sum_{r=k_0}^{k} \|\epsilon(r)\|^2 + J_2(\alpha^{k_0} - \alpha^{k+1}) \end{aligned} \tag{5.58}$$

where $J_2 = \frac{N\bar{\Gamma}\beta}{1-\alpha}\left(\bar{\psi}\|(I \otimes V)\|^2 + 2\|(\mathcal{H} \otimes H)\|^2\right)$. Since $V(k)$ is always positive, rearranging (5.58) gives an upper bound for the sum of disagreement vector $\|\epsilon(k)\|$ as follows:

$$\sum_{r=k_0}^{k} \|\epsilon(r)\|^2 \leq \frac{V(k_0) + J_2(\alpha^{k_0} - \alpha^{k+1})}{J_1} \tag{5.59}$$

This implies that $\sum_{r=k_0}^{k} \|\epsilon(r)\|^2$ converges absolutely. Absolute convergence implies that $\|\epsilon(r)\| \to 0$ for complete normed spaces which proves that the consensus is achieved.

5.3.3 Observer Based Leader following Consensus

Observer based leader-following consensus is investigated in this part under the following consensus protocol:

$$u_i(k) = -\hat{K}\hat{z}_i\left(k_{l^i}^i\right) \quad k \in [k_{l^i}^i, k_{l^i+1}^i) \tag{5.60}$$

where

$$\hat{z}_i\left(k_{l^i}^i\right) = \left[\sum_{j \in N_i} a_{ij}\left(\hat{x}_i\left(k_{l^i}^i\right) - \hat{x}_j\left(k_{l^j}^j\right)\right) + d_i\left(\hat{x}_i\left(k_{l^i}^i\right) - x_0(k)\right)\right] \tag{5.61}$$

and $\hat{x}_i(k)$ represents the estimate of the state of i-th agent. We consider the same dynamics of the observer of the leaderless (5.8) but with different protocol which is given in (5.60). Let the vectors $\hat{x}(k)$, $\zeta(k)$, $\varpi_i(k)$, and $\hat{z}\left(k_l\right)$ are defined similar to the

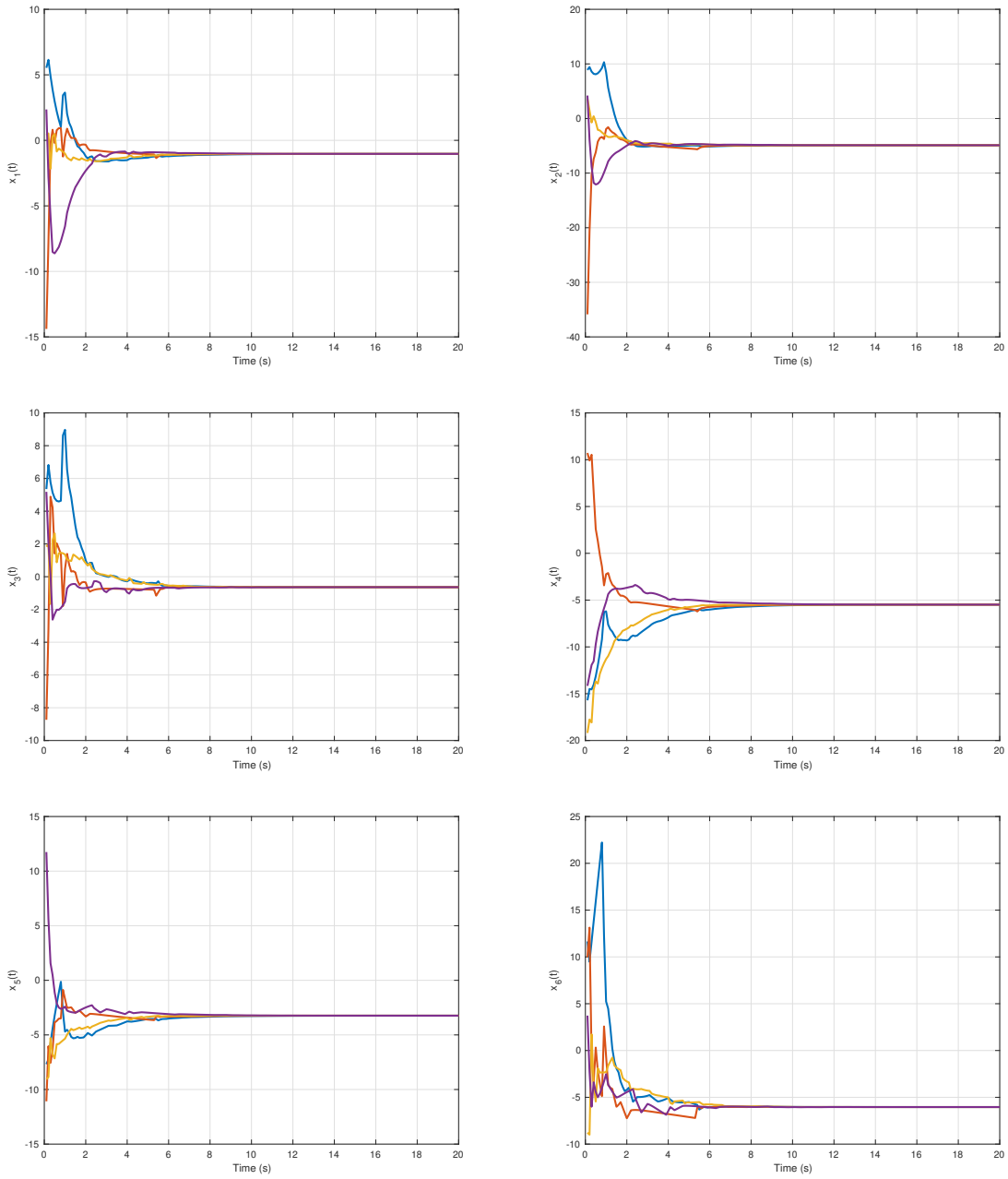

Figure 5.1: State trajectories of the six satellites, leaderless case.

leaderless case. Denote $\hat{\epsilon}_i(k) = \hat{x}_i(k) - x_0(k)$,, then based on the multiagent dynamics (5.1), (5.2), and the observer dynamics, we obtain the following dynamics:

$$\hat{\epsilon}(k+1) = (I_N \otimes A)\,\hat{\epsilon}(k) - (I \otimes B\hat{K})\hat{z}\,(k_l) + (I \otimes LC)\zeta(k) \tag{5.62}$$

$$\zeta(k+1) = [I_N \otimes (A - LC)]\,\zeta(k) \tag{5.63}$$

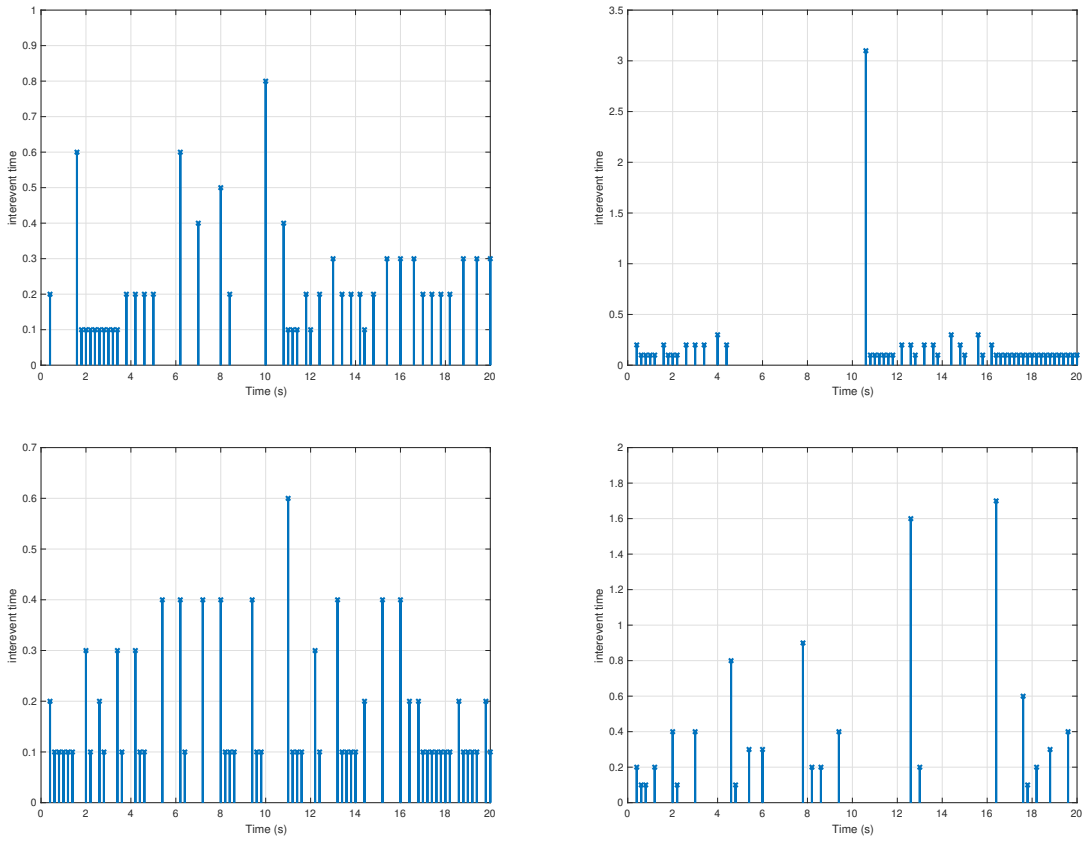

Figure 5.2: Triggering instants of the leaderless consensus.

Based on the decoupled estimation error (5.63), if we select an observer gain L such that the matrix $A - LC$ is Schur stable, then the estimation error will vanish and the disagreement vector is obtained as follows:

$$\hat{\epsilon}(k+1) = (I_N \otimes A)\,\hat{\epsilon}(k) - \left(I \otimes B\hat{K}\right)\hat{z}\,(k_l) \tag{5.64}$$

Theorem 5.4 *Consider the multiagent system (5.1) with undirected topology G. If there exist symmetric matrices $P > 0$, $Q > 0$ and $S > 0$ with appropriate dimensions satisfying the conditions in Theorem 5.3 and $A - LC$ is Schur stable, then the multiagent system (5.1) achieves leaderless-following consensus under the observer-based controller with the following modified event-triggering function (5.9).*

Proof: The proof follows parallel line of Theorem 5.3.

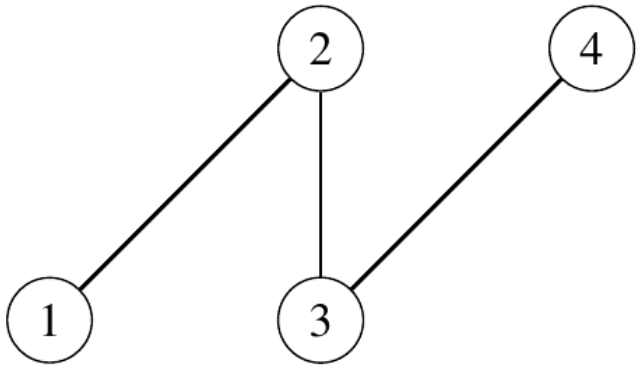

Figure 5.3: Connected topology of the multiagent system.

5.4 Simulation Results

In this section, two simulation examples are given to demonstrate the effectiveness of proposed theoretical findings. The first example is devoted to leaderless consensus and the second example will be related to leader-following consensus.

5.4.1 Example 1

Consider a multiagent system comprises four agents with the graph topology shown in Figure 5.3. The corresponding matrices are given as

$$A = \begin{bmatrix} .8906 & .0596 & -.297 & .0572 & -.270 & .0947 \\ .0745 & .8808 & -.481 & .1183 & -.380 & .2315 \\ .0341 & .0195 & .5786 & -.026 & -.171 & .1394 \\ -.120 & .1096 & -.295 & .8696 & .0366 & .0617 \\ -.032 & .0694 & -.216 & .1144 & .6789 & .0405 \\ -.095 & .1354 & -.402 & .1145 & -.209 & .9576 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -2.974 & -0.357 & -4.043 & -1.416 & 3.891 & 12.5 \end{bmatrix}$$

The Graph $\mathcal{G}$ is connected and the pair (A, B) is controllable. Consider the triggering parameters: $\alpha = 0.6$, $\beta = 0.1$, $\kappa_1 = 0.02$, $\kappa_2 = 0.0751$, $\kappa_1 = 00.05$, and $\kappa_1 = 0.01$. We assign the initial states as $x_1(0) = \begin{bmatrix} 5.5 & 8.8 & 5.3 & -15.7 & -7.6 & 11.6 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} -14.3 & -36 & -8.7 & 10.7 & -11 & 10 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 0.38 & 3.63 & 1.83 & -19 & -9 & -8.8 \end{bmatrix}^T$, $x_4(0) = \begin{bmatrix} 2.35 & 4.2 & 5.2 & -14.1 & 11.7 & 3.7 \end{bmatrix}^T$ The consensus matrix K is obtained by solving the conditions presented in *Theorem 5.1* as follows: $K = \begin{bmatrix} -.0058 & .0162 & -.0378 & .0138 & -.0116 & .0268 \end{bmatrix}$. We use the event-triggered function (5.4) to consider this model. The trajectories of the four agents are shown in Fig. 5.1. The triggering instants under the modified-type triggering (5.4) are plotted in Fig. 5.2.

5.4.2 Example 2

Consider 4 agents and every agent $x_i(t) \in \mathbb{R}^2$, $i = 1, 2, 3, 4$. Their interaction topology is shown in Fig. 5.3. The dynamics of the agents and the followers are given as

$$\dot{x}_i(t) = \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950 \end{bmatrix} x_i(t) + \begin{bmatrix} 0.0998 \\ 0.9950 \end{bmatrix} u_i(t)$$

Set the triggering parameters as the previous example and the initial condition to $x_1(0) = [6.5, 2.9]^T$, $x_2(0) = [5.4, -2.7]^T$, $x_3(0) = [2.7, 3.7]^T$, and $x_4(0) = [-1.4, 3.9]^T$. According to *Theorem 5.3*, the leader-following consensus is obtained $\tilde{K} = [0.1027, 0.3907]^T$. Figure 5.4 shows the agents response and the trajectory of the leader. It can be seen form the response that agents track the leader asymptotically and the leader-following is achieved.

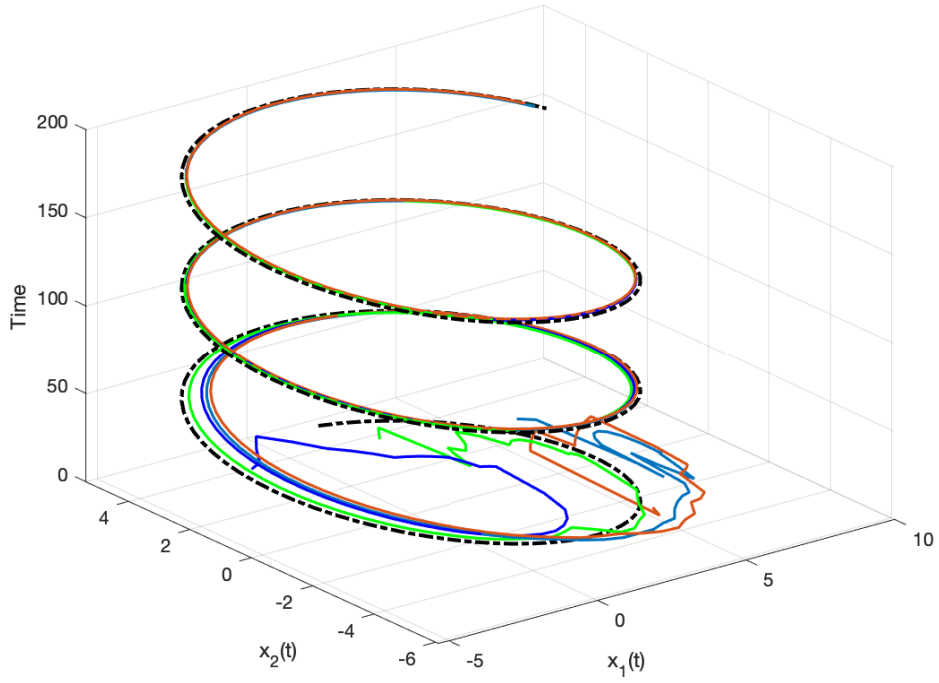

Figure 5.4: Phase plane plots of the leader and its followers that demonstrate that the leader-consensus is being achieved.

CHAPTER 6

LEADER-FOLLOWING CONSENSUS WITH TIME-VARYING DELAY

This Chapter studies the consensus problem for discrete time nonlinear multiagent systems with time delay. Based on an asynchronous event-triggered mechanism where local information of a communication graph is involved, a distributed consensus protocol is proposed to reduce the network communication congestion for achieving state consensus for multiagent nonlinear systems. To avoid complications involving data fusion, this distributed consensus protocol allows every agent to broadcast its own state at its own triggering condition instead of using continuous updates and without continuous monitoring of the triggering condition. To save the energy consumption involved in a consensus protocol, the proposed event-triggering guaranteed cost is investigated in this Chapter for multiagent systems. We select an agent as leader

that is constrained by its neighbors and controlled by the proposed protocol as other agents. It has been theoretically demonstrated that the consensus problem of multi-agent systems with non- linear perturbation and time delays can be achieved if the control gains and the triggering parameters are appropriately selected. Sufficient conditions are given to guarantee that the agents achieve consensus. Moreover, the gain matrix and triggering parameters of the proposed protocol are cast into matrix inequalities conditions. A simulation example is given to illustrate the effectiveness of the theoretical results.

6.1 Introduction

Consensus of multiagent systems is one of the challenging problems in cooperative control methodologies and has a wide variety of scientific applications, such as formation problem [37], [38], [39], wireless sensor networks [40], intelligent traffic management [41], rendezvous of spacecrafts [42]. Thus, multiagent systems have been researched extensively by various scientific communities in automatic control, computer science, and other fields [47]. As a result, a profound results on the properties of multi-agent systems have been established [48], [49]. Generally speaking, consensus objective is guaranteed by finding a dynamic interaction protocol that drive agents to reach an agreement on a common target. It is a fundamental demand for investigating cooperative control methodologies of multiagent control systems [50]. Based on Lyapunov methods and Lasalle's invariance principle, together with the algebraic graph and matrix theories, consensus methodologies have been reported in the literature, see,

e.g. [51]. For instance, it is demonstrated that the consensus of position and velocity second-order challenge is investigated for multiagent systems in [52]. The leader follower consensus of linear and nonlinear multiagent systems are addressed in [61], [62], where the states are updated in accordance with neighboring information exchange in order to track the behavior of the leader.

For achieving the consensus of multiagent system, continuous information exchange of agents through neighbor-based interactions may lead to a data congestion in the network. When capacity limitation of the communication network and energy resources are limited, an effective mechanism called the event-triggered strategy should be adopted to reduce unnecessary sampling and data transmission among agents [175], [176], [177]. Unlike the classical time-triggered control mechanism, sampled data is transmitted only by occurrence of a predefined condition instead of periodic broadcasting among agents. Motivated by these features, a considerable number of researchers have extended event-triggered control mechanism to the consensus of multiagent systems earlier by the group guided by Dimarogonas [178],[179], and so far, a substantial amount of research has been reported on this issue [180], [181]. For instance, output feedback consensus methodology is addressed to control disturbed multiagent systems via event-triggered mechanism in [182]. Furthermore, as proposed in the work of Yang et al [183], a distributed event-triggered strategy for nonlinear multiagent systems is addressed that effectively reduces the information exchange and actuation updates. Most of the above works focus on multiagents with certain dynamics and without considering the time delay. However, several event-triggered methodologies on uncertain multiagent systems with time-delay have been investigated [184]. In [185] and [186],

leader-following consensus via event triggering scheme is investigated by considering the leader that is not controlled by the consensus protocol and can not receive information from the followers. On the other hand, in various practical applications, the leader should be updated and controlled by the state of other agents. For example, the leader should receive information from the control center in order to solve the formation of aircraft problem. Hence, one might select an agent as a "Similar Leader". Unlike the results in [185], [186], "Similar Leader" is selected, constrained and controlled by the same consensus protocol of other agents. Based on consensus protocol designed, agents reach consensus with "Similar Leader".

It is worth mentioning that most of the aforementioned literatures on event-triggered consensus control of multiagent focus on achieving consensus when agents are governed by first/second-order dynamics. However, in some cases, for example, the agents have limited energy resources to carry out specific tasks such as movement and communication. Taking into account the energy consumption, the event-based consensus guaranteed cost is addressed, with a performance index involving energy consumption. Energy-consumption is a very appealing topic in multiagent systems that ought to be addressed simultaneously with the consensus problem [187]. In [188], [189], a cost performance index is introduced to obtain tradeoff between consensus regulating performance and energy consumption. A powerful methodology for guaranteed performance based control can accommodate an energy index with an upper bound. In [190], guaranteed performance consensus protocol has been addressed for multiagent systems with Lipschitz nonlinear dynamics. Few findings have been stated on guaranteed performance of multiagent systems under event-triggered strategies.

State feedback and event-triggered guaranteed cost based control are addressed for uncertain switched linear systems in [191] and [192], respectively. However, it worth to mention that no time delay is considered in the dynamics of the agents investigated in [191] and [192]. On the other hand, time-delay among agents is inevitable in real applications. A guaranteed performance is studied for a class of higher-order linear multiagent control systems with sampled-data information [193]. Suboptimal output consensus for high dimensional multiagent system with constant delay is addressed in [194], and time-varying delay [195]. Guaranteed cost consensus is also addressed in [196] for second-order multiagents with directed graphs where communication delays are introduced. However, the aforementioned research on guaranteed cost consensus is based on time-triggered mechanism rather than distributed event-triggered protocol. It is found that guaranteed cost control has gained more attention for time delay multiagent systems with time triggered strategy. Although it has growing attention in recent years, the problem is rarely investigated for event triggered consensus protocols of nonlinear and uncertain multiagent systems.

Our main contribution is the analysis and synthesis of event-triggered based protocols to solve the multiagent consensus problem. The consensus protocols are designed for a class of nonlinear multi-agent high order multiagent systems. The consensus problem is transformed into the stabilization problem and by constructing appropriate Lyapunov functionals, some sufficient conditions are obtained for achieving guaranteed performance consensus. Moreover, an upper bound of the cost index function is provided which depends on the initial conditions of the disagreement vector. The proposed consensus protocol does not require individual agents to continuously measure

their states and do not have access to the state of their neighbors continuously. The triggering scheme in this article is distributed and different from the triggering policy investigated in [191], [197]. The protocol is fully distributed in the sense that agent does not require global knowledge by neighbors to execute the triggering strategy. The triggering scheme only requires the error at triggered instants rather than the error of all the neighboring agents. Moreover, the triggering mechanism excludes the phenomenon that an agent could be out of control due to large errors. The proposed methodology gives a good way to reduce the energy consumption and the waste of communication resources simultaneously.

6.2 Problem Formulation

Consider a nonlinear discrete-time multiagent system compromises N agents, where the dynamical description of each agent is given by the following difference time-delay equation:

$$x_i(k+1) = Ax_i(k) + A_d x_i(k-\tau) + Bu_i(k) + f_i\big(x_i(k), x_i(k-\tau)\big) \tag{6.1}$$

where $x_i(k) \in \mathbb{R}^n$, and $u_i(k) \in \mathbb{R}^m$ are the state and the control law of the agent i, respectively. The control signal $u_i(k)$ can only reach to a local information of neighbors' states. $\phi_0(k)$ is a compatible initial condition and $\tau(k)$ is a positive integer that represents the time delay and assumed to be time varying for all agents such that it is bounded

$$\underline{\tau} \leq \tau(k) \leq \bar{\tau}$$

where $\underline{\tau}$ and $\bar{\tau}$ are positive integers. The nonlinear function $f_i\big(x_i(k), x_i(k-\tau)\big) \in \mathbb{R}^n$

is a real unknown vector function representing the uncertainty and disturbances which are the result of unmodeled and neglected dynamics.

The following assumption on the nonlinear function $f(.,.)$ is imposed for technical convenience:

1. $f_i(0,0) = 0$
2. $\frac{\partial}{\partial\theta} f_i(\theta,\alpha) = EF(k)N_1$
3. $\frac{\partial}{\partial\alpha} f_i(\theta,\alpha) = EF(k)N_2$

where $F(k)$ represents unknown matrix functions with where its elements are discrete-time functions satisfying $F^T(k)F(k) \leq I$. Moreover, the matrices E, N_1 and N_2 are real constant with appropriate dimensions which represent how the uncertain parameters in $F_i(k)$ enter to the nominal matrices. For simplicity, consider the following notation of the partial derivatives of the nonlinearity:

$$\Delta A(\sigma) = \frac{\partial f_i}{\partial\theta}\bigg|_{(\sigma x_i(k), x_i(k-\tau))} \quad \Delta A_d(\sigma) = \frac{\partial f_i}{\partial\alpha}\bigg|_{(x_i(k), \sigma x_i(k-\tau))}$$

Then, we can easily verify the following representation of the nonlinear function:

$$f\big(x_i(k), x_i(k-\tau)\big) = \int_{\sigma=0}^{1} \Big[\Delta A(\sigma)x_i(k) + \Delta A_d(\sigma)x_i(k-\tau)\Big] d\sigma \tag{6.2}$$

Definition 6.1 *The multiagent system represented in (6.1) is considered to achieve consensus if the following condition holds:*

$$\lim_{k\to\infty} \|x_i(k) - x_j(k)\| = 0,$$

for all $i, j \in \mathcal{V}$.

With the event-triggered scheme, to determine when the communication action is

activated, the error between the last action and current instant is introduced:

$$e_i(k) = x_i(k_l^i) - x_i(k) \quad \forall k \in [k_l^i, k_{l+1}^i) \tag{6.3}$$

Now, consider the following event-triggered protocol designed for the muliagent system (6.1):

$$u_i(k) = K \sum_{j \in \mathcal{N}_i} a_{ij} \Big[x_i(k_l^i) - x_j(k_l^j) \Big], \quad \forall i \in \mathcal{V} \tag{6.4}$$

where $k_{l^i}^i$ is the last transmission instant of the agent i, and $K \in \mathbb{R}^{m \times N}$ is a feedback constant matrix to be determined. Denote the disagreement vector by $\delta_i(k) = x_i(k) - x_1(k)$ where the consensus is reached if $\lim_{k\to\infty} \|\delta_i(k)\| = 0, i = 2, 3, ..., N$. Substituting the disagreement vector $\delta_i(k)$ and the triggering error in (6.3) into the consesus protocol (6.4), we come up with the following representation of the protocol:

$$\begin{aligned} u_i(k) =& K \sum_{j \in \mathcal{N}_i} a_{ij} \Big[x_i(k) + e_i(k) - x_i(k) - e_j(k) \Big] \\ =& K \sum_{j \in \mathcal{N}_i} a_{ij} \Big[\delta_i(k) + e_i(k) - \delta_j(k) - e_j(k) \Big] \qquad (6.5) \\ =& d_i K \delta_i(k) + d_i K e_i(k) - a_{i1} K e_1(k) - K \sum_{j=2}^{N} a_{ij} \Big[\delta_j(k) + e_j(k) \Big], \quad \forall i = 2, 3, ..., N. \end{aligned}$$

(6.6)

where K is the controller gain matrix to be determined. Since $\delta_1(k) = 0, \forall k$ and $a_{11} = 0$, then the similar leader has the following protocol form:

$$u_1(k) = d_1 K e_1(k) - K \sum_{j=2}^{N} a_{1j} \Big[\delta_j(k) + e_j(k) \Big] \tag{6.7}$$

and

$$\begin{aligned} u_i(k) - u_1(k) =& d_i K \delta_i(k) + d_i K e_i(k) - (a_{i1} + d_1) K e_1(k) \\ & - K \sum_{j=2}^{N} (a_{ij} - a_{1j}) \Big[\delta_j(k) + e_j(k) \Big] \end{aligned} \tag{6.8}$$

Denote

$\delta(k) = col\{\delta_2(k), ..., \delta_N(k)\}, \bar{e}(k) = col\{e_2(k), ..., e_N(k)\}$

$\bar{e}_1(k) = col\{e_1(k), ..., e_1(k)\}, e(k) = col\{e_1(k), ..., e_N(k)\}$

Then the overall dynamics of the multiagent system can be represented as follows

$$\delta(k+1) = \int_{\sigma=0}^{1} \Big[\big[\Pi + \Delta\Pi(\sigma)\big]\delta(k) + \big[\Pi_d + \Delta\Pi_d(\sigma)\big]\delta(k-\tau) + \mathcal{W}_1\bar{e}(k) + \mathcal{W}_2\bar{e}_1(k) \Big] d\sigma \tag{6.9}$$

where

$\Pi = (I_{N-1} \otimes A) + (\mathcal{L}_{22} + 1_{N-1}a_1^*) \otimes (BK)$

$\Delta\Pi(\sigma) = I_{N-1} \otimes \Delta A(\sigma), \Pi_d = (I_{N-1} \otimes A_d)$

$\Delta\Pi_d(\sigma) = I_{N-1} \otimes \Delta A_d(\sigma), a_1^* = (a_{12}, a_{13}, \ldots, a_{1N})\,,$

$\mathcal{W}_1 = (\mathcal{L}_{22} + 1_{N-1}a_1^*) \otimes (BK), \mathcal{W}_2 = -D^* \otimes (BK)$

$D^* = \text{diag}\,\{d_1 + a_{21}, d_1 + a_{31}, \ldots, d_1 + a_{N1}\}$

$$\mathcal{L}_{22} = \begin{bmatrix} d_2 & -a_{23} & \ldots & -a_{2N} \\ -a_{32} & d_3 & \cdots & -a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & d_N \end{bmatrix}$$

The following representation of the triggering-error terms $\bar{e}_1(k)$ and $\bar{e}(k)$ in (6.9)will be used in Theorem 6.2:

$$\bar{e}_1(k) = (G \otimes I_n)e(k), \quad \bar{e}(k) = (H \otimes I_n)e(k) \tag{6.10}$$

where

$$H = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \ldots & 0 \end{bmatrix},$$

and the overall triggering error is given by:

$$\mathcal{W}_1\bar{e}(k) + \mathcal{W}_2\bar{e}_1(k) = \Pi_e e(k) \tag{6.11}$$

where $\Pi_e = \big(\mathcal{W}_1(H\otimes I_n)+\mathcal{W}_2(G\otimes I_n)\big)$. As stated in [230], there exists a similar transformation $T_i\,(T_i\otimes I_n)^{-1}\,(I_{N-1}\otimes A+(d_i+1_{N-1}a_i^*-\mathcal{A}_i^*)\otimes B\;K)\,(T_i\otimes I_n)$=$I_{N-1}\otimes A+J_i\otimes BK$, where the principal diagonal elements of J_k^i are $\lambda_s(\mathcal{L})(s=2,3,\ldots,N)$. Moreover, the eigenvalues of $(I_{N-1}\otimes A)+(\mathcal{L}_{22}+1_{N-1}a_1^*)\otimes(BK)$ are given by those of the matrices $A+\lambda_s(\mathcal{L})BK$,$(s=2,3,\ldots,N)$. Applying the same strategy to the rest of matrices in Equation (6.9), then the consensus problem is equivalent to the the stability of the following multiagent system:

$$\begin{aligned}\delta_i(k+1) = \int_{\sigma=0}^{1}\Big[&(A+\Delta A(\sigma)+\lambda_i(\mathcal{L})BK)\,\delta_i(k)\\ &+(A_d+\Delta A_d(\sigma))\,\delta_i(k-\tau)+\lambda_i(\mathcal{L})BKe_i(k)-\lambda_i\,(D_1)\,BKe_1(k)\Big]d\sigma\end{aligned} \tag{6.12}$$

where $k\in[k_{l^i}^i, k_{(l+1)^i}^i)$, $i=2,3,...,N$. and $D_1=\mathrm{diag}(0,D^\star)$.

Consider an event triggering condition for sensor i as follows

$$\begin{aligned}k_{l+1}^i = \inf\Bigg\{k>k_l^i : e_i^T(k)\tilde{M}e_i(k)\geq\mu_i\sum_{j\in\mathcal{N}_i}a_{ij}\Big[x_i(k_l^i)\\ -x_j(k_l^j)\Big]^T Q\sum_{j\in\mathcal{N}_i}a_{ij}\Big[x_i(k_l^i)-x_j(k_l^j)\Big]\Bigg\}\end{aligned} \tag{6.13}$$

where μ_i are given positive threshold parameters, and $\tilde{M}$ and Q are positive definite weight matrices. When the parameters $\mu_i=0$, all samples are transmitted and it becomes the traditional time-triggered policy and this case is excluded in this Chapter. However, two cases will be considered, the first case appears when $\mu_i=\mu_2$, for all $i=3,...,N$ and $\mu_1=\mu_2/N$ whereas μ_2, $\tilde{M}$ and Q are freely chosen triggering-parameters. In the second case, $\tilde{M}=Q=I$ whereas threshold parameters μ_i are

selected freely by the designer. The former case is more general and will be explited for the guaranteed cost scheme. The overall triggering error of the latter case can be bounded as follows:

$$\begin{aligned}\|e(k)\|^2 =& \sum_{i=1}^{N} \|e_i(k)\|^2 \\ \leq& \sum_{i=1}^{N} \mu_i^2 \left\| \sum_{j\in\mathcal{N}_i} a_{ij} \left[x_i(k_l^i) - x_j(k_l^j) \right] \right\|^2 \\ =& \sum_{i=1}^{N} \mu_i^2 \left\| \sum_{j\in\mathcal{N}_i} a_{ij} \left[\delta_i(k) + e_i(k) - \delta_j(k) - e_j(k) \right] \right\|^2 \\ =& \sum_{i=1}^{N} \mu_i^2 \left\| \left[\mathcal{L}_i\delta(k) + \mathcal{L}_i e(k) \right] \right\|^2 \\ \leq& \sum_{i=1}^{N} \mu_i^2 \left[\frac{1+\epsilon_i}{\epsilon_i} \delta^T(k)\mathcal{L}_i^T\mathcal{L}_i\delta(k) + (1+\epsilon_i) e^T(k)\mathcal{L}_i^T\mathcal{L}_i e(k) \right]\end{aligned}$$

where $\mathcal{L}_i$ is the i-th row of the Laplacian matrix. Then the following holds:

$$\|e(k)\|^2 \leq \frac{\sum_{i=1}^{N} \mu_i^2 \frac{1+\epsilon_i}{\epsilon_i} \lambda_{\max}(\mathcal{L}_i^T\mathcal{L}_i)}{1 - \sum_{i=1}^{N} \mu_i^2 (1+\epsilon_i) \lambda_{\max}(\mathcal{L}_i^T\mathcal{L}_i)} \delta^T(k)\delta(k) \tag{6.14}$$

if

$$\sum_{i=1}^{N} \mu_i^2 (1+\epsilon_i) \lambda_{\max}(\mathcal{L}_i^T\mathcal{L}_i) < 1$$

The guaranteed cost index associated with the multi-agent system (6.1) is defined as follows:

$$J_\infty = \sum_{k=0}^{\infty} \left[\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \left(x_i(k) - x_j(k) \right)^T \frac{J_1}{2} \left(x_i(k) - x_j(k) \right) + \sum_{i=1}^{N} u_i^T(k) J_2^{-1} u_i(k) \right] \tag{6.15}$$

where J_1 and J_2 are positive definite matrices.

Remark 10 *For multiagent system with parameter uncertainty, a guaranteed cost consensus is investigated in [196], [210]. Compared with the guaranteed performance index in [211], [231] where the cost index were solely defined based on the dynamic performances of agents, our cost index comprises the dynamic performance and control*

energy consumption. This leads to a trade-off between the dynamic performance and energy consumption by selecting J_1 and J_2. Moreover, this article investigates the guaranteed cost performance under event triggered mechanism for a class of nonlinear multiagent systems.

Before we state our analysis, we need the following Lemmas.

Lemma 6.1 *[232] For any constant real matrices with the appropriate dimension X and Y, the following inequality is true:*

$Y^T F(k) X^T + X F^T(k) Y \leq \epsilon X X^T + \frac{1}{\epsilon} Y^T Y$

for every matrix function $F(k)$ satisfying $\|F(k)\| \leq 1$ and a scalar $\epsilon > 0$.

.

Lemma 6.2 *[184] For the Laplacian matrix $\mathcal{L}$, we have*

$$x^T(k)\mathcal{L}x(k) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(x_i(k) - x_j(k)\right)^T\left(x_i(k) - x_j(k)\right)$$

and L is positive semi-definite. In addition, if $1^T x(k) = 0$, then $\lambda_2(\mathcal{L})x^T(k)x(k) \leq x^T(k)\mathcal{L}x(k) \leq \lambda_{\max}(\mathcal{L})x^T(k)x(k)$

.

Lemma 6.3 *[233] Let $x(k) \in \mathbf{R}^n$ be a vector function. Then, for any positive definite matrix $M > 0$, and integers $m > n$ the following inequality holds:*

$$-(m-n+1)\sum_{r=n}^{m} x^T(r)Mx(r) \leq -\sum_{r=n}^{m} x^T(r)M\sum_{r=n}^{m} x(r)$$

Theorem 6.1 *[234] Let $f_1, f_2, \ldots, f_N : \mathbf{R}^m \mapsto \mathbf{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbf{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies*

$$\min_{\substack{\{\alpha_i|\alpha_i>0,\sum_i \\ \alpha_i=1\}}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{\mathbb{Q}_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \mathbf{R}^m \mapsto \mathbf{R}, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}$$

6.3 Guaranteed Performance Consensus

This section establishes sufficient conditions to achieve the consensus of time-delay nonlinear multiagent systems based on state feedback protocol. Moreover, guaranteed performance consensus is investigated. The Lyapunov methodology is used to investigate the stability of the disagreement dynamics under event-triggering scheme. Upper bounds of performance indexes J_∞ are acquired and the design method to obtain the control matrix is given by matrix inequalities.

Theorem 6.2 *Consider the multiagent system (6.1) under the event triggering mechanism (6.13) with the triggering parameters:* $\mu_i = \mu_2$, *for all* $i = 3, ..., N$ *and* $\mu_1 = \mu_2/N$. *Given scalars* $\epsilon_1 > 0$ *and* $\epsilon_2 > 0$, *if there exit matrices* $\mathcal{X} > 0$, $\mathcal{Y} > 0$, $\mathcal{Z} > 0$, $\mathcal{M} > 0$, *and* $\mathcal{W}$ *with appropriate dimensions satisfying the following LMI:*

$$\Upsilon = \begin{bmatrix} -\mathcal{X}+\mathcal{Y}+\bar{\tau}\mathcal{Z} & 0 & 0 & 0 & \mathcal{X}A^T+\lambda_i(\mathcal{L})\mathcal{W}^TB^T \\ \bullet & -\mathcal{Y}-\bar{\tau}\mathcal{Z} & 0 & 0 & \mathcal{X}A_d^T \\ \bullet & \bullet & -\mathcal{M} & 0 & \lambda_i(\mathcal{L})\mathcal{W}^TB^T \\ \bullet & \bullet & \bullet & -\mathcal{M} & \lambda_i(D_1)\mathcal{W}^TB^T \\ \bullet & \bullet & \bullet & \bullet & -\mathcal{X} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1\mathcal{X}N_1^T & 0 & \mathcal{X}E^T & \mathcal{X}E^T & \lambda_{\max}^2(\mathcal{L})\mathcal{X} \\ \epsilon_2\mathcal{X}N_2^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\epsilon_1 & 0 & 0 & 0 & 0 \\ \bullet & -\epsilon_2 & 0 & 0 & 0 \\ \bullet & \bullet & -\epsilon_1 & 0 & 0 \\ \bullet & \bullet & \bullet & -\epsilon_2 & 0 \\ \bullet & \bullet & \bullet & \bullet & \frac{-I}{\mu_2\lambda_{\max}(Q)} \end{bmatrix} < 0 \qquad (6.16)$$

Then the multiagent system (6.1) reaches consensus. Moreover, the consensus gain is given by $K = \mathcal{W}\mathcal{X}^{-1}$.

Proof. Consider the following Lyapunov candidate functional:

$$V_i(k) = \delta_i^T(k)P_1\delta_i(k) + \sum_{l=k-\bar{\tau}}^{k-1} \delta_i^T(l)P_2\delta_i(l) + \sum_{\theta=-\bar{\tau}+1}^{0} \sum_{l=k-1+\theta}^{k-1} \delta_i^T(l)P_3\delta_i(l) \tag{6.17}$$

where $\delta_i(k)$ is the disagreement vector given in the dynamical system (6.12), $i = 2, 3, ..., N$, and P_1, P_2 and P_3 are symmetric positive definite variables with appropriate dimensions.

The time difference of the Lyapunov funcational can be given by:

$$\begin{aligned}
\Delta V_i(k) &= \delta_i^T(k+1)P_1\delta_i(k+1) - \delta_i^T(k)P_1\delta_i(k) + \delta_i^T(k)P_2\delta_i(k) \\
&\quad - \delta_i^T(k-\bar{\tau})P_2\delta_i(k-\bar{\tau}) + \bar{\tau}\delta_i^T(k)P_3\delta_i(k) - \sum_{l=k-\bar{\tau}}^{k-1} \delta_i^T(l)P_3\delta_i(l) \\
&\leq \int_{\sigma=0}^{1} \left[\xi_i^T(k) \left[\Gamma^T P_1 \Gamma + \Xi \right] \xi_i(k) \right] d\sigma + e_i^T(k)\tilde{M}e_i(k) \\
&\quad + e_1^T(k)\tilde{M}e_1(k) - \mu_2\lambda_{\max}^2(\mathcal{L})\delta_i^T(k)Q\delta_i(k)
\end{aligned} \tag{6.18}$$

where $\xi_i^T = \begin{bmatrix} \delta_i^T(k) & \delta_i^T(k-\tau) & e_1^T(k) & e_i^T(k) \end{bmatrix}$, $\Gamma = \begin{bmatrix} A + \Delta A + \lambda_i(\mathcal{L})BK, & A_d + \Delta A_d, & \lambda_i(\mathcal{L})BK, & \lambda_i(D_1)BK \end{bmatrix}$ and

$$\Xi = \begin{bmatrix} \begin{bmatrix} -P_1 + P_2 + \bar{\tau}P_3 \\ +\mu_2\lambda_{\max}^2(\mathcal{L})\lambda_{\max}(Q) \end{bmatrix} & 0 & 0 & 0 \\ 0 & -P_2 - \bar{\tau}P_3 & 0 & 0 \\ 0 & 0 & -\tilde{M} & 0 \\ 0 & 0 & 0 & -\tilde{M} \end{bmatrix}$$

. From the Kronecker product properties, we come up with the following upper bound: $-\sum_{i=2}^{N} \mu_2\lambda_{\max}^2(\mathcal{L})\delta_i^T(k)Q\delta_i(k) \leq -\mu_2\hat{\delta}^T(k)\left(L^2 \otimes Q\right)\hat{\delta}(k)$ with the representation of

the error terms given in (6.10), we get

$$\begin{aligned}
\Delta V(k) &= \sum_{i=2}^{N} \Delta V_i(k) \\
&\leq \int_{\sigma=0}^{1} \left[\sum_{i=2}^{N} \xi_i^T(k) \left[\Gamma^T P_1 \Gamma + \Xi \right] \xi_i(k) \right] d\sigma \\
&+ \bar{e}^T(k)(I_{N-1} \otimes \tilde{M})\bar{e}(k) + \bar{e}_1^T(k)(I_{N-1} \otimes \tilde{M})\bar{e}_1(k) \\
&- \sum_{i=2}^{N} \mu_2 \lambda_{\max}^2(\mathcal{L}) \delta_i^T(k) Q \delta_i(k) \\
&\leq \int_{\sigma=0}^{1} \left[\sum_{i=2}^{N} \xi_i^T(k) \left[\Gamma^T P_1 \Gamma + \Xi \right] \xi_i(k) \right] d\sigma \\
&+ e^T(k)([H^T H + G^T G] \otimes \tilde{M}) e(k) - \mu_2 \hat{\delta}^T(k) \left(\mathcal{L}^2 \otimes Q \right) \hat{\delta}(k)
\end{aligned} \tag{6.19}$$

where $\hat{\delta}^T(k) = [\delta_1^T(k), \delta^T(k)]$. Straightforward calculation on G and H gives the following matrix $H^T H + G^T G = \begin{bmatrix} N & 0 \\ 0 & I_{N-1} \end{bmatrix}$ This allows us to rewrite the upper bound of the functional difference as follows:

$$\begin{aligned}
\Delta V(k) \leq & \int_{\sigma=0}^{1} \left[\sum_{i=2}^{N} \xi_i^T(k) \left[\Gamma^T P_1 \Gamma + \Xi \right] \xi_i(k) \right] d\sigma \\
& + N \left[e_1^T(k) \tilde{M} e_1(k) - \frac{\mu_2}{N} \sum_{j \in N_1} a_{1j} \left(x_1(k) - x_j(k) \right)^T Q \sum_{j \in N_1} a_{1j} \left(x_1(k) - x_j(k) \right) \right] \\
& + \sum_{i=2}^{N} \left[e_i^T(k) \tilde{M} e_i(k) - \mu_2 \sum_{j \in N_i} a_{ij} \left(x_i(k) - x_j(k) \right)^T Q \sum_{j \in N_i} a_{ij} \left(x_i(k) - x_j(k) \right) \right]
\end{aligned} \tag{6.20}$$

The triggering strategy guarantees that the last two terms are negative. Therefore, when the event triggering policy (6.13) is activated then $\Delta V < 0$ if the inequality $\Gamma^T P_1 \Gamma + \Xi < 0$ holds. By Schur complement, the last inequality is equivalent to the following matrix inequality:

$$\left[\begin{array}{c|c} \Xi & \begin{array}{c} A^T + \lambda_i(\mathcal{L})K^T B^T \\ A_d^T \\ \lambda_i(\mathcal{L})K^T B^T \\ \lambda_i(D_1)K^T B^T \end{array} \\ \hline \bullet & -P_1^{-1} \end{array}\right] + \bar{N}^T \bar{F}^T \bar{E}^T + \bar{E}\bar{F}\bar{N} < 0$$

where $\bar{N}^T = \begin{bmatrix} N_1^T & 0 \\ 0 & N_2^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \bar{F} = \begin{bmatrix} F^T & 0 \\ 0 & F^T \end{bmatrix}$ and $\bar{E} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ E & E \end{bmatrix}$.

Using Lemma 6.2 and Schur complements, we get the following sufficient condition:

$$\begin{bmatrix}
-P_1+P_2+\bar{\tau}P_3 & 0 & 0 & 0 & A^T+\lambda_i(\mathcal{L})K^TB^T \\
\bullet & -P_2-\bar{\tau}P_3 & 0 & 0 & A_d^T \\
\bullet & \bullet & -\tilde{M} & 0 & \lambda_i(\mathcal{L})K^TB^T \\
\bullet & \bullet & \bullet & -\tilde{M} & \lambda_i(D_1)K^TB^T \\
\bullet & \bullet & \bullet & \bullet & -P_1^{-1} \\
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet
\end{bmatrix}$$

$$\begin{bmatrix}
\epsilon_1 N_1^T & 0 & E^T & E^T & \lambda_{\max}^2(\mathcal{L})I \\
\epsilon_2 N_2^T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\epsilon_1 & 0 & 0 & 0 & 0 \\
\bullet & -\epsilon_2 & 0 & 0 & 0 \\
\bullet & \bullet & -\epsilon_1 & 0 & 0 \\
\bullet & \bullet & \bullet & -\epsilon_2 & 0 \\
\bullet & \bullet & \bullet & \bullet & \frac{-I}{\mu_2\lambda_{\max}(Q)}
\end{bmatrix} < 0 \qquad (6.21)$$

Multiplying both sides of the inequality (6.21) by the congruent transformation diag $\{P_1^{-1}, P_1^{-1}, P_1^{-1}, P_1^{-1}, I, I, I, I, \}$, where $\mathcal{X} = P_1^{-1}$, $\mathcal{Y} = P_1^{-1}P_2P_1^{-1}$, $\mathcal{Z} =$

$P_1^{-1}P_3P_1^{-1}$, $\mathcal{W} = KP_1^{-1}$ and $\mathcal{M} = P_1^{-1}\tilde{M}P_1^{-1}$ gives its equivalent inequality in (6.16) which completes the proof. ∎

The following Theorem establishes sufficient conditions to achieve guaranteed cost consensus.

Theorem 6.3 *For the multiagent system (6.1) under the event triggering mechanism (6.13) with the triggering parameters: $\mu_i = \mu_2$, for all $i = 3, ..., N$ and $\mu_1 = \mu_2/N$. Given scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$, if there exit matrices $\mathcal{X} > 0$, $\mathcal{Y} > 0$, $\mathcal{Z} > 0$, $\mathcal{M} > 0$, and $\mathcal{W}$ satisfying the following LMI:*

$$\left[\begin{array}{c|c|c} \Upsilon & \begin{bmatrix} \mathcal{X}J_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} & \begin{bmatrix} \mathcal{W}^T \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ \hline \bullet & -J_1 & 0 \\ \hline \bullet & \bullet & -J_2 \end{array}\right] < 0 \tag{6.22}$$

for all $i = 2, 3, ..., N$, where Υ is given in (6.16). Then the multiagent system achieves consensus with an upper bound $\hat{\delta}(0)(I_n \otimes \left[P_1 + \bar{\tau}P_2 + \frac{\bar{\tau}-\bar{\tau}^2}{2}P_3\right])\hat{\delta}(0)$ of the performance index J_∞. Moreover, the consensus gain matrix is given by $K = \mathcal{W}\mathcal{X}^{-1}$.

Proof. The finite sum of the performance index (6.15) can be written as follows

$$\begin{aligned}
J_m =& \sum_{k=0}^{m} \Bigg[\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \left(\delta_i(k) - \delta_j(k)\right)^T \frac{J_1}{2} \left(\delta_i(k) - \delta_j(k)\right) \\
&+ \sum_{i=1}^{N} u_i^T(k) J_2^{-1} u_i(k) \Bigg] \\
&= \sum_{k=0}^{m} \left[\hat{\delta}^T(k) \left(\mathcal{L} \otimes J_1\right) \hat{\delta}(k) + \hat{\delta}^T(k) \left(\mathcal{L}^2 \otimes K^T J_2^{-1} K\right) \hat{\delta}(k) \right] \\
&\leq \sum_{k=0}^{m} \left[\lambda_{\max}(L) \hat{\delta}^T(k) \left(I_N \otimes J_1\right) \hat{\delta}(k) + \lambda_{\max}(L)^2 \hat{\delta}^T(k) \left(I_N \otimes K^T J_2^{-1} K\right) \hat{\delta}(k) \right] \\
&= \sum_{k=0}^{m} \left[\sum_{i=2}^{N} \lambda_{\max}(L) \delta_i^T(k) J_1 \delta_i(k) + \sum_{i=2}^{N} \lambda_{\max}(L)^2 \delta_i^T(k) K^T J_2^{-1} K \delta_i(k) \right]
\end{aligned}$$

Using the Lyapunov functional $V_i(k)$ in (6.17) and following the same proof of Theorem 6.2 with Schur complements, then the LMI (6.22) becomes a sufficient condition to the following inequality:

$$\sum_{k=0}^{m} \Delta V(k) + J_m = \sum_{k=0}^{m} \sum_{i=2}^{N} \Delta V_i(k) + J_m \leq 0 \tag{6.23}$$

Since the Lyapunov functional is positive definite, then

$$J_m \leq \sum_{i=2}^{N} V_i(0) \qquad \forall\, m \geq 0 \tag{6.24}$$

Taking m goes to inifinty and due to $\delta_i(0) = \delta_i(k)$, $k \in [\bar{\tau}, 0]$, $i = 1, 2, ..., N$, it can be deduced that

$$J_\infty \leq \sum_{i=2}^{N} V_i(0) = \hat{\delta}(0)(I_n \otimes \left[P_1 + \bar{\tau} P_2 + \frac{\bar{\tau} - \bar{\tau}^2}{2} P_3\right])\hat{\delta}(0) \tag{6.25}$$

which completes the proof. ∎

Remark 11 *Theorem 6.2 provides sufficient condition to achieve consensus only, whereas Theorem 6.3 is used to design guarantee guaranteed-cost consensus. Both of them achieve consensus under the proposed event triggered scheme. However, the conditions presented in Theorem 6.3 is more conservative than the inequalities of Theorem 6.2. This is the cost of guaranteeing upper bound of the performance index.*

Now, we establish the distributed consensus problem for the multiagent system using the overall dynamics of the disagreement vector given in (6.9). The triggering mechanism will be a special case of the triggering control policy given in Theorems 6.2 and 6.3. This may lead to conservative triggering policy. However, it allows us to use more Lyapunov functional that decease this conservativeness.

Theorem 6.4 *Given positive integers $\bar{\tau}$ and $\underline{\tau}$, positive scalars μ_i, and weighting matrices $Q = \tilde{M} = I$. The multiagent system (6.1) is said to achieve event based consensus under the distributed protocol (6.4) and the event triggering policy (6.13), if there exist positive scalars γ, ϵ_i, δ_i $i = 1, ..., N$, and matrices $\mathcal{P} > 0$, $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{Q}_3 > 0$, $\mathcal{Z}_1 > 0$, $\mathcal{Z}_2 > 0$, $\mathcal{Z}_3 > 0$, $\mathcal{N}$, S, M, $\mathcal{R}$, and $L_j > 0$, $j = 1, 2, ..., 6$ such that the following inequalities hold:*

$$\begin{bmatrix} \Omega & U_1 & \epsilon U_2 \\ \bullet & -\epsilon I & 0 \\ \bullet & \bullet & -\epsilon \end{bmatrix} < 0 \tag{6.26}$$

$$\begin{bmatrix} \mathcal{Z}_3 & \mathcal{R} \\ \mathcal{R} & \mathcal{Z}_3 \end{bmatrix} \geq 0 \tag{6.27}$$

$$C = 1 - \gamma^2 \frac{\sum_{i=1}^{N} \mu_i^2 \frac{1+\epsilon_i}{\epsilon_i} \lambda_{\max}(\mathcal{L}_i^T \mathcal{L}_i)}{1 - \sum_{i=1}^{N} \mu_i^2 (1+\epsilon_i) \lambda_{\max}(\mathcal{L}_i^T \mathcal{L}_i)} > 0 \tag{6.28}$$

for all $i = 1, 2, ..., N$, *where*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ \bullet & \Omega_{22} & \Omega_{23} & \mathcal{R} & -L_2 & L_2B \\ \bullet & \bullet & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} \\ \bullet & \bullet & \bullet & \Omega_{44} & -L_4 & L_4B \\ \bullet & \bullet & \bullet & \bullet & \Omega_{55} & \Omega_{56} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \Omega_{66} \end{bmatrix} \tag{6.29}$$

$$\Omega_{11} = L_1(\Pi - I) + (\Pi - I)^T L_1^T + (\tau^\star + 1)\mathcal{Q}_3$$

$$+ \mathcal{Q}_1 + \mathcal{Q}_2 - \mathcal{Z}_1 - \mathcal{Z}_2 + I$$

$$\Omega_{12} = \mathcal{Z}_2 + (\Pi - I)^T L_2^T, \Omega_{22} = \mathcal{Q}_1 - \mathcal{Z}_2 - \mathcal{Z}_3$$

$$\Omega_{13} = L_1\Pi_d + (\Pi - I)^T L_3^T + I, \Omega_{23} = -R^T + \mathcal{Z}_3^T + L_2\Pi_d$$

$$\Omega_{33} = -\mathcal{Q}_3 + L_3\Pi_d + \Pi_d^T L_3^T - 2\mathcal{Z}_3 + R + R^T + I$$

$$\Omega_{14} = \mathcal{Z}_1 + (\Pi - I)^T L_4^T, \Omega_{34} = -R + \mathcal{Z}_3 + \Pi_d^T L_4^T$$

$$\Omega_{44} = -\mathcal{Q}_2 - \mathcal{Z}_1 - \mathcal{Z}_3,\ \Omega_{35} = L_3 + \Pi_d^T L_5^T$$

$$\Omega_{15} = \mathcal{P} - L_1^T + (\Pi - I)^T L_5^T - L_2$$

$$\Omega_{55} = \bar{\tau}^2\mathcal{Z}_1 + \underline{\tau}^2\mathcal{Z}_2 + \tau^\star\mathcal{Z}_3 - L_5 - L_5^T + \mathcal{P}$$

$$\Omega_{16} = L_1B + (\Pi - I)^T L_6,\ \Omega_{36} = \Pi_d^T L_6^T + L_3B$$

$$\Omega_{56} = L_5B + L_6^T,\ \Omega_{66} = -\gamma^2 I + L_5B + B^T L_5^T$$

$$U_1 = \begin{bmatrix} E^T L_1^T, & E^T L_2^T, & \ldots, & E^T L_6^T \end{bmatrix}^T$$

$$U_2 = \begin{bmatrix} N_1 & 0 & N_2 & 0 & 0 & 0 \end{bmatrix}, \textit{and } \tau^\star = \bar{\tau} - \underline{\tau}.$$

Then the multiagent system (6.1) achieves consensus with the following performance

bound:

$$J_\infty = \sum_0^\infty \delta^T(k)\delta(k) \leq \frac{1}{C}V(0)$$

Proof. A Lyapunov functional candidate is constructed as follows:

$$V(k) = \int_0^1 \sum_{i=1}^N V_i(k)d\sigma$$

where

$$\begin{aligned}
V_1(k) &= \delta^T(k)\mathcal{P}\delta(k) \\
V_2(k) &= \sum_{l=k-\underline{\tau}}^{k-1} \delta^T(l)\mathcal{Q}_1\delta(l) + \sum_{l=k-\bar{\tau}}^{k-1} \delta^T(l)\mathcal{Q}_2\delta(l) \\
V_3(k) &= \sum_{\theta=-\bar{\tau}}^{-\underline{\tau}} \sum_{l=k+\theta}^{k-1} \delta^T(l)\mathcal{Q}_3\delta(l) \\
V_4(k) &= \bar{\tau} \sum_{\theta=-\bar{\tau}}^{-1} \sum_{l=k+\theta}^{k-1} \vartheta^T(l)^T\mathcal{Z}_1\vartheta(l) + \underline{\tau} \sum_{\theta=-\underline{\tau}}^{-1} \sum_{l=k+\theta}^{k-1} \vartheta^T(l)^T\mathcal{Z}_2\vartheta(l) \\
&\quad + \tau^\star \sum_{\theta=-\bar{\tau}}^{\underline{\tau}-1} \sum_{l=k+\theta}^{k-1} \vartheta^T(l)^T\mathcal{Z}_3\vartheta(l)
\end{aligned}$$

where $\tau^\star = \bar{\tau} - \underline{\tau}$ and $\vartheta(k) = \delta(k+1) - \delta(k)$. The following equations give the functional difference $\Delta V_i(k) = V_i(k+1) - V_i(k)$ along the trajectory of multiagent

dynamics (6.9) :

$$\Delta V_1(k) = \delta^T(k+1)^T\mathcal{P}\delta(k+1) - \delta^T(k)^T\mathcal{P}\delta(k)$$

$$= \vartheta^T(k)^T\mathcal{P}\vartheta(k) + 2\delta^T(k)^T\mathcal{P}\vartheta(k) \tag{6.30}$$

$$\Delta V_2(k) = \delta^T(k)[\mathcal{Q}_1 + \mathcal{Q}_2]\delta(k) - \delta^T(k-\underline{\tau})\mathcal{Q}_1\delta(k-\underline{\tau})$$

$$- \delta^T(k-\bar{\tau})\mathcal{Q}_2\delta(k-\bar{\tau}) \tag{6.31}$$

$$\Delta V_3(k) = (\tau^\star+1)\delta^T(k)\mathcal{Q}_3\delta(k) - \sum_{l=k-\bar{\tau}}^{k-\underline{\tau}} \delta^T(l)\mathcal{Q}_3\delta(l)$$

$$\leq (\tau^\star+1)\delta^T(k)\mathcal{Q}_3\delta(k) - \delta^T(k-\tau(k)Q_3\delta(k-\tau(k) \tag{6.32}$$

$$\Delta V_4(k) = \vartheta^T(k)^T\left[\bar{\tau}^2\mathcal{Z}_1 + \underline{\tau}^2\mathcal{Z}_2 + \tau^\star\mathcal{Z}_3\right]\vartheta(k)$$

$$- \bar{\tau}\sum_{l=k-\bar{\tau}}^{k-1} \vartheta^T(l)^T\mathcal{Z}_1\vartheta(l) - \underline{\tau}\sum_{l=k-\underline{\tau}}^{k-1} \vartheta^T(l)^T\mathcal{Z}_2\vartheta(l)$$

$$- \tau^\star\sum_{l=k-\bar{\tau}}^{k-\underline{\tau}-1} \vartheta^T(l)^T\mathcal{Z}_3\vartheta(l) \tag{6.33}$$

According to Lemma 6.3, we come up with

$$-\bar{\tau}\sum_{l=k-\bar{\tau}}^{k-1} \vartheta^T(l)^T\mathcal{Z}_1\vartheta(l) \leq -[\delta(k)-\delta(k-\bar{\tau})]^T\mathcal{Z}_1[\delta(k)-\delta(k-\bar{\tau})] \tag{6.34}$$

$$-\underline{\tau}\sum_{l=k-\underline{\tau}}^{k-1} \vartheta^T(l)^T\mathcal{Z}_2\vartheta(l) \leq -[\delta(k)-\delta(k-\underline{\tau})]^T\mathcal{Z}_2[\delta(k)-\delta(k-\underline{\tau})] \tag{6.35}$$

Using Lemma 6.3 and Theorem 6.1, the following inequality is satisfied

$$\begin{aligned} -\tau^\star\sum_{l=k-\bar{\tau}}^{k-\underline{\tau}-1} \vartheta^T(l)\mathcal{Z}_3\vartheta(l) &= -\tau^\star\sum_{l=k-\bar{\tau}}^{k-\tau(k)-1} \vartheta^T(l)\mathcal{Z}_3\vartheta(l) - \tau^\star\sum_{l=k-\tau(k)}^{k-\underline{\tau}-1} \vartheta^T(l)\mathcal{Z}_3\vartheta(l) \\ &\leq -\frac{1}{\alpha_1}\Psi_1^T(k)\mathcal{Z}_3\Psi_1(k) - \frac{1}{\alpha_1}\Psi_2^T(k)^T\mathcal{Z}_3\Psi_2(k) \\ &\leq -\begin{bmatrix} \Psi_1(k) \\ \Psi_2(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{Z}_3 & \mathcal{R} \\ \mathcal{R} & \mathcal{Z}_3 \end{bmatrix} \begin{bmatrix} \Psi_1(k) \\ \Psi_2(k) \end{bmatrix} \end{aligned} \tag{6.36}$$

whenever the following inequality holds:

$$\begin{bmatrix} \mathcal{Z}_3 & \mathcal{R} \\ \mathcal{R} & \mathcal{Z}_3 \end{bmatrix} \geq 0$$

where α_1, α_2, $\Psi_1(k)$, and $\Psi_2(k)$ are given as follows

$$\alpha_1 = \frac{\bar{\tau} - \tau(k)}{\bar{\tau} - \underline{\tau}}, \quad \Psi_1(k) = \delta(k - \tau(k)) - \delta(k - \bar{\tau})$$
$$\alpha_2 = \frac{\tau(k) - \underline{\tau}}{\bar{\tau} - \underline{\tau}}, \quad \Psi_1(k) = \delta(k - \underline{\tau}) - \delta(k - \tau(k))$$

From the dynamics (6.9), it is easy to see that the following equation holds for any slack variables $L_1, L_2, L_3, L_4, L_5, L_6$ with appropriate dimensions:

$$2\xi^T(k)L^T\Big[(\Pi - I)\delta(k) + \Pi_d\delta(k - \tau(k)) + \Pi_e e(k) - \vartheta(k)\Big] = 0 \tag{6.37}$$

where $L^T = \begin{bmatrix} L_1^T & L_2^T & L_3^T & L_4^T & L_5^T & L_6^T \end{bmatrix}$. Let

$$\xi^T(k) = \begin{bmatrix} \delta^T(k) & \delta^T(k - \underline{\tau}) & \delta^T(k - \tau(k)) & \delta^T(k - \bar{\tau}) & \vartheta^T(k) & e^T(k) \end{bmatrix}$$

Using Equations (6.30)-(6.37), we get the following bound:

$$\Delta V + \delta^T(k)\delta(k) - \gamma^2 e^T(k)e(k) \leq \int_0^1 \xi^T(k)\Theta\xi(k)d\sigma \tag{6.38}$$

where $\Theta = \Omega + U_1^T F(k)U_2 + U_2^T F(k)U_1$. According to Schur complements and Lemma 6.1, Θ is negative definite if inequality (6.26) is satisfied. Since $Q = \tilde{M} = I$ then Equation (6.14) holds and inequality (6.26) becomes a sufficient condition to the following inequality:

$$\begin{aligned} \Delta V &\leq -\delta^T(k)\delta(k) + \gamma^2 e^T(k)e(k) \\ &\leq -\left[1 - \gamma^2 \frac{\sum_{i=1}^N \mu_i^2 \frac{1+\epsilon_i}{\epsilon_i}\lambda_{\max}(\mathcal{L}_i^T\mathcal{L}_i)}{1 - \sum_{i=1}^N \mu_i^2(1+\epsilon_i)\lambda_{\max}(\mathcal{L}_i^T\mathcal{L}_i)}\right]\delta^T(k)\delta(k) \\ &\leq -C\delta^T(k)\delta(k) \end{aligned} \tag{6.39}$$

Hence $\Delta V(k)$ is negative, by condition (6.28), one can conclude that consensus is achieved. Since $V(k)$ is positive and $V(\infty) = 0$. Taking the infinite sum for both

sides of inequality (6.39) gives the following guaranteed performance

$$J_\infty = \sum_0^\infty \delta^T(k)\delta(k) \leq \frac{1}{C}V(0)$$

This completes the proof. ∎

Theorem 6.5 *Given positive integers $\bar{\tau}$ and $\underline{\tau}$, positive scalars μ_i, and weighting matrices $Q = \tilde{M} = I$. The multiagent system (6.1) is said to achieve event based consensus under the distributed protocol (6.4) and the event triggering scheme (6.13), if there exist positive scalars γ, ϵ_i, δ_i and matrices $\mathcal{P} > 0$, $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{Q}_3 > 0$, $\mathcal{Z}_1 > 0$, $\mathcal{Z}_2 > 0$, $\mathcal{Z}_3 > 0$, $\mathcal{R}$, $\mathcal{X}_1$, $\mathcal{X}_2$, amd $L_j > 0$, $i = 1, ..., N$, $j = 1, 2, ..., 6$ such that the following inequalities hold:*

$$\begin{bmatrix} \Phi & \Omega_1^{\mathrm{T}}(I\otimes\mathcal{P}) & \sqrt{\bar{\tau}}\Omega_2^{\mathrm{T}}(I\otimes\mathcal{Z}_1) & \sqrt{\tau^\star}\,\Omega_2^{\mathrm{T}}(I\otimes\mathcal{Z}_2) & \epsilon\bar{E} & \bar{N}^T \\ \bullet & -(I\otimes\mathcal{P}) & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -(I\otimes\mathcal{Z}_1) & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -(I\otimes\mathcal{Z}_2) & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\epsilon I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\epsilon I \end{bmatrix} < 0 \tag{6.40}$$

$$\Psi_1 = \begin{bmatrix} \mathcal{X}_1 & \mathcal{N} \\ * & \mathcal{Z}_1 \end{bmatrix} \geqslant 0 \tag{6.41}$$

$$\Psi_2 = \begin{bmatrix} \mathcal{X}_2 & S \\ * & \mathcal{Z}_2 \end{bmatrix} \geqslant 0 \tag{6.42}$$

$$\Psi_3 = \begin{bmatrix} \mathcal{X}_1 + \mathcal{X}_2 & M \\ * & \mathcal{Z}_1 + \mathcal{Z}_2 \end{bmatrix} \geqslant 0 \tag{6.43}$$

$$C = 1 - \gamma^2 \frac{\sum_{i=1}^{N} \mu_i^2 \frac{1+\epsilon_i}{\epsilon_i} \lambda_{\max}(\mathcal{L}_i^T \mathcal{L}_i)}{1 - \sum_{i=1}^{N} \mu_i^2 (1+\epsilon_i) \lambda_{\max}(\mathcal{L}_i^T \mathcal{L}_i)} > 0$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & S_1 & -M_1 & 0 \\ \bullet & \Phi_{22} & S_2 & -M_2 & 0 \\ \bullet & \bullet & -(I \otimes \mathcal{Q}_1) & 0 & 0 \\ \bullet & \bullet & \bullet & -(I \otimes \mathcal{Q}_2) & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} I \otimes E & 0 \\ 0 & I \otimes E \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{X}_1 = \begin{bmatrix} \mathcal{X}_{11}^1 & \mathcal{X}_{12}^1 \\ \bullet & \mathcal{X}_{22}^1 \end{bmatrix}, \quad \mathcal{X}_2 = \begin{bmatrix} \mathcal{X}_{11}^2 & \mathcal{X}_{12}^2 \\ \bullet & \mathcal{X}_{22}^2 \end{bmatrix}$$

$$\bar{N} = \begin{bmatrix} 0 & (I \otimes N_1 \mathcal{P}) & (I \otimes N_1 \mathcal{P}) & (I \otimes N_1 \mathcal{P}) \\ 0 & (I \otimes N_2 P) & (I \otimes N_2 \mathcal{P}) & (I \otimes N_2 \mathcal{P}) \end{bmatrix}$$

$$\Phi_{11} = I \otimes (I + \mathcal{Q}_1 + \mathcal{Q}_2 + (\tau^\star + 1)\, \mathcal{Q}_3 - \mathcal{P}) + \mathcal{N}_1 + \mathcal{N}_1^{\mathrm{T}} + \bar{\tau} \mathcal{X}_{11}^1 + \tau^\star \mathcal{X}_{11}^2$$

$$\Phi_{12} = \mathcal{N}_2^{\mathrm{T}} - \mathcal{N}_1 + M_1 - S_1 + \bar{\tau} \mathcal{X}_{12}^1 + \tau^\star \mathcal{X}_{12}^2$$

$$\Phi_{22} = -\mathcal{Q}_3 - \mathcal{N}_2 - \mathcal{N}_2^{\mathrm{T}} + M_2 + M_2^{\mathrm{T}} - S_2 - S_2^{\mathrm{T}} + \bar{\tau} \mathcal{X}_{22}^1 + \tau^\star \mathcal{X}_{22}^2$$

$$\Omega_1 = \begin{bmatrix} \Pi & \Pi_d & 0 & 0 & \Pi_e \end{bmatrix}$$

$$\Omega_2 = \begin{bmatrix} (\Pi - I) & \Pi_d & 0 & 0 & \Pi_e \end{bmatrix}$$

$\tau^\star = \bar{\tau} - \underline{\tau}$

$\mathcal{N} = \left[\mathcal{N}_1^{\mathrm{T}} \mathcal{N}_2^{\mathrm{T}}\right]^{\mathrm{T}}, S = \left[S_1^{\mathrm{T}} S_2^{\mathrm{T}}\right]^{\mathrm{T}}$, *and* $M = \left[M_1^{\mathrm{T}} M_2^{\mathrm{T}}\right]^{\mathrm{T}}$ *Then the multiagent system*

(6.1) achieves consensus with the following performance bound:

$$J_\infty = \sum_0^\infty \delta^T(k)\delta(k) \le \frac{1}{C}V(0)$$

Proof. A Lyapunov functional candidate is constructed as follows:

$$V(k) = \sum_{i=2}^{\mathcal{N}} \Bigg[\delta_i^{\mathrm{T}}(k)\mathcal{P}\delta_i(k) + \sum_{\theta=-\bar{\tau}+1}^{0} \sum_{l=k-1+\theta}^{k-1} \vartheta_i^{\mathrm{T}}(l)\mathcal{Z}_1\vartheta_i(l) + \sum_{\theta=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k-1+\theta}^{k-1} \vartheta_i^{\mathrm{T}}(l)\mathcal{Z}_2\vartheta_i(l)$$
$$+ \sum_{l=k-\underline{\tau}}^{k-1} \delta_i^{\mathrm{T}}(l)\mathcal{Q}_1\delta_i(l) + \sum_{l=k-\bar{\tau}}^{k-1} \delta_i^{\mathrm{T}}(l)\mathcal{Q}_2\delta_i(l) + \sum_{\theta=-\bar{\tau}+1}^{-\underline{\tau}+1} \sum_{l=k-1+\theta}^{k-1} \delta_i^{\mathrm{T}}(l)\mathcal{Q}_3\delta_i(l) \Bigg] \tag{6.44}$$

where $\vartheta_i(k) = \delta_i(k+1) - \delta_i(k)$. The following inequity is given by obtaining the functional difference $\Delta V(k) = V(k+1) - V(k)$ along the trajectory of multiagent dynamics (6.9):

$$\Delta V(k) + \delta^T(k)\delta(k) - \gamma^2 e^T(k)e(k) \le \int_{\sigma=0}^{1} \Bigg[\zeta_1^{\mathrm{T}}(k)\Xi_1^{\mathrm{T}}(I\otimes\mathcal{P})\Xi_1\zeta_1(k) - \delta^{\mathrm{T}}(k)(I\otimes\mathcal{P})\delta(k)$$
$$- \sum_{l=k-d(k)}^{k-1} \vartheta^{\mathrm{T}}(l)(I\otimes\mathcal{Z}_1)\vartheta(l) + \zeta_1^{\mathrm{T}}(k)\Xi_2^{\mathrm{T}}\left(\bar{\tau}(I\otimes\mathcal{Z}_1) + \tau^\star(I\otimes\mathcal{Z}_2)\right)\Xi_2\zeta_1(k)$$
$$- \sum_{l=k-d(k)}^{k-\underline{\tau}-1} \vartheta^{\mathrm{T}}(l)(I\otimes\mathcal{Z}_2)\vartheta(l) - \sum_{l=k-\bar{\tau}}^{k-d(k)-1} \vartheta^{\mathrm{T}}(l)\left(I\otimes(\mathcal{Z}_1+\mathcal{Z}_2)\right)\vartheta(l)$$
$$+ \delta^{\mathrm{T}}(k)\left((I\otimes(\mathcal{Q}_1+\mathcal{Q}_2))\,\delta(k) - \delta^{\mathrm{T}}\left(k-\underline{\tau}\right)(I\otimes\mathcal{Q}_1)\delta\left(k-\underline{\tau}\right)\right.$$
$$- \delta^{\mathrm{T}}\left(k-\bar{\tau}\right)(I\otimes\mathcal{Q}_2)\delta\left(k-\bar{\tau}\right) + (\tau^\star+1)\,\delta^{\mathrm{T}}(k)(I\otimes\mathcal{Q}_3)\delta(k$$
$$\left. - \delta^{\mathrm{T}}(k-d(k))(I\otimes\mathcal{Q}_3)\delta(k-d(k)) + \delta^T(k)\delta(k) - \gamma^2 e^T(k)e(k)\right] d\sigma \tag{6.45}$$

where

$$\Xi_1(s) = \Omega_1 + \begin{bmatrix} \Delta\Pi(s) & \Delta\Pi_d(s) & 0 & 0 & 0 \end{bmatrix} \tag{6.46}$$

$$\Xi_2(s) = \Omega_1 + \begin{bmatrix} \Delta\Pi(s) & \Delta\Pi_d(s) & 0 & 0 & 0 \end{bmatrix} \tag{6.47}$$

Consider the following augmented vector:

$$\zeta_1(k) = \left[\delta^{\mathrm{T}}(k), \delta^{\mathrm{T}}(k-d(k)), \delta^{\mathrm{T}}\left(k-\underline{\tau}\right), \delta^{\mathrm{T}}\left(k-\bar{\tau}\right), e^{T}(k)\right]^{\mathrm{T}}$$

then the following equations hold

$$0 = \bar{\tau}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_1\zeta_1(k) - \textstyle\sum_{l=k-d(k)}^{k-1}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_1\zeta_1(k) - \sum_{l=k-\bar{\tau}}^{k-d(k)-1}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_1\zeta_1(k) \quad (6.48)$$

$$0 = \bar{\tau}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_2\zeta_1(k) - \textstyle\sum_{l=k-d(k)}^{k-1}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_2\zeta_1(k) - \sum_{l=k-\bar{\tau}}^{k-d(k)-1}\zeta_1^{\mathrm{T}}(k)\bar{\mathcal{X}}_2\zeta_1(k) \quad (6.49)$$

$$0 = 2\zeta_1^{\mathrm{T}}(k)R\mathcal{N}[\delta(k) - \delta(k-d(k)) - \textstyle\sum_{l=k-d(k)}^{k-1}\vartheta(l)] \quad (6.50)$$

$$0 = 2\zeta_1^{\mathrm{T}}(k)RM[\delta(k-d(k)) - \delta\left(k-\bar{\tau}\right) - \textstyle\sum_{l=k-\bar{\tau}}^{k-d(k)-1}\vartheta(l)] \quad (6.51)$$

$$0 = 2\zeta_1^{\mathrm{T}}(k)RS[\delta\left(k-\underline{\tau}\right) - \delta(k-d(k)) - \textstyle\sum_{l=k-d(k)}^{k-\underline{\tau}-1}\vartheta(l)] \quad (6.52)$$

where $\bar{\mathcal{X}}^i = R\mathcal{X}_i R^T$, and $R = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix}$. Exploiting the equations of slack variables appears in (6.48)-(6.52), then inequality (6.45) can be expressed as follows:

$$\begin{aligned}\Delta V(k) + \delta^T(k)\delta(k) - \gamma^2 e^T(k)e(k) \leq \int_{\sigma=0}^{1} \Bigg[& \zeta_1^{\mathrm{T}}(k)\left[\Phi\right. \\ & \left. + \Xi_1^{\mathrm{T}}(I\otimes\mathcal{P})\Xi_1 + \Xi_2^{\mathrm{T}}\left(\bar{\tau}(I\otimes\mathcal{Z}_1) + \tau^{\star}(I\otimes\mathcal{Z}_2)\right)\Xi_2\right]\zeta_1(k) \\ & - \sum_{l=k-d(k)}^{k-1}\zeta_2^{\mathrm{T}}(k,l)\Psi_1\zeta_2(k,l) - \sum_{l=k-d(k)}^{k-\underline{\tau}-1}\zeta_2^{\mathrm{T}}(k,l)\Psi_2\zeta_2(k,l) \\ & - \sum_{l=k-\bar{\tau}}^{k-d(k)-1}\zeta_2^{\mathrm{T}}(k,l)\Psi_3\zeta_2(k,l)\Bigg] d\sigma\end{aligned}$$

where $\zeta_2(k,l) = \left[\zeta_2^{\mathrm{T}}(k), \vartheta^{\mathrm{T}}(l)\right]^{\mathrm{T}}$. Since $\Psi_i \geqslant 0, i = 1,2,3$ guaranteed by the inequalities given in (6.41)-(6.43), then $\Delta V(k) + \delta^T(k)\delta(k) - \gamma^2 e^T(k)e(k)$ is negative if $W = \Phi + \Xi_1^{\mathrm{T}}\mathcal{P}\Xi_1 + \Xi_2^{\mathrm{T}}\left(\bar{\tau}\mathcal{Z}_1 + \tau^{\star}\mathcal{Z}_2\right)\Xi_2$ is guaranteed to be negative. Using the Kronecker product properties, the following equations holds: $\Delta\Pi(I\otimes\mathcal{P}) = (I\otimes E)(I\otimes F)(I\otimes N_1\mathcal{P})$ and $\Delta\Pi_d(I\otimes\mathcal{P}) = (I\otimes E)(I\otimes F)(I\otimes N_2\mathcal{P})$. This allows us to

decompose W as follows:

$$W = \begin{bmatrix} \Phi & \Omega_1^{\mathrm{T}}(I \otimes \mathcal{P}) & \sqrt{\bar{\tau}}\Omega_2^{\mathrm{T}}(I \otimes \mathcal{Z}_1) & \sqrt{\tau^\star}\Omega_2^{\mathrm{T}}(I \otimes \mathcal{Z}_2) \\ \bullet & -(I \otimes \mathcal{P}) & 0 & 0 \\ \bullet & \bullet & -(I \otimes \mathcal{Z}_1) & 0 \\ \bullet & \bullet & \bullet & -(I \otimes \mathcal{Z}_2) \end{bmatrix}$$

$$+ \begin{bmatrix} \bar{E} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (I \otimes F(\sigma)) & 0 \\ 0 & (I \otimes F(\sigma)) \end{bmatrix} \begin{bmatrix} \bar{N} & 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \bar{N}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (I \otimes F^T(\sigma)) & 0 \\ 0 & (I \otimes F^T(\sigma)) \end{bmatrix} \begin{bmatrix} \bar{E}^T & 0 & 0 & 0 \end{bmatrix} \tag{6.53}$$

Applying Lemma 6.1 and Schur complement to the last inequality gives the condition (6.40). The rest of the proof follows parallel details to Theorem 6.4. ∎

6.4 Simulation Results

6.4.1 Spacecraft Flying in Low Earth Orbits

In this section, simulation results are presented to demonstrate the effectiveness of the proposed discrete-time consensus protocol for multi-agent systems under the effect of nonlinear perturbation and time delay. Consider the discretized version of a spacecraft flying in low Earth orbit [235] with nonlinear perturbation, where its dynamics is given

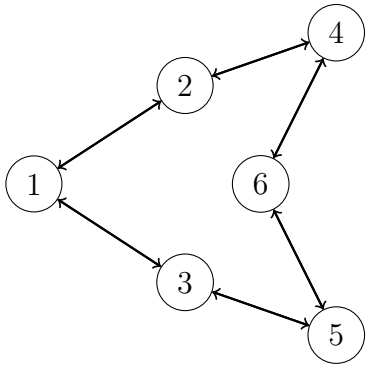

Figure 6.1: Connected topology of the multiagent system.

in terms of the discrete dynamical model (6.1) and its parameters are given as follows:

$$A = \begin{bmatrix} 1.1052 & 0 & 0 & 0.0111 & 0 & 0 \\ 0 & 1.1052 & 0 & 0 & 0.0111 & 0 \\ 0 & 0 & 1.1052 & 0 & 0 & 0.0111 \\ 0 & 0 & 0 & 1.1052 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.1052 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.1052 \end{bmatrix}$$

$$A_d = \begin{bmatrix} -0.03 & -0.09 & -0.47 & -0.19 & -0.07 & 0.3 \\ 0.02 & -0.26 & -0.3 & -0.2 & 0.08 & 0.24 \\ 0.03 & -0.08 & -0.4 & -0.15 & -0.01 & 0.24 \\ 0.08 & -0.11 & -0.3 & -0.3 & -0.1 & 0.2 \\ 0.06 & -0.08 & -0.3 & -0.1 & -0.2 & 0.26 \\ 0.06 & -0.18 & -0.4 & -0.2 & 0 & 0.2 \end{bmatrix}$$

$$f(x_i(k), x_i(k-\tau(k)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ sin(c^T x_i(k) + \frac{1}{2} c^T x_i(k-\tau)) \\ 0 \\ \frac{1}{2} tan^{-1}(c^T x_i(k-\tau)) + sin(c^T x_i(k)) \end{bmatrix}$$

$$c = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 0.0005 & 0 & 0 \\ 0 & 0.0005 & 0 \\ 0 & 0 & 0.0005 \\ 0.1052 & 0 & 0 \\ 0 & 0.1052 & 0 \\ 0 & 0 & 0.1052 \end{bmatrix}$$

For the nonlinear perturbation, the matrices E, N_1 and N_2 are obtained by simple calculations as follows:

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and $N_1 = 2N_2 = \text{diag}\{[1,\ 1,\ 0,\ 0,\ 0,\ 1]\}$. Suppose that there are six agents with an undirected communication graph G shown in Fig.6.1 where Laplacian matrix $\mathcal{L}$ is

given as follows

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

For such a Laplacian matrix, the eigenvalues are $\lambda_2 = \lambda_3 = 1$, $\lambda_4 = \lambda_5 = 3$, and $\lambda_6 = 4$, whereas the eigenvalues of $D^\star$ are $\lambda_2(D^\star) = \lambda_3(D^\star) = \lambda_4(D^\star) = 2$ and $\lambda_5(D^\star) = \lambda_6(D^\star) = 3$. Select $\epsilon_1 = 0.2$ and $\epsilon_2 = 0.25$ and solving the resulting LMI (6.16). A feasible event triggered control gain matrix is obtained as follows:

$$K = \begin{bmatrix} -1.3680 & 0.0007 & 0 & -2.0684 & -0.0002 & 0 \\ -0.0007 & -1.3680 & 0 & 0.0002 & -2.0684 & 0 \\ 0 & 0 & -1.3680 & 0 & 0 & -2.0684 \end{bmatrix}$$

The state trajectories of the six satellites are shown in Fig. 7.4 for the discrete-time multi-agent system (6.1) with the event-triggered consensus protocol (6.13). It can be seen that the consensus is achieved asymptotically using the proposed event triggered scheme. Moreover, the update instants are given in Fig.6.2, from which it can be easily concluded that the consensus protocol policy effectively reduces the transmissions as compared with the classical time-triggered schemes. The presented simulation has confirmed that the designed consensus protocol performs very well.

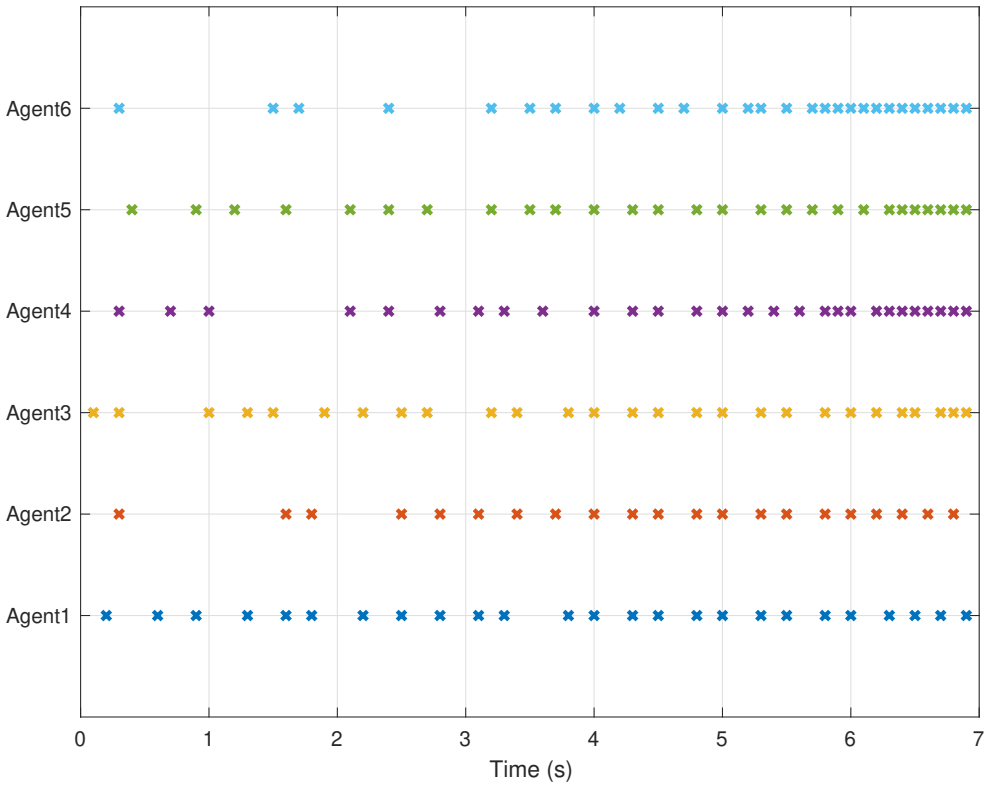

Figure 6.2: Update times of controllers.

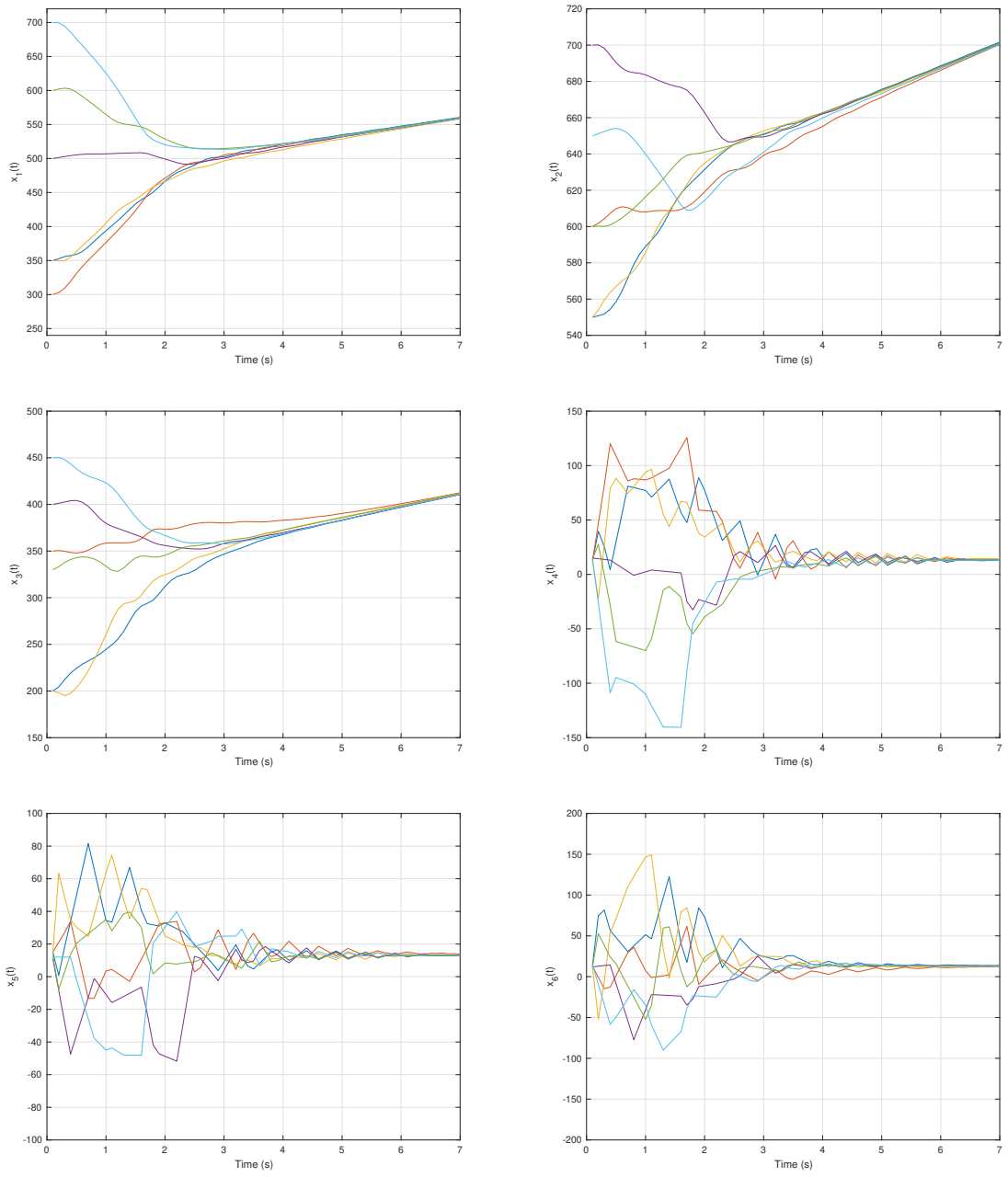

Figure 6.3: State trajectories of the six satellites.

CHAPTER 7

OUTPUT-SYNCHRONIZATION

This Chapter investigates output synchronization of heterogeneous time-delay discrete-time multiagent systems via output couplings. Firstly, sufficient conditions are provided to guarantee dissipativity for an individual agent modeled by the descriptor dynamics. A distributed consensus protocol based on the outputs of the neighboring agents is proposed to show that the individual dissipativity leads to output synchronization of the overall multiagent system. The proposed synchronization scheme exhibits a strong robustness facing norm-bounded uncertainties. Moreover, the counterpart of the result of the cooperation based on dissipativity for the multiagent system with event triggering mechanism is also investigated. The proposed triggering mechanism and the consensus protocol solve the synchronization for all admissible uncertainties and time varying delay. Finally, a numerical heterogeneous model is carried out to illustrate the effectiveness of the proposed strategies.

7.1 Introduction

Cooperation among agents without centralized controller while each agent can only access to local/neighboring information is one of the main challenges in multiagent systems. This problem is one of the most fundamental issues of cooperation control, whose main duty is to implement a distributed protocol via local and neighboring information so that the states or the outputs of agents can reach a prescribed agreement [72]. Analytical and numerical methodologies to solve the synchronization problem have been reported in literature, which is evident from several monographs ([73], [74]), and numerous articles in different settings, such as consensus ([75]) and the references therein. However, most of the attention had been paid to cooperation among homogeneous agents rather than heterogeneous systems. Whereas synchronization methodologies for homogeneous multiagent systems have been thoroughly investigated, synchronization of non-identical agents still remains a challenging problem. Interesting results have been addressed for the multiagent synchronization problems for heterogeneous dynamics, including descriptor system [135], [136].

Consensus protocols for linear and nonlinear multiagent systems is investigated with the help of Lyapunov method, algebraic graph theory and matrix theory in [74]. Moreover, various consensus problem of second-order multiagent systems is addressed where the consensus of velocity and position can be achieved [103]. Dissipativity as well as its special case passivity has been exploited to analyze the stability of multiagent systems. The finite time synchronization problem is investigated based on passivity approach by exploiting an adaptive state feedback with linear and nonlinear

couplings [104]. Sufficient delay dependent conditions are derived to realize cooperation of a class of nonlinear multiagent systems such that the trajectories converge to the convex hull with mixed $\mathcal{H}_\infty$ and passivity index [105]. Under switching topologies, consensus protocol with the help of the dissipativity is proposed to ensure that the agents can reach an agreement [106]. In [107], distributed suboptimal $\mathcal{H}_\infty$ consensus problem was established based on dissipativity of uncertain systems.

Most of the synchronization passivity-based methods have considered continuous time multiagent systems; however, implementation of cooperative discrete-time multiagent systems is needed. Discrete time output consensus for heterogeneous multiagent systems is addressed using internal model principle [145]. The internal model is exploited to convert the synchronization problem into an output stabilization problem. Two types of distributed protocols to solve the synchronization problem for discrete time dynamical system is proposed using observer based methodology [146]. The aforementioned results focus on continuously updated protocols such that every agent need to continuously employs its state and neighbors' states. On the other hand, to avoid this problem, researchers have investigated the triggering algorithms to reduce communication congestion over networks. Since continuous communication between agents usually leads to the waste of energy and resources, event- triggered mechanisms are usually used to reduce unnecessary transmissions in practical applications [147], [148], [149]. Event triggering mechanisms have been tailored to fit synchronization over networks [150], [151], and so far, a number of results have been widely developed to address this issue [152], [153], [154], [155]. Some theoretical approaches have been investigated to address event-triggering consensus control for time delay multiagent

systems [132], [133], [134]. Moreover, a decentralized event-triggered protocol based on dissipativity approach is proposed for systems with multiple sensors to sample agents outputs [156]. This chapter focuses on fully distributed discrete-time protocols using a disipassivity-based approach. In addition, unmodeled dynamics and imperfection of networks such as time delays must be taken into consideration. Motivated by the aforementioned discussions, this chapter concentrates on dissipativity-based synchronization problems for discrete time multiagent systems with delays.

In recent decade, a number of methodologies on guaranteed cost synchronization have been reported for time delayed multiagent systems. A distributed event-triggered guaranteed cost synchronization for uncertain multi-agents is achieved based on the Lyapunov theory [236]. However, the guaranteed cost synchronization of uncertain heterogeneous systems via event-triggered of multiagent systems with time delay is not fully investigated, which is the main motivation of this chapter. Comparing this chapter with some relevant results, the main contributions are:

1. The synchronization problem is investigated for heterogeneous systems that are not passive but satisfies a weaker dissipative condition. In this chapter, the considered dynamical system is not required to be passive nor stable. Sufficient conditions on the consensus are presented to obtain the control low gains.

2. The problem of guaranteed cost synchronization control for a class of uncertain discrete time descriptor multi-agent systems with time-delays is addressed. To the best of our knowledge, there is few research on this issue for such systems.

3. Due to economic costs and practical constraints, states of agents can not be

obtained directly. We assume that the output measurement but not the state measurement of the agents is available, where the static output coupling is used to achieve synchronization rather than observer based approach. Continuous communication between neighboring agents is avoided.

4. We first establish simple time triggered synchronization protocol. Then a distributed event triggered policy id presented, where the measurement of each agent will be transmitted only when its threshold is violated.

7.2 Problem Formulation and Preliminaries

Consider a group of N heterogeneous nth order agents represented by the following descriptor state space representation with uncertainties for each agent:

$$\begin{cases} E_i x_i(k+1) = (A_i + \Delta A_i) x_i(k) + (A_{di} + \Delta A_{di}) x_i(k - \tau(k)) + B_i u_i(k) \\ z_i(k) = C_i x(k) + C_{di} x_i(k - \tau(k)) \\ x_i(k) = \phi_{i0}(k), \quad k \in [-\bar{\tau}, 0] \end{cases} \tag{7.1}$$

where $x_i(k) \in \mathbb{R}^n$ is the agent i's state, $z_i(k) \in \mathbb{R}^p$ represents the agent's output, and $u_i(k)$ is the control law of the agent i which can only use the local information of neighbors' outputs. $\phi_0(k)$ is a compatible initial condition. The nominal constant matrices $E_i \in \mathbb{R}^{n\times n}$, $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{n\times p}$, $C_{id} \in \mathbb{R}^{n\times p}$ might be not identical for different agents. The real matrix $E_i \in \mathbb{R}^{n\times n}$ could be singular i.e. $\text{rank}(E_i) = r < n$, which constitutes a generalization of state space systems. $\tau(k)$ is a positive integer that represents the time delay and assumed to be

time varying for all agents such that

$$\underline{\tau} \leq \tau(k) \leq \bar{\tau}$$

where $\underline{\tau}$ and $\bar{\tau}$ are positive integers representing the upper and lower bounds of the time delay. ΔA_i, ΔA_{di}, are real unknown time varying matrix functions representing the parametric uncertainties which are the result of unmodeled dynamics or/and model linearizion. Without loss of generality, the following assumption is introduced for technical convenience

$$\Delta A_i = M_i F_i(k) N_i \quad \Delta A_{di} = M_i F_i(k) N_{di}$$

where $F_i(k)$ represents unknown matrix functions with where its elements are Lebesgue-measurable functions satisfying $F_i^T(k)F_i(k) \leq I$. Moreover, the matrices M_i, N_i and N_{di} are real constant with appropriate dimensions which represent how the uncertain parameters in $F_i(k)$ enter to the nominal matrices. The following assumptions play a central role to get feasible solutions to the developed conditions on consensus.

Assumption 9 *The pair (A, B) is assumed to be controllable.*

Remark 12 *Note that the controllability is a mild assumption. In addition, no conditions have been imposed on the eigenvalues of the state matrix A.*

Assumption 10 *The graph G is assumed to be undirected and connected.*

Definition 7.1 *The heterogeneous multiagent dynamical system is considered to achieve output synchronization if for every initial state $x_i(k_0) \in \mathbb{R}^n$, the following holds*

$$\lim_{k\to\infty} \|z_i(k) - z_j(k)\| = 0$$

for all $i, j \in \mathcal{V}$.

Firstly, we use time triggered protocol and each agent uses its own state to guarantee the dissipativity with respect to the new distributed control law. The following control law is considered:

$$u_i(k) = K_i x_i(k) + \tilde{u}_i(k) \tag{7.2}$$

A constant gain matrix K_i should be designed such that each agent becomes dissipative with respect the input $\tilde{u}_i(k)$. The control law $\tilde{u}_i(k)$ will be designed based on local measurements of neighboring agents only, i.e. the cooperation approach is distributed rather than centralized. Then the discrete time consensus protocol (7.3) for the heterogeneous system (7.1) will be analyzed for both time triggered and event triggered mechanisms based on dissipativity analysis. Consider the following consensus protocol:

$$\tilde{u}_i(k) = -c_i \sum_{j \in \mathcal{N}_i} (z_i(k) - z_j(k)) \tag{7.3}$$

A general event-triggered control framework for each agent can be illustrated in Fig. 7.1. In classical consensus protocol settings, it is assumed that agents can access to its continuous measurements and neighboring information. Such an assumption requires enormous computing power and an ideal communication settings. This makes the discrete time protocols more realistic and inevitable. Note that the discrete time protocol (7.3) must be updated at each time instant $k = 1, 2, ...$ However, this kind of consensus protocols might lead to a congestion in the communication network particularly due to the performance requirements. Based on the previous results, we focus on developing an event triggered scheme such that the multiagent system (7.8)

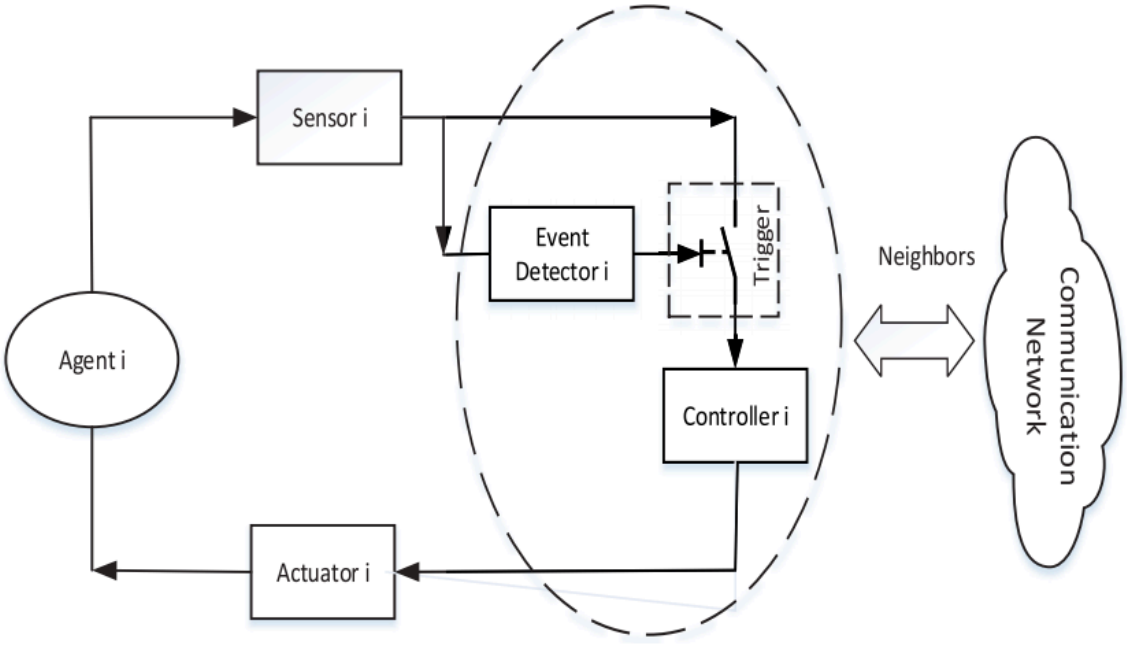

Figure 7.1: Schematic diagram of event-triggered control for agent i .

is synchronized. The distributed event triggering policy for agent i is designed as follows:

$$k_{l+1}^i = \inf\left\{k : k_l^i \middle| \left\|e_i(k)\right\| > \gamma_i \left\| \sum_{j\in\mathcal{N}_i} z_i(k_l^i) - z_j(k_{l'}^j) \right\| \right\} \tag{7.4}$$

where $\{k_{l+1}^i\}$ represents the event-triggered time sequence; $\{k_l^i\}$ and$\{k_{l+1}^i\}$ are the adjacent sampling instants, γ_i is a positive scalar to be designed, and $l' \triangleq \arg\min_{l\in\mathbb{N}:k\geq t_l^j} \left\{k - k_l^j\right\}$. Thus for each $k \in [k_l^i, k_{l+1}^i)$, $k_{l'}^j$ is the last broadcasting time of agent j. The event -triggering error between the current and last updated measurements for agent i can be represented as:

$$e_i(k) = z_i(k_l^i) - z_i(k)$$

where $z_i(k_l^i)$ is the system output at time k_l^i. The event triggered protocol for the

descriptor system (7.1) is governed by:

$$\tilde{u}_i(k) = -c_i \sum_{j \in \mathcal{N}_i} (z_i(k_l^i) - z_j(k_{l'}^j)), \quad k \in [k_l^i, k_{l+1}^i) \tag{7.5}$$

The graph is assumed to be connected in order to allow agents cooperate over the network. If the communication topology is not connected, the problem becomes unsolvable. Before proceeding, the following definitions and lemmas are introduced to be used in deriving our main results.

Definition 7.2 *([237]) The following are some essential properties of descriptor systems:*

1. *Pair (E_i, A_i) is said to be regular if the determinant of the matrix $(zE_i - A_i)$ is not identically zero.*

2. *If the pair (E_i, A_i) is regular and deg $det(zE_i - A_i)$ = rank(E_i), then it is said to be causal.*

3. *For given positive scalars $\underline{\tau}$ and$\bar{\tau}$, discrete-time descriptor system (7.1) is said to be regular and causal for any time delay $\tau(k)$ satisfying $\underline{\tau} \leq \tau(k) \leq \bar{\tau}$, if the pair (E_i, A_i) is regular and causal. Finally, the dynamical system (7.1) is admissible if it is causal, regular and stable.*

Definition 7.3 *The dynamical system of an agent i is called $\mathcal{QSR}$ dissipative if for any integer N, there exist symmetric matrices $\mathcal{Q}_i, \mathcal{S}_i$, and $\mathcal{R}_i$ such that the following inequality holds:*

$$\sum_{k=0}^{N} z_i^T(k)\mathcal{Q}_i z_i(k) + 2z_i^T(k)\mathcal{S}_i u_i(k) + u_i^T(k)\mathcal{R}_i u_i(k) \geq 0$$

Remark 13 *The notion of passivity index* $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ *comprises the* $\mathcal{H}_\infty$*-performance, strict passivity, and shortage of passivity as special cases by selecting different values of the index parameters. Obviously, if* $\mathcal{Q}_i = -I$*,* $\mathcal{R}_i = \gamma^2 I$*, and* $\mathcal{S}_i = 0$*, then dissipative inequality reduces to an* $\mathcal{H}_\infty$*-performance index. If* $\mathcal{Q}_i = 0$*,* $\mathcal{R}_i < 0$*, and* $\mathcal{S}_i = I$*, then the system is called input excess passive. Moreover, If* $\mathcal{Q}_i = 0$*,* $\mathcal{R}_i > 0$*, and* $\mathcal{S}_i = I$*, then the system is called input shortage passive. In this chapter, we consider the index with a trade off between* $\mathcal{H}_\infty$ *performance and the shortage passivity index.*

Lemma 7.1 *[238] Let* $x(k) \in \mathbf{R}^n$ *be a vector function. Then, for any positive definite matrix* $M > 0$*, and integers* $m > n$ *the following inequality holds:*

$$-(m-n+1)\sum_{r=n}^{m} x^T(r)Mx(r) \leq -\sum_{r=n}^{m} x^T(r)M\sum_{r=n}^{m} x(r)$$

Lemma 7.2 *Let* N *be the number of agents and a negative scalar* $\beta < 0$*, then the following inequality holds:*

$$\beta\sum_{i=1}^{N}\|z_i(k)\|^2 \leq \beta\frac{1}{4N}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}^{N}\left\|z_i(k)-z_j(k)\right\|^2$$

Proof. It can be easily shown that the sum can be rewritten in the following form:

$$\sum_{i=1}^{N}\|z_i(k)\|^2 = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\|z_i(k)\|^2 = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\|z_j(k)\|^2 \tag{7.6}$$

Based on the parallelogram law in inner product spaces and the representation in (7.6), one has

$$\begin{aligned}\sum_{i=1}^{N}\|z_i(k)\|^2 &= \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\|z_i(k)\|^2+\|z_j(k)\|^2\right]\\ &= \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\frac{\|z_i(k)+z_j(k)\|^2}{2}+\frac{\|z_i(k)-z_j(k)\|^2}{2}\right]\\ &\geq \frac{1}{4N}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\|z_i(k)-z_j(k)\right\|^2 \end{aligned}\tag{7.7}$$

Since the number of agents connected to the $i-$th agent is less than or equal to N and the right side has positve terms, we come up with

$$\sum_{i=1}^{N} \|z_i(k)\|^2 \geq \frac{1}{4N} \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i}^{N} \left\| z_i(k) - z_j(k) \right\|^2$$

multiplying both sides by the negative scalar β completes the proof. ∎

Fact 1 *Given any constant matrices M, N and Q with appropriat dimensions, where Q a is symmetric matrix, then the followng holds*

$$Q + MF(k)N + N^T F^T(k) M^T < 0$$

for every matrix function $F(k)$ satisfying $\|F(k)\| \leq 1$, if and only if there exists a positive scalar ϵ such that

$$Q + \epsilon M M^T + \frac{1}{\epsilon} N^T N < 0$$

The following inequality will be used in the sequel:

$$\begin{aligned} z_i^T(k)\tilde{u}_i(k) &= -\frac{1}{2\delta_i}(\tilde{u}_i(k) - \delta_i z_i(k))^T(\tilde{u}_i(k) - \delta_i z_i(k)) \\ &+ \frac{1}{2\delta_i}\tilde{u}_i^T(k)\tilde{u}_i(k) + \frac{\delta_i}{2} z_i^T(k) z_i(k) \\ &\leq \frac{1}{2\delta_i}\tilde{u}_i^T(k)\tilde{u}_i(k) + \frac{\delta_i}{2} z_i^T(k) z_i(k) \end{aligned} \tag{7.8}$$

7.3 Synchronization Results: Dissipative Approach

In this section, analysis and synthesis of the proposed consensus protocol are addressed for the descriptor multiagent system (7.1). In the centralized consensus problem, all agents should be aware of the global information of the multiagent system to achieve the synchronization. We investigate the distributed counterpart. In

particular, every agent now execute its control law based on the measurements received from its neighboring agents only.

7.3.1 Sampled-Time Synchronization

Firstly, we introduce the following result regarding the time -triggered consensus protocol (7.3).

Theorem 7.1 *Consider the multiagent system (7.1) with the time -triggered consensus protocol (7.3). For given positive integers* $\bar{\tau}$ *and* $\underline{\tau}$*, and positive scalars* c_i*, the agents reach consensus robustly and every agent is dissipative with* $(\beta, 1, \alpha_i)$*-index, if there exist positive scalars* ϵ_i*,* δ_i *and matrices* $\mathcal{P}_i > 0$*,* $\mathcal{Q}_1^i > 0$*,* $\mathcal{Q}_2^i > 0$*,* $\mathcal{Q}_3^i > 0$*,* $\mathcal{Z}_1^i > 0$*,* $\mathcal{Z}_2^i > 0$*,* $\mathcal{Z}_3^i > 0$*,* $\mathcal{R}_i$*,* L_j^i*,* $i = 1, ..., N$*,* $j = 1, 2, ..., 6$ *such that the following inequalities hold:*

$$\Theta = \begin{bmatrix} \Gamma^i & U_1^i & \epsilon_i U_2^i \\ \bullet & -\epsilon_i I & 0 \\ \bullet & \bullet & -\epsilon_i I \end{bmatrix} < 0 \tag{7.9}$$

$$\begin{bmatrix} \mathcal{Z}_3^i & \mathcal{R}_i \\ \mathcal{R}_i & \mathcal{Z}_3^i \end{bmatrix} \geq 0 \tag{7.10}$$

$$(\alpha_i + \frac{1}{2\delta_i}) c_i^2 d_i + (\beta + \frac{\delta_i}{2}) \frac{1}{4N} < 0 \tag{7.11}$$

where

$$\Gamma^i = \begin{bmatrix} \Gamma^i_{11} & \Gamma^i_{12} & \Gamma^i_{13} & \Gamma_{14} & \Gamma^i_{15} & \Gamma^i_{16} \\ \bullet & \Gamma^i_{22} & \Gamma^i_{23} & E_i^T R_i E_i & -L^i_2 & L^i_2 B_i \\ \bullet & \bullet & \Gamma^i_{33} & \Gamma^i_{34} & \Gamma^i_{35} & \Gamma^i_{36} \\ \bullet & \bullet & \bullet & \Gamma^i_{44} & -L^i_4 & L^i_4 B_i \\ \bullet & \bullet & \bullet & \bullet & \Gamma^i_{55} & \Gamma^i_{56} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \Gamma^i_{66} \end{bmatrix} \tag{7.12}$$

$$\Gamma_{11}^i = L_1^i(A_i - E_i) + (A_i - E_i)^T L_1^{iT} + (\tau^\star + 1)\mathcal{Q}_3^i$$

$$\mathcal{Q}_1^i + \mathcal{Q}_2^i - E_i^T \mathcal{Z}_1^i E_i - E_i^T \mathcal{Z}_2^i E_i - \beta C_i^T C_i$$

$$\Gamma_{12}^i = E_i^T \mathcal{Z}_2^i E + (A_i - E_i)^T L_2^{iT}$$

$$\Gamma_{22}^i = \mathcal{Q}_1^i - E_i^T \mathcal{Z}_2 E_i - E_i^T \mathcal{Z}_3 E_i$$

$$\Gamma_{13}^i = L_1^i A_{di} + (A_i - E_i)^T L_3^{iT} - \beta C_i^T C_{di}$$

$$\Gamma_{23}^i = -E_i^T R_i^T E_i + E_i^T \mathcal{Z}_3^{iT} E_i + L_2^i A_{di}$$

$$\Gamma_{33}^i = -\mathcal{Q}_3^i + L_3^i A_{di} + A_{di}^T L_3^{iT} - 2E_i^T \mathcal{Z}_3^i E_i$$

$$+ E_i^T R_i E_i + E_i^T R_i^T E_i - \beta C_{di}^T C_{di}$$

$$\Gamma_{14}^i = E_i^T \mathcal{Z}_1^i E_i + (A_i - E_i)^T L_4^{iT}$$

$$\Gamma_{34}^i = -E_i^T R_i E_i + E_i^T \mathcal{Z}_3^i E_i + A_{di}^T L_4^{iT}$$

$$\Gamma_{44}^i = -\mathcal{Q}_2^i - E_i^T \mathcal{Z}_1^i E_i - E_i^T \mathcal{Z}_3^i E_i,\ \Gamma_{35}^i = L_3^i + A_{di}^T L_5^{iT}$$

$$\Gamma_{15}^i = E_i^T \mathcal{P}_i - L_1^{iT} + (A - E)^T L_5^{iT} - L_2^i$$

$$\Gamma_{55}^i = \bar{\tau}^2 \mathcal{Z}_1^i + \underline{\tau}^2 \mathcal{Z}_2^i + \tau^\star \mathcal{Z}_3^i - L_5^i - L_5^{iT} + \mathcal{P}_i$$

$$\Gamma_{16}^i = -C_i^T + L_1^i B_i + (A_i - E_i)^T L_6^i$$

$$\Gamma_{36}^i = -C_{di}^T + A_{di}^T L_6^i + L_3^i B_i$$

$$\Gamma_{56}^i = L_5^i B_i + L_6^i,\ \Gamma_{66}^i = -\alpha_i I + L_6^{iT} B_i + B_i^T L_6^i$$

$$U_1 = \begin{bmatrix} M_i^T L_1^{iT} & M_i^T L_2^{iT} & \dots & M_i^T L_6^{iT} \end{bmatrix}^T$$

$$U_2 = \begin{bmatrix} N_i & 0 & N_{di} & 0 & 0 & 0 \end{bmatrix}$$

and $\tau^\star = \bar{\tau} - \underline{\tau}$.

Proof. The detailed proof is provided in Appendix 7.5.1. ∎

Remark 14 *The coupling strength c_i can be designed for each agent independently of the other agents couplings, i.e. the coupling strength only depends on its own dissipativity parameters and the number of other agents as well. Moreover, the protocol can be fully implemented in a distributed manner in the sense that an agent does not require global information by neighbors to execute the consensus policy. Since β is negative, inequality (7.11) admits a solution of c_i if the linear matrix inequalities in Theorem 7.1 are feasible. The agents reach consensus robustly and every agent is dissipative with $(\beta, 1, \alpha_i)$-index.*

7.3.2 Event-Triggered Protocol

It is worth noting that the triggering rule (7.4) is verified by each agent only based on its own measurement and its neighboring agents' information, which compromises only the relative output measurement of agent 's neighbors. The following theorem investigates event-triggered protocol (7.5) under the limited communication constraint:

Theorem 7.2 *Given positive integers $\bar{\tau}$ and $\underline{\tau}$, and positive scalars c_i. If there exist positive scalars ϵ_i and matrices $\mathcal{P}_i > 0$, $\mathcal{Q}_1^i > 0$, $\mathcal{Q}_2^i > 0$, $\mathcal{Q}_3^i > 0$, $\mathcal{Z}_1^i > 0$, $\mathcal{Z}_2^i > 0$, $\mathcal{Z}_3^i > 0$, $\mathcal{R}_i$, $L_j^i > 0$, $i = 1, ..., N$, $j = 1, 2, ..., 6$, then the multiagent system (7.8) reaches consensus via the event triggered protocol (7.5) with the triggering condition (7.4) if the inequalities (7.9), (7.10) and*

$$(\alpha_i + \frac{1}{2\delta_i} + \frac{1}{2r} + \gamma_i \frac{1+s+rs/2}{sc_i^2}) c_i^2 d_i + (\beta + s + \frac{\delta_i}{2}) \frac{1}{4N} < 0 \tag{7.13}$$

hold for positive scalars γ_i, δ_i, s, and r.

Proof. The detailed proof is provided in Appendix 7.5.2. ∎

Remark 15 *The triggering mechanism does not require global knowledge by neighbors. It solely requires the error at the last triggered instant rather than the error of the neighboring agents at the execution time. This indicates that the implementation of the proposed event-triggered protocol more realizable to be implemented over networks. In particular, no continuous communication is required at all. In addition, the smallest interevent time is bounded below by the sampling-time. It is not required to prove the nonexistence of Zeno behavior for discrete systems.*

In multiagent applications, it is important to check whether the descriptor multiagent dynamics is solvable for synchronization-protocols and every initial value that is consistent with the protocol. The following proposition provides a sufficient condition to investigate the regularity of the overall multiagent system.

Proposition 7.1 *Given full column matrices $\Omega_i \in \mathbb{R}^{n\times n-r_i}$ satisfying $E_i^T\Omega_i = 0$ for all $i = 1, 2, ..., N$. If the conditions of Theorem 7.1 are satisfied with Γ_{15}^i is replaced by*

$$\Gamma_{15}^i = S_i\Omega_i^T + E_i^T\mathcal{P}_i - L_1^{iT} + (A_i - E_i)^T L_5^{iT} - L_2^i \tag{7.14}$$

Then, the pair (E_i, A_i) is regular, causal for all $i = 1, 2, \ldots, N$, and the consensus is guaranteed.

Proof. The detailed proof is provided in Appendix 7.5.3. ∎

Based on the previous Theorems, we concentrate on investigating a methodology to synthesize controller gains such that the multiagent (7.1) is synchronized and dissipative. It should be noted that the dissipative gain K_i appearing in (7.2) is generally

not known in advance. This makes the matrix inequalities in Theorem 7.2 to be bi-linear inequalities and obtaining such gain could become a very hard task. Thus we consider the following theorem to obtain the dissipative gain and coupling strength simultaneously.

Theorem 7.3 *Given positive integers $\bar{\tau}$ and $\underline{\tau}$, and positive scalars c_i. If there exist positive scalars ϵ_i and matrices $\mathcal{P}_i > 0$, $\mathcal{Q}_1^i > 0$, $\mathcal{Q}_2^i > 0$, $\mathcal{Q}_3^i > 0$, $\mathcal{Z}_1^i > 0$, $\mathcal{Z}_2^i > 0$, $\mathcal{Z}_3^i > 0$, $\mathcal{R}_i$, $L_j^i > 0$, $i = 1, ..., N$, $j = 1, 2, ..., 6$, then the multiagent system (7.8) reaches consensus via the event triggered protocol (7.5) with the triggering condition (7.4) if the inequalities*

$$\begin{bmatrix} \Gamma^i & U_1^i & \epsilon_i U_2^i \\ \bullet & -\epsilon_i I & 0 \\ \bullet & \bullet & -\epsilon_i I \end{bmatrix} < 0 \tag{7.15}$$

$$\begin{bmatrix} \mathcal{Z}_3^i & \mathcal{R}_i \\ \mathcal{R}_i & \mathcal{Z}_3^i \end{bmatrix} \geq 0 \tag{7.16}$$

and

$$(\alpha_i + \frac{1}{2\delta_i} + \frac{1}{2r} + \gamma_i \frac{1+s+rs/2}{sc_i^2}) c_i^2 d_i + (\beta + s + \frac{\delta_i}{2}) \frac{1}{4N} < 0 \tag{7.17}$$

hold for positive scalars γ_i, δ_i, s, and r. where

$$\Gamma^i = \begin{bmatrix} \Gamma_{11}^i & \Gamma_{12}^i & \Gamma_{13}^i & \Gamma_{14} & \Gamma_{15}^i & \Gamma_{16}^i \\ \bullet & \Gamma_{22}^i & \Gamma_{23}^i & E_i^T R_i E_i & -L_2^i & L_2^i B_i \\ \bullet & \bullet & \Gamma_{33}^i & \Gamma_{34}^i & \Gamma_{35}^i & \Gamma_{36}^i \\ \bullet & \bullet & \bullet & \Gamma_{44}^i & -L_4^i & L_4^i B_i \\ \bullet & \bullet & \bullet & \bullet & \Gamma_{55}^i & \Gamma_{56}^i \\ \bullet & \bullet & \bullet & \bullet & \bullet & \Gamma_{66}^i \end{bmatrix} \tag{7.18}$$

$$U_1^i = \begin{bmatrix} M_i^1 & M_i^2 & \dots & M_i^6 \end{bmatrix}^T$$

$$U_2^i = \begin{bmatrix} N_i & 0 & N_{di} & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_{11}^i = A_i^1 + A_i^{1T} + (\tau^\star + 1)\mathcal{Q}_3^i + \mathcal{Q}_1^i + \mathcal{Q}_2^i - E_i^T \mathcal{Z}_1^i E_i$$

$$- E_i^T \mathcal{Z}_2^i E_i - \beta \tilde{C}_i^T \tilde{C}_i$$

$$\Gamma_{12}^i = E_i^T \mathcal{Z}_2^i E + A_i^2,\ \Gamma_{22}^i = \mathcal{Q}_1^i - E_i^T \mathcal{Z}_2 E_i - E_i^T \mathcal{Z}_3 E_i$$

$$\Gamma_{13}^i = A_{di}^1 + A_i^3 - \beta C_i^T C_{di},$$

$$\Gamma_{23}^i = -E_i^T R_i^T E_i + E_i^T \mathcal{Z}_3^{iT} E_i + A_{di}^2$$

$$\Gamma_{33}^i = -\mathcal{Q}_3^i + A_{di}^3 + A_{di}^{3T} - 2E_i^T \mathcal{Z}_3^i E_i + E_i^T R_i E_i$$

$$+ E_i^T R_i^T E_i - \beta C_{di}^T C_{di},\ \Gamma_{14}^i = E_i^T \mathcal{Z}_1^i E_i + A_i^{4T}$$

$$\Gamma_{34}^i = -E_i^T R_i E_i + E_i^T \mathcal{Z}_3^i E_i + A_{di}^{4T}$$

$$\Gamma_{44}^i = -\mathcal{Q}_2^i - E_i^T \mathcal{Z}_1^i E_i - E_i^T \mathcal{Z}_3^i E_i,\ \Gamma_{35}^i = L_3^i + A_{di}^{5T}$$

$$\Gamma_{15}^i = E_i^T \mathcal{P}_i - L_1^{iT} + A_i^{5T} - L_2^i$$

$$\Gamma_{55}^i = \bar{\tau}^2 \mathcal{Z}_1^i + \underline{\tau}^2 \mathcal{Z}_2^i + \tau^\star \mathcal{Z}_3^i - L_5^i - L_5^{iT} + \mathcal{P}_i$$

$$\Gamma_{16}^i = -\tilde{C}_i^T + B_i^1 + A_i^{6T},\ \Gamma_{36}^i = -C_{di}^T + A_{di}^{6T} + B_i^3$$

$$\Gamma_{56}^i = B_i^5 + L_6^{iT},\ \Gamma_{66}^i = -\alpha_i I + B_i^5 + B_i^{5T}$$

$$A_i^j = \begin{bmatrix} L_{j1}^i (A_i - E_i) + \lambda_j \mathbb{I} Y_i & L_{j1}^i B_i - \lambda_j \mathbb{I} F_i \\ L_{j2}^i (A_i - E_i) + \lambda_j Y_i & L_{j1}^i B_i - \lambda_j F_i \end{bmatrix},$$

$$A_{di}^j = \begin{bmatrix} L_{j1}^i A_{di} & 0 \\ L_{j2}^i A_{di} & 0 \end{bmatrix},$$

$$\mathbb{I} = \begin{bmatrix} I_m \\ 0_{(n-m)\times m} \end{bmatrix},\ B_i^j = \begin{bmatrix} L_{j1}^i B_i \\ L_{j2}^i B_i \end{bmatrix},\ M_i^j = \begin{bmatrix} L_{j1}^i M_i \\ L_{j2}^i M_i \end{bmatrix},$$

$i = 1, 2, \ldots, N$ $j = 1, 2, \ldots, 6$ *and* $\tau^\star = \bar{\tau} - \underline{\tau}$. *Furthermore,* $K_i = F_i^{-1} Y_i$.

Proof. The detailed proof is provided in Appendix 7.5.4. ∎

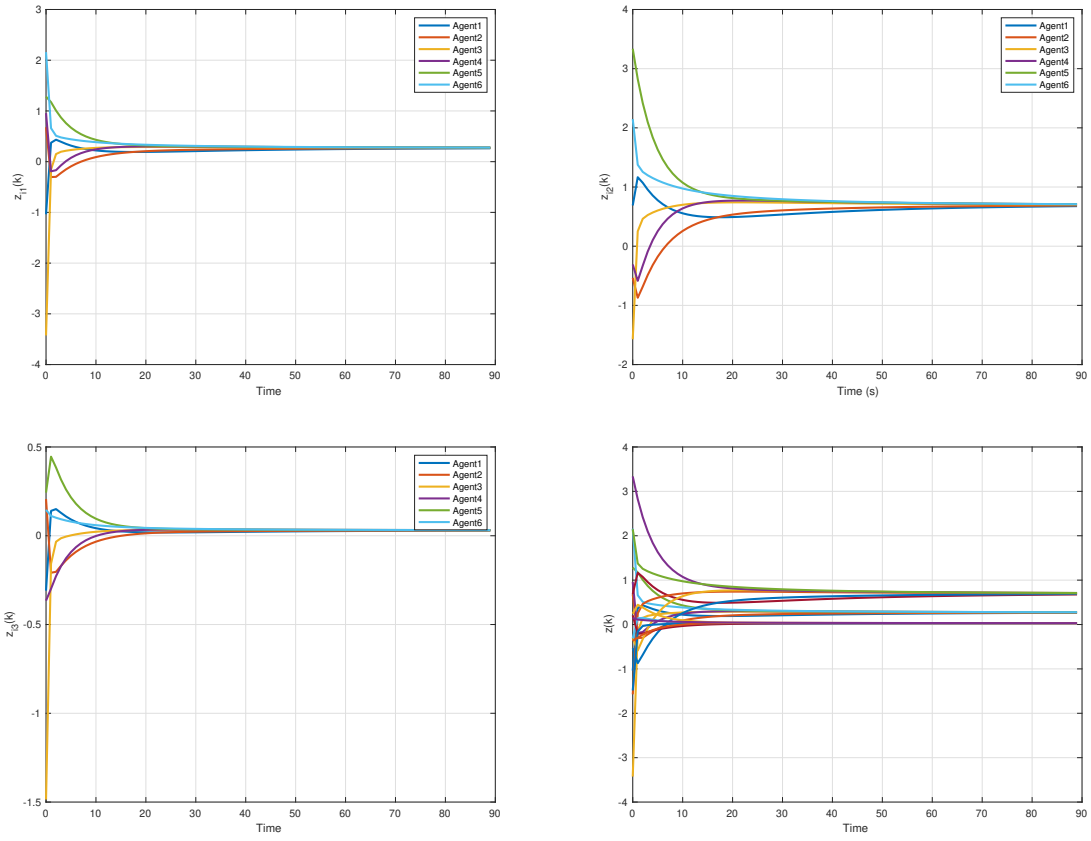

Figure 7.2: Outputs trajectories of the agents of time-triggered synchronization.

Remark 16 *Compared with most exiting results dealing with the consensus problem of multiagent dynamical systems [239],[209], the proposed results generalize the consensus problem to synchronization of agents outputs, which covers a state consensus as as a special case.*

Although previous Theorems are allocated to single output synchronization, i.e. $z_i(k) \in \mathbb{R}$, the proposed methods can be easily extended to multi-output synchro-

nization $z_i(k) \in \mathbb{R}^p$ by portioning the output matrix of each agent into a set of p rows $C_i^T = \begin{bmatrix} C_{i1}^T, ..., C_{ip}^T \end{bmatrix}$. In addition, denote c_{ij} to represent the coupling strength associated with the j-th row of the output matrix C_i of the i-th agent. Moreover, the consensus protocol can be expressed in the following form:

$$\tilde{u}_i(k) = -\bar{K}_i \sum_{j \in \mathcal{N}_i} (z_i(k_l^i) - z_j(k_{l'}^j)), \quad k \in [k_l^i, k_{l+1}^i) \tag{7.19}$$

where $\bar{K}_i = \begin{bmatrix} c_{i1} & \dots & c_{ip} \end{bmatrix}$, $i = 1, 2, ..., N$. Compared with the available results on the LMI-based distributed synchronization [240], [241], the proposed methodology does not require dynamic controllers. In addition, the presented conditions are flexible and less conservative due to the inserted slack variables L_j^i.

7.4 Simulation Results

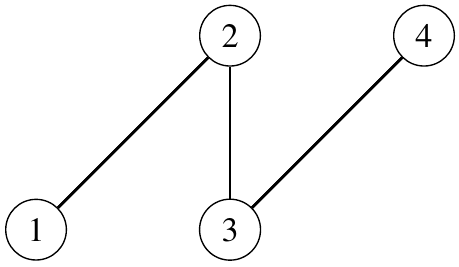

Figure 7.3: Connected topology of the multiagent system.

For the time-triggered protocol, we consider a network which consists of 6 homogeneous agents whose communication topology is shown in Figure 7.3. We assumed that only the edge output information is available and the state information can not

be obtained. The corresponding matrices and parameters are given as follows

$$A_i = \begin{bmatrix} 0.9307 & 0.9360 & 0.5102 \\ 0.4732 & 0.7668 & 0.0992 \\ 0.7223 & 0.6461 & 0.0883 \end{bmatrix}, B_i = \begin{bmatrix} 0.6228 \\ 0.4134 \\ 0.5094 \end{bmatrix}$$

$$A_{di} = \begin{bmatrix} 0.1 & 0.2 & 0 \\ 0 & -0.05 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_i = \begin{bmatrix} 0.385 & 0.07 & 0.07 \\ 0.07 & 0.455 & 0.035 \\ 0.07 & 0 & 0.315 \end{bmatrix}$$

$$E_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_{di} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The initial conditions for agents are given as

$$x_{01} = \begin{bmatrix} -3 \\ 2 \\ -0.608 \end{bmatrix}, x_{02} = \begin{bmatrix} 2 \\ -1 \\ 0.414 \end{bmatrix}, x_{03} = \begin{bmatrix} -8 \\ -2 \\ -1.249 \end{bmatrix},$$

$$x_{04} = \begin{bmatrix} 3 \\ -1 \\ 0.557 \end{bmatrix}, \quad x_{05} = \begin{bmatrix} 2 \\ 7 \\ -0.018 \end{bmatrix}, x_{06} = \begin{bmatrix} 5 \\ 4 \\ 0.64 \end{bmatrix},$$

The time delay is asumed to be bounded by the values $\bar{\tau} = 10$ and $\underline{\tau} = 2$ for all agents. Based on the communication topology, we obtain the following in-degree values for each agent: $d_1 = d_6 = 1$, $d_2 = d_4 = d_5 = 2$, and $d_3 = 4$. By using the LMI Toolbox in Matlab, it can be solved that exist coupling strengths c_{ij} satisfy the matrix inequalities of Theorem 7.1 $c_{11} = c_{61} = 0.0824, c_{21} = c_{41} = c_{51} = 0.0589, c_{21} = 0.0412$, $c_{12} = c_{62} = 0.4932, c_{22} = c_{42} = c_{52} = 0.03487, c_{32} = 0.2466,\ c_{13} = c_{63} = 0.05, c_{23} =$

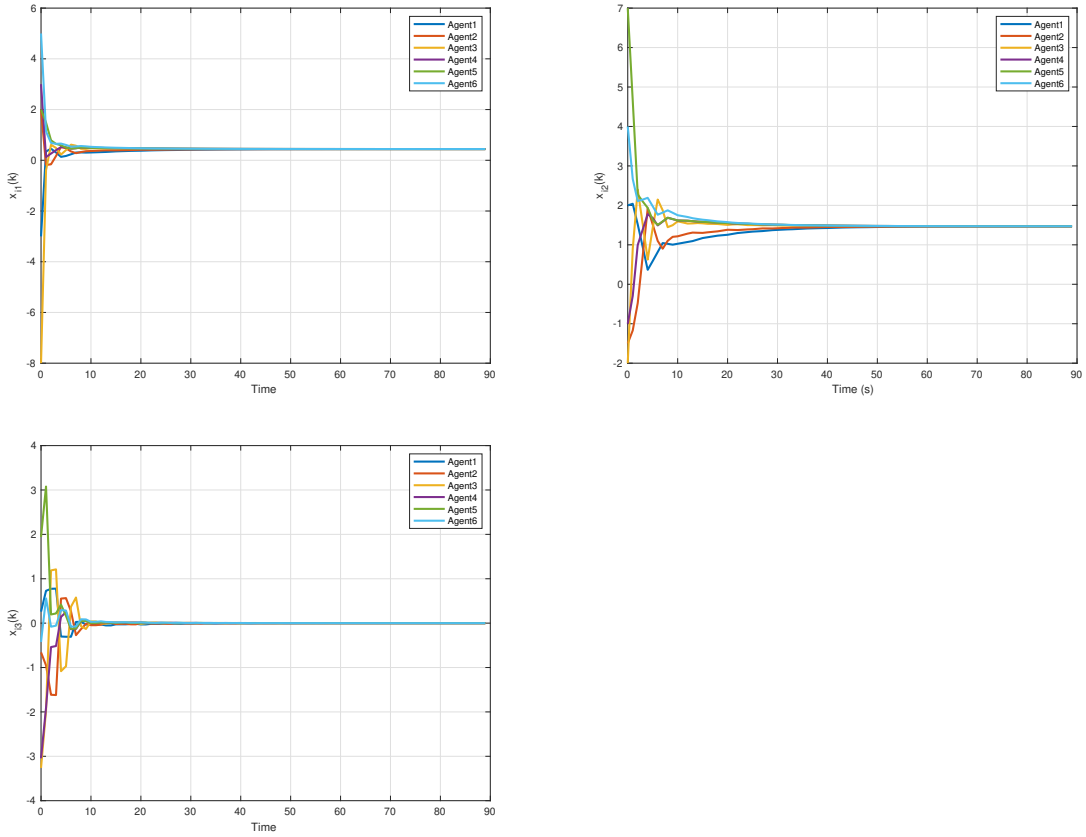

Figure 7.4: Outputs trajectories of the agents with event-triggered synchronization.

$c_{43} = c_{531} = 0.0353, c_{33} = 0.025$, and adopting the controllers:

$$\bar{K}_1 = \bar{K}_6 = \begin{bmatrix} 0.0824 & 0.4932 & 0.05 \end{bmatrix}$$

$$\bar{K}_3 = \begin{bmatrix} 0.0589 & 0.2466 & 0.025 \end{bmatrix}$$

$$\bar{K}_2 = \bar{K}_4 = \bar{K}_5 = \begin{bmatrix} 0.0412 & 0.3487 & 0.0353 \end{bmatrix}$$

Applying the time triggered protocol, the agents (7.1) achieves consensus asymptotically (see Figure 7.2). One can clearly decide that all agent's outputs are synchronized, which clearly demonstrate the effectiveness of the considered methodology.

For the event-triggered case, consider the same descriptor multiagent dynamical system, where the interaction topology among agents is given in Figure. 7.3. Applying

the same procedure, using the LMI Toolbox in Matlab, it can be solved that exist coupling strengths c_{ij} satisfy the matrix inequalities in Theorem 7.3, we get $\beta = -300.5945$ and $\alpha = 244.8650$. The negative sign of β means that there exists a solution to (7.17) by selecting appropriate coupling strengths c_{ij} such that

$$\bar{K}_1 = \bar{K}_6 = \begin{bmatrix} 0.1120 & 0.6708 & 0.0680 \end{bmatrix}$$

$$\bar{K}_3 = \begin{bmatrix} 0.4636 & 0.4249 & 0.0780 \end{bmatrix}$$

$$\bar{K}_2 = \bar{K}_4 = \bar{K}_5 = \begin{bmatrix} 0.052 & 0.374 & 0.0333 \end{bmatrix}$$

and the triggering parameters are given as $\gamma_1 = \gamma_6 = 0.6327319$, $\gamma_2 = \gamma_4 = \gamma_5 = 0.5328$, and $\gamma_3 = 0.4327307$. The synchronized event triggered outputs are shown in Figure 7.4. Figure 7.5 shows releasing instants of each agent. This demonstrates that event-triggered policy can reduce the frequency of agents broadcasts over networks and reduce CPU usage effectively. The matrix inequality are not feasible for time delay bound $\bar{\tau} = 10$ if the lower bound $\underline{\tau} = 0$. On one hand, this imply that the conditions treat interval delays appropriately even if the upper bound of the delay is large. On the other hand, this does not imply that the multiagent can not be synchronized because the condition are sufficient and not necessary. Time delay in multiagents leads to sever behavior which makes the synchronization more difficult. Since the proposed methodology is proposed for output synchronization, we observe that the state consensus appeared in literature can be obtained by replacing the output matrix C by identity. Moreover, the homogeneous multiagent system becomes a special case of the presented scheme.

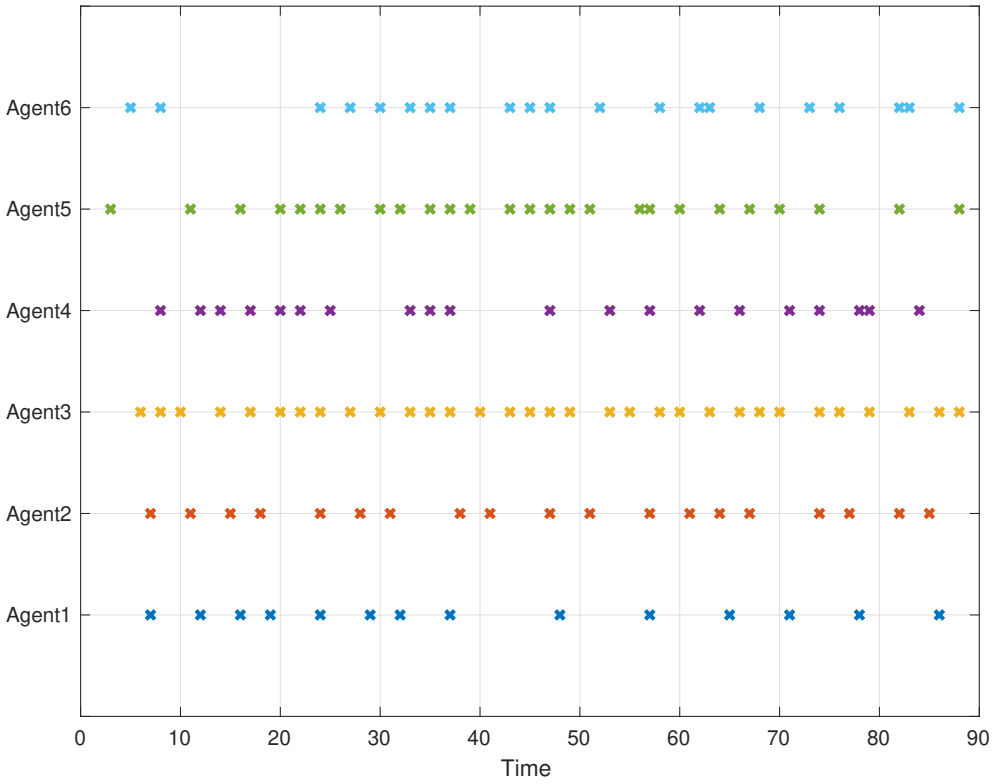

Figure 7.5: Triggering instants of each agent.

7.5 Appendix. Proofs of the Theorems

7.5.1 Proof of Theorem 7.1

Consider the following Lyapunov functional as

$$V(k) = \sum_{i=1}^{N} \sum_{i=1}^{4} V_j^i(k)$$

$$
\begin{aligned}
V_1^i(k) &= x_i^T(k) E_i^T \mathcal{P}_i E_i x_i(k) \\
V_2^i(k) &= \sum_{l=k-\underline{\tau}}^{k-1} x_i^T(l) \mathcal{Q}_1^i x_i(l) + \sum_{l=k-\bar{\tau}}^{k-1} x_i^T(l) \mathcal{Q}_2^i x_i(l) \\
V_3^i(k) &= \sum_{\theta=-\bar{\tau}}^{-\underline{\tau}} \sum_{l=k+\theta}^{k-1} x_i^T(l) \mathcal{Q}_3^i x_i(l) \\
V_4^i(k) &= \bar{\tau} \sum_{\theta=-\bar{\tau}}^{-1} \sum_{l=k+\theta}^{k-1} \eta_i^T(l) E_i^T \mathcal{Z}_1^i E_i \eta_i(l) \\
&\quad + \underline{\tau} \sum_{\theta=-\underline{\tau}}^{-1} \sum_{l=k+\theta}^{k-1} \eta_i^T(l) E_i^T \mathcal{Z}_2^i E_i \eta_i(l) \\
&\quad + \tau^\star \sum_{\theta=-\bar{\tau}}^{\underline{\tau}-1} \sum_{l=k+\theta}^{k-1} \eta_i^T(l) E_i^T \mathcal{Z}_3^i E_i \eta_i(l)
\end{aligned}
$$

and $\tau^\star = \bar{\tau} - \underline{\tau}$ and $\eta_i(k) = x_i(k+1) - x_i(k)$. The difference of Lyapunov functional $V(k)$ can be derived as follows:

$$
\begin{aligned}
\Delta V_1^i(k) &= x_i^T(k+1) E_i^T \mathcal{P}_i E_i x_i(k+1) - x_i^T(k) E_i^T \mathcal{P}_i E_i x_i(k) \\
&= \eta_i^T(k) E_i^T \mathcal{P}_i E \eta_i(k) + 2 x_i^T(k) E_i^T \mathcal{P}_i E \eta_i(k) \qquad (7.20) \\
\Delta V_2^i(k) &= x_i^T(k) [\mathcal{Q}_1^i + \mathcal{Q}_2^i] x_i(k) - x_i^T(k-\underline{\tau}) \mathcal{Q}_1^i x_i(k-\underline{\tau}) \\
&\quad - x_i^T(k-\bar{\tau}) \mathcal{Q}_2^i x_i(k-\bar{\tau}) \qquad (7.21) \\
\Delta V_3^i(k) &= (\tau^\star + 1) x_i^T(k) \mathcal{Q}_3^i x_i(k) - \sum_{l=k-\bar{\tau}}^{k-\underline{\tau}} x_i^T(l) \mathcal{Q}_3^i x_i(l) \\
&\leq (\tau^\star + 1) x_i^T(k) \mathcal{Q}_3^i x_i(k) - x_i^T(k-\tau(k) Q_3^i x_i(k-\tau(k)) \qquad (7.22) \\
\Delta V_4^i(k) &= \eta_i^T(k) E_i^T \left[\bar{\tau}^2 \mathcal{Z}_1^i + \underline{\tau}^2 \mathcal{Z}_2^i + \tau^\star \mathcal{Z}_3^i \right] E \eta_i(k) \\
&\quad - \bar{\tau} \sum_{l=k-\bar{\tau}}^{k-1} \eta_i^T(l) E_i^T \mathcal{Z}_1^i E_i \eta_i(l) - \underline{\tau} \sum_{l=k-\underline{\tau}}^{k-1} \eta_i^T(l) E_i^T \mathcal{Z}_2^i E_i \eta_i(l) \\
&\quad - \tau^\star \sum_{l=k-\bar{\tau}}^{k-\underline{\tau}-1} \eta_i^T(l) E_i^T \mathcal{Z}_3^i E_i \eta_i(l) \qquad (7.23)
\end{aligned}
$$

According to Lemma 7.1, we come up with

$$-\bar{\tau}\sum_{l=k-\bar{\tau}}^{k-1}\eta_i^T(l)E_i^T\mathcal{Z}_1^iE_i\eta_i(l)\leq\ -[x_i(k)-x_i(k-\bar{\tau})]^TE_i^T\mathcal{Z}_1^iE_i[x_i(k)-\ x_i(k-\bar{\tau})] \tag{7.24}$$

$$-\underline{\tau}\sum_{l=k-\underline{\tau}}^{k-1}\eta_i^T(l)E_i^T\mathcal{Z}_2^iE_i\eta_i(l)\leq\ -[x_i(k)-x_i(k-\underline{\tau})]^TE_i^T\mathcal{Z}_2^iE_i[x_i(k)-\ x_i(k-\underline{\tau})] \tag{7.25}$$

Using Lemma 7.1 and 7.2, the following bound holds:

$$\begin{aligned}
&-\tau^\star\sum_{l=k-\bar{\tau}}^{k-\underline{\tau}-1}\eta_i^T(l)E_i^T\mathcal{Z}_3^iE_i\eta_i(l)=-\tau^\star\sum_{l=k-\bar{\tau}}^{k-\tau(k)-1}\eta_i^T(l)\times\\
&E_i^T\mathcal{Z}_3^iE_i\eta_i(l)-\tau^\star\sum_{l=k-\tau(k)}^{k-\underline{\tau}-1}\eta_i^T(l)E_i^T\mathcal{Z}_3^iE_i\eta_i(l)\\
&\leq-\frac{1}{\alpha_1}\Psi_1^{iT}(k)E_i^T\mathcal{Z}_3^iE_i\Psi_1^i(k)-\frac{1}{\alpha_1}\Psi_2^{iT}(k)E_i^T\mathcal{Z}_3^iE_i\Psi_2^i(k)\\
&\leq-\begin{bmatrix}\Psi_1^i(k)\\ \Psi_2^i(k)\end{bmatrix}^T\begin{bmatrix}E_i^T\mathcal{Z}_3^iE_i & E_i^T\mathcal{R}_iE_i\\ E_i^T\mathcal{R}_iE_i & E_i^T\mathcal{Z}_3^iE_i\end{bmatrix}\begin{bmatrix}\Psi_1^i(k)\\ \Psi_2^i(k)\end{bmatrix}
\end{aligned} \tag{7.26}$$

such that: $\begin{bmatrix}E_i^T\mathcal{Z}_3^iE_i & E_i^T\mathcal{R}^iE_i\\ E_i^T\mathcal{R}^iE_i & E_i^T\mathcal{Z}_3^iE_i\end{bmatrix}\geq 0$, where

$$\alpha_1=\frac{\bar{\tau}-\tau(k)}{\bar{\tau}-\underline{\tau}},\quad \Psi_1^i(k)=x_i(k-\tau(k))-x_i(k-\bar{\tau})$$

$$\alpha_2=\frac{\tau(k)-\underline{\tau}}{\bar{\tau}-\underline{\tau}},\quad \Psi_2^i(k)=x_i(k-\underline{\tau})-x_i(k-\tau(k))$$

$$\text{Let } \xi_i(k) = \begin{bmatrix} x_i(k) \\ x_i(k-\underline{\tau}) \\ x_i(k-\tau(k)) \\ x_i(k-\bar{\tau}) \\ \eta_i(k)E_i \\ \tilde{u}_i(k) \end{bmatrix}, L_i = \begin{bmatrix} L_1^i \\ L_2^i \\ L_3^i \\ L_4^i \\ L_5^i \\ L_6^i \end{bmatrix}$$

$$2 \times \xi_i^T(k) L_i \Big[(A_i - E_i)x_i(k) + A_{di}x_i(k-\tau(k)) + B_i\tilde{u}_i(k) - E_i\eta_i(k) \Big] = 0 \qquad (7.27)$$

From the dynamics (7.1), and Equations (7.20)-(7.27), it is straight forward to see that the following equation holds for any matrices $L_1^i, L_2^i, L_3^i, L_4^i, L_5^i, L_6^i$ with appropriate dimensions:

$$\Delta V^i - z_i^T(k)\tilde{u}_i(k) - \alpha_i \tilde{u}_i^T(k)\tilde{u}_i(k) - \beta z_i^T(k) z_i(k) \leq \xi_i^T(k)\Theta\xi_i(k) \qquad (7.28)$$

where Θ is given in (7.9). negative definite Using Lemma 7.2, Cauchy-Shwartz inequality, inequalities (7.8), (7.28), and (7.9) and the consensus protocol (7.3), we

come up with the following upper bound of the proposed Lyapunov functional:

$$\begin{aligned}
\Delta V(k) &= \sum_{i=1}^{N} \Delta V^i(k) \\
&\leq \sum_{i=1}^{N} \left[z_i^T(k)\tilde{u}_i(k) + \alpha_i \tilde{u}_i^T(k)\tilde{u}_i(k) \ + \beta z_i^T(k) z_i(k) \right] \\
&\leq \sum_{i=1}^{N} \left[(\alpha_i + \frac{1}{2\delta_i}) \|\tilde{u}_i(k)\|^2 + (\beta + \frac{\delta_i}{2}) \|z_i(k)\|^2 \right] \\
&\leq \sum_{i=1}^{N} \left[(\alpha_i + \frac{1}{2\delta_i}) \left\| \sum_{j \in \mathcal{N}_i} c_i (z_i(k) - z_j(k)) \right\|^2 \right. \\
&\qquad \left. + (\beta + \frac{\delta_i}{2}) \frac{1}{4N} \sum_{j=\mathcal{N}_i} \left\| z_i(k) - z_i(k) \right\|^2 \right] \\
&\leq \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \left[(\alpha_i + \frac{1}{2\delta_i}) c_i^2 d_i + (\beta + \frac{\delta_i}{2}) \frac{1}{4N} \right] \left\| z_i(k) - z_i(k) \right\|^2 \qquad (7.29)
\end{aligned}$$

Thus, choosing the coupling strength c_i such that the following inequality holds

$$(\alpha_i + \frac{1}{2\delta_i}) c_i^2 d_i + (\beta + \frac{\delta_i}{2}) \frac{1}{4N} < 0 \qquad (7.30)$$

we have that $\|z_i(k) - z_i(k)\|$ converges to zero for all $i, j = 1, 2, ..., N$ which implies that the nominal multiagent system achieves the output synchronization. In the light of the same philosophy, it can be concluded that the multiagent system (7.8) is robustly admissible and reaches consensus for all norm-bounded uncertainties. By considering the uncertainty of the multiagent system, we can establish that

$$\Gamma_i + U_1^{iT} F_i(k) U_2^i + U_2^{iT} F_i(k) U_1^i \leq 0$$

Then, according to Lemma 7.2, Fact 1, and applying the Schur complement completes the proof.

7.5.2 Proof of Theorem 7.2

Consider the same Lyapunov functional in Theorem 7.1. Using the linear matrix inequalities (7.9), (7.28) and (7.10) and the protocol (7.5), we get

$$\begin{aligned}\Delta V(k) &= \sum_{i=1}^{N} \Delta V^i(k) \\ &\leq \sum_{i=1}^{N} \left[z_i^T(k)\tilde{u}_i(k) + \alpha_i \tilde{u}_i^T(k)\tilde{u}_i(k) + \beta z_i^T(k) z_i(k) \right] \\ &= \sum_{i=1}^{N} \Big[\big[z_i(k_l^i) - e_i(k)\big]^T \tilde{u}_i(k) + \alpha_i \tilde{u}_i^T(k)\tilde{u}_i(k) \\ &\quad + \beta \big[z_i(k_l^i) - e_i(k)\big]^T \big[z_i(k_l^i) - e_i(k)\big] \Big] \end{aligned} \tag{7.31}$$

Using Fact 1, the following inequalities hold:

$$e_i^T(k)u_i(k) \leq \frac{r}{2} e_i^T(k)e_i(k) + \frac{1}{2r} u_i^T(k)u_i(k) \tag{7.32}$$

$$2z_i^T(k_l^i)e_i(k) \leq s z_i^T(k_l^i) z_i(k_l^i) + \frac{1}{s} e_i^T(k)e_i(k) \tag{7.33}$$

for any scalars $s, r > 0$. With Cauchy-Schwartz inequality, we come up with:

$$\left\| \sum_{j\in\mathcal{N}_i} c_i(z_i(k) - z_j(k)) \right\|^2 \leq d_i c_i^2 \sum_{j\in\mathcal{N}_i} \left\| (z_i(k) - z_j(k)) \right\|^2 \tag{7.34}$$

and

$$\left\| \sum_{j\in\mathcal{N}_i} (z_i(k) - z_j(k)) \right\|^2 \leq d_i \sum_{j\in\mathcal{N}_i} \left\| (z_i(k) - z_j(k)) \right\|^2 \tag{7.35}$$

From (7.31)–(7.35), we have

$$
\begin{aligned}
\Delta V(k) \leq \sum_{i=1}^{N} \Bigg[& z_i^T(k_l^i)\tilde{u}_i(k) + (\alpha_i + \frac{1}{2r})\tilde{u}_i^T(k)\tilde{u}_i(k) \\
& + (\beta+s) z_i^T(k_l^i) z_i(k_l^i) + \frac{1+s+rs/2}{s} e_i^T(k) e_i(k) \Bigg] \\
& \leq \sum_{i=1}^{N} \Bigg[(\alpha_i + \frac{1}{2\delta_i} + \frac{1}{2r}) \Big\| \sum_{j \in \mathcal{N}_i} c_i (z_i(k) - z_j(k)) \Big\|^2 \\
& + (\beta + s + \frac{\delta_i}{2}) \frac{1}{2N} \sum_{j=\mathcal{N}_i} \frac{\|z_i(k) - z_i(k)\|^2}{2} \Bigg] \\
& + \gamma_i \frac{1+s+rs/2}{s} \Big\| \sum_{j \in \mathcal{N}_i} (z_i(k_l^i) - z_j(k_l^j)) \Big\| \\
& \leq \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \Bigg[[(\alpha_i + \frac{1}{2\delta_i} + \frac{1}{2r} \\
& + \gamma_i \frac{1+s+rs/2}{s c_i^2}) c_i^2 d_i + (\beta + s + \frac{\delta_i}{2}) \frac{1}{4N}] \|z_i(k) - z_j(k)\|^2 \Bigg]
\end{aligned} \tag{7.36}
$$

Thus, choosing the coupling strength c_i such that (7.13) is guaranteed, forces $\Delta V(k)$ to be negative whenever $\|z_i(k) - z_j(k)\|$ is positive. Then $\|z_i(k) - z_j(k)\|$ converges to zero for all $i, j = 1, 2, ..., N$ which implies that the multiagent system achieves the output synchronization.

7.5.3 Proof of Proposition 7.1

First of all, it is clear that $2x^T(k) S_i \Omega_i^T E_i \eta_i(k) = 0$ which can be added to the difference of the Lyapunove function in Equation (7.29) such that Γ_{15}^i in Equation (7.14) is modified without affecting the results of Theorem 7.1. Since $rank(E_i) = r_i < n$, there always exist non-singular matrices H_i and $F_i \in \mathbb{R}^{n \times n}$ such that

$$
\bar{E}_i = H_i E_i F_i = \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix} \tag{7.37}
$$

Then for any nonsingular matrix Φ_i, we have $\Omega_i = H_i \begin{bmatrix} 0 \\ \Phi_i \end{bmatrix}$ that satisfies $E_i^T \Omega_i = 0$. Define the following partitioned matrices

$$\mathbb{A}_i = H_i A_i F_i = \begin{bmatrix} \mathbb{A}_{11}^i & \mathbb{A}_{12}^i \\ \mathbb{A}_{21}^i & \mathbb{A}_{22}^i \end{bmatrix} \tag{7.38}$$

$$\mathbb{A}_{di} = H_i A_{di} F_i = \begin{bmatrix} \mathbb{A}_{d11}^i & \mathbb{A}_{d12}^i \\ \mathbb{A}_{d21}^i & \mathbb{A}_{d22}^i \end{bmatrix} \tag{7.39}$$

$$\mathbb{S}_i = F_i^T S_i = \begin{bmatrix} \mathbb{S}_{11}^i \\ \mathbb{S}_{21}^i \end{bmatrix} \tag{7.40}$$

It follows from the inequality (7.9) with the modified Γ_{15}^i that

$$\Gamma^i = \begin{bmatrix} \Gamma_{11}^i & \Gamma_{15}^i \\ \bullet & \Gamma_{55}^i \end{bmatrix} < 0$$

which implies the following inequality:

$$\Psi^i = \begin{bmatrix} \Psi_{11}^i & \Psi_{15}^i \\ \bullet & \Psi_{55}^i \end{bmatrix} < 0 \tag{7.41}$$

where

$$\Psi_{11}^i = L_1^i(\bar{A}_i - E_i) + (\bar{A}_i - E_i)^T L_1^{iT} - E_i^T \mathcal{Z}_1^i E_i - E_i^T \mathcal{Z}_2^i E_i$$

$$\Psi_{12}^i = S_i \Omega_i^T + E_i^T \mathcal{P}_i - L_1^{iT} + (\bar{A}_i - E)^T L_5^{iT} - L_2^i$$

$$\Psi_{22}^i = -L_5^i - L_5^{iT}$$

Multiplying both sides of (7.41) by $\begin{bmatrix} I & A_i^T \end{bmatrix}$ and its transpose, respectively, yields

$$E_i^T(\mathcal{P}_i - L_3^{iT} - L_1^{iT})A_i + A_i^T(\mathcal{P}_i^T - L_3^i - L_1^i)E_i^T - L_1^{iT} E_i$$

$$-E_i^T L_1^i + S_i \Omega_i^T A + A_i^T \Omega_i S_i^T - E_i^T \mathcal{Z}_1 E_i - E_i^T \mathcal{Z}_1 E_i < 0 \tag{7.42}$$

Multiply both sides of (7.42) by F_i^T andF_i, respectively, and then using equations

(7.37), (7.38), and (7.40), we obtain

$$\begin{bmatrix} ? & ? \\ \bullet & S_{21}^i \Phi_i^T \mathbb{A}_{21}^i + \mathbb{A}_{21}^{iT} \Phi_i S_{21}^{iT} \end{bmatrix} < 0 \tag{7.43}$$

The question mark represents elements which are not needed since they will be eliminated as follows. Let v be a non zero vector such that $\mathbb{A}_{21}^i v = 0$, i.e. $\mathbb{A}_{21}^i$ is singular. Then, multiplying both sides of (7.43) by $\begin{bmatrix} 0 & v^T \end{bmatrix}$ and its transpose yields a zero value which is a contradiction. This implies that $\mathbb{A}_{21}^i$ is nonsingular and the pair (E_i, A_i) is regular and causal for all $i = 1, 2, ..., N$. This completes the proof.

7.5.4 Proof of Theorem 7.3

If we view agents input $u_i(k)$ as a state component, the multiagent dynamics (7.1) can be represented as follows

$$\begin{aligned} \tilde{E}_i \xi_i(k+1) =& (\tilde{A}_i + \Delta\tilde{A}_i)\xi_i(k) + (\tilde{A}_{di} + \Delta\tilde{A}_{di})\xi_i(k-\tau(k)) + \tilde{B}_i \tilde{u}_i(k) \\ \tilde{z}_i(k) =& \tilde{C}_i \xi(k) + \tilde{C}_{di} \xi_i(k-\tau(k)) \\ \xi_i(k) =& \phi_{i0}(k), \quad k \in [-\bar{\tau}, 0] \end{aligned} \tag{7.44}$$

where $\xi_i(t) = \begin{bmatrix} x_i^T(t) & u_i^T(t) \end{bmatrix}^T$, $\Delta\tilde{A}_i = \tilde{M}_i F(k) \tilde{N}_i$, $\Delta\tilde{A}_{id} = \tilde{M}_i F(k) \tilde{N}_{id}$, $\tilde{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}$, $\tilde{C}_i = \begin{bmatrix} C_{di} & 0 \end{bmatrix}$, $\tilde{M}_i = \begin{bmatrix} M_i^T & 0 \end{bmatrix}^T$, $\tilde{N}_i = \begin{bmatrix} N_i & 0 \end{bmatrix}$

$$\tilde{E}_i = \begin{bmatrix} E_i & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{A}_i = \begin{bmatrix} A_i & B_i \\ K_i & -I \end{bmatrix}$$

$$\tilde{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{B}_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix}$$

Applying Theorem 7.2 to system (7.44) with the following particular configuration of real matrices $L_j^i, (j{=}1,2,3,4,5,6),\ i = 1,2,...,N$

$$L_j^i = \begin{bmatrix} L_{j1}^i & \lambda_j \mathbb{I} F \\ \\ L_{j2}^i & \lambda_j F \end{bmatrix}, \quad \mathbb{I} = \begin{bmatrix} I_m \\ \\ 0_{(n-m)\times m} \end{bmatrix}$$

condition (7.9) holds by replacing $Y_i = F_i K_i$. It is clear from the inequalities in Theorem 7.2 that a feasible solution fulfills the inequality $\Gamma_{55}^i < 0$ for all $i = 1, \ldots, N$. This implies that L_5^i are non-singular matrices for all $i = 1, \ldots, N$ and thus F_i is invertible. This completes the proof.

CHAPTER 8

CONCLUSIONS

In this dissertation, we investigated various types of cooperative problems for multi-agent systems. Firstly, we investigated the state and point formation protocols for discrete-time multi-agent systems subject to DoS attacks. The discrete-time formation problem is transformed into a stability problem of a switching system. Besides, multiple DoS attacks with independent characteristics for different transmission channels were investigated. Sufficient and necessary conditions in terms of matrix inequalities were derived to solve the formation problem. Secondly, the scaled-consensus problem has been investigated for multiagent systems over non-ideal networks where DoS attack is considered where adversaries can exclude agents totally form the network. Distributed protocols have been developed in order to achieve Leaderless and leader-follower scaled-consensus. The problem is converted to mean square stability and analysis of the disagreement error has been addressed. Then, we dealt with event-based consensus protocols of multiagent systems with a class of nonlinear dynamics. We showed a framework of designing distributed event triggered consensus protocol for multagent systems with time delay, which include linear dynamics of any order

as a special case. It has been theoretically demonstrated that the consensus problem of multiagent systems with nonlinear perturbation and time delays can be achieved if the control gains and the triggering parameters are appropriately selected. The multiagent system is not only cooperative but also satisfies some level of performance and the energy consumption. Moreover, the proposed results are extended to observer based consensus methodology under the triggering policy. Finally, a new distributed event-triggered protocol based on dissipativity approach has been investigated to solve the synchronization problem for discrete time uncertain linear multiagent systems.

Compared to the previous related results, the main contribution is that we have proposed both time triggered and event-triggered consensus protocols that do not require continuous monitoring of measurement nor continuous communication among agents. The proposed triggering mechanisms do not reveal the Zeno behavior due to the discrete nature of the multiagent dynamics. Furthermore, the event/time triggering controller gains are solved by obtaining a feasible solution for a set of linear matrix inequalities. Lastly, the effectiveness of the synchronization protocols have been demonstrated by several simulation examples.

8.1 Future Challenges, Suggested Problems, and New Directions

There are still several important problems and challenges that deserve more investigation, since so far the current cooperative control algorithms still susceptible to perturbation, cyber attacks, time varying structure, impulsive behavior, networked

problems, etc.

- A large amount of the existing research about cooperative control addresses simple cases where the agents acquire information in real time sensing. However, in real applications, the agents broadcast their own information to the neighbors over a digital communication network. Due to practical consideration, the cooperative schemes should take into account the time delay, packet losses, cyber attacks, band limited channel, and quantization error of the communication networks.

- Various papers discuss the robustness and model uncertainty issues of cooperative control systems. Most of the vehicle dynamics are complex and their effects can not be ignored or simplified by linearizion for the control design of multi-vehicle systems. However, the results that have devoted to the robust cooperative control still limited and most of the existing work is devoted to simple dynamics such as single integrator and double integrator dynamics. Moreover, the adaptive cooperative control of time varying dynamics is promising.

- It is of interest to further investigate time varying formation control with switching topology for general linear dynamics of multiagent systems. Moreover, another future work could be about the tracking control schemes in which all the trajectories of the agents form a predefined time varying formation as well as the formation itself tracks a formation trajectory.

- Most of the existing work on consensus problem focus on the standard consensus while the the practical problems require an adequate level of performance on the

positions and the process that achieves the consensus. Performance assessment within multi-agent systems should be more investigated.

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