PROCESS MONITORING USING NEW ASSORTED AND PROGRESSIVE APPROACHES

BY

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A Thesis Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

APPLIED STATISTICS

MAY 2017

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN- 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

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To my parents, wife, children’s and siblings

ACKNOWLEDGMENTS

In the name of ALLAH, the Most Gracious, the Most Merciful.

All the praises and thanks to ALLAH ALMIGHTY, Lord of all the worlds, the most Affectionate and the Generous. Thanks to ALLAH, Who bestowed me energy and wisdom to complete this thesis. I have no words to say thanks, ALLAH ALMIGHTY beloved Prophet MOHAMMAD (صلى الله عليه وسلم), who is the eternal source of guidance and knowledge to all the humanity.

The last two years played a significant role in my life, which I spent in King Fahd University of Petroleum & Minerals. Generally, when the work load is over it impacts on the performance but due to guidance and encouragement of some peoples, I learned how to perform well in this situation. Now, this is time to pay thanks to all of them.

It is a great honor for me to work under the kind supervision of **Prof. Dr. Muhammad Riaz**. All credit of this study and shaping my future career goes to him. I would like to appreciate and acknowledge him for his continuous support, valuable suggestions and encouragement throughout my research work.

Thanks to all my teachers, **Prof. Dr. Muhammad Riaz, Dr. Mohammad Hafidz Omar, Dr. Nasir Abbas, Dr. Saddam Akber Abbasi, Dr. Sohail Chand (PU), Mr. Zubair Anwar Dr. Tauhidur Rehman and Prof. Dr. Taimur ul Hassan (Dean SMC, BNU)**.

I would like to pay my gratitude to committee members, **Dr. Walid S. Al-Sabah and Dr. Nasir Abbas.**

I am thankful to Ministry of Education and King Fahd University of Petroleum and Minerals (KFUPM) for providing me a chance to study in this reputed institution. I would like to thank honorable Chairman (Dr. Husain Al- Attas), instructors and admin staff (especially Mr. Haider Ali, Mr. Iftikhar Sheikh, Mr. Muzzafar Khan and Mr. Anas Uddin) at the Mathematics & Statistics Department. The Information Technology Centre at KFUPM provide us world class facility such as High-Performance Computing (HPC), I acknowledge and appreciated their efforts.

Last but not the least, I would like to pay thanks and tribute to my mother (Zubaida Begum), wife (AIM), children's (Adeen Usman, Mohammad Talal, and Jahaniya Usman) and siblings for their kind prayers, love and endless support from beginning to the end of my study in KFUPM. I am also thankful to my friends especially (Usama Bin Irshad, Tahir Mehmood, Hafiz Afzal and Naveed Butt) and Pakistani community in KFUPM for making my stay delightful and pleasant.

Usman Saeed

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$\hat{\beta}_{0j}$	Estimate of β_0 for j^{th} random sample
$\hat{\beta}_{1j}$	Estimate of β_1 for j^{th} random sample
B_0	Intercept of the transformed model
B_1	Slope of the transformed model
LCL_E	Lower control limit for EWMA chart
LCL_{EE}	Lower control limit for error variance parameter in EWMA_3 chart
LCL_{EI}	Lower control limit for intercept parameter in EWMA_3 chart
LCL_{ES}	Lower control limit for slope parameter in EWMA_3 chart
LCL_H	Lower control limit of Hotelling T^2 chart
LCL_R	Lower control limit for R chart
LCL_{SE}	Lower control limit for error variance parameter in Shewhart_3 chart
LCL_{SI}	Lower control limit for intercept parameter in Shewhart_3 chart

LCL_{SS}	Lower control limit for slope parameter in Shewhart_3 chart
L_E	Charting constant for error variance parameter in PM_3 chart
L_{EE}	Charting constant for error variance parameter in EWMA_3 chart
L_{EI}	Charting constant for intercept parameter in EWMA_3 chart
L_{ES}	Charting constant for slope parameter in EWMA_3 chart
L_I	Charting constant for intercept parameter in PM_3 chart
L_S	Charting constant for slope parameter in PM_3 chart
$PM_{E(i)}$	Progressive mean statistic for error variance of PM_3 chart
$PM_{I(i)}$	Progressive mean statistic for intercept of PM_3 chart
$PM_{S(i)}$	Progressive mean statistic for slope of PM_3 chart
UCL_E	Upper control limit for EWMA chart

UCL_{EE}	Upper control limit for error variance parameter in EWMA_3 chart
UCL_{EI}	Upper control limit for intercept parameter in EWMA_3 chart
UCL_{ES}	Upper control limit for slope parameter in EWMA_3 chart
UCL_{H}	Upper control limit of Hotelling T^2 chart
UCL_{R}	Upper control limit for R chart
UCL_{SE}	Lower control limit for error variance parameter in Shewhart_3 chart
UCL_{SI}	Upper control limit for intercept parameter in Shewhart_3 chart
UCL_{SS}	Upper control limit for slope parameter in Shewhart_3 chart
$Z_{\alpha/2}$	$(\alpha/2)^{th}$ quantile point of standard normal distribution
b_{0j}	Estimate of B_0 for j^{th} random sample
b_{1j}	Estimate of B_1 for j^{th} random sample

β_0	Intercept of the original model
β_1	Slope of the original model
$\chi^2_{(\alpha/2),(n-2)}$	Upper quantile point of χ^2distribution with $(n-2)$ degrees of freedom
$\chi^2_{(1-\alpha/2),(n-2)}$	Lower quantile point of χ^2distribution with $(n-2)$ degrees of freedom.
ϵ_{ij}	Error of the original model
ARL	Average run length
CUSUM	Cumulative sum
EQL	Extra quadratic loss
EWMA	Exponentially weighted moving average
MSE	Mean square error
N	Sample size
IC	In-control
OOC	Out-of-control
PCC	Pressure of cold compaction
PM	Progressive mean

RARL	Relative average run length
SEQL	Sequential extra quadratic loss
SRARL	Sequential relative average run length
L	Charting constant for EWMA
β	Shifts for the slope in original model
γ	Shifts for the error variance in original model
δ	Shifts for the slope in transformed model
λ	Smoothing parameter
φ	Shifts for the intercept in transformed model
K	Charting constant for Shewhart control chart
h	Charting constant for CUSUM control chart

ABSTRACT

Full Name : [Usman Saeed]

Thesis Title : [Process Monitoring Using New Assorted and Progressive Approaches]

Major Field : [Applied Statistics]

Date of Degree : [May 2017]

The performance of a process is overseen by numerous quality features that may be classified as characteristics of interest and auxiliary characteristics. These quality characteristics may be operated separately or there may exist a relationship among these quality characteristics/variables in real processes. These relationships are enumerated by models termed as profiles that may be linear or nonlinear. The simple linear profiles are determined by three parameters namely intercept, slope, and residual variance.

In this thesis, we have proposed two new approaches for an efficient monitoring of process parameters. These approaches include assorted and progressive schemes for location, dispersion and profile parameters. Under progressive setup, we have proposed a Shewhart control chart for the simultaneous monitoring of simple linear profile parameters. Under assorted approach, we have designed control charts for location and scale parameters. Moreover, the assorted approach is extended to monitor simple linear profile parameters (intercept, slope, and residual variance). The performance of proposed charts and its counterparts is evaluated and compared using some useful measures such as average run length, standard deviation run length, relative average run length, sequential relative average run length, extra quadratic loss, sequential extra quadratic loss and performance comparison index. The comparative analysis, using run length properties, revealed that the

proposed assorted and progressive approaches are very efficient at detecting shifts of varying amounts in process parameters. In addition, we have presented the implementation of our proposals on some real datasets from different disciplines including computers sciences, environmental sciences and material sciences.

ملخص الرسالة

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عنوان الرسالة: مراقبة العمليات باستخدام أساليب جديدة متنوعة ومتقدمة

التخصص: الإحصاء التطبيقي

تاريخ الدرجة العلمية: مايو 2017

يتم الإشراف على أداء عملية من قبل العديد من الميزات الجودة التي يمكن تصنيفها على أنها خصائص الفائدة والخصائص المساعدة. ويمكن تشغيل خصائص الجودة هذه بشكل منفصل أو قد توجد علاقة بين خصائص / متغيرات الجودة هذه في العمليات الحقيقية. يتم تعداد هذه العلاقات من قبل نماذج وصفت بأنها ملامح التي قد تكون خطية أو غير الخطية. يتم تحديد ملامح الخطية بسيطة من قبل ثلاث معلمات وهي اعتراض، المنحدر، والتباين المتبقي.

وفي هذه الأطروحة، اقترحنا نهجين جديدين للرصد الفعال لمعايير العملية. وتشمل هذه النهج مخططات متنوعة ومتقدمة للموقع، والتشتت، والمعلمات الشخصية. تحت الإعداد التدريجي، اقترحنا مخطط التحكم شيوهارت لرصد في وقت واحد من المعلمات الشخصية الخطية بسيطة. تحت نهج متنوعة، قمنا بتصميم مخططات التحكم للموقع والمعلمات مقياس. وعلاوة على ذلك، يتم توسيع نطاق النهج المتنوع لرصد المعلمات الشخصية الخطية بسيطة (اعتراض، المنحدر، والتباين المتبقية). ويتم تقييم ومقارنة أداء المخططات المقترحة ونظيراتها باستخدام بعض التدابير المفيدة مثل متوسط طول المدى وطول تشغيل الانحراف المعياري ومتوسط طول المدى النسبي ومتوسط طول المدى النسبي المتتابع وفقدان التربيع الإضافي وفقدان التربيع الإضافية المتتالية ومؤشر مقارنة الأداء . وكشف التحليل المقارن، باستخدام خصائص طول المدى، أن النهج المقترحة والتقدمية المقترحة فعالة جدا في الكشف عن تحولات متفاوتة في معلمات العملية. وبالإضافة إلى ذلك، قدمنا تنفيذ مقترحاتنا على بعض مجموعات البيانات الحقيقية من مختلف التخصصات بما في ذلك علوم الكمبيوتر والعلوم البيئية وعلوم المواد

Chapter 1

INTRODUCTION

In this study two main subjects are addressed, i.e. monitoring of simple linear profile parameters through progressive and assorted approaches. Firstly, a brief introduction to Statistical Process Control (SPC), classical and modified control charting techniques and monitoring of linear profile parameters are discussed.

1.1 Statistical Process Control

In the modern era, new technology is systematically emerging all around us, particularly in the field of consumer behavior. Buyers are becoming more technology smart with the passage of time. There is competition among manufacturers to fulfill the demands of their loyal customers. On the other hand, customers want a high-quality product with the cheapest price. It is an uphill task for organizations to retain customers and delivered according to their wish. To make high-quality products manufacturing process should be the state of the art. Generally, two types of variations occur in a process, named as common cause variation and special cause of variation. Common causes are the inherent part of the process, natural and uncontrollable. Special causes are un-natural and need to be controlled immediately. The existence of special causes may defect in the outputs of the process. Statistical Process Control (SPC) is used to control the special causes of variation in manufacturing or service processes. Further, in SPC seven tools are available to improve the quality of a process such as (i) Flow Chart (ii) Check Sheet (iii) Pareto Analysis (iv) Histogram (v) Cause-and-Effect (Fishbone) Diagram (vi) Scatter Diagram and (vii) Control Charts.

1.2 Control Charts

Control Chart is the most efficient and widely employed technique of SPC. Control charts are used a graphical tool to monitor the variable of interest in the process. Control chart has upper and lower limit. The process said to be in-control if observations are lies in the control limits. A process is said to be statistically out-of-control when observations crossed the control limits. Which indicates that there must be some special cause occur in the process. The expectation of control chart is that it must prompt the process engineer about variations in the process. There are many real-life applications of control charts exists in the field of manufacturing engineering, nuclear engineering, health care, economics, education and analytical labs. Control charts are employed in service industries such as in banking, restaurant and gasoline pumps etc.

- ✓ Monitoring the relationship between interest rate and unemployment
- ✓ Monitoring the waiting time of pizza delivery
- ✓ Monitoring the calling time of customer relationship officer
- ✓ Monitoring the transactions time of ATM users

and so on. Control charts consist on three lines: the center line (CL), upper control limit (UCL) and lower control limit (LCL). A process is said to be in-control (IC) when any plotted statistics (e.g. $\bar{X}$, S^2, P) lies within the LCL and UCL and if plotted statistics fall outside the control limits is considered an out-of-control (OOC).

1.2.1. Classical Control Charts

The three well known classical control charts are Shewhart, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA).

The Shewhart's control chart

The Shewhart's $\bar{X}$ control chart, originated by Shewhart (1920), is most important and commonly used technique of the SPC tool box. It is very useful to detect large shift in the process mean. $\bar{X}$ chart consists of two decision lines known as Upper Control Limit (UCL) and Lower Control Limit (LCL). Suppose that X is a normally distributed random variable with mean or process target value μ_0 and known standard deviation σ_0 i.e. $X_{ij} \sim N(\mu_0, \sigma_0^2)$ where $i = 1,2,3, \ldots \ldots, j = 1,2, \ldots \ldots, n$ and n is the size of i^{th} sample. The plotting statistic and the control limits for Shewhart's $\bar{X}$ chart are given below:

$$\bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \tag{1.1}$$

$$\left.\begin{aligned} LCL &= \mu_0 - K\frac{\sigma_0}{\sqrt{n}} \\ UCL &= \mu_0 + K\frac{\sigma_0}{\sqrt{n}} \end{aligned}\right\} \tag{1.2}$$

where K denotes the control limit coefficient.

CUSUM Control Chart

CUSUM control chart was initiated by Page (1954). The plotting statistics of classical CUSUM denoted by C_i^+ and C_i^- are given as follow:

$$\left.\begin{aligned} C_i^+ &= \max[0, (\bar{X}_i - \mu_0) - k\frac{\sigma_0}{\sqrt{n}} + C_{i-1}^+] \\ C_i^- &= \min[0, (\bar{X}_i - \mu_0) + k\frac{\sigma_0}{\sqrt{n}} + C_{i-1}^-] \end{aligned}\right\} \tag{1.3}$$

where k is the sensitivity parameter also called reference value and is chosen about half of the shift (δ) in the process location, $\delta = \frac{|\mu_1 - \mu_0|}{\sigma_0 / \sqrt{n}}$, μ_1 is the disturbed/dislocated mean. The starting values of C_i^+ and C_i^- are taken equal to zero. Finally, the statistics in Equation (1.3) are plotted against $\pm h\frac{\sigma_0}{\sqrt{n}}$, respectively, where h is the control limit coefficient and selected according to the choice of k and ARL_0. More details on the choice of k and h are available in Hawkins and Olwell (1998).

A value of $C_i^+ > h\frac{\sigma_0}{\sqrt{n}}$ indicates a possible upward shift in the process location whereas a value of $C_i^- < -h\frac{\sigma_0}{\sqrt{n}}$ means that the process location has possibly shifted downwards.

EWMA control chart

The EWMA control chart (like CUSUM chart) is also used to detect small to moderate shifts in process location. The plotting statistic and the control limits of EWMA control chart by Roberts (1959) are given as follows:

$$\boldsymbol{Z_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1}} \tag{1.4}$$

$$\left.\begin{aligned} \boldsymbol{LCL_i = \mu_0 - L\left(\frac{\sigma_0}{\sqrt{n}}\right)\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}} \\ \boldsymbol{UCL_i = \mu_0 + L\left(\frac{\sigma_0}{\sqrt{n}}\right)\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}} \end{aligned}\right\}, \tag{1.5}$$

where $0 < \lambda \leq 1$ is the sensitivity parameter, the starting value $\mathrm{Z_i}$ is set equal to the process target μ_0 and L is the control limit coefficient that is selected according to the choice of λ and ARL_0. More details on the choice of λ and L are available in Crowder (1989).

A value of $\mathrm{Z_i} > UCL_i$ indicates a possible upward shift in the process location whereas a value of $\mathrm{Z_i} < LCL_i$ means that the process location has possibly shifted downwards.

1.2.2. Modified Control Charts

Several modifications of classical control charts have been addressed such as combined Shewhart- CUSUM (CSC) by Lucas (1982), fast initial response (FIR) CUSUM by Lucas and Crosier (1982), combined Shewhart- EWMA (CSE) by Lucas and Saccucci (1990), FIR EWMA by Steinar (1999) , Adaptive EWMA by Capizzi and Masarotto (2003), Adaptive CUSUM by Jiang et al. (2008), an enhanced Adaptive CUSUM by Wu et al. (2009), enhancing the performance of EWMA charts by Abbas et al. (2010), improving the performance of CUSUM by Riaz et al. (2011), mixed EWMA-CUSUM by Abbas et al. (2013) and mixed CUSUM-EWM (MCE) control chart by Zaman et al. (2015). Further, details and structure of some modified control chart will discuss in Chapter 3.

1.3 Simple Linear Profile

In many manufacturing and services process, the quality of a product or process is categorized (qualitative and quantitative) and briefed by an association (linear or nonlinear) between response variable (dependent variable) and one or more explanatory variable (independent variable). This relationship is termed as a simple linear profile. Generally, simple linear profiles have three parameters of interest i.e. slope, intercept and error variance. Recently, there has been an interest developed to monitor processes by simple linear profiles. In quality control applications, the monitoring of simple linear profile is a new domain of interest. The framework of simple linear profiles data-set is given in Figure 1.1.

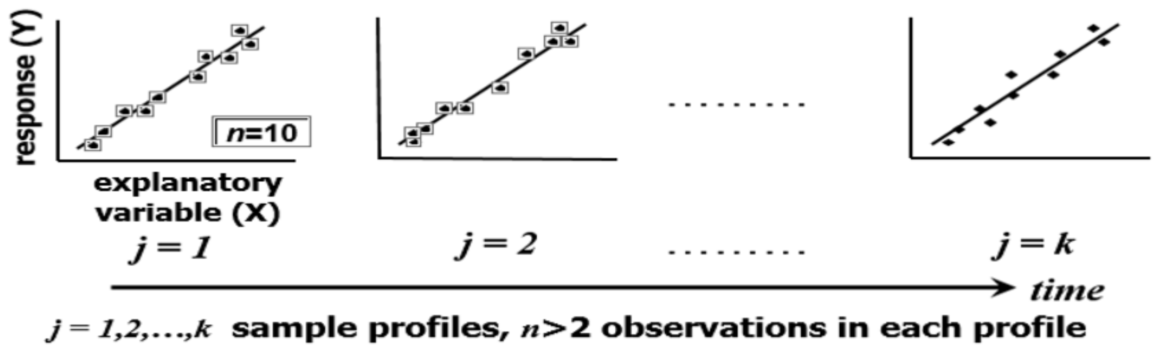

Figure 1.1: Description of simple linear profile data set

In literature, several practical applications of linear profiles are discussed by different authors such as Woodall et al. (2004) present the literature review and framework of simple linear profiles. The study of calibration curves is well known in linear profiles. A multivariate control chart to monitor calibration curves of Fe^{3+} (photometric determination) with sulfosalicylic acid discussed by Mestek et al. (1994) while Stover and Brill (1998) monitor ion chromatography calibrations frequency by a multivariate control chart. The practical application of linear profiles in manufacturing process discussed by Lawless et al. (1999) . Kang and Albin (2000) described the semiconductor manufacturing example while Ajmani (2003) discussed the Intel Corp. semiconductor manufacturing without calibration example. Jin and Shi (1999) and Walker and Wright (2002) discussed the non-linear profile applications in literature.

The detail discussion on simple linear profile monitoring will discuss in chapter 2.

1.4 Performance Measure

The well-known measures used to evaluate the performance of control charts are given below

Average run length (ARL): ARL is the most commonly used performance measure in control charts. It is defined as the average number of sample points until the first out of control signal occurs. Further, there are two types of ARL, IC ARL (ARL_0) needs to be maximized that the false alarm is delayed as much as possible the process in control while OOC ARL (ARL_1) needs to be minimized so that the signal is given as early as possible when the process goes out of control. In this thesis, the run length distribution of several charts is evaluated by using Monte Carlo simulation with 100,000 replications.

Extra quadratic loss (EQL): EQL is defined as the weighted average ARL with respect to range of shift (∇_{max} to ∇_{min}) by considering square of shift (∇^2) as weight. Mathematically, EQL is described as

$$\boldsymbol{EQL} = \frac{\mathbf{1}}{\nabla_{max}-\nabla_{min}} \int_{\nabla_{min}}^{\nabla_{max}} \nabla^2 \boldsymbol{ARL}(\nabla) \boldsymbol{d}\nabla \tag{1.6}$$

Sequential extra quadratic loss ($SEQL$): $SEQL$ refers to the EQL up to a certain shift (say ∇_i), mathematically defined as

$$\boldsymbol{SEQL_i} = \frac{\mathbf{1}}{\nabla_i-\nabla_{min}} \int_{\nabla_{min}}^{\nabla_i} \nabla^2 \boldsymbol{ARL}(\nabla) \boldsymbol{d}\nabla, \tag{1.7}$$

where $i = 2,3, \dots \dots, \nabla_{max}$

Relative average run length ($RARL$): The $RARL$ is described as the efficiency of a specific control chart relative to a benchmark control chart. It observe the performance of a chart with respect to its benchmark chart in terms of ARL for each value of shift (cf. Wu *et al.* (2009)). Mathematically, it is defined as

$$\boldsymbol{RARL} = \frac{\mathbf{1}}{\nabla_{max}-\nabla_{min}} \int_{\nabla_{min}}^{\nabla_{max}} \frac{\boldsymbol{ARL}(\nabla)}{\boldsymbol{ARL_{benchmark}}(\nabla)} \boldsymbol{d}\nabla, \tag{1.8}$$

Sequential relative average run length ($SRARL$): $SRARL$ refers to the $RARL$ up to a certain shift (say ∇_i), mathematically defined as

$$SRARL_i = \frac{1}{\nabla_i - \nabla_{min}} \int_{\nabla_{min}}^{\nabla_i} \frac{ARL(\nabla)}{ARL_{benchmark}(\nabla)} d\nabla, \qquad (1.9)$$

where $i = 2,3, \ldots \ldots, \nabla_{max}$

where $ARL(\nabla)$ and $ARL_{benchmark}(\nabla)$ are the average run lengths of a particular chart and benchmark chart at shift ∇ respectively. In general, a chart having minimum EQL value is considered as a benchmark chart and it is noted that at ∇_{max} , the $SEQL$ and $SRARL$ are said to be EQL and $RARL$. The $RARL$ value of a benchmark chart is equal to 1 and for other charts $RARL$ >1. The value of RARL >1 of any chart shows the inferior performance as compared to the benchmark chart.

Performance comparison index (PCI): The PCI is the ratio of EQL of a chart and of chart having least EQL ($EQL_{benchmark}$).

$$PCI = \frac{EQL}{EQL_{benchmark}}, \qquad (1.10)$$

A chart is considered best if it has PCI is equal to 1.

1.5 Objectives of the thesis

The objectives of this studies are given below:

- ✓ Simultaneous monitoring of simple linear profile under progressive setup
- ✓ An assorted control chart for monitoring the process location
- ✓ An assorted control chart for monitoring the process dispersion
- ✓ An assorted approach for monitoring simple linear profiles

1.6 Outline of the thesis

In Chapter 2, a new control chart is proposed for the simultaneous monitoring of simple linear profile parameters under progressive setup. The performance of the proposed chart is evaluated in terms of ARL, standard deviation run length ($SDRL$), EQL, $SEQL$, $RARL$ and $SRARL$ on four types of shifts. The comparative analysis of proposed chart with its counterpart charts is also in tabular and graphical forms. The findings reveal that the performance of proposed chart is better than all its competitor charts. The proposed chart is applied on N-Queen size problem solution.

In chapter 3, a new control chart named as (Assorted control chart) proposed for monitoring of process location. The said approach is used to monitor small, medium and large disturbances in a single control chart at the same time. Several performance measures are used such as $ARL, SDRL, EQL, RARL$ and PCI for the evaluation of the proposed chart. A tabular and graphical comparison of proposed versus classical and modified control charts is presented. A real-life example is also discussed to monitor the pH value of water.

In chapter 4, a new control chart named as (S^2Assorted control chart) proposed for monitoring of process dispersion. The said approach is used to monitor small, medium and large disturbances in a single control chart at the same time. Several performance measures are used such as $ARL,\ EQL,\ SEQL, RARL$ and $SRARL$ for the evaluation of the proposed chart. A tabular and graphical comparison of proposed versus classical and modified control charts is presented. A real-life example is also discussed to monitor the Flow Width Measurement for Hard-Bake Process.

Chapter 5 incorporates chapter 2, chapter 3 and chapter 4. A new control chart proposed for the monitoring of simple linear profile parameters including slope, intercept and error variance. The performance of proposed chart is evaluated through A$RL, EQL, SEQL, RARL$ and $SRARL$. The comparison analysis of proposed and its counterpart is also presented.

Finally, conclusion, summaries, and recommendations of the work are presented in Chapter 6.

1.7 Limitations of the thesis

This study has some limitations which are described as follow:

- ✓ In chapter 2 the independent variable is considered as a fixed level.
- ✓ The study in chapter 3 is based on univariate case.
- ✓ The study in chapter 4 is monitored only univariate statistic (S^2)
- ✓ The proposed chart in chapter 5 is used to monitor simple linear profile parameters.

Chapter 2

SIMULTANEOUS MONITORING OF LINEAR PROFILE PARAMETERS UNDER PROGRESSIVE SETUP

In many manufacturing or service processes, we come across different quality characteristics that govern the process behavior. These characteristics are categorized as the main quality characteristics (study variables) and the supporting or explanatory characteristics. There is always a possibility that some of the explanatory variables offer a relationship with the study variable which is known as profiles. The monitoring of study variable which is linearly associated with an explanatory variable is termed as simple linear profiles. In this chapter, we intend to design an efficient memory type structure based on progressive mean for the simultaneous monitoring of linear profile parameters. The performance of proposed scheme (PM_3) and its counterparts (i.e. EWMA_3 chart, bivariate T^2 chart, EWMA/R chart and Shewhart_3 chart) are evaluated using some useful performance measures such as average run length (ARL), relative average run length (RARL), sequential relative average run length (SRARL), extra quadratic loss (EQL) and sequential extra quadratic loss (SEQL). In the presence of shifts in linear profile parameters, the findings depict that PM_3 has better detection ability as compared to counterpart charts. A case study related to Queen size problem is also discussed to highlight the importance of newly proposed control chart.

2.1 Introduction

In this modern era, new technology is consistently coming forth all around us, especially in the domain of consumer behavior. With the passage of time, customers are seemly more tech-smart and want products as per their needs with high quality and low cost. Nowadays, many companies upping efforts to satisfy such desires of customer's. In general, every process has some cause of variations namely natural and un-natural variations. Natural variations are the inherent part of the process and cannot be completely eradicated while un-natural variations occur due to some assignable factors that affect the performance of any process. A control chart is a key tool among seven magnificent tools of statistical process control, mainly used to differentiate the aforementioned variations in any process.

In many manufacturing processes, control charts are used to monitor a single quality characteristic (qualitative or quantitative) but in some processes, the quality characteristic has an association (linear or nonlinear) with other ancillary variable(s) in the process. To monitor such quality characteristic which has a linear relationship with another explanatory variable is termed as simple linear profiles. In general, three parameters are considered to express the state of simple linear profiles such as slope, intercept and error variance. In literature, many studies are developed to monitor simple linear profile parameters. A regression control chart proposed by Mandel (1969) was applied on manpower scheduling whereas, Hawkins (1991 and 1993) proposed multivariate charting structures based on regression adjusted variables. Hauck et al. (1999) proposed control chart for multivariate monitoring of grouped adjusted variables. Kang and Albin (2000) proposed two control chart structures for the monitoring of linear profile parameters. They used the multivariate T^2 chart to monitor intercept and slope while EWMA/R chart for the monitoring of error variance. A well-known control chart (EWMA_3) was proposed by Kim et al. (2003), which is used for the monitoring of small/moderate shifts in linear profile parameters.

Moreover, comparative study on the performance of EWMA_3 chart, multivariate $T^{2,}$ and EWMA/R charts are also reported in their study.

For the monitoring of simple linear profile parameters, R and Multivariate CUSUM charts were developed by Noorossana et al. (2004) while a new Phase I approach based on general linear F-test was introduced by Mahmoud and Woodall (2004). Zou et al. (2006) proposed a Phase II study based on change point model for the monitoring of simple linear profile parameters. Gupta et al. (2006) proposed Shewhart_3 chart for the monitoring of intercept, slope and error variance and compared the performance of Shewhart_3 chart with NIST and EWMA_3 charts. A Phase I analysis of change point model based on segmented regression technique was discussed by Mahmoud et al. (2007). Further, Zou et al. (2007) proposed Multivariate EWMA control chart based on general linear profile model and enhanced the proposed method by implementing variable sampling interval (VSI), self-starting scheme and paramedic diagnostic technique.

A Phase II approach based on first order autoregressive model was proposed by Noorossana et al. (2008) and the CUSUM_3 chart was proposed by Saghaei et al. (2009) to enhance the detection ability of simple linear profiles. Soleimani et al. (2009) discussed a transformation study to overcome the issue of within autocorrelation in simple linear profiles. Yeh and Zerehsaz (2013) developed two control charts for simple linear profile parameters such as; a chart based on likelihood ratio test was designed for monitoring of intercept and slope and another chart based on recursive residuals was constructed to monitor the error variance. Most of the current literature on linear profile parameters are discussed under fixed effect model while an approach based on random effect model was designed by Noorossana et al. (2014). In simple linear profiles, the effect of estimation error under fixed effect model was discussed by Aly et al. (2014) while under random effect model was studied by Noorossana et al. (2016). Recently, monitoring of simple linear profile parameters are enhanced by incorporating several run rules (cf. Riaz and Touqeer (2015)) and different ranked set sampling techniques (cf. Riaz et al. (2017)).

Generally, memory type control charts are used to monitor small and moderate shifts in process parameter(s). Recently, Abbas et al. (2013) proposed a new memory type control chart based on progressive mean (PM) which provides better detection ability as compared to classical charts (EWMA and CUSUM). The progressive version for the variability was introduced by Zafar et al. (2014) and more modification in the progressive setup was discussed by Abbasi et al. (2013). In this study, we intend to use the progressive setup for the monitoring of simple linear profile parameters. Further, the outline of the study is as follow; in Section. 2.2, we will briefly describe the existing charts and proposed structures used to monitor simple linear profile parameters. In Section. 2.3, discussion on the performance evaluation of the proposed and other competing charts. In Section. 2.4, comparative analysis of PM_3 chart with its counterpart are discussed. In Section. 2.5, a case study about N-Queen size problem has been discussed to highlight the importance of proposed method. Finally, summary, conclusions, and recommendations are described in Section. 2.6.

2.2 Simple Linear Profile Methods

Simple linear profiles play a key role in the monitoring of process parameters when study variable is linearly associated with another auxiliary variable. In this section, we will provide a brief introduction about simple linear profiles and a memory type structure for the monitoring of linear profile parameters under progressive setup.

Simple Linear Profiles

In some practical applications, the quality of a process or product is described by a relationship (linear or nonlinear) between two or more variables instead of single study variable. For example, in semiconductor manufacturing, the pressure of the gasses is dependent on the flow of the gasses in tank (cf. Kang and Albin (2000)) and in electrical engineering, the voltage of a photovoltaic system is inversely related to capacitance of a capacitor (cf. Riaz et al. (2017)). Generally, the monitoring of study variable when it is linearly related to an explanatory variable is termed as simple linear profiles. There exist three parameters to be monitored in simple linear profiles such as slope, intercept and error variance.

Assume that we have paired observation (X_i, Y_{ij}) for the j[th] random sample collected with respect to time. Then the simple linear regression model with intercept (β_0) and slope (β_1) is defined as:

$$\boldsymbol{Y_{ij} = \beta_0 + \beta_1 X_i + \epsilon_{ij}} \tag{2.1}$$

where $i = 1,2, \dots \dots, n$ and ϵ_{ij} is random error term which follows normal distribution with mean (μ) zero and unit variance (σ^2) of Equation (2.1) (referred as original model). The OLS estimates of the linear regression parameters are defined as:

$$\hat{\beta}_{1j} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})Y_{ij}}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{S_{XY(j)}}{S_{XX}},$$

$$\hat{\beta}_{0j} = \bar{Y}_j - \hat{\beta}_{1j}\bar{X},$$

where $\bar{Y}_j = \sum_{i=1}^{n} Y_{ij}/n$, $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S_{XX} = \sum(X - \bar{X})^2$ while the means, variances and co-variance term of $\hat{\beta}_{0j}$ and $\hat{\beta}_{1j}$ are computed as follow:

$$E(\hat{\beta}_{1j}|X) = \beta_1; \quad E(\hat{\beta}_{0j}|X) = \beta_0,$$

$$Var(\hat{\beta}_{1j}|X) = \frac{\sigma^2}{S_{XX}}; \quad Var(\hat{\beta}_{0j}|X) = \sigma^2 \left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right],$$

$$Cov(\hat{\beta}_{1j}, \hat{\beta}_{0j}|X) = -\sigma^2 \frac{\bar{X}}{S_{XX}},$$

It is noted that the mean square error (MSE) is an unbiased estimator of error variance (σ^2), which is computed by

$$MSE_j = \frac{\sum_{i=1}^{n}(Y_{ij} - \hat{Y}_{ij})^2}{n-2} = \frac{\sum_{i=1}^{n} e_{ij}^2}{n-2},$$

where $\widehat{Y}_{ij}$ is the i^th^ predicted value for j^th^ random sample. Usually, simple linear profile parameters are monitored in simultaneous structure which requires the assumption of independence between the parameters. To meet such assumption, coded method is an effective way which requires a transformation on X_i values (i.e. $X_i^* = X_i - \bar{X}$). The coded form of Equation (2.1) is defined as:

$$\boldsymbol{Y_{ij} = B_0 + B_1 X_i^* + \epsilon_{ij}} \tag{2.2}$$

where $\boldsymbol{i = 1, 2, \ldots \ldots, n}$.

It is noted that Equation (2.2) is referred as transformed model, where intercept of transformed model is $B_0 = \beta_0 + \beta_1\bar{X} + \beta\sigma\bar{X}$ and slope of transformed model is estimated by $B_1 = (\beta_1 + \beta\sigma)X_i^*$, where the shifts in the slope (β) of Equation (2.1) are considered in terms of σ. Further, in the same line, one may obtain OLS estimates of transformed model (b_{0j}, b_{1j}) and their properties. In recent literature, several studies are available about the monitoring of linear profile parameters, some are briefly described below.

2.2.1 The Hotelling T^2 chart

Kang and Albin (2000) proposed a multivariate control chart for the monitoring of slope and intercept. The j^th statistic of Hotelling T^2 control chart is estimated by

$$\boldsymbol{T_j^2} = (\boldsymbol{Z_j} - \boldsymbol{U})^{\boldsymbol{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{Z_j} - \boldsymbol{U}) \tag{2.3}$$

where the terms in Equation (2.3) are defined as follows:

$$Z_j = \left(\hat{\beta}_{0j}\,, \hat{\beta}_{1j}\,\right)^T; U = (\beta_0, \beta_1)^T,$$

$$\Sigma = \begin{bmatrix} \sigma^2 \left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right] & -\sigma^2 \frac{\bar{X}}{S_{XX}} \\ -\sigma^2 \frac{\bar{X}}{S_{XX}} & \frac{\sigma^2}{S_{XX}} \end{bmatrix},$$

The Hotelling T^2 statistic follows χ^2 distribution with 2 degrees of freedom and the upper control limit ($UCL_H = {\chi^2}_{2,\alpha}$) is the α^{th} quantile of χ^2 distribution while the lower control limit is equal to zero ($LCL_H = 0$). When process is unstable then the Hotelling T^2 statistic follows non-central χ^2 distribution with non-centrality parameter (τ), which is obtain as:

$$\tau = n(\varphi\sigma + \beta\sigma\bar{X})^2 + (\beta\sigma)^2 S_{XX},$$

where φ is the amounts of shifts in intercept and β is the measure of shift in the slope for Equation (2.1)

2.2.2 The EWMA/R chart

Kang and Albin (2000) also proposed a combined structure based on EWMA and R chart for the monitoring of linear profile parameters. Basically, EWMA chart has some limitations which are covered by incorporating the R chart. The j^th statistic of EWMA chart is estimated by

$$Z_j = \lambda \bar{e}_j + (1-\lambda) Z_{j-1},$$

where λ is the smoothing parameter which ranges from 0 to 1, $\bar{e}_j = \frac{\sum_{i=1}^{n} e_{ij}}{n}$ and the initial value of EWMA statistic is zero. (i.e. $Z_0 = 0$). The process is said to be OOC when Z_j is less than LCL or greater than UCL. The control limits (LCL and UCL) based on charting constant (L) for EWMA chart are given as follow:

$$LCL_E = -L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[\frac{1}{n}\right]}; \; UCL_E = L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} \left[\frac{1}{n}\right]},$$

There exist two causes to combine R chart with EWMA chart, (i) to detect shifts in error variance under model (1) and (ii) to tackle the unusual situation of error variance. Further, the jth statistic and control limits of R chart are defined as

$$R_j = max_i(e_{ij}) - min_i(e_{ij})$$

$$LCL_R = \sigma(d_2 - Ld_3); \quad UCL_R = \sigma(d_2 + Ld_3)$$

where d_2 and d_3 are, unbiased constants reported in Montgomery (2012).

2.2.3 The EWMA_3 control chart

A memory type approach based on EWMA structure was designed by Kim et al. (2003). This chart is efficient for the monitoring of small or moderate shifts in slope, intercept and error variance. The structure of EWMA_3 chart depends on the transformed model given in (2) and the three individual EWMA statistics for each linear profile parameter are defined as:

$$EWMA_{I(j)} = \lambda b_{0j} + (1-\lambda) EWMA_{I(j-1)},$$

$$EWMA_{S(j)} = \lambda b_{1j} + (1-\lambda)EWMA_{S(j-1)},$$

$$EWMA_{E(j)} = \max\{\lambda \ln\left(MSE_j\right) + (1-\lambda)EWMA_{E(j-1)}, \ln(\sigma_0^2)\},$$

where $EWMA_{I(j)}$ is the j[th] EWMA statistic for intercept while $EWMA_{S(j)}$ and $EWMA_{E(j)}$ are the j[th] EWMA statistics for slope and error variance respectively. Further, the control limits for each EWMA statistic are as follow:

$$\text{For intercept:} \begin{cases} LCL_{EI} = B_0 - L_{EI}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]} \\ UCL_{EI} = B_0 + L_{EI}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]} \end{cases},$$

$$\text{For slope:} \begin{cases} LCL_{ES} = B_1 - L_{ES}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\frac{\sigma^2}{S_{XX}}} \\ UCL_{ES} = B_1 + L_{ES}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\frac{\sigma^2}{S_{XX}}} \end{cases},$$

$$\text{For error variance:} \begin{cases} LCL_{EE} = 0 \\ UCL_{EE} = L_{EE}\sqrt{\frac{\lambda}{(2-\lambda)}Var\left[\ln\left(MSE_j\right)\right]} \end{cases},$$

where L_{EI}, L_{ES} and L_{EE} are the charting constants which describes the width of control limits. The asymptotic variance of logarithmic mean square error was derived by Crowder and Hamilton (1992) which is estimated by

$$Var\left[\ln\left(MSE_j\right)\right] \approx \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{2}{3(n-2)^3} - \frac{16}{15(n-2)^5}$$

2.2.4 The Shewhat_3 control chart

Gupta et al. (2006) proposed a control charting scheme based on Shewhart structure for the monitoring of linear profile parameters. The control limits for each parameter are defined as

$$\text{For intercept:} \begin{cases} LCL_{SI} = \beta_0 - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} \\ UCL_{SI} = \beta_0 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} \end{cases},$$

$$\text{For slope:} \begin{cases} LCL_{SS} = \beta_1 - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{XX}}} \\ UCL_{SS} = \beta_1 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{XX}}} \end{cases},$$

$$\text{For error variance:} \begin{cases} LCL_{SE} = \frac{\sigma^2}{n-2}\chi^2_{(1-\alpha/2),(n-2)} \\ UCL_{SE} = \frac{\sigma^2}{n-2}\chi^2_{(\alpha/2),(n-2)} \end{cases},$$

where $Z_{\alpha/2}$ is the $(\alpha/2)^{th}$ quantile point of standard normal distribution while $\chi^2_{(1-\alpha/2),(n-2)}$ and $\chi^2_{(\alpha/2),(n-2)}$ are the lower and upper quantile points of χ^2distribution with $(n-2)$ degrees of freedom.

2.2.5 The Progressive Mean (PM_3) control chart

Recently, Abbas et al. (2013) proposed a memory type structure based on progressive mean (PM) to monitor the location parameter of a process. The progressive mean is a special case of EWMA structure as discussed by Abbas (2015). Let $Y_i \sim N(\mu, \sigma^2)$; $i = 1, 2, \dots., n$ and Y_i is the sequence of i.i.d observations then the i^{th} progressive mean is defined as the cumulative average over time,

$$PM_{(i)} = \frac{\sum_{j=1}^{i} Y_j}{i},$$

where the progressive mean is an unbiased estimator of the population mean (μ) and have a minimum variance (σ^2/i) as compared to arithmetic mean ($\bar{Y}$). The PM_3 chart is specially designed to monitor the linear profile parameters based on progressive setup. So, the progressive statistics for linear profile parameters are defined as follows (cf. Appendix A):

$$\textbf{Progressive statistics}: \begin{cases} \boldsymbol{PM_{I(i)}} = \frac{\sum_{j=1}^{i} \boldsymbol{b_{0j}}}{\boldsymbol{i}} \\ \boldsymbol{PM_{S(i)}} = \frac{\sum_{j=1}^{i} \boldsymbol{b_{1j}}}{\boldsymbol{i}} \\ \boldsymbol{PM_{E(i)}} = \frac{\sum_{j=1}^{i} \boldsymbol{MSE_j}}{\boldsymbol{i}} \end{cases}, \tag{2.4}$$

where $PM_{I(i)}$, $PM_{S(i)}$ and $PM_{E(i)}$ are the progressive statistics for intercept, slope and error variance respectively. Further, the averages, variances and control limits for each progressive statistic are as follows:

$$\textbf{Averges}: \begin{cases} \boldsymbol{E(PM_{I(i)}) = B_0} \\ \boldsymbol{E(PM_{S(i)}) = B_1} \\ \boldsymbol{E(PM_{E(i)}) = \sigma^2} \end{cases} \tag{2.5}$$

$$\boldsymbol{Variances}: \begin{cases} \boldsymbol{Var(PM_{I(i)}) = \frac{\sigma^2}{i}\left(\frac{1}{n}\right)} \\ \boldsymbol{Var(PM_{S(i)}) = \frac{\sigma^2}{i}\left(\frac{1}{S_{XX}}\right)} \\ \boldsymbol{Var(PM_{E(i)}) = \frac{2}{n-2}\sigma^4} \end{cases} \tag{2.6}$$

$$\textbf{For intercept}: \begin{cases} \boldsymbol{LCL_i = B_0 - L_I\sigma\sqrt{\frac{1}{n}}\, f(i)} \\ \boldsymbol{UCL_i = B_0 + L_I\sigma\sqrt{\frac{1}{n}}\, f(i)} \end{cases} \tag{2.7}$$

$$\textbf{For slope}: \begin{cases} \boldsymbol{LCL_i = B_1 - L_S\sigma\sqrt{\frac{1}{S_{XX}}}\, f(i)} \\ \boldsymbol{UCL_i = B_1 + L_S\sigma\sqrt{\frac{1}{S_{XX}}}\, f(i)} \end{cases} \tag{2.8}$$

$$\textbf{For error variance}: \begin{cases} \boldsymbol{LCL_i = \sigma^2 - L_E \sigma^2 \sqrt{\frac{2}{n-2}}\, f(i)} \\ \boldsymbol{UCL_i = \sigma^2 + L_E \sigma^2 \sqrt{\frac{2}{n-2}}\, f(i)} \end{cases} \quad (2.9)$$

where L_I, L_S and L_E are the charting constants of intercept, slope and error variance of PM_3 chart respectively, while $f(i) = i^{-q}$ is a penalty function used to stabilize the control limits of the PM_3 chart (cf. Zafar et al. (2014).

2.3 Performance evaluations

In this section, we will briefly describe the performance measures used to analyzed the proposed and existing simple linear profile techniques. Further, the IC design parameters of simple linear profiles are also discussed.

2.3.1 Performance measures

There exist several measures to describe the performance of linear profile parameters such as ARL, $SDRL$, median run length ($MDRL$), EQL, $SEQL$, $RARL$ and $SRARL$. The detail discussion on performance measures is available in Section. 1.4

2.3.2 Design of in-control parameters

In this study, we have utilized IC linear profile model $(i.e. Y_{ij} = 3 + 2X_i + \epsilon_{ij})$ by following Kang and Albin (2000) with fixed sample size $(n = 4)$ and values of independent variable $(X_i = 2, 4, 6, 8)$. Further, the transformed model given in equation (2.2) with $B_0 = 13 + 5(\beta\sigma)$ and $B_1 = (2 + \beta\sigma)X_i^*$ is defined as $Y_{ij} = B_0 + B_1 X_i^* + \epsilon_{ij}$. The transformed value of X_i are $X_i^* = -3, -1, 1, 3,$ having the mean is equal to zero. Keeping in mind the end goal to settle ARL_0 at a prefixed level, we require to set control limit coefficients for different combination of design parameters used in given charting methods. One may get results for various combination of the design parameters at different values of ARL_0. We have assessed the outcomes for some specific options of these design parameters, and the outcomes are accounted in Table 2.1 to accomplish an overall $ARL_0 = 200$. For calculations, we have utilized Monte Carlo simulation study with 10^6 iterations.

Table 2.1: Control charting constant for each method at fixed $ARL_0 = 200$

Parameter	PM_3	Shewhat_3	EWMA_3	EWMA/R
Intercept	L_I=4.54	$Z_{\alpha/2}$= 3.14	L_{EI}=3.0156	L=3
Slope	L_S=4.54	$Z_{\alpha/2}$= 3.14	L_{ES}=3.0109	L=3
Error variance	L_E=4.535	${\chi^2}_{LCL}$= 0.001, ${\chi^2}_{UCL}$= 14.17	L_{EE}=1.3723	L=3.1151
Design	q=0.2		λ=0.2	λ=0.2

Similarly, the charting constants for proposed chart for different overall ARL_0 are given in Table 2.2.

Table 2.2: Control charting constants for proposed method (PM_3)

Parameters	$ARL_0 = 200$	$ARL_0 = 370$	$ARL_0 = 500$
Intercept	L_I=4.54	L_I=5.20	L_I=5.52
Slope	L_S=4.54	L_S=5.20	L_S=5.52
Error variance	L_E=4.535	L_E=5.25	L_E=5.525
Design	q=0.2	q=0.2	q=0.2

2.3.3 Sensitivity analysis

For the selection of optimal choice of q in penalty function $f(i) = i^{-(0.5+q)}$, sensitivity analysis has done on the base of ARL, SDRL, MDRL and different percentile points. The overall $ARL_0 = 200$, minimum SDRL and approximately equal individual ARL_0 are the criteria to select the optimal value of q. In, nine cases with different values of control charting constants are reported in Table 2.3.

Table 2.3: Charting parameters for the optimal choice of q

Case	q	Constants	ARL	SDRL	MDRL	P_5	P_{25}	P_{50}	P_{75}	P_{95}	P_{99}
1	0.15	L_I=2.950	590.18	1612.28	138.00	7.00	38.00	138.00	485.00	2516.05	7092.18
		L_S=2.950	599.49	1717.61	137.00	7.00	39.00	137.00	489.00	2535.45	7055.34
		L_E=7.100	579.04	1611.05	136.00	4.00	38.00	136.00	470.00	2451.10	6866.03
		Overall	140.09	311.91	49.00	4.00	17.00	49.00	138.00	557.05	1357.02
2	0.20	L_I=4.200	775.75	1725.20	277.50	21.00	89.00	277.50	797.25	3032.05	7082.29
		L_S=4.200	774.82	1445.40	282.00	20.00	90.00	282.00	823.25	3210.10	6787.01
		L_E=4.120	757.11	1514.22	264.00	16.00	86.00	264.00	763.00	3118.25	6999.04
		Overall	134.44	183.57	72.00	9.00	32.00	72.00	162.00	469.00	910.01
3	0.20	L_I=4.500	1112.22	2236.97	393.50	27.00	129.00	393.50	1129.00	4604.10	10453.21
		L_S=4.500	1126.52	2326.89	392.00	24.00	127.00	392.00	1157.25	4537.15	11011.15
		L_E=4.500	1122.96	2309.34	386.50	23.00	127.00	386.50	1135.00	4466.35	11060.61
		Overall	188.23	283.13	104.00	13.00	47.00	104.00	235.00	686.00	1463.10
4	0.20	L_I=4.530	1146.14	2312.91	408.00	28.00	134.00	408.00	1170.00	4653.10	10793.07
		L_S=4.530	1143.78	2314.61	396.00	28.00	129.00	396.00	1191.00	4532.20	11171.64
		L_E=4.530	1160.49	2388.54	402.00	25.00	131.00	402.00	1181.00	4730.10	11173.15
		Overall	190.61	281.58	104.00	13.00	46.00	104.00	233.00	677.00	1382.06
5	0.20	L_I=4.540	1145.62	2308.13	404.00	29.00	133.00	404.00	1176.00	4624.05	10825.19
		L_S=4.540	1147.83	2260.33	403.50	30.00	133.00	403.50	1189.00	4647.20	10907.08
		L_E=4.535	1152.93	2266.01	411.00	26.00	137.00	411.00	1182.25	4723.05	10583.67
		Overall	200.66	301.54	105.00	13.00	46.00	105.00	232.00	696.00	1413.00
6	0.25	L_I=5.800	816.60	1215.22	379.00	42.00	148.00	379.00	972.00	3036.20	6031.08
		L_S=5.800	835.84	1284.35	398.00	40.00	153.00	398.00	977.25	3068.05	6176.49
		L_E=5.700	791.15	1185.51	367.00	37.00	141.00	367.00	927.25	2975.15	5731.04
		Overall	194.75	223.76	124.00	21.00	61.00	124.00	240.00	614.00	1120.03
7	0.25	L_I=5.700	785.94	1218.89	361.00	38.00	138.00	361.00	896.00	2999.20	6175.00
		L_S=5.700	791.87	1252.35	367.00	39.00	142.00	367.00	911.00	2972.45	5872.02
		L_E=5.710	791.15	1185.51	367.00	37.00	141.00	367.00	927.25	2975.15	5731.04
		Overall	186.19	227.33	116.00	20.00	58.00	116.00	230.00	574.05	1081.00
8	0.25	L_I=5.750	824.09	1300.51	384.00	43.00	147.75	384.00	957.00	3042.05	6252.23
		L_S=5.750	791.87	1252.35	367.00	39.00	142.00	367.00	911.00	2972.45	5872.02
		L_E=5.710	817.77	1262.04	381.00	37.00	148.00	381.00	956.25	3048.00	5996.11
		overall	186.19	227.33	116.00	20.00	58.00	116.00	230.00	574.05	1081.00
9	0.25	L_I=5.741	799.78	1242.45	376.00	39.00	144.00	376.00	934.00	2967.00	5755.46
		L_S=5.747	800.25	1254.17	368.00	42.00	146.00	368.00	912.00	2952.05	6386.01
		L_E=5.705	797.84	1198.41	383.00	37.00	145.00	383.00	947.00	2914.40	5593.48
		overall	187.46	213.16	118.00	21.00	58.75	118.00	231.00	599.05	1045.01

Although two cases (2 and 9) are near to qualify the criteria used for the selection of q value in the penalty function such as both cases have relatively minimum SDRL and an approximately equal individual value of ARL_0 but both cases (2 and 9) does not have overall $ARL_0 = 200$. So, only case 5 meets satisfactory conditions to select the value of q which is equals to 0.2 in the penalty function.

2.4 Comparative analysis

With the aim to assess the performance of PM_3 chart to monitor simple linear profile parameters such as slope, intercept and error variance, different kind of shifts introduced in profile parameters. The details of these shifts are described in Table 2.4.

Table 2.4: Amounts of shift introduced in linear profile parameters

Type of Shifts	Notation	Amounts of Shift
In intercept of transformed model	B_0 to $B_0 + \varphi\sigma$	$\varphi = 0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8,2.0$
In slope of original model	β_1 to $\beta_1 + \beta\sigma$	$\beta = 0.025,0.05,0.075,0.1,0.125,0.15,0.175,0.2,0.225,0.25$
In slope of transformed model	B_1 to $B_1 + \delta\sigma$	$\delta = -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9, -1.0$ and for joint monitoring $\delta = 0.025,0.05,0.075,0.1,0.125,0.15,0.175,0.2,0.225,0.25$
In error variance of original model	σ to $\gamma\sigma$	$\gamma = 1.2,1.4,1.6,1.8,2,2.2,2.4,2.6,2.8,3$

We have compared the performance of PM_3 chart with some existing control charts in terms of $ARL, EQL, SEQL, RARL$ and $SRARL$. The control charts selected for comparison purpose include the Hotelling T^2, EWMA/R, EWMA_3, and Shewhart_3. The ARL comparison for intercept shifts in the transformed model, slope shifts in the original model, error variance shifts in the original model and joint shifts (intercept and slope) in the transformed model are portrayed in following tables. The results are reported in terms of percentage decrease in ARL at certain shift which is obtained by,

$$\text{Percentage decrease } = \left(\frac{ARL_0 - ARL_1}{ARL_0}\right) \times 100$$

2.4.1 Shifts in intercept parameter:

The results for the shifts in intercept parameter are reported in Table 2.5 while ARL curves are portrayed in Figure 2.1. The 10% increment in intercept parameter ($\varphi = 0.20$) may cause 66.75% reduction in the ARL of EWMA/R chart, 31.15% reduction in the ARL of Hotelling T^2 chart, 70.45% reduction in the ARL of EWMA_3 chart, 24.3% reduction in the ARL of Shewhart_3 chart and 84.88% reduction in the ARL of PM_3 chart. Further, shift in intercept ($\varphi = 1.60$) may result 2.30, 1.80, 2.30, in 1.90, 2.22 unit ARL for EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively. The 80% increment in intercept parameter ($\varphi = 0.80$) may resulted 5.40, 13.20, 5.10, 15.50 and 5.09 unit SEQL values for EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively. Further, 1.21, 2.25, 1.15, 2.57 and 1 are the values of RARL with respect to EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively. The minimum ARL, EQL and RARL values are the, evident that PM_3 chart has better detection ability as compared to other charts.

Table 2.5: ARL comparison for intercept shifts in transformed model (B_0 to $B_0 + \varphi\sigma$)

Chart		φ									
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMA/R	ARL	66.5	17.7	8.4	5.4	3.9	3.2	2.7	2.3	2.1	1.9
	SEQL	1.33	2.04	2.33	2.56	2.78	3.03	3.3	3.59	3.9	4.23
	SRARL	1.6	1.7	1.56	1.44	1.36	1.31	1.27	1.24	1.22	1.21
Hotelling T^2	ARL	137.7	63.5	28	13.2	6.9	4	2.6	1.8	1.5	1.2
	SEQL	2.75	5.29	6.9	7.49	7.53	7.33	7.06	6.78	6.55	6.38
	SRARL	2.77	3.79	4	3.8	3.48	3.16	2.87	2.63	2.42	2.25
EWMA_3	ARL	59.1	16.2	7.9	5.1	3.8	3.1	2.6	2.3	2.1	1.9
	SEQL	1.18	1.83	2.13	2.36	2.59	2.85	3.13	3.42	3.75	4.09
	SRARL	1.47	1.55	1.43	1.33	1.26	1.22	1.19	1.17	1.15	1.15
Shewhart_3	ARL	151.4	77.9	33.8	15.5	7.7	4.3	2.7	1.9	1.5	1.2
	SEQL	3.03	6.14	8.2	8.91	8.89	8.57	8.16	7.78	7.45	7.19
	SRARL	2.99	4.3	4.67	4.46	4.07	3.67	3.32	3.03	2.78	2.57
PM_3	ARL	30.34	12.53	7.36	5.09	3.86	3.09	2.58	2.22	1.95	1.75
	SEQL	0.61	1.11	1.51	1.87	2.21	2.53	2.85	3.17	3.48	3.8
	SRARL	1	1	1	1	1	1	1	1	1	1

2.4.2 Shifts in the slope of original model:

The results for the shifts in slope parameter are reported in Table 2.6 The 20% increase in slope parameter ($\beta = 0.050$) may cause 78.05% drop in the ARL of EWMA/R chart, 47.20% drop in the ARL of Hotelling T^2 chart, 81.75% drop in the ARL of EWMA_3 chart, 37.50% drop in the ARL of Shewhart_3 chart and 89.28% drop in the ARL of PM_3 chart. Further, shift in slope ($\beta = 0.225$) may result 196.6, 196.3, 196.7, 195.0, 196.7 unit decrease in ARL for EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively.

When the shift is increased by 50% in slope parameter ($\beta = 0.125$) then 7.70, 20.10, 7.20, 27.90 and 6.83 unit ARL values are reported for EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively. Further, the values of EQL of EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts are 0.12, 0.24, 0.11, 0.31 and 0.10 respectively. Similarly, the values of RARL of these charts are 1.36, 2.72, 1.23, 3.50 and 1. So, PM_3 chart has least values of ARL, EQL and RARL as compared to other charts (cf. Figure 2.1) therefore the detection ability of PM_3 is better than EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 charts.

Table 2.6: ARL comparison for slope shifts in original model (β_1 to $\beta_1 + \beta\sigma$)

Chart		β									
		0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
EWMA/R	ARL	119.00	43.90	19.80	11.30	7.70	5.80	4.70	3.90	3.40	3.00
	SEQL	0.04	0.07	0.08	0.09	0.09	0.10	0.10	0.11	0.12	0.12
	SRARL	1.73	2.00	1.93	1.79	1.67	1.58	1.50	1.44	1.40	1.36
Hotelling T^2	ARL	166.00	105.60	60.70	34.50	20.10	12.20	7.80	5.20	3.70	2.70
	SEQL	0.05	0.12	0.18	0.22	0.24	0.25	0.25	0.25	0.24	0.24
	SRARL	2.22	3.20	3.74	3.87	3.77	3.57	3.35	3.12	2.91	2.72
EWMA_3	ARL	101.60	36.50	17.00	10.30	7.20	5.50	4.50	3.80	3.30	2.90
	SEQL	0.03	0.06	0.07	0.08	0.08	0.09	0.09	0.10	0.11	0.11
	SRARL	1.55	1.73	1.66	1.55	1.46	1.39	1.33	1.29	1.26	1.23
Shewhart_3	ARL	178.30	125.00	79.20	46.70	27.90	17.10	10.90	7.10	5.00	3.60
	SEQL	0.06	0.13	0.22	0.28	0.31	0.33	0.33	0.33	0.32	0.31
	SRARL	2.35	3.56	4.36	4.68	4.67	4.49	4.25	3.99	3.73	3.50
PM_3	ARL	48.20	21.43	12.94	9.07	6.83	5.43	4.49	3.82	3.30	2.92
	SEQL	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.09	0.10
	SRARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

2.4.3 Shifts in error variance:

In Table 2.7 and Figure 2.1, the results are reported for the shifts in error variance parameter. At the small amount of shift (ranges from 1.2 to 2.00) the PM_ 3 chart have lowest ARL values as compared to other charts while for moderate to large shift (ranges from 2.2 to 3.0) Shewhart_3 chart has least ARL values. The largest shift in error variance ($\gamma = 3.0$) may resulted 1.40, 1.80, 2.10, 1.40 and 1.50 unit ARL for the EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively. When two or more charts have equal values of ARL, then their performance assessed on the basis of EQL and RARL. The EQL values of EWMA/R, Hotelling T^2, EWMA_3, Shewhart_3 and PM_3 charts are 26.44, 30.57, 29.97, 27.49 and 24.94 respectively. Since, EQL of PM_3 is fewer than all other charts. The RALR values of these charts are 1.03, 1.31, 1.32, 1.07 and 1.00. Hence, the smallest EQL and RARL values are showed that PM_3 chart has better detection ability as compared to other charts.

Table 2.7: ARL comparison for error variance shifts in original model (σ to $\gamma\sigma$)

Chart		γ									
		1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00
EWMA/R	ARL	34.30	12.00	6.10	3.90	2.90	2.30	1.90	1.70	1.50	1.40
	SEQL	124.70	80.58	60.24	48.71	41.39	36.39	32.77	30.07	28.02	26.44
	SRARL	1.19	1.25	1.22	1.17	1.14	1.11	1.08	1.06	1.05	1.03
Hotelling T^2	ARL	39.60	14.90	7.90	5.10	3.80	3.00	2.50	2.20	2.00	1.80
	SEQL	128.51	85.81	65.45	53.68	46.12	40.91	37.13	34.32	32.20	30.57
	SRARL	1.30	1.43	1.44	1.42	1.39	1.37	1.35	1.33	1.32	1.31
EWMA_3	ARL	33.50	12.70	7.20	5.10	3.90	3.20	2.80	2.50	2.30	2.10
	SEQL	124.12	80.34	60.78	49.96	43.18	38.57	35.32	32.97	31.25	29.97
	SRARL	1.18	1.25	1.26	1.27	1.28	1.28	1.29	1.30	1.31	1.32
Shewhart_3	ARL	40.10	13.50	6.50	4.00	2.80	2.20	1.80	1.60	1.50	1.40
	SEQL	128.87	85.49	64.17	51.83	43.88	38.39	34.41	31.43	29.19	27.49
	SRARL	1.31	1.40	1.36	1.29	1.23	1.18	1.14	1.10	1.08	1.07
PM_3	ARL	24.81	9.82	5.61	3.90	2.99	2.42	2.06	1.81	1.65	1.50
	SEQL	117.86	72.67	54.05	43.91	37.59	33.30	30.22	27.95	26.25	24.94
	SRARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

2.4.4 Shifts in the slope of transformed model:

The results for the shifts in slope parameter of transformed model are reported in Table 2.8 while ARL curves are portrayed in Figure 2.1. The 20% decrement in slope parameter ($\delta = -0.2$) may cause 61.65% loss in the ARL of EWMA/R chart, 73.9% in T^2 chart, 93.45% in EWMA_3 chart, 67.85% in Shewhart_3 chart and 94.58%in PM_3 chart. Further, shift in slope of transformed model ($\delta = -0.7$) may resulted 2.6, 1.9, 2.3, 2.03 and 2.27 unit ARL for EWMA/R, T^2, EWMA_3, Shewhart_3 and PM_3 charts respectively.

When shift is decreased by 60% in slope parameter of transformed model ($\delta = -0.6$) than SEQL of EWMA/R, T^2, EWMA_3, Shewhart_3 and PM_3 charts are 2.12, 1.39, 0.57, 1.63 and 0.54 respectively0. Further, the values of EQL of these five charts are 1.76, 1.23, 0.87, 1.38 and 0.83. Similarly, the values of RARL of EWMA/R, T^2, EWMA_3, Shewhart_3 and PM_3 charts are 2.73, 1.88, 1.05, 2.18 and 1.00. Hence, based on minimum ARL, EQL and RARL the detection ability of PM_3 chart is superior than other charts.

Table 2.8: ARL comparison for slope shifts in transformed model (B_1 to $B_1 + \delta\sigma$)

Chart		δ								
		-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0
EWMA/R	ARL	76.7	33.7	15.3	7.5	4.2	2.6	1.8	1.4	1.2
	SEQL	1.53	2.04	2.21	2.2	2.12	2.02	1.92	1.83	1.76
	SRARL	4.04	4.75	4.65	4.29	3.89	3.52	3.21	2.95	2.73
T^2	ARL	52.2	21.2	9.6	4.9	2.9	1.9	1.5	1.2	1.1
	SEQL	1.04	1.36	1.45	1.44	1.39	1.33	1.28	1.25	1.23
	SRARL	2.91	3.29	3.15	2.88	2.61	2.37	2.18	2.02	1.88
EWMA_3	ARL	13.1	6.6	4.4	3.3	2.7	2.3	2.1	1.9	1.7
	SEQL	0.26	0.36	0.43	0.5	0.57	0.63	0.71	0.79	0.87
	SRARL	1.10	1.11	1.08	1.06	1.05	1.04	1.04	1.05	1.05
Shewhart_3	ARL	64.29	25.29	11.08	5.42	3.06	2.03	1.49	1.24	1.10
	SEQL	1.28	1.66	1.75	1.72	1.63	1.55	1.47	1.42	1.38
	SRARL	3.46	3.96	3.77	3.43	3.08	2.79	2.54	2.34	2.18
PM_3	ARL	10.83	6.4	4.43	3.36	2.7	2.27	1.96	1.73	1.53
	SEQL	0.22	0.31	0.4	0.47	0.54	0.61	0.69	0.76	**0.83**
	SRARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

2.4.5 Joint Shifts in intercept and slope of transformed model:

In Table 2.9, At fixed $\varphi = 0$ and shift in the slope (i.e. $\delta = 0.075$) may cause loss of 19.19%, 61.65%,16.55% and 81.32% in the ARL of EWMA/R, EWMA_3, Shewhart_3 and PM_3 charts respectively. The ARL at fixed $\delta = 0$ and shift in intercept (i.e. $\varphi = 0.15$ and $\varphi = 0.45$) are reported as (110.25, 88.66, 170.71 and 56.97) and (21.94, 13.01, 63.40

and 16.92) in EWMA/R, EWMA_3, Shewhart_3 and PM_3 respectively. At $\varphi = 0.20$ and $\delta = 0.15$ the unit loss in ARL of EWMA/ R, EWMA_3, Shewhart_3 and PM_3 charts are 147.1, 180.5, 112.79 and 185.79 respectively. Similarly, at $\delta = 0.125$ and $\varphi = 0.20$ unit loss in ARL are 143.4, 174.6, 98.49 and 183.15 of EWMA/R, EWMA_3, Shewhart_3 and PM_3 respectively. So, based on ARL results reported in joint shift (intercept and slope) the performance of PM_3 chart is better than other charts.

Table 2.9: ARL comparison for joint (intercept and slope) shift in transformed model (B_0 to $B_0 + \varphi\sigma$) and (B_1 to $B_1 + \delta\sigma$)

φ	Chart	δ										
		0	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
0.0	EWMA/R	200	182.54	174.65	161.62	146.45	134.65	115.62	95.36	78.65	69.85	55.45
	EWMA_3	200	172.50	119.40	76.70	49.10	32.40	23.00	16.70	13.20	10.60	8.80
	Shewhart_3	200	195.00	181.80	166.90	142.10	120.80	99.20	81.20	63.80	51.00	41.00
	PM_3	200	108.37	58.72	37.36	26.35	19.93	15.75	12.89	10.83	9.29	8.11
0.05	EWMA/R	189.65	179.10	169.90	156.60	140.80	123.20	105.10	88.80	73.40	60.20	49.60
	EWMA_3	175.61	157.60	114.70	74.80	48.30	32.20	22.50	16.90	13.20	10.70	8.90
	Shewhart_3	196.28	193.18	180.64	162.76	139.73	119.05	98.79	79.91	63.73	50.47	40.90
	PM_3	138.98	87.56	53.82	36.09	25.87	19.73	15.65	12.91	10.78	9.31	8.03
0.10	EWMA/R	167.85	139.50	133.60	125.40	115.50	103.50	90.40	78.30	65.70	55.60	46.30
	EWMA_3	137.48	122.10	94.60	66.40	44.90	30.70	21.90	16.60	13.10	10.60	8.90
	Shewhart_3	186.73	184.63	172.96	154.66	137.51	115.32	95.28	77.85	62.38	50.83	40.31
	PM_3	84.16	57.44	44.69	32.75	24.49	19.14	15.37	12.69	10.69	9.28	8.09
0.15	EWMA/R	110.25	96.80	94.20	90.30	85.10	78.50	70.90	63.00	55.30	47.70	40.90
	EWMA_3	88.66	84.60	70.80	54.50	39.60	28.50	20.90	16.10	12.80	10.40	8.80
	Shewhart_3	170.71	165.25	160.43	143.16	128.64	109.11	91.59	74.65	60.80	49.55	39.75
	PM_3	56.97	40.41	34.51	28.06	22.61	18.23	15.01	12.50	10.67	9.15	8.00
0.20	EWMA/R	84.20	64.80	63.80	62.10	59.70	56.60	52.90	48.50	44.00	39.20	34.60
	EWMA_3	59.37	57.10	51.10	42.40	33.30	25.40	19.50	15.40	12.40	10.20	8.70
	Shewhart_3	151.90	148.30	143.85	132.16	116.66	101.51	87.21	70.79	57.63	47.31	38.28
	PM_3	42.10	29.64	27.20	23.68	20.06	16.85	14.21	12.04	10.45	9.08	7.93
0.25	EWMA/R	64.50	44.30	43.80	42.90	41.80	40.30	38.40	36.10	33.60	30.80	28.10
	EWMA_3	39.74	39.50	36.50	32.30	27.10	22.00	17.80	14.40	11.90	10.00	8.50
	Shewhart_3	132.50	130.54	125.22	114.09	103.59	91.89	78.02	67.20	55.43	45.70	36.84
	PM_3	32.77	21.46	19.02	17.86	16.30	14.50	13.31	11.5	10.05	8.90	7.80
0.30	EWMA/R	54.51	31.00	30.80	30.50	29.90	29.20	28.30	27.10	25.70	24.20	22.50
	EWMA_3	28.50	28.20	26.90	24.70	22.00	18.80	15.70	13.20	11.20	9.60	8.30
	Shewhart_3	112.90	112.33	107.85	100.24	91.56	81.92	71.51	61.72	50.87	42.88	35.20
	PM_3	26.74	17.89	17.34	16.70	15.22	13.88	12.33	10.98	9.71	8.62	7.68
0.35	EWMA/R	38.53	22.90	22.80	22.60	22.20	21.90	21.40	20.70	19.90	19.10	18.00
	EWMA_3	20.75	20.90	20.20	19.10	17.60	15.80	13.90	12.10	10.50	9.10	8.00
	Shewhart_3	93.75	93.36	90.25	85.38	79.35	71.53	63.79	54.97	47.05	40.37	33.23
	PM_3	22.50	14.85	14.53	14.07	13.33	12.37	11.33	10.23	9.30	8.31	7.46
0.40	EWMA/R	29.8	17.40	17.30	17.20	17.10	16.80	16.60	16.20	15.80	15.30	14.70
	EWMA_3	16.27	16.20	15.90	15.30	14.50	13.50	12.10	10.90	9.70	8.60	7.60
	Shewhart_3	77.67	76.11	75.17	71.33	66.92	62.19	55.44	49.13	43.38	36.36	30.71
	PM_3	22.48	12.46	12.31	12.05	11.61	11.01	10.27	9.52	8.65	7.96	7.25
0.45	EWMA/R	21.94	13.90	13.90	13.90	13.80	13.60	13.50	13.30	13.00	12.60	12.20
	EWMA_3	13.01	13.10	12.90	12.60	12.10	11.40	10.60	9.80	8.90	8.00	7.30
	Shewhart_3	63.40	63.05	62.15	59.40	56.24	52.19	49.02	43.19	38.39	33.32	28.66
	PM_3	16.92	10.73	10.66	10.45	10.22	9.80	9.36	8.76	8.17	7.56	6.99
0.50	EWMA/R	18.98	11.50	11.50	11.40	11.30	11.30	11.10	11.00	10.80	10.60	10.30
	EWMA_3	10.69	10.80	10.80	10.60	10.30	9.90	9.30	8.70	8.10	7.50	6.90
	Shewhart_3	51.24	51.32	50.88	49.25	46.96	44.14	40.69	37.55	33.57	29.96	25.94
	PM_3	14.99	9.31	9.34	9.17	8.98	8.77	8.48	8.05	7.60	7.14	6.62

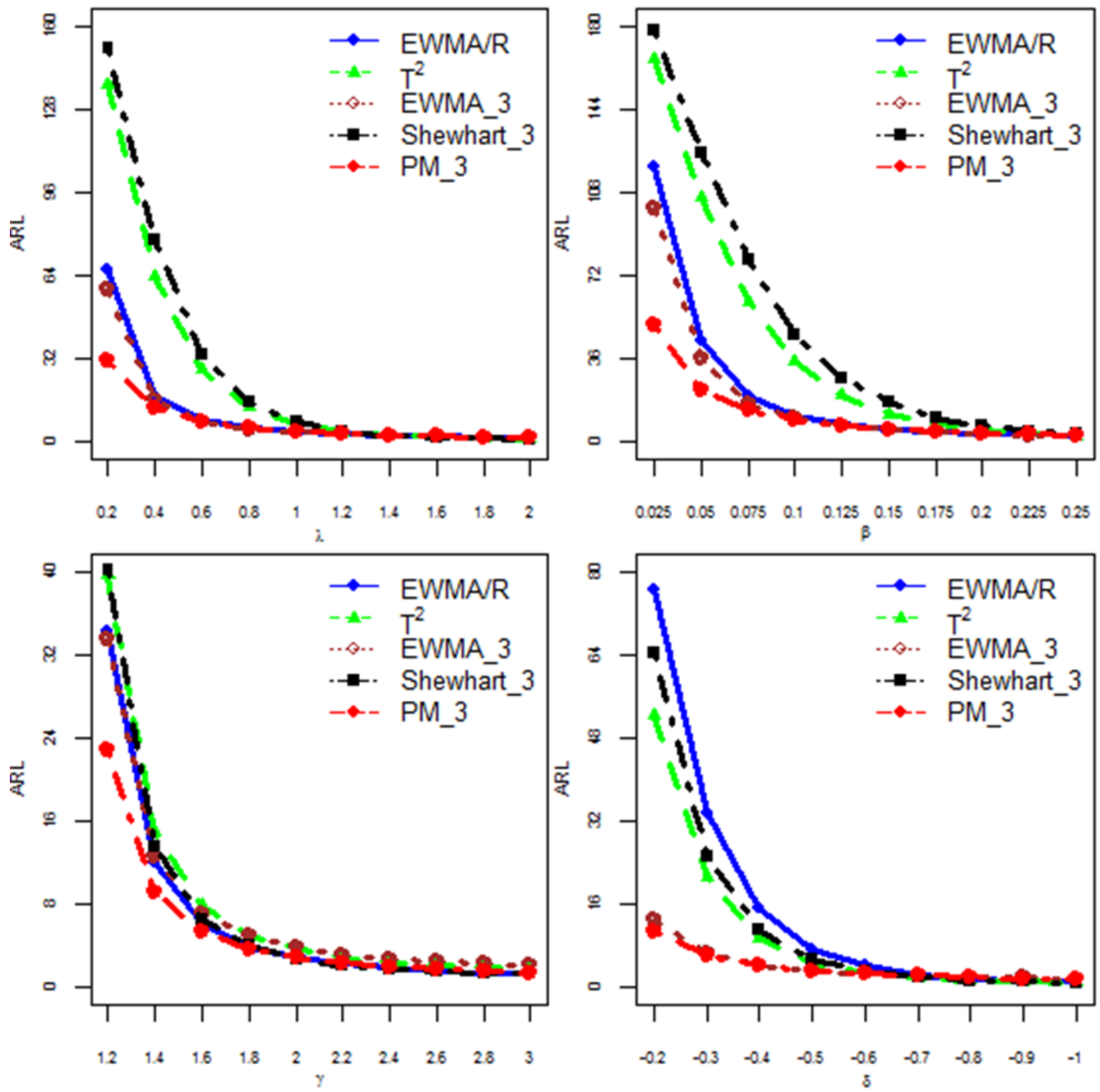

Figure 2.1: ARL curves with respect to different amounts of shifts

2.5 A case study: N-Queens size problems

The N-Queen problem is a computational/multithreading program used for benchmarking. This problem involves by placing N-queens on a $N \times N$ chessboard (given in Figure 2.2) such that no Queen repeated in diagonal, row and in column.

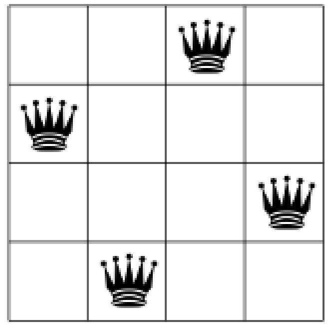

Figure 2.2: Solution of 4-Queen size

There exist different numbers of an N-Queen problem with their possible solutions. In Table 2.10, two machines having different specification shows the time to solve the solution of different Queen size problem. The machine has a 2GHz processor and 24 central processing unit (CPU) takes significantly less time to solve the Queen size problem at the different level of N (ranging from 4 to 19). It is also noted that as the number of the N-Queen problems increased the possible solutions and elapsed time also increased.

In the first case, we used one virtual machine/operating system (VM/OS) and 2-VCPU (virtual CPU) to run the N-Queen problems. When two CPU are working than rest of the 22 CPU are in idle condition. In the next experiment, we increase the of VCPU from 2 to 4 to reduce the elapsed time and utilize more CPU. In this case, 4-VCPU are used and 20 CPU is in idle condition. We gradually increase the number of VCPU up to 24 and then increase it from 24 to 32 to check to see its effect of elapsed time. Once all 24 CPU is used, the best result produced (i.e. less elapsed time). When we cross the limit of 24 CPU the elapsed time increased for 28 & 32 CPU. After that, we apply virtualization technology to run more than one VM/OS on the top of our powerful server and run the N-Queen benchmarking application. The layout of our experimental study is portrayed in Figure 2.3.

Table 2.10: Comparison of two machines for N-Queens problem

Board Size (N x N chessboard)	Number of Solutions to N-Queens Problem	Time to find a solution. PC specification 2GHz with 24 CPU	Time to find a solution. PC specification 800 MHz
4	2	10 milliseconds	13 milliseconds
5	10	10 milliseconds	13 milliseconds
6	4	10 milliseconds	13 milliseconds
7	40	10 milliseconds	13 milliseconds
8	92	10 milliseconds	13 milliseconds
9	352	20 milliseconds	28 milliseconds
10	724	20 milliseconds	60 milliseconds
11	2680	30 milliseconds	130 milliseconds
12	14200	50 milliseconds	550 milliseconds
13	73712	165 milliseconds	860 milliseconds
14	365596	385 milliseconds	1 seconds
15	2279184	605 milliseconds	4 seconds
16	14772512	970 milliseconds	23 seconds
17	95815104	3.5 seconds	2 min 38 seconds
18	666090624	23.5 seconds	19 min 26 seconds
19	4968057848	3.3 min	2 hour 31 min 24 seconds

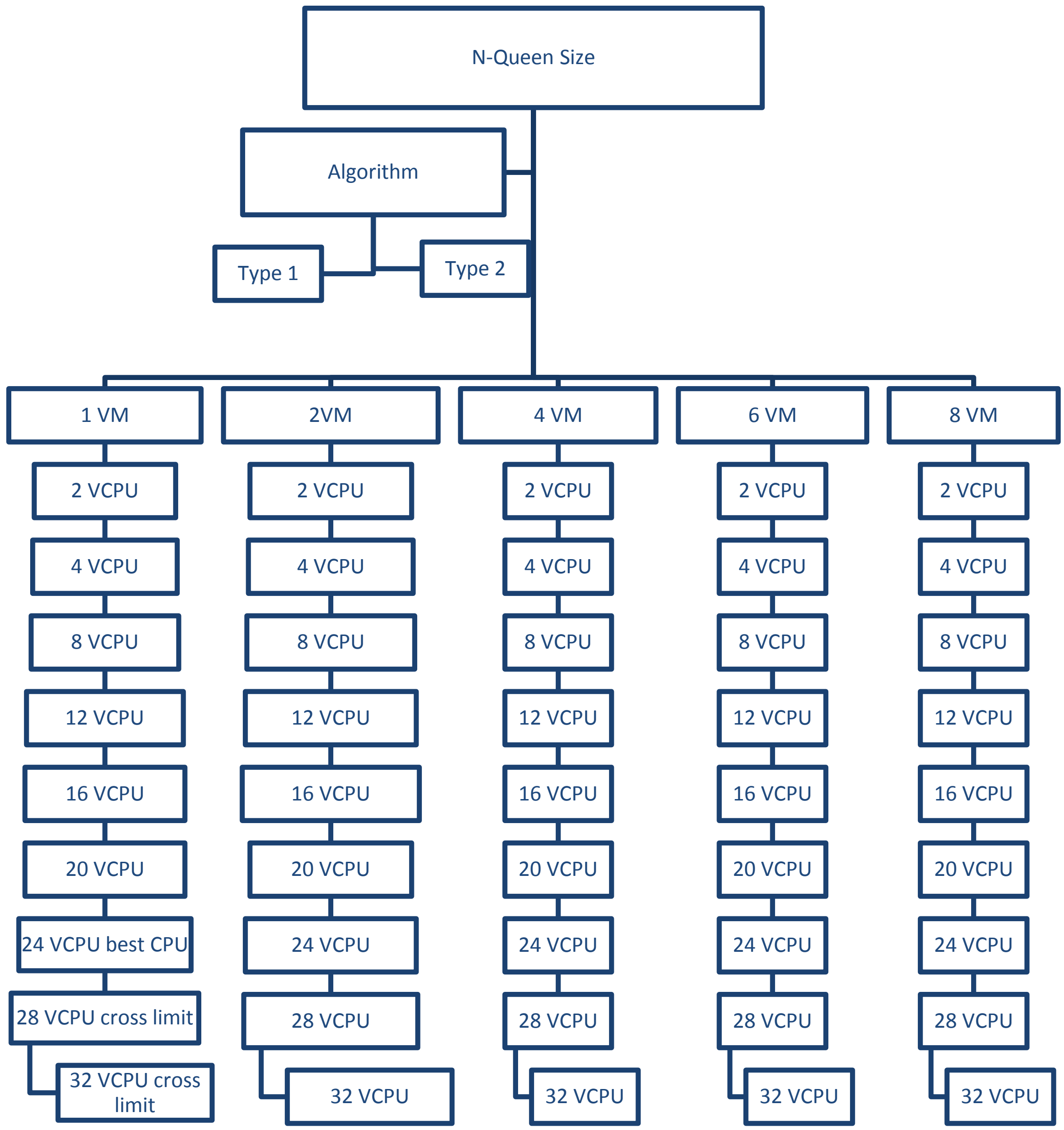

Figure 2.3: Experimental layout of N-Queen size

In this case study, we have selected 16 Queen problems ranging from (chessboard size 4 to 19), two type of algorithm, five VM/OS and nine different level of VCPU. Thus, we have $1440\,(16 \times 2 \times 5 \times 9)$ possible combinations of the solution. So, each combination of Queen size has 90 elapsed time (in sec) values against a VM/OS. The total values of elapsed time are $90 \times 5 = 450$. We are considering problem size 17 as a benchmark and applied our proposed progressive mean control chart (PM_3).

2.5.1 Implementation of PM_3 and Shewhart_3 chart

We have considered VCPU value as independent variable ($X = 2,4,8,12,16,20,24,28,32$) and elapsed time ($Y$) as a dependent variable. The following steps are described the implementation of Shewhart_3 and PM_3 charts.

Step 1: We have total 450 sample values (50 profiles). The IC regression model based on 50 profiles is

$$Y = 19.367 - 0.506X + \varepsilon. \text{ (original model)}$$

Step 2: Further, to gain the assumption of independence between parameters, we transformed the X values in X' by using $X' = X - \bar{X}$,

$$X' = -14.22, -12.22, -8.22, -4.22, -0.22, 3.78, 7.78, 11.78, 15.78$$

$$Y = 11.367 - 0.506X' + \varepsilon. \text{ (transformed model)}$$

Step 3: The selected charting constants for each chart is given below

$$\text{For PM\_3:} \begin{cases} L_I = 16.54 \\ L_S = 4.54 \\ L_E = 4.54 \end{cases},$$

$$\text{For Shewhart\_3:} \begin{cases} Z_{\alpha/2} = 3.14 \\ {\chi^2}_{LCL} = 0.001 \\ {\chi^2}_{UCL} = 14.17 \end{cases},$$

Step 4: We have plotted our proposed statistics for each parameter (i.e. intercept, slope and error variance) against their control limits which are displayed in Figure 2.4 and Figure 2.5. The process is declared IC (shaded in pink color) for the first 25 points while OOC region presented in white color. The points which are lying outside the limits (UCL or LCL) shows that the process is out of control.

Step 5: When a turbulence occurs in the data set due to Queen size (16 problem) after 25th sample, we can see that the elapsed time of new data has significantly changed in terms of intercept, slope and error variance. The PM_3 and Shewhart_3 charts for Queen size (16 problems) are portrayed in Figure 2.4 and Figure 2.5. The findings depict that for shifted intercept parameter, PM_3 detect 6 OOC points while Shewhart_3 detects 10 OOC signals. For the shifted slope and error variance parameters, PM_3 declares 7 and 9 OOC points while Shewhart_3 detects 0 and 10 OOC signals respectively.

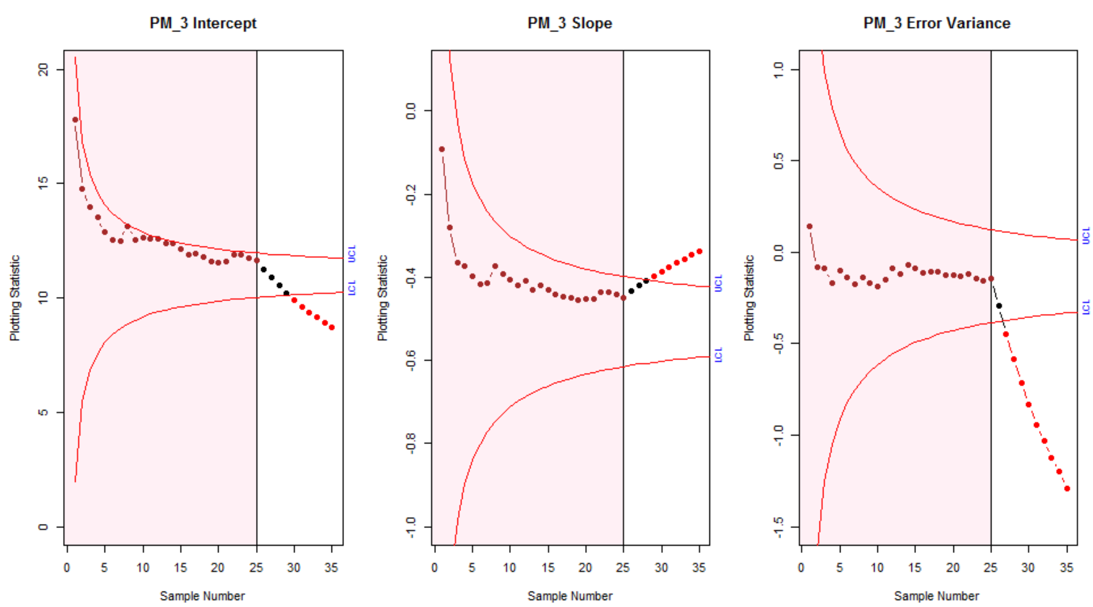

Figure 2.4: PM_3 chart for Queen size problem 16

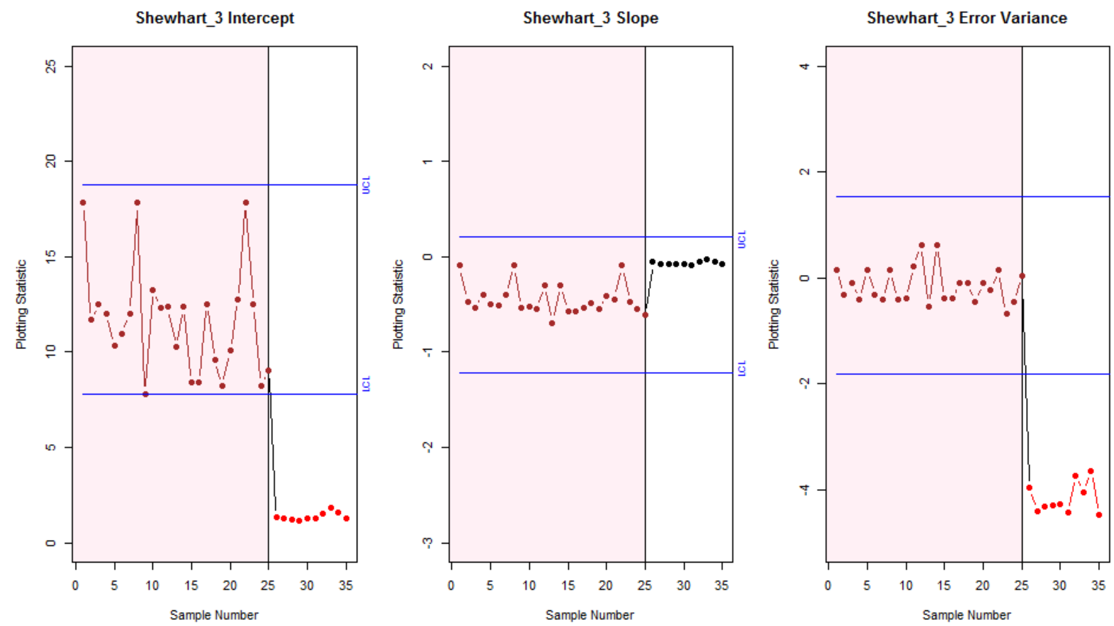

Figure 2.5: Shewhart_3 chart Queen size problem 16

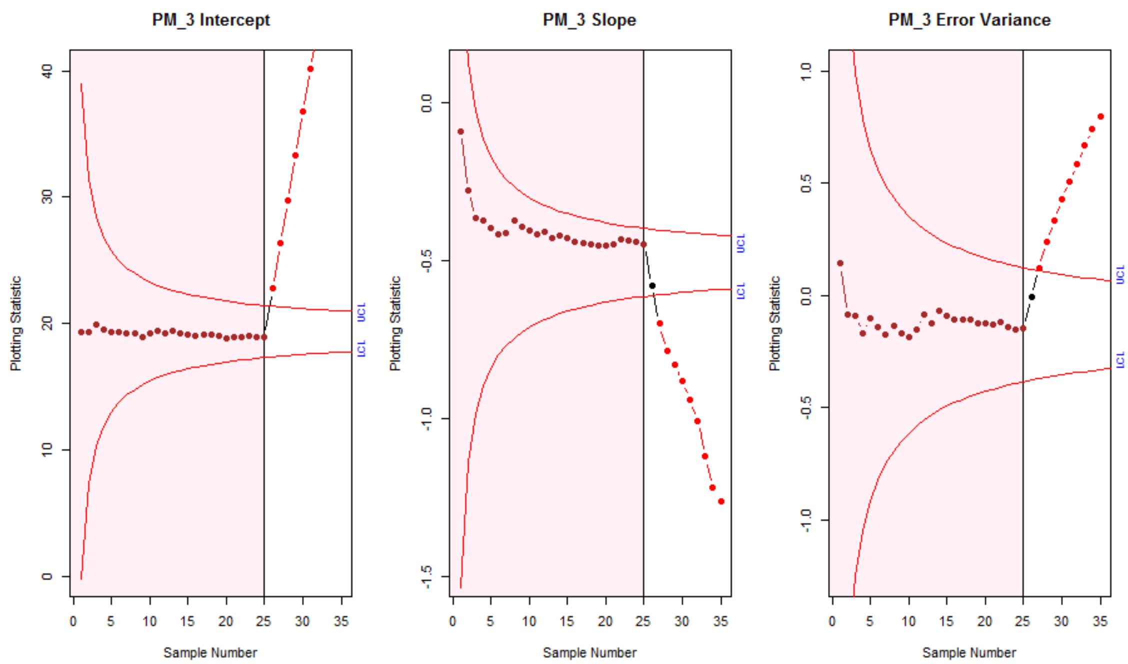

Figure 2.6: PM_3 charts for Queen size 18

Further, to explore the detection ability of PM_3 chart, we also introduced large shifts by considering last 25 profiles of Queen size 18. The graphical display of PM_3 chart for

Queen size 18 is portrayed in Figure 2.6, which shows more out of control signals as compared to a data set having Queen size 16.

2.5.2 Implementation of charts on perturbing the data

In literature, data perturbation approaches are classified into two categories such as value distortion approach and probability distribution approach. In distortion technique, data elements are perturbed by several methods that include additive noise, multiplicative noise, or other randomization methods. The probability distribution approach substitutes the data set with the sample from the own distribution. Recently, Muralidhar and Sarathy (2006) discussed four different perturbation approaches on linear models. In this study, an additive noise model was used which was proposed by Kim (1986) and modified by Tendick and Matloff (1994). The linear profile parameters of the regression model (OOC model) obtained by perturbating data along with IC model are described in Table 2.11.

Table 2.11: Comparison of in-control and out-of-control models

IC Model	OOC Model
Intercept = 19.364	Intercept = 33.711
Slope = -0.506	Slope = -0.844
Standard error = 8.535	Standard error = 16.107

Rate of Change in intercept = 100*(33.711 -19.364) / 19.364 = 74.09%

Rate of Change in slope = 100*(-0.844+0.506) / -0.506 = 66.79%

Rate of Change in error = 100*(16.107 - 8.535) / 8.535 = 88.71%

The performance of PM_3 chart is still better than Shewhart_3 when the shift is introduced by using distorted the data. The PM_3 chart detects 10 OOC points in intercept, 6 OOC points in slope and 7 OOC points in error variance in perturbated data as shown in Figure 2.7. On the other hand, Shewhart_3 is detecting 10 OOC points in intercept, 4 in slope and 6 OOC points in error variance as shown in Figure 2.8. So, based on data perturbation PM_3 chart has more detection ability as compare to Shewhart_3 chart.

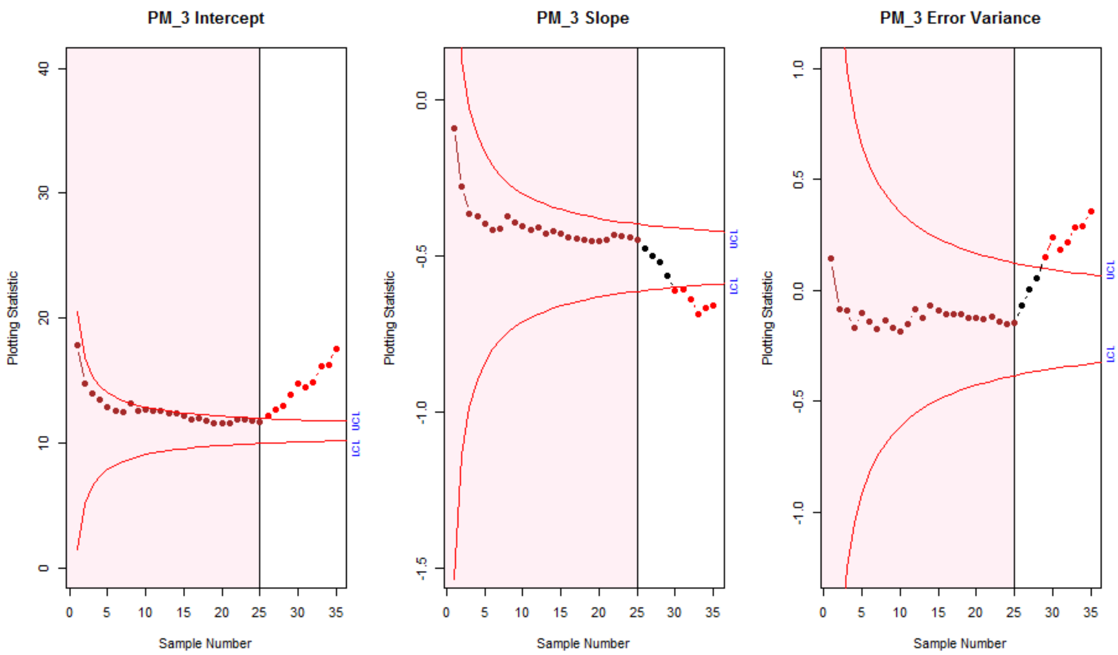

Figure 2.7: PM_3 chart for perturbated data

The detection ability of PM_3 chart is better when the shift is introduced by perturbation technique. PM_3. Hence, in decreasing shift (Queen size problem 16) and perturbation shift PM_3 chart has outperformed well as compared to Shewhart_3 chart

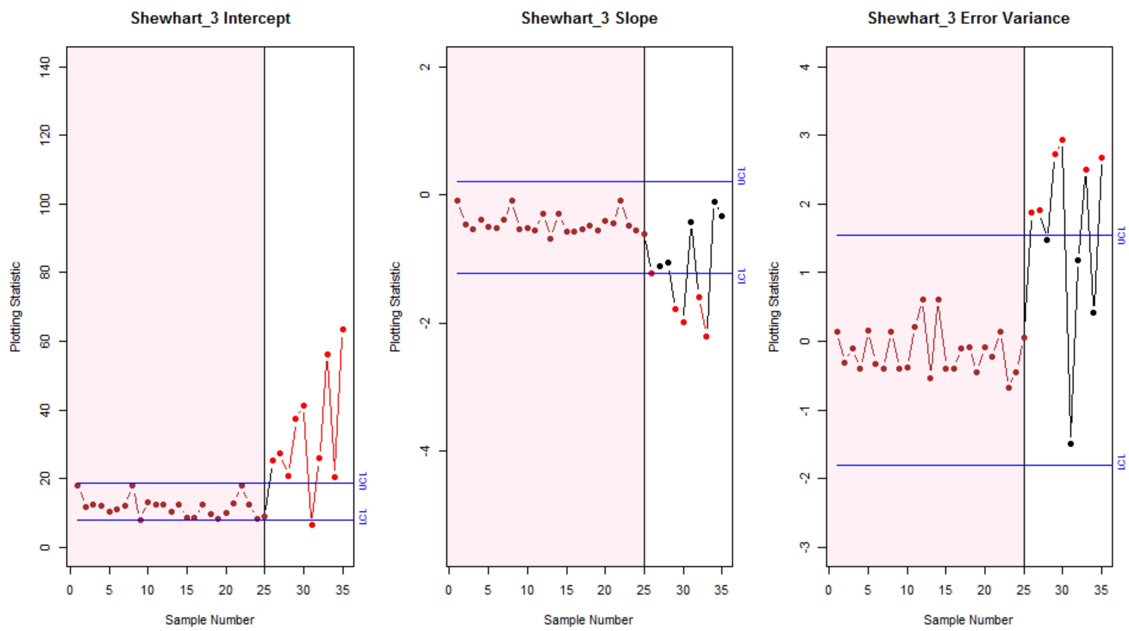

Figure 2.8: Shewhart_3 chart perturbated data

2.6 Conclusions

In this study, we proposed a new charting method based on progressive mean for the simultaneous monitoring of simple linear profile parameters. For the comparative study, we have introduced several amounts of shifts in linear profile parameters such as intercept, slope and error variance. Further, we have used several performance measures such as $ARL, EQL, SEQL, RARL$ and $SRARL$ for determining the detection ability of PM_3 chart and its counterparts. In the presence of shifts in linear profile parameters, the findings depict that PM_3 chart has better detection ability as compared to other competing charts. Moreover, the proposed chart PM_3 is applied on a case study from computer engineering field and results depict the importance of PM_3 chart for controlling the real systems.

Chapter 3

AN ASSORTED CONTROL CHART FOR LOCATION PARAMETER

Primarily, three types of control charts are employed to observe the disturbances in the process parameters. Large turbulences are detected efficiently by Shewhart's control chart whereas, for small and medium instabilities, cumulative sum and exponentially weighted moving average control charts are used. This chapter proposes an assorted approach to monitor small, medium and large disturbances in a single control charting procedure. The said objective is met by using the well-known max approach. For the evaluation of the proposed assorted control chart, we have used various measures like average run length, standard deviation run length, extra quadratic loss, relative average run length and performance comparison index. A comparison of the assorted control chart is presented with some typical charts including the Shewhart's, cumulative sum and exponentially weighted moving average control charts. Finally, a real-life example is presented to monitor the pH level of water in Ecotoxicology lab.

3.1. Introduction

Generally, the control charts are designed to detect three types of shifts in the process parameter(s) (i.e. small, medium and large). Shewhart's control charts are efficient in detecting large shifts whereas CUSUM and EWMA charts are used to detect small to moderate shifts in the process. In literature, several classical and enhanced schemes are available to detect different amounts of shift in process location parameter. Some of these control charting strategies are as follows: Fast Initial Response (FIR) CUSUM by Lucas and Crosier (1982), FIR EWMA by Lucas and Saccucci (1990), new CUSUM by Croiser (1986), generalized weighted moving average control chart by Sheu and Lin (2003), a run-rules scheme by Abbas et al. (2011), mixed EWMA-CUSUM (MEC) by Abbas et al. (2013) and mixed CUSUM-EWMA by Zaman et al. (2015).

All these enhanced and modified control charts focus on detecting a specific size of the shift. This makes these charts insensitive to the others shift sizes. The said drawback is addressed by some further modifications (focusing on detecting a range of shifts) in Shewhart's, CUSUM and EWMA charts. Some of these modifications are combined Shewhart-CUSUM(CSC) by Lucas (1982), Adaptive EWMA (AEWMA) by Capizzi and Masarotto (2003), Adaptive CUSUM by Alippi and Roveri (2006) and Wu et al. (2009), Riaz et al. (2011) ,Abujiya et al. (2013a) and combined Shewhart-EWMA(CSE) by Abujiya et al. (2013) and Koshti (2016). Taking inspiration from these enhanced and modified control charts, this chapter proposes a new assorted control charting approach that can be used to detect the small, medium and large shifts (in process location) simultaneously. Before getting into the details of proposed assorted control chart, the next few subsections provide the structural details of some classical, modified and enhanced control charts. The outlined of rest of the chapter is as follows: in sub section 3.1.1- 3.1.6, a brief discussion on existing methods to monitor process location parameter. In section 3.2, proposed structure of Assorted control chart is discussed. The performance and comparative analysis of proposed with the existing control charting strategies are discussed in Section 3.3. An implementation of proposed and competing charts on a real-life application is discussed in Section 3.4. The concluding remarks are given in the Section 3.5.

3.1.1. The Classical Control Charts

A detailed discussion on classical control charts has been described in Section 1.2.1.

3.1.2. The FIR EWMA control chart

The FIR EWMA control chart was proposed by Steiner (1999). The structure and control limits of FIR EWMA are as follow:

$$\boldsymbol{FIR_{adj} = 1 - (1 - f)^{1+\alpha(1-t)}}$$

$$\left.\begin{aligned} LCL_{i(FIR)} &= \mu_0 - L\left(\frac{\sigma_0}{\sqrt{n}}\right)\left(1 - (1 - f)^{1+\alpha(1-t)}\right)\sqrt{\frac{\lambda}{(2-\lambda)^n}\left[1 - (1 - \lambda)^{2i}\right]} \\ UCL_{i\,(FIR)} &= \mu_0 + L\left(\frac{\sigma_0}{\sqrt{n}}\right)\left(1 - (1 - f)^{1+\alpha(1-t)}\right)\sqrt{\frac{\lambda}{(2-\lambda)^n}\left[1 - (1 - \lambda)^{2i}\right]} \end{aligned}\right\},$$

where $\alpha = \left(\frac{-2}{\log(1-f)} - 1\right)/19$,

3.1.3. The Combined Shewhart- CUSUM (CSC) control chart

The CSC control chart is the combination of Shewhart and CUSUM control charts: Shewhart control chart is used to detect a large amount of shift, while CUSUM control chart is used to observe small and moderate shifts. The structure of CSC control chart proposed by Lucas (1982) is as follow:

$$\boldsymbol{Z_i = \frac{\bar{X}_i - \mu_0}{\frac{\sigma_0}{\sqrt{n}}}}$$

The standardized CUSUM statistics are

$$\left.\begin{aligned} C_i^+ &= \max[0, Z_i - k + C_{i-1}^+] \\ C_i^- &= \max[0, -k - Z_i + C_{i-1}^-] \end{aligned}\right\},$$

The CSC scheme indicates out of control signal if C_i^+ and/or C_i^- have exceeded decision interval value h.

3.1.4. The Combined Shewhart- EWMA (CSE) control chart

The CSE control chart scheme is obtained by some modification in EWMA scheme probability matrix by Lucas (1982). To detect small and large shifts in a process, CSE control chart performs well. The structure of CSE is obtained by merging Shewhart limits with EWMA scheme. In this case, the process is considered as IC if EWMA statistic lies within the control limit of Shewhart and declared as an OOC if EWMA statistic falls outside the limits. Koshti (2016) compared the performance of CSE control chart with Shewhart chart limits between 4 to 4.5 standard deviation.

3.1.5. The Adaptive EWMA (AEWMA) control chart

Capizzi and Masarotto (2003) was proposed AEWMA control chart, detect the small and large shifts in the process parameters. The design structure of AEWMA control chart is as follow:

$$\mathrm{X_i} = \mathrm{X_{i-1}} + \phi(f_i)$$

where $\mathrm{X_0} = \mu_0$, $f_i = y_i - \mathrm{X_{i-1}}$ and $\phi(f_i)$ is a score function. When $y_i \neq \mathrm{X_{i-1}}$, the above equation can be rewritten as

$$\mathrm{X_i} = (1 - \mathrm{w}(f_i))\mathrm{X_{i-1}} + w(f_i)y_i$$

where $w(f) = \phi(f)/f$ i.e. an EWMA statistic varying weights

$$\boldsymbol{\phi}(\boldsymbol{f}) = \begin{cases} \boldsymbol{f} + (\mathbf{1} - \boldsymbol{\lambda})\boldsymbol{\tau}, & \boldsymbol{f} < -\boldsymbol{\tau} \\ \boldsymbol{\lambda f}, & |\boldsymbol{f}| \leq \boldsymbol{\tau} \\ \boldsymbol{f} + (\mathbf{1} - \boldsymbol{\lambda})\boldsymbol{\tau}, & \boldsymbol{f} > \quad \boldsymbol{\tau} \end{cases}$$

where λ is a smoothing constant ranging $0 < \lambda \leq 1$ and $\tau \geq 0$.

3.1.6. The Mixed EWMA-CUSUM (MEC) control chart

Abbas et al. (2013) proposed MEC control chart for the monitoring of process location. The two statistics of MEC chart and UCL are as follows

$$\left.\begin{aligned} M_i^+ &= \max[0, (Q_i - \mu_0) - a_i + M_{i-1}^+] \\ M_i^- &= \max[0, -(Q_i - \mu_0) - a_i + M_{i-1}^-] \end{aligned}\right\}$$

where $M_i^+$and M_i^- are the upper and lower CUSUM statistics and a_i time varying reference value. Further, M_i^+= $M_i^- = 0$ at $i = 1$. The $EWMA$ statistic and control limits are defined as

$$Q_i = \lambda_q X_i + (1 - \lambda_q) Q_{i-1}$$

$$LCL_i = a^* \sigma_X \sqrt{\frac{\lambda_q}{2-\lambda_q}\left[1 - \left(1 - \lambda_q\right)^{2i}\right]}$$

$$UCL_i = b^* \sigma_X \sqrt{\frac{\lambda_q}{2-\lambda_q}\left[1 - \left(1 - \lambda_q\right)^{2i}\right]},$$

where a^* and b^* are constant value like k and h.The further details may be seen in Abbas et al. (2013). On the same lines, a mixed CUSUM-EWMA (MCE) control chart for process monitoring location was proposed by Zaman et al. (2015). The details of MCE charts may be seen in the said paper.

3.2. The design structure of $\boldsymbol{Assorted_{k,\lambda}}$ control chart for location

In this segment, we propose an assorted approach to detect large, medium and small shifts in a single control chart namely an Assorted control chart. Assume that X is normally distributed random variable $X_{ij} \sim N\left(\mu_0 + \delta \frac{\sigma_0}{\sqrt{n}}, \sigma_0\right)$, $i = 1,2, \dots \dots$ and $j = 1,2, \dots, n$

$\delta = 0$ corresponds to an IC situation.

$\delta \neq 0$ means that process has shifted to a new location.

Mathematically, shift can be defined as

$$\delta = \frac{\mu_1 - \mu_0}{\sigma_0/\sqrt{n}}$$

where μ_0 is IC mean.

μ_1 is OOC mean.

σ_0 is IC standard deviation and n is sample size.

The following statistics of Shewhart, CUSUM, and EWMA are used in our proposed chart respectively.

$$\bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}$$

$$\left.\begin{aligned} C_i^+ &= \max[0, (\bar{X}_i - \mu_0) - k\frac{\sigma_0}{\sqrt{n}} + C_{i-1}^+] \\ C_i^- &= \min[0, (\bar{X}_i - \mu_0) + k\frac{\sigma_0}{\sqrt{n}} + C_{i-1}^-] \end{aligned}\right\},$$

$$Z_i = \lambda\bar{X}_i + (1-\lambda)Z_{i-1}$$

Let $\boldsymbol{T_1}$ is first statistic of the proposed chart to detect large shift in the process location. It is defined as:

$$\boldsymbol{T_1} = \left|\frac{\bar{X}_i - \mu_0}{c}\right|, \tag{3.1}$$

where $c = c_s\frac{\sigma_0}{\sqrt{n}}$ and c_s is the charting constant for Shewhart chart.

The following two statistics are used to detect the moderate shift in the process location

$$\boldsymbol{T_2^+} = \frac{C_i^+}{h_c\frac{\sigma_0}{\sqrt{n}}} \quad , \boldsymbol{T_2^-} = \frac{C_i^-}{h_c\frac{\sigma_0}{\sqrt{n}}}, \tag{3.2}$$

where h_c is the charting constant for CUSUM chart.

Similarly, the following statistic is used to detect the small shift in the process location

$$\boldsymbol{T_3} = \left|\frac{Z_i - \mu_0}{L}\right|, \tag{3.3}$$

where $L = L_e \frac{\sigma_0}{\sqrt{n}}\left[\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)^{2i}\}}\right]$, $0 < \lambda \leq 1$ and L_e is the charting constant for EWMA chart.

The plotting statistic of proposed chart is defined as:

$$T = \max(T_1, {T_2}^{+}, {T_2}^{-}, T_3). \quad (3.4)$$

In Eq. (3.4) T is the maximum value of four statistics as discussed above and plotted with respect to time. Because T is the function of standardized max statistics, therefore, it will always have positive value. The upper control limit of T is defined as:

$$\boldsymbol{UCL = T > 1}. \quad (3.5)$$

The sensitivity of the proposed Assorted control chart depends on the selection of (k, λ). For the said reason, we will use the notation $Assorted_{k,\lambda}$ for our proposed control chart. Different combinations of sensitivity parameters (k, λ) are used in the proposed $Assorted_{k,\lambda}$ chart. To detect large, medium and small shift in process location three types of charting constants are incorporated in this study. Table 3.1 Ranges of sensitivity parameters for different categories of shift portrays the ranges of sensitivity parameters for different categories of shifts.

Table 3.1 Ranges of sensitivity parameters for different categories of shift

Sensitivity Parameter	**Category of shift**		
	Small	Medium	Large
λ	0.03 to 0.2	0.21 to 0.5	0.51 to 1
k	0.1 to 0.75	0.76 to 1.5	more than 1.5

When the process is in IC state (i.e. $\delta = 0$) we fix ARL_0 at specific level such as 370. In order to fix the ARL_0 of the proposed $Assorted_{k,\lambda}$ control chart we need to set the control limit coefficients (h_c, L_e, c_s) used with reference to Eqs. 3.1-3.3. For the said purpose, we have used several combinations of sensitivity parameters (k, λ) and worked out the triplets

(h_c, L_e, c_s) for our proposed control chart. The resulting control charting constants/coefficients (h_c, L_e, c_s) are provided in Table 3.2 at some useful combinations of (k, λ) for two commonly used choices of ARL_0=370 and ARL_0 =500. One may work out the same for other choices of ARL_0.

Table 3.2: Charting Constant at $ARL_0 = 370$ and $ARL_0 = 500$

Case	k	λ	$ARL_0 = 370$			ARL_0=500		
			h_c	L_e	c_s	h_c	L_e	c_s
1	0.25	0.25	9.7787	3.1932	3.2685	10.3997	3.2891	3.3571
2		0.38	9.7403	3.2306	3.2629	10.3369	3.3196	3.3483
3		0.55	9.6924	3.2457	3.2559	10.2884	3.3326	3.3414
4	0.5	0.25	5.5842	3.1634	3.2411	5.8962	3.2594	3.3296
5		0.38	5.5960	3.2114	3.2445	5.9133	3.3052	3.3344
6		0.55	5.6018	3.2357	3.2461	5.9048	3.3231	3.3320
7	0.75	0.05	3.9749	2.8705	3.2804	4.1611	2.9725	3.3587
8		0.13	3.9113	3.0695	3.2534	4.1069	3.1655	3.3361
9		0.25	3.8503	3.1483	3.2272	4.0673	3.2484	3.3195
10		0.38	3.8461	3.1914	3.2254	4.0564	3.2849	3.3149
11		0.55	3.8419	3.2127	3.2236	4.0601	3.3072	3.3164
12	1	0.05	2.9838	2.8647	3.2760	3.1466	2.9835	3.3672
13		0.13	2.9456	3.0706	3.2543	3.0948	3.1682	3.3384
14		0.25	2.9044	3.1521	3.2307	3.0554	3.2451	3.3164
15	1.25	0.05	2.3487	2.8556	3.2691	2.4721	2.9700	3.3567
16		0.13	2.3237	3.0668	3.2511	2.4444	3.1669	3.3372
17		0.25	2.2817	3.1412	3.2207	2.4136	3.2440	3.3154

3.3. Performance evaluations and comparisons

In this section, performance evaluations and comparisons of the proposed and some other competing charts are discussed. The competing charts include the classical (Shewhart, EWMA, and CUSUM) and some modified charts (CSE and MEC). We have used different performance measure based on run length including $ARL, SDRL, EQL, SEQL, RARL,$ and

PCI. In order to evaluate these measures, we have used Monte Carlo simulations (for ARL) and numerical integration for other measures ($EQL, SEQL, RARL$ and PCI).

The computational algorithm for these measures is given as: (i) generate random samples from the parent probability model (normal); (ii) compute the sample statistics ; (iii) set the control limits using the description given in Section 3.2; (iv) using steps (i)–(iii), implement the procedural steps of ARL depending on the choices of λ and k (cf. Table 3.2); (v) based on the results of step (iv) for ARL as function of δ: integrate the ARL values over the entire δ range by using an appropriate numerical integration technique (like Simpson or Trapezoidal) (this results into EQL value); (vi) repeat steps (iv) and (v) for all the charts; (vii) based on the results of step (vi), take the ratio of the ARL of a particular chart by the ARL of the benchmark chart (the usual one in this study), divide with the range of δ values and then integrate the output over the entire δ range using an appropriate numerical integration technique (like Simpson or Trapezoidal) (this results into RARL values).

3.3.1. Performance analysis of $Assorted_{k,\lambda}$ control chart

The performance of proposed $Assorted_{k,\lambda}$ control chart is evaluated in terms of ARL and SDRL for varying combination of k, λ and δ. The resulting outcomes are presented in Tables 3.3-3.6 at ARL_0=370 and ARL_0 =500. In addition to the tabular results, we have produced some useful graphical displays based on ARLs and are provided in Figure 3.1 The results advocate the following:

- The proposed chart is sensitive for all types of shifts i.e. small, moderate and large (cf. Table 3.3 and Table 3.5).

- The sensitivity of the proposed chart increases with a decrease in λ at a specific choice of k and it is true for all values of k.
- The sensitivity of the proposed chart increases with a decrease in k at a specific choice of λ and it is true for all values of λ (cf. Table 3.3 and Table 3.5).
- The SDRL behavior of the proposed chart is quite stable for different combinations of sensitivity parameter (k, λ) at varying values of δ (cf. Table 3.4 and Table 3.6).
- The case 15 is stated the optimal choice of charting constants ($h_c = 2.3487, L_e = 2.8556, c_s = 3.2691$) and ($h_c = 2.4721, L_e = 2.9700, c_s = 3.3567$) with sensitivity parameter $k = 1.25$ and λ=0.05 at ARL_0=370 and ARL_0 =500 respectively(cf. Table 3.2).
- The selection of charting constant at ARL_0=370 and ARL_0 =500 are based on minimum ARLs highlighted in bold (cf. Table 3.3 and Table 3.5).
- Four different type of charts is portrayed in Figure 3.1. Graph (a) shows the comparison of ARL values at $ARL_0 = 370$ with fixed value k=0.75 and varying λ for different amounts of shifts ranging from 0.25 to 3. Graph (b) shows the comparison of ARL values at $ARL_0 = 370$ with fixed value λ =0.25 and varying k for different amounts of shifts ranging from 0.25 to 3. The results depicted that the $Assorted_{0.75,0.05}$ has minimum ARL at λ=0.05. The ARL comparison of $Assorted_{k,\lambda}$ control charts at $ARL_0 = 370$ with varying k and fixed λ=0.25 is described that at smaller value $k = 0.25$ has minimum ARL. Similarly, the same results are portrayed in graph (c) and (d) at $ARL_0 = 500$.

Table 3.3: ARLs of $Assorted_{k,\lambda}$ Chart at $ARL_0 = 370$

	k	λ	δ								
			0	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{k,\lambda}$	0.25	0.25	366.988	98.278	31.789	16.917	10.637	5.294	3.223	2.266	1.739
		0.38	365.026	98.871	32.520	17.598	11.438	5.808	3.458	2.360	1.785
		0.55	370.578	99.582	32.979	18.112	12.043	6.440	3.844	2.520	1.850
	0.5	0.25	371.645	139.138	39.554	17.338	10.204	5.124	3.174	2.235	1.720
		0.38	368.862	142.539	40.654	17.778	10.552	5.369	3.342	2.329	1.768
		0.55	374.478	147.456	41.621	18.162	10.818	5.624	3.550	2.456	1.832
	0.75	0.05	374.061	91.623	27.778	13.706	8.380	4.272	2.717	1.960	1.533
		0.13	37.995	133.161	37.771	16.630	9.636	4.762	2.991	2.134	1.654
		0.25	369.287	167.580	51.394	20.796	11.072	5.071	3.143	2.221	1.709
		0.38	371.107	179.890	56.878	22.380	11.650	5.286	3.251	2.295	1.758
		0.55	370.373	185.415	58.935	23.146	11.900	5.437	3.351	2.365	1.799
	1	0.05	368.292	90.376	27.437	13.672	8.353	4.250	2.705	1.955	1.530
		0.13	371.649	133.443	37.882	16.625	9.604	4.758	2.991	2.133	1.655
		0.25	374.543	176.335	55.012	22.392	11.686	5.193	3.165	2.226	1.714
	1.25	0.05	**370.067**	**90.133**	**27.522**	**13.527**	**8.260**	**4.231**	**2.692**	**1.948**	**1.525**
		0.13	375.813	133.171	37.838	16.553	9.626	4.736	2.980	2.129	1.653
		0.25	369.060	174.856	54.966	22.098	11.594	5.182	3.143	2.216	1.703

Table 3.4: SDRL at $ARL_0 = 370$ of $Assorted_{k,\lambda}$ Chart

	k	λ	δ								
			0	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{k,\lambda}$	0.25	0.25	359.507	82.761	20.145	9.614	6.010	2.997	1.651	1.065	0.759
		0.38	358.634	83.503	19.924	9.346	6.014	3.274	1.874	1.157	0.798
		0.55	363.612	82.862	19.765	9.027	5.777	3.400	2.170	1.349	0.891
	0.5	0.25	367.759	134.249	33.124	12.102	6.176	2.775	1.608	1.046	0.749
		0.38	366.224	136.895	34.030	12.093	6.192	2.814	1.702	1.123	0.791
		0.55	372.186	141.425	34.732	12.217	6.124	2.791	1.772	1.219	0.864
	0.75	0.05	378.113	82.205	20.559	9.025	5.188	2.407	1.401	0.926	0.665
		0.13	378.547	128.893	32.009	12.007	6.149	2.612	1.502	0.993	0.719
		0.25	367.081	164.943	47.875	17.147	7.906	2.872	1.587	1.041	0.749
		0.38	367.817	176.272	53.451	18.808	8.302	2.985	1.631	1.089	0.779
		0.55	368.461	181.865	55.548	19.335	8.433	2.997	1.645	1.118	0.816
	1	0.05	370.531	81.205	20.302	9.070	5.211	2.390	1.396	0.926	0.662
		0.13	369.543	128.535	32.250	12.052	6.142	2.628	1.503	0.995	0.719
		0.25	375.425	174.604	51.514	18.953	8.736	3.075	1.625	1.046	0.744
	1.25	0.05	379.470	81.027	20.457	8.979	5.116	2.392	1.387	0.924	0.661
		0.13	376.160	128.397	32.402	12.020	6.172	2.614	1.491	0.994	0.713
		0.25	369.472	172.997	51.352	18.758	8.565	3.066	1.622	1.045	0.745

Table 3.5: ARLs of $Assorted_{k,\lambda}$ Chart at $ARL_0 = 500$

	k	λ	δ								
			0	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{k,\lambda}$	0.25	0.25	503.693	112.726	34.735	18.288	11.489	5.625	3.402	2.376	1.810
		0.38	500.498	112.237	35.272	18.923	12.251	6.201	3.658	2.469	1.856
		0.55	500.181	113.525	35.384	19.398	12.864	6.890	4.090	2.657	1.924
	0.5	0.25	498.034	170.702	43.845	18.613	10.845	5.395	3.333	2.340	1.790
		0.38	498.731	174.436	45.349	19.204	11.223	5.693	3.522	2.437	1.838
		0.55	499.040	177.536	46.000	19.457	11.498	5.928	3.743	2.572	1.906
	0.75	0.05	499.840	106.328	30.258	14.712	8.935	4.526	2.862	2.043	1.597
		0.13	502.063	164.421	42.939	18.164	10.392	5.021	3.140	2.231	1.717
		0.25	501.029	211.937	60.354	23.022	11.946	5.404	3.310	2.323	1.784
		0.38	499.039	225.219	66.028	24.589	12.532	5.597	3.424	2.402	1.819
		0.55	502.470	235.656	68.588	25.192	12.797	5.714	3.530	2.479	1.870
	1	0.05	502.486	107.911	30.643	14.822	9.010	4.549	2.865	2.058	1.602
		0.13	498.778	163.351	43.126	18.174	10.375	5.021	3.134	2.223	1.718
		0.25	499.586	223.552	66.069	25.245	12.808	5.506	3.303	2.320	1.783
	1.25	0.05	**501.378**	**106.911**	**30.358**	**14.666**	**8.927**	**4.512**	**2.841**	**2.046**	**1.591**
		0.13	500.405	164.215	43.069	18.134	10.376	5.028	3.124	2.227	1.721
		0.25	500.039	224.814	66.671	25.371	12.856	5.517	3.319	2.324	1.777

Table 3.6: SDRL at $ARL_0 = 500$ of $Assorted_{k,\lambda}$ Chart

	k	λ	δ								
			0	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{k,\lambda}$	0.25	0.25	499.811	96.755	21.217	9.929	6.370	3.251	1.807	1.144	0.810
		0.38	494.894	95.831	21.154	9.721	6.239	3.494	2.018	1.233	0.848
		0.55	492.696	94.648	20.983	9.329	5.965	3.562	2.306	1.446	0.939
	0.5	0.25	496.601	169.257	37.984	12.961	6.444	2.902	1.719	1.130	0.806
		0.38	501.622	172.344	38.426	12.950	6.397	2.898	1.778	1.189	0.838
		0.55	496.429	171.864	38.495	12.897	6.375	2.869	1.822	1.281	0.903
	0.75	0.05	498.695	162.349	32.398	12.401	6.725	3.031	1.775	1.195	0.857
		0.13	495.176	198.513	47.329	15.925	7.462	2.989	1.702	1.130	0.814
		0.25	503.809	222.782	60.697	20.126	8.702	3.059	1.686	1.118	0.803
		0.38	507.219	229.406	63.603	20.901	8.916	3.088	1.711	1.134	0.817
		0.55	500.078	233.662	64.427	21.287	9.023	3.095	1.697	1.159	0.851
	1	0.05	504.963	164.511	32.410	12.415	6.783	3.029	1.777	1.193	0.856
		0.13	499.019	200.150	47.658	15.951	7.574	3.020	1.699	1.127	0.810
		0.25	499.417	240.509	69.663	23.969	10.437	3.390	1.748	1.107	0.798
	1.25	0.05	508.963	164.258	32.324	12.336	6.794	3.049	1.778	1.190	0.846
		0.13	502.983	202.466	47.937	16.003	7.507	3.003	1.706	1.128	0.809
		0.25	500.448	241.577	70.292	23.923	10.358	3.465	1.760	1.121	0.800

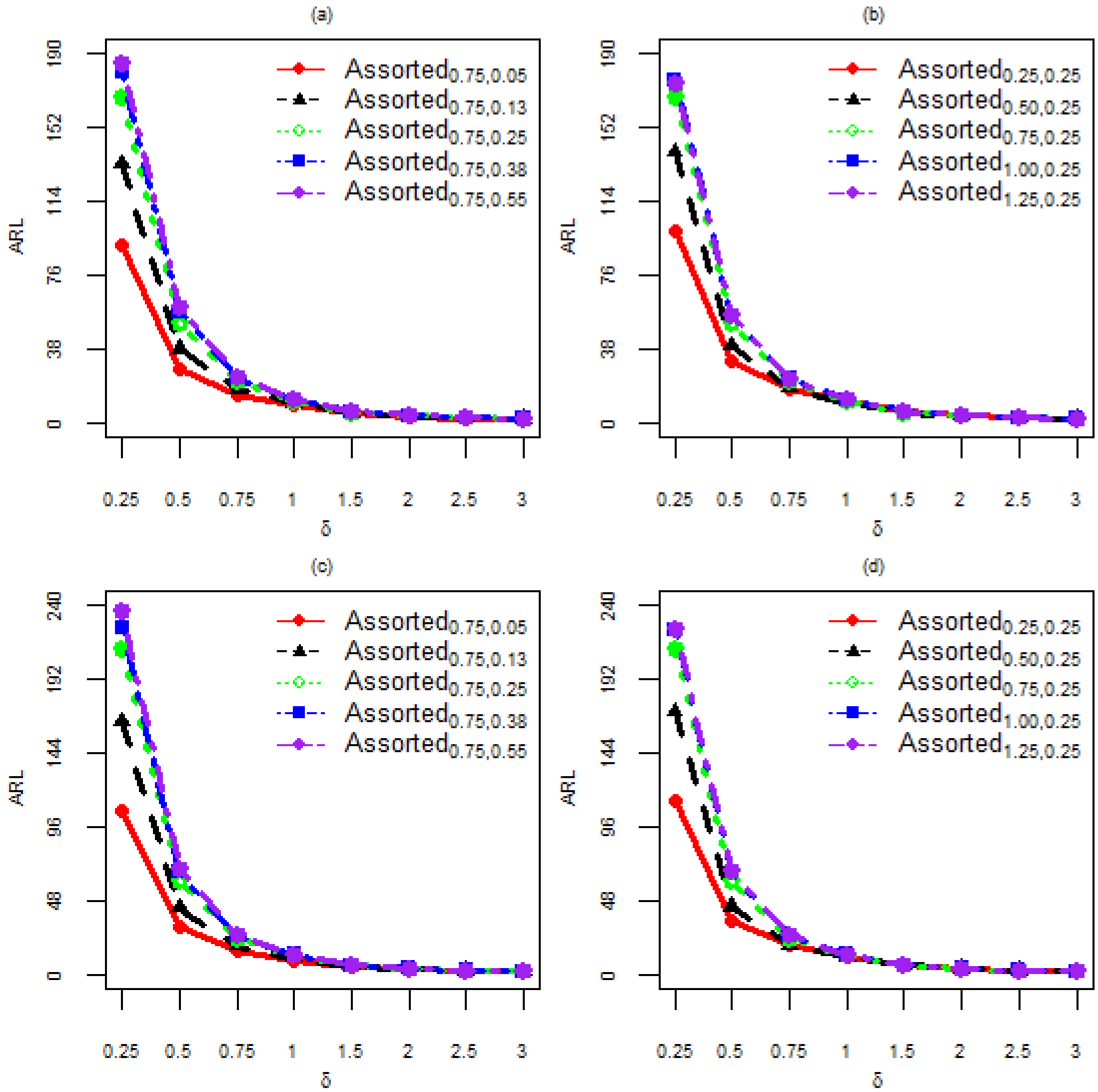

Figure 3.1: ARL comparison of Assorted Chart for: (a) varying values of λ and fixed k at $ARL_0 = 370$; (b) varying values of k and fixed λ at $ARL_0 = 370$; (c) varying values of λ and fixed k at $ARL_0 = 500$; (d) varying values of k and fixed λ at $ARL_0 = 500$;

3.3.2. Comparative analysis

In this section, we provide a comparative analysis of the proposed $Assorted_{k,\lambda}$ chart with the classical (Shewhart, EWMA and CUSUM) charts and some modified charts (including CSE and MEC charts) at $ARL_0 = 500$. The ARL results of all aforementioned competing charts, along with the proposed chart, are compile in the form of tabular display (cf. Table 3.7). This table helps in carrying out ARL comparison of the proposed $Assorted_{k,\lambda}$ chart with the existing counterparts charts including Shewhart_L, $\text{EWMA}_{\lambda,L}$, $\text{CUSUM}_{k,h}$, $\text{CSE}_{L(sd)}$ and $\text{MEC}_{\lambda_q,\, a^*,\, b^*}$. In addition to the tabular results, we have also produced some useful comparative graphical displays based on $ARLs$ and are provided in Figure 3.2. The results advocate the following:

- The proposed chart has minimum ARL at k=1.25 and λ=0.05 for all types of shifts ranges from 0.25 to 3 (cf. Table 3.7).
- The sensitivity of the proposed $Assorted_{1.25,0.05}$ chart is significantly better than Shewhart, EWMA and CUSUM control charts at small amount of shift. For example, at δ=0.25 the ARL_1 of proposed, Shewhart, EWMA and CUSUM charts are 106.911, 373.66, 171.90 and 143.90 respectively.
- The proposed chart has minimum ARL_1 at δ=0.75 (i.e. 14.67) as compared to CSE (19.41) and MEC (16.63).
- Four different type of charts are portrayed in Figure 3.2. Graph (a) shows the comparison of ARL values at $ARL_0 = 500$ and different amounts of shifts ranging from 0.25 to 3. The $Assorted_{1.25,0.05}$ has minimum ARL values among the classical approaches of control charting techniques. The ARL comparison of $Assorted_{1.25,0.05}$ and CSE chart are highlighted in graph (b). In graph (c), the ARL values of $Assorted_{1.25,0.05}$ are less than MEC at different combinations of λ, a^* and b^* for small and moderate shifts. The graph (d) shows ARL of $Assorted_{k,\lambda}$ by using dissimilar combination of k and λ. The proposed $Assorted_{1.25,0.05}$ chart has lowest ARL.

Table 3.7: ARL comparisons at $ARL_0 = 500$

Control Charts	δ								
	0	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{1.25,0.05}$	501.37	106.91	30.35	14.67	8.92	4.51	2.84	2.04	1.59
$\text{Shewhart}_{3.09}$	499.87	373.66	201.30	103.03	54.52	17.79	7.26	3.59	2.16
$\text{EWMA}_{0.25,2.998}$	497.51	171.90	48.51	20.29	7.45	4.4	3.12	2.24	1.94
$\text{CUSUM}_{0.5,5.06}$	500	143.90	38.71	17.29	10.53	5.77	3.67	3.11	2.57
$\text{CSE}_{4(sd)}$	492.62	166.30	49.12	19.41	7.42	4.27	3.54	2.15	1.77
$\text{CSE}_{4.5(sd)}$	499.18	171.50	48.06	20.10	7.43	4.33	3.61	2.21	1.88
$\text{CSE}_{5(sd)}$	509.09	172.40	48.35	19.76	7.42	4.37	3.61	2.26	1.96
$\text{MEC}_{0.1,0.5,37.42}$	498.38	80.13	35.52	24.05	18.86	13.79	11.19	9.55	8.41
$\text{MEC}_{0.25,0.5,20.18}$	502.01	83.75	30.88	18.87	13.88	9.60	7.59	6.40	5.59
$\text{MEC}_{0.5,0.5,11.2}$	507.95	100.26	30.74	16.63	11.45	7.29	5.52	4.54	3.91

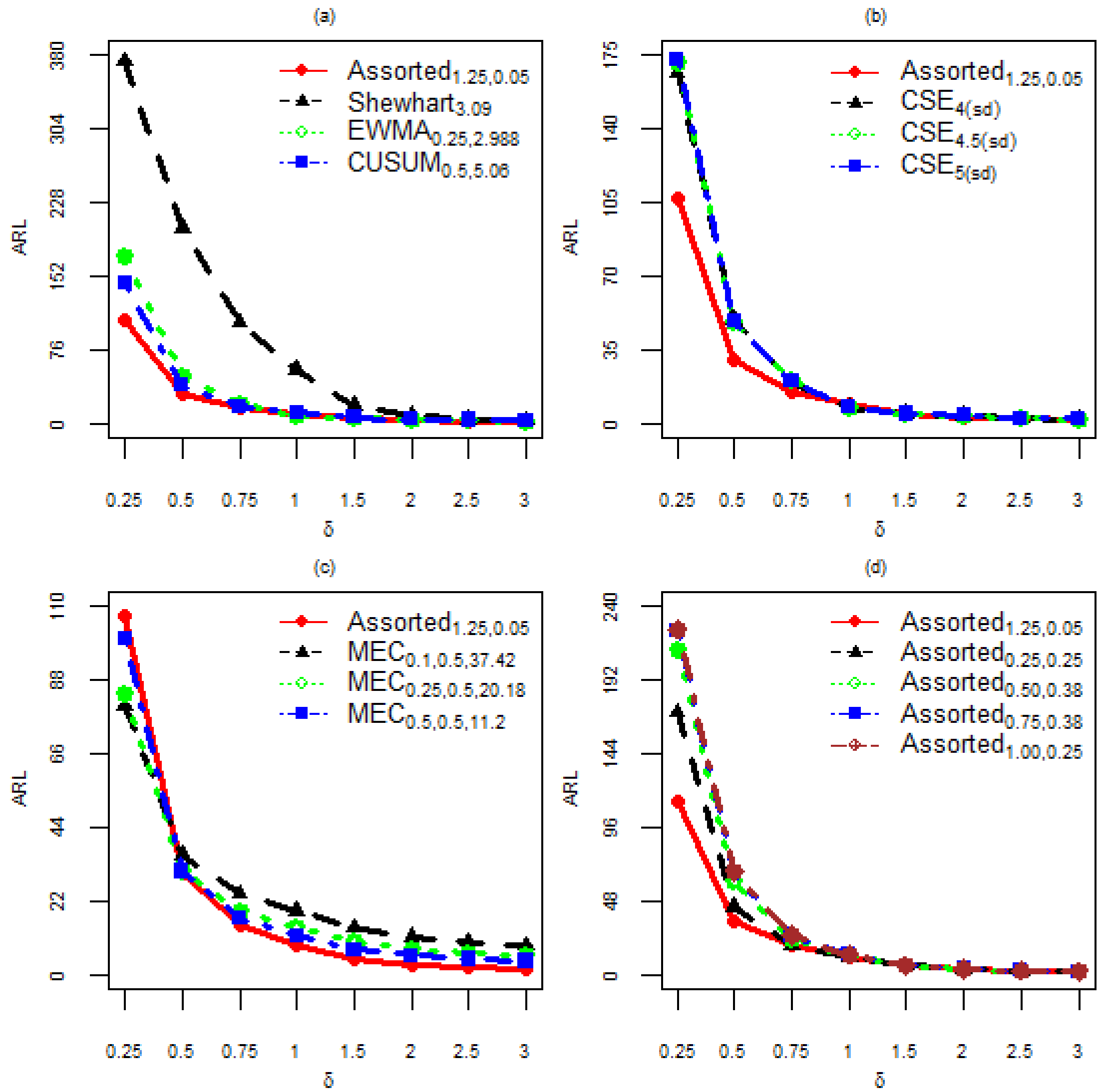

Figure 3.2: ARL comparison of Assorted Chart with (a) Shewhart, EWMA, and CUSUM; (b) CSE; (c) MEC; (d) varying k and λ;

3.3.3. Performance Analysis based on overall measures

Besides ARL (used as performance measure at a particular shift), there are some important measures such as EQL, $RARL$ and PCI that are used to evaluate overall performance. The details of these performance measure have been discussed in Section 1.4. A comparative analysis (among proposed, classical and modified control charting strategies) based on these measures is presented in Table 3.8.

- The assorted chart with $k = 1.25$ and $\lambda = 0.05$ is considered as benchmark chart based on minimum EQL result (i.e. 10.50). For the other competing charts, the EQLs are 36.74, 11.85, 14.27,11.81 and 16.47.
- The $RARL$ of assorted chart is equal to 1 while the $RARL$ of contending charts are 3.82, 1.14, 1.32, 1.13 and 1.19. These results depict that the performance of the proposed chart is better than all other competing control charting strategies.
- As the proposed $Assorted_{1.25,0.05}$ chart has minimum EQL (i.e.10.50) so it is considered as benchmark chart. The PCI is defined as the ratio between the EQL of a chart and EQL of benchmark chart.
- The proposed $Assorted_{1.25,0.05}$ chart has PCI is equal to 1 while all others competing charts have PCI greater than 1 (3.50, 1.13, 1.36, 1.12 and 1.57) which shows the superiority of the proposed $Assorted_{1.25,0.05}$ chart.
- To check the sensitivity of the proposed $Assorted_{1.25,0.05}$ chart and competing charts on different ranges of shifts from 0.25 to 3, we have calculated Sequential Extra Quadratic Loss ($SEQL$) for proposed and competing charts. The results depict that the detection ability of proposed $Assorted_{1.25,0.05}$ chart is better than all others contending charts (cf. Table 3.9 and Figure 3.3).

Table 3.8: Comparison of EQL, RARL and PCI

	k	λ	EQL	RARL	PCI
$Assorted_{k,\lambda}$	0.25	0.25	11.81	1.19	1.19
		0.38	12.47	1.25	1.26
		0.55	13.37	1.33	1.35
	0.5	0.25	12.06	1.23	1.22
		0.38	12.56	1.28	1.27
		0.55	13.06	1.33	1.32
	0.75	0.05	9.92	1.001	1.002
		0.13	11.49	1.18	1.16
		0.25	12.92	1.35	1.31
		0.38	13.52	1.41	1.36
		0.55	13.92	1.45	1.41
	1	0.05	9.98	1.01	1.01
		0.13	11.47	1.18	1.16
		0.25	13.35	1.40	1.35
	1.25	**0.05**	**9.90**	**1.00**	**1.00**
		0.13	11.48	1.18	1.16
		0.25	13.39	1.40	1.35
	$\text{Shewhart}_{3.09}$		34.66	3.82	3.50
	$\text{EWMA}_{0.25,2.998}$		11.31	1.14	1.14
	$\text{CUSUM}_{0.5,5.06}$		13.46	1.32	1.36
	$\text{CSE}_{4(sd)}$		11.26	1.13	1.14
	$\text{CSE}_{4.5(sd)}$		11.51	1.15	1.16
	$\text{CSE}_{5(sd)}$		11.63	1.16	1.17
	$\text{MEC}_{0.1,0.5,37.42}$		22.85	1.73	3.39
	$\text{MEC}_{0.25,0.5,20.18}$		16.47	1.35	2.34
	$\text{MEC}_{0.5,0.5,11.2}$		13.17	1.19	1.76

Table 3.9: SEQL comparison at $ARL_0 = 500$

Control Charts	δ							
	0.25	0.5	0.75	1	1.5	2	2.5	3
$Assorted_{1.25,0.05}$	3.34	5.23	6.13	6.74	7.67	8.44	9.16	9.90
$\text{Shewhart}_{3.09}$	11.67	24.25	34.21	39.72	42.24	40.31	37.40	34.66
$\text{EWMA}_{0.25,2.998}$	5.37	8.40	9.53	9.50	9.22	9.71	10.42	11.31
$\text{CUSUM}_{0.5,5.06}$	4.50	6.91	7.84	8.41	9.52	10.60	11.89	13.46
$\text{CSE}_{4.5(sd)}$	5.35	8.36	9.46	9.44	9.15	9.89	10.73	11.51
$\text{MEC}_{0.5,0.5,11.2}$	3.13	5.05	6.20	7.26	9.48	11.92	14.58	17.45

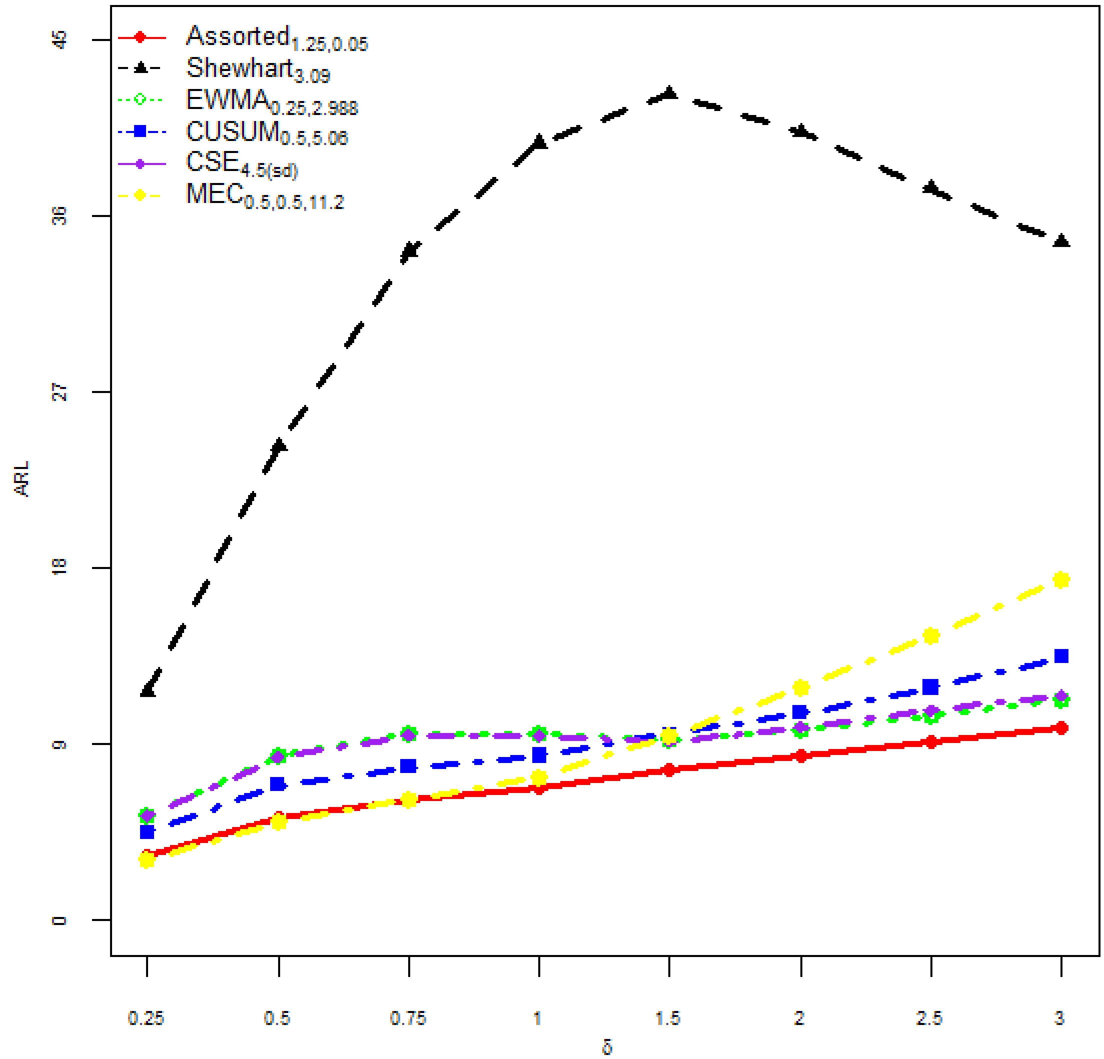

Figure 3.3: SEQL comparison of proposed and competing charts

3.4. Application: Monitoring the amount of pH characteristics

In this section, an application of our proposed chart to monitor the amount of potential Hydrogen (pH) characteristic in water at Aquatic Ecotoxicology laboratory is illustrated. The pH values of water were recorded regularly from 1st April to 30th May 2016 and resulting data are provided in Table 3.10.

Table 3.10: pH values

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
pH value	8.2	8.2	8.2	8.19	8.25	8.19	8.19	8.34	8.12	8.29	8.25	8.12	8.19	8.3	8.25
Days	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
pH value	8.2	8.2	8.2	8.15	8.25	8.29	8.3	8.3	8.28	8.27	8.19	8.28	8.3	8.4	8.3
Days	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
pH value	8.11	8.35	8.22	8.24	8.29	8.3	8.15	8.14	8.24	8.11	8.13	8.11	8.15	8.14	8.2
Days	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
pH value	8.36	8.26	8.11	8.32	8.11	8.33	8.42	8.32	8.41	8.39	8.37	8.31	8.29	8.26	8.27

3.4.1. Data description

The toxic effects of chemicals on aquatic organisms especially on mysids are examined and tested in ecotoxicology lab. The lab has its own sophisticated environment and functionality. The living environment of mysids consists of four characteristic parameters including lab temperature, pH, salinity and dissolved oxygen (DO) of circulating water. Mysids are small shrimps having carapace and usually, their length is about 1 cm. Mysids inhibit in fresh and salt water. Extensively, Mysids have been used as an indicator of spices in water toxicity tests Miller et al. (1990) for many years and are commonly brought up or cultured in the lab. In the Ecotoxicology lab mysids are growing up in water tanks (4x3x2 cubic feet) in fresh and salted water. The culture system of mysids is showing Figure 3.4. The fresh & salted water is circulated through supply lines. The suction and filtration pump work 22 hours/day to keep water fresh and free from algae. Mysids are being fed by lab supervisor twice a day (morning and afternoon). The average values of water characteristic in the lab are (cf. Marini (2003):

- Temperature 23.8 ^{0}C / 75^{0}F
- Light 75 foot -candle / 941.775 LUX
- pH 8.2

- Ammonia(NH_3) 0.1 mg/Liter
- Salinity 20 - 22ppt

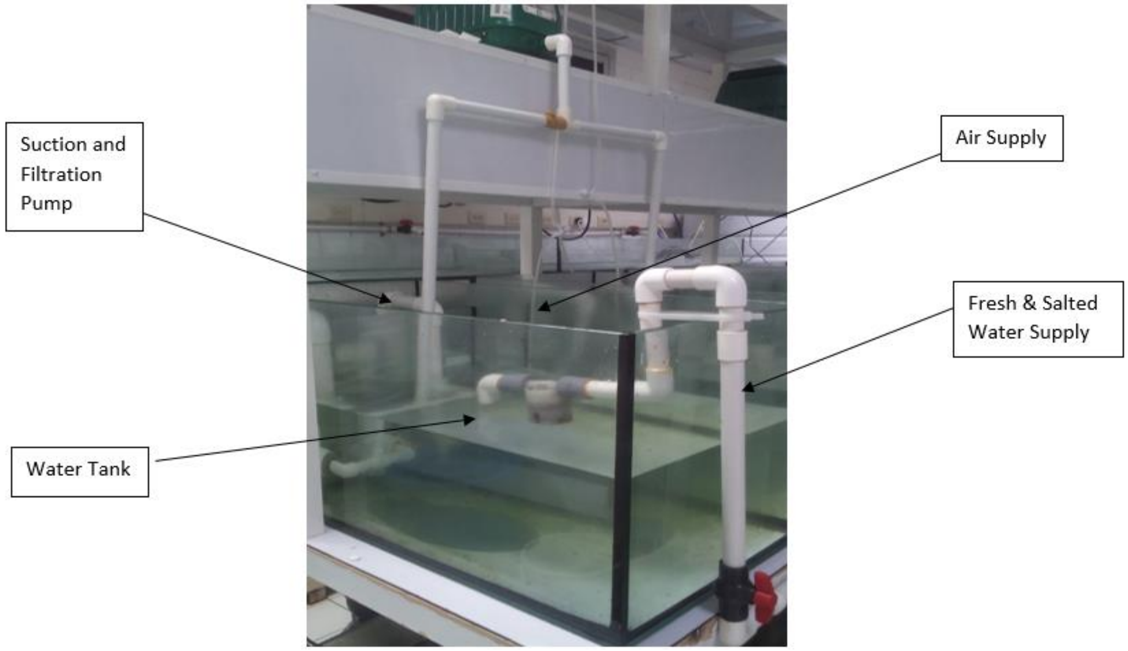

Figure 3.4: Mysids culture system

3.4.2. Application of different control chart on pH characteristics

The reproduction system of mysids depends upon four main factors of water (Salinity, Temperature, Dissolved oxygen (DO) and pH). Our aim here is to monitor the pH value of water in lab using the proposed and other competing charts of this study. The average value of pH (cf. Marini (2003)) for mysids is 8.2 with standard deviations 0.1. Using this information and the data given in Table 3.10, we have constructed the following control charts with their respective settings as listed below:

- The proposed chart ($Assorted_{1.25,0.05}$) with charting constant ($h_c = 2.4721$, $L_e = 2.9700$ and $c_s = 3.3567$) and $UCL = 1$;
- The Shewhart chart with charting constant ($K = 3.0892$) and control limits ($LCL = 7.8917$ and $UCL = 8.5098$);
- The CUSUM chart with sensitivity parameter ($k = 1.25$), control limit coefficient ($h = 2.1053$) and ($UCL = 0.2105$);
- The EWMA chart with sensitivity parameter ($\lambda = 0.05$) and control limit coefficient ($L = 2.6150$) with varying limits.

The implementation of these charts on pH data of Table 3.10 is portrayed in the form of Figure 3.5 ($Assorted$, Shewhart, CUSUM and EWMA). The IC region contains first 50 number of days and OOC region starting from 51st to 60th number of days as shown in graphical representation. The OOC points are indicated by red color in all figures. The detection summary of these charts is given as:

Table 3.11: Detection Summary

Control Chart	OOC detections	False Alarms
The Proposed Assorted	6	0
Shewhart	0	0
CUSUM	4	0
EWMA	7	4

It is evident from the detection ability of the charts that Shewhart appeared as the least efficient chart, followed by CUSUM and EWMA. It is to be noted that EWMA chart has detected seven but at the cost of high false alarms as may be seen in the summary Table 3.11. The proposed chart ($Assorted_{1.25,0.05}$) perfumed the best in detecting OOC points. The reason for this superiority order relates to the amount of shift in the real process. As the aim of proposed chart is to detect small, medium and large shift in the process so it takes edge over other charts in detecting OOC scenarios.

For these OOC signals, we investigated the process in search of the assignable cause(s) and found that water suction and filtration pumps were not functioning properly. The variations in the pH value of water affect the reproduction system of mysids. Usually, the daily production of juveniles in each water tank should be between 80 to 90. But, due to the high value of pH, the production rate was decreased by almost 15%.

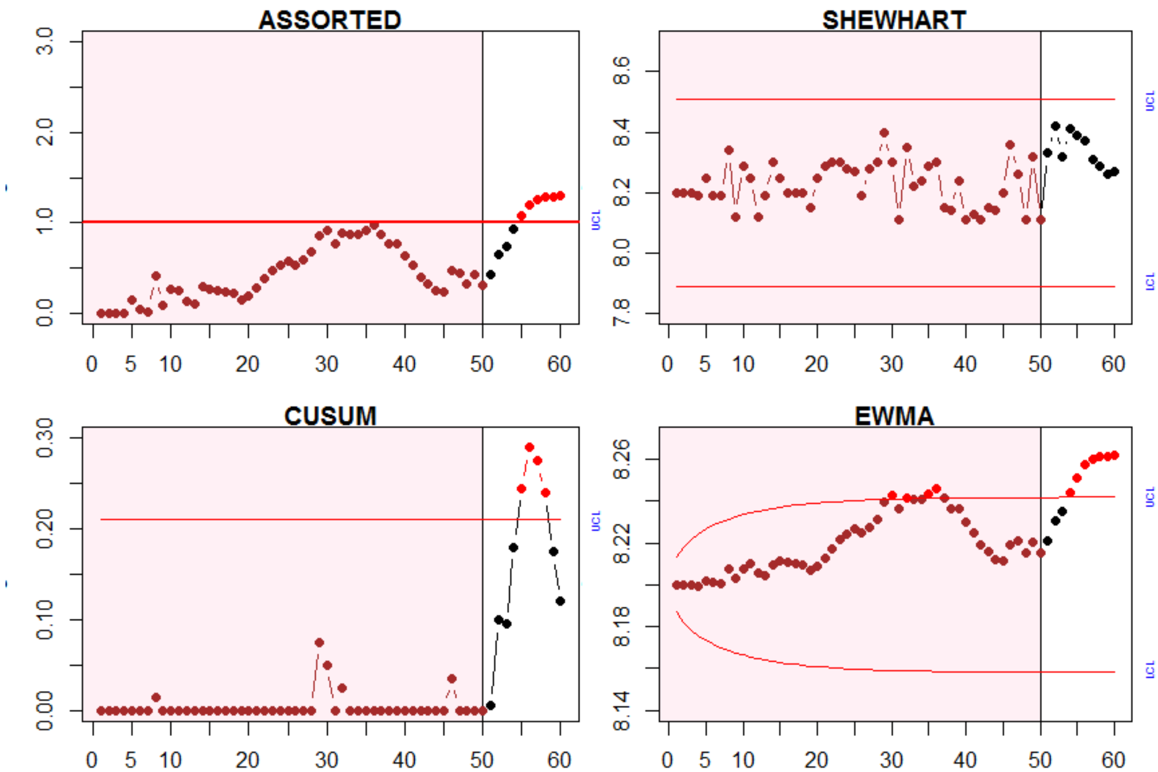

Figure 3.5: Graphical representation of Assorted, Shewhart, CUSUM and EWMA chart on pH data

3.4.3. Application through data perturbation on pH data.

There may be one or more sources of assignable causes (such as water suction and filtration pumps not functioning properly, cleaning of the water tanks and salinity of the water) to generate OOC points in the process. These causes may lead to small, medium and/or large amounts of shifts depending on their intensity. In order to cover different potential causes of OOC scenarios, we have distorted the given data-set through data perturbation (cf. Liu and Kargupta (2006) and Kargupta et al. (2005)). We have perturbed the data using small, moderate and large amounts of distortions and applied the proposed assorted and other classical charts of this study. The graphical and tabular representation of the resulting charts and their detection abilities are presented in Figures 3.6 - 3.8 and Table 3.12. From these results, it is obvious that the performance of the proposed Assorted chart is better than the competing Shewhart, CUSUM and EWMA control charts for to detect all type of shifts (0.75σ, 1.4σ and 3σ) as may be seen form Table 3.12 and Figures 3.6-3.8.

Table 3.12: Detection summary through data perturbation

Control Chart	**OOC detections**	**False Alarms**	**Shift**
The Proposed Assorted	8	0	0.75σ
Shewhart	0	0	0.75σ
CUSUM	0	0	0.75σ
EWMA	7	3	0.75σ
The Proposed Assorted	15	0	1.4σ
Shewhart	0	0	1.4σ
CUSUM	3	0	1.4σ
EWMA	15	3	1.4σ
The Proposed Assorted	15	0	3σ
Shewhart	5	0	3σ
CUSUM	15	0	3σ
EWMA	15	3	3σ

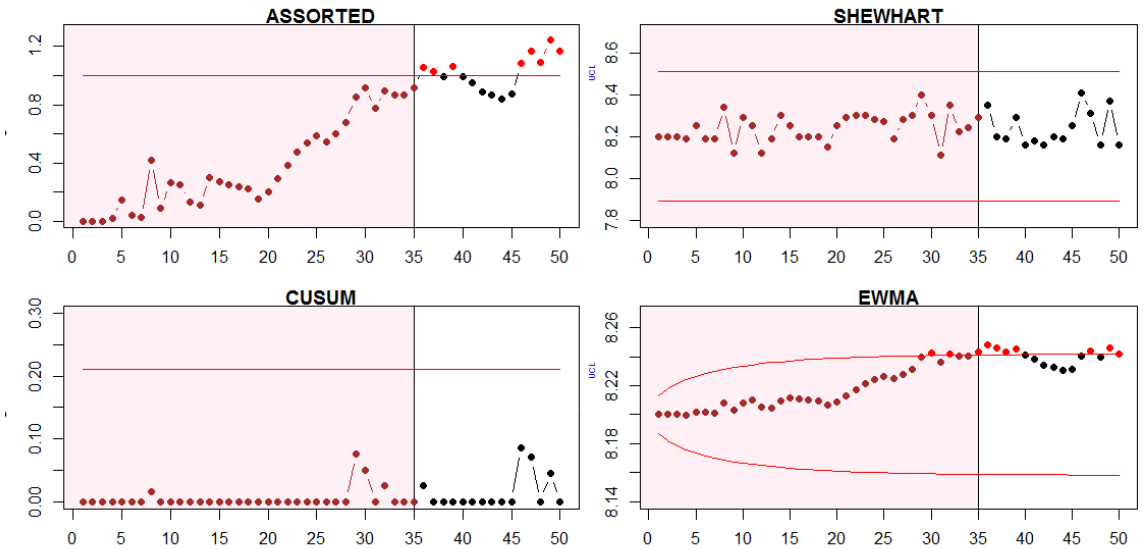

Figure 3.6: Graphical comparison at shift=0.75σ

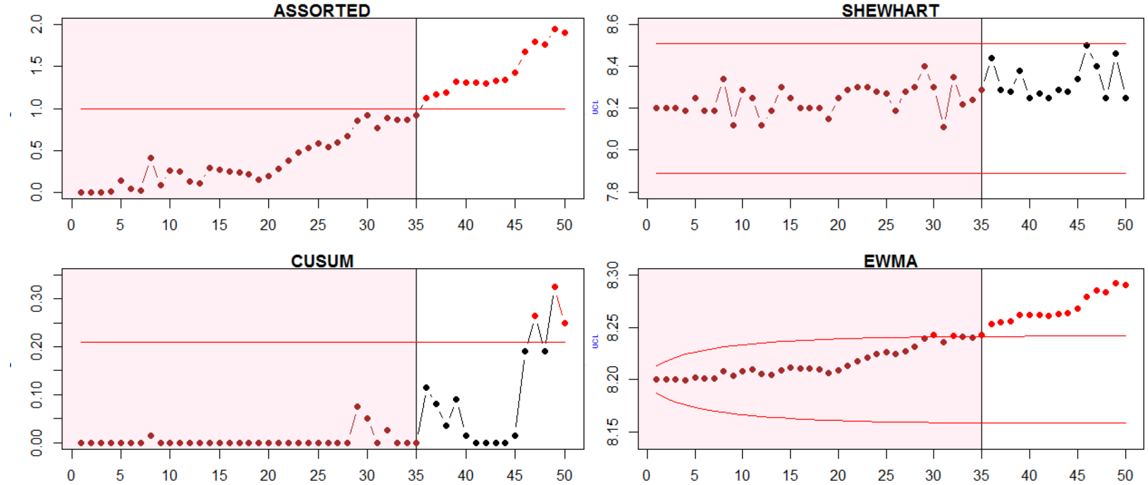

Figure 3.7: Graphical comparison at shift=1.4σ

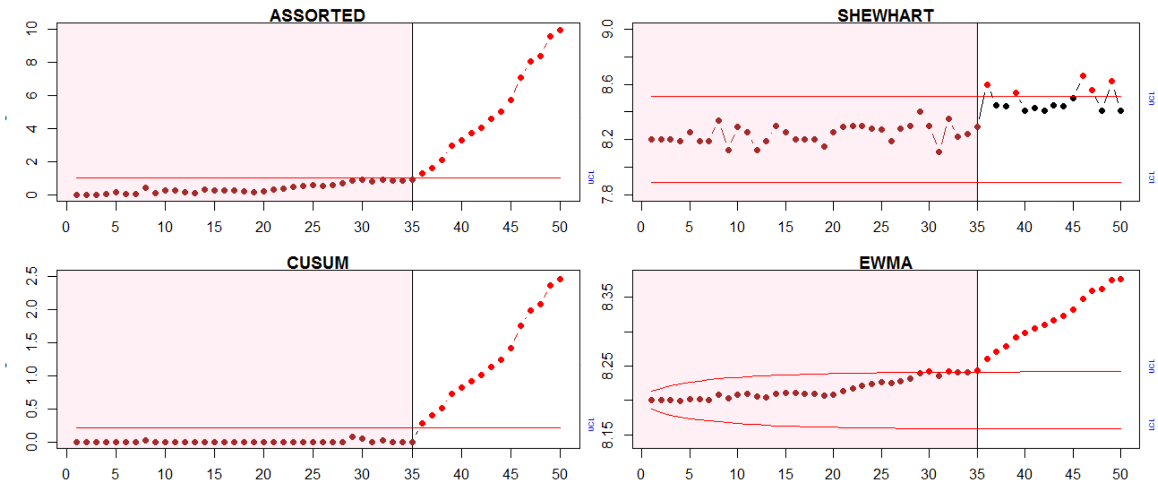

Figure 3.8: Graphical comparison at shift=3σ

3.5. Summary and Concluding Remarks

The classical Shewhart control chart is a memoryless control chart that is used to detect large shift while CUSUM and EWMA are memory charts that are used to detect a moderate and small shift in process parameters. We have proposed an assorted approach to detect a small, medium and large shift in a single control chart. Using the performance measures $ARL, SDRL, EQL, SEQL, RARL$ and PCI we have evaluated the performance of the proposed chart. We have compared the proposed assorted chart with some existing counterparts including the traditional charts (Shewhart, CUSUM and EWMA) and some modified charts (CSE and MEC).

A detailed performance analysis advocated that the proposed chart is sensitive for all types of shifts i.e. small, moderate and large. The sensitivity of the proposed assorted chart depends on k and λ. The ability of the proposed chart increases with decrease in λ at a specific choice of k and it is true for all values of k and vice versa. We have noticed that the performance of proposed $Assorted_{k,\lambda}$ control chart at $k = 1.25$ and $\lambda = 0.05$ is best in terms of different run length properties. The Assorted chart with $k = 1.25$ and $\lambda = 0.05$ is considered as benchmark chart based on minimum EQL as compared to others competing charts. The $RARL$ and for PCI of contending charts are greater than 1 which shows that the performance of proposed chart is best among Shewhart, CUSUM, EWMA, CSE and MEC charts. Further, $SEQL$ is calculated to investigate the performance of the aforementioned charts at different amounts of shifts and it also supports the proposed chart. A real application of the proposed and other competing charts is presented in ecotoxicology lab to monitor pH value. The said application also supports the findings in favor of our proposed assorted technique to monitor location parameter.

Chapter 4

AN ASSORTED CONTROL CHART FOR MONITORING DISPERSION

The monitoring of process variability is very important to get optimal output from any process. In Statistical Process Control toolkit, control charts are one of the important tools to monitor process variability. Mostly, three types of control charts are applied to observe the disturbances in the process variations. Large turbulences are detected efficiently by Shewhart R and Shewhart S control charts whereas, for small and medium instabilities, cumulative sum and exponentially weighted moving average control charts with some transformation are used. This chapter proposes an assorted approach to monitor small, medium and large disturbances in process variability. The said objective is met by using the well-known max approach. For the evaluation of the proposed assorted control chart, we have used various measures like average run length, sequential extra quadratic loss, extra quadratic loss, sequential relative average run length and relative average run length. A comparison of the assorted control chart is presented with some typical charts including the Shewhart R, Shewhart S, the EWMA of lnS^2**,** the CUSUM of lnS^2, the CUSUM R, the χ CUSUM, the P_σ CUSUM**,** and the CUSUM S charts.

4.1. Introduction

The quality of a process is determined by different parameters such as location, shape, and dispersion. The dispersion parameter is of prime importance as the stability of other parameters (like location) depends on dispersion. Generally, dispersion charts are used for two main reasons (i) if the variation in process increases, there is a possibility that more defective units will be produced (ii) if the variation in the process decreases than more units will be near the target value and hence process capability will also increase. These changes can be quickly detected by dispersion charts. These charts are also important while interpreting the results of a location chart because they assume that standard deviation remains constant.

The Shewhart range R chart and Shewhart S chart are used to monitor process variability of small subgroup sizes. Page (1963) introduced CUSUM chart to monitor process variability among different subgroups. Many authors have evaluated the performance of CUSUM and EWMA control charts that were based on the subgroup standard deviation (cf. Truph and Ncube (1987) and Ng and Case (1989)). One sided EWMA control chart based on the natural log was suggested by Crowder and Hamilton (1992) to monitor subgroup variance. Chang and Gen (1995) have proposed CUSUM chart based on the logarithmic transformation of the subgroup variance. Amin et al. (1999) proposed a MaxMin EWMA chart to monitor process variability. Acosta-Mejia et al. (1999) have discussed and compared several control charts to monitor variation in the process. Castagliola (2005) proposed a new two sided S^2 chart based on logarithmic transformation for monitoring variation in the process. A new CUSUM-S^2 to monitor the process variation was proposed by Castagliola (2009).

As we have seen the aforementioned assorted approach is very effective to monitor process location, the same may be true for other parameters. Now with the same spirit of assorted structure, we propose a new one-sided control chart called "$S^2 - Assorted_{k,\lambda}$" control chart to monitor the process variability. The aim of this chapter is to enhance the detection ability of simple linear profile parameters by a newly assorted control chart based on Max statistics. The outlined of rest of the chapter is as follows: in sub section 4.1.1- 4.1.8, a brief discussion on existing methods to monitor process dispersion. In section 4.2, proposed structure of one sided S^2 Assorted control chart. The performance and comparative analysis of proposed with the existing control charting strategies are discussed in Section 4.3. An implementation of proposed and competing charts on a real-life application is discussed in Section4.4. The concluding remarks are given in the Section 4.5.

4.1.1. The Shewhart R Chart

The Shewhart R chart (cf. Montgomery (2012)) is used to control the process variability for small sample size group (sample size less than or equal to 10). Let $R_1, R_2, \dots \dots, R_m$ be the ranges of m samples.

Since $R = W\sigma$, the standard deviation of R is defined as

$$\sigma_R = d_3\sigma,$$

where d_3 is the standard deviation of W. The 3-sgima control limit of R chart is

$$UCL = \bar{R} + 3d_3 \frac{\bar{R}}{d_2}$$

4.1.2. The Shewhart S Chart

The Shewhart S chart (cf. Montgomery (2012)) is used to monitor the standard deviation (σ) in the process. Assume that at disposition there are m prelimnary samples each of size $n,$ and let s_i be the standard deviation of ith sample. Then the average of m standard deviation is defined as

$$\bar{s} = \frac{1}{m}\sum_{i=i}^{m} s_i$$

The upper control limit of Shewhart S chart is

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1 - {c_4}^2},$$

4.1.3. The EWMA lnS^2 control chart

For the monitoring of process variance, Crowder and Hamilton (1992) were apply the EWMA scheme to the normal approximation of natural logarithmic $\left(\frac{S^2}{{\sigma_0}^2}\right)$ where ${\sigma_0}^2$ is the IC process variance. To enhance the efficiency for monitoring the process variability, they readjust the EWMA statistic to 0 if it is less than 0. The readjustment of smaller EWMA statistics to 0 definitely may improve the EWMA statistic inertia problem and increase its detection ability. They used the following EWMA statistic

$$EWMA_i \ = \ max\,\{(1 \ - \ \lambda)EWMA_{i-1} \ + \ \lambda ln(S_i^2), ln(\ \sigma_0^2)\}$$

where $EWMA_0 = \ ln(\ \sigma_0^2)$, λ is the smoothing constant, $\sigma_0^2 = 1$ and S_i^2 is the sample variance. The upper control limit of EWMA statistic is

$$UCL = L\sigma_{EWMA}$$

where L is the charting constant and $\sigma_{EWMA} = \sqrt{\frac{\lambda}{2-\lambda}\left[\frac{2}{n-1} + \frac{2}{(n-1)^2} + \frac{4}{3(n-1)^3} - \frac{16}{15(n-1)^5}\right]}$

4.1.4. The CUSUM lnS^2 control chart

Chang and Gan (1995) proposed one sided CUSUM lnS^2 control chart to monitor process variance. The CUSUM statistic used in this study is given by

$$C_i = max\{0, lnS_i^2 - k + C_{i-1}\}, i = 1,2, ...$$

where $C_0 = u$ for $0 \leq u < h$ and S_i^2 is the sample variance. The out-of-control signal is issues at the first i $C_i \geq h$.

4.1.5. The $\chi -$ CUSUM control chart

Wilson and Hilferty (1931) was proposed a CUSUM control chart based on a transformation for the monitoring of process variability. They proved that $\left(\frac{\chi_n^2}{n}\right)^{\frac{1}{3}}$ is approximately follow normal distribution with mean $1 - 2/(9n)$ and variance $2/(9n)$. Further, if the observations are independent and identical distributed $N(\mu, \sigma)$ then

$$\chi_i = \frac{\left(\frac{S_i^2}{\sigma_0^2}\right)^{\frac{1}{3}} - \left(1 - \frac{2}{9(n-1)}\right)}{\sqrt{\frac{2}{9(n-1)}}},$$

will follow an approximately standard normal distribution when $\sigma = \sigma_0$. The CUSUM statistic used in this study is

$$C_i^+ = max\{0, \chi_i - k + C_{i-1}^+\}$$

where $C_0^+ \geq 0$ and k is the reference value. The control limit of this statistic greater than h. For a specific (i.e. $ARL_0 = 200$) the value of $h = 4.28$ and $k = 0.38$.

4.1.6. The P_σ CUSUM control chart

The same approximation which was applied in $\chi -$ CUSUM, also used for P_σ CUSUM control chart. The following statistic is used in this study

$$C_i^+ = max\{0, \chi_i - k + C_{i-1}^+\}$$

The reference value for P_σ CUSUM can be obtained as

$$k = \frac{1}{2}\left[\left\{\left(\sigma_{0^+}^2/\sigma_0^2\right)^{1/3} - 1\right\}\left\{1 - \frac{2}{9(n-1)}\right\}\Big/\sqrt{\frac{2}{9(n-1)}}\right]$$

where $C_0^+ \geq 0$ and control limit of this statistic is greater than h. For a specific (i.e. $ARL_0 = 200$) the value of $h = 4.28$ and $k = 0.38$.

4.1.7. The CUSUM R control chart

Page (1963) proposed CUSUM chart on subgroup range to monitor the process variability.

The plotting statistic used in this study is $S_r = \sum_{i=1}^{r}(x_i - k)$. The quantity k is called reference value and h is the control limit.

4.1.8. The CUSUM S control chart

Tuprah and Ncube (1987) proposed CUSUM S control chart to monitor the process dispersion. The statistic used in this study is given below

$$C_i = max\{0, S_i - k + C_{i-1}\}, i = 1,2, \ldots.$$

where S_i is the sample standard deviation, $C_0 = 0$ and k is the reference value. Immediate corrective action is taken if $C_i > h$, where h is the decision interval.

4.2. The design structure of one-sided $S^2 - Assorted_{k,\lambda}$ chart

In this segment, we proposed an assorted approach to detect large, medium and small variations in the process in a single control chart namely one-sided $S^2 - Assorted_{k,\lambda}$ control chart. The proposed $S^2 - Assorted_{k,\lambda}$ is designed for the upward detection in the process variability. Assume that X is normally distributed random variable $X_{ij} \sim N(\mu_0, (\delta_1 \sigma_0))$, $i = 1,2, \dots \dots$ and $j = 1,2, \dots, n$

$\delta_1 = 1$ corresponds to an IC situation.

$\delta_1 \neq 1$ means that some variations exist in the process.

Mathematically, shift can be defined as

$$\delta_1 = \frac{\sigma_1}{\sigma_0}$$

where σ_0 is IC standard deviation.

σ_1 is OOC standard deviation.

The sample variance is defined as

$$S_i^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X}_i)^2}{n-1},$$

We define the following statistic that may be used for the detection of the large, medium and small amount of shift in the process variance.

$$V_i = \phi^{-1} \left[H \left\{ \frac{(n-1)s^2}{\sigma_0^2}, n-1 \right\} \right] \sim \text{N } (0,1)$$

where H is CDF of chi-square distribution and n denotes the sample size (n=5 is used in this study).

Let $\boldsymbol{U}$ be the statistic of the $\boldsymbol{S^2 - Assorted_{k,\lambda}}$ chart to detect large shift in the process variability. It is defined as:

$$\boldsymbol{U_{1i}} = \frac{\boldsymbol{V_i}}{\boldsymbol{C_s}} \tag{4.1}$$

where c_s is the control limit coefficient for Shewhart control chart.

To detect the moderate shift in the process variability the following statistic is used

$$\boldsymbol{U_{2i}^{+}} = \mathbf{max}[\mathbf{0}, \boldsymbol{V_i} - \boldsymbol{k} + \boldsymbol{U_{2i-1}^{+}}] / \boldsymbol{h_c} \ , \tag{4.2}$$

where h_c is the control limit coefficient for CUSUM control chart.

Similarly, the following statistic is used to detect the small shift in the process variance

$$\boldsymbol{U_{3i}} = (\boldsymbol{\lambda V_i} + (\mathbf{1} - \boldsymbol{\lambda})\boldsymbol{U_{3i-1}}) \Big/ \boldsymbol{L_e}\sqrt{\frac{\boldsymbol{\lambda}}{\mathbf{2}-\boldsymbol{\lambda}}[\mathbf{1} - (\mathbf{1} - \boldsymbol{\lambda})^{\mathbf{2}\boldsymbol{i}}]} \tag{4.3}$$

where L_e is the control limit coefficient for EWMA. The value sensitivity parameter λ is lies between 0 and 1.

The plotting statistic of proposed chart is defined as:

$$U = \max(U_{1i}, U_{2i}^{+}, U_{3i}) \tag{4.4}$$

In Eq. (4.4) U is the maximum value of three statistics as discussed above and plotted with respect to time. Because U is the function of standardized max statistics, therefore, it will always have positive value. The upper control limit of U is defined as:

$$\boldsymbol{UCL} = \boldsymbol{U} > \mathbf{1}. \tag{4.5}$$

The sensitivity of the $S^2 - Assorted$ control chart depends on the selection of (k, λ). Different combinations of sensitivity parameters (k, λ) are used in the proposed $Assorted_{k,\lambda}$ chart. To detect large, medium and small shift in process location three types of charting constants are incorporated in this study. Table 4.1 portrays the ranges of sensitivity parameters for different categories of shifts.

Table 4.1: Ranges of sensitivity parameters for different categories of shift for $S^2 - Assorted$ chart

Sensitivity Parameter	**Category of shift**		
	Small	Medium	Large
λ	0.05 to 0.15	0.16 to 0.25	0.4 to 1
k	0.1 to 0.25	0.26 to 0.5	More than 0.5

When the process is in IC state (i.e. $\delta_1 = 1$) we fix ARL_0 at a specific level such as 200. In order to fix the ARL_0 of the proposed $S^2 - Assorted_{k,\lambda}$ control chart we need to set the control limit coefficients (h_c, L_e, c_s) used with reference to Eqs. 4.1-4.3. For the said purpose, we have used several combinations of sensitivity parameters (k, λ) and worked out the triplets (h_c, L_e, c_s) for our proposed control chart. The resulting control charting constants/coefficients (h_c, L_e, c_s) are provided in Table 4.2 at some useful combination of (k, λ) for ARL_0=200. One may work out the same for other choices of ARL_0. The charting constants highlighted in bold in Table 4.2 are selected as an optimum choice because it has lowest EQL (i.e. 13.21) (cf. Table 4.3). The graphical representation of $ARL_0 = 200$ with different combination of (k, λ) is portrayed in Figure 4.1.

Table 4.2: Charting Constant at $ARL_0 = 200$

Case	k	λ	$ARL_0 = 200$		
			h_c	L_e	c_s
1	0.1	0.25	11.3000	2.7000	2.8230
2		0.40	11.3000	2.7900	2.8300
3		0.55	11.3000	2.8000	2.8300
4	0.25	0.25	6.9500	2.7000	2.8300
5		0.40	6.9500	2.7900	2.8300
6		0.55	6.9500	2.8020	2.8300
7	0.5	0.05	4.2490	2.2150	2.8350
8		0.4	4.2470	2.7900	2.8300
9		0.55	4.2100	2.7950	2.8200
10	**1**	**0.05**	**2.2298**	**2.2100**	**2.8295**
11		0.15	2.2260	2.5700	2.8100
12		0.55	2.2160	2.7400	2.7600
13	1.5	0.05	1.3950	2.5200	2.8000
14		0.15	1.3900	2.5400	2.8000
15		0.25	1.3900	2.6000	2.8100

Table 4.3: ARL and EQL of $S^2 - Assorted$ chart for Case 1 to Case 15.

Shift	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1.05	74.01	77.01	77.13	79.35	81.89	80.29	74.06	91.45	90.83	**74.31**	81.80	98.48	76.28	84.04	87.44
1.10	39.77	40.54	40.74	40.63	41.21	41.24	38.32	47.76	47.02	**37.88**	41.96	52.81	38.18	41.59	45.39
1.15	25.22	25.88	26.58	24.65	24.72	25.44	23.80	27.75	27.45	**23.15**	25.32	32.01	23.90	25.16	26.92
1.20	17.59	18.36	19.01	17.06	17.49	17.66	16.61	18.31	18.09	**16.27**	17.33	21.38	16.70	16.92	17.80
1.25	13.25	13.74	14.22	12.70	12.93	13.06	12.49	13.24	13.44	**12.49**	12.39	14.82	12.55	12.25	12.88
1.30	10.36	10.76	11.34	9.85	10.21	10.38	9.99	10.08	10.27	**9.74**	9.54	11.03	10.04	9.77	10.07
1.35	8.31	8.72	9.30	8.14	8.22	8.51	8.21	8.15	8.03	**8.15**	7.82	8.68	8.35	7.83	8.03
1.40	6.93	7.28	7.67	6.81	6.82	7.08	6.98	6.69	6.72	**6.89**	6.55	6.99	7.04	6.59	6.58
1.45	5.88	6.18	6.42	5.80	5.89	6.06	5.94	5.79	5.83	**5.85**	5.67	5.80	6.06	5.66	5.59
1.50	5.13	5.30	5.49	5.10	5.13	5.27	5.26	5.02	4.94	**5.09**	4.94	5.09	5.34	4.98	4.89
1.55	4.51	4.69	4.79	4.49	4.50	4.65	4.63	4.43	4.42	**4.58**	4.36	4.42	4.67	4.36	4.32
1.60	4.01	4.08	4.21	4.06	4.05	4.13	4.17	3.97	3.96	**4.04**	3.92	3.96	4.22	4.01	3.88
1.65	3.69	3.69	3.84	3.63	3.66	3.72	3.81	3.62	3.53	**3.64**	3.56	3.52	3.76	3.61	3.53
1.70	3.36	3.36	3.43	3.35	3.31	3.36	3.49	3.31	3.26	**3.31**	3.24	3.20	3.44	3.31	3.22
1.75	3.06	3.10	3.15	3.08	3.04	3.08	3.17	3.00	3.00	**3.06**	2.97	2.94	3.14	3.05	2.97
1.80	2.87	2.87	2.88	2.88	2.80	2.87	2.96	2.84	2.79	**2.84**	2.73	2.73	2.98	2.81	2.79
1.85	2.69	2.68	2.63	2.67	2.62	2.65	2.78	2.64	2.61	**2.63**	2.58	2.54	2.73	2.61	2.57
1.90	2.50	2.45	2.56	2.52	2.50	2.50	2.58	2.46	2.46	**2.48**	2.41	2.37	2.54	2.48	2.44
1.95	2.40	2.34	2.37	2.38	2.33	2.34	2.47	2.33	2.29	**2.32**	2.32	2.24	2.40	2.32	2.28
2.00	2.24	2.22	2.27	2.23	2.18	2.21	2.31	2.19	2.21	**2.22**	2.19	2.12	2.26	2.20	2.17
2.05	2.12	2.14	2.11	2.13	2.10	2.11	2.19	2.12	2.11	**2.11**	2.10	2.03	2.14	2.12	2.08
2.10	2.02	2.02	2.01	2.04	2.01	2.02	2.06	1.99	2.00	**1.99**	2.01	1.94	2.06	2.00	1.99
2.15	1.94	1.92	1.94	1.96	1.94	1.94	2.00	1.92	1.91	**1.92**	1.91	1.86	1.95	1.94	1.91
2.30	1.76	1.72	1.74	1.75	1.72	1.74	1.79	1.73	1.71	**1.73**	1.69	1.69	1.76	1.74	1.71
2.35	1.71	1.68	1.67	1.69	1.67	1.67	1.71	1.67	1.66	**1.65**	1.66	1.63	1.70	1.67	1.67
2.40	1.64	1.62	1.64	1.62	1.63	1.63	1.66	1.61	1.61	**1.61**	1.62	1.58	1.62	1.62	1.62
2.45	1.59	1.59	1.58	1.58	1.57	1.58	1.63	1.57	1.57	**1.58**	1.58	1.54	1.61	1.59	1.57
2.50	1.55	1.53	1.54	1.55	1.53	1.53	1.58	1.54	1.54	**1.53**	1.51	1.51	1.55	1.53	1.53
2.55	1.52	1.50	1.51	1.50	1.50	1.51	1.54	1.50	1.49	**1.50**	1.49	1.46	1.52	1.48	1.51
2.60	1.48	1.48	1.47	1.49	1.47	1.48	1.51	1.47	1.48	**1.47**	1.46	1.45	1.47	1.48	1.46
2.70	1.42	1.41	1.41	1.44	1.41	1.42	1.44	1.41	1.40	**1.40**	1.41	1.39	1.43	1.42	1.39
2.75	1.40	1.38	1.39	1.39	1.38	1.38	1.42	1.39	1.38	**1.38**	1.38	1.36	1.38	1.39	1.38
2.80	1.36	1.37	1.37	1.37	1.36	1.36	1.38	1.36	1.36	**1.37**	1.35	1.34	1.36	1.36	1.37
2.85	1.36	1.35	1.34	1.35	1.35	1.34	1.37	1.34	1.34	**1.34**	1.33	1.32	1.33	1.33	1.34
2.90	1.32	1.32	1.32	1.32	1.32	1.33	1.33	1.33	1.31	**1.32**	1.32	1.30	1.31	1.32	1.30
2.95	1.30	1.30	1.29	1.31	1.30	1.30	1.33	1.30	1.30	**1.30**	1.30	1.29	1.30	1.30	1.30
3.00	1.30	1.29	1.29	1.30	1.29	1.29	1.30	1.28	1.29	**1.27**	1.28	1.27	1.29	1.27	1.29
EQL	13.52	13.72	13.93	13.52	13.57	13.69	13.52	14.01	13.93	**13.21**	13.41	14.49	13.47	13.48	13.71

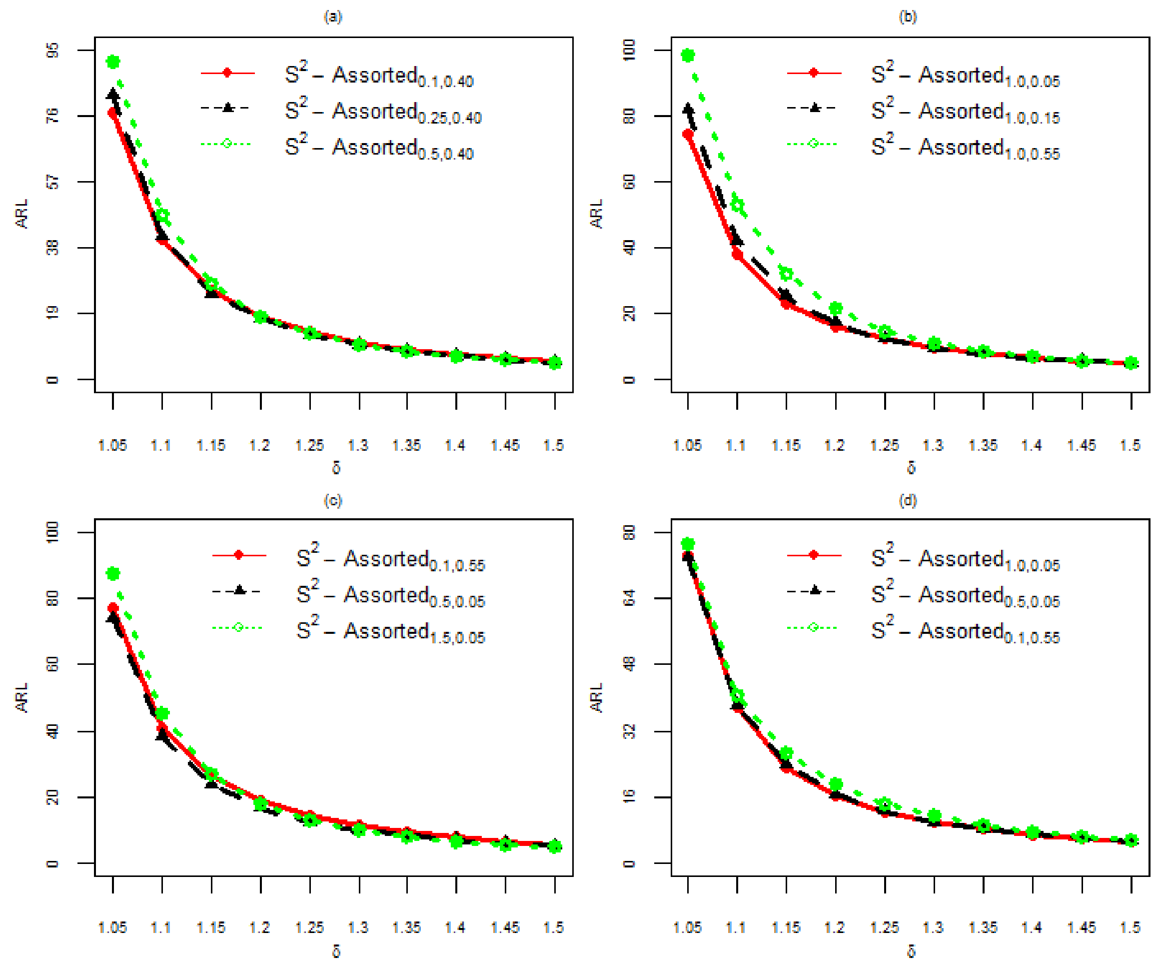

Figure 4.1: ARL comparison of $S^2 - Assorted_{k,\lambda}$ Chart for: (a) varying values of k and fixed λ at $ARL_0 = 200$; (b) varying values of λ and fixed k at $ARL_0 = 200$; (c) varying values of λ and k at $ARL_0 = 200$; (d) varying values of k and λ at $ARL_0 = 200$;

4.3. Performance evaluations and comparisons

In this section, performance evaluations and comparisons of the proposed $S^2 - Assorted_{k,\lambda}$ chart and other competing charts are discussed. The competing charts include the *Shewhart R*, and *Shewhart S*, *the EWMA of* lnS^2, *the CUSUM of* lnS^2, *the CUSUM R*, *the* χ *CUSUM*, *the* P_σ *CUSUM*, and *the CUSUM S charts*. We have used different performance measure based on run length including ARL, EQL, $SEQL$, $RARL$, and $SRARL$. In order to evaluate these measures, we have used Monte Carlo simulations (for

ARL) and numerical integration for other measures ($EQL, SEQL,$ and $RARL$). The computational algorithm for these measures is given as: (i) Generation of random sample from normal distribution with sample size (i.e. $n = 5$); (ii) computation of the sample statistics (i.e. sample variance and then V_i) ; (iii) set the control limits using the description given in Section 4.2; (iv) using steps (i)–(iii), implement the procedural steps of ARL depending on the choices of λ and k (cf. Table 4.3); (v) based on the results of step (iv) for ARL as function of δ_1: integrate the ARL values over the entire δ range by using an appropriate numerical integration technique (like Simpson or Trapezoidal) (this results into EQL value); (vi) repeat steps (iv) and (v) for all the charts; (vii) based on the results of step (vi), take the ratio of the ARL of a particular chart by the ARL of the benchmark chart (the usual one in this study), divide with the range of δ_1 values and then integrate the output over the entire δ_1 range using an appropriate numerical integration technique (like Simpson or Trapezoidal) (this results into RARL values).

4.3.1. Performance analysis of $S^2 - Assorted_{k,\lambda}$ control chart

The performance of proposed $S^2 - Assorted_{k,\lambda}$ control chart is evaluated in terms of ARL and EQL for varying combination of k, λ and δ_1. The resulting outcomes are presented in Tables 4.3 at ARL_0=200. In addition to the tabular results, we have produced some useful graphical displays based on ARLs and are provided in Figure 4.1 The results advocate the following:

- The proposed chart is sensitive for all types of shifts i.e. small, moderate and large (cf. Table 4.3).

- The sensitivity of the proposed chart increases with a decrease in λ at a specific choice of k and it is true for all values of k.
- The sensitivity of the proposed chart increases with a decrease in k at a specific choice of λ and it is true for all values of λ (cf. Table 4.3).
- The case 10 is the optimal choice because it has minimum ARL (cf. Figure 4.1 (d)) and EQL (i.e.13.21). The charting constants of this case are ($h_c = 2.2298, L_e = 2.2100\,, c_s = 2.8295$) with sensitivity parameter $k = 1.00$ and λ=0.05 at ARL_0=200.
- Four different types of charts are portrayed in Graph (a) shows the comparison of ARL values at $ARL_0 = 200$ with fixed value λ=0.40 and varying k for different amounts of shifts ranging from 1.05 to 1.5. Graph (b) shows the comparison of ARL values at $ARL_0 = 200$ with fixed value k=1.00 and varying λ for different amounts of shifts ranging from 1.05 to 1.5. In Graph (c) and (d) shows that varying k and λ are used to detect small and moderate amount of shift and vice versa. The results depicted that the $S^2 - Assorted_{1.00,0.05}$ has minimum ARL.

4.3.2. Comparative analysis

In this section, performance evaluations and comparisons of the $S^2 - Assorted_{k,\lambda}$ and some other competing charts are discussed. The competing charts include the Shewhart R, Shewhart S, the EWMA of lnS^2**,** the CUSUM of lnS^2, the CUSUM R, the χ CUSUM, the P_σ CUSUM**,** and the CUSUM S charts. We have used different performance measures based on run length including $ARL, SEQL, EQL, RARL$ and $SRARL$. In order to evaluate these measures, we have covered different OOC situations by considering varying values of shift (δ_1) given in Table 4.4

- The $ARLs$ comparison of the proposed $S^2 - Assorted_{k,\lambda}$ chart with different competing charting charts is described in Figure 4.2.
- The $S^2 - Assorted_{k,\lambda}$ chart with $k = 1.00$ and $\lambda = 0.05$ is considered as benchmark chart based on minimum EQL result (i.e. 25.56). The $EQLs$ of other competing charts are 34.65, 32.96, 28.40, 28.32, 28.14, 27.34, 27.42 and 27.33.
- Because $S^2 - Assorted$ is conceived as a benchmark chart so it $RARL$ is equal to 1. All contending charts have RARLs (1.42, 1.32, 1.21, 1.21, 1.20, 1.13,1.14 and 1.16) greater than 1, which shows the superiority of the proposed charts.
- As we have seen that proposed chart has lowest EQL. To check the sensitivity of the $S^2 - Assorted_{1,0.05}$ chart and competing charts on each amount of shift. We should determine Sequential Extra Quadratic Loss ($SEQL$). The results in Table 4.4 and in Figure 4.3 depicts that the performance of proposed chart at each amount of shift is better than all competing charts.
- The results advocate that the detection ability of $S^2 - Assorted_{1,0.05}$ chart based on $ARL, SEQL, EQL, SRARL$ and $RARL$ is better than competing charts discussed in this study.

Table 4.4: Performance comparison based on ARL, EQL, and RARL of $S^2 - Assorted$ and others competing charts

Chart		δ_1					
		1.10	1.20	1.30	1.40	1.50	2.00
Shewhart- R	ARL	68.75	30.72	16.55	10.20	6.96	2.40
	SEQL	141.68	102.69	80.50	66.37	56.66	34.65
	SRARL	1.40	1.63	1.68	1.66	1.61	1.42
Shewhart- S	ARL	65.10	28.30	15.10	9.20	6.30	2.40
	SEQL	139.43	99.59	77.44	63.52	54.04	32.96
	SRARL	1.36	1.54	1.57	1.54	1.49	1.32
EWMA lnS^2	ARL	43.00	18.10	11.00	7.60	6.00	3.20
	SEQL	126.01	82.53	62.46	51.03	43.66	28.40
	SRARL	1.06	1.09	1.10	1.11	1.11	1.21
CUSUM lnS^2	ARL	42.94	18.07	10.75	7.63	5.98	3.18
	SEQL	125.94	82.46	62.34	50.89	43.55	28.32
	SRARL	1.07	1.09	1.09	1.10	1.11	1.21
CUSUM R	ARL	40.40	17.60	10.82	7.81	6.13	3.13
	SEQL	125.34	81.22	61.42	50.27	43.12	28.14
	SRARL	1.03	1.05	1.06	1.08	1.09	1.20
CUSUM S	ARL	38.80	16.85	10.36	7.50	5.85	3.01
	SEQL	123.77	79.69	60.08	49.09	42.06	27.33
	SRARL	1.01	1.02	1.03	1.04	1.05	1.16
χ CUSUM	ARL	41.04	17.17	10.23	7.26	5.66	2.90
	SEQL	125.17	81.18	61.12	49.78	42.52	27.34
	SRARL	1.04	1.06	1.05	1.05	1.06	1.13
P_σ CUSUM	ARL	41.04	17.15	10.21	7.24	5.65	2.98
	SEQL	125.37	81.27	61.17	49.81	42.54	27.42
	SRARL	1.04	1.06	1.05	1.05	1.06	1.14
$S^2 - Assorted$	ARL	**37.87**	**16.27**	**9.74**	**6.89**	**5.09**	**2.21**
	SEQL	**122.91**	**78.77**	**59.16**	**48.11**	**40.99**	**25.56**
	SRARL	1	1	1	1	1	1

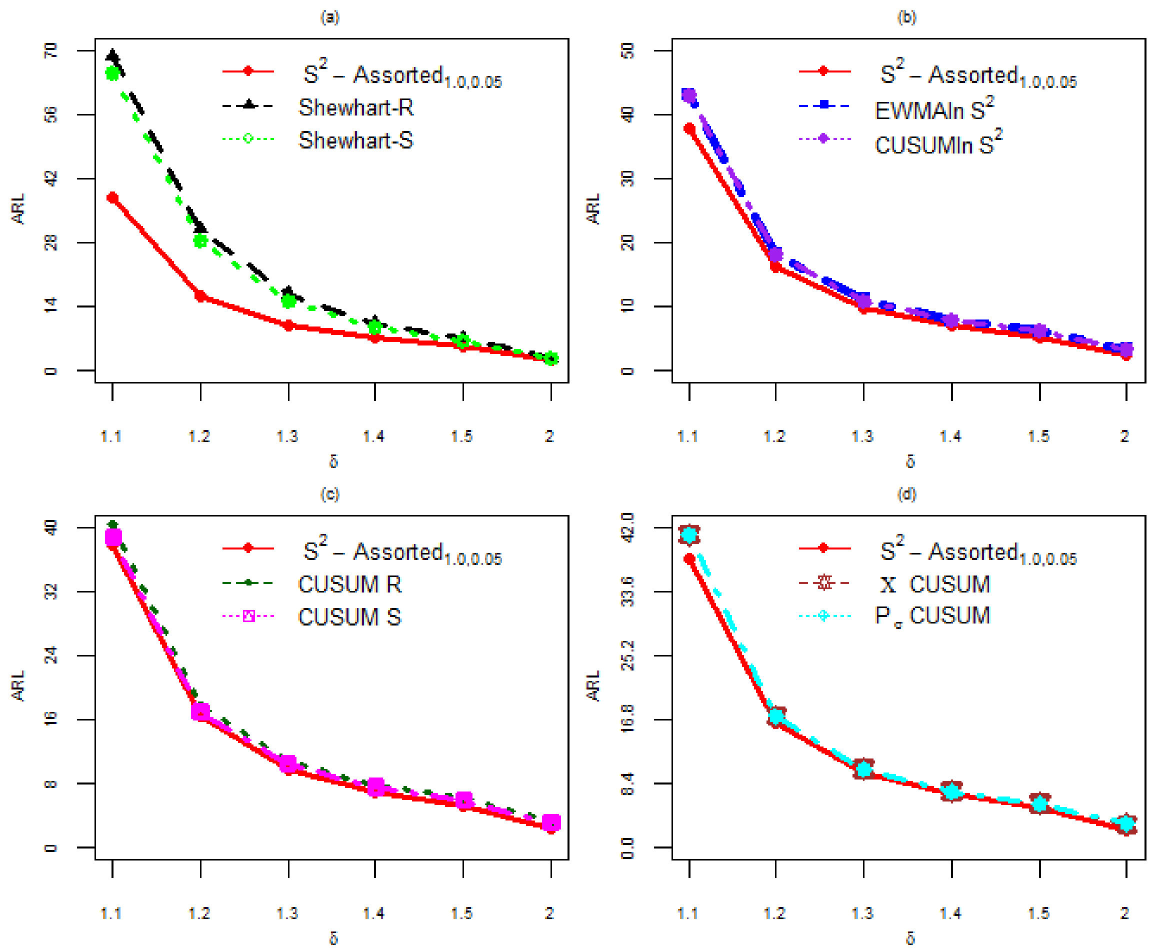

Figure 4.2: ARL comparison of $S^2 - Assorted_{\ 1.0,0.05}$ Chart with: (a) Shewhart- R and Shewhart- S; (b) EWMA lnS^2 and CUSUM lnS^2; (c) CUSUM R and CUSUM S; (d)χ CUSUM and P_σ CUSUM;

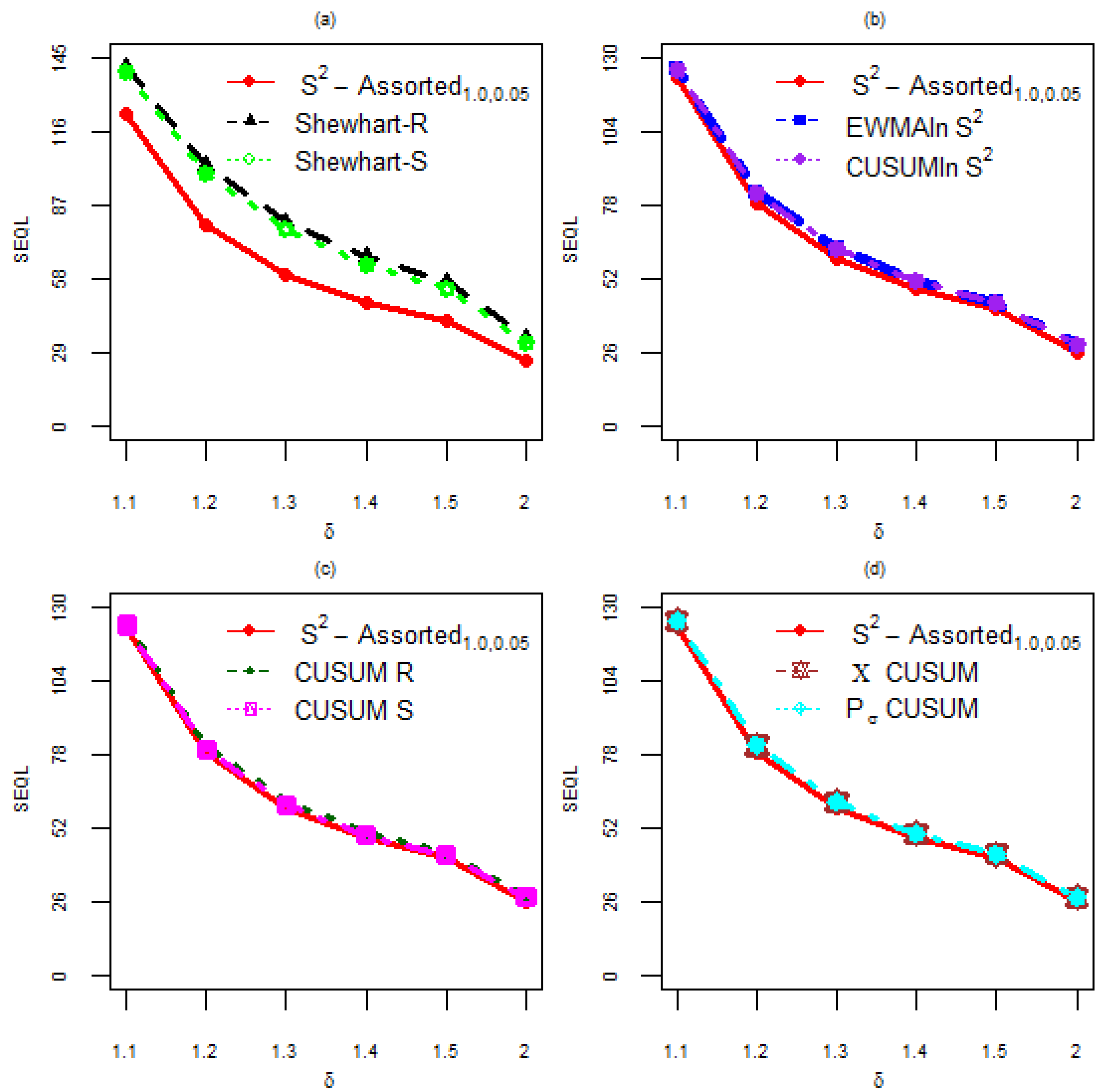

Figure 4.3: SEQL comparison of $S^2 - Assorted_{\ 1.0,0.05}$ Chart with: (a) Shewhart- R and Shewhart- S; (b) EWMA lnS^2 and CUSUM lnS^2; (c) CUSUM R and CUSUM S; (d)χ CUSUM and P_σ CUSUM;

4.4. Application: Monitoring the Flow Width Measurements

In this section, an application of our proposed chart to monitor the flow width measurements (in microns) for the Hard-Bake Process is illustrated. For the IC process, 25 samples, each of size five is taken. (cf. Montgomery (2012)).

4.4.1. Application of Proposed and other charts

We have constructed the following control charts of Hard-Bake measurements process data (cf. Montgomery (2012)) with their respective settings (such that $ARL_0 = 200$) as listed below:

- The proposed $S^2\ Assorted_{1.0,0.05}$ chart with charting constant ($h_c = 2.2298$, $L_e = 2.2100$ and $c_s = 2.8295$) and $UCL = 1$;
- The S^2 Shewhart chart with charting constant ($K = 3.84$) and upper control limits ($UCL = 0.07277$);
- The S^2 CUSUM chart with sensitivity parameter ($k = 1.0$), control limit coefficient ($h = 1.88$) and ($UCL = 1.88$);
- The S^2 EWMA chart with sensitivity parameter ($\lambda = 0.05$) and control limit coefficient ($L = 1.81$) and ($UCL = 0.2898$);.

The graphical implementation of these charts on Hard-Break measurements data is portrayed in Figure 4.4. The IC region contains first 25 and we have seen that neither a single OOC point nor a false alarm exist.

Table 4.5: Detection Summary

Control Chart	OOC detections	False Alarms
S^2Assorted	0	0
S^2Shewhart	0	0
S^2CUSUM	0	0
S^2EWMA	0	0

It is evident from the detection ability of the proposed S^2Assorted, S^2 Shewhart, S^2 CUSUM and S^2 EWMA charts have equal detection ability when process is IC.

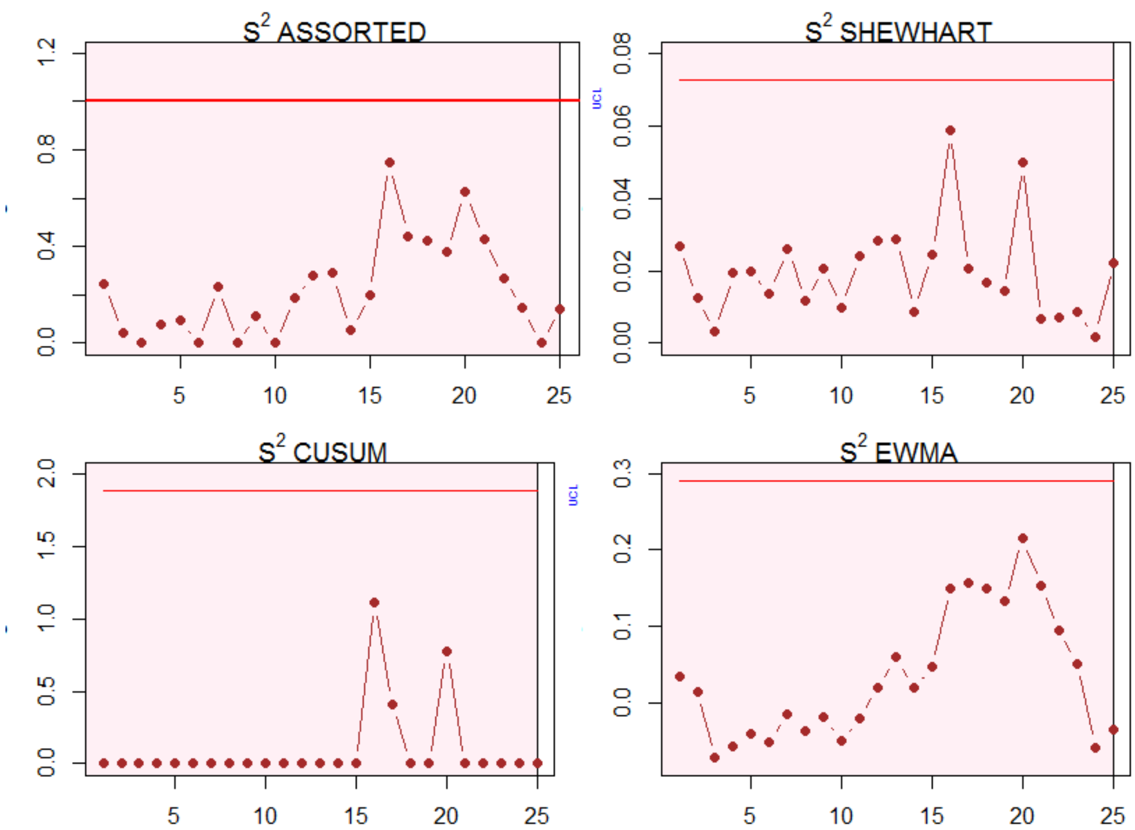

Figure 4.4: Graphical representation of S^2Assorted, S^2 Shewhart, S^2 CUSUM and S^2 EWMA charts

4.4.2. Application through data perturbation

The data perturbation technique is used for monitoring the future production of the control charts. It will use after established a set of reliable control limits. From Hard-Bake measurement process 25 new sample were collected and plotted immediately after IC region. We have perturbed the data using small (1.1σ), moderate (1.5σ), and large (2σ). amounts of distortions and applied the proposed assorted and other charts of this study. The graphical and tabular representation of the resulting charts and their detection abilities are presented in Figures 4.5 - 4.7 and Table 4.6. Form these results, it is obvious that the performance of the proposed S^2Assorted chart is better than the competing S^2 Shewhart and S^2 CUSUM charts while S^2 EWMA detect more OOC points because it has $\lambda = 0.05$ which is targeted small amount of shift (1.1σ). At medium amount of shift (1.5σ) the detection ability of S^2Assorted chart is better than its competing charts. At large shift the proposed S^2Assorted chart detect OOC point on first sample while EWMA detect first OOC point on third sample. From graphical and tabular results, we have seen that our proposed S^2Assorted chart performed well in all types of shifts.

Table 4.6: Detection summary through data perturbation

Control Chart	OOC detections	False Alarms	Shift
S^2Assorted	1	0	1.1σ
S^2Shewhart	0	0	1.1σ
S^2CUSUM	0	0	1.1σ
S^2EWMA	5	0	1.1σ
S^2Assorted	19	0	1.5σ
S^2Shewhart	2	0	1.5σ
S^2CUSUM	17	0	1.5σ
S^2EWMA	19	0	1.5σ
S^2Assorted	24	0	2σ
S^2Shewhart	13	0	2σ
S^2CUSUM	25	0	2σ
S^2EWMA	22	0	2σ

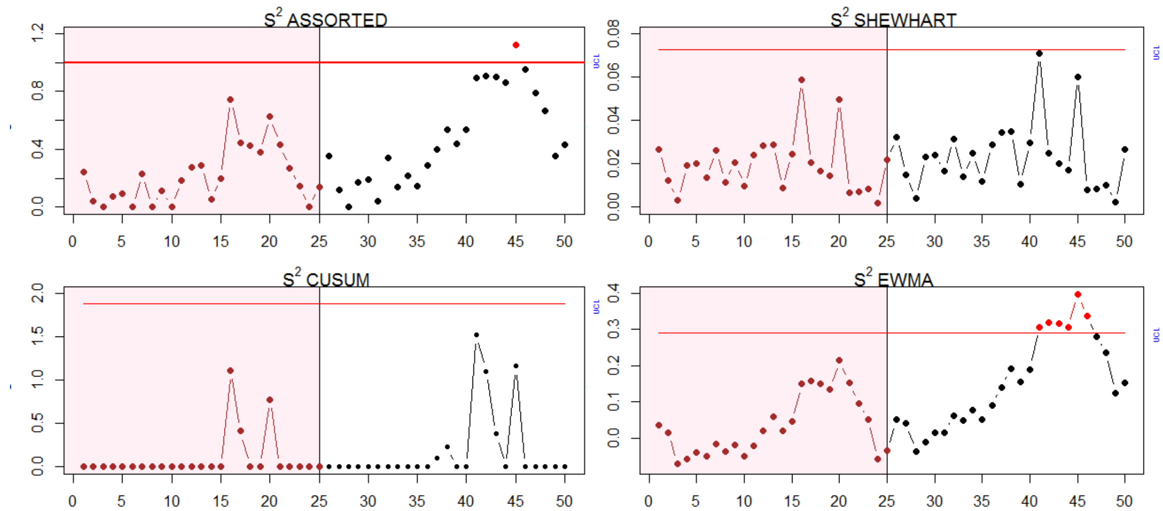

Figure 4.5: Graphical comparison at shift=1.1σ

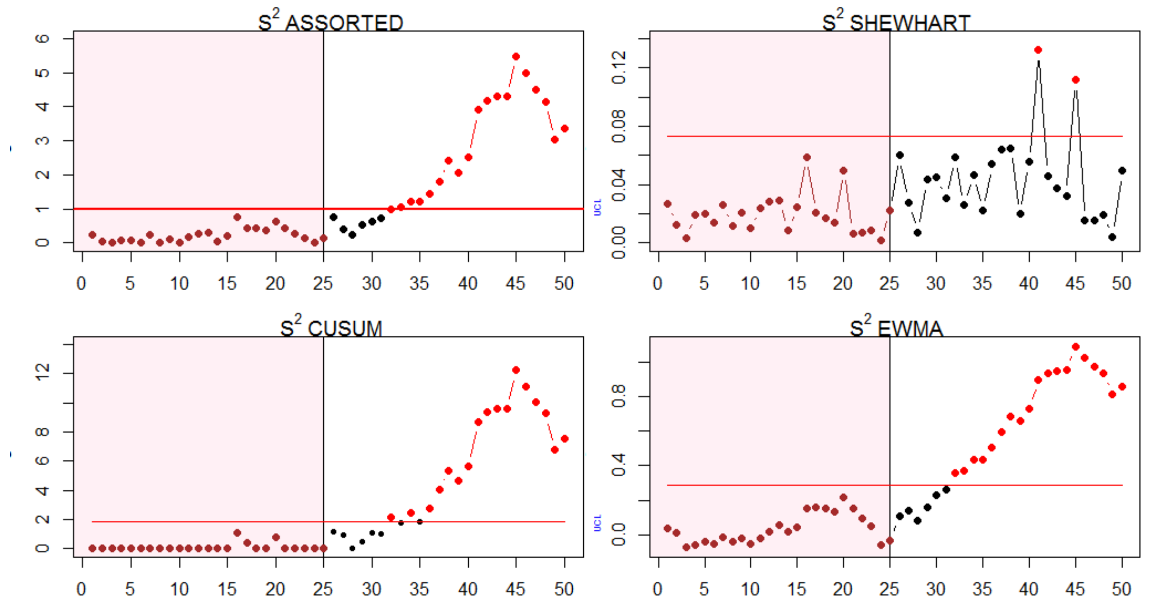

Figure 4.6: Graphical comparison at shift=1.5σ

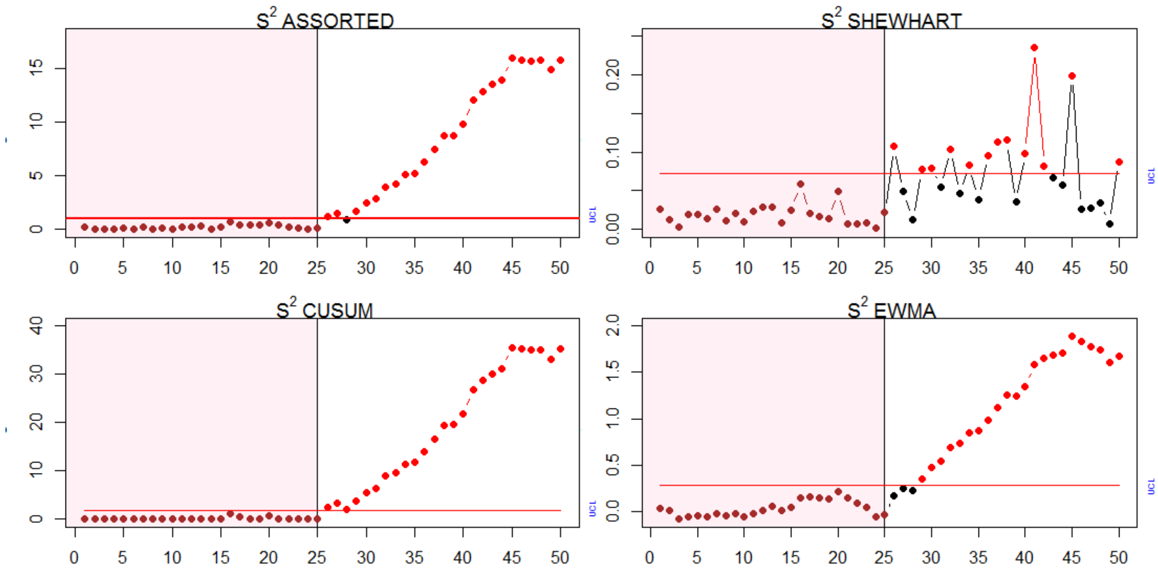

Figure 4.7: Graphical comparison at shift=2σ

4.5. Summary and Concluding Remarks

The Shewhart R and Shewhart S are the basic control charts used to detect large shift while CUSUM lnS^2 and EWMA lnS^2 charts are used to detect moderate and small shift in process variation. We have proposed an assorted approach ($S^2 - Assorted_{k,\lambda}$) to detect small, medium and large variability in the process in a single control chart. We have evaluated the performance of the proposed $S^2 - Assorted_{k,\lambda}$ chart by using the well-known measures such as ARL, EQL, $SEQL$, and $RARL$. We have compared the proposed assorted chart with some existing counterparts including the Shewhart R, Shewhart S, the EWMA of lnS^2, the CUSUM of lnS^2, the CUSUM R, the χ CUSUM, the P_σ CUSUM, and the CUSUM S charts.

A detailed performance analysis advocated that the proposed $S^2 - Assorted_{k,\lambda}$ chart is sensitive for all types of shifts i.e. small, moderate and large. The sensitivity of the proposed $S^2 - Assorted_{k,\lambda}$ chart depends on k and λ. The ability of the proposed chart increases with decrease in λ at a specific choice of k and it is true for all values of k and vice versa. We have noticed that the performance of proposed $S^2 - Assorted_{k,\lambda}$ control chart at $k = 1.0$ and $\lambda = 0.05$ is best in terms of different run length properties. The $S^2 - Assorted_{k,\lambda}$ chart with $k = 1.0$ and $\lambda = 0.05$ is considered as benchmark chart based on minimum EQL as compared to others competing charts. The $RARL$ of contending charts are greater than for 1 which shows that the performance of proposed chart is best among the Shewhart R, Shewhart S, the EWMA of lnS^2**,** the CUSUM of lnS^2, the CUSUM R, the χ CUSUM, the P_σ CUSUM**,** and the CUSUM S charts. Further, $SEQL$ is calculated to investigate the performance of the aforementioned charts at different amounts of shifts and it also supports the proposed chart.

Chapter 5

AN ASSORTED APPROACH FOR MONITORING SIMPLE LINEAR PROFILES

In manufacturing and nonmanufacturing processes, there exist a relationship between quality and quality characteristics of a product and process. This association is known as a profile. The nature of profile depends on the variable of interest which may be linear or nonlinear. So far, a lot of studies have been done to monitor linear profile parameters by different researchers. The well-known study for monitoring the linear profile parameters including intercept, slope and error variance are multivariate T^2, Shewhart_3, EWMA_3, CUSUM_3 and EWMA/R charts. This chapter proposes a new assorted control chart to monitor linear profile parameters. The performance and comparison of proposed chart with existing approaches are evaluated using some useful performance measures such as $ARL, RARL, SRARL, EQL$ and $SEQL$.

5.1. Introduction

In the modern era, new technology is systematically emerging all around us particularly in the field of consumer behavior. Buyers are becoming more technology smart with the passage of time. There is competition among manufacturers to fulfill the demands of their loyal customers. On the other hand, customers want high-quality product with cheapest price. It is an uphill task for organizations to retain customers and deliver according to their wish. There is an inverse relationship between quality of a product and variations in the product.

In SPC, control charts play a significant role to monitor the variable of interest in the process. Sometimes, the nature of a process is described in a connection between a dependent variable and at least one independent variable, which is alluded to as a profile. Several examples of linear profiles are discussed by different researchers such as Kang and Albin (2000), Woodall et al. (2004), Mahmood and Woodall (2004), Wang and Tsung (2005), Zou et al. (2007) and Riaz et al. (2017). Different strategies are produced to monitor simple linear profiles in both Phases I and II. In Phase I, one assesses the process stability and estimates its parameters on the basis of historical data. However, the purpose of Phase II examination is to identify disturbance in the process parameters at the earliest.

The aim of this chapter is to enhance the detection ability of simple linear profile parameters by a newly assorted control chart based on Max statistics. The outline of rest of the chapter is as follows: in section 5.2, a brief discussion on existing methods to monitor simple linear profile parameters is given; in section 5.3, proposed structure of assorted control chart is provided; the performance and comparative analysis of proposed with the existing control charting strategies are discussed in section 5.4; the implementation of Assorted_3 chart is demonstrated in section 5.5 and concluding remarks are given in the section 5.6.

5.2. Simple linear profile methods

In this chapter, equation (2.1) and (2.2) with its properties are used as original and transformed model respectively. The detail discussion on simple linear profiles and different methods to monitor simple linear profile parameters are described in section 2.2.

5.2.1. The CUSUM_3 chart

Saghaei et al. (2009) proposed three distinct CUSUM control charts to monitor intercept, slope and error variance separately. The proposed method is known as CUSUM_3. The three individual statistics are given as

$$\text{For intercept:}\begin{cases} CUSUM_{I(j)}^{+} = \max[0, b_{0j} - (B_0 + k_I) + CUSUM_{I(j-1)}^{+}] \\ CUSUM_{I(j)}^{-} = \max[0, (B_0 + k_I) - b_{0j} + CUSUM_{I(j-1)}^{-}] \end{cases},$$

$$\text{For slope:}\begin{cases} CUSUM_{S(j)}^{+} = \max[0, b_{1j} - (B_1 + k_S) + CUSUM_{S(j-1)}^{+}] \\ CUSUM_{S(j)}^{-} = \max[0, (B_1 + k_S) - b_{1j} + CUSUM_{S(j-1)}^{-}] \end{cases},$$

$$\text{For error variance:}\begin{cases} CUSUM_{E(j)}^{+} = \max[0, MSE_j - k_E + CUSUM_{E(j-1)}^{+}] \\ CUSUM_{E(j)}^{-} = \min[0, MSE_j - k_E + CUSUM_{E(j-1)}^{-}] \end{cases},$$

where

$$CUSUM_{I(0)}^{+} = CUSUM_{I(0)}^{-} = CUSUM_{S(0)}^{+} = CUSUM_{S(0)}^{-} = CUSUM_{E(0)}^{+} = CUSUM_{E(0)}^{-} = 0$$

k_I: intercept reference value, k_S: slope reference value and k_E: error variance reference value.

The decision interval for intercept, slope and error variance is same as of classical CUSUM.

5.3. Structure of Assorted_3 control chart and Computation of ARL

Assume that we have paired observation (X_i, Y_{ij}) for the j^th^ random sample collected with respect to time. Then the simple linear regression model with intercept (β_0) and slope (β_1) (already have discussed in Section 2.2) will have the following original model

$$Y_{ij} = \beta_0 + \beta_1 X_i + \epsilon_{ij} \tag{55.1}$$

where $i = 1,2, \dots \dots, n$ and ϵ_{ij} is random error term that follows normal distribution with mean (μ) zero and variance (σ^2). The OLS estimates of the linear regression parameters are described in Section 2.2. Usually, simple linear profile parameters are monitored in simultaneous structure which requires the assumption of independence between the parameters. To meet such assumption, coded method is an effective way which requires a transformation on X_i values (i.e. $X_i^* = X_i - \bar{X}$). The coded form of equation (5.1) is defined as:

$$\boldsymbol{Y_{ij} = B_0 + B_1 X_i^* + \epsilon_{ij}} \tag{55.2}$$

where $\boldsymbol{i = 1, 2, \dots \dots, n}$.

It is noted that Equation (5.2) is referred as transformed model, where intercept of transformed model is $B_0 = \beta_0 + \beta_1 \bar{X} + \beta\sigma\bar{X}$ and slope of transformed model is estimated by $B_1 = (\beta_1 + \beta\sigma)X_i^*$, where the shifts in the slope (β) of Equation (5.1) are considered in terms of σ. Further, in the same line, one may obtain OLS estimates of transformed model (b_{0j}, b_{1j}) and their properties.

In the Assorted_3 control chart for monitoring the intercept of transformed model, the estimate of intercept, b_{0j}, is used to compute the assorted statistics.

$$\text{Assorted statistic for intercept:} \begin{cases} T_{1(I)} = \frac{1}{c_s}\left| \frac{b_{0j} - B_0}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]}} \right| \\ T_{2(I)}^{+} = \frac{1}{h_c}\left(\frac{C_i^{+}}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]}} \right), T_{2(I)}^{-} = \frac{1}{h_c}\left(\frac{C_i^{-}}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]}} \right), \\ T_{3(I)} = \frac{1}{L_e}\left| \frac{Z_i - B_0}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]}\left[\sqrt{\frac{\lambda}{2-\lambda}\{1 - (1-\lambda)^{2i}\}}\right]} \right| \end{cases}$$

where c_s , h_c and L_e are the charting constants for the Shewhart, CUSUM and EWMA control chart. The statistics of CUSUM are $C^+_{i(I)}$ and $C^-_{i(I)}$ where k is the reference value.

$$C^+_{i(I)} = max\left[0, b_{0j} - B_0 - k\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]} + C^+_{i-1(I)}\right],$$

$$C^-_{i(I)} = max\left[0, -(b_{0j} - B_0) - k\sigma\sqrt{\left[\frac{1}{n} + \frac{\overline{X'}^2}{S_{XX}}\right]} + C^-_{i-1(I)}\right],$$

The EWMA statistic is given below and λ is sensitivity parameter

$Z_{i(I)} = \lambda b_{0j} + (1-\lambda)Z_{i-1(I)}$,

The overall statistic of assorted control chart for intercept is denoted by $T_{(I)}$ is given below

$$\boldsymbol{T_{(I)} = max[T_{1(I)}, T^+_{2(I)}, T^-_{2(I)}, T_{3(I)}]} \tag{5.3}$$

For monitoring the slope of transformed model, the estimate of slope, b_{1j}, is used to compute the assorted statistics.

Assorted statistic for slope: $$\begin{cases} T_{1(S)} = \frac{1}{C_s}\left|\frac{b_{1j}-B_1}{\sigma\sqrt{\frac{1}{S_{X'X'}}}}\right| \\ T^+_{2(S)} = \frac{1}{h_c}\left(\frac{C^+_i}{\sigma\sqrt{\frac{1}{S_{X'X'}}}}\right), T^-_{2(S)} = \frac{1}{h_c}\left(\frac{C^-_i}{\sigma\sqrt{\frac{1}{S_{X'X'}}}}\right), \\ T_{3(I)} = \frac{1}{L_e}\left|\frac{Z_i-B_1}{\sigma\sqrt{\frac{1}{S_{X'X'}}}\left[\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)^{2i}\}}\right]}\right| \end{cases}$$

,

where $$C^+_{i(S)} = max\left[0, b_{1j} - B_1 - k\sigma\sqrt{\frac{1}{S_{X'X'}}} + C^+_{i-1(S)}\right],$$

$$C_{i(S)}^{-} = max\left[0, -(b_{1j} - B_1) - k\sigma\sqrt{\frac{1}{S_{X'X'}}} + C_{i-1(S)}^{-}\right],$$

and $Z_{i(S)} = \lambda b_{1j} + (1-\lambda)Z_{i-1(S)}$.

The overall statistic of assorted control chart for slope is denoted by $T_{(S)}$ is given below

$$\boldsymbol{T}_{(\boldsymbol{S})} = \boldsymbol{max}\left[\boldsymbol{T}_{\boldsymbol{1}(\boldsymbol{S})}, \boldsymbol{T}_{\boldsymbol{2}(\boldsymbol{S})}^{+}, \boldsymbol{T}_{\boldsymbol{2}(\boldsymbol{S})}^{-}, \boldsymbol{T}_{\boldsymbol{3}(\boldsymbol{S})}\right] \tag{5.4}$$

For monitoring the error variance, the estimate of slope, ε_{ij}, is used to compute the assorted statistics.

Assorted statistic for error variance: $\begin{cases} T_{1(E)} = \frac{1}{C_s}\left(\frac{MSE}{\sigma}\right) \\ T_{2(E)}^{+} = \frac{1}{h_c}\left(\frac{C_i^{+}}{\sigma}\right), T_{2(E)}^{-} = \frac{1}{h_c}\left(\frac{C_i^{-}}{\sigma}\right) \\ T_{3(E)} = \frac{1}{L_e}\left|\frac{Z_i - B_1}{\sigma\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)^{2i}\}}}\right| \end{cases}$,

where $MSE = -0.7882 + 2.1089 \times \log_e\left(\frac{\varepsilon_{ij}^2}{n-2} + 0.6261\right)$ is the transformed mean square error, CUSUM and EWMA statistics are given below

$$C_{i(E)}^{+} = max\left[0, MSE - k\sigma + C_{i-1(E)}^{+}\right]$$

$$C_{i(E)}^{-} = max\left[0, -MSE - k\sigma + C_{i-1(E)}^{-}\right]$$

$$Z_{i(E)} = \lambda MSE + (1-\lambda)Z_{i-1(E)}$$

The overall statistic of assorted control chart for error variance is denoted by $T_{(E)}$ is given below

$$\boldsymbol{T}_{(\boldsymbol{E})} = \boldsymbol{max}\left[\boldsymbol{T}_{\boldsymbol{1}(\boldsymbol{E})}, \boldsymbol{T}_{\boldsymbol{2}(\boldsymbol{E})}^{+}, \boldsymbol{T}_{\boldsymbol{2}(\boldsymbol{E})}^{-}, \boldsymbol{T}_{\boldsymbol{3}(\boldsymbol{E})}\right] \tag{5.5}$$

Hence, the final plotting statistics for assorted control chart is given below

$$\boldsymbol{T}_{(\boldsymbol{overll})} = \boldsymbol{max}\left[\boldsymbol{T}_{(\boldsymbol{I})}, \boldsymbol{T}_{(\boldsymbol{S})}, \boldsymbol{T}_{(\boldsymbol{E})}\right] \tag{5.6}$$

In Eq. (5.4) $T_{(overll)}$ is the maximum value of three assorted statistics as discussed above and plotted with respect to time. Because $T_{(overll)}$ is the function of standardized max statistics, therefore, it will always have positive value. The upper control limit of $\boldsymbol{T}_{(\boldsymbol{overll})}$ is defined as:

$$\boldsymbol{UCL} = \boldsymbol{T}_{(\boldsymbol{overll})} > \mathbf{1}. \quad (5.5)$$

The sensitivity of the $Assorted\_3$ control chart depends on the selection of (k, λ). Different combinations of sensitivity parameters (k, λ) are used in the proposed $Assorted\_3$ chart. To detect large, medium and small shift in linear profile parameters three types of charting constants are incorporated in this study.

Table 3.1 Ranges of sensitivity parameters for different categories of shift portrays the ranges of sensitivity parameters for different categories of shifts.

Table 5.1: Ranges of sensitivity parameters for different categories of shift

Sensitivity Parameter	**Category of shift**		
	Small	Medium	Large
λ	0.03 to 0.2	0.21 to 0.5	0.51 to 1
k	0.1 to 0.75	0.76 to 1.5	more than 1.5

When the linear profile model is in IC state (i.e. $\varphi = \beta = \delta = 0$ and $\gamma = 1$) we fix overall ARL_0 at specific level such as 200. In order to fix the overall ARL_0 of the proposed $Assorted\_3$ control chart we need to determine the control limit coefficients (h_c, L_e, c_s) with reference to Eqs 5.1-5.4. For the said purpose, we have used several combinations of sensitivity parameters (k, λ) and worked out the triplets (h_c, L_e, c_s) for our proposed control chart. The resulting control charting constants/coefficients (h_c, L_e, c_s) are provided in Table 5.2 at some useful combination of (k, λ) for overall ARL_0=200. The outcomes of proposed charts are described as:

- The case 15 having sensitivity parameters (i.e. $k = 1.25$ and $\lambda = 0.05$) with charting constants $(h_c = 2.722548, L_e = 3.188036\ , c_s = 3.528191)$ is an optimal choice because it has minimum EQL (3.340).
- The ARL and EQL of all cases are portrayed in Table 5.3.
- The sensitivity of the proposed Assorted_3 chart increases with a decrease in λ at a specific choice of k and it is true for all values of k.
- The sensitivity of the proposed Assorted_3 chart increases with a decrease in k at a specific choice of λ and it is true for all values of λ (cf. Table 5.3).

Table 5.2: Charting constant for $ARL_0 = 200$

Case	k	λ	h_c	L_e	c_s
1		0.25	11.57075	3.461273	3.518018
2	0.25	0.38	11.57075	3.495503	3.518018
3		0.55	11.57075	3.511677	3.518018
4		0.25	6.421674	3.414323	3.473969
5	0.5	0.38	6.431839	3.452829	3.476706
6		0.55	6.431839	3.469855	3.476706
7		0.05	4.566855	3.182446	3.523721
8		0.13	4.439271	3.321801	3.472589
9	0.75	0.25	4.370697	3.383178	3.444825
10		0.38	4.370697	3.419843	3.444825
11		0.55	4.380133	3.441441	3.448658
12		0.05	3.446544	3.189418	3.529296
13	1	0.13	3.367723	3.338557	3.487365
14		0.25	3.281446	3.379014	3.440934
15		**0.05**	**2.722548**	**3.188036**	**3.528191**
16	**1.25**	0.13	2.65931	3.336629	3.485664
17		0.25	2.594751	3.379852	3.441717

Table 5.3: ARL and EQL of proposed Assorted_3 chart

Case	ARL										EQL
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	
1	51.842	18.303	9.394	5.560	3.702	2.733	2.147	1.765	1.501	1.310	3.861
2	52.326	19.120	10.323	6.182	4.038	2.889	2.222	1.811	1.525	1.328	4.060
3	53.236	19.829	11.172	7.053	4.624	3.203	2.373	1.875	1.554	1.341	4.350
4	68.363	17.603	8.535	5.237	3.600	2.663	2.102	1.732	1.475	1.289	3.770
5	69.446	18.053	8.872	5.512	3.801	2.797	2.175	1.776	1.498	1.308	3.900
6	70.259	18.382	9.165	5.824	4.055	2.984	2.291	1.831	1.532	1.320	4.050
7	49.246	15.633	7.533	4.517	3.161	2.385	1.891	1.581	1.362	1.212	3.360
8	68.679	17.428	8.062	4.867	3.369	2.533	2.011	1.669	1.431	1.258	3.610
9	87.020	21.154	8.774	5.143	3.525	2.623	2.075	1.709	1.459	1.280	3.840
10	91.783	22.193	9.076	5.304	3.658	2.717	2.143	1.746	1.481	1.297	3.960
11	94.632	22.807	9.323	5.467	3.789	2.817	2.209	1.796	1.510	1.309	4.070
12	49.701	15.701	7.532	4.523	3.157	2.384	1.905	1.590	1.367	1.213	3.370
13	70.171	17.706	8.140	4.918	3.401	2.550	2.023	1.676	1.434	1.263	3.640
14	93.492	23.822	9.353	5.241	3.517	2.625	2.071	1.705	1.454	1.279	3.930
15	**48.717**	**14.696**	**7.337**	**4.511**	**3.154**	**2.383**	**1.905**	**1.585**	**1.367**	**1.215**	**3.340**
16	69.939	17.721	8.147	4.904	3.394	2.545	2.022	1.677	1.434	1.264	3.640
17	95.000	24.040	9.447	5.287	3.547	2.626	2.072	1.710	1.458	1.280	3.950

5.4. Performance evaluations and comparisons

In this section, the performance of assorted control chart is evaluated and compared with existing control charts in the literature. An IC linear profile model $(i.e. Y_{ij} = 3 + 2X_i + \epsilon_{ij})$ discussed by Kang and Albin (2000) utilized in this study with fixed sample size $(n = 4)$ and $(X_i = 2, 4, 6, 8)$. The performance of proposed chart is compared with Shewhart_3, T^2, CUSUM_3, EWMA/R, EWMA_3 and PM_3. Further, the transformed model given in Equation (2.2) with $B_0 = 13 + 5(\beta\sigma)$ and $B_1 = (2 + \beta\sigma)X_i^*$ is defined as $Y_{ij} = B_0 + B_1 X_i^* + \epsilon_{ij}$. where $X_i^* = -3, -1, 1, 3$. To calculate ARL values by Monte Carlo simulation with 10^6 iterations have done in R-language. For performance evaluations, we have considered four types of shifts in different parameters as listed below in Table 5.4.

Table 5.4: Four types of Shifts introduced in proposed study

Type of Shifts	Notation	Amounts of Shifts
In intercept of transformed model	B_0 to $B_0 + \varphi\sigma$	$\varphi = 0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8,2.0$
In slope of original model	β_1 to $\beta_1 + \beta\sigma$	$\beta = 0.025,0.05,0.075,0.1,0.125,0.15,0.175,0.2,0.225,0.25$
In slope of transformed model	B_1 to $B_1 + \delta\sigma$	$\delta = 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0$
In error variance of original model	σ to $\gamma\sigma$	$\gamma = 1.2,1.4,1.6,1.8,2,2.2,2.4,2.6,2.8,3.0$

5.4.1. Shift in intercept of transformed model

The OOC ARL (ARL_1) values of proposed and its counterpart charts are shown in Table 5.4 for shift in intercept **($\boldsymbol{B_0}$ to $\boldsymbol{B_0 + \varphi\sigma}$)**. The performance of PM_3 chart is best to detect small shifts (i.e. φ=0.2 and φ=0.4) while the detection ability of proposed chart at moderate and large shift is most proficient. The minimum ARL_1 values at different amounts of shifts in intercept is highlighted in bold numbers. CUSUM_3 has the minimum value (**3.10**) of ARL_1 at φ=1.0. The aim of this study is to see the overall best detection ability of a chart. The chart has minimum EQL value is considered as a best chart. So, based on EQL proposed chart has minimum EQL value (**3.11**) and shown graphical in Figure 5.1. Hence, proposed chart is considered as a benchmark chart. Also, the $RARL$ value of proposed chart is equal to 1. As all other charts have $RARL > 1$ which shows that the detection ability of the Assorted_3 chart is superior from all other charts.

Table 5.5 Performance comparison based on ARL under shift in intercept ($\boldsymbol{B_0}$ to $\boldsymbol{B_0 + \varphi\sigma}$)

Chart		φ									
		0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
Shewhart_3	ARL	151.40	77.90	33.80	15.50	7.70	4.30	2.70	1.90	1.50	1.20
	SEQL	3.03	6.14	8.20	8.91	8.89	8.57	8.16	7.78	7.45	7.19
	SRARL	2.05	3.13	3.74	3.81	3.64	3.39	3.13	2.90	2.71	2.54
T^2	ARL	137.70	63.50	28.00	13.20	6.90	4.00	2.60	1.80	1.50	1.20
	SEQL	2.75	5.29	6.90	7.49	7.53	7.33	7.06	6.78	6.55	6.38
	SRARL	1.91	2.75	3.19	3.24	3.10	2.90	2.71	2.53	2.37	2.24
CUSUM_3	ARL	72.10	20.30	8.20	4.60	**3.10**	2.40	1.91	1.60	1.40	1.30
	SEQL	1.44	2.25	2.54	2.64	2.72	2.81	2.92	3.04	3.19	3.35
	SRARL	1.24	1.34	1.31	1.25	1.20	1.16	1.14	1.12	1.11	1.11
EWMA/R	ARL	66.50	17.70	8.40	5.40	3.90	3.20	2.70	2.30	2.10	1.90
	SEQL	1.33	2.04	2.33	2.56	2.78	3.03	3.30	3.59	3.90	4.23
	SRARL	1.18	1.23	1.22	1.20	1.21	1.22	1.24	1.27	1.29	1.32
EWMA_3	ARL	59.10	16.20	7.90	5.10	3.80	3.10	2.60	2.30	2.10	1.90
	SEQL	1.18	1.83	2.13	2.36	2.59	2.85	3.13	3.42	3.75	4.09
	SRARL	1.11	1.13	1.12	1.12	1.13	1.15	1.17	1.20	1.24	1.27
PM_3	ARL	**30.34**	**12.53**	7.36	5.09	3.86	3.09	2.58	2.22	1.95	1.75
	SEQL	0.61	1.11	1.51	1.87	2.21	2.53	2.85	3.17	3.48	3.80
	SRARL	0.81	0.77	0.83	0.89	0.94	1.00	1.04	1.09	1.12	1.15
Assorted_3	ARL	48.70	14.68	**7.31**	**4.52**	3.16	**2.39**	**1.90**	**1.58**	**1.37**	**1.21**
	SEQL	0.97	1.56	1.87	2.09	2.28	2.45	2.61	2.77	2.93	**3.11**
	SRARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	**1.00**

5.4.2. Shift in slope of original model

The results of OOC ARL (ARL_1), $SEQL$ and $SRARL$ are portrayed in Table 5.5 under shift in slope of original model. The results of mentioned performance measure are quite interested. From small to moderate shift in slope (i.e. $\beta = 0.025\ to\ 0.10$) the detection ability of PM_3 chart is better than all other charts while when shift is increased from 0.10 to onwards up to 0.25 the Assorted_3 charts performed well. The Shewhart_3 and Hoteling T^2 charts have worst detection ability. Again, the Assorted_3 chart has the minimum EQL

value (see Figure 5.1) so it is considered as a benchmark chart. The $RARL$ values show that approximately PM_3 and Assorted_3 charts have equal detection ability because both have same $RARL$=1

Table 5.6 Performance comparison based on ARL under shift in slope ($\boldsymbol{\beta_1}$ to $\boldsymbol{\beta_1 + \beta\sigma}$)

Chart		β									
		0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
Shewhart_3	ARL	178.300	125.000	79.200	46.700	27.900	17.100	10.900	7.100	5.000	3.600
	SEQL	0.056	0.134	0.216	0.276	0.311	0.327	0.332	0.329	0.322	0.314
	SRARL	1.481	2.236	3.010	3.509	3.724	3.753	3.676	3.542	3.387	3.228
T^2	ARL	166.000	105.600	60.700	34.500	20.100	12.200	7.800	5.200	3.700	2.700
	SEQL	0.052	0.118	0.179	0.220	0.242	0.251	0.252	0.248	0.243	0.236
	SRARL	1.414	2.012	2.558	2.861	2.958	2.931	2.840	2.721	2.594	2.469
EWMA/R	ARL	119.000	43.900	19.800	11.300	7.700	5.800	4.700	3.900	3.400	3.000
	SEQL	0.037	0.065	0.080	0.088	0.094	0.099	0.104	0.110	0.116	0.123
	SRARL	1.414	1.258	1.286	1.272	1.254	1.243	1.240	1.243	1.250	1.260
EWMA_3	ARL	101.600	36.500	17.000	10.300	7.200	5.500	4.500	3.800	3.300	2.900
	SEQL	0.032	0.055	0.068	0.075	0.082	0.088	0.094	0.100	0.107	0.114
	SRARL	1.059	1.103	1.113	1.107	1.104	1.106	1.115	1.128	1.143	1.160
CUSUM_3	ARL	85.700	37.800	19.000	11.100	7.200	5.000	3.900	3.100	2.600	2.300
	SEQL	0.027	0.050	0.067	0.078	0.084	0.089	0.093	0.097	0.100	0.104
	SRARL	0.972	1.025	1.090	1.116	1.120	1.111	1.100	1.090	1.082	1.078
PM_3	ARL	**48.200**	**21.430**	**12.940**	**9.070**	6.830	5.430	4.490	3.820	3.300	2.920
	SEQL	0.015	0.028	0.040	0.050	0.060	0.069	0.078	0.086	0.094	0.102
	SRARL	0.765	0.687	0.712	0.757	0.805	0.852	0.895	0.936	0.973	1.007
Assorted_3	ARL	90.841	31.112	15.533	9.509	**6.552**	**4.834**	**3.771**	**3.055**	**2.552**	**2.186**
	SEQL	0.028	0.048	0.059	0.067	0.074	0.079	0.084	0.088	0.092	**0.096**
	SRARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	**1.000**

5.4.3. Shift in error variance of original model

Table 5.6 presented the results of ARL_1, $SEQL$ and $SRARL$ under shift in error variance. The detection ability of Assorted_3 chart at small and moderate shift is significantly better among all other mentioned charts. Although, at large amount of shift the performance of CUSUM_3 is slightly better than Assorted_3 chart. The minimum ARL_1 is highlighted in bold numbers. Assorted_3 chart has minimum EQL value so it is considered as benchmark chart. The detection ability of CUSUM_3 chart better than all other charts except Assorted_3 chart. The graphical presentation of $SEQL$ is portrayed in Figure 5.1.

Table 5.7 Performance comparison based on ARL under shift in error variance (σ to $\gamma\sigma$)

Chart		γ									
		1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.00
Shewhart_3	ARL	40.10	13.50	6.50	4.00	2.80	2.20	1.80	1.60	1.50	1.40
	SEQL	128.87	85.49	64.17	51.83	43.88	38.39	34.41	31.43	29.19	27.49
	SRARL	1.25	1.38	1.40	1.39	1.36	1.32	1.29	1.26	1.24	1.22
T^2	ARL	39.60	14.90	7.90	5.10	3.80	3.00	2.50	2.20	2.00	1.80
	SEQL	128.51	85.81	65.45	53.68	46.12	40.91	37.13	34.32	32.20	30.57
	SRARL	1.24	1.41	1.50	1.54	1.56	1.56	1.55	1.54	1.53	1.52
EWMA/R	ARL	34.30	12.00	6.10	3.90	2.90	2.30	1.90	1.70	1.50	1.40
	SEQL	124.70	80.58	60.24	48.71	41.39	36.39	32.77	30.07	28.02	26.44
	SRARL	1.14	1.23	1.26	1.26	1.25	1.25	1.24	1.22	1.21	1.19
EWMA_3	ARL	33.50	12.70	7.20	5.10	3.90	3.20	2.80	2.50	2.30	2.10
	SEQL	124.12	80.34	60.78	49.96	43.18	38.57	35.32	32.97	31.25	29.97
	SRARL	1.12	1.23	1.32	1.38	1.44	1.47	1.50	1.52	1.53	1.54
CUSUM_3	ARL	31.20	9.40	4.80	3.20	2.40	2.00	1.70	**1.50**	1.40	**1.30**
	SEQL	122.46	77.07	56.49	45.20	38.16	33.40	30.02	27.51	25.63	24.20
	SRARL	1.08	1.10	1.08	1.07	1.06	1.05	1.05	1.04	1.03	1.03
PM_3	ARL	**24.81**	9.82	5.61	3.90	2.99	2.42	2.06	1.81	1.65	1.50
	SEQL	117.86	72.67	54.05	43.91	37.59	33.30	30.22	27.95	26.25	24.94
	SRARL	0.96	0.99	1.04	1.09	1.12	1.14	1.16	1.16	1.17	1.17
Assorted_3	ARL	26.90	**8.84**	**4.70**	**3.11**	**2.37**	**1.95**	**1.69**	1.52	**1.39**	1.31
	SEQL	119.37	73.69	54.02	43.28	36.58	32.06	28.85	26.49	24.72	**23.38**
	SRARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	**1.00**

5.4.4. Shift in slope of transformed model

In this study, the positive amount of shift is used in the slope of transformed model. Kang and Albin (2000) used the same amount of shift in negative numbers. The OOC ARL values and other performance measures such as EQL and $RARL$ of proposed and others chart are given in Table 5.7. Although, the PM_3 has minimum ARL_1 (**10.83**) at $\delta = 0.2$. But, for all the amount of shifts the performance of Assorted_3 chart is best. The EQL of proposed chart is equal to 0.65 which is minimum from all other charts (see Figure 5.1). So, it is considered as benchmark chart. The $RARL$ of all other charts is not less than 1 which shows the inferiority of these charts.

Table 5.8 Shift in slope transformed model

Chart		δ								
		0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
EWMA/R	ARL	76.7	33.7	15.3	7.5	4.2	2.6	1.8	1.4	1.2
	SEQL	1.53	2.04	2.21	2.2	2.12	2.02	1.92	1.83	1.76
	SRARL	3.67	4.43	4.53	4.31	4	3.69	3.41	3.17	2.96
Shewhart_3	ARL	64.29	25.29	11.08	5.42	3.06	2.03	1.49	1.24	1.10
	SEQL	1.28	1.66	1.75	1.72	1.63	1.55	1.47	1.42	1.38
	SRARL	3.15	3.68	3.65	3.42	3.15	2.89	2.67	2.50	2.34
T^2	ARL	52.2	21.2	9.6	4.9	2.9	1.9	1.5	1.2	1.1
	SEQL	1.04	1.36	1.45	1.44	1.39	1.33	1.28	1.25	1.23
	SRARL	2.66	3.07	3.06	2.89	2.68	2.48	2.31	2.17	2.05
EWMA_3	ARL	13.1	6.6	4.4	3.3	2.7	2.3	2.1	1.9	1.7
	SEQL	0.26	0.36	0.43	0.5	0.57	0.63	0.71	0.79	0.87
	SRARL	1.04	1.06	1.07	1.1	1.13	1.17	1.2	1.24	1.27
CUSUM_3	ARL	12.4	7.9	5.8	4.6	3.8	3.3	2.9	2.6	2.4
	SEQL	0.26	0.37	0.48	0.59	0.7	0.82	0.93	1.05	1.17
	SRARL	1.01	1.06	1.15	1.25	1.34	1.43	1.51	1.58	1.64
PM_3	ARL	**10.83**	6.4	4.43	3.36	2.7	2.27	1.96	1.73	1.53
	SEQL	0.22	0.31	0.4	0.47	0.54	0.61	0.69	0.76	0.83
	SRARL	0.95	0.96	0.99	1.04	1.09	1.13	1.16	1.19	1.21
Assorted_3	ARL	12.1	**6.05**	**3.76**	**2.66**	**2.02**	**1.63**	**1.38**	**1.21**	**1.1**
	SEQL	0.24	0.33	0.39	0.44	0.48	0.52	0.56	0.6	**0.65**
	SRARL	1	1	1	1	1	1	1	1	**1**

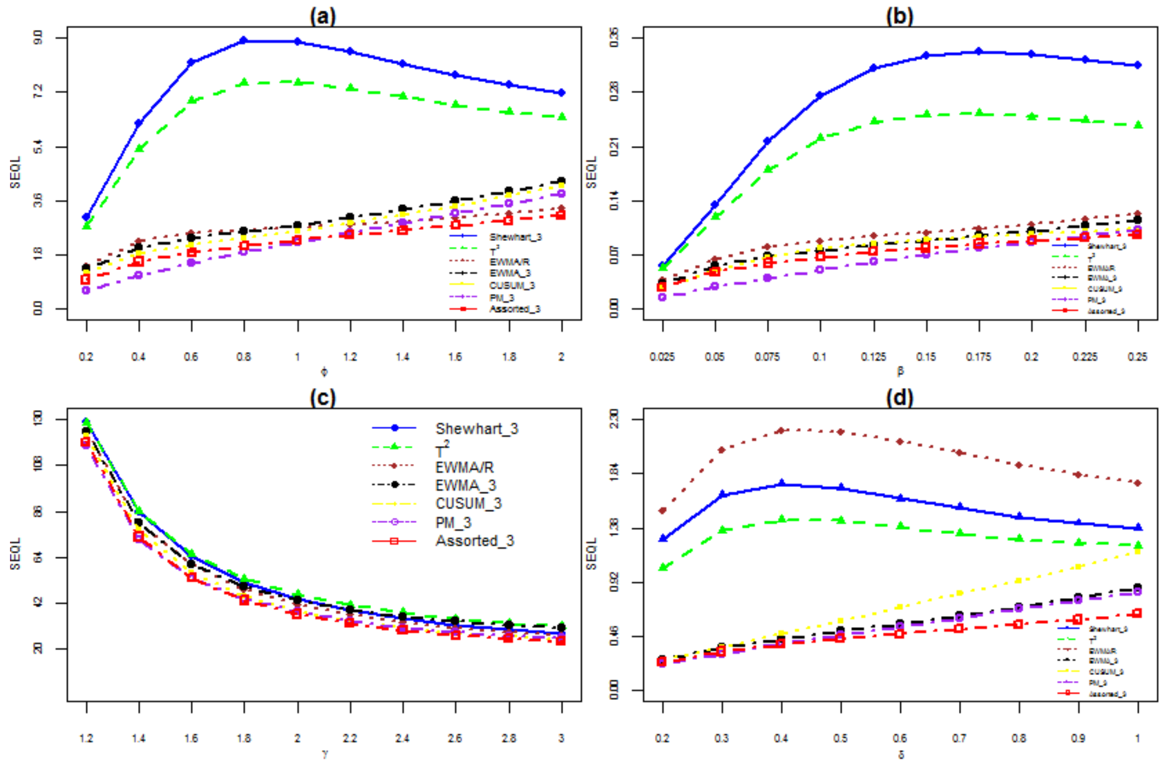

Figure 5.1: SEQL of four types of shifts

5.5. Application of proposed chart on Thermal Management of Diamond-Copper composites

Thermal management of high-performance electronic devices is the key to their efficient and continued working. The average size of electronic devices is decreasing day by day with the decreasing size of the transistor. Each electronic process produces waste heat in the component. In overall, thermal density (heat produced per unit area) of electronic devices is increasing with the level of miniaturization. The waste heat produced must be carried away from the component without disturbing the electronic operation of the electronic component and the device.

One solution to this problem is to have a high thermal conductivity substrate for the device. Its high thermal conductivity will enable fast extraction of waste heat from the device. So, high thermal conductivity metals like copper, silver or aluminum (for copper ~ 400 W/mK) seems a good solution for the substrate material. However, the thermal expansion behavior of the substrate should be comparable to that of the parent device. So that any rise or decrease in temperature of the system produce comparable thermal expansion, otherwise large difference in thermal expansion will produce stresses at the interface and may lead to delamination. The thermal expansion coefficient of electronic devices has low value (for silicon ~ 5 m/mK) while that of the mentioned metals is very high in comparison (for copper ~ 16 m/mK). Therefore, metals alone are not compatible with most of the electronic devices. Hence for the substrate, a high thermal conductivity and a low thermal expansion are desired.

Ceramic materials are generally very low in their thermal expansion coefficient plus they have a wide range of thermal conductivity available. For example, diamond has a very high thermal conductivity of 2000 W/mK with a thermal expansion coefficient of only ~ 2 m/mK. A composite of diamond particles and copper metal may produce a combination of high thermal conductivity and a thermal expansion coefficient comparable to that of electronic devices. Mostly, the researchers have adopted powder metallurgy route for making the diamond-copper composites.

Briefly, in powder metallurgy, powders are consolidated with a combination of mechanical pressure and high temperature. The treatment of powders at high temperature is called as sintering. Conventional sintering is the type of powder metallurgy route in which the powders are mixed, cold compacted at room temperature by pressing the mixed powders in a die and then sintered at high temperature in the desired environment. The effective thermal conductivity and thermal expansion coefficients are mainly affected by the volume fraction of diamond and the densification of the composite. Densification is the ratio of actual density to the theoretical density of the composite sample. In other words, densification is an inverse measure of the porosity in the composite sample. A densification closer to 100% means the lower volume of porosity and improved effective thermal properties.

In this study, diamond-copper composites were produced by conventional sintering route. The pressure of cold compaction (PCC) is an important parameter which affects the final properties of the composite. The composite samples were sintered following the same sintering cycle. The volume fraction of diamond particles was 10% and the sintering was carried out at 900 °C for 2 hours in a vacuum environment. The only independent variable was PCC. The composite samples were cold compacted at five different levels of pressure i.e. 425, 450, 475, 500 and 525 MPa. The dependent variable was the densification of the diamond-copper composite. The densification was measured 24 times by an apparatus based on Archimedes' principle.

5.5.1. Implementation of Assorted_3 chart

In this study, we have considered the independent variable (PCC) ($X = 425,450,475,500$ and 525) and densification (Y) as a dependent variable. The implementation of Assorted_3 chart needs the following steps.

Step 1: We have total 120 sample values (24 profiles). The IC regression model based on 24 profiles is

$$Y = 86.899 + 0.00972X + \varepsilon. \text{ (original model)}$$

Step 2: Further, to gain the assumption of independence between parameters, we transformed the X values in X' by using $X' = X - \bar{X}$,

$$X' = -50, -25, 0, 25, 50$$

$$Y = 91.518 + 0.00972X' + \varepsilon. \text{ (transformed model)}$$

Step 3: The selected charting constants for proposed chart is given below

$$\text{For Assorted\_3:}\begin{cases} k = 1.25\ , \lambda = 0.05 \\ h_c = 2.7225 \\ L_e = 3.1880 \\ c_s = 3.5281 \end{cases},$$

Step 4: We have plotted our proposed statistics for each parameter (i.e. Intercept, slope and error variance) against their upper control limit (i.e. $UCL = 1$). The proposed $Assorted\_3$ chart for IC process (for 24 profiles) is portrayed in Figure 5.2.

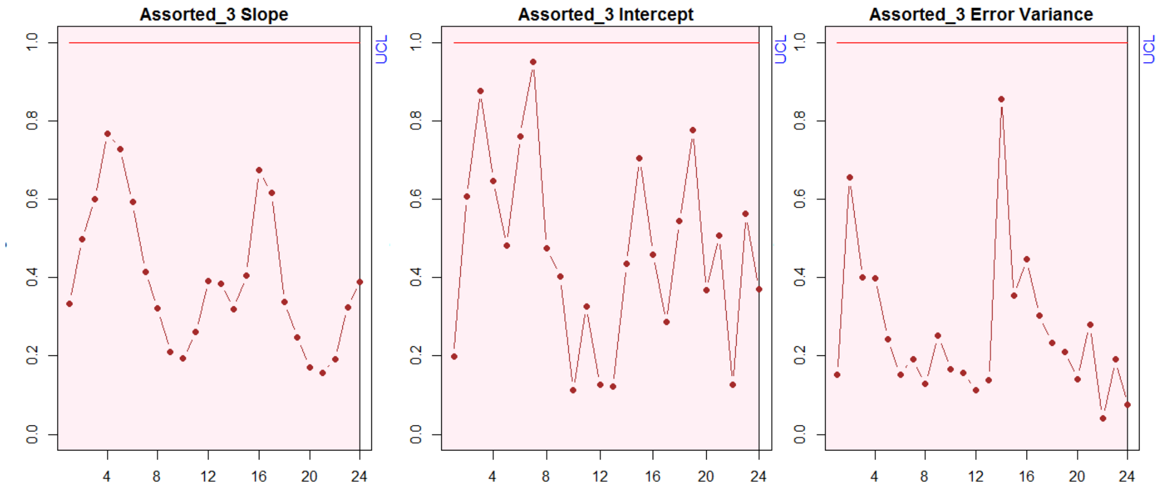

Figure 5.2: The performance of Assorted_3 chart for IC process

Step 5: When a turbulence occurs in the data set due to increase the PCC after 16th sample profile. The densification of the diamond-copper composite is affected. We can see the performance of our proposed Assorted_3 chart versus CUSUM_3, EWMA_3 and Shewhart_3 chart in Figure 5.3-5.6 respectively. The summary of detection ability of these charts is presented in Table 5.9. The following results reveal that our proposed Assorted_3 chart have better detection ability to monitor simple linear profile parameters.

It is evident from the detection ability of the charts that CUSUM_3 and EWMA_3 appeared as the least efficient chart. It is to be noted that Shewhart_3 chart has detected one OOC point but at the cost of four false alarms as may be seen in the summary Table 5.9. The proposed Assorted_3 chart perfumed the best in detecting OOC points. The reason for this superiority order relates to the amount of shift in the real process. As the aim of proposed chart is to detect small, medium and large shift in the process so it takes edge over other charts in detecting OOC scenarios.

For these OOC signals, we investigated the process in search of the assignable cause(s) and found that there are some technical issues (voltage ampere) occur in PCC. Due to this reason densification is changed and some OOC points detected by our proposed chart.

Table 5.9: Detection Summary

Control Charts	Slope		Intercept		Error Variance	
	OOC Detection	False Alarms	OOC Detection	False Alarms	OOC Detection	False Alarms
Assorted_3	0	0	5	0	0	0
Shewhart_3	1	4	0	0	0	0
CUSUM_3	0	0	0	0	0	0
EWMA_3	0	0	0	0	0	0

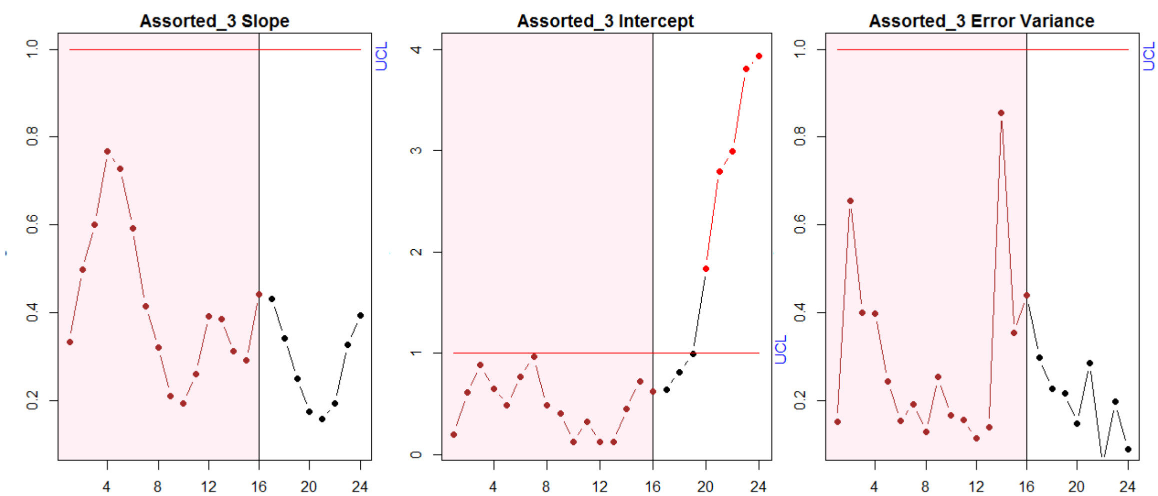

Figure 5.3: The performance of Assorted_3 chart for OOC process

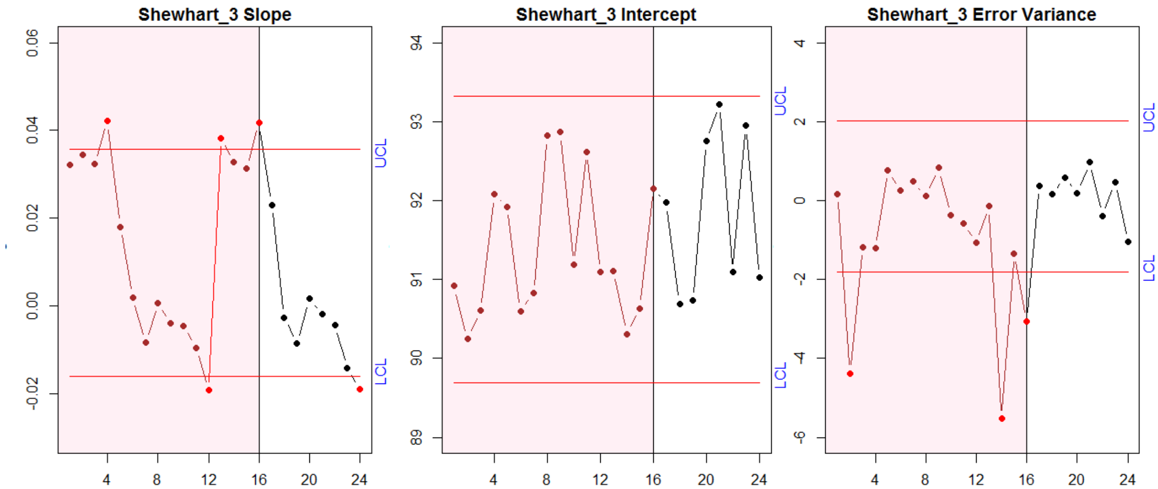

Figure 5.4: The performance of Shewhart_3 Chart for OOC process

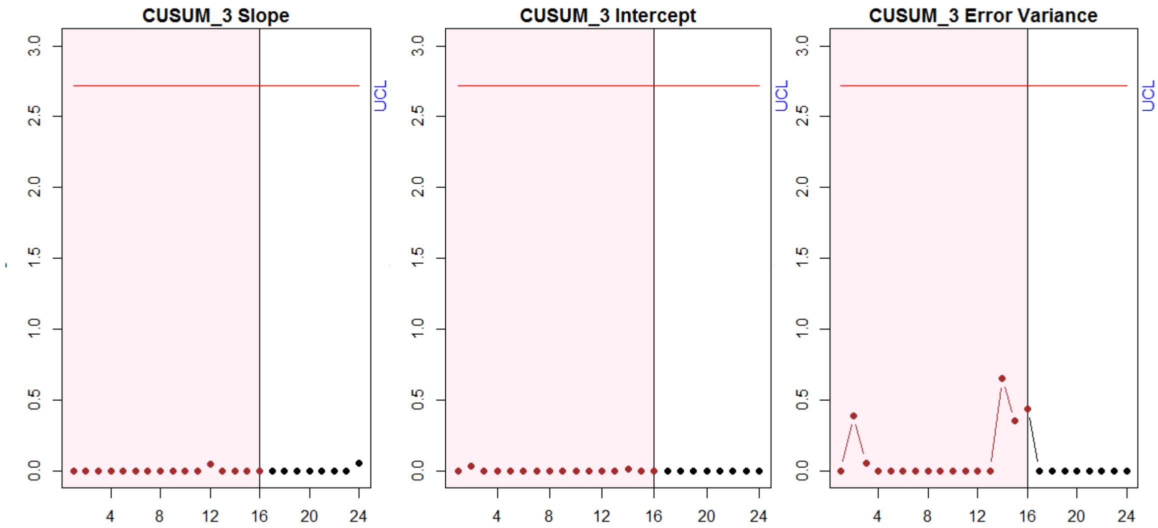

Figure 5.5: The performance of CUSUM_3 Chart for OOC process

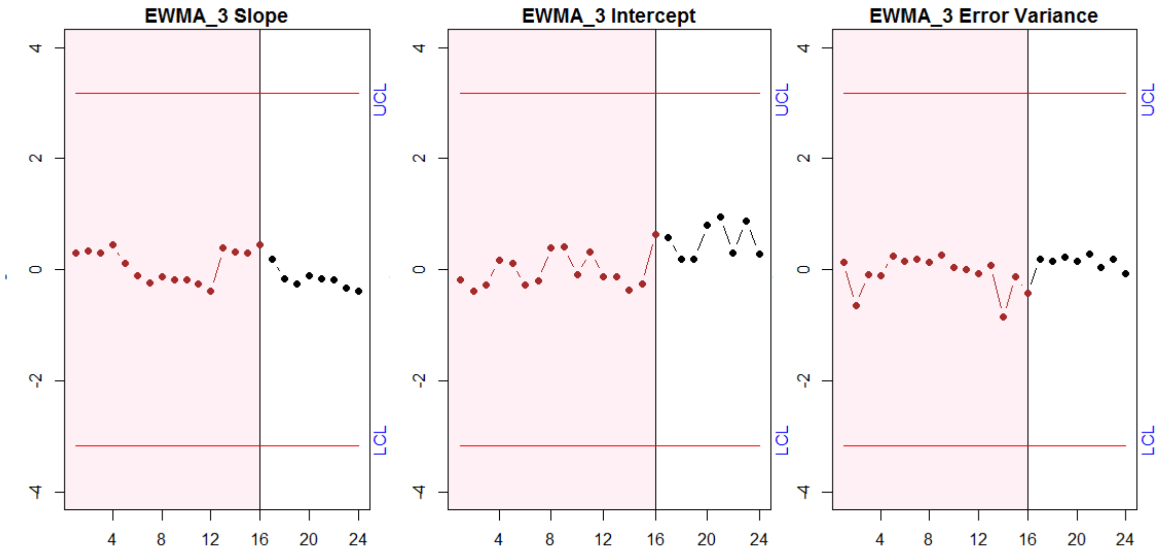

Figure 5.6: The performance of EWMA_3 Chart for OOC process

The diamond-copper composite is portrayed in Figure 5.7;(a) at 500 PCC while when PCC is increased we can observe, a blister on the diamond-copper composite in Figure 5.7 (b). Further, this blister is investigated by scanning electron microscopy (SEM) (cf. Figure 5.7 (c and d))

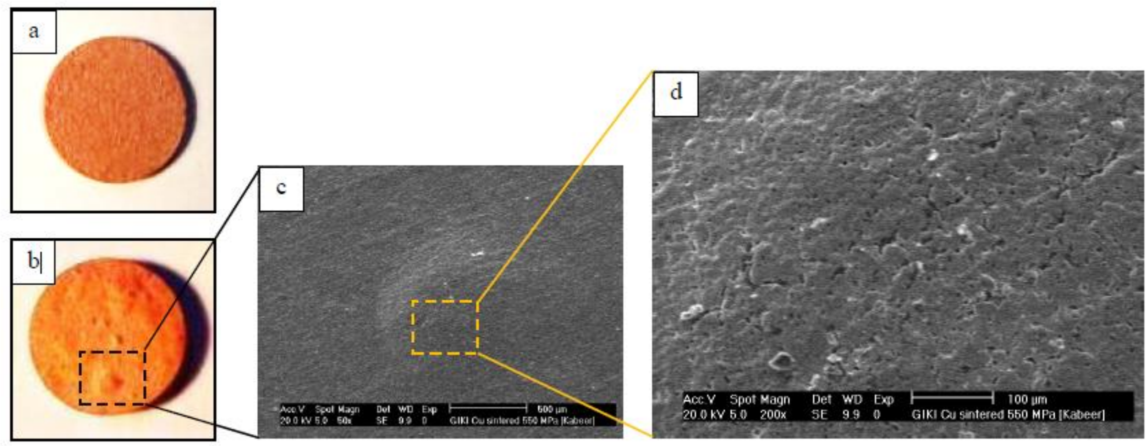

Figure 5.7: Impact of increasing PCC

5.6. Conclusions

The monitoring of simple linear profile parameters is an emerging field in SPC. Several approaches have been developed to monitor the simple linear profile parameters such as EWMA/R, Hotelling T^2, EWMA_3, CUSUM_3 and PM_3. We have proposed an Assorted_3 approach to monitor simple linear profile parameters. Using the performance measures such as $ARL, EQL, SEQL, RARL$ and $SRARL$, we have evaluated and compared the performance of the proposed Assorted_3 chart with some existing counterparts charts aforementioned.

A detailed performance analysis urged that the proposed Assorted_3 chart is sensitive for monitoring linear profile parameters including intercept, slope and error variance at different amounts of shifts. We have found that the performance of proposed Assorted_3 control chart at $k = 1.25$ and $\lambda = 0.05$ is best in terms of different run length properties. The $RARL$ of contending charts are greater than 1 which shows that the performance of proposed Assorted_3 chart is best among EWMA/R, Hotelling T^2, EWMA_3, CUSUM_3 and PM_3 charts. Further, $SEQL$ is calculated to investigate the performance of the said charts at different amounts of shifts and it also supports the proposed Assorted_3 chart. A real application of the Assorted_3 chart is also presented to affirms the findings in favor of our proposed Assorted_3 chart to monitor simple linear profile parameters.

Chapter 6

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The monitoring of study variable which is linearly related to another ancillary variable is known as simple linear profiles. A number of control charting structures based on memory or memoryless type structures are available for the simultaneous monitoring of linear profile parameters. The most famous simple linear profile monitoring approaches are (EWMA/R, the Hotelling T^2, EWMA_3, Shewhart_3, CUSUM_3). Recently, a memory type structure based on progressive mean was introduced as an efficient chart as compared to existing memory type structures (EWMA) and CUSUM).

So, in Chapter 2 with the inspiration of progressive mean structure, a control chart is developed to monitor simple linear profile parameters under progressive mean setup. The performance of proposed chart is evaluated on the basis of well-known performance measures such as $ARL, EQL, SEQL, RARL$ and $SRARL$. The results depict that the detection ability of PM_3 chart is best among the existing approaches to monitor simple linear profile parameters such as slope, intercept and error variance.

There are two approaches of control charts exists in literature recognized as classical (Shewhart, CUSUM, and EWMA) and modified control charts. To enhance the detection ability of classical approaches at different amount of shift (i.e. small, medium and large) several modifications are available in the literature. In Chapter 3, a new control chart technique is proposed to detect large, medium and small turbulences in the process location based on max approach in a sole control chart. The performance of $Assorted_{k,\lambda}$ control chart is evaluated by $ARL, RARL, EQL$ and PCI . The comparison of proposed $Assorted_{k,\lambda}$

chart versus classical and modified control charts are also portrayed. The comparative analysis concludes that the $Assorted_{k,\lambda}$ chart outperform well to detect turbulences as compared to classical and some modified approaches.

In Chapter 4, a new control chart technique is proposed to detect large, medium and small turbulences in the process dispersion based on max approach in a sole control chart. The performance of $S^2 - Assorted_{k,\lambda}$ control chart is evaluated by $ARL, SEQL, EQL,\ RARL,$ and SRARL. The comparison of proposed $S^2 - Assorted_{k,\lambda}$ chart versus competing charts are presented and results depicted that the performance of proposed $S^2 - Assorted_{k,\lambda}$ chart is better than all others contending chats.

In Chapter 5, the idea of assorted control chart is employed to monitor simple linear profile parameters. So, each parameter of linear profile parameter is shaped in max approach and finally the overall assorted statistic plotted against its control limits. There were existing 12 individual statistics existing. The proposed control chart than compare with existing approaches to monitor simple linear profile parameters and results conclude that the detection ability of assorted control chart to monitor simple linear profile parameter is outclass from all other existing approaches such as Shewhart_3, Hotelling T^2, EWMA/R, EWMA_3, CUSUM_3 and PM_3.

6.1. Future recommendations

- ✓ In future, one may extend simultaneous monitoring of linear profiles under progressive setup study to multiple linear/nonlinear regression models.
- ✓ The $Assorted_{k,\lambda}$ control chart can also be applied for the monitoring of process location and dispersion in multivariate control charts.

- ✓ Joint monitoring of linear profile.
- ✓ The study in Chapter 5 based on univariate and linear profile monitoring, one may extend this idea into multivariate and nonlinear profile monitoring.

Appendix A

Properties of linear profile parameters under progressive setup

As discussed in Section 2.2.1.5 that the progressive mean is the cumulative mean of the variable so the progressive mean of the Intercept term may be obtained by,

$$PM_{I(i)} = \frac{\sum_{j=1}^{i} b_{0j}}{i} = \frac{b_{01} + b_{02} + \cdots + b_{0i}}{i}$$

by taking expectation

$$E(PM_{I(i)}) = E\left(\frac{b_{01} + b_{02} + \cdots + b_{0i}}{i}\right) = \frac{E(b_{01}) + E(b_{02}) + \cdots + E(b_{0i})}{i}$$

as we know that $E(b_{01}) = B_0$, so

$$E(PM_{I(i)}) = \frac{B_0 + B_0 + \cdots + B_0}{i}$$

$$E(PM_{I(i)}) = \frac{i(B_0)}{i} = B_0$$

Similarly, one may also get the mean of slope and mean square error under progressive mean setup such as,

$$E(PM_{S(i)}) = B_1; \qquad E(PM_{E(i)}) = \sigma^2$$

The variance of the slope parameter under progressive mean setup is obtain by

$$Var(PM_{S(i)}) = Var\left(\frac{\sum_{j=1}^{i} b_{1j}}{i}\right)$$

$$Var(PM_{S(i)}) = \frac{1}{i^2}\left[\sum_{j=1}^{i} Var(b_{1j})\right] + \sum_{j \neq i} Cov(b_{1j}, b_{1i})$$

As we know that

$$Var\left(b_{1j}\right) = \frac{\sigma^2}{S_{XX}} \text{ and } Cov\left(b_{1j}, b_{1i}\right) = 0$$

Then the variance of the slope parameter under progressive mean structure is

$$Var(PM_{S(i)}) = \frac{1}{i^2}\left\{\frac{\sigma^2}{S_{XX}} + \frac{\sigma^2}{S_{XX}} + \cdots + \frac{\sigma^2}{S_{XX}}\right\}$$

$$Var(PM_{S(i)}) = \frac{1}{i^2}\left\{i\left(\frac{\sigma^2}{S_{XX}}\right)\right\} = \frac{\sigma^2}{i}\left(\frac{1}{S_{XX}}\right)$$

Similarly, the variance of the progressive intercept term may be obtained as

$$Var(PM_{I(i)}) = \frac{\sigma^2}{i}\left(\frac{1}{n}\right)$$

As we know that the mean square error (MSE) is an unbiased estimator of error variance which has following properties.

$$MSE \sim \frac{\sigma^2}{n-2}\chi^2{}_{(n-2)},$$

$$E(MSE) = \frac{\sigma^2}{n-2}E\left[\chi^2{}_{(n-2)}\right] = \frac{\sigma^2}{n-2}(n-2) = \sigma^2,$$

$$Var(MSE) = \left(\frac{\sigma^2}{n-2}\right)^2 Var\left[\chi^2{}_{(n-2)}\right] = \left(\frac{\sigma^2}{n-2}\right)^2 2(n-2) = \frac{2}{n-2}\sigma^4,$$

So, the variance of $PM_{E(i)}$ is obtained as follows

$$Var(PM_{E(i)}) = \frac{2}{n-2}\sigma^4$$

Appendix B

The properties of pH characteristic value of data are given below. The probability plot in Figure B-1 portrayed that the dataset satisfied the normality assumption. Also, the value of Anderson-Darling (AD) test statistic for normality is equal to 0.589 with P-value=0.119.

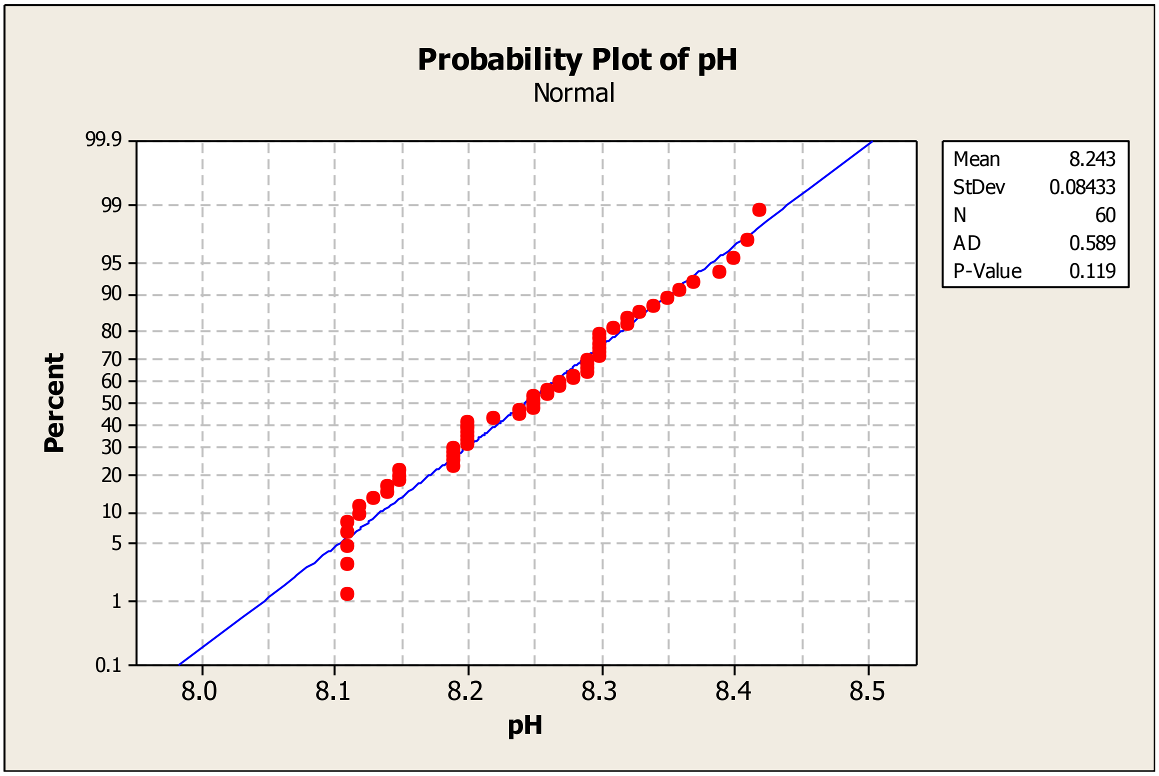

Figure B-6.1: Probability plot of pH value of water

The descriptive summary of given data is described in Figure B-2. The histogram suggested that the distribution of the data is bell shaped (i.e. normal).

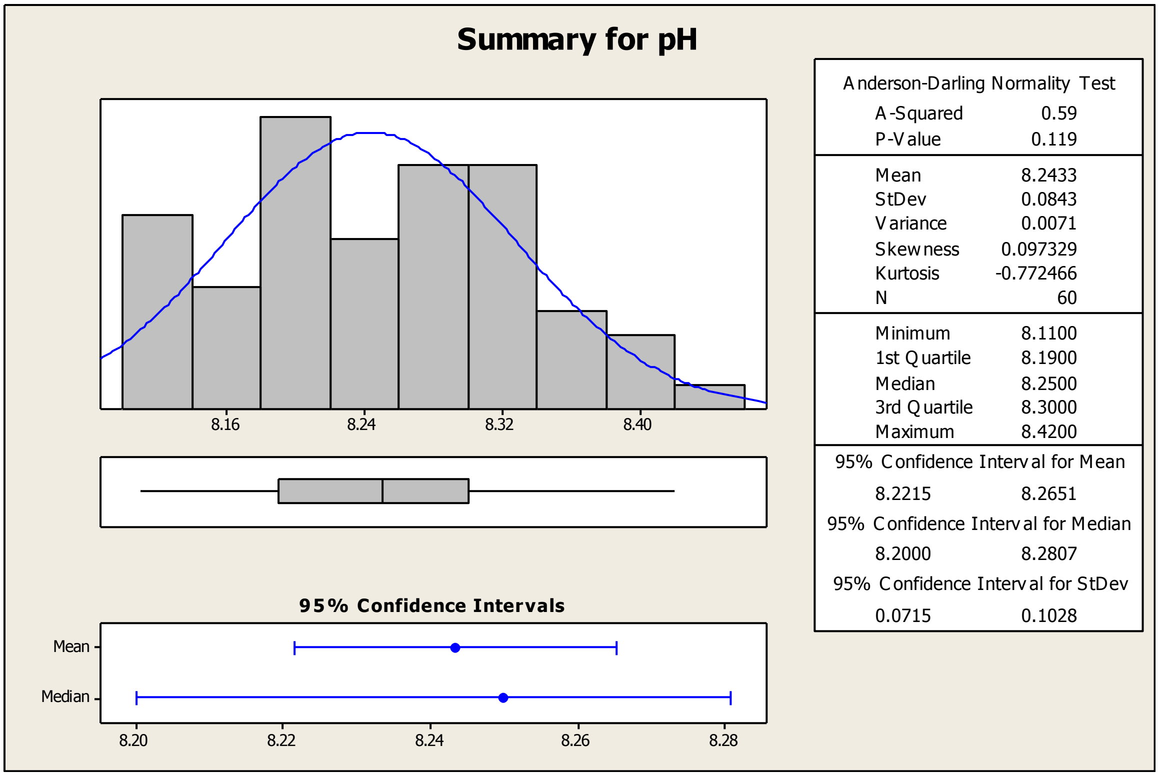

Figure B-6.2: Graphical and descriptive summary of pH value of water

Appendix C

The probability plot of diamond-copper composite densification is portrayed in Figure C-1. The value of Anderson-Darling (AD) test statistic for normality is equal to 0.336 with P-value=0.504 which shows that the given dataset satisfies the normality assumption.

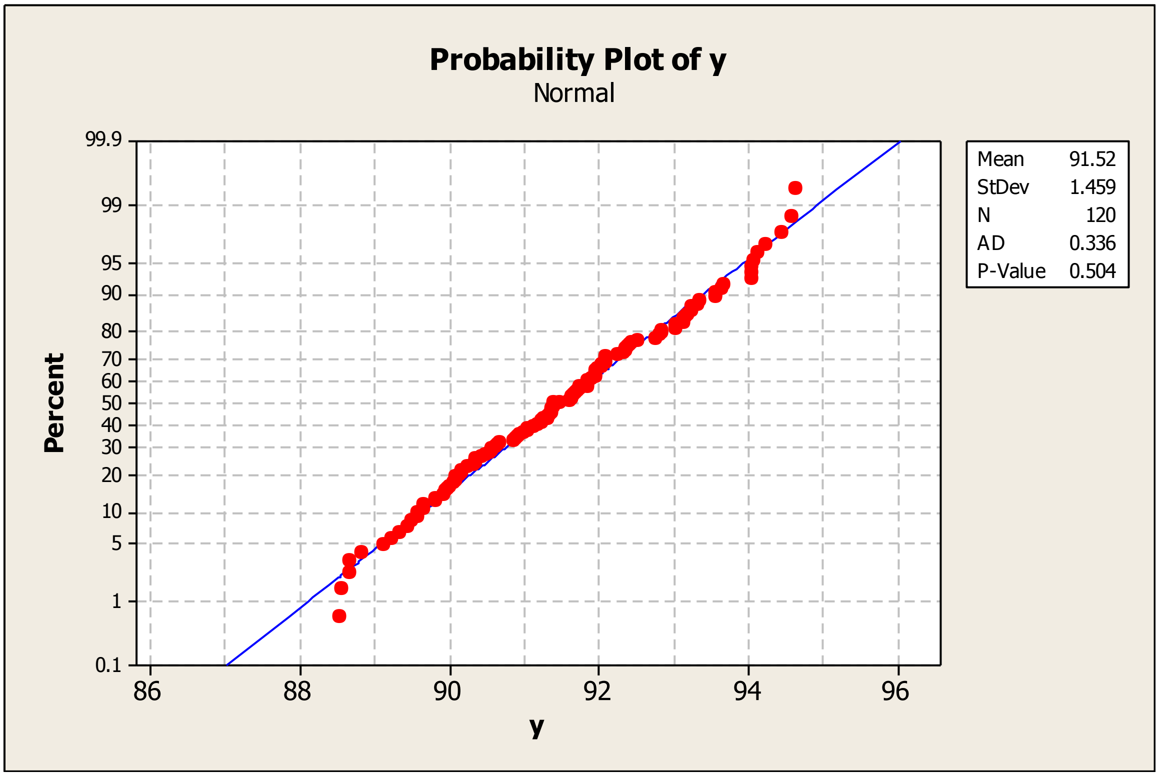

Figure C-1: Diamond-copper composite densification

The descriptive summary of given data is described in Figure C-2. The histogram suggested that the distribution of the data is bell shaped (i.e. normal).

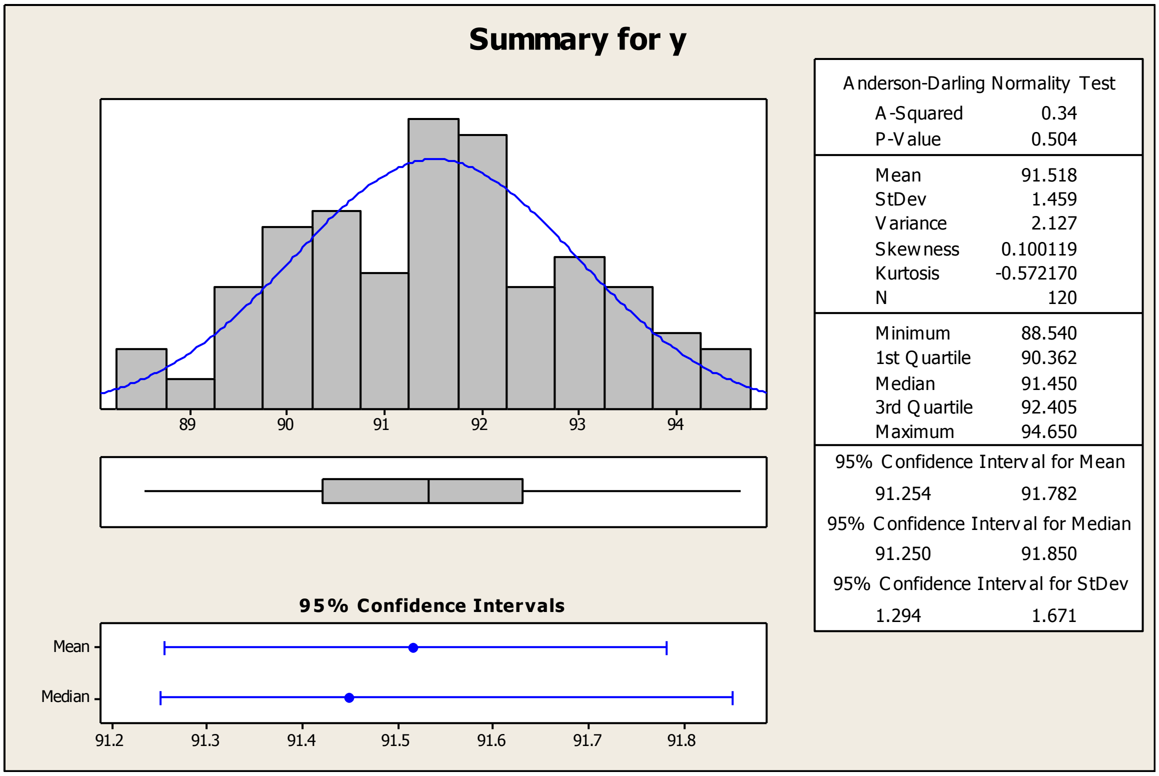

Figure C-2: Descriptive Summary

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