CONTROL AND ESTIMATION OVER UNRELIABLE COMMUNICATION NETWORKS

BY

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THESIS ABSTRACT

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The topic of this thesis is control and estimation over unreliable communication networks such as wireless network. It is assumed that the plant and control unit are connected though unreliable channels. We considered the problems of estimation and control under two different protocols. In the TCP-like protocol, where the control unit provides acknowledgments successfully delivered of the packets, while the acknowledgments are absent in case of UDP-like protocol. This thesis investigates techniques for designing linear quadratic Gaussian LQG controller and estimation schemes subject to packet dropout using state and output feedback. Firstly, LQG optimal controller is designed using optimal theory based on Linear quadratic regulator and a discrete Kalman filter with packet dropout according to Bernoulli process. Necessary and sufficient conditions to guaranty stability are stated. Then estimation schemes are elaborated for a class of networked control system with nonstationary data lost. Two observer based stabilizing controller of networked control systems (NCSs) are designed in case of zero input and hold input strategies. Sufficient conditions for stability are de-

rived in terms of using linear matrix inequality (LMIs). Theoretical analysis and simulation results are presented using MATLAB software for several numerical examples.

ملخص الرسالة

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عنوان هذه الرساله السيطره و التقدير عبر الشبكات الغير موثوق بها مثل الشبكات الاسلكيه . حيث يتم افتراض ان البيانات المرسله من المصنع الي المتحكم و من ثم الى المصنع تمر خلال شبكات اتصال غير موثوقه مما يعرض الرسال المرسله للفقد. تم دراسه المشكله خلال بروتوكولين مختلفيه هما اليو دي بي او بروتوكول ال تي سي بي . حيث ان بروتوكول ال تي سي بي مزود بتقنيه الاشعار باللاستلام حيث يستلم بينات تخبره بنجاح استلام البينات المرسله او لا . اما بروتوكول يو دي بي فلم يزود بهذه التقنيه. اولا في هذه الأطروحة ندرس تقنيات لتصميم تحكم ال كيو جي عندما تكون البيانات المرسله معرضه للفقد التغذيه الخلفيه للخرج و المتغيرات . حيث تم افتراض ان البيانات تفقد وفقا لحزمه برنولي و بأستخدام استراتجيات تعويض البيانات المفقوده. الشروط الاساسيه و الضروريه لا ستقرار النظام تأخذ بعين الاعتبار . ثانيا يتم وضع مخططات تقدير لفئة من نظام مراقبة الشبكات مع المنظمات غير ثابتة البيانات المفقودة . في هذه الحالات يتم تصميم مراقبين في حاله تعويض حزمات التحكم المفقود بأستخدام بأحدى الاستراتيجيتين: اما نعوض الحزمه المفقوده بصفر تحكم او بتطبيق التحكم السابق عليها. يتم استخدام تقنيه ال ال ام آي لاستنتاج الشروط الاساسيه و المناسبه لاستقرار النظام . يتم استخدام برنامج المحكاه المات لاب لمحاكات النتائج و تطبيق الامثله الرقميه.

Chapter 1

INTRODUCTION

1.1 Overview

Communications and control theories are extremely attractive topics with a slight intersection. However, these two problems had been addressed independently up to 1990s due to the fact that the assumption of signal transmission through the communication channel was implemented with infinite precision in the value for state estimation and control. Hence, the system is operating well usually when the design and the analytic have been made easier over large bandwidth systems. In its traditional, control design considers with assumption that system observations, which are observed by a sensor are feedback, without failure through infinite bandwidth transmission channels to estimation/control unit where an estimator estimates the state of the operation process. This estimate is directed to a controller to optimize a quadratic cost function. Then a control packet is transmitted to an actuator in the process side to make actions, see Fig 1.1. In particular, control and observation packets are sent via a communication path with a limited capacity according to system constraints and limitations. This becomes a problem for large systems that have a massive number of packets need to be transmitted immediately. For instance, the communication bandwidth of large-scale control systems for platoons of underwater vehicles design is severely limited, see [2].

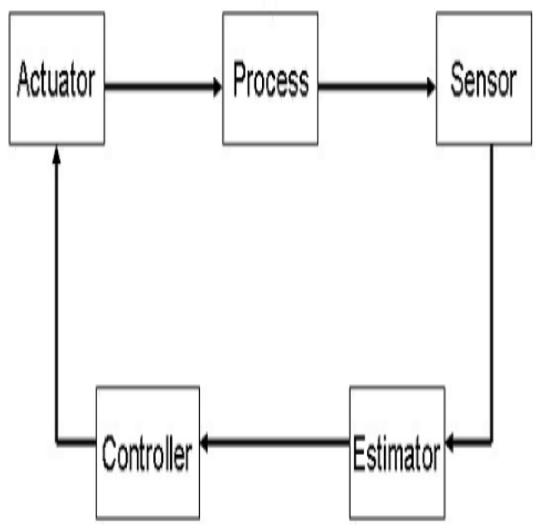

Figure 1.1: A basic traditional control systems structure

In addition, these issues rising and become difficult in the state estimation/control theory with numerous sensors and actuators sending and receiving signals though the same communication network due to the fact that the controller/state estimator only can be observed the transmitted consequence of finite-valued. As a result, complex control systems usually severe from data time delayed, irregular, time-varying, and packet dropout. Moreover, data transmitted might be dropout because congestion, unreliable nature of the link or protocol malfunctions. These are considered as weakness of the control and estimation theory, which assumption that the signal processing and transmission need to be performed instantaneously [5]. In addition, the effect of time delay and data loss on the performance of the networked control system is studied by many researchers. The bandwidth limitation and packet size constraints is considered for the communication network [28]. It is known that large size packets are separated into small packet according to the packet switched networks. As a result, multiple packets must be transmitted via the communication path. However, for better performance sensing or actuation packets should be combined in large packet and transmitted as single packet. Moreover, the suitable and the correct problem formulation of networked control systems (NCSs) model is very important in state estimation and control design through NCS systems. In particular, several problems of state estimation, control design, and stability analysis over communication links are studied and solved for discrete time model in most of the research.

Networked Control Systems (NCSs) are closed loop control systems, composed of various types of actuators, controllers, and sensors connected together through any communication network. This communication media may be wired or wireless. In general, controllers and actuator are event driven while sensors are time driven. Although, event driven nodes reduce the time delay, but generate a difficulty in the analysis result in time varying systems.

Many approaches have been used to derive the optimal control law for discrete time systems with quadratic objective function when the NCSs have Lossy links. For particular information structures, most of the previous works developed the optimal control law based on dynamic programming approach for different protocols Transmission Control Protocol TCP and User Datagram Protocol UDP such as in [1, 2, 10, 12, 15, 16]. In addition, some estimation algorithms for a class of networked control systems for unreliable communication channels have been investigated for UDP-like protocol with random data lost,[37, 41, 39, 38]. Examination of the key issues involved in controller and estimator design, when measurements and control packets are randomly lost due to unreliability of communication links. It is known that there are two actions for the actuator when the control packets lost. In the first case, the "last available control" is applied, called hold-input strategy [17, 42, 45, 46], while in the second case, "zero control" is applied, called zero-input strategy [1, 17, 41, 44].

In this thesis, sensors, actuators, and controller are assumed to be time driven rather than even driven [21]. Moreover, this thesis presents some approaches, in one chapter, to design linear quadratic Gaussian LQG problems for both TCP-like and UDP-like protocols, where LQGs are used to optimize the infinite horizon cost function based on standard LQR optimal techniques. In addition, our approach differs drastically, in which the data transmission is assumed to be nonstationary packet dropout. Researchers also studied the problem of nonstationary delay, see [71, 72].

1.2 Fundamental Issues of NCSs

1.2.1 Band-Limited

It is well known that there is specified bandwidth of any communication channel as result of that packet size, bit rate and amount of transmission are limited. These constraints affect the stability and the overall performance when large size packet or huge amount of information need to be transmitted per unit of time. In addition, the quantization effects on NCS stability and performance are ignored.[36]

1.2.2 Time Delay

Transmission data through communication channels suffers from networking delay, processing delay, and waiting delay. These factors has salient impacts on the system performance and can be gone to instability.

1.2.3 Packet Dropouts

This issue is our work mainly concerned. Usually packet suffer from transmission delay during transmission and sometimes Long transmission delay is considered as packet dropout. Packet loss has worse impacts on the stability and the performance of the control system. Data transmitted may be lost as result of congestion on the path, unreliable nature of the link or protocol malfunctions. In TCP-like protocol, acknowledgment of packet reception is used, the transmitter knows if the sending packet is received or not. Even though retransmission mechanisms is used in TCP case, but sometimes it is not enough to deliver the lost packet due to time limitations. However, in UDP-like protocol the acknowledgment of packet reception is absent. So the receiver and the transmitter do not know if the sending packet is received or not, [34].

1.3 Why Networked Control Systems (NCSs)?

Networked control systems is a rising area in modern control engineering and applications, that give flexibility and the opportunity for companies to reduce the overall operation cost, where the companies can site their most expert control and maintenance engineers in one control facility like in London, and their control systems located abroad such as in India, China, or Korea. Although, NCSs are still an academic research, however, the interaction between communication and control is an interesting subject. NCSs are already used in some manufactures such as aircraft, automobile, and health care systems, see [3][5]. In mathematical electrical engineering, a new chapter is concerned about the communication and control systems. In fact, numeric of control strategies emerging from classical control theory are considered for NCS systems like PID control, adaptive control, optimal control, robust and intelligent control as well, that might make the control over network challenge as given in [4]. This thesis presents that at each time k the plant send measurement (sensing) packets over a lossy communication links and the control unit send a control (actuation) packet to the plant over a lossy network but with different probability of link failure. Packets dropping network have studied when the transmission media has probability to fail because of the congestion, unreliable of the link or protocol malfunctions with negligible quantization effects. In addition, the control/estimation could be performed instantaneously. It is known that packet dropout normally degrades the performance of the NCS and result to instability. The unreliable nature of the link from the sensor to the estimation /control unit and from the later to the actuator are modeled by two independent identically distributed (i.i.d) Bernoulli processes. The stability of the control system for the NCS is approved in finite and infinite horizon cases of both transmission control protocol TCP and user datagram protocol UDP, as given in [1][6][15] [8], and the state-feedback gains are mode-dependent, where many control techniques are used to calculate the state-feedback gains.

1.4 Thesis Objectives

The main objectives of the thesis are to

- To design Linear Quadratic Gaussian (LQG) controller using optimal theory. In addition, the analysis of a discrete Kalman filter will be considered in details.
- To design an observer based stabilizing controller with nonstationary packet drops out on both directions. Furthermore, two compensation strategies will be used to compensate the control packets dropouts. In addition, a modified feedback controller will be used in our case.

1.5 Problem Statement

1.5.1 Plant Model

In this thesis, we consider a discrete linear time-invariant (LTI) system. The plant consists of one or more output elements (sensors) and one or more input elements (actuators). The state-space model of the system subject to packet dropout is described by

$$x_{k+1} = Ax_k + \alpha_k B u_k + w_k; \quad k = 0, 1, 2.. \tag{1.1}$$

$$y_k \quad = \quad \beta_k C_k x_k + v_k \tag{1.2}$$

where $x_k \in \Re^n$ is the state vector, $y_k \in \Re^p$ is the measured output by the sensors, $u_k \in \Re^m$; is the control input that applied by the actuator. $w_k \in \Re^q$ is the input disturbance while $v_k \in \Re^p$ is the measurement disturbance, which are independent zero mean second-order random vectors and also independent of α_k and β_k. The unreliable of the links from sensor to the controller/estimator unit and from the later to the actuator are modeled by two i.i.d. Bernoulli processes β_k and α_k respectively. The model matrices are, the dynamic matrix $A \in \Re^{n\times n}$, the control input matrix $B \in \Re^{n\times m}$, and the output observation matrix $C \in \Re^{p\times n}$.

$$Pr(\alpha_k) = \begin{cases} \hat{\alpha}_k; & \text{if } \alpha_k = 1; \\ 1 - \hat{\alpha}_k = \bar{\alpha}_k; & \text{if } \alpha_k = 0. \end{cases}$$

$$Pr(\beta_k) = \begin{cases} \hat{\beta}_k; & \text{if } \beta_k = 1; \\ 1 - \hat{\beta}_k = \bar{\beta}_k; & \text{if } \beta_k = 0. \end{cases}$$

The observer form is given as

$$\hat{x}_{k+1} \quad = \quad A\hat{x}_k + \hat{\alpha}_k B u_k + K_k(y_k - \hat{y}_k) \tag{1.3}$$

where K_k is the Kalman filter gain while $\hat{x}_k$ and $\hat{y}_k$ define the estimated state

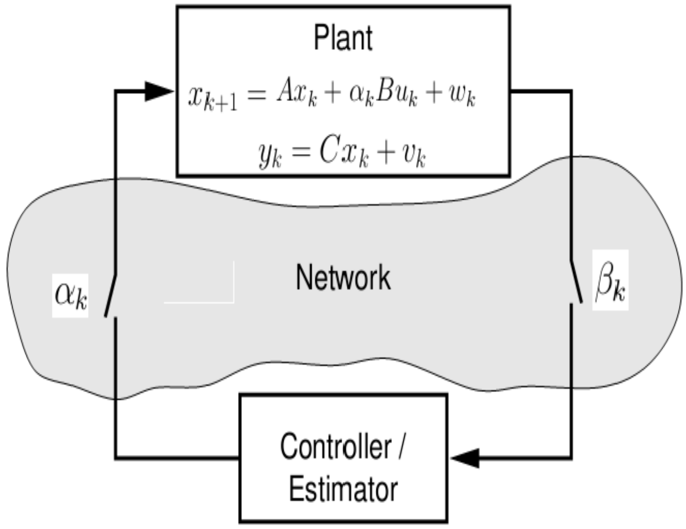

Figure 1.2: Lossy networked control systems structure [52]

and measurement respectability. The estimated measurement is given as

$$\hat{y}_k = \hat{\beta}_k C_k \hat{x}_k$$

We consider the quadratic cost function in finite horizon as where $N > 0$ is a finite horizon, and the weighted matrices $Q = Q^T \geq 0$, where $Q \in \Re^{n\times n}$, $R = R^T > 0$, where $R \in \Re^{m\times m}$, and $F = F^T \geq 0$.

1.5.2 Hold Input Scheme Model

To compensate the loss of the control packets using last applied control, the plant model can be presented by

$$x_{k+1} = Ax_k + \alpha_k B u_k + (1-\alpha_k)B\xi_k + w_k \quad (1.4)$$

$$\xi_{k+1} = \alpha_k \xi_k + (1-\alpha_k)u_k \quad (1.5)$$

The state variable ξ_k keeps track the last available control applied by the actuator. The applied control input by the actuator in hold scheme according to α_k is

$$u_k = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ u_{k-1}; & \text{if } \alpha_k = 0. \end{cases}$$

The state equation is

$$x_{k+1} = \begin{cases} Ax_k + Bu_k + w_k; & \text{if } \alpha_k = 1; \\ Ax_k + Bu_{k-1} + w_k; & \text{if } \alpha_k = 0. \end{cases}$$

and the observer based is considered as following for hold input method

$$\hat{x}_{k+1} = \begin{cases} A\hat{x}_k + Bu_k + K_k(y_k - \hat{y}_k); & \text{if } \alpha_k = 1; \\ A\hat{x}_k + Bu_{k-1} + K_k(y_k - \hat{y}_k); & \text{if } \alpha_k = 0. \end{cases}$$

1.5.3 Zero Input Scheme Model

To compensate the loss of the control packets by zero input, the plant model can be presented by

$$x_{k+1} = Ax_k + \alpha_k Bu_k + w_k \tag{1.6}$$

the plant will receive a control input from the actuator according to the random values of α_k according to

$$u_k = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ 0; & \text{if } \alpha_k = 0. \end{cases}$$

Then, the state equation is written as

$$x_{k+1} = \begin{cases} Ax_k + Bu_k + w_k; & \text{if } \alpha_k = 1; \\ Ax_k + w_k; & \text{if } \alpha_k = 0. \end{cases}$$

The observer based in terms of the zero input scheme can be written as

$$\hat{x}_{k+1} = \begin{cases} A\hat{x}_k + Bu_k + K_k(y_k - \hat{y}_k); & \text{if } \alpha_k = 1; \\ A\hat{x}_k + K_k(y_k - \hat{y}_k); & \text{if } \alpha_k = 0. \end{cases}$$

As can be seen from the above equation, the compensation schemes are applied when the control signal get lost. However, the compensation schemes are not used to compensate the lost of the measurement signals

1.6 Thesis Organization

This thesis contains several chapters, the first of which is the introduction. Chapter 2 showing the previous work on NCSs plants subject to estimation and control theory that were developed. Chapter 3 contains linear quadratic Gaussian controller design over lossy communication channels when observation noise is available for both transmission control protocol TCP and user datagram protocol UDP. Chapter 4 is focused on a class of an observer based networked control system with nonstationary packet Loss in terms of zero and hold input strategies. In chapters conclusions will be drawn and directions for future research will be presented.

Notations: The Euclidean norm $|.|$ is used for vectors in the n-dimensional space $\Re^n$ and we denote by $||.||$ the corresponding induced matrix norm in $\Re^n$. The notation W^t, W^{-1}, $\lambda_m(W)$ and $\lambda_M(W)$ denote the transpose, the inverse, the minimum eigenvalue and the maximum eigenvalue of any square matrix W, respectively. We use $W < 0$ (≤ 0) to denote a symmetric negative definite (negative semidefinite) matrix W and I_j to denote the $n_j \times n_j$ identity matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In symmetric block matrices or complex matrix expressions, we use the symbol $\bullet$ to represent a term that is induced by symmetry. Sometimes, the arguments of a function will be omitted when no confusion can arise.

Chapter 2

LITERATURE SURVEY

2.1 Literature Review

With the rapid advance in the control engineering and its applications, networked control systems (NCS) acquired a significant attention currently because of the advantages that NCS provide. The purpose of this section is to provide background and motivation that make control and estimation problems over communication networks subject to packet lost an interesting area.

In 2004s, Gupta Vijay [18], investigated an optimal linear quadratic Gaussian (LQG) control problem when the sensor to the controller is unreliable link where the measurements packets drops randomly according to a Markov chain. Packet dropout was modeled by a switch. It was shown that, the separation principle exists between the optimal estimate and the optimal control law. The stability and performance analysis of the algorithm were compared to other methods presented in the literature.

In [20], a mutual analysis of control and coding for stability and performance of remotely an LTI plant over communication links was studied. Further, the authors considered stability of the differential entropy and mean-square stability of the state estimation error with the requisite communication rate. It was shown, the optimal control law which optimizes a quadratic performance objective function is linear function of the estimated states. The separation principle is hold as a result of that. Its solution was dependent on the individual design of the estimator and the LQR controller. Researchers also reported that the communication rate requirements was impacted by the estimation problem when the measured state was quantized.

In [21], different policies for a stochastic control with a fixed end-time were investigated. It was focused on distinction between feedback and closed-loop policies in stochastic control. When the control has a dual effect the feedback policy can only be actively adaptively. In addition, the separation principle holds has been expanded if the certainty equivalence property formerly known. A controller and an estimator can be designed separately.

Great attention has been paid to a solution of the infinite horizon control. Generally, a discrete time linear system and a performance index were studied with independent stochastic parameters. Compared with properties of the deterministic systems, authors introduced that mean square m.s. stabilizability property was a stronger condition while m.s. observability property was a weaker condition. It is presented that, if the system is m.s stabilizability the infinite time problem will has a solution. Furthermore, the solution will be unique even when it is m.s. observability as well. On the other hand, m.s. observability does not guarantee the existence of the solution, see [24].

In [19], the authors proposed an LQG optimal control problem for scalar models under network limitations, where the quadratic objective function was extended to involve a quadratic penalty for communication where the optimization problem has to be quasi-convex in the output matrix C. However, with assumption that, the system was unstable, the problem was convex. The problem was figured out in a computationally efficient way using semidefinite programming.

In [12], they discussed the problem of a discrete LQG under transfer control protocol (TCP) where measurements and control packets can get lost over sensor to estimation/control unit and from the later to the actuator respectively. Packets dropout are on both direction because of unreliable features of the communication links. It is will known that in TCP-like protocol case, the transmitter receive an acknowledgment whether the sending packet received or not. However, authors investigated the problem when the acknowledgment was always available for the control packet reception. Besides, LQG optimal control was established to be a linear function of the state. It was shown, the existence of critical arrival probabilities could not able to stabilize the system. Moreover, a stochastic Linear system is studied with intermittent measurements. This model is described by

$$\begin{aligned} x_{k+1} &= Ax_k + \nu B u_k + w_k \\ y_k &= \gamma(Cx_k + v_k) \end{aligned}$$

where x_k is the state vector, y_k is the measured output by the sensors, u_k; is the control input that applied by the actuator. w_k and v_k are the process and the observation disturbance. Two i.i.d. Bernoulli random processes, γ and ν , are used to model the unreliable of the communication channels from sensor to the controller/estimator unit and from the later to the actuator respectively.

In [16], authers extended the work of [12] to the case of LQG optimal controller over user datagram protocol (UDP), and they proved that the control law was generally nonlinear. Furthermore, the separation principle is not hold any more whereas the acknowledgment of the control packet reception was missing. It was present that, UDP-like protocols provided a much more complex scenario than TCP-like protocol with impractical solution.[23, 1] However, the UDP optimal controller is linear when the output matrix C is invertible and there is no output noise as a special case.

In [1], linear quadratic regulator control problem for a discrete-time linear system was introduced when several of the observation and the control packets were lost. It was assumed that the packet may be lost with two simple independent Bernoulli random processes, and there was no observation noise. Moreover the structure of the controller counted on the features of the communication networks. Researchers focused on design of LQR optimal controller using dynamic programming approach with quadratic cost function in finite and infinite horizon. The control law is linear with state when the acknowledgment was always available. Sufficient and necessary conditions of mean square stability were derived. The plant is presented by a discrete LTI model as

$$\begin{aligned} x_{k+1} &= Ax_k + \alpha_k B u_k + w_k \\ y_k &= \beta_k C x_k \end{aligned}$$

where $x_k \in \Re^n$ is the state, $y_k \in \Re^p$ is the output, $u_k \in \Re^m$; is the control input. $w_k \in \Re^q$ the input noise. However, the authors assumed that there was no observation noise. The random variable α_k is the probability of the control signals to be lost while β_k is the probability of the observation signals to get lost.

Generally speaking, an LQG optimal control for networks with a lack of packet acknowledgment such as the UDP-like case cause analytical complications. In fact, the controller design is affected by the shortage of delivered acknowledgment. On the other hand, the LQG controller is considered linear if assuming there are no measurements noise sent whenever the measurement packet arrives, i.e. output matrix C is invertible and weighted R = 0. Additionally, for UDP-like protocols the critical arrival probabilities for the control and observation channels were coupled. Otherwise, the critical arrival probabilities for the control and observation channels are independent for TCP-like protocols. In particular, the UDP protocols were considered as the exclusive solution for extremely lossy channels that guarantee successful delivery of acknowledgment, see [23].

Besides the models established above, single input single output SISO, LTI system was investigated with one-degree-of-freedom control architectures via lossy communications links. Also, signals might be lost across erasure links with i.i.d. Bernoulli random process. It was shown that the result of [73] was extended by considering additional i.i.d. noise channel instead of the the analog erasure channel, where the controller was designed consequently. A necessary and sufficient conditions were derived to ensure m.s. stability. In fact, output feedback controller was designed over UDP like protocols, see [11]

In [14], optimal estimation problem over lossy communication channels were studied. The control packets, the observation packets in addition to the acknowledgment packets were subject to loss. Packet drop properties were considered as unknown i.i.d. Bernoulli random variables. Furthermore, the focus of that research was to design suboptimal control in terms of linear matrix inequalities (LMIs) with the present of uncertain networks constraints.

In [15], an LQG optimal via TCP-like unreliable communication networks was considered. Also, all the transmission packets which were damaged sited as i.i.d. Bernoulli processes, see e.g. [43]. Moreover the work in [41] was extended to multi -input -multi -output (MIMO) communication channels. It was shown that the separation principle was hold, and the optimal control law was linear function of the estimated state. Sufficient and necessary conditions for systems stability for infinite horizon LQG control were given in form of LMIs.

In [17], zero input and hold input schemes were investigated, which are compensation schemes that normally used to compensate the fate of the control input packets. Bernoulli random processes were used to model the fate of the transmission packets. In zero input scheme, in which zero control is applied by the actuator when the control input is lost. Nevertheless, the last available control input is applied in case of hold input scheme. For control applications, the linear quadratic performance for both compensation schemes using a static feedback were analyzed. As a conclusion of that, although zero input strategy was used widely to simplify mathematical problems, but no one of these compensation scheme was considered as the best strategy for most of the systems.

In [10], authors proposed an LQG discrete problem in two events using dynamic programming approach. Firstly, the acknowledgment are available constantly, where the optimal control law considered using hold input strategy was linear. In the second the event, the acknowledgment can be lost, in which zero input strategy was considered for simplifying the optimal control problem due to the fact that the optimal control law was nonlinear and the separation principle was not hold. As result, a suboptimal LTI approach was investigated. In addition, the plant was assumed in the following form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k^a + w_k \\ y_k &= \gamma_k(Cx_k + v_k) \end{aligned}$$

u_k^a is the actual control input applied by the actuator under zero and hold strategies.

An estimation LTI stochastic problem over erase communication channels for both TCP and UDP protocols was studied. For TCP-like protocol, the impact of packet retransmissions and acknowledgment mechanisms on the system stability and performance were discussed. A comparison between UDP and TCP protocols was shown in two cases. In the first case, while a single sensor device communicated via a single path, TCP protocol provided better exhibit than UDP. In the second case, multi sensors communicated across communication channels, UDP protocol provided better exhibit than TCP protocol, see [39]

In [37] and [38], estimation strategies over unreliable networked without packet acknowledgment (e.g. UDP) were studied. It was shown that the standard observer strategy could not be used in case for user datagram protocol due to the lack of acknowledgment packets. The main of that works was focused on design of simple estimator involved of mode and state observer where the measurement packets were assumed to be regularly delivered to the control/estimation unit. Furthermore, a discrete time has been modeled using Jump Linear System to pick up the lost control packets. Moreover in [38], the system was supported by an additional control input to ensure recovering the lost control packets . After that, in [52] the works of [37]and [38] were extended in case the measurement packets randomly dropped out through a lossy network. Hence, the discrete linear system is described by

$$\begin{aligned} x_{k+1} &= Ax_k + \beta_k B u_k + w_k \\ y_k &= \gamma_k (C x_k + v_k) \end{aligned}$$

Moreover, they assumed that there were two estimators, one to estimate the state and the other to estimate the fate of the control packets. The state estimator is considered as

$$\hat{x}_{k+1} = Ax_k + \beta_k B u_k + \gamma_k L_k (y_{k+1} - \hat{y}_{k+1})$$

where $\hat{y}_k$ is the estimated output. Furthermore, the fate estimator is given by

$$\hat{\beta}_k = \arg\min \|y_{k+1} - \hat{y}_{k+1}\|^2$$

this fate observer is used to determine the value of $\hat{\beta}_k$

In [55], the authors have considered a model predictive control MPC technique over lossy communication network. Interestingly, MPC method has the feature to be a good explanation for the networked control systems constraints, in which future control packets are sending at the last control input. In fact, the compensation strategy here was totally different in its compensate to the fate of the control packet by the future control input packet. Moreover a switching strategy was used to swap various control laws.

In [40] the focus of the research was on the important of packet acknowledgment and its impact on the communication between the controller/estimator unit and the plant. To present these affects, a discrete LQG control was investigated across a lossy channels with two strategies of packet acknowledgment. Firstly, packet delivered is acknowledged under TCP-like protocol. Hence, the control system is linear and the separation principle between the controller and the estimator is hold. In the second case, packet received does not acknowledged under UDP-like protocol. It is shown that the separation principle between the controller and the estimator does not hold. The performance and stability are affected due to the control system nonlinearity.

Recently, In [30] a problem of kalman filter state estimation over wireless sensor networks (WSNs) with the effect of unmatched sensors connected via NCS. These sensors usually generate measurements disturbances,that transported with measurements packets to a fusion center. This research was focused on using partial broadcasting policy to optimize the estimation error covariance. Moreover due to the issues of data lost, battery power, and bandwidth limitation, partial broadcasting policy could not be an optimal method. As result of that, a good-sensor-late-broadcasting was investigated using finite horizon LTI discrete system in case of perfect packet transmission without lost. There are several researches about the optimal estimation and kalman filter estimation, see ([13],[47],[48], [49], [50], [51], [53], and [54])

2.2 Stability Criteria of NCSs

Generally speaking, stability analysis of the networked control systems under networking constraints is a fundamental problem. This section addresses the stability of feedback loops that are closed over a network with time delay and package dropout constraints. In fact, there were various technique developed to establish the stability condition for NCS's with different assumptions. As given in [25], time delay normally exists in most communication network. In particular, transmission delay takes place on both paths from sensors to controllers as well as from controllers to actuators. It is well know that time delay can debase the performance of control systems and may take the system to unstable side. In addition, long delay of a packet was considered as packet dropout after a specified time. On the other hand, packet dropout might be occurred on an NCS fitfully when there are link failures or congestion result in buffering problem. In order to avoid these problems, the vast majority of transmission protocols were supplied by retransmission mechanisms, in which the lost packet was retransmitted until it successfully delivered such as in TCP -like. However, there are limited time for retransmission process, otherwise the packet was considered as dropped.

In [27], the conditions mean square m.s. stable were studied of lossy undisturbed NCS. In the main of this work, authors concentrated on data lost because networking errors such as network congestion. It was shown that the uncertainty threshold principle was employed to clarify the stability under certain conditions. In particular, they discussed the impacts of applying the retransmission strategy and do not applying on the system stability when some packets got lost.

In [28], a stabilization strategy for a class of unreliable networked control systems was investigated. The unreliable features of the communication channels were considered as erase channels. It is well known that control gains affected by the trade off between packets dropout and instability parameters. Particularly, researchers extended the work in [22] for addressing upper dimensional dynamic system under satisfied conditions.

More specifically, an exponential stability with a dynamic output feedback control was discussed under the influence of time varying delays and data lost . Data lost may occur on both direction sensor to controller and the latter to the plant. The asynchronous dynamic system theory , Lyapunov principle, and LMI methods were accomplished for the control system stability analysis. Furthermore, the conditions of negative semi-definite matrix with exponential stable and control design were studied, see [31].

In [32], try-once-discard (TOD) control protocol was considered for MIMO networked control systems, in which several autonomously sensors and actuator were connected to the network. Primarily, mathematical analysis was done for global exponential stability for TOD protocol and the traditional protocols (e.g. statically scheduled protocol) as well. First of all, authors established a controller design for TOD protocol to study the impacts of networks constraints on the performance of system. After that, the system performances for both TOD control protocol and statically scheduled protocol were compared.

In [33], stability analysis of NCS via output feedback control was studied when data packet lost and packet time delay could be occurred through the networked control system links. Based on Lyapunov function method, the sufficient conditions of stability of NCSs were investigated by using asynchronous dynamical system (ADS) approach. The idea of this could be found in Nilsson's Ph.D. dissertation [26].

In [42], the problem of stability analysis and controller design were studied based on a new model of the NCSs with single and multiple packets transmission. As considered in the previous works, transmission data may be get lost when the packet sending from sensor-to-controller as well as from the later-to-actuator. However, the feature of data dropout characterized by two different independent Markov chains. Sufficient and necessary conditions for stochastic stability were obtained via LMIs.

In [6], the author investigated a discrete LQG optimal problem across analog erase communication links to study the effects of the acknowledgments (e.g. TCP) using smart actuator and global actuator which do not have direct access to the plant. They concluded that, although the acknowledgment with smart actuator can provided several processing alone, it could not upgraded the stability performance region better than that for the global actuator, see also [5]and [7].

[29] considered a NCS architecture where the plant normally was nonlinear, in which a packetized predictive controller uses over lossy network affected by signal loss. It was shown that the input to state stability was guaranteed by determining the value of the turning parameters of the control system design, in which packet lost became limited. Furthermore, the effect of the disturbance on the constrained nonlinear system has been investigated.

In the next chapter we will consider problems of discrete linear quadratic Gaussian when the transmission data packets prone to failure because of the unreliable communication channels using hold and zero input strategies.

Chapter 3

LINEAR QUADRATIC GAUSSIAN (LQG) DESIGN

3.1 Introduction

In this chapter, we introduce an optimal control problem of discrete LTI system with a quadratic cost objective function. As noted in the literature review, linear quadratic (LQ) methods are one of the most useful linear optimal control approaches. It is well known that in LQ methods, a quadratic objective function is optimized to get a suitable state feedback gain G_k through minimizing the estimation error covariance. In fact, the linear quadratic regulator (LQR) is used when all the true state available for the control. This problem increases for high order systems and when the control systems contain noise due to the fact that we can not find the exact states. Hence, the linear quadratic Gaussian (LQG) solves that problem by estimating states using a kalman filter, so there is need to evaluate all the states. Consequently, the classical LQG is LQR linked together with a kalman filter to eliminate input Gaussian white noise and to estimate state, see [56][57]. Broadly speaking, we are concerned in the design of linear quadratic Gaussian over unreliable communication networks. The unreliable features of the channels are modeled by a Bernoulli stochastic process. The stability of the control system for the NCS is proved in finite and infinite horizon cases of both transmission control protocol TCP and user datagram protocol UDP, also see [10, 12, 1, 2]. In particular, we extend the work of [1] and [10] further to the case where measurement noise is present. Our analytical approach is based on the linear-quadratic optimal approach [57, 47, 48]. Furthermore, we discuss the optimal control problem design in terms of both compensation schemes hold-input and zero-input.

3.2 Problem Formulation

In this chapter, we are specifically interested in the design of linear quadratic Gaussian (LQG) controller using optimal theory. In addition, the analysis of a discrete Kalman filter will be considered in details. It is known that there are two actions for the actuator when the control packets lost in terms of LQG optimal control approach. In the first case, the "previous available control" is applied, while the second case, "zero control" is applied. In this work, we assume the previous available control input will be applied by the actuator to compensate the fate of control packets. The plant process given by a discrete-time state-space model, is described in our work by

$$x_{k+1} = Ax_k + \alpha_k B u_k + (1-\alpha_k) B \xi_k + w_k \tag{3.1}$$

$$\xi_{k+1} = \alpha_k \xi_k + (1-\alpha_k) u_k \tag{3.2}$$

The state variable ξ_k keeps track the last available control applied by the actuator. We can write this system in a new state space form as follows

$$\begin{aligned} \bar{x}_{k+1} &= \begin{bmatrix} x_k \\ \xi_k \end{bmatrix} \\ &= \begin{bmatrix} A & (1-\alpha_k)B \\ 0 & \alpha_k I \end{bmatrix} \bar{x}_k + \begin{bmatrix} \alpha_k B \\ (1-\alpha_k) I \end{bmatrix} u_k \\ &\quad + \begin{bmatrix} I \\ 0 \end{bmatrix} w_k \end{aligned} \tag{3.3}$$

In compact form, this is described by the discrete-time model

$$\bar{x}_{k+1} = \bar{A}_k \bar{x}_k + \bar{B}_k u_k + \bar{I} w_k$$
$$y_k = \beta_k C_k x_k + v_k \tag{3.4}$$

where

$$\bar{A} = \begin{bmatrix} A & (1-\alpha_k)B \\ 0 & \alpha_k I \end{bmatrix},\ \bar{B} = \begin{bmatrix} (1-\alpha_k)B \\ \alpha_k I \end{bmatrix},\ \bar{I} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

where $x_k \in \Re^n$ is the state vector, $y_k \in \Re^p$ is the measured output, $u_k \in \Re^m$ is the control input, $w_k \in \Re^q$, $v_k \in \Re^p$ are respectively, the input and measurement disturbances, which are independent zero mean second-order random vectors and also independent of α_k and β_k. The stochastic processes α_k and β_k are Bernoulli processes represent the lossy nature of the controller- actuator and sensor- controller links respectively. Associated with system (3.4), the quadratic performance function:

$$J = E[\bar{x}_N^T F \bar{x}_N + \sum_{K=0}^{N-1} \bar{x}_k^T Q \bar{x}_k + u_k^T R u_k]$$

where $N > 0$ is a finite horizon, and the weighted matrices $Q = Q^T \geq 0$, where $Q \in \Re^{n\times n}$, $R = R^T > 0$, where $R \in \Re^{m\times m}$, and $F = F^T \geq 0$.

3.3 LQG Optimal Control Over TCP Protocol

Consider the networked control system presented in figure 3.1. We aim to design an optimal control policy that optimize the cost function for the best estimation of error covariance.

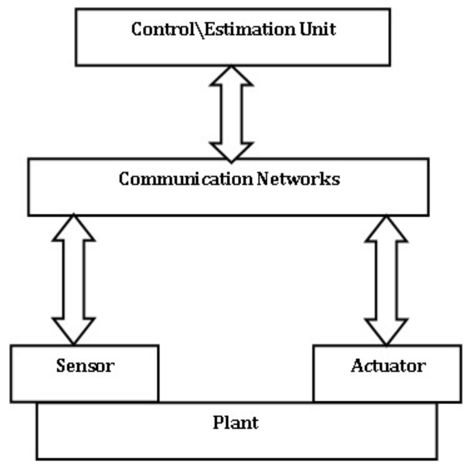

Figure 3.1: An NCS structure with TCP-like protocol

3.3.1 Estimator Design

Tthe separation principle is hold and the optimal control policy is linear in case of TCP-like protocol. As result of that the estimation unit of the controller can be designed individually. For simplification, we assume $\bar{C}_k$=$\beta_k C_k$,and $\bar{w}_k$= $\bar{I}w_k$ then equation (3.4) becomes

$$\bar{x}_{k+1} = \bar{A}_k \bar{x}_k + \bar{B}_k u_k + \bar{w}_k \tag{3.5}$$

$$y_k = \bar{C}_k x_k + v_k \tag{3.6}$$

First, define some variables as

$$\hat{\bar{x}}_k = E[\bar{x}_k] \tag{3.7}$$

where the estimation error is

$$e_k = \bar{x}_k - \hat{\bar{x}}_k \tag{3.8}$$

The estimation error covariance matrix is given as The noises $\bar{w}_k$, and v_k are uncorrelated i.i.d. processes with zero mean, and the covariance matrices corresponding to the orthogonality principle is deduced as

$$\begin{aligned} E[\bar{w}_k^T \bar{w}_k] &= S_w, \;\; E[v_k^T v_k] = S_v \\ E[\bar{w}_k^T v_k] &= 0, \;\; E[\bar{x}_k v_k^T] = 0 \\ E[\bar{w}_k^T \bar{x}_0] &= 0 \end{aligned} \tag{3.9}$$

The a priori estimate of the process model is given as

$$E[\bar{x}_{k+1}] = \bar{A}_k E[\bar{x}_k] + \bar{B}_k E[u_k] \quad (3.10)$$

$$\hat{\bar{x}}_{k+1|k} = \bar{A}_k \hat{\bar{x}}_{k|k} + \bar{B}_k u_k \quad (3.11)$$

from estimation error equation, we have

$$\begin{aligned} e_{k+1|k} &= \bar{x}_{k+1} - \hat{\bar{x}}_{k+1} \\ &= \bar{A}_k \bar{x}_k + \bar{B}_k u_k + \bar{w}_k - (\bar{A}_k \hat{\bar{x}}_k + \bar{B}_k u_k) \\ &= \bar{A}_k e_k + \bar{w}_k \end{aligned} \quad (3.12)$$

The a priori estimation error covariance can be calculated as

$$\begin{aligned} \Sigma_{k+1|k} &= E[e_{k+1} e_{k+1}^T] = E[[\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}][\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}]^T] \\ &= E[[\bar{A}_k e_k + \bar{w}_k][\bar{A}_k e_k + \bar{w}_k]^T] \\ &= \bar{A}_k \Sigma_k \bar{A}_k^T + S_w \end{aligned} \quad (3.13)$$

where the control input u_k is considered as deterministic function. It is shown, there are no dual effect between the estimator and the controller over the TCP networks according to the separation principle, see [1, 12, 41].

Lemma 3.3.1 *[41]: By using the algebraic operations , we have:*

1.

$$\begin{aligned} E[(\bar{x}_k - \hat{\bar{x}}_k)\hat{\bar{x}}_k^T] &= E[e_k \hat{\bar{x}}_k^T] \\ &= 0 \end{aligned}$$

2. For $\forall\, T > 0$, *the following fact is true*

$$E[\bar{x}_k^T T \bar{x}_k] = E[\hat{\bar{x}}_k^T T \hat{\bar{x}}_k] + tr(T\Sigma_k)$$

proof 3.3.1 *1.*

$$\begin{aligned} E[(\bar{x}_k - \hat{\bar{x}}_k)\hat{\bar{x}}_k^T] &= E[\bar{x}_k \hat{\bar{x}}_k^T - \hat{\bar{x}}_k \hat{\bar{x}}_k^T] \\ &= E[\bar{x}_k]\hat{\bar{x}}_k^T - \hat{\bar{x}}_k \hat{\bar{x}}_k^T \\ &= \hat{\bar{x}}_k \hat{\bar{x}}_k^T - \hat{\bar{x}}_k \hat{\bar{x}}_k^T; \quad (from (3.7)) \\ &= 0 \end{aligned}$$

2.

$$\begin{aligned} E[\bar{x}_k^T T \bar{x}_k] &= E[(e_k + \hat{\bar{x}}_k)^T T (e_k + \hat{\bar{x}}_k)] \\ &= \hat{\bar{x}}_k^T T \hat{\bar{x}}_k + 2tr(T\ E[e_k \hat{\bar{x}}_k^T]) \\ &\quad + tr(T\ E[e_k e_k^T]) \\ &= \hat{\bar{x}}_k^T T \hat{\bar{x}}_k + tr(T\Sigma_k) \end{aligned}$$

The observer form of the Kalman filter is given as

$$\hat{\bar{x}}_{k+1} = \bar{A}_k\hat{\bar{x}}_k + \bar{B}_k u_k + K_{k+1}[y_{k+1} - \bar{C}_{k+1}\hat{\bar{x}}_{k+1}] \tag{3.14}$$

where

$$y_{k+1} = \bar{C}_{k+1}x_{k+1} + v_{k+1}$$

Rearranging the observer form we get

$$\hat{\bar{x}}_{k+1} = \bar{F}_{k+1}\hat{\bar{x}}_{k+1} + \bar{B}_{k+1}u_{k+1} + K_{k+1}y_{k+1} \tag{3.15}$$

where $\bar{F}_{k+1} = \bar{A}_{k+1} - K_{k+1}C_{k+1}$. The main idea here, we use kalman filter to filter the observations through the part $(y_{k+1} - \bar{C}_{k+1}\hat{\bar{x}}_{k+1})$ so -called observation innovation so as estimate the state $\hat{\bar{x}}_k$ to minimize the effects of the process and observation distributions w_k and v_k respectively. The correction step of TCP is given as

$$\hat{\bar{x}}_{k+1|k+1} = \hat{\bar{x}}_{k+1|k} + K_{k+1}(y_{k+1} - \bar{C}_{k+1}\hat{\bar{x}}_{k+1}) \tag{3.16}$$

$$e_{k+1|k+1} = \bar{x}_{k+1|k+1} - \hat{\bar{x}}_{k+1|k+1} \tag{3.17}$$

For the prediction updating, we supposed that, the updating prediction is a weighed linear function of the discrete system given in (3.12), and an observation.

$$\begin{aligned} \hat{\bar{x}}_{k+1|k+1} &= K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}y_{k+1} \\ &= K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}\bar{C}_{k+1}\bar{x}_{k+1|k} \\ &\quad +K_{k+1}v_{k+1|k} \end{aligned} \tag{3.18}$$

K'_{k+1} is gain matrix with different size from the Kalman gain matrix K_{k+1}. when the prediction is unbiased:

$$E[\hat{\bar{x}}_{k+1|k+1}] = E[\bar{x}_{k+1|k}]$$

$$E[\hat{\bar{x}}_{k+1|k+1}] = K'_{k+1}E[\hat{\bar{x}}_{k+1|k}] + K_{k+1}\bar{C}_{k+1}E[\bar{x}_{k+1|k}] \tag{3.19}$$

According to 3.20, and by assumption $E[\hat{\bar{x}}_{k+1|k+1}] = E[\bar{x}_{k+1|k}]$, we have

$$\begin{aligned} I &= K'_{k+1} + K_{k+1}\bar{C}_{k+1} \\ K'_{k+1} &= I - K_{k+1}\bar{C}_{k+1} \end{aligned} \tag{3.20}$$

then the updating of the prediction can be arranged as

$$\begin{aligned} \hat{\bar{x}}_{k+1|k+1} &= (I - K_{k+1}\bar{C}_{k+1})\hat{\bar{x}}_{k+1|k} + K_{k+1}y_{k+1} \\ &= (I - K_{k+1}\bar{C}_{k+1})\hat{\bar{x}}_{k+1|k} + K_{k+1}\bar{C}_{k+1} \\ &\quad \hat{\bar{x}}_{k+1|k} - K_{k+1}v_{k+1} \end{aligned} \tag{3.21}$$

The principle unbiased prediction is represented in Appendix A. The a posteriori estimation error covariance is described by

$$\begin{aligned}
\Sigma_{k+1|k+1} &= E[e_{k+1|k+1}e^T_{k+1|k+1}] \\
&= E[[\bar{x}_{k+1} - \hat{\bar{x}}_{k+1|k+1}][\bar{x}_{k+1} - \hat{\bar{x}}_{k+1|k+1}]^T] \\
&= (I - K_{k+1}\bar{C}_{k+1})E[e_{k+1|k}e^T_{k+1|k}](I - K_{k+1}\bar{C}_{k+1})^T \\
&\quad +K_{k+1}E[v_{k+1|k}v^T_{k+1|k}]K'_{k+1} \\
&\quad +2(I - K_{k+1}\bar{C}_{k+1})E[e_{k+1|k}v^T_{k+1|k}]K'_{k+1} \\
&= (I - K_{k+1}\bar{C}_{k+1})\Sigma_{k+1|k}(I - K_{k+1}\bar{C}_{k+1})^T \\
&\quad +K_{k+1}S_vK'_{k+1} \qquad (3.22)
\end{aligned}$$

We can get the Kalman gain by differentiating the trace of the estimation error covariance matrix with respect to K.

$$\begin{aligned}
\mathcal{L} &= min_{K_{K+1}} tracE[\Sigma_{k+1|k+1}] \\
&= min_{K_{K+1}} tracE[(I - K_{k+1}\bar{C}_{k+1})\Sigma_{k+1|k}(I - K_{k+1}\bar{C}_{k+1})^T \\
&\quad +K_{k+1}S_vK'_{k+1}] \qquad (3.23)
\end{aligned}$$

and setting the differentiation of $\mathcal{L}$ equal to zero, we obtain

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{K+1}} &= -2(I - K_{k+1}\bar{C}_{k+1})\Sigma_{k+1|k}\bar{C}^T_{k+1} \\
&\quad +2K_{k+1}S_v = 0
\end{aligned}$$

Then, the Kalman gain matrix is

$$K_{K+1} = \Sigma_{k+1|k}\bar{C}_{k+1}^T[\bar{C}_{k+1}\Sigma_{k+1|k}\bar{C}_{k+1}^T + S_v]^{-1} \quad (3.24)$$

To minimize the error covariance, we firstly rearrange the error covariance equation (3.23) as

$$\begin{aligned}\Sigma_{k+1|k+1} &= (I - K_{k+1}\bar{C}_{k+1})\Sigma_{k+1|k}(I - K_{k+1}\bar{C}_{k+1})^T \\ &\quad + K_{k+1}S_vK'_{k+1} \\ &= \Sigma_{k+1|k} - K_{k+1}\bar{C}_{k+1}\Sigma_{k+1|k} - \Sigma_{k+1|k}\bar{C}_{k+1}^TK'_{k+1} \\ &\quad + K_{k+1}\bar{C}_{k+1}\Sigma_{k+1|k}\bar{C}_{k+1}^TK'_{k+1} + K_{k+1}S_vK'_{k+1} \end{aligned} \quad (3.25)$$

Substitute the Kalman gain matrix into the posteriori estimation error covariance (3.26), we deduce

$$\Sigma_{k+1|k+1} = \Sigma_{k+1|k+1} - K_{k+1}\bar{C}_{k+1}\Sigma_{k+1|k+1} \quad (3.26)$$

3.3.2 Controller Design

In this section, we investigate the optimal LQR control law by means of the quadratic performance index

$$J = E[\bar{x}_N^TF\bar{x}_N + \sum_{K=0}^{N-1}\bar{x}_k^TQ\bar{x}_k + u_k^TRu_k] \quad (3.27)$$

Applying Lemma 3.2.1 on the performance index function, we obtain

$$\begin{aligned} J &= E[\hat{\hat{x}}_N^T F \hat{\hat{x}}_N + \sum_{K=0}^{N-1} \hat{\hat{x}}_k^T Q \hat{\hat{x}}_k + u_k^T R u_k] \\ &\quad + \sum_{K=0}^{N-1} [tr(F\Sigma_k) + tr(Q\Sigma_k)] \\ &= \min_u E[\hat{\hat{x}}_N^T F \hat{\hat{x}}_N + \sum_{K=0}^{N-1} \hat{\hat{x}}_k^T Q \hat{\hat{x}}_k + u_k^T R u_k] \end{aligned} \tag{3.28}$$

Because of the trace terms are independent of the control input, so they will be canceled when the quadratic function is minimizing, and we have assuming

$$\hat{\hat{x}}_N = \hat{\hat{x}}_{k+1}$$

$$J = \min_u E[\hat{\hat{x}}_{k+1}^T F \hat{\hat{x}}_{k+1} + \sum_{K=0}^{N-1} \hat{\hat{x}}_k^T Q \hat{\hat{x}}_k + u_k^T R u_k] \tag{3.29}$$

We have

$$u_k^* = G_k \hat{\hat{x}}_k$$

$$\begin{aligned} J &= \min_u E[(\bar{A}_k \hat{\hat{x}}_k + \bar{B}_k u_k)^T F (\bar{A}_k \hat{\hat{x}}_k + \bar{B}_k u_k) \\ &\quad + \hat{\hat{x}}_k^T Q \hat{\hat{x}}_k + u_k^T R u_k] \\ &= \min_u E[(\hat{\hat{x}}_k^T \bar{A}_k^T F \bar{A}_k \hat{\hat{x}}_k + u_k^T \bar{B}_k^T F \bar{B}_k u_k \\ &\quad + 2\hat{\hat{x}}_k^T \bar{A}_k^T F \bar{B}_k u_k + \hat{\hat{x}}_k^T Q \hat{\hat{x}}_k + u_k^T R u_k] \end{aligned}$$

By differentiating with respect the control input gain and make it equal to zero, we can get the state feedback G_k as

$$0 = \bar{B}_k^T F \bar{B}_k u_k^* + \bar{B}_k^T F \bar{A}_k \hat{\bar{x}}_k + R u_k^*$$

$$u_k^* = -(R + \bar{B}_k^T F \bar{B}_k)^{-1} \bar{B}_k^T F \bar{A}_k \bar{x}_k$$

Then the state feedback gain is

$$G_k = -(R + \bar{B}_k^T F \bar{B}_k)^{-1} \bar{B}_k^T F \bar{A}_k \quad (3.30)$$

3.3.3 Finite and Infinite Horizon LQG control

The following theorems summarize the results for the finite and the infinite LQG optimal control problem over TCP protocol:

Theorem 3.3.1 *Consider the system in (3.4) and (3.28), and assuming that* $(\bar{A}, S_w^{\frac{1}{2}})$ *to be controllable,(* $\bar{A}$, $\bar{C}$ *) to be observable, and* $\bar{A}$ *to be unstable matrix. Then there exists a critical observation arrival probability* β_c *such that the expectation of estimator error covariance is bounded if and only if the observation arrival probability is greater than the critical arrival probability, i.e.* $E[\Sigma_{k|k}] \leq M \quad \forall k \ , \ \beta_c < \bar{\beta}$. *where* M *is a positive definite matrix possibly* *dependent on* P_0*, see [58]*

The proof is available in Appendix B. For infinite horizon, where $k \mapsto \infty$ as $N \mapsto \infty$, we assume the matrices $\bar{A}_k$, $\bar{B}_k$,$\bar{C}_k$ are time-invariant , and we deduce

$$
\begin{aligned}
\bar{A}_k &= \bar{A}_\infty &= \bar{A},\ \ \bar{B}_k = \bar{B}_\infty = \bar{B},\ \ \bar{C}_k = \bar{C}_\infty = \bar{C} \\
\bar{x}_k &= & \bar{x}_\infty = x,\ \ u_k = u_\infty = u,\ \ v_k = v_\infty = v \\
w_k &= w_\infty &= w,\ \ F_k = F_\infty = F,\ \ K_k = K \\
G_k &= & G_\infty = G,\ \ Q_N = Q_k = Q_\infty = Q,\ \ P_k = P_\infty = P \\
\Sigma_k &= & \Sigma_\infty = \Sigma
\end{aligned}
$$

In fact, there are difficulty to calculate the minimal of the objective function analytically without a limit and the exception of the estimation error covariance matrices $E[\Sigma_k]$. It is shown that the estimator gain does not converge to a steady state value, however it is tightly time-varying result since it is a function of a stochastic parameter β_k. It is well known that, the optimal LQG regulator stabilize the system usually in case packets lost. The system become unstable if the packets lost become below some certain threshold.

$$
\begin{aligned}
\bar{x}_{k+1} &= \bar{x}_k \\
&= \bar{x}_\infty = \bar{x}
\end{aligned}
$$

$$\begin{aligned} e_\infty &= \bar{x}_\infty - \hat{\bar{x}}_\infty \\ e &= \bar{x} - \hat{\bar{x}} \\ &= \bar{A}\hat{\bar{x}} + \bar{B}u + w - (\bar{A}\hat{\bar{x}} + \bar{B}u) \\ &= \bar{A}e + w \end{aligned} \tag{3.31}$$

$$\begin{aligned} \Sigma_\infty &= E[e_{k+1}e_{k+1}^T] \ = E[[\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}][\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}]^T] \\ &= [\bar{A}e + w][\bar{A}e + w]^T \\ &= \bar{A}\Sigma_\infty\bar{A} + S_w \end{aligned} \tag{3.32}$$

The observer form of the Kalman filter for the infinite horizon system Σ_∞ is given as

$$\hat{\bar{x}}_\infty = \bar{A}_\infty\hat{\bar{x}}_\infty + \bar{B}_\infty u_\infty + K_\infty[y_\infty - \bar{C}_\infty\hat{\bar{x}}_\infty] \tag{3.33}$$

$$\hat{\bar{x}} = \bar{A}\hat{\bar{x}} + \bar{B}u + K[y - \bar{C}\hat{\bar{x}}] \tag{3.34}$$

Rearranging the observer form we have

$$\hat{\bar{x}} = \bar{F}\hat{\bar{x}} + \bar{B}u + Ky \tag{3.35}$$

Where $\bar{F} = \bar{A} - K\bar{C}$. The updating prediction at infinite horizon gives as

$$\begin{aligned} E[\hat{\bar{x}}_\infty] &= E[K'_\infty \hat{\bar{x}}_\infty \\ &\quad + K_\infty \bar{C}_\infty \bar{x}_\infty + K_\infty v_\infty] \end{aligned}$$

$$\begin{aligned} \hat{\bar{x}}_\infty &= K'_\infty \hat{\bar{x}}_\infty \\ &\quad + K_\infty \bar{C}_\infty \bar{x}_\infty + K_\infty v_\infty \end{aligned}$$

where K is the kalman filter gain

$$\begin{aligned} \Sigma_\infty &= (I - K\bar{C})E[ee^T](I - K\bar{C})^T + KE[vv^T]K' \\ &\quad +2(I - K\bar{C})E[ev^T]K' \\ &= (I - K\bar{C})\Sigma_\infty(I - K\bar{C})^T + KS_vK' \end{aligned} \tag{3.36}$$

We can get the Kalman gain by differentiating the trace of the estimation error covariance matrix and make it equal to zero, we obtain:

$$\begin{aligned} \mathcal{L} &= \min_{K_\infty} tracE[\Sigma_\infty] \\ &= \min_{K_\infty} tracE[(I - K\bar{C})\Sigma_\infty(I - K\bar{C})^T + KS_vK'] \end{aligned} \tag{3.37}$$

$$\frac{\partial \mathcal{L}}{\partial K} = -2(I - K\bar{C})\Sigma_\infty \bar{C}^T + 2KS_v = 0 \tag{3.38}$$

Then, the Kalman gain matrix is

$$K \;\; = \;\; \Sigma_\infty \bar{C}^T[\bar{C}\Sigma_\infty \bar{C}^T + S_v]^{-1} \tag{3.39}$$

The quadratic performance index is

$$\begin{aligned} J_\infty &= \min_u E[\hat{\bar{x}}^T F \hat{\bar{x}} + \lim_{k\to\infty} \sum_{K=0}^{\infty} \bar{x}^T Q \bar{x} + u^T R u] \\ &= \min_u E[\hat{\bar{x}}^T_{K+1} F \hat{\bar{x}}_{k+1} \lim_{k\to\infty} \sum_{K=0}^{\infty} \hat{\bar{x}}^T Q \hat{\bar{x}} + u^T R u] \\ &\quad + \sum_{K=0}^{\infty} [tr(Q\Sigma_k)] \end{aligned} \tag{3.40}$$

Assuming

$$\hat{\bar{x}}_N \;\; = \;\; \hat{\bar{x}}_\infty = \hat{\bar{x}}$$

$$J_\infty \;\; = \;\; \min_u E[\hat{\bar{x}}^T F \hat{\bar{x}} + \lim_{k\to\infty} \sum_{K=0}^{\infty} \hat{\bar{x}}^T Q \hat{\bar{x}} + u^T R u] \tag{3.41}$$

The trace terms are independent of the control input, so they will be canceled when the quadratic function is minimizing, and we have

$$J \;\; = \;\; \min_u E[\hat{\bar{x}}^T F \hat{\bar{x}} + \sum_{K=0}^{\infty} \hat{\bar{x}}^T Q \hat{\bar{x}} + u^T R u] \tag{3.42}$$

Therefore,

$$
\begin{aligned}
J_\infty &= \lim_{N\to\infty} \min_u E[(\bar{A}_k\hat{\bar{x}} + \bar{B}_k u)^T F(\bar{A}_k\hat{\bar{x}} + \bar{B}_k u) \\
&\quad +\hat{\bar{x}}^T Q\hat{\bar{x}} + u^T R u] \\
&= \lim_{N\to\infty} \min_u E[(\hat{\bar{x}}_k^T \bar{A}_k^T F \bar{A}_k \hat{\bar{x}}_k + u_k^T \bar{B}_k^T F \bar{B}_k u_k \\
&\quad -2\hat{\bar{x}}_k^T \bar{A}_k^T F \bar{B}_k u_k + \hat{\bar{x}}_k^T Q \hat{\bar{x}}_k + u_k^T R u_k]
\end{aligned}
$$

By differentiating with respect the control input gain and make it equal to zero, we can get the state feedback gain G as

$$0 = \bar{B}_k^T F \bar{B}_k u^* + \bar{B}_k^T F \bar{A}_k \hat{\bar{x}} + R u^*$$

$$u_\infty^* = -(R + \bar{B}_k^T F_\infty \bar{B}_k)^{-1} \bar{B}_k^T F_\infty \bar{A}_k \bar{x}_\infty$$

$$u^* = -(R + \bar{B}_k^T F \bar{B}_k)^{-1} \bar{B}_k^T F \bar{A}_k \bar{x}$$

Then the state feedback gain is given as

$$G_\infty = -(R + \bar{B}_\infty^T F_\infty \bar{B}_k)^{-1} \bar{B}_\infty^T F_\infty \bar{A}_\infty \tag{3.43}$$

and

$$G = -(R + \bar{B}^T F \bar{B})^{-1} \bar{B}^T F \bar{A} \tag{3.44}$$

Theorem 3.3.2 *Studying the equations that are defined in (3.4) and (3.28) with the assumptions* $F_N = F_k = F$, $Q_N = Q_k = Q$, *and* $R_N = R_k = R$, *In addition, let* ($\bar{A}$,$\bar{B}$) *and* $(\bar{A}, S_w^{\frac{1}{2}})$ *be controllable, and* ($\bar{A}$,$\bar{C}$) $(\bar{A}, S_w^{\frac{1}{2}})$ *be observable. Besides,* $\beta_{max} < \bar{\beta}$ *and* $\alpha_c < \bar{\alpha}$ *Then we have the following.*

1. *The optimal controller gain at infinite horizon be a constant.*

$$\lim_{k\to\infty} G = -(R + \bar{B}^T F \bar{B})^{-1} \bar{B}^T F \bar{A} \tag{3.45}$$

2. *The optimal estimator gain* K_k, *is stochastic and time-varying result in it depends on the previous observation packets, that arrival sequence* β_j *when* $j = 1 \to k$

3. *The expected minimum cost can be bounded. This theorem is available at [41].*

The proof of this theorem can be found in Appendix B. In this section, equations the optimal LQG control over TCP-like protocol were derived in which the estimator and the controller is designed independently due to the fact that the separation principle is hold here using the standard Kalman filter. In the next section, we are going to derive the same results for UDP-like protocol.

3.4 LQG Optimal Control Over UDP Protocol

In this section, we investigate the LQG problem under UDP-like protocol. Figure 3.1. introduced a simply structure of NCS over UDP protocol. Hence, our goal is to design an optimal control policy that optimize the cost function that claim for the best estimation of error covariance. However, because of acknowledgment missing in UDP-like protocol and due to the quantization impacts into the networked control systems, there will be a dual effect between the estimator and the controller. So the separation principle does not hold here. In particular, the quantization impacts will not be considered. To show the difference between protocols with and without acknowledgments, we add a stochastic process σ in UDP-like case, and the expectation of this element $E[\sigma]$ define as $\hat{\sigma}$.

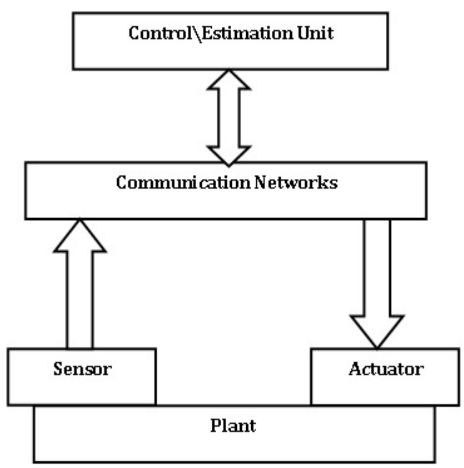

Figure 3.2: An NCS structure with UDP-like protocol

3.4.1 Estimator Design

We assume the discrete system as

$$\bar{x}_{k+1} = \bar{A}_k \bar{x}_k + \sigma \bar{B}_k u_k + \bar{w}_k \tag{3.46}$$

The Kalman filter estimates of the UDP discrete system is given by

$$E[\bar{x}_{k+1}] = \bar{A}_k E[\bar{x}_k] + \sigma \bar{B}_k E[u_k] + E[\bar{w}_k] \tag{3.47}$$

$$\hat{\bar{x}}_{k+1} = \bar{A}_k \hat{\bar{x}}_k + \bar{\sigma} \bar{B}_k u_k \tag{3.48}$$

The estimation error is written as

$$\begin{aligned} e_{k+1} &= \bar{x}_{k+1} - \hat{\bar{x}}_{k+1} \\ e_{k+1} &= \bar{A}_k \hat{\bar{x}}_k + \sigma \bar{B}_k u_k + w_k - (\bar{A}_k \hat{\bar{x}}_k \\ &\quad + \bar{\sigma} \bar{B}_k u_k) \end{aligned} \tag{3.49}$$

$$e_{k+1} = \bar{A}_k e_k + (\sigma - \bar{\sigma}) \bar{B}_k u_k + \bar{w}_k \tag{3.50}$$

the difference $(\sigma - \bar{\sigma})$ is available at UDP-like protocol due to the fact that the estimated value $E[\alpha_k] \neq \alpha_k$ and $E[\beta_k] \neq \beta_k$ result in the acknowledgment is not available here. Therefore, we assume a random variable σ at UDP-like protocol case. In addition, the standard Kalman filter is not used anymore in case of measurement loss and the absent of the acknowledgment. Hence, the estimation error covariance for UDP-like protocol is

$$\begin{aligned}\Sigma_{k+1} &= E[e_{k+1}e_{k+1}^T] \ = E[[\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}][\bar{x}_{k+1} - \hat{\bar{x}}_{k+1}]^T] \\ &= [\bar{A}_k e_k + (\sigma - \bar{\sigma})\bar{B}_k u_k + \bar{w}_k][\bar{A}_k e_k + (\sigma - \bar{\sigma})\bar{B}_k u_k + \bar{w}_k]^T \\ &= \bar{A}_k \Sigma_k \bar{A}_k^T + (1 - \bar{\sigma})\bar{\sigma}\bar{B}_k u_k u_k^T \bar{B}_k^T + S_w \end{aligned} \tag{3.51}$$

From the previous equation (3.51) and (3.52), It is found that the estimation error and its covariance depend on the input control u_k. As a result, there is a dual effect between the estimator and the controller over the UDP networks, and the separation principle is not valid anymore. However, we find that the correction step of the UDP-like protocol is the same as that for the TCP-like protocol.

$$\begin{aligned}\hat{\bar{x}}_{k+1|k+1} &= \hat{\bar{x}}_{k+1|k} + K_{k+1}(y_{k+1} - \bar{C}_{k+1}\hat{\bar{x}}_{k+1}) \\ e_{k+1|k+1} &= \bar{x}_{k+1|k+1} - \hat{\bar{x}}_{k+1|k+1}\end{aligned}$$

and

$$\Sigma_{k+1|k+1} = \Sigma_{k+1|k} - K_{k+1}\bar{C}_{k+1}\Sigma_{k+1|k}$$

The prediction updating is given as

$$\begin{aligned} E[\hat{\bar{x}}_{k+1|k+1}] &= E[K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}\bar{C}_{k+1}\bar{x}_{k+1|k} \\ &\quad +K_{k+1}v_{k+1|k}] \\ &= K'_{k+1}E[\hat{\bar{x}}_{k+1|k}] + K_{k+1}\bar{C}_{k+1}E[\bar{x}_{k+1|k}] \end{aligned}$$

when the prediction is unbiased we have:

$$\begin{aligned} \hat{\bar{x}}_{k+1|k+1} &= (I - K_{k+1}C_{k+1})\hat{\bar{x}}_{k+1|k} + K_{k+1}y_{k+1} \\ &= (I - K_{k+1}C_{k+1})\hat{\bar{x}}_{k+1|k} + K_{k+1}C_{k+1}\hat{\bar{x}}_{k+1|k} \\ &\quad -K_{k+1}v_{k+1} \end{aligned}$$

$$E[\hat{\bar{x}}_{k+1|k+1}] = (K'_{k+1} + K_{k+1}\bar{C}_{k+1})E[\bar{x}_{k+1|k}] \tag{3.52}$$

Then we will deduce that:

$$\begin{aligned} I &= K'_{k+1} + K_{k+1}\bar{C}_{k+1} \\ K'_{k+1} &= I - K_{k+1}\bar{C}_{k+1} \end{aligned} \tag{3.53}$$

where K_k is the Kalman filter gain. We update the estimation error covariance, and we get

$$\begin{aligned}
\Sigma_{k+1|k+1} &= E[e_{k+1|k+1}e^T_{k+1|k+1}] \\
&= E[[\bar{x}_{k+1} - \hat{\bar{x}}_{k+1|k+1}][\bar{x}_{k+1} - \hat{\bar{x}}_{k+1|k+1}]^T] \\
&= (I - K_{k+1}\bar{C}_{k+1})E[e_{k+1|k}e^T_{k+1|k}](I - K_{k+1}\bar{C}_{k+1})^T \\
&\quad +K_{k+1}E[v_{k+1|k}v^T_{k+1|k}]K'_{k+1} + 2(I - K_{k+1}\bar{C}_{k+1}) \\
&\quad E[e_{k+1|k}v^T_{k+1|k}]K'_{k+1}
\end{aligned}$$

Then, we deduce a difference Riccate equation (DRE) as

$$\begin{aligned}
\Sigma_{k+1|k+1} &= (I - K_{k+1}\bar{C}_{k+1})\Sigma_{k+1|k}(I - K_{k+1}\bar{C}_{k+1})^T \\
&\quad +K_{k+1}S_vK'_{k+1}
\end{aligned}$$

Following the same procedure of TCP-like protocol to derive the Kalman filter gain of UDP-like protocol, we have

$$K_{K+1} = \Sigma_{k+1|k}\bar{C}^T_{k+1}[\bar{C}_{k+1}\Sigma_{k+1|k}\bar{C}^T_{k+1} + S_v]^{-1} \tag{3.54}$$

While the control and the estimator could not be separated in UDP-like protocol, an LQG suboptimal control strategy will be used. We already have the plant and the Kalman equation are given respectively as:

$$\bar{x}_{k+1} = \bar{A}_k\bar{x}_k + \bar{B}_ku_k + \bar{w}_k$$

$$\hat{\hat{x}}_{k+1} = \bar{A}_k\hat{\hat{x}}_k + \bar{B}_k u_{esk} + K_{k+1}[y_{k+1} - \bar{C}_{k+1}\hat{\hat{x}}_{k+1}]$$

The error dynamic is written as

$$\begin{aligned} e_{k+1} &= \bar{x}_{k+1} - \hat{\hat{x}}_{k+1} \\ &= (\bar{A}_k - K_k\bar{C}_k)e_k + \bar{B}_k u_k - \bar{B}_k u_{esk} + w_k - K_k v_k \end{aligned}$$

where we assume that u_{esk} is the estimated input, and u_k is the actual input. The estimation error covariance of the error dynamic is

$$\begin{aligned} \Sigma_{k+1} &= E[e_{k+1|k}e^T_{k+1|k}] \\ &= (\bar{A}_k - K_k\bar{C}_k)\Sigma_k(\bar{A}_k - K_k\bar{C}_k)^T + \bar{B}_k u_k u_k^T \bar{B}_k^T \\ &\quad +\bar{B}_k u_{esk} u_{esk}^T \bar{B}_k^T - 2\bar{B}_k u_k u_{esk}^T \bar{B}_k^T \\ &\quad +S_w + K_k S_v K_k' \end{aligned}$$

Then, the estimation error covariance is minimized with respect to u_k and set the derivative equal to zero to get the control law. In addition, using trace operator properties on the the estimation error covariance, and eliminate the parts that is independent from the u_k, we deduce

$$tr[\bar{B}_k u_{esk} u_{esk}^T \bar{B}_k^T - 2\bar{B}_k u_k u_{esk}^T \bar{B}_k^T + \bar{B}_k u_k u_k^T \bar{B}_k^T]$$

Differentiating trace of the estimation error covariance with respect u_k , we obtain

$$0 = \bar{B}_k \bar{B}_k^T u_k + 0 - \bar{B}_k \bar{B}_k^T u_{esk}$$

Then the control law of the predictive estimation is

$$u_{esk}^* = u_k \tag{3.55}$$

For the case of packet missing on the communication links, there are two cases to be considered. Zero input strategy where we have

$$u_{esk}^* = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ 0; & \text{if } \alpha_k = 0. \end{cases}$$

and hold input strategy where

$$u_{esk}^* = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ u_{k-1}; & \text{if } \alpha_k = 0. \end{cases}$$

3.4.2 Controller Design

The quadratic performance index is given as

$$\begin{aligned} J &= \min_{u_{esk}} E[\bar{x}_N^T F \bar{x}_N + \sum_{K=0}^{N-1} \bar{x}_k^T Q \bar{x}_k + u_{esk}^T R u_{esk}] \\ &= \min_{u_{esk}} E[\hat{\bar{x}}_N^T F \hat{\bar{x}}_N + \sum_{K=0}^{N-1} \hat{\bar{x}}_k^T Q \hat{\bar{x}}_k + u_{esk}^T R u_{esk}] \\ &\quad + \sum_{K=0}^{N-1} [tr(F\Sigma_k) + tr(Q\Sigma_k)] \end{aligned} \tag{3.56}$$

$$J = \min_{u_{esk}} E[\hat{\bar{x}}_N^T F \hat{\bar{x}}_N + \sum_{K=0}^{N-1} \hat{\bar{x}}_k^T Q \hat{\bar{x}}_k + u_{esk}^T R u_{esk}] \tag{3.57}$$

The trace terms are independent of the control input, so they will be canceled when the quadratic function is minimized, and we have assuming

$$\hat{\bar{x}}_N = \hat{\bar{x}}_{k+1}$$

Then,

$$J = \min_{u_{esk}} E[\hat{\bar{x}}_{K+1}^T F \hat{\bar{x}}_{k+1} + \sum_{K=0}^{N-1} \hat{\bar{x}}_k^T Q \hat{\bar{x}}_k + u_{esk}^T R u_{esk}]$$

We have

$$u_{esk}^* = G_k \hat{\bar{x}}_k$$

The objective cost function is given by

$$\begin{aligned} J &= \min_u E[(\bar{A}_k\hat{\bar{x}}_k + \bar{B}_k u_{esk})^T F(\bar{A}_k\hat{\bar{x}}_k + \bar{B}_k u_{esk}) \\ &\quad +\hat{\bar{x}}_k^T Q\hat{\bar{x}}_k + u_{esk}^T R u_{esk}] \\ &= \min_u E[(\hat{\bar{x}}_k^T \bar{A}_k^T F\bar{A}_k\hat{\bar{x}}_k + u_{esk}^T \bar{B}_k^T F\bar{B}_k u_{esk} \\ &\quad +2\hat{\bar{x}}_k^T \bar{A}_k^T F\bar{B}_k u_{esk} + \hat{\bar{x}}_k^T Q\hat{\bar{x}}_k + u_{esk}^T R u_{esk}] \end{aligned}$$

By differentiating with respect to u_{esk} and make it equal to zero, we can get the state feedback G_k as

$$0 = \bar{B}_k^T F\bar{B}_k u_{esk}^* + \bar{B}_k^T F\bar{A}_k\hat{\bar{x}}_k + R u_{esk}^*$$

$$u_{esk}^* = -(R + \bar{B}_k^T F\bar{B}_k)^{-1}\bar{B}_k^T F\bar{A}_k\bar{x}_k \quad (3.58)$$

Then the state feedback gain is giving as

$$G_k = -(R + \bar{B}_k^T F\bar{B}_k)^{-1}\bar{B}_k^T F\bar{A}_k \quad (3.59)$$

For infinite time, we can replace k by ∞ in the estimation error covariance. A time-invariant controller could be found at a steady-state using equation (3.59).

3.5 Numerical Example

In this section, numerical examples and simulations are considered to illustrate the effectiveness of the proposed approaches and to verify the design method developed in terms of hold input strategy.

Example 3.5.1

Consider MIMO system represented by the following state space model:

$$A = \begin{bmatrix} 0.3679 & 0 & 0 & 0 \\ 0 & 0.3679 & 0 & 0 \\ 0.2864 & -0.1432 & 0.6065 & 0 \\ 0 & 0.0239 & 0 & 0.6065 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.6321 & 0 \\ 0 & 0.6321 \\ 0.1858 & -0.0929 \\ 0 & 0.0155 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We consider the state feedback $u = G\hat{\hat{x}}_k$, where the weighted matrices are chosen as

$$R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

and the positive definite matrix F is stated as

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

With the initial state $x_0 = [0, 0, 0, 0, 0]^T$. The fate of the control packets is chosen as α =0.45, while the measurements loss with probability β =0.25. Further, the time horizon is given as N =300. In TCP-like protocol, and for equations (3.18–3.31), the Kalman filter gain is

$$K_{TCP} = \begin{bmatrix} 11.4856 & -9.9471 \\ 11.4856 & -9.9471 \\ 13.2903 & -11.7519 \\ 9.8631 & -8.3247 \\ 5.8631 & -4.3247 \\ 5.8631 & -4.3247 \end{bmatrix}$$

the performance index is J_{TCP} =0.8136 , and state feedback gain matrix is given as

$$G_{TCP} = \begin{bmatrix} 0.7536 & -0.5832 & 0.3937 & -0.0198 & 0.6028 & -0.1257 \\ -0.6669 & 0.6616 & -0.3757 & 0.0231 & -0.5258 & 0.9627 \end{bmatrix}$$

In UDP-like protocol case, using equations (3.47–3.60), the Kalman filter gain is deduced as

$$K_{UDP} = \begin{bmatrix} 6.5734 & -4.7953 \\ 6.9555 & -5.2018 \\ 9.2986 & -7.5411 \\ 6.3577 & -4.6528 \\ 2.3577 & -0.6528 \\ 2.3577 & -0.6528 \end{bmatrix}$$

As result of our approach, the minimum of the cost function is $J_{UDP} = 0.2930$, and state feedback gain matrix is given as

$$G_{UDP} = \begin{bmatrix} 0.7536 & -0.5832 & 0.3937 & -0.0198 & 0.6028 & -0.1257 \\ -0.6669 & 0.6616 & -0.3757 & 0.0231 & -0.5258 & 0.9627 \end{bmatrix}$$

Figures 3.3– 3.8 show the comparison of the estimated states of LQG under UDP-like protocol and LQG under TCP-like protocol according to our approach designed in this chapter.

Example 3.5.2

In this example, we investigate single input single output SISO LTI system. To see the efficiency of our proposed approach in designing different LTI system

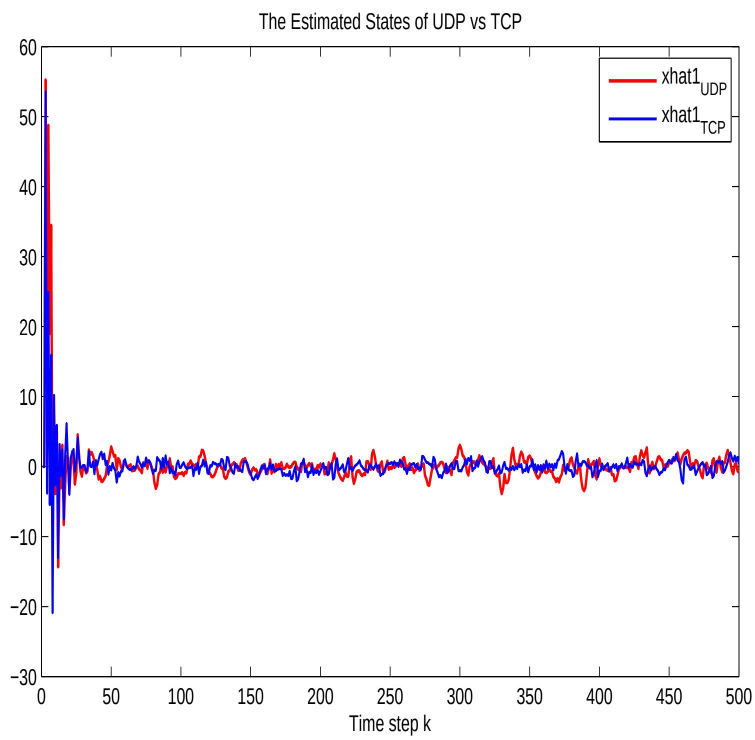

Figure 3.3: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_1$ for UDP and TCP.

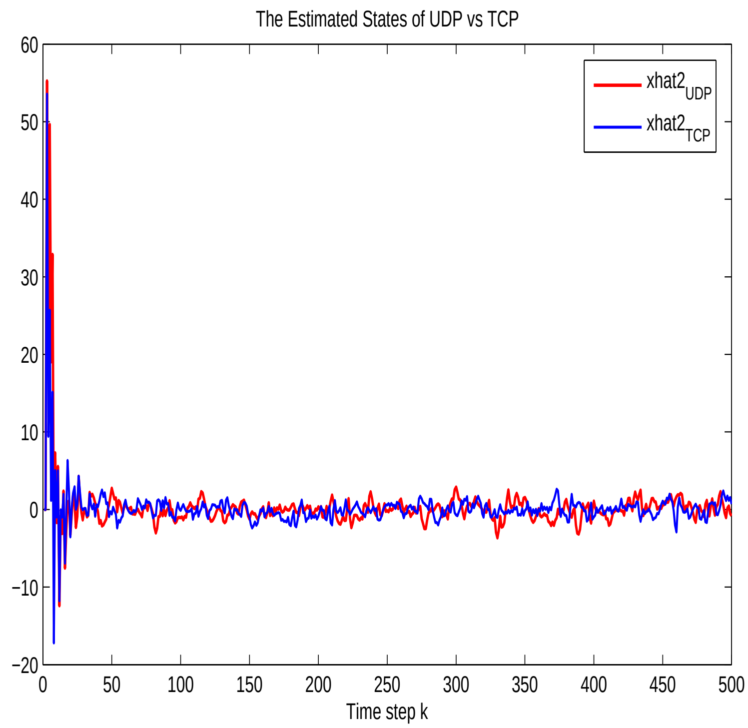

Figure 3.4: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_2$ for UDP and TCP.

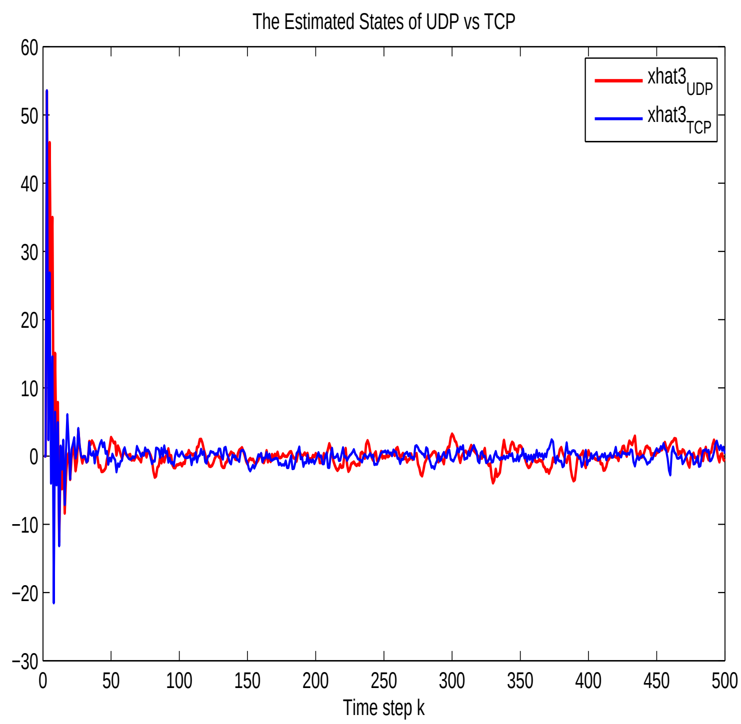

Figure 3.5: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_3$ for UDP and TCP.

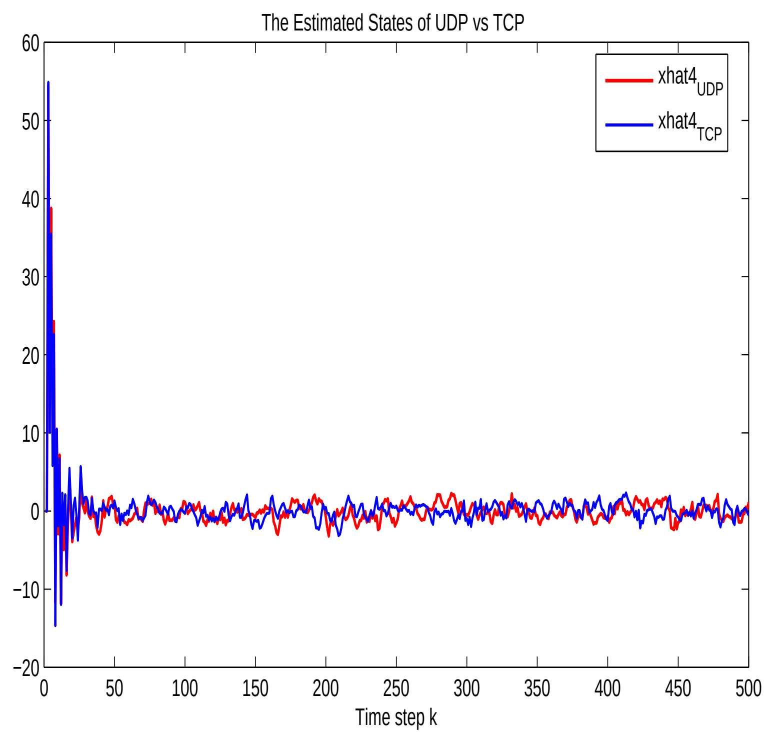

Figure 3.6: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_4$ for UDP and TCP.

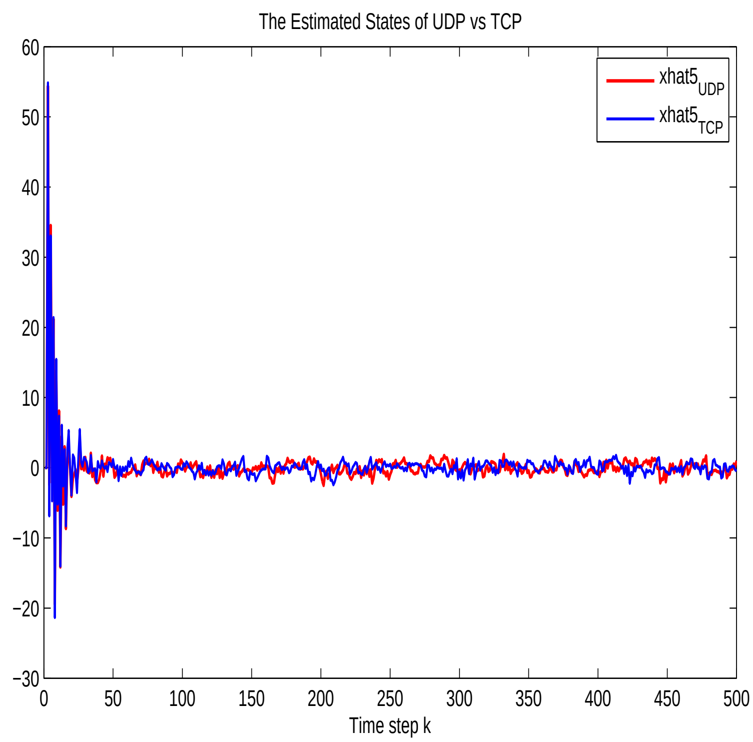

Figure 3.7: Comparison of the trajectories of the sixth estimated states $\hat{\hat{x}}_5$ for UDP and TCP.

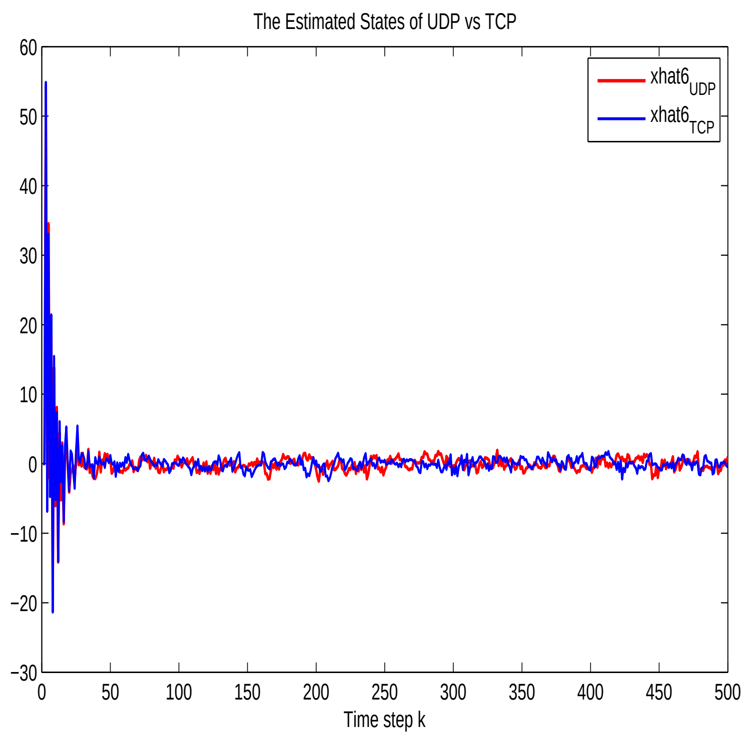

Figure 3.8: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_6$ for UDP and TCP.

structures. The SISO control system is described by

$$A = \begin{bmatrix} 0.66 & 0.209 \\ -0.123 & -0.5 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

The LQ state feedback control is $u = G\hat{\hat{x}}_k$, where the weighted matrices are given as $R = 1.00e^{-03}$ and

$$Q = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

and the positive definite matrix F is selected as

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With the initial state x_0 =[0,0]^T. Assuming that, the fate of the control packets is α =0.6, and the measurements dropped with probability β =0.75. Moreover the time horizon is chosen as N =800. In TCP-like protocol, we get the followings: The Kalman filter gain is

$$K_{TCP} = \begin{bmatrix} 1.1388 \\ 1.0641 \\ 1.1316 \end{bmatrix}$$

The performance index as J_{TCP} = 0.4865 , and state feedback gain matrix is given as

$$G_{TCP} = \begin{bmatrix} -0.5333 & -0.4829 & 0.8173 \end{bmatrix}$$

In UDP-like protocol case, the Kalman filter gain is given by

$$K_{UDP} = \begin{bmatrix} 1.1844 \\ 0.6653 \\ 1.1475 \end{bmatrix}$$

As result of our approach, the minimum of the cost function is J_{UDP} = 0.2911 , and state feedback gain matrix is given as

$$G_{UDP} = \begin{bmatrix} -0.5333 & -0.4829 & 0.8173 \end{bmatrix}$$

Figure 3.9–3.11. represent the estimated states of UDP-like and TCP-like protocols.

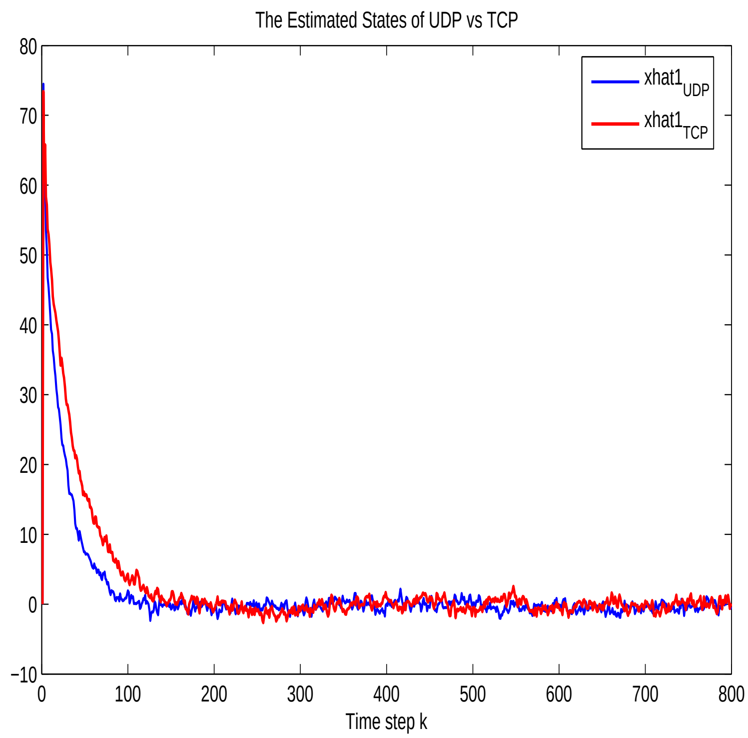

Figure 3.9: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_1$ for UDP and TCP

3.6 Conclusion

In this chapter, new results to solve the an LQG state feedback optimal control problem over network were provided. We extended the works of [1] and [10] to the case where observation noise is available. We study problems of estimation and control under two different protocols. In the TCP-like protocol, where the

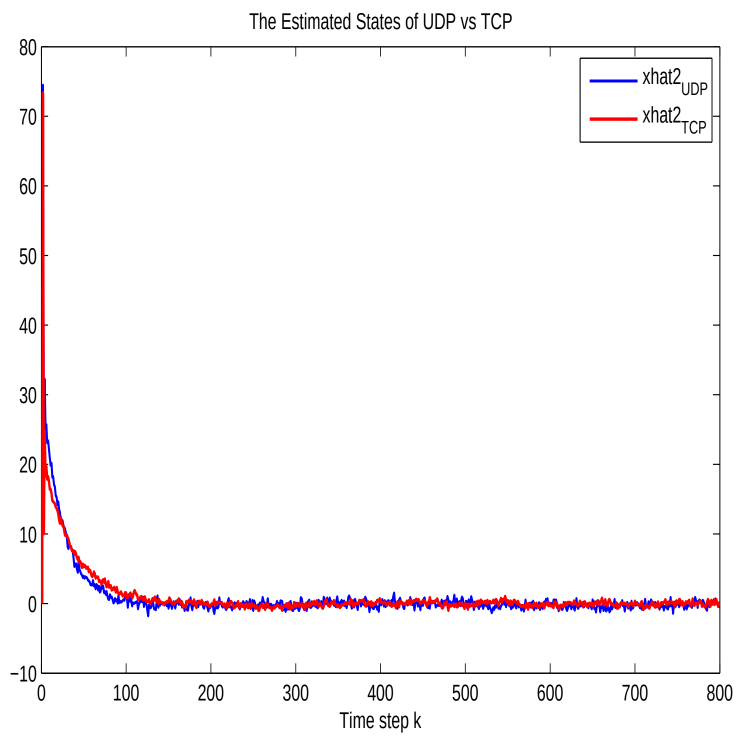

Figure 3.10: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_2$ for UDP and TCP

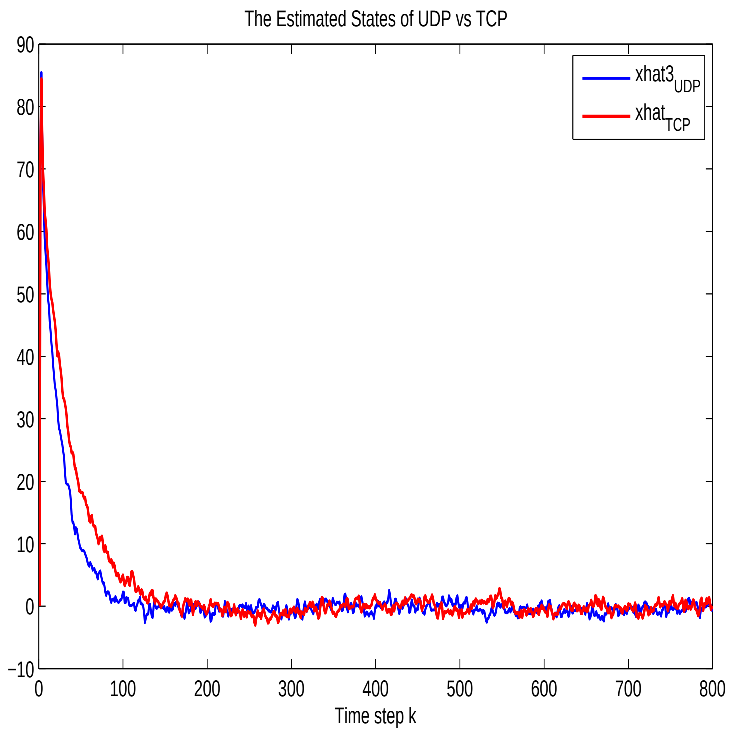

Figure 3.11: Comparison of the trajectories of the estimated states $\hat{\hat{x}}_3$ for UDP and TCP

estimation/ control unit provides acknowledgments when successfully the packet are delivered. On the other-hand, there is no acknowledgments are permitted in the UDP-like case. We addressed the nonlinearity for UDP protocol using suboptimal approach.

It is worth while to note that from our examples, the suboptimal controller design for UDP like protocol using zero input strategy gives better performance than the standard linear quadratic Gaussian approach in case of TCP-like protocol using hold input strategy. Moreover the steady state response of the suboptimal approach for UDP protocol convergence faster than standard LQG approach for TCP-like protocol as Fig. (3.3–3.11).

Chapter 4

OBSERVER BASED NETWORKED CONTROL SYSTEMS WITH NONSTATIONARY PACKET DROPOUT

4.1 Introduction

Due to the rapid evolution of communication and industrial technologies, many industrial control applications have been devoted more attention to networked control systems. It becomes increasingly apparent that stabilization problem of networked control systems has been addressed in the presence of networks constraints such as time delay and data lost as given in [25] and [59]. Moreover necessary and sufficient conditions have been studied for NCSs stability by applying state and output feedback [60]-[62].

It is well known that data can be sent across communication networks in packet form, this makes it possible in NCSs to cover packets time delay and packet lost by sending a sequence control predirection in one packet for routing control according the last network condition. However, transmission packet may be completely lost due to unreliable nature of the communication channels. As result, the packets transmitted from a sensor to a controller/estimator (S/C) and from the controller to an actuator (C/A) are randomly loss. In practice, these communication properties can affect the estimation and control performance. The effect of packet dropout is proposed by many authors, see [52] and [37].

This chapter investigates an estimation schemes in networked control systems (NCS). A class of observer based stabilizing controller of networked control systems (NCSs) is considered with nonstationary packet dropped out. Furthermore, the fate of control and observation signals are modeled by two mutually independent random variables. In particular, there are two compensation strategies that are used to compensate the control packet fates. Hold input strategy, at which the last control packet is used for compensation if the control packet get lost, while in zero input strategy zero control is applied when the control (actuation) packet failed to reach the actuator. It is shown that the observer based controller is designed to stabilize Networked systems in means of exponential stability, where a sufficient condition for stability is driven in terms of using linear matrix inequality (LMIs).

In this chapter, we present an extension the work of [63] by designing an observer based stabilizing control scheme for state and control estimation with nonstationary packet lost, where data loss occurs due to unreliable nature of the link, transmission errors and/or congestion or long time delays. We assume that time delay and quantization error are avoided. It is shown that the observer is figured out to stabilize NCS in the sense of exponential stability using linear matrix inequality (LMI). We formulate the design problem on hold-input and zero-input strategies [17].

4.2 Hold Input Strategy

In this section, we consider a networked control system with a packet drop over both directions. In addition, probabilities of the control input packet and the observation packets to get lost are modeled by two mutually independent random variables. The hold input strategy, where the last control packet is used for compensation if the control input packet is lost, will be applied. The plants we studied are discrete linear time invariant LTI systems given as following:

$$\begin{aligned} x_{k+1} &= Ax_k + \alpha_k B u_k + w_k; \quad k = 0, 1, 2.. \\ y_k &= \beta_k C x_k + w_k \\ g_k &= G x_k + w_k \end{aligned} \tag{4.1}$$

where $x_k \in \Re^n$ is the state vector, $y_k \in \Re^p$ is the measured output by the sensors, $u_k \in \Re^m$ is the control input that applied by the actuator. $w_k \in \Re^q$ is input disturbance in which $w_k \in \ell_2[0, \infty]$. The model matrices are, the dynamic matrix $A \in \Re^{n\times n}$, the control input matrix $B \in \Re^{n\times m}$, and the output observation matrix $C \in \Re^{p\times n}$. The controlled output is $g_k \in \Re^q$, which is the signal to be estimated. The unreliable of the links from sensor to the controller/estimator unit and from the later to the actuator is modeled by two i.i.d. Bernoulli processes β_k and α_k respectively. Where

$$P(\alpha_k) = \begin{cases} p_k; & \text{if } \alpha_k = 1; \\ 1 - p_k = \bar{p}_k; & \text{if } \alpha_k = 0. \end{cases}$$

we assume p_k discrete values, they take different values revolved the following figures, where $P\{p_k = q_k\} = r_k$. Usually, probability mass function that shows the probability for a discrete random variable, in which $q_r - q_{r-1} = constant$ and $r = 2, 3, ...n$. These include the case of uniform discrete distribution when we do not know probabilities of some p_k, where we have $r_i = \frac{1}{n}, i = 1, 2, 3, ..., n$, and the case when different probabilities of p_k take value adopted a symmetric triangle distribution. This is subsumed of two cases: Symmetric triangle distribution for n even, $r_i = a + jd, \quad j = 0, 1, 2, 3, .., n/2$, and $r_i = a + (n - j)d, \quad j = 0, 1, 2, ..., n/2 + 1, n/2 + 2, n, \quad$ also $na + dn(n - 1)/4 = 1$ and Probabilities of p_k follow a symmetric triangle distribution for n odd, $r_i = a + jd, \quad j = 0, 1, 2, 3, .., (n-1)/2$, along with $r_i = a+(n-j)d, \;\; j = 0, 1, 2, 3, ..., (n+1)/2, (n+2)/2, n, \quad$ also $na + dn(n - 1)^2/4 = 1$. In the cases when the different values of probabilities of p_k follow a decreasing linear function and $r_i = a - jd, \;\; j = 0, 1, 2, ..., n$, where $na - dn(n-1)/2 = 1$. In addition, the different values of probabilities of p_k may follow an increasing linear function, and $r_i = a - (n - j)d, \;\; j = 0, 1, 2, ..., n$, where $na - dn(n-1)/2 = 1$. Finally, likelihood of random value of p_k follow Binomial distribution, it is shown that $p_k = X/n, n > 0$ and $0 \leq X \leq n$ where $q > 0$, we have: $Prob(p_k = (ax + b)) = \binom{n}{x} q^x (1 - q)^{n-x}; \;\; if \;\; b > 0$ where $x = 0, 1, 2, ..., n, \quad an + b < n$. By considering that the measurement packets are prone to loss, where β_k and α_k are Bernoulli distributed white sequence independent of each other. We assume that

$$y_k = \begin{cases} Cx_k + w_k; & \text{if } \beta_k = 1; \\ w_k; & \text{if } \beta_k = 0. \end{cases}$$

where

$$P(\beta_k) = \begin{cases} m_k; & \text{if } \beta_k = 1; \\ 1 - m_k = \bar{m}_k; & \text{if } \beta_k = 0. \end{cases}$$

The observer is presented by

$$\hat{x}_{k+1} = A\hat{x}_k + \alpha_k B u_k + K(y_k - \hat{y}_k) \tag{4.2}$$

Assuming the estimated output $\hat{y}_k$ is described by

$$\hat{y}_k = \beta C \hat{x}_k \tag{4.3}$$

where $\hat{x}_k \in \Re^n$ is the state estimation vector of the system state, $\hat{y}_k \in \Re^p$ is the estimation output measurement and $K \in \Re^{n \times p}$ is the observer gain. When we got the state estimation and the actual state. We can define the estimation error between the states by $e_k = x_k - \hat{x}_k$ at time k, and $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$ at time $k+1$.

We rewrite this system according to the random values of β_k and α_k where the control input is described by:

$$u_k = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ u_{k-1}; & \text{if } \alpha_k = 0. \end{cases}$$

when $\alpha_k = 1$ we have

$$x_{k+1} = Ax_k + Bu_k + w_k$$

In case of $\alpha_k = 0$, the system is

$$x_{k+1} = Ax_k + Bu_{k-1} + w_k$$

Then, the discrete time LTI is identified as

$$x_{k+1} = \begin{cases} Ax_k + Bu_k + w_k; & \text{if } \alpha_k = 1; \\ Ax_k + Bu_{k-1} + w_k; & \text{if } \alpha_k = 0. \end{cases}$$

Applying the hold input scheme, the observer based is considered as following

$$\hat{x}_{k+1} = \begin{cases} A\hat{x}_k + Bu_k + K(y_k - \hat{y}_k); & \text{if } \alpha_k = 1; \\ A\hat{x}_k + Bu_{k-1} + K(y_k - \hat{y}_k); & \text{if } \alpha_k = 0. \end{cases}$$

The estimation error at time $k + 1$ comes as $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Then the estimation error when $\alpha_k = 0$ is given by

$$e_{k+1} = \begin{cases} (A - KC)e_k + w_k - Kw_k; & \text{if } \beta_k = 1; \\ Ae_k + w_k - Kw_k; & \text{if } \beta_k = 0. \end{cases}$$

We apply modified state feedback

$$u_k = \begin{cases} L\hat{x}_k; & \text{if } \alpha_k = 1; \\ L\hat{x}_{k-1}; & \text{if } \alpha_k = 0. \end{cases}$$

where L is the state feedback gain. Lets define a new augmented state $\xi_k = [x_k^T \; e_k^T]^T$. According to the fact that α_k and β_k are mutually independent, as result of that we have four Likelihoods or pairs as $\big($ $(\alpha_k = 1, \beta_k = 1)$; $(\alpha_k = 1, \beta_k = 0)$; $(\alpha_k = 0, \beta_k = 0)$; and $(\alpha_k = 0, \beta_k = 1)\big)$, which imply new subsystems. For the first case, when $\alpha_k = 1$ and $\beta_k = 1$.

$$A_1 = \begin{bmatrix} A+BL & -BL \\ 0 & A-KC \end{bmatrix}; \; B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

secondly, when $\alpha_k = 1$ and $\beta_k = 0$:

$$A_2 = \begin{bmatrix} A+BL & -BL \\ 0 & A \end{bmatrix}; \; B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

for the third case $\alpha_k = 0$ and $\beta_k = 1$:

$$A_3 = \begin{bmatrix} A & 0 \\ 0 & A - KC \end{bmatrix}; \; B_3 = \begin{bmatrix} BL & -BL \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

for the last pair $\alpha_k = 0$ and $\beta_k = 0$

$$A_4 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}; \; B_4 = \begin{bmatrix} BL & -BL \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

The new discrete model is considering as

$$\xi_{k+1} = \begin{bmatrix} x_k \\ e_k \end{bmatrix} = \mathbb{A}_j \xi_k + \mathbb{B}_j \xi_{k-1} + D \tag{4.4}$$

where $\mathbb{A}_j = \{A_i; i = 1, 2, 3, 4\}$, and $\mathbb{B}_j = \{B_i; i = 1, 2, 3, 4\}$

$$\begin{aligned} g_k &= \begin{bmatrix} G & 0 \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + w_k \\ &= \bar{G}\xi_k + w_k \end{aligned} \tag{4.5}$$

The goal of this work is to design an observer based stabilizing controller, in which the equation (4.2) with a compensation input scheme for control packets, such that the closed loop system is exponential stable. Basing on the switched time delay systems [74], we present

$\sigma_1 = Prob\{\alpha_k = 1, \beta_k = 1\}, \quad E[\sigma_1] = \hat{\sigma}_1$

$\sigma_2 = Prob\{\alpha_k = 0, \beta_k = 1\}, \quad E[\sigma_2] = \hat{\sigma}_2$

$\sigma_3 = Prob\{\alpha_k = 1, \beta_k = 0\}, \quad E[\sigma_3] = \hat{\sigma}_3$

$\sigma_4 = Prob\{\alpha_k = 0, \beta_k = 0\}, \quad E[\sigma_4] = \hat{\sigma}_4$

4.2.1 Main Results

In this section, we analyze the stability property for nonstationary packet dropout process, and closed loop problems are considered, where a necessary and a sufficient stability conditions for the closed loop system is derived using adopted packet dropout independent Lyapunov functions, see [75].

$$V(\xi_k) = \sum_{i=1}^{2} V_i(\xi_k) \tag{4.6}$$

$$V_1(\xi_k) \;=\; \sum_{j=1}^{4} \sigma_j \; \xi_k^T P_j \xi_k, \quad P_j = P_j^T > 0$$

$$V_2(\xi_k) \;=\; \sum_{j=1}^{4} \sum_{i=0}^{k-1} \sigma_j \; \xi_k^T Q_j \xi_k, \quad Q_j = Q_j^T > 0$$

It is not difficult to show that there exist real scalars $\mu > 0$ and $\nu > 0$ such that $\mu \left\|\xi\right\|^2 \leq V(\xi_k) \leq \nu \left\|\xi\right\|^2$

Remark 4.2.1:

It is proper to take a matrix P to be the same for all operational modes, hence independent of j, while keeping matrix Q_j dependent on model j. To show the system (4.4) is exponentially stable, we propose the following theorem

Theorem 4.2.1 *Assuming the controller and observer gain matrices K and L to be known. Then the closed loop system (4.4) is exponentially stable if there exist matrices* $0 < P = P^T$, *and* $0 < Q = Q^T$, $j = 1, .., 4$ *and matrices R, S, and M such that linear matrix inequality holds*

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & \Lambda_{2j} \\ \bullet & \Lambda_{3j} \end{bmatrix} < 0; \tag{4.7}$$

$$\Lambda_{1j} = \begin{bmatrix} \Psi_j + \Phi_{j1} & -R + S^T \\ \bullet & -S - S^T \end{bmatrix}$$

$$\Lambda_{2j} = \begin{bmatrix} -R + M^T + \Phi_{j2} & \Phi_{j3} \\ -S - M^T & 0 \end{bmatrix}$$

$$\Lambda_{3j} = \begin{bmatrix} -M - M^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & \Phi_{j6} \end{bmatrix} \tag{4.8}$$

where

$\Psi_j = -P + R + R^T + \hat{\sigma}_j Q_j;\ \Phi_{j1} = (A_j + B_j)^T P(A_j + B_j)$

$\Phi_{j2} = (A_j + B_j)^T P B_j;\ \Phi_{j3} = (A_j + B_j)^T P$

$\Phi_{j4} = B_j^T P B_j;\ \Phi_{j5} = B_j^T P\ ;\ \Phi_{j6} = -P$

by S-procedure, we assume

$\Lambda_{3j} = \Lambda_{3j} - \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$

Then,

$$\Lambda_{3j} = \begin{bmatrix} -M - M^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & -P - I \end{bmatrix} \tag{4.9}$$

proof 4.2.1 *Defining* $y_k = x_{k+1} - x_k$, *one has*

$$\xi_{k-1} = \xi_k - y_{k-1} \tag{4.10}$$

where

$\lambda_k = y_{k-1}$; $D = \omega_k$

then we have

$$\xi_k - \xi_{k-1} - \lambda_k = 0 \tag{4.11}$$

After that, the systems can be transmitted into:

$$\xi_{k+1} = (A_j + B_j)\xi_k - B_j\lambda_k + \omega_k \tag{4.12}$$

For some matrices R, S, and M, we deduce

$$2[\xi_k^T R + \xi_{k-1}^T S + \lambda_k^T M][\xi_k - \xi_{k-1} - \lambda_k] = 0 \tag{4.13}$$

Then,

$$\begin{aligned}\Delta V_1(\xi_k) &= V_1(\xi_{k+1}) - V_1(\xi_k)\\ &= \sum_{j=1}^{4}\sigma_j\big[\xi_k^T[\Phi_{j1}+\Psi_j]\xi_k - 2\xi_k^T R\xi_{k-1}\\ &\quad +2\xi_{k-1}^T S\xi_k - 2\xi_k^T\Phi_{j2}\lambda_k - 2\xi_k^T R\lambda_k\\ &\quad -2\xi_k^T\Phi_{j3}\omega_k - 2\xi_{k-1}^T S\xi_{k-1} + \lambda_k^T\Phi_{j4}\lambda_k\\ &\quad +2\lambda_k^T\Phi_{j5}\omega_k - 2\xi_{k-1}^T S\lambda_k + 2\lambda_k^T M\xi_k\\ &\quad -2\lambda_k^T M\xi_{k-1} - 2\lambda_k^T M\lambda_k + \omega_k^T\Phi_{j6}\omega_k\big]\end{aligned} \tag{4.14}$$

The difference of V_2 *is described by*

$$\begin{aligned}\Delta V_2(\xi_k) &= \sum_{j=1}^{4}\sigma_j\big[\sum_{i=1}^{k}\xi_k^T Q_j\xi_k - \sum_{i=0}^{k-1}\xi_k^T Q_j\xi_k\big]\\ &\leq \xi_k^T Q_j\xi_k - \xi_{k-1}^T Q_j\xi_{k-1}\\ &\quad +\sum_{i=0}^{k-1}\xi_i^T Q_j\xi_i\end{aligned} \tag{4.15}$$

$$\begin{aligned}\Delta V_2(\xi_k) &= \sum_{j=1}^{4}\sigma_j\big[\sum_{i=1}^{k}\xi_k^T Q_j\xi_k - \sum_{i=0}^{k-1}\xi_k^T Q_j\xi_k\big]\\ &\leq \xi_k^T Q_j\xi_k - \xi_{k-1}^T Q_j\xi_{k-1}\\ &\quad +\sum_{i=0}^{k-1^-}\xi_i^T Q_j\xi_i\end{aligned} \tag{4.16}$$

On combining equation from (4.6)-(4.15), we get

$$
\begin{aligned}
\Delta V(\xi_k) &= V(\xi_{k+1}) - V(\xi_k) \\
&\leq \sum_{j=1}^{4} \sigma_j \big[\xi_k^T[\Psi_j + \Phi_{j1}]\xi_k - 2\xi_K^T R\xi_{k-1} \\
&\quad +2\xi_{k-1}^T S\xi_k - 2\xi_k^T \Phi_{j2}\lambda_k - 2\xi_k^T R\lambda_k \\
&\quad -2\xi_k^T \Phi_{j3}\omega_k - 2\xi_{k-1}^T[S + \hat{\sigma}_j Q_j]\xi_{k-1} \\
&\quad +\lambda_k^T \Phi_{j4}\lambda_k + 2\lambda_k^T \Phi_{j5}\omega_k - 2\xi_{k-1}^T S\lambda_k \\
&\quad +2\lambda_k^T M\xi_k - 2\lambda_k^T M\xi_{k-1} - 2\lambda_k^T M\lambda_k \\
&\quad +\omega_k^T \Phi_{j6}\omega_k\big] \\
&= \sum_{j=1}^{4} \sigma_j [\Theta_k^T \widetilde{\Lambda}_j \Theta_k]
\end{aligned} \tag{4.17}
$$

where

$$
\Theta_k^T = \begin{bmatrix} \Theta_1 & \Theta_2 \end{bmatrix};
$$

$$
\Theta_1^T = \begin{bmatrix} \xi_k & \xi_{k-1} \end{bmatrix}; \;\; \Theta_2^T = \begin{bmatrix} \lambda_k & \omega_k \end{bmatrix} \tag{4.18}
$$

and $\widetilde{\Lambda}_j$*, by Schur complements, is similar to (4.7). If* $\Lambda_j < 0,\ j = 1, .., 4$ *holds* *then*

$$\begin{aligned} \Delta V(\xi_k) &= V(\xi_{k+1}) - V(\xi_k) \\ &= \sum_{j=1}^{4} \sigma_j [\xi_k^T \widetilde{\Lambda}_j \xi_k] \\ &\leq \sum_{j=1}^{4} \sigma_j [-\widetilde{\Lambda}_{min}(\widetilde{\Lambda}_j) \xi_k^T \xi_k] \\ &\quad - \sum_{j=1}^{4} \sigma_j [\eta_j \xi_k^T \xi_k] \end{aligned} \tag{4.19}$$

where $0 < \eta_j < min[\lambda_{min}(\widetilde{\Lambda}_j), max\{\lambda_{max}(P), \lambda_{max}(O_j)\}]$ *[63]. It clear that an inequality (4.18) indicates* $V(\xi_{k+1}) - V(\xi_k) < -\phi V(\xi_k)$, *where* ϕ *is bounded variable define as* $0 < \phi < 1$. *In manner of [64], we get*

$\|\xi_k\|^2 \leq \frac{\nu}{\kappa} \|\xi_0\|^2 (1-\phi)^k + \frac{\lambda}{\mu\phi}$ *consider the objective function*

$$J_k = \sum_{\kappa=0}^{k} (g_k^T g_k - \gamma^2 w_k^T w_k) \tag{4.20}$$

For $w_k \in \ell[0;\ \infty) \neq 0$, *with zero initial condition, we deuce*

$$\begin{aligned} J_k = \sum_{\kappa=0}^{k} (g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(x)|_1 - \Delta V(\xi_k)|_1) \\ \leq \sum_{\kappa=0}^{k} (g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(x)|_1) \end{aligned}$$

where $\Delta V(x)|_1$ *describes the difference of the Lyapunov functions along the anal-*

ysis of the system (4.1), we have

$$g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(\xi_k)|_1 = \sum_{j=1}^{4} \sigma_j [\xi_k^T \bar{\Lambda}_j \xi_k] \tag{4.21}$$

where $\bar{\Lambda}_j$ *corresponds to the* $\widetilde{\Lambda}_j$ *in (4.16) by Schur complements. It is shown that*

$$g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(\xi_k)|_1 < 0$$

Basing on $\kappa \in [0, k]$, *which denotes for any* $w_k \in \ell\ [0, \infty] \neq 0$ *that* $J < 0$ *result* $\|g_k\|_2 < \gamma \|w_k\|_2$

Therefore, it can be proved that the closed loop system (4.4) is exponential stable. This completes the proof.

It is shown that a solution of problem design of the observer based stabilizing controller has been provided by the following theorem:

Theorem 4.2.2 *The closed loop system (4.6) is exponentially stable if there exist matrices* $0 < X, Y_1, Y_2,\ 0 < \Xi_j,\ j = 1, .., 4$ *such that the following matrices holds for* $j = 1, .., 4$: *and also* Π_i, Υ_i *and* Γ_i *where* $i = 1, 2.$ *that satisfied the following:*

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j & \widehat{\Omega}_j & 0 & \widehat{X}\widehat{G}^T \\ \bullet & \widehat{\Lambda}_{3j} & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\widehat{X} & \widehat{X}\ \widehat{\Gamma}^T & 0 & 0 \\ \bullet & \bullet & \bullet & -\widehat{X} & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0; \tag{4.22}$$

$$\widehat{X} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \tag{4.23}$$

$$\widehat{\Psi}_j = -\widehat{X} + \Pi_1 + \Pi_1^T + \Xi_j$$

$$\widehat{\Lambda}_{1j} = \begin{bmatrix} \widehat{\Psi}_j & -\Pi_1 + \Upsilon_1 \\ \bullet & -\Upsilon_1 - \Upsilon_1^T \end{bmatrix}$$

$$\widehat{\Lambda}_{2j} = \begin{bmatrix} -\Pi_1 + \Gamma_1^T & 0 \\ -\Upsilon_1 - \Gamma_1^T & 0 \end{bmatrix}$$

$$\widehat{\Lambda}_{3j} = \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & 0 \end{bmatrix}$$

by S-procedure, we assume

$$\widehat{\Lambda}_{3j} = \widehat{\Lambda}_{3j} - \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

Then,

$$\widehat{\Lambda}_{3j} = \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & -I \end{bmatrix} \tag{4.24}$$

$$\widehat{\Omega}_j^T = \begin{bmatrix} \widehat{\Omega}_{1j} & 0 & 0 & -\widehat{\Omega}_{4j} & -\widehat{\Omega}_{5j} \end{bmatrix}$$

$$\widehat{\Omega}_{1j} = \begin{bmatrix} XA^T + Y_1^T B^T & 0 \\ XA^T & XA^T - Y_2^T \end{bmatrix}, \quad \forall j$$

$$\widehat{\Omega}_{4j} = \begin{bmatrix} Y_1^T B^T & 0 \\ 0 & 0 \end{bmatrix}; \quad j = 1, 4$$

$$\widehat{\Omega}_{5j} = \begin{bmatrix} 0 & 0 \\ 0 & -Y_2^T \end{bmatrix}; \quad j = 1, 2$$

$$\widehat{\Omega}_{4j} = 0; \quad j = 2, 3; \quad \widehat{\Omega}_{5j} = 0; \quad j = 3, 4 \tag{4.25}$$

where the controller gain matrix L and the observer gain matrix K are given by

$$L = Y_1 X^{-1}; \quad K = Y_2 X^{-1} C^{\dagger}$$

proof 4.2.2 *Let*

$$\Omega_j = \begin{bmatrix} (A_j + B_j) & 0 & -B_j & 0 \end{bmatrix}$$

we can display the inequality (4.7) as the following form

$$\Lambda_j = \widetilde{\Lambda} + \Omega_j P \Omega_j^T < 0 \tag{4.26}$$

$$\widehat{\Lambda}_j = \begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} \\ \bullet & \widehat{\Lambda}_{3j} \end{bmatrix} < 0; \tag{4.27}$$

$$\widetilde{\Lambda}_{1j} = \begin{bmatrix} \Psi_j & -R + S^T \\ \bullet & -S - S^T \end{bmatrix}$$

$$\widetilde{\Lambda}_{2j} = \begin{bmatrix} -R_{+}M^T & 0 \\ -S - M^T & 0 \end{bmatrix}$$

$$\widetilde{\Lambda}_{3j} = \begin{bmatrix} -M_{+}M^T & 0 \\ \bullet & -I \end{bmatrix}$$

Put $\widehat{X} = P^{-1}$, *using Schur complements, the matrix* Λ_j *in (4.24) can be represented as*

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j & \widehat{\Omega}_j & 0 & \widehat{X}\widehat{G}^T \\ \bullet & \widehat{\Lambda}_{3j} & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\widehat{X} & -\widehat{\Gamma}^T P & 0 & 0 \\ \bullet & \bullet & \bullet & -P & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0; \qquad (4.28)$$

Employing the congruence transformation

$$T_j = diag[\ \widehat{X}\ , \widehat{X}\ , \widehat{X},\ \widehat{X},\ I\ ,\ \widehat{X},\ I,\ I]$$

this apply for matrix inequality in (4.26) and manipulating using (4.22), so we deduce

$$\Xi_j = \widehat{X} Q_j \widehat{X}, \ \ \Pi_j = \widehat{X} R_j \widehat{X}, \ \ \Gamma_j = \widehat{X} M_j \widehat{X} \ \ \omega_j = \widehat{X} S_j \widehat{X}$$

Remark 4.2.2:

Setting $\widehat{X}$ as represent at (4.22) has benefit that the solution of bilinear matrix inequalities is converted that linear matrix inequalities. It is shown that the LMI (4.21) subject to dropout models that characterized by $E[\beta_k]$, $E[\alpha_k]$ and $E[\sigma_i]$, which are quite useful in illustrating different operating conditions of the communications network.

4.2.2 Numerical Examples

In this section, numerical examples and simulations are considered to illustrate the effectiveness of the proposed approaches in terms of previous input strategy. We assumed $\hat{\beta}_k = 0.1$ and $\hat{\alpha}_k = 0.1$. Our purpose is to design an observer-based stabilizing controller in the form of (4.2).

Example 4.2.1:

We study the uninterruptible power system UPS, in which we try to control the pulsewidth-modulated inverter to guarantee the output ac voltage is kept at the desired setting and undistorted, with the discrete-time model (1) as follows [63]:

$$A = \begin{bmatrix} 0.9226 & -0.633 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 23.737 & 20.287 & 0 \end{bmatrix}$$

we obtain the controller and the observer gain matrices as follows:

$$L = \begin{bmatrix} 0.0668 & -0.0001 & -0.0008 \end{bmatrix}; \quad and \quad \|L\| = 0.0668,$$

$$K = \begin{bmatrix} 0.0064 & 0.0041 & 0 \end{bmatrix}^T; \quad and \quad \|K\| = 0.0076,$$

the simulation results of the states and the estimated states responses are given in Fig. 4.1 and Fig. 4.2 respectability, using the proposed the control algorithm. It is shown from the above figures that the performance of the control and estimation systems are regarded with nonstationary packet loss. The Bernoulli distribution of the random values α_k and β_k are described in Figs. 4.3 –4.5

Example 4.2.2:

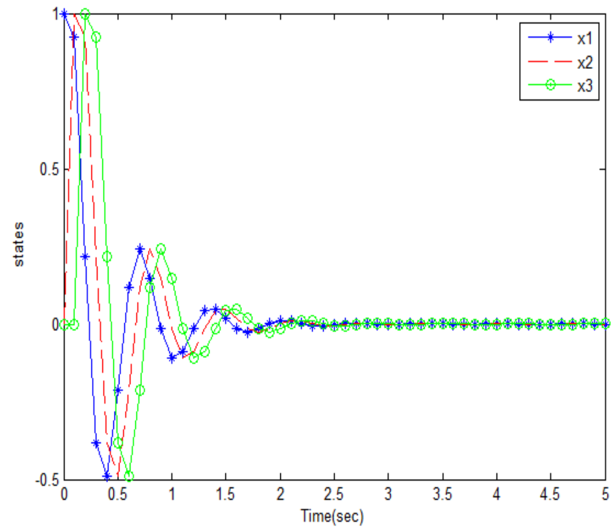

Figure 4.1: Trajectories of system states for nonstationary packet loss using hold input scheme

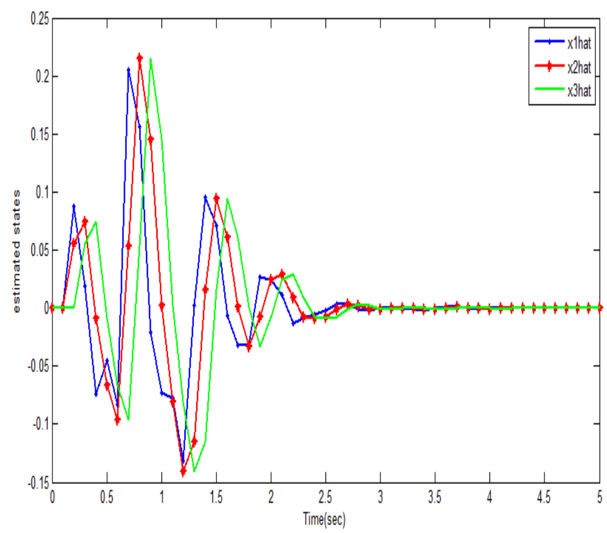

Figure 4.2: Trajectories of the estimated states for nonstationary packet loss using hold input scheme

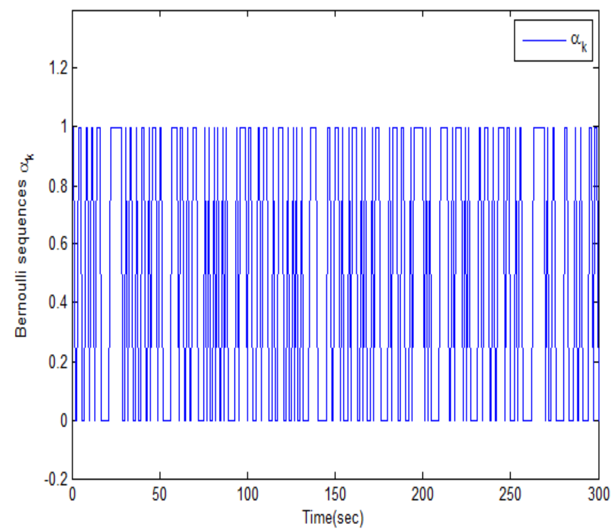

Figure 4.3: Bernoulli sequences α_k

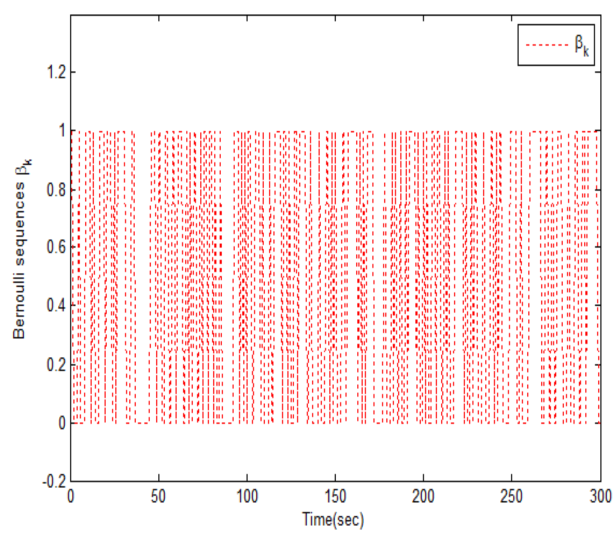

Figure 4.4: Bernoulli sequences β_k

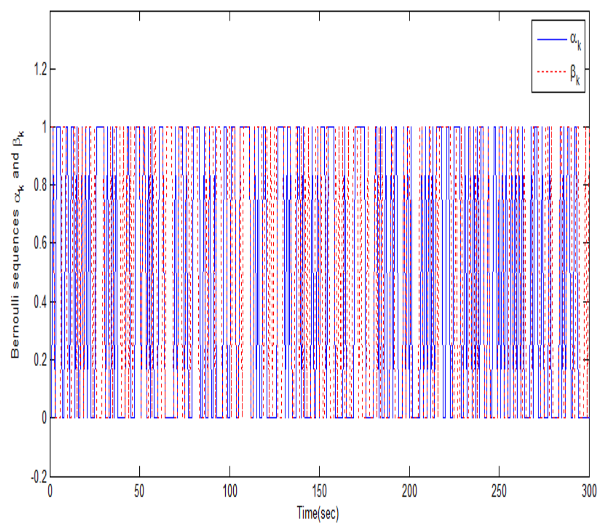

Figure 4.5: Bernoulli sequences α_k and β_k

We consider the following system with the plant parameters

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.0625 & -0.5 & 1.25 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0.0625 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -7 & 5 \end{bmatrix}$$

the eigenvalues of the plant were $[0.25, 0.5 - 0i, 0.5 + 0i]$. At each instant k, the sensor send an output vector y_k to the controller/estimator unit, then the input vector u_k was sent to actuator through networked control systems.

we obtain the state feedback controller and observer gain matrices as follows:

$$L = \begin{bmatrix} -0.0001 & 0.0683 & 0.0055 \end{bmatrix}; \ \ and \ \ \|L\| = 0.0686,$$

$$K = \begin{bmatrix} 0.0227 & -0.0053 & -0.0015 \end{bmatrix}^T; \ \ and \ \ \|K\| = 0.0233,$$

the simulation results of the states and the estimated states responses are given

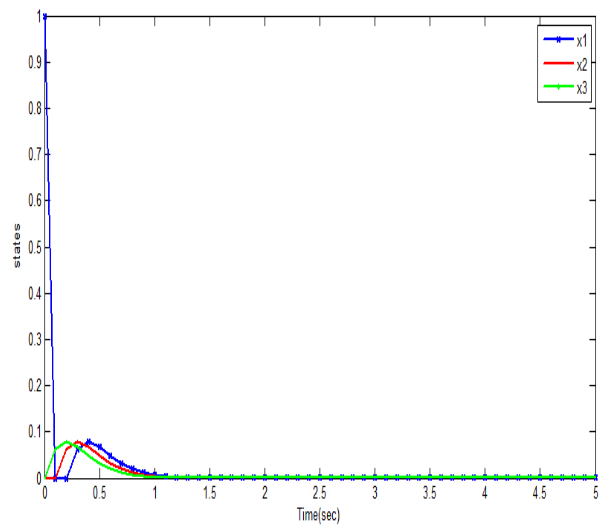

Figure 4.6: States trajectories under nonstationary random dropout using hold input scheme

in Fig. 4.6 and Fig. 4.7 respectability, using the proposed the control algorithm. It is shown from the above figures that the performance of the control and estimation systems are regarded with nonstationary packet loss.

Example 4.2.3:

In this example, we investigate a second order linear system. The state space

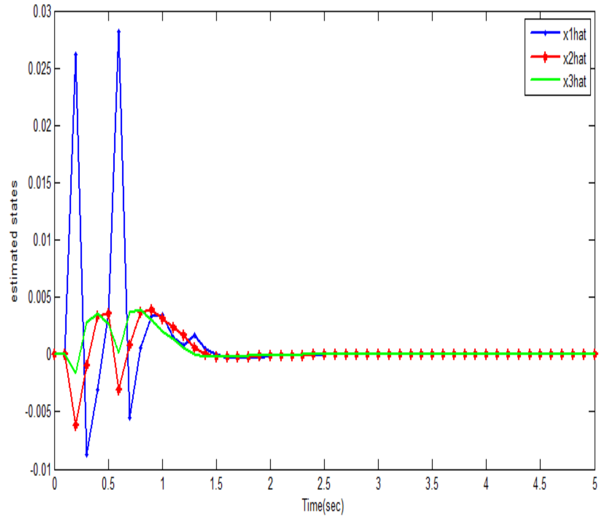

Figure 4.7: Estimated states trajectories under nonstationary random dropout using hold input scheme

model is described by

$$A = \begin{bmatrix} 0.66 & 0.209 \\ -0.123 & -0.5 \end{bmatrix}; \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

we obtain the state feedback controller and observer gain matrices as follows:

$$L = \begin{bmatrix} 0.1154 & 0.0824 \end{bmatrix}; \quad and \quad \|L\| = 0.1418,$$

$$K = \begin{bmatrix} 0 & -0.0237 \end{bmatrix}^T; and \quad \|K\| = 0.0237,$$

the real and estimated states trajectories are explained in Fig. 4.8-4.9 respectability. It is shown that the performance of the control and estimation systems are suffered when the measurements and control packet are sending over NCSs with nonstationary packet dropout.

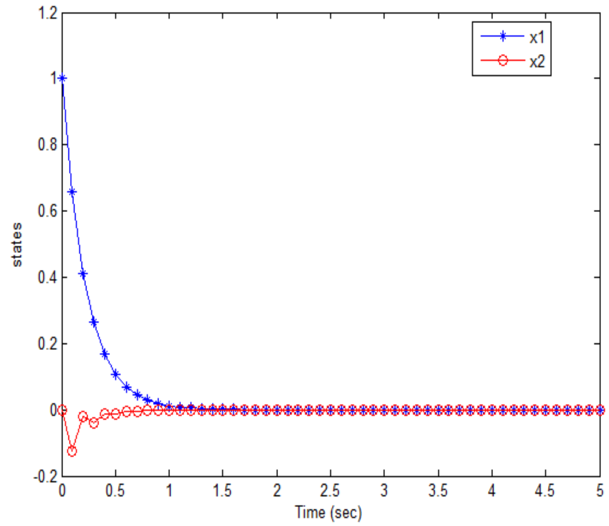

Figure 4.8: Trajectories of system states for nonstationary packet dropout using hold input scheme

4.3 Zero Input Strategy

This section investigate an NCS with nonstationary packet loss in both direction from the sensor to the estimation/control unit and between the latter and the actuator, in which the lost of the control packets are compensated by zero input

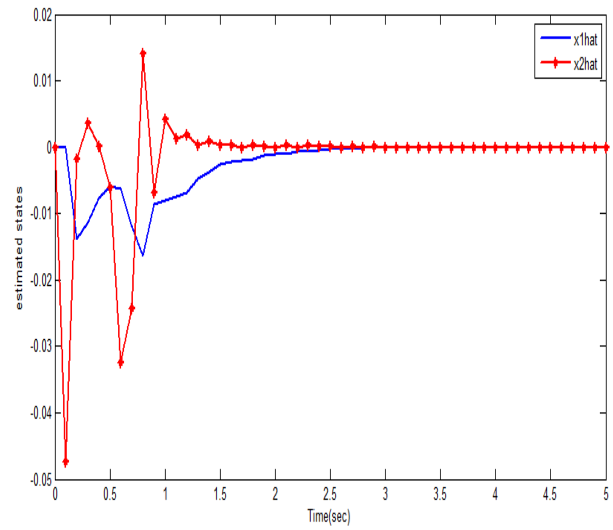

Figure 4.9: Trajectories of the estimated states for nonstationary packet dropout using hold input scheme

control packets. The discrete plant of NCS is described by

$$\begin{aligned} x_{k+1} &= Ax_k + \alpha_k B u_k + w_k; \quad k = 0, 1, 2.. \\ y_k &= \beta_k C x_k + w_k \\ g_k &= G x_k + w_k \end{aligned} \tag{4.29}$$

where $x_k \in \Re^n$ is the state vector, $y_k \in \Re^p$ is the measured output by the sensors, $u_k \in \Re^m$ is the control input that applied by the actuator. $w_k \in \Re^q$ is input disturbance in which $w_k \in \ell_2[0, \infty]$. The model matrices are, the dynamic matrix $A \in \Re^{n \times n}$, the control input matrix $B \in \Re^{n \times m}$, and the output observation matrix $C \in \Re^{p \times n}$. The controlled output is $g_k \in \Re^q$, which is the signal to be estimated. The unreliable of the links from sensor to the controller/estimator unit and from the later to the actuator is modeled by two i.i.d.

Bernoulli processes β_k and α_k respectively. We assume that, the control packet can get loss with random probability, which satisfies Bernoulli distribution as

$$Prob(\alpha_k) = \begin{cases} p_k; & \text{if } \alpha_k = 1; \\ 1 - p_k; & \text{if } \alpha_k = 0. \end{cases}$$

p_k take discrete values, they take different values revolved two classes, where $P\{p_k = q_k\} = r_k$.

1. **Class 1:** p_k has a probability mass function that shows the probability for a discrete random variable, in which $q_r - q_{r-1} = constant$ and $r = 2,3, ...n$. This class covering Uniform discrete distribution, symmetric triangle distribution, increasing linear function, and decreasing linear function.

2. **Class 2:** it is shown that the random values p_k revolve as $p_k = X/n$, $n > 0$ and $0 \leq X \leq n$ where $q > 0$. This follows a Binomial distribution.

In this section, we apple zero input procedure when any packet got lost:

$$u_k = \begin{cases} u_k; & \text{if } \alpha_k = 1; \\ 0; & \text{if } \alpha_k = 0. \end{cases}$$

then, the discrete time LTI is identified as

$$x_{k+1} = \begin{cases} Ax_k + Bu_k + w_k; & \text{if } \alpha_k = 1; \\ Ax_k + w_k; & \text{if } \alpha_k = 0. \end{cases}$$

It is shown that the measurement packets are prone to loss, where β_k and α_k are Bernoulli distributed white sequence independent of each other. We assume that the probability of measurements to get lost

$$Prob(\beta_k) = \begin{cases} m_k; & \text{if } \beta_k = 1; \\ 1 - m_k; & \text{if } \beta_k = 0. \end{cases}$$

then

$$y_k = \begin{cases} Cx_k + w_k; & \text{if } \beta_k = 1; \\ w_k; & \text{if } \beta_k = 0. \end{cases}$$

When some of states are missing with sensing and actuation data might be lost due to unreliable nature of the links, the observer is design for this desire is gives by

$$\hat{x}_{k+1} = A\hat{x}_k + \hat{\alpha}_k Bu_k + K(y_k - \hat{y}_k) \tag{4.30}$$

and the estimation of the measurements equation specific as

$$\hat{y}_k = \hat{\beta}_k C\hat{x}_k \tag{4.31}$$

where $\hat{x}_k \in \Re^n$ is the state estimation vector of the system state, $\hat{y}_k \in \Re^p$ is the estimation output measurement, and $K \in \Re^{n\times p}$ is the observer gain. When we got the state estimation and the actual state, we can define the estimation error between them by $e_k = x_k - \hat{x}_k$ at time k, and $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$ at time $k+1$.

and the observer based is considered as following for zero input method

$$\hat{x}_{k+1} = \begin{cases} A\hat{x}_k + Bu_k + K(y_k - \hat{y}_k); & \text{if } \alpha_k = 1; \\ A\hat{x}_k + K(y_k - \hat{y}_k); & \text{if } \alpha_k = 0. \end{cases}$$

and the estimation error at time $k+1$ comes as $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$, we figure out that the estimation error when $\alpha_k = 0$ gives as:

$$e_{k+1} = \begin{cases} (A - KC)e_k + w_k - Kw_k; & \text{if } \beta_k = 1; \\ Ae_k + w_k - Kw_k; & \text{if } \beta_k = 0. \end{cases}$$

the state feedback in case of using zero input scheme specifies as

$$u_k = L\hat{x}_k \tag{4.32}$$

when zero input strategy is applied

$$u_k = \begin{cases} L\hat{x}_k; & \text{if } \alpha_k = 1; \\ 0; & \text{if } \alpha_k = 0. \end{cases}$$

for the first case, when $\alpha_k = 1$ and $\beta_k = 1$,

$$A_1 = \begin{bmatrix} A + BL & -BL \\ 0 & A - KC \end{bmatrix};$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

secondly, when $\alpha_k = 1$ and $\beta_k = 0$:

$$A_2 = \begin{bmatrix} A + BL & -BL \\ 0 & A \end{bmatrix};$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

for the third case $\alpha_k = 0$ and $\beta_k = 1$:

$$A_3 = \begin{bmatrix} A & 0 \\ 0 & A - KC \end{bmatrix};$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

for the last pair $\alpha_k = 0$ and $\beta_k = 0$

$$A_4 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix};$$

$$D = \begin{bmatrix} w_k & 0 \\ 0 & w_k - Kw_k \end{bmatrix}$$

the new discrete model is considering as:

$$\xi_{k+1} = \begin{bmatrix} x_k \\ e_k \end{bmatrix} = \mathbb{A}_j \xi_k + D \tag{4.33}$$

where $\mathbb{A}_j = \{A_i; i = 1, 2, 3, 4\}$.

$$\begin{aligned} g_k &= \begin{bmatrix} G & 0 \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix} + w_k \\ &= \bar{G}\xi_k + w_k \end{aligned} \tag{4.34}$$

The aim of this paper is design an observer based stabilizing controller corresponding to equations (4.28) and (4.30) such that the closed loop system (4.31) is exponential stable in sense of mean square see [74], we present

$\sigma_1 = Prob\{\alpha_k = 1, \beta_k = 1\}, \quad E[\sigma_1] = \hat{\sigma}_1$

$\sigma_2 = Prob\{\alpha_k = 0, \beta_k = 1\}, \quad E[\sigma_2] = \hat{\sigma}_2$

$\sigma_3 = Prob\{\alpha_k = 1, \beta_k = 0\}, \quad E[\sigma_3] = \hat{\sigma}_3$

$\sigma_4 = Prob\{\alpha_k = 0, \beta_k = 0\}, \quad E[\sigma_4] = \hat{\sigma}_4$

4.3.1 Main Results

In this section, we analyze the stability property for nonstationary packet dropout process, and closed loop problems are considered, where a necessary and a sufficient stability conditions for the closed loop system is derived using adopted packet dropout independent Lyapunov functions, see [75].

$$V(\xi_k) = \sum_{j=1}^{4} \sigma_j \, \xi_k^T P_j \xi_k, \quad P_j = P_j^T > 0 \tag{4.35}$$

I t is not difficult to show that there exist real scalars $\mu > 0$ and $\nu > 0$ such that

$\mu \left\|\xi\right\|^2 \leq V(\xi_k) \leq \nu \left\|\xi\right\|^2$, see lemma 3.1 in [63].

Theorem 4.3.1 *Assuming, we know the observer and the controller gains, the closed loop system (4.31) is exponential stable if there exist matrix* $0 < P = P^T$, *and matrices R, S, and M such that the following matrix inequality holds.*

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & \Lambda_{2j} \\ \bullet & \Lambda_{3j} \end{bmatrix} < 0; \tag{4.36}$$

$$\Lambda_{1j} = \begin{bmatrix} \Psi_j + \Phi_{j1} & -R + S^T \\ \bullet & -S - S^T \end{bmatrix}$$

$$\Lambda_{2j} = \begin{bmatrix} -R + M^T + \Phi_{j2} & \Phi_{j3} \\ -S - M^T & 0 \end{bmatrix};$$

$$\Lambda_{3j} = \begin{bmatrix} -M + M^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & \Phi_{j6} \end{bmatrix} \tag{4.37}$$

where $\Psi_j = -P + R + R^T$; $\Phi_{j1} = A_j^T P A_j$

$\Phi_{j2} = A_j^T P$; $\Phi_{j3} = P$; $\Phi_{j4} = \Phi_{j5} = \Phi_{j6} = 0$

by S-procedure, we assume

$$\Lambda_{3j} = \Lambda_{3j} - \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

Then,

$$\Lambda_{3j} = \begin{bmatrix} -M + M^T & 0 \\ \bullet & -I \end{bmatrix} \tag{4.38}$$

proof 4.3.1 *Defining* $\lambda_k = y_{k-1}$; $D = \omega_k$ *then we find that*

$$\xi_k - \xi_{k-1} - \lambda_k = 0 \tag{4.39}$$

For the matrices R, S, and M

$$2[\xi_k^T R + \xi_{k-1}^T S + \lambda_k^T M][\xi_k - \xi_{k-1} - \lambda_k] = 0 \tag{4.40}$$

we have that

$$\begin{aligned}
\Delta V(\xi_k) &= V(\xi_{k+1}) - V(\xi_k) \\
&= \sum_{j=1}^{4} \sigma_j \big[\xi_k^T [\Phi_{j1} + \Psi_j] \xi_k - 2\xi_K^T R \xi_{k-1} \\
&\quad +2\xi_{k-1}^T S \xi_k + 2\xi_k^T \Phi_{j2} \omega_k - 2\xi_{k-1}^T S \lambda_k \\
&\quad +\omega_k^T \Phi_{j3} \omega_k - 2\xi_{k-1}^T S \lambda_k + 2\lambda_k^T M \xi_k - 2\xi_K^T R \lambda_k \\
&\quad -2\lambda_k^T M \xi_{k-1} - 2\xi_{k-1}^T S \xi_{k-1} - 2\lambda_k^T M \lambda_k \big] \\
&= \sum_{j=1}^{4} \sigma_j [\Theta_k^T \widetilde{\Lambda}_j \Theta_k] \qquad (4.41)
\end{aligned}$$

where

$$\Theta_k^T = \begin{bmatrix} \Theta_1 & \Theta_2 \end{bmatrix};$$

$$\Theta_1^T = \begin{bmatrix} \xi_k & \xi_{k-1} \end{bmatrix}; \;\; \Theta_2^T = \begin{bmatrix} \lambda_k & \omega_k \end{bmatrix} \qquad (4.42)$$

and $\widetilde{\Lambda}_j$ *corresponds to* Λ_j *by Schur complements. If* $\Lambda_j < 0, j = 1, .., 4$ *holds* *then*

$$\begin{aligned}\Delta V(\xi_k) &= V(\xi_{k+1}) - V(\xi_k)\\ &= \sum_{j=1}^{4}\sigma_j[\xi_k^T\widetilde{\Lambda}_j\xi_k]\\ &\leq \sum_{j=1}^{4}\sigma_j[-\widetilde{\Lambda}_{min}(\widetilde{\Lambda}_j)\xi_k^T\xi_k]\\ &\quad -\sum_{j=1}^{4}\sigma_j[\eta_j\xi_k^T\xi_k] \qquad (4.43)\end{aligned}$$

where

$0 < \eta_j < min[\lambda_{min}(\widetilde{\Lambda}_j), max\{\lambda_{max}(P), \lambda_{max}(O_j)\}]$

Inequality (4.40) implies that $V(\xi_{k+1}) - V(\xi_k) < -\phi V(\xi_k,$

where $0 < \phi < 1$. *In manner of [64], we get*

$\|\xi_k\|^2 \leq \frac{\nu}{\kappa}\|\xi_0\|^2(1-\phi)^k + \frac{\lambda}{\mu\phi}$

consider the objective function

$$J_k = \sum_{\kappa=0}^{k}(g_k^T g_k - \gamma^2 w_k^T w_k) \qquad (4.44)$$

For $w_k \in \ell[0;\ \infty) \neq 0$, *with zero initial condition, we deuce*

$$J_k = \sum_{\kappa=0}^{k}(g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(x)|_1 - \Delta V(\xi_k)|_1)$$
$$\leq \sum_{\kappa=0}^{k}(g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(x)|_1)$$

where $\Delta V(x)|_1$ *describes the difference of the Lyapunov functions along the analysis of the system (4.27), we have*

$$g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(\xi_k)|_1 = \sum_{j=1}^{4} \sigma_j[\xi_k^T \bar{\Lambda}_j \xi_k] \tag{4.45}$$

where $\bar{\Lambda}_j$ *corresponds to the* $\widetilde{\Lambda}_j$ *in (4.38) by Schur complements. It is shown that*

$$g_k^T g_k - \gamma^2 w_k^T w_k + \Delta V(\xi_k)|_1 < 0$$

Basing on $\kappa \in [0, k]$, *which denotes for any* $w_k \in \ell\ [0, \infty] \neq 0$ *that* $J < 0$ *result* $\|g_k\|_2 < \gamma \|w_k\|_2$ *Therefore, it can be verified that the closed loop system (4.31) is exponential stable. This completes the proof.*

It is shown that we can solve the observer-based stabilizing controller problem by using the following theorem

Theorem 4.3.2 *The closed loop system (4.31) is exponentially stable if there exist matrices* $0 < X$, $Y_{1,2}$, $j = 1, .., 4$ *such that the following matrices holds for* $j = 1, 2., 4$.

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j & \widehat{\Omega}_j & 0 & \widehat{X}\widehat{G}^T \\ \bullet & \widehat{\Lambda}_{3j} & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\widehat{X} & \widehat{X}\ \widehat{\Gamma}^T & 0 & 0 \\ \bullet & \bullet & \bullet & -\widehat{X} & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0; \tag{4.46}$$

$$\widehat{X} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \tag{4.47}$$

$\widehat{\Psi}_j = -X + \Pi_1 + \Pi_1^T$

$$\widehat{\Lambda}_{1j} = \begin{bmatrix} \widehat{\Psi}_j & -\Pi_1 + \Upsilon_1 \\ \bullet & -\Upsilon_1 - \Upsilon_1^T \end{bmatrix}$$

$$\widehat{\Lambda}_{2j} = \begin{bmatrix} -\Pi_1 + \Gamma_1^T & 0 \\ -\Upsilon_1 - \Gamma_1^T & 0 \end{bmatrix}$$

$$\widehat{\Lambda}_{3j} = \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & 0 \end{bmatrix}$$

Using S-procedure, we obtain

$$\widehat{\Lambda}_{3j} = \widehat{\Lambda}_{3j} - \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

Then,

$$\widehat{\Lambda}_{3j} = \begin{bmatrix} -M + M^T & 0 \\ \bullet & -I \end{bmatrix} \tag{4.48}$$

$$\widehat{\Omega}_j^T = \begin{bmatrix} \widehat{\Omega}_{1j} & 0 & 0 & -\widehat{\Omega}_{4j} & -\widehat{\Omega}_{5j} \end{bmatrix}$$

$$\widehat{\Omega}_{1j} = \begin{bmatrix} XA^T + Y_1^T B^T & 0 \\ XA^T & XA^T - Y_2^T \end{bmatrix}, \quad \forall j$$

$$\widehat{\Omega}_{4j} = \begin{bmatrix} Y_1^T B^T & 0 \\ 0 & 0 \end{bmatrix}; \quad j = 1, 4$$

$$\widehat{\Omega}_{5j} = \begin{bmatrix} 0 & 0 \\ 0 & -Y_2^T \end{bmatrix}; \quad j = 1, 2$$

$$\widehat{\Omega}_{4j} = 0; \quad j = 2, 3; \quad \widehat{\Omega}_{5j} = 0; \quad j = 3, 4 \tag{4.49}$$

where the gain matrices are given by

$$L = Y_1 X^{-1}; \quad K = Y_2 X^{-1} C^{\dagger}$$

proof 4.3.2 *Set*

$$\Omega_j = \begin{bmatrix} A_j & 0 & 0 & 0 \end{bmatrix}$$

we can display the inequality (4.34) can be represent as

$$\Lambda_j = \widetilde{\Lambda} + \Omega_j P \Omega_j^T < 0 \tag{4.50}$$

$$\widehat{\Lambda}_j = \begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} \\ \bullet & \widehat{\Lambda}_{3j} \end{bmatrix} < 0; \tag{4.51}$$

$$\widetilde{\Lambda}_{1j} = \begin{bmatrix} \Psi_j & -R + S^T \\ \bullet & -S - S^T \end{bmatrix}$$

$$\widetilde{\Lambda}_{2j} = \begin{bmatrix} -R_+ M^T & 0 \\ -S - M^T & 0 \end{bmatrix}$$

$$\widetilde{\Lambda}_{3j} = \begin{bmatrix} -M_+ M^T & 0 \\ \bullet & -I \end{bmatrix}$$

Setting $\widehat{X} = P^{-1}$*, invoking Schur complements, we write matrix* Λ_j *in (4.42) equivalently as*

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j & \widehat{\Omega}_j & 0 & \widehat{X}\widehat{G}^T \\ \bullet & \widehat{\Lambda}_{3j} & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\widehat{X} & -\widehat{\Gamma}^T P & 0 & 0 \\ \bullet & \bullet & \bullet & -P & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0; \qquad (4.52)$$

Applying the congruence transformation

$$T_j = diag[\ \widehat{X}\ , \widehat{X}\ , \widehat{X},\ \widehat{X},\ I\ ,\ \widehat{X},\ I,\ I]$$

to matrix inequality in (4.48) and manipulating using (4.44) and

$$\Xi_j = \widehat{X} Q_j \widehat{X},\ \ \Pi_j = \widehat{X} R_j \widehat{X},\ \ \Gamma_j = \widehat{X} M_j \widehat{X}\ \ \omega_j = \widehat{X} S_j \widehat{X}$$

to matrix inequality in (4.43) and handling by (4.45), we deduce
$\Pi = \widehat{X} R \widehat{X}$, $\gamma = \widehat{X} S \widehat{X}$, $\Gamma = \widehat{X} M \widehat{X}$

4.3.2 Numerical Examples

In this section, a numerical example and some simulations are given to confirm the effectiveness of the proposed approaches in terms of zero input strategy[17]. We assumed $\hat{\beta}_k = 0.1$ and $\hat{\alpha}_k = 0.1$. Our purpose is to design an observer-based controller in the form of equation (4.31).

Example 4.3.1:

We consider the uninterruptible power system UPS, in which we try to control the pulsewidth-modulated inverter to guarantee the output ac voltage is kept at the desired setting and undistorted, the discrete-time model was presented into [63]:

$$A = \begin{bmatrix} 0.9226 & -0.633 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C^T = \begin{bmatrix} 23.737 \\ 20.287 \\ 0 \end{bmatrix}$$

we obtain the controller and observer gain as

$$L = \begin{bmatrix} 0.0000 & -0.0000 & 0.2329 \end{bmatrix}; \quad and \quad \|L\| = 0.2329,$$

$$K = \begin{bmatrix} -0.0009 & -0.0020 & -0.0000 \end{bmatrix}^T; \quad and \quad \|K\| = 0.0022,$$

the estimated states of the UPS model are presented by Fig. 4.10. using zero input scheme.

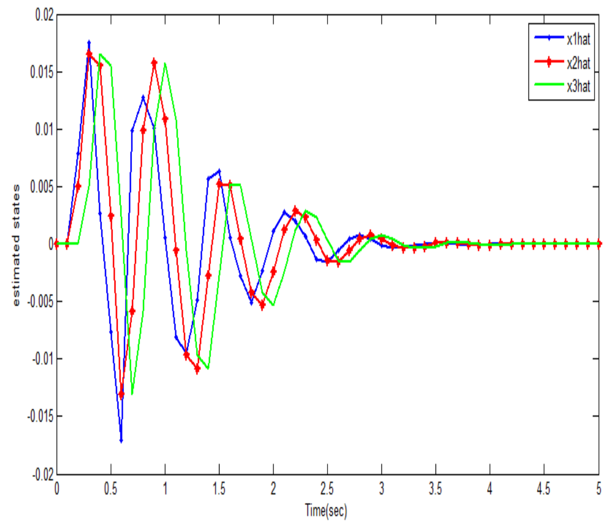

Figure 4.10: Trajectories of the estimated states for nonstationary packet loss using zero input scheme

Example 4.3.2:

We consider the following system with the plant parameters

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.0625 & -0.5 & 1.25 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0.0625 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -7 & 5 \end{bmatrix}$$

the eigenvalues of the plant were $[0.25, 0.5 - 0i, 0.5 + 0i]$. At each instant k, the sensor send an output vector y_k to the controller/estimator unit, then the input vector u_k was sent to actuator through networked control systems.

we obtain the controller and observer gain matrices as follows:

$$L = \begin{bmatrix} 0.0231 & 0.0389 & -0.0001 \end{bmatrix}; \quad and \quad \|L\| = 0.0453,$$

$$K = \begin{bmatrix} 0.0016 & -0.0001 & 0.0274 \end{bmatrix}^T; \quad and \quad \|K\| = 0.0275,$$

the simulation results of the states and the estimated states responses are given in Fig. 4.11 and Fig. 4.12 respectability, using the proposed the control algorithm. It is shown from the above figures that the performance of the control and estimation systems are regarded with nonstationary packet loss.

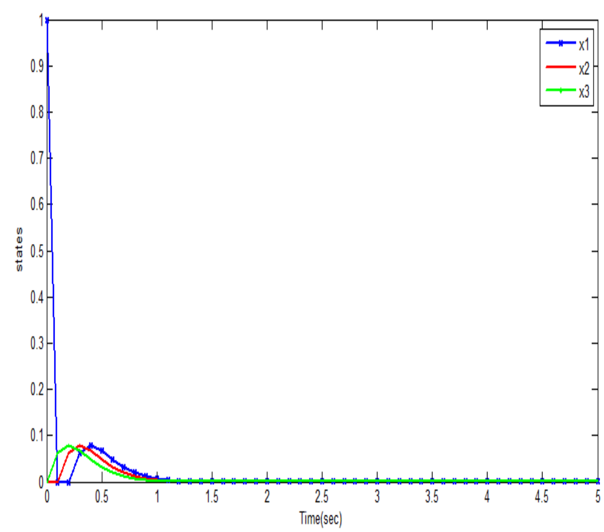

Figure 4.11: Trajectories of system states for nonstationary packet loss using zero input scheme

Example 4.3.3:

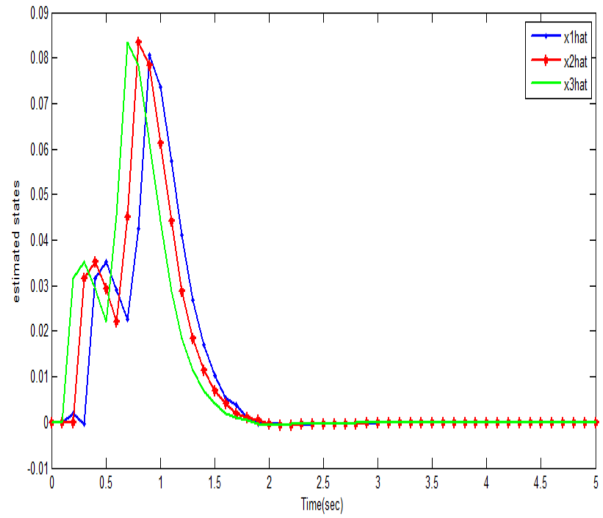

Figure 4.12: Trajectories of the estimated states for nonstationary packet loss using zero input scheme

Consider a discrete NCS second order model as following

$$A = \begin{bmatrix} 0.66 & 0.209 \\ -0.123 & -0.5 \end{bmatrix}; \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

the eigenvalues of the open loop system [0.6374,−0.4774] we obtain the state feedback controller and observer gain matrices as follows:

$$L = \begin{bmatrix} 0.1241 & 0.1179 \end{bmatrix}; \quad and \quad \|L\| = 0.1712,$$

$$K = \begin{bmatrix} 0 & -0.0338 \end{bmatrix}^T; \quad and \quad \|K\| = 0.0338,$$

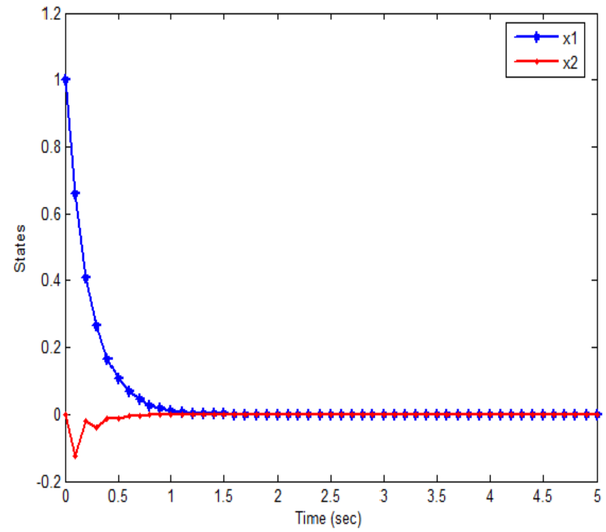

Figure 4.13: Trajectories of system states for nonstationary packet dropout using zero input scheme

Fig. 4.13. and Fig. 4.14. describe the trajectories of the real and estimated states respectability. It is shown that the performance of the control and estimation systems are regarded when the measurements and control packet are sending over NCSs with nonstationary packet dropout.

4.4 Conclusion

This chapter investigates the estimation problems over unreliable communication channels with nonstationary packet loss. Two classes of observers based stabilizing controller over networked control systems have been considered in

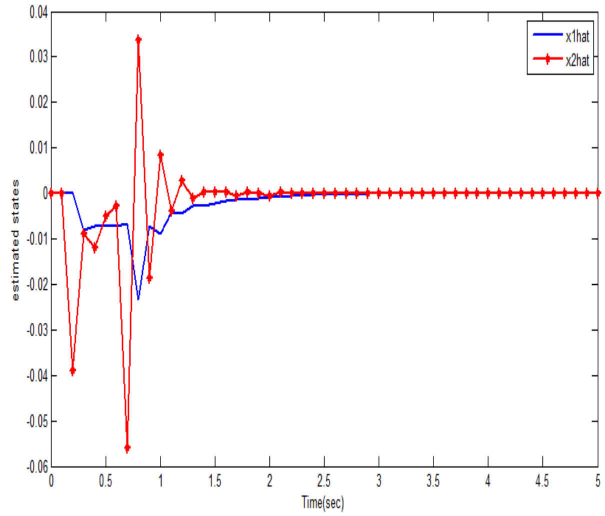

Figure 4.14: Trajectories of the estimated states for nonstationary packet dropout using zero input scheme

terms of different compensation strategies. Firstly, observer based controller problem is addressed based on hold input strategy, where the last available control input applied by the actuator is used when control packet lost. The other strategy, we employ zero input strategy to design observer based controller where zero control input is applied by the actuator when the control packet dropped out. It is shown that the former scheme gives better performance than the later when compared on the basis of the proposed examples where the response of the second strategy so-called zero input has more vibration, where the hold input response converge faster with higher gains. Furthermore, a necessary and sufficient conditions for observer stabilizability are addressed using linear matrix inequalities LMIs. As a result, the proposed schemes have been successfully evaluated on the given numerical examples.

Chapter 5

CONCLUSIONS AND FUTURE WORK

In this thesis, we have been interesting in control and estimation over unreliable communication networks. Problems of packet dropped in the control loops under two different protocols are discussed. TCP-like protocol, where the control node received acknowledgments for successfully delivered of the control input packets, while the acknowledgments are absent in case of UDP-like protocol. In both cases, observations, control, and acknowledgments packets are assumed to be subject to failure with different random probabilities. The system performance and stability may be effected by data lost. We consider the famous two compensation strategies, hold input and zero input, in which they used to compensate the lost of control packets.

In general, Chapter 1 was dedicated to an motivation, an introduction, objectives and thesis contribution of this thesis. A discrete linear time invariant LTI system was considered, where data transmitted via lossy communication links with input and measurement disturbances.

More specifically, in Chapter 2, literature review has been introduced for some classes of networked control system, in which lots of problems of estimation and control over NCSs subject to difference communication networks constraints were presented.

In Chapter 3, we studied the problems of discrete linear quadratic Gaussian LQG when data packets transmitted over lossy networks. It is shown that at the absent of the acknowledgments such as (UDP protocol case) the separation principle not hold more. Moreover, new suboptimal approach was developed for UDP-like protocol, while the separation principle in hold in TCP-like protocol where the estimator and controller can design separately. In addition, we assumed that the packets dropped out across networked control systems according to different independent Bernoulli processes. Moreover, the solution of the infinite horizon LQG control was derived.

In Chapter 4, two estimation schemes were derived for class of networked control systems with nonstationary data packets dropped out over both communication channels, from a sensor to a control/estimation unit and from the latter to an actuator. Furthermore, the fate of control and observation signals are modeled by two mutually independent random variables. In particular, two observer based stabilizing controller of networked control systems (NCSs) are designed to stabilize networked systems in means of exponential stable for both zero input and hold input strategies. Essentially, in both problems, we derived sufficient conditions for stability by using linear matrix inequality (LMIs).

In different chapters, numerical examples and simulations were used to illustrate the effectiveness of the proposed approaches subjects to packet lost. The simulation results clearly indicated that

- New results to the an LQG state feedback optimal control problem were provided for both TCP-like and UDP-like protocols. It is worth while to note that from our examples that, the suboptimal controller design for UDP like protocol gives better performance that the standard linear quadratic Gaussian approach in case of TCP-like protocol using hold input strategy using a quadratic cost function.

- Problems of estimation over loss networked control systems were addressed with stationary packet dropout for both compensation strategies. Two classes of observer based controller for NCS with output feedback control gains were designed using linear matrix inequalities (LMIs). A sufficient condition for stabilization by has been derived using a packet drop dependent Lyapunov function approach.

Suggestions for future work would be

- Designing LQG controllers over networked control systems with nonstationary packet drop out and quantization effects.
- Designing LQG controllers when data packet lost according to Markovian random process with nonlinearity and disturbance.
- Designing an estimation scheme for Markovian random process packet drop out with modified feedback.
- Designing an observer based stabilizing controller by applying the hold-zero strategies when the measurements packets get lost as well.

Chapter 6

Appendix

6.1 Appendix A

6.1.1 The Principle Unbiased Prediction: Chapter 3

In this part, we aim to present that, when the prediction is unbiased, the unbiased minimum variance estimator that given in equation 3.20 is developed expression of K'_{k+1} to get equation 3.21 as

$$\hat{\bar{x}}_{k+1|k+1} = K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}y_{k+1}$$

Substituting $\bar{x}_{k+1|k+1}$ on both sides

$$\hat{\bar{x}}_{k+1|k+1} - \bar{x}_{k+1|k+1} = K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}y_{k+1} - \bar{x}_{k+1|k+1}$$

Then, we add and substitute two terms, we deduce

$$\begin{aligned} \hat{\bar{x}}_{k+1|k+1} - \bar{x}_{k+1|k+1} &= K'_{k+1}\hat{\bar{x}}_{k+1|k} + K_{k+1}(\bar{C}_k\bar{x}_{k+1|k} + v_{k+1}) - \bar{x}_{k+1|k+1} \\ &\quad -K'_{k+1}\bar{x}_{k+1|k} + K'_{k+1}\bar{x}_{k+1|k} \end{aligned}$$

Rearranging terms gives

$$\begin{aligned} &= K'_{k+1}(\hat{\bar{x}}_{k+1|k} - \bar{x}_{k+1|k}) + K_{k+1}\bar{C}_{k+1}\bar{x}_{k+1|k} + K_{k+1}v_{k+1} - \bar{x}_{k+1|k+1} \\ &\quad +K'_{k+1}\bar{x}_{k+1|k} \end{aligned}$$

For $\bar{x}_{k+1}$ to be unbiased, the estimate must meet with $E[\hat{\bar{x}}_{k+1|k+1} - \bar{x}_{k+1|k+1}] = 0$.

By taking expectation at both sides, and setting this equal to zero, we obtain

$$0 = K_{k+1}\bar{C}_{k+1}\bar{x}_{k+1|k} - \bar{x}_{k+1|k+1} + K'_{k+1}\bar{x}_{k+1|k}$$

where

$E[K'_{k+1}(\hat{\bar{x}}_{k+1|k} - \bar{x}_{k+1|k})] = 0$; $E[K_{k+1}v_{k+1}] = 0$

the above expression implies

$$0 = [K_{k+1}\bar{C}_{k+1} - I + K'_{k+1}]\bar{x}_{k+1|k}$$

Then, according to 3.20, we have

$$\begin{aligned} I &= K'_{k+1} + K_{k+1}\bar{C}_k \\ K'_{k+1} &= I - K_{k+1}\bar{C}_k \end{aligned}$$

6.2 Appendix B

6.2.1 Proof Theorem 3.2.1: Chapter 3

This lemma will be used to prove the following theorem see [58]

Lemma 6.2.1 *Assuming, we have*

$$\phi(K, X) = (1-\beta)(\bar{A}_k X \bar{A}_k^T + S_w) + \beta(FXF^t + V) \tag{6.1}$$

where $F = \bar{A}_k + K\bar{C}_k$, $V = KS_vK^T$. *Let* $X \in \mathbb{S} = S \in \mathbb{R}^{n\times n}|S \geq 0$, $S_v > 0$, $S_w \geq 0$, *and* $(\bar{A}_k, S_w^{\frac{1}{2}})$ *is controllable. Then, following facts are true.*

1. *With* $K_x = -\bar{A}_k X \bar{C}_k^T (\bar{C}_k X \bar{C}_k^T + S_v)^{-1}, g_\beta(X) = \phi(K_X, X)$.

2. $g_\beta(X) = min_K \phi(K, X) \leq \phi(K, X), \forall K$.

3. *If* $\beta_1 \leq \beta_2, then g_{\beta_1}(X) \geq g_{\beta_2}(X)$

proof 6.2.1 *In this part, we proof the previous facts:*

1. *Define* $F = \bar{A}_k + K_x \bar{C}_k$, *and observe that*

$$\begin{aligned} F_x X \bar{C}_k + K_x S_v &= (\bar{A}_k + K_x \bar{C}_k) X \bar{C}_k^T + K_x S_v \\ &= \bar{A}_k X \bar{C}_k^T + K_x (\bar{C}_k X \bar{C}_k^T + S_v) = 0 \end{aligned}$$

Then, we deduce

$$\begin{aligned} g_\beta(X) &= (1-\beta)(\bar{A}_k X \bar{A}_k^T + S_w) + \beta(\bar{A}_k X \bar{A}_k^T \\ &\quad + S_w - \bar{A}_k X \bar{C}_k^T (\bar{C}_k X \bar{C}_k^T + S_v)^{-1} \bar{C}_k X \bar{C}_k^T) \\ &= (1-\beta)(\bar{A}_k X \bar{A}_k^T + S_w) + \beta(\bar{A}_k X \bar{A}_k^T + S_w + K_x \bar{C}_k X \bar{A}_k^T \\ &= (1-\beta)(\bar{A}_k X \bar{A}_k^T + S_w) + \beta(F_x X \bar{A}_k + S_w) \\ &= (1-\beta)(\bar{A}_k X \bar{A}_k^T + S_w) + \beta(F_x X \bar{A}_k + S_w) + (F_x X \bar{C}_k + K_X S_v) K_X^T \\ &= \phi(K_X, X). \end{aligned}$$

2. *Assume* $\varphi(K, X) = (\bar{A}_k + K\bar{C}_k)X(\bar{A}_k + K\bar{C}_k)^T + KS_vK^T + S_w$. *Note that*

$$\begin{aligned} \min_K \phi(K, X) &= \min_K FXF^T + V \\ &= \min_K \varphi(K, X) \end{aligned}$$

Since $X, S_v \geq 0, \phi(K, X)$ *is quadratic and convex in the variable* K, *therefore, the minimizer might be found by solving the differentiation given by* $(\partial\varphi(K, X))/\partial K = 0$, *which holds* $2(\bar{A}_k + K\bar{C}_k)X\bar{C}_k + 2KS_v = 0 \Rightarrow K = -\bar{A}_kX\bar{C}_k^T(\bar{C}_kX\bar{C}_k^T + S_v)^{-1}$. *As result of the fact that the minimizer present according to* K_X, *the proof of fact 2 runs after fact 1.*

3. *Note that* $\bar{A}_kX\bar{C}_k^T(\bar{C}_kX\bar{C}_k^T + S_v)^{-1} \geq 0$.

$$\begin{aligned} g_{\beta_1}(X) &= \bar{A}_kX\bar{A}_k^T + S_w - \beta_1\bar{A}_kX\bar{C}_k^T(\bar{C}_kX\bar{C}_k^T + S_v)^{-1}\bar{C}_kX\bar{A}_k \\ &\geq \bar{A}_kX\bar{A}_k^T + S_w - \beta_2\bar{A}_kX\bar{C}_k^T(\bar{C}_kX\bar{C}_k^T + S_v)^{-1}\bar{C}_kX\bar{A}_k \\ &= g_{\beta_2}(X). \end{aligned}$$

This completes the proof of the lemma.

proof 6.2.2 *This theorem is shown, there are two conditions available. First of all, if* $\beta_c = 1$, *we get a standard Riccati difference equation. As result, it is converge to a fixed point, under the theorem's assumption. Consequently, the estimator error covariance matrix is bounded for all initial conditions* $\Sigma_0 \geq 0$. *If* $\beta_c = 1$, *the prediction becomes open loop prediction, and if the matrix* A *is unstable, then the estimator error covariance matrix* Σ_k *diverges for some* *initial condition* $\Sigma_0 \geq 0$ *. After that we prove the existence of a single point of transition between both cases. Set* $0 < \beta_1 < 1$ *such that* $E_{\beta_1}[\Sigma_k]$ *is bounded for any initial condition . More what, for any* $\beta_2 \geq \beta_1$*, it is shown that* $E_{\beta_2}[\Sigma_k]$ *is also bounded for any initial condition* $\Sigma_0 \geq 0$ *as well. Actually, we have*

$$\begin{aligned} E_{\beta_1}[\Sigma_{k+1}] &= \bar{A}_k \Sigma_k \bar{A}_k^T + S_w - \beta_{k+1} \bar{A}_k \Sigma_k \bar{C}_k^T (\bar{C}_k \Sigma_k \bar{C}_k^T + S_v)^{-1} \\ &\quad \bar{C}_k \Sigma_k \bar{A}_k] \\ &= \bar{A}_k \Sigma_k \bar{A}_k^T + S_w - \beta_1 \bar{A}_k \Sigma_k \bar{C}_k^T (\bar{C}_k \Sigma_k \bar{C}_k^T + S_v)^{-1} \\ &\quad \bar{C}_k \Sigma_k \bar{A}_k] \\ &= E[g_{\beta_1}(\Sigma_k)] \\ &\geq E[g_{\beta_2}(\Sigma_k)] \\ &= E_{\beta_2}[(\Sigma_{k+1})] \end{aligned}$$

We use part 3 of lemma 6.2.1 to rewrite the last inequality. By choosing $\beta_c = \{inf \ \ \beta^* : \beta > \beta^* \Rightarrow E_\beta[(\Sigma_k)], \ \ \forall \Sigma_0 \geq 0\}$ *is bounded any initial condition The proof is complete.*

6.2.2 Proof theorem 3.3.2: Chapter 3

proof 6.2.3 *1. According to the fact that if* $\alpha > \alpha_c$*, then* $\lim_{k \to \infty} F_k = F_\infty : \forall F_0 \geq 0$*. As result, equation 3.47 follows from equation 3.46.*

2. *Due to the fact that the optimal state estimator gain* K_k *was derived according the random variable of arrival sequences* β *and* α*, as given (3.5), (3.14), (3.15), (3.23), and (3.25).*

3. *For* $\beta_{max} < \bar{\beta}$ *and* $\alpha_c < \bar{\alpha}$*, we have* $\lim_{k \to +\infty} \Sigma_k = \Sigma_{+\infty}$*, and* $\lim_{k \to -\infty} \Sigma_k = \Sigma_{-\infty}$*, the estimation error covariance* Σ_k *was defined in (3.14). In addition,* $\lim_{k \to \infty} F_k = F_\infty : \quad \forall F_0 \geq 0$*. Final result, from equation (3.28) - (3.42), we conclude that the expected minimum cost function can be bounded.*

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