DYNAMIC SPECTRUM ALLOCATION FOR HETEROGENEOUS COGNITIVE RADIO NETWORKS WITH FEMTOCELLS

BY

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I would like to dedicate this work to my parents, my brothers and sisters for their support and love.

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LIST OF ACRONYMS

CR	Cognitive Radio
PU	Primary User
SU	Secondary User
CBS	Cognitive Base Station
FBS	Femtocell Base Station
FSU	Femtocell Secondary User
MSU	Macrocell Secondary User
MBS	Macrocell Base Station
NE	Nash Equilibrium
OFDMA	Orthogonal Frequency Division Multiple-Access
Wi-Fi	Wireless Fidelity
MAP	Multi-Auctioneer Progressive
QoS	Quality of Service
FFR	Fractional Frequency Reuse
VC	Virtual Cluster
SINR	Signal to Noise and Interference Ratio
SIR	Signal to Interference Ratio

FFI	Femto-Femto Interference
AWGN	Additive White Gaussian Noise
DSL	Digital Subscriber Line
FDD	Frequency Division Duplexing
TDD	Time Division Duplexing
GPS	Global Positioning System
PPP	Point Poisson Process
CDF	Cumulative Distribution Function
r.v.	Random Variable
BER	Bit Error Rate
i.i.d.	Independent Identically Distributed
i.n.d.	Independent Non-Identically Distributed

THESIS ABSTRACT

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TITLE OF STUDY: Dynamic Spectrum Allocation for Heterogeneous Cognitive Radio Networks with Femtocells

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The frequency spectrum is the scarcest radio resource in wireless communications. Cognitive radio is an emergent technology which has been proposed to deal with the spectrum scarcity problem. One of the major problems that may limit the emergence of cognitive radio networks is high spectrum costs. Several marketing environments (e.g. Monopoly market) may result in high spectrum costs for the secondary user (SU) operators. In this work, a novel GPS-assisted scheme is proposed to enhance spectrum efficiency and reduce the operational cost of cognitive radio networks with femtocells. The SU network, represented by the cognitive base station (CBS), determines the minimum number of channels needed for its operation before it starts purchasing spectrum from the primary user (PU) networks. This can be achieved by grouping the femtocells into non-interfering groups

based on the distances between them.

Two approaches for implementing the grouping scheme are proposed; namely, the femtocell secondary user (FSU)-based and the femtocell base station (FBS)-based grouping algorithms. The complexity of each grouping algorithm is evaluated and both are compared in terms of the required time to simulate and the resulting number of spectrum channels to be purchased. Results showed that the FBS-based grouping is much less complex, but results in larger number of spectrum channels to be purchased. Additionally, the complexity of the update process for each method is derived, where results show that sorting the groups in an ascent order, according to the number of members in each group for FSU-based grouping and the category for FBS-based grouping, will generally reduce the complexity of the update process. Moreover, the outage performance of both the FSU-based and the the FBS-based grouping algorithms is compared, and it was shown that the FSU-based grouping algorithm results in a better outage performance (lower outage probability) than the FBS-based grouping one.

In addition, several methods for optimizing and enhancing the grouping algorithms are proposed, and their performances have been evaluated. Results show that the CBS profit maximization algorithm achieves higher expected profits than the distance-based grouping with the minimization of the distance threshold, but with no QoS guarantees. Finally, an extension for the grouping scheme to the co-channel deployment scenario is presented, which helps to further reduce the number of purchased channels, but at the cost of worse outage performance.

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الطيف الترددي هو المورد الأكثر ندرة في الاتصالات اللاسلكية. الراديو الذكي هو تقنية حديثة تم اقتراحها للتعامل مع مشكلة ندرة الطيف. أحد المشاكل الرئيسية التي قد تحد من انتشار شبكات الراديو الذكي هي ارتفاع أسعار الطيف الترددي. عدة بيئات تسويقية (مثل سوق الاحتكار) قد ينتج عنها ارتفاع في أسعار الطيف الترددي على مشغلي شبكات المستخدمين الفرعيين. في هذه الأطروحة، طريقة جديدة تقترح لتحسين فاعلية الطيف و تقليل تكاليف شرائه من شبكات الراديو الذكي. شبكة المستخدمين الفرعيين، ممثلة في محطة البث الذكية، تحدد عدد القنوات المطلوب شراؤها لخدمة مستخدميها قبل الشروع في شراء الطيف من المستخدمين الأساسيين. يمكن تحقيق هذا الأمر عن طريق تصنيف المستخدمين الفرعيين ضمن مجموعات وفقا للمسافات بينهم. تظهر النتائج التجريبية أن الطريقة المقترحة تقلل عدد القنوات المطلوب شراؤها من المستخدمين الأساسيين و بالتالي تعمل على تحسين فاعلية الطيف و تقليل كلفته على المستخدمين الفرعيين.

CHAPTER 1

INTRODUCTION

1.1 Motivation

The trend in wireless communications industry has been always towards enhancing the spectrum efficiency and energy efficiency of system operation [1]. The drive for enhancing energy efficiency is mainly to satisfy the requirements of "green communications" and to extend the battery life time of user equipments (UE's) [2]. On the other hand, the need to promote the spectrum efficiency of future communications systems is driven by the spectrum scarcity problem and, at the same time, to allow for very high data rate transmission to satisfy the needs of the new applications and services [3]. Cognitive radio (CR) [4] has been considered as a novel approach to solve the spectrum scarcity problem. The idea of cognitive radio was originally proposed to allow the unlicensed users (called secondary users (SU's)) to utilize the spectrum reserved for the primary users (PU's) when this spectrum is idle [5, 6]. Of course, this requires the SU to identify the idle spectrum before exploiting it which is not a simple task [5]. Recently, a new paradigm for implementing CR has been proposed, which allows the PU operators to identify the holes in their spectrum and rent these holes to the SU's. This new paradigm is called the spectrum trading mechanism [7].

The spectrum trading mechanism, defined as the process of selling and buying spectrum between the PU operators and the SU operators, is one of the key enablers for spectrum sharing in CR networks [7]. In a spectrum trading scenario, the SU network (represented by the cognitive base station (CBS)) should try to reduce the number of required channels to be purchased from PU operators. This can be achieved by determining how many SU's can use a certain channel at the same time without harming each others. After that, the CBS can purchase spectrum bands/channels and re-use them to reduce operation costs.

The remaining of this chapter is organized as follows. In Section 1.2, some basic introductory subjects, which constitute the basic knowledge needed in the next chapters, are demonstrated. Section 1.3 provides the literature review of the previous works in this area. Section 1.4 discusses the motivations behind the contributions of this thesis. Section 1.5 summarizes the contributions of this thesis work, and Section 1.6 illustrates the organization of this thesis document.

1.2 Background

In this Section, an introduction to some basic concepts is presented. At first, the concept of the emergent cognitive radio technology and the motivation of using it is illustrated. After that, another novel technology, namely femtocells, is introduced and motivated. The basics of convex optimization are then explained, and finally, a brief demonstration of the notions of the well-known game theory is provided.

1.2.1 Cognitive Radio Networks

Cognitive radio is an emerging technology that was proposed as a solution for spectrum scarcity problem which has been considered as a consequence of fixed spectrum allocation [8]. The cognitive radio is a radio that can sense the spectrum, select the suitable channel to use, transfer from a spectrum band to another band when necessary and share the spectrum with other radios. To provide all of these capabilities, a cognitive radio should be self-organized [9], i.e., the radio should be able to adapt itself according to the surrounding environment without human intervention. In cognitive radio networks, two types of users exist [3]

- Primary users (PU's): users whom own the spectrum, and they have higher priority to use it.
- Secondary users (SU's): Users whom can use the spectrum only if it is not used by the PU's. The cognitive radio is needed for these users.

Several scenarios for the implementation of cognitive radios have been investigated in the literature. One of these scenarios [10, 11] assumes that there are different PU operators serving their own users and competing (or colluding) to serve the secondary users to maximize their profits. Another possible scenario for cognitive radio networks is the so called trading mechanism [12]. In this scenario, the PU networks have surplus spectrum (i.e., the PU networks are not using the whole spectrum to serve their users) which can be rented to the SU networks to gain more revenue for PU operators at the cost of service degradation for the PU's. The existence of several PU networks in the same area of the SU network will trigger a price competition among the PU networks in an attempt to sell more spectrum to the SU network. The cognitive base station (CBS) is supposed to purchase the spectrum from the PU networks.

1.2.2 Femtocells

Femtocells, also called femtocell base stations (FBS's), are small, inexpensive, short range and low-power base stations that are usually deployed on customer premises [13], and connected to the macrocell through wired backhaul [14]. Because of the fact that 50 percent of voice calls and more than 70 percent of the data traffic are originated indoors [15], femtocells' deployment has been considered as a key technique to enhance the capacity of wireless networks [16]. Moreover, because of the unavoidably increasing demand for green wireless communications [1], energy efficiency is considered one of the most important aspects of heterogeneous networks (HetNets). The existence of femtocells in the network alongside the macrocell has been shown to have significant influence on the reduction of energy consumption as compared to macrocell-centric networks [17]. Additionally, in [18], femtocells was adopted as an energy-efficient solution for indoor coverage in the LTE-Advanced standard.

Femtocells have been introduced the first time in the 3rd generation partnership project (3GPP) Release 8 [19] for the universal mobile telecommunications system (UMTS), and extended for the long term evolution (LTE) in Release 9 [20]. The importance of femtocells can be summarized in the following [19]

- Serving the indoor users where the macrocell can not reach and help the macrocell base station (MBS) to focus on the mobile (outdoor) users.
- Improving network capacity by reusing the radio spectrum indoor if the interference between femtocells and macrocells is successfully managed.
- Providing high data rates to indoor users.
- Reducing energy consumption of UE's, which prolongs the battery life for the UE's.

In the literature, the words femtocell, femtocell base station (FBS), and femtocell access point (FAP) have been used interchangeably to denote the base station itself [21]. In this work, the word femtocell is used to denote the coverage area, and FBS is used to denote the base station.

1.2.3 Convex Optimization

The use of convex optimization methods is very common in communications, especially for the problems modelled using game theory (involving convex or concave utility functions defined on convex sets)[22], which is to be introduced in the next part. This part presents the basics of convex optimization which will be needed when discussing the Stackelberg game in Chapter 5.

Before introducing convex optimization, we will start by illustrating the basic optimization problem. An optimization problem is a mathematical problem of the following form [22, 23]

$$\begin{aligned} &\text{maximize}_x \ f(x), \\ &\text{subject to } x \in \mathcal{X}; \end{aligned} \tag{1.1}$$

where $f(x)$ is the object/cost function, x is the optimization variable of the problem, and $\mathcal{X}$ is the constraint set. An optimization problem is said to be convex if and only if [22]:

- $\mathcal{X}$ is closed and convex.

- f is concave and continuously differentiable on $\mathcal{X}$.

A closed set is a set that contains all of its limit points. A convex set is the one that meets the following condition

$$\alpha x + (1-\alpha)y \in \mathcal{X}, \quad \forall x, y \in \mathcal{X}, \alpha \in [0,1]. \tag{1.2}$$

In other words, the set $\mathcal{X} \in R^n$ is said to be convex if and only if for any two points x and y belonging to that set, the segment connecting the two points is inside the set.

A function f is said to be convex on the set $\mathcal{X}$ if it satisfies the following condition

$$f(\alpha x + (1-\alpha)y) \le \alpha f(x) + (1-\alpha)f(y), \ \ \forall x, y \in \mathcal{X}, \alpha \in [0,1]. \tag{1.3}$$

The expression in (1.3) indicates that for any points x and y belonging to the convex set $\mathcal{X}$, the function f is convex if it is always below the segment connecting $f(x)$ and $f(y)$. Contrarily, a concave function f is always above the segment connecting x to y, i.e.,

$$f(\alpha x + (1-\alpha)y) \geq \alpha f(x) + (1-\alpha)f(y), \quad \forall x, y \in \mathcal{X}, \alpha \in [0,1]. \tag{1.4}$$

A function f is strictly concave if (1.4) is strict (i.e., there is no equality sign). Additionally, a function f is strongly concave if it has only one global maxima which is not equal to infinity.

1.2.4 Game Theory

People usually have different interests. Each one has his own reasons and aims, but at the end, they should interact with each other. When people interact, their actions are determined by their interests. Because of the their different interests, the interaction between people may raise conflicts of interests. A game is a theoretical model of the conflict of interests, and game theory provides the mathematical tools to model and analyze games [24].

One of the famous games which has been used to model warfare is Chess. In Chess, each player is trying to defend his king and, at the same time, kill the king of the other player. So, in Chess, we have two **players**. Each player puts a **strategy** which corresponds to his next **move** depending on the strategies/moves of the other player. At the end of the Chess game, the result for each player is either win (+1), lose (-1) or draw (0). The number associated with each result is called a **payoff**.

The players try to maximize their own payoffs. In the Chess case, a player is trying to win or to get the maximum payoff of +1. Each player has preferences which affect his strategies. Some players prefer to attack, and other players prefer to defend and wait for the mistakes of others. The preferences are characterized by the **utility** of the player.

After demonstrating the terminology of game theory, the next question to answer is: Why game theory is used in optimization? We have already implicitly answered this question when we talked about the payoffs. Each player tries to **maximize** his payoffs **subject to** the strategies/moves taken by the other players. *"Roughly speaking, a game can be represented as a set of coupled optimization problems."* [22]. Complex optimization problems which consist of different entities with different optimization indices can be decomposed into multi-stage games.

The recent trend in research towards cooperative networks, self-organized or autonomous networks and cognitive radio networks raised the necessity to analyze the interactions between the nodes in future wireless communication networks [25]. Game theory provides all kinds of tools to study these interactions, and gives low-complexity optimal or sub-optimal solutions which can not be provided by traditional optimization methods.

A game is usually solved by finding the Nash Equilibrium (NE) point [26]. NE is defined as the point at which if a player changes his actions, his payoffs will not increase if the other players do not take actions [12]. This means that no one is interested in taking actions at the NE point and hence, NE is considered as a stable point after which no improvements are observed in the game. Nevertheless, NE does not always correspond to an optimal solution for all players. Sometime it is optimal, and sometimes it represents a compromise solution which no one can deviate from. This depends on the type of the game being played.

A Stackelberg game is a game consisting of leaders and followers [27]. The leader, as can be inferred from the name, leads the game by playing first moves which are to be followed by the followers. Taking the CR trading mechanism scenario as an example, the PU networks are considered the leaders here, because the SU network do not take any action until the prices are offered by the PU networks. We have two levels of followers: those who directly follow the leader (the CBS) and those who follow the followers (the FBS's). In this case, the optimization problem is formulated as a three-stage Stackelberg game [28]. To ensure the existence of equilibrium solution for the Stackelberg multi-stage game, the NE at each stage should exist.

1.3 Literature Review

Several models of spectrum trading have been studied in the literature such as the competitive pricing model [12, 29, 10, 28], the bargaining model [30, 31], the auctioning model [32, 33, 6], and the agent-based spectrum trading models [34, 35]. In [34], it has been shown that the success of the SU network operators depends on achieving lower transaction costs. However, this may not be attainable under some marketing environments (e.g., when there is one seller, or when collusion is established between multiple sellers) which puts the chances of the SU network to accomplish the expected profits at risk.

To reduce spectrum costs, the SU network needs to use the spectrum more efficiently. Many solutions have been suggested in the literature for enhancing the spectrum efficiency of two-tier networks while reducing interference by utilizing the concept of CR [36, 37, 38, 39, 40]. All of the works existing in the literature are designed for the case when the macrocell base station (MBS) and the FBS's are operating on the spectrum licensed for their operator. On the other hand, the scheme proposed in this Thesis can be implemented with both licensed and unlicensed spectrum to alleviate the problems of spectrum insufficiency and high spectrum costs.

In the following, a review on the previous work in the areas of spectrum trading and interference mitigation in femtocells networks is conducted, which forms the basis for the work presented in the next chapters of this document.

1.3.1 Spectrum Trading

Spectrum trading, which is defined as the process of selling and buying radio resources, is one of the essential mechanisms for future heterogeneous cognitive radio networks, because it enables spectrum sharing [7]. A model based on market games for spectrum trading between PU's and SU's was introduced in [7]. The authors proposed an iterative learning algorithm to adjust spectrum price and demand, and the algorithm has been shown to converge to the NE.

The dynamics of SU's (the buyers) and PU's (the sellers) behaviors have been modelled in [29] for a scenario consisting of multiple-seller and multiple-buyer. Each one of the buyers and sellers is trying to maximize its payoff by taking into account the moves of the other players and adapting accordingly. An iterative algorithm was presented to find the solution of the game modelling the spectrum trading problem. The dynamics of the competitive spectrum sharing among the SU's have been modelled using game theory in [41]. The problem of spectrum sharing was modelled as an oligopoly market, and the NE was presented as the solution of the game.

In [12], the authors explored the effects of collusion games, established (and maintained) by the PU's, on their profit, and they compared that profit to the one gained as a result of the NE price in the competitive pricing scenario. They showed that the NE is not the price that maximizes the profit of the PU networks. Assuming that all the PU networks will be committed to the collusion game, they can achieve the Pareto price which represents the price that maximizes the profit of each PU network on the long term. The problem with the Pareto price is that it is not stable. That is, any PU network can deviate from this price to get a higher profit at the cost of lower profits for the other PU networks. To ensure that this will not happen, a punishment strategy can be played by all of the other PU networks against any deviating PU by returning to the NE price.

A dynamic spectrum allocation scheme was proposed in [42] with the aim of combating the collusive behavior of the SU's. By doing so, the scheme can maximize the overall spectrum efficiency of the network. The collusive behavior (called the bidding ring) of the SU's will help to minify the cost on each one of them by submitting only one bid for the whole group. The solution is simply that the PU should consider the size of the ring when it performs spectrum pricing.

A different paradigm for spectrum trading was investigated in [10]. In this paradigm, the PU networks will not sell spectrum opportunities to the SU networks, but instead, they will be competing to serve the SU's in addition to their users to increase their revenues. Two approaches of spectrum sharing under this paradigm were considered; namely, coordinated and uncoordinated access. In coordinated access, the PU network has the right to reject the purchase request of an SU if this will help the PU operator to maximize its revenue (i.e., if serving the PU's will provide higher revenues). In uncoordinated access, the PU's and SU's have equal priority to share the spectrum. The pricing problem was modelled as a non-cooperative game in which each operator chooses a price slightly lower than the price put by the other operators as long as the chosen price is higher than the breakeven price (the price above which the PU is earning profit regardless of the spectrum demand). When the breakeven prices of the PU networks are different, the operator with the lowest break even price will dominate the market. On the other hand, if the breakeven prices of the PU networks are equal, then the pricing game reaches the NE.

In [43], an interference-constrained pricing game scenario has been investigated. In this scenario, any SU which contributes to the total interference by a value higher than a certain threshold will be penalized by additional costs. The equilibrium of the network was defined as one of two cases: either when the interference conditions are satisfied, or when the SU's has no interest in changing their transmit power levels. The equilibrium was found for two different network configurations; namely, a centralized network with access point, and a distributed network where the SU's are transceiver pairs. For the second network configuration, there is a constraint on the value of the channel gain to find the equilibrium.

The evolution of spectrum trading market consisting of a heterogeneous of local area (LA) and wide area (WA) operators has been explored in [34]. In a trial to find a win-win situation for all operators, different spectrum management schemes were discussed, and different market scenarios were evaluated using agent-based modelling. Results showed the importance of the low transaction costs for the success of SU operators. Furthermore, LA networks, especially Wi-Fi capacity markets, was proven to be a better choice for SU operators than WA networks.

An agent-based model for spectrum trading under spectrum demand uncertainty conditions was presented in [35]. The objective here is to maximize the profit of the agent. With the increase of the number of agents, the profit gained by each agent is decreased. Furthermore, the profit of agents is higher when they work independently than when they play a competition game.

A monopoly scenario which encompasses one PU network selling its underutilized spectrum bands to several SU's was discussed in [44]. The trading process has been modelled as a monopoly market game. In such a game, the seller determines the quality versus price for the spectrum to be sold. The only role for SU's here is to choose between the offers of quality-price what is the best for their operation requirements. The optimal contract that is feasible for the SU's and maximizes the PU profit was derived for this scenario.

Re-visiting the monopoly market, the work in [45] was focused on finding the maximum expected profit for the PU's under network uncertainty. A hybrid spectrum market was considered which consists of two types of markets, the futures and the spot markets. In futures market, there is a contract being agreed upon between PU's and SU's which determines demand and price over a given period. On the other hand, in spot market, the spectrum is being traded in the real time depending on the SU demand. The hybrid market combines the advantages of both markets (i.e., reliability of futures market and flexibility of spot market). Depending on whether the PU can observe the SU's private information or not, the optimal pricing is either perfect price discrimination mechanism or an integrated contract and auction design, respectively.

The effect of the risk associated with imperfect spectrum sensing on auction-based trading has been considered in the method proposed in [33]. The method was shown to outperform its conventional counterparts that do not consider the risk of imperfect spectrum sensing in terms of revenue for both PU and SU networks.

The scenario of multi-auctioneers offering their (idle) spectrum channels to get more revenue was studied in [6]. A new mechanism (called multi-auctioneer progressive (MAP) auction) was proposed which involves raising the trading price by each auctioneer, and each bidder chooses one auctioneer for bidding. Results revealed that MAP converges to NE with spectrum efficiency close to the optimal one depending on the step size.

In [31], a bargaining two-stage game model was advised. In this model, the market constitutes of two tiers. In the first tier, one PU network offers part of its spectrum to the SU's, whereas in the second tier, the SU's redistribute the channels between themselves. The NE was derived for the two-stage bargaining game. Furthermore, the effects of some parameters (e.g., availability of channels and bargain partners) have been investigated. The study revealed the minority of some parameters (e.g., bargain partner), and the advantage that can be gained for the SU's by maintaining some other parameters (e.g., current traffic demand).

The authors in [28] focused on the energy-efficient aspect of cognitive radio networks with femtocells. As for any competitive pricing model, each PU network offers part of its spectrum at a given price to increase its revenues, but the SU network (represented by the CBS) takes into account the energy efficiency of user transmission when the spectrum is allocated. That is, the CBS will purchase the spectrum with the lowest price and allocate it to the user with the highest energy efficiency to maximize its profit. A user here could be a macrocell secondary user (MSU) or a FBS which was assumed to serve only one user. The problem was formulated as a three-stage Stackelberg game which can be solved by finding the NE at each stage.

As was mentioned earlier, the NE is not the price that maximizes the profit of the PU networks, and the maximum-profit price is Pareto-optmal [12]. Assuming that no PU network will deviate from the optimal price, the CBS will be exposed to very high prices. In addition, the spectrum offered by the PU networks may not be enough to satisfy the requirements of the SU's in terms of capacity. In these two cases, the CBS should be able to manage the spectrum in a more efficient way to be able to survive in this environment.

The collusion problem was addressed in [11]. The interaction between the PU networks has been modelled using a coalition game, and the dynamics of users under an uncertain market environment was studied using the evolutionary game model. The authors developed mathematical tools that will help regulators to determine upper limits on spectrum price that should be enforced to prohibit collusion of operators.

1.3.2 Interference Mitigation using CR

The majority of the existing literature that merges femtocells and CR was focused on the utilization of CR concepts in two-tier networks (consisting of macrocells and femtocells) to mitigate interference and increase spectrum efficiency. The work of [36] presented a scheme to mitigate interference in femtocell networks relying on cognitive radio idea (femtocells treat macrocells as PU's). The scheme was shown to provide good quality-of-service (QoS) while maintaining a fairly high spectral efficiency.

Similarly, the idea of cognitive radio was exploited in [37], but combined with interference cancellation and multi-antenna techniques. The proposed system is built upon the assumption that femtocells can decode control channels of macrocells and, accordingly, build their transmission schedule in line with the schedule of the macrocells.

An optimization scheme to enhance the spectrum efficiency and energy efficiency of two-tier networks was proposed in [38]. The basic idea of this scheme is to analyze the interference resulting from the dense deployment of femtocells and putting a constraint on the resultant outage capacity. It was found that the best approach to optimize energy and spectrum utilization under low outage constraint is the round-robin scheduling approach.

In [39], the authors proposed an interference mitigation scheme for open-access femtocell-macrocell networks that allows the end user, which is requesting to receive some downlink information, to sense the available channels and non-coherently detect and accumulate the interference levels on each sensed channel to find the first channel with an interference level higher than some threshold. The scheme performance was studied for the cases of underloaded and overloaded channels. The scheme was shown to provide better performance than the random selection one. Furthermore, the scheme was shown to reduce the processing load as compared with the minimum interference channel selection scenario.

The same authors presented another scheme in [40] for a different scenario, where the coordination between femtocells is allowed but no feedback from the end user. Here, the FBS's are assumed to acquire the sensing capability to detect the interference levels on each available channel. The FBS utilizes the first channel which has an interference power level above some interference threshold. Again, the scheme was compared to the random selection and the minimum interference selection schemes, and it was shown to over-perform the first one and to reduce the complexity of the second one under overloaded channels conditions.

A framework for interference avoidance in OFDMA femtocells was introduced in [46] which basically depends on macrocell frequency scheduling information (assumed to be available) and spectrum sensing. The aim of the femtocell here is to avoid using the resource blocks occupied by mobile stations (MS) close to that femtocell. The framework was found to be helpful to solve interference problems in closed-subscriber-group mode of femtocell operation.

In [47], spectrum sensing and spectrum sharing ideas have been exploited to enhance the interference coordination capability of femtocells. The efficiency of these two ideas in handling cross-tier interference coordination for femtocell networks was confirmed by the authors.

The effect of the information sensed at the FBS on the network capacity has been investigated in [48]. Based on that, a spectrum sharing scheme was proposed between the femtocells themselves and the femtocells and macrocells, which has been shown to significantly enhance spectrum efficiency by utilizing location and channel information.

In [49], the scenario of the femtocell sharing the spectrum bands with macrocells and with other licensed communication systems (e.g., TV bands) was investigated. Results showed that these (cognitive) femtocells can achieve a considerably higher capacity than the femtocells operating only on the macrocell spectrum.

Three CR-enabled approaches to mitigate interference have been studied in [50]; namely, opportunistic interference avoidance, interference cancellation and interference alignment. As a result, a joint opportunistic interference avoidance scheme with Gale-Shapley spectrum sharing was proposed. A significant enhancement in throughput was noticed as a result of applying the proposed scheme when compared to random access schemes.

The idea of virtual clustering was introduced in [51], where each group of femtocells separated by a safety distance are grouped into a virtual cluster (VC). The safety distance is predefined and is fixed for all VC's. The number VC's is also fixed, and any femtocell that can not be assigned to a VC will be assigned to the reserve set which includes femtocells that can not uphold the safety distance.

An enhancement on the virtual clustering idea was suggested in [52] by adding range expansion capability to it. The effect of femtocell coverage expansion on the performance of closed-access femtocell network was analyzed, and an enhanced virtual clustering formation method was shown to significantly improve signal-to-noise-and-interference-ratio (SINR).

In [53], a novel scheme for interference mitigation in two-tier OFDMA networks was proposed. The scheme is based on reducing co-tier interference by clustering femtocells and assigning those which interfere with each other to different clusters. Cross-tier interference is mitigated by allowing femtocells to sense the sub-bands and find the suitable ones which do not interfere with macrocell users. The interference among the macrocells is mitigated using fractional frequency reuse (FFR).

A similar idea was utilized in [54], where the femtocells are clustered, and then femtocells in different clusters are allowed to use the same channels. The technique is called virtual clustering, and is combinted with cognitive sensing capability to enhance spectrum efficiency by allowing the femtocells to exploit the channels underutilized by the macrocells and by other systems like TV systems.

In [55], a graph-theoretic approach is followed to minimized co-tier interference between femtrocells. A cluster-based sub-channel allocation scheme is proposed to maximize capacity by dynamically adapting to the femto-femto interference (FFI) distribution. The scheme starts by deducing the maximum cluster size from the downlink outage probability constraint, and the FBS's are clustered under the maximum cluster size constraint. After that, the capacity in each cluster is maximized.

1.4 Thesis Motivation

Motivated by the importance of increasing the spectrum efficiency of wireless systems to deal with the spectrum scarcity problem, we propose a GPS-assisted spectrum allocation scheme to reduce the number of channels to be purchased from the PU networks. This scheme is designed to be used along with the spectrum trading mechanism to maximize the profit of the SU network, but it can be also utilized by the licensed operators to enhance the spectrum efficiency of their networks.

The Pareto-optimal price, which is maintained by the collusion game, was claimed to be the price that maximizes the profit of the PU operator in [12], and in [11], the authors developed mathematical tools that help the regulators to prohibit collusion by controlling the maximum price that can be set by the PU operators. In this thesis work, the question of the price that maximizes the profit of the PU networks is further investigated using the model presented in [28] that includes femtocells, which are becoming an essential part of the present wireless communications networks [15, 56], in the game model. Furthermore, the price is not the only parameter affecting the profit of the SU network, and hence controlling the price may not be the best way to ensure the success of the SU operators.

1.5 Thesis Contributions

The first contribution of this thesis is a new scheme that can be utilized by the SU network to reduce the number of channels to be purchased from the PU networks. Two methods, namely the FSU-based and the FBS-based grouping methods, to implement the scheme are proposed, and their performance in terms of spectrum efficiency and complexity, is compared. Moreover, the complexity of the update process, defined as the average number of examined groups before finding a suitable one for a new FSU/FBS, for each method is analyzed using sequential modelling.

The second contribution of this thesis is the derivation of the uplink outage probability resulting from the grouping scheme. The uplink outage probability is derived for both the FSU-based and the FBS-based grouping schemes. Furthermore, several modifications and enhancements are proposed to optimize the performance of the scheme in terms of satisfying one of two objectives; namely, either to minimize the number of channels to be purchased from the PU networks while satisfying some outage probability constraint or to maximize the profit of the SU network. Moreover, each one of the optimization methods is extended to the co-channel deployment scenario, where the FBS's share part of the spectrum allocated to the MSU's.

The last contribution of the work in this thesis is an investigation of the model presented in [28], which is mainly aimed at answering two questions: The first question is regarding the price that maximizes the profit of the PU networks. In [12], it has been argued that the Pareto-optimal price is the one that maximizes the profit of the PU networks. This argument is examined using the general model with femtocells in [28]. The second question is that, even if the highest price that can be achieved is known, will limiting the price ensure a better payoff for the SU network? In other words, is the minimum profit that can be attained by the SU network associated with the highest price that can be offered by the PU networks?

1.6 Thesis Organization

In Chapter 2, a scheme for the reduction of the number of channels to be purchased from the PU networks is suggested, which depends on grouping the SU's into groups of non-interferers based on the distances between them. Two different approaches are proposed to implement this scheme, namely, the FSU-based and the FBS-based grouping approaches. The performance of the two approaches in terms of spectrum efficiency and complexity is compared by implementing the two schemes and observing the average number of channels to be purchased and the average time to simulate for each one. Finally the complexity of the update process is derived for each approach.

Chapter 3 presents the performance analysis for both the FSU-based and the FBS-based grouping schemes. The general uplink outage probability is derived for each scheme, and several scenarios are considered for finding the uplink outage probability of each scheme under some worst-case interference assumptions.

The worst-case uplink outage probability expression will be used in Chapter 4 to optimize the grouping scheme by finding a minimum value of the threshold distance to be used in grouping, and by adding the MSU's to the groups of FSU's/FBS's. Moreover, an algorithm for maximizing the profit of the SU network, which is based on the greedy approach, is proposed.

In Chapter 5, the stability of the algorithm presented in [28] is analyzed, and the optimal step size is derived. Furthermore, the Nash and the Pareto-optimal prices are derived and compared under a variety of parameters to get some conclusion about the price that maximizes the profit of the PU networks. This will lead to the conclusion about whether the minimum profit of the SU network is associated with the price that maximizes the PU profit or not. Finally, Chapter 6 provides the conclusions and proposed future work.

CHAPTER 2

GROUPING-BASED SPECTRUM ALLOCATION SCHEME

2.1 Introduction

In this chapter, a novel scheme to increase spectrum efficiency of cognitive radio networks and to reduce the required spectrum costs is proposed. The idea here is to add a scheme before spectrum trading which involves determining the minimum required number of orthogonal channels to be purchased from the PU networks. The scheme is applicable for the case when the SU operator serves macrocell SU's (MSU's) and femtocell base stations (FBS's). The basic idea is to classify the femotcells into non-interfering groups based on the distances between them, and then to purchase the number of channels required to serve the groups. Two types of the grouping scheme will be presented in this chapter, namely, FSU-based and FBS-based grouping.

This chapter is organized as follows. Section 2.2 describes the system model. The FSU-based grouping scheme is presented in Section 2.3, and the FBS-based grouping scheme is presented in Section 2.4. Section 2.5 provides the simulation results, and Section 2.6 concludes the chapter.

2.2 System Model

The general network model is shown in Fig. 2.1, where one SU macrocell is considered so that there is one CBS serving I MSU's and K FBS's in the network. The k^{th} FBS serves N_k FSU's (where $k = 1, ..., K$). The FBS's are connected to the CBS using broadband connection (e.g., DSL) [18] or optical fiber. The coverage radius of the macrocell and each femtocell is assumed to be ideally circular, centred at the CBS and the FBS, respectively. All the femtocells are assumed to have the same coverage radius. Closed-access is assumed where the femtocell serves only registered users [14]. It is also assumed that the MSU's are allocated orthogonal channels to those assigned to the FBS's.

The spectrum allocation process is illustrated in Fig. 5.1. Each FBS sends its position $\{G_k\}_{k=1}^{K}$ (determined using a built-in GPS receiver [57]) and the number of users it serves to the CBS through the wired backhaul. The CBS performs grouping of FSU's/FBS's (to be described in Sections 2.3 and 2.4) based on the location information. After that, the CBS purchases a channel l from a PU net-

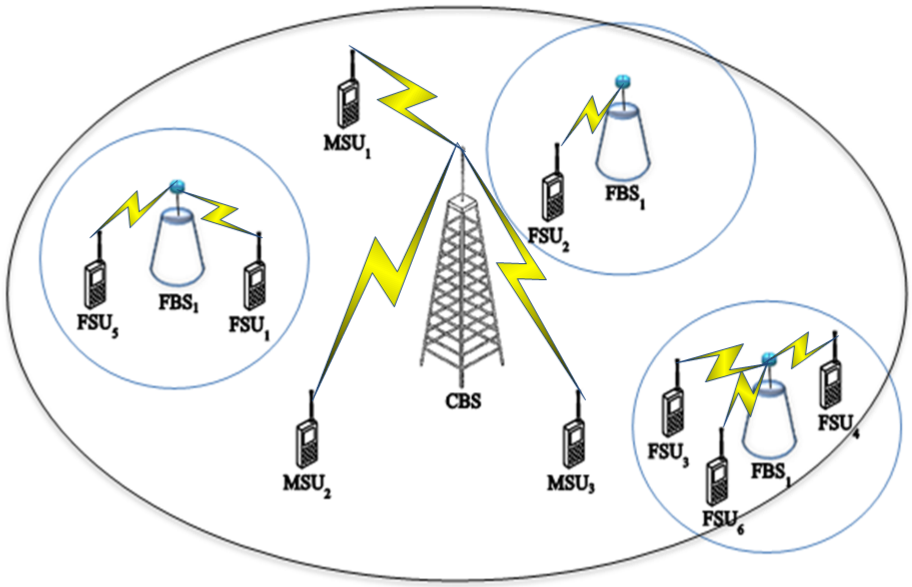

Figure 2.1: System model for the cognitive radio netowrk with femtocells. One macrocell with arbitrary number of MSU's and FBS's is assumed. Each FBS serves arbitrary number of FSU's.

work and allocates it directly to an MSU (by setting the spectrum allocation index of that MSU x_{li} to 1), or to a FBS by setting its allocation index y_{lk} to 1. The assignment of channels to different MSU's or FBS groups can be done by the CBS randomly, or according to their energy efficiencies as proposed in [28]. L PU networks are assumed to exist each of which offers several channels/bands which are not being used by its users at some time instant [1]. It is assumed that all the channels offered by the PU networks have the same bandwidth regardless to the access scheme. The spectrum channels purchased from different PU networks or from the same PU network are assumed to be perfectly orthogonal (i.e., interference may only result when the same channel is being used by more than one

[1]The whole spectrum trading process is operated in a time-slotted manner. That is, in each time slot the offers from each PU network may differ depending on the PU load. This necessitates that the PU networks and the SU network be perfectly synchronized [28, 58]

FSU/MSU at the same time).

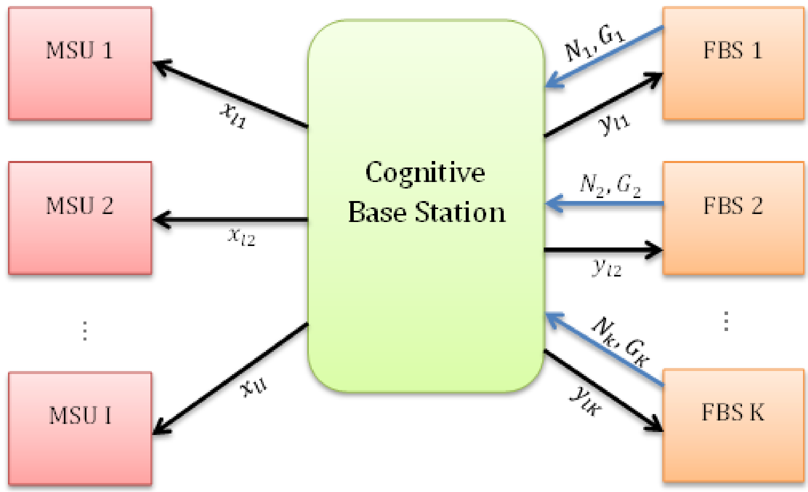

Figure 2.2: The general system model for the spectrum allocation process.

2.3 FSU-Based Grouping

In this Section, the FSU-based grouping scheme is explained, where each FSU in a femtocell belongs to a different group, and each group is assigned only one channel. The algorithm is presented, and its complexity is analyzed.

2.3.1 Algorithm

In order to reduce the number of spectrum channels to be purchased from the PU networks, the CBS needs to group the FSU's into non-interfering groups. This requires that the distance from a FSU to a FBS in another femtocell should be found in order to determine whether the FSU is interfering with that femtocell or not. However, since the coverage radius of a femtocell is usually very small (from 10 to 30 meters [59]), the distance from the i^{th} FSU to the k^{th} FBS, D_{ik}, can be well approximated by the distance between the two FBS's [16]. It should be emphasized here that grouping a FSU requires the CBS to assign its FBS to that group, so in general, a FBS may be assigned to several groups depending on the number of FSU's served by that FBS.

The FSU-based grouping scheme is performed as follows. First, the CBS starts with the first FBS with ungrouped FSU's and assigns it to the first group. Then, it finds the distance between the first FBS and the second FBS with ungrouped FSU's. If this distance is larger than a certain threshold distance (D_{th}) then the second FBS is also assigned to the first group. Otherwise, it is not be assigned to that group, and the CBS examines the third FBS with ungrouped FSU's and so on. When all the FBS's are examined, the CBS restarts from the first FBS with ungrouped FSU's and assigns it to the second group. Each time, the CBS checks all the FBS's with ungrouped FSU's to examine whether they can be assigned to a certain group or not. This process is repeated until all the FBS's have their FSU's grouped, where a FBS with ungrouped FSU's is assigned to a group if its distance to all the members of that group is larger than D_{th}. The whole process is illustrated in Algorithm 1.

It is assumed that for each FSU, the FBS needs one channel to maintain some minimum required QoS. Therefore, for N_k users served by the same FBS, N_k channels are required. Of course, this is associated with maintaining some outage performance at the FBS for each FSU (to be discussed in Chapter 4). The grouping index of the k^{th} FBS (g_k) is set to one when all the FSU's served by this

Algorithm 1 FSU-Based Grouping:

Initialization: the set of FBS's $\Omega_K = \{1, 2, ..., K\}$*, the set of FBS's locations* $\{G_k\}_{k=1}^{K}$*, and the set of grouping indices* $\mathcal{G} = g_1, g_2, ..., g_K$*. Each FBS serves* N_k *ungrouped users.*
Set $s = 1$*.*
repeat
    Denote the number of FBS's in group s *by* M_s*, and set* M_s *to zero.*
    Find the first FBS k *with grouping index* $g_k = 0$*, and put it in group* s*.*
    Set $n = k + 1$*,* $M_s = M_s + 1$*, and* $N_k = N_k - 1$*.*
    if $N_k == 0$ **then**
        Set $g_k = 0$;
    end if.
    repeat
        if $g_n == 0$ **then**
            Find the distance between the FBS n and all the FBS's in group s, $\{D_{ni}\}_{i=1}^{M_s}$;
            if $\{D_{ni}\}_{i=1}^{M_s} \geq D_{th}$ **then**
                Put FBS n in group s,
                Set $M_s = M_s + 1$, and $N_n = N_n - 1$,
                if $N_n == 0$ **then**
                    Set $g_n = 1$;
                end if;
            end if.
        end if.
        Set $n = n + 1$.
    until $n = K + 1$, *end repeat.*
    Set $s = s + 1$.
until all elements in $\mathcal{G}$ equal 1, *end repeat.*
Output the number of groups, and the members of each group.

FBS are grouped. So, this FBS will not be considered for further grouping.

After the grouping process is performed, the CBS will purchase channels from PU networks (through spectrum trading mechanism) and assign them to the MSU's and the groups of FBS's. The assignment of channels to different MSU's or FBS groups can be done by the CBS randomly, or according to their energy efficiencies as proposed in [28]. Since serving macrocell users has a higher priority for operators [60], it makes sense to assume that the CBS will purchase spectrum for MSU's first, and then for the groups of FBS's. As described in the introductory part of this chapter, several models have been studied in the literature for spectrum trading. However, a few studies describe the whole trading process between SU and PU networks in the presence of MSU's and FBS's in the network. The framework presented in [28] is of particular interest in this context.

If a new FSU appears in one of the existing femtocells, its FBS will report to the CBS that it has a new FSU. The CBS searches for a group to assign the new FSU to after making sure that this group satisfies the distance threshold condition. Similarly, if a new FBS, with arbitrary number of FSU's, appears in the network, then it sends its location and the number of its FSU's to the CBS, which in turn groups the new FSU's. In both scenarios, either the new FSU is assigned to one of the existing groups that satisfies the distance threshold condition or it is assigned to a new group. If an FSU is removed from the network, its FBS will report to the CBS the group number of the channel assigned to that user, and the CBS will remove this FBS from that group. Similarly, if an FBS is removed from the network, then the CBS will remove it from all the groups it is participating in.

2.3.2 Complexity

The first step in the FSU-based grouping scheme is to assign the first ungrouped FSU to the group. Starting from the first group, the first FSU will be assigned to this group and the other FSU's will be compared to this FSU. If a FSU is assigned to a group (i.e., it satisfies the distance threshold condition), it will not be examined for the next groups. The worst-case occurs when each FSU is assigned to a separate group. In this case, each FSU will be examined with all the FSU's with an index higher than its one. Assuming that N FSU's exists in the network, at most, the CBS will need $N-1$ operations for the first group, $N-2$ operations for the second group, and so on. An operation here is defined as the the processes required to examine whether a FSU can be assigned to a certain group or not. This includes finding the distance between this FSU and the FSU assigned to the group, comparing this distance to D_{th}, and all the accompanying assignment and counting operations. For N FSU's, the CBS will need $\sum_{i=1}^{N}(N-i) = \frac{1}{2}N(N-1)$ operations. So the complexity of the FSU-based grouping scheme is in the order of $O(N^2)$.

Similarly, the complexity of the update process is defined as the average number of groups to be examined before finding a suitable group for a new FSU. Referring to the discussion in the previous subsection, the i^{th} FSU (for $i = 1, \ldots, N$) will be assigned to the first group that satisfies the condition $\{D_{ij}\}_{j=1}^{M_s} \geq D_{th}$ (where D_{ij} is the distance from the i^{th} FSU to the j^{th} FSU member in Group s, and M_s is the number of members in Groups s). This process has a lower complexity than the optimum one where the distances from the k^{th} FBS to the members of all groups are found, and the group with the highest sum of distances to that FSU is chosen. The optimum approach passes over all the groups each time a new FSU appears in any femtocell.

To find the average number of groups to be examined before a suitable group is found in the FSU-based scheme, let $l = 1, \ldots, S$ be a random variable representing the number of groups to be examined before finding the group that satisfies the condition $\{D_{kj}\}_{j=1}^{M_s} \geq D_{th}$ (where M_s is the number of members in Group s). The probability that one group will be examined is the probability that the first group will satisfy the condition (i.e., $\Pr\{l = 1\} = \prod_{i=1}^{M_1}[\Pr\{D_{ik} \geq D_{th}\}]$). In the same way, the probability that 2 groups will be examined is the probability that the first group will fail to satisfy the condition and the second group will do so (i.e., $\Pr\{l = 2\} = [1 - \prod_{i=1}^{M_1}[\Pr\{D_{ik} \geq D_{th}\}]] \prod_{i=1}^{M_2}[\Pr\{D_{ik} \geq D_{th}\}]$). Generally, the probability that s groups will be examined is

$$\begin{aligned} \Pr\{l = s\} = \prod_{i=1}^{s-1} &\left\{ 1 - \prod_{j=1}^{M_i} \left[\Pr\left\{D_{jk} \geq D_{th}\right\}\right] \right\} \\ &\times \prod_{v=1}^{M_s} \left[\Pr\left\{D_{vk} \geq D_{th}\right\}\right]. \end{aligned} \tag{2.1}$$

Note that for the event $\{l = S\}$, (2.1) gives the probability that the last group satisfies the condition. There is another event that can be denoted as $\{l = S\}$, which occurs when no group can satisfy the condition (i.e., when the FSU is assigned to a new group). Based on the aforementioned discussion, the average number of examined groups before a suitable group is found ($\bar{l}$) can be written as

$$\begin{aligned} \bar{l} = &\sum_{s=1}^{S} \left[s \prod_{i=1}^{s-1} \left[1 - \prod_{j=1}^{M_i} \Pr\left\{D_{jk} \geq D_{th}\right\} \right] \times \prod_{v=1}^{M_s} \Pr\{D_{vk} \geq D_{th}\} \right] \\ &+ S \prod_{a=1}^{S} \left[1 - \prod_{n=1}^{M_1} [\Pr\{D_{nk} \geq D_{th}\}] \right]. \end{aligned} \tag{2.2}$$

The expressions in (2.1) and (2.2) depend on the probability that the distance between two FBS's is larger than D_{th}. When the FBS's are distributed using the Point Poisson Process (PPP) [2], their locations will be uniformly and independently distributed in the macrocell region [62]. In this case, the probability that the distance between two FBS's inside the circular range of the macrocell (with radius R_M) is larger than D_{th} can be written as [63]

$$\begin{aligned}\Pr\{D \leq D_{th}\} = 1 + \frac{2}{\pi}\left(\frac{D_{th}^2}{R_M^2} - 1\right)\cos^{-1}\left(\frac{D_{th}}{2R_M}\right) - \frac{D_{th}}{\pi R_M} \\ \times \left(1 + \frac{D_{th}^2}{2R_M^2}\right)\sqrt{1 - \frac{D_{th}^2}{4R_M^2}}.\end{aligned} \tag{2.3}$$

Hence, the probability that two FBS's are at a distance of at least D_{th} is just the complementary cdf of (2.3). As will be mentioned in Chapter 4, another constraint should be put on the distance between two FSU's/FBS's for optimization purposes. This constraint is that the distance between two FBS's should be greater or equal to double the radius of the FBS (let us denote this radius by R_F). Applying this constraint, the probability that two FBS's inside the circular range of the macrocell are at a distance of at least D_{th} given that the distance between them is at least $2R_F$ is written as

$$\begin{aligned}\Pr\{D \geq D_{th} \mid D \geq 2R_F\} &= \frac{\Pr\{D \geq D_{th}, D \geq 2R_F\}}{\Pr\{D \geq 2R_F\}} = \frac{\Pr\{D \geq D_{th}\}}{\Pr\{D \geq 2R_F\}} \\ &= \frac{\frac{D_{th}}{\pi R_M}(1 + \frac{D_{th}^2}{2R_M^2})\sqrt{1 - \frac{D_{th}^2}{4R_M^2}} - \frac{2}{\pi}(\frac{D_{th}^2}{R_M^2} - 1)\cos^{-1}(\frac{D_{th}}{2R_M})}{\frac{2R_F}{\pi R_M}(1 + \frac{2R_F^2}{R_M^2})\sqrt{1 - \frac{R_F^2}{R_M^2}} - \frac{2}{\pi}(\frac{4R_F^2}{R_M^2} - 1)\cos^{-1}(\frac{R_F}{R_M})}.\end{aligned} \tag{2.4}$$

[2] The Poisson Point Process (PPP) is a widely popular process to model the locations of the randomly distributed nodes in wireless communications networks [61].

Since (2.4) does not depend on the location of the FBS (i.e., the locations are identically distributed), the expression in (2.2) can be simplified as

$$\begin{aligned}\bar{l} = &\sum_{s=1}^{S}\left[s\prod_{i=1}^{s-1}\left(1-[\Pr\{D \geq D_{th}\}]^{M_i}\right)\times[\Pr\{D \geq D_{th}\}]^{M_s}\right] \\ &+ S\prod_{a=1}^{S}\left(1-[\Pr\{D \geq D_{th}\}]^{M_a}\right).\end{aligned} \tag{2.5}$$

2.4 FBS-Based Grouping

Another approach that can be used to reduce the number of spectrum channels to be purchased from the PU networks, is that the CBS groups each FBS only once, and it puts a constraint on the number of FSU's served by an FBS to make it a candidate for a certain group depending on the number of channels that will be purchased for that group. As in the FSU-based grouping scheme, it is assumed that for each FSU, the FBS needs one channel to maintain some minimum required QoS regardless to the application. Therefore, for N_k users served by the same FBS, N_k channels are required.

2.4.1 Algorithm

The whole FBS-based grouping scheme can be implemented as follow. At first, each FBS determines its location using GPS and sends it to the CBS. The CBS finds the distances between the femtocells and stores them. Starting with the first FBS, the CBS assigns that FBS to the first group, and it stores the number of FSU's served by that FBS as the category of the group. The category of the group here is defined as the maximum allowed number of FSU's per one FBS member, which corresponds to the number of channels needed to be assigned to that group. The second FBS is then examined by the CBS, and if the distance between the second and the first FBS's is larger than D_{th} and the number of FSU's served by the second FBS is less or equal to the category of the group, then the CBS assigns the second FBS to the first group. Otherwise, if any of the previously described conditions is not satisfied, then the second FBS is not grouped and the CBS examines the third FBS and so on. When all the FBS's are examined, the CBS restarts from the first ungrouped FBS and assigns it to the second group. Each time, the CBS will check all the ungrouped FBS's to examine whether they can be assigned to a certain group or not. This process is repeated until all the FBS's are grouped, where a FBS will be assigned to a group if its distance to all the members of that group is larger than D_{th} and the number of FSU's it serves is less than or equal to the category of the group. The whole operation is illustrated in Algorithm 2.

The grouping index of the k^{th} FBS (g_k) is set to one when this FBS is grouped, and it will not be considered for further grouping. If a new FSU appears in one of the existing femtocells, its FBS will report to the CBS that it has a new FSU. The CBS will check whether the FBS still satisfies the category condition and if not, then the CBS increases the category of the group by one and purchase a new channel for it. Further, if a new FBS, with arbitrary number of FSU's, appears in the network, it sends its location and the number of its FSU's to the CBS which

Algorithm 2 FBS-Based Grouping:

Initialization: the set of FBS's $\Omega_K = \{1, 2, ..., K\}$, *the set of FBS's locations* $\{G_k\}_{k=1}^{K}$, *and the set of grouping indices* $\mathcal{G} = g_1, g_2, ..., g_K$. *Each FBS serves* N_k *users. The category of each Group* s *is* C_s *which is set to 0 for all groups.*

Set $s = 1$.

repeat

  Denote the number of FBS's in group s *by* M_s, *and set* M_s *to zero.*

  Find the first FBS (with index k*) with grouping index* $g_k = 0$, *and put it in group* s.

  Set $n = k + 1$, $M_s = M_s + 1$, *and* $C_s = N_k$.

  repeat

    if $g_n == 0$ **then**

      Find the distance between the n^{th} FBS and all the FBS's in group s, $\{D_{ni}\}_{i=1}^{M_s}$;

      if $\{D_{ni}\}_{i=1}^{M_s} \geq D_{th}$ **then**

        if $N_n \leq C_s$ **then**

          Put FBS n in group s,

          Set $M_s = M_s + 1$, and $g_n = 1$

        end if.

      end if.

    end if.

    Set $n = n + 1$.

  until $n = K + 1$, *end repeat.*

  Set $s = s + 1$.

until all elements in $\mathcal{G}$ equal 1, *end repeat.*

Output the number of groups, and the members of each group.

in turn groups the new FBS.

2.4.2 Complexity

Similar to the FSU-based scheme, the worst-case complexity, defined as the number of operations required to group K FBS's when each FBS is assigned to a different group from the other FBS's (i.e., when no FBS satisfies both the distance and the category conditions for any previously formed group). The first FBS is assigned to the first group and then all the remaining $K - 1$ FBS's are to be examined. For the second group, $K-2$ FBS's are to be examined. For K FBS's, the total required number of operations is $\sum_{i=1}^{K}(K-i) = \frac{1}{2}K(K-1)$, and hence, the FBS-based grouping scheme has order of K^2 time complexity. Comparing this result to the one found in the previous Subsection, it is concluded that the FSU-based scheme is much more complex than the FBS-based grouping scheme. This is because the number of FBS's in the network (K) is generally smaller than the number of FSU's in the network (N), since each FBS usually serves more than one FSU.

For the case when a new FBS appears in the network, the complexity is defined as the average number of groups to be examined before finding a suitable group for the new FBS given that S groups have been already formed by the grouping scheme with M_s FBS members per each group. The new FBS will be assigned to Group s if it satisfies the distance threshold condition $\{D_{kj}\}_{j=1}^{M_s} \geq D_{th}$ (where D_{kj} is the distance from the k^{th} FBS to the j^{th} FBS member in Group s) and the category condition $N_k \leq C_s$. The category of each group (C) is determined by the number of FSU's in the first femtocell member of that group.

To find the average number of groups to be examined before a suitable group is found in the FBS-based scheme, let $l = 1, \dots, S$ be a random variable representing the number of groups to be examined before finding the group that satisfies the distance condition ($\{D_{kj}\}_{j=1}^{M_s} \geq D_{th}$) and the category condition ($N_k \leq C_s$). The probability that one group will be examined is the probability that the first group will satisfy the two conditions (i.e., $\Pr\{l = 1\} = \prod_{i=1}^{M_1} \Pr\{D_{ik} \geq D_{th}\} \times$

$\Pr\{N_k \leq C_1\})$. In the same way, the probability that 2 groups will be examined is the probability that the first group will fail to satisfy the condition and the second group will do so (i.e., $\Pr\{l = 2\} = [1 - \prod_{i=1}^{M_1}[\Pr\{D_{ik} \geq D_{th}\}] \times \Pr\{N_k \leq C_1\}] \prod_{i=1}^{M_2}[\Pr\{D_{ik} \geq D_{th}\}] \times \Pr\{N_k \leq C_2\})$. Generally, the probability that s groups will be examined can be written as

$$\begin{aligned} \Pr\{l = s\} = \prod_{i=1}^{s-1} \left[1 - \prod_{j=1}^{M_i}[\Pr\{D_{jk} \geq D_{th}\}] \times Pr\{N_k \leq C_j\}\right] \\ \times \prod_{v=1}^{M_s}[\Pr\{D_{vk} \geq D_{th}\}] \times Pr\{N_k \leq C_s\}. \end{aligned} \tag{2.6}$$

Based on the aforementioned discussion, the average number of examined groups before a suitable group is found ($\bar{l}$) can be written as

$$\begin{aligned} \bar{l} = \sum_{s=1}^{S} s \prod_{i=1}^{s-1} \left[1 - \prod_{j=1}^{M_i}[\Pr\{D_{jk} \geq D_{th}\}] \times Pr\{N_k \leq C_j\}\right] \times \\ \prod_{v=1}^{M_s}[\Pr\{D_{vk} \geq D_{th}\}] \times Pr\{N_k \leq C_s\} + S \prod_{a=1}^{S} \left[1 - \prod_{n=1}^{M_a}[\Pr\{D_{nk} \geq D_{th}\}] \times Pr\{N_k \leq C_n\}\right]. \end{aligned} \tag{2.7}$$

The expression in (2.7) depends on the probability that the distance between two FBS's is higher than D_{th}. When the FBS's are distributed using PPP, their locations will be uniformly and independently distributed in the macrocell region [62]. In this case, the probability that the distance between two FBS's inside the circular range of the macrocell (with radius R_M) is larger than D_{th} can be expressed as in (2.3), and is modified as in (2.4) to consider the constraint on D_{th}.

For FBS-based grouping, in addition to the distance distribution, a model is needed for the number of FSU's per femtocell. Assuming that the number of FSU's in a femtocell can be modelled as a Poisson r.v. with parameter λ representing the average number of FSU's per femtocell, the probability that the number of FSU's in a femtocell is lower or equal to some value is just the cdf of the Poisson r.v. which can be approximated using the χ^2 cdf [64] as follows

$$Pr\{N \leq m\} = 1 - F_{\chi^2}(2\lambda, 2(m+1)), \quad m \text{ is integer}, \tag{2.8}$$

where the cdf of the χ^2 r.v. is [65]

$$F_{\chi^2}(x, a) = \frac{\gamma(\frac{a}{2}, \frac{x}{2})}{\Gamma(\frac{a}{2})}, \tag{2.9}$$

where $\gamma(k, u) = \int_0^u t^{k-1} e^{-t} dt$ is the incomplete Gamma function.

Since there is a maximum number of users (N_{max}) that can be served by a FBS which depends on the architecture of the FBS itself, (2.9) should be modified to consider this condition as follows

$$\begin{aligned} Pr\{N \leq m | N \leq N_{max}\} = \frac{Pr\{N \leq m\}}{Pr\{N \leq N_{max}\}} = \\ \frac{1 - F_{\chi^2}(2\lambda, 2(m+1))}{1 - F_{\chi^2}(2\lambda, 2(N_{max}+1))}, \quad m \text{ is integer}. \end{aligned} \tag{2.10}$$

2.5 Simulation Results

In order to compare the FSU-based grouping to the FBS-based grouping, simulations are performed to find the average time needed to group per FBS and the required number of channels to be purchased per FBS for both schemes. The coverage radius of the macrocell is 100m, and is 20m for each femtocell The FBS's are distributed using PPP with a parameter defined as the FBS density in the network which is set to 0.05 FBS's/m. The number of FSU's in each FBS is modelled using a Poisson r.v. with parameter (λ) which depends on the maximum number of FSU's per FBS (for $N_{max} = 4, \lambda = 2$, and for $N_{max} = 8, \lambda = 4$). Algorithms 1 and 2 are implemented over a range of D_{th} values. Figure 2.3 shows the average number of channels needed to be purchased per one FBS for both the FSU-based and the FBS-based grouping schemes. At large values of D_{th}, the distance threshold condition becomes very difficult to be satisfied, but since in the FBS-based scheme an FBS which is assigned to a new group requires a number of channels equal to the number of FSU's served by it, this results in a slightly larger average number of channels to be purchased for the FBS-based method than for the FSU-based method. As D_{th} approaches double the coverage radius of the macrocell, both the FBS-based and the FSU-based schemes will converge to the maximum number of channels to purchased per FBS which is equal to the average number of FSU's served by each FBS.

For the average time to group, each algorithm is simulated separately using the same computer, and the memory is cleared after each implementation to ensure that the attained results serve the comparison between the two schemes. As shown in Fig. 2.4, the FBS-based scheme is much less-complex than the FSU-based scheme, and the time to group in the FBS-based scheme does not depend on the maximum number of FSU's per FBS, which is not the case for the FSU-

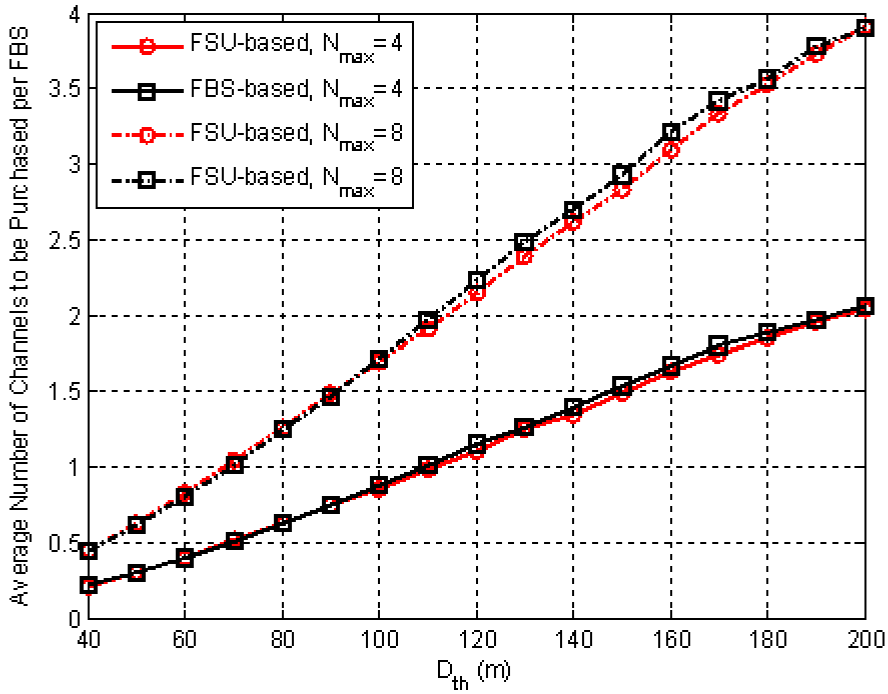

Figure 2.3: The average number of channels to be purchased per one FBS as a result of applying the FSU-based and the FBS-based grouping schemes versus D_{th} for FBS density in the network $\lambda = 0.05$.

based grouping scheme. Again, as D_{th} approaches double the coverage radius of the macrocell, both the FBS-based and the FSU-based approaches the maximum average time to group per FBS.

Next, the complexity of the update process, defined as the average number of examined groups before a suitable group is found for a new FSU/FBS, is plotted against D_{th} using (2.2) and (2.7) for the FSU-based and the FBS-based grouping schemes, respectively. Starting with the FSU-based grouping scheme, the relation between D_{th} and the average number of examined groups under different number of FSU's per group (M) is shown in Fig. 2.5. The number of groups is assumed to be 10. It is intuitive that with the increase of D_{th}, the average number of examined groups will increase, because it becomes harder to find a group that satisfies the

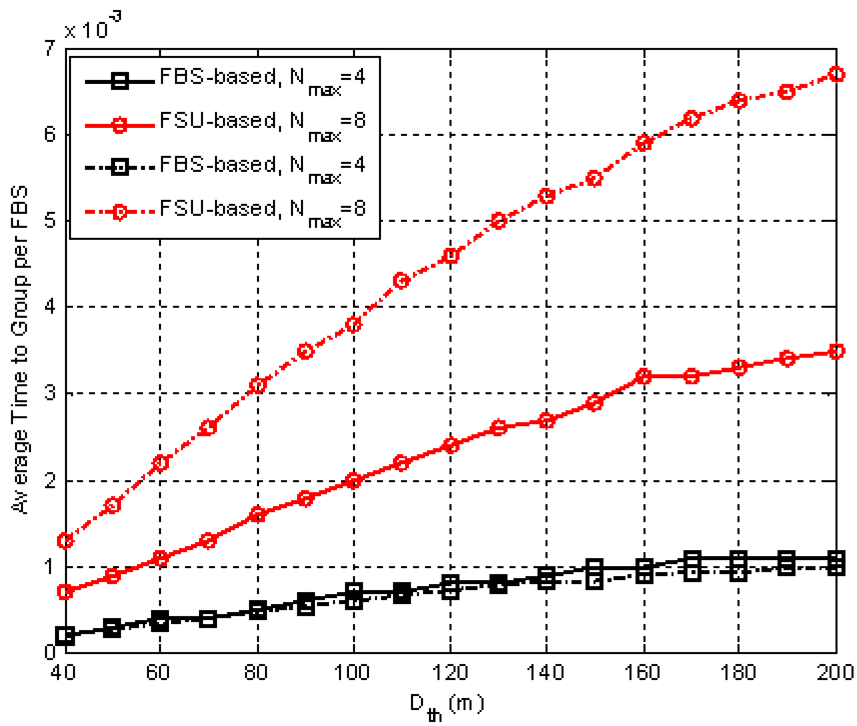

Figure 2.4: The average time to group per FBS for the FSU-based and the FBS-based grouping schemes versus D_{th} for FBS density in the network $\lambda = 0.05$.

condition $\{D_{ik} > D_{th}\}, \forall i$. The same intuition can be used to describe the raise of the curve as M increases.

What is more interesting is the relation between the density of groups and the resultant number of examined groups which is shown in Figure 2.6. The total number of groups is 5, and three cases have been studied here. In case 1, the process starts with the denser groups and ends with the less dense ones (M=[10 8 6 4 2]). In case 2, each group contains 6 FBS's. In case 3, in contrast to case 1, the process starts from the group with the lowest density and ends with the one of the highest density of FBS's.

Even though the total number of FBS's is the same for all cases (30 FBSs), the reduction in the average number of examined groups is very significant between

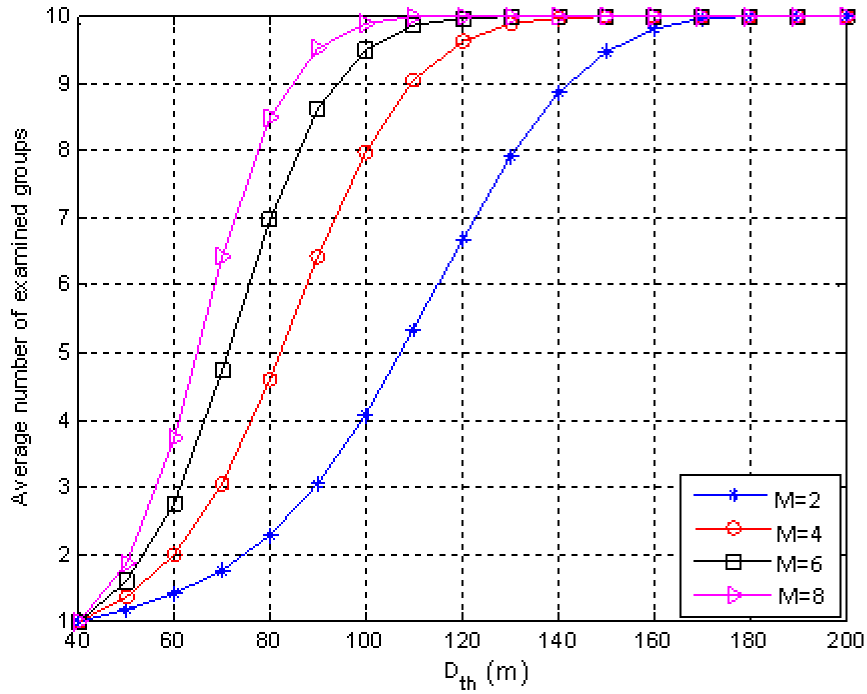

Figure 2.5: The average number of examined groups versus D_{th} for different number of FBS's per one group.

case 3 and case 1 especially for low values of D_{th}. This means that the performance of the scheme (in terms of complexity) can be greatly enhanced if the CBS sorts the groups in an ascending order after each grouping process. Of course, the overhead associated with sorting is not trivial when the algorithm is applied to all FBS's, but this happens only once. After that, only the groups that have a change in their number of members will be compared to the groups before and after until the right place for each group is arranged. This process needs number of operations that is equal to the number of groups in the worst-case which does not add a lot of complexity.

The effect of sorting the groups on the complexity of the FBS-based grouping scheme for three different cases is shown in Fig. 2.7. The first case is when

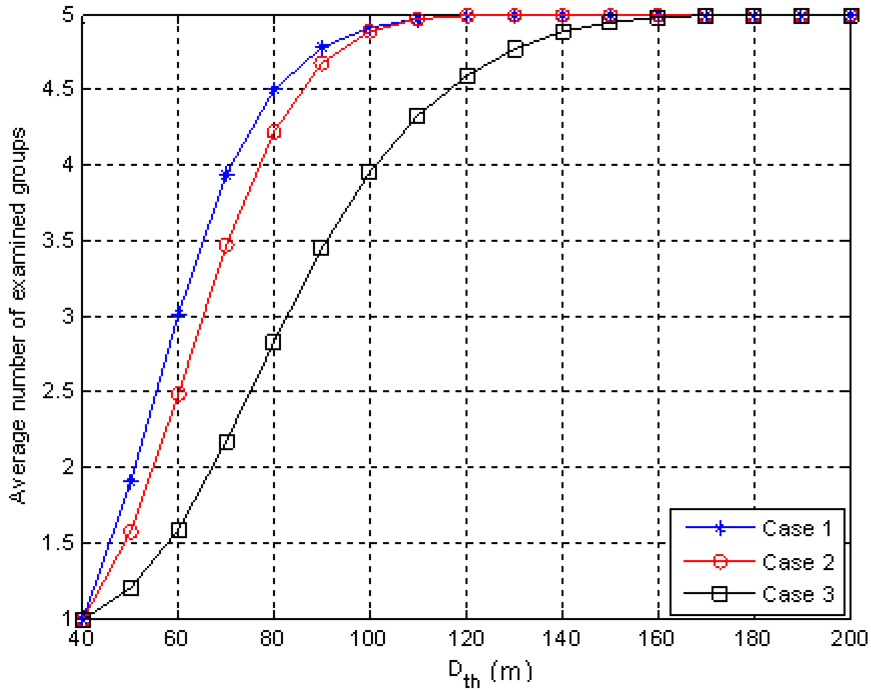

Figure 2.6: The average number of examined groups as a function of D_{th} for different cases. Case 1: M=[10 8 6 4 2]. Case 2: M=[6 6 6 6 6 6]. Case 3: M=[2 4 6 8 10].

the CBS sorts the groups according to their categories from the group with the smallest category to the one with the largest category (in an ascending order). The second case is when all the groups have the same category, and the third case is when the CBS sorts the groups in a descending order. The number of members is chosen such that the group with a higher category will have greater number of members than the group with smaller number of members which is likely to happen since with the increase of the category, the probability that the number of FSU's served by a FBS is lower or equal to the category becomes larger. It can be noticed here that for very small values of D_{th} the complexity of the grouping scheme is higher when the groups are being sorted in an ascending order than when they are being sorted in a descending order. This happens because the probability that the number of FSU's served by the FBS is lower than or equal to 1 is very small, so the CBS will need to examine over more groups on average.

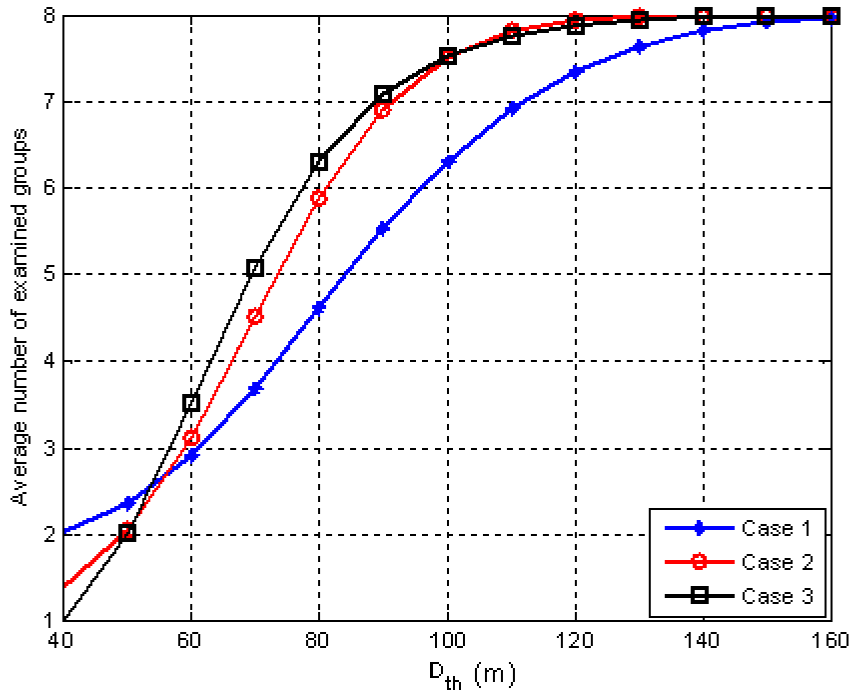

Figure 2.7: The average number of examined groups as a function of D_{th} for different cases of group categories. Case 1: C=[1 1 2 2 2 3 3 4], M=[2 2 3 4 5 7 7 10]. Case 2: C=[2 2 2 2 2 2 2 2], M=[5 5 5 5 5 5 5 5]. Case 3: C=[4 3 3 2 2 2 1 1], M=[10 7 7 5 4 3 2 2].

2.6 Summary and Conclusions

In this Chapter, two methods for implementing the grouping scheme are presented, namely the FSU-based and the FBS-based grouping methods. It was shown by simulations that the FSU-based scheme is slight more spectrum-efficient than the FBS-based scheme. On the other hand, the FBS-based scheme was shown to be much less-complex than the FSU-based scheme. Furthermore, the complexity of the update process for both the FSU-based and the FBS-based schemes was derived and analyzed, and it was shown by simulations that sorting the groups, according to their number of members for the FSU-based method and their category for the FBS-based method, in an ascent order generally reduces the complexity of the update process.

CHAPTER 3

PERFORMANCE ANALYSIS

3.1 Introduction

In this chapter, the average uplink outage probability for the FSU-based and the FBS-based grouping schemes is derived based on modelling the grouping process. The outage probability is defined as the probability that the signal-to-interference-and-noise ratio (SINR) at the FBS is lower than a certain threshold. Furthermore, the worst-case interference scenarios, upon which the optimization described in the next chapter will be performed, is illustrated.

This chapter is organized as follows: Section 3.2 illustrates the signal and the channel models under which the performance of the grouping schemes is to be examined. In Section 3.3, the general expression for the uplink outage probability is derived for both the FSU-based and the FBS-based grouping schemes. The worst-case interference scenario is described in Section 3.4. Simulations are provided in Section 3.5, and Section 3.6 summarizes and concludes the chapter.

3.2 Signal and Channel Models

In this chapter and the next chapters, the performance of the uplink channel is considered, an approach which has been adopted in many previous works due to the severity of the uplink channel in two-tier networks [60, 66, 67, 68]. On uplink, the received signal at the FBS from a desired FSU is defined as follows

$$P_d = r_d^{-n}\zeta, \tag{3.1}$$

where r_d is the distance from the desired FSU to the desired FBS, n is the path loss exponent, and ζ is a random variable (r.v.) modelling the channel gain from the desired FSU to the FBS. Similarly, the received signal from the i^{th} interfering FSU to the FBS is defined as follows

$$P_i = L_i r_i^{-n}\chi_i, \tag{3.2}$$

where L_i is the penetration loss representing the obstacles between the i^{th} interfering FSU and the FBS, r_i is the distance between the i^{th} interfering FSU and the FBS, and χ_i is a r.v. modelling the channel gain from the i^{th} interfering FSU to the FBS.

The performance analysis are conducted under two channel models. The first channel model, namely the only-shadowing model, is suited for suburban areas [66]. The second model, which is the composite shadowing-fading model, is more suited for urban areas [69]. The two channel models will be demonstrated here-after.

3.2.1 Suburban Areas with Low-Mobility

Suburban areas with low mobility are usually modelled as a slow fading environment where only path loss and shadowing are considered in the channel gain. Shadowing is usually modelled using the log-normal model, but recently the Gamma model started to gain wide popularity because of its analytical tractability and showing a good approximation for the log-normal shadowing model over a wide range of shadowing severity levels [65].

Referring to the Gamma model, the variations of the local mean received power (ξ) can be modelled as [65]

$$f_\xi(x) = \frac{1}{\Gamma(m_s)} \left(\frac{m_s}{\Omega_0}\right)^{m_s} x^{m_s-1} \exp\left(-\frac{m_s}{\Omega_0}x\right), \quad x > 0, m_s > 0, \tag{3.3}$$

where m_s is the shadowing parameter which indicates the severity of the shadowing effect (the larger m_s, the less the effect of shadowing), and Ω_0 is the average of the local mean received power at the desired FBS. The penetration and path losses can be incorporated in the means of the received local powers of the desired FSU (Ω_{0d}) and of the i^{th} interfering FSU (Ω_{0i}) as follows

$$\Omega_{0d} = r_d^{-n}, \quad \Omega_{0i} = L_i \times r_i^{-n}. \tag{3.4}$$

3.2.2 Urban Areas with Low-Mobility

Due to the simultaneous effect of both multipath fading and shadowing, the channel of low-mobility users in urban areas will suffer from composite fading [69]. Composite fading channels have been usually modelled by log-normal shadowing and Nakagami fading (Gamma power distribution). However, recently, the Gamma-Gamma (also called the generalized-K) model started to gain more attention due to its tractability and the availability of approximations with high accuracy for it [65, 69].

Referring to Nakagami's model, the instantaneous received power under small-scale multipath fading conditioned on the average local power Ω is modelled as a Gamma r.v. [70]

$$f_\gamma(y) = \frac{1}{\Gamma(m_m)} \left(\frac{m_m}{\Omega}\right)^{m_m} x^{m_m-1} \exp\left(-\frac{m_m}{\Omega} y\right), \quad y > 0, m_m > 0.5, \tag{3.5}$$

where m_m is the multipath fading parameter. The average local power is modelled as in (3.3), and the composite fading effect is approximated (by matching the first two moments) using a Gamma r.v. with the following parameters [65]

$$\kappa = \frac{m_m m_s}{m_m + m_s + 1 - m_m m_s \epsilon}, \quad \theta = \frac{\Omega}{\kappa}, \tag{3.6}$$

where κ and θ are the scale and the shape parameters of the composite fading r.v. respectively, and ϵ is the adjustment factor.

3.3 Uplink Outage Probability

3.3.1 FSU-Based Grouping

For each FSU served by the k^{th} FBS, the CBS will find a suitable group to assign that FBS to, and the channel of that group will be allocated to the FSU. Therefore, the outage event at the FBS can be defined as the event that the uplink SIR at the FBS from the FSU under consideration using the channel assigned to Group s will fall below a certain threshold, given that S groups have been formed by the grouping scheme, with M_s FSU members belonging to Group s, $\ s = 1, \dots, S$. So to find the outage probability, we start by finding the probability that a FSU is utilizing the uplink channel assigned to Group s. The probability that the FSU served by the k^{th} FBS is utilizing the channel belonging to the first group is the probability that the first group will satisfy the distance threshold condition (i.e., $\Pr\{s = 1\} = p_{k1} = \prod_{i=1}^{M_1}[\Pr\{D_{ik} \geq D_{th}\}]$). In the same way, the probability that the FSU is assigned to the second group is the probability that the first group does not satisfy the condition and the second group does so (i.e., $\Pr\{s = 2\} = p_{k2} = [1 - \prod_{i=1}^{M_1}[\Pr\{D_{1k} \geq D_{th}\}]] \prod_{i=1}^{M_2}[\Pr\{D_{2k} \geq D_{th}\}]$). Generally, the outage probability averaged over all the possible groups to which the FSU is likely to be assigned can be written as

$$
\begin{aligned}
P_{out}^{(k)} &= \sum_{s=1}^{S} \left[P_{out|s}^{(k)} \times p_{ks} \right] \\
&= \sum_{s=1}^{S} \prod_{i=1}^{s-1} \left(1 - \left(\Pr \left\{ D \geq D_{th} \right\} \right)^{M_i} \right) \qquad (3.7) \\
&\times \left(\Pr \left\{ D \geq D_{th} \right\} \right)^{M_s} \times P_{out|s}^{(k)},
\end{aligned}
$$

where, p_{ks} is the probability that the FSU served by the k^{th} FBS is utilizing the channel assigned to Group s, and $P_{out|s}^{(k)}$ is the outage probability given that the FSU under consideration is utilizing the channel assigned to Group s (depends on the number of the members of Group s, and on their distances from the k^{th} FBS). Hence

$$
P_{out|s}^{(k)} = \Pr \left\{ \frac{P_k}{\sum_{i \in V_s} P_i + \sigma^2} < a \right\}, \qquad (3.8)
$$

where P_k is the received power from the desired FSU, P_i is the received power from the i^{th} interferer, a is the SINR threshold, σ^2 is the noise power assuming an additive white Gaussian noise (AWGN), and V_s is a vector containing the indices of the FBS's serving the FSU members of Group s, which correspond to the indices of the interferers since only one FSU from each femtocell will interfere with the FSU's in other femtocells belonging to the same group.

The outage expression in (3.7) can be interpreted in two different ways. First, it is the average uplink outage probability resulting from the grouping scheme when applied over an arbitrary number of FSU's given that S groups have been formed by the grouping scheme, with M_s members in Group s. Second, this is the average outage probability on the uplink channel of a new FSU appearing in one of the femtocells.

3.3.2 FBS-Based Grouping

After assigning the k^{th} FBS the to Group s, the channel reserved for this group will be utilized by one of the FSU's served by that FBS. Therefore, the outage event at the FBS can be defined as the probability that the uplink SIR of a FSU using the channel belonging to Group s will go below a certain threshold given that S groups have been formed as a result of the grouping scheme, with M_s members in Group s. To find the outage probability, the probability that a FSU is utilizing the channel of Group s should be found. The probability that the FSU served by the k^{th} FBS is utilizing the uplink channel belonging to the first group is the probability that the first group satisfies the distance threshold condition ($\{D_{kj}\}_{j=1}^{M_s} \geq D_{th}$) and the category condition ($N_k \leq C_s$, where C_s is the category of Group s). The probability that one group will be examined is the probability that the first group satisfies the two conditions (i.e. $\Pr\{s = 1\} = p_{k1} = \prod_{i=1}^{M_1} \Pr\{D_{ik} \geq D_{th}\} \times \Pr\{N_k \leq C_1\}$). In the same way, the probability that 2 groups will be examined is the probability that the first group does not satisfy the two conditions and the second group does so (i.e. $\Pr\{s = 2\} = p_{k2} = [1 - \prod_{i=1}^{M_1}[\Pr\{D_{ik} \geq D_{th}\}] \times \Pr\{N_k \leq C_1\}] \prod_{i=1}^{M_2}[\Pr\{D_{ik} \geq D_{th}\}] \times \Pr\{N_k \leq C_2\}$). Generally, the uplink outage probability averaged over all the possible groups to which the FBS is likely to be assigned can be written as

$$
\begin{aligned}
P_{out}^{(k)} &= \sum_{s=1}^{S} [P_{out|s}^{(k)} \times p_{ks}] \\
&= \sum_{s=1}^{S} \prod_{i=1}^{s-1} \left[1 - (\Pr\{D \geq D_{th}\})^{M_i} \times \Pr\{N_k \leq C_i\}\right] \\
&\times (\Pr\{D \geq D_{th}\})^{M_s} \times \Pr\{N_k \leq C_s\} \times P_{out|s}^{(k)},
\end{aligned} \tag{3.9}
$$

where, $P_{out|s}$ is the outage probability for the FSU utilizing the channel belonging to Group s which is expressed as in (3.8).

As for the FSU-based grouping scheme, (3.9) can be interpreted as the average uplink outage probability resulting from applying the FBS-based grouping over an arbitrary number of FBS's, as it is the average outage probability on the uplink channel of a FSU belonging to a new FBS appearing in the network.

The remaining in deriving the average uplink outage probability is to derive (3.8), which depends on the distribution of interferers and their number around the desired FBS. This is discussed in details in the following section.

3.4 Worst-Case Interference Scenario

In this section, several worst-case scenarios are considered for finding (3.8). The worst-case assumptions are as follows. First, it is assumed that the FSU served by the desired FBS and all the FSU's served by the FBS's belonging to the same group are transmitting at the same time. Second, the interfering FSU's exist at the edge of their respective femtocells towards the desired FBS, and the desired

FSU is at the edge of its femtocell. Under this scenario, two cases are discussed, namely, the first-tier assumption used in [66], and the generalized worst-case.

3.4.1 First-Tier Assumption

In this case, only the first tier of interfering femtocells around the desired femtocell is considered. This is because other tiers can be ignored since they are at a distance of at least $2D_{th}$ from the desired FBS and more penetration losses are expected. All the first-tier interfering FBS's are placed at a distance equal to D_{th} from the desired FBS.

The worst-case interference scenario is shown in Fig. 3.1, where the distance from the desired FSU to its FBS is equal to the radius of the femtocell (R_F), and the distance from the interfering FSU to the desired FBS is $D_{th} - R_F$. Due to the distance threshold condition, the maximum possible number of interfering FBS's that can be placed at a distance of D_{th} from the desired FBS is six. Since a Gamma distribution is assumed to model shadowing/composite fading for both the desired and interfering FSU's, and taking into account the SIR expression in 3.8, it is required to find the distribution of the ratio of a Gamma r.v. to the sum of six i.i.d. Gamma r.v.'s which can be found using the expression in [71, p. 206]

$$\begin{aligned} p_{SIR}(z) = L^{\kappa_d} \left(\frac{R_F}{D_{th} - R_F} \right)^{n\kappa_d} \frac{\Gamma(\kappa_d + \sum_{i=1}^{6} \kappa_i)}{\Gamma(\kappa_d)\Gamma(\sum_{i=1}^{6} \kappa_i)} \\ \times \left(L \left(\frac{R_F}{D_{th} - R_F} \right)^{n} z + 1 \right)^{-(\kappa_d + \sum_{i=1}^{6} \kappa_i)} z^{\kappa_d - 1}, \quad z > 0. \end{aligned} \tag{3.10}$$

where κ_d and κ_i are the shape parameters of the composite fading r.v. for the desired and the interfering users' channels respectively (can be found from the shadowing and the multipath parameters using (3.6)), and L is the penetration loss which is assumed to be constant for all the interferers. The outage probability is just the cdf of the SIR

$$\begin{aligned}
&P_{out} = \Pr(SIR < b) = \int_{-\infty}^{b} p_{SIR}(z)dz \\
&= \frac{\Gamma(\kappa_d + \sum_{i=1}^{6} \kappa_i)}{\Gamma(\kappa_d)\Gamma(\sum_{i=1}^{6} \kappa_i)} \times b^{\kappa_d} L^{\kappa_d} \left(\frac{R_F}{D_{th} - R_F} \right)^{n\kappa_d} \times \\
&\frac{{}_2F_1 \left(\kappa_d, \kappa_d + \sum_{i=1}^{6} \kappa_i, 1 + \kappa_d, -bL(\frac{R_F}{D_{th}-R_F})^n \right)}{\kappa_d},
\end{aligned} \tag{3.11}$$

where ${}_2F_1$ is the hypergeometric function which represents the solution of the hypergeometric series [72], where the numbers before and after the F denote the number of parameters in the numerator and the denominator of the hypergeometric series, respectively; so ${}_2F_1$ implies two parameters in the numerator and one parameter in the denominator of the hypergeometric series [73].

Since the interferers are assumed to be placed at D_{th} from the desired FBS, it is expected that the optimization based on (3.11) will result in a conservative value of D_{th}.

3.4.2 Generalized Worst-Case

To perform less conservative optimization, the distance assumption is relaxed, and all the interfering FBS's are considered. Again, when uplink power control is assumed, placing the FSU's at the edge of their respective femtocells implies that they are using the maximum allowed power to transmit. Since a Gamma

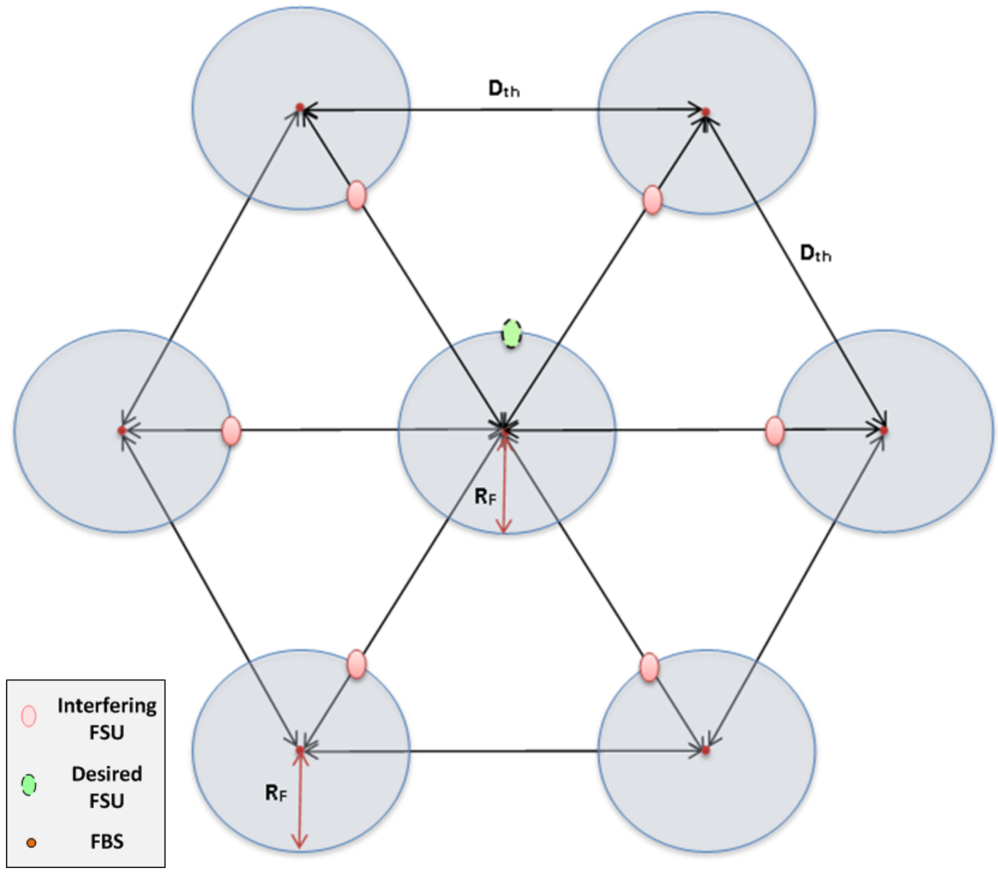

Figure 3.1: The worst case scenario with six interferers around the desired FBS, and both the desired FSU and the interfering FSU's placed at the edge of their femtocells. The maximum number of first-tier interferers at D_{th} is six.

distribution is assumed to model shadowing/composite fading for both the desired and interfering FSU's, it is needed to find the distribution of the ratio of a Gamma r.v. to the sum of independent non-identically distributed (i.n.d.) Gamma r.v.'s. In [74], a general closed-form expression for the sum of i.n.d. Gamma distribution was derived. The problem with that expression is that it puts a constraint on the shadowing parameter to be an integer, and the scaling parameter of any two interferers should be different. This makes the expression very limited and hardly useful for performance analysis, not to mention optimization.

Alternatively, an asymptotic expression for the distribution of the (maximum) of the sum of i.n.d. Gamma r.v.'s was derived in [75]. The problem with this expression is that it can not be used to find the distribution of the SINR in closed-form (the distribution of the ratio is in integral form), which makes the expression of the outage probability very complex to calculate (double integral expression).

In [76], an exact expression for the case of the ratio of Gamma and sum of i.n.d. Gamma r.v.'s was derived in one integral form. The expression will be modified here to incorporate noise by adding it to the outage expression as follows

$$P_{out|s} = \Pr\{a\xi_I - \xi_d > -a\sigma^2\} = \Pr\{\alpha > -a\sigma^2\}, \tag{3.12}$$

where $\alpha = b\xi_I - \xi_d$ is the test statistic (ξ_I is a r.v. representing the sum of Gamma interferers). Following the same approach used in [76], the outage probability is derived by finding the cdf of α ($F(x)$) and then finding the outage probability as $1 - F(-a\sigma^2)$. The cdf of α is written as [76]

$$\begin{aligned} &F_\alpha(x) = \Pr\{\alpha \le x\} = \frac{1}{2} - \frac{1}{\pi}\times \\ &\int_0^\infty \frac{\sin\left[\sum_{i\in V_s} m_{si}\tan^{-1}(at\theta_i) - m_{sd}\tan^{-1}(t\theta_d) - tx\right]}{[1+(t\theta_d)^2]^{m_{sd}/2}\prod_{i\in V_s}[1+(at\theta_i)^2]^{m_{si}/2}}dt, \end{aligned} \tag{3.13}$$

where m_{si} and θ_i are the shadowing and the scale parameters of the i^{th} interferer, respectively. Hence, the outage probability for the event when a suitable group is found for the new user is written as

$$
\begin{aligned}
&P_{out|s}(b) = 1 - F(-a\sigma^2) = \frac{1}{2} + \frac{1}{\pi}\times \\
&\int_0^\infty \frac{\sin[\sum_{i\in V_s} m_{si}\tan^{-1}(at\theta_i) - m_{sd}\tan^{-1}(t\theta_d) + a\sigma^2 t]}{[1+(t\theta_d)^2]^{m_{sd}/2}\prod_{i\in V_s}[1+(at\theta_i)^2]^{m_{si}/2}}dt,
\end{aligned}
\tag{3.14}
$$

Since the path loss and penetration loss are incorporated in the average received local power, the scale parameters of the desired user and the interfering user can be written, respectively as

$$
\theta_d = \frac{\Omega_d}{m_{sd}} = \frac{r_d^{-n}}{m_{sd}}, \qquad \theta_i = \frac{\Omega_i}{m_{si}} = \frac{L_i r_i^{-n}}{m_{si}}, \tag{3.15}
$$

where n is the path loss exponent, r_d and r_i are the distances from the desired FBS to the desired and interfering FSU's, respectively, and L_i is the penetration loss resulting from obstacles between the i^{th} interferer and the desired FBS.

Even though the expression in (3.14) can be utilized for performance analysis, it is very complex to be utilized for optimization purposes. In [77], an approximation for the distribution of the ratio of a Gamma r.v. to the sum of i.n.d. Gamma r.v.'s was obtained which can be written as

$$
\begin{aligned}
p_\gamma(\gamma) =& \frac{\Gamma\left(m_{sd}+m_{se}\right)}{\Gamma(m_{sd})\Gamma(m_{se})}\left(\frac{m_{se}}{\Omega_{0e}}\right)^{m_{se}}\left(\frac{m_{sd}}{\Omega_{0d}}\right)^{m_{sd}}\gamma^{m_{se}-1}\times \\
&\left(\frac{m_{se}}{\Omega_{0e}} + \frac{m_{sd}}{\Omega_{0d}}\gamma\right)^{-(m_{se}+m_{sd})} \qquad \gamma \geq 0,
\end{aligned}
\tag{3.16}
$$

where m_{sd} is the shadowing parameter of the desired FSU's channel, m_{se} and Ω_{0e} are the parameters of the approximate distribution for the sum of i.n.d. Gamma r.v.'s given, respectively as [77]

$$\Omega_{0e} = \sum_{i \in V_s} \Omega_{0i}, \qquad m_{se} \simeq \frac{\left(\sum_{i \in V_s} \Omega_{0i}\right)^2}{\sum_{i \in V_s} \frac{\Omega_{0i}^2}{m_{si}}}. \tag{3.17}$$

The outage probability is just the cdf of γ which can be expressed as

$$\begin{aligned} P_{out|s} = \Pr\{\gamma \leq a\} = \frac{\Gamma\left(m_{sd} + m_{se}\right)}{\Gamma(m_{sd})\Gamma(m_{se})} \left(\frac{\Omega_{0e}}{\Omega_{0d}}\right)^{m_{sd}} \times a^{m_{sd}} \times \\ \frac{{}_2F_1\left(m_{sd}, m_{sd} + m_{se}, 1 + m_{sd}, -a\left(\frac{\Omega_{0e}}{\Omega_{0d}}\right)\right)}{m_{sd}}, \end{aligned} \tag{3.18}$$

where ${}_2F_1(.)$ denotes the hypergeometric function. Since the distances from the desired FBS to the desired and the interfering FSU's are incorporated in the mean of the received power, it is important to emphasize here that the outage expression in (3.18) is conditioned on the number of members in Group s and their distances from the desired FBS.

3.5 Numerical Results

In this Section, the coverage radius of the macrocell and of each femtocell is assumed to be circular with coverage radii $R_m = 100m, R_F = 20m$, respectively. The FBS's are dispersed using PPP with a parameter denoting the FBS density in the network fixed at 4. The number of FSU's per one femtocell is modelled using a Poisson r.v. with an average equal to $\lambda = 2$ and a maximum number equal to 4. The SIR threshold is fixed at 10dB, the outage threshold is fixed at 10^{-3}, and the penetration loss is assumed to be 15dB. The noise spectral density is fixed at -174dB/Hz and the bandwidth of each channel is 200KHz. If not mentioned, the channels from an interfering FSU to FBS and desired FSU to FBS are modelled using a gamma r.v. with shadowing and fading parameters $(m_s, m_m) = [(1, 1), (4.23, 4)]$, for the composite fading environment, and with shadowing parameters $m_s = 1, 4.23$, for the shadowing environment.

Figure 3.2 shows the uplink outage probability found using the first-tier worst-case expression in (3.11) , the general exact (3.14) and the approximate (3.18) expressions for different values of the path loss exponent n. The first observation is the monotonic reduction in the uplink outage probability with the increase of the value of D_{th} which is expected since, with the increase of the value of D_{th}, the number of members in each group is smaller and thus, less interference is expected. It should be emphasized here that the outage probability value will saturate at values of D_{th} larger or equal to the radius of the macrocell (R_M) since each FSU/FBS will definitely be assigned to a distinct group in this case. Thus, the outage probability is generally a non-increasing function of D_{th}, but is monotonically decreasing with $D_{th} \in [0, R_M]$. This fact will be used in Chapter 4 to find the minimum value of D_{th} that achieves a certain (maximum) expected outage probability.

The second observation is the reduction in the uplink outage probability with the increase of n. This happens because, since the interfering FSU's are at a relatively large distance from the FBS when compared to the desired FSU, the path loss exponent has greater influence on the channel gains from the interfering FSU's to the desired FBS. Furthermore, it can be noticed that the general approximation is more accurate than the first-tier expression when compared to the general exact outage probability expression. The performance of the group-

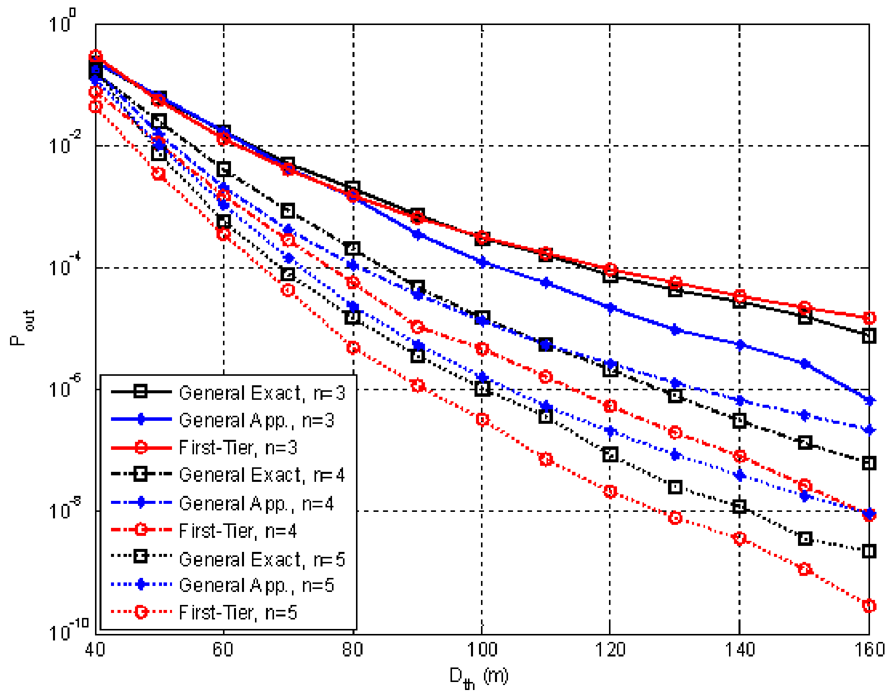

Figure 3.2: The uplink outage proability as a function of D_{th} for different values of the path loss exponent.

ing scheme is examined under the composite fading channel and the shadowing channel for different values of n as shown in Fig. 3.3. It can be noticed that the grouping scheme performs much better under the shadowing environment which represents a less-severe environment than the composite fading one. The performance of the FSU-based grouping scheme in terms of the resultant uplink outage probability is compared to that of the FBS-based grouping scheme under shadowing and composite fading channels in Fig.'s 3.4 and 3.5, respectively. As shown in the figures, the FSU-based and the FBS-based grouping schemes achieve the

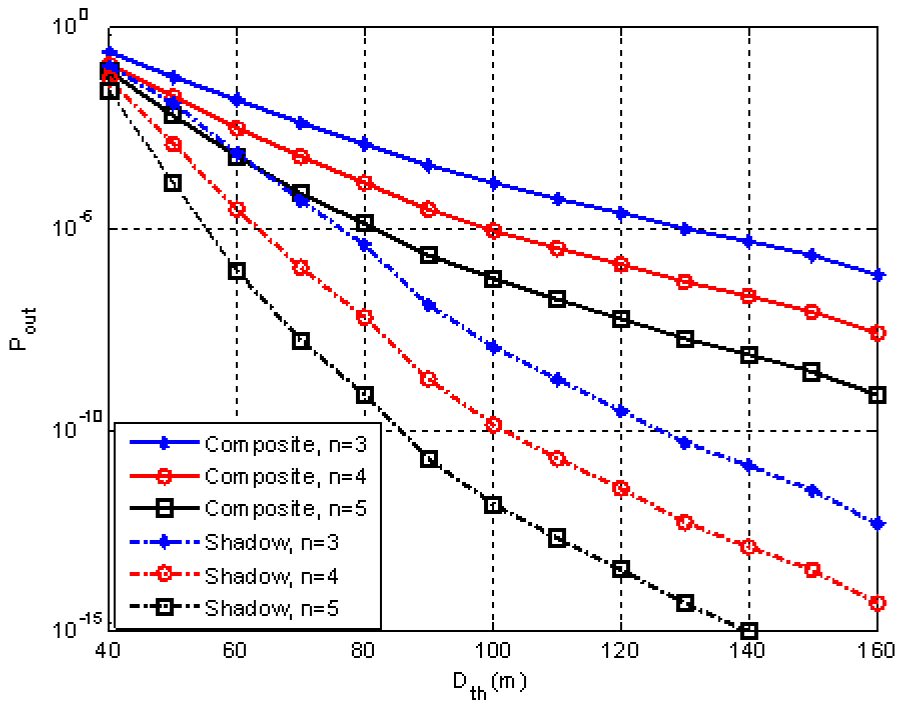

Figure 3.3: The uplink outage proaibility as a function of D_{th} for different values of the path loss exponent under shadow-only and composite shadow-multipath fading channels.

same uplink outage probability performance.

To examine the outage performance of the update process for the FSU-based grouping scheme, semi-analytic analysis are performed under composite fading environment assuming the same shadowing and multipath fading parameters for both the desired FSU and the interfering FSU's ($m_s = 1, m_m = 1$). The path loss exponent n is fixed at 3, and the penetration loss is fixed at 10 dB. The uplink outage probability as a function of D_{th} is shown in Fig. 3.6 for three cases of the distribution of the FSU's among the groups. The first case is when the CBS starts from the group with the largest number of members and ends with the group that has the smallest number of members. The second case is when M is the same for all groups, and the last case is when the CBS starts from the group with the

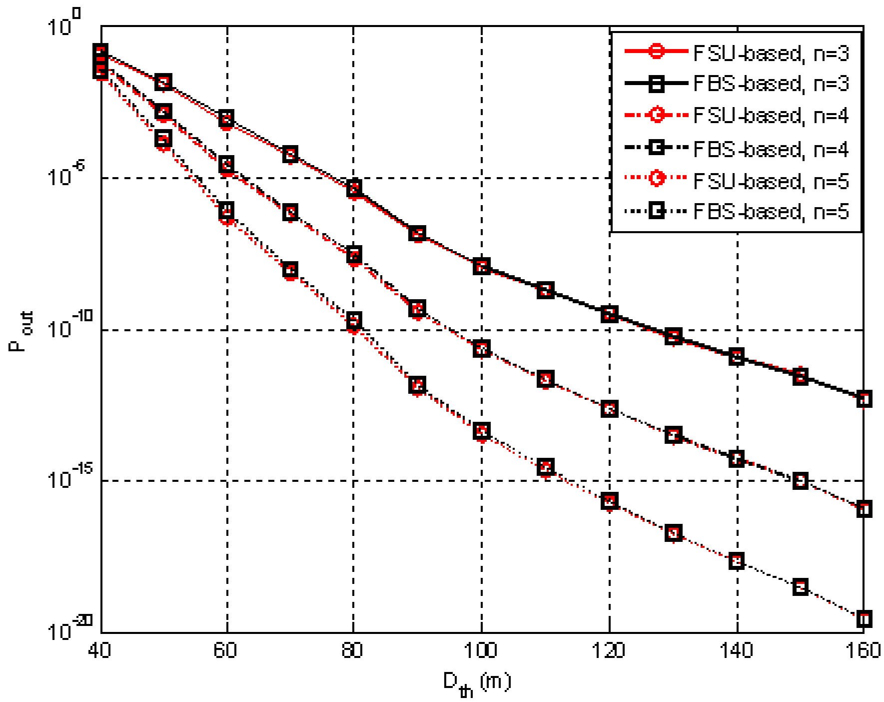

Figure 3.4: The uplink outage probability against D_{th} for the FBS-based and the FSU-based schemes under shadow fading channels.

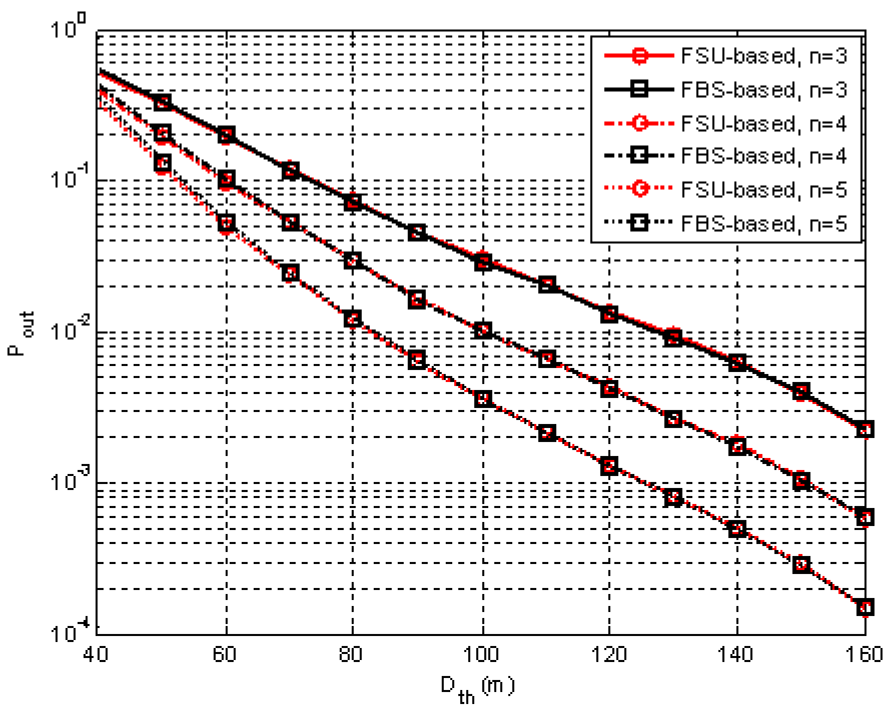

Figure 3.5: The uplink outage probability against D_{th} for the FBS-based and the FSU-based schemes under composite fading channels.

smallest number of members up to the one with the largest number of members. The first and the third cases can be seen as different ways of implementing the update process by the CBS, where the first case represents sorting the groups according to their number of members in a descending order, and the third case represents sorting the groups in an ascending order.

As shown in the figure, sorting the groups in an ascending order will result in a lower outage probability at small values of D_{th} as compared to sorting them in a descending order. For large values of D_{th}, the new FSU is much more likely to be assigned to the groups with smaller number of members regardless to the sorting order since the probability that the distance will be larger than D_{th} is very low, and this will result in the same outage probability for both the first and the third cases. For the second case, when the value of D_{th} is large, it is not likely that the new FSU will be assigned to any group since all of them has the same, relatively large, number of members. Instead, the FSU will be assigned to a new group which will result in a lower average probability.

For the FBS-based grouping, each FBS is assumed to serve only one FSU, and the path loss exponent is fixed at 4. The outage probability versus D_{th} for three cases of performing the FBS-based grouping scheme is shown in Fig. 3.7. The first case is when the CBS sorts the groups according to their categories from the group with the smallest category to the one with the largest category (in an ascending order). The second case is when all the groups have the same category, and the third case is when the CBS sorts the groups in a descending order. The

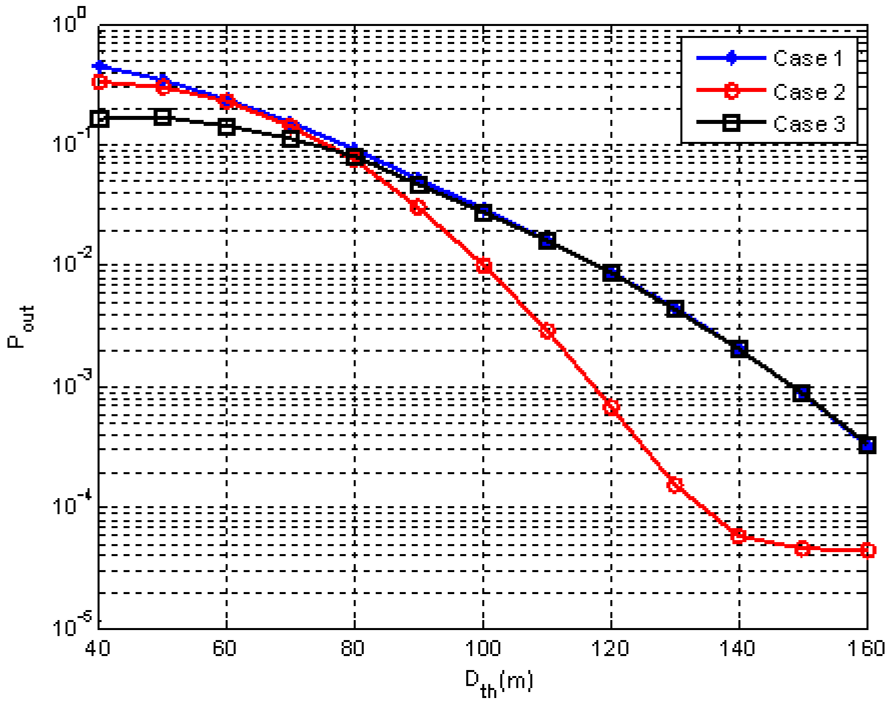

Figure 3.6: The uplink outage proability as a function of D_{th} for different cases. Case 1: N=[10 8 6 4 2]. Case 2: N=[6 6 6 6 6 6]. Case 3: N=[2 4 6 8 10]. $n = 3$.

number of members is chosen such that the group with a larger category will have greater number of members than the group with smaller category which is likely to happen since with the increase of the category, the probability that the number of FSU's served by a FBS is smaller or equal to the category becomes larger. The same observation can be made on the effect of sorting as for the FSU-based scheme, where sorting the groups in a descending order according to their categories (equivalent to sorting the groups in an ascending order according their number of members) results in a better outage performance than sorting them in an ascending order (equivalent to sorting the groups in a descending order according their number of members) for small values of D_{th}.

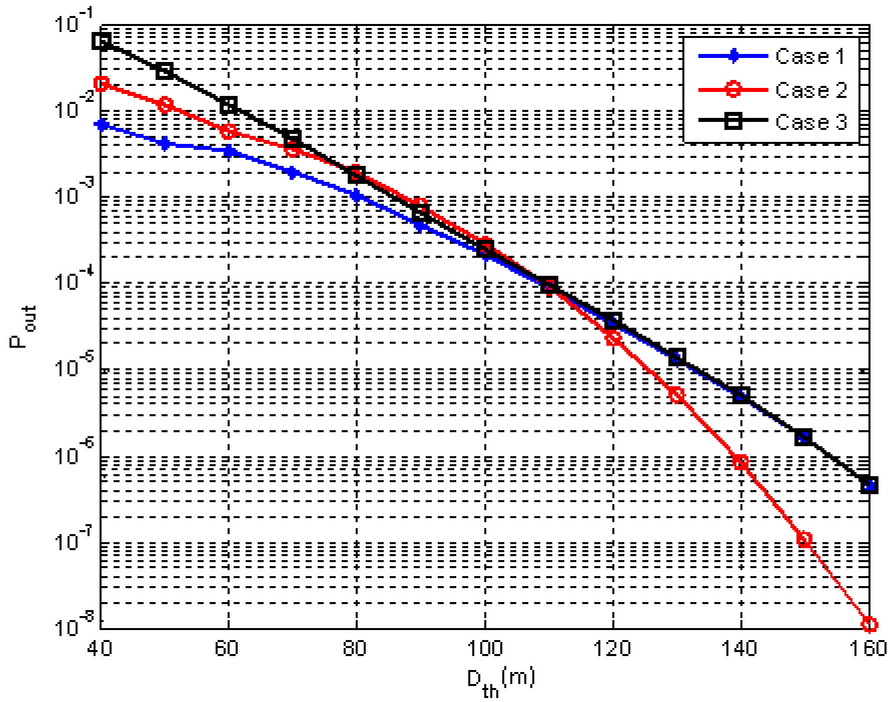

Figure 3.7: The uplink outage proabibility as a function of D_{th} for different cases of group categories. Case 1: C=[1 1 2 2 2 3 3 4], N=[2 2 3 4 5 7 7 10]. Case 2: C=[2 2 2 2 2 2 2 2], N=[5 5 5 5 5 5 5 5]. Case 3: C=[4 3 3 2 2 2 1 1], N=[10 7 7 5 4 3 2 2]. $n = 4$.

3.6 Summary and Conclusions

In this Chapter, the FSU-based and the FBS-based grouping schemes are compared in terms of the resultant uplink outage probability, where the FSU-based grouping scheme shows better outage performance than the FBS-based one. Furthermore, the performance of the update process, defined as the uplink outage probability of a new FSU appearing in the network, is derived and simulated for both schemes, and it is shown by simulations that sorting the groups according to their number of members in an ascending order results in a lower outage probability than when they are being sorted in a descending order for small values of D_{th}.

CHAPTER 4

OPTIMIZATION OF THE GROUPING SCHEME

4.1 Introduction

The grouping schemes presented in Chapter 2 assume fixed value of D_{th} for each group, and in Chapter 3, it was shown that the outage probability is a monotonically decreasing function of D_{th}. Since our objective is to maximize the profit of the CBS, then it is intuitive to choose the minimum value of D_{th}. However, if there is a minimum QoS that should be guaranteed by the SU operator for the SU's, then this puts a constraint on the outage probability and accordingly, it puts a constraint on the value of D_{th} that can be chosen. On the other hand, if the objective is just to maximize the profit of the CBS without any QoS constraints, then other approaches other than the distance-based grouping can be used by the CBS to attain higher profits.

In this chapter, several methods for optimizing and enhancing the grouping scheme are proposed. Namely, the distance-based optimization method based on the worst-case interference scenario, the CBS profit maximization method based on the greedy approach, and the co-channel deployment extensions. Simulations are provided to evaluate the performance of each method.

This Chapter is organized as follows. Section 4.2 presents the D_{th} minimization algorithm. The CBS profit maximization algorithm is demonstrated in 4.3. The co-channel deployment extensions are illustrated in 4.4. The simulation results are shown and discussed in Section 4.5, and Section 4.6 provides the summary and conclusions.

4.2 Minimization of D_{th}

The grouping scheme described in Chapter 2 is done at a fixed value of D_{th} while in this Section, a different (minimum) value of D_{th} is determined for each group to optimize the grouping scheme performance under the QoS constraint. The optimization problem to be solved here is to find the minimum D_{th} while maintaining some desired outage performance which depends on the data rate requirements of the FSU's.

With the increase of D_{th}, it becomes more difficult to find FBS's that satisfy the distance threshold condition described in Chapter 2. This results in increasing the resultant number of groups, and hence increasing the number of channels to be purchased from the PU network. Therefore, to minimize the number of channels to be purchased, the minimum value of D_{th} required for certain QoS needs to be found. The smaller the value of D_{th}, the greater the number of members per group, which means more interference on each channel. In other words, fixing the required QoS dictates the minimum SIR to be maintained which dictates the interference level allowed in each group, which determines the number of allowed group members. Thus, some constraint should be put on D_{th} such that the minimum QoS is maintained on each channel.

Here, the uplink outage probability is considered as the constraint, so the optimization problem to be solved is to find the minimum value of D_{th} while maintaining some desired uplink outage performance which depends on the uplink data rate requirements of the FSU's. For each group, the SIR threshold is chosen such that the minimum QoS can be provided by the FBS to all of its FSU's. Therefore, the optimization problem for Group s is formulated as follows

$$\begin{aligned} &\text{minimize} \quad D_{th}^{(s)} \\ &\text{subject to} \;\; P_{out|s}^{(max)} \leq P_{out}^{(th)}, \end{aligned} \tag{4.1}$$

where $D_{th}^{(s)}$ is the threshold distance of Group s, $P_{out}^{(th)}$ is the minimum outage threshold needed for the set QoS, and $P_{out|s}^{(max)}$ is the maximum uplink outage probability on the channels belonging to Group s.

The value of $P_{out|s}^{(max)}$ can be found using (3.18) by performing grouping with a certain value of D_{th} for Group s, and calculating the outage probability at each FBS and, after that, finding the maximum outage probability. Since the outage probability is a monotonically decreasing function with D_{th}, then the well-known bisection method can be utilized to find the optimum value of D_{th} for Group s as follows. The algorithm searches for a value of D_{th} at the left of the optimum one (with higher outage probability) and for another value of D_{th} at the right to the optimum (with lower outage probability). The CBS starts with a small value of D_{th} (can be chosen to be any value larger than 0, but recommended to be larger than R_F) and performs grouping for the first group, and it determines the maximum outage probability among the members of the first group. If this outage probability is lower than the outage threshold, the CBS decreases the first value by some increment and re-group until the value becomes larger than the outage threshold and set the last two values of D_{th} as the range for applying the bisection method. If the maximum outage probability is larger than the outage threshold, the CBS increases the first value by the same increment, performs grouping, and finds the maximum outage probability. If this outage probability is larger than the outage threshold, then the CBS will keep increasing and grouping until the maximum outage probability becomes lower than the outage threshold.

After finding the range, the bisection method, combined with iterative grouping to find the maximum outage probability, can be utilized to find the optimum value of D_{th}. The bisection method is based on finding the average of the two limits of the range and examining whether the average should replace the value at the left or at the right of the range depending on whether the resultant maximum outage probability is larger or smaller than the outage threshold. The bisection algorithm terminates when the difference between the maximum outage probability and the outage probability threshold is less than some small value. Finally, the CBS will use the optimum D_{th} to find the members of Group s. This process is repeated for each group. The whole operation is summarized in Algorithm 3.

Algorithm 3 D_{th} Minimization Using Iterative Grouping and Bisection:

1: *Initialization: the set of FBS's* $\Psi_K = \{1, 2, ..., K\}$, *the set of FBS's locations* $\{G_k\}_{k=1}^{K}$, *and the set of grouping indices* $\mathcal{G} = g_1, g_2, ..., g_K$. *Each FBS serves* N_k *ungrouped users. The indices of the members of each group* s *belongs to the vector* (V_s) *which is empty at start.*
2: *Set* $s = 1$.
3: **repeat**
4: *Find the value* A *of* D_{th} *for which* $P_{out}^{(max)}$ *is larger than the threshold outage probability.*
5: *Find the value* B *of* D_{th} *for which* $P_{out}^{(max)}$ *is lower than the threshold outage probability.*
6: *Use bisection method and iterative grouping to find the optimum* D_{th} *from the range* $[A, B]$.
7: *Perform grouping for Group* s *using the optimum threshold* $D_{th}^{(s)}$.
8: Set $s = s + 1$.
9: **until** all elements in $\mathcal{G}$ equal 1, *end repeat.*
10: *Output the number of groups, and the members of each group.*

4.3 Maximization of CBS Profit

The aim in the previous Section was to minimize to number of purchased channels while maintaining a target maximum outage probability. This can serve the situation when the offers from PU networks are insufficient for the needs of the SU's. However, it is not necessary that this strategy will maximize the CBS profit. This is because minimizing the number of purchased channels will result in a smaller value of the SIR for each FSU, resulting in a lower sum rate (profit) for the CBS.

Here, we assume that the CBS profit is directly proportional to the sum rate of the FSU's. To illustrate, let us consider a simplified version of the CBS utility function (represents the profit of the CBS on the l^{th} channel) presented in [58]

$$\pi_l = \sum_{k=1}^{N} c_b w_l \varrho_{lk} x_{lk} - c_l w_l, \tag{4.2}$$

where N is the total number of FSU's in the network, c_b is the cost paid by a FSU for using the channel l which is assumed to be the same for all FSU's, w_l is the bandwidth of channel l which represents the channel bandwidth required to serve a FSU regardless of the application, ϱ_{lk} is the spectrum efficiency [12] or the energy efficiency [58] of user transmission on channel l, $x_{lk} \in \{0, 1\}$ is the spectrum allocation index which is set to 1 when the l^{th} channel is allocated to the k^{th} FSU, and c_l is the price offered by the PU network for purchasing/renting the l^{th} channel. When adaptive modulation is utilized, the spectrum efficiency of user transmission can be obtained as follows [12]

$$\varrho = \log_2(1 + J\gamma), \quad \text{where} \quad J = \frac{1.5}{\ln(0.2/BER^{(t)})}, \tag{4.3}$$

where γ is the SINR (SIR when there are interferers, and SNR when no interfering FSU's), and $BER^{(t)}$ is the target bit error rate (BER).

The utility function in (4.2) represents the profit of the CBS from one channel. Therefore, two choices are available for each channel; the first choice is to allocate the channel only to one FSU and in this case the maximum CBS profit can be attained on each channel but more channels should be purchased. The other choice is to allocate the channel to more than one FSU and in this case the number of channels is reduced, but the revenue gained from each user is lower due to interference which affects the spectrum/energy efficiency of user transmission. Therefore, the problem that needs to be solved here is to determine how many groups should be formed and how many FSU's should be put in each group such that the CBS profit is maximized on the channel assigned on each group, which will result in maximizing the total CBS profit over all the channels.

The direct way to solve such a problem is by performing an extensive search over all the possible set of FSU's for each group and choosing the set that results in the maximum expected profit for the CBS. However, the complexity of this solution is $O(2^N)$ where N is the number of FSU's. For large number of FSU's, such a solution will be time consuming since the complexity of the algorithm increases exponentially with the number of FSU's. Instead, we propose the use of the greedy approach, which is a very well-known approach in the context of optimization and resource allocation in femtocell networks [78, 79, 80], to reduce the complexity of the algorithm to $O(N^2)$ which is a polynomial-time complexity.

The proposed algorithm is similar to the distance-based grouping algorithms described in Sections 2.3 and 2.4. The CBS starts with the first FSU and assigns it to the first group. Then, the CBS finds the expected profit from assigning the second FSU to the first group and compares it to the profit of the first FSU being alone in the group. If the first quantity is larger, the CBS assigns the second FSU to the first group, and sets its expected profit as the maximum profit which will be the benchmark for the next comparisons. Otherwise, the CBS examines the third FSU and so on, till the last FSU is examined. The process is repeated until all the FSU's are grouped, where a FSU is assigned to a group if the following condition is satisfied

$$\pi_{l|M} > \pi_{l|M-1}^{(max)}, \tag{4.4}$$

where $\pi_{l|M}$ is the expected sum profit of the CBS from assigning M FSU's to the group, and $\pi_{l|M-1}^{(max)}$ is the maximum expected sum profit given that $M-1$ FSU's have been already assigned to the group. The whole operation is illustrated in Algorithm 4 where π_{NI} is the CBS profit resulting from assigning the channel to only one FSU (i.e., interference-free conditions) and is found as follows

$$\pi_{NI} = c_b \varrho_{NI} - c = c_b \log_2(1 + J\ \mathrm{SNR}) - c, \tag{4.5}$$

where ϱ_{NI} is the spectrum efficiency for interference-free conditions (depends only on the distance between the FSU and the desired FBS which affects the uplink SNR at the FBS). It is assumed here that all the channels with the same bandwidth have the same price, so the profit is normalized by the bandwidth of the channel.

It is important to point out here that the greedy algorithm does not usually find the ultimate optimal point, but in many cases it ends up finding a local optimum rather than getting the ultimate one. The optimal point can be found by using the N-path greedy solution [81] where for each group, the CBS finds

Algorithm 4 Greedy Algorithm for the Maximization of the CBS Sum Profit:

Initialization: the set of FBS's $\Omega_K = \{1, 2, ..., K\}$, *the set of FBS's locations* $\{G_k\}_{k=1}^{K}$, *and the set of grouping indices* $\mathcal{G} = g_1, g_2, ..., g_K$. *Each FBS serves* N_k *ungrouped users.*

Set $s = 1$.

repeat

  Denote the number of FSU's in Group s *by* U_s, *and set* U_s *to zero.*

  Find the first FBS k *with grouping index* $g_k = 0$, *and assign it to Group* s.

  Set $n = k + 1$, $U_s = U_s + 1$, *and* $N_k = N_k - 1$.

  Set $\pi^{(max)} == \pi^{(NI)}$.

  if $N_k == 0$ **then**

    Set $g_k = 0$;

  end if.

  repeat

    if $g_n == 0$ **then**

      Find the distance between the FBS n and all the FBS's in Group s, $\{D_{ni}\}_{i=1}^{U_s}$;

      Find the expected profit of the CBS from assigning the FSU served by the FBS n to Group s, π_{U_s+1}.

      if $\pi_{U_s+1} > \pi^{(max)}$ **then**

        Assign FBS n to group s,

        Set $U_s = U_s + 1$, $N_n = N_n - 1$, and $\pi^{(max)} = \pi_{U_s+1}$

        if $N_n == 0$ **then**

          Set $g_n = 1$;

        end if;

      end if.

    end if.

    Set $n = n + 1$.

  until $n = K + 1$, *end repeat.*

  Set $s = s + 1$.

until all elements in G equal 1, *end repeat.*

Output the number of groups, and the members of each group.

the optimum set of members starting from each ungrouped FSU. It then chooses the set that has the maximum expected profit. Nevertheless, this enhancement in profit is attained at the cost of more processing time since the complexity of the N-path solution is $O(N^3)$ since it is just the greedy algorithm repeated N times (for each FSU).

The problem in finding the spectrum efficiency in (4.3) is that, since the CBS still does not have the channels, the SIR cannot be measured before the channel is purchased from the PU network. To overcome this problem, the CBS should try to estimate the SIR under the worst-case scenario described in Chapter 3. Under composite fading channels, and considering the worst-case conditions, the pdf of the SIR is just the pdf of the ratio of a gamma r.v. to the sum of i.n.d. gamma r.v.'s which can be found using the approximation derived in [77], and the expected SIR for the approximated pdf is written as

$$E\{\mathrm{SIR}\} = \frac{\Omega_d}{\Omega_e}\frac{m_e}{m_e - 1}, \quad m_e > 1, \tag{4.6}$$

where Ω_e and m_e are as defined in Eq. (3.17).

In addition to (4.2), which is linear in the spectrum demand, we consider the use of the quadratic utility function used in [58] to evaluate the performance of the greedy algorithm, but with the following modifications:

- The FBS's and MSU's cannot switch spectrum. This assumption is used to simplify the equation by excluding the part which corresponds to the risk aversion (by setting the substitutability parameter to 1), since we are focusing on the spectrum allocation problem.
- To completely ignore the price competition between the PU networks, it is assumed that the spectrum bands/channels offered have the same bandwidth at the same high price (assuming that collusion is established and maintained by the PU networks [12]).

- Assuming that the FSU's are to be grouped, each channel may be utilized by several FSU's simultaneously.

Based on these changes, the profit of the CBS, on the channel assigned to Group s, can be written as

$$\pi_{CBS} = \sum_{k=1}^{M_s} w \varrho_{ks} c_b - \frac{1}{2} w^2 - cw, \tag{4.7}$$

where w is the spectrum demand which is expressed in terms of bandwidth or number of channels, ϱ_{ks} is the spectrum efficiency of FSU k utilizing the channel assigned to Group s, c is the price of one unit of spectrum, and M_s is the number of FSU's utilizing the channel assigned to Group s (for the case of no grouping, this is equal to one, whereas, for the case of grouping, it is equal to the number of members in the group). Due to the assumption of the bandwidth per channel, the spectrum demand w is a common factor in the equation. Hence, normalizing (4.7) by the spectrum demand (w) we get the average profit per unit bandwidth as follows

$$\bar{\pi}_{CBS} = \sum_{k=1}^{M_s} \varrho_{ks} c_b - \frac{1}{2} w - c. \tag{4.8}$$

Furthermore, we consider the practical scenario in which the number of channels offered by the PU networks is limited, and compare the CBS expected profit of the greedy algorithm to that of the distance-based grouping scheme. Therefore, assuming that the total number of channels offered by the PU networks is fixed at B, the expected total CBS profit can be written as

$$\bar{\pi}_{CBS}^{total} = \begin{cases} \sum_{s=1}^{S} \sum_{k=1}^{M_s} \varrho_{ks} c_b - \frac{1}{2} wS - cS, & S \leq B; \\ \sum_{s=1}^{B} \sum_{k=1}^{M_s} \varrho_{ks} c_b - \frac{1}{2} wB - cB, & S > B. \end{cases} \tag{4.9}$$

4.4 Grouping with Co-Channel Deployment

To further reduce the number of channels to be purchased by the CBS from the PU networks, the groups of FBS's/FSU's should be allowed to use some of the spectrum allocated to the MSU's. This can be achieved by adding the MSU's to the FBS/FSU groups after making sure that the MSU's will not cause harmful uplink interference to any of the FSU's and that the FSU's belonging to the same group will not cause harmful uplink interference to the MSU's. It should be emphasized here that assigning a MSU to a group means that the channel allocated to that MSU can be utilized by all the members of the group. Therefore, assuming that each group has a pool of channels which is empty at the start, each MSU assigned to the group will add one channel to the pool of that group. Two methods for the extension of the grouping scheme to the co-channel deployment scenario will be examined in the following two subsections. The first one, namely the distance-based co-channel deployment grouping, is less complex, but it does not ensure the QoS of the MSU's. The other method, which is the outage-based grouping, is reliable and maintains the QoS for both the FSU's and the MSU's, but at the cost of more complexity.

Conventional uplink power control is assumed to be utilized by the MSU's [82], where the MSU transmits either at a power level enough to compensate for the channel gain or at the maximum allowed power if it cannot compensate for the channel gain. That is, the Transmit power of the MSU is written as [83]

$$P_t = \min(P_{max}, P_0/\gamma), \tag{4.10}$$

where P_{max} is the maximum allowed power for the MSU's, P_0 is design parameter used to set some desired SINR at the base station [82], and γ is the composite fading channel gain modelled as in (3.5).

4.4.1 Distance-Based Grouping

To ensure that no harmful interference is posed by either the MSU or the FSU, and assuming that uplink power control is used by both of them, two conditions need to be satisfied. The first condition is that the distance between the FSU and the CBS (D_F) should be larger than the radius of the femtocell (R_F). The second condition is that the distance between the MSU and the CBS (D_M) should be smaller than the distance between the MSU and FBS (D_{MF}). The first condition ensures that the virtual range of the FSU does not reach the CBS even when the FSU is at the edge of its femtocell. The second condition ensures that the virtual range of the MSU will not reach the FBS. A third condition to be satisfied is $D_{MF} < D_F$; that is, the MSU should be closer to the CBS than all the FSU's served by the FBS's belonging to the same group. This condition is needed for two reasons; first, the efficiency of the first condition in limiting the interference depends on the maximum transmit power allowed for the FSU, the severity of the composite fading channel for the FSU, and the coverage radius of the femtocell. Second, even if the signal received at the CBS from one FSU is very weak, the aggregate interference from the FSU's served by all the members of the group may be significant. An example of a MSU and a group of FBS's satisfying the three conditions is illustrated in Fig. 4.1.

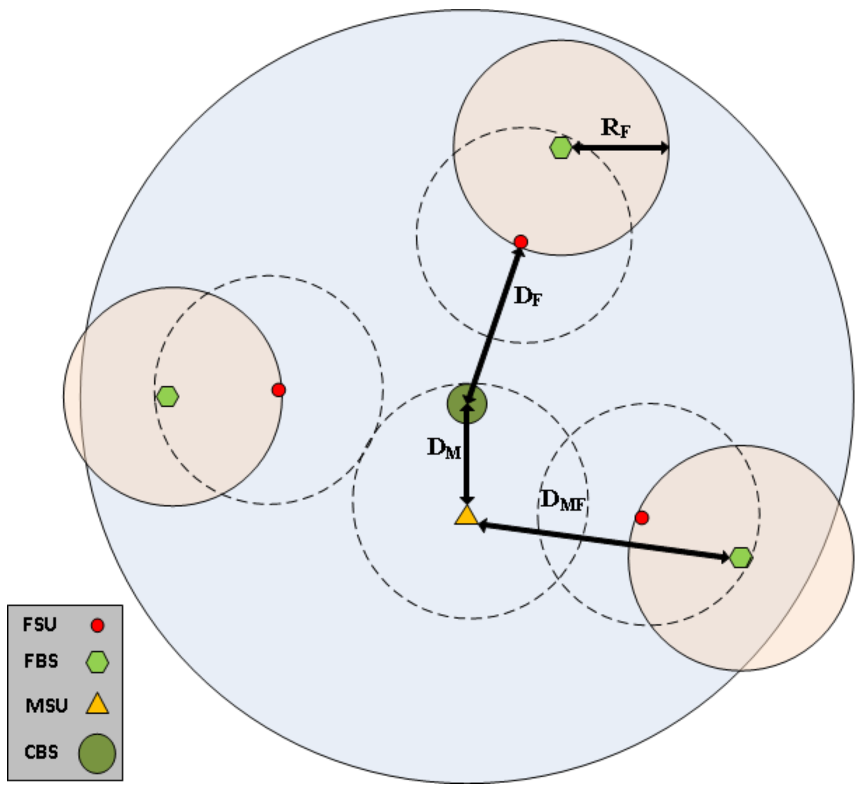

Figure 4.1: An example of a MSU and a group of FBS's that satisfy the three distance conditions.

The distance-based MSU grouping algorithm is initialized with the groups formed using the grouping scheme described in chapter 2 (FSU-based or FBS-based). The CBS starts with the first MSU, and searches for the first group that satisfies the three distance conditions for that MSU and assigns the MSU to that group. The channel of the MSU will be added to the channel pool of the group. This operation is repeated for all MSU's. The whole process is illustrated in Algorithm 5.

Algorithm 5 MSU's Distance-Based Grouping:

Initialization: S groups, each has a set of FBS members V_s, and the channel pool of each group which is empty at start. The set of MSU's $\Psi_I = \{1, 2, ..., I\}$, and the set of their grouping indices $\mathcal{I} = g_1, g_2, ..., g_I$.
Set $i = 1$.
while $i \neq I$ **do**
    Search for a group that satisfies the three distance conditions for MSU i and assign the MSU to this group. Otherwise assign MSU i to a new group and set the category of the group to 1. Update $\mathcal{I}$ and V_s.
    $i = i + 1$.
end while
Output the pool of channels and the category of each group.

4.4.2 Outage-Based Grouping

To maintain the QoS of both the MSU's and the FSU's, the CBS should make sure that adding a MSU to a group does not result in an average uplink outage probability, for both the FSU's and the MSU, larger than the outage probability threshold. This is not ensured by the distance-based MSU grouping scheme. The scheme is initialized with the groups formed using the (optimized) FBS/FSU-based grouping scheme described in Chapter 2, and the CBS starts with the first MSU. The CBS searches for a suitable group and assigns the MSU to that group. A group is considered suitable for the MSU if the resultant outage probability for both the MSU and the FSU's in the group, assuming they are transmitting simultaneously, is less than the outage threshold. The channel of the MSU will be added to the channel pool of the group. This operation is repeated for all MSU's. The whole process is illustrated in Algorithm 6.

Algorithm 6 MSU's Outage-Based Grouping:

1: *Initialization: S groups, each has a set of FBS members V_s, and the channel pool of each group which is empty at start. The set of MSU's* $\Psi_I = \{1, 2, ..., I\}$, *and the set of their grouping indices* $\mathcal{I} = g_1, g_2, ..., g_I$.
2: *Set* $i = 1$.
3: **while** $i \neq I$ **do**
4:     *Search for a group that satisfies the outage threshold condition for MSU i and assign the MSU to this group. Otherwise assign MSU i to a new group and set the category of the group to 1. Update $\mathcal{I}$ and V_s.*
5:     $i = i + 1$.
6: **end while**
7: *Output the pool of channels and the category of each group.*

4.4.3 CBS Profit Maximization

As in the distance-based co-channel deployment extensions described in the previous two parts, the MSU's will be added to the groups of FSU's which have been formed in Algorithm 4. However, the condition that needs to be satisfied has nothing to do with the QoS or the outage probability. For each MSU and each FBS group, the CBS has two choices: either to purchase two orthogonal channels for the MSU and the group of FSU's or to add the MSU to the group of FSU's and purchase one channel only. The CBS will add a MSU to a group of FSU's only if the sharing the channel will result in a higher expected profit for the CBS than when the MSU is assigned an orthogonal channel. Therefore, the condition to be satisfied by the MSU to assign it to a certain group is

$$\bar{\pi}_{Co} \geq \bar{\pi}_{NI}, \tag{4.11}$$

where $\bar{\pi}_{Co}$ is the expected sum profit when one channel is purchased and shared among the MSU and the group of FSU's, and $\bar{\pi}_{NI}$ is the expected sum profit when two orthogonal channels are purchased (i.e., when there is no interference between the MSU and the group of FSU's). The whole process is summarized in Algorithm 7

Algorithm 7 MSU's Profit Maximization Grouping:

1: *Initialization: S groups, each has a set of FSU members V_s, and the channel pool of each group which is empty at start. The set of MSU's* $\Psi_I = \{1, 2, ..., I\}$, *and the set of their grouping indices* $\mathcal{I} = g_1, g_2, ..., g_I$.
2: *Set* $i = 1$.
3: **while** $i \neq I$ **do**
4:     *Search for a group of FSU's that satisfies the condition* $\bar{\pi}_{Co} \geq \bar{\pi}_{NI}$ *for MSU* i *and assign the MSU to this group. Update* $\mathcal{I}$ *and* V_s.
5:     $i = i + 1$.
6: **end while**
7: *Output the pool of channels for each group.*

An MSU which is not added to a group of FSU's is assigned to a separate group. After adding the MSU's to the groups of FSU's, the CBS purchases the number of channels enough to serve the groups if the offered spectrum is sufficient. If the spectrum is insufficient to serve all the groups (i.e., the number of channels is equal to $B < S$ where S is the number of groups), the CBS sorts the groups in a descending order according to the sum of spectrum efficiencies of the members of each group and allocates the channels to the first B groups.

4.5 Simulation Results

4.5.1 Maximization of CBS Profit

In order to compare the greedy algorithm with the case of no grouping and grouping by D_{th} minimization, system-level simulation is performed assuming the same path loss exponent for both the desired FSU and the interfering FSU's. The shadowing parameter for the desired FSU is $m_{sd} = 4.23$ (corresponds to a shadow spread of 4 dB) and the multipath fading parameter for the desired FSU is $m_{md} = 2$. The shadowing and the multipath fading parameters for each interfering FSU are assumed to be $m_{si} = 2, m_{mi} = 4$ respectively. The penetration loss is fixed at 15 dB, and the target BER is 10^{-6}. The offered price per spectrum channel is $c = 20$, and the cost paid by each FSU to the CBS is $c_b = 1$. Only one FSU per FBS is assumed in the simulations. The noise spectral density is -174dBm/Hz, and the bandwidth of each channel is 200 KHz. The outage threshold is fixed at 10^{-3}, and the SIR threshold is fixed at 10 dB.

Figures 4.2, 4.3, and 4.4 show the expected CBS profit, normalized by the channel bandwidth, against the FBS density in the network for n=3, 4, and 5 respectively. For n=3 (implies severe interference conditions and high interference-free spectrum efficiency), the cases of no grouping and grouping using the greedy algorithm perform much better than the case of grouping by D_{th} minimization. This happens because, since the D_{th} minimization is concerned with minimizing the required number of channels while ensuring some minimum required QoS, and does not observe the CBS profit. In particular, it tries to reduce the number of groups by assigning more members to each group which, under sever interference conditions, will result in a very low profit sum for each group. For less severe interference conditions, as shown in Fig. 4.3, the D_{th} minimization method outperforms the case of no grouping in terms of profit since adding more members does not affect the SIR significantly in this case, but still the greedy algorithm achieves the best profit. Finally, for $n = 5$, the D_{th} minimization algorithm outperforms the greedy algorithm because it is focused at reducing the number of groups and the interference is not a big issue in this case.

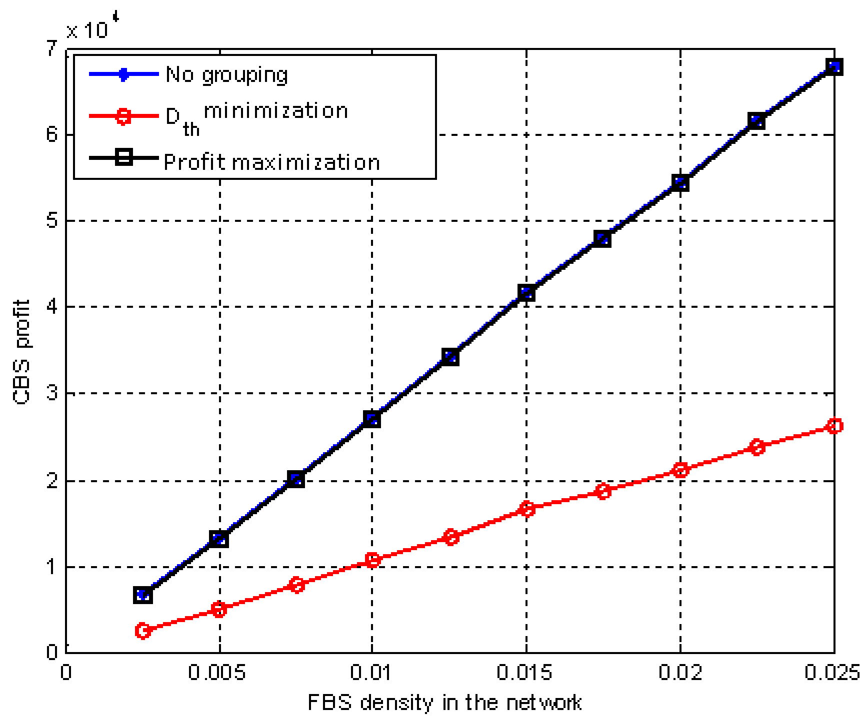

Figure 4.2: CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 3$.

To compare the performance of the CBS profit maximization algorithm to the distance-based grouping scheme under spectrum insufficiency environment, (4.9) is utilized to find the profit of the CBS with the total number of spectrum channels

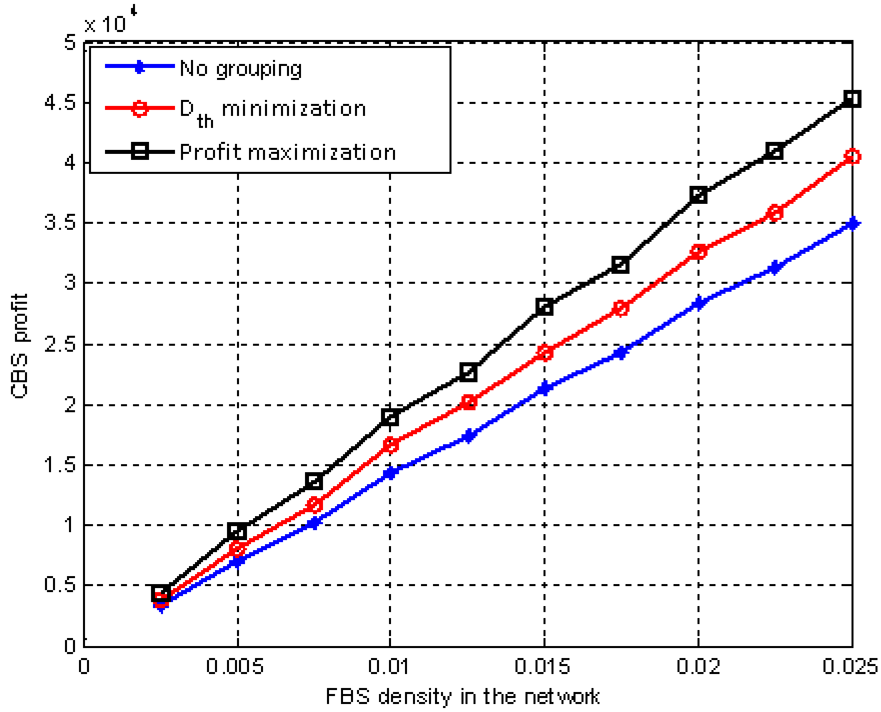

Figure 4.3: Expected CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 4$.

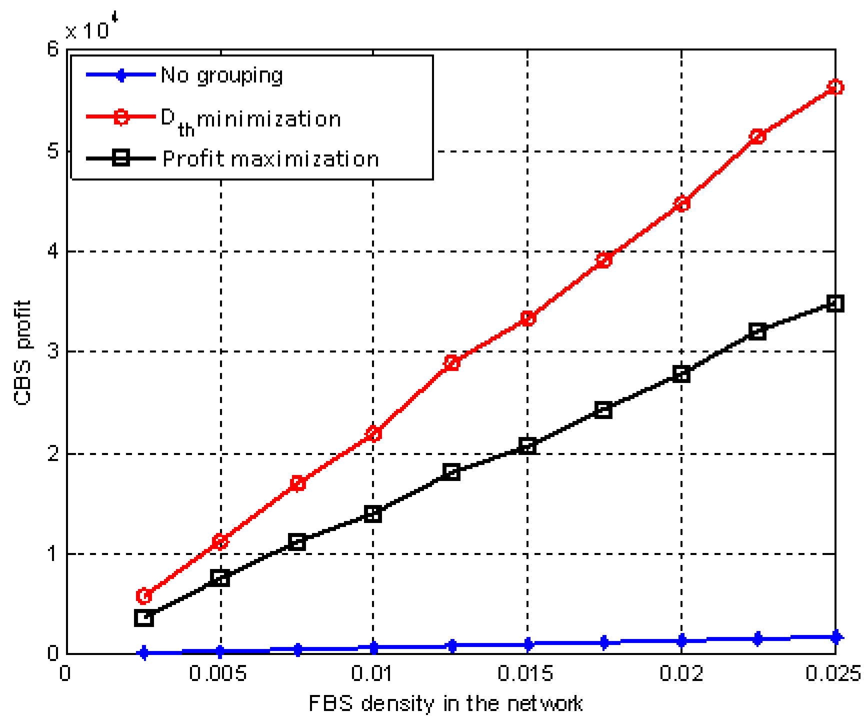

Figure 4.4: Expected CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 5$.

offered by the PU networks is $B = 100$, and the price of each channel is raised to 25. Fig. 4.5 shows the normalized CBS profit for $n = 3$, wherein the same can be observed regarding the resultant CBS profit from the distance-based grouping as compared to the one resulting from the greedy algorithm and the no-grouping case except for the limit on the profit of both the greedy algorithm and the no-grouping case which is caused by the limit of the number of channels offered from the PU networks.

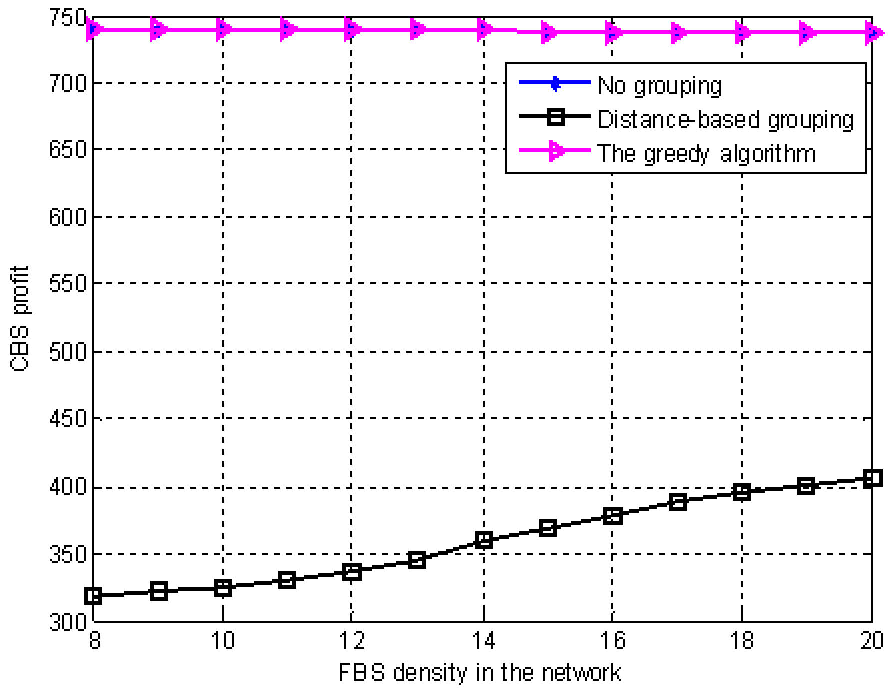

Figure 4.5: Expected CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 3$.

What is really interesting is what can be observed for the cases of $n = 4$ and $n = 5$. As expected, since the no-grouping profit depends on the spectrum efficiency of each FSU which is affected by the path loss exponent. So for large path loss exponent, and due to the limit on the number of channels to be purchased, the CBS will get a relatively small profit or it will lose under sever path loss environments. Furthermore, it can be noticed that the resultant number of groups for the distance-based grouping scheme is larger, which is reflected on the faster saturation of the CBS profit for the distance-based grouping scheme as compared to the greedy algorithm.

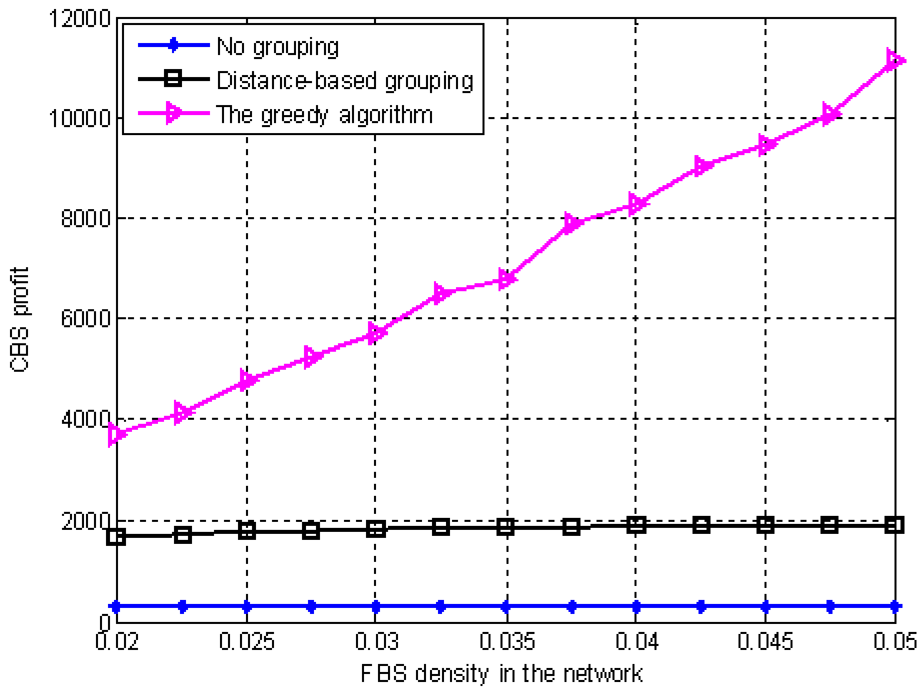

Figure 4.6: Expected CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 4$.

So the greedy algorithm seems to provide better performance in terms of the CBS profit under high spectrum costs and spectrum insufficiency conditions. However, two drawbacks of the greedy algorithm may prohibit the use of it:

- The greedy algorithm does not guarantee any QoS, so it cannot be applied to QoS-guaranteed services.

- The usage of the expression in (4.6) to find the expected SIR is limited with

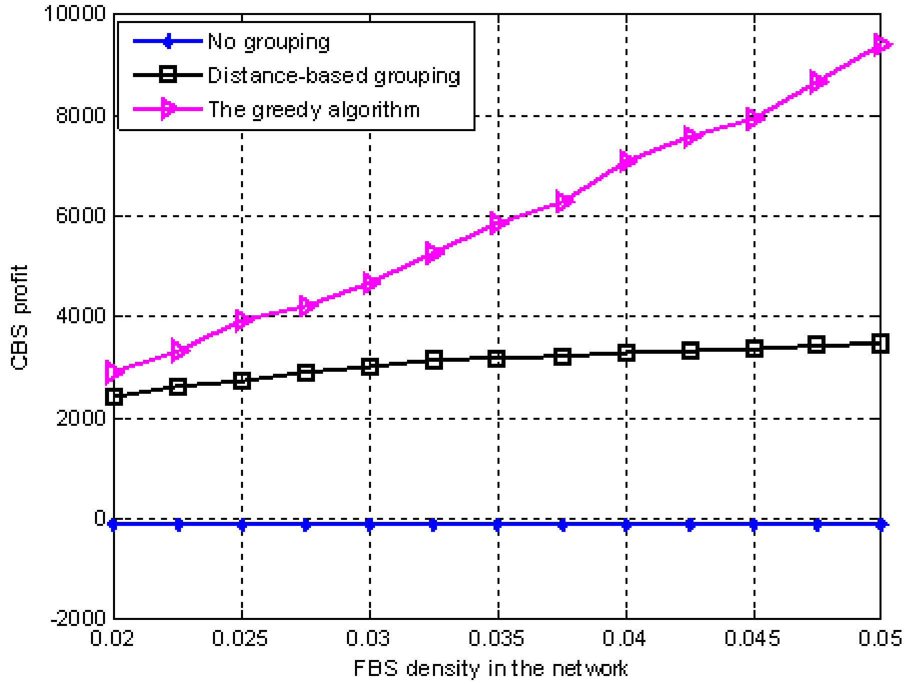

Figure 4.7: Expected CBS profit for the cases of no grouping, grouping by D_{th} minimization, and the profit maximization algorithm. $n = 5$.

the constraint that m_e should be larger than one, and this renders the greedy algorithm inapplicable under severe composite fading channel environments (i.e., when m_e is smaller than one).

Figure 4.8 shows the CBS profit maximization algorithm with orthogonal channel deployment and co-channel deployment (i.e., when the MSU's are added to the groups of FSU's) for different values of the path loss exponent n. The same set of parameters is used as in the previous figures except for the existence of the MSU's. The FBS's constitutes 60 percent of the total number of MSU's and FBS's in the network. It can be noticed that adding the MSU's to the groups of FSU's does not always help to increase the profit of the CBS.

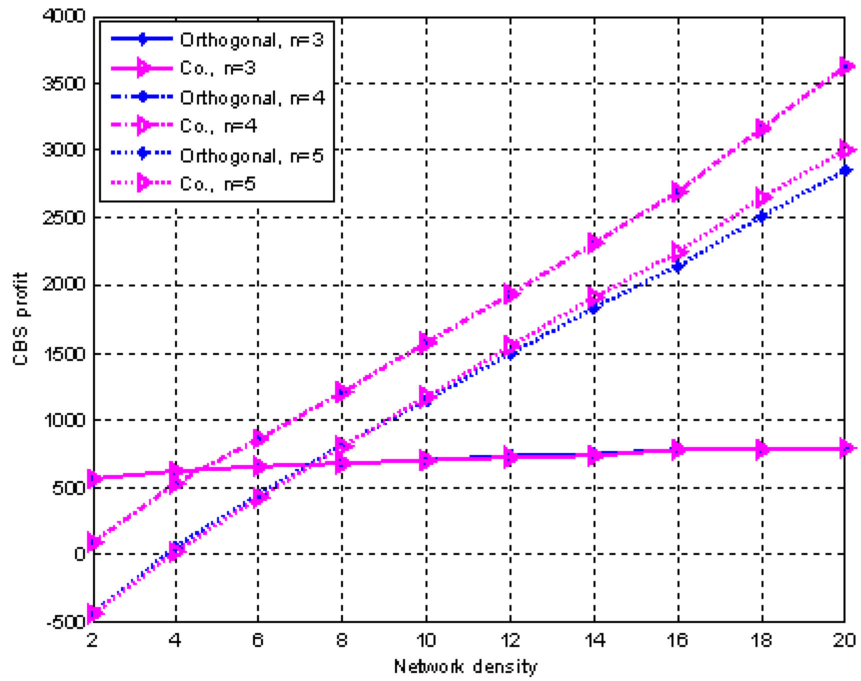

Figure 4.8: Expected CBS profit for the cases of orthogonal channel deployment and co-channel deployment.

4.5.2 Distance-Based Grouping with Co-Channel Deployment

In this part, the performance of the co-channel deployment extension is compared to the orthogonal scenario, which is based on the optimization of the grouping scheme using Algorithm 3, in terms of the average resulting number of channels to be purchased from the PU networks and the average resultant outage probability. We start with the distance-based co-channel deployment extension method, and then move to the outage-based method. In the simulations, the coverage radius of the macrocell is 100m, and the coverage radius of each femtocell is 20m. The FBS's and the MSU's are distributed using the Point Poisson Process with an average number defined as the network density, and the number of FSU's per femtocell follows a Poisson distribution with parameter λ=2 and maximum number of four FSU's per femtocell. The FBS's constitute 40% of the total number of MSU's and FBS's in the network. The SIR threshold is fixed at 10dB, the outage threshold is fixed at 10^{-3}, and the penetration loss is assumed to be 15dB. The noise spectral density is fixed at -174dB/Hz and the bandwidth of each channel is 200KHz.

The channels from an interfering FSU to FBS, desired FSU to FBS, interfering FSU to CBS, and MSU to CBS are modelled using a gamma r.v. with shadowing and fading parameters $(m_s, m_m) = [(1,1),(4.23,4),(1,1),(2.5,4)]$ (shadowing parameter m_s=$1, 2.5, 4.23$ correspond to shadow spread σ_s=$7, 5, 4$dB). To evaluate the outage performance of the scheme, both Eq.'s (3.14) and (3.18) can be used.

Figure 4.9 shows the average number of channels to be purchased from the primary networks as a function of the network density (the average number of FBS's and MSU's per meter). The figure compares the case of no grouping with the cases of grouping with and without using co-channel deployment under different values of the path loss exponent n which is used to quantify different levels of interference severity (the larger the path loss exponent, the lower the interference). A significant reduction in the number of spectrum channels to be purchased when grouping can be observed regardless to the path loss exponent value. Furthermore, it can be noticed that adding MSU's to the groups can help to further reduce the number of channel to be purchased from the PU networks.

To examine whether the scheme maintains the outage performance or not, the average uplink outage probability is plotted as a function of the network density

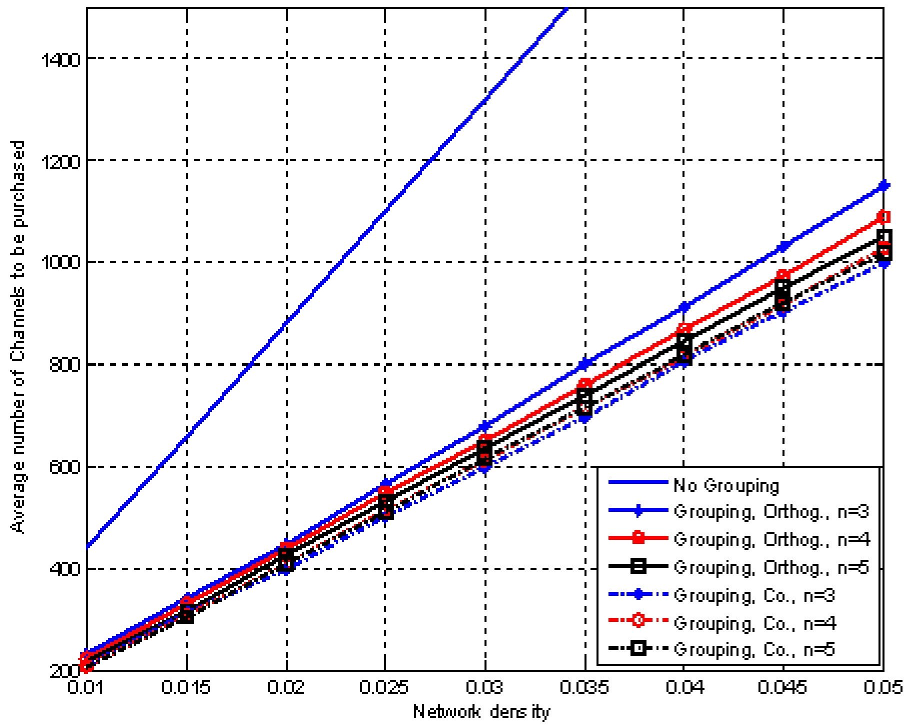

Figure 4.9: Average number of groups resulting from the grouping scheme, with and without the use of the distance-based co-channel-deployment, as a function of the FBS density in the network over three values of the path loss exponent, compared to the case of no grouping.

for the same set of parameters described for the previous plot. The result is shown in Figure 4.10 where it can be noticed that the (optimized) grouping scheme can maintain the required outage probability threshold for the FSU's, but because of co-channel deployment, the MSU's will suffer from a larger outage probability which, depending on the channel environment and the SIR threshold, may degrade their uplink data rates.

For the outage-based co-channel deployment scheme, the maximum allowed transmit power for the FSU's is fixed at 0.1W, and the maximum allowed transmit power for the MSU's is equal to 1W. The design parameter $P0$ is assumed to be -40dB.

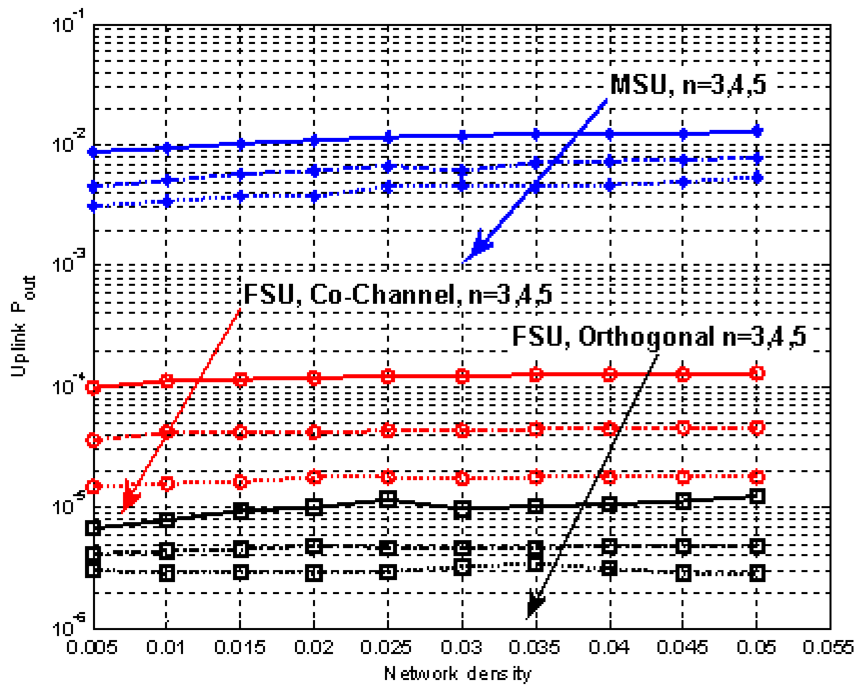

Figure 4.10: The average uplink outage probability resulting from the grouping scheme, with and without the use of the distance-based co-channel deployment extension, as a function of the FBS density in the network over three values of the path loss exponent.

Fig. 4.11 shows the average number of channels to be purchased versus the network density for the cases of no grouping, grouping with orthogonal channel deployment, and grouping with co-channel deployment under several interference conditions characterized by the value of the path loss exponent n (the larger is n, the less-severe is the interference environment). The first observation here is the significant reduction in the number of channels to be purchased from the PU networks as a result of applying the grouping scheme when compared to the case of no grouping. The second observation is that the (optimized) grouping scheme can exploit the less-severe interference environments to further reduce the number of groups by grouping at a smaller value of D_{th}. The third observation is the reduction in the number of channels to be purchased as a result of applying the co-channel deployment extension which improves the cost-efficiency of the scheme. The last observation is that, with the increase of n (implying a less-sever interference environment), the capability of the co-channel deployment becomes limited. This is expected since in a low interference environment, a smaller number of FBS groups will be formed which corresponds to a larger number of members per group. This makes it more difficult for the CBS to find a MSU that satisfies the outage threshold condition for any group and hence, the number of channels to be purchased resulting from the co-channel deployment extension becomes closer to the one resulting from the orthogonal channel deployment.

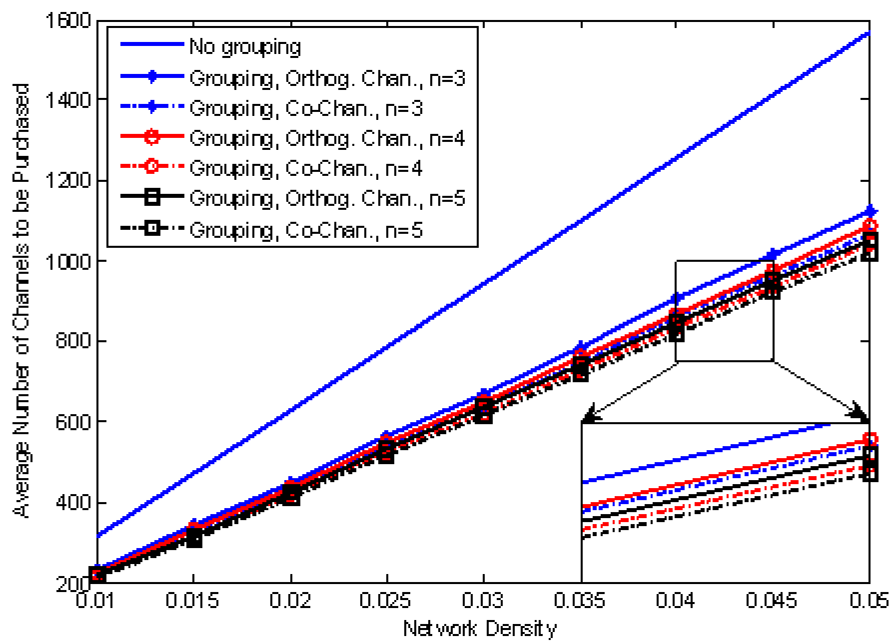

Figure 4.11: The average number of groups resulting from the grouping scheme, with and without the use of the outage-based co-channel deployment extension, as a function of the FBS density in the network over three values of the path loss exponent, compared to the case of no grouping.

The reduction in the number of channels to be purchased resulting from the co-channel deployment extension is at the cost of worse outage performance for both the FSU's and MSU's. The average MSU's and FSU's uplink outage probabilities versus the network density are shown in Fig. 4.12 for different values of n. As it can be noticed from the figure, the scheme with co-channel deployment compromises the QoS of the FSU's and the MSU's to further reduce the number of purchased channels. Since the outdoor environment is usually more sever than the indoor one, which shows in the MSU uplink outage as compared to the FSU's uplink outage, the use of the co-channel deployment extension is limited with the QoS requirements of the MSU's.

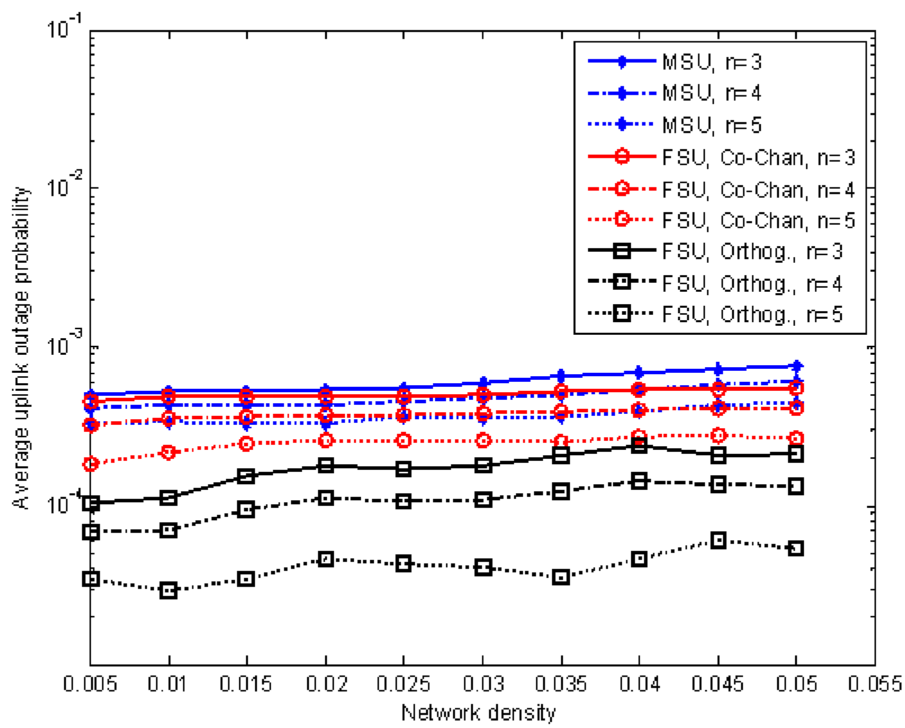

Figure 4.12: The average uplink outage probability resulting from the grouping scheme, with and without the use of the outage-based co-channel deployment extension, as a function of the FBS density in the network over three values of the path loss exponent.

Comparing the two co-channel deployment extensions to each other, it can be observed that the distance-based co-channel deployment extension method results in a smaller number of groups than the outage-based one, but at the cost of worse outage performance, especially for the MSU's.

4.6 Summary and Conclusions

In this chapter, several methods for optimizing and enhancing the grouping scheme are suggested. First, an algorithm to find the minimum value of D_{th} that maintains the QoS of the FSU's uplink channel is presented. Next, an algorithm for maximizing the CBS profit based on the greedy approach is proposed. Simulations show that the suboptimal CBS profit maximization algorithm achieves higher profit for the SU network than the distance-based grouping scheme, especially when the offered spectrum is not sufficient for the needs of the SU's. Moreover, an algorithm for the extension of the CBS profit maximization algorithm to co-channel deployment is suggested and compared to the orthogonal deployment algorithm. It is shown that implementing the co-channel deployment extension for the CBS profit maximization algorithm does not always help increasing the profit of the CBS. Finally, two algorithms for grouping with co-channel deployment are presented, namely, the distance-based and the outage-based co-channel deployment grouping methods. Simulations show that the outage-based grouping method maintains the uplink outage performance of both the FSU's and the MSU's, while the distance-based grouping method maintains the uplink outage performance of the FSU's only.

CHAPTER 5

MARKET GAMES: COMPETITION AND COLLUSION

5.1 Introduction

A game-theoretical model for the price competition between the primary user (PU) networks in selling spectrum to a two-tier cognitive radio network (i.e., the cognitive base station (CBS) with its users, and the femtocell base stations (FBS's) with their users)was presented in

In this chapter, we compare the collusion game to the price-competition game for the scenario presented in [28] in terms of the profits of the PU networks and of the CBS. Before doing so, we perform stability analysis for the three-stage Stackelberg algorithm and find the conditions for this algorithm to be stable for the general case of multiple PU networks. The optimum step size for the algorithm in terms of speed of convergence is also determined. We also study the effect of the substitutability parameter and the energy efficiency of the macrocell SU's (MSU's) and the FBS's on the resultant Nash and Pareto prices. Furthermore, we prove that the Pareto-optimal price is not necessarily the price that will result in the highest profit for the PU's as argued in [12]. Finally, it is shown by simulation that the minimum CBS profit is not associated with the maximum price offered by the PU networks.

This chapter is organized as follows. In Section 5.2, the system model is illustrated. The stability analysis is presented in Section 5.3. The equations of the Pareto price and the NE price are derived, compared, and the effect of some parameters on both of them is discussed in Section 5.4. The simulation results and discussions are provided in Section 5.5, and Section 5.6 contains the summary and conclusions.

5.2 System Model

As shown in Fig. 5.1, there exist L PU networks, K SU's (K_F FBS's and K_M MSU's, where $K_F + K_M = K$) in the networks. Each FBS serves one femtocell SU (FSU) in each time slot. PU network l offers a price c_l to the CBS and determines the amount of the offered spectrum. Each FBS performs the energy-efficient power allocation for its FSU, finds its energy efficiency, and sends this energy efficiency to the CBS. In the same way, the CBS performs the energy-efficient power allocation for its MSU's and find the energy efficiency of each one. The energy-efficient power allocation is the transmit power of each user (MSU or FSU) that maximizes its energy efficiency. The energy efficiency of each MSU/FSU ($\eta_k, k = 1, 2, \ldots, K$) is defined as [28]

$$\eta_k = \frac{\log_2\left(1 + \frac{h_k^2 p_k}{\sigma^2}\right)}{p_a + p_k}, \tag{5.1}$$

where p_k and h_k are the energy-efficient power allocation of the k^{th} MSU/FSU and the channel gain from that MSU/FSU to its base station (CBS or FBS), respectively, p_a is the additional circuit power consumption due to MSU/FSU transmission [84], and σ^2 is the noise power assuming an additive white Gaussian noise (AWGN) with zero mean. It is assumed here that the price information sent by each PU is given per unit bandwidth. So, the CBS knows the maximum spectrum bandwidth that can be purchased from PU networks, and it will inform the FBS's to use that bandwidth to calculate the energy efficiency of the transmission of their FSU's. The CBS will allocate the spectrum band with the lowest price to the FBS/MSU with the highest energy efficiency to maximize its revenue. This is done by using the spectrum allocation index x_{lk} as follows. For each FBS/MSU k, the spectrum allocation index on the l^{th} channel (x_{lk}) is initially set to zero. Setting this index to one by the CBS indicates that the l^{th} channel is allocated to the k^{th} MSU/FBS. Since each channel is allocated for only one MSU/FBS to avoid interference, the sum of the spectrum allocation indices of all MSU's/FSU's on the l^{th} channel equal to one (i.e., $\sum_{k=1}^{K} x_{lk} = 1$).

We assume a static environment in which the channel conditions of users do

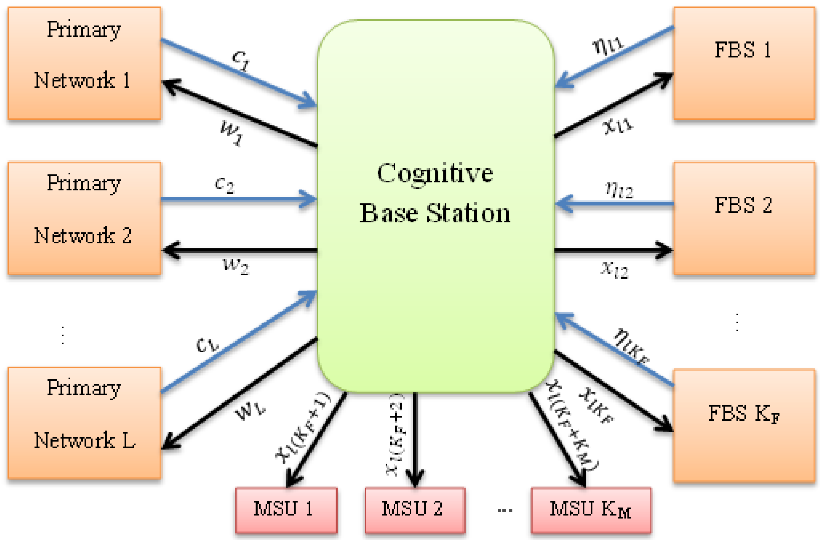

Figure 5.1: System model for the two-tier cognitive radio network.

not change over the trading period. The CBS purchases the spectrum bands that maximize its profit from the spectrum offered by L PU operators. Then, the CBS uses that spectrum to serve the MSU's and FBS's according to their energy efficiencies to maximize its profit (i.e. the cheapest spectrum band is allocated to the MSU/FBS with the highest energy efficiency).

Each spectrum band purchased by the CBS from a PU network is allocated only to one MSU/FBS. We assume that the number of SU's (MSU's and FBS's) that can be accessed to the CBS is equal to the number of bands offered by the PU networks ($K = L$). The FBS's are connected to the CBS via broadband connection (e.g., digital subscriber line (DSL)) or optical fiber link.

5.3 Stability Anlysis for the Stackelberg Algoriothm

In this section, the stability of the Stackelberg algorithm proposed in [28] is analyzed, the stability conditions are derived, and the optimal step size is determined. First of all, we start this section by revising the Stackelberg algorithm and presenting the equations upon which the work in the rest of this chapter will be based.

5.3.1 The Stackelberg Algorithm

As mentioned in the previous section, the algorithm starts with the users sending their energy efficiencies and the PU networks sending their offers (prices/bandwidth) to the CBS. The CBS determines the spectrum demand from each PU network that will maximize its profit. The CBS profit (P_{CBS}) can be calculated using the following utility function [28, 12]

$$P_{CBS}(w) = \sum_{l=1}^{L} w_l R_{\eta l} - \frac{1}{2}\left\{\sum_{l=1}^{L} w_l^2 + 2v \sum_{q \neq l} w_l w_q\right\} - \sum_{l=1}^{L} c_l w_l, \tag{5.2}$$

where c_l w_l are the offered price and the spectrum demand (in MHz) from the l_{th} PU network, respectively, $R_{\eta l} = \sum_{k=1}^{K} \zeta_k x_{lk} \eta_k$ is the revenue of the CBS from spectrum band l, ζ_k is the cost paid by the k^{th} user (MSU/FBS) to the CBS, x_{lk} is the spectrum allocation index. When $x_{lk} = 1$, it indicates that the spectrum band purchased from PU network l is allocated to the user k. In addition, $v \in [-1, 1]$

is the substitutability parameter. When v is larger than 0, the CBS can switch between the spectrum bands of different PU networks; whereas when it is equal to 0, the CBS can not switch among the spectrum because of the large penalty set by the PU operator on doing so [12]. When v is less than 0, this means that the purchased band is complementary, i.e., the CBS should purchase another band to make use of this band. For example, when the CBS purchases the downlink band, it needs to purchase the uplink band also [12]. The motivations behind using this utility function can be summarized as follows:

- The function is concave, so it can represent user satisfaction from the bandwidth offered by the PU networks.

- When the function is differentiated, it results in a linear spectrum demand function, which simplifies further analysis.

- The effect of spectrum quality and the spectrum substitutability factor is incorporated in this function.

Differentiating (5.2) with respect to the spectrum demand results in the following set of equations

$$\frac{\partial P_{CBS}(w)}{\partial w_l} = R_{\eta l} - w_l - v \sum_{q \neq l} w_q - c_l. \tag{5.3}$$

Solving the set of equations in (5.3), the optimal spectrum demand from the l^{th} PU network is obtained as follows

$$w_l^* = \frac{1}{M} \left[N(R_{\eta l} - c_l) - v \sum_{q \neq l} (R_{\eta q} - c_q) \right]. \tag{5.4}$$

where $N = v(L-2)+1$, and $M = (1-v)(v(L-1)+1)$. Once the CBS has determined its spectrum demand from each PU network, each PU network can calculate its expected profit by substituting (5.4) in the following utility function

$$P_{PU_l}(c) = a\varepsilon_l(B_l - w_l) + c_l w_l, \tag{5.5}$$

where B_l is the total bandwidth of the l^{th} PU network, a represents the revenue paid by the PU to the PU networks, and ε_l is the spectrum efficiency of the PU transmission. Since the demand in (5.4) depends on the prices offered by each PU network, by using this value, the PU network takes into account the prices offered by all the other PU networks. Therefore, the PU network can find the price that maximizes its profit taking into account the prices offered by all the other PU networks by differentiating (5.5) with respect to c_l

$$\frac{\partial P_{PU_l}(c)}{\partial c_l} = \frac{1}{M}\left[N(a\varepsilon_l + R_{\eta l} - 2c_l) - v\sum_{q\neq l}(R_{\eta q} - c_q)\right]. \tag{5.6}$$

The second derivative of (5.6)

$$\frac{\partial^2 P_{PU_l}(c)}{\partial c_l^2} = \frac{-2N}{M}. \tag{5.7}$$

which is clearly negative (N and M are both positive or both negative for $L \geq 2$ and $v \in [-1, 1]$). So by definition, the utility function in (5.5) is concave. The following self-mapping function is proposed in [28] to iteratively obtain three-stage

Stackelberg game equilibrium

$$c_l(t+1) = c_l(t) + \mu \frac{\partial P_{PU_l}(c)}{\partial c_l}, \tag{5.8}$$

where μ denotes the step size of the price, and t is the iteration number. The algorithm will terminate when the difference between the prices from two consecutive iterations is lower than a certain value ϵ. The closer is ϵ to 0, the lower the error in the resultant optimal price.

5.3.2 Stability of the Self-Mapping Function

In order to examine the stability of the Stackelberg algorithm, it is necessary to check whether the self-mapping function converges or not. This can be done by finding the Jacobean matrix of the self-mapping function and determining its eigenvalues. If all the eigenvalues of the Jacobean matrix are inside the unit circle, this means that the self-mapping function is stable and thus, the algorithm is stable [12]. The Jacobean matrix of the self-mapping function for L PU networks is defined as follows

$$J = \begin{pmatrix} \frac{\partial c_1(t+1)}{\partial c_1(t)} & \frac{\partial c_1(t+1)}{\partial c_2(t)} & \cdots & \frac{\partial c_1(t+1)}{\partial c_L(t)} \\ \frac{\partial c_2(t+1)}{\partial c_1(t)} & \frac{\partial c_2(t+1)}{\partial c_2(t)} & \cdots & \frac{\partial c_2(t+1)}{\partial c_L(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial c_L(t+1)}{\partial c_1(t)} & \frac{\partial c_L(t+1)}{\partial c_2(t)} & \cdots & \frac{\partial c_L(t+1)}{\partial c_L(t)} \end{pmatrix}. \tag{5.9}$$

By differentiating all the self-mapping functions in (5.8) with respect to all prices $\{c_l\}_{l=1}^{L}$, the resultant Jacobean matrix can be written as

$$J = \begin{pmatrix} 1-\frac{2\mu N}{M} & \frac{\mu v}{M} & \cdots & \frac{\mu v}{M} \\ \frac{\mu v}{M} & 1-\frac{2\mu N}{M} & \cdots & \frac{\mu v}{M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\mu v}{M} & \frac{\mu v}{M} & \cdots & 1-\frac{2\mu N}{M} \end{pmatrix}. \tag{5.10}$$

This matrix is a special case of the Toeplitz circulant matrix [85], where the values around the diagonal are identical. The Toeplitz circulant matrix can be written as

$$J = \begin{pmatrix} \rho_0 & \rho_{L-1} & \cdots & \rho_1 \\ \rho_1 & \rho_0 & \cdots & \rho_2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{L-1} & \rho_{L-2} & \cdots & \rho_0 \end{pmatrix}. \tag{5.11}$$

The eigenvalues of an $L \times L$ circulant matrix can be found as follows [86]

$$\lambda_l = \rho_0 + \rho_{L-1}\omega_l + \rho_{L-2}\omega_l^2 + \ldots + \rho_1\omega_l^{L-1}, \tag{5.12}$$

where $\omega_l = \exp\left(\frac{j2\pi l}{L}\right)$, $l = 0, \ldots, L-1$. It should be noticed here that $\rho_1, \ldots, \rho_{L-1}$, which are the values around the diagonal, are identical and equal to $\frac{\mu v}{M}$ in our case. Let $\rho = \rho_1 = \rho_2 = \cdots = \rho_{L-1}$ denotes the coefficients outside the diagonal then, (5.12) becomes

$$\lambda_l = \rho_0 + \rho \sum_{n=1}^{L-1} \exp\left(\tfrac{j2\pi ln}{L}\right). \tag{5.13}$$

The summation in (5.13) can be simplified further

$$\begin{aligned}\sum_{n=1}^{L-1} \exp\left(\tfrac{j2\pi ln}{L}\right) &= -1 + \sum_{n=0}^{L-1} \exp\left(\tfrac{j2\pi ln}{L}\right) \\ &= \begin{cases} L-1, & l = 0; \\ -1 + \frac{1-\exp(j2\pi l)}{1-\exp\left(\frac{j2\pi l}{L}\right)}, & l > 0. \end{cases}\end{aligned} \tag{5.14}$$

Since $l = 1, \ldots, L-1$ is an integer, $\exp(j2\pi l) = 1$, and since $l < L$, $\exp\left(\tfrac{j2\pi l}{L}\right)$ does not equal to one, and therefore, the term including exponentials in (5.14) is equal to zero. Hence

$$\sum_{n=1}^{L-1} \exp\left(\tfrac{j2\pi ln}{L}\right) = \begin{cases} L-1, & l = 0; \\ -1, & l \neq 0. \end{cases} \tag{5.15}$$

So we have $\lambda_0 = \rho_0 + \rho(L-1)$ and the $L-1$ eigenvalues $\lambda_l = \rho_0 - \rho$. Substituting the values of ρ_0 and ρ and simplifying, the eigenvalues become

$$\begin{aligned} \lambda_0 &= 1 - \mu\left[\frac{2N + v(L-1)}{M}\right] = 1 - \mu\left[\frac{v(L-3)+2}{M}\right], \\ \lambda_l &= 1 - \mu\left[\frac{2N - v}{M}\right], \quad l = 1 \ldots L-1. \end{aligned} \tag{5.16}$$

To find the stability conditions, the inequality $|\lambda| < 1$ should be solved for both terms in (5.16). By solving the inequality, the stability conditions are as follows

$$0 < \mu < min\left(\frac{2M}{2N - v}, \frac{2M}{N - v + 1}\right). \tag{5.17}$$

Figure 5.2 shows v against the upper limit in (5.17). It can be noticed that the stability range of the Stackelberg algorithm tends to shrink when v increases with the increase of L except for small values of v. This means that maintaining a low value of v will give the PU networks a wider range of iterative step sizes to choose from while staying in the stable region.

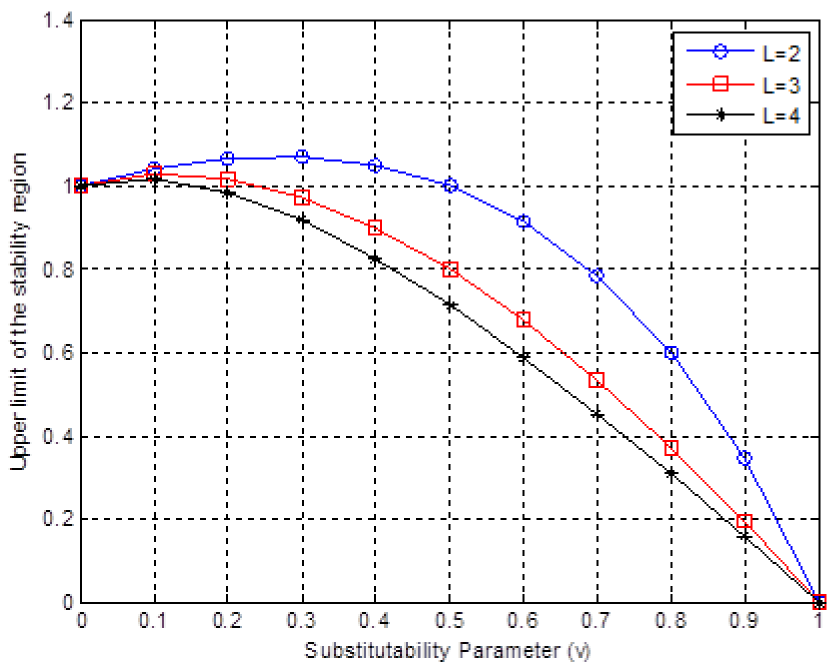

Figure 5.2: The substitutabilty parameter (v) against the upper limit of (5.17) for different values of L.

Lemma 5.1 *The optimal step size for the Stackelberg algorithm in terms of convergence speed is* $\mu = \frac{M}{2N}$.

Proof. We start by taking two consecutive prices, $c_l(t)$ and $c_l(t+1) = c_l(t) + \mu\frac{\partial P_{PU_l}(c_l(t))}{\partial c_l}$. Substituting $c_l(t+1)$ in (5.6), the derivative of the updated price can be expressed as

$$\frac{\partial P_{PU_l}(c_l(t+1))}{\partial c_l} =$$
$$\frac{N}{M}\left[\left(a\varepsilon_l + R_{\eta l} - 2\left[c_1 + \mu\left[\frac{N(a\varepsilon_l + R_{\eta l} - 2c_1) - v\sum_{q\neq l}(R_{\eta q} - c_q)}{M}\right]\right]\right) - v\sum_{q\neq l}(R_{\eta q} - c_q)\right]. \tag{5.18}$$

To find μ, the equation to be solved is given by

$$\frac{\partial P_{PU_l}(c_l(t+1))}{\partial c_l} = \epsilon. \tag{5.19}$$

Solving (5.19) for μ and observing as $\epsilon \to 0$, we find that $\mu \to \frac{M}{2N}$. When ϵ approaches 0 this means two things. First, the two prices are approaching each other, and at $\epsilon = 0$ they become identical. This means the price found by the algorithm is one hundred percent accurate. Second, having zero at this step indicates that the prices have reached any other value of $\epsilon > 0$ before this step. This means that at any other value of ϵ larger than zero, the lowest number of steps to reach ϵ is achievable at $\mu = \frac{M}{2N}$. Hence, the optimal iterative step size is $\mu = \frac{M}{2N}$. ∎

5.4 Collusion Game and the Maximum Price for PU Networks

In this section, the equations for the NE and the Pareto-optimal price are presented, and we compare between the two equations in terms of the effect of the substitutability parameter and the energy efficiency of SU transmission on each of them.

5.4.1 Nash and Pareto Prices

The Nash price is the price that maximizes the profit of each PU network taking into account the moves of the other PU networks. Therefore, to find the Nash price, it is required to solve $\frac{\partial P_{PU_l}(c)}{\partial c_l} = 0$ for each PU network. Assuming all the SU's have the same energy efficiency and all the PU's have the same spectrum efficiency, substituting c_l instead of c_q in (5.6), the equation to be solved to find the Nash price becomes

$$N(a\varepsilon_l + R_{\eta l} - 2c_l) - v\sum_{q \neq l} R_{\eta q} + v(L-1)c_l = 0. \tag{5.20}$$

Solving (5.20) for c_l, the Nash price for any number L of PUs can be written as

$$c_{Nash} = \frac{N(a\varepsilon_l + R_{\eta l} - v\sum_{q \neq l} R_{\eta q})}{2 + v(L-3)}. \tag{5.21}$$

An important point to note is that the Nash price decreases with the increase of the number of PU networks, L, because when the number of PU networks increases, the competition among them will increase since the spectrum demand will be distributed over a larger number of operators. Hence, each PU network needs to take into account a larger number of other moves, and this will cause the price to be more conservative with the increase of L. It should be noticed here that we are interested in the positive values of (5.21) since it is a price (i.e., $L \geq 2$ for $v \in [-1, 1]$).

For the Pareto-optimal price, the sum of the profits of all PU operators should be maximized [12]. Hence, we need to solve $\frac{\partial \sum_{l=1}^{L} P_{PU_l}(c)}{\partial c_l} = 0$. Again, due to the assumption of equal energy efficiencies of the SU's and equal spectrum efficiencies of the PU's, all the optimal prices will be the same. After differentiating the sum, substituting c_l instead of c_q, and finally solving for c_l, the Pareto price is written as

$$c_{Pareto} = \frac{a\varepsilon_l + R_{\eta l}}{2}. \tag{5.22}$$

Note that the Pareto price does not depend on the substitutability parameter or the number of PU networks; but rather, it depends on the energy efficiency of the SU's and the spectrum efficiency of the transmission of PU's.

The substitutability parameter has no effect on the Pareto point, but it helps the PU operators to reach the Pareto point in the competition game by decreasing it (when it is positive). On the other hand, the energy efficiency changes the Pareto point. That is, when the energy efficiency of the SU's increases, they are expected to use the spectrum more extensively. Thus, the spectrum demand of the CBS is expected to increase and hence increases the maximum price that can be offered by the PU operators.

5.4.2 The Maximum Pricing

To understand the relation between Nash and Pareto prices, we take the difference between Equations (5.21) and (5.22) and investigate when this difference is larger than zero

$$c_{Nash} - c_{Pareto} > 0. \tag{5.23}$$

Substituting and simplifying, the inequality in (5.23) becomes

$$v(1-2L)K_{\eta l} + v(L-1)a\varepsilon_l > 0. \tag{5.24}$$

When the inequality in (5.24) is satisfied, the Nash price will be larger than the Pareto price. At first, we need to point out that the condition $c_l > a\varepsilon_l$ should be satisfied in order for the PU network to prefer selling the spectrum to the CBS [28]. But $K_{\eta l}$ should be larger than c_l for the CBS to prefer buying the spectrum from the PU network. Hence, $K_{\eta l} > a\varepsilon_l$. The term $1-2L$ is larger than $L-1$ but in negative for $L \geq 2$. This means that when the substitutability parameter v is larger than zero, the Pareto price is always larger than the Nash price. On the contrary, when v is lower than zero, the Nash price is always larger than the Pareto price.

Based on the aforementioned discussion, the Nash price is always larger than Pareto price when v is negative, but what does it mean when v is negative? To understand, we need to know more about competitive pricing. The type of the game played here is called Bertrand game [24], in which the firm (the PU operator in our case) determines the price and the user (the CBS in our case) determines the spectrum demand. Such a game is usually preferred by the firms when the good is a complement [87] which is, as illustrated in the revision at the start of Section 5.3, the case of v being negative. To demonstrate the meaning of complement spectrum, we re-evoke the example given in [12] with some modifications to make it suitable for our scenario. When the CBS needs two spectrum bands, one for the uplink and the other one for the downlink, each one of these bands is considered a complement (i.e., it can not be used alone). A practical example of this case is LTE-FDD [88] which needs two separate bands for the uplink and downlink channels. On the other hand, LTE-TDD can use the same band for the two channels.

5.5 Numerical Results

If not mentioned, it is assumed in the simulations that the number of PU networks is 2, the spectral efficiency of PU transmission is set to be 2 bits/s/Hz, the energy efficiency of SU transmission is 22.1444 bits/Hz/Joule, the additional circuit power consumption is 0.1W, and the cost paid by the PU to its operator and by the SU to the CBS is set to 1. It is assumed here that the number of users is equal to the number of the PU networks (i.e., $L = K$).

The profit of PU networks versus the Nash price is shown in Fig. 5.3 for different (positive) values of v. The values pointed by the circles have been analytically found using (5.22). It is obvious that the Pareto point achieves the highest profit for the PU operators.

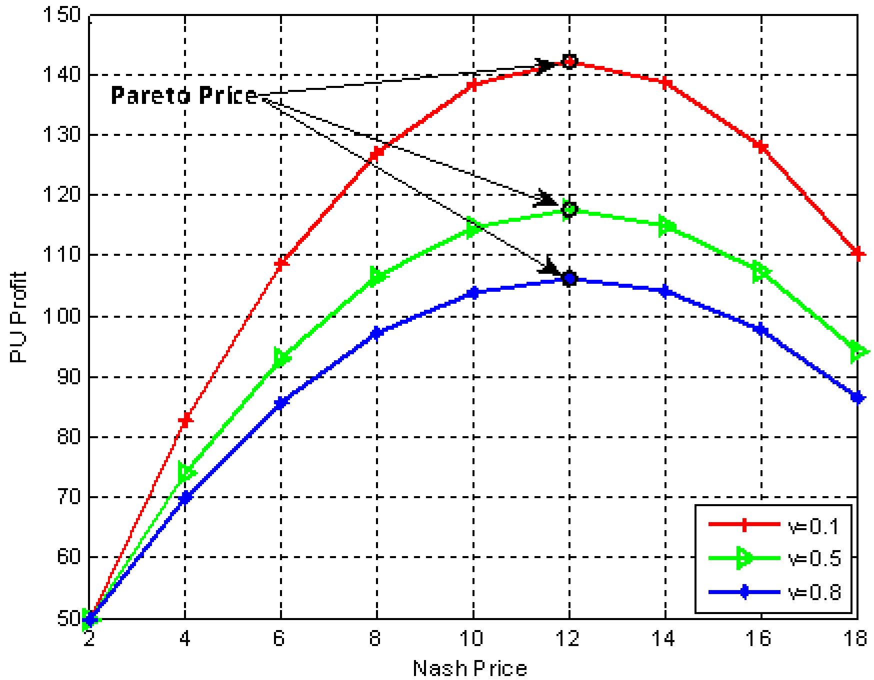

Figure 5.3: The profit of PUs versus Nash price for different values of the substitutability parameter (v).

The approach of Nash price to Pareto point with the decrease of the (positive) value of v is shown in Figure 5.4. Here, the Pareto point represents the highest price, while for negative values of v, Pareto is the lowest price that can be offered by the PU's, and this is shown in Fig. 5.5.

Fig. 5.6 shows the maximum profit that can be achieved by the CBS against the substitutability parameter v for different numbers of the PU networks. The first note is that the minimum (maximum) profit the CBS can achieve depends on the number of PU networks. This is the worst case for the CBS in terms of profit. The important note here is that the worst case is not associated with negative substitutability parameter (which corresponds to highest PU price). This is due to the structure of the utility function used to represent the profit of the CBS.

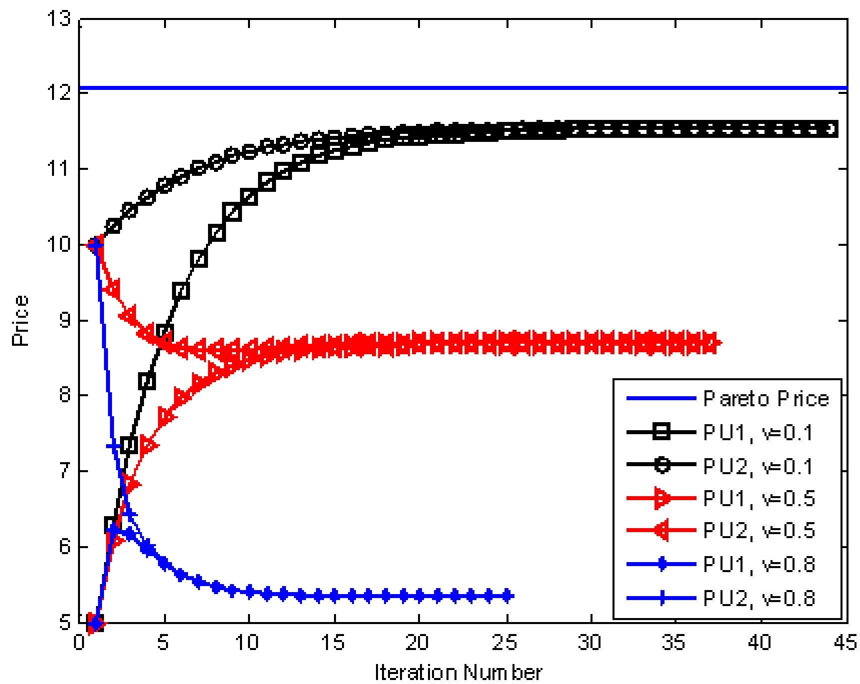

Figure 5.4: The convergence of Nash price to Pareto price with the decrease of the (positive) value of v.

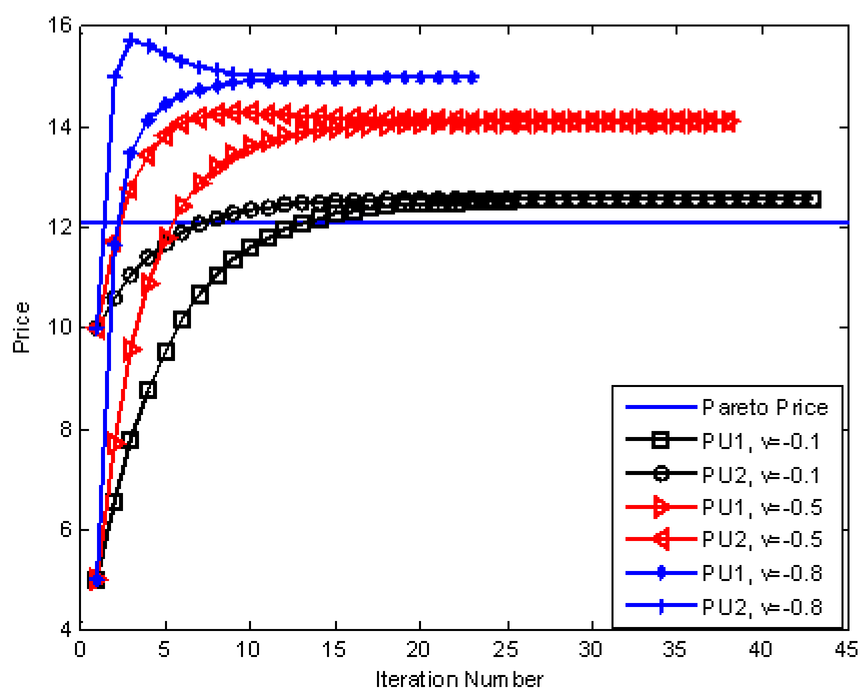

Figure 5.5: The superiority of Nash price to Pareto price with the decrease of the (negative) value of v.

Returning to (5.2), it can be noticed that when v is negative, the third term becomes positive and add to the profit. This means that, when the penalty is put by the PU networks on spectrum switching, the profit of CBS is expected to increase! The rationale behind this attitude of the quadratic utility function at negative values of v is related to what is called risk aversion. In a nutshell, the utility function in (5.2) takes into account not only the profit of CBS, but also the preferences of CBS. The term multiplied by v indicates that the CBS would prefer to get lower (certain) profit by staying at the same spectrum instead of getting a larger (uncertain) profit by switching to another spectrum which may cause additional unexpected costs to CBS. When the penalty is increased, the CBS is more inclined to stay in the same spectrum which, according to the structure of its utility function, is preferred by the CBS. For $v \in [-1, 0]$, the CBS will not switch among the spectrum, so with the increase of L, the utility increases since higher risk is avoided. While for $v \in [0, 1]$, the CBS will be switch among the spectrum which will reduce its utility with the increase of L since the larger the number of PU networks, the more the CBS will be willing to switch (due to the availability of more offers from the PU networks).

5.6 Summary and Conclusions

In this chapter, the stability conditions for the Stackelberg algorithm for the scenario of heterogeneous cognitive network with femtocells was derived. Furthermore, the effect of the collusion game established and maintained by the PU

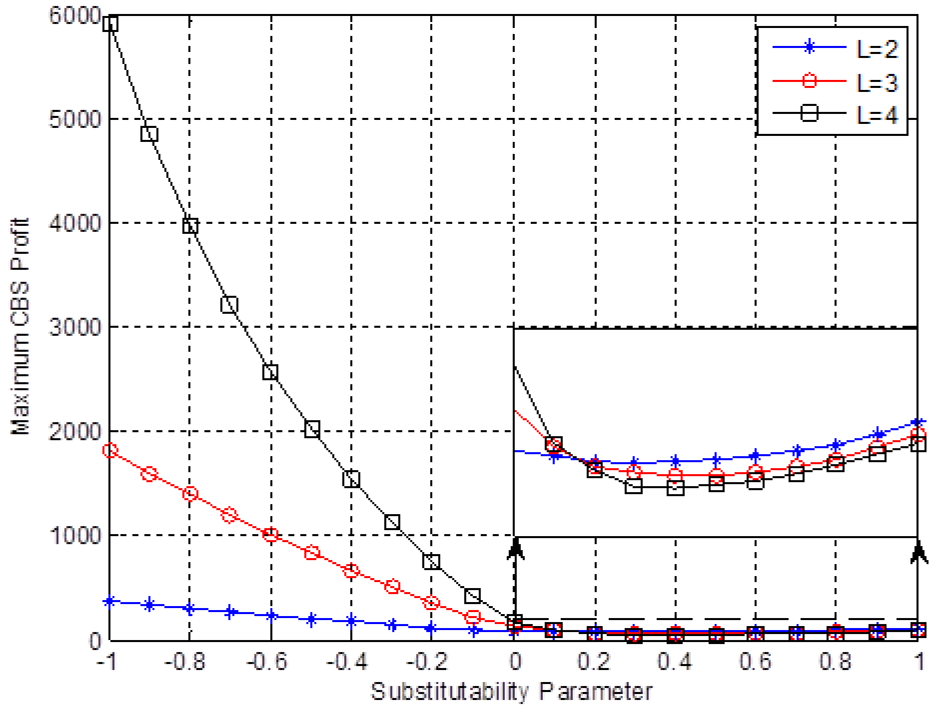

Figure 5.6: The profit of CBS versus spectrum demand for different positive and negative values of v.

operators on the profit of both the PU networks and the CBS has been investigated. It was found that the collusion price (Pareto price) does not represent the highest price that can be offered by the PU operators. The worst case for the CBS has been also investigated, and simulations show the minimum (maximum) profit achieved by the CBS is not associated with the highest price that can be offered by the PU networks. Depending on the aforementioned results, it is recommended to use TDD in any deployment for a cognitive radio network, that depends on spectrum trading to provide (or to enhance) the services to its users, because it is more flexible and reduces the number of spectrum bands required to maintain the service. Moreover, the CBS should be designed such that it can (automatically) reduce the number of spectrum bands/channels required to be purchased from the

PU networks by using the available spectrum more efficiently.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusions

In this thesis, a new scheme for enhancing spectrum efficiency and thus reducing spectrum costs for cognitive radio networks is proposed. The scheme is based on using the location information of the FBS's to determine which FSU's can utilize the same channel based on some distance threshold criterion. Two approaches for implementing the grouping scheme were presented, namely, the FSU-based and the FBS-based grouping methods. The FBS-based grouping method appears to be much less complex but slightly less spectrum-efficient than the FSU-based grouping method. Moreover, sorting the groups, according to their number of members or their categories, in an increasing order seems to reduce the complexity of the update process for the FSU-based and the FBS-based grouping schemes, respectively.

The uplink outage probability is derived for both the FSU-based and the FBS-based grouping schemes, and simulations show that the FSU-based grouping scheme achieves better outage performance than the FBS-based scheme. In addition, the effect of sorting the groups in an increasing order according to their number of groups or their categories is examined for both the FSU-based and the FBS-based, respectively. It is shown that sorting the groups in an ascent order will result in a reduced uplink outage probability at small values of D_{th}.

Furthermore, two approaches to optimize the grouping scheme are proposed. The first approach is the distance-based approach, which depends on the worst interference scenario and guarantees the desired outage performance of the FSU's. The second approach is the CBS profit maximization approach, which depends on the greedy algorithm to find a suboptimal grouping of the FSU's in terms of maximizing the CBS profit. The CBS profit maximization approach shows better performance than the distance-based approach in terms of the expected sum profit, but with no QoS guarantees. Finally, three methods to extend the operation of the grouping scheme to the co-channel deployment scenario have been proposed. Two of the methods are for the distance-based grouping scheme which were shown to further reduce the number of channels to be purchased but at the cost of worse uplink outage performance for both the MSU's and the FSU's. The third method is for the CBS profit maximization algorithm which was shown to result in a higher profit for the CBS under some interference conditions.

6.2 Future Work

This section is allocated to discuss some proposed solutions which can be considered as future works:

- 2D GPS positioning can not distinguish between multiple floors in the same building. If it happens that the distance between two floors is larger than D_{th} and both floors have FBS's, the two FBS's will be put in different groups though they can belong to the same group. In this work, only 2D-positioning is considered. The process can be enhanced by considering 3D-positioning but at the cost of higher complexity. 3D-positioning requires the detection of 5 satellites at the receiver to achieve acceptable accuracy [89].

- The grouping process is a centralized process. The ultimate solution should be a hybrid between centralized and distributed processes. That is, the CBS performs grouping of FSU's/FBS's and adds MSU's to the groups. After that, if a new FSU appears in an FBS, the CBS tries to find a group to serve it. If it can not find such a group, then it sends the available channels to the FBS, and the FBS performs spectrum sensing on these channels to determine which channels can be utilized. If no channel can be used, then the FBS requests the CBS to purchase a new channel.

- The work presented in this thesis is based on the assumption that only one CBS exists in the SU network. This can be generalized to the case when several CBS's existing in the network. The question here is how the grouping should be implemented in this case? Two choices will be of interest, either each CBS performs grouping and spectrum trading alone, or all the CBS's cooperate and perform the grouping together and assign one of them to perform the spectrum trading with the PU networks. Intuition says that the second approach will result in lower number of groups than the first approach. Still, how should the joint grouping be performed to reduce the complexity and achieve significant reduction in spectrum costs is an interesting problem to investigate in the future.

- The solutions presented in this work are aimed at maintaining the QoS or maximizing the sum rate on the uplink channel. This can be extended by considering the downlink channel and observing how the grouping scheme should be modified to maximize the sum rate or maintain QoS on both the uplink and the downlink channels.

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