New Models in Inventory Control

BY

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This thesis, written by **Ali Muhammad Abudari** under the direction of his thesis advisor and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE** in **Industrial & Systems Engineering**.

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Dedicated

To

My Dear Parents, Brothers and Sisters

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All praise belongs to Allah, the lord of the universe. May blessings and greetings be to the master of the creatures, Muhammad Al-Mustafa, and his blessed family and his noble companions until the Day of Reckoning.

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ABSTRACT

Full Name : Ali Muhammad Salim Abudari

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An Economic Production Quantity model (EPQ) aims to minimize the total production-inventory cost by balancing between multiple conflicting costs. In this thesis, we introduce two new models in this area; both of them are developed under the conditions of linear process deterioration and machine breakdown. Additionally, corrective and preventive maintenance actions are performed according to a specific policy. In one model, time to failure and time to deterioration are assumed independent, while in the other model the two variables are assumed dependent where machine failure can happen only if preceded by process deterioration. The proposed two models are formulated under general probability distributions, but optimality is proved under selected distributions. The two models' behavior is investigated thoroughly for numerical examples.

Another two models are developed for determining the optimal quantity to be ordered by a retailer from his supplier, in addition to the optimal credit period to be offered by the retailer to his customers. Both models are developed under the presence of two-levels of trade credit periods, and in which the demand is linked to the credit period offered by the retailer to his customers. The two models differ in the payment procedure from the

retailer to the supplier. In both models, we made two main assumptions. Firstly, we assumed non-instantaneous replenishment from the supplier to the retailer, and secondly we assumed a percentage of retailer's sales are considered bad debt. Two numerical examples are solved for joint determination of the optimal order quantity and the optimal credit period.

ملخص الرسالة

الاسم الكامل: علي محمد سليم ابو داري

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يهدف نموذج كمية الانتاج المثلى الى تقليل الكلفة الكلية لنظام الانتاج و التخزين عن طريق تحقيق التوازن بين عدد من الكلف المتناقضة. نقدم في هذه الرسالة نموذجين جديدين في هذا المجال, كلا النموذجين تم بنائهما في ظل احتمالية انحراف العملية الانتاجية بشكل تدريجي خطي, بالاضافة الى احتمالية عطل الماكينة. أضافة الى ذلك يتم عمل صيانة تصحيحية و وقائية وفقا لسياسة محددة. في أحد النموذجين تم افتراض استقلالية عطل الماكينة عن انحراف العملية الانتاجية, بينما في النموذج الاخر تم افتراض وجود علاقة بينهما بحيث أنه لا يمكن ان يحدث عطل ماكينة الا اذا سبقه انحراف في العملية الانتاجية. كلا النموذجين المقدمين تم بنائهما بافتراض توزيعات احتمالية عامة, بينما تم اثبات الأمثلية لتوزيعات احتمالية مختارة. تم تقصي سلوك كلا النموذجين بالتفصيل عن طريق امثلة رقمية.

تم بناء نموذجين اخرين لتحديد الكمية المثلى التي ينبغي لتاجر التجزئة طلبها من تاجر الجملة بالاضافة الى فترة الاقراض المثلى الممنوحة من تاجر التجزئة لزبائنه. كلا النموذجين تم بنائهما في ظل وجود مستويين من فترات الاقراض, بالاضافة الى اعتمادية مستوى الطلب من المستهلكين على فترة الاقراض الممنوحة من قبل تاجر التجزئة. يتمثل الفرق الاساسي بين النموذجين في الطريقة التي يتم بها الدفع من تاجر التجزئة الى تاجر الجملة. افتراضين رئيسيين تم اعتمادهما في كلا النموذجين. أولا افترضنا ان التزويد من تاجر الجملة الى تاجر التجزئة يتم بشكل تدريجي و ليس لحظي, ثانيا افترضنا ان جزءا من مبيعات تاجر التجزئة هو دين ميؤوس منه ولن يسترد ابدا. تم حل مثالين رقميين لغاية تحديد كمية الطلب المثلى و فترة الاقراض المثلى بشكل متزامن.

CHAPTER 1

INTRODUCTION

We start this document with an introduction to the area of our work. Two main problems in the field of inventory control are investigated. The first problem is about finding the optimal production lot sizing policy that would minimize the overall production and inventory cost under the conditions of process deterioration and machine breakdown. The second problem aims to find the optimal policy in terms of inventory cycle length and the length of credit period offered to customers in retailing industry under credit-linked demand and two-level credit system. In the following sections, we introduce the two problems in detail.

1.1 Production Lot Sizing and Economic Ordering

The production lot-sizing problem is originated from the Economic Order Quantity (EOQ) model. When first developed by Ford W. Harris at 1913, and later extensively applied by the consultant R. H. Wilson, the EOQ model was used to determine the order quantity that minimize the total inventory holding costs and the ordering costs. The early EOQ model was simple and built on a number of assumptions:

a) The ordering cost is constant, regardless of the order quantity.

b) The demand rate is known and fixed.

c) The lead-time is known and fixed.

d) The purchase price of the item is constant, i.e. no volume discount.

e) The replenishment is made instantaneously; the whole batch is delivered at once.

f) Only one product is involved.

The EOQ model aims to minimize the total cost, which is composed of the purchasing cost, the ordering cost, and the inventory holding cost, the total cost function is given by:

$$\mathrm{TC} = \mathrm{PD} + \frac{\mathrm{DS}}{\mathrm{Q}} + \frac{\mathrm{HQ}}{2}$$

The following notation applies:

- TC is the total cost.
- P is the purchase unit price
- D is the annual demand
- S is the ordering cost
- Q is the order quantity
- H is the annual holding cost per unit.

All variables are assumed constant in the model except the order quantity Q. The total cost function for this simple model attains its minimum at:

$$Q^* = \sqrt{\frac{2DS}{H}}$$

The EOQ model is extended to serve in the area of production scheduling, in which the model is used in determining the optimal quantity to be produced in order to minimize the total production and inventory costs. The early Economic Production Quantity (EPQ) model is very simple in nature and does not include many features of modern production systems; also, it lacks the capability of handling the inherent randomness of the involved variables.

A practically useful EPQ model must consider various aspects and variables in relation to the modern production systems; quality issues, machine breakdown, random repair times, deteriorating items, variable demand, finite production horizon, learning effects, and imperfect processes are just few examples. Production lot sizing, or alternatively EPQ models mostly share the same objective of minimizing a total cost function by determining the optimal production quantity, or alternatively the production run time. In the literature review section, we will present some of the production lot sizing models.

1.2 Process Deterioration and Production Lot Sizing

Variation is inherent in any process, and manufacturing processes are no exception. There are two basic sources of variation in a manufacturing process.

- Common Cause variation

- Special Cause variation

Common cause variation is created by multiple factors that are commonly part of the process, and they are acting at random and in an independent manner. Their origin can usually be traced to the key elements of the system in which the process operates. (Materials, Equipment, People, Environment, Methods). If only common causes of variation are present, the output of a process forms a distribution that is stable over time.

Special Cause variation is created by a non-random event leading to an unexpected change in the process output. The effects are intermittent and unpredictable. If special causes of variation are present, the process output is not stable over time and is not predictable. All processes must be brought into statistical control by first detecting and removing the Special Cause variation.

Process deterioration, or alternatively process drift is a common occurrence in many manufacturing processes where processing parameters degrade, negatively affecting production system performance characterized by producing more nonconforming items. Common causes of process drift include corrosion, fatigue and cumulative wear (Fei et al 2009).

Statistical process control (SPC) tools are used to track process quality to determine when the process has gone out of control; i.e. has drifted beyond its specifications. SPC depends upon inspecting the parts produced, measuring critical attributes of the parts, and using these to determine process quality (Chincholkar et al 2004).

In some industries, e.g. drug manufacturing, process drift is not acceptable, and strict inspection procedures are established in order to instantaneously detect any drift and fix it immediately. Some other industries, such as soft drink filling operations, are to some extent tolerable toward process drift as the later can only affect profitability.

Process drift has a great influence on the production lot sizing decisions as it directly contributes to producing more items that are defective. The cost of producing defective items will be added to a number of conflicting components in determining the optimal production lot size.

Three aspects of any production process need to be clearly defined and distinguished; namely, process deterioration, out-of-control and system failure:

- A production process is said to be in the out-of-control state if it experiences special or assignable cause variation in its output, and hence produces defective items in greater rate compared to the rate when it is in-control.
- Process deterioration is the event of shifting from in-control state to out-of-control state due to some special cause. After some time of the production run, process parameters start to change, i.e. increase or decrease in process mean or variance.

- Failure means stoppage of the production process. In some systems failure is simply the extreme end of deterioration, while in others, failure is independent from deterioration and totally resulted from different causes.

1.3 Trade Credit Financing and Economic Ordering

Economic ordering decisions play a vital rule in business success especially in retailing industry. In the traditional EOQ model it is assumed that the retailer pays the purchasing cost of the products as soon as the products are received which contradicts the reality in which the supplier (wholesaler or manufacturer) usually offers a delay period, known as trade credit period, to encourage the retailer to order more quantity.

In cases that the supplier is the manufacturer of the product, and for the sake of better production and inventory control, manufacturers prefer less frequent orders with larger order sizes to frequent orders with smaller order sizes. In such situation, they offer a longer credit period for larger amount of purchase. Their policies are meant to motivate the retailer to make order size large enough to avail for a credit period (Soni et al 2010).

Usually it is assumed that the supplier would offer a fixed credit period to the retailer but the retailer in turn would not offer any credit period to its customers, which is unrealistic, because in real practice retailer might offer a credit period to his customers in order to stimulate his own demand (Jaggi et al 2008).

The supply chain system in which the supplier offers trade credit to his customer (retailer) and the retailer also offers trade credit to his customers is referred to as two-levels of trade credit system. Trade credits can be viewed as a kind of price discount, since paying later indirectly reduces the purchase cost. In the literature review section, we will present some of the work done in the economic ordering problem in the presence of one-level and two-levels of trade credit financing.

1.4 Thesis Objectives

This research aims to develop four new models in the area of inventory control. The first two models are designed to determine the optimal production quantity for an unreliable production system, which is subject to random linear process deterioration and random machine breakdown. Additionally, preventive and corrective maintenance actions are performed according to a specific policy, and their durations are random as well. In one model, failure and deterioration are assumed independent, while in the other model, the two events are assumed dependent where failure can happen only if preceded by deterioration. In both model, process deterioration starts after some random time, at which the rate of producing defectives increases linearly with time.

The other two models are designed to determine the optimal order quantity in retailing industry in addition to the optimal credit period offered by the retailer to his customers. Both models are designed in the presence of two-level of trade credit periods, in addition to the assumption of credit-linked demand. In two-level trade credit systems, the wholesaler offers the retailer a period to settle the due payment. Similarly, the retailer offers each of his customers a period to settle their payments. In both models, replenishment from the supplier to the retailer is assumed non-instantaneous, additionally a percentage of the retailer's sales are considered as bad debt. The two models differ in the payment procedure from the retailer to his supplier.

1.5 Thesis Organization

This thesis is presented in seven chapters. In chapter 2, we present the relevant literature on the production lot sizing models, and the Economic Order Quantity (EOQ) models in retailing industry.

Chapter 3 presents the first production lot-sizing model, its description, assumptions, notation, mathematical formulation, optimality under selected probability distributions, and finally numerical results and conclusions.

Chapter 4 presents the second production lot-sizing model, its description, assumptions, mathematical formulation and numerical results and conclusions.

Chapter 5 presents the first EOQ model in retailing industry, its description, assumptions, notation, mathematical formulation, optimality, solution procedure, and finally numerical results and conclusions.

Chapter 6 presents the second EOQ model in retailing industry, its description, assumptions, mathematical formulation and numerical results and conclusions.

Chapter 7 presents thesis conclusions and gives directions for future research.

CHAPTER 2

LITERATURE REVIEW

In this chapter we present some of the notable researches carried out on the two problems of our interest; namely the production lot sizing under process deterioration and/or machine breakdown, and the economic ordering under one-level and two-level credit financing.

2.1 Production Lot Sizing Models

As stated earlier, the lot-sizing problem is originated from the classical Economic Order Quantity (EOQ) model invented by Ford W. Harris in the year 1913. A tremendous amount of research can be found in the literature about this important problem in the area of production and inventory planning and control. Our focus is on the lot sizing models for unreliable production systems in which machine failure and/or process deterioration is present.

While both process deterioration and machine breakdowns have great influence on Economic Manufacturing Quantity (EMQ) decisions, most of the research considers only one of the two factors while ignoring the effect of the other. Rahim and Lashkari (1985) developed a model for determining the optimal production run time in an industrial process in which process mean and process variance are likely to shift and assume different values compared to the initial ones. Arcelus and Banerjee (1987) developed optimal production policies for processes where the quality characteristic of the product exhibits non-negative shifts in both its mean and its variance and where different rewards exist for acceptable, undersized and oversized parts. Rahim and Banerjee (1988) considered the problem of selecting the optimal production run for a process with random linear drifts. Al-Sultan and Al-Fawzan (1997) extended Rahim and Banerjee (1988) by introducing lower and upper specification limits to the model. The new model aims to find the optimal initial process mean in addition to the optimal production cycle length.

Al-Sultan and Raouf (1998) considered a production process with a continuous drift in the mean of the quality characteristic of the product. They developed models for the problem in which process drift is either, known in advance and constant, or it occurs in a random fashion. Kim and Hong (1999) presented an EMQ model that determines the optimal production run length in a deteriorating production process. It is assumed that the process is subject to random deterioration from an in-control state to an out-of-control state with an arbitrary distribution, and thus produces some proportion of defective items. Three patterns of process deterioration are considered; constant, linearly increasing and exponentially increasing. Chung and Hou (2003) developed a model to determine the optimal run time for a deteriorating production system under allowable shortage. It is assumed that the elapsed time until the production process shift is arbitrarily distributed.

Ben-Daya (2002) dealt with an integrated model for the joint determination of economic production quantity and Preventive Maintenance (PM) level for an imperfect process having a general deterioration distribution with increasing hazard rate. The effect of PM activities on the deterioration pattern of the process is modeled using the imperfect maintenance concept. Hsieh and Lee (2005) considered two EMQ models with un-repairable and repairable standby key modules. They determined the economic production run length and the economic number of standbys in a deteriorating production process. Chiu et al. (2007) studied the optimal lot-sizing decision for a production system with rework, random scrap rate, and service level constraint.

Dagpunar (1996) examined the lot sizing problem with machine time to failure following a Weibull distribution; the machine is minimally repaired until the interrupted lot is completed; at the end of the production cycle, the machine is restored to as-good-as-new condition and a new cycle is started. Kim and Hong (1997) presented an EMQ model that determines the optimal production lot size in failure prone machine. It is assumed that time between failures is generally distributed, and machine is repaired instantaneously when it fails. Kuhn (1997) suggested a stochastic dynamic programming model to determine the optimal lot sizing decision where the equipment is subject to stochastic breakdowns. The analysis considered two cases; first, it is assumed that, after the machine breakdown, the setup is totally lost and new setup cost is incurred. The second case considers the situation in which the cost of resuming the production run after a failure might be substantially lower than the production set-up cost.

Moini and Murthy (2000) developed a production-sizing model for unreliable system with machine breakdown and quality variations under alternative repair option strategies. Chung (2003) showed that the long-run average cost function per unit of time for the case of exponential breakdowns is unimodal but neither convex nor concave, and he obtained an approximation for lower and upper bounds on lot sizing under this condition.

Giri and Dohi (2004) considered the Net Present Value (NPV) approach to determine the economic manufacturing quantities for an unreliable production system over an infinite planning horizon. The NPV of the expected total cost was obtained under general breakdown time and general repair time. The criteria for the existence and uniqueness of the optimal production time were derived under exponential breakdown and constant/zero repair time.

Giri and Yun (2005) considered an economic manufacturing quantity problem for an unreliable manufacturing system where the machine is subject to random breakdown and at most two failures can occur in a production cycle. Upon the first failure; the repair action is started immediately and the demand is met first from the on-hand inventory. If shortages take place due to long repair time, then they are backlogged partially by resuming the production run after machine repair. If failure occurs again during the backlog period, then the accumulated shortages until completion of the second repair are assumed lost. The model was formulated under general breakdown and general repair time distributions.

Chiu et al (2007) considered the economic production quantity (EPQ) model with scrap, rework, and stochastic machine breakdowns. El-Ferik (2008) studied the joint determination of both economic production quantity and preventive maintenance schedules, under the realistic assumption that the production facility is subject to random breakdown and the maintenance is imperfect. The manufacturing system was assumed to deteriorate while in operation, with an increasing failure rate. The system undergoes PM either upon failure or after having reached a predetermined age, whichever of them occurs first.

Chiu et al (2011) developed a model for solving manufacturing run time problem with random defective rate and stochastic machine breakdown under no-resumption inventory policy. Widyadana and Wee (2011) developed a production inventory model with random machine breakdown and stochastic repair time for deteriorating items. The model assumes the machine repair time is independent of the machine breakdown rate. Das et al (2011) developed an economic production lot-sizing model for an item with imperfect quality and by considering random machine failure. Jeang (2012) developed a model for jointly determine the optimal production lot size and process parameters under the possibility of process deterioration and breakdown.

Boone et al. (2000) was the first to model a production lot sizing problem taking into consideration both machine breakdowns and process deterioration. The proposed model provided guidelines to choose the appropriate production run times to buffer against both the production of defective items and stoppages occurring due to machine breakdowns. The model assumed exponential time to breakdown, uniform time for the process to shift from in-control to out-of-control state, and constant rate of producing defectives when the process is out-of-control.

Chakraborty et al. (2008) presented a generalized economic manufacturing quantity model for an unreliable production system in which the production facility may shift from an 'in-control' state to an 'out-of-control' state at any random time (when it starts producing defective items) and may ultimately break down afterwards. If a machine breakdown occurs during a production run, then corrective repair is done; otherwise, preventive repair is performed at the end of the production run to enhance the system reliability. The proposed model is formulated assuming that the time to machine breakdown, corrective and preventive repair times follow arbitrary probability distributions. However, the criteria for the existence and uniqueness of the optimal production time are derived under general breakdown and uniform repair time (corrective and preventive) distributions.

Chakraborty et al. (2009) developed an integrated production, inventory and maintenance models for a deteriorating production system in which the production facility may not only shift from an 'in-control' state to an 'out-of-control' state but also may break down at any random point in time during a production run. In case of machine breakdown, production of the interrupted lot is aborted and a new production lot is started when the on-hand inventory is depleted after corrective repair. The process is inspected during each production run to examine the state of the production process. If it is found in the ‘in-control’ state then either (a) no action is taken except at the time of last inspection where preventive maintenance is done or (b) preventive maintenance is performed. If, however, the process is found to be in the ‘out-of-control’ state at any inspection then restoration is done. The proposed models are formulated under general shift, breakdown and repair time distributions.

Our work for this problem is an extension and modification to Chakraborty et al. (2008) model.

2.2 EOQ Models under Credit Financing

As mentioned earlier the EOQ model was first introduced by Ford W. Harris in the year1913 and gained researchers attention since then, and it continues to have the same level of interest for being a key problem in the area of inventory planning and control as it directly affects business success. While many extensions have been made to the original EOQ model in order to serve as a decision making tool, our focus in this work is the relation between the EOQ and trade credit financing in the retailing industry.

It is a common practice in business transactions for suppliers to allow a specified credit period to the retailers for payment without penalty to stimulate the demand of their products. This credit term in financial management is denoted as ‘‘net 30’’. Many research papers have appeared in the literature trying to build inventory models in which trade credit financing is involved. Some of the work done in this area is presented below.

Haley and Higgins (1973) proposed the first model that considers the EOQ under conditions of permissible delay in payments with deterministic demand, no shortages, and instantaneous delivery. Goyal (1985) established a single item inventory model for determining the economic ordering quantity in the case that the supplier offers the retailer the opportunity to delay his payment within a fixed period. Chung (1989) presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of trade credit.

Aggarwal and Jaggi (1995) extended Goyal (1985) by introducing deterioration to the model and assuming exponential deterioration rate. Jamal et al (2000) generalized the model to allow shortages. Teng (2002) revisited Goyal (1985) model and assumed that the selling price is not equal to the purchasing price (actually, it can be seen as a correction to Goyal's model as the proposed modification reflects the reality).

Huang (2003) extended one-level trade credit into two-level trade credit to develop the retailer's replenishment model from the viewpoint of the supply chain. He assumed that not only the supplier offers the retailer trade credit but also the retailer offers the trade credit to his customers. Huang (2006) incorporated both models of Teng (2002) and Huang (2003) by considering two-level trade credit and limitation on retailer's storage space to reflect the real-life situations. Chung and Huang (2007) proposed a twowarehouse inventory model for deteriorating items under two-level trade credit. Liao (2008) proposed an EOQ model with non-instantaneous delivery and exponentially deteriorating items under two-level trade credit.

Jaggi et al (2008) incorporated the concept of credit-linked demand and developed an inventory model under two levels of trade credit policy. In this model, the demand is assumed to be positively influenced by the credit period offered by the retailer. Thangam and Uthayakumar (2009) extended Jaggi et al (2008) model by assuming that demand depends on both the selling price and the credit period. Chen and Kang (2010) also assumed sensitivity of demand to retailer’s price in their two-level trade credit inventory model and they developed a recursive solution procedure to determine the optimal pricing and production/order strategy.

Ho (2011) proposed a generalized two-level trade-credit inventory model, in which the demand rate is a function of both retail price and credit period. Kreng and Tan (2011) proposes a production model for a lot-size inventory system with finite production rate and defective quality under the condition of two-level trade credit policy and the condition that defective items involve both imperfect quality and scrap items. Lin et al. (2012) proposed an integrated supplier–retailer inventory model in which both supplier and retailer have adopted trade credit policies, and the retailer receives an arriving lot containing some defective items. The customer’s market demand rate depends on the length of the credit period offered by retailer. The model objectives is to determine the retailer's optimal order cycle length, the order quantity, and the optimal number of shipments per production run from the supplier to the retailer so that the entire supply system has maximum profit. Su (2012) proposed a single-supplier, single-retailer integrated inventory model that accounts for defective items that arrive in a retailer's order under a full-lot inspection policy. Shortages are allowed and are fully backlogged. Only supplier offers trade credit to the retailer.

Teng et al. (2012) extended the constant demand to a linear non-decreasing demand function of time in building their EOQ model with trade credit. Chung (2012) introduced the transportation cost in developing a new supplier–retailer inventory model under the condition that both supplier and retailer offer trade credits. Zhou et al. (2012) proposed an EOQ under conditions of trade credit, inventory dependent demand, and limited displayed-shelf space. Thangam (2012) considered a supply chain where the supplier offers the retailer a full trade credit period for payments whereas the retailer offers a partial trade credit to his customers in addition to another option of price discount if advance payment is made. Model objective is to find the optimal price discount and the optimal lot-sizing policies for perishable items. Jaggi et al. (2012) proposed a model to determine the retailer's optimal replenishment and credit policies under tow-level of credit policy when demand is influenced by credit period.

Our work for this problem is an extension and modification to Jaggi et al. (2008) model and will be presented in the following chapters.

2.3 Conclusion

The surveyed literature on the production lot-sizing problem reveals the scarcity of models that jointly considers process deterioration and machine breakdown in determining the optimal production quantity. Boone et al. (2000) was the first to devise such model followed by Chakraborty et al. (2008) who evolved the model in some directions. Our work on this problem is an extension and modification to Chakraborty et al. (2008). Our main contribution summarized in introducing the linear process deterioration concept, in addition to the assumption of independent deterioration and failure events. Please refer to thesis objectives, section 1.4 for more information.

A tremendous amount of literature found on the EOQ in retailing industry under trade credit financing, but none of them covers the case in which replenishment from supplier to retailer is non-instantaneous, neither the case of bad debt. Our work on this problem, which is an extension and modification to Jaggi et al. (2008), will incorporate those two aspects and devise two new models. Please refer to thesis objectives, section 1.4 for more information.

CHAPTER 3

PRODUCTION LOT SIZING MODEL-I

3.1 Introduction

Production systems are unreliable to a significant degree in real life; one source of unreliability is process deterioration. Process deterioration might be stated as the process of shifting from an in-control state to an out-of-control state where the production system starts to produce more items that are defective. In addition to deterioration, there is the possibility of machine breakdown that causes the abortion of the production lot before completion. Obviously, any breakdown will severely affect plans for meeting customer demand. Building on the previous argument, the need for more realistic modeling of the Economic Manufacturing Quantity (EMQ) problem is rising in the manufacturing field. Such models should take into consideration many attributes of real life production systems including but not limited to,

a. Time to shift from in-control state to out-of control state, and its probability distribution.
b. Time to machine breakdown, and its probability distribution.

c. Corrective maintenance time and its probability distribution (performed after each breakdown).

d. Preventive maintenance time and its probability distribution (performed at the end of each successfully completed production run, in our model).

e. Rate of producing defective items, we consider production of defectives to take place before process deterioration, and we assume that the rate of producing defectives increases linearly with time after deterioration is started.

f. In addition, we should consider a variety of costs corresponding to production, inventory and maintenance such as; corrective and preventive repair costs, inventory holding cost, shortage penalty cost, cost of producing defective items and finally production set-up cost.

The following notations are used in building model-I:

t	None-negative random variable denoting time to machine breakdown.
$f(t)$	Time to breakdown probability density function.
λ	Failure rate (parameter for $f(t)$ when it is exponentially distributed).
τ	Random variable denoting the time taken by the machine to shift from "in-control" state to "out-of-control" state.
$h(\tau)$	The probability density function of the time to shift from in-control to out-of-control state.
γ	Deterioration rate (parameter for $h(\tau)$ when it is exponentially distributed).

t_o	Production run time; a decision variable.
$\underline{t_o}$	The lower bound on t_o.
$\overline{t_o}$	The upper bound on t_o.
l_1	None-negative random variable denoting corrective repair time.
$g_1(l_1)$	Corrective repair time probability density function.
b_1	The upper bound on l_1 when it's uniformly distributed.
l_2	None-negative random variable denoting preventive repair time.
$g_2(l_2)$	Preventive repair time probability density function.
b_2	The upper bound on l_2 when it's uniformly distributed.
d	Demand rate, $(d > 0)$.
p	Production rate, $(p > d)$
c_O	Set up cost for each production run, $(c_O > 0)$.
c_1	Corrective repair cost per unit time, $(c_1 > 0)$.
c_2	Preventive repair cost per unit time, $(c_2 < c_1)$.
c_i	Inventory holding cost per unit product per unit time, $(c_i > 0)$.
c_s	Shortage penalty cost per unit product, $(c_s > 0)$.
c_D	Cost of producing a defective item, $(c_D > 0)$.
C_{cycle}	Expected total cost per production-inventory cycle.
T_{cycle}	Expected length of a production-inventory cycle.
W	Average cost per unit time.

α_I Proportion of defectives while process is in-control (before deterioration).

α_y Proportion of defectives while process is out of control (after deterioration).

β Process deterioration factor.

y Process deterioration timer.

3.2 Model Formulation

The model developed in this work is an extension of that in Chakraborty et al (2008). In that model, a process starts in control with no defectives generated. After a random period, the process deteriorates and defectives are generated at a constant rate. The machine may fail only after deterioration; i.e. every failure is preceded by process deterioration. The time to deteriorate follows a uniform distribution and it is dependent on the time to failure or the unknown production cycle time, whichever is shorter. If machine breakdown takes place during a production run, then the interrupted lot is aborted and a new lot is started after corrective maintenance is finished and all available inventory is depleted (no resumption policy). On the other hand, if machine breakdown does not occur until the end of the planned production run time t_o; then preventive maintenance is carried out after production run completion to get the machine back to "as good as new" condition before the start of the next production run. Again, the next production run will not start until available inventory is totally depleted even if repair has finished earlier. During machine repair, either corrective or preventive, the demand is met from the accumulated on-hand inventory. Shortages may occur due to longer corrective/preventive repair times. If shortages occur, they are not delivered after the machine repair; actually, they are considered as lost sales.

The model considered in this work extends that work in several directions and replaces some assumptions with more practical ones. We consider a production process which may shift from an in-control-state to an out-of-control state at any random time τ during the planned production run time t_o. In both states; in-control and out-of-control, defective items are produced at different rates. Once a shift to the out-of-control state has occurred at time τ, it is assumed that the proportion of defectives will continue to increase following a linear pattern as in the following equation;

$$\alpha_y = \alpha_I + \beta y \qquad (3\text{-}1)$$

α_I is the proportion of defectives before deterioration is started, $\beta > 0$ is a known scalar and y is time that quantifies the period while process is out of control.

The increase in defectives' rate is continued with time until the whole lot has been produced or machine breakdown takes place. We also assume that process deterioration and machine failure are independent events; hence, time to failure, t, and time to deterioration, τ, are two independent random variables. Therefore, not every machine failure is coming after deterioration, and similarly; not every process deterioration occurrence is followed by machine failure. In the case of machine tools, deterioration in the process might result from cutting tool wear while machine failure is a result of motor or any other mechanical or electrical part failure.

Figures 1 to 8 show all possible scenarios that any single production cycle may encounter.

Figure 1 shows the case where both deterioration and failure take place but no shortage. In this case, time to deterioration is less than time to failure, which is less than the planned production run time; $\tau \leq t \leq t_o$ and hence the production process encounters deterioration followed by failure. In addition, the corrective repair time is less than the time needed to consume the accumulated inventory; $l_1 \leq \frac{(p-d)t}{d}$ and hence no shortage is encountered.

Figure 2 shows the case in which deterioration, failure and shortage are encountered. Shortage happens because corrective repair action extended for longer time beyond the zero-inventory point, $l_1 > \frac{(p-d)t}{d}$.

In figure 3, deterioration takes place but neither failure nor shortage is encountered. In this case, time to deterioration is less than the planned production run time, which is less than time to failure; $\tau \leq t_o \leq t$ and hence the production process encounters deterioration but ends successfully without failure. In addition, the preventive repair time is less than the time needed to consume the accumulated inventory; $l_2 \leq \frac{(p-d)t_o}{d}$ and hence no shortage is encountered.

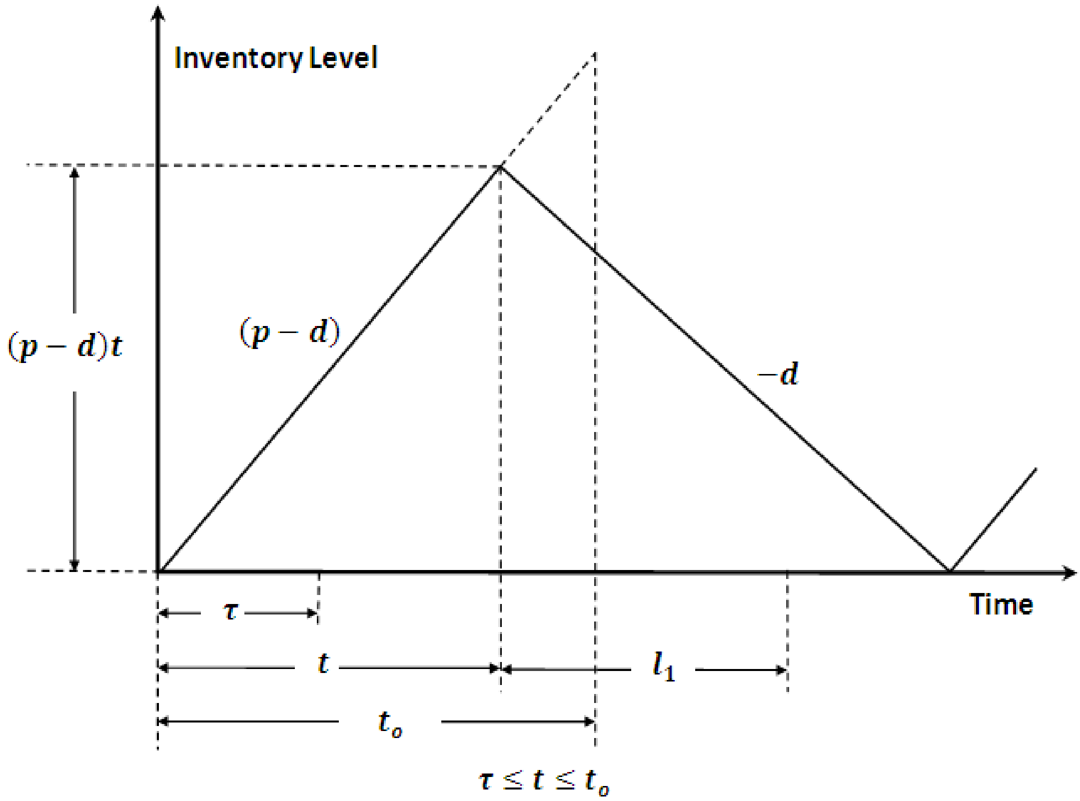


Figure 1 Deterioration-Failure-No Shortage Case in Model-I

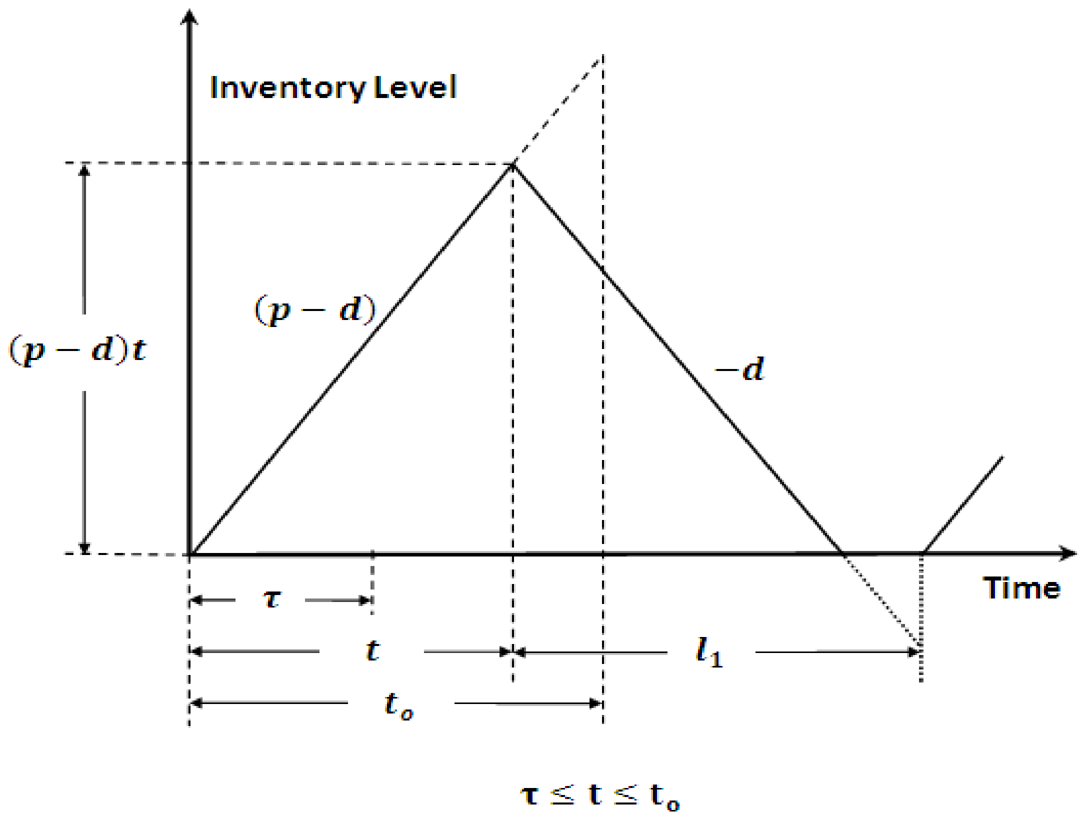


Figure 2 Deterioration-Failure-Shortage Case in Model-I

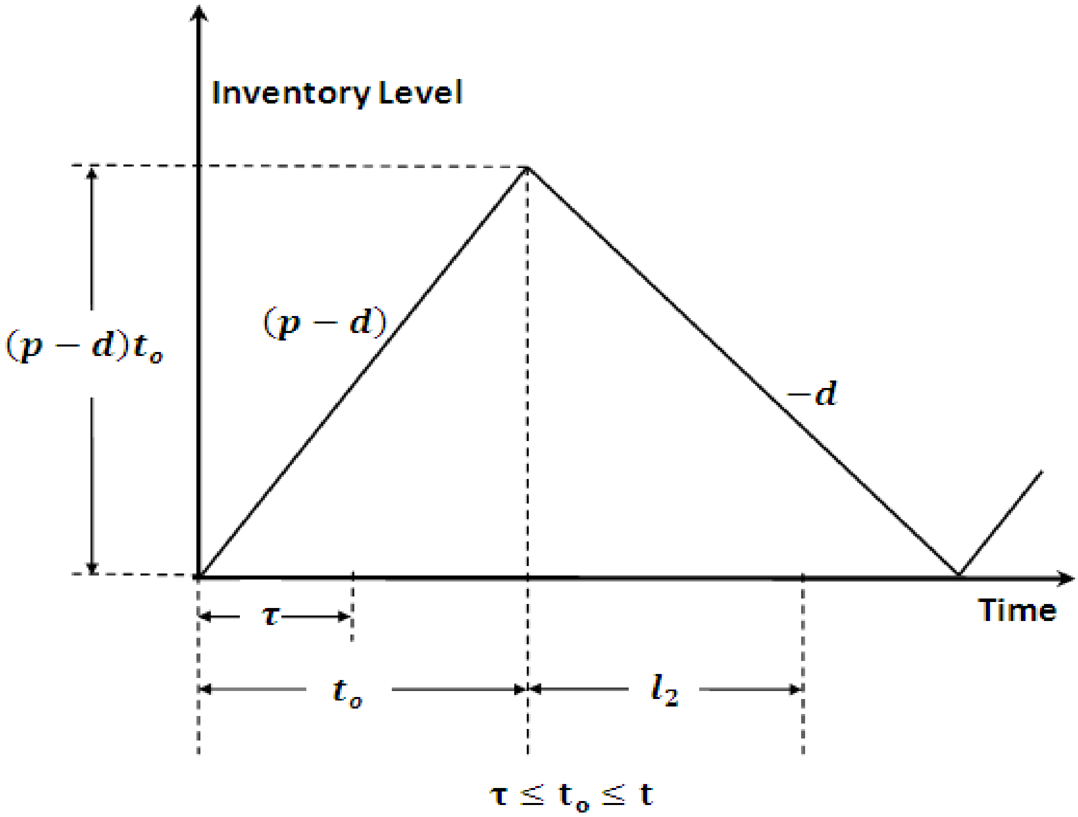


Figure 3 Deterioration-No Failure-No Shortage Case in Model-I

In figure 4, deterioration and shortage are encountered but no failure takes place. Shortage happens because preventive repair action extended for longer time beyond the zero-inventory point, $l_2 > \frac{(p-d)t_o}{d}$.

In figure 5, failure takes place but neither deterioration nor shortage is encountered. In this case, time to failure is less than both; the planned production run time and the time to deterioration, $t \leq \tau \leq t_o \; or \; t \leq t_o \leq \tau$ and hence failure is encountered before process is deteriorated. In addition, the corrective repair time is less than the time needed to consume the accumulated inventory; $l_1 \leq \frac{(p-d)t}{d}$ and hence no shortage is encountered.

In figure 6, failure and shortage are encountered but no deterioration. Shortage happens because corrective repair action extended for longer time beyond the zero-inventory point, $l_1 > \frac{(p-d)t}{d}$.

In figure 7, no deterioration, no failure and no shortage are encountered. In this case, the planned production run time is less than both; the time to deterioration and the time to failure, $t_o \leq \tau \leq t \; or \; t_o \leq t \leq \tau$ and hence the production run ends successfully before experiencing deterioration or failure. In addition, the preventive repair time is less than the time needed to consume the accumulated inventory; $l_2 \leq \frac{(p-d)t_o}{d}$ and hence no shortage is encountered.

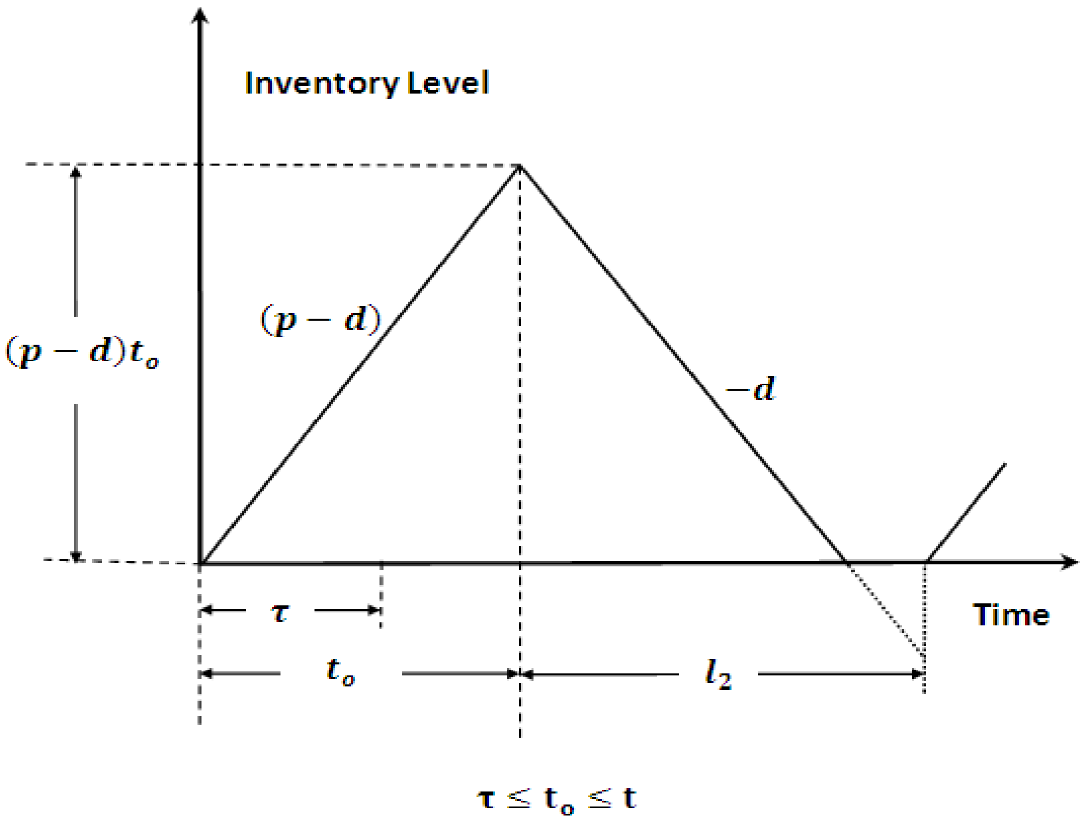


Figure 4 Deterioration-No Failure-Shortage Case in Model-I

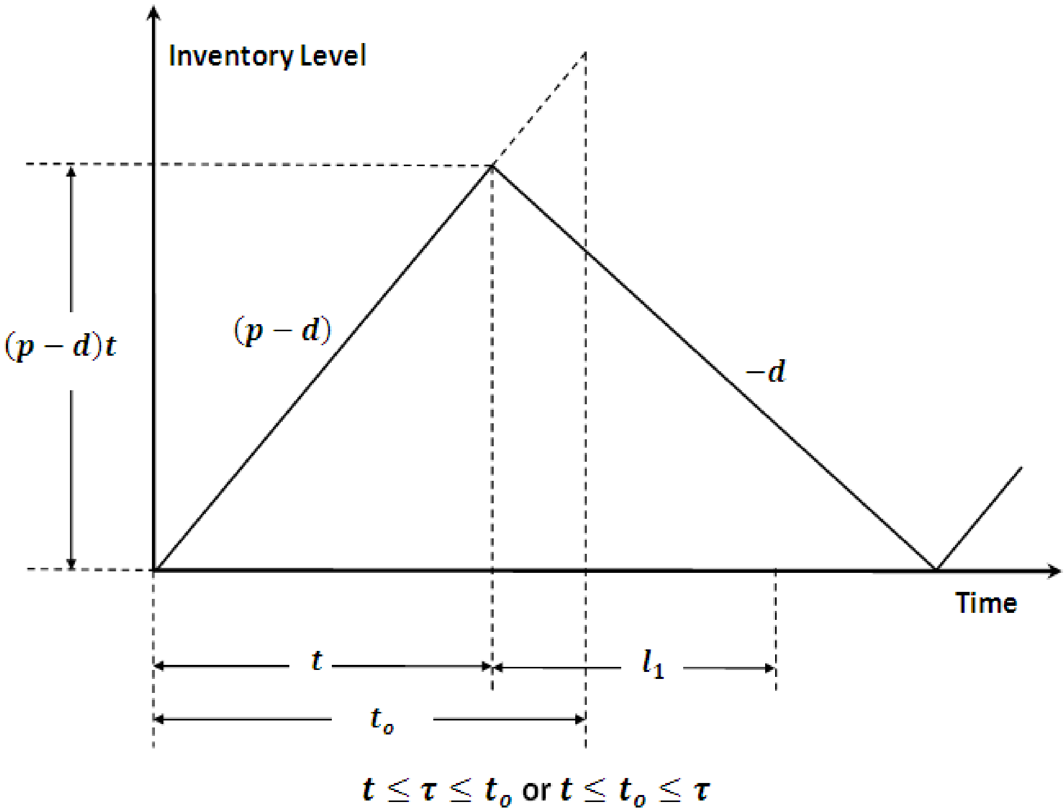


Figure 5 No Deterioration-Failure-No Shortage Case in Model-I

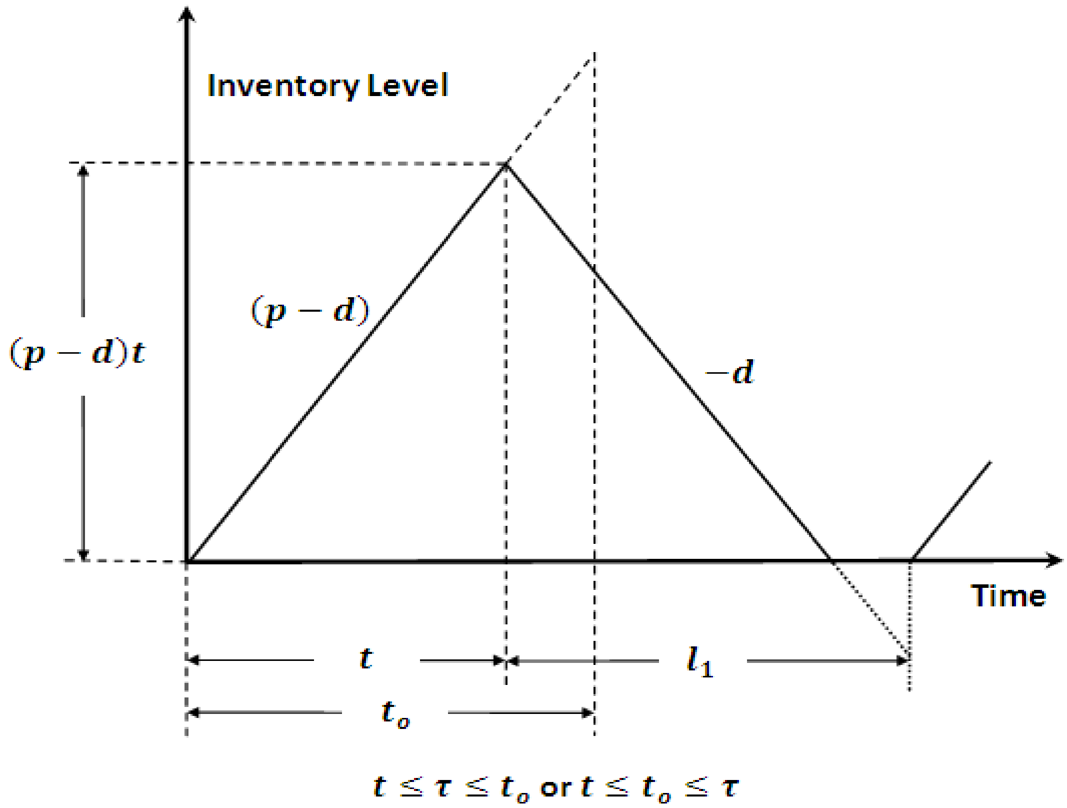


Figure 6 No Deterioration-Failure-Shortage Case in Model-I

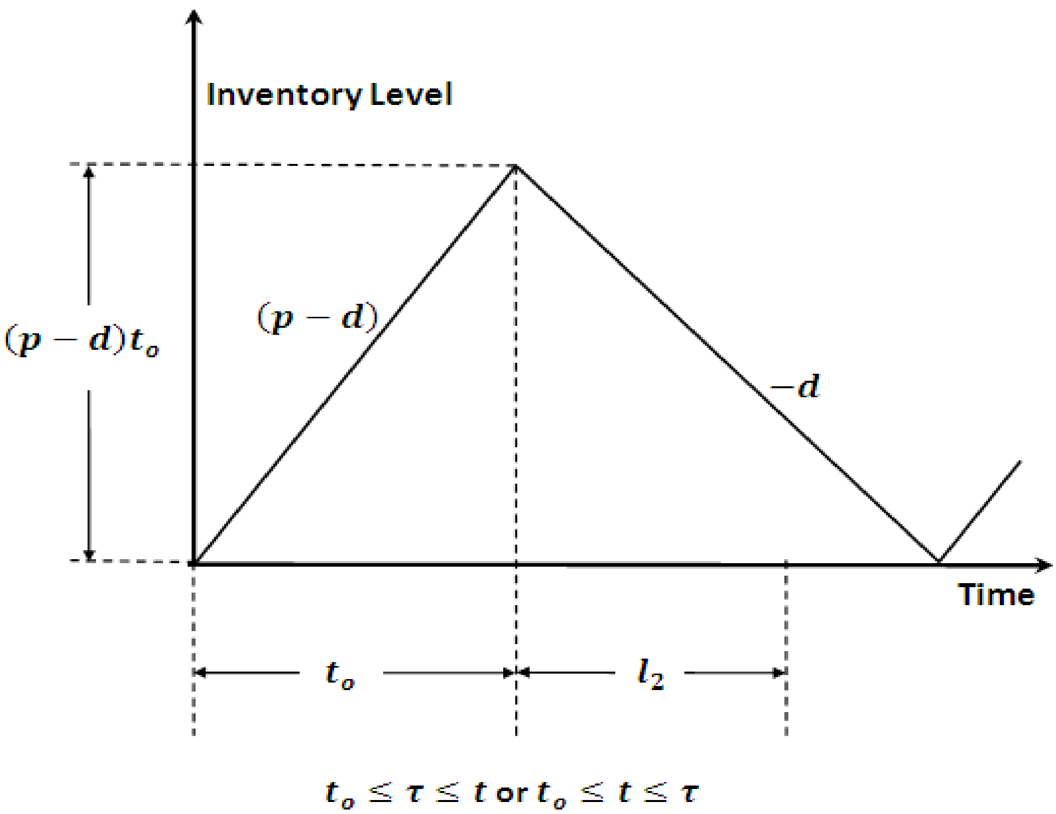
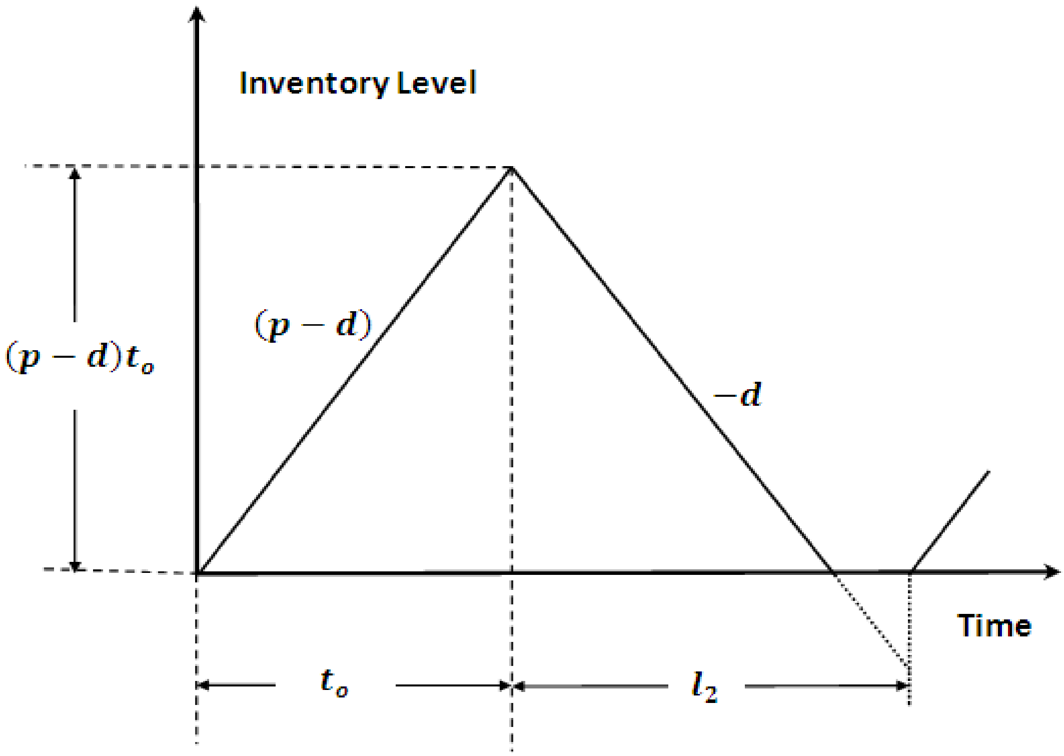


Figure 7 No Deterioration-No Failure-No Shortage Case in Model-I

In figure 8, no deterioration and no failure take place, but shortage is encountered. Shortage happens because preventive repair action extended for longer time beyond the zero-inventory point, $l_2 > \frac{(p-d)t_o}{d}$.

Those 8 figures presented above show all possible scenarios resulting from the randomness of time to deterioration, τ, time to failure, t, corrective repair duration, l_1, and preventive repair duration, l_2.



$t_o \leq \tau \leq t$ or $t_o \leq t \leq \tau$

Figure 8 No Deterioration-No Failure-Shortage Case in Model-I

By conditioning on the time to machine breakdown, the expected length of the production-inventory cycle is given by:

$$
\begin{aligned}
T_{cycle} = & \int_{t=0}^{t_o} \int_{l_1=0}^{(p-d)t/d} \left(t + \frac{(p-d)t}{d}\right). g_1(l_1). f(t). dl_1. dt \\
& + \int_{t=0}^{t_o} \int_{l_1=(p-d)t/d}^{\infty} (t + l_1). g_1(l_1). f(t). dl_1. dt \\
& + \int_{t=t_o}^{\infty} \int_{l_2=0}^{(p-d)t_o/d} \left(t_o + \frac{(p-d)t_o}{d}\right). g_2(l_2). f(t). dl_2. dt \\
& + \int_{t=t_o}^{\infty} \int_{l_2=(p-d)t_o/d}^{\infty} (t_o + l_2). g_2(l_2). f(t). dl_2 . dt
\end{aligned}
$$

The first term in the expected cycle length represents those cycles that will encounter failure but no shortage (Figures 1 and 5). The second term represents the cycles with failure and shortage (Figures 2 and 6). The third term represents the cycles with no failure and no shortage (Figures 3 and 7). Finally, the last term represents the cycles with no failure but with shortage (Figures 4 and 8). The expected production-inventory cycle length can be reduced to,

$$T_{cycle} = \int_{t=0}^{t_o}\int_{l_1=0}^{(p-d)t/d}\left(\frac{pt}{d}\right).g_1(l_1).f(t).dl_1.dt$$
$$+\int_{t=0}^{t_o}\int_{l_1=(p-d)t/d}^{\infty}(t+l_1).g_1(l_1).f(t).dl_1.dt$$
$$+\int_{t=t_o}^{\infty}\int_{l_2=0}^{(p-d)t_o/d}\left(\frac{pt_o}{d}\right).g_2(l_2).f(t).dl_2.dt$$
$$+\int_{t=t_o}^{\infty}\int_{l_2=(p-d)t_o/d}^{\infty}(t_o+l_2).g_2(l_2).f(t).dl_2\,.dt$$

The total expected cost per production-inventory cycle is composed of setup cost, repair costs (corrective and preventive), inventory holding cost, shortage cost, and the cost due to producing defective items:

$$C_{cycle} = Setup\ Cost + Corrective\ Maintenance\ Cost$$
$$+ Preventive\ Maintenance\ Cost + Inventory\ Holding\ Cost$$
$$+ Shortage\ Cost + Defectives'\ Cost$$

The setup cost, c_o, is the cost incurred at the start of each production run to get the machine ready. Examples of setup costs include the cost of changing tools or dies, moving materials or components, testing the initial production output to ensure meeting specs, in addition to labor cost of setting up the machine. The measuring unit of c_o is \$/cycle.

The corrective maintenance cost is the cost incurred to bring the machine back to as good as it was before failure. It is calculated as the expected corrective repair time multiplied by the cost per unit time, c_1, and finally multiplied by the probability of encountering failure $(t \leq t_o)$, and is given by:

$$Corrective\ Repair\ Cost = c_1.\int_{t=0}^{t_o}\int_{l_1=0}^{\infty} l_1.g_1(l_1).f(t).dl_1.dt$$

The preventive maintenance cost is the cost incurred at the end of each successful production run to enhance machine reliability. It is calculated as the expected preventive repair time multiplied by the cost per unit time, c_2, and finally multiplied by the probability of finishing the production cycle successfully with no failure $(t \geq t_o)$, and is given by:

$$Preventive\ Repair\ Cost = c_2.\int_{t=t_o}^{\infty}\int_{l_2=0}^{\infty} l_2.g_2(l_2).f(t).dl_2.dt$$

The measuring unit of c_1 and c_2 is $ per unit time.

It is worthy to notice that both corrective and preventive maintenance costs consider only the cost of time spent in performing maintenance actions, while the cost of material and spare parts is not included. In section 3.5, we modify this assumption by including material and spare parts' cost.

The inventory holding cost is the cost incurred to keep and maintain stock in storage; examples include space rent, handling, labor, insurance, security and opportunity loss. Inventory holding cost is given by:

$$Inventory\ Holding\ Cost$$
$$= c_i \int_{t=0}^{t_o} \frac{1}{2}\left(t + \frac{(p-d)t}{d}\right).(p-d).t.f(t).dt$$
$$+ c_i \int_{t=t_o}^{\infty} \frac{1}{2}\left(t_o + \frac{(p-d)t_o}{d}\right).(p-d).t_o.f(t).dt$$

The first term in the inventory holding cost expression is the average on hand inventory for cycles with failure multiplied by the inventory holding cost per unit product per unit time. The second term represents the cycles with no failure $(t \geq t_o)$. The inventory holding cost can be reduced to:

$$Inventory\ Holding\ Cost$$
$$= \frac{c_i p(p-d)}{2d}.\left[\int_{t=0}^{t_o} t^2.f(t).dt + {t_o}^2.\int_{t=t_o}^{\infty} f(t).dt\right]$$

The measuring unit of c_i is \$ per unit product per unit time.

The shortage cost is the cost of stock-out situation when there is demand but cannot be satisfied. In our model, shortages are considered lost sales and they are not backlogged. Shortage cost is given by:

$$Shortage\ Cost = c_s d.\left[\int_{t=0}^{t_o}\int_{l_1=\frac{(p-d)t}{d}}^{\infty}\left(l_1-\frac{(p-d)t}{d}\right).g_1(l_1).f(t).dl_1.dt\right] + c_s d.\left[\int_{t=t_o}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}\left(l_2-\frac{(p-d)t_o}{d}\right).g_2(l_2).f(t).dl_2.dt\right]$$

The first term in the shortage cost expression is the expected length of the shortage period multiplied by the demand rate and finally multiplied by the shortage cost per unit product for cycles with failure. The second term represents cycles with no failure $(t \geq t_o)$. Shortages in our model are considered lost sales; accordingly, shortage cost is calculated based on the maximum shortage in units of the product rather than the average. The measuring unit of c_s is \$ per unit product.

Defectives' cost is the cost incurred due to producing less-quality items. Types of cost under this category include discounted price and rework. In our model, we consider defective parts to be used in filling the demand.

As stated earlier and due to the adoption of different assumptions than those in Chakraborty et al (2008), the cost of defectives is given by:

$$Defectives\ Cost$$

$$= pc_D\alpha_I.\int_{t=0}^{t_o}\int_{\tau=0}^{t}\tau.h(\tau).f(t).d\tau.dt$$

$$+ pc_D.\int_{t=0}^{t_0}\int_{\tau=0}^{t}\int_{y=0}^{t-\tau}\alpha_y\,.dy.h(\tau).f(t)\,.d\tau.dt$$

$$+ pc_D\alpha_I.\int_{t=t_o}^{\infty}\int_{\tau=0}^{t_o}\tau.h(\tau).f(t).d\tau.dt$$

$$+ pc_D.\int_{t=t_0}^{\infty}\int_{\tau=0}^{t_0}\int_{y=0}^{t_0-\tau}\alpha_y\,.dy\,.h(\tau).f(t).d\tau.dt$$

$$+ pc_D\alpha_I.\int_{\tau=0}^{t_o}h(\tau).\left(\int_{t=0}^{\tau}t.f(t).dt\right).d\tau$$

$$+ pc_D\alpha_I.\int_{\tau=t_o}^{\infty}h(\tau).d\tau\,.\int_{t=0}^{t_o}t.f(t).dt$$

$$+ pc_D\alpha_I.t_o.\int_{\tau=t_o}^{\infty}h(\tau).d\tau\,.\int_{t=t_o}^{\infty}f(t).dt$$

The first term in the defectives cost expression gives the cost of defectives produced during the in-control state for cycles in which deterioration is followed by failure (Figure 1 and 2). The second term gives the cost of defectives produced during the out-of-control state for cycles in which deterioration is followed by failure (Figure 1 and 2). α_y is defined by equation 3-1. The third term gives the cost of defectives produced during the in-control state for cycles in which only deterioration is encountered but no failure (Figure 3 and 4). The forth term gives the cost of defectives produced during the out-of-control state for cycles in which only deterioration is encountered but no failure (Figure 3 and 4). The fifth expression gives the cost of defectives produced during the cycles in which failure is encountered but no deterioration, and in which the following inequality applies $t \leq \tau \leq t_o$ (Figure 5 and 6). The sixth expression gives the cost of defectives produced during the cycles in which failure is encountered but no deterioration, and in which the following inequality applies $t \leq t_o \leq \tau$ (Figure 5 and 6). Finally, the last expression gives the cost of defectives produced during the cycles in which neither deterioration nor failure is encountered, $t_o \leq \tau \leq t \; or \; t_o \leq t \leq \tau$ (Figure 7 and 8). The measuring unit of c_D is \$ per unit product.

From the renewal reward theorem, the average cost per unit time is given by;

$$W(t_o) = \frac{C_{cycle}(t_o)}{T_{cycle}(t_o)}$$

In the next section, we consider special cases of the distribution functions and simplify the above expressions accordingly. We also show that the average cost per unit time is a quasi-convex function in specific interval for the selected distributions.

3.3 Optimality under Selected Distributions

In this section, we will assume that the time to failure follows an exponential distribution with failure rate λ,

$$f(t) = \lambda e^{-\lambda t}$$

The use of exponential failure can be justified by noticing that the planning horizon in our model is only few hours, i.e. the production run time. On the other hand, the aging effect, which results in increasing hazard rate over time, affects the manufacturing equipment only on the long run, i.e. months or years. Hence, within the planning horizon, the risk of having failure can be assumed constant.

Additionally, we assume time to shift follows an exponential distribution with rate γ,

$$h(\tau) = \gamma e^{-\gamma\tau}$$

Finally, we assume the corrective and preventive repair times to follow uniform distributions,

$$g_1(l_1) = \frac{1}{b_1}, 0 \le l_1 \le b_1$$

and,

$$g_2(l_2) = \frac{1}{b_2}, 0 \le l_2 \le b_2$$

After substitution and simplification, the expected production cycle cost can be expressed as:

$$C_{cycle} = z_1 {t_o}^2 e^{-t_o\lambda} + z_2 t_o e^{-t_o\lambda} + z_3 {t_o}^2 e^{-t_o(\gamma+\lambda)} - z_4 t_o e^{-t_o(\gamma+\lambda)} - z_5 e^{-t_o(\gamma+\lambda)} + z_6 e^{-t_o\lambda} + z_7$$

The constants z_1 to z_7 are given by:

$$z_1 = \frac{c_S(b_1 - b_2)(d-p)^2}{2b_1b_2d}$$

$$z_2 = \frac{\frac{(d-p)[b_1c_Ip + c_S(p-d)]}{b_1d} - c_Dp\beta}{\lambda}$$

$$z_3 = c_Dp\alpha_I$$

$$z_4 = \frac{c_Dp\alpha_I(\lambda - 1)}{\lambda}$$

$$z_5 = \frac{c_Dp\beta}{\gamma(\gamma + \lambda)}$$

$$z_6 = \frac{1}{2}[-b_1(c_1 + c_Sd) + b_2(c_2 + c_Sd)] + \frac{(d-p)[b_1c_Ip + c_S(p-d)]}{b_1d\lambda^2} + \frac{c_S(p-d)}{\lambda} - \frac{c_Dp[\beta(\gamma - \lambda) + \alpha_I\gamma\lambda]}{\gamma\lambda^2}$$

$$z_7 = c_O + \frac{1}{2}b_1(c_1 + c_Sd) + \frac{(d-p)[c_Sd - (b_1c_I + c_S)p]}{b_1d\lambda^2} + \frac{c_S(d-p)}{\lambda} + \frac{c_Dp[\beta\gamma + \alpha_I\lambda(\gamma + \lambda)]}{\lambda^2(\gamma + \lambda)}$$

The expected cycle length can be expressed as:

$$T_{cycle} = v_1{t_o}^2e^{-t_o\lambda} - v_2t_oe^{-t_o\lambda} - v_3e^{-t_o\lambda} + v_4$$

The constants v_1 to v_4 are expressed as:

$$v_1 = \frac{(b_1 - b_2)(d - p)^2}{2b_1 b_2 d^2}$$

$$v_2 = \frac{(d - p)^2}{b_1 d^2 \lambda}$$

$$v_3 = \frac{b_1 - b_2}{2} + \frac{(d - p)^2}{b_1 d^2 \lambda^2} + \frac{1}{\lambda}$$

$$\mathrm{v}_4 = \frac{\mathrm{b}_1}{2} + \frac{(\mathrm{d} - \mathrm{p})^2}{\mathrm{b}_1 \mathrm{d}^2 \lambda^2} + \frac{1}{\lambda}$$

Our objective is to determine the optimal production run time t_o^* which minimizes the average cost per unit time, $\mathrm{W}(\mathrm{t}_o)$. In order to avoid unrealistic values of the optimal run time, we will assume that $\underline{\mathrm{t}_o} \leq \mathrm{t}_o^* \leq \overline{\mathrm{t}_o}$ where $\underline{\mathrm{t}_o}$ and $\overline{\mathrm{t}_o}$ are the lower and the upper bounds on t_o, respectively and they are known in advance. The assumption of known lower and upper bonds of production run time can be justified due to many aspects including: machine design which prevents prolonged continuous run time to ensure safety, and expected customer's demand which suggests minimum amount to be produced.

In order to prove the existence and uniqueness of the solution for our model, we will prove that our cost function per unit time is quasi-convex. For Quasi-Convex functions, every local minimum is a global minimum, or otherwise the function is flat (constant) in the neighborhood of the local minimum (Greenberg and Pierskalla, 1971). If the function $\mathrm{W}(\mathrm{t}_o)$ is quasi-convex over the set $\{ \mathrm{t} \in [\underline{\mathrm{t}_o}, \overline{\mathrm{t}_o}]\}$ then:

$$W[\theta t_{o1} + (1-\theta)t_{o2}] \leq max[W(t_{o1}), w(t_{o2})]$$

For all $\theta \in [0,1]$ and all $t_{o1}, t_{o2} \in [\underline{t_o}, \overline{t_o}]$.

$W(t_o)$ is a Quasi-Convex function over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$ if any of the following conditions holds, (Greenberg and Pierskalla, 1971):

1. C_{cycle} is convex and $T_{cycle} > 0$ all over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$.
2. C_{cycle} is concave and $T_{cycle} < 0$ all over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$.
3. T_{cycle} is linear and $C_{cycle} \leq 0$ all over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$.
4. T_{cycle} is convex and $C_{cycle} \leq 0$ all over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$.
5. T_{cycle} is concave and $C_{cycle} \geq 0$ all over $\{t_o \in [\underline{t_o}, \overline{t_o}]\}$.

In what follows, we prove condition 5:

To prove that the expected production-inventory cycle length; T_{cycle}, is concave we need to find its second derivative and check its negativeness;

$$\frac{d^2 T_{cycle}}{dt_o^2} = e^{-\lambda t_o}[v_1\lambda^2 t_o^2 - (4v_1\lambda + v_2\lambda^2)t_o + 2v_1 + 2v_2\lambda - v_3\lambda^2]$$

The sign of the second derivative depends on the sign of the quadratic function in the square brackets. The coefficient of the quadratic term, $v_1\lambda^2$ is positive since the time to perform corrective repair is always greater than that needed to perform preventive maintenance, i.e. $b_1 > b_2$. Hence, the quadratic function is convex. The minimum of this function is given by:

$$-2v_1 - \frac{\lambda^2(v_2^2 + 4v_1v_3)}{4v_1}$$

This is a negative value, hence there is an interval (r_1, r_2) over which the second derivative of T_{cycle} is negative, and hence T_{cycle} is concave over this interval. In fact, r_1 and r_2 are the roots of the quadratic function of the second derivative, and they are given by:

$$r_1 = \frac{4v_1\lambda + v_2\lambda^2 - \lambda\sqrt{8v_1^2 + \lambda^2(v_2^2 + 4v_1v_3)}}{2v_1\lambda^2}$$

and;

$$r_2 = \frac{4v_1\lambda + v_2\lambda^2 + \lambda\sqrt{8v_1^2 + \lambda^2(v_2^2 + 4v_1v_3)}}{2v_1\lambda^2}$$

In conclusion, a unique solution is guaranteed if the following condition is satisfied:

$$r_1 < \underline{t_o} < \overline{t_o} < r_2 \tag{3-2}$$

Figure 9 shows the quasi-convexity of the function $W(t_o)$ for selected set of parameters.

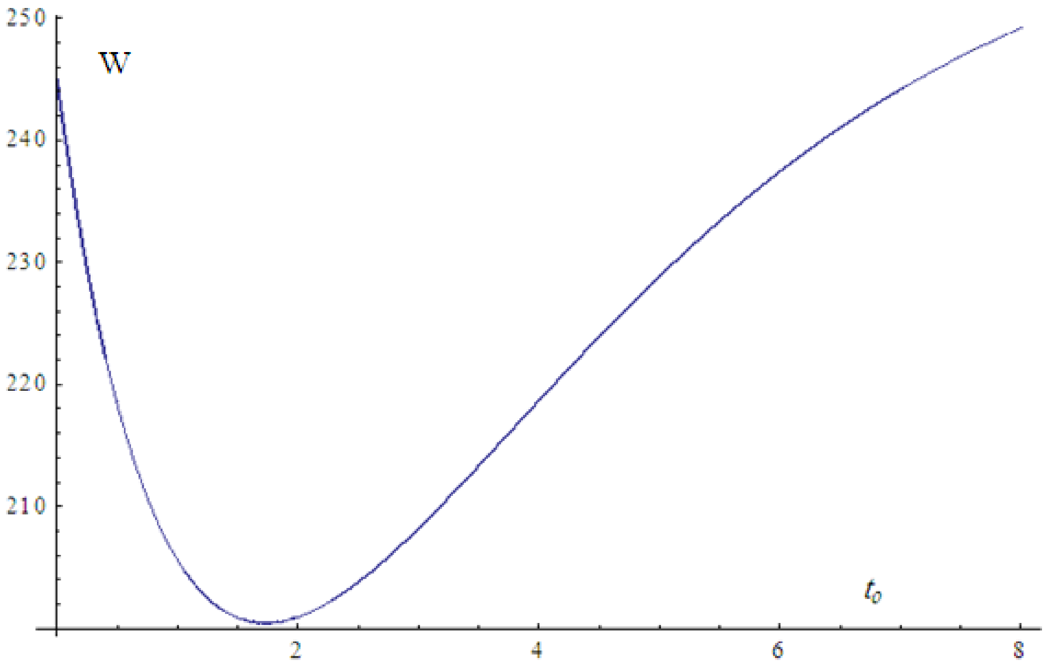


Figure 9 Quasi-Convexity of the average cost function for model-I

3.4 Results and conclusion

In this section, we use Wolfram Mathematica 8 in order to solve for the optimal production run time t_o and its associated average cost. In all calculations below, failure and deterioration are assumed to follow the exponential distribution with rates λ and γ respectively. In addition, corrective and preventive repairs are assumed to follow the uniform distribution.

The lower and upper bounds on the production run time t_o are assumed to imitate the normal work shift that extends to 8 hours:

$$0 \leq t_o \leq 8$$

The model parameters are: $p = 180$, $d = 90$, $c_O = 300$, $c_1 = 30$, $c_2 = 5$, $b_1 = 12$, $b_2 = 10$, $c_I = 0.5$, $c_S = 2$, $c_D = 3$, $\beta = 0.1$ and $\alpha_I = 0.05$.

Table 1 shows the dependency of the optimal production run time and the corresponding average cost on failure and deterioration rates; λ and γ respectively.

Results show that under low deterioration rate ($\gamma = 0.1$); the optimal production run time increases when the failure rate λ increases. This trend is justified because under low deterioration rate, the chances are low for the system to experience process deterioration, and hence defectives are unlikely to be generated; accordingly longer production run times are suggested by the model even with increasing failure rate. On the other hand, the average cost decreases as the failure rate increases for relatively low values of the failure rate ($\lambda < 0.4$), but it starts to increase when failure rate assumes relatively higher values.

Under relatively medium and high values of the deterioration rate ($\gamma \geq 0.3$); the optimal production run time decreases when the failure rate increases. This trend is justified because under higher deterioration rates, the chances are higher to experience process deterioration, and hence defectives are expected to be produced in a higher rate; accordingly shorter production run times are suggested by the model in order to reduce the instances of process deterioration and machine breakdown. On the other hand, and under medium values of the deterioration rate ($0.3 \leq \gamma \leq 0.5$); the average cost decreases as the failure rate increases for relatively low values of the failure rate, but it starts to increase when failure rate assumes higher values. Under high values of the deterioration rate ($\gamma \geq 0.7$); the average cost consistently increases as the failure rate increases.

Table number 2 shows that the condition in equation 3-2 is satisfied and all results in Table 1 are indeed the unique solutions for the model under different values of failure rate λ.

TABLE 1 Dependency of the optimal production policy on λ and γ in model-I

λ	$\gamma = 0.1$		$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.7$		$\gamma = 0.9$	
	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$
0.1	1.90	223.4	3.20	201.1	3.18	187.7	3.00	181.2	2.84	177.9
0.2	2.54	199.8	2.97	190.3	2.91	185.4	2.79	183.1	2.67	182.0
0.3	2.74	196.2	2.92	191.2	2.82	188.9	2.69	187.9	2.59	187.6
0.4	2.89	197.3	2.91	194.5	2.76	193.3	2.63	192.9	2.53	192.8
0.5	3.04	199.8	2.90	198.3	2.72	197.7	2.58	197.6	2.48	197.6
0.6	3.22	202.8	2.90	202.0	2.69	201.8	2.54	201.8	2.43	201.9
0.7	3.47	205.9	2.90	205.5	2.65	205.5	2.50	205.6	2.39	205.7
0.8	3.95	208.8	2.90	208.8	2.62	208.9	2.46	209.0	2.34	209.2
0.9	7.98	211.6	2.90	211.8	2.59	211.9	2.41	212.1	2.30	212.2

TABLE 2 Satisfying the optimality condition in model-I

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
r_1	-0.85	-5.56	-7.18	-8.02	-8.54	-8.89	-9.16	-9.36	-9.51
r_2	140.85	75.56	53.85	43.02	36.54	32.23	29.16	26.86	25.07

Figure 10 shows how the optimal production run time is affected by changing the deterioration rate γ and the deterioration factor β, under a fixed value of failure rate λ = 0.5. As expected, the optimal production run time tends to decrease when the deterioration factor β is increased while fixing the deterioration rate γ. Similarly, the optimal production run time decreases when the deterioration rate γ is increased while fixing the deterioration factor β. In both cases, the model is trying to reduce the cost of producing defective items by shortening the production run time, and hence reducing the time interval while the system is in the out-of-control state.

Again optimality is guaranteed for calculations in figure 10 based on table 2 as both γ and β have no effect on r_1 and r_2.

Figure 11 shows how the optimal cost is influenced by changing the deterioration rate γ and the deterioration factor β. It is obvious that increasing any of the two parameters will surely result in increasing the total cost. This result is fairly expected as increasing either the deterioration rate or the deterioration factor will increase the rate of producing defectives, which has direct effect on the cost function.

Figure 12 exhibits the same trend as in figure 10, but this time for different values of failure rate λ and under a fixed value of the deterioration rate γ = 0.5.

Figure 13 exhibits the same trend as in figure 11, but this time for different values of failure rate λ and under a fixed value of the deterioration rate γ = 0.5.

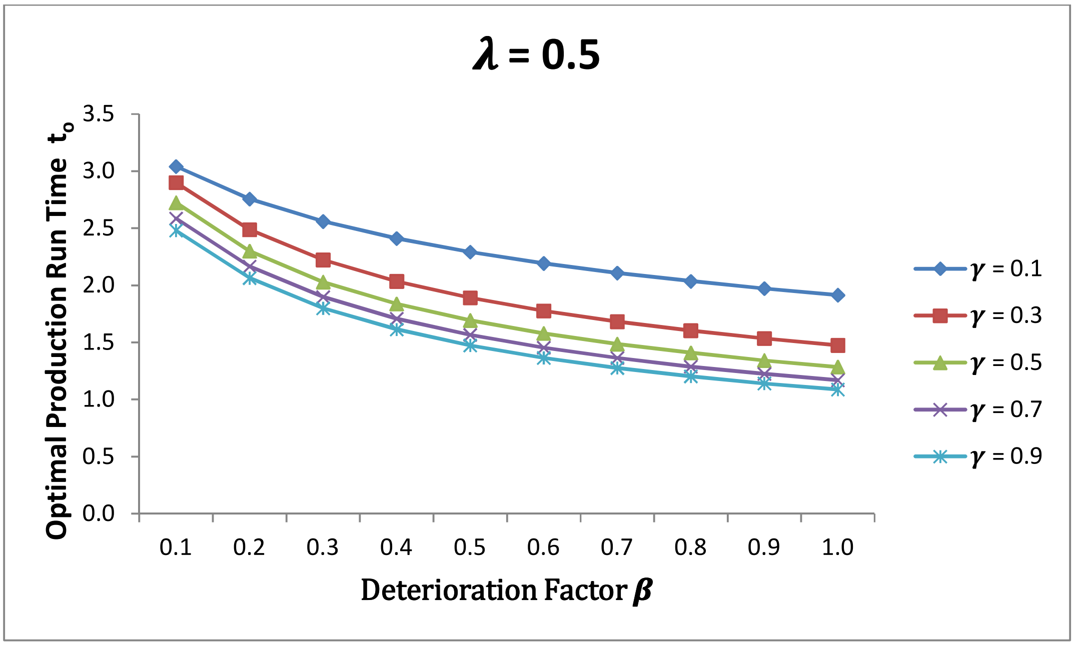


Figure 10 Dependency of the optimal production run time on β under fixed λ

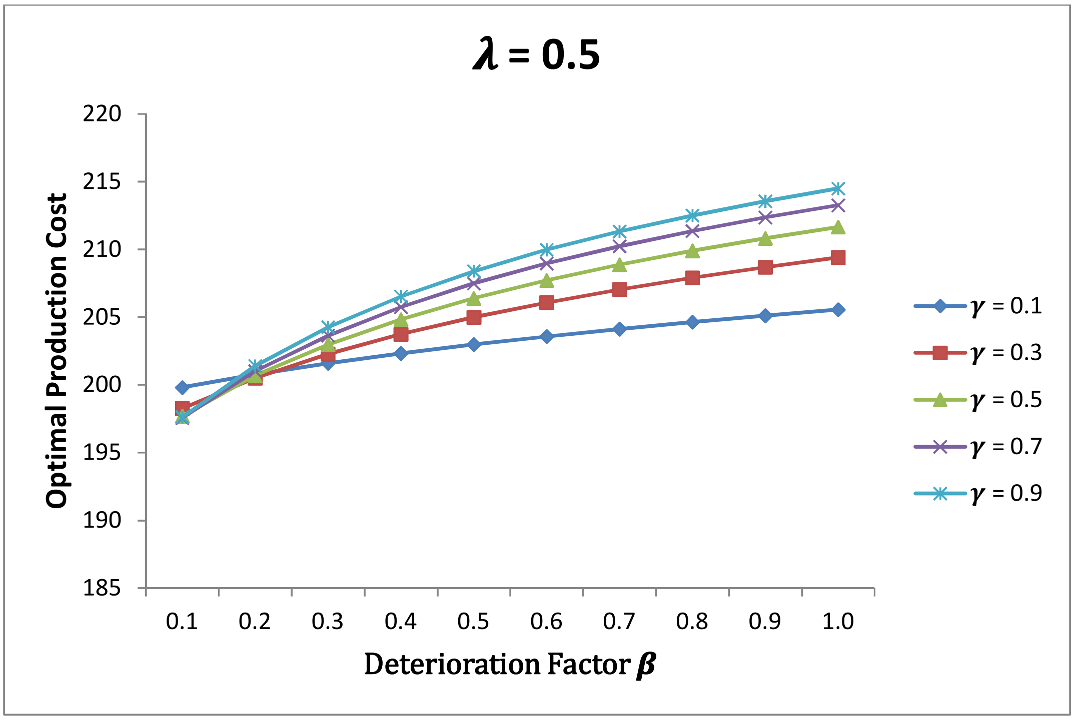


Figure 11 Dependency of the optimal production cost on β under fixed λ

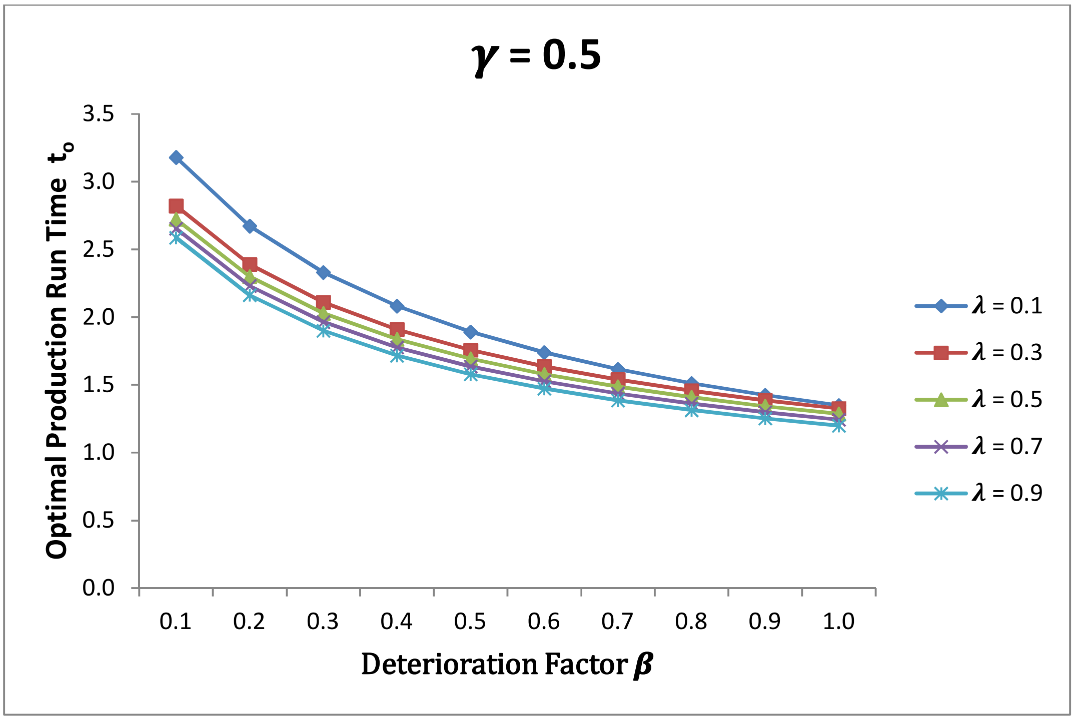


Figure 12 Dependency of the optimal production run time on β under fixed γ

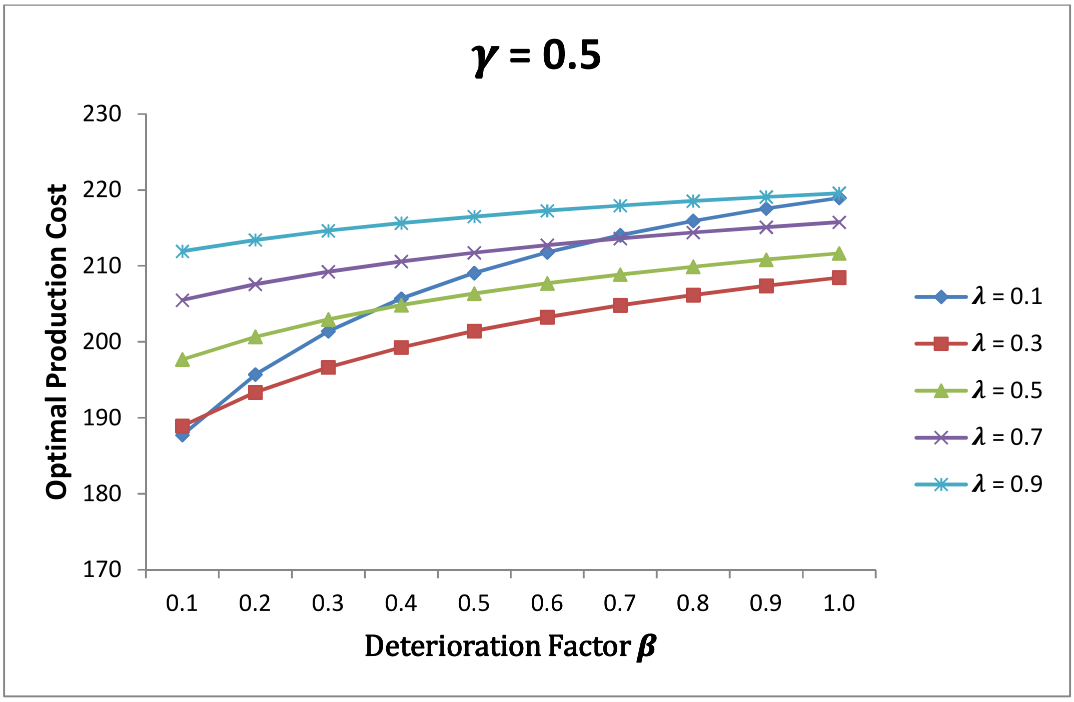


Figure 13 Dependency of the optimal production cost on β under fixed γ

Table 3 shows how the optimal production run time is affected by changing the corrective repair cost c_1 under fixed preventive repair cost c_2. An increase in the corrective repair cost results in a decrease in the optimal production run time. Shorter run times mean lower possibility of encountering failure, and hence the risk of incurring corrective repair cost is minimized.

Table 4 shows how the optimal production run time is affected by changing the preventive repair cost c_2 under fixed corrective repair cost c_1. Increasing the preventive repair cost results in increasing the optimal production run time. Longer run times mean lower possibility of successful completion with no failure; in this case, the risk of incurring preventive repair cost is minimized.

Again optimality is guaranteed for calculations in tables 3 and 4 based on table 2; c_1, c_2 and γ have no effect on r_1 and r_2.

TABLE 3 Sensitivity analysis by changing c_1 in model-I

$c_2 = 5$						
c_1	$\lambda = 0.1$ $\gamma = 0.9$		$\lambda = 0.5$ $\gamma = 0.5$		$\lambda = 0.9$ $\gamma = 0.1$	
	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$
10	2.890	174.169	3.060	185.389	8.000	194.946
15	2.879	175.098	2.979	188.504	8.000	199.103
20	2.867	176.025	2.895	191.596	8.000	203.259
25	2.856	176.950	2.811	194.662	8.000	207.416
30	2.844	177.873	2.724	197.700	7.983	211.572
35	2.833	178.794	2.635	200.709	3.768	215.695
40	2.821	179.713	2.544	203.686	2.874	219.701

TABLE 4 Sensitivity analysis by changing c_2 in model-I

$c_1 = 30$						
c_2	$\lambda = 0.1$ $\gamma = 0.9$		$\lambda = 0.5$ $\gamma = 0.5$		$\lambda = 0.9$ $\gamma = 0.1$	
	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$
5	2.844	177.873	2.724	197.700	7.983	211.572
9	2.875	179.735	2.824	198.376	8.000	211.574
13	2.905	181.584	2.922	199.014	8.000	211.576
17	2.936	183.420	3.018	199.620	8.000	211.578
21	2.966	185.244	3.113	200.194	8.000	211.580
25	2.996	187.056	3.206	200.740	8.000	211.583
29	3.026	188.855	3.297	201.258	8.000	211.585

In this chapter, we presented an EMQ model in which process deterioration and machine breakdown jointly affects the optimal production policy. Model is built for general failure, deterioration, corrective and preventive repair time distributions, but optimality is proved for exponential failure and deterioration times, and uniform corrective and preventive repair times. Process deterioration is assumed to take place gradually where the rate of producing defective items starts, at some point, to increase with time following a linear pattern. Process deterioration and machine breakdown events are assumed independent of each other. Numerical results supported the capability of the proposed model to be used as a decision making tool in finding the optimal production policy. The work presented here can be extended in many directions; for instance, the process drift can be assumed to follow an exponential pattern rather than a linear pattern that might suit some applications. Additionally, inspection process can be incorporated in the model especially that we already have defectives production and in increasing rate.

One interesting extension to this model is by considering that machine failure, if happened, is always preceded by process deterioration, and hence the system will never encounter production cycles with failure but no deterioration. In the following chapter, we develop this model and study its behavior.

3.5 Maintenance material and spare parts costs

In the primary model, the assumption is made that the cost of maintenance is solely due to the time spent in performing it. This assumption might be acceptable in some circumstances in which maintenance does not require spare parts and other consumables. In many situations, cost of material required to perform maintenance is significant, accordingly we can modify the corrective and preventive maintenance costs as in the following:

$$Corrective\ maintenance\ Cost$$
$$= c_1.\int_{t=0}^{t_o}\int_{l_1=0}^{\infty} l_1.g_1(l_1).f(t).dl_1.dt + c_{1m}\int_{t=0}^{t_o} f(t).dt$$

Similarly, the preventive maintenance cost is expressed as:

$$Preventive\ maintenance\ Cost$$
$$= c_2.\int_{t=t_o}^{\infty}\int_{l_2=0}^{\infty} l_2.g_2(l_2).f(t).dl_2.dt + c_{2m}\int_{t=t_o}^{\infty} f(t).dt$$

c_{1m} and c_{2m} represent the cost of material needed in performing one corrective or preventive maintenance action respectively. It is important to emphasize on the fact that in a single production run, either corrective or preventive maintenance action is performed once.

CHAPTER 4

PRODUCTION LOT SIZING MODEL-II

4.1 Introduction

In the previous model (Model-I) we assumed that the process deterioration and machine failure are two independent events; hence the two random variables t and τ are independent. Therefore, not every machine failure is coming after deterioration, and similarly; not every process deterioration occurrence is followed by machine failure. In this chapter, we will develop a production lot sizing model in which process deterioration and machine failure are correlated; meaning that machine failure can happen only if preceded by process deterioration.

Same notation as in chapter three will be used here; the only difference is that the time to failure t will be considered to start from the time when deterioration starts; i.e. after τ, instead of starting from the beginning of the production-inventory cycle.

4.2 Model Formulation

The model developed in this chapter is similar to that developed in chapter three, accordingly we will not repeat model description and we go directly to building the mathematical model.

The rate of producing defectives starts to increase linearly with time after some random time τ. Process transition from in-control to out-of-control state follows the following linear equation:

$$\alpha_y = \alpha_I + \beta y$$

Figures 14 to 19 show all possible scenarios that any single production cycle may encounter.

Fig.14 shows the case in which both process deterioration and failure take place but no shortage is encountered because corrective repair is finished before the inventory is completely depleted, $l_1 \leq \frac{(p-d)(\tau+t)}{d}$.

Figure 15 shows the case where process deterioration, failure and shortage are encountered. Shortage happens because corrective repair extended for longer time beyond the point of zero-inventory, $l_1 > \frac{(p-d)(\tau+t)}{d}$.

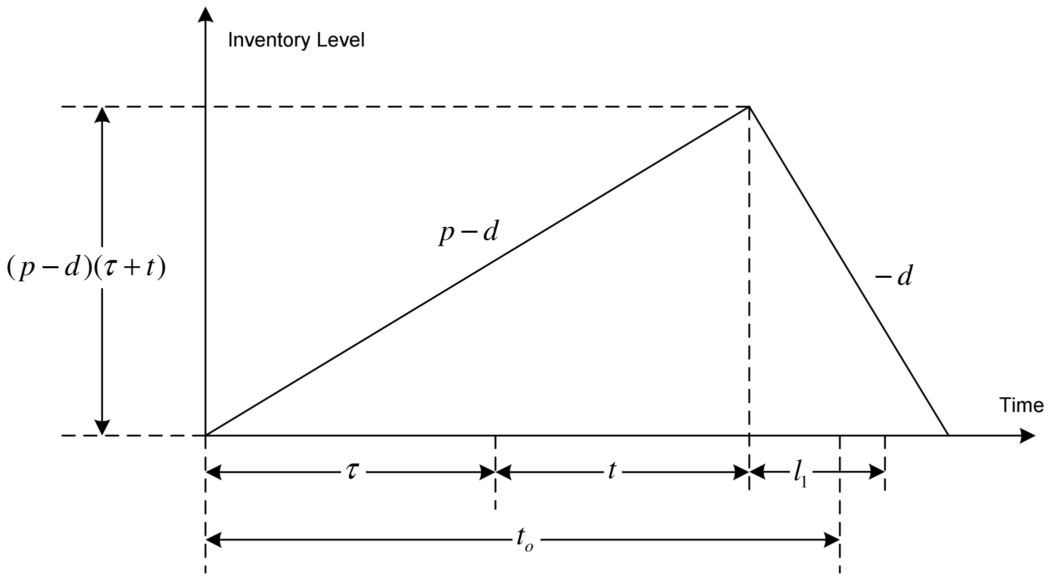


Figure 14 Deterioration-Failure-No Shortage Case in Model-II

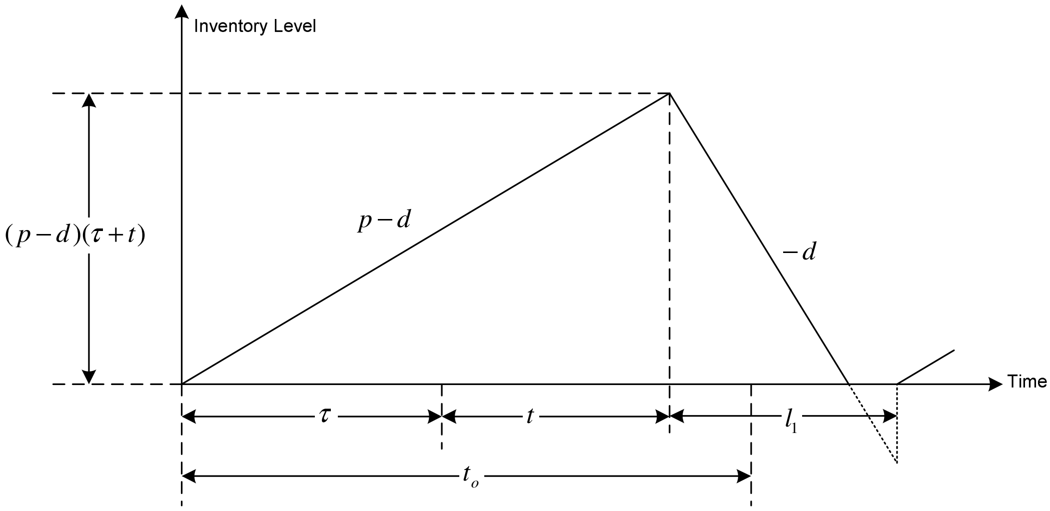


Figure 15 Deterioration-Failure-Shortage Case in Model-II

In figure 16, deterioration takes place, but neither failure nor shortage is encountered. In this case, the preventive maintenance action is finished before the inventory is completely depleted, $l_2 \leq \frac{(p-d)t_o}{d}$.

In figure 17, deterioration and shortage are encountered but no failure takes place. Shortage happens because the preventive maintenance extended for longer time beyond the point of zero-inventory, $l_2 > \frac{(p-d)t_o}{d}$.

In figure 18 no deterioration, no failure and no shortage are encountered. In this case, the preventive maintenance action is finished before the inventory is completely depleted, $l_2 \leq \frac{(p-d)t_o}{d}$.

In figure 19, no deterioration and no failure take place, but shortage is encountered. Shortage happens because the preventive maintenance extended for longer time beyond the point of zero-inventory, $l_2 > \frac{(p-d)t_o}{d}$.

Those 6 figures show all possible scenarios resulting from the randomness of time to shift τ, time to failure t, corrective maintenance duration, l_1, and preventive maintenance duration, l_2. We note that, no case in which failure happens without being preceded by process deterioration as we stated earlier as an assumption for this model.

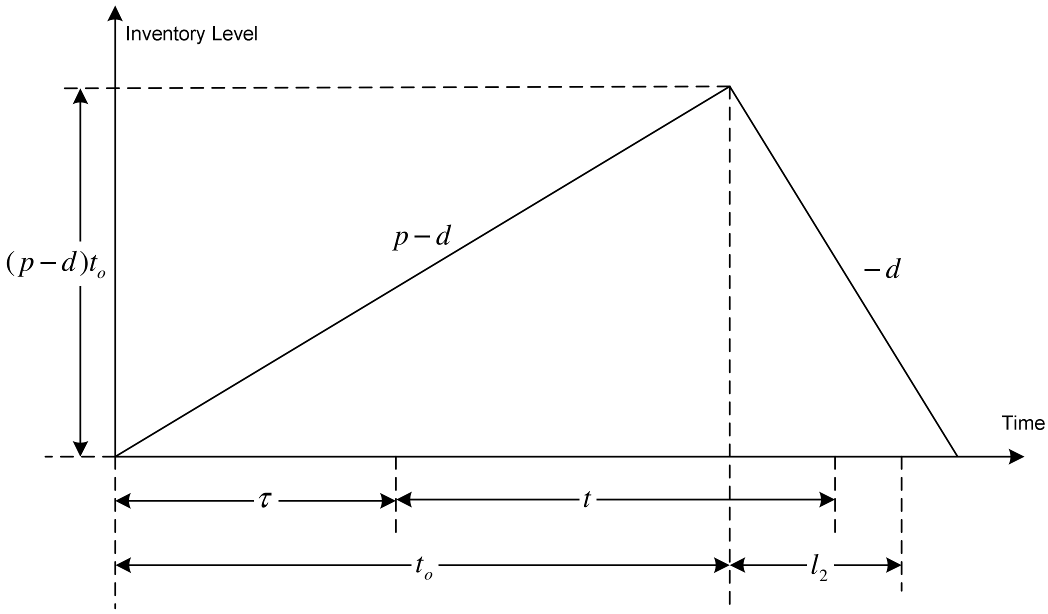


Figure 16 Deterioration-No Failure-No Shortage Case in Model-II

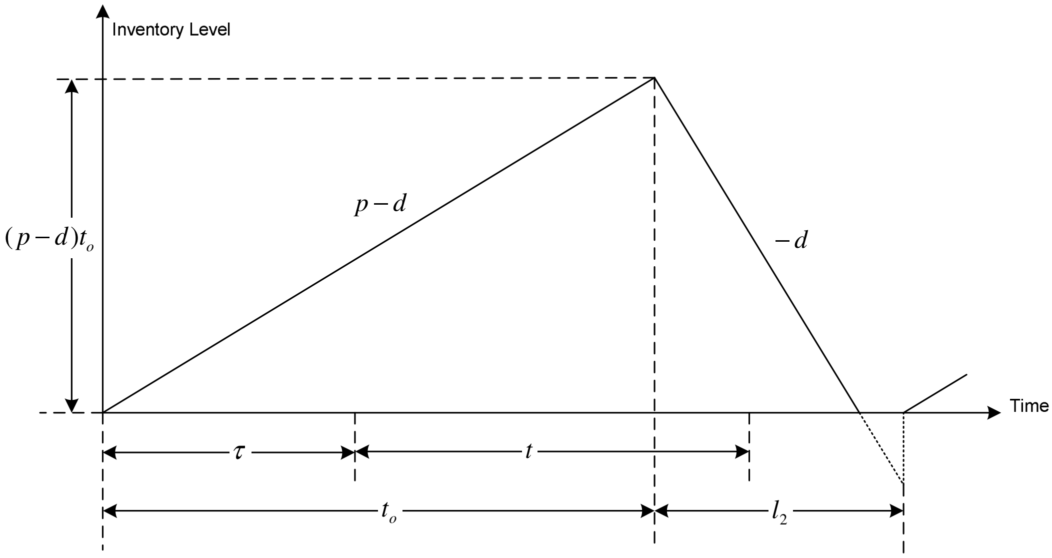


Figure 17 Deterioration-No Failure-Shortage Case in Model-II

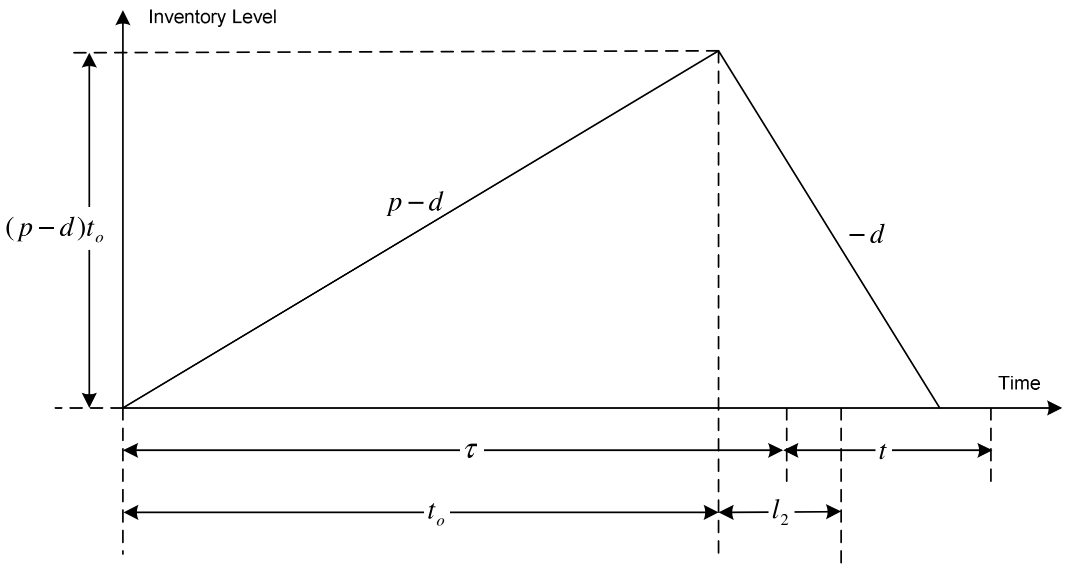


Figure 18 No Deterioration-No Failure-No Shortage Case in Model-II

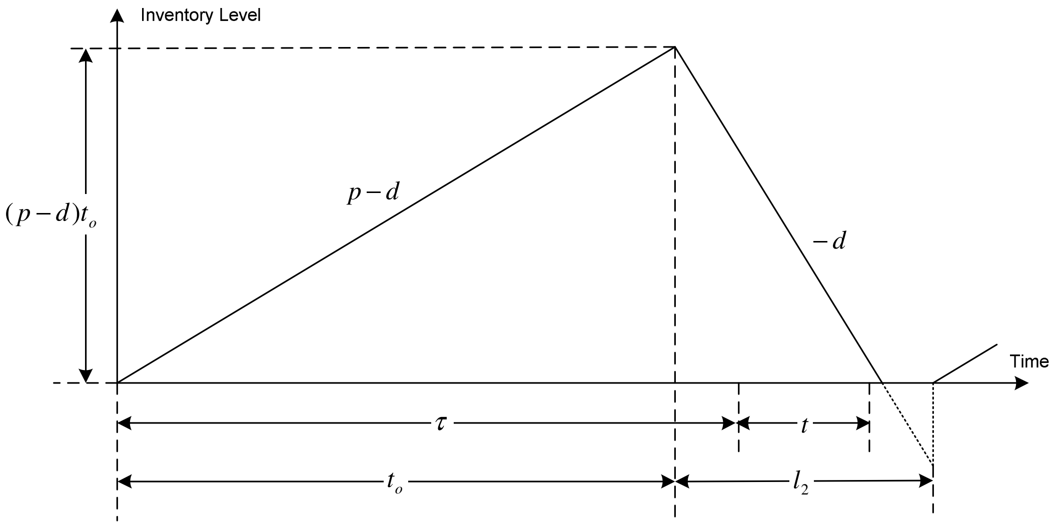


Figure 19 No Deterioration-No Failure-Shortage Case in Model-II

The expected length of a production-inventory cycle is given by:

$$T_{cycle} = \int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{l_1=0}^{\frac{(p-d)(\tau+t)}{d}}\left(\tau + t + \frac{(p-d)(\tau+t)}{d}\right).g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{l_1=\frac{(p-d)(\tau+t)}{d}}^{\infty}(\tau + t + l_1).g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=0}^{\frac{(p-d)t_o}{d}}\left(t_o + \frac{(p-d)t_o}{d}\right).g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}(t_o + l_2).g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$

$$+\int_{\tau=t_o}^{\infty}\int_{l_2=0}^{\frac{(p-d)t_o}{d}}\left(t_o + \frac{(p-d)t_o}{d}\right).g_2(l_2).h(\tau).dl_2.d\tau$$

$$+\int_{\tau=t_o}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}(t_o + l_2).g_2(l_2).h(\tau).dl_2.d\tau$$

The first term in the expected cycle length represents those cycles that will encounter deterioration followed by failure but with no shortage (Figure 14). The second term represents cycles with deterioration followed by failure and in which shortage is encountered due to prolonged corrective repair (Figure 15). The third term represents cycles with deterioration but no failure and no shortage (Figure 16). The forth term represents cycles with deterioration and shortage but no failure (Figure 17). The fifth term represents cycles with no deterioration, no failure and no shortage (Figure 18). The last term represents cycles with no deterioration and no failure, but in which shortage is encountered (Figure 19). The expected length of the production-inventory cycle can be reduced to:

$$T_{cycle} = \int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{l_1=0}^{\frac{(p-d)(\tau+t)}{d}} \frac{p}{d}.(\tau+t).g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{l_1=\frac{(p-d)(\tau+t)}{d}}^{\infty}(\tau+t+l_1).g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=0}^{\frac{(p-d)t_o}{d}} \frac{p}{d}.t_o.g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$

$$+\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}(t_o+l_2).g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$

$$+\int_{\tau=t_o}^{\infty}\int_{l_2=0}^{\frac{(p-d)t_o}{d}} \frac{p}{d}.t_o.g_2(l_2).h(\tau).dl_2.d\tau$$

$$+\int_{\tau=t_o}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}(t_o+l_2).g_2(l_2).h(\tau).dl_2.d\tau$$

The total expected cost per production-inventory cycle is composed of setup cost, repair costs (corrective and preventive), inventory holding cost, shortage cost, and the cost due to producing defective items:

$$C_{cycle} = Setup\ Cost + Corrective\ Repair\ Cost$$
$$+ Preventive\ Repair\ Cost + Inventory\ Holding\ Cost$$
$$+ Shortage\ Cost + Defectives'\ Cost$$

The corrective repair cost is simply the expected corrective repair time multiplied by the cost per unit time and finally multiplied by the probability of encountering deterioration followed by failure (Figures 14 and 15). The corrective repair cost is given by:

$$\mathit{Corrective\ Repair\ Cost}$$
$$= c_1.\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{l_1=0}^{\infty} l_1.g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

The preventive repair cost is given by:

$$\mathit{Preventive\ Repair\ Cost}$$
$$= c_2.\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=0}^{\infty} l_2.g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$
$$+ c_2.\int_{\tau=t_o}^{\infty}\int_{l_2=0}^{\infty} l_2.g_2(l_2).h(\tau).dl_2.d\tau$$

The first term in the preventive repair cost is simply the expected preventive repair time multiplied by the cost per unit time and finally multiplied by the probability of encountering deterioration but no failure (Figures 16 and 17). The second term represents cycles with no deterioration and no failure (Figures 18 and 19).

The inventory holding cost is given by:

$Inventory\ Holding\ Cost$

$$= c_i.\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\frac{1}{2}.\left(\tau + t + \frac{(p-d).(\tau+t)}{d}\right)(p-d).(\tau+t).f(t).h(\tau).dt.d\tau$$

$$+ c_i.\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\frac{1}{2}.\left(t_o + \frac{(p-d)t_o}{d}\right).(p-d).t_o.f(t).h(\tau).dt.d\tau$$

$$+ c_i.\int_{\tau=t_o}^{\infty}\frac{1}{2}.\left(t_o + \frac{(p-d)t_o}{d}\right).(p-d)t_o.h(\tau).d\tau$$

The first term in the inventory holding cost expression is the inventory holding cost per unit product per unit time multiplied by the average on hand inventory for cycles with deterioration followed by failure (Figures 14 and 15). The second term represents cycles with deterioration but no failure, $t \geq t_o - \tau$ (Figures 16 and 17). The last term represents cycles in which neither deterioration nor failure is encountered. The inventory holding cost can be reduced to,

$Inventory\ Holding\ Cost$

$$= c_i.\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\frac{1}{2}.\frac{p}{d}.(p-d).(\tau+t)^2.f(t).h(\tau).dt.d\tau$$

$$+ c_i.\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\frac{1}{2}.\frac{p}{d}.(p-d).t_o{}^2.f(t).h(\tau).dt.d\tau$$

$$+ c_i.\int_{\tau=t_o}^{\infty}\frac{1}{2}.\frac{p}{d}.(p-d).t_o{}^2.h(\tau).d\tau$$

The shortage cost is given by,

$Shortage\ Cost$

$$= c_s.d.\int_{\tau=o}^{t_o}\int_{t=o}^{t_o-\tau}\int_{l_1=\frac{(p-d)(\tau+t)}{d}}^{\infty}\left(l_1-\frac{(p-d)(\tau+t)}{d}\right).g_1(l_1).f(t).h(\tau).dl_1.dt.d\tau$$

$$+ c_s.d.\int_{\tau=o}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}\left(l_2-\frac{(p-d)t_o}{d}\right).g_2(l_2).f(t).h(\tau).dl_2.dt.d\tau$$

$$+ c_s.d.\int_{\tau=t_o}^{\infty}\int_{l_2=\frac{(p-d)t_o}{d}}^{\infty}\left(l_2-\frac{(p-d)t_o}{d}\right).g_2(l_2).h(\tau).dl_2.d\tau$$

The first term in the shortage cost expression is the expected length of the shortage period multiplied by the demand rate and finally multiplied by the shortage cost per unit product for cycles with deterioration followed by failure and in which shortage is encountered (Figure 15). The second term represents cycles with deterioration and shortage, but no failure (Figure 17). The last term represents cycles in which shortage is encountered but neither deterioration nor failure is encountered (Figure 19). Shortages in our model are considered as lost sales; accordingly, shortage cost is calculated based on the maximum shortage in units of the product rather than the average.

The defectives cost is given by;

$$Defectives\ Cost$$

$$= p.c_D.\alpha_I.\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\tau.f(t).h(\tau).dt.d\tau$$

$$+ p.c_D.\int_{\tau=0}^{t_o}\int_{t=0}^{t_o-\tau}\int_{y=0}^{t}\alpha_y.dy.f(t).h(\tau).dt.d\tau$$

$$+ p.c_D.\alpha_I.\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\tau.f(t).h(\tau).dt.d\tau$$

$$+ p.c_D.\int_{\tau=0}^{t_o}\int_{t=t_o-\tau}^{\infty}\int_{y=0}^{t_o-\tau}\alpha_y.dy.f(t).h(\tau).dt.d\tau$$

$$+ p.c_D.\alpha_I.t_o.\int_{\tau=t_o}^{\infty}h(\tau).d\tau$$

The first term in the defectives cost expression gives the cost of defectives produced during the in-control state for cycles in which deterioration is followed by failure (Figures 14 and 15). The second term gives the cost of defectives produced during the out-of-control state for cycles in which deterioration is followed by failure (Figures 14 and 15). The third term gives the cost of defectives produced during the in-control state for cycles in which only deterioration is encountered but no failure (Figures 16 and 17). The forth term gives the cost of defectives produced during the out-of-control state for cycles in which only deterioration is encountered but no failure (Figure 16 and 17). Finally, the last term gives the cost of defective produced during cycles in which neither deterioration nor failure is encountered (Figures 18 and 19).

We assume the following probability distributions for failure, deterioration, corrective repair and preventive repair times respectively as given below:

$$f(t) = \lambda e^{-\lambda t}$$

$$h(\tau) = \gamma e^{-\gamma\tau}$$

$$g_1(l_1) = \frac{1}{b_1}, 0 \le l_1 \le b_1$$

$$g_2(l_2) = \frac{1}{b_2}, 0 \le l_2 \le b_2$$

From renewal theory, the average cost per unit time is given by,

$$W(t_o) = \frac{C_{cycle}(t_o)}{T_{cycle}(t_o)}$$

4.3 Results and Conclusion

In this section, we use Wolfram Mathematica 8 in order to solve for the optimal production run time t_o and the corresponding average cost. In all calculations below, failure and deterioration are assumed to follow the exponential distribution with rates λ and γ respectively. In addition, corrective and preventive repairs are assumed to follow the uniform distribution.

The lower and upper bounds on the production run time t_o are assumed to imitate the normal work shift that extends to 8 hours:

$$0 \le t_o \le 8$$

We chose the following parameters: $p = 180$, $d = 90$, $c_O = 300$, $c_1 = 30$, $c_2 = 5$, $b_1 = 12$, $b_2 = 10$, $c_I = 0.5$, $c_S = 2$, $c_D = 3$, $\beta = 0.1$ and $\alpha_I = 0.05$.

Table 5 shows the dependency of the optimal production run time and the corresponding average cost on failure and deterioration rates, λ and γ respectively.

In general, table five shows that the average cost always increases by increasing the failure rate λ regardless of the value of deterioration rate γ, but it is worthy to notice that as γ getting larger; the average cost increased in greater rate over the given range of λ. For instance; under $\gamma = 0.1$ the average cost increases only by 3.7 over λ changing between 0.1 and 0.9; on the other hand, the average cost increased by 19.6 under $\gamma = 0.9$ over the same range of λ. This behavior can be explained by noticing that in this model we assume that failure, if happened, is always preceded by deterioration, and hence when γ assumes larger values not only the chances for deterioration are increased but also the chances for failure increases as well.

Results from table 5 also show that under low deterioration rate (γ = 0.1) the optimal production run time increases when the failure rate λ increases. This trend is justified because under low deterioration rate, the chances are low for the system to experience process deterioration, and hence defectives are unlikely to be generated; accordingly longer production run times are suggested by the model even with increasing failure rate. Another point of view comes again from the assumption of failure is always preceded by deterioration, and hence as deterioration is unlikely to happen, failure is even more unlikely to happen, and so the model suggests longer run times even with increasing the failure rate.

Table 5 also shows that under high values of the deterioration rate (γ = 0.9); the optimal production run time consistently decreases when the failure rate increases. This trend is justified because under higher deterioration rates, the chances are higher for the system to experience process deterioration, and hence defectives are expected to be produced in a higher rate; accordingly shorter production run times are suggested by the model in order to reduce the instances of process deterioration and machine failure.

Finally, table 5 shows that for intermediate values of deterioration rate (γ = 0.3 and 0.5); the optimal production run time decreases then start to increase by increasing the failure rate λ. In this case, the model is trying to balance between the effect of process deterioration and machine failure and their associated costs.

Table 6 shows how the optimal production run time and the corresponding average cost are affected by changing the corrective repair cost c_1. Results show that an increase in the corrective repair cost leads to consistent decrease in the optimal production run time. Shorter run times mean lower chances of encountering failure, and hence the risk of incurring corrective repair cost is minimized. Increasing the corrective repair cost results in increasing the average cost, which is fairly expected.

TABLE 5 Dependency of the optimal production policy on λ and γ in model-II

λ	$\gamma = 0.1$		$\gamma = 0.3$		$\gamma = 0.5$		$\gamma = 0.7$		$\gamma = 0.9$	
	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*
0.1	151.71	3.03	157.07	2.79	160.72	2.64	163.39	2.55	165.43	2.48
0.2	152.44	3.04	158.96	2.78	163.41	2.62	166.68	2.52	169.17	2.44
0.3	153.07	3.05	160.57	2.78	165.74	2.61	169.51	2.49	172.39	2.41
0.4	153.61	3.05	161.97	2.79	167.75	2.60	171.98	2.47	175.20	2.38
0.5	154.07	3.06	163.19	2.79	169.51	2.60	174.14	2.45	177.66	2.35
0.6	154.47	3.07	164.25	2.80	171.06	2.60	176.04	2.44	179.82	2.33
0.7	154.82	3.08	165.19	2.82	172.43	2.60	177.72	2.44	181.74	2.31
0.8	155.13	3.09	166.01	2.83	173.64	2.61	179.22	2.43	183.45	2.30
0.9	155.40	3.10	166.74	2.84	174.72	2.62	180.55	2.43	184.98	2.29

TABLE 6 Sensitivity analysis by changing c_1 in model-II

$c_2 = 5$						
c_1	$\lambda = 0.1$ $\gamma = 0.9$		$\lambda = 0.5$ $\gamma = 0.5$		$\lambda = 0.9$ $\gamma = 0.1$	
	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*
10	163.32	2.54	163.59	2.81	152.78	3.18
15	163.85	2.52	165.11	2.76	153.44	3.16
20	164.38	2.51	166.61	2.70	154.10	3.14
25	164.91	2.49	168.07	2.65	154.75	3.12
30	165.43	2.48	169.51	2.60	155.40	3.10
35	165.96	2.47	170.93	2.54	156.04	3.08
40	166.47	2.45	172.32	2.49	156.68	3.06

Table 7 shows how the optimal production run time and the corresponding average cost are affected by changing the preventive repair cost c_2. Increasing the preventive repair cost results in increasing the optimal production run time. Longer run times mean lower possibility of successful completion with no failure; in this case, the risk of incurring preventive repair cost is minimized. Again and as expected, increasing the preventive repair cost results in increasing the average cost.

TABLE 7 Sensitivity analysis by changing c_2 in model-II

$c_1 = 30$						
c_2	$\lambda = 0.1$ $\gamma = 0.9$		$\lambda = 0.5$ $\gamma = 0.5$		$\lambda = 0.9$ $\gamma = 0.1$	
	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*	$W(t_o^*)$	t_o^*
5	165.43	2.48	169.51	2.60	155.40	3.10
9	167.65	2.51	171.10	2.66	157.32	3.15
13	169.85	2.55	172.64	2.73	159.21	3.20
17	172.04	2.58	174.13	2.80	161.09	3.24
21	174.20	2.62	175.59	2.87	162.95	3.29
25	176.35	2.65	177.00	2.94	164.79	3.34
29	178.48	2.68	178.38	3.01	166.61	3.39

In this chapter, we presented an EMQ model in which process deterioration and machine breakdown jointly affects the optimal production policy. Model is built for general failure, deterioration, corrective and preventive repair time distributions, but numerical analysis is carried out under exponential failure and deterioration times, and uniform corrective and preventive repair times. Process deterioration is assumed to take place gradually where the rate of producing defective items starts, at some point, to increase with time following a linear pattern. Process deterioration and machine breakdown events are assumed dependent in the sense that machine failure, if happened, is preceded by process deterioration and it cannot happen alone. Numerical results supported the capability of the proposed model to be used as a decision making tool in finding the optimal production policy.

CHAPTER 5

EOQ IN RETAILING INDUSTRY MODEL-I

5.1 Introduction

In retailing industry, suppliers tend to offer a fixed credit period to settle the account in order to stimulate retailer's demand. During the credit period, retailers start selling to their direct customers and accumulate revenue. If the retailer fails to settle the account by the due time, the supplier charges interest. This "buy now and pay later" agreement is beneficial for both parties involved. From supplier's point of view; trade credits encourage the retailer to buy more and it can be seen as an effective promotional tool that attracts new customers (retailers). On the other hand, trade credits help retailers in lowering their overall cost and increasing profit thru earning interest on revenue collected during the credit period. Credit period is also offered by retailers to their direct customers in order to positively influence the demand.

In this chapter, we will develop an EOQ model in which the supplier offers credit period to his customer (retailer); also, the retailer offers a credit period to his direct customers (end consumers). Demand is assumed to depend on the length of the credit period offered by the retailer to his customers. The purpose of the model is to determine the optimal order quantity that would maximize the retailer's profit.

The following notations are used in developing the mathematical model:

D	Demand rate
R	Replenishment rate
N	Credit period offered by the retailer to his customers
M	Credit period offered by the supplier to the retailer
T	Inventory cycle length
A	Ordering cost per order
C	Unit purchase price of the item
P	Unit selling price of the item
I	Inventory carrying cost per unit of the item per unit time
I_e	Interest rate that can be earned per unit currency per unit time
I_p	Interest rate payable per unit currency per unit time
α	Bad debt ratio
r	Rate of saturation of demand
D_{min}	Minimum demand
D_{max}	Maximum demand
$Z(T,N)$	Retailer's profit per unit time

The following assumptions are made to develop the mathematical model:

1. The replenishment rate R form the supplier to the retailer is finite and it is greater than the maximum demand rate D_{max}.

2. The supplier offers a fixed credit period M to the retailer to settle the accounts. As the relenishemet rate is finite; we assume counting for M to start at the point when delivery of the whole lot is completed.
3. The retailer offers a fixed credit period N to each of his customers to settle their acounts. However we assume a percentage of sales α will never be collected and it is considered as bad debts.
4. The demand rate is a function of the customer's credit period offered by the retailer (N), and is given by (Jaggi et al 2008):

$$D = D_{max} - (D_{max} - D_{min})(1 - r)^N$$

In which D_{max} is the maximum possible demand, D_{min} is the minimum demand and r is the saturation rate of demand, and all are constant quantities and can be estimated using market conditions and past data. Figure 20 shows the demand function for different values of saturation rate r with $D_{min} = 50$ and $D_{max} = 100$.
5. Shortages are not allowed.
6. Sales revenue after deducting the bad debts is still greater than the purchasing cost, i.e. $(1 - \alpha)P \geq C$

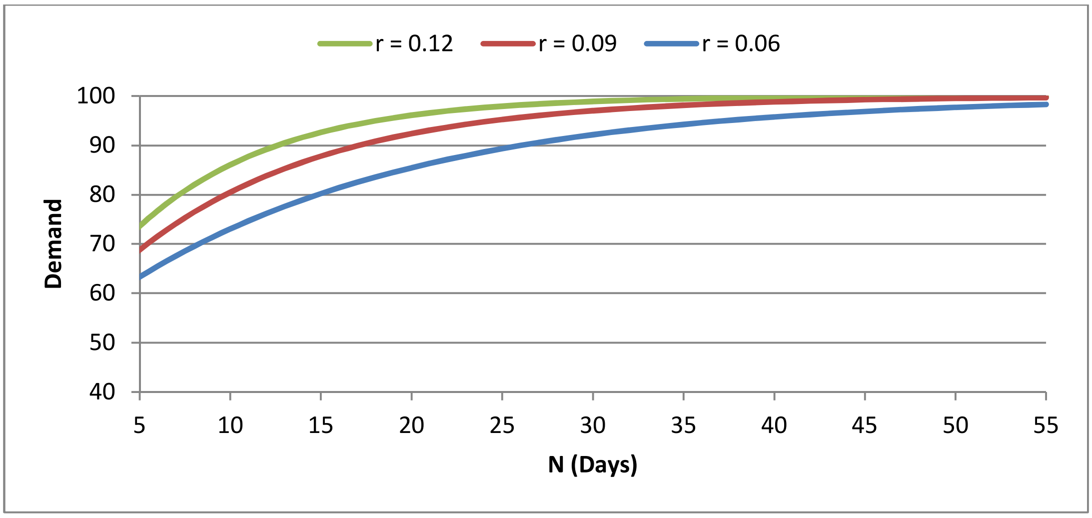


Figure 20 Credit linked demand function

5.2 Model Formulation

The retailer receives the whole lot by time $\frac{DT}{R}$ (Figure 21). The payment of the whole lot to the supplier is due at time $\frac{DT}{R} + M$. The retailer starts to collect revenue from his customers at time N and it continues until time $N + T$. There are three cases to be considered here. In the first case, the payment to the supplier is due some time after the retailer has already started to collect revenue i.e. $N \leq \frac{DT}{R} + M$ and before the retailer receives all the revenue i.e. $\frac{DT}{R} + M \leq T + N$. In the second case the payment to the supplier is due after the retialer has received all revenue from sales to his customers, i.e. $T + N \leq \frac{DT}{R} + M$. The last case is when the payment to the supplier is due before the retailer receives any revenue, i.e. $\frac{DT}{R} + M \leq N$. We discuss these cases in the following sections.

The retailer's profit per unit time is defined by sales revenue, ordering cost, purchasing cost, inventory-holding cost, and finally interest earned and/or paid depending on the time at which supplier payment is due.

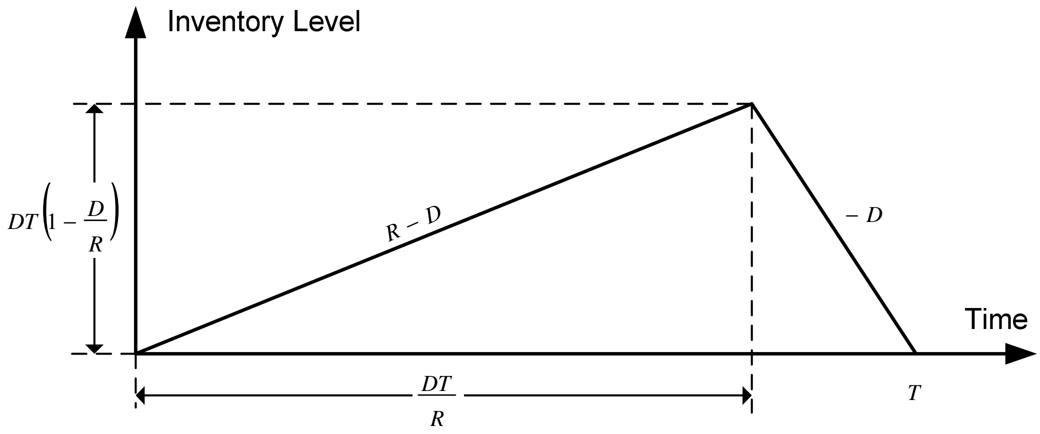


Figure 21 Inventory Level

Sales revenue per unit time is the demand rate multiplied by the unit-selling price, and finally multiplied by $(1-\alpha)$ to exclude bad debts. Sales revenue is expressed as:

$$Sales\ Revenue = (1-\alpha)DP \tag{5-1}$$

Ordering cost per unit time is simply the cost of ordering divided by the inventory cycle length:

$$Ordering\ Cost = \frac{A}{T} \tag{5-2}$$

Purchasing cost per unit time is simply the unit-purchasing price multiplied by the demand rate:

$$Purchasing\ Cost = CD \tag{5-3}$$

Inventory holding cost per unit time is the average inventory multiplied by the inventory holding cost per unit of the product per unit time; I (Figure 21):

$$Inventory\ Holding\ Cost = \frac{1}{2}IDT\left(1-\frac{D}{R}\right) \tag{5-4}$$

As mentioned earlier, different cases arise depending on the due time for the supplier payment. All three cases have the terms in equations (5-1) to (5-4) in their profit functions, accordingly we define Z_o as:

$$Z_o = (1-\alpha)DP - \frac{A}{T} - CD - \frac{1}{2}IDT\left(1 - \frac{D}{R}\right)$$

Interest earned and interest payable differs for each of the three cases and they are explained in the following sub-sections.

5.2.1 Case I

In this case (Figure 22), the retailer starts getting actual sales revenue at time N, until time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ retailer earns interest on average sales revenue for the time period $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{N}$. From time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ until time $T + N$ supplier charges interest on (a) the average quantity of items with their debt successfully collected from end users, and (b) the full quantity of items considered as bad debt.

Case I happens when the following condition applies:

$$\left(N \leq \frac{DT}{R} + M\right) \& \left(\frac{DT}{R} + M \leq T + N\right)$$

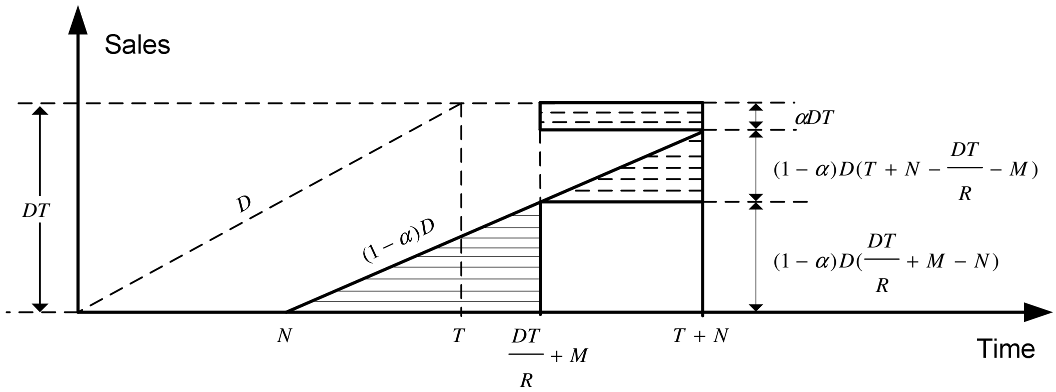


Figure 22 Case I

It should be clear that under case I an assumption is made implying that the retailer pays the supplier according to the following procedure:

a. At time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$, the retailer pays the supplier for the quantity that their debt is collected successfully by that time.

b. During the time period $T + N - \frac{\mathrm{DT}}{\mathrm{R}} - \mathrm{M}$, the retailer continuously (i.e. at the end of each day) pays the supplier for quantities that their debt is successfully collected, in addition to interests due on their value.

c. At the end of debt collection period; i.e. $(T + N)$, the retailer pays the supplier for the bad debt quantity in addition to interests due on their value.

Interest earned per unit time is given by:

$$\frac{I_e(1-\alpha)PD\left(\frac{DT}{R} + M - N\right)^2}{2T}$$

Interest payable per unit time by the retailer to the supplier is given by:

$$\frac{\frac{1}{2}I_p(1-\alpha)CD\left(T + N - \frac{DT}{R} - M\right)^2 + I_p\alpha CDT\left(T + N - \frac{DT}{R} - M\right)}{T}$$

Interest payable can be reduced to:

$$\frac{CDI_p\left(\frac{R-D}{R}T+N-M\right)\left((1-\alpha)\left[\frac{R-D}{R}T+N-M\right]+2\alpha T\right)}{2T}$$

Accordingly, the retailer's profit per unit time in this case is given by:

$$Z_1(T,N)=Z_o+Interest\ Earned-Interest\ Payable$$

$$Z_1(T,N)\ =Z_o+\frac{I_e(1-\alpha)PD\left(\frac{DT}{R}+M-N\right)^2}{2T}$$
$$-\frac{CDI_p\left(\frac{R-D}{R}T+N-M\right)\left((1-\alpha)\left[\frac{R-D}{R}T+N-M\right]+2\alpha T\right)}{2T}$$

5.2.2 Case II

In this case (Figure 23), the retailer earns interest on average sales revenue during the period $(\mathrm{N},\mathrm{T}+\mathrm{N})$ and on full sales revenue for the time period $\left(\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}-\mathrm{T}-\mathrm{N}\right)$. Under case II, the retailer makes a single payment to the supplier at time $\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}$ of value CDT with no extra interest as he makes the payment on the due time with no delay.

Case II happens when the following condition applies:

$$T+N\leq\frac{DT}{R}+M$$

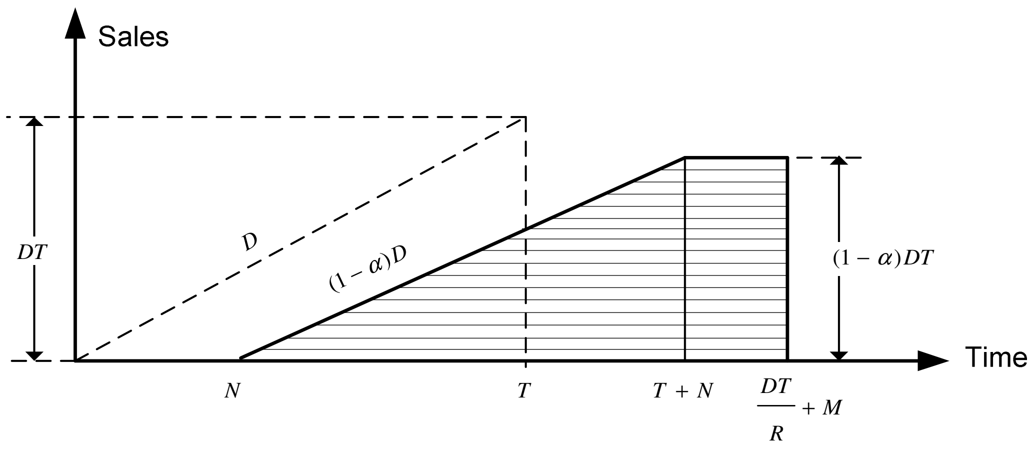


Figure 23 Case II

Interest earned per unit time is given by:

$$\frac{\frac{1}{2}(1-\alpha)I_e PDT^2 + (1-\alpha)I_e PDT\left(\frac{DT}{R} + M - T - N\right)}{T}$$

Interest earned can be reduced to:

$$(1-\alpha)DI_e P\left(\frac{2D-R}{2R}T + M - N\right)$$

Accordingly, the retailer's profit per unit time in this case is given by:

$$Z_2(T,N) = Z_o + Interest\ Earned$$

$$Z_2(T,N) = Z_o + (1-\alpha)DI_e P\left(\frac{2D-R}{2R}T + M - N\right)$$

5.2.3 Case III

In this case, (Figure 24), the supplier payment is due even before the retailer start collecting debt from his customers. In this case the retailer earns no interest but pays interest on full order quantity for a period of $N - \frac{DT}{R} - M$, and for a period of T he pays interest on (a) average quantity of items with their debt successfully collected, and (b) the full quantity of items considered as bad debt.

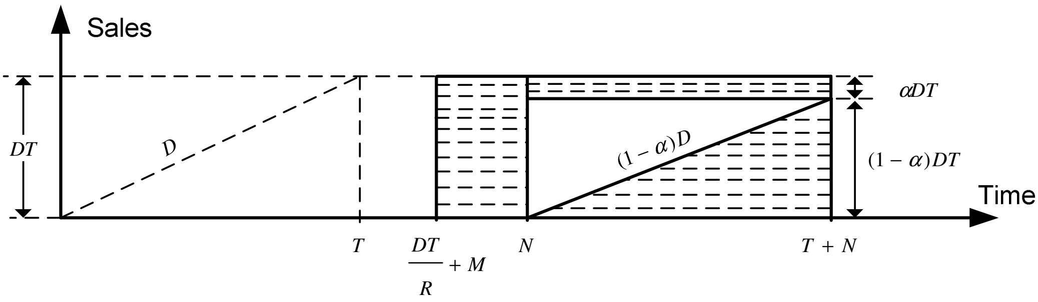


Figure 24 Case III

Case III happens if the following condition applies:

$$\frac{DT}{R} + M \leq N$$

Under case III, the retailer pays the supplier according to the following procedure:

a. During the time period extending from N to $T + N$, the retailer continuously (i.e. at the end of each day) pays the supplier for quantities that their debt is successfully collected, in addition to interests due on their value.
b. At the end of debt collection period; i.e. $(T + N)$, the retailer pays the supplier for the bad debt quantity in addition to interests due on their value.

Interest payable per unit time by the retailer to the supplier is given by:

$$\frac{I_p CDT\left(N - \frac{DT}{R} - M\right) + \alpha I_p CDT^2 + \frac{1}{2}(1-\alpha) I_p CDT^2}{T}$$

Interest payable is reduced to,

$$CDI_p\left(\frac{(1+\alpha)R - 2D}{2R} T + N - M\right)$$

Accordingly, the retailer's profit per unit time in this case is given by:

$$Z_3(T, N) = Z_o - Interest\ Payable$$

$$Z_3(T,N) = Z_o - CDI_p\left(\frac{(1+\alpha)R - 2D}{2R}T + N - M\right)$$

5.2.4 Retailer's Profit Function

Combining the results from the three cases discussed above, the retailer's profit function is given by:

$$Z(T,N) = \begin{cases} Z_1(T,N), & N \leq \frac{DT}{R} + M \leq T + N \\ Z_2(T,N), & T + N \leq \frac{DT}{R} + M \\ Z_3(T,N), & \frac{DT}{R} + M \leq N \end{cases}$$

which is a function of two variables T and N where T is continuous and N is discrete.

5.3 Optimality

Our problem is to determine the optimum values of T and N which maximizes the retailer's profit $Z(T,N)$. For a fixed value of N, we find the second derivatives of $Z_1(T,N)$, $Z_2(T,N)$ and $Z_3(T,N)$ with respect to T, we get:

$$Z_1''(T,N) = \frac{-2A - (1-\alpha)D(M-N)^2(CI_p - PI_e)}{T^3}$$

and:

$$Z_2''(T,N) = \frac{-2A}{T^3}$$

and,

$$Z_3''(T,N) = \frac{-2A}{T^3}$$

For a fixed N, $Z_2(T,N)$ and $Z_3(T,N)$ are concave on $T > 0$. However $Z_1(T,N)$ is concave on $T > 0$ if the following condition applies:

$$(1-\alpha)D(M-N)^2\left(CI_p - PI_e\right) > -2A \tag{5-5}$$

It is worthy to notice that if $CI_p > PI_e$, then the concavity of $Z_1(T,N)$ is guaranteed, otherwise condition 5-5 should be tested to conclude if $Z_1(T,N)$ is concave or not.

5.4 Solution Procedure

In order to jointly optimize T and N, we propose the following algorithm:

1. Set $N = 1$.
2. Search for the optimal values of T $(i.e. T_1^*,\ T_2^*\ and\ T_3^*)$ which maximize $Z_1(T,N)$, $Z_2(T,N)$ and $Z_3(T,N)$ respectively on $T > 0$.
3. If $\left(N \leq \frac{DT_1^*}{R} + M \leq T_1^* + N\right)$; set $T^* = T_1^*$ and $Z^* = Z_1^*$ then go to step 4. If the condition is not satisfied go to step 5.
4. If $Z^*(T,N) > Z^*(T,N-1)$, increment the value of N by 1 and go to step 2, else previous value of N $(i.e. N-1)$ is optimal and its corresponding values of T and $Z(T,N)$ are retrieved and algorithm is terminated.

5. If $\left(T_2^* + N \leq \frac{DT_2^*}{R} + M\right)$; set $T^* = T_2^*$ and $Z^* = Z_2^*$ then go to step 4. If the condition is not satisfied go to step 6.

6. If $\left(\frac{DT_3^*}{R} + M \leq N\right)$; set $T^* = T_3^*$ and $Z^* = Z_3^*$ then go to step 4.

The aforementioned algorithm is coded in Mathematica 8 to produce numerical results presented in the following section.

5.5 Results and Conclusion

We chose the following values of model parameters: $\mathrm{R} = 150$, $\mathrm{D_{max}} = 100$, $\mathrm{D_{min}} = 30$, $\mathrm{r} = 0.12$, $\mathrm{A} = 1000$, $\mathrm{C} = 30$, $\mathrm{P} = 40$, $\mathrm{I_e} = 10\%$, $\mathrm{I_p} = 15\%$, $\mathrm{I} = 20$ and $\alpha = 0.05$.

Table 8 shows the effect of changing M on the optimal policy ($\mathrm{T^*}$ and $\mathrm{N^*}$) and the associated cost. The optimal cycle length is slightly affected by increasing M. Mainly three types of cost affect the behavior of the model. Ordering-cost pushes the model for higher values of cycle length. Inventory holding cost demands shorter cycle lengths. Interest payable is defined by the location of supplier payment due time $\frac{\mathrm{DT^*}}{\mathrm{R}} + \mathrm{M}$ which is governed by the cycle length and the supplier credit period M. Increasing M helps in deferring the supplier payment and hence the cycle length can stay almost unchanged to keep the balance between ordering cost and inventory holding cost.

The retailer's credit period N is also slightly affected by increasing M. Increasing N positively affects the demand which is in turn helps in increasing sales revenue and deferring the supplier payment due time $\left(\frac{DT^*}{R} + M\right)$. But that effect is limited as explained by the demand function, and the demand rate almost attains its maximum at $N = 32$ and there is no tangible benefit by increasing N. Also increasing N in larger magnitudes would render the model in case III, which is not preferable in terms of interest payable as the supplier payment might be due even before the retailer start collecting debt from his customers.

Finally the retailer's profit increases consistently by increasing M which is self explained as this leads to deferring supplier payment due time with no extra interest.

Table 9 shows the effect of changing the ordering cost A on the optimal policy. Obviously increasing A results in increasing the optimal cycle length in order to distribute the ordering cost over larger quantity. On the other hand, retailer's credit period is not affected by increasing A. As expected, retailer's profit decreases by increasing the ordering cost.

TABLE 8 Effect of changing M on the optimal policy

M	T^*	N^*	D	$\frac{DT^*}{R}+M$	Case	$Z(T^*,N^*)$
0	36.26	32	98.83	23.89	III	696.48
10	36.16	32	98.83	33.83	I	708.66
20	35.86	33	98.97	43.66	I	720.62
30	35.68	33	98.97	53.54	I	732.26
40	35.72	33	98.97	63.57	I	743.54
50	36.38	34	99.10	74.03	II	754.27

TABLE 9 Effect of changing A on the optimal policy

A	T^*	N^*	$Z(T^*, N^*)$
500	25.236	33	748.679
1000	35.6788	33	732.263
1500	43.6932	33	719.664

Table 10 shows the effect of changing the saturation rate r on the optimal policy. Increasing r leads to decreasing N; larger values of r requires smaller values of N in order to achieve the maximum demand as explained by the demand function. Increasing r has tiny effect on the optimal cycle length. Retailer's profit increases with r; higher values of r means high demand rate while still offering relatively short credit period. Shorter credit periods mean early collection of debt from retailer's customers.

Table 11 shows the effect of changing R on the optimal policy. Increasing the replenishment rate leads to a decrease in the optimal cycle length. Increasing R leads to an increase in the inventory holding cost, and hence the model suggests shorter values of cycle length in order to overcome this effect. Increasing R has tiny effect on N. Retailer' profit decreases by increasing R which is fairly expected as supplier payment due time $\left(\frac{\mathrm{DT}^*}{\mathrm{R}} + \mathrm{M}\right)$ is becoming earlier.

Table 12 shows the effect of changing the bad debt ratio α on the optimal policy. Increasing α results in a marginal decrease in both T and N. Retailer's profit decreases by increasing α which is expected as sales revenue decreases.

TABLE 10 Effect of changing r on the optimal policy

r	T^*	N^*	$Z(T^*, N^*)$
0.09	35.6337	41	719.283
0.12	35.6788	33	732.263
0.15	35.7911	28	740.468

TABLE 11 Effect of changing R on the optimal policy

R	T^*	N^*	$Z(T^*, N^*)$
125	60.2143	34	755.019
150	35.6788	33	732.263
175	29.5076	33	720.496

TABLE 12 Effect of changing α on the optimal policy

α	T^*	N^*	$Z(T^*, N^*)$
0.025	36.0731	34	831.8
0.05	35.6788	33	732.263
0.075	35.2937	32	632.85

The model in this chapter is designed to determine the optimal order quantity and the optimal trade credit period that can be followed in the retailing industry to maximize the retailer's profit. In the following chapter, we present a modified version of this model; in which, the retailer pays the wholesaler according to a different procedure.

CHAPTER 6

EOQ IN RETAILING INDUSTRY MODEL-II

6.1 Introduction

In this chapter, we develop a model similar to that in chapter 5. The main difference in this model is the procedure in which the retailer pays the supplier; and this is thoroughly explained in the following sections. Same assumptions apply as those in chapter five, with the exception of the procedure of paying the supplier as mentioned before. Concerning notation, same used as in chapter 5 with addition of the following 2:

S	Supplier profit per unit product
W	Supplier's profit per unit time

6.2 Model Formulation

The retailer receives the whole lot by time $\frac{\mathrm{DT}}{\mathrm{R}}$ (Figure 21, Ch5). The payment of the whole lot to the supplier is due at time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$. The retailer starts to collect revenue from his customers at time N and it continues until time $\mathrm{N} + \mathrm{T}$. There are three cases to be considered here. In the first case, the payment to the supplier is due sometime after the retailer has already started to collect revenue i.e. $\mathrm{N} \leq \frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ and before the retailer receives all the revenue i.e. $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} \leq \mathrm{T} + \mathrm{N}$. In the second case the payment to the supplier is due after the retailer has received all revenue from sales to his customers, i.e. $\mathrm{T} + \mathrm{N} \leq \frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$. The last case is when the payment to the supplier is due before the retailer receives any revenue, i.e. $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} \leq \mathrm{N}$. We discuss these cases and their sub-cases in the following sections.

The retailer's profit per unit time is defined by sales revenue, ordering cost, purchasing cost, inventory-holding cost, and finally interest earned and/or paid depending on the time at which supplier payment is due.

Sales revenue, ordering cost, purchasing cost and inventory holding cost in this model are similar to those in chapter 5, and they are combined together in one expression:

$$Z_o = (1-\alpha)DP - \frac{A}{T} - CD - \frac{1}{2}IDT\left(1 - \frac{D}{R}\right)$$

Interest earned and interest payable differs for each case and they are explained in the following sections.

6.2.1 Case 1

In this case (Figure 25), the retailer starts getting actual sales revenue from time N to $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ and earns interest on average sales revenue for the time period $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{N}$. At time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ accounts should be settled with the supplier; total purchasing cost of value

CDT is due at this time. If the sum of sales revenue and interest earned accumulated by that time is less than the purchasing cost, then the retailer pays the accumulated cash and the rest of the payment is considered as a loan. This loan to be paid off with interest at the end of debt collection period $(T+N)$. On the other hand; if the accumulated cash equals or exceeds the purchasing cost, then the retailer pays to the supplier in full.

Case 1 happens when the following condition applies:

$$\left(N \le \frac{DT}{R} + M\right) \& \left(\frac{DT}{R} + M \le T + N\right)$$

Accumulated sales revenue (\$) at time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ is given by:

$$(1-\alpha)PD\left(\frac{DT}{R} + M - N\right)$$

Accumulated interest earned (\$) at time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ is given by:

$$\frac{1}{2}(1-\alpha)I_e PD\left(\frac{DT}{R} + M - N\right)^2$$

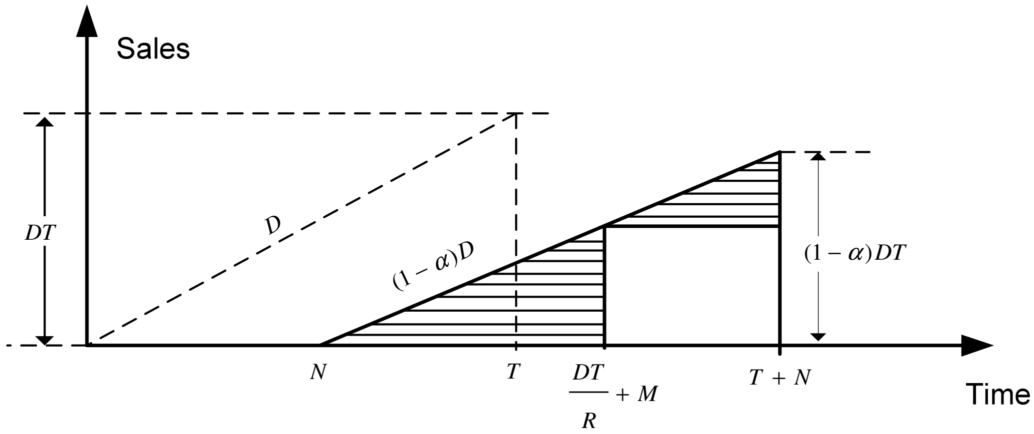


Figure 25 Case 1

Accordingly, the loan value (if needed) is given by:

$$CDT - (1-\alpha)PD\left(\frac{DT}{R} + M - N\right) - \frac{1}{2}(1-\alpha)I_e PD\left(\frac{DT}{R} + M - N\right)^2$$

The loan expression can be reduced to,

$$CDT - (1-\alpha)PD\left(\frac{DT}{R} + M - N\right)\left(1 + \frac{1}{2}I_e\left[\frac{DT}{R} + M - N\right]\right)$$

The loan function above is quadratic in T and can be expressed as:

$$Loan = v_1T^2 + v_2T + v_3$$

v_1, v_2 and v_3 are expressed as:

$$v_1 = -\frac{(1-\alpha)PI_eD^3}{2R^2}$$

$$v_2 = \frac{DRC - (1-\alpha)PD^2(1 + I_e[M-N])}{R}$$

$$v_3 = -\frac{(1-\alpha)}{2}PD(M-N)(2 + I_e[M-N])$$

Since v_1 is a negative quantity, the quadratic function is concave. The roots of this quadratic function are given by:

$$r_1 = \frac{-v_2 + \sqrt{{v_2}^2 - 4v_1v_3}}{2v_1}$$

and

$$r_2 = \frac{-v_2 - \sqrt{{v_2}^2 - 4v_1v_3}}{2v_1}$$

Not that $r_1 < r_2$ since v_1 is negative, also r_2 can be negative; and hence both roots can be negative when both v_2 and v_3 are negative (which is already possible). Figure 26 shows all possible scenarios for the loan function in terms of the location of its roots. In case (a), both roots are negative; and hence the loan function is never positive over positive values of T. In (b) the loan is positive over some period from zero to r_2. In (c) the loan is positive over the period from r_1 to r_2. Finally, in (d) the loan function is always negative and there is no real roots for the loan function.

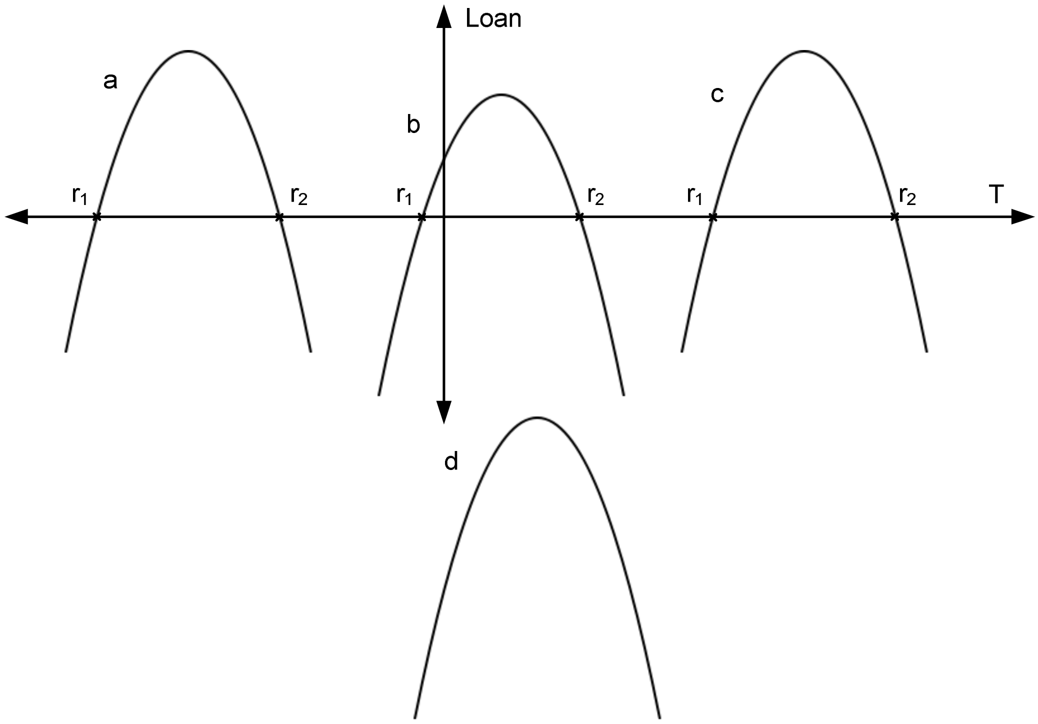


Figure 26 The loan function

Based on the loan function, two sub-cases under case one are considered and they are explained below in the following two sub-sections.

6.2.1.1 Case 1.1

In this sub-case the accumulated cash at retailer's hand at time $\frac{DT}{R}+M$ is less than the purchasing cost (CDT), accordingly a loan should be arranged. Case 1.1 happens when the following condition applies:

$$(4v_1v_3 < {v_2}^2)\&\,(r_1 \leq T \leq r_2)$$

The loan is paid off with its interests when all debt is collected from customers at time $\mathrm{T+N}$ (excluding bad debts). Consequently, interest payable per unit time is given by:

$$\frac{I_p\left(T+N-\frac{DT}{R}-M\right).Loan}{T}$$

The retailer earns interest on average sales revenue from time N to $\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}$. After paying the accumulated cash at time $\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}$, the retailer earns interest on average sales revenue from time $\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}$ until the end of debt collection period $(\mathrm{T+N})$. Consequently, interest earned per unit time is given by:

$$\frac{(1-\alpha)I_ePD}{2\mathrm{T}}\left(\left(\frac{\mathrm{DT}}{\mathrm{R}}+\mathrm{M}-\mathrm{N}\right)^2+\left(T+N-\frac{DT}{R}-M\right)^2\right)$$

The retailer's profit per unit time in this sub-case is given by:

$$Z_{1.1} = Z_o + Interest\ Earned - Interest\ Payable$$

$$Z_{1.1} = Z_o + \frac{(1-\alpha)I_e PD}{2\mathrm{T}}\left(\left(\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{N}\right)^2 + \left(T + N - \frac{DT}{R} - M\right)^2\right)$$

$$- \frac{I_p\left(T + N - \frac{DT}{R} - M\right).Loan}{T}$$

Under this sub-case, the supplier profit per unit time is given by:

$$W_{1.1} = SD + \frac{I_p\left(T + N - \frac{DT}{R} - M\right).Loan}{T}$$

S is the supplier profit per unit product. The supplier profit comes from selling his product to the retailer; the first term, and from interest paid by the retailer in case a loan is needed; the second term.

6.2.1.2 Case 1.2

In this case the accumulated cash at retailer's hand at time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ equals or exceeds the purchasing cost (CDT), accordingly no loan is needed and the supplier is paid in full from sales revenue and interest earned generated by that time. Case 1.2 happens when the following condition applies:

$$(4v_1v_3 \geq {v_2}^2)\ Or\ (T \leq r_1)\ Or\ (T \geq r_2)$$

The retailer earns interest on average sales revenue from time N to $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$. After settling the accounts with the supplier the retailer earns interest on average sales revenue from time $\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M}$ until the end of debt collection period $(\mathrm{T} + \mathrm{N})$. Additionally the retailer earns interest on cash amount which remains after settling the accounts over the period $\left(\mathrm{T} + \mathrm{N} - \frac{\mathrm{DT}}{\mathrm{R}} - \mathrm{M}\right)$. Interest earned per unit time in this sub-case is given by:

$$\frac{(1-\alpha)I_ePD}{2\mathrm{T}}\left(\left(\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{N}\right)^2 + \left(T + N - \frac{DT}{R} - M\right)^2\right)$$
$$+ \frac{I_e\left(T + N - \frac{DT}{R} - M\right)\left((1-\alpha)PD\left(\frac{DT}{R} + M - N\right)\left(1 + \frac{1}{2}I_e\left[\frac{DT}{R} + M - N\right]\right) - CDT\right)}{T}$$

Moreover, the retailers profit per unit time is given by:

$$Z_{1.2} = Z_o + Interest\ Earned$$

$$Z_{1.2} = Z_o + \frac{(1-\alpha)I_ePD}{2\mathrm{T}}\left(\left(\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{N}\right)^2 + \left(T + N - \frac{DT}{R} - M\right)^2\right)$$
$$+ \frac{I_e\left(T + N - \frac{DT}{R} - M\right)\left((1-\alpha)PD\left(\frac{DT}{R} + M - N\right)\left(1 + \frac{1}{2}I_e\left[\frac{DT}{R} + M - N\right]\right) - CDT\right)}{T}$$

Under this sub-case, the supplier profit per unit time is given by:

$$W_{1.2} = SD$$

The supplier earns no iterest from the retailer in this sub-case.

6.2.2 Case 2

In this case (Figure 27), the retailer earns interest on average sales revenue collected during the period $(\mathrm{N}, \mathrm{T} + \mathrm{N})$ and on full sales revenue for a period of $\left(\frac{\mathrm{DT}}{\mathrm{R}} + \mathrm{M} - \mathrm{T} - \mathrm{N}\right)$. Case 2 happens when the following condition applies:

$$T + N \leq \frac{DT}{R} + M$$

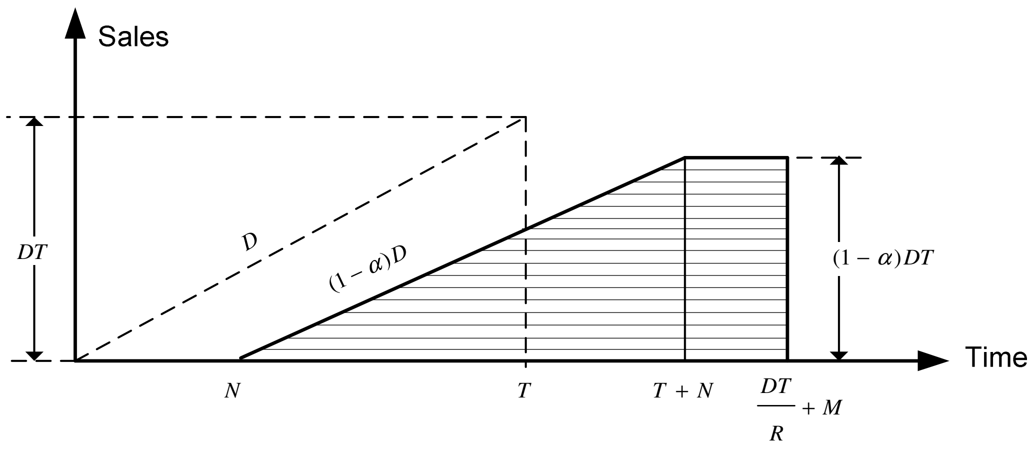


Figure 27 Case 2

Interest earned per unit time under case 2 is given by:

$$\frac{\frac{1}{2}(1-\alpha)I_e PDT^2 + (1-\alpha)I_e PDT\left(\frac{DT}{R} + M - T - N\right)}{T}$$

This reduces to:

$$(1-\alpha)DI_e P\left(\frac{2D-R}{2R}T + M - N\right)$$

the retailer profit per unit time in this case is given by:

$$Z_2 = Z_o + Interest\ Earned$$

$$Z_2 = Z_o + (1-\alpha)DI_e P\left(\frac{2D-R}{2R}T + M - N\right)$$

The supplier profit per unit time under this case is given by:

$$W_2 = SD$$

Again, the supplier earns no interest from the retailer under this case.

6.2.3 Case 3

In this case (Figure 28), the supplier payment is due even before the retailer start collecting debt from his customers, accordingly a loan of value CDT (total purchasing cost) is arranged and to be paid off with its interest when debt collection period is over (i.e. at time $T + N$). Case 3 happens when the following condition applies:

$$\frac{DT}{R} + M \leq N$$

Interest payable per unit time in this case is given by:

$$\frac{I_p CDT\left(T + N - \frac{DT}{R} - M\right)}{T}$$

On the other hand, retailer earns interest on average sales revenue over the debt collection period $(N\,, T + N)$. Interest earned per unit time is given by:

$$\frac{(1 - \alpha) I_e PD\mathrm{T}}{2}$$

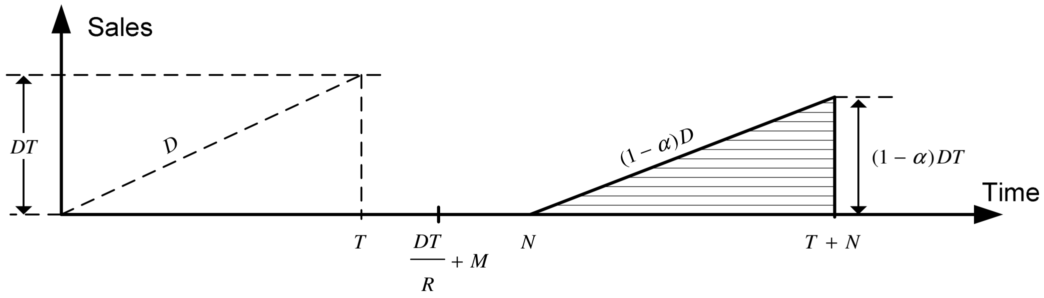


Figure 28 Case 3

The retailer profit per unit time is given by:

$$Z_3 = Z_o + Interest\ Earned - Interest\ Payable$$

$$Z_3 = Z_o + \frac{(1-\alpha)I_e PD\mathrm{T}}{2} - \frac{I_p CDT\left(T + N - \frac{DT}{R} - M\right)}{T}$$

The supplier profit function in this case is given by:

$$W_3 = SD + \frac{I_p CDT\left(T + N - \frac{DT}{R} - M\right)}{T}$$

In this case, the supplier earns interest from the retailer.

6.2.4 Retailer and supplier's profit functions

Combining results from the previous cases and sub-cases, the retailer's profit function per unit time is given by:

$$Z = \begin{cases} Z_{1.1}, & \left(N \le \frac{DT}{R} + M \le T + N\right) \ \& \ (4v_1v_3 < {v_2}^2) \ \& \ (r_1 \le T \le r_2) \\ Z_{1.2}, & \left(N \le \frac{DT}{R} + M \le T + N\right) \ \& \ \{(4v_1v_3 \ge {v_2}^2)\ OR\ (T \le r_1)\ OR\ (T \ge r_2)\} \\ Z_2, & T + N \le \frac{DT}{R} + M \\ Z_3, & \frac{DT}{R} + M \le N \end{cases}$$

and the supplier profit function per unit time is given by:

$$W = \begin{cases} W_{1.1}, & \left(N \leq \frac{DT}{R} + M \leq T + N\right) \ \& \ (4v_1v_3 < v_2{}^2) \ \& \ (r_1 \leq T \leq r_2) \\ W_{1.2}, & \left(N \leq \frac{DT}{R} + M \leq T + N\right) \ \& \ \{(4v_1v_3 \geq v_2{}^2) \ OR \ (T \leq r_1) \ OR \ (T \geq r_2)\} \\ W_2, & T + N \leq \frac{DT}{R} + M \\ W_3, & \frac{DT}{R} + M \leq N \end{cases}$$

In order to investigate the concavity of the retailer's profit function, we find its second derivative:

$$Z_{1.1}^{''} = \frac{(1-\alpha)I_eI_pPD^3(R-D)T^3 - (1-\alpha)PD\big(2I_p - 2I_e + MI_pI_e - NI_pI_e\big)(M-N)^2R^3 - 2AR^3}{R^3T^3}$$

$$Z_{1.2}^{''} = \frac{(1-\alpha)PD^3I_e{}^2(R-D)T^3 - (1-\alpha)PDI_e{}^2(M-N)^3R^3 - 2AR^3}{R^3T^3}$$

$$Z_2^{''} = Z_3^{''} = \frac{-2A}{T^3}$$

Both Z_2 and Z_3 are concave on $T > 0$. However, the concavity of $Z_{1.1}$ and $Z_{1.2}$ is highly sensitive to model parameters, accordingly we propose a different approach for numerical analysis other than trying to maximize the retailer's profit function.

6.3 Results and Conclusion

We will try to help both the retailer and the supplier in finding an efficient solution for both of them. Under a specific value of supplier credit period M, and for each $N = \{1{:}\,120\}$, we generate two arrays of profit function values for $T = \{1{:}\,120\}$. One array represent the supplier's profit and the other represent the retailer's profit. Combining supplier's profit arrays in one matrix, and retailer's profit arrays in another matrix, we create two matrices of dimension 120×120 each. Analogous entries in the supplier and the retailer's profit matrices share the same values of N and T, and certainly same value of M.

From each pair of profit matrices (linked to a specific M), we extract the list of efficient points. An efficient point is a pair of supplier and retailer's profit. For an efficient point, no improvement is possible on one of the two profits without worsening the other. The list of efficient points is plotted for each M. Supplier's profit is shown on the horizontal axis while the retailer's profit is on the vertical axis.

We chose the following set of parameters: $\mathrm{R} = 120$, $\mathrm{D_{max}} = 80$, $\mathrm{D_{min}} = 30$, $\mathrm{r} = 0.12$, $\mathrm{A} = 1000$, $\mathrm{C} = 30$, $\mathrm{P} = 40$, $\mathrm{I_e} = 10\%$, $\mathrm{I_p} = 15\%$, $\mathrm{I} = 20$, $\alpha = 0.05$ and $S = 5$.

Figure 29 shows the efficient front under $\mathrm{M} = 5$. At $\mathrm{N} = 120$ and $\mathrm{T} = 120$, the supplier earns the maximum possible profit of 553 while the retailer earns only 441. At $\mathrm{N} = 30$ and $\mathrm{T} = 47$, the retailer earns the maximum possible profit of 557 while the supplier earns only 428. A total of 975 efficient points found under $\mathrm{M} = 5$. Supplier and retailer's profit differs for each efficient point. Both parties should agree on the most suitable point on which they should operate. For instance, they might chose the point at which they earn equal profit of 502 with $\mathrm{N} = 90$ and $\mathrm{T} = 56$.

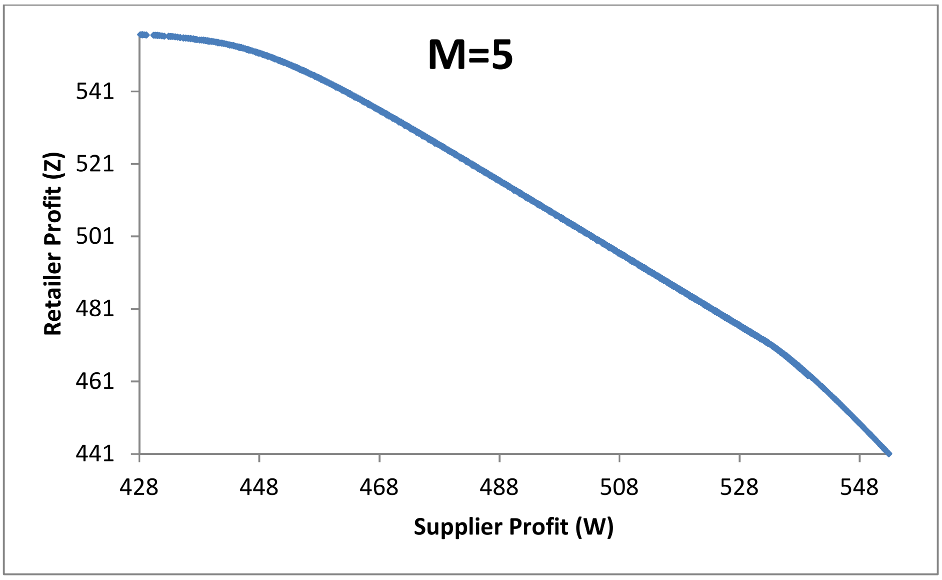


Figure 29 Efficient front (M=5)

Figure 30 shows the efficient fronts for different values of M. As M increases, the maximum profit that would be gained by the supplier decreases, and the maximum possible profit for the retailer increases. This observation is fairly expected as increasing M allows the retailer to earn more interest on sales revenue before the due time of supplier payment.

The model developed in this chapter provides the supplier and the retailer with a tool that would help in achieving an optimal integrated policy of ordering (i.e.T) and offering trade credits (i.e. N and M).

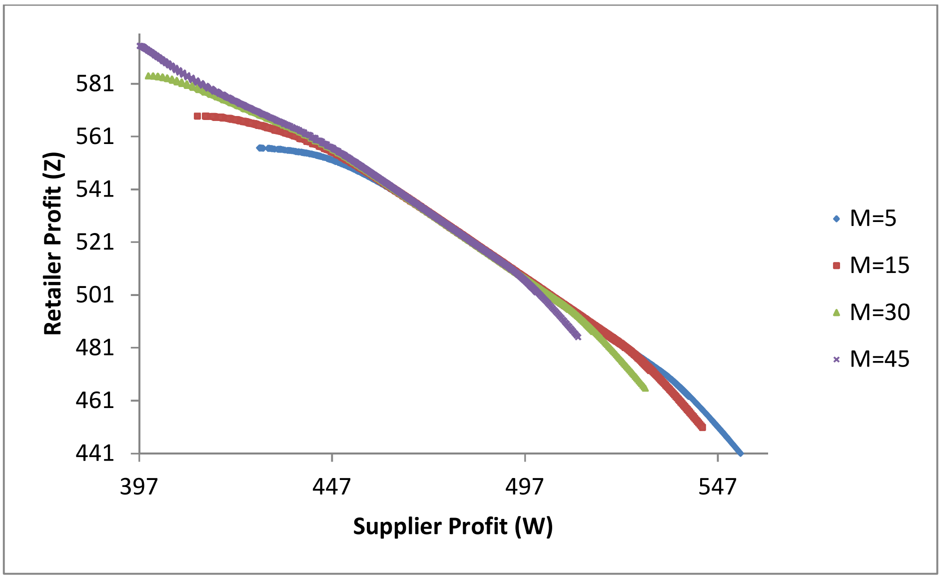


Figure 30 Efficient fronts for different M's

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH

In chapter 3, we developed an EPQ model in which process deterioration and machine breakdown jointly affects the optimal production policy. The model is built for general failure, deterioration, corrective and preventive repair time distributions, but optimality is proved for exponential failure and deterioration times, and uniform corrective and preventive repair times. Process deterioration is assumed to take place gradually where the rate of producing defective items starts, at some point, to increase with time following a linear pattern. Process deterioration and machine breakdown events are assumed independent of each other. Numerical results supported the capability of the proposed model to be used as a decision making tool in finding the optimal production policy.

In chapter 4, we developed an EPQ model similar to that in chapter 3. The proposed model assumes that process deterioration and machine breakdown are dependent events, in the sense that machine failure, if happened, should be preceded by process deterioration.

The two models in chapters 3 and 4 can be extended in many directions; for instance, the process drift can be assumed to follow an exponential pattern rather than a linear pattern, which might suit some applications. Additionally, inspection process can be incorporated in the model, especially that the model already assumes an increasing rate of defectives' production. Hence, the decision maker might be interested in interrupting the production process if defectives' rate reaches a predefined level. Moreover, numerical examples for the two models can be solved under failure time following the Weibull distribution instead of the exponential distribution.

The other two models in chapters 5 and 6 are designed to determine the optimal order quantity in retailing industry, in addition to the optimal credit period offered by the retailer to his customers. Both models are designed in the presence of two-level of trade credit periods, in addition to the assumption of credit-linked demand. In both models, replenishment from the supplier to the retailer is assumed non-instantaneous, additionally a percentage of the retailer's sales are considered as bad debt. The two models differ in the payment procedure from the retailer to his supplier.

The two models in chapters 5 and 6 can be extended in several directions. One interesting extension is by allowing shortage. Under this assumption, shortages can be considered either as lost sales or to be backlogged. Another realistic extension is to consider demand to depend on both the credit period and the selling price to the end consumers. Moreover, randomness can be introduced to the model, as by now all model variables are assumed deterministic. For instance, demand can be assumed random following some probability distribution.

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Vita

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