ENHANCING THE MONITORING OF LINEAR PROFILE PARAMETERS

BY

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The thesis is dedicated to my beloved parents, uncle, fiance, sisters, brother, teachers & friends for their endless love, support and encouragement in all stages of my life.

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Tahir Mahmood

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LIST OF ABBREVIATIONS

β_0	Intercept of original model
β_1	Slope of original model
λ	Smoothing parameter of EWMA chart
ρ	Correlation coefficent
ε	Error term
AB	Ansari bradley
AC	Alternating current
ARL	Average run length
ARL_0	In-control average run length
ARL_1	Out-of-control average run length
B_0	Intercept of transformed model
B_1	Slope of transformed model
C	Capacitance
CL	Central limit

CSI	Current source inverter
$CUSUM$	Cummulative sum
DC	Direct current
$DERSS$	Double extreme ranked set sampling
$DMRSS$	Double median ranked set sampling
$DRSS$	Double ranked set sampling
EQL	Extra quadretic loss
$ERSS$	Extreme ranked set sampling
$EWMA$	Exponentially weighted moving average
FAR	False alarm rate
IC	In-control
k	Rational subgroups
LCL	Lower control limit
m_0	Stable subgroups
m_1	In-consistance subgroups
Max	Maximum
$Max - p$	Max Progressive
$MRSS$	Median ranked set sampling
MSE	Mean square error
MSS	Modified succesive sampling

n	Sample size
OOC	Out-of-control
PCI	Performance comparison index
PTS	Probability to signal
PV	Photovoltaic
Q	Charge
r	Number of cycles in RSS
$RARL$	Relative average run length
RSS	Ranked set sampling
SC	Shewhart Cucconi
$SDRL$	Standard deviation of run length
SL	Shewhart Lepage
SPC	Statistical process control
SRS	Simple random sampling
SS	Sum of squares
UCL	Upper control limit
VSI	Voltage source inverter
WRS	Wilcoxon rank sum

THESIS ABSTRACT

NAME:	Tahir Mahmood
TITLE OF STUDY:	Enhancing the Monitoring of Linear Profile Parameters
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The behavior of a process is governed by several quality characteristics that may be classified as characteristics of interest and ancillary characteristics. In most of the real processes there exists relationships among these quality characteristics/variables. These relationships are quantified by models termed as profiles that may be linear or non-linear. In this study, we have focused on simple linear profiles that are described by three parameters namely slope, intercept and error variance. Control charts play a key role to monitor any possible variations in the parameters of interest. In this dissertation, we have investigated the performance of the existing linear profile charts in Phase I and Phase II under some useful variants of sampling schemes. These include a variety of ranked set sampling schemes and modified successive sampling schemes. Moreover, we have covered joint monitoring approaches to control location and scale parameters under parametric and

non-parametric setups.

We have investigated the performance of our newly proposed charts using different measures including probability to signal and run length properties. We have compared our results with the well-known existing methodologies under different settings of design parameters. The comparative analysis revealed that our study proposals outshine the existing methods under different amounts of shifts in the process parameters. In addition, we have used practical datasets from industrial and electrical engineering and implemented our proposed techniques to show their application in real processes.

ملخص الرسالة

الاسم: طاهر محمود

عنوان الرسالة: تحسين مراقبة معالم المظهر الخطي (linear profile)

التخصص الرئيسي: الأحصاء التطبيقي

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سلوك العملية تحدد بواسطة العديد من خصائص الجودة و التي يمكن تصنيفها إلي خصائص مطلوبة و خصائص مساعدة. في العديد من العمليات الواقعية هناك علاقة تكمن بين هذه الخصائص أو المتغيرات. هذه العلاقات يتم تحديدها عن طريق نماذج ترمز لها بالمظهر (profile) و التي يمكن أن تكون خطية أو غير خطية. في هذه الدراسة ركزنا على المظاهر الخطية البسيطة (linear profiles) و التي يتم وصفها بثلاث معالم هي: الميل, القاطع و خطأ التباين. مخططات التحكم تلعب دوراً مهما لمراقبة أي أنحرافات في المعالم المستهدفة. في هذه الاطروحة, ناقشنا اداء مخططات المظهر الخطية في المرحلة الاولى و المرحلة الثانية تحت بعض طرق المعاينة المفيدة. هذه تشمل مجموعة المخططات المتنوعة لعينات أُخذت من معاينة المجموعة الرتبية و من مخطط المعاينة المتتابعة المعدلة .علاوة على ذلك، قمنا بتغطية طريقة المراقبة المشتركة لإدارة معلمة المكان ومعلمة المستوى تحت الظروف المعلمية و اللا معلمية.

ناقشنا أداء مخططاتنا الجديدة بإستخدم معايير مختلفة تتضمن خصائص إحتمال الإشارة و طول الشغل. قارنا نتائجنا مع نتائج الطرق الموجودة تحت إعدادات مختلفة لمعالم العلمية. المقارنة أظهرت أن دراستنا تتفوق على الطرق الموجودة تحت قيم مختلفة للإنحرافات فى معالم العملية. بالأضافة, إستخدمنا بيانات عملية من الصناعة و الهندسة الكهربائية و تم تطبيق تقنياتنا المقترحة لإظهار تطبيقاتها فى العمليات الحقيقية.

CHAPTER 1

INTRODUCTION

This chapter introduces statistical process control (SPC) and more specifically control charts such as memory less chart (Shewhart) and memory type chart (EWMA). The structures of simple linear profiles and its special case (joint/simultaneous monitoring of mean and variability) are also discussed in the following sections. Finally, the outlines of the thesis are also reported.

1.1 Statistical process control

World is a global village, where the super markets are filled with the variety of products or services. In our days, customers do not only purchase a product to fulfill their need but also consider the quality and cost efficiency of the product/service. Quality, in manufacturing perspective, is a measure of excellence or a state of being free from defects, deficiencies and significant variation. Generally, there are two causes of variation that affect the performance of the process; chance cause or natural cause of variation that cannot be properly eliminated unless there is major change in the equipment or material used in the process and other is the special or assignable cause of variation that can be divided further in two categories namely transient and persistent variations. These causes of variation can be precisely identified, eliminated or reduced by investing the problem and finding the causes results in process improvement. Statistical process control (SPC), a set of the well-known tool kits, is used to monitor the variations in a process. SPC tool-kit contains seven magnificent tools that are used to differentiate the aforementioned variations. These tools are known as histogram, box-plot, pareto chart, check sheet, defect concentration diagram, scatter plot and control charts (for brief discussion one may see [1–3]).

1.1.1 Control charts

Control chart, one of the major tool of SPC, is commonly applied to monitor the performance of process with respect to time. In control chart, there are two decision lines named as lower control limit (LCL) and upper control limit (UCL), which allows us to decide whether the process is working under in-control (IC) or out-of-control (OOC) situation. If the control chart identifies that process is out-of-control, there is a need to diagnose the cause behind this abrupt change in the process. Generally, Control charts are worked into two main stages named as retrospective stage (Phase I) and prospective stage (Phase II). The objective of the retrospective analysis is to find the optimal choice of process parameters and control limits for the monitoring phase (Phase II). In Phase I analysis historical data is used to estimate the in-control state of the process while Phase II depends on present data to analyze current state of the process. Usually, in retrospective stage, practitioner expect some source of variations in the process and a higher probability of their detection, whereas monitoring stage is used for quick detection of changes in the process parameters. Further, Control charts are divided into two main classes: memory less control charts (e.g. Shewhart) and memory type (EWMA) control charts which are briefly discussed in the following subsections.

Shewhart control chart

In the earlier of 19^{th} century, [4] proposed a memory less control chart named as Shewhart control chart which is used to detect abrupt change in the process parameters (location or/and scale). The Shewhart chart is dependent on the current sample information, thats why its effective to detect the large or transient shifts in the process. The structure of Shewhart control chart consist of two decision lines known as upper control limit (UCL), lower control limit (LCL) and a central limit (CL). The aforementioned limits for a statistic (θ) are defined as:

$$UCL = \mu_0 + k_1 \ (\sigma_0)\,; \qquad CL = \mu_0; \qquad LCL = \mu_0 - k_1 \ (\sigma_0)$$

where, $\theta \sim N(\mu_0, \sigma_0^2)$ and the limits of the Shewhart chart depends on charting constant (k_1) which is selected against the fixed IC average run length (ARL_0). Generally, when the distribution of the statistic follows standard normal distribution then $k_1 = 3$ is used to gain $ARL_0 = 370$ and hence the limits are known as

3σ limits. Further, the Shewhart chart declares an OOC signal when (θ) plotted outside of the band (i.e. LCL to UCL) otherwise chart declares IC state of the process.

As mentioned above that Shewhart charts are not efficient for the detection of small and moderate shifts. So, there have been numerous attempts to patch this deficiency. One attempt is the application of sensitizing rules discussed by [5, 6] and another is the use of variable sampling interval (VSI) control charts in place of fixed sampling interval (FSI) (cf. [7–9]).

Exponentially weighted moving average control chart

The exponentially weighted moving average (EWMA) control chart was introduced by [10], which is also used to monitor the small or moderate shifts in the process parameters. The EWMA chart is a memory type structure because it utilizes the past information along current information. The EWMA statistic for a statistic (θ) is defined as:

$$Z_i = \lambda\theta_i + (1-\lambda)\, Z_{i-1}$$

where i is the sample number and λ is the constant which have a range between zero and one (*i.e.* $0 < \lambda \leq 1$). The starting value of aforementioned EWMA statistic is taken equal to zero (*i.e.* $Z_1 = \mu_0$). The time varying limits of the EWMA charts are given as:

$$UCL_i = \mu_0 + k_2\sigma_0\sqrt{\tfrac{\lambda}{2-\lambda}(1-(1-\lambda))^{2i}}$$

$$CL = \mu_0$$

$$LCL_i = \mu_0 - k_2\sigma_0\sqrt{\tfrac{\lambda}{2-\lambda}(1-(1-\lambda))^{2i}}$$

The EWMA chart have two parameters λ and k_3 . λ determines the decline of weights while k_2 determines the width of control limits. These two parameters are carefully chosen against fixed IC average run length (ARL_0) to determine the performance of EWMA chart. On large values of i ,the aforementioned time varying limits converges to constant limits which are given as:

$$UCL = \mu_0 + k_2\sigma_0\sqrt{\frac{\lambda}{2-\lambda}}; \quad CL = \mu_0; \quad LCL = \mu_0 - k_2\sigma_0\sqrt{\frac{\lambda}{2-\lambda}}$$

Hence, the term $(1-(1-\lambda))^{2i}$ tends to 1 if the sample tends to ∞. Some modifications on EWMA charts may see in [11–17]

1.2 Linear profiling

Usually, control charts are designed to monitor single quality characteristic (e.g. qualitative or quantitative) of a process but in many manufacturing processes, quality characteristics have a relationship with other auxiliary variable(s). For example, in semiconductor manufacturing, flow of gasses is dependent on pressure of mass flow controller and in electrical engineering, charge of a capacitor is dependent on the capacitance level. When such quality characteristic is linearly associated with another explanatory variable then it is termed as simple linear profile and the monitoring of simple linear profile parameters (i.e. slope, intercept and error variance) is known as liner profiling.

Assume that for the j^{th} random sample collected over time, we have the paired observation $(Y_{ij},\ X_i)$; $i = 1, 2, \ldots\ldots.., n$, then simple linear profile model used in linear profiling is defined as:

$$Y_{ij} = \beta_0 + \beta_1 X_i + \varepsilon_{ij}\ ;\ j = 1, 2, 3, \ldots\ldots.., m; \tag{1.1}$$

where the terms appearing in model 1.1 are β_0 (Intercept), β_1 (Slope) and ε is the error term. We have also assumed that $\varepsilon_{ij} \sim N(\mu, \sigma^2)$. The least square estimates of the parameters are given by the following expressions:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(X_{(i)} - \bar{X})Y_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{m}(X_{(i)} - \bar{X})^2} = \frac{S_{xy}}{S_{xx}}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X}$$

where $\bar{Y} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} Y_{ij}}{nm}$, $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ and the conditional mean, variance and covariance of $\hat{\beta}_0,\ \hat{\beta}_1$ are defined as:

$$E\left[\hat{\beta}_0|X\right] = \beta_0; \qquad E\left[\hat{\beta}_1|X\right] = \beta_1$$

$$var\left[\hat{\beta}_0|X\right] = \sigma^2\left[\frac{1}{nm} + \frac{\bar{X}^2}{S_{xx}}\ \right];\ var\left[\hat{\beta}_1|X\right] = \frac{\sigma^2}{S_{xx}};\ cov\left[\hat{\beta}_0, \hat{\beta}_1|X\right] = -\frac{\sigma^2\ \bar{X}}{S_{xx}}$$

Mean square error is an unbiased estimator of the variance of error term σ_e^2, which is defined as:

$$MSE = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}^2}{nm - 2}$$

where $e_{ij} = y_{ij} - \hat{y}_{ij}$ is the i^{th} residual of j^{th} sample and $\hat{y}_{ij}$ is the i^{th} fitted regression line in the j^{th} sample. It is to be mentioned that model 1.1 may be transformed using the transformation $X_i{}^* = X_i - \bar{X}$ to gain the zero covariance of $\hat{b}_0$ and $\hat{b}_1$. After transforming X_i, we obtain a modified form (derived later in Section A.1) of the aforementioned model 1.1 named as transformed model which is defined as:

$$Y_{ij} = (B_0) + (B_1)\, X_i{}^* + \varepsilon_{ij} \tag{1.2}$$

where $B_0 = \ \beta_0 + \beta_1 \bar{X} + (\beta\sigma)\, \bar{X}$ and $B_1 = \ (\beta_1 + \beta\sigma)\, X_i{}^*$. Here, shift is defined in σ units with reference to slope of model 1.1 (i.e.$\beta\sigma$). One may define the expressions of means, variances and covariance of $\hat{b}_0$ and $\hat{b}_1$. It is to be noted that the covariance of $\hat{b}_0$ and $\hat{b}_1$ will be zero as the average of $X_i{}^*$ is zero. It is to be mentioned that the estimated intercept and slope of transformed model will be denoted by $\hat{b}_0$ and $\hat{b}_1$ respectively.

In literature, many researchers addressed several studies on linear profiling (cf. [18–20]) but a popular proposal named as $Shewhart-3$ chart was introduced by [21]. In $Shewhart - 3$ chart, individual chart for each parameter (i.e. slope, intercept and error variance) are combined to evaluate the joint/simultaneous monitoring of the process. The structure of the individual chart for each parameter are define below:

$$\text{for intercept}: \left\{ \begin{array}{l} UCL = B_0 + L_{\alpha/2} \sqrt{\sigma^2 \left[\frac{1}{nm}\right]} \\ LCL = B_0 - L_{\alpha/2} \sqrt{\sigma^2 \left[\frac{1}{nm}\right]} \end{array} \right\}$$

$$\text{for slope} : \left\{ \begin{array}{c} UCL = B_1 + L_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}} \\ LCL = B_1 - L_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}} \end{array} \right\}$$

$$\text{for error variance} : \left\{ \begin{array}{c} UCL = \frac{\sigma^2}{n-2} + \chi^2_{\frac{\alpha}{2},nm-2} \\ LCL = \frac{\sigma^2}{n-2} - \chi^2_{1-\frac{\alpha}{2},nm-2} \end{array} \right\}$$

where $L_{\alpha/2}$ is the $\alpha/2^{th}$ quantile of students t distribution whereas, $\chi^2_{\alpha/2}$ and $\chi^2_{1-(\alpha/2)}$ are the upper and lower $\alpha/2^{th}$ quantiles of chi-square distribution having *nm-2* degree of freedom. The level of significance (α) is obtained by the definition of overall level of significance (i.e. $\alpha_{overall} = 1 - (1-\alpha)^3$).

As discussed above that Shewhart charts are only useful for the detection of large shifts in process parameters while memory type structure (EWMA) are suitable to detect small or moderate shifts in process parameters. Recently, [22] proposed a memory type structure for the joint monitoring of linear profile parameters named as *EWMA-3* control chart. The structure of the *EWMA-3* chart is defined as:

$$EWMA_{Ij} = \lambda\left(\hat{b}_0\right) + (1-\lambda)\, EWMA_{I[j-1]}$$

$$EWMA_{Sj} = \lambda\left(\hat{b}_1\right) + (1-\lambda)\, EWMA_{S[j-1]}$$

$$EWMA_{Ej} = \max\left\{\ \lambda\, ln\,(MSE) + (1-\lambda)\, EWMA_{E[j-1]},\ ln\left({\sigma^2}_0\right)\right\}$$

where $EWMA_{Ij}$ is the j^{th} EWMA statistic for intercept; $EWMA_{Sj}$ and $EWMA_{Ej}$ are the j^{th} EWMA statistics for slope and error variance respectively;λ is the smoothing parameter that ranges between zero and one (i.e. $0 < \lambda \leq 1$).

The mean and variance of each of the three EWMA statistic are given as:

$$E\left(EWMA_{Ij}\right) = B_0; \quad E\left(EWMA_{Sj}\right) = B_1; \quad E\left(EWMA_{Ej}\right) = ln\left(\sigma^2\right)$$

$$Var\left(EWMA_{Ij}\right) = \frac{\lambda}{2-\lambda}\sigma^2\left[\frac{1}{nm}\right]; \quad Var\left(EWMA_{Sj}\right) = \frac{\lambda}{2-\lambda}\frac{\sigma^2}{S_{xx}}$$

$$Var\left(EWMA_{Ej}\right) = Var\left(ln\left(MSE\right)\right) \cong \frac{2}{n-2}+\frac{2}{\left(n-2\right)^2}+\frac{4}{3\left(n-2\right)^3}-\frac{16}{15\left(n-2\right)^5}$$

Based on the above mentioned properties of the EWMA statistics, the asymptotic limits for each EWMA plotting statistic are given as:

$$\text{for } EWMA_{Ij}: \left\{\begin{array}{l} UCL_I = B_0 + L_I\sqrt{\frac{\lambda}{2-\lambda}\sigma^2\left[\frac{1}{nm}\right]} \\ LCL_I = B_0 - L_I\sqrt{\frac{\lambda}{2-\lambda}\sigma^2\left[\frac{1}{nm}\right]} \end{array}\right\}$$

$$\text{for } EWMA_{Sj}: \left\{\begin{array}{l} UCL_S = B_1 + L_S\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2}{S_{xx}}} \\ LCL_S = B_1 - L_S\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2}{S_{xx}}} \end{array}\right\}$$

$$\text{for } EWMA_{Ej}: \left\{UCL_E = ln\left(\sigma^2\right) + L_E\sqrt{\frac{\lambda}{2-\lambda}Var\left(ln\left(MSE\right)\right)}\right\}$$

where L_I, L_I and L_E , are the control limits coefficients for intercept, slope and standard deviation of error term, which are carefully choosen against the prespecified IC average run length. In this dissertation, we have designed several studies to enhanced the performance of aforementioned methods.

1.2.1 Special case of simple linear profiles

In simple linear profiles, control charts are used to monitor the study variable which is linearly associated with another explanatory variable. In simple linear profiles, three parameters are considered for the monitoring purpose such as slope, intercept and error variance. Slope is an important parameter which provides the estimate of average rate of change between study and explanatory variable. If the slope of the model is zero (i.e. $\beta_1 = 0$) then the effect of explanatory variable is eliminated from the process and the IC simple linear profile model (given in equation (1.1)) is defined as:

$$Y_{ij} = \beta_0 + \varepsilon_{ij} \tag{1.3}$$

where β_0 is the arithmetic mean of Y while the variance of error term (σ^2) is the simple variance of Y . In literature,$\bar{Y}$ control chart is a famous technique used for the monitoring of process mean and S^2 chart is a well-known method for the monitoring of process variability (σ^2).

Joint monitoring of process mean and variability

Usually, control charts are used to monitor a single process parameter such as location or scale. Before monitoring location parameter, it is important to make sure that the process scale or dispersion is in-control (IC). Variation in scale parameter may affect the performance of specific control chat in two ways; an increase in scale parameter may cause increase in False Alarm Rate (FAR) while reduction in scale parameter may cause decrease in the probability of detecting a shift. So, it seems more appealing to monitor both parameters together. For example, in the manufacturing process of circuit, a shift may be observed in both mean and variance of the thickness of the solder paste printed onto circuit boards due to improper fixation of the stencil.

In literature, two well-known terms named joint monitoring and simultaneous monitoring are used for the monitoring of mean and dispersion parameters together. It is noted that the joint/simultaneous monitoring of mean and variability may say a special case of simple linear profiles when the slope of the simple linear profile model is zero (i.e. $\beta_1 = 0$). For the joint/simultaneous monitoring of mean and variability several studies are designed by [23–25] but the most popular proposals based on maximum (Max) and sum of square (SS) statistics are proposed by [12, 26]. These popular charts include EWMA-Max, EWMA-SS, SS-EWMA and Max-EWMA charts which are briefly discussed in chapter 4.

1.3 Brief literature review

In this section, we provide a comprehensive literature on simple linear profiles and the existing joint monitoring methods.

1.3.1 Simple linear profiles

In many production processes, variable of interest can be modeled by a relation between a predicted variable and one or more predictor variables. The functional relationship among these variables is referred as profile and is addressed by different authors with fixed and random explanatory variables. For monitoring of simple linear profiles that is related to the control charting of regression adjusted variables was proposed by [18,27–30]. [31] developed EWMA/R and Hotelings T^2 charts to monitor the parameters of simple linear profiles. [22] proposed simultaneous scheme named as EWMA-3 for the monitoring of intercept, slope and error variance while [32] examined the quality characteristics of linear profile through multivariate cumulative sum control chart.

In simple linear profiles, effects of non-normal environments are studied by [33] and a control chart based on change point model was discussed by [34]. However, a comparative study between Shewhart methods (cf. [22] and [35]) was discussed by [36], whereas χ^2 and integrated MCUSUM control charts are proposed by [37] for the monitoring of linear profile parameters. A comprehensive overview on linear profiles was given by [38] and a control chart for the monitoring of recursive residuals was proposed by [39]. Control chart based on likelihood ratio in linear profiles was discussed by [40]. Moreover, [41] proposed a study based on CUSUM approach and a study related to small sample size (one or two) in linear profiles was discussed by [42]. [20] proposed a Phase II study about linear profile parameters under random effect model.

In linear profiles monitoring, there is a limited literature available regarding Phase I analysis. To mention a few of these we have: [43] proposed two multivariate control charts for the stability of linear calibration curves; [44] discussed two Phase I control charts for multilevel ion chromatography linear calibrations; [19]

proposed a Phase I study based on F-test in the simple linear profiles; The retrospective studies based on change point model was discussed by [45, 46]; for mixed model in linear profiles was discussed by [47], whereas Problem of within autocorrelation in the model of [47] was eliminated by [48]; [49] discussed the effect of Phase I estimation for the linear profile parameters (i.e. intercept, slope and error variance) under EWMA-3 structure proposed by [22].

The aforementioned studies have used the idea of linear profiling by using simple random sampling (SRS). In this study, we intend to use different ranked set strategies and modified succesive sampling to enhance the detection ability of control charts used to monitor linear profile parameters.

1.3.2 Joint monitoring methods

In the literature, two well-known terms named joint monitoring and simultaneous monitoring are used for the monitoring of location and dispersion parameters together. Joint monitoring is a term that alludes to monitoring both parameters through a single plotting statistic plotted against a pair of control limits. In simultaneous monitoring, parameters are monitored through separate plotting statistics plotted against distinct pair of control limits.

There is a variety of literature addressing simultaneous/joint monitoring of process location and dispersion parameters. [23] initiated simultaneous monitoring and used two independent plotting statistics on the same chart. [50] used an EWMA based simultaneous scheme using absolute value of standardize sample mean. [24] also discussed simultaneous monitoring of mean and variance under EWMA structure. Keeping in view some limitations of simultaneous monitoring (such as it requires independence of plotting statistics, interpretation of an out-of-control (OOC) signal is not straight forward), [51] proposed semicircle control chart based on the root mean square statistic which is further improved in the form of Max chart by [52]. [53] also designed a simultaneous scheme for variable control chart. [25] proposed a maximum generally weighted moving average (MaxGWMA) control chart for the simultaneous monitoring of process parameters.

For the joint monitoring of location and scale parameters, [12, 26] used some memory-type charts based on single statistic. These charts are based on the maximum (Max) and sum of square (SS) statistics and include EWMA-Max, EWMA-SS, SS-EWMA and Max-EWMA charts. Later, some other approaches were explored that include: a Max-CUSUM chart [54], an EWMA-SC chart [14], a non-central chi-square chart for joint monitoring [55], an SS-CUSUM chart [56], a likelihood ratio based approach [57], a modified Max-EWMA chart using range statistic instead of variance [13], a Max-DEWMA chart [58, 59], an SS-DEWMA approach [60], a change point approach [61], non-parametric approaches [62–66], a joint Shewhart approach for finite horizons [67]. In addition, [68, 69] and the references therein may be seen for an overview in this direction.

1.4 Objectives of the thesis

Simple linear profiles play a key role in many manufacturing processes used for the monitoring of study variable which is linearly related with another ancillary variable. In this dissertation, we plan to invertigate and further enhance the performance of existing control charts related to the monitoring of linear profile parameters and its special cases. The specific objectives of our study are listed below.

(i) Proposing Phase I and Phase II studies for the monitoring of simple linear profiles under several ranked set sampling schemes.

(ii) Enhancing the performance of Shewhart structure using modified successive sampling scheme for the monitoring of linear profile parameters.

(iii) Designing new joint monitoring scheme under EWMA structure for linear profile parameters and its special case under progressive setup.

(iv) Investigating non-parametric control charting setup for the joint monitoring of process parameters (location and dispersion).

1.5 Outline of the thesis

The investigation of linear profile parameters in retrospective stage (Phase I) and prospective stage (Phase II) has been done only under simple random sampling (SRS). In chapter 2, we have examined both Phases for the monitoring of linear profiles parameters (i.e. intercept, slope and variance of error term) under different ranked set strategies such as ranked set sampling (RSS), median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), double ranked set sampling (DRSS), double median ranked set sampling (DMRSS) and double extreme ranked set sampling (DERSS). The comparative study on the performance of existing and our proposed schemes has been discussed in terms of probability to signal (PTS) for Phase I and in terms of average run length (ARL) for Phase II. Moreover, an illustrative example from electrical engineering is also used to highlight the importance of the proposed method in real applications.

The modified successive sampling (MSS) is a cost-effective scheme as compared to simple random sampling (SRS), also useful when data consist of missing observations. Chapter 3 offers $Shewhart-3$ control chart under MSS to enhanced the monitoring of linear profile parameters. Moreover, the special cases of the linear profiling are also discussed in this chapter. For the performance analysis, we have used an extensive comparative study which is expressed in form of run length properties. The illustrative examples with real-life data sets are also included to highlight the importance of the proposed charts.

Most of the recent literature on simple liner profiles was discussed under simultaneous structure for the monitoring of linear profile parmeters such as intercept, slope and error variance. In simulteneous methods each parameter have individual chart which consist of respective pair of limits such as for the simultaneous monitoring of linear profile parameters three individual charts based on each parameter are designed in a mechanism to obtain overall performance of the process. In such cases, if a process is designed to target overall $ARL_0 = 200$ then each individual chart have to bear $ARL_0 = 584.5$ which is a tedious mechanism and impractical. In Chapter 4, we have designed an alternative approach (joint monitoring) to the simultaneous monitoring of linear profile parameters. Moreover, special case of simple linear profiles under progressive setup (which is a special case of EWMA chart) is also discussed in this study. The study has provided an extensive comparison between the proposed charts and some existing schemes in terms of run length properties. Further, real-life application about electrical engineering is also included to highlight the importance of the proposed methods.

Process monitoring is a continuous process and it needs careful attention for an improved quality of output. Location and dispersion parameters play a vital role in regulating every process and it requires a timely detection of any change in their stable behaviors. Nowadays, practitioners prefer a single charting setup that offers better ability to detect joint shifts in the process parameters. In Chapter 5, we have designed a comprehensive study based on non-parametric charting structures for the joint monitoring of location and dispersion parameters. The study has provided an extensive comparison about proposed charts under different environments and the performance analysis is discussed in terms of run length properties. Real-life application about strikes in US manufacturing industries is also included to highlight the importance of the proposed methods.

CHAPTER 2

LINEAR PROFILE MONITORING UNDER RANKED SET SCHEMES

In statistical process control (SPC), control chart is a dynamic tool which works under two different phases (Phase I or Phase II). Retrospective stage (Phase I) is mainly used to estimate the unknown parameters of the process while prospective stage (Phase II) focuses on the monitoring of process based on the estimated control limits from Phase I. For more discussion on Phase I and Phase II, see [70–72]).

The investigation of linear profile parameters in these Phases has been done only under simple random sampling [19, 22]. In this chapter, we will examine these Phases for the monitoring of linear profiles parameters (i.e. intercept, slope and variance of error term) under different ranked set strategies such as ranked set sampling (RSS), median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), double ranked set sampling (DRSS), double median ranked set sampling (DMRSS) and double extreme ranked set sampling (DERSS). Moreover, an illustrative example from electrical engineering is also used to highlight the importance of the proposed method in real applications.

2.1 Phase I analysis

Energy is a critical enabler, which have essential demand all over the world. It is a basic need for all living organisms to perform several activities such as breathing, movement, metabolism etc. In this modern era, energy is used as a source for cooking, heating, lighting, transport, telecommunications and mechanical power. Electricity is the ideal form of energy which is generated through coal, oil, fossils fuels, solar, wind and nuclear energy. Nowadays, in term of cost, cheap electricity is also generated through the solar medium. The solar system (given in Figure 2.1) consists of photovoltaic (PV) panels that converts sunlight into DC electricity. Further, solar panels are connected through inverters that are used to convert the DC voltage into AC voltage. The bi-directional meter is used as a medium between solar supply and power grid supply to exchange the rate of electricity from one end to another end.

In solar panel system, several inverters are used to convert the DC voltage into AC voltage such as voltage source invertor (VSI), current source inverter (CSI) and Z-source inverter. The VSI is a buck inverter which gives less output

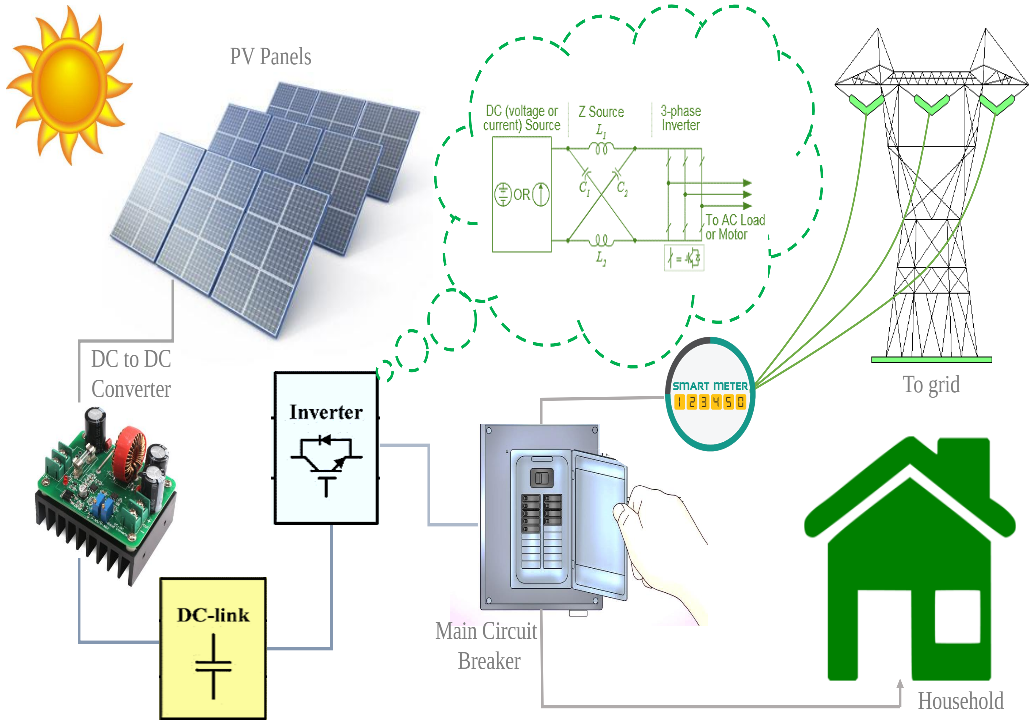

Figure 2.1: Portrait of solar panel system for households

voltage as compare to DC input voltage while CSI is a boost inverter and it provides more output voltage as compare to DC input voltage. However, Z-source inverter consists of both abilities (buck and boost) which is responsible to overcome various problems of VSI and CSI. The structure of Z-source inverter consists of two capacitors, two switches (series or anti-parallel) and a $3-\varphi$ bridge inverter. A three phase $(3-\varphi)$ bridge inverter is an electronic device which is used to converts the DC into three phase AC. Usually, parallel plate capacitors are used as a DC link in grid-connected PV system and its capacitance is inversely related to the distance between two conductive plates while direct relationship exists in the capacitance and the surface area of the conductive plates.

Generally, electrical engineers are concerned about the variations of output voltage in power grid system. Whereas, monitoring of the voltage using control chart is not possible until practitioner considers the capacitance of the process which is inversely associated with the voltage. Similarly, monitoring the dissolve amount of aspartame (an artificial sweetener) have no worth without considering effect of temperature in the study. However, in the semiconductor manufacturing application, monitoring of pressure in chamber also needed the information about flow of the gases in the chamber. So, to overcome such problems one needs to use the concept of linear profiles which can handle the monitoring of study variable when it is linearly associated with another (explanatory) variable.

In the literature, several studies (discussed in Section 1.3.1) are avaible for the monitoring of simple linear profile parameters (i.e. intercept, slope and error variance) under Phsae I, such as [22] recommended their prospective method into retrospective method, by replacing the three EWMA structures with three Shewhart structures. Further, [19] compared their Phase I study based on F-test with the three Phase I approaches proposed by [22, 43, 44]. In this study, we intend to use different ranked set strategies (RSS) to enhance the detection ability of control charts used to monitor linear profile parameters in Phase I. The performance of proposed schemes will be compared to the Phase I method originally introduced by [22] and elaborated by [19] under simple random sampling.

2.1.1 Modification of linear profiles under Phase I

In this section, we provide the background of ranked set schemes and linear profiles. These are required to develop our modified Phase I structure for simple linear profile parameters under several ranked set sampling schemes.

Ranked set sampling schemes

The efficient scheme as compare to simple random sampling (SRS) named as Ranked set sampling (RSS) was proposed by [73]. RSS is defined as: select n random samples for each of n sets and sort them in each set with respect to concomitant variable. In each cycle, choose the minimum value from the 1^{st} set, then 2^{nd} smallest from the 2^{nd} set and the largest sample from n^{th} set. The cycle may be repeated r times until nr samples have been measured. These nr samples thus, form the RSS.

[74] suggested another type of ranked set scheme termed as median ranked set sampling (MRSS). MRSS is defined as: randomly select n samples for each of n sets and ranked them in each set using the concomitant variable. In MRSS selection of n samples are dependent on even or odd set size. For even set size, select the 1^{st} half samples from the smallest rank of $(n/2)^{th}$ order and the smallest rank of $(n+1/2)^{th}$ order in the 2^{nd} half. For odd set size, choose the median value of each ranked set (i.e. $(n+1/2)^{th}$ ranked value). The cycle may be repeated r times until nr samples have been measured. These nr samples thus, form the MRSS.

Another modified scheme named as extreme ranked set sampling (ERSS) is defined as (cf. [75]); select n random sets each of siz en and sort each set with respect to concomitant variable. When set size is even, select the smallest sample from the first $(n/2)^{th}$ set and from the other $(n/2)^{th}$ set the largest sample for actual measurement. For the odd set size, select the smallest sample from the first $(n+1/2)^{th}$ set and the largest sample from the last $(n+1/2)^{th}$ sets; median of the remaining set, for actual measurement. The cycle may be repeated r time until nr samples have been measured. These nr samples thus, form the ERSS.

The outline of double ranked set sampling (DRSS) was provided by [76], which is defined as: randomly select n^3 samples and further split them into n sets each of n^2 samples. Apply the aforementioned RSS on each set having n^2 samples and form the new n sets each of size n then again apply the RSS technique to obtain the second stage samples. The cycle may be repeated r time until nr samples have been measured. These nr samples thus, form the DRSS. However, the second stage of MRSS and ERSS is termed as double median ranked set sampling (DMRSS) and double extreme ranked set (DERSS) respectively. Which may also be obtained by the double implementation of their procedures (MRSS and ERSS), following the same steps as mentioned above for DRSS. For more details, one may see [77].

The ranked set sampling and its modifications such as MRSS, ERSS, DRSS, DMRSS, DERSS are considered as perfect if there exist extreme positive correlation ($\rho = 1$) between study variable and concomitant variable. Otherwise, they are categorized as imperfect.

Linear profiles under RSS schemes

The technique used to monitor the study variable which is associated with other explanatory variable(s) is termed as linear profiles. In this subsection, we briefly describe the theoretical background of linear profiles under different ranked set strategies (Later denoted by (τ) in stated study). We have covered several choices of (τ) named RSS, MRSS, ERSS, DRSS, DMRSS and DERSS. For more discussion about simple linear profiles model under RSS strategies, one may see [78,79]. The simple linear profile model with intercept (B_0) and slope (B_1) having sample size (n), subgroups (k) and number of cycles (r) under ranked set strategies is defined as:

$$Y_{[i]jl} = B_0 + B_1 X_{(i)} + \varepsilon_{[i]jl} \; ; \; i = 1, 2, 3, .., n \; ; j = 1, 2, 3, \ldots m.., k; \quad l = 1, 2, 3, .., r \tag{2.1}$$

where $Y_{[i]jl}$ is the explained variable for i^{th} ordered sample in j^{th} subgroup and l^{th} cycle, $X_{(i)}$ is fixed explanatory variable with i^{th} random sample and $\varepsilon_{[i]jl}$ is the error term for i^{th} ordered sample in j^{th} subgroup and l^{th} cycle. It is to be noted that If $l = 1$ then one may get the above terms under SRS. The least square estimates of intercept (B_0), slope (B_1) and their properties are also estimated by following Section 1.2 and the least square estimates of transformed model are denoted by $b_{0[i]jl}$ and $b_{1[i]jl}$. It is also to be noted that X variable is fixed in our study and is used for the estimation of profile parameters. However, a different variable is used for the ranking of errors used in our profile model.

The Shewhart $-\ 3_{[\tau]}$ method

[22] although introduced retrospective method under SRS but the performance of his scheme was examined by [19]. In this study, we will introduce different ranked set strategies (τ) in the said scheme to enhance its performance. The Shewhart control chart for each linear profile parameter (i.e. intercept, slope and error variance) under Phase I on the base of transformed model are defined as;

$$\text{for } \tilde{b}_{0[i]jl}; \qquad \begin{array}{l} LCL = B_0 - L_{I[\tau]}\sqrt{\frac{(m-1)MSE_{[i]jl}}{mn}} \\ UCL = B_0 + L_{I[\tau]}\sqrt{\frac{(m-1)MSE_{[i]jl}}{mn}} \end{array}$$

$$\text{for } \tilde{b}_{1[i]jl}; \qquad \begin{array}{l} LCL = B_1 - L_{S[\tau]}\sqrt{\frac{(m-1)MSE_{[i]jl}}{mS_{xx}}} \\ UCL = B_1 + L_{S[\tau]}\sqrt{\frac{(m-1)MSE_{[i]jl}}{mS_{xx}}} \end{array}$$

where $L_{I[\tau]}$ and $L_{S[\tau]}$ are the control charting constant for intercept and slope respectively (c.f. Table 2.1). For the monitoring of error variance σ^2, $F_{[i]jl}$ is used which is defined as

$$F_{[i]jl} = \frac{MSE_{[i]l}}{MSE_{[i](-j)l}}$$

where $MSE_{[i]jl} = \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\sum_{i=1}^{n} e^2_{[i]jl}}{rk(n-2)}$ and $MSE_{[i](-j)l} = \frac{\sum_{i\neq j}^{k}\sum_{l=1}^{r}\sum_{i=1}^{n} e^2_{[i]jl}}{r(n-2)(k-1)}$. The control limits for monitoring of error variance ((LCL_f and(UCL_f)are also provided in Table 2.1.

The aforementioned three plotting statistics are combined in such a way to evaluate simultaneous monitoring of the linear profile parameters (intercept, slope and error variance). The said combined scheme is designed under different ranked set strategies which is named as $Shewhart-3_{[\tau]}$ chart in the later part of this study.

2.1.2 Performance evaluation and comparison

In this section, we will evaluate the performance of $Shewhart-3_{[\tau]}$ chart which is used to monitor simple linear profile parameters namely intercept, slope and standard deviation of error term. Moreover, we will also provide the comparison between the proposed schemes and the existing scheme under simple random sampling.

IC parameters and charting constants

In this study, we assumed IC simple linear profile with $B_0 = 0$ and $B_1 = 1$ following [19] *i.e.* $Y_{[i]jl} = X^*_{(i)} + \varepsilon_{[i]jl}$. we fixed sample size $(n = 10)$ and $X_{(i)} = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6,$ and 1.8 while corresponding transformed values of $X_{(i)}$ are $X^*{}_{(i)} = -0.9, -0.7, -0.5, -0.3, -0.1, 0.1, 0.3, 0.5, 0.7,$ and 0.9 with average equals to zero. The transformed model is as follows: $Y_{[i]jl} = (B_0) + (B_1)\, X^*{}_{(i)} + \varepsilon_{[i]jl}$, where $B_0 = \bar{X} + (\beta\sigma)\,\bar{X}$, $B_1 = (1+\beta\sigma)\, X^*{}_{(i)}$ and $\varepsilon_{[i]jl} \backsim BN\,(s, t;\ \mu_s = 0,\ \sigma_s = 1,\ \mu_t = 0,\ \sigma_t = 1, \rho))$. Several choices of ρ has been consider in our study. $\rho = 0.25, 0.50$ and 0.75 are used to represents imperfect ranked set samplings whereas $\rho = 1$ represents imperfect ranked set samplings.

For the Phase I study, samples are collected in the form of rational subgroups. Subgroups are introduced in such way that in the presence of instable values, the chances of variation with in subgroups will be minimized while chances of variation between subgroups will be maximized. We have considered different choices of subgroups (i.e. $k = 20, 30, 50, 100, 200$) each of sample size ($n = 3, 5, 8, 10$). Further, subgroups are categorized as m_0 (stable subgroups) and m_1 (inconsistent subgroups) (i.e. $k = m_0 + m_1$), whereas we also investigate the performance of proposed structure with different pairs of subgroups such as ($k = 20, m_1 = 2, 5, 10$), ($k = 30, m_1 = 3, 5, 10$) and ($k = 50, m_1 = 5, 10$).

In this study, the performance of proposed schemes with its counterparts is evaluated in terms of overall probability to signal (PTS) (α). PTS is defined as the detection ability of a chart in terms of probability when the process is actually OOC. The charting constants and control limits are chosen in such a way that individual PTS (α^*) may be set to achieve a specified value α. We have fixed $\alpha = 0.04$ (following [19]) and by using the relationship $\alpha = 1 - (1 - \alpha^*)^3 k$, we get $\alpha^* = 0.00068, 0.0004530, 0.0002721, 0.0001360641, 0.00006803434$ for $k = 20, 30, 50, 100$ and 200 respectively. For the computation of control charting constants (intercept and slope) and control limits for error variance (given in Table 2.1), we have carried an extensive Monte Carlo simulation study with $1e^6$ iteration.

Performance evaluations for Shewhart $-$ $3_{[\tau]}$ method

In order to monitor the performance of $Shewhart - 3_{[\tau]}$ method we have considered several shifts in the linear profiles parameters which are as follows:

(i) Shifts introduced in m_1 ($\theta = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5,$ and 5.0) for the intercept of transformed model.

(ii) Shifts presented in m_1 ($\beta = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5,$ and 5.0) for the slope of original model.

(iii) Shifts in m_1 ($\delta = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5,$ and 5.0) for the slope of transformed model.

(iv) Simultaneous shifts ($\theta, \beta = 1.0, 2.0, 3.0, 4.0,$ and 5.0) in m_1 for intercept of transformed model and slope of original model.

(v) Simultaneous shifts ($\theta, \delta = 1.0, 2.0, 3.0, 4.0,$ and 5.0) in m_1 for both intercept and slope of transformed model.

(vi) Shifts existing in m_1 ($\gamma = 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8$ and 3.0) for the error variance of original model.

The performance of $Shewhart - 3_{[\tau]}$, has been evaluated using overall PTS by carrying out extensive simulation study. The results are given in Tables 2.2 to 2.6 and some useful graphs of selective cases are also portrayed in Figures 2.2 to 2.6.

Comparative analysis of Shewhart $-$ $\mathbf{3}_{[\tau]}$ method

The overall probability to signal (with respect to several shifts) of Phase I method under different strategies (i.e. SRS, RSS, MRSS, ERSS, DRSS, DMRSS, DERSS) at fixed $k = 20$ and $m_1 = 2$ are reported in Tables 2.2 to 2.6 and Figures 2.2 and 2.3. Moreover, the selective cases regarding effect of inconsistent subgroups (m_1) and rational subgroups (k) are portrayed in Figures 2.4 and 2.5. In the following discussion, the term $Shewhart - 3_{[\tau]}$ is used for the proposed method

Table 2.1: Control charting constants and limits on fixed $k = 20$,$n = 10$,$m_1 = 2$, and $\alpha = 0.04$

τ	ρ	L_I	L_S	LCL_f	UCL_f
SRS	0	3.465302	3.465302	0.078703	3.885346
RSS	0.25	3.46045	3.476658	0.078676	3.886953
	0.5	3.458631	3.499682	0.078759	3.892813
	0.75	3.461062	3.598332	0.077852	3.931364
	1	3.428133	4.184676	0.073483	4.888926
MRSS	0.25	3.463541	3.459926	0.07853	3.883322
	0.5	3.458447	3.459738	0.078512	3.886022
	0.75	3.459945	3.458302	0.078896	3.88036
	1	3.418342	3.426893	0.078209	3.927688
ERSS	0.25	3.385554	3.386755	0.078628	3.880239
	0.5	3.144039	3.154674	0.079882	3.806181
	0.75	2.699666	2.741504	0.087807	3.577494
	1	1.9695	2.127761	0.15448	3.129479
DRSS	0.25	3.462009	3.476124	0.078945	3.877121
	0.5	3.456355	3.505408	0.078061	3.891441
	0.75	3.440969	3.605198	0.077809	3.951971
	1	3.330267	4.737909	0.059595	7.21733
DMRSS	0.25	3.485839	3.437171	0.079421	3.856836
	0.5	3.483514	3.475506	0.078659	3.919555
	0.75	3.436684	3.480446	0.078883	3.885405
	1	3.413136	3.421338	0.078984	3.939591
DERSS	0.25	3.302964	3.300814	0.08075	3.847935
	0.5	2.850913	2.840614	0.083236	3.613461
	0.75	2.168248	2.16968	0.121724	3.063981
	1	1.214729	1.361864	0.372063	2.276139

with fixed order of strategies (τ) such as SRS, RSS, MRSS, ERSS, DRSS, DMRSS, DERSS.

Shifts in intercept parameter:

Probability to signal for the shifts in intercept are reported in Table 2.2. The results depict that for fixed $\rho = 0.25$, the shift in intercept ($\theta = 1.00$) may cause 0.0135 unit increase in the overall PTS of $Shewhart - 3_{[SRS]}$ while 0.0119, 0.0109, 0.0112, 0.0122, 0.0146 and 0.0119 units increase in the overall PTS are reported in $Shewhart - 3_{[\tau]}$ with respect to RSS, MRSS, ERSS, DRSS, DMRSS and DERSS schemes. When shift ($\theta = 2.50$) is added in intercept then 0.2237, 0.2434, 0.2440, 0.2359, 0.2467, 0.2419 and 0.2393 units increase in term of overall PTS are reported for $Shewhart - 3_{[\tau]}$ respectively. Further, for $Shewhart - 3_{[\tau]}$ on fixed shift ($\theta = 4.50$), the overall PTS are reported as: 0.7559, 0.7694, 0.7681, 0.7648, 0.7713, 0.7689 and 0.7682.

In the presence of shifts in intercept at fixed $\rho = 0.75$, results of $Shewhart - 3_{[\tau]}$ are portrayed in Figure 2.2(A). The comparison reveled that proposed method under DRSS and DMRSS outperforms all other schemes. However, the comparative study regarding different choices of ρ was described in Figure 2.3(A). Shifts in intercept for $Shewhart - 3_{[MRSS]}$, revealed a direct relation between ρ and performance of the method.

Shifts in slope of original model:

Table 2.3 represents the comparative analysis of $Shewhart - 3_{[\tau]}$ in the presence of shifts in slope of original model. The results reveals that on fixed $\rho = 0.5$, increase in slope ($\beta = 1.00$) may cause 0.0545, 0.0788, 0.0894, 0.0714, 0.0848, 0.0896 and 0.0763 units increase in term of overall PTS for $Shewhart - 3_{[\tau]}$ respectively. When shift ($\beta = 2.50$) is added in slope then 0.7292 unit increase in the overall PTS was reported for$Shewhart - 3_{[SRS]}$ and 0.8075, 0.8160, 0.7959, 0.8201, 0.8287 and 0.8122 units increase in the overall PTS are reported for $Shewhart - 3_{[\tau]}$ with respect to RSS, MRSS, ERSS, DRSS, DMRSS and DERSS schemes. However, for $Shewhart - 3_{[\tau]}$ at fixed shift ($\beta = 4.50$), the overall PTS are reported as: 0.9366, 0.9401, 0.9417, 0.9397, 0.9432, 0.9447 and 0.9414.

For fixed $\rho = 0.75$, results of $Shewhart - 3_{[\tau]}$ with shifts in the slope of original model are represented in Figure 2.2(B). The assessment revealed that proposed method under DRSS and DMRSS performs comparatively better than others. Moreover, the analysis for different selections of ρ was defined in Figure 2.3(B). Shifts in slope of original model for $Shewhart - 3_{[DRSS]}$, revealed a direct relation between ρ and the performance of method.

Shifts in slope of transformed model:

Shifts in the slope of transformed model are described in Table 2.4. The results depict that at fixed $\rho = 1$, the shift in slope ($\delta = 1.00$) may cause 0.0122 unit increase in the overall PTS of $Shewhart - 3_{[SRS]}$ while 0.0596, 0.2368, 0.0351, 0.1058,

Table 2.2: Comparison of $Shewhart-3_{[\tau]}$ method for intercept shifts (B_0 to $B_0+\theta\sigma/\sqrt{n}$)

τ	**SRS**		**RSS**				**MRSS**				**ERSS**		
θ	$\rho=0.00$	$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$
0.00	0.0394	0.0416	0.0404	0.0385	0.0354	0.0405	0.0395	0.0395	0.0392	0.0410	0.0386	0.0375	0.0336
0.50	0.0433	0.0411	0.0410	0.0444	0.0482	0.0422	0.0416	0.0426	0.0638	0.0415	0.0417	0.0377	0.0433
1.00	0.0529	0.0535	0.0556	0.0644	0.1762	0.0514	0.0561	0.0690	0.2819	0.0523	0.0529	0.0597	0.0928
1.50	0.0798	0.0839	0.1001	0.1492	0.5110	0.0815	0.0996	0.1576	0.6830	0.0813	0.0964	0.1262	0.2731
2.00	0.1439	0.1545	0.1944	0.3147	0.7765	0.1503	0.2011	0.3448	0.8488	0.1507	0.1801	0.2676	0.5529
2.50	0.2632	0.285	0.3573	0.5358	0.8618	0.2846	0.3622	0.5737	0.8778	0.2770	0.3335	0.4763	0.7637
3.00	0.4262	0.4508	0.5402	0.7192	0.8780	0.4491	0.5486	0.7428	0.8827	0.4461	0.5180	0.6642	0.8468
3.50	0.5795	0.6131	0.6926	0.8166	0.8825	0.6150	0.7021	0.8284	0.8870	0.6088	0.6721	0.7833	0.8722
4.00	0.7149	0.7351	0.7907	0.8581	0.8847	0.7355	0.7967	0.8639	0.8919	0.729	0.7779	0.8427	0.8788
4.50	0.7953	0.8109	0.8417	0.8736	0.8898	0.8086	0.8435	0.8763	0.8996	0.8058	0.8337	0.8673	0.8812
5.00	0.8391	0.8473	0.8661	0.8792	0.8956	0.8477	0.8664	0.8801	0.9087	0.8472	0.8624	0.8766	0.8845
τ		**DRSS**				**DMRSS**				**DERSS**			
θ		$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$
0.00		0.0407	0.0406	0.0386	0.0336	0.0402	0.0388	0.0393	0.0398	0.0405	0.0401	0.0411	0.033
0.50		0.0426	0.0431	0.0416	0.0768	0.0411	0.0404	0.0444	0.5096	0.0422	0.0422	0.0411	0.0452
1.00		0.0528	0.0575	0.0702	0.4369	0.0548	0.0530	0.0822	0.8786	0.0524	0.0574	0.0674	0.1652
1.50		0.0826	0.1028	0.1613	0.8042	0.0836	0.0983	0.2012	0.8899	0.0861	0.0945	0.1381	0.4902
2.00		0.1563	0.1996	0.3610	0.8730	0.1533	0.2046	0.4282	0.9103	0.1550	0.1884	0.3152	0.7675
2.50		0.2874	0.3696	0.5994	0.8816	0.2822	0.3677	0.6548	0.9386	0.2798	0.3516	0.5374	0.8590
3.00		0.4528	0.5571	0.7603	0.8868	0.4492	0.5652	0.7959	0.9700	0.4476	0.5337	0.7182	0.8766
3.50		0.6158	0.7071	0.8376	0.8938	0.6088	0.7113	0.8537	0.9903	0.6120	0.6927	0.8159	0.8815
4.00		0.7366	0.7992	0.8680	0.9029	0.7366	0.8037	0.8741	0.9982	0.7334	0.7905	0.8588	0.8841
4.50		0.8119	0.8471	0.8778	0.9151	0.8092	0.8484	0.8799	0.9998	0.8087	0.8421	0.8735	0.8875
5.00		0.8497	0.8676	0.8805	0.9289	0.8478	0.8689	0.8823	1.00	0.8482	0.8647	0.8790	0.8934

Table 2.3: Comparison of $Shewhart-3_{[\tau]}$ method for slope shifts (β_1 to $\beta_1+\beta\sigma/\sqrt{S_{xx}}$)

τ	**SRS**		**RSS**				**MRSS**				**ERSS**		
β	$\rho=0.00$	$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$
0.00	0.0394	0.0412	0.0415	0.0392	0.0348	0.0417	0.0407	0.0422	0.0394	0.0403	0.0413	0.0366	0.0327
0.50	0.0474	0.0473	0.0508	0.0538	0.1060	0.0502	0.0516	0.0560	0.1654	0.0499	0.0477	0.0514	0.0650
1.00	0.0938	0.1054	0.1203	0.1821	0.5892	0.1079	0.1301	0.2035	0.7893	0.1029	0.1127	0.1587	0.3387
1.50	0.2524	0.2714	0.3379	0.5191	0.8884	0.2737	0.3549	0.5643	0.9591	0.2671	0.3242	0.4586	0.7625
2.00	0.5248	0.5532	0.6498	0.8147	0.9508	0.5570	0.6630	0.8433	0.9828	0.5536	0.6237	0.7687	0.9193
2.50	0.7685	0.7896	0.8490	0.9237	0.9766	0.7904	0.8567	0.9394	0.9871	0.7842	0.8372	0.9071	0.9630
3.00	0.8914	0.9006	0.9297	0.9605	0.9854	0.9039	0.9341	0.9685	0.9890	0.9012	0.9247	0.9547	0.9796
3.50	0.9418	0.9467	0.9600	0.9764	0.9881	0.9488	0.9626	0.9804	0.9912	0.9465	0.9575	0.9734	0.9852
4.00	0.9647	0.9675	0.9746	0.9830	0.9899	0.9682	0.9761	0.9850	0.9938	0.9669	0.9732	0.9816	0.9872
4.50	0.9760	0.9774	0.9816	0.9860	0.9919	0.9778	0.9824	0.9868	0.9960	0.9773	0.9810	0.9852	0.9882
5.00	0.9818	0.9826	0.9851	0.9874	0.9940	0.9828	0.9853	0.9879	0.9979	0.9825	0.9845	0.9868	0.9895

τ		**DRSS**				**DMRSS**				**DERSS**			
β		$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.50$	$\rho=0.75$	$\rho=1.00$	$\rho=0.25$	$\rho=0.5$	$\rho=0.75$	$\rho=1$
0.00		0.0389	0.0391	0.0379	0.0341	0.0431	0.0382	0.0397	0.0389	0.0397	0.0405	0.0382	0.0339
0.50		0.0507	0.0516	0.0560	0.2356	0.0510	0.0478	0.0681	0.9220	0.0482	0.0487	0.0545	0.0973
1.00		0.1028	0.1240	0.2089	0.8433	0.1043	0.1278	0.2524	0.9874	0.1033	0.1169	0.1884	0.5716
1.50		0.2730	0.3551	0.5895	0.9337	0.2698	0.3629	0.6568	0.9927	0.2745	0.3432	0.5277	0.8940
2.00		0.5634	0.6630	0.8500	0.9744	0.5636	0.6792	0.8894	0.9981	0.5566	0.6498	0.8214	0.9590
2.50		0.7900	0.8592	0.9373	0.9870	0.7921	0.8669	0.9538	0.9999	0.7878	0.8527	0.9300	0.9808
3.00		0.9038	0.9339	0.9669	0.9904	0.9054	0.9381	0.9751	1.0000	0.9019	0.9315	0.9648	0.9862
3.50		0.9474	0.9626	0.9794	0.9933	0.9497	0.9650	0.9835	1.0000	0.9479	0.9617	0.9783	0.9878
4.00		0.9677	0.9759	0.9846	0.9961	0.9688	0.9770	0.9864	1.0000	0.9675	0.9755	0.9840	0.9894
4.50		0.9776	0.9823	0.9869	0.9982	0.9783	0.9830	0.9878	1.0000	0.9778	0.9820	0.9863	0.9912
5.00		0.9829	0.9853	0.9880	0.9994	0.9833	0.9856	0.9890	1.0000	0.9828	0.9852	0.9875	0.9933

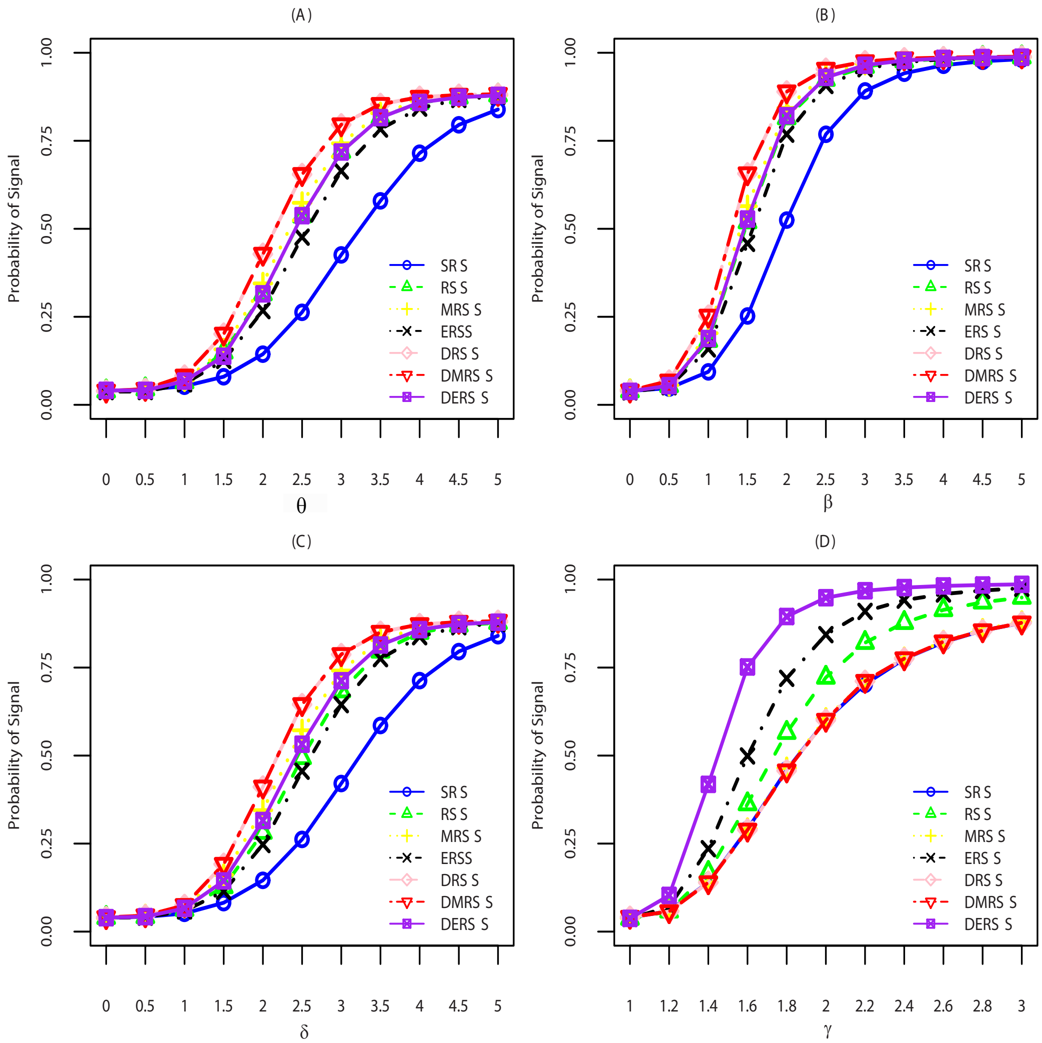

Figure 2.2: Comparison of $Shewhart - 3_{[\tau]}$ at fixed $\rho = 0.75$

0.8365 and 0.0655 units increase in the overall PTS are reported in $Shewhart - 3_{[\tau]}$ with respect to RSS, MRSS, ERSS, DRSS, DMRSS and DERSS schemes. When shift ($\delta = 2.50$) added in the slope then 0.2226, 0.7579, 0.8376, 0.6606, 0.8274, 0.8971 and 0.8023 units increase in term of overall PTS are reported for $Shewhart - 3_{[\tau]}$ respectively. Further, for $Shewhart - 3_{[\tau]}$ at fixed shift ($\delta = 4.50$), the overall PTS are reported as: 0.7564, 0.8431, 0.8590, 0.8401, 0.8413, 0.9582 and 0.84428.

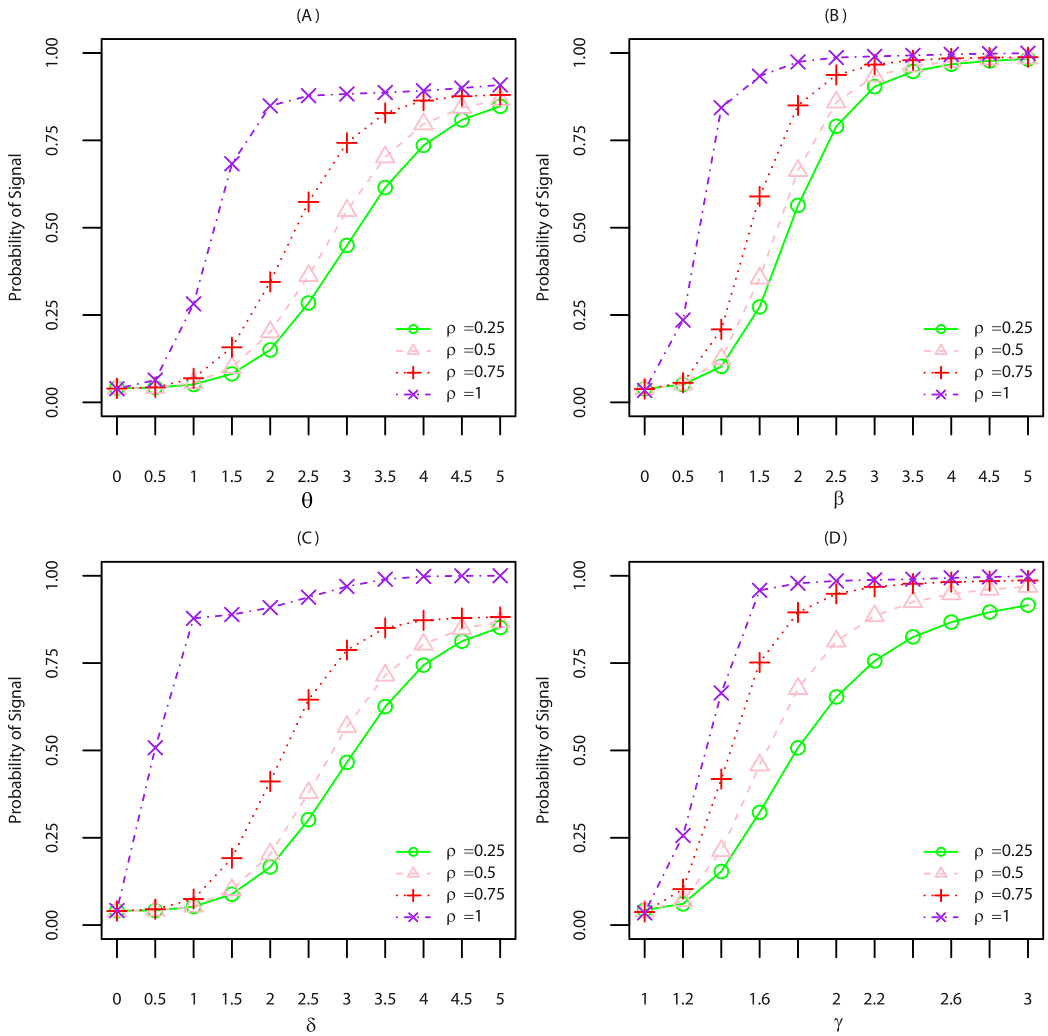

Figure 2.3: Performance of $Shewhart - 3_{[\tau]}$ with different choices of ρ

In the presence of shifts in the slope of transformed model, results of $Shewhart - 3_{[\tau]}$ are portrayed in Figure 2.2(C). The comparison revealed that proposed method under DRSS and DMRSS beats all other schemes under study. Further, the comparative study for different choices of ρ was examined in Figure 2.3(C). Shifts in the slope of transformed model for $Shewhart - 3_{[DMRSS]}$, revealed a direct relationship between ρ and performance of the method.

Table 2.4: Comparison of $Shewhart - 3_{[\tau]}$ method for slope shifts (B_1 to $B_1 + \delta\sigma/\sqrt{S_{xx}}$)

τ	SRS	RSS				MRSS				ERSS			
δ	$\rho = 0.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 1$
0.00	0.0394	0.0410	0.0401	0.0394	0.0339	0.0380	0.0393	0.0382	0.0398	0.0389	0.0398	0.0391	0.0349
0.50	0.0419	0.0433	0.0422	0.0419	0.0396	0.0424	0.0425	0.0464	0.0625	0.0429	0.0422	0.0409	0.0366
1.00	0.0516	0.0509	0.0573	0.0622	0.0934	0.0515	0.0577	0.0698	0.2766	0.0542	0.0556	0.0588	0.0700
1.50	0.0820	0.0832	0.0966	0.1268	0.2875	0.0874	0.1028	0.1611	0.6793	0.0854	0.0886	0.1142	0.1931
2.00	0.1457	0.1545	0.1906	0.2812	0.5934	0.1579	0.1980	0.3448	0.8475	0.1545	0.1746	0.2473	0.4429
2.50	0.2619	0.2776	0.3422	0.4936	0.7917	0.2846	0.3615	0.5718	0.8774	0.2761	0.3225	0.4556	0.6955
3.00	0.4208	0.4445	0.5240	0.6833	0.8598	0.4498	0.5502	0.7416	0.8822	0.4468	0.5136	0.6446	0.8229
3.50	0.5853	0.6091	0.6854	0.7959	0.8734	0.6122	0.7012	0.8281	0.8868	0.6064	0.6696	0.7753	0.8645
4.00	0.7128	0.7297	0.7858	0.8491	0.8755	0.7340	0.7962	0.8644	0.8918	0.7284	0.7780	0.8377	0.8736
4.50	0.7957	0.8067	0.8390	0.8698	0.8770	0.8095	0.8442	0.8762	0.8988	0.8070	0.8335	0.8659	0.8750
5.00	0.8404	0.8471	0.8639	0.8774	0.8784	0.8486	0.8664	0.8800	0.9076	0.8467	0.8615	0.8752	0.8759

τ	DRSS				DMRSS				DERSS			
δ	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 1$
0.00	0.0405	0.0410	0.0378	0.0336	0.0410	0.0401	0.0402	0.0416	0.0391	0.0383	0.0396	0.0303
0.50	0.0551	0.0434	0.0436	0.0447	0.0420	0.0423	0.0454	0.5076	0.0438	0.0410	0.0425	0.0388
1.00	0.1438	0.0535	0.0675	0.1394	0.0527	0.0537	0.0746	0.8781	0.0532	0.0555	0.0660	0.0958
1.50	0.2959	0.0963	0.1458	0.4613	0.0885	0.1012	0.1916	0.8889	0.0849	0.0982	0.1439	0.3306
2.00	0.4702	0.1960	0.3200	0.7656	0.1663	0.2030	0.4112	0.9087	0.1576	0.1956	0.3156	0.6622
2.50	0.6126	0.3537	0.5507	0.8610	0.3015	0.3784	0.6454	0.9386	0.2799	0.3589	0.5323	0.8327
3.00	0.7193	0.5415	0.7243	0.8728	0.4658	0.5677	0.7874	0.9693	0.4499	0.5378	0.7127	0.8702
3.50	0.7839	0.6933	0.8209	0.8736	0.6250	0.7152	0.8507	0.9901	0.6142	0.6944	0.8141	0.8739
4.00	0.8350	0.7911	0.8604	0.8742	0.7444	0.8037	0.8726	0.9981	0.7348	0.7915	0.8578	0.8739
4.50	0.8666	0.8427	0.8749	0.8749	0.8128	0.8490	0.8791	0.9998	0.8089	0.8433	0.8734	0.8746
5.00	0.8879	0.8666	0.8791	0.8767	0.8521	0.8677	0.8819	1.0000	0.8483	0.8655	0.8789	0.8756

Simultaneous shifts in intercept and slope:

The performance of $Shewhart - 3_{[\tau]}$ under DMRSS and DERSS, in the presence of joint shifts in intercept and slope of original or transformed model are discussed in Table 2.6. In the following comparative study, we fixed rational subgroups ($k = 20$), inconsistent subgroups ($m_1 = 5$) and $\rho = 0.75$. The results depict that on fixed IC intercept ($\theta = 0$)and IC slope of transformed model ($\theta = 0$), the shift in slope of original model ($\beta = 1.00$) may cause 0.2803 and 0.2009 units increase in the overall PTS of $Shewhart - 3_{[DMRSS]}$ and $Shewhart - 3_{[DERSS]}$ respectively, while shift in slope of transformed model ($\delta = 1.00$) when intercept ($\theta = 0$) and slope of original model ($\beta = 0$) are fixed may cause 0.0600 and 0.0386 units increase in the overall PTS of $Shewhart - 3_{[\tau]}$ with respect to DMRSS and DERSS.

For fixed slope of original model ($\beta = 0$) and slope of transformed model ($\delta = 0$), a shift in intercept ($\theta = 3$) may cause almost 0.8926 and 0.8282 units increase in the overall PTS of $Shewhart - 3_{[DMRSS]}$ and $Shewhart - 3_{[DERSS]}$ respectively. Further, on fixed IC slope of transformed model ($\delta = 0$), a shift in both slope of original model ($\beta = 2$) and intercept ($\theta = 2$) probably cause 0.9589 and 0.9578 units increase in the overall PTS of $Shewhart - 3_{[\tau]}$ with respect to DMRSS and DERSS. A shift in both slope of transformed model ($\delta = 2$) and intercept ($\theta = 2$) at fixed IC slope of original model ($\beta = 0$) may cause 0.7202 and 0.5689 units increase in the overall PTS of $Shewhart - 3_{[DMRSS]}$ and $Shewhart - 3_{[DERSS]}$ respectively.

Overall, at IC intercept, our proposed schemes are more effective to monitor shift in slope of original model as compared to shift in the slope of transformed model while $Shewhart-3_{[DMRSS]}$ have relatively good performance when there is a shift in intercept and slope of original model or transformed model are IC or OOC.

Shifts in the variance of error term:

Table 2.5 represents the comparative analysis of $Shewhart-3_{[\tau]}$ in the presence of shifts in error variance. The results revealed that on fixed $\rho = 0.75$, increase in error variance ($\gamma = 1.40$) may cause 0.1006, 0.1305, 0.1017, 0.1982, 0.7059, 0.5597 and 0.9109 units increase in term of overall PTS for $Shewhart-3_{[\tau]}$ respectively. When shift ($\gamma = 2.00$) is added in error variance then 0.5614 unit increase in the overall PTS was reported for $Shewhart-3_{[SRS]}$ and 0.6849, 0.5662, 0.8062, 0.7059, 0.5597 and 0.9109 units increase in the overall PTS are reported for $Shewhart-3_{[\tau]}$ with respect to RSS, MRSS, ERSS, DRSS, DMRSS and DERSS schemes. However, for $Shewhart-3_{[\tau]}$ on fixed shift ($\gamma = 2.80$), the overall PTS are reported as: 0.8159, 0.8985, 0.8194, 0.9316, 0.9063, 0.8139 and 0.9470.

The findings of $Shewhart-3_{[\tau]}$ with shifts in the error variance are represented in Figure 2.2(D). The assessment reveled that proposed method under DERSS performs comparatively better than others. Moreover, the analysis for different selections of was portrayed in Figure 2.3(D). Shifts in error variance for $Shewhart-3_{[DERSS]}$, revealed an increasing relation between and the performance of method.

Table 2.5: Comparison of $Shewhart - 3_{[\tau]}$ method for error variance (σ to $\gamma\sigma$)

τ	**SRS**		**RSS**				**MRSS**			**ERSS**		
γ	$\rho = 0.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 1$
1.00	0.0394	0.0425	0.0404	0.0370	0.0348	0.0381	0.0418	0.0386	0.0394	0.0393	0.0409	0.0373	0.0309
1.20	0.0587	0.0564	0.0580	0.0577	0.0668	0.0599	0.0580	0.0586	0.0587	0.0554	0.0604	0.0702	0.0905
1.40	0.1400	0.1385	0.1451	0.1674	0.1724	0.1367	0.1376	0.1403	0.1523	0.1443	0.1648	0.2355	0.2535
1.60	0.2907	0.2915	0.3102	0.3624	0.4940	0.2885	0.2858	0.2916	0.2925	0.2984	0.3540	0.4989	0.7646
1.80	0.4592	0.4713	0.4932	0.5651	0.7159	0.4591	0.4568	0.4644	0.4649	0.4777	0.5588	0.7195	0.9063
2.00	0.6008	0.6118	0.6479	0.7219	0.8371	0.6000	0.6011	0.6048	0.6088	0.6243	0.7089	0.8435	0.9534
2.20	0.7027	0.7157	0.7493	0.8207	0.8412	0.7058	0.7063	0.7110	0.7202	0.7291	0.8046	0.9095	0.9595
2.40	0.7746	0.7858	0.8171	0.8774	0.8822	0.7736	0.7762	0.7769	0.7819	0.7985	0.8640	0.9415	0.9615
2.60	0.8219	0.8320	0.8637	0.9139	0.9530	0.8232	0.8230	0.8245	0.8333	0.8432	0.9010	0.9590	0.9830
2.80	0.8553	0.8633	0.8919	0.9355	0.9639	0.8553	0.8563	0.8579	0.8648	0.8749	0.9267	0.9689	0.9854
3.00	0.8766	0.8858	0.9132	0.9489	0.9711	0.8789	0.8791	0.8798	0.8884	0.8968	0.9408	0.9753	0.9875

τ		**DRSS**				**DMRSS**				**DERSS**			
γ		$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 1$
1.00		0.0398	0.0377	0.0382	0.0332	0.0415	0.0373	0.0411	0.0379	0.0434	0.0399	0.0375	0.0349
1.20		0.0421	0.0590	0.0593	0.0691	0.0589	0.0516	0.0572	0.0549	0.0609	0.0698	0.1029	0.2564
1.40		0.0520	0.1477	0.1775	0.1835	0.1378	0.1307	0.1393	0.1499	0.1537	0.2126	0.4182	0.6645
1.60		0.0839	0.3207	0.3817	0.6235	0.2900	0.2771	0.2887	0.2899	0.3228	0.4580	0.7517	0.9583
1.80		0.1534	0.5027	0.5909	0.8017	0.4645	0.4496	0.4569	0.4622	0.5077	0.6765	0.8951	0.9785
2.00		0.2769	0.6496	0.7441	0.8735	0.6099	0.5934	0.6008	0.6118	0.6529	0.8127	0.9484	0.9853
2.20		0.4479	0.7544	0.8377	0.8823	0.7092	0.7021	0.7113	0.7345	0.7566	0.8849	0.9683	0.9883
2.40		0.6089	0.8239	0.8935	0.9025	0.7799	0.7708	0.7754	0.7921	0.8247	0.9253	0.9772	0.9902
2.60		0.7348	0.8660	0.9233	0.9336	0.8250	0.8199	0.8235	0.8316	0.8670	0.9477	0.9819	0.9940
2.80		0.8085	0.8969	0.9445	0.9431	0.8573	0.8526	0.8550	0.8668	0.8960	0.9610	0.9846	0.9966
3.00		0.8482	0.9167	0.9559	0.9508	0.8784	0.8746	0.8773	0.8905	0.9156	0.9690	0.9864	0.9985

Table 2.6: Joint shifts in intercept (B_0 to $B_0 + \lambda\sigma/\sqrt{n}$) and slope ($\beta_1$ to $\beta_1 + \beta\sigma/\sqrt{S_{xx}}$) or ($B_1$ to $B_1 + \delta\sigma/\sqrt{S_{xx}}$)

		$\theta = 0.00$							$\theta = 1.00$				
β	DMRSS	DERSS	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS
0.00	0.0403	0.0374	0.00	0.0434	0.0393	0.00		0.0994	0.0864	0.00		0.0969	0.0878
1.00	0.3207	0.2414	1.00	0.1004	0.0779	1.00		0.8255	0.7116	1.00		0.1629	0.1228
2.00	0.9776	0.9371	2.00	0.5245	0.3697	2.00		0.9975	0.9930	2.00		0.5599	0.4032
3.00	0.9999	0.9994	3.00	0.9367	0.8456	3.00		1.0000	0.9998	3.00		0.9419	0.8550
4.00	1.0000	1.0000	4.00	0.9936	0.9833	4.00		1.0000	1.0000	4.00		0.9939	0.9839
5.00	1.0000	1.0000	5.00	0.9981	0.9964	5.00		1.0000	1.0000	5.00		0.9983	0.9965
		$\theta = 2.00$							$\theta = 3.00$				
β	DMRSS	DERSS	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS
0.00	0.5115	0.4044	0.00	0.5092	0.4026	0.00		0.9330	0.8687	0.00		0.9339	0.8686
1.00	0.9847	0.9654	1.00	0.5435	0.4328	1.00		0.9971	0.9948	1.00		0.9379	0.8745
2.00	0.9992	0.9983	2.00	0.7606	0.6082	2.00		0.9998	0.9993	2.00		0.9675	0.9167
3.00	1.0000	0.9999	3.00	0.9689	0.9071	3.00		1.0000	1.0000	3.00		0.9961	0.9810
4.00	1.0000	1.0000	4.00	0.9969	0.9900	4.00		1.0000	1.0000	4.00		0.9996	0.9980
5.00	1.0000	1.0000	5.00	0.9991	0.9979	5.00		1.0000	1.0000	5.00		0.9999	0.9996
		$\theta = 2.00$							$\theta = 3.00$				
β	DMRSS	DERSS	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS	β	δ	DMRSS	DERSS
0.00	0.9931	0.9856	0.00	0.9932	0.9859	0.00		0.9980	0.9968	0.00		0.9980	0.9968
1.00	0.9990	0.9982	1.00	0.9936	0.9865	1.00		0.9997	0.9993	1.00		0.9982	0.9970
2.00	1.0000	0.9998	2.00	0.9969	0.9909	2.00		1.0000	1.0000	2.00		0.9991	0.9980
3.00	1.0000	1.0000	3.00	0.9996	0.9980	3.00		1.0000	1.0000	3.00		0.9999	0.9996
4.00	1.0000	1.0000	4.00	1.0000	0.9998	4.00		1.0000	1.0000	4.00		1.0000	1.0000
5.00	1.0000	1.0000	5.00	1.0000	1.0000	5.00		1.0000	1.0000	5.00		1.0000	1.0000

2.1.3 Effect of different design parameters

In this subsection, we will briefly discuss the effect of inconsistent subgroups (m_1), rational subgroups (k) and sample size (n) on probability to signal for detecting shifts in intercept, slope and error variance.

Effect of inconsistent subgroups ($\mathbf{m_1}$):

The effects of OOC subgroups in linear profile parameter such as intercept, slope and error variance are portrayed in Figure 2.4. In case of shifts in intercept, the effect of m_1 on fixed $\rho = 1$ are reported in Figure 2.4(A). The results reveal that on fixed intercept shift ($\theta = 1.5$), 0.8501, 0.9591, 0.9611 increase in overall PTS are reported for $Shewhart - 3_{[DMRSS]}$ with respect to $m_1 = 2, 5$ and 10. The results of different m_1 for shift in slope of original model at $\rho = 0.25$ are discussed in Figure 2.4(B). On fixed slope shift ($\beta = 2.5$), the increasing rate of overall PTS are reported 0.7511, 0.8511 and 0.8715 for the $Shewhart - 3_{[DMRSS]}$ with respect to m_1=2,5 and 10. In terms of shift in slope of transformed model, the effect of m_1 on fixed $\rho = 0.75$ are described in Figure 2.4(C). The results depict that on fixed slope shift ($\delta = 3.5$), 0.8105, 0.9425 and 0.9460 units increase in overall PTS are reported for the $Shewhart - 3_{[DMRSS]}$ with respect to m_1=2,5 and 10. The findings of several m_1 for shift in error variance at $\rho = 0.5$ are portrayed in Figure 2.4(D). On fixed error variance shift ($\gamma = 2.8$), the increase in overall PTS are reported about 0.8335, 0.8809, and 0.9210 units for the $Shewhart - 3_{[DERSS]}$ with respect to m_1=2,5 and 10.

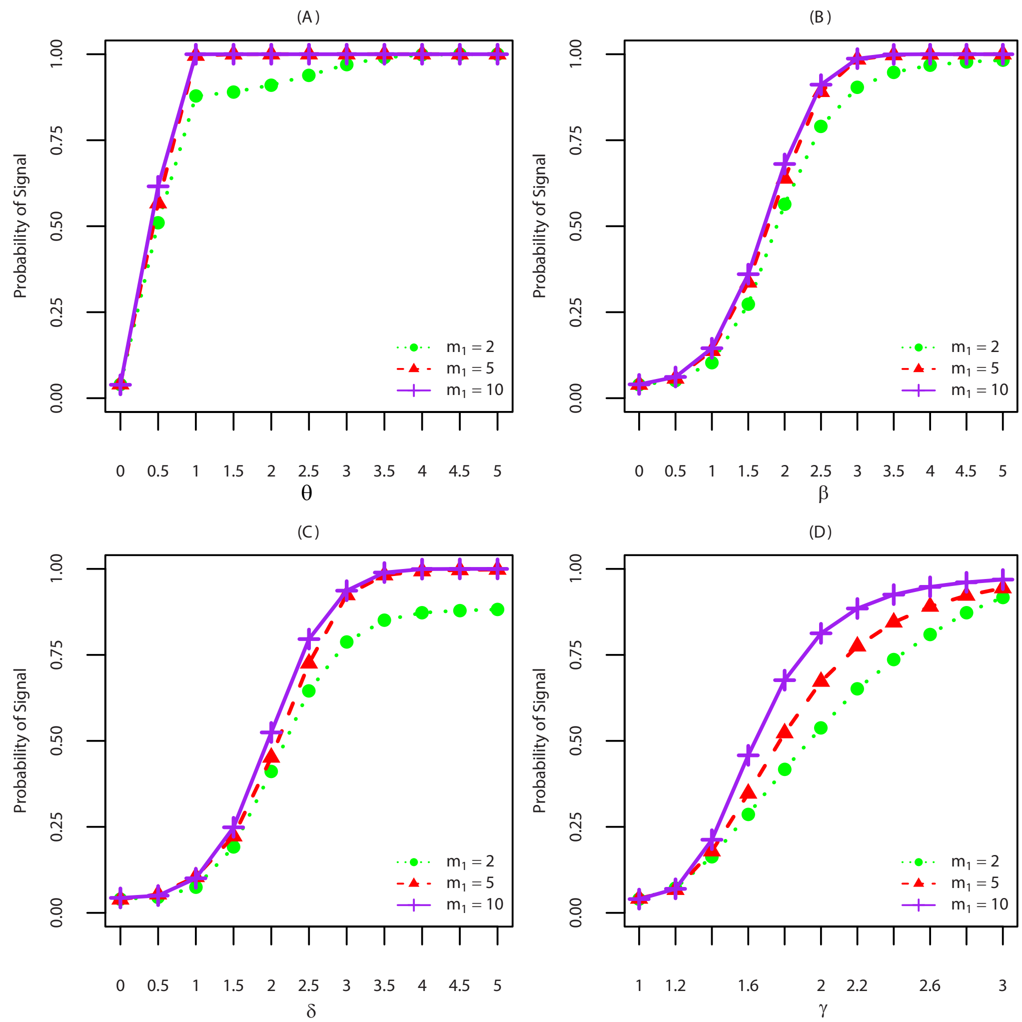

Figure 2.4: Comparison of m_1 with respect to shifts in linear profile parameters

Effect of rational subgroups (k):

The comparative analysis of subgroups with respect to linear profiles parameters (i.e. intercept, slope and error variance) are discussed in Figure 2.5. For the appropriate analysis, we fixed 10% ratio between the rational subgroups (k) and inconsistent subgroups (m_1). For example, when $k = 20$ then we fixed $m_1 = 2$ and when $k = 30, 50, 100$ and 200 then we fixed m_1=3, 5, 10 and 20 respectively.

In terms of shifts in intercept, the effect of k on fixed $\rho = 0.25$ are reported in Figure 2.5(A). The results depict that on fixed intercept shift ($\theta = 1$), 0.0146, 0.0120, 0.0172, 0.0172, 0.0182, unit increase in overall PTS are reported for the $Shewhart - 3_{[DMRSS]}$ with respect to k=20, 30, 50, 100 and 200. The results of different k for the shift in slope of original model at ρ=0.5 are discussed in Figure 2.5(B). On fixed slope shift (β=2), the increasing rate of overall PTS are reported 0.6410, 0.7330, 0.8357, 0.8742 and 0.9131 for the $Shewhart - 3_{[DMRSS]}$ with respect to k=20, 30, 50, 100 and 200.

In case of shift in slope of transformed model, the effect of k on fixed $\rho = 1$ are described in Figure 2.5(C). The results reveal that on fixed slope shift ($\delta = 3$), 0.9278, 0.9502, 0.9623 , 0.9666 and 0.9720 units increase in overall PTS are reported for the $Shewhart - 3_{[DMRSS]}$ with respect to k=20, 30, 50, 100 and 200. The findings of several k for shift in error variance at ρ=0.75 are portrayed in Figure 2.5(D). On fixed error variance shift (γ=2.6), the increase in overall PTS are reported about 0.9443, 0.9612, 0.9623, 0.9661 and 0.9661 units for the $Shewhart - 3_{[DERSS]}$ with respect to k=20, 30, 50, 100 and 200. Overall, it is depicted from the stated simulated study that the performance of our proposed schemes increased due to increase of rational subgroups (k) and inconsistent subgroups (m_1).

Effect of sample size (n):

The effect of sample size in the monitoring of linear profile parameters are discussed in Figure 2.6. For the comparative analysis, we fixed rational subgroups (k=20) and inconsistent subgroups (m_1=2) while we consider several choices of

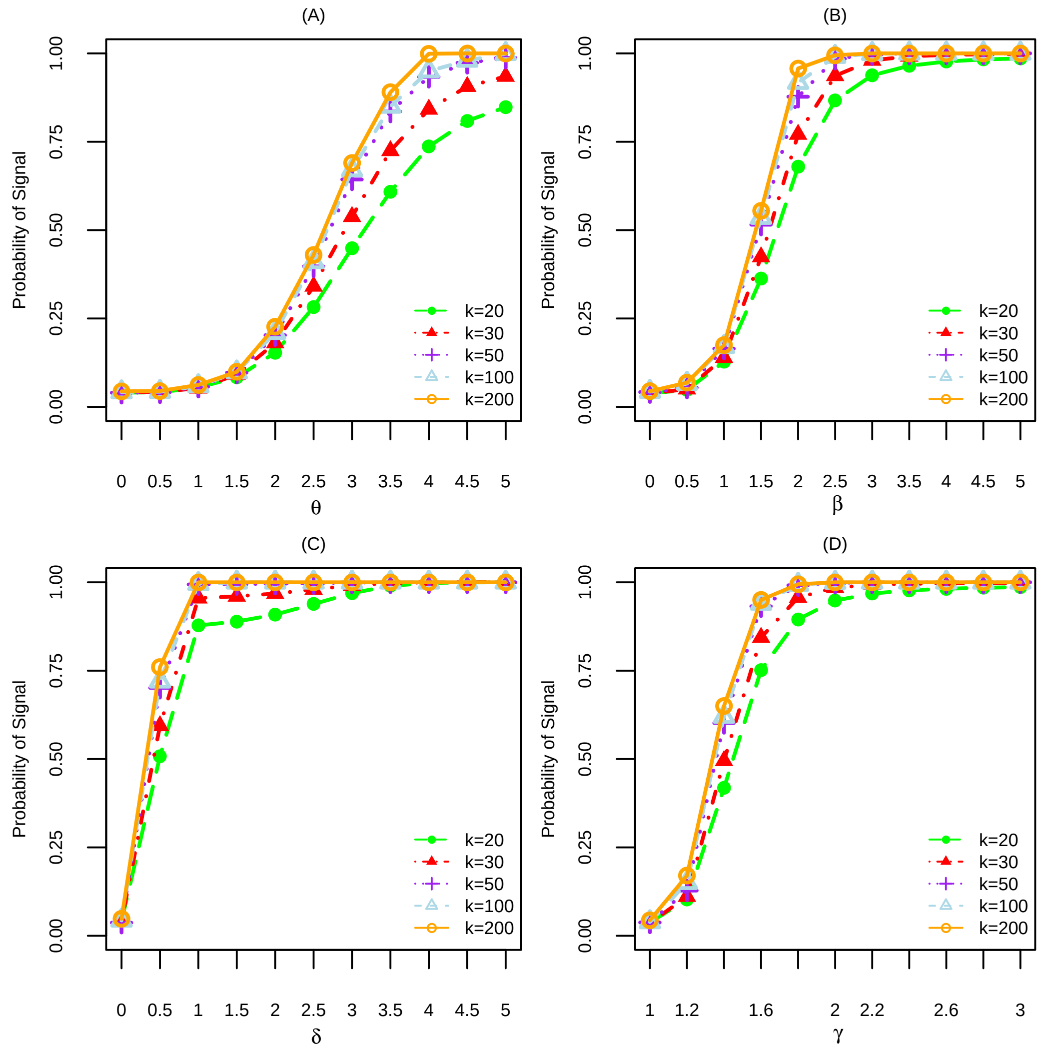

Figure 2.5: Comparison of k with respect to shifts in linear profile parameters

sample size (i.e n= 3,5,8 and 10) for the analysis purpose. On n=3, we fixed $X_{(i)}$=0,0.9,1.8 and used $X_{(i)}$=0,0.2,0.9,1.6,1.8 for the sample size n=5. In case of sample size n= 8, $X_{(i)}$=0,0.2,0.4,0.6,1.2,1.4,1.6,1.8 is used for the said study. In case of shifts in intercept, the effect of n on fixed ρ=0.25 are reported in Figure 2.6(A). The results depict that on fixed intercept shift ($\theta = 3$), 0.2851, 0.3782, 0.4039 and 0.4090, unit increase in overall PTS are reported for the $Shewhart - 3_{[DMRSS]}$ with respect to n=3,5, 8 and 10. The results of different n for the shift in slope of original model at =0.5 are discussed in Figure 2.6(B). On fixed slope shift (β=1.5), the increasing rate of overall PTS are reported almost 0.1035, 0.1712, 0.2405 and 0.3247 units for the $Shewhart-3_{[DMRSS]}$ with respect to n=3,5,8 and 10.

For the amount of shift in slope of transformed model, the effect of n on fixed ρ=1 are described in Figure 2.6(C). The results reveal that on fixed slope shift (δ=4), 0.8423, 0.8753, 0.9365 and 0.9565 units increase in overall PTS are reported for the $Shewhart-3_{[DMRSS]}$ with respect to n=3,5,8 and 10. The findings of several n for shift in error variance at ρ=0.75 are portrayed in Figure 2.6(D). On fixed error variance shift (γ=3.0), the increase in overall PTS are reported about 0.5074, 0.8412, 0.9375 and 0.9489 units for the $Shewhart-3_{[DERSS]}$ with respect to n=3,5,8 and 10. Overall, it shows that the performance of $Shewhart-3_{[\tau]}$ schemes increased due to increase in sample size (n) at fixed rational subgroups (k=20) and inconsistent subgroups (m_1=2).

2.1.4 A real life application

As discussed above that for fixed charge (Q), capacitance (C) is inversely related to voltage (V). So, for the monitoring of voltage (V) generated through Z-source inverter in grid connected PV system, we used a data set having values of V at each level of C. The implementation of $Shewhart-3_{[\tau]}$ scheme on aforementioned data set is discussed in the following steps:

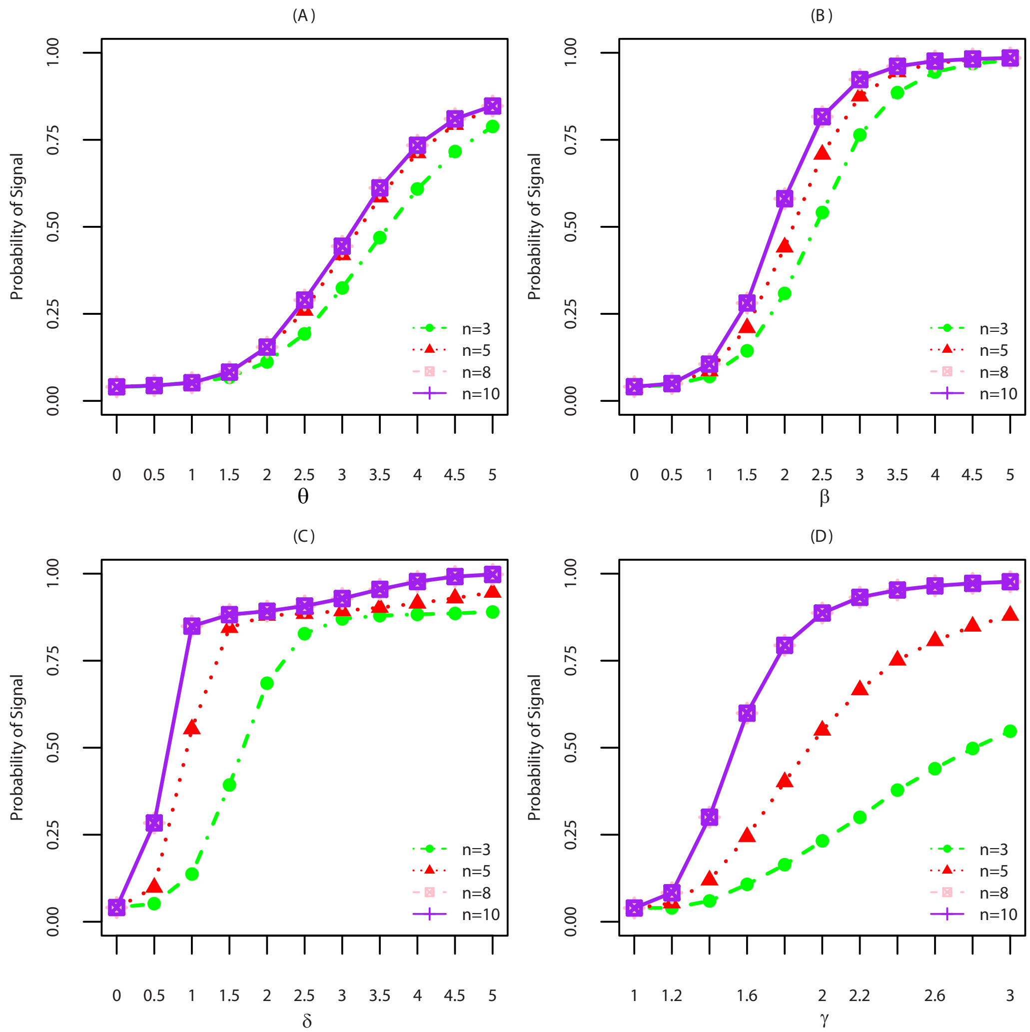

Figure 2.6: Comparison of n with respect to shifts in linear profile parameters

Step 1: Run the 75456 profiles to get the following IC regression model:

$$\hat{V} = \ 402.3512 - 0.01983691\ C$$

Further, the properties of linear regression model are reported in Appendix A.6

Step 2: Apply SRS, DMRSS and DERSS techniques on 75456 samples of V at each level of C. Finally, 1533 samples of V at each level of C are compiled though the aforementioned schemes. By using selected 1533 data sets and transformed capacitance ($C^* = -150\mu F, -100\mu F, -50\mu F, 0\mu F, 50\mu F, 100\mu F$ and$150\mu F$), we calculate 1533 profiles.

Step 3: On the base of 1533 profiles, we calculate control limits at fixed k=200 and α=0.05 for each scheme which are

$$Shewhart-3_{[SRS]} = \begin{matrix} LCL_{B_0} = 788.5457 & LCL_{B_1} = -0.1138149 & LCL_f = 0.02875877 \\ UCL_{B_0} = 803.6243 & LCL_{B_1} = 0.09670317 & UCL_f = 3.79974000 \end{matrix}$$

$$Shewhart-3_{[DMRSS]} = \begin{matrix} LCL_{B_0} = 786.4855 & LCL_{B_1} = -0.1157737 & LCL_f = 0.02933718 \\ UCL_{B_0} = 804.2855 & LCL_{B_1} = 0.10163490 & UCL_f = 3.44160200 \end{matrix}$$

$$Shewhart-3_{[DERSS]} = \begin{matrix} LCL_{B_0} = 785.6046 & LCL_{B_1} = -0.1236473 & LCL_f = 0.0114879 \\ UCL_{B_0} = 804.6087 & LCL_{B_1} = 0.11757860 & UCL_f = 3.4597220 \end{matrix}$$

The linear profile parameters under SRS schemes are plotted against their control limits in Figure 2.7. The figure shows that 3 OOC signals are reported in intercept parameter while 2 and 3 signals are reported in the slope and error variance respectively. For the $Shewhart-3_{[DMRSS]}$, shows 2 OOC signals in intercept while 3 signals are reported in both slope and error variance respectively (cf. Figure 2.8).

For the $Shewhart-3_{[DERSS]}$, statistics of linear profile parameters are plotted against limits in Figure 2.9. Where only 1 OOC signal is captured in intercept while 7 signals are reported in slope parameter. Percentages of OOC signals in IC situation are provided in Table 2.7

Step 4: For the diagnosis analysis, we used data perturbation aproach (cf.

Section A.7) to introduced intercept shift in first 50 samples, next 50 samples with index 51-100 have slope shift, next 101-150 index have joint shift in intercept and slope and finally, last 50 have error variance shift. We introduced shifts in following way:

(i) For the intercept shift, we used $C^* = -550, -500, -450, -400, -350, -300, -250$.

(ii) For the slope shift, we used $C^* = 75, 50, 25, 0, -25, -50, -75$.

(iii) For both shifts in slope and intercept, we used $C^* = -25, -50, -75, -100, -125, -150, -175$.

(iv) For the shift in error variance, we multiply each observation of voltage data set with 1.5.

The percentages of OOC signals are given in Table 2.7. In case of $Shewhart - 3_{[SRS]}$, shifts in linear profile parameters are plotted in Figure 2.10. The findings depict that 83 OOC points are reported in intercept chart while 28, 41, 2, 4 and 2 OOC points are reported with respect to slope, both intercept and slope, error variance, both intercept and error and in all parameters. Figure 2.11 represents the portrayed of OOC situation under $Shewhart - 3_{[DMRSS]}$. The results depicts that 80, 25, 53, 2, 2 and 3 OOC points are reported with respect to intercept, slope, both intercept and slope, error variance, both intercept and error and in all parameters.

The results of $Shewhart - 3_{[DERSS]}$ with respect to different shifts in linear profile parameters are plotted in Figure 2.12. The findings reveal 70 OOC points in intercept chart while 15, 49, 3, 2 and 5 OOC points are reported with respect to slope, both intercept and slope, error variance, both intercept and error and in all parameters.

In this illustrative example, the overall results depict that in case of shifts in intercept, slope and their joint shifts (intercept and slope), $Shewhart - 3_{[DMRSS]}$ detecting large number of OOC signals while $Shewhart - 3_{[DERSS]}$ have relatively good detection in the presence of shifts in error variance.

Table 2.7: The percentages of OOC points with respect to linear profile parameters

Parameters		$\mathbf{B_0}$	$\mathbf{B_1}$	$\mathbf{B_0+B_1}$	**F**	$\mathbf{B_0+F}$	$\mathbf{B_0+B_1+F}$	**Overall**
In control	**SRS**	1.5	1	-	1.5	-	-	4
	DMRSS	1	1.5	-	1.5	-	-	4
	DERSS	0.5	3.5	-	-	-	-	4
Special Cause	**SRS**	41.5	14	20.5	1	2	1	80
	DMRSS	40	12.5	26.5	1	1	1.5	82.5
	DERSS	35	7.5	24.5	1.5	1	2.5	81

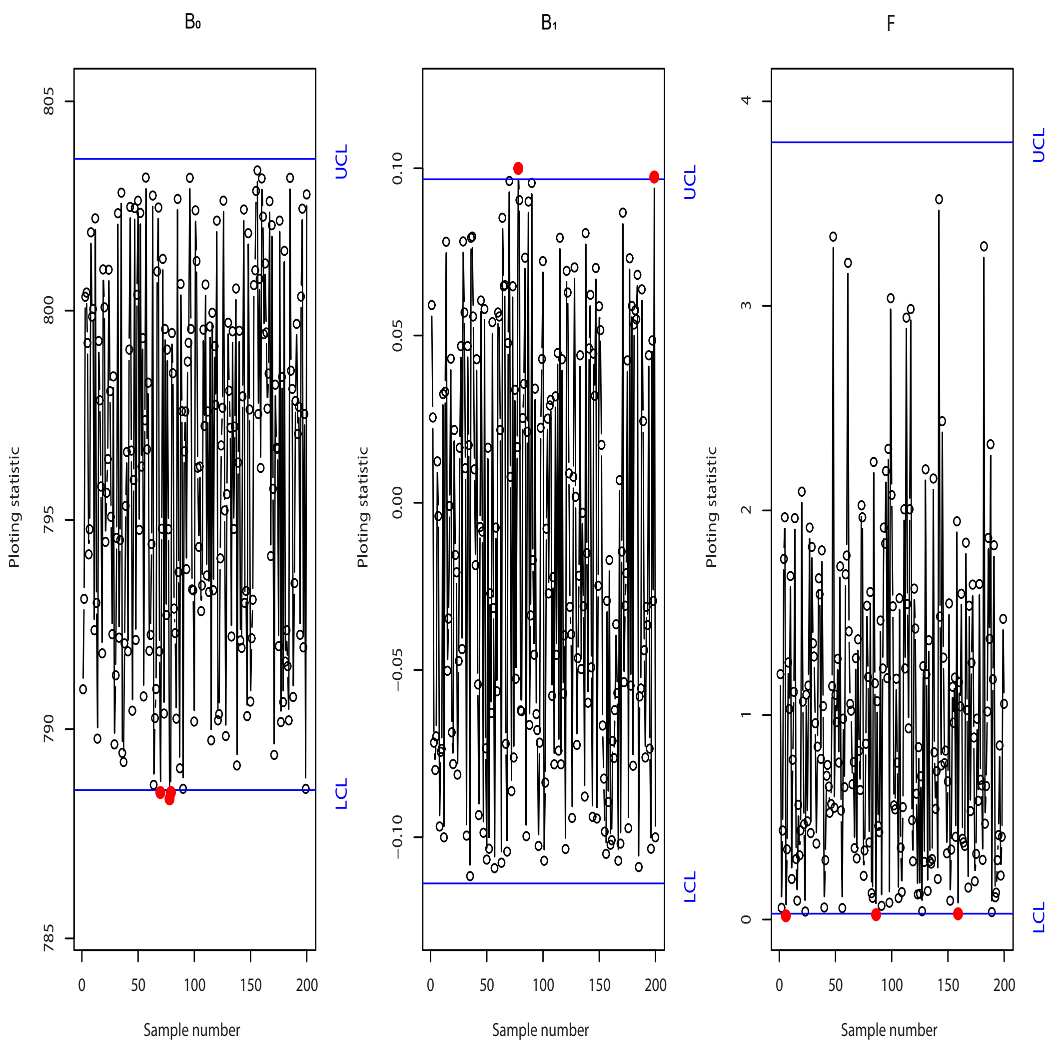

Figure 2.7: In-control situation for $Shewhart - 3_{[SRS]}$

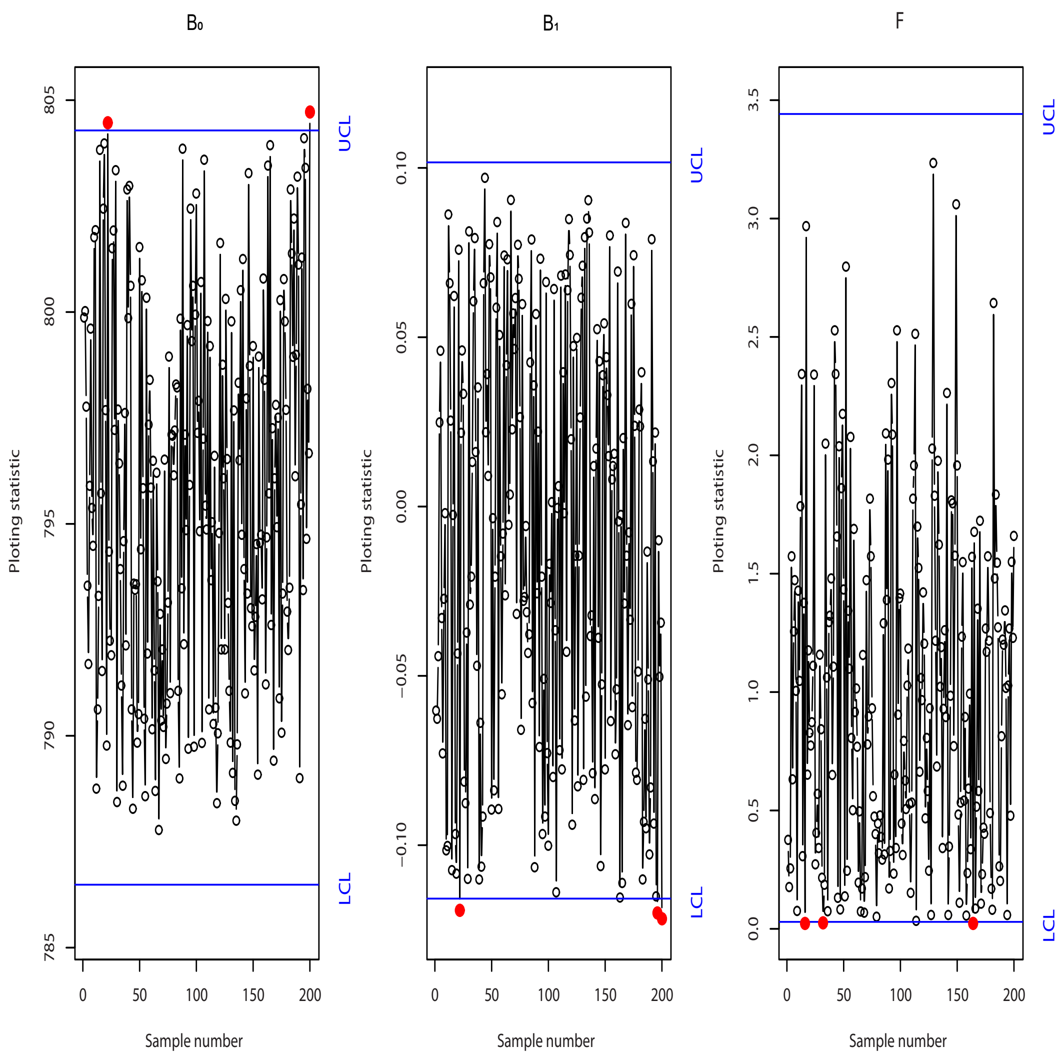

Figure 2.8: In-control situation for $Shewhart - 3_{[DMRSS]}$

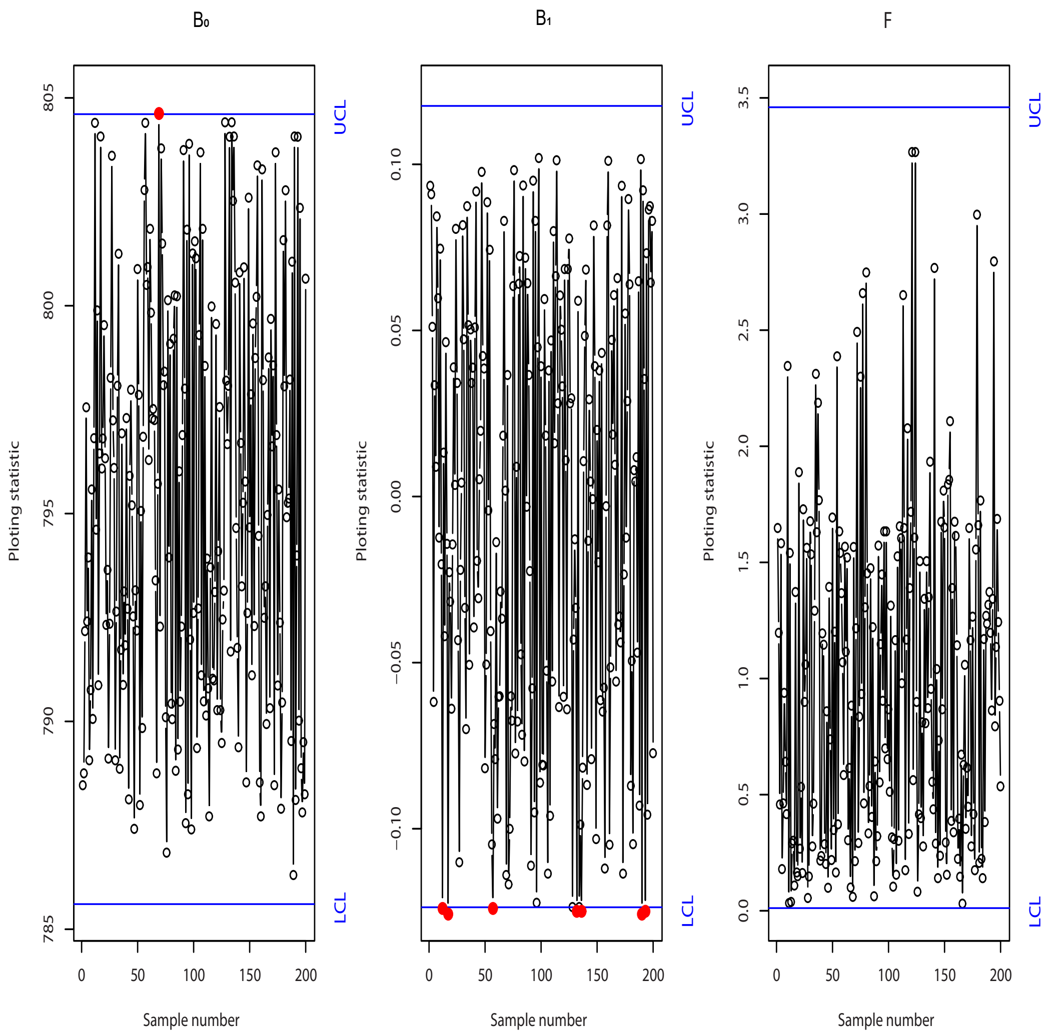

Figure 2.9: In-control situation for $Shewhart - 3_{[DERSS]}$

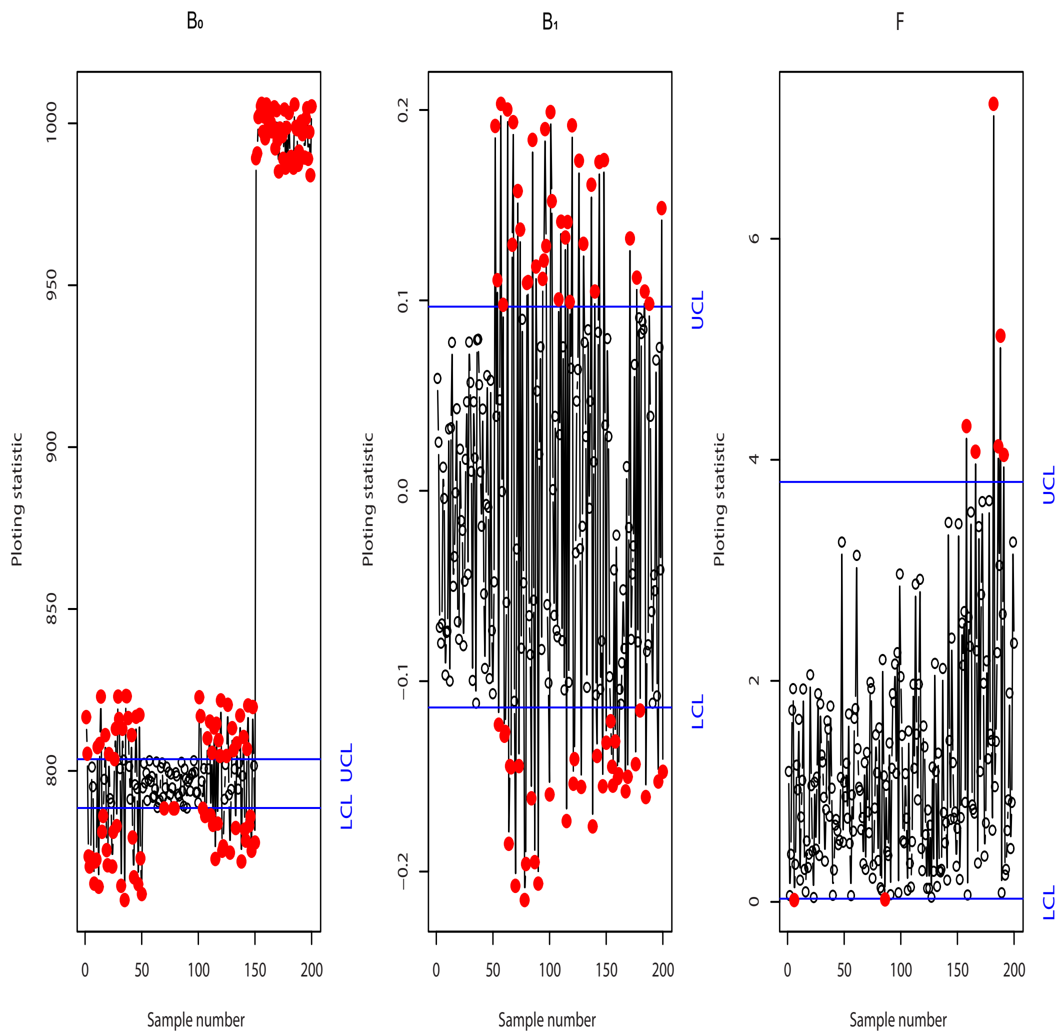

Figure 2.10: Diagnosis analysis for $Shewhart - 3_{[SRS]}$

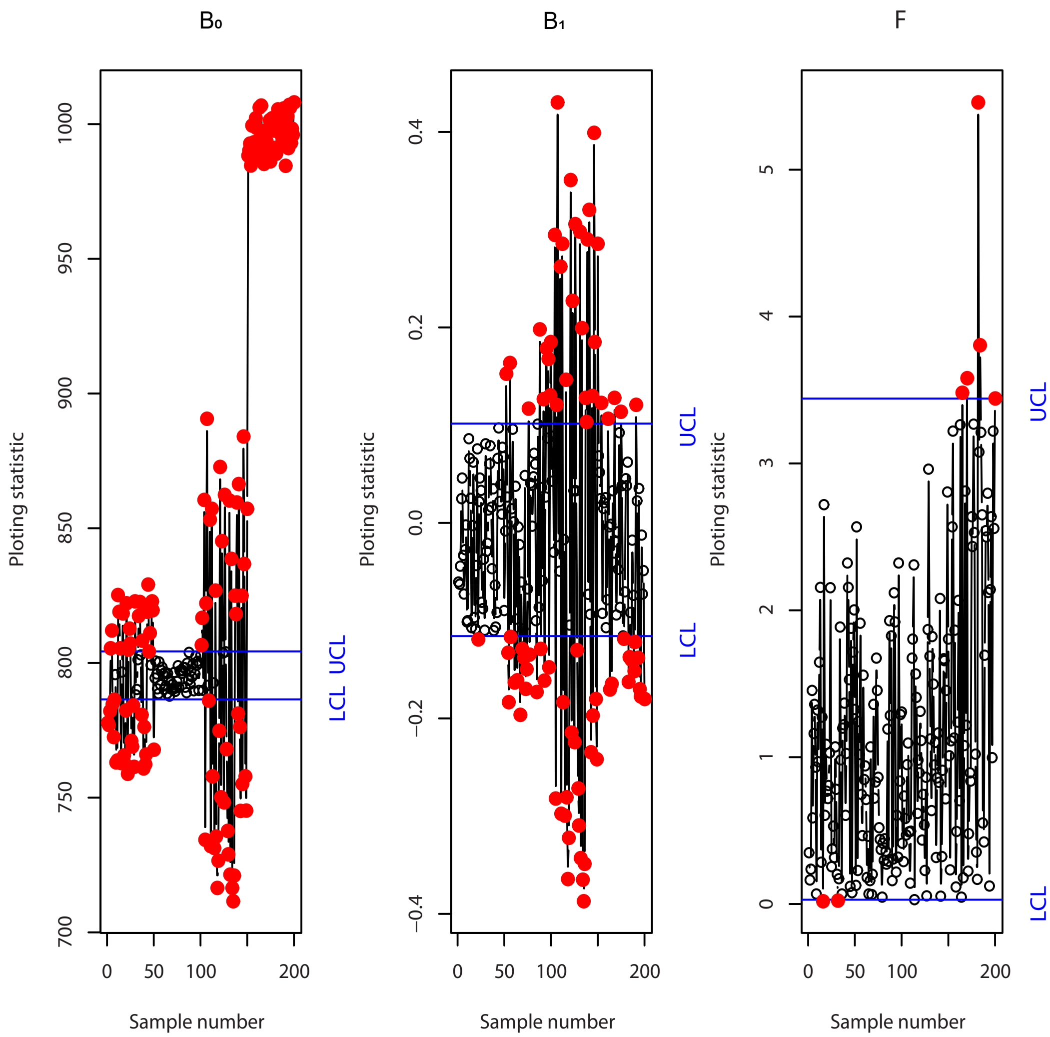

Figure 2.11: Diagnosis analysis for $Shewhart - 3_{[DMRSS]}$

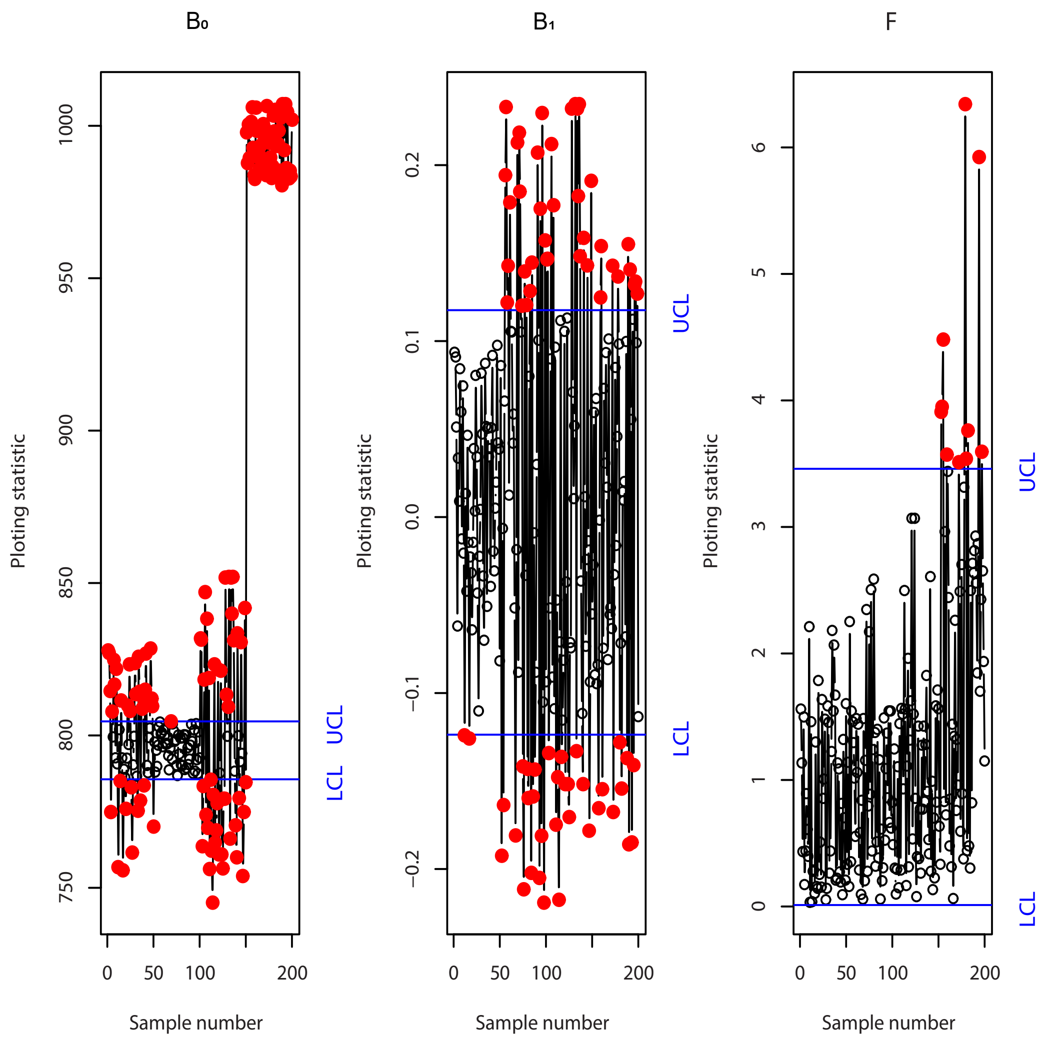

Figure 2.12: Diagnosis analysis for $Shewhart - 3_{[DERSS]}$

2.2 Phase II analysis

In recent literature (given in Section 1.2), the idea of linear profiling under Phase II was discussed using simple random sampling (SRS). The concept of ranked set sampling (RSS) was introduced by [73] and more mathematical modifications were developed by [80]. Many of the researchers used ranked set sampling in control charts to make them more sensitive against different type of shifts (cf. [76,81–86]).

[22] introduced an $EWMA-3$ control chart for simultaneous monitoring of shifts in intercept, slope and standard deviation of disturbance term. They used simple random sampling (SRS) in their study for process monitoring. In this study, we intend to use different ranked set sampling techniques to enhance the performance of the aforementioned $EWMA-3$ control chart.

EWMA − $3_{[\tau]}$ charting structure

[22] introduced $EWMA-3$ control chart under SRS ($EWMA-3_{[SRS]}$) for simultaneous monitoring of shifts in linear profile parameters including intercept, slope and standard deviation of disturbance term. We introduce here the $EWMA-3$ charting structure under different ranked set strategies (τ) on the base of transformed model (cf. equation (1.2) is defined as;

$$EWMA_{I[i][\tau]} = \lambda\left(b_{0[i]l}\right) + (1-\lambda)EWMA_{I[i-1][\tau]}$$

$$EWMA_{S[i][\tau]} = \lambda\left(b_{1[i]l}\right) + (1-\lambda)EWMA_{S[i-1][\tau]}$$

$$EWMA_{E[i][\tau]} = \max\{\ \lambda\ ln(MSE_{[i]l}) + (1-\lambda)$$

$$EWMA_{E[i-1][\tau]},\ ln({\sigma^2}_0)\}$$

where $EWMA_{I[i][\tau]}$ is the ith EWMA statistic for intercept under different samplings (τ) ; $EWMA_{S[i][\tau]}$ and $EWMA_{E[i][\tau]}$ are the ith EWMA statistics for slope and error variance respectively under different strategies (τ); λ is the smoothing parameter that ranges between zero and one (i.e. $0 < \lambda \leq 1$). The popular choices of λ fall in the interval $0.05 \leq \lambda \leq 0.25$ (cf. [87]).

The mean and variance of each of the three $EWMA_{[\tau]}$ statistic are given as;

$$E\left(EWMA_{I[i][\tau]}\right) = \ B_0,$$

$$E\left(EWMA_{S[i][\tau]}\right) = \ B_1,$$

$$E\left(EWMA_{E[i][\tau]}\right) = \ ln\left({\sigma^2}_0\right),$$

$$Var\left(EWMA_{I[i][\tau]}\right) = \ \frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}\left[\frac{1}{nr}\right],$$

$$Var\left(EWMA_{S[i][\tau]}\right) = \frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx}},$$

$$Var(EWMA_{E[i][\tau]}) = \ Var(ln(MSE_{[i]l})) \cong \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}.$$

(cf. [11]) Based on the above mentioned properties of the $EWMA_{[\tau]}$ statistics, the asymptotic limits for each $EWMA_{[\tau]}$ plotting statistic are given as:

(i) For EWMA$_{I[i][\tau]}$:

$$UCL_I = B_0 + L_{EI[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}[\frac{1}{nr}\ ]}$$
$$LCL_I = B_0 - L_{I[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\sigma^2_{e[\tau]}[\frac{1}{nr}\ ]}$$

(ii) For EWMA$_{S[i][\tau]}$:

$$UCL_S = B_1 + L_{ES[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx}}}$$
$$LCL_S = B_1 - L_{ES[\tau]}\sqrt{\frac{\lambda}{2-\lambda}\frac{\sigma^2_{e[\tau]}}{S_{xx}}}$$

(iii) For EWMA$_{E[i][\tau]}$:

$$UCL_E = ln\left({\sigma^2}_0\right) + L_{EE[\tau]}\sqrt{\frac{\lambda}{2-\lambda}Var(ln\left(MSE_{[i]l}\right))}$$

where $L_{EI[\tau]}$, $L_{ES[\tau]}$ and $L_{EE[\tau]}$ are the control limits coefficients for intercept, slope and standard deviation of error term respectively under different sampling strategies (SRS, RSS, MRSS, ERSS, DRSS, DMRSS and DERSS); $\sigma^2_{e[r]}$ is the error variance of RSS and $\sigma^2_{e[dr]}$, $\sigma^2_{e[m]}$, $\sigma^2_{e[dm]}$, $\sigma^2_{e[e1,en]}$ and $\sigma^2_{e[de1,den]}$ are the error variances of DRSS, MRSS, DMRSS, ERSS and DERSS respectively. The error variances for different ranked set samplings ($\sigma^2_{e[\tau]}$) are given in the Appendix A.2. The above mentioned three $EWMA_{[\tau]}$ statistics are combined in such a way to evaluate simultaneous monitoring of the three parameters of interest namely intercept, slope and error variance.

The findings of the $EWMA_{[\tau]}$ are reported in [17]. For the diagnostic analysis, we consider different shifts for the monitoring of linear profile parameters including intercept, slope and error variance. In case of several shifts in intercept term (θ), it is concluded that $EWMA-3_{[DMRSS]}$ scheme outperforms all other schemes under consideration. The same may also be noticed where shifts in term of σ units are considered for the slope in original model (β). The detection of shifts in error variance (γ) reveals that $EWMA-3_{[DERSS]}$ is marginally better than others while $EWMA-3_{[DMRSS]}$ is a poor performer, especially at larger values of shifts and ρ. Further, negative shifts in the slope of transformed model (δ) depicts that $EWMA-3_{[DMRSS]}$ scheme surpass all other schemes under consideration and referring to different values of independent variable exhibits that $EWMA-3_{[DMRSS]}$ scheme beats all the other schemes under consideration.

Overall in monitoring phase, when ranked set sampling is imperfect ($\rho = 0.25$), no real change is experienced in the performance $EWMA-3_{[\tau]}$ schemes and With the increase of ρ, we have observed significant improvement in the performance of $EWMA-3_{[\tau]}$ schemes. Further, as the smoothing parameter λ decrease performance of $EWMA-3_{[\tau]}$ also increased but in some cases reverse results are observed and as the sample size n increase, performance of $EWMA-3_{[\tau]}$ also increased.

2.3 Concluding remarks

In statistical process control, control chart is a key device and its implementation involves two different Phases (retrospective phase or prospective phase). Usually, retrospective phase is used to estimate the unknown parameters of the process while prospective phase emphasizes on the monitoring of process based on the estimated parameters from retrospective phase. In this chapter, we used different ranked set strategies τ such as RSS, MRSS, ERSS, DRSS, DMRSS and DERSS instead of SRS to enhance the performance of Phase I method and Phase II method.

In Phase I method, we have used overall probability to signal (PTS) as a performance measure to compare the $Shewhart-3_{[\tau]}$ and existing $Shewhart-3_{[SRS]}$ charts. The findings reveal that the proposed method $Shewhart-3_{[\tau]}$ outperforms the existing $Shewhart-3_{[SRS]}$ scheme to timely detect assignable causes in process parameters. In case of shifts in intercept and slope of original or transformed model, $Shewhart-3_{[DRSS]}$ and $Shewhart-3_{[DMRSS]}$ outperforms all the other schemes while $Shewhart-3_{[DERSS]}$ exhibits relatively better performance in the presence of shifts in error variance.

In case of simultaneous monitoring, $Shewhart-3_{[\tau]}$ takes an edge for original model as compared to transformed model to monitor slope, given process intercept is IC. For OOC intercept, $Shewhart-3_{[DMRSS]}$ offers relatively superior detection ability for shifts in slope for original/ transformed model. Moreover, the efficiency of $Shewhart-3_{[\tau]}$ scheme increases with the increasing levels of correlation used in ranked set samples (ρ), rational subgroups (k), inconsistent subgroups (m_1) and sample size (n).

In Phase II, we intend to check the performance of $EWMA - 3_{[\tau]}$ chart by introducing different amounts of shifts in linear profile parameters. The findings depict that when we introduce several amounts of shifts in intercept (B_0), slope of orignal model (β_1) and slope of tranformed model (B_1), $EWMA - 3_{[DMRSS]}$ scheme outperforms all other schemes under consideration. In the presence of shifts in error variance parameter, the detection abiliy of $EWMA - 3_{[DMRSS]}$ is very poor while $EWMA - 3_{[DERSS]}$ is marginally better than others. Overall in monitoring phase, with the increase of design parameters (correlation (ρ) and sample size (n)), we have observed significant improvement in the performance of $EWMA - 3_{[\tau]}$ schemes. Further, as the smoothing parameter λ decrease performance of $EWMA - 3_{[\tau]}$ also increased.

CHAPTER 3

LINEAR PROFILING UNDER MODIFIED SUCCESSIVE SAMPLING

In the literature, the term simple linear profiles is used for the monitoring of linear profile parameters (slope, intercept and error variance) when the study variable is linearly associated with a single explanatory variable. Most of the present literature on simple linear profiles utilizes the simple random sampling. In this chapter, we intend to enhance the monitoring of simple linear profile parameters by considering the modified successive sampling scheme which is not only cost-effective but also efficient as compared to simple random sampling scheme. Moreover, special cases of the simple linear profile monitoring are also discussed in this chapter.

3.1 Introduction

All recent studies about simple linear profiling (reported in Section 1.3.1) are designed under simple random sampling (SRS) and ranked set samplings (RSS) (cf. Chapter 2). Generally, for the single occasion inventory problem, simple random sampling (SRS) is referred in most surveys while [88] suggested the successive sampling for various occasions. The design of successive sampling considers the first sample taken at first occasion and second sample (including some points from first sample) taken at the next occasion. The design of MSS scheme consists of small number of observations which makes it cost efficient scheme as well as it is a useful technique in the presence of missing observations. Some modifications in the successive sampling can be seen in [89–92].

Recently, [93] proposed the modified form of successive sampling (MSS) for the quality characteristic variable which is defined in following steps;

Step 1: Take first sample $(Y_{1,1},\ Y_{1,2},\ Y_{1,3}, \ldots\ldots, Y_{1,n})$ of size n by using the SRS.

Step 2: Take second sample $(Y_{2,1},\ Y_{2,2},\ Y_{2,3}, \ldots\ldots, Y_{2,n-c})$ of size n-c by using the SRS and the remaining c observations are picked as percentiles points of first sample in the following way: $Y_{2,n-c+1} = P_1\left(Y_{1,1},\ Y_{1,2}, \ldots\ldots, Y_{1,n}\right)$, $Y_{2,n-c+2} = P_2\left(Y_{1,1},\ Y_{1,2}, \ldots\ldots, Y_{1,n}\right)$ and so on, up to $Y_{2,n} = P_c\left(Y_{1,1},\ Y_{1,2}, \ldots\ldots, Y_{1,n}\right)$.

Step 3: Similarly, third sample consist of n-c new observations by using the SRS and remaining c observations from the percentile points of second sample, and this procedure is repeated for the specific run of production.

In existing literature, [21] originated a phase II study named as Shewhart$-$3 chart (given in Section 1.2) for the monitoring of simple linear profile parameters under simple random sampling (SRS). In this study, we have designed similar phase II study based on successive sampling technique (cf. [88]). It is noted that modified Shewhart structure under MSS named as $Shewhart-3_{[MSS]}$ chart in the later part of this study.

3.1.1 The $Shewhart-3_{MSS}$ charting structure

The Shewhart control chart for each linear profile parameter (i.e. intercept, slope and error variance) under MSS on the base of transformed model given in equation (1.2) are defined as:

$$\text{for } b_{0[i]}: \left\{ \begin{array}{c} UCL_I = B_0 + L_{I1[MSS]}\sqrt{\sigma^2_{e[MSS]}\left[\frac{1}{nm}\right]} \\ LCL_I = B_0 - L_{I2[MSS]}\sqrt{\sigma^2_{e[MSS]}\left[\frac{1}{nm}\right]} \end{array} \right\}$$

$$\text{for } b_{1[i]}: \left\{ \begin{array}{c} UCL_S = B_1 + L_{S1[MSS]}\sqrt{\frac{\sigma^2_{e[MSS]}}{S_{xx}}} \\ LCL_S = B_1 - L_{S2[MSS]}\sqrt{\frac{\sigma^2_{e[MSS]}}{S_{xx}}} \end{array} \right\}$$

$$\text{for } \hat{\sigma}^2_{e[i]}: \left\{ \begin{array}{c} LCL_E = ln\left(\sigma_0^2\right) - L_{E1[MSS]}\sqrt{Var\left(ln\left(MSE_{[i]j}\right)\right)} \\ UCL_E = ln\left(\sigma_0^2\right) + L_{E2[MSS]}\sqrt{Var\left(ln\left(MSE_{[i]j}\right)\right)} \end{array} \right\}$$

where $Var\left(ln\left(MSE_{[i]j}\right)\right) \cong \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}$, and $L_{I1[MSS]}$, $L_{I2[MSS]}$, $L_{S1[MSS]}$, $L_{S2[MSS]}$, $L_{E1[MSS]}$ and $L_{E2[MSS]}$ are the control limits coefficients for intercept, slope and variance of error term under modified successive sampling.

In this chapter, modified successive sampling is symbolized as $MSS_{n,c,P_1,\, P_2,.......,P_c}$ where n represents sample size, the number of observations from previous sample is represented by c and $P_q \forall\, q = 1, 2, 3, \ldots\ldots, c$. is the percentile picked from the previous sample. Although, several possible values of c and $P_{q,1-q}$ can be taken but the current study considers the following two situations.

i $MSS_{n,2,P_1,\, P_2}$, where *n-2* observations are generated by using SRS and the remaining two observations are taken from the specific percentile pairs $(P_1,\ P_2)$ of the previous sample. In this study, the choice of percentile pairs $(P_1,\ P_2)$ are $(P_{0.25},\ \ P_{0.75})$, $(P_{0.30},\ \ P_{0.70})$,$(P_{0.35},\ \ P_{0.65})$, $(P_{0.40},\ \ P_{0.60})$ and $(P_{0.45},\ \ P_{0.55})$.

ii $MSS_{n,3,P_1,\, P_2,P_3}$, where *n-3* observations are generated by using SRS and the remaining three observations are taken from the specific percentile pairs $(P_1,\ P_2,\ P_3)$ of the previous sample. In this study, the choice of percentile pairs $(P_1,\ P_2,\ P_3)$ are $(P_{0.25},\ P_{0.50},\ P_{0.75})$, $(P_{0.30},\ P_{0.50},\ \ P_{0.70})$, $(P_{0.35},\ P_{0.50},\ P_{0.65})$, $(P_{0.40},\ P_{0.50},\ P_{0.60})$ and $(P_{0.45},\ P_{0.50},\ P_{0.55})$.

It is to be noted that the MSS schemes $(MSS_{n,2,P_1,\, P_2},\ MSS_{n,3,P_1,\, P_2,P_3})$ are said to be simple random sampling (SRS) when observations are not taken from of the previous sample $(i.e.\ SRS = MSS_{n,2,P_0,\, P_0}$ or $MSS_{n,3,P_0,P_0,\, P_0})$.

3.1.2 Performance evaluations

In this subsection, we provide a brief discussion on the IC parameters of proposed charts. Moreover, we will discuss the performance evaluation of the stated study.

Designing of in-control parameters and control limits

For the original IC simple linear model given in equation (1.1), we assumed $\beta_0 = 3$ and $\beta_1 = 2$ by following [21] $\left(i.e.\ Y_{[i]k} = 3 + 2X_{(i)} + \varepsilon_{[i]k}\right)$. Where the fixed values of explanatory variable are $X_{(i)} = 2,\ 4,\ 6,$ and 8 , sample size $(n = 4)$ and the error term is $\varepsilon_{[i]k} \sim N\left(s;\ \mu_s = 0,\ \sigma_s = 1\right)$. Moreover, the transformed model given in equation (1.2) is obtained by substituting the $B_0 = 3 + 2\bar{X} + (\beta\sigma)\,\bar{X}$ and $B_1 = (2 + \beta\sigma)\, X^{*}_{(i)}$. whereas, the fixed transformed values of explanatory variable are $X^{*}_{(i)} = -3,\ -1,\ 1,$ and 3 with average equals to zero.

The performance of $Shewhart - 3_{[MSS]}$ charts is evaluated in terms of average run length (ARL) which is defined as the number of samples until a signal occurs. ARL is categorized into two types, in-control average run length (ARL_0) and out-of-control average run length (ARL_1). For the fixed overall (ARL_0), we need to set the control limits coefficients including $L_{I1[MSS]}, L_{I2[MSS]}, L_{S1[MSS]}, L_{S2[MSS]}, L_{E1[MSS]}$ and $L_{E2[MSS]}$ with respect to different combinations of design parameters (n, c) and choices of percentiles. In this study, we have evaluated the results for some selective choices of these design parameters (n=5, 7 and c=2,3) and the results are reported in Table 3.1 to achieve an overall $ARL_0 = 200$. For computations, we used Monte Carlo simulation study with 10^5 iterations.

Table 3.1: Control charting constants of Shewhart-3 chart under MSS schemes

n	Scheme	$L_{I1[MSS]}$	$L_{I2[MSS]}$	$L_{S1[MSS]}$	$L_{S2[MSS]}$	$L_{E1[MSS]}$	$L_{E2[MSS]}$
5	$MSS_{n,2,P_{0.25},\,P_{0.75}}$	-2.65051	2.63689	-2.19630	2.78229	0.08673	5.43789
	$MSS_{n,2,P_{0.30},\,P_{0.70}}$	-2.66279	2.66568	-2.32100	2.71618	0.02304	5.04384
	$MSS_{n,2,P_{0.35},\,P_{0.65}}$	-2.66348	2.67022	-2.38273	2.64063	0.00769	4.85906
	$MSS_{n,2,P_{0.40},\,P_{0.60}}$	-2.63955	2.63672	-2.38533	2.55926	0.00425	4.71889
	$MSS_{n,2,P_{0.45},\,P_{0.55}}$	-2.61524	2.61429	-2.41571	2.48039	0.00270	4.71316
	$MSS_{n,3,P_{0.25},\,P_{0.50},P_{0.75}}$	-2.49249	2.50122	-2.01845	2.98639	0.00480	4.39014
	$MSS_{n,3,P_{0.30},\,P_{0.50},P_{0.70}}$	-2.42583	2.43451	-2.16765	2.76051	0.00169	4.03714
	$MSS_{n,3,P_{0.35},\,P_{0.50},P_{0.65}}$	-2.31189	2.30833	-2.24453	2.55740	0.00073	3.90108
	$MSS_{n,3,P_{0.40},\,P_{0.50},P_{0.60}}$	-2.24524	2.21959	-2.26650	2.42236	0.00051	3.83348
	$MSS_{n,3,P_{0.45},\,P_{0.50},P_{0.55}}$	-2.12171	2.10866	-2.24848	2.30943	0.00036	3.81815
7	$MSS_{n,2,P_{0.25},\,P_{0.75}}$	-2.76920	2.78558	-2.37381	2.69893	0.13854	5.59925
	$MSS_{n,2,P_{0.30},\,P_{0.70}}$	-2.78380	2.77136	-2.41385	2.63938	0.09414	5.40458
	$MSS_{n,2,P_{0.35},\,P_{0.65}}$	-2.78116	2.78970	-2.43796	2.58164	0.05909	5.28626
	$MSS_{n,2,P_{0.40},\,P_{0.60}}$	-2.75744	2.76380	-2.45170	2.56195	0.03939	5.16095
	$MSS_{n,2,P_{0.45},\,P_{0.55}}$	-2.76222	2.74351	-2.47242	2.50960	0.02792	5.18708
	$MSS_{n,3,P_{0.25},\,P_{0.50},P_{0.75}}$	-2.63886	2.63644	-2.25238	2.77901	0.05541	5.00933
	$MSS_{n,3,P_{0.30},\,P_{0.50},P_{0.70}}$	-2.62606	2.61153	-2.29772	2.67295	0.03349	4.81843
	$MSS_{n,3,P_{0.35},\,P_{0.50},P_{0.65}}$	-2.57092	2.57175	-2.32866	2.55189	0.01955	4.69043
	$MSS_{n,3,P_{0.40},\,P_{0.50},P_{0.60}}$	-2.55048	2.54438	-2.35341	2.49075	0.01366	4.66880
	$MSS_{n,3,P_{0.45},\,P_{0.50},P_{0.55}}$	-2.52279	2.53287	-2.34390	2.41319	0.01221	4.65537

Shifts for performance evaluation

In order to evaluate the performance of $Shewhart-3_{[MSS]}$ charts, we have considered several amount of shifts in linear profile parameters (ψ). The description of shifts in linear profile parameters are given as follows:

(i) Shifts in intercept parameter $\left(B_0 \; to \; B_0 + \theta\left(\sigma_{e[\mathrm{MSS}]}/\sqrt{n}\right)\right)$,

(ii) Shifts in slope parameter $\left(B_1 \; to \; B_1 + \delta\left(\sigma_{e[\mathrm{MSS}]}/\sqrt{S_{\mathrm{xx}}}\right)\right)$,

(iii) Shifts in slope parameter $\left(\beta_1 \; to \; \beta_1 + \beta\left(\sigma_{e[\mathrm{MSS}]}/\sqrt{S_{\mathrm{xx}}}\right)\right)$,

(iv) Shifts in error variance $\left(\sigma^2_{e[\mathrm{MSS}]} \; to \; \gamma^2\sigma^2_{e[\mathrm{MSS}]}\right)$,

where the size of shifts in intercept parameter are quantified as ($\theta =$ 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0), for slope parameter (δ and $\beta =$ 0.025, 0.05, 0.075, 0.1, 0.125, 0.150, 0.175, 0.2, 0.225 and 0.25) and for the variance of disturbance term shifts are enumerated as (γ = 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8 and 3.0). It is noted that process is said to be IC when θ, β and δ are equal to zero and $\gamma = 1$ otherwise, process is said to be OOC.

3.2 Comparative analysis

In this section, we discuss the comparative results of $Shewhart-3$ chart under different sampling environments. The average run length *(ARL)*, standard deviation of run length *SDRL* and different percentiles (25^{th}, 75^{th} and 95^{th}) of run length distribution are provided in Tables 3.2-3.10 and ARL curves are plotted in Figures 3.1-3.4. Further, the performance of $Shewhart-3$ chart under different sampling environments is discussed in terms of percentage change in the ARL_1 which is obtained as:

$$Percentage\ change\ = \frac{ARL_0 - ARL_1}{ARL_0}$$

3.2.1 Shifts in intercept parameter:

The results for $Shewhart-3_{[SRS]}$ chart, $Shewhart-3_{[MSS]}$ charts at fixed $c = 2$ and $c = 3$ under shifted intercept parameter are reported in Tables 3.2-3.4 respectively. Which shows that (100%) upward shift in intercept parameter, may decrease 97.31% and 98.41% ARL_1 of $Shewhart-3_{[SRS]}$ chart, 98.08% and 98.72%

ARL_1 of $Shewhart-3_{[MSS_{n,2,P_{0.25},\ P_{0.75}}]}$ chart and 98.00% and of 98.83%, ARL_1 of $Shewhart-3_{[MSS_{n,2,P_{0.25},\ P_{0.75}}]}$ chart for the both cases of n (i.e. n=5 and 7) respectively. Moreover, the ARL curves for shifted intercept parameter are plotted in Figure 3.1, which reveals that $Shewhart{-}3_{[MSS]}$ charts have better performance as compared to $Shewhart{-}3_{[SRS]}$ chart. Specifically, $Shewhart{-}3_{[MSS]}$ chart with percentile choices $(P_{0.45},\ P_{0.55})$ and $(P_{0.45}, P_{0.50},\ P_{0.55})$ outperforms all others except in case when design parameters are n=*7*and c=*2*.

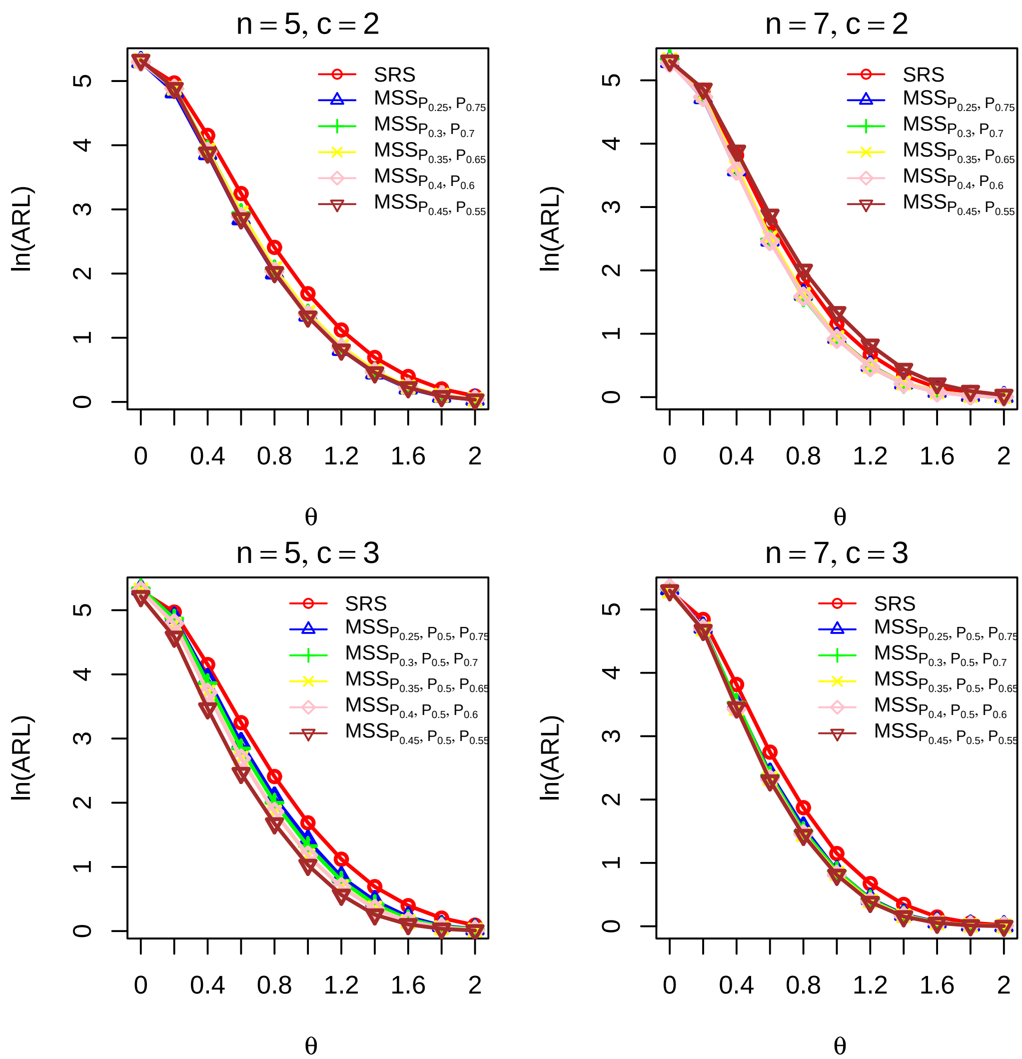

Figure 3.1: ARL curves of $Shewhart-3$ chart under different sampling schemes for intercept shifts $\left(B_0\ to\ B_0+\theta\left(\sigma_{e[\mathbf{MSS}]}/\sqrt{n}\right)\right)$

Table 3.2: Performance of linear profile parameters under SRS

ψ	Shifts	n=5					n=7				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
θ	0.000	200.80	199.53	58.00	276.00	597.00	199.34	198.77	57.00	276.00	598.00
	0.200	144.24	141.98	42.75	200.00	425.05	126.75	126.97	36.00	176.00	383.05
	0.400	63.62	62.80	19.00	89.00	193.00	45.44	44.53	13.00	63.00	134.00
	0.600	25.72	25.41	8.00	36.00	77.00	15.62	15.28	5.00	21.00	46.00
	0.800	11.11	10.71	4.00	15.00	33.00	6.51	6.03	2.00	9.00	18.00
	1.000	5.41	4.88	2.00	7.00	15.00	3.16	2.62	1.00	4.00	8.00
	1.200	3.07	2.52	1.00	4.00	8.00	1.96	1.35	1.00	2.00	5.00
	1.400	2.00	1.41	1.00	3.00	5.00	1.41	0.76	1.00	2.00	3.00
	1.600	1.49	0.86	1.00	2.00	3.00	1.16	0.43	1.00	1.00	2.00
	1.800	1.22	0.52	1.00	1.00	2.00	1.06	0.24	1.00	1.00	2.00
	2.000	1.10	0.32	1.00	1.00	2.00	1.02	0.13	1.00	1.00	1.00
δ	0.000	200.72	199.02	57.00	276.00	606.00	200.14	201.41	57.00	277.00	608.00
	0.025	191.12	191.52	54.00	263.00	578.05	176.52	177.91	51.00	245.00	530.05
	0.050	168.22	169.77	48.00	234.00	505.00	127.51	127.41	37.00	177.00	382.05
	0.075	135.74	136.21	39.00	188.00	402.05	78.49	78.25	23.00	108.00	237.05
	0.100	108.38	107.57	30.00	150.25	325.05	45.89	45.15	14.00	64.00	138.00
	0.125	77.93	75.92	23.00	109.00	228.00	26.86	26.54	8.00	37.00	80.00
	0.150	58.35	57.36	17.00	80.00	172.05	16.03	15.49	5.00	22.00	47.00
	0.175	40.82	40.04	12.00	56.00	121.00	9.85	9.14	3.00	14.00	28.00
	0.200	30.19	29.32	9.00	42.00	89.00	6.47	5.85	2.00	9.00	18.00
	0.225	21.95	21.35	7.00	30.00	65.00	4.44	3.93	2.00	6.00	12.00
	0.250	15.82	15.17	5.00	22.00	46.00	3.21	2.67	1.00	4.00	8.00
β	0.000	201.67	200.37	59.00	278.00	602.05	202.40	201.99	58.00	278.00	609.00
	0.025	159.61	160.09	44.00	222.00	476.05	116.79	116.37	33.00	162.00	345.00
	0.050	89.93	89.91	26.00	124.00	270.05	40.63	39.74	12.00	56.00	121.00
	0.075	46.81	45.96	14.00	65.00	138.00	14.18	13.60	4.00	19.00	41.00
	0.100	23.08	22.37	7.00	32.00	68.00	5.97	5.44	2.00	8.00	17.00
	0.125	12.10	11.47	4.00	17.00	35.00	2.97	2.47	1.00	4.00	8.00
	0.150	6.96	6.51	2.00	9.00	20.00	1.84	1.23	1.00	2.00	4.00
	0.175	4.30	3.71	2.00	6.00	12.00	1.35	0.70	1.00	2.00	3.00
	0.200	2.89	2.35	1.00	4.00	8.00	1.14	0.41	1.00	1.00	2.00
	0.225	2.12	1.54	1.00	3.00	5.00	1.04	0.21	1.00	1.00	1.00
	0.250	1.65	1.05	1.00	2.00	4.00	1.01	0.11	1.00	1.00	1.00
γ	1.000	199.32	199.82	56.00	276.00	610.00	198.78	196.16	57.00	279.00	596.00
	1.200	37.45	36.97	11.00	52.00	112.00	33.19	33.30	10.00	45.25	100.00
	1.400	11.76	11.32	4.00	16.00	34.05	9.48	9.04	3.00	13.00	27.00
	1.600	5.60	5.13	2.00	8.00	16.00	4.39	3.90	2.00	6.00	12.00
	1.800	3.40	2.85	1.00	5.00	9.00	2.67	2.11	1.00	3.00	7.00
	2.000	2.42	1.84	1.00	3.00	6.00	1.93	1.36	1.00	2.00	5.00
	2.200	1.95	1.37	1.00	2.00	5.00	1.54	0.91	1.00	2.00	3.00
	2.400	1.65	1.05	1.00	2.00	4.00	1.34	0.68	1.00	2.00	3.00
	2.600	1.44	0.80	1.00	2.00	3.00	1.23	0.54	1.00	1.00	2.00
	2.800	1.33	0.66	1.00	2.00	3.00	1.15	0.42	1.00	1.00	2.00
	3.000	1.25	0.56	1.00	1.00	2.00	1.11	0.35	1.00	1.00	2.00

Table 3.3: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in intercept parameter at fixed c=2

Schemes	θ	n=5					n=7				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,2,P_{0.25},P_{0.75}}$	0.00	200.34	212.24	51.00	279.00	625.05	199.01	205.28	54.00	274.00	604.00
	0.20	124.99	129.52	31.00	178.00	384.00	112.65	114.15	31.00	157.00	338.00
	0.40	47.73	49.62	12.00	68.00	148.00	36.05	36.35	10.00	50.00	110.00
	0.60	17.30	18.38	4.00	24.00	54.00	11.88	12.02	3.00	17.00	36.00
	0.80	7.44	7.81	2.00	10.00	23.00	5.05	4.87	1.00	7.00	15.00
	1.00	3.85	3.89	1.00	5.00	12.00	2.56	2.21	1.00	3.00	7.00
	1.20	2.26	1.99	1.00	3.00	6.00	1.64	1.18	1.00	2.00	4.00
	1.40	1.57	1.12	1.00	2.00	4.00	1.25	0.60	1.00	1.00	2.00
	1.60	1.24	0.64	1.00	1.00	3.00	1.09	0.34	1.00	1.00	2.00
	1.80	1.09	0.35	1.00	1.00	2.00	1.02	0.16	1.00	1.00	1.00
	2.00	1.04	0.21	1.00	1.00	1.00	1.01	0.08	1.00	1.00	1.00
$MSS_{n,2,P_{0.30},P_{0.70}}$	0.00	199.52	203.67	54.75	279.00	598.05	207.77	211.47	58.00	289.25	631.05
	0.20	131.78	134.53	35.00	185.00	398.00	114.03	119.27	31.00	158.00	342.05
	0.40	50.94	52.62	13.00	71.00	154.00	35.75	36.50	10.00	50.00	108.00
	0.60	19.00	19.90	5.00	26.00	58.00	12.06	12.27	3.00	17.00	37.00
	0.80	7.89	8.04	2.00	11.00	24.00	4.83	4.67	1.00	7.00	14.00
	1.00	3.96	3.93	1.00	5.00	12.00	2.52	2.19	1.00	3.00	7.00
	1.20	2.32	2.05	1.00	3.00	7.00	1.61	1.10	1.00	2.00	4.00
	1.40	1.60	1.17	1.00	2.00	4.00	1.23	0.60	1.00	1.00	2.00
	1.60	1.26	0.67	1.00	1.00	3.00	1.09	0.33	1.00	1.00	2.00
	1.80	1.10	0.38	1.00	1.00	2.00	1.03	0.17	1.00	1.00	1.00
	2.00	1.04	0.22	1.00	1.00	1.00	1.01	0.07	1.00	1.00	1.00
$MSS_{n,2,P_{0.35},P_{0.65}}$	0.00	204.37	204.88	56.00	287.00	611.05	204.67	206.68	55.75	288.00	621.00
	0.20	132.68	135.41	36.00	184.00	409.00	117.14	118.93	32.00	166.00	357.05
	0.40	50.53	52.81	13.00	69.00	155.05	37.76	37.93	10.00	53.00	114.00
	0.60	19.01	20.29	5.00	26.00	59.00	12.35	12.45	3.00	17.00	38.00
	0.80	7.98	8.28	2.00	11.00	24.00	5.05	4.83	1.00	7.00	15.00
	1.00	4.02	3.92	1.00	5.00	12.00	2.55	2.24	1.00	3.00	7.00
	1.20	2.37	2.13	1.00	3.00	7.00	1.63	1.13	1.00	2.00	4.00
	1.40	1.62	1.22	1.00	2.00	4.00	1.25	0.62	1.00	1.00	3.00
	1.60	1.26	0.68	1.00	1.00	3.00	1.09	0.32	1.00	1.00	2.00
	1.80	1.11	0.41	1.00	1.00	2.00	1.03	0.17	1.00	1.00	1.00
	2.00	1.04	0.22	1.00	1.00	1.00	1.01	0.08	1.00	1.00	1.00
$MSS_{n,2,P_{0.40},P_{0.60}}$	0.00	199.29	201.20	54.00	281.00	597.00	195.05	198.28	54.00	273.00	595.10
	0.20	131.97	135.10	36.00	182.00	403.00	113.88	114.28	32.00	159.00	337.05
	0.40	49.27	50.92	13.00	69.00	150.00	35.68	36.52	9.00	50.00	109.00
	0.60	17.87	18.39	5.00	24.00	54.00	11.70	11.77	3.00	16.00	35.00
	0.80	7.63	7.82	2.00	11.00	23.00	4.90	4.77	1.00	7.00	15.00
	1.00	3.88	3.82	1.00	5.00	12.00	2.51	2.16	1.00	3.00	7.00
	1.20	2.30	2.05	1.00	3.00	7.00	1.60	1.11	1.00	2.00	4.00
	1.40	1.58	1.15	1.00	2.00	4.00	1.24	0.60	1.00	1.00	2.00
	1.60	1.25	0.66	1.00	1.00	3.00	1.08	0.31	1.00	1.00	2.00
	1.80	1.10	0.40	1.00	1.00	2.00	1.02	0.17	1.00	1.00	1.00
	2.00	1.04	0.22	1.00	1.00	1.00	1.01	0.08	1.00	1.00	1.00
$MSS_{n,2,P_{0.45},P_{0.55}}$	0.00	205.58	208.93	54.00	291.00	620.00	201.76	207.39	53.75	280.00	612.05
	0.20	132.52	135.00	36.00	186.00	404.00	129.48	131.41	34.00	181.00	396.00
	0.40	48.46	49.68	13.00	67.00	149.00	48.68	50.05	13.00	68.00	148.00
	0.60	17.39	18.41	4.00	24.00	54.00	17.72	18.61	4.00	25.00	55.00
	0.80	7.52	7.94	2.00	10.00	23.00	7.43	7.79	2.00	10.00	23.00
	1.00	3.77	3.80	1.00	5.00	12.00	3.81	3.77	1.00	5.00	11.00
	1.20	2.25	1.99	1.00	3.00	6.00	2.28	2.05	1.00	3.00	6.00
	1.40	1.58	1.16	1.00	2.00	4.00	1.55	1.13	1.00	2.00	4.00
	1.60	1.25	0.69	1.00	1.00	3.00	1.23	0.64	1.00	1.00	3.00
	1.80	1.09	0.36	1.00	1.00	2.00	1.10	0.39	1.00	1.00	2.00
	2.00	1.03	0.21	1.00	1.00	1.00	1.03	0.20	1.00	1.00	1.00

Table 3.4: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in intercept parameter at fixed c=3

Schemes	θ	n=5					n=7				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,3,P_{0.25},P_{0.50},P_{0.75}}$	0.00	205.62	213.73	53.00	287.00	638.05	205.05	215.09	54.00	288.00	643.00
	0.20	132.50	136.90	34.00	186.00	405.00	111.81	114.73	29.00	158.00	336.00
	0.40	51.03	54.52	12.00	72.00	162.00	33.59	34.55	9.00	47.00	103.00
	0.60	18.80	20.30	4.00	27.00	60.00	11.16	11.72	3.00	15.00	34.00
	0.80	8.04	8.77	1.00	11.00	26.00	4.78	4.81	1.00	7.00	15.00
	1.00	4.10	4.42	1.00	6.00	13.00	2.40	2.22	1.00	3.00	7.00
	1.20	2.32	2.32	1.00	3.00	7.00	1.54	1.10	1.00	2.00	4.00
	1.40	1.60	1.35	1.00	2.00	4.00	1.20	0.59	1.00	1.00	2.00
	1.60	1.25	0.75	1.00	1.00	3.00	1.07	0.30	1.00	1.00	2.00
	1.80	1.09	0.39	1.00	1.00	2.00	1.02	0.14	1.00	1.00	1.00
	2.00	1.03	0.21	1.00	1.00	1.00	1.00	0.06	1.00	1.00	1.00
$MSS_{n,3,P_{0.30},P_{0.50},P_{0.70}}$	0.00	208.87	219.15	54.00	289.00	642.00	204.30	210.23	55.00	285.00	626.00
	0.20	130.32	136.79	33.00	185.00	402.00	109.96	111.97	29.00	154.00	336.00
	0.40	47.64	51.81	11.00	66.00	150.00	34.11	35.78	9.00	48.00	103.00
	0.60	17.21	19.08	3.00	25.00	54.00	10.81	11.35	3.00	15.00	33.00
	0.80	7.38	8.36	1.00	10.00	24.00	4.50	4.52	1.00	6.00	14.00
	1.00	3.76	4.15	1.00	5.00	12.00	2.39	2.21	1.00	3.00	7.00
	1.20	2.19	2.15	1.00	3.00	7.00	1.53	1.09	1.00	2.00	4.00
	1.40	1.52	1.21	1.00	1.00	4.00	1.19	0.55	1.00	1.00	2.00
	1.60	1.20	0.66	1.00	1.00	2.00	1.06	0.28	1.00	1.00	2.00
	1.80	1.07	0.36	1.00	1.00	2.00	1.02	0.15	1.00	1.00	1.00
	2.00	1.03	0.21	1.00	1.00	1.00	1.00	0.07	1.00	1.00	1.00
$MSS_{n,3,P_{0.35},P_{0.50},P_{0.65}}$	0.00	202.91	212.07	50.00	292.00	628.05	197.83	202.26	52.00	278.00	605.00
	0.20	119.34	127.77	27.00	167.00	378.05	109.16	114.29	28.00	151.00	340.00
	0.40	41.32	45.75	8.00	58.00	136.00	31.15	32.45	8.00	44.00	97.00
	0.60	14.83	17.06	2.00	21.00	49.00	10.34	10.93	2.00	14.00	32.00
	0.80	6.38	7.46	1.00	9.00	22.00	4.27	4.29	1.00	6.00	13.00
	1.00	3.24	3.65	1.00	4.00	10.00	2.30	2.08	1.00	3.00	7.00
	1.20	1.99	1.98	1.00	2.00	6.00	1.49	1.04	1.00	2.00	4.00
	1.40	1.43	1.13	1.00	1.00	4.00	1.18	0.55	1.00	1.00	2.00
	1.60	1.16	0.62	1.00	1.00	2.00	1.06	0.27	1.00	1.00	1.00
	1.80	1.05	0.29	1.00	1.00	1.00	1.02	0.14	1.00	1.00	1.00
	2.00	1.02	0.15	1.00	1.00	1.00	1.00	0.07	1.00	1.00	1.00
$MSS_{n,3,P_{0.40},P_{0.50},P_{0.60}}$	0.00	200.90	211.95	47.00	281.25	633.00	207.65	213.21	57.00	290.00	631.05
	0.20	121.01	128.88	28.00	170.00	381.05	109.47	114.62	28.00	153.00	335.00
	0.40	41.10	45.17	8.00	58.00	135.00	30.87	32.35	7.00	44.00	95.00
	0.60	14.73	16.87	2.00	21.00	49.00	10.18	10.88	2.00	14.00	32.00
	0.80	6.41	7.37	1.00	9.00	22.00	4.28	4.42	1.00	6.00	13.00
	1.00	3.25	3.66	1.00	4.00	11.00	2.26	2.08	1.00	3.00	7.00
	1.20	1.98	2.01	1.00	2.00	6.00	1.47	1.03	1.00	2.00	4.00
	1.40	1.41	1.07	1.00	1.00	4.00	1.17	0.52	1.00	1.00	2.00
	1.60	1.16	0.60	1.00	1.00	2.00	1.05	0.26	1.00	1.00	1.00
	1.80	1.06	0.31	1.00	1.00	1.00	1.01	0.11	1.00	1.00	1.00
	2.00	1.02	0.15	1.00	1.00	1.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,3,P_{0.45},P_{0.50},P_{0.55}}$	0.00	183.98	210.57	30.00	262.00	615.00	200.15	207.45	52.00	279.00	618.05
	0.20	97.81	111.77	15.00	141.00	321.05	106.89	113.02	26.00	151.00	334.05
	0.40	32.03	37.42	4.00	47.00	107.00	31.49	33.55	7.00	45.00	98.00
	0.60	11.70	14.37	2.00	17.00	42.00	9.96	10.88	2.00	14.00	32.00
	0.80	5.34	6.65	1.00	7.00	20.00	4.21	4.38	1.00	6.00	13.00
	1.00	2.80	3.42	1.00	3.00	10.00	2.25	2.11	1.00	3.00	7.00
	1.20	1.76	1.80	1.00	2.00	5.00	1.46	1.03	1.00	2.00	4.00
	1.40	1.29	0.92	1.00	1.00	3.00	1.17	0.53	1.00	1.00	2.00
	1.60	1.11	0.49	1.00	1.00	2.00	1.05	0.27	1.00	1.00	1.00
	1.80	1.04	0.24	1.00	1.00	1.00	1.01	0.13	1.00	1.00	1.00
	2.00	1.01	0.11	1.00	1.00	1.00	1.00	0.06	1.00	1.00	1.00

3.2.2 Shifts in slope parameter of transformed model:

The Table 3.2, 3.5 and 3.6 are about the results for shifted slope parameter of transformed model in $Shewhart-3_{[SRS]}$ chart, $Shewhart-3_{[MSS]}$ charts at fixed $c=2$ and 3 respectively. Which reveals that (17.5%) upward shift in slope parameter of transformed model, may decrease 79.67% and 95.08% ARL_1 of $Shewhart-3_{[SRS]}$ chart, 88.74% and 97.74% ARL_1 of $Shewhart-3_{\left[MSS_{n,2,P_{0.30},\ P_{0.70}}\right]}$ chart and 89.58% and 97.90% ARL_1 of $Shewhart-3_{\left[MSS_{n,3,P_{0.30},P_{0.5},\ P_{0.70}}\right]}$ chart for n=5 and 7 respectively. However, the ARL curves for shifted slope parameter of transformed model are plotted in Figure 3.2, which shows that $Shewhart-3_{[MSS]}$ charts have relatively better performance as compared to $Shewhart-3_{[SRS]}$ chart. Specifically, $Shewhart-3_{[MSS]}$ chart under percentile choices $(P_{0.45},\ P_{0.55})$ and $(P_{0.45}, P_{0.50},\ P_{0.55})$ outperforms all others except in case when design parameters are n=*7* and c=*2*.

3.2.3 Shifts in slope parameter of original model:

The results for $Shewhart-3_{[SRS]}$ chart, $Shewhart-3_{[MSS]}$ charts at fixed c=2 and 3, under shifted slope parameter of orignal model are reported in Tables 3.2, 3.7 and 3.8 respectively. Which shows that (7.5%) upward shift in intercept parameter, may decrease 76.79% and 92.99% ARL_1 of $Shewhart-3_{[SRS]}$ chart, 83.84% and 83.81% ARL_1 of $Shewhart-3_{\left[MSS_{n,2,P_{0.45},\ P_{0.55}}\right]}$ chart and 88.76% and 95.67% ARL_1 of $Shewhart-3_{\left[MSS_{n,3,P_{0.45},P_{0.5},\ P_{0.55}}\right]}$ chart for the both cases of n (i.e. n=5 and 7 respectively. Moreover, the ARL curves for shifted intercept parameter are

Table 3.5: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the slope of transformed model parameter at fixed *c=2*

Schemes	δ	***n=5***					***n=7***				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,2,P_{0.25},P_{0.75}}$	0.000	194.12	205.30	48.00	272.00	596.05	195.65	200.48	54.00	273.00	597.00
	0.025	184.70	192.26	46.00	261.00	567.00	168.13	172.86	45.00	234.00	501.05
	0.050	148.82	156.08	37.00	210.00	453.00	105.03	108.26	28.00	146.00	320.05
	0.075	110.80	114.49	28.75	156.00	334.00	54.60	55.69	15.00	76.00	166.00
	0.100	74.61	78.86	18.00	106.00	234.00	27.53	27.57	8.00	38.00	83.00
	0.125	49.33	50.62	13.00	69.00	153.00	13.95	13.80	4.00	20.00	42.00
	0.150	32.54	33.92	8.00	45.00	100.00	8.07	7.70	3.00	11.00	24.00
	0.175	21.59	21.97	6.00	30.00	65.05	4.88	4.30	2.00	7.00	13.00
	0.200	14.60	14.50	4.00	20.00	44.00	3.28	2.61	1.00	4.00	8.00
	0.225	10.09	9.90	3.00	14.00	29.00	2.39	1.64	1.00	3.00	6.00
	0.250	7.19	6.58	2.00	10.00	20.00	1.91	1.16	1.00	2.00	4.00
$MSS_{n,2,P_{0.30},P_{0.70}}$	0.000	203.54	205.49	57.00	286.00	620.00	202.33	202.60	57.00	283.00	614.00
	0.025	190.96	195.04	51.00	265.00	576.00	171.70	172.95	48.00	239.00	527.05
	0.050	155.99	158.73	43.00	219.25	469.00	104.14	107.24	27.00	146.00	315.05
	0.075	116.43	118.84	32.00	164.00	353.00	54.42	55.55	15.00	75.00	167.05
	0.100	80.22	81.36	22.00	113.00	243.05	26.55	26.91	7.00	37.00	81.05
	0.125	53.22	54.52	15.00	74.00	161.00	14.05	13.93	4.00	19.00	42.00
	0.150	35.23	35.91	10.00	49.00	105.05	7.48	7.04	2.00	10.00	22.00
	0.175	22.92	22.97	6.00	32.00	69.00	4.59	3.97	2.00	6.00	13.00
	0.200	15.71	15.88	4.00	22.00	47.00	3.06	2.38	1.00	4.00	8.00
	0.225	10.75	10.27	3.00	15.00	31.00	2.30	1.60	1.00	3.00	5.00
	0.250	7.44	6.84	2.00	10.00	21.00	1.85	1.13	1.00	2.00	4.00
$MSS_{n,2,P_{0.35},P_{0.65}}$	0.000	205.68	211.46	57.00	284.00	625.00	204.77	202.62	58.00	284.00	622.00
	0.025	191.70	192.66	54.00	268.00	573.00	168.97	172.53	47.00	234.00	521.00
	0.050	158.48	161.67	43.00	221.00	479.00	99.66	99.61	28.00	140.00	297.00
	0.075	117.33	120.60	32.00	163.00	363.00	51.64	52.26	14.00	73.00	156.00
	0.100	81.87	83.59	23.00	114.00	244.00	24.85	25.56	7.00	34.00	76.00
	0.125	53.62	54.84	14.00	74.00	165.05	13.24	13.37	4.00	18.00	40.00
	0.150	34.92	35.83	9.00	49.00	105.00	7.16	6.91	2.00	10.00	21.00
	0.175	22.96	23.76	6.00	32.00	71.00	4.49	4.05	2.00	6.00	12.00
	0.200	15.44	15.55	4.00	22.00	47.00	3.01	2.43	1.00	4.00	8.00
	0.225	10.48	10.37	3.00	15.00	31.00	2.22	1.54	1.00	3.00	5.00
	0.250	7.43	7.08	2.00	10.00	22.00	1.80	1.10	1.00	2.00	4.00
$MSS_{n,2,P_{0.40},P_{0.60}}$	0.000	199.13	200.86	55.00	280.00	600.05	195.70	198.35	52.00	275.00	590.00
	0.025	190.19	198.40	51.00	266.00	570.00	168.12	171.02	49.00	234.00	515.00
	0.050	155.29	155.86	44.00	215.00	460.00	103.58	106.50	27.00	145.00	316.00
	0.075	113.85	117.57	31.00	161.00	349.00	54.61	55.87	15.00	76.00	166.00
	0.100	80.15	83.31	22.00	113.00	245.00	26.04	26.59	7.00	36.00	80.00
	0.125	52.05	54.72	13.00	72.00	164.00	13.41	13.72	4.00	19.00	41.00
	0.150	34.05	35.39	9.00	48.00	105.00	7.41	7.24	2.00	10.00	22.00
	0.175	22.08	22.84	6.00	31.00	68.00	4.55	4.07	2.00	6.00	13.00
	0.200	14.55	14.79	4.00	20.00	44.00	3.02	2.42	1.00	4.00	8.00
	0.225	10.01	10.17	3.00	14.00	30.00	2.26	1.60	1.00	3.00	5.00
	0.250	7.13	6.84	2.00	10.00	21.00	1.80	1.11	1.00	2.00	4.00
$MSS_{n,2,P_{0.45},P_{0.55}}$	0.000	205.32	207.29	56.00	286.00	616.00	200.77	207.55	53.00	281.00	618.05
	0.025	188.59	195.35	49.00	262.00	584.00	190.86	194.25	52.00	268.00	582.05
	0.050	151.90	155.91	40.00	214.00	463.00	151.52	154.51	41.00	211.00	456.05
	0.075	112.49	116.56	30.00	157.00	346.00	112.45	115.82	30.00	156.00	344.00
	0.100	74.78	77.20	20.00	104.00	227.00	75.99	78.06	20.00	107.00	231.00
	0.125	49.09	50.56	13.00	69.00	151.00	49.22	50.85	13.00	69.00	152.00
	0.150	31.33	33.44	8.00	44.00	99.00	31.86	33.41	8.00	44.00	97.00
	0.175	20.57	21.83	5.00	28.00	63.00	20.24	21.12	5.00	29.00	62.00
	0.200	13.64	14.21	3.00	19.00	41.00	13.80	14.48	3.00	19.00	43.00
	0.225	9.22	9.34	2.00	13.00	29.00	9.31	9.30	3.00	13.00	28.00
	0.250	6.50	6.24	2.00	9.00	19.00	6.65	6.52	2.00	9.00	20.00

Table 3.6: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the slope of transformed model parameter at fixed c=*3*

Schemes	δ	***n=5***					***n=7***				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,3,P_{0.25},P_{0.50},P_{0.75}}$	0.000	208.02	217.60	53.00	290.00	637.00	206.06	209.33	55.00	290.00	629.10
	0.025	191.94	199.92	49.00	271.00	589.00	168.04	169.00	46.00	234.00	498.05
	0.050	155.21	157.85	42.00	215.00	472.00	102.51	105.83	28.00	141.00	309.00
	0.075	118.49	122.45	32.00	165.00	361.00	52.81	53.92	14.00	72.00	163.00
	0.100	78.94	80.91	21.00	110.00	243.00	25.87	26.04	7.00	36.00	78.00
	0.125	52.20	53.32	14.00	72.00	159.00	13.56	13.35	4.00	19.00	40.00
	0.150	35.31	35.26	10.00	49.00	106.00	7.35	6.78	2.00	10.00	21.00
	0.175	22.77	22.67	6.00	32.00	68.00	4.60	3.88	2.00	6.00	12.00
	0.200	15.39	15.29	5.00	21.00	46.00	3.16	2.29	1.00	4.00	8.00
	0.225	10.79	10.33	3.00	15.00	31.00	2.37	1.56	1.00	3.00	5.00
	0.250	7.71	7.05	3.00	10.00	22.00	1.92	1.12	1.00	2.00	4.00
$MSS_{n,3,P_{0.30},P_{0.50},P_{0.70}}$	0.000	206.14	216.10	52.00	289.00	644.00	204.71	210.05	55.00	285.00	621.05
	0.025	190.81	200.96	49.00	268.00	584.05	169.91	176.82	44.00	237.00	521.05
	0.050	161.24	169.75	39.00	225.00	503.00	105.56	108.92	28.00	146.00	322.00
	0.075	115.39	120.91	29.00	162.00	359.05	52.26	53.90	14.00	73.00	158.00
	0.100	77.52	82.49	19.00	107.00	240.00	24.97	25.89	7.00	35.00	76.00
	0.125	51.66	54.42	13.00	72.00	162.05	12.74	12.73	4.00	17.00	39.00
	0.150	33.17	34.33	8.00	47.00	102.00	7.11	6.69	2.00	10.00	21.00
	0.175	21.48	22.77	5.00	30.00	67.00	4.30	3.69	2.00	6.00	12.00
	0.200	14.49	14.90	4.00	20.00	45.00	2.95	2.19	1.00	4.00	7.00
	0.225	9.76	9.71	3.00	14.00	29.00	2.29	1.50	1.00	3.00	5.00
	0.250	6.97	6.76	2.00	9.00	21.00	1.83	1.08	1.00	2.00	4.00
$MSS_{n,3,P_{0.35},P_{0.50},P_{0.65}}$	0.000	199.65	210.88	48.00	285.00	622.00	199.48	204.68	53.00	282.00	599.00
	0.025	191.08	200.92	46.00	269.00	591.05	160.14	164.01	42.00	224.00	493.00
	0.050	157.39	169.14	36.75	217.00	499.05	95.72	99.86	24.00	133.00	294.00
	0.075	113.77	119.71	25.00	163.00	348.00	47.95	49.80	12.00	67.00	146.00
	0.100	74.30	80.14	16.00	107.00	232.05	23.19	23.69	6.00	33.00	71.00
	0.125	47.62	51.48	10.00	68.00	153.00	11.38	11.71	3.00	16.00	35.00
	0.150	30.40	33.16	6.00	43.00	97.00	6.53	6.36	2.00	9.00	19.00
	0.175	20.08	22.46	4.00	28.00	66.00	3.97	3.50	1.00	5.00	11.00
	0.200	13.18	14.43	3.00	18.00	43.00	2.78	2.17	1.00	4.00	7.00
	0.225	8.69	9.42	2.00	12.00	28.00	2.12	1.42	1.00	3.00	5.00
	0.250	6.34	6.52	2.00	9.00	20.00	1.73	1.03	1.00	2.00	4.00
$MSS_{n,3,P_{0.40},P_{0.50},P_{0.60}}$	0.000	202.28	219.55	46.00	285.00	650.05	210.35	219.07	55.00	294.00	642.00
	0.025	186.54	202.94	41.00	263.00	598.00	173.18	178.31	45.00	243.00	534.05
	0.050	146.65	160.18	31.00	207.25	464.00	101.12	105.70	26.00	143.25	313.05
	0.075	104.15	115.98	20.00	147.25	339.05	49.71	52.73	12.00	70.00	155.00
	0.100	68.06	74.61	14.00	97.00	218.00	23.56	24.96	5.00	33.00	74.05
	0.125	43.02	48.28	8.00	61.00	141.00	11.43	12.12	3.00	16.00	36.00
	0.150	27.23	30.40	5.00	39.00	89.00	6.44	6.43	2.00	9.00	19.00
	0.175	17.20	19.41	3.00	24.00	57.00	4.06	3.75	1.00	5.00	12.00
	0.200	11.63	12.78	2.00	16.00	37.00	2.76	2.24	1.00	4.00	7.00
	0.225	7.82	8.44	2.00	11.00	26.00	2.10	1.49	1.00	3.00	5.00
	0.250	5.54	5.76	2.00	7.00	18.00	1.70	1.06	1.00	2.00	4.00
$MSS_{n,3,P_{0.45},P_{0.50},P_{0.55}}$	0.000	186.73	212.49	33.00	265.00	622.05	197.92	202.17	52.00	277.00	603.00
	0.025	167.95	191.66	27.00	242.00	558.00	160.57	167.95	40.00	224.00	494.00
	0.050	129.90	149.42	20.00	188.00	433.05	95.22	99.74	23.00	134.00	293.00
	0.075	90.89	105.74	12.00	132.00	301.00	45.08	47.62	10.00	63.00	139.00
	0.100	57.89	67.41	9.00	83.00	193.00	21.25	23.09	4.00	30.00	68.00
	0.125	35.75	41.83	5.00	51.00	119.00	10.77	11.31	2.00	15.00	34.00
	0.150	23.76	27.95	4.00	34.00	79.05	6.10	6.00	2.00	9.00	18.00
	0.175	15.42	17.71	3.00	22.00	53.00	3.87	3.64	1.00	5.00	11.00
	0.200	10.38	11.91	2.00	14.00	35.00	2.71	2.27	1.00	3.00	7.00
	0.225	7.66	8.56	2.00	11.00	26.00	2.03	1.52	1.00	2.00	5.00
	0.250	5.48	6.11	1.00	7.00	18.00	1.63	1.01	1.00	2.00	4.00

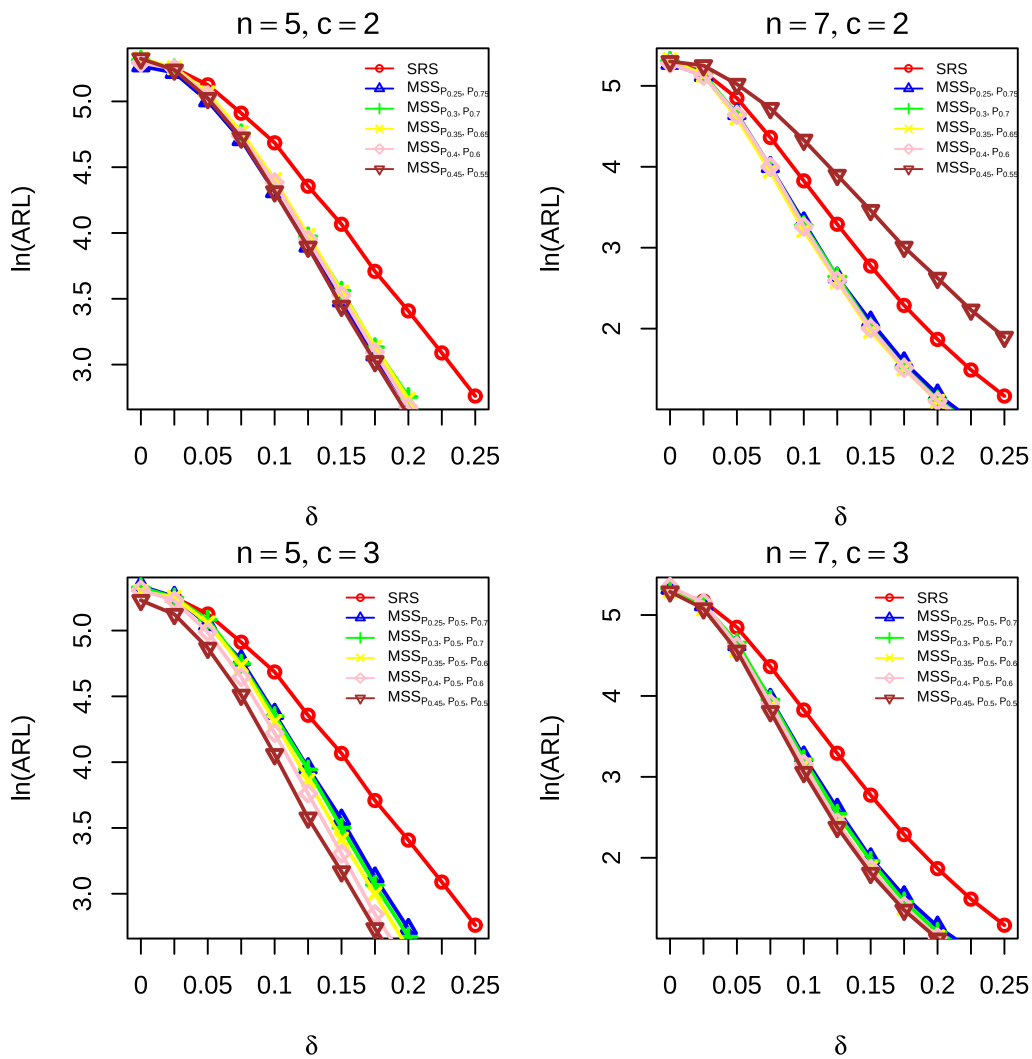

Figure 3.2: ARL curves of $Shewhart-3$ chart under different sampling schemes for slope shifts $\left(B_1\ to\ B_1+\delta\left(\sigma_{e[\mathbf{MSS}]}/\sqrt{S_{\mathbf{xx}}}\right)\right)$

plotted in Figure 3.3, which reveals that $Shewhart-3_{[MSS]}$ charts have better performance as compared to $Shewhart-3_{[SRS]}$ chart. Specifically,$Shewhart-3_{[MSS]}$ chart with percentile choices $(P_{0.45},\ P_{0.55})$ and $(P_{0.45}, P_{0.50},\ P_{0.55})$ outperforms all others except in case when design parameters are n=*7* and c=*2*.

Table 3.7: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the slope of original model parameter at fixed c=2

Schemes	β	n=5					n=7				
		ARL	***SDRL***	**$Q_{0.25}$**	**$Q_{0.75}$**	**$Q_{0.95}$**	***ARL***	***SDRL***	**$Q_{0.25}$**	**$Q_{0.75}$**	**$Q_{0.95}$**
$MSS_{n,2,P_{0.25},P_{0.75}}$	0.000	194.76	208.14	47.00	273.00	617.00	199.81	202.71	52.00	281.00	602.00
	0.025	142.50	147.78	37.00	202.00	433.00	103.37	104.74	28.00	146.00	312.05
	0.050	71.10	75.55	17.00	100.00	221.00	31.83	32.64	8.00	45.00	96.00
	0.075	32.21	34.27	8.00	45.00	102.05	10.57	10.69	3.00	15.00	32.00
	0.100	15.65	16.53	4.00	22.00	49.00	4.38	4.31	1.00	6.00	13.00
	0.125	8.12	8.46	2.00	12.00	25.00	2.34	1.97	1.00	3.00	6.00
	0.150	4.72	4.84	1.00	7.00	15.00	1.51	0.96	1.00	2.00	4.00
	0.175	3.02	2.90	1.00	4.00	9.00	1.19	0.53	1.00	1.00	2.00
	0.200	2.12	1.84	1.00	3.00	6.00	1.06	0.25	1.00	1.00	2.00
	0.225	1.62	1.20	1.00	2.00	4.00	1.01	0.12	1.00	1.00	1.00
	0.250	1.33	0.78	1.00	1.00	3.00	1.00	0.06	1.00	1.00	1.00
$MSS_{n,2,P_{0.30},P_{0.70}}$	0.000	201.07	208.54	54.00	279.00	613.00	208.39	209.68	58.00	293.00	624.00
	0.025	147.22	149.87	39.00	208.00	443.00	104.75	106.13	30.00	145.00	320.00
	0.050	75.30	76.72	20.00	106.00	224.00	30.92	31.03	8.00	44.00	92.00
	0.075	34.55	36.35	9.00	48.00	107.00	10.46	10.70	3.00	14.00	32.00
	0.100	16.68	17.22	4.00	23.00	51.00	4.35	4.22	1.00	6.00	13.00
	0.125	8.64	8.95	2.00	12.00	27.00	2.26	1.92	1.00	3.00	6.00
	0.150	5.07	5.17	1.00	7.00	15.00	1.51	0.98	1.00	2.00	4.00
	0.175	3.18	3.01	1.00	4.00	9.00	1.18	0.49	1.00	1.00	2.00
	0.200	2.18	1.88	1.00	3.00	6.00	1.05	0.25	1.00	1.00	1.00
	0.225	1.66	1.25	1.00	2.00	4.00	1.02	0.13	1.00	1.00	1.00
	0.250	1.35	0.79	1.00	1.00	3.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,2,P_{0.35},P_{0.65}}$	0.000	201.64	203.97	57.00	284.00	604.00	206.41	208.32	57.00	289.00	617.05
	0.025	152.19	157.36	41.00	208.00	467.05	104.77	106.76	29.00	147.00	314.00
	0.050	76.67	79.26	20.00	107.00	233.00	32.60	33.11	9.00	46.00	99.00
	0.075	35.18	36.05	9.00	49.00	105.00	10.47	10.59	3.00	15.00	32.00
	0.100	16.98	17.63	4.00	24.00	53.00	4.35	4.20	1.00	6.00	13.00
	0.125	8.76	9.14	2.00	12.00	27.00	2.34	1.97	1.00	3.00	6.00
	0.150	5.02	5.06	1.00	7.00	15.00	1.48	0.96	1.00	2.00	3.00
	0.175	3.14	3.03	1.00	4.00	9.00	1.18	0.50	1.00	1.00	2.00
	0.200	2.17	1.90	1.00	3.00	6.00	1.06	0.25	1.00	1.00	2.00
	0.225	1.64	1.20	1.00	2.00	4.00	1.01	0.12	1.00	1.00	1.00
	0.250	1.34	0.80	1.00	1.00	3.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,2,P_{0.40},P_{0.60}}$	0.000	206.14	211.34	57.00	292.00	627.00	194.50	197.06	54.00	270.00	591.05
	0.025	151.13	154.54	41.00	210.00	459.00	103.19	102.88	29.00	146.00	304.00
	0.050	75.24	77.39	20.00	103.00	229.00	30.44	31.21	8.00	42.00	95.00
	0.075	34.13	35.94	8.00	48.00	106.00	10.05	10.36	3.00	14.00	31.00
	0.100	16.32	16.98	4.00	23.00	52.00	4.31	4.12	1.00	6.00	13.00
	0.125	8.40	8.74	2.00	12.00	26.00	2.25	1.90	1.00	3.00	6.00
	0.150	4.77	4.88	1.00	7.00	15.00	1.48	0.97	1.00	2.00	3.00
	0.175	3.01	2.92	1.00	4.00	9.00	1.16	0.46	1.00	1.00	2.00
	0.200	2.08	1.77	1.00	2.00	6.00	1.06	0.26	1.00	1.00	1.00
	0.225	1.61	1.20	1.00	2.00	4.00	1.01	0.11	1.00	1.00	1.00
	0.250	1.31	0.75	1.00	1.00	3.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,2,P_{0.45},P_{0.55}}$	0.000	204.11	209.24	55.00	286.00	624.00	203.73	208.07	55.00	283.00	621.00
	0.025	148.51	154.89	39.00	205.00	455.00	148.33	150.14	41.00	206.00	449.00
	0.050	73.84	77.19	19.00	102.00	231.00	74.25	77.39	19.00	104.00	224.00
	0.075	32.93	34.65	8.00	46.00	101.05	32.98	35.01	8.00	47.00	102.00
	0.100	15.62	16.67	3.00	22.00	50.00	15.58	16.52	4.00	22.00	48.00
	0.125	8.01	8.51	2.00	11.00	25.00	8.04	8.56	2.00	11.00	25.00
	0.150	4.67	4.81	1.00	6.00	14.00	4.59	4.75	1.00	6.00	14.00
	0.175	2.90	2.84	1.00	4.00	9.00	2.93	2.85	1.00	4.00	9.00
	0.200	2.05	1.81	1.00	2.00	6.00	2.03	1.77	1.00	2.00	6.00
	0.225	1.54	1.09	1.00	2.00	4.00	1.56	1.13	1.00	2.00	4.00
	0.250	1.27	0.68	1.00	1.00	3.00	1.28	0.71	1.00	1.00	3.00

Table 3.8: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the slope of original model parameter at fixed c=3

Schemes	β	*n*=5					*n*=7				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,3,P_{0.25},P_{0.50},P_{0.75}}$	0.000	206.15	212.78	53.00	289.00	639.00	202.97	208.73	55.00	280.00	609.05
	0.025	153.63	158.54	40.00	216.00	470.00	100.28	101.84	27.00	140.00	302.00
	0.050	76.37	79.49	20.00	106.00	233.00	29.80	31.17	8.00	42.00	91.00
	0.075	34.82	37.07	8.00	49.00	109.00	9.96	10.39	2.00	14.00	30.05
	0.100	16.90	17.95	4.00	24.00	52.00	4.20	4.33	1.00	6.00	13.00
	0.125	8.82	9.47	2.00	12.00	28.00	2.23	1.98	1.00	3.00	6.00
	0.150	5.11	5.51	1.00	7.00	16.00	1.45	0.97	1.00	2.00	3.00
	0.175	3.16	3.31	1.00	4.00	10.00	1.16	0.49	1.00	1.00	2.00
	0.200	2.19	2.11	1.00	3.00	7.00	1.05	0.26	1.00	1.00	1.00
	0.225	1.64	1.34	1.00	2.00	4.00	1.01	0.12	1.00	1.00	1.00
	0.250	1.32	0.86	1.00	1.00	3.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,3,P_{0.30},P_{0.50},P_{0.70}}$	0.000	205.97	214.10	53.00	287.25	632.00	205.08	207.42	54.00	290.00	627.00
	0.025	149.15	157.12	37.00	210.00	462.05	102.87	105.94	28.00	144.00	313.00
	0.050	73.13	77.71	17.00	104.00	223.00	29.26	30.72	7.00	41.00	92.00
	0.075	32.28	35.73	7.00	46.00	101.00	9.58	10.09	2.00	13.00	29.00
	0.100	15.23	16.78	3.00	22.00	50.00	4.03	4.07	1.00	5.00	12.00
	0.125	8.01	9.07	1.00	11.00	27.00	2.12	1.84	1.00	3.00	6.00
	0.150	4.69	5.26	1.00	6.00	15.00	1.41	0.90	1.00	1.00	3.00
	0.175	2.90	3.03	1.00	4.00	9.00	1.14	0.46	1.00	1.00	2.00
	0.200	2.02	1.98	1.00	2.00	6.00	1.04	0.23	1.00	1.00	1.00
	0.225	1.52	1.21	1.00	2.00	4.00	1.01	0.10	1.00	1.00	1.00
	0.250	1.26	0.76	1.00	1.00	3.00	1.00	0.05	1.00	1.00	1.00
$MSS_{n,3,P_{0.35},P_{0.50},P_{0.65}}$	0.000	203.12	215.52	48.00	287.00	630.00	198.12	204.56	53.00	276.00	608.05
	0.025	144.19	154.78	33.00	202.00	457.00	95.21	99.81	25.00	133.00	293.00
	0.050	65.97	72.81	13.00	94.00	213.00	27.63	29.31	6.00	39.00	85.05
	0.075	28.21	32.14	5.00	40.00	91.00	9.03	9.53	2.00	13.00	28.00
	0.100	13.16	15.24	2.00	18.00	45.00	3.81	3.92	1.00	5.00	12.00
	0.125	6.81	8.02	1.00	9.00	23.00	2.00	1.72	1.00	2.00	6.00
	0.150	3.93	4.52	1.00	5.00	13.00	1.37	0.85	1.00	1.00	3.00
	0.175	2.51	2.69	1.00	3.00	8.00	1.12	0.42	1.00	1.00	2.00
	0.200	1.77	1.62	1.00	2.00	5.00	1.04	0.22	1.00	1.00	1.00
	0.225	1.40	1.03	1.00	1.00	3.00	1.01	0.10	1.00	1.00	1.00
	0.250	1.20	0.63	1.00	1.00	2.00	1.00	0.04	1.00	1.00	1.00
$MMSS_{n,3,P_{0.40},P_{0.50},P_{0.60}}$	0.000	206.24	222.07	47.00	290.00	649.00	205.07	211.94	54.00	288.00	629.10
	0.025	137.13	149.25	28.00	195.00	445.00	100.49	106.34	24.00	142.00	317.00
	0.050	60.00	67.46	10.00	86.00	194.00	27.66	29.76	6.00	39.00	88.00
	0.075	25.22	29.83	3.00	36.00	86.00	8.76	9.63	2.00	12.00	28.00
	0.100	11.78	14.52	2.00	17.00	41.00	3.62	3.80	1.00	5.00	11.00
	0.125	5.98	7.37	1.00	8.00	21.00	1.98	1.73	1.00	2.00	6.00
	0.150	3.46	4.13	1.00	4.00	12.00	1.33	0.78	1.00	1.00	3.00
	0.175	2.28	2.39	1.00	3.00	7.00	1.12	0.43	1.00	1.00	2.00
	0.200	1.63	1.40	1.00	2.00	4.00	1.03	0.19	1.00	1.00	1.00
	0.225	1.32	0.89	1.00	1.00	3.00	1.01	0.09	1.00	1.00	1.00
	0.250	1.16	0.56	1.00	1.00	2.00	1.00	0.04	1.00	1.00	1.00
$MSS_{n,3,P_{0.45},P_{0.50},P_{0.55}}$	0.000	186.02	211.88	31.00	266.00	613.00	197.65	205.69	51.00	278.00	611.00
	0.025	118.26	138.38	17.00	171.00	393.00	97.44	105.26	22.00	135.00	310.00
	0.050	49.69	60.96	6.00	72.00	173.00	26.48	29.44	5.00	37.00	86.00
	0.075	20.91	26.22	2.00	30.00	72.00	8.54	9.49	2.00	12.00	28.00
	0.100	9.76	12.62	1.00	13.00	37.00	3.52	3.67	1.00	5.00	11.00
	0.125	5.09	6.58	1.00	6.00	18.00	1.94	1.73	1.00	2.00	5.00
	0.150	2.99	3.59	1.00	3.00	10.00	1.33	0.78	1.00	1.00	3.00
	0.175	1.99	2.05	1.00	2.00	6.00	1.11	0.39	1.00	1.00	2.00
	0.200	1.49	1.22	1.00	1.00	4.00	1.03	0.18	1.00	1.00	1.00
	0.225	1.25	0.72	1.00	1.00	3.00	1.01	0.08	1.00	1.00	1.00
	0.250	1.12	0.45	1.00	1.00	2.00	1.00	0.02	1.00	1.00	1.00

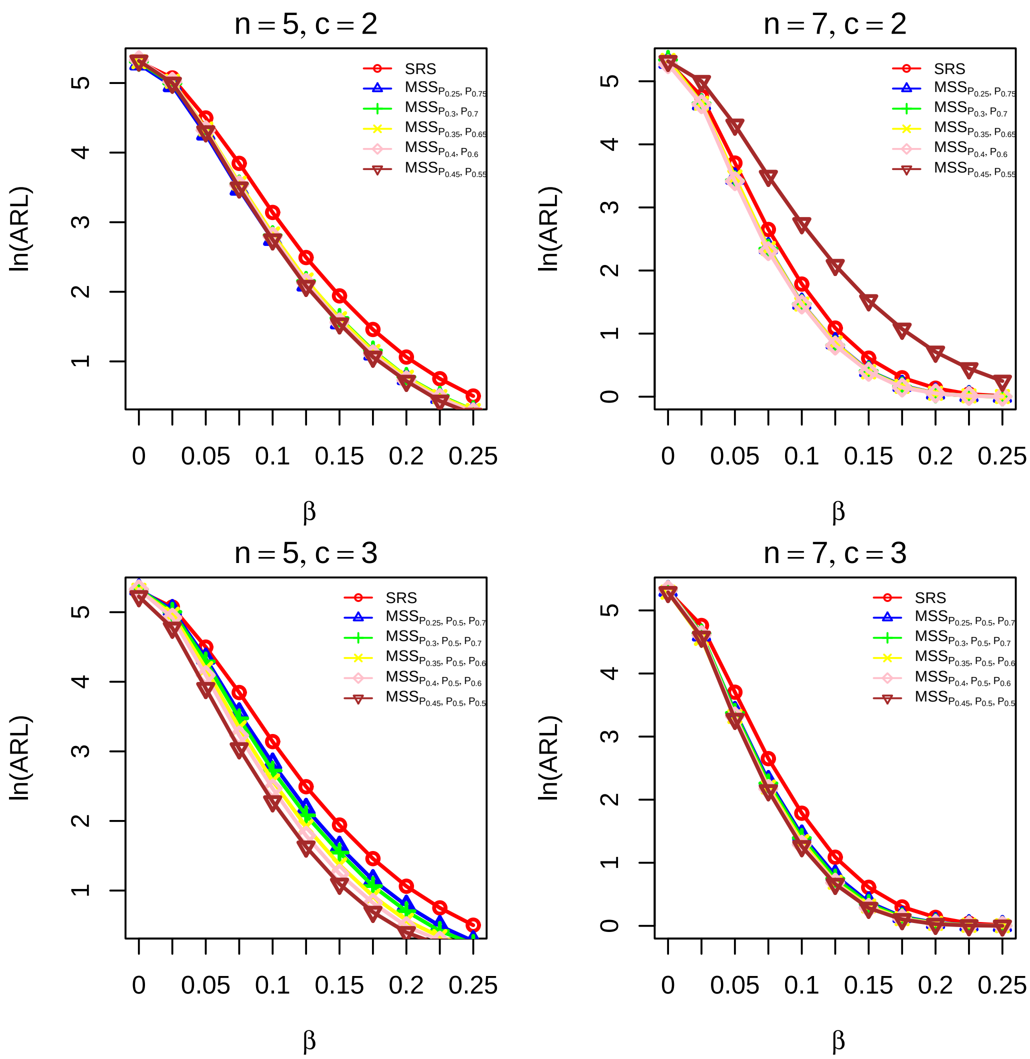

Figure 3.3: ARL curves of $Shewhart-3$ chart under different sampling schemes for slope shifts $\left(\beta_1 \; to \; \beta_1+\beta\left(\sigma_{e[\mathbf{MSS}]}/\sqrt{S_{\mathbf{xx}}}\right)\right)$

3.2.4 Shifts in error variance of disturbance term:

Tables 3.2,3.9 and 3.10 are about the results for shifted error variance parameter in $Shewhart-3_{[SRS]}$ chart, $Shewhart-3_{[MSS]}$ charts at fixed c=2 and 3 respectively. Which reveals that (60%) upward shift in error variance parameter, may decrease 97.19% and 97.79% ARL_1 of $Shewhart-3_{[SRS]}$ chart, 97.12% and 97.16% ARL_1 of $Shewhart-3_{\left[MSS_{n,2,P_{0.40},\,P_{0.60}}\right]}$ chart and 97.84% and 97.91% ARL_1 of

$Shewhart-3_{\left[MSS_{n,3,P_{0.40},P_{0.5},\ P_{0.60}}\right]}$ chart for n=5 and 7 respectively. However, the curves for shifted error variance parameter are plotted in Figure 3.4, which shows that $Shewhart-3_{[MSS]}$ charts have relatively better performance as compared to $Shewhart-3_{[SRS]}$ chart. Specifically,$Shewhart-3_{[MSS]}$ chart under percentile choices $(P_{0.45},\ P_{0.55})$ and $(P_{0.45}, P_{0.50},\ P_{0.55})$ outperforms all others while in case of n=7 and c=2,$Shewhart-3_{[MSS]}$ chart under percentile choice $(P_{0.40},\ P_{0.60})$ have relatively good performance among all others.

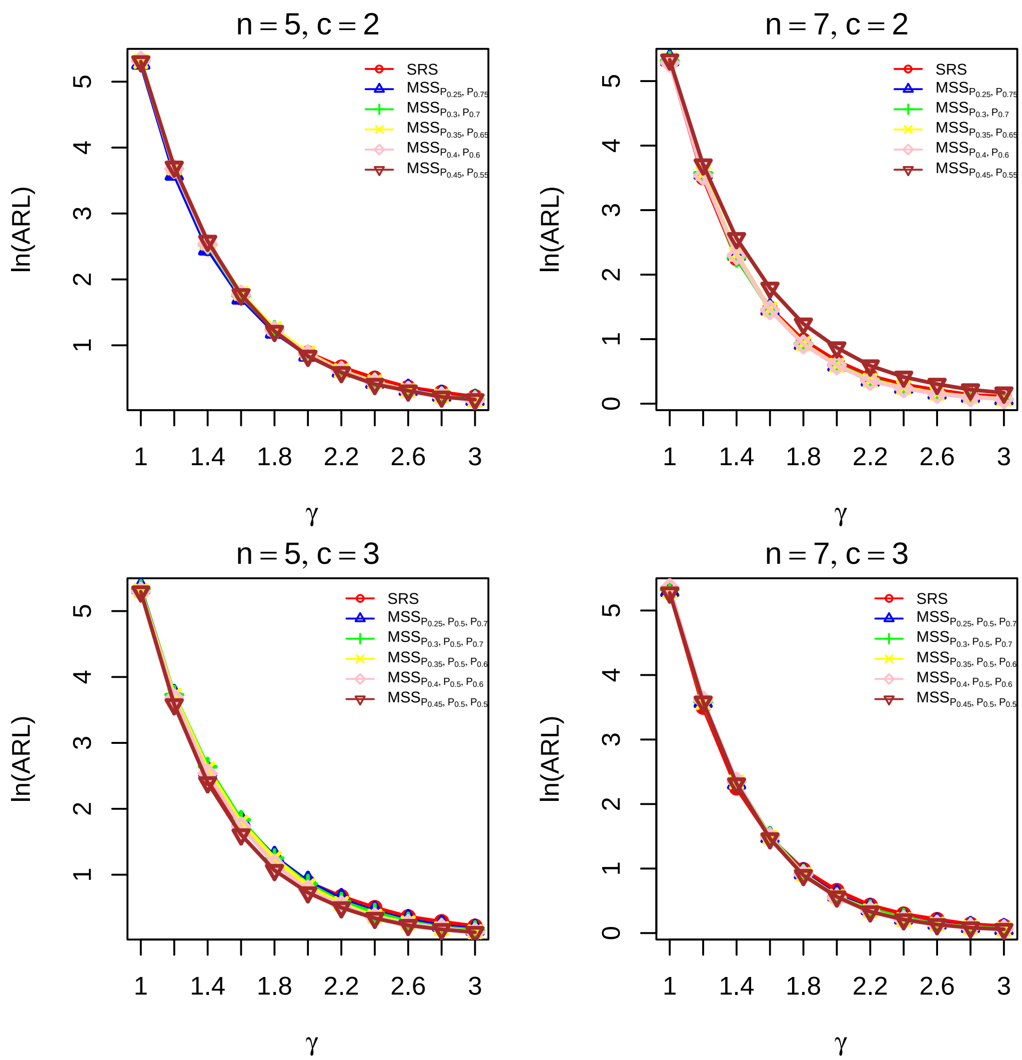

Figure 3.4: ARL curves of $Shewhart-3$ chart under different sampling schemes for error variamce shifts $\left(\sigma^2_{e[\mathbf{MSS}]}\ to\ \gamma\sigma^2_{e[\mathbf{MSS}]}\right)$

Table 3.9: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the error variance parameter at fixed c=2

Schemes	γ	n=5					n=7				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,2,P_{0.25},P_{0.75}}$	1.00	196.15	204.03	48.00	280.00	606.05	205.80	209.06	57.00	285.00	620.00
	1.20	36.36	38.51	8.00	51.00	113.05	35.51	36.91	9.00	50.00	110.00
	1.40	11.69	12.50	2.00	17.00	37.00	10.13	10.94	2.00	14.00	32.00
	1.60	5.55	5.79	1.00	8.00	17.00	4.32	4.56	1.00	6.00	14.00
	1.80	3.31	3.34	1.00	4.00	10.00	2.53	2.42	1.00	3.00	8.00
	2.00	2.33	2.19	1.00	3.00	7.00	1.81	1.51	1.00	2.00	5.00
	2.20	1.83	1.53	1.00	2.00	5.00	1.44	0.97	1.00	1.00	3.00
	2.40	1.55	1.13	1.00	2.00	4.00	1.29	0.72	1.00	1.00	3.00
	2.60	1.39	0.89	1.00	1.00	3.00	1.18	0.53	1.00	1.00	2.00
	2.80	1.27	0.70	1.00	1.00	3.00	1.12	0.42	1.00	1.00	2.00
	3.00	1.19	0.57	1.00	1.00	2.00	1.07	0.32	1.00	1.00	2.00
$MSS_{n,2,P_{0.30},P_{0.70}}$	1.00	203.32	204.20	55.00	287.00	620.05	204.05	207.67	57.00	284.00	607.00
	1.20	39.47	41.41	10.00	56.00	122.00	35.78	36.99	9.00	50.00	109.00
	1.40	12.50	13.44	3.00	17.00	40.00	9.60	10.11	2.00	13.00	30.00
	1.60	5.75	6.31	1.00	8.00	19.00	4.28	4.37	1.00	6.00	13.00
	1.80	3.43	3.55	1.00	5.00	11.00	2.54	2.42	1.00	3.00	7.00
	2.00	2.37	2.27	1.00	3.00	7.00	1.83	1.51	1.00	2.00	5.00
	2.20	1.85	1.60	1.00	2.00	5.00	1.46	0.99	1.00	2.00	3.00
	2.40	1.56	1.18	1.00	2.00	4.00	1.28	0.71	1.00	1.00	3.00
	2.60	1.36	0.90	1.00	1.00	3.00	1.17	0.52	1.00	1.00	2.00
	2.80	1.26	0.71	1.00	1.00	3.00	1.12	0.41	1.00	1.00	2.00
	3.00	1.19	0.58	1.00	1.00	2.00	1.07	0.30	1.00	1.00	2.00
$MSS_{n,2,P_{0.35},P_{0.65}}$	1.00	201.74	204.08	55.00	282.00	609.00	205.80	209.06	57.00	285.00	620.00
	1.20	40.10	41.95	10.00	56.00	123.00	35.51	36.91	9.00	50.00	110.00
	1.40	12.84	13.98	2.00	18.00	41.00	10.13	10.94	2.00	14.00	32.00
	1.60	5.98	6.62	1.00	8.00	19.00	4.32	4.56	1.00	6.00	14.00
	1.80	3.50	3.75	1.00	5.00	11.00	2.53	2.42	1.00	3.00	8.00
	2.00	2.42	2.42	1.00	3.00	8.00	1.81	1.51	1.00	2.00	5.00
	2.20	1.85	1.62	1.00	2.00	5.00	1.44	0.97	1.00	1.00	3.00
	2.40	1.56	1.22	1.00	2.00	4.00	1.29	0.72	1.00	1.00	3.00
	2.60	1.37	0.93	1.00	1.00	3.00	1.18	0.53	1.00	1.00	2.00
	2.80	1.26	0.74	1.00	1.00	3.00	1.12	0.42	1.00	1.00	2.00
	3.00	1.18	0.58	1.00	1.00	2.00	1.07	0.32	1.00	1.00	2.00
$MSS_{n,2,P_{0.40},P_{0.60}}$	1.00	203.73	208.43	56.00	285.00	624.00	197.02	198.10	54.00	279.00	589.05
	1.20	39.50	42.16	9.00	56.00	122.00	34.22	36.04	8.00	48.00	106.00
	1.40	12.65	14.06	2.00	18.00	41.00	9.97	10.63	2.00	14.00	31.00
	1.60	5.86	6.55	1.00	8.00	19.00	4.26	4.47	1.00	6.00	13.00
	1.80	3.39	3.66	1.00	4.00	11.00	2.51	2.45	1.00	3.00	8.00
	2.00	2.37	2.42	1.00	3.00	7.00	1.81	1.51	1.00	2.00	5.00
	2.20	1.84	1.70	1.00	2.00	5.00	1.42	0.96	1.00	1.00	3.00
	2.40	1.54	1.20	1.00	2.00	4.00	1.27	0.73	1.00	1.00	3.00
	2.60	1.37	0.93	1.00	1.00	3.00	1.16	0.51	1.00	1.00	2.00
	2.80	1.25	0.70	1.00	1.00	3.00	1.11	0.39	1.00	1.00	2.00
	3.00	1.18	0.56	1.00	1.00	2.00	1.07	0.32	1.00	1.00	2.00
$MSS_{n,2,P_{0.45},P_{0.55}}$	1.00	201.22	206.64	54.00	279.00	609.00	206.29	212.88	55.00	287.00	622.00
	1.20	40.69	43.64	9.00	58.00	127.00	40.55	43.11	9.00	57.00	127.00
	1.40	13.20	15.01	2.00	18.00	43.00	12.95	14.54	2.00	18.00	42.00
	1.60	5.87	6.65	1.00	8.00	19.00	6.00	6.80	1.00	8.00	20.00
	1.80	3.37	3.68	1.00	4.00	11.00	3.44	3.79	1.00	4.00	11.00
	2.00	2.31	2.36	1.00	3.00	7.00	2.38	2.40	1.00	3.00	7.00
	2.20	1.80	1.63	1.00	2.00	5.00	1.80	1.63	1.00	2.00	5.00
	2.40	1.51	1.14	1.00	2.00	4.00	1.51	1.18	1.00	1.00	4.00
	2.60	1.36	0.91	1.00	1.00	3.00	1.35	0.92	1.00	1.00	3.00
	2.80	1.24	0.70	1.00	1.00	3.00	1.24	0.70	1.00	1.00	3.00
	3.00	1.19	0.61	1.00	1.00	2.00	1.18	0.58	1.00	1.00	2.00

Table 3.10: Performance of $Shewhart-3_{[MSS]}$ in the presence of shifts in the error variance parameter at fixed c=*3*

Schemes	γ	*n=5*					*n=7*				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,3,P_{0.25},P_{0.50},P_{0.75}}$	1.00	210.29	218.85	55.00	294.00	650.00	201.79	207.27	55.00	279.00	605.05
	1.20	41.69	45.97	8.00	59.00	134.00	36.10	38.54	8.00	50.00	115.00
	1.40	13.17	15.66	2.00	19.00	45.00	10.20	11.30	2.00	14.00	34.00
	1.60	6.14	7.41	1.00	9.00	21.00	4.45	4.89	1.00	6.00	14.00
	1.80	3.57	4.25	1.00	5.00	13.00	2.54	2.63	1.00	3.00	8.00
	2.00	2.45	2.76	1.00	3.00	8.00	1.80	1.60	1.00	2.00	5.00
	2.20	1.88	1.89	1.00	2.00	6.00	1.45	1.08	1.00	1.00	4.00
	2.40	1.55	1.36	1.00	1.00	4.00	1.26	0.74	1.00	1.00	3.00
	2.60	1.37	1.06	1.00	1.00	3.00	1.16	0.55	1.00	1.00	2.00
	2.80	1.26	0.82	1.00	1.00	3.00	1.10	0.41	1.00	1.00	2.00
	3.00	1.18	0.67	1.00	1.00	2.00	1.07	0.33	1.00	1.00	2.00
$MSS_{n,3,P_{0.30},P_{0.50},P_{0.70}}$	1.00	207.19	215.18	53.00	293.00	639.00	205.43	205.62	57.00	290.00	618.05
	1.20	41.79	46.40	8.00	60.00	135.00	35.98	38.84	8.00	51.00	115.00
	1.40	14.01	16.71	2.00	20.00	48.00	10.41	12.16	1.75	15.00	35.00
	1.60	6.25	8.00	1.00	8.00	23.00	4.50	5.13	1.00	6.00	15.00
	1.80	3.49	4.39	1.00	4.00	13.00	2.52	2.68	1.00	3.00	8.00
	2.00	2.37	2.81	1.00	3.00	8.00	1.75	1.60	1.00	2.00	5.00
	2.20	1.81	1.85	1.00	2.00	6.00	1.45	1.11	1.00	1.00	4.00
	2.40	1.53	1.42	1.00	1.00	4.00	1.27	0.78	1.00	1.00	3.00
	2.60	1.34	1.04	1.00	1.00	3.00	1.14	0.51	1.00	1.00	2.00
	2.80	1.22	0.78	1.00	1.00	3.00	1.10	0.43	1.00	1.00	2.00
	3.00	1.17	0.66	1.00	1.00	2.00	1.07	0.33	1.00	1.00	1.00
$MSS_{n,3,P_{0.35},P_{0.50},P_{0.65}}$	1.00	202.30	211.20	48.75	285.00	630.05	200.14	203.33	54.00	281.00	605.00
	1.20	41.42	47.95	6.00	60.00	139.05	35.63	38.63	7.00	51.00	112.00
	1.40	13.54	17.81	1.00	19.00	50.00	10.41	12.29	1.00	15.00	35.00
	1.60	6.01	8.03	1.00	8.00	22.00	4.41	5.21	1.00	6.00	15.00
	1.80	3.49	4.57	1.00	4.00	13.00	2.48	2.72	1.00	3.00	8.00
	2.00	2.30	2.85	1.00	2.00	8.00	1.77	1.66	1.00	2.00	5.00
	2.20	1.75	1.82	1.00	2.00	6.00	1.41	1.08	1.00	1.00	4.00
	2.40	1.45	1.25	1.00	1.00	4.00	1.23	0.72	1.00	1.00	3.00
	2.60	1.31	1.00	1.00	1.00	3.00	1.15	0.54	1.00	1.00	2.00
	2.80	1.20	0.73	1.00	1.00	2.00	1.09	0.39	1.00	1.00	2.00
	3.00	1.14	0.58	1.00	1.00	2.00	1.06	0.29	1.00	1.00	1.00
$MSS_{n,3,P_{0.40},P_{0.50},P_{0.60}}$	1.00	199.41	219.38	44.00	279.00	634.05	211.07	216.12	55.00	299.25	642.00
	1.20	39.52	47.63	5.00	56.00	138.00	36.87	40.86	7.00	53.00	119.00
	1.40	12.62	16.48	1.00	18.00	48.00	10.48	12.48	1.00	15.00	36.00
	1.60	5.65	7.70	1.00	7.00	22.00	4.41	5.27	1.00	6.00	15.00
	1.80	3.20	4.17	1.00	4.00	12.00	2.48	2.76	1.00	3.00	8.00
	2.00	2.16	2.63	1.00	2.00	7.00	1.72	1.61	1.00	2.00	5.00
	2.20	1.71	1.78	1.00	1.00	5.00	1.41	1.07	1.00	1.00	4.00
	2.40	1.42	1.24	1.00	1.00	4.00	1.22	0.70	1.00	1.00	3.00
	2.60	1.29	0.96	1.00	1.00	3.00	1.14	0.51	1.00	1.00	2.00
	2.80	1.21	0.77	1.00	1.00	3.00	1.09	0.38	1.00	1.00	2.00
	3.00	1.13	0.55	1.00	1.00	2.00	1.06	0.31	1.00	1.00	1.00
$MSS_{n,3,P_{0.45},P_{0.50},P_{0.55}}$	1.00	182.04	209.76	30.00	259.00	605.00	195.21	200.87	52.00	271.00	601.00
	1.20	35.92	46.76	3.00	51.00	132.05	35.99	40.48	6.00	51.00	119.05
	1.40	11.00	15.41	1.00	14.00	43.00	10.13	12.09	1.00	14.00	35.00
	1.60	4.99	7.16	1.00	6.00	19.00	4.33	5.26	1.00	6.00	15.00
	1.80	2.91	3.85	1.00	3.00	11.00	2.45	2.69	1.00	3.00	8.00
	2.00	2.07	2.52	1.00	2.00	7.00	1.76	1.69	1.00	2.00	5.00
	2.20	1.65	1.70	1.00	1.00	5.00	1.39	1.02	1.00	1.00	4.00
	2.40	1.41	1.19	1.00	1.00	4.00	1.23	0.73	1.00	1.00	3.00
	2.60	1.26	0.89	1.00	1.00	3.00	1.14	0.54	1.00	1.00	2.00
	2.80	1.19	0.70	1.00	1.00	2.00	1.09	0.40	1.00	1.00	2.00
	3.00	1.14	0.57	1.00	1.00	2.00	1.06	0.32	1.00	1.00	1.00

3.3 A case study

Generally, electrical engineers are engaged to monitor the variations of voltage in the system. As discussed in Section 2.1 that capacitance *(C)* has inverse relation with voltage *(V)* at fixed charge *(Q)*. So, we used 75456 sample values of V against each level of C ($i.e.$ $C = 50\mu F, 100\mu F, 150\mu F, 200\mu F, 250\mu F, 300\mu F$ and $350\mu F$) reported in [94]. In this study, we consider *(V)* as dependent variable and *(C)* as an explanatory variable. The implementation of $Shewhart-3_{[SRS]}$ chart and $Shewhart-3_{[MSS]}$ chart on the real data set is discussed with the following steps;

Step 1: For the IC regression model, we run 75456 sample values of V against fixed values of C and get a following model

$$\hat{V} = 402.3512 - 0.01983691\ C$$

Further, the properties of aforementioned linear regression model are reported in Appendix A.6.

Step 2: For the analysis, we have fixed overall ARL ($i.e.$ $ARL_0 = 200$) to obtain the charting constants of the $Shewhart-3_{[SRS]}$ chart and $Shewhart-3_{[MSS]}$ chart. These constants are computed by an extensive Monte Carlo simulation study with 10^6 iteration. The control limits are

$$\text{For } Shewhart-3_{[SRS]}: \begin{cases} LCL_I = 764.6225 \quad UCL_I = 827.4706 \\ LCL_S = -0.2972 \quad UCL_S = 0.2877 \\ LCL_E = 35.7496 \quad UCL_E = 1873.7680 \end{cases}$$

$$\text{For } Shewhart - 3_{[MSS]}: \quad \begin{cases} LCL_I = 769.1012 & UCL_I = 823.3839 \\ LCL_S = -0.2462 & UCL_S = 0.2440 \\ LCL_E = 13.5944 & UCL_E = 1294.5400 \end{cases}$$

Step 3: Once, we estimate the control limits, we used only 100 profiles as IC profiles shaded pink in Figures 3.5 and 3.6 for $Shewhart - 3_{[SRS]}$ chart and $Shewhart - 3_{[MSS]}$ chart. Further, following phases for several shifts are made by using the data perturbation approach later discussed in Section A.7:

i For the detection of shifts in intercept, we used $C^* = -350, -300, -250, -200, -150, -100 \text{ and } -50$ against 25 sets of V and the resulted 25 profiles with index 101 to 125 for $Shewhart - 3_{[SRS]}$ chart and $Shewhart - 3_{[MSS]}$ chart are portrayed in Figures 3.5 and 3.6.

ii For the detection of shifts in slope parameter, we used $C^* = -12.5, -25, -37.5, -50, -62.5, -75 \text{ and } -87.5$ against 25 sets of V and the resulted 25 profiles with index 126 to 150 for $Shewhart - 3_{[SRS]}$ chart and $Shewhart - 3_{[MSS]}$ chart are portrayed in Figures 3.5 and 3.6.

iii For the detection of shifts in variance of disturbance term, we multiply 25 sets of V with $\sqrt{2.25}$ and the resulted 25 profiles with index 151 to 175 for $Shewhart - 3_{[SRS]}$ chart and $Shewhart - 3_{[MSS]}$ chart are portrayed in Figures 3.5 and 3.6.

For $Shewhart - 3_{[SRS]}$ chart and $Shewhart - 3_{[MSS]}$ chart, the number of OOC profiles with their index are reported in Table 3.11. In the presence of shifts

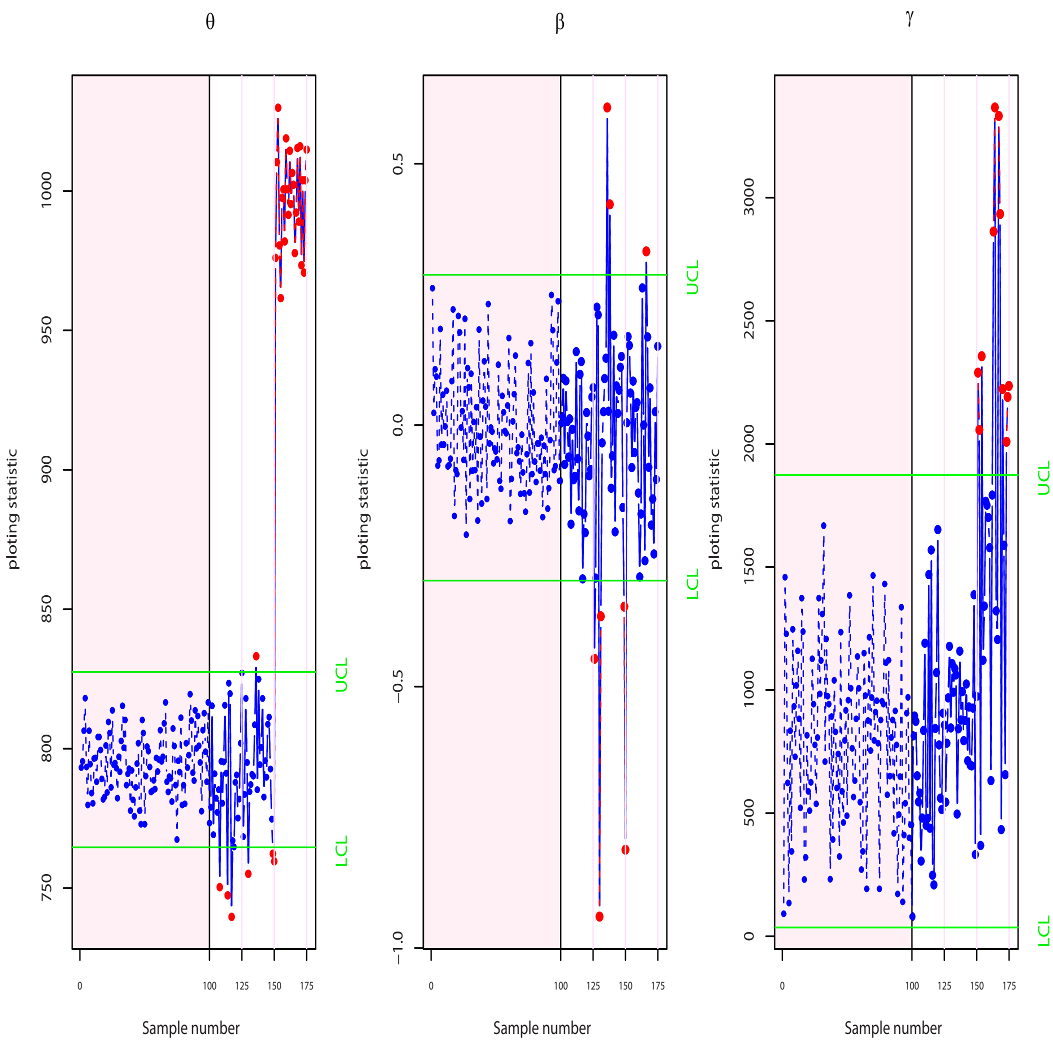

Figure 3.5: $Shewhart - 3_{[SRS]}$ chart for different phases of illustrative example

in linear profile parameters such as intercept, slope and error variance, the findings reveal that $Shewhart - 3_{[MSS]}$ chart have better detection ability relative to $Shewhart - 3_{[SRS]}$ chart. In precise, the implementation of modified successive sampling in $Shewhart{-}3$ chart enhanced its performance for the detection of OOC linear profile parameters. In grid-connected PV system, the $Shewhart - 3_{[MSS]}$ appeared as efficient chart to detect the variations in the voltage which is linearly associated with capacitance. Moreover, it may be useful for the practitioners who

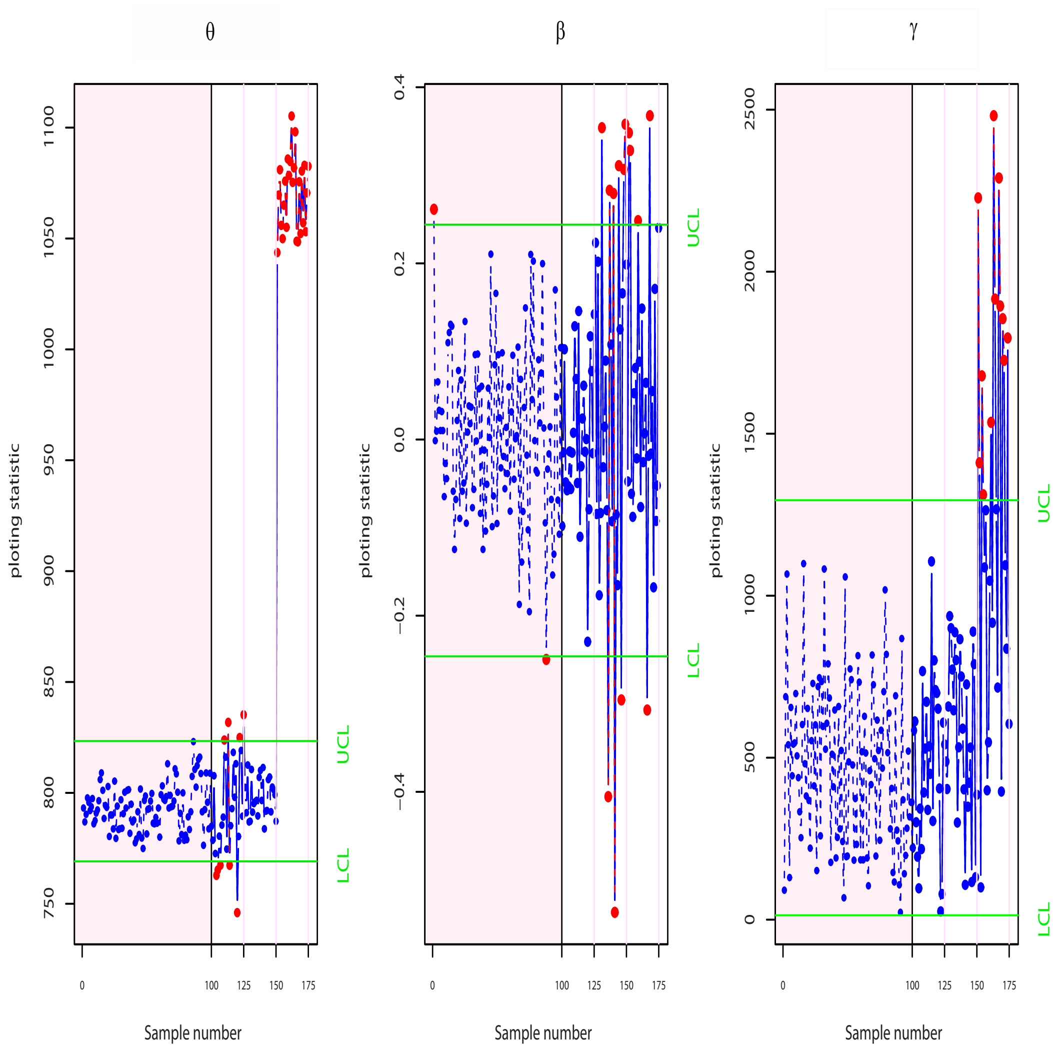

Figure 3.6: $Shewhart-3_{[MSS]}$ chart for different phases of illustrative example

are engaged in the monitoring of simple linear profile parameters.

3.4 Special case of simple linear profiles

In the literature, many studies based on statistical quality control charts are designed to monitor the mean and variability of a single study variable. In simple linear profiles, control charts are used to monitor the study variable which is linearly associated with explanatory variable. The monitoring of mean and vari-

Table 3.11: The number of OOC profiles (*index*) with respect to different phases

Parameters	Shewhart-3	Phases			
		1-100	**101-125**	**126-150**	**151-175**
Intercept	SRS	**0**	**3** *108,114,117*	**2** *130,136*	**27** *149-175*
	MSS	**0**	**9** *104,105,107, 110,113,114, 120,122,125*	**0**	**26** *150-175*
Slope	SRS	**0**	**0**	**7** *126,130,131, 136,138,149,150*	**1** *166*
	MSS	**2** 1,88	**0**	**9** *131,136,137,140, 141,144,146,148,149*	**5** *152,153, 159,166,168*
Error variance	SRS	**0**	**0**	**0**	**10** *151,152,154,163, 164,167,168, 170,173-174*
	MSS	**0**	**0**	**0**	**12** *151,152,154,155,161,163, 164,167,168,170,171,174*

ability may say a special case of simple linear profiles when the slope of the simple linear profile model is zero (i.e. $\beta_1 = 0$). The IC simple linear profile model given in equation (1.1) with $\beta_1 = 0$ is defined as:

$$Y_{[i]k} = \beta_0 + \varepsilon_{[i]k} \tag{3.1}$$

where $\beta_1 = 0$ is the arithmetic mean of Y while the variance of error term (σ_e^2) is the simple variance of Y. In literature, $\bar{Y}$ control chart is a famous technique used for the monitoring of process mean while S^2 chart is a well-known method for the monitoring of process variability. Further, these two classical charts under modified successive sampling are discussed in following subsections.

3.4.1 $\bar{Y}$ control chart under MSS

The Shewhart type charts are widely used to monitor the location in many manufacturing processes. Many of the researchers are still engaged to improve these control charts. The location charts under different sampling plans are discussed in [85, 86, 95] while some studies on other type of modifications are discussed by [96–100].

Recently, [93] used a cost efficient sampling strategy (Modified successive sampling (MSS)) to design Shewhart chart to monitor the shifts in location parameter. The plotting statistic and control limits of the Shewhart chart under MSS are defined as:

$$\overline{Y_i} = \frac{\sum_{j=1}^{n} Y_{i,j}}{n}$$

$$LCL_{MSS-M} = \mu_0 - L_{MSS-M}\sqrt{\frac{\sigma_0^2}{n}}; UCL_{MSS-M} = \mu_0 - L_{MSS-M}\sqrt{\frac{\sigma_0^2}{n}}$$

where μ_0 is the population mean,σ_0^2 is the variance of the population and L_{MSS-M} is the charting constant on the specific IC average run length (ARL_0). They have proposed four Shewhart charts based on MSS scheme and the performance of proposed chart is measured using average run length. The findings of their study depicts that the proposed scheme outperforms the existing scheme (Shewhart $\bar{Y}$ control chart under SRS) in the presence of both positive and negative amount of shifts in location parameter of the process.

3.4.2 S^2 control chart under MSS

Dispersion charts are used to monitor with in samples variability while location charts are used to monitor between samples variability. Sometime practitioners are unable to interpret an OOC signal in location chart due to instability in the variance of process. Moreover, variance is also the part of the control limits for location chart that is why it is preferable to monitor the process dispersion before location of the process.

The Shewhart type charts such as S, R and S^2 charts are widely used to monitor the dispersion in many manufacturing processes. Many of the researchers are still engaged to improve these control charts. The dispersion charts under different sampling plans are discussed in [101–105] while other type of modifications are studied by [42, 106–113]. In this subsection, we have designed a study to improve the existing S^2 chart by implementing successive sampling technique (cf. [88]). The plotting statistic and control limits of the Shewhart S^2 chart under MSS are defined as:

$$S_i^2 = \frac{\sum_{j=1}^{n} \left(Y_{i,j} - \bar{Y}_i\right)^2}{n-1} \tag{3.2}$$

$$LCL_{MSS} = \hat{\mu}_{S^2} - L_{MSS}\hat{\sigma}_{S^2} = \bar{S}_{S^2} - L_{MSS}MSE_{S^2} \tag{3.3}$$

$$UCL_{MSS} = \hat{\mu}_{S^2} + L_{MSS}\hat{\sigma}_{S^2} = \bar{S}_{S^2} + L_{MSS}MSE_{S^2} \tag{3.4}$$

where L_{MSS} is the charting constant on the specific IC average run length (ARL_0),$\bar{S}_{S^2}$ and MSE_{S^2} are the mean and mean square error of S^2 under MSS (cf. Table 3.12).

Performance evaluation and comparisons

Design of proposed control chart depends on the sample size *(n)*, number of observations from previous sample *(c)* and percentile function ($P_q \,\forall\; q = 1, 2, 3, \ldots\ldots, c$) which are used to pick observations from previous sample. We consider the samples of size n=5 and 7 while c=2 and 3 for the current study. The prefixed $ARL_0 = 370$ is used to search the appropriate L_{MSS} values which are given in Table 3.12.

Table 3.12: Properties of S^2 under MSS

c	Scheme	n=5			n=7		
		$\tilde{S}_{S^2}$	MSE_{S^2}	L_{MSS}	$\tilde{S}_{S^2}$	MSE_{S^2}	L_{MSS}
2	$MSS_{P_{0.25},\ P_{0.75}}$	1.084	0.3678	4.29	0.9731	0.258	4.13
	$MSS_{P_{0.30},\ P_{0.70}}$	0.8552	0.3415	4.39	0.8608	0.2604	4.07
	$MSS_{P_{0.35},\ P_{0.65}}$	0.7294	0.3758	4.16	0.7972	0.2775	3.92
	$MSS_{P_{0.40},\ P_{0.60}}$	0.6689	0.4004	4.006	0.7601	0.2922	3.83
	$MSS_{P_{0.45},\ P_{0.55}}$	0.6397	0.4159	3.92	0.7417	0.2992	3.78
3	$MSS_{P_{0.25}, P_{0.50},\ P_{0.75}}$	0.7565	0.2996	4.16	0.7768	0.2533	3.86
	$MSS_{P_{0.30}, P_{0.50},\ P_{0.70}}$	0.5514	0.4035	3.49	0.6827	0.2917	3.55
	$MSS_{P_{0.35}, P_{0.50},\ P_{0.65}}$	0.46	0.4823	3.15	0.6277	0.3247	3.34
	$MSS_{P_{0.40}, P_{0.50},\ P_{0.60}}$	0.4291	0.5059	3.045	0.6016	0.342	3.25
	$MSS_{P_{0.45}, P_{0.50},\ P_{0.55}}$	0.4126	0.5211	2.97	0.5901	0.3499	3.21

The dispersion charts such as S^2 chart are important and applicable to detect the degree of change in the variation of process. Along with explaining the IC properties of the charts, it is useful to examine the OOC performance of the charts. The OOC average run length (ARL_1), standard deviation of run length *SDRL* and different percentiles (25^{th}, 75^{th} and 95^{th}) of run length distribution are given in Tables 3.13-3.15 for S^2 chart under SRS and MSS respectively. To check the OOC performance of the proposed charts, shifts of different size are introduced in dispersion parameter. That shift parameter is denoted by γ and is equal to $\gamma = \frac{\sigma_1}{\sigma_0}$ where σ_1 is the OOC process standard deviation. The run length study for S^2 chart under SRS is reported in Table 3.13. The findings indicate that an upward shift (20%) in dispersion parameter from the in-control situation resulted about 55.04% and 60.36% decrease in (ARL_1) of S^2 chart under SRS for both cases (i.e.n=5 and 7) respectively.

Table 3.13: Run length properties of S^2 chart under SRS

γ	n=5					n=7				
	ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
1.00	369.90	370.87	106.00	513.00	1110.00	373.15	374.60	108.00	516.00	1119.00
1.10	253.75	253.87	73.00	353.00	758.05	245.81	244.15	71.00	342.00	732.00
1.20	166.31	166.58	48.00	231.00	497.00	147.92	147.50	43.00	205.00	443.00
1.30	109.23	108.91	32.00	151.00	325.00	90.80	90.53	26.00	125.00	271.00
1.40	74.33	73.69	22.00	103.00	222.00	59.09	58.74	17.00	82.00	176.00
1.50	52.68	52.23	15.00	73.00	157.00	40.15	39.65	12.00	56.00	119.00
1.60	38.86	38.56	11.00	54.00	115.00	28.57	28.14	8.00	39.00	85.00
1.70	29.79	29.36	9.00	41.00	89.00	21.22	20.72	6.00	29.00	62.00
1.80	23.43	22.87	7.00	32.00	70.00	16.61	16.11	5.00	23.00	49.00
1.90	18.93	18.46	6.00	26.00	56.00	13.21	12.76	4.00	18.00	39.00
2.00	15.58	15.00	5.00	21.00	46.00	10.78	10.32	3.00	15.00	31.00

The results for S^2 chart under MSS at fixed c=2 are given in Table 3.14. If the choice of percentiles pair is ($P_{0.25}$, $P_{0.75}$) then (30%) upward shift in dispersion parameter, may decrease 80.47% and 83.83% ARL_1 of said S^2 chart for both cases i.e. n=5 and 7 respectively. Moreover, if the choice of percentiles pair is then upward shift in dispersion parameter, may decrease 88.42% and 91.73% ARL_1 of S^2 chart for both cases i.e. n=5 and 7 respectively. Finally, the run length study for S^2 chart under MSS at fixed c=3 is reported in Table 3.15. If the choice of percentiles is ($P_{0.30}$, $P_{0.50}$, $P_{0.70}$) then an upward 40% shift in dispersion parameter decreases the ARL_1 of said S^2 chart for both cases (n=5 and 7) to 61.64 and 45.49 respectively. Further, when choice of ercentiles is ($P_{0.40}$, $P_{0.50}$, $P_{0.60}$)

then an upward 60% shift in dispersion parameter may decrease up to 36.65 and 24.51 ARL_1 of the said S^2 chart for both cases i.e. n=5 and 7 respectively.

Considering the different sample sizes (i.e. n=5 and 7), number of observations from previous sample (i.e. c=2 and 3), shifts in dispersion parameter γ (on horizontal axis) and log average run length ($\ln(ARL)$) (on vertical axis), we have portrayed the display in Figure 3.7. The results depict that the performance of charts increase with the increase of shift in dispersion parameter $\theta = 1.00$ up to 2. It is also observed that S^2 chart under MSS with choice of percentile pairs ($P_{0.25}$, $P_{0.75}$) and ($P_{0.25}$, $P_{0.50}$, $P_{0.75}$), outperforms then the other S^2 charts under different schemes.

Illustrative example

Nowadays, electrical engineers take interest in the Z-source inverter for a grid connected PV system instead of conventional voltage source inverter *(VSI)* and conventional current source inverter *(CSI)* (for more detail see Section 2.1). For an illustrative example, we get 75456 sample values of Voltage *(V)* against each level of Capacitance *(C)* given in [94]. In the stated study, we consider 75455 values of Voltage *(V)* against 150 , 250 and 350 capacitance level which are further divided into 15091 subgroups each of size 5.

For the classical Shewhart S^2 chart, we estimate sample variance of each subgroup belongs to $350\mu F$ capacitance level and through it we calculate the lower control limit $LCL_{SRS} = 0.0144$ and upper control limit $UCL_{SRS} = 2.463$. On the other hand, for the Shewhart S^2 chart under ($MSS_{5,2,Q_{0.25},\, Q_{0.75}}$), we estimate sam-

Table 3.14: Run length properties of S^2 chart under MSS at fixed $c = 2$

Schemes	γ	***n=5***					***n=7***				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,2,P_{0.25},P_{0.75}}$	1	369.56	369.08	106	513	1109	369.21	371.56	105	511.25	1110
	1.1	193.53	194.28	55	269	579	177.7	177.88	51	247	536
	1.2	113.28	113.41	32	157	340	97.62	97.61	28	136	292
	1.3	72.18	72.09	21	100	217	59.7	60.09	17	83	179
	1.4	49.45	49.48	14	69	148	39.16	39.31	11	54	118
	1.5	35.53	35.46	10	49	106	27.24	27.26	8	38	81
	1.6	26.97	26.9	8	37	81	20.18	20.37	6	28	61
	1.7	21.23	21.13	6	29	63	15.36	15.33	4	21	46
	1.8	17.04	16.96	5	24	51	12.23	12.15	3	17	37
	1.9	14.09	14	4	19	42	9.94	9.81	3	14	30
	2	11.88	11.78	3	16	35	8.32	8.22	2	11	25
$MSS_{n,2,P_{0.30},P_{0.70}}$	1	371.31	371.98	106	514	1110	371.72	375.07	105	515.25	1120
	1.1	200.19	202.15	56	278	607	183.68	185.5	52	255	551
	1.2	119.88	121.58	33	166	362	102.47	103.13	29	143	309
	1.3	77.33	78.68	21	108	234	62.62	63.46	17	87	189
	1.4	53.3	54.19	14	74	161	41.59	42.56	11	58	126
	1.5	38.63	39.71	10	54	118	29.24	29.94	8	41	89
	1.6	29.29	30.11	8	41	89	21.33	21.85	6	30	65
	1.7	22.69	23.32	6	32	70	16.33	16.61	4	23	50
	1.8	18.49	19.18	5	26	57	12.93	13.27	3	18	39
	1.9	15.15	15.69	4	21	46	10.51	10.77	3	15	32
	2	12.7	13.19	3	18	39	8.65	8.82	2	12	26
$MSS_{n,2,P_{0.35},P_{0.65}}$	1	374.12	375.77	106	521	1122	365.67	367.15	104	509	1096
	1.1	206.36	208.55	57	288	623	184.49	185.84	52	256	558
	1.2	124.14	126.43	34	173	376	103.32	105.21	29	143	313
	1.3	81.44	83.01	22	114	246	63.94	65.25	17	89	194
	1.4	56.46	58.17	15	79	173	42.17	43.11	11	59	128
	1.5	41.1	42.65	11	58	126	29.71	30.59	8	42	91
	1.6	31.1	32.49	8	44	96	22.06	22.76	6	31	68
	1.7	24.32	25.58	6	34	76	16.7	17.38	4	23	52
	1.8	19.49	20.61	5	27	61	13.2	13.84	3	18	41
	1.9	16	16.99	4	23	50	10.63	11.12	3	15	33
	2	13.4	14.31	3	19	42	8.89	9.28	2	12	27
$MSS_{n,2,P_{0.40},P_{0.60}}$	1	376.03	379.63	106	521	1134	375.33	378.75	105	522	1129
	1.1	207.91	211	57	290	628	189.03	191.7	53	262	573
	1.2	127.41	129.93	35	178	386	107.02	108.71	29	149	324
	1.3	84.13	86.42	22	118	257	66.26	67.76	18	92	202
	1.4	58.41	60.58	15	82	179.05	44.02	45.16	12	61	134
	1.5	42.51	44.1	11	60	131	30.86	31.93	8	43	95
	1.6	32.41	34.13	8	45	101	22.66	23.57	6	32	70
	1.7	25.32	26.93	6	36	79	17.35	18.33	4	24	54
	1.8	20.21	21.49	5	28	63	13.68	14.42	3	19	42
	1.9	16.72	17.94	4	24	53	11.1	11.72	3	16	35
	2	14.02	15.26	3	20	45	9.09	9.62	2	13	29
$MSS_{n,2,P_{0.45},P_{0.55}}$	1	371.87	373.74	105	518	1122	376.04	377.2	107	525	1120
	1.1	208.89	211.68	58	291	637	188.64	190.19	53	262	570
	1.2	128.23	131.36	35	179	390	107.9	109.52	30	150	327
	1.3	85.32	88.56	22	120	261	66.78	68.19	18	93	201
	1.4	59.26	61.36	15	83	182	44.49	46.04	12	62	136
	1.5	43.07	45.14	11	60	134	31.1	32.17	8	44	96
	1.6	32.6	34.48	8	46	102	23.1	24.01	6	32	71
	1.7	25.61	27.48	6	36	81	17.55	18.42	4	25	55
	1.8	20.54	22.18	4	29	65	13.87	14.59	3	19	43
	1.9	16.79	18.27	3	24	54	11.22	11.9	3	16	35
	2	14.06	15.44	3	20	45	9.22	9.81	2	13	29

Table 3.15: Run length properties of S^2 chart under MSS at fixed $c = 3$

Schemes	γ	***n=5***					***n=7***				
		ARL	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$	***ARL***	***SDRL***	$\mathbf{Q_{0.25}}$	$\mathbf{Q_{0.75}}$	$\mathbf{Q_{0.95}}$
$MSS_{n,2,P_{0.25},P_{0.50},P_{0.75}}$	1	373.83	379.94	102	520	1134	374.13	377.8	105	521	1129
	1.1	208.43	213.7	56	290	636	187.6	191.13	52	260	571
	1.2	126.35	130.14	33	177	386	106.39	108.87	29	149	322
	1.3	82.62	85.98	21	116	254	65.75	68.32	17	92	203
	1.4	58.02	61.31	14	81	182	43.29	45.25	11	61	134
	1.5	42.32	45.48	10	60	133	30.7	32.42	7	43	95
	1.6	31.92	34.57	7	45	101	22.47	23.8	5	32	71
	1.7	25.01	27.42	5	35	80	17.15	18.35	4	24	54
	1.8	20.22	22.22	4	29	65	13.51	14.64	3	19	43
	1.9	16.63	18.44	3	24	54	10.92	11.96	2	15	35
	2	13.88	15.63	2	20	45	9.09	9.88	2	13	29
$MSS_{n,2,P_{0.30},P_{0.50},P_{0.70}}$	1	368.52	376.5	99	515	1121	374.1	379.64	104	522	1129
	1.1	211.02	219.56	55	296	650	190.71	195.74	51	265	580
	1.2	131.91	139.26	32	186	409	108.75	112.75	28	152	335
	1.3	87.46	93.8	20	123	276	68.32	71.99	17	96	212
	1.4	61.64	66.89	13	87	196	45.49	48.13	11	64	141
	1.5	45.13	50.2	9	64	145	32.17	34.72	7	46	102
	1.6	34.21	38.58	6	49	112	23.62	25.91	5	33	75
	1.7	26.68	30.66	4	38	88	17.98	19.77	3	26	57
	1.8	21.61	25.21	3	31	73	14.12	15.83	2	20	46
	1.9	17.65	20.85	2	25	60	11.36	12.81	2	16	37
	2	14.78	17.64	2	21	50	9.4	10.69	1	13	31
$MSS_{n,2,P_{0.35},P_{0.50},P_{0.65}}$	1	375.11	385.43	99	524	1151	367.13	372.65	102	511	1111
	1.1	217.49	228.07	55	306	673	190.47	195.58	51	265	582
	1.2	135.84	145.65	32	191	428	110.47	114.5	29	155	340
	1.3	91.23	99.84	19	129	292	69.39	73.47	17	97	216
	1.4	64.46	71.81	12	92	209	46.38	49.53	11	66	146
	1.5	47.28	53.58	8	67	155	32.68	35.63	7	47	105
	1.6	35.71	41.39	5	51	119	24.33	26.87	5	34	78
	1.7	28.14	33.37	4	40	95	18.45	20.73	3	26	60
	1.8	22.69	27.22	3	32	78	14.44	16.4	2	21	47
	1.9	18.46	22.64	2	26	65	11.61	13.32	2	16	39
	2	15.32	19.01	1	22	54	9.58	11.14	1	14	32
$MSS_{n,2,P_{0.40},P_{0.50},P_{0.60}}$	1	376.46	388.17	98	529	1151	368.15	375.18	101	512	1116
	1.1	219.78	232.8	53	308	685	193.73	198.72	52	269.25	592
	1.2	138.21	149.28	31	195	438	112.05	116.81	28	157	345.05
	1.3	92.89	101.98	19	132	297	70.82	74.91	17	100	219
	1.4	65.66	73.55	12	93	213	47.19	50.86	10	67	149
	1.5	48.34	55.43	8	69	159	33.43	36.61	7	48	107
	1.6	36.65	43.04	5	53	123	24.51	27.41	4	35	79
	1.7	28.73	34.63	3	41	98	18.72	21.27	3	27	62
	1.8	22.88	28.14	2	33	80	14.62	16.8	2	21	48
	1.9	18.65	23.12	2	27	65	11.83	13.71	2	17	40
	2	15.66	19.72	1	22	55	9.68	11.36	1	14	33
$MSS_{n,2,P_{0.45},P_{0.50},P_{0.55}}$	1	370.2	386.05	94	519	1144	369.79	376.85	101	517	1123
	1.1	215.3	229.16	51	304	675	193.1	197.78	51	271	588
	1.2	136.48	148.08	29	194	432	113.43	118.53	29	159	350
	1.3	91.67	102.09	17	131	297	71.11	75.48	17	101	222
	1.4	65.05	73.81	11	93	214	47.94	51.99	10	68	152
	1.5	47.89	55.95	7	69	161	33.56	36.85	7	48	107
	1.6	36.22	43.49	5	52	124	24.99	27.98	5	35	81
	1.7	28.32	34.58	3	40	99	18.88	21.49	3	27	62
	1.8	22.61	28.12	2	32	80	14.85	17.18	2	21	49
	1.9	18.39	23.37	1	26	66	11.85	13.94	1	17	40
	2	15.15	19.54	1	21	55	9.73	11.51	1	14	33

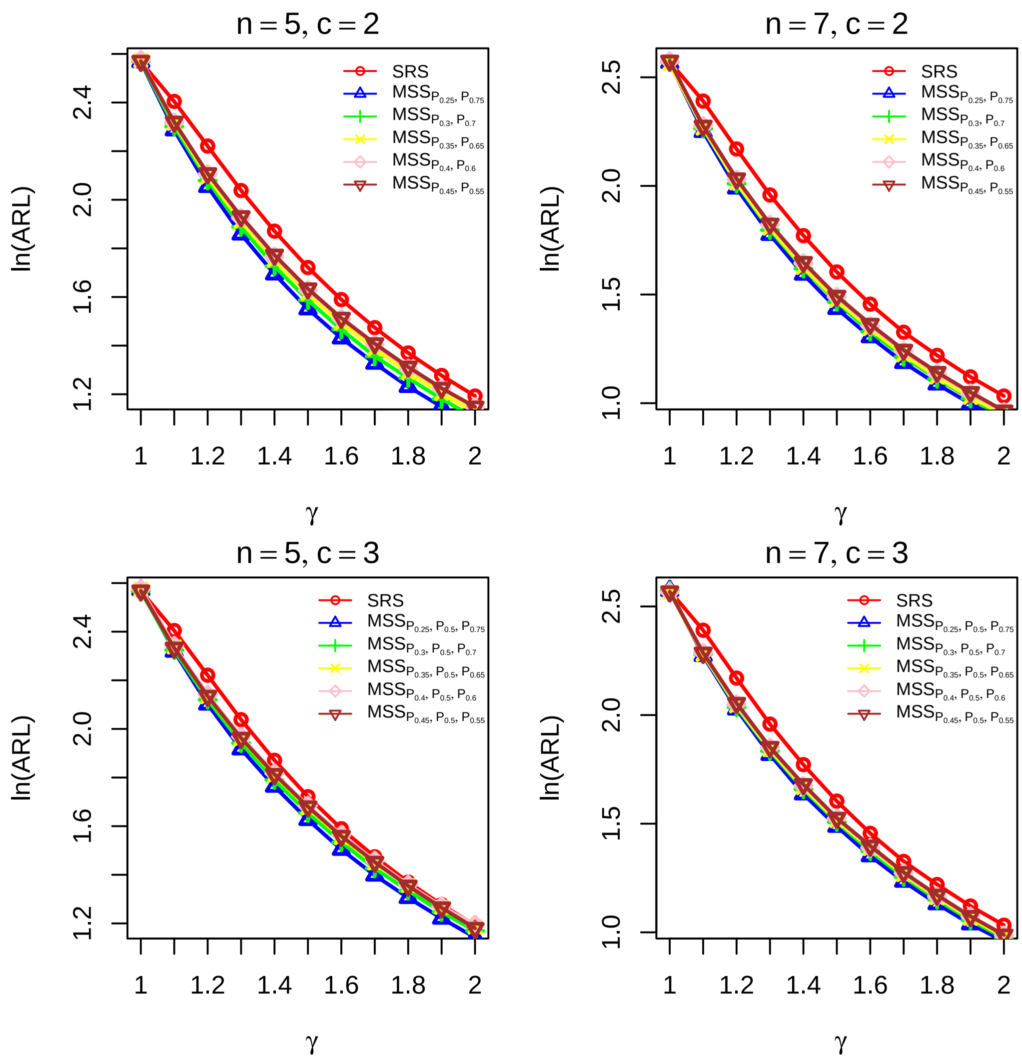

Figure 3.7: Comparative analysis of S^2 charts under different schemes

ple variances of subgroups after implementing the modified successive sampling on existing subgroups and calculate the control limits (i.e. $LCL_{MSS} = 0.0183$ and $UCL_{MSS} = 1.9868$). Further, if plotting statistic S^2_{SRS} or S^2_{MSS} fall outside of their corresponding limits then the process is declared OOC. This could imply that the generation of voltage has been disturbed and engineers have to look for the factor(s) behind OOC condition.

For the diagnosis purpose, we select first 100 in-control subgroups from $350\mu F$, second 100 shifted subgroups from $250\mu F$ and finally last 100 shifted subgroups from $150\mu F$. We calculate sample variances (S^2_{SRS} and S^2_{MSS}) of selected 300 subgroups which are plotted against the control limits in Figures 3.8 and 3.8. The classical Shewhart S^2 chart depicts that there exist no OOC point in first 100 subgroups, in next 100 subgroups 10 OOC signals are received and in last 100 subgroups 70 points are declared OOC. However, the Shewhart S^2 chart under MSS reveals no OOC point in first 100 subgroups, 14 OOC points in the next 100 subgroups and 76 OOC signals in last 100 samples. This shows that our proposed schemes perform well in the detection of voltage converted by Z-source inverter in 3-φ grid connected PV system.

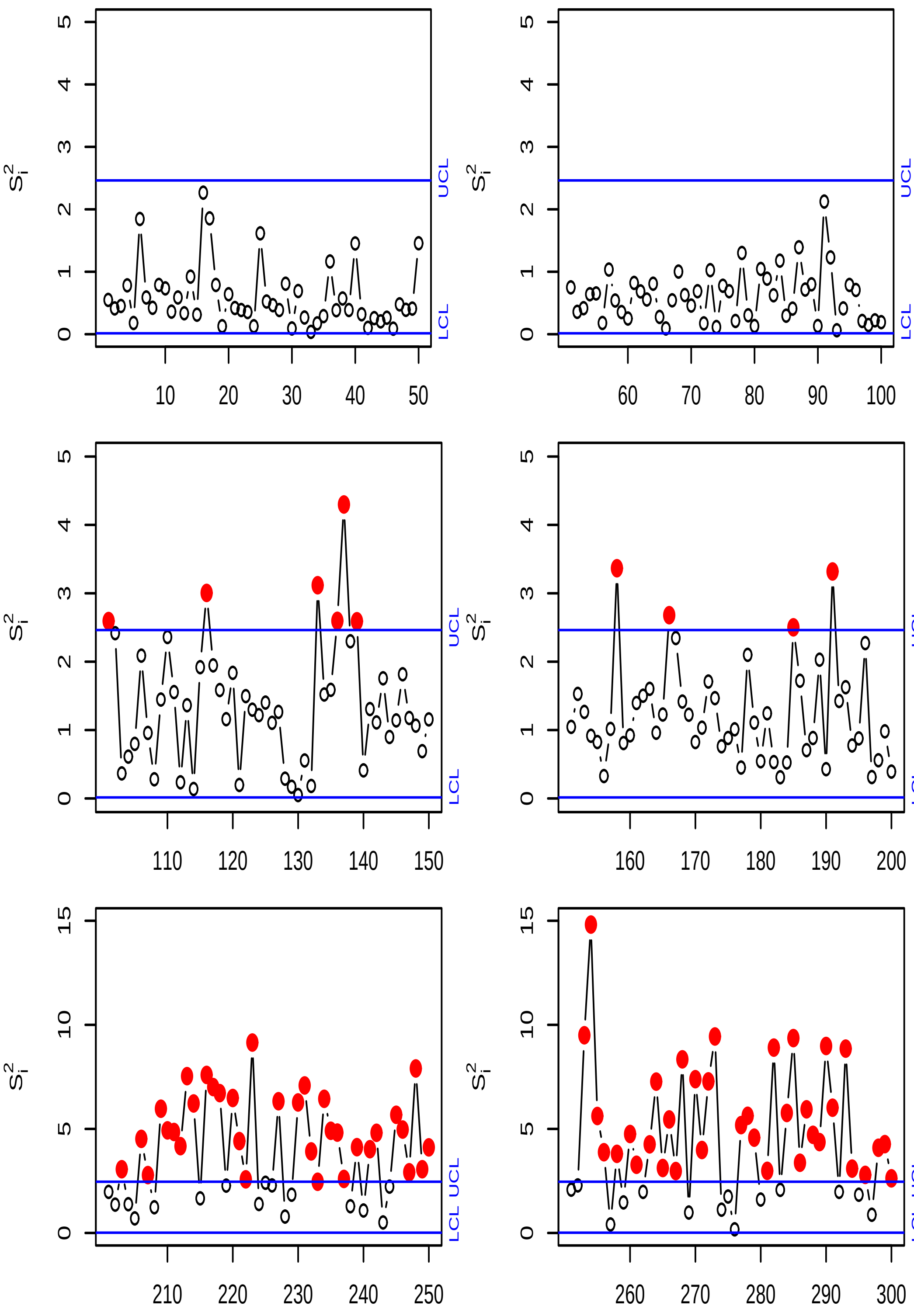

Figure 3.8: Shewhart $S^2{}_{[SRS]}$ chart for the detection of voltage in 3-φ grid connected PV system

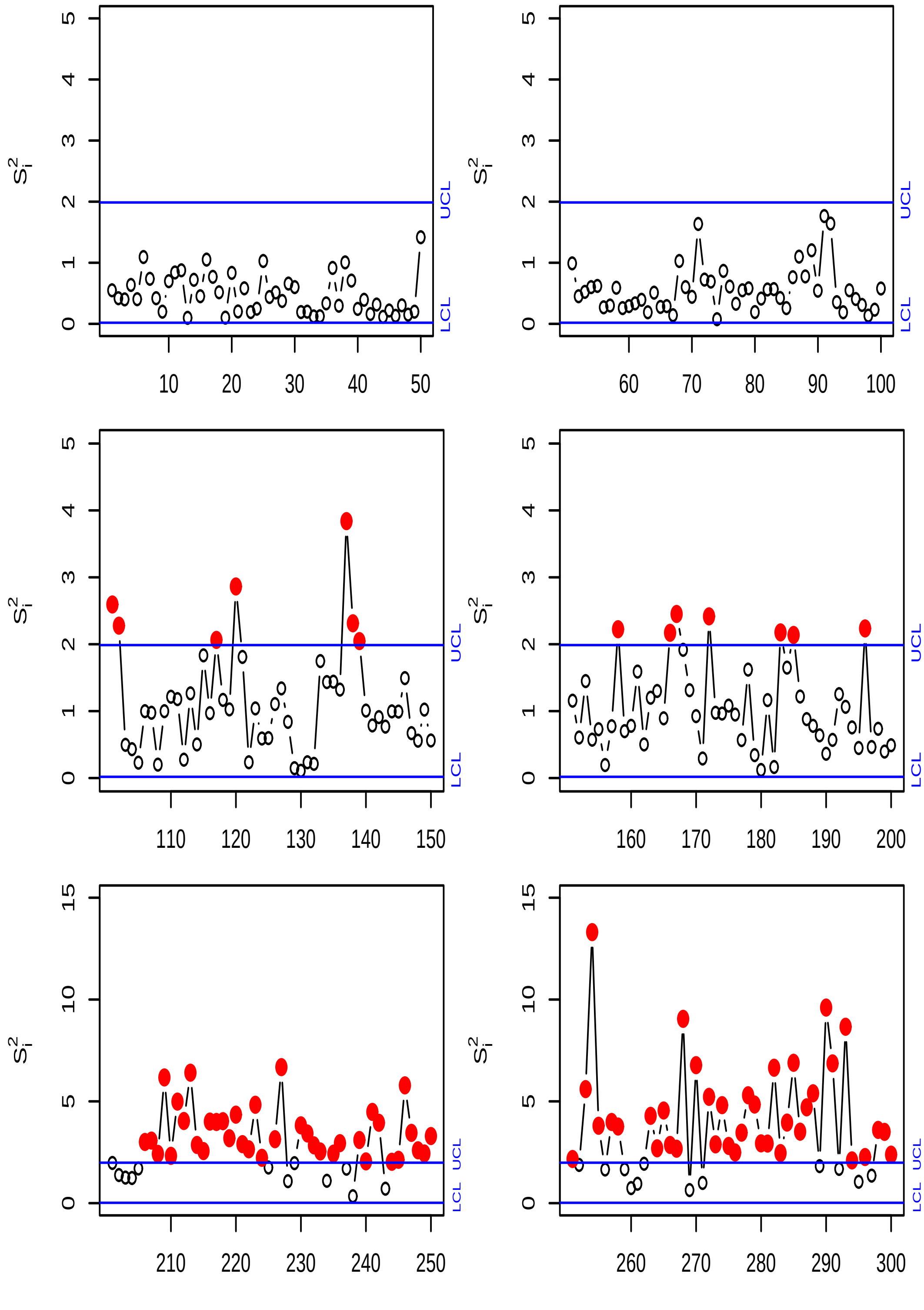

Figure 3.9: Shewhart $S^2{}_{[MSS]}$ chart for the detection of voltage in 3-φ grid connected PV system

3.5 Concluding Remarks

In any process, when the study variable is linearly associated with a single explanatory variable then the monitoring of such process is known as simple linear profiling. In simple linear profiling three parameters (slope, intercept and error variance) are considered for the monitoring purpose. Most of the recent literature on simple linear profiling has been done under simple random sampling but in this chapter, we implemented modified successive sampling to enhanced the existing Shewhart chart. Moreover, the monitoring of mean and variability may say a special case of simple linear profiles when the slope of the simple linear profile model is zero (i.e. $\beta_1 = 0$). Practically, detection of dispersion shift is important before the detection of location shift in the process. The classical S^2 chart is the best choice from the literature for the monitoring of dispersion parameter. This chapter also proposes an improved S^2 chart by using modified successive sampling. The run length properties are used for performance measure which indicates that all the proposed charts under MSS outperforms the classical charts which are based on SRS.

CHAPTER 4

AN ALTERNATIVE APPROACH TO SIMULTANEOUS MONITORING OF LINEAR PROFILE PARAMETERS

Often control charts are designed to monitor single parameter of the process but very few studies are available related to simultaneous and joint monitoring of the process parameters. In simultaneous monitoring, parameters are monitored through separate plotting statistics plotted against distinct pair of control limits while joint monitoring is a term that alludes to monitoring parameters through a single plotting statistic plotted against a pair of control limits. One may see brief literature on simultaneous and joint monitoring in Section 1.3.2. In this chapter, we have designed joint structures for the monitoring of simple linear profile parameters and discussed the special case of joint linear profiling and EWMA structure.

4.1 Joint monitoring of linear profile parameter

The recent studies for simple linear profiles (given in Section 1.3.1) are based on simultaneous structure which is a tedious method such as distinct pair of control limits required individual charting constants. For example, in case of linear profile parameters such as slope, intercept and error variance, on fixed overall average run length 200, one my need 584.7 average run length for each individual chart of simultaneous structure which is a tedious method for the practitioner. EWMA-3 chart proposed by [22] is well-known methodology based on simultaneous structure for the monitoring of linear profile parameters (i.e. intercept, slope and error variance). In this study, we have designed new control charts based on joint structures for the monitoring of simple linear profile parameters which is a simple procedure and easy to applicable.

4.1.1 Simple linear profile structures

In this subsection, we provide the theoretical structure of our proposed joint methodologies and the existing charts such as Hoteling T2 chart, EWMA/R chart, Shewhart-3 chart and EWMA-3 chart.

Proposed control charts

Control chart based on simple linear profiles play a key role in any process where the study variable is linearly associated with another explanatory variable. The simple linear model given in equation (1.1) is a basic model used in linear profiling but due to the limitation (e.g. independence of parameters) model (1.2) (transformed model) was preferred in many studies. The least square estimates of the parameters of transformed model are represented by $\hat{b}_0$, $\hat{b}_1$ and $\widehat{MSE}$. For the joint monitoring, we transformed the estimates to get the normality such as:

$$Z_{\hat{b}_0} = \frac{\hat{b}_0 - B_0}{\sqrt{\sigma^2 \left({}^1\!/_{nm} \right)}}$$

$$Z_{\hat{b}_1} = \frac{\hat{b}_1 - B_1}{\sqrt{\left({}^{\sigma^2}\!/_{S_{xx}} \right)}}$$

$$Z_{\widehat{MSE}} = \Phi^{-1} \left[H \left\{ \frac{(n-2)\,\widehat{MSE}}{\sigma_0^2}; n-2 \right\} \right]$$

where $\Phi^{-1}[.]$ is inverse standard normal distribution function and $H\{.;(n-2)\}$ is termed as chi-square distribution function having $(n-2)$ degree of freedom. In recent literature, two more transformations are used for the dispersion parameter to gain approximate normal results such as three-parameter logarithmic transformation (cf. [114]) and Johnson SB transformation (cf. [115]). The description of these two transformations for mean square error is

$$T_{\widehat{MSE}} = a_T + b_T \ln\left(\widehat{MSE} + c_T\right)$$

$$U_{\widehat{MSE}} = a_U + b_U \ln\left(\frac{\widehat{MSE} - c_U}{d_U + c_U - \widehat{MSE}}\right)$$

where $a_T = A_T(n) - 2B_T(n)\ln\sigma_0, b_T = B_T(n), c_T = C_T(n)\sigma_0^2, a_U = A_U(n), b_U = B_U(n), c_U = C_U(n)\sigma_0^2$ and $d_U = D_U(n)\sigma_0^2$. The values of these constants are reported in Table 4.1 for $n = 3,\ 4,\ 5, \ldots,\ 15$.

Table 4.1: Constant for transformations (three-parameter logarithmic transformation and Johnson S_B transformation)

n	a_T	b_T	c_T	a_U	b_U	c_U	d_U
3	-0.6627	1.8136	0.6777	3.1936	1.1952	-0.2588	15.0770
4	-0.7882	2.1089	0.6261	3.3657	1.3983	-0.2438	12.5910
5	-0.8969	2.3647	0.5979	3.5402	1.5727	-0.2352	11.3120
6	-0.9940	2.5941	0.5801	3.7111	1.7281	-0.2295	10.5300
7	-1.0827	2.8042	0.5678	3.8768	1.8698	-0.2254	10.0000
8	-1.1647	2.9992	0.5588	4.0369	2.0010	-0.2224	9.6180
9	-1.2413	3.1820	0.5519	4.1918	2.1238	-0.2200	9.3280
10	-1.3135	3.3548	0.5465	4.3417	2.2396	-0.2181	9.1000
11	-1.3820	3.5189	0.5421	4.4869	2.3495	-0.2166	8.9170
12	-1.4473	3.6757	0.5384	4.6279	2.4544	-0.2152	8.7660
13	-1.5097	3.8260	0.5354	4.7648	2.5549	-0.2141	8.6400
14	-1.5697	3.9705	0.5327	4.8981	2.6515	-0.2132	8.5320
15	-1.6275	4.1100	0.5305	5.0279	2.7446	-0.2123	8.4400

The Max-EWMA-3 charting structures

The exponentially weighted moving average chart (EWMA) was firstly originated by [10] which is an effective technique to monitor small or moderate shifts in the process. Further, [12] proposed a modified EWMA chart termed as Max-

EWMA for the joint monitoring of two parameters (location and scale). We have used similar Max-EWMA approach to monitor the linear profile parameters (i.e. intercept, slope and error variance) which is further referred as Max-EWMA-3 chart. The structure of Max-EWMA-3 depends on EWMA statistics which are based on $Z_{\hat{b}_0}$, $Z_{\hat{b}_1}$ and transformed mean square error $\left(Z_{\widehat{MSE}},\ T_{\widehat{MSE}} \text{ and } U_{\widehat{MSE}}\right)$,

$$M_i = \lambda Z_{\hat{b}_0} + (1-\lambda)\, M_{i-1}$$

$$N_i = \lambda Z_{\hat{b}_1} + (1-\lambda)\, N_{i-1}$$

$$O_i = \lambda Z_{\widehat{MSE}} + (1-\lambda)\, O_{i-1}$$

$$P_i = \lambda T_{\widehat{MSE}} + (1-\lambda)\, P_{i-1}$$

$$Q_i = \lambda U_{\widehat{MSE}} + (1-\lambda)\, Q_{i-1}$$

where $M_0,\ N_0,\ O_0,\ P_0$ and Q_0 are used as initial values and λ is a smoothing (weight) parameter having range $(0 < \lambda \leq 1)$. As discussed above that there exist three transformations to obtain the normality of error variance. So, based on these three transformations, three separate Max-EWMA-3 charts with their limits are given below:

$$Max - EWMA - 3 - A_i : \begin{cases} Statistic & Max\left(|M_i|, |N_i|, |O_i|\right), \\ UCL_{MO_i} & \sqrt{\frac{\lambda}{2-\lambda}}\left(1.32639 + 0.5859607 L_{Max}\right) \end{cases}$$

$$Max - EWMA - 3 - B_i : \begin{cases} Statistic & Max\left(|M_i|, |N_i|, |P_i|\right), \\ UCL_{MP_i} & \sqrt{\frac{\lambda}{2-\lambda}}\left(1.32639 + 0.5859607 L_{Max}\right) \end{cases}$$

$$Max - EWMA - 3 - C_i : \begin{cases} Statistic & Max\left(|M_i|, |N_i|, |Q_i|\right), \\ UCL_{MQ_i} & \sqrt{\frac{\lambda}{2-\lambda}}\left(1.32639 + 0.5859607 L_{Max}\right) \end{cases}$$

where L_{Max} is the control limits coefficient that is used to control the IC run length behavior of the chart.

The SS-EWMA-3 charting structures

Another approach for the joint monitoring of process parameters based on sum of square of EWMA statistics was proposed by [26]. We used this concept for the monitoring of linear profile parameters and referred as SS-EWMA-3 chart. The structure of SS-EWMA-3 chart depends on aforementioned EWMA statistics which are based on $Z_{\hat{b}_0}$, $Z_{\hat{b}_1}$ and transformed mean square error $\left(Z_{\widehat{MSE}},\ T_{\widehat{MSE}} \text{ and } U_{\widehat{MSE}}\right)$,

$$SS - EWMA - 3 - A_i : \begin{cases} Statistic & M_i^2 + N_i^2 + O_i^2 \\ UCL_{SSO_i} & \frac{\lambda\left(3 + L_{SS}\sqrt{6}\right)}{2-\lambda} \end{cases}$$

$$SS - EWMA - 3 - B_i : \begin{cases} Statistic & M_i^2 + N_i^2 + P_i^2 \\ UCL_{SSP_i} & \frac{\lambda\left(3 + L_{SS}\sqrt{6}\right)}{2-\lambda} \end{cases}$$

$$SS - EWMA - 3 - C_i : \begin{cases} Statistic & M_i^2 + N_i^2 + Q_i^2 \\ UCL_{SSQ_i} & \frac{\lambda\left(3 + L_{SS}\sqrt{6}\right)}{2-\lambda} \end{cases}$$

where L_{SS} is the control limits coefficient that is used to control the IC run length behavior of the chart.

Existing control charts

For the comparison, we have considered several existing simple linear profile methods such as Shewhart-3 chart, EWMA-3 chart, T^2 chart and EWMA/R chart. The Shewhart-3 chart was proposed by [21] while EWMA-3 chart was originated by [22] which are already discussed in Section 1.2. Further, the structures of T^2 chart and EWMA/R chart are given below.

The T^2 chart

[31] proposed a multivariate control chart for the monitoring of slope and intercept. The j^{th} statistic of T^2 control chart is estimated by

$$T_j^2 = (Z_j - U)^T \Sigma^{-1} (Z_j - U)$$

where

$$Z_j = \left(\hat{\beta}_{0j}\ ,\ \hat{\beta}_{1j}\right)^T;\ U = (\beta_0, \beta_1)^T$$

$$\Sigma = \begin{bmatrix} \sigma^2\left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right] & -\sigma^2\frac{\bar{X}}{S_{XX}} \\ -\sigma^2\frac{\bar{X}}{S_{XX}} & \frac{\sigma^2}{S_{XX}} \end{bmatrix}$$

The T^2 statistic follows χ^2 distribution with 2 degree of freedom and the upper control limit ($UCL_H = {\chi^2}_{2,\alpha}$) is the α^{th} quantile of χ^2 distribution while lower control limit ($LCL_H = 0$). When process is unstable then the T^2 statistic follows non-central χ^2 distribution with non-centrality parameter (τ), which is obtain as:

$$\tau = n\left(\varphi\sigma + \beta\sigma\bar{X}\right)^2 + (\beta\sigma)^2 S_{XX}$$

where φ is the amount of shift in intercept for model (1.1) and β is the measure of shift in the slope of model (1.2).

The EWMA/R chart

[31] also proposed a combined structure based on EWMA and R chart for the monitoring of linear profile parameters. Basically, EWMA chart has some limitations which are covered by incorporating the R chart. The j^{th} statistic of EWMA chart is estimated by

$$Z_j = \lambda\bar{e}_j + (1-\lambda)\, Z_{j-1}$$

where, λ is the smoothing parameter which ranges from 0 to 1, $\bar{e}_j = \frac{\sum_{i=1}^{n} e_{ij}}{n}$ and the initial value of EWMA statistic is zero. (i.e. $Z_0 = 0$). The process is said to be out-of-control (OOC) when Z_j is less than LCL or greater than UCL. The control limits (LCL and UCL) based on charting constant $(L_E R)$ for EWMA chart are given as follow:

$$LCL_E = -L_{ER}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}; UCL_E = L_{ER}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}$$

There exist two causes to combine R chart with EWMA chart, (i) to detect shifts in error variance under model (1.1) and (ii) to tackle the unusual situation of error variance. Further, the j^{th} statistic and control limits of R chart are defined as

$$R_j = max_i\left(e_{ij}\right) - min_i\left(e_{ij}\right)$$

$$LCL_R = \sigma\left(d_2 - L_{ER}d_3\right); UCL_R = \sigma\left(d_2 + L_{ER}d_3\right)$$

where d_2 and d_3 are unbiased constants reported in [1].

4.1.2 Performance evaluations

In this subsection, we provide a brief discussion on the IC parameters of proposed charts. Moreover, we will discuss the performance evaluation of the stated study.

Designing of in-control parameters and control limits

For the original IC simple linear model given in equation (1.1), we assumed $\beta_0 = 3$ and $\beta_0 = 2$ by following [22] ($i.e.$ $Y_{ij} = 3 + 2X_i + \varepsilon_{ij}$). Where the fixed values of explanatory variable are $X_{\text{i}} = 2,\ 4,\ 6,\ \text{ and } 8$, sample size $(n = 4)$ and the error term is $\varepsilon_{ij} \sim N\left(s;\ \mu_s = 0,\ \sigma_s = 1\right)$. Moreover, the transformed model given in equation (1.2) is obtained by substituting the $B_0 = 3 + 2\bar{X} + \left(\beta\sigma\right)\bar{X}$ and $B_1 = \left(2 + \beta\sigma\right)X_i^*$. whereas, the fixed transformed values of explanatory variable are $X_i^* = -3,\ -1,\ 1,\ \text{ and } 3$ with average equals to zero.

The performance of proposed charts is evaluated in terms of average run length (ARL) which is defined as the number of samples until a signal occurs.(ARL) is categorized into two types, in-control average run length (ARL_0) and out-of-control average run length (ARL_1). For the fixed overall $ARL_0 = 200$, we need to set the control limits coefficients including L_{Max}, L_{SS}, $L\alpha_{/2}$, L_I, L_S and L_E which are reported in Table 4.2. For computations, we used Monte Carlo simulation study with 10^5 iterations.

Table 4.2: In-control design parameters for each chart at fixed $ARL_0 = 200$

Parameters	Max-EWMA-3	SS-EWMA-3	Shewhat_3	EWMA_3	EWMA/R
Intercet	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$Z_{\alpha/2} = 3.14$	$L_I = 3.0156$	$L_{ER} = 3.1151$
Slope	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$Z_{\alpha/2} = 3.14$	$L_S = 3.0109$	$L_{ER} = 3.1151$
Error variance	$L_{Max} = 2.91$	$L_{SS} = 3.63$	$LCL_H = 0.001$ $UCL_H = 14.17$	$L_E = 1.3723$	$L_{ER} = 3.1151$
Smoothing parameter	$\lambda = 0.2$	$\lambda = 0.2$	-	$\lambda = 0.2$	$\lambda = 0.2$

Shifts for performance evaluation

In order to evaluate the performance of charts under consideration, we have considered several amounts of shifts in linear profile parameters. The description of shifts in linear profile parameters are given as follows:

(i) Shifts in intercept parameter $(B_0 \ to \ B_0 + \theta\left(\sigma_e/\sqrt{n}\right))$

(ii) Shifts in slope parameter $\left(\beta_1 \ to \ \beta_1 + \beta\left(\sigma_e/\sqrt{S_{\text{xx}}}\right)\right)$

(iii) Shifts in slope parameter $\left(B_1 \ to \ B_1 + \delta\left(\sigma_e/\sqrt{S_{\text{xx}}}\right)\right)$

(iv) Joint shifts in intercept $(B_0 \ to \ B_0 + \theta\left(\sigma_e/\sqrt{n}\right))$ and slope parameter $\left(B_1 \ to \ B_1 + \delta\left(\sigma_e/\sqrt{S_{\text{xx}}}\right)\right)$

(v) Shifts in error variance $(\sigma_e^2 \ to \ \gamma\sigma_e^2)$

It is noted that process is said to be IC when λ, β and δ are equal to zero and $\gamma = 1$ otherwise, process is said to be OOC.

4.1.3 Comparative analysis

In this section, we discuss the comparative results of proposed and existing charts in terms of average run length (ARL). Further, the performance of charts under consideration is discussed in terms of percentage change in the (ARL_1) which is obtained as:

$$Percentage\ change\ = \frac{ARL_0 - ARL_1}{ARL_0}$$

Shifts in intercept parameter

The results for charts under consideration at shifted intercept parameter are reported in Table 4.3. Which shows that (40%) upward shift in intercept parameter ($\theta = 0.40$), may decrease 68.3% and 61.1% ARL_1 of T^2 and Shewhart-3 charts while all other charts have approximately 92.0% decrease in the ARL_1. Moreover, the ARL curves for shifted intercept parameter are plotted in Figure 4.1(A), which reveals that joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts outperforms all others charts under consideration.

Shifts in slope parameter of original model

The Table 4.4 is about the results for shifted slope parameter of original model given in equation (1.1). Which reveals that upward shift in slope parameter of original model ($\beta = 0.075$), may decrease 90.1%, 69.7% and 60.4% ARL_1 of EWMA/R, T^2 and Shewhart-3 charts while 92.0% decrease in ARL_1 was reported

Table 4.3: ARL comparison of control charts under intercept shifts

Chart	θ									
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMA/R	66.50	17.70	8.40	5.40	3.90	3.20	2.70	2.30	2.10	1.90
T^2	137.70	63.50	28.00	13.20	6.90	4.00	2.60	1.80	1.50	1.20
Shewhart-3	151.40	77.90	33.80	15.50	7.70	4.30	2.70	1.90	1.50	1.20
EWMA-3	59.10	16.20	7.90	5.10	3.80	3.10	2.60	2.30	2.10	1.90
Max-EWMA-3-A	61.29	16.69	7.99	5.14	3.84	3.10	2.60	2.28	2.07	1.90
Max-EWMA-3-B	60.89	16.33	7.98	5.16	3.83	3.09	2.63	2.29	2.08	1.89
Max-EWMA-3-C	58.87	16.17	7.89	5.13	3.80	3.06	2.59	2.26	2.04	1.89
SS-EWMA-3-A	59.99	17.07	8.49	5.51	4.13	3.33	2.81	2.44	2.20	2.03
SS-EWMA-3-B	59.17	17.51	8.45	5.56	4.14	3.32	2.82	2.45	2.19	2.03
SS-EWMA-3-C	61.34	17.38	8.48	5.52	4.13	3.31	2.81	2.44	2.19	2.03

in EWMA-3, Max-EWMA-3 and SS-EWMA-3 charts. However, the ARL curves for shifted slope parameter of original model are plotted in Figure 4.1(B), which shows that joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts outperforms all others charts under consideration.

Table 4.4: ARL comparison of control charts under shifts in slope of orignal model

Chart	β									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMA/R	119.00	43.90	19.80	11.30	7.70	5.80	4.70	3.90	3.40	3.00
T^2	166.00	105.60	60.70	34.50	20.10	12.20	7.80	5.20	3.70	2.70
Shewhart-3	178.30	125.00	79.20	46.70	27.90	17.10	10.90	7.10	5.00	3.60
EWMA-3	101.60	36.50	17.00	10.30	7.20	5.50	4.50	3.80	3.30	2.90
Max-EWMA-3-A	107.27	37.50	17.39	10.30	7.23	5.58	4.49	3.77	3.26	2.91
Max-EWMA-3-B	104.17	37.38	17.47	10.27	7.19	5.51	4.49	3.79	3.29	2.93
Max-EWMA-3-C	102.87	35.90	17.02	10.16	7.06	5.42	4.43	3.73	3.25	2.88
SS-EWMA-3-A	98.17	35.01	16.43	9.87	6.98	5.36	4.35	3.70	3.21	2.88
SS-EWMA-3-B	101.11	35.39	16.43	9.89	6.90	5.33	4.36	3.72	3.23	2.87
SS-EWMA-3-C	101.03	34.78	16.66	9.89	6.94	5.35	4.36	3.69	3.22	2.87

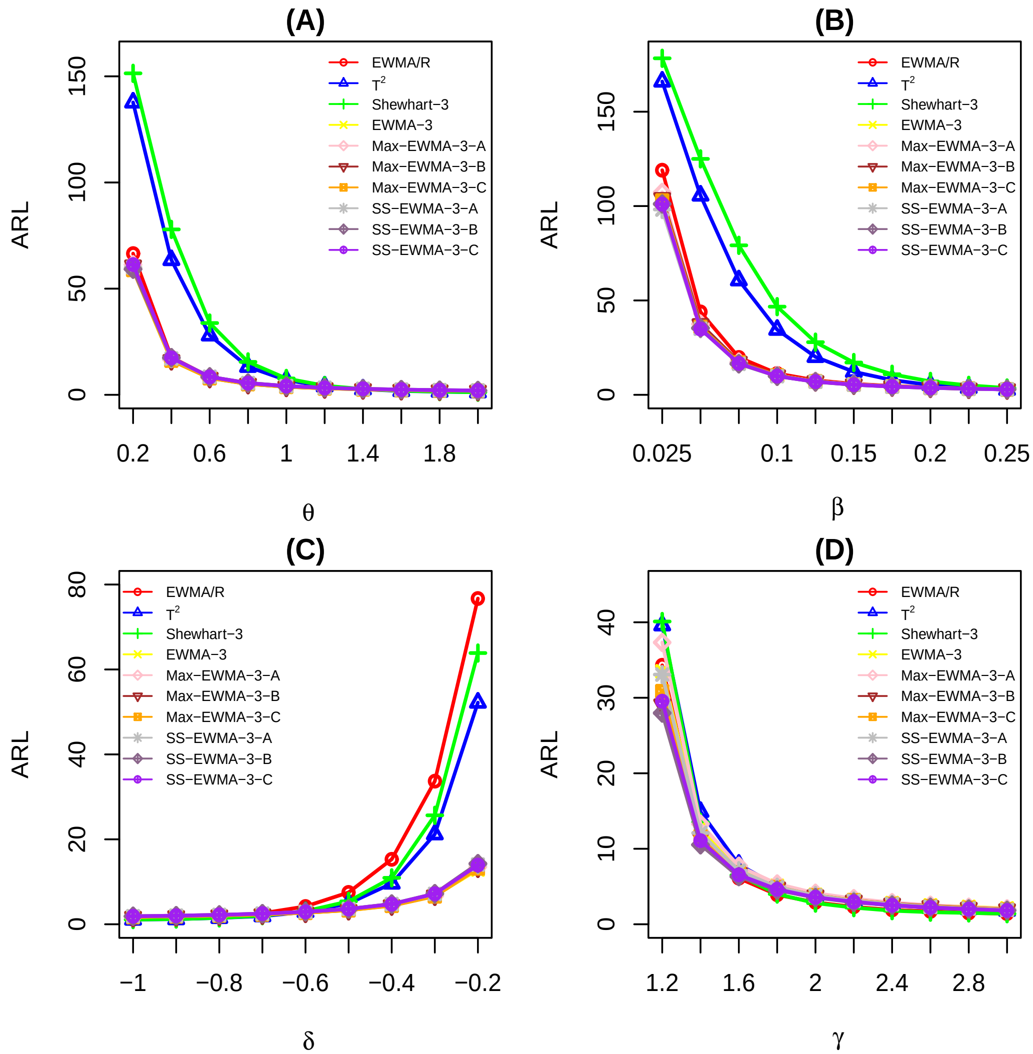

Figure 4.1: ARL curves of control charts with respect to different shifts in parameters

Shifts in slope parameter of transformed model

The results for charts under consideration at shifted slope parameter of transformed model are reported in Table 4.5. Which shows that downward shift in slope parameter of transformed model ($\delta = -0.4$), may decrease 92.4%, 95.2% and 94.5% ARL_1 of EWMA/R, T^2 and Shewhart-3 charts. However, the joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have approximately 97.7% decrease in ARL_1. Moreover, the ARL curves for shifted slope parameter of transformed model are plotted in Figure 4.1(C), which reveals that Max-EWMA-3, SS-EWMA-3 and EWMA-3 charts have similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts outperforms all others charts under consideration.

Table 4.5: ARL comparison of control charts under shifts in slope of transformed model

Chart	δ								
	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2
EWMA/R	1.20	1.40	1.80	2.60	4.20	7.50	15.30	33.70	76.70
T^2	1.10	1.20	1.50	1.90	2.90	4.90	9.60	21.20	52.20
Shewhart-3	1.10	1.23	1.49	2.03	3.07	5.40	10.93	25.65	63.87
EWMA-3	1.71	1.87	2.06	2.32	2.73	3.32	4.39	6.70	12.90
Max-EWMA-3-A	1.73	1.89	2.08	2.34	2.74	3.36	4.48	6.71	13.52
Max-EWMA-3-B	1.73	1.89	2.08	2.33	2.73	3.34	4.46	6.76	13.36
Max-EWMA-3-C	1.71	1.88	2.06	2.31	2.71	3.31	4.37	6.60	13.18
SS-EWMA-3-A	1.89	2.03	2.21	2.50	2.93	3.61	4.75	7.13	14.04
SS-EWMA-3-B	1.89	2.02	2.20	2.49	2.95	3.61	4.77	7.21	14.27
SS-EWMA-3-C	1.88	2.02	2.21	2.50	2.93	3.61	4.74	7.18	13.93

Shifts in error variance of disturbance term

The Table 4.6 is about the results for shifted error variance parameter in charts under consideration. Which reveals that upward shift in error variance parameter ($\gamma = 1.6$), may decrease 97.0%, 96.1%, 96.8%, 96.4%, 96.2%, 96.6%, 96.6%, 96.5%, 96.8% and 96.7% ARL_1 of EWMA/R, T^2, Shewhart-3, EWMA-3, Max-EWMA-3-A, Max-EWMA-3-B, Max-EWMA-3-C, SS-EWMA-3-A, SS-EWMA-3-B and SEWMA-3-C charts respectively. However, the ARL curves for shifted error variance parameter are plotted in Figure 4.1(D), which shows that joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts have relatively good performance among all others.

Table 4.6: ARL comparison of control charts under shifts in error variance

Chart	γ									
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMA/R	34.30	12.00	6.10	3.90	2.90	2.30	1.90	1.70	1.50	1.40
T^2	39.60	14.90	7.90	5.10	3.80	3.00	2.50	2.20	2.00	1.80
Shewhart-3	40.10	13.50	6.50	4.00	2.80	2.20	1.80	1.60	1.50	1.40
EWMA-3	33.50	12.70	7.20	5.10	3.90	3.20	2.80	2.50	2.30	2.10
Max-EWMA-3-A	37.33	13.28	7.70	5.35	4.12	3.40	2.87	2.52	2.26	2.06
Max-EWMA-3-B	29.11	11.53	6.77	4.86	3.81	3.13	2.72	2.41	2.17	1.97
Max-EWMA-3-C	30.86	11.68	6.86	4.92	3.82	3.16	2.68	2.36	2.14	1.94
SS-EWMA-3-A	33.07	12.09	7.09	4.90	3.81	3.10	2.67	2.31	2.08	1.90
SS-EWMA-3-B	27.98	10.53	6.32	4.52	3.53	2.91	2.52	2.23	2.01	1.83
SS-EWMA-3-C	29.55	11.08	6.55	4.63	3.57	2.98	2.52	2.22	2.01	1.82

Joint shifts in intercept and slope of transformed model

The Table 4.7 is about the results of all charts under consideration for the joint shifts in intercept and slope of transformed model. As discussed above that joint and simultaneous charts have similar performance but Max-EWMA-3-C and SS-EWMA-3-C charts have relatively good performance as compared to others. At fixed shift in slope of transformed model ($\delta = 0.1$), shift in intercept parameter ($\theta = 0.05$) may resulted 29.6%, 30.1%, 75.9%, 76.0% and 75.5% decrease in the ARL_1 of EWMA/R, T^2, Shewhart-3, EWMA-3, Max-EWMA-3-C, SS-EWMA-3-C charts respectively. At fixed shift in intercept parameter ($\theta = 0.25$), shift in slope of transformed model ($\delta = 0.15$), may resulted 80.8%, 61.0%, 91.1%, 91.3% and 92.1% decrease in the ARL_1 of EWMA/R, T^2, Shewhart-3, EWMA-3, Max-EWMA-3-C, SS-EWMA-3-C charts respectively. In conclusion, the joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts have relatively good performance among all others.

Table 4.7: ARL comparison of control charts under shifts in error variance

θ	Chart	δ									
		0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
0.05	EWMA/R	179.10	169.90	156.60	140.80	123.20	105.10	88.80	73.40	60.20	49.60
	Shewhart-3	193.18	180.64	162.76	139.73	119.05	98.79	79.91	63.73	50.47	40.90
	EWMA-3	157.60	114.70	74.80	48.30	32.20	22.50	16.90	13.20	10.70	8.90
	Max-EWMA-3-C	156.63	113.18	73.85	48.06	31.53	22.40	16.70	13.13	10.55	8.77
	SS-EWMA-3-C	157.46	109.89	75.16	49.01	33.63	23.30	17.74	13.96	11.36	9.44
0.1	EWMA/R	139.50	133.60	125.76	115.50	103.50	90.40	78.30	65.70	55.60	46.30
	Shewhart-3	184.63	172.96	154.66	137.51	115.32	95.28	77.85	62.38	50.83	40.31
	EWMA-3	122.10	94.60	66.40	44.90	30.70	21.90	16.60	13.10	10.60	8.90
	Max-EWMA-3-C	118.71	94.15	64.80	44.09	30.10	21.96	16.28	12.74	10.57	8.74
	SS-EWMA-3-C	118.33	90.65	62.10	43.18	30.03	22.50	17.05	13.39	10.98	9.32
0.15	EWMA/R	96.80	94.20	90.30	85.10	78.50	70.90	63.00	55.30	47.70	40.90
	Shewhart-3	165.25	160.43	143.16	128.64	109.11	91.59	74.65	60.80	49.55	39.75
	EWMA-3	84.60	70.80	54.50	39.60	28.50	20.90	16.10	12.80	10.40	8.80
	Max-EWMA-3-C	83.85	69.37	54.43	38.85	28.13	20.98	15.90	12.71	10.24	8.75
	SS-EWMA-3-C	83.70	67.61	49.40	36.63	26.81	20.51	15.94	12.78	10.71	9.03
0.2	EWMA/R	64.80	63.80	62.10	59.70	56.60	52.90	48.50	44.00	39.20	34.60
	Shewhart-3	148.30	143.85	132.16	116.66	101.51	87.21	70.79	57.63	47.31	38.28
	EWMA-3	57.10	51.10	42.40	33.30	25.40	19.50	15.40	12.40	10.20	8.70
	Max-EWMA-3-C	56.41	50.39	41.34	32.61	25.06	19.41	15.27	12.31	10.21	8.57
	SS-EWMA-3-C	56.89	48.89	38.62	30.20	22.91	18.34	14.39	12.00	10.09	8.60
0.25	EWMA/R	44.30	43.80	42.90	41.80	40.30	38.40	36.10	33.60	30.80	28.10
	Shewhart-3	130.54	125.22	114.09	103.59	91.89	78.02	67.20	55.43	45.70	36.84
	EWMA-3	39.50	36.50	32.30	27.10	22.00	17.80	14.40	11.90	10.00	8.50
	Max-EWMA-3-C	39.34	35.99	31.89	26.62	21.71	17.44	14.30	11.68	9.96	8.43
	SS-EWMA-3-C	40.08	35.29	29.26	24.28	19.45	15.85	13.19	11.10	9.57	8.27
0.3	EWMA/R	31.00	30.80	30.50	29.90	29.20	28.30	27.10	25.70	24.20	22.50
	Shewhart-3	112.33	107.85	100.24	91.56	81.92	71.51	61.72	50.87	42.88	35.20
	EWMA-3	28.20	26.90	24.70	22.00	18.80	15.70	13.20	11.20	9.60	8.30
	Max-EWMA-3-C	27.85	26.03	24.31	21.52	18.66	15.43	13.26	11.12	9.48	8.18
	SS-EWMA-3-C	28.78	26.37	22.81	19.61	16.56	13.92	11.97	10.18	8.86	7.78

4.2 Special case of joint linear profiling

As discussed in Section 3.4 that the monitoring of mean and variability may say a special case of simple linear profiles when the slope of the simple linear profile model (given in equation 1.1) is zero (i.e. $\beta_1 = 0$). In literature there exist several methods (discussed in Section 1.3.2) for the joint or simultenious monitoring of process parameters. Max-EWMA and SS-EWMA are well-known methods used for the joint monitoring of process parameters (location and dispersion).

Recently, [116] proposed a new memory-type procedure named progressive mean (PM) control chart. PM chart is a special case of EWMA chart [117] which is not only simple but also dominates existing memory-type charts and most of their modifications. In this study, we have proposed a new memory-type control chart based on progressive mean under max statistic, namely Max-P chart, for the joint monitoring of location and dispersion parameters.

4.2.1 Control charts for joint monitoring of location and dispersion

Let Y be the quality characteristic of a process which is used to monitor the parameters of stated process (e.g. location (μ_0) and scale (σ_0^2)). Assume, $Y_{ij} \sim N\left(\mu_0 + \theta\sigma_0,\ \gamma\sigma_0^2\right)$ where subgroup number and sample size of each subgroup are represented by $i = 1, 2, 3, \ldots,\ m$ and $j = 1, 2, 3, \ldots,\ n$ respectively. The process is said to be stable or IC if $\theta = 0$ and $\gamma = 1$. However, if $\theta \neq 0$ and $\gamma > 1$, the process is deemed OOC or unstable. Generally, $\overline{Y}_i = \sum_{j=1}^{n} Y_{ij}/n$ is used to monitor the location parameter and ${S_i}^2 = \sum_{j=1}^{n} \left(Y_{ij} - \overline{Y}_i\right)^2 / n-1$ is used to monitor variations in the process. The estimator $\overline{Y}_i$ is a complete sufficient statistic and $(n-1){S_i}^2$is an ancillary statistic because its distribution is free from parental parameters. Hence, by the use of Basu's theorem (given in Appendix A.3) both $\overline{Y}_i$ and ${S_i}^2$ are independent (for more details see, [118]). These two statistics have their own different sampling distributions namely: $\overline{Y}_i \sim N\left(\mu_0, \sigma_0^2/n\ \right)$ and ${S_i}^2 \sim \left(\sigma_0^2/(n-1)\right)\chi_{n-1}^2$. However, we can transform them to a single distribution using the following transformations:

$$U_i = \frac{\overline{Y}_i - \mu_0}{\sqrt{\frac{\sigma_0^2}{n}}}, \tag{4.1}$$

$$V_i = \Phi^{-1}\left[H\left\{\frac{(n-1){S_i}^2}{\sigma_0^2}; n-1\right\}\right], \tag{4.2}$$

where $\Phi^{-1}[.]$ is inverse standard normal distribution function and $H\{.;(n-1)\}$ is termed as chi-square distribution function having $(n-1)$ degree of freedom. The statistics $\overline{Y}_i$ and ${S_i}^2$are independent and U_i and V_i respectively are their one-to-one transformation so this implies that U_i and V_i are also independent. Here, both U_i and V_i follow a standard normal distribution.

Based on the above mentioned equations (4.1) and (4.2), we provide mathematical structures of some existing and the proposed charting structures. We have covered four existing and one new proposed Max Progressive (Max-P) control charts in this study.

Existing control charts

This subsection provides some memory type control charts that are used to monitor small or transient shifts in the process parameters. We have covered Max-EWMA, Max-DEWMA, SS-EWMA and SS-DEWMA charts for our study purposes.

The Max-EWMA chart

A memory type control chart named as exponentially weighted moving average (EWMA) control chart was proposed by [10]. Later, [12] proposed a modified EWMA chart termed as Max-EWMA for the joint monitoring of location and scale parameters. The structure of Max-EWMA depends on two EWMA statistics which are based on U_i and V_i given in equation (4.1) and (4.2),

$$W_i = \lambda U_i + (1-\lambda) W_{i-1}, \tag{4.3}$$

$$Z_i = \lambda V_i + (1-\lambda) Z_{i-1}, \tag{4.4}$$

where U_0 and V_0 are used as initial values and λ is a smoothing (weight) parameter having range $(0 < \lambda \leq 1)$. The structure of Max-EWMA chart is given as:

$$Max - EWMA_i = Max(|W_i|, |Z_i|),$$

$$UCL_{Max-EWMA_i} = \sqrt{\frac{\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}}(1.128379 + 0.602810L_3),$$

$$UCL_{Max-EWMA_i} = \sqrt{\frac{\lambda}{2-\lambda}}(1.128379 + 0.602810L_3),$$

where L_3 is the control limits coefficient that is used to control the IC run length behavior of the chart.

The SS-EWMA chart

For the joint monitoring of process parameters (location and scale), [26] proposed a scheme based on classical EWMA chart named as SS-EWMA. The structure of SS-EWMA depends on two EWMA statistics W_i and Z_i given in equation (4.3) and (4.4). The SS-EWMA statistic and its UCL are defined as:

$$SS - EWMA_i = W_i^2 + Z_i^2$$

$$UCL_{SS-EWMA_i} = \frac{2\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}(1 + L_4)$$

$$UCL_{SS-EWMA_i} = \frac{2\lambda}{2-\lambda}(1 + L_4),$$

where L_4 is the control limits coefficient that is used to control the IC run length behavior of the chart.

The Max-DEWMA chart

[119] proposed an extended version of EWMA chart named as double exponentially weighted moving average (DEWMA) control chart. [58] developed a modification in DEWMA chart named as Max-DEWMA. The Max-DEWMA technique is very useful for the joint monitoring of location and scale. The structure of Max-DEWMA depends on two new EWMA statistics S_i and T_i that depend on

two EWMA statistics W_i and Z_i given in equation (4.3) and (4.4).

$$S_i = \lambda W_i + (1-\lambda) S_{i-1}, \tag{4.5}$$

$$T_i = \lambda X_i + (1-\lambda) T_{i-1}, \tag{4.6}$$

Further, the Max-DEWMA statistic and its UCL are defined as:

$$Max - DEWMA_i = Max(|S_i|, |T_i|),$$

$$UCL_{Max-DEWMA_i} = (1.128379 + 0.602810 L_5) \sqrt{\frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3} \left\{ \begin{array}{c} 1 + (1-\lambda)^2 \\ -\left(i^2 + 2i + 1\right)(1-\lambda)^{2i} \\ + \left(2i^2 + 2i - 1\right)(1-\lambda)^{2i+2} \\ -\left(i^2\right)(1-\lambda)^{2i+4} \end{array} \right\}},$$

where L_5 is the control limits coefficient that is used to control the IC run length behavior of the chart.

The SS-DEWMA chart

[60] proposed a new SS-DEWMA chart (similar to Max-DEWMA chart) used for the joint monitoring of process parameters. The structure of SS-DEWMA depends on two DEWMA statistics S_i and T_i (cf. (4.6) and (4.6)). The plotting statistic and UCL of SS-DEWMA are defined as:

$$SS - DEWMA_i = S_i^2 + T_i^2$$

$$UCL_{SS-DEWMA_i} = 2\left(1 + L_6\right) \frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3} \left\{ \begin{array}{c} 1 + (1-\lambda)^2 \\ -\left(i^2 + 2i + 1\right)(1-\lambda)^{2i} \\ +\left(2i^2 + 2i - 1\right)(1-\lambda)^{2i+2} \\ -\left(i^2\right)(1-\lambda)^{2i+4} \end{array} \right\}$$

where L_6 is the control limits coefficient that is used to control the IC run length behavior of the chart.

A new max progressive (Max-P) control chart

[116] used a statistic for control charting named as progressive mean (PM) which is defined as;

$$PM_i = \frac{\sum_{k=1}^{i} \overline{Y}_k}{i} \tag{4.7}$$

where PM_i is an unbiased estimator of population mean μ_0 and its variance for a given i is given as σ_0^2/ni. If we consider $n = 1$ then the progressive mean can be viewed as a cumulative average of samples and (4.7) reduces to

$$PM_i = \frac{\sum_{k=1}^{i} Y_k}{i} \tag{4.8}$$

The PM statistics based on U_i's and V_i's given in (4.1) and (4.2) can now be defined as:

$$PM_{1i} = \frac{\sum_{k=1}^{i} U_k}{i} \tag{4.9}$$

$$PM_{2i} = \frac{\sum_{k=1}^{i} V_k}{i} \tag{4.10}$$

The mean and the variance of PM statistics in (4.9) and (4.10) are given as:

$$\mu_{PM_{1i}} = \mu_{PM_{2i}} = 0 \tag{4.11}$$

$$\sigma^2_{PM_{1i}} = \sigma^2_{PM_{2i}} = \frac{1}{i} \tag{4.12}$$

Further, the aforementioned progressive mean statistics are plugged into Max statistic which is the plotting statistic of our new proposed charting scheme. Mathematically, it is given as,

$$Max - P_i = Max(|PM_{1i}|, |PM_{2i}|), \tag{4.13}$$

The independence of U_i and V_i also ensures that PM_{1i} and $PM_{2i}$are independent. Hence, the cumulative distribution function (CDF) of $Max - P_i$ under IC situation is derived as:

$$\begin{aligned} F(g; \sigma_{PM_{1i}}) &= P(MaxP_i \leq g), \\ &= P(|PM_{1i}| \leq g. |PM_{2i}| \leq g), \\ &= P(|PM_{1i}| \leq g).P(|PM_{2i}| \leq g), \\ &= \left\{2\Phi\left(\frac{g}{\sigma_{PM_{1i}}}\right)\right\}^2; g \geq 0 \end{aligned} \tag{4.14}$$

and the probability density function (pdf) of $Max-P_i$ is derived as,

$$f\left(g;\sigma_{PM_{1i}}\right)=\frac{d}{dg}F\left(g;\sigma^2_{PM_{1i}}\right)=\frac{4}{\sigma^2_{PM_{1i}}}\ \phi\left(\frac{g}{\sigma_{PM_{1i}}}\right)\left\{2\Phi\left(\frac{g}{\sigma_{PM_{1i}}}\right)-1\right\} \quad (4.15)$$

where $\Phi(.)$ and $\phi\,(.)$ are known as standard normal CDF and standard normal pdf respectively. Moreover, by using the numerical computation, mean and the variance of $Max-P_i$ are defined as,

$$\mu_{Max-P_i}=\int_0^\infty g\ f\left(g;\sigma_{PM_{1i}}\right)\ dg=\frac{2}{\sqrt{\pi}}\left(\sigma_{PM_{1i}}\right) \quad (4.16)$$

$$\sigma^2_{Max-P_i}=\int_0^\infty g^2\ f\left(g;\sigma_{PM_{1i}}\right)\ dg=\left(1-\frac{2}{\pi}\right)\left(\sigma^2_{PM_{1i}}\right) \quad (4.17)$$

Finally, based on (4.15) and (4.16), the control limits of the proposed chart can be defined as:

$$UCL_{1-MaxP_i}{=}\mu_{Max-P_i}+L\ \sigma^2_{Max-P_i} \quad (4.18)$$

where L is the constant that determines the width of control limits. For a fixed value L, (4.18) produces fairly wide control limits for large values of i which may cause the deprivation in false alarm rate (FAR). To overcome such problem, [120] introduced a penalty function which results into narrower limits. The updated $UCL_{1Max-P_i}$is given as:

$$UCL_{Max-P_i}{=}\mu_{Max-P_i}+L\ \sigma^2_{Max-P_i}\frac{L_1}{i^q}=\mu_{Max-P_i}+L_2\ \sigma^2_{Max-P_i}\frac{1}{i^q} \quad (4.19)$$

where L_2 ($= L * L_1$) is the control limits coefficient that is used to control the IC run length behavior of the proposed Max-P chart using penalized limits and $f\,(i) = i^q$ is treated as penalty function. It is to be noted that the performance of proposed chart is affected (positively or negatively) by any change in the value of q. Also, $q = 0$ leads us back to un-panelized limits given in (4.18).

4.2.2 Performance evaluations

In this section, we will discuss various performance measures used in this study to evaluate the ability of different charting structures. For the proposed chart, we will also derive the charting constants and evaluate the performance ability of the proposed Max-P chart.

Performance measures

The performance of a control chart is measured through run length (RL) that is defined as the number of samples until an OOC signal is received. The RL properties are summarized using some useful properties including average run length (ARL), standard deviation of run length ($SDRL$), relative average run length ($RARL$), extra quadratic loss (EQL) and performance comparison index (PCI). ARL is a well-known measure which is defined as the average number of plotting statistics until process is declared as OOC. We denote ARL by ARL_0 (when the process parameters are IC) while for case of OOC situation it is denoted by ARL_1. $SDRL$ is defined as the standard deviation of the run length distribution. Further,

$RARL$, EQL and PCI are defined as follow (for more details see [121–123]).

$$RARL = \frac{1}{\Psi_{max} - \Psi_{min}} \int_{\Psi_{min}}^{\Psi_{max}} \frac{ARL(\Psi)}{ARL_{bmk}(\Psi)} d\Psi, \tag{4.20}$$

$$EQL = \frac{1}{\Psi_{max} - \Psi_{min}} \int_{\Psi_{min}}^{\Psi_{max}} \Psi^2 ARL(\Psi) d\Psi, \tag{4.21}$$

$$PCI = \frac{EQL}{EQL_{best\ chart}}, \tag{4.22}$$

where $ARL(\Psi)$ is the ARL_1 of a particular chart at shift Ψ (i.e. θ and γ) and $ARL_{bmk}(\Psi)$is the ARL_1 of the benchmark chart (we consider Max-P as a benchmark for our study purposes) at shift Ψ.

Charting constants for the proposed Max-P chart

As mentioned above, the UCL of Max-P chart depends on control limits coefficient parameter L_2 and panelizing function $f(i) = i^q$. We have to carefully choose the values of L_2 in order to fix ARL_0 at a pre-specified level. We have tested several choices of q and found that the optimal value is $q = 0.1$ (we will use this choice through this study i.e. $f(i) = i^{0.1}$).The procedure to find control charting parameter (L_2) for Max-P control chart is illustrated in the following steps:

(i) Generate a subgroup of a fixed size n from normal distribution and calculate sample mean $\overline{Y}_1$ and sample variance S_1^2. By using the $\overline{Y}_1$ and S_1^2, calculate U_1 and V_1 given in equation (4.1) and (4.2) and save them in respective vectors for U_i and V_i. Calculate the progressive means PM_{11}based on U_1 and PM_{21} based on V_1 using equations (4.9) and (4.10) respectively. Further, for the calculation of first Max progressive $Max - p_1$ plotting statistic, find the maximum of the absolute PM_{11} and absolute PM_{21} using equations (4.13).

(ii) Plot $Max - p_1$ against a preset control limit (UCL_{Max-P_1}) using an arbitrary value of L_2. If $Max - p_1$ exceeds UCL_{Max-P_1} then the process is declared OOC and the corresponding subgroup number is saved as a run length. On the other hand, we proceed to next step (iii), if the $Max - p_1$ remains inside the zero and UCL_{Max-P_1}.

(iii) We keep doing these iterations, as demonstrated in (i)-(ii), until a value of $Max - p_i$ exceeds from UCL_{Max-P_i} and the process is declared OOC. The corresponding sample number (which is the minimum value of i for which the process goes OOC is saved as a value of run length.

(iv) Repeat steps (i)-(iii) a large number of times in order to get a complete empirical behavior of distribution of run lengths and calculate the average of that distribution. That is our observed ARL_0.

Following steps (i)-(iv), we search the value of L_2 such that our observed ARL_0 is equal to the prefixed ARL_0. We have carried out extensive Monte Carlo simulations to work out the values of L_2 at prefixed choices of ARL_0.

For our study purposes we have derived the values of L_2 for $n = 5$ and $i = 0.1$ at some useful choices of ARL_0 such as 168, 250 or 370, as given below:

$$L_2 = 2.16 \text{ at prefixed } ARL_0 = 168;$$

$$L_2 = 2.33 \ \text{ at prefixed } ARL_0 = 250;$$

$$L_2 = 2.52 \ \text{ at prefixed } ARL_0 = 370;$$

For other combinations of ARL_0 and n one may follow the same lines as above to derive the appropriate values of control limits coefficient L_2.

Performance evaluation for the proposed Max-P chart

In order to examine the OOC performance of the proposed Max-P control chart, we have considered several amounts of shifts in location and scale parameters. The specific choices are listed as:

$$\theta = 0.00,\ 0.25,\ 0.50,\ 0.75,\ 1.00,\ 1.25,\ 1.50,\ 1.75,\ 2.00;$$

$$\gamma = 0.00,\ 0.25,\ 0.50,\ 0.75,\ 1.00,\ 1.25,\ 1.50,\ 1.75,\ 2.00.$$

In the presence of different shifts in the process parameters, performance of the proposed Max-P chart, at prefixed $ARL_0's$, is evaluated in the form of ARL and $SDRL$ and is reported as a tabular display (cf. Table 4.8). We have used the same design parameters as finalized in above section. The results show that the proposed Max-P chart exhibits attractive detection ability in presence of shifts in location and/or scale parameter as may be seen form Table 4.8.

For a quantitative discussion of the ARL results, we define a measure referring to the percentage change in ARL_1 relative to the prefixed ARL_0, mathematically given as:

$$Percentage\ change = \left(\frac{ARL_0 - ARL_1}{ARL_0}\right) \times 100.$$

Using this measure we can discuss the results in terms of percentage gain (decrease in in ARL_1 relative to the prefixed ARL_0). For instance, at $ARL_0 = 370$, the results of the proposed Max-P chart revealed that:

- when $\theta = 0$, 25% increase in scale parameter (*i.e.* $\gamma = 1.25$) produces 97.48% decrease in the ARL_1; 75% increase in scale parameter (*i.e.* $\gamma = 1.75$) produces 99.35% decrease in ARL_1.

- when $\gamma = 1$, 25% increase in location parameter ($\theta = 1.25$) produces 96.41% decrease in ARL_1; 75% increase in location parameter ($\theta = 1.75$) produces 99.21% decrease in ARL_1.

- when $\theta = 0.25$, 75% increase in scale parameter (*i.e.* $\gamma = 1.75$) produces 99.37% decrease in the ARL_1;

- when $\gamma = 1.25$, 75% increase in location parameter ($\theta = 1.25$) produces 99.25% decrease in ARL_1;

- when $\theta = 0.25$ and $\gamma = 1.25$, 25% increase in both parameters causes 98.09% decrease in the ARL_1.

The similar findings may be observed at other combinations of the design parameters $(n,\ i,\ L_2)$ for our proposed Max-P chart.

Table 4.8: Run length properties of Max-P control chart at different ARL_0

γ	θ	$ARL_0 = 168$		$ARL_0 = 250$		$ARL_0 = 370$		γ	θ	$ARL_0 = 168$		$ARL_0 = 250$		$ARL_0 = 370$	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**			**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	0	1.39	0.49	1.48	0.51	1.57	0.51		0	7.54	7.03	8.2	7.4	8.96	7.84
	0.25	1.4	0.49	1.48	0.51	1.57	0.51		0.25	5.63	4.74	6.17	5.12	6.77	5.45
	0.5	1.4	0.49	1.47	0.5	1.57	0.51		0.5	3.55	2.54	3.76	2.69	4.04	2.79
	0.75	1.39	0.49	1.47	0.5	1.57	0.5		0.75	2.4	1.47	2.51	1.51	2.65	1.6
0.25	1	1.3	0.46	1.41	0.49	1.54	0.5	1.25	1	1.78	0.94	1.86	0.99	1.95	1.04
	1.25	1.03	0.16	1.07	0.25	1.16	0.36		1.25	1.45	0.66	1.49	0.69	1.55	0.73
	1.5	1	0	1	0.02	1	0.04		1.5	1.24	0.47	1.27	0.49	1.3	0.52
	1.75	1	0	1	0	1	0		1.75	1.12	0.33	1.14	0.36	1.16	0.38
	2	1	0	1	0	1	0		2	1.05	0.22	1.06	0.23	1.07	0.26
	0	2.83	1.14	2.98	1.19	3.17	1.24		0	3.19	2.45	3.42	2.58	3.69	2.81
	0.25	2.84	1.16	2.99	1.18	3.14	1.22		0.25	3	2.25	3.2	2.38	3.41	2.5
	0.5	2.63	0.95	2.78	1	2.96	1.03		0.5	2.48	1.72	2.66	1.85	2.82	1.93
	0.75	2.1	0.66	2.23	0.68	2.37	0.7		0.75	2.02	1.26	2.12	1.32	2.23	1.38
0.5	1	1.61	0.53	1.71	0.53	1.81	0.52	1.5	1	1.66	0.92	1.74	0.96	1.82	1.02
	1.25	1.22	0.41	1.28	0.45	1.36	0.48		1.25	1.42	0.68	1.46	0.71	1.52	0.75
	1.5	1.03	0.17	1.05	0.21	1.07	0.26		1.5	1.25	0.51	1.28	0.53	1.32	0.56
	1.75	1	0.04	1	0.05	1.01	0.07		1.75	1.14	0.37	1.16	0.4	1.19	0.42
	2	1	0	1	0	1	0.01		2	1.07	0.27	1.09	0.29	1.1	0.31
	0	8.49	5.36	9.04	5.52	9.13	5.59		0	2.11	1.41	2.2	1.48	2.32	1.56
	0.25	6.74	3.84	7.24	4.05	7.26	3.99		0.25	2.03	1.34	2.16	1.45	2.26	1.5
	0.5	3.86	1.86	4.13	1.94	4.1	1.94		0.5	1.87	1.17	1.96	1.23	2.08	1.33
	0.75	2.4	1.03	2.54	1.06	2.54	1.06		0.75	1.68	0.97	1.74	1.03	1.82	1.08
0.75	1	1.7	0.68	1.78	0.7	1.79	0.69	1.75	1	1.49	0.78	1.54	0.82	1.62	0.88
	1.25	1.32	0.49	1.37	0.51	1.38	0.52		1.25	1.34	0.63	1.37	0.64	1.43	0.69
	1.5	1.11	0.31	1.13	0.34	1.14	0.35		1.5	1.23	0.49	1.26	0.52	1.29	0.56
	1.75	1.02	0.15	1.03	0.18	1.03	0.17		1.75	1.14	0.38	1.16	0.4	1.18	0.42
	2	1	0.06	1	0.07	1	0.07		2	1.09	0.29	1.1	0.31	1.11	0.33
	0	172.65	646.86	254.47	995.59	355.32	1352.94		0	1.63	0.96	1.68	0.99	1.76	1.05
	0.25	10.68	8.73	11.59	9.28	12.74	9.65		0.25	1.61	0.93	1.65	0.98	1.73	1.03
	0.5	4.27	2.74	4.53	2.81	4.83	2.9		0.5	1.53	0.85	1.61	0.92	1.66	0.96
	0.75	2.52	1.35	2.68	1.42	2.82	1.46		0.75	1.44	0.75	1.49	0.79	1.55	0.84
1	1	1.79	0.84	1.86	0.88	1.95	0.91	2	1	1.36	0.65	1.4	0.69	1.44	0.73
	1.25	1.4	0.58	1.45	0.61	1.51	0.64		1.25	1.26	0.54	1.29	0.58	1.33	0.62
	1.5	1.18	0.4	1.21	0.42	1.24	0.45		1.5	1.19	0.45	1.21	0.48	1.23	0.5
	1.75	1.07	0.26	1.08	0.28	1.1	0.31		1.75	1.13	0.37	1.14	0.39	1.16	0.41
	2	1.02	0.14	1.03	0.16	1.03	0.18		2	1.08	0.29	1.09	0.31	1.11	0.33

4.2.3 Comparative analysis

In this section, we provide a comparative analysis of the proposed Max-P chart with some other competing counterparts namely Max-EWMA, Max-DEWMA, SS-EWMA and SS-DEWMA charts. We have evaluated the performance of all the said charts at several combinations of θ and γ using their respective design parameters. For comparison purposes, the specific design parameters of different charts used in this study are listed below:

$$\textit{Max-EWMA:}\ \lambda = 0.10,\ L_3 = 2.79,\ n = 5,\ ARL_0 = 250;$$

$$\textit{SS-EWMA:}\ \lambda = 0.10,\ L_4 = 3.57,\ n = 5,\ ARL_0 = 250;$$

$$\textit{Max-DEWMA:}\ \lambda = 0.10,\ L_5 = 2.082,\ n = 5,\ ARL_0 = 250;$$

$$\textit{SS-DEWMA:}\ \lambda = 0.10,\ L_6 = 2.348,\ n = 5,\ ARL_0 = 250;$$

$$\textit{Max-P:}i = 0.10,\ L_2 = 2.33,\ n = 5,\ ARL_0 = 250;$$

The results obtained at these design parameters are shown in the form of table and graph (cf. Table 4.9 and Figure 4.2). The comparative reveals that the proposed Max-P chart offers better run length features relative to other competing charts in the presence of shifts in scale and/or location parameter(s), as may be seen from Table 4.9 and Figure 4.2. Moreover, the DEWMA version based on both Max and SS charts perform better than their corresponding EWMA version based on Max and SS charting schemes. Some specific observations at the above mentioned design parameters are listed below:

- when $\theta = 0$, 50% increase in scale parameter ($\gamma = 1.50$) causes 98.66% decrease in the ARL_1 for Max-P chart while 97.08%, 97.06%, 98.23% and

98.29% decreases are observed in the ARL_1s for Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA charts respectively.

- when $\gamma = 1$, 50% increase in location parameter ($\theta = 0.50$) causes 95.44%, 90.00%, 89.76%, 92.52% and 92.53% decrease in ARL_1s of Max-P, Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA charts respectively.

- when $\theta = 0.25$, 50% increase in scale parameter ($\gamma = 1.50$) causes 98.74% decrease in the ARL_1 of Max-P chart while 97.25%, 97.32%, 98.40% and 98.46% decrease reported in the ARL_1s of Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA charts respectively.

- for 25% shift in scale parameter ($\gamma = 1.25$) and 75% increase in location parameter ($\theta = 0.75$), we observe 99.01%, 97.93%, 97.94%, 98.84% and 98.89% reductions in the ARL_1s of Max-P, Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA charts respectively.

- In case of 50% increase in both parameters ($\theta = 0.5$ and $\gamma = 1.5$), the reductions in ARL_1s of Max-P, Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA charts are respectively 98.95%, 97.68%, 97.82%, 98.73% and 98.82%.

It is obvious from the analysis of our results that the proposed Max-P chart offers relatively better ARL properties and outperforms the other competing charts for varying amounts of shifts in location and/or scale parameter(s).

In addition, we have also evaluated the overall performances, in the form of EQL, RARL and PCI, of all the charts (under discussion in this study) using equations (4.20)-(4.22). These performance measures are reported in Table 4.9. The smaller values of EQL, RARL and PCI for the proposed Max-P chart relative to other competing counterparts also advocates the superiority of the proposal of this study.

Table 4.9: Comparative analysis of Max-P chart with existing charts at $ARL_0 = 250$

γ	θ	**Max-P**		**Max-EWMA**		**SS-EWMA**		**Max-DEWMA**		**SS-DEWMA**	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	0	1.48	0.51	3.22	0.46	3.47	0.53	1.36	0.5	1.53	0.54
	0.25	1.48	0.51	3.23	0.47	3.38	0.51	1.37	0.5	1.47	0.52
	0.5	1.47	0.5	3.22	0.46	3.15	0.4	1.36	0.5	1.31	0.47
	0.75	1.47	0.5	3.23	0.46	2.96	0.25	1.36	0.49	1.12	0.32
0.25	**1**	1.41	0.49	3.18	0.43	2.77	0.42	1.26	0.44	1.01	0.11
	1.25	1.07	0.25	2.98	0.15	2.28	0.45	1.02	0.13	1	0.01
	1.5	1	0.02	2.58	0.49	2.01	0.08	1	0	1	0
	1.75	1	0	2	0.04	2	0	1	0	1	0
	2	1	0	2	0	2	0.01	1	0	1	0
EQL		1.66		3.78		3.36		1.62		1.56	
RARL		1		2.3		2.06		0.95		0.9	
PCI		1		2.28		2.02		0.97		0.94	
	0	2.98	1.19	5.89	1.43	6.36	1.54	3.44	1.65	3.83	1.77
	0.25	2.99	1.18	5.9	1.43	5.87	1.31	3.44	1.64	3.4	1.51
	0.5	2.78	1	5.78	1.28	4.88	0.94	3.18	1.42	2.56	1.08
	0.75	2.23	0.68	4.85	0.8	3.98	0.66	2.33	0.91	1.87	0.74
0.5	**1**	1.71	0.53	3.74	0.57	3.29	0.5	1.61	0.61	1.39	0.52
	1.25	1.28	0.45	3.03	0.34	2.89	0.37	1.19	0.39	1.11	0.32
	1.5	1.05	0.21	2.55	0.5	2.44	0.5	1.02	0.15	1.02	0.12
	1.75	1	0.05	2.07	0.26	2.06	0.24	1	0.03	1	0.03
	2	1	0	2	0.04	2	0.04	1	0	1	0.01
EQL		1.88		4.13		3.89		1.86		1.75	
RARL		1		2.18		2.01		1.02		0.93	
PCI		1		2.19		2.07		0.99		0.93	
	0	9.04	5.52	18.37	9.94	19.86	11.01	13.44	8.23	14.37	8.64
	0.25	7.24	4.05	15.76	7.29	13.06	5.66	10.91	6.31	9.17	5.3
	0.5	4.13	1.94	8.47	2.68	7.48	2.25	5.29	2.91	4.66	2.55
	0.75	2.54	1.06	5.23	1.29	5.06	1.19	2.82	1.47	2.67	1.39
0.75											

	1	1.78	0.7	3.81	0.8	3.82	0.78	1.78	0.84	1.76	0.83
	1.25	1.37	0.51	3.04	0.56	3.12	0.55	1.32	0.53	1.32	0.53
	1.5	1.13	0.34	2.54	0.52	2.65	0.52	1.1	0.3	1.1	0.31
	1.75	1.03	0.18	2.17	0.37	2.24	0.43	1.02	0.14	1.02	0.15
	2	1	0.07	2.02	0.15	2.04	0.2	1	0.05	1	0.06
EQL		2.04		4.36		4.39		2.1		2.05	
RARL		1		2.13		2.1		1.08		1.03	
PCI		1		2.13		2.15		1.03		1	
	0	254.47	995.59	252.32	245.61	247.07	239.23	251.32	271.24	252.95	268.67
	0.25	11.59	9.28	25.22	16.84	25.3	16.97	18.8	14.62	18.88	14.8
	0.5	4.53	2.81	8.84	3.75	9.1	3.82	5.95	4.09	6.11	4.24
	0.75	2.68	1.42	5.33	1.72	5.54	1.75	3.04	1.97	3.08	2.03
1	**1**	1.86	0.88	3.85	1.03	4.03	1.06	1.93	1.1	1.97	1.15
	1.25	1.45	0.61	3.07	0.74	3.22	0.74	1.43	0.68	1.45	0.71
	1.5	1.21	0.42	2.56	0.58	2.69	0.6	1.18	0.42	1.19	0.44
	1.75	1.08	0.28	2.22	0.44	2.34	0.5	1.07	0.25	1.07	0.26
	2	1.03	0.16	2.04	0.29	2.1	0.32	1.02	0.13	1.02	0.15
EQL		2.17		4.47		4.66		2.27		2.3	
RARL		1		2.05		2.14		1.11		1.12	
PCI		1		2.06		2.15		1.04		1.06	
	0	8.2	7.4	17.61	11.44	17.24	11.02	13.01	11.69	12.95	11.67
	0.25	6.17	5.12	13.2	7.44	12.14	6.7	9.26	8.07	8.63	7.45
	0.5	3.76	2.69	7.96	3.63	7.43	3.29	4.96	4.07	4.6	3.73
	0.75	2.51	1.51	5.23	1.99	5.08	1.84	2.9	2.16	2.8	2.09
1.25	**1**	1.86	0.99	3.88	1.25	3.88	1.2	1.99	1.31	1.94	1.27
	1.25	1.49	0.69	3.11	0.89	3.14	0.86	1.49	0.81	1.47	0.79
	1.5	1.27	0.49	2.61	0.68	2.66	0.68	1.24	0.53	1.23	0.52
	1.75	1.14	0.36	2.27	0.52	2.33	0.53	1.11	0.34	1.11	0.34
	2	1.06	0.23	2.06	0.42	2.11	0.41	1.04	0.21	1.05	0.21
EQL		2.17		4.43		4.46		2.25		2.21	
RARL		1		2.07		2.04		1.11		1.07	
PCI		1		2.04		2.05		1.04		1.02	
	0	3.42	2.58	7.38	3.52	7.26	3.44	4.45	3.93	4.32	3.92
	0.25	3.2	2.38	6.93	3.18	6.61	3.06	4.03	3.53	3.89	3.45
	0.5	2.66	1.85	5.86	2.48	5.39	2.27	3.2	2.73	2.98	2.5
	0.75	2.12	1.32	4.67	1.77	4.28	1.59	2.42	1.89	2.24	1.72
1.5	**1**	1.74	0.96	3.75	1.31	3.49	1.18	1.85	1.29	1.73	1.14
	1.25	1.46	0.71	3.09	0.99	2.96	0.91	1.48	0.85	1.43	0.81
	1.5	1.28	0.53	2.63	0.78	2.56	0.71	1.26	0.58	1.23	0.54
	1.75	1.16	0.4	2.29	0.61	2.28	0.58	1.14	0.4	1.12	0.37
	2	1.09	0.29	2.06	0.51	2.08	0.49	1.07	0.27	1.06	0.26
EQL		2.08		4.28		4.14		2.13		2.06	
RARL		1		2.1		2		1.07		1.02	
PCI		1		2.05		1.99		1.02		0.99	
	0	2.2	1.48	4.78	2.01	4.72	1.97	2.51	2.08	2.4	2.04
	0.25	2.16	1.45	4.65	1.91	4.51	1.85	2.41	1.99	2.34	1.96

1.75

	0.5	1.96	1.23	4.35	1.7	4.08	1.63	2.17	1.73	2.07	1.63
	0.75	1.74	1.03	3.87	1.44	3.57	1.33	1.89	1.42	1.78	1.29
	1	1.54	0.82	3.38	1.19	3.11	1.07	1.61	1.08	1.53	0.97
	1.25	1.37	0.64	2.94	0.98	2.72	0.87	1.4	0.8	1.34	0.72
	1.5	1.26	0.52	2.57	0.81	2.43	0.72	1.25	0.59	1.22	0.53
	1.75	1.16	0.4	2.29	0.68	2.2	0.61	1.15	0.43	1.13	0.4
	2	1.1	0.31	2.07	0.59	2	0.54	1.08	0.3	1.07	0.29
EQL		1.97		4.05		3.82		2		1.94	
RARL		1		2.12		1.99		1.04		1	
PCI		1		2.05		1.94		1.01		0.98	
	0	1.68	0.99	3.62	1.42	3.57	1.41	1.8	1.32	2.42	2.06
	0.25	1.65	0.98	3.58	1.39	3.5	1.34	1.73	1.24	2.32	1.93
	0.5	1.61	0.92	3.45	1.3	3.29	1.24	1.67	1.18	2.08	1.64
	0.75	1.49	0.79	3.23	1.17	3.04	1.11	1.54	1.01	1.77	1.28
2	**1**	1.4	0.69	2.98	1.05	2.75	0.96	1.4	0.83	1.53	0.96
	1.25	1.29	0.58	2.71	0.92	2.51	0.84	1.31	0.69	1.34	0.71
	1.5	1.21	0.48	2.44	0.8	2.27	0.72	1.21	0.53	1.21	0.53
	1.75	1.14	0.39	2.22	0.7	2.08	0.65	1.13	0.41	1.13	0.4
	2	1.09	0.31	2.03	0.63	1.92	0.58	1.09	0.32	1.07	0.28
EQL		1.88		3.77		3.52		1.88		1.93	
RARL		1		2.07		1.95		1.01		1.11	
PCI		1		2.01		1.88		1		1.03	

4.2.4 Diagnostic ability of charts

Usually, when the process is declared OOC then it is important to diagnose the source of shift (e.g. due to location parameter, dispersion parameter or with respect to both parameters). For the diagnostic analysis we choose three charts namely $Max{-}P_i$, $Max{-}EWMA_i$ and $Max{-}DEWMA_i$ charts. The specific symbols we will use in this diagnostic analysis are reported in the form a table (cf. Table 4.10). The structure of the stated diagnosis analysis is given as: Plot the plotting statistics (i.e. $Max{-}P_i$, $Max{-}EWMA_i$ and $Max{-}DEWMA_i$) against their respective control limits UCL_{Max-P_i}, $UCL_{Max-EWMA_i}$ and $UCL_{Max-DEWMA_i}$. Plot a dot against i when plotting statis-

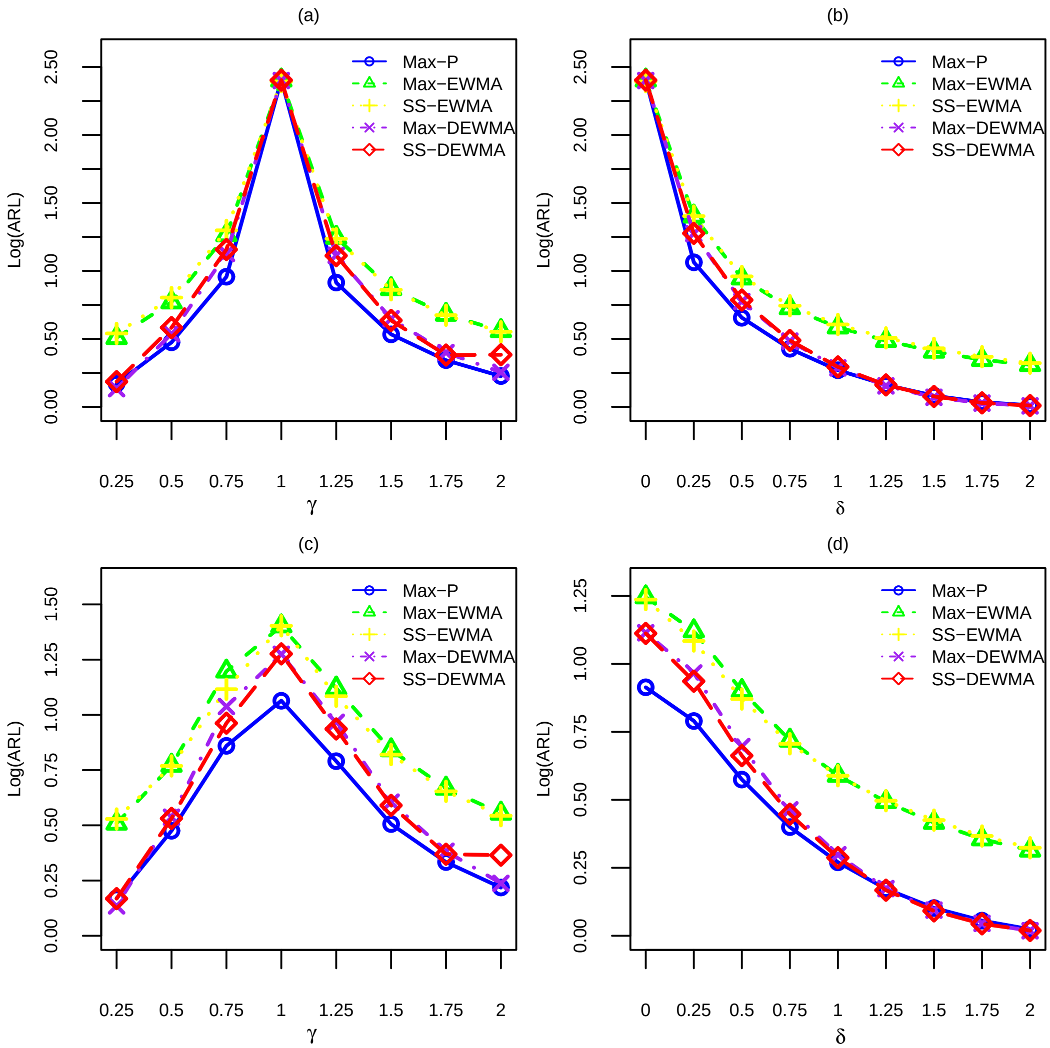

Figure 4.2: ARL curves (on logarithmic scale) for the proposed and some counterpart charts under (a) shifts in scale parameter at fixed IC location parameter; (b) shifts in location parameter at fixed IC scale parameter; (c) shifts in scale parameter at a shifted location parameter; (d) shifts in location parameter at a shifted scale parameter.

tics do not exceed their limits, otherwise, label the plotted points accordingly using the symbols shown in Table. In case of Max-P chart, when the plotting statistic $Max{-}P_i$ exceeds UCL_{Max-P_i}, check both $|PM_{1i}|$ and $|PM_{2i}|$against UCL_{Max-P_i}. If $U_i > 0$ and only $|PM_{1i}|$ is greater than UCL_{Max-P_i} then label it with symbol "$m+$" which indicates that only process mean increased. However, decrease in the process mean will be shown when $U_i < 0$ and only $|PM_{1i}|$ is greater than UCL_{Max-P_i} which is labeled by "$m-''$. On the other hand, in case of increase in process dispersion, symbol "$v+''$ can be assigned when $V_i > 0$ and only $|PM_{2i}|$ is greater than UCL_{Max-P_i} whereas, when $V_i < 0$ and only $|PM_{2i}|$ is greater than UCL_{Max-P_i} then label with symbol "$v-''$ indicates a deprivation in process dispersion. Further, when both $|PM_{1i}|$ and $|PM_{2i}|$ exceedUCL_{Max-P_i} and U_i, $V_i > 0$, the symbol "$++$" depicts a positive shift in both parameters. Similarly, symbol "$--$" shows a decreasing shift in both parameters, "$+-$" depicts an increasing shift in location and decreasing shift in dispersion and symbol "$-+$" shows an increasing shift in dispersion parameter and decreasing shift in location parameter. A similar structure may also be used to analyze the diagnosis ability of other competing Max-EWMA and Max-DEWMA charts.

The results obtained for the diagnostic abilities of Max-P, Max-EWMA and Max-DEWMA control charts are reported in a tabular form (cf. Table 4.11). The results advocate that the proposed Max-P chart outshines the competing Max-EWMA and Max-DEWMA charts, in general, in diagnosing location and/or dispersion shifts. For instance, when $\gamma = 0.25$ and $\theta = 1$ (for the case of $\boldsymbol{m+}$) the proposed Max-P chart produces 63 signals, whereas 45 and 6 signals are reported in Max-EWMA and Max-DEWMA charts respectively. Moreover, when $\gamma = 0.25$ and $\theta = 1$ (for the case of $\boldsymbol{+-}$) the proposed Max-P chart produces 448 signals, whereas 297 and 470 signals are reported in Max-EWMA and Max-DEWMA charts respectively. Furthermore, when $\gamma = 0.25$ and $\theta = 2$ (for the case of $\boldsymbol{+-}$)

the proposed Max-P chart produces 545 signals, whereas 15 and 0 signals are reported in Max-EWMA and Max-DEWMA charts respectively.

Table 4.10: Assignment of the symbols to different situations

			$\lvert PM_{2i}\rvert > UCL_{Max-P_i}$ $\lvert Z_i\rvert > UCL_{Max-EWMA_i}$ $\lvert T_i\rvert > UCL_{Max-\text{DEWMA}_i}$	
		$\lvert PM_{2i}\rvert < UCL_{Max-P_i}$ $\lvert Z_i\rvert < UCL_{Max-EWMA_i}$ $\lvert T_i\rvert < UCL_{Max-\text{DEWMA}_i}$	$V_i > 0$	$V_i < 0$
$\lvert PM_{1i}\rvert < UCL_{Max-P_i}$ $\lvert W_i\rvert < UCL_{Max-EWMA_i}$ $\lvert S_i\rvert < UCL_{Max-\text{DEWMA}_i}$			$v+$	$v-$
$\lvert PM_{1i}\rvert > UCL_{Max-P_i}$ $\lvert W_i\rvert > UCL_{Max-EWMA_i}$ $\lvert S_i\rvert > UCL_{Max-\text{DEWMA}_i}$	$U_i > 0$	$m+$	$++$	$+-$
	$U_i < 0$	$m-$	$-+$	$--$

4.2.5 A real application in electrical engineering

In this section, we describe a real phenomena related to electrical engineering and the implementation of Max-P, Max-EWMA and Max-DEWMA charts to monitor the voltage of the photovoltaic (PV) system in the said electrical engineering process. The description of the PV system is already dicussed in Section 2.1.

Implementation of the proposed and existing charts

Usually, electrical engineers are engaged to monitor the variations of voltage in the system. As discussed above that voltage (V) has inverse relation with capacitance (C) at fixed charge (Q). In this illustrative example, the monitoring of voltage (V) generated through Z-source inverter in grid connected PV system, we get a data set having two capacitance levels ($C = 250\mu F$ and $350\mu F$) each of 75455

Table 4.11: Diagnostic abilities of the proposed and some counterpart charts

		Max-P					**Max-EWMA**					**Max-DEWMA**				
γ		θ					θ					θ				
		0	**0.25**	**0.5**	**1**	**2**	**0**	**0.25**	**0.5**	**1**	**2**	**0**	**0.25**	**0.5**	**1**	**2**
0.25	$m+$	0	0	0	63	455	0	0	0	45	985	0	0	0	6	1000
	$m-$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$v+$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$v-$	1000	1000	1000	489	0	1000	1000	1000	658	0	1000	1000	1000	524	0
	$++$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$+-$	0	0	0	448	545	0	0	0	297	15	0	0	0	470	0
	$-+$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$--$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	$m+$	0	2	145	693	919	0	0	68	864	1000	0	0	35	940	1000
	$m-$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$v+$	0	0	0	0	0	0	0	0	0	0	6	6	5	0	0
	$v-$	1000	988	705	66	0	1000	1000	864	22	0	994	994	882	4	0
	$++$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$+-$	0	10	150	241	81	0	0	68	114	0	0	0	78	56	0
	$-+$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$--$	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1	$m+$	257	888	943	983	989	272	976	993	1000	1000	239	852	919	990	1000
	$m-$	218	2	0	0	0	245	0	0	0	0	255	120	76	10	0
	$v+$	263	45	27	4	1	238	10	4	0	0	236	11	1	0	0
	$v-$	261	58	15	7	0	244	13	0	0	0	270	16	3	0	0
	$++$	0	6	10	4	8	1	0	3	0	0	0	1	0	0	0
	$+-$	0	1	5	2	2	0	1	0	0	0	0	0	1	0	0
	$-+$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$--$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	$m+$	81	270	406	671	805	27	169	405	794	977	5	72	310	843	997
	$m-$	91	31	16	0	0	37	3	0	0	0	11	18	51	36	2
	$v+$	775	626	448	159	23	923	788	526	129	4	884	797	476	48	0
	$v-$	1	3	2	0	0	0	0	0	0	0	95	82	64	8	0
	$++$	36	61	124	170	170	6	40	69	86	19	0	18	80	52	1
	$+-$	0	0	0	0	2	0	0	0	0	0	2	0	8	10	0
	$-+$	16	9	4	0	0	7	0	0	0	0	3	11	11	3	0
	$--$	0	0	0	0	0	0	3	0	0	0	0	2	0	0	0
2	$m+$	80	142	184	330	475	29	60	150	383	758	2	14	52	319	892
	$m-$	80	42	23	6	0	31	6	1	0	0	7	4	12	26	7
	$v+$	683	674	582	374	72	910	872	747	412	46	933	915	839	443	22
	$v-$	1	0	0	0	0	0	0	0	0	0	52	48	46	20	0
	$++$	71	98	185	288	453	14	59	100	204	96	0	9	41	165	73
	$+-$	0	0	0	0	0	0	0	0	0	0	0	2	0	7	6
	$-+$	85	44	26	2	0	16	3	2	1	0	5	7	9	20	0
	$--$	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0

values of V reported in [94] and also used by [17].

In this study, we consider first 150 subgroups each of size $n = 5$ from IC capacitance level ($C = 350\mu F$) while next 250 OOC subgroups are taken from the $C = 250\mu F$. By using 15091 subgroups of $C = 350\mu F$, we compute population mean ($\mu_0 = 397.0841$) and variance ($\sigma_0^2 = 0.4367731$). On the fixed $ARL_0 = 250$ and some specific design parameters we have computed the control limits' coefficients for the three competing charts and the outcomes are listed below (to be used with this dataset):

$$\textit{Max-P Chart: } f(i) = i^{0.1} \textit{ and } \; L_2 = 3.98$$

$$\textit{Max-EWMA Chart: } \lambda = 0.1 \textit{ and } L_3 = 4.50$$

$$\textit{Max-DEWMA Chart: } = 0.1 \textit{ and } L_5 = 3.40$$

Using these quantities, we have constructed all the three charts for the aforementioned dataset. The resulting charting displays are given in graphical form in Figures 4.3-4.5. The plots contain brown shaded points with pink background from IC subgroups while white shaded area is the OOC area where red shaded point are the OOC signals. The diagnostic abilities of the three charts are also evaluated for this dataset and the results are reported in Table 4.12.

The implementation of Max-P chart (presented in Figure 4.3) reveals that 232 points are declared OOC. Out of 232 OOC (indices reported in Table 4.12) points, 168 points are declared OOC due to increase in scale while 53 points are declared OOC due to decrease in the scale parameter. Moreover, 5 points are OOC due to increase in both location and scale parameters while 4 points are OOC due to decrease in location and increase in scale parameter. However, only one point is declared OOC due to increase in location and decrease in scale parameter and the same detections due to decrease in both location and scale parameters. For Max-EWMA chart (presented in Figure 4.4), 114 points are declared OOC (indices reported in Table 4.12). The case wise diagnosis depicts that 15 points are declared OOC due to increase in location while only 3 points are declared OOC due to decrease in the location parameter. However, 86 OOC points are reported due to increase in scale parameter and only 10 OOC points are reported due to decrease in scale parameter. For Max-DEWMA chart (shown in Figure 4.5), 132 OOC points are detected (indices stated in Table 4.12). The diagnostic analysis shows that 16 OOC points are captured due to increase in location parameter and only 11 OOC points are reported due to decrease in scale parameter. Moreover, 91 points are declared OOC due to increase in scale parameter while only 14 points are declared OOC due to decrease in the scale parameter.

In precise, the implementation of our proposed chart and existing charts depict that Max-P chart outperforms other counterparts for the detection of joint shift in process parameters (location and scale). The proposed Max-P chart appeared as an efficient scheme to detect the variation in voltage converted by Z-source inverter in 3-φ grid connected PV system and may be useful for the practitioners who are engaged in the joint monitoring of such kind of parameters.

Table 4.12: Indices of case wise diagnosis for the proposed and some counterpart charts

Chart	**MAX-P**	**Max-EWMA**	**Max-DEWMA**
$\boldsymbol{m+}$	NA	267-271, 273-275, 277-279, 281, 283-285	269-271, 273-275, 277-279, 281, 283-285, 287, 290, 293
$\boldsymbol{m-}$	NA	272, 280, 282	272, 276, 280, 282, 286, 288, 289, 291, 292, 294, 295
$\boldsymbol{v+}$	168, 170-174, 176, 177, 182-186, 189-196, 199-210, 213, 216, 217, 219, 221, 223, 224, 226, 228-233, 235, 237-243, 245, 247, 248, 250, 252, 256, 259, 261, 263, 267, 268, 270, 271, 276-278, 281, 282, 292, 293, 295, 297, 298, 301-306, 308-310, 312-328, 330, 332-339, 341-345, 348-352, 354-381, 384-400	305, 306, 308-310, 312-328, 330, 332-339, 341-345, 348-381, 384-400	170-174, 176, 177, 308-310, 312-328, 330, 332-339, 341-345, 348-352, 354-381, 384-400
$\boldsymbol{v-}$	175, 178-181, 187, 188, 197, 198, 211, 212, 214, 215, 218, 220, 222, 225, 227, 234, 236, 244, 246, 249, 251, 253-255, 257, 258, 260, 262, 264-266, 269, 272- 275, 279, 280, 294, 296, 307, 311, 329, 331, 340, 346, 347, 353, 382, 383	307, 311, 329, 331, 340, 346, 347, 353, 382, 383	175, 178-180, 307, 311, 329, 331, 340, 346, 347, 353, 382, 383
$++$	283, 284, 287, 290, 299	NA	NA
$+-$	285	NA	NA
$-+$	286, 288, 289, 300	NA	NA
$--$	291	NA	NA

*NA=not available

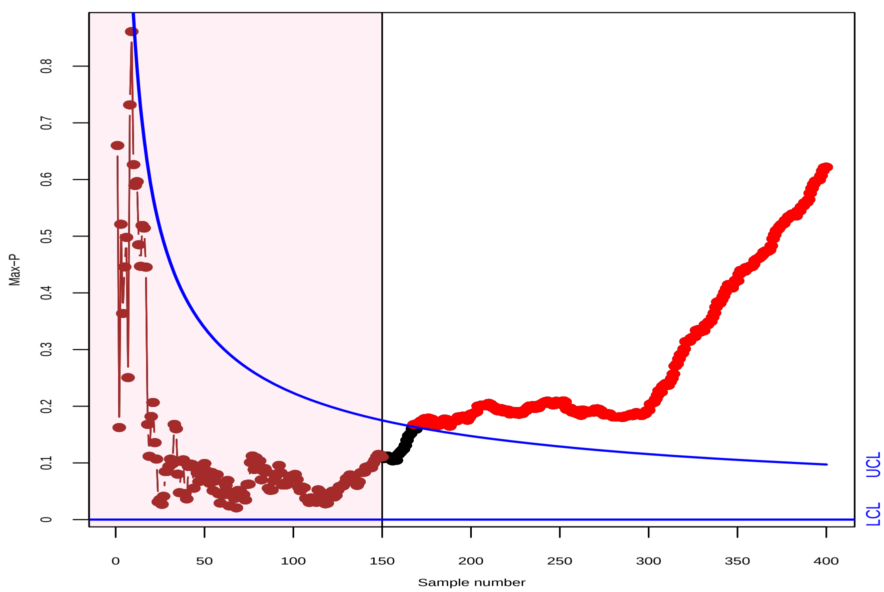

Figure 4.3: Max-P chart for IC and OOC states for the illustrative example

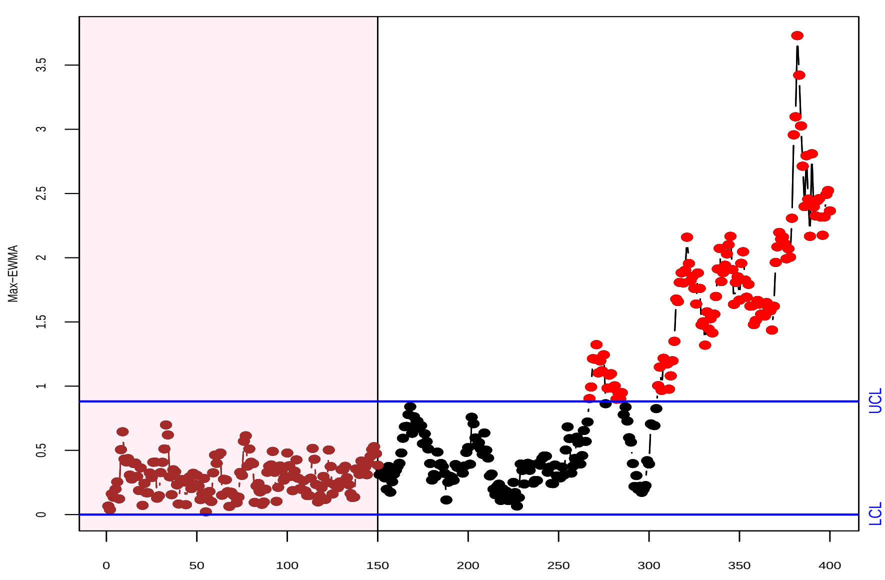

Figure 4.4: Max-EWMA chart for IC and OOC states for the illustrative example

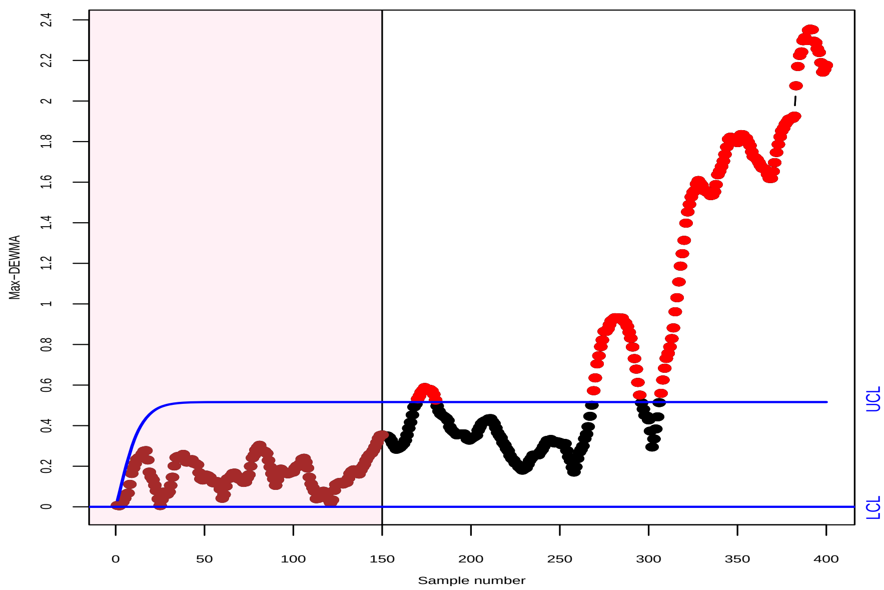

Figure 4.5: Max-DEWMA chart for IC and OOC states for the illustrative example

4.3 Concluding remarks

Recent literature about simple linear profiles was designed under simultaneous structure for the monitoring of linear profile parmeters (i.e. intercept, slope and error variance). In simulteneous methods each parameter have individual chart with its respective pair of limits. In simultaneous monitoring of linear profile parameters three individual charts based on each parameter are designed in a mechanism to obtain overall performance of the process. The joint monitoring is an alternative approch to simultaneous monitoring which has single charting methedology for all parameters under consideration. In this study, we proposed joint structures (Max-EWMA-3 and SS-EWMA-3) for the monitoring of linear profile parameters. The results concludes that joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have similar performance but joint structure offers simple design and paractical to use. Moreover, Max-EWMA-3, SS-EWMA-3 and EWMA-3 charts have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. In precise, Max-EWMA-3-C and SS-EWMA-3-C charts have relatively better performance (in terms of efficency and simplicity) among all others.

The joint monitoring of location and dispersion are the special case of joint linear profiling. In this chapter, we proposed Max-P chart to monitor both process parameters in a single charting setup. This study comprises an extensive comparison among Max-p chart and some existing joint monitoring schemes including Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA. The study findings reveal that the newly proposed Max-P monitoring scheme has relatively better performance in the presence of shifts in process parameter(s). Therefore, it may used as a powerful tool by quality control researchers and practitioners in the monitoring of joint shifts in manufacturing processes. A real application of the proposed scheme has also offered attractive detection ability to monitor the variations in the voltage of a photovoltaic (PV) system in an electrical engineering process.

CHAPTER 5

NON-PARAMETRIC APPROACH FOR JOINT MONITORING OF PROCESS PARAMETERS

Control charts are often designed and used to monitor single process parameter such as location and dispersion. There exist several studies (discussed in Section 1.3.2) for the joint monitoring of process parameters. Usually, normality is a typical assumption needed for parametric charts while non-parametric charts are free from any such constraints. The literature in this direction may see in [64–66, 124]. Moreover, a traditional approach used in SPC is to monitor each parameter separately, however simultaneous monitoring of more than one parameters is also getting popular in industry. [62, 63, 69, 125] and the references therein may be seen for literature on simultaneous charts.

Recently, [62] proposed a Shewhart type distribution free chart for joint monitoring of the process parameters. It is based on the Lepage test, a combination of Wilcoxon rank sum test for location and Ansari Bradley test for scale (cf. [126]) and this chart hereafter named as Shewhart-Lepage (SL) chart. On the same lines, [63] developed a distribution free Shewhart chart for joint monitoring that utilizes Cucconi test proposed by [127] and hereafter referred as Shewhart Cucconi (SC) chart. [128] provided a comparative analysis of Cucconi test versus Lepage test under some distributional setups and favored Cucconi test over Lepage.

This study intends to investigate the impact of the light and heavy tailed distributions on the performance of SL and SC charts. In addition, the effect of reference/test samples is included in this study.

5.1 Description of SC and SL charts

Let $U_1, U_2, \ldots\ldots\ldots, U_m$ and $V_1, V_2, \ldots\ldots\ldots\ldots, V_n$ be independent random samples from their respective populations with continuous cumulative distribution functions: $F(U) = Q\left(\frac{U-\theta}{\gamma}\right)$ and $G(V) = Q\left(\frac{V-\theta}{\gamma}\right)$; $\theta \in \Re$; $\gamma > 0$; where Q is some unknown continuous functions. The constants θ and γ represent the unknown location and scale parameters respectively. Let us introduce an indicator variable $I_k = 0$ or 1 depending on whether or not the k^{th} order statistic of the combined sample of $N = m + n$ observations belongs to U or V. It is to be mentioned that m is reference sample (phase I) and n is the test sample (phase

II). Further, we assume that k be the linear rank of k^{th} order variable.

The popular nonparametric Wilcoxon rank sum (WRS) test statistic T_1 is defined as

$$T_1 = \sum_{k=1}^{N} k I_k$$

For the equality of two scale parameters, Ansari Bradley (AB) is an efficient non-parametric test whose statistic T_2 is defined as

$$T_2 = \sum_{k=1}^{N} \left| k - \frac{1}{2}(N+1) \right| I_k$$

Consider S_1 as the sum of the square of the ranks of V_i's in the combined sample i.e.

$$S_1 = \sum_{k=1}^{N} k^2 I_k$$

Further, note that the quantities $(N + 1 - k)I_k$, for $k = 1, 2, ..., N$, may be considered as the contrary ranks of V_i's. The sum of squares of contrary ranks of V_i's in the combined sample, say S_2, is given by

$$S_2 = \sum_{k=1}^{N} (N + 1 - k)^2 I_k = n(N+1)^2 - 2(N+1) T_1 + S_1$$

Assuming $\theta = 0$ and $\gamma = 1$ refer to IC state ($F = G$), we have the following properties (cf. Appendix A.4).

$$E(T_1 \mid IC) = \frac{1}{2} n (N+1) \quad Var(T_1 \mid IC) = \frac{1}{12} mn (N+1)$$

$$E\left(T_2 \mid IC\right) = \begin{cases} \frac{m(N+2)}{4} & when\ N\ is\ even \\ \frac{m}{N}\left[\frac{N+1}{2}\right]^2 & when\ N\ is\ odd \end{cases}$$

$$Var\left(T_2 \mid IC\right) = \begin{cases} \frac{mn\left(N^2-4\right)}{48(N-1)} & when\ N\ is\ even \\ \frac{mn(N+1)\left(N^2+3\right)}{48N^2} & when\ N\ is\ odd \end{cases}$$

$$E\left(S_1 \mid IC\right) = E\left(S_2 \mid IC\right) = \frac{n\left(N+1\right)\left(2N+1\right)}{6}$$

$$Var\left(S_1 \mid IC\right) = Var\left(S_2 \mid IC\right) = \frac{mn}{180}\left(N+1\right)\ \left(2N+1\right)\left(8N+11\right)$$

The combination of AB and WRS is known as Lepage statistic [126] and is given as

$$L = \frac{\left(T_1 - E\left(T_1|IC\right)\right)^2}{VAR\left(T_1 \mid IC\right)} + \frac{\left(T_2 - E\left(T_2|IC\right)\right)^2}{VAR\left(T_2 \mid IC\right)} \tag{5.1}$$

and Cucconi statistic [127] for testing both location and scale is defined by

$$C = \frac{W^2 + Z^2 - 2WZ\rho}{2\left(1-\rho^2\right)} \tag{5.2}$$

where W and Z are the standardized statistics given as

$$W = \frac{S_1 - E\left(S_1|IC\right)}{\sqrt{VAR\left(S_1 \mid IC\right)}} = \frac{6S_1 - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{mn}{5}\left(N+1\right)\ \left(2N+1\right)\left(8N+11\right)}}$$

$$Z = \frac{S_2 - E\left(S_2|IC\right)}{\sqrt{VAR\left(S_2 \mid IC\right)}} = \frac{6S_2 - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{mn}{5}\left(N+1\right)\ \left(2N+1\right)\left(8N+11\right)}}\ ,$$

when $\theta > 0$ and $\gamma = 1$, $E(W) > 0$ and $E(Z) < 0$; when $\theta = 0$ and $\gamma > 1$,

$E(W) > 0$ and $E(Z) > 0$; and in general, when $\theta \neq 0$ and $\gamma \neq 1$, $E(W) \neq 0$ and $E(Z) \neq 0$. Similar inequalities may be observed in other possible cases, when either θ differs from 0, or γ differs from 1, in any direction. Also, note that $E(W|IC) = E(Z|IC) = 0$ and $V(W|IC) = V(Z|IC) = 1$ (cf. Appendix A.5). Moreover, when $F = G$, the correlation coefficient between W and Z is given as [128]:

$$\rho = Corr\,(W, Z \mid IC) = \frac{2\,(N^2 - 4)}{(2N+1)\,(8N+11)} - 1$$

5.2 Design of control charting constants of distribution free charts

Construction and design of both SC and SL charts depend on the distributions of the statistics given in (5.1) and (5.2). The lower control limit of both charts is zero as both statistics can never be negative (cf. [62,63]) and the upper control limits of both charts, say H, used to make decision. The values of H are provided in [62,63] for some selective values of n and m. We have covered more combinations of n and m to find the upper control limit say H for both charts, using a simulation study with 100,000 replicates (in R 3.1.1). We have taken the retrospective samples i.e. m = 30, 50, 100, 150, 500 and 1000 while prospective samples i.e. n = 5, 8, 11, 16 and 25 for this study, fixing $ARL_0 = 500$. The results are reported in Table 5.1 for SL and SC charts.

The decision procedure for the two charts is given as:

Table 5.1: Constant H for SC and SL charts at ARL_0=500

	m=30	**m=50**	**m=100**	**m=150**	**m=500**	**m=150**
			SC			
n=5	4.48	5.25	5.98	6.25	6.65	6.73
n=8	4.31	4.77	5.56	5.91	6.42	6.54
n=11	4.45	4.8	5.34	5.67	6.29	6.42
n=16	4.47	4.85	5.31	5.56	6.11	6.28
n=25	4.18	4.7	5.25	5.49	6	6.16
			SL			
n=5	9.4	10.32	11.25	11.5	12.02	12.14
n=8	9.28	10.22	11.15	11.53	12.1	12.24
n=11	9.24	10.1	11.07	11.45	12.06	12.22
n=16	9.11	9.95	10.9	11.32	12.04	12.21
n=25	8.4	9.5	10.74	11.17	12.02	12.2

***SL chart*:** The statistic L is used for plotting in SL chart. If L is greater than H, then the process is declared OOC. For the follow up analysis, we compute the p-values of the WRS test for location and AB test for scale with the phase I sample and the i^{th} test sample and are denoted as p_1 and p_2 respectively. If p_1 is very low but not p_2,a shift of location is detected or if p_2 is very low and p_1 is relatively high,a shift in scale parameter is detected. When both WRS and AB p-values are very low, a joint shift in the location and scale is considered.

***SC chart*:** The statistic C is used for plotting in SL chart. If C exceeds H, the process is declared OOC. For the follow up analysis, we compute the p-values for Wilcoxon test (p_3) and Mood test (p_4) based on two samples (reference sample and test sample), [63]. The shift in location is noted, when p_3 is very low but not p_4, and if p_3 is relatively high but not p_4 then there is the indication about shift in scale. If both p_3 and p_4 are very low, shift is noted in both location and scale. Sometimes neither p_3 nor p_4 are very low though the plotting statistic C is high, in this situation the effect is due to the relation between location and scale changes or due to false alarm. So to overcome this problem combine $i^{th}$and $(i-1)^{th}$ prospective samples and recalculate the (p_3) and (p_4) for further decision.

5.3 Performance analysis of SL and SC charts

In this section, we will investigate the performance of SL and SC charts under different distributional environments. We will also examine the effects of reference and test samples on the performance of these charts. We will use average run length (ARL) and standard deviation run length ($SDRL$) as performance measures. The ARL value is denoted by ARL_0 for in-control situation and ARL_1 for out-of-control situation. The distributional setups covered in this study include: Uniform: U ($\sqrt{-3}$, $\sqrt{3}$), Student's t: t_4 , Lognormal: *LN(1,1)* , Gamma: *G(1,1)* , and contaminated normal (*C1*: with 10% contaminations; *C2*: with 30% contaminations). The first two are symmetric and light tailed, next two are skewed and heavy tailed, and last two are contaminated distributions. [15, 128–131] are some useful references about the said distributional environments. The graphical displays of these distributions are given in Figure 5.1.

5.4 OOC performance

In order to examine the OOC performance of SL and SC charts, we have considered shifts in location and scale for these choices: $\theta = 0,\ 0.25,\ 0.50,\ 0.75,\ 1.00,\ 1.50,$

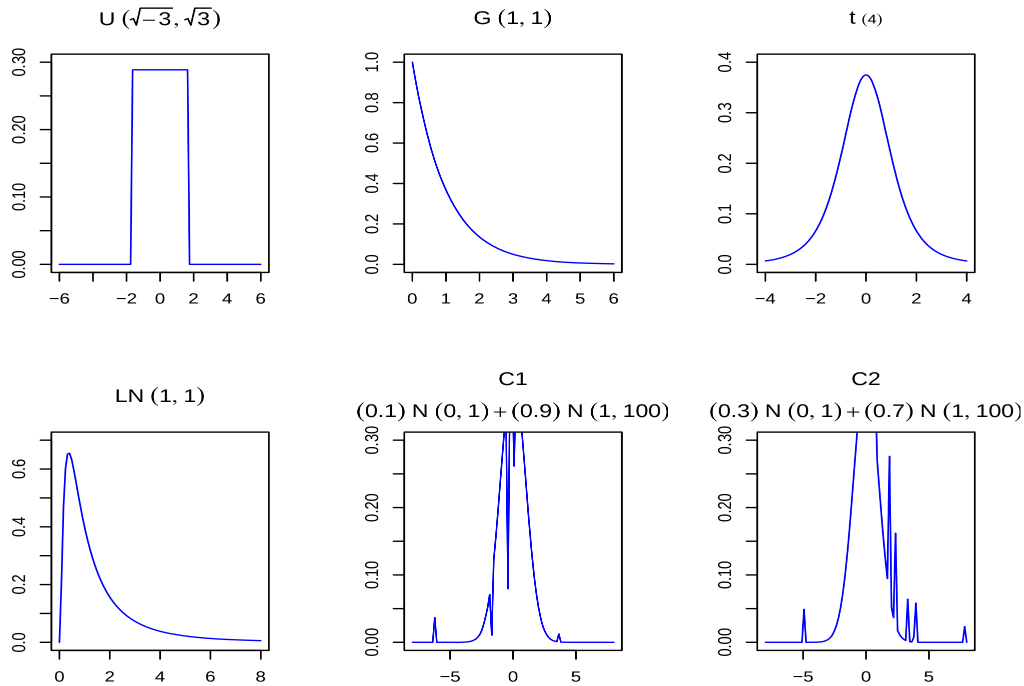

Figure 5.1: Probability density plots of different distributions

2.00 and $\gamma = 0.50,\ 0.75,\ 1,\ 1.25,\ 1.5,\ 1.75$ and 2. We have chosen m= 30, 50, 100, 150, 500 and 1000 aand n = 5, 8, 11, 16 and 25. It makes a total of 30 pairs (m, n). The properties of SL and SC charts, in terms of ARL and $SDRL$, are evaluated for different combinations θ and γ. These results are provided in Tables 5.2 and 5.3 under different distributions. For the sake of brevity, we only discuss the results of the pair (100, 5). Moreover, some useful ARL curves are also produced and are provided in Figures 5.2 and 5.3.

The useful findings about the two charts are listed as:

- In *general*, the run length follows right skewed distribution; the run length distributions of both charts decrease with the increase in the location and scale shifts; shift in the scale parameter is detected faster than the shift

Table 5.2: *ARLs* of SC and SL Charts under Different Distributions using $m = 100$ and $n = 5$

		$U\ (\sqrt{-3}\ ,\ \sqrt{3})$		*t4*		*LN(1,1)*		*G(1,1)*		*C1*		*C2*	
γ	θ	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**
	0.00	133.60	1171.98	15598.55	4741.50	1958.81	1351.42	14130.31	332.10	4244.22	771.56	753.65	374.45
	0.25	153.60	1184.99	15053.16	4721.17	4446.38	1405.95	12805.36	2288.34	3179.54	652.55	717.98	342.99
	0.50	4596.41	990.82	3651.45	6725.76	9842.98	1701.95	5629.85	1595.17	1644.35	479.31	655.09	314.40
0.5	**0.75**	2185.38	182.76	271.02	3925.29	12483.72	1703.05	1238.17	296.32	625.48	226.86	560.32	281.93
	1.00	16.18	11.66	13.11	2888.23	9251.59	1183.41	218.56	35.65	215.57	71.55	453.31	238.19
	1.50	1.59	1.39	1.14	1607.68	1044.82	164.16	3.66	1.32	22.81	5.46	288.71	133.39
	2.00	1.00	1.00	1.01	795.95	47.68	11.86	1.05	1.00	3.79	1.49	185.64	68.50
	0.00	3639.70	5250.33	5776.59	9287.89	12139.71	1784.44	10500.39	1406.33	1497.88	692.08	631.99	360.24
	0.25	10453.54	1930.49	2348.37	8441.62	6892.77	1146.05	3579.71	290.75	1198.41	535.60	612.85	345.37
	0.50	201.87	122.63	285.72	6249.00	1657.41	405.53	866.22	150.12	645.93	270.15	552.64	314.30
0.75	**0.75**	25.75	21.42	34.85	4173.70	269.82	104.72	183.01	55.13	287.55	107.38	483.19	257.58
	1.00	8.05	7.39	6.00	3025.55	55.96	28.35	38.16	11.77	105.60	36.96	412.44	207.27
	1.50	2.07	1.96	1.32	1435.98	5.97	4.12	2.86	1.43	17.23	5.48	273.54	118.28
	2.00	1.11	1.07	1.03	589.88	1.80	1.57	1.08	1.01	3.91	1.71	185.71	71.33
	0.00	506.23	499.45	511.47	503.09	503.04	505.16	503.96	506.55	511.82	500.85	506.78	507.24
	0.25	111.64	133.09	267.71	245.33	261.97	259.18	782.65	534.14	423.98	404.52	494.57	490.28
	0.50	31.67	37.89	66.51	55.74	71.47	69.17	240.77	162.23	266.94	216.13	455.29	450.29
1	**0.75**	13.19	15.55	16.09	13.54	20.79	20.23	72.60	53.58	133.49	96.20	411.23	387.42
	1.00	6.75	7.89	5.17	4.64	7.64	7.64	22.60	16.62	63.85	39.75	359.26	321.20
	1.50	2.52	2.85	1.51	1.49	2.10	2.16	3.02	2.23	13.94	8.47	260.57	207.84
	2.00	1.36	1.41	1.07	1.09	1.20	1.23	1.14	1.06	4.40	2.86	190.03	139.47
	0.00	22.81	39.65	112.22	136.30	76.10	102.55	18.35	28.47	203.03	198.85	410.70	389.44
	0.25	19.97	31.65	73.29	84.49	45.91	57.89	177.22	199.10	180.53	172.20	401.98	380.56
	0.50	14.46	19.18	28.97	29.93	17.54	20.25	121.88	110.31	123.96	109.05	383.55	359.37
1.25	**0.75**	9.47	11.07	10.80	10.74	7.31	7.97	48.95	43.15	73.42	54.95	353.64	317.61
	1.00	6.06	6.96	4.71	4.68	3.62	3.85	19.29	16.75	41.02	28.50	315.83	276.75
	1.50	2.90	3.25	1.68	1.72	1.50	1.57	3.62	2.85	12.83	8.47	244.72	197.99
	2.00	1.66	1.81	1.14	1.17	1.09	1.11	1.26	1.14	4.65	3.24	186.30	135.86
	0.00	8.09	14.67	40.09	56.25	24.25	37.13	7.06	11.07	100.81	97.56	340.70	307.79
	0.25	7.83	13.64	30.78	40.54	17.07	23.69	18.87	31.48	88.99	89.52	342.21	301.97
	0.50	7.01	11.14	16.53	19.25	8.28	10.24	61.19	69.09	68.48	63.88	324.87	286.24
1.5	**0.75**	6.17	8.41	8.05	9.00	4.20	4.81	38.18	37.91	46.47	38.58	301.81	268.04
	1.00	5.18	6.29	4.32	4.63	2.45	2.71	18.40	17.78	29.86	22.70	281.01	238.85
	1.50	3.14	3.49	1.82	1.91	1.31	1.36	4.38	3.83	11.48	8.05	229.81	181.89
	2.00	1.97	2.14	1.23	1.27	1.05	1.07	1.49	1.32	5.08	3.72	186.46	135.55
	0.00	4.74	8.46	19.39	28.97	11.72	18.94	4.31	6.69	59.37	55.99	288.24	250.96
	0.25	4.64	8.15	16.38	23.38	9.06	13.34	7.47	12.63	50.90	54.32	283.87	246.25
	0.50	4.42	7.33	10.88	13.94	5.19	6.62	15.58	25.51	43.14	42.68	271.38	237.93
1.75	**0.75**	4.20	6.29	6.43	7.71	3.03	3.51	27.12	30.59	32.22	29.04	264.18	227.34
	1.00	3.80	5.29	3.96	4.50	1.97	2.18	17.83	18.30	22.62	19.19	247.84	208.32
	1.50	3.12	3.60	1.92	2.08	1.21	1.26	5.24	5.07	10.84	8.22	214.13	168.13
	2.00	2.20	2.39	1.31	1.37	1.03	1.04	1.84	1.65	5.39	4.10	180.64	132.26
	0.00	3.38	5.81	11.54	18.11	7.12	11.84	3.20	4.87	40.46	35.55	248.38	209.49
	0.25	3.35	5.68	10.24	15.29	5.86	8.79	4.55	7.44	33.66	37.23	250.30	207.66
	0.50	3.24	5.41	7.75	10.65	3.77	4.89	7.11	12.18	29.54	31.10	242.79	200.31
2	**0.75**	3.17	4.97	5.25	6.69	2.42	2.88	11.64	17.69	24.08	23.29	234.72	193.50
	1.00	2.99	4.40	3.63	4.32	1.70	1.89	14.87	16.88	18.16	16.44	225.19	184.62
	1.50	2.68	3.41	1.98	2.20	1.16	1.20	6.14	6.36	9.79	8.20	195.52	153.90
	2.00	2.32	2.55	1.38	1.47	1.03	1.03	2.29	2.15	5.54	4.51	170.17	132.57

Table 5.3: *SDRLs* of SC and SL Charts under Different Distributions using $m = 100$ and $n = 5$

		***U* ($\sqrt{-3}$, $\sqrt{3}$)**		***t4***		***LN(1,1)***		***G(1,1)***		***C1***		***C2***	
γ	$\boldsymbol{\theta}$	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**	**SC**	**SL**
0.5	**0.00**	1523.32	3438.77	26154.47	15223.50	10820.01	3922.19	24104.39	1212.42	5762.79	1152.54	1165.85	462.47
	0.25	3023.32	3506.40	22552.20	14967.63	15555.80	3377.95	20772.47	7520.43	4863.72	973.78	1135.92	419.56
	0.50	14639.87	2769.44	10064.97	14773.53	21235.22	4046.90	13380.19	5877.11	3319.82	767.23	1063.13	417.88
	0.75	7716.73	499.28	2384.35	13315.89	21560.04	3839.40	5596.81	2573.27	1796.22	496.24	1013.83	394.96
	1.00	22.29	15.20	106.34	11305.25	17742.19	2831.89	1910.50	475.51	818.40	281.56	823.35	410.21
	1.50	1.07	0.83	0.71	8864.32	4978.54	777.76	35.83	2.47	149.53	22.06	655.10	319.56
	2.00	0.07	0.05	0.07	6302.91	408.96	41.36	0.64	0.08	30.15	2.89	494.31	185.81
0.75	**0.00**	13916.04	9698.16	9485.65	19606.95	17472.98	2883.92	16745.08	2724.92	2298.94	865.22	1005.81	461.84
	0.25	18810.79	4380.18	5963.97	19060.68	12867.26	1896.86	8592.19	523.63	2021.06	744.40	989.55	432.95
	0.50	327.14	185.02	1039.44	16231.99	4985.29	859.17	3075.42	329.09	1381.87	467.73	918.59	438.71
	0.75	30.13	24.54	221.31	13388.02	942.35	262.07	908.14	165.33	907.08	363.98	859.89	395.57
	1.00	8.37	7.76	17.96	11673.86	136.19	54.03	186.15	34.21	427.66	123.80	785.62	370.67
	1.50	1.58	1.49	0.79	8030.86	8.67	4.79	9.81	1.87	134.33	17.72	615.42	264.77
	2.00	0.36	0.29	0.17	4801.92	1.43	1.10	0.60	0.14	27.68	2.10	452.18	203.34
1	**0.00**	851.89	702.26	853.95	712.30	818.90	720.95	836.01	711.49	723.14	804.96	827.01	729.03
	0.25	134.50	162.99	535.68	417.24	461.83	424.46	1915.06	1037.74	760.01	646.67	871.64	711.38
	0.50	34.76	42.01	146.08	130.01	124.03	111.20	662.72	313.50	617.50	437.40	765.56	703.94
	0.75	13.56	16.23	33.98	21.43	31.48	27.53	188.94	137.30	386.99	268.98	738.65	669.15
	1.00	6.61	7.78	8.02	5.66	9.10	8.94	52.61	36.38	256.72	147.65	723.87	629.51
	1.50	2.04	2.42	1.04	0.95	1.70	1.76	5.51	3.76	99.17	54.36	590.70	493.13
	2.00	0.73	0.79	0.29	0.32	0.51	0.55	0.69	0.39	35.49	27.53	495.19	424.78
1.25	**0.00**	23.81	42.05	163.10	174.78	97.38	126.44	18.82	29.69	286.95	335.96	728.38	570.31
	0.25	20.59	33.27	109.80	123.10	59.77	72.91	281.84	283.74	327.52	271.34	733.74	587.17
	0.50	14.58	19.67	44.99	44.47	21.03	24.47	222.75	182.89	271.87	214.53	701.67	579.38
	0.75	9.29	11.07	15.30	14.19	7.96	8.59	101.57	69.38	200.18	126.31	674.40	563.25
	1.00	5.77	6.69	5.48	5.09	3.41	3.63	33.07	27.11	157.51	85.21	611.01	529.24
	1.50	2.42	2.78	1.21	1.20	0.91	0.99	5.50	3.74	60.55	43.06	548.87	476.71
	2.00	1.08	1.25	0.42	0.46	0.31	0.35	0.85	0.57	24.00	15.72	470.69	373.64
1.5	**0.00**	7.77	14.68	51.18	67.60	27.76	41.45	6.73	10.82	138.08	168.01	686.31	478.92
	0.25	7.45	13.42	41.65	51.14	19.44	26.69	20.10	33.57	160.81	137.43	601.55	485.66
	0.50	6.64	10.96	21.10	23.69	8.71	10.88	84.64	96.83	167.35	113.32	668.38	463.00
	0.75	5.79	8.12	9.63	10.33	4.00	4.58	55.69	53.46	111.48	75.83	576.60	472.61
	1.00	4.76	5.90	4.59	4.73	1.99	2.29	26.45	25.01	97.94	53.64	547.22	470.79
	1.50	2.62	3.00	1.35	1.43	0.65	0.72	5.75	4.69	55.60	21.25	511.73	434.71
	2.00	1.39	1.59	0.55	0.60	0.23	0.27	1.19	0.91	27.12	8.49	493.54	391.42
1.75	**0.00**	4.26	8.03	22.75	32.97	12.28	19.95	3.85	6.29	80.30	97.01	568.63	394.23
	0.25	4.16	7.76	19.15	26.84	9.33	13.94	7.27	12.66	95.33	76.66	506.45	404.74
	0.50	3.90	6.98	12.77	16.22	5.03	6.52	16.98	28.03	93.15	69.49	487.01	398.63
	0.75	3.71	5.85	7.11	8.44	2.59	3.11	34.93	38.86	75.73	48.14	481.79	418.78
	1.00	3.31	4.83	3.93	4.47	1.43	1.65	22.75	22.82	62.09	38.74	472.90	405.47
	1.50	2.61	3.10	1.42	1.60	0.51	0.57	6.26	5.95	41.53	19.02	442.68	385.83
	2.00	1.64	1.82	0.65	0.74	0.19	0.21	1.62	1.36	26.37	6.29	443.42	341.43
2	**0.00**	2.87	5.39	12.63	19.82	7.01	12.18	2.70	4.41	58.03	64.65	436.40	347.44
	0.25	2.82	5.22	11.26	16.66	5.68	8.62	4.14	7.13	57.65	51.27	478.36	342.76
	0.50	2.70	4.92	8.42	11.68	3.37	4.58	6.94	12.39	53.91	46.07	449.94	332.65
	0.75	2.65	4.52	5.36	6.95	1.91	2.42	12.64	19.66	50.35	39.54	457.56	345.47
	1.00	2.46	3.88	3.42	4.17	1.11	1.33	17.57	19.59	45.03	26.37	439.34	356.90
	1.50	2.13	2.89	1.47	1.72	0.43	0.49	6.84	7.10	27.69	15.50	430.07	348.88
	2.00	1.76	2.02	0.73	0.85	0.16	0.18	2.11	1.96	31.29	7.96	387.64	351.30

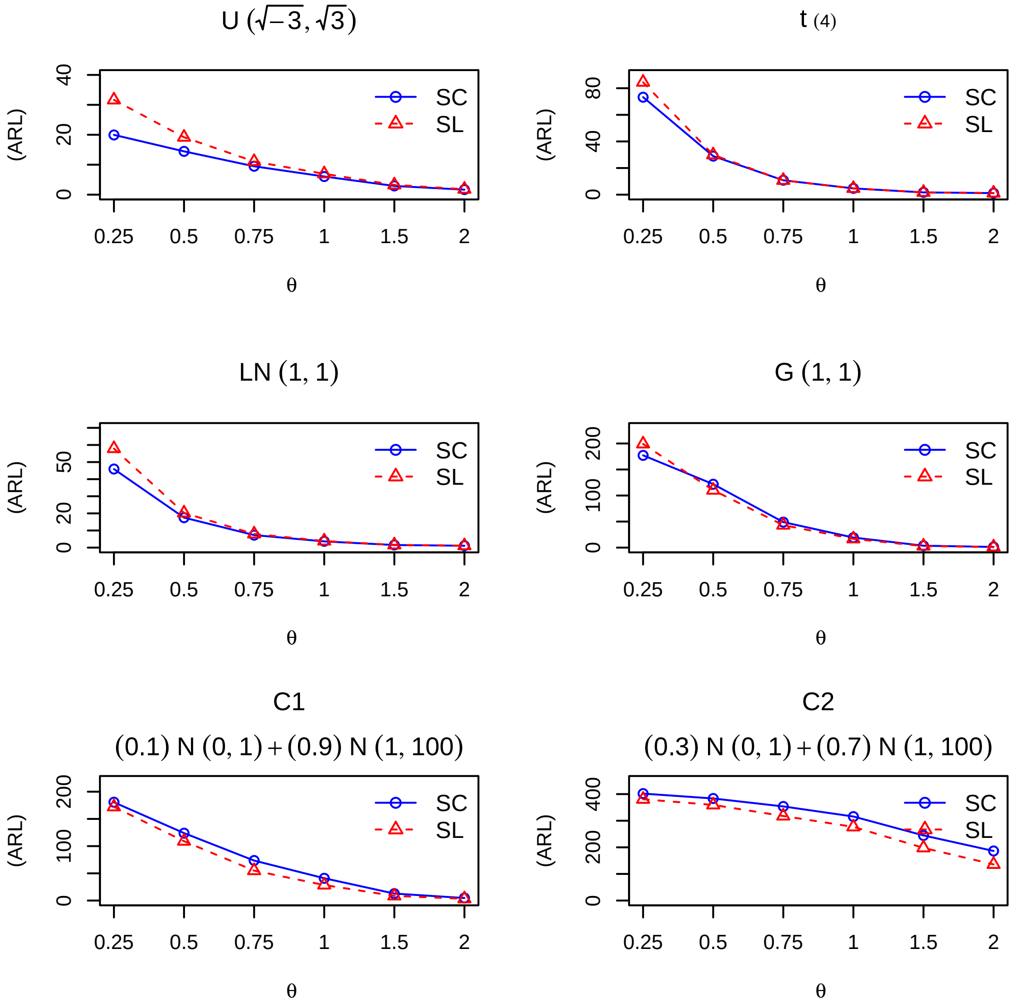

Figure 5.2: ARL_1 curve with varying location shifts θ and fixed $\delta = 1.25$

in the location parameter; both charts are sensitive to shifts in location and scale but both charts react more quickly to detect a shift in standard deviation rather than mean.

- For the case of *uniform* distribution, SC chart performs slightly better than SL chart. For instance: when $\theta = 0.25$ and $\gamma = 1.25$ ARL_1 values of SC and SL charts are 19.97 and 31.espectively; when $\theta = 0.0$ and $\gamma = 1.25$ ARL_1 values of SC and SL charts are 22.81 and 39.65 respectively; when $\theta = 0.25$

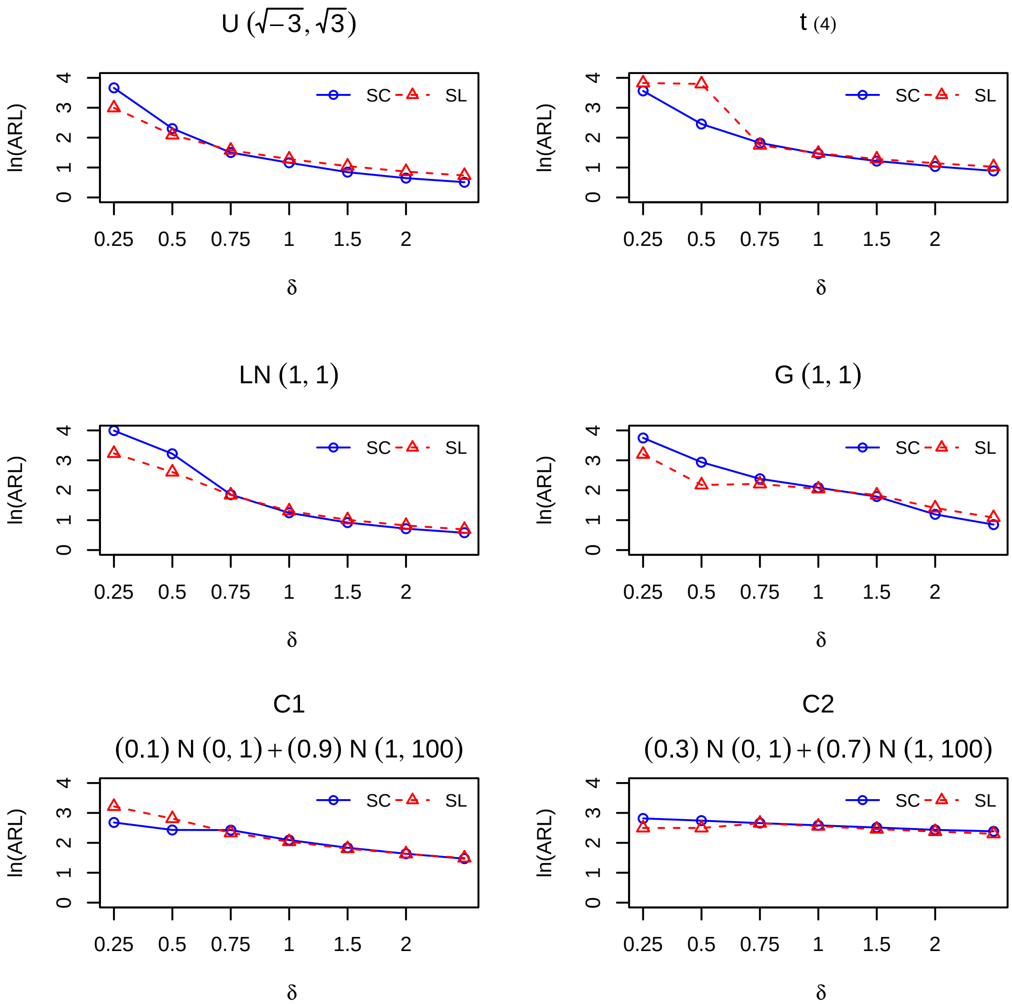

Figure 5.3: ($lnARL_1$) profile with respect to scale shift (γ) on fixed $\theta = 0.5$

and $\gamma = 1.00$ ARL_1 values of SC and SL charts are 111.64 and 133.09.

- The SC chart performs slightly better than SL chart under t_4. For example: when $\theta = 0.25$ and $\gamma = 1.25$ ARL_1 values of SC and SL charts are 73.29 and 84.49 respectively; when $\theta = 0.0$ and $\gamma = 1.25$ ARL_1 values of SC and SL charts are 112.36 and 136.30 respectively while when $\theta = 0.25$ and $\gamma = 1.00$ ARL_1 values of SC and SL charts are 267.71 and 245.33.

- For the case of *lognormal* distribution, SC chart performs slightly better than SL chart. Due to an upward shift θ= 0.25 and γ= 1.25 ARL_1 decreases 45.91 of SC and 57.89 of SL chart. when θ= 0.0 and γ= 1.25 ARL_1 values of SC and SL charts are 76.10 and 102.55 respectively while when θ= 0.25 and γ= 1.00 ARL_1 values of both SC and SL charts decreases approximately 48%.

- *Gamma (1, 1)* provides substantial results when θ= 0.0 and γ= 1.0. When θ= 0.25 and γ= 1.00, ARL_1 of both charts is greater than the intended ARL_0, which makes both charts less effective and ARL biased for such shift. By varying the γ we observe the same effect on the results of said charts. Moreover, having θ= 1.5 and 2 with γ= 1.25 shows increasing trend as compared to the results when γ remains IC. Similar type of the finding for the exponential distribution was also noted by [132].

- In contaminated environment (*C1* and *C2*), effectiveness of detecting the shift in location and scale is affected for both SC and SL charts as compared to other environments. SL chart performs slightly better than SC chart. In *C1*, reduction in ARL_1 values of SC and SL charts are reported as: 64% and 66% on θ= 0.25 and γ= 1.25, 59% and 60% on θ= 0.0 and γ= 1.25 and approximately 15% and 19% on θ= 0.25 and γ= 1.00. On the other hand in *C2*, reduction in ARL_1 values of SC and SL charts are as: 20% and 24% on θ= 0.25 and γ= 1.25, 18% and 22% on θ= 0.0 and γ= 1.25 and approximately 1.08% and 1.9% on θ= 0.25 and δ= 1.00.

- Consider the effect of specific shift γ =1.25 on the charts with respect to different environments. The shifts in θ (on horizontal axis) and ARL_1 (on vertical axis) are portrayed in Figure 5.2. The results revealed better performance of SC and SL chart with the increase of θ. Further, results from Figure 5.3 show better performance of SC and SL chart with increase in γ on fixed θ= 0.5. Moreover, in light tailed distributions SC chart performs well while in heavy tailed environments SL chart is superior, and both charts lose their performance in case of *C2*.

5.5 Effect of reference sample and test sample on the performance of charts

Control limits of nonparametric charts are estimated from reference sample (m) and this may have a significant effect on the performance of the phase-II chart which is reported in Table 5.4. In general, increasing m produces decreasing trend in ARL_1 of both charts under all environments. Specifically, at fixed $\gamma = 1.5$, the ARL_1 of the SC chart under *G (1,1)* decreases about 44.5% due to increase in m from 30 to 50 at fixed $\theta = 0.75$ while it decreases 64.1%, 68.4%, 70.7% and 71.8% from the 30 to 100, 150, 500 and 1000 samples respectively. On the other hand in SL chart 23.5% , 32.9%, 36.3%, 40.1% and 40.7% fall out is reported in ARL_1 from $m = 30$ to 1000 respectively on the fixed location parameter $\theta = 0.25$. Moreover, the same findings are examined for different γ at fixed $\theta = 1$.

The test sample (n) also exhibits significant effects on the performance of the phase-II chart and its profile study is given in Table 5.5. At fixed $\gamma = 1.25$, ARL_1 of the SC chart under t_4 environment decreases about 53.9% due to increase in n from 5 to 8 at fixed $\theta = 0.75$ while it decreases 66.5%, 76.6% and 84.7% from the 5 to 11, 16 and 25 samples respectively. On the other hand, a decrease of 39.6%, 66.4%, 74.5% and 84.7% in ARL_1 of SL chart is reported with $n = 5$ to 25 respectively on the fixed location parameter $\theta = 0.75$. The same findings are also observed at fixed $\theta = 1$ and varying values of γ. In general, increasing the test sample size produces decreasing trend in ARL_1 of both charts under all environments.

5.6 Illustrative example

In this section, we apply our SC and SL charts on a dataset containing duration of contract strikes in US manufacturing industries (cf. [133]). A strike is a refusal of employees to perform work as a form of protest. In industries, strikes may cause the losses in manufacturing and production departments. So, administration and human resource management always try to avoid it. In case of a strike they monitor the strike duration to minimize loss. From the said data, we have considered the data from January 1968 to October 1976. Further (following [66]) we have considered 100 observations between January 1968 and February 1969 as a reference sample and remaining 460 data points as test samples (each of size 10). The control limits for SC and SL charts are obtained by the same simulation procedure as mentioned for Table 5.1, and are given as: 5.37 for SC chart and 11.1

Table 5.4: Profile of ARL and $SDRL$ using $n = 11$ and $\gamma = 1.5$

Dist.	Chart	m	30			50			100			150			500			1000		
		θ	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00
G(1,1)	**SC**	**ARL**	14.69	37.63	1.03	10.22	20.90	1.01	8.66	13.51	1.00	8.71	11.90	1.00	9.06	11.01	1.00	8.88	10.58	1.00
		SDRL	21.98	112.93	0.32	12.77	53.01	0.11	9.44	23.26	0.03	8.97	16.00	0.02	8.83	11.69	0.00	8.53	10.58	0.00
	SL	**ARL**	27.32	41.05	1.04	20.89	24.80	1.01	18.32	15.71	1.00	17.41	13.58	1.00	16.36	10.78	1.00	16.20	10.49	1.00
		SDRL	42.83	137.68	0.36	27.62	57.12	0.17	21.20	24.12	0.03	18.94	18.23	0.02	16.28	11.28	0.00	15.97	10.59	0.00
$U\ (\sqrt{-3}\ ,\sqrt{3})$	**SC**	**ARL**	15.85	5.90	1.10	11.16	4.54	1.06	9.71	4.04	1.05	9.74	3.99	1.04	3.60	2.73	1.12	3.61	2.70	1.11
		SDRL	21.68	6.55	0.34	12.84	4.45	0.26	10.32	3.73	0.22	9.91	3.62	0.21	3.10	2.18	0.36	3.10	2.15	0.36
	SL	**ARL**	28.72	7.48	1.13	22.31	6.15	1.10	19.19	5.49	1.08	18.40	5.27	1.07	6.36	3.76	1.17	6.35	3.75	1.16
		SDRL	42.41	8.67	0.42	28.36	6.52	0.35	21.62	5.34	0.30	19.79	4.99	0.28	5.89	3.26	0.44	5.86	3.21	0.44
t_4	**SC**	**ARL**	38.16	6.44	1.02	18.66	3.52	1.01	12.88	2.73	1.00	12.06	2.60	1.00	11.49	2.52	1.00	11.39	2.50	1.00
		SDRL	136.96	32.75	0.16	41.36	5.34	0.09	18.81	2.72	0.07	15.26	2.34	0.06	11.86	2.04	0.06	11.26	1.99	0.05
	SL	**ARL**	50.84	7.63	1.03	18.66	3.52	1.01	20.40	3.31	1.01	18.26	3.07	1.01	15.91	2.78	1.01	15.43	2.77	1.01
		SDRL	164.23	27.94	0.18	41.36	5.34	0.09	29.15	3.47	0.09	22.99	2.88	0.08	16.36	2.29	0.08	15.41	2.25	0.07
LN(1,1)	**SC**	**ARL**	14.56	2.44	1.00	8.81	1.91	1.00	6.84	1.73	1.00	6.73	1.71	1.00	6.72	1.69	1.00	6.67	1.69	1.00
		SDRL	34.97	3.00	0.02	13.34	1.55	0.01	7.84	1.21	0.01	7.18	1.15	0.01	6.40	1.11	0.01	6.23	1.09	0.01
	SL	**ARL**	21.92	2.81	1.00	14.54	2.28	1.00	11.46	2.06	1.00	10.92	1.99	1.00	9.72	1.89	1.00	9.57	1.87	1.00
		SDRL	52.03	3.30	0.03	21.84	2.08	0.02	13.95	1.62	0.01	12.20	1.48	0.01	9.55	1.32	0.01	9.27	1.29	0.01
C1	**SC**	**ARL**	152.50	58.14	2.17	74.16	28.52	1.58	40.24	12.77	1.30	35.42	10.52	1.25	30.63	8.64	1.21	29.52	8.21	1.20
		SDRL	531.70	235.39	4.01	266.60	128.20	2.33	92.66	31.75	0.81	58.63	17.81	0.63	36.22	9.37	0.52	31.30	8.15	0.50
	SL	**ARL**	157.89	75.30	3.23	88.63	34.24	1.95	51.95	14.30	1.34	43.38	11.37	1.29	35.06	8.83	1.23	33.30	8.38	1.23
		SDRL	550.94	334.08	31.30	293.00	202.80	51.15	102.50	30.53	0.81	64.53	19.93	0.67	37.99	9.03	0.53	35.21	8.33	0.54
C2	**SC**	**ARL**	435.72	241.76	19.27	350.80	240.10	27.12	233.60	159.90	24.68	215.70	133.61	17.44	198.37	102.18	9.73	192.37	92.92	8.73
		SDRL	1018.67	609.59	40.09	950.00	663.20	82.12	528.30	444.70	143.30	393.70	315.30	67.56	250.32	139.36	13.56	216.89	109.03	9.89
	SL	**ARL**	480.23	331.12	53.29	385.60	299.00	92.69	269.80	178.50	26.97	224.90	132.70	13.54	174.28	78.53	6.53	171.01	69.12	5.76
		SDRL	1175.49	846.16	171.27	923.10	806.40	573.60	518.30	428.20	143.90	370.10	287.60	49.11	208.62	106.99	7.89	194.29	79.72	6.44

Table 5.5: Profile of ARL and $SDRL$ using $m = 50$ and $\gamma = 1.25$

Distributions	Chart		5		8		11		16		25	
			0.75	**2**	**0.75**	**2**	**0.75**	**2**	**0.75**	**2**	**0.75**	**2**
G(1,1)	**SC**	***ARL***	66.23	1.53	30.14	1.03	15.81	1.00	6.20	1.00	1.88	1.00
		SDRL	206.14	2.53	111.94	0.30	37.51	0.09	17.84	0.00	7.67	0.00
	SL	***ARL***	59.59	1.35	30.20	1.03	16.91	1.00	5.94	1.00	1.86	1.00
		SDRL	171.20	1.68	63.90	0.31	36.08	0.10	17.68	0.00	5.49	0.00
$\boldsymbol{U}$ $(\sqrt{-\mathbf{3}}\ ,\ \sqrt{\mathbf{3}})$	**SC**	***ARL***	5.81	1.94	5.45	1.17	4.52	1.14	3.65	1.01	1.68	1.00
		SDRL	5.55	1.39	5.37	0.46	4.46	0.41	3.60	0.10	1.18	0.06
	SL	***ARL***	8.28	2.16	8.12	1.29	6.19	1.22	4.42	1.01	2.10	1.00
		SDRL	8.18	1.64	8.05	0.66	6.53	0.54	4.68	0.13	1.72	0.07
t_4	**SC**	***ARL***	13.29	1.17	6.12	1.02	4.46	1.00	3.11	1.00	2.03	1.00
		SDRL	33.41	0.49	13.75	0.14	9.94	0.05	5.49	0.02	4.60	0.00
	SL	***ARL***	13.29	1.17	8.03	1.03	4.46	1.00	3.39	1.00	2.03	1.00
		SDRL	33.41	0.49	23.69	0.18	9.94	0.05	6.81	0.02	3.15	0.00
LN(1,1)	**SC**	***ARL***	7.57	1.09	4.06	1.01	3.14	1.00	2.33	1.00	1.61	1.00
		SDRL	9.77	0.32	5.19	0.09	3.86	0.02	2.79	0.00	1.62	0.00
	SL	***ARL***	8.72	1.12	5.38	1.01	3.82	1.00	2.58	1.00	1.69	1.00
		SDRL	11.01	0.38	7.40	0.12	4.93	0.04	3.15	0.01	1.68	0.00
C1	**SC**	***ARL***	121.24	21.19	61.43	2.81	43.57	1.38	27.96	1.13	16.04	1.03
		SDRL	541.81	317.18	369.53	46.72	197.72	1.59	110.00	0.54	62.73	0.21
	SL	***ARL***	101.05	13.09	71.54	3.88	53.65	1.66	28.68	1.14	17.18	1.03
		SDRL	370.64	225.37	347.72	28.89	235.46	22.57	121.34	0.59	77.65	0.22
C2	**SC**	***ARL***	371.56	246.12	305.37	137.23	257.64	21.62	145.77	6.97	105.54	3.54
		SDRL	1008.69	934.20	1021.67	962.15	662.90	65.37	311.41	11.49	269.41	5.15
	SL	***ARL***	388.05	242.54	374.93	182.64	325.04	75.86	178.23	8.41	121.87	3.79
		SDRL	943.40	854.49	994.99	976.19	786.37	409.76	444.82	15.83	318.78	6.16

for SL chart at $ARL_0 = 500$. The values of the plotting statistics for SC and SL charts, along with test samples, are reported in Table 5.6 and their corresponding control charts are given in Figure 5.4.

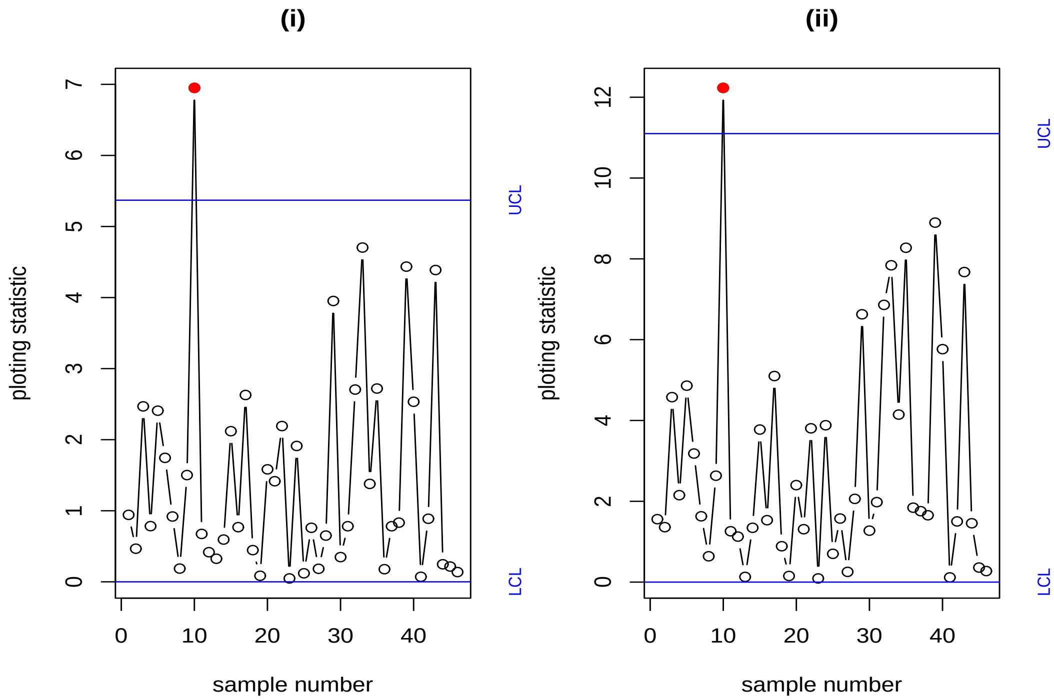

Figure 5.4: Control chart displays i) SC chart, ii) SL chart

It is evident that both SC and SL charts indicate an OOC signal at 10^{th}point. For the follow up diagnosis of shift by SL chart, we have computed the p-values for Wilcoxon test (p_1= 0.001684) and Ansari Bradley test (p_2= 0.1267), indicating a shift in location parameter. Similarly for SC chart, we got the p-values for Wilcoxon test (p_3= 0.001684) and Mood test (p_4= 0.04445), referring to locational shift. The results of this example are also in line with [66] which concluded that there is no scale shift in the process.

Table 5.6: Contract strikes, test samples and corresponding SC and SL Statistics

Serial no.				**Test samples** (n)							**SC**	**SL**
1	5	18	44	44	59	60	7	14	31	32	0.942	1.557
2	77	1	2	7	10	18	23	25	36	42	0.466	1.361
3	46	47	50	77	9	37	41	49	52	119	2.47	4.58
4	2	13	25	31	31	35	44	45	53	111	0.784	2.149
5	3	4	5	6	7	9	14	23	26	37	2.407	4.86
6	46	47	77	2	11	16	147	2	2	4	1.745	3.181
7	6	16	18	31	42	6	7	32	44	70	0.92	1.628
8	32	71	7	27	14	26	4	4	43	60	0.186	0.639
9	62	64	68	82	3	13	30	154	3	17	1.502	2.632
10	19	28	72	99	104	114	152	153	216	15	6.95	12.23
11	21	52	109	3	5	9	26	52	61	148	0.674	1.258
12	168	2	11	19	26	30	36	47	50	87	0.416	1.124
13	3	5	7	17	23	30	104	108	192	18	0.326	0.129
14	40	47	57	1	5	10	15	19	28	42	0.594	1.346
15	64	148	4	6	12	12	28	105	112	163	2.12	3.774
16	11	12	29	50	235	10	19	41	52	100	0.769	1.534
17	3	4	10	12	34	88	101	102	104	124	2.629	5.101
18	15	61	98	22	24	38	64	84	5	6	0.445	0.886
19	70	70	1	11	18	19	50	90	9	15	0.084	0.149
20	20	24	84	117	1	23	25	59	63	179	1.582	2.399
21	92	153	17	226	13	23	2	38	3	3	1.417	1.311
22	6	139	2	25	85	13	125	4	54	91	2.193	3.807
23	38	2	6	61	18	64	122	11	16	31	0.046	0.089
24	39	41	2	4	5	7	9	13	38	3	1.911	3.883
25	10	4	5	22	27	28	36	39	85	191	0.119	0.7
26	5	44	56	6	21	33	109	125	127	8	0.762	1.574
27	9	13	14	15	28	50	60	135	5	7	0.184	0.253
28	16	21	37	41	2	2	20	24	57	8	0.651	2.062
29	16	24	59	115	123	141	146	146	3	15	3.952	6.63
30	15	18	20	26	34	84	122	174	4	14	0.347	1.274
31	15	17	22	24	39	53	107	5	9	10	0.782	1.978
32	16	22	24	31	31	34	38	42	65	74	2.706	6.862
33	101	130	1	2	2	3	4	8	11	22	4.704	7.843
34	23	27	32	33	35	43	43	44	100	2	1.378	4.147
35	19	20	20	20	23	24	33	33	63	67	2.719	8.275
36	94	116	1	8	15	15	22	23	26	27	0.176	1.844
37	55	160	5	8	13	20	42	53	59	83	0.782	1.755
38	101	8	11	15	22	58	60	108	31	42	0.832	1.656
39	45	50	61	106	142	36	52	99	38	47	4.435	8.895
40	62	38	51	98	133	9	86	141	9	5	2.534	5.763
41	49	8	13	2	6	37	28	36	48	136	0.07	0.118
42	139	2	14	15	33	143	42	8	122	56	0.886	1.503
43	14	14	106	127	131	140	141	163	22	23	4.387	7.672
44	29	99	118	2	12	12	21	21	27	38	0.247	1.458
45	42	117	2	12	19	22	75	126	8	36	0.216	0.36
46	107	5	5	29	151	9	16	29	35	65	0.137	0.274

5.7 Concluding remarks

Control charts are often designed and used to monitor single process parameter such as location and dispersion but an attractive approach is to monitor both parameters together. In this study, we investigate the two nonparametric SC and SL charts for the joint monitoring of location and scale parameters. The performance analysis has revealed that SC takes an edge over SL under light tailed distributions while SL is a good alternative under heavy tailed distributions. Moreover, a reasonably larger reference and test samples produce better ARL performance of these charts.

CHAPTER 6

SUMMARY, CONCLUSIONS AND FUTURE RECOMMENDATIONS

In this chapter, the findings of this thesis along future recomendations are reported. Which are given in the following sections:

6.1 Summary and conclusions

In many manufacturing/production processes, control charts are used to monitor the quality characteristic of the process whereas, in some processes, quality characteristic has relationship (linear or non-linear) with other explanatory variable(s). For example, the dissolve amount of aspartame (an artificial sweetener) is reliant on the temperature, in semiconductor manufacturing application; pressure in the chamber depends on the flow of gases in the chamber and in electrical process; capacitance of the capacitor has inverse relation with the voltage at fixed charge. In literature, the term simple linear profiles is referred for the methods which are used to monitor such quality characteristic that has linear association with another ancillary variable. Usually, in simple linear profiles, three parameters are considered to study the state of any process such as slope, intercept and error variance.

Generally, control charts are worked into two main stages named as retrospective stage (Phase I) and prospective stage (Phase II). The objective of the retrospective analysis is to find the optimal choice of process parameters and control limits for the monitoring phase (Phase II). In this dissertation, we have designed and investigated Phase I and Phase II simple linear profile methods under the different ranked set sampling strategies such as ranked set sampling (RSS), median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), double ranked set sampling (DRSS), double median ranked set sampling (DMRSS) and double extreme ranked set sampling (DERSS). The results indicated that the proposed methods under RSS and its modified forms have superior detection ability as compared to the existing schemes. Particularly, DMRSS and DERSS offers superior performance as compared to the other schemes of interest. In addition to RSS samplings, we have also used modified version of successive sampling scheme (MSS) to enhance the performance of simple linear profiles method named as $Shewhart-3$ chart. The run length properties are used as performance measure which indicates that all the proposed charts under MSS outperforms the classical chart which is based on SRS.

In literature, simple liner profiles was evaluated under simultaneous structure for the monitoring of linear profile parmeters such as intercept, slope and error variance. In simulteneous structure each parameter have individual chart which consist of respective pair of limits. For example, in simultaneous monitoring of linear profile parameters three individual charts based on each parameter are designed in a mechanism to obtain overall performance of the process. We have designed joint structures (which depends on single charting structure) for the monitoring of linear profile parameters. The results reveals that the joint (Max-EWMA-3 and SS-EWMA-3) and simultaneous (EWMA-3) charts have almost similar performance but they have better performance as compared to EWMA/R, T^2 and Shewhart-3 charts. Specifically, Max-EWMA-3-C and SS-EWMA-3-C charts have relatively good performance among all others.

Slope is an important parameter which provides the estimate of average rate of change between study and explanatory variable. If the slope of the simple linear model is zero (i.e. $\beta_1 = 0$) then the effect of explanatory variable is eliminated from the process and the joint monitoring of mean and variability becomes a special case of simple linear profiles. In this dissertation, we proposed a parametric control chart named Max progressive (Max-p) chart for joint monitoring of shifts in process parameter(s). The results reveal that the newly proposed chart has better performance to detect shifts in the process parameter(s) as compared to popular proposals such as Max-EWMA, SS-EWMA, Max-DEWMA and SS-DEWMA.

Further, we also investigate the two nonparametric SC and SL charts for the joint monitoring of location and scale parameters. The performance analysis has revealed that SC takes an edge over SL under light tailed distributions while SL is a good alternative under heavy tailed distributions. Moreover, a reasonably larger reference and test samples produce better ARL performance of these charts.

6.2 Limitations of the study

The limitations about our study are given in the following points:

(i) This dissertation is designed to monitor the parameters of simple linear model named slope, intercept and error variance.

(ii) Generally, in simple linear profiles two well-known models are used such as fixed effect model and random effect model. This study comprises fixed effect model in simple linear profiles.

(iii) This study is focusing on normal distributional setups for the quality characteristics of interest.

6.3 Future recommendations

The future recommendations on our proposed methods are given in the following points:

(i) The current study has investigated the impact of variant sampling schemes in simple linear profile analysis. However, the scope of this study may be extended to multiple linear profiles and non-linear profiles.

(ii) The complete dissertation consists of methods that are used to enhanced the monitoring of linear profile parameters under fixed effect model. One may use these methods to enhanced the monitoring of linear profile parameters under random effect model.

(iii) Mostly, present study has covered the normal behavior in simple linear profiles. However, the scope of this study may be extended to cover non-normal behaviors in simple linear profiles.

(iv) The Shewhart structures for simple linear profiles may be extended in other directions such as implementation of run rules and addition of fast initial response (FIR) feature.

(v) Some interesting future research directions might include studying the performance of these charts under multiple structural breaks and when a shift occurs at steady-state.

APPENDIX A

A.1 Transformed linear model

The simple linear profile model under ranked set strategies is defined as:

$$Y_{ij} = \beta_0 + \beta_1 X_i + \varepsilon_{ij} \ ; \ i = 1, 2, 3, \ldots\ldots.., n \ ; \quad j = 1, 2, 3, \ldots\ldots.., m$$

writing the shifted β_1 such as $\beta_1 = \beta_1 + \beta\sigma$ in the linear regression model given in above model, we obtain

$$Y_{ij} = \beta_0 + (\beta_1 + \beta\sigma)X_i + \varepsilon_{ij}$$

where β is the shift for slope and by adding or subtracting with $(\beta_1 + \beta\sigma)\overline{X}$, we get

$$Y_{ij} = \beta_0 + (\beta_1 + \beta\sigma)X_i + \varepsilon_{ij} + (\beta_1 + \beta\sigma)\overline{X} - (\beta_1 + \beta\sigma)\overline{X}$$

$$Y_{ij} = [\beta_0 + (\beta_1 + \beta\sigma)\overline{X}] + [(\beta_1 + \beta\sigma)X_i - (\beta_1 + \beta\sigma)\overline{X}] + \varepsilon_{ij}$$

$$Y_{ij} = [(\beta_0 + \beta_1\overline{X}) + (\beta\sigma)\overline{X}] + [(\beta_1 + \beta\sigma)(X_i - \overline{X})] + \varepsilon_{ij}$$

Since, average of ${X_{(i)}}^*$ is zero so the covariance will be also zero and assumed $A_0 = \beta_0 + \beta_1 \overline{X}$, $A_1 = \beta_1$ and ${X_i}^* = X_i - \overline{X}$ then the above equation is written as

$$Y_{ij} = \left(A_0 + (\beta\sigma)\,\overline{X}\right) + (A_1 + \beta\sigma){X_i}^* + \varepsilon_{ij}$$

$$Y_{ij} = (B_0) + (B_1){X_i}^* + \varepsilon_{ij}$$

A.2 Properties of error term in different ranked set samplings

In the simple regression, we assumed that the error term is normally distributed having the mean zero and constant variance (i.e. $\varepsilon \sim \ N\ (0, \sigma^2)$). So the standardized form of error is defined as

$$w = \frac{\varepsilon - 0}{\sigma} = \frac{\varepsilon}{\sigma}$$

In this study, we are focusing on different strategies (τ) named RSS, MRSS, ERSS, DRSS, DMRSS and DERSS so the probability density function $f(.)$, mean $E(.)$ and variance $Var(.)$ for error term under RSS $(\varepsilon_{(r)})$ and DRSS $\left(\varepsilon^*_{(r)}\right)$ are defined as:

$$f\left(\varepsilon_{(r)}; \sigma^2\right) = n! \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!}\left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{r-1}\left\{1 - F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-r} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}\right]$$

$$E\left(\varepsilon_{(r)}\right) = \sigma \int_{-\infty}^{+\infty} w\ n!\ \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!} \{F\,(w)\}^{r-1} \{1 - F\,(w)\}^{n-r} f\,(w)\,dw \ \right]$$

$$E\left(\varepsilon_{(r)}\right) = \sigma\ D_{r1}$$

$$Var\left(\varepsilon_{(r)}\right) = \sigma^2 \int_{-\infty}^{+\infty} w^2\ n!\ \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!} \{F\,(w)\}^{r-1} \{1 - F\,(w)\}^{n-r} f\,(w)\,dw \ \right]$$

$$\text{-}\left(E\left(\varepsilon_{(r)}\right)\right)^2$$

$$\sigma^2_{e[r]} = Var\left(\varepsilon_{(r)}\right) = \sigma^2\ D_{r2} - (\sigma\ D_{r1})^2 = \sigma^2\ (\ D_{r2} - D_{r1}^2)$$

$$f\left(\varepsilon^*_{(r)};\sigma^2\right) = n!\ \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!} \left\{F\left(\frac{\varepsilon_{(r)}}{\sigma}\right)\right\}^{r-1} \left\{1 - F\left(\frac{\varepsilon_{(r)}}{\sigma}\right)\right\}^{n-r} f\left(\frac{\varepsilon_{(r)}}{\sigma}\right) \frac{1}{\sigma}\ \right]$$

$$E\left(\varepsilon^*_{(r)}\right) = \sigma \int_{-\infty}^{+\infty} w_{(r)}\ n!\ \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!} \left\{F\left(w_{(r)}\right)\right\}^{r-1} \left\{1 - F\left(w_{(r)}\right)\right\}^{n-r} f\left(w_{(r)}\right) dw_{(r)}\ \right]$$

$$E\left(\varepsilon_{(r)}\right) = \sigma D^*_{r1}$$

$$Var\left(\varepsilon^*_{(r)}\right) = \sigma^2 \int_{-\infty}^{+\infty} w^2_{(r)}\ n!\ \prod_{r=1}^{n} \left[\frac{n!}{(r-1)!\,(n-r)!} \left\{F\left(w_{(r)}\right)\right\}^{r-1} \left\{1 - F\left(w_{(r)}\right)\right\}^{n-r}\right.$$

$$\mathrm{f}\left(w_{(r)}\right) dw_{(r)}\ - \left(E\left(\varepsilon^*_{(r)}\right)\right)^2$$

$$\sigma^2_{e[dr]} = Var\left(\varepsilon^*_{(r)}\right) = \sigma^2\ D^*_{r2} - (\sigma D^*_{r1})^2 = \sigma^2\ (D^*_{r2} - D^{*2}_{r1})$$

For odd set $(n = 2m - 1)$, the probability density function $f(.)$, mean $E(.)$ and variance $Var(.)$ for error term under MRSS $(\varepsilon_{(m)})$ and DMRSS $(\varepsilon^*_{(m)})$ are defined

$$f\left(\varepsilon_{(m)};\sigma^2\right) = \frac{(2m-1)!}{(m-1)!\,(m-1)!} \left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{m-1} \left\{1 - F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{m-1} f\left(\frac{\varepsilon}{\sigma}\right) \frac{1}{\sigma}$$

$$E\left(\varepsilon_{(m)}\right)=\sigma\int_{-\infty}^{+\infty} w \ \frac{(2m-1)!}{(m-1)!\,(m-1)!}\{F(w)\}^{m-1}\{1-F(w)\}^{m-1} f(w)\, dw$$

$$E\left(\varepsilon_{(m)}\right)=\sigma \ D_{m1}$$

$$Var\left(\varepsilon_{(m)}\right)=\sigma^2 \ \int_{-\infty}^{+\infty} w^2 \ \frac{(2m-1)!}{(m-1)!\,(m-1)!}\{F(w)\}^{m-1}\{1-F(w)\}^{m-1} f(w)\, dw-\left(E\left(\varepsilon_{(m)}\right)\right)^2$$

$$\sigma_{e[m]}^2=Var\left(\varepsilon_{(m)}\right)=\sigma^2 \ \ D_{m2}-(\sigma \ D_{m1})^2=\sigma^2 \ (\ D_{m2}-D_{m1}^2)$$

$$f\left(\varepsilon_{(m)}^*;\sigma^2\right)=\frac{(2m-1)!}{(m-1)!\,(m-1)!}\left\{F\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\right\}^{m-1}\left\{1-F\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\right\}^{m-1} f\left(\frac{\varepsilon_{(m)}}{\sigma}\right)\frac{1}{\sigma}$$

$$E\left(\varepsilon_{(m)}^*\right)=\sigma\int_{-\infty}^{+\infty} w_{(m)} \ \frac{(2m-1)!}{(m-1)!\,(m-1)!}\left\{F\left(w_{(m)}\right)\right\}^{m-1}\left\{1-F\left(w_{(m)}\right)\right\}^{m-1} f\left(w_{(m)}\right) dw_{(m)}$$

$$E\left(\varepsilon_{(m)}^*\right)=\sigma D_{m1}^*$$

$$Var\left(\varepsilon_{(m)}^*\right)=\sigma^2 \ \int_{-\infty}^{+\infty} w_{(m)}^2 \ \frac{(2m-1)!}{(m-1)!\,(m-1)!}\left\{F\left(w_{(m)}\right)\right\}^{m-1}\left\{1-F\left(w_{(m)}\right)\right\}^{m-1}$$

$$\mathrm{f}\left(w_{(m)}\right) dw_{(m)}-\left(E\left(\varepsilon_{(m)}\right)\right)^2$$

$$\sigma_{e[dm]}^2=Var\left(\varepsilon_{(m)}^*\right)=\sigma^2 \ D_{m2}^*-(\sigma D_{m1}^*)^2=\sigma^2 \ (D_{m2}^*-D_{m1}^{*2})$$

Let ε_1 is the error of smallest sample and ε_n is the error of largest sample then for odd sets, the probability density function $f(.)$, mean $E(.)$ and variance $Var(.)$ for error term under ERSS $(\varepsilon_{(1)},\varepsilon_{(n)})$ and DERSS $(\varepsilon_{(1)}^*,\varepsilon_{(n)}^*)$ are defined

$$f\left(\varepsilon_{(1)};\sigma^2\right)=n\left\{1-F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}$$

$$f\left(\varepsilon_{(n)};\sigma^2\right)=n\left\{F\left(\frac{\varepsilon}{\sigma}\right)\right\}^{n-1} f\left(\frac{\varepsilon}{\sigma}\right)\frac{1}{\sigma}$$

$$E\left(\varepsilon_{(1)}\right)=\sigma\int_{-\infty}^{+\infty} w\ n\ \{1-F\left(w\right)\}^{n-1}f\left(w\right)dw$$

$$E\left(\varepsilon_{(1)}\right)=\sigma\ D_{11}$$

$$E\left(\varepsilon_{(n)}\right)=\sigma\int_{-\infty}^{+\infty} w\ n\ \{F\left(w\right)\}^{n-1}f\left(w\right)dw$$

$$E\left(\varepsilon_{(n)}\right)=\sigma\ D_{n1}$$

$$Var\left(\varepsilon_{(1)}\right)=\sigma^2\ \int_{-\infty}^{+\infty} w^2\ n\ \{1-F\left(w\right)\}^{n-1}f\left(w\right)dw-\left(E\left(\varepsilon_{(1)}\right)\right)^2$$

$$\sigma^2_{e[e1]}=Var\left(\varepsilon_{(1)}\right)=\sigma^2\ \ D_{12}-\left(\sigma\ D_{11}\right)^2=\sigma^2\ (\ D_{12}-D_{11}^2)$$

$$Var\left(\varepsilon_{(n)}\right)=\sigma^2\ \int_{-\infty}^{+\infty} w^2\ n\ \{F\left(w\right)\}^{n-1}f\left(w\right)dw-\left(E\left(\varepsilon_{(n)}\right)\right)^2$$

$$\sigma^2_{e[en]}=Var\left(\varepsilon_{(n)}\right)=\sigma^2\ D_{n2}-\left(\sigma\ D_{n1}\right)^2=\sigma^2\ (\ D_{n2}-D_{n1}^2)$$

$$f\left(\varepsilon^*_{(1)};\sigma^2\right)=n\Big\{1-F\left(\frac{\varepsilon_{(1)}}{\sigma}\right)\Big\}^{n-1}f\left(\frac{\varepsilon_{(1)}}{\sigma}\right)\frac{1}{\sigma}$$

$$f\left(\varepsilon^*_{(n)};\sigma^2\right)=n\Big\{F\left(\frac{\varepsilon_{(n)}}{\sigma}\right)\Big\}^{n-1}f\left(\frac{\varepsilon_{(n)}}{\sigma}\right)\frac{1}{\sigma}$$

$$E\left(\varepsilon^*_{(1)}\right)=\sigma\int_{-\infty}^{+\infty} w_{(1)}\ n\ \left\{1-F\left(w_{(1)}\right)\right\}^{n-1}f\left(w_{(1)}\right)dw_{(1)}$$

$$E\left(\varepsilon_{(1)}\right)=\sigma D^*_{11}$$

$$E\left(\varepsilon^*_{(n)}\right)=\sigma\int_{-\infty}^{+\infty} w_{(n)}\ n\ \left\{F\left(w_{(n)}\right)\right\}^{n-1}f\left(w_{(n)}\right)dw_{(n)}$$

$$E\left(\varepsilon^*_{(n)}\right)=\sigma D^*_{n1}$$

$$Var\left(\varepsilon^*_{(1)}\right)=\sigma^2\int_{-\infty}^{+\infty} w^2_{(1)}\ n\ \left\{1-F\left(w_{(1)}\right)\right\}^{n-1}f\left(w_{(1)}\right)dw-\left(E\left(\varepsilon^*_{(1)}\right)\right)^2$$

$$\sigma^2_{e[de1]}=Var\left(\varepsilon^*_{(1)}\right)=\sigma^2 D^*_{12}-\left(\sigma D^*_{11}\right)^2=\sigma^2\ (D^*_{12}-D^{*2}_{11})$$

$$Var\left(\varepsilon^*_{(n)}\right) = \sigma^2 \int_{-\infty}^{+\infty} w^2_{(n)}\ n \left\{F\left(w_{(n)}\right)\right\}^{n-1} f\left(w_{(n)}\right) dw - \left(E\left(\varepsilon^*_{(n)}\right)\right)^2$$

$$\sigma^2_{e[den]} = Var\left(\varepsilon^*_{(n)}\right) = \sigma^2 D^*_{n2} - \left(\sigma D^*_{n1}\right)^2 = \sigma^2\ \left(D^*_{n2} - D^{*2}_{n2}\right)$$

A.3 Independence of sample mean $\left(\bar{Y}\right)$ and sample variance $\left(S^2\right)$

Let Y $= (Y_1, Y_2, \ldots.., Y_n)$ be the normal random variable with parameters μ and σ^2. The sample mean and variance of Y are defined as

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n};\ S^2 = \frac{\sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2}{n-1}$$

To prove the independence of $\bar{Y}$ and S^2, following theorems are used.

Theorem 1: Complete sufficient statistic through exponential family distribution.

Let $\{f_\theta :\ \theta \in \Theta\}$ be a k-parameter exponential family given by

$$f_\theta\left(y\right) = \exp\left[\sum_{i=1}^{k} Q_i \theta T_i\left(y\right) + D\left(\theta\right) + S\left(y\right)\right]$$

where $\theta = (\theta_1, \theta_2, \ldots.., \theta_k) \in \Theta$, an interval in $\mathcal{R}_k$, $T_1, T_2,\ \ldots.,\ T_k$ and S are defined on $\mathcal{R}_n$, $T = (T_1, T_2,\ \ldots.,\ T_k)$ and y $= (y_1, y_2, \ldots.., y_n)$, $k \leq n$. Let $Q = (Q_1, Q_2,\ \ldots.,\ Q_k)$, and suppose the range of Q contains an open set in $\mathcal{R}_k$, then $T = \left(T_1\left(Y\right), T_2\left(Y\right),\ \ldots.,\ T_k\left(Y\right)\right)$ is a complete sufficient statistic.

Suppose Y $\sim$ N (μ, σ^2) and the both parameters are unknown. We know that the family of distribution of X $= (Y_1, Y_2, \ldots.., Y_n)$ is a two-parameter exponential family with $T(Y_1, Y_2, \ldots.., Y_n) = (\sum_{i=1}^{n} Y_i,\ \sum_{i=1}^{n} Y_i^2)$. Then by the aforementioned theorem $\sum_{i=1}^{n} Y_i$ is a complete sufficient statistic of parameter μ. We also know that any 1-1 transformation of complete sufficient statistic is also complete sufficient statistic (i.e. $\bar{Y}$ is a complete sufficient statistic).

Theorem 2: Ancillary statistic.

A statistic $B(Y)$ is said to be ancillary statistics if its distribution does not depend on the under-laying model parameter θ. For example, let $Y_1, Y_2, \ldots.., Y_n$ be a random sample from $N(\mu, 1)$. Then the statistic $B(Y) = (n-1)S^2 = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$ is an ancillary statistic because $(n-1)S^2 \sim \chi^2_{n-1}$ and free from parental parameter μ.

Theorem 3: Basus theorem.

Statement: if $A(Y)$ is a complete sufficient statistic for θ, then any ancillary statistics $B(Y)$ is independent of A. Proof: as mentioned in Rohatgi that if B is an ancillary statistic, then $P_\theta = \{B(Y) \le b\}$ is free of θ for all b. Let the conditional probability $g_b(A) = \{B(Y) \le b | A(Y) = a\}$, then

$$E_\theta\{g_b(A(Y))\} = P_\theta\{B(Y) \le b\}$$

Thus

$$E_\theta(g_b(A) - P\{B(Y) \le b\}) = 0$$

For all θ . By completeness of A it follows that

$$P_\theta\left(g_b\left(A\right) - P\left\{B \leq b\right\} = 0\right) = 1$$

That is,

$$P_\theta\left\{B\left(Y\right) \leq b | A\left(Y\right) = a\right\} = P\left\{B\left(Y\right) \leq b\right\}$$

with probability equals to 1. Hence A and B are independent.

As discussed in theorem 1 that $\bar{Y}$ is a complete sufficient statistic and S^2 is an ancillary statistic (cf. theorem 2). So, by theorem 3 (Basu's theorem) it is proved that $\bar{Y}$ and S^2 are independent.

A.4 Statistical framework for Lepage

The Lepage statistic depends on two well-known test statistics namely Wilcoxon Rank-sum (T_1) and Ansari Bradley (T_2). Which are defined as

$$T_1 = \sum_{k=1}^{N} k I_K$$

$$T_2 = \sum_{k=1}^{N} \left| k - \frac{1}{2}\left(N+1\right) \right| I_K$$

where I_k is an indicator variable used to assign zero when sample belongs to reference set (U) otherwise the value of I_k is 1. The derivation of mean and variance of Wilcoxon Rank-sum statistic are

$$E\left(T_1\right)=\sum_{k=1}^{N} kE\left(I_K\right)$$

$$E\left(T_1\right)=\sum_{k=1}^{N} k\frac{n}{N}$$

$$E\left(T_1\right)=\frac{N\left(N+1\right)}{2}\frac{n}{N}$$

$$E\left(T_1\right)=\frac{n\left(N+1\right)}{2}$$

$$Var\left(T_1\right)=\sum_{k=1}^{N} k^2 Var\left(I_K\right)$$

$$Var\left(T_1\right)=\sum_{k=1}^{N} k^2\frac{mn}{N^2}$$

$$Var\left(T_1\right)=\frac{N\left(N+1\right)\left(2N+1\right)}{6}\frac{mn}{N^2}$$

$$Var\left(T_1\right)=\frac{mn\left(N+1\right)\left(2N+1\right)}{6N}$$

The derivation of mean and variance for Ansari Bradley statistic when total sample size is even are

$$E\left(T_2\right)=2m\sum_{K=1}^{\frac{N}{2}}\frac{K}{N}$$

$$E\left(T_2\right)=2m\frac{\frac{N}{2}\left(\frac{N}{2}+1\right)}{2}\frac{1}{N}$$

$$E\left(T_2\right)=m\frac{N}{2}\left(\frac{N}{2}+1\right)\frac{1}{N}$$

$$E\left(T_2\right)=\frac{m}{2}\left(\frac{N}{2}+1\right)$$

$$E\left(T_2\right)=\frac{m}{2}\left(\frac{N+2}{2}\right)$$

$$E\left(T_2\right)=\frac{m\left(N+2\right)}{4}$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[N\sum_{k=1}^{N}\left(k-\frac{1}{2}\left(N+1\right)\right)^2-\sum_{k=1}^{N}\left(\left|k-\frac{1}{2}\left(N+1\right)\right|\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[\frac{N^2\left(N^2-1\right)}{12}-\left(\frac{N}{m}E\left(T_2\right)\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[\frac{N^2\left(N^2-1\right)}{12}-\left(\frac{N^2}{4}\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn\left(N^2-4\right)}{48\left(N-1\right)}$$

and in case of odd total sample size (N), the mean and variance are

$$E\left(T_2\right)=\frac{m\left(2\sum_{k=1}^{\frac{N-1}{2}}k+\frac{N+1}{2}\right)}{N}$$

$$E\left(T_2\right)=m\left(2\left\{\frac{\frac{(N-1)}{2}\left[\frac{(N-1)}{2}+1\right]}{2}\right\}+\frac{N+1}{2}\right)\frac{1}{N}$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N-1}{2}\left(\frac{N+1}{2}\right)+\frac{N+1}{2}\right]$$

$$E\left(T_2\right)=\frac{m}{N}\left(\frac{\left(N-1\right)\left(N+1\right)}{4}+\frac{N+1}{2}\right)$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N^2+N-N-1}{4}+\frac{N+1}{2}\right]$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N^2-1}{4}+\frac{N+1}{2}\right]$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N^2-1+2N+2}{4}\right]$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N^2+2N+1}{4}\right]$$

$$E\left(T_2\right)=\frac{m}{N}\left[\frac{N+1}{2}\right]^2$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[N\sum_{k=1}^{N}\left(k-\frac{1}{2}\left(N+1\right)\right)^2-\sum_{k=1}^{N}\left(\left|k-\frac{1}{2}\left(N+1\right)\right|\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[\frac{N^2\left(N^2-1\right)}{12}-\left(\frac{N}{m}E\left(T_2\right)\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn}{N^2\left(N-1\right)}\left[\frac{N^2\left(N^2-1\right)}{12}-\left(\frac{N^2-1}{4}\right)^2\right]$$

$$Var\left(T_2\right)=\frac{mn\left(N+1\right)\left(N^2+3\right)}{48N^2}$$

A.5 Statistical framework for Cucconi

The Cucconi statistic depends on two statistics namely W and Z. For the properties of W and Z statistics, mean and variance of S_1 and S_2 statistics are required which are derived as follow:

$$E\left(S_1\right)=E\left(\sum_{k=1}^{N}k^2I_k\right)$$

where I_k is an indicator variable used to assign zero when sample belongs to reference set (U) otherwise the value of I_k is 1.

$$E\left(S_1\right)=\sum_{k=1}^{N}k^2E\left(I_k\right)$$

as mentioned by [134] that the sum of square of first N natural number is

$$\sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}$$

and the expected value, variance and correlation of indicator variable are

$$E(I_k) = \frac{n}{N}, \quad Var(I_k) = \frac{nm}{N^2}, \quad Cov(I_j, I_k) = \frac{-nm}{N^2(N-1)}$$

$$E(S_1) = \frac{n}{N}\left(\frac{N(N+1)(2N+1)}{6}\right)$$

$$E(S_1) = \frac{n(N+1)(2N+1)}{6}$$

by using similar estimation one may easily derived the mean S_2 whhich is

$$E(S_2) = E\left(\sum_{k=1}^{N} (N+1-k)^2 I_k\right) = \frac{n(N+1)(2N+1)}{6}$$

The derivations of variances for S_1 and S_2 are as follows:

$$Var(S_1) = Var\left(\sum_{k=1}^{N} k^2 I_k\right)$$

$$Var(S_1) = \sum_{k=1}^{N} k^4 Var(I_k) + \sum_{i=1}^{N}\sum_{j=1}^{N} k_i^2 k_j^2 Cov(I_i, I_j)$$

$$Var(S_1) = \sum_{k=1}^{N} k^4 \frac{nm}{N^2} + \sum_{i=1}^{N}\sum_{j=1}^{N} k_i^2 k_j^2 \left(-\frac{nm}{N^2(N-1)}\right)$$

$$Var(S_1) = \frac{(N-1)\sum_{k=1}^{N} k^4 nm - nm\sum_{i=1}^{N}\sum_{j=1}^{N} k_i^2 k_j^2}{N^2(N-1)}$$

$$Var\left(S_1\right) = \frac{nm}{N^2\left(N-1\right)}\left[N\sum_{k=1}^{N} R^4 - \sum_{k=1}^{N} R^4 - \sum_{i=1}^{N}\sum_{j=1}^{N} k_i^2 k_j^2\right]$$

$$Var\left(S_1\right) = \frac{nm}{N^2\left(N-1\right)}\left[N\sum_{k=1}^{N} R^4 - \sum_{k=1}^{N} R^4 - \sum_{i=1}^{N}\sum_{j=1}^{N} k_i^2 k_j^2\right]$$

$$Var\left(S_1\right) = \frac{nm}{N^2\left(N-1\right)}\left[N\sum_{k=1}^{N} R^4 - \left(\sum_{i=j=1}^{N} R_j^2\right)^2\right]$$

$$Var\left(S_1\right) = \frac{nm}{N^2\left(N-1\right)}\left[\frac{NN\left(N+1\right)\left(2N+1\right)\left(3N^2+3N-1\right)}{30} - \frac{N^2\left(N+1\right)^2\left(2N+1\right)^2}{36}\right]$$

$$Var\left(S_1\right) = \frac{N^2nm\left(N+1\right)\left(2N+1\right)}{N^2\left(N-1\right)}\left[\frac{3N^2+3N-1}{30} - \frac{\left(N+1\right)\left(2N+1\right)}{36}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{6\left(N-1\right)}\left[\frac{3N^2+3N-1}{5} - \frac{2N^2+3N+1}{6}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{6\left(N-1\right)}\left[\frac{18N^2+18N-6-10N^2-15N-5}{30}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{6\left(N-1\right)}\left[\frac{8N^2+3N-11}{30}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{180}\left[\frac{8N^2+11N-8N-11}{\left(N-1\right)}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{180}\left[\frac{N\left(8N+11\right)-1\left(8N+11\right)}{\left(N-1\right)}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)}{180}\left[\frac{N\left(8N+11\right)-1\left(8N+11\right)}{\left(N-1\right)}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)\left(8N+11\right)}{180}\left[\frac{\left(N-1\right)}{\left(N-1\right)}\right]$$

$$Var\left(S_1\right) = \frac{nm\left(N+1\right)\left(2N+1\right)\left(8N+11\right)}{180}$$

by using similar estimation one may easily derived the variance of S_2 statistic

$$Var\left(S_2\right) = Var\left(\sum_{k=1}^{N}\left(N+1-k\right)^2 I_k\right) = \frac{nm\left(N+1\right)\left(2N+1\right)\left(8N+11\right)}{180}$$

Further, the means and variances of W and Z are derived below:

$$E\left(W\right) = \frac{6E\left(\sum_{k=1}^{N} k^2 I_k\right) - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}}$$

$$E\left(W\right) = \frac{6\left(\frac{n(N+1)(2N+1)}{6}\right) - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}}$$

$$E\left(W\right) = \frac{n\left(N+1\right)\left(2N+1\right) - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}}$$

$$E\left(W\right) = \frac{0}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}}$$

$$E\left(W\right) = 0$$

by using similar estimation one may easily derived the mean of Z statistic

$$E\left(Z\right) = \frac{6E\left(\sum_{k=1}^{N}\left(N+1-k\right)^2 I_k\right) - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}} = 0$$

The derivations of variances for W and Z are as follows:

$$Var\left(W\right) = \frac{6Var\left(\sum_{k=1}^{N} K^2 I_k\right) - n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\mathrm{nm(N+1)(2N+1)(8N+11)}}{5}}}$$

$$Var\left(W\right)=\frac{36\left[\frac{nm(N+1)(2N+1)(8N+11)}{180}\right]-0}{\frac{\text{nm(N+1)(2N+1)(8N+11)}}{5}}$$

$$Var\left(W\right)=\left[\frac{\frac{nm(N+1)(2N+1)(8N+11)}{5}}{\frac{\text{nm(N+1)(2N+1)(8N+11)}}{5}}\right]$$

$$Var\left(W\right)=1$$

by using similar estimation one may easily derived the variance of Z statistic

$$Var\left(Z\right)=\frac{6Var\left(\sum_{k=1}^{N}\left(N+1-k\right)^{2}I_{k}\right)-n\left(N+1\right)\left(2N+1\right)}{\sqrt{\frac{\text{nm(N+1)(2N+1)(8N+11)}}{5}}}=1$$

A.6 Description of real data set

In grid connected PV system, parallel plate capacitors are used as a DC link which consists of two conductive plates separated by a dielectric material (as discussed in Section 2.1). Usually, in a parallel plate capacitor, capacitance (C) is directly related to the surface area of the conductive plates and inversely associated to the potential difference between the plates (V). For the illustrative example, we get 75456 sample values of Voltage (V) against each level of Capacitance (C) given in [94]. There exist 7 different capacitance levels such as, $50\mu F$, $100\mu F$, $150\mu F$, $200\mu F$, $250\mu F$, $300\mu F$ and $250\mu F$. In the stated study, we consider Voltage (V) as a dependent variable and Capacitance (C) as an independent variable. For the IC regression model, we run 75456 sample values of

V against fixed values of C and get a following model

$$\hat{V} = \ 402.3512 - 0.01983691\ C$$

$$SE = (0.02465) \quad (0.0001102)$$

$$t = (16321.5) \qquad (-179.9) \qquad R^2 = 0.5301$$

Based on the above simple linear regression model, as the one μF capacitance increases indefinitely, we expect 402.3313 Voltage in the gird PV connected system. We used t-test approach to access that whether there is any significant relationship between C and V. Assuming the null hypothesis that $\beta = 0$, t-test value having p-value less than 0.05 reveals that there is a significant relationship between the variables used in the linear regression model. Further, $R^2 = 0.5301$ depicts that 53% variation in the Voltage (V) is explained by the capacitance (C).

The diagnosis analysis of simple linear regression is presented in Figure A.1. Normality of the residuals is an important assumption of simple linear regression. For the normality checking, we used both graphical and testing approaches. The normal QQ plot and Lilliefors test having statistic values 0.191 (p-value = 0.2628) shows that there is no issue with the normality of residuals. Further, the plot about fitted values and capacitance depicts that linearity assumption is also satisfied. The plots about fitted versus residuals and standardized residuals also depicts that the residuals have constant variances while the Breusch-Pagan test having statistic value 3.2149 (p-value = 0.07297) is also the evident that there

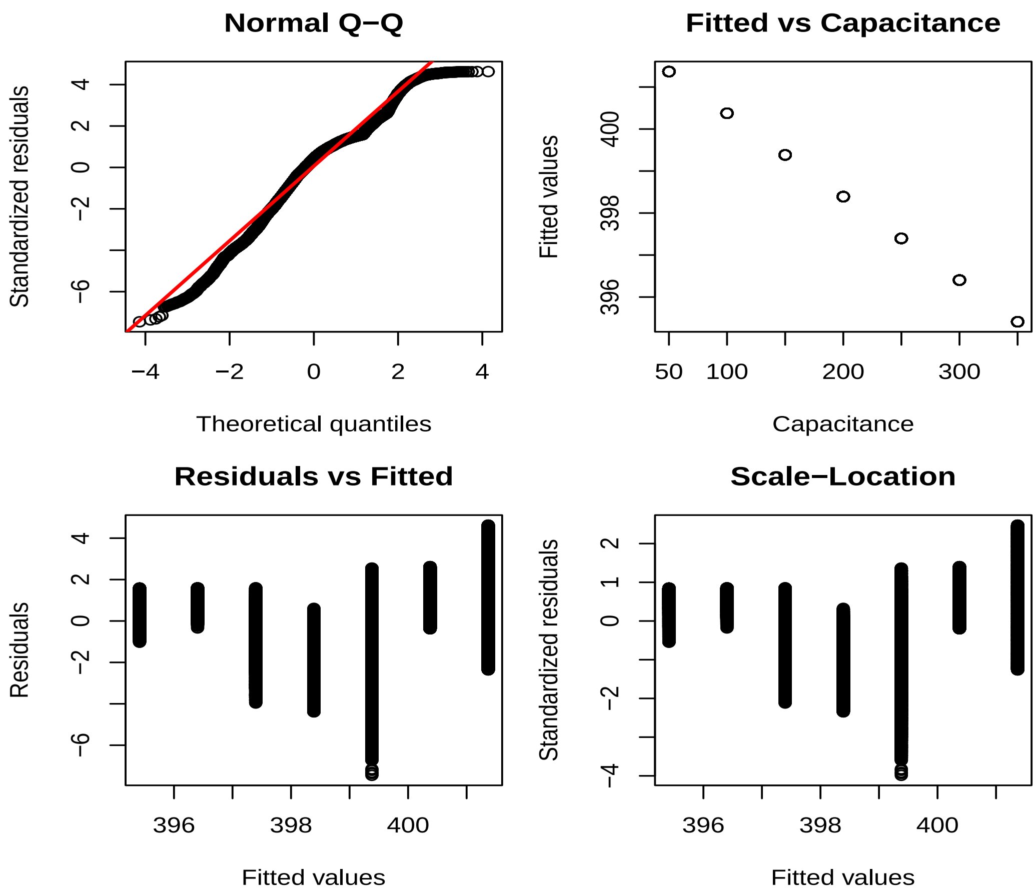

Figure A.1: Diagnosis analysis of simple linear model between Voltage and Capacitance

exists no issue with the homoscedasticity of the residuals.

A.7 Data perturbation

In literature, data perturbation approaches are classified into two categories such as value distortion approach and probability distribution approach [135, 136]. In distortion technique, data elements are perturbed by several methods that includes additive noise, multiplicative noise, or other randomization methods [137]. The probability distribution approach substitutes the data set with the sample from own distribution [138] or by another sample from same (or estimated) distribution (cf. [139]). In linear perturbation, the additive noise method is the simplest one perturbation method having model,

$$Y = X + \varepsilon$$

where ε is the random noise and independent from X, with zero mean and co-variance matrix $\Sigma_{\varepsilon\varepsilon}$. $\Sigma_{\varepsilon\varepsilon}$ has non-zero diagonal terms and the off-diagonal terms are equals to zero which is the evident that all ε's are independent for each other (for more details see [140–145]. Further, [146] enhanced the basic additive noise method by considering following model

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

This model is like the basic additive noise model but the covariance matrix of the noise term is defined by $\Sigma_{\varepsilon\varepsilon} = d\Sigma_{XX}$. Where d is the scaler quantity and Σ_{XX} is the covariance matrix of X. More modifications on the additive noise models are briefly describe in [147]. Further, there exist several perturbation techniques for non-linear models such as: multiplicative model proposed by [148], Sullivans Model addressed by [149], Copula model initiated by [150] and data shuffling discussed by [151]. Usually, additive and multiplicative perturbations are used for numeric data while some perturbation techniques are also used for categorical data set (for more detail see [152–155]

A.7.1 Implication of data perturbation in regression analysis

Regression is a well-known statistical tool used to estimate the association between explained (Y) and one or more explanatory variable (X). The classical normal linear regression model (CNLR) is defined as follow

$$Y = \beta'_{(1\times k)}\, X_{(k\times 1)} + \varepsilon$$

Where ε is the disturbance term and the normality and independence of CNLR model imply that

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left[\begin{pmatrix} \mu \\ \beta'\mu \end{pmatrix}\begin{pmatrix} \Sigma & \Sigma\beta \\ \beta'\Sigma & \beta'\Sigma\beta + \sigma^2 \end{pmatrix}\right]$$

where μ is the vector of means of X, Σ is the $(K \times K)$ covariance matrix of X and σ^2 is the variance of disturbance terms. Further, the masked data (M) may be defined as

$$M_{(K\times 1)} = X_{(K\times 1)} + N_{(K\times 1)}$$

where N is the vector normally distributed zero mean additive noise with covariance matrix Θ. Assume X and N are independent then it follows

$$\begin{pmatrix} M \\ Y \end{pmatrix} \sim N\left[\begin{pmatrix} \mu \\ \beta'\mu \end{pmatrix}\begin{pmatrix} \Sigma\Theta & \Sigma\beta \\ \beta'\Sigma & \beta'\Sigma\beta + \sigma^2 \end{pmatrix}\right]$$

It is noted that noise addition schemes may be characterized by the covariance Θ and it is defined as $\Theta = \Omega\Sigma$ (cf. [146, 156]. Ω is a proportionality constant and under the assumption about the independence of noise components, Ω is a matrix. In common practice, variances of the noise components are the proportional to the variance of corresponding attributes then Ω may defined as $\Theta = \text{diagonal}\ (\Omega\Sigma)$. When the subset of sensitive attributes is masked then the noise addition scheme may be characterized by a partitioned matrix. For the first P masked of the K attributed the variance covariance matrix is defined as follow

$$\Theta = \begin{pmatrix} \Theta_{11} & 0 \\ 0 & 0 \end{pmatrix}$$

where the Θ_{11} is $(P \times P)$ sub matrix and Tendicks method required $\Theta_{11} = \Omega\Sigma_{11}$. Moreover, independent noise components required $\Theta_{11} = \Omega\,\text{diagonal}\,(\Sigma_{11})$ and Σ_{11} is the $(P \times P)$ sub matrix of Σ. It is noted that when Y is regressed on true values of the attribute X then the expected value of the regression coefficients is β while when masked attribute are used then it may obtained as follow

$$E\,(Y|M) = \beta' \left[I - \Sigma(\Sigma - \Theta)^{-1}\right] \mu + \beta'\Sigma(\Sigma - \Theta)^{-1} M$$

Hence, the expected value of the coefficients for Y regressed on M is obtained by $b = (\Sigma - \Theta)^{-1}\Sigma\beta$. One may obtain more details on perturbation used in regression model in [157].

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