

**NUMERICAL MODELING FOR PERFORMANCE
PREDICTION OF STANDING WAVE
THERMOACOUSTIC SYSTEMS**

BY

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In

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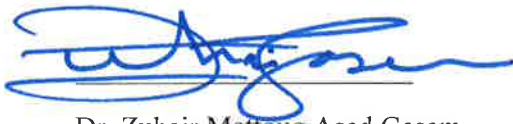
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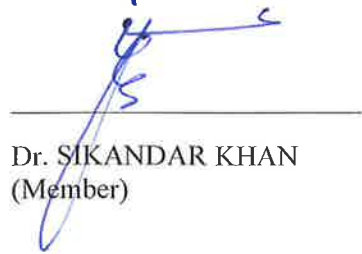
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To my beloved parents, wife and son.

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LIST OF ABBREVIATIONS

CFD	:	Computational Fluid Dynamics
COP	:	Coefficient of Performance
COPC	:	Carnot's Coefficient of Performance
COPR	:	Relative Coefficient of Performance
NSC	:	Non-Dimensionalized Sensitivity Coefficient
PD	:	Particle Displacement Length
REV	:	Representative Element Volume
SWTAD	:	Standing Wave Thermoacoustic Device
SWTAE	:	Standing Wave Thermoacoustic Engine
SWTAR	:	Standing Wave Thermoacoustic Refrigerator
TADTAR	:	Thermoacoustically Driven Thermoacoustic Refrigerator
TAE	:	Thermoacoustic Engine
TAR	:	Thermoacoustic Refrigerator
TAPM	:	Thermoacoustic Prime Mover

ABSTRACT

Full Name : Umar Nawaz Bhatti

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Standing wave thermoacoustic devices utilize high intensity sound waves and heat in the form of temperature gradient across a porous medium for the transfer of energy and can be categorized as engines or refrigerators. These devices are limited in their efficiency or performance due to high degree of irreversibility inherited in their operation and it is primary hindrance in industrialization of these systems. This has led to concentrated efforts in the recent past to analyze and improve the performance of these devices. Numerical modeling tools have proven their ability to accurately predict the performance of these devices by successfully capturing the inherent non-linearities and are used extensively to model these devices.

The current work aims at performance prediction of standing wave thermoacoustic devices using the numerical modeling techniques. At first, a detailed review of analytical and numerical modeling concerning the standing wave thermoacoustic devices has been carried out. In the next phase, one dimensional numerical model of a Standing Wave Thermoacoustic Refrigerator (SWTAR), based on linear thermoacoustic theory, has been developed. The developed SWTAR achieved a temperature drop of 34 K for cooling load of 10 W with coefficient of performance (COP) of 1.264. Based on the results from the developed model a sensitivity study using Non-dimensionalized Sensitivity Coefficient

(NSC) method has been conducted for three important performance parameters i.e. temperature drop achieved (ΔT), COP and Relative Coefficient of Performance (COPR). It has been found that all the output parameters are most sensitive to the small changes in the oscillating pressure amplitude (p_1). Largest values of NSC are recorded with respect to p_1 for all the cases with a maximum value of 24.12 obtained for ΔT and value of 17.92 for COPR.

In the third phase, simplified two dimensional CFD models of Standing Wave Thermoacoustic Engine (SWTAE) and SWTAR have been developed. Their performance have been simulated using ANSYS Fluent and results are validated by comparing them with the published results. In the last phase, effect of heterogeneous stack on the performance of SWTAR have been analyzed using enhanced solution domain of the developed simplified model. Nine cases have been simulated to analyze the performance. Two different aspects of heterogeneous stack, namely different material combinations and different lengths of stacked layers have been investigated in detail in terms of temperature drop achieved at cold heat exchanger ΔT_{CHx} , time averaged energy flux $\langle \dot{E}_y \rangle_t$ absorbed by stack and COP. Among all the material combinations of the heterogeneous stack investigated, combinations having Recirculated Vitreous Carbon (RVC) as side stacked layers produced the best results. The performance of Celcor based RVC combination, when compared with the homogeneous Celcor stack, showed an increase in the ΔT_{CHx} by 26.14 %, reduction in the $\langle \dot{E}_y \rangle_t$ consumption by 4.55 % and enhancement of COP by 5.12 %. Furthermore, an increase in length of side stacked material layer resulted in an increase in values of both ΔT_{CHx} and COP. As the length of the side stacked material is increased from half to three particle displacements, the results for ΔT_{CHx} and COP improved by 72.91 %

and 14.12 % respectively. Thus it can be inferred that the presence of layers of different materials, resulting in an anisotropic arrangement of thermal conductivities, inside the stack improves the SWTAR performance significantly. Heterogeneous stacks provide lower refrigeration temperatures, consume less energy and have a higher COP value than their homogeneous counterparts.

ملخص الرسالة

الاسم الكامل: عمر نواز بهاتي

عنوان الرسالة: نمذجة رقمية وتقييم لأنظمة المحركات الصوتية الحرارية ذات الموجة الموقوفة

التخصص: الهندسة الميكانيكية

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تستخدم الأجهزة الصوتية الحرارية موجات صوتية عالية الكثافة وحرارة في شكل تدرج حراري عبر وسط مسامي لنقل الطاقة، ويمكن تصنيفها كمحركات أو مبردات. هذه الأجهزة محدودة في كفاءتها أو أدائها بسبب احتوائها على درجة عالية متأصلة من عدم القابلية للانعكاس في عملها وهو عائق رئيسي في تصنيع هذه الأنظمة. وقد أدى ذلك إلى بذل جهود مركزة في الماضي القريب لتحليل وتحسين أداء هذه الأجهزة. أثبتت أدوات النمذجة العددية قدرتها على التنبؤ بدقة بأداء هذه الأجهزة من خلال التقاط العناصر اللاخطية المتأصلة بنجاح وتستخدم على نطاق واسع لتصميم هذه الأجهزة.

يهدف العمل الحالي إلى التنبؤ بأداء الأجهزة الصوتية الحرارية ذات الموجة الموقوفة باستخدام تقنيات النمذجة الرقمية. في البداية ، تم إجراء مراجعة تفصيلية للنمذجة التحليلية والعديد المتعلقة بالأجهزة الصوتية الحرارية ذات الموجة الموقوفة . في المرحلة التالية ، تم تطوير نموذج عددي أحادي الأبعاد للثلاجة الحرارية ذات الموجة الموقوفة (SWTAR) ، استنادًا إلى النظرية الصوتية الحرارية الخطية. حققت SWTAR المطورة انخفاضًا في درجة الحرارة قدره 34 كلفن و 10 واط حمولة مع معامل الأداء (COP) يبلغ 1.264 بناءً على نتائج النموذج المطور ، أجريت دراسة حساسية باستخدام طريقة معامل الحساسية عديم الأبعاد (NSC) لثلاثة معايير أداء مهمة ، مثل حاصل انخفاض درجة الحرارة المتحقق (ΔT) ومعامل الأداء COP ومعامل الأداء النسبي (COPR). لقد وجد أن جميع المعايير الناتجة هي الأكثر حساسية للتغيرات الصغيرة في سعة الضغط المتذبذب (p_1). تم تسجيل أكبر قيمة لمعامل الحساسية عديم الأبعاد NSC فيما يتعلق بسعة الضغط المتذبذب p_1 لجميع الحالات مع أقصى قيمة تم الحصول عليها للتدرج الحراري

ΔT 24.12 وقيمة 17.92 لمعامل الأداء النسبي COPR.

في المرحلة الثالثة ، تم تطوير نموذجين CFD مبسطين للمحرك الصوتي الحراري ذي الموجة الموقوفة (SWTAE) و SWTAR. تمت محاكاة أدائهما باستخدام برنامج ANSYS Fluent وتم التحقق من صحة النتائج من خلال مقارنتها بالنتائج المنشورة. في المرحلة الأخيرة ، تم تحليل تأثير الحزمة غير المتجانسة على أداء SWTAR باستخدام مجال حل تم تحسينه للنموذج المبسط المطور. وقد تم محاكاة تسع حالات لتحليل الأداء ثم تم التحقق بالتفصيل في جانبيين مختلفين من الحزمة غير المتجانسة ، وهما مجموعات المواد المختلفة والأطوال المختلفة للطبقات التي بداخل الحزمة من حيث انخفاض درجة الحرارة الحاصلة في المبادل الحراري البارد ΔT_{CHx} و متوسط معدل تدفق الطاقة $\langle \dot{E}_y \rangle_t$ التي تم امتصاصها بواسطة الحزمة ومعامل الأداء (COP). من بين جميع مجموعات مواد الحزمة الغير المتجانسة التي تم فحصها ، أعطت التوليفات التي تحتوي على الكربون المعاد تدويره (RVC) بصفقتها طبقات محزمة جانبية أفضل النتائج. أظهر أداء Celcor المستندة إلى توليفة RVC ، مقارنةً بحزمة Celcor المتجانسة ، زيادة في ΔT_{CHx} بنسبة 26.14٪ ، وانخفاض في استهلاك $\langle \dot{E}_y \rangle_t$ بنسبة 4.55٪ وتحسن في معامل الأداء COP بنسبة 5.12٪. علاوة على ذلك ، أدت الزيادة في طول طبقة مواد الحزمة الجانبية إلى زيادة في قيم كل من ΔT_{CHx} و COP. نظراً لزيادة طول مادة الحزمة الجانبية من نصف إلى ثلاثة من إزاحات الجسيم ، فقد تحسنت نتائج ΔT_{CHx} و COP بنسبة 72.91٪ و 14.12٪ على التوالي.

وبالتالي يمكن استنتاج أن وجود طبقات من مواد مختلفة داخل الحزمة يقوم بتحسين أداء SWTAR بشكل ملحوظ ، مما يؤدي إلى ترتيب متباينة الخواص للناقلية الحرارية. الحزم غير المتجانسة تقوم بتوفير درجات حرارة أقل لعملية التبريد ، وتستهلك طاقة أقل وتكون قيمة معامل الأداء COP أعلى من نظيراتها المتجانسة.

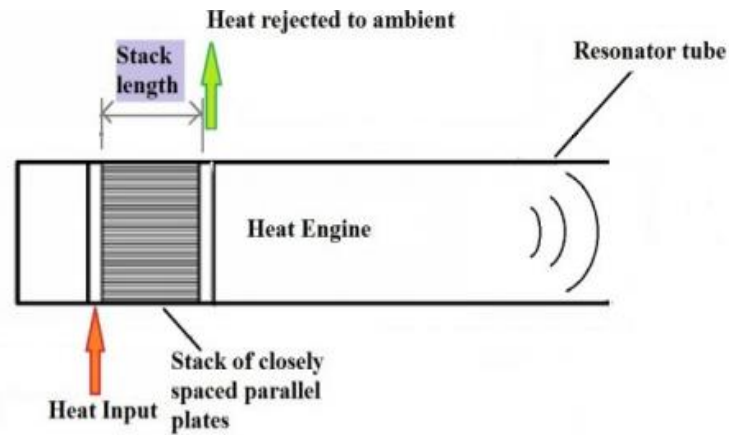
CHAPTER 1

Introduction

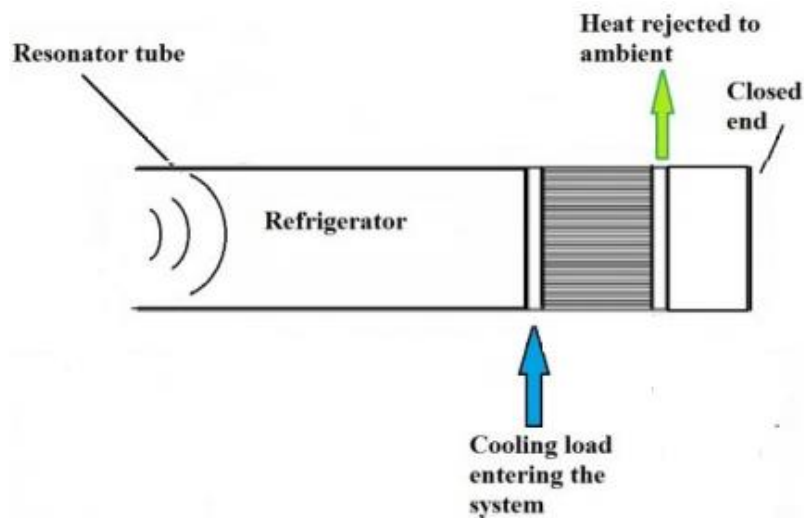
1.1 Background

The drastic changes in the climate and depletion of conventional energy resources have posed serious concerns among the global community and strict legislations are being imposed to overcome the challenge of global warming. This has brought the challenge of developing the renewable energy technologies to cope up the current energy needs of world. Conventional refrigeration/air conditioning systems use CFCs and HFCs that pose a major threat to environment and their use has been banned in the Montreal Protocol in 1996 [1]. These challenges have motivated the development of thermoacoustic refrigeration systems as a new environment friendly cooling technology, as these systems contain no refrigerants and can operate using air or noble gases [2]. Thermoacoustic systems utilize high intensity sound waves for transfer of energy and can thus be categorized as engines or refrigerators depending upon the desired output i.e. production of desired work/cooling effect. A typical thermoacoustic device, shown in Figure 1.1, consist of working fluid, a core in the form of stack or regenerator, heat exchangers at either side of core and a resonator tube housing the whole components. An additional component known as driver is present in the refrigeration devices. Driver act as a sound source that generates acoustic power resulting in the pressure pulsations and oscillatory motion of gas which creates a temperature gradient across the stack while passing through it and transfers

energy in the form of heat. The resulting heat is then transferred to outside load with the help of heat exchangers to generate the desired cooling effect [3]. The working fluid used in these devices are either noble gases or air that do not pose any threat to environment and thus this technology is environment friendly.



(a)



(b)

Figure 1.1: Basic schematics of a thermoacoustic (a) engine (b) refrigerator

Thermoacoustic devices offer added advantages of simplicity in the design for low temperature applications, less maintenance cost due to absence of moving/rotating components, and thus, eliminating the need of lubrication system. These devices can also

work on solar energy [4] and waste heat [5]. Thermoacoustic refrigeration system can also provide continuous cooling control in contrast to conventional refrigeration systems which have on/off control.

Keeping these benefits in mind, thermoacoustic refrigeration system present themselves as an ideal candidate to replace the conventional refrigeration systems. Hence, they have been at focal point of research during the last two decades. The fundamental factor hindering the commercialization of these systems is their low coefficient of performance (COP) compared to conventional vapor compression refrigeration systems. Considerable efforts are being made to improve the COP of the system.

1.2 History

Thermoacoustics is the branch of science that involves interaction between the thermal and acoustic energy. Net work flow or heat flow along (or opposite to) the direction of propagating sound waves is produced whenever the oscillating fluid is interacted with solid walls in the presence of temperature gradient [6].

History of thermoacoustics dates back to 1777 when Byron Higgins first noticed that the presence of hydrogen flame inside a tube results in the production of acoustic oscillations [7]. First ever thermoacoustic machines could be attributed to Carl Sondhauss, 1850 [8] and Peter Rijke, 1859 [9] now known as Soundhass tube and Rijke tube, shown in Figure 1.2(a) and Figure 1.2(b) respectively. Soundhass quantified the thermoacoustic effects of heated glass tube closed at one end while Rijke was able to produce strong acoustic oscillations through the placement of heated wire gauze inside the tube. Lord Rayleigh in 1877 [10] for the first time qualitatively explained the thermoacoustic effect in

these devices. According to his explanation, vibrations are produced if heat is supplied to waves at time of maximum condensation or removed at the time of maximum rarefaction. Carter et. al. [11] considerably improved the performance of Soundhass tube by the placement of “stack” inside the tube for the first time.

Nicolas Rott [12] laid down the theoretical formulation that serve as foundation of today’s modern thermoacoustics. He devised 1-D linearized thermoacoustics equations describing the physics of thermoacoustic devices and are known as Rott’s thermoacoustic equations. Swift and his team could be considered as pioneers of most of the modern practical thermoacoustic refrigerators/engines and he has summarized most of his work in [6]. He has also published a text book [3] in this area which is the first ever book in this field.

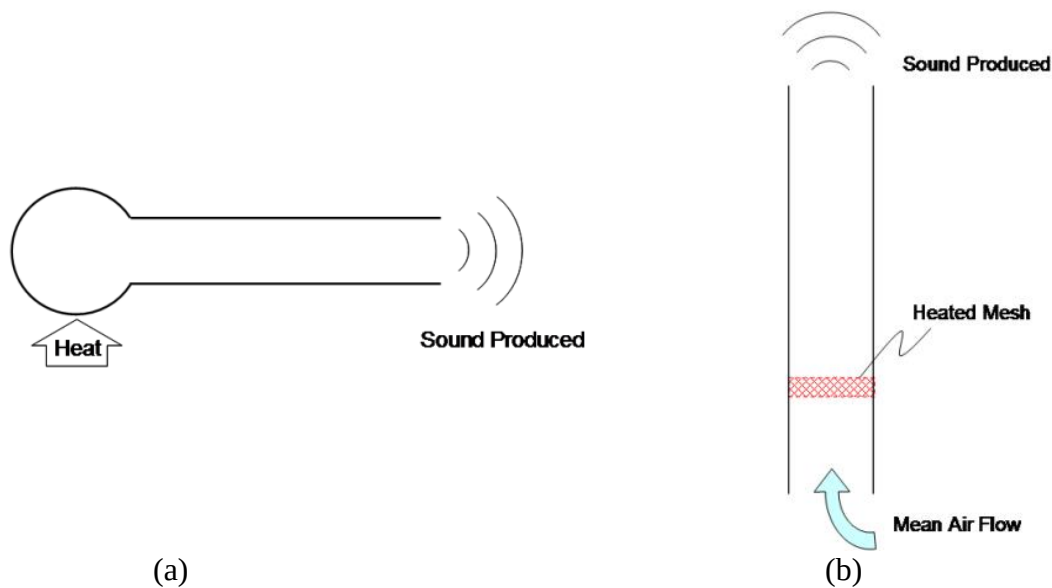


Figure 1.2: (a) Soundhass Tube (b) Rijke Tube

1.3 Classification of thermoacoustic devices

Thermoacoustic devices can be classified in following two categories depending on the type of function they intend to perform:

1.3.1 Thermoacoustic engine (TAE)

TAE converts heat energy into acoustic power which can further be utilized to perform the useful work. The temperature gradient is applied across the stack and heat flows from hot source to cold sink resulting in the production of oscillating pressure amplification thus acoustic power output.

1.3.2 Thermoacoustic refrigerator (TAR)

In TAR acoustic power is utilized to pump the heat across the stack from cold heat exchanger to hot heat exchanger thus producing a refrigeration or heat pumping effect.

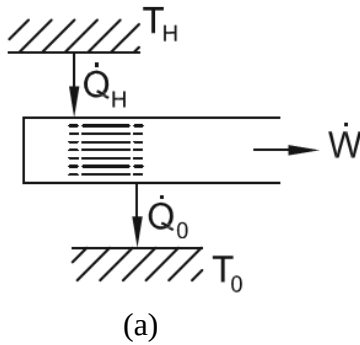
From construction point of view both TAE and TAR are effectively same. In TAE one end is open and acoustic power produced can be utilized from this end by placing some power output mechanism while in TAR a loudspeaker or any other acoustic power input device is placed at this end. The basic schematic of both devices is shown in Figure 1.3

Thermoacoustic devices can further be classified according to the nature of wave being generated in the resonated tube.

1.3.3 Standing wave thermoacoustic system

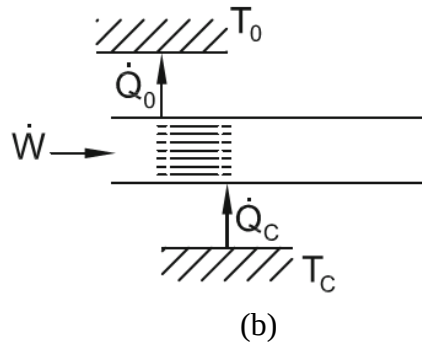
The geometry of the standing wave thermoacoustic devices is designed in such a way that a standing wave is generated inside the resonator and the acoustic power is generated/utilized by producing a phase difference of 90° between the pressure and the velocity. The spacing between the stack plates in these devices is designed in such a way to have an imperfect thermal contact between the fluid and solid thus resulting in the phase difference.

Engine:



(a)

Refrigerator:



(b)

Figure 1.3: Basic schematics of thermoacoustic (a) engine (b) refrigerator [2]

1.3.4 Travelling wave thermoacoustic system

In the traveling wave thermoacoustic devices, thermoacoustic phenomenon results in the development of pressure and velocity waveforms which are in phase and are allowed to travel in a loop. Stack is replaced by a regenerator and a perfect thermal contact is developed between the fluid and solid plate inside the regenerator. Due to the presence of perfect thermal contact, these devices are usually more efficient than their standing counterparts.

1.4 Thermodynamic cycle of a standing wave thermoacoustic device

Thermoacoustic cycle of a standing wave refrigeration can be explained effectively using a Lagrangian perspective. Let us consider one fluid particle moving inside a pair of stack plates along its length and is acted upon by a standing wave. The following four step process completely defines the thermodynamic cycle of fluid particle, see Figure 1.4.

A-B: Adiabatic compression

B-C: Isobaric heat loss

C-D: Adiabatic expansion

D-A: Isobaric heat gain

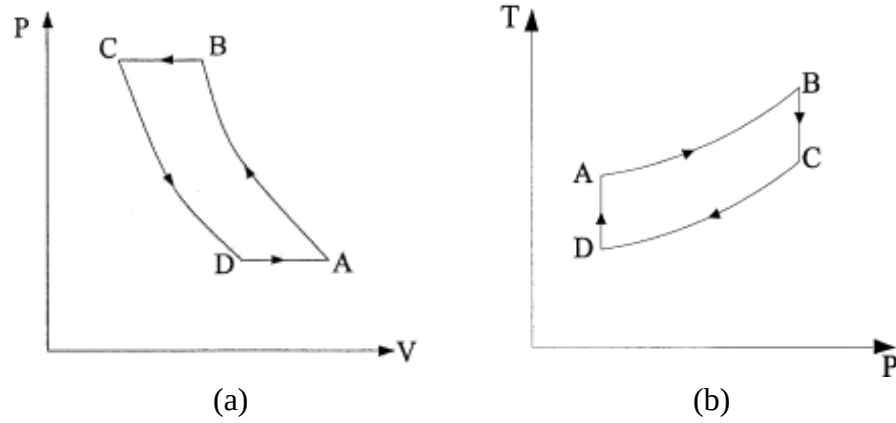


Figure 1.4: Thermodynamic cycle of gas particle (a) P-V diagram (b) T-P diagram

A small temperature gradient is also present along the stack, as a result the right side of surface is hot and the left side is cold. Figure 1.5 is showing the complete cycle followed by the particle.

In the first step, particle goes through adiabatic compression and its pressure and correspondingly the temperature increases. The fluid particle at the same time travels along the stack by an amount equivalent to gas displacement amplitude. When it reaches the right side, at its maximum displacement, it is slightly hotter than the solid plate and as a result, rejects heat to the plate thus performing the second step of isobaric heat loss. Adiabatic expansion take place in the third step and both the pressure and the temperature of fluid decreases. Meanwhile, particle travels back to the left side of stack by an amount of gas displacement amplitude and when it reaches the left side it is slightly cooler than the solid plate. Thus it gains heat isobarically completing the fourth step and the cycle continues with a small amount of heat pumped across the stack in each cycle. To enhance the amount of heat transfer, stack length is significantly increased to multiple gas displacement amplitudes and large no. of plates are placed inside the resonator.

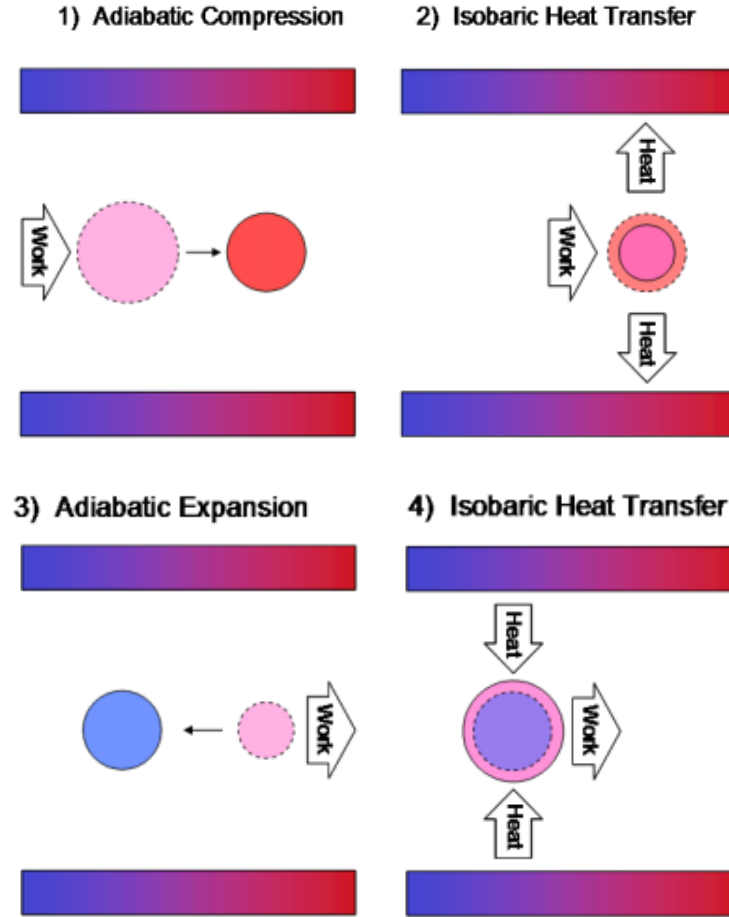


Figure 1.5: Thermodynamic cycle followed by fluid particle in thermoacoustic refrigerator

1.5 Important length scales

Following length scales, shown in Figure 1.6, are associated with the thermoacoustic devices and play a critical role in the design and performance of these devices:

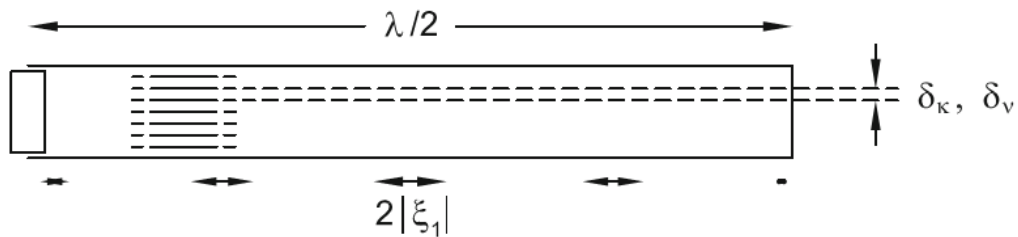


Figure 1.6: Important length scales involved in thermoacoustic devices

1.5.1 Wavelength (λ)

Wavelength of sound wave in longitudinal direction is given by:

$$\lambda = \frac{a}{f} \quad (1.1)$$

where, a is the speed of sound while f is the frequency of the acoustic wave.

Length of the whole apparatus in most of the standing wave devices are either half or quarter wavelength. The size of components where heat transfer occurs, i.e. stack or regenerator, is always kept at a fraction of the wavelength in all the thermoacoustic devices.

1.5.2 Gas displacement amplitude ($|\xi_1|$)

Gas displacement amplitude is the distance travelled by gas particle in one cycle and is given by:

$$|\xi_1| = \frac{|u_1|}{\omega} \quad (1.2)$$

where, $|u_1|$ is the velocity amplitude and ω is the angular frequency of the wave. The length of the heat exchangers at each end of the stack are either of the same magnitude or smaller than this distance. However, the length of the stack or regenerator is often a very large multiple of this amplitude.

1.5.3 Thermal (δ_k) and viscous (δ_v) penetration depth

These two length scales are perpendicular to the direction of the gas flow and dictate the heat transfer process between the fluid and solid stack/regenerator plates and are given by:

$$\delta_k = \sqrt{\frac{2K}{\rho C_p \omega}} \quad (1.3)$$

$$\delta_v = \sqrt{\frac{2\mu}{\rho\omega}} \quad (1.4)$$

where, K is thermal conductivity of gas, C_p is the specific heat capacity at constant pressure, ρ is the density of fluid at T_m , ω is the angular velocity and μ is the dynamic viscosity of the gas.

Thermal penetration depth defines the vertical distance from the solid plates where heat transfer could take place, smaller the distance effective is the heat transfer. Viscous penetration depth defines the vertical distance from the plate where viscous effects are stronger, larger the distance smaller are the viscous effects. Viscous effects reduce the efficiency of thermoacoustic devices and thus are undesirable. Thus a trade-off between δ_k and δ_v is required for an efficient design.

1.6 Critical performance parameters

Various parameters play a pivotal role in the performance estimation/optimization of thermoacoustic devices. These parameters are highly non-linear having a strong dependence on each other thus making the evaluation of coupled non-linear effects of parameters on each other very difficult. Numerical solvers required to evaluate these effects results in high computational cost and time. The key performance parameters are as follows:

1.6.1 Critical temperature gradient (∇T_{cric})

The temperature gradient across the length of stack that matches the adiabatic temperature change in the gas is known as critical temperature gradient and is given by equation:

$$\nabla T_{cric} = \frac{\omega A |p_1|}{\rho_m C_{p,s} |U_1|} \quad (1.5)$$

where, ω is the angular frequency of the acoustic wave, A is the cross sectional area of stack, p_1 is the oscillating pressure, ρ_m is the mean density of the fluid, $C_{p,s}$ is the specific heat at constant pressure of solid plate, U_1 is the complex volume flow rate of the fluid. If applied ∇T_m is the applied temperature gradient across the stack then a non-dimensional parameter Γ can be defined as:

$$\Gamma = \frac{\nabla T_m}{\nabla T_{cric}} \quad (1.6)$$

If $\Gamma = 1$, then no heat transfer would occur inside the stack and if $\Gamma \geq 1$ then the device would act as thermoacoustic engine and if $\Gamma \leq 1$ then it would work as refrigerator. The same process has been shown in the Figure 1.7, where the shaded region represent the hysteresis loop experienced by gas during a complete acoustic cycle.

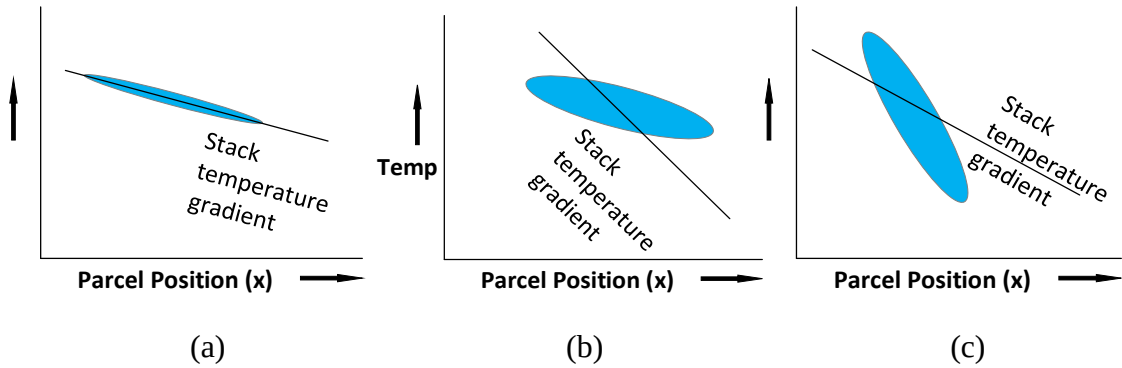


Figure 1.7: Applied temperature gradients (a) ∇T_{cric} ($\Gamma = 1$) (b) thermoacoustic engine ($\Gamma \geq 1$) (c) thermoacoustic refrigerator ($\Gamma \leq 1$)

1.6.2 Stack geometry

Stack is considered as the heart of thermoacoustic devices and typical geometries for stack, shown in Figure 1.8, are either parallel plates, spiral plate or pin array. The geometry is

important as it act as thermal storage and defines the hydraulic radius for the working fluid thus dictating the heat transfer process.

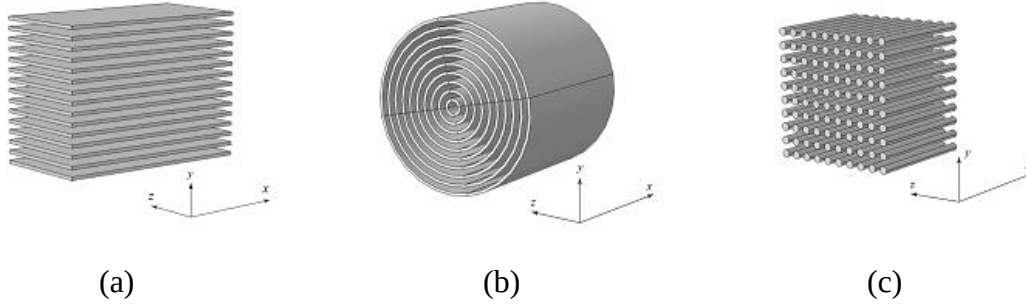


Figure 1.8: Different geometries of stack (a) parallel plate (b) Concentric spiral (c) Pin Array

1.6.3 Stack Spacing

It is a very crucial parameter as its value defines the way a thermoacoustic device would work. For an efficient performance, the spacing between the plates should have a best compromise between the thermal and the viscous penetration depths. For standing wave devices it is normally a small multiple of δ_k or δ_v . However for traveling wave devices, it is only a fraction of δ_k or δ_v , as a perfect thermal contact is required.

1.6.4 Working fluid properties

Properties of the selected working fluid significantly affect the performance and efficiency of thermoacoustic devices. Typically a fluid with a high speed of sound (lighter gases) and high thermal conductivity is recommended. It yield the highest power and a high value of δ_k allowing a large spacing between the stack plates.

1.6.5 Frequency

Both the power density and the δ_k of a thermoacoustic devices are dependent on resonance frequency of the system and thus it is one of the fundamental design parameters. The power

density is a linear function of frequency while δ_k varies inversely with its square root [3], so the selection of appropriate frequency is extremely important.

1.6.6 Pressure amplitude or drive ratio

Drive ratio is the ratio of peak pressure amplitude to mean pressure of the device. Pressure amplitude obtained and drive ratio are key performance parameters for both engines and refrigerators respectively.

1.6.7 Mean pressure

Mean pressure like frequency is also a fundamental design parameter of thermoacoustic devices and studies have shown that higher mean pressure leads to an increase in the pressure amplitudes of these devices and the onset temperature of TAE also increases.

1.7 Motivation

Thermoacoustic refrigerators present themselves as a viable alternative to the conventional refrigeration systems as their use can considerably reduce the global warming potential. Standing wave thermoacoustic refrigerators, because of their simplicity in design, flexibility to be developed both at small & large scale and ability to be operated by waste heat or solar energy, find their potential in various application areas. Some of the potential application areas include cooling in remote areas, cooling of the on-board electronics, ice cream coolers, supplementary cooling systems in addition to the conventional systems to support additional cooling loads etc. The primary challenge in their industrial acceptance is their lower efficiencies and less refrigeration capacity compared to the conventional systems. Detailed understanding of inherent loss mechanisms in these devices is of utmost importance if these challenges are to be met in the near future.

All these factors have motivated the researchers to focus on the performance optimization of these refrigerators using both experimental and numerical methods. Great deal of research has focused on the optimization of stack as it is considered as heart of these devices. Recently researchers have focused their attention towards CFD methods to simulate and investigate the performance of these devices as they have proven their ability to accurately model these systems. Literature review suggests that two-dimensional CFD model is enough to accurately model and capture the losses and performance parameters associated with these systems. Further, there is a great potential for improvement of these devices at all scales more specifically at small scales.

Current research would focus on building the understanding of application of CFD methods to the standing wave thermoacoustic devices especially refrigerators. An attempt would also be made to carry out a sensitivity study to explore and analyze the effect of key performance parameters. At the end an effort would be made to develop a novel 2-D numerical scheme to investigate the performance of heterogeneous stack. Since this is the first effort to model the thermoacoustic systems in KFUPM a start would be made by developing a 1-D model.

1.8 Objectives

As outlined above, this work aims to investigate the performance of standing wave thermoacoustic devices using numerical modeling tools thus following are the main objectives of the current work:

1. Understanding the concepts and state of the art concerning the numerical modeling of thermoacoustic devices.

2. Development of a working 2-D numerical model of standing wave thermoacoustic refrigerator/engine.
3. Performing the sensitivity study of standing wave thermoacoustic refrigerator.
4. Investigation of effect of heterogeneous stack on the performance of thermoacoustic refrigerator using the developed numerical model.

1.9 Outline of thesis

The thesis consists of six chapters. In this chapter, after a brief description of ideas pertaining to thermoacoustic devices, general thermoacoustics, history, classification, important length scales and critical performance parameters, the motivation and objective of current research have been presented. A detailed review of analytical and numerical modeling concerning the standing wave thermoacoustic devices has been carried out in Chapter 2. Sensitivity analysis of standing wave thermoacoustic refrigerator using non-dimensionalized sensitivity coefficients has been presented in Chapter 3. Chapter 4 covers the details of two numerical models developed for performance simulation of standing wave thermoacoustic engine and refrigerator respectively. Performance of standing wave thermoacoustic refrigerator under the effect of heterogeneous stack has been analyzed in detail in Chapter 5. Finally conclusions and recommendations are presented in Chapter 6.

CHAPTER 2

Literature Review

2.1 Introduction

Thermoacoustic devices have gained much attention in recent decades as an alternative energy technology and can be categorized as standing and travelling wave devices. Standing Wave Thermoacoustic Devices (SWTAD) are advantageous because of simplicity in the design, absence of moving/rotating components, ability to work on solar energy [4] and waste heat [5], but have lower efficiencies than their travelling counterpart. It is the main factor hindering the commercialization of these systems. Considerable efforts are being made to improve their performance by minimizing the associated losses and numerical modeling tools are actively used in this regard as they provide a useful insight by modeling the thermoacoustic mechanisms inside these devices in detail.

Operation of SWTAD involve complex flows utilizing the effects of acoustics, thermodynamics and heat transfer mechanism. Analytical solution expressing the physics of these devices is very difficult and models developed so far are based on linearized wave equation. They are limited to low amplitude flows and calculations are based on steady flow data. Detailed understanding of the flow field and heat transfer mechanism, for oscillatory flow conditions, within the internal structures of standing wave thermoacoustic systems is a key factor in efficient development of these devices. Experimental studies have also confirmed the presence of nonlinearities which cannot be addressed sufficiently by the available analytical models. Numerical modeling is a good tool to capture these

complex flows and associated nonlinearities. Thus these techniques are being actively utilized by researchers for advancement in this field.

2.2 Analytical modeling

Rott's [2, 12-15] linear thermoacoustic model lay down the formulation for the analytical solution of acoustic wave equation applicable to thermoacoustic devices. The model was later on reviewed and expanded by G.W. Swift [6] to include the various effects associated with the flow inside thermoacoustic devices. The solution is based on the assumptions of one dimensional flow with sinusoidal temporal variation, that can be neglected, and ideal working fluid. Further, a first order approximation for all variables except acoustic power and heat flux is assumed to be enough while deriving the relations. The momentum and continuity equations are linearized based on these assumptions and give rise to following two equations.

$$\frac{dp_1}{dx} = -\frac{j\omega\rho_m}{1-f_v} \frac{U_1}{A} \quad (2.1)$$

$$\frac{dU_1}{dx} = -\frac{j\omega}{\rho_m a^2} \left[1 + \frac{(\gamma-1)f_\kappa}{1+\xi} \right] p_1 + \frac{(f_\kappa - f_v)}{(1-f_v)(1-\text{Pr})(1+\xi)} \frac{U_1}{T_m} \frac{dT_m}{dx} \quad (2.2)$$

where

$$\frac{dT_m}{dx} = \frac{\dot{H}_2 - \frac{1}{2} \text{Re} \left[p_1 \tilde{U}_1 \left(1 - \frac{(f_\kappa - \tilde{f}_v)}{(1-f_v)(1+\text{Pr})(1+\xi)} \right) \right]}{\frac{\rho_m C_p |U_1|^2}{2A\omega(1-\text{Pr})|1-f_v|^2} \text{Im} \left[\tilde{f}_v - \frac{(f_\kappa - \tilde{f}_v)(1+\xi f_v/f_\kappa)}{(1+\text{Pr})(1+\xi)} \right] - (AK + A_s K_s)} \quad (2.3)$$

The equations (2-1) and (2-2) could be combined to give a linearized form of the acoustic wave equation given by:

$$[1 + (\gamma - 1)f_k]p_1 + \frac{\gamma\rho_m}{\omega^2} \frac{d}{dx} \left(\frac{1 - f_v}{\rho_m} \frac{dp_1}{dx} \right) - \frac{a^2}{\omega^2} \frac{f_k - f_v}{1 - \text{Pr}} \frac{1}{T_m} \frac{dT_m}{dx} \frac{dp_1}{dx} = 0 \quad (2.4)$$

The equations (2.1) to (2.3) are the foundation of general thermoacoustic theory and could be solved to provide analytical relations for oscillating pressure (p_1), velocity (u_1), temperature (T_1), acoustic power ($\dot{H}$) and energy flux ($\dot{W}$). The oscillating velocity (u_1) is given by relation:

$$u_1 = \frac{i}{\omega\rho_m} [1 - h_v(y, z)] \frac{\partial p_1}{\partial x} \quad (2.5)$$

The oscillating temperature (T_1) and its spaced averaged value ($\langle T_1 \rangle$) are given by:

$$T_1 = \frac{1}{\rho_m C_p} (1 - h_k) p_1 - \frac{1}{i\omega A} \frac{dT_m}{dx} \frac{(1 - h_k) - \text{Pr}(1 - h_v)}{(1 - f_v)(1 - \text{Pr})} U_1 \quad (2.6)$$

$$\langle T_1 \rangle = \frac{1}{\rho_m C_p} (1 - f_k) p_1 - \frac{1}{i\omega A} \frac{dT_m}{dx} \frac{(1 - f_k) - \text{Pr}(1 - f_v)}{(1 - f_v)(1 - \text{Pr})} U_1 \quad (2.7)$$

where h_k , h_v , f_k and f_v are defined as

$$h_{k,v} = \frac{\cosh[(1 + i)y/\delta_{k,v}]}{\cosh[(1 + i)y_0/\delta_{k,v}]} \quad (2.8)$$

$$f_{k,v} = \frac{\tanh[(1 + i)y/\delta_{k,v}]}{[(1 + i)y_0/\delta_{k,v}]} \quad (2.9)$$

Complete set of equations are capable of demonstrating the thermoacoustic stacks and can also be applied to other components with little modifications. Swift has contributed the most by introducing the relations accounting for porous media [16], local losses, turbulence, entrance and joining effects [17]. Another milestone achieved in the domain of analytical modeling is by Arnott [18]. He provided the general formulae for stacks of various geometries and linked thermoacoustics to the physics of porous media. Complete

details of analytical modeling of thermoacoustic devices can be found in the book written by Swift [3].

First order linear analytical model, although limited to low amplitude flows with steady state conditions, is still being utilized for design, validation and optimization of thermoacoustic devices especially in the linear regime.

Poignand G. et al. [19] have utilized analytical relations based on linear thermoacoustic theory to develop independent relations for optimal velocity and phase shift in a half wave length SWTAR. It has been concluded that the acoustic characteristics of optimal design is different than usual half wavelength design. A compact SWTAR has been developed using the developed relationships to create an optimal acoustic field inside it. A systematic algorithm has been devised by Babaei H. & K. Siddique [20] for performance prediction and optimization of Thermoacoustically Driven Thermoacoustic Refrigerator (TADTAR). The algorithm is based on analytical formulation of linear thermoacoustics and is specific to TADTAR but can also be used for individual Thermoacoustic Refrigerators (TAR) and Thermoacoustic Engines (TAE). Analytical model has also been used by Lotton P. et al. [21] to find solution for transient temperature profile inside stack of SWTAR. The solution is built upon classical analytical theory and is based on extraction of normal modes using eigenvalue problem. The optimal solution for the design of stack of SWTAR has been achieved using constructional theory [22]. Expressions for optimal number of plates and optimal plate spacing have been derived with an approximation of short stack. Analytical formulation has been used by Tasnim S.H. et al. [23] to gauge the performance and characterize the major sources of loss inside the stack of porous medium using the concept of entropy generation. The analysis is based on compressible, transient flow inside porous

parallel plate stack with isotropic properties. Alahmer et al. [24] has made use of classical linear theory to design and construct a SWTAR whose performance is later on simulated using Solidworks.

2.3 Numerical modeling

Numerical modeling techniques employed for the performance evaluation of SWTAD can be broadly classified as linear and non-linear thermoacoustic modeling. Further classification of these two categories are shown in Figure 2.1. This section provides a detailed review of all these categories. Detailed summary of various geometric and simulation parameters examined by researchers in numerical modeling of SWTADs has been given in Table A-1 and Table A-2 in appendix A. In section 2.3.1, numerical modeling based on linear thermoacoustic theory is reviewed and section 2.3.2 covers a detailed review of two and three dimensional CFD models employed for performance evaluation. Sections 2.3.3 to 2.3.9 cover different aspects of two dimensional CFD models i.e. configurations, capabilities, tools, application areas and validation schemes. Challenges associated with the CFD modeling of SWTAD are discussed in section 2.3.10.

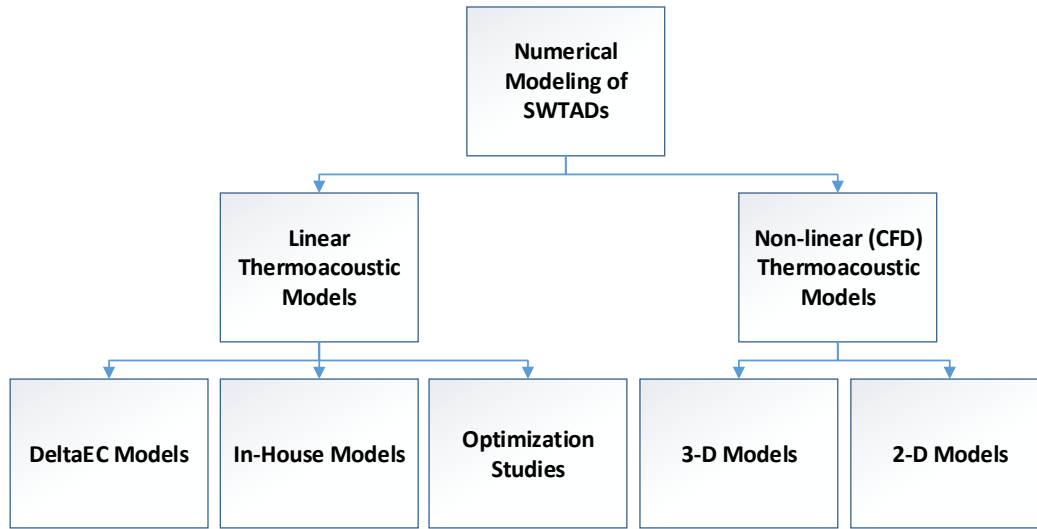


Figure 2.1: Classification of numerical modeling work carried out for SWTADs

2.3.1 Modeling based on linear thermoacoustics

Numerical modeling based on linear thermoacoustic theory is one dimensional (1-D) and often work in the frequency domain thus simulating the steady state conditions. Almost all of the 1-D numerical simulations have been carried out using DeltaEC software which was developed by Ward and Swift [25]. It is considered as benchmark for the design & optimization of low amplitude devices and principally used for validation of numerical simulations or empirical correlations. Nonlinear effects such as boundary layer turbulence, acoustic streaming and wave steepening are neglected by DeltaEC. Most of current literature still use the software results to make some comparison with the numerical and experimental results.

2.3.1.1 Modeling using DeltaEC

Tijani et.al [26] have utilized DeltaEC to design SWTAR driven by the goals of meeting the required lower temperature and cooling load. The device performance is evaluated and compared for different stack center positions, with same resonance frequency in terms of relative coefficient of performance (COPR) and cold heat exchanger temperature. Results from DeltaEC are utilized to optimize the geometric parameters of inertance segments of a Heat Driven Thermoacoustic Refrigerator (HDTAR) [27]. The inertance is modeled using AC electrical element analogy and performance is gauged with respect to throat radius for various output parameters. The evaluation is done for various working fluid properties. A two stage Standing Wave Thermoacoustic Electricity Generator (SWTAEG) has been designed and its performance is evaluated by A. Piccolo [28] using DeltaEC software. Target output power of 100W is used to calculate frequency, pressure amplitude difference, volume flow rate and electroacoustic efficiency for a variable load resistance. Detailed

parametric studies using DeltaEC have been carried out by Alamir et.al. [29, 30] to investigate the performance of loud speaker driven heat pump and SWTAR. The effect of operating conditions and stack geometrical parameters on the performance have been analyzed. Optimized parameters producing highest COP and temperature difference have been proposed based on the study. A novel design of Traveling Wave Thermoacoustic Cooler (TWTAC) driven by Standing Wave Thermoacoustic Engine (SWTAE) is designed and optimized using DeltaEC [31]. The device is intended for fulfilling medical storage needs in rural areas and its performance is monitored using overall cooling efficiency parameter. Optimal parameters of the device are explored using fourth Runga-Kutta scheme in the software.

DeltaEC has been used to design small capacity SWTAR capable of producing 10W cooling power by Kajurek et al. [32]. The effect of drive ratio, for variation in cold heat exchanger temperature, on COP and COPR of proposed device is also evaluated.

2.3.1.2 Modeling using in-built codes

Numerical studies conducted for SWTAD are not limited to use of DeltaEC for simulation and various researchers have developed different numerical methods based on 1-D linear thermoacoustics. Single step Euler finite difference scheme has been employed by Raspet R. et.al [33] to simulate the performance of thermoacoustic stacks of SWTAD. The developed scheme is not limited to standing wave devices and can be used for either traveling or standing wave, engine or refrigerator but with an approximation that stack length is very short compared to acoustic wavelength. Jebali F. et al. [34] have come up with a computational model for performance simulation of SWTAR that make use of electrical network analogy. The proposed model is 1-D and make use of transfer matrix for

performance prediction. Numerical scheme is developed and solved numerically [35], using Runge Kutta method, for simulating temperature profile inside a 1-D stack. The temperature field inside inert gas environment is calculated using two different approaches for enthalpy i.e. enthalpy is conserved and constant enthalpy by making its spatial derivative equal to zero.

2.3.1.3 Numerical optimization studies

B.L. Minner et al. [36] developed a mathematical tool based on direct search optimization method to optimize the performance of SWTAR in terms of maximum overall efficiency achieved. Results from DeltaEC are used for purpose of optimization. Two different case studies based on experimental Hofler prototype have been optimized to show the benefits gained by systematic optimization carried out by this tool. Modified Genetic algorithm has been proposed by Srikituwan S. et.al [37] to optimize the Relative Coefficient of Performance (COPR) of SWTAR. The modification in terms of soothing method is proposed to resolve the disparity in normal genetic algorithm to solve the two point boundary value problem. The proposed model is based on 1-D linear theory and can simultaneously solve the governing equations and perform optimization. Work carried out by Paek et al. [38], focus on the development of SWTAR with optimized operating temperature ranges. The optimization routine is based on non-linear function minimization using successive multi-dimensional golden section search and build upon the DeltaEC results. The program is written in Matlab and optimize the results in terms of COPR and drive efficiency. Response Surface Methodology (RSM) has been utilized by Hariharan et al. [39] to obtain optimized values of stack position and length, frequency and plate spacing. DeltaEC is used for generation of results and optimization is carried out for

SWTAR. Analysis of Variance (ANOVA) analysis has been carried out to test the significance of proposed model. Zolpakar et.al [40-42] in a series of publications have utilized Multi Objective Genetic Algorithm (MOGA) to optimize the performance of SWTAR in terms of maximum COP and minimum acoustic power required across the stack. Four parameters i.e. stack center position, drive ratio, blockage ratio and stack length are all optimized simultaneously. The results are compared with experimental prototype of Tijani et al. [26] and have shown to increase the performance of device by 30.5% in terms of cooling load and 5.4% in terms of COP. Tartibu et.al [43] have carried out multi-objective Lexicographic optimization using GAMS software to reach an optimal stack geometric parameters for SWTAR. Global optima has been achieved for conflicting objectives like maximum COPR, cooling power and minimum acoustic power using the said technique.

2.3.2 Nonlinear CFD modeling

Owing to lesser efficiencies and small COPs of SWTAD research focus has shifted towards the development of high amplitude and high frequency devices. Nonlinear effects such as acoustic streaming, asymmetrical temperature distribution, phase shifting, flow transition etc. become more significant for these high power density devices. Linear thermoacoustic model doesn't remain valid and significant deviation has been reported. Two dimensional (2-D) and three dimensional (3-D) CFD numerical models based on fully compressible Navier-Stokes, continuity and energy equation can accurately capture these nonlinear phenomenon. They are governed by unsteady fully compressible Navier-Stokes, continuity and energy equation given by:

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \mu \nabla^2 \mathbf{u} + \left(\xi + \frac{\mu}{3} \right) \nabla (\nabla \cdot \mathbf{u}) \quad (2.10)$$

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (2.11)$$

and

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho \mathbf{u}^2 + \rho \varepsilon \right) = -\nabla \cdot [\rho \mathbf{u} \left(\frac{1}{2} \mathbf{u}^2 + h \right) - K \nabla T - \mathbf{u} \cdot \boldsymbol{\mu}] \quad (2.12)$$

where μ and ξ are shear and bulk viscosity, respectively; ε and h are internal energy and enthalpy per unit mass, respectively; K is the gas's thermal conductivity; and $\boldsymbol{\mu}$ is the viscous stress tensor with components given by:

$$\mu_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left(\xi - \frac{2}{3} \mu \right) \delta_{ij} \frac{\partial u_k}{\partial x_k} \quad (2.13)$$

The governing equation for heat transfer across the solid domain is given by:

$$\rho C_p \frac{\partial T}{\partial t} - \nabla \cdot (K \nabla T) = S \quad (2.14)$$

where K , T , ρ & C_p are the thermal conductivity, temperature, density and specific heat at pressure of solid while S is the heat source term.

These tools are also useful as they can provide a detailed insight in to physical mechanisms associated with losses which is impossible through the use of analytical or experimental methods. Thus they have been utilized extensively in recent past for design, optimization and performance prediction of SWTADs.

2.3.2.1 Three dimensional CFD modeling

CFD modeling concerned with the simulation of SWTAD is limited to 2-D models and to date, only two research publications have utilized 3-D models to address the issue of

oscillatory flow inside these devices. A 3-D study of a single stack unit of high amplitude SWTAR has been carried out by Ahmed [44]. ANSYS Fluent has been used to model a square stack channel with helium as working fluid. Only one unit cell with no heat exchangers and acoustically matching boundary conditions at each end of computational domain have been modeled for simplicity and Figure 2.2(a) shows a meshed domain chosen for analysis. Laminar flow is considered and it has been reported that flow inside stack could be considered as 1-D even at drive ratios of 2% despite the previous results which suggest deviation from linear theory even at drive ratio $\geq 1\%$. Another work simulating the thermoacoustic effects in 3-D has been carried out by Olusegun et al. [45]. They have investigated the effect of minor losses on the heat transfer and pressure drop characteristics of oscillatory flows inside the tubular heat exchangers. A 3D-CFD model with symmetric arrangement of cold, hot and cold heat exchangers has been developed and stack has been excluded for simplicity, shown in Figure 2.2(b).

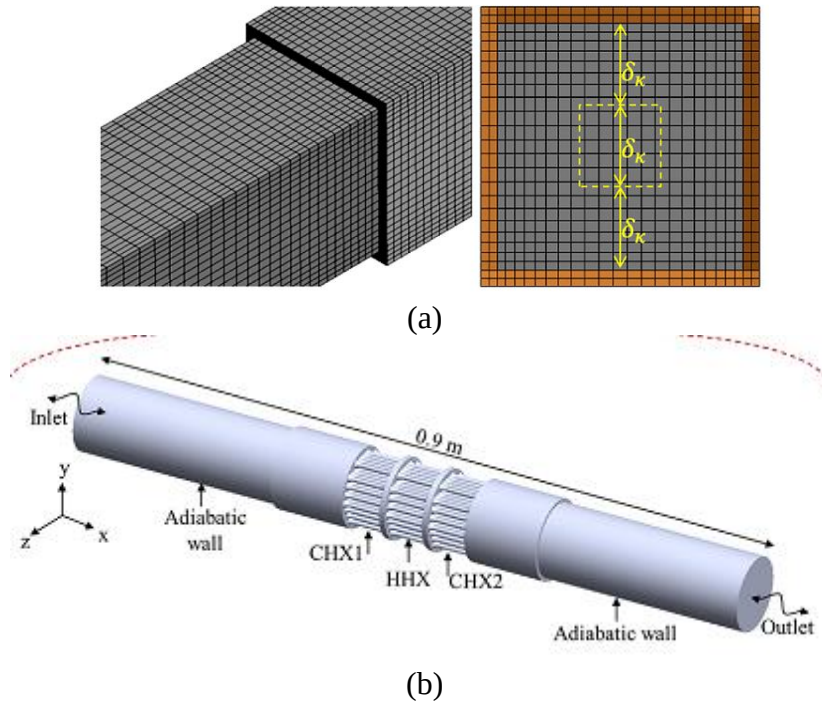


Figure 2.2: Computational domain of 3-D studies conducted for SWTADs (a) 3-D square stack [44] (b) 3-D heat exchanger setup [45]

ANSYS Fluent software has been used and k- SST turbulence model has been utilized for turbulence modeling. The simulation model is then further extended to incorporate the aerodynamic edge shape of tubes for improvement in pressure drop losses.

2.3.2.2 Two dimensional CFD modeling

CFD simulations are computationally very intensive due to complexity of thermoacoustic flows and disparity of the length scales involved. This combined with axial symmetry and the flow dominance in a 2-D plane inside SWTADs has forced the research community to limit the CFD modeling to 2-D. It can broadly be classified into three different categories in terms of computational domain chosen for simulation i.e. Representative Element Volume (REV) (a) single stack models (b) multiple stack models (c) full stack models.

A further segregation can be done in terms of components chosen inside the computational domain for simulation i.e. models with (a) stack only (b) stack and heat exchangers (c) heat exchangers only. A graphical view of this classification has been presented in Figure 2.3. Single stack models can greatly reduce the computational cost as the performance is evaluated inside a very short domain but are unable to capture the effects such as flow asymmetries. They are ideal for modeling refrigeration application where time required to reach steady state is very long and prime objective is to improve the overall performance in terms of COP and lowest temperature achieved. Full stack models can accurately predict the flow asymmetries and the effect of neighboring stacks on each other but are computationally expensive. Thus they are mostly used for engines and TADTAR where detailed insight into the loss mechanisms resulting in the reduction of power and pressure amplitude achieved is more important. Multiple stack models have been proposed to provide a compromise between benefits of single and full stack models. Figure 2.4(a), (b)

and (c) show one example of a single, multiple and full stack models adopted by different researchers.

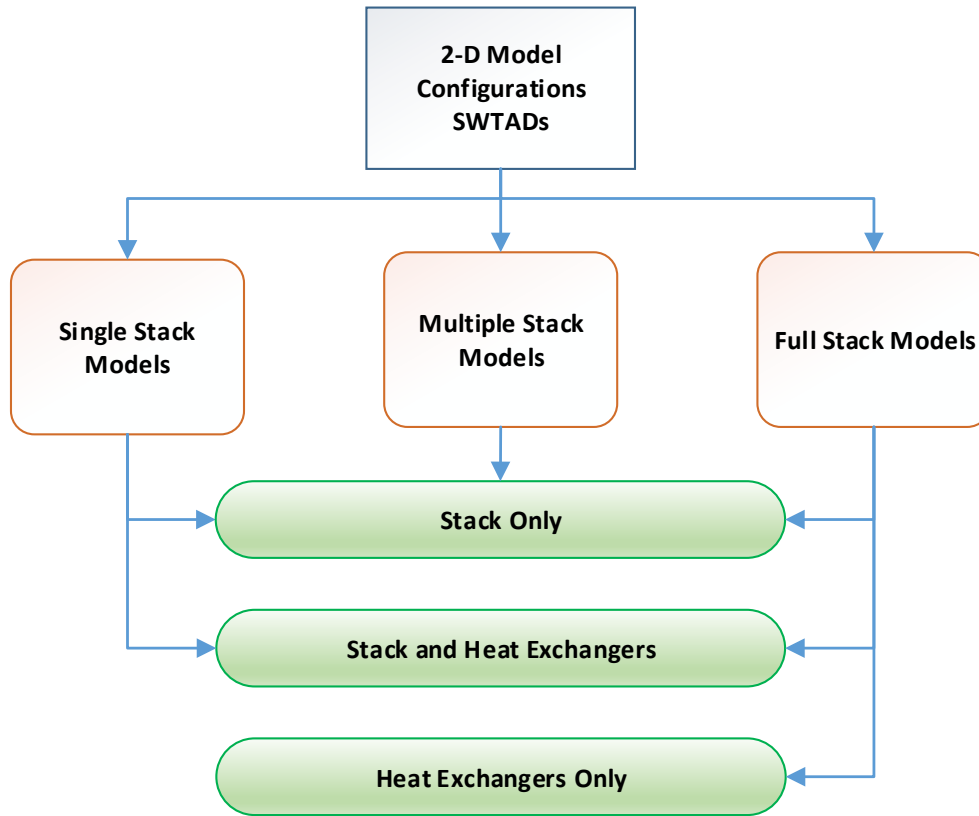


Figure 2.3: Different configurations of 2-D CFD models of SWTADs

Stack only models are mostly used for engine or engine driven applications where the temperature gradient is input and thus heat exchanger performance is not of much importance. Stack with heat exchangers models are often used for refrigeration or heat pumping applications where different parameters of heat exchangers i.e. position, length, thickness etc. can have significant effect on the overall temperature drop and COP. Models having heat exchangers only have specifically been used to investigate the heat transfer characteristics under oscillatory flows in these devices.

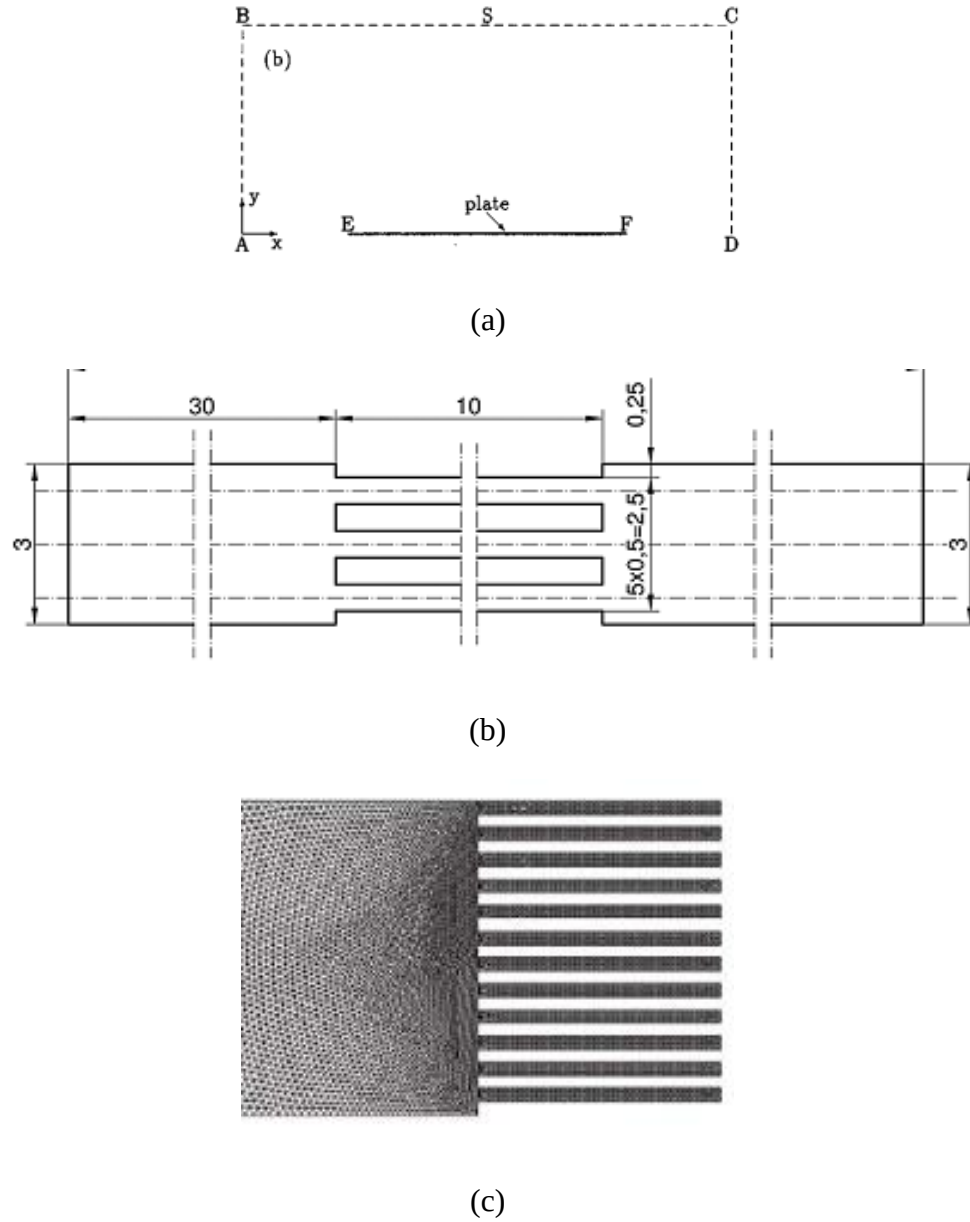


Figure 2.4: Different computational domains adopted for simulation of SWTAD (a) Single stack [46] (b) Multiple stack [47] (c) Full stack [48]

2.3.3 Single stack CFD models

Among single stack models researchers have opted for different variants while selecting the computational domain. Some of these variants have been shown in Figure 2.5 where Figure 2.5(a) shows a computational domain enclosing stack plate only, Figure 2.5(b) has

domain enclosing stack and one end of resonator while Figure 2.5(c) shows domain covering whole space of a single stack channel.

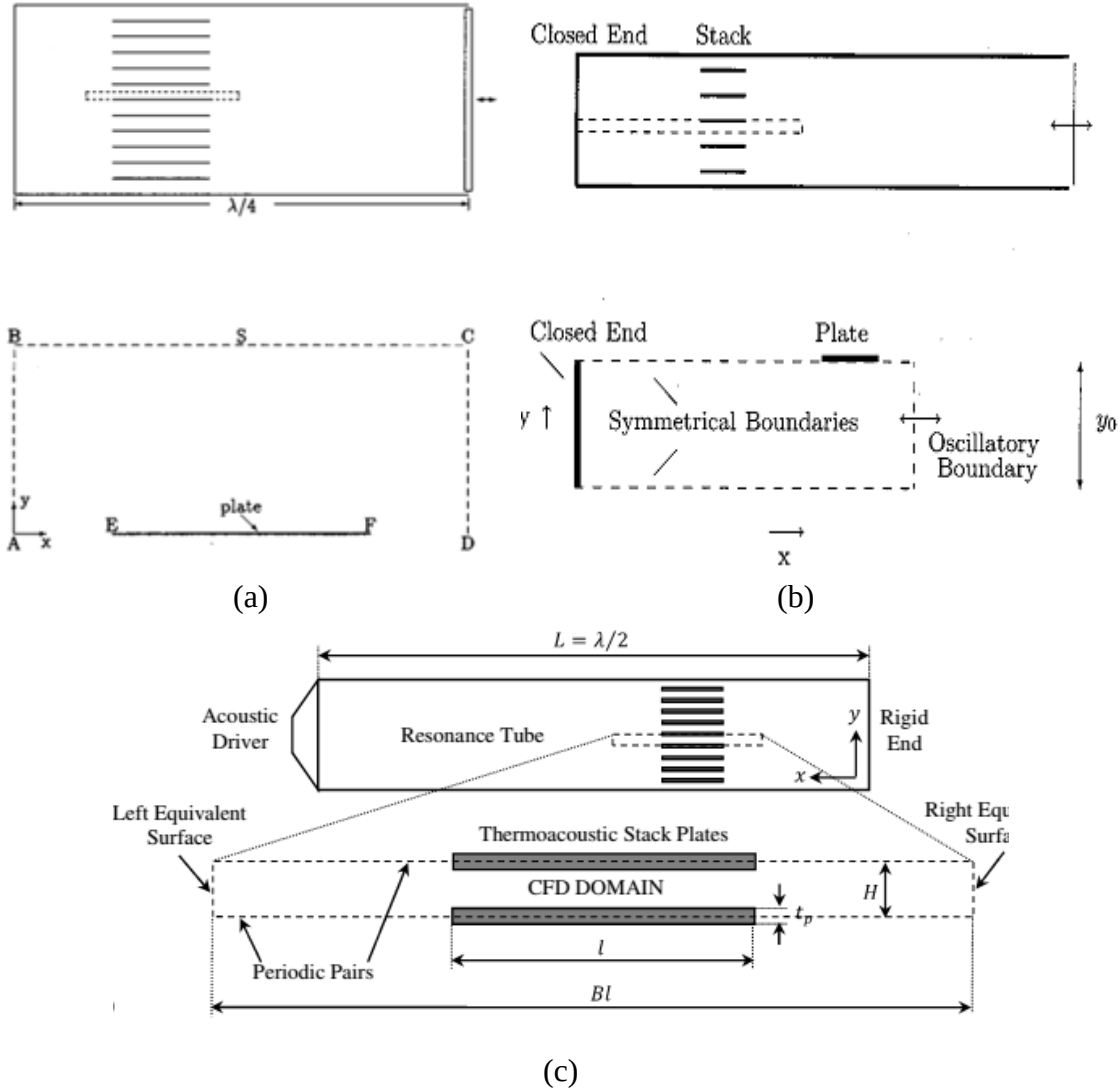


Figure 2.5: Different variants of single stack computational domains (a) Cao N. [46] (b) Ishikawa H. [49] (c) Worlikar [50] & Abd EI Rahman [51]

2.3.3.1 Single stack models without heat exchangers

Cao N. et.al [46] were the first one to introduce the concept of single stack known as thermoacoustic couple to numerically simulate the heat and energy fluxes at ends of stack of SWTAD. The stack in their model was constructed as short (compared to acoustic wavelength) parallel plates with isothermal conditions and computational domain consists

of stack with no heat exchangers. The chosen domain was reduced to region near the stack to save the computational resources. Velocity boundary conditions imposing the desired standing wave inside the domain were applied at transverse boundaries. Worlikar et al. [50, 52] extended the work of Cao. et al. [46] by considering the low Mach no. approximation for fully compressible Navier stokes equations and also solved for vorticity equations. Isothermal stacks with no heat exchangers were considered in their work and were able to capture the formation of vortices at the edges of the stack plate. Ishikawa and Mee [49] extended the thermoacoustic couple analysis proposed by Coa et al. [46] to include long stack plates and analyzed the effect of plate spacing using PHOENICS finite volume software. It was shown that the heat transfer process take place at the ends of stack within one particle displacement length. The computational domain proposed by them include both the stack plate of zero thickness and closed end. Isothermal stack plate without heat exchangers have been modeled for simulation. Marx D. and Blanc Benon [53-55] utilized computational domain enclosing only a single stack plate with a closed end to simulate SWTAR. They were able to compute mean velocity field above stack plate and observed the vortical regions at plate ends. Temperature difference across the stack extremities has also been calculated numerically. Further, it has been identified that temperature distortion is observed over the entire plate length if its length is smaller than or equal to four particle displacements. Zoontjens L. et al. [56] for the first time utilized Fluent software to model a stack of finite thickness and thermal conductivity to simulate the energy and flow fields inside a single stack unit of SWTAR without heat exchangers. Performance is investigated for symmetrical flow conditions and wide range of drive ratios (1.7-8.5%) with frequency of 100Hz. First order discretization scheme has been utilized which affected the accuracy

of the obtained results. Another instance of modeling using single stack without any heat exchangers is by Remiorz et.al [57] where CFX software is employed to simulate flow inside open ended Rijke tube. STAR-CD software has been used by Tasnim and Fraser [58] to visualize the flow and thermal fields inside a single stack of finite thickness of SWTAR. Visualization is done for laminar flow at very low drive ratio (0.7%) and heat exchangers are not included in the analysis. Tasnim et al. [59] have utilized similar computational domain as the one proposed by Ishikawa & Mee [49] i.e. single isothermal stack plate with zero thickness. Performance of SWTAR in terms of COP, entropy generation and acoustic power is evaluated by varying Prandtl no., drive ratio, stack spacing and mean pressure. Abd El Rahman & Abel Rahman [51] has simulated the performance of SWTAR using FLUENT and computational domain similar to Worlikar [50] has been chosen. However, left and right hand boundaries are modeled as moving rigid wall using layer dynamic meshing. The transient simulation was able to successfully demonstrate the relationship of acoustic power dissipation and non-linear streaming intensity with increasing drive ratio. Effect of variation of computational domain length on the performance characteristics of SWTAR has been evaluated by Mergen S. et al. [60] . The computational domain is non-dimensionalized with respect to wavelength. It has been shown that the pressure distribution applied at boundaries follow the theoretical standing wave behaviour more closely as the domain length is increased. A minimum value of length/wavelength ratio has also been reported in order to obtain reliable simulation results for these refrigeration devices. This study excludes heat exchangers and only stack is modeled for analysis.

2.3.3.2 Single stack models with heat exchangers

Hireche O. et al. [61] has developed a numerical code that simulates the SWTAE performance by coupling 2-D viscous compressible model inside the stack and heat exchangers with 1-D linear acoustic model inside the resonator. Developed code is based on low Mach flows and uses multiple scale technique and performs much faster than traditional fully compressible model utilized throughout the device. A single stack unit with heat exchangers has been simulated for performance comparison. Namdar et al. [62] have utilized OpenFoam software to simulate SWTAR performance against weak and large amplitudes. Computational domain consist of single parallel stack plate with heat exchanger encompassing full resonator length. It is concluded that at large amplitudes, position of cold heat exchanger becomes critical as the pressure displacement node is shifted due to presence of nonlinearities. ANSYS CFX has been used by Ramdane et.al [63] to simulate the effect of heat exchanger's length on the performance of SWTAE. The computational domain consists of single cell of stack and heat exchangers. Both the acoustic power and specific work have been observed to increase with an increase in length. Effect of heat exchanger blockage ratio on the cooling performance of SWTAR has been investigated by Nehar Belaid [64]. Helium filled resonator and single stack model with heat exchangers has been utilized in the study. Low Mach no. flow is simulated and results are compared with DeltaEC results and a difference of 10% is reported. The temperature of cold side heat exchanger is reported to increase with the increase in the blockage ratio thus negatively effecting the cooling performance. Rahpiema R. [65] has employed COMSOL to analyze the effect of various geometrical and thermo-physical parameters of stack of SWTAR on three key performance parameters i.e. developed temperature gradient,

lowest temperature achieved and COP. Single stack channel with heat exchangers is modeled. Based on their investigation, thickness of stack plates should be minimal, material with lowest thermal conductivity is preferred and optimal value for length and spacing of stacks is required.

Few studies have targeted the performance optimization using 2-D single stack models. Finite volume based numerical method has been developed by Han Bing et.al [66] and applied on a single stack of parallel plate including the heat exchangers. Performance of SWTAR at large amplitudes has been simulated and parametric optimization has been carried out using the developed method. Single stack unit with heat exchangers of a SWTAR has been utilized by Piccolo A. [67] to develop an entropy based computational model. Optimal length for heat exchangers has been reported to be between one and two particle displacement lengths using the results of simulation.

2.3.4 Full stack CFD models

2.3.4.1 Full stack model without heat exchangers

Florian Zink et al. [48, 68] proposed a CFD based methodology that successfully simulated the performance of SWTAE. The developed 2-D model utilized a full stack of parallel plates without heat exchangers. The proposed methodology involves the initialization of small velocity distribution inside the stack by applying a temperature gradient across stack and using a small pressure gradient across the resonator in a steady state. Then these initialized settings are transferred to transient analysis for simulation of SWTAE. The simulation is continued until pressure amplification has reached stable Limit Cycle Oscillations (LCO). The same team in another work [69] has applied the same methodology for the performance simulation of TADTAR. Heat exchangers have been

excluded in this study as well. Since then, the same model and methodology has been adopted by various other researchers to validate and simulate their work, both for thermoacoustic engines and refrigerators. A high frequency SWTAE has been simulated by Yu Guoyao et al. [70] using Fluent 6.1. Two different methods i.e. specific heating load & applied surface temperature have been used to impose temperature gradient at hot side. Parallel plate stack is replaced with spiral one for computational purpose and heat exchangers are excluded. Based on the results, it is suggested that porous media model is insufficient for stack modeling and instead a fluid-solid model with finite heat exchange should be used. Skaria et.al. [71] have simulated the performance of twin Standing Wave Thermoacoustic Prime Mover (SWTAPM) and TADTAR using a full stack model without heat exchangers. The effect of average pressure and working fluid has been analyzed and it has been found that their selection play a critical role in determining the performance through numerical simulations. For TADTAR helium gas is considered only and thus effect of average pressure alone is studied. CFD simulations, based on methodology of Zink F., have been utilized by Badi A.P. et al. [72] to study the non-linear interaction of unsteady heat releases and acoustical disturbances in a multistage SWTAE. Results of two different configurations, i.e. series and parallel, of these engines are simulated and validated by comparing them to earlier published data. Serial configuration showed promising results in comparison to parallel one in terms of achieved pressure amplitudes and heat input required. Hariharan et.al [73, 74] have investigated the effect of resonator length, operating pressure, noble gases and their binary mixtures on performance of twin SWTAPM. The performance is measured in terms of operating frequency and pressure amplitude achieved. Methodology of Zink. F. et al. [48] has again been utilized by Hussain N.M. [75] to carry

out a sensitivity study on SWTAE. The study analyzes the effect of applied temperature gradient and resonator length on the developed frequency and pressure amplitude for standing wave thermoacoustic engine. Fatimah et al. [76] have numerically investigated the turbulence effects of vortex structures created at stack ends of a SWTAR using a 2D full stack model. Heat exchangers are not modeled in this analysis. It has been proposed that turbulence models must be incorporated for efficient modeling of oscillatory flows and turbulence behavior has been found to take effect even at drive ratios of less than 1%. Further, velocity profile inside stack is affected by returning vortices and effect grows stronger with higher drive ratios. Volatility of vortex structure fades away as frequency is increased.

2.3.4.2 Full stack model with heat exchangers

Kuzuu K. et al. [77] has modeled the flow inside the full stack with heat exchangers of a SWTAE by utilizing 2D CFD model. The developed model captured the non-linearities inside the stack and resulted in the formation of asymmetrical temperature field near the stack ends. The CFD results, when compared with those obtained from linear theory model, reported 9% less power output. Later on, based on their findings, a modification to existing linear model has been proposed to improve its efficiency. Same authors in another work [78] have analyzed the effect of asymmetrical temperature formulation on the heat transfer characteristics of heat exchangers inside the SWTAE. The CFD results when compared with the results obtained from numerical heat transfer analysis model showed considerable difference as the heat transfer method was unable to capture the non-linear behaviour. Novel converging design of stack has been proposed by Liu et al. [79] and two different variants have been simulated through 2-D CFD using full stack model with heat

exchangers. Performance parameters are gauged in terms of ratio between the plate spacing and thermal penetration depth at cold side of heat exchangers. Converging stack has shown promising results over parallel one in terms of material cost.

2.3.4.3 Full stack model with heat exchangers only

Mohd Saat F. [80] has proposed a 2-D CFD model in ANSYS Fluent that captures the physics of flow across two parallel plate heat exchangers in a SWTAD. Stack has been excluded and simulated heat exchanger model spans over full resonator diameter. A symmetrical temperature and velocity profiles have been seen due to nonlinear effect of temperature driven oscillatory flow and thus net streaming flow is visualized. In addition, model has also been utilized to study gravity and orientation effects on flow and heat transfer characteristics. The work has been further extended by Mohd. Saat & Jaworski [81] to analyze the flow fields inside heat exchangers under the effect of various turbulence models. Various turbulence models i.e. $k-\epsilon$, $k-\omega$ and $k-\omega$ SST have been investigated and it is concluded that turbulence starts very early compared to critical Stokes Reynold no. value. It has been found that $k-\omega$ SST best depicts the oscillatory flows when compared with experimental results.

2.3.5 Multiple stack CFD models

Multiple stack models have been utilized in research so far to simulate the performance of SWTAE only and simulations using these models are limited to stack component alone. Concept developed by Florian [48] has been utilized by both Rulik et al. [82] and Nowak I. et al. [83] to simulate the flow fields inside quarter wavelength SWTAE. Parallel plate stack without any heat exchangers has been simulated in both studies. Two different CFD models, a simplified one simulating only one stack with symmetry boundary conditions

and detailed one simulating multiple stacks with no-slip boundary conditions, have been investigated. In both studies the velocity and pressure amplitudes calculated by detailed model are less than the simplified one and are attributed to non-linear vortices at stack end captured by this model. Nowak I. et al. [83] also developed an analytical model and compared its results with the simplified one which were found to be in close agreement. However simplified model showed non-linear velocity distribution inside stack whereas it is assumed linear in analytical one. M. Ghazali N.& Lew K. Fa [84] also have performed similar study and obtained similar results. But the comparison has been made between single and two stack models and have used their own numerical code for simulation. Lin J. et al. [85] has developed a high fidelity axisymmetric numerical model, taking advantage of axisymmetric conditions of stack geometry and time domain impedance boundary conditions, that can simulate the performance of Standing Wave Thermoacoustic Piezoelectric Engine (SWTAPE). The engine performance is simulated with and without the acoustic power extraction and from startup phase to limit cycle oscillation. Start-up phase is simulated with no acoustic power absorption. The Rozosinki et al. [47] have published an article where the results of Nowak I. [83] have been extended to include two different detail models i.e. two and three stack plates respectively. Turbulent $k-\omega$ SST model is utilized instead of $k-\epsilon$ model used in the previous work. Contours of non-linear vortices captured by the detailed models have been plotted for different phases of thermoacoustic cycle. It has been concluded that by applying the simplifications such as symmetry boundary conditions affect the solution obtained.

SWTAE stack has been optimized by Zink F. et.al [86] considering thermal losses to surrounding. 2-D axisymmetric stack has been modeled using COMSOL and Matlab and

geometry of stack has been optimized. Multi-objective Nelder Mead Simplex method has been utilized for optimization with weights assigned to all four objectives. Trapp A.C. et al. [87] have formulated a multi-objective optimization problem in 2-D for SWTAE by taking advantage of symmetric stack conditions. The program takes into account all three forms of heat flows inside the stack i.e. conductive, convective and radiative and attempts to reduce heat losses and maximize work output simultaneously via implementation in Matlab.

2.3.6 CFD modeling capabilities

2.3.6.1 Flow modeling

Flow inside SWTADs are generally categorized as laminar or turbulent based on the value of critical Reynold no. (Re_{cr}) proposed by Ohmi et al. [88] for oscillatory flows inside the pipe. Value of $Re_{cr} = 400$ has been given for the transition of flow from laminar to turbulent and it agrees well with the critical value suggested by Merkli & Thomann [89]. However, recent investigations carried out by Mohd. Saat et al. [76] states that turbulence effects inside thermoacoustic devices start taking effect even at values as low as $Re_{cr} = 70-100$ and it must be incorporated into the analysis for accurate predictions. Based on the above review of simulation studies of SWTADs a summary of flow modeling techniques adopted is as follows:

Laminar flow conditions are modeled in all studies involving the use of single stack unit for simulation and turbulent flow conditions are adopted in research with the use of multiple or full stack models. Turbulence models utilized for flow simulations are k- ϵ , k- ω and k- ω SST. Further, the performance simulation of SWTAR can be broadly associated with laminar flow conditions while turbulent models are limited to modeling of SWTAE,

SWTAPM and TADTAR. Lin J. et al. [85] are the only one who have utilized laminar flow conditions for the simulation of SWTAPE since the Stokes Reynold no. in their model is less than 100. Mohd. Saat et al. [76] work specifically focuses on the effects of turbulence on the performance of SWTAR and thus it is the only research that utilized turbulence model for performance prediction of SWTAR.

2.3.6.2 Heat Transfer Modeling

Different heat transfer models have been utilized over the years to analyze the heat transfer characteristics of both stack and heat exchangers. Heat transfer through the horizontal walls of stack have been focus of all studies and only conductive heat transfer is considered. Many of studies have applied isothermal conditions on the stack to calculate the energy fluxes and heat transfer across it [45, 46, 49, 53, 55, 60]. Apart from using isothermal conditions some researchers have applied adiabatic conditions on stacks for performance evaluation [50, 53, 87]. The concept of applying a fixed sinusoidal temperature profile with constant heat transfer coefficient on the horizontal walls of stack was first introduced by Zink et al. [48, 68, 69] and later on the same concept has been adopted by many other researchers to simulate the performance of the thermoacoustic devices [47, 72-75]. Rest of studies have used conjugate heat transfer model to evaluate the heat transfer characteristics between the solid stack wall and fluid [56, 64, 65].

In addition, a common technique to model the ambient heat exchanger especially in SWTAR application is by fixing the solid temperature to a fixed value, usually the ambient temperature i.e. Piccolo [67], Nehar Belaid et al. [64], Rahpeima et al. [65], Yu Guoyao et al. [70]. This is based on the assumption that ambient heat exchanger is fully capable of transferring all the applied heating load. .

2.3.6.3 Material modeling

Materials of stack and heat exchangers, in all simulation studies of SWTADs, have been modeled with constant material and thermal properties. Stack materials in the order of their usage are Stainless Steel, Glass, Cordierite, Kapton, Mylar and PMMA. Ke Han-Bing et al. [66] have used anisotropic material properties for stack using different values of thermal conductivity in x and y axes. Most commonly used material for heat exchangers is Copper and some studies have used Aluminum as well.

Working fluid is modeled as compressible ideal gas with density given by the equation of state and has temperature dependent dynamic viscosity, thermal conductivity and usually given by power law as proposed by Cao N. et al. [46] and Ishikawa & Mee [49]. The most commonly used working fluids are Air and He. However Nitrogen, Xeon and Argon have also been used in various studies.

2.3.6.4 Thermoacoustic modeling

With regard to pressure amplitude SWTADs can be broadly classified in to two main types i.e. “high or large Amplitude” devices and “low or weak amplitude” devices. The amplitude term is frequently represented in terms of drive ratio which is the ratio of pressure ratio and mean pressure of device. Devices where the dynamic pressure is high and Drive Ratio (DR) becomes $> 1\%$ are considered as high amplitude or high DR devices [62]. At such conditions, nonlinear effects have a considerable impact on the performance of the device. When the DR. $\leq 1\%$ then the device is considered as low amplitude device. Examples of CFD studies of SWTADs involving low amplitude are [46, 50, 52, 53]. Devices that have modeled high amplitude flows are [48, 49, 54-56, 68, 69, 86, 87]

Power density of thermoacoustic devices is linearly related to resonance frequency so ideally a higher frequency is desired. However it is also inversely proportional to square of thermal penetration depth resulting in very small stack plate spacing. So a compromise has to be made between the desired power and stack manufacturing constraints. As a results researchers worldwide have used wide range of frequencies for simulation of SWTADs i.e. 13 -860 Hz. A division can be made as low frequency and high frequency devices if operating frequency is above or below 300Hz. CFD studies with resonance frequency $< 300\text{Hz}$ are [45, 49, 56-62, 65-67, 70, 71, 73, 76-80] and those utilizing frequencies $\geq 300\text{Hz}$ are [44, 48, 51, 53-55, 64, 68, 69, 72, 75, 82-87].

Mean operating pressure (P_m) also has same relationship with the power density and thermal penetration depth as resonance frequency. So a high pressure device is preferred but the primary limitations are resonance tube strength and device manufacturing issues. Keeping these in mind, most of the research studies have used atmospheric pressure (1.0 bar) for performance simulation [44, 47, 48, 55, 58, 59, 62, 63, 66-69, 72, 75-78, 80-87]. However few studies have used 10 kPa (0.1 bar) [49, 56, 60, 65] and researches involving parametric studies have target higher P_m [71, 73]. Operating pressures as high as 10 bar [64, 79] and 40 bar [70] have also been used.

2.3.7 Validation of CFD models

Numerical models utilized in research have been validated in three different ways (a) comparison with analytical results or DelatEC results (b) comparison with other published numerical simualtions (c) Experimental results. The most widely used CFD models for validation of simulations are one developed by Cao N. [46], Ishikawa H. [49] and Zink F. [48]

Results of experimental setup developed by Atchley et al. [90] & Shi et al. [91, 92] have been used multiple times for validation of numerical simulation. Other experimental results that have been used once or twice in the research for validation of numerical results are Hamilton et al. [93], Piccolo A. [94], Hantschx C.C. [95], Saenger R.A. [96], Kamble B. et al. [97], Kim Fa L. [98], Smoker et al. [99], Duthil et al. [100]

2.3.8 Applications targeted in CFD modeling

Main focus of research using CFD modeling is either stack or heat exchangers or both. Primary application areas targeted are SWTAE [45, 47, 48, 61, 63, 68, 70, 72, 75, 77-79, 82, 83, 85-87] and SWTAR [44, 46, 49-52, 55, 56, 58-60, 62, 64-67, 76]. Another application area is evaluation of performance of TADTAR where a thermoacoustic engine is used to drive the thermoacoustic refrigeration unit [69, 71]. Other novel configurations of STWTADs such as twin TAPM [71, 73, 74], serial parallel configurations [72], gravity effects [81], thermoacoustic device with converging stack [79] have also been investigated. Figure 2.6 shows all variants of SWTADs targeted in numerical modeling.

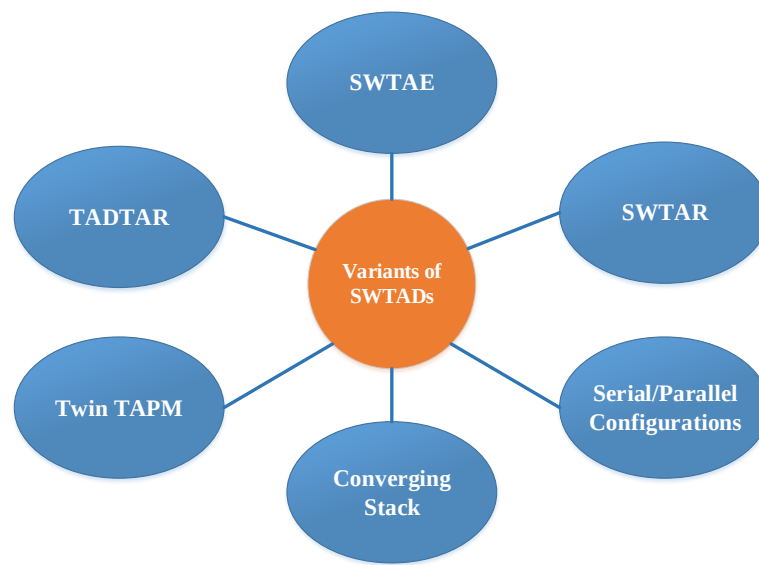


Figure 2.6: Different variants of SWTADs developed and analyzed in numerical modeling.

2.3.9 CFD modeling tools

Various computational codes and commercial CFD modeling tools have been utilized in the simulation of SWTADs and a common feature in almost all the tools is that they are finite volume tools utilizing compressible flow solvers. The most frequently used tool are ANSYS Fluent [44, 45, 48, 51, 56, 68-76, 79-81] and ANSYS CFX [47, 57, 63, 82, 83] . Recent studies have used COMSOL for the performance evaluation [60, 65, 86]. Other commercial tools include PHEONICS [49], STAR-CD [58, 59], SOLA-ICE [46], Le Quere [64], OpenFoam [62], LS-Flow [77, 78], CharlesLES [85].

2.3.10 Challenges in CFD modeling

Thermoacoustic phenomenon inside SWTADs is very complex and involve length scales ranging from order of meter inside the resonator to few millimeters inside the stack. Furthermore, parameters playing a pivotal role in performance estimation of these devices are highly non-linear having a strong dependence on each other thus making the evaluation of coupled non-linear effects of parameters on each other very difficult. Numerical solvers required to evaluate these effects results in high computational cost and time. Typical timesteps reported for the transient simulation are $1e-5$ secs to $2.5e-5$ secs. Thus it is the primary challenge in detailed simulation of these systems and has primarily restricted research effort to be limited to two-dimensions.

Another challenge is the lack of standardization while selecting the simulation parameters for performance evaluations. A wide range of simulation settings and flow/heat transfer models have been employed. In addition, these devices have been design and evaluated for

a broad range of geometric and operating parameters thus making it difficult for naïve researchers to work in this area and also hindering its industrial commercialization.

2.4 Conclusions

A detailed review of numerical modeling work that has been conducted in the field of SWTADs has been made in this chapter. Details regarding the various models utilized, configurations adopted for performance simulations, modeling capabilities, and application areas targeted have been summarized. The most important findings are:

- First order linear analytical model, although limited to low amplitude flows with steady state conditions, is still being utilized for design and optimization of SWTADs especially stack. It has also been used as a tool to either validate or compare the results of developed numerical models.
- Numerical models based on linear thermoacoustic theory are 1-D and can only predict steady state results thus mostly operating in frequency domain. DeltaEC software is considered as benchmark for carrying out these studies. In recent studies, linear thermoacoustic theory has been used to carry out multi-objective optimization of SWTAR using evolutionary and other nonlinear programming methods.
- CFD modeling has proved to be an effective tool in capturing the nonlinearities by simulating detailed physical mechanisms especially inside and around the stack plate extremities. CFD simulations are computationally very intensive due to complexity of thermoacoustic flows and disparity of the length scales involved. This combined with axial symmetry and the flow dominance in a 2-D plane inside SWTADs has forced the research community to limit the CFD modeling to 2-D.

Only two cited literatures, one related to stack and other with heat exchangers, have investigated the performance of SWTADs in 3-D.

- Prime focus of performance evaluation of SWTADs using CFD modeling is stack and region around the stack. Various computational domains have been proposed and can be categorized into three major types (a) single stack models (b) multiple stack models and (c) full stack models. Further segregation can be made in terms of components of SWTAD considered for simulation e.g. (a) stack only (b) stack and heat exchangers (c) heat exchangers only.
- Both laminar and turbulent flow models have been utilized and $k-\omega$, $k-\epsilon$ and $k-\omega$ SST turbulence models have been utilized. Laminar flow modeling can be broadly associated with single stack domains and simulation of SWTARs. Multiple and full stack domains have been simulated using turbulent models and generally covers applications areas like, SWTAE, TADTAR and SWTAPM.
- Stack and heat exchangers are modeled with constant material and thermal properties. Working fluid in all simulation studies is considered as ideal gas with temperature dependent properties where density is calculated by equation of state and viscosity and thermal conductivity using power law.
- Results obtained from analytical, numerical and experimental setups have been used for the validation of proposed simulation models.
- Various applications of SWTADs targeted in the numerical simulations are SWTAR, SWTAE, twin TAPM, TADTAR and studies simulating various configurations i.e. serial/parallel arrangement and horizontal/vertical arrangements for gravity effects.

CHAPTER 3

Sensitivity Study of Standing Wave Thermoacoustic Refrigerator Using Non-Dimensionalized Sensitivity Coefficients

3.1 Introduction

A key strategy to improve performance of thermoacoustic devices, especially for designs leading to physical development, is to have a detailed insight of relationships that exist between various input and output parameters. Various devices i.e. loudspeaker/amplifiers, heating wire etc. and instrumentations i.e. thermocouples, pressure sensor, sound meters etc. are used to gauge the performance of these systems in terms of different input and output variables. Uncertainties in such devices/instrumentation lead to variation in measured value and can significantly alter the final output results if a given parameter/variable, associated with these devices, is highly sensitive to a given input. It is of utmost importance to calculate the sensitivities of key performance parameters to various design variables prior to any experimental development. It will lead to appropriate selection of devices/instrumentation that will have a minimal effect on the final calculated outputs. To date, no such study has been reported that covers this aspect. Most of studies in the literature are parametric studies that are mainly focused on studying the effect of

various parameters on the performance variables i.e. Hussain N.M. et al. [75], Alamir et al. [29, 30], Kajurek et al. [32, 101], Skaria et al. [71], Rahpiema et al. [65].

Sensitivity analysis is not limited to availability of direct experimental information and can also be carried out using appropriate validated model even in design phase [102] as it helps us to isolate single answers to single perturbed parameter which is not possible in experimentation. The objective of this chapter is to carry out sensitivity study using Normalized Sensitivity Coefficient (NSC) method. The method can be used to identify important model parameters affecting the device performance by allowing one to one comparison.

In this chapter, sensitivity analysis of a small capacity Standing Wave Thermoacoustic Refrigerator (SWTAR) has been performed using NSC. Specialized tool DeltaEC has been used to generate the results for subsequent analysis. Three key performance parameters i.e. temperature difference achieved, COP and COPR have been analyzed for perturbations in one device variable namely oscillating pressure amplitude and three stack geometric parameters i.e. stack length, stack center position and half stack spacing. Sensitivity results have been obtained for a wide range of operating pressures, operating temperature and drive ratios. It has been found that performance parameters are most sensitive to perturbations in oscillating pressure amplitude and are least affected by perturbations in stack length.

3.2 Sensitivity analysis

Sensitivity analysis help us to gain insight into a system by identifying the most responsive model parameters, precision level required for measurement of these key input parameters

and the key future research areas [103]. Generally the sensitivity analysis is performed by measuring the change in the model output for given change in the model rate coefficients and thus calculates the derivatives (change rates) of system responses w.r.t. design variables [104, 105]. Normalized Sensitivity Coefficients (NSC's) are utilized for efficient comparison of various parameters having different units and especially having different orders of magnitude [106]. Uncertainties in measurement of different input parameters can have a significant effect on the final values of output variables. NSC analysis provides a platform to convert all parameters to same measurement system. Sensitivity analysis using NSC method gives a clear picture of how sensitive given output parameters are to change in the values of input variables for a range of design variable and allows direct comparison of parameters.

For instance, any independent variable X can be represented in terms of its uncertainty U_X as:

$$X = \bar{X} \pm U_X \quad (3.1)$$

where $\bar{X}$ is the nominal value and $\pm U_X$ defines the bandwidth within which a variable can be ascertained with a certain degree of confidence [107]. The uncertainty in an output function $Y(X)$ can then be represented by

$$U_Y = \frac{dY}{dX} U_X \quad (3.2)$$

For a multivariable function $Y = Y(X_1, X_2, \dots, X_N)$ the uncertainty is given by root sum square product of individual uncertainties

$$U_Y = \left[\sum_{i=1}^N \left(\frac{\partial Y}{\partial X_i} U_{X_i} \right)^2 \right]^{1/2} \quad (3.3)$$

Physically, partial derivative terms represent the sensitivity of Y with respect to slight changes in X_i and are defined as sensitivity coefficients. The response parameter Y and all other input variables can be normalized with respect to their nominal values and Eq. (3-3) takes the following form

$$\left(\frac{U_Y}{\bar{Y}} \right) = \left\{ \sum_{i=1}^N \left[\left(\frac{\partial Y}{\bar{Y}} \frac{\bar{X}_i}{\partial X_i} \right) \left(\frac{U_{X_i}}{\bar{X}_i} \right) \right]^2 \right\}^{1/2} \quad (3.4)$$

The term $\left(\frac{\partial Y}{\bar{Y}} \frac{\bar{X}_i}{\partial X_i} \right)$ of Eq. (3.4) is the respective dimensionless sensitivity coefficient referred to as normalized sensitivity coefficient (NSC) while second term $\left(\frac{U_{X_i}}{\bar{X}_i} \right)$ is normalized uncertainty (NUs). Eq. (3.4) can be rephrased as

$$\left(\frac{U_Y}{\bar{Y}} \right) = \left\{ \sum_{i=1}^N [NSC_{X_i} \cdot NU_{X_i}]^2 \right\}^{1/2} \quad (3.5)$$

The current study is concerned only with NSC, therefore, partial derivatives are replaced by discrete variables. The NSC can be rewritten as

$$NSC_{X_i} = \left(\frac{\Delta Y_i}{\bar{Y}} \cdot \frac{\bar{X}_i}{\Delta X_i} \right)^2 \quad (3.6)$$

Since all output responses and input variables are normalized relative to their nominal values, a one to one comparison can be made between all the coefficients. Practically, NSC describe the expected variation of analyzed function, in terms of order of magnitude, for one order magnitude change in a certain design parameter [102].

3.3 DeltaEC model

A numerical model of SWTAR has been developed using DeltaEC and results obtained are utilized to perform the sensitivity study. DeltaEC is a numerical modeling software that solves pressure and flow rate equations obtained from the one-dimensional linearization of wave equation, momentum and continuity equations [108]. Energy equation is also integrated and solved for various segments. Complicated forms of momentum and energy equations are employed to account for the thermal and viscous losses resulting from acoustic dissipation inside each segment. The model numerically integrates the equations in the x-direction through the whole range. Shooting method is used and several iterations are conducted to ensure that the obtained solution of $p_1(x)$ and $U_1(x)$ reach the desired targets for a given values of guesses.

The generalized form of pressure and flow rate equation are as follows [108]:

$$\frac{dp_1}{dx} = F_{Momentum}(P_m, U_1, T_m, \omega, \theta, x \text{ geometry, gas/solid properties etc.}) \quad (3.7)$$

$$\frac{dU_1}{dx} = F_{Continuity}(P_m, U_1, T_m, \omega, \theta, x \text{ geometry, gas/solid properties etc.}) \quad (3.8)$$

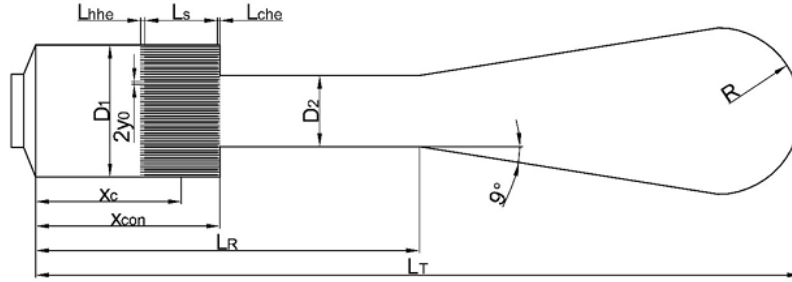
Numerical results obtained from DeltaEC model are compared with the analytical design results of Kajurek J. et al. [32] to validate the developed simulation model. The operating and geometric properties of the design are summarized in Table 3-1 and schematics of SWTAR are shown in Figure 3.1.

For the current DeltaEC model, a constant cooling load (10 W) is applied while the incoming acoustic power is variable depending on the operating and geometric parameters. The boundary conditions applied at the first segment ($x = 0$) are as follows:

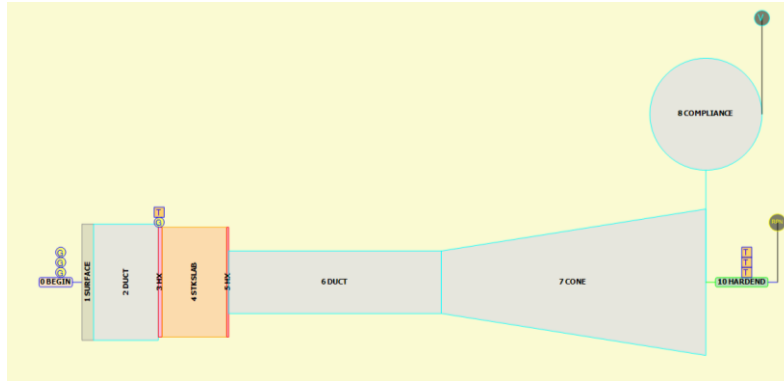
- Resonance conditions are enforced i.e. phases of oscillating pressure and flow rate are equal $\theta(p_1) = \theta(U_1) = 0$.
- Pressure amplitude is constant for each run i.e. nominal value = 10 kPa.

Table 3-1: Operating and geometric parameters of Compact SWTAR

Parameter	Nominal Value
Mean pressure, P_m (bar)	10.0
Mean temperature, T_m (K)	300
Drive ratio, DR (%)	1.0
Total Length, L (mm)	636
Stack length, L_s (mm)	60
Stack center position, X_s (mm)	120
Half stack Spacing, y_0 (mm)	0.2
Half plate thickness, L_{plate} (mm)	0.05
Stack material	Mylar
Working fluid	Helium



(a)



(b)

Figure 3.1: SWTAR model (a) Schematic illustration [32] (b) DeltaEC model

Four guesses and four targets are imposed for each run of the current model. The four unknown guesses are: hot heat exchanger power required for each run, the resonance frequency, volume flow rate and mean temperature. Three targets are applied at bottom end of the resonator while the fourth is applied at the hot heat exchanger. The four targets are as follows:

1 & 2: complex pressure amplitude is zero (open end of the tube),

3: outside energy flow is zero (insulated model) and

4: constant hot heat exchanger temperature = 300 K.

Table 3-2 presents the comparison between the simulated device performance and the analytical design results obtained by Kajurek J. et al. [32] . The results obtained by the current model are in close agreement with the reported data, hence, the model is suitable for subsequent sensitivity analysis. The values of temperature difference developed across stack, COP and COPR are recorded for each run in DeltaEC. Normalized sensitivity coefficients are calculated based on the obtained results.

Table 3-2: Comparison of theoretical and simulation results

Kajurek J. et al. [32] design	DeltaEC design
$f = 500 \text{ Hz}$	$f = 500.06 \text{ Hz}$
$T_{\text{chx}} = 270 \text{ K}$	$T_{\text{chx}} = 266 \text{ K}$
$T_{\text{hhx}} = 300 \text{ K}$	$T_{\text{hhx}} = 300 \text{ K}$
$\dot{Q}_c = 10 \text{ W}$	$\dot{Q}_c = 10 \text{ W}$
$\dot{W} = 7.76 \text{ W}$	$\dot{W} = 7.92 \text{ W}$
$\text{COP} = 1.29$	$\text{COP} = 1.264$

3.4 Normalized sensitivity coefficient: solution procedure

DeltaEC model has been used to generate the numerical results required for carrying out the sensitivity study. Three key performance parameters of thermoacoustic refrigerator, i.e. temperature difference achieved (ΔT) across the stack, coefficient of performance (COP)

and relative coefficient of performance (COPR) have been selected as the response variables. COP is the ratio of obtained cooling power to consumed acoustic power. COPR is the ratio of COP and COPC where COPC is coefficient of performance relative to Carnot. The relations for COP, COPC and COPR are given by relation.

$$COP = \frac{\dot{Q}_c}{\dot{W}} \quad (3.9)$$

$$COPC = \frac{T_c}{T_H - T_c} \quad (3.10)$$

$$COPR = \frac{COP}{COPC} \quad (3.11)$$

where, $\dot{Q}_c$ is the desired cooling load and $\dot{W}$ is the consumed acoustic power. The sensitivity analysis has been performed for a range of following operating parameters, namely, mean temperature (T_m), mean pressure (P_m) and drive ratio (DR) respectively. The drive ratio is defined as

$$DR = \frac{|P_1|}{P_m} \quad (3.12)$$

The chosen independent parameters for the NSC sensitivity analysis are: pressure amplitude (p_1), stack length (L_s), stack center position (X_s), half stack spacing (y_0) and stack plate thickness (L_{plate}) respectively. A sequential perturbation method can be applied for accomplishment of sensitivity analysis and schematic diagram has been shown in Figure 3.2(a) and Figure 3.2(b) and detailed as follows:

- Result for the output parameter Y is calculated form the input data ($\overline{X_{1,...,N}}$) and is designated as nominal value $\bar{Y}$.
- Value of i_{th} input variable is increased by its interval (U_{X_i}) and response variable is calculated with all other input parameters are kept fixed at their nominal values.

These values are recorded as X_{i+} and Y_{i+} . Similarly X_{i-} and Y_{i-} are recorded by decreasing the value by interval for i_{th} variable.

- Let $\Delta Y_i = |Y_{i+} - Y_{i-}|$ and $\Delta X_i = |X_{i+} - X_{i-}|$.
- Required NSC of X_i is calculated by using the Eq. (6) and plugging in the appropriate values.

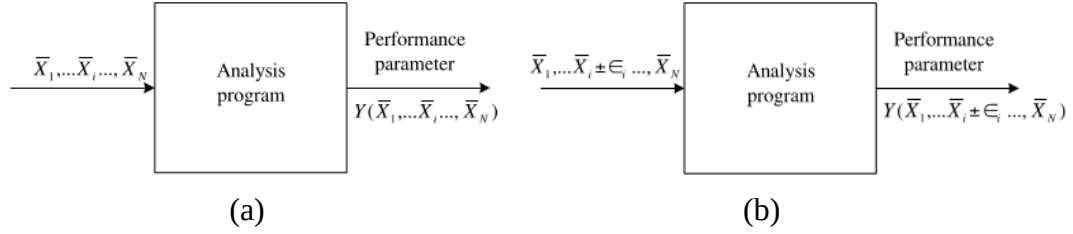


Figure 3.2: Block diagram of sensitivity analysis procedure (a) nominal values (b) perturbed values.

The sensitivity analysis method adopted in the current study has been proposed by Hodge B.K. and Taylor R.P. [109]. Either known ranges of uncertainty in the independent variables or arbitrarily selected variation ranges can be applied for accomplishment of uncertainty [110]. To have a common ground for comparison, all the independent variables have been perturbed for $\pm 1\%$ (arbitrary) of their nominal values and NSC of response variables have calculated for variation within this range.

To demonstrate the procedure for the calculation of NSC, an illustrative numerical example is presented below for a single value of inlet mean pressure (P_m) of 10 bar (refer to Figure 3.3). Independent and response variables are pressure amplitude ($Y = p_1$) and temperature gradient achieved ($X_i = T_{diff}$). Following steps are followed to calculate the values of sensitivity coefficients:

Step (a): For a given pressure amplitude of 10 kPa the temperature gradient calculated by the DeltaEC is 34.0°C . So $\bar{p}_1 = 10 \text{ kPa}$ and $\bar{T}_{diff} = 34^\circ\text{C}$.

Step (b): When the pressure amplitude is increased by 1% then resulting temperature difference is 34.835°C . So $p_{1+} = 10.1 \text{ kPa}$ and $T_{diff+} = 34.835^{\circ}\text{C}$. Similarly with $p_{1-} = 9.9 \text{ kPa}$ the temperature difference is $T_{diff-} = 33.145^{\circ}\text{C}$.

Step (c): Thus $\Delta p_1 = 0.2 \text{ kPa}$ and $\Delta T_{diff} = 1.69^{\circ}\text{C}$

Step (d): Substituting the nominal and delta (Δ) values in Eq. (3.6), a value of 6.18 is obtained for NSC and can be seen in Figure 3.3.

It is worth mentioning that the label of y-axis of Figure 3.3 to Figure 3.10 is NSC (Response variable, Independent variable) where the second parameter is the one whose value is perturbed. The limits of values of various input parameters used in the next section can be seen in Table 3-3.

3.5 Results and discussion

Sensitivity results obtained using DeltaEC and the procedure described above for a wide range of three selected operating parameters i.e. P_m , T_m and DR are elaborated in this section.

Table 3-3: Numeric limits of various variables

Variable	Limits
Pressure amplitude, p_1 (kPa)	9.9 – 10.1
Stack length, L_s (mm)	59.4 – 60.6
Stack centre position, X_s (mm)	118.8 – 121.2
Half stack spacing, y_0 (mm)	0.198 – 0.202
Stack plate thickness, L_{plate} (mm)	0.0495 – 0.0505

The ranges investigated are 5-10 bar, 290-310 K and 1-3 % for P_m , T_m and DR respectively.

The preliminary results show that variation in the independent variable stack plate

thickness (L_{plate}) within the selected range of $\pm 1\%$ produced absolutely no change in the device performance and response variables. Thus SWTAR design using DeltaEC is insensitive to perturbation in L_{plate} and its plots for sensitive coefficients are not displayed in the subsequent subsections.

3.5.1 Sensitivity study for mean pressure, P_m

Figure 3.1 shows the normalized forms of achieved temperature difference (ΔT) sensitivity coefficients with respect to selected design variables, i.e. $(\partial \Delta T / \partial p_1)$, $(\partial \Delta T / \partial L_s)$, $(\partial \Delta T / \partial X_s)$, $(\partial \Delta T / \partial y_0)$, versus the mean operating pressure (P_m) of the refrigerator. The figure shows that ΔT is most sensitive to variation in pressure amplitude as it produced the largest values of NSC. As the mean pressure is increased the sensitivities decreases for p_1 , X_s and y_0 with p_1 and X_s decreasing in similar fashion while a slight linear decrease is observed for y_0 . The sensitivity coefficients for L_s increase slightly with the increase of P_m but the coefficient values are too small (from 0.0018 to 0.124) to be clearly seen in the comparative figure. A NSC value of 24.12 has been obtained for oscillating pressure amplitude (p_1) at mean pressure of 5 bar and it decreases to 6.18 at 10 bar. The oscillating pressure amplitude is directly related to acoustic power produced inside the device thus it dictates the overall performance, compared to other selected design variables. Therefore the achieved temperature drop is significantly affected by its change for all values of mean pressures. The temperature drop (ΔT) parameter becomes less sensitive to variations in independent input variables at higher values of mean operating pressures (P_m). The trend can be explained by the fact that at higher operating pressures the amount of temperature drop (ΔT) that can be achieved from the device is significantly increased. However, the

variation in ΔT parameter corresponding to perturbations in independent variables is decreased resulting in a net decrease in NSC values.

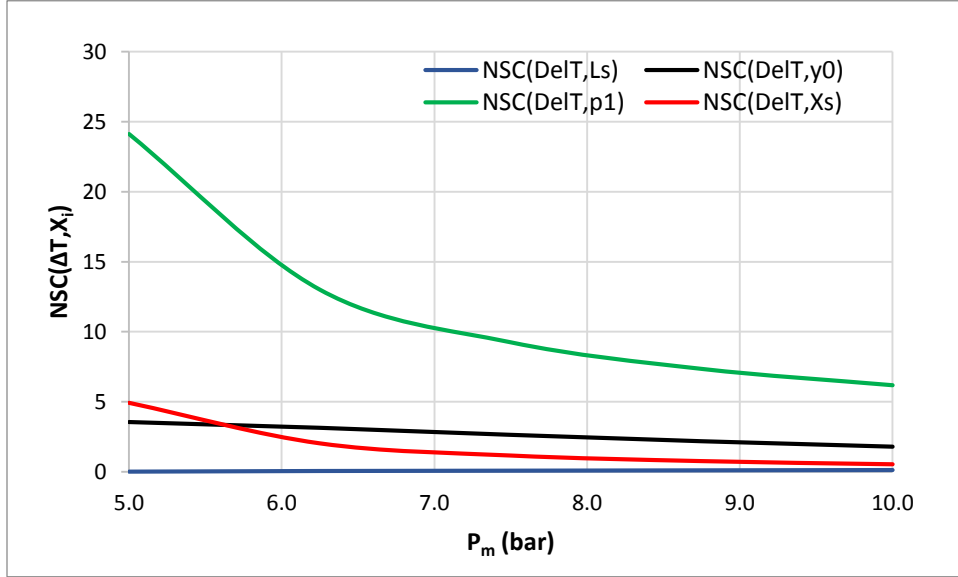


Figure 3.3: Variation of ΔT NSC with respect to independent variables for a range of operating pressures

Figure 3.4 shows the normalized forms of device coefficient of performance (COP) sensitivity coefficients with respect to selected design variables, i.e. $(\partial COP / \partial p_1)$, $(\partial COP / \partial L_s)$, $(\partial COP / \partial X_s)$, $(\partial COP / \partial y_0)$, versus the mean operating pressure (P_m) of the refrigerator. It was noticed, again that COP of the device has been found to be most sensitive to oscillating pressure amplitude (p_1) and produce largest values of NSC throughout the investigated range. However, for the case of COP, the sensitivity coefficients increase with an increase in the mean pressure. Sensitivity values for other variables are fairly constant in the selected range with L_s and y_0 decreasing slightly and X_s increasing slightly. COP is the ratio of desired cooling load to acoustic power absorbed by stack. Hence, acoustic power is directly dependent on pressure amplitude (p_1) making COP most sensitive to it. As the cooling load of the SWTAR is constant therefore the COP is a function of acoustic power alone. With an increase in the mean pressure (P_m) the value of

achieved ΔT is increased making stack absorb more acoustic power thus decreasing nominal value of COP and explains the increase in value of NSC with respect to p_1 .

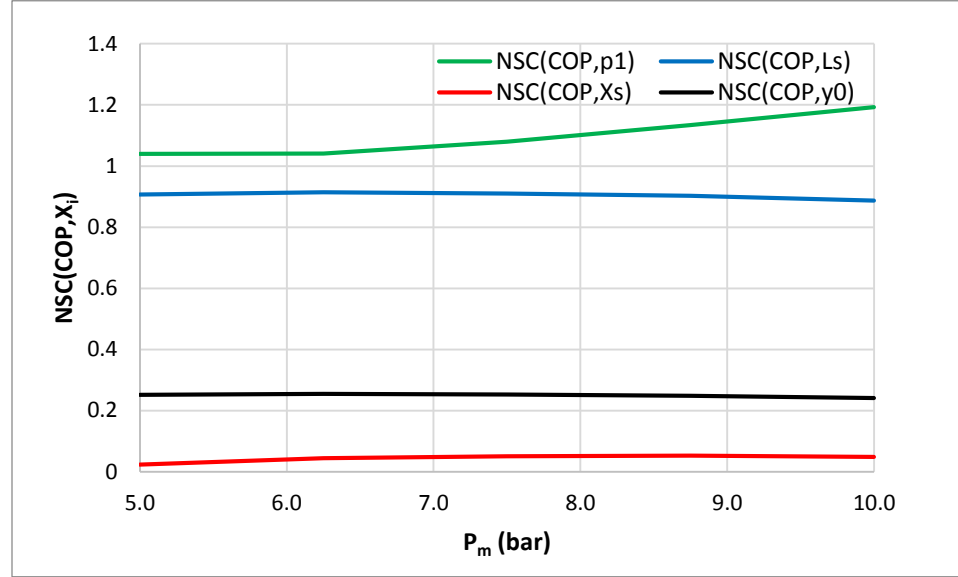


Figure 3.4: Variation of COP NSC with respect to independent variables for a range of operating pressures

Figure 3.5 shows the normalized forms of device relative coefficient of performance (COPR) sensitivity coefficients with respect to selected design variables, i.e. $(\partial COPR / \partial p_1)$, $(\partial COPR / \partial L_s)$, $(\partial COPR / \partial X_s)$, $(\partial COPR / \partial y_0)$, versus the mean operating pressure (P_m) of the refrigerator. In all cases, normalized sensitivity coefficients (NSC) decrease with an increase in the value of mean pressure. Oscillating pressure amplitude (p_1) has largest values of NSC followed by stack center position (X_s), stack spacing (y_0) and stack length (L_s), respectively. As clearly shown in the Figure 3.5, the NSC corresponding to oscillating pressure amplitude dominate in the whole range of mean pressure with a maximum value of 17.92 at 5 bar and lowest value of 2.93 at 10 bar. COPR is the ratio of device Coefficient of Performance (COP) and Carnot's Coefficient of Performance (COPC). As the temperature of hot heat exchanger is kept fixed for each design iteration thus an increase in the mean pressure results in a lower value of cold heat

exchanger temperature. This ultimately lowers the COPC and hence nominal values of COPR are increased while the changes in COPR (ΔCOPR) (due to perturbation in independent variables) are decreased. The net combined effect is a decrease of NSC values with increasing mean pressure.

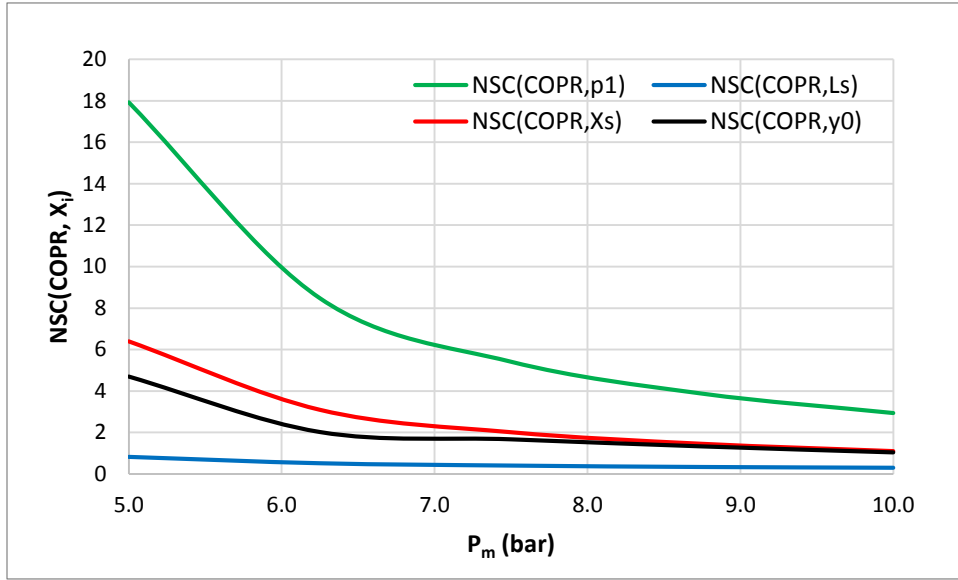


Figure 3.5: Variation of COPR NSCs with respect to independent variables for a range of operating pressures

3.5.2 Sensitivity study for mean temperature, T_m

Figure 3.6 shows the normalized forms of achieved temperature difference (ΔT) sensitivity coefficients with respect to selected design variables, i.e. $(\partial\Delta T/\partial p_1)$, $(\partial\Delta T/\partial L_s)$, $(\partial\Delta T/\partial X_s)$, $(\partial\Delta T/\partial y_0)$, versus the mean temperature (T_m) of the refrigerator. For all four cases normalized sensitivity coefficient (NSC) of achieved temperature difference (ΔT) decreases with an increase in the value of mean temperature (T_m). Oscillating pressure amplitude (p_1) is the most dominating variable at all mean temperatures. Increase in mean temperature (T_m) of refrigerator results in an increase in the value of achieved temperature difference (ΔT) meanwhile the variation in ΔT values (due to perturbation in independent variables) is decreased. The combination of all these factors cause a decrease in the value

of NSC. It should be noted that the ratio of NSC $(\Delta T, p_1)/\text{NSC}(\Delta T, y_0)$, where y_0 is the second most dominating variable, is approximately 3.4. The ratio indicates that for all values of device mean temperature the ΔT of refrigerator is approximately 3.4 times more sensitive to oscillating pressure (p_1) than stack plate spacing (y_0). The ratio of oscillating pressure NSC with respect to other parameters is even higher.

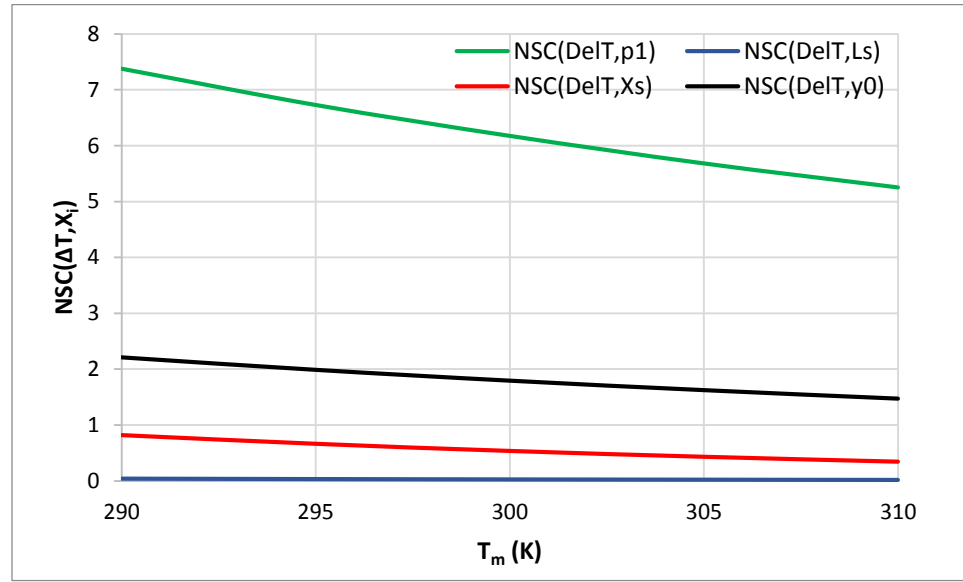


Figure 3.6: Variation of ΔT NSC with respect to independent variables for a range of operating temperatures

Figure 3.7 shows the normalized forms of device Coefficient of Performance (COP) sensitivity coefficients with respect to selected design variables, i.e. $(\partial \text{COP} / \partial p_1)$, $(\partial \text{COP} / \partial L_s)$, $(\partial \text{COP} / \partial X_s)$, $(\partial \text{COP} / \partial y_0)$, versus the mean temperature (T_m) of the refrigerator. Sensitivity coefficients of all independent variables almost remain constant throughout the range of mean temperatures. However, with an increasing mean temperature, NSC for oscillating pressure amplitude (p_1) and stack spacing (y_0) increase slightly while NSC values for stack center position (X_s) and stack length (L_s) show a slight decrease. The decreasing pattern is not visible in the combined figure as variation of NSC

values is very small. Once again, oscillating pressure is the most dominating variable with ratio $NSC(\Delta T, p_1)/NSC(\Delta T, y_0)$ is nearly 5.0 at all mean temperatures.

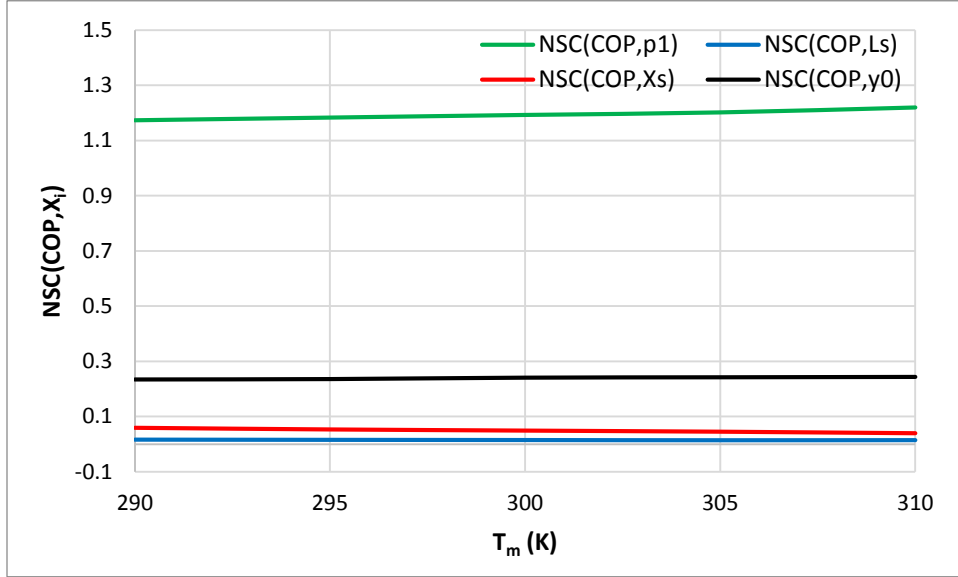


Figure 3.7: Variation of COP NSC with respect to independent variables for a range of operating temperatures

Figure 3.8 shows the normalized forms of device Relative Coefficient of Performance (COPR) sensitivity coefficients with respect to selected design variables, i.e. $(\partial COPR/\partial p_1)$, $(\partial COPR/\partial L_s)$, $(\partial COPR/\partial X_s)$, $(\partial COPR/\partial y_0)$, versus the mean temperature (T_m) of the refrigerator. COPR NSC with respect to all four independent variables decreases with an increase in mean temperature (T_m). Similar pattern is observed in all variables and the pattern is not observable for stack length (L_s) due to small values of NSC compared to other variables. Reasons for decrease in the values of the COPR NSCs with an increasing temperature are similar to the one given in the explanation of COPR NSCs in section 5.1. Sensitivities concerned with oscillating pressure amplitude (p_1) are dominating all other variables with a ratio $NSC(\Delta T, p_1)/NSC(\Delta T, y_0)$ around 2.7 for this case.

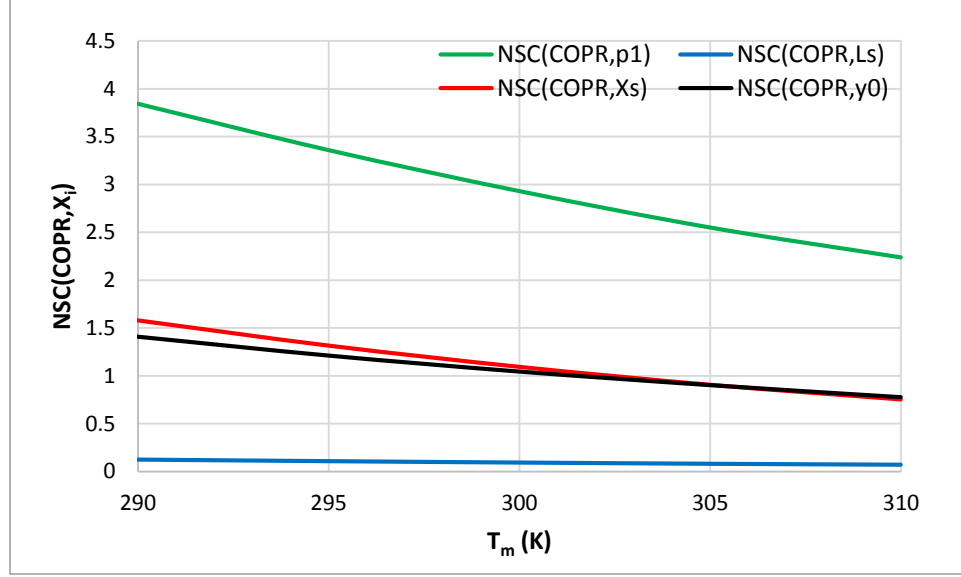


Figure 3.8: Variation of COPR NSCs with respect to independent variables for a range of operating temperatures

By comparing Figure 3.6-Figure 3.8, it can be easily ratiocinated that for variation in mean temperature (T_m), NSCs of all response variables are most sensitive to oscillating pressure (p_1) followed by stack spacing (y_0), stack center position (X_s) and stack length (L_s) respectively in the order of decreasing NSC values. The oscillating pressure amplitude (p_1) is the most dominating parameter where a unit change in its value, for the case of ΔT , COP and COPR respectively, produces an impact of approximately 3.4, 5.0 and 2.7 times higher than second most dominating parameter.

3.5.3 Sensitivity study for drive ratio, DR (%)

Figure 3.9 shows the normalized forms of achieved temperature difference (ΔT) sensitivity coefficients with respect to selected design variables, i.e. $(\partial \Delta T / \partial p_1)$, $(\partial \Delta T / \partial L_s)$, $(\partial \Delta T / \partial X_s)$, $(\partial \Delta T / \partial y_0)$, versus the Drive Ratio (DR) of the refrigerator. Sensitivity coefficients of ΔT with respect to oscillating pressure amplitude (p_1) and stack spacing (y_0) decreased with an increasing drive ratio and almost approach to zero at higher drive ratios especially beyond 2%. NSC with respect to stack center position (X_s) increased as the drive

ratio increases. NSC with respect to stack length (L_s) also show a slight increase but the values are minute to have a visible effect on a comparative scale. It can be concluded that at higher drive ratios, achieved temperature difference (ΔT) is only affected by changes in stack center position (X_s) and is insensitive to all other parameters.

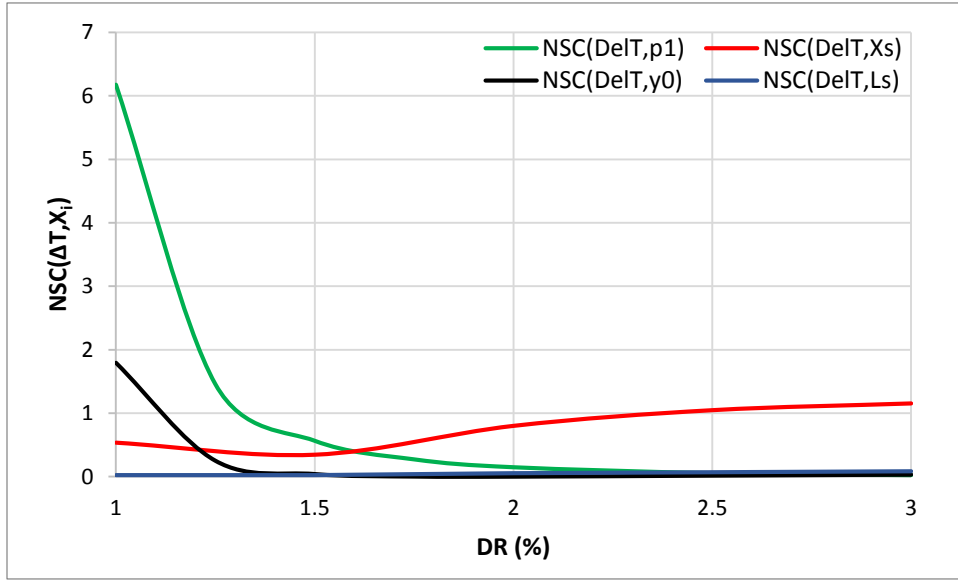


Figure 3.9: Variation of ΔT NSC with respect to independent variables for a range of drive ratios

Figure 3.10 shows the normalized forms of device Coefficient of Performance (COP) sensitivity coefficients with respect to selected design variables, i.e. $(\partial COP / \partial p_1)$, $(\partial COP / \partial L_s)$, $(\partial COP / \partial X_s)$, $(\partial \Delta T / \partial y_0)$, versus the Drive Ratio (DR) of the refrigerator. With an increase in the drive ratio (DR), an increase in the values of NSC is observed with respect to oscillating pressure amplitude (p_1), Stack center position (X_s) and stack spacing (y_0). Values of NSC with respect to stack length are again very small to have a visible trend however a slight decrease is noted. Oscillating pressure amplitude has the most dominating characteristics having largest values at all drive ratios.

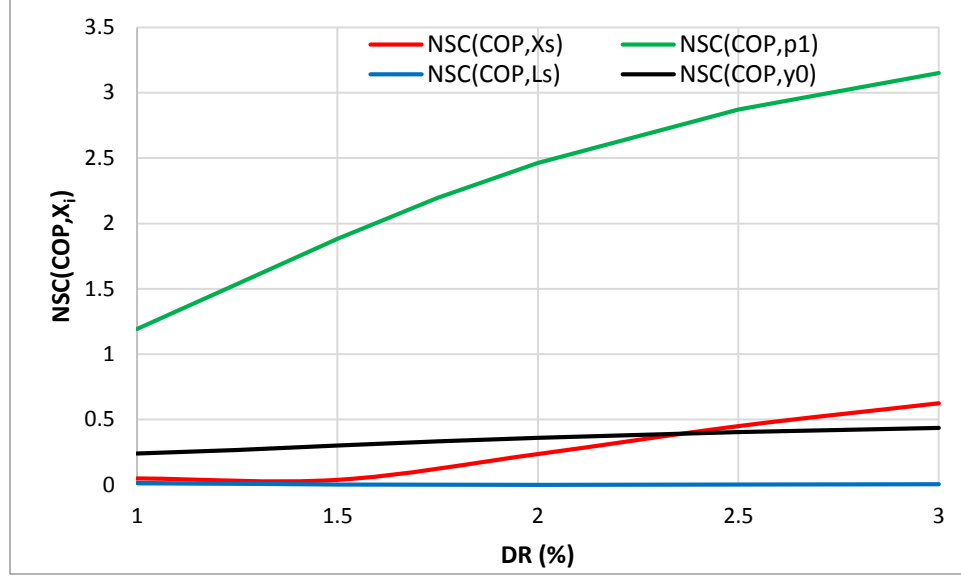


Figure 3.10: Variation of COP NSC with respect to independent variables for a range of operating drive ratios

For variation in Drive Ratio (DR) the normalized sensitivity coefficient values obtained for device Relative Coefficient of Performance (COPR) are extremely small i.e. in the range of $5e-07$ for all selected design variables. Thus highlighting the fact that COPR of SWTAR is insensitive to changes in the design variables at all drive ratios within selected ranges.

3.6 Conclusions

Sensitivity analysis of important response variables, namely ΔT , COP and COPR, of a small capacity compact SWTAR has been performed using DeltaEC. Normalized sensitivity coefficients are calculated to evaluate the sensitivity with respect to one device and three stack geometric variables. Initially a numerical model has been developed based on a design from literature and results have been verified against available analytical results. The developed model has been selected as nominal case for further evaluation of NSC.

The results of current work provide an insight about the effect of small variations in all considered input parameters on selected outputs of SWTAR in terms of normalized

coefficients. The benefit of such a coefficient is that it helps to classify various input variables according to order of variation produced in output parameter for an equal variation in all input variables. All output response parameters have been found to be most sensitive to variation in oscillating pressure amplitude (p_1) and values of ΔT , COP and COPR vary significantly for unit variation in p_1 . The only exception is for ΔT NSC verses the drive ratio where ΔT becomes almost insensitive to p_1 variation at higher drive ratios. COPR parameter is insensitive to changes in chosen independent variables at all drive ratios. Largest values of NSC are recorded with respect to oscillating pressure amplitude (p_1) for all cases with a maximum value of 24.12 for ΔT operating at P_m of 5 bar. Least values are noted for stack length (L_s) with an exception of COP NSC where L_s has significant effect after p_1 . The conclusions drawn from this study can be summarized as:

- Oscillating pressure amplitude (p_1) has the most significant effect on performance of SWTARs. As acoustic driver is responsible for this pressure, care should be taken in its selection as its accuracy can significantly affect the performance.
- Variation in stack center position (X_s) and stack spacing (y_0) also have considerable effect, though not as significant as p_1 . Thus, stack should be manufactured accurately and placed inside device carefully for improved performance.
- Small variations in stack length (L_s), which is also a manufacturing issue, does not greatly affect the device performance.
- Perturbations in stack plate thickness (L_{plate}) has no effect on the refrigerator's performance.

CHAPTER 4

Development of a Numerical Model of Thermoacoustic Devices

4.1 Introduction

In this chapter, simplified 2D numerical models of thermoacoustic devices have been modeled using Computational Fluid Dynamics (CFD). In the first step, a two dimensional (2-D) model of Standing Wave Thermoacoustic Engine (SWTAE) has been developed. The developed model only consists of stack with a specified temperature distribution and heat exchangers are neglected. In second phase performance of Standing Wave Thermoacoustic Refrigerator (SWTAR) has been simulated using another 2-D numerical model. Once again a single stack plate with zero thickness and isothermal conditions without any heat exchangers has been considered. The simulation results for both devices are validated by comparing them with the published results.

4.2 Governing equations

The governing equations utilized for development of simplified models of both SWTAE and SWTAR are generalized mass, momentum and energy conservation equations. These are given by unsteady fully compressible continuity, Navier-Stokes and energy equations and are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (4.1)$$

$$\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \nabla \cdot (\boldsymbol{\mu} \cdot \nabla \mathbf{u}) + S_{mi} \quad (4.2)$$

and

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho u^2 + \rho \varepsilon \right) = -\nabla \cdot [\rho \mathbf{u} \left(\frac{1}{2} u^2 + h \right) - K \nabla T - \mathbf{u} \cdot \boldsymbol{\mu}] \quad (4.3)$$

where ρ is the density, $\mathbf{u}$ is velocity, p is the pressure, S_{mi} accounts for momentum sources in each direction, ε is the internal energy, h is the enthalpy per unit mass, K is the gas's thermal conductivity and $\boldsymbol{\mu}$ is the viscous stress tensor.

4.3 Numerical model of Standing Wave Thermoacoustic Engine (SWTAE)

Methodology and model proposed by Florian Zink [48] has been used to develop a basic 2-D model of SWTAE. The model utilizes CFD based methodology that can successfully simulate the performance of thermoacoustic engine. Since its inception, the same model and methodology has been adopted by various other researchers to validate and simulate their work, both for thermoacoustic engines and refrigerators i.e. Yu G. et al. [70] for thermoacoustic engine, Hariharan et al. [73, 74] for twin spool TAPM, Skaria et al. [71] for standing wave thermoacoustically driven refrigerator, Hussain M.N. et al. [75] for sensitivity study etc.

4.3.1 Geometric details and operating conditions

A quarter wavelength SWTAE having a parallel plate stack inside the resonator with air as working fluid has been used in the current study. Simulation domain consist of complete resonator with full stack of thermoacoustic engine and operates with left end closed and right end open. Figure 4.1 shows the computational domain of SWTAE used for numerical simulation. It consist of 2-D representation of complete resonator where the physical

geometry of resonator has been reduced to 2-D by assuming stack plates of infinite width. The left end of domain is modeled as closed adiabatic wall while the right end is open where acoustic power is observed. Stack made of series of parallel plates is placed at distance of 30 mm from the closed end and length of stack is 10 mm. Resonator has a height of 12 mm. Spacing between the stack plates and plate thickness are taken as 0.5 mm. Stack material is taken Cordierite and air has been taken as working fluid. Details of operating parameters and materials utilized for simulation of SWTAE model are tabulated in Table 4-1 while the geometric parameters of simulation domain are given in

Table 4-2.

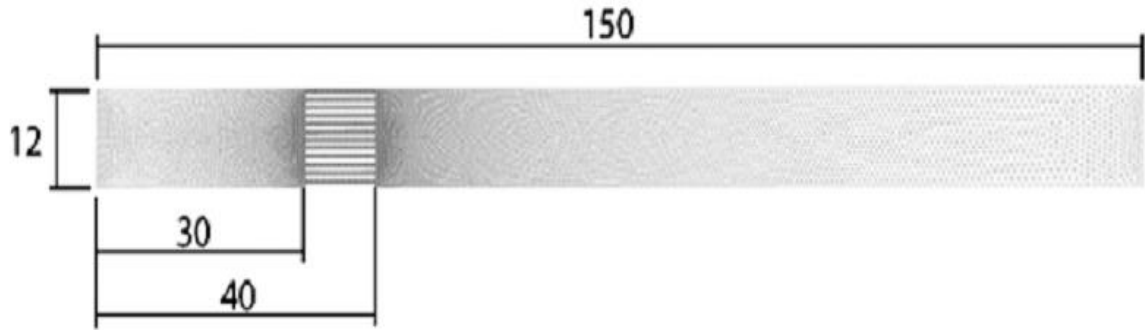


Figure 4.1: Geometry of SWTAE simulation model

Table 4-1: Operating parameters and materials used for SWTAE simulation

Mean Pressure, p_m	1.01bar
Frequency, f_{res}	600 Hz
Mean Temperature, T_m	300 K
Temperature Gradient	400K
Temperature Profile	Sinusoidal
Working Fluid	Air
Stack Plate	Cordierite
Resonator Tube	Pyrex glass

Table 4-2: Geometric Parameters of SWTAE

Stack Plate Thickness	0.5 mm
Stack Plate Spacing	0.5 mm
Height of Resonator	12 mm
Length of Resonator	150 mm
Length of stack plates	10 mm
Distance of stack from closed end	30 mm

4.3.2 Mesh details

Unstructured mesh with triangular elements have been utilized to mesh the entire simulation domain. Size of triangular cells has been refined for region inside and surrounding the stack plates and mesh size is increase gradually towards the ends. Total no. of elements in mesh are 35790 and total no. of nodes are 19870. Final shape of mesh utilized for simulation is shown in Figure 4.2.

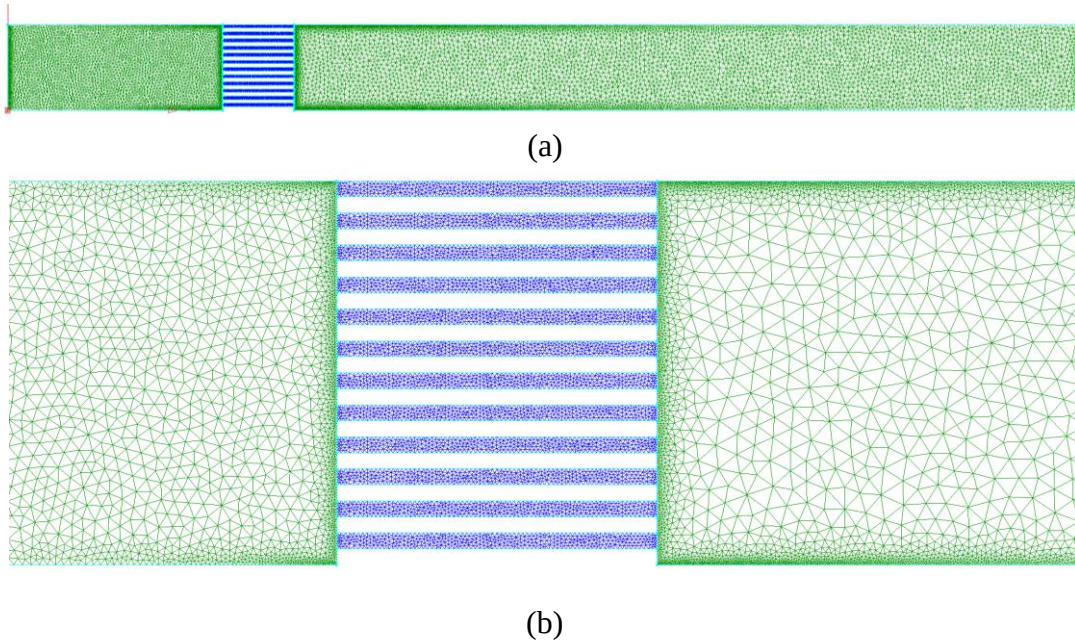


Figure 4.2: Preview of mesh used for SWTAE (a) complete domain (b) magnified view of stack region

4.3.3 Boundary conditions

Horizontal walls of stack are modeled as walls of non-zero thickness and are given a specified temperature distribution with a gradient from 700K to 300K. The relation used to specify the distribution is given by

$$T(x) = 500 + \left[200 \cos \left(\frac{\pi x}{0.01} \right) \right] \quad (4.4)$$

All other edges of simulation domain are modeled as adiabatic walls except for closed and open end of domain. The boundary conditions applied at closed and open end are given as follows:

Closed end: $\begin{cases} \text{Pressure inlet} = 101325 \text{ Pa} & \text{(steady state)} \\ \text{Adiabatic wall} & \text{(transient analysis)} \end{cases}$

Open end: Pressure outlet (Gauge pressure = 0 Pa)

4.3.4 Solution strategy

Simulation of thermoacoustic phenomenon has been done using ANSYS Fluent and involves the initialization of small velocity distribution inside the stack by applying a specified temperature gradient across stack and using a small pressure gradient across the resonator in a steady state. Then these initialized settings are transferred to transient analysis for simulation of thermoacoustic engine involving the initiation of pressure amplification process. The analysis is carried out until pressure amplification has sustained stable Limit Cycle Oscillations (LCO).

Steady state analysis has been conducted to develop an initial conditions inside simulation domain and for said purpose, inlet gauge pressure (P_{inlet}) of 10 Pa has been applied at the

closed end while an outlet gauge pressure (P_{outlet}) of 0 Pa has been applied at the open end. The analysis has given the initial distribution of pressure, velocity and temperature inside the fluid domain and stack which are then used for start of transient analysis. Transient analysis has been conducted with similar conditions except that closed end has been changed to wall form pressure inlet. With the passage of time, thermoacoustic effect inside the stack will excite pressure amplification and amplification will continued until it reaches steady state results in terms of sustained pressure oscillation also known as Limit Cycle Oscillations (LCO).

4.3.5 Analysis details

Pressure based solver has been used, absolute velocity formulation with green Gauss node based method for gradients has been applied, PRESTO scheme is used for pressure discretization, while pressure velocity coupling is done through PISO. Turbulence has been modeled and k- ϵ realizable model has been used to capture the turbulence effects. Second order upwind discretization scheme with implicit formulation has been used for density, momentum, energy, turbulent KE and dissipation rate equations. First order implicit formulation and a fixed time step of 1e-05 sec has been selected for carrying out the transient simulation.

4.3.6 Simulation results and its validation

Steady state analysis has been conducted to develop an initial conditions which can further be utilized in transient simulation to model the thermoacoustic phenomenon. As a result of steady state a pressure difference of 10 Pa across the stack has been developed inside the domain, as shown in Figure 4.3(a). Due to the application of specified sinusoidal

temperature distribution, a temperature gradient ranging from 700 K to 300 K has developed inside the stack, as depicted in Figure 4.3(b), while rest of domain is at mean temperature of 300 K. As a result of pressure and temperature gradient across the stack, a small velocity distribution is also present inside the domain with a maximum value of 1.89 m/s inside the stack, see Figure 4.3(c).

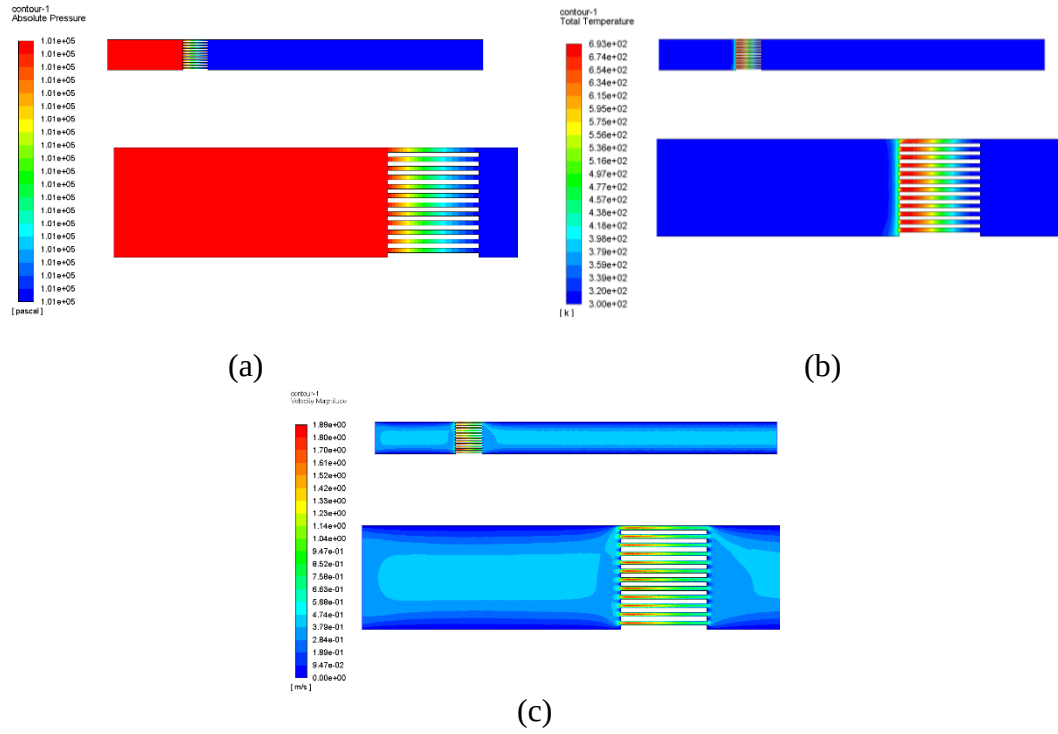


Figure 4.3: Contours of (a) absolute pressure (b) total temperature (c) velocity

Absolute pressure development inside the SWTAE has been plotted in Figure 4.4. For comparison and validation of current model, published results of Zink F. et al. has also been given in the same figure. Figure 4.4(a) shows that onset of pressure amplification starts around 1×10^4 time steps and sustained oscillations are reached around 2×10^4 time steps. When compared with published results for straight resonator, shown in Figure 4.4(b), there is a slight difference in the start of pressure amplification process but the period and amplitude of absolute pressure matches quite closely thus validating the current simulation.

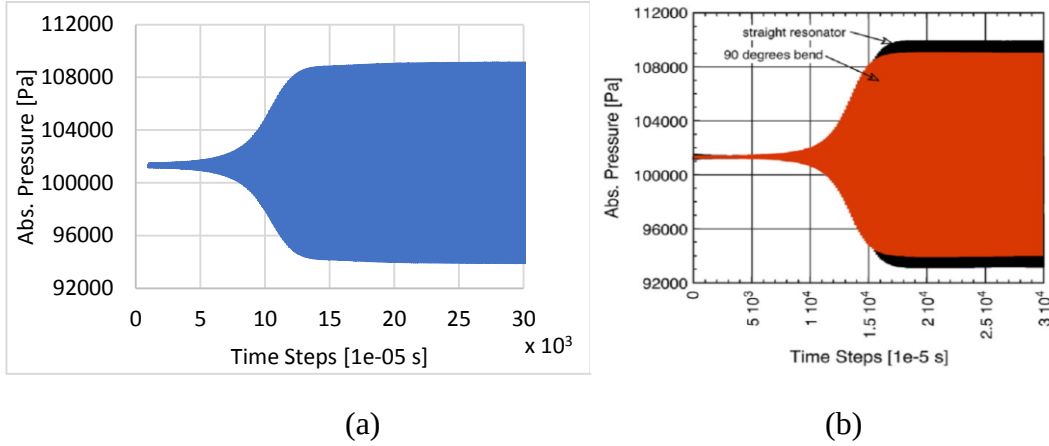


Figure 4.4: Comparison of development of absolute pressure inside SWTAE

Figure 4.5(a) shows the variation of absolute pressure after the achievement of sustained oscillations. Published results for the same parameter are displayed in Figure 4.5(b) for comparison purpose. The pressure amplitude attained and frequency of oscillations are clearly visible in Figure 4.5 which confirms that there is very small difference between this simulation and published results. Table 4-3 compares the values of pressure amplitude achieved and frequency attained with the published one.

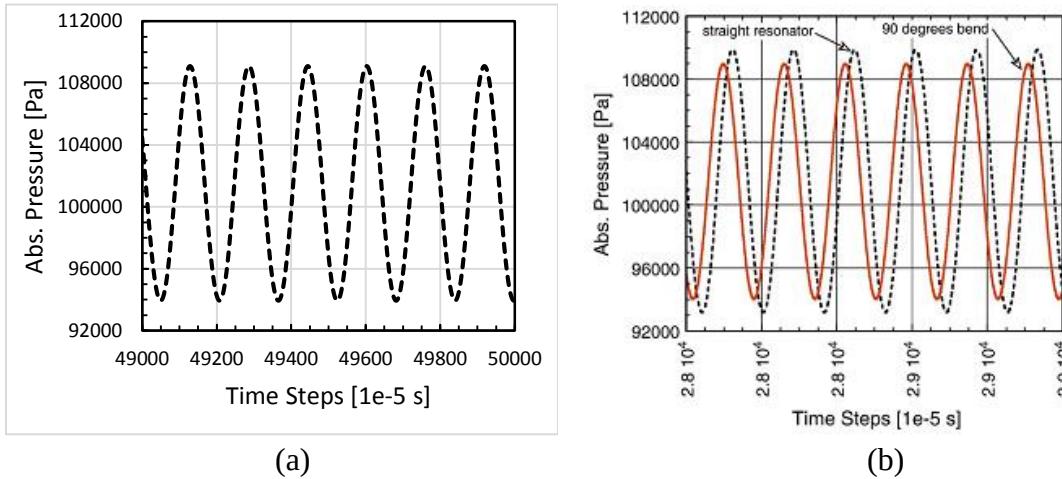


Figure 4.5 Comparison of simulation with published results (a) Current simulation (b) Zink F. et al. [48]

The static temperature distribution inside SWTAE, after reaching steady state conditions, is shown in Figure 4.6. Asymmetrical temperature distribution is seen at the hot side of

resonator and thermal hot buffer has formed beside the hot end of stack. The results clearly show the ability of CFD simulations to capture the nonlinear phenomenon associated with thermoacoustic devices.

Table 4-3: Comparison of main output parameters of SWTAE

Parameter	Simulation	Published [48]	Difference
Pressure Amplitude, P_A (Pa)	7772	8584	9.4 %
Frequency Achieved, f (Hz)	625	614	1.8%

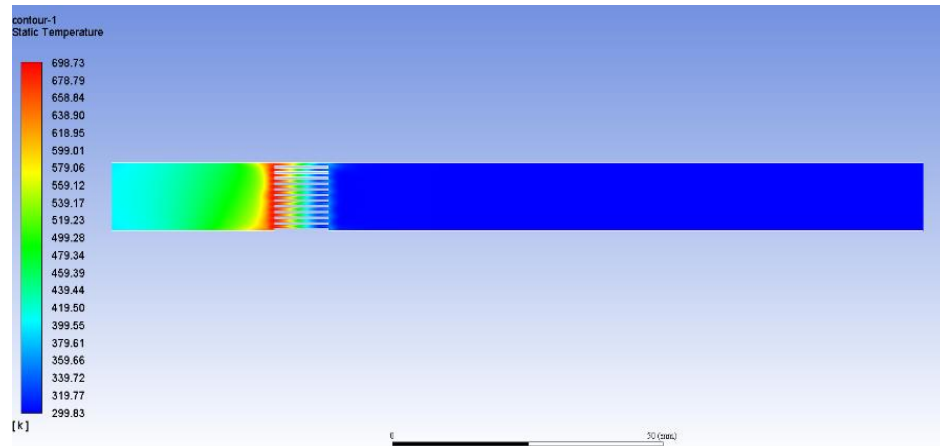


Figure 4.6: Static temperature contours after achieving sustained oscillations

4.4 Numerical model of Standing Wave Thermoacoustic Refrigerator (SWTAR)

A 2-D numerical model of SWTAR has been developed and validated using the work flow adopted by Ishikawa et al. [49].

4.4.1 Operating parameters and geometric details

In the work of Ishikawa et al. [49], performance of a quarter wavelength SWTAR consisting of parallel plate stacks has been simulated using a single thermoacoustic couple. Stack plate's right end is placed at one-eighth of wavelength. Resonator tube is filled with Helium and a standing wave is generated inside it. The left end of resonator is closed while

acoustic power is supplied at right end. The concept of thermoacoustic couple i.e. one stack channel with truncated domain, has been modeled and analyzed in almost every research carried out for performance prediction of refrigerators.

Computational domain of SWTAR consist of a thermoacoustic couple enclosing the closed end of resonator and stack. To reduce the computational cost, simulation domain is truncated at a distance of $1/8\lambda+0.0085\lambda$. For simplification, stack plate thickness is not modeled, the resulting simulation model consist of half stack spacing with stack plate subjected to isothermal conditions. The details of a thermoacoustic couple and computational domain has been shown in Figure 4.7.

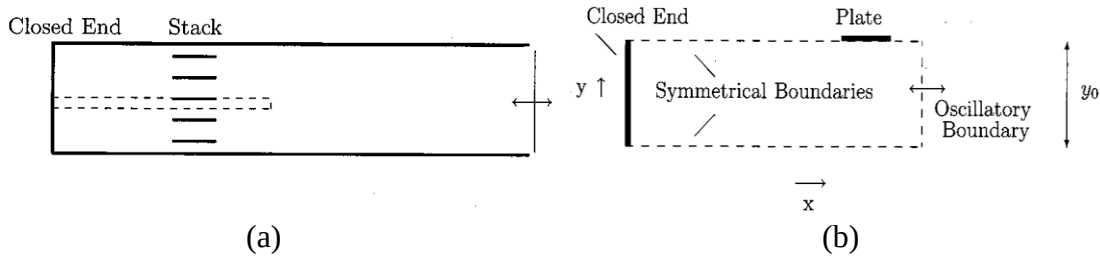


Figure 4.7: Simulation model of SWTAR [49] (a) thermoacoustic couple (b) computational domain

Operating parameters and properties that are used in the simulation of SWTAR are given in Table 4-4. The geometric details of chosen simulation domain are provided in Table 4-5.

Table 4-4: Operating parameters and properties of SWTAR model

Working Fluid	Helium
Mean Pressure, p_m	10 kPa
Frequency, f_{res}	100 Hz
Mean Temperature, T_m	300 K
Wavelength, λ	1.008 m
Drive Ratio, P_A/p_m	1.7%
Thermal Penetration Depth, δ_k	2.4 mm
Mach No.	0.01

Stack Plate Temperature, $T = T_m$	300 K
------------------------------------	-------

Table 4-5: Geometric parameters of SWTAR simulation model

Stack Plate Thickness	None
Half Plate Spacing, y_0	4 mm
Stack Length, L_s	253 mm
Distance of Stack Right End from Closed End	$1/8\lambda$
Distance of Osc. Boundary from Stack End	0.0085λ

4.4.2 Mesh details

Structured mesh with grid spacing of $\Delta x = 2.55\text{mm}$ in x-direction and $\Delta y = 0.211\text{mm}$ in y-direction has been used to mesh the whole simulation domain. The resulting mesh has 9820 mesh elements and 8759 nodes, a preview of mesh has been given in Figure 4.8.



Figure 4.8: Preview of mesh used for thermoacoustic couple of SWTAR

4.4.3 Boundary conditions

Stack plates are modeled as walls with constant temperature ($T = T_m = 300\text{K}$). Closed end of resonator has been modeled as adiabatic wall thus no heat can flow cross that boundary. All other boundaries have been given symmetry conditions except for oscillatory end where special boundary condition in terms of oscillating pressure has been implemented to force the standing wave conditions inside the domain. The boundary conditions at oscillatory end are given by:

$$P = P_m + P_A \cos(kx) \cos(\omega t) \quad (4.5)$$

$$u_1 = \frac{P_A}{\rho_m a} \sin(kx) \sin(\omega t) \quad (4.6)$$

$$T = T_m + \frac{P_A}{\rho_m C_p} \cos(kx) \cos(\omega t) \quad (4.7)$$

where, P_m , ρ_m and T_m are mean pressure, density and temperature, u_1 is oscillating velocity, P_A is oscillating pressure amplitude, C_p is fluid's specific heat capacity at constant pressure, ω is angular frequency, a is speed of sound inside fluid and k is wave no. given by $2\pi/\lambda$.

4.4.4 Analysis details

ANSYS Fluent software has been used for simulation of results. Laminar flow conditions are modeled and a transient analysis with a fixed time step of 1e-05 secs has been carried out. Working fluid is considered as an ideal gas and density of fluid is given by equation of state. Viscosity and thermal conductivity of fluid has been using power law where power factor $n = 0.647$ has been used.

Pressure based solver has been used with an absolute velocity formulation and green gauss node based method has been selected for gradients. Pressure discretization is done through PRESTO, while PISO has been adopted for pressure velocity coupling. Second order upwind discretization scheme has been used for all flow and energy variables i.e. density, momentum & energy while first order implicit formulation is used for transient analysis.

4.4.5 Simulation results and validation

Energy flux density is an important parameter as it explain the direction in which heat is flowing i.e. positive value of time average energy flux density means heat flux is leaving the domain while a negative value indicates heat flux entering the domain. The time averaged plot for y component of energy flux density (e_y) over isothermal stack plate calculated using the current simulation has been shown in Figure 4.9 and the obtained

results have been validated by comparing them with published results of same parameter. Both results are in close agreement thus validating the developed simulation model of SWTAR. Further, the results explain that energy flux density is positive at left edge of stack and negative at right end while its values is zero for central part of stack. Thus despite the application of isothermal conditions there will be a net heat flow from right to left end resulting in a refrigeration process making the right end cooler and left end warmer.

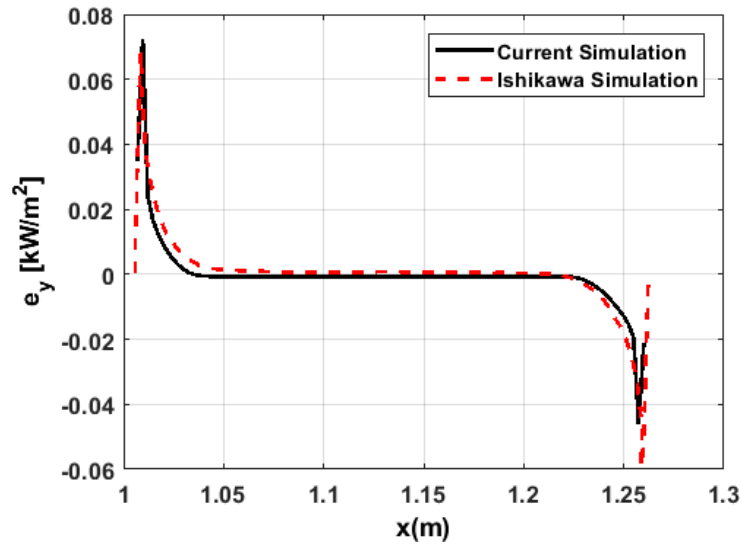


Figure 4.9: Time averaged energy flux density over stack plate of SWTAR

Figure 4.10 show static temperature variation at either end of stacks confirming the presence of refrigeration process as there is a difference of approximately 1°C between mean temperature values at the cold and hot sides of stack.

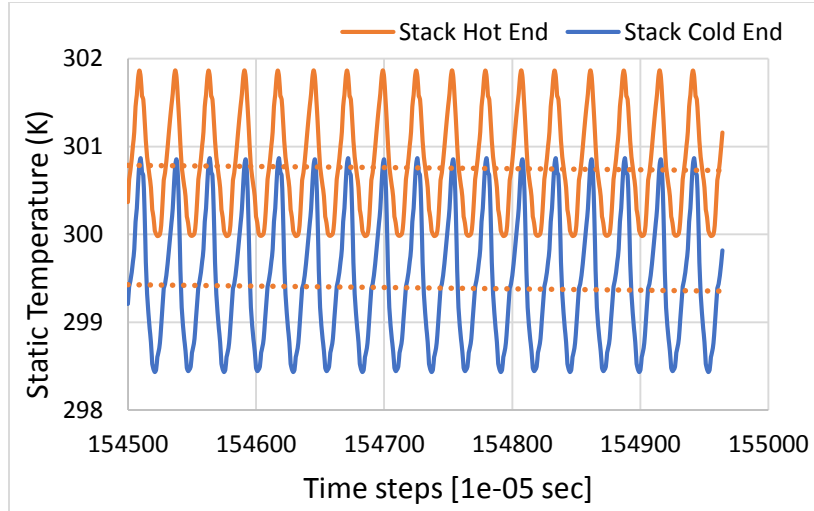


Figure 4.10: Static fluid temperature at stack ends of SWTAR

4.5 Conclusions

Numerical simulation models of SWTAE and SWTAR has been developed and their performance has been simulated using ANSYS Fluent software. The performance results of both devices has been compared with the published results for their validation and a good agreement has been found between the two. For the case of SWTAE results obtained for oscillating pressure amplitude and frequency of oscillations has been compared with results of Zink F. et al. [48] and difference of 9.4% and 1.8% has been found respectively. For SWTAR results of energy flux density distribution along the length of plates has been calculated and compared with results of Ishikawa et al. [49] and very close agreement has been found between the two distributions. The validated simulation model of SWTAR developed in this chapter will further be enhanced and utilized for performance estimation of heterogeneous stacks in the next chapter.

CHAPTER 5

Numerical Simulations for Performance Evaluation of SWTAR using Heterogeneous Stack

Performance of thermoacoustic devices can be significantly improved by making an improvement in the power density of stack. One way to achieve this goal is to have an arrangement resulting in anisotropic thermal conductivities inside the stack. For the case of Standing Wave Thermoacoustic Refrigerator (SWTAR), the arrangement can result in the reduction of heat/energy transfers taking place between the fluid parcel and solid plate thus reducing the inherent irreversibilities associated with the stack materials. The arrangement can be done by having a heterogeneous stack composed of alternative insulating and conductive materials. The present study explore the effect of having a heterogeneous stack inside a Standing Wave Thermoacoustic Refrigerator (SWTAR) on the device performance. Performance is evaluated using a validated numerical model. Effect of various heterogeneous stack material combinations and effect of variation in length of layered material has been analyzed in terms of temperature difference achieved by the refrigerator, energy flux consumed by the stack and COP attained by the SWTAR.

5.1 Numerical Model

Figure 5.1(a) shows the basic sketch of a computational domain used for the performance evaluation of current work. It represents a two-dimensional model of a quarter-wavelength resonator consisting of stack and heat exchangers made of parallel plates. It approximates

the performance of SWTAR where one (left) end of resonator is closed while other (right) end is open and acoustic power is supplied at this end. The numerical model has been reduced to two dimensions by considering stack plates of infinite width.

To enable comparison and validation with previous studies the numerical model of current work utilizes similar operating conditions and geometric parameters as proposed by Ishikawa [49]. Thus, the resonator is filled with Helium at 10 kPa absolute pressure. Mean temperature of the device is 298 K and speed of sound is 1008 m/s. The operating frequency of device is fixed at 100 Hz and total length of resonator is 2.52 m.

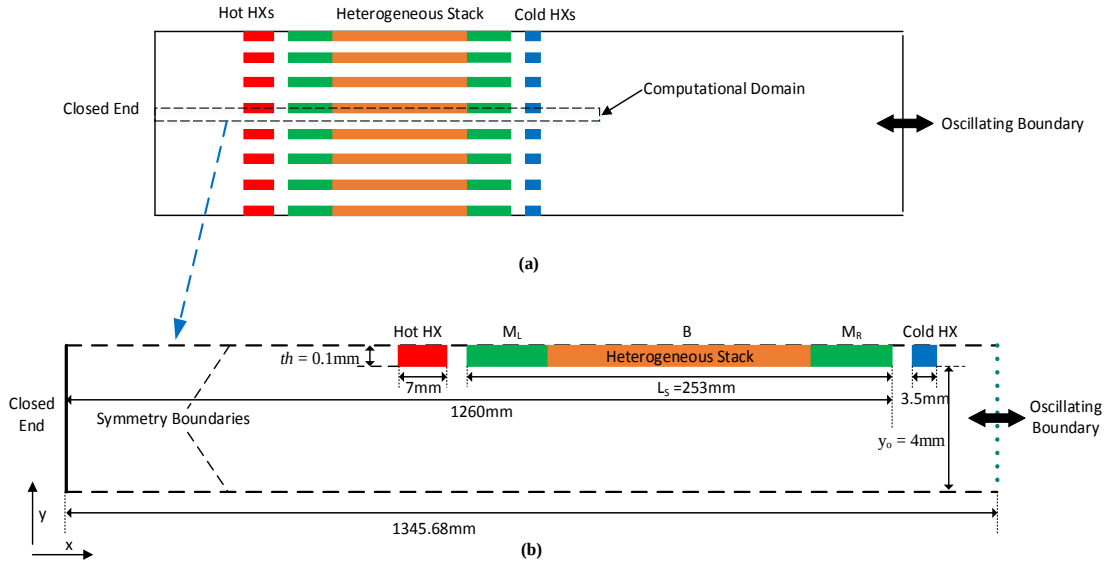


Figure 5.1: Geometric details of SWTAR (a) Schematic model (b) Computational domain

5.1.1 Mathematical modeling and governing equations

Complete thermoacoustic phenomenon is solved numerically using fully developed compressible continuity, Navier-Stokes and energy equations inside the fluid domain. Only transient heat conduction equation is solved for solid domains. Since the developed numerical model is 2-D thus the governing equations, neglecting the body forces, for both fluid and solid domain are given by:

Fluid domain:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (5.1)$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \vec{V}) = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} \quad (5.2)$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v \vec{V}) = -\frac{\partial p}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \quad (5.3)$$

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\rho \left(\epsilon + \frac{V^2}{2} \right) \right] + \nabla \cdot \left[\rho \left(h + \frac{V^2}{2} \right) \vec{V} \right] \\ &= \rho \dot{q} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) - \frac{\partial(Up)}{\partial x} - \frac{\partial(vp)}{\partial y} \\ &+ \frac{\partial(u\sigma_{xx})}{\partial x} + \frac{\partial(u\sigma_{yx})}{\partial y} + \frac{\partial(v\sigma_{xy})}{\partial x} + \frac{\partial(v\sigma_{yy})}{\partial y} \end{aligned} \quad (5.4)$$

Solid domain:

$$\rho C_p \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S \quad (5.5)$$

where ρ , u , v , $\vec{V}$, p , ϵ , h , V , $\dot{q}$, k , C_p , T and S are density, x-velocity, y-velocity, velocity vector, pressure, internal energy, enthalpy of fluid, velocity magnitude, heat transfer rate, thermal conductivity coefficient, constant pressure heat capacity, temperature and heat source respectively. σ_{ij} is stress tensor where i specifies the stress component's direction and j specifies the surface orientation on which stress is acting.

Energy flux density:

For problems involving convection heat transfer energy flux density ($\dot{e}$) is a useful parameter for visualizing the amount and direction of energy transfer. The x and y components of $\dot{e}$ can be represented by following equations:

$$\dot{e}_x = \rho u \left(\frac{1}{2} V^2 + h \right) - k \frac{\partial T}{\partial x} - (u \sigma_{xx} + v \sigma_{yx}) \quad (5.6)$$

$$\dot{e}_y = \rho v \left(\frac{1}{2} V^2 + h \right) - k \frac{\partial T}{\partial y} - (u \sigma_{xy} + v \sigma_{yy}) \quad (5.7)$$

and viscous stress tensors, σ_{ij} , in the above equations are defined as:

$$\sigma_{xx} = \mu \frac{2}{3} \left[2 \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right] \quad (5.8)$$

$$\sigma_{yy} = \mu \frac{2}{3} \left[2 \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right] \quad (5.9)$$

$$\sigma_{xy} = \sigma_{yx} = \mu \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] \quad (5.10)$$

where, μ is the dynamic viscosity, h is the specific enthalpy and rest of variables are same as defined in equations (5.1) to (5.5).

Acoustic power:

Acoustic power applied at inlet is given by relation:

$$\dot{W}_{ac} = \frac{1}{2} P_1 u_1 \quad (5.11)$$

Heat flux:

Heat flux across the cold and hot heat exchangers are calculated by using the equation:

$$\dot{q} = -k \nabla T \quad (5.12)$$

All the fluid properties are changing periodically over one complete cycle of a acoustic wave. Thus for a better evaluation all major performance parameters are averaged over one

acoustic cycle and the values are calculated. The time averaged values for $\dot{e}_y$, $\dot{W}_{ac}$ and $\dot{q}$ can be calculated by using following formulation:

$$\langle \dot{E}_y \rangle_t = \frac{1}{\tau} \int_0^\tau \int_0^l \dot{e}_y dx dt \quad (5.13)$$

$$\langle \dot{W} \rangle_t = \frac{1}{\tau} \int_0^\tau \int_y^{y_0} \dot{W}_{ac} dy dt \quad (5.14)$$

$$\langle \dot{Q} \rangle_t = \frac{1}{\tau} \int_0^\tau \int_0^l \dot{q} dx dt \quad (5.15)$$

5.1.2 Model Space

The computational domain used for the current work comprise of a thermoacoustic couple consisting of single plate of stack, similar to one proposed by Ishikawa [49] but with enhanced solution domain. The chosen computational domain is shown in Figure 5.1(b). It includes hot and a cold heat exchanger placed at either side of stack. Both the stack and heat exchangers have been modeled as solids with non-zero thickness. A gap of 0.5 mm is placed between each heat exchanger and stack end. For simplicity of analysis, the thickness of stack and heat exchangers have been kept same. Total length of computational domain is $1/8 \lambda + 0.0085\lambda$ where λ , is the wavelength of acoustic wave. The cold (right) end of stack is placed at a distance of $1/8 \lambda$ from the closed end. The width of domain is $(y_0 + th)$ where y_0 is half of stack spacing and th is half of the thickness of stack and heat exchangers. Left end of the domain is closed while oscillating boundary conditions are applied at right end of domain. Stack plate can be classified as either homogeneous stack made of single layer of material or heterogeneous stack made of different layers of materials stacked together. The heterogeneous stack is divided into three layers i.e. material layer at the right

side (M_R), base material in the middle (B) and material layer at left side (M_L). Heat exchangers material is taken as copper and lengths of hot and cold exchangers are taken as 7 mm and 3.5 mm respectively.

Nine different cases are run for comparison and evaluation of SWTAR performance under various configurations and combinations of heterogeneous stacks. It includes different combinations of stacked materials for same length and different lengths of stacked materials for same combination. Overall length of stack (L_s) and width of domain (y_0+th) is kept constant in all simulations and are given by $L_s=253$ mm and $y_0+th = 4.1$ mm. Two base materials namely Celcor and Kapton are considered in the analysis. For materials at right and left side, two different materials namely RVC and Aluminum (Al) foam are considered. Details of these cases are summarized in Table 5-1. The simulation process is carried out from the onset of process until the steady state is reached while the temperature of hot exchanger is kept fixed. Looking at Table 5-1, all simulations can be categorized into three main group. At first, two cases are run with homogeneous stacks consisting of base materials only thus comprising the first group with prefix B. The second group having prefix C consists of four different combinations of base and side stacked materials. The combinations are made such that alternating layers of stacks act as conducting and insulating materials inside the stack. Third group with Prefix PD analyzes the effect of varying the length of different layers on the performance of the device. For said purpose, the length of stacked materials at either side is varied uniformly as multiple of particle displacement and base material length is adjusted in such a way that the total length of stack remains constant at 11PD.

Table 5-1: Details of all test cases simulated in the current study. $PD = 23$ mm, $\delta_k = 2.4$ mm, half stack spacing, $y_o = 4$ mm, half plate thickness, $th = 0.1$ mm, overall stack length, $L_s = 253$ mm

Sample Name	Stack Materials	Length in terms of PD		
		Base	M_R	M_L
B1	Celcor	11.0	-	-
B2	Kapton	11.0	-	-
C1	RVC,Celcor,RVC	9.0	1.0	1.0
C2	Al,Celcor, Al	9.0	1.0	1.0
C3	RVC,Kapton,RVC	9.0	1.0	1.0
C4	Al,Kapton,Al	9.0	1.0	1.0
PD0.5	RVC,Celcor,RVC	10.0	0.5	0.5
PD2	RVC,Celcor,RVC	7.0	2.0	2.0
PD3	RVC,Celcor,RVC	5.0	3.0	3.0

5.1.3 Boundary conditions

Boundary conditions applied at each boundary of computational domain are defined as:

$$\left. \begin{array}{l} u = 0 \\ v = 0 \\ \frac{dT}{dx} = 0 \end{array} \right\} \quad \text{at closed end.}$$

$$\left. \begin{array}{l} \frac{du}{dy} = 0 \\ v = 0 \\ \frac{dT}{dx} = 0 \end{array} \right\} \quad \text{at fluid symmetry boundaries.}$$

$$\left. \frac{dT}{dy} = 0 \right\} \quad \text{at solid symmetry boundaries}$$

$$\left. \begin{array}{l} u = 0 \\ v = 0 \\ T_{fluid} = T_{solid} \end{array} \right\} \quad \text{at surface of solids.}$$

Oscillatory boundary requires special treatment where standing wave conditions corresponding to an open end needs to be implemented at truncated domain. The pressure (P), temperature (T) and velocity (u) at oscillating end are given by:

$$P = P_m + \text{Re}\{P_1 e^{i\omega t}\} \quad (5.16)$$

$$u = \text{Re}\{u_1 e^{i\omega t}\} \quad (5.17)$$

$$T = T_m + \text{Re}\{T_1 e^{i\omega t}\} \quad (5.18)$$

where, ReXXX represents the real part of oscillating pressure P_1 , oscillating temperature T_1 and oscillating velocity u_1 respectively. T_m and P_m are mean temperature and pressure. Time dependence of all parameters is given by $e^{i\omega t}$ term where ω is the angular frequency given by $\omega = 2\pi f$ and f is the normal frequency. P_1 , u_1 and T_1 for a standing wave are further defined as:

$$P_1 = P_A \cos(kx) \quad (5.19)$$

$$u_1 = -i \frac{P_A}{\rho_m a} \sin(kx) \quad (5.20)$$

$$T_1 = \frac{P_A}{\rho_m C_p} \cos(kx) \quad (5.21)$$

where, P_A is the pressure amplitude at pressure antinode, k is the wave no. given by $2\pi/\lambda$, ρ_m is the mean density, a is the speed of sound and C_p is the specific heat at constant pressure. Thus, the finalized boundary conditions at oscillatory end are given by:

$$P = P_m + P_A \cos(kx) \cos(\omega t) \quad (5.22)$$

$$u_1 = \frac{P_A}{\rho_m a} \sin(kx) \sin(\omega t) \quad (5.23)$$

$$T = T_m + \frac{P_A}{\rho_m C_p} \cos(kx) \cos(\omega t) \quad (5.24)$$

5.1.4 Operating conditions

Initial conditions at start of all test cases are given as follows:

$$\left. \begin{array}{l} u = 0 \\ v = 0 \\ T = T_m \end{array} \right\} \quad \text{at } t = 0 \text{ at all } x \text{ and } y \text{ cells.}$$

As thermal penetration depth, δ_k and particle displacement length, PD are defined as:

$$\delta_k = \sqrt{2k_f / \rho_m C_p \omega} \quad (5.25)$$

$$PD = 2u_1 / \omega \quad (5.26)$$

Based on the values given in Table 5-2 for operating frequency (f), mean pressure (P_m) and mean temperature (T_m), reference thermal conductivity (k_f') and heat capacity (C_p) the calculated values of δ_k and PD are 2.3 mm and 24 mm respectively.

5.1.5 Material Properties

Helium gas has been used as working fluid and operating conditions and gas properties common to all simulations have been summarized in Table 5-2. Solid materials in all simulated test cases are modeled with constant material properties and values considered for various solids are tabulated in Table 5-3.

Table 5-2: Operating conditions and properties common to all simulations

Property	Value	Unit
Working fluid	Helium	-
Mean Temperature, T_m	298	K
Mean pressure, P_m	10	kPa
Operating frequency, f	100	Hz
Oscillating pressure amplitude, P_A	170	Pa
Drive Ratio, $DR = P_A/P_m$	1.7	%
Wavelength, λ	10.08	m
Hot heat exchanger's temperature (constant)	298	K
Gas properties:		
Reference thermal conductivity, k'	0.152	W/m-K
Reference dynamic viscosity, μ'	1.98×10^{-5}	kg/m-s
Heat capacity, C_p	5190	J/kg-K
Prandtl no., σ	0.69	-

Table 5-3: Properties of various solid materials used in simulations

Material	C_p (J/kg-K)	k (W/m-K)	ρ (kg/m ³)
Celcor	1000	1.46	2300
Kapton	1090	0.12	1420
RVC	1260	0.033	49.5
Al Foam	895	5.8	216
Copper	385	400	8960

Working fluid, Helium, is considered as an ideal gas where density is governed by equation of state. Viscosity and thermal conductivity of Helium has been modeled using power law, as proposed by Ishikawa [49], with $n = 0.647$. Thus, dynamic viscosity and thermal conductivity of gas are given by:

$$\mu = \mu' \left(\frac{T}{T'} \right)^n \quad (5.27)$$

$$k = k' \left(\frac{T}{T'} \right)^n \quad (5.28)$$

where, μ, k are dynamic viscosity, thermal conductivity at temperature T of fluid respectively. While μ' and k' are dynamic viscosity and thermal conductivity at the reference temperature, T' .

5.1.6 Numerical Implementation

ANSYS Fluent CFD package has been used to perform the simulations. Double precision parallel computation with unsteady formulation has been utilized for the analysis. Both flow and energy equations are solved for capturing the physical thermoacoustic phenomena while conjugate heat transfer algorithm has been utilized to model the heat transfer between the fluid and solid boundaries. As the Mach No. of current simulation setup is 0.01 and critical Reynold no. ($Re = 2u_1/\sqrt{\nu\omega}$) is always below 400 so turbulence is neglected and laminar flow conditions are modeled.

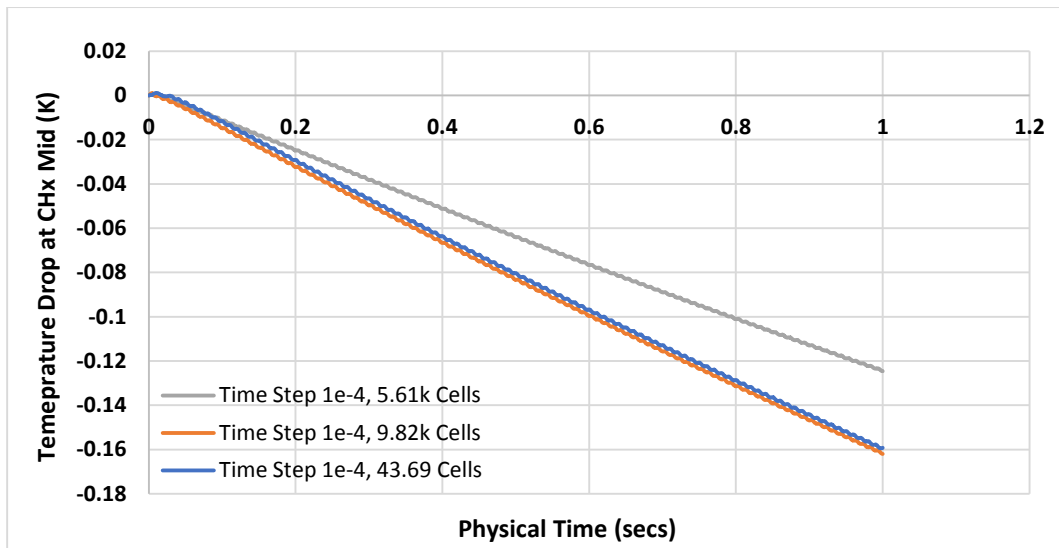
Pressure based solver with absolute velocity formulation has been selected for completing the transient analysis. First order implicit scheme has been used for temporal discretization and spatial discretization has been carried out using second order upwind schemes. For pressure interpolation, high accuracy pressure staggering scheme (PRESTO) is selected while pressure implicit with splitting of operators (PISO) is enabled for pressure velocity coupling.

Selection of appropriate time step is of utmost importance to ensure fast convergence and accurately capturing the cyclic thermoacoustic refrigeration process. Thus sufficient

resolution of each waveform is ensured by selecting a fixed time step of 5×10^{-5} secs for all simulations. As a result, every single oscillation period (τ) is captured in 200 time steps ($\Delta t = \tau/200$). Justification for selection of this time step is presented in detail in the mesh independence study section. Maximum no. of allowable iterations in each time step is 125 however convergence is achieved in between 25 to 30 iterations with the selected time step.

5.1.7 Mesh independence study

A mesh independence study has been conducted to make the simulation model independent of grid spacing and time step effects. For said purpose, simulations have been carried out with homogeneous stack made of stainless steel and profile of temperature drop produced at middle of cold heat exchanger has been recorded for a time period of 1 sec in each simulation. Three different spatial grids with mesh elements of 43.59k, 9.82k and 5.17k cells and three different time steps namely 5×10^{-4} secs, 1×10^{-4} secs and 1×10^{-5} secs have been analyzed. The results of these cases are shown in Figure 5.2(a) and Figure 5.2(b).



(a)

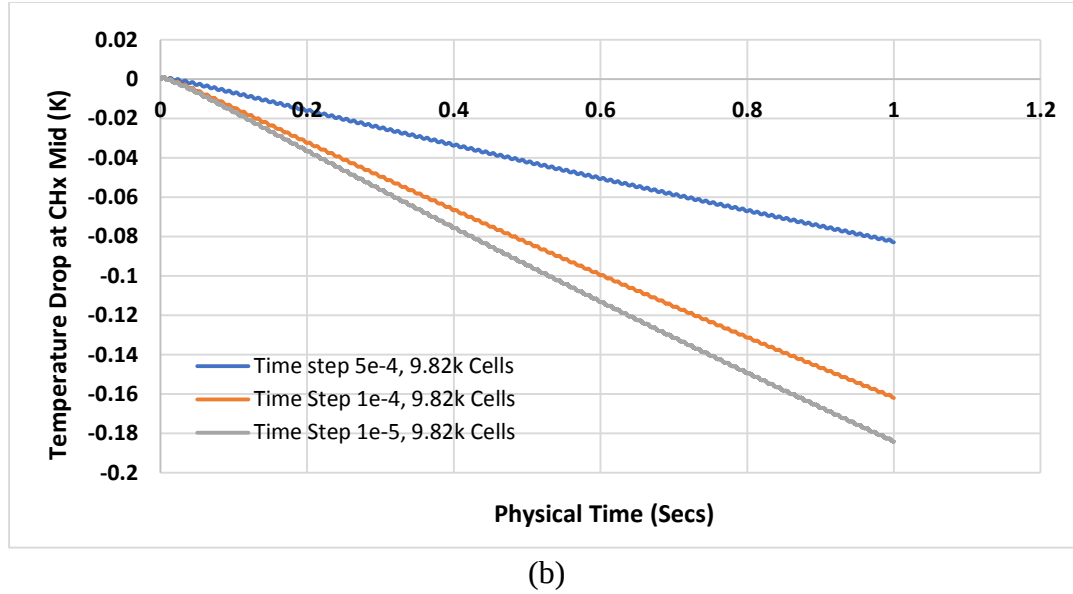


Figure 5.2: Mesh independence study with respect to (a) grid spacing (b) time increment

To have a convergent solution independent of spatial and temporal resolutions and save computational time a simulation setup with 9.82k mesh elements and time step of 5×10^{-5} secs, which is two hundredth of applied time period ($\Delta t = \tau/200$), has been chosen for conducting every simulation.

5.1.8 Monitor points

For the evaluation of results thirteen monitor points have been added in the simulation domain and details are shown in Figure 5.3. Nine points (3-11) are located within the stack region of simulation domain with three points located along the solid surface of plate, three in the gas stream at distance of 0.5 mm from the plate and three along the symmetry line. Span wise location of each of these three set of points are as follows: points 3, 6 & 10 at start (hot side), points 4, 7 & 10 at mid and points 5, 8 & 11 at end (cold side) of stack plate. Rest of four points are located outside stack region with two points (1& 2) in hot heat exchanger region and two (12 & 13) in the cold heat exchanger region. Points 1 & 12, are

located at mid of horizontal solid surfaces of hot and cold heat exchanger while points 2 & 13 are located in the gas stream at distance of 0.5 mm from the solid surface. The main purpose of the points along the solid surfaces of plates is to monitor the temperatures/heat transfer rates of solid domain interacting with the oscillating fluid, while points in the fluid region can be utilized for measuring temperatures of gas stream near the solid walls and along the symmetry line of stack channel.

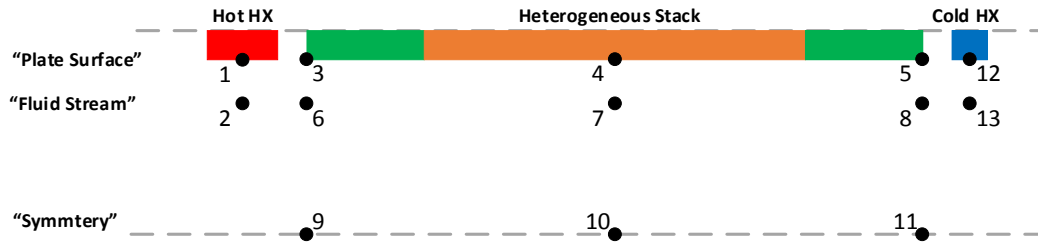


Figure 5.3: Monitors points for evaluation of results

5.1.9 Performance Scales

Performance metrics has been developed for the evaluation and comparison of performance of all cases. The development of refrigeration process from onset until reaching steady state is an important parameter as it depicts the lowest possible temperature achievable and also the rate at which a device would deliver the desired cooling. Very few numerical studies have addressed this phenomenon and most of the studies have performed simple calculations analyzing the SWTAR performance within the stack region only. The Root Mean Square (RMS) value of temperature difference achieved at mid of cold heat exchanger (ΔT_{CHx}), monitor point 12, has been used to gauge this parameter. ΔT_{CHx} is given by relation:

$$\Delta T_{CHx} = T_{RMS|CHx} - T_m \quad (5.29)$$

where, $T_{RMS|CHx}$ is the root mean squared value of cold exchanger temperature at monitor point 12 and its value has been averaged over complete acoustic cycle and is given by equation:

$$T_{RMS|CHx} = \sqrt{\frac{1}{\tau} \int_{t_0}^{t_0+\tau} T_{CHx}^2 dt} \quad (5.30)$$

Coefficient of performance (COP) and time averaged total energy flux in y-direction across the stack ($\langle \dot{E}_y \rangle_t$) has also been calculated for each case. COP of SWATR is given by the equation:

$$COP = \frac{\langle \dot{Q}_{CHx} \rangle_t}{\langle \dot{W} \rangle_t} \quad (5.31)$$

where, $\langle \dot{Q}_{CHx} \rangle_t$ is the time averaged heat flux across the cold heat exchanger surface and $\langle \dot{W} \rangle_t$ is the time averaged acoustic power applied at inlet.

Second major performance metrics employs the distribution of time averaged values at various segments of domain where the averaging has been done over complete cycle for every parameter under investigation. Time averaged temperature distribution over the stack plate, $\langle T_{stack} \rangle_t$, has been utilized to evaluate the axial distribution of temperature for various material and length combination of stacks. Time averaged energy flux density distribution across the stack plate's fluid solid boundaries, $\langle \dot{e} \rangle_t$ is a very useful parameter and it has been plotted in majority of SWATR numerical studies. As the current work involves stacks of non-zero thickness so both the horizontal and vertical surfaces will experience the energy flux. Thus the quantity $\langle \dot{e}_y \rangle_t$ is monitored along the horizontal stack surfaces to calculate the energy flux density in y-direction.

Similarly the time averaged distribution of heat fluxes through the cold and hot exchanger surfaces i.e. $\langle \dot{q}_{CHx} \rangle_t$ & $\langle \dot{q}_{HHx} \rangle_t$, have been monitored to evaluate the device performance in terms of heat transfer rates at heat exchangers.

Temperature difference between the solid surface of stack and gas stream is also an important parameter as it dictates the heat transfer mechanism between solid and gas and can be defined as:

$$\Delta T_{1|gas \& solid} = T|_{y=y_0} - T|_{y=y_0-0.2\delta_k} \quad (5.32)$$

For effective evaluation of heat transfer process the profile of $\Delta T_{1|gas \& solid}$ parameter over a complete period of acoustic cycle has been monitored at two different locations i.e. stack cold end ($x = 1.26$ m) & mid of cold exchanger ($x = 1.2625$ m). The monitor points at each locations are 5 & 8 and 12 & 13 respectively.

Time required to reach the steady state for each simulation is estimated to be at least ≥ 300 secs (5 mins) which corresponds to computational time of 207.82 days on Intel Xeon E52650, 2.60 GHz CPUs. As the accomplishment of steady state at cold heat exchanger requires a very lengthy computational time and resources. Therefore, all the measurements are recorded for a physical time of 30 secs for all test cases and simulation is extended to 90 secs for the selected cases and performance evaluation has been carried out at these time scales. Steady state values of ΔT_{CHx} for each case are estimated using exponential extrapolation. In the first stage, for validation of the exponential function an exponential fitting was performed on graphical data provided in the published work of Rahpiema et al. [65]. For said purpose, fitting was conducted on the time history plot of ΔT_{CHx} produced for sample case T0.1. The exponential function utilized to reproduce the results is given by equation:

$$\Delta T = a + b.(1 - e^{c.t}) + d.(1 - e^{f.t}) \quad (5.33)$$

where a , b , c , d and f are fitting constant and t is the global time for the simulation. The proposed function reproduced the graph with a Root Mean Square Error (RMSE) of 0.48%. Since the current setup is exactly similar to the one used by Rahpiema et al. [65] with the exception that homogeneous stack has been replaced with heterogeneous stack. Thus, this exponential function has been further utilized to extrapolate the results for cases of current work.

5.2 Results and discussion

Application of standing wave conditions at right side (oscillating end) of truncated domain resulted in a sinusoidal oscillating pressure (P_1) with an amplitude of 113.6 Pa while the amplitude of oscillating velocity (u_1) achieved is around 7.955 m/s. The development of pressure and velocity at start of simulation is shown in Figure 5.4. From the Figure 5.4 it is apparent that velocity profile shows a transient behaviour at start but after three to four cycles it becomes periodic and have a mean velocity component of approximately 0.111 m/s. These conditions of pressure and velocity at inlet will result in constant acoustic power inside the domain as the major geometric and operating parameters, i.e. L_s , y_0 , th , stack center position (x_n), P_m and DR , are constant in all the cases. The time averaged value of this acoustic power for one complete cycle is calculated as, $\langle \dot{W} \rangle_t = 263$ W.

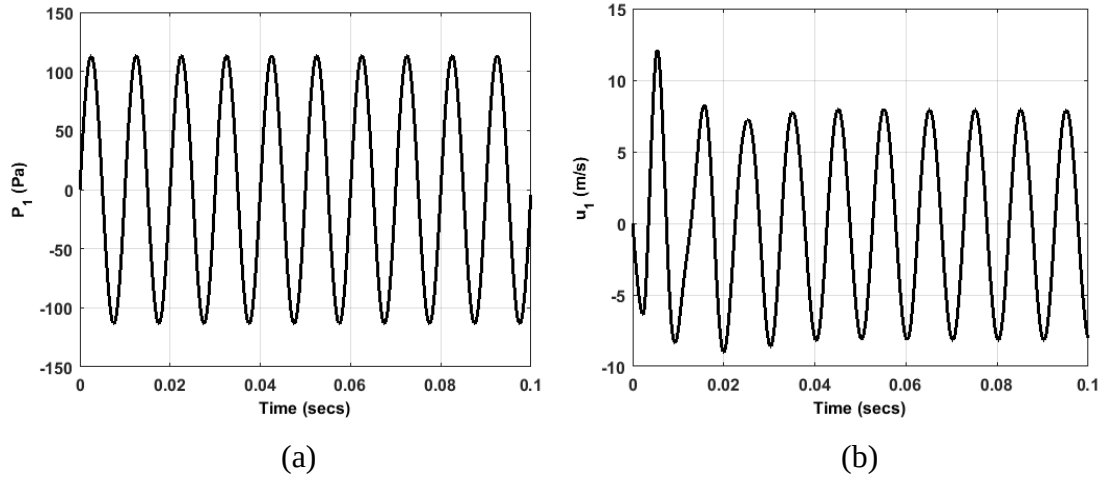


Figure 5.4: Amplitudes of (a) Pressure, P_1 (b) Velocity, u_1 at oscillating boundary

5.2.1 Effect of different material combinations of heterogeneous stack

5.2.1.1 Celcor based material combinations

Performance results of SWTAR for two material combinations of stacks having Celcor as base material i.e. C1 and C2 have been tabulated in Table 5-4. For comparison and better evaluation the results obtained from homogenous stack made of Celcor (B1) have also been tabulated. It is worth mentioning that length of stacked layer at either side, in both C1 and C2, is 23 mm (equivalent to 1PD) and the length of base material is 207 mm.

Table 5-4: Results for Celcor based material combination of stack

Case	ΔT_{CHx} (K)		$\langle \dot{E}_y \rangle_t$ (W)	COP
	30 secs	Steady state		
B1	2.42	4.59	79.26	0.254
C1	3.63	5.79	75.64	0.261
C2	3.47	5.69	76.89	0.255

The results obtained clearly depicts that the presence of heterogeneous layers in the stack have a significant effect on the refrigeration performance. Temperature drop, ΔT_{CHx} , values calculated for cases B1, C1 and C2, after 30 secs, are 2.42 K, 3.63 K and 3.47 K respectively and their transient profiles are depicted in Figure 5.5 (a). The cooling process is extrapolated for said cases using the function given by Eqn. (5.34) and profiles of ΔT_{CHx} obtained as result are shown in Figure 5.5 (b). The values of fitting constants used to generate these results are provided in Table 5-7.

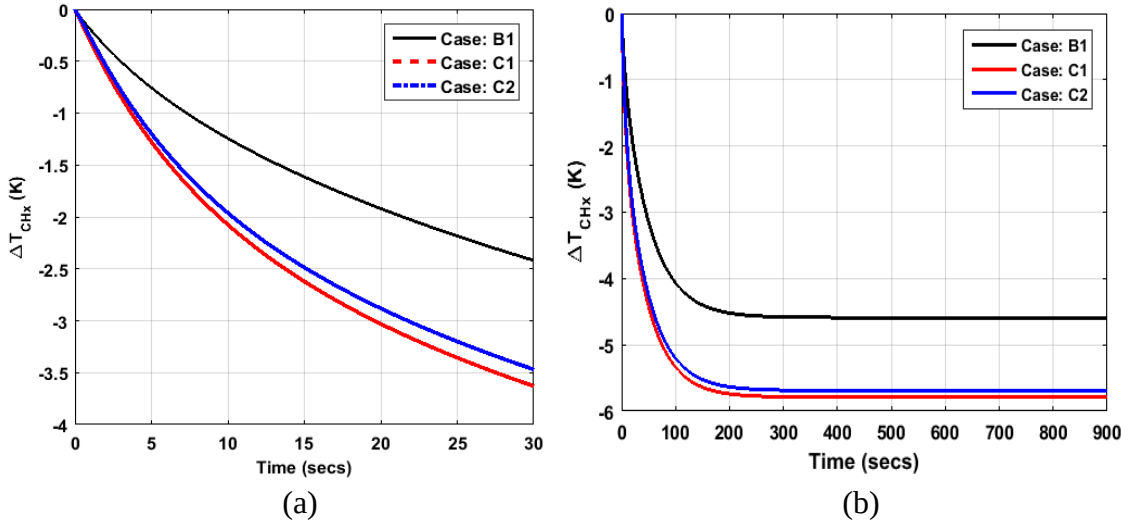


Figure 5.5 ΔT_{CHx} calculated for Celcor based material combinations of heterogeneous stack (a) 30 secs data (b) steady state value

It is clearly evident that, based on 30 secs data, the time required by the cooling process to reach stability is at least 300 secs. The estimated steady state values for cases B1, C1 and C2 are 4.59 K, 5.79 K and 5.69 K respectively. Thus, heterogeneous stacks C1 and C2 produced approximately 26.14 % and 23.96 % higher refrigeration temperatures than homogenous stack B1.

Ideally stack should act as perfect thermal barrier between hot and cold sides to allow maximum heat transfer to take place at heat exchangers. Energy flux density is a useful

parameter in analyzing the energy transfer and parameter $\langle \dot{E}_y \rangle_t$ indicates the time averaged energy absorbed by stack during a complete cycle. Larger the value of $\langle \dot{E}_y \rangle_t$ lower is the performance of stack and from the results given in Table 5-4 it can be seen that case C1 has the lowest value followed by C2 and then B1. The values of $\langle \dot{E}_y \rangle_t$ obtained for C1, C2 and B1 are 75.64 W, 76.89 and 79.24 W respectively. Thus $\langle \dot{E}_y \rangle_t$ consumed by the stacks of C1 and C2 are less than stack of B1 by an amount of 4.55 % and 2.97 % respectively.

COP values for current simulation setup is indicative of amount heat transfer taking place at cold heat exchanger as the acoustic power, $\langle \dot{W} \rangle_t$ is constant for all cases. Higher the heat transfer at cold heat exchanger, $\langle \dot{Q}_{CHx} \rangle_t$ higher is the COP of the device and better is the performance of SWTAR. The COP values obtained for cases B1, C1 and C2 are 0.254, 0.267 and 0.255 respectively again proving C1 as the best combination. COP of SWTAR has been enhanced by an amount of 5.12 % and 0.39 % for cases C1 and C2 respectively when compared with case B1.

Ideally the material of the stack should have low thermal conductivity and high specific heat capacity so that it could act as a perfect thermal barrier between the hot and cold side. Among the three materials considered in the above combinations, RVC foam has the lowest thermal conductivity, followed by Celcor and Aluminum (Al) foam while specific heat capacity of RVC is highest followed by Celcor and Al foam. Thus the combination C1 has the best properties, suitable for an ideal stack, among all the three cases i.e. B1, C1 and C2, resulting in highest value of temperature drop (ΔT_{CHx}), lowest value of $\langle \dot{E}_y \rangle_t$ consumed and highest COP value for SWTAR.

The effect of having an insulating material layers at either side of stack can be best visualized and evaluated by distribution of energy flux density over whole length of stack. Time averaged energy flux is positive for heat leaving the fluid and entering the plate surface and negative for heat leaving the plate surface. Figure 5.6 shows the distribution of time averaged energy flux, $\langle \dot{e}_y \rangle_t$ for the three Celcor based material combinations i.e. B1, C1 and C2. For homogeneous Celcor stack (B1), a negative peak is observed at cold end of stack ($x = 1.260$ m) resulting in transfer of heat from the solid plate to fluid at this end. This peak shifts towards left by an amount equal to 1PD for the cases C1 and C2 due to the presence of different material at the end. The width of the peak is quite large for all the cases accounting to almost 60 % of stack length ($x = 1.160$ m or $0.6 L_s$). Despite the shifting of the negative peak towards left for heterogeneous stacks the change in the direction of energy from negative to positive occurs at almost the same location along the stack for all three cases. The transition location is at $x = 1.105$ m from the origin which is approximately $0.38 L_s$ depicting asymmetric distribution of energy flux along the plate. This type of asymmetry is in fact present in every thermoacoustic couple due to the variable velocity amplitude along the stack length but it is not captured by models having stacks with zero thickness. Since the simulation hasn't reached steady state where the total time averaged energy flux becomes zero the transition from the positive to negative energy flux is occurring at value higher than zero.

For the case of C1 where Celcor has been replaced by RVC foam at the stack ends the negative peak is still observed at the right end (cold side) of middle Celcor layer but it dies out quickly for RVC part. Negligible heat transfer take place along the RVC length and insulation zone is formed at the end. As a result during the expansion phase of acoustic

wave when the gas particle would travel towards the right side along the cold end of stack it will absorb minimal heat from the stack and will maintain its lower temperature. Upon reaching the cold heat exchanger there will be a maximum temperature difference available between the solid and gas resulting in a higher heat transfer. The same is true for the compressive cycle but for compressive phase the gas particle will now travel towards left. Thus it will enter the stack plate and will maintain its high temperature due to presence of insulation zone. The net result of formation of insulation zone is the decrease in the overall energy flux of stack which allows more heat transfer to take place at cold heat exchanger location thus enhancing the performance of device.

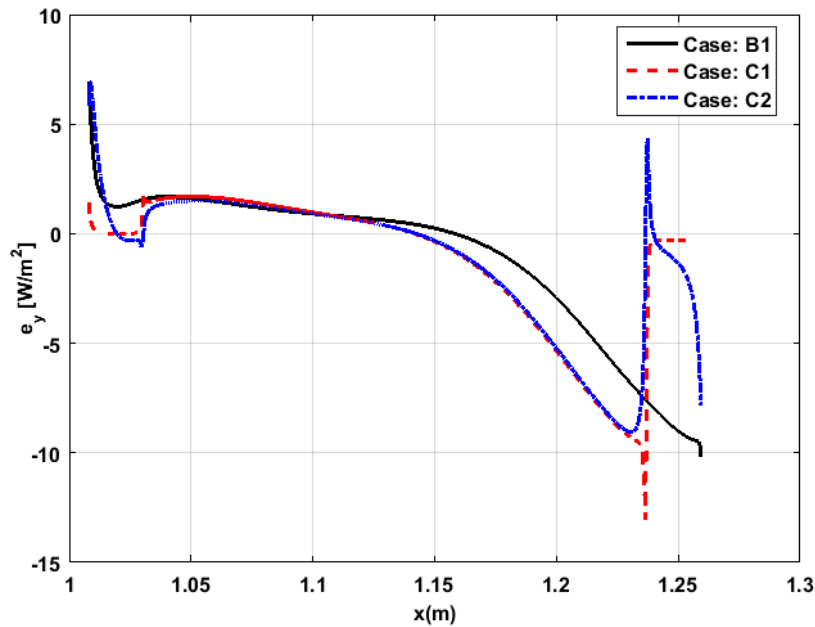


Figure 5.6: $\langle \dot{e}_y \rangle_t$ distribution, after 30 secs, for Celcor based material combinations

For combination C2 in which Al foam has been placed at the ends of stack similar phenomenon in the energy flux distribution is observed. However in this case, another energy flux peak is observed at end of Al layer (cold end) having a lower amplitude compared to end of Celcor layer. Thus, resulting in a small heat transfer taking between the fluid and solid inside the stack in this region. It will raise the temperature of gas particle

a little compared to RVC case and will lower the refrigeration performance of device compared to performance of combination C1. Presence of interface between two materials at location of 1PD results in the formation of extra sharp peak due to numerical inconsistencies.

Time averaged axial temperature distribution over stack plate surface for cases B1, C1 and C2 has been shown in Figure 5.7. It shows that at the temperature of plate is fairly constant around mean temperature value in the region of $0.3 L_s$ to $0.55 L_s$ and it starts decreasing along the plate length where the energy flux density becomes negative. Although the temperature of the hot exchanger is kept fixed at $T_m = 298$ K in the boundary conditions. A high temperature region is observed at hot side of stack due to presence of positive energy flux in that region, as shown in Figure 5.6. As the results are plotted for only 30 secs data so the temperature distribution along the plate is not saturated and uniform temperature gradient will develop across the stack plate once the steady state has been reached.

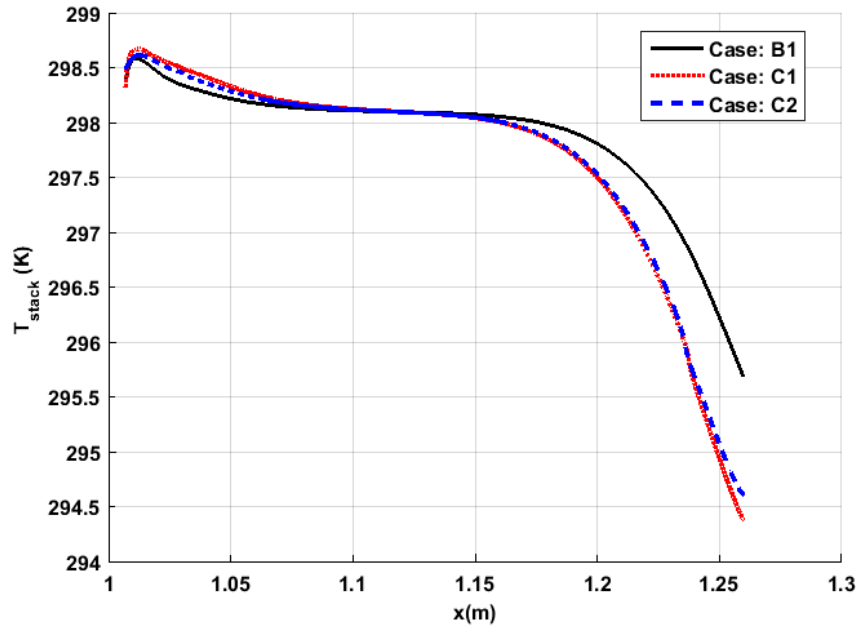
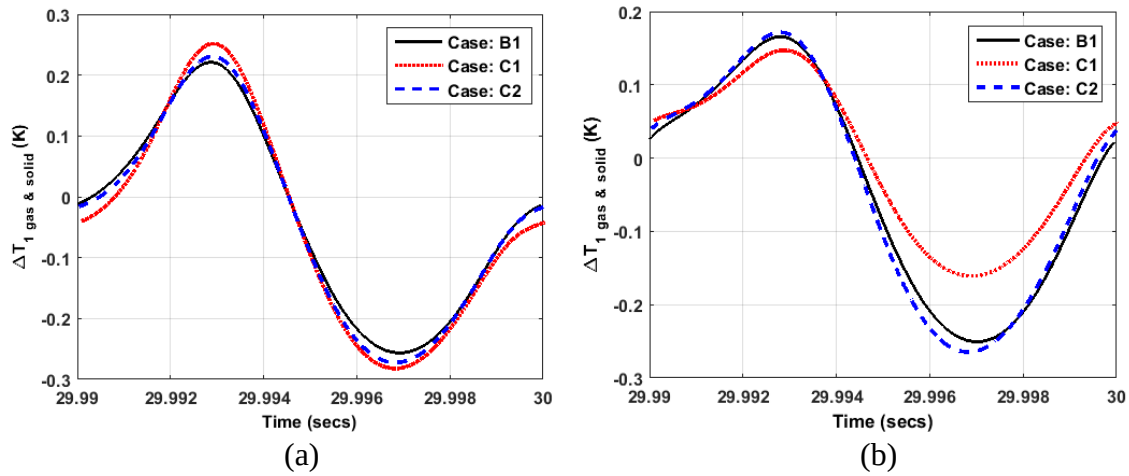


Figure 5.7: Time averaged temperature distribution over stack plate, $\langle T_{stack} \rangle_t$, after 30 secs for Celcor based material combinations

Temperature difference between the solid surface of stack and gas stream is also an important parameter as it dictates the heat transfer mechanism between solid and gas particles. Figure 5.8(a) and Figure 5.8(b) compares the profiles of $\Delta T_{1|_{gas \& solid}}$, for one complete cycle, for cases B1, C1 and C2 at two different location i.e. at mid of cold exchanger and at cold end of stack. Higher the peak of $\Delta T_{1|_{gas \& solid}}$ higher is the heat transfer at that location. By looking at Figure 5.8(a) it can be inferred that, at the mid of cold exchanger, the maximum value of $\Delta T_{1|_{gas \& solid}}$ is recorded for combination C1 and minimum for B1 while combination C2 lies in between the two. Thus, C1 has the highest heat transfer rate resulting in lowest temperature achieved for this case. Further for an efficient refrigeration, stack must act as perfect insulated barrier as much as possible i.e. its temperature should remain close to gas temperature so that minimum heat transfer take place inside the stack. Therefore a complete reverse trend is observed at cold end of stack, as shown in Figure 5.8(b).



**Figure 5.8: Comparison of $\Delta T_{1|_{gas \& solid}}$ for Celcor based combinations after 30 secs at (a) cold heat exchanger
(b) stack cold end**

Thus the combination C1 is having lowest values of $\Delta T_{1|_{gas \& solid}}$ at cold end of the stack. These findings further confirm the reasoning provided in the explanation of energy flux distribution at stack ends for heterogeneous stack.

5.2.1.2 Kapton based material combinations

Performance results obtained for Kapton based material combinations, i.e. B2, C3 and C4, are given in Table 5-5 and calculation of ΔT_{CHx} has been shown in Figure 5.9(a). The plots of the steady state values achieved for ΔT_{CHx} , extrapolated using exponential function given by Eqn. (5.33), are shown in Figure 5.9(b). The values of fitting constants used to generate the results are provided in Table 5-7. The extrapolated data once again predicted that the stability conditions are achieved after 300 secs of simulation. The pattern of results obtained is similar to one achieved for Celcor based combinations however the overall performance for each case is slightly better than its Celcor based counterpart. It can be attributed to the fact that Kapton has lower thermal conductivity and higher specific heat capacity value compared to Celcor.

Table 5-5: Results for Kapton based material combinations of stack

Case	ΔT_{CHx} (K)		$\langle \dot{E}_y \rangle_t$ (W)	COP
	30 secs	Steady State		
B2	2.81	5.57	77.23	0.247
C3	3.92	6.36	74.07	0.268
C4	3.73	6.33	75.37	0.261

Combination C3 has produced the best results with a value of 3.92 K, achieved in 30 secs, for ΔT_{CHx} . The total energy flux density absorbed by stack $\langle \dot{E}_y \rangle_t$ is 74.07 W and value of

COP achieved is 0.268. Based on the steady state results both combinations of heterogeneous stacks i.e. C3 and C4 reached the refrigeration temperatures that are approximately 14.18 % and 13.64 % lower than homogenous Kapton stack B2. Similarly the values of $\langle \dot{E}_y \rangle_t$ absorbed by C3 and C4 are lower than B2 by an amount of 4.09 % and 2.41 % respectively. Device COP has been increased by 8.50 % and 5.67 % for cases C3 and C4 respectively. The explanation for case C3 being the best one is similar to one given for C1 except that the base material has now changed from Celcor to Kapton.

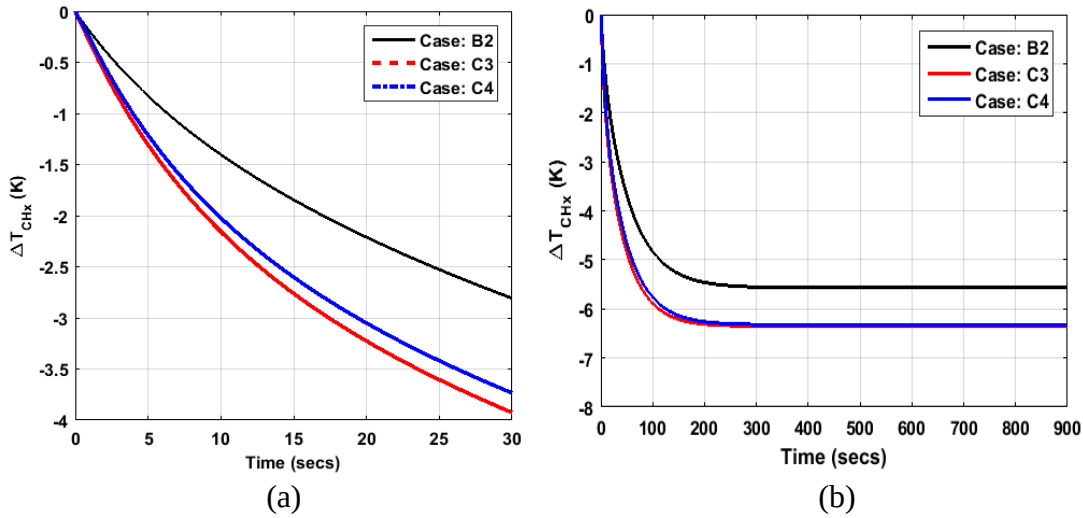


Figure 5.9: ΔT_{CHx} calculated for Kapton based material combinations of heterogeneous stack (a) 30 secs data (b) steady state value

Distribution of time averaged energy flux density, $\langle \dot{e}_y \rangle_t$ for the cases B2, C3 and C4 have been plotted in Figure 5.10. The results obtained are similar to cases investigated previously with the negative peak amplitudes smaller than one obtained through Celcor based cases. The negative peak of energy flux density for case B1 ends at cold end of stack and shifts towards the left, at the interface between the two materials, for cases C3 and C4. $\langle \dot{e}_y \rangle_t$ becomes approximately zero for RVC layer in C3 while for Al foam of C4 a second negative peak is observed. As a result, combination C3 produced the best refrigeration

performance. The energy flux density distribution is again asymmetric and transition from positive to negative direction takes place around $0.31 L_s$ from the hot end.

Time averaged temperature distribution for Kapton based material combinations, shown in Figure 5.11, also depicts a small rise in temperature at hot end due to positive energy flux available in that area. Central portion of stack is approximately at mean temperature value and the stack plate gradually cools down from $x = 1.122 \text{ m}$ ($0.45 L_s$) onwards until the cold end is reached where the temperatures are closer to values obtained at cold heat exchanger.

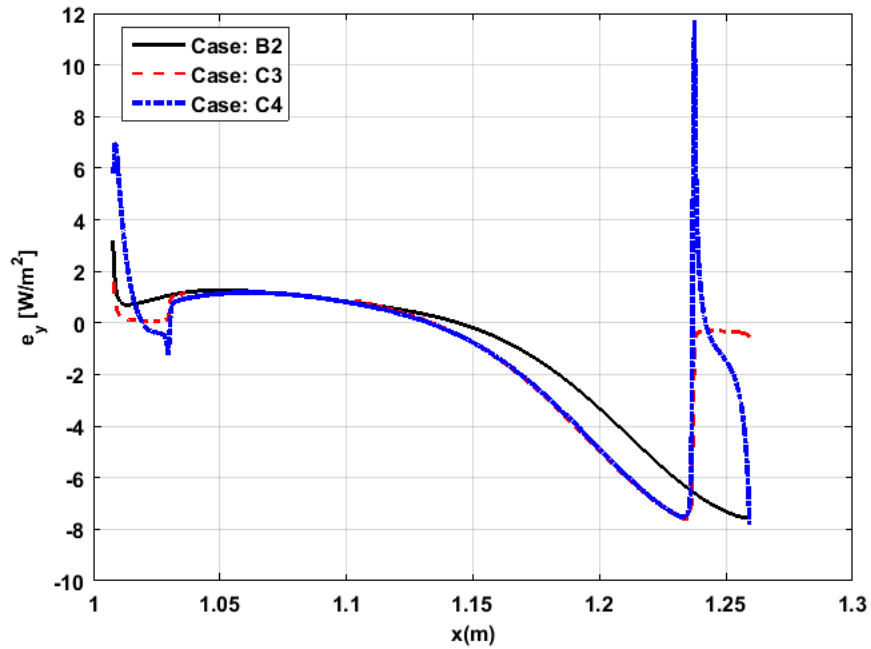


Figure 5.10: $\langle \dot{e}_y \rangle_t$ distribution after 30 secs for Kapton based material combinations

A comparison of $\Delta T_{1|_{gas \& solid}}$ at two different locations i.e. mid of cold exchanger and at stack cold end are plotted in Figure 5.12(a) and Figure 5.12(b). The findings are similar to one attained in Figure 5.8 i.e. Combination C3, having Kapton as the base and RVC foam at the ends, has maximum temperature difference and subsequently highest heat transfer rate resulting in maximum refrigeration performance. Thus the performance results

displayed in Table 5-5 for the cases B2, C3 and C4 are confirmed with reasoning similar to the one given earlier for Celcor based combinations.

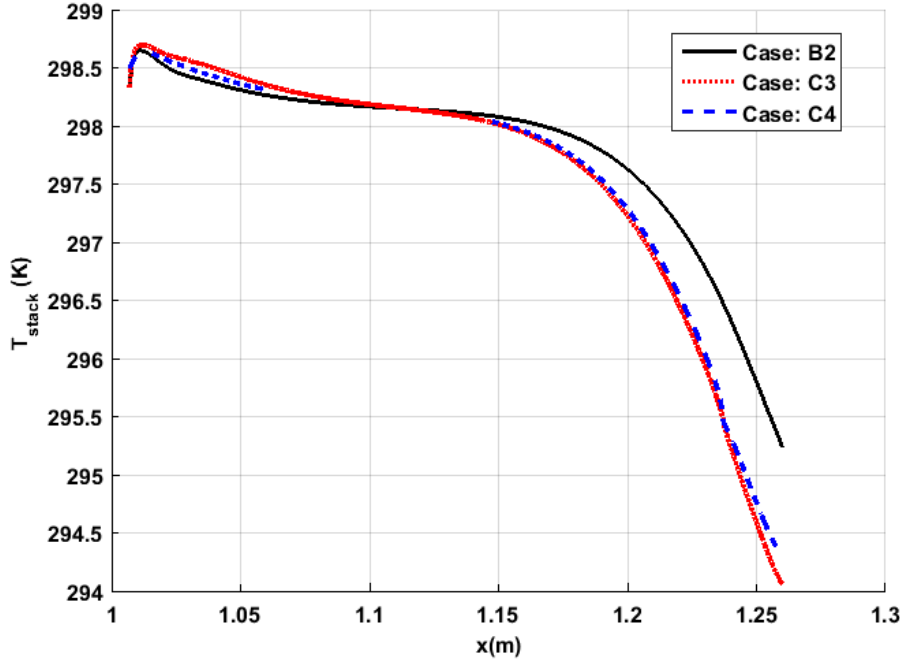


Figure 5.11: Time averaged temperature distribution over stack plate, $\langle T_{stack} \rangle_t$, after 30 secs for Kapton based material combinations

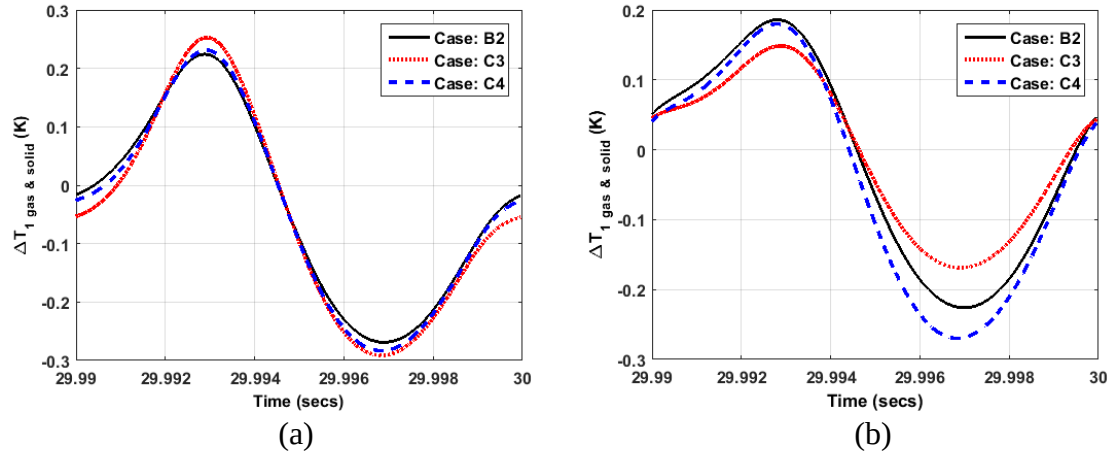


Figure 5.12: Comparison of $\Delta T_{1|_{gas \& solid}}$ for Kapton based combinations after 30 secs at (a) cold heat exchanger (b) stack cold end

For effective assessment of performance of different material combinations of the stack the performance parameters such as time averaged energy flux, $\langle \dot{E}_y \rangle_t$ and COP are calculated for same temperature drop produced by each case. For said purpose, value of ΔT_{CHx}

obtained for case B1 after 30 secs, i.e. 2.42 K, has been selected. The results obtained are then normalized with respect to values obtained for case B1 for comparative study and are depicted in Figure 5.13 in the form of bar graph.

Results, displayed in Figure 5.13, in terms of $\langle \dot{E}_y \rangle_t$, show that for given cooling load all stack combinations consume less energy flux when compared with the value absorbed by Celcor stack (case B1). The trend obtained is same as one obtained for 30 secs data. $\langle \dot{E}_y \rangle_t$ values for combinations C1 and C2 1.64 % and 0.39 % less than their homogeneous counterpart B1. Combination C1 showed the best performance among Celcor based stacks followed by C2. Similarly for Kapton based stacks, combination C3 produced the best results followed by C4 and then B2. The values of $\langle \dot{E}_y \rangle_t$ obtained for B2, C3 and C4 are less than $\langle \dot{E}_y \rangle_t$ value of the case B1 by 1.54 %, 2.95 % and 2.13 % respectively.

Similarly, for a given temperature drop, combinations C1 and C3 have the highest COP values among Celcor and Kapton based stacks. The COP value for C1 is 12.81 % higher than COP value of B1 and this margin increases to 17.18 % for C3. Performance improvement in terms of COP, taking B1 as base case, for cases C2, B2 and C4 are 8.08 %, 3.35 % and 13.63 % respectively.

To conclude, the presence of combination of layers of different materials inside the stack improve the SWTAR performance significantly. Heterogeneous stacks produce lower refrigeration temperatures, consume less energy and have higher COP value than their homogeneous counterparts. Among all the material combinations of heterogeneous stack investigated, stacks having a material with a lower thermal conductivity at ends i.e. RVC foam produced the best performance.

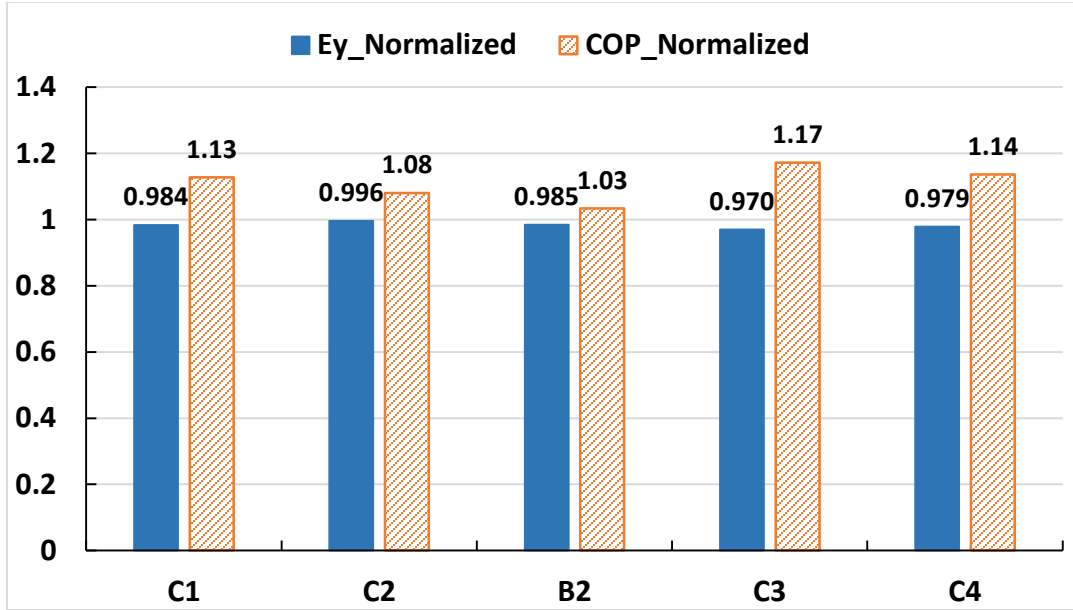


Figure 5.13: Values of $\langle \dot{E}_y \rangle_t$ and COP normalized w.r.t values of case B1 for $\Delta T = 2.42$ K

5.2.2 Effect of variation in length of layers of heterogeneous stack

RVC based heterogeneous stacks have produced the best performance among all material combinations evaluated. Thus heterogeneous stack combination made of Celcor as base and RVC foam at both ends (RVC, Cel, RVC) have been selected for further evaluation of variation in length of individual segments of stacks. It is worth mentioning that the overall length of stack (L_s) has been kept constant i.e. 253 mm and length of individual layers have been varied as function of PD which is equivalent to 23 mm for current setup. Based on the naming conventions defined in Table 5-1 following length cases are investigated: PD0.5, C1 or (PD1), PD2 and PD3.

5.2.2.1 Performance after 30 secs

Performance results obtained, based on 30 secs data, for all the length cases are summarized in Table 5-6. It can easily be seen that as the length of side stack layer is increased gradually from 0.5 PD to 3PD the value of ΔT_{CHx} achieved is increased. Case

PD0.5 observed the smallest temperature drop with a value of 3.05 K and largest drop is calculated for PD3 which is 5.33 K. The values of ΔT_{CHx} obtained for C1 and PD2 are 3.63 K and 4.61 K respectively.

The temporal development of ΔT_{CHx} has been shown in Figure 5.14(a) while the steady state values are extrapolated using the exponential function given in Eqn. (5.33) and are depicted in Figure 5.14(b). Values of fitting constant used are given in Table 5-7. Estimated steady state values, based on 30 secs data, are 5.02 K, 5.79 K, 7.10 K and 8.68 K for cases PD0.5, C1, PD2 and PD3 respectively. Thus the refrigeration capacity of SWTAR is enhanced by almost 72.91 % as the length of RVC layer is increased from 0.5PD to 3PD.

Table 5-6: Results for different length combinations of stack

Case	ΔT_{CHx} (K)		$\langle \dot{E}_y \rangle_t$ (W)	COP
	30 secs	Steady State		
PD0.5	3.05	5.02	82.90	0.255
C1	3.63	5.79	75.64	0.261
PD2	4.61	7.10	77.60	0.276
PD3	5.33	8.68	69.69	0.291

COP value also increases with an increase in the length of side stacked layers meaning that as the length of RVC portion is increased the device performance is enhanced by allowing higher heat transfers to take place at cold heat exchanger location. The values obtained for cases PD0.5, C1, PD2 and PD3 are 0.255, 0.261, 0.276 and 0.291 respectively. Performance enhancement of device in terms of COP is 14.12 % as RVC length is increased from 0.5 PD to 3PD.

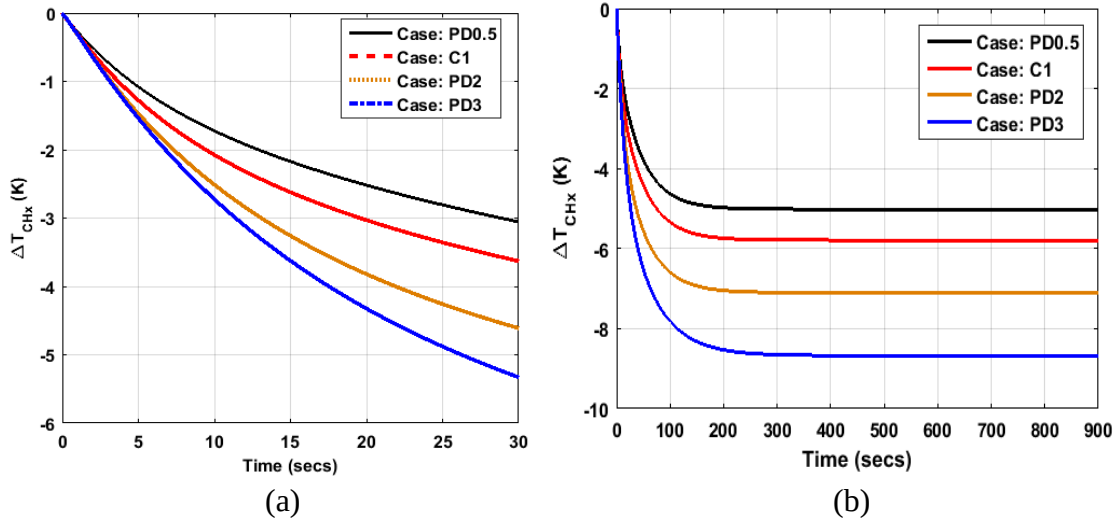


Figure 5.14: ΔT_{CHx} calculated for different length combinations of heterogeneous stack (a) 30 secs data (b) steady state value

Performance results obtained for $\langle \dot{E}_y \rangle_t$ show a peculiar behaviour where highest energy flux is absorbed by stack of PD0.5 followed by PD2 stack and then C1 and PD3 in the decreasing order. The value of energy flux absorbed by PD0.5 is even higher than value observed for homogeneous stack case B1 and difference is 3.64 W which accounts for 4.1% more energy flux for the length case PD0.5. The time averaged values of total absorbed energy flux are 82.9 W, 75.64 W, 77.6 W and 69.69 W for cases PD0.5, C1, PD2 and PD3 respectively. Thus combinations PD0.5 and PD2, despite having better performance than B1 and C1, consumed more energy flux in the same time span of 30 secs. Based on the results of all three parameters, combinations C1 and PD3 have better overall performance than PD0.5 and PD2 with combination C3 having the best results.

Axial distribution of time averaged energy flux density, $\langle \dot{e}_y \rangle_t$, over entire length of stack plate has been shown in Figure 5.15(a). It can be seen that the negative energy peaks observed at the right end of Celcor layer die out and energy flux density becomes almost negligible in the entire length of RVC layer for every length case. Thus larger the length

of RVC layer larger is the insulation zone created inside the stack and higher is the performance of SWTAR. The enhanced device performance is once again attributed to capacity of stack to have minimal heat transfer with the fluid allowing the fluid parcel to extract maximum heat transfer from cold heat exchanger during the expansion phase. The peculiarity observed in the values of $\langle \dot{E}_y \rangle_t$ tabulated in Table 5-6 is also observed here and it is concerned with the energy transfer behaviour of Celcor component of stack. Highest negative peak amplitude is observed for PD0.5 stack followed by PD2 and the case C3 has the lowest peak. Peak amplitude for the case C1 is lower than PD0.5 and PD2 but higher than C3. The behaviour can easily be seen in Figure 5.15(b) which shows the zoom in of the area around cold end of stack. Further detailed investigation is required for complete assessment of different length arrangements that can be made inside the stack for better performance.

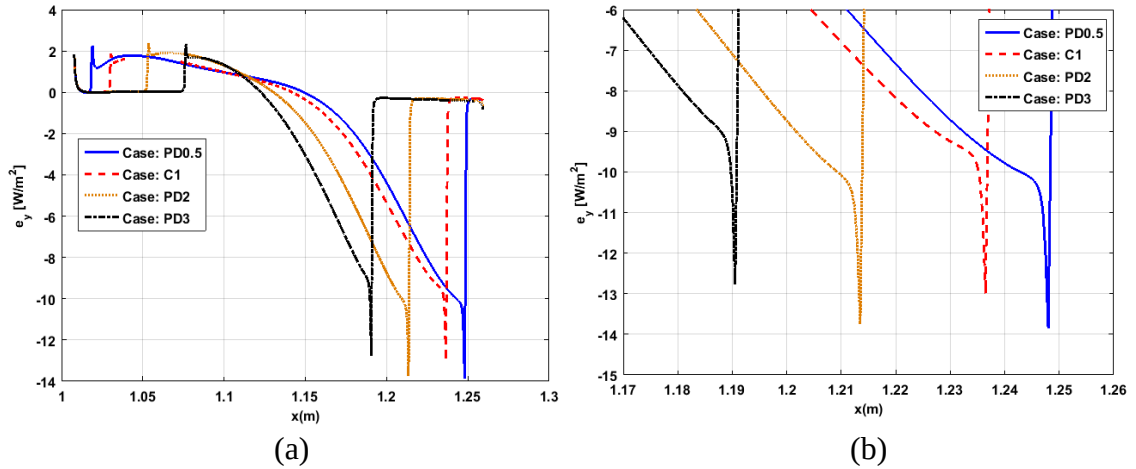


Figure 5.15: $\langle \dot{E}_y \rangle_t$ distribution after 30 secs for different lengths combinations of individual layers of heterogeneous stack (a) entire stack length (b) zoom in at cold end

Figure 5.16 show the axial distribution of time averaged temperature along the stack plate. The central area of stack has fairly constant temperature with value equivalent to T_m while the temperature drops gradually from $x = 1.122\text{m}$ (or $0.45 L_s$) until the cold end of stack.

Hot end of stack has slightly higher temperatures due to presence of positive energy flux in that region and the values are almost constant for entire length of RVC layer highlighting the effect of low thermal conductivity material.

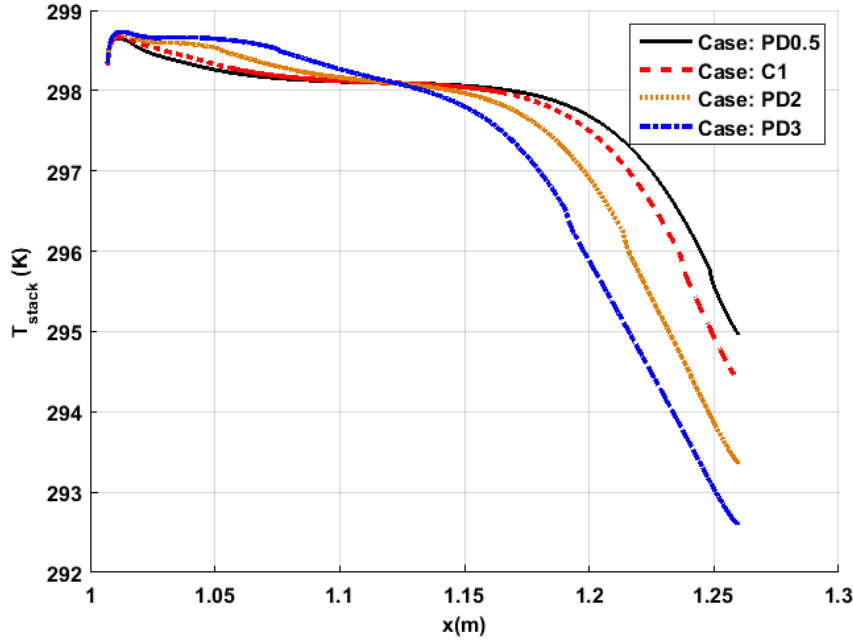


Figure 5.16: Time averaged temperature distribution over stack plate, $\langle T_{stack} \rangle_t$, after 30 secs for different length combinations

Plots in Figure 5.17 show the profile of $\Delta T_{1|_{gas \& solid}}$ for all the four cases with length variation i.e. PD0.5, C1, PD2 and PD3 respectively. Results are again plotted for one acoustical cycle and at same two locations i.e. mid of cold heat exchanger and cold end of stack. At mid of cold exchanger, Figure 5.17(a), maximum temperature difference is observed for case PD3 having largest length of RVC placed at both sides of the stack. The difference is decreased as these lengths are reduced and a minimum value is obtained for PD0.5. The peculiar behavior of PD2 and PD0.5 stacks observed in the results of energy flux is also observed in the profiles of $\Delta T_{1|_{gas \& solid}}$ at the cold end of stack shown in the Figure 5.17(b). Highest value of $\Delta T_{1|_{gas \& solid}}$ is recorded for case PD2 followed by PD0.5, PD3 and C1 respectively in a decreasing fashion.

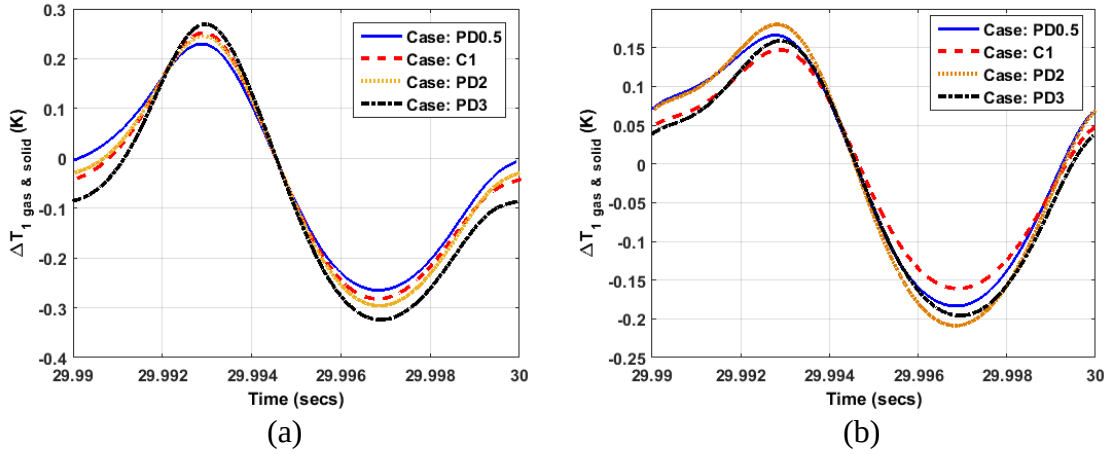


Figure 5.17: Comparison of $\Delta T_{1|gas \& solid}$ for different length combinations after 30 secs at (a) cold heat exchanger (b) stack cold end

For an effective performance comparison of all length combinations the results of $\langle \dot{E}_y \rangle_t$ and COP are evaluated for same temperature drop achieved by each case at cold heat exchanger ΔT_{CHx} . The value of 2.42 K achieved for case B1 in 30 secs is selected as reference and performance of all length cases is evaluated for this cooling load. Once, again results for $\langle \dot{E}_y \rangle_t$ and COP are normalized w.r.t values of case B1 and presented in Figure 5.18. The findings of results are similar to one obtained for 30 secs data. It can be seen that for equal cooling load, PD0.5 and PD2 consume higher energy flux than B1 while the values obtained for C1 and C3 are lower. The energy absorbed by PD0.5 exceeds the value of B1 by 6.36% and by 3.24 % for PD2. Length combinations C1 and C3 consumed 1.63 % and 5.49 % less energy flux than B1. COP of SWTAR is increased as the length of RVC layer inside the stack is increased. When compared with results of homogeneous Celcor stack (B1) COP of PD0.5 case showed 5.06 % improvement and increase noted for C1, PD2 and PD3 is 12.80 %, 32.79 % and 45.67 % respectively.

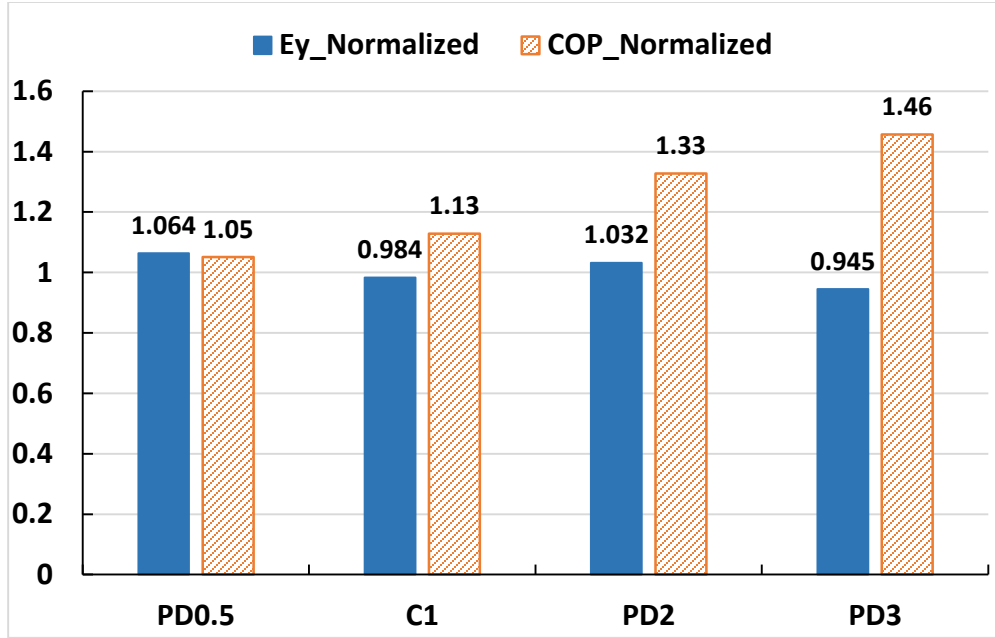


Figure 5.18: Values of $\langle E_y \rangle_t$ and COP normalized w.r.t to values of case B1 ($\Delta T_{CHx} = 2.42$ K) for various length combinations

Table 5-7: Fitting constants for exponential extrapolation

Case / Fitting constant	a	b	c	d	e
B1	-0.01396	-0.5908	-0.1857	-3.985	-0.02029
B2	-0.006375	-0.7127	-0.1563	-4.853	-0.01884
C1	0.005943	-1.628	-0.1462	-4.168	-0.02222
C2	0.03373	-1.525	-0.1499	-4.201	-0.02147
C3	0.008403	-1.487	-0.147	-4.876	-0.02347
C4	0.03299	-1.411	-0.1473	-4.947	-0.02177
PD0.5	-0.004335	-1.094	-0.1786	-3.919	-0.02313
PD2	0.03284	-2.535	-0.1038	-4.602	-0.02197
PD3	0.04365	-3.659	-0.0758	-5.061	-0.01776

5.2.2.2 Performance after 90 secs

Extended data is required for the better estimation of steady state values and evaluation of performance trends. Thus the simulations of selected cases have been continued to obtain extended data. Since length cases were having some peculiar behaviour and required further investigation for confirmation. To cater this aspect and keeping in mind the limiting computational resources available simulations for cases B1, C1, PD2 and PD3 have been continued to obtain the extended performance data.

The performance results obtained after 90 secs of simulation are provided in Table 5-8. ΔT_{CHx} and COP results obtained for 90 secs data reconfirmed the findings of 30 secs data by showing that the performance of SWTAR is increased as length of RVC layer is increased from 1 to 3 PD. The transient development of calculated ΔT_{CHx} is shown in Figure 5.19(a). Values of ΔT_{CHx} obtained for the cases B1, C1, PD1 and PD3, after 90 secs, are 4.27 K, 5.53 K, 6.69 K and 7.71 K respectively and COP values calculated are 0.252, 0.265, 0.275 and 0.284.

In terms of $\langle \dot{E}_y \rangle_t$, homogeneous Celcor stack (B1) absorbed highest energy flux, with a value of 80.61W, thus resulting in the lowest refrigeration performance. As predicted previously in the 30 secs data, PD2 stack once again showed different trend compared to results of ΔT_{CHx} and COP and is having highest value among the three selected length combinations of heterogeneous stacks. The values of time averaged total energy flux, $\langle \dot{E}_y \rangle_t$, calculated for cases PD2, C1 and C3 are 73.08 W, 71.62 W and 65.13 W.

Table 5-8: Results based on extended data for different length combinations of stack

Case	ΔT_{CHx} (K)		$\langle \dot{E}_y \rangle_t$ (W)	COP
	90 secs	Steady State		
B1	4.27	6.44	80.61	0.252
C1	5.53	7.27	71.62	0.265
PD2	6.69	8.35	73.08	0.275
PD3	7.71	9.26	65.13	0.284

The plots of ΔT_{CHx} extrapolated using Eq. (5.33) are displayed in Figure 5.19(b) and fitting constants utilized in generation of these plots are given in Table 5-9. The values of ΔT_{CHx} estimated for steady state conditions are higher than one estimated for 30 secs data. The primary reason is the availability of more data points that can be utilize by curve fitting tool to predict the transient behaviour and estimate the time constants. Thus the predictions based on this data are more realistic and reported values of steady state conditions are 6.44 K, 7.27 K, 8.35 K and 9.26 K for cases B1, C1, PD1 and PD3 respectively. Replacement of Celcor material with RVC layer of 3PD length at stack ends resulted in net increase of 43.79 % in the refrigeration performance.

Time averaged energy flux density distribution over the stack plate for extended simulation are shown in Figure 5.20. The energy flux becomes negligible for RVC part of stack resulting in the formation of insulation zone at the ends. Larger the length of this zone lower is the energy flux occurring across the stack surface and higher is the performance of SWTAR. The negative energy peak observed for PD2 is once again higher than peaks of C1 and C3 thus confirming the results tabulated in Table 5-8.

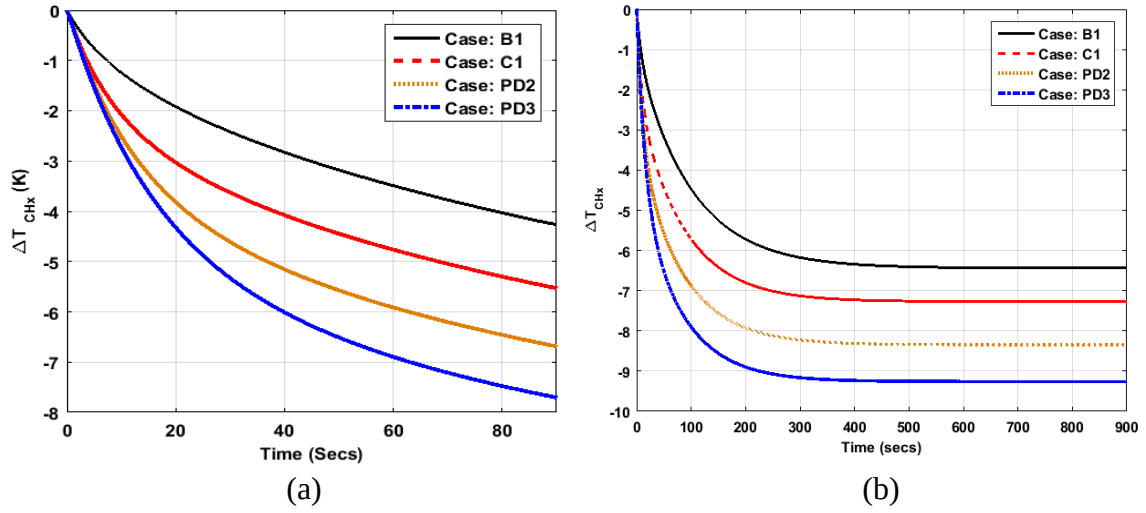


Figure 5.19: ΔT_{CHx} calculated for different length combinations of heterogeneous stack (a) 90 secs data (b) steady state value

The distribution of energy flux inside the Celcor component of stack have now straightened out resulting in the decrease in the peak values in both positive and negative direction. As the length of RVC component is increased the amount of positive energy flux occurring near hot end of stack is decreased and PD3 case has virtually zero positive energy flux. Further, when compared with 30 secs results, negative energy flux is occurring at larger proportions of stack. The distribution based on the extended data is indicating the saturation of thermoacoustic effect and approaching of stability conditions.

Axial distribution of time averaged temperature along the stack length is depicted in Figure 5.21. For all length cases, the central portion of stack that was having temperatures equivalent to T_m , seen in 30 secs results, have diminished. The hot end of stack has attained a constant temperature around 298.7 K, little higher than T_m and this temperature is maintained uptill the length of RVC layer for each case. After that it start decreasing gradually until the cold end of stack. The distribution once again highlights the

advancement of simulations towards the steady state at which the stacks would develop a constant temperature gradient across the entire length.

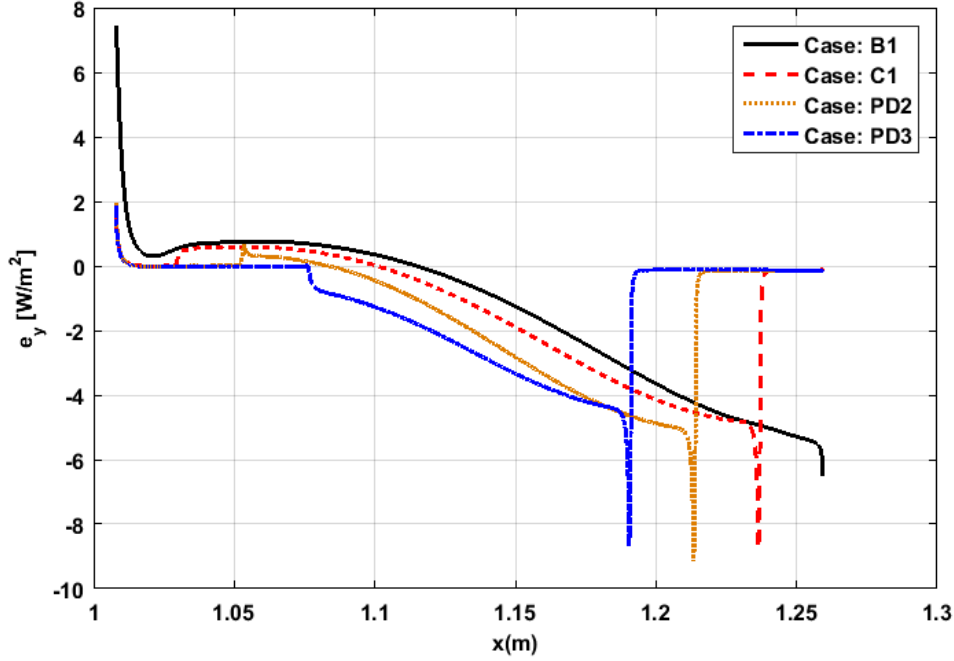


Figure 5.20: $\langle \dot{e}_y \rangle_t$ distribution after 90 secs for different lengths combinations of individual layers of heterogeneous stack

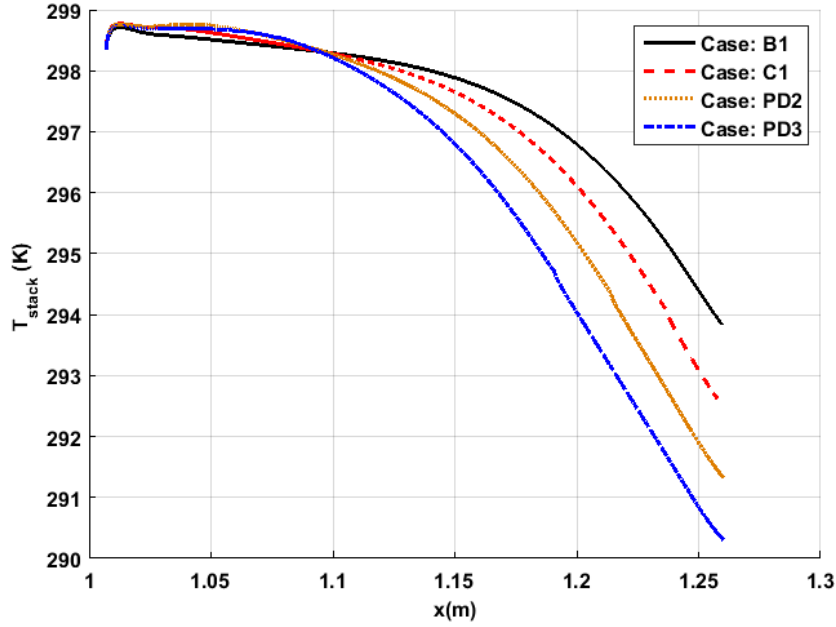


Figure 5.21: Time averaged temperature distribution over stack plate, $\langle T_{stack} \rangle_t$, after 90 secs for different length combinations

Figure 5.22 displays the profile of $\Delta T_{1|_{gas \& solid}}$ for one complete acoustic cycle at two different locations i.e. mid of cold exchanger, Figure 5.22(a), and cold end of stack, Figure 5.22(b). At cold heat exchanger location highest temperature difference is observed for PD3 followed by PD2, C1 and B1. These results reiterate the outcomes of stack performance parameters as highest heat transfer will take place for case having maximum $\Delta T_{1|_{gas \& solid}}$. As the stack should have minimal heat transfers with fluid parcels so values of $\Delta T_{1|_{gas \& solid}}$ obtained are lowest for C3 followed by C1, PD2 and B1. PD2 case has higher $\Delta T_{1|_{gas \& solid}}$ values than C1 and C3 at stack cold end again confirming the trend observed for $\langle \dot{E}_y \rangle_t$ values.

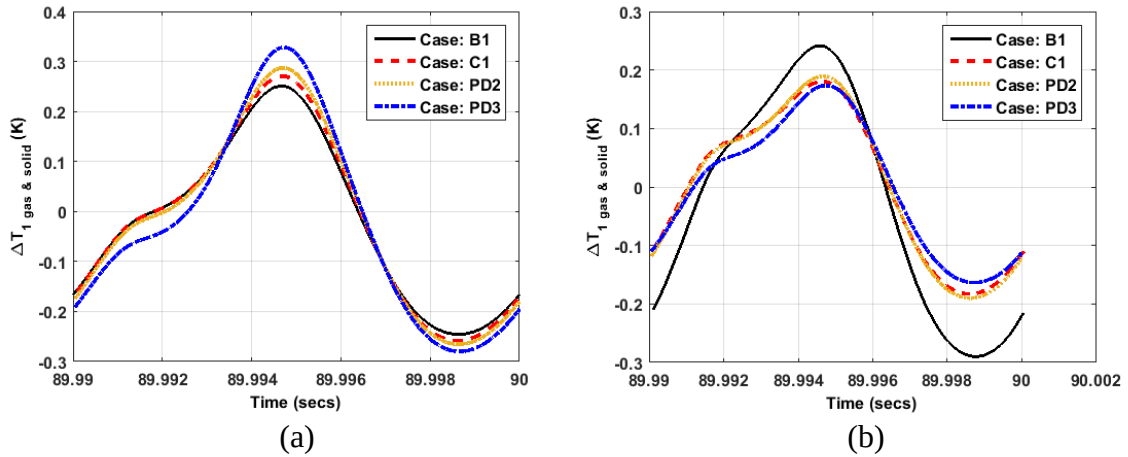


Figure 5.22: Comparison of $\Delta T_{1|_{gas \& solid}}$ for different length combinations after 90 secs at (a) cold heat exchanger (b) stack cold end

Table 5-9: Fitting constants for exponential extrapolation for extended results

Case / Fitting constant	a	b	c	d	e
B1	-0.06661	-5.387	-0.01007	-0.9842	-0.1099
C1	-0.03323	-5.117	-0.01192	-2.121	-0.1170
PD2	0.01577	-5.109	-0.01248	-3.252	-0.08966
PD3	0.03041	-5.067	-0.01317	-4.224	-0.07003

To conclude, addition of layer of insulative material at stack ends, even for lengths as small as half of particle displacement length, has a beneficial effect on the refrigeration performance of SWTAR. However maximum benefit could be obtained if the lengths of end layers are kept around 1PD and 3PD as the length combinations PD0.5 and PD2 consume much higher energy flux for the same amount cooling.

5.3 Conclusions

Effect of heterogeneous stack on the refrigeration performance of a SWTAR has been simulated numerically. Two different aspects of heterogeneous stack namely different material combinations and different lengths of stacked layers have been investigated in detail in terms of ΔT_{CHx} , $\langle \dot{E}_y \rangle_t$ and COP. Nine cases including two homogeneous stacks, four material combinations, four different length combinations have been analyzed.

It can be concluded, the presence of combination of layers of different materials inside the stack improve the SWTAR performance significantly. Heterogeneous stacks produce lower refrigeration temperatures, consume less energy and have higher COP value than their homogeneous counterparts. Among all the material combinations of heterogeneous stack investigated, stacks having a material layer with lower thermal conductivity at ends, i.e. RVC foam, produced the best performance.

The values of ΔT_{CHx} , $\langle \dot{E}_y \rangle_t$ and COP achieved using Celcor based RVC combination C1 are 5.79 K, 75.64 W and 0.261 respectively. For Kapton based RVC combination C3 these values are 6.36 K, 74.07 W and 0.268 respectively. The performance of C1 and C3, when compared with their homogeneous counterparts B1 and B2, show that: ΔT_{CHx} values have

increased by 26.14 % and 14.18 %, $\langle \dot{E}_y \rangle_t$ consumption has been reduced by 4.55 % and 4.09 % and COP of SWTAR has been enhanced by 5.12 % and 8.50 %

For the case of variation in individual lengths of stacked layers it can be concluded that, addition of layer of insulative material at stack ends, even for lengths as small as half of particle displacement length, has a beneficial effect on the refrigeration performance of SWTAR. However maximum benefit could be obtained if the lengths of end layers are kept around 1PD and 3PD as the length combinations PD0.5 and PD2 consume much higher energy flux for the same amount of cooling.

It has been found that, for a constant overall length, an increase in length of side stacked material layer results in an increase in values of both ΔT_{CHx} and COP. The maximum temperature drop, ΔT_{CHx} , of 8.68 K has been achieved for case PD3 with an improvement of 72.91 % when compared to value achieved for PD0.5. Similarly COP of SWTAR for PD3 is 0.291 which is 14.12 % higher than the value of length combination PD0.5.

Time averaged energy flux consumed by the stacks for cases PD0.5 and PD2 are higher when compared to other length combinations C1 and PD3. The values of $\langle \dot{E}_y \rangle_t$ obtained for PD0.5 is 82.90 W and for PD2 it is 77.60 W while stacks of cases C1 and C3 consumed 75.64 W and 66.69 W respectively in the same time span This trend is evident for both scenarios i.e. results obtained for same time span and results obtained for same temperature drop.

For the visualization and explanation of above results the distribution of time averaged energy flux density over stack plate ($\langle \dot{e}_y \rangle_t$) and difference in oscillating temperature of

solid surfaces and gas stream ($\Delta T_{1|gas \& solid}$) has also been plotted. The findings of distribution confirmed and elaborated the trends obtained in performance results described above. $\langle \dot{e}_y \rangle_t$ distribution showed that cases having largest negative peaks consumed highest energy flux while $\Delta T_{1|gas \& solid}$ profiles showed that cases having highest $\Delta T_{1|gas \& solid}$ values at cold exchanger location i.e. C1, C3 and PD3, have the maximum performance in terms of ΔT_{CHx} and $\langle \dot{E}_y \rangle_t$.

CHAPTER 6

Conclusions and Recommendations

6.1 Conclusions

The thesis focuses on the numerical modeling of SWTADs with a prime focus on SWTAR.

The research work conducted in the thesis covers three major areas which are as follows:

- Detailed review of the numerical modeling work that has been conducted in the field of standing wave thermoacoustic devices (SWTAD).
- Development of 1-D numerical model of SWTAR and its sensitivity study using non-dimensionlized sensitivity coefficient (NSC) method.
- Development of 2-D CFD model of SWTAR and its detailed performance evaluation for various material and length combinations of heterogeneous stack

Numerical Modeling Review: Major findings extracted from the review of numerical modeling techniques applied in the field of SWTADs are as follows:

1. Numerical models of SWTADs can broadly be divided in to 1-D linear thermoacoustic models and 2-D nonlinear CFD models.
2. 1-D linear models are limited to steady state and can only be used for initial design or general performance prediction of SWTADs. These models have been outdated with development of CFD models due to their ability to model the transient

behaviour and precise capturing of inherent nonlinearities associated with these devices.

3. CFD models are primarily focused on the stack and regions around the stack. These models have the ability to accurately simulate various nonlinear phenomenon's accounting for losses inside SWTADs i.e. asymmetrical temperature distribution, thermal buffer formation and axial heat transfer at stack ends etc.
4. Various computational domains have been proposed for CFD analysis of SWTADs. They can be categorized into three major types (a) single stack models (b) multiple stack models and (c) full stack models. Further segregation can be made in terms of components of SWTAD considered for simulation e.g. (a) stack only (b) stack and heat exchangers (c) heat exchangers only.

Sensitivity Analysis: One dimensional numerical model of small compact SWTAR has been developed using DeltaEC software and NSC method has been applied to analyze the sensitivity of the three main output parameters namely, temperature drop achieved (ΔT), COP and COPR. Five input variables i.e. oscillating pressure (p_1), stack length (L_s), stack center position (X_s), stack spacing (y_0) and stack plate thickness (L_{plate}) have been perturbed to obtain sensitivities for a wide range of three design variables namely mean pressure (P_m), mean temperature (T_m) and drive ratio (DR). The major outcomes extracted from the sensitivity analysis are as follows:

1. Oscillating pressure amplitude (p_1) has the most significant effect on performance of SWTARs and the performance is least affected by the stack length (L_s).
2. All the output parameters are found to be insensitive to small changes in L_{plate} and most sensitive to p_1 . Largest values of NSC are recorded with respect to p_1 for all

the cases with a maximum value of 24.12 obtained for ΔT and value of 17.92 for COPR.

3. COPR has been found to insensitive to perturbations in all the chosen input variables at all drive ratios.
4. Based on the findings from sensitivity analysis it can be concluded that extreme care should be taken while selecting acoustic driver for SWTAR as its accuracy has a significant effect on the overall performance of device. Further, placement of stack inside the resonator and spacing between the stack also have a significant effect thus stack must be manufactured accurately and placed precisely inside the resonator.

Simplified CFD Model: Simplified two dimensional CFD models of SWTAE and SWTAR have been developed and the simulation results have been validated with the existing published results. Heat exchangers are neglected in both numerical models and only stack component has been modeled. For performance evaluation, a specified temperature gradient has been applied on the stack plates in SWTAE and isothermal plate with no thickness has been considered for SWTAR. The major conclusions drawn from simplified CFD analysis are:

1. Despite all the simplifications the developed CFD models have been able to capture the formation of asymmetrical temperature distribution at hot side of stack for SWTAE and presence of energy flux density at stack edges for SWTAR. The results highlight the presence of hot buffer zone in the stack and heat transfer taking place at the stack ends.

2. These findings elucidate the ability of CFD solvers to capture the thermoacoustic mechanism inside SWTADs in detail.

Heterogeneous Stack: The performance of SWTAR in the presence of heterogeneous stack made of different materials with anisotropic thermal conductivity have been analyzed by enhancing the solution space of developed simplified model. The effect of various materials combinations and various lengths of individual stacked layers of heterogeneous stack has been analyzed and comparison has been made with traditional homogeneous stack for efficient evaluation. The key results of the work are as follows:

1. It can be concluded that presence of anisotropic combination of layers of different materials inside the stack significantly improves the performance of SWTAR. Heterogeneous stacks provide lower refrigeration temperatures, consume less energy and have higher COP values than their homogeneous counterparts.
2. Among all the material combinations of heterogeneous stack investigated, stacks having a material layer with lower thermal conductivity at ends, i.e. RVC foam, produced the best performance
3. The values of ΔT_{CHx} , $\langle \dot{E}_y \rangle_t$ and COP achieved using the Celcor based RVC combination C1 are 5.79 K, 75.64 W and 0.261 respectively. For the Kapton based RVC combination C3 these values are 6.36 K, 74.07 W and 0.268 respectively. The performance of C1 and C3, when compared with their homogeneous counterparts B1 and B2, show an increase in the ΔT_{CHx} values by 26.14 % and 14.18 %, reduction in the $\langle \dot{E}_y \rangle_t$ consumption by 4.55 % and 4.09 % and enhancement of SWTAR's COP by 5.12 % and 8.50 % respectively.

4. For the case of variation in the individual lengths of stacked layers, it can be concluded that an increase in the length of insulative material layer at the stack ends has a beneficial effect on the refrigeration performance of SWTAR. However maximum benefit could be obtained if the lengths of end layers are kept around 1PD and 3PD as the length combinations PD0.5 and PD2 consumed much higher energy flux for the same amount of cooling.
5. It has been found that, for a constant overall length, an increase in length of side stacked material layer results in an increase in the values of both ΔT_{CHx} and COP. The maximum temperature drop, ΔT_{CHx} , of 8.68 K has been achieved for the case PD3 with an improvement of 72.91 % when compared to the value achieved for PD0.5. Similarly COP of SWTAR for PD3 is 0.291 which is 14.12 % higher than the value of length combination PD0.5.
6. Time averaged energy flux consumed by the stacks for the cases PD0.5 and PD2 are higher when compared to the other length combinations C1 and PD3. The values of $\langle \dot{E}_y \rangle_t$ obtained for PD0.5 is 82.90 W and for PD2 it is 77.60 W while the stacks of cases C1 and C3 consumed 75.64 W and 66.69 W respectively in the same time span. This trend is evident for both scenarios i.e. results obtained for the same time span and results obtained for the same temperature drop.

6.2 Recommendations

Several recommendations for future work are as follows:

1. Operating frequency of SWTAR is an important design parameter which can have effect on the output performance measures. For the case of sensitivity study, in the

current model, the change in frequency required change in length of resonator. For simplicity, the sensitivity study of SWTAR for a range of operating frequencies has been neglected and could be incorporated for more comprehensive analysis.

2. In heterogeneous stack performance evaluation, the side stacked materials considered are porous materials such as RVC and Aluminum foam. The porosity of these material is much less than regular stack designs and should be evaluated for more accurate prediction of the heterogeneous stack performance. Thus the developed computational model can be further enhanced to analyze the effect of having different porosity in the side stacked layers.
3. In the current work, identical thickness has been considered for both the heat exchangers and the stack. However in practical heat exchangers have different plate thickness than the stack resulting in a different blockage ratios for both components. This blockage ratio further effects the flow around the stacks and thus can also be incorporated in the performance evaluation of heterogeneous stack.

APPENDIX A

Additional Tables: Literature review

Table A-1: Simulation parameters for numerical studies of SWTADs

Authors	Year	Dimension	Components	REV	Software	Flow Model	Application	Outcome
Swift [25]	1994	–	–	–	DeltaEC	–	–	DeltaEC software guide
Cao N. et al. [46]	1996	2-D	Stack Only	Single Stack	SOLA-ICE	Laminar	TAR	Heat pumping phenomenon inside TAR
Minner BL [36]	1996	1-D	Stack + HX	–	DeltaEC	–	TAR	C.O.P optimization
Worlikar [50]	1996	2-D	Stack Only	Single Stack	–	–	TAR	Compressible flow model
Raspet [33]	1998	1-D	Stack Only	–	–	–	TAE,TAR	Finite difference numerical model
Worlikar [52]	1998	2-D	Stack Only	Single Stack	–	–	TAR	Compressible flow model
Ishikawa H. et al. [49]	2002	2-D	Stack Only	Single Stack	PHOENICS	Laminar	TAR	Explanation of heat transfer phenomenon in stack
Tijani [26]	2002	1-D	Stack + HX	–	DeltaEC	–	TAR	Design of TAR
Jebali F. [34]	2004	1-D	Stack + HX	–	–	–	TAR	Performance estimation
Marx D. [53]	2004	2-D	Stack Only	Single Stack	Own Code	–	TAD	Velocity field along stack

Marx D. [54]	2005	2-D	Stack Only	Single Stack	Own Code	–	TAD	Temperature difference at stack extremities
Marx D. [55]	2005	2-D	Stack Only	Single Stack	–	–	TAR	Temperature distortion inside stack plate
Jensen [35]	2006	1-D	Stack + HX	–	Own Code	–	TAE, TAR	Novel design method for temperature calculation
Zoontjens L. [27]	2006	1-D	Stack + HX	–	ANSYS, DeltaEC	–	TAR	Inertance optimization
Paek Insu [38]	2007	1-D	Stack + HX	–	Matlab, DeltaEC	–	TAR	COPR optimization
Poignand [19]	2007	1-D	Stack Only	–	–	–	TAR	Novel standing wave like design
Srikitsuwan [37]	2007	1-D	Stack + HX	–	–	–	TAR	COPR Optimization using GA
Babaei H. [20]	2008	1-D	Stack + HX	–	–	–	TADTAR	Design and optimization
Zink F. et. al. [68]	2008	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k-ε)	TAE	CFD model for simulating onset conditions and curvature effect on performance
Lotton [21]	2009	1-D	Stack + HX	Full Stack Model	–	–	TAR	Analytical model of TAR
Wu-Feng [22]	2009	1-D	Stack + HX	–	–	–	TAR	Stack optimization
Zink F. [86]	2009	2-D	Stack Only	Axisym-metric	COMSOL/ Matlab	–	TAE	Optimization w.r.t thermal losses to surrounding
Zoontjens L. [56]	2009	2-D	Stack Only	Single Stack	Fluent	Laminar	TAR	Thickness and spacing effect on performance
Hireche O. [61]	2010	2-D	Stack + HX	Single Stack	Own Code	–	TAE	Numerical code for simulation
Ke Han Bing [66]	2010	2-D	Stack + HX	Single Stack	Own Model	Laminar	TAR	Parametric optimization

Remiorz L. [57]	2010	2-D	Stack Only	Single Stack	CFX	–	Rijke Tube	Thermoacoustic phenomenon using CFD
Tasnim [58]	2010	2-D	Stack Only	Single Stack	Star CD	Laminar	TAR	Flow field visualization around stack
Yu Guoyao [70]	2010	2-D	Stack only	Full Stack Model	Fluent	Turbulent (LES)	TAE	Utility of fluid solid coupled model over porous media model
Zink F. [48]	2010	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k-ε)	TAE	CFD model for simulating onset conditions and curvature effect on performance
Zink F. [69]	2010	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k- ε)	TADTAR	Cooling achieved through heat input
Rulik et. al [82]	2011	2-D	Stack Only	Single Stack, Multiple Stack	CFX	Turbulent (k-ε)	TAE	Device performance simulation
Tasnim [23]	2011	1-D	Stack Only	–	–	–	TAD	Porous material performance evaluation
Trapp A.C. [87]	2011	2-D	Stack Only	–	Matlab	–	TAE	Multi-objective optimization
Tasnim [59]	2012	2-D	Stack Only	Single Stack	Star CD	Laminar	TAR	Performance improvement in terms of COP
Abd El-Rahman [51]	2013	2-D	Stack only	Single Stack	Fluent	Laminar	TAR	Non-linear phenomenon capturing in high DR design.
Alahmer et al. [24]	2013	1-D	Stack + HX	–	–	–	TAR	Performance simulation of TAR
Hariharan [39]	2013	1-D	Stack + HX	Full Stack Model	DeltaEC, ANNOVA	–	TAR	Optimization using response surface methodology
Piccolo [67]	2013	2-D	Stack + HX	Half Stack	Own Code	–	TAR	Optimization
Nowak I. [83]	2014	2-D	Stack Only	Single Stack,	CFX	Turbulent (k-ε)	TAE	Effect of various stack models on performance

				Full Stack Model				
Zolpakar [40]	2014	1-D	Stack + HX	–	Matlab	–	TAR	MOGA Optimization for COP
Zolpakar [41]	2014	1-D	Stack + HX	–	Matlab	–	TAR	MOGA Optimization for COP
Namdar A. [62]	2015	2-D	Stack + HX	Single Stack	OpenFoam	Laminar	TAR	HXs parameters effect on device performance
Ramdane M. [63]	2015	2-D	Stack + HX	Single Stack	CFX	–	TAE	HX length effect on performance
Skaria M. [71]	2015	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k-ε)	TADTAR +Twin TAPM+ TWTAPM	Working fluid and P_{avg} effect on device performance
Tartibu L.K. [43]	2015	1-D	Stack Only	Single Stack	GAMS	–	TAR	Optimization using Lexicographic scheme
Alamir A. et. al [29]	2016	1-D	Stack + HX	–	DelatEC	–	TAR	Parametric performance study & working fluid effect
Badi A.P. [72]	2016	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k-ε)	TAE	Serial/Parallel configuration for multistage TAE
Lin J. [85]	2016	2-D	Stack Only	Axisymmetric	CharLES	Laminar	TAPE	Optimized design for energy harvesting
Mohd. Ghazali [84]	2016	2-D	Stack only	Single Stack, Double Stack	In-Built code	Laminar	TAD	Selection of symmetry line for model
Abd El-Rahman [44]	2017	3-D	Stack Only	Single Stack	Fluent	Laminar	TAR	Prediction of limits for transition from 1-D to multidimensional flow
Hussain N.M. [75]	2017	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k-ε)	TAE	Sensitivity study

Kuzuu K. [77]	2017	2-D	Stack + HX	Full Stack Model	LS-FLOW	–	TAE	Asymmetric thermal buffer and modified convective term in linear analytical solution
Kuzuu K. [78]	2017	2-D	Stack + HX	Full Stack Model	LS-FLOW	–	TAE	Asymmetrical temperature distribution at HHX
Mohd. Saat F. A. Z. [80]	2017	2-D	HXs only	Full Stack Model	Fluent	Laminar	TAD	Gravity effects on temperature and velocity profiles
Mohd. Saat F.A.Z [81]	2017	2-D	HXs only	Full Stack Model	Fluent	Turbulent (k- ϵ), (k- ω), (k- ω SST)	TAD	Early stage turbulence inside oscillatory flows
Mothilal Hariharan [73]	2017	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k- ϵ realizable)	Twin TAPM	L_{res} and P_m on frequency and P_a
Rogozinski K. et. al [47]	2017	2-D	Stack + HX	Single Stack, Double Stack, Three Stack	CFX	Turbulent (k- ω SST)	TAE	Effect of different REV on the performance
Zolpakar [42]	2017	1-D	Stack + HX	–	Matlab	–	TAR	MOGA Optimization for COP
Alamir A. [30]	2018	1-D	Stack + HX	–	DelatEC	–	TAR	Parametric performance study
Hariharan N.M. [74]	2018	2-D	Stack Only	Full Stack Model	Fluent	Turbulent (k- ϵ)	Twin TAPM	Binary gas mixture effect on performance
Ilori O. M. [45]	2018	3-D	HXs Only	Full Stack Model	Fluent	Turbulent (k- ω SST)	TAE	Tubular HX characterization and aerodynamic shape effect on heat transfer characteristics
Nehar Belaid [64]	2018	2-D	Stack + HX	Single Stack	Le Quéré	Laminar	TAR	HXs blockage ratio effect on device performance

Piccolo A. [28]	2018	1-D	Stack + HX	–	DeltaEC	–	2 stage TAE generator	Loud speaker driven SWTAE generator having equivalent performance to TWTAE generator
Kajurek J. [32]	2019	1-D	Stack + HX	–	DeltaEC	–	TAR	Design and sensitivity study
Liu Liu [79]	2019	2-D	Stack + HX	Full Stack Model	Fluent	Turbulent (k- ϵ)	TAE	Converging stack
Mergen S. [60]	2019	2-D	Stack only	Single Stack	COMSOL	Laminar	TAR	Computational domain length selection
Mohd. Saat F.A.Z [76]	2019	2-D	Stack only	Full Stack Model	Fluent	Turbulent (k- ω SST)	TAR	Importance and criticality of turbulence effect on performance
Rahpeima R. [65]	2019	2-D	Stack + HX	Single Stack	COMSOL	Laminar	TAR	Stack geometric & thermo-physical parameter effect on performance
Saechan P. [31]	2019	1-D	Stack + HX	–	DeltaEC	–	TWTAR/ SWTAE	Performance optimization

Table A-2: Operating & geometric parameters utilized for numerical studies of SWTADs

Authors	Year	Stack geometry	Stack material	Resonator length	Working fluid	Frequency (Hz)	P_m (bar)	T_m (K)	Validation/ Compared With
Swift [25]	1994	–	–	–	–	–	–	–	–
Cao N. et al. [46]	1996	Parallel	Zero Thickness	$\lambda/4$	Mono-atomic	–	–	–	Linear thermoacoustic theory
Minner BL [36]	1996	Parallel	Kapton	–	He, He-Xe	64-505	10,50	300	Hofler T.J. [111]
Worlikar [50]	1996	Parallel	–	$\lambda/2$	–	–	–	–	Atchley [90]
Raspet [33]	1998	Parallel	–	–	He	–	1.7,3.76	290	Atchley [112]
Worlikar [52]	1998	Parallel	Stainless Steel, Fibre Glass	$\lambda/2$	–	–	–	–	Atchley [90]
Ishikawa H. et al. [49]	2002	Parallel	Zero Thickness	$\lambda/4$	Helium	100	0.1	298	Cao N. [46]
Tijani [26]	2002	Parallel	Mylar	–	He	400	10	250	–
Jebali F. [34]	2004	Parallel	Mylar	–	–	40,53	2.5	300	Own experimental setup
Marx D. [53]	2004	Parallel	–	$\lambda/2$	Air	20k	–	–	Hamilton [93]
Marx D. [54]	2005	Parallel	–	$\lambda/2$	Air	20k	–	–	Linear Theory
Marx D. [55]	2005	Parallel	–	$\lambda/2$	Air	–	1	298	Linear theory
Jensen [35]	2006	–	Stainless Steel Mylar	$\lambda/2$	–	178.2,320	40,20.7	300	–
Zoontjens L. [27]	2006	Rectangular Pore	–	–	Air	114	1.01	300	Wetzel [113]
Paek Insu [38]	2007	–	–	$\lambda/4$	He-Ar	15.4-111.1	30	300	Poese [114], T.J. Hofler [4]

Poignand [19]	2007	Parallel	PVC	$\lambda/2$	Air	160	1.01	295	Patent
Srikitsuwan [37]	2007	Parallel	–	–	He	500	–	300	–
Babaei H. [20]	2008	–	–	$\lambda/2, \lambda/4$	He	400	7	300	DeltaEC
Zink F. et. al. [68]	2008	Parallel	–	$\lambda/4$	Air	580	1.01	300	–
Lotton [21]	2009	Square channel	Cordierite	$\lambda/2$	Air	200	1.01	302	Own experimental setup
Wu-Feng [22]	2009	Parallel	–	$\lambda/2, \lambda/4$	–	314-440	10	–	Theoretical calculation
Zink F. [86]	2009	Parallel	Steel, PMMA, SiO ₂ , Al, Cu, Titanium	–	Air	600	1.01	300	–
Zoontjens L. [56]	2009	Parallel	Hypothetical	$\lambda/2$	Helium	100	0.1	300	Cao N. [46], Ishikawa H. [49], Piccolo [94]
Hireche O. [61]	2010	Parallel	Stainless Steel	–	Helium	64	10	293	–
Ke Han Bing [66]	2010	Parallel	–	$\lambda/2$	Ar, Air	100.6	1.01	293	Saenger [96]
Remiorz L. [57]	2010	Mesh	–	$\lambda/2$	Air	200	–	300	–
Tasnim [58]	2010	Parallel	Glass	$\lambda/2$	Air	200	1	300	Cao N. et al. [46], Worlikar et al. [50, 52]
Yu Guoyao [70]	2010	Co-axial Tube	Stainless Steel	–	Helium	293	40	300	Experimental results
Zink F. [48]	2010	Parallel	–	$\lambda/4$	Air	600	1.01	300	Hantschk, C.C. [95]
Zink F. [69]	2010	Parallel	–	$\lambda/4$	Air	614	1.01	300	Experimental Setup
Rulik et. al [82]	2011	Parallel	–	$\lambda/4$	Air	600	1	300	–

Tasnim [23]	2011	Porous	RVC,Cu,Al	$\lambda/4$	He, Air	350	1	300	Analytical& experimental
Trapp A.C. [87]	2011	Parallel	Corderite	$\lambda/4$	Air	580	1.01	300	–
Tasnim [59]	2012	Parallel	–	–	He–Xe, He	100,48.89	1	300	Cao N. [46], Ishikawa H. [49]
Abd El-Rahman [51]	2013	Parallel	Fibreglass epoxied to Stainless steel	$\lambda/2$	Helium	696	1.441	298.2	Worlikar et al. [50, 52], Atchely et. al [90]
Alahmer et al. [24]	2013	Parallel	Plastic	$\lambda/4$	He	350	1.01	280	Experimental work
Hariharan [39]	2013	Spiral	Mylar	–	Air	254	1.01	298	DeltaEC & experimental setup
Piccolo [67]	2013	Parallel	Stainless Steel	$\lambda/2$	He	200	1.01	300	Analytical & Ishikawa H. [49]
Nowak I. [83]	2014	Parallel	–	$\lambda/4$	Air	600	1	300	Analytical solution
Zolpakar [40]	2014	Parallel	Mylar	$\lambda/4$	He	–	–	–	Tijani [7]
Zolpakar [41]	2014	Parallel	Mylar	$\lambda/4$	He	400	10	250	Tijani [7]
Namdar A. [62]	2015	Parallel	Glass	$\lambda/2$	Air	200	1	300	Tasnim & Fraser [58]
Ramdane M. [63]	2015	Parallel	Stainless Steel	–	Helium	–	1	300	–
Skaria M. [71]	2015	Parallel	Stainless Steel	$\lambda/2$	He, Ar, N	120, 40	3.0-7.0	301	Kamble B.[97]
Tartibu L.K. [43]	2015	–	Mylar	–	He	24752	5	–	–
Alamir A. et. al [29]	2016	Parallel	–	$\lambda/2$	Air,Helium	500	2	303	Tijani M.E.H [115]
Badi A.P. [72]	2016	Parallel	–	$\lambda/2, \lambda/4$	Air	600	1	300	Zink F. [48]

Lin J. [85]	2016	Co-axial cylindrical	–	$\lambda/2$	Air	388	1.01	300	Smoker et. al [99]
Mohd. Ghazali [84]	2016	Parallel	–	$\lambda/4$	Air	400	1.01	298	Kim Fa [98]
Abd El-Rahman [44]	2017	Square Channel	Cordierite	$\lambda/2$	Helium	592.5	1.0	300	Arnott [18], Abd El-Rahman [51]
Hussain N.M. [75]	2017	Parallel	–	$\lambda/4$	Air	600	1	300	Zink F. [68]
Kuzuu K. [77]	2017	Parallel	–	$\lambda/4$	Air	21.2	1	298	Linear thermoacoustic theory
Kuzuu K. [78]	2017	Parallel	–	$\lambda/4$	Air	21.2	1	298	Theoretical linear heat transfer analysis
Mohd. Saat F. A. Z. [80]	2017	Parallel	Aluminum	$\lambda/4$	Nitrogen	13.1	1	300	Shi et.al [91, 92]
Mohd. Saat F.A.Z [81]	2017	Parallel	Aluminum	$\lambda/4$	Nitrogen	13.1	1	300	Shi et.al [91, 92]
Mothilal Hariharan [73]	2017	Parallel	Mylar	–	Nitrogen	120-230	1.0-5.0	300	Zink F. [48]
Rogozinski K. et. al [47]	2017	Parallel	–	$\lambda/4$	Air	600	1.01	300	Analytical model own
Zolpakar [42]	2017	Parallel	Mylar	$\lambda/4$	He	400	10	250	Tijani [7]
Alamir A. [30]	2018	Parallel	Mylar	$\lambda/2$	Helium	500	2	300	DeltaEC
Hariharan N.M. [74]	2018	Parallel	Mylar	–	He, Ar,N, He-Ar, He-N	160-495	5	300	Zink F. [48]
Ilori O. M. [45]	2018	Circular pores (HX)	Aluminum (HX)	$\lambda/2$	Helium	53.6	5	300	Own experimental setup
Nehar Belaid [64]	2018	Parallel	Polyethylene	$\lambda/4$	Helium	858.5	10	300	Experimental Setup [100]
Piccolo A. [28]	2018	Square Pore	–	$\lambda/2$	Air	194.5	1	–	DeltaEC

Kajurek J. [32]	2019	Parallel	Mylar	$\lambda/4$	Helium	500	10	300	DeltaEC
Liu Liu [79]	2019	Parallel+ Converging	Stainless Steel	$\lambda/2$	Helium	100-113	10	300	DeltaEC
Mergen S. [60]	2019	Parallel	Zero Thickness	$\lambda/4$	Helium	100	0.1	300	Cao N.et al. [46]
Mohd. Saat F.A.Z [76]	2019	Parallel	Aluminum	$\lambda/4$	Nitrogen	13.1,23.1	1.0 1	300	Own experimental setup
Rahpeima R. [65]	2019	Parallel	Kapton , Stainless Steel, Glass	$\lambda/4$	Helium	100	0.1	293	Ishikawa H. et al.[49]
Saechan P. [31]	2019	Parallel	Stainless steel	$\lambda/2$	Air	50	10	300	Own experimental setup

REFERENCES

- [1] Wuebbles, D.J., *The role of refrigerants in climate change*. International Journal of Refrigeration, 1994. **17**(1): p. 7-17.doi: [https://doi.org/10.1016/0140-7007\(94\)90082-5](https://doi.org/10.1016/0140-7007(94)90082-5).
- [2] Rott, N., *Thermoacoustics*, in *Advances in Applied Mechanics*, C.-S. Yih, Editor. 1980, Elsevier. p. 135-175, doi: [https://doi.org/10.1016/S0065-2156\(08\)70233-3](https://doi.org/10.1016/S0065-2156(08)70233-3).
- [3] Swift, G.W., *Thermoacoustics : a unifying perspective for some engines and refrigerators*. 2002, Acoustical Society of America through the American Institute of Physics: Melville, NY :, doi.
- [4] T. J. Hofler, J.A.A., *Design and construction of a solar-powered, thermoacoustically driven, thermoacoustic refrigerator*. Calhoun: The NPS Institutional Archive DSpace Repository, 2000.doi.
- [5] Garrett, S.L., J.A. Adeff, and T.J. Hofler, *Thermoacoustic refrigerator for space applications*. Journal of Thermophysics and Heat Transfer, 1993. **7**(4): p. 595-599.doi: 10.2514/3.466.
- [6] Swift, G.W., *Thermoacoustic engines*. The Journal of the Acoustical Society of America, 1988. **84**(4): p. 1145-1180.doi: 10.1121/1.396617.
- [7] M.E.H. Tijani, J.C.H.Z., A.T.A.M. de Waele, *Construction and performance of a thermoacoustic refrigerator*. Cryogenics, 2002. **42**: p. 59-66.doi.
- [8] K T Feldman, H.H., and R L Carier, *Experiments on the Sondhauss Thermoacoustical Phenomenon*. The Journal of the Acoustical Society of America, 1966. **39**(6): p. 1236.doi.
- [9] Rijke, P.L., *LXXI. Notice of a new method of causing a vibration of the air contained in a tube open at both ends*. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1859. **17**(116): p. 419-422.doi: 10.1080/14786445908642701.
- [10] Rayleigh, B.J.W.S., *The theory of sound*. 1896. **2**.doi.
- [11] P. Mortell, M., *Resonant thermal-acoustic oscillations*. Vol. 9. 1971. 175-192, doi: 10.1016/0020-7225(71)90019-X.
- [12] Rott, N., *Thermally driven acoustic oscillations, part III: Second-order heat flux*. Zeitschrift für angewandte Mathematik und Physik ZAMP, 1975. **26**(1): p. 43-49.doi: 10.1007/BF01596277.
- [13] Rott, N., *Damped and thermally driven acoustic oscillations in wide and narrow tubes*. Zeitschrift für angewandte Mathematik und Physik ZAMP, 1969. **20**(2): p. 230-243.doi: 10.1007/BF01595562.
- [14] Rott, N., *Thermally driven acoustic oscillations. Part II: Stability limit for helium*. Zeitschrift für angewandte Mathematik und Physik ZAMP, 1973. **24**(1): p. 54-72.doi: 10.1007/BF01593998.
- [15] Rott, N., *The heating effect connected with non-linear oscillations in a resonance tube*. Zeitschrift für angewandte Mathematik und Physik ZAMP, 1974. **25**(5): p. 619-634.doi: 10.1007/BF01596123.

- [16] Swift, G.W. and W.C. Ward, *Simple harmonic analysis of regenerators*. Journal of Thermophysics and Heat Transfer, 1996. **10**(4): p. 652-662.doi: 10.2514/3.842.
- [17] Smith, B. and G. Swift, *Power dissipation and time-averaged pressure in oscillating flow through a sudden area change*. The Journal of the Acoustical Society of America, 2003. **113**: p. 2455-63.doi: 10.1121/1.1564022.
- [18] Arnott, W.P., H.E. Bass, and R. Raspet, *General formulation of thermoacoustics for stacks having arbitrarily shaped pore cross sections*. The Journal of the Acoustical Society of America, 1991. **90**(6): p. 3228-3237.doi: 10.1121/1.401432.
- [19] Poignand, G., et al., *Optimal acoustic fields in compact thermoacoustic refrigerators*. Applied Acoustics, 2007. **68**(6): p. 642-659.doi: <https://doi.org/10.1016/j.apacoust.2006.03.009>.
- [20] Babaei, H. and K. Siddiqui, *Design and optimization of thermoacoustic devices*. Energy Conversion and Management, 2008. **49**(12): p. 3585-3598.doi: <https://doi.org/10.1016/j.enconman.2008.07.002>.
- [21] Lotton, P., et al., *Transient temperature profile inside thermoacoustic refrigerators*. International Journal of Heat and Mass Transfer, 2009. **52**(21): p. 4986-4996.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2009.03.075>.
- [22] Wu, F., et al., *Constructal design of stack filled with parallel plates in standing-wave thermo-acoustic cooler*. Cryogenics, 2009. **49**(3): p. 107-111.doi: <https://doi.org/10.1016/j.cryogenics.2008.09.009>.
- [23] Tasnim, S.H., S. Mahmud, and R.A. Fraser, *Second law analysis of porous thermoacoustic stack systems*. Applied Thermal Engineering, 2011. **31**(14): p. 2301-2311.doi: <https://doi.org/10.1016/j.applthermaleng.2011.03.028>.
- [24] Alahmer, A., Omar, M., Al-Zubi, M., *Demonstrating of a standing wave thermoacoustic refrigerator*. International journal of thermal and environmental engineering, 2013. **62**: p. 75-81.doi: 10.5383/ijtee.06.02.005.
- [25] Ward, W.C. and G.W. Swift, *Design environment for low-amplitude thermoacoustic engines*. The Journal of the Acoustical Society of America, 1994. **95**(6): p. 3671-3672.doi: 10.1121/1.409938.
- [26] Tijani, M.E.H., J.C.H. Zeegers, and d. Waele, A.T.A.M., *Design of thermoacoustic refrigerators*. Cryogenics, 2002. **42**(1): p. 49-57.doi: 10.1016/S0011-2275(01)00179-5.
- [27] Zoontjens, L., et al. *Modelling and optimisation of acoustic inertance segments for thermoacoustic devices*. 2006. doi.
- [28] Piccolo, A., *Design issues and performance analysis of a two-stage standing wave thermoacoustic electricity generator*. Sustainable Energy Technologies and Assessments, 2018. **26**: p. 17-27.doi: 10.1016/j.seta.2016.10.011.
- [29] Alamir, M., E. Elnegiry, and H. Eltahan, *Optimizing the Performance of a Standing Wave Loudspeaker Driven Thermoacoustic Heat Pump*. International Journal of Scientific and Engineering Research, 2016. **7**: p. 460-465.doi.
- [30] Alamir, M.A. and A.A. Elamer, *A compromise between the temperature difference and performance in a standing wave thermoacoustic refrigerator*. International Journal of Ambient Energy, 2018: p. 1-13.doi: 10.1080/01430750.2018.1517673.
- [31] Saechan, P. and A.J. Jaworski, *Numerical studies of co-axial travelling-wave thermoacoustic cooler powered by standing-wave thermoacoustic engine*.

- Renewable Energy, 2019. **139**: p. 600-610.doi: <https://doi.org/10.1016/j.renene.2019.02.059>.
- [32] Kajurek, J., A. Rusowicz, and A. Grzebielec, *Design and simulation of a small capacity thermoacoustic refrigerator*. SN Applied Sciences, 2019. **1**(6): p. 579.doi: [10.1007/s42452-019-0569-2](https://doi.org/10.1007/s42452-019-0569-2).
- [33] Raspet, R., J. Brewster, and H.E. Bass, *A new approximation method for thermoacoustic calculations*. The Journal of the Acoustical Society of America, 1998. **103**(5): p. 2395-2402.doi: [10.1121/1.422758](https://doi.org/10.1121/1.422758).
- [34] Jebali, F., J.V. Lubiez, and M.-X. François, *Response of a thermoacoustic refrigerator to the variation of the driving frequency and loading*. International Journal of Refrigeration, 2004. **27**(2): p. 165-175.doi: [https://doi.org/10.1016/S0140-7007\(03\)00100-2](https://doi.org/10.1016/S0140-7007(03)00100-2).
- [35] Jensen, C., R. Raspet, and W. Slaton, *Temperature gradient integration in thermoacoustic stacks*. Applied Acoustics, 2006. **67**(7): p. 689-699.doi: <https://doi.org/10.1016/j.apacoust.2005.11.004>.
- [36] B. L. Minner, J.E.B., L. Mongeau. *Optimizing the Design of a Thermoacoustic Refrigerator*. in *International Refrigeration and Air Conditioning Conference*. 1996. Purdue University, USA, doi.
- [37] Srikitsuwan, S., S. Kuntanapreeda, and P. Vallikul, *A genetic algorithm for optimization design of thermoacoustic refrigerators*. 2007.doi.
- [38] Paek, I., J.E. Braun, and L. Mongeau, *Evaluation of standing-wave thermoacoustic cycles for cooling applications*. International Journal of Refrigeration, 2007. **30**(6): p. 1059-1071.doi: <https://doi.org/10.1016/j.ijrefrig.2006.12.014>.
- [39] Hariharan, N.M., P. Sivashanmugam, and S. Kasthuriengan, *Optimization of thermoacoustic refrigerator using response surface methodology*. Journal of Hydrodynamics, Ser. B, 2013. **25**(1): p. 72-82.doi: [https://doi.org/10.1016/S1001-6058\(13\)60340-6](https://doi.org/10.1016/S1001-6058(13)60340-6).
- [40] Zolpakar, N.A., N. Mohd-Ghazali, and R. Ahmad, *Analysis of Increasing the Optimized Parameters in Improving the Performance of a Thermoacoustic Refrigerator*. Energy Procedia, 2014. **61**: p. 33-36.doi: <https://doi.org/10.1016/j.egypro.2014.11.899>.
- [41] Zolpakar, N.A., N. Mohd-Ghazali, and R. Ahmad, *SIMULTANEOUS OPTIMIZATION OF FOUR PARAMETERS IN THE STACK UNIT OF A THERMOACOUSTIC REFRIGERATOR*. International Journal of Air-Conditioning and Refrigeration, 2014. **22**(02): p. 1450011.doi: [10.1142/S2010132514500114](https://doi.org/10.1142/S2010132514500114).
- [42] Atiqah Zolpakar, N., N. Mohd-Ghazali, and R. Ahmad, *Optimization of the Stack Unit in a Thermoacoustic Refrigerator*. Heat Transfer Engineering, 2017. **38**(4): p. 431-437.doi: [10.1080/01457632.2016.1195138](https://doi.org/10.1080/01457632.2016.1195138).
- [43] Tartibu, L.K., B. Sun, and M.A.E. Kaunda, *Lexicographic multi-objective optimization of thermoacoustic refrigerator's stack*. Heat and Mass Transfer, 2015. **51**(5): p. 649-660.doi: [10.1007/s00231-014-1440-z](https://doi.org/10.1007/s00231-014-1440-z).
- [44] Abd El-Rahman, A.I., W.A. Abdelfattah, and M.A. Fouad, *A 3D investigation of thermoacoustic fields in a square stack*. International Journal of Heat and Mass Transfer, 2017. **108**: p. 292-300.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2016.12.015>.

- [45] Ilori, O.M., A.J. Jaworski, and X. Mao, *Experimental and numerical investigations of thermal characteristics of heat exchangers in oscillatory flow*. Applied Thermal Engineering, 2018. **144**: p. 910-925.doi: <https://doi.org/10.1016/j.applthermaleng.2018.07.073>.
- [46] Cao, N., et al., *Energy flux density in a thermoacoustic couple*. The Journal of the Acoustical Society of America, 1996. **99**(6): p. 3456-3464.doi: 10.1121/1.414992.
- [47] Rogoziński, K., I. Nowak, and G. Nowak, *Modeling the operation of a thermoacoustic engine*. Energy, 2017. **138**: p. 249-256.doi: 10.1016/j.energy.2017.07.058.
- [48] Zink, F., J. Vipperman, and L. Schaefer, *CFD simulation of a thermoacoustic engine with coiled resonator*. International Communications in Heat and Mass Transfer, 2010. **37**: p. 226-229.doi: 10.1016/j.icheatmasstransfer.2009.09.001.
- [49] Ishikawa, H. and D.J. Mee, *Numerical investigations of flow and energy fields near a thermoacoustic couple*. The Journal of the Acoustical Society of America, 2002. **111**(2): p. 831-839.doi: 10.1121/1.1430687.
- [50] Worlikar, A.S. and O.M. Knio, *Numerical Simulation of a Thermoacoustic Refrigerator: I. Unsteady Adiabatic Flow around the Stack*. Journal of Computational Physics, 1996. **127**(2): p. 424-451.doi: <https://doi.org/10.1006/jcph.1996.0185>.
- [51] Abd El-Rahman, A.I. and E. Abdel-Rahman, *Computational Fluid Dynamics Simulation of a Thermoacoustic Refrigerator*. Journal of Thermophysics and Heat Transfer, 2013. **28**(1): p. 78-86.doi: 10.2514/1.T4150.
- [52] Worlikar, A.S., O.M. Knio, and R. Klein, *Numerical Simulation of a Thermoacoustic Refrigerator: II. Stratified Flow around the Stack*. Journal of Computational Physics, 1998. **144**(2): p. 299-324.doi: <https://doi.org/10.1006/jcph.1997.5816>.
- [53] Marx, D. and P. Blanc-Benon, *Computation of the mean velocity field above a stack plate in a thermoacoustic refrigerator*. Comptes Rendus Mécanique, 2004. **332**(11): p. 867-874.doi: <https://doi.org/10.1016/j.crme.2004.07.010>.
- [54] Marx, D. and P. Blanc-Benon, *Numerical calculation of the temperature difference between the extremities of a thermoacoustic stack plate*. Cryogenics, 2005. **45**(3): p. 163-172.doi: <https://doi.org/10.1016/j.cryogenics.2004.08.007>.
- [55] Marx, D. and P. Blanc-Benon, *Computation of the temperature distortion in the stack of a standing-wave thermoacoustic refrigerator*. The Journal of the Acoustical Society of America, 2005. **118**(5): p. 2993-2999.doi: 10.1121/1.2063087.
- [56] Zoontjens, L., et al., *Numerical study of flow and energy fields in thermoacoustic couples of non-zero thickness*. International Journal of Thermal Sciences, 2009. **48**(4): p. 733-746.doi: <https://doi.org/10.1016/j.ijthermalsci.2008.06.007>.
- [57] Remiorz, L., Dykas, S., Rulik, S., *Numerical Modelling of Thermoacoustic Phenomenon as Contribution to Thermoacoustic Engine Model*. TASK Quarterly : scientific bulletin of Academic Computer Centre in Gdansk, 2010. **14**(3): p. 257-269.doi.
- [58] Tasnim, S.H. and R.A. Fraser, *Computation of the flow and thermal fields in a thermoacoustic refrigerator*. International Communications in Heat and Mass

- Transfer, 2010. **37**(7): p. 748-755.doi: <https://doi.org/10.1016/j.icheatmasstransfer.2010.04.006>.
- [59] Tasnim, S., S. Mahmud, and R. Fraser, *Effects of variation in working fluids and operating conditions on the performance of a thermoacoustic refrigerator*. International Communications in Heat and Mass Transfer, 2012. **39**: p. 762–768.doi: 10.1016/j.icheatmasstransfer.2012.04.013.
- [60] Mergen, S., E. Yildirim, and H. Turkoglu, *Numerical study on effects of computational domain length on flow field in standing wave thermoacoustic couple*. Cryogenics, 2019. **98**: p. 139-147.doi: <https://doi.org/10.1016/j.cryogenics.2018.09.012>.
- [61] Hireche, O., et al., *Numerical model of a thermoacoustic engine*. Comptes Rendus Mécanique, 2010. **338**(1): p. 18-23.doi: <https://doi.org/10.1016/j.crme.2009.12.006>.
- [62] Namdar, A., A. Kianifar, and E. Roohi, *Numerical investigation of thermoacoustic refrigerator at weak and large amplitudes considering cooling effect*. Cryogenics, 2015. **67**: p. 36-44.doi: <https://doi.org/10.1016/j.cryogenics.2015.01.005>.
- [63] Ramdane, M., M. Abidat, and M. Hamel, *Numerical Simulation of Heat Exchanger's Length's Effect in a Thermoacoustic Engine*. Journal of Physical Science and Application, 2015. **5**.doi: 10.17265/2159-5348/2015.01.009.
- [64] Nehar Belaid, K. and O. Hireche, *Influence of heat exchangers blockage ratio on the performance of thermoacoustic refrigerator*. International Journal of Heat and Mass Transfer, 2018. **127**: p. 834-842.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.144>.
- [65] Rahpeima, R. and R. Ebrahimi, *Numerical investigation of the effect of stack geometrical parameters and thermo-physical properties on performance of a standing wave thermoacoustic refrigerator*. Applied Thermal Engineering, 2019. **149**: p. 1203-1214.doi: <https://doi.org/10.1016/j.applthermaleng.2018.12.093>.
- [66] Ke, H.-B., et al., *Numerical simulation and parameter optimization of thermoacoustic refrigerator driven at large amplitude*. Cryogenics, 2010. **50**(1): p. 28-35.doi: <https://doi.org/10.1016/j.cryogenics.2009.10.005>.
- [67] Piccolo, A., *Optimization of thermoacoustic refrigerators using second law analysis*. Applied Energy, 2013. **103**: p. 358-367.doi: <https://doi.org/10.1016/j.apenergy.2012.09.044>.
- [68] Zink, F., J. Vipperman, and L. Schaefer. *Advancing Thermoacoustics Through CFD Simulation Using Fluent*. in *ASME International Mechanical Engineering Congress and Exposition, Proceedings*. 2008. Boston, USA, doi: 10.1115/IMECE2008-66510.
- [69] Zink, F., J. Vipperman, and L. Schaefer, *CFD simulation of thermoacoustic cooling*. International Journal of Heat and Mass Transfer, 2010. **53**(19): p. 3940-3946.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2010.05.012>.
- [70] Yu, G., W. Dai, and E. Luo, *CFD simulation of a 300Hz thermoacoustic standing wave engine*. Cryogenics, 2010. **50**(9): p. 615-622.doi: <https://doi.org/10.1016/j.cryogenics.2010.02.011>.
- [71] Skaria, M., et al., *Simulation studies on the performance of thermoacoustic prime movers and refrigerator*. Computers & Fluids, 2015. **111**: p. 127-136.doi: <https://doi.org/10.1016/j.compfluid.2015.01.011>.

- [72] Badi, A.P. and H. Beheshti, *Numerical Study of Multistage Serial and Parallel Configurations of Thermoacoustic Engines*. Journal of Thermal Science and Engineering Applications, 2016. **8**(2): p. 0210251-0210256.doi: 10.1115/1.4032710.
- [73] Mothilal, H., et al., *CFD simulation on the performance of twin thermoacoustic prime mover for various resonator lengths and operating pressures*. Heat Transfer-Asian Research, 2017. **47**: p. 337-346.doi: <https://doi.org/10.1002/htj.21306>.
- [74] Hariharan, N.M. and P. Sivashanmugam, *CFD Simulation of Twin Thermoacoustic Prime Mover for Binary Gas Mixtures*. High Temperature, 2018. **56**(2): p. 309-311.doi: 10.1134/S0018151X18020232.
- [75] Hussain, M.N. and I. Janajreh, *Analysis of Pressure Wave Development in a Thermo-acoustic Engine and Sensitivity Study*. Energy Procedia, 2017. **142**: p. 1488-1495.doi: <https://doi.org/10.1016/j.egypro.2017.12.597>.
- [76] Mohd Saat, F.A.Z., et al., *Numerical study of turbulence related to vortex shedding structures of an oscillatory flow in thermoacoustic energy system*. Energy Procedia, 2019. **156**: p. 239-243.doi: <https://doi.org/10.1016/j.egypro.2018.11.135>.
- [77] Kuzuu, K. and S. Hasegawa, *Numerical investigation of heated gas flow in a thermoacoustic device*. Applied Thermal Engineering, 2017. **110**: p. 1283-1293.doi: <https://doi.org/10.1016/j.applthermaleng.2016.08.093>.
- [78] Kuzuu, K. and S. Hasegawa, *Effect of non-linear flow behavior on heat transfer in a thermoacoustic engine core*. International Journal of Heat and Mass Transfer, 2017. **108**: p. 1591-1601.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2016.12.064>.
- [79] Liu, L., P. Yang, and Y. Liu, *Comprehensive performance improvement of standing wave thermoacoustic engine with converging stack: Thermodynamic analysis and optimization*. Applied Thermal Engineering, 2019. **160**: p. 114096.doi: <https://doi.org/10.1016/j.applthermaleng.2019.114096>.
- [80] Mohd Saat, F. and A. Jaworski, *The Effect of Temperature Field on Low Amplitude Oscillatory Flow within a Parallel-Plate Heat Exchanger in a Standing Wave Thermoacoustic System*. Applied Sciences, 2017. **7**(4): p. 417.doi: 10.3390/app7040417.
- [81] Mohd Saat, F. and A. Jaworski, *Numerical Predictions of Early Stage Turbulence in Oscillatory Flow across Parallel-Plate Heat Exchangers of a Thermoacoustic System*. Applied Sciences, 2017. **7**: p. 673.doi: 10.3390/app7070673.
- [82] Rulik, S., L. Remiorz, and S. Dykas, *Application of CFD technique for modelling of the thermoacoustic engine*. Archives of Thermodynamics, 2011. **32**: p. 175-190.doi: 10.2478/v10173-011-0021-5.
- [83] Nowak, I., et al., *Analytical and numerical approach in the simple modelling of thermoacoustic engines*. International Journal of Heat and Mass Transfer, 2014. **77**: p. 369-376.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2014.05.018>.
- [84] Mohd-Ghazali, N. and L. Fa, *Numerical simulation of flow surrounding a thermoacoustic stack: Single-stack against double-stack plate domain*. Jurnal Teknologi, 2016. **78**: p. 119-125.doi: 10.11113/jt.v78.9593.
- [85] Lin, J., C. Scalo, and L. Hesselink, *High-fidelity simulation of a standing-wave thermoacoustic-piezoelectric engine*. Journal of Fluid Mechanics, 2016. **808**: p. 19-60.doi: 10.1017/jfm.2016.609.

- [86] Zink, F., et al., *Geometric optimization of a thermoacoustic regenerator*. International Journal of Thermal Sciences, 2009. **48**(12): p. 2309-2322.doi: <https://doi.org/10.1016/j.ijthermalsci.2009.05.007>.
- [87] Trapp, A.C., et al., *Thermoacoustic heat engine modeling and design optimization*. Applied Thermal Engineering, 2011. **31**(14): p. 2518-2528.doi: <https://doi.org/10.1016/j.applthermaleng.2011.04.017>.
- [88] Ohmi, M., et al., *Transition to Turbulence and Velocity Distribution in an Oscillating Pipe Flow*. Bulletin of JSME, 1982. **25**(201): p. 365-371.doi: 10.1299/jsme1958.25.365.
- [89] Merkli, P. and H. Thomann, *Transition to turbulence in oscillating pipe flow*. Journal of Fluid Mechanics, 2006. **68**(3): p. 567-576.doi: 10.1017/S0022112075001826.
- [90] Atchley, A.A., et al., *Acoustically generated temperature gradients in short plates*. Journal of the Acoustical Society of America, 1990. **88**(1): p. 251-263.doi: 10.1121/1.399947.
- [91] Shi, L., X. Mao, and A. Jaworski, *Application of planar laser-induced fluorescence measurement techniques to study the heat transfer characteristics of parallel-plate heat exchangers in thermoacoustic devices*. Measurement Science and Technology, 2010. **21**: p. 115405.doi: 10.1088/0957-0233/21/11/115405.
- [92] Shi, L., Z. Yu, and A.J. Jaworski, *Application of laser-based instrumentation for measurement of time-resolved temperature and velocity fields in the thermoacoustic system*. International Journal of Thermal Sciences, 2010. **49**(9): p. 1688-1701.doi: <https://doi.org/10.1016/j.ijthermalsci.2010.03.015>.
- [93] Hamilton, M.F., Y.A. Ilinskii, and E.A. Zabolotskaya, *Acoustic streaming generated by standing waves in two-dimensional channels of arbitrary width*. The Journal of the Acoustical Society of America, 2003. **113**(1): p. 153-160.doi: 10.1121/1.1528928.
- [94] Piccolo, A. and G. Pistone, *Estimation of heat transfer coefficients in oscillating flows: The thermoacoustic case*. International Journal of Heat and Mass Transfer, 2006. **49**(9): p. 1631-1642.doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2005.11.009>.
- [95] Hantschk, C.C. and D. Vortmeyer, *Numerical simulation of self-excited thermoacoustic instabilities in a Rijke tube*. Journal of Sound and Vibration, 1999. **227**(3): p. 511-522.doi: <https://doi.org/10.1006/jsvi.1999.2296>.
- [96] Saenger, R.A. and G.E. Hudson, *Periodic Shock Waves in Resonating Gas Columns*. The Journal of the Acoustical Society of America, 1960. **32**(8): p. 961-970.doi: 10.1121/1.1908343.
- [97] Kamble, B., et al., *Experimental and simulation studies on the performance of standing wave thermoacoustic prime mover for pulse tube refrigerator*. International Journal of Refrigeration, 2013. **36**: p. 2410-2419.doi: 10.1016/j.ijrefrig.2013.08.001.
- [98] Kim Fa, L. and N. Mohd-Ghazali, *Simulation of Flow and Temperature Development in a Thermoacoustic Resonator*. International Journal of Technology, 2015. **6**(4): p. 573-583.doi.

- [99] Smoker, J., et al., *Energy harvesting from a standing wave thermoacoustic-piezoelectric resonator*. Journal of Applied Physics, 2012. **111**(10): p. 104901.doi: 10.1063/1.4712630.
- [100] Duthil, P., et al., *Experimental and numerical investigation of temperature differences within a thermoacoustic cell*. 2019.doi.
- [101] Jakub, K., R. Artur, and G. Andrzej, *The Influence of Stack Position and Acoustic Frequency on the Performance of Thermoacoustic Refrigerator with the Standing Wave*. Archives of Thermodynamics, 2017. **38**(4): p. 89-107.doi: <https://doi.org/10.1515/aoter-2017-0026>.
- [102] Masi, M., S. Fogliani, and S. Carra, *Sensitivity Analysis on Indium Phosphide Liquid Encapsulated Czochralski Growth*. Crystal Research and Technology, 1999. **34**(9): p. 1157-1167.doi: 10.1002/(SICI)1521-4079(199911)34:9<1157::AID-CRAT1157>3.0.CO;2-V.
- [103] Kitchell, J.F., D.J. Stewart, and D. Weininger, *Applications of a Bioenergetics Model to Yellow Perch (*Perca flavescens*) and Walleye (*Stizostedion vitreum vitreum*)*. Journal of the Fisheries Research Board of Canada, 1977. **34**(10): p. 1922-1935.doi: 10.1139/f77-258.
- [104] Chen, B. and L. Tong, *Sensitivity analysis of heat conduction for functionally graded materials*. Materials & Design, 2004. **25**: p. 663-672.doi: 10.1016/j.matdes.2004.03.007.
- [105] James, C.A., R.P. Taylor, and B.K. Hodge, *The Application of Uncertainty Analysis to Cross-Flow Heat Exchanger Performance Predictions*. Heat Transfer Engineering, 1995. **16**(4): p. 50-62.doi: 10.1080/01457639508939863.
- [106] Qureshi, B.A. and S.M. Zubair, *A comprehensive design and rating study of evaporative coolers and condensers. Part II. Sensitivity analysis*. International Journal of Refrigeration, 2006. **29**(4): p. 659-668.doi: <https://doi.org/10.1016/j.ijrefrig.2005.09.015>.
- [107] Kim, J.H., T.W. Simon, and R. Viskanta, *Journal of Heat Transfer Policy on Reporting Uncertainties in Experimental Measurements and Results*. Journal of Heat Transfer, 1993. **115**(1): p. 5-6.doi: 10.1115/1.2910670.
- [108] Bill Ward, J.C., and Greg Swift *Design Environment for Low-amplitude Thermoacoustic Energy Conversion (DeltaEC)*. Version 6.4b2.7, Users Guide, 2017.
- [109] Hodge, B.K. and R.P. Taylor, *Analysis and design of energy systems*. 1999, Upper Saddle River, N.J.: Prentice Hall, doi.
- [110] Ardente, F., et al., *Life cycle assessment of a solar thermal collector: Sensitivity analysis, energy and environmental balances*. Renewable Energy, 2005. **30**: p. 109-130.doi: 10.1016/j.renene.2004.05.006.
- [111] T.J., H., *Thermoacoustic Refrigerator Design and Performance*, in *Physics*. 1986, University of California: San Diego, doi.
- [112] Atchley, A.A., et al., *Study of a thermoacoustic prime mover below onset of self-oscillation*. The Journal of the Acoustical Society of America, 1992. **91**(2): p. 734-743.doi: 10.1121/1.402535.
- [113] Wetzel, M. and C. Herman, *Design optimization of thermoacoustic refrigerators*. International Journal of Refrigeration, 1997. **20**(1): p. 3-21.doi: [https://doi.org/10.1016/S0140-7007\(96\)00064-3](https://doi.org/10.1016/S0140-7007(96)00064-3).

- [114] Poese, M.E. and S.L. Garrett, *Performance measurements on a thermoacoustic refrigerator driven at high amplitudes*. The Journal of the Acoustical Society of America, 2000. **107**(5): p. 2480-2486.doi: 10.1121/1.428635.
- [115] M.E.H, T., *Loudspeaker-driven thermo-acoustic refrigeration*, in *Department of Applied Physics*. 2001, Technische Universiteit Eindhoven: Eindhoven, Netherlands, doi: 10.6100/IR547542.

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2. Umar Nawaz Bhatti, Salem Bashmel, Rached Ben Mansour, Sikandar Khan, “Numerical Modeling of Standing Wave Thermoacoustic Devices – A review”, In-Process.

3. Umar Nawaz Bhatti, Salem Bashmel, Rached Ben Mansour, Sikandar Khan,
“Performance Evaluation of Standing Wave Thermoacoustic Refrigerator using
Heterogeneous Stack”, In-Process.