

**VIBROACOUSTIC CONTROL USING
METAMATERIAL RESONATORS**

BY

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In

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*In the name of Allah, the Most
Gracious and the
Most Merciful*

KING FAHD UNIVERSITY OF PETROLEUM and MINERALS

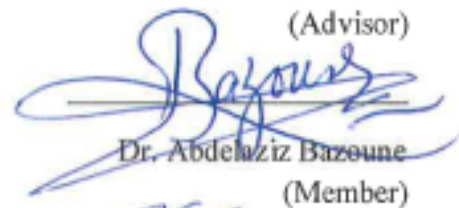
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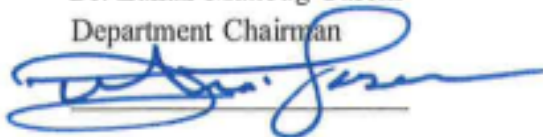


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Dedicated

to

My Beloved Parents, Brothers, and

friends

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NOMENCLATURE

A	Cross section area of the beam
b	Width of beam
m	Mass of resonator
I	Moment of inertia
K	Stiffness matrix
E	Modulus of elasticity
L	Length of the unit cell
h	Height of the beam
M	Mass of beam per unit length
k_b	Wave number
m^*	Mass of beam and resonator per unit length
ω	Frequency
Q	Force
δ	Attenuation constant
ϵ	Phase constant
μ	Propagation constant
K_θ	Torsional spring
D_o	Dynamic stiffness
f	Force vector

u	Displacement vector
M	Moment
w	Displacement
Θ	Angle of deformation
u_L	Displacement vector for the left side of the beam
u_R	Displacement vector for right side of the beam
f_L	Force vector for the left side of the beam
f_R	Force vector for right side of the beam
T_N	Transverse vibration transmission

THESIS ABSTRACT (ENGLISH)

NAME: AYMAN HUSSAM SHEHDAH ALDARABIE

TITLE: VIBROACOUSTIC CONTROL USING METAMATERIAL RESONATORS

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Metematerials are unique structural materials , which when developed from locally resonant arrays, shows a very effective attenuation behavior at low frequency vibroacoustic applications.

This thesis deals with Euler Bernoulli-Beams resting on periodic supports as a host structure .

Local resonators and/or torsional springs are periodically attached to the periodic beam in order to provide effective control of vibration transmission.

The approach used in analysing these periodic structures is an exact analytical one, which is based on the combined theory of spectral element method and transfer matrix method.

Mathematical expressions are developed for calculating attenuation constants and investigating complex banded response. Non-Classical boundary conditions are utilized to see the investigate effect on bandgaps and it is seen that torsional springs used with local resonators can enlarge the bandgaps, over which elastic waves cannot freely propagate.

This configuration shows very interesting aspects in guiding the wave and providing remarkable filtering properties.

Expressions are further developed to analyse the effect of various system's parameters on the bandgaps patterns, including wave attenuation performance, width and position of all the bandgaps. Bandgap maps and vibration transmission curves are developed to analyse vibration reduction in finite periodic beams. It is shown that utilizing this control strategy induces large band gaps in finite periodic beams providing significant attenuation to the vibroacoustic waves transmission by absorbing the necessary transmitted energy.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Saudi Arabia, Dhahran

THESIS ABSTRACT (ARABIC)

ملخص الرسالة

الإسم: أيمن حسام شحده الدرابيع

عنوان الرسالة: التحكم الإهتزازي الصوتي باستخدام المواد غير الإعتيادية

التخصص: الهندسة الميكانيكية

تاريخ التخرج: (كانون الأول ، 2018 م)

المواد غير الإعتيادية هي مواد فريدة تم تطويرها من مصفوفات رنانة محلية ، حيث أظهرت تأثيراً فعالاً في التقليل من الاهتزاز والصوت عند الترددات المنخفضة في المجالات المختلفة.

وفي هذه الرسالة سوف يتم دراسة الذراع المتذبذب ايلر بيرنولي ، والتي هي بنية دورية ذات طول غير محدود ، حيث تم ربط الذراع المتذبذب في المصفوفات الرنانة المحلية الديناميكية والضاغط الإلتوائي ، و التي تعمل على التحكم في الاهتزاز الديناميكي ، وكما تعمل هذه الماصات كنظام فعال للتحكم بإنقال الموجات ، مما يوفر التحكم في عامل انتقال الاهتزاز . وقد لوحظ مدى فاعليتها في التحكم بخصائص الصوت للاهتمام في توجيه الموجات و ترشيحها ، كما أنه هناك مناطق لا تستطيع الموجات الانتشار بها وتسمى هذه المناطق فجوات النطاق .

النهج المستخدم هو النهج التحليلي الدقيق الذي يقوم على نظرية الجمع بين البنية الدورية وطريقة العنصر الطيفي.

واستناداً إلى هذه الطرق ، تم تطوير تعبيرات توفر لنا ثوابت التوهين وحسابات بنية النطاق المعقد.

وقد تم استخدام الحدود غير الكلاسيكية جنباً إلى جنب مع النهج المذكور اعلاه لمعرفة التأثير على نطاق الفجوات المعقدة وقد

لوحظ زيادة هذه النطاقات عند اضافة الملفات الالتروائية عند تحليل الذراع المتذبذب.

وقد تم تطوير هذه التعبيرات بشكل أكبر لتحليل تأثير معاملات النظام على سلوك الفجوات في النطاق التي تتضمن أداء توهين الموجة وعرضها وموضع جميع فجوات النطاق.

ولهذا تم تطوير خرائط الفجوات في النطاق ومنحنيات انتقال الاهتزازات لتحليل الاهتزاز وتقليله في البنية المحدودة من الذراع المتذبذب من خلال عامل الانتقال . كما يتبين أن استخدام إستراتيجية التحكم هذه يؤدي إلى وجود ثغرات كبيرة في النطاق في الذراع المتذبذب المحدود، حيث توفر توهيناً كبيراً في انتقال الموجات الصوتية عن طريق امتصاص طاقة الانتقال الضرورية.

شهادة ماجستير علوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

Chapter 1 : INTRODUCTION

In modern technological and industrial requirements, focusing on the environmental and occupant health protection is a main part in research and development. The environmental and health protection criteria along with other factors emphasize on noise reduction or elimination of sources of sound pollution. As a result, various scientific and research-based techniques have been developed to minimize or eliminate the effect of acoustic pollution on human hearing. Of all the methods, the most common and effective applied techniques are based on the use of particular type of materials either metamaterial resonators and electro-magnetic type of absorbers. Our study is focused on metamaterials resonators for providing acoustic isolation through absorbing the acoustical energy.

This work is primarily based on the principle of acoustic wave propagation, as it is known fact, that when any sound wave impinges on the surface, it results in three phenomenon: reflection, transmission, and absorption. However, we have more concern about transmission as it allows the waves downstream, resulting in spreading of the sound wave.

The two factors of concern are Sound Transmission Coefficient and Sound Isolation Effectiveness . Metamaterial resonators are employed to control these parameters. In general, the law defining the resonator's function to lessen or eliminate wave transmission in an acoustic field is established on how the inertia energy is being exchanged with the elastic energy .

From other view pointing , metamaterial resonators can be looked at as an advance combinations of the spring-mass systems, for achieving both active and passive wave controls.

Considering practical applicability, there are beam structures that are attached with a particular kind of acoustical type metamaterial resonators having same properties in all directions and generally termed as isotropic metamaterials. These metamaterials have spring-mass vibration absorbers of the built-in type.

Since the directional absorption of waves is based on how the resonating units are attached. There can be a unidirectional absorption of the wave (e.g., longitudinal waves) or bi-directional absorption of the wave (for example coupled transversal and longitudinal waves).

In the case of beams , simple spring-mass resonators lead to producing only one or two frequency bandgaps. More sophisticated, spring mass combination, however to induce multiple frequency bandgaps in the beam frequency response.

1.1 Background

Sound, its propagation and control has been studied through ages. These studies have been given the name acoustics. In general terminology, acoustics is which branch of science that deals explicitly with amplitude, frequency, transmission/propagation and complexity of sound waves, that are the intrinsic properties of the sounds.[1]

Acoustics is subcategorized into many branches based on applications. These include

- Noise control ,
- SONAR: Use of sound waves for underwater marine technologies (Navigation of Submarines),
- Medical imaging.

- Thermo-acoustics,
- Earthquake studies,
- Biological acoustics and communication using electro-acoustics [2].

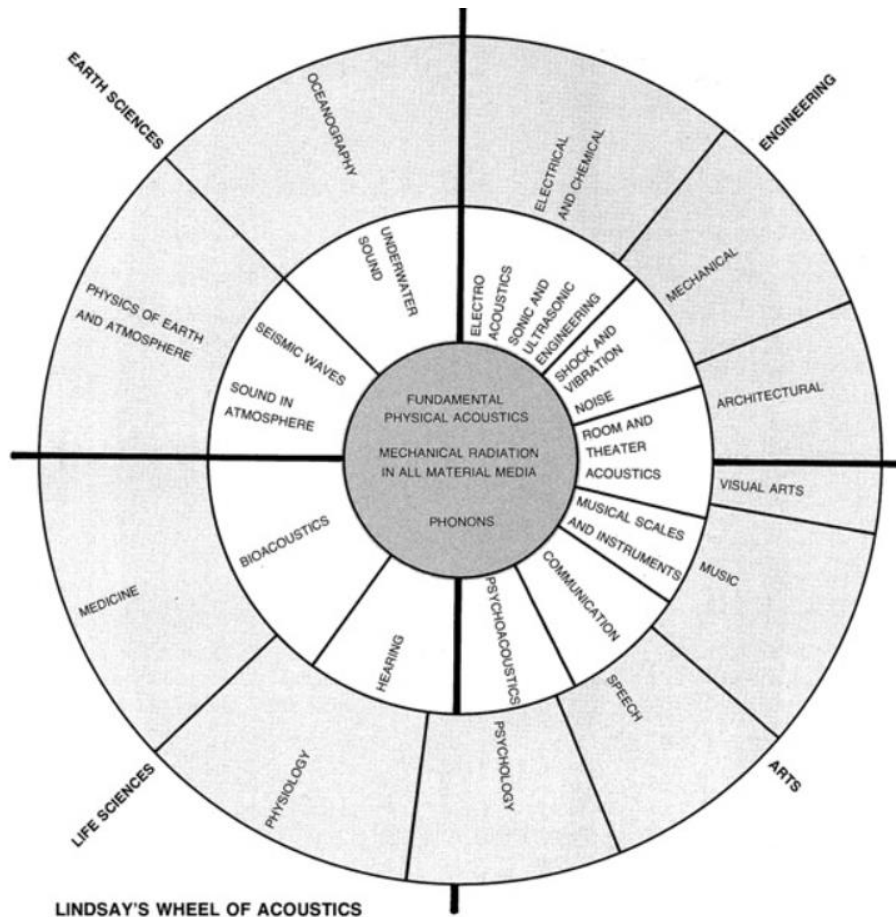


Figure 1-1 LINDSAY'S Wheel of Acoustics

Just like light, the sound is also a mixture of many acoustical waves including many different frequencies.

The unit of measurement for acoustical signal amplitude is decibel (dB), which can be defined as the logarithm of the ratio of the actual sound pressure level to a specific sound pressure level. In

addition, the gains of sound amplifiers, attenuation of acoustic signals, and signal-to-noise ratios are often expressed in decibels [3].

1.2 Metamaterials

The term Metamaterial was first coined when four seminar papers used it for the first time to describe unusual wave motion characteristics. Veselago [4] was the first one to write on a metamaterial calling it a left hand material. In [4] a material with both negative permittivity and permeability, without giving details on its construction at that time. Another paper by Smith et al. [5] presented a physical experiment showing a composite structure with periodic array of split ring resonators, which were non-magnetic and inter-space conducting. The composite showed that the frequencies in the microwave zones exist with simultaneous negative values of both permeability and permittivity. A third paper by Shelby. [6] and a fourth paper by Pendry [7] demonstrated a negative refractive on materials hosting resonators. Many researchers then studied metamaterial unusual electrical properties. Later studies on mechanical metamaterials came into existence. Jichun [8] in his paper worked on mechanical metamaterial resonators and defined their stiffness, rigidity and compressibility properties. Mechanical materials are man-made structures with unique geometries that is capable of showing unusual behaviour .

Metamaterials are specially designed materials with combinations of the spring-mass resonators , and have been found to be quite useful in the reduction of noise and vibration. A metamaterial does not exist naturally with the same properties and is produced in the form of composites [9].

1.3 Periodic structures

A structure that has periodicity consist of periodic elements or units, that are categorize based on how these unit cells or elements are joined together. Some periodic structure are joined in side-to-side configuration and some have end-to-end configuration.

These periodic structures are being used in many practical applications, which include multi span beams in bridges and multi storey buildings, multi-blade turbines and compressors, chemical pipelines and space trusses.[10]

Rayleigh in 1887 was the first to consider continuous periodic structures. Then researchers performed analysis on the increasingly complicated systems. These efforts led lead to the development of the general theory for the periodic structures. [11-12]

One of the basic and unique properties for the periodic structures is that they have impedance mismatch, which leads to wave propagation changes. In comparison, it is usually expected that waves in the non-periodic structures travel without any change until they reach the edge or boundary.

When a discontinuity exists in the structure either in its geometry or material properties, the wave travelling through it, splits into two parts, one part continues its travel in the next part of the medium and the other is reflected back to the same part . As a result, the interaction occurs between the incident and reflected waves, which produces constructive or destructive interferences. These interferences are due to the periodic impedance mismatch, generating

frequency bands where waves do not propagate freely. These frequency bands are call stop bands or band gaps. Another name for these band gaps are Bragg band gaps. [13]

In order to study wave propagation in periodic structures , researchers have utilized the concept of propagation constant , μ

$$\mu = \delta + i\epsilon$$

Where μ has both a real and an imaginary parts . The real part δ is the attenuation constant (Np/m) and the imaginary part ϵ is the phase constant(rev/m). Thus , the real part represent the amplitude decay rate off the wave and the imaginary part is representing the difference of wave motion between two adjacent unit cells.

In order to describe wave motion in the total periodic structures , researchers have utilized the receptance method, the transfer matrix method and the phased array methods. [14-19]

Take example of transfer matrix method that is based on compatibility relation at the boundaries between the structural elements. This approach also implies a matrix based relation between forces and displacements of the two-end of a substructure.[20]

If certain structures are periodically connected with the main one, the total transfer matrix is obtained as the product of the single transfer matrices. Thus , the analysis of a unit cell transfer matrix is enough to describe the dynamics of whole structure.[21]

In the analysis of periodic structures in this thesis, the method adopted is the spectral element (SE) formulation, and it is considered a form of the receptance matrix. In this method, the formulations for SE is derived or executed based on the SE matrices of the structural

components for the unit cell. The considerable advantage of the SE method is that it is well suited to be utilized for establishing the total transfer matrix, and it is applicable for large variety of the structures including beams. [22]

1.4 Flexural Wave Propagation in Beams

A number of theories for flexural motions in beams have been developed considering the beam theories and methods.

In this work, the Euler-Bernoulli beam theory is adopted because it does not include the shear displacement factor, which makes simplifying the equation of motion [23].

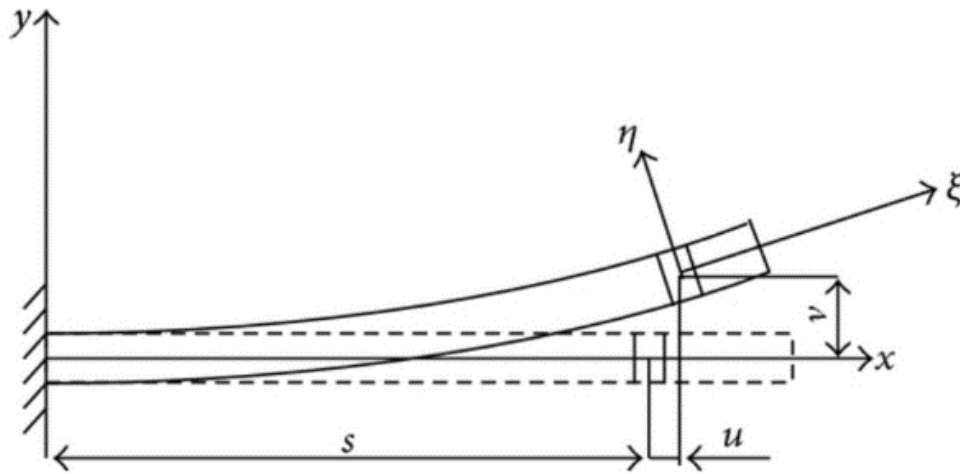


Figure 1-2 Euler-Bernoulli beam

For the Euler-Bernoulli theory certain[24] , several standard assumptions are noted, including

- A. The material is elastic, homogeneous and isotropic.
- B. The beam is prismatic and symmetric about the principal axis.

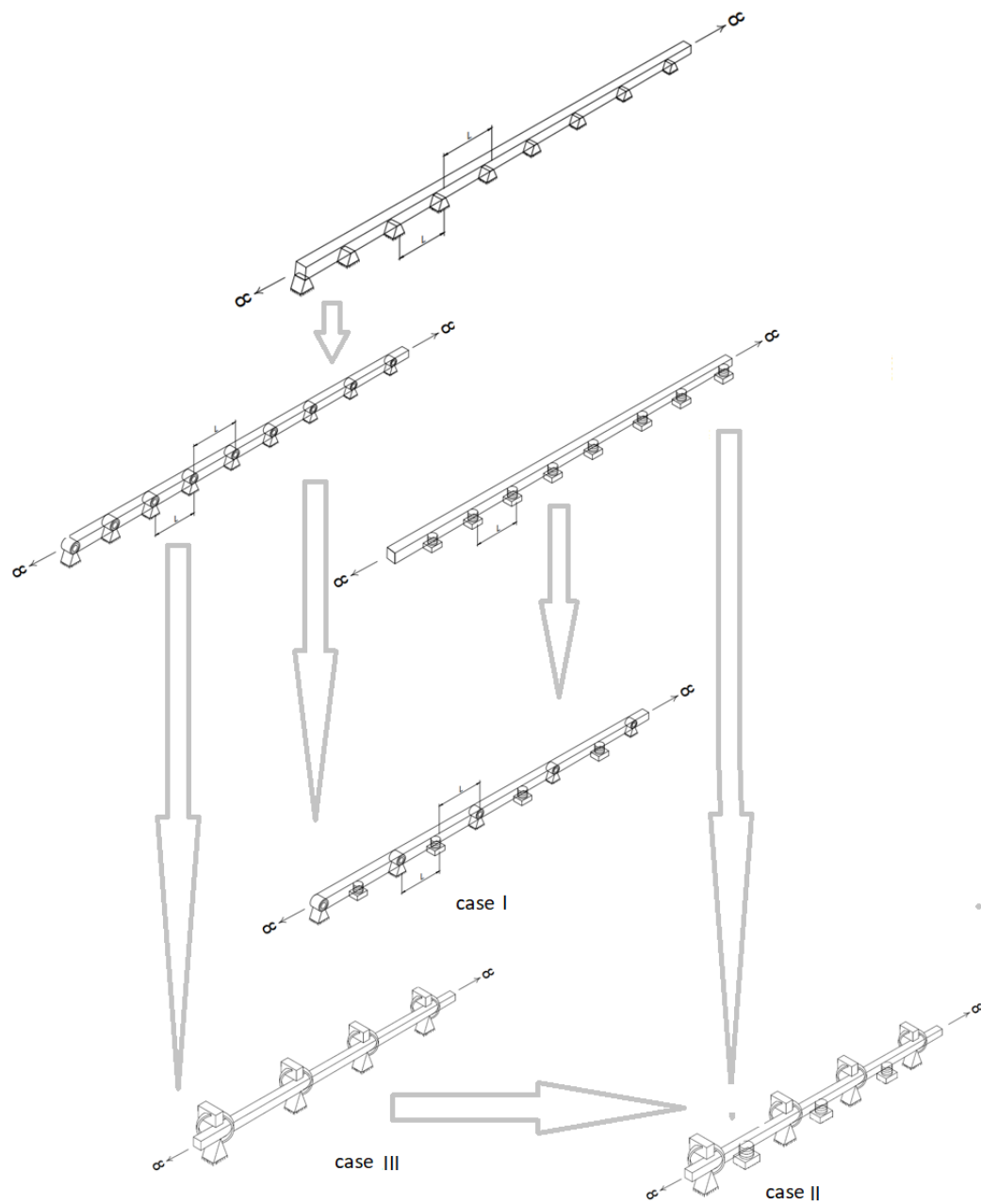
C. Transverse deflections are small.

These assumptions allow for simplifications within the upcoming derivations, such as being able to pull constant material and geometric properties out of integrals.

1.5 Thesis Objectives

To study the effect of metamaterial resonators, namely local spring-mass systems on Euler-Bernoulli periodic beams assuming the following conditions, study the effect of local resonators when they are attached to the Euler Bernoulli beam along with the non-classical boundary conditions, represented by torsional spring attachment.

- A. Study the variations of the propagation constant and its relation with the dynamic response of infinite beams resting on periodic supports.
- B. Study the characteristics of bandgaps including their position and width in the frequency response along with wave attenuation.
- C. Study the effect on band gap maps by varying beam unit cell length “lattice constant”, mass of local resonator, local resonator stiffness and gamma “mass ratio between the absorber and the beam”.
- D. Study the vibration transmission in finite periodic beams.
- E. Repeat A,B,C,D for the cases I,II,III.



1.6 Organization of the work

In this thesis, chapter 1 describes metamaterial resonators, periodic lattice structures, flexural wave propagation and their dependence on different acoustical parameters of wave propagation. In chapter 2, literature review is done for the Euler Bernoulli beam, periodic structures, band gaps and metamaterial resonators. In Chapter 3, modelling is done for different cases of Euler Bernoulli beam. In the first case, wave propagation Periodic Euler Bernoulli beams with no attached resonators is analysed. The band gaps are observed and results are obtained. After that, to study the effect of non-classical boundary conditions, torsional springs are added at the periodic support of the beam and characteristics of bandgaps are observed. In the next case, local resonators are attached to the beam to observe the bandgaps and attenuation properties. For the final case, both torsional spring and local resonators are attached to beams as metamaterial and above-mentioned characteristics are observed.

In chapters 4 and 5, comparisons are done to understand how these configurations affect flexural wave propagation. The results also includes effect on the band gap maps by variation in various parameters, along with the effect on vibration transmission through the periodic structures.

Chapter 2 LITERATURE REVIEW

2.1 Euler-Bernoulli Beam

The Euler-Bernoulli beam theory is the simplest theory compared to other beam theories. In Euler Bernoulli Beam theory, we neglect the shear deformation, as a result constraints are lifted to have a convenient analysis, and the plane sections remain planes and perpendicular to the longitudinal axis. From a matrix view point , Euler Bernoulli beam has 2 degrees of freedom at the beam section level, and 4 degrees of freedom at nodal element level. The constitutive law is applicable for 4 x 4 matrix of Euler Bernoulli beam in solving for the above degrees of freedom. [25]

2.2 Metamaterial Resonators

Electromagnetic wave propagation researchers first introduced metamaterial concepts. In latest development, acoustics field have taken a keen interest in understanding and utilizing the principles of metamaterials in research and further development [26].

Inspiring things have been observed in metamaterials when studied in the context of acoustics and Vibroacoustic . It was found that acoustic metamaterials in their structure generally embrace particular internal design , which includes subwavelength arrays of the resonating unit. Phenomena such as negative mass density, double negative characteristics and negative modulus were observed. The negative parameters of these metamaterials are different from these generally existing as natural materials.

Enhancements in the field of acoustic metamaterials have been helping in controlling the wave transmission and manipulation of wave motion in acoustic media. Some of the examples on utilizing these ideas are the acoustical cloaks and low-frequency sound insulation materials [27].

In recent going on researches on acoustical materials, one that gained much of the interest is the resonant sonic material. The reason is that it has a particular design, containing arrays that exist in the subwavelength region. In one case the array consists of coated spheres (the coating is an elastically soft material, whereas the core is made of dense solid) . The array of sphere are submerged in a medium of matrix based structure [28].

Resonant sonic materials have one unique characteristic that they can introduce sonic band gaps even at low frequencies. Experimental studies show that significant sound suppression was observed in the low-frequency region. [29].

This idea has been used in designing of the certain types of beams, rods and plates and to suppress any of the unwanted elastic waves propagating in the structure [30].

New development in these materials has led to the formation of membrane type of structure that is light in weight as an acoustic metamaterial. The membrane is capable of achieving high sound transmission loss (STL) in a broad low-frequency range. The design of these metamaterials includes one rigid frame for each resonating unit with a small mass elastically attached to the center of each frame cell [31].

The use of secondary resonators (e.g., dynamic absorbers), which are attached with the vibrating master (host) structure are used to reduce sound from vibrating structures in low frequency zones [32].

For thin structures having partitions, some researchers used the Spectral Element method to evaluate distributed vibration absorbers used for minimizing the sound wave transmission and improving STL factor. [33]

In the investigations, mostly a sound generating structure or master structure is finite, and the resonance frequencies for the auxiliary absorbers attached to the master structure are tuned/adjusted to selected modes to suppress. Thus, the vibration of generating structure and hence the sound transmission, are reduced.[34]

Many designs of distributed vibration absorbers have been so far proposed. One of these was put forward by Usik [35], in which he used vibration absorbers in the form of regular array of beams mounted on a thin plate. In his work, all absorbers were tuned to different natural frequencies. These absorbers were spring-mass resonators of the single degree of freedom type.

In comparison to this work, Kidner [36] used the different type of resonators, which were acoustical blankets attached to a thin plate. These blankets were made of a poro-elastic material with masses attached to them to induce effect of special hetero-blankets. They were modelled as a spring-mass resonator arrays for simplification.

Carl [37] based on the existing environmental and economic requirements showed that lightweight structures are needed. Fulfilling the requirements of efficient attenuation, favourable behavior, and lightweight is a challenging task. However, vibroacoustic metamaterial can play an active role in meeting the said requirements. Moreover, they have stop bands referred to as tuneable frequency ranges which provide useful attenuation properties in noise and vibration for specified frequencies ranges. The structure of vibroacoustic meta-materials includes unit cells

assembly (most of the time-periodic) with non-homogenous material topology. They investigated different material layouts and analysed a duct with a square section. Both 1D waveguide and 2D infinite periodic structure were compared. It was observed that resonant structures targeting different frequencies could be merged in different patterns either regular, checkers or sequential to obtain the desired set of stop bands.

Sheng [38] utilized spectral element method to have insight into the wave propagation and transmission control achieved by the using periodic attached vibration absorbers, which provide wave filtering. In their work they utilized finite and infinite beams and derived the expression for obtaining initial propagation constants, along with complex band structures.

Taking periodicity into account in [39], it was shown that it helped in analysing all band edge frequencies without the hassle of calculating propagation constants. Explicit equations, providing insight into band gap opening mechanism, were useful in finding transition and local resonances.

Yong [40] performed the remarkable job in showing that what limitation possessed by the photonic electromagnetic material that does not exist in the locally resonant metamaterials. The limitation was that low-frequency bandgap can only be created in a vast structure, but by successful research work they showed that band gaps whose wavelengths were more significant than the lattice size do exist, and it resulted in the generation of low-frequency band gaps in relatively small structures. The metamaterial also has an additional feature of storing and transferring energy in its mechanical and electromechanical resonant structures. They also analysed the rigid framed structure with rubber coated resonators and derived analytical expressions for the effective mass densities of the 3-D plots and the 2-D type of resonant

materials. In they how surprisingly found that the effective mass become negative near the resonant frequency.

Huang[41] observed that the effective mass density, which was a critical parameter becomes frequency dependent and reduced near resonant frequency. However, in the case of microstructures, it was more convenient to describe the dispersive based behavior of the wave propagation and the bandgap structure.

The left-handed metamaterials are one with negative electric permittivity and magnetic permeability and they got much attention from researches since many of them tried exploring the negative index metamaterial .With the mathematical analogy between acoustics and electromagnetic physics, negative mass density metamaterials were developed. But Romeo and Lungo [42] have realized that naturally existing metamaterials do not possess these negative mass characteristics unlike negative permittivity in left-hand electromagnetic metamaterials.

Certain metamaterials with fabricated microstructures possess the negative mass density near resonant frequency if treated as classic resonant beams. Many papers[43-48] investigated this concept and realized that the actual frequency region, which was resonant in connection with the negative mass density, created a bandgap effects.

One-dimensional modelling analysis was done for the spring-mass system to see how it affected the wave propagation, but it was not possible to analyse the micro-motion in this type of structure using the classical /conventional continuum theory [49].

Casalotti [50] analysed a nonlinear case of metamaterial beam and studied the characteristics of non-linear metamaterial resonators attached to the beam. In their analysis, they used Euler

Bernoulli beam that was connected with non-linear spring mass subsystems which were introduced to act as local resonators or vibration absorbers.

Wang [51] have considered multi cells frequency bands by using the attached array of non-linear local resonators . They utilized an optimization algorithm to control the oscillations the lowest sets of vibration modes .

Wang [52] studied both an infinite beam and a finite beam with four links resonators to understand the effect of geometries changes on wave propagation control.

Using Hamilton Principle and Bloch theorem, they found that the flexural waves were partially transformed into the longitudinal waves while other waves were entirely locked.

Thus band gaps existing in the frequency spectrum could be tuned by having geometrical changes. They [53] showed that substantial flexural wave attenuation was possible within the band gaps. They also proved that active damping by local lateral resonators was possible through lowering the response peaks at the expense of bandgap strength .

Valentine [54] studied the modification of existing conventional metamaterials through the adjustment of acoustic metamaterial by introducing carbon-fibre reinforcement.

These laminations were integrated with periodic arrays of mass-spring-damper systems. It was observed through wider stopbands were observed in the dispersion response.

Due to the superior strength to weight ratio of these integrated composites, these laminated acoustic metamaterials can be suitable materials to be used in designing vehicles for effective vibration suppression with the lightweight advantage .

2.3 Boundary Conditions for Euler Beam

Flexible Boundary conditions have shown effect on the band gaps frequency and wave attenuation.

Zhiwie[55] to examine the band gap properties of flexural wave in compound plates, which are periodic in nature, had used the flexible boundary conditions. In their work, they adopted the general model of the boundary conditions by using the specific combination of linear and torsional springs. These combinations were used to analyze all non-dissipative boundary conditions.

Similarly, Zhnag [56] examined modal bandgap attenuation properties of a plate with simply supported boundary conditions.

Chapter 3 MATHEMATICAL MODELING

3.1 Infinite Euler-Bernoulli Beam on Periodic Supports

The behaviour of the metamaterial resonators discussed in the previous section is utilized here in the form of Euler Bernoulli beam equation model.

Consider the infinite Euler Bernoulli beam with a unit cell of length L , we examine the elastic wave propagation by using the spectral stiffness matrix and transfer matrix approach

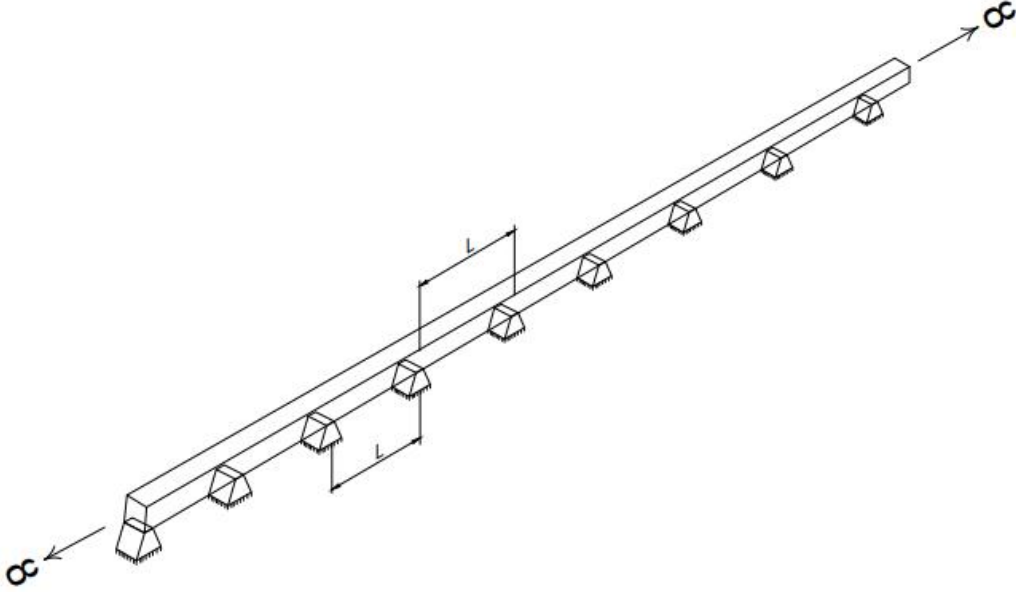


Figure 3-1 infinite Euler Bernoulli Beam

The free flexural vibration of Euler Bernoulli beam at the unit cell is represented by

$$EIw'''' + \rho A\ddot{w} = 0 \quad (1)$$

where $w(x, t)$ is the transverse displacement, E is the modulus of elasticity, I is the area moment of inertia, ρ is density of the material, A is cross sectional area of the beam

Assuming a solution in spectral form [57]

$$w(x, t) = \frac{1}{N} \sum_{n=0}^{N-1} W_n(x, \omega_n) e^{-i\omega_n t} \quad (2)$$

where N is the number of spans , n is integer numbers (1,2,...), ω_n is a specific discrete frequency

Substituting Equation 2 in to Equation 1 yields gives for a specific discrete frequency like ($\omega = \omega_n$) as

$$EIW'''' - \omega^2 \rho A W = 0 \quad (3)$$

Assuming the general solution of Equation 3 as

$$W(x) = a e^{-ik\omega x} \quad (4)$$

Where (W) the amplitude displacement wave, (a) is constant amplitude , k is wavenumber, ω is the circular frequency.

In addition, substituting equation (4) into (3) leads to following dispersion equation :

$$k^4 - k_b^4 = 0 \quad (5)$$

where k_b is the flexural wavenumber of the beam materials, which is

$$k_b = \sqrt[4]{\frac{\rho A \omega^2}{EI}} \quad (6)$$

The dispersion relation gives the following two real roots and two imaginary roots

$$k_1 = -k_2 = k_b, \quad (7)$$

$$k_3 = -k_4 = ik_b$$

Then , the general solution can be rewritten in the form

$$W(x, \omega) = e(x, \omega)a \quad (8)$$

where

$$e(x, \omega) = [e^{-ik_b x} \quad e^{-k_b x} \quad e^{ik_b x} \quad e^{k_b x}] \quad (9)$$

and

$$a = \{a_1 \quad a_2 \quad a_3 \quad a_4\}^T \quad (10)$$

Displacements and slopes at the unit cell ends (0 , L) can be related to the displacement field

$$u = \begin{Bmatrix} W_1 \\ \theta_1 \\ W_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} W(0) \\ W'(0) \\ W(L) \\ W'(L) \end{Bmatrix} \quad (11)$$

Reorganizing the Equation (11)

$$u = \begin{Bmatrix} e(0, \omega) \\ e'(0, \omega) \\ e(L, \omega) \\ e'(L, \omega) \end{Bmatrix} a = H_B(\omega) a \quad (12)$$

where

$$H_B(\omega) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -ik_b & -k_b & ik_b & k_b \\ e^{-ik_bL} & e^{-k_bL} & e^{ik_bL} & e^{k_bL} \\ -ik_b e^{-ik_bL} & -k_b e^{-k_bL} & ik_b e^{ik_bL} & k_b e^{k_bL} \end{bmatrix} \quad (13)$$

Then

$$W(x) = N_B(x, \omega) u \quad (14)$$

where

$$N_B(x, \omega) = e(x, \omega) H_B^{-1}(\omega) \quad (15)$$

Shear forces and bending moment can be related to displacement field too by the following relations:

$$Q(x) = -EIW'''(x) \quad (16)$$

$$M(x) = EIW''(x) \quad (17)$$

The spectral nodal transverse shear forces and bending moments at the unit cell ends (0, L) can be related to the corresponding shear forces and bending moments as

$$f_c = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} -Q(0) \\ -M(0) \\ Q(L) \\ M(L) \end{Bmatrix} \quad (18)$$

Eventually, the spectral element matrix is obtained as

$$[K(\omega)]\{u\} = \{f_c\} \quad (19)$$

Where $K(\omega)$ is dynamic stiffness matrix of the beam given by

$$K(\omega) = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad (20)$$

where

$$K_{11} = \frac{EI}{L^3} \begin{bmatrix} a_1 & c_2 L \\ c_2 L & b_1 L^2 \end{bmatrix} \quad (21)$$

$$K_{12} = K_{21}^T = \frac{EI}{L^3} \begin{bmatrix} -a_2 & c_1 L \\ -c_1 L & b_2 L^2 \end{bmatrix} \quad (22)$$

$$K_{22} = \frac{EI}{L^3} \begin{bmatrix} a_1 & -c_2 L \\ -c_2 L & b_1 L^2 \end{bmatrix} \quad (23)$$

with

$$a_1 = [\sin k_b L \cosh k_b L + \cos k_b L \sinh k_b L] \frac{(k_b L)^3}{\Delta} \quad (24)$$

$$a_2 = [\sinh k_b L + \sin k_b L] \frac{(k_b L)^3}{\Delta} \quad (25)$$

$$b_1 = [\sin k_b L \cosh k_b L - \cos k_b L \sinh k_b L] \frac{k_b L}{\Delta} \quad (26)$$

$$b_2 = [\sinh k_b L - \sin k_b L] \frac{k_b L}{\Delta} \quad (27)$$

$$c_1 = [\cosh k_b L - \cos k_b L] \frac{(k_b L)^2}{\Delta} \quad (28)$$

$$c_2 = [\sinh k_b L \sin k_b L] \frac{(k_b L)^2}{\Delta} \quad (29)$$

$$\Delta = 1 - \cosh k_b L \cos k_b L \quad (30)$$

3.2 Periodic Beam with Attached Local Resonator

After obtaining the dynamic stiffness matrix (20) for a unit cell , Let us get the stiffness matrix of the beam and the local Resonator attached to the span of the beam unit cell shown in ,

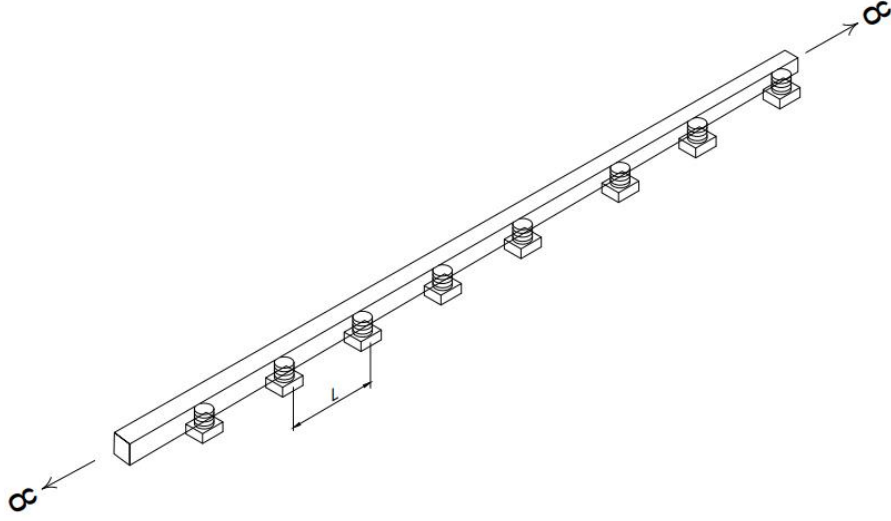


Figure 3-2 Infinite Euler Bernoulli Beam With Local Resonator Spring-Mass System

Using the formula

$$D u = f \quad (31)$$

where D is the stiffness matrix including the resonator and f is the corresponding force function, and u is the displacement function given respectively as

$$f = \begin{Bmatrix} f_L \\ f_R \end{Bmatrix} \quad (32)$$

$$, u = \begin{Bmatrix} u_L \\ u_R \end{Bmatrix} \quad (33)$$

Let us establish the relationships between physical variables at the left and the right ends of the beam unit cell i.e. ,

$$u_L = \begin{Bmatrix} w_L \\ \theta_L \end{Bmatrix} \text{ and } u_R = \begin{Bmatrix} w_R \\ \theta_R \end{Bmatrix} \quad (34)$$

It is now possible to utilize the Bloch theorem [48] for wave propagation in the infinitely periodic beams as

$$\{u_L\} = \{u_R\}e^{\mu} \quad (35)$$

$$\{f_L\} = -\{f_R\}e^{\mu} \quad (36)$$

where μ is the wave propagation constant

Then the total dynamic can be written as

$$\begin{bmatrix} D_{LL} & D_{LR} \\ D_{RL} & D_{RR} \end{bmatrix} = \begin{bmatrix} K_{11} + D_0 & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad (37)$$

where D_0 is the dynamic stiffness matrix of the local resonator

$$D_0 = \begin{bmatrix} D_0 & 0 \\ 0 & 0 \end{bmatrix} \quad (38)$$

Substituting Equations 35 and 36 in equation 31 and the following second order eigenvalue problem in e^{μ} is obtain as

$$[D_{RL} + (D_{LL} + D_{RR})e^{\mu} + D_{LR}e^{2\mu}]u_L = 0 \quad (39)$$

Equation 39 is a quadratic eigenvalue problem, for which we can obtain two pairs of propagation constants $\pm \mu_1$ and $\pm \mu_2$. These pairs represent waves travelling in opposite direction. The real part of μ is the attenuation constant, which is defined as the value representing the amplitude decay rate experienced by the wave during its travel from one unit cell to the next. The

imaginary part of μ is the phase constant, which is defined as the value that relates the phase change of a wave at the two ends of unit cell. When considering these definitions , one finds that when either μ_1 or μ_2 is purely imaginary, wave propagation in the beam is possible .

3.3 Effect of Torsional Springs at Supports

Let us considering the case of a beam with torsional spring at the periodic supports without the resonators as shown in schematic Figure 3-3 . Mead [58] also performed this analysis.

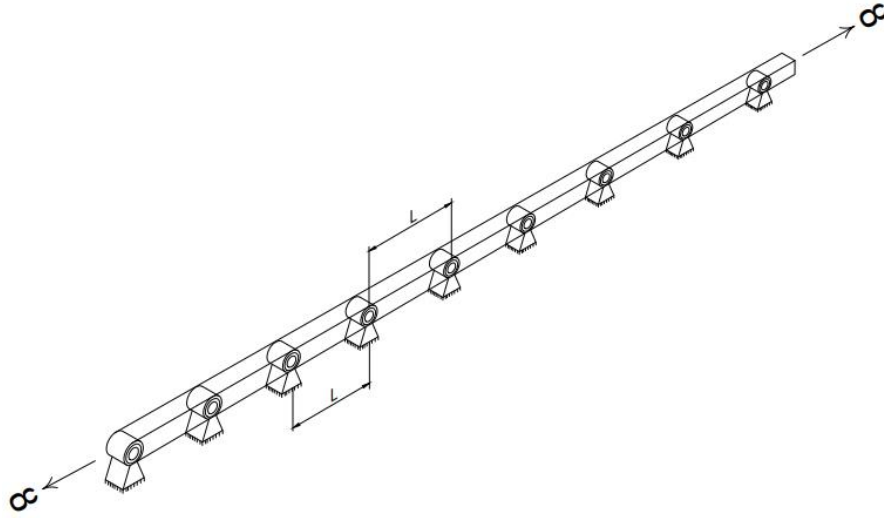


Figure 3-3 Infinite Euler Bernoulli Beam With Torsional Spring

$$D_0 = \begin{bmatrix} K_\theta & 0 \\ 0 & 0 \end{bmatrix} \quad (40)$$

Substituting the dynamic stiffness D_0 from Eq.40 in Eq. 37 and Eq. 39 and solving the equation as shown in Appendix A, we obtain the final equations as follows

$$\begin{aligned} & \cosh^2(\mu) + \frac{(b_2(2a_1 + K_\theta) - 2b_1a_2)}{(c_1^2 - b_1a_2)} \cosh(\mu) \\ & + \frac{(2b_1(2a_1 + K_\theta) - 2a_2b_2 - 2c_1^2))}{2(c_1^2 - b_1a_2)} = 0 \end{aligned} \quad (41)$$

3.4 Periodic Beam Torsional Spring and Local Resonators

Let us consider an infinite Euler Bernoulli Beam resting on torsional completion support with attached local resonator as shown in Figure 3-4 . In this case , the dynamic stiffness matrix is

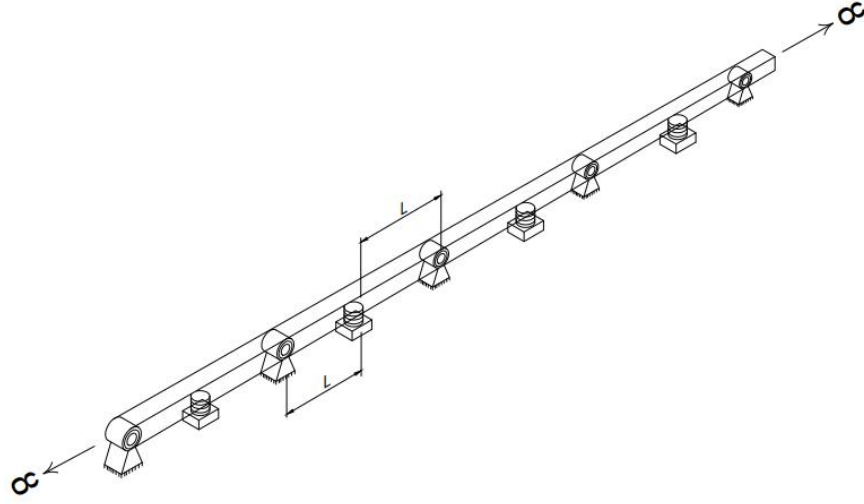


Figure 3-4 Infinite Euler Bernoulli Beam With The Local Resonators and Torsional Springs

$$D_0 = \begin{bmatrix} \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta & 0 \\ 0 & 0 \end{bmatrix} \quad (44)$$

Substituting the dynamic stiffness D_0 from Eq.44 in Eq. 37 and Eq. 39 on obtains the following eigen value problem :

$$\begin{aligned}
 & \cosh^2(\mu) \tag{45} \\
 & + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} \cosh(\mu) \\
 & + \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2a_2b_2 - 2c_1^2\right)}{2(c_1^2 - b_1a_2)} = 0
 \end{aligned}$$

Derivation are given in Appendix C .

3.5 Results

The derived equations 41, 43 and 45 are used to study periodic beams with different configurations.

The Euler Bernoulli beam parameters properties as follows:

Table 3-1 Properties of Beam (Aluminium)

E	ρ	L/b	M	h
69G Pa	2700Kg/m ³	4.5	0.135 Kg/m	0.05 m

The following are the properties for the local resonators and torsional springs under investigation:

Table 3-2 Properties of Local Resonators and Torsional Spring

m resonator	K resonator	ω resonator	K_θ
0.01 kg	1.5*10 ⁶ N/m	50 rad/s	(50, 100 or 150)N.m/rad

Then ,

$$I = \frac{1}{12}bh^3, A = bh \quad (77)$$

Table 3-3 Moment of Inertia and Area of Beam

A beam section area	I moment of inertia	EI
0.05 m ²	1.041666*10 ⁻⁵ m ⁴	718750 Pa.m ⁴

3.5.1 Case 1 : Periodic Euler Bernoulli Beam without Local Resonators and Torsional Springs

As a starting case of modelling , let's consider a periodic beam with no resonator and torsional springs . Figure 3-5 shows that 3 bandgaps exist at [1] , [2] ,and [3] where the attenuation constant grow sharply and decay sharply. Phase constant , exist in the same frequency ranges of the bandgaps , as shown in Figure 3-6 .

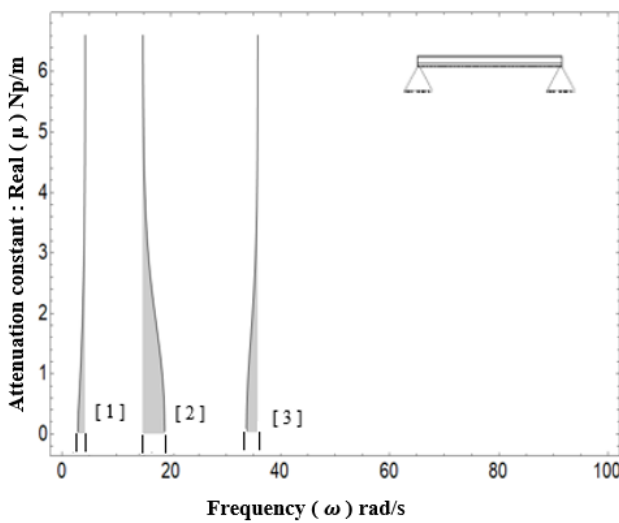


Figure 3-5 Propagation Constant of Euler Bernoulli Beam Real Part without Local resonator Resonators (Aluminium)

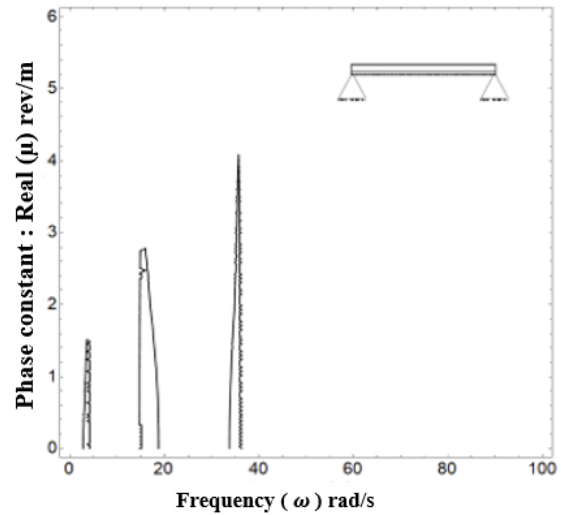


Figure 3-6 Propagation Constant of Euler Bernoulli Beam Imaginary Part without Local Resonators (Aluminium)

Two figures are plotted for the Real Part (Attenuation Constant) and Imaginary Part (Phase Constant) vs Frequency to analyse the effect of band gaps and attenuations over the frequency response of the beam .

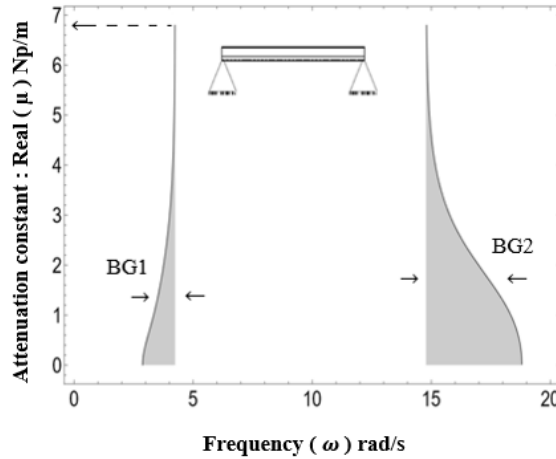


Figure 3-7 Propagation Constant of Euler Bernoulli Beam Real Part without Local Resonators

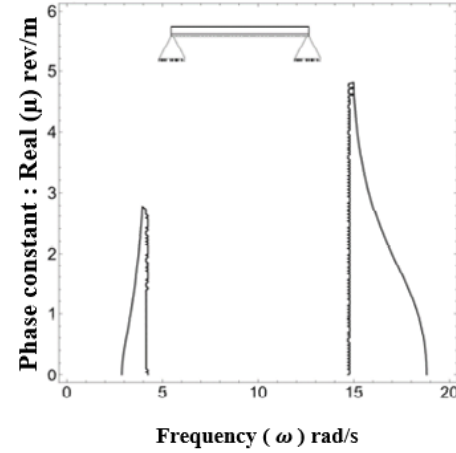


Figure 3-8 Propagation Constant of Euler Bernoulli Beam Imaginary Part without Local Resonators

Focusing on frequency range [0 – 20] rad/s , Figure 3-7 shaded area is showing first bandgaps with a maximum value that is reached for the attenuation constant is > 6 Np /m for the frequency range between < 3 and > 4 rad/s, also the bandgap is covering lesser frequency region. Second bandgap is in the shaded area over frequency range between < 15 and > 20 rad/s, the attenuation

constant has the same maximum value of 6.8 Np /m but second bandgap is larger and covering wider range of frequency.

3.5.2 Case 2 : Periodic Euler-Bernoulli Beam with Torsional Springs without Local Resonators

In this section, we are using the torsional springs with three different stiffnesses investigate the influence of support stiffnesses on the bandgaps formation phenomena , and hence ,on waves propagation through the periodic beam .

3.5.2.1 Effect of Torsional Spring with $K_{\theta} = 50 \text{ N.m/rad}$

After analysing the periodic beam resting support , let us introduce torsional springs at the intermediate support and investigate their influence on the bandgap structure .

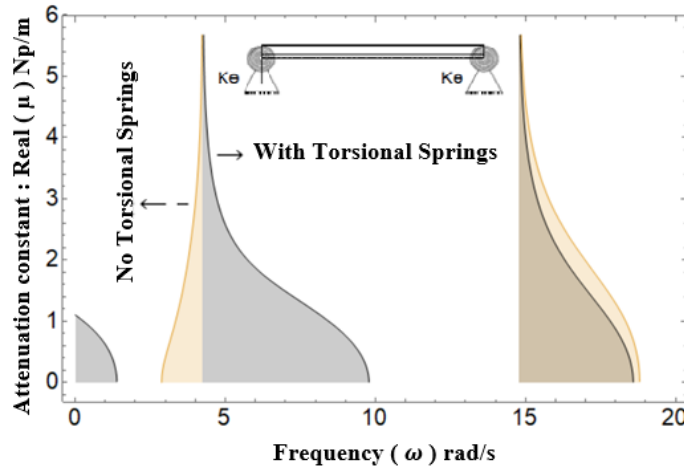


Figure 3-9 Propagation Constant of Euler Bernoulli Beam Real Part without Torsional Springs and with Torsional Spring $K_{\theta} = 50 \text{ N.m/rad}$

In this modified system, we also change torsional spring stiffness to analyse its effect on bandgap regions. When graphs are plotted for this scenario in which $K_\theta = 50 \text{ N.m/rad}$, it is that the bandgaps are cover more frequency region compared to the case without torsional springs . It is also noted that the value of attenuation constant is reduced by 1.9 Np /m , whereas the width of bandgap is increased as shows in Figure 3-9 compared with Fig 3-7.

It is interesting to note that a “ new ” low frequency bandgap is formed , however , with considerably lower attenuation level .

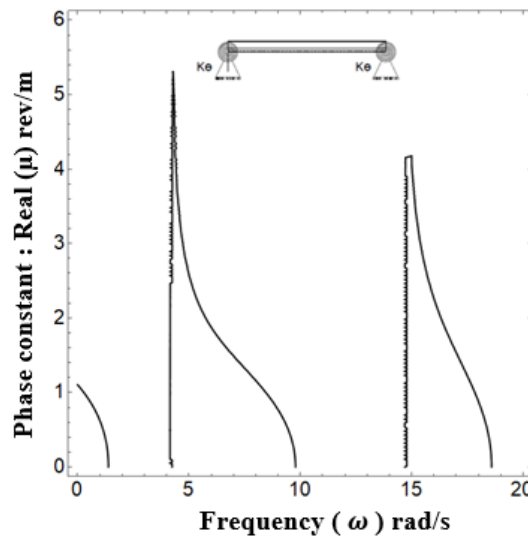


Figure 3-10 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Torsional Spring $K_\theta = 50 \text{ N.m/rad}$

3.5.3 Effect of Torsional Spring with $K_\theta = 100$ N.m/rad

In previous section, we observed an improvement increase in band gap due to imposing torsional springs .

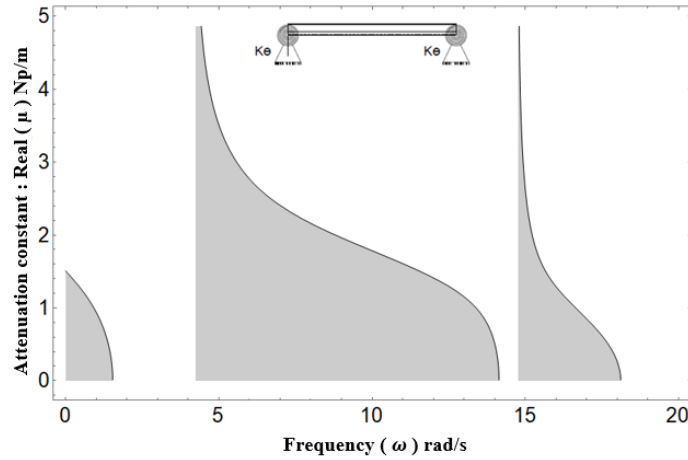


Figure 3-11 Propagation Constant of Euler Bernoulli Beam Real Part with Local Resonator Torsional Spring $K_\theta = 100$ N.m/rad

Let us increase the torsional stiffness to $K_\theta = 100$ N.m/rad .

Figure 3-11 shows an increase of bandgap frequency domains , while the attenuation constant remain unchanged .

For the phase constant plot, the behaviour observed in Figure 3-12 is almost similar Figure 3-12. However , the value of phase constant is reduced by 1.2 rev/m at the bandgap of higher frequency.

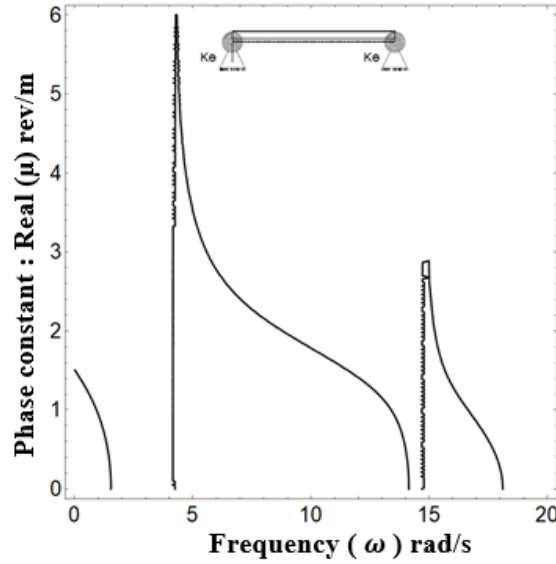


Figure 3-12 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Torsional Spring $K_{\theta} = 100 \text{ N.m/rad}$

3.5.4 Effect of Torsional Spring with $K_{\theta} = 150 \text{ N.m/rad}$

Let us now have the torsional spring stiffness further increased to $K_{\theta} = 150 \text{ N.m/rad}$, Figure 3-13 shows that one band gap extending from 4.5-15 rad/s. As if, two bandgaps have been fused together into a single bigger one, with a slight reduction of 0.15 Np/m in attenuation constant value.

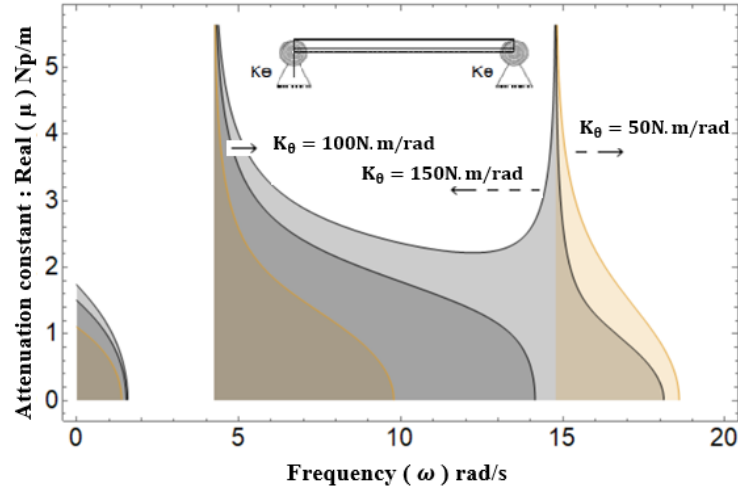


Figure 3-13 Propagation Constant of Euler Bernoulli Beam Real Part with Torsional Spring $K_{\theta} = (50, 100, 150) \text{ N.m/rad}$

For the phase constant plot , Figure 3-14, shows that for the region covering 5 rad/s to 15 rad/s, the value of phase constant has been increased by 0.45 rev/m .

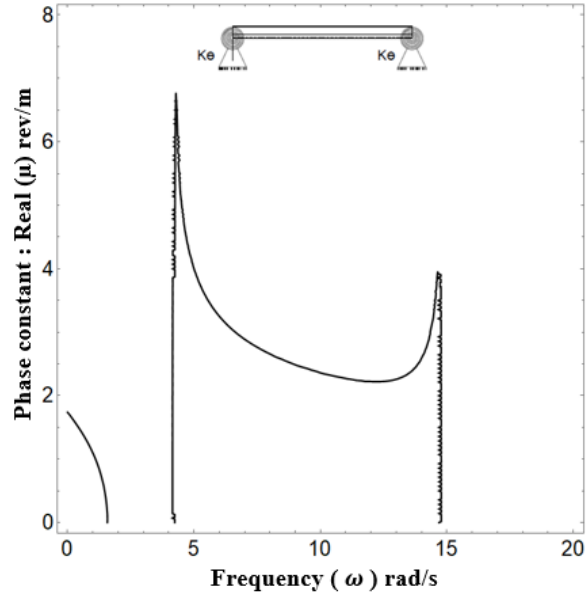


Figure 3-14 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Torsional Spring $K_{\theta} = 150 \text{ N.m/rad}$

3.6 Case 3 : Periodic Euler Bernoulli with Local Resonators

In this section , we start to investigate the effect of attached local resonators to the periodic infinite beam , Thus , mass-spring resonators are added to the Euler Bernoulli beam resting on periodic supports with no torsional spring attachments .

Fig 3-15 shows the attenuation –frequency graph with continuous bandgap having minor unattached wave propagation spaces.

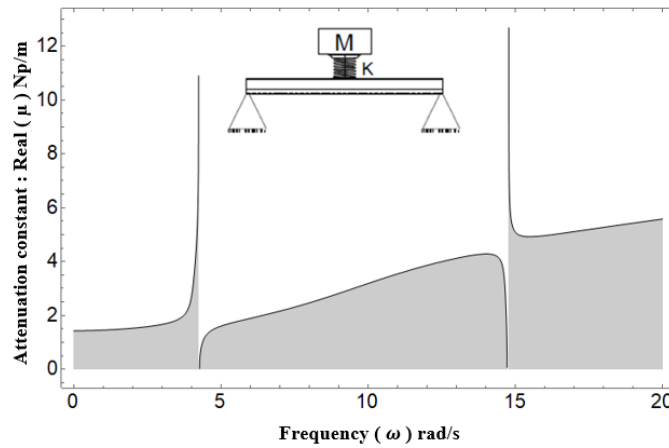


Figure 3-15 Propagation Constant of Euler Bernoulli Beam Real Part with Local Resonator Spring-Mass System

In addition, area along the bandgap regions has now widen covering more frequency zones, which is our primary concern in this study. When seen the value of attenuation constant along the y-axis, it has also been increased.

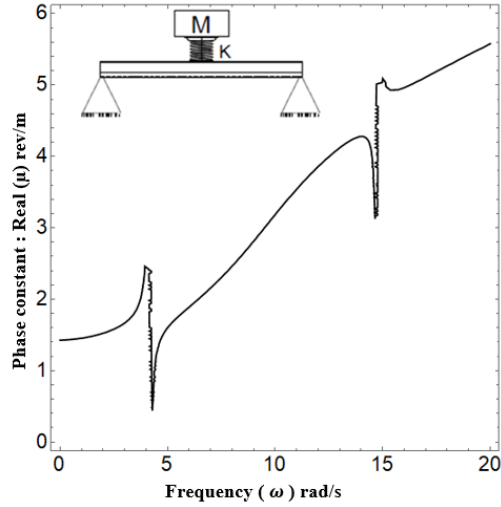


Figure 3-16 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Local Resonator Spring-Mass System

For the phase constant values, as the frequency increases there is a gradual increase in its value. In comparison with beam configuration without local resonators , we see that in the lower frequency range, the phase constant value is lower by 3.2 rev/m and for the high frequency zone the phase constant value is higher by 0.9 rev/m.

3.7 Case 4 : Periodic Euler Bernoulli with Local Resonator and Torsional Springs

Let us now consider a periodic Euler Bernoulli beam with both local and torsional springs at support .

The stiffness of torsional springs are changed [50,100,150] and the stiffness of local resonator spring is kept constant at k and m.

3.7.1 Periodic Euler Bernoulli beam with Local resonator and torsional springs ($K_\theta = 50 \text{ N.m/rad}$)

The torsional spring stiffness in this case is taken as $K_\theta = 50 \text{ N.m/rad}$, while keeping the mass and spring stiffness of the local resonators at $k = 1.5 \times 10^6 \text{ N/m}$ and $m = 0.01 \text{ kg}$, respectively. When compared with the periodic beam with no local resonators, we observe that the bandgap structure has changed and the flexural wave suffers attenuation over most of the frequency spectrum. The attenuation free zones is reduced further.

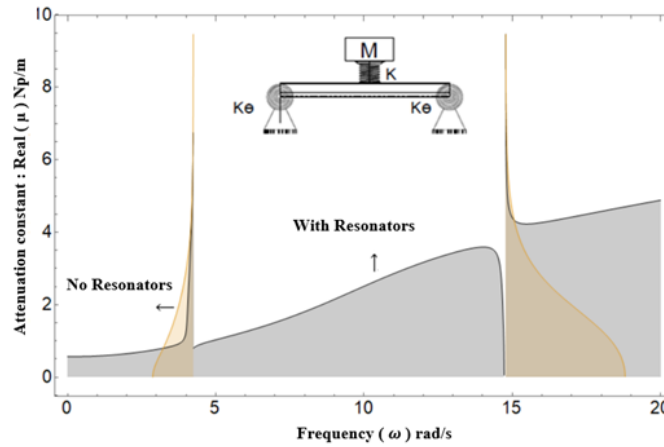


Figure 3-17 Propagation Constant of Euler Bernoulli Beam Real Part with Local Resonator Spring-Mass System and Torsional Spring $K_\theta = 50 \text{ N.m/rad}$

It is also observed that a reduction in the attenuation constant at lower bandgap, while at the high frequency bandgap value and for the high frequency zone, the attenuation constant remains unchanged.

For the phase constant vs frequency plot shown in Fig 3-18, the curve behaviour is similar to that of the beam with no resonator , but the discontinuity peaks along the curve has are reduced by.

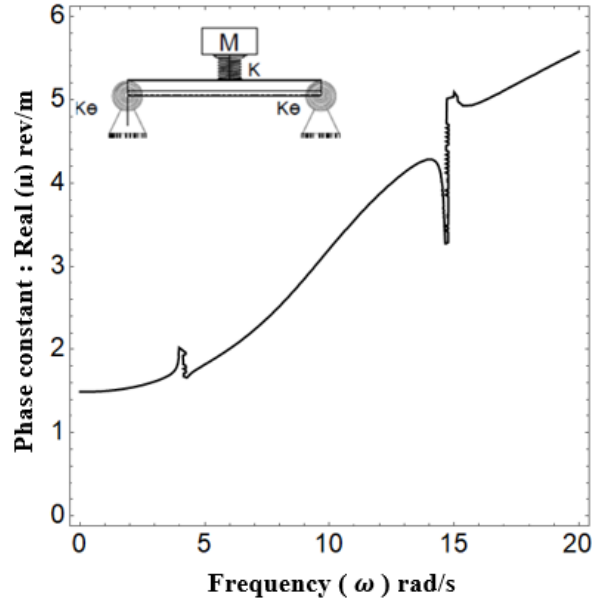


Figure 3-18 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Local Resonator Spring-Mass System and Torsional Spring $K_{\theta} = 50$ N.m/rad

3.7.2 Periodic Euler Bernoulli beam with Local resonator and torsional springs ($K_{\theta} = 100$ N.m/rad)

The torsional spring stiffness in this case is taken as $K_{\theta} = 100$ N.m/rad , while keeping the mass and spring stiffness of the local resonators at $k = 1.5 \times 10^6$ N/m and $m = 0.01$ kg , respectively.

When compared with the periodic beam with no local resonators, we observe that the bandgap structure has changed and the flexural wave suffers attenuation over most of the frequency spectrum .The attenuation free zones is reduced further.

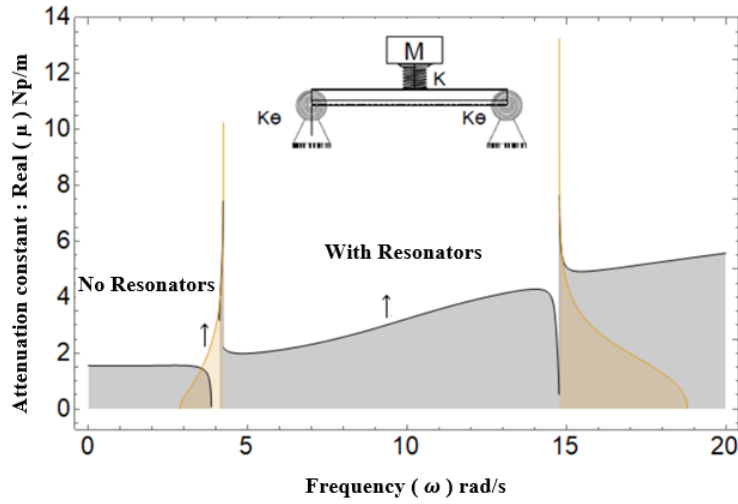


Figure 3-19 Propagation Constant of Euler Bernoulli Beam Real Part With The Local Resonator Spring-Mass System and Torsional Spring $K_{\theta} = 100$ N.m/rad

It is also observed that a reduction in the attenuation constant at lower bandgap, while at the high frequency bandgap value and for the high frequency zone, the attenuation constant remains unchanged.

For the phase constant vs frequency plot shown in Fig 3-20, the curve behaviour is similar to that of the beam with no resonator, but the discontinuity peaks along the curve has are reduced by.

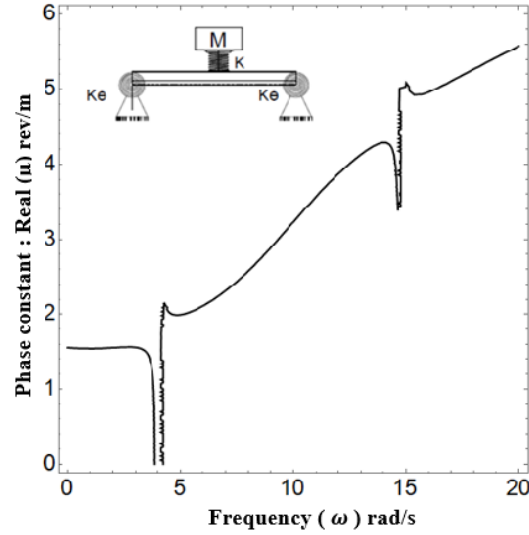


Figure 3-20 Propagation Constant of Euler Bernoulli Beam Imaginary Part With Local Resonator Spring-Mass System and Torsional Spring $K_{\theta} = 100 \text{ N.m/rad}$

3.7.3 Periodic Euler Bernoulli beam with Local resonator and torsional springs ($K_{\theta} = 150 \text{ N.m/rad}$)

The torsional spring stiffness in this case is taken as $K_{\theta} = 150 \text{ N.m/rad}$, while keeping the mass and spring stiffness of the local resonators at $k = 1.5 \times 10^6 \text{ N/m}$ and $m = 0.01 \text{ kg}$, respectively.

When compared with the periodic beam with no local resonators, we observe that the bandgap structure has changed and the flexural wave suffers attenuation over most of the frequency spectrum. The attenuation free zones are reduced further.

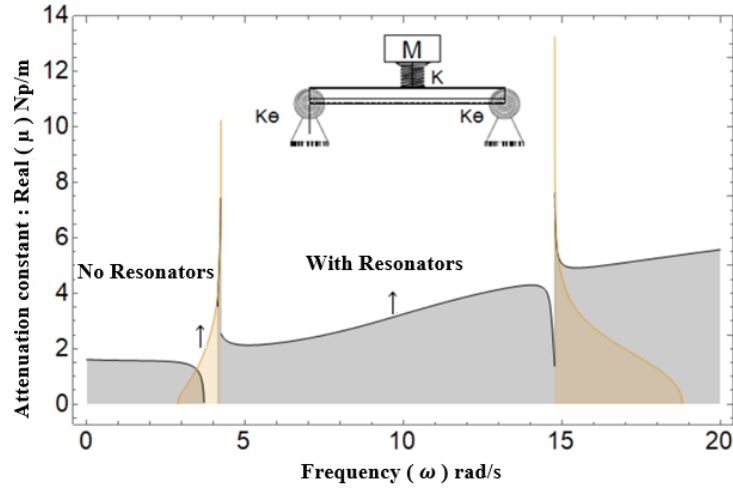


Figure 3-21 Propagation Constant of Euler Bernoulli Beam Real Part With Local Resonator Spring-Mass System and Torsional Spring $K_{\theta} = 150 \text{ N.m/rad}$

It is also observed that a reduction in the attenuation constant at lower band gap, while at the high frequency band gap value and for the high frequency zone, the attenuation constant remains unchanged .

For the phase constant vs frequency plot shown in Fig 3-22, the curve behaviour is similar to that of the beam with no resonator, but the discontinuity peaks along the curve are reduced.

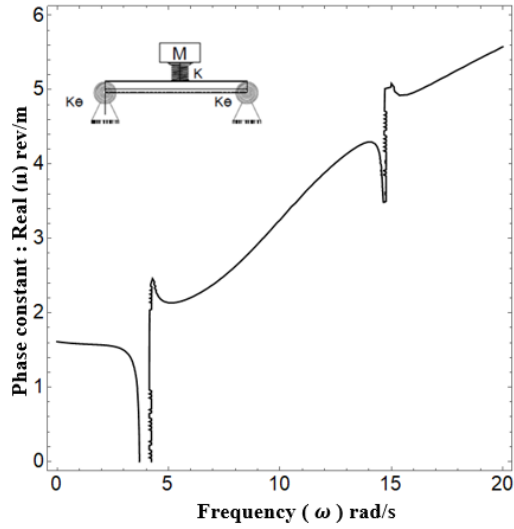


Figure 3-22 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Local Resonator Spring-Mass System and Torsional Spring $K_{\theta} = 150 \text{ N.m/rad}$

3.8 Case 5: Rotational Spring-Lumped Rotary Inertia Resonator

In this case, we have used rotational spring-lumped rotary inertia resonator. The results has been plotted between the frequency and attenuation constant.

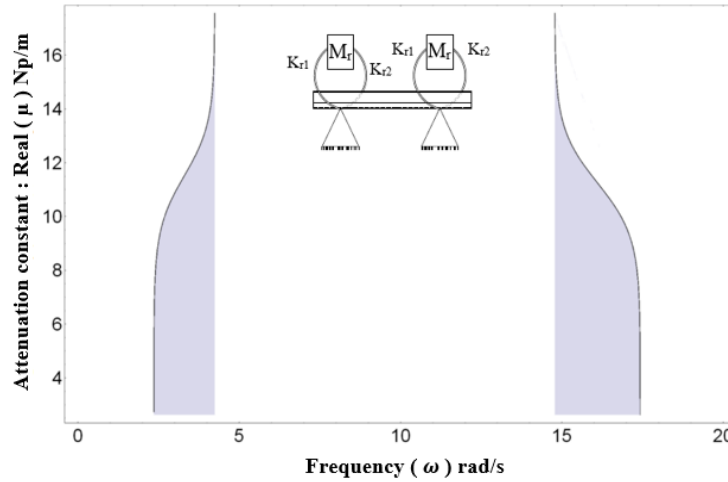


Figure 3-23 Propagation Constant of Euler Bernoulli Beam Real Part with Rotational Spring-Lumped Rotary Inertia Resonator

It can be seen that the attenuation constants values has been increased and the shaded band gap region is covering less spectrum of frequency region which is approximately from 3 – 5 rad/s and between 15-17 rad/s.

It is also observed that the attenuation constant value is the highest in comparison to the previous cases and similar increase is observed for the phase constant curves when compared with the previous cases of beams.

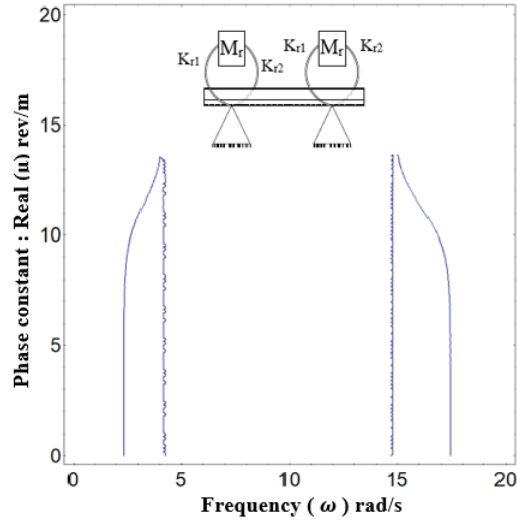


Figure 3-24 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Rotational Spring-Lumped Rotary Inertia Resonator

3.9 Case 6: Rotational Spring-Lumped Rotary Inertia Resonator and linear spring mass Resonator

In this case, we have used rotational spring-lumped rotary inertia resonator with linear mass spring resonators. The results has been plotted between the frequency and attenuation constant & phase constant.

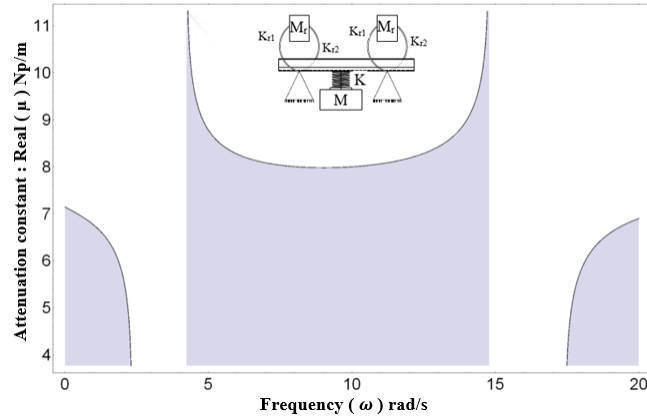


Figure 3-25 Propagation Constant of Euler Bernoulli Beam Real Part with Rotational Spring-Lumped Rotary Inertia Resonator and linear spring mass Resonator

It can be seen that the attenuation constants values has been decreased and the shaded band gap region is now covering more spectrum of frequency region which is approximately from 5 – 15 rad/s. For phase constant the values has been decreases but the size of frequency spectrum has been increased.

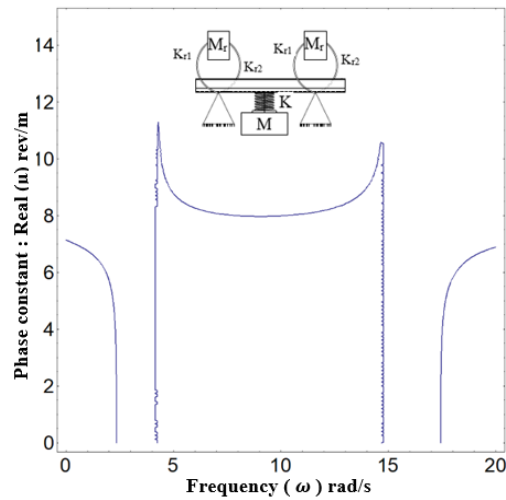
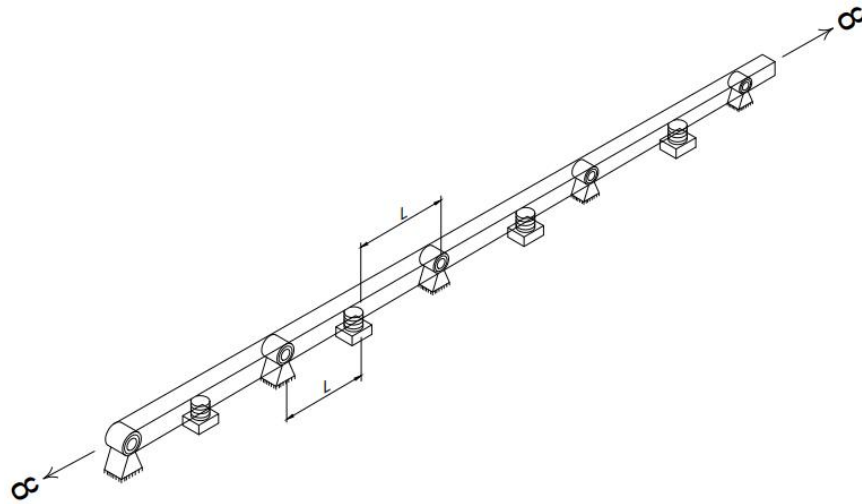


Figure 3-26 Propagation Constant of Euler Bernoulli Beam Imaginary Part with Rotational Spring-Lumped Rotary Inertia Resonator and linear spring mass Resonator

Chapter 4 Bandgap maps

Bandgap maps are developed to perform a parametric study on vibration reduction in the infinite periodic structures. In describing the effect of design parameters, bandgap maps play a very critical role since $|Re(\mu)| = F(\omega_r, L, m, k, \gamma)$.



**Figure 4-1 Infinite Euler Bernoulli Beam With The Local Resonator Spring-Mass System
And Torsional Spring**

In following analysis, the periodic beam configuration includes local resonators with $k=1.5 \times 10^6$ N/m, $m=0.1$ kg and torsional springs $K_\theta = 150$ N.m/rad, and $L = 4.5$ m.

4.1 Dependence of Attenuation Constant on the Local Resonator Frequency

The variables defined are local resonators stiffness (k), and mass (m), lattice constant (L) which is the distance between local resonator and support and mass ratio between mass of unit cell to the mass of beam (γ).

Each parameter is selected individually for variation and other parameters are kept fixed to study the effects on attenuation constant over the frequency spectrum .

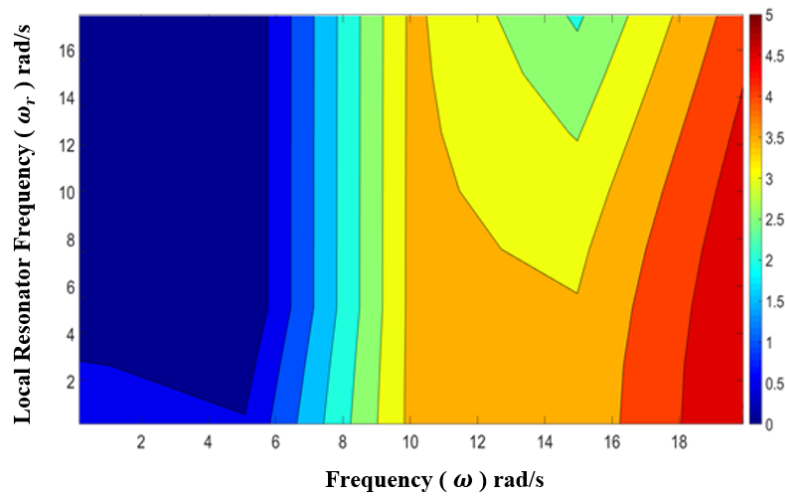


Figure 4-2 Dependence of Attenuation Constant on Local Resonator Frequency

It is noted from Figure 4-2 that as the frequency approaches that of the bandgap, the attenuation constant value is increased.

4.2 Dependence of Attenuation Constant on Lattice Constant

In Fig 4-3 , we observe the effect of L on the attenuation constant . Three parameters has been made constant which are Mass Ratio (γ), Local Resonator frequency and Torsional springs stiffness

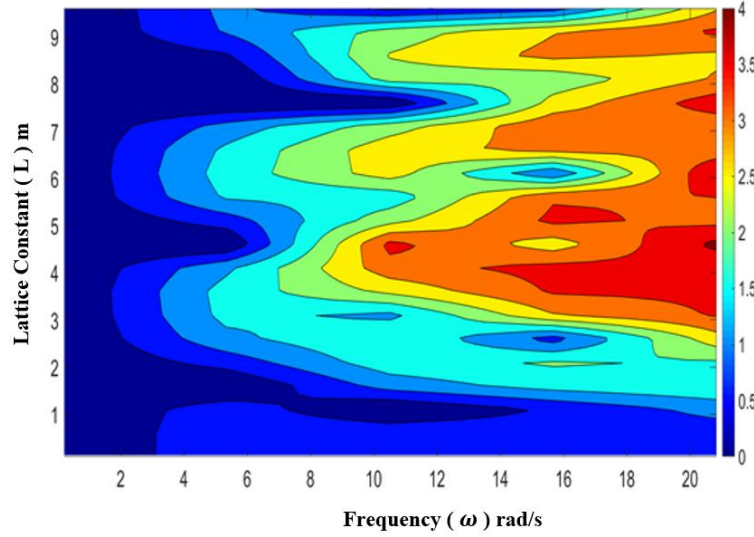


Figure 4-3 Dependence of Attenuation Constant with Lattice Constant

It is observed that the highest value of attenuation constant is achieved when $L = 4.5$ m. It is concluded from this figure that as the lattice constant value increases, higher attenuation constant values are achieved at the higher beam frequency range.

4.3 Dependence of Attenuation Constant on Local Resonator Mass

In Fig 4-4 , the resonator mass is increased and the attenuation constant effect is visualized. In this case, all the other parameters are kept constant.

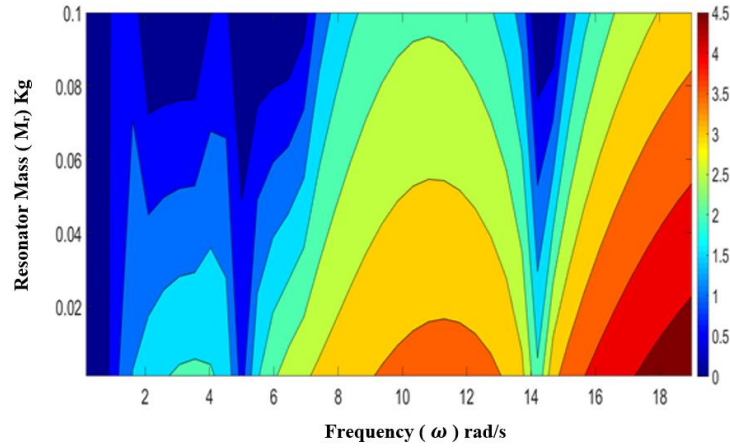


Figure 4-4 Dependence of Attenuation Constant with Resonator Mass

It is observed, that the band gap effective region increases as the resonators mass decreases.

4.4 Dependence of Attenuation Constant with Mass Ration between Mass of Unit Cell to the Mass of Beam

In Fig 4-5, the ratio of mass of unit cell to the mass of beam (γ) is varied and its on the attenuation constant is observed .

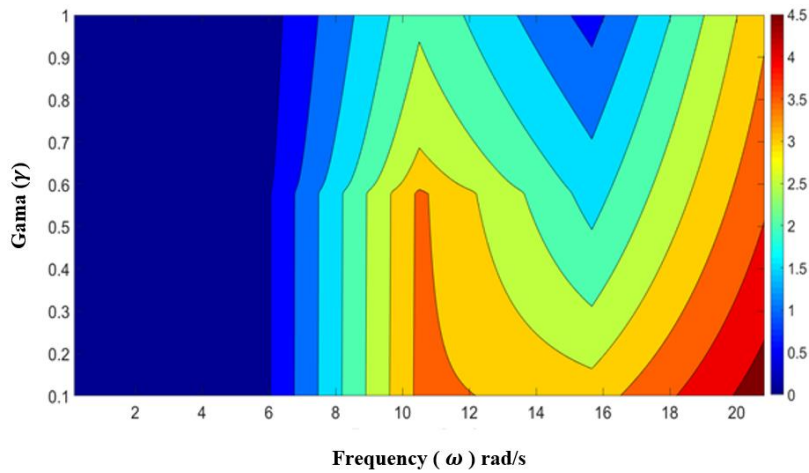


Figure 4-5 Dependence of Attenuation Constant with Mass Ration Gama

As the value of Gamma is increased, the attenuation is decreased and the frequency of the shift slightly towards the lower frequency.

4.5 Dependence of Attenuation Constant on Resonator Stiffness

In Fig 4-6 the bandgap map plot, we see that band gaps with lattice constant and torsional spring parameters has no effect due to change in the resonator stiffness.

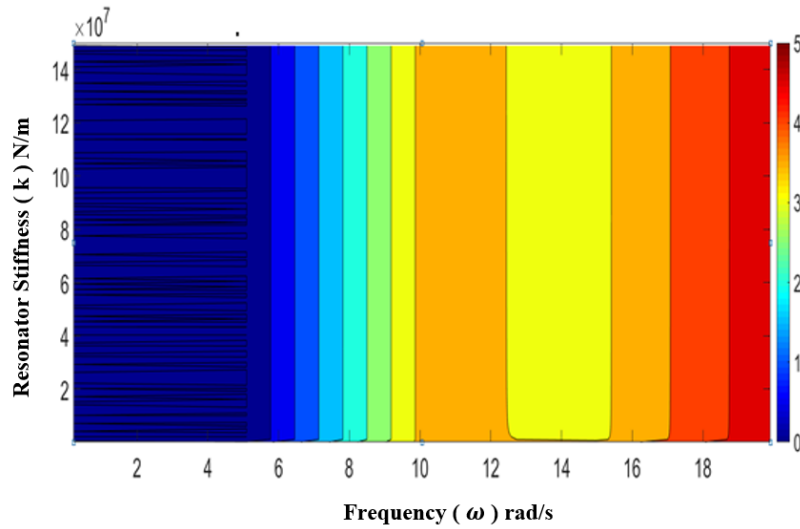


Figure 4-6 Dependence of Attenuation Constant with Resonator Stiffness

However, the band gaps which have variation due to vibration absorbers stiffness showed shift towards the higher frequency region as the value of resonator stiffness is increased.

Chapter 5 Vibration Transmission through a Finite Periodic Structure

In this part, the vibration transmission occurring across a finite periodic structure is investigated .

As an initial step, SEM is used to derive the reacceptance matrix that links the displacements and forces at each end and it helps in modelling the unit cell structure. The SEM technique is more efficient in comparison to conventional FEM, because for unit cell to establish the dynamic stiffness matrix, only two beam elements are required, [49] .

A schematic representation of the problem is displayed in Figure 5-1(a)

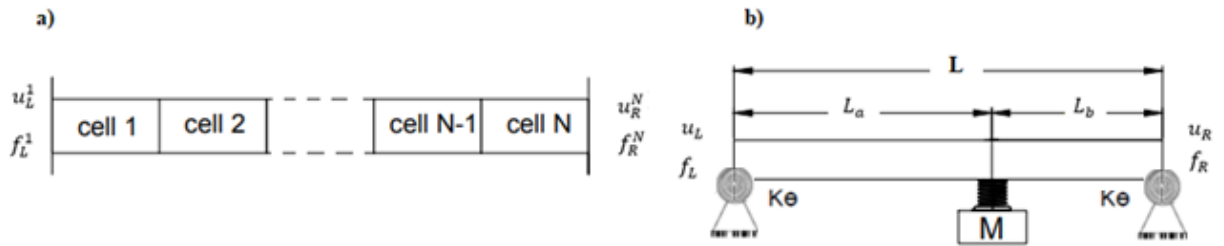


Figure 5-1(a) Schematic representation of a finite periodic structure with N unit cells. (b)

One unit cell of the structure

Using the SEM, the dynamic equation of the unit cell shown in Figure 5-1 (b) can be written as

$$\begin{bmatrix} \bar{D}_{LL} & \bar{D}_{LI} & 0 \\ \bar{D}_{IL} & \bar{D}_{II} & \bar{D}_{IR} \\ 0 & \bar{D}_{RI} & \bar{D}_{RR} \end{bmatrix} \begin{Bmatrix} u_L \\ u_I \\ u_R \end{Bmatrix} = \begin{Bmatrix} f_L \\ 0 \\ f_R \end{Bmatrix} \quad (46)$$

where u_I represents the displacement of the attachment point of local resonator with the beam and

$$\begin{aligned}\bar{D}_{LL} &= K_{11}(L_a), \quad \bar{D}_{LI} = K_{12}(L_a), \quad \bar{D}_{IL} = K_{21}(L_a), \\ \bar{D}_{II} &= K_{22}(L_a) + K_{11}(L_b) + D_0 \\ \bar{D}_{IR} &= K_{12}(L_b), \quad \bar{D}_{RI} = K_{21}(L_b), \quad \bar{D}_{RR} = K_{22}(L_b),\end{aligned}\tag{47}$$

Note that u_I can be eliminated as

$$u_I = -\bar{D}^{-1}(\bar{D}_{IL}u_L + \bar{D}_{IR}u_R)\tag{48}$$

Then , Equation 46 reduces to

$$\begin{bmatrix} \tilde{D}_{LL} & \tilde{D}_{LR} \\ \tilde{D}_{RL} & \tilde{D}_{RR} \end{bmatrix} \begin{Bmatrix} u_L \\ u_R \end{Bmatrix} = \begin{Bmatrix} f_L \\ f_R \end{Bmatrix}\tag{49}$$

where,

$$\tilde{D}_{LL} = \bar{D}_{LL} - \bar{D}_{LI}\bar{D}_{II}^{-1}\bar{D}_{IL}, \quad \tilde{D}_{LR} = -\bar{D}_{LI}\bar{D}_{II}^{-1}\bar{D}_{IR}\tag{50}$$

$$\tilde{D}_{RR} = \bar{D}_{RR} - \bar{D}_{RI}\bar{D}_{II}^{-1}\bar{D}_{RL}, \quad \tilde{D}_{RL} = -\bar{D}_{RI}\bar{D}_{II}^{-1}\bar{D}_{IL}\tag{51}$$

Introducing the Bloch condition for wave propagation in one-dimensional periodic structure, a quadratic eigenvalue problem is derived

$$[\tilde{D}_{RL} + (\tilde{D}_{LL} + \tilde{D}_{RR})e^\mu + \tilde{D}_{LR}e^{2\mu}]u_L = 0\tag{52}$$

Equation 52 is manipulated and reformulated in to

$$\left(\begin{bmatrix} \tilde{D}_{RL} & \tilde{D}_{RR} \\ 0 & I \end{bmatrix} - \begin{bmatrix} -\tilde{D}_{LL} & -\tilde{D}_{LR} \\ I & 0 \end{bmatrix} \right) \begin{Bmatrix} u_L \\ \tilde{u}_L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (53)$$

where

$$\tilde{u}_L = e^\mu u_L \quad (54)$$

From Equation 53, two sets of eigenvalues associated with eigenvectors of u_L are obtained as

$$\left(e^{-\mu j}, \phi_{u_j}^+ \right), (j = 1, 2) \quad (55)$$

$$\left(e^{+\mu j}, \phi_{u_j}^- \right), (j = 1, 2) \quad (56)$$

These eigen pairs characterize a pair of a positive-going and a negative-going characteristic waves. The associated force eigenvectors are given by

$$\phi_{f_j}^+ = (\tilde{D}_{LL} + e^{-\mu j} \tilde{D}_{LR}) \phi_{u_j}^+, \quad \phi_{f_j}^- = (\tilde{D}_{LL} + e^{\mu j} \tilde{D}_{LR}) \phi_{u_j}^- \quad (57)$$

The displacements and forces at a junction between two unit cells of the periodic system can be represented by the sum of the positive and negative-going waves with amplitudes a^+ and a^- as

$$u = \phi_u^+ a^+ + \phi_u^- a^- \quad , \quad f = \phi_f^+ a^+ + \phi_f^- a^- \quad (58)$$

where

$$\phi_u^+ = [\phi_{u_1}^+, \phi_{u_2}^+] \quad , \quad \phi_u^- = [\phi_{u_1}^-, \phi_{u_2}^-] \quad (59)$$

$$\phi_f^+ = [\phi_{f_1}^+, \phi_{f_2}^+] \quad , \quad \phi_f^- = [\phi_{f_1}^-, \phi_{f_2}^-] \quad (60)$$

and

$$a^+ = \{a_1^+, a_2^+\}^T, \quad a^- = \{a_1^-, a_2^-\}^T \quad (61)$$

Hence, the displacement at the two ends of the finite periodic structure depicted in Figure 5-1 are expressed by

$$u_L^1 = \sum_{j=1}^2 (a_j^+ \phi_{u_j}^+ + a_j^- \phi_{u_j}^-) = \varphi_u^+ a^+ + \varphi_u^- a^- \quad (62)$$

$$u_R^N = \sum_{j=1}^2 (b_j^+ \phi_{u_j}^+ + b_j^- \phi_{u_j}^-) = \varphi_u^+ b^+ + \varphi_u^- b^- \quad (63)$$

The wave amplitudes at the two ends are then related by

$$b^+ = E^N a^+, \quad a^- = E^N b^- \quad (64)$$

where

$$E = \begin{bmatrix} e^{-\mu_1} & 0 \\ 0 & e^{-\mu_2} \end{bmatrix} \quad (65)$$

Substituting Equation 64 into Equations 62 and 63 gives

$$u_L^1 = \varphi_u^+ a^+ + \varphi_u^- E^N b^- \quad (66)$$

$$u_R^N = \varphi_u^+ E^N a^+ + \varphi_u^- b^- \quad (67)$$

In a matrix form, Equation 66 and 67 are written as

$$\begin{Bmatrix} u_L^1 \\ u_R^N \end{Bmatrix} = \begin{bmatrix} \varphi_u^+ & \varphi_u^- E^N \\ \varphi_u^+ E^N & \varphi_u^- \end{bmatrix} \begin{Bmatrix} a^+ \\ b^- \end{Bmatrix} \quad (68)$$

In a similar way, the associated force vector at the ends of the finite periodic structure are obtained as

$$\begin{Bmatrix} f_L^1 \\ f_R^N \end{Bmatrix} = \begin{bmatrix} \varphi_f^+ & \varphi_f^- E^N \\ \varphi_f^+ E^N & \varphi_f^- \end{bmatrix} \begin{Bmatrix} a^+ \\ b^- \end{Bmatrix} \quad (69)$$

Combining Equations 68 and 69 leads to

$$\begin{Bmatrix} u_L^1 \\ u_R^N \end{Bmatrix} = \alpha_N \begin{Bmatrix} f_L^1 \\ f_R^N \end{Bmatrix} \quad (70)$$

where α_N is the receptance matrix of the whole periodic structure given by

$$\alpha_N = \begin{bmatrix} \alpha_{N11} & \alpha_{N12} \\ \alpha_{N21} & \alpha_{N22} \end{bmatrix} = \begin{bmatrix} \varphi_u^+ & \varphi_u^- E^N \\ \varphi_u^+ E^N & \varphi_u^- \end{bmatrix} \begin{bmatrix} \varphi_f^+ & \varphi_f^- E^N \\ \varphi_f^+ E^N & \varphi_f^- \end{bmatrix}^{-1} \quad (71)$$

Referring back to the Figure 5-1, it is assumed that the finite structure is excited by a harmonic transverse displacement w_L^1 at its left end and no other load is applied to the structure. This means that

$$u_L^1 = \{w_L^1 \quad , \quad \theta_L^1\}^T \quad , \quad f_L^1 = \{Q_L^1 \quad , \quad 0\}^T \quad , \quad f_R^N = 0 \quad (72)$$

Therefore from Equation 70 , one finds that

$$u_L^1 = A_T f_L^1 \quad , \quad u_R^N = B_T u_L^1 \quad (73)$$

where

$$A_T = \alpha_{N11} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (74)$$

$$B_T = \alpha_{N21} \alpha_{N11}^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (75)$$

The transverse vibration transmittance of the finite periodic structure is obtained as

$$T_N = \left| \frac{w_R^N}{w_L^1} \right| = |B_{11} + B_{12} A_{21} A_{11}^{-1}| \quad (76)$$

This standard method for transmission as utilized by [60].

5.1 Vibration Transmission

Let us consider the finite Euler-Bernoulli beam with five repeated spans, as shown in Fig 5-2

The purpose here is to study vibration transmission through the beam in different configurations , including (1) finite Euler Bernoulli beam local resting on simple supports , (2) finite Euler Bernoulli beam with torsion springs at supports , (3) finite beam with torsional springs and local resonator .

5.1.1 Vibration Transmission Through Finite Euler Bernoulli Beams

Wave transmission , in dB is depicted versus frequency in Fig 5-3 . Equation (76) is used to calculate the transmittance factor .

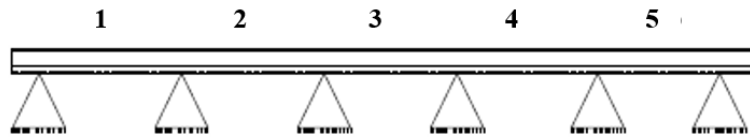


Figure 5-2 Finite Euler Bernoulli Beam with five unit cells

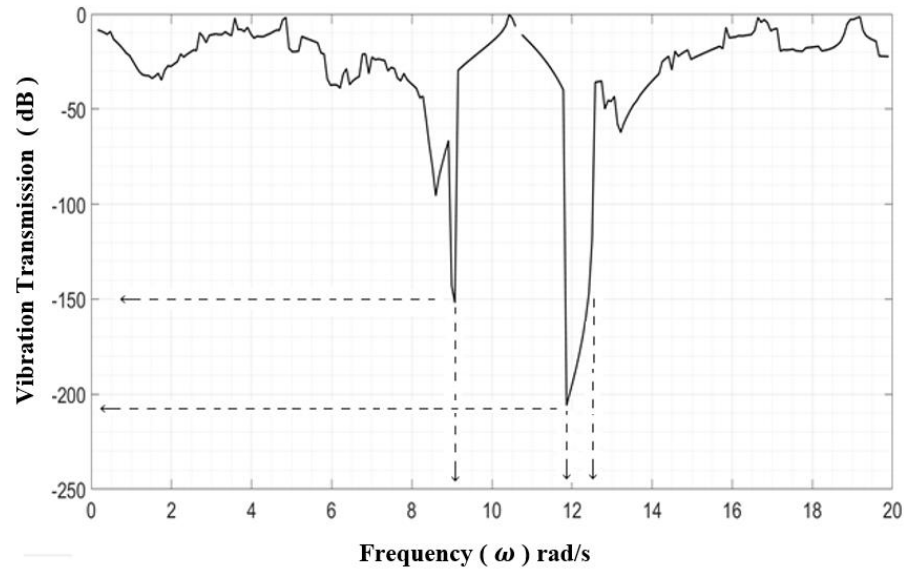


Figure 5-3 Vibration Transmission of Finite Euler Bernoulli Beam

The highest attenuation can be seen in tow frequency ranger [8+ , 9-]rad/s and [12-, 13-] rad /s with of -150 dB and -220 dB, respectively .The vibration transmission observe a sharp downfall peak where there is high band gap shaded region. We see that vibration transmission is sharply reduced at frequency of 9 rad/s and 12 rad/s .

5.1.2 Vibration Transmission Through Finite Euler Bernoulli Beam with Torsional Springs

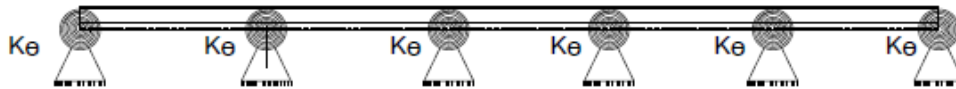


Figure 5-4 Finite Euler Bernoulli Beam With Torsional Spring

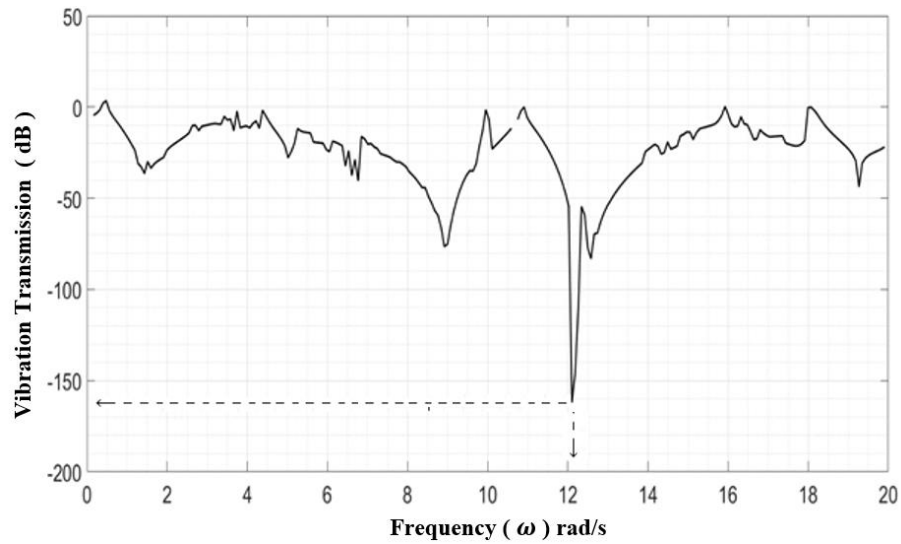


Figure 5-5 Vibration Transmission of Finite Euler Bernoulli Beam with Torsional Spring

In this case, torsional springs are added at the support of the beam. Using Equation 76 the transmittance coefficient is calculated . In Fig 5-5 It can be seen that the least vibration transmission value occurs at $[12-, 12+]$ rad /s , with a value of 170 dBs .In this case the band gap region is providing high attenuation at the value of 12 rad/s, so a sharp peak is observed in the negative zone of vibration transmission. This is also in agreement with Figure 3-9 in which band gaps are shown in shaded region along this frequency spectrum.

5.1.3 Vibration Transmission Through Finite Euler Bernoulli Beam with Local Resonator and Torsional Springs

In this case, local resonators are added as attachment to spans , in addition to the torsional springs as shows in Fig5-6 .

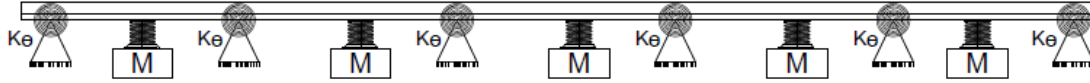


Figure 5-6 Finite Euler Bernoulli Beam With The Local Resonator Spring-Mass System and Torsional Spring

It is shown in Fig 5-7 that the addition of local resonators has not only increased the band gap but also reduced the vibration transmission resulting in the increase of attenuation constant.

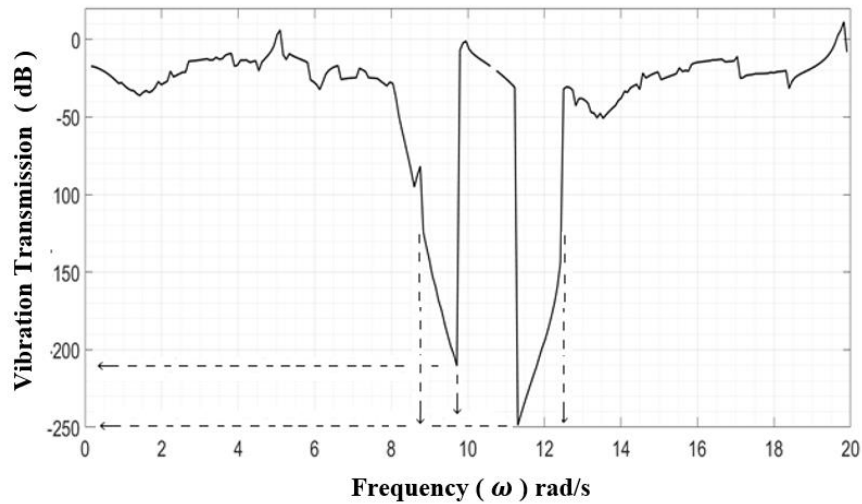


Figure 5-7 Vibration Transmission of Finite Euler Bernoulli Beam with Local Resonator and Torsional Spring

The least transmission value has reduced further, in comparison to the first case discussed for vibration transmission. The region of frequencies from the $[8+, 10+]$ rad/s and $[11+ \text{ to } 12+]$ rad/s with values of -210 dB and -250 dB, respectively.

In comparison with band gaps Figure 3-19 for this case of beam with both torsional spring and local resonators, we observe sharp negative peak for vibration transmission in the same region of frequency spectrum which is in agreement the equations derived and curves obtained.

Chapter 6 CONCLUSIONS

6.1 Conclusions

The work done in this thesis is primarily focused on the significance of the metamaterial resonators in the periodic beams. Different configurations has been employed by using only beam, using beam with local resonators, using beam with torsional spring and using beam with metamaterial (combination of local resonator and torsional spring). Each of these configurations has proved its advantage in developing bandgaps and provided non-dispersed wave propagation due to its lattice-based periodic structure.

Following results were concluded using the Euler Bernoulli beam with spectral stiffness matrix and transfer matrix approach:

When considering only beam system on periodic supports without any resonators and torsional spring, the value for attenuation constant is seen smaller in value and the curves showed less width of band gaps along the frequency region selected for this study. When the torsional springs are added, the width of bandgap has increased in value as evident from the curves plotted. In addition, attenuation constant value reduced, so peak with less band gap has lowered in value along the y-axis of attenuation constant. Though still exist large width of attenuation free zones but is lower than beam on periodic supports system configuration. The stiffness of torsional spring is also varied to analyse the effect of material parameter on bandgaps. It is found that as torsional stiffness increased, the attenuation free zones reduced because of “fusion of curves”. Though attenuation constant value remained same and bandgaps shifted to left side of curve but still we have lower attenuation constant and more continuous band gap curves. When used local

resonators one the beam on stiffness periodic supports, the width of band gap curve extended to almost the entire region with very small gaps of attenuation free zone and lower value of attenuation constant. The peaks at which band gaps are negligible has increased in value along the y-axis of attenuation constant due to this configuration. But bandgap still existing in higher frequency region of the range given and the peak has higher attenuation constant value that is adjusted by varying the spring stiffness to lower the value of attenuation constant. Thus it is concluded that metamaterial resonators play a dominant role in bandgaps enhancement for the wave attenuation.

It is also seen that band gap maps in finite periodic structures have provided us better understanding for different parameters and their effect on the attenuation value, width and position of bandgaps. The shift of bandgaps regions and values of attenuation constants showed significant change for the parameter of spring mass resonator and “lattice constant (length of unit cell of structure” when compared with other parameters.

The vibration transmission in the between unit cells in periodic beams reduced to the minimum value when Metamaterials resonators beams are used, as seen from the curves and maximum band gap obtained. The trends of these plots are validating the previous analysis done for the propagation constant. The additional information we obtained from the vibration transmission plots is the width of the band gap and the amplitude of the attenuation.

With the use of Rotational Spring-Lumped Rotary Inertia Resonator, the attenuation reached the maximum value in comparison to the previous cases observed but the reduction in size of frequency spectrum for band gap regions has been observed. When the linear spring mass

resonator is added with Rotational Spring resonator, the attenuation constant reduced but the band gap region has been increased covering the wide range of frequency spectrum validating our results.

The purpose is to validate our point that Metamaterials resonators are more effective in providing control on wave transmission, attenuation, position and width of bandgaps for low frequency regions.

Appendix

Appendix A

Substituting Eq. 20-26 in the Eq. 37, which provides us with different Stiffness Matrix D value. Substituting this in Eq. 39, we obtain the below equation.

$$\left[\frac{EI}{L^3} \left(\begin{bmatrix} -a_2 & -c_1 L \\ c_1 L & b_2 L^2 \end{bmatrix} + \begin{bmatrix} 2a_1 + K_\theta & 0 \\ 0 & 2b_1 L^2 \end{bmatrix} \right) e^\mu + \begin{bmatrix} -a_2 & c_1 L \\ -c_1 L & b_2 L^2 \end{bmatrix} e^{2\mu} \right] u_L = 0 \quad (78)$$

Now finding the determinant for the above equation and simplifying,

$$\begin{vmatrix} -a_2 + (2a_1 + K_\theta)e^\mu - a_2 e^{2\mu} & -c_1 L + c_1 L e^{2\mu} \\ c_1 L - c_1 L e^{2\mu} & b_2 L^2 + 2b_1 L^2 e^\mu + b_2 L^2 e^{2\mu} \end{vmatrix} = 0 \quad (79)$$

The determinant for Equation 79 as

$$\begin{aligned} e^{4\mu} + \frac{(b_2(2a_1 + K_\theta) - 2b_1 a_2)}{(c_1^2 - b_1 a_2)} e^{3\mu} + \frac{(2b_1(2a_1 + K_\theta) - 2a_2 b_2 - 2c_1^2)}{(c_1^2 - b_1 a_2)} e^{2\mu} \\ + \frac{(b_2(2a_1 + K_\theta) - 2b_1 a_2)}{(c_1^2 - b_1 a_2)} e^\mu + 1 = 0 \end{aligned} \quad (80)$$

Divided all terms by $e^{2\mu}$

$$\begin{aligned}
e^{2\mu} + \frac{(b_2(2a_1 + K_\theta) - 2b_1a_2)}{(c_1^2 - b_1a_2)}e^\mu & \quad (81) \\
+ \frac{(2b_1(2a_1 + K_\theta) - a_2b_2 - a_2b_2 - 2c_1^2))}{(c_1^2 - b_1a_2)} \\
+ \frac{(b_2(2a_1 + K_\theta) - 2b_1a_2)}{(c_1^2 - b_1a_2)}e^{-\mu} + e^{-2\mu} = 0
\end{aligned}$$

then α defined as

$$\alpha = \frac{(b_2(2a_1 + K_\theta) - 2b_1a_2)}{(c_1^2 - b_1a_2)} \quad (82)$$

and β as

$$\beta = \frac{(2b_1(2a_1 + K_\theta) - a_2b_2 - 2c_1^2))}{(c_1^2 - b_1a_2)} \quad (83)$$

Then substitute equation 82 and 83 in 84

$$\cosh^2(\mu) + \alpha \cosh(\mu) + \beta = 0 \quad (84)$$

Now re-arranging the equation in terms of Cosh function, it gives us the equation 26.

Appendix B

Similarly again substituting Eq. 20-26 in the Eq. 37, it provides us with different stiffness matrix D value. Substituting this in Eq. 39, we obtain the below equation.

$$\left[\frac{EI}{L^3} \left(\begin{bmatrix} -a_2 & -c_1L \\ c_1L & b_2L^2 \end{bmatrix} + \left(\begin{bmatrix} 2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} & 0 \\ 0 & 2b_1L^2 \end{bmatrix} e^\mu + \begin{bmatrix} -a_2 & c_1L \\ -c_1L & b_2L^2 \end{bmatrix} e^{2\mu} \right) \right] u_L = 0 \quad (85)$$

Now finding the determinant for the above equation and simplifying,

$$\begin{vmatrix} -a_2 + \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} \right) e^\mu - a_2 e^{2\mu} & -c_1L + c_1L e^{2\mu} \\ c_1L - c_1L e^{2\mu} & b_2L^2 + 2b_1L^2 e^\mu + b_2L^2 e^{2\mu} \end{vmatrix} = 0 \quad (86)$$

The determinant for Equation 86 is as follows,

$$\begin{aligned}
& e^{4\mu} + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}}\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^{3\mu} \\
& + \frac{\left(2b_1 \left(2a_1 - \frac{1}{\frac{1}{K} + \frac{1}{m\omega^2}}\right) - 2a_2b_2 - 2c_1^2\right)}{(c_1^2 - b_1a_2)} e^{2\mu} \\
& + \frac{\left(b_2 \left(2a_1 - \frac{1}{\frac{1}{K} + \frac{1}{m\omega^2}}\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^\mu + 1 = 0
\end{aligned} \tag{87}$$

Dividing all terms by $e^{2\mu}$

$$\begin{aligned}
& e^{2\mu} + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}}\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^\mu \\
& + \frac{\left(2b_1 \left(2a_1 - \frac{1}{\frac{1}{K} + \frac{1}{m\omega^2}}\right) - 2a_2b_2 - 2c_1^2\right)}{(c_1^2 - b_1a_2)} \\
& + \frac{\left(b_2 \left(2a_1 - \frac{1}{\frac{1}{K} + \frac{1}{m\omega^2}}\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^{-\mu} + e^{-2\mu} = 0
\end{aligned} \tag{88}$$

then α defined as

$$\alpha = \frac{(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} \right) - 2b_1a_2)}{(c_1^2 - b_1a_2)} \quad (89)$$

$$\beta = \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} \right) - 2a_2b_2 - 2c_1^2 \right)}{(c_1^2 - b_1a_2)} \quad (90)$$

Now re-arranging the equation in terms of Cosh function from Equation 84, it provides us the equation 43.

Appendix C

Substituting Eq. 20-26 in the Eq. 37, it provides us with Stiffness Matrix D. Substituting this in Eq. 39, we obtain below equation.

$$\left[\frac{EI}{L^3} \left(\begin{bmatrix} -a_2 & -c_1L \\ c_1L & b_2L^2 \end{bmatrix} + \left(\begin{bmatrix} 2a_1 + \frac{1}{\bar{K}} - \frac{1}{m\omega^2} + K_\theta & 0 \\ 0 & 2b_1L^2 \end{bmatrix} \right) e^\mu + \begin{bmatrix} -a_2 & c_1L \\ -c_1L & b_2L^2 \end{bmatrix} e^{2\mu} \right) \right] u_L = 0 \quad (91)$$

Now finding the determinant for the above equation and simplifying,

$$\begin{vmatrix} -a_2 + \left(2a_1 + \frac{1}{\bar{K}} - \frac{1}{m\omega^2} + K_\theta \right) e^\mu - a_2 e^{2\mu} & -c_1L + c_1L e^{2\mu} \\ c_1L - c_1L e^{2\mu} & b_2L^2 + 2b_1L^2 e^\mu + b_2L^2 e^{2\mu} \end{vmatrix} = 0 \quad (92)$$

Further simplifying and substituting necessary values in above equation we get,

$$\begin{aligned}
& (c_1^2 - b_2 a_2) e^{4\mu} + \left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2b_1 a_2 \right) e^{3\mu} \\
& + \left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2a_2 b_2 - 2c_1^2 \right) e^{2\mu} \\
& + \left(\left(-2a_2 b_1 + b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) \right) e^\mu + c_1^2 \right. \\
& \left. + -a_2 b_2 \right) = 0
\end{aligned} \tag{93}$$

Rearranging the above terms,

$$\begin{aligned}
& e^{4\mu} + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2b_1 a_2 \right) e^{3\mu}}{(c_1^2 - b_1 a_2)} \\
& + \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2a_2 b_2 - 2c_1^2 \right) e^{2\mu}}{(c_1^2 - b_1 a_2)} \\
& + \frac{\left(-2a_2 b_1 + b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) \right) e^\mu}{(c_1^2 - b_1 a_2)} + 1 = 0
\end{aligned} \tag{94}$$

Simplifying ,

$$\begin{aligned}
 & e^{4\mu} + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^{3\mu} \\
 & + \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2a_2b_2 - 2c_1^2\right)}{(c_1^2 - b_1a_2)} e^{2\mu} \\
 & + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^\mu + 1 = 0
 \end{aligned} \tag{95}$$

Divided all terms by $e^{2\mu}$

$$\begin{aligned}
 & e^{2\mu} + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^\mu \\
 & + \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2a_2b_2 - 2c_1^2\right)}{(c_1^2 - b_1a_2)} \\
 & + \frac{\left(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta\right) - 2b_1a_2\right)}{(c_1^2 - b_1a_2)} e^{-\mu} + e^{-2\mu} = 0
 \end{aligned} \tag{96}$$

then α defined as

$$\alpha = \frac{(b_2 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2b_1a_2)}{(c_1^2 - b_1a_2)} \quad (97)$$

$$\beta = \frac{\left(2b_1 \left(2a_1 + \frac{1}{\frac{1}{K} - \frac{1}{m\omega^2}} + K_\theta \right) - 2a_2b_2 - 2c_1^2 \right)}{(c_1^2 - b_1a_2)} \quad (98)$$

Now re-arranging the equation in terms of Cosh function equation 84, it gives us equation 45.

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