

**A NEW FRAMEWORK FOR PRECISE PERFORMANCE
ANALYSIS OF MIXED RF/FSO RELAYING SCHEMES
EMPLOYING BOTH HD & IM-DD RECEIVERS**

BY

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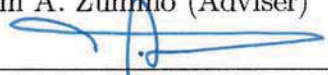
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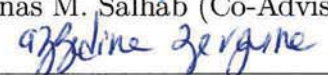
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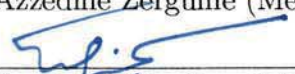
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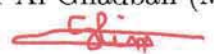
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Dedicated to my parents, wife, siblings, friends and children.

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In the name of Allah, the Entirely Merciful, the Especially Merciful. All praise and thanks be to Allah, the One and the Only Lord of Âlamîn (mankind, jinn and all that exists, i.e. all the universe), its Creator, Owner, Organizer, Provider, Master, Planner, Sustainer, Cherisher, Giver of security, the Most Gracious, the Most Merciful, who alone we worship, and who alone we ask for help (for each and everything), who alone give me strength and ability for everything I do including my post-graduate studies and gaining this degree.

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LIST OF ABBREVIATIONS

AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit error rate
BTS	base transceiver station
CDF	cumulative distribution function
CSI	channel state information
DF	decode-and-forward
DoF	degrees of freedom
Duo GG	Double Generalized Gamma
FSO	free space optical
G-G	Gamma-Gamma
HD	heterodyne detection
IA	interference alignment
IC	interference channel
IM-DD	intensity modulation direct detection
LAN	local area network

LED	light emitting diode
LO	local oscillator
LOS	line-of-sight
MAN	metropolitan area network
MIMO	multiple-input multiple-output
MS	mobile user
NLOS	non line-of-sight
OWC	optical wireless communication
pdf	probability density function
RF	radio frequency
SER	symbol error rate
SIM	subcarrier intensity modulation
SNR	signal-to-noise ratio

DISSERTATION ABSTRACT

NAME: Omer Mahmoud Salih Elhag

TITLE OF STUDY: A New Framework for Precise Performance Analysis of Mixed RF/FSO Relaying Schemes Employing both HD & IM-DD Receivers

MAJOR FIELD: Electrical Engineering

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In this dissertation a precise performance analysis is provided for the dual-hop mixed radio frequency (RF)/unified free space optical (FSO) decode-and-forward (DF) relaying system, in which the heterodyne detection (HD) and the intensity modulation-direct detection (IM-DD) are considered in a unified analysis manner for FSO detection. To this end, we derive closed-form expressions for the outage probability, average bit error rate (BER), and the ergodic channel capacity of this system assuming generalized fading models, i.e., Nakagami- m and Málaga ($\mathcal{M}$) turbulence with pointing error for the RF and FSO channels, respectively. In this analysis, we utilize, for the first time as per our knowledge, a precise channel capacity result for the IM-DD channel. Moreover, this is the first time that not only the IM-DD input-independent but also the IM-DD

cost-dependent AWGN channel is considered in such system analyses. Also, in BER derivation, we assume that the modulation schemes in the two hops can be different, since not all modulation schemes are suitable for IM-DD FSO links. In addition, the system performance is investigated asymptotically at high signal to noise ratios (SNRs). This investigation leads to new non-reported coding gain and diversity order analyses of such system. Interestingly, we find that in the FSO hop, at high transmitted powers, all the considered FSO detectors result in the same diversity order, whereas their coding gains are different. Also, at high transmitted powers, we find that, in the FSO hop, the ergodic capacities pre-logs for HD and IM-DD input-independent are identical and equal twice the one of the IM-DD cost-dependent. Additionally, we show the overestimations which occur in the outage and ergodic capacity of the system when the well-known Shannon's capacity formula for the power-constrained AWGN channel is used for IM-DD instead of its precise capacity approx. These overestimations lead to misconception in the comparison between HD and IM-DD. Furthermore, we provide simulation results for this system that verify the accuracy of the obtained analytical and asymptotic expressions.

One more relaying scheme that its performance is studied, for the first time as per our knowledge, in this dissertation is the one in which the first hop is an interference-aligned multiple-input multiple-output (MIMO) RF interference channel (IC), while the second hop is an FSO channel in which HD as well as IM-DD are considered in a unified analysis manner. In this work, the Interference alignment (IA) technique is utilized in the MIMO RF IC hop to eliminate the interference, and hence this leads to enhance the multiplexing gain in this hop, and accordingly the overall

system performance is improved. The design of the IA precoding matrix and receiving interference eliminating matrix is performed by using orthogonalized minimum leakage (OR-ML) algorithm. In this mixed (IA MIMO RF IC)/(FSO) relaying, it is assumed that the MIMO RF links follow Rayleigh fading, whilst the FSO link encounters Málaga ($\mathcal{M}$) fading with pointing errors. For this scheme, exact closed-form expressions are derived for the outage probability and ergodic capacity, in these expressions the best-known FSO channel capacity results are utilized. Moreover, both the IM-DD input-independent AWGN and the IM-DD cost-dependent AWGN channel are considered in this derivation. In addition, the system performance is studied asymptotically at the high SNRs region, where the diversity order and coding gain analyses are illustrated. It is found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2). Alternatively, under FSO hop domination, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors. Also, It is observed that the overall system ergodic capacity enhances by increasing “d” when the RF hop dominates the system. Further, we allocate simulation results for this scheme that verify the deduced analytical and asymptotic expressions.

Furthermore, we explore the performance deterioration in the dual hop mixed (imperfect IA MIMO RF IC)/(FSO) relaying due to the consideration of the imperfect channel state information (CSI). Considering imperfect CSI means that the interference will not be totally eliminated in this case. For this scheme, closed-form expressions are derived for the outage probability in terms of Whittaker and Meijer functions and for the ergodic capacity in terms of the bivariate Meijer’s G-function. It is found that

imperfect CSI results in zero diversity order causing a floor in the outage probability at high SNRs. This floor worsen with either the reduction in the CSI accuracy, or the decrement in “d”. Also under imperfect CSI, It is found that there is a ceil in the ergodic capacity at high SNRs, this ceil becomes worse with more interferers or when the CSI accuracy or the number of RF info streams (d) decreases.

ملخص الرسالة

الاسم:

عمر محمود صالح الحاج

عنوان الدراسة:

إطار جديد لتحليل دقيق لأداء أنظمة الترحيل اللاسلكي المخلطة من الترددات الراديوية (RF) / والترددات البصرية

(FSO) والتي تستخدم كلا من مستقبلات HD و IM-DD

التخصص:

الهندسة الكهربائية

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هذه الرسالة تقدم تحليل دقيق لأداء نظام الترحيل اللاسلكي المبني على فك التشفير ثم إعادة الإرسال (DF) ذو المرحلتين المخلط من الترددات الراديوية (RF) / والترددات البصرية (FSO). في هذه الدراسة يتم بصورة تحليلية موحدة اعتبار كل من الكشف المتسق (HD) والكشف المباشر لشدة الاضاءة المعدلة (IM-DD) في استقبال اشارات ال FSO. لتحليل هذا النظام نقوم باشتقاق الصيغ الرياضية لكل من: احتمال الانقطاع، ومتوسط معدل الخطأ في استقبال البيانات الثنائية BER، والسعة الإرفودية لقناة النظام. هذه الاشتقاقات مبنية على افتراض نماذج عامة للخبو (تحديداً) نستخدم Nakagami-m و Málaga ($\mathcal{M}$) turbulence with pointing error لخبو قنوات ال RF وال FSO على التوالي). في هذه الدراسة التحليلية يتم -لاول مرة حسب علمنا- إستعمال نتائج دقيقة لسعة قناة ال IM-DD. علاوة على ذلك هذه هي المرة الاولى التي لا يقتصر فيها على اعتبار قناة ال IM-DD input-independent AWGN ولكن ايضا يتم اعتبار قناة ال IM-DD cost-dependent AWGN في تحليل مثل هذا النظام. ايضا في اشتقاق ال BER نقوم بافتراض ان نوعي التعديل في مرحلتي الإرسال يمكن ان يكونا مختلفين، وهذا مهم لانه ليست كل انواع التعديل مناسبة لوصلات ال IM-DD FSO. بالإضافة الى ذلك يتم فحص أداء النظام بشكل تقريبي دقيق عند النسب العالية لمتوسط قدرة الإشارة الى الضجيج (high SNRs). هذا الفحص يظهر تحليلات جديدة -لم يتم نشرها- عن درجة التنوع وكسب التشفير لهذا النظام. ومن المثير للاهتمام أننا نجد في وصلة ال FSO، عند قدرات الإرسال العالية، أن جميع كاشفات اشارات ال FSO -التي تم دراستها- تؤدي الى نفس درجة التنوع في

حين ان كسوب تشفيرها مختلفة. ايضا عند قدرات الارسال العالية، في وصلة ال FSO، نجد ان معامل الدالة اللوغريتمية للسعة الإرقودية في حالة ال HD يساوي نظيره في حالة ال IM-DD input-independent، بينما يساوي ضعف نظيره في حالة ال IM-DD cost-dependent. إضافة لذلك يتم عرض الخطأ في تقدير احتمال الانقطاع و السعة الإرقودية للنظام عند استخدام صيغة شانون المشهورة لسعة قناة ال AWGN ذات القدرة المحدودة لقناة ال IM-DD AWGN بدلا من السعة التقريبية الدقيقة للاخيرة. و يؤدي هذا الخطأ في التقدير إلى صورة مغلوطة عند مقارنة ادائي ال HD و ال IM-DD. علاوة على ذلك يتم تقديم نتائج محاكاة لهذا النظام والتي بها تتحقق من صحة التعبيرات التحليلية الدقيقة والتقريبية التي تم الحصول عليها.

هنالك نظام ترحيل لاسلكي اخر تتم دراسة ادائه في هذه الرسالة و لأول مرة حسب علمنا. في ذلك النظام المرحلة الاولى هي قناة ترددات راديوية (RF) مشوشة (IC) متعددة المدخلات و المخرجات (MIMO) ذات تداخل مرصوف، بينما المرحلة الثانية هي قناة ترددات بصرية (FSO) يتم فيها اعتبار كل من ال HD و ال IM-DD بطريقة تحليل موحدة. و في هذا العمل تتم الاستفادة من تقنية رصف (محاذاة) التداخل (IA) في مرحلة ال MIMO RF IC للقضاء على التداخل (التشويش) و بالتالي تحسين كسب تعدد الإرسال في تلك المرحلة، و تبعا لذلك يتحسن اداء النظام ككل. تصميم مصفوفة ما قبل التشفير -في المرسل- و مصفوفة التخلص من التداخل -في المستقبل- لتقنية ال IA يتم باستخدام خوارزمية OR-ML. في هذا النظام للترحيل المخلط من (FSO) / (IA MIMO RF IC) نقوم بافتراض ان وصلات ال MIMO RF تتبع الخبو Rayleigh في حين نفترض ان وصلة ال FSO تواجه الخبو Málaga ($\mathcal{M}$) turbulence with pointing error. بالنسبة لهذا النظام يتم اشتقاق الصيغ الرياضية لاحتمال الانقطاع و السعة الإرقودية، و في هذه الصيغ تتم الاستفادة من ادق النتائج -المعروفة حاليا- لسعة قناة ال FSO. علاوة على ذلك يتم اعتبار كل من قناة ال IM-DD input-independent AWGN و قناة ال IM-DD cost-dependent AWGN في هذه الاشتقاقات. بالإضافة إلى ذلك تتم دراسة أداء هذا النظام بطريقة تقريبية دقيقة في حالة النسب العالية لمتوسط قدرة الإشارة الى الضجيج (high SNRs)، حيث يتم تحليل درجة تنوع النظام و كسب تشفيره. و يظهر هذا التحليل انه عند سيطرة مرحلة ال RF على اداء النظام فان درجة التنوع تساوي مربع عدد مدخلات معلومات التردد الراديوي التي تم تنقيتها من التداخل (d^2). في المقابل فانه عند سيطرة مرحلة ال FSO على اداء النظام فان درجة التنوع تعتمد على معاملات الاضطرابات الجوية و انحرافات التوجيه. من الملاحظ أيضا ان السعة الارقودية الكلية للنظام تتحسن بزيادة "d" عند سيطرة مرحلة ال RF على اداء النظام. علاوة على ذلك نقوم بعرض نتائج محاكاة لهذا النظام لظهار صحة التعبيرات الرياضية الدقيقة و التقريبية المتحصل عليها. من جهة اخرى يتم استكشاف التدهور في أداء نظام الترحيل اللاسلكي -ذو المرحلتين- المخلط من (FSO) / (imperfect IA MIMO RF IC) نتيجة للخطأ في معرفة حالة القناة (CSI). إعتبار الخطأ في ال CSI يعني أنه لن يتم القضاء على التداخل تماما في هذه الحالة. بالنسبة لهذا النظام يتم اشتقاق الصيغة الرياضية لاحتمال الانقطاع بدلالة دالة Whittaker و دالة Meijer، و ايضا يتم اشتقاق الصيغة الرياضية للسعة الإرقودية بدلالة

دالة Meijer ثنائية المتغيرات. و من الملاحظ ان الخطأ في ال CSI يؤدي إلى عدم وجود أي تباين (يجعل درجة التنوع تساوي صفر) ويظهر ذلك في صورة وجود حد ادنى ثابت -لا يتغير- لاحتمال الانقطاع عند النسب العالية لمتوسط قدرة الإشارة الى الضجيج (high SNRs)، يسوء هذا الحد الادنى كلما زاد الخطأ في ال CSI او قلت " d ". ايضا تحت تأثير الخطأ في ال CSI نجد ان هنالك سقفا -حد اعلى ثابت لا يتغير- للسعة الإرقودية عند النسب العالية لمتوسط قدرة الإشارة الى الضجيج (high SNRs)، يصبح هذا السقف اسوء كلما زاد عدد اشارات التشويش او قلت دقة ال CSI او نقص عدد مدخلات معلومات التردد الراديوي المنقاة من التداخل (d).

CHAPTER 1

INTRODUCTION

Nowadays, it has been observed that there is a huge growing demand for high speed wireless data communications. This increases the requirements for high bandwidth/capacity wireless communication systems which leads to congestion in the available radio frequency (RF) spectrum. One way to overcome this problem and to guarantee high bandwidth is to use optical wireless communication (OWC), which is a technology that utilizes an optical frequency carrier to convey data from a laser (outdoor applications) or a LED (indoor applications) transmitter to an optical detector receiver [1], [2]. Free space optical (FSO) communication is the term mostly used for outdoor OWC in which information is sent from a transmitter to a receiver through a line-of-sight (LOS) unguided channel such as the free space.

In addition to providing much higher data rates, due to its high bandwidth, FSO systems have many other advantages in comparison to RF ones. Some of these advantages are as follows: FSO systems are license-free spectrum, quick and easy to deploy. Also due to the use of high-directive optical beams FSO communications are more secure, and less demanding in terms of power consumption, mass requirements

and electromagnetic interference isolation. For these reasons, FSO has been implemented in many applications such as: radio astronomy, remote sensing, back-haul for cellular networks, extension of metropolitan area network (MAN), local area network (LAN)-to-LAN communications, disaster recovery, high definition TV and medical video/image transmission, last mile access, wireless video surveillance, and several more. Also FSO is excellent for transmission to/from airplanes, spacecraft and in hospitals where electromagnetic interference reduction is mandatory.

1.1 Motivation

Due to these distinctive features of FSO communications, there has been a great number of researches in this field recently, see [1], [2] and the references therein. One of the main challenges in FSO communications is that it can offer transmission only over relatively short distances due to the optical propagation limiting factors [2]. To be specific these factors are mainly the atmospheric path loss, atmospheric turbulence-induced fading, beam dispersion, the pointing instability, and the sensitivity to weather conditions such as cloud, snow, fog, and smog. However an energy-efficient way to overcome this problem and to extend the coverage area in wireless communications is to use the relay networks [3], [4]. Therefore FSO relay networks have recently gained a considerable quantity of researches in literature. In these networks, a relay or a group of relays are used to support a source node in transmitting the data to a destination.

One of the very important scenarios in the literature is the mixed RF/FSO dual-hop relay network. The significance of this scenario comes from its valuable appli-

cations such as in mobile cellular systems and femtocells networks where the mobile stations (sources) are multiplexed to transmit their data to a relay station (or an access point) via an RF link in the first hop, while in the second hop the relay forwards the data to the macro base station (destination) via an FSO connection [5]. Additionally an advantage of this mixed relaying configuration is that it economizes the resources by preventing unnecessary installations for optical fiber, for example in suburban and downtown areas, and simultaneously saves the RF spectrum while providing high data rates by exploiting the FSO link. Another advantage of the mixed RF/FSO relaying is that applying it in mobile cellular networks leads to overcome the limitation of applying the pure FSO schemes in mobile cellular networks. This limitation comes from the requirement of the FSO schemes for the LOS link which is usually not guaranteed in mobile cellular networks in the link between the mobile station and the base station [5].

The mixed RF/FSO relaying gives rise to other valuable scenarios such as the scenario in which the second hop is an FSO link, while the first hop is a multiple-input multiple-output (MIMO) RF interference channel (IC). In this scenario the interference alignment (IA) technique could be used to eliminate the interference in the first (MIMO RF IC) hop, and hence this leads to enhance the multiplexing gain (degrees of freedom (DoF)) in that hop. Due to the use of the IA in this scenario we will call it the mixed IA MIMO RF IC/FSO relaying scheme.

An important application of this scheme is the one in which the first hop is a MIMO uplink IC where K multiantennas-equipped mobile users (MSs) communicate with their corresponding K multiantennas-equipped base transceiver stations (BTSS)

via RF links, specially when those MSs are located at the coverage border of their neighboring BTSs. While in the second hop these BTSs are connected with the backbone network via FSO links.

Due to the significance of these mixed RF/FSO schemes, we are motivated to contribute in this research field by analyzing their performances in a new precise framework in this study.

1.2 Literature Survey

1.2.1 FSO Single Relay Schemes

As stated in the Introduction, FSO relay networks have gained a considerable quantity of researches in the literature because they extend the coverage area and offer diversity gain without the requirement for high power transmitters. As a result in the single relay systems, the performance of the FSO relay network was inspected in [30], [31] over log normal fading channel model which is commonly employed to model the FSO link in weak atmospheric turbulence situations. Specifically, the work in [30] studied the outage probability of the decode-and-forward (DF) and amplify-and-forward (AF) FSO relay network considering the existence of a direct link between the source and destination. The performance of the single relay non-coherent FSO relaying schemes using IM-DD was analyzed in [32] by deriving closed-form and semi-analytical expressions for the error probability in the existence and absence of background radiations with the adoption of the log-normal and Rayleigh turbulence-induced fading channel models. Alternatively in [33]–[37] the performance of FSO relay network was inves-

tigated assuming Gamma-Gamma fading channel model which is more precise and has been employed to model the FSO link in both strong and weak turbulence situations. Specifically, in [33] an exact closed-form expression was derived for the average bit error rate (BER) of DF FSO relay schemes. The same system analyzed in [33] was also investigated in [34] but with differential modulation. Additionally in [35], a closed form expression for the average BER of the dual-hop AF FSO relay systems was developed. Moreover some approximations and bounds for the capacity of the AF FSO relay systems were derived in [36] considering the effect of atmospheric loss, turbulence induced fading, and pointing errors. Furthermore, in [37], closed-form expressions were developed for the outage and symbol error probabilities of the two way FSO relaying systems.

1.2.2 FSO Multi-Relay Networks

Beside the FSO single relay schemes, the FSO multiple relay networks have attained the interest of many researchers. Particularly in the parallel relaying field, the performance of the dual-hop FSO relaying through the log-normal fading channel was investigated in [40], [41] for AF relaying and in [38], [39] for DF relaying. In [42] the performance of FSO relay schemes having two relays beside the direct link, between the source and destination, was analyzed with the assumption of inter-relay cooperation. The performance of the dual-hop FSO selective-relay scheme when the source data is forwarded to the destination through the direct link or via the best relay was examined in [43]. In that work asymptotic and closed-form expressions were developed for the average BER assuming log-normal and Rayleigh fading channel

models. In [44] a closed-form expression was derived for the outage probability of DF FSO relay schemes over the Gamma-Gamma fading channel with the pointing error. Also exact closed-form expressions were derived in [45] for the outage probability of DF FSO relaying systems with all-active and select-max relaying strategies assuming Gamma-Gamma fading channel model. In the all-active relaying strategy the source information is forwarded to the destination by all active relays; while in the select-max strategy only the best active relay forwards the source information to the destination. A similar system as the one in [45] was also investigated in [46] but with the addition of the distributed switch-and-stay relay selection strategy and moreover the asymptotic analysis of the outage probability was provided at the high SNR regime. The outage probability of the CSI-assisted and semi-blind AF opportunistic FSO parallel relaying scheme was inspected in [47] under the assumption of composite channel which consists of atmospheric loss, atmospheric turbulence-induced fading, and pointing error effects.

In the serial or multihop relaying field the outage and bit error probability of DF and AF FSO relay schemes over Gamma-Gamma fading channel were inspected in [48]. A similar system was also analyzed in [49] for fixed-gain as well as CSI-assisted AF relaying scheme where closed form expressions were derived for the outage probability and the average BER beside the derivation of the asymptotic average BER. The work in [50] studies the outage probability for DF and AF parallel and serial FSO relaying systems over the log-normal fading channel with the consideration of the path loss. The erbium-doped fiber amplifier was proposed in [51] for serial multihop and parallel dual-hop AF FSO relay systems with fixed and variable gain, where

in that work the outage probability was analyzed. These amplifiers do not require optical-to-electrical and electrical-to-optical conversions at the relay. The authors in [52] proposed an optimal relay location selection scheme to minimize the outage probability in multi-relay parallel and serial DF FSO relay systems assuming log-normal fading channel. In addition to that they conducted a diversity order analysis for those systems. Additionally the error probability for the serial relay M-ary pulse-position modulation (M-PPM) FSO schemes with partially coherent Gaussian beam was investigated in [53] over the Gamma-Gamma fading channel with the pointing error. An inter-relay cooperation was proposed in [54] to enhance the FSO relaying systems over Gamma-Gamma fading channel. That approach utilizes the existence of FSO links between the adjacent relays to perform an inter-relay negotiation which leads to improve the reliability in the reconstructed signal at the relays before being forwarded to the destination. The outage probability and error rate of multihop fixed-gain and CSI-assisted AF FSO relaying systems were analyzed in [55] over weak-to-moderate and moderate-to-strong turbulence regimes using HD. The work in [56] investigated the effect of boresight pointing error on the outage probability of serial and parallel FSO relaying network. The boresight error comes from the thermal expansion of the buildings. The outage performance of parallel multihop DF FSO relay network where the k th best path is chosen was studied in [57] assuming log-normal fading channel. This selection method is useful when the best path is busy in some scheduling or load balance duties in other parts of the network.

1.2.3 Dual-Hop Mixed RF/FSO Relaying Schemes

In the dual-hop mixed RF/FSO relaying schemes multiple RF-capable users can be multiplexed into a single FSO link through a relay node [58]–[60]. Such schemes attract a significant quantity of researches due to its valuable applications (see the Introduction), the economic resource saving which it provides by preventing unnecessary installations for optical fiber, e.g. in suburban & downtown areas, and the saving in the RF spectrum it offers while providing high data rates by exploiting the FSO link.

The research works on these relaying networks are summarized in Table 1.1. In this table R, Nm, Rc, G-G, Duo GG, denote Rayleigh, Nakagami- m , Rician, Gamma-Gamma, and Double Generalized Gamma fading models respectively.

Table 1.1: Literature review on the dual-hop mixed RF/FSO relaying schemes.

Ref	RF Link	FSO Link	Relaying Type	Pointing Error	FSO Detector	Metrics
[58]	R	G-G	AF, fixed gain	No	IM-DD	OP
[59]	R	G-G	AF, fixed gain	No	IM-DD	BER
[60]	R	G-G	AF, variable gain	Yes	HD, IM-DD	OP, BER
[61]	R	G-G	AF, fixed gain	Yes	IM-DD	OP, BER
[62]	R	G-G	AF, fixed gain+direct link	Yes	HD, IM-DD	OP, BER
[63]	R	G-G	DF, multiuser	Yes	IM-DD	OP, SER
[64]	Nm	G-G	AF, fixed & variable gain	Yes	HD, IM-DD	OP, BER
[65]	Nm	G-G	AF, fixed gain + direct link	Yes	HD, IM-DD	OP, BER
[66]	R	G-G	AF, variable gain + direct link	Yes	HD, IM-DD	OP, BER
[67]	R	$\mathcal{M}$	AF, fixed gain	Yes	IM-DD	OP
[68]	Rc	G-G	AF, fixed gain	No	IM-DD	SER
[69]	Nm	G-G	AF, fixed gain	Yes	IM-DD	OP, BER
[70]	Nm	G-G	DF	Yes	IM-DD	OP, BER
[71]	R	G-G	AF, variable gain, multiuser	Yes	HD, IM-DD	OP, BER
[72]	Nm	Duo GG	AF, fixed& variable gain+direct link	Yes	HD, IM-DD	OP, BER

Specifically in [60] closed-form expressions were developed for the outage probability, the average bit error rate and the channel capacity of AF variable gain mixed

RF/FSO dual-hop relay scheme. It is assumed there that the RF link follows Rayleigh fading while the FSO link follows Gamma-Gamma fading with pointing error, and both IM-DD as well as HD are considered for FSO detection. A similar scheme to that in [60] was investigated in [61] but for the fixed-gain AF relaying system, and in [62] but this time for the fixed-gain AF relaying system in the existence of the direct link from the source to the destination. Also an analogous system to these ones was studied in [64] for mixed Nakagami- m and Gamma-Gamma fading. Another similar scheme to the one in [60] was inspected in [71] but this time for multiusers (sources) opportunistic scheduling. Additionally the outage probability and BER for AF (fixed and variable gain) mixed RF/FSO schemes were investigated in [72] assuming Nakagami- m and Double Generalized Gamma fading models for the RF and FSO links respectively, and considering pointing errors, co-channel interference and the existence of the direct RF link.

From Table 1.1, we can see that the RF link is considered to encounter Rayleigh fading in most of the works, whereas Nakagami- m fading (which contains the Rayleigh fading as a special case) is assumed in [64], [65], [69], [70], and [72]. Alternatively, Rician fading is assumed only in [68].

Additionally Table 1.1 shows that the most assumed atmospheric turbulence-induced fading model for the FSO link is the G-G fading, while the Duo GG fading (which contains G-G fading and others as special cases) is assumed only in [72]. On the other hand, the $\mathcal{M}$ -distributed fading (which contains G-G fading and others as special cases) is assumed only in [67] with Rayleigh faded RF link in AF relaying.

Actually although G-G is better than historically earlier tractable turbulence fading

ing models, still it deviates from experimental data in tails, see [23], [24]. Since the detection probabilities are mainly based on the tails of the distribution, overestimation or underestimation of this part considerably affects the performance analysis accuracy. Therefore this motivated the development of the Duo GG and $\mathcal{M}$ -distributed fadings which are more fit to the experimental data, and hence lead to attain more precise performance analysis.

On the other hand, the merits of the $\mathcal{M}$ -distributed fading over the Duo GG fading are its fewer parameters, its novel physical model and its better fitness to the experimental data.

However a common considerable problem in the mixed RF/FSO AF schemes, in Table 1.1, is that they do not satisfy the non-negativity of the IM-DD FSO transmitted signal, even if the subcarrier intensity modulation (SIM) is used in some of them. Particularly, in these AF schemes the relay receives an unbounded bipolar signal, which can not be made positive, for the sake of IM-DD transmission, by mere amplification and/or applying a DC bias. Therefore, the mathematical model and the results of the above AF works do not reflect the correct performance of those mixed RF/FSO AF systems when IM-DD is used. This non-negativity of the IM-DD transmitted signal can be satisfied by using a DF mixed RF/FSO relaying system like the one employed in [63], [70]. Namely, in [63] the performance of multiuser mixed RF/FSO dual hop DF relaying system was investigated using V-BLAST technique, IM-DD, and G-G fading channels with pointing error.

Nevertheless, the problem in the works in [63], [70] is that they did not use the correct capacity results for IM-DD channel. Instead, they used the well-known Shan-

non's capacity formula for the power-constrained AWGN channel [73] which does not apply for IM-DD systems, because it does not consider the non-negativity constraint in the IM-DD transmitted signal.

1.3 Dissertation Contributions

The major contributions in this study are as follows:

- We utilize, for the first time to the best of our knowledge, in such systems outage and capacity analysis, a precise channel capacity approximation for the IM-DD channel. As a Consequence, this work presents precise quantitative comparison between HD and IM-DD performances, and hence helps the planners in deciding whether to deploy HD (higher rate) or IM-DD (lower cost/complexity).
- Moreover, to the best of our knowledge, this is the first time that not only the IM-DD input-independent but also the IM-DD cost-dependent AWGN channel is considered in such systems investigations. The difference between these IM-DD channel models is in the AWGN variance which does not depend on the input signal in the first model, while in the second one it is proportionate to the constraint (cost) of the input signal (optical average power constraint). It is important also to state that the IM-DD input-independent AWGN channel model is only valid wherever the thermal noise dominates the total receiver noise which is the case in the receivers that use the PIN-photodiode or in case of low optical power. Alternatively, IM-DD cost-dependent AWGN channel model is useful when the shot-noise dominates the total receiver noise which is the case

in the receivers based on the avalanche photodiode (APD), or in case of high optical power.

- Also, in BER analysis we assume that the two hops do not necessarily use the same modulation scheme. This assumption is valid in the DF relaying schemes, which is the case in our work, and it is one of their advantages. Especially, this is very important in mixed RF/FSO relaying as not all modulations are suitable for IM-DD FSO. Specifically, in the IM-DD systems we modulate the optical intensity (non-negative quantity) which does not have a phase, and hence all kinds of phase modulations are not applicable there.
- Additionally, the performance of the mixed RF/FSO system is studied asymptotically at the high SNRs. This asymptotic study results in new non-reported coding gain and diversity order analyses of such system. Interestingly, we find that in the FSO hop, at high transmitted powers all the FSO detectors result in the same diversity order, whereas their coding gains are different.
- Further, at high transmitted powers, we find that, in the FSO hop, the ergodic capacities pre-logs for HD and IM-DD input-independent are identical and equal twice the one of the IM-DD cost-dependent.
- Beside that, we show the overestimations which occur in the outage and ergodic capacity of the system when the well-known Shannon's capacity formula for the power-constrained AWGN channel is used for IM-DD instead of its precise capacity approx. These overestimations lead to misconception in the comparison

between HD and IM-DD.

- All the analyses is done in a unified manner for the FSO hop which combines HD, IM-DD input-independent, and (for the first time) IM-DD cost-dependent in a unified expression for each statistical characteristic or performance indicator. Note that obtaining these unified expressions is not straightforward. In addition, the above work assumes generalized fading models, i.e., Nakagami- m and $\mathcal{M}$ fading with pointings errors for the RF and FSO links, respectively.
- One more scheme that its performance was studied, for the first time as per our knowledge, in this dissertation is the dual-hop DF mixed IA MIMO RF IC/FSO relaying scheme in which the HD and IM-DD were considered in a unified analysis manner in the FSO hop. In the performance analysis of this scheme, the best-known FSO channel capacity results are utilized also both the IM-DD input-independent and the IM-DD cost-dependent AWGN channel are considered.
- In addition, the mixed IA MIMO RF IC/FSO system performance is asymptotically studied at high SNRs, where the diversity order and coding gain analyses are illustrated. It is found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2). Alternatively, under FSO hop domination, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors. Also, It is observed that the overall system ergodic capacity enhances by increasing “ d ” when the RF hop dominates the system.

- Furthermore, we explore the performance deterioration in the dual hop mixed (imperfect IA MIMO RF IC)/(FSO) relaying due to the consideration of the imperfect CSI. Considering imperfect CSI means that the interference will not be totally eliminated in this case. For this scheme, closed-form expressions are derived for the outage probability in terms of Whittaker and Meijer functions and for the ergodic capacity in terms of the bivariate Meijer's G-function.
- For the above system it is found that imperfect CSI results in zero diversity order causing a floor in the outage probability at high SNRs. This floor worsen with either the reduction in the CSI accuracy, or the decrement in " d ". Also under imperfect CSI, It is found that there is a ceil in the ergodic capacity at high SNRs, this ceil becomes worse with more interferers or when the CSI accuracy or the number of RF info streams (d) decreases.

CHAPTER 2

THEORETICAL BACKGROUND

2.1 Transceiver in Free Space Optical Systems

In an FSO transmission scheme, a source generates information signals, then these signals modulate an optical frequency carrier. The produced optical field is then propagated thru the atmosphere to a faraway destination. At the receiver side, the optical field is detected by a photo-detector which converts this optical field to an electrical current. Then the receiver processes this detected electrical current to decode the sent data [2].

2.1.1 The FSO Transmitter

As shown in Figure 2.1, the FSO transmitter contains an optical source, an encoder, a modulator, an optical amplifier (if needed), and beam forming optics [2]. Usually the source data is first encoded, before being modulated. After that the optical intensity of the modulated laser beam is amplified by passing it thru the optical amplifier. After that and before the transmission the optical beam is refocused by means of

beam forming optics. In FSO schemes the usual optical source is the laser diode (LD) [6], however some manufacturing companies employ high power light emitting diodes (LEDs) with beam collimators to make the optical source [7].

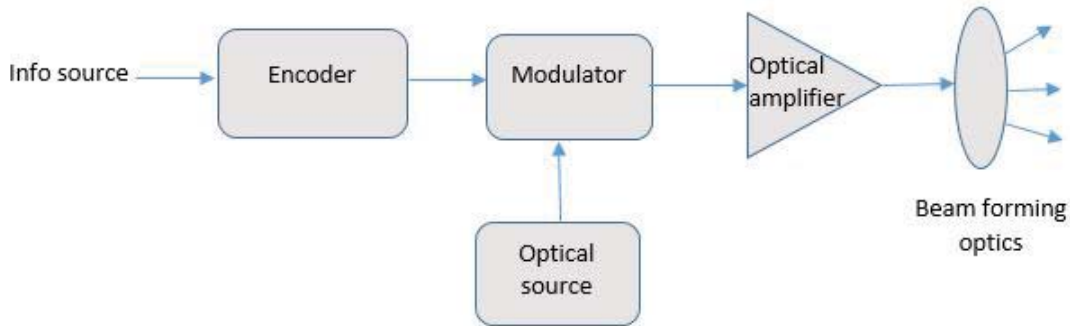


Figure 2.1: Typical block diagram of the FSO transmitter.

2.1.2 The FSO Receiver and Its Noise Models

Based on the detection method, FSO systems can be classified into two categories: non-coherent and coherent systems. In general the term “Heterodyne Detection” (HD) is used to refer to the coherent systems, while the optical non-coherent systems are also well-known as intensity modulation direct detection (IM-DD) systems. An important advantage of FSO HD systems is that they achieve higher data rates than the FSO IM-DD ones. This is due to the fact that the HD permits amplitude, frequency, or phase modulation, however the IM-DD systems transfer the information by modulating the optical intensity which does not have a phase, and thus the frequency and phase modulations are not valid there. Another significant advantage of HD is its ability to overcome the thermal noise. On the other hand the IM-DD systems are simpler and cheaper than the HD ones [2], [8]. Hence whether it is beneficial to deploy HD or not is determined by the gain attained by this deployment, in comparison with the

additional cost. Therefore, it is interesting to compare the performance of the two systems, and quantify the differences between them.

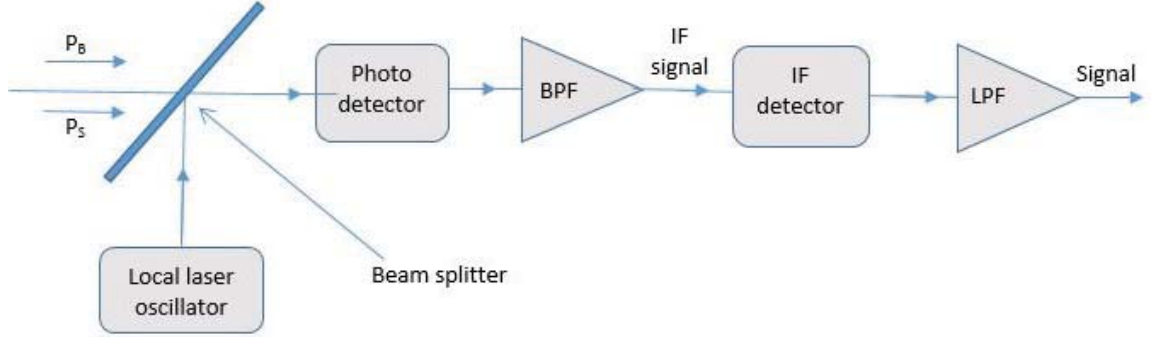


Figure 2.2: Simple optical heterodyne detector block diagram.

Heterodyne Detection

The optical heterodyne receiver is described in detail in [8]. As shown in Figure 2.2, the basic idea is the mixing of the optical received electric field (signal), before photo-detection, with an optical local oscillator (LO) to transform the optical received signal to a lower intermediate frequency at which detection is done. Let the received optical electric field be given by $a \cos(\omega t + \theta)$ where a , θ , and ω are respectively the amplitude, phase, and frequency of the optical received signal. Note that a and/or θ are modulated by the source data. Additionally, let the signal of the LO be given by $a_o \cos(\omega_o t)$ where a_o and ω_o are respectively the amplitude and frequency of the LO. Also denote the powers of these two signals by $P = a^2/2$ and $P_o = a_o^2/2$. Then after mixing the two signals, the detected photodiode (PD) output current is given by $i_x = \rho a a_o \cos((\omega - \omega_o)t + \theta)$ where ρ is a constant which depends on the efficiency of the detector among other parameters and known as the effective photo-electric conversion ratio. The corresponding detected signal electrical power is $2\rho^2 P P_o$ (load

resistance 1 Ohm). This operation induces a noise current i_z consists of thermal noise and shot noise. The shot noise can be considered to be Gaussian¹ with variance $2qBI$ where B is the bandwidth, q is the electron charge, and I is the sum of all shot noise induced currents [10]. This current is given by $I = \rho(P_o + P + P_b)$, where P_b represents the power of the background radiation. Adding thermal noise of variance σ_{th}^2 leads to an overall noise of variance $2qB\rho(P_o + P) + (2qB\rho P_b + \sigma_{th}^2)$. Let us combine the background noise (usually negligible as compared to the other) and the thermal noise to one noise with variance $4\rho^2\sigma_1^2$ where $\sigma_1^2 = \frac{qBP_b}{2\rho} + \frac{\sigma_{th}^2}{4\rho^2}$.

The resultant electrical signal-to-noise ratio (SNR) is $\gamma_{HD} = \frac{PP_o}{\sigma_2^2(P_o+P)+2\sigma_1^2}$ where $\sigma_2^2 = qB/\rho$. It is clear that this SNR depends on the power of the optical local oscillator. To guarantee a fair comparison between the IM-DD and HD, this dependence on the LO power has to be eliminated, which is possible by using sufficiently high LO power. Observe that increasing the LO signal power P_o leads to increase the SNR and makes the LO shot noise dominates all the other noise sources, resulting in an SNR of $\gamma_{HD} = \frac{P}{\sigma_2^2}$. Hence the channel in this case can be modeled as a complex AWGN channel with an SNR of γ_{HD} . In other words, the HD channel, with sufficiently high LO power, can be modeled by $Y = X + Z$ where X has power P and $Z \sim \mathcal{CN}(0, \sigma_2^2)$.

¹This is valid if the number of received photons in the symbol duration is large enough, which is the case in terrestrial FSO communication systems whose coverage distances are up to several kilometers. However in deep space FSO systems, where very small number of photons are received in the symbol duration, it is more realistic to model the detected current by a Poisson random process [9].

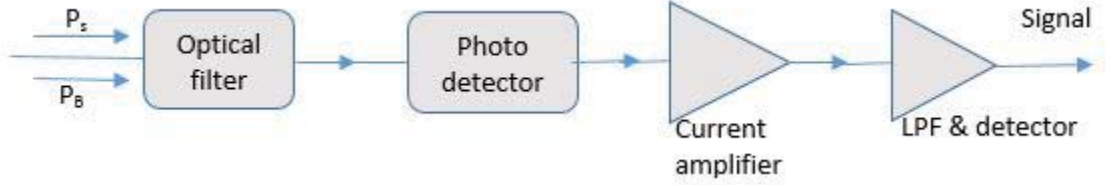


Figure 2.3: Simple optical direct detector block diagram.

Intensity Modulation Direct-Detection (IM-DD)

On the other hand, consider an IM-DD scheme with an optical received electric field $a \cos(\omega t + \theta)$. As shown in Figure 3, this signal is detected by a detection device (e.g. photodiode), which results in a detected current $i_x = \rho a^2/2 = \rho P$ and the corresponding electrical power is $\rho^2 P^2$. Similar to the HD scheme, this operation produces an additive noise current $\tilde{i}_z$ which consists of two independent noises namely the thermal noise and the shot noise. This total noise is a Gaussian one with variance $\tilde{\sigma}_z^2 = 2\sigma_2^2 \rho^2 P + 4\rho^2 \sigma_1^2$. Thus in this case, the channel can be modeled by an AWGN channel with input-dependent noise with output $\tilde{Y} = X + \tilde{Z}$, where $X = P \geq 0$ has power P^2 , and $\tilde{Z} = \tilde{i}_z/\rho$ is a real zero mean Gaussian noise with variance depends on X (optical intensity) and given by $\tilde{\sigma}^2 = 2\sigma_2^2 X + 4\sigma_1^2$. This model is an IM-DD AWGN channel with input-dependent noise, and was discussed in [11].

In some cases as in the receivers based on the PIN-photodiode, or in case of low SNR, the thermal noise dominates the noise $\tilde{Z}$. Hence in such cases the channel can be modeled by an IM-DD AWGN channel with input-independent noise $Z_{imdd}^{ind} \sim \mathcal{N}(0, 4\sigma_1^2)$. This channel model is simple and convenient to deal with, and it was studied in [12]–[14].

Alternatively in other cases as in the receivers based on the avalanche photodi-

ode (APD), or in case of high SNR, the shot-noise dominates the noise $\tilde{Z}$. Thus in such cases the channel model is an IM-DD AWGN channel with input-dependent noise $Z_{imdd}^{dep} \sim \mathcal{N}(0, 2\sigma_2^2 X)$. The analysis of this channel model is more complicated. However there is a simple channel model whose capacity is a lower bound for the capacity of the IM-DD input-dependent AWGN channel at high SNR. Actually this simple model captures (or matches) the high SNR performance (capacity scale) of the IM-DD input-dependent Gaussian channel model, and hence it can be used as a very good simple approximation for it at high SNR. This simple model was invented in [15] and known as “IM-DD AWGN Channel with Cost-dependent Noise”. In this model the noise is a real zero mean Gaussian with variance depends on the constraint (cost) of the input signal (optical intensity here). In our work we will consider the input average constraint (allowed optical average intensity, $\mathbb{E}[X] \leq \mathcal{E}$). So the noise in the cost-dependent channel is $Z_{imdd}^{cost} \sim \mathcal{N}(0, 2\sigma_2^2 \mathcal{E})$.

2.2 Fading in RF and FSO Channels

Electromagnetic wave propagation thru a wireless channel is a complex phenomenon that can be modeled as a random process. Thus to deal with this phenomenon some statistical models and characterizations are required. Therefore in this section we highlight some of these statistical models for both RF and FSO channels.

2.2.1 Multipath Fading in RF Channels

The wireless propagation of the Radio wave is characterized by a number of properties such as the time-varying multipath environment [16], [17]. The term multipath is used because the sent signal from the transmitter reaches the receiver via different several paths at the same time, this is due to the existence of some objects in the vicinity of the radio link which form several reflected paths. Moreover this multipath propagation has a time varying nature, i.e. the number of paths and the path length is changing from time to time. This time varying nature happens as a result of the movement of the receiver, or the transmitter, or some objects that form the reflections. This movement leads to the so called Doppler effect which causes a frequency shift on each component of the received signal.

Due to the time-varying multipath propagation, the received signal consists of a very large sum of attenuated, delayed, and phase-shifted replicas of the transmitted signal, each influencing each other [18]. According to the phase of each component, the superposition can be constructive or destructive and it is changing randomly for the same transmitted signal from time to time due to the time-varying propagation. Therefore the time-varying multipath propagation gives rise to random variations in both the phase and the amplitude of the received signal. These variations (fluctuations) have been called *channel fading*. To deal with these amplitude and phase random processes, in the analysis of wireless communication systems, their statistical models are required, i.e., their probability density functions (pdfs) at each time instant and at each frequency component are required, and also the correlations between

them from time instant to the other and from one frequency component to the other are required.

For the systems which use coherent modulations, the change in the received signal phase can severely degrade the system performance. In such systems, some measures are taken to compensate for such effects. In systems where noncoherent modulations are used, the changes in signal phase due to fading do not affect the system performance. The coherent systems where the phase variations are assumed to be fully compensated for are called ideal coherent systems. Hence, in ideal coherent and non-coherent modulation systems which experience fading, only the statistical model of the received signal envelope (amplitude) is required for evaluating the system performance².

Depending on the nature of the fading environment, several models have been proposed in the literature to characterize the distribution of the envelope of the multipath fading channel. One of these models is the Rayleigh distribution which is usually employed to model channels where there is no direct LOS path between the receiver and the transmitter [17]. In this case, the amplitude of the fading channel is a random variable and has the following distribution

$$f_{\alpha}(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0, \quad (2.1)$$

and thus, the instantaneous SNR per symbol of the fading channel γ has an exponential

²In general in coherent modulation systems, when needed, the random phase is modeled by the uniform distribution between 0 to 2π .

distribution given by

$$f_{\gamma}(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0, \quad \text{where } \bar{\gamma} = \Omega. \quad (2.2)$$

The Rayleigh fading model usually coincides very well with the experimentally measured data in mobile communication systems where normally no LOS exists between the transmitter and the receiver. It also applies to the propagation of refracted and reflected paths via the ionosphere and troposphere, and in case of ship-to-ship radio communications.

Another proposed model for the distribution of the envelope of the multipath fading channel is the Nakagami- m distribution. This distribution is basically a chi-square pdf which is given by

$$f_{\alpha}(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right), \quad \alpha \geq 0, \quad (2.3)$$

where m is the Nakagami- m fading parameter, which ranges from $\frac{1}{2}$ to ∞ . The SNR per symbol γ is distributed in this case according to the gamma distribution which is given by

$$f_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0, \quad \bar{\gamma} = \Omega, \quad (2.4)$$

which reduces to the Rayleigh fading when $m=1$.

The Nakagami- m distribution often provides the best fit to the multipath channel fading in indoor-mobile and land-mobile systems, as well as scintillating ionospheric

radio communications.

An alternative model which is proposed for the envelope distribution of the multipath fading channel is the Rician distribution which is suitable to model the propagation that consist of one dominant direct LOS path in addition to many other weaker paths. Here, the fading channel amplitude has a distribution given by

$$f_{\alpha}(\alpha) = \frac{2(1+K)e^{-K}\alpha}{\Omega} \exp\left(-\frac{(1+K)\alpha^2}{\Omega}\right) I_0\left(2\alpha\sqrt{\frac{K(1+K)}{\Omega}}\right), \quad \alpha \geq 0, \quad (2.5)$$

where I_0 denotes the modified Bessel function of the first type and order 0, and K is the Rician fading parameter that takes value from 0 to ∞ , and it represents the ratio between the power of the LOS path to the average power of the scattered paths.

In this case the SNR per symbol γ is a noncentral chi-square random variable whose pdf is given by

$$f_{\gamma}(\gamma) = \frac{(1+K)e^{-K}}{\bar{\gamma}} \exp\left(-\frac{(1+K)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right), \quad \gamma \geq 0. \quad (2.6)$$

The Rician distribution covers the range from non fading case (constant amplitude) ($K=\infty$) to Rayleigh fading ($K=0$). This kind of channel fading is usually noticed in the first resolvable LOS paths of the microcellular suburban and urban mobile systems, and the picocellular indoor systems. It is also valid for the dominant LOS satellite and ship-to-ship RF communication systems.

As mentioned previously a fading channel is characterized by the pdfs of the com-

plex (amplitude and phase) random process, which represents this fading channel, at each time instant and at each frequency component. Beside that the fading channel is also characterized by the correlations of this complex random process from time instant to the other and from one frequency component to the other. With regard to the above mentioned time domain correlation, the fading can be classified into two types, fast and slow fading. Distinguishing these two types is significant for modeling the fading channel mathematically and for evaluating the system performance. This concept is related to the coherence time T_c of the fading channel, which represents the time period after which the correlation function between two samples of the channel response collected at the same frequency but at different instants of time decreases under a certain determined threshold. The fading is considered slow if the channel coherence time T_c is larger than the signal symbol time duration T_s ; otherwise, it is said to be fast fading. In the slow fading, any fading level will affect several consecutive signal symbols and this will result in burst errors. While, in the fast fading, the fade effect will be different from one symbol to the other.

On the other hand according to the correlation of the fading channel random process in the frequency domain, the fading channel is classified to either a frequency-nonselective (flat) or frequency selective channel. If the fading channel affects all the frequency components of a transmitted signal in the same manner, in this case, the channel is considered frequency-flat or frequency-nonselective fading channel. This occurs in the narrowband schemes, where the bandwidth of the transmitted signal is much smaller than the coherence bandwidth f_c of the channel. This bandwidth represents the bandwidth during which the correlation function between two samples of the

fading channel response collected at the same time but by using different frequencies within this f_c , falls above a certain threshold. On the other hand, if the fading channel affects the frequency components of a transmitted signal in different manners, in this case, the fading channel is considered to be frequency-selective. This usually happens in the wideband systems where the bandwidth of the transmitted signal is bigger than the channel coherence bandwidth.

2.2.2 Atmospheric Turbulence Fading and Pointing Error in FSO Channels

As the RF links, the FSO links also suffer from the channel fading. However the reason which causes the channel fading in FSO links is different than the one which causes it in RF links. In fact the inhomogeneities in the pressure and the temperature of the atmosphere, occurred due to wind and solar heating, result in fluctuated air refractive index along the FSO channel [19]. The resultant atmospheric turbulence leads to arbitrary variations in the phase and the amplitude of the received signal. These variations (fluctuations) have been termed channel fading or scintillation. These effects produce a significant degradation in the performance of the FSO systems, particularly in the long distance transmissions that cover several kilometers.

As mentioned in the RF channel fading, to deal with these amplitude and phase random processes, in the analysis of FSO communication systems, their statistical models are required. Therefore several statistical models have been proposed in the literature to characterize the distribution of the atmospheric turbulence-induced chan-

nel fading in FSO communications. One of the commonly used models in case of weak atmospheric turbulence is the lognormal model. This model was developed on the base of the first-order Rytov approximation [20]. It is valid in the FSO systems that are used to cover relatively short distances. However, experimentally measured data in long-range FSO systems have demonstrated that the log-normal model is not applicable for the moderate-to-strong atmospheric turbulence. Another proposed model is the Rayleigh distribution model which has been suggested in case of very strong atmospheric turbulence conditions [21]. Also the so called K distribution has been proposed in [22] to model the strong atmospheric turbulence fading. The pdf of the received optical intensity I in this model is given by

$$f_I(I) = \frac{2m(mI)^{(m-1)/2}}{\Gamma(m)} K_{m-1} \left(2\sqrt{mI} \right), \quad \alpha > 0, I > 0, \quad (2.7)$$

where $K_n(.)$ is the n^{th} -order second-kind modified Bessel function, and m is a distribution parameter.

Furthermore there have been considerable works done to develop a general model that is appropriate to any kind of atmospheric turbulence. These research works are basically based on the doubly-stochastic scintillation theory. In this theory the small and large scale turbulence cells (eddies) are assumed to create diffractive and refractive effects on the optical beam, respectively [20]. One of the doubly-stochastic scintillation model is the widely used Gamma-Gamma distribution which models both strong and weak atmospheric turbulence fadings. In this model the received optical intensity I is assumed to be a product of two independent Gamma random variables

Y and X which denote the intensity fluctuations caused by the small and large scale atmospheric turbulence eddies, respectively. The pdf of the received intensity I in this case is given by

$$f_I(I) = \frac{2(\alpha\beta I)^{(\alpha+\beta)/2}}{I \Gamma(\alpha)\Gamma(\beta)} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta I} \right), \quad I > 0, \quad (2.8)$$

where the distribution parameters α and β are related to the numbers of effective large and small scale atmospheric turbulence eddies respectively, and $\Gamma(.)$ is the Gamma function.

The Double Generalized Gamma (Duo GG) distribution is another doubly-stochastic atmospheric turbulent channel model which was proposed in [23]. In this model again $I = YX$, however Y , and X are assumed to be two independent Generalized Gamma random variables, and hence this model contains Gamma-Gamma fading and others as special cases.

Another doubly-stochastic scintillation model is the $\mathcal{M}$ (Málaga) distribution which has been recently developed in [24], [25]. The novelty in the $\mathcal{M}$ distribution comes from the way (physical model) by which the small-scale eddies' fluctuation is modeled. Additionally this distribution contains most of the proposed statistical scintillation models, such as the K and the Gamma-Gamma distributions, as special cases. Moreover the simulation data showed the validity of the $\mathcal{M}$ distribution over a wide range of atmospheric turbulence conditions (strong to weak).

Beside modeling the distribution of the fluctuation in the optical intensity, another important issue is the spectral and temporal description of this fluctuation. Actually

almost all practical FSO systems are considered to have a very slow fading (scintillation) channel. This is because the FSO channel coherence time ranges normally from 0.1 to 10 msec, while the FSO symbol time duration is in the range from 1 to 100 nsec (high transmission rates from tens Mbps upto some Gbps). This means that the fading channel coefficient stays the same during the transmission of thousands upto millions successive bits. On the other hand the coherence bandwidth f_c in typical FSO systems ranges from 20 to 100 GHz. In fact this f_c is much bigger than the typical FSO signal bandwidth which is approximately equal the transmission rate. Therefore the practical FSO channels can be efficiently considered as frequency non-selective channels.

Another factor that affects the optical wave propagation, beside the atmospheric turbulence fading, is the misalignment losses or the so called pointing errors. This pointing error is a random variable and it normally happens as a result of the building swing or the optical beam wandering. Optical beam wandering is caused by inhomogeneities of large scale atmosphere eddies which lead to random deviations in the beam path. This is particularly significant in long distance FSO links. Alternatively the building swing is occurred due to several factors, such as the wind, thermal expansion, vibrations, and small earthquakes.

Various statistical models have been suggested for the pointing error as a random variable to analyze its effect in FSO systems. One of the commonly used models is the one provided in [26]. In developing this model it is assumed that the optical beam has a Gaussian beam profile, and the radial displacement at the detector has a Rayleigh distribution function.

2.3 Diversity

In wireless systems, having a channel in a deep fade results in an erroneous received signal. One way to combat such a problem is to provide the receiver with multiple copies of the same signal which is known as diversity. If p is the probability that the transmitted signal will fade under a critical threshold, then, p^L is the probability that L independent copies of that same transmitted signal will fade under that critical threshold. There are many ways by which the receiver can be provided with a set of independently faded copies of the same transmitted signal.

One popular method is the frequency diversity. In this method, several copies of the signal are sent on a set of independently faded carriers with a frequency separation between any two successive carriers being larger than the channel coherence bandwidth f_c . A second method is to send multiple copies of the same signal on a set of independently faded time slots with a time separation between the consecutive time slots being greater than the channel coherence time T_c . This method is known as time diversity. One may view the transmission of the same data either at different frequencies or different time slots or at both as a simple form of repetition coding. The separation between carriers f_c and between time slots T_c is essentially a form of block-interleaving of the bits in the repetition code in an attempt to break up the error bursts and, hence, to get independent errors. Another commonly used method is the space or antenna diversity. In this technique, two or more antennas are used to send and/or to receive multiple replicas of the same signal. It is based on the idea that sending or receiving a signal using geographically-spaced antennas will result in

independently faded copies of this signal. This method can be implemented at the transmit side (transmit space diversity) or at the receive side (receive space diversity) or at both. The antenna elements must be spaced sufficiently far apart so that the signals have significantly different propagation paths between the transmitter and receiver antennas. Typically, a separation of a few wavelengths is needed between two antennas in order to get signals that fade independently.

2.4 Relaying Networks

As mentioned previously, an energy-efficient way to extend the coverage area in wireless communications is to use the relaying networks (cooperative networks) [3]. Several areas in relay or cooperative networks have been opened for research work. Among these topics are the relaying schemes, relaying protocols, relay selection, multiple antennas cooperative systems, multiuser cooperative networks, parallel relaying (multi-relay) networks, and combining techniques at the receive side. A main difference between the relaying scheme and the relaying protocol is that the relaying scheme specifies the way the relay cooperates in a cooperative system. Whereas the relaying protocols answer the question about when and how should the relay cooperate in a relay system.

The most common relaying schemes are the amplify-and-forward (AF) and the decode-and-forward (DF) [4]. In AF scheme, after receiving the signal from the source, the relay simply amplifies it and then retransmits to the destination without any decoding. In DF scheme, the relay first decodes the source signal and then retransmits

it to the destination and hence it is a more complicated system when compared to the AF scheme. In general, the AF systems are classified into two subcategories, the channel-state information (CSI)-based gain (variable gain) relays, which use the instantaneous CSI from the previous hop, and the fixed-gain relays, which introduce a fixed gain in forwarding the source signals. The CSI-assisted AF relay system requires a continuous estimation of the channel fading amplitudes to produce its gain and to limit the output power of the relay. In contrast, the fixed-gain relay system introduces a fixed scale to the received signal regardless of the fading amplitudes which leads to a variable signal power at the relay output. This subcategory has a lower complexity as compared to the former one.

Among the relaying protocols is the fixed relaying where a set of relays are used to forward the source signal, the selection relaying where a relay or a number of relays are chosen to do the relaying process, and the incremental relaying where the relay or a number of relays are asked to cooperate according to the quality of the direct link [3]. In systems where multiple relays are used, several selections schemes have been proposed in the literature. One popular scheme is the opportunistic relaying where only the best relay is chosen to do the relaying process. The relay selection criterion may depend on the quality of the first hop, second hop, or it could depend on a certain threshold. Moreover some authors have introduced the technique of multiple antennas to cooperative systems at either the source, the relay, the destination, or any combination of them. Recently, the topic of parallel relaying (multi-relay) networks has been studied by many authors. In these networks, the relays cooperate in receiving or forwarding signals to the other nodes and this results in a diversity gain. Finally, the

way the signals on both the direct link and the relayed path or pathes are combined at the destination has been studied in many papers. Among these techniques are the selection combining (SC), maximal ratio combining (MRC), and the equal gain combining (EGC).

2.5 Interference Alignment

Interference alignment (IA) is a technique to eliminate the interference in interference channels³ [28]. To do that IA exploits the knowledge of the channel state information of all transmission links to form precoding (beamforming) at the transmitters as well as interference elimination combining at the receivers. In other words, the main idea of the IA is to build (beamform) the transmitted signals in a way such that the interference signals at any receiver overlay on the same subspace while the desired signal, at that receiver, remains separable at another orthogonal subspace [28]. If this occurs, interference elimination is possible by projecting the total received signal, using interference elimination combining matrix, on the desired signal subspace (zero-forcing the interference). The core of this technique is essentially located in the design of the transmitters' beamforming matrices.

Eliminating the interference in ICs by the IA leads to enhance (or to optimize) the multiplexing gain (also known as capacity pre-log or DoF) in those channels. Actually

³The interference channel [27] is a telecommunication network in which the communication medium is shared by several transmitter–receiver pairs, where each transmitter sends desired info only to its receiver while causing interference to the other receivers.

by using IA it has been shown that the interference channel is not basically interference limited, because it is proved by IA that in an IC each transmitter-receiver pair can simultaneously communicate with a rate equal to half of its interference-free channel capacity, even if the number of users K become arbitrarily big [28].

To illustrate the IA concept we use the K -pairs MIMO interference channel shown in Figure 2.4 where transmitter i has N_T antennas and needs to send d_i independent info streams, each one is equivalent to one DoF, to receiver i which has N_R antennas. In this channel for the IA to be feasible there should exist $N_T \times d_i$ transmission precoding matrices V_i , with $\text{rank}(V_i) = d_i$, and $N_R \times d_i$ receiving interference elimination combining matrices U_i , with $\text{rank}(U_i) = d_i$, such that for all $i, j \in \{1, 2, \dots, K\}, i \neq j$, we have [29]

$$U_i^\odot H_{ij} V_j = \mathbf{0}_{d_i \times d_j}, \quad (2.9)$$

$$\text{rank}(U_i^\odot H_{ii} V_i) = d_i, \quad (2.10)$$

where $U_i^\odot$ is the complex transpose of U_i , and H_{ij} is $N_R \times N_T$ matrix which represents the channel between receiver i and transmitter j . Therefore using the above notation the interference eliminated received $d_i \times 1$ signal vector at receiver i is given by

$$y_i = U_i^\odot H_{ii} V_i x_i + \underbrace{\sum_{j=1, j \neq i}^K U_i^\odot H_{ij} V_j x_j}_{\text{Zero due to (2.9)}} + U_i^\odot n_i, \quad (2.11)$$

$$y_i = U_i^\odot H_{ii} V_i x_i + U_i^\odot n_i, \quad (2.12)$$

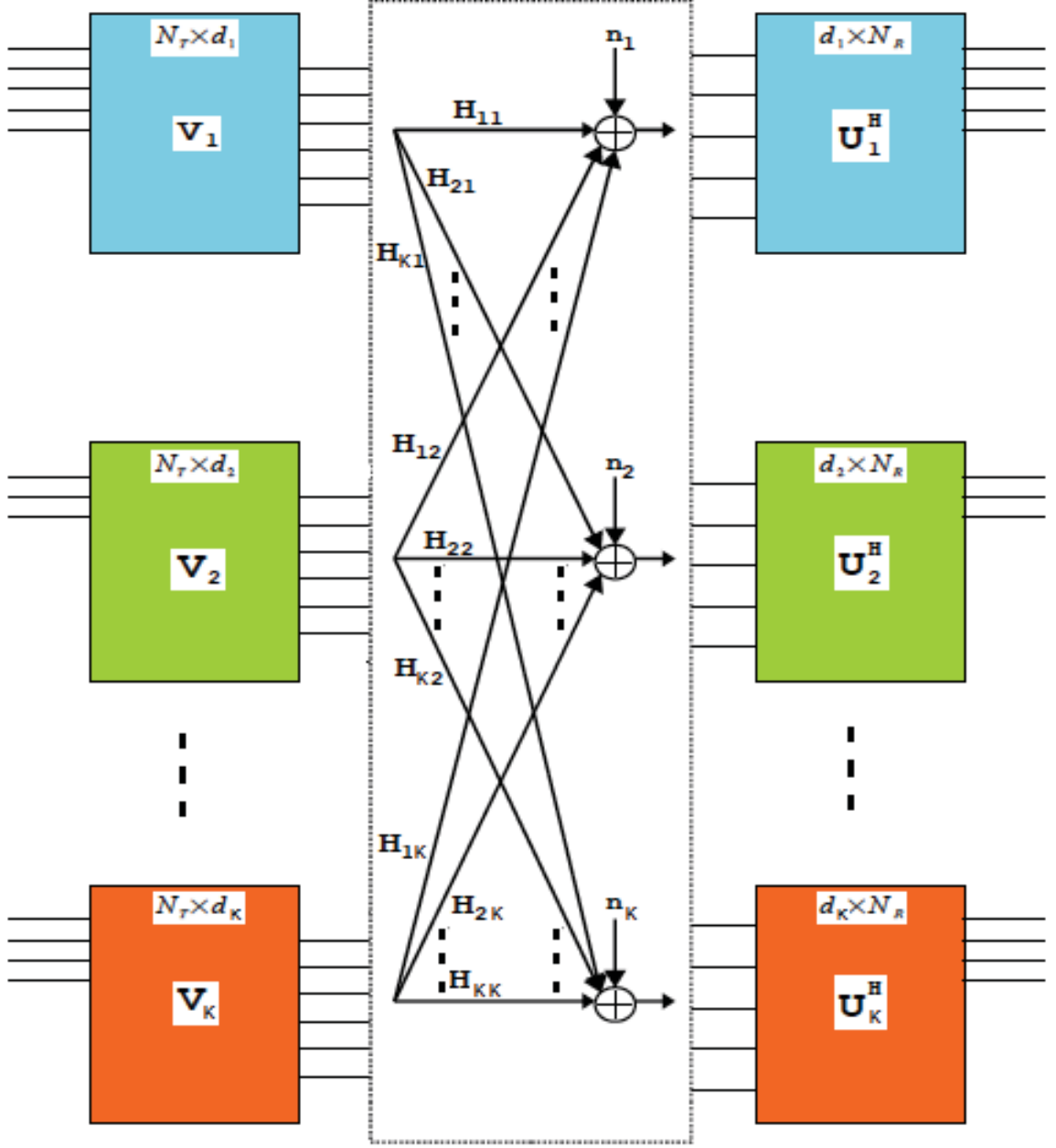


Figure 2.4: K -pairs MIMO interference channel with IA.

where x_i is the sent $d_i \times 1$ info vector from transmitter i , and n_i is the $N_R \times 1$ AWGN vector at receiver i .

CHAPTER 3

DUAL-HOP MIXED RF/UNIFIED-FSO RELAYING SCHEMES

3.1 Introduction

Due to the significance of the mixed RF/FSO relaying and to overcome the mentioned problems in its analysis, see Section 1.2.3, we investigate in this chapter the performance of dual-hop mixed RF/unified FSO DF relaying system, where both HD and IM-DD are considered in the FSO hop. To this end, we derive closed-form expressions for the outage probability, BER, and ergodic channel capacity for this scheme with the following contributions:

- We correctly utilize, for the first time to the best of our knowledge, in such system outage and capacity analysis, a precise channel capacity approximation

for the IM-DD channel deduced in [12]–[15], [77]. This precise approximation is tight at high-SNR, and hence it can serve as the channel capacity in that regime. Consequently, this work presents precise quantitative comparison between HD and IM-DD performances, and thus helps the designers in deciding whether to deploy HD (higher rate) or IM-DD (lower cost/complexity).

- Moreover, to the best of our knowledge, this is the first time that not only the IM-DD input-independent but also the IM-DD cost-dependent AWGN channel is taken into account in such system analyses. The difference between these two IM-DD channel models is in the AWGN variance which is independent of the input signal in the first model, while in the second model it is proportional to the constraint (cost) of the input signal (optical average power constraint here). Also, it is worthy to mention that the IM-DD input-independent AWGN channel model [12], [14] is only applicable wherever the thermal noise dominates the total receiver noise which is the case in the receiver that uses the PIN-photodiode or in case of low optical powers. Alternatively, IM-DD cost-dependent AWGN channel model [15], [77] is useful when the shot-noise dominates the total receiver noise which is the case in the receiver based on the avalanche photodiode (APD), or in case of high optical powers.
- Also, we run the BER analysis under the assumption that the two hops do not necessarily use the same modulation scheme. This assumption is valid in the DF relaying schemes, which is the case in our work, and it is one of their advantages. Especially, this is very important in mixed RF/ FSO relaying as not all mod-

ulation schemes are suitable for IM-DD FSO communications. Specifically, in the IM-DD systems we modulate the optical intensity (non-negative quantity) which does not have a phase, and hence all kinds of phase modulations are not applicable there.

- Additionally, the performance of the system is studied asymptotically at the high SNRs regime. This asymptotic study results in new non-reported coding gain and diversity order analyses of such system. Interestingly, we find that in the FSO hop, based on the average SNRs, the HD or IM-DD cost-dependent results in the same diversity order which is twice the one of IM-DD input-independent. However, each FSO detector has a distinct relation between its average SNR and the transmitted power, thus based on the transmitted power all the FSO detectors result in the same diversity order, whereas the coding gain is different for each FSO detector.
- Further, at high transmitted powers, we find that, in the FSO hop, the ergodic capacities pre-logs for HD and IM-DD input-independent are identical and equal twice the one of the IM-DD cost-dependent.
- Furthermore, we show the overestimations which occur in the outage and ergodic capacity of the system when the well-known Shannon's capacity formula for the power-constrained AWGN channel is used for IM-DD instead of its precise capacity approx. These overestimations lead to misconception in the comparison between HD and IM-DD.
- All the above analysis is done in a unified manner for the FSO hop which

combines HD, IM-DD input-independent, and (for the first time) IM-DD cost-dependent in a unified expression for each statistical characteristic or performance indicator. Note that obtaining these unified expressions is not straightforward.

In doing this work, we assume that the FSO link follows Málaga ($\mathcal{M}$) fading with pointing errors, whereas the RF link follows Nakagami- m fading which can be used to model the LOS transmission (see e.g. [64], [76]) between the source and relay. Note that Nakagami- m also models the non LOS (NLOS) transmission (i.e. Rayleigh fading) as a special case. Also, notice that the $\mathcal{M}$ fading model contains other FSO fading models (including G-G) as special cases. Therefore, our performance analyses can be seen as a generalized one from the FSO and RF fading models point of view.

The remainder of this chapter is arranged as follows. The system model is presented in Section 3.2. Section 3.3 analyzes the system performance. Numerical and simulation results are carried out and discussed in Section 3.4, these results confirm the accuracy of the derived analytical and asymptotic expressions. Finally, conclusions are provided in Section 3.5.

3.2 System and Channel Models

The system under study is a dual-hop mixed RF/FSO relaying system which contains a source S, a DF relay R, and a destination D as shown in Fig. 3.1. The source communicates with the relay over an RF link and the relay communicates with the destination over an FSO link. The direct S-D link is assumed to be infeasible.



Figure 3.1: Dual-hop mixed RF/FSO DF relaying scheme

The transmission from S to D happens over two phases (i.e. two channel uses): S $\rightarrow$ R and R $\rightarrow$ D. In the first phase, the received RF signal at R from S can be given by

$$y_{\text{RF}} = \sqrt{P_s} h_{\text{RF}} x_{\text{RF}} + n_r, \quad (3.1)$$

where x_{RF} is the transmitted signal from S with $\mathbb{E}\{|x_{\text{RF}}|^2\} = 1$, P_s is S transmitted power, h_{RF} is the coefficient of the fading channel from S to R, $n_r \sim \mathcal{CN}(0, N_0)$ is an AWGN, and $\mathbb{E}\{\cdot\}$ denotes the expectation operator. Hence, the SNR at R can be expressed as $\gamma_{\text{RF}} = \frac{P_s}{N_0} |h_{\text{RF}}|^2$. We assume that the channel coefficient of the RF link follow Nakagami- m fading model, where in this work we use k to denote the parameter m and restrict it to be an integer value. Thus, the probability density function (pdf) of γ_{RF} is given by [76]

$$f_{\gamma_{\text{RF}}}(\gamma) = \left(\frac{k}{\bar{\gamma}_{\text{RF}}} \right)^k \frac{\gamma^{k-1}}{(k-1)!} \exp \left(-\frac{k}{\bar{\gamma}_{\text{RF}}} \gamma \right), \quad (3.2)$$

where $\bar{\gamma}_{\text{RF}}$ is the average received RF SNR. Namely, $\bar{\gamma}_{\text{RF}} = \frac{P_s}{N_0} \mathbb{E}\{|h_{\text{RF}}|^2\} = \frac{P_s}{N_0} \Omega_{\text{RF}}$, where we define Ω_{RF} as $\Omega_{\text{RF}} = \mathbb{E}\{|h_{\text{RF}}|^2\}$. Note that $(P_s \Omega_{\text{RF}})$ designates the average received power at R, and hence Ω_{RF} can represent the multiplication of the RF link losses (L_{RF}), gains (K_{RF}) and design margins (M_{RF}). These losses include RF transmitter

losses ($L_{t_{\text{RF}}}$), RF receiver losses ($L_{r_{\text{RF}}}$), path loss ($L_{s_{\text{RF}}}$) (This loss is constant for a given link distance and weather condition), etc. Thus, for example, we can have

$$\Omega_{\text{RF}} = L_{\text{RF}} K_{\text{RF}} M_{\text{RF}} = L_{t_{\text{RF}}} L_{r_{\text{RF}}} L_{s_{\text{RF}}} K_{\text{RF}} M_{\text{RF}}.$$

It is worthy to say that when ($k=1$) the RF channel model reduces to the Rayleigh fading model as a special case.

After receiving the signal, R forwards a decoded version of it to the destination in the second transmission phase. Let x_{HD} and x_{IMDD} represent the transmitted signal from R to D in case of HD and IM-DD, respectively, with average power constraints $\mathbb{E}\{|x_{\text{HD}}|^2\} = 1$ and $\mathbb{E}\{x_{\text{IMDD}}\} = 1$. Note that the average power constraint in the IM-DD is on the expected value of the signal and not on its square because the signal in this case represents an optical intensity. Consequently, the received FSO signal at D from R in case of HD and IM-DD can be, respectively, given by

$$y_{\text{HD}} = \sqrt{P_r} g_{\text{FSO}} x_{\text{HD}} + n_{\text{HD}}, \quad (3.3)$$

$$y_{\text{IMDD}} = P_r I_{\text{FSO}} x_{\text{IMDD}} + n_{\text{IMDD}}, \quad (3.4)$$

where P_r is R transmitted power, g_{FSO} and $I_{\text{FSO}} = |g_{\text{FSO}}|^2$ represent the fluctuation in the received electric field and irradiance, respectively. This fluctuation is due to the atmospheric turbulence in the channel between R and D. In (3.3), the noise $n_{\text{HD}} \sim \mathcal{CN}(0, N_{\text{HD}})$ is a complex AWGN. While in (3.4), the noise n_{IMDD} is a real AWGN where $n_{\text{IMDD}} \sim \mathcal{N}(0, N_{\text{IMDD}})$ in case of IM-DD input-independent AWGN channel, and $n_{\text{IMDD}} \sim \mathcal{N}(0, 2P_r I_{\text{FSO}} N_{\text{HD}} \mathbb{E}\{x_{\text{IMDD}}\})$ in case of IM-DD cost-dependent AWGN channel assuming relatively high P_r as compared to the thermal plus background noise power.

For the details of the HD and both types of IM-DD and their noise variances, the reader is referred to [15], [77]. Thus the SNRs at D, upon which the outage, capacity, and BER are based, are introduced as follows for HD, IM-DD input-independent, and IM-DD cost-dependent, respectively,

$$\begin{aligned}\gamma_{\text{HD}} &= \frac{P_r |g_{\text{FSO}}|^2}{N_{\text{HD}}} \mathbb{E}\{|x_{\text{HD}}|^2\} = \frac{P_r I_{\text{FSO}}}{N_{\text{HD}}}, \\ \gamma_{\text{IMDD}_{\text{indp}}} &= \frac{P_r^2 I_{\text{FSO}}^2}{N_{\text{IMDD}}} \mathbb{E}^2\{x_{\text{IMDD}}\} = \frac{P_r^2 I_{\text{FSO}}^2}{N_{\text{IMDD}}}, \\ \gamma_{\text{IMDD}_{\text{cost}}} &= \frac{P_r^2 I_{\text{FSO}}^2 \mathbb{E}^2\{x_{\text{IMDD}}\}}{2P_r I_{\text{FSO}} N_{\text{HD}} \mathbb{E}\{x_{\text{IMDD}}\}} = \frac{P_r I_{\text{FSO}}}{2N_{\text{HD}}}.\end{aligned}\tag{3.5}$$

In this work, we assume that the R→D FSO link experiences $\mathcal{M}$ fading model with the pointing error effect. In developing the $\mathcal{M}$ fading model [24], it is assumed that the received signal consists of a LOS component (U_L), a component scattered by eddies on the propagation axis and coupled to the LOS (U_S^C), and a component scattered by off-axis eddies (U_S^G) which is statistically independent from U_L and U_S^C . So as a result of adopting the $\mathcal{M}$ fading model with pointing error for the FSO link, the pdf of the received irradiance I_{FSO} is given by [25, Eq. (21)]

$$f_{I_{\text{FSO}}}^{\mathcal{M}}(I) = \frac{\zeta^2 A}{2I} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{\alpha \beta}{g \beta + \Omega'} \frac{I}{A_0} \middle| \zeta^2 + 1 \right]_{\zeta^2, \alpha, m}, \tag{3.6}$$

where

$$A = \frac{2\alpha^{\alpha/2}}{g^{1+\alpha/2}\Gamma(\alpha)} \left(\frac{g \beta}{g \beta + \Omega'} \right)^{\beta+\alpha/2}, \quad b_m = a_m [\alpha \beta / (g \beta + \Omega')]^{-(\alpha+m)/2},$$

$$a_m = \binom{\beta-1}{m-1} \frac{(g\beta + \Omega')^{1-m/2}}{(m-1)!} \left(\frac{\Omega'}{g}\right)^{m-1} \left(\frac{\alpha}{\beta}\right)^{m/2},$$

α is a positive parameter related to the effective number of large-scale cells of the scattering (fluctuation) process, β is a natural number¹ which represents the amount of fading related to the effective number of small-scale cells of the fluctuation process², $\Omega' = \Omega + 2b_0\rho + 2\sqrt{2b_0\rho\Omega} \cos(\Phi_A - \Phi_B)$ is the average power of the coherent contributions, $\Omega = \mathbb{E}\{|U_L|^2\}$, $\mathbb{E}\{|U_S^C|^2\} = 2b_0\rho$, Φ_A and Φ_B are the deterministic phases of the LOS and the coupled-to-LOS signal components, respectively, $2b_0 = \mathbb{E}\{|U_S^C|^2 + |U_S^G|^2\}$ is the total average power of the scattered components, $g = \mathbb{E}\{|U_S^G|^2\} = 2b_0(1 - \rho)$, the parameter $0 \leq \rho \leq 1$ indicates the amount of the scattered power coupled to the LOS³, ζ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver (i.e. $\zeta \rightarrow \infty$,

¹In this work we restrict β to be a natural number. This restriction leads to a closed-form expression with a finite summation ($\sum_{m=1}^{\beta}$) for the pdf of I_{FSO} as in (3.6), and hence more evident analytical tractability is assured. Also, due to the high degree of freedom of the $\mathcal{M}$ distribution, as shown in [24], the aforementioned restriction does not prevent it to offer an excellent fitting to the experimental data, in all turbulent conditions, without the need for the infinite summation ($\sum_{m=1}^{\infty}$) which appears in the pdf of I_{FSO} , as given by [25, Eq. (23)], in the generalized case when β is a real number.

²The fading parameters α and β are related to the atmospheric turbulence conditions (Rytov variance) with lower values generally representing severe (strong) atmospheric turbulence conditions having the other parameters fixed.

³The details of the $\mathcal{M}$ distribution and the generation of its random variable can be drawn out from [24, Eqs. (13-21)].

represents the non-pointing error case)⁴, A_0 is a constant which defines the pointing loss (or generally the FSO link losses and design margins), $\Gamma(\cdot)$ is the Gamma function as defined in [74, Eq. (8.310)], and $G[\cdot]$ is the Meijer's G-function as defined in [74, Eq. (9.301)].

It is worthy to mention that, in general the constant A_0 can represent the multiplication of the FSO link losses (L_{FSO}), gains (K_{FSO}) and design margins (M_{FSO}). These losses include FSO transmitter losses (Lt_{FSO}), FSO receiver losses (Lr_{FSO}), pointing loss (Lp_{FSO}), path loss (Ls_{FSO}) (This loss is constant for a given link distance and weather condition), etc. Thence, for example, we can have $A_0 = L_{\text{FSO}}K_{\text{FSO}}M_{\text{FSO}} = Lt_{\text{FSO}}Lr_{\text{FSO}}Lp_{\text{FSO}}Ls_{\text{FSO}}K_{\text{FSO}}M_{\text{FSO}}$.

From the pdf in (3.6) (i.e. $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$) and by using random variable transformation, we obtain the pdfs of the SNRs in (3.5) in the following unified form

$$f_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) = \frac{\zeta^2 A}{2^s \gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[B \left(\frac{w\gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, m \end{matrix} \right], \quad (3.7)$$

where γ_{FSO} is used to indicate the unified SNR of the FSO link, $B = \zeta^2 \alpha \beta (g + \Omega') / [(\zeta^2 + 1)(g\beta + \Omega')]$, s and w are the parameters that identify the type of the FSO detection, i.e., $(s, w) = (1, 1)$, $(2, 1)$ or $(1, 2)$ for HD, IM-DD input-independent, and IM-DD cost-dependent respectively, and where $\mu_1 \triangleq P_r \mathbb{E}\{I_{\text{FSO}}\} / N_{\text{HD}} = A_0 P_r \zeta^2 (g + \Omega') / [(\zeta^2 + 1)N_{\text{HD}}]$,⁵ and for normalization purposes $\mu_2 \triangleq P_r^2 \mathbb{E}\{I_{\text{FSO}}^2\} / N_{\text{IMDD}}$ as previously defined in [79] and references therein. This definition results in $\mu_2 = A_0^2 P_r^2 \zeta^4 (g + \Omega')^2 / [(\zeta^2 + 1)N_{\text{IMDD}}]$.

⁴For more details on the pointing error model, the reader is referred to [78].

⁵ $\mathbb{E}\{I_{\text{FSO}}\}$ and $\mathbb{E}\{I_{\text{FSO}}^2\}$ are derived from [25, Eq. (33)].

$1)^2 N_{\text{IMDD}}]$. From these expressions of μ_1 , μ_2 , and (3.5), it is clear that μ_s is proportional to the average of the unified FSO SNR ($\bar{\gamma}_{\text{FSO}}$). Specifically, $\mu_1 = \bar{\gamma}_{\text{HD}} = 2\bar{\gamma}_{\text{IMDD}_{\text{cost}}}$, however $\mu_2 = \bar{\gamma}_{\text{IMDD}_{\text{indp}}} \frac{\mathbb{E}\{I_{\text{FSO}}\}}{\mathbb{E}\{I_{\text{FSO}}^2\}} = \frac{\zeta^2(\zeta^2+1)^{-2}(\zeta^2+2)(g+\Omega')^2}{\alpha^{-1}(\alpha+1)[2g(g+2\Omega')+\Omega'^2(1+1/\beta)]} \bar{\gamma}_{\text{IMDD}_{\text{indp}}}$.⁵

Notice that when $\rho=1$ (i.e. $g=0$), we get $Ab_m = 2/(\Gamma(\alpha)\Gamma(\beta))$ for $m=\beta$ and $Ab_m=0$ for all other m . Thus, when $\rho=1$, and $\Omega'=1$ (for normalization), the pdf $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (3.6) turns to the G-G distribution with pointing error, given by [80, Eq. (1)], as a special case. Actually, having $\rho=1$ means that the power $\mathbb{E}\{|U_S^G|^2\} \triangleq g = 0$, i.e., in the G-G fading channel model, the received signal component scattered by off-axis eddies (U_S^G) is neglected. Therefore, by setting $\Omega'=1$ and $\rho=1$ in (3.7), the unified pdf of the FSO SNR in G-G fading with pointing error can be given by

$$f_{\gamma_{\text{FSO}}}^{\text{G-G}}(\gamma) = \frac{\zeta^2}{s\gamma\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2\alpha\beta}{(\zeta^2+1)} \left(\frac{w\gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, \beta \end{matrix} \right]. \quad (3.8)$$

3.3 System Performance Analysis

3.3.1 Outage Probability

The outage probability (P_{out}) is an important performance metric in slow fading channels which are the channels in most RF and FSO systems [2], [81] including the system here. It is defined, as in [81], by $P_{\text{out}} = \Pr[C < R]$, where $\Pr[\cdot]$ represents the probability operator, R is the end-to-end (e2e) desired transmission rate in bits/channel-use, C is a function on the random equivalent-SNR of the e2e channel. Therefore, C is a random variable whose realization represents the e2e channel capacity given the

corresponding realization of the e2e channel equivalent-SNR. As in any dual-hop DF relaying scheme, in the system under study, the outage event occurs if either the first hop or the second hop is in outage, i.e., the e2e outage event is the union of the outage events of the two hops. Thus the outage probability in our system is given by ⁶

$$P_{\text{out}} = \Pr \left[C_{\text{RF}} < 2R \cup C_{\text{FSO}} < 2R \right], \quad (3.9)$$

$$= \Pr \left[\frac{1}{2} \min [C_{\text{RF}}, C_{\text{FSO}}] < R \right], \quad (3.10)$$

where C_{RF} and C_{FSO} are related to the channel capacity of the first hop (RF link) and the second hop (FSO link), respectively.

Specifically, C_{RF} is a function on the random SNR of the first hop. Hence, C_{RF} is a random quantity whose realization represents the channel capacity of the first hop given the corresponding realization of the first hop SNR (γ_{RF}). However, given γ_{RF} , the first hop is an AWGN channel as represented by (3.1). So, utilizing the AWGN channel capacity expression, given in [73], results in⁷

$$C_{\text{RF}} = \log(1 + \gamma_{\text{RF}}). \quad (3.11)$$

⁶ R is the e2e desired transmission rate in bits/channel-use. Also, here the desired rates of the two hops are assumed equal, and hence the factor 2 is due to the assumption that the e2e transmission (from S to D) happens over two equal phases (i.e. two equal channel uses) which means that each hop rate is twice the e2e rate.

⁷In this work, unless otherwise specified, the log base is 2.

On the other hand, C_{FSO} is a function on the random SNR of the second hop, and hence it is also a random quantity. Moreover, in case of HD the realization of C_{FSO} represents the channel capacity of the second hop given the corresponding realization of the second hop SNR (γ_{HD}). Alternatively, in case of IM-DD we relax the manner and let the realization of C_{FSO} represents an excellent approximation (just above the lower bound)⁸ of the channel capacity of the second hop given the corresponding realization of the second hop SNR ($\gamma_{\text{IMDD}_{\text{indp}}}$ or $\gamma_{\text{IMDD}_{\text{cost}}}$). However, given γ_{HD} , $\gamma_{\text{IMDD}_{\text{indp}}}$ or $\gamma_{\text{IMDD}_{\text{cost}}}$ the second hop is, respectively, an AWGN channel, IM-DD input-independent AWGN channel, or IM-DD cost-dependent AWGN channel as represented by (3.3) or (3.4). Hence, using the expressions of the AWGN channel capacity [73] and the approximations of the two IM-DD AWGN channel capacities [12], [15] results in

$$C_{\text{FSO}} = \begin{cases} \log(1 + \gamma_{\text{HD}}) & \text{for HD} \\ \log\left(1 + \sqrt{\frac{e}{2\pi}} \gamma_{\text{IMDD}_{\text{indp}}}\right) & \text{for IM-DD input-independent} \\ \log\left(1 + \sqrt{\frac{e}{2\pi}} \gamma_{\text{IMDD}_{\text{cost}}}\right) & \text{for IM-DD cost-dependent} \end{cases}, \quad (3.12)$$

or in a unified form

$$C_{\text{FSO}} = \log(1 + \gamma_{\text{FSO}}^{\text{eff}}), \quad (3.13)$$

where $\gamma_{\text{FSO}}^{\text{eff}}$ is used to represent the unified effective SNR of the FSO link, i.e., $\gamma_{\text{FSO}}^{\text{eff}}$

⁸We use this approx. (just above the lower bound) because the IM-DD channel capacity is yet unknown in closed-form [14]. However, this approx. is very tight at high-SNR, and hence it can serve as the IM-DD channel capacity in that regime.

equals either γ_{HD} , $\sqrt{\frac{c}{2\pi}\gamma_{\text{IMDD}_{\text{indp}}}}$, or $\sqrt{\frac{c}{2\pi}\gamma_{\text{IMDD}_{\text{cost}}}}$ based on the used FSO transmission scheme. Due to its importance in what follows, let us calculate the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$. From $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (3.6), with the help of (3.5), and by using random variable transformation, the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ can be given in a unified form as

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) = \frac{w \zeta^2 A}{2\gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{B (\gamma/c)^w}{\mu_s^{1/s}} \middle| \zeta^2+1 \right]_{\zeta^2, \alpha, m}, \quad (3.14)$$

where $c = 1, \sqrt{\frac{c}{2\pi}}$, or $\sqrt{\frac{c}{4\pi}}$ for HD, IM-DD input-independent, or IM-DD cost-dependent, respectively, and the rest of parameters are as in (3.6) and (3.7). Note that by setting $\Omega'=1$ and $\rho=1$ in (3.14), we get the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ in the special case of G-G fading with pointing error as follows

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma) = \frac{w \zeta^2}{\gamma \Gamma(\alpha) \Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| \zeta^2+1 \right]_{\zeta^2, \alpha, \beta}. \quad (3.15)$$

Now let's go back to P_{out} and substitute (3.11) and (3.13) in (3.10) to get

$$P_{\text{out}} = \Pr \left[\min [\gamma_{\text{RF}}, \gamma_{\text{FSO}}^{\text{eff}}] < (2^{2R} - 1) \right]. \quad (3.16)$$

From (3.16) we realize that P_{out} is nothing but the cumulative distribution function (CDF) of $\gamma_{eq} \triangleq \min[\gamma_{\text{RF}}, \gamma_{\text{FSO}}^{\text{eff}}]$ evaluated at $\gamma_{\text{out}} \triangleq (2^{2R} - 1)$, this CDF, and hence P_{out} , can be expressed as [82]

$$P_{\text{out}} = F_{\gamma_{eq}}(\gamma_{\text{out}}) = F_{\gamma_{\text{RF}}}(\gamma_{\text{out}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out}}) - F_{\gamma_{\text{RF}}}(\gamma_{\text{out}}) F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out}}), \quad (3.17)$$

where $F_{\gamma_{\text{RF}}}(\gamma_{\text{out}})$ and $F_{\gamma_{\text{FSO}}^{\mathcal{M}}}(\gamma_{\text{out}})$ are the CDFs of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$, respectively. These CDFs are, respectively, given, by integrating their corresponding pdfs in (3.2) and (3.14) with respect to γ , as follows

$$F_{\gamma_{\text{RF}}}(\gamma) = 1 - \exp\left(-\frac{k\gamma}{\bar{\gamma}_{\text{RF}}}\right) \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{k\gamma}{\bar{\gamma}_{\text{RF}}}\right)^i, \quad (3.18)$$

$$F_{\gamma_{\text{FSO}}^{\mathcal{M}}}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right]. \quad (3.19)$$

Upon substituting the CDFs (3.18) and (3.19) in (3.17) and after simple algebraic manipulations, we get

$$P_{\text{out}} = F_{\gamma_{\text{eq}}}(\gamma_{\text{out}}) = 1 - \left(\exp\left(-\frac{k\gamma_{\text{out}}}{\bar{\gamma}_{\text{RF}}}\right) \sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{k\gamma_{\text{out}}}{\bar{\gamma}_{\text{RF}}}\right)^i \right) \left(1 - \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma_{\text{out}}/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right] \right). \quad (3.20)$$

Remember that the P_{out} in (3.20) is for the case of $\mathcal{M}$ fading in the FSO link. Alternatively, to attain the P_{out} for the special case of G-G fading in the FSO link, the CDF $F_{\gamma_{\text{FSO}}^{\mathcal{M}}}(\gamma)$ in (3.17) should be replaced by the CDF $F_{\gamma_{\text{FSO}}^{\text{G-G}}}(\gamma)$ which can be given, after integrating its pdf in (3.15), as follows

$$F_{\gamma_{\text{FSO}}^{\text{G-G}}}(\gamma) = \frac{\zeta^2}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,1} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, \beta, 0 \end{matrix} \right]. \quad (3.21)$$

3.3.2 Asymptotic Outage Probability

Due to the complexity of the derived expression in (3.20), it is not easy to know the impact of the different system parameters on its P_{out} . Therefore, in this section we derive more simple (asymptotic) expression for the system outage probability in the high SNR regime. This asymptotic P_{out} provides more help in understanding and analyzing the system performance by offering some system outage performance indicators clearly in its expression.

Namely, in the high SNR regime, the asymptotic outage probability can be normally given as $P_{\text{out}}^{(a)} \simeq (G_c \overline{\text{SNR}})^{-G_d}$, where G_d and G_c are outage performance indicators which are, respectively, termed the diversity order and the coding gain related to the P_{out} of the system [76]. The diversity order G_d specifies the slope of the log-log P_{out} versus average SNR plot in the high SNR regime, while $\log G_c$ specifies the horizontal shift of this plot as compared to a benchmark outage probability plot of $(\overline{\text{SNR}})^{-G_d}$.

Thus, to highlight G_d and G_c of our system, we need to reduce (3.20) to the typical $P_{\text{out}}^{(a)}$ expression. To this end, we start by modifying (3.17) as follows: first we replace the used CDFs by their asymptotic versions at the high SNR regime, then we drop the last term because it is negligible at the high SNRs. After these modifications, we obtain

$$P_{\text{out}}^{(a)} = F_{\gamma_{eq}}^{(a)}(\gamma_{\text{out}}) = F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma_{\text{out}}), \quad (3.22)$$

where $F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out}})$ and $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma_{\text{out}})$ are, respectively, the asymptotic CDFs of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$ in the high SNR regime. Next, we are left with determining these asymptotic

CDFs. Let's begin with $F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out}})$ which is derived by first using the high SNR limit ($\lim_{\bar{\gamma}_{\text{RF}} \rightarrow \infty} \exp(-\frac{k\gamma}{\bar{\gamma}_{\text{RF}}}) = 1$) in (3.2) to get $f_{\gamma_{\text{RF}}}^{(a)}(\gamma) = (\frac{k}{\bar{\gamma}_{\text{RF}}})^k \frac{\gamma^{k-1}}{(k-1)!}$, then integrating this asymptotic pdf leads to

$$F_{\gamma_{\text{RF}}}^{(a)}(\gamma) = (k/\bar{\gamma}_{\text{RF}})^k \gamma^k / (k)! . \quad (3.23)$$

Then again to determine $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma_{\text{out}})$, we adjust (3.19) by applying [84, Eq. (07.34.06.0006.01)] which is the Meijer G-function series representation for $\frac{1}{\mu_s} \rightarrow 0$, i.e.,⁹ $\bar{\gamma}_{\text{FSO}} \rightarrow \infty$ (high SNR). Doing this adjustment results in

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \sum_{l=1}^3 \frac{\prod_{j=1, j \neq l}^3 \Gamma(b'_j - b'_l)}{\Gamma(\zeta^2 + 1 - b'_l) b'_l} \left(\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \right)^{b'_l}, \quad (3.24)$$

where $b'_1 = \zeta^2, b'_2 = \alpha$ and $b'_3 = m$,¹⁰. Moreover, (3.24) can be simplified more by keeping only the dominant term in the second summation ($\sum_{l=1}^3$) at high FSO SNR ($\frac{1}{\mu_s} \rightarrow 0$), i.e., the term which contains the minimum b'_l ($b'_{\min} \triangleq \min[\zeta^2, \alpha, m]$). By this simplification¹¹, we get

⁹Because by definition, μ_s is proportional to $\bar{\gamma}_{\text{FSO}}$ (see the definitions after (3.7)).

¹⁰Note that [84, Eq.(07.34.06.0006.01)] requires $b'_j - b'_l \neq 0, \pm 1, \pm 2, \dots; (j, l = 1, 2, 3; j \neq l)$.

This condition can be fulfilled in all channels, and without affecting the physical channel model, by adding different infinitesimal numbers (ϵ_l) to any b'_l which does not satisfy it. All the other conditions, given in [84], for [84, Eq.(07.34.06.0006.01)] to be valid are already satisfied here in our work.

¹¹The deeper in high SNR regime we go, the more precise this simplification is. However generally speaking, in the beginning of the high SNR regime this simplification will be acceptable if $b'_{\min}$ is less than the other b'_l 's by about 1 or more, otherwise the second summation in (3.24) ($\sum_{l=1}^3$) will be dominated by the terms which have the least b'_l 's where the difference among these least b'_l 's is less than about 1, and the difference between these

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \frac{\prod_{j=1, b'_j \neq b'_{\min}}^3 \Gamma(b'_j - b'_{\min})}{\Gamma(\zeta^2 + 1 - b'_{\min}) b'_{\min}} \left(\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \right)^{b'_{\min}}. \quad (3.25)$$

For the case of¹² $\rho \neq 1$, and by utilizing the definition of $b'_{\min}$, (3.25) can be expressed as

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \Lambda (B(\gamma/c)^w / \mu_s^{1/s})^{b''_{\min}}, \quad (3.26)$$

where $b''_{\min} = \min[\zeta^2, \alpha, 1]$, and

$$\Lambda \triangleq \begin{cases} \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \frac{\prod_{j=1, b'_j \neq b'_{\min}}^3 \Gamma(b'_j - b'_{\min})}{\Gamma(\zeta^2 + 1 - b'_{\min}) b'_{\min}} & \text{if } \zeta^2 \text{ or } \alpha < 1, \Rightarrow b'_{\min} = \min[\zeta^2, \alpha] \\ \frac{\zeta^2 A}{2} b_1 \frac{\Gamma(\zeta^2 - 1) \Gamma(\alpha - 1)}{\Gamma(\zeta^2) \times 1} & \text{if } \zeta^2 \text{ \& } \alpha > 1, \Rightarrow b'_{\min} = 1 \end{cases}, \quad (3.27)$$

where in case of $b'_{\min} = 1$, we keep only the first term ($m=1$) in the summation with respect to m , because it dominates the other terms at high FSO SNR.

Now upon substituting (3.23) and (3.26) in (3.22), we obtain

$$P_{\text{out}}^{(a)} = F_{\gamma_{\text{eq}}}^{(a)}(\gamma_{\text{out}}) = \left(\frac{(k!)^{1/k} \bar{\gamma}_{\text{RF}}}{k \gamma_{\text{out}}} \right)^{-k} + \left(\frac{\Lambda^{-s/b''_{\min}} B^{-s} \mu_s}{(\gamma_{\text{out}}/c)^{ws}} \right)^{-b''_{\min}/s}. \quad (3.28)$$

For the $P_{\text{out}}^{(a)}$ in (3.28), let us focus on two different cases:

Case 1: The first hop dominates the system outage probability

This is the case in which the first summand in (3.28) is much larger than the second, i.e., the 1st hop (RF link) is in worse conditions than the 2nd hop (FSO least b'_l 's and the other b'_l 's is about 1 or more.

¹²The case of $\rho=1$ (i.e. $g=0$), in which $Ab_m=0$ for all m except $m=\beta$, is analyzed after (3.29b).

link). This case happens for example when the FSO link experiences relatively weak turbulence conditions and small pointing errors, i.e., when α , β , and ζ have relatively large values. This case can also happen when the average RF SNR is relatively low.

By referring to the standard $P_{\text{out}}^{(a)}$ expression, it can be seen, in this case, that our considered system can achieve a diversity order of k , ($G_d^{\text{RF}} = k$), and a coding gain which is given by $G_c^{\text{RF}} = (k!)^{1/k} / (k\gamma_{\text{out}})$. So in this case the coding gain is a function of k and γ_{out} .

Case 2: The second hop dominates the system outage probability

In this case, the second summand in (3.28) is much larger than the first, i.e., the 2nd hop (FSO link) is in worse conditions than the 1st hop (RF link) in this case. As an example, this case occurs when the FSO link suffers from relatively strong turbulence and significant pointing errors, i.e., when α , β , and ζ have relatively small values. This case can also happen when the average RF SNR is relatively large.

Then again, by referring to the standard $P_{\text{out}}^{(a)}$ expression, it can be seen, in this case, that the achievable diversity order and coding gain of our system are, respectively, given by

$$G_d^{\mathcal{M}} = b_{\min}''/s = \min[\zeta^2, \alpha, 1]/s, \quad (3.29a) \quad G_c^{\mathcal{M}} = \Lambda^{-s/b_{\min}''} B^{-s} / (\gamma_{\text{out}}/c)^{ws}. \quad (3.29b)$$

Recall that the above $P_{\text{out}}^{(a)}$ in (3.28), and hence ((3.29a), (3.29b)) are when the FSO link encounters $\mathcal{M}$ fading with $\rho \neq 1$. Alternatively, to obtain the $P_{\text{out}}^{(a)}$ for the special case of G-G fading ($\rho = 1$) in the FSO link, the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma)$ in (3.22) should be replaced by the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma)$ which is given, by setting $\Omega' = 1$ (for normalization)

and $\rho=1$ in (3.25), as follows

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma) = \left(\Delta^{1/b_{\min}^{\star}} (\gamma/c)^w / \mu_s^{1/s} \right)^{b_{\min}^{\star}}, \quad (3.30)$$

where $b_1^{\star} = \zeta^2$, $b_2^{\star} = \alpha$, $b_3^{\star} = \beta$, $b_{\min}^{\star} = \min[\zeta^2, \alpha, \beta]$, and $\Delta \triangleq \frac{\zeta^2}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\zeta^2 \alpha \beta}{\zeta^2 + 1} \right)^{b_{\min}^{\star}} \frac{\prod_{j=1, b_j^{\star} \neq b_{\min}^{\star}}^3 \Gamma(b_j^{\star} - b_{\min}^{\star})}{\Gamma(\zeta^2 + 1 - b_{\min}^{\star}) b_{\min}^{\star}}.$

Consequently, by using (3.23) and (3.30), we get the $P_{\text{out}}^{(a)}$ for our system in the case of mixed Nakagami- m /G-G fading as follows

$$P_{\text{out}}^{(a)} = F_{\gamma_{\text{eq}}}^{(a)}(\gamma_{\text{out}}) = F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma_{\text{out}}) \quad (3.31)$$

$$= \left(\frac{(k!)^{1/k} \bar{\gamma}_{\text{RF}}}{k \gamma_{\text{out}}} \right)^{-k} + \left(\frac{\Delta^{-s/b_{\min}^{\star}} \mu_s}{(\gamma_{\text{out}}/c)^{w/s}} \right)^{-b_{\min}^{\star}/s}. \quad (3.32)$$

Here again for the $P_{\text{out}}^{(a)}$ in (3.32), we focus on the previous two cases. Regarding case 1, nothing is changed, and so we can deduce the same results as the previous ones. However, for case 2, i.e., when the 2nd summand dominates in (3.32), we can notice that, related to P_{out} , our system achieves a coding gain and a diversity order which are, respectively, given by

$$G_c^{\text{G-G}} = \Delta^{-s/b_{\min}^{\star}} / (\gamma_{\text{out}}/c)^{w/s}, \quad (3.33a) \quad G_d^{\text{G-G}} = b_{\min}^{\star}/s = \min[\zeta^2, \alpha, \beta]/s. \quad (3.33b)$$

It is worthy to observe that it is possible for $G_d^{\text{G-G}}$ to be greater than $1/s$, whereas this is not possible for G_d^{M} . Another observation is that in the FSO hop, G_c is different for each FSO detector. However, based on the previous high SNR analysis,

using HD or IM-DD cost-dependent results in the same G_d which is twice the G_d of IM-DD input-independent. While, if the asymptotic analysis is based on high transmitted power regime, all these optical detectors will result in the same P_{out} -based diversity order (call it G_{dp}). To illustrate this consider the aforementioned “case 2”, and by referring to (3.28), (3.32), and recalling that μ_s is proportional to P_r^s (see definitions after (3.7)), it can be shown that, for all considered FSO detectors, we have

$$G_{\text{dp}}^{\mathcal{M}} = \min[\zeta^2, \alpha, 1], \quad (3.34a) \quad G_{\text{dp}}^{\text{G-G}} = \min[\zeta^2, \alpha, \beta]. \quad (3.34b)$$

Additional significant note here is that, to the best of our knowledge, no work in the literature (see e.g. [63], [64], [70], [85]) utilizes a precise IM-DD channel capacity approx., as (3.12), in such system outage analysis. Instead there has been an unsound adoption of the form $(\log(1 + \gamma_{\text{IMDD}_{\text{indp}}}))$, let's denote it by IM-DD_capX, for IM-DD input-independent capacity in outage analysis. This leads to misconception in the comparison between HD and IM-DD, see Fig. 3.7. Hence, to overcome this we utilize Eqs. (3.12), (3.13) here to derive P_{out} and its asymptotic analysis.

Actually, by using the CDF of γ_{FSO} (given in (3.42) on next page) instead of the CDF of $\gamma_{\text{FSO}}^{\text{eff}}$ (given in (3.19)) in the outage analysis, it can be shown that, the unsound adoption of IM-DD_capX results in the same previous $G_d^{\mathcal{M}}$ and $G_d^{\text{G-G}}$ for IM-DD input-independent. Then again, by using (3.42) instead of (3.19) in P_{out} analysis, it can be seen that, the use of IM-DD_capX leads to the following P_{out} -based coding gains in case of $\mathcal{M}$ (with $\rho \neq 1$) and G-G ($\rho = 1$) fadings, respectively,

$$G_{\text{c-X}}^{\mathcal{M}} = \Lambda^{-2/b_{\text{min}}''} B^{-2}/\gamma_{\text{out}}, \quad (3.35a) \quad G_{\text{c-X}}^{\text{G-G}} = \Delta^{-2/b_{\text{min}}^{\star}}/\gamma_{\text{out}}. \quad (3.35b)$$

Therefore, by comparing (3.35a), (3.35b) with (3.29b), (3.33a) (when $s=2$, $w=1$, and $c=\sqrt{\frac{e}{2\pi}}$), we observe that there is a coding gain overestimation by using IM-DD_capX instead of our precise IM-DD input-independent capacity approx. (3.12). This overestimation in the P_{out} -based coding gain is given, in dB for both $\mathcal{M}$ (with $\rho \neq 1$) and G-G ($\rho=1$), by

$$\delta G_c = 10 \log_{10}(2\pi\gamma_{\text{out}}/e) = 10 \log_{10}(\gamma_{\text{out}}) + 3.639 \text{ dB}. \quad (3.36)$$

The δG_c in (3.36) is based on high SNR analysis. However, by comparing the 2^{nd} summands in (3.28), and (3.32) with a P_{out} of the form $(G_{\text{cp}}P_r)^{-G_{\text{dp}}}$, after recalling that μ_2 is proportional to P_r^2 , it can be shown that, based on high Tx power regime, the aforementioned overestimation in G_c is given by

$$\delta G_{\text{cp}} = \delta G_c / 2 = 5 \log_{10}(\gamma_{\text{out}}) + 1.819 \text{ dB}. \quad (3.37)$$

3.3.3 Bit Error Rate

In general, for a dual-hop DF relaying scheme over independent fading channels, the average BER can be introduced as follows [48, Eq. (14)]

$$P_{\text{BER}} = P_{\text{BER}}^{hp1} + P_{\text{BER}}^{hp2} - 2P_{\text{BER}}^{hp1}P_{\text{BER}}^{hp2}, \quad (3.38)$$

where P_{BER}^{hp1} and P_{BER}^{hp2} are the average BERs for the first and second hop, respectively.

In terms of their corresponding CDFs, these average BERs can be expressed as [75]

$$P_{\text{BER}}^{hp1} = \frac{a_1 \sqrt{b_1}}{2\sqrt{\pi}} \int_0^\infty \frac{\exp(-b_1 \gamma)}{\sqrt{\gamma}} F_{\gamma_{\text{RF}}}(\gamma) d\gamma, \quad (3.39)$$

$$P_{\text{BER}}^{hp2} = \frac{a_2 \sqrt{b_2}}{2\sqrt{\pi}} \int_0^\infty \frac{\exp(-b_2 \gamma)}{\sqrt{\gamma}} F_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) d\gamma, \quad (3.40)$$

where a_1, b_1, a_2 , and b_2 are the parameters which specify the modulation type in the corresponding hop¹³. For a possible list of the modulation types indicated by these parameters, the reader is referred to [75]. Now to obtain P_{BER}^{hp1} , we carry out the integration in (3.39), after substituting (3.18) in it, with the help of [74, Eq. (3.381.4)] to get

$$P_{\text{BER}}^{hp1} = \frac{a_1 \sqrt{b_1}}{2\sqrt{\pi}} \left[\frac{\Gamma(0.5)}{b_1^{0.5}} - \sum_{i=0}^{k-1} \left(\frac{k}{\bar{\gamma}_{\text{RF}}} \right)^i \frac{\Gamma(i+0.5)}{i! (b_1 + k/\bar{\gamma}_{\text{RF}})^{(i+0.5)}} \right]. \quad (3.41)$$

Alternatively, to solve the integral in (3.40) we need the CDF $F_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma)$ which can be given, by integrating its pdf in (3.7) with the help of [84, Eq.(07.34.21.0084.01)], as follows

¹³Eqs. (3.39) and (3.40) represent the symbol error rate. So they give the BER in case of binary modulations, while they give an upper bound for the BER of M-ary modulations. Moreover if they are divided by the #bits/symbol, they will provide a lower bound for the BER of M-ary modulations which is an excellent BER approx. at high SNR when gray mapping is used [16].

$$F_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) = D \sum_{m=1}^{\beta} c_m G_{s+1,3s+1}^{3s,1} \left[E \frac{\gamma}{\mu_s} \middle| \begin{matrix} 1, \mathbf{x}_1 \\ \mathbf{x}_2, 0 \end{matrix} \right], \quad (3.42)$$

where $D = \zeta^2 A / [2^s (2\pi)^{s-1}]$, $c_m = b_m s^{\alpha+m-1}$, $E = B^s w / s^{2s}$, $\mathbf{x}_1 = \frac{\zeta^2+1}{s}, \dots, \frac{\zeta^2+s}{s}$ includes s terms, and $\mathbf{x}_2 = \frac{\zeta^2}{s}, \dots, \frac{\zeta^2+s-1}{s}, \frac{\alpha}{s}, \dots, \frac{\alpha+s-1}{s}, \frac{m}{s}, \dots, \frac{m+s-1}{s}$ includes $3s$ terms. Now the P_{BER}^{hp2} is given, after substituting (3.42) in (3.40), with the help of [74, Eq. (7.813.1)] as follows

$$P_{\text{BER}}^{hp2} = \frac{a_2 D}{2\sqrt{\pi}} \sum_{m=1}^{\beta} c_m G_{s+2,3s+1}^{3s,2} \left[\frac{E}{b_2 \mu_s} \middle| \begin{matrix} 0.5, 1, \mathbf{x}_1 \\ \mathbf{x}_2, 0 \end{matrix} \right]. \quad (3.43)$$

Subsequently, upon substituting (3.41) and (3.43) in (3.38), we get

$$\begin{aligned} P_{\text{BER}} = & \frac{a_1 \sqrt{b_1}}{2\sqrt{\pi}} \left[\frac{\Gamma(0.5)}{b_1^{0.5}} - \sum_{i=0}^{k-1} \left(\frac{k}{\bar{\gamma}_{\text{RF}}} \right)^i \frac{\Gamma(i+0.5)}{i! (b_1 + k/\bar{\gamma}_{\text{RF}})^{(i+0.5)}} \right] \\ & \times \left[1 - \frac{a_2 D}{\sqrt{\pi}} \sum_{m=1}^{\beta} c_m G_{s+2,3s+1}^{3s,2} \left[\frac{E}{b_2 \mu_s} \middle| \begin{matrix} 0.5, 1, \mathbf{x}_1 \\ \mathbf{x}_2, 0 \end{matrix} \right] \right] + \frac{a_2 D}{2\sqrt{\pi}} \sum_{m=1}^{\beta} c_m G_{s+2,3s+1}^{3s,2} \left[\frac{E}{b_2 \mu_s} \middle| \begin{matrix} 0.5, 1, \mathbf{x}_1 \\ \mathbf{x}_2, 0 \end{matrix} \right]. \end{aligned} \quad (3.44)$$

Notice that the P_{BER}^{hp2} in (3.43), and hence the P_{BER} in (3.44), is when the FSO link (2^{nd} hop) faces $\mathcal{M}$ fading. Nevertheless, the same previous procedures can be followed to attain the P_{BER}^{hp2} , and its related P_{BER} , for the G-G faded FSO link, however the CDF $F_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma)$ in (3.40) should be changed by $F_{\gamma_{\text{FSO}}}^{\text{G-G}}(\gamma)$ which is given, by setting $\Omega' = 1$ and $\rho = 1$ in (3.42), as follows

$$F_{\gamma_{\text{FSO}}}^{\text{G-G}}(\gamma) = \mathcal{K} G_{s+1,3s+1}^{3s,1} \left[\mathcal{L} \frac{\gamma}{\mu_s} \middle| \begin{matrix} 1, \mathbf{x}_1 \\ \mathbf{x}_3, 0 \end{matrix} \right], \quad (3.45)$$

where $\mathcal{K} = \zeta^2 s^{\alpha+\beta-1} / [(4\pi)^{s-1} \Gamma(\alpha) \Gamma(\beta)]$, $\mathcal{L} = w(\zeta^2 \alpha \beta)^s / [(\zeta^2 + 1)s^2]^s$, and $\mathbf{x}_3 = \frac{\zeta^2}{s}, \dots, \frac{\zeta^2+s-1}{s}, \frac{\alpha}{s}, \dots, \frac{\alpha+s-1}{s}, \frac{\beta}{s}, \dots, \frac{\beta+s-1}{s}$ includes $3s$ terms.

3.3.4 Asymptotic Bit Error Rate

The derived average BER in (3.44) is complicated, and hence it is hard to analyze how this BER is influenced by our system parameters. Therefore, to have more insight about the effects of our system parameters on its BER, let us drive the asymptotic average BER (P_{BER}^a) in the high SNR regime. To do that, we begin by omitting the last term in (3.38) because it is negligible at the high SNRs. Then, we modify the rest of (3.38) to get the asymptotic average BER as follows

$$P_{\text{BER}}^a = P_{\text{BER}}^{a^{hp1}} + P_{\text{BER}}^{a^{hp2}}, \quad (3.46)$$

where $P_{\text{BER}}^{a^{hp1}}$ and $P_{\text{BER}}^{a^{hp2}}$ are, respectively, the asymptotic average BERs for the first and second hops in the high SNR regime. Next, to obtain $P_{\text{BER}}^{a^{hp1}}$ we solve the integration in (3.39), but after replacing $F_{\gamma_{\text{RF}}}(\gamma)$ with $F_{\gamma_{\text{RF}}}^{(a)}(\gamma)$ from (3.23) and with the help of [74, Eq. (3.381.4)], to get

$$P_{\text{BER}}^{a^{hp1}} = \frac{a_1 \Gamma(k+0.5)}{2\sqrt{\pi} k! b_1^k} \left(\frac{k}{\bar{\gamma}_{\text{RF}}} \right)^k. \quad (3.47)$$

On the other hand, $P_{\text{BER}}^{a^{hp2}}$ can be derived from (3.43) by using [84, Eq.(07.34.06.0006.01)], and mimicking the same method which is followed in deriving

$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma)$ (Eqs. (3.24) and (3.25)), to get

$$P_{\text{BER}}^{a_{hp2}} = \frac{a_2 D}{2\sqrt{\pi}} \sum_{m=1}^{\beta} c_m \frac{\Gamma(b'_{\min}/s + 0.5) \prod_{j=1, s \text{ } x_{2j} \neq b'_{\min}}^{3s} \Gamma(x_{2j} - b'_{\min}/s)}{(b'_{\min}/s) \prod_{j=1}^s \Gamma(x_{1j} - b'_{\min}/s)} \left(\frac{E}{b_2 \mu_s} \right)^{b'_{\min}/s}. \quad (3.48)$$

For the case of $\rho \neq 1$, and by utilizing the definition of $b'_{\min}$, (3.48) can be compactly expressed as

$$P_{\text{BER}}^{a_{hp2}} = \frac{a_2 D \Gamma(b''_{\min}/s + 0.5) J}{2\sqrt{\pi} (b''_{\min}/s) \prod_{j=1}^s \Gamma(x_{1j} - b''_{\min}/s)} \left(\frac{E}{b_2 \mu_s} \right)^{b''_{\min}/s}, \quad (3.49)$$

where x_{1j} is the j^{th} term in $\mathbf{x}_1$,

$$J \triangleq \begin{cases} \sum_{m=1}^{\beta} c_m \prod_{j=1, s \text{ } x_{2j} \neq b'_{\min}}^{3s} \Gamma(x_{2j} - b'_{\min}/s) & \text{if } \zeta^2 \text{ or } \alpha < 1, \Rightarrow b'_{\min} = \min[\zeta^2, \alpha] \\ \sum_{m=1}^1 c_m \prod_{j=1, s \text{ } x_{2j} \neq 1}^{3s} \Gamma(x_{2j} - 1/s) & \text{if } \zeta^2 \text{ \& } \alpha > 1, \Rightarrow b'_{\min} = 1 \end{cases}, \quad (3.50)$$

and x_{2j} is the j^{th} term in $\mathbf{x}_2$.

Thus, by substituting (3.47) and (3.49) in (3.46), we attain

$$P_{\text{BER}}^a = \frac{a_1 \Gamma(k + 0.5)}{2\sqrt{\pi} k! b_1^k} \left(\frac{\tilde{\gamma}_{\text{RF}}}{k} \right)^{-k} + \frac{a_2 D \Gamma(b''_{\min}/s + 0.5) J}{2\sqrt{\pi} (b''_{\min}/s) \prod_{j=1}^s \Gamma(x_{1j} - b''_{\min}/s)} \left(\frac{b_2 \mu_s}{E} \right)^{-b''_{\min}/s}. \quad (3.51)$$

Of course, the $P_{\text{BER}}^{a_{hp2}}$ in (3.49), and accordingly the P_{BER}^a in (3.51), is when the FSO link meets $\mathcal{M}$ fading with $\rho \neq 1$. However, by setting $\rho = 1$, and for normalization $\Omega' = 1$, in (3.48), we can get the $P_{\text{BER}}^{a_{hp2}}$, and its correspondent P_{BER}^a , for the special case of G-G faded FSO link as

$$P_{\text{BER}}^{a_{hp2}} = \frac{a_2 \mathcal{K} \Gamma(b_{\min}^*/s + 0.5) \prod_{j=1, s}^{3s} \Gamma(x_{3j} - b_{\min}^*/s)}{2\sqrt{\pi} (b_{\min}^*/s) \prod_{j=1}^s \Gamma(x_{1j} - b_{\min}^*/s)} \left(\frac{\mathcal{L}}{b_2 \mu_s} \right)^{b_{\min}^*/s}, \quad (3.52)$$

$$P_{\text{BER}}^a = \frac{a_1 \Gamma(k + 0.5)}{2\sqrt{\pi} k! b_1^k} \left(\frac{\bar{\gamma}_{\text{RF}}}{k} \right)^{-k} + \frac{a_2 \mathcal{K} \Gamma(b_{\min}^*/s + 0.5) \prod_{j=1, s}^{3s} \Gamma(x_{3j} - b_{\min}^*/s)}{2\sqrt{\pi} (b_{\min}^*/s) \prod_{j=1}^s \Gamma(x_{1j} - b_{\min}^*/s)} \left(\frac{b_2 \mu_s}{\mathcal{L}} \right)^{-b_{\min}^*/s}. \quad (3.53)$$

In fact as the $P_{\text{out}}^{(a)}$, in the high SNR regime, the asymptotic average BER can be typically given as $P_{\text{BER}}^a \simeq (\mathcal{G}_c \text{SNR})^{-\mathcal{G}_d}$, where $\mathcal{G}_d$ and $\mathcal{G}_c$ are BER performance indicators which, respectively, designate the diversity order and the coding gain related to the BER of the system [16]. Therefore, from Eqs. (3.51) and (3.53), and by keeping the standard P_{BER}^a in mind, it can be noticed that our system has BER-based diversity orders $(\mathcal{G}_d^{\text{RF}}, \mathcal{G}_d^{\mathcal{M}}, \mathcal{G}_d^{\text{G-G}})$ which, respectively, equal its P_{out} -based diversity orders $(G_d^{\text{RF}}, G_d^{\mathcal{M}}, G_d^{\text{G-G}})$ in each one of the previous analyzed cases in Section 3.3.2. However, this is not the case for the coding gains (i.e. $\mathcal{G}_c^{\text{RF}}, \mathcal{G}_c^{\mathcal{M}}, \mathcal{G}_c^{\text{G-G}}$ are, respectively, not equal to $G_c^{\text{RF}}, G_c^{\mathcal{M}}, G_c^{\text{G-G}}$)¹⁴.

Also, from (3.51) and (3.53), we can perceive that the used modulation type affects

¹⁴This diversity-coding gains analysis, for both P_{out} and P_{BER} , has not been done in the open literature as per our knowledge. Recently a related diversity-coding gains analysis was done in [85]. However, the outcomes drawn from that analysis are different from our $\mathcal{G}_d^{\mathcal{M}}, \mathcal{G}_c^{\mathcal{M}}, \mathcal{G}_c^{\text{G-G}}, G_d^{\mathcal{M}}, G_c^{\mathcal{M}}$, and $G_c^{\text{G-G}}$ which are obtained here from the detailed diversity-coding gains derivations given in ((3.22)–(3.32), (3.46)–(3.53)).

only the $\mathcal{G}_c$, while the encountered fading model (Nakagami- m , $\mathcal{M}$, G-G) affects both the $\mathcal{G}_d$ and $\mathcal{G}_c$.

3.3.5 Ergodic Channel Capacity

Ergodic channel capacity (C_{erg}) is the term used for information theoretical channel capacity in case of fast fading channels [81]. As a dual hop DF system, our system¹⁵ has the following ergodic capacity

$$C_{\text{erg}} = \frac{1}{2} \min \left[\mathbb{E}\{C_{\text{RF}}\}, \mathbb{E}\{C_{\text{FSO}}\} \right], \quad (3.54)$$

where C_{RF} and C_{FSO} are defined in (3.11) and (3.13), respectively, and thus the above expectations are given by

$$\mathbb{E}\{C_{\text{RF}}\} = \frac{1}{\ln(2)} \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{RF}}}(\gamma) d\gamma. \quad (3.55)$$

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{1}{\ln(2)} \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) d\gamma. \quad (3.56)$$

Now upon substituting (3.2) in (3.55), while utilizing $\ln(1 + \gamma) = G_{2,2}^{1,2} \left[\gamma \middle|_{1,0}^{1,1} \right]$

¹⁵In general, the RF and FSO faded channels are a slow fading channels. However, they can be considered as fast fading channels, if long enough interleaving is used. This interleaving-based fast fading consideration can be especially valid in some situations such as in windy conditions where RF and FSO channels (particularly FSO pointing errors) fluctuate relatively fast.

and $\exp(-z) = G_{0,1}^{1,0}\left[z\left|_{0}^{-}\right.\right]$, and with the help of [84, Eq. (07.34.21.0013.01), Eq. (07.34.17.0011.01)], we get

$$\mathbb{E}\{C_{\text{RF}}\} = \frac{1}{(k-1)! \ln 2} G_{2,3}^{3,1} \left[\frac{k}{\bar{\gamma}_{\text{RF}}} \left| \begin{matrix} 0, 1 \\ k, 0, 0 \end{matrix} \right. \right]. \quad (3.57)$$

Then again, after substituting (3.14) in (3.56) and utilizing $\ln(1+\gamma) = G_{2,2}^{1,2}\left[\gamma\left|_{1,0}^{1,1}\right.\right]$, $\mathbb{E}\{C_{\text{FSO}}\}$ can be attained by using [84, Eq. (07.34.21.0013.01)]. By doing that, and making some simplifications by exploiting the Meijer's G-function definition [74, Eq. (9.301)], we obtain

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2 A/2}{(2\pi)^{w-1} \ln 2} \sum_{m=1}^{\beta} b_m G_{w+2,w+4}^{w+4,w} \left[\frac{B/c^w}{\mu_s^{1/s}} \left| \begin{matrix} \mathbf{x}_4, 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0, \mathbf{x}_4 \end{matrix} \right. \right], \quad (3.58)$$

where $\mathbf{x}_4 = \frac{0}{w}, \dots, \frac{w-1}{w}$ includes w terms.

Finally, by substituting (3.57) and (3.58) in (3.54), we get our system Ergodic capacity.

Recall that the $\mathbb{E}\{C_{\text{FSO}}\}$ in (3.58) is for the $\mathcal{M}$ faded FSO link. Instead, to obtain the $\mathbb{E}\{C_{\text{FSO}}\}$ for the special case of G-G faded FSO link, the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ in (3.56) should be replaced by the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma)$ given in (3.15). Doing that, and following the same previous steps, results in

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2/(\Gamma(\alpha)\Gamma(\beta))}{(2\pi)^{w-1} \ln 2} G_{w+2,w+4}^{w+4,w} \left[\frac{\zeta^2 \alpha \beta / c^w}{(\zeta^2+1) \mu_s^{1/s}} \left| \begin{matrix} \mathbf{x}_4, 1, \zeta^2+1 \\ \zeta^2, \alpha, \beta, 0, \mathbf{x}_4 \end{matrix} \right. \right]. \quad (3.59)$$

3.4 Simulation and Numerical Results

In this section, we present numerical examples to illustrate the analytical and asymptotic expressions of the previous section and also to show the effect of the different fading parameters and pointing error on the system performance.¹⁶ Additionally, this section validates the derived analytical and asymptotic expressions by Monte-Carlo simulations. In these simulations, 10^6 realizations of the SNR random variable are used except at the results which are less than 10^{-6} where 10^8 realizations are used. Also, BPSK ($a_1 = b_1 = 1$) and OOK ($a_2 = 1, b_2 = 0.5$) are used in the results of the BER as the modulation schemes for the RF and IMDD-FSO hops, respectively, while BPSK ($a_2 = b_2 = 1$) is used for HD-FSO hop.

Fig. 3.2 demonstrates the effect of the atmospheric turbulence parameters (α and β) on our system P_{out} for a specific pointing error ($\zeta=1.5$).¹⁷ From this figure, it can be noticed that the lower values of α and β , i.e., stronger atmospheric turbulence, result in higher (worse) P_{out} and vice versa.¹⁸

¹⁶Following the same strategy as in related literature, see e.g. [71], [63], [24], [25], [85], we chose the values of the system parameters to validate the derived expressions, and thus other parameters' values can be used by planning engineers to get the results in their particular scenarios.

¹⁷ In all results, without loss of generality, we use $\gamma_{\text{out}}=0$ dB, $\Omega=0.78$, and $b_0=0.11$.

¹⁸Note that $\bar{\gamma}_{\text{HD}}$ and $\bar{\gamma}_{\text{IMDD}_{\text{cost}}}$ are proportional to P_r , whereas $\bar{\gamma}_{\text{IMDD}_{\text{indp}}}$ is proportional to P_r^2 . Therefore the results plotted versus the average SNR should not be used in comparing the different FSO detectors, otherwise this will lead to unfair comparison from P_r point of view.

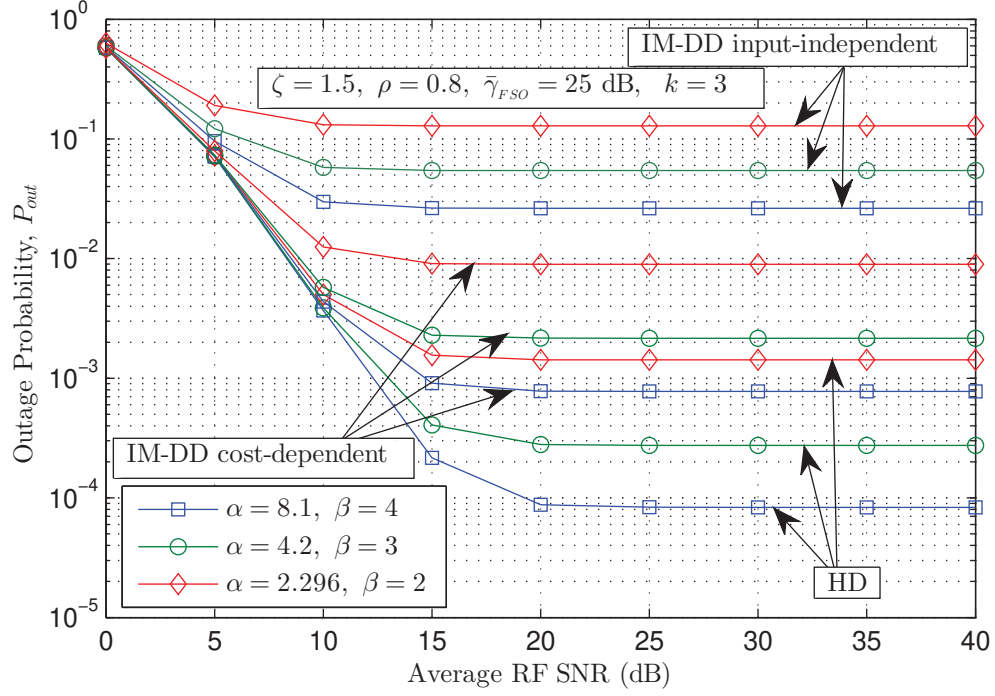


Figure 3.2: P_{out} for different values of α and β considering all FSO detection techniques.

The effect of different values of k , ρ , and ζ (pointing error) on our system outage probability (P_{out}) is demonstrated in Fig. 3.3 for fixed atmospheric turbulence conditions considering IM-DD cost-dependent. As expected, the higher the values of k , ρ , or ζ (smaller pointing error), the lower is P_{out} , and hence the better is the attained performance. Also, based on the used parameters, we can perceive from Fig. 3.2 and Fig. 3.3 that in the low RF SNRs, the RF hop dominates P_{out} , i.e., P_{out} varies when $\bar{\gamma}_{RF}$ changes although $\bar{\gamma}_{FSO}$ is constant here. However, in these figures when the RF SNR becomes relatively high, the P_{out} becomes constant because in this case it is dominated by the FSO hop whose $\bar{\gamma}_{FSO}$ is set constant in these figures.

Figs. 3.4, 3.5, and 3.6 evince that the asymptotic P_{out} expression converges to the exact P_{out} expression at the high SNRs. Additionally, these figures show the diversity

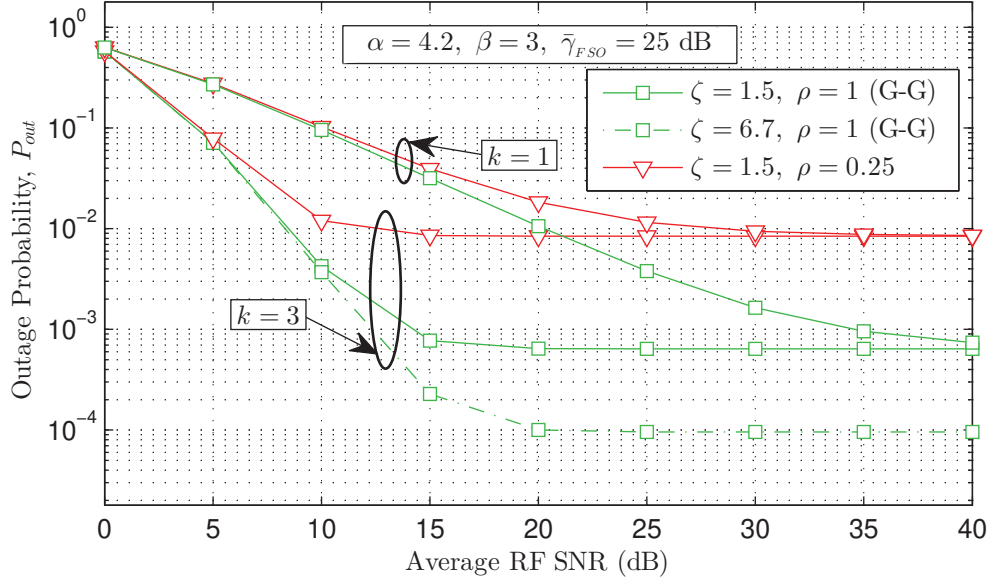


Figure 3.3: P_{out} for different values of k , ρ and ζ considering IM-DD cost-dependent FSO link.

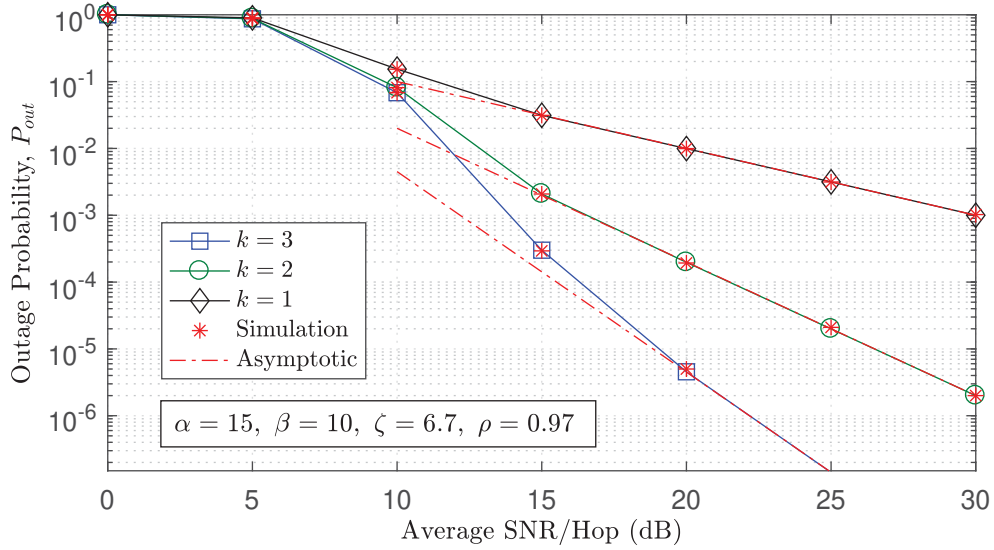


Figure 3.4: P_{out} dominated by the RF hop when IM-DD cost-dependent is used in the FSO hop.

order and coding gain of our system outage probability at high SNRs in different cases. More specifically, Fig. 3.4 confirms that $G_d^{\text{RF}} = k$ as derived in case 1 of Section 3.3.2, i.e., the measured slopes of the log-log curves in the high SNR from Fig. 3.4 equal the derived G_d^{RF} .

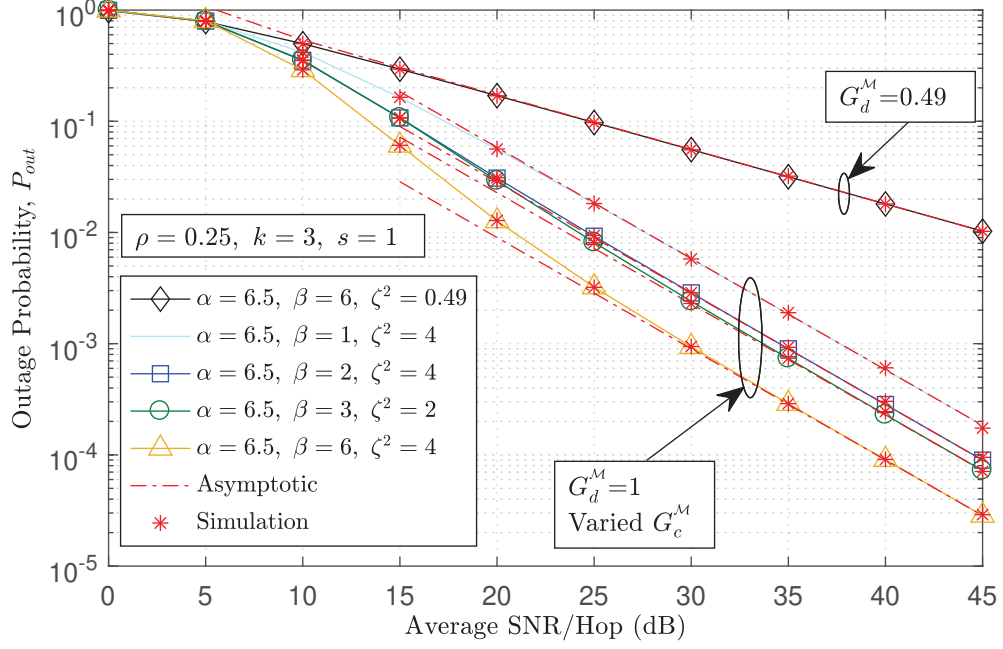


Figure 3.5: P_{out} dominated by IM-DD cost-dependent FSO hop.

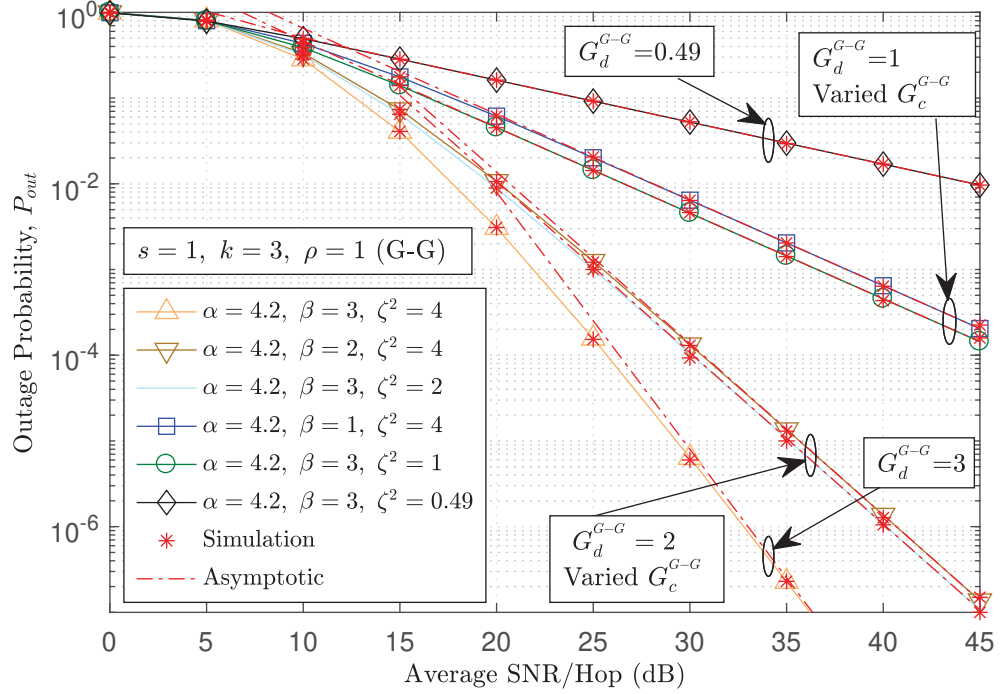


Figure 3.6: P_{out} dominated by IM-DD cost-dependent G-G FSO hop.

On the other hand, Fig. 3.5 confirms that, given $s=1$, $G_d^M = b_{\min}''/s = \min[\zeta^2, \alpha, 1]$ as derived in (3.29a). To explain this, we labeled two groups in this figure. In the group

of $G_d^M=1$, the values of β, ζ are different among this group curves but they still have $\min[\zeta^2, \alpha, 1]=1 \Rightarrow G_d^M=1$ which is identical to their log-log slope in the figure. However, the difference in β, ζ among this group leads to different G_c^M in each of its curves. The other group has $\min[\zeta^2, \alpha, 1]=\zeta^2 \Rightarrow G_d^M=0.49$ which is identical to its log-log slope. Alternatively, Fig. 3.6 confirms that, given $s=1$, $G_d^{G-G}=b_{\min}^*/s=\min[\zeta^2, \alpha, \beta]$ as derived previously, i.e., the labeled measured log-log slopes from Fig. 3.6 are identical to the derived G_d^{G-G} in (3.33b).

Moreover, Figs. 3.4, 3.5, and 3.6 confirm the validity of the derived analytical and asymptotic P_{out} expressions by showing the perfect match between them and the Monte-Carlo simulations.

Fig. 3.7 shows that the asymptotic P_{out} expression converges to the exact P_{out} at the high transmitted powers. Also, this figure validates that, at high transmitted powers, all FSO detectors result in the same diversity order which is given by $G_{\text{dp}}^M = \min[\zeta^2, \alpha, 1]$ or $G_{\text{dp}}^{G-G} = \min[\zeta^2, \alpha, \beta]$ as deduced in (3.34), i.e., in Fig. 3.7 the labeled measured log-log slopes are the same as the earlier deduced ones. Additionally, from this figure, due to the different coding gains, we can see that the HD offers better outage performance than both IM-DD techniques, and IM-DD input-independent reveals less P_{out} than IM-DD cost-dependent. However, based on the used parameters in Fig. 3.7, this IM-DD input-independent outage is overestimated because the IM-DD input-independent ignores the shot-noise which is considerable in this example.

For comparison, Fig. 3.7 also includes a curve labeled *IM-DD X*. This curve reflects the overestimation (misconception) in P_{out} when we unsoundly use Shannon's capacity $(\log(1+\gamma_{\text{IMDD}_{\text{indp}}}))$ for IM-DD input-independent, see e.g. [63], [64], [70], [85], instead

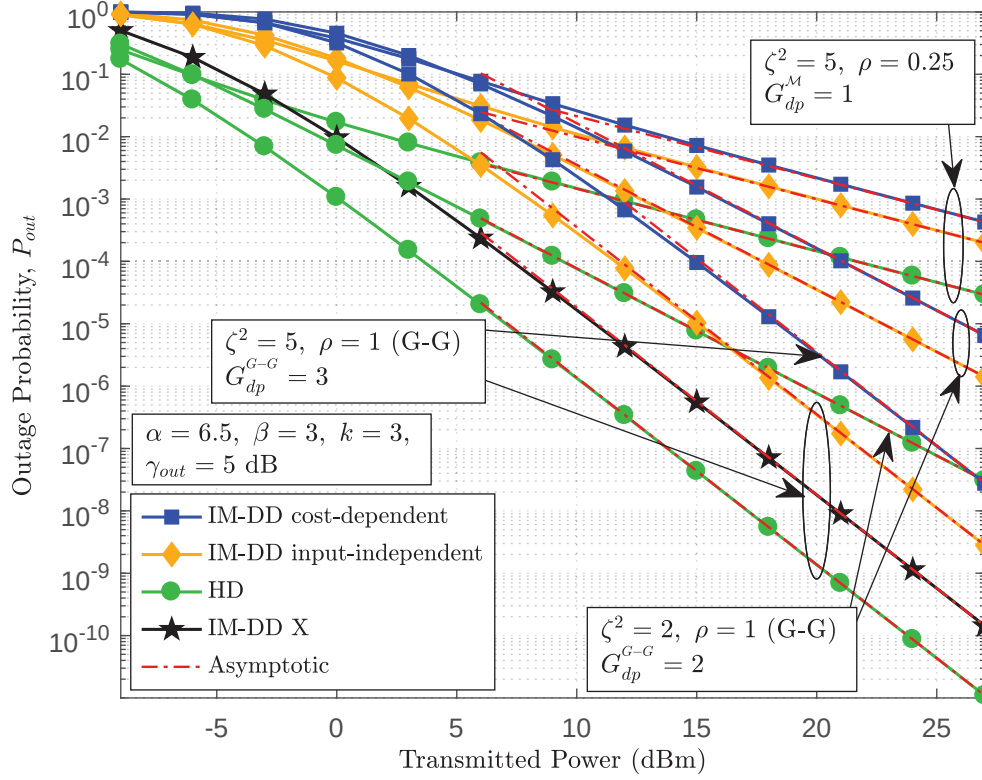


Figure 3.7: P_{out} dominated by the FSO hop vs transmitted power.

of its precise capacity approx. $\left(\log\left(1+\sqrt{\frac{e}{2\pi}\gamma_{\text{IMDD}_{\text{indp}}}}\right)\right)$ which is used here in our work. Specifically, as deduced in Section 3.3.2, Fig. 3.7 proves that, under the same parameters, IM-DD X holds the same G_d as all FSO detectors, including IM-DD input-independent. Alternatively, this figure evinces that, under the same parameters, there is an overestimation in G_c reflected by IM-DD X as compared to the precise IM-DD input-independent curve. It can be seen that, given $\gamma_{\text{out}}=5$ dB, this overestimation is consistent with the drawn one in (3.37) (i.e. $\delta G_{\text{cp}}=5\log(\gamma_{\text{out}})+1.819=4.319$ dB).

Alternatively, Fig. 3.8 illustrates the influence of the atmospheric turbulence parameters (α and β) on our system P_{BER} for a fixed pointing error ($\zeta=1.5$). From this figure, it can be observed that higher values of α and β , i.e., weaker atmospheric

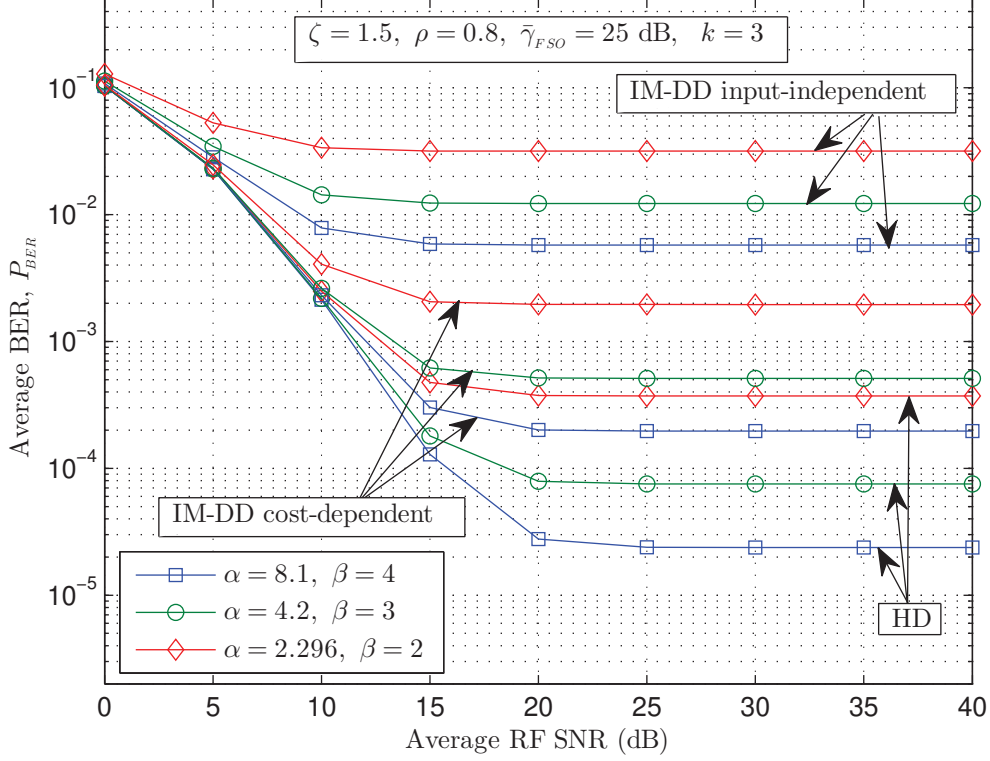


Figure 3.8: P_{BER} for different values of α and β considering all FSO detection techniques.

turbulence, result in a lower (better) P_{BER} and vice versa. Another note, based on the used parameters, in Fig. 3.8 is that in the relatively high RF SNR regime, the P_{BER} is constant, since in this regime it is dominated by the FSO hop whose $\bar{\gamma}_{FSO}$ is kept constant in this figure, however in the low RF SNRs the P_{BER} is dominated by the RF hop.

The impact of the different values of ρ , k , and ζ on the system BER (P_{BER}) is shown in Fig. 3.9 for IM-DD cost-dependent. As expected, an increase in the value of ρ , k , or ζ (smaller pointing error) leads to a decrease in the P_{BER} , and hence results in a better performance. Beside that, Fig. 3.9 reveals the convergence between the asymptotic and exact P_{BER} expressions. Furthermore, Fig. 3.9 validates that, given

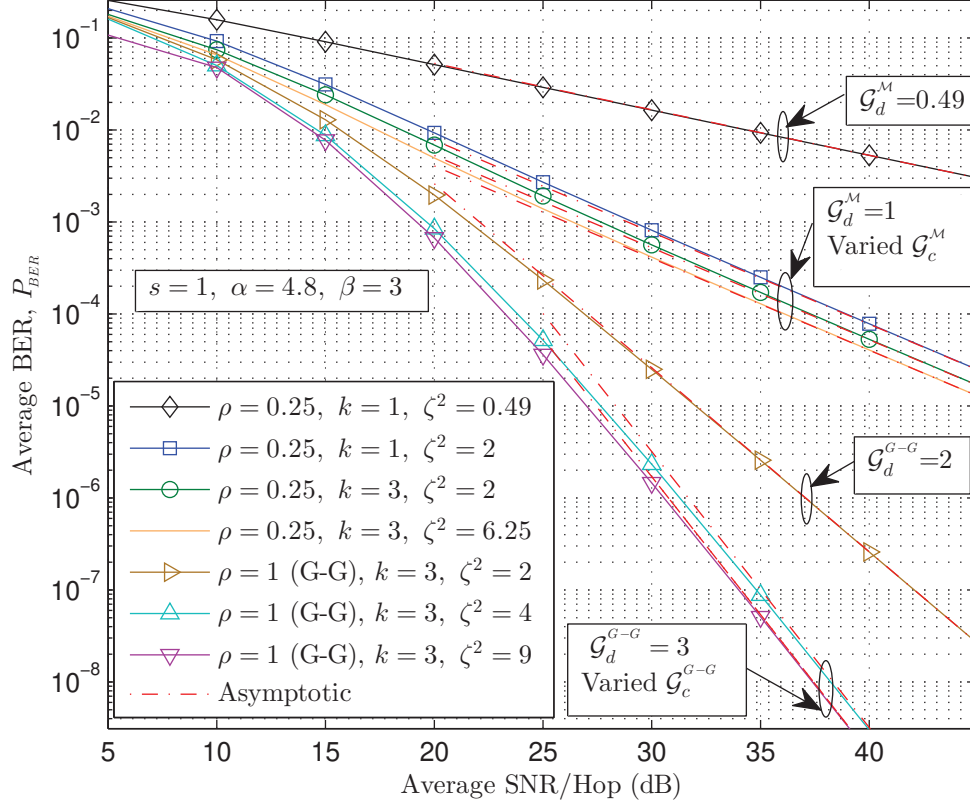


Figure 3.9: P_{BER} dominated by IM-DD cost-dependent FSO hop for different values of k , ρ and ζ .

$s=1$, $\mathcal{G}_d^{\mathcal{M}} = b_{\min}''/s = \min[\zeta^2, \alpha, 1]$ and $\mathcal{G}_d^{G-G} = b_{\min}^*/s = \min[\zeta^2, \alpha, \beta]$ as drawn in Section 3.3.4, i.e., in Fig. 3.9 the labeled measured log-log slopes are identical to the formerly drawn ones.

The ergodic channel capacity (C_{erg}) versus the RF SNR ($\bar{\gamma}_{\text{RF}}$) is plotted in Fig. 3.10 for different values of the atmospheric turbulence parameters (α and β), while the FSO SNR ($\bar{\gamma}_{\text{FSO}}$) and the pointing error (ζ) are kept fixed in this figure. It can be seen, from this figure, that lower values of α and β , i.e., stronger atmospheric turbulence, result in lower (worse) C_{erg} and vice versa. Moreover, based on the used parameters, we can notice from Fig. 3.10 that in the relatively low RF SNRs the RF hop dominates the C_{erg} , i.e., C_{erg} varies with the change in $\bar{\gamma}_{\text{RF}}$ although $\bar{\gamma}_{\text{FSO}}$ is set

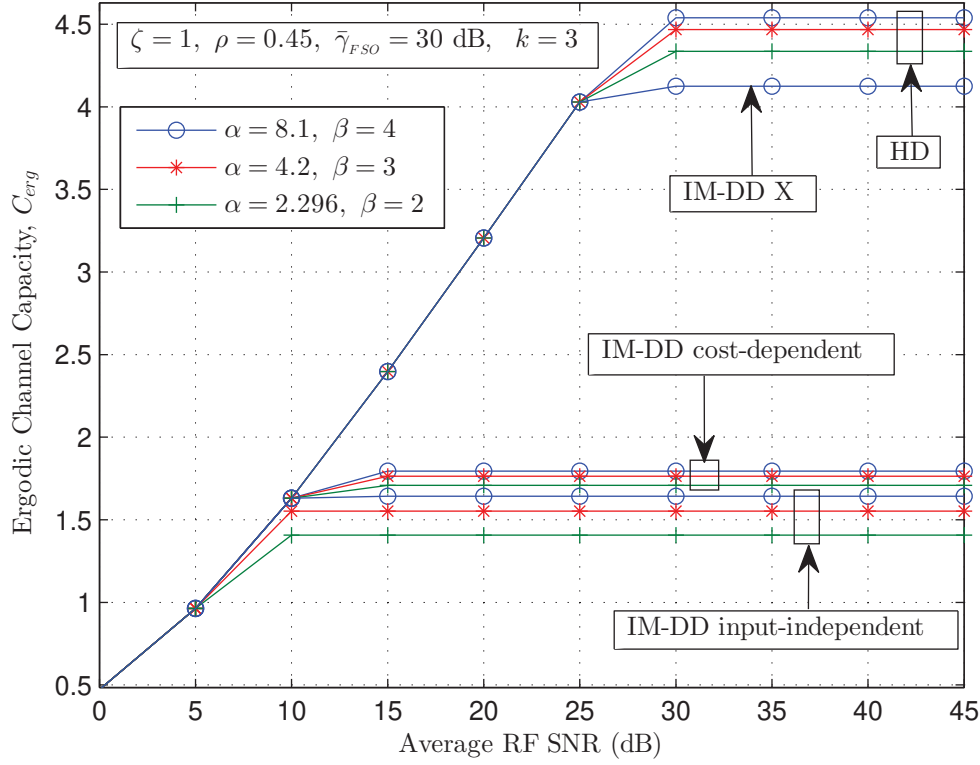


Figure 3.10: C_{erg} for different values of α and β considering all FSO detection techniques.

constant here. However, if the RF SNR becomes relatively high, C_{erg} will be constant, because in this case it will be dominated by the FSO hop whose $\bar{\gamma}_{\text{FSO}}$ is set constant in this figure.

Fig. 3.11 provides the influence of the different values of ρ and ζ on the system ergodic channel capacity (C_{erg}) for all FSO detectors. Clearly, C_{erg} increases by the increase in the value of ρ or ζ (smaller pointing error). In addition, this figure demonstrates that the HD provides better ergodic capacity than both IM-DD techniques, and IM-DD input-independent reveals higher C_{erg} than IM-DD cost-dependent. However, based on the used parameters in this figure, this IM-DD input-independent ergodic capacity is overestimated because again the IM-DD input-independent ignores the

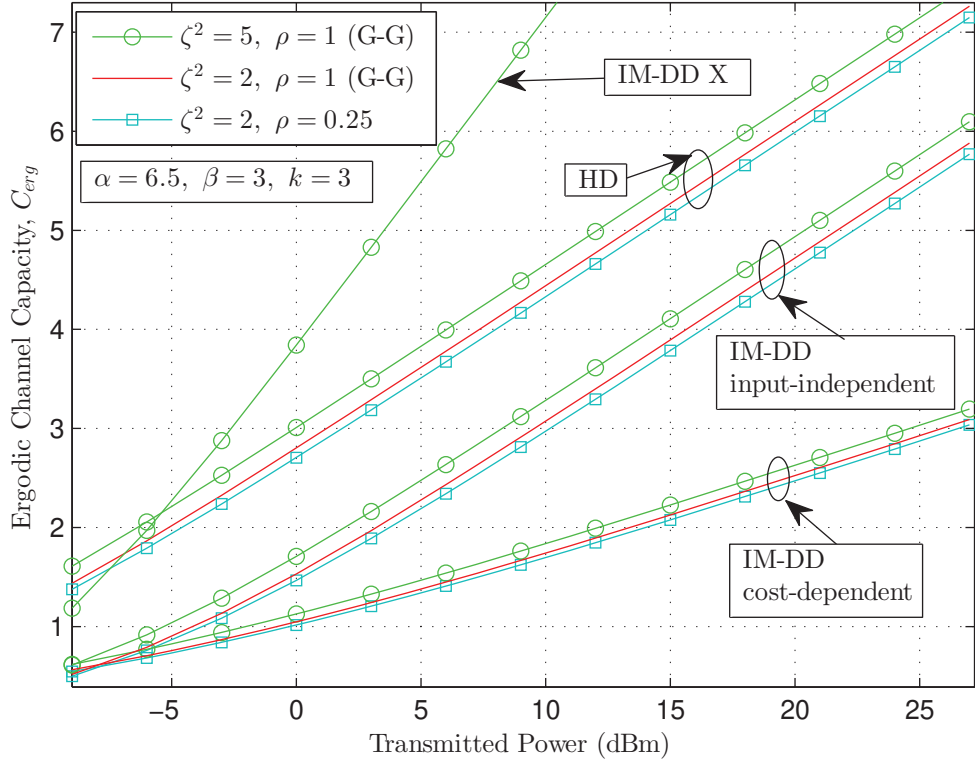


Figure 3.11: C_{erg} dominated by the FSO hop vs transmitted power.

shot-noise which is considerable in this example. Furthermore, from Fig. 3.11 it can be seen that the capacity pre-log equals 0.5 for HD and IM-DD input-independent, while it is 0.25 for IM-DD cost-dependent. This confirms the capacity pre-log, assuming Tx over 2 channel uses, which can be deduced, at high transmitted powers, from $\mathbb{E}\{C_{\text{FSO}}\}$ with the help of (3.13), (3.12) and (3.5).

Then again for comparison, Figs. 3.10 and 3.11 contain the curve IM-DD X which reveals the overestimation in C_{erg} if the capacity form IM-DD_capX ($\log(1 + \gamma_{\text{IMDD}_{\text{indp}}})$) is unsoundly used for IM-DD input-independent instead of its precise capacity approx. ($\log(1 + \sqrt{\frac{e}{2\pi}} \gamma_{\text{IMDD}_{\text{indp}}})$) given in (3.12). This overestimation, in C_{erg} (and its pre-log) of IM-DD input-independent, is mainly due to the missed sqrt. in the IM-DD_capX form as compared to the one in (3.12), i.e., even if a form such as

$(\log(1 + \frac{\epsilon}{2\pi}\gamma_{\text{IMDD}_{\text{indp}}}))$ is inaccurately used for IM-DD input-independent capacity, the same above indicated overestimation, in the pre-log of C_{erg} , will appear.

3.5 Conclusion

In this work, we studied the performance of dual-hop mixed RF/FSO DF relaying, where HD and IM-DD were considered in a unified manner for FSO detection. Specifically, we derived closed-form expressions for the outage probability, average BER, and ergodic capacity of this system assuming generalized fading models, i.e., Nakagami- m and $\mathcal{M}$ fading with pointing error for the RF and FSO links, respectively. In doing that, we utilized, for the first time as per our knowledge, a precise IM-DD channel capacity result. Moreover, this was the first time that not only the IM-DD input-independent but also the IM-DD cost-dependent AWGN channel is considered in such system analysis. Also, in BER analysis, we assumed that the modulation schemes in the two hops can be different, as not all modulations are suitable for IM-DD FSO links. Additionally, the system performance was asymptotically investigated at high SNRs where a new non-reported diversity order and coding gain analysis was shown. Interestingly, we found that in the FSO hop, at high transmitted powers, all the optical detectors result in the same diversity order, whereas their coding gains are different. Also, for HD and IM-DD input-independent we found that, at high transmitted powers, their capacities pre-logs are identical and equal twice the one of IM-DD cost-dependent. Moreover, we showed the overestimations which occur in the outage and capacity of the system when the well-known Shannon's capacity formula for the

power-constrained AWGN channel is used for IM-DD instead of its precise capacity approx. These overestimations lead to misconception in the comparison between HD and IM-DD. Furthermore, we offered simulation results which confirm the derived exact and asymptotic expressions.

CHAPTER 4

DUAL-HOP MIXED INTERFERENCE ALIGNED MIMO RF/UNIFIED-FSO RELAYING SCHEMES

4.1 Introduction

Despite its worthy applications no work in the literature, as per our knowledge, investigated the performance of dual-hop mixed interference aligned MIMO RF interference channel (IC)/FSO DF relaying system. Therefore, this encourage us to analyze the performance of such system in which both HD and IM-DD are considered in the FSO link. In this work the Interference alignment (IA) technique is utilized in the MIMO RF IC hop to eliminate the interference, and hence this leads to enhance the

multiplexing gain (degrees-of-freedom (DoF)) in this hop [28], and as a result the overall system performance is improved. To do that IA exploits the knowledge of the channel state information (CSI) of all transmission links to build (beamform) the transmitted signals in a way that the interference signals at any receiver overlies on the same subspace while the desired signal, at that receiver, remains separable at another orthogonal subspace [28]. If this happens, interference elimination is possible by projecting the total received signal, using interference elimination combining matrix, on the desired signal subspace (zero-forcing the interference).

An important application of this scheme is the one in which the first hop is a MIMO uplink IC where K multiantennas-equipped mobile users (MSs) communicate with their corresponding K multiantennas-equipped base transceiver stations (BTSs) via RF links, specially when those MSs are located at the coverage border of their neighboring BTSs. While in the second hop these BTSs are connected with the backbone network via FSO links.

In this study, we utilize, in the system outage and capacity analyses, an accurate approximation for the IM-DD channel capacity drawn in [12]–[15], [77]. This rigorous approximation is tight in high SNRs, and thus it can act as the channel capacity at that region. Therefore, our analysis offers a rigorous quantitative comparison between IM-DD and HD performances, and hence assists the planners in choosing whether to use HD (better info rates) or IM-DD (less costs/complexity).

Additionally, not only the IM-DD input-independent AWGN but also the IM-DD cost-dependent AWGN channel is analyzed in these investigations. The difference between those two IM-DD channels resides in the variance of the AWGN which does

not depend on the input signal in the first model, however in the second model it is proportionate to the cost (constraint) of the input signal (average optical power). It is important also to state that the input-independent AWGN IM-DD channel model [12], [14] is only valid where the receiver noises are dominated by thermal noise, and this is the case in the receivers which employ PIN-photodiode or in case of low optical powers. On the other hand, the cost-dependent AWGN IM-DD channel model [15], [77] is useful wherever the receiver noises are dominated by shot noise, and this is the case in the receivers which employ avalanche photodiode (APD) or in case of high optical power.

In doing this analysis, we assume that the FSO links follow Málaga ($\mathcal{M}$) fading with pointing errors, whilst the MIMO RF links follow Rayleigh fading. Note that the $\mathcal{M}$ fading model, as shown in [24], is valid under the whole range of atmospheric turbulence scenarios and it gives a very accurate fit to the experimental data. Also, the $\mathcal{M}$ fading contains most of the FSO fading models (comprising G-G) as special cases. Consequently, our analysis can be viewed as a generic one from the point of view of the FSO fading models.

Moreover, the system performance is studied asymptotically at the high SNRs region, where the diversity order and coding gain analyses are illustrated. It is found that when the system is dominated by the RF hop, the diversity order equals the square of the number of the interference-eliminated RF info streams. Alternatively, when the system is dominated by the FSO hop, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors.

The analysis in this chapter is accomplished in a unified way for the FSO connec-

tion which merges HD, IM-DD input-independent, and IM-DD cost-dependent in a unified expression for each statistical characteristic and/or performance measure.

Notation: $\det(\mathbf{Q})$ represents the determinant of matrix $\mathbf{Q}$. $(\mathbf{Q})^H$ represents the conjugate transpose of matrix $\mathbf{Q}$. $\text{Tr}(\mathbf{Q})$ denotes the trace of matrix $\mathbf{Q}$ and $I_{d \times d}$ is the identity matrix of size $d \times d$. $\mathbb{E}\{\cdot\}$ is the expectation operator. $\Gamma(\cdot)$ is the Gamma function as defined in [74, Eq. (8.310)], and $G[\cdot]$ is the Meijer's G-function as defined in [74, Eq. (9.301)]. $\text{Pr}[\cdot]$ denotes the probability operator.

The rest of this chapter is organized as follows. Section 4.2 presents the system model. The system analyses are offered in Section 4.3. Section 4.4 carries out and discusses the numerical and simulation results. Those results validate the deduced analytical and asymptotic expressions. Lastly, the conclusion is given in Section 4.5.

4.2 System and Channel Models

As shown in Fig. 4.1, the system under study is a dual-hop mixed MIMO RF IC/FSO DF relaying. In this system, K source-relay (transmitter-receiver) pairs form a MIMO RF interference channel in the first hop, where the transmitting MS user i (source i) has N_T antennas and needs to send d independent info streams, each one is equivalent to one DoF, to the receiving BTS i (relay i) which has N_R antennas. On the other hand, in the second hop the relays forward the sources messages to the corresponding destinations in the backbone network through FSO links.

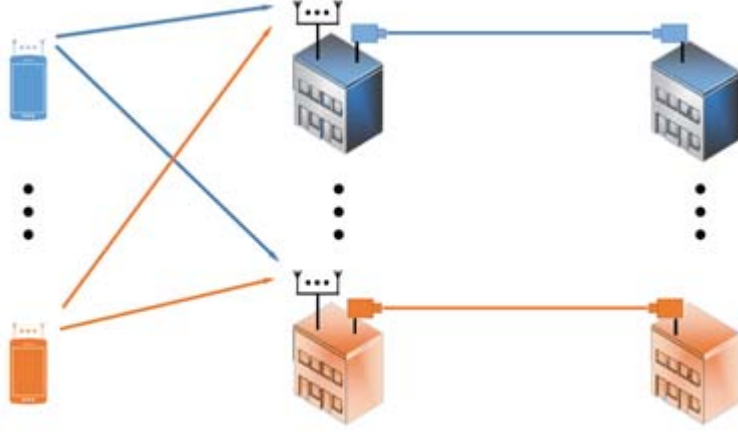


Figure 4.1: Dual-hop mixed MIMO RF IC/FSO DF relaying system

4.2.1 The MIMO RF IC hop

In this work the IA technique is utilized in the MIMO RF IC hop to eliminate the interference, and hence results in enhancing the multiplexing gain (DoF) in this hop. The IA is feasible if there exist transmission precoding matrix $\mathbf{V}_i \in \mathbb{C}^{N_T \times d}$ at the transmitting MS user i (source i), and receiving interference elimination combining matrix $\mathbf{U}_i \in \mathbb{C}^{N_R \times d}$ at the receiving BTS i (relay i) with

$$\text{rank}(\mathbf{V}_i) = d, \quad \mathbf{V}_i^H \mathbf{V}_i = \mathbf{I}_{d \times d}, \quad (4.1)$$

$$\text{rank}(\mathbf{U}_i) = d, \quad \mathbf{U}_i^H \mathbf{U}_i = \mathbf{I}_{d \times d}, \quad (4.2)$$

such that [29]

$$\text{rank}(\mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i) = d, \quad \forall i \in \{1, 2, \dots, K\}, \quad (4.3)$$

$$\mathbf{U}_i^H \mathbf{H}_{ij} \mathbf{V}_j = \mathbf{0}_{d \times d}, \quad \forall j \neq i, \quad i, j \in \{1, 2, \dots, K\}, \quad (4.4)$$

where $\mathbf{H}_{ij} \in \mathbb{C}^{N_R \times N_T}$ is the matrix which represents the MIMO RF faded channel between relay i and source j . We assume that the elements of $\mathbf{H}_{ij}$ are i.i.d. $\mathcal{CN}(0, \Omega_{\text{RF}})$, i.e., Rayleigh faded MIMO RF channel. Also, as mentioned in [29], the IA in $(N_R \times N_T, d)^K$ MIMO IC is feasible when

$$N_R + N_T - d(K + 1) \geq 0. \quad (4.5)$$

Assuming perfect CSI at each node, the interference eliminated received $d \times 1$ signal vector at the receiving BTS i is given by

$$y_i = \sqrt{\frac{P_s}{d}} \mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i x_i + \underbrace{\sum_{j=1, j \neq i}^K \sqrt{\frac{P_s}{d}} \mathbf{U}_i^H \mathbf{H}_{ij} \mathbf{V}_j x_j}_{\text{Zero due to (4.4)}} + \mathbf{U}_i^H n_i, \quad (4.6)$$

$$= \sqrt{\frac{P_s}{d}} \tilde{\mathbf{H}}_{ii} x_i + \tilde{n}_i, \quad (4.7)$$

where x_i is the sent $d \times 1$ info vector from the transmitting MS user i , and $n_i \sim \mathcal{CN}(0, N_0 \mathbf{I})$ is the $N_R \times 1$ AWGN vector at the receiving BTS i , $\tilde{\mathbf{H}}_{ii} = \mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i$, and $\tilde{n}_i = \mathbf{U}_i^H n_i$.

4.2.2 Design of the precoding and the receiving interference eliminating matrices

The design of the b^{th} user precoding matrix $\mathbf{V}_b$ and the interference suppression matrix $\mathbf{U}_b$ associated to its corresponding b^{th} BTS is performed using the orthogonalized minimum leakage (**OR-ML**) algorithm. In this algorithm, the interference I_b at each b^{th} BTS is minimized at the forward direction, where the interference is written as

$$I_b = \text{Tr}(\mathbf{U}_b^H \mathbf{Q}_b \mathbf{U}_b), \quad (4.8)$$

where $\mathbf{Q}_b$ is the interference covariance matrix for that b^{th} BTS modeled as

$$\mathbf{Q}_b = \sum_{g=1, g \neq b}^K \frac{P_s}{d} \mathbf{H}_{bg} \mathbf{V}_g \mathbf{V}_g^H \mathbf{H}_{bg}^H. \quad (4.9)$$

To minimize the interference in this direction, $\mathbf{U}_b$ is composed from the eigenvectors associated with the smallest eigenvalues of $\mathbf{Q}_b$.

In the reverse direction, the interference $\overleftarrow{I}_k$ for the k^{th} user is minimized where

$$\overleftarrow{I}_k = \text{Tr}(\overleftarrow{\mathbf{U}}_k^H \overleftarrow{\mathbf{Q}}_k \overleftarrow{\mathbf{U}}_k), \quad (4.10)$$

and the reverse interference covariance matrix is

$$\overleftarrow{\mathbf{Q}}_k = \sum_{b=1, b \neq k}^K \frac{P_s}{d} \overleftarrow{\mathbf{H}}_{kb} \overleftarrow{\mathbf{V}}_b \overleftarrow{\mathbf{V}}_b^H \overleftarrow{\mathbf{H}}_{kb}^H, \quad (4.11)$$

Algorithm 1 Orthogonalized Minimum Leakage (*OR-ML*) Algorithm

- 1: **Start** with arbitrary $N_T \times d$ precoding matrices $\mathbf{V}_k$ with $\mathbf{V}_k^H \mathbf{V}_k = \mathbf{I}_{d \times d} \quad \forall k \in \{1, \dots, K\}$.
 - 2: **Repeat**
 - 3: **Calculate** the interference covariance matrices $\mathbf{Q}_b \quad \forall b \in \{1, \dots, K\}$ according to (4.9).
 - 4: **Compute** the interference suppression vector $\mathbf{U}_b^{l*}$ for the l^{th} data stream at b^{th} BTS where $\mathbf{U}_b^{l*} = v_l[\mathbf{Q}_b] \quad \forall b \in \{1, \dots, K\}$, where $v_l[\mathbf{Q}_b]$ is the eigenvector of the l^{th} smallest eigenvalue of $\mathbf{Q}_b$.
 - 5: **Combine** the interference suppression vectors $\mathbf{U}_b^{l*}$ to form the interference suppression matrix $\mathbf{U}_b \quad \forall b \in \{1, \dots, K\}$.
 - 6: **Reverse** the direction and set the reverse precoding matrix as the interference suppression matrix $\overleftarrow{\mathbf{V}}_b = \mathbf{U}_b \quad \forall b \in \{1, \dots, K\}$.
 - 7: **Calculate** the reverse interference covariance matrices $\overleftarrow{\mathbf{Q}}_k \quad \forall k \in \{1, \dots, K\}$ according to (4.11).
 - 8: **Compute** the reverse interference suppression vector $\overleftarrow{\mathbf{U}}_k^{l*}$ for the l^{th} data stream at k^{th} user where $\overleftarrow{\mathbf{U}}_k^{l*} = v_l[\overleftarrow{\mathbf{Q}}_k] \quad \forall k \in \{1, \dots, K\}$.
 - 9: **Combine** the interference suppression vectors $\overleftarrow{\mathbf{U}}_k^{l*}$ to form the reverse interference suppression matrix $\overleftarrow{\mathbf{U}}_k \quad \forall k \in \{1, \dots, K\}$.
 - 10: **Reverse** the direction and set the precoding matrix as the reverse interference suppression matrix $\mathbf{V}_k = \overleftarrow{\mathbf{U}}_k \quad \forall k \in \{1, \dots, K\}$.
 - 11: **Until** a pre-defined number of iterations.
 - 12: **Output** $\mathbf{U}_b \quad \forall b \in \{1, \dots, K\}$ and $\mathbf{V}_k \quad \forall k \in \{1, \dots, K\}$.
-

where $\overleftarrow{\mathbf{H}}_{kb} = \mathbf{H}_{bk}^H$, $\overleftarrow{\mathbf{V}}_b = \mathbf{U}_b$ and $\overleftarrow{\mathbf{U}}_k = \mathbf{V}_k$. To minimize the interference in the reverse direction, $\overleftarrow{\mathbf{U}}_k$ (i.e. $\mathbf{V}_k$) is composed from the eigenvectors associated with the smallest eigenvalues of $\overleftarrow{\mathbf{Q}}_k$.

4.2.3 The FSO hop

After receiving the signal, the relay (BTS) forwards a decoded version of it to the destination in the second phase of transmission. Let's use x_{HD} and x_{IMDD} to designate the transmitted signal from the relay for the case of HD and IM-DD, with the constraints $\mathbb{E}\{|x_{\text{HD}}|^2\}=1$ and $\mathbb{E}\{x_{\text{IMDD}}\}=1$ respectively. It is worthy noticing that in the IM-DD case the average power constraint is on the expected signal value and not on its expected square as the signal in the IM-DD technique is an optical intensity. So, the

received signal at the destination for the case of HD and IM-DD can be, respectively, expressed by

$$y_{\text{HD}} = \sqrt{P_r} g_{\text{FSO}} x_{\text{HD}} + n_{\text{HD}}, \quad (4.12)$$

$$y_{\text{IMDD}} = P_r I_{\text{FSO}} x_{\text{IMDD}} + n_{\text{IMDD}}, \quad (4.13)$$

where P_r is the relay transmitted power, g_{FSO} and $I_{\text{FSO}} = |g_{\text{FSO}}|^2$ designate the fading in the received electric field and irradiance, respectively. This fading is a result of the atmospheric turbulence in the relay-destination channel. In (4.12), $n_{\text{HD}} \sim \mathcal{CN}(0, N_{\text{HD}})$ is a complex AWGN. Whilst in (4.13), n_{IMDD} is a real AWGN where $n_{\text{IMDD}} \sim \mathcal{N}(0, N_{\text{IMDD}})$ for the IM-DD input-independent AWGN model, and $n_{\text{IMDD}} \sim \mathcal{N}(0, 2P_r I_{\text{FSO}} N_{\text{HD}} \mathbb{E}\{x_{\text{IMDD}}\})$ for the IM-DD cost-dependent AWGN channel under the assumption of relatively high P_r in comparison with the thermal and/or background noises power. The reader is referred to [15], [77] for details of the HD and the two IM-DDs and their noises. So the SNRs at the destination are given as follows for the HD, IM-DD input-independent, and IM-DD cost-dependent,

$$\gamma_{\text{HD}} = \frac{P_r I_{\text{FSO}}}{N_{\text{HD}}}, \quad \gamma_{\text{IMDD}_{\text{indp}}} = \frac{P_r^2 I_{\text{FSO}}^2}{N_{\text{IMDD}}}, \quad \gamma_{\text{IMDD}_{\text{cost}}} = \frac{P_r I_{\text{FSO}}}{2N_{\text{HD}}}. \quad (4.14)$$

In this study, it is assumed that the FSO connection encounters $\mathcal{M}$ fading with pointing error. In the development of the $\mathcal{M}$ fading channel [24], the received signal is assumed to be consisted of a LOS component (U_L), a component coupled to the LOS and scattered by eddies on the propagation axis (U_S^C), and a component scattered by off-axis eddies (U_S^G) that is statistically independent from U_L and U_S^C . Thus by

assuming the $\mathcal{M}$ fading with pointing mismatch in the FSO channel, the probability density function (pdf) of I_{FSO} can be represented by [25, Eq. (21)]

$$f_{I_{\text{FSO}}}^{\mathcal{M}}(I) = \frac{\zeta^2 A}{2I} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{\alpha \beta}{g \beta + \Omega'} \frac{I}{A_0} \middle| \zeta^{2+1} \right], \quad (4.15)$$

where

$$A = \frac{2\alpha^{\alpha/2}}{g^{1+\alpha/2}\Gamma(\alpha)} \left(\frac{g \beta}{g \beta + \Omega'} \right)^{\beta+\alpha/2}, \quad b_m = a_m [\alpha \beta / (g \beta + \Omega')]^{-(\alpha+m)/2},$$

$$a_m = \binom{\beta-1}{m-1} \frac{(g \beta + \Omega')^{1-m/2}}{(m-1)!} \left(\frac{\Omega'}{g} \right)^{m-1} \left(\frac{\alpha}{\beta} \right)^{m/2},$$

α is positive parameter and it is related to the effective number of the large-scale cells of the scattering (fading) process, β is a natural number and it is related to the effective number of the small-scale cells of the fading process ¹, $\Omega' = \Omega + 2b_0\rho + 2\sqrt{2b_0\rho\Omega} \cos(\Phi_A - \Phi_B)$ is the average power of the coherent contribution, $\Omega = \mathbb{E}\{|U_L|^2\}$, $\mathbb{E}\{|U_S^C|^2\} = 2b_0\rho$, Φ_A and Φ_B are deterministic phases of the LOS and the coupled-to-LOS signal components, respectively, $2b_0 = \mathbb{E}\{|U_S^C|^2 + |U_S^G|^2\}$ is the average power of all scattered components, $g = \mathbb{E}\{|U_S^G|^2\} = 2b_0(1 - \rho)$, the parameter $0 \leq \rho \leq 1$ denotes the part of the scattered power coupled to the LOS, ζ is the ratio between the equivalent beam radius and the standard deviation of the pointing error displacement at the receiver (i.e. $\zeta \rightarrow \infty$, indicates the nonpointing error case) [78], A_0 is a parameter that represents the pointing losses (or typically the FSO link losses).

¹The fading parameters α and β reflect the atmospheric turbulences condition (The Rytov variance) with low values basically denoting strong (severe) atmospheric turbulent situation.

From the pdf in (4.15) ($f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$) and by random variable transformations, we attain the pdfs of the SNRs in (4.14) in the following unified formula

$$f_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) = \frac{\zeta^2 A}{2^s \gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[B \left(\frac{w\gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, m \end{matrix} \right], \quad (4.16)$$

where γ_{FSO} represents the unified SNR of the FSO channel, $B = \zeta^2 \alpha \beta (g + \Omega') / [(\zeta^2 + 1)(g\beta + \Omega')]$, s and w are the parameters which determine the sort of the FSO detections, i.e., $(s, w) = (1, 1)$, $(2, 1)$ or $(1, 2)$ for the HD, IM-DD input-independent, and IM-DD cost-dependent respectively, and where $\mu_1 \triangleq P_r \mathbb{E}\{I_{\text{FSO}}\} / N_{\text{HD}} = A_0 P_r \zeta^2 (g + \Omega') / [(\zeta^2 + 1)N_{\text{HD}}]$,² and for normalization purposes $\mu_2 \triangleq P_r^2 \mathbb{E}^2\{I_{\text{FSO}}\} / N_{\text{IMDD}}$ as previously defined in [79] and references therein. This definition results in $\mu_2 = A_0^2 P_r^2 \zeta^4 (g + \Omega')^2 / [(\zeta^2 + 1)^2 N_{\text{IMDD}}]$. From these expressions of μ_1 , μ_2 , and (4.14), it is clear that μ_s is proportional to the average of the unified FSO SNR ($\bar{\gamma}_{\text{FSO}}$). Specifically, $\mu_1 = \bar{\gamma}_{\text{HD}} = 2\bar{\gamma}_{\text{IMDD}_{\text{cost}}}$, however $\mu_2 = \bar{\gamma}_{\text{IMDD}_{\text{indp}}} \frac{\mathbb{E}^2\{I_{\text{FSO}}\}}{\mathbb{E}\{I_{\text{FSO}}^2\}} = \frac{\zeta^2 (\zeta^2 + 1)^{-2} (\zeta^2 + 2)(g + \Omega')^2}{\alpha^{-1}(\alpha + 1)[2g(g + 2\Omega') + \Omega'^2(1 + 1/\beta)]} \bar{\gamma}_{\text{IMDD}_{\text{indp}}}$.²

Note that having $\rho = 1$ (i.e. $g = 0$) results in $Ab_m = 2/(\Gamma(\alpha)\Gamma(\beta))$ when $m = \beta$ and $Ab_m = 0$ for all other m . So in this case, by having $\rho = 1$, and $\Omega' = 1$ (for normalization), the pdf $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (4.15) turns into the special case of the G-G distribution with pointing errors [25]. By having $\rho = 1$, the power $\mathbb{E}\{|U_S^G|^2\} \triangleq g = 0$, this means that the G-G fading channel neglects the component of the received signal scattered by the off-axis eddies (U_S^G). Thus, by using $\Omega' = 1$ and $\rho = 1$ in (4.16), the pdf of the $^2\mathbb{E}\{I_{\text{FSO}}\}$ and $\mathbb{E}\{I_{\text{FSO}}^2\}$ are derived from [25, Eq. (33)].

unified FSO SNR in the G-G fading with pointing error can be presented by

$$f_{\gamma_{\text{FSO}}}^{\text{G-G}}(\gamma) = \frac{\zeta^2}{s \gamma \Gamma(\alpha) \Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2 \alpha \beta}{(\zeta^2 + 1)} \left(\frac{w \gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2 + 1 \\ \zeta^2, \alpha, \beta \end{matrix} \right]. \quad (4.17)$$

4.3 System Performance Analysis

4.3.1 Outage Probability

The outage probability (P_{out}) is a substantial performance indicator in slow fading channels³. It can be presented, as in [81], by $P_{\text{out}} = \Pr[C < R]$, where R is the end to end (e2e) required information rate in bits/channel-use, C is a random variable and it is a function on the random e2e-equivalent SNR of the channel. The realization of C designates the e2e channel capacity given the realization of the e2e channel equivalent SNR. As in any DF dual-hop system, in the considered system, the outage event takes place whenever the first hop or the second hop falls in an outage. Therefore, the outage probability in the system under study can be expressed as⁴

$$P_{\text{out}} = \Pr \left[C_{\text{RF}} < 2R \bigcup C_{\text{FSO}} < 2R \right], \quad (4.18)$$

³Note that the channels in most of the RF and FSO schemes, including our system in this work, are slow fading channels [2], [81].

⁴Here the required data rates in the two hops are assumed to be identical, and thence the factor 2 results by assuming that the e2e transmission (from the source to destination) takes place on two equal phases (two equal channel uses). This lets each hop data rate to be twice the e2e data rate.

where C_{RF} and C_{FSO} are related to the channel capacity of the first hop (RF hop) and the second hop (FSO hop), respectively.

Particularly, C_{RF} is a random variable and its realization is the channel capacity of the RF hop given the related realization of the effective channel fading matrix ($\tilde{H}_{ii}$) of the first hop. However by utilizing the IA, the first hop is effectively a MIMO AWGN channel as represented by (4.7). Thus, by utilizing the MIMO AWGN channel capacity expression, as given in [81], we obtain

$$C_{\text{RF}} = \log \left[\det \left(\frac{P_s}{N_o d} \tilde{H}_{ii} \tilde{H}_{ii}^H + I_{d \times d} \right) \right] \quad (4.19)$$

However, since the RF channel matrix has i.i.d. $\mathcal{CN}(0, \Omega_{\text{RF}})$ elements, then with probability 1 this matrix has full rank, and it is well-conditioned [81]. Due to these properties, we get $\tilde{H}_{ii} \tilde{H}_{ii}^H = \lambda_{hh} I_{d \times d}$. Furthermore, by using the eigenvalues identities, it can be shown that $\lambda_{hh} d = \text{Tr}(\tilde{H}_{ii} \tilde{H}_{ii}^H)$, and this means that $\tilde{H}_{ii} \tilde{H}_{ii}^H = \frac{\text{Tr}(\tilde{H}_{ii} \tilde{H}_{ii}^H)}{d} I_{d \times d}$. Then, employing the last expression of $\tilde{H}_{ii} \tilde{H}_{ii}^H$ in (4.19) leads to

$$C_{\text{RF}} = d \log \left(1 + \frac{P_s}{N_o d^2} \text{Tr}(\tilde{H}_{ii} \tilde{H}_{ii}^H) \right). \quad (4.20)$$

In a compact way, (4.20) can be written as

$$C_{\text{RF}} = d \log(1 + \gamma_{\text{RF}}), \quad (4.21)$$

where γ_{RF} is defined by $\gamma_{\text{RF}} = \frac{P_s}{N_o d^2} \text{Tr}(\tilde{H}_{ii} \tilde{H}_{ii}^H)$, and it is used to represent the effective SNR of the MIMO RF link (or in short we may call it the RF SNR). As a result of

assuming i.i.d. $\mathcal{CN}(0, \Omega_{\text{RF}})$ elements of the RF channel matrix, γ_{RF} has a gamma pdf which is given by [16]

$$f_{\gamma_{\text{RF}}}(\gamma) = \left(\frac{d^2}{\bar{\gamma}_{\text{RF}}} \right)^{d^2} \frac{\gamma^{d^2-1}}{(d^2-1)!} \exp \left(-\frac{d^2}{\bar{\gamma}_{\text{RF}}} \gamma \right), \quad (4.22)$$

where $\bar{\gamma}_{\text{RF}}$ denotes the average effective SNR of the MIMO RF link (or shortly the average RF SNR). Namely, $\bar{\gamma}_{\text{RF}} = \mathbb{E}\{\gamma_{\text{RF}}\} = \frac{P_s}{N_0} \Omega_{\text{RF}}$.

On the other side, C_{FSO} is a function on the random SNR of the FSO hop, and thence it is a random variable. Additionally, in case of HD the realization of C_{FSO} designates the channel capacity of the second hop given the realization of the second hop SNR (γ_{HD}). Whereas, in case of IM-DD we let the realization of C_{FSO} designates a precise approximation ⁵ of the channel capacity of the second hop given the related realization of the second hop SNR ($\gamma_{\text{IMDD}_{\text{indp}}}$ or $\gamma_{\text{IMDD}_{\text{cost}}}$). But, given γ_{HD} , $\gamma_{\text{IMDD}_{\text{indp}}}$ or $\gamma_{\text{IMDD}_{\text{cost}}}$ the FSO hop is, respectively, an AWGN channel, IM-DD input-independent AWGN channel, or IM-DD cost-dependent AWGN channel as expressed by (4.12) or (4.13). So, by utilizing the expressions of the AWGN channel capacity [73] and the approximations of both IM-DD AWGN channel capacities [12], [15] we get a unified C_{FSO} expressed as

$$C_{\text{FSO}} = \log(1 + \gamma_{\text{FSO}}^{\text{eff}}), \quad (4.23)$$

where $\gamma_{\text{FSO}}^{\text{eff}}$ is used to represent the unified effective SNR of the FSO channel, i.e., $\gamma_{\text{FSO}}^{\text{eff}}$

⁵This approx. is used as the IM-DD channel capacity is so far unknown [14]. Nonetheless, this approx. is very rigorous at high SNRs, and thus it can act as the IM-DD channel capacity in that region.

equals either γ_{HD} , $\sqrt{\frac{c}{2\pi}\gamma_{\text{IMDD}_{\text{indp}}}}$, or $\sqrt{\frac{c}{2\pi}\gamma_{\text{IMDD}_{\text{cost}}}}$ based on the used FSO transmission type. Because it is important in what follows, let's derive the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$. From $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (4.15), with the help of (4.14), and by random variable transformation, the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ can be expressed in a unified form as

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) = \frac{w \zeta^2 A}{2\gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{B (\gamma/c)^w}{\mu_s^{1/s}} \middle| \zeta^2+1 \right]_{\zeta^2, \alpha, m}, \quad (4.24)$$

where the constant $c = 1$, $\sqrt{\frac{c}{2\pi}}$, or $\sqrt{\frac{c}{4\pi}}$ for HD, IM-DD input-independent, or IM-DD cost-dependent, respectively, and the remainder of the parameters are as in (4.15) and (4.16). Notice that using $\Omega'=1$ and $\rho=1$ in (4.24), results in the following pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ in the special case of G-G fading with pointing errors

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma) = \frac{w \zeta^2}{\gamma \Gamma(\alpha) \Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| \zeta^2+1 \right]_{\zeta^2, \alpha, \beta}. \quad (4.25)$$

Now with the help of (4.21) and (4.23), the P_{out} in (4.18) can be given by [83]

$$P_{\text{out}} = \Pr[\gamma_{\text{RF}} < (2^{2R/d} - 1)] + \Pr[\gamma_{\text{FSO}}^{\text{eff}} < (2^{2R} - 1)] - \Pr[\gamma_{\text{RF}} < (2^{2R/d} - 1)] \Pr[\gamma_{\text{FSO}}^{\text{eff}} < (2^{2R} - 1)]. \quad (4.26)$$

From (4.26) P_{out} can be expressed in terms of the cumulative distribution functions (CDFs) of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$ evaluated at $\gamma_{\text{out1}} \triangleq (2^{2R/d} - 1)$ and $\gamma_{\text{out2}} \triangleq (2^{2R} - 1)$, respectively. Consequently, we get

$$P_{\text{out}} = F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out2}}) - F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out2}}), \quad (4.27)$$

where $F_{\gamma_{\text{RF}}}(\gamma)$ and $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ are the CDFs of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$, respectively. Those CDFs are, respectively, expressed, after integrating their pdfs in (4.22) and (4.24) with respect to γ , as follows

$$F_{\gamma_{\text{RF}}}(\gamma) = 1 - \exp\left(-\frac{d^2\gamma}{\bar{\gamma}_{\text{RF}}}\right) \sum_{i=0}^{d^2-1} \frac{1}{i!} \left(\frac{d^2\gamma}{\bar{\gamma}_{\text{RF}}}\right)^i, \quad (4.28)$$

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right]. \quad (4.29)$$

Then, by exploiting the CDFs (4.28) and (4.29) in (4.27) and after some algebraic simplifications, we get

$$P_{\text{out}} = 1 - \left(\exp\left(-\frac{d^2\gamma_{\text{out1}}}{\bar{\gamma}_{\text{RF}}}\right) \sum_{i=0}^{d^2-1} \frac{1}{i!} \left(\frac{d^2\gamma_{\text{out1}}}{\bar{\gamma}_{\text{RF}}}\right)^i \right) \left(1 - \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma_{\text{out2}}/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right] \right). \quad (4.30)$$

Recall that the P_{out} in (4.30) is in case of $\mathcal{M}$ faded FSO hop. However, to have P_{out} in the special case of the G-G faded FSO hop, the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ in (4.27) must be changed by the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma)$ that can be given, by integrating its corresponding pdf in (4.25), as follows

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma) = \frac{\zeta^2}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,1} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, \beta, 0 \end{matrix} \right]. \quad (4.31)$$

4.3.2 Asymptotic Outage Probability

The expression of the outage in (4.30) is complicated, and hence, from it, we can not easily see the influence of the system parameters on the system outage. Hence, in this section a simpler (asymptotic) expression is derived for the system outage probability in the high SNRs region. This asymptotic P_{out} helps more in analyzing and understanding the system performance by clearly containing some outage performance indicators in its expression.

Particularly, at high SNRs, the asymptotic outage probability can be typically represented as $P_{\text{out}}^{(a)} \simeq (G_c \overline{\text{SNR}})^{-G_d}$, where G_c and G_d are outage performance indicators that are, respectively, named the coding gain and diversity order associated with the P_{out} of the system [76]. The diversity order G_d identifies the slope of the P_{out} versus the average SNR log-log curve at high SNRs, whilst $\log G_c$ identifies the horizontal shift of this curve in comparison with a curve of benchmark outage probability given by $(\overline{\text{SNR}})^{-G_d}$.

Therefore, to analyze G_d and G_c of our system, we should reduce (4.30) to the standard $P_{\text{out}}^{(a)}$ formula. For this purpose, we modify Eq. (4.27) as follows: firstly we change the used CDFs by their asymptotic versions on high SNRs, then we neglect the last term as it is ignorable on high SNRs. These modifications result in

$$P_{\text{out}}^{(a)} = F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out1}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma_{\text{out2}}), \quad (4.32)$$

where $F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out1}})$ and $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma_{\text{out2}})$ are, respectively, the asymptotic CDFs of γ_{RF} and

$\gamma_{\text{FSO}}^{\text{eff}}$ at high SNRs. Next, Let's determine those asymptotic CDFs. To obtain $F_{\gamma_{\text{RF}}}^{(a)}(\gamma)$

we begin by applying the high SNR limit ($\lim_{\gamma_{\text{RF}} \rightarrow \infty} \exp(-\frac{d^2 \gamma}{\bar{\gamma}_{\text{RF}}}) = 1$) in (4.22) to attain

$f_{\gamma_{\text{RF}}}^{(a)}(\gamma) = (\frac{d^2}{\bar{\gamma}_{\text{RF}}})^{d^2} \frac{\gamma^{d^2-1}}{(d^2-1)!}$, then by integrating this asymptotic pdf we get

$$F_{\gamma_{\text{RF}}}^{(a)}(\gamma) = (d^2/\bar{\gamma}_{\text{RF}})^{d^2} \gamma^{d^2}/(d^2)! . \quad (4.33)$$

Then again to derive $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma)$, we adjust (4.29) by using [84, Eq. (07.34.06.0006.01)]

which is the Meijer G-function series representation for $\frac{1}{\mu_s} \rightarrow 0$, i.e., ⁶ $\bar{\gamma}_{\text{FSO}} \rightarrow \infty$ (on high SNRs). By doing this we obtain

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \sum_{l=1}^3 \frac{\prod_{j=1, j \neq l}^3 \Gamma(b'_j - b'_l)}{\Gamma(\zeta^2 + 1 - b'_l) b'_l} \left(\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \right)^{b'_l}, \quad (4.34)$$

where $b'_1 = \zeta^2, b'_2 = \alpha$ and $b'_3 = m$. Further, (4.34) can be reduced more by keeping the dominant term only in the second summation ($\sum_{l=1}^3$) on high FSO SNR ($\frac{1}{\mu_s} \rightarrow 0$),

i.e., the term which has the smallest b'_l ($b'_{\min} \triangleq \min[\zeta^2, \alpha, m]$). This simplification leads

to

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \frac{\prod_{j=1, b'_j \neq b'_{\min}}^3 \Gamma(b'_j - b'_{\min})}{\Gamma(\zeta^2 + 1 - b'_{\min}) b'_{\min}} \left(\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \right)^{b'_{\min}}. \quad (4.35)$$

For the case of ⁷ $\rho \neq 1$, and by using the definition of $b'_{\min}$, (4.35) can be represented as

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma) = \Lambda \left(B(\gamma/c)^w / \mu_s^{1/s} \right)^{b''_{\min}}, \quad (4.36)$$

⁶Since by definition, μ_s is proportional to $\bar{\gamma}_{\text{FSO}}$ (see the definitions after (4.16)).

⁷The case of $\rho = 1$ (i.e $g = 0$), which leads to $Ab_m = 0$ for all m except $m = \beta$, is considered after (4.40b).

where $b''_{\min} = \min[\zeta^2, \alpha, 1]$, and

$$\Lambda \triangleq \begin{cases} \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m \frac{\prod_{j=1, b'_j \neq b'_{\min}}^3 \Gamma(b'_j - b'_{\min})}{\Gamma(\zeta^2 + 1 - b'_{\min}) b'_{\min}} & \text{if } \zeta^2 \text{ or } \alpha < 1, \Rightarrow b'_{\min} = \min[\zeta^2, \alpha] \\ \frac{\zeta^2 A}{2} b_1 \frac{\Gamma(\zeta^2 - 1) \Gamma(\alpha - 1)}{\Gamma(\zeta^2) \times 1} & \text{if } \zeta^2 \text{ \& } \alpha > 1, \Rightarrow b'_{\min} = 1 \end{cases}, \quad (4.37)$$

where in case of $b'_{\min} = 1$, we keep only the term which has $m = 1$ in the summation with respect to m , since this is the term which dominates on the high FSO SNRs.

Now by using (4.33) and (4.36) in (4.32), we attain

$$P_{\text{out}}^{(a)} = \left(\frac{(d^2!)^{1/d^2} \bar{\gamma}_{\text{RF}}}{d^2 \gamma_{\text{out1}}} \right)^{-d^2} + \left(\frac{\Lambda^{-s/b''_{\min}} B^{-s} \mu_s}{(\gamma_{\text{out2}}/c)^{ws}} \right)^{-b''_{\min}/s}. \quad (4.38)$$

For the $P_{\text{out}}^{(a)}$ in (4.38), let's focus on the following cases:

Case 1: The 1st hop dominates the P_{out} of the system

This is the case whenever the first summand in (4.38) is much bigger than the second one, i.e., the 1st hop (RF link) is in worse situations than the 2nd hop (FSO link). This case occurs for example if the FSO link meets relatively weak turbulence condition and insignificant pointing errors, i.e., where α, β , and ζ possess relatively high values. Also, this case can occur wherever the MIMO RF link possess relatively low average effective SNR.

By referring to the standard $P_{\text{out}}^{(a)}$ formula, it can be noticed that, in this case, the system under consideration can attain the following diversity order and coding gain, respectively,

$$G_d^{\text{RF}} = d^2, \quad (4.39a) \quad G_c^{\text{RF}} = (d^2!)^{1/d^2} / (d^2 \gamma_{\text{out}1}). \quad (4.39b)$$

Case 2: The 2^{nd} hop dominates the P_{out} of the system

This is the case whenever the second summand in (4.38) is much greater than the first term, i.e., the 2^{nd} hop (FSO link) faces worse situations than the 1^{st} hop (RF link). As an example, this case takes place if the FSO link meets relatively strong turbulent condition and considerable pointing mismatches, i.e., where α , β , and ζ possess relatively smaller values. This case can occur also wherever the MIMO RF link possess relatively big average effective SNR.

Then again, by referring to the standard $P_{\text{out}}^{(a)}$ formula, it can be noticed that, in this case, the achieved diversity order and coding gain of our system are, respectively,

$$G_d^{\mathcal{M}} = b_{\min}''/s = \min[\zeta^2, \alpha, 1]/s, \quad (4.40a) \quad G_c^{\mathcal{M}} = \Lambda^{-s/b_{\min}''} B^{-s}/(\gamma_{\text{out}2}/c)^{ws}. \quad (4.40b)$$

The above $P_{\text{out}}^{(a)}$ in (4.38), and so ((4.40a), (4.40b)) are wherever the FSO hop faces $\mathcal{M}$ fadings with $\rho \neq 1$. However, to obtain the $P_{\text{out}}^{(a)}$ at the special case of G-G fadings ($\rho=1$) on the FSO hop, the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}(a)}(\gamma)$ in (4.32) must be exchanged by the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma)$ which is expressed, after applying $\Omega'=1$ (for normalization) and $\rho=1$ in (4.35), as

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma) = \left(\Delta^{1/b_{\min}^{\star}} (\gamma/c)^w / \mu_s^{1/s} \right)^{b_{\min}^{\star}}, \quad (4.41)$$

where $b_1^\star = \zeta^2$, $b_2^\star = \alpha$, $b_3^\star = \beta$, $b_{\min}^\star = \min[\zeta^2, \alpha, \beta]$, and $\Delta \triangleq \frac{\zeta^2}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\zeta^2 \alpha \beta}{\zeta^2 + 1} \right)^{b_{\min}^\star} \frac{\prod_{j=1, b_j^\star \neq b_{\min}^\star}^3 \Gamma(b_j^\star - b_{\min}^\star)}{\Gamma(\zeta^2 + 1 - b_{\min}^\star) b_{\min}^\star}$.

Therefore, by exploiting (4.33) and (4.41), we attain the $P_{\text{out}}^{(a)}$ in case of G-G fading on the FSO hop as follows

$$P_{\text{out}}^{(a)} = F_{\gamma_{\text{RF}}}^{(a)}(\gamma_{\text{out1}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}(a)}(\gamma_{\text{out2}}) \quad (4.42)$$

$$= \left(\frac{(d^2!)^{1/d^2} \bar{\gamma}_{\text{RF}}}{d^2 \gamma_{\text{out1}}} \right)^{-d^2} + \left(\frac{\Delta^{-s/b_{\min}^\star} \mu_s}{(\gamma_{\text{out2}}/c)^{ws}} \right)^{-b_{\min}^\star/s}. \quad (4.43)$$

Then again for $P_{\text{out}}^{(a)}$ in (4.43), we explore the prior mentioned two cases. In regard to case 1, nothing changes, and thus we can infer the same previous results. Whereas, for case 2, i.e., wherever the 2nd summand is dominant in (4.43), we can observe that, related to P_{out} , our considered system can attain the following diversity order and coding gain, respectively,

$$G_d^{\text{G-G}} = b_{\min}^\star/s = \min[\zeta^2, \alpha, \beta]/s, \quad (4.44a) \quad G_c^{\text{G-G}} = \Delta^{-s/b_{\min}^\star} / (\gamma_{\text{out2}}/c)^{ws}. \quad (4.44b)$$

It is important to notice that it is possible for $G_d^{\text{G-G}}$ to be higher than $1/s$, however this is not possible for G_d^{M} . Another note is that in the FSO link, G_c is different for each FSO detector. Whereas, based on the previous high SNR analyses, using HD or IM-DD cost-dependent leads to the same G_d which is twice the G_d of IM-DD input-independent. Nevertheless, if the asymptotic analyses is based on high transmitted power region, all these FSO detectors will lead to the same P_{out} -based diversity

order (call it G_{dp}). To elucidate this consider the aforementioned “case 2”, and by referring to (4.38), (4.43), and remembering that μ_s is proportional to P_r^s (see the definitions after (4.16)), it can be shown that, for all considered FSO detectors, we get

$$G_{\text{dp}}^{\mathcal{M}} = \min[\zeta^2, \alpha, 1], \quad (4.45a)$$

$$G_{\text{dp}}^{\text{G-G}} = \min[\zeta^2, \alpha, \beta]. \quad (4.45b)$$

4.3.3 Ergodic Channel Capacity

The ergodic channel capacity (C_{erg}) is the information theoretical channel capacity in the case of the fast fadings channel [81]. As in dualhop DF relaying systems, the ergodic capacity in our considered system ⁸ is given by

$$C_{\text{erg}} = \frac{1}{2} \min \left[\mathbb{E}\{C_{\text{RF}}\}, \mathbb{E}\{C_{\text{FSO}}\} \right], \quad (4.46)$$

where C_{RF} and C_{FSO} are presented in (4.21) and (4.23), respectively, and hence the above expectations are obtained by

$$\mathbb{E}\{C_{\text{RF}}\} = \frac{1}{\ln(2)} \int_0^\infty d \ln(1 + \gamma) f_{\gamma_{\text{RF}}}(\gamma) d\gamma. \quad (4.47)$$

⁸Generally, the RF and FSO fadings channels are slow fadings channels. Nonetheless, they can be seen as fast fadings channels when long enough interleaving is employed especially in some conditions such as in the windy condition when the RF and FSO channels (especially the FSO pointing error) fluctuate in a fast manner.

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{1}{\ln(2)} \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) d\gamma. \quad (4.48)$$

Then by substituting (4.22) in (4.47), while using $\ln(1 + \gamma) = G_{2,2}^{1,2}\left[\gamma\right]_{1,0}^{1,1}$ and $\exp(-z) = G_{0,1}^{1,0}\left[z\right]_0^{-}$, and with the helps of [84, Eq. (07.34.21.0013.01), Eq. (07.34.17.0011.01)], we attain

$$\mathbb{E}\{C_{\text{RF}}\} = d \frac{(d^2/\bar{\gamma}_{\text{RF}})^{d^2}}{(d^2 - 1)! \ln 2} G_{2,3}^{3,1} \left[\frac{d^2}{\bar{\gamma}_{\text{RF}}} \middle| \begin{matrix} -d^2, 1-d^2 \\ 0, -d^2, -d^2 \end{matrix} \right]. \quad (4.49)$$

On the other hand, by substituting (4.24) in (4.48) and using $\ln(1 + \gamma) = G_{2,2}^{1,2}\left[\gamma\right]_{1,0}^{1,1}$, $\mathbb{E}\{C_{\text{FSO}}\}$ can be obtained by exploiting [84, Eq. (07.34.21.0013.01)]. By doing that, and after some manipulations by utilizing the Meijer's G-function definition [74, Eq. (9.301)], we get

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2 A/2}{(2\pi)^{w-1} \ln 2} \sum_{m=1}^{\beta} b_m G_{w+2, w+4}^{w+4, w} \left[\frac{B/c^w}{\mu_s^{1/s}} \middle| \begin{matrix} \mathbf{x}_4, 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0, \mathbf{x}_4 \end{matrix} \right], \quad (4.50)$$

where $\mathbf{x}_4 = \frac{0}{w}, \dots, \frac{w-1}{w}$ which has w terms.

Eventually, substituting (4.49) and (4.50) in (4.46) leads to the ergodic capacity of the considered system.

Note that the $\mathbb{E}\{C_{\text{FSO}}\}$ in (4.50) is for the $\mathcal{M}$ fadings FSO channel. Alternatively, to attain the $\mathbb{E}\{C_{\text{FSO}}\}$ at the special case of G-G fadings FSO channel, the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ in (4.48) must be changed by the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma)$ given in (4.25). By doing that, and

following the same steps as the prior ones, we obtain

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2/(\Gamma(\alpha)\Gamma(\beta))}{(2\pi)^{w-1}\ln 2} G_{w+2,w+4}^{w+4,w} \left[\frac{\zeta^2\alpha\beta/c^w}{(\zeta^2+1)\mu_s^{1/s}} \middle| \mathbf{x}_4, 1, \zeta^2+1 \right]. \quad (4.51)$$

4.4 Simulation and Numerical Results

In this section, we present numerical examples to demonstrate the analytical exact and asymptotic expressions of the previous section and also to illustrate the impact of the various system and channel parameters on the performance of our system.⁹

Additionally, this section verifies the obtained analytical and asymptotic expressions through Monte-Carlo simulations. In these simulations, 10^6 realizations of the SNR random variable are used.¹⁰

The effects of various values of the DoF (d), ρ , and ζ (the pointing errors) on the system outage probability (P_{out}) is shown in Fig. 4.2 for fixed atmospheric turbulences scenario in case of IM-DD cost-dependent. From this figure, it can be perceived that the higher values of ρ , d (more DoF), or ζ (smaller pointing mismatches) give lower value of P_{out} , and so better performance is gotten. In addition, according to the used parameters, we can see from this figure that at low average FSO SNRs the P_{out} is dominated by the FSO hop, i.e., P_{out} changes with the change in $\bar{\gamma}_{\text{FSO}}$ even though

⁹Following the same approach as in related literature, e.g. [71], [63], [24], [25], the values of the system parameters were chosen to confirm the deduced expressions, and thence other parameters values can be used by the designer to attain the needed result in specific schemes.

¹⁰ In these results, without loss of generality, we set $\Omega=0.78$ and $b_0=0.11$.

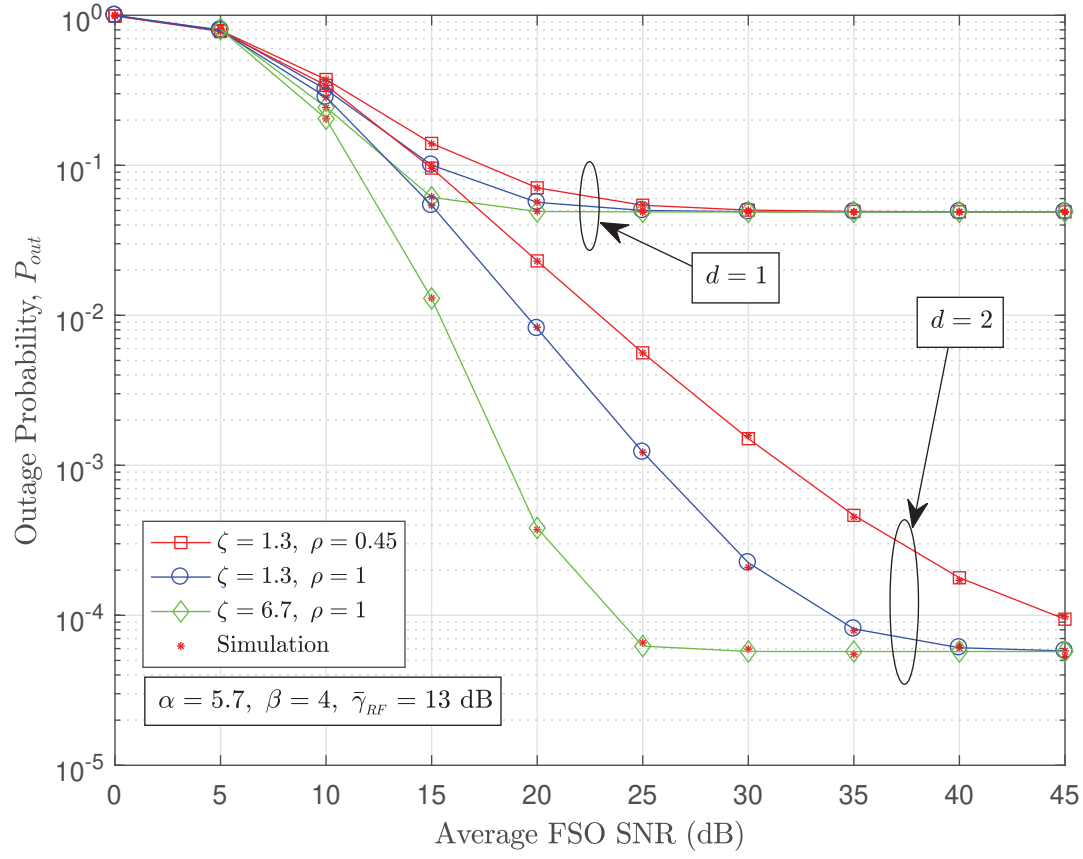


Figure 4.2: P_{out} for different values of the DoF (d), ζ and ρ considering IM-DD cost-dependent FSO link.

$\bar{\gamma}_{\text{RF}}$ is fixed here. Where as, in this figure at relatively high FSO SNRs the P_{out} is constant since it is dominated by the RF hop for which $\bar{\gamma}_{\text{RF}}$ is kept constant in this case. Moreover, Fig. 4.2 shows that, from power allocation point of view, there is no benefit in increasing $\bar{\gamma}_{\text{FSO}}$ when the system outage is dominated by the RF hop, this is specially clear in case of small values of DoF (e.g. $d = 1$).

Fig. 4.3 illustrates the influences of the atmospheric turbulence parameters (α and β) on P_{out} for a fixed pointing errors ($\zeta = 2$). It can be clearly seen that that the smaller values of α and β , i.e., tougher atmospheric turbulences, give bigger (worse) P_{out} and the higher values of α and β result in lower outage probability. Beside that, we can

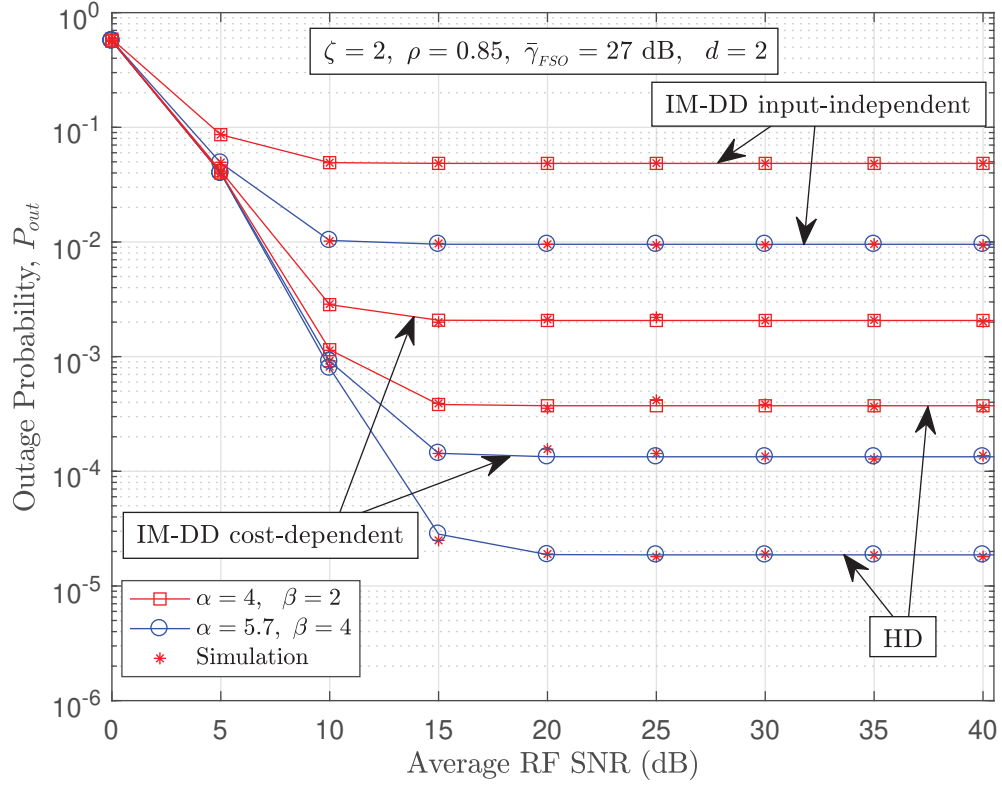


Figure 4.3: P_{out} for various values of α and β considering all FSO detection methods. perceive from Fig. 4.3 that at lower average RF SNRs the RF link dominates our P_{out} , i.e., P_{out} changes when $\bar{\gamma}_{RF}$ varies though $\bar{\gamma}_{FSO}$ is set constant in this figure. Whereas, as seen from this figure at relatively high RF SNRs the P_{out} becomes constant, as it is dominated by the FSO link for which $\bar{\gamma}_{FSO}$ is kept fixed in this case. Then again, from the power allocation point of view there is no benefit, as seen from Fig. 4.3, to increase $\bar{\gamma}_{RF}$ when the FSO channel dominates the system outage, this is particularly clear in the case of the IM-DD input-independent.

Fig. 4.4 and Fig. 4.5 prove that the asymptotic expression of P_{out} converges to the exact P_{out} expression in the high SNR region. Moreover, these figures explain the coding gain and diversity order of the system outage probability in the high SNR for

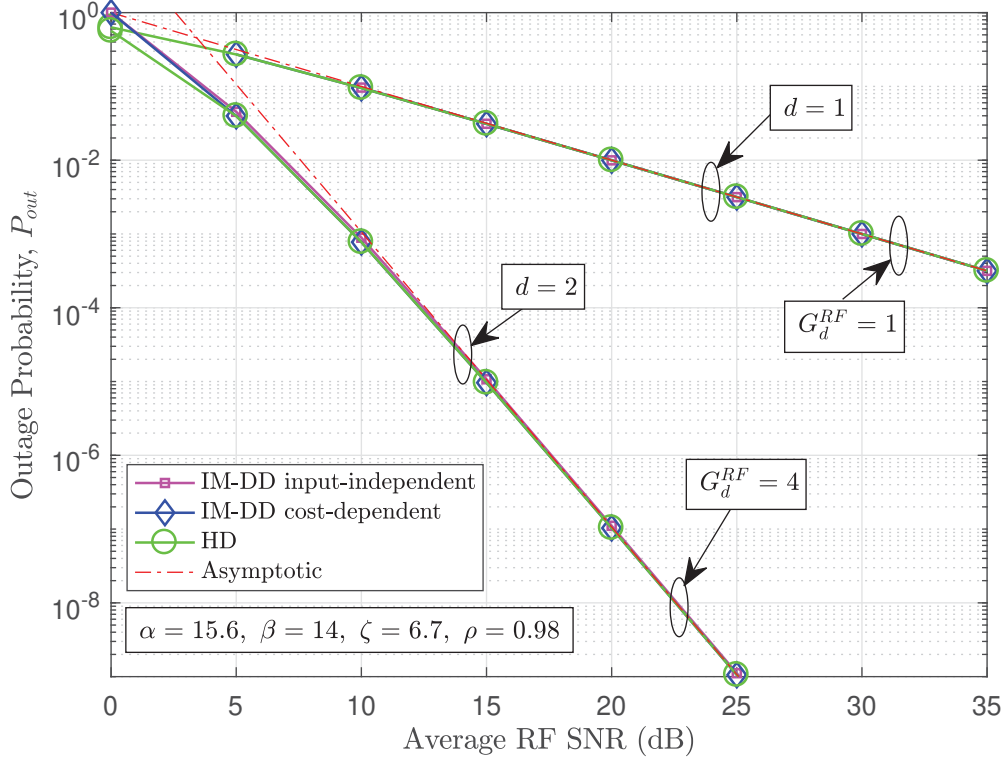


Figure 4.4: P_{out} dominated by the RF hop with different DoF (d) considering all FSO detection techniques.

various cases. Particularly, Fig. 4.4 evinces that $G_d^{RF} = d^2$ as derived before in Section 4.3.2, i.e., the measured slopes of the log-log curves at high SNRs in Fig. 4.4 equal the derived G_d^{RF} .

On the other hand, Fig. 4.5 validates that, given $s=1$, $G_d^{\mathcal{M}} = b_{\min}''/s = \min[\zeta^2, \alpha, 1]$ and $G_d^{G-G} = b_{\min}^*/s = \min[\zeta^2, \alpha, \beta]$ as drawn previously in Section 4.3.2, i.e., on the high SNR region in Fig. 4.5 the labeled measured log-log slopes are identical to the former derived ones.

Fig. 4.6 relates the ergodic channel capacity (C_{erg}) in the y axis with the average RF SNR ($\bar{\gamma}_{RF}$) in the x axis for different values of the atmospheric turbulence parameters (α and β), when the pointing error (ζ), the average FSO SNR ($\bar{\gamma}_{FSO}$) and the DoF

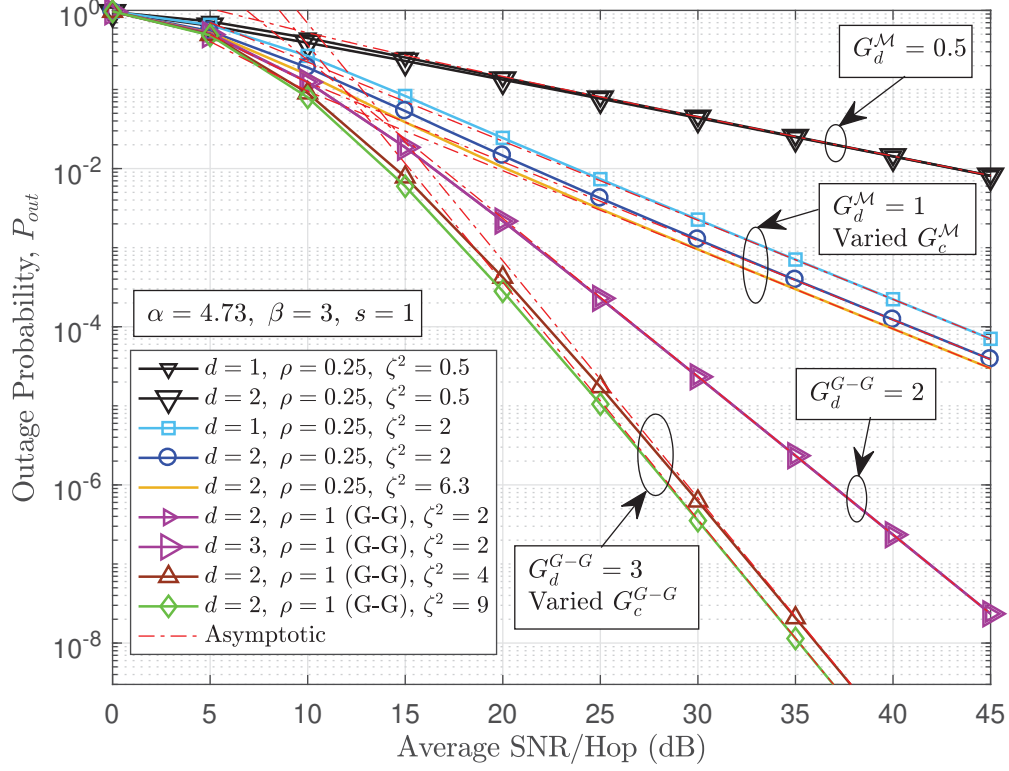


Figure 4.5: P_{out} dominated by FSO IM-DD cost-dependent hop with different values of the DoF (d), ρ and ζ .

(d) are kept constant. It can be seen that, stronger atmospheric turbulence, i.e., lower values of α and β , gives lower (worse) C_{erg} and higher values of α and β result in higher C_{erg} . In addition, based on the used parameters, we can also note that in the relatively low RF SNRs the RF hop dominates the C_{erg} , i.e., C_{erg} varies with the change in $\bar{\gamma}_{RF}$ though $\bar{\gamma}_{FSO}$ is set constant here. Whereas, C_{erg} will be constant, if the RF SNR becomes relatively high, because it will be dominated in this case by the FSO hop whose $\bar{\gamma}_{FSO}$ is set constant.

The impact of the number of the interference-eliminated RF info streams (d) on the system ergodic channel capacity (C_{erg}) is provided in Fig. 4.7 for all considered FSO detectors. It can be observed that the overall system ergodic capacity enhances

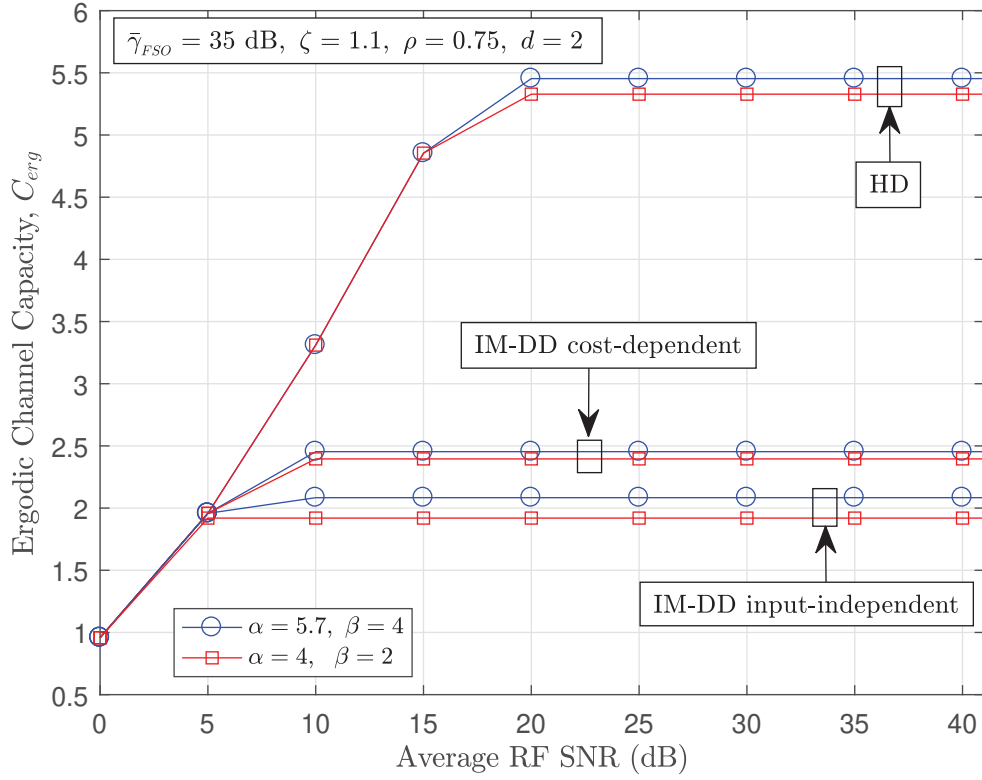


Figure 4.6: C_{erg} for various values of α and β considering all FSO detection methods. by increasing d when the RF hop dominates the C_{erg} (i.e. at relatively low RF SNRs). We can also see from this figure that the capacity pre-log (DoF) equals d in the RF domination region. Moreover, it is clear from Fig. 4.7 that increasing $\bar{\gamma}_{RF}$ or d will not improve the C_{erg} in the FSO domination region (i.e. at relatively high RF SNRs and specially when IM-DD is used). Further, Fig. 4.7 reflects that there is no benefit in changing the FSO detector when C_{erg} is dominated by the RF hop (i.e. at relatively low RF SNRs and specially with $d = 1$).

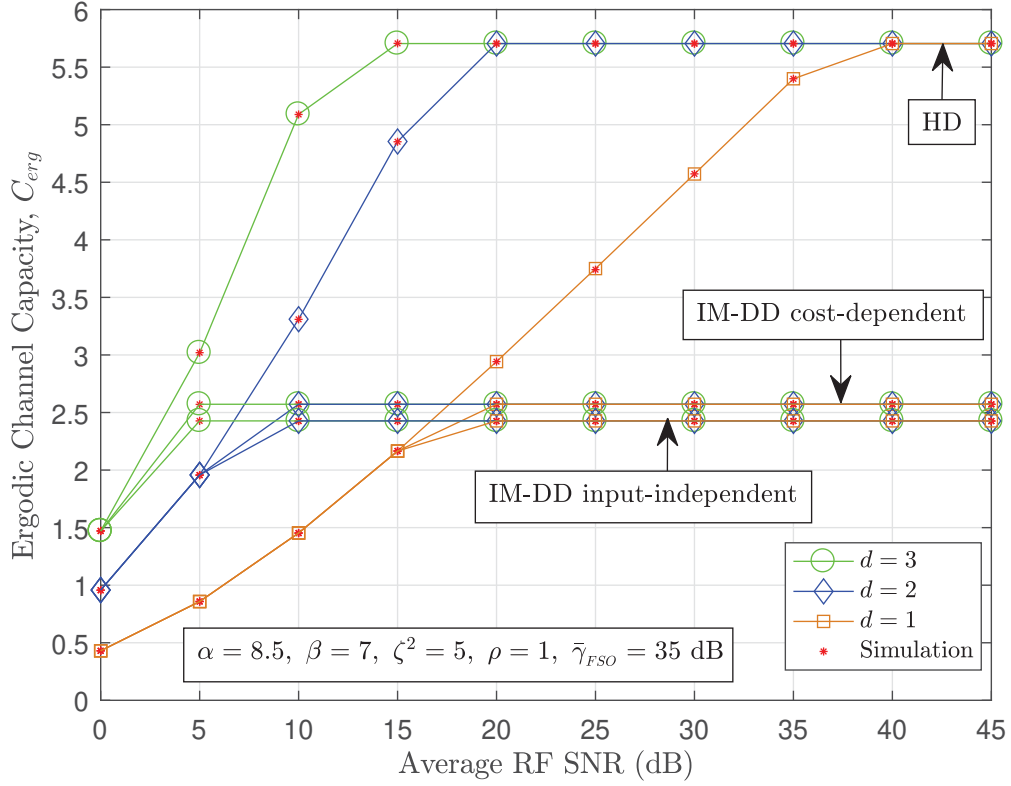


Figure 4.7: C_{erg} for different DoF (d) in the RF hop considering all FSO detection methods in the FSO hop.

4.5 Conclusion

This chapter studied a dual-hop DF mixed (IA MIMO RF IC)/(FSO) relaying scheme in which HD as well as IM-DD were considered in a unified manner in the FSO hop.

It was assumed that the MIMO RF links follow Rayleigh fading, while the FSO link suffers $\mathcal{M}$ fading with pointing error. In the MIMO RF IC hop, the IA technique was utilized to eliminate the interference, and hence this leads to enhance the multiplexing gain in this hop, and accordingly the overall system performance is improved.

The IA precoding and receiving interference eliminating matrices were designed by using OR-ML algorithm. For this scheme, closed-form expressions were derived for

the outage probability and ergodic capacity, in these expressions the best-known FSO channel capacity results were utilized. Moreover, not only the IM-DD inputindependent AWGN but also the IM-DD costdependent AWGN channel was considered in this derivation. Additionally, the system performance was asymptotically studied on the high SNRs region, where the diversity order and coding gain analyses were illustrated. It was found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2). Alternatively, under FSO hop domination, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors. Also, It was observed that the overall system ergodic capacity enhances by increasing d when the RF hop dominates the system. Further, we provided simulation results that verify the deduced analytical and asymptotic expressions.

CHAPTER 5

DUAL-HOP MIXED IMPERFECT INTERFERENCE ALIGNED MIMO RF/UNIFIED-FSO RELAYING SCHEMES

5.1 Introduction

This chapter studies a dual-hop DF mixed (imperfect IA MIMO RF IC)/(FSO) relaying system in which HD as well as IM-DD are considered in a unified analysis manner in the FSO hop. It is assumed that the MIMO RF links follow Rayleigh fading, while the FSO link suffers $\mathcal{M}$ fading with pointing error.

In the MIMO RF IC hop, the IA technique is utilized however with imperfect CSI, since it is usually the case in practical systems due to the limitation in the feedback to inform the transmitters the CSI. Considering imperfect CSI means that the interference will not be totally eliminated in this case.

For this scheme, closed-form expressions are derived for the outage probability in terms of Whittaker and Meijer functions and for the ergodic capacity in terms of the bivariate Meijer's G-function. In those expressions the best-known FSO channel capacity results are utilized. Moreover, both the IM-DD input-independent AWGN as well as the IM-DD cost-dependent AWGN channel are considered in the derivations.

It is found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2) in case of perfect CSI. Alternatively, under FSO hop domination, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors in case of perfect CSI. However the case of imperfect CSI results in zero diversity order causing a floor in the outage probability curve at high SNRs. This floor worsen with either the reduction in the CSI accuracy, or the decrement in " d ".

Also under imperfect CSI, It is found that there is a ceil in the ergodic capacity at high SNRs, this ceil becomes worse with more interferers or when the CSI accuracy or the number of RF info streams (d) decreases.

5.2 System and Channel Models

As shown in Fig. 4.1, the system under study is a dual-hop mixed MIMO RF IC/FSO DF relaying. In this system, K source-relay (transmitter-receiver) pairs form a MIMO RF interference channel in the first hop, where the transmitting MS user i (source i) has N_T antennas and needs to send d independent info streams, each one is equivalent to one DoF, to the receiving BTS i (relay i) which has N_R antennas. On the other hand, in the second hop the relays forward the sources messages to the corresponding destinations in the backbone network through FSO links.

5.2.1 The MIMO RF IC hop

In this scheme the IA technique is utilized in the MIMO RF IC hop to eliminate the interference, and hence results in enhancing the multiplexing gain (DoF) in that hop. To do that the IA exploits the knowledge of the CSI of all transmission links to form the transmission precoding matrix and the receiving interference elimination combining matrix. In this chapter, we consider the imperfect CSI scenario, since it is usually the case in practical systems due to the limitation in the feedback to inform the transmitters the CSI. Therefore in our IA MIMO RF IC hop, the relationship between the imperfect CSI matrix (denoted by $\hat{\mathbf{H}}_{ij} \in \mathbb{C}^{N_R \times N_T}$) and the real (perfect) CSI matrix (denoted by $\mathbf{H}_{ij} \in \mathbb{C}^{N_R \times N_T}$) is assumed to be

$$\mathbf{H}_{ij} = \sqrt{1 - \varrho} \hat{\mathbf{H}}_{ij} + \sqrt{\varrho} \mathbf{E}_{ij} \quad (5.1)$$

where $\mathbf{E}_{ij} \in \mathbb{C}^{N_R \times N_T}$ is the quantization error matrix between relay i and source j . We assume that the elements of $\mathbf{E}_{ij}$ are i.i.d. $\mathcal{CN}(0, 1)$ and $\mathbf{E}_{ij}$ is independent of $\hat{\mathbf{H}}_{ij}$. The parameter ϱ represents the RF CSI accuracy, and it is scaling from 0 to 1, with $\varrho = 0$ denoting the perfect CSI case. It is also assumed that the elements of $\mathbf{H}_{ij}$ are i.i.d. $\mathcal{CN}(0, 1)$, i.e., Rayleigh faded MIMO RF channel.

The imperfect CSI matrix $\hat{\mathbf{H}}_{ij}$ is used to design the transmission precoding matrix $\hat{\mathbf{V}}_i \in \mathbb{C}^{N_T \times d}$ at the transmitting MS user i (source i), and the receiving interference elimination combining matrix $\hat{\mathbf{U}}_i \in \mathbb{C}^{N_R \times d}$ at the receiving BTS i (relay i) such that

$$\text{rank}(\hat{\mathbf{V}}_i) = d, \quad \hat{\mathbf{V}}_i^H \hat{\mathbf{V}}_i = \mathbf{I}_{d \times d}, \quad (5.2)$$

$$\text{rank}(\hat{\mathbf{U}}_i) = d, \quad \hat{\mathbf{U}}_i^H \hat{\mathbf{U}}_i = \mathbf{I}_{d \times d}, \quad (5.3)$$

such that [29]

$$\text{rank}(\hat{\mathbf{U}}_i^H \hat{\mathbf{H}}_{ii} \hat{\mathbf{V}}_i) = d, \quad \forall i \in \{1, 2, \dots, K\}, \quad (5.4)$$

$$\hat{\mathbf{U}}_i^H \hat{\mathbf{H}}_{ij} \hat{\mathbf{V}}_j = \mathbf{0}_{d \times d}, \quad \forall j \neq i, \quad i, j \in \{1, 2, \dots, K\}, \quad (5.5)$$

Assuming imperfect CSI, the received $d \times 1$ signal vector at the receiving BTS i is

given by

$$y_i = \sqrt{\frac{P_s}{d}} \hat{\mathbf{U}}_i^H \mathbf{H}_{ii} \hat{\mathbf{V}}_i x_i + \underbrace{\sum_{j=1, j \neq i}^K \sqrt{\frac{P_s}{d}} \hat{\mathbf{U}}_i^H \mathbf{H}_{ij} \hat{\mathbf{V}}_j x_j}_{\text{In imperfect CSI this is not zero, see (5.1) and (5.5)}} + \hat{\mathbf{U}}_i^H n_i. \quad (5.6)$$

In imperfect CSI this is not zero, see (5.1) and (5.5)

Utilizing (5.1) and (5.5) in (5.6) results in

$$y_i = \sqrt{\frac{P_s}{d}} \hat{\mathbf{U}}_i^H \mathbf{H}_{ii} \hat{\mathbf{V}}_i x_i + \sum_{j=1, j \neq i}^K \sqrt{\frac{P_s}{d}} \sqrt{\varrho} \hat{\mathbf{U}}_i^H \mathbf{E}_{ij} \hat{\mathbf{V}}_j x_j + \hat{\mathbf{U}}_i^H n_i. \quad (5.7)$$

$$= \sqrt{\frac{P_s}{d}} \hat{\mathbf{H}}_{ii} x_i + \sum_{j=1, j \neq i}^K \sqrt{\frac{P_s}{d}} \sqrt{\varrho} \hat{\mathbf{E}}_{ij} x_j + \acute{n}_i, \quad (5.8)$$

where x_i is the sent $d \times 1$ info vector from the transmitting MS user i , and $n_i \sim \mathcal{CN}(0, N_0 \mathbf{I})$ is the $N_R \times 1$ AWGN vector at the receiving BTS i , $\hat{\mathbf{H}}_{ii} = \hat{\mathbf{U}}_i^H \mathbf{H}_{ii} \hat{\mathbf{V}}_i$, $\hat{\mathbf{E}}_{ij} = \hat{\mathbf{U}}_i^H \mathbf{E}_{ij} \hat{\mathbf{V}}_j$ and $\acute{n}_i = \hat{\mathbf{U}}_i^H n_i$.

5.2.2 The FSO hop

After receiving the signal, the relay (BTS) forwards a decoded version of it to the destination in the second phase of transmission. Let's use x_{HD} and x_{IMDD} to designate the transmitted signal from the relay for the case of HD and IM-DD, with the constraints $\mathbb{E}\{|x_{\text{HD}}|^2\} = 1$ and $\mathbb{E}\{x_{\text{IMDD}}\} = 1$ respectively. It is worthy noticing that in the IM-DD case the average power constraint is on the expected signal value and not on its expected square as the signal in the IM-DD technique is an optical intensity. So, the received signal at the destination for the case of HD and IM-DD can be, respectively,

expressed by

$$y_{\text{HD}} = \sqrt{P_r} g_{\text{FSO}} x_{\text{HD}} + n_{\text{HD}}, \quad (5.9)$$

$$y_{\text{IMDD}} = P_r I_{\text{FSO}} x_{\text{IMDD}} + n_{\text{IMDD}}, \quad (5.10)$$

where P_r is the relay transmitted power, g_{FSO} and $I_{\text{FSO}} = |g_{\text{FSO}}|^2$ designate the fading in the received electric field and irradiance, respectively. This fading is a result of the atmospheric turbulence in the relay-destination channel. In (5.9), $n_{\text{HD}} \sim \mathcal{CN}(0, N_{\text{HD}})$ is a complex AWGN. Whilst in (5.10), n_{IMDD} is a real AWGN where $n_{\text{IMDD}} \sim \mathcal{N}(0, N_{\text{IMDD}})$ for the IM-DD input-independent AWGN model, and $n_{\text{IMDD}} \sim \mathcal{N}(0, 2P_r I_{\text{FSO}} N_{\text{HD}} \mathbb{E}\{x_{\text{IMDD}}\})$ for the IM-DD cost-dependent AWGN channel under the assumption of relatively high P_r in comparison with the thermal and/or background noises power. The reader is referred to [15], [77] for details of the HD and the two IM-DDs and their noises. So the SNRs at the destination are given as follows for the HD, IM-DD input-independent, and IM-DD cost-dependent,

$$\gamma_{\text{HD}} = \frac{P_r I_{\text{FSO}}}{N_{\text{HD}}}, \quad \gamma_{\text{IMDD}_{\text{indp}}} = \frac{P_r^2 I_{\text{FSO}}^2}{N_{\text{IMDD}}}, \quad \gamma_{\text{IMDD}_{\text{cost}}} = \frac{P_r I_{\text{FSO}}}{2N_{\text{HD}}}. \quad (5.11)$$

In this study, it is assumed that the FSO connection encounters $\mathcal{M}$ fading with pointing error. In the development of the $\mathcal{M}$ fading channel [24], the received signal is assumed to be consisted of a LOS component (U_L), a component coupled to the LOS and scattered by eddies on the propagation axis (U_S^C), and a component scattered by off-axis eddies (U_S^G) that is statistically independent from U_L and U_S^C . Thus by assuming the $\mathcal{M}$ fading with pointing mismatch in the FSO channel, the probability

density function (pdf) of I_{FSO} can be represented by [25, Eq. (21)]

$$f_{I_{\text{FSO}}}^{\mathcal{M}}(I) = \frac{\zeta^2 A}{2I} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{\alpha \beta}{g \beta + \Omega'} \frac{I}{A_0} \middle| \frac{\zeta^2 + 1}{\zeta^2, \alpha, m} \right], \quad (5.12)$$

where

$$A = \frac{2\alpha^{\alpha/2}}{g^{1+\alpha/2}\Gamma(\alpha)} \left(\frac{g \beta}{g \beta + \Omega'} \right)^{\beta+\alpha/2}, \quad b_m = a_m [\alpha \beta / (g \beta + \Omega')]^{-(\alpha+m)/2},$$

$$a_m = \binom{\beta-1}{m-1} \frac{(g \beta + \Omega')^{1-m/2}}{(m-1)!} \left(\frac{\Omega'}{g} \right)^{m-1} \left(\frac{\alpha}{\beta} \right)^{m/2},$$

α is positive parameter and it is related to the effective number of the large-scale cells of the scattering (fading) process, β is a natural number and it is related to the effective number of the small-scale cells of the fading process ¹, $\Omega' = \Omega + 2b_0\rho + 2\sqrt{2b_0\rho\Omega} \cos(\Phi_A - \Phi_B)$ is the average power of the coherent contribution, $\Omega = \mathbb{E}\{|U_L|^2\}$, $\mathbb{E}\{|U_S^C|^2\} = 2b_0\rho$, Φ_A and Φ_B are deterministic phases of the LOS and the coupled-to-LOS signal components, respectively, $2b_0 = \mathbb{E}\{|U_S^C|^2 + |U_S^G|^2\}$ is the average power of all scattered components, $g = \mathbb{E}\{|U_S^G|^2\} = 2b_0(1 - \rho)$, the parameter $0 \leq \rho \leq 1$ denotes the part of the scattered power coupled to the LOS, ζ is the ratio between the equivalent beam radius and the standard deviation of the pointing error displacement at the receiver (i.e. $\zeta \rightarrow \infty$, indicates the nonpointing error case) [78], A_0 is a parameter that represents the pointing losses (or typically the FSO link losses).

From the pdf in (5.12) ($f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$) and by random variable transformations, we

¹The fading parameters α and β reflect the atmospheric turbulences condition (The Rytov variance) with low values basically denoting strong (severe) atmospheric turbulent situation.

attain the pdfs of the SNRs in (5.11) in the following unified formula

$$f_{\gamma_{\text{FSO}}}^{\mathcal{M}}(\gamma) = \frac{\zeta^2 A}{2^s \gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[B \left(\frac{w\gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, m \end{matrix} \right], \quad (5.13)$$

where γ_{FSO} represents the unified SNR of the FSO channel, $B = \zeta^2 \alpha \beta (g + \Omega') / [(\zeta^2 + 1)(g\beta + \Omega')]$, s and w are the parameters which determine the sort of the FSO detections, i.e., $(s, w) = (1, 1)$, $(2, 1)$ or $(1, 2)$ for the HD, IM-DD input-independent, and IM-DD cost-dependent respectively, and where $\mu_1 \triangleq P_r \mathbb{E}\{I_{\text{FSO}}\} / N_{\text{HD}} = A_0 P_r \zeta^2 (g + \Omega') / [(\zeta^2 + 1)N_{\text{HD}}]^2$, and for normalization purposes $\mu_2 \triangleq P_r^2 \mathbb{E}\{I_{\text{FSO}}^2\} / N_{\text{IMDD}}$ as previously defined in [79] and references therein. This definition results in $\mu_2 = A_0^2 P_r^2 \zeta^4 (g + \Omega')^2 / [(\zeta^2 + 1)^2 N_{\text{IMDD}}]$. From these expressions of μ_1 , μ_2 , and (5.11), it is clear that μ_s is proportional to the average of the unified FSO SNR ($\bar{\gamma}_{\text{FSO}}$). Specifically, $\mu_1 = \bar{\gamma}_{\text{HD}} = 2\bar{\gamma}_{\text{IMDD}_{\text{cost}}}$, however $\mu_2 = \bar{\gamma}_{\text{IMDD}_{\text{indp}}} \frac{\mathbb{E}\{I_{\text{FSO}}^2\}}{\mathbb{E}\{I_{\text{FSO}}^2\}} = \frac{\zeta^2 (\zeta^2 + 1)^{-2} (\zeta^2 + 2)(g + \Omega')^2}{\alpha^{-1}(\alpha + 1) [2g(g + 2\Omega') + \Omega'^2 (1 + 1/\beta)]} \bar{\gamma}_{\text{IMDD}_{\text{indp}}}$.²

Note that having $\rho=1$ (i.e. $g=0$) results in $Ab_m = 2/(\Gamma(\alpha)\Gamma(\beta))$ when $m=\beta$ and $Ab_m=0$ for all other m . So in this case, by having $\rho=1$, and $\Omega'=1$ (for normalization), the pdf $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (5.12) turns into the special case of the G-G distribution with pointing errors [25]. By having $\rho=1$, the power $\mathbb{E}\{|U_S^G|^2\} \triangleq g = 0$, this means that the G-G fading channel neglects the component of the received signal scattered by the off-axis eddies (U_S^G). Thus, by using $\Omega'=1$ and $\rho=1$ in (5.13), the pdf of the unified FSO SNR in the G-G fading with pointings error can be presented by

$$f_{\gamma_{\text{FSO}}}^{\text{G-G}}(\gamma) = \frac{\zeta^2}{s \gamma \Gamma(\alpha) \Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2 \alpha \beta}{(\zeta^2 + 1)} \left(\frac{w\gamma}{\mu_s} \right)^{1/s} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, \beta \end{matrix} \right]. \quad (5.14)$$

² $\mathbb{E}\{I_{\text{FSO}}\}$ and $\mathbb{E}\{I_{\text{FSO}}^2\}$ are derived from [25, Eq. (33)].

5.3 System Performance Analysis

5.3.1 Outage Probability

The outage probability (P_{out}) is a substantial performance indicator in slow fading channels³. It can be presented, as in [81], by $P_{\text{out}} = \Pr [C < R]$, where R is the end to end (e2e) required information rate in bits/channel-use, C is a random variable and it is a function on the random e2e-equivalent SNR of the channel. The realization of C designates the e2e channel capacity given the realization of the e2e channel equivalent SNR. As in any DF dual-hop system, in the considered system, the outage event takes place whenever the first hop or the second hop falls in an outage. Therefore, the outage probability in the system under study can be expressed as⁴

$$P_{\text{out}} = \Pr \left[C_{\text{RF}} < 2R \cup C_{\text{FSO}} < 2R \right], \quad (5.15)$$

where C_{RF} and C_{FSO} are related to the channel capacity of the first hop (RF hop) and the second hop (FSO hop), respectively.

Particularly, C_{RF} is a random variable and its realization is the channel capacity of the RF hop given the related realization of the effective channels fading matrices $\hat{\mathbf{H}}_{ii}$

³Note that the channels in most of the RF and FSO schemes, including our system in this work, are slow fading channels [2], [81].

⁴Here the required data rates in the two hops are assumed to be identical, and thence the factor 2 results by assuming that the e2e transmission (from the source to destination) takes place on two equal phases (two equal channel uses). This lets each hop data rate to be twice the e2e data rate.

and $\dot{\mathbf{E}}_{ij}$ of the first hop. However due to the imperfect IA, the first hop is effectively a MIMO AWGN channel with the remaining interference as represented by (5.8). Thus, by utilizing the MIMO AWGN channel capacity expression in the presence of interference, as given in [81], we obtain ⁵

$$C_{\text{RF}} = \log \left[\frac{\det \left(\frac{P_s}{N_o d} \dot{H}_{ii} \dot{H}_{ii}^H + \sum_{j=1, j \neq i}^K \varrho \frac{P_s}{N_o d} \dot{E}_{ij} \dot{E}_{ij}^H + I_{d \times d} \right)}{\det \left(\sum_{j=1, j \neq i}^K \varrho \frac{P_s}{N_o d} \dot{E}_{ij} \dot{E}_{ij}^H + I_{d \times d} \right)} \right]. \quad (5.16)$$

However, since the RF channel matrices have i.i.d. $\mathcal{CN}(0, 1)$ elements, then with probability 1 these matrices have full rank, and they are well-conditioned [81]. Due to these properties, we get $\dot{H}_{ii} \dot{H}_{ii}^H = \lambda_{hii} I_{d \times d}$ and $\dot{E}_{ij} \dot{E}_{ij}^H = \lambda_{eij} I_{d \times d}$, and this results in

$$C_{\text{RF}} = \log \left[\frac{\det \left(\left(\frac{P_s}{N_o d} \lambda_{hii} + \sum_{j=1, j \neq i}^K \varrho \frac{P_s}{N_o d} \lambda_{eij} + 1 \right) I_{d \times d} \right)}{\det \left(\left(\sum_{j=1, j \neq i}^K \varrho \frac{P_s}{N_o d} \lambda_{eij} + 1 \right) I_{d \times d} \right)} \right]. \quad (5.17)$$

Furthermore, by using the eigenvalues identities, it can be shown that $\lambda_{hii} d = \text{Tr}(\dot{H}_{ii} \dot{H}_{ii}^H)$, $\lambda_{eij} d = \text{Tr}(\dot{E}_{ij} \dot{E}_{ij}^H)$ and this means that

$$C_{\text{RF}} = d \log \left(1 + \frac{\frac{P_s}{N_o d^2} \text{Tr}(\dot{H}_{ii} \dot{H}_{ii}^H)}{1 + \sum_{j=1, j \neq i}^K \frac{\varrho P_s}{N_o d^2} \text{Tr}(\dot{E}_{ij} \dot{E}_{ij}^H)} \right). \quad (5.18)$$

In a compact way, (5.18) can be written as

$$C_{\text{RF}} = d \log \left(1 + \frac{Y}{1 + X} \right) = d \log(1 + \gamma_{\text{RF}}), \quad (5.19)$$

⁵Unless otherwise specified, in this work, the log base is 2.

where $Y = \frac{P_s}{N_o d^2} \text{Tr}(\dot{H}_{ii} \dot{H}_{ii}^H)$, $X = \sum_{j=1, j \neq i}^K \frac{\varrho P_s}{N_o d^2} \text{Tr}(\dot{E}_{ij} \dot{E}_{ij}^H)$, and γ_{RF} is defined as $\gamma_{\text{RF}} = \frac{Y}{1+X}$.

As a result of assuming i.i.d. $\mathcal{CN}(0, 1)$ elements of the RF channel matrices, Y and X are gamma random variables with the following pdfs [16]

$$f_Y(y) = \frac{1}{(d^2 - 1)!} \left(\frac{N_o d^2}{P_s} \right)^{d^2} (y)^{d^2-1} \times \exp \left(\frac{-d^2 N_o y}{P_s} \right), \quad (5.20)$$

$$f_X(x) = \frac{1}{(d^2(K-1)-1)!} \left(\frac{d^2 N_o}{\varrho P_s} \right)^{d^2(K-1)} (x)^{d^2(K-1)-1} \times \exp \left(\frac{-d^2 N_o x}{\varrho P_s} \right), \quad (5.21)$$

and the CDF of Y is given by

$$F_Y(y) = 1 - \exp \left(\frac{-y N_o d^2}{P_s} \right) \sum_{i=0}^{d^2-1} \frac{1}{i!} \left(\frac{d^2 N_o}{P_s} \right)^i y^i. \quad (5.22)$$

On the other side, C_{FSO} is a function on the random SNR of the FSO hop, and thence it is a random variable. Additionally, in case of HD the realization of C_{FSO} designates the channel capacity of the second hop given the realization of the second hop SNR (γ_{HD}). Whereas, in case of IM-DD we let the realization of C_{FSO} designates a precise approximation ⁶ of the channel capacity of the second hop given the related realization of the second hop SNR ($\gamma_{\text{IMDD}_{\text{indp}}}$ or $\gamma_{\text{IMDD}_{\text{cost}}}$). But, given γ_{HD} , $\gamma_{\text{IMDD}_{\text{indp}}}$ or

⁶This approx. is used as the IM-DD channel capacity is so far unknown [14]. Nonetheless, this approx. is very rigorous at high SNRs, and thus it can act as the IM-DD channel capacity in that region.

$\gamma_{\text{IMDD}_{cost}}$ the FSO hop is, respectively, an AWGN channel, IM-DD input-independent AWGN channel, or IM-DD cost-dependent AWGN channel as expressed by (5.9) or (5.10). So, by utilizing the expressions of the AWGN channel capacity [73] and the approximations of both IM-DD AWGN channel capacities [12], [15] we get a unified C_{FSO} expressed as

$$C_{\text{FSO}} = \log(1 + \gamma_{\text{FSO}}^{\text{eff}}), \quad (5.23)$$

where $\gamma_{\text{FSO}}^{\text{eff}}$ is used to represent the unified effective SNR of the FSO channel, i.e., $\gamma_{\text{FSO}}^{\text{eff}}$ equals either γ_{HD} , $\sqrt{\frac{e}{2\pi}\gamma_{\text{IMDD}_{indp}}}$, or $\sqrt{\frac{e}{2\pi}\gamma_{\text{IMDD}_{cost}}}$ based on the used FSO transmission type. Because it is important in what follows, let's derive the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$. From $f_{I_{\text{FSO}}}^{\mathcal{M}}(I)$ in (5.12), with the help of (5.11), and by random variable transformation, the pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ can be expressed in a unified form as

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) = \frac{w \zeta^2 A}{2 \gamma} \sum_{m=1}^{\beta} b_m G_{1,3}^{3,0} \left[\frac{B (\gamma/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, m \end{matrix} \right], \quad (5.24)$$

where the constant $c = 1$, $\sqrt{\frac{e}{2\pi}}$, or $\sqrt{\frac{e}{4\pi}}$ for HD, IM-DD input-independent, or IM-DD cost-dependent, respectively, and the remainder of the parameters are as in (5.12) and (5.13). Notice that using $\Omega'=1$ and $\rho=1$ in (5.24), results in the following pdf of $\gamma_{\text{FSO}}^{\text{eff}}$ in the special case of G-G fading with pointing errors

$$f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma) = \frac{w \zeta^2}{\gamma \Gamma(\alpha) \Gamma(\beta)} G_{1,3}^{3,0} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| \begin{matrix} \zeta^2+1 \\ \zeta^2, \alpha, \beta \end{matrix} \right]. \quad (5.25)$$

Now with the help of (5.19) and (5.23), the P_{out} in (5.15) can be given by [83]

$$P_{\text{out}} = \Pr[\gamma_{\text{RF}} < (2^{2R/d} - 1)] + \Pr[\gamma_{\text{FSO}}^{\text{eff}} < (2^{2R} - 1)] - \Pr[\gamma_{\text{RF}} < (2^{2R/d} - 1)] \Pr[\gamma_{\text{FSO}}^{\text{eff}} < (2^{2R} - 1)]. \quad (5.26)$$

From (5.26) P_{out} can be expressed in terms of the cumulative distribution functions (CDFs) of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$ evaluated at $\gamma_{\text{out1}} \triangleq (2^{2R/d} - 1)$ and $\gamma_{\text{out2}} \triangleq (2^{2R} - 1)$, respectively. Consequently, we get

$$P_{\text{out}} = F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) + F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out2}}) - F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma_{\text{out2}}), \quad (5.27)$$

where $F_{\gamma_{\text{RF}}}(\gamma)$ and $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ are the CDFs of γ_{RF} and $\gamma_{\text{FSO}}^{\text{eff}}$, respectively.

By utilizing (5.21) and (5.22), the CDF $F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}})$ can be given by

$$F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) = \int_0^\infty F_Y(\gamma_{\text{out1}}(x+1)) f_X(x) dx \quad (5.28)$$

$$\begin{aligned} &= 1 - \left(\frac{\Lambda_{ij}}{\varrho}\right)^{d^2(K-1)} \int_0^\infty \frac{\exp(-\Lambda_{ii}\gamma_{\text{out1}})}{[d^2(K-1)-1]!} \exp\left(-\left(\Lambda_{ii}\gamma_{\text{out1}} + \frac{\Lambda_{ij}}{\varrho}\right)x\right) x^{[d^2(K-1)-1]} \\ &\quad \times \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \gamma_{\text{out1}}^i [x+1]^i dx. \end{aligned} \quad (5.29)$$

Using [74, Eq. (3.384.3)] in (5.29) leads to

$$\begin{aligned} F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) &= 1 - \left(\frac{\Lambda_{ij}}{\varrho}\right)^{d^2(K-1)} \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \gamma_{\text{out1}}^i \frac{\exp\left(-\frac{1}{2}\left(\Lambda_{ii}\gamma_{\text{out1}} - \frac{\Lambda_{ij}}{\varrho}\right)\right)}{\left(\Lambda_{ii}\gamma_{\text{out1}} + \frac{\Lambda_{ij}}{\varrho}\right)^{\frac{i+1}{2} + \frac{d^2(K-1)}{2}}} \\ &\quad \times W_{\frac{i+1-d^2(K-1)}{2}, \frac{i+d^2(K-1)}{2}} \left[\Lambda_{ii}\gamma_{\text{out1}} + \frac{\Lambda_{ij}}{\varrho}\right], \end{aligned} \quad (5.30)$$

where $W[\cdot]$ is the Whittaker's W-function as defined in [74, Eq. (9.222)]. The CDF

$F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}})$ can be given in another expression which is useful in deriving the ergodic

rate. To obtain this expression we employ the binomial expansion for the term $[x+1]^i$ in (5.29) to get

$$F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) = 1 - \left(\frac{\Lambda_{ij}}{\varrho}\right)^{d^2(K-1)} \frac{e^{-\Lambda_{ii}\gamma_{\text{out1}}}}{[d^2(K-1)-1]!} \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \gamma_{\text{out1}}^i \\ \times \sum_{q=0}^i \binom{i}{q} \int_0^\infty x^{q+[d^2(K-1)]-1} e^{-(\Lambda_{ii}\gamma_{\text{out1}} + \frac{\Lambda_{ij}}{\varrho})x} dx. \quad (5.31)$$

Then by utilizing [74, Eq. (3.381.4)] in (5.31) we obtain

$$F_{\gamma_{\text{RF}}}(\gamma_{\text{out1}}) = 1 - \left(\frac{\Lambda_{ij}}{\varrho}\right)^{d^2(K-1)} \frac{e^{-\Lambda_{ii}\gamma_{\text{out1}}}}{[d^2(K-1)-1]!} \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \gamma_{\text{out1}}^i \\ \times \sum_{q=0}^i \binom{i}{q} \frac{\Gamma(q+d^2(K-1))}{(\Lambda_{ii}\gamma_{\text{out1}} + \Lambda_{ij}/\varrho)^{q+d^2(K-1)}}. \quad (5.32)$$

On the other side, the CDF of $\gamma_{\text{FSO}}^{\text{eff}}$, is expressed, after integrating the pdf in (5.24) with respect to γ , as follows

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) = \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right]. \quad (5.33)$$

Then, by exploiting the CDFs (5.30) and (5.33) in (5.27) and after some algebraic simplifications, we get

$$P_{\text{out}} = 1 - \left(\left(\frac{\Lambda_{ij}}{\varrho}\right)^{d^2(K-1)} \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \gamma_{\text{out1}}^i \frac{e^{-\frac{1}{2}(\Lambda_{ii}\gamma_{\text{out1}} - \Lambda_{ij}/\varrho)}}{(\Lambda_{ii}\gamma_{\text{out1}} + \Lambda_{ij}/\varrho)^{\frac{i+1}{2} + \frac{d^2(K-1)}{2}}} \right. \\ \left. \times W_{\frac{i+1-d^2(K-1)}{2}, \frac{i+d^2(K-1)}{2}} \left[\Lambda_{ii}\gamma_{\text{out1}} + \frac{\Lambda_{ij}}{\varrho} \right] \right) \left(1 - \frac{\zeta^2 A}{2} \sum_{m=1}^{\beta} b_m G_{2,4}^{3,1} \left[\frac{B(\gamma_{\text{out2}}/c)^w}{\mu_s^{1/s}} \middle| \begin{matrix} 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0 \end{matrix} \right] \right). \quad (5.34)$$

Recall that the P_{out} in (5.34) is in case of $\mathcal{M}$ faded FSO hop. However, to have P_{out} in the special case of the G-G faded FSO hop, the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ in (5.27) must be changed by the CDF $F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma)$ that can be given, by integrating its corresponding pdf in (5.25), as follows

$$F_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma) = \frac{\zeta^2}{\Gamma(\alpha)\Gamma(\beta)} \text{G}_{2,4}^{3,1} \left[\frac{\zeta^2 \alpha \beta (\gamma/c)^w}{(\zeta^2 + 1) \mu_s^{1/s}} \middle| 1, \zeta^2 + 1 \right]_{\zeta^2, \alpha, \beta, 0}. \quad (5.35)$$

5.3.2 Ergodic Capacity

As in dual-hop DF relaying systems, the ergodic capacity in our considered system is given by

$$C_{\text{erg}} = \frac{1}{2} \min \left[\mathbb{E}\{C_{\text{RF}}\}, \mathbb{E}\{C_{\text{FSO}}\} \right], \quad (5.36)$$

where C_{RF} and C_{FSO} are presented in (5.19) and (5.23), respectively, and hence the above expectations are obtained by

$$\mathbb{E}\{C_{\text{RF}}\} = \frac{1}{\ln(2)} \int_0^\infty d \ln(1 + \gamma) f_{\gamma_{\text{RF}}}(\gamma) d\gamma. \quad (5.37)$$

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{1}{\ln(2)} \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma) d\gamma. \quad (5.38)$$

Then by using the integration by parts in (5.37), while utilizing $\ln(1 + \gamma) = \text{G}_{2,2}^{1,2} \left[\gamma \middle| \begin{smallmatrix} 1, 1 \\ 1, 0 \end{smallmatrix} \right]$ and $\exp(-z) = \text{G}_{0,1}^{1,0} \left[z \middle| \begin{smallmatrix} - \\ 0 \end{smallmatrix} \right]$, and with the helps of (5.32), [84, Eq.

(07.34.17.0011.01), Eq. (07.34.20.0011.02), Eq. (07.34.21.0081.01)], we attain

$$\begin{aligned} \mathbb{E}\{C_{\text{RF}}\} &= \frac{d}{\ln 2} \left(\frac{\Lambda_{ij}}{\varrho} \right)^{d^2(K-1)} \frac{1}{[d^2(K-1) - 1]!} \sum_{i=0}^{d^2-1} \frac{(\Lambda_{ii})^i}{i!} \sum_{q=0}^i (-1)^{q+d^2(K-1)-1} \binom{i}{q} \\ &\quad \times (\Lambda_{ii})^{-(q+d^2(K-1))} G_{3,3:0,1:3,3}^{3,1:1,0:1,3} \left[\Lambda_{ii}, \frac{\Lambda_{ii}}{\Lambda_{ij}/\varrho} \left| \begin{matrix} -\vartheta, -\vartheta, 1-\vartheta \\ 1-\vartheta, -\vartheta, -\vartheta \end{matrix} \right| \begin{matrix} 0, 1, 1 \\ 1, [q+d^2(K-1)], 0 \end{matrix} \right], \end{aligned} \quad (5.39)$$

where $\vartheta = i - (q + d^2(K-1))$.

On the other hand, by substituting (5.24) in (5.38) and using $\ln(1 + \gamma) = G_{2,2}^{1,2} \left[\gamma \left| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right. \right]$, $\mathbb{E}\{C_{\text{FSO}}\}$ can be obtained by exploiting [84, Eq. (07.34.21.0013.01)]. By doing that, and after some manipulations by utilizing the Meijer's G-function definition [74, Eq. (9.301)], we get

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2 A/2}{(2\pi)^{w-1} \ln 2} \sum_{m=1}^{\beta} b_m G_{w+2, w+4}^{w+4, w} \left[\frac{B/c^w}{\mu_s^{1/s}} \left| \begin{matrix} \mathbf{x}_4, 1, \zeta^2+1 \\ \zeta^2, \alpha, m, 0, \mathbf{x}_4 \end{matrix} \right. \right], \quad (5.40)$$

where $\mathbf{x}_4 = \frac{0}{w}, \dots, \frac{w-1}{w}$ which has w terms.

Eventually, substituting (5.39) and (5.40) in (5.36) leads to the ergodic capacity of the considered system.

Note that the $\mathbb{E}\{C_{\text{FSO}}\}$ in (5.40) is for the $\mathcal{M}$ fadings FSO channel. Alternatively, to attain the $\mathbb{E}\{C_{\text{FSO}}\}$ at the special case of G-G fadings FSO channel, the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\mathcal{M}}(\gamma)$ in (5.38) must be changed by the pdf $f_{\gamma_{\text{FSO}}^{\text{eff}}}^{\text{G-G}}(\gamma)$ given in (5.25). By doing that, and following the same steps as the prior ones, we obtain

$$\mathbb{E}\{C_{\text{FSO}}\} = \frac{\zeta^2/(\Gamma(\alpha)\Gamma(\beta))}{(2\pi)^{w-1} \ln 2} G_{w+2, w+4}^{w+4, w} \left[\frac{\zeta^2 \alpha \beta / c^w}{(\zeta^2+1) \mu_s^{1/s}} \left| \begin{matrix} \mathbf{x}_4, 1, \zeta^2+1 \\ \zeta^2, \alpha, \beta, 0, \mathbf{x}_4 \end{matrix} \right. \right]. \quad (5.41)$$

5.4 Simulation and Numerical Results

In this section, we present numerical examples to demonstrate the analytical expressions of the previous section and also to illustrate the impact of the various system and channel parameters on the performance of our system.⁷ Additionally, this section verifies the obtained analytical expressions through Monte-Carlo simulations. In these simulations, 10^6 realizations of the SNR random variable are used except in some results which are close to or less than 10^{-6} where 10^8 realizations are used.⁸

In Fig. 5.1 the outage probability P_{out} is plotted for perfect and imperfect RF CSI accuracy and all of the three FSO detection types. From this figure it can be perceived that for IM-DD input independent, the outage probability is limited/dominated by the FSO hop at high SNRs, and changing the CSI accuracy will not change the outage performance. Alternatively, For the heterodyne and IM-DD cost dependent detections, enhancing the CSI accuracy (i.e. decreasing ρ) in the RF hop improves the outage probability performance, i.e., the system P_{out} is limited by the RF CSI accuracy in this case.

From Fig. 5.2 it can be observed that, at relatively good FSO channel conditions, there is a room to enhance the outage probability by improving the RF CSI accuracy. While, as seen from this figure, the perfect and moderate CSI accuracies result in

⁷Following the same approach as in related literature, e.g. [71], [63], [24], [25], the values of the system parameters were chosen to confirm the deduced expressions, and thence other parameters values can be used by the designer to attain the needed result in specific schemes.

⁸ In these results, without loss of generality, we set $\Omega=0.78$ and $b_0=0.11$.

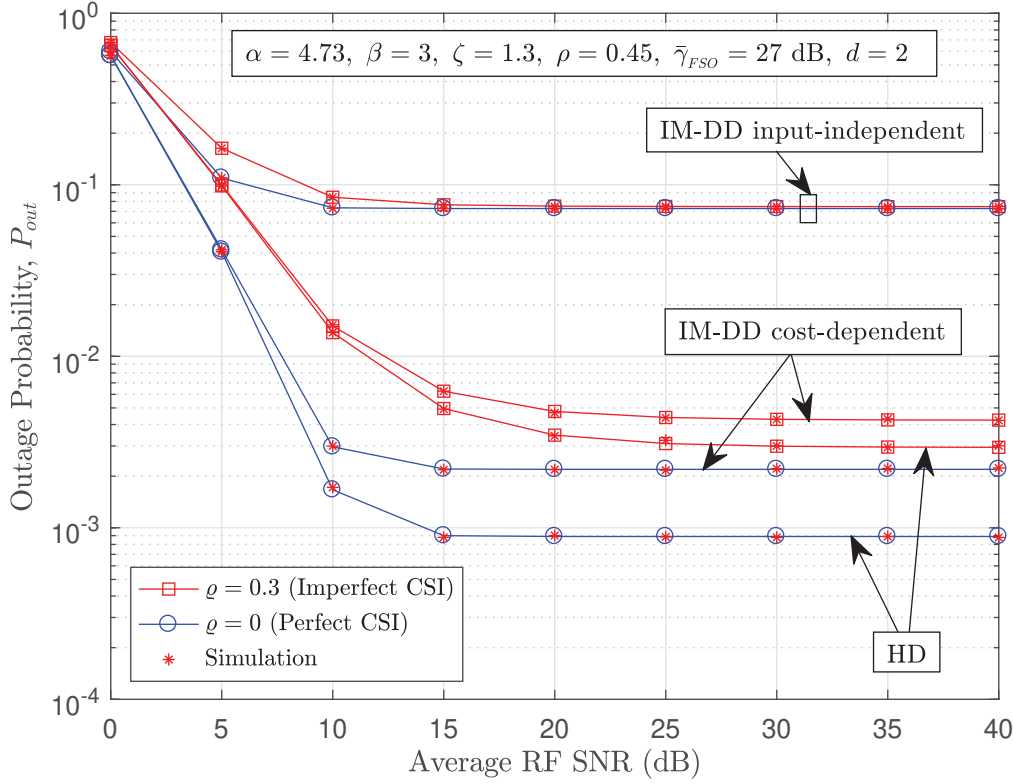


Figure 5.1: P_{out} for different values of the RF CSI accuracy (ρ), considering all FSO detection methods.

almost the same outage probability at relatively bad FSO channel conditions, as a consequence there is no advantage in increasing the RF CSI accuracy too much in this case. Moreover, Fig. 5.2 illustrates that the change in the FSO channel conditions leads to inconsiderable change in the outage probability at relatively bad CSI accuracy.

Fig. 5.3 and Fig. 5.4 present the diversity order for the outage probability of the system. Additionally, these figures demonstrate that the performance loss due to the imperfect CSI increases with the increase in the SNR. Specifically, Fig. 5.3 shows that, under the RF domination, we get $G_d^{RF} = d^2$ for the perfect CSI case, i.e. when $\rho = 0$, which matches the derived diversity order. However, the imperfect CSI effect takes over the outage performance and results in zero diversity order causing a floor in the

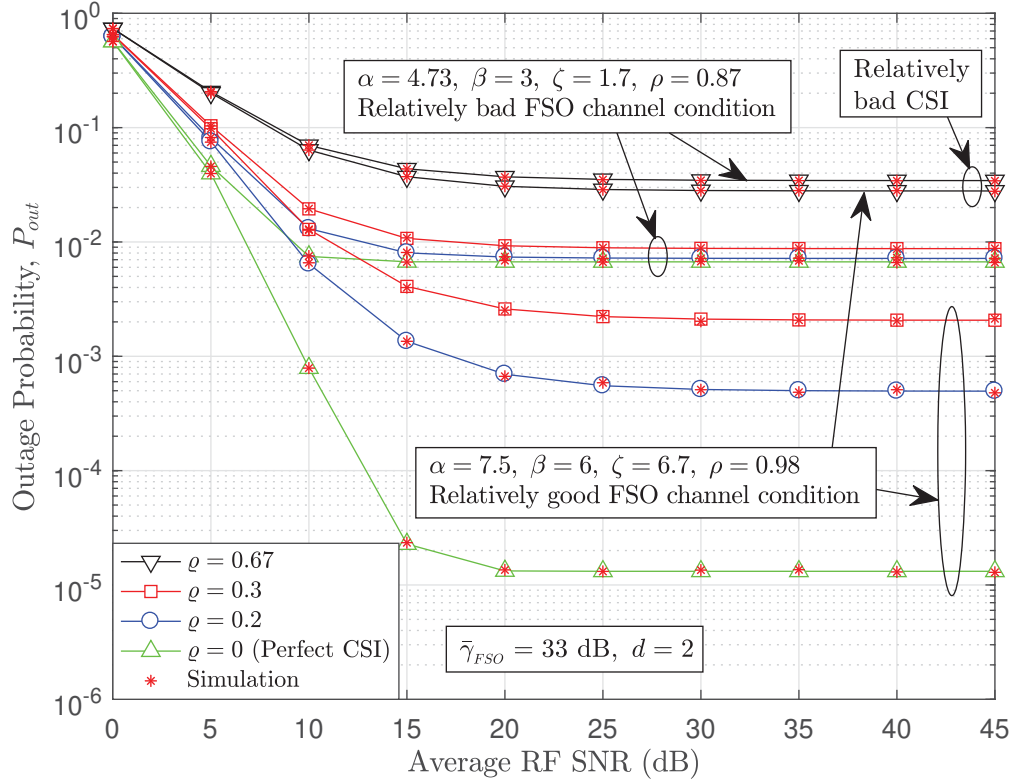


Figure 5.2: P_{out} for various FSO channel conditions and different RF CSI accuracy considering the IM-DD input-independent FSO.

outage probability curve at high SNRs. This floor worsen with either the increment of ϱ (the reduction in the RF CSI accuracy), or the decrement in the number of RF info streams (d).

On the other hand, Fig. 5.4 validates the derived diversity order (i.e., given $s=1$, $G_d^{\mathcal{M}} = b_{\min}''/s = \min[\zeta^2, \alpha, 1]$ and $G_d^{\text{G-G}} = b_{\min}^*/s = \min[\zeta^2, \alpha, \beta]$) when the FSO hop dominates the outage performance with perfect RF CSI. Whereas in case of imperfect RF CSI, the RF hop takes the domination over the outage performance and leads to floor (zero diversity order) at high SNRs.

In Fig. 5.5 the ergodic channel capacity C_{erg} is shown for perfect and imperfect RF CSI accuracy considering the three types of FSO detection. From this figure it can

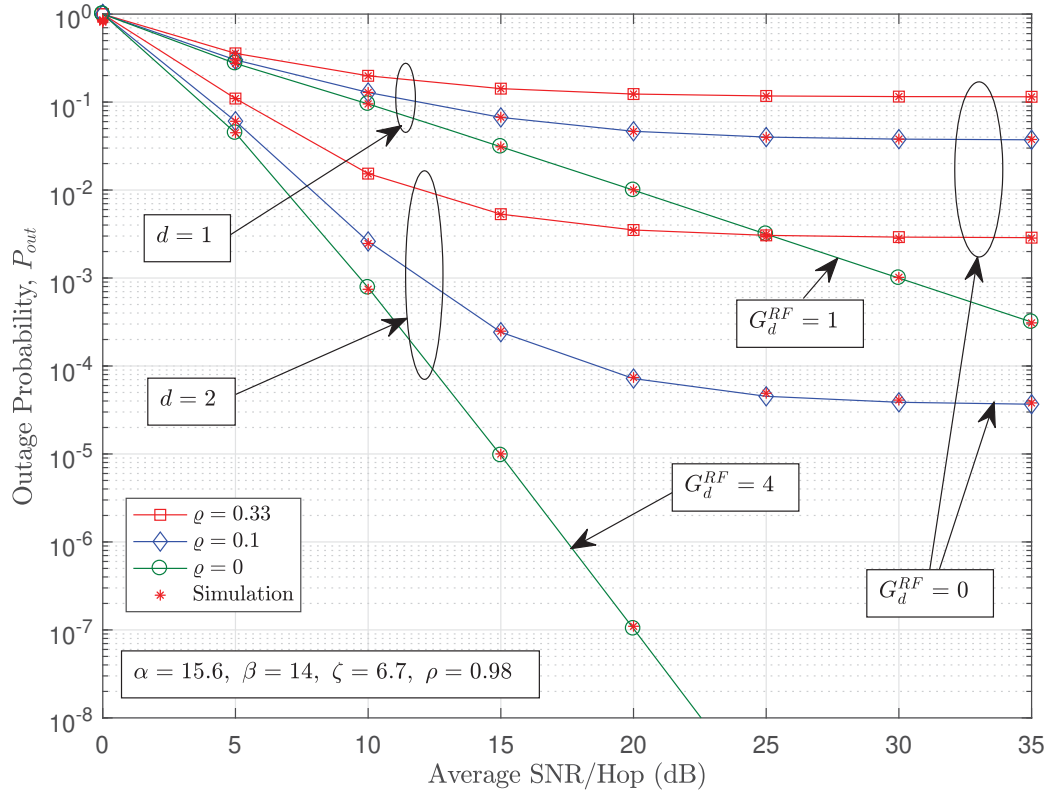


Figure 5.3: P_{out} dominated by the RF hop with different DoF (d) and CSI accuracy (ρ) considering HD in the FSO hop.

be seen that for IM-DD input independent and IM-DD cost dependent, the ergodic capacity is dominated/limited at high SNRs by the FSO hop, and changing the CSI accuracy does not change the system capacity. Alternatively, For the heterodyne detection, enhancing the RF CSI accuracy (i.e. decreasing ρ) increases the ergodic capacity, i.e., in this case the ergodic capacity is limited by the RF CSI accuracy.

Fig. 5.6 and Fig. 5.7 reveal that the system ergodic capacity loss due to the imperfect RF CSI increases with the increase in the average SNR. This is because the imperfect CSI leads to a ceiling in the capacity curve at high SNRs, while under perfect CSI the capacity is proportional to the SNR and it has a pre-log (DoF) equals d . Fig. 5.6 also shows that the performance ceiling, in the imperfect CSI case, becomes worse

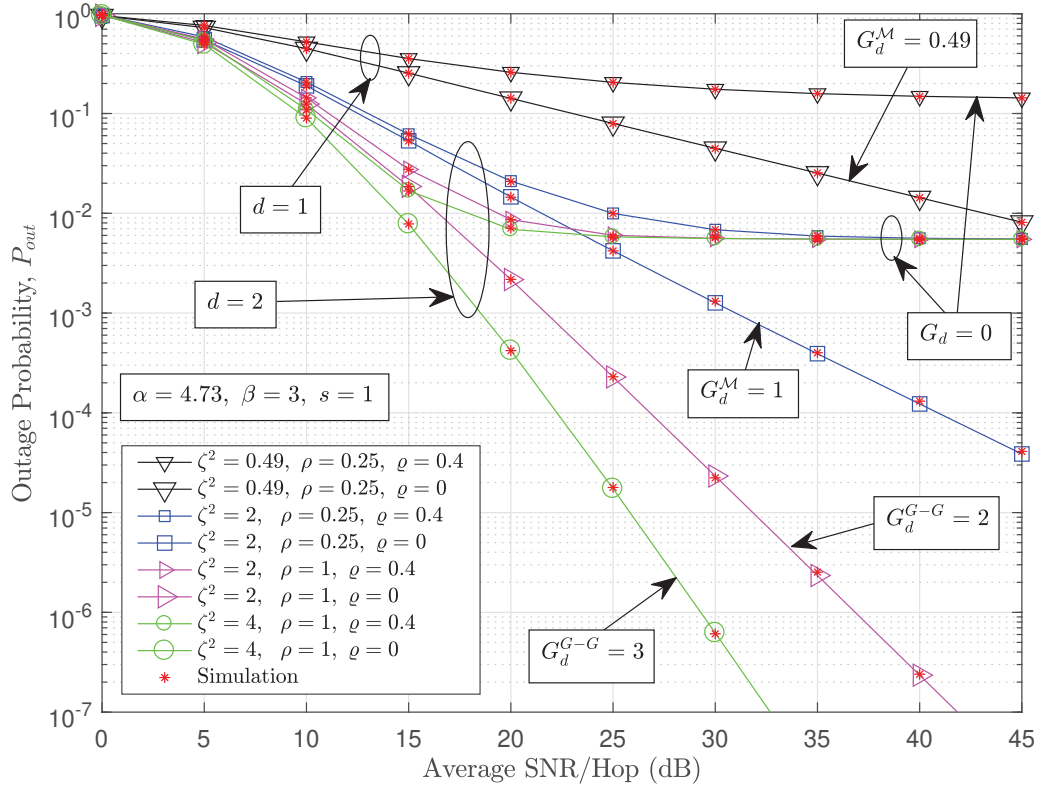


Figure 5.4: P_{out} dominated by FSO IM-DD cost-dependent hop with different values of the CSI accuracy (ϱ), DoF (d), ρ and ζ .

when the CSI accuracy or the number of RF info streams (d) decreases. Beside that, Fig. 5.7 elucidates that under perfect CSI the number of interferers does not change the ergodic rate performance since the interference is perfectly eliminated in this case, however under imperfect CSI increasing the interferers number deteriorate the ergodic rate.

5.5 Conclusion

This chapter studied a dual-hop DF mixed (imperfect IA MIMO RF IC)/(FSO) relaying scheme in which HD as well as IM-DD were considered in a unified manner in the FSO hop. It was assumed that the MIMO RF links follow Rayleigh fading, while

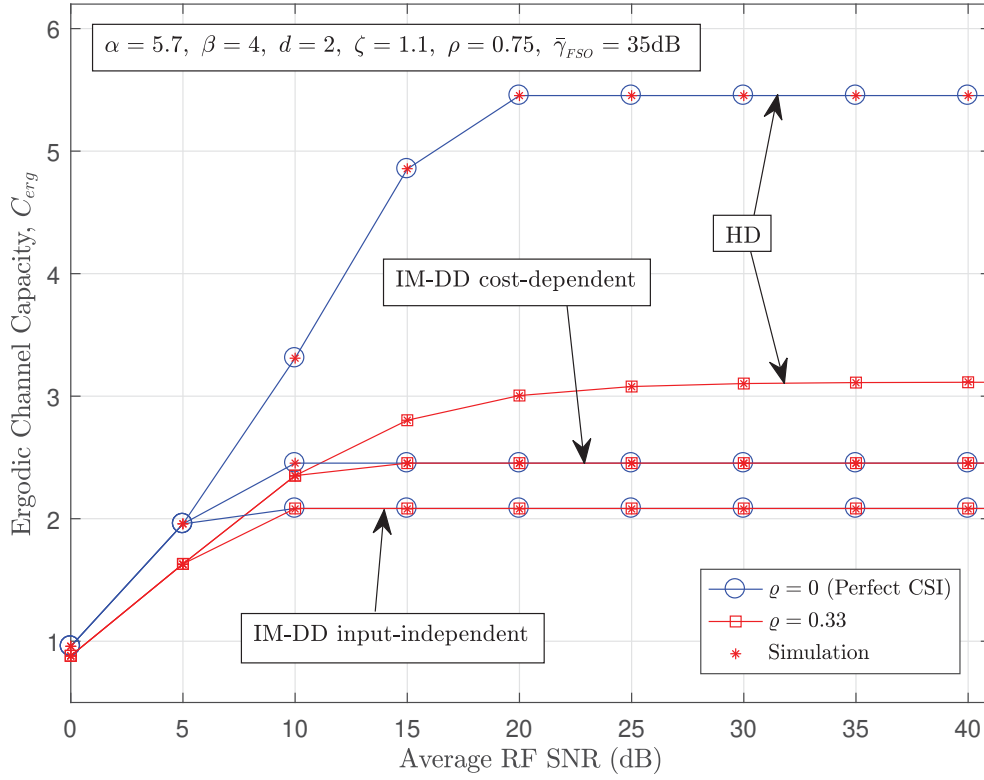


Figure 5.5: C_{erg} for various values of the RF CSI accuracy (ρ) considering all FSO detection methods.

the FSO link suffers $\mathcal{M}$ fading with pointing error. In the MIMO RF IC hop, the IA technique was utilized however with imperfect CSI which means that the interference will not be totally eliminated in this case. For this scheme, closed-form expressions were derived for the outage probability in terms of Whittaker and Meijer functions and for the ergodic capacity in terms of the bivariate Meijer's G-function. In those expressions the best-known FSO channel capacity results were utilized. Moreover, not only the IM-DD inputindependent AWGN but also the IM-DD costdependent AWGN channel was considered in the derivations. It was found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2) in case of perfect CSI. Alternatively, under FSO hop domina-

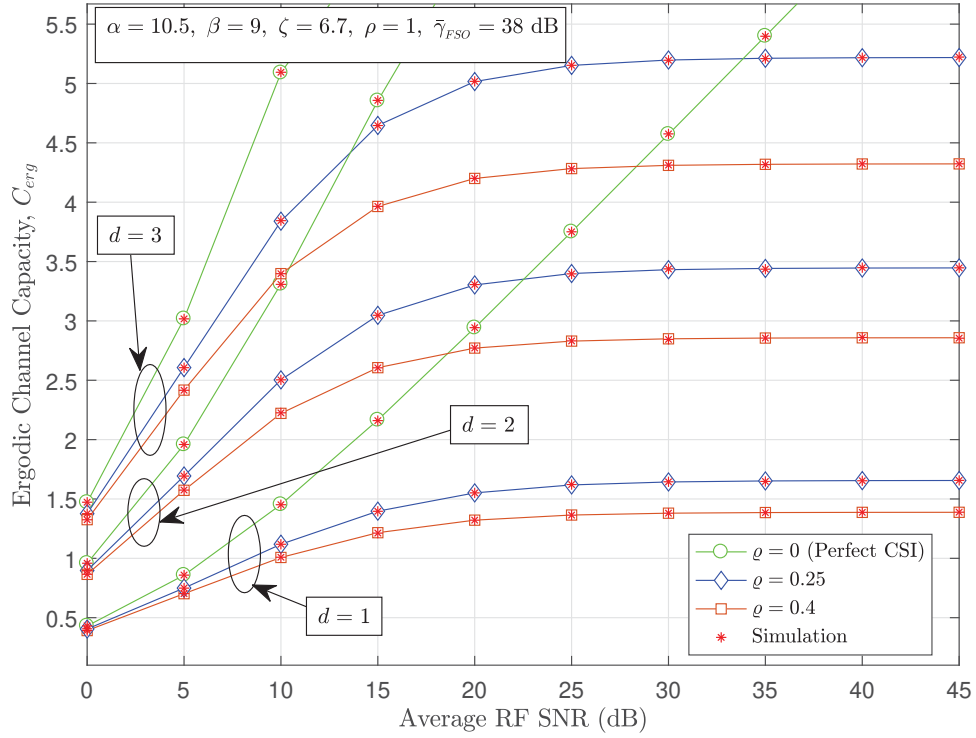


Figure 5.6: C_{erg} dominated by the RF hop for different DoF (d) and CSI accuracy (ϱ) considering HD in the FSO hop.

tion, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors in case of perfect CSI. However the case of imperfect CSI results in zero diversity order causing a floor in the outage probability curve at high SNRs. This floor worsen with either the reduction in the CSI accuracy, or the decrement in “ d ”. Also under imperfect CSI, It was found that there is a ceil in the ergodic capacity at high SNRs, this ceil becomes worse with more interferers or when the CSI accuracy or the number of RF info streams (d) decreases. Further, we provided Monte-Carlo simulation results that verify the deduced expressions.

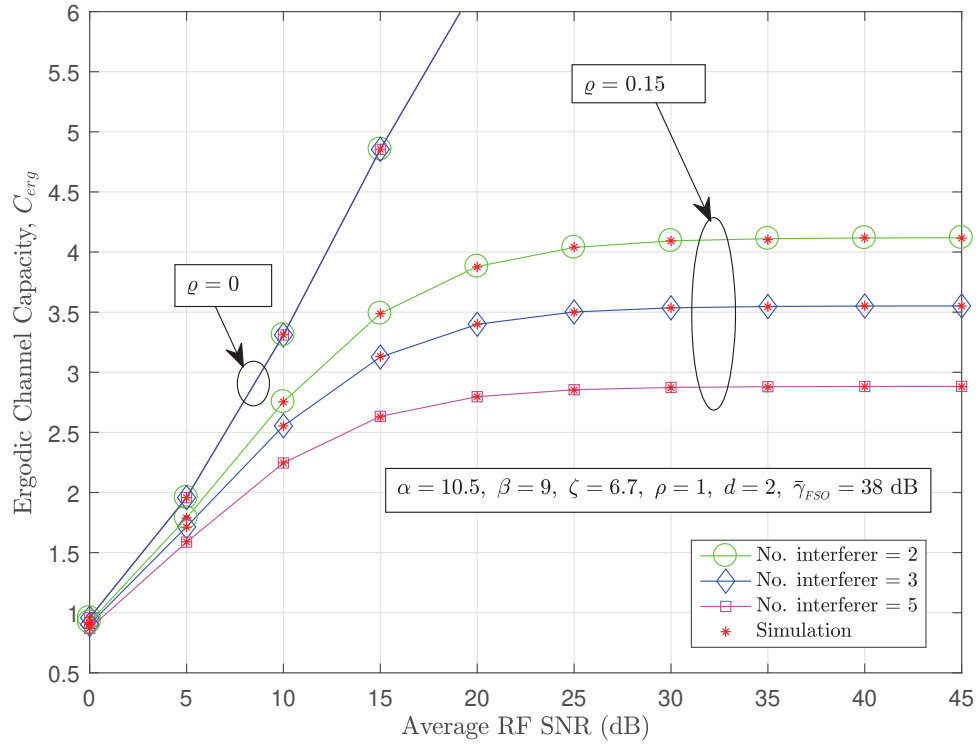


Figure 5.7: C_{erg} dominated by the RF hop for different number of interferers and CSI accuracy considering HD in the FSO hop.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

In this dissertation, we studied the performance of dual-hop mixed RF/FSO DF relaying, where HD and IM-DD were considered in a unified manner for FSO detection. Specifically, we derived closed-form expressions for the outage probability, average BER, and ergodic capacity of this system assuming generalized fading models, i.e., Nakagami- m and $\mathcal{M}$ fading with pointing error for the RF and FSO channels, respectively.

In doing that, we utilized, for the first time as per our knowledge, a precise IM-DD channel capacity approximation. Consequently, this work presents precise quantitative comparison between HD and IM-DD performances, and thus helps the designers in deciding whether to deploy HD (higher rate) or IM-DD (lower cost/complexity).

Moreover, this was the first time that not only the IM-DD input-independent but

also the IM-DD cost-dependent AWGN channel is considered in such system analysis. The difference between those two IM-DD channels resides in the variance of the AWGN which does not depend on the input signal in the first model, however in the second model it is proportionate to the cost (constraint) of the input signal (average optical power). It is important also to state that the input-independent AWGN IM-DD channel model is only valid where the receiver noises are dominated by thermal noise, and this is the case in the receivers which employ PIN-photodiode or in case of low optical powers. On the other hand, the cost-dependent AWGN IM-DD channel model is useful wherever the receiver noises are dominated by shot noise, and this is the case in the receivers which employ avalanche photodiode (APD) or in case of high optical power.

Also, in BER analysis, we assumed that the modulation schemes in the two hops can be different, as not all modulation schemes are suitable for FSO IM-DD links. Specifically, in the IM-DD systems we modulate the optical intensity (nonnegative quantity) which does not have a phase, and hence all kinds of phase modulations are not applicable there.

Additionally, the system performance was asymptotically investigated at high SNR where new non-reported diversity order and coding gain analyses was shown. Interestingly, we found that in the FSO hop, at high transmitted powers, all the optical detectors result in the same diversity order, whereas their coding gains are different.

Also, for HD and IM-DD input-independent we found that, at high transmitted powers, their capacities pre-logs are identical and equal twice the one of IM-DD cost-dependent.

Moreover, we showed the overestimations which occur in the outage and capacity of the system when the well-known Shannon's capacity formula for the power-constrained AWGN channel is used for IM-DD channel instead of its precise capacity approx. These overestimations lead to misconception in the comparison between HD and IM-DD. Furthermore, we offered simulation results that confirm the derived exact and asymptotic expressions.

One more scheme that its performance was studied, for the first time as per our knowledge, in this dissertation is the dual-hop DF mixed (IA MIMO RF IC)/(FSO) relaying scheme in which the HD and IM-DD were considered in a unified manner in the FSO hop. In this system, it was assumed that each one of the MIMO RF links follows Rayleigh fading, while the FSO link suffers $\mathcal{M}$ fading with pointing error.

Additionally, in the MIMO RF IC hop the IA technique was utilized to eliminate the interference, and hence this leads to enhance the multiplexing gain in this hop, and accordingly the overall system performance is improved. The IA precoding and receiving interference eliminating matrices were designed by using OR-ML algorithm.

For this scheme, exact closed-form expressions were derived for the outage probability and ergodic capacity, in these expressions the best-known FSO channel capacity results were utilized. Moreover, both the IM-DD input-independent AWGN and the IM-DD cost-dependent AWGN channel were considered in this derivation.

In addition, the system performance was asymptotically studied on the high SNRs region, where the diversity order and coding gain analyses were illustrated. It was found that under RF hop domination, the diversity order equals the square of the number of the interference-eliminated RF info streams (d^2). Alternatively, under FSO

hop domination, the diversity order depends on the parameters of the atmospheric turbulences and pointing errors. Also, It was observed that the overall system ergodic capacity enhances by increasing “ d ” when the RF hop dominates the system. Further, we provided simulation results that verify the deduced analytical and asymptotic expressions.

Furthermore, we inspected the performance deterioration in the dual hop mixed (imperfect IA MIMO RF IC)/(FSO) relaying due to the consideration of the imperfect CSI. Considering imperfect CSI means that the interference was not totally eliminated in this case. For this scheme, closed-form expressions were derived for the outage probability in terms of Whittaker and Meijer functions and for the ergodic capacity in terms of the bivariate Meijer’s G-function.

For this system it was found that imperfect CSI results in zero diversity order causing a floor in the outage probability at high SNRs. This floor worsen with either the reduction in the CSI accuracy, or the decrement in “ d ”. Also under imperfect CSI, It was found that there is a ceil in the ergodic capacity at high SNRs, this ceil becomes worse with more interferers or when the CSI accuracy or the number of RF info streams (d) decreases.

6.2 Future Work

In future research works, the considered mixed RF/FSO relaying schemes in this dissertation are proposed to be investigated with the inclusion of the cognitive radio, parallel relayings, physical layer security and/or buffer-aided relaying concepts.

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